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ANALYSIS AND DESIGN OF MULTISTORY
STEEL UNBRACED FRAMES FOR SEISMIC LOADING

by

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B. Eng., M. Tech.

A THESIS

Presented to the Faculty of Graduate Studies
of Carleton University in partial
fulfillment of the degree of
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ABSTRACT

The behaviour of multistory steel frames under the action of seismic forces is investigated by analytical means, and several alternative methods of design for seismic forces are examined.

In preparation for the main work of the thesis, it is shown that the existing methods of numerical analysis applicable to problems in structural dynamics all belong to the category of "predictor-corrector" methods and can be derived by a common systematic approach. This approach is then used to derive a multipoint formula whose efficiency is demonstrated by a series of examples of response calculation.

Ten story steel frames are analyzed for selected earthquake ground motions, and the effect of certain parameters on the response is studied. It is shown that column axial deformations may be neglected in analysis unless the building is very slender. It is also shown that the effect of live load on response may be neglected except probably for storage structures.

Selected set-back frame models are analyzed for earthquake motion, and effects of two parameters on the response of the frames are investigated. It is shown that, for set-back frames with slender towers, the tower response may be greatly accentuated. The design provisions of the SEAOC code on set-back frames are compared with the results of analysis for recorded ground motions, and it is demonstrated that, under certain situations, the code provisions may underestimate the tower forces.

The requirements of ductility in real steel frames subjected to seismic forces are examined; a definition of ductility ratio and a method of computing utilized ductility ratios are presented, that make it possible to compare the utilized ductility with experimentally determined ductility capacity. The existing test results on inelastic rotation capacity of beams under moment gradient are then reviewed.

It is shown that the existing code provisions on the distribution of seismic shears throughout the height of a building structure may underestimate the shears in the upper stories. A more realistic distribution of seismic shear forces and overturning moments is obtained, through the use of a uniform vertical cantilever model and a rational superposition of the modal contributions. New empirical relations are developed to represent the maximum shear and moment envelopes, for use with the "equivalent static lateral load" method of design.

Methods of design for seismic forces based on an elastic analysis of the structure are described. It is shown that the time-dependant response method of analysis and design is a viable alternative to the modal superposition method, which avoids some of the inaccuracies inherent in the latter method. It is further shown that, when force reduction factors are used in conjunction with an elastic analysis to compute the design forces in a ductile structure, the use of a uniform force reduction factor for both the girders and the columns is not compatible with the commonly accepted design objective of forcing the inelastic action to

occur in the girders rather than in the columns. It is more appropriate to use a lower reduction factor for column forces than for girder forces. A detailed procedure for the design of a frame building, using the time dependant response method of analysis and the plasticity force reduction factors, is developed and illustrated with the aid of a design example.

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CHAPTER 1

INTRODUCTION

1.1 THE PROBLEM

Over the years, earthquakes have been the cause of a very large loss of life and property. It is estimated that, in the last 4000 years in all parts of the world, over thirteen million people have been killed by earthquakes. The figures available for the value of the property destroyed by earthquakes are equally staggering, and the physical suffering and the anguish of the survivors cannot be estimated. The enormity of the problem has provided a strong incentive for man to learn more about the causes, nature and the effects of earthquakes, permitting him to select safer building sites and to design his structures with improved resistance to seismic forces.

Some of the effects of earthquakes are very spectacular. Earthquakes often cause landslides, land subsidence, land liquefaction, faulting and huge seismic sea waves. However, the single most important cause of the loss of life and property during an earthquake is the destruction of buildings. This has stimulated engineers to improve their ability to design buildings with improved resistance to earthquakes. The resulting buildings have, in fact, performed much better during actual earthquakes than buildings not so designed.

Earthquake resistant building design demands a knowledge of the phenomena of seismic ground motion, and also the characteristics of

building structures and their behaviour during earthquakes. Much has been learnt, but the unknowns and complexities are still so great that earthquake-resistant design remains an art well as a science. The practice of earthquake-resistant construction, therefore, relies heavily upon observations and experience. The codes of practice for earthquake-resistant building design typically reduce the complex dynamic problem actually encountered to the much simpler problem of a set of equivalent static lateral forces. The amount and distribution of these forces, although influenced by analytical studies of the dynamic behaviour of buildings, have substantially been arrived at by empirical means and the use of experience and judgement.

The above observations apply to most of the present codes on earthquake-resistant design. However, in this study, the remarks relating to the codes are in specific reference to the seismic codes followed in Canada and the United States. In Canada, the provisions on earthquake-resistant design of buildings are included in the National Building Code of Canada, 1970 (NBC 1970). In the United States, the Uniform Building Code, which is widely used for the design of buildings, contains a section on earthquake-resistant design. The provisions of this section are more or less the same as the lateral force requirements of the Seismology Committee of the Structural Engineers Association of California (SEAOC). These latter requirements, briefly referred to as the SEAOC code, are applicable to the seismic conditions of California. Throughout this study, when a general reference is made to earthquake-resistant design

codes, the NBC 1970 and the SEAOC, 1973 are implied. Whenever a design aspect is treated differently in the two codes, the specific code being referred to is named.

Analytical studies of the elastic response of undamped buildings to the ground motions recorded during actual earthquakes show that the theoretical forces induced in the building are several times those obtained from the equivalent static loads prescribed by the codes [1, 2, 3, 4, 5, 6][†]. The discrepancy is so great that it can not be reconciled by taking into account the factor of safety, damping or other conventional explanations. Yet, the buildings designed in accordance with the provision of the code have withstood some severe earthquakes reasonably well. The explanation of this apparent paradox lies in the fact that, during a severe or even moderate earthquake, structural members designed according to the requirements of the codes are strained beyond the elastic limit and absorb a very substantial amount of energy through inelastic deformations [3, 7, 8]. This capacity of the structure to absorb seismic energy reduces the induced seismic forces, and frequently saves the building from severe damage or collapse. The codes implicitly account for this capacity of the structure to absorb energy through inelastic deformation by stipulating a reduced design shear.

[†] Numbers in square brackets refer to references given at the end of this study.

In the present codes, the provision for inelastic energy absorption capacity is not separately identified. Also, there are inherent difficulties in using even the simplified elastic approach to predict the behaviour under actual earthquake. In particular, the behaviour of structures with dynamic discontinuities and asymmetry is so complex that the effect of these variables can not, even approximately, be accounted for in a simplified design approach. Analytical studies of response are, therefore, important not only for understanding the dynamic behaviours of structures subject to earthquakes, but also as design tools in certain situations.

1.2 REVIEW OF THE LITERATURE

The accurate analytical study of the inelastic response of a typical engineering structure to seismic load is a highly complex mathematical problem. A number of investigators have, therefore, directed their studies to single-degree-of-freedom systems. Notable among those who have used this approach are: Housner [2], Newmark and Veletos [3], Berg and Thomaides [7] and Jennings [9, 10, 11]. Based on this research, efforts have been made to predict the inelastic response of structures with several degrees of freedom. However, it has not been always possible to find a direct correlation between the response of a single-degree-of-freedom system and a system with many degrees of freedom [12, 13].

Several studies are concerned with the direct analyses of the inelastic response of multistory buildings. Based on a method proposed by Bleich in 1955 [14], Di Maggio in 1958 [15] developed a method of

calculating the dynamic elasto-plastic response of a rigid frame using mode superposition. Every time a new plastic hinge was formed or an existing plastic hinge was removed, a new set of mode shapes was calculated. These mode shapes were used in evaluating the response for further time steps until another change occurred in structural configuration. Since a multistory building may have a large number of plastic hinges and they may be introduced or removed very frequently, Di Maggio's method is not very practicable for such structures. In 1960, Berg and Da Peppo [16] used a numerical integration technique for calculating the elasto-plastic response of multistory buildings. With this method, all possible locations of plastic hinges were first identified. Influence coefficient matrices were then developed relating the rotation of the plastic hinges to the forces developed at other degrees of freedom. Changes during a time step were made using the original frame. This induced moments greater than the plastic moments at certain plastic hinges. These hinges were then rotated back so that moments became equal to the plastic moments. An iterative scheme was used to evaluate the corrector rotations.

In 1961, Berg [17] carried out limited studies on the elastic and elasto-plastic models of a four story building frame using the method developed in reference 16. The ground motions used were the Helena Montana earthquake of October 31, 1935 and the first 10 seconds of the N-S components of the El-Centro earthquake of May 18, 1940. The study was too limited to draw any conclusion about behaviour of tall buildings under earthquakes and the effects of the various parameters involved.

In 1960, Penzien [8] used numerical integration of the equations of motion to study the elasto-plastic response of lumped mass shear type buildings, where the floor girders were considered to be infinitely rigid and only sidesway degrees of freedom were permitted. This assumption limited the applicability of the results since, in practical buildings, the girders are flexible and absorb a large amount of energy through inelastic deformation.

In 1964, Heidebrecht, Lee and Fleming [19] used the Conjugate Frame Method, earlier developed by Lee [18] to calculate the elasto-plastic response of building frames. They did not, however, study the response of building frames to earthquake motion.

Using a method developed earlier by themselves [18, 19] Saul, Fleming, Lee and Perelman published studies on the dynamic analysis of multistory shear type buildings constructed of bilinear elastic inelastic material [20, 21]. Several four story shear type buildings were analyzed and it was found that both the elastic and inelastic shears induced by earthquakes were larger than the equivalent static loads prescribed by the codes. The study was, however, limited in scope. Its applicability was also limited by the assumption of infinitely rigid floor girders.

A very useful study of the inelastic response of multistory frames, that had direct applicability to practical structures, was published by Clough, Benuska and Wilson in 1965 [12], and again in 1967 [22]. A step by step numerical integration procedure based on earlier work by

Newmark [23, 24, 25] was used. The stiffness matrix of the structure was assumed to be constant during a time interval, at the end of which the moments and deformations in the structure were examined. Where necessary, plastic hinges were introduced or removed. The stiffness matrix was then revised and used to calculate the incremental displacements and forces during the next time interval. Due to the large amount of computer time involved in the analysis, the response was calculated only for the first four seconds of the El-Centro earthquake. Also, the effect of certain important parameters such as column deformations, the $P-\Delta$ effect and damping were not studied. Studies on similar lines were also published by Walpole and Shepherd [26] and by Guru and Heidebrecht in 1969 [27]; the first of these studies dealt with the response of five story reinforced concrete frame and the second with the response of a ten story steel frame.

A significant study of inelastic structural response to seismic load was presented by Goel and Berg in 1968 [28], and by Goel in 1969 [29].

This investigation was more comprehensive and covered the effect of a number of parameters on the inelastic response of frames. A Ramberg Osgood type force-deflection relationship was used for the beam members. Such a relationship had been made use of earlier by Jennings in 1964 [9]. It was assumed that the columns remained elastic and that the real building frame could be represented by an equivalent single bay frame in which the girder end rotations were antisymmetric. For a typical regular building frame, designed according to the code, analysis shows that most of the columns remain elastic during an earthquake motion. The columns

in the lowest and top stories of a regular frame and several additional columns in an irregular frame may, however, develop plastic hinges.

Pekau [30] conducted a study of the inelastic behaviour of a typical building frame system subjected to earthquake forces. A bilinear hysteresis model was adopted for member behaviour. It was assumed that the height of the structure and axial deformation of the columns had no significant effect on the response. The effects of a number of parameters on the inelastic earthquake response were investigated.

A method of calculating the inelastic response of multistory frames was published more recently by Adams and Suko [31]. The proposed procedure takes into account the effect of a bilinear force deflection relationship, secondary beam-column effects and the effect of semi-rigid joints. However, it is based upon two important assumptions that limit its applicability as follows: (1) the column axial loads do not change during the response history, (2) the points of inflexion remain fixed during the entire response history. Analysis shows that the maximum column axial loads under earthquake motion are considerably higher than those produced by static vertical loads [12, 28]. Also, the location of the points of inflexion, particularly those in the lowest and topmost stories, vary considerably; the assumption of fixed points of inflexion is likely to lead to serious errors in some cases [5, 32].

A number of multistory buildings have set-back towers. The dynamic behaviour of such a building is very significantly affected by the relative weight and stiffness of the tower. In fact, the tower structure

may be subject to lateral forces considerably larger than those at the corresponding levels of a comparable uniform building. Berg [33] made an exploratory study of the effect of the tower set-backs on structural response, in which the building was modelled by a vertical stepped cantilever shear beam. An approximate method for the analysis of the earthquake response of buildings supporting light appendages was presented by Penzien and Chopra [34]. Similar studies were made by Skinner et al [35]. In the two studies made by Penzien, Chopra and Skinner, the tower portion or appendage was treated as a single-degree-of-freedom system having a very small weight as compared to the building on which it was mounted. It was found that the tower response was greatly accentuated when its natural period was close to one of the lower mode periods of the building. Penzien later extended his approximate method to a light weight tower with more than one degree of freedom [36]. A more comprehensive study of set-back structures was made by Jhaveri [37] in 1967. This study related to the response calculations of elastic rectangular shaped vertical cantilever shear beams and also elastic multistory buildings having infinitely rigid floor girders. A limited number of examples of inelastic response of set-back buildings were presented by Pekau [30].

1.3 SCOPE OF THE PRESENT STUDY

The present study covers some important aspects of the analysis and design of multistory steel frames for seismic loading. It proceeds in the following steps.

1. The development of improved numerical procedures for the integration of the equations of motion.
2. The development of efficient special purpose computer programs for the elastic and inelastic analysis of multistory steel building frames.
3. The selection of the regular and set-back building examples and the earthquake ground motion to be used in the studies of the thesis.
4. The study of the effect of a number of important parameters on the response of building frames to earthquake ground motions.
5. The study of the seismic response of set-back frames.
6. The study of the ductility requirements in building frames subject to earthquake loads.
7. A review of some aspects of the current practice in the earthquake-resistant design of multistory steel buildings, and development of new methods of design making use of information accumulated in steps 4 through 6.

The elastic and inelastic analysis of structures subjected to random forcing function such as those resulting from earthquakes, blast or wind loads requires the use of a numerical integration technique. The efficiency and accuracy of the technique employed is of great importance for both research and practical design. The first stage of this work, found in Chapter 2, is therefore devoted to the refinements of the

techniques of numerical analysis to obtain improved computational efficiency. The following significant contributions are presented in Chapter 2.

1. It is shown by the author, it is believed, for the first time, that the various methods of numerical analysis developed for application to the problems of structural dynamics all belong to the category of predictor corrector methods and can be derived by one unified approach. This approach, though standard in the mathematical theory of numerical analysis, does not seem to have been used in structural engineering.
2. Using the approach referred to above, an improved multi-point formula is derived by the author and its truncation error and conditions of stability are examined. It is then demonstrated by a series of examples that, under certain situations, appropriate multi-point formulae can give much better accuracy in the computation of elastic response of structures than the conventional single point formulae.
3. It is shown that, for most problems in structural dynamics, iteration of the corrector is not necessary; a direct solution of the simultaneous difference equations by a process of elimination is not only possible, but also offers considerable savings in computation time.

Chapters 3 and 4 prepare the ground for much of the analytical.

work contained in the following chapters. Chapter 3 presents the assumptions made by the author in modelling the behaviour of steel members and frames. The hysteretic moment - curvature and moment-rotation relationships for individual members are discussed and the validity of the assumption of an elasto-plastic hysteretic relationship is examined. Chapter 3 also presents the conventions used in numbering the degrees of freedom and introduces and defines several terms used later in the study, such as: plastic hinge rotations, ductility ratios and shear coefficients. The computer programs used in the present study were developed by the author. The success of the studies depended very heavily upon the availability of special purpose computer programs for elastic and inelastic analysis, with good computation efficiency. Since the author was not able to discover suitable existing programs at the beginning of his research, he had to develop his own programs which included the numerical techniques referred to above and also a number of other computational devices and carefully selected approximations which greatly improved the computation speed. A brief description of the programs appears in Chapter 3. Chapter 4 describes the multistory steel building frames selected for study by the author, and the strong motion accelerograms used as input base motions.

Chapter 5 presents the results of an investigation by the author on the effect of a series of input parameters on the dynamic response of multistory frames. Though a number of studies have been carried out in the past on the effects of several input parameters, only a limited amount of work has been done on the effect of axial deformation of columns and

the effect of the contribution of live load to the mass of the building. These aspects are covered in some detail in the present study. The following important conclusions can be drawn from the studies described in Chapter 5.

1. The effect of axial deformation of columns may be neglected in seismic analysis of multistory frames unless the frame is very slender.
2. The effect of building live load on response may be neglected except possibly for storage structures.

Multistory buildings with set-backs are used frequently in practice. Code provisions do exist for the design of such buildings. However, the dynamic response of set-back buildings is so complex that analytical studies of response become very important. Only a limited amount of work has been done in the past on the seismic response of set-back frames with flexible girders. The following contributions are made by the analytical studies of set-back frames carried out for the thesis by the author and reported in Chapter 6.

1. A significant amount of new data is added to that already available in the area of the seismic behaviour of set-back frames.
2. For the first time, it is believed, the current code provisions on set-back frames are compared with the results of analytical studies; it is demonstrated that in some cases the code provisions may not be entirely adequate.

The satisfactory performance of a building subject to a severe earthquake depends, to a large extent, upon the amount of inelastic deformation which the structural members are able to tolerate. In other words, it is important to know whether the structural elements can undergo the computed inelastic deformations without distress. It is customary to measure these deformations in terms of ductility ratios. However, the term "ductility ratio" has been used in several different senses in earthquake engineering. Many of the definitions of ductility ratio do not have any direct relation to a measurable physical behaviour of the member, so that it is difficult to assess whether the actual ductility requirement is less than the ductility capacity. Keeping in view the considerations just cited, the question of ductility is examined in Chapter 7, and the following contributions are made:

- 1: A definition of ductility that does have a direct relation to measurable physical behaviour is presented and methods of estimating the actual ductility requirements from the results of a dynamic analysis are developed.
2. A review is conducted of the existing experimental studies dealing with ductility capacity as it applies to seismic loading.

As stated earlier, the present codes for a seismic design of structures prescribe a set of "equivalent lateral static loads" for which the structure is required to be designed. This approach is generally satisfactory for a large number of structures that are fairly regular in

shape and in the distribution of mass and stiffness. The author, however, believes that the code provisions dealing with the distribution of shears and overturning moments throughout the height of a building could be based on a more rational approach. Also, the code method of relating the contribution of higher modes to the slenderness of the building is not supported by analytical studies of response, particularly in the case of buildings that derive their resistance to lateral forces predominantly by a shear type of deflection. A new approach to the distribution of shear forces and overturning moments throughout the height of the building, to be used with the equivalent lateral load method of design, has been developed by the author and is presented in Chapter 8. Also presented are new empirical relations developed to approximate the true shapes of the seismic shear and moment envelopes. The author believes that the suggested improvements make the equivalent lateral load method closer to the results of a dynamic analysis.

While the equivalent-lateral-load method of design is satisfactory for fairly regular structures, some form of dynamic approach must be used in the design of irregular or unusual buildings or special structures. The dynamic analysis approach now commonly favoured is the use of modal superposition in conjunction with a structural design spectrum to obtain the elastic seismic shears. The effect of inelasticity is taken into account by reducing the elastic shears so obtained through the use of a suitable reduction factor. The reduced shears are then resolved into a set of equivalent lateral forces and the building is designed to carry these lateral forces.

Two new viewpoints on aspects of aseismic design through dynamic analysis are offered in this study along with supporting evidence. Hopefully, they represent significant contributions to the present thinking on the subject.

1. The time dependant response method of analysis is a viable alternative to the modal superposition method which avoids some of the inaccuracies inherent in the latter method, and hence may be preferable in some situations.
2. When force reduction factors are used in conjunction with an elastic analysis to compute the design forces in a ductile structure, the use of a uniform force reduction factor for both the girders and the columns is not compatible with the commonly accepted design objective of forcing the inelastic action to occur in the girders rather than in the columns. It is more appropriate to use a lower reduction factor for column forces than for girder forces.

These concepts are presented in the latter part of Chapter 8. Also presented are a description of the proposed time dependant response method of design and a design example to illustrate the method.

A summary of the work done in the present study and the conclusions arrived at are contained in Chapter 9.

CHAPTER 2

NUMERICAL METHODS FOR THE CALCULATION OF STRUCTURAL RESPONSE

2.1 INTRODUCTION

The research work for this thesis relies heavily upon an efficient means for carrying out the numerical integration of the equations of motion. Also, in situations where it can be carried out with a sufficient degree of efficiency, such integration becomes useful not only for research but also as a practical design tool. For these reasons, it is most desirable to obtain whatever computational efficiency is possible in the numerical integration process. This chapter deals with the author's research in this area.

The methods available for the numerical integration of the differential equations associated with analysis of dynamic response can be divided into two broad categories, the methods based on Taylor series, and the "predictor-corrector" methods. The Taylor series methods, which also include the well-known Runge Kutta methods, have received extensive treatment in the literature [38, 39, 40]. The Runge Kutta methods can be applied to problems involving random forcing functions. It is, however, difficult to estimate the errors involved; besides the time required in the solution of a problem may be rather large when compared to that required with certain other methods of numerical integration.

The predictor-corrector methods are iterative methods that use the information at one or more previous points to assist in evaluating

the dependant variable at each successive location. These methods offer several advantages over the Taylor series type methods.

The Euler predictor corrector formula [41] is a simple example of this class of method. The use of this formula can be illustrated by applying it to the following simple single first order differential equation:

$$\dot{y} = f(t, y) \quad (2.1)$$

where t is the independant variable, y is the dependant variable, and $\dot{y}$ is the first derivative of y with respect to t . If t_n and t_{n+1} are two successive time intervals, $h = t_{n+1} - t_n$ is the time step and y_n represents the value of displacement y at t_n , then y_n is approximately given by:

$$\dot{y}_n = \frac{y_{n+1} - y_n}{h} \quad (2.2)$$

provided the time step h is sufficiently small.

A first approximation to y_{n+1} is given by:

$$y_{n+1}^p = y_n + h f(y_n, t_n) \quad (2.3)$$

where superscript p signifies a predicted value of y_{n+1} . A better estimate of y_{n+1} is obtained by:

$$y_{n+1}^c = y_n + \frac{h}{2} [f(y_n, t_n) + f(y_{n+1}^p, t_{n+1})] \quad (2.4)$$

where the superscript c indicates a corrected value of y_{n+1} .

The value of y_{n+1} so obtained can be used to evaluate $f(y_{n+1}, t_{n+1})$ for the next iteration which will hopefully give a further

improved value of y_{n+1} . The use of the corrector formula, Eq. 2.4, is repeated as many times as is desired. The accuracy of the final answer will depend upon the degree of convergence of the iteration process and the adequacy of the integration interval h . The method just described is an example of a "single point" method which makes use only of information at location n and not, for example, at location $n-1$.

Numerical methods developed for the dynamic analysis of structures have been variously called the "Difference Equation Methods" and "Acceleration Methods" [23, 24]. The latter include the wellknown Newmark's β method [23, 24, 25] and the Acceleration Pulse method developed at M.I.T. [38]. It will be shown herein that all these methods, in fact, belong to the category of predictor-corrector methods and can be derived by one unified approach. This approach, though standard in the mathematical theory of numerical analysis, does not seem to have been used in structural engineering. The author believes that the identification of the various methods with the general category of the predictor-corrector methods should lead to a better appreciation of their relative merits and to added facility in the derivation of suitable multipoint formulae.

Multipoint formulae, which make use of information at more than one previous point in evaluating the dependant variable at a successive location, generally have a smaller truncation error than the single point formulae. One might therefore expect that their use would result in lower errors in practical numerical solutions. This has not always been found to be the case. On the contrary, it has been observed that, with

many multi-point methods, the errors have a tendency to grow with time t [38, 39]; i.e. the solutions are not stable. An attempt has been made in this study to show that it is possible to derive multipoint methods that show good performance with respect to stability and give more accurate solutions than do the single point methods for the same amount of computation time. One such multipoint formula will be presented in this Chapter and its truncation error and condition of stability will be derived. It will then be demonstrated by a series of examples that, when used for appropriate problems, multipoint formulae give much better accuracy in the computation of elastic response of structures than do the single point formulae.

This study will also show that, for the type of differential equations involved in structural dynamics, iteration of the corrector is not necessary. On the contrary, a direct solution of the simultaneous difference equations by a process of elimination is not only feasible, but also offers the definite advantages of (1) conserving computing time and (2) eliminating the need for any limitation on the magnitude of the step size h to ensure convergence of the iterative process.

2.2 THE GENERAL PREDICTOR CORRECTOR FORMULA

A general corrector formula should give the value of y_{n+1} in terms of the information at several previous points $n-k, n-k+1, \dots, n$. It may be written in the form:

$$y_{n+1} = \sum_{e=n-k}^n A_e y_e + \sum_{e=n-k}^{n+1} B_e y_e + \sum_{e=n-k}^{n+1} C_e y_e + R \quad (2.5)$$

where R is a remainder term and A_e , B_e , C_e are constants, some of which may equal zero. The total number of these constants, including possible zero terms, is

$$m = 5 + 3k \quad (2.6)$$

Higher order derivatives could be included but the general principles will remain unchanged. However, when the differential equation being solved is of the second order, as is usually the case with structural dynamics problems, derivatives higher than the second may be difficult to obtain.

If Eq. 2.5 has m free parameters, A_e , B_e , C_e , then by a suitable choice of the values for these parameters the equation can be made exact in the special case where y is in the form of a polynomial of order $m-1$. When the formula is exact, the remainder term R will equal zero. Now, if Eq. 2.5 is exact for a polynomial of order $(m-1)$, it will also be exact when y takes on any one of the following values:

$$1, t, t^2, \dots, t^{m-1}$$

Therefore, to evaluate the m free parameters, y can be successively set equal to each of these values in turn, and in each case a substitution made into Eq. 2.5. This procedure yields m simultaneous equations involving the set of coefficients A_e , B_e , C_e , which are also m in number. A solution of these equations provides the required values of these parameters.

Now if $y = t^m$ is substituted into Eq. 2.5, R cannot be expected to automatically equal zero. The resulting value of R will be designated

as E_m . It will be evident that, when Eq. 2.5 is used to estimate the value of y_{n+1} for a high order polynomial, a larger number of terms in the equation will have the effect of reducing the magnitude of the estimated truncation error R . Equations having many terms may, however, encounter problems of spurious roots and instability. Therefore, it may not be desirable to use all m free parameters to reduce the truncation error. An approach having practical usefulness is to employ Eq. 2.5 to represent exactly a polynomial of order $p-1$, p being an integer smaller than m by an appropriate number of terms. Then $(m-p)$ slack parameters become available which can be assigned arbitrary values chosen so as to improve the stability or the convergence characteristics of the resulting predictor-corrector formula.

As an example of a solution of a differential equation using a corrector formula, the following second order differential equation may be considered:

$$\ddot{y} = f(t, y, \dot{y}) \quad (2.7)$$

A possible second order single point corrector formula is:

$$y_{n+1} = a_1 y_n + a_2 \dot{y}_n + a_3 \ddot{y}_n + a_4 \ddot{y}_{n+1} \quad (2.8)$$

where lower case letters are used here for the constant coefficients to distinguish from the convention used for Eq. 2.5. In this formula, the constant coefficient of $\ddot{y}_{n+1}$ has in effect been set equal to zero.

Since there are four free parameters, the formula can be used to represent exactly a polynomial of the third order. If one chooses

to make the formula capable of representing exactly only a second order polynomial, one slack parameter is obtained. Selecting a_4 as the slack parameter to be used in this latter case, and substituting in turn

$$y = 1, t, t^2$$

in Eq. 2.8, one obtains:

$$\begin{aligned} a_1 &= 1 \\ a_2 &= h \\ 2a_3 + 2a_4 &= h^2 \end{aligned} \tag{2.9}$$

Equation 2.8 then reduces to:

$$y_{n+1} = y_n + h \dot{y}_n + \left[\frac{h^2}{2} - a_4 \right] \ddot{y}_n + a_4 \ddot{y}_{n+1} \tag{2.10}$$

where a_4 is an arbitrary constant which can be selected to optimize the performance of Eq. 2.10. Setting $a_4 = \beta h^2$, where β is now the arbitrary constant which can be assigned an appropriate value, one obtains:

$$y_{n+1} = y_n + h \dot{y}_n + h^2 \ddot{y}_n (1/2 - \beta) + h^2 \beta \ddot{y}_{n+1} \tag{2.11}$$

The above formula is basic to Newmark's β method [25]. In Table 2.1 are shown the formulae obtained by substituting into Eq. 2.11 the following values of β : 0, 1/4, 1/6, 1/8. The Table also presents the associated truncation errors and conditions of stability. These topics are discussed in Sections 2.3 and 2.5, respectively; reference is made to Table 2.1 again in those sections.

TABLE 2.1
A COMPARISON OF SOME OF THE METHODS OF NUMERICAL ANALYSIS USED IN STRUCTURAL DYNAMICS

Method	Truncation Error Term For Velocity	Truncation Error Term For Displacement	Criterion for Stability of Displacement	Remarks
Constant Acceleration Method $y_{n+1} = y_n + h \dot{y}_n$ $\ddot{y}_{n+1} = \ddot{y}_n + h \ddot{y}_n + \frac{h^2}{2} \ddot{\ddot{y}}_n$	$\frac{h^2}{2} \ddot{y}^{(3)}(\theta)$	$\frac{h^3}{6} \ddot{y}^{(3)}(\theta)$	$h\omega < 4$	Solution diverges with time
Newmark's β Method (i) $\beta = 1/4$ in Eq. 2.11 Timoshenko Method $\ddot{y}_{n+1} = \ddot{y}_n + \frac{h}{2} (\ddot{y}_n + \ddot{y}_{n+1})$ $\dot{y}_{n+1} = \dot{y}_n + h \dot{y}_n + \frac{h^2}{4} \ddot{y}_n + \frac{h^2}{4} \ddot{\ddot{y}}_{n+1}$	$-\frac{h^3}{12} \ddot{y}^{(4)}(\theta)$	$0.152h^3 \ddot{y}^{(3)}(\theta)$	Unconditionally stable	The displacement truncation error term represents an upper bound
(ii) $\beta = 1/6$ in Eq. 2.11 Linear Acceleration Method $\ddot{y}_{n+1} = \ddot{y}_n + \frac{h}{2} (\ddot{y}_n + \ddot{y}_{n+1})$ $\dot{y}_{n+1} = \dot{y}_n + h \dot{y}_n + \frac{h^2}{3} \ddot{y}_n + \frac{h^2}{6} \ddot{\ddot{y}}_{n+1}$	$-\frac{h^3}{12} \ddot{y}^{(4)}(\theta)$	$-\frac{h^4}{24} \ddot{y}^{(4)}(\theta)$	$h\omega < \sqrt{12}$	

TABLE 2.1 (cont'd)

Method	Truncation Error Term For Velocity	Truncation Error Term For Displacement	Criterion for Stability of Displacement	Remarks
<p>(iii) $\theta = 1/8$ in Eq. 2.11</p> $\dot{y}_{n+1} = \dot{y}_n + \frac{h}{2} (\ddot{y}_n + \ddot{y}_{n+1})$ $y_{n+1} = y_n + h \dot{y}_n + \frac{3}{8} h^2 \ddot{y}_n + \frac{h^2}{8} \ddot{y}_{n+1}$	$\frac{h^3}{12} y^{(4)}(\theta)$	$\frac{h^3}{8} y^{(3)}(\theta)$	$h\omega < \sqrt{8}$	The displacement truncation error term represents an upper bound
<p>Acceleration Pulse Method</p> $\dot{y}_{n+1} = \dot{y}_n + h \ddot{y}_n$ $y_{n+1} = -y_{n-1} + 2y_n + h^2 \ddot{y}_n$	$\frac{h^2}{2} y^{(3)}(\theta)$	$\frac{h^4}{12} y^{(4)}(\theta)$	$h\omega < 2$	
<p>Parabolic Acceleration Method</p> $\dot{y}_{n+1} = \dot{y}_n + \frac{h}{12} (-\ddot{y}_{n-1} + 8\ddot{y}_n + 5\ddot{y}_{n+1})$ $y_{n+1} = y_n + h \dot{y}_n + \frac{h^2}{24} (-\ddot{y}_{n-1} + 10\ddot{y}_n + 3\ddot{y}_{n+1})$	$-\frac{h^4}{24} y^{(5)}(\theta)$	$-\frac{7}{360} h^5 y^{(5)}(\theta)$	$h\omega < 3.358$	Solution diverges with time

Equation 2.11 can be made capable of representing a third order polynomial exactly by assigning the appropriate value to β , or in effect eliminating the slack parameter. This is done by setting $\gamma = 1$ in the equation, with the result that β becomes $1/6$. Eq. 2.11 then reduces to

$$y_{n+1} = y_n + h \dot{y}_n + \frac{h^2}{6} \ddot{y}_n + \frac{h^3}{6} \ddot{\ddot{y}}_n \quad (2.12)$$

which is Newmark's linear acceleration formula. The predictor to be used with Eq. 2.12 is

$$y_{n+1}^p = y_n + h \dot{y}_n + \frac{h^2}{2} \ddot{y}_n \quad (2.13)$$

obtained by setting $\gamma_{n+1} = \gamma_n$. Newmark [25] has presented the criterion for stability and convergence of Eq. 2.11 for various values of β .

Corresponding to Eq. 2.8, the general second order single point corrector formula for velocity is

$$\dot{y}_{n+1} = b_1 \dot{y}_n + b_2 \dot{y} + b_3 \dot{y}_{n+1} \quad (2.14)$$

A velocity formula compatible with the corresponding displacement formula is obtained by differentiating the latter with respect to the interval h . The following expression for velocity

$$\dot{y}_{n+1} = \dot{y}_n + \frac{h}{2} (\ddot{y}_n + \ddot{y}_{n+1}) \quad (2.15)$$

is compatible with Eq. 2.11 for displacement in the special case where $\beta = 1/6$. In this case, $\dot{y}_{n+1} = \dot{y}_n + a h$, where a is a constant. In practice, the expressions for displacement and velocity need not be compatible. In the Newmark method, Eq. 2.15 is used consistently for

velocity although it is compatible with the displacement equation only when $\beta = 1/6$ (see Table 2.1).

2.3 ERRORS IN THE COMPUTATIONS

The errors introduced into the numerical solution of a differential equation can be classified into three types:

1. Round-off errors introduced by repeated computations using a small step size.
2. Truncation errors involved in representing y_{n+1} by a finite number of terms in the Taylor series expansion. This is the error term represented by R in Eq. 2.5.
3. Propagated error introduced by replacing the differential equation by a particular finite difference equivalent.

Round-off errors are not considered in this study. They are random in nature and therefore must be treated by statistical methods. Error bounds obtained by statistical means often grossly overestimate the real errors. In computer calculations, it is often possible to reduce the round-off errors by using a higher precision in the computations, say, double precision with IBM System/360 and System 370 equipment.

Truncation errors are accumulated locally at each step; unless the integration method being used is totally or partially unstable, the size of the truncation error is the best indication of the accuracy of the numerical solution. Truncation error terms, therefore, provide a useful criterion for assessing the relative accuracy of the various methods of numerical integration. The method of obtaining the truncation

error term for the corrector formula is explained in a number of books on the numerical solution of differential equations [40].

The truncation error terms for some of the presently used formulae for displacement and velocity are summarized in Table 2.1, where the superscript in parentheses indicates differentiation with respect to time t , and θ represents a particular value of t located in the range $t_n \leq \theta \leq t_{n+1}$. The value of θ is not known, but an estimate of the maximum value of the applicable derivative of y in the time range under consideration can be used to obtain an upper bound for the error term.

Propagated errors are discussed later herein under the heading Stability of Computational Method.

2.4 MULTIPOINT FORMULAE

In Section 2.2, a general procedure for deriving predictor-corrector formulae was presented. It was also shown, by an example, that the existing methods of numerical analysis used in the field of structural dynamics can all be derived by this procedure. A similar approach will be used in this section to derive several multipoint formulae for the displacement and velocity of a single-degree-of-freedom system.

A possible multipoint corrector formula for the displacement of a single-degree-of-freedom system is as follows:

$$y_{n+1} = a_1 y_{n-1} + a_2 y_n + a_3 \ddot{y}_{n-1} + a_4 \ddot{y}_n + a_5 \ddot{y}_{n-1} + a_6 \ddot{y}_n + a_7 \ddot{y}_{n+1} \quad (2.16)$$

The formula has seven free parameters and hence can be made exact for

$$y = 1, t, t^2, t^3, t^4, t^5 \text{ and } t^6$$

A whole class of formulae can be obtained by making the equation exact for only

$$y = 1, t, t^2, t^3 \text{ and } t^4$$

leaving two slack parameters, which are selected herein as a_1 and a_3 .

By successively assigning the five values, $1 \dots t^4$ to y , expressions are obtained for a_1 through a_7 . These are summarized in Table 2.2 which gives five different possible formulae obtained by assigning different values to a_1 and a_3 .

The expressions for velocity are derived in a similar manner starting from the general corrector formula:

$$\dot{y}_{n+1} = b_1 \dot{y}_{n-1} + b_2 \dot{y}_n + b_3 \ddot{y}_{n-1} + b_4 \ddot{y}_n + b_5 \ddot{y}_{n-1} \quad (2.17)$$

Eq. 2.17 is made exact for

$$\dot{y} = 1, t, t^2$$

leaving two slack parameters to be chosen as b_1 and b_3 . Some of the possible resulting formulae are presented in Table 2.3.

Of the formulae presented in Tables 2.2 and 2.3, the following (No. 1 in Table 2.2 and No. 2 in Table 2.3, respectively) have been presented by Newmark [23, 24], and were derived by assuming a parabolic variation of acceleration over the time intervals t_{n-1} , t_n , t_{n+1} .

TABLE 2.2
HIGH ORDER CORRECTOR FORMULA, DISPLACEMENT

		(1)	(2)	(3)	(4)	(5)
Coefficient	Expression	$a_1=0$ $a_3=0$	$a_1=0$ $a_3=\frac{7}{15}h$	$a_1=-1$ $a_3=0$	$a_1=0$ $a_3=\frac{h}{10}$	$a_1=1$ $a_3=\frac{14}{15}h$
a_1	a_1	0	0	-1	0	1
a_2	$1-a_1$	1	1	2	1	0
a_3	a_3	0	$\frac{7}{15}h$	0	$\frac{h}{10}$	$\frac{14}{15}h$
a_4	$h+a_1h-a_3$	h	$\frac{8}{15}h$	0	$\frac{9h}{10}$	$\frac{16}{15}h$
a_5	$\frac{1}{24}[-h^2-3a_1h^2+10a_3h]$	$-\frac{h^2}{24}$	$\frac{11}{72}h^2$	$\frac{h^2}{12}$	0	$\frac{2}{9}h^2$
a_6	$\frac{1}{24}[10h^2-10a_1h^2+16a_3h]$	$\frac{10}{24}h^2$	$\frac{262}{360}h^2$	$\frac{10}{12}h^2$	$\frac{29}{60}h^2$	$\frac{28}{45}h^2$
a_7	$\frac{1}{24}[3h^2+a_1h^2-2a_3h]$	$\frac{3h^2}{24}$	$\frac{31}{360}h^2$	$\frac{h^2}{12}$	$\frac{7}{60}h^2$	$\frac{4}{45}h^2$
E_5	$-\frac{7}{3}h^5 - \frac{7}{3}a_1h^5 + 5a_3h^4$	$-\frac{7}{3}h^5$	0	0	$-\frac{11}{6}h^5$	0
E_6	$-\frac{3}{2}h^6 + \frac{3}{2}a_1h^6 - 4a_3h^5$	$\frac{7}{360}h$	$-\frac{101}{30}h^6$	$-3h^6$		$\frac{56}{15}h^6$
Truncation Error		$\frac{7}{360}h^5 y^{(5)}(\theta)$		$-\frac{1}{240}h^6 y^{(6)}(\theta)$		

TABLE 2.3
HIGH ORDER CORRECTOR FORMULA - VELOCITY

		1	2	3	4
Coefficient	Expression	$b_1=0$ $b_3=0$	$b_1=0$ $b_3=-\frac{h}{12}$	$b_1=1$ $b_3=\frac{h}{3}$	$b_3=0$ $b_1=\frac{1}{5}$
b_1	b_1	0	0	1	$\frac{1}{5}$
b_2	$1-b_1$	1	1	0	$\frac{4}{5}$
b_3	b_3	0	$-\frac{h}{12}$	$\frac{h}{3}$	0
b_4	$\frac{h}{2} + \frac{3}{2} b_1 h - 2b_3$	$\frac{h}{2}$	$\frac{8h}{12}$	$\frac{4}{3} h$	$\frac{4}{5} h$
b_5	$\frac{h}{2} - b_1 \frac{h}{2} + b_3$	$\frac{h}{2}$	$\frac{5h}{12}$	$\frac{h}{3}$	$\frac{2}{5} h$
E_3	$-\frac{h^3}{2} + \frac{5}{2} b_1 h^3 - 6b_3 h^2$	$-\frac{h^3}{2}$	0	0	0
E_4	$-h^4 + b_1 h^4$		$-h^4$	0	$-\frac{4}{5} h^4$
Truncation Error		$\frac{h^3}{12} y^{(4)}(\theta)$	$-\frac{h^4}{24} y^{(5)}(\theta)$		

$$y_{n+1} = y_n + h \dot{y}_n + \frac{h^2}{24} [-\ddot{y}_{n-1} + 10\ddot{y}_n + 3\ddot{y}_{n+1}] - \frac{7}{360} h^5 y^{(5)}(\theta) \quad (2.18)$$

$$\dot{y}_{n+1} = \dot{y}_n + \frac{h}{12} [-\ddot{y}_{n-1} + 8\ddot{y}_n + 5\ddot{y}_{n+1}] - \frac{h^4}{24} y^{(5)}(\theta) \quad (2.19)$$

where

$$t_n \leq \theta \leq t_{n+1}$$

Formula No. 1 in Table 2.3 will be recognized as the expression for velocity used in association with the Newmark's β method. It can also be derived on the assumption that acceleration varies linearly over a time step.

Of the formulae shown in Table 2.2, the following (No. 3 in the table) has the minimum truncation error:

$$y_{n+1} = -y_{n-1} + 2y_n + \frac{h^2}{12} [\ddot{y}_{n-1} + 10\ddot{y}_n + \ddot{y}_{n+1}] - \frac{h^6}{240} y^{(6)}(\theta) \quad (2.20)$$

where

$$t_{n-1} \leq \theta \leq t_{n+1}$$

Equation 2.20 has been previously presented in the mathematical theory of the numerical solution of differential equations as a formula ~~useful for the numerical solution of second order differential equations~~ that do not contain the first derivative [39, 40]. The present study shows that it is also useful for the solution of second order differential equations of motion that may involve the velocity term, for example, for vibration with viscous damping. The stability of this formula has been studied by the author; the results of the study are contained in Section 2.5. The performance of the formula with respect to stability is

quite good. It also gives significant improvement in the accuracy of the solution when compared with the conventional single point formulae. This is demonstrated by a series of examples of response calculations presented towards the end of this Chapter, where this formula is used in conjunction with a multipoint expression for velocity given by Eq. 2.19.

It may be noted that the Acceleration Pulse Method [38] developed at M.I.T. uses a lower order formula rather similar to that given in Eq. 2.20.

2.5 STABILITY OF THE COMPUTATIONAL METHOD

As discussed earlier in this Chapter, errors are introduced into the numerical solution due to truncation and due to the finite difference approximation to the differential equation. It is important to know the effect of the error introduced at one step upon the computations at the next step. If the errors have a tendency to grow, the solution will soon become meaningless. In such a situation, the computational method is said to be unstable.

As discussed earlier, instability may be total or partial.

Total instability may be caused by the presence of spurious roots in the solution of a difference equation while partial instability results from progressive divergence of the numerical solution from the real solution. This may be explained by considering Newmark's parabolic acceleration method. A detailed examination of the stability of this method has been presented elsewhere [23, 24]; a summary of the treatment is presented directly below.

The differential equation

$$\ddot{y} + \omega^2 y = 0 \quad (2.21)$$

has an exact solution

$$y = D \sin \omega t + E \cos \omega t \quad (2.22)$$

in which D and E are constants of integration.

In a numerical solution with a step size h, the elapsed time t at any stage in the analysis is given by

$$t = n h$$

where n is the number of time intervals already considered. The displacement at time t, as obtained by the numerical solution, is denoted by y_n to distinguish it from the corresponding exact value of the displacement at time t.

With the parabolic acceleration method, the following difference equation is used for the numerical integration of Eq. 2.21.

$$a_1 y_{n+2} + a_2 y_{n+1} + a_3 y_n + a_4 y_{n-1} = 0 \quad (2.23)$$

Provided ωh is less than 3.358 [24], the solution of this difference equation can be written as:

$$y_n = A \rho^n + \gamma^n (B \cos n\phi + C \sin n\phi) \quad (2.24)$$

where A, B, and C are constants of integration and ρ , γ and ϕ are functions of ωh . On comparison with the exact solution which has two roots, it will be evident that ρ^n represents an extraneous root. Fortunately, in this case, ρ works out to less than unity and the extraneous

solution dies out very soon. If the numerical solution were exact, γ would equal 1; also $n\phi$ would equal ωt or ϕ would equal ωh . In fact, γ turns out to be slightly greater than 1 and for large values of n the solution starts oscillating farther and farther from the true solution. If h is larger than 3.358, all roots of the difference equation become real and the method is rendered unstable.

In the above example, although the extraneous root does not give rise to instability since ρ is less than one, the method is stable only as long as ωh is less than 3.358. Also, since γ is slightly greater than unity, partial instability also results because of progressive divergence of the numerical solution from the true solution.

The stability of Eq. 2.20 may also be examined for the case when it is used to integrate Eq. 2.21. The substitution of Eq. 2.21 into Eq. 2.20, with the truncation error term omitted, gives:

$$y_{n+1} - \alpha y_n + y_{n-1} = 0 \quad (2.25)$$

where

$$\alpha = \frac{2 - 5/6 h^2 \omega^2}{1 + \frac{h^2 \omega^2}{12}}$$

The difference equation Eq. 2.25 has a solution of the form:

$$y_n = A_1 \rho_1^n + A_2 \rho_2^n$$

where A_1 and A_2 are constants whose value depend upon the initial conditions and ρ_1 and ρ_2 are roots of the characteristic equation:

$$\rho^2 - \rho \alpha + 1 = 0 \quad (2.26)$$

The roots of Eq. 2.26 are:

$$\rho_{1,2} = \frac{1}{2} (\alpha \pm \sqrt{\alpha^2 - 4}) \quad (2.27)$$

For the solution of Eq. 2.25 to be periodic so that it corresponds to the exact solution of Eq. 2.21, the roots ρ_1 and ρ_2 must be complex or the quantity $(\alpha^2 - 4)$ must be less than zero. For this condition to be satisfied, $h^2 \omega^2$ must be less than 6. Thus, the maximum step size which will permit stability is established. Assuming that the condition for stability is satisfied, the solution of Eq. 2.25 reduces to the form:

$$y_n = A \cos n\phi + B \sin n\phi$$

where A and B are constants and

$$\cos \phi = \frac{1 - \frac{5}{12} \omega^2 h^2}{1 + \frac{\omega^2 h^2}{12}} \quad \sin \phi = \frac{\omega h \sqrt{1 - \frac{\omega^2 h^2}{6}}}{1 + \frac{\omega^2 h^2}{12}} \quad (2.28)$$

The value of A will be equal to y_0 , the initial displacement.

However, B will depend upon the method used to start the integration process, that is upon how y_1 is obtained from y_0 and $\dot{y}_0$. If Newmark's β method is used for starting the integration, then

$$B = \frac{y_0}{\omega} \frac{1 + \frac{\omega^2 h^2}{12}}{(1 + \beta h^2 \omega^2) (1 - \frac{\omega^2 h^2}{6})^2} - y_0 \frac{\frac{h^3 \omega^3}{24} - \beta \frac{h^3 \omega^3}{2}}{(1 + \beta h^2 \omega^2) (1 - \frac{\omega^2 h^2}{6})^2} \quad (2.29)$$

Thus, when ωh tends to zero, the numerical solution approaches the true solution. The method does not then give any extraneous roots; also, the numerical solution never diverges from the true solution provided $h^2 \omega^2$ is less than 6.

As a further indication of the relative accuracy of this method, one may compare the period T and amplitude A in the true solution with the comparable quantities T_s and A_s which are obtained by the numerical solution. The period obtained by the numerical solution is given by:

$$T_s = \frac{2\pi}{\phi/h} \quad (2.30)$$

Thus

$$T_s/T = \frac{\omega h}{\phi}$$

where

$$\phi = \arccos \frac{1 - 5/12 \omega^2 h^2}{1 + \frac{\omega^2 h^2}{12}}$$

The ratio T_s/T is plotted against ωh in Fig. 2.1. The ratio of the peak amplitudes in the numerical solution and true solution for $\beta = 1/6$ and $y_0 = 0$ is plotted in Fig. 2.2.

The conditions of stability of the displacement formulae in Table 2.1 have been discussed in relation to the solution of the simple second order differential equation of motion, Eq. 2.21, in references 23, 24, 25 and 38. These conditions are also presented in Table 2.1 in terms of the parameter ωh , the natural frequency multiplied by the time step.

FIG. 2.1 RATIO OF PERIOD OBTAINED BY NUMERICAL
SOLUTION TO EXACT PERIOD

FIG. 2.2 RATIO OF PEAK AMPLITUDE OBTAINED BY NUMERICAL
SOLUTION TO EXACT PEAK AMPLITUDE

2.6 SOLUTION OF THE DIFFERENTIAL EQUATIONS

A second order differential equation of motion of a single-degree-of-freedom mechanical system subjected to a force which varies with time may take the form:

$$m \ddot{y} + c \dot{y} + k y = F(t) \quad (2.32)$$

where m is mass, c is the viscous damping coefficient, k is stiffness of the system, and F is an exciting force which is a function of time.

Alternatively, the system may be subjected to base motion in which case the equation of motion may take the form:

$$m \ddot{z} + c \dot{z} + k z = -m \ddot{x} \quad (2.33)$$

where x is a foundation displacement and $z = y - x$. If k remains constant, both equations are linear with constant coefficients.

Considering Eq. 2.32, the following relationship will hold at point $n+1$:

$$m \ddot{y}_{n+1} + c \dot{y}_{n+1} + k y_{n+1} = F(t_{n+1}) \quad (2.34)$$

Suppose that the following expressions are used for the displacement and velocity at successive time steps:

$$y_{n+1} = -y_{n-1} + 2y_n + \frac{h^2}{12} \{\ddot{y}_{n-1} + 10\ddot{y}_n + \ddot{y}_{n+1}\} \quad (2.35)$$

$$\dot{y}_{n+1} = \dot{y}_n + \frac{h}{12} \{-\ddot{y}_{n-1} + 8\ddot{y}_n + 5\ddot{y}_{n+1}\} \quad (2.36)$$

If the solution for each new time step is to be obtained by iteration, an estimated value must be used for $\ddot{y}_{n+1}$, the first approxi-

mation being usually taken as equal to y_n ; then, $\dot{y}_{n+1}$ and $\ddot{y}_{n+1}$ are evaluated from Eqs. 2.35 and 2.36. A better approximation for $\ddot{y}_{n+1}$ is now obtained by substituting these values into Eq. 2.32; this improved approximation is used in the next iteration.

However, since Eq. 2.34 is linear, Eqs. 2.34, 2.35 and 2.36 can all be solved simultaneously to obtain the values of the three unknowns y_{n+1} , $\dot{y}_{n+1}$ and $\ddot{y}_{n+1}$. This direct solution by elimination makes it possible to achieve a large economy in computation time.

Equations 2.35 and 2.36 and also other multipoint methods discussed in Section 2.4 make use of information at two previous points, that is at t_{n-1} and t_n . At the beginning of the solution, the values of y , $\dot{y}$ and $\ddot{y}$ are available only at one point, usually at $t=0$. The solution is, therefore, not self-starting until y_1 , $\dot{y}_1$ and $\ddot{y}_2$ are evaluated by some other means. This, however, does not present any special problems since several methods are available to start the solution. Any one of the Taylor Series, Runge-Kutta or Linear Acceleration Methods can be used to obtain the values of y , $\dot{y}$ and $\ddot{y}$ at time t_1 .

As will be discussed in more detail in Section 3.13 under the topic of Inelastic Analysis, there are cases where the structural stiffness k is not constant but instead depends upon the structural displacement y . Although equations of motion are then no longer linear, a numerical solution can be obtained by assuming that the structural stiffness remains constant during each individual time step. Multipoint methods cannot be used to advantage in this situation, first because the differ-

ential equation changes from time step to time step due to changes in the value of k , and secondly because it may be desirable to reduce the time step size whenever the structural stiffness changes. However, it is still possible to use the process of elimination to obtain the numerical solution by using the equations in their incremental form. The elimination method again makes it possible to achieve considerable economy in computation time.

2.6.1 Selection of Time Step Size

The step size to be adopted for a numerical solution using the predictor-corrector method depends upon the following:

- (1) the convergence characteristics of the corrector
- (2) truncation errors
- (3) the stability of the solution.

In general, an upper limit is imposed upon the possible time step size by the need to ensure convergence and stability and to reduce the truncation errors. However, as discussed above, an iterative solution of the corrector is not necessary for problems in the dynamic analysis of structures. Therefore, in structural dynamics, convergence need not be a criterion for limiting the step size. The step size can then be selected on the basis of the other two considerations.

In all the formulae presented in this study, the truncation error term is of the form:

$$C h^p y^{(p)}(0)$$

where C is a constant and p is the degree of the formula. Since the p th derivative of y will contain a factor ω^p , the magnitude of truncation error depends on the value of $(\omega h)^p$. It is evident that, to keep the truncation errors low, the time step size h should be chosen so that ωh is less than 1.

For systems with only a few degrees of freedom, the criteria for stability are not usually as restrictive as those for truncation error. For systems with a larger number of degrees of freedom, the truncation error term is usually important only for the first few low frequency modes, with only a small contribution from the higher modes. However, the stability criterion must be based on the highest mode frequency because instability even in the highest mode may result in a progressively dominating contribution from that mode, ultimately rendering the solution unstable. In such situations, the stability criterion may become more restrictive. This can be seen very clearly in Example 3 presented in this Chapter.

2.6.2 Solution of Sets of Simultaneous Equations

Up to this point, the discussion has been confined to a system having a single degree of freedom. For a multi-degree-of-freedom system, such as would be used to model a high rise building under dynamic load or seismic acceleration, the procedure is essentially similar. An r degree of freedom system requires the solution of $3r$ simultaneous equations at each time step, instead of the three for a single-degree-of-freedom system. Typical sets of equations are similar to Eqs. 2.34,

2.35 and 2.36, except that they are now used in their vector form.

2.7 EXAMPLES OF RESPONSE CALCULATION

To provide an indication of the comparative accuracy of some of the predictor-corrector methods described in this Chapter, three examples of response calculation are presented.

Example No. 1

In this example, the response of a single degree of freedom system to a sinusoidal base excitation is calculated using the linear acceleration methods and the formulae of Eqs. 2.18, 2.19 and 2.20. The single degree of freedom oscillator shown in Fig. 2.3 has the following properties:

$$\text{Natural frequency} = \frac{\omega}{2\pi} = 1.0 \text{ cycles per sec}$$

$$\text{Ratio of damping to critical damping} = 0.0$$

The oscillator, initially at rest, is excited by a base motion given by:

$$x = Q \sin \Omega t$$

where

$$Q = \text{amplitude} = 1.00 \text{ inches}$$

$$\Omega = \text{exciting frequency in radians per second} = 1.5708$$

$$t = \text{time in seconds.}$$

The results of the calculations are compared in Table 2.4 with an accurate solution given by:

$$z = Q \frac{\sin \Omega t - \frac{\omega}{\Omega} \sin \omega t}{\left(\frac{\omega}{\Omega}\right)^2 - 1}$$

FIG. 2 3 SINGLE DEGREE-OF-FREEDOM OSCILLATOR
SUBJECT TO BASE MOTION

TABLE 2.4
RESPONSE OF SINGLE-DEGREE-OF-FREEDOM OSCILLATOR

Time secs	Exact	Relative Displacement, inches									
		Newmark's Method $\beta = 1/6$					Formulae				
		$h=0.02\text{sec}$					$h=0.04\text{sec}$				
		Displace.	Error	Displace.	Error	Displace.	Displace.	Error	Displace.	Error	Formulae 2.20, 2.19 $h=0.04\text{sec}$
1.0	.06667	.06727	-60	.07056	-389	.06622	.06616	+45	.06616	+51	
1.2	-.19021	-.19010	-11	-.18934	-87	-.19075	-.19036	+74	-.19036	+15	
1.4	-.10281	-.10362	+81	-.10763	+482	-.10279	-.10227	-2	-.10227	-54	
1.6	.19593	.19513	+80	.19114	+479	.19690	.19648	-97	.19648	-55	
1.8	.27422	.27465	-43	.27690	-268	.27485	.27388	-63	.27388	+34	
2.0	.00000	.00029	-29	.00786	-786	-.00051	-.00074	+81	-.00074	+94	
10.0	.00001	.00643	-642	.03919	-3918	-.00411	-.00472	+412	-.00472	+473	
10.2	-.27422	-.27219	-203	-.25959	-1463	-.28078	-.27559	+656	-.27559	+137	
10.4	-.19594	-.20133	+539	-.22741	+3147	-.19578	-.19188	-16	-.19188	-406	
10.6	.10279	.09732	+547	.06770	+3509	.10975	.10682	-696	.10682	-403	
10.8	.19022	.19240	-218	.20071	-1049	.19447	.18857	-425	.18857	+165	
11.0	-.06665	-.05955	-710	-.02355	-4310	-.07115	-.07182	+450	-.07182	+517	
14.0	.00002	.00901	-899	.05465	-5466	-.00579	-.00661	+581	-.00661	+663	
14.2	-.27421	-.27133	-288	-.25214	-2287	-.28341	-.27614	+920	-.27614	+193	
14.4	-.19595	-.20337	+742	-.23825	+4230	-.19572	-.19033	-23	-.19033	-562	
14.6	.10279	.09519	+760	.05348	+4931	.11243	.10832	-964	.10832	-553	
14.8	.19022	.19312	-290	.20268	-1246	.19606	.18794	-584	.18794	+278	
15.0	-.06665	-.05697	-965	-.00810	-5855	-.07285	-.07370	+620	-.07370	+705	

where ω is the natural frequency of the oscillator.

Example No. 2

In this example, a two story single bay frame is analyzed for a sinusoidal base motion. The frame shown in Fig. 2.4 has the following properties:

- (1) The masses are lumped at the floor levels. The mass matrix, defined in terms of the two degrees of freedom of the structure shown in Fig. 2.4, is:

$$[m] = \begin{bmatrix} 0.445 & 0.000 \\ 0.000 & 0.445 \end{bmatrix} \frac{\text{lbs-sec}^2}{\text{in}}$$

- (2) The stiffness matrix, defined in terms of the two degrees of freedom shown in Fig. 2.4, is:

$$[k] = \begin{bmatrix} 1966.7 & -946.02 \\ -946.02 & 850.67 \end{bmatrix} \text{ lbs/in}$$

- (3) The natural frequencies of the system are as follows:

$$\omega_1 = 26.41 \text{ rad/sec}$$

$$\omega_2 = 75.06 \text{ rad/sec}$$

It is assumed that there is 5% critical damping in each mode.

The corresponding damping matrix works out to:

$$[c] = \begin{bmatrix} 2.810 & -0.935 \\ -0.935 & 1.710 \end{bmatrix} \frac{\text{lbs-sec}}{\text{in}}$$

The structure, initially at rest, is excited by a base motion given by:

$$x = Q \sin \Omega t$$

FIG. 2.4 TWO STORY STEEL FRAME

SUBJECTED TO BASE MOTION

where

Q = amplitude = 1 in.

Ω = exciting frequency = 36.0 rad/sec

An exact solution for the response is given by:

$$\{z\} = [\phi] \{n\}$$

where $\{z\}$ is the vector of the displacement relative to base, $[\phi]$ is the modal matrix, and the components of vector $\{n\}$ are given by:

$$n_r = \lambda_r Q \frac{e^{-\zeta_r \omega_r t} N_r - 2 \zeta_r \frac{\omega_r}{\Omega} \cos \Omega t + \left[\left(\frac{\omega_r}{\Omega} \right)^2 - 1 \right] \sin \Omega t}{\left[\left(\frac{\omega_r}{\Omega} \right)^2 - 1 \right]^2 + 4 \zeta_r^2 \left(\frac{\omega_r}{\Omega} \right)^2}$$

where

$$N_r = 2 \zeta_r \frac{\omega_r}{\Omega} \cos \omega_{dr} t - \frac{\omega_r}{\Omega \sqrt{1 - \zeta_r^2}} \left[\left(\frac{\omega_r}{\Omega} \right)^2 - 1 + 2 \zeta_r^2 \right] \sin \omega_{dr} t$$

$$\omega_{dr} = \omega_r \sqrt{1 - \zeta_r^2} \quad (r=1,2)$$

and

$$\lambda_r = \frac{\{a^r\}^T [m] \{1\}}{\{a^r\}^T [m] \{a^r\}}$$

The quantity λ_r is the modal participation factor for the r th mode, $\{a^r\}$ is the r th mode shape, and ζ_r is the fraction of critical damping for the r th mode.

The results of the calculations are shown in Table 2.5.

TABLE 2.5
RESPONSE OF TWO-DEGREE-OF-FREEDOM
SYSTEM

Time sec	Story	Relative Displacement, inches									
		Exact		Newmark's Method $\beta = 1/6$				Formulae 2.18, 2.19			
		h=0.005sec		h=0.01sec		h=0.01sec		h=0.01sec		Formulae 2.20, 2.19	
		Displace.	Error	Displace.	Error	Displace.	Error	Displace.	Error	Displace.	Error
0.00	1	.0000	0	.0000	0	.0000	0	.0000	0	.0000	0
	2	.0000	0	.0000	0	.0000	0	.0000	0	.0000	0
0.04	1	-.4186	+47	-.4371	+185	-.4223	+185	-.4223	+37	-.4186	0
	2	-.8836	+9	-.8875	+39	-.8865	+39	-.8865	+29	-.8849	+13
0.08	1	.6234	+5	.6210	+24	.6244	+24	.6244	-10	.6222	+12
	2	.9127	+136	.8588	+539	.9065	+539	.9065	+62	.9130	-3
0.12	1	1.0492	+109	1.0066	+426	1.0468	+426	1.0468	+24	1.0509	-17
	2	2.1701	+31	2.1572	+129	2.1790	+129	2.1790	-89	2.1701	0
0.16	1	-.3009	-140	-.2466	-543	-.2870	-543	-.2870	-139	-.3022	+13
	2	-.5483	-77	-.5172	-311	-.5421	-311	-.5421	-62	-.5473	-10
0.20	1	-1.8452	-37	-1.8283	-169	-1.8559	-169	-1.8559	+107	-1.8443	-9
	2	-3.2500	-232	-3.1591	-909	-3.2459	-909	-3.2459	-41	-3.2514	+14
0.24	1	-.5129	-53	-.4955	-174	-.5097	-174	-.5097	-32	-.5140	+11
	2	-1.2130	+67	-1.2370	+240	-1.2341	+240	-1.2341	+211	-1.2124	-6
0.28	1	1.6472	+199	1.5728	+744	1.6311	+744	1.6311	+161	1.6488	-16
	2	3.1493	+216	3.0614	+879	3.1504	+879	3.1504	-11	3.1498	-5
0.32	1	1.5773	+22	1.5624	+149	1.5980	+149	1.5980	-207	1.5768	+5
	2	2.8324	+143	2.7786	+538	2.8379	+538	2.8379	-55	2.8332	-8
0.36	1	-.9629	-81	-.9242	-387	-.9688	-387	-.9688	+59	-.9634	+5
	2	-1.6417	-230	-1.5596	-821	-1.6207	-821	-1.6207	-210	-1.6432	+15

Example No. 3

A ten story single bay steel rigid frame, shown in Fig. 2.5, is analyzed for the first four seconds of the north-south component of the El-Centro earthquake of May 18, 1940.

The weight of each story level is 44.0 kips. The moments of inertias shown in Fig. 2.5 for the beams and columns each apply to a group of members represented by a single line in the figure. The frequencies of the first three modes of vibration of the structure are as follows:

$$\omega_1 = 4.41 \text{ radians per sec}$$

$$\omega_2 = 12.56 \text{ radians per sec}$$

$$\omega_3 = 22.40 \text{ radians per sec}$$

The results of numerical solutions using different sizes of time step are presented in Tables 2.6 and 2.7. Since an exact analysis is not possible for this frame, the response values obtained from the higher order formula with $h = .0025 \text{ sec.}$, which are in fact very close to the values obtained from Newmark's linear acceleration method with the same size of time step, are assumed to be accurate, and the errors are evaluated on that basis. The additional accuracy achieved by using the higher order formula is more evident in the values obtained for the floor level accelerations.

The highest mode (tenth mode) frequency for the frame is 122.80 radians per second, corresponding to a period of 0.05 sec. For

FIG. 2.5 TEN STORY STEEL RIGID FRAME

TABLE 2.6
RESPONSE OF TEN STORY FRAME, RELATIVE DISPLACEMENTS

		Relative Displacement, inches											
		Newmark's Method $\beta = 1/6$						Formulae 2.20, 2.19					
Time sec	Story	h=0.0025sec		h=0.005sec		h=0.01sec		h=0.025sec		h=0.005sec		h=0.01sec	
		Displace.	Error	Displace.	Error	Displace.	Error	Displace.	Error	Displace.	Error	Displace.	Error
1.2	1	.0879	-2	.0881	-2	.0882	-3	.0879	0	.0879	0	.0879	0
	4	.8018	+3	.8016	+3	.8009	+10	.8019	-1	.8020	-1	.8022	-3
	7	1.9692	+4	1.9689	+4	1.9675	+18	1.9693	-2	1.9695	-2	1.9702	-9
	10	2.8208	-2	2.8209	-2	2.8213	-6	2.8207	-2	2.8209	-2	2.8213	-6
1.4	1	.2379	-4	.2382	-4	.2387	-9	.2378	0	.2378	0	.2377	+1
	4	1.2951	+2	1.2950	+2	1.2944	+8	1.2952	0	1.2952	0	1.2956	-4
	7	1.9415	+5	1.9411	+5	1.9402	+14	1.9414	+2	1.9414	+2	1.9410	+6
	10	2.7108	-8	2.7114	-8	2.7138	-32	2.7106	0	2.7106	0	2.7107 ₆	-1
1.6	1	.2801	-1	.2801	-1	.2785	+15	.2800	-1	.2801	-1	.2802	-2
	4	1.7453	-5	1.7457	-5	1.7462	-10	1.7452	-3	1.7455	-3	1.7466	-14
	7	2.7504	-14	2.7515	-14	2.7542	-41	2.7501	-1	2.7502	-1	2.7510	-9
	10	3.2341	+18	3.2328	+18	3.2276	+70	3.2346	-1	3.2347	-1	3.2354	-8
1.8	1	.3373	-5	.3370	-5	.3364	-11	.3375	+1	.3376	+1	.3379	+4
	4	1.3748	+6	1.3752	+6	1.3771	+25	1.3746	+2	1.3748	+2	1.3755	+9
	7	0.4186	-14	.4175	-14	.4133	-56	.4189	-2	.4187	-2	.4182	-7
	10	1.1677	+4	1.1675	+4	1.1670	+9	1.1679	-4	1.1683	-4	1.1700 ₉	-21
2.0	1	.0825	+2	.0828	+2	.0824	-2	.0826	0	.0826	0	.0825	-1
	4	1.3730	-3	1.3727	-3	1.3689	-41	1.3730	+1	1.3731	+1	1.3734	+4
	7	4.4778	+29	4.4801	+29	4.4867	+95	4.4772	+3	4.4775	+3	4.4785	+13
	10	7.9427	-16	7.9446	-16	7.9374	-58	7.9432	+8	7.9440	+8	7.9468	+36

TABLE 2.7
RESPONSE OF TEN STORY FRAME, RELATIVE ACCELERATIONS

Time sec	Story	Relative Acceleration, inches per sec ²											
		Newmark's Method $\beta = 1/6$						Formulae 2.20, 2.19					
		h=0.0025sec		h=0.005sec		h=0.01sec		h=0.025sec		h=0.05sec		h=0.10sec	
		Acceleration	Acceler.	Error	Acceler.	Error	Acceler.	Acceler.	Error	Acceler.	Error	Acceler.	Error
1.2	1	-8.217	-9.310	+1.373	-7.827	-.11	-7.937	-7.928	-.084	-8.000	+.06	-8.000	+.06
	4	-31.177	-28.669	-3.003	-18.689	-12.983	-31.672	-31.716	+.044	-32.000	+.32	-32.000	+.32
	7	-113.050	-111.850	-1.652	-107.110	-6.392	-113.502	-113.530	+.028	-113.800	+.29	-113.800	+.29
	10	-97.448	-97.974	+.714	-99.740	+2.480	-97.260	-97.233	-.027	-97.082	-.17	-97.082	-.17
1.4	1	2.191	-1.511	+4.893	-.994	+4.376	3.381	3.510	-.129	4.810	-1.42	4.810	-1.42
	4	-61.411	-60.233	-1.453	-56.101	-5.585	-61.686	-61.905	+.219	-62.962	+1.27	-62.962	+1.27
	7	70.724	71.594	-1.231	69.524	+0.839	70.363	70.360	+.003	70.156	+.20	70.156	+.20
	10	-21.545	-22.071	+.683	-24.619	+3.231	-21.388	-21.375	-.012	-21.288	-.10	-21.288	-.10
1.6	1	-33.019	-36.914	+5.618	-24.400	-6.896	-31.296	-31.174	-.122	-28.926	-2.37	-28.926	-2.37
	4	-161.100	-162.630	+1.510	-164.740	+3.580	-161.160	-161.250	+.090	-162.130	+.97	-162.130	+.97
	7	-114.200	-116.210	+2.860	-119.030	+5.680	-113.350	-113.340	-.010	-113.020	-.33	-113.020	-.33
	10	-85.015	-83.643	-1.854	-78.915	-6.586	-85.501	-85.569	+.068	-85.930	+.42	-85.930	+.42
1.8	1	23.021	22.237	+2.630	27.739	-2.872	24.867	24.862	+.005	27.094	-2.22	27.094	-2.22
	4	145.050	144.260	+.403	137.520	+7.143	144.663	144.877	-.214	144.440	+.22	144.440	+.22
	7	-84.503	-87.078	+3.818	-96.574	+13.314	-83.260	-83.197	-.063	-82.412	-.84	-82.412	-.84
	10	-239.990	-239.020	-1.280	-235.300	-5.006	-240.300	240.360	-.060	-240.410	+.11	-240.410	+.11
2.0	1	-23.682	-14.799	-9.401	-14.543	-9.660	-24.200	-24.423	+.223	-24.454	+.25	-24.454	+.25
	4	77.768	76.832	-.697	61.113	+15.022	76.135	76.296	-.161	74.768	+1.36	74.768	+1.36
	7	209.530	214.190	-6.280	224.520	-16.610	207.910	207.820	+.090	207.500	+.61	207.500	+.61
	10	473.520	473.970	-0.530	477.620	-4.180	473.440	473.530	-.090	473.890	+.45	473.890	+.45

the higher order formula, instability is expected to result when $h\omega$ is greater than $\sqrt{6}$, or for h greater than 0.019 sec. It is found that the method does become unstable for $h = 0.02$ sec.

With Newmark's linear acceleration method, the maximum value of h beyond which the method becomes unstable is given by:

$$h = 0.551 \times T_{\min} \\ = 0.0282 \text{ sec.}$$

where $T_{\min}$ is the period of the highest frequency mode of the structure. A test run with $h = 0.03$ sec shows that the method does indeed become unstable.

The examples presented here show quite clearly that, provided the conditions of stability are satisfied, the multipoint method recommended in this study gives considerably more accurate results than the single point linear acceleration method when the same time step is used for both of them. In fact, even with a time step twice as large as that used in the single point method the accuracy obtained with the multipoint method is better.

CHAPTER 3

METHOD OF ANALYSIS FOR A MULTISTORY RIGID FRAME BUILDING STRUCTURE

3.1 INTRODUCTION

A greater part of the supporting evidence for the research of this thesis is derived from analytical studies of multistory rigid frame building structures. This Chapter discusses the development of a mathematical model of the building, which would permit a numerical integration of the equations of motion in an efficient manner. It also presents the approximations and assumptions made in modelling the building, the details of conventions used in defining the degrees of freedom, the details of the analytical techniques used, and a brief description of the computer programs developed for the computations.

The multistory frame modelled is essentially planar, all out of plane motions and torsion being inhibited. Although the assumption of in-plane displacements has, no doubt, limited the generality of the model, nevertheless the availability of this model has permitted a number of important questions to be investigated.

Since a greater part of this Chapter is devoted to preparing the ground for analytical studies that follow, and to introducing and defining subtopics such as plastic hinge rotations, ductility ratios and shear coefficients, not many original ideas will be presented at this stage. However, the methods of analysis described in Section 3.13 are of interest. This Section demonstrates the use of a multipoint method

developed in Chapter 2, and shows that a process of elimination for solving the numerical equation is more efficient than a process of iteration. It also presents, what the author believes to be an efficient procedure for solving the matrix equations encountered in response computation by the use of Cholesky's method.

3.2 ASSUMPTIONS MADE IN THE ANALYSIS

The following assumptions have been made in the analysis.

1. The moment curvature relationship for the frame members is assumed to be perfectly elasto-plastic. The effect of this assumption is to confine inelastic behaviour to point hinges; extended regions of inelasticity are excluded from consideration.

2. The plastic hinges form only at the ends of members.

3. The effect of axial deformation of beams can be neglected.

To examine the effect of column deformations on response, two separate sets of computer programs have been developed. In the first set of programs, the columns are considered axially rigid, while in the other set, axial column deformations are taken into account.

4. The effect of axial loads on member stiffness properties is negligible except for the effect of vertical loads acting through sideways deflections, normally termed the P- Δ effect, which can be taken into account in formulating the equations of equilibrium.

5. Local buckling or out of plane buckling of members is prevented.

6. The effect of the rate of straining on material property is small enough that the strain rate can be considered to be very low.
7. The material has a stable hysteretic moment curvature relationship under cyclic loading.
8. The effect of axial load on the yield and plastic moments of beams is small. For columns, the effect of axial load on the plastic moment capacity can be accounted for by a simple empirical equation.
9. Member deformations due to shear are negligible.
10. The frame is plane, rigid-jointed and free from joint translational restraint. First story columns are fixed to a rigid foundation.
11. The frame undergoes vibration only in its own plane.
12. The mass of the structure is lumped at the floor levels.
13. Inertia forces associated with lateral floor displacements only are considered. Inertia effects due to the angular acceleration of floors and the vertical motion resulting from horizontal sway are negligible.
14. The damping in the structure can be adequately represented by a viscous type damping, i.e. the damping force is assumed to be proportional to velocity.
15. The base excitation is a horizontal motion of the rigid foundation in the plane of the frame.

3.3 MOMENT CURVATURE RELATIONSHIP FOR A STEEL MEMBER

Experiments have shown that, under cyclic loading, the moment curvature relationship for a steel section forms a closed hysteresis loop

like the one shown in Fig. 3.1(a) in a non-dimensional form. Several idealized forms of this relationship have been used in dynamic analysis; one that represents the relationship mathematically to a very adequate level of accuracy is the Ramberg-Osgood function. The initial loading curve in tension and compression, called the skeleton curve or the virgin curve, is given by the equation:

$$\frac{\phi}{\phi_y} = \frac{M}{M_y} \left\{ 1 + \left| \frac{M}{M_y} \right|^{r-1} \right\} \quad (3.1)$$

where ϕ is the curvature, M is the moment and ϕ_y , M_y and r are parameters dependant on the material properties. The unloading and reloading curves called the branch curves are given by:

$$\frac{\phi - \phi_o}{2\phi_y} = \frac{M - M_o}{2M_y} \left\{ 1 + \left| \frac{M - M_o}{2M_y} \right|^{r-1} \right\} \quad (3.2)$$

where M_o/M_y , ϕ_o/ϕ_y is the origin of the branch curve.

Though the Ramberg-Osgood functions have been used for the dynamic analysis of certain idealized frames [28], they are not very convenient to handle in computations. Simple bilinear relationships like the one shown in Fig. 3.1(c) have, therefore, been used in some investigations [12, 30]. In a series of cyclic load tests on wide flange steel cantilever beams, Popov and Bertero [43], in fact, found that the load displacement hysteretic curves computed from a bilinear moment curvature relationship, where the second branch had a slope of 1/4% of the initial loading branch, were reasonably close to the experimentally obtained curves.

FIG. 3-1 MOMENT-CURVATURE RELATIONSHIP

The easiest to handle in computations is the elasto-plastic moment curvature relationship like the one shown in Fig. 3.1(b). There is also some evidence to show that, for problems in dynamic analysis, such a relationship gives fairly good results. Goel [28] has compared the results of response calculations using a Ramberg-Osgood moment curvature relationship with $r=5$ and $r=10$ with those obtained using a perfectly elasto-plastic relationship (equivalent to $r=\infty$). The response values obtained from all the three analyses were quite close. The results of the elasto-plastic analysis were, in fact, slightly on the conservative side. Similar relationships appear to exist between the results obtained from a simple bilinear model and an elasto-plastic model in Pekau's studies [30].

In practical building frames, lack of perfect rigidity in the beam column joints and shear deformation in the column web at a joint add to the flexibility of the joint. These factors can be approximately accounted for by a modification of the moment curvature relationship of the members [31]. This modification has the effect of decreasing the slope of the second branch of the bilinear moment curvature relationship, bringing the latter closer to an elasto-plastic curve.

For the reasons stated above, the moment curvature hysteresis curves have been assumed as perfectly elasto-plastic in the present study.

3.4 STABILITY OF THE HYSTERESIS LOOP UNDER CYCLIC LOADING

The moment curvature hysteresis curves for members were initially derived on the basis of static load tests. Naturally, there have been doubts whether these relationships are applicable to dynamic cyclic loading of the type experienced by the frame members during an earthquake induced vibration. There has been concern that the material properties may deteriorate under sustained cyclic loading in the post-elastic range.

Recently, a number of experimental investigations have been carried out to determine the characteristics of the hysteresis curve under dynamic cyclic loading and large deformations [43, 44, 45, 46, 47, 48]. The results of these investigations are quite encouraging. They show that hysteresis curves are remarkably stable under dynamic cyclic loading, that is, there is no evident deterioration or change in material properties. Further, the dynamic hysteresis curves are found to be very close to similar curves obtained under static loading.

3.5 MEMBER STIFFNESS MATRICES

When the columns of a structure confined to a plane are considered to be axially deformable, there are four degrees of freedom associated with each girder and six degrees of freedom associated with each column. The convention used in numbering these degrees of freedom is shown in Figs. 3.2(a) and (b). The figures also show the positive directions of the degrees of freedom. When the columns are considered to be axially rigid, the degrees of freedom are respectively: two for a girder and four for a column, as shown in Figs. 3.3(a) and (b).

(a) BEAM

(b) COLUMN

FIG. 32 MEMBER DEGREES OF FREEDOM WITH AXIALLY DEFORMABLE COLUMNS

(a) BEAM

(b) COLUMN

FIG. 33 MEMBER DEGREES OF FREEDOM WITH AXIALLY RIGID COLUMNS

Once the degrees of freedom are defined and it is assumed that only flexural deformations are important and localized plastic hinges occur at member ends, the derivation of member stiffness matrices is quite straight-forward.

3.6 THE STRUCTURAL STIFFNESS MATRIX

Fig. 3.4 shows the convention used in numbering the structure coordinates when the columns are axially deformable; Fig. 3.5 shows the convention when columns are considered axially rigid. In either case, since the beams are considered axially rigid, there is only one sideways degree of freedom associated with each story. Figs. 3.4 and 3.5 also show the positive directions of the degrees of freedom. Forces and displacements acting along these directions are considered positive. The sequence adopted for numbering the structural coordinates gives the minimum bandwidth for the structural stiffness matrix. The individual member stiffness matrices are computed taking into account the location of plastic hinges if any. The structural stiffness matrix is then assembled using the correspondence of member degrees of freedom to structural coordinates.

3.7 P- Δ EFFECT

In the analysis of a frame for vertical and lateral loads, the non-linearities associated with (1) the deterioration of individual member stiffness due to axial load, and (2) the effect of gravity load acting through the sidesway of the frame, termed the P- Δ effect, can be

FIG 35 STRUCTURE COORDINATES WITH COLUMNS AXIALLY RIGID

taken into account by a suitable formulation of the column stiffness matrix. Corresponding to the degrees of freedom defined in Fig. 3.3(b), such a modified stiffness matrix for a column works out to [49]:

$$\frac{EI}{L} \begin{bmatrix} C & S & \frac{C+S}{L} & -\frac{C+S}{L} \\ S & C & \frac{C+S}{L} & -\frac{C+S}{L} \\ \frac{C+S}{L} & \frac{C+S}{L} & \frac{2(C+S)}{L^2} - \frac{P}{EI} & -\frac{2(C+S)}{L^2} + \frac{P}{EI} \\ -\frac{C+S}{L} & -\frac{C+S}{L} & -\frac{2(C+S)}{L^2} + \frac{P}{EI} & \frac{2(C+S)}{L^2} - \frac{P}{EI} \end{bmatrix} \quad (3.3)$$

where

$$C = \frac{c^2}{c^2 - s^2}$$

$$S = \frac{s^2}{c^2 - s^2}$$

and

$$c = \frac{1 - F_r L / \tan F_r L}{(F_r L)^2} \quad (3.4)$$

$$s = \frac{F_r L / \sin F_r L - 1}{(F_r L)^2}$$

$$F_r = \sqrt{\frac{P}{EI}}$$

P = axial load

L = length of the member

E = Young's Modulus of Elasticity

I = moment of inertia of the section

The stability functions C and S in matrix, Eq. 3.3 involve transcendental functions of P/L . It is difficult to handle the stiffness matrix in this form in the computation, because P , the individual column axial load, not being a constant, the column stiffness matrix has to be re-evaluated at each step of the numerical solution. By assuming that the deflected shape of the beam-column is given by a cubic polynomial satisfying the differential equation for bending deflection without the axial load, the effects of the axial load can be separately calculated to give the "stability coefficient matrix" to be added to the conventional bending stiffness matrix. The stability coefficient matrix so obtained is shown in Eq. 3.5; although approximate, it has been found to give excellent results in buckling problems [50].

$$-\frac{P}{L} \begin{bmatrix} \frac{2}{15} L^2 & -\frac{1}{30} L^2 & \frac{1}{10} L & -\frac{1}{10} L \\ \frac{1}{30} L^2 & \frac{2}{15} L^2 & -\frac{1}{10} L & \frac{1}{10} L \\ \frac{1}{10} L & -\frac{1}{10} L & \frac{6}{5} & -\frac{6}{5} \\ -\frac{1}{10} L & \frac{1}{10} L & -\frac{6}{5} & \frac{6}{5} \end{bmatrix} \quad (3.5)$$

Previous studies [51] have been shown that, for frames subject to large sidesway deflections, a major portion of the non-linearity is accounted for by the P- Δ effect; the deterioration of member stiffness

due to axial load has a small effect. If the effect of axial load on member stiffness is neglected, the P- Δ effect can be represented by a "geometric stiffness matrix", Eq. 3.6, to be added to the conventional bending stiffness matrix.

$$P \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad (3.6)$$

In the computer programs developed for this study, the analysis for gravity loads is performed neglecting the P- Δ effect. The values of column axial loads obtained from such an analysis are then used to modify the column stiffness matrices by adding the appropriate geometric stiffness matrix, Eq. 3.6; the modified matrices are used in the seismic analysis. During the deformation of structure due to earthquake excitation, the individual column loads do change, but the sum of the axial loads in all the columns of any one story remains constant. Thus, the contribution of the P- Δ effect to the structural stiffness matrix remains unaffected by seismic loading.

The modification of structural stiffness matrix for the P- Δ effect is carried out in a subroutine of the computer programs written for this study. Hence, if it is desired to use the stability coefficient matrix, Eq. 3.5 in place of the geometric stiffness matrix, Eq. 3.6, it is only necessary to replace the present subroutines by one that is appropriate.

3.8. CONDENSATION OF THE STRUCTURAL STIFFNESS MATRIX

Since the mass of a building frame is considered as lumped at floor levels, the effective dynamic degrees of freedom are the same as the sideways degrees of freedom, one associated with each story. Thus, for an elastic analysis, the structure stiffness matrix, which remains constant during the entire period of response, can be condensed so that the number of degrees of freedom is equal to the number of stories in the frame. The frame stiffness matrix can be written in the following form:

$$\begin{bmatrix}
 A_1 & B_1 & 0 & 0 & - & - & - & C_1 \\
 B_1^T & A_2 & B_2 & 0 & - & - & - & C_2 \\
 0 & B_2^T & A_3 & B_3 & - & - & - & C_3 \\
 - & - & - & - & - & - & - & - \\
 - & - & - & - & - & B_{n-1}^T & A_n & C_n \\
 \hline
 C_1^T & C_2^T & C_3^T & - & - & C_{n-1}^T & C_n^T & X
 \end{bmatrix} \quad (3.7)$$

where A_1, A_2 , etc. are submatrices relating a group of degrees of freedom all pertaining to one story except for one corresponding to sideways deflection. Thus, referring to Fig. 3.4, A_1 is an 8x8 matrix that relates to each other degrees of freedom 1 through 4 and 6 through 9. Submatrices B relate the degrees of freedom of one story to those of the next above. For the frame of Fig. 3.4, B_1 is an 8x8 matrix relating degrees of freedom 1 through 4 and 6 through 9, to degrees of freedom 10 through 13 and 15 through 18. Submatrices C relate degrees of freedom of one story to the sideways deflections of all stories. C_1 is thus an

8x4 matrix for the frame of Fig. 3.4. Lastly, submatrix X relates, to each other, the degrees of freedom corresponding to sideways. It is a 4x4 matrix for the frame of Fig. 3.4.

The frame stiffness matrix can be decomposed into the product of an upper triangular matrix and a lower triangular matrix as below:

$$\begin{bmatrix} A_1 & & & & & \\ B_1^T & A_2^* & & & & \\ 0 & B_2^T & A_3^* & & & \\ & & & \ddots & & \\ & & & & B_{n-1}^T & A_n^* \\ C_1^T & C_2^{*T} & C_3^{*T} & \dots & C_{n-1}^{*T} & C_n^{*T} | X^* \end{bmatrix} = \begin{bmatrix} I & A_1^{-1} B_1 & - & - & - & - | A_1^{-1} C_1 \\ & I & A_2^{*-1} B_2 & & & & | A_2^{*-1} C_2 \\ & & I & A_3^{*-1} B_3 & - & - & | A_3^{*-1} C_3 \\ & & & & \ddots & & | \\ & & & & & I & A_n^{*-1} C_n \\ & & & & & & I \end{bmatrix} \quad (3.8)$$

where

$$A_n^* = A_n - B_{n-1}^T [A_{n-1}^*]^{-1} B_{n-1} \quad (3.9)$$

$$C_n^* = C_n - B_{n-1}^T [A_{n-1}^*]^{-1} C_{n-1} \quad (3.10)$$

and

$$X^* = X - C_1^T A_1^{-1} C_1 - C_2^{*T} A_2^{*-1} C_2 - \dots - C_n^{*T} A_n^{*-1} C_n \quad (3.11)$$

I = Identity Matrix

Representing the lower triangular matrix by L and the upper triangular matrix by U, one gets:

$$[L] [U] \begin{Bmatrix} \delta \\ A \end{Bmatrix} = \begin{Bmatrix} 0 \\ P \end{Bmatrix} \quad (3.12)$$

in which δ are the displacements associated with all degrees of freedom other than sideways degrees of freedom, Δ are the sideways displacements and P are the inertial forces associated with sideways degrees of freedom other inertial forces being zero. It can then be shown that

$$\{\Delta\} = [X^*]^{-1} \{P\}$$

or

$$[X^*] \{\Delta\} = \{P\} \quad (3.13)$$

Thus, X^* is a condensed stiffness matrix that directly relates dynamic degrees of freedom to inertial forces. Recursion formulae 3.9 and 3.10 and Eq. 3.11 are used to obtain the matrix X^* .

3.9 PLASTIC HINGE ROTATIONS

The positive sense of the plastic hinge rotations has to be defined with some care. It is useful in the computation if the hinge rotations taking place in the positive sense of the plastic moments can be considered as positive. Since hinges are assumed to occur only at the ends of the members, and the positive sense of the moments at the ends is the same as the positive directions of the member degrees of freedom, the sign convention shown in Fig. 3.6 is used for plastic hinges. Thus, at End No. 1 of a beam, the hinge rotation that causes the included angle to be located above the beam centre line is positive, while at End No. 2 hinge rotation that causes the included angle to be located below the beam centreline is positive.

(a) PLASTIC HINGES IN THE GIRDERS

(b) PLASTIC HINGES IN THE COLUMNS

FIG 3.6 POSITIVE DIRECTIONS OF PLASTIC HINGE ROTATIONS

With these sign conventions, the incremental plastic rotations are given by the following expressions:

For Beams:

With hinge at End No. 1 alone,

$$\delta p_1 = \delta \theta_1 + \frac{1}{2} \delta \theta_2 - 1.5 \frac{\delta \Delta_3 - \delta \Delta_4}{L} \quad (3.14)$$

With hinge at End No. 2 alone,

$$\delta p_2 = \delta \theta_2 + \frac{1}{2} \delta \theta_1 - 1.5 \frac{\delta \Delta_3 - \delta \Delta_4}{L} \quad (3.15)$$

With hinges at both ends,

$$\delta p_1 = \delta \theta_1 + \frac{\delta \Delta_4 - \delta \Delta_3}{L} \quad (3.16)$$

$$\delta p_2 = \delta \theta_2 + \frac{\delta \Delta_4 - \delta \Delta_3}{L} \quad (3.17)$$

where

δp_i = incremental plastic rotation corresponding to degree of freedom i

$\delta \theta_i, \delta \Delta_i$ = incremental rotation and displacement, respectively, corresponding to degree of freedom i

If columns are considered axially rigid, $\delta \Delta_i = 0$ in Eqs. 3.14 through 3.17.

For columns:

With hinge at End No. 1 alone,

$$\delta p_1 = \delta \theta_1 + \frac{1}{2} \delta \theta_2 - 1.5 \frac{\delta \Delta_4 - \delta \Delta_3}{L} \quad (3.18)$$

With hinge at End, No. 2 alone,

$$\delta p_2 = \delta \theta_2 + \frac{1}{2} \delta \theta_1 - 1.5 \frac{\delta \Delta_4 - \delta \Delta_3}{L} \quad (3.19)$$

With hinges at both ends,

$$\delta p_1 = \delta \theta_1 + \frac{\delta \Delta_3 - \delta \Delta_4}{L} \quad (3.20)$$

$$\delta p_2 = \delta \theta_2 + \frac{\delta \Delta_3 - \delta \Delta_4}{L} \quad (3.21)$$

3.10 MAXIMUM HINGE ROTATIONS AND DUCTILITY

Fig. 3.7 shows the moment rotation relationship for the end of a typical member strained into yielding. The net residual plastic hinge rotation after the member has undergone the cycles of loading shown is given by:

$$p_{\text{net}} = \delta p_1 - \delta p_2 + \delta p_3 - \delta p_4$$

while the maximum hinge rotation is given by:

$$\delta p_{\text{max}} = \delta p_1 - \delta p_2 + \delta p_3$$

The maximum hinge rotations are numerically expressed in a dimensionless form designated as the "ductility ratio". One must distinguish here between utilized ductility ratio which is the measure of actual plastic hinge rotation in frame member under seismic loading and the ductility capacity ratio which is a measure of the maximum inelastic

FIG 37 MOMENT ROTATION RELATIONSHIP

deformation the member can undergo without distress. Ductility capacity ratios are normally determined from experiments on simply supported beams subjected to cyclic loading where the load is applied in such a way that the beam is subject to a moment gradient as shown in Fig. 3.8(a). The ductility capacity ratio is then given by:

$$\mu = \frac{\theta}{\theta_p} = 1 + \frac{\theta_H}{\theta_p} \quad (3.22)$$

where θ is the total end rotation up to distress, θ_p is the elastic component of end rotation assuming an idealized perfectly elastic-plastic moment rotation relationship, and θ_H is the plastic component of end rotation, so that

$$\theta_H = \theta - \theta_p$$

Since

$$\theta_p = \frac{M_p L}{3EI}$$

$$\mu = 1 + \frac{3EI \theta_H}{M_p L} \quad (3.23)$$

where

M_p = plastic moment of resistance of the section.

Obviously, one must ensure that the utilized ductility ratios computed from analysis do not exceed the comparable ductility capacity ratios. Keeping this in view, methods of computing utilized ductility ratios have been derived in this study and are described below. A more complete description will be found in Chapter 7.

(a) BEAM UNDER MOMENT GRADIENT

(b) BEAM MOMENTS WITH THE FRAME UNDER LATERAL LOADS

(c) BEAM MOMENTS WITH THE FRAME UNDER LATERAL AND GRAVITY LOADS

FIG. 3 8 DISTRIBUTION OF MOMENTS THROUGHOUT THE LENGTH OF A BEAM

If a member is free of gravity loads, or if such loads are comparatively small, the distribution of moment throughout the length of a beam is as shown in Fig. 3.8(b). During elastic and plastic rotation at the two end hinges, the point of inflexion remains stationary at or close to the centre of the member. The utilized ductility ratio is then given by:

$$\mu = 1 + \frac{3EI p_{\max}}{M_p \cdot \frac{L}{2}} \quad (3.24)$$

When gravity loads are important, the point of inflexion is not stationary. Also, the moment graph does not have a uniform gradient. In such cases, the ductility ratio is approximately determined by taking the effective length as equal to the minimum distance of the point of inflexion from the end, as shown in Fig. 3.8(c). The ductility ratio is then given by:

$$\mu = 1 + \frac{3EI p_{\max}}{M_p (kL)}$$

where kL is the distance of the point of inflexion from the end of the beam. The value of k can be determined if the distribution of gravity moment and the value of M_p is known.

When a member is not strained into the inelastic range, the term ductility ratio as defined above does not have any significance. However, to provide more information about the behaviour of the structure, the ductility ratio is, in such cases, defined as the ratio of maximum

end moment in the member reached during the entire history of the response to the plastic moment of resistance of the member cross-section. This definition is used because it is consistent with that used by several other authors [12, 28, 30].

3.11 SHEAR COEFFICIENTS

The lateral forces in a building structure subjected to seismic ground motion result from the inertia of the building mass. An inertial force is obviously equal to the product of the mass and its acceleration. Conceptually, it is therefore convenient to think of the maximum shear at any level of a building structure as being equal to the product of the total weight of the building above that level multiplied by a coefficient which is, in some way, related to the accelerations of the floor masses. The "shear coefficient" at any level of a building structure is thus defined as the maximum shear at that level divided by the total weight of the building above that level. In particular, the base shear coefficient, C_b , is equal to the maximum base shear divided by the total weight of the building.

3.12 MODAL SHEAR COEFFICIENTS

The equations of motion for an N degree-of-freedom system subject to a base acceleration, $\ddot{x}(t)$ are of the form:

$$[m]\{\ddot{z}\} + [c]\{\dot{z}\} + [k]\{z\} = -\ddot{x}(t)[m]\{1\} \quad (3.26)$$

where

$[m]$ = mass matrix

$[c]$ = viscous damping matrix

$[k]$ = stiffness matrix

$\{z\}$ = vector of displacement relative to the base along degrees of freedom of the system

If matrix c is of the type:

$$[c] = \alpha[m] + \beta[k] \quad (3.27)$$

α and β being constants, the transformation $\{z\} = [\phi]\{\eta\}$ where $[\phi]$ is a matrix formed from modal vectors reduces Eqs. 3.26 to N independent single degree of freedom equations. The r th equation is:

$$\ddot{\eta}_r + 2\zeta_r \omega_r \dot{\eta}_r + \omega_r^2 \eta_r = -\lambda_r \ddot{x}(t) \quad (3.28)$$

where

ω_r = undamped natural frequency in r th mode

ζ_r = fraction of critical damping in r th mode and λ_r is the modal participation factor for the r th mode given by

$$\lambda_r = \frac{[\phi^r]^T [m] \{1\}}{[\phi^r]^T [m] \{\phi^r\}} \quad (3.29)$$

If $\psi_r = -\lambda_r \ddot{\eta}_r$ in Eq. 3.28, $\psi_r(t) + \ddot{x}(t)$ gives the value of the absolute acceleration of a single-degree-of-freedom system with natural frequency ω_r and fraction of critical damping ζ_r , subject to a base acceleration $\ddot{x}(t)$. The base shear in an N story building frame due to its r th mode of vibration, V_b^r , is therefore given by:

$$V_b^r = \lambda_r \sum_{i=1}^N \phi_i^r m_i [\psi_r(t) + \ddot{x}(t)] \quad (3.30)$$

Following the definition given in Section 3.11, the base shear coefficient due to the r th mode, $C_b^r(t)$, is given by:

$$C_b^r(t) = \frac{\lambda_r \sum_{i=1}^N \phi_i^r m_i}{\sum_{i=1}^N m_i} [\ddot{\psi}_r(t) + \ddot{x}(t)]/g \quad (3.31)$$

The value of the function $[\ddot{\psi}_r(t) + \ddot{x}(t)]/g$ depends on ω_r , ζ_r and the nature of base excitation. The maximum value of this function for a particular base excitation is termed the spectral acceleration. The non-dimensional term,

$$C_b^r = \frac{\lambda_r \sum_{i=1}^N \phi_i^r m_i}{\sum_{i=1}^N m_i} \quad (3.32)$$

is, however, independent of the type of base excitation and, depends completely on the characteristics of the frame, that is, upon its mass and stiffness distribution. It represents the base shear coefficient in r th mode when the spectral acceleration value for that mode is equal to $1.0g$. Values of C_b for various modes thus provide a comparison of the contributions made by these modes to the total response when they are equally excited by an accelerogram.

3.13 TIME DEPENDANT RESPONSE ANALYSIS

Analysis of a multi-degree-of-freedom system by the superposition of modes is possible only when the system is elastic and the damping

2.4

OF/DE

matrix satisfies Eq. 3.27. On the other hand, analysis of response by a direct numerical integration of the equations of motion is a more general method applicable to both elastic and inelastic damped and undamped systems. The latter method has therefore been used throughout this study.

3.13.1 Elastic Analysis

For the elastic analysis of an N degree-of-freedom system subject to a base acceleration, $\ddot{x}(t)$, the set of differential equations, Eq. 3.26, must be solved. These equations can be solved by one of the appropriate methods of numerical analysis presented in Chapter 2. The choice of a particular method will depend upon the considerations cited in that chapter. If requirements of stability can be met, the multipoint formula given by Eqs. 2.19 and 2.20 will be the most suitable. These formulae are written below in their vector form:

$$\{\dot{z}_{n+1}\} = \{\dot{z}_n\} + \frac{h}{12} [-\{\ddot{z}_{n-1}\} + 8\{\ddot{z}_n\} + 5\{\ddot{z}_{n+1}\}] \quad (3.33)$$

$$\{z_{n+1}\} = -\{z_{n-1}\} + 2\{z_n\} + \frac{h^2}{12} [\{\ddot{z}_{n-1}\} + 10\{\ddot{z}_n\} + \{\ddot{z}_{n+1}\}] \quad (3.34)$$

Starting from known values of vectors $\{z_{n-1}\}$, $\{\dot{z}_{n-1}\}$, $\{z_n\}$, $\{\dot{z}_n\}$ and $\{\ddot{z}_n\}$, the three sets of equations 3.26, 3.33 and 3.34 can be solved simultaneously to obtain $\{z_{n+1}\}$, $\{\dot{z}_{n+1}\}$ and $\{\ddot{z}_{n+1}\}$. At the beginning of the solution, $\{z_1\}$, $\{\dot{z}_1\}$ and $\{\ddot{z}_1\}$ must be obtained by some other means. This, however, presents no special problems as explained in Section 2.6.

The general procedure for the solution of equations is independent of the choice of the corrector formulae for velocity and displacement. In every case, the problem ultimately reduces to the solution of the following matrix equation:

$$[k^*] \{z_{n+1}''\} = \{f_{n+1}^*\} \quad (3.35)$$

where the star has a significance explained directly below.

When Eqs. 3.33 and 3.34 are used in the solution,

$$[k^*] = [m] + \frac{5}{12} h[c] + \frac{1}{12} h^2[k] \quad (3.36)$$

$$\{f_{n+1}^*\} = \{f_{n+1}\} - [c]\{A\} - [k]\{B\}$$

where

$$\{A\} = \{z_n\} - \frac{1}{12} h\{z_{n-1}''\} + \frac{8}{12} h\{z_n''\}$$

$$\{B\} = -\{z_{n-1}\} + 2\{z_n\} + \frac{1}{12} h^2\{z_{n-1}''\} + \frac{10}{12} h^2\{z_n''\}$$

Matrix Eq. 3.35 is solved using Cholesky's method. Since $[k^*]$ is a symmetric, positive definite matrix, it can be decomposed into the product of a lower triangular matrix and its transpose, thus:

$$[L][L]^T \{z_{n+1}''\} = \{f_{n+1}^*\} \quad (3.37)$$

This equation is solved in two steps. First, the equation

$$[L]\{C\} = \{f_{n+1}^*\} \quad (3.38)$$

is solved for $\{C\}$ and then

$$[L]^T \{\ddot{z}_{n+1}\} = \{C\} \quad (3.39)$$

is solved for $\{\ddot{z}_{n+1}\}$.

It will be observed that matrix $[k^*]$ does not change with time, provided h , the time interval, is constant. The time consuming decomposition of matrix $[k^*]$ has, therefore, to be carried out only once. For each time interval, the vector $\{f^*\}$ is worked out and Eqs. 3.38, 3.39 are solved in a segment of the Cholesky subroutine to obtain $\{\ddot{z}\}$.

The advantage of a solution by elimination over one obtained by an iterative process is evident. The fact that decomposition of the $[k^*]$ matrix has to be carried out only once, makes the elimination process much less time consuming than the iteration scheme where iterations have to be repeated at each time step. Besides, the elimination process is certainly more accurate than the iterative scheme. Also, the problem of convergence of iteration has not to be considered while using the elimination scheme. The saving of computation time is, by far, the most important of the advantages mentioned above. This was very evident in comparative test runs carried out by the author using both methods of solution.

3.13.2 Inelastic Analysis

The solution of differential equations follows the same lines as in elastic analysis, except that only single step methods are used and all equations are used in incremental form. Also, since $[k]$ varies with time, there is no advantage in carrying out a matrix condensation. The mass, stiffness and damping matrices are, therefore, used in their full

form. It will be observed that matrix $[m]$ is a diagonal matrix and only those elements on the diagonal that correspond to the sideways degrees of freedom are non-zero.

The equations of motion in their incremental form are:

$$[m]\{\ddot{\Delta z}\} + [c]\{\dot{\Delta z}\} + [k]\{\Delta z\} = -\ddot{\Delta x}[m]\{1\} \quad (3.40)$$

where

$$\{z_{n+1}\} = \{z_n\} + \{\Delta z_n\}, \text{ and so on.}$$

Ultimately, the problem reduces to the solution of an equation of the form:

$$[k^*]\{\ddot{\Delta z}\} = \{\Delta f^*\} \quad (3.41)$$

The Cholesky method is again used to solve the above matrix equation. This time, however, the matrix $[k^*]$ is not constant but varies with the number of plastic hinges. Every time $[k^*]$ changes, the decomposition into two triangular matrices has to be carried out afresh. But since $[k^*]$ does not change at each step and, in fact, the number of changes during an entire response history may not be very large, there is still considerable economy in computation time if the elimination process is used instead of an iterative scheme.

3.14 COMPUTER PROGRAMS

Several computer programs have been developed to carry out the considerable volume of computation work required for this study. These programs use the numerical techniques and methods of analysis described in Chapter 2 and Section 3.13 of this chapter. Specifically, for the

elastic analysis of frames, the numerical method of integration used is either the Newmark's linear acceleration method or the set of multipoint formulae given by Eqs. 2.19 and 2.20. For inelastic analysis, the linear acceleration method is used throughout. In the development of these programs, it has been the intention to make them capable of being used by any interested user and, where possible, in a design office. To this end, self-contained program user manuals have been prepared. Program specifications, flow charts, program listings and list of variables appear in these manuals. A brief description of the programs follows.

3.14.1 Elastic Analysis Programs

Three different programs have been developed for the elastic analysis of multistory rigid jointed frames.

(a) Elastic Analysis Program No. 1

This program calculates the natural frequencies, mode shapes and time history of the elastic response of a multistory frame with rigid joints when subject to a specified base motion. The program can analyze rectangular frames (including set-back frames) with horizontal beams, vertical columns and fully rigid joints.

The columns are considered to be axially deformable; however, frames with axially rigid columns can be analyzed by specifying a large value for the cross-sectional area. The P- Δ effect can be taken into account if desired.

The program first develops the stiffness matrix of the frame and then carries out a condensation of this matrix to obtain an N by N

stiffness matrix where N is the number of stories in the frame. Having obtained the condensed stiffness matrix, the natural frequencies and mode shapes are calculated by a solution of the "eigenvalue problem". The program uses the matrix iterative scheme which has been described in several excellent texts [52, 53].

If desired, the response to a given base excitation, specified in the form of the acceleration of base at equally spaced discrete intervals of time, can be calculated.

(b) Elastic Analysis Program No. 2

This program has been developed to calculate the time history of the elastic response of a rectangular rigid jointed building frame subject to a specified base motion. The response values supplied by the program include story displacements, interstory displacements, floor accelerations and velocities, story forces and shears, girder end moments, column end moments and column axial loads. At the end of the response history, the program also outputs the maximum values of the above quantities.

The columns are considered axially deformable, and column-cross sectional areas are supplied in the input.

(c) Elastic Analysis Program No. 3

This program is exactly similar to Program No. 2 except that the columns are considered to be axially rigid. The degrees of freedom corresponding to axial column deformations are, therefore, omitted.

3.14.2 Inelastic Analysis Programs

Two programs have been developed for the inelastic analysis of multistory rigid jointed frames.

(a) Inelastic Analysis Program No. 1

The program has been developed to calculate the time history of the elasto-plastic response of a rectangular, rigid jointed building frame subject to a specified base motion. The columns are considered to be axially deformable. The response values supplied by the program include story displacements, interstory relative displacements, floor acceleration and velocities, story forces and shears, girder end moments, column end moments, column axial loads and the plastic hinge rotations. At the end of the response history, the program outputs the maximum values of the above quantities.

Test for plastic hinges

It is assumed that plastic hinges are located only at the ends of the members. The test for yielding at ends of members is carried out at the end of each interval of time. At every point where the moment equals or exceeds the plastic moment of resistance, a hinge is introduced. In the case of columns, the plastic moment of resistance is modified for the presence of axial load, using a simple approximate formula applicable to steel wide flange shapes [49].

$$M_{pc} = 1.18 M_p \left(1 - \frac{P}{A\sigma_y} \right) \leq M_p \quad (3.42)$$

where M_{pc} is the modified plastic moment of resistance of column,

M_p is the plastic moment of resistance of wide flange shape used,

P is the axial load, and

σ_y is the yield stress,

The program also provides an option whereby the above modification may be omitted. In that case, a suitable input value should be used for

M_p

At the location of existing plastic hinges, a check is made for the direction of rotation during the time interval under consideration. If this rotation is opposite to the sense of plastic moment, the hinge is removed. Thus, the possibility of hinge formation is examined at the end of each interval of time, and hinges are introduced or removed where necessary. Normally, the structural configuration will change during an interval of time and not at the end of it. The structural stiffness matrix will not, therefore, be constant during an interval at the end of which a change in hinge formation has been detected. This is illustrated in Fig. 3.9. The error introduced due to the above will not be important if the time interval used in the numerical integration is sufficiently small; on the other hand, in some cases, the accumulated error over the entire period of response history may become significant.

The program has, therefore, been so designed that whenever a change in hinge formation is detected at the end of an interval of time, the step by step integration reverts back to the position at the beginning of the interval. The integration over one normal period is then retraced.

FIG 3-9 VARIATION OF MEMBER END-MOMENT WITH TIME

in smaller sub-steps, the size of which is specified by the program user in his input data.

Where a hinge is introduced in a member at any location, the moment in that location remains constant until the rotation at the corresponding point hinge reverses its sense. On such reversal, the moment curvature relationship becomes an inclined line parallel to the elastic portion of the moment curvature curve. Due to the fact that the check for yield condition is made only at the ends of discrete intervals of time, the value of maximum moment will always be slightly greater than the plastic moment of resistance. This can be stated as:

$$|M| = M_p + \epsilon_1$$

where M is the member end moment at yield, M_p is the plastic moment of resistance of the member and ϵ_1 is a small error introduced because a step by step numerical integration process is being used.

The magnitude of ϵ_1 will depend upon the size of the time interval used in the integration; the smaller the time interval, the smaller is the magnitude of ϵ_1 .

As soon as the hinge at a particular location is removed in the manner just described, the moment there starts decreasing. At the end of the time interval immediately after the hinge has been removed, it will be reduced to:

$$M_p + \epsilon_1 - \epsilon_2$$

where ϵ_2 is the change in the moment taking place during the time interval immediately after the hinge has been removed. It is possible that in

certain situations ϵ_2 is smaller than ϵ_1 . If that is the case, then when a check is made for the yielding moment at the above location, the moment will be found larger than M_p , and it would appear that a hinge should be introduced. This is obviously incorrect and the error is purely a result of the numerical procedure used. To avoid this contingency, a hinge is introduced at any point only when the following two criteria are simultaneously satisfied:

- (i) the absolute value of moment is larger than M_p , the plastic moment of resistance
- (ii) the absolute value of the moment is also larger than its absolute value at the beginning of the interval of time under consideration.

(b) Inelastic Analysis Program No. 2

This is exactly similar to Inelastic Program No. 1 except that columns are considered axially rigid; the degrees of freedom corresponding to axial column deformations are therefore omitted.

CHAPTER 4

SELECTION OF BUILDING FRAME MODELS AND EARTHQUAKE GROUND MOTIONS FOR STUDY

4.1 INTRODUCTION

The specific objective of this study is to examine the behaviour of multistory steel unbraced frames subjected to earthquake induced ground motion with the help of response analyses for these frames subjected to one or more specified base motions. The problem of analysis is completely defined once the physical properties of the structural system, that is, its mass and stiffness distribution and the damping characteristics are known and the time-varying displacement at the base or foundation is specified.

This chapter describes the frame models and earthquake accelerograms selected for study. Two standard steel frames are used; one of these is a fifteen story single bay frame and the other a ten story single bay frame. The fifteen story frame is used for deriving the set-back frame models which are analyzed for earthquake response. The earthquake inputs used in this study are ground acceleration histories recorded during several earthquakes that occurred in California. The digitized values of the earthquake accelerations are suitably scaled before they are used in the analysis. The details of the accelerogram used and the scale factors adopted are described in this chapter.

4.2 STANDARD SINGLE BAY FRAMES

Two standard single bay frames, a ten story frame, and a fifteen story frame were designed in accordance with the National Building Code of Canada, 1970. To ensure that the results of the analytical study were not obscured by irrelevant details, certain simplifying assumptions were made in selecting the frame dimensions and the loading. These assumptions do not, however, alter the representative character of the frames. Thus, the story heights and the floor and roof dead loads and live loads were assumed to be equal for all the stories. The dead and live loads were taken as uniformly distributed over the floor girder lengths and were assumed to include an allowance for the weights of the beams, columns and partitions. It was further assumed that the wind loads were such that they did not govern the design. The frame stiffness and member stiffnesses and strengths were thus completely dependant on the dead, live and earthquake loads. The following standard data was used in the design:

Column spacing, i.e. the width of frame	= 24 ft.
Bay spacing	= 24 ft.
Live load	= 100 p.s.f.
Dead load	= 120 p.s.f.
Seismic zone	= zone 3 of the N.B.C.
Importance factor I	= 1.0
Foundation factor F	= 1.0
Steel type	= ASTM A36

For the type of frames selected, it is the common practice to use rolled steel wide flange shapes for the beams and columns. A study of the properties of wide flange shapes showed that for these shapes the following approximate empirical relationships could be used to relate the section modulus and the moment of inertia about the major axis,

$$\begin{aligned} Z &= \frac{I}{0.45 D}, \text{ which leads to} \\ S &= \frac{Z}{1.11} \end{aligned} \quad (4.1)$$

where Z = the plastic section modulus in inch³

S = the elastic section modulus in inch³

I = the moment of inertia about the major axis in inch⁴

D = the depth of the section in inches.

Design calculations indicated that 21 inch wide flange shapes would be suitable for girders. The following relations, obtained by substituting 21" in Eqs. 4.1, were therefore used in deriving the properties of the girders:

$$\begin{aligned} I &= 0.452 M \\ M_p &= 3.81 I \end{aligned} \quad (4.2)$$

where M = design moment in kip-in

M_p = plastic moment of resistance in kip-inches.

The girder cross-sectional area appropriate to the selected value of I was obtained from the tables of the properties of wide flange sections by interpolation.

Figures against
members show
I in inch⁴

FIG 41 FIFTEEN STORY FRAME

The design of the columns was carried out in accordance with the provisions of the National Building Code of Canada, using the wide flange shapes as trial sections.

For the ten story and the fifteen story standard frames, it was observed that the girder and column stiffnesses could be assumed to decrease linearly with height from the base. The column and girder properties were slightly adjusted to conform to a linear variation. Adjustments in girder properties were carried out in accordance with Eq. 4.2. For columns, the design indicated that 14 WF shapes would be suitable. Column strengths were, therefore, derived from the moments of inertia using the following relationships:

$$Z = \frac{I}{0.45D} = \frac{I}{0.45 \times 14.0} = \frac{I}{6.3} \quad (4.3)$$

$$M_p = 36.0 Z = 5.7 I$$

As for the girders, the column cross-sectional areas were obtained from the tables of the properties of wide flange shapes by interpolation.

The fifteen story standard frame is illustrated in Fig. 4.1. The upper ten stories of this frame represent the ten story frame.

4.3 SET-BACK FRAME MODELS

The objectives in the selection of set-back frame models were the same as those adopted in the selection of standard single bay frames; the frames should have properties similar to practical multi-story

building frames and the number of variables should be kept to a minimum so that important parameters can be isolated and their effects on the dynamic response of the structure can be studied. With this in view, all the frame models were derived from the fifteen story single bay frame of Fig. 4.1 termed the "Basic Frame". A frame termed the "Related Uniform Frame" is a three bay fifteen story frame with the member properties the same as for the basic frame, except that the internal columns are twice as strong as the columns of the basic frame.

The set-back frame consists of a base portion, three bays wide, with the same member properties as the related uniform frame, and a tower portion comprised of a single bay frame continuous with the central bay of the base portion. It is derived from the related uniform frame by varying two parameters, termed the level of set-back ratio, L_s , and the tower base plan area ratio, C_s . These parameters have also been used in earlier studies [30, 37] and are given by the following expressions:

$$C_s = \frac{A_t}{A_b} \quad (4.4)$$

$$L_s = \frac{h}{H}$$

where A_t is the plan area of the tower, A_b is the plan area of the base, h is the height of the base portion and H is the total height of the building.

It is assumed that the mass of each floor of the related uniform frame is similar. In the set-back frames, the mass of each floor in the base portion, designated as m_b , is taken as equal to the mass of a typical floor of the Related Uniform Frame. The floor mass in the tower portion of the set-back frame, designated as m_t , is derived on the assumption that

the mass is proportional to the floor plan area, giving:

$$m_t = C_s \times m_b \quad (4.5)$$

In a similar manner, the building stiffness is assumed to be proportional to the plan area, so that the combined column or beam stiffness properties at any level of the tower frame are equal respectively to the same combined properties at the corresponding level of the Related Uniform Frame multiplied by the tower-base-plan-area ratio, C_s . The Basic Frame, the Related Uniform Frame and a set-back frame are shown in (a), (b) and (c) of Fig. 4.2.

In the research of this thesis, four different values are signed to the parameters C_s . For $C_s = 1$, the set-back building is identical to the related uniform building. The three other values assigned to C_s are 0.111, 0.333 and 0.667. These values are applicable to the building plans shown, respectively, in Figs. 4.3 (a), (b) and (c). The three bay frame of Fig. 4.2(c) can directly represent a possible structural framing for set-back buildings with plan views shown in Figs. 4.3 (a) and (b); this, however, is not true of the building shown in Fig. 4.3 (c). One possible structural framing for this latter building will, for example, have six equal bays in the base portion and four bays of the same size in the tower portion. Since the bay widths are equal and the internal columns are well considered to be of the same size and twice as strong as the external columns, the three bay frame of Fig. 4.2(c) will have exactly the same seismic response as the set-back building represented by Fig. 4.3(c), provided the columns can be considered axially rigid. For this

(a) BASIC FRAME

(b) RELATED UNIFORM FRAME

(c) SET-BACK FRAME

FIG. 4.2 UNIFORM AND SET-BACK FRAMES

(a) $C_s = 0.111$

(b) $C_s = 0.333$

(c) $C_s = 0.667$

FIG. 4.3 FLOOR PLANS OF SET-BACK BUILDINGS

reason, the three bay frame is used throughout for the study of the response of set-back buildings. The reduction of a building to a single equivalent frame is discussed in greater detail in Section 8.8.

The values assigned to L_s are 1.0, 0.8, 0.6, 0.4 and 0.2 corresponding respectively to a uniform building and a building with three, six, nine, and twelve story towers.

4.4 EARTHQUAKE GROUND MOTION INPUT

Based on the type of base displacement history used, the seismic response analysis of structures is divided in the following two categories:

1. deterministic dynamic analysis, using measured ground acceleration histories
2. non-deterministic or probabilistic analysis, using ground motion inputs that are described in a statistical sense only.

The present study is based on deterministic dynamic analysis, using real earthquake accelerograms in the form of digitized values of ground acceleration at appropriate intervals of time.

Reference [54] gives the digitized values of ground accelerations obtained from several of the original accelerogram records of U.S. earthquakes. The digitized values have been corrected for the errors introduced in the process of recording and digitization, and are believed to be the most accurate form of input data for structural response calculations that it is feasible to obtain from the original seismic records. Of the accelerograms presented in reference [54], the following have been selected for use in the present study:

1. El-Centro, California, May 18, 1940, N.S. Component
2. Taft, California, July 21, 1952, N21E Component
3. Taft, California, July 21, 1952, S69E Component
4. San Francisco Southern Pacific Building, Basement, March 22, 1957, N45E Component.

The acceleration time records for the motion are shown in Fig.

4.4. The undamped velocity response spectra for the first 20 seconds of each of these ground motions were obtained by the author by a process of step by step integration of the differential equation of motion of a single-degree-of-freedom oscillator subject to a base motion, using the computer programs developed in this study for the elastic analysis of frames. The resulting velocity spectra are shown in Fig. 4.5.

4.5 DURATION AND INTENSITY OF EARTHQUAKE MOTION

Strong ground motion accelerograms recorded on firm ground at close epicentral distances are known to have certain well defined properties. Such accelerograms typically have a primary phase of strong motion lasting from 12 to 25 seconds followed by a secondary phase of less intense motion that gradually dies away. The maximum excitation of a structure with damping is, in general, determined by the strong phase of the ground motion; hence, it is customary to use only the strong phase for calculating the response of the building frames. In the present study, all response calculations have been carried out for the first 20 seconds of the selected ground motion.

FIG. 4-4(o) EARTHQUAKE ACCELEROGRAMS

SAN FRANCISCO S.P. BUILDING MARCH 22, 1957 N45E

FIG. 4.4(b) EARTHQUAKE ACCELEROGRAMS

FIG 4-5 UNDAMPED VELOCITY SPECTRA

FIG 4.5 UNDAMPED VELOCITY SPECTRA

The damage potential of an earthquake is measured by some form of intensity scale. The one used in this study is the Spectrum Intensity, defined as the area under the velocity spectrum curve over a range of periods from 0.1 seconds to 2.5 seconds [55], and given by:

$$S_1 = \int_{0.1}^{2.5} S_v(\zeta, \frac{2\pi}{T}, T_p) dt$$

where

ζ = viscous damping ratio

T_p = duration of ground motion

T = undamped natural period

$S_v(\zeta, \frac{2\pi}{T}, T_p)$ = pseudo velocity response spectrum

Housner [55] has given the values of spectrum intensities for the four ground motions selected in this study. These values are shown in Table 4.1. The El-Centro, 1940 N-S accelerogram has been used as the standard and appropriate scale factors have been applied to the other accelerograms to bring their intensities up to the level of El-Centro. These scale factors are also shown in Table 4.1.

TABLE 4.1
SPECTRUM INTENSITIES AND SCALE FACTORS FOR SELECTED GROUND MOTIONS

Description of Ground Motion	Spectrum Intensity with Zero Damping	Scale Factor
El-Centro, California May 18, 1940 N-S component	8.94	1.00
Taft, California July 21, 1952 N21E component	4.53	1.975
Taft, California July 21, 1952 S69E component.	4.84	1.85
San Francisco, Southern Pacific Building Basement, March 22, 1957 N45E component.	1.32	6.77

CHAPTER 5

THE EFFECT OF CERTAIN INPUT PARAMETERS ON THE SEISMIC RESPONSE OF FRAMES

5.1 INTRODUCTION

The effects of the following input or design parameters on the seismic response of multistory rigid jointed steel framed structures are examined in this chapter.

1. Axial column deformations
2. Intensity of excitation
3. Period of vibration
4. Contribution of live load to the effective mass

A number of similar studies have been carried out in the past [12, 22, 28, 29, 30]. Parameters such as the axial column deformations and the contribution of live load have, however, been given only a limited amount of attention. These are dealt with in some detail in the present study; others are included to provide corroboration of the reported results of the past studies.

The approach used in the response study is to carry out a deterministic analysis of typical multistory frames for certain recorded ground motions. All the frames analyzed are derived from the standard ten story single bay frame described in Section 4.2. The earthquake accelerograms used in the study are those described in Sections 4.4 and 4.5. To study the effect of a certain input parameter on the response, that parameter is varied while all other significant input parameters are

kept constant. The maximum response envelopes obtained for different values of the parameter being studied are plotted and examined for definite patterns of influence on the response.

The values adopted for the relevant control data are summarized in Table 5.1. It will be observed that the response has been calculated for zero value of damping. This is not to signify that, in practice, building frames without damping can exist. However, recent measurements on multi-story steel structures have shown that the values of damping for such structures are quite low [56]. For a nine story steel building with concrete encased columns and welded trussed floor girders, designed to carry the lateral loads, the damping in the first mode measured by ambient tests varied between 0.4 and 0.6% [57]. Even during a real earthquake, the effective damping in the first mode of the building was estimated to be about 3 to 4% [58]. This figure, in fact, included the effect of energy absorption due to yielding in the structure, implying that the structural or viscous damping was even less. For such low values of damping, the shape of the response spectrum does not change significantly, though the absolute value of the response decreases.

For the method of analysis used in this study, the damping in individual modes can be defined explicitly only if the damping matrix is of the form given by Eq. 3.27. Since there are two free parameters α and β in this equation, specified values of damping can be assigned for only two of the modes. Once such values are assigned, the damping in other modes becomes dependant on the selected values of α and β . This may, at

TABLE 5.1
CONTROL DATA FOR THE ANALYSIS OF TEN STORY FRAME

Description of Control Block	Type of Parametric Response Analysis			Effect of the Effective Mass
	Effect of Column Deformations	Effect of the Intensity of Excitation	Effect of the Period of Vibration	
Damping	nil	nil	nil	nil
P-Δ Effect	nil	nil	nil	nil
Gravity Load	dead plus live load	dead load	dead load	dead load
Floor Mass	dead load	dead load	dead load	varying
Earthquake Motion	E1-Centro, Taft S69E	E1-Centro varying intensity	E1-Centro, Taft S69E	E1-Centro, Taft S69E, Taft N21E

times, result in an artificial suppression or accentuation of the contributions of the remaining modes, caused by the nature of the method of analysis used, and not because such an effect was consciously desired.

For the reasons stated above, response calculations have been carried out for a zero value of damping. Since the purpose of the study presented in this Chapter is to examine the effect of the variation of a certain parameter on response, and not to determine the absolute value of response, the selection of zero damping does not affect the conclusions.

5.1.1 Response Parameters

The results of analyses made by the author are presented in the form of envelopes of the absolute maximum values of response parameters described below. As detailed in Chapter 3, the computer programs used in the analytical study were designed to output the absolute maximum values that these parameters attained during the entire history of the response. The conventions used for presenting the response parameters are as follows:

1. Maximum Story Displacements

The absolute maximum values of the story displacements are plotted directly in inches. A displacement value shown against story i refers to the maximum displacement of the i th floor, where the base of the frame is designated as floor no. Zero.

2. Maximum Interstory Drifts

The absolute maximum values of the relative story displacements are also plotted directly in inches. A relative displacement value shown against story i refers to the maximum displacement of the i th floor rela-

tive to floor no. $i-1$.

3. Maximum Story Shears

The absolute maximum values of the story shears are plotted in kips. The shear shown against story i refers to the maximum shear in the story bounded by floors i and $i+1$.

4. Maximum Column Axial Loads

The absolute maximum column loads due to earthquake forces and the gravity loads present on the frame are plotted directly in kips. The axial load shown against story i refers to the maximum axial load in the columns running from floor i to $i+1$.

5. Maximum Utilized Girder Ductility Ratios

The girder ductility ratios shown (defined in Section 3.10) are the absolute maximum rotation ductility ratios utilized during the entire response history. The ductility ratio shown against story level i is the maximum ductility ratio attained in any of the girders at floor i .

6. Maximum Utilized Column Ductility Ratios

The column ductility ratios are also the rotation ductility ratios utilized during the response history. It is assumed that there are no horizontal loads present along the length of the columns. The ductility ratio shown against story i refers to the maximum value of the ductility ratio attained in any of the columns running from floor $i-1$ to i .

5.2 THE EFFECT OF AXIAL COLUMN DEFORMATIONS ON RESPONSE

The effect of axial column deformations on the seismic response of frames has not been studied extensively. Rubinstein [59] and Blume [60] have investigated the effect of the axial deformations of columns on the periods of multistory frames. Goel [29] has presented a few examples of the effect of axial column deformations on the seismic response of multistory frames.

If the axial column deformations are taken into account, the number of degrees of freedom used in the analysis increases considerably. For example, a ten story two bay frame has 40 degrees of freedom if axial column and beam deformations are neglected; if axial column deformations are taken into account, the number of degrees of freedom increases to 70. This may be expected to add considerably to the computer time involved in the analysis. Since the frame masses are assumed to be lumped at the floor levels, the number of effective dynamic degrees of freedom of a multistory frame is equal to the number of stories. Therefore, in an elastic analysis, once the frame stiffness matrix has been condensed as described in Section 3.8, one has to deal with a stiffness matrix whose size is equal only to the number of stories; the extra degrees of freedom for the axial column deformations should not add much to the computation time. However, when a complete building structure having a number of different types of frames is being analyzed, it is desirable to reduce the building structure to a single equivalent frame to economize on the computer time required for the elastic dynamic analysis (see Chapter 8).

This reduction to a single equivalent frame is justified only when it is possible to make certain simplifying assumptions, one of which is that the effect of column deformations on the seismic response is negligible. Thus, even for an elastic analysis, it is important to know whether it is reasonable to neglect the effect of axial column deformations on the seismic response of a frame.

The importance of the above simplifying assumption is even greater in the case of an inelastic seismic analysis of a frame. Unlike in an elastic structure, the stiffness matrix in an inelastic frame is not constant but depends upon the displacements. Every time the stiffness matrix changes, matrix condensation has to be repeated. Alternatively, the complete stiffness matrix has to be used in the analysis and has to be decomposed into a lower and an upper triangular matrix every time there is a change in its elements. The size of the complete stiffness matrix, therefore, determines, to a large extent, the total time involved in the analysis.

Since the height to width ratio of a frame can be expected to have an important bearing on the effect of axial column deformations on response, a ten story two-bay frame and a ten story three bay frame were developed for study, in addition to the standard ten story single bay frame described in Chapter 4. The two and three bay frames are similar in all respects to the standard ten story single bay frame except that their internal columns are twice as strong as the columns in the corresponding stories of the single bay frame. The three frames were

analyzed for the El-Centro and the S69E component of the Taft accelerogram. The effective floor mass was assumed as equal to the floor dead load. It was further assumed that full gravity loads, dead and live, were present so that the column axial loads were maximum. For both the elastic and the inelastic cases, two sets of analyses were carried out, one taking the column deformations into account and the other with column deformations neglected, providing in effect two values of this particular parameter.

The effect of axial column deformations on the period of the frames is seen from the results presented in Table 5.2. It is observed that with decreasing ratio of the height to the width of the frame, the effect of axial column deformations on the periods and the seismic response of the frame decreases. The fundamental period of the single bay frame, with a height to width ratio of 5, increases by 4.78% due to the axial deformation of columns. The corresponding increase in the period of the three bay frame, with a height to width ratio of 1.67, is only 1.43%.

The results of the response calculations for the El-Centro accelerogram are shown in Figs. 5.1 through 5.3. For the single bay frame, the errors introduced in response values of the maximum story displacements, the maximum interstory drifts and the maximum story shears by neglecting the axial deformations of columns are on an average of the order of 10%. The errors in axial column loads are a little less. For the three bay frame, the magnitude of the errors reduces to about 5%.

TABLE 5.2
THE EFFECT OF COLUMN DEFORMATIONS ON THE PERIODS OF THE FRAME

Description	Period with Columns Axially Rigid sec.	Period with Columns Axially Deformable sec.	Percentage Increase in the Period %
Ten story single bay frame $H/B = 5$			
Fundamental period	2.090	2.19	4.78
Second mode period	0.712	0.734	3.09
Third mode period	0.410	0.413	0.73
Ten story two bay frame $H/B = 2.5$			
Fundamental period	2.090	2.14	2.39
Second mode period	0.712	0.724	1.68
Third mode period	0.410	0.412	0.49
Ten story three bay frame $H/B = 1.67$			
Fundamental period	2.090	2.12	1.43
Second mode period	0.712	0.719	0.98
Third mode period	0.410	0.411	0.24

--- COLUMNS AXIALLY RIGID
 - - - COLUMNS AXIALLY DEFORMABLE

FIG 51 SINGLE BAY FRAME RESPONSE TO EL CENTRO

— COLUMNS AXIALLY RIGID
 --- COLUMNS AXIALLY DEFORMABLE

FIG 52 TWO BAY FRAME RESPONSE TO EL-CENTRO

— COLUMNS AXIALLY RIGID
 --- COLUMNS AXIALLY DEFORMABLE

FIG. 53 THREE BAY FRAME RESPONSE TO EL CENTRO

The results of an analysis of the single bay frame for the scaled Taft S69E accelerogram are shown in Fig. 5.4. The errors in the elastic values of interstory drifts and shears are slightly larger than in the case of El-Centro earthquake. For both the El-Centro and the Taft accelerograms, the effect of the axial column deformations seem to be slightly less pronounced on the inelastic response.

5.2.1 Conclusions on the Effect of Neglecting Column Axial Deformations

From the results presented here, it appears that the effect of axial deformations on the dynamic response can be neglected in analysis and design except for very slender buildings. Even with a height to width ratio of 5, the errors introduced by neglecting the axial deformations of the columns are of the order of 10%. For smaller height to width ratios, the error decreases. The results of elastic and inelastic analyses presented in the remaining part of this study are, therefore, based on the assumption that columns are axially rigid.

5.3 THE EFFECT OF THE INTENSITY OF EXCITATION ON RESPONSE

To examine the effect of the intensity of excitation on the response of the frame, the standard ten story frame, taken as an elasto-plastic structure, was subjected to four accelerograms of different intensities. All the accelerograms were derived from the El-Centro ground motion by applying different scale factors to the acceleration amplitudes. The scale factors used were 25, 50, 100 and 150%. An earlier analysis had shown that the first girder yielding commenced at

— COLUMNS AXIALLY RIGID
 --- COLUMNS AXIALLY DEFORMABLE

FIG 54 SINGLE BAY FRAME RESPONSE TO TAFT S69E

an intensity slightly higher than 25% of the El-Centro accelerometer.

As the intensity increases beyond this level, the girders are subjected to progressive yielding.

The results of the analysis are presented in Fig. 5.5. It is seen that the maximum story displacement and the maximum interstory displacements do increase with increase in the intensity of excitation, but at a lesser rate than in proportion. The shape of the envelope of interstory drifts also changes with the intensity. At an intensity of 50%, the interstory drifts and the girder ductility requirements are comparatively higher in the upper stories. With a further increase in the intensity, the maximum yielding shifts towards the lower stories as evident from the distribution of both the interstory drifts and the girder ductility ratios. At an intensity of 100%, the girder ductility requirements are largest at the mid-height of the building. At an intensity of 150%, the ductility requirements are largest in the lower stories. Most of the columns remain elastic even up to an intensity of 100%. With further increase in intensity, the uppermost and the lowest columns start yielding.

The story shears also increase continuously with increase in the intensity of excitation. However, the relative increase in the shears with increased seismic intensity is less than that in the lateral displacements and interstory drifts. The increase in shears between the intensities of 50 and 100% is the same as the increase between 25 and 50%. The increase between 100 and 150% is only about half the increase

FIG. 5-5 INTENSITY OF EXCITATION AND INELASTIC RESPONSE

between 50 and 100%. This is to be expected, since as girders progressively yield the stiffness of the frame and hence the rate of increase in shears reduces. A similar trend is seen in the column axial loads. In fact, there is hardly any increase in the column axial loads while the intensity of excitation increases from 100 to 150%. The axial loads in columns are dependant upon the girder end shears. Since the girder end shears due to earthquake motion are proportional to the sum of the end moments in the girders, the limiting values of girder end shears and hence of the column axial loads are reached when almost all girders start developing plastic hinges at their ends at about the same time.

A response parameter which increased rapidly with the seismic intensity was the utilized ductility ratio, especially for the girders. This shows that the safety of the building when it is subject to a severe earthquake depends upon the capacity of its members to undergo inelastic rotation without distress.

5.4 THE EFFECT OF THE PERIOD OF VIBRATION ON RESPONSE

To study the effect of the fundamental period of the frame upon its seismic response, the stiffnesses of the members of the standard ten story frames were scaled proportionately to give three different fundamental periods: 1.0 seconds, 1.5 seconds and 2.5 seconds. All other properties of the frame, including the strength of the members, were kept unchanged.

Fig. 5.6 shows the results of an elastic analysis of the three frames for the El-Centro accelerogram. The results of a dynamic analysis of the standard ten story frame having a fundamental period of 2.1 seconds are also shown. The effect of the period on the response is strongly influenced by the characteristics of the ground motion used in the analysis. From the undamped elastic response spectrum of the El-Centro accelerogram, Fig. 4.5, it is observed that the spectral velocities for the four values of periods, 1.0 sec., 1.5 sec., 2.1 sec., and 2.5 sec. are, respectively: 46.0 in/sec, 24.0 in/sec., 32 in/sec, and 39.0 in/sec. The relative values of spectral displacements obtained by multiplying the spectral velocities by the corresponding periods are, respectively: 46.0, 36.0, 67.2 and 97.5. From Fig. 5.6, it is seen that the maximum story displacements follow a similar relationship - the displacements for a fundamental period of 1.5 seconds are indeed the smallest - showing the predominant influence of the fundamental mode on the displacements.

On the other hand, the maximum interstory drifts are affected by the higher modes also. In cases where the higher modes make a significant contribution, the response in the upper stories of the frame should be expected to show a relative increase. Such an increase, often termed the whiplash effect, is least evident in the interstory drifts of the frames with fundamental periods of 1.0 sec and 1.5 sec. For frames with longer periods, the second and third mode periods fall under the peaks of the response spectrum; the contribution of the higher modes is therefore more pronounced, resulting in interstory drifts in the upper

FIG 5-6 PERIOD OF VIBRATION AND ELASTIC RESPONSE

FIG. 5.7. PERIOD OF VIBRATION AND INELASTIC RESPONSE

stories of the same order or somewhat higher than in the lower stories.

Since the effective floor masses are the same in all the four frames, the maximum story shears and the maximum axial loads due to the earthquake forces should be proportional to the maximum floor accelerations. The relative values of spectral accelerations obtained by dividing the spectral velocities by the corresponding periods are, respectively: 46, 16, 15.2 and 15.6. The maximum story shears are seen to bear a similar relationship. They are of the same order for frames with periods of 1.5 sec, 2.1 sec and 2.5 sec and are considerably higher for the frame with a period of 1.0 sec. As would be expected, the column axial loads follow a similar pattern.

The results of an elasto-plastic analysis of the four frames for the El-Centro earthquake are shown in Fig. 5.7. The maximum floor displacements and interstory drifts follow patterns similar to those in the elastic frames. However, slight changes in the behaviour occur due to the elongation of period caused by inelasticity. The interstory drifts and the girder ductility ratios are influenced by the higher modes also. For the frame with a fundamental period of 1.0 sec, the girder ductility ratios and the interstory drifts are the highest in the lower stories, indicating that the fundamental mode dominates the response of the frame. For the other frames, both the maximum girder ductility ratios and the maximum interstory drifts are almost of the same order in the upper and the lower stories, indicating that the contribution of the higher modes to the response is significant. In the case of the

frame with a fundamental period of 1.5 sec, the whiplash effect seen in the inelastic response did not appear in the elastic response. The difference in the response is presumably a result of the elongation of the period due to inelasticity.

The major difference between the elastic and the inelastic response is in the relative values of the maximum story shears and the maximum axial loads. For the elasto-plastic frames, there is only a small change in these response parameters with a change in the period. This is so because once the girder starts yielding there can be very little further increase in the story shears and column axial loads. The maximum values of story shears and column axial loads are thus primarily dependant upon the girder strengths which are the same in all the four frames.

It should be expected that where there is a large difference between the maximum elastic and elasto-plastic shears, the girders in the real frame should be called upon to undergo large inelastic deformations when the frame is subjected to the seismic motion. That this is so will be evident from the large girder ductility ratios utilized for the frame with a fundamental period of 1.0 sec.

Fig. 5.8 shows the results of the elastic response of the four frames to the Taft S69E accelerogram. As in the case of El-Centro accelerogram, the response is strongly influenced by the characteristics of the ground motion. The spectral velocities for the four values of fundamental periods: 1.0 sec, 1.5 sec, 2.1 sec and 2.5 sec, as obtained from the undamped response spectrum of the Taft accelerogram, Fig. 4.5

FIG 5.8 PERIOD OF VIBRATION AND ELASTIC RESPONSE

are, respectively: 42.0 in/sec, 38.0 in/sec, 24.0 in/sec and 20 in/sec. The relative spectral displacements work out respectively to: 42.0, 37.0, 50.4 and 50.0. The maximum story displacements follow a similar pattern. The pronounced whiplash effect, seen in the interstory drifts for the frame with a period of 2.5 sec, is caused because the response spectrum has a peak at 0.85 sec which is the second mode period of the frame.

The relative spectral accelerations for the four periods of 1.0 sec, 1.5 sec, 2.1 sec and 2.5 sec work out respectively to 42.0, 25.3, 11.4 and 8.0. The maximum story shears and maximum column axial loads follow a similar pattern except that for the frame with a period of 2.5 sec they are somewhat higher than expected, presumably because of the large contribution from the second mode.

Fig. 5.9 shows the results of an elasto-plastic analysis of the four frames for the Taft accelerogram. The maximum displacements and interstory drifts follow patterns that are similar to those for the elastic response. The whiplash effect is more pronounced for the fundamental period of 2.5 sec. Again, the shears and axial loads do not change much with change in the period.

5.5 THE EFFECT OF THE CONTRIBUTION OF LIVE LOAD TO THE EFFECTIVE MASS ON RESPONSE

Before a building frame can be analyzed to obtain the seismic design forces, the building mass must be determined. The building mass is formed of the dead load and the live load present at the time of the earthquake. The dead loads are known relatively accurately but the

FIG. 5-9 PERIOD OF VIBRATION AND INELASTIC RESPONSE

incidence of live load is indeterminate. Besides, only a portion of the live load present contributes to the effective mass, unless the live load is rigidly mounted on structure, which is seldom the case. Lastly, the larger mass does not necessarily result in increased design forces. The contribution of live load to the seismic response thus depends on the following three factors:

1. the incidence of live load
2. contribution of the live load to the effective mass
3. the effect of mass on the seismic response of the structure.

In a majority of cases, increased floor mass does result in increased design forces. In such cases, therefore, a conservative approach will be to consider the mass as equivalent to factored dead and live loads. Since the probability of occurrence of the most severe earthquake at the same time as the maximum dead and live loads is rather remote, such an approach is not economically justified. In the absence of statistical data, a decision on the incidence of live load has to be a subjective engineering decision.

The contribution of the live load present to the effective mass of the structure can be approximately determined by rational analysis. A limited amount of work has been done in the past on this aspect [61]. It is briefly discussed in Subsection 5.5.1. The effect of the building mass on its seismic response depends upon the characteristics of the ground motion and can be studied by carrying out elastic and elasto-plastic analyses of the building. Subsection 5.5.2 is devoted mainly to

the presentation and discussion of the results of such analyses.

5.5.1 The Contribution of Live Load to the Effective Mass of the Structure

The contribution of the live load to the effective mass depends upon the manner in which the live load is mounted on the structure. The mounting is rarely rigid; it can, in fact, be represented by a spring and a dash-pot where the latter may contribute a viscous type damping or a Coulomb friction. In reference [61], the effect of the type of mounting on response has been studied by idealizing the primary system and the live-load system as single-degree-of-freedom systems shown in Fig. 5.10.

The maximum lateral shear, F in a single-degree-of-freedom system subjected to an earthquake ground motion, is given by:

$$F = S_A \cdot m \quad (5.1)$$

where S_A is the spectral acceleration and m is the mass of the system.

The spectral acceleration for a particular ground motion is a function of the period T . Since T is given by:

$$T = 2\pi \sqrt{\frac{m}{k}} \quad (5.2)$$

where k is the spring-stiffness, S_A becomes a function of m provided k is a constant. When the live load is rigidly mounted, the maximum lateral shear is given by

$$F_1 = S_A(m_t) \cdot m_t \quad (5.3)$$

$$m_t = m_d + m_l \quad (5.4)$$

FIG 510 THE EFFECT OF LIVE LOAD ON SEISMIC RESPONSE

where m_t is the total mass, m_d is the dead load mass, m_l is the live load mass and the spectral acceleration S_A is a function of m_t . When the live load is not rigidly mounted, the maximum shear is given by:

$$F_2 = \epsilon \cdot S_A(m_t) \cdot m_t \quad (5.5)$$

in which ϵ is a modifying factor termed the live load response factor.

Instead of working with the live load response factor, it may be more convenient to express the shear in terms of the effective mass m_e which consists of the dead load mass m_d and a portion of the live load mass, say θm_l , θ being termed the live load contribution factor. Thus, the effective mass is given by:

$$m_e = m_d + \theta m_l \quad (5.6)$$

and

$$F_2 = S_A(m_e) \cdot m_e \quad (5.7)$$

Thus, using Eqs. 5.6 and 5.7, θ can be related to ϵ provided S_A can be expressed as a function of the mass. Setting

$$\begin{aligned} S_A &= K' T^n \\ &= K m^{n/2} \end{aligned} \quad (5.8)$$

where K' and K are constants, one obtains

$$\theta = \frac{1}{\alpha} \left[(1+\alpha) \epsilon^{2/2+n} - 1 \right] \quad (5.9)$$

where

$$\alpha = m_l / m_d$$

In Reference [61], values of ϵ have been worked out for a range of practical values of Coulomb friction or viscous damping and the mounting

spring-stiffness. The primary system had a period of 0.1 sec or 0.2 sec or 0.5 sec and the response computations were carried out for two recorded earthquakes - the El-Centro and the Taft. By choosing a function of m to approximately represent the response spectrum of these ground motions, it is possible to obtain some idea of the order of live load contribution factor θ . The value of ϵ reported in reference [61] for the systems studied ranged between 0.4 and 0.6 when α was equal to 1. If the acceleration spectra for the ground motions referred to above could be represented by $S_A = K m^{1/2}$ (the corresponding value of n being equal to 1) in the range of periods studied, a value of $\epsilon = 0.5$ will correspond to a value of $\theta = 0.26$. On the other hand, if the spectral accelerations are considered constant over the range of periods studied so that $n = 0$, θ will work out to zero.

Although the study reported in reference [61] is rather limited and does not cover the range of practical periods of multistory buildings, it does, when interpreted as above, indicate that even where the presence of live load increases the response, the increase is only a fraction of what it would be if the whole of the live load were contributing to the effective mass.

5.5.2 The Effect of the Mass on the Response of the Structure

To study the effect of mass on the seismic response, the standard ten story single bay frame was analyzed for four different values of the effective masses using three different ground motions. The effective floor masses were selected to give four different values of the fundamental

period: 1.0 sec, 1.5 sec, 2.1 sec and 2.5 sec. If the distribution of stiffness and mass and the dimensions of a frame are constant, the elastic seismic response of the frame will depend only upon the ratio of its floor mass to stiffness and hence upon its fundamental period. It should, therefore, be expected that the frames having the same periods will have the same floor displacements, velocities and accelerations. This is confirmed by comparing the results of the elastic analysis of the frames for the El-Centro and the Taft accelerograms, presented respectively in Figs. 5.11 and 5.13, with similar results presented in Figs. 5.6 and 5.8 where the period of the frame is varied by varying the member stiffnesses. The frames having the same values of periods have exactly the same values of maximum floor displacements and maximum interstory drifts. The maximum story shears in the two cases are proportional to the effective masses, implying that the maximum floor accelerations are exactly the same.

It is clear that the change in the maximum floor displacements and accelerations and the maximum interstory drifts, due to a change in the effective mass, is really a consequence of the attendant change in the period of the frame. The maximum story shears and maximum axial loads, on the other hand, depend upon the product of the maximum accelerations and the corresponding effective floor masses. For the El-Centro accelerogram, the relative shears obtained by taking the products of the spectral accelerations and the corresponding effective masses work out to 18.9, 14.8, 27.3 and 39.9, for the corresponding fundamental periods of 1.0 sec, 1.5 sec, 2.1 sec and 2.5 sec. Fig. 5.11 shows that the maximum story

PERIOD
 O 10 SEC
 ● 15 SEC
 ▲ 21 SEC
 ■ 25 SEC

EL CENTRO, 1940 N-S

FIG 5.11 EFFECTIVE MASS AND ELASTIC RESPONSE

FIG. 8-12 EFFECTIVE MASS AND INELASTIC RESPONSE

PERIOD
 O 10 SEC
 □ 15 SEC
 △ 21 SEC
 X 25 SEC

PART, 1052 5000

FIG 5.13 EFFECTIVE MASS AND ELASTIC RESPONSE

FIG. 5-14 EFFECTIVE MASS AND INELASTIC RESPONSE

shears and the maximum column axial loads vary with the period in a similar manner.

For the Taft S69E accelerogram, the relative shears for the frames with fundamental periods of 1.0 sec, 1.5 sec, 2.1 sec and 2.5 sec, work out respectively to 17.2, 23.2, 20.5 and 20.5. As seen by the results presented in Fig. 5.13, the maximum story shears and the maximum column axial loads do vary with the period in a similar manner except in the case of the frame with a fundamental period of 2.5 sec. This frame has a large spectral response corresponding to its second mode period of 0.85 sec. This causes a pronounced whiplash and also an increase in the overall response.

An examination of the response spectrum for the Taft N21E accelerogram, shown in Fig. 4.5, will show that the spectral velocities are considerably higher in the short period range, from about 0.4 sec to 0.9 sec, than in the range of comparatively longer periods. In this respect, the response spectrum for the Taft N21E accelerogram differs markedly from the response spectra for the El-Centro and the Taft S69E accelerograms. A consequence of this characteristic of the Taft N21E accelerogram is that the contribution of the second modes of the building frames to the total response is very substantial. The spectral velocities corresponding to the fundamental mode periods of 1.0 sec, 1.5 sec, 2.1 sec and 2.5 sec are respectively 28.0, 25.0, 26.0 and 25.5 in/sec. The spectral velocities corresponding to the second mode periods of these frames, .339 sec, .507 sec, .712 sec and .847 sec, are respectively 48.0, 72.0, 82.0 and 73.0 in/sec.

Fig. 5.15 shows the elastic response of the four frames to the Taft N21E accelerograms. The relative spectral displacement values for the fundamental modes of the frames work out respectively to 28, 37.5, 54.6 and 63.8. The maximum story displacement do vary with the period in a similar manner except that for the frame with a period of 2.1 sec the response is somewhat accentuated, presumably because of the very high second mode spectral response of this frame. The frames with fundamental periods of 1.5 sec, 2.1 sec and 2.5 sec show pronounced whiplash effects as seen from the maximum interstory drifts and the maximum story shears. The relative spectral acceleration values corresponding to the fundamental periods of the four frames work out to 28.0, 16.7, 12.4 and 10.2. The products of the spectral accelerations and the corresponding effective masses give relative shear values of 11.5, 15.2, 22.2 and 26.1, respectively. The maximum story shears and the maximum axial loads vary in a similar manner with the period, except that the response of the frame with a fundamental period of 2.1 sec is somewhat accentuated.

Fig. 5.12 shows the elasto-plastic response of the four frames studied to the El-Centro accelerogram. The maximum story displacements, the maximum interstory drifts and the maximum story shears follow a pattern similar to the elastic response values. However, the increase in the story shears with an increase in the effective mass, as observed in the responses of the frames with fundamental periods of 2.1 sec and 2.5 sec, is proportionately less when compared to the corresponding increases in the elastic case. This is because once the girders start yielding the

PERIOD
 0 1.0 SEC
 1 1.5 SEC
 2 2.0 SEC
 3 2.5 SEC

TAFT 1950 WIDE

FIG. 515 EFFECTIVE MASS AND ELASTIC RESPONSE.

FIG. 5-16 EFFECTIVE MASS AND INELASTIC RESPONSE

rate of increases of shear with increase in the displacement reduces substantially. This effect is even more pronounced on the column axial loads. The curves of girder ductility ratios show substantial girder yielding for the frames with fundamental periods of 2.1 sec and 2.5 sec. This limits the increase in the story shears and the column axial loads for these frames.

The above observations relating to the elasto-plastic response apply in general terms to the responses to the Taft S69E and N21E accelerograms, shown respectively in Figs. 5.14 and 5.16. In Fig. 5.14, the pronounced whiplash effect for the periods of 2.1 sec and 2.5 sec is again evident in the girder and column ductility ratios. For the Taft N21E accelerogram, whiplash occurs for the three frames with the fundamental periods of 1.5 sec, 2.1 sec as seen in Fig. 5.16.

5.5.3 Conclusions on the Contribution of Live Load to the Seismic Response.

The evidence presented in this section supports the view that the presence of live load in a building structure does not necessarily cause an increase in the response. Whether or not such an increase occurs depends upon the characteristics of the ground motion. Even where an increase in response takes place, typically only a fraction of the live load present contributes to the inertia forces because it is not rigidly attached to the structure. These considerations, coupled with the fact that the probability of a high level of live load at the same time as the occurrence of an extreme earthquake is very unlikely, suggest

that it may be quite reasonable to neglect the contribution of the live load to seismic response except perhaps for storage structures or warehouses. In cases where the presence of live load decreases the response, taking the effective mass as equivalent to the dead load will be the safest course. For other cases, the increase in response due to the live load is likely to be small due to the combined influence of the factors discussed above.

CHAPTER 6

RESPONSE OF SET-BACK FRAMES TO EARTHQUAKE MOTION

6.1 INTRODUCTION

A brief outline of the work done in the past on the seismic response of set-back building was presented in Chapter 1. This Chapter presents the results of an investigation of the elastic and inelastic response of symmetric set-back frames to seismic forces. The methods of analysis used in the investigation have been described in Chapters 2 and 3. Chapter 3 also lists the assumptions made in modelling the member and the frame behaviour. Based on these assumptions, symmetric set-back building models were developed for analytical study. These models are described in Chapter 4 which also contains details of the earthquake accelerograms used in the analysis.

The earlier parts of this Chapter deal with the results of a study of the effects of the characteristics of the frame on its seismic response. The investigation covers the effects of certain parameters of the set-back frame on its periods, mode shapes and modal shear coefficients. Several set-back frames are then analyzed for two recorded ground motions and the effect of the same parameters on the response of the frame is studied. Towards the end of the Chapter, the computed responses to the recorded earthquakes are compared with the specifications of the seismic building codes in regard to the effect of set-backs.

6.2 THE EFFECTS OF THE CHARACTERISTICS OF THE FRAME ON ITS RESPONSE

6.2.1 The Effects of L_s and C_s on the Natural Periods of the Building

The level of set-back ratio, L_s , and the tower-base-plan area ratio, C_s , have been defined in Chapter 4. Fig. 6.1 and Table 6.1 show the effect of the value of C_s on the periods of set-back buildings with different levels of set-back. For the purpose of comparison, the natural periods of the tower portion considered as a separate building of its own height are also given. It is observed that the fundamental period of the set-back building decreases with decreasing value of C_s for all levels of set-back. The decrease is more pronounced when the level of set-back is near mid-height of the frame. For $L_s = 0.2$ and 0.8 , the fundamental period is closer to the period of the related uniform building.

For a set-back near mid-height, the second mode period increases with decreasing tower-base-plan area ratio C_s . But, for $L_s = 0.2$ and 0.8 , the second mode period decreases with a decreasing value of C_s . The change in period for the second mode is not so pronounced as for the fundamental mode. For the third mode, the change in period due to a change in C_s is even smaller.

6.2.2 The Effects of L_s and C_s on the Mode Shapes

The first, second and third mode shapes, normalized so that the value of modal displacement at the top floor is unity, have been plotted in Fig. 6.2. It is observed that in the fundamental mode the response of the tower portion is generally accentuated in comparison to the base, as

FIG 61 EFFECT OF TOWER-BASE-PLAN-AREA RATIO ON THE PERIODS
OF A SET-BACK BUILDING

TABLE 6.1
THE EFFECT OF L_s and C_s ON THE PERIODS OF A SET-BACK BUILDING

L_s	C_s	Building Period, secs.			Tower Period, secs.		
		1st Mode	2nd Mode	3rd Mode	1st Mode	2nd Mode	3rd Mode
1.0	1.000	2.94	1.030	.605			
0.8	0.667	2.74	1.000	.622			
	0.333	2.52	.935	.640	.71	.217	.119
	0.111	2.37	.858	.667			
0.6	0.667	2.63	1.070	.595			
	0.333	2.26	1.140	.580	1.33	.436	.242
	0.111	1.92	1.230	.565			
0.4	0.667	2.67	1.040	.618			
	0.333	2.34	1.050	.630	1.91	.645	.369
	0.111	2.07	1.070	.642			
0.2	0.667	2.80	.987	.597			
	0.333	2.61	.920	.575	2.44	.842	.490
	0.111	2.51	.875	.555

THIRD MODE

THIRD MODE

SECOND MODE

SECOND MODE

FIRST MODE

FIRST MODE

FIG. 6.20 SET-BACK FRAME MODE SHAPES

FIG. 62b SET-BACK FRAME MODE SHAPES

C_s decreases. For $L_s = 0.8$, however, the behaviour is different. In that case, the tower response in the fundamental mode decreases with decreasing value of C_s , although the change is not very large. It will be shown later that higher modes make a large contribution to the response of low slender towers. When the contributions of all modes are superimposed, the response of such towers is found to be much larger than that encountered at the corresponding levels of the related uniform building. In general, for all levels of set-back, the tower response in the second and third modes is accentuated with respect to the base, and the accentuation is more pronounced with decreasing values of C_s .

Some exceptions can be noted in the characteristic behaviour just described. Thus, for some values of L_s , a particular mode shape may not change significantly with a change in value of C_s . In the present example, this is the case for the second mode when $C_s = 0.4$.

It can be stated that, with some exceptions, the modal tower response in set-back frames increases in relationship to the base response, when compared to the response of a related uniform building. This increase is more pronounced for smaller values of C_s .

6.2.3 The Modal Shear Coefficients

A definition of the modal shear coefficients and the modal base shear coefficients, C_b^r , was given in Section 3.12. As explained there, the base shear coefficients for the various modes of a building provide a comparison of the contributions made by these modes to the total response of the building, when it is subjected to a ground motion which

excites its individual modes equally. For the purpose of obtaining the base shear, the N story uniform building of total weight W can be thought of as comprising of N single-degree-of-freedom systems with periods $T_1, T_2, T_3 \dots T_N$, being the natural periods of the building. The effective weight of each system is simply the appropriate modal base shear coefficient multiplied by the total weight W ; the sum of the effective weights assigned to all N modes is W . Fig. 6.3 shows the weight assigned to the first three modes of the ten story uniform building. The effective weight of the first mode is 77.6% of the total weight W , while the first three modes account for 92.08% of the total weight.

Values of C_b have been plotted in Fig. 6.4 for set-back buildings with different values of L_s and varying tower-base-plan-area-ratios. The plots of Fig. 6.4 show that for buildings with set-backs the contribution of the higher modes to the base shear generally increases with decreasing C_s . For $L_s = 0.4$ and $C_s = .111$, the contributions of the first three modes are respectively 0.258, 0.547 and 0.0216 and the sum of the effective weights of these modes is 82.66% of W . For $L_s = 0.2$ and $C_s = 0.111$, the contributions of the first three modes are respectively 0.278, 0.095 and 0.3165, and the sum of effective weights in the first three modes is 68.95% of W .

Like the base shear coefficient, the tower base shear coefficient is defined as the shear at the base of tower divided by the tower weight. The r th mode tower base shear coefficient is given by:

FIG. 6.3 RELATIVE EFFECTIVE WEIGHTS FOR THE FIRST THREE MODES OF THE UNIFORM BUILDING

O L1.0.8
 □ L1.0.6
 △ L1.0.4
 X L1.0.2

FIG 64 SET-BACK FRAME MODAL BASE SHEARS

$$C_t^r = \frac{\lambda_r \sum_{i=M+1}^N m_i \phi_i^r}{\sum_{i=M+1}^N m_i} \quad (6.1)$$

where $M = L_s \times N$.

Values of tower base shear coefficient have been plotted in Fig. 6.5 for different levels of set-back ratio and varying tower-base-plan-area-ratio. In general, the value of C_t increases for decreasing degree of set-back indicating an accentuated tower response when compared to the uniform building.

6.3 RESPONSE TO RECORDED EARTHQUAKE MOTIONS

The periods, mode shapes, and modal shear coefficients are completely dependent upon the characteristics of the building, that is, upon its mass-stiffness ratio and the distribution of stiffness and mass. The earthquake response, however, depends also upon the characteristics of the ground motion. The effect of L_s and C_s on the earthquake response has, therefore, been studied by analyzing set-back frames subject to El-Centro and Taft ground motions.

6.3.1 Maximum Floor Displacements and Maximum Interstory Drifts, Elastic Response

Figs. 6.6(a) and 6.6(b) show the effect of the L_s and C_s on the maximum floor displacements and the maximum interstory drifts of twelve different elastic set-back buildings subject to El-Centro Earthquake. In the same figure, the behaviour of a related uniform building

O — $L_1 = 0.8$
 □ — $L_1 = 0.6$
 Δ — $L_1 = 0.4$
 X — $L_1 = 0.2$

FIG 6.5 SET-BACK FRAME TOWER BASE SHEAR

O -- $C_g = 1.000$
 □ -- $C_g = 0.667$
 △ -- $C_g = 0.333$
 × -- $C_g = 0.111$

EL CENTRO, 1940 N. S.

FIG 6 60 SET-BACK FRAME ELASTIC RESPONSE

- — $C_s = 1.000$
- — $C_s = 0.667$
- △ — $C_s = 0.333$
- × — $C_s = 0.111$

EL-CENTRO, 1940 N-S

FIG 6 6b SET-BACK FRAME ELASTIC RESPONSE

is shown for comparison. The maximum floor displacements of the set-back buildings are smaller than those of the related uniform building; a decrease in C_s usually decreases the maximum displacements for all levels of set-back.

The interstory drifts in the base portions of the set-back buildings are smaller than those in the related uniform building and, in general, decrease with decreasing values of C_s . The decrease is more pronounced when the level of set-back is near mid-height. Since the stiffness of base portion is the same for all the set-back building models analyzed, a smaller interstory drift implies a smaller story shear. However, it has to be remembered that, with decreasing values of C_s , the total mass of the tower portion decreases. Since the shear coefficient at any level is inversely proportional to the total building mass above that level, the smaller shear in the base portion does not necessarily mean a smaller shear coefficient. The shear coefficient, in fact, increases with decreasing C_s as will be discussed later in this Chapter.

In the tower portion, the maximum interstory drifts are substantially larger than those at the corresponding levels of the uniform building. For slender towers, the increase is very large. For $L_s = 0.8$ and $C_s = 0.411$, the maximum interstory drift in tower portion is almost four times that at the corresponding level of the related uniform building. For a value of $C_s = 0.667$, which for base and tower portions square in plan corresponds to a tower-to-base-plan-dimension ratio of approximately 0.8, the response is of the same order as that of the uniform building.

A number of set-back frames have also been analyzed for the Taft accelerogram, for one level of set-back ratio, $L_s = 0.4$ and varying values of C_s . The maximum story displacements and maximum interstory drifts are shown in Fig. 6.7. The behaviour is substantially the same as noted in the case of the El-Centro accelerogram.

Inelastic Response

Figs. 6.8(a) and 6.8(b) show the maximum floor displacements and maximum interstory drifts for the set-back buildings subject to El-Centro ground motion, when inelasticity is taken into account. The response is smaller than the corresponding elastic response. However, the effects of varying values of L_s and C_s are very similar to what is encountered with an elastic set-back building.

For a better comparison of the elastic and inelastic responses, the maximum floor displacements and maximum interstory drifts for both elastic and inelastic buildings, for $L_s = 0.6$ and varying values of C_s have been plotted in Figs. 6.9(a) and 6.9(b). The striking decrease in the magnitude of response caused by the presence of inelasticity is at once apparent. It is also observed that there is definite qualitative similarity between the elastic and inelastic responses. This suggests that, for the purpose of design, for inelastic behaviour, it should be possible to use the response values obtained from the elastic analysis by applying a suitable reduction factor.

The inelastic response of set-back building frames to the Taft accelerogram has also been calculated for $L_s = 0.4$ and varying values of

a, b INELASTIC

c, d ELASTIC

○ — $C_s = 1.000$
 □ — $C_s = 0.667$
 △ — $C_s = 0.333$
 × — $C_s = 0.111$

FIG 67 SET-BACK FRAME RESPONSE TO S69E

- — $C = 1.000$
- — $C = 0.887$
- △ — $C = 0.333$
- × — $C = 0.111$

EL CENTRO, 1940 N-S

FIG. 6 B a SET-BACK FRAME INELASTIC RESPONSE.

EL-CENTRO 1940 N S

O - CS-1000
 □ - CS-0.667
 Δ - CS-0.333
 X - CS-0.111

FIG 6.8b SET-BACK FRAME INELASTIC RESPONSE

$L_s = 0.8$

— ELASTIC
- - - INELASTIC

FIG. 6.9a SET-BACK FRAME RESPONSE TO EL-CENTRO

$L_s = 0.6$

— ELASTIC
- - - INELASTIC

FIG. 6.9b SET-BACK FRAME RESPONSE TO EL-CENTRO

C_s . The maximum story displacement and maximum interstory drifts are shown in Fig. 6.7. The behaviour is similar to that noted for the El-Centro accelerogram.

Again, the elastic and inelastic values have been plotted side by side for comparison in Figs. 6.10(a) and 6.10(b). The plots show that the inelastic response is always smaller than the elastic response. Also, the qualitative similarity between the two sets of response values is apparent.

6.3.2 Shear Coefficients

An important response quantity of direct applicability in earthquake resistant building design is the maximum value of story shear. This is best expressed in the form of story shear coefficient as defined earlier.

The story shear coefficients for the inelastic set-back buildings subject to the El-Centro ground motion are shown in Fig. 6.11. These plots illustrate several important aspects of the behaviour. In the uniform building, the shear coefficients increase with height and the increase is very much pronounced in the top floors. This is apparently due to the contribution of higher modes. The plots of Fig. 6.11 also show that the shear coefficient at any level of a set-back building is larger than that at the same level of the related uniform building. This behaviour is noted throughout the height of the building. Also, the shear coefficients increase as C_s decreases. The marked effect of higher modes on the tower response is apparent from the large values of shear coefficients for the top stories.

$L = 0.4$

— ELASTIC
- - - INELASTIC

FIG 610a SET-BACK FRAME RESPONSE TO TAFT, S69E

$L_s = 0.4$

— ELASTIC
 --- INELASTIC

FIG 6.10b SET-BACK FRAME RESPONSE TO TAFT, S69E

FIG. 6 II SET-BACK FRAME INELASTIC RESPONSE

For buildings with a relatively small value of C_s , the shear coefficient shows a sudden and marked increase in the transition region between the base and the tower. This behaviour may appropriately be termed the notch effect.

Shear coefficients obtained for the Taft accelerogram for a building with $L_s = 0.4$ and a range of values of C_s have been plotted in Fig. 6.12. The behaviour is similar to that for El-Centro accelerogram. The shear coefficients increase with decreasing values of C_s and with very slender towers a notch effect is present at the level of set-back.

6.3.3 Utilized Ductility Ratios

The girder and column ductility ratios utilized by the El-Centro accelerogram for a building with $L_s = 0.6$, and a range of values for C_s have been plotted in Fig. 6.13. The results obtained are based on the assumption that the girders carry the full design dead load but no live load. The plots of Fig. 6.13 again show the accentuation of the tower response. The girder ductility ratios in the tower portion show a large increase with an increase in the slenderness of the tower. Similarly, the column ductility ratios in the tower portion increase with an increase in the slenderness of the tower. For the related uniform building, no plastic hinges develop in the columns. But, for the set-back building with a value of $C_s = 0.333$, the lowest and topmost columns in the tower portion develop plastic hinges. For $C_s = 0.111$, most of the columns in the tower portion develop plastic hinges.

O $C_s = 1.000$

□ $C_s = 0.667$

Δ $C_s = 0.333$

X $C_s = 0.111$

TAFT 1952 S69E

FIG. 6-12 SET-BACK FRAME INELASTIC RESPONSE

EL-CENTRO, 1940 NS

0 Cs = 1.000
 □ Cs = 0.667
 △ Cs = 0.333
 x Cs = 0.111

FIG. 613 SET-BACK FRAME INELASTIC RESPONSE

3

4

OF/DE

It is generally considered advisable to limit the utilized ductility ratios for the girders to a maximum of about 4 to 6. Also, it is considered desirable to avoid inelasticity in the columns because of the likelihood of the collapse of the frame as a mechanism and the more drastic consequences of a failure of the columns. The set-back buildings selected for study more or less conform to the earthquake design provisions of the present codes. It is then evident that set-back buildings with slender towers, designed according to the code, may undergo serious distress in the tower portion when subjected to a severe earthquake.

6.4 COMPARISON OF THE RESULTS OF ANALYSIS WITH THE CODE PROVISIONS

The present SEAOC Code [62] and the National Building Code of Canada [63] make specific provisions for the design of set-back frames. According to these codes, where the plan dimension of the tower in each direction is at least 75% of the corresponding plan dimension of the base, the effect of set-back may be neglected. For buildings with a smaller size of tower, the tower seismic shears are to be evaluated by considering the tower either as a separate building of its own height or as a part of the whole building, whichever gives the larger values of shears. The SEAOC Code further requires that the tower base shear so obtained be applied to the top of the base portion, which may otherwise be considered as a separate building of its own height. For many set-back buildings, including those with tower plan dimensions substantially smaller than 75% of the plan dimension of the base, these provisions do not result in any increase in the design shears.

The Commentary to the SEAOC Code contains more elaborate provisions for the design of set-back buildings. These provisions place set-back buildings into several different categories depending upon: (1) the ratio of the height of the tower to the total height of the building; and (2) the ratio of the width of the tower parallel to the direction under consideration to the width of the base in the same direction. The method of design to be used for a particular set-back building depends upon the category to which the building belongs. The set-back building design requirements in the Commentary to the SEAOC Code are the only detailed set of provisions for such buildings available in any code. These requirements are also presently under consideration for inclusion in the 1975 edition of the National Building Code of Canada. The author has made a comparison of these code provisions with the results of analytical studies of set-back buildings.

The code requirements relate the set-back building response to the plan dimensions of the tower and the base in the direction of the earthquake motion. The results of analyses carried out in this study have shown that in the frequently occurring case where the mass and stiffness can be taken as substantially proportional to the plan area, the set-back building response is, in fact, dependant upon the ratio of the tower plan area to the base plan area and not upon the ratio of the plan dimensions of the tower and the base in one direction. Thus, the two set-back buildings shown in Figs. 6.14(a) and (b) respectively will have the same elastic response to a particular base excitation since the tower

ELEVATION

ELEVATION

PLAN

(a)

PLAN

(b)

FIG 614 SET-BACK BUILDINGS WITH EQUAL TOWER-BASE-PLAN-AREA RATIOS

base area in each case bears the same ratio to the base plan area.

However, in accordance with the code, the building shown in Fig. 6.14(a) will be considered a set-back building for vibrations in the N-S direction while the building shown in Fig. 6.14(b) will be considered to be a uniform building.

6.4.1 The Natural Periods of Set-Back Buildings

The code provisions referred to above require that the weighted average width of the set-back building be used for deriving its fundamental period. The fundamental periods of twelve different set-back buildings derived on this basis have been shown in Table 6.2 along with the periods computed by eigenvalue solutions. For the purpose of comparison, all periods have been normalized by dividing them by the fundamental periods of the related uniform building. The results show that the periods computed according to the code bear no relation to the periods found from analysis.

A simple procedure for finding the fundamental period of a set-back building has been developed by the author and is presented below. This procedure is found to give fairly good approximations to the periods of rigid jointed framed set-back buildings, provided the stiffnesses and the masses of the tower and the base are reasonably proportionate to their respective plan areas. For set-back buildings with level of set-back ratio L_s greater than 0.5, find the weighted height $\bar{H}$ so that the volume of a building with plan area A_0 and height $\bar{H}$ is the same as the total volume of the set-back building. Thus, $\bar{H}$ is given by:

TABLE 6.2
THE NATURAL PERIODS OF THE SET-BACK FRAMES

No.	L _s	C _s	Normalized Building Periods		Suggested Procedure
			Exact, from an eigenvalue solution	Code Prescribed Method	
1	1.000	1.000	1.000	1.000	1.000
2	0.800	0.667	0.932	1.035	0.932
3		0.333	0.857	1.075	0.857
4		0.111	0.807	1.075	0.807
5	0.600	0.667	0.895	1.075	0.895
6		0.333	0.769	1.170	0.770
7		0.111	0.653	1.170	0.653
8	0.400	0.667	0.910	1.120	0.895
9		0.333	0.795	1.292	0.770
10		0.111	0.705	1.292	0.653
11	0.200	0.667	0.952	1.170	0.932
12		0.333	0.887	1.380	0.857
13		0.111	0.855	1.380	0.807

$$\bar{H} = \frac{A_b \cdot h + A_t (H-h)}{A_b} \quad (6.2)$$

The period of the set-back building is then related to the period T_o of the related uniform building by the expression:

$$T_s = T_o \cdot \frac{\bar{H}}{H} \quad (6.3)$$

The period T_o can be determined by the relevant code provisions for the determination of the periods of regular buildings. For set-back buildings with a level of set-back ratio smaller than 0.5, extend the base portion and reduce the tower height so that the total height of the set-back building remains unaltered but the level of set-back ratio changes to $(1-L_s)$. Now compute the set-back building period as above.

The fundamental periods of the set-back buildings derived in accordance with the suggested procedure are also shown in Table 6.2. The procedure as described above is applicable only to buildings with a single set-back but it can be extended for application to the case of multiple set-backs.

6.4.2 Comparison of the Elastic Response to Recorded Earthquakes with the Provisions of the Code

The elastic response of the twelve set-back buildings to the El-Centro earthquake were computed and the maximum story shears obtained were compared with their code-prescribed values. To facilitate comparison, all the shears were first converted into shear coefficients which

were then normalized by dividing them by the shear coefficients at the corresponding story of the related uniform building. The code shear coefficients for the set-back buildings were similarly normalized by dividing them by the code shear coefficient at the corresponding story of the related uniform building. The normalized shear coefficients represent in each case the effect of set-back on the story shears. The results are shown in Fig. 6.15. It may be noted that since the shear coefficients have been normalized the results are not influenced by the difference in the distribution of shears in a uniform building as derived from the code and as obtained by analysis. The results show the wide fluctuations between the provisions of the code and the El-Centro response. In the El-Centro response, the shear coefficients show a pronounced increase at the level of set-back and in the upper stories of the tower. It is also seen that the code values underestimate the set-back building response when the tower is slender and the level of set-back ratio is either very high or very low. In fact, for $L_s = 0.2$, the code permits the use of Procedure C [62] for evaluating the design seismic shears. This procedure will give values of tower shear coefficients that are the same as in the related uniform building. The analysis, however, shows a substantial increase in the tower shears of the set-back building. If Procedure D of the code is used to derive the relative tower shear coefficients, the values obtained will be higher than those for the uniform building but less than the relative shear coefficients obtained from analysis.

CODE PRESCRIBED
COMPUTED

0-0-0 007
0-0-0 003
0-0-0 011

EL-CENTRO, 1940 N-S

FIG. 6.15 CODE PRESCRIBED AND COMPUTED SHEAR COEFFICIENTS FOR A SET-BACK FRAME

EL CENTRO, 1940 N.S.

FIG 6.15 CONTO.

Fig. 6.16 compares the relative shear coefficients obtained from the code with those obtained from an analysis for the Taft S69E accelerogram. The values shown in the figure are for three set-back buildings, all with a level of set-back ratio equal to 0.4 but with different tower base plan area ratios. As in the case of El-Centro accelerogram, the results show a marked increase in the shear coefficients at the level of set-back.

One reason for the discrepancy between the results of analysis and the code specifications is that the code relates the set-back building response to the ratio of the plan dimensions of the tower and the base in the direction of the vibrations, while the computed results show that the response is really dependant upon the ratio of the plan areas of the tower and the base. But, the primary cause for the difference is that no simple provision can take into account all the complex factors that affect the seismic response of irregular structures such as the set-back frames. Within these limitations, the provisions in the Commentary to the SEAOC Code seem to be adequate for set-backs near the building mid-height except when the tower is very slender. The code provisions may, however, underestimate the response when the tower is either relatively low or relatively tall ($L_s = 0.8$ or $L_s = 0.2$), particularly when it is slender at the same time.

TAFT, 1952 S69E

--- CODE PRESCRIBED
 ——— COMPUTED

□--□ □--□ □--□ $C_s = 0.667$
 ○--○ ○--○ ○--○ $C_s = 0.333$
 x--x x--x x--x $C_s = 0.111$

FIG 616 CODE PRESCRIBED AND COMPUTED SHEAR COEFFICIENTS FOR A SET-BACK FRAME

CHAPTER 7

DUCTILITY REQUIREMENTS IN FRAMES SUBJECTED TO SEISMIC FORCES

7.1 INTRODUCTION

The satisfactory performance during an earthquake of building frames in general, and steel frames in particular, depends to a large extent upon the amount of inelastic deformations that the frame members can undergo without distress. In the analysis procedure used for the work of this thesis, it is assumed that the inelastic member deformation is concentrated at point type plastic hinges at ends of members. It is further assumed that a member undergoes rotation at the plastic hinge, while the moment in that location remains constant at a value equal to the plastic moment of resistance. For these assumptions to be valid, it is necessary that the plastic moment-carrying capacity not be impaired by local or lateral buckling up to a magnitude of rotation equal to or more than the maximum expected. A reduction in the moment carrying capacity of a plastic hinge will give rise to a deteriorating hysteresis loop and a possible instability in the frame.

The inelastic member rotation capacity is thus a quantity of primary interest in earthquake-resistant design. The actual inelastic rotations in frame members are often measured in terms of ductility ratio. However, the term "ductility ratio" is used in several different senses in earthquake engineering. Many of the definitions of ductility ratio do not have any direct relation to a measurable physical behaviour of the

member, so that it is difficult to assess whether the actual ductility requirement is less than the ductility capacity. A definition of rotation ductility ratio and methods of computing the utilized rotation ductility from the results of a dynamic analysis are proposed by the author in this chapter. The utilized rotation ductility ratios computed by these methods can be compared directly with safe values of rotation ductility capacity ratios, reliable values for which can, at present, be determined only from experiments.

In earthquake engineering, the term ductility is also used for the ratio of total lateral displacement of a structure to the elastic component of such displacement. In the present study, this ratio is termed "deflection ductility ratio" or simply "deflection ductility" to distinguish it from the other usage of the term ductility.

As a part of the research for this thesis, the moment rotation relationship for a wide flange beam subject to an end-moment was obtained by numerical integration of the curvature. Such a relationship gives some idea of the maximum fibre strains caused by inelastic rotation at a plastic hinge. Details of the procedure used in deriving the moment rotation relationship are presented in Section 7.5 of this Chapter. Towards the end of the chapter, a review is made of the existing experimental work on the rotation capacity of beams, in order to arrive at some quantitative appreciation of the ductility capacity of steel members.

7.2 DEFLECTION DUCTILITY

Fig. 7.1(a) shows a single story frame, in which the beam is assumed to be infinitely stiff. The lateral force deflection diagram of Fig. 7.1(c) follows from the assumption of an elasto-plastic moment-rotation relationship for the columns. The frame is essentially a single degree of freedom system and can be represented by the model shown in Fig. 7.1(b). When vibrating, the mass m may undergo a total deflection δ_{cp} . The deflection ductility μ is then defined as:

$$\mu = \frac{\delta_{cp}}{\delta_y} \quad (7.1)$$

where δ_y is the elastic component of the total deflection.

Based on investigations of single-degree-of-freedom elastic plastic systems, approximations to the inelastic response of multi-degree-of-freedom systems have been derived, so that the forces and displacements in an elastic system subject to seismic excitation are related to the corresponding values for an elastic system through a known or selected value of the deflection ductility [6, 64]. This is a useful concept because inelastic dynamic analysis is far more complex than an elastic analysis. However, it is based on the premise that a suitable value for deflection ductility is known or can be determined.

The calculation of the deflection ductility capacity of a frame is a very difficult non-deterministic problem. However, the inelastic rotation capacity of beams can be determined by experiment. It will be found that the beam rotation capacity depends primarily upon the properties

(a) SINGLE STORY FRAME

(b) DYNAMIC MODEL

(c) SPRING FORCE DEFLECTION RELATIONSHIP

FIG 71 LATERAL VIBRATIONS OF A SINGLE STORY FRAME

of the material, section dimensions, the type of beam to column connection, and the distribution of load on the beam. If experiments are performed under conditions reasonably similar to those existing in the structure, the experimental results can be used to obtain the values of rotation capacities of the members of the frame. On the other hand, the frame deflection ductility capacity depends not only upon the behaviour of and loading on the individual members but also upon their structural configuration in the structure. It is not, therefore, possible to obtain in a direct manner any unique value for the deflection ductility capacity.

In the frame under consideration, the plastic sidesway deflection results from the fact that the plastic hinges in the columns undergo rotation at practically constant moment. During this inelastic stage of the deformation, the fibres at the locations of the plastic hinges are subject to increasing strains, and the members may undergo local or lateral torsional buckling at these hinge locations. The allowable inelastic rotation should then be limited sufficiently to ensure that these kinds of buckling do not take place. Fortuitously, in the frame under consideration, the rotation ductility of the columns, that is the ratio of total column rotation at the plastic hinges to the elastic component of that rotation, is equal to the deflection ductility ratio as defined above. Thus, a limit imposed on inelastic hinge rotation provides the limit on deflection ductility in a direct manner. Such direct correlation between the deflection and rotation ductilities does not exist for multistory frames, or even for single story frames where beams are

flexible and also carry gravity loads.

Fig. 7.2(a) shows a single story frame with flexible girder, the stiffness of the girder being K_B . The column stiffness is represented by K_C ; M_{pC} and M_{pB} represent, respectively, the column plastic moment of resistance and beam plastic moment of resistance. Fig. 7.2(b) shows the lateral force deflection relationship for this frame. Curve, oab, applies when $K_B = \infty$. If the girder strength M_{pB} is smaller than column strength M_{pC} , the force-deflection relationship follows the path, ocde. In this case, symmetrical plastic hinges are formed at the ends of the girder when F , the lateral force, reaches its value at point c; plastic hinges form at column bases when F reaches its value at point d. Path, ofgb, applies when the beam strength is large so that plastic hinges form first at the base of the column and next at top of columns. The relationships shown in Fig. 7.2 are based on the assumption that the girder carries no gravity loads. The force deflection graph becomes even more complex when the girder carries such loads. Obviously, no simple correlation can be found between deflection ductility and the extent of inelastic rotation at the plastic hinges.

7.3. LOCATION OF THE PLASTIC HINGES IN A BUILDING FRAME

If a building frame is subjected to seismic forces and the girder moments due to gravity loads are small in comparison to the seismic moments, the distribution of moments throughout the length of a typical

(a)

(b)

FIG. 72 LATERAL FORCE DEFLECTION RELATIONSHIP FOR A SINGLE STORY FRAME

girder is as shown in Fig. 7.3(a). In this case, the plastic hinges are formed at the ends of the girders. When gravity loads are important, the moment diagram is as shown in Fig. 7.3(b). In this case, a plastic hinge is formed first at one end of the girder, the left hand end if the story is swaying to the left as shown in the figure. If the frame continues to sway in the same direction, another plastic hinge is formed, either within the span or at the right hand end, depending on the comparative values of the lateral loads and gravity loads.

Assuming that the vertical load is a uniformly distributed load of w units per unit of length, the gravity load moment, M_G , at the centre of a simply supposed span is given by:

$$M_G = \frac{w \cdot L^2}{8} \quad (7.2)$$

If $M_p \geq 2M_G$ the plastic hinge will form at the end of the girder. On the other hand, if $M_p < 2M_G$, the hinge will form within the span. For most steel framed building structures, it can be assumed that the plastic hinges form at or close to the ends of the girders. In some investigations [27, 65], the possibility of the formation of hinges near the midspans of the girders has been taken into account. It has been found that plastic hinges hardly ever form near the midspan.

7.4 ROTATION DUCTILITY

As discussed in Section 7.2, it is very difficult in most practical cases to find a close correlation between the deflection ductility

(a) WITHOUT GRAVITY LOADS

(b) WITH GRAVITY LOADS

FIG. 7.3 DISTRIBUTION OF MOMENT IN A TYPICAL BEAM OF A MULTISTORY FRAME

of a frame and inelastic rotations of the members of which it is composed. On the other hand, it is the inelastic rotation of the members for which a safe limit can be determined on the basis of experiments. It is, therefore, most desirable to define ductility in terms of member rotation at plastic hinges.

Fig. 7.4(a) shows a simply supported steel wide flange beam of length L subject to a moment at one end. The end moment versus rotation relationship is shown in Fig. 7.4(b). The relationship is linear until the yield moment M_y is reached. The rate of increase of moment drops rapidly after M_y is exceeded and the moment is still very nearly equal to the plastic moment at a rotation of several times θ_y , the rotation at the beginning of yield. The variation of moment with rotation is given to a good approximation by OAB in Fig. 7.4(b). This elastic plastic idealization has been used throughout this study. According to this idealization, it is assumed that moment is proportional to rotation until M_p is reached; rotation θ_p at that instant is therefore termed the elastic component of rotation. The rotation ductility μ is then defined as:

$$\begin{aligned}\mu &= \frac{\theta}{\theta_p} \\ &= 1 + \frac{\theta_H}{\theta_p}\end{aligned}\tag{7.3}$$

where θ is the total end rotation and θ_H is the plastic component of end rotation so that:

$$\theta_H = \theta - \theta_p\tag{7.4}$$

(a)

(b)

FIG 7 4 .MOMENT ROTATION RELATIONSHIP FOR A BEAM

Since

$$\theta_p = \frac{M_p L}{3EI} \quad (7.5)$$

$$\mu = 1 + \frac{3EI\theta_H}{M_p L} \quad (7.6)$$

From experimental investigations on the rotation capacity of beams under moment gradient, lower safe bounds can be obtained on the rotation ductility capacity of a flexural member as defined above. If the maximum utilized rotation ductility in each important member of a structure is lower than the ductility capacity as determined above, the structure is satisfactory.

In a seismic analysis, if there are no gravity moments in a member, or if they are comparatively small, the moment diagram is as shown in Fig. 7.3(a). Plastic hinges form at the two ends of the member almost simultaneously and the point of inflexion lies close to the mid-length. When the member undergoes further rotation at the plastic hinges, the moment diagram remains unchanged. Since the point of inflexion is at a distance $L/2$ from the end, the utilized ductility ratio is given by:

$$\mu = 1 + \frac{3EI\theta_H}{M_p (L/2)} \quad (7.7)$$

where θ_H in this case is the maximum plastic rotation at the hinges.

When the gravity moments are substantial, ductility requirement cannot be determined in such a simple manner. Fig. 7.5 shows the distribution of moment in a beam due to gravity and lateral loads. When

(a)

(b)

(c)

FIG. 7.5 DISTRIBUTION OF MOMENT IN A BEAM

the frame is swaying to the left, the first plastic hinge forms at the left end. The distribution of the moment throughout the member length at that instant is similar to that shown in Fig. 7.5(a). The point of inflexion is to the left of the centre. As the frame sways further, the moment at the right hand end continues to increase until a plastic hinge has formed either at that end or within the span. The moment diagram is then similar to that shown in Fig. 7.5(c). During this process, the point of inflexion does not remain stationary but moves away from the midspan and towards the left. The effective length of the beam to be used in determining the ductility is not, therefore, easily determined. An approximate value of the ductility requirement can be estimated by taking the effective length as equal to the minimum distance of the point of inflexion from the beam end. This minimum distance occurs when the two plastic hinges have formed. Its value kL depends upon the plastic moment of resistance of the beam and the magnitude and distribution of the gravity loads. Suppose that the gravity load is represented by a uniformly distributed load of w lbs per running foot, and the plastic moment of resistance M_P is related to the midspan gravity load moment M_G by the equation:

$$M_P = c.M_G \quad (7.8)$$

Then, k is given by:

$$k = \frac{1}{2} \left[1 + \frac{c}{2} - \sqrt{1 + \frac{c^2}{4}} \right] \quad (7.9)$$

For example, when $c = 2.0$, k will be equal to 0.293. Once k is known, the utilized rotation ductility can be worked out from the equation:

$$\mu = 1 + \frac{3EI \theta_H}{M_p (kL)} \quad (7.10)$$

7.5 THE RELATIONSHIP BETWEEN ROTATION DUCTILITY AND THE MAXIMUM FIBRE STRAIN IN A BEAM UNDER MOMENT GRADIENT

For the beam shown in Fig. 7.6(a), the maximum moment and therefore the maximum fibre strain will occur at location A. If the moment-curvature relationship is perfectly elasto-plastic with no strain hardening, the realization of full plastic moment will theoretically require infinite strain in the fibres at location A. However, due to strain hardening, the plastic moment is realized at a finite value of maximum strain. The plastic rotation requirement at a location where a plastic hinge is present, such as at location A, can be realized only through additional strain in the member fibres beyond the strain at which the plastic moment is realized. Such large strains may lead to local or lateral torsional buckling which will impair the moment resisting capacity of the section. The limit of the safe inelastic rotation is, therefore, reached when the maximum strain approaches a point where local instability may result. It will be useful, therefore, to obtain a relationship between the inelastic rotation and the maximum fibre strain.

Such a relationship can be obtained theoretically by integrating the curvature of the beam shown in Fig. 7.6(a). A closed form solution can be obtained only for a regular section shape, such as a rectangle; for

(a)

WIDE FLANGE SECTION

STRAIN

STRESS

(b)

(c)

FIG 7 6 A WIDE FLANGE BEAM SUBJECTED TO AN END MOMENT

wide flange shape, a numerical procedure of integration has to be employed. Using this approach, the moment rotation relationship for a typical wide flange beam was derived by the author. Details of the procedure used are presented below.

An idealized wide flange is shown in Fig. 7.6(b) where the various dimensions of the section have also been indicated. Fig. 7.6(c), the stress-strain diagram for the steel used in the wide flange shape. The

• notations used in the discussion that follows are defined below:

$\epsilon_{\max}$ = maximum extreme fibre strain

ϵ_s = strain at onset of strain hardening

ϵ_y = strain at beginning of yield

σ_y = yield stress

E_s = modulus of elasticity in strain hardening range

y_s = distance from neutral axis of a fibre with strain ϵ_s

y_e = distance from neutral axis of a fibre with strain ϵ_y

M_p = plastic moment of resistance

M_y = moment when the extreme fibre strain is ϵ_y

$$\beta = \frac{\epsilon_s}{\epsilon_y}, \quad \alpha = \frac{\epsilon_{\max}}{\epsilon_s}, \quad \lambda = \frac{\epsilon_{\max}}{\epsilon_y} = \frac{D/2}{y_e}$$

M_y and M_p are given by the following equations:

$$M_y = \frac{\sigma_y}{6D} [2 Bt(3D^2 - 6Dt + 4t^2) + w(D - 2t)^3] \quad (7.11)$$

$$M_p = \sigma_y \left[\frac{BD^2}{4} - (B-w) \left(\frac{D}{2} - t \right)^2 \right] \quad (7.12)$$

The relationship between moment and curvature for various conditions of strain is as follows:

For $\lambda < 1$,

$$M = M_y \lambda \quad (7.13)$$

For $1 \leq \lambda \leq \frac{D/2}{D/2-t}$

$$M = B \sigma_y \left(\frac{D}{2} \right)^2 \left\{ 1 - \frac{1}{3\lambda^2} \right\} - \frac{2}{3} \sigma_y (B-w) \left(\frac{D}{2} - t \right)^3 \frac{\lambda}{(D/2)} \quad (7.14)$$

For $\frac{D/2}{D/2-t} \leq \lambda \leq \beta$

$$M = \sigma_y (B-w) (D-t)t + \frac{wD^2}{12} \sigma_y (3-1/\lambda^2) \quad (7.15)$$

For $\beta \leq \lambda \leq \frac{D/2}{D/2-t} \beta$

$$M = \sigma_y Bt(D-t) + \sigma_y \frac{E_s}{E} (\lambda-\beta) \frac{BD^2}{6} [1-1/\alpha][1/2\alpha+1] \\ + w \sigma_y [(D/2-t)^2 - 1/3 (D/2)^2/\lambda^2] \quad (7.16)$$

For $\lambda > \frac{D/2}{D/2-t} \beta$

$$M = w \sigma_y [(D/2-t)^2 - 1/3 (D/2)^2/\lambda^2] + \lambda \sigma_y \frac{E_s}{E} Bt(D/2-t/3)t/D/2 \\ + \frac{1}{3} w \lambda \sigma_y \frac{E_s}{E} \left[\frac{D/2-t}{D/2} - \frac{1}{\alpha} \right] \left[\frac{D}{2} - t - \frac{D/2}{\alpha} \right] \left[\frac{D/2}{\alpha} + D - 2t \right] \\ + \sigma_y Bt(D-t) \left[1 + \lambda \frac{E_s}{E} \left(\frac{D/2-t}{D/2} - \frac{1}{\alpha} \right) \right] \quad (7.17)$$

From the above relations, a table of moments for various values of curvature can be constructed. Such a table can be used to find the value of curvature for a given value of moment by means of linear interpolation. A numerical integration of the curvature graph, after adjustment for boundary conditions, gives the rotation at section A.

A set of calculations has been performed for the following data:

$$D = 21.13 \text{ in.}$$

$$B = 8.27 \text{ in.}$$

$$t = .615 \text{ in.}$$

$$w = .40 \text{ in.}$$

$$E = 30,000.0 \text{ ksi}$$

$$\sigma_y = 36.0 \text{ ksi}$$

$$E_s = 720.0 \text{ ksi}$$

$$\beta = 12.0$$

Fig. 7.7 shows the relationship between end rotation and end moment in a non-dimensional form. Fig. 7.8 shows the relation between end rotation and maximum fibre strain, again in a non-dimensional form. These relationships are independent of the length of the member. It will be observed that any significant inelastic rotation will involve a maximum fibre strain considerably in excess of the value at onset of strain hardening. Also, the moment capacity is somewhat larger than M_p on account of the strain hardening. Experimental evidence tends to support these conclusions. Experiments also show that wide flange sections

FIG. 7.7 MOMENT ROTATION RELATIONSHIP FOR A WIDE FLANGE BEAM

FIG. 7.8 RELATIONSHIP BETWEEN END ROTATION AND MAXIMUM FIBRE STRAIN IN A WIDE FLANGE BEAM

that meet the requirements for plastic design, as specified in the AISC specifications for steel structures or the CSA Standard S-16, are capable of withstanding quite large fibre strains before they buckle locally. Some of the available experimental results on the rotation capacities of steel beams are reviewed in the next section.

7.6 ALLOWABLE INELASTIC ROTATION

The large strains accompanying inelastic rotation may cause lateral torsional buckling of the member or local buckling of the section, impairing the capacity of the member section to resist moment. The delivered rotation, therefore, terminates at the point where the reduction in capacity takes place. Theoretical and experimental investigations of inelastic local buckling phenomenon in wide flange beams have been reported in references [66] through [69]. References [69] through [76] contain most of the important work done on the lateral-torsional buckling phenomenon.

The theoretical and experimental work mentioned above relates to the inelastic rotation capacities of steel beams under static loading. In a structure subjected to seismic excitation, the member forces are essentially dynamic in nature. Also, portions of the structure may be subjected to alternating cycles of plasticity. Evidently, one cannot expect the rotation capacities obtained for static loading to be entirely valid for conditions of dynamic loading and alternating plasticity. Rotation capacities under such conditions of loading can, therefore, be determined only by dynamic or quasi-static load tests.

Investigations made for the case of beams under moment gradient are relevant to seismic analysis. Static tests under moment gradient have been reported by Sawyer [77], and by Lukey and Adams [78]. In the tests conducted by Lukey and Adams, simply supported wide flange beams with various ratios of flange width to flange thickness were subjected to a central load and curves of load versus end rotation were obtained. Lateral bracing was provided only at the point of support and the load point. The unsupported length was so adjusted that the length to slenderness ratio was 35. In all cases, it was observed that lateral-torsional buckling did not take place, but that after a substantial inelastic hinge rotation, local buckling of the compression flange occurred, followed by a reduction in the strength of the member. The total angle of rotation up to a point when the strength fell below a value equal to the calculated plastic moment of resistance was measured. The test setup is schematically shown in Fig. 7.9.

In the above experiments, the total inelastic hinge rotation θ_H was given by $\theta - \theta_p$ where $\theta_p = M_p L / EI$. The rotation capacity R was defined as:

$$R = \frac{\theta}{\theta_p} - 1 \quad (7.18)$$

The rotation ductility capacity, μ , defined earlier in this chapter, is related to the rotation capacity R as follows:

$$\mu = 1 + \frac{\frac{1}{2} \theta_H}{\frac{M_p L}{3EI}} \quad (7.19)$$

FIG 7.9 TEST ON INELASTIC ROTATION CAPACITY OF A SIMPLY SUPPORTED BEAM

FIG 7.10 TEST ON INELASTIC ROTATION CAPACITY OF A CANTILEVER BEAM

$$\mu = 1 + 1.5R$$

(7.20)

The test results are shown in Table 7.1 where M_o is the ultimate strength of the joint, b is the flange width and t is the thickness of flange. Certain important conclusions can be drawn from the results. In all cases, the measured ultimate strength was greater than the calculated plastic moment of resistance, indicating that considerably strain hardening took place before the ultimate moment was reached. The current AISC specifications require that for compact sections to be used in plastic design, the ratio $b/t \cdot \sqrt{\sigma_y/36}$, where σ_y is the yield stress, should not exceed 18. The results of the tests described above show that sections which meet this requirement have large rotation capacity. Sections which have larger width-to-thickness ratios for the flange and do not meet the requirement mentioned above have smaller rotation capacities.

Tests on inelastic rotation capacity have also been reported by Popov and Pinkney [47] and by Popov and Bertero [43]. These tests were designed to examine the performance of column-to-beam connections and the connecting beams in the proximity of the connection when subjected to cyclic loading. The model used was a cantilever beam connected to the flanges or the webs of a rigid column stub as shown in Fig. 7.10. The tip of the cantilever was loaded with a concentrated cyclic load and the tip deflection was measured. From the value of measured deflection, the rotation at the plastic hinge at column face could be computed. The normalized hinge rotation capacity was defined as:

$$R = \frac{\theta_H}{\phi_p L}$$

(7.21)

TABLE 7.1
TESTS ON ROTATION CAPACITIES OF BEAMS

Sl. No.	Test	M_o/M_p	$\frac{b}{t} \times \sqrt{\frac{EY}{36}}$	θ_H Radians	R	μ	Remarks
1	A-1	1.37	20.2	0.260	11.8	18.80	Tests No. 1 to 12 by Adams and Lukey, Static load tests, b/t varied
2	A-2	1.40	17.45	0.272	13.6	21.40	
3	B-1	1.12	23.8	0.042	2.9	5.35	
4	B-2	1.16	17.15	0.120	10.4	16.60	
5	B-3	1.14	19.95	0.092	6.7	11.10	
6	B-4	1.06	21.80	0.052	3.4	6.10	
7	B-5	1.04	22.40	0.048	3.2	5.80	
8	C-1	1.12	23.75	0.049	4.2	7.30	
9	C-2	1.26	17.15	0.112	13.7	21.60	
10	C-3	1.16	19.95	0.078	8.0	13.00	
11	C-4	1.12	21.80	0.045	4.2	7.30	
12	C-5	1.14	20.95	0.068	6.5	10.75	
13	F1S	1.42	13.9	.1640	6.96	21.90	
14	F1C1	1.38	13.9	.0650	2.77	9.30	
15	F1C2	1.41	13.9	.0694	2.78	9.33	
16	F1C3	1.35	13.9	.0548	2.17	7.50	
17	F1C4	1.33	13.9	.0457	2.08	7.23	
18	F1C6	1.35	13.9	.0626	2.84	9.50	

TABLE 7.1 (cont'd)

Sl. No.	Test	M_o/M_p	$b' \times \sqrt{\frac{p_y}{36}}$	θ_H radians	R	μ	Remarks
19	W1C1	1.17	13.9	.0218	.87	3.61	Connections Five 7/8 in bolts. All welded Five 3/4 in bolts Four 3/4 in bolts Seven 7/8 in bolts Seven 1 in bolts All welded
20	W1C7	1.21	13.9	.0256	.94	3.82	
21	W1C9	1.21	13.9	.0314	1.14	4.42	
22	W2AC7	1.16	13.9	.0235	.85	3.55	
23	W2BC10	1.22	13.9	.0236	.86	3.58	
24	1	1.33	14.75	.0301	1.85	6.55	
25	2	1.43	14.75	.0478	2.93	9.80	
26	3	1.39	14.75	.0429	2.63	8.90	
27	4	1.41	14.75	.0306	1.88	6.63	
28	5	1.54	13.20	.0538	5.31	16.90	
29	6	1.34	13.20	.0207	2.05	7.15	
30	7	1.60	13.20	.0576	5.70	18.10	

where θ_H is the plastic hinge rotation and

$$\phi_p = \frac{M_p}{EI} \quad (7.22)$$

The rotation ductility capacity, μ , is related to R as follows:

$$\begin{aligned} \mu &= 1 + \frac{H}{M_p L / 3EI} \\ &= 1 + 3R \end{aligned} \quad (7.23)$$

Tests No. 13 to 23 in Table 7.1 were carried out by Popov and Pinkney. In these tests, two different types of connections were used. In Series F, a W8x20 wide flange cantilever beam was connected to the flange of a W8x48 column stub. The connection was designed so that the entire capacity of the member was developed by means of full penetration single bevel groove welds applied to both flanges and webs. Details of the connection are shown in Fig. 7.11(a). In Series W, the W8x20 cantilever beam was connected to the web of the W8x48 column stub as shown in Fig. 7.11(b) and (c). In these connections, stiffener plates, welded to both flanges and web of the column provided for a butt-welded connection to the beam. The stiffener plates were either flush as in Fig. 7.11(b) or tapered or shaped as in Fig. 7.11(c). Quasi-static cyclic loading was applied to the tip of the cantilever, except in the case of test F1S, which was a static load test with monotonically increasing load.

Test Nos. 24 to 29 were carried out by Popov and Bertero. In these tests, a W18x50 or W24x76 cantilever beam was connected to the flange of a W12x106 column stub. Two major types of connections were

(a) SPECIMEN TYPE F1

(b) CONNECTION TYPE WF

FIG. 711 TESTS ON BEAM TO COLUMN CONNECTIONS

(c) CONNECTION TYPE W2A

used - all welded connections and connections with welded flanges and bolted webs.

In the tests reported by Popov and Pinkney and Popov and Bertero, the effect of the variation of flange width to thickness ratio was not studied, but all sections tested met the requirement of AISC specifications on flange width-thickness ratio for plastic design. From a comparison of these results with those reported by Lukey and Adams, it is seen that for comparable width-thickness ratios the rotation capacity of the monotonically loaded specimens is considerably higher than the capacity of the specimens subjected to cyclic loading.

The results of Table 7.1 and the descriptions of failure, reproduced in Table 7.2 from References [43] and [47], show the effect of the type of connection on its ductility. Welded connections to column flange are found to have the highest ductility. With such connections, the failure is seen to take place generally by local buckling of the beam. Bolted connections are less ductile and may, at times, lead to abrupt failures. Least ductile of all are the flange to web connections of the type shown in Figs. 7.11(b) and (c).

The available tests on rotation capacity under cyclic loading are quite limited and the effects of certain variables have not been fully studied. They, however, indicate what features are desirable in joints required to be ductile. Thus, for example, all welded flange to flange connections made with sections that meet the requirements of AISC for compact sections are likely to exhibit good ductility. The values

TABLE 7.2
DESCRIPTION OF THE FAILURE OF BEAMS
SUBJECTED TO CYCLIC LOADING

Specimen	Cycles to Failure	Description of Failure
F1C1	28	Flange buckling; crack at buckle in bottom flange
F1C2	22-1/2	Flange buckling; crack near bottom flange weld
F1C3	120	Flange buckling; crack at top flange weld
F1C4	39-1/2	Flange buckling; crack at stud at bottom flange buckle
F1C6	32	Flange buckling; crack at top flange buckle
W1C1	5	Crack at top flange weld; defective welding
W1C7	37	Crack from end of top flange weld into plate
W1C4	51-1/2	Crack from end of bottom flange weld into plate
W2AC7	46-1/2	Buckling of bottom plate; crack initiated at cutting torch gouge in bottom plate
W2BC10	30	Crack at weld in top plate
1	not reported	Abrupt with noise
2	"	Ductile with buckling
3	"	Fracture through flange
4	"	Abrupt
5	"	Ductile with buckling
6	"	Abrupt through weld
7	"	Ductile with buckling

of rotation ductility capacity, as determined from these tests, are distributed over a wide range. This is to be expected because these values depend upon a number of factors such as the loading history, type of connection and workmanship. If sufficient number of test results are available, a reasonable lower bound for rotation ductility capacity can be selected for the purpose of design.

The foregoing description of the available test data establishes the need for more extensive testing of the ductility capacities of beams and columns. Obviously, the tests should cover all important variables that affect ductility capacities. At the same time, it may well be that member behaviour depends not just on ductility ratio but upon certain other aspects of the inelastic history of the member, such as cumulative hinge rotations, number of cycles of alternating plasticity, etc. If research on the behaviour of beams under inelastic cyclic loading shows that some variable other than the ductility ratio is important, it will be a simple step to program the computers to collect the relevant information as a response parameter.

CHAPTER 8

PROCEDURES FOR THE SEISMIC DESIGN OF MULTISTORY BUILDINGS

8.1 INTRODUCTION

The dynamic behaviour of a structure subject to an earthquake motion is a very complex phenomenon. On the other hand, the design codes which make provision for earthquake loads must, of necessity, be kept as simple as possible. To satisfy these two conflicting considerations, the seismic codes stipulate two different approaches to the design of structures for earthquake loads. The first approach converts the dynamic problem into a problem of static analysis by prescribing a set of "equivalent lateral static loads" to be used in the design of the structure. This approach is applicable to the large number of structures that are rather regular in shape and in the distribution of mass and stiffness, for example, conventional multistory buildings. The second approach employs a dynamic analysis of the structure, using a suitably selected seismic response spectrum or a suitably selected recorded or synthetic ground motion. The dynamic approach is required for irregular or unusual buildings and special structures.

8.1.1 The Equivalent Lateral Static Load Approach to Aseismic Design

In this simplified method, the structure is designed for a set of "equivalent static lateral forces". The magnitudes and distribution of these equivalent forces are related to the dynamic characteristics of the building. Essentially, the present US and Canadian codes (SEAOC, 1973

and NBC, 1970) require that a building be designed for a certain base shear, the value of which has been determined on the basis of past experience and engineering judgement. The equivalent lateral forces producing the base shear are then distributed over the height of the building on the assumption that they arise predominantly from the fundamental mode vibration of the building. The effect of higher modes of vibration is taken into account to some extent by requiring that a certain percentage of the base shear be assigned to the top of the building.

The dynamic elastic response of an elastic building structure to ground motion can be obtained by an appropriate superposition of the responses of the several independent modes of vibration. Although the fundamental mode makes the major contribution, several of the higher modes may also be of consequence. A procedure for estimating the distribution of the shears and overturning moments in a multistory building structure, based on a rational method of superposition of the modal contributions, has been developed by the author, for use with the "equivalent static force" method of seismic analysis and design. New empirical relations have been developed to approximate the shape of the shear and moment envelopes derived by the above procedure. The author believes that this new approach to the aseismic design of fairly regular buildings removes certain anomalies in the current provisions of the codes and makes the equivalent static load method of analysis closer to the results of a dynamic analysis. The suggested method is, at the same time, simple enough to be used in the routine design of regular buildings. Much of the

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earlier part of this chapter is devoted to a description of this procedure.

8.1.2 The Dynamic Analysis Approach to Aseismic Design

The Canadian and the U.S. codes (NBC, 1970 and SEAOC, 1973) recommend the use of dynamic analysis for tall or unusual buildings, buildings with irregular shapes or distribution of stiffness and mass, and for critical industrial structures such as nuclear reactors, power plants, stacks, etc. The building code of the city of Los Angeles [79], in fact, makes it mandatory that dynamic analyses be used for the design of structures over 160 feet in height or of irregular shape or stiffness. There seems to be little doubt that dynamic analysis is the only rational approach to aseismic design of irregular or unusual buildings. At present, the following two different methods of aseismic design through dynamic analysis are available:

- (a) the spectral modal analysis method
- (b) the time dependant response method

In the spectral modal analysis method, a "Structural Design Spectrum" is used to determine the maximum responses of the individual modes of vibration. The modal responses are then superimposed in a suitable manner to obtain the resultant response values; one method of superposition frequently used is to take the resultant response as equal to the square root of the sum of the squares of the modal responses. It is intrinsic in the modal superposition method of analysis that the structure be treated as elastic.

In the time dependant response method, the structural response is calculated for a base motion given by an accelerogram of an actual or a synthetic earthquake, using one of the methods of numerical integration of the equations of motion described in Chapter 2. Although it is possible with this method to account for inelasticity in the structure in a direct manner, an elastic analysis is usually preferred because it is much simpler than the corresponding inelastic analysis.

Always in the first method and also in the second method when an elastic analysis is used, member forces are obtained initially on the assumption that the structure remains elastic. The capacity of the structure for inelastic deformation is then taken into account indirectly by reducing the elastic member forces through the use of suitable force reduction factors.

At present, most of the development work is being done on the spectral modal analysis method of design, and code revisions are under consideration which will recognize the use of this method. It is shown in this study that the time dependant response method of analysis and design is not only a viable alternative to the spectral modal analysis method, it may also be the more preferable method for the framed buildings because it does not involve some of the uncertainties or inaccuracies inherent in the modal superposition method. Further, methods are now available by which spectrum compatible ground motions can be generated, so that it is as simple or as difficult to specify a design ground motion as it is to specify a structural design spectrum.

It is also shown in the study that for multistory rigid frame buildings when the design objective is to ensure that the columns remain elastic while the girders yield under seismic excitation, the use of a uniform force reduction factor for both the girders and the columns to obtain the design forces from the results of an elastic analysis is not justified. Analytical evidence tends to show that the elastic columns will carry additional forces beyond the stage at which the girders start yielding. It is, therefore, more appropriate to use a lower force reduction factor for column moments and column axial loads than for girder moments.

In the latter part of Chapter 8, evidence is presented which supports the considerations just cited. Also presented is a design example using the time dependant response method of analysis and two different force reduction factors, one for the girder forces and the other for the column forces.

8.2 EVOLUTION OF THE PRESENT CANADIAN AND U.S. SEISMIC CODES

A rational approach to the equivalent lateral load method of design really began with the development of the concept of "response spectrum". Using an experimental method, M.A. Biot in the early 1940's determined the acceleration spectra for a number of recorded ground motions [80, 81]. It was observed that these spectra had remarkably similar shapes and had their peaks in the range of periods from 0.20 to 0.25 seconds. A similar spectral curve was obtained by E.C. Robinson for

the El-Centro record [82]. Fig. 8.1 shows the average response spectrum obtained by Biot and also the El-Centro spectrum obtained by Robinson. The response values shown in Fig. 8.1 have been normalized by expressing them as a fraction of the peak response value.

Biot's average spectral curve of Fig. 8.1, with some modifications, has for some time formed the basis for the provisions for earthquake-resistant design in the U.S. and the Canadian codes. In 1952, a Joint Committee appointed by the San Francisco Section of the American Society of Civil Engineers and the Structural Engineers Association of the Northern California recommended a set of code provisions for earthquake-resistant design [83]. Following the result of Biot's work, the committee recommended that the design base shear for a building structure due to seismic load, be obtained as follows:

$$V_b = CW \quad (8.1)$$

$$C = \frac{K}{T} \quad (8.2)$$

where K is a coefficient with a magnitude depending upon the maximum intensity of ground motion for which the structure is to be designed, W is the total effective weight of the structure including the live load likely to be present at the time of an earthquake and T is the fundamental period of the building. The "Standard Spectrum" curve shown in Fig. 8.1 represents the normalized response obtained from Eq. 8.2 and is derived as follows:

FIG. 8.1 ACCELERATION SPECTRA

$$S = \frac{C}{C'} = \frac{0.25}{T} \quad (8.3)$$

where C' is the value of C for a building having a natural period of 0.25 sec.

8.2.1 Distribution of Base Shear

The Joint Committee's recommendation on the distribution of the design seismic shear at the upper levels of a building structure was based on a simplified representation of the fundamental mode of vibration of the building, in which the corresponding mode shape was assumed to be a straight line with zero deflection at the base and the maximum deflection at the top. The base shear was accordingly presumed to be produced by a set of equivalent static lateral forces distributed in the shape of an inverted triangle. For the more general case of varying masses, the Committee recommended the following formula:

$$F_x = V_b \frac{w_x h_x}{\sum w h} \quad (8.4)$$

where F_x is the force assigned to level x

h_x is the height of the building up to level x

w_x is the weight assigned to level x

$\sum w h$ is the sum of the products of the weight at a level and the corresponding height, the summation being taken over the entire height of the building.

Eqs. 8.1, 8.2 and 8.4 form the basis of the lateral force provisions in the present Canadian and U.S. codes for earthquake-resistant design; according to these codes, the base shear is given by:

$$V_b = C K W \quad (8.5)$$

$$C = \frac{0.05}{T^{1/3}} \text{ but } \leq 0.1 \quad (8.6)$$

Eqn. 8.5 is similar to Eq. 8.1 except that a new factor K is introduced with a different meaning from the K factor appearing in Eq. 8.2. Here it provides for the fact that building structures having higher ductility are more earthquake-resistant; the value of K is, therefore, smaller for such structures. In Eq. 8.6, coefficient C is made less sensitive than in Eq. 8.2 to the value of period T, partly to account for the contribution of the higher modes to the base shear and partly because there is an inherent difficulty in making a good estimate of the fundamental period T. A normalized response spectrum can be obtained from Eq. 8.6 as follows:

$$S = \frac{C}{C'} = \frac{0.628}{T^{1/3}} \quad (8.7)$$

where C' is the value of C for a building having a natural period of 0.25 sec. The "Code Spectrum" shown in Fig. 8.1 is a plot of Eq. 8.7.

8.2.2 Overturning Moments

The seismic overturning moment at any level of a building structure could be obtained by assuming that the building behaves like a

vertical cantilever subjected to the set of equivalent lateral forces described earlier. If the structure did respond in the fundamental mode exclusively, the above method of estimating the overturning moments would be correct. But, since the maximum seismic shears throughout the height of the structure represent a superposition of the shears in various modes, it is clear that the cantilever overturning moments consistent with the maximum shears would overestimate the true moments, and that suitable reduction factors should be applied to obtain the moments to be used in design.

Values of the base overturning moment reduction factor, J , were obtained in reference 84 by a logical assignment of the base shears to the different modes. The following empirical expression was proposed to approximate the computed values of J :

$$J = \frac{0.5}{T^{2/3}} \quad (8.8)$$

Eq. 8.8 formed a part of the requirements of the 1960 SEAOC code. Subsequently, a modification of this analysis was made, using a standard 5% of critical damped spectrum and the probabilistic combination of root-mean square response [85]. Based on the results of the new analysis, Eq. 8.8 was revised as follows:

$$J = \frac{0.6}{T^{1/3}} \quad (8.9)$$

Subsequent studies showed that the J factor, while less than unity, would not be as low as that previously specified [62]. Also,

observations of building damage in the Alaska earthquake of 1964 and the Venezuela earthquake of 1967 [86] highlighted the need for being more conservative in estimating the overturning moments. In the 1969 revision of the SEAOC code, therefore, the J factor was eliminated and the buildings were required to be designed for the full overturning moment obtained from lateral forces equivalent to the maximum shear envelope. Elimination of J factor was considered as an interim measure subject to revision with further study and research.

The National Building Code, 1970, still provides a reduction factor for the overturning moments, though its value is more conservative than that in the 1968 version of the SEAOC code. The specified reduction in the overturning is midway between nil ($J = 1.0$) and that given by Eq. 8.8. The corresponding expression for J works out to:

$$J = \frac{1}{2} \left(1 + \frac{0.5}{T^{3/2}} \right) \quad (8.10)$$

8.3 REPRESENTATION OF A REGULAR BUILDING FRAME BY AN IDEALIZED CANTILEVER

A building of uniform mass and stiffness can be idealized as a uniform vertical cantilever, with either flexure type or shear type lateral deformations, such as shown in Fig. 8.2. Its structural form will determine whether such a building will deform predominantly, in flexure or in shear. In the development of a rational approach to the distribution of design shears throughout the height of a building, presented in the following sections of this chapter, the building is

(a) Uniform Cantilever

(b) Mode Shapes

(c) Mode Shapes

FIG. 8.2 FUNDAMENTAL MODE SHAPE OF A UNIFORM CANTILEVER

represented by an idealized cantilever whose mode shapes and modal periods are the arithmetic means of the corresponding values for a shear-deflecting and a moment-deflecting cantilever. This is rather an arbitrary way of defining the properties of a building structure, but one which appears reasonable for the purpose of design. Also, the validity of this assumption is, to some extent, supported by the fact that the relative values of the measured periods of the different modes of buildings have been found to be close to the arithmetic averages of the relative modal periods of the shear-deflecting and the moment-deflecting uniform cantilevers [87]. For brevity of reference, the idealized cantilever model referred to above will be called the "uniform hybrid cantilever".

The mode shapes of the uniform hybrid cantilevers are assumed to be substantially the arithmetic averages of the mode shapes for shear-deflecting and moment-deflecting cantilevers. A triangular shape is assumed to represent the first mode shape since it is a very close approximation to the average for the moment-deflecting and shear-deflecting cantilever as shown in Fig. 8.2(c). Mode shape No. 2 is an exact arithmetic average of the corresponding shapes for the moment-deflecting and shear-deflecting cantilevers. Mode 3 is taken as similar to that for a shear-deflecting cantilever; the third mode shape for a moment-deflecting cantilever is not substantially different.

8.4 THE EFFECT OF BUILDING SLANDERNESS ON THE DISTRIBUTION OF HORIZONTAL SHEAR

As stated earlier, present U.S. and Canadian codes provide for an inverted-triangle type of distribution of the lateral forces producing the base shear. This distribution is based on the assumption that, for uniform structures, the lateral deflections vary linearly with height, being zero at the base and maximum at the top. For slender structures, it has been argued that lateral deflections due to flexure may be predominant, causing the shape of the fundamental mode deflection curve to depart from the straight lines, with increasing deflections at the top as shown in Fig. 8.2(b) [88]. To allow for this effect, the 1960 SEAOC code required an empirical and arbitrary 10% of the base shear to be assigned as a special force to the top of any building with a height-to-width ratio of 5 or more. Later, it was assumed that this provision would also take care of the contribution of the higher modes which are known to be more significant for flexible structures. The 1973 SEAOC code [62] provides a smoother transition in the range of height-to-width ratios from 3.00 (at which no base shear is arbitrarily assigned to the top) to 6.13 (at and above which a maximum of 15% of the base shear is assigned to the top as a special concentration). In this range, the magnitude of the special top force is given by the equation:

$$F_t = 0.004 V_b \left(\frac{H}{D_s} \right)^2 \quad (8.11)$$

where F_t = force to be assigned to the top of the building

D_s = width of the lateral force resisting system of the building

H = total height of the building.

To study the effect of slenderness on the fundamental mode shape and the period of a rigid jointed multistory frame, the two frames shown in Figs. 8.3(a) and 8.3(b) were analyzed by the author. One of these frames is a single bay, ten story frame with a height-to-width ratio of 5 and a fundamental period of 2.19 sec. The second frame is a three bay frame with the same mass and stiffness properties as the single bay frame, except that the internal columns are twice as strong as the columns of the single bay frame. Both frames have a uniform distribution of mass throughout the height of the building.

Figs. 8.4(a) and 8.4(b) show respectively the distributions of inertia forces and the shears, both throughout the building height, for the two frames of Fig. 8.3 vibrating in their fundamental modes. The story forces and shears, obtained on the basis of an assumed triangular distribution of the forces producing the base shear, have also been plotted. It is observed that an inverted triangular distribution is a very good approximation to the fundamental mode forces for this type of building. On the other hand, the height-to-width ratio has very little effect upon the mode shape. In fact, the shear distribution curves for the three bay frame and the single bay frame agree so closely that the difference is hardly perceptible. Curve No. 4, Fig. 8.4(b), shows the shear distribution obtained from the code requirement (SEACO, 1973 and NBC, 1970) that 15% of the base shear be assigned to the top of the

FLOOR MASS = 0.179 LB.-SEC.² IN
FUNDAMENTAL PERIOD = 2.19 SEC.

(a)

FLOOR MASS = 0.537 LB.-SEC.² IN
FUNDAMENTAL PERIOD = 2.12 SEC.

(b)

FIG. 8.3 TEN STORY ONE BAY AND THREE BAY FRAMES

FIG. 8.4 DISTRIBUTION OF LATERAL FORCES IN A BUILDING FRAME

building. For the type of building frames analyzed, it is obvious that the increased lateral deflections in upper stories, resulting from the increased slenderness of the building, do not cause any significant increase in the upper story shears.

• 8.5 DEVELOPMENT OF A RATIONAL METHOD OF ESTIMATING THE DISTRIBUTION OF SEISMIC SHEARS

As noted in Section 8.4, the present code method of accounting for the effect of higher modes of vibration on the distribution of seismic shears is more or less arbitrary. Further, the procedure of relating the contribution of higher modes to the slenderness of the building does not appear to be supported by the results of analysis, especially in the case of framed buildings. A rational method of estimating the distribution of seismic shears, by a logical assignment of the base shear among the various significant modes, and a superposition of the corresponding modal shears throughout the height of the building, has been developed by the author and is presented in this and the following sections. It should be noted that the absolute magnitude of the base shear is not in question; the value of the design base shear as prescribed by the present code is presumed to be correct. The building is modelled by a uniform hybrid cantilever described earlier, and it is assumed that the design ground motion is represented by the "Standard Spectrum" of Fig. 8.1.

Following the method described in Section 6.2, a cantilever of uniform weight can be represented by a series of equivalent single mass systems. Figs. 8.5(a) and 8.5(b) show the equivalent mass systems

(a) MOMENT DEFLECTING CANTILEVER

(b) SHEAR DEFLECTING CANTILEVER

FIG. 8.5 MODAL EFFECTIVE WEIGHTS FOR A UNIFORM CANTILEVER

representing respectively a uniform moment-deflecting cantilever and a uniform shear-deflecting cantilever [83]. The data presented in these figures have been used to obtain the relative modal base shears in the first three modes of a uniform hybrid cantilever with a fundamental period of 1.03 sec. The computations are shown in Table 8.1. Fig. 8.6 shows the corresponding distribution of shears throughout the height of the cantilever. The maximum total shear at any level can now be obtained by an appropriate superimposition of the modal values. In the presentation that follows, the total shears have been obtained by taking the square root of the sum of squares (R.S.S.) of the individual modal contributions.

Table 8.2 shows the computations for obtaining the resultant total shear distributions for uniform buildings with different fundamental periods. The uniform hybrid cantilever model described directly above was used for these buildings. These shear distributions have been plotted in Fig. 8.7 along with the shear distribution obtained from a force diagram represented by an inverted triangle. Also shown is the shear distribution obtained from the code provision requiring 15% base shear to be assigned to the top.

From the plots of Fig. 8.7, and the computations shown in Table 8.2, it is observed that the contribution of higher modes increases with an increase in the fundamental period of the building until the period reaches a value of 2 secs. The result is a relative increase in the upper story shears for buildings with longer periods.

FIG. 8.6. DISTRIBUTION OF RELATIVE SHEARS IN THE FIRST THREE MODES OF A UNIFORM HYBRID CANTILEVER

FIG. 8.7 DISTRIBUTION OF RESULTANT SHEAR IN A UNIFORM HYBRID CANTILEVER

TABLE 8.1
RELATIVE BASE SHEARS FOR UNIFORM CANTILEVER: $T = 1.03$ sec.

	(a) Ratio W_e/W			(b) Ratio T_i/T			(c) Relative Base Shears	
	Moment Defl. Type	Shear Defl. Type	Average	Moment Defl. Type	Shear Defl. Type	Average	Relative Shear = Spectral Response * $x W_e/W$	
Fundamental	0.613	0.810	0.712	1.000	1.000	1.000	$V_1 = 0.247 \times 0.712 = .176$	
Second	0.188	0.090	0.139	0.160	0.333	0.247	$V_2 = 1.000 \times 0.139 = .139$	
Third	0.065	0.032	0.049	0.057	0.200	0.129	$V_3 = 0.67 \times 0.049 = .033$	
All higher modes	0.134	0.068	0.100					

Periods of first three modes are 1.03 sec, 0.252 sec, and 0.132 sec.

Relative shears $V_1:V_2:V_3 = 1.000:0.790:0.188$

* Spectral response obtained from the standard spectrum of Fig. 8.1.

TABLE 8.2
SHEAR COEFFICIENTS C FOR UNIFORM CANTILEVER
BUILDINGS

Period secs.	Height h_x/H	Relative Shears			Root Sum Square Combination of Shears	Normalized Shears V_x/V_b	Normalized Shear Coefficients C_x/C_b
		1st Mode	2nd Mode	3rd Mode			
$T_1 = 1.500$ $T_2 = .123$ $T_3 = .064$	0.0	1.00	.252	.066	1.035	1.000	1.000
	0.1	.99	.233	.047	1.015	.980	1.089
	0.2	.96	.191	.000	.978	.945	1.181
	0.3	.91	.113	-.047	.920	.888	1.270
	0.4	.84	.000	-.066	.840	.810	1.350
	0.5	.75	-.081	-.047	.755	.730	1.460
	0.6	.64	-.148	.000	.657	.635	1.587
	0.7	.51	-.177	.047	.539	.520	1.730
	0.8	.36	-.161	.066	.400	.386	1.930
	0.9	.19	-.100	.047	.220	.212	2.120
$T_1 = 1.03$ $T_2 = .252$ $T_3 = .132$	1.0	.00	.000	.000	.000	.000	-
	0.0	1.00	.78	.187	1.283	1.000	1.000
	0.1	.99	.72	.134	1.230	.960	1.067
	0.2	.96	.59	.000	1.128	.878	1.098
	0.3	.91	.35	-.134	.985	.767	1.096
	0.4	.84	.00	-.187	.860	.670	1.116
	0.5	.75	-.25	-.136	.802	.625	1.250
	0.6	.64	-.46	.000	.790	.615	1.537
	0.7	.51	-.55	.136	.762	.595	1.983
	0.8	.36	-.50	.187	.643	.501	2.505
	0.9	.19	.31	.134	.388	.303	3.030
	1.0	.00	.00	.000	.000	.000	-

TABLE 8.2 (cont'd)

Period secs:	Height h_x/H	Relative Shears			Root Sum Square Combination of Shears	Normalized Shears V_x/V_b	Normalized Shear Coefficients C_x/C_b
		1st Mode	2nd Mode	3rd Mode			
$T_1 = 1.50$	0.0	1.00	.78	.35	1.315	1.000	1.000
$T_2 = 3.71$	0.1	0.99	.72	.25	1.250	.950	1.050
$T_3 = .194$	0.2	.96	.59	.00	1.127	.855	1.070
	0.3	.91	.35	-.25	1.005	.765	1.090
	0.4	.84	.00	-.35	.910	.692	1.150
	0.5	.75	-.25	-.25	.829	.630	1.260
	0.6	.64	-.46	.00	.790	.600	1.500
	0.7	.51	-.55	.25	.790	.600	2.000
	0.8	.36	-.50	.35	.710	.540	2.700
	0.9	.19	-.31	.25	.441	.334	3.340
	1.0	.00	.00	.00	.000	.000	-
$T_1 = 2.00$	0.0	1.00	.78	.534	1.375	1.000	1.000
$T_2 = .494$	0.1	.99	.72	.381	1.283	.933	1.037
$T_3 = .258$	0.2	.96	.59	.000	1.128	.820	1.025
	0.3	.91	.35	-.381	1.045	.760	1.086
	0.4	.84	.00	-.534	.995	.724	1.207
	0.5	.75	-.25	-.381	.877	.637	1.274
	0.6	.64	-.46	.000	.790	.575	1.438
	0.7	.51	-.55	.381	.840	.610	2.033
	0.8	.36	-.50	.534	.815	.592	2.960
	0.9	.19	-.31	.381	.526	.383	3.830
	1.0	.00	.00	.000	.000	.000	-

For a hybrid cantilever model with a fundamental period of 2.0 seconds, the third-mode period works out to 0.258 seconds. On the "Standard Spectrum" of Fig. 8.1, this value of the third mode period lies just to the right of the peak of the spectral response curve. Hence, for a fundamental period greater than 2.0 seconds, the modal spectral response values for the first three modes are uniformly given by the expression:

$$S = \frac{0.25}{T}$$

This implies that since the relative values of the modal periods are unaffected by a change in the value of the fundamental period, the relative shears for the first three modes will also remain unaffected by a change in period for fundamental periods greater than 2.0 seconds.

The considerations cited in this section strongly suggest that the code provisions do not truly represent the distribution of the seismic shear throughout the height of a building structure. Presuming that the magnitude of the design base shear specified by the code is correct, the distribution of shears throughout the height of the building, made in accordance with the code, in general, results in an underestimation of the upper story shears and an overestimation of the lower story shears.

The evidence cited up to this point has been based upon studies made on a uniform hybrid cantilever model, using the "Standard Spectrum" curve shown in Fig. 8.1 and a root-mean square summation of the contribution of individual vibration modes. To further examine the seismic shear distribution in uniform multistory buildings, the author subjected three

different realistic building frames, as contrasted to hybrid cantilevers models, to the El-Centro ground motion. The three buildings studied were respectively five stories, ten stories and fifteen stories in height. They were designed in accordance with the NBC, 1970 with earthquake forces corresponding to Seismic Zone 3. Both elastic and elasto-plastic behaviour was investigated. The resulting shear distributions, plotted in Fig. 8.8, confirm that the triangular force distribution results in an underestimation of the shears in the upper stories.

8.6 MAGNITUDES OF THE SEISMIC SHEAR COEFFICIENTS

The seismic shear coefficient at any level of a building frame is defined as the ratio of the seismic shear at that level to the total weight of the frame above that level (see Section 3). The shear coefficients C_x for four uniform hybrid cantilever model type buildings with different fundamental periods, subjected to an earthquake ground motion whose acceleration spectrum is represented by the "Standard Spectrum" of Fig. 8.1, are shown in Tables 8.2 where h_x is the height to the level where the shear coefficient C_x applies, and H is the total height of the building. In Fig. 8.9 are plotted the values of C_x/C_b , obtained from Table 8.2, against the height of the building where C_b is the base shear coefficient. The plots show that the shear coefficient at about 0.6 of the total height from the base is almost the same for all frames and is equal to the value obtained from triangular distribution of forces. Above this level, the shear coefficient increases approximately linearly with height. For a particular value of the fundamental period of the

FIG. 8-8 (a) DISTRIBUTION OF ELASTIC SHEARS IN FRAMES SUBJECTED TO EL-CENTRO GROUND MOTION

FIG. 8.8 (b) DISTRIBUTION OF INELASTIC SHEARS IN FRAMES SUBJECTED TO EL-CENTRO GROUND MOTION

FIG. 8-9 SHEAR COEFFICIENTS FOR A UNIFORM HYBRID CANTILEVER

building, the variation of the shear coefficient throughout the height of the building can be represented by two empirical curves, one applicable to the case when h_x is less than $0.6H$ and the other applicable when h_x is greater than $0.6H$. This latter curve is very nearly a straight line. For buildings with short periods, this straight line coincides with the inverted triangular distribution of forces assumed in the seismic codes and given by Eq. 8.4. As the period of the building increases, the contribution of the higher modes increases causing the shear coefficients in the upper portions of the buildings to increase. As a consequence, the slope of the straight line approximating the value of shear coefficients decreases, i.e. the shears increase more rapidly with height than provided for by Eq. 8.4. When the fundamental period has reached a value of about 2 seconds, there is no further change in the shape of the graph of shear coefficients. Between the two limits of fundamental period, i.e. for periods between approximately 0.26 secs. and 2.00 secs., the slope s of the straight line approximating the values of shear coefficients varies with the fundamental period. The relationship between s and T , shown in Fig. 8.10, is given by the empirical equation:

$$s = \frac{0.26}{T} \quad (8.12)$$

Using this relationship, the shear coefficients throughout the height of the building can be approximated by the following empirical equations:

FIG. 8.10 SLOPE OF THE STRAIGHT LINE DEFINING THE SHEAR COEFFICIENT
CURVE FOR A UNIFORM HYBRID CANTILEVER

For $h_x \leq 0.6 H$,

$$\frac{C_x}{C_b} = 1 + \frac{0.156 \frac{h_x}{H}}{0.6T - (T-0.26) \frac{h_x}{H}} \quad (8.13)$$

For $h_x \geq 0.6H$,

$$\frac{C_x}{C_b} = 1.6 + 3.85 T \left(\frac{h_x}{H} - 0.6 \right) \quad (8.13)$$

where, in the above equations, the minimum and maximum values of T to be used are 0.26 and 2.0 secs., respectively.

Fig. 8.11 compares the values of shear coefficients given by Eqs. 8.13 with the exact values computed in Table 8.2.

Since Eqs. 8.13 are in dimensionless form, they can be used to plot convenient design graphs of C_x/C_b against h_x/H for various values of T .

8.7 DISTRIBUTION OF OVERTURNING MOMENT

As noted in Section 8.2, the base overturning moment reduction factors stipulated in the 1960 SEAOC code were based on the approach presented in reference 84. Table 8.3 has been reproduced from reference 84. It shows the computations for J factors for a shear deflecting cantilever; similar computations were carried out for a moment deflecting cantilever, and the average of the two sets of values were used for the code.

FIG. 8.11 SHEAR COEFFICIENTS FOR A UNIFORM HYBRID CANTILEVER

TABLE 8.3
MOMENT RESPONSE FOR SHEAR-DEFLECTING STRUCTURE

Row No.	Item	Description	Fundamental Period T_1 (sec)					
			0.25	0.75	1.00	1.25	2.00	3.00
1	S_1	Response	1.0000	0.3300	0.2500	0.2000	0.1250	0.0830
2	W_{e1}	Equiv. Mass	0.8690	0.8690	0.8690	0.8690	0.8690	0.8690
3	V_1'	$S_1 W_{e1}$	0.8690	0.2900	0.2170	0.1740	0.1085	0.0727
4	T_2	$0.333 T_1$	0.0830	0.2500	0.3330	0.4170	0.6670	1.0000
5	S_2	Response	0.5320	1.0000	0.7570	0.6000	0.3750	0.2500
6	W_{e2}	Equiv. Mass	0.0960	0.0960	0.0960	0.0960	0.0960	0.0960
7	V_2'	$S_2 W_{e2}$	0.0510	0.0960	0.0728	0.0576	0.0360	0.0240
8	T_3	$0.200 T_1$	0.0500	0.1500	0.2000	0.2500	0.4000	0.6000
9	S_3	Response	0.4400	0.7200	0.8600	1.0000	0.6250	0.4160
10	W_{e3}	Equiv. Mass	0.0350	0.0350	0.0350	0.0350	0.0350	0.0350
11	V_3'	$S_3 W_{e3}$	0.0154	0.0253	0.0301	0.0350	0.0219	0.0146
12	V'	$V_1' + V_2' + V_3'$	0.9354	0.4112	0.3199	0.2666	0.1664	0.1107
13	V_1	$V_1'/0.9354$	0.9290	0.3100	0.2320	0.1860	0.1160	0.0770
14	V_2	$V_2'/0.9354$	0.0550	0.1020	0.0775	0.0620	0.0386	0.0256
15	V_3	$V_3'/0.9354$	0.0160	0.0268	0.0321	0.0374	0.0232	0.0155
16	V	$V_1 + V_2 + V_3$	1.0000	0.4388	0.3416	0.2854	0.1778	0.1181
17	M_1/H	$0.636 V_1$	0.5910	0.1970	0.1480	0.1180	0.0740	0.0490
18	M_2/H	$0.212 V_2$	0.0117	0.0216	0.0164	0.0131	0.0082	0.0054
19	M_3/H	$0.127 V_3$	0.0020	0.0034	0.0041	0.0047	0.0029	0.0020
20	M/H	$(M_1 + M_2 + M_3)/H$	0.6047	0.2220	0.1685	0.1358	0.0851	0.0564
21	M'/H	$0.667 V$	0.6670	0.2920	0.2275	0.1900	0.1185	0.0787

TABLE 8.3 (cont'd)

Row No.	Item	Description	Fundamental Period T_1 (sec)					
			0.25	0.75	1.00	1.25	2.00	3.00
22	J'	M/M'	0.9050	0.7250	0.7400	0.7170	0.7170	0.7170
23	V_d	Design Shear $0.628(T_1)^{1/3}$	1.0000	0.6920	0.6280	0.5840	0.4980	0.4360
24	k	V/V_d	1.0000	0.6350	0.5430	0.4870	0.3570	0.2710
25	J	$J' \times k$	0.9050	0.4600	0.4010	0.3480	0.2570	0.1940

The normalized values, S , entered in the table are taken from the "Standard Spectrum" of Fig. 8.1. The modal overturning moments in Rows 17, 18 and 19 are obtained by taking the product of the modal shears and the corresponding modal moment arms for a shear deflecting structure as shown in Fig. 8.5(b). In Row 22 of this table, values of J' have been worked out for various periods of the cantilever, by taking the ratio of the moment M , obtained by superposition of modes (Row 20) and the moment M' , obtained from a triangular distribution. The spectral shears shown in Row 16 are smaller than the code prescribed shears entered in Row 23 since the "Code Spectrum" lies above the "Standard Spectrum" in Fig. 8.1. The ratios of spectral shears to code prescribed shears are shown in Row 24. The value of J specified in 1960 SEAOC code were obtained by multiplying Row 22 by Row 24. The moment reduction factor J of that code was thus a product of two factors which took into account the following considerations: (1) the story shears given by the maximum shear envelope do not occur concurrently; (2) the base shears obtained from the "Standard Spectrum" are smaller than those obtained from the "Code Spectrum". Such a code provision implies that, while the building frame is designed for shears in accordance with the "Code Spectrum" curve of Fig. 8.1, the overturning moments correspond to the "Standard Spectrum" curve shown therein. This provision is obviously inconsistent, and in part; accounts for the rather low values of J stipulated in the earlier version of the SEAOC code.

Following an approach rather similar to the one used in Sections 8.5 and 8.6 in developing the expressions for distribution of shears,

values of the moment reduction factors have been obtained by the author for use with the cantilever moments obtained from the equivalent lateral forces representing the shear envelope of Eqs. 8.13. A description of the procedure used follows.

Fig. 8.12 shows a plot of the relative overturning moments in the first three modes of a uniform hybrid cantilever with a fundamental period of 1.03 secs. The resultant overturning moment at any level can be obtained by a probabilistic combination of the modal contribution. A reasonable approximation to the true maximum is thus obtained by taking the square root of the sum of squares of modal contributions.

Table 8.4 shows the author's computations for obtaining the resultant distribution of moment for uniform buildings with different fundamental periods. Moments consistent with the shear envelopes of Fig. 8.7 have also been computed by an integration of the shear graph of that figure. The ratio of these two different moments is shown in the last column of Table 8.4 as $\bar{J}_x$ where the bar is used to distinguish the author's coefficient from the code reduction coefficient J_x .

In Fig. 8.13, the author's base overturning moment reduction factor $\bar{J}$ is plotted against the period. An approximate value of $\bar{J}$ can be obtained from the following empirical equation:

$$\bar{J} = .675 + \frac{.15}{T} \quad (8.14)$$

where in the above equation the minimum and maximum values of T to be used are 0.46 sec and 2.0 sec, corresponding respectively to a maximum value of $\bar{J} = 1.00$ and a minimum value of $\bar{J} = 0.75$.

FIG. 8-12 DISTRIBUTION OF RELATIVE MOMENTS IN THE FIRST THREE MODES OF A UNIFORM HYBRID CANTILEVER

TABLE 84

MOMENT REDUCTION FACTOR $\bar{J}_x$ FOR A UNIFORM CANTILEVER

h_x/H	R.S.S. Shear (Table 82)	Moment				Normal. Shear V_x/V_b	Normal. Moment for $V_b=1.0$	Moment Consist with Shear	Resulting $\bar{J}_x$
		Mode 1	Mode 2	Mode 3	RSS				
Fundamental Period = 0.5 sec.									
0	1.035	.667	-.0003	.0086	.667	1.000	.646	.662	.975
0.1	1.015	.567	-.0252	.0025	.567	.980	.549	.563	.976
0.2	0.978	.470	-.0460	.0000	.472	.945	.457	.467	.978
0.3	0.920	.376	-.0614	.0025	.381	.888	.369	.375	.985
0.4	0.840	.288	-.0675	.0086	.296	.810	.286	.289	.989
0.5	0.755	.209	-.0646	.0146	.219	.730	.212	.213	.997
0.6	0.657	.139	-.0530	.0171	.150	.635	.144	.144	1.000
0.7	0.539	.078	-.0362	.0146	.087	.520	.086	.086	1.000
0.8	0.400	.038	-.0191	.0086	.043	.386	.041	.041	1.000
0.9	0.220	.010	-.0055	.0025	.012	.212	.011	.011	1.000
1.0	0.000	.000	.0000	.0000	.000	.000	.000	.000	-
Fundamental Period = 1.03 sec.									
0	1.283	.667	-.001	.0242	.667	1.000	.520	.644	.807
0.1	1.230	.567	-.078	.0071	.572	.960	.446	.545	.818
0.2	1.128	.470	-.144	.0000	.491	.878	.383	.453	.845
0.3	0.985	.376	-.190	.0071	.421	.767	.328	.371	.884
0.4	0.860	.288	-.209	.0242	.357	.670	.278	.299	.928
0.5	0.802	.209	-.200	.0413	.292	.625	.228	.235	.970
0.6	0.790	.139	-.164	.0485	.220	.615	.172	.173	.992
0.7	0.762	.078	-.122	.0413	.150	.595	.112	.112	1.000
0.8	0.643	.038	-.059	.0242	.074	.501	.057	.057	1.000
0.9	0.388	.010	-.017	.0071	.021	.303	.015	.015	1.000
1.0	0.000	.000	.000	.0000	.000	.000	.000	.000	-
Fundamental Period = 1.50 sec.									
0	1.315	.667	-.001	.0452	.668	1.000	.508	.649	.783
0.1	1.250	.567	-.078	.0132	.572	.950	.435	.551	.790
0.2	1.127	.470	-.144	.0000	.491	.855	.373	.461	.811
0.3	1.005	.376	-.190	.0132	.421	.765	.320	.380	.843
0.4	.910	.288	-.209	.0452	.359	.692	.273	.307	.890
0.5	.828	.209	-.200	.0771	.299	.630	.227	.240	.946
0.6	.790	.139	-.164	.0905	.233	.600	.177	.181	.982
0.7	.790	.078	-.112	.0771	.157	.600	.119	.120	.997
0.8	.710	.038	-.059	.0452	.083	.540	.062	.062	1.000
0.9	.441	.010	-.017	.0132	.024	.334	.018	.018	1.000
1.0	.000	.000	.000	.0000	.000	.000	.000	.000	-

TABLE 84(cont'd)

Fundamental Period = 2.00 sec.									
h_x/h	R.S.S. Shear (Table 82)	Moment				Normal Shear V_x/V_b	Normal Moment for $V_b=1.0$	Moment from Shear Graph	Resulting J_x
		Mode 1	Mode 2	Mode 3	RSS				
0	1.375	.667	-.001	.0689	.670	1.000	.487	.655	.743
0.1	1.283	.567	-.078	.0202	.573	.933	.417	.558	.746
0.2	1.128	.470	-.144	.0000	.491	.820	.357	.471	.758
0.3	1.045	.376	-.190	.0202	.422	.760	.307	.392	.782
0.4	.995	.288	-.209	.0689	.362	.724	.263	.318	.827
0.5	.877	.209	-.200	.1175	.312	.637	.227	.250	.908
0.6	.790	.139	-.164	.1380	.255	.575	.185	.190	.975
0.7	.840	.078	-.112	.1175	.180	.610	.131	.131	.998
0.8	.815	.038	-.059	.0689	.098	.592	.069	.069	1.000
0.9	.526	.010	-.017	.0202	.028	.383	.020	.020	1.000
1.0	.000	.000	.000	.0000	.000	.000	.000	.000	-

FIG. 813 BASE-MOMENT REDUCTION FACTOR

The plots of Fig. 8.14 show the values of the overturning moment reduction factors at different heights of the building for different fundamental periods. The value $\bar{J}_x$ at a height h_x from the base can be approximated by the following equation:

$$\bar{J}_x = \bar{J} + 2.78 (1 - \bar{J}) \left(\frac{h_x}{H} \right)^2 \quad (8.15)$$

$$\bar{J}_x = 1 \text{ for } \frac{h_x}{H} \geq 0.6$$

$\bar{J}_x$ as obtained by the formula is shown in dashed lines in the figure.

8.8 REDUCTION OF THE BUILDING STRUCTURE TO A SINGLE EQUIVALENT FRAME

The reduction of a building structure to a single equivalent frame or a frame-wall system has been tried in the past for the static lateral load analysis of frame and shear wall structures. However, the possibility of using an equivalent frame for the dynamic seismic analysis of a building has not been fully explored. A limited attempt has been made in the present study to investigate this possibility for framed structures; the results of the investigation are presented in this section.

The reduction of a building structure to a single frame is based upon the assumption that the beams and columns are axially rigid and that the floors are infinitely stiff in their own planes. In addition, certain assumptions must be made relating to the beam-to-column joints. The significance of these assumptions and their effect on the method of analysis are illustrated here with the aid of an analysis of the three story steel framed building shown in Fig. 8.15. The building is required

FIG. 8.14 DISTRIBUTION OF MOMENT REDUCTION FACTOR
THROUGHOUT THE HEIGHT OF A UNIFORM CANTILEVER

FIG 8 15 a STRUCTURAL LAYOUT OF THREE STORY BUILDING

FRAME TYPE I

FRAME TYPE II

FRAME TYPE III

FIG 8-15 b FRAME DETAILS AND STIFFNESS MATRICES FOR THE THREE STORY BUILDING

to be analyzed for vibrations in the north-south direction. The floor plan and the structural framing detail are shown in the figure. The building consists of three different types of frames as shown. In an accurate analysis, the structural stiffness matrix is worked out for each frame and condensed to a 3×3 matrix corresponding to the three side-way degrees of freedom (see Section 3.8). The condensed stiffness matrices so obtained are added together to get the stiffness matrix for the complete building. Once that is done, the time history of the response of the building to a given excitation can be computed.

In the above method, the frames of which the structure is composed, do not lose their individuality. The method may involve much computation time for a large building with many different types of frames. However, on the basis of certain assumptions relating to the rotations of the beam column joints at any particular story, the building structure can be represented by a single frame which will be termed the "Equivalent Frame", or even by one single bay frame which will be termed the "Condensed Single Bay Frame".

(a) The reduction to an equivalent frame is based on the assumption that at any particular floor level, the rotations of all the beam-to-column joints lying on the same column line are equal. For the building of Fig. 8.15, this implies that at any floor level the rotations of all joints along a particular column line, No. 1, 2, 3 or 4 are equal. The equivalent frame derived on this assumption is shown in Fig. 8.16(a).

The column stiffnesses for the equivalent frame are derived by summing up

$$\begin{bmatrix} 3384.6 & -1792.5 & 324.7 \\ -1792.5 & 2445.1 & -1100.1 \\ 324.7 & -1100.1 & 828.9 \end{bmatrix} \text{ kip/in}$$

(a) EQUIVALENT FRAME

$$\begin{bmatrix} 3385.1 & -1792.8 & 324.7 \\ -1792.8 & 2446.9 & -1101.4 \\ 324.7 & -1101.4 & 830.2 \end{bmatrix} \text{ kip/in}$$

(b) CONDENSED SINGLE BAY FRAME

$$\begin{bmatrix} 3359.1 & -1783.6 & 328.8 \\ -1783.6 & 2350.4 & -1029.0 \\ 328.8 & -1029.0 & 758.8 \end{bmatrix} \text{ kip/in}$$

(c) EXACT VALUE OF BUILDING STIFFNESS MATRIX

FIG 816 REDUCED FRAMES AND STIFFNESS MATRICES FOR THE THREE

the stiffnesses of all columns lying on the same column line. Similarly, the stiffnesses for the beams of the equivalent frame are derived by summing the stiffnesses of all the beams lying in a bay formed by two adjacent column lines. Since the story heights and bay sizes in the equivalent frame are the same as those in the main building structure, summing up of the stiffnesses leads to the same results as the summing up of the moments of inertias of the member cross-sections.

The equivalent frame can be analyzed for an earthquake-induced ground motion by means of an appropriate method of dynamic analysis. The computed shears and moments in the members of the equivalent frame are then distributed among the members of the constituent frames in proportion to the stiffnesses of these members.

(b) The equivalent frame can be further reduced to a condensed single bay frame on the basis of the assumption that at any particular floor level, the rotations of all the beam-to-column joints are equal. The condensed single bay frame derived on the basis of this assumption is shown in Fig. 8.16(b). In every story, the stiffness of each column of the condensed frame is equal to half the sum of the stiffnesses of all the columns in the corresponding story of the equivalent frame. Since the story heights in the condensed frame and the equivalent frame are equal, the summing up of the stiffnesses is equivalent to the summing up of the moments of inertia. The beam stiffnesses in the condensed frame are simply the sum of the stiffnesses of all the beams at the corresponding floors of the equivalent frame. Any reasonable value can be assigned

to the size of the single bay in the condensed frame; when the equivalent frame consists of a number of equal bays, the bay widths in the equivalent frame and the condensed frame can be made equal.

To explore the effect of these assumptions on the accuracy of a dynamic analysis of the building, the mode shapes and frequencies of the three story building were calculated. The results are shown in Fig. 8.17. The mode shapes obtained from the equivalent frame and the condensed frame are so close to the exact mode shapes that the difference is hardly perceptible. The error in the approximate frequencies is no more than 3.5%. The results of similar calculations on a 19 story building have been presented by Hurty and Rubinstein in reference 52. The approximate mode shapes are found to be very close to the exact; the maximum error in fundamental frequency is less than 6%.

As a further check on the errors introduced due to the above assumptions in an elastic response analysis of the building, the equivalent frame and the condensed frame were analyzed for the first 20 secs of the El-Centro ground motion, and the results were compared with an exact analysis of the building response. The maximum values of story displacements, interstory drifts and story shears obtained for three different values of damping are shown in Table 8.5. It is observed that the errors in story shears are, in general, of the order of 5-6% while the errors in the maximum story displacements and maximum interstory drifts are of the order of 10%. In one case for the undamped response, the error in the story shear is as large as 25%. This is presumably because of the

STORY

Exact Shape

Based on the Equivalent Frame

FREQUENCIES RADS / SEC

Exact 8.4047

Assumption a 8.6566

Assumption b 8.6608

26.9412

27.8800

27.8963

49.9079

50.3169

50.3249

(a) FIRST MODE

(b) SECOND MODE

(c) THIRD MODE

FIG. 8-17 MODE SHAPES OF THE THREE STORY BUILDING

TABLE 8.5.
ELASTIC SEISMIC RESPONSE OF THREE STORY BUILDING, EL-CENTRO 1940 N-S

Story	Max. Displacement, in.			Max. Error %	Max. Relative Story Displacement, in.			Max. Error %	Max. Shear, kips			Max. Error %
	Exact	From Equiv. Frame	From Cond. Frame		Exact	From Equiv. Frame	From Cond. Frame		Exact	From Equiv. Frame	From Cond. Frame	
1	2.456	2.410	2.419	1.88	2.458	2.410	2.419	1.88	2646.5	2586.9	2605.7	2.23
2	5.229	5.766	5.758	10.25	2.923	3.511	3.508	20.1	1743.2	2190.1	2188.5	25.6
3	7.016	8.502	8.497	21.15	2.944	2.838	2.848	3.6	1596.9	1566.8	1575.1	1.88
1	Damping - 3% of the			critical	value for 1st Mode			7.15	1468.9	1415.6	1401.1	4.02
2	1.441	1.349	1.338	7.15	1.441	1.349	1.338	9.15	1150.4	1139.1	1134.7	1.36
3	3.397	3.138	3.114	8.32	1.991	1.818	1.809	12.7	800.2	765.7	769.1	4.31
1	4.881	4.488	4.474	8.35	1.647	1.438	1.438					
2	Damping - 5% of the			critical	value for 1st Mode			1.17	1189.4	1255.1	1259.3	5.87
3	1.199	1.182	1.185	1.17	1.199	1.182	1.185	9.55	973.8	941.2	944.2	3.35
1	2.870	2.685	2.692	6.45	1.675	1.515	1.518	14.9	656.6	625.6	626.3	4.90
2	4.094	3.630	3.611	11.80	1.361	1.162	1.158					

fact that the undamped response spectrum for the El-Centro ground motion exhibits sharp peaks and a slight shift in the frequency may therefore cause a considerable change in the response. However, with a reasonable amount of damping, the sharp peaks in the spectrum curve are smoothed out to some extent and violent variations in the response due to a slight change in the frequency are not so likely.

Conclusions

The simplified method of analysis presented above can lead to significant economies in the time and costs of the computations. It will give good results provided the assumptions regarding the joint rotations are reasonably applicable to the structure being analyzed. It is, therefore, of importance to know the conditions under which the validity of the assumptions can be presumed.

A multibay frame is said to be "proportioned" when it can be divided into a series of one-bay symmetrical frames, with the bending stiffness at all corresponding points bearing a constant ratio to each other [89]. When such a division is possible, all the joints of the multibay frame at any one floor level have the same rotation under lateral load. To analyze a proportioned frame, it is only necessary to consider the equivalent single bay frame. In real frames, a large departure from the proportioned ideal is possible. When such is the case, the condensed single bay frame should not be used.

In a similar manner, the building structure can be reduced to a single equivalent frame provided the lateral stiffnesses of the constituent

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frames are reasonably equal. When there are wide variations in individual frame stiffnesses, the single equivalent frame may not provide a good estimate of the response of the building.

It should be expected that the simplified method presented above will, in general, give good results for framed buildings that are fairly regular in shape and in the distribution of stiffnesses. A reasonable amount of non-uniformity will not, however, invalidate the results of the simplified analysis. Thus, for the example building considered in this section, the lateral stiffnesses of the constituent frames are not quite equal, nor is the single equivalent frame itself proportioned, yet the condensed single bay frame gives fairly good estimate of the elastic response.

8.9 DESIGN BASED ON THE MODE SUPERPOSITION METHOD OF DYNAMIC ANALYSIS

8.9.1 Selection of the Structural Design Spectrum

A large number of strong ground motion records have been obtained from earthquakes in many countries of the world and response spectra have been calculated for many of these ground motions. Earthquake-induced ground motion is essentially a random process, and like all random processes, any single earthquake motion is an accident, never likely to occur again. But this indeterminacy of a single record is coupled with strong statistical features of a large collection of records. Similarly, a collection of response spectra exhibits certain statistical features. A structural design spectrum should therefore be some form of

statistical mean of a large collection of response spectra for recorded earthquake motions. Following this principle, recommended design spectra for various values of damping have been derived in reference 90, for the seismic conditions of the west coast of the United States.

8.9.2 Combining Modal Responses to Obtain Probable Response of the Structure

The maximum response in each of the significant modes of the building, computed from a structural response spectrum for the selected value of damping, must be superimposed in some manner to obtain the total response. A direct summation of the absolute values of the maximum modal responses will give a rather conservative estimate of the total response. It has been suggested that a square root of the sum of the squares of the maximum modal responses should provide a good estimate of the total response. These approximate methods of combining the responses of individual modes introduces a substantial uncertainty into the results obtained by a modal superposition method. Comparisons of the modal and the exact solutions for the seismic response of multi-degree of freedom systems have been reported in reference 64. It is observed that the errors involved in the approximate superposition of modes may be large as 40%, but in general, are not much larger than 20%.

8.9.3 Computation of Member Forces

The method usually employed to obtain the design seismic forces in the members of a building structure, from the maximum story shears

computed using a modal analysis, is to resolve the shears into a set of equivalent lateral forces and to carry out a static analysis of the structure for this set of forces. The member forces obtained by this method are not accurate; girder moments are usually overestimated, the column moments may be larger or smaller than their true value, and the column axial loads are substantially overestimated.

Girder end moments calculated from the maximum shear envelope are very likely to be conservative because these moments depend mainly upon the shears in the two stories adjacent to the girder, and the shears in two adjacent stories may not obtain their maximum values at the same instance of time. On the other hand, the sum of the maximum column end moments in any story is entirely dependant upon the maximum value of shear in that story. The difference between the exact and the estimated maximum values of column end moments therefore results from a difference in the way in which the sum of the end moments is distributed between the tops and bottoms of the columns. Obviously, the difference between the exact and the estimated values may be positive or negative. The column axial loads may be substantially overestimated because the cantilever overturning moments obtained from the maximum shear envelope are often substantially larger than the true maximum overturning moments (see Section 8.2). Some form of correction factors, which can only be approximate, must be used with the estimated column axial loads to obtain their true design values.

8.10 EFFECT OF INELASTICITY ON THE DESIGN FORCES

In general, a structure designed for seismic forces in accordance with the codes is expected to undergo substantial inelastic deformation during a major earthquake. This inelastic deformation represents the principal difference between the elastic response spectrum analysis and the "equivalent static load" analysis. The recognition of the capacity of structures possessing adequate strength and ductility to resist major earthquakes by responding inelastically to them lies behind the relatively low forces specified by the codes for structures with substantial ductility. Following a similar reasoning, the shears and forces obtained from an elastic analysis for a ground motion representing an expected major earthquake are suitably scaled down to obtain their design value. The reduction factors used are often termed the ductility factors. However, the terms ductility factor and ductility ratio are also used to measure the inelastic deformations of structures and members as explained in Sections 7.2 and 7.4. The force reduction factor mentioned above is equal to the ductility ratio only in certain simple situations and should be differentiated from the latter. In the present study, it is called the "plasticity force reduction factor" and is defined as the ratio of the maximum force in a member obtained from an elastic analysis of the structure for a particular ground motion to that obtained from an inelastic analysis for the same ground motion. The safe values of plasticity force reduction factors that can be used in a certain design situation are, of course, dependant upon the capacity of the structural system for

inelastic deformation; at present, they must be based mainly on engineering judgement.

8.10.1 The Use of the Plasticity Force Reduction Factor in Design

In the method of dynamic analysis proposed for inclusion in the new editions of some codes (SEAOC and NBC), the design seismic force in the members of an inelastic structure are derived from the related elastic analysis by applying a uniform force reduction factor to the elastic seismic forces. This approach tacitly assumes that either all the members start yielding at the same level of lateral forces or that once yielding begins in any location of the structure it inhibits further increase in the forces in any other part of the structure. In real multistory rigid jointed frames, most of the inelastic deformation is restricted to the girders. In fact, designers consciously try to force the inelasticity into the girders because of the more hazardous consequences of a possible column failure. As a result, even when the girders start yielding, the columns are usually still elastic and can carry additional forces.

As an example, the results of a seismic analysis of the standard ten story frame described in Section 4.2 may be considered. The frame was designed for the equivalent lateral forces prescribed in the NBC, 1970 for seismic zone no. 3. The results of an elastic and an elastoplastic analysis of the frame for the first 20 seconds of the El-Centro earthquake are shown in Fig. 8.18. The results indicate that most of the columns remain elastic, and the ductility requirements for the girders

FIG 8-18 TEN STORY FRAME RESPONSE TO EL CENTRO

are within acceptable limits. The ratios of the maximum girder end moments, column end moments and column axial loads due to seismic forces in the elastic frame to their corresponding values in the elasto-plastic frame are shown in Fig. 8.19. It will be observed that the elastic to inelastic force ratios for the girder moments are considerably larger than those for the column end moments, column axial loads and the story shears. An opposite trend is noticed only in the topmost story, presumably because the design of members in that story is ruled by gravity loads and not by earthquake forces.

The concentration of inelastic deformation in the girders as evident in the example presented above seems to be the general trend in regular multistory rigid jointed frames designed according to the present codes. In many other structures, particularly in structures that are irregular in shape and in the distribution of stiffnesses, this may not be so unless conscious effort is made in the design to confine the inelasticity to the girders. It is evident that, if the columns are to remain elastic, while the girders yield, the plasticity force reduction factors to be applied to the column and girder forces cannot be the same. This suggests a new approach to the design of framed building structures. Essentially, this approach consists of carrying out an elastic analysis of the structure by any one of the two methods of dynamic analysis, and then scaling down the member forces so obtained by applying appropriate plasticity force reduction factors to obtain the seismic design forces. A uniform reduction factor say of the order of 4 may be applied to the beam moments. Another uniform reduction factor, say about 2 to 2.5, may

FIG. 8-19 MEMBER-FORCE REDUCTIONS DUE TO INELASTICITY IN THE TEN STORY FRAME

be applied to the column end moments and column axial loads. Until more experimental and theoretical evidence is available on the capacity of frame members for inelastic deformation, the values of plasticity force reduction factors will have to be based on judgement. A design method and example using this approach are described in the following sections.

8.11 DESIGN BASED ON THE TIME-DEPENDANT RESPONSE METHOD OF DYNAMIC ANALYSIS

The first step in any method of dynamic analysis of a building structure is the derivation of a condensed structural stiffness matrix, either by an exact method or by reducing the structure to a single equivalent frame. From this point onwards, the mode superposition and the time-dependant response methods follow two different routes. The approximations involved in the mode superposition method have been described in Section 8.9. On the other hand, the time-dependant response method is free of these approximations. The extra accuracy of the time dependant response method is, of course, achieved at the cost of some additional computation time. However, since a digital computer must be used both for the mode superposition and the time-dependant response method, a small amount of extra computer time required for the latter will not mean any significant disadvantage. As a method of dynamic analysis, therefore, the time-dependant response method offers certain advantages and would be the more appropriate to use provided a suitable design ground motion can be selected. The selection of the design ground

motion and the details of the method of design to be used with a time-dependant response analysis are discussed in this section. In the last section of the chapter, an example of the design of a set-back building using the time-dependant response method is described.

8.11.1 Selection of the Design Ground Motion

As stated earlier, earthquake-induced ground motion is a random process. Each record is different from the other, although a collection of records may show a strong statistical character. A particular record may contain a larger energy input in a certain frequency band than the average over the group, while it may have less energy input in other frequency bands. Because of this possible bias, different records will excite a particular structure in different manners; a single record may not be satisfactory as an input ground motion to a time-dependant response analysis carried out for the purpose of establishing design forces. One way of getting over this difficulty is to subject the structure to a number of selected ground motions, possibly two or three or more, to obtain the maximum design forces. This method is not entirely satisfactory and it also involves a large amount of computation time. It would be more desirable to have a single ground motion which is some form of average of the available past records and shares their statistical characteristics. Specifically, a ground motion that will produce the design spectra described in Section 8.9 should be quite satisfactory. Two different methods are available for the generation of spectrum compatible ground motions. In one method [91], an existing ground motion

record having a response spectrum roughly similar in shape to the structural design spectrum is selected and then progressively modified till its response spectrum is fairly close to the design spectrum. In general, the response spectrum of the selected ground motion, before it is modified, will lie above the design spectrum for certain values of the frequency and below it for certain others. It will, therefore, have to be suppressed at some points and raised at others. Suppression is achieved by passing the selected ground motion successively through a set of frequency suppressing filters. Raising is achieved by superposing, on the selected ground motion, some sinusoidal motions of suitable amplitudes and phase angles.

The methods of generating simulated earthquake records can also be adapted to produce spectrum compatible ground motions. Reference 92 describes a program developed by Watabe for generating simulated earthquakes with arbitrary or prescribed spectral properties. The method of generating the simulated earthquake is based on the concept of random phase angle.

Since a spectrum compatible artificial record was not available to the author, the El-Centro accelerogram was used in the design example presented in Section 8.12. Selection of a particular input ground motion, however, does not alter the essentials of the design method.

8.11.2 Method of Design Using the Time-Dependant Response

The following steps are involved in the design of a rigid jointed frame structure using a time-dependant method of seismic analysis:

1. Determine approximate member sizes by means of a preliminary design for gravity loads and the code-prescribed lateral forces due to wind and earthquake.

2. When justified, reduce the building structure to a single equivalent frame.

3. Carry out a dynamic analysis of the structure or the equivalent frame for the design ground motion using an appropriate computer program. The floor masses used in design may be those due to dead load for a normal structure and dead plus a certain portion of the live load for a warehouse or storage structure.

4. Scale down the member forces obtained from the elastic analysis by applying appropriate plasticity force reduction factor; for steel frames, a reduction factor of 4 may be applied to the girder end moments and a reduction factor of 2.5 may be applied to the column end moments and column axial loads.

5. If an equivalent frame has been used in the analysis, distribute the design forces obtained in step 4 among the members of the constituent frames by a method that is consistent with the assumptions made in deriving the equivalent frame. ✓

6. Design the girders for a plastic moment resistance capacity equal to the sum of the earthquake, dead load and live load moments. Design the columns for the combination of dead load, live load and seismic forces so that the plastic moment of resistance, reduced for the presence of axial loads, is equal to the maximum design moment.

7. Check the frame so designed for gravity and wind loads in accordance with the code. Include a check on the displacements due to wind.

8. The elastic analysis for earthquake motion gives rather conservative estimates of the maximum displacements and maximum inter-story displacements. The maximum values of such displacements in the elasto-plastic frame are, in fact, smaller than those in the elastic frame. Where necessary, check that the maximum displacements as given by the elastic analysis are within desired limits.

8.12 DESIGN EXAMPLE USING TIME-DEPENDANT RESPONSE ANALYSIS

The ten story set-back frame shown in Fig. 8.20 is designed by the time-dependent response analysis method of design. The figure also shows the floor dead and live loads used in design. The El-Centro 1940 N-S earthquake motion is selected as the design accelerogram. A design for gravity and earthquake loads carried out in accordance with the National Building Code of Canada, 1970, assuming that the building is located in seismic zone No. 3, gives preliminary member sizes shown in Figs. 8.20(b) and (c). It is assumed that wind loads are small and do not govern the design.

The building selected for design has a rather large set-back. It belongs to one of the several types of structures where a dynamic analysis method of seismic design is preferable to the equivalent static lateral load method. In fact, from the studies reported in Chapter 6, it should be expected that if the building were designed according to

(a) PLAN

(b) SECTION A-A AND PRELIMINARY DESIGN OF FRAME I

FIG. 8-20 TEN STORY SET-BACK BUILDING

Figures against Members show I in inch⁴

(c) SECTION B-B AND PRELIMINARY DESIGN OF FRAME II

DESIGN DATA

Dead load including allowance for member weights and fixed partitions

120 psf, all floors

Live load including allowance for movable partitions

100 psf, all floors

FIG. 8-20 CONTD.

the equivalent lateral method of the code, the tower portion of the building will be in severe distress when the building is subjected to a major earthquake. The set-back building as designed by the code is reduced to the single equivalent frame shown in Fig. 8.21. The equivalent frame is analyzed for the first 20 seconds of the El-Centro accelerogram. The results of both an elastic analysis and an elasto-plastic analysis are shown in Fig. 8.22. The accentuated response of the tower is evident from the results. The rotation ductility ratios in the tower beams are rather high. Also, the lowest columns in the tower develop plastic hinges at their base. The increased response of the tower also causes substantial increase in the axial load in those columns in the base portion which directly support the tower. As a result of this, plastic hinges are formed in these columns. It is clear that the beams and columns in the tower portion and those columns in the base portion that directly support the tower need to be strengthened.

The building is redesigned, using the time-dependant response method of analysis and the plasticity force reduction factors described in the preceding sections of this chapter. The member forces obtained from the elastic analysis are scaled down by applying a reduction factor of 4 to the beam moments and 2.5 to the column axial loads and column moments. The scaled down forces are distributed among the members of the constituent frames. Individual frame members are then designed for the combination of the gravity loads and the earthquake design forces. Details of the redesigned frame are shown in Fig. 8.23; Fig. 8.24 shows

Figures against Members show I in Inch^4

FIG. 8-21 EQUIVALENT FRAME FOR THE TEN STORY SET-BACK BUILDING

FIG 8-22 TEN STORY SET BACK FRAME RESPONSE TO EL CENTRO

(a) FRAME II

(b) FRAME I

FIG. 8-23 FINAL DESIGN OF THE TEN STORY SET-BACK BUILDING

FIG. 824 TEN STORY SET BACK FRAME RESPONSE TO EL CENTRO

the results of an elastic and an elasto-plastic analysis of the redesigned frame for the El-Centro-earthquake. As expected, the members in the tower portion have been strengthened as also those columns in the base portion which directly support the tower. There is no change in the sizes of most of the other members. The ductility ratios in the tower have been reduced to acceptable values. Also, there are no plastic hinges in the columns except at the base of the building which do not cause any loss in the structural integrity.

CHAPTER 9

SUMMARY AND CONCLUSIONS

9.1 GENERAL REMARKS

This study was concerned with the analysis and design of multi-story rigid jointed steel frames subjected to seismic forces. To meet its objective, the study was approached in the following stages:

- i) the development of the methods of analysis;
- ii) the selection of appropriate structural models and earthquake ground motions for study;
- iii) an analytical study of the seismic response of the selected model frames, including the effects of certain parameters on response;
- iv) an assessment of the ductility requirements and ductility capacities of multistory steel rigid frames subject to seismic loads
- v) a review of existing seismic design procedures and the development of methods of design based on the information obtained from steps iii) and iv) above.

The important conclusions drawn from this study are summarized in this chapter. Although some of the conclusions are of general applicability to the seismic response of structures, in a majority of cases, they apply primarily to steel frame buildings having complete moment-resisting joints. It is realized that many of the conclusions

will need further study and corroboration before they can be accepted for broader application.

9.2 DEVELOPMENT OF THE METHODS OF ANALYSIS

In many practical engineering situations, the analysis of structures subjected to dynamic loads requires the use of a numerical method of integration. The more effective methods of numerical analysis belong to the category designated as "Predictor Corrector Methods". A systematic method for the derivation of single point and multi-point formulae under the above category has been described in this study, and an efficient multi-point formula has been derived using the method just cited. The truncation error and the stability of this formula have been studied and its usefulness in calculation of dynamic response has been demonstrated by a series of examples. Some of the important conclusions drawn are summarized below:

1. The various methods of numerical analysis developed for the purpose of structural dynamics all belong to the category of predictor-corrector methods and can be derived by one unified approach. The same approach can be used for the derivation of multi-point formulae suitable for application in the field of structural dynamics.

2. For the dynamic analysis of structures that are not strained into the inelastic range, i.e. where the stiffness matrix remains constant, the choice of a good multipoint method of numerical integration will give a definite improvement in the accuracy of the solution. For comparable accuracy, the time step size which can be used with a properly selected

multipoint formula is considerably larger than that which is possible with a single point formula. The main consideration in the selection of a multipoint method is the requirement of stability. Multipoint methods that do not give rise to extraneous roots or to partial instability can be used to advantage. The corrector formulae presented in Eqs. 2.19 and 2.20 of this study are very suitable for use in dynamic analysis of structures subject to exciting forces such as wind, blast or earthquakes, provided the condition of stability can be met, i.e. provided h can be so selected that $\omega^2 h^2$ is less than 6.

3. For most problems encountered in structural dynamics, the iteration of the corrector is not necessary. Instead, a process of elimination described in Sections 2.6 and 3.13 can be used for the solution, resulting in considerable economy in the computation time. Though this is true of both elastic and inelastic systems, the economy will be most evident in the case of elastic systems.

4. For the reasons stated in 3 above, convergence of the corrector need not be a criterion for selecting the time step size. To reduce the truncation error, the step size should be so chosen that $h\omega$ is substantially less than 1. The need to maintain stability of the solution will impose different limitations on the step size, depending on the formula selected. Some formulae are, in fact, unconditionally stable.

5. For systems with a few degrees of freedom, the limitation imposed on the step size by the need to minimize the truncation error is

more restrictive than that imposed by the requirement of stability of the solution. However, for systems with a large number of degrees of freedom, the stability criterion may become more restrictive. In cases where it is possible to select a reasonable step size and still meet the requirements of stability, multipoint formulae like the one in Eq. 2.20 can be used with advantage.

9.3 ANALYTICAL STUDIES OF THE RESPONSE TO SEISMIC GROUND MOTION

In Chapter 5, the effects of certain parameters on the seismic response of the frames have been studied by reference to the results of elastic and elasto-plastic analyses of frames for recorded ground motions. The following conclusions can be drawn from these studies:

1. The effect of axial column deformations on the response depends upon the height-to-width ratio of the frame. Except for very slender frames, the errors introduced in the response values by ignoring the axial column deformations are small and may be neglected. For the frames studied, such errors in the response were of the order of 10% for a height-to-width ratio of 5. The errors decreased with decreasing height-to-width ratio.
2. An increase in the intensity of the base excitation results in an increase in the maximum story displacements and maximum interstory drifts, but at a rate that is less than proportionate. The story shears and the column axial loads also increase with an increase in the intensity of the base motion, but once the girders start yielding, the rate of increase reduces considerably.

3. The effect of the period of vibration on the elastic response depends primarily upon the characteristics of the ground motion as reflected in its elastic response spectrum curve. The variation of the maximum story displacement is controlled mainly by the fundamental mode response of the frame. On the other hand, the distribution of the maximum interstory drifts and maximum story shears depends not only upon the fundamental mode response but also upon the responses of the first few higher modes. The inelastic response is similar to the elastic response except that the maximum story shears and maximum column axial loads in the inelastic frame do not vary much with a change in the period of the frame.

4. The effect of the live load on the response of the frame depends upon: (1) the incidence of the live load at the time of the earthquake, (2) the contribution of the live load to the effective mass, (3) the effect of the mass of the frame on its response. A study carried out in the past has shown that the contribution of live load to the effective mass depends upon the manner in which the live load is mounted on the structure. Unless the live load is rigidly mounted, which is seldom the case, only a fraction of it contributes to the effective mass of the frame. The work carried out in the present study shows that the effect of a variation in the mass on the response of the frame is really a consequence of the attendant change in the period of the frame, and therefore depends upon the characteristics of the ground motion. For some combinations of ground motion and period of the frame, magnitude of

the response parameters including the story shears may, in fact, decrease with an increase in the effective mass. The incidence of a high level of live load at the same time as an extreme earthquake is very unlikely. On account of this and the facts that only a fraction of the live load contributes to the effective mass and that an increased mass does not necessarily result in an increased response, it appears reasonable to neglect the presence of live load in working out seismic design forces. However, for storage and warehouse structures, it may be desirable to include a part of the live load in the effective mass.

Chapter 6 deals with the response of set-back frames to seismic ground motion. The effects of two important parameters: (1) the level of set-back ratio, and (2) the tower base plan area ratio, on the response of the frame have been studied. The conclusions drawn from the study are summarized below:

1. For buildings with set-backs, the relative contribution of higher modes to the base shear in general increases with decreasing tower-base plan area ratio. In some cases, the contribution of the second or the third mode may be higher than that of the fundamental. A direct consequence of this behaviour is that the approximate code method of distributing the base shear on the assumption that the building vibrates predominantly in the fundamental mode cannot be applied to set-back frame buildings with slender towers.

2. The maximum floor displacements of a set-back building subjected to an earthquake excitation does not show a marked change when

compared to those of the related uniform building. In general, the maximum floor displacements decrease, but only slightly, with a decrease in the tower-base area ratio.

3. The tower response of a typical set-back building, as measured by maximum interstory drifts, maximum shear coefficients and maximum girder ductility ratios, is substantially accentuated when compared to the related uniform building. Therefore, unless the tower portion is suitably designed for the increased response, it may show severe distress under a strong earthquake. Also, plastic hinges may form in several of the columns in the tower portion.

4. The distribution of base shears as provided for by the present code underestimates the shears in the upper one-third of the building even for a uniform building. For a set-back building, the discrepancy is larger.

5. In set-back buildings with very slender towers, the shear coefficient shows a sudden and marked increase in the transition region between the base and the tower, a behaviour that can be appropriately termed a "notch effect".

6. The elastic analysis of set-back frames subject to recorded ground motion can be accomplished at a reasonable cost with the help of high speed digital computers. The inelastic analysis, although more time consuming and expensive, is still possible. In some exceptional cases, inelastic analysis can even now be considered as a possible design tool, and with reduced computing costs in the future, its usefulness as a design

tool should improve. However, there is a definite qualitative correlation between the elastic and inelastic response, which should permit the less expensive elastic analysis to be used in most cases provided the appropriate correction factors are known.

7. The seismic response of set-back building, whose mass and stiffness can be taken as substantially proportional to the plan area, depends upon the ratio of the tower plan area to the base plan area, and not upon the ratio of the plan dimensions of the tower and the base in the direction of the vibration, as specified in the present SEAOC code. When the plan area ratio and the plan dimension ratio referred to above are equal, the provisions in the commentary to the SEAOC code for the design of set-back building seem to be adequate for set-backs near the building mid-height except when the tower is very slender. The code provisions may, however, underestimate the response when the tower is either relatively low or relatively tall, particularly when it is slender at the same time.

9.4 DUCTILITY REQUIREMENT IN FRAMES SUBJECT TO SEISMIC GROUND MOTION

The satisfactory performance of building structures during an earthquake depends, to a large extent, upon the amount of inelastic deformation that the frame members can undergo without distress. This inelastic deformation is often measured in terms of the ductility ratio. The methods for determining the ductility requirements in frames subjected to earthquake ground motion have been discussed in Chapter 7. It has been

shown that the ductility requirements should be measured in terms of the inelastic rotation at the plastic hinges because safe values for such rotation can be determined through experiments. The following are some of the important conclusions drawn in Chapter 7 mainly on the basis of experimental work carried out by other researchers and reviewed in the chapter.

1. Any significant inelastic rotation at the plastic hinges involves maximum fibre strains considerably in excess of the value at the onset of strain hardening.
2. Sections that meet AISC width-to-thickness ratio requirements for compact section in plastic design have larger rotation capacities than sections that do not meet these requirements.
3. For comparable width-to-thickness ratios, the rotation capacities of beams subjected to cyclic loading are considerably less than the capacities of beams monotonically loaded.
4. If compact sections are used and the connections are adequately designed for ductility, a large amount of ductility capacity can be obtained from multistory rigid frames, which should make such frames very effective in resisting seismic loads.
5. Although there is good reason to believe that the ductility capacity of multistory steel rigid frames is large, its actual magnitude under seismic loads cannot be calculated at the present time, due to the lack of information about the available ductility ratios of members under repeated/reversed loading into the inelastic stage. It is most desirable

that experimental work be carried out to more thoroughly explore this topic.

9.5 PROCEDURES FOR THE SEISMIC DESIGN OF MULTISTORY BUILDINGS

The "equivalent lateral static load" method of design for seismic forces provided in the codes is a reasonable design procedure for a large number of structures that are fairly regular in shape and in the distribution of mass and stiffness. For this reason, and because the alternative method of design by dynamic analysis is rather complex, the equivalent lateral load method should remain in wide use for aseismic design. An attempt has been made in this study to relate the distribution of shears and moments determined by the equivalent lateral load method to the true dynamic behaviour. New empirical relations have been derived to approximate the shapes of the shear and overturning moment envelopes. Some of the important conclusions drawn are summarized below:

1. While the fundamental mode of vibration makes the major contribution to the response of a building, several higher modes also make a significant contribution and should be taken into account in obtaining the distribution of shears and overturning moments.
2. Assuming that the design base shear provided for by the design code is correct, the present codes underestimate the shears, in say, the upper third of the building, and overestimate the shears in the lower stories.
3. As a consequence of the contribution of higher modes, the shape of the envelope of maximum shears depends directly upon the period

of the building and only indirectly upon the building height-to-width ratio. In fact, for a shear-deforming building, the height-to-width ratio has little or no effect on the period, quite contrary to the recommendations of the present code.

4. Experience of building column failures during some of the recent earthquakes have led to the realization that the 1968 SEAOC code provisions for overturning moments were probably unconservative. The NBC 1970 therefore specified values of J that were higher than those in the earlier versions of the SEAOC code. Also, in the 1969 revision of the SEAOC code, the J factor was completely eliminated as an interim measure, awaiting the results of further study and research. Thus there still is a need for a realistic appraisal of the overturning moment reduction factors.

5. The empirical relations developed in this study and contained in Eqs. 8.13, 8.14 and 8.15 can be employed to correct the deficiencies noted above in the equivalent-lateral-force provisions of the present codes. These relationships are simple enough to be employed in design without entailing more work than that involved in the present code. It is believed that, for fairly regular buildings, the only cases to which the code provisions are really applicable, the empirical relations developed in this study can serve as a basis for a set of design provisions which would be substantially more consistent with the result of dynamic analyses than are provisions of the present codes.

While the equivalent lateral load method of design is satisfactory

for fairly regular structures, some form of dynamic approach must be used in the design of tall or unusual buildings, buildings with irregular layout, large set-backs or unusual taper. The available methods of seismic design through dynamic analysis have been studied in this work and a design method using the time-dependant response analysis and plasticity force reduction factors has been developed. The following important conclusions can be drawn from this work:

1. The dynamic analysis of a large building with a number of frames of different types may involve a large amount of computer time. However, under certain situations, it is possible to reduce the building structure to a single equivalent frame. The member design forces obtained from the dynamic analysis of this frame are reasonably accurate, provided the building is fairly regular in shape and in the distribution of stiffnesses.

2. In the modal superposition method of dynamic analysis, some form of approximation has to be used in combining the modal responses to obtain the total response. This introduces an uncertainty in the resulting response values. There is some evidence that the errors in the response values obtained by an approximate superposition of modes are generally less than 20% but may be as high as 40%.

3. In the mode superposition method of analysis, the member forces are usually obtained by resolving the maximum story shears into a set of equivalent lateral forces and carrying out a static analysis of the building frame for these forces. The beam moments obtained by

this method are generally conservative while the column moments may err on either side; the column axial loads are grossly overestimated and can be used only after appropriate correction factors are applied.

4. The time-dependant response method of analysis does not involve the above uncertainties of the mode superposition method. It gives more accurate values for the maximum member forces in a building subject to earthquake motion, at only a little larger investment in the computer time.

5. Structural design spectra obtained by taking statistical averages of the recorded ground motion can be directly used in the mode superposition method of design. The time-dependant response method of design, on the other hand, requires the time history of an appropriate ground motion as an input. It is possible to artificially generate design ground motions that share the statistical properties of the recorded ground motions and give response spectra that are similar to the structural design spectra just mentioned.

6. In accordance with code proposals on the dynamic analysis approach to design, the design seismic forces in the members of a ductile structure are derived from the related elastic analysis by applying a uniform reduction factor. However, analyses of practical multistory rigid jointed frames show that if the columns are to remain elastic the reduction factors applicable to the beam moments should be larger than those applicable to the column moments and the column axial loads. It is, therefore, appropriate to use two different reduction factors in design; a larger

one for the girder moments and a smaller one for the column moments and column axial loads. This will ensure that most of the inelastic deformation is confined to the girders leaving the columns elastic and will provide a safeguard against the more hazardous consequences of column failures.

9.6 RECOMMENDED FUTURE STUDIES

During the course of this research, certain topics of significance in the field of earthquake engineering have become identified on which there is scope for additional work. Some of the potential areas of study are described below.

1. Additional studies should be made on the effect of axial column deformations on the seismic response of multistory building structures with shear walls, braced frames or the combination of these with moment resisting frames.
2. There is a scope for additional study on the effect of the type of mounting of live load on the seismic response of building structures.
3. There is need for more extensive studies on the behaviour of members under inelastic cyclic loading and on the ductility capacity of members under such loading.
4. The investigations on the behaviour of set-back frames should be expanded to cover buildings that derive their resistance to lateral forces from shear walls or braced frames or a combination of these with rigid frames.

5. A part of the present research was devoted to the development of more rational methods for the distribution of shears and overturning moments throughout the height of a building for use with the equivalent lateral load method of seismic design. These procedures were developed on the basis of analyses of the seismic response of a uniform hybrid cantilever model supported by studies of realistic rigid frame buildings. The applicability of these procedures to a broad range of structures can be established only through further studies on the different structural systems. These studies should include investigations on the effect of the periods and the height to width ratios of buildings with different structural forms such as those with braced frames and shear walls on the distribution of seismic shears and overturning moments.

6. Further studies are required to establish suitable values for the plasticity force reduction factors to be used with a dynamic analysis approach to the design of buildings with different structural forms including those with moment resisting frames. The force reduction factors will of course have to be related to the ductility capacities of the members. The effect of damping on the magnitude of the force reduction factors should also be studied.

NOMENCLATURE

Symbols are defined where they first appear in the text. In addition, they are defined below in alphabetical order. Wherever possible, the same definition of a symbol was retained throughout; when a symbol has been used in more than one sense, all the alternative definitions used are given below.

A = constant of integration; cross-sectional area; amplitude of oscillation

A_b = plan area of the base in a set-back building

A_s = amplitude of oscillation obtained from a numerical solution

A_t = plan area of the tower in a set-back building

A_e, B_e, C_e = coefficients in the corrector formula for displacement

A_n, B_n, C_n = submatrices in the frame stiffness matrix

a_i = coefficients in the corrector formula for displacement

a^r = rth mode shape

B = constant of integration; width of the frame

B, b = flange width

b_i = coefficients in the corrector formula for velocity

$[C], [c]$ = damping matrix

C = constant of integration; stability coefficient; shear coefficient

C_b^r = base shear coefficient in the rth mode

C_t^r	= tower base shear coefficient in the r th mode
C_s	= tower base plan area ratio
C_r	= damping coefficient in the r th mode
C_x	= shear coefficient applicable to level x
c	= damping coefficient; stability coefficient
D	= constant of integration; depth of wide-flange section
D_s	= width of the lateral force resisting system
E	= constant of integration; Youngs Modulus of Elasticity
E_m	= error term in a corrector formula
E_s	= Modulus of Elasticity in the strain-hardening range
$\{F\}$	= vector of lateral forces
F	= lateral force; shear; foundation factor in seismic design
$F(t)$	= force which is a function of time
$f(t,y)$	= function of time and variable y
G	= shear modules of rigidity
g	= acceleration due to gravity
H	= total height of the building
h	= interval of time used in numerical integration; height of the base portion in a set-back building
h_x	= height up to level x of the building
$[I]$	= identity matrix
I	= moment of inertia of the cross-section; importance factor in seismic design

- $J, \bar{J}$ = base moment reduction factor
- $J_x, \bar{J}_x$ = moment reduction factor applicable to level x of the building
- $[K], [k]$ = stiffness matrix
- K = a constant; a constant factor in the formula for base shear to account for the ductility of the structure
- K_B = girder stiffness
- K_C = column stiffness
- k = an integer; stiffness; a constant
- k_r = a constant; effective stiffness in the rth mode
- $[L]$ = a lower triangular matrix
- L = length of member
- L_s = level of set-back ratio
- ℓ = length
- $[M], [m]$ = mass matrix
- M = moment
- M_G = mid-span moment in a simply supported beam carrying a uniformly distributed load
- M_p = plastic moment of resistance
- M_{pB} = plastic moment of resistance of the beam
- M_{pC} = plastic moment of resistance of the column
- M_y = moment at the beginning of yield
- m = an integer; mass

m_b	= floor mass in the base portion of a set-back building
m_d	= mass corresponding to the dead load
m_e	= effective mass
m_l	= mass corresponding to the live load
m_t	= total mass; floor mass in the tower portion of a set-back building
N	= number of stories; number of degrees of freedom
n	= subscript indicating the location of interval of time at which the subscripted variable is evaluated
P	= axial load
p	= an integer
P_{net}	= net residual plastic hinge rotation
P_{max}	= maximum plastic hinge rotation
R	= remainder term in integration formula; rotation capacity; seismic regionalization factor
r	= parameter in the Ramberg Osgood curve representing a material property
r_y	= radius of gyration
S	= stability coefficient; elastic section modulus; spectral response
S_A	= spectral pseudo acceleration
S_v	= spectral pseudo velocity
SI_{ξ}	= spectrum intensity for fraction of critical damping equal to ξ

- s = stability coefficient; slope of straight line in shear coefficient graph
- T = undamped fundamental natural period
- T_d = duration of the earthquake motion
- T_i = period of the i th mode of vibration
- T_s = period obtained from a numerical solution; period of set-back building
- t = elapsed time; flange thickness of wide flange section
- $[U]$ = upper triangular matrix
- V, V_b = base shear
- W = total effective weight of the building
- W_i^e = effective weight associated with the i th mode of vibration
- w = web thickness of a wide flange section
- w_x = weight assigned to level x
- $[X]$ = submatrix in the frame stiffness matrix
- $[X^*]$ = condensed stiffness matrix
- x = coordinate along the length or the height of the building; ground displacement
- $\ddot{x}$ = ground acceleration
- $y, \dot{y}, \ddot{y}$ = absolute value of displacement; velocity and acceleration relative to the base
- y_e = distance from the neutral axis of a fibre with a strain equal to yield strain

- y_s = distance from the neutral axis of a fibre with a strain equal to that at the onset of strain-hardening
- y^c = corrected value of displacement
- y^p = predicted value of displacement
- Z = plastic section modulus
- $z, \dot{z}, \ddot{z}$ = values of displacement, velocity and acceleration relative to the base
- α = a constant; live load ratio = m_l/m_d ; ratio of maximum fibre strain to the strain at onset of strain hardening
- β = a constant; parameter in Newmark's method of numerical integration; ratio of strain at onset of strain hardening to the yield strain
- $\{\Delta\}$ = vector of sideways displacements
- Δ = displacement
- $\Delta x, \Delta y, \Delta z$ = incremental values of x , y and z respectively
- $\{\delta\}$ = vector of displacements other than sideways displacements
- δ_{el} = elastic deflection
- δ_{ep} = total deflection, elastic and plastic
- δ_{pi} = incremental plastic rotation corresponding to degree of freedom i
- δ_y = deflection at the onset of yield
- $\delta \Delta_i$ = incremental displacement corresponding to degree of freedom i
- $\delta \theta_i$ = incremental rotation corresponding to degree of freedom i

- ϵ = live load response factor; strain
- ϵ_1 = error
- ϵ_2 = change in end moment over a time step
- $\epsilon_{\max}$ = maximum fibre strain
- ϵ_s = strain at the onset of strain-hardening
- ϵ_y = strain at the beginning of yield
- ξ_r = fraction of critical damping in the rth mode
- η_r = variable representing displacement in the rth mode
- θ = a particular value of time t located somewhere in the interval h between t_n and t_{n+1} ; live load contribution factor; total end rotation
- θ_H = total plastic end rotation
- θ_p = elastic component of the total end rotation
- λ = ratio of the maximum fibre strain to the yield strain
- λ_r = modal participation factor for the rth mode
- μ = ductility ratio
- ρ = mass per unit volume
- σ_y = yield stress
- $[\phi]$ = modal matrix
- $\{\phi^r\}$ = modal vector in the rth mode
- ϕ = curvature
- ϕ_y = curvature at the beginning of yield

ψ_r = variable representing displacement in the r th mode

Ω = frequency of exciting base motion

ω = natural frequency

ω_{dr} = damped natural frequency in the r th mode

ω_r = natural frequency in the r th mode

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