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CIRCUIT FACTOR COMPENSATION FOR SAW FILTERS
USING MODAL ANALYSIS

by

Thomas P. Cameron, B.Sc.

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of
Master of Engineering

Department of Electronics

Carleton University

Ottawa, Ontario

April 1988

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USING MODAL ANALYSIS"

submitted by Thomas P. Cameron
in partial fulfilment of the requirements for
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ABSTRACT

SAW filters used in the IF section of a digital microwave radio system must be designed to very high standards to ensure the signal fidelity for high level modulation schemes.

In an effort to maintain strict control over the designed frequency response from filter to filter, a highly accurate SAW filter Y-matrix model has been developed based on the modal analysis theory. Combining this with models for the buffering electronics surrounding the SAW filter, the overall circuit response has been predicted to give very close agreement with experimental results.

The application of this model in the SAW filter design cycle allows the undesired frequency response distortion arising from the interaction between the interdigital transducers and the buffering electronics to be identified in the design stages. An iterative correction algorithm has been developed which predistorts the SAW filter frequency response to correct for these undesired second-order effects.

The advantage of this iterative design technique is that the second-order effects are identified prior to device fabrication, reducing the need for experimental design iterations. The work presented in this thesis will provide a substantial decrease in the design cycle time for prototype SAW filters and increase the level of quality which may be achieved.

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TABLE OF CONTENTS

	Page
Acceptance	ii
Abstract	iii
Acknowledgements	iv
Table of Contents	v
List of Figures	vii
List of Symbols	xv
 Chapter 1: Introduction	 1
1.1 Surface Acoustic Wave Technology	1
1.2 Analysis and Synthesis of SAW Devices	3
1.3 Thesis Objectives	6
1.4 Thesis Outline	9
 Chapter 2: Physics of SAW Devices	 14
2.1 Surface Acoustic Waves and SAW Materials	14
2.2 The Interdigital Transducer	19
2.3 Second-Order Effects	26
2.3.1 Loss Mechanisms	26
2.3.2 Triple Transit Echo	29
2.3.3 Edge Reflections	30
2.3.4 Electromagnetic Feedthrough	33
2.3.5 Bulk Wave Coupling	33
2.3.6 Diffraction and Beam Steering	34
2.4 Circuit Theory for the IDT	36
2.4.1 An Equivalent Circuit Representation of the IDT	36
2.4.2 Matching Problems	39
2.4.3 The Circuit Factor	42
 Chapter 3: SAW Filter Design	 45
3.1 SAW Filter Design	45
3.2 The Fourier Transform Design Method	46
3.3 Design Example: A 70 MHz Bandpass Filter	54

Chapter 4: The Admittance Matrix of a SAW Filter	60
4.1 Admittance Models for SAW IDTs	60
4.2 A 3-D Modal Analysis of SAW Filters	62
4.3 Solution of the Propagating Modes in a SAW Filter	66
4.4 Solution of the SAW Potential in the Filter Structure	71
4.5 The Admittance Matrix of a SAW Filter	82
Chapter 5: A Wideband Buffer Amplifier For Saw Filters	100
5.1 Amplifier Specifications	100
5.2 Design of a Wideband Buffer Amplifier	101
5.3 Modeling the Amplifier	103
5.4 Results	104
Chapter 6: The SAW/Buffer Circuit	109
6.1 Circuit Layout Considerations	109
6.2 The Matching Network	112
6.3 Circuit Model	116
6.4 Results	119
Chapter 7: Circuit Factor Compensation	122
7.1 Circuit Factor Compensation Algorithm	122
7.2 Compensated Design Example	124
7.3 Experimental Results of the Compensation Algorithm	132
Chapter 8: Summary and Conclusions	138
References	142
Appendix I : Derivation of Wavenumber Relations	147
Appendix II : Derivation of k_s	153
Appendix III : Modal Distribution Functions	161
Appendix IV : Derivation of Linear Coefficients of Modal Function Superposition	167

LIST OF FIGURES

Figure	Description	Page
1.1	A simple nondispersive delay line	3
1.2	Schematic of cascade circuit	8
2.1	The propagation of a Rayleigh wave on the surface of an isotropic solid.	14
2.2	Surface wave velocity versus direction of propagation for 128° lithium niobate (LiNbO_3) [13]	16
2.3	The use of interdigital transducers for launching and detecting surface acoustic waves.	20
2.4	<p>a) The SAW excitation by the electric field between adjacent fingers may be modelled by discrete delta ultrasonic sources.</p> <p>b) The impulse response of the IDT is the cumulative effect of all delta sources at some arbitrary time t.</p>	22
2.5	<p>a) Infinite sinusoidal SAW train</p> <p>b) Rectangular truncation representing the finite length of the IDT</p> <p>c) Discrete sampling points representing fingers</p> <p>d) Resulting impulse response</p>	23
2.6	<p>a) The apodized weighting generates a wave train with spatial variation corresponding to the overlap function.</p> <p>b) For withdrawal weighting, the desired impulse is approximated by the removal of selective fingers.</p>	25
2.7	<p>a) The three-port representation of an IDT. V_s is the electrical source and G_0 represents the acoustic conductance of the substrate.</p> <p>b) A SAW filter is represented by two three-port IDT's connected by a lossless line.</p>	27
2.8	Sources of reflections on the SAW substrate.	30

2.9 (a) The array factor for an IDT with a non-integer number of finger pairs superimposed on the element factor.	32
(b) The product of the array factor and the element factor yields a modified frequency response with a benign slope across the IDT passband.	
2.10 Spurious signal generation: Bulk acoustic waves (BAW) are launched in various directions into the substrate and reappear at the output IDT. Electromagnetic (EM) feedthrough travels directly from the input to the output to interfere with the main signal.	34
2.11 Diffraction and beam steering effects.	35
2.12 The basic equivalent electrical circuit model of the IDT consists of a radiation conductance $G_a(f)$ and a radiation susceptance $B_a(f)$ in parallel with the static capacitance C_T .	37
2.13 Extended equivalent circuit model including the electrode resistance R_e , the lead inductance L_e and the stray capacitance C_e .	38
2.14 A shunt inductor is used to tune out the capacitance at f_0 and $G_a(f_0)$ is set equal to R_0^{-1} .	39
2.15 The response of the IDT is the product of the circuit factor $C_1(f)$ and the FIR response of the IDT $A_1(f)$.	42
2.16 The overall response of the SAW filter may be represented as the product of the circuit factors $C_1(f)$ and $C_2(f)$, and the electroacoustic response of the filter $H_{SAW}(f)$.	44
3.1 Typical bandpass filter specifications are:	46
1) f_0 - the center frequency	
2) B_1 - the transition bandwidth	
3) B_2 - the pass bandwidth	
4) R_1 - the pass band ripple maximum amplitude	
5) R_2 - the maximum stop band ripple (side lobe) amplitude	

3.2	For a SAW filter with one apodized and one unapodized IDT, the filter response $H_F(f)$ is the product of the responses of the individual IDT's, $H_1(f) \cdot H_2(f)$.	47
3.3	The $\text{SIN}(X)/X$ frequency response of an unapodized IDT showing the 4dB beamwidth and the position of the nulls.	48
3.5	Truncation of the impulse response by a rectangular window function.	52
3.6	Truncation of the impulse by the use of a smoother window function. Note that although the ripple has decreased, the transition bandwidth has broadened somewhat.	53
3.7	The specification for a 70 MHz raised cosine ($\alpha = 0.12$) bandpass filter.	54
3.8	a) Linear magnitude of desired response $H_D(f)$ b) Response of unapodized IDT, $H_1(f)$ c) Required response of the output IDT, $H_2(f) / H_1(f)$	57
3.9	Filter design compared to original specification a) designed sidelobe level is at -57dB b) designed passband ripple is less than 0.015dB	58
3.10	Measured response of the filter compared to the designed filter response. The solid line represents the designed magnitude and phase. The dashed line is the response of the filter TPCA11-0306 measured on a HP3577 network analyzer.	59
4.1	SAW filter layout comprising of an apodized IDT and an unapodized IDT with a section of grounded "dummy" fingers in between them.	63
4.2	Representation of the substrate as a thin film waveguide with longitudinal layers of material with fast, slow, and intermediate SAW velocities. The SAW wave propagates in the Z direction.	64

4.3	An example of a typical modal distribution function. Notice that for $ x < b$, the function is trigonometric and it decays to zero outside the metallized region.	68
4.4	(a) The five lowest-order symmetric trigonometric modes in a structure $5500 \mu\text{m}$ wide at 70MHz created by a source $4500 \mu\text{m}$ wide centered about $x=0$. (b) The symmetric hyperbolic mode launched by the same source as in (a)	75
4.5	The sum of the modes launched by the source in Figure 4.4	76
4.6	(a) The five lowest-order symmetric trigonometric modes launched by a source $1000 \mu\text{m}$ long centered at $x=500\mu\text{m}$. (b) The five lowest-order antisymmetric trigonometric modes launched by the same source.	78
4.7	(a) The symmetric hyperbolic mode launched by the source of Figure 4.6 (b) The corresponding antisymmetric hyperbolic mode.	79
4.8	The sum of the modes launched by the source of Figure 4.6.	80
4.9	(a) The magnitude profile of the propagating wave launched by the line source of Figure 4.4. (b) The magnitude profile of the propagating wave launched by the line source of Figure 4.6.	81
4.10	(a) The electric field between adjacent fingers. (b) The acoustic line source approximation for the excitation of SAW. The arrows on the sources indicate that the two sources are 180° out of phase. Notice also that the length of the source is directly proportional to the finger overlap.	83
4.11	The superposition of N line sources at a point (x, z_q) .	85
4.12	The superposition of induced SAW potential and induced charge of a uniform IDT with 2.5 periods of split fingers at (a) 70MHz , (b) 60MHz and (c) 42MHz .	88

4.13 The predicted radiation conductance and susceptance of a uniform IDT with 2.5 split fingers.	90
4.14 The radiation admittance of the apodized IDT of the filter designed in Chapter 3 with 312 fingers.	90
4.15 Y-matrix representation of the SAW filter as it is simulated in this modal analysis model.	91
4.16 The equivalent circuit for the radiation Y-matrix of a SAW filter.	92
4.17 Equivalent circuit representation of a SAW filter including parasitic elements.	95
4.18 The predicted input admittance of an unapodized IDT with 2.5 split finger pairs and a 4500 μ m aperture (____), compared to the experimental input admittance measured on a HP3577 network analyser(----).	95
4.19 The predicted input admittance of an inline apodized IDT with 312 fingers and a 4500 μ m aperture(____), compared to the experimental input admittance measured on a HP3577 network analyser (----).	96
4.20 The predicted input admittance of an incline apodized IDT with 312 fingers and a 4500 μ m aperture (____) compared to the experimental input admittance measured on a HP3577 network analyser (----).	96
4.21 The predicted input admittance of a V- line apodized IDT with 312 fingers and a 4500 μ m aperture (____) compared to the experimental input admittance measured on a HP3577 network analyser (----).	97
4.22 The predicted response of filter(____) TPCA011-0306 compared to the response measured on the HP3577, network analyser(----).	99
5.1 Basic schematic of the wideband buffer amplifier.	103

5.2	The equivalent circuit model for the amplifier circuit of Figure 5.1. All transmission lines have $\epsilon=2.2$ and substrate thickness of 60mils. All dimensions for w , the width of a transmission line and p , the physical length of a transmission line are in mils.	105
5.3	The modeled magnitude and phase response of the amplifier compared to the measured response (indicated by the dots)	107
5.4	The modeled input impedance (plotted as S_{11}) of the amplifier compared to the measured input impedance (indicated by the dots).	107
5.5	The modeled output impedance (plotted as S_{22}) of the amplifier compared to the measured output impedance (indicated by the dots).	108
6.1	Block diagram of the SAW/buffer circuit.	110
6.2	Response of the input amplifier taken at test point TP-1. Note the ripples at 60 and 80 MHz due to the radiation admittance of the IDT.	111
6.3	The response of the output amplifier plus the matching circuit. The response looks like the tank circuit with 8dB of gain.	111
6.4	a) The response of the circuit with no matching inductor on the output of the SAW filter. b) The response of the circuit with a 410nH matching inductor on the output IDT.	113
6.5	The response of the circuit with a 510 Ω damping resistor in the matching circuit. On a large scale (a) the response looks flat but on a 1dB scale (b) some rounding is seen in the magnitude response.	114
6.6	The desired(——) filter response compared to the measured (----) filter response.	116
6.7	a) The equivalent circuit of the input amplifier b) The equivalent circuit of the output amplifier	117
6.8	The equivalent circuit of the SAW/buffer circuit and the transmission model implementation.	118

6.9 The response predicted by the transmission matrix model of the circuit (____) compared to the measured response (----).	120
6.10 The modelled response(____) with a 10Ω shunt matching resistor shows <u>good</u> agreement in the phase reponse but a slightly greater slope across the passband than measured (----).	120
6.11 The predicted response (____) with a $1K\Omega$ shunt matching resistor shows good agreement to the measured response (----).	121
7.1 The SAW filter design algorithm including compensation for the modelled second-order effects.	123
7.2 The original filter specification for the filter design program.	124
7.3 The designed response (____) shows nearly exact agreement to the specification (----) to -45dB.	125
7.4 The modal analysis (____) of the designed filter shows that the design deviates somewhat from the filter specification (----).	126
7.5 The predistorted filter specification derived from the error function (----) superimposed on the analysed response(____).	126
7.6 The designed response (----) meets the predistorted specification (____) closely in magnitude but deviates somewhat in phase.	128
7.7 The analysed response (____) appears to match the desired filter response (----) resonably well in magnitude but the phase still shows some deviation from the zero degree specification.	129
7.9 The resulting response of the third design iteration (dashed line) meets the specification (solid) nearly exactly.	131
7.10 The analysed response (solid line) compares quite closely with the desired filter response	131

7.11 (a) On a coarse scale the predicted response(____) shows excellent agreement in both the magnitude and phase with the measured response(----).	133
(b) On a finer scale the experimental magnitude(----) is flatter than predicted but the phase matches the prediction (____) within a few degrees.	
7.12 (a) On a coarse scale the measured response (----) shows excellent agreement in both the magnitude and phase response with the desired filter response(____)	134
(b) On a finer scale the experimental magnitude(----) is within 0.3 dB of the desired passband response (____) and the phase matches within 2 degrees.	
7.13 The input admittance of the unapodized IDT matches that of the iterated design very closely.	135
7.14 The predicted input admittance (____) of the iterated apodized IDT shows excellent agreement with experimental measured admittance (----).	136
7.15 Comparison of the input admittance of the original (____) and iterated(----) apodized IDT's.	136
A1.1 Projection of the index ellipsoid on the plane of incidence.	147
A1.2 Slowness curve on an anisotropic substrate with the Z-axis as the pure mode axis.	150
A2.1 An infinite array of metal strips on the surface of a piezoelectric substrate.	156

LIST OF SYMBOLS

α	the roll-off factor of a raised cosine function
α_m	the anisotropic constant on a metallized surface
α_s	the anisotropic constant on a semi-metallized surface
α_f	the anisotropic constant on a free surface
a, b, c	the waveguide boundaries of Chapter 4
A	the amplitude of the mode function in the semi-metallized region
$A(f)$	the array factor
$A_1(f)$	the array factor of the input IDT
$A_2(f)$	the array factor of the output IDT
$A_i(x, z)$	the potential of a wave generated by the i th line source at a point (x, z)
A_q	the potential of a wave in the q th gap
B	the amplitude of the mode function in the metallized region
B_1	the transition bandwidth of a filter specification
B_2	the pass bandwidth of a filter specification
$B_a(f)$	the radiation susceptance of an IDT
$B_{a1}(\omega)$	the radiation susceptance of the input IDT
$B_{a2}(\omega)$	the radiation susceptance of the output IDT
B_n	the linear coefficient of superposition for the n th mode launched by a line source
B_{ni}	the linear coefficient of superposition for the n th mode launched by the i th line source

B_{ni1}	the linear coefficient of superposition for the n th mode launched by the i th line source of the input IDT
B_{nq2}	the linear coefficient of superposition for the n th mode received by the q th line source of the output IDT
B_T	the full bandwidth of a raised cosine function
C	the amplitude of the mode function in the free region
$C(f)$	the generalized circuit factor
C_1	the response of the input matching circuit
C_2	the response of the output matching circuit
C_{12}	the response of the output matching circuit
C_e	the parasitic capacitance in an IDT
C_{e1}	the parasitic capacitance of the input IDT
C_{e2}	the parasitic capacitance of the output IDT
C_{FF}	the interelectrode capacitance per unit length of one period an IDT
C_{Ti}	the interelectrode capacitance of an IDT
C_{T1}	the interelectrode capacitance of the input IDT
C_{T2}	the interelectrode capacitance of the output IDT
δ	Dirac delta function
Δ	the finger width to period ratio
D	a proportionality constant encompassing H and H'
$D(f)$	the designed frequency response in the iteration algorithm
D_x	the finger width
$e(f)$	the element factor

ϵ_{11}	the permittivity constant in the direction of propagation
ϵ_{13}	the cross-axis permittivity constant between the direction of propagation and the normal to the direction of propagation
ϵ_{33}	the permittivity constant along the normal to the direction of propagation
ϵ_0	the permittivity constant in free space
ϵ_{PR}^T	the relative dielectric constant
$E(f)$	the error function used in the iteration process
f	frequency
f_0	the synchronous frequency of an IDT
f_1	the flat bandwidth of a raised cosine function
f_t	the trial function in the Galerkin method
$f_n(x)$	the modal distribution function
$G_a(f)$	the radiation conductance of an IDT
$G_{a1}(\omega)$	the radiation conductance of the input IDT
$G_{a2}(\omega)$	the radiation conductance of the output IDT
G_0	the peak radiation conductance of an unapodized IDT
$h(t)$	the impulse response of the IDT
$h_2(t)$	the truncated impulse of a SAW filter
H	the proportionality constant representing the electro-acoustic energy transformation
H'	the proportionality constant representing the acousto-electric energy transformation
$H(f)$	the frequency response of the IDT

$H_1(f)$	the overall frequency response of the input IDT
$H_2(f)$	the overall frequency response of the output IDT
H_{a1}	response of the input buffer amplifier
H_{a2}	response of the output buffer amplifier
$H_D(f)$	the desired frequency response of a SAW filter
H_F	the product of the FIR responses of the input and the output IDTs
$H_{SAW}(f)$	the acousto-electric frequency response of a SAW filter
H_T	response of the SAW/buffer circuit
i_1	position of the i th line source in the input IDT
I_1	the current in the input IDT
I_2	the current in the output IDT
I_n	amplitude and sign of the n th delta source
k_{mx}	the propagation constant in the x direction on a metallized surface
k_{sx}	the propagation constant in the x direction on a semi-metallized surface
k_{fx}	the propagation constant in the x direction on a free surface
k_z	the propagation constant in the z direction in the waveguide structure
k_{m0}	the propagation constant in the z direction on a metallized surface
k_{s0}	the propagation constant in the z direction on a semi-metallized surface
k_{f0}	the propagation constant in the z direction on a free surface

k_{nmx}	the propagation constant in the x direction on a metallized surface for the n th mode
k_{nsx}	the propagation constant in the x direction on a semi-metallized surface for the n th mode
k_{nfx}	the propagation constant in the x direction on a free surface for the n th mode
k_{zn}	the propagation constant in the z direction in the waveguide structure for the n th mode
K^2	the acousto-electric coupling coefficient
l	the electrode length
$\tilde{L}$	the acoustic aperture of an IDT in wavelengths
L	the distance between the IDT fingers
L_e	the parasitic inductance in an IDT
L_x	the finger period
λ_0	the wavelength corresponding to f_0
M	number of modes under consideration
n	mode index
N	the number of gaps in an IDT
N_1	the number of line sources in the input IDT
N_2	the number of line receivers in the output IDT
N_p	the number of finger periods in an IDT
NP	the number of finger pairs in a single finger IDT
ω	angular frequency
Φ	the power flow angle
$P(f)$	the raised cosine function

P_s	the Legendre function of the first kind of order s
q_2	position of the q th gap in the output IDT
Q_q	the charge induced on the q th finger by a voltage V
Q_T	the charge induced on an IDT by a voltage V
ρ	resistivity
R_1	the maximum passband ripple of a filter specification
R_2	the maximum stopband ripple of a filter specification
R_e	the electrode resistance in the IDT
R_{e1}	the electrode resistance in the input IDT
R_{e2}	the electrode resistance in the output IDT
R_0	the load resistance on an IDT
R_L	the load resistance
R_s	the source resistance
$S(f)$	the filter specification used in iteration process
$S'(f)$	the predistorted specification used in iteration process
θ	the angle between the propagation vector and the crystal axis
t_n	time corresponding to the n th delta function source
T_{nq}	the superposition of the n th mode launched from all line sources in the q th gap
T_0	time period of the IDT fingers
$u_n(x,z)$	potential of the a wave launched by the n th line source at a point (x,z)
v_f	the surface wave velocity on a free surface

v_m	the surface wave velocity on a metalized surface
v_s	the surface wave velocity in the IDT
V	the voltage applied to an IDT
V_1	the voltage across the input IDT
V_2	the voltage across the output IDT
$V_s(f)$	the source voltage into an IDT
$V_1(f)$	the available voltage across an IDT
w	the aperture of an unapodized IDT in metres
$w(t)$	the window function used to truncated the impulse response of the apodized IDT
x_q, x_{q+1}	define the ends of the line source in the qth gap
ψ	amplitude of the rectangular crested wave launched by a line source
$Y_{11}^{rad}(\omega)$	the input radiation admittance of the input IDT
$Y_{12}^{rad}(\omega)$	the reverse radiation transadmittance of an pair of IDTs
$Y_{22}^{rad}(\omega)$	the input radiation admittance of the output IDT
$Y_{21}^{rad}(\omega)$	the forward radiation transadmittance of an pair of IDTs
Y_{in}	the input admittance of an IDT
Y_m	the admittance of a matching circuit on an IDT
z_i	the position of the ith line source
z_q	the position of the qth line source

CHAPTER 1

INTRODUCTION

1.1 Surface Acoustic Wave Technology

Surface Acoustic Wave technology has matured to the point where a wide variety of SAW devices are available for commercial and military use [1,2]. Over the last decade the global SAW components market has increased by approximately 25% per year from \$10M to more than \$100M and it is predicted by Hartmann [3] that a \$1B market is feasible by the end of the next decade.

The widespread growth of the SAW components market may be attributed to the many attractions offered by SAW technology. Typical SAW devices operate at a center frequency between 10 MHz and 2 GHz, with bandwidths from approximately 10 KHz to greater than 500 Mhz [1]. SAW technology excels in this frequency range where conventional lumped components and microwave technologies display limited capabilities.

The signal energy (in acoustic form) propagates along a path on the surface of a SAW device substrate, allowing the signal to be simultaneously sampled at several points along the propagation path. This property results in a unique flexibility of device design enabling many signal processing functions to be achieved.

Also, the planar nature of SAW technology enables the circuit pattern to

be deposited entirely on the substrate surface. By exploiting photolithographic manufacturing techniques which have been developed in the semiconductor industry, very reproducible and rugged devices may be produced at a reasonable cost.

SAW technology shares some common characteristics with thick/thin film hybrid technology. To exploit these similarities, it has been suggested that these two compatible technologies be integrated into one package [4]. This type of integration will enhance the reliability and economy of SAW devices, and this trend is predicted for all future SAW based systems.

Although this level of integration has not been attempted here, the models which have been developed will make it possible to reliably integrate SAW devices and active electronics in future work.

1.2 Analysis and Synthesis of SAW Devices

The simplest SAW device is a delay line such as that shown in Figure 1.1. A typical SAW device consists of two electro-acoustic transducers on a polished piezoelectric substrate.

Figure 1.1 A simple nondispersive delay line

The electro-acoustic transducers convert electrical signal energy to acoustic energy and vice versa. Each transducer acts as a multi-element antenna in such a way that the geometry demonstrates a direct first-order correspondence to the impulse response. To a good approximation, the impulse response of the SAW filter is the convolution of the impulse responses of the input and output transducers. Subsequently, the frequency response of the filter is the product of the frequency responses of the individual transducers [5]. This relationship is the basis for the

design of all SAW devices. This concept is developed rigourously later in the thesis.

Due to technological advances in the communications industry, there exists a strong motivation to develop SAW filters to meet very stringent specifications. For typical applications in the IF section of microwave radio links, the passband response must be within 0.1 dB of the design specification. The phase ripple over the passband must be less than one degree and the out of band rejection greater than 60 dB [6].

In order to maintain strict control over the frequency response from device to device, a highly accurate design procedure must be developed. The design procedure can be based on simple finite impulse response theory, but in order to achieve the aforementioned design requirements, all second-order effects must be accounted for and, if necessary, compensated for in the filter design.

In many cases, the design procedure consists of repetitive physical iterations. That is, an initial design is fabricated based on a simple model. The filter is then measured and the design is modified to compensate for the deviation from the desired response caused by second-order effects. This iteration process continues until the achieved response falls within the given specifications. Needless to say, such an iterative procedure is time consuming and the development cost increases with the number of

iterations.

An ideal design procedure must be based on a rigorous computer model which will accurately predict the response of any SAW filter when inserted in a network. This will allow the physical iterations to be replaced by faster, less costly computer iterations.

At the present time there appears to be no model which is comprehensive enough to completely describe the complicated behavior of a SAW filter. The most accurate models, which account for some second-order effects are, by necessity, unwieldy and limited to analysis [7].

For economic reasons, the design process is usually limited to a simple first order model. As previously mentioned, to a good approximation, the frequency response of a transducer may be designed using the Fourier transform of the impulse response based on its relationship to the transducer geometry. However, the frequency response of a transducer is also dependent on the non-ideal interaction between the transducer and the electrical source via the intervening matching circuit. This "circuit factor" [8] creates some distortion from the ideal magnitude and phase responses of the filter.

1.3 Thesis Objectives

The two main objectives of this thesis are to model the effects of the embedding network (i.e. the circuit factor) on the SAW filter performance and to derive a compensation algorithm to eliminate the distortion which it introduces.

The major stumbling block in modelling the circuit factor is the ability to accurately model the input admittance of a SAW transducer. To a first-order approximation, the conductance of a SAW transducer varies as the square of the theoretical frequency response [9]. This simple model has been implemented to yield excellent results, however it is limited to narrowband filters on quartz substrates [6].

In this thesis, a three dimensional modal analysis [10,11] has been used to more reliably model the input admittance of a SAW transducer. This theory is based on the solution of the propagating surface wave modes which are supported by the planar transducer structure. The model, implemented in FORTRAN code, produces very good agreement with experiment for several different transducer geometries.

This model of the input admittance of the SAW transducer combined with the matching circuit components can then be analyzed to predict the circuit factor. By cascading the input and output circuit factors with the response of the SAW filter, the overall response may be predicted to a very

high degree of accuracy.

Although the ability to predict the response of a SAW filter with a given source and load is very advantageous, it does not seem very practical or economical to redesign a SAW filter to compensate for the different circuit factors encountered in individual systems. For this reason, the third objective of this thesis is to design a circuit which will enable SAW filters to operate independent of the surrounding network.

The network shown in Figure 1.2 consists of a SAW filter surrounded by wideband buffer amplifiers and the necessary matching structures. The buffer amplifiers compensate for insertion loss through the SAW filter and allow the input and output ports to be matched to 75Ω .

The overall response of the circuit H_T , is the product of the responses of the amplifiers H_{a1} and H_{a2} , the matching circuits C_1 and C_2 , and the SAW filter. The amplifiers are designed with a wide bandwidth so that they have very little undesired effect on the overall response. Therefore, given the ability to accurately model the response of the SAW filter and the matching circuits, the overall circuit response may be accurately predicted.

Figure 1.2 Schematic of cascade circuit

The SAW filter response can then be designed to compensate for non-ideal circuit factor distortion to give the desired overall circuit response. Establishing such a design procedure is the fourth objective of this thesis.

This type of design procedure will provide the ability to manufacture reliable SAW filters with low insertion loss and no need for tuning of the matching networks. From the point of view of a system designer it will be advantageous to have the ability to specify a desired SAW filter response and have the confidence that it will operate to the given specification independent of the system configuration.

This should not only increase the usage of SAW filters, but should also make SAW technology a more competitive alternative to conventional technologies.

1.4 Thesis Outline

Chapter 2 provides an introduction to the physical principles of SAW filters. To ensure proper background, material characteristics of substrates and microwave acoustics are discussed. This includes terms and definitions commonly encountered as well as a brief summary of the physical operation of a SAW filter.

As previously mentioned, the electro-acoustic transducer is the fundamental component of all SAW devices. The interdigital transducer (IDT) is developed in some detail as it is important to have a clear understanding of its operation.

Pertinent second-order effects arising from undesired reflections and spurious signals in the actual filter are described and illustrated. There have been a variety of methods developed to compensate for most of the second-order effects over the past several years. The most successful of these will be explained and others will be mentioned for completeness.

The second-order effects of greatest interest in this text arise from circuit considerations. The basis of SAW circuit theory is the equivalent circuit model developed by Smith et. al. [12]. Starting with this simple model, extensions are added to account for metallization loss and parasitic elements.

To achieve minimum insertion loss a SAW transducer may be matched to the source and the load. Due to the fact that the transducer is predominantly capacitive, a series or shunt inductance is usually sufficient to match the transducer at the filter's center frequency. The issues to be considered when matching will be illustrated in an example of a shunt-matched IDT.

The circuit factor will be mathematically derived and the effects on the filter response will be explained. To illustrate these effects, the circuit factor for a shunt matching structure will be provided. From the given example the circuit factor effects should become obvious.

Chapter 3 covers the basics of SAW filter design. In this thesis a filter is designed as a finite impulse response (FIR) filter using a Fourier Transform approach. Another design method called the Remez exchange algorithm will be mentioned to provide a complete background.

The Fourier Transform method stems from the fact that the impulse response of a transducer has a direct relationship to its physical geometry. Combining this with the property that frequency response is the Fourier transform of the impulse response, the transducer geometries may be determined to produce the desired frequency response.

Due to the finite length of an IDT, some undesired ripple may result. In some cases a smooth window function is necessary to achieve the desired

response. The use of a smooth window function is described although only a simple rectangular window was actually used to design the filters in this thesis.

To illustrate the Fourier Transform design algorithm, a 70 MHz bandpass filter is designed as an example. The filter is specified to have 30% relative bandwidth, inband ripple of less than 0.01 dB and out-of-band rejection to 60 dB.

Chapter 4 commences with a survey of the notable works published with regards to the modelling of the input admittance of an IDT.

The most impressive work is that of [11] presented at the 1985 Ultrasonics Symposium. This modal analysis is developed in detail beginning with the solution of propagating surface wave modes. From the determination of modes, expressions are derived for the charge induced on an IDT by an applied voltage. From the calculated charge, expressions for induced current and admittance have been derived.

The model has been applied to various transducers and results have been compiled demonstrating excellent agreement with experiment.

As previously mentioned, a circuit has been designed combining a SAW filter with buffer amplifiers. Chapter 5 discusses the design and implementation of a D.C. coupled buffer amplifier with a 3 dB bandwidth in excess of 500 MHz. The amplifier exhibits high input impedance and very

low output impedance as well as very small gain ripple.

The amplifier was designed with the aid of a SUPER-COMPACT¹ computer model to identify the parasitic elements and to optimize the frequency response. The actual circuit was constructed using surface mount components mounted on a DUROID² substrate, and placed in a metal package.

Chapter 6 begins with the design of a test circuit comprised of a SAW filter buffered by two amplifiers on a printed circuit board. Several design considerations are presented which improve the circuit performance by reducing second-order effects.

A computer model has been designed to predict the response of this circuit. The theoretical model based on the circuit factor has been described and the results compared with experiment.

Given this ability to model the response of the cascade circuit, a compensation algorithm was developed to predistort the SAW filter response to produce the desired circuit response.

1. Product of COMPACT software.

2. Trademark of Rogers Corp.

Chapter 7 presents the development and implementation of the algorithm. Simulated results are presented for a new design which eliminates the undesired effects modeled in the original SAW filter. The experimental results of this design are presented in comparison to the response predicted by the model.

Conclusions are drawn and improvements for future work are suggested in Chapter 8.

CHAPTER 2

PHYSICS OF SAW DEVICES

2.1 Surface Acoustic Waves and SAW Materials

A mechanical perturbation applied to an isotropic solid will distort the surface in such a way that a "slow" wave will form and propagate. The slow wave of particular interest here is the Rayleigh wave, commonly known as the surface acoustic wave (SAW) [1].

Generally, the energy of a SAW is confined to within a few wavelengths of the surface (Figure 2.1). Also, for a forward propagating wave, the particle motion follows a retrograde elliptical path. Typical phase velocities are in the order of 3000 to 4000 m/s.

Figure 2.1 The propagation of a Rayleigh wave on the surface of an isotropic solid.

For SAW devices, single crystal piezoelectric substrates are used. In a piezoelectric material, an applied electric field has associated with it a mechanical stress or strain. The inherent reciprocity of this piezoelectric effect implies that a propagating SAW has components of electric field as well as physical displacement.

In general, piezoelectric crystals are anisotropic with respect to phase velocity. Consequently, when identifying a crystal it is necessary to specify both the plane of the crystal surface as well as the direction of SAW propagation. For example, the term Y-Z lithium niobate [LiNbO_3] describes a crystal with its surface cut normal to the Y crystalline axis and the Z axis as the direction of propagation.

The phase velocity of a SAW on a particular substrate is usually presented as a continuous function of the propagation angle. Figure 2.2 displays such curves for a SAW on 128° rotated YX LiNbO_3 . Cases for both a free surface [open] and a surface coated with an infinitesimally thin conductor [shorted] have been included. Similar curves for many substrate materials have been documented in [13].

As may be observed, the velocity curves exhibit both maxima and minima over the angular range. At each of these extrema, the power flow vector of the SAW is parallel to the propagation vector. The vector which corresponds to such an angle is known as a pure mode axis [15].

Figure 2.2 Surface wave velocity versus direction of propagation for 128° lithium niobate (LiNbO_3)

If, due to some unintentional misalignment of the transducer, the SAW is not launched along a pure mode axis the acoustic beam (power flow vector) will diverge from the propagation vector. Such a condition is referred to as beam steering [14].

The power flow angle, Φ , is defined as the angle between the power flow vector and the propagation vector. More important is the slope of the

power flow angle with the angle of propagation, $d\Phi/d\theta$, as it is an indication of the extremity of the beam steering condition [14].

Perhaps the single most important SAW material parameter is the acousto-electric coupling coefficient, K^2 . Implied by its name, the coupling coefficient is a measure of a material's ability to transform electrical energy to acoustic energy and vice versa. A good approximation to K^2 is defined in terms of SAW velocity as [12]:

$$K^2 = -2 \frac{\Delta v}{v} = -2 \left(\frac{v_s - v_f}{v_s} \right) \quad (2.1)$$

Where v_f is the velocity on a free surface and v_s is the velocity on a shorted surface. Tables of coupling coefficients for several substrate materials have been tabulated in [25].

In certain SAW materials, the phase velocity is very sensitive to temperature fluctuations. Any change in velocity will alter the time delay through the SAW device. Also, because the wavelength of the propagating SAW varies as velocity, the operating center frequency of the SAW device will shift slightly. (This will become evident in Section 2.2). For many applications, these temperature effects are unimportant. However for applications which allow little tolerance in center frequency or time delay,

the temperature coefficient has a direct influence on the choice of substrate material [15].

Ideally, the SAW device designer desires a substrate with both a high coupling coefficient and a low temperature coefficient. Unfortunately, no substrate material has yet been developed which simultaneously satisfies both requirements. The two most commonly used SAW materials are quartz and LiNbO_3 . ST-X quartz demonstrates a zero first-order temperature coefficient of delay but has a low coupling coefficient of 0.0016. Conversely YZ LiNbO_3 has a relatively large coupling coefficient of 0.048, but has a larger temperature coefficient of $-94 \text{ ppm}/^\circ\text{C}$. Therefore in choosing a substrate the SAW designer must consider the trade-offs between coupling and temperature coefficients. In general, quartz is used for narrowband filters and LiNbO_3 is used for wideband filters as it has been shown that the maximum bandwidth varies directly as the coupling coefficient [12].

2.2 The Interdigital Transducer

The most effective method of launching and detecting surface acoustic waves is through the use of an interdigital transducer (IDT) [16]. A typical IDT, as shown in Figure 2.3, consists of a series of interleaved metallic electrodes (fingers) alternately connected to bus bars of opposite polarity. If a time varying voltage is applied across the bus bars, the resultant electric field between adjacent fingers creates a spatially varying wave via the piezoelectric effect. Inversely, as the launched wave passes under the output IDT, a time varying voltage is induced on the load connected across the bus bars.

At a certain synchronous frequency f_0 , the SAW wavelength ($\lambda_0 = v/f_0$) is exactly equal to the spatial period ($2L$) of the IDT. The wavelets launched by each finger pair interfere constructively, producing a strong excitation of SAW's. At arbitrarily chosen planes to the left and to the right of the IDT, envelopes of SAW with periodicity $2L$ and amplitude proportional to the voltage applied across the fingers may be observed to propagate away from the IDT.

As the frequency departs from f_0 , the constructive interference decreases or the wavelets begin to interfere destructively. At certain frequencies, total destructive interference occurs so that no energy is launched by the IDT. These points appear as nulls in the frequency response of the IDT.

Figure 2.3 The use of interdigital transducers for launching and detecting surface acoustic waves.

The surface wave launched by adjacent fingers may be approximated by a sinusoidal function with period $2L$ as shown in Figure 2.3. Although the IDT is inherently an analog device, the SAW excitation may be modeled as Dirac delta sources located between the fingers as shown in Figure 2.4. At some arbitrary observation point to the right of the IDT in Figure 2.4 the cumulative effect of these delta functions defines the IDT impulse response as:

$$h(t) = \sum_n I_n \delta(t - t_n) \quad (2.2)$$

where I_n indicates the amplitude and the sign of the n^{th} delta source

and t_n indicates the position of the n^{th} delta function as illustrated in Figure 2.4.

The corresponding frequency response of this IDT is the Fourier transform of the impulse response:

$$H(f) = \sum_n I_n e^{-j2\pi f t_n} \quad (2.3)$$

For example, the impulse response of the IDT in Figure 2.4 is a train of sine waves sampled at intervals of $T_0/2$ with the length set by the number of finger pairs in the IDT. This function may be thought of as the product of an infinite sinusoid multiplied by a rectangular function and an infinite sampling function as shown in Figure 2.5. The well known Fourier transforms of each of these three functions are shown adjacent to the time domain functions in this figure.

Now, because multiplication in the time domain corresponds to convolution in the frequency domain, the frequency response is the convolution of the three frequency functions of Figure 2.5. As illustrated, the frequency response of this uniform IDT may be represented as a discrete function with a SINC $((\sin x)/x)$ shape about the centre frequency f_0 .

(a)

(b)

Figure 2.4

- a) The SAW excitation by the electric field between adjacent fingers may be modeled by discrete delta ultrasonic sources.
- b) The impulse response of the IDT is the cumulative effect of all delta sources at some arbitrary time t .

Figure 2.5

- a) Infinite sinusoidal SAW train
- b) Rectangular truncation representing the finite length of the IDT
- c) Discrete sampling points representing fingers
- d) Resulting impulse response

(X denotes multiplication, * denotes convolution)

The frequency response of an IDT may be altered by adjusting the impulse response. This may be accomplished by weighting the fingers of the IDT either through overlap apodization or the selective withdrawal of fingers.

Apodization refers to the variation of the overlap of finger pairs along the length of the IDT. As shown in Figure 2.6(a), the overlap variation creates a spatial variation in the train of launched wavelets. Effectively, this controls the amplitude of the delta function sources so that a very complex time domain function may be achieved.

The impulse response may also be varied using a selected withdrawal of electrodes as shown in Figure 2.6(b). Based on the assumption that a cluster of electrodes acts collectively, the SAW amplitude launched by a cluster is taken to be proportional to the number of electrodes which comprise it. Therefore the resultant weighting is a step function approximation to the desired impulse response. This geometry eliminates diffraction effects related to small overlaps but due to the fact that withdrawal weighting requires a large number of fingers to be efficient, it is recommended that it be used only for filters with less than 20% relative bandwidth [20].

(a)

(b)

Figure 2.6

- (a) The apodized weighting generates a wave train with spatial variation corresponding to the overlap function.
- (b) For withdrawal weighting, the desired impulse is approximated by the removal of selective fingers.

2.3 Second-Order Effects

The second-order effects discussed in this section are loss mechanisms, triple transit echo, edge reflections, electromagnetic feedthrough, bulk wave coupling, diffraction and beam steering. Each of these will be described in turn, as well as methods of dealing with these effects.

2.3.1 Loss Mechanisms

Insertion loss is primarily caused by mismatching the IDTs to the source and load. However, even if the ports of the SAW device are perfectly matched, some insertion loss occurs due to the launching and propagation of the SAW.

An IDT is represented as the three port device shown in Figure 2.7 (a). One port is the electrical input (output) port and the other two ports account for the bidirectionally launched acoustic waves. The entire SAW device is modeled as a pair of three ports joined by a lossless acoustic transmission line. The resistive loads on port three of each IDT represent the absorption of the SAW energy at the ends of the substrate.

Due to the bidirectional nature of the device, only half of the energy available at the input IDT is launched toward the output IDT. Similarly at the output, half of the power passes through the IDT to be absorbed at the substrate end. Since at each transducer a 3dB loss occurs, then the

(a)

(b)

Figure 2.7

- (a) The three-port representation of an IDT. V_s is the electrical source and G_0 represents the acoustic conductance of the substrate.
- (b) A SAW filter is represented by two three-port IDTs connected by a lossless line.

minimum insertion loss through the device is 6dB [20].

Another source of insertion loss is the loss in energy which the SAW suffers while propagating along the substrate. Propagation loss [17] stems from three different mechanisms. Some of the energy is transferred into the substrate in the form of thermally excited elastic waves. Crystalline defects and the surface scratches scatter some surface wave energy and some energy is coupled to longitudinal modes in the air adjacent to the surface. Fortunately these effects only begin to appear at frequencies in excess of 500 MHz and are negligible in the 70MHz range which is of interest here.

Ohmic losses in the metal fingers are observable to some extent in all IDT's. It is only in high frequency devices with long fingers ($>500\lambda$) that these effects cause severe distortions in device response [18].

The finger resistance of a uniform IDT is calculated in a straightforward manner by:

$$R_e = \rho l / 2N_p A \quad (2.4)$$

where ρ is the sheet resistance of aluminum

l is the length of the fingers

N_p is the number of finger periods

A is the cross-sectional area of a finger

2.3.2 Triple Transit Echo

It is impossible to simultaneously match all three ports in a lossless symmetrical three port device [19]. In the IDT three port, if the electrical port and one acoustic port are matched, the other acoustic port must be mismatched. Consequently, a wave launched toward an output IDT with an electrically matched load will be partially reflected at the mismatched acoustic port. This signal, shown in Figure 2.8, propagates back to the input IDT where it is re-reflected toward the output IDT. This attenuated, delayed signal, called triple transit echo (TTE), interferes with the main signal to create distortions in the frequency response of the device.

TTE is related to the main signal by the following expression [20]:

$$\frac{\text{TTE POWER}}{\text{MAIN SIGNAL POWER}} = 6(\text{dB}) + 2 * \text{IL (dB)} \quad (2.5)$$

where IL is insertion loss expressed in dB.

By mismatching the electrical ports, the reflection is reduced but the insertion loss is increased. Hence, as demonstrated by (2.5), to achieve a high degree of triple transit suppression, insertion loss must be increased by mismatching to the electrical ports.

Figure 2.8 Sources of reflections on the SAW substrate.

2.3.3 Edge Reflections

Acoustic reflections at the interdigital electrode edges are caused by the shorting of the electric field under the metal area. Since the electrodes are spaced at half wavelength centers at the synchronous frequency f_0 , the reflections add constructively and, on transducers with a large number of electrodes, these reflections will cause distortions in the filter response.

A double electrode geometry, commonly called a split finger, geometry, provides an effective compensation for these reflections [21]. The electrodes are placed at quarter wavelength centers so that reflections

from successive strips tend to cancel. In uniform overlap IDTs previously discussed, the frequency response was the array factor (i.e. the superposition of all the fingers). In split finger designs, the response of individual fingers must be taken into account. The element factor $e(f)$ for split finger designs is of the form [8]:

$$e(f) = \cos (f / (4f_0)) \quad (2.6)$$

The effect of this factor is illustrated in Figure 2.9. As may be seen, the array factor previously discussed has a SINC shape about f_0 . The element factor introduces a downward slope in the vicinity of f_0 and creates a null at $2f_0$. Therefore, although the split finger geometry compensates for edge reflections, the frequency response of the IDT deviates from the SINC function of the single finger IDT.

Acoustic reflections are also created by the discontinuities at the substrate ends as shown in Figure 2.8. These reflections may be reduced to an acceptable level through the use of an acoustic absorbing material on the ends of the substrate. Alternatively, the ends of the chip can be cut at an angle to the IDT fingers in order to reflect the signal away from the IDT. Both techniques may be used simultaneously to reduce the interference caused by the end reflections.

(a)

(b)

Figure 2.9

- (a) The array factor for an IDT with a non-integer number of finger pairs superimposed on the element factor.
- (b) The product of the array factor and the element factor yields a modified frequency response with a benign slope across the IDT passband.

2.3.4 Electromagnetic Feedthrough

Electromagnetic feedthrough (shown in Figure 2.10) is the direct coupling of an electrical signal from the input of the SAW filter to the output. The feedthrough signal arrives instantaneously at the output where it interferes with the main signal to cause ripples in the frequency response. Through the use of careful circuit board layout and SAW device packaging techniques, the electromagnetic feedthrough can be reduced to 60 dB below the main signal.

2.3.5 Bulk Wave Coupling

Along with the generation of surface waves, the IDT also excites various modes of bulk acoustic waves. These bulk waves (as shown in Figure 2.10) radiate into the substrate and reflect off the bottom surface to reappear at the output transducer. The velocities of bulk modes are higher than surface modes so that the effects appear at frequencies higher than the synchronous frequency of the filter. Hence in some cases the upper stopband becomes seriously degraded due to the bulk wave coupling.

By choosing crystal cuts such as 128° rotated Y-cut LiNbO_3 which does

not support shear bulk mode propagation and by roughing the bottom surface of the substrate to absorb longitudinal bulk waves, these effects may be reduced to an acceptable level [22].

Figure 2.10 Spurious signal generation: Bulk acoustic waves (BAW) are launched in various directions into the substrate and reappear at the output IDT. Electromagnetic (EM) feedthrough travels directly from the input to the output to interfere with the main signal.

2.3.6 Diffraction and Beam Steering

Beam steering, as described in Section 2.1, is the divergence of the acoustic beam from the transducer axis caused by a misalignment between the IDT axis and a pure mode axis.

Diffraction is the spreading of the acoustic beam energy with propagation distance. Figure 2.11 illustrates the combined effects of diffraction and

beam steering. In severe cases, very little energy will be transmitted through the device.

Diffraction has been thoroughly studied and compensation methods have been developed to correct for diffraction related effects [23]. Recent work [11] proposes the use of surface waveguides to contain the energy of the propagating wave.

Figure 2.11 Diffraction and beam steering effects.

2.4 Circuit Theory for the IDT

In this section, circuit theory for the IDT is developed based on an equivalent electrical circuit model. To minimize the insertion loss through the device, it is desirable to match the electrical ports of the IDTs. The problems encountered in attempting to match an IDT are described and an example of a shunt match is provided. Finally, the circuit factor is mathematically described and the effects on the performance of the SAW filter are illustrated.

2.4.1 An Equivalent Circuit Representation of the IDT

An equivalent electrical circuit for the IDT is shown in Figure 2.12. The equivalent circuit consists of a frequency dependent radiation conductance (G_a) and a radiation susceptance (B_a) in parallel with an inter-electrode static capacitance (C_T). For an unapodized IDT, $G_a(f)$ has a SINC^2 shape about the synchronous frequency. The corresponding susceptance is the Hilbert transform of the conductance [24].

Figure 2.12 The basic equivalent electrical circuit model of the IDT consists of a radiation conductance $G_a(f)$ and a radiation susceptance $B_a(f)$ in parallel with the static capacitance C_T .

The static capacitance, C_T has been defined as [25] :

$$C_T = \frac{C_{FF} L N_p v_s}{f_0} \quad (2.7)$$

where $\tilde{L}$ = the acoustic aperture (length of the finger overlap) in terms of λ_0

N_p = the number of finger periods

v_s = the surface wave velocity

f_0 = the synchronous frequency

The parameter C_{FF} is the capacitance per unit length of a single IDT finger period given by [25] :

$$C_{FF} = 2(\epsilon_{PR}^T + 1) \left[6.5(D_x/L_x)^2 + 1.8(D_x/L_x) + 2.37 \right] \text{ pF/m} \quad (2.8)$$

where D_x = the finger width

L_x = the finger period

and ϵ_{PR}^T is the relative dielectric constant given by [25] :

$$\epsilon_{PR}^T = \frac{(\epsilon_{11} \epsilon_{33} - \epsilon_{13}^2)}{\epsilon_0} \quad (2.9)$$

where ϵ_0 is the dielectric constant of free space and ϵ_{11} , ϵ_{33} and ϵ_{13} [26] are the permittivity constants of the crystal (at constant stress) with direction 1 being the direction of propagation and direction 3 being normal to the surface.

In this thesis an extension of the simple circuit model has been used to account for ohmic losses and parasitic elements. With reference to Figure 2.13, R_e represents the series electrode resistance, L_e is the inductance of leads and bond wires and C_e represents stray package and printed circuit board capacitances.

Figure 2.13 Extended equivalent circuit model including the electrode resistance R_e , the lead inductance L_e and the stray capacitance C_e .

2.4.2 Matching Problems

In order to meet insertion loss requirements, it is often necessary to match the IDTs of a SAW filter to a given source and load. To do this, the IDT is designed so that at the synchronous frequency, the radiation resistance ($1/G_a$) matches the source (or load) resistance. The static capacitance C_T is tuned out at f_0 by means of a series or shunt matching inductor.

As an illustrative example, consider matching an unapodized IDT with four finger pairs to a 500 Ohm load using the shunt inductor, L_m shown in Figure 2.14. The IDT is to be fabricated on YZ LiNbO_3 with $f_0 = 100$ MHz. Assume for simplicity that parasitics and ohmic losses are negligible.

Figure 2.14 A shunt inductor is used to tune out the capacitance at f_0 and $G_a(f_0)$ is set equal to R_0^{-1} .

For an unapodized single finger (as opposed to split finger) IDT, the radiation conductance at f_0 , may be reliably taken to be [12];

$$G_o = 8 f_o K^2 C_{FF} N_p^2 w \quad (2.10)$$

where w is the acoustic aperture in meters.

Mathematically the matched condition at f_0 is described:

$$R_o = \frac{1}{G_o + \left(j \omega C_T - \frac{1}{\omega L_m} \right)} \quad (2.11)$$

At f_0 , $\omega C_T = \frac{1}{\omega L_m}$ so that (2.11) reduces to :

$$R_o = \frac{1}{8 f_o K^2 C_{FF} N_p^2 w} \quad (2.12)$$

For YZ LiNbO₃, the coupling coefficient K^2 and C_{FF} are given to be 0.0482 and 464 pF/m, respectively [25]. Therefore, by adjusting the aperture w , the radiation conductance is altered to achieve a resistive match at f_0 :

$$w = (8 f_o K^2 C_{FF} N_p^2 R_o)^{-1} = 7 \text{ mm}$$

The static capacitance C_T for a 7mm aperture is calculated to be :

$$C_T = N_p C_{FF} w = 13 \text{ pF}$$

so that a tuning inductor of $1.9\mu\text{H}$ must be used to match at f_0 .

The advantage of this type of shunt matching is that it is simple and requires few components. More exotic matching structures may be created for specific device requirements but are beyond the scope of this thesis. A number of matching methods are thoroughly described in [20].

2.4.3 The Circuit Factor

The circuit factor is defined here as the complex voltage transfer function of the matching circuit of an IDT. With reference to Figure 2.15 the circuit factor may be defined as V_1/V_s , the ratio of the available voltage at the IDT (V_1) to the total available voltage at the electrical source (V_s).

As an example, consider the IDT of Figure 2.15 matched by a shunt admittance Y_m . In this case the circuit factor is:

$$C(f) = V_1(f)/V_s(f) = (1 + R_s (Y_m + Y_{in}))^{-1} \quad (2.13)$$

Figure 2.15 The response of the IDT is the product of the circuit factor $C_1(f)$ and the FIR response of the IDT $A_1(f)$.

Now, defining the acousto-electric transfer function as $A_1(f)$, the overall transfer function of the IDT is:

$$H_1(f) = C_1(f) \cdot A_1(f) \quad (2.14)$$

Consider the SAW filter of Figure 2.16 with two electrically matched IDT's. The overall frequency response, $H_T(f)$, of the filter is the product of the IDT responses :

$$\begin{aligned} H_T(f) &= H_1(f) \cdot H_2(f) \\ &= [C_1(f) \cdot A_1(f)] \cdot [A_2(f) \cdot C_2(f)] \\ &= C_1(f) \cdot [A_1(f) \cdot A_2(f)] \cdot C_2(f) \\ &= C_1(f) \cdot H_{SAW}(f) \cdot C_2(f) \end{aligned} \quad (2.15)$$

where $H_{SAW}(f) = A_1(f) \cdot A_2(f)$ is the response of the filter due to the acousto-electric interactions.

Therefore, given the acousto-electric response of the filter $H_{SAW}(f)$, the overall response of the matched device may be calculated as the cascade of the two circuit factors and the SAW filter.

Figure 2.16 The overall response of the SAW filter may be represented as the product of the circuit factors $C_1(f)$ and $C_2(f)$, and the electro-acoustic response of the filter $H_{SAW}(f)$.

If the total response $H_T(f)$ from (2.15) is set equal to the desired response

$H_D(f)$, then the SAW filter response $H_{SAW}(f)$ is defined as :

$$H_{SAW}(f) = H_D(f) / [C_1(f) \cdot C_2(f)] \quad (2.16)$$

Thus it is obvious that if the circuit factors are known, then it should be possible to predistort the SAW filter response to compensate for this undesired effect.

CHAPTER 3

SAW FILTER DESIGN

3.1 SAW Filter Design

A typical bandpass filter specification is illustrated in Figure 3.1. The filter is defined in terms of the center frequency, the pass bandwidth, the transition bandwidth and the maximum ripple in both the passband and the stopband. It is the task of a SAW filter designer to derive IDT finger patterns which will produce this specified frequency response.

Two methods most often used in SAW filter design are an analytic method based on Fourier Transforms [27], and the Remez Exchange Algorithm, a numerically iterative procedure [28]. The Remez Exchange Algorithm produces an "optimum" design through numerical methods. It is termed an optimum design in the sense that the smallest transition bandwidth may be achieved for a specified maximum passband ripple.

The Fourier Transform method, which is used in this thesis does not produce a numerically optimum design. However it does provide the designer with analytic relations between the IDT length, the ripple and the transition bandwidth, which the Remez Exchange method cannot do.

Figure 3.1 Typical bandpass filter specifications are:

- 1) f_0 - the center frequency
- 2) B_1 - the transition bandwidth
- 3) B_2 - the pass bandwidth
- 4) R_1 - the pass band ripple maximum amplitude
- 5) R_2 - the maximum stop band ripple (side lobe) amplitude

3.2 The Fourier Transform Design Procedure

In developing a SAW filter, the desired frequency response is first divided into two factors which will correspond to the frequency responses of the input and output IDTs.

If one IDT is apodized and the other is unapodized as in the filter of Figure 3.2, then the overall impulse response of the filter is the convolution of the impulse responses of the individual IDTs [5]. Correspondingly, the

filter frequency response is the multiplication of the individual IDT frequency responses.

In this analysis, the input IDT is chosen to be unapodized. As shown in Figure 3.3, the 4dB bandwidth of the frequency response $H_1(f)$ of the unapodized IDT is inversely proportional to the number of finger pairs, NP, in IDT. Therefore, for a given specification, the number of finger pairs is simply derived from the 4dB bandwidth of the filter specification.

Figure 3.2 For a SAW filter with one apodized and one unapodized IDT, the filter response $H_F(f)$ is the product of the responses of the individual IDT's, $H_1(f) \cdot H_2(f)$.

Figure 3.3 The $\text{SIN}(X)/X$ frequency response of an unapodized IDT showing the 4dB beamwidth and the position of the nulls.

Given the number of finger pairs, the n^{th} null of the response appears at frequency:

$$f_n = \left(1 + \frac{n}{NP}\right) f_0 \quad (n = \pm 1, \pm 2, \pm 3 \dots) \quad (3.1)$$

These nulls must be placed outside the passband of the filter.

Now with reference to Figure 3.2, the filter frequency response is described as:

$$H_F(f) = H_1(f) \cdot H_2(f) \quad (3.2)$$

Setting the filter response equal to the desired response $H_D(f)$, then the response of the output IDT is :

$$H_2(f) = \frac{H_D(f)}{H_1(f)} \quad (3.3)$$

as illustrated in figure 3.4(c) Notice the horns on the required response $H_2(f)$ so that the resulting product of $H_1(f) \cdot H_2(f)$ gives the ideal filter amplitude response.

Taking the inverse Fourier transform of $H_2(f)$ yields the infinite impulse response $h_2(t)$ shown in Figure 3.4(d). Due to physical limitations, the impulse response must be truncated to a finite duration. The finite impulse response, Figure 3.4(f), is obtained by multiplying the infinite impulse response by a rectangular weighting function, Figure 3.4(e). The weighting function is termed a "window" as it determines how much of the impulse response is actually seen. Equivalently in the frequency domain the desired frequency response $H_D(f)$ is convolved with the oscillatory Fourier transform of the window function. In addition to causing

Figure 3.4

a) Frequency response of input IDT

b) Desired response

c) Response of output IDT

d) Infinite impulse response

e) Window function

f) Truncated impulse response

undesired ripples in the passband, this convolving function also determines the transition bandwidth as shown in Figure 3.5.

These oscillations, known as the Gibbs phenomenon [20] may be reduced by using a window function which is a slowly varying time function. The convolving function then has reduced oscillations but a broader bandwidth as shown in Figure 3.6. Therefore the resultant response of the filter has a smaller ripple amplitude but a larger transition bandwidth. The design problem now becomes a matter of finding the optimum window function to give the narrowest transition width and the minimum ripple. There are a number of window functions which may be used. These include the Hamming, Hanning, Blackman and Dolph-Chebyshev functions outlined in [29, 30].

As will be seen in Section 3.3, no window function has been used in the filter design procedure. The filter designed here has been defined in terms of a raised cosine function [29]:

$$P(f) = \begin{cases} \frac{1}{2B_T} & |f| < f_1 \\ \frac{1}{4B_T} \left\{ 1 + \cos \left[\frac{\pi(|f| - f_1)}{2B_T - 2f_1} \right] \right\} & f_1 < |f| < 2B_T - f_1 \\ 0 & |f| > 2B_T - f_1 \end{cases} \quad (3.4)$$

The frequency f_1 and the bandwidth B_T are related by :

$$\alpha = 1 - \frac{f_1}{B_T} \quad (3.5)$$

which is called the roll-off factor.

It was found in designing a filter of this type that the use of a smooth window function was not required to produce the desired inband ripple since a gradual transition has been specified.

Figure 3.5 Truncation of the impulse response by a rectangular window function.

Figure 3.6 Truncation of the impulse by the use of a smoother window function. Note that although the ripple has decreased, the transition bandwidth has broadened somewhat.

3.3 Design Example: A 70 MHz Bandpass Filter

As an illustrative example, a 70 MHz $\alpha = 25\%$ raised cosine bandpass filter with $f_1 = 27$ MHz has been designed. The specification for this filter is shown in Figure 3.7. The filter is to be designed using split fingers on 128° LiNbO₃.

In choosing the number of fingers for the input IDT, note that the -4 dB points are at 57 MHz and 83 MHz, giving a 4 dB bandwidth of 26 MHz. This corresponds to an IDT length of 2.7 finger pairs. Because the fingers must be specified in multiples of half finger pairs, this is rounded to $NP = 2.5$ finger pairs.

Figure 3.7 The specification for a 70 MHz raised cosine ($\alpha = 0.25$) bandpass filter.

In split finger IDTs, the response of individual fingers must be taken into account. The element factor $e(f)$ for split finger designs was defined in equation (2.5). The slope which the element factor introduces into the response of the IDT which was shown in Figure 2.9 is quite evident in the unapodized response of Figure 3.8(b).

The response of the output IDT necessary to produce the desired filter response is shown in Figure 3.8(c). Note the positive slope on this response which compensates for the negative slope of the input IDT.

Hermitian design procedures [31] are used so that the impulse response of the output IDT consists of purely real equally spaced time domain samples.

For a frequency response to be Hermitian, the amplitude response is specified as symmetric about $2f_0$, and the phase response is antisymmetric. Taking the inverse Fourier Transform of the Hermitian specification and experimentally truncating the impulse response yielded the amplitude response in Figure 3.9. An apodized IDT with 312 fingers was found to give a filter response with the specified shape and a passband ripple of less than 0.015 dB pk-pk. As shown in Figure 3.9 the designed response meets the specification almost perfectly down to the first sidelobe peak occurring at -59 dB.

The filter was fabricated on 128° LiNbO_3 and packaged in a 16 pin metal DIP package. Figure 3.10 compares the experimental filter response measured in a $75\ \Omega$ system with the designed response of Figure 3.9. This filter design includes techniques for the suppression of spurious signals as discussed in Chapter 2 [32]. However, there are some discrepancies in the magnitude response as well as a large, unexplained swing in the phase response. Most of these discrepancies will be accounted for and corrected in the circuit factor analysis.

Figure 3.8

- a) Linear magnitude of desired response $H_D(f)$
- b) Response of unapodized IDT, $H_1(f)$
- c) Required response of the output IDT, $H_2(f) / H_1(f)$

(a)

(b)

Figure 3.9 Filter design compared to original specification
 a) designed sidelobe level is at -57 dB
 b) designed passband ripple is less than 0.015 dB pk-pk

Figure 3.10 Measured response of the filter compared to the designed filter response. The solid line represents the designed magnitude and phase. The dashed line is the response of the filter TPCA11-0306 measured on a HP3577 network analyzer.

CHAPTER 4

THE ADMITTANCE MATRIX OF A SAW FILTER

4.1 Admittance Models for SAW IDTs

Since the advent of the first SAW device, much effort has been devoted to modelling the complicated physical behavior of SAW IDTs [7]. The publications available on this subject are too numerous to mention here, but a summary of notable works has been included to provide a background for the work developed in this chapter.

In 1969, Smith et. al [12] published the original SAW IDT model. By extending the Mason model [33] for bulk acoustic wave transducers to SAW transducers, each electrode period was modeled by an equivalent electrical circuit model. Expressions were derived for the Y matrix of each period and these individual matrices were cascaded to produce the overall response of the SAW filter. The most important contribution made was the development of a closed form approximation for IDT input admittance. This approximation is still widely used for uniform IDTs but does not account for apodized patterns.

In 1971, Tancrrell and Holland [5] published the classic delta function model to describe the frequency response of a SAW filter. The acoustic excitation of each finger was modeled as a delta function related to the finger overlap. These delta functions comprised the impulse response from

which the frequency response was derived. It was shown that arbitrary frequency responses could be obtained through apodization. The input admittance for uniform overlap IDTs was modeled by direct application of Smith's theory [12]. The admittance of an apodized IDT was calculated by horizontally "slicing" the pattern into many parallel uniform IDTs to which Smith's model applied. By combining the transfer response and the input admittance each IDT was represented by a three port with a common port connected by an acoustic transmission line. Reasonable results were presented for various filters but the model did not account for any second-order effects.

In 1973, the delta function model (also called the impulse model) was extended by Hartmann et. al. [9] to account for the input admittance. It was shown that, in general, the IDT radiation conductance was directly related to the impulse response and that the radiation susceptance was the Hilbert transform of the conductance. Closed form expressions for the radiation admittance of a uniform IDT were derived to agree exactly with those developed by Smith [12]. However, although this simple impulse model provided a very good approximation to experiment, the theory did not account for physical second-order effects such as diffraction or metal conductor losses.

In 1982, Mader [6] simplified the impulse model approach and included

the effects of diffraction in the model. Results were presented for narrow-band, low shape function filters which closely matched the theory. However, this model was not extended to address filters with wider bandwidths or steeper roll-off factors.

In this thesis the modal analysis of [11] has been adopted to provide an accurate Y-matrix representation of a SAW filter. By modelling the wavefronts created by the IDT fingers excellent agreement may be achieved with the experimental response. The unique property of this theory is the ability to accurately model the radiation conductance of wideband apodized IDTs. It has been implemented in this thesis to produce a SAW filter Y-matrix offering excellent results for the wideband filter designed in Section 3.3.

4.2 A 3-D Modal Analysis of SAW Filters

The three dimensional (3-D) modal analysis of a SAW filter [11] involves the representation of the metallized IDT structures as a thin film acoustic waveguide. This work is a direct extension of the investigations of metal strip piezoelectric waveguides used in SAW convolvers to contain the launched acoustic energy [34-36].

As shown in Section 2.1, the SAW velocity on a metallized surface is slower than on a non-metallized (free) surface. This velocity difference

tends to guide the wave along a deposited metal strip, creating an acoustic waveguide. ♦

An IDT consists of three regions of varying degrees of metallization. Referring to Figure 4.1 it may be seen that the IDT consists of three velocity regions. The bus-bars, which are completely metallized (slow velocity), contain a region of semi-metallized surface (intermediate velocity). The entire IDT is surrounded by a free surface (fast velocity) so that energy should be contained within the IDT region. Therefore, this structure is simply an extension of the simple metal strip waveguide of [34] with an extra velocity region to be considered.

Figure 4.1 SAW filter layout comprising of an apodized IDT and an unapodized IDT with a section of grounded "dummy" fingers in between them.

Notice in Figure 4.1 that a section of grounded fingers has been included between the IDTs. The IDTs must be separated by some distance to minimize electromagnetic feed through, so this grounded section has been included to provide a continuous medium between the input and output IDTs. Although the primary purpose of this grounded section is guiding, it has been observed to eliminate spurious reflections caused by the impedance discontinuity of the IDT edges [32].

Figure 4.2 Representation of the substrate as a thin film waveguide with longitudinal layers of material with fast, slow, and intermediate SAW velocities. The SAW wave propagates in the Z direction.

Figure 4.2 illustrates further this idea of different velocity regions. The guiding theory of [37] has been extended to solve for the propagating SAW modes in such a structure. However, the problem is complicated by the

fact that three different velocity regions exist. The cross section of the IDT, shown in Figure 4.2, consists of "fast" unmetallized regions, "slow" metallized regions and a semimetallized region with an "intermediate" velocity. It should be noted here that the axis convention shown in Figure 4.2 is intended for a general study and these should not be confused with the crystal axes for any particular material.

The wave numbers in each of these regions can be shown to be related by [Appendix I].

$$\begin{aligned} k_z^2 + (1 + \alpha_m) k_{mx}^2 &= k_{mo}^2 \\ k_z^2 + (1 + \alpha_s) k_{sx}^2 &= k_{so}^2 \\ k_z^2 + (1 + \alpha_f) k_{fx}^2 &= k_{fo}^2 \end{aligned} \quad (4.1)$$

where α_m , α_s , α_f are the anisotropic constants on metallized, semimetallized and free surfaces respectively,

k_{mx} , k_{sx} , k_{fx} refer to the wavenumber components along the x-axis for each of the three regions (as shown in Figure 4.2),

k_{mo} , k_{so} , k_{fo} are the Rayleigh wave numbers in the z-direction for the three regions, respectively,

and k_z is the propagation constant in the z direction.

The numerical values for k_{mo} and k_{fo} are readily calculated from the SAW velocity (v) and the given frequency (f) by:

$$k_{mo} = \frac{2\pi f}{v_s} \quad \text{and} \quad k_{fo} = \frac{2\pi f}{v_f} \quad (4.2)$$

In the semimetallized finger region, a propagating wave encounters both metallized (slow) and free (fast) regions. From studies on the behavior of SAWs on a metallic grating it has been shown that the wave number k_{so} may be described by the function [Appendix II]:

$$k_{so}^2 = k_{fo}^2 \left\{ 1 + \frac{K^2}{4} \left[1 + \frac{P_s(-\cos \Delta)}{P_{s-1}(-\cos \Delta)} \right] \right\}^2 \quad (4.3)$$

where P_s is the Legendre function of the first kind of order s .

K^2 is the acousto-electric coupling coefficient

and $\Delta = (\text{finger width}) / (\text{finger period})$

4.3 Solution of the Propagating Modes in a SAW Filter

It has been shown that the energy potential of a SAW propagating in a

metal strip waveguide may be accurately approximated in terms of scalar functions. The analysis developed here is based on the assumption that four possible mode types may exist in the SAW filter structure [11]. From their mathematical definitions, these modes have been labelled symmetric and antisymmetric trigonometric (cos and sin) and hyperbolic (cosh and sinh) modes.

Similar to the development of [38], the symmetric trigonometric mode has been defined in terms of cosine functions. As shown in Figure 4.3, the modal energy distribution function $f_n(x)$, is a cosine function in the metallized and semi-metallized regions. Outside this region the energy potential function decays exponentially to zero.

Mathematically expressed the function $f_n(x)$ is defined:

$$f_n(x) = \begin{cases} A \cos(k_{nsx}x) & 0 < |x| \leq a \\ B \cos(k_{nmx}x) & a < |x| \leq b \\ Ce^{-k_{nfx}(x-b)} & b < |x| \leq c \end{cases} \quad (4.4)$$

where: A, B, C are amplitude constants. k_{nsx} , k_{nmx} and k_{nfx} are the wave numbers in the x direction for the nth mode ($n = 1, 2, \dots$) in the semi-metal, metal and free regions respectively.

and a, b, c are the boundaries as shown in Figure 4.3.

Figure 4.3 An example of a typical modal distribution function. Notice that for $|x| < b$, the function is trigonometric and it decays to zero outside the metallized region.

For a mode to exist, $f_n(x)$ must be continuous across the substrate ($|x| \leq c$). The derivative $df_n(x)/dx$ must also be continuous.

Applying these boundary conditions to $f_n(x)$ at $x = -b, -a, a, b$ yields eight equations which must be solved simultaneously. Due to the symmetry of the structure about the z -axis it suffices to consider only one half of the structure.

Applying the boundary conditions at $x = a$ yields:

$$A \cos(k_{nsx}a) = B \cos(k_{nmx}a + \theta) \quad (4.5a)$$

and

$$-k_{nsx} A \sin(k_{nsx}a) = -k_{nmx} B \sin(k_{nmx}a + \theta) \quad (4.5b)$$

Rearranging equation (4.5a), we obtain:

$$B = \frac{A \cos(k_{nsx}a)}{\cos(k_{nmx}a + \theta)} \quad (4.6)$$

Substituting (4.6) into (4.5b) for B :

$$k_{nsx} \tan(k_{nsx}a) = k_{nmx} \tan[k_{nmx}a + \theta] \quad (4.7)$$

From (4.7) θ may be determined:

$$\theta = \tan^{-1} \left[\frac{k_{nsx}}{k_{nmx}} \tan(k_{nsx}a) \right] - k_{nmx}a \quad (4.8)$$

In a similar fashion applying the boundary conditions at $x=b$ gives two equations:

$$B \cos(k_{nmx}b + \theta) = C e^{-k_{nfx}(b-b)} \quad (4.9a)$$

$$\text{and } -k_{nmx} B \sin(k_{nmx}b + \theta) = -k_{nfx} C \quad (4.9b)$$

The amplitude constant C is then determined to be:

$$C = B \cos (k_{nmx} b + \theta) \quad (4.10)$$

From (4.9) and (4.10):

$$k_{nmx} = \frac{k_{nfx}}{\tan (k_{nmx} b + \theta)} \quad (4.11)$$

Now substituting (4.11) into (4.7) yields a dispersion relation for the waveguide.

$$k_{nsx} \tan (k_{nsx} a) - \frac{k_{nfx} \tan (k_{nmx} a + \theta)}{\tan (k_{nmx} b + \theta)} = 0 \quad (4.12)$$

This equation must be further simplified as it contains three unknowns:

k_{nsx} , k_{nmx} and k_{nfx} . Rearranging (4.1) and substituting into (4.12) for k_{nsx} ,

k_{nmx} and k_{nfx} gives one transcendental equation in terms of the propagation constant k_{zn} :

$$\frac{\sqrt{k_{so}^2 - k_{zn}^2}}{\sqrt{(1 + \alpha_s)}} \tan \left[\frac{\sqrt{k_{so}^2 - k_{zn}^2}}{\sqrt{(1 + \alpha_s)}} a \right] - \frac{\sqrt{k_{fo}^2 - k_{zn}^2}}{\sqrt{(1 + \alpha_f)}} \tan \left[\frac{\sqrt{k_{mo}^2 - k_{zn}^2}}{\sqrt{(1 + \alpha_m)}} a + \theta \right] = 0$$

$$\frac{\sqrt{k_{mo}^2 - k_{zn}^2}}{\sqrt{(1 + \alpha_m)}} b + \theta$$

$$(4.13)$$

where k_{fo} and k_{mo} are defined in (4.2)

k_{so} is defined in (4.3)

and k_{zn} is k_z of equation (4.2) subscripted to denote the corresponding mode. k_{zn} is defined as:

$$k_{zn} = \frac{2\pi f}{v_n} \quad (1 \leq n \leq \infty) \quad (4.14)$$

where f is the frequency of operation

and v_n is the phase velocity of the n th mode in the z -direction.

For a selected frequency f , equation (4.13) is iteratively solved to determine all possible values of k_{zn} . Consequently, the transverse wave numbers k_{nfx} , k_{nsx} , and $k_{nm x}$ may be calculated from (4.1) and the modal distribution function f_n follows by numerically evaluating (4.4).

For completeness, similar derivations to the above are presented in Appendix III for the remaining three mode types.

4.4 Solution of the SAW Potential in the Filter Structure

Given the allowed values of k_{zn} and the related modal function $f_n(x)$, the potential u_n of the n th excited SAW mode at an arbitrary point (x, z) on the substrate surface is defined by:

$$u_n(x,z) = f_n(x) e^{-jk_z z} \quad (4.15)$$

where $f_n(x)$ is the modal function defined in (4.2).

Consider an ideal line source of acoustic radiation located at $z = 0$ on the substrate surface and bounded by $x_1 \leq x \leq x_2$. Assuming that all possible modes are excited by this line source, then the SAW potential at any point (x, z) may be expressed as the linear superposition of all excited modes. That is:

$$u_n(x,z) = \sum_{n=1}^{\infty} B_n f_n(x) e^{-jk_z z} \quad (4.16)$$

where B_n are the linear weighting coefficients (not related to B of equation (4.4)).

Now, based on the assumption of an infinite number of modes being excited, it is possible that all energy launched by the ideal line source will be coupled into acoustic form. If the excitation of the line source is constant along its length ($x_1 \leq x \leq x_2$) and zero elsewhere, then the SAW potential $u_n(x, 0)$ launched by this line source is:

$$u_n(x,0) = \begin{cases} \psi & (x_1 \leq |x| \leq x_2) \\ 0 & (x < x_1, x > x_2) \end{cases} \quad (4.17)$$

where ψ is the amplitude of the launched wave

From (4.16) this becomes:

$$\sum_{n=1}^{\infty} B_n f_n(x) = \begin{cases} \psi & (x_1 \leq x \leq x_2) \\ 0 & (x < x_1, x > x_2) \end{cases} \quad (4.18)$$

Truncating the series to a finite number of modes M , and considering only that region along the length of the source gives:

$$\sum_{n=1}^M B_n f_n(x) = \psi \quad (x_1 \leq x \leq x_2) \quad (4.19)$$

The values of the linear coefficients B_n may be obtained through the use of the Galerkin method [39]. In this method the set of M simultaneous equations to be solved for B_n ($n = 1, \dots, M$) is multiplied by a function $f_l(x)$ ($l = 1, 2, \dots, M$) and integrated over all x .

The result is:

$$\int_{-\infty}^{\infty} \sum_{n=1}^M B_n f_n(x) f_l(x) dx = \psi \int_{x_1}^{x_2} f_l(x) dx \quad (l = 1, 2, \dots, M) \quad (4.20)$$

It has been found [11] that in practice these modes are orthogonal. That is:

$$\sum_{n=1}^{\infty} f_n(x) f_l(x) = 0 \quad (l \neq n)$$

So (4.20) reduces to a single analytic equation from which the linear coefficients B_n may be evaluated:

$$B_n = \psi \frac{\int_{x_1}^{x_2} f_n(x) dx}{\int_{-\infty}^{\infty} f_n^2(x) dx} \quad (4.21)$$

For completeness, expressions for the linear coefficients, B_n , have been derived for each of the four mode types in Appendix IV.

From this analysis it is obvious that a line source of given length $(x_1 \leq x \leq x_2)$ will preferentially excite different modes to various degrees.

To illustrate this point two examples have been included here.

The first example shows the simulated modes excited by a line source 4500 μm long centered in the semi-metallized IDT region for $f=70\text{MHz}$.

2

(a)

(b)

Figure 4.4

(a) The five lowest-order symmetric trigonometric modes in a structure 5500 μm wide at 70 MHz created by a source 4500 μm wide centered about $x=0$.

(b) The symmetric hyperbolic-mode launched by the same source as in (a)

(where $\square = 1$, $\circ = 2$, $\times = 3$, $\Delta = 4$, $\diamond = 5$)

Figure 4.5 The sum of the modes launched by the source in Figure 4.4

With reference to Figure 4.4(a), notice that in this case, the lowest-order symmetric trigonometric mode has been preferentially excited. The second-order mode is excited to a lesser degree and high orders are excited only slightly. As shown in Figure 4.4(b), only one symmetric hyperbolic mode is excited, and the energy of this mode is confined almost entirely to the metallized region. Also, it is interesting to note that because the source is perfectly symmetric about $x=0$, no antisymmetric modes have been excited.

The superposition of all excited modes shown in Figure 4.5 approaches the theoretical rectangular crested excitation. If an infinite

number of modes were considered then the rectangular shape would result.

The second example illustrates the modes launched by a source $1000\text{ }\mu\text{m}$ long centered at $x=500\text{ }\mu\text{m}$. As shown in Figure 4.6(a) the two lowest-order symmetric trigonometric modes are strongly excited. In Figure 4.6(b), only the lowest-order antisymmetric trigonometric mode is strongly coupled with the higher order modes excited to a lesser degree. One interesting point to note in this example is that due to the large distance separating the source from the bus-bar region, the coupling to the hyperbolic modes shown in Figure 4.7 is extremely low. This tends to suggest that the beam should be kept away from the bus-bars in practice to increase the guiding of the launched wavefront.

The sum of the coupled modes shown in Figure 4.8 is approaching a square crested wave centered at $x=500\text{ }\mu\text{m}$. As in the previous example, this function should approach a rectangular function if an infinite number of modes were considered.

(a)

(b)

Figure 4.6

- (a) The five lowest-order symmetric trigonometric modes launched by a source $1000\ \mu\text{m}$ long centered at $x=500\ \mu\text{m}$.
 (b) The five lowest-order antisymmetric trigonometric modes launched by the same source.

(a)

(b)

Figure 4.7

- (a) The symmetric hyperbolic mode launched by the source of Figure 4.6
 (b) The corresponding antisymmetric hyperbolic mode.

Figure 4.8 The sum of the modes launched by the source of Figure 4.6.

The propagating wavefront launched by the wide line source of Figure 4.4 is shown in Figure 4.9(a). At $z=0$, the magnitude of the wavefront appears as in Figure 4.5. As the wave propagates, it may be seen that the energy has spread slightly. Over the distance of 80λ (which is approximately the distance between the IDTs of a typical filter) the wave launched by a wide source does not appear to be greatly affected by diffraction effects.

For the narrower source of Figure 4.6, the wave profile is plotted in Figure 4.9(b). As may be seen, the energy of the wavefront has spread

(a)

(b)

Figure 4.9

- (a) The magnitude profile of the propagating wave launched by the line source of Figure 4.4.
- (b) The magnitude profile of the propagating wave launched by the line source of Figure 4.6.

quite significantly over a propagation distance of 80λ . This behavior agrees closely with classical diffraction theory profiles presented in [40]. Although the beam spreads to a large extent over the plotted distance, it may be seen that most of the launched energy is still confined to within the metal IDT structure illustrating the guiding nature of the device.

4.5 The Admittance Matrix of a SAW Filter

Many references have been made to the IDT structure in the previous analysis. This section examines the excitation of SAW modes by individual finger pairs and derives from the superposition of these launched modes the expression for the Y matrix of the filter.

Consider first a single pair of metal electrodes of opposite electrical polarity on the surface of a piezoelectric substrate. The electric field between the two electrodes is illustrated in Figure 4.10(a). According to the analysis of [11], the excited ultrasonic radiation may be represented by a line source proportional to the length of the finger overlap. As shown in Figure 4.10(b), the line source has been extended midway into the finger break region to account for the fringing effects between finger ends [41].

Figure 4.10

- The electric field between adjacent fingers.
- The acoustic line source approximation for the excitation of SAW. The arrows on the sources indicate that the two sources are 180° out of phase. Notice also that the length of the source is directly proportional to the finger overlap.

The acoustic wave potential A_i generated by the i^{th} line source observed at some arbitrary point $(\bar{x}, z)$ is, from (4.16):

$$A_i(x, z) = -jHV \sum_{n=1}^M B_{ni} f_n(x) e^{\pm jk_{nz}(z-z_i)} \quad (4.22)$$

where: H is the proportionality constant representing the transformation from electric to acoustic energy.

V is the applied voltage

B_{ni} is the linear coefficient for the n^{th} mode launched by the i^{th} line source.

and z_i is the position of the i^{th} line source.

NOTE: the $\pm$ sign of k_{nz} depends on whether $z < z_i$ or $z > z_i$, respectively.

Extending this result to an IDT with $N+1$ fingers (or N line sources between the N fingers) as in Figure 4.11, the total potential $A(x,z)$ may be expressed as the superposition of the contributions from each line source located at z_i ($i = 1, 2, \dots, N$):

$$A(x,z) = \sum_{i=1}^N \left[-j H V \sum_{n=1}^M B_{ni} f_n(x) e^{\pm j k_{nz} (z-z_i)} \right] \quad (4.23)$$

Assuming that the applied voltage has unity amplitude and is a sinusoidal function of time ($V = e^{j\omega t}$) the potential takes the form:

$$A(x,z) = -j H e^{j\omega t} \sum_{i=1}^N \sum_{n=1}^M B_{ni} f_n(x) e^{\pm j k_{nz} (z-z_i)} \quad (4.24)$$

where ω is the angular frequency under investigation.

Figure 4.11 The superposition of N line sources at a point (x, z_q) .

Now, if it is assumed that there are no other sources of SAW radiation on the substrate, then the acoustic potential in a gap at position z_q ($q=1,2,\dots,N$) is the superposition of SAW from all line sources as in (4.24):

$$A_q(x, z_q) = -jHe^{j\omega t} \sum_{n=1}^M f_n(x) \left[\sum_{i=1}^{q-1} B_{ni} e^{-jk_{nz}(z_q - z_i)} + \sum_{i=q}^N B_{ni} e^{+jk_{nz}(z_q - z_i)} \right] \quad (4.25)$$

It has been shown in [42] that the acoustic potential $A_q(x, z)$ will piezoelectrically induce charges of equal magnitude and opposite polarity on the fingers adjacent to the q^{th} gap. Integrating the charge over the

overlap length on the positively polarized finger yields the total charge Q_q induced on the q^{th} positive finger:

$$Q_q = \int_{x_q}^{x_{q+1}} H' A_q(x, z_q) dx \quad (4.26)$$

$$= -jDe^{j\omega t} \sum_{n=1}^M \int_{x_q}^{x_{q+1}} f_n(x) dx \quad T_{nq}$$

where H' is a constant representing the acoustic to electric transformation.

D is a new proportionality constant encompassing H and H' .

x_q, x_{q+1} define the line source in the q^{th} gap as in Figure 4.10.

$$\text{and } T_{nq} = \sum_{i=1}^{q-1} B_{ni} e^{-jk_{nz}(z_q - z_i)} + \sum_{i=q}^N B_{ni} e^{+jk_{nz}(z_q - z_i)}$$

Substituting from (4.21) into (4.26) for $\int_{x_q}^{x_{q+1}} f_n(x) dx$ simplifies to:

$$Q_q = -j D e^{j\omega t} \sum_{n=1}^M \int_{x_q}^{x_{q+1}} f_n^2(x) dx B_{nq} T_{nq} \quad (4.27)$$

Summing the charge induced on the positive fingers yields the total

complex charge induced on the IDT by the voltage V :

$$Q^T = \sum_{q=1}^N \left[-jQe^{j\omega t} \sum_{n=1}^M \int_{-\infty}^{\infty} f_n^2(x) dx B_{nq} T_{nq} \right] \quad (4.28)$$

Rearranging (4.28) gives:

$$Q^T = -je^{j\omega t} D \sum_{n=1}^M \int_{-\infty}^{\infty} f_n^2(x) dx \sum_{q=1}^N B_{nq} T_{nq} \quad (4.29)$$

From this the complex input radiation admittance may be determined to be:

$$\begin{aligned} Y_{in} &= \frac{I}{V} = \left(\frac{dQ^T}{dt} \right) \frac{1}{V} \\ &= j \frac{\omega Q^T}{V} \\ &= \omega D \sum_{n=1}^M \int_{-\infty}^{\infty} f_n^2(x) dx \sum_{q=1}^N B_{nq} T_{nq} \end{aligned} \quad (4.30)$$

where D has been experimentally determined to be $\frac{2\pi K^2}{C_{FF}}$ [11].

As an illustrative example consider a uniform overlap IDT with 2.5 split finger pairs as shown in Figure 4.12. The polarity of the fingers is shown along with the generated SAW potential. For fingers of similar polarity it is assumed that no SAW excitation occurs.

(a)

(b)

(c)

Figure 4.12

The superposition of induced SAW potential and induced charge of a uniform IDT with 2.5 periods of split fingers at (a) 70 MHz, (b) 60 MHz and (c) 42 MHz.

For $f = 70$ MHz in Figure 4.12(a), it can be seen that both in-phase (real) and quadrature (imaginary) components of SAW potential result from the excitation. However, in the gaps with line sources, the superposition of the SAW potential is purely real so that the induced charge is also real.

In Figure 4.7(b), it can be observed that in the excited gaps both in-phase and quadrature components of SAW potential at 60MHz are present and in this case appear almost equal.

At $f = 42$ MHz almost no real charge has been induced indicating that no energy is launched which tends to agree with the predicted null position of a uniform IDT with 2.5 split fingers from [12].

Performing this calculation at 1MHz intervals from 40 MHz to 100 MHz the admittance $Y_{in}(f)$ was calculated and plotted in Figure 4.13. As expected, the radiation conductance $G_a(f)$ displays a skewed $\text{Sin}(X)/X$ shape. However, from the impulse model [9] the radiation conductance was expected to peak at slightly less than 70 MHz similar to the designed amplitude response shown in Figure 3.8. At first glance it was thought that the IDT model predicted an excessively high center frequency, but as shown later, this prediction agrees closely with experiment.

Figure 4.13 The predicted radiation conductance and susceptance of a uniform IDT with 2.5 split fingers.

Figure 4.14 The radiation admittance of the apodized IDT of the filter designed in Chapter 3 with 312 fingers.

Figure 4.14 shows the simulated radiation admittance of the apodized IDT of the filter TPCA11-0306 designed in Chapter 3. With reference to Figure 3.8, note the expected correlation between the FIR designed frequency response and the radiation conductance. From this simulation, it may be seen that the radiation conductance peaks correspond to the horns of the predicted frequency response of the apodized IDT.

Now, consider the two-port representation of the SAW filter in Figure 4.15.

Figure 4.15 Y-matrix representation of the SAW filter as it is simulated in this modal analysis model.

In general, Y_{in} is defined as:

$$Y_{in} = Y_{11} - \frac{Y_{12} Y_{21}}{Y_{22} + Y_L} \quad (4.31)$$

To determine Y_{11} , the output ports of the filter are electrically shorted. For

$Y_L = \infty$ equation (4.31) reduces to:

$$Y_{in} = Y_{11} \quad (4.32)$$

From this it may be concluded that the radiation admittances $Y_{11}^{\text{rad}}(\omega)$ and $Y_{22}^{\text{rad}}(\omega)$ of the SAW filter may be reliably determined using the procedure to calculate input admittances for each IDT. The equivalent circuit representation of the acousto-electric operation of the IDTs is shown in Figure 4.16.

The transadmittance Y_{21} is calculated in a manner similar to that used for input admittance. Assuming that all sources of acoustic radiation are located in the input IDT (i.e. the output ports are shorted) then $Y_{21}^{\text{rad}}(\omega)$ may be realized in a treatment similar to that for $Y_{11}^{\text{rad}}(\omega)$.

Figure 4.16 The equivalent circuit for the radiation Y-matrix of a SAW filter.

With N_1 launching line sources in the input IDT and N_2 line detectors in

the output IDT the charge induced on the output IDT by an applied voltage

$V_1 = e^{j\omega t}$ on the input IDT is:

$$Q_2^T = -j D \sum_{n=1}^M \int_{-\infty}^{\infty} f_n^2(x) dx \sum_{q_2=1}^{N_2} B_{nq_2} T_{nq_2} \quad (4.33)$$

$$\text{where } T_{nq_2} = \sum_{i=1}^{N_1} B_{ni_1} e^{-k_{nz}(z_{q_2} - z_{i_1})}$$

Then $Y_{21}^{\text{rad}}(\omega) = j\omega Q_2^T$.

From reciprocity Y_{21}^{rad} and Y_{12}^{rad} have been set as equal for the derivation of the admittance matrix.

Therefore from the previous derivations, it may be concluded that the acoustoelectric behavior has been represented by a radiation admittance matrix as illustrated by the equivalent electric circuit model of Figure 4.16.

$${}^{\text{rad}}Y(\omega) = \begin{bmatrix} {}^{\text{rad}}Y_{11}(\omega) & {}^{\text{rad}}Y_{12}(\omega) \\ {}^{\text{rad}}Y_{21}(\omega) & {}^{\text{rad}}Y_{22}(\omega) \end{bmatrix} \quad (4.34)$$

This matrix representation is idealized as it does not account for the electrical parasitics as described in Chapter 2. Including these parasitics with the circuit of Figure 4.16, the SAW filter may be represented by the

equivalent circuit of Figure 4.17. Recalling the definitions of C_e , R_e and C_T from Section 2.4, then Y_{11} and Y_{22} of the circuit are readily evaluated.

Simulated results for the apodized and unapodized IDTs are compared to measured input admittances in Figures 4.18-4.21. The input admittance was calculated for three different apodization geometries, i.e.: inline, incline and V-line. The finger overlaps on each apodization are identical, but the transverse finger positions affect the admittance (hence frequency response) as shown. These geometries will not be described here but a detailed study has recently been carried out on this topic in [43].

In all cases, excellent agreement has been achieved between theoretical and experimental input admittance.

Figure 4.17 Equivalent circuit representation of a SAW filter including parasitic elements.

Figure 4.18 The predicted input admittance of an unapodized IDT with 2.5 split finger pairs and a $4500\mu\text{m}$ aperture compared with the input admittance of filter TPCA011-0309 measured on a HP3577 network analyser.

Figure 4.19 The predicted input admittance of an inline apodized IDT with 312 fingers and a 4500 μ m aperture(—), compared to the experimental input admittance measured on a HP3577 network analyser(---).

Figure 4.20 The predicted input admittance of an incline apodized IDT with 312 fingers and a 4500 μ m aperture(—), compared to the experimental input admittance measured on a HP3577 network analyser(---).

Figure 4.21 The predicted input admittance of a V-line apodized IDT with 312 fingers and a 4500 μm aperture (—), compared to the experimental input admittance measured on a HP3577 network analyser(---).

To predict the transfer response of the filter for comparison to experiment, the circuit representation of Figure 4.17 was converted into ABCD matrix form using the following transformations [19]

$$A(\omega) = -Y_{22}^{\text{rad}}(\omega) / Y_{21}^{\text{rad}}(\omega)$$

$$B(\omega) = -1.0 / Y_{21}^{\text{rad}}(\omega)$$

$$C(\omega) = -\Delta / Y_{21}^{\text{rad}}(\omega)$$

(4.35)

$$D(\omega) = -Y_{11}^{\text{rad}}(\omega) / Y_{12}^{\text{rad}}(\omega)$$

$$\text{where } \Delta = Y_{11}^{\text{rad}}(\omega) \cdot Y_{22}^{\text{rad}}(\omega) - Y_{12}^{\text{rad}}(\omega) \cdot Y_{21}^{\text{rad}}(\omega)$$

Also from [19] the parasitic elements were each represented by individual matrices using the transformations for shunt admittances Y :

$$\begin{aligned} A_Y &= 1 & B_Y &= 0 \\ C_Y &= Y & D_Y &= 1 \end{aligned} \quad (4.36)$$

and series impedances Z :

$$\begin{aligned} A_Z &= 1 & B_Z &= Z \\ C_Z &= 0 & D_Z &= 1 \end{aligned} \quad (4.37)$$

Cascading the ABCD matrices for the circuit of Figure 4.17 and converting to S-parameters, the overall transfer response of the filter was predicted by the transformation [19].

$$S_{21}(\omega) = R_L / (A(\omega) * R_L + B(\omega) + C(\omega) * R_S + D(\omega) * R_L) \quad (4.38)$$

where R_L and R_S are the load and source resistances, respectively.

The modeled S_{21} is compared to experiment in Figure 4.22 for filter TPC0306 with $R_S = R_L = 75 \Omega$. With the exception of some yet unexplained slope in the modeled magnitude response, a good match to the measured magnitude response may be observed. The modeled deviation from linear

phase matches experiment to within a few degrees in the passband. The deviation between the theory and experiment is attributed to second-order effects not taken into account in this model.

Figure 4.22 The predicted response of filter(____) TPCA011-0306 compared to the response measured on the HP3577 network analyser(----).

CHAPTER 5

A WIDEBAND BUFFER AMPLIFIER FOR SAW FILTERS

5.1 Amplifier Specifications

As previously described in Section 2.3, it is possible to minimize the insertion loss through a SAW filter by electrically matching the input IDT to the source impedance and the output IDT to the load impedance. However, as discussed in Section 2.2, an electrically matched IDT creates an acoustic mismatch leading to triple transit echo appearing as ripple in the frequency response of the SAW filter. It has been shown that if the input IDT is driven by an ideal voltage source (zero source resistance) the triple transit signal may be eliminated [32]. By virtually shorting the terminals of the input IDT, the double transit signal reflected from the output IDT passes undetected through the shorted fingers of the input IDT to be totally absorbed in the silicon absorber. Therefore it is possible to reduce the insertion loss through the SAW filter by electrically matching the output port without the associated triple transit ripple.

In order to accomplish such a scheme it is necessary to present a low source impedance at the input port of the SAW filter and a high load impedance at the output port. Buffer amplifiers placed on the input and the output of the SAW filter can present the necessary low and high impedances, respectively.

In addition the gain of each buffer amplifier can be designed to compensate for the insertion loss (15 - 20 dB) through the SAW filter.

The objective set was to design a single buffer amplifier which could be used on both the input and the output of the SAW filter. The specifications decided upon for this design were a flat gain of approximately 8 dB (± 0.1 dB) as well as a minimum real input impedance of 2 k Ω and a maximum output impedance of 5 Ω from D.C. to 200 MHz. The maximum output power level capability was specified at 0 dBm and the given power supplies were ± 10 volts.

It should be clarified that although the actual frequency range of interest in this thesis is from 50 MHz to 90 MHz, the amplifier bandwidth was specified from D.C. to 200 MHz to ensure a very flat group delay across the passband of the buffered filter. Another reason for this choice was to keep the design general enough to be used for other I.F. applications.

5.2 Design of a Wideband Buffer Amplifier

The constraints placed upon this design were to minimize the overall component count and to avoid the use of reactive components entirely. The reasoning here was to minimize the size and cost of the device and to leave the design open to possible future integration using thin-film or thick-film integrated circuit technology.

The final design schematic for the buffer amplifier is shown in Figure 5.1. The amplifier consists of a differential pair wideband gain block between two emitter-follower stages to provide the required high input impedance and low output impedance.

The differential pair stage is configured for single-ended inverting gain with feedback applied to the inverting input. The feedback loop is a simple voltage divider with a parallel lead-lag network in parallel with the upper half of the divider for frequency shaping. This network also provides D.C. bias stability throughout the amplifier.

Notice the inclusion of the diode pair D1 in the output stage. In order for the amplifier to be D.C. coupled, the collector-emitter voltage of Q3 would have been 1.4 volts. To increase the linearity of the amplifier D1 was inserted to raise this voltage by another 1.4 volts. Also, to reduce the current through the D.C. coupled feedback network, a transistor wired as a diode has been placed on the base of Q3. Note that although only the base-emitter junction was used it makes a good diode at the low current being applied. The reason that the collector was not shorted to the base of the transistor was that the parasitic inductance and capacitance associated with the transistor package caused the device to oscillate. Also this configuration needs approximately 0.1 mA to achieve the necessary 1.4 volt drop.

The transistors were modeled as three-port devices with S-parameters supplied in the data bank of the program. These S-parameters were derived from the manufacturer's two-port parameters [44] measured in a common-emitter configuration. This conversion is relatively straight forward based on the characterization of bipolar transistors in [45].

The pins of the metal package and the gold wire bonds were modeled as small inductors and based on their physical geometry, the copper interconnects between the surface mount components were modeled as short sections of transmission line.

5.4 Results

The circuit of Figure 5.1 was implemented using surface-mount NE85633 NPN transistors (SOT-23 and SOT-89 packages) and chip resistors (1206 type) on a 62 mil low-loss ($\epsilon_r = 2.2$) RT/Duroid[®] substrate. The total area of the circuit was approximately 0.5 square inches and it was packaged in a 24 pin metal DIP package.

The frequency response of the amplifier measured on an HP8505 network analyser, is shown in Figure 5.3. The frequency response predicted by the equivalent circuit model is also shown and very good agreement can be seen.

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A gain of 8.5 dB $\pm$ 0.1 dB is maintained from D.C. to 300 MHz in a matched configuration.

The measured input and output impedances are plotted in Figures 5.4 and 5.5 respectively. For comparison, the impedances predicted by the equivalent circuit are also shown. Very good agreement was achieved for the input impedance up to approximately 900 MHz. The output impedance showed good agreement up to 200 MHz but deviated from prediction at the higher frequencies.

In conclusion it can be seen that a very high performance amplifier may be designed by careful modeling and the elimination of parasitic elements. It is predicted that a further reduction in size would lead to even better performance of the amplifier.

In the following chapter the cascade circuit of the amplifiers and a SAW filter is presented along with theoretical predictions for the circuit factors and the response of the overall circuit.

Figure 5.3 The modeled magnitude and phase response of the amplifier compared to the measured response (indicated by the dots).

Figure 5.4 The modeled input impedance (plotted as S11) of the amplifier compared to the measured input impedance (indicated by the dots).

Figure 5.5 The modeled output impedance (plotted as S22) of the amplifier compared to the measured output impedance (indicated by the dots).

CHAPTER 6

THE SAW / BUFFER CIRCUIT

In this chapter the SAW/buffer circuit is presented, along with circuit considerations, equivalent circuit representations and comparisons between theory and experiment.

6.1 Circuit Layout Considerations

One of the thesis objectives stated in Section 1.3 was to build and model a circuit consisting of a SAW filter buffered by two amplifiers. In this section the layout of this circuit is described using the wideband buffer amplifier of Chapter 5.

The schematic of the circuit is illustrated in Figure 6.1. The input amplifier has feedback necessary to produce a flat response from D.C. to approximately 130 MHz while driving the capacitive SAW IDT. As with any amplifier of this configuration, driving a capacitive load will cause the response of the amplifier to peak. It was found that by inserting a small series damping resistor (22Ω) between the amplifier and the SAW filter, the response could be made flat with some reduction in bandwidth and a small increase in output impedance.

Figure 6.1 Block diagram of the SAW/buffer circuit.

Shown in Figure 6.2 is the response of the amplifier taken at test point TP-1 attenuated 20 dB so as not to interfere with the amplifier-SAW interaction. As may be seen small ripples appear in both the magnitude and delay responses of the amplifier. These correspond to the peaks in the input admittance of the apodized IDT (as shown in Chapter 4) at 60 MHz and 80 MHz. In this case, because a small source resistance has been used this effect is nearly negligible, but will introduce some small amount of distortion into the SAW filter response.

On the output amplifier, the feedback remains unchanged from that shown in Figure 5.1. The response of the output amplifier with the RL matching network appears to be a shunt tuned circuit with 14 dB of gain as shown in Figure 6.3.

Figure 6.2 Response of the input amplifier taken at test point TP-1. Note the ripples at 60 MHz and 80 MHz due to the radiation admittance of the IDT.

Figure 6.3 The response of the output amplifier plus the matching circuit. The response looks like the tank circuit with 14 dB of gain.

Certain layout precautions were taken to ensure optimum performance of the circuit. The foil side was used to mount components and the amplifier packages were soldered to the circuit board so that a "good" ground was provided for the circuitry inside the metal packages. The RF connectors were mounted on the metal jig under the circuit board and a metal shield was placed across the middle of the jig between the input and the output to reduce EM feedthrough. Isolation between the two amplifiers was provided by using an LC network on the supply rails, and careful use of decoupling capacitors on each amplifier ensured the elimination of jitter on the supply lines. Through the use of these techniques, the EM feedthrough on this circuit was suppressed to a level greater than 70 dB below the main signal.

6.2 The Matching Network

Initially, it was hoped that the shunt tuning network on the output of the SAW filter would be unnecessary. The response of the circuit with no matching on the output of the SAW is shown in Figure 6.4(a). Note that the measured gain of approximately 1.2 dB through the circuit is the sum of the gain of the amplifiers and the SAW filter insertion loss. Although the response is reasonably flat the insertion loss of 23 dB (measured with a

REF LEVEL /DIV MARKER 70 100 000.000Hz
 1.170dB 5.000dB MAG(S21) 1.175dB

CENTER 70 000 000.000Hz SPAN 40 000 000.000Hz
 AMPTD 0.1dBm

(a)

REF LEVEL /DIV MARKER 70 100 000.000Hz
 15.460dB 5.000dB MAG(S21) 15.638dB

CENTER 70 000 000.000Hz SPAN 40 000 000.000Hz
 AMPTD 0.0dBm

(b)

Figure 6.4

- a) The response of the circuit with no matching inductor on the output of the SAW filter.
- b) The response of the circuit with a 410 nH matching inductor on the output IDT

REF LEVEL /DIV MARKER 70 100 000.000Hz
 8.700dB 5.000dB MAG(S21) 8.719dB

(a)

REF LEVEL /DIV MARKER 70 000 000.000Hz
 8.500dB 1.000dB MAG(S21) 8.543dB

(b)

Figure 6.5 The response of the circuit with a 510 Ω damping resistor in the matching circuit. On a large scale (a) the response looks flat but on a 1 dB scale (b) some rounding is seen in the magnitude response.

FET probe) through the SAW filter was thought to be excessive. In Figure 6.4(b), the response of the circuit with a matching inductor $L_m = 410$ nH is shown. In this case, the insertion loss was ~ 8 dB but the Q of the shunt tuning circuit is much too large and the tank circuit dominates the circuit response.

As a compromise, a 510Ω damping resistor was added to the matching network to increase the bandwidth of the match as well as provide reasonable insertion loss (14 dB) shown in Figure 6.5(a). Although the passband response looks reasonably flat on a coarse scale, the finer scale of Figure 6.5(b) reveals that the magnitude is slightly rounded across the passband and the ripples at 60 MHz and 80 MHz seen in Figure 6.2 are evident.

The comparison to the desired response provided in Figure 6.6 shows the deviation of the tuned SAW filter response from the desired specification. As shown, the amplitude response deviates from the desired response at the band edges and the sidelobes are excessively high. Also the center frequency is 200 kHz high and the bandwidth appears slightly greater than desired. The measured phase response is parabolic in nature due to the tuning circuit. As shown, the phase response deviates from the specified linear phase by as much as 15 degrees at the upper band edge.

Figure 6.6 The desired (—) filter response compared to the measured (----) filter response.

6.3. Circuit Model

To simplify the analysis of the circuit the input amplifier was modeled as a simple voltage source with a small source resistance and some associated parasitic elements. Similarly, the impedance presented by the output amplifier was assumed to be some large resistance in parallel with a small capacitance.

The output impedance of the input amplifier was measured on the network analyser and the equivalent circuit model of Figure 6.7(a) was

derived using **SUPER-COMPACT**. The amplifier was then taken to be a voltage source with a $10\ \Omega$ source resistance, a shunt parasitic capacitance of $11\ \text{pF}$ and a series lead inductance of $20\ \text{nH}$ for the circuit model.

(a)

(b)

Figure 6.7

- a) The equivalent circuit model for the output impedance of the driver amplifier.
- b) The equivalent circuit model of the input impedance of the post amplifier.

Similarly, the load presented by the output amplifier was modeled as the shunt circuit of Figure 6.7(b). The amplifier plus the tuned load may be represented as $510\ \Omega$ in parallel with $7\ \text{pF}$ and a shunt inductance of approximately $410\ \text{nH}$. It should be noted that in both cases an empty SAW

package was placed in the socket for the measurements so that the parasitic elements associated with the package were accounted for.

Given the models for the input and output amplifiers, the overall circuit response was calculated as a cascade of transmission matrices (commonly called ABCD matrices) as illustrated in Figure 6.8.

Figure 6.8 The equivalent circuit of the SAW/buffer circuit and the transmission model implementation.

6.4 Results

The results of this cascade shown in Figure 6.9 demonstrate very good agreement with the measured circuit response. Figures 6.10 and 6.11 support this agreement with different load resistors. In Figure 6.10 the measured data for a $10\ \Omega$ load resistor shows good agreement with the cascade matrix model. In this case, the tuning network is essentially shorted so that the response is due almost entirely to the electro-acoustic interaction of the SAW filter. As may be seen the model predicts very closely the phase response but the model predicts a larger slope across the passband than actually measured. It is suspected that this deviation in the magnitude response is due to the charge distribution on the fingers which has not been accounted for in this thesis [52]. This phenomenon will be investigated in future work. Figure 6.11 shows the measured response with a $1\ \text{k}\Omega$ resistor in the matching network. As shown, the model predicts the amplitude distortion quite closely, but the predicted phase deviates from experiment by as much as five degrees at 60 MHz.

Given this ability to model second-order effects it seems reasonable that the desired filter response should now be obtainable through computer iterations. In Chapter 7, a compensation algorithm is developed to overcome these predicted second-order effects and the results of this process are examined.

Figure 6.9 The response predicted by the transmission matrix model of the circuit (—) compared to the measured response (----).

Figure 6.10 The modeled response(—) with a $10\ \Omega$ shunt matching resistor shows good agreement in the phase response but a slightly greater slope across the passband than measured (----).

Figure 6.11 The predicted response (____) with a 1 k Ω shunt matching resistor shows good agreement to the measured response (----).

CHAPTER 7

CIRCUIT FACTOR COMPENSATION

7.1 Circuit Factor Compensation Algorithm

Once it was demonstrated that a reasonable computer model had been established, a compensation algorithm was developed to predistort the SAW filter response to correct for the deviations from the desired response created by the circuit factor.

The algorithm, as shown in Figure 7.1, is relatively straightforward. First the desired SAW filter frequency response is specified in the terms outlined in Section 2.1. The filter is then designed to meet the specification using the Fourier Transform technique described in Chapter 3. When an acceptable mathematical response is achieved the design is analysed using modal analysis. If the result of this analysis is close enough to the original specification the filter is fabricated. Otherwise an error function $E(f)$, is derived by dividing the analysed response $A(f)$ into the designed response

$$D(f) : \quad E(f) = D(f) / A(f) \quad (7.1)$$

A predistorted specification $S'(f)$ is then calculated as the product of the original specification $S(f)$ and the error function $E(f)$:

$$S'(f) = E(f) \cdot S(f) \quad (7.2)$$

$S'(f)$ is then used as the new filter specification and the design procedure is repeated until a satisfactory design is achieved.

Figure 7.1 The SAW filter design algorithm including compensation for the modeled second-order effects.

7.2 Compensated Design Example

As an illustrative example, consider the SAW filter circuit modeled in Figure 6.8 with the tuning circuit consisting of a $510\ \Omega$ resistor and a $410\ \text{nH}$ inductor. Figure 6.6 shows the deviation in both the magnitude and phase responses of the desired filter response. The following series of graphs illustrates the iterative design process used to compensate for these deviations from the desired response. The compensation algorithm begins with specification of a filter such as that shown in Figure 7.2.

Figure 7.2 The original filter specification for the filter design program.

The product of the apodized and unapodized IDT responses resulting from the Fourier transform design algorithm of Chapter 3 is shown superimposed on the filter specification in Figure 7.3. As may be seen the design matches the specification nearly exactly. At this point the modal analysis is applied to the designed finger patterns to include the second-order effects for evaluation. Figure 7.4 shows the comparison of the analysed response with the filter specification in both the magnitude and the phase response. This large deviation requires that an error function be derived and a predistorted filter specification be calculated.

Figure 7.3 The designed response (____) shows nearly exact agreement to the specification (----) to -45 dB.

Figure 7.4 The modal analysis (____) of the designed filter shows that the design deviates somewhat from the filter specification (----).

Figure 7.5 The predistorted filter specification derived from the error function (----) superimposed on the analysed response (____).

The product of the error function and the original filter specification gives the predistorted filter specification shown in Figure 7.5. As may be seen the predistorted specification is just a mirror image of the analysed response about the desired response. That is, where the analysis predicts a deviation from the desired response, the predistorted response shows an equal deviation of the opposite sign.

The predistorted specification is substituted for the original filter specification and the design loop is repeated. Figure 7.6 shows the designed response compared to the predistorted specification. The designed magnitude appears to meet the specification nearly exactly. The designed phase appears to have the proper shape but demonstrates some linear deviation from the specification. This may just be an effect of the method used to remove the linear phase component from the response.

The analysed response compared to the desired filter response in Figure 7.7 shows a closer match than in Figure 7.5, but some deviation is still present. A close match to the specification has been achieved in the magnitude shape with an expected upslope across the passband (ie: the same deviation seen in the comparison to experiment in Figure 6.4). The sidelobe in the lower passband has been decreased by approximately 3dB and the sidelobe in the upper passband appears to have been replaced by a softer transition.

Figure 7.6 The designed response (----) meets the predistorted specification (____) closely in magnitude but deviates somewhat in phase.

Figure 7.7 The analysed response (____) appears to match the desired filter response (----) reasonably well in magnitude but the phase still shows some deviation from the zero degree specification.

A second error function was calculated and a second predistorted response, shown in Figure 7.8, was adopted as the new specification for a third pass through the design algorithm. The comparison of the second predistorted response to the analysed response of the second iteration illustrates the iteration process. Essentially the deviations from the desired response in the new specification have been increased from that of the previous specification. The new specification has been predistorted to a greater degree than the first iteration because the first iteration did not achieve the desired response.

Figure 7.8 The second predistorted specification (----) compared to the analysed response (—). As may be seen the predistortion has been increased to overcome the deviations from the desired response which the first iteration failed to remove.

Figure 7.9 shows the third iteration design compared to the predistorted specification. In this case the designed response (dashed line) meets the specification almost exactly in both magnitude and phase. The analysis of this design shown in Figure 7.10 meets the specification closely in magnitude and the phase deviation has been minimized over the passband with a 15 degree spike near the upper passband limit. Notice also that the upper transition bandwidth has been reduced to meet the specification more closely than the previous iteration shown in Figure 7.7.

The result of the third iteration shown in Figure 7.10 was accepted as being close enough to the desired filter response and this filter was fabricated on 128°LiNbO_3 .

Although more iterations may have produced a better filter, it must be kept in mind that the primary purpose of this experiment was to prove the validity of the modal analysis model and compensation algorithm. As a precautionary move the iteration process was stopped at the third iteration. This was considered to be sufficient to demonstrate the theory.

Figure 7.9 The resulting response of the third design iteration (---) meets the specification (____) nearly exactly.

Figure 7.10 The analysed response (____) compares quite closely with the desired filter response.

7.3 Experimental Results of the Compensation Algorithm

The filter SAW03I was mounted in a 16 pin metal DIP identical to the original filter, TPCA011-0309, and tested in the same set of electronics with the modeled matching network. Figure 7.11 presents a comparison of the predicted response shown previously in Figure 7.10 with that measured on the HP3577 network analyser. On the coarse scale of Figure 7.11(a) the measured response appears to give excellent agreement to the predicted response. On the finer scale in Figure 7.11(b) the passband response shows agreement within 0.1 dB at the midband with a larger deviation of 0.3 dB near the band edges. The phase response matches nearly exactly to the predicted response. Note that even the ripples at 80 MHz and 84 MHz appear to have been predicted very closely to the actual response.

Figure 7.12 shows the experimental response compared to the filter specification. The coarse scale comparison of Figure 7.12(a) shows good agreement in both magnitude and phase with the exception of the deviation in the upper stopband transition which was predicted in Figure 7.10. On a fine scale in Figure 7.12(b) the magnitude response shows agreement within 0.1 dB in magnitude across the passband (60 MHz to 80 MHz) and less than two degrees of phase distortion.

(a)

(b)

Figure 7.11

- (a) On a coarse scale the predicted response(____) shows excellent agreement in both the magnitude and phase with the measured response(-----).
- (b) On a finer scale the experimental magnitude(-----) is flatter than predicted but the phase matches the prediction (____) within a few degrees.

(a)

(b)

Figure 7.12

- (a) On a coarse scale the measured response (----) shows excellent agreement in both the magnitude and phase response with the desired filter response (____)
- (b) On a finer scale the experimental magnitude (----) is within 0.3 dB of the desired passband response (____) and the phase matches within two degrees.

To further illustrate the effectiveness of this model, the predicted input admittances of the unapodized and apodized IDT's are shown in Figures 7.13 and 7.14, respectively. The unapodized IDT admittance does not differ from the original design as it was not changed from the previous design. The predicted apodized IDT admittance for the filter SAW03C shows very good agreement to that measured on the network analyser.

Figure 7.13 The input admittance of the unapodized IDT matches that of the iterated design very closely.

Figure 7.14 The predicted input admittance (____) of the iterated apodized IDT shows excellent agreement to experimental measured admittance (----).

Figure 7.15 Comparison of the input conductance of the original (____) and iterated(----) apodized IDT's.

Comparing the conductance of the iterated design to the original design in Figure 7.15, the difference in the frequency responses of the two filters becomes evident. The iterated conductance has more equal peaks and a lower sidelobe on the upper stopband than the original design. This reduced slope in the conductance versus frequency characteristic reflects the downslope in the third iteration design shown in Figure 7.9. The reduced sidelobe level is evident in the low sidelobe level of the measured frequency of the iterated filter shown in Figure 7.14(a).

The results of this experiment have demonstrated both the reliability of the modal analysis model and the improved filter response which may be obtained on a first pass using the circuit factor compensation algorithm.

CHAPTER 8

CONCLUSIONS AND FURTHER WORK

8.1 General Conclusions

An accurate Y-matrix representation of a SAW filter has been achieved based on the modal analysis theory. The model predicted very closely the input admittances of both apodized and unapodized IDTs and very accurately demonstrated the effect of the apodization geometry on the input admittance of an IDT. Although the model was originally intended to be used to predict the input admittances of the SAW filter, it was found that the transadmittance was also accurately predicted. The model for the SAW filter cascaded with the surrounding buffer electronics implemented in this thesis gives a very good approximation to the measured frequency response in both magnitude and phase.

The electronics intended to buffer the SAW filter were successfully designed and implemented on a printed circuit board. The wideband buffer amplifier operated beyond the original specification in a matched configuration. Some reduction in bandwidth was necessary for the input buffer amplifier in order to drive the SAW filter, but the overall circuit performance was satisfactory for this project.

Based on the Y-matrix model of this SAW/buffer circuit, a compensation algorithm has been derived and a design procedure has been implemented

to correct for the second-order circuit factor distortion.

As a demonstration of the design procedure, a filter based on three computer iterations of the compensation algorithm was fabricated. As shown in Section 7.3, the experimental results of this iterated filter design showed great improvements over the uncorrected benchmark design in both the magnitude and phase responses. Also, the iterated filter response was very close to the predicted filter response demonstrating the accuracy and reliability of the computer model.

With some small improvements this design program will prove to be not only a cost effective design tool for SAW filter production but also a tool to predict and correct problems encountered in SAW filter design.

8.2 Further Work

There remain many unexplored avenues which this thesis has opened. The prime area in which further work is required is in the SAW filter model. As seen throughout this thesis the modal analysis model predicts an upslope in the amplitude response which does not exist in the experimental data. The cause of this discrepancy has not yet been discovered but it is postulated that this slope comes from an improper representation of the charge distribution on the IDT fingers [52]. Given that this slope could be removed, then it is predicted that the model would

produce nearly exact agreement with experiment.

As seen in the iteration example the upper transition of the iterated filter became less steep after iteration. It is thought that this phenomenon results from the fact that the truncated impulse is too short to accurately achieve the complicated predistorted filter specification. Some simulations were performed with an increased number of fingers and it was found that the addition of 100 fingers in the apodized IDT greatly remedied this problem. In future designs this approach will be used to produce a better iterated frequency response.

Although the compensation algorithm used in this thesis produced reasonable results, it did not work as well as originally expected. It was thought that if enough iterations were used then eventually the desired filter could be achieved. In practice, the algorithm failed to produce a filter to exactly meet the specified response, independent of the number of iterations. Some investigation into this problem is necessary to guarantee that the algorithm gives reliable results.

The algorithm is based on the assumption that the overall frequency response of the filter circuit is a multiplicative product of the transfer functions of the SAW filter and the surrounding buffer circuits. However, the SAW filter is not exactly matched at either port, so this relationship does not hold true. This was the reason that the transmission matrix

representation was adopted in the model of Chapter 4.

If the filter specification is defined in terms of the ABCD matrix then the predistorted SAW filter frequency response will be the inverse of the input circuit factor matrix cascaded with the specification matrix, cascaded with the inverse of the output circuit factor matrix. This method should show some improvement as the basis is mathematically valid. Because the input admittances do not significantly change in the iteration process, then the specification should be achievable in the first iteration.

In this thesis the model was applied to a single filter design. To prove the robustness of the model, it is intended that the model be applied to filters of various center frequencies and shape factors. Also a study of narrow aperture filters, similar to that presented in [12] should be investigated to identify the effect of the modal energy distribution on the frequency response of the filter. From an economic point of view it would be advantageous to use a smaller aperture as the number of filters per wafer could be significantly increased. Some transverse mode compensation may be necessary as in [31], but with the use of the model developed in this thesis this should be easily achieved.

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APPENDIX I

DERIVATION OF WAVENUMBER RELATIONS

Consider the surface shown in Figure A1.1 which has elliptical anisotropy. The ellipse results from the projection of the optical index ellipsoid onto the plane of incidence and the major and minor axes refer to the ordinary and extraordinary indices n_o and n_e respectively. The index $n(\theta)$, appropriate to an extraordinary wave with wavevector propagation at an angle θ to the optic axis is [46] :

$$\frac{1}{n^2(\theta)} = \frac{\cos^2(\theta)}{n_o^2} + \frac{\sin^2(\theta)}{n_e^2} \quad (\text{A1.1})$$

Figure A1.1 Projection of the index ellipsoid on the plane of incidence.

The ordinary and extraordinary wave vectors are:

$$k_o = \frac{2\pi n_o}{\lambda} \quad \text{and} \quad k_e = \frac{2\pi n_e}{\lambda} \quad (\text{A1.2})$$

and the incident wavevector is defined as:

$$k(\theta) = \frac{2\pi n(\theta)}{\lambda} \quad (\text{A1.3})$$

where λ = the optical wavelength in free space.

Substituting into equation (A1.1) gives [47]:

$$\frac{1}{k^2(\theta)} = \frac{\cos^2(\theta)}{k_o^2} + \frac{\sin^2(\theta)}{k_e^2} \quad (\text{A1.4})$$

Rearranging equation (A1.4):

$$k(\theta) = \frac{k_o}{\cos(\theta)} \left[1 + \left(\frac{k_o}{k_e} \right)^2 \tan^2(\theta) \right]^{-\frac{1}{2}} \quad (\text{A1.5})$$

Using the small angle approximation :

$$\cos(\theta) = 1 - \frac{\theta^2}{2}$$

$$\text{and } \tan(\theta) = \theta$$

Then (A1.5) becomes :

$$k(\theta) = k_o \left(1 - \frac{\theta^2}{2} \right)^{-1} \left[1 + \left(\frac{k_o}{k_e} \right)^2 \theta^2 \right]^{-\frac{1}{2}} \quad (\text{A1.6})$$

Using the binomial theorem for small arguments:

$$\left(1 - \frac{\theta^2}{2}\right)^{-1} = 1 + \frac{\theta^2}{2}$$

and

$$\left[1 + \left(\frac{k_0}{k_\theta}\right)^2 \theta^2\right]^{-\frac{1}{2}} = 1 - \frac{1}{2} \left(\frac{k_0}{k_\theta}\right)^2 \theta^2 \quad (\text{A1.7})$$

Then equation (A1.6) becomes:

$$\begin{aligned} k(\theta) &= k_0 \left(1 + \frac{\theta^2}{2}\right) \left[1 - \frac{1}{2} \left(\frac{k_0}{k_\theta}\right)^2 \theta^2\right] \\ &= k_0 \left[1 + \frac{\theta^2}{2} - \frac{\theta^2}{2} \left(\frac{k_0}{k_\theta}\right)^2 - \frac{\theta^4}{4} \left(\frac{k_0}{k_\theta}\right)^2\right] \\ &= k_0 \left\{1 - \frac{\theta^2}{2} \left[\left(\frac{k_0}{k_\theta}\right)^2 - 1\right]\right\} \quad (\text{dropping } \theta^4 \text{ term}) \\ &= k_0 \left\{1 - \gamma \frac{\theta^2}{2}\right\} \quad (\text{A1.8}) \end{aligned}$$

$$\text{where } \gamma = \left(\frac{k_0}{k_\theta}\right)^2 - 1$$

The anisotropy constant γ is determined from velocity curves such as that shown in Figure 2.2. From these curves it is apparent that near the pure mode axis the velocity curves are approximately parabolic.

Mathematically, this parabolic approximation to velocity anisotropy [40] may be expressed by rearranging equation (A1.8):

$$v(\theta) = v_0 \left[1 + \gamma \frac{\theta^2}{2} \right] \quad (\text{A1.9})$$

where v_0 is the phase velocity along a pure mode axis

and $v(\theta)$ is the phase velocity in along some vector at an angle θ to the pure mode axis.

With reference to the slowness curve of Figure A1.2, the pure mode axis is defined as the Z-axis. Then k_0 may be defined as k_{z0} and k_e maps to k_{x0} where k_{z0} and k_{x0} are defined in equation (4.1).

Figure A1.2 Slowness curve on an anisotropic substrate with the Z-axis as the pure mode axis.

Rewriting equation (A1.4) in these wavenumbers gives:

$$k^2(\theta) \left[\frac{\cos^2(\theta)}{k_{z0}^2} + \frac{\sin^2(\theta)}{k_{x0}^2} \right] = 1$$

$$\text{or} \quad \frac{k_z^2}{k_{z0}^2} + \frac{k_x^2}{k_{x0}^2} = 1 \quad (\text{A1.10})$$

$$\text{where} \quad k_z^2 = k^2(\theta) \cos^2(\theta)$$

$$\text{and} \quad k_x^2 = k^2(\theta) \sin^2(\theta)$$

Now in equation (A1.8), γ may be written:

$$\gamma = \left[\frac{k_{z0}}{k_{x0}} \right]^2 - 1$$

so that

$$k_{x0} = k_{z0} (1 + \gamma)^{-1/2} \quad (\text{A1.11})$$

Substituting (A1.11) into (A1.10) gives :

$$\frac{k_z^2}{k_{z0}^2} + \frac{(1 + \gamma) k_x^2}{k_{z0}^2} = 1$$

$$\text{or } k_z^2 + (1 + \gamma) k_x^2 = k_{z0}^2 \quad (\text{A1.12})$$

Equation (A1.12) is the required relation between the wavenumbers. Equation (4.1) has been derived by substituting the wave numbers for the given media and γ has been replaced by $\alpha_s, \alpha_m, \alpha_f$ where s. represents the semi-metallized region, m is the metal region and f is the free region.

APPENDIX II

DERIVATION OF k_{so}

Floquet's theorem states that in an infinite periodic structure, the field of a Bloch wave repeats at every terminal plane except for a propagation factor $e^{-\gamma d}$, where d is the length of one periodic cell [48]. The electric field in a periodic structure such as that shown in Figure A2.1 may be expressed as an infinite sum [49]:

$$E_z = \sum_{n=-\infty}^{\infty} A_n e^{-j(k+2\pi n/p)z} \quad (A2.1)$$

where A_n is a constant

and k is the wavenumber of the fundamental spatial mode

Each space harmonic results in a normal component D_x given by [49]:

$$D_{x,x} = -jS(k) \frac{x}{|x|} \epsilon A e^{-jkz} e^{-S(k) k|x|} \quad (A2.2)$$

where $S(k)$ is the sign of k .

The superposition of these harmonics at the boundary $x=0$ is :

$$D_{x,I,II} = \pm \sum_{n=-\infty}^{\infty} A_n S(k + 2\pi n/p) \epsilon_{I,II} e^{-j(k+2\pi n/p)z} \quad (A2.3)$$

where $\epsilon_{I,II}$ are the dielectric constants of regions I and II

and the +/- refer to media I and II respectively.

Figure A2.1 An infinite array of metal strips on the surface of piezoelectric substrate.

To simplify notation, let:

$$\theta = 2\pi z / p$$

$$s = kp / 2\pi$$

and define

$$S_n = \begin{cases} +1 & n \geq 0 \\ -1 & n < 0 \end{cases}$$

Then equations (A2.1) and (A2.3) become:

$$E_z = \sum_{n=-\infty}^{\infty} A_n e^{-j(s+n)\theta} \quad (A2.4)$$

and

$$D_{x\,I,II} = \pm \sum_{n=-\infty}^{\infty} A_n \epsilon_{I,II} S_{n+N} e^{-j(s+n)\theta} \quad (A2.5)$$

where N is the largest integer less than Re(s).

The boundary conditions at $x=0$ require $E_z=0$ on the strips. That is, $E_z=0$ for

$|\theta| < \Delta$ where $\Delta = \pi w/p$. Also D_x must be continuous between the strips

(ie: $\Delta < |\theta| < \pi$). Therefore:

$$E_z = \sum_{n=-\infty}^{\infty} A_n e^{-j(s+n)\theta} = 0 \quad |\theta| < \Delta \quad (\text{A2.6})$$

$$\text{and } D_x = \sum_{n=-\infty}^{\infty} A_n \epsilon_n S_{n+N} e^{-j(s+n)\theta} = 0 \quad \Delta < |\theta| < \pi \quad (\text{A2.7})$$

It may be shown that a solution which simultaneously satisfies both of these conditions is [50]:

$$A_n = \sum_{m=N_1}^{N_2+1} \alpha_m S_{n-m} P_{n-m}(\cos \Delta) \quad (\text{A2.8})$$

where α_m is a constant

P_{n-m} is the Legendre function of the first kind [51]

and N_1, N_2 are constants

The voltage on a strip is found by integrating E_z :

$$V = -j \frac{p}{2\pi} \sum_{n=-\infty}^{\infty} \frac{A_n}{n+s} e^{-j(s+n)\theta} \quad (\text{A2.9})$$

As the metal strip is an equipotential surface, it suffices to calculate $V(z=0)$:

$$V = -j \frac{\rho}{2\pi} \sum_{n=-\infty}^{\infty} \frac{A_n}{n+s} \quad (\text{A2.10})$$

Substituting (A2.8) into (A2.10) for A_n :

$$\begin{aligned} V &= -j \frac{\rho}{2\pi} \sum_{n=-\infty}^{\infty} \frac{1}{n+s} \sum_{m=N_1}^{N_2+1} \alpha_m S_{n-m} P_{n-m}(\cos \Delta) \\ &= -j \frac{\rho}{2\pi} \sum_{m=N_1}^{N_2+1} \alpha_m V_m \end{aligned} \quad (\text{A2.11})$$

where:

$$\begin{aligned} V_m &= \sum_{n=-\infty}^{\infty} \frac{S_{n-m} P_{n-m}(\cos \Delta)}{n+s} \\ &= \sum_{n=-\infty}^{\infty} \frac{S_{(n-m)+m} P_{(n-m)+m}(\cos \Delta)}{(n+s)+m} \\ &= \sum_{n=-\infty}^{\infty} \frac{S_n P_n(\cos \Delta)}{n+s+m} \\ &= \sum_{n=0}^{\infty} \frac{P_n(\cos \Delta)}{n+s+m} + \sum_{n=1}^{\infty} \frac{P_{n-1}(\cos \Delta)}{n-s-m} \\ &= \sum_{n=0}^{\infty} \frac{P_n(\cos \Delta)}{n+s+m} + \sum_{n=0}^{\infty} \frac{P_n(\cos \Delta)}{n+1-s-m} \\ &= \sum_{n=0}^{\infty} P_n(\cos \Delta) \left[\frac{1}{n+s+m} + \frac{1}{n+1-s-m} \right] \end{aligned} \quad (\text{A2.12})$$

Note also that for a Legendre function $P_{-n} = P_{n-1}$ [53].

Dougall's expansion states that [51]:

$$P_v^{-\mu}(\cos \theta) = \frac{\sin(v\pi)}{\pi} \sum_{n=0}^{\infty} (-1)^n \left[\frac{1}{v-n} - \frac{1}{v+n+1} \right] P_n^{-\mu}(\cos \theta)$$

where $-\mu$ denotes the function type.

Note also that for a Legendre function [51]:

$$P_n(-x) = (-1)^n P_n(x)$$

Using this property:

$$P_v^{-\mu}(-\cos \theta) = \frac{\sin(v\pi)}{\pi} \sum_{n=0}^{\infty} \left[\frac{1}{v-n} - \frac{1}{v+n+1} \right] P_n^{-\mu}(-\cos \theta)$$

Then letting $v = m-1+s$, V_m may be expressed as:

$$\begin{aligned} V_m &= \frac{\pi P_{m-1+s}(-\cos \Delta)}{\sin((m-1+s)\pi)} \\ &= \frac{\pi P_{m-1+s}(-\cos \Delta)}{\sin((m-1)\pi) \cos(s\pi) + \cos((m-1)\pi) \sin(s\pi)} \\ &= \frac{(-1)^m \pi P_{m-1+s}(-\cos \Delta)}{\sin(s\pi)} \quad (s \neq \text{integer}) \end{aligned} \quad (\text{A2.13})$$

Substituting equation (A2.13) into (A2.11) for V_m gives :

$$V = \frac{-j p}{2 \sin(s \pi)} \sum_{m=N_1}^{N_2+1} (-1)^m \alpha_m P_{m-1+s}(-\cos \Delta) \quad (\text{A2.14})$$

For practical purposes, if $0 < s < 1$, then N_1 and N_2 are both taken to be zero

[51]. Equation (A2.14) then becomes:

$$\begin{aligned} V &= \frac{-j p}{2 \sin(s \pi)} \sum_{m=0}^1 (-1)^m \alpha_m P_{m-1+s}(-\cos \Delta) \\ &= \frac{-j p}{2 \sin(s \pi)} \left[\alpha_0 P_{s-1}(-\cos \Delta) - \alpha_1 P_s(-\cos \Delta) \right] \quad (\text{A2.15}) \end{aligned}$$

$$\text{where [51] : } \alpha_0 = A_0 \left[\frac{k - k_v + \frac{1}{2}(k_0 - k_v)}{k - k_0} \right]$$

$$\text{and } \alpha_1 = \frac{1}{2} A_0 \left[\frac{k_0 - k_v}{k - k_0} \right]$$

where k_v is the wavenumber on a free surface

k_0 is the wavenumber on a shorted surface

A_0 is the amplitude of the electric field

For shorted fingers, which is the case under study for the SAW filter structure presented in Chapter 4, $V=0$ so (A2.13) becomes:

$$\alpha_0 P_{s-1}(-\cos\Delta) - \alpha_1 P_s(-\cos\Delta) = 0$$

$$\Rightarrow A_0 \left[\frac{k - k_v - \frac{1}{2}(k_0 - k_v)}{k - k_0} \right] P_{s-1}(-\cos\Delta) - \frac{1}{2} A_0 \left[\frac{k_0 - k_v}{k - k_0} \right] P_s(-\cos\Delta) = 0$$

$$\Rightarrow \left[k - k_v - \frac{1}{2}(k_0 - k_v) \right] P_{s-1}(-\cos\Delta) - \frac{1}{2} [k_0 - k_v] P_s(-\cos\Delta) = 0$$

$$\Rightarrow [k - k_v] P_{s-1}(-\cos\Delta) - \frac{1}{2} [k_0 - k_v] [P_{s-1}(-\cos\Delta) + P_s(-\cos\Delta)] = 0$$

$$\Rightarrow k - k_v - \frac{1}{2} [k_0 - k_v] \left[1 + \frac{P_s(-\cos\Delta)}{P_{s-1}(-\cos\Delta)} \right] = 0$$

$$\Rightarrow k = k_v + \frac{1}{2} [k_0 - k_v] \left[1 + \frac{P_s(-\cos\Delta)}{P_{s-1}(-\cos\Delta)} \right] \quad (\text{A2.16})$$

Recall now the definition of the electroacoustic coupling coefficient defined in Section 2.1:

$$K^2 = \frac{-2\Delta v}{v} = -2 \left(\frac{v_0 - v_v}{v_0} \right) \quad (\text{A2.17})$$

Substituting $v = \frac{\omega}{k}$ into (A2.17):

$$K^2 = 2 \left(\frac{k_0 - k_v}{k_v} \right) \quad (\text{A2.18})$$

Rearranging equation (A2.18) :

$$\frac{1}{2} (k_0 - k_v) = \frac{k_v K^2}{4} \quad (\text{A2.19})$$

Substituting equation (A2.19) into (A2.16) and solving for k:

$$\begin{aligned}
 k &= k_v + \frac{k_v K^2}{4} \left[1 + \frac{P_s (-\cos \Delta)}{P_{s-1} (-\cos \Delta)} \right] \\
 &\cong k_v \left\{ 1 + \frac{K^2}{4} \left[1 + \frac{P_s (-\cos \Delta)}{P_{s-1} (-\cos \Delta)} \right] \right\} \quad (A2.20)
 \end{aligned}$$

Changing the notation $k \rightarrow k_{s0}$ and $k_v \rightarrow k_{f0}$ and squaring both sides of equation (A.2.20) gives the required relationship between k_{f0} and k_{s0} :

$$k_{s0}^2 = k_{f0}^2 \left\{ 1 + \frac{K^2}{4} \left[1 + \frac{P_s (-\cos \Delta)}{P_{s-1} (-\cos \Delta)} \right] \right\}^2 \quad (A2.21)$$

APPENDIX III

Modal Distribution Functions

Derivations similar to that performed in Chapter 4 for the symmetric trigonometric mode have been performed for the antisymmetric mode and the hyperbolic modes. The results of these derivations have been presented here.

A.3.1 The Antisymmetric Trigonometric Mode

The antisymmetric trigonometric modal function is defined here as:

$$f_n(x) = \begin{cases} A \sin(k_{nsx}x) & 0 < |x| \leq a \\ B \sin(k_{nmx}x + \theta) & a < |x| \leq b \\ C e^{-k_{ntx}(x-b)} & b < |x| \leq c \end{cases} \quad (A3.1)$$

Applying the boundary conditions outlined in Chapter 4 at $x=a$ yields :

$$A \sin(k_{nsx}a) = B \sin(k_{nmx}a + \theta) \quad (A3.2a)$$

$$\text{and } k_{nsx} A \cos(k_{nsx}a) = k_{nmx} B \cos(k_{nmx}a + \theta) \quad (A3.2b)$$

From equation (A3.2a) the constant B is defined:

$$B = A \frac{\sin(k_{nsx}a)}{\sin(k_{nmx}a + \theta)} \quad (A3.3)$$

Substituting equation (A3.3) into (A3.2b) gives:

$$k_{nsx} \tan(k_{nmx}a + \theta) = k_{nmx} \tan(k_{nsx}a) \quad (A3.4)$$

and it follows that θ is :

$$\theta = \tan^{-1} \left[\frac{k_{nmx} \tan(k_{nsx} a)}{k_{nsx}} \right] - k_{nmx} a \quad (A3.5)$$

The boundary conditions at $x=b$ dictate that:

$$B \sin(k_{nmx} x + \theta) = C e^{-k_{nfx} (x-b)} \quad (A3.6a)$$

and

$$k_{nmx} B \cos(k_{nmx} x + \theta) = -k_{nfx} C e^{-k_{nfx} (x-b)} \quad (A3.6b)$$

From equation (A3.6a):

$$C = B \sin(k_{nmx} b + \theta) \quad (A3.7)$$

and substituting equation (A3.7) into (A3.6b) gives:

$$k_{nmx} = -k_{nfx} \tan(k_{nmx} b + \theta) \quad (A3.8)$$

Substituting (A3.8) into (A3.4) yields the dispersion equation in terms of

k_{nmx} , k_{nfx} and k_{nsx} :

$$k_{nfx} \tan(k_{nsx} a) + k_{nsx} \left[\frac{\tan(k_{nmx} a + \theta)}{\tan(k_{nmx} b + \theta)} \right] = 0 \quad (A3.9)$$

By substituting equation (4.1) into equation (A3.9) an equation similar to equation (4.13) has been derived and iteratively solved for k_{nz} .

A.3.2 The Symmetric Hyperbolic Mode

The symmetric hyperbolic modal function is defined similarly to the symmetric trigonometric function with the exception that the cosine dependence in the semi-metallized region is defined in terms of a hyperbolic cosine function:

$$f_n(x) = \begin{cases} A \cosh(k_{nsx}x) & 0 < |x| \leq a \\ B \cos(k_{nmx}x + \theta) & a < |x| \leq b \\ C e^{-k_{nfx}(x-b)} & b < |x| \leq c \end{cases} \quad (\text{A3.10})$$

From the boundary conditions at $x=a$:

$$A \cosh(k_{nsx}a) = B \cos(k_{nmx}a + \theta) \quad (\text{A3.11a})$$

$$\text{and } k_{nsx} A \sinh(k_{nsx}a) = -k_{nmx} B \sin(k_{nmx}a + \theta) \quad (\text{A3.11b})$$

From equation (A3.11a) the constant B is defined:

$$B = A \frac{\cosh(k_{nsx}a)}{\cos(k_{nmx}a + \theta)} \quad (\text{A3.12})$$

Substituting equation (A3.12) into (A3.11b) gives:

$$k_{nsx} \tanh(k_{nsx}a) = -k_{nmx} \tan(k_{nmx}a + \theta) \quad (\text{A3.13})$$

and it follows that θ is:

$$\theta = \tan^{-1} \left[\frac{-k_{nsx} \tanh(k_{nsx}a)}{k_{nmx}} \right] - k_{nmx}a \quad (\text{A3.14})$$

The boundary conditions at $x=b$ dictate that:

$$B \cos(k_{nmx}x + \theta) = -C e^{-k_{nfx}(x-b)} \quad (A3.15a)$$

and

$$-k_{nmx} B \sin(k_{nmx}x + \theta) = -k_{nfx} C e^{-k_{nfx}(x-b)} \quad (A3.15b)$$

From equation (A3.15a):

$$C = B \cos(k_{nmx}b + \theta) \quad (A3.16)$$

and substituting equation (A3.16) into (A3.15b) gives:

$$k_{nmx} = k_{nfx} / \tan(k_{nmx}b + \theta) \quad (A3.17)$$

Substituting (A3.17) into (A3.13) yields the dispersion equation in terms of

k_{nmx} , k_{nfx} and k_{nsx} :

$$k_{nsx} \tanh(k_{nsx}a) + k_{nfx} \left[\frac{\tan(k_{nmx}a + \theta)}{\tan(k_{nmx}b + \theta)} \right] = 0 \quad (A3.18)$$

By substituting equation (4.1) into equation (A3.18) an equation similar to equation (4.13) has been derived and iteratively solved for k_{nz} .

A.3.3 The Antisymmetric Hyperbolic Mode

The antisymmetric hyperbolic modal function is defined similarly to the symmetric hyperbolic function with the exception that the even cosine dependencies in the semi-metallized and metallized region are defined in terms of the odd sinh and sin function:

$$f_n(x) = \begin{cases} A \sinh(k_{nsx} x) & 0 < |x| \leq a \\ B \sin(k_{nmx} x + \theta) & a < |x| \leq b \\ C e^{-k_{nfx} (x-b)} & b < |x| \leq c \end{cases} \quad (A3.19)$$

From the boundary conditions at $x=a$:

$$A \sinh(k_{nsx} a) = B \sin(k_{nmx} a + \theta) \quad (A3.20a)$$

$$\text{and } k_{nsx} A \cosh(k_{nsx} a) = k_{nmx} B \cos(k_{nmx} a + \theta) \quad (A3.20b)$$

From equation (A3.20a) the constant B is defined:

$$B = A \frac{\sinh(k_{nsx} a)}{\sin(k_{nmx} a + \theta)} \quad (A3.21)$$

Substituting equation (A3.21) into (A3.20b) gives:

$$k_{nmx} \tanh(k_{nsx} a) = k_{nsx} \tan(k_{nmx} a + \theta) \quad (A3.22)$$

and it follows that θ is :

$$\theta = \tan^{-1} \left[\frac{k_{nmx} \tanh(k_{nsx} a)}{k_{nsx}} \right] - k_{nmx} a \quad (A3.23)$$

The boundary conditions at $x=b$ dictate that:

$$B \sin(k_{nmx} x + \theta) = C e^{-k_{nfx} (x-b)} \quad (A3.24a)$$

and

$$k_{nmx} B \cos(k_{nmx} x + \theta) = -k_{nfx} C e^{-k_{nfx} (x-b)} \quad (A3.24b)$$

From equation (A3.24a):

$$C = B \sin(k_{nmx} b + \theta) \quad (A3.25)$$

and substituting equation (A3.25) into (A3.24b) gives:

$$k_{nmx} = -k_{nfx} \tan(k_{nmx} b + \theta) \quad (A3.26)$$

Substituting (A3.26) into (A3.22) yields the dispersion equation in terms of

k_{nmx} , k_{nfx} and k_{nsx} :

$$k_{nfx} \tanh(k_{nsx} a) + k_{nsx} \left[\frac{\tan(k_{nmx} a + \theta)}{\tan(k_{nmx} b + \theta)} \right] = 0 \quad (A3.27)$$

By substituting equation (4.1) into equation (A3.27) an equation similar to equation (4.13) has been derived and iteratively solved for k_{nz} .

APPENDIX IV

Derivation of Linear Coefficients of Modal Function Superposition.

In equation (4.4) the symmetric trigonometric function was defined as:

$$f_n(x) = \begin{cases} A \cos(k_{nsx}x) & 0 < |x| \leq a \\ B \cos(k_{nm}x + \theta) & a < |x| \leq b \\ Ce^{-k_{nt}x}(x-b) & b < |x| \leq c \end{cases} \quad (A4.1)$$

Setting the amplitude constant Ψ in equation (4.21) to unity, the linear coefficients for modal function superposition are defined as:

$$B_n = \frac{\int_{x_1}^{x_2} f_n(x) dx}{\int_{-\infty}^{\infty} f_n^2(x) dx} \quad (A4.2)$$

Now since all excitation of SAW modes is confined to the semi-metallized electrode region of the IDT structure, it suffices to evaluate the numerator of equation (A4.2) in the interval $0 \leq |x| < a$ as follows:

$$\begin{aligned} \int_{x_1}^{x_2} f_n(x) dx &= \int_{x_1}^{x_2} A \cos(k_{nsx}x) dx \\ &= \frac{A}{k_{nsx}} \left[\sin(k_{nsx}x_2) - \sin(k_{nsx}x_1) \right] \end{aligned} \quad (A4.3)$$

In the case of the denominator, since the function $f_n^2(x)$ is symmetric about

$x=0$ then:

$$\int_{-\infty}^{\infty} f_n^2(x) dx = 2 \int_0^{\infty} f_n^2(x) dx \quad (A4.4)$$

Evaluating equation (A4.4) gives:

$$\begin{aligned} 2 \int_0^{\infty} f_n^2(x) dx &= 2 \int_0^a A^2 \cos^2(k_{nsx} x) dx \\ &\quad + 2 \int_a^b B^2 \cos^2(k_{nmx} x + \theta) dx \\ &\quad + 2 \int_b^{\infty} C^2 e^{-2k_{nfx} (x-b)} dx \\ &= A^2 \left[a + \frac{1}{2k_{nsx}} \sin(2k_{nsx} a) \right] \\ &\quad + B^2 \left[(b-a) + \frac{1}{2k_{nmx}} (\sin(2k_{nmx} b + \theta) - \sin(2k_{nmx} a + \theta)) \right] \\ &\quad + \frac{C^2}{k_{nfx}} \end{aligned} \quad (A4.5)$$

Given that the values, for the constants A , B , C and θ and the wavenumbers k_{nsx} , k_{nmx} and k_{nfx} have been determined as in Appendix

III, the evaluation of equation (A4.5) is simply a matter of substituting in these numerical values and performing the calculation.

A similar treatment for the antisymmetric mode yields:

$$\begin{aligned} \int_{x_1}^{x_2} f_n(x) dx &= \int_{x_1}^{x_2} A \sin(k_{nsx} x) dx \\ &= \frac{A}{k_{nsx}} \left[\cos(k_{nsx} x_2) - \cos(k_{nsx} x_1) \right] \end{aligned} \quad (A4.6)$$

and

$$\begin{aligned} \int_{-\infty}^{\infty} f_n^2(x) dx &= A^2 \left[a - \frac{1}{2k_{nsx}} \sin(2k_{nsx} a) \right] \\ &+ B^2 \left[(b-a) - \frac{1}{2k_{nmx}} (\sin(2k_{nmx} b + \theta) - \sin(2k_{nmx} a + \theta)) \right] \\ &+ \frac{C^2}{k_{ntx}} \end{aligned} \quad (A4.7)$$

Similarly, for the symmetric hyperbolic mode:

$$\begin{aligned}
\int_{x_1}^{x_2} f_n(x) dx &= \int_{x_1}^{x_2} A \cosh(k_{nsx} x) dx \\
&= \frac{A}{k_{nsx}} \left[\sinh(k_{nsx} x_2) - \sinh(k_{nsx} x_1) \right] \quad (A4.8)
\end{aligned}$$

and

$$\begin{aligned}
\int_{-\infty}^{\infty} f_n^2(x) dx &= A^2 \left[a + \frac{1}{2k_{nsx}} \sinh(2k_{nsx} a) \right] \\
&\quad + B^2 \left[(b-a) + \frac{1}{2k_{nmx}} (\sin(2k_{nmx} b + \theta) - \sin(2k_{nmx} a + \theta)) \right] \\
&\quad + \frac{C^2}{k_{ntx}} \quad (A4.9)
\end{aligned}$$

Finally for the antisymmetric hyperbolic mode:

$$\begin{aligned}
\int_{x_1}^{x_2} f_n(x) dx &= \int_{x_1}^{x_2} A \sinh(k_{nsx} x) dx \\
&= \frac{A}{k_{nsx}} \left[\cosh(k_{nsx} x_2) - \cosh(k_{nsx} x_1) \right] \quad (A4.10)
\end{aligned}$$

and

$$\begin{aligned}
 \int_{-\infty}^{\infty} f_n^2(x) dx &= A^2 \left[a + \frac{1}{2k_{nsx}} \sinh(2k_{nsx} a) \right] \\
 &+ \frac{1}{2k_{nmx}} \left[(b-a) - \frac{1}{2k_{nmx}} (\sin(2k_{nmx} b + \theta) - \sin(2k_{nmx} a + \theta)) \right] \\
 &+ \frac{C^2}{k_{nfx}} \quad (A4.11)
 \end{aligned}$$

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