MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

National Library
of Canada

Acquisitions and
Bibliographic Services Branch

395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale
du Canada

Direction des acquisitions et
des services bibliographiques

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Author - Auteur

Title - Titre

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

Canada

**The Numerical Prediction
of Strain Distribution
in
Sheet Metal Forming Operations**

by

Kevin Vaughan Williams

B. Eng. (Mechanical Engineering)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of

Master of Engineering
in Mechanical Engineering

**Department of Mechanical and Aerospace Engineering
Ottawa-Carleton Institute
for Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
December, 1993**

©Copyright

1993, Kevin V. Williams

National Library
of Canada

Acquisitions and
Bibliographic Services Branch

395 Wellington Street
Ottawa, Ontario
K1A 0N4

Bibliothèque nationale
du Canada

Direction des acquisitions et
des services bibliographiques

395, rue Wellington
Ottawa (Ontario)
K1A 0N4

Your title / Votre référence

Your title / Votre référence

The author has granted an irrevocable non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of his/her thesis by any means and in any form or format, making this thesis available to interested persons.

L'auteur a accordé une licence irrévocable et non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de sa thèse de quelque manière et sous quelque forme que ce soit pour mettre des exemplaires de cette thèse à la disposition des personnes intéressées.

The author retains ownership of the copyright in his/her thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without his/her permission.

L'auteur conserve la propriété du droit d'auteur qui protège sa thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

ISBN 0-315-89907-7

Canada

Name Kevin V. Williams

Dissertation Abstracts International is arranged by broad, general subject categories. Please select the one subject which most nearly describes the content of your dissertation. Enter the corresponding four-digit code in the spaces provided.

Mechanical Engineering

SUBJECT TERM

0548

SUBJECT CODE

U·M·I

Subject Categories

THE HUMANITIES AND SOCIAL SCIENCES

COMMUNICATIONS AND THE ARTS

Architecture 0729
Art History 0377
Cinema 0900
Dance 0378
Fine Arts 0357
Information Science 0723
Journalism 0391
Library Science 0399
Mass Communications 0708
Music 0413
Speech Communication 0459
Theater 0465

EDUCATION

General 0515
Administration 0514
Adult and Continuing 0516
Agricultural 0517
Art 0273
Bilingual and Multicultural 0282
Business 0688
Community College 0275
Curriculum and Instruction 0727
Early Childhood 0518
Elementary 0524
Finance 0277
Guidance and Counseling 0519
Health 0680
Higher 0745
History of 0520
Home Economics 0279
Industrial 0521
Language and Literature 0279
Mathematics 0280
Music 0522
Philosophy of 0998
Physical 0523

Psychology 0525
Reading 0535
Religious 0527
Sciences 0714
Secondary 0533
Social Sciences 0534
Sociology of 0340
Special 0529
Teacher Training 0530
Technology 0710
Tests and Measurements 0288
Vocational 0747

LANGUAGE, LITERATURE AND LINGUISTICS

Language 0679
General 0289
Ancient 0290
Linguistics 0291
Modern
Literature 0401
General 0294
Classical 0295
Comparative 0297
Medieval 0298
Modern 0316
African 0591
American 0305
Asian 0352
Canadian (English) 0355
Canadian (French) 0593
English 0311
Germanic 0312
Latin American 0315
Middle Eastern 0313
Romance 0314
Slavic and East European

PHILOSOPHY, RELIGION AND THEOLOGY

Philosophy 0422
Religion
General 0318
Biblical Studies 0321
Clergy 0319
History of 0320
Philosophy of 0322
Theology 0469

SOCIAL SCIENCES

American Studies 0323
Anthropology
Archaeology 0324
Cultural 0326
Physical 0327
Business Administration
General 0310
Accounting 0272
Banking 0770
Management 0454
Marketing 0338
Canadian Studies 0385
Economics
General 0501
Agricultural 0503
Commerce Business 0505
Finance 0508
History 0509
Labor 0510
Theory 0511
Foreign 0358
Geography 0366
Gerontology 0351
History
General 0578

Ancient 0579
Medieval 0581
Modern 0582
Black 0328
African 0331
Asia, Australia and Oceania 0332
Canadian 0334
European 0335
Latin American 0336
Middle Eastern 0333
United States 0337
History of Science 0585
Law 0398
Political Science
General 0615
International Law and Relations 0616
Public Administration 0617
Recreation 0814
Social Work 0452
Sociology
General 0626
Criminology and Penology 0627
Demography 0938
Ethnic and Racial Studies 0631
Individual and Family Studies 0628
Industrial and Labor Relations 0629
Public and Social Welfare 0630
Social Structure and Development 0700
Theory and Methods 0344
Transportation 0709
Urban and Regional Planning 0999
Women's Studies 0453

THE SCIENCES AND ENGINEERING

BIOLOGICAL SCIENCES

Agriculture 0473
General 0285
Agronomy
Animal Culture and Nutrition 0475
Animal Pathology 0476
Food Science and Technology 0359
Forestry and Wildlife 0478
Plant Culture 0479
Plant Pathology 0480
Plant Physiology 0817
Range Management 0777
Wood Technology 0746
Biology
General 0306
Anatomy 0287
Biostatistics 0308
Botany 0709
Cell 0379
Ecology 0329
Entomology 0353
Genetics 0369
Limnology 0793
Microbiology 0410
Molecular 0307
Neuroscience 0317
Oceanography 0416
Physiology 0437
Radiation 0821
Veterinary Science 0778
Zoology 0722
Biophysics
General 0786
Medical 0760

EARTH SCIENCES

Biogeochemistry 0420
Geochemistry 0996

Geodesy 0370
Geology 0372
Geophysics 0373
Hydrology 0388
Mineralogy 0411
Paleobotany 0345
Paleoecology 0426
Paleontology 0418
Paleozoology 0985
Palynology 0427
Physical Geography 0368
Physical Oceanography 0415

HEALTH AND ENVIRONMENTAL SCIENCES

Environmental Sciences 0768
Health Sciences
General 0566
Audiology 0300
Chemotherapy 0992
Dentistry 0567
Education 0350
Hospital Management 0769
Human Development 0758
Immunology 0982
Medicine and Surgery 0564
Mental Health 0347
Nursing 0569
Nutrition 0570
Obstetrics and Gynecology 0380
Occupational Health and Therapy 0354
Ophthalmology 0381
Pathology 0571
Pharmacology 0419
Pharmacy 0572
Physical Therapy 0382
Public Health 0573
Radiology 0574
Recreation 0575

Speech Pathology 0460
Toxicology 0383
Home Economics 0386

PHYSICAL SCIENCES

Pure Sciences
Chemistry
General 0485
Agricultural 0749
Analytical 0486
Biochemistry 0487
Inorganic 0488
Nuclear 0738
Organic 0490
Pharmaceutical 0491
Physical 0494
Polymer 0495
Radiation 0754
Mathematics 0405
Physics
General 0605
Acoustics 0986
Astronomy and Astrophysics 0606
Atmospheric Science 0608
Atomic 0748
Electronics and Electricity 0607
Elementary Particles and High Energy 0798
Fluid and Plasma 0759
Molecular 0609
Nuclear 0610
Optics 0752
Radiation 0756
Solid State 0611
Statistics 0463

Applied Sciences

Applied Mechanics 0346
Computer Science 0984

Engineering
General 0537
Aerospace 0538
Agricultural 0539
Automotive 0540
Biomedical 0541
Chemical 0542
Civil 0543
Electronics and Electrical 0544
Heat and Thermodynamics 0348
Hydraulic 0545
Industrial 0546
Marine 0547
Materials Science 0794
Mechanical 0548
Metallurgy 0743
Mining 0551
Nuclear 0552
Packaging 0549
Petroleum 0765
Sanitary and Municipal 0554
System Science 0790
Geotechnology 0428
Operations Research 0796
Plastics Technology 0795
Textile Technology 0994

PSYCHOLOGY

General 0621
Behavioral 0384
Clinical 0622
Developmental 0620
Experimental 0623
Industrial 0624
Personality 0625
Physiological 0989
Psychobiology 0349
Psychometrics 0632
Social 0451

Carleton University
Ottawa, Canada K1S 5J7

Thesis contains black and white and/or coloured graphs/tables/photographs which when microfilmed may lose their significance. The hardcopy of the thesis is available upon request from Carleton University Library.

The undersigned recommend to the Faculty of Graduate Studies
and Research acceptance of the thesis

**The Numerical Prediction of Strain Distribution
in
Sheet Metal Forming Operations**

submitted by Kevin Vaughan Williams
in partial fulfilment of the requirements for
the degree of Master of Engineering
in Mechanical Engineering

Thesis Supervisor M.J. Worswick

Chair, Department of Mechanical and Aerospace Engineering

Carleton University
December, 1993

Abstract

A numerical simulation of the limiting dome height test, due to Hecker and Ghosh, has been implemented using a commercial FEM code. The model incorporates transverse anisotropic effects through the use of Hill's quadratic yield criterion. The validation of the model was based on a series of interrupted dome height tests on two AK steels and one experimental aluminum alloy sheet. Three geometries, representative of the three primary forming conditions (stretching, plane strain, and drawing), were tested for each material. Circle grid analysis was used to obtain measured Green's strain distributions for the tested specimens.

Numerical results indicate reasonable agreement between the predicted and measured results for steel. The implementation of the anisotropic material model is seen to significantly improve the predicted strain distributions. Poor agreement between numerical and measured strain results was observed for the aluminum. The trends observed in the aluminum FEM model were consistent with previous analytical studies of Hill's yield criteria which indicate that the quadratic yield criteria does not provide an accurate prediction of the flow characteristics of a material with $r < 1$.

An examination of the effects of anisotropy and frictional coefficient was carried out in the form of a parameter study of r and μ . Predicted trends for variations in r are consistent with other solutions using the quadratic yield criteria. The numerical model, which incorporates the Coulomb formulation for frictional forces, was shown to be very sensitive to variations in μ .

Acknowledgments

I would like to thank Prof. M. Worswick for his endless patience and support during this research. To him I owe the greatest debt. There were many times when I would have been lost without his wisdom, knowledge, and understanding.

My work at Carleton University would not have been half as enjoyable had it not been for the staff of the Faculty of Engineering and, in particular, the faculty and staff of the Department of Mechanical and Aerospace Engineering. Special thanks to Ian Jefferson and Bruce Johnson for allowing me to test the limits of the computer network and for picking up the pieces after I was done.

I owe my eternal gratitude to Dr. Pierre Martin without whose support this project would not have been possible. Above all, I thank him for sharing his wisdom and knowledge of the field of metal forming in our many long discussions. I would also like to thank the staff of the Metal Forming Technologies Laboratories of CANMET for their assistance in the experimental portion of the work.

I would like to acknowledge the assistance of the ALCAN Kingston Research and Development Centre for providing the aluminum used in this research. Special thanks also to Chris Galbraith and Mark Finn of ALCAN for setting me straight on some of the finer points of LS-DYNA3D.

I would like to thank my parents for their unending love and support throughout my studies and for giving me the opportunity to pursue my dreams. Without their encouragement I know that I would not have been able to achieve all the goals that I have.

Finally, I would like to thank the Natural Sciences and Engineering Research Council of Canada for providing financial support for this project.

Contents

Abstract	iii
Acknowledgement	iv
Contents	v
List of Figures	ix
List of Tables	xvii
List of Symbols	xviii
1 Introduction	1
1.1 Current Work	3
1.2 The Forming Limit Diagram	4
1.3 Formability Tests	6
1.4 Experimental Strain Evaluation	8
1.5 Effects of Process Variables on Sheet Metal Forming	10
1.5.1 Lubrication and Friction	10
1.5.2 Binder Force	13
1.6 Material Constitutive Behaviour	14

1.6.1	Stress-Strain-Strain Rate Measures	14
1.6.2	Von Mises Yield Criterion	15
1.6.3	Anisotropy	16
1.6.4	Hill's Quadratic Yield Function	19
1.6.5	Hill's Non-Quadratic Yield Function	24
1.6.6	Other Anisotropic Yield Functions	27
1.7	Summary of Material Property Effects on the Formability of Sheet Metal	28
1.7.1	Anisotropy	28
1.7.2	Thickness	29
1.7.3	Friction	29
1.8	Application of FEM to the Simulation of Sheet Forming	30
1.8.1	Trends in the FEM Simulation of Metal Forming Operations .	30
1.8.2	Major Issues	31
1.9	Current Research	35
2	Experimental Programme	37
2.1	Material Selection	37
2.2	Material Characterization	38
2.3	LDH Specimen Preparation	39
2.3.1	Blank Cutting	39
2.3.2	Surface Preparation	40
2.4	Experimental Setup	41
2.5	Specimen Test Procedure	43
2.6	Grid Measurement	45
2.7	Determination of the Forming Limit Diagram	45
2.8	Interrupted Dome Tests	47
3	Numerical Model	49
3.1	Finite Element Mesh	50
3.1.1	Mesh Description	50
3.1.2	Model Refinement	51
3.2	Element Description	52
3.2.1	Choice of Element Formulation	52

3.3	The Belytschko-Lin-Tsay Four Node Quadrilateral Shell Element . . .	54
3.3.1	Thickness Update Option	55
3.3.2	Hourglass Control	55
3.4	Constitutive Model	56
3.5	Boundary Conditions	57
3.5.1	Displacement Boundary Conditions	57
3.5.2	Applied Loads	58
3.6	Contact Modelling	59
3.6.1	Contact Surfaces	59
3.6.2	Interface Stiffness	61
3.7	Frictional Boundary Conditions	62
3.8	Mesh Optimization Studies	63
3.8.1	Load Curve Optimization	63
3.8.2	Initial Mesh Investigation of Frictional Forces	65
3.8.3	Convergence Check	66
4	Discussion of Numerical and Experimental Results	67
4.1	Experimental Results	67
4.1.1	Comparison of Material Formabilities	67
4.1.2	Experimentally Measured Strain Distributions	68
4.1.3	Sheet AK1	69
4.1.4	Sheet AK2	71
4.1.5	Sheet AL	72
4.2	Numerical Results	73
4.2.1	Predicted Strain Distribution	73
4.2.2	Comparison Between Predictions of Numerical and Experimental Strain Distributions	76
4.2.3	Effects of Small Variations in r	80
4.3	Parametric Studies	81
4.3.1	Effect of Large Variations in r	81
4.3.2	Effect of Frictional Variations	82

5 Conclusion	84
5.1 Future Work	85
References	87

List of Figures

1.1	Example of the time requirements for the various stages of a car body development and production tooling cycle.	109
1.2	The position of formability in the die-face design cycle.	110
1.3	Factors affecting the material behaviour in press forming.	111
1.4	Limit strain loci for simple sheet forming operations.	112
1.5	The representation of the main deformation modes in strain space and their relation to the position of the forming limit diagram.	113
1.6	Comparison of the failure strain predicted at the failure (Goodwin curve) and adjacent to the failure (Keeler curve).	114
1.7	Schematic of the Swift drawing test.	115
1.8	Schematic of the Erichsen punch test.	116
1.9	Schematic of the punch and drawbead arrangement of the Hecker and Ghosh punch test.	117
1.10	Effect of the lubrication condition on the major strain distribution of a limiting dome height test.	118
1.11	Schematic of a draw bead simulator used to assess the frictional characteristic of a material being drawn through a set of draw beads. . . .	119
1.12	Elements of a metalworking lubrication system.	120
1.13	Variation of draw-in and punch depth with specimen width for a clamping forces of 125 and 250 kN.	121

1.14	Variation of draw in with clamping force showing the minimum required clamping force for a maximum draw-in of 0.25 mm.	122
1.15	The predicted effect of r value on the yield locus.	123
1.16	Experimentally measured variation of r with the orientation to the rolling direction.	124
1.17	Experimental and analytically predicted work hardening characteristics of a steel with $\bar{r} = 1.48$ using Hill's quadratic yield criterion. . . .	125
1.18	Uniaxial and biaxial stress-strain curves for commercial-purity aluminum in the annealed ($\bar{r} = 0.68$) and hard rolled ($\bar{r} = 0.43$) conditions.	126
1.19	Uniaxial and biaxial stress-strain curves for annealed copper-containing steel ($\bar{r} = 1.62$).	127
1.20	Predicted effect of r on the FLD according to Marciniak and Kuczyński theory.	128
1.21	Qualitative non-mathematical interpretation of the Marciniak and Kuczyński predictions using the predicted effect of the r value on the yield locus.	129
1.22	Effect of the exponent, M , of Hill's non-quadratic yield function on the yield locus.	130
1.23	Predicted influence of the exponent, M , of Hill's non-quadratic yield criterion on the right hand side of the FLD.	131
1.24	Effect of anisotropy, r , on the strain distribution in a hemispherical dome test specimen.	132
1.25	Effect of the strain hardening exponent, n , on the FLD.	133
2.1	Orientation of tensile test specimens relative to the sheet rolling direction.	134
2.2	Experimental true stress versus true strain curves for sheet IF (Dofasco IF steel) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.	135

2.3	Experimental true stress versus true strain curves for sheet AK1 (Dofasco AK steel) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.	135
2.4	Experimental true stress versus true strain curves for sheet AK2 (commercial grade AK steel) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.	136
2.5	Experimental true stress versus true strain curves for sheet AL (experimental aluminum X544.0 supplied by Alcan) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.	136
2.6	Close up of a deformed 203.2 mm specimen (AK1) showing the circle grid pattern etched on the blanks.	137
2.7	Limiting dome height press facility at CANMET.	138
2.8	View of the clamping die and punch arrangement.	139
2.9	Schematic view of the limiting dome height press facility.	146
2.10	Schematic view of the limiting dome height test data acquisition setup.	141
2.11	Schematic showing the positioning of the blank in the punch test setup using the location pins.	142
2.12	Example of a load-displacement plot of a blank deformed past necking indicating the approximate punch load and displacement at failure.	143
2.13	Schematic of a deformed circle grid showing the grid measurement technique.	144
2.14	Geometry of the narrow, notched specimens used for the 25.4 mm, 50.8 mm, and 76.2 mm blank sizes.	145
2.15	Photograph of a failed 127.0 mm AK1 blank showing the neck and the surrounding grids.	146
2.16	Experimentally measured forming limit curve for sheet IF.	147
2.17	Experimentally measured forming limit curve for sheet AK1.	147
2.18	Experimentally measured forming limit curve for sheet AK2.	148

2.19	Example of a series of interrupted dome test specimens (127.0 mm blanks of sheet AK1).	149
2.20	Schematic cross section of the deformed blank showing the pole of the dome, the blank to punch tangency point (TP) and the coordinate frame for dome height (y) and strain measurement position (x).	150
3.1	Exploded view of the numerical model of the 127.0 mm wide blank.	151
3.2	Schematic of the numerical models showing the symmetry used for the 127.0 mm and 203.2 mm blank geometries.	152
3.3	Schematic showing the reduced tooling required to model the 50.8 mm blank geometry.	153
3.4	Plot of the deformed 127.0 mm wide blank indicating the region of severe deformation which combined stretching, bending, and shear.	154
3.5	Indications of drawing parallel to the draw bead in the 127.0 mm wide blank of AK1 material observed experimentally and predicted numerically.	155
3.6	FEM mesh plot of the 50.8 mm wide blank model.	156
3.7	FEM mesh plot of the 127.0 mm wide blank model.	156
3.8	FEM mesh plot of the 203.2 mm wide blank model.	157
3.9	Quadrilateral shell element displacement modes and hourglass modes.	158
3.10	Example of the elastic-plastic material model with isotropic hardening represented by the hardening modulus, E_T , fit to actual true stress versus true strain curve.	159
3.11	Graphical representation of the stages of the modelled process and the load histories used.	160
3.12	Comparison of predicted strain distribution using step function load curves and sinusoidal load curves.	161
3.13	Refined mesh of the 203.2 mm wide blank used in the convergence check.	162
3.14	Comparison of predicted strain distributions for varying mesh densities.	163

4.1	Comparison of measured forming limit curves for the AK1 and AK2 steels with published results for Al 1050-0.	164
4.2	Measured major and minor principal Green's strain for the 203.2 mm wide blank size of AK1.	165
4.3	Measured major and minor principal Green's strain for the 127.0 mm wide blank size of AK1.	166
4.4	Measured major and minor principal Green's strain for the 50.8 mm wide blank size of AK1.	167
4.5	Measured major and minor principal Green's strain for the 203.2 mm wide blank size of AK2.	168
4.6	Measured major and minor principal Green's strain for the 127.0 mm wide blank size of AK2.	169
4.7	Measured major and minor principal Green's strain for the 50.8 mm wide blank size of AK2.	170
4.8	Measured major and minor principal Green's strain for the 203.2 mm wide blank size of AL.	171
4.9	Measured major and minor principal Green's strain for the 127.0 mm wide blank size of AL.	172
4.10	Measured major and minor principal Green's strain for the 50.8 mm wide blank size of AL.	173
4.11	Fringe plot of predicted major principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 0 mm.	174
4.12	Fringe plot of predicted major principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 19.0 mm.	175
4.13	Fringe plot of predicted major principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$). at a punch depth of 35.9 mm.	176
4.14	Fringe plot of predicted minor principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 0 mm.	177

4.15	Fringe plot of predicted minor principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 19.0 mm.	178
4.16	Fringe plot of predicted minor principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$). at a punch depth of 35.9 mm.	179
4.17	Plot of predicted major and minor principal Green's strain versus position relative to the pole of the dome ($\bar{r} = 2.21$ and $\mu = 0.18$) for the 203.2 mm wide AK1 blank.	180
4.18	Fringe plot of predicted thickness distribution for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 0 mm. . .	181
4.19	Fringe plot of predicted thickness distribution for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 19.0 mm. .	182
4.20	Fringe plot of predicted thickness distribution for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 35.9 mm. .	183
4.21	Plot of predicted thickness distribution versus position relative to the pole of the dome ($\bar{r} = 2.21$ and $\mu = 0.18$) for the 203.2 mm wide AK1 blank.	184
4.22	Plot of predicted major and minor principal Green's strain versus position relative to the pole showing the variation in the predicted strain distribution across the sheet.	185
4.23	Fringe plot of the predicted major principal Green's strain for the 127.0 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 37.1 mm.	186
4.24	Fringe plot of the predicted minor principal Green's strain for the 127.0 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 37.1 mm.	187
4.25	Fringe plot of predicted thickness distribution for the 127.0 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 37.1 mm. .	188

4.26	Fringe plot of the predicted major principal Green's strain for the 50.8 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 38.0 mm.	189
4.27	Fringe plot of the predicted minor principal Green's strain for the 50.8 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 38.0 mm.	190
4.28	Fringe plot of predicted thickness distribution for the 50.8 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 38.0 mm. .	191
4.29	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 203.2 mm wide AK1 blank.	192
4.30	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 127.0 mm wide AK1 blank.	193
4.31	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 50.8 mm wide AK1 blank.	194
4.32	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 203.2 mm wide AK2 blank.	195
4.33	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for 127.0 mm wide AK2 blank. .	196
4.34	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for 50.8 mm wide AK2 blank. .	197

4.35	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 203.2 mm wide AL blank.	198
4.36	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for 127.0 mm wide AL blank. . .	199
4.37	Comparison of the measured and predicted strain distributions for an isotropic material and a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 50.8 mm wide AL blank.	200
4.38	Predicted effect of small variation in r on the strain distribution. AK1 material 50.8 mm and 127.0 mm blank sizes.	201
4.39	Predicted effect of large variations in r on the strain distribution of the 203.2 mm wide AK1 blank.	202
4.40	Predicted effect of variations in r on the material thickness distribution for the 203.2 mm wide blank of AK1 material.	203
4.41	Predicted effect of variations in μ for the 203.2 mm wide blank of AK1 material.	204
4.42	Predicted effect of variations in μ for the 127.0 mm wide blank of AK1 material.	205
4.43	Predicted effect of variations in μ for the 50.8 mm wide blank of AK1 material.	206

List of Tables

1.1	Summary of frictional coefficients quoted in published numerical and analytical studies of sheet metal forming	99
1.2	Experimentally determined coefficients of friction.	100
2.1	Chemical composition of the sheet steels.	101
2.2	Chemical composition of the aluminum sheet.	101
2.3	Uniaxial tensile test data.	102
2.4	Summary of material property data.	103
2.5	Typical limit minor strains for various blank sizes	104
2.6	Interrupted dome test dome heights.	105
3.1	Outline of analysis mesh sizes used in the investigation.	106
3.2	Comparison of shell element formulations implemented in LS-DYNA3D.	106
3.3	Summary of material property data for the numerical material models.	107
3.4	Breakdown of model run time showing the three stages of load application for the step load curves.	107
3.5	Summary of the sinusoidal load curve trials.	108

List of Symbols

F_{frict}	frictional contact force
F_n	normal surface contact force
K	strength constant
LDH_0	limiting dome height at plane strain
m, η	strain rate sensitivity
n	strain hardening exponent
$F, G, H,$ L, M, N	parameters used in Hill's quadratic yield criterion
M	exponent in Hills non-quadratic yield criterion
r	Lankford's constant used to characterize anisotropy
$\bar{r}$	average anisotropic parameter
R, S, T	yield stresses in shear in the principal axis of anisotropy
X, Y, Z	tensile yield stresses in the principal axis of anisotropy
ϵ	strain
$\bar{\epsilon}^p$	effective plastic strain
μ	Coulomb coefficient of friction
σ	stress
σ_Y	yield stress
σ'_Y	yield stress in a tangent modulus fit to the stress-strain curve
$\bar{\sigma}$	effective stress

Additional Subscripts

0, 45, 90	measured at 0°, 45°, and 90° to the rolling direction of the sheet
1	major principal value
2	minor principal value

Chapter 1

Introduction

The need for advances in sheet metal research stems from the increasing pressures that are being placed on the existing design system. In the automobile industry, arguably the largest utiliser of sheet metal forming, economic factors, safety standards, competition from alternate production techniques, styling characteristics, and environmental requirements [1] are all driving the need for forming increasingly complex parts made with thinner, higher strength materials [1, 2, 3, 4]. The field of die design and stamping, until recently, has been one of experience, artisanship, and trial and error [1, 5, 6]. In a discussion of the breakdown of the artisan system of die design, Keeler [1] cites the following factors as driving the sudden need for change to an engineering approach:

1. Lack of young people willing to learn the skill of the artisan through the apprenticeship or any other long-term training system
2. Earlier retirement opportunities for those skilled artisans who could be the teachers of future generations
3. Reduced lead times, including die tryout, due to constantly changing targets and requirements
- 4 Introduction of newer materials for which no experience bank is available

5. Greater emphasis on cost effectiveness

6. Increased complexity of parts.

The first two items are sociological and must be accepted as they are. The latter items represent a technological challenge. Figure 1.1 from Yoshida [3] demonstrates the time required for the various stages of design of the body and tooling for a car. The design and engineering involved with the presses for the body panels, in his example, is roughly 60% of the time taken for the whole design. The introduction of Computer Aided Design (CAD) has greatly enhanced the speed of the initial design process as well as the modification and refinement of the final design. Once the production run has started, however, a significant amount of time is needed to make adjustments before sound parts can be produced. These requirements absorb large amounts of labour associated with refinement of the dies, not to mention the expense of the machining and the materials consumed in the design and refinement cycles. A method of determining sound production techniques before the money and time has been spent on dies and trials is therefore of great value [1], especially if significant savings in time can also be realized. In the example mentioned above, Yoshida [3] envisages a reduction of the time to first production by almost one half with a significant portion of the savings coming from the introduction of numerical methods in the die tryout and modification stage.

Item 4, the introduction of new materials, indicates a need for a better understanding of the response of materials to different forming processes. Simple analysis and experimental assessments of material properties have little relation to practical forming processes where the tool forms and strain gradients are completely different from those observed in a simple tensile test. In particular, the interaction of the various material and press parameters cannot be incorporated into simple analytical or experimental tools.

Item 5 is self explanatory. Increased competition and the expense of lead time (i.e. any time not spend in production) are both critical in today's highly competitive

markets.

Finally, item 6 is of particular interest in terms of the breakdown of the artisan system. The trial and error approach is simply not capable of incorporating all of the increased complexities while satisfying reduced lead time requirements. As a result, an engineering approach is necessary.

The characteristics of the system necessary to replace the artisan system of design [1] point towards some form of mathematical model; an interactive system which incorporates all aspects of the forming process and which allows for the interaction of any or all of these parameters. There is some interest in small, approximate personal computer (PC) based simulations [7]. These allow investigation of particular aspects of a forming operation such as the binder pressure, die radii, or sheet thickness. A parametric study can be carried out in a matter of hours but the results are only rough approximations. The limitations in computational power and technical expertise available to small press shops make these tools attractive but any significant contributions to the field require a significantly more complex simulation. Currently, efforts are concentrated on the use of finite element method (FEM) techniques as a means of simulating forming processes largely due to the ability of FEM to handle complex problems of arbitrary shape [8]. The advent of improved numerical tools capable of accurately modelling materials used in forming processes and the order of magnitude increase in computer power available every few years has made the FEM a realistic tool.

1.1 Current Work

Of particular interest is the evaluation of material formability which, as shown in Figure 1.2, is the controlling factor in most areas of the die design. The formability of a material is a measure of its ability to be plastically formed under various loading conditions and strain paths. In general terms, the formability will be influenced by the material properties, the boundary conditions (including friction), and the

failure criterion as shown in Figure 1.3 [4, 9]. Successful analysis of the forming process requires a complete and accurate description of the material characteristics and response during the forming operation. The present research is concerned with the accuracy of constitutive models of sheet materials and their use in finite element simulation of forming operations. In particular, the effect of anisotropy as predicted by an implementation of Hill's quadratic yield surface is investigated.

The assessment of the formability of a material is made using a simulative test. Although it is nearly impossible to investigate every possible forming condition, over one hundred quantitative formability tests have been proposed [9]. The more successful of these have become standards upon which the formability of materials is compared. Accepted formability tests [10] include the Erichsen and Olsen test for stretchability, the Swift test for deep drawing, and the Hecker and Ghosh test for assessing formability under various strain states, all of which are discussed later in Section 1.3. Each of these tests uses a simplified geometry which generates a stress or strain state in the material that is representative of a forming condition seen in a more complex part. The current work is based on the Ghosh formability test described in detail in Section 1.3 below. A numerical model of this test was created and the predicted strain distributions and load-displacement histories are compared to experimental measurements.

1.2 The Forming Limit Diagram

Initial attempts to model sheet metal forming conditions were made using what is termed a formability test. The most common method of presenting the results of such experimental measurements is in terms of the forming limit strains [11]. The identification of a critical level of strain for a given strain state as a means of characterising the formability of a material represented a significant breakthrough in the field of metal forming when it was introduced by Gensamer [11]. When plotted in strain space, as shown in Figure 1.4, with the minor strain along the abscissa and

major strain along the ordinate, the locus of the limit strains form a curve commonly referred to as the forming limit diagram (FLD), the forming limit curve (FLC) or the Keeler-Goodwin diagram [12]. This FLD is one of a number of limit strain loci which define the safe region for sheet forming in strain space (refer to Figure 1.5) and represents the limit strains for the most common failure mode, tearing.

The original work by Goodwin involved the strain at the fracture point whereas Keeler concentrated on the strain just adjacent to the point of fracture. There is a significant difference between the two strain measures as shown in Figure 1.6. The area between the two curves is an uncertainty band where, depending on the failure criterion, the material may or may not be considered to have failed. The FLD is based on the Keeler curve as it is easier to measure and has more meaning in practical applications since it is more desirable to design against necking rather than fracture.

Three general strain conditions are commonly observed in a forming operation. Biaxial stretching (eg. bulging), characterized by a tensile major and minor strain, is represented on the right hand side of the forming limit diagram. Drawing, where the material is allowed to draw-in, resulting in a positive major strain and a negative minor strain, is represented by the left side of the FLD. Finally plane strain deformation is characterized by a zero in-plane minor strain. This strain condition forms the tip of the FLD, the point of lowest formability, and is usually the limiting case in a complex forming problem (see Figure 1.5).

The forming limit diagram is a measure of the local limit surface strains which occur at the onset of a localization, neck, or fracture. The understanding of the strain distribution in a part, is therefore critical to a study of the formability of a material. Erichsen, Olsen, Fukui, and Swift followed the work by Gensamer with simulative tests [11].

1.3 Formability Tests

For many years, a form of cup test was used by press shops and stamping plants to evaluate the formability of sheet metal. It was recognized that drawing and stretching were important mechanisms used for sheet metal forming. Because of the diversity of forming operations, a number of different simulative cup tests were developed including the Swift cup test [13] for deep drawing and the Olsen and Erichsen cup tests [14, 15] for stretching.

The Swift test, shown in Figure 1.7, was based on a hemispherical indenter, of radius 50.8 to 101.6 mm (2 to 4 inches), pressing through a circular sheet of metal clamped between two dies. As this test was meant to investigate deep-drawing, the clamping force was sufficient only to prevent wrinkling. The punch load-displacement history and the thickness strain were used to evaluate the drawing ratio.

The Erichsen test (see Figure 1.8), in contrast with the Swift test, used a smaller indenter in the form of a 20mm diameter steel ball. The die opening was 27 mm in diameter and the clamping force was fixed at 1 tonne. Although modifications were made to the clamping dies in the form of serrations on the die surface, no attempt was made to completely clamp the blank. The deformation was largely stretching but there was drawing through the clamping dies as well. The Olsen test, mentioned in the discussion above, was the U.S. equivalent of the Erichsen test using a 22.23 mm (7/8 inch) diameter steel ball forming the blank through a 25.4 mm (1 inch) circular opening. The depth of the punch penetration, at the point of fracture, was used as the limiting parameter and was termed the Erichsen number.

These tests were the subject of a number of investigations. Of particular interest were the suitability of these tests as measures of formability, the effects of material parameters and test characteristics, comparisons with other formability tests, and the repeatability of the results [14, 16]. Although these studies generally supported the idea of the Erichsen test as a method of measuring the formability of the sheet, a number of questions were raised about the repeatability of the test results. In

particular, the combination of both stretching and drawing were identified as a major problem as it was impossible to predict the contribution of each to the formability of the material.

Hecker [17] cited many of these short comings when he presented the basis for a new form of the cup or punch test in 1974. Experimental scatter from one lab to another and even between one test and another were often larger than the material effects which were to be measured [17]. He suggested four major modifications to the existing Erichsen style tests. These included an increase in the size of the punch over that used in the Erichsen test. The small indenter used by Erichsen gave rise to significant bending stresses and can influence the mode of fracture. To this end, Hecker proposed using a 4 inch diameter punch. An attempt to eliminate draw-in through the use of draw beads was made (see Figure 1.9). Previous tests allowed too many variables to affect the amount of draw-in expected in a test. The variables included the surface finish of the tooling, lubricant, clamping pressure, and specimen size, for example. Unexpected draw-in artificially increases the cup height and over-predicts the formability of a material. The third major modification came in the form of establishing a consistent standard for lubrication. Changes in the lubrication used can mask differences due to the material properties. Even through careful application of lubricants, Hecker found a large amount of scatter in the results. An unlubricated or dry test was therefore found to provide the best repeatability as it allows the measurement of the materials ability to distribute strain under a given stress state independent of any lubrication effects [17, 18]. Finally, the definition of the fracture point in the Erichsen test was left up to some interpretation of the operator. By introducing a monitoring system to the test to record punch depth and load, he hoped to eliminate operator induced errors and a somewhat subjective identification of the failure criterion. Hecker noted that the most critical strain state in a forming operation falls within a wide band around plain strain ($\epsilon_2 = 0$). As the Hecker test provides a slightly biaxial strain, Hecker argued that the relative heights of the failed

tests specimens should give an indication of the formability of a given material.

Although the Hecker test provided a good basis for a measure of a materials formability, the test allowed the investigation of only one strain state in the material, biaxial stretching. Furthermore, the strain state produced in a test was not repeatable between materials because the strain path was material dependent. It was therefore impossible to use the Hecker test, as it was proposed, to obtain a quantitative comparison of formability between materials. The final major modification of the experimental procedure, which allowed the investigation of virtually all strain conditions that can be expected in a forming operation, was introduced by Ghosh [19]. Ghosh varied the sample widths while maintaining the same punch diameter thereby controlling the amount of lateral drawing in the blank which, in turn, controls the strain state at the neck [10, 19]. The result of this study was the observation of a minimum dome height when the material is close to or at plane strain conditions. This minimum dome height is denoted as LDH_0 , hence the name limiting dome height (LDH) test.

More recent studies using the limiting dome height test have been concerned with the prediction of effects of material properties on the formability of the material. In addition, a significant amount of work has been carried out on the application of analytical and numerical models to the forming process.

1.4 Experimental Strain Evaluation

One of the most widely used methods of strain determination in formed parts is the circle grid analysis [4, 20, 21] originally proposed in 1968 by Keeler [22]. A regular circular grid pattern, typically 0.1 inches in diameter, is physically deposited or chemically etched on the sheet before deformation. After deformation, the major and minor axis of the elliptically deformed grid are measured and divided by the original circle diameter to obtain a measure of the principal surface strains. Measuring a series of these grids then gives a distribution of the strain and measuring the variation of

the strain in a given grid during the deformation (using an interrupted test or a continuous real time photographic technique) gives a measure of the strain path of a particular element of the blank. Although this method gives a macroscopic view of the strain distribution and strain gradients in a part, steep gradients will not be picked up accurately. Some analytical methods based on the initial and final shape of the deformed grid have been developed to account for micro strain gradients but the assumption of a zero strain gradient across the grid is usually made [21]. The other major limitation of this method has to do with the importance of strain path. Grid measurement techniques generally involve deriving the strain in an element of material from the initial and final geometry of a grid embedded in that element of material. As a result, two different strain paths which result in the same deformed grid shape will result in the same strain measure even though the equivalent strains may be quite different depending on the deformation process [21]. Generally, strain paths are ignored when grid measurements are made and the resulting strain measures are principal Green's strains.

Not only is circle grid analysis used to predict failure in experimental investigations, but it can also be a useful tool for troubleshooting production setups [4, 23, 24]. Variations on the original circular grid pattern include a square grid pattern, overlapping grids to capture high strain gradients, and a combination of circle and square patterns to provide a better image of the magnitude and direction of material flow. Recent developments in digital imaging techniques have provided a means of automatically determining the strain distribution through the use of nodal or grid point measurements [21]. Computerized techniques greatly reduce the effort associated with experimental strain measurements and provide a reasonable method for determining the strain distribution in an entire part.

1.5 Effects of Process Variables on Sheet Metal Forming

There are many factors outside of the material properties which influence the formability of a sheet which come under the general heading of process variables. These include blank size, lubrication type, punch speed, press rigidity, blank thickness, die surface finish, die radii, and wear, to name a few [6, 25]. It is clear that a complete sensitivity analysis involves a significant amount of time and work and it is beyond the scope of the current research to investigate all the process variables. Many of them can be neglected when considering an experimental process as they only become significant on a large scale presses (eg. die rigidity) or during high volume production (eg. wear). There are a few variables, however, which are critical to the simulation of the experimental test setup. These include the lubrication and the binder or clamping force, both of which change the drawing characteristics of the material through the clamping dies and over the punch thereby changing the strain condition in the part. The blank material properties are, of course, critical to the forming operation and the effects of these on the formability of a material will be covered in more detail in Section 1.7.

1.5.1 Lubrication and Friction

The frictional forces acting on the blank during forming represent additional boundary conditions to the process. The frictional interaction of the die and workpiece surfaces, and any lubricants used during the forming operation controls the stress field to which the part is subjected and the resulting strain distribution (see Figure 1.10). A high frictional coefficient results in a high frictional force at the tooling-blank interface, resisting any relative motion between the two surfaces and limiting lateral draw-in of the material under the punch. This restraint will tend to push the strain state towards the plain strain condition and will move the localization away from the pole

of the punch. A lower frictional force allows the material to stretch or draw over the punch, moving the strain state away from plane strain. In a narrow sheet, this allows lateral drawing resulting in a stress state closer to uniaxial tension. For axisymmetric drawing, the stress state tends towards biaxial stretching and in the limit equi-biaxial tension.

A common method for determining the frictional characteristics of a material is the use of a simulative test. The purpose of such simulative tests is to control certain process variables and to allow the accurate measurement of the frictional forces. There are numerous simulative tests for measuring the coefficient of friction [18]. Many of these tests investigate specific aspects of either process or material effects on the frictional coefficient. One of the most commonly used tests for predicting the frictional coefficient in drawing operations uses an apparatus known as the draw bead simulator [8, 18, 26], shown in Figure 1.11.

These simulative tests, however, are limited in their application and do not necessarily give realistic values of the frictional conditions under the punch. The frictional coefficient is largely based on the surface characteristics of the material and its interaction with the lubricant and tooling surface [8, 18, 27]. The surface characteristics, however, evolve as the forming operation proceeds [18, 28, 29] due to deformation of the material through changing crystallographic structure, plastic deformation of the surface roughness features, the relative velocity of the surfaces, and intermittent contact of the sheet with the tooling. Under these varying conditions, there are four different lubrication regimes which can occur: the thick-film regime, thin-film regime, boundary regime, and the mixed lubrication regime [27]. Each of these is characterised by a different inter-dependence on the surface topography, pressure, and lubricant.

In addition to the material characteristics, atmospheric conditions (eg. pressure and ambient temperature), local heating due to deformation, the dissipation of heat produced by friction, wear, and surface contamination, to name a few, also affect the frictional condition which exists at the tooling-blank interface. An appreciation of the

complex dependencies involved with friction can be gained from Figure 1.12 which shows the various elements of a metalworking lubrication system. Schey [18] provides an excellent reference for detailed information on all aspects of tribology including it's effects in sheet metal forming.

It has become apparent that the coefficient of friction can no longer be considered to be a fixed quantity but it must be considered to be a state variable which varies with time. The number of parameters affecting the frictional characteristics of the material, however, make a complete mathematical description impractical.

Within the framework of the finite element analysis, the correct modelling of the frictional effect is critical to the accuracy of the results [30, 31]. In fact, a parametric sensitivity study by Burford et al. [10] using rigid viscoplastic FEM routine with Hill's quadratic yield function and Coulomb friction determined that μ was the most important of the tested parameters in terms of the accuracy of the simulation.

The most commonly adopted mathematical representation of friction is the Coulomb frictional coefficient [8, 10, 18, 27].

$$F_{frict} = \mu \cdot F_n \quad (1.1)$$

where F_{frict} is the frictional force, μ is Coulomb's frictional coefficient, and F_n is the normal contact force. This simplified representation of the friction can give reasonable results but it is common practice [8, 10, 12, 32] to fit the frictional value to give a good comparison of predicted results with experimentally obtained measurements.

Despite the recognition that there are a number of factors influencing the frictional properties of a material, the lack of a representative mathematical model limits the choice of how to model the friction. This situation is slowly changing with a number of investigations aimed at developing frictional models for application in FEM analysis [30, 31, 33], but the Coulomb formulation remains by far the most commonly used representation of friction.

Table 1.1 gives a summary of some of the frictional coefficients assumed, measured

or quoted in a variety of studies. Although numerous references were made to the effects of friction, only specific references to a value assumed or measured are included as the purpose of the table is to establish a basis for the friction coefficients assumed in the present numerical analysis. Table 1.2 from Schey [18] also gives an outline of some experimentally measured friction coefficients for various materials and lubricants.

1.5.2 Binder Force

The wide variation in LDH test results from one laboratory to another has led to standardization of the test procedure. Schey and Fisher [34] outline a series of tests in which ten laboratories were given comparable material blanks for the purposes of producing a FLD. Significant variations in the sample size and test results were observed despite the fact that the recommended practice was followed as closely as possible. Although some of the differences could be attributed to variations in test procedure, the range of LDH values was higher than the standard deviation of the individual results indicating that variations in the test equipment were significant. One variation, the subject of Schey and Fisher's investigation, was the clamping force. The use of the dome test as a means of characterizing the formability of a material is based on the ability to control the clamping boundary condition and therefore the strain state in the blank, by varying the width of the blank. Any other variation in the clamping boundary condition would introduce another variable, negating the comparative nature of the test. As mentioned previously, the LDH test is based on a fully clamped specimen. The investigation of Schey and Fisher found little published experimental data on the effect of clamping force on the LDH test [34]. Their investigation had as a goal, the quantification of the effect of clamping force and suggested some guidelines applicable to punch tests. The study used a range of specimen widths deformed by a 101.6 mm diameter hemispherical punch (e.g. the Hecker and Ghosh dome tests). Figures 1.13 and 1.14 shows the results of their experimental work. The conclusion was that a clamping force of 250 kN is suggested

as a minimum necessary to keep the amount of draw-in to less than 0.25 mm, as recommended by the North American Deep Drawing Research Group [34].

1.6 Material Constitutive Behaviour

1.6.1 Stress-Strain-Strain Rate Measures

Several relations exist to relate the stress, strain, and strain rate of a material undergoing plastic flow. So long as the deformation is not taking place at high temperatures, most metals deform in a uniaxial stress state following the stress-strain-strain rate relation given by Equation 1.2. The strain dependence, $f(\epsilon)$ may be one of many simple relations usually accounting for the strain hardening of the material. Hollomon, for example, proposed [35]:

$$\sigma_1 = f(\epsilon_1) \cdot g(\dot{\epsilon}_1) \quad (1.2)$$

$$f(\epsilon_1) = K \epsilon_1^n \quad (1.3)$$

where K is the strength constant and n is the strain hardening exponent.

$$n = \left(\frac{\partial \ln \sigma_1}{\partial \ln \epsilon_1} \right)_{\epsilon, T} \quad (1.4)$$

n and K are material parameters derived from a uniaxial tension test. The most common strain-rate dependence of stress, $g(\dot{\epsilon}_1)$, is a power-law dependence:

$$g(\dot{\epsilon}_1) \propto \dot{\epsilon}_1^m \quad (1.5)$$

where m is the strain rate sensitivity exponent. These equations give the material response under a uniaxial stress state. The majority of metal forming operations, however, are performed under a more general stress state characterised by some arbitrary loading condition and complex strain path. The parameters used to extrapolate the material behaviour from a simple loading condition to a more general one are the

effective stress and effective strain [35]. In order to be able to define the effective stress and effective strain, it is necessary to first consider the yield locus which is defined as “the locus of all possible combinations of stress state that will give rise to yielding or plastic flow in a material characterized by a given mechanical or hardness state” [35].

1.6.2 Von Mises Yield Criterion

The von Mises yield criterion, or shear strain energy criterion, is the most widely accepted yield criterion for isotropic materials. The criterion is based on the concept put forward by Huber [36] and Maxwell and Hencky [37] that an element of material subjected to a general stress state would yield when a critical level of distortion energy is reached. The criterion is generally referred to as the Huber-Mises or more commonly the von Mises yield criterion.

In terms of a general stress state, the von Mises criterion is:

$$(\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{xx} - \sigma_{zz})^2 + (\sigma_{yy} - \sigma_{zz})^2 + 6\sigma_{xy}^2 + 6\sigma_{xz}^2 + 6\sigma_{yz}^2 = 2\sigma_Y^2 \quad (1.6)$$

For plane stress, in terms of principal stresses, Equation 1.6 reduces to:

$$\sigma_1^2 + (\sigma_1 - \sigma_2)^2 + \sigma_2^2 = 2\sigma_Y^2 \quad (1.7)$$

The effective stress can now be defined as:

$$\bar{\sigma} = \sqrt{\frac{1}{2} \{ \sigma_1^2 + (\sigma_1 - \sigma_2)^2 + \sigma_2^2 \}} \quad (1.8)$$

Employing the Lévy-Mises relation, the effective plastic strain increment for plane stress is:

$$d\bar{\epsilon}^p = \sqrt{\frac{2}{9} \{ d\epsilon_1^2 + (d\epsilon_1 - d\epsilon_2)^2 + d\epsilon_2^2 \}} \quad (1.9)$$

Similarly it is possible to define the effective plastic strain rate as:

$$d\dot{\bar{\epsilon}}^p = \sqrt{\frac{2}{9} \{ d\dot{\epsilon}_1^2 + (d\dot{\epsilon}_1 - d\dot{\epsilon}_2)^2 + d\dot{\epsilon}_2^2 \}} \quad (1.10)$$

These relations are valid for any arbitrary loading state. If plane stress is assumed, Equations 1.8, 1.9, and 1.10 can be used directly with the stress, strain, strain-rate relation of the form of Equation 1.2 by substituting the effective stress, the effective strain (work conjugate strain of the effective stress), and effective strain rate for σ_1 , ϵ_1 , and $\dot{\epsilon}_1$, respectively. For a general loading case, the differential equations, Equations 1.8, 1.9, and 1.10 are used in the differential form of Equation 1.2 by integrating over the strain path. This differential or incremental form is the form of the equations commonly employed in non-linear finite element analysis routines.

1.6.3 Anisotropy

A major assumption that is made by more traditional yield functions, such as those by von Mises, Tresca, Lévy-von Mises, and Reuss for example, is that all materials are isotropic and remain so during deformation. In sheet metals in particular, a texture develops in the material as a result of the plastic deformation imposed by rolling. Texture manifests itself in the material as an anisotropy [38] and this anisotropy plays a significant role in determining the deformation characteristics of the material.

Single crystals, undergoing tensile elongation, tend to elongate in the direction of the tensile load through a mechanism of glide and rigid body rotation. As outlined by Taylor [39] in his early work on active slip systems in polycrystalline metals, grains of a polycrystalline material also tend to rotate to a preferred orientation through the same mechanism, although the rotation is not as much as would be observed in a single crystal because of the mutual constraints of neighbouring crystals. An initially isotropic metal, with a random distribution of grain orientations, will have a non-random distribution of grains after plastic deformation. A larger percentage of the grains will have a similar orientation as defined by the direction of the load the material was subjected to. If the resulting texture is strongly oriented in one particular direction, it is referred to as a preferred orientation [40].

Anisotropy is commonly characterised by Lankford's coefficient, r , which, in a

tensile test, represents the ratio of width strain increment to thickness strain increment.

$$r = \frac{d\epsilon_2}{d\epsilon_3} \quad (1.11)$$

The r value is predicted to have the effect of stretching the yield locus (represented in plane stress space) into an ellipse along the equiaxial stretching line as shown in Figure 1.15.

The number of active slip systems or potentially active slip systems in a material has a significant effect on the level of strain ratio or anisotropy [41]. A material with a single slip system aligned with the plane of the sheet, for example, will have an infinite r value. In the other extreme, a material with an infinite number of active slip systems will be isotropic [38] ($r = 1.0$). Face centred cubic materials have twelve potential slip systems and, in commercial sheet materials, $r_{max} = 1$ can be obtained with the average being about 0.6 [38]. In body centred cubic (bcc) metals, more slip systems can be active as there exists four slip directions on a number of similar slip planes. The possible slip systems, however, are more constraining than those which exist in an fcc metal. Depending upon which slip system is active, higher or lower strain ratios are obtainable. Anisotropy values from 0.3 to 2.2 have been measured in bcc metals [38]. Close packed hexagonal metals have a single slip direction in one of three slip planes. Again, depending on the direction of the load (i.e. sheet orientation with respect to the crystallographic planes) relative to the slip plane either a high or low r value is possible. Because of the limitation in the number of slip systems which can be active, slip will either be heavily constrained or heavily favoured resulting in extreme r values. Pearce [38] reports values as low as 0.12 in zinc and as high as 9.0 in zirconium.

An experimental study by Bramly and Mellor [42] showed that the r value can vary quite widely in a sheet of material depending on the orientation with respect to the rolling direction (see Figure 1.16). This variation in r in the plane of the sheet is denoted Δr . It is obvious that a material model incorporating the continuous

variation of the r value across the sheet would be prohibitively complicated. As a result, models of anisotropy incorporate one, two, or three parameters describing the anisotropy. Commonly, an average r value is determined in order to simplify the treatment of anisotropy. The r_{avg} or $\bar{r}$ can be found either from the area under experimentally determined curves such as Figure 1.16 or as defined by:

$$\bar{r} = \frac{r_0 + 2r_{45} + r_{90}}{4} \quad (1.12)$$

where r_0 , r_{45} , and r_{90} are the anisotropies measured along the rolling direction (RD), at 45° to the RD, and transverse to the RD. Equation 1.12 is the most commonly used definition because of the relative simplicity in determining the three r values used in the equation through a series of tensile tests of specimens cut along the three directions relative to the rolling direction. Deriving $\bar{r}$ from Figure 1.16 requires an enormous effort and a significant volume of material. Note that the r value is found using a standard tensile test by measuring the transverse strain and the thickness strain. The r value is then determined by taking the ratio of these strains at a given longitudinal strain, usually about 15% strain. It is important to point out that the use of $\bar{r}$ as a representative measure of anisotropy has been challenged [42, 43] as most materials do not exhibit pure planar isotropy.

A reasonable and more accurate representation has yet to be put forward and there is some evidence [42] that the use of $\bar{r}$ does give reasonable results but only for simple load cases. The assumption of a constant state of anisotropy is also not valid for large deformation problems as the anisotropy is closely related to texture which in turn develops and changes during deformation [38, 44, 45]. Again, a simplified approach cannot be used to account for these variations although this assumption is made in most analytical and numerical solutions. More powerful computational techniques are presently being developed which will, in the future, allow the inclusion of a larger number of state variables into the problem. Anisotropy is one of those state variables, friction is possibly another. For the purposes of the following discussion, we will accept the notion of $\bar{r}$ as a measure of the anisotropy in a material.

1.6.4 Hill's Quadratic Yield Function

Hill's quadratic yield criteria (also known as Hill's 1948 yield criterion) [46] is one of the most commonly used anisotropic yield criteria currently implemented in finite element codes. A significant effort has been made to verify and define the limitations of Hill's 1948 yield function and its applications [10, 38, 42, 47] but despite the presentation of newer anisotropic yield criteria, as discussed in the next section, Hill's quadratic yield function is still commonly used.

Using the observed anisotropic behaviour of most material as a framework, and ignoring internal stresses that may result from the texture of the material. Hill defined a yield criterion for materials with three orthogonal planes of anisotropy [46]. That is, the anisotropy of the material has three mutually perpendicular principal axes. In a rolled sheet, these principal directions will be coincident with the rolling direction, the transverse direction and the through thickness direction (assuming any non-uniformities which may occur at the edge of the sheet are small). To summarize the assumptions made in Hill's theory [36]:

- the material possesses three principal anisotropy axes which are orthogonal
- no Bauschinger effect is exhibited by the material, as a result linear terms ignored
- internal stresses resulting from the differential orientation of the grains associated with the anisotropy are ignored
- hydrostatic stresses do not influence yielding
- the yield criterion is quadratic in the stress components (i.e. as with the von Mises criterion)
- the yield criterion should reduce to the von Mises (isotropic) yield criterion for "vanishing" anisotropy

Based on these assumptions, Hill proposed the following yield criterion:

$$2f(\sigma_{ij})^2 = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 = 1 \quad (1.13)$$

where $f(\sigma_{ij})$ is the plastic potential or flow stress

The parameters F, G, H, L, M , and N are characteristic of the state of anisotropy. If, following Hill notation, X, Y , and Z are the tensile yield stresses and R, S , and T are the yield stresses in shear (all taken in the principal axis of the anisotropy), the following relations hold:

$$\begin{aligned} \frac{1}{X^2} &= G + F & 2F &= \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \\ \frac{1}{Y^2} &= H + F & 2G &= \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2} \\ \frac{1}{Z^2} &= F + G & 2H &= \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \\ 2L &= \frac{1}{R^2} & 2M &= \frac{1}{S^2} & 2N &= \frac{1}{T^2} \end{aligned} \quad (1.14)$$

Note that the form of the equation given above only holds if the xyz reference axes coincides with the principal axis of anisotropy. For an arbitrary reference axis system, a simple coordinate transformation is required to put the stresses in terms of the axis of anisotropy.

For planar isotropy, that is, the anisotropy is symmetric about the thickness direction or z axis, the following hold true:

$$N = F + 2H = G + 2H \quad \text{and} \quad L = M \quad (1.15)$$

For complete symmetry:

$$L = M = N = 3F = 3G = 3H \quad (1.16)$$

and the yield function reduces to the von Mises criterion which is consistent with Hill's assumption that the yield criterion should reduce to the von Mises criterion when the material approaches isotropy.

We now take the general equation and apply it to a rolled sheet. The sheet is assumed to deform in plane stress ($\sigma_z = 0$) and if the through thickness shear stresses are assumed to be small, Equation 1.13 reduces to:

$$2f(\sigma_{ij})^2 = (G + H)\sigma_x^2 - 2H\sigma_x\sigma_y + (H + F)\sigma_y^2 + 2N\tau_{xy}^2 = 1 \quad \text{at yield} \quad (1.17)$$

A further simplification can be made if the plate is assumed to be loaded by two stresses σ_1 and σ_2 lying along the principal axes of anisotropy and if we assume through-thickness anisotropy (Equation 1.15), characterised by Lankford's coefficient (Equation 1.11).

$$\sigma_0^2 = \frac{3(1+r)}{2} (2+r) \left\{ \sigma_1^2 + \sigma_2^2 - \left(\frac{2r}{(1+r)} \right) \sigma_1 \sigma_2 + 2 \frac{2r+1}{r+1} \sigma_{12}^2 \right\} \quad (1.18)$$

If the in-plane shears are assumed to be small, this can be rewritten as:

$$\sigma_0^2 = \frac{3(1+r)}{2} (2+r) \left\{ \sigma_1^2 + \sigma_2^2 - \left(\frac{2r}{(1+r)} \right) \sigma_1 \sigma_2 \right\} \quad (1.19)$$

Using the Lévy-Mises relation, the corresponding effective plastic strain increment is:

$$d\bar{\epsilon}^p = \sqrt{\frac{2(2+r)}{3(1+2r)}} \left\{ (d\epsilon_1 - r d\epsilon_3)^2 (\epsilon_2 - r d\epsilon_3)^2 + r (d\epsilon_1 - d\epsilon_2)^2 \right\}^{\frac{1}{2}} \quad (1.20)$$

Hill's original papers presenting his quadratic yield criterion [46, 48] gave only qualitative evidence to support his theory. Referring to experimental observations such as the presence of a zero extension line and trends in the necking angle versus the orientation of the rolling direction for tensile tests of strips cut from rolled sheets of steel, Hill was able to support the characteristics of the yield function.

Experimental Investigations of Hill's Quadratic Criterion

Responding to a general lack of experimental testing of Hill's quadratic yield criterion, Bramley and Mellor [42], carried out a study which included an investigation of the predictions of Hill's yield criterion compared to experimental results. An example of the results of simple tension and biaxial bulge tests carried out on a steel (type unspecified) with $\bar{r} = 1.48$ on are shown in Figure 1.17 in terms of true stress versus natural strain. Curves 1 through 3 are the tensile test curves for sheet orientations of 0°, 45°, and 90°. Curve 4 is from a biaxial bulge test and curves 5 through 7 represent theoretical predictions. This curve is representative of the qualitative agreement which Bramely and Mellor found.

An investigation by Pearce [38] concentrated on the effect of the level of anisotropy on the yield and flow of a metal subjected to uniaxial and biaxial tension. As with the study by Bramley and Mellor, uniaxial tensile tests of sheets cut at 0° , 45° , and 90° relative to the rolling direction were carried out to determine the stress strain curves. Biaxial tests were carried out using a bulge test. Figures 1.18 and 1.19 show results from the study for aluminum and steel respectively, presented in a similar manner to those from Bramley and Mellor. As with the results presented by Bramley and Mellor, the predicted agreement for materials with $r > 1$ are quite good with the predicted curve falling between the uniaxial and biaxial experimental curves. The prediction was particularly good for materials exhibiting high r values (eg. zirconium and titanium). For materials with $r < 1$, however, the prediction was quite far off and the predicted biaxial curve fell below the uniaxial curve. The exact opposite was shown to be the case by the experimental data.

Pearce argued that the effects of microstructural elements of the metals, especially for materials with low r values, could account for the differences between the predicted and experimental results. The void formation in zinc and the effects of dislocation entanglement, initial strain hardening of metals with low r values, and the variation of the r value in materials undergoing deformation as a result of grain rotation were cited as possible causes for the differences. Plasticity theory is limited in its ability to account for such mechanisms. The poor results for low r value materials showed that more work was needed in this area and later prompted Hill to produce a modified version of his yield criterion in an attempt to provide a more general formulation suitable for materials with wider range of r values..

Analytical Investigations of Hill's Quadratic Criterion

The theory presented by Marciniak and Kuczyński in 1967 [49] for predicting the localization of strain in a sheet became the starting point for many analytical investigations of strain localization [50, 51, 52, 53]. It also proved to be a vehicle for the

investigation of Hills' quadratic yield criterion upon which their analysis was based. The initial aim of the approach was to predict a localization mechanism for strain states in the regime of biaxial stretching. The major component of the work was the assumption of a local inhomogeneity in the material stemming from either variations in the thickness, distribution of impurities, or a local variation in the orientation of grains for example. The effect of r value on the limit strain predicted by the M-K analysis is shown in Figure 1.20. The increase in r actually decreases the limit strain. This is in contrast to its effect during drawing where a high r value tends to contribute to a resistance to strain localization, increasing the drawability of the material.

Sowerby and Duncan [53] interpreted the M-K analysis non-mathematically by observing the effect of r on the yield surface as shown in Figure 1.21. Increasing the value of r stretches the elliptical yield locus along the equi-biaxial load path, OQ as shown previously in Figure 1.21. As this occurs, the plane strain line, OS , also moves towards the equibiaxial line and the material with the higher r is more likely to reach failure (i.e. plane strain in the groove) than the material with an r value of 1. Sowerby and Duncan point out that although the yield theory predicts the reverse for $r < 1$, experimental evidence does not support the shrinking of the yield locus towards a circle for materials with an r value less than 1. The small effect of variations in r near plane strain can also be deduced from Figure 1.21. As B_0 , the stress state in the developing localization at initial yield, approaches the plane strain condition, the strain increment to failure of both cases ($r = 1$ and $r > 1$) becomes the same.

Venter *et al.* [54] aimed to provide experimental support to the extended M-K analysis through tensile testing, torsional testing, circular bulge tests, and elliptical bulge testing on grooved and ungrooved annealed aluminum sheet. The purpose of the groove was to simulate the initial inhomogeneity proposed by Marciniak and Kuczyński [49]. Venter *et al.* calculated an average r value ($\bar{r} = 0.54$) from the experimental strain measurements. They showed, however, that the predicted yield

locus did not fit the experimental data. One possible explanation for the poor fit obtained was cited as being the inability of Hill quadratic yield surface to correctly predict the behaviour of materials with r values near to or less than unity.

The results of these investigations indicated that Hill's quadratic yield surface should be used with some caution. It is clear that for materials with $r \leq 1$, Hill's 1948 criterion does not capture the correct response of the material. This precludes its application to materials such as aluminum which exhibit r values on the order of 0.4 to 0.8. For steels and most other materials with r values greater than 1, the quadratic yield criterion does give fairly reasonable results.

1.6.5 Hill's Non-Quadratic Yield Function

Although Hill's quadratic yield criterion demonstrated a fairly good correlation with experimental data for steels and other materials with $\bar{r} > 1$, there were serious discrepancies between predicted stress-strain relationships and experimental observations for materials with $\bar{r} \leq 1$ [47, 55]. These discrepancies were particularly evident in biaxial stress-strain predictions of materials such as aluminum [38]. In 1979 Hill proposed a new form for his anisotropic yield criterion in an effort to correct for these deficiencies in his original theory.

$$f |\sigma_2 - \sigma_3|^M + g |\sigma_3 - \sigma_1|^M + h |\sigma_1 - \sigma_2|^M + a |2\sigma_1 - \sigma_2 - \sigma_3|^M \\ + b |2\sigma_2 - \sigma_3 - \sigma_1|^M + c |2\sigma_3 - \sigma_1 - \sigma_2|^M = \bar{\sigma}^M \quad (1.21)$$

where the value of the exponent, M (not to be confused with the strain hardening rate exponent, m), is determined experimentally from uniaxial and biaxial tests.

It can be shown that Equation 1.21 reduces to the following form for deformation under plane stress in a material exhibiting planar isotropy:

$$\bar{\sigma} = \left\{ \frac{1}{2(1+r)} \left[(1+2r) |\sigma_1 - \sigma_2|^M |\sigma_1 + \sigma_2|^M \right] \right\}^{\frac{1}{M}} \quad (1.22)$$

The corresponding effective plastic strain is:

$$d\bar{\epsilon}^p = \frac{2(1+r)^{\frac{1}{M}}}{2} \left[\frac{1}{(1+2r)^{\frac{1}{(M-1)}}} |d\epsilon_1 - d\epsilon_2|^{\frac{M}{(M-1)}} + |d\epsilon_1 + d\epsilon_2|^{\frac{M}{(M-1)}} \right]^{\frac{(M-1)}{M}} \quad (1.23)$$

The exponent M changes the shape of the yield locus as shown in Figure 1.22. For $M = 1$, the yield locus is convex. For $M < 2$, the yield locus is elongated in the direction of balanced biaxial tension [55], and for $M = 2$, Equation 1.22 reduces to Hill's original equation Equation 1.18.

Experimental Investigations of Hill's Non-Quadratic Criterion

An analytical/experimental investigation by Kobayashi *et al.* [43] of the validity of Hill's non-quadratic yield criterion demonstrated reasonable agreement between predicted and experimental stress strain responses for a variety of materials (steel, copper, brass, and aluminum). A value of M was calculated for each material using uniaxial and biaxial stress-strain data. The stress-strain relations were then derived for each material using Hill's non-quadratic yield criterion and compared to experimental data. Kobayashi *et al.* were able to show that the predicted stress-strain curves for all four materials were very close to the experimentally determined curves. The variations that did exist were attributed to two factors. First, a constant value of M was assumed even though experimental data showed that M varies with strain. The choice of an M value at an arbitrary strain ($\epsilon = 0.100$ in their study) is necessary in order to simplify the analysis but a more accurate approach would be to provide a relation for M in terms of the equivalent uniaxial tensile strain. The second contributor to the error was indicated to be the assumption of planar isotropy in the material through the use of an average r value (Equation 1.12):

$$\bar{r} = \frac{r_0 + 2r_{45} + r_{90}}{4}$$

The use of $\bar{r}$ is the most common approach to dealing with planar anisotropy but there are indications, as discussed in Section 1.6.3 above, that this method can lead to

errors in the predicted material response. The original formulation allows for planar anisotropy although the axes of the principal stresses are assumed to be coincident with the principal axes of anisotropy. The implementation of the full formulation in a case where this assumption is not valid would be very difficult if not impossible [8].

Analytical Investigations of Hill's Non-Quadratic Criterion

A study of Hill's new yield criterion by Parmar and Mellor [55] investigated some aspects of the non-quadratic yield criterion. The work centres around the failure of Hill's quadratic yield criterion to correctly predict the behaviour of materials with r values less than unity. They found reasonable correlation for materials with r values in the order of 1-2 (eg. killed steel, stainless steel) but there were serious discrepancies for materials with r values under 1 (eg. aluminum). Experimental results [42] showed that curves for balanced biaxial tension always fall above uniaxial curves for a given material. Predicted uniaxial curves, however, fell well above the biaxial curves. Dillamore [47] argued from a crystallographic point of view that the quadratic yield criterion could only give reasonable results for materials with r values between 1 and 2 and this seems to be supported by the comparative studies which were carried out. The study by Parmar and Mellor gave preliminary indications that the new Hill yield criterion did provide a better description of materials with r values less than unity.

By implementing Hill's non-quadratic criterion in the Marciniak and Kuczyński [49] approach, Parmar and Mellor were able to show trends in the predictions which would improve the M-K analysis for materials with r values less than one. The results of the effect of variations of the index M on the limit strains is shown in Figure 1.23 for a material with $r=0.5$. As noted above, a value of 2 for M gives Hill's original quadratic yield surface. Reducing the index M results in a significant reduction in the limit strain for a given stress ratio. Also note that variations in M do not effect the limit strain in plane strain. As M decreases, the stress ratio to give plane strain,

and hence a localization, approaches one [55]. A typical M value for aluminum is 1.8. If this value is assumed in the analysis of Parmar and Mellor, it is easy to see from Figure 1.23 that there would be a reduction in the limit strain over that predicted by using $M=2$ (i.e. over the original M-K and Sowerby and Duncan approach) because of the stretching of the yield locus similar to the effect of an increased r value. This trend is supported by the comparison of experimental observations and predictions made using the M-K analysis [56].

1.6.6 Other Anisotropic Yield Functions

Other methods of modelling the anisotropy of sheet metal have been proposed. These include a generalized quadratic flow law applicable to sheet metals presented by Jonas and Gillis [57], Hosford's special case of Hill's non-quadratic yield criterion [58], and the continuum mechanics of textured polycrystals (CMTP) method discussed by Lequeu and Jonas [59]. All try to give a more accurate description of the deformation characteristics of anisotropic materials but in general they have had limited use in the field of analytical prediction of sheet formability. Hill's quadratic function seems to be the most popular in terms of both analytical and numerical work. Hill's non-quadratic yield function is the second choice, next to his original 1948 yield function, but it seems to be gaining wider acceptance. The one major stumbling block seems to be the determination of the exponent M which is dependent on the level of strain [43]. Accurate matching of the yield function requires the M value to be updated depending on the strain state. Nevertheless, Hill's new formulation seems to fill the weakness of the original formulation by providing a means to accurately capture the yield characteristics of materials with low r values.

1.7 Summary of Material Property Effects on the Formability of Sheet Metal

The following is a brief summary the effects of variations in the material parameters considered in this study. A brief mention of the effect of thickness is also included with the discussion of the effects of r and μ .

1.7.1 Anisotropy

The level of anisotropy (defined above in Section 1.6.3) present in a sheet metal has a significant effect on the formability and drawability for most materials [47]. As mentioned previously, the anisotropy of a material is a measure of the variation of the width strain to a variation of the thickness strain which in turn is a measure of the crystallographic preferred orientations of the material.

It is generally agreed that a higher r value has a negative effect on the FLD in the stretching regime (i.e. positive minor strain) and that the limit strain at plane strain is insensitive to r [51, 53, 55]. Pearce [38] states that metals with $r > 1$ demonstrate an increased resistance to yielding or texture hardening in biaxial tension. For materials with $r < 1$, texture softening or a decrease in the resistance to yielding is observed. This observation is supported by analytical work by Parmar and Mellor [55] and Sowerby and Duncan [53].

Predictions of the effects of r on the limit strain at negative minor strains was examined analytically by Chan *et al.* [51]. The study indicated that an increased r value can significantly delay the onset of a localization, increasing the FLD (i.e. the opposite effect as in stretching). Lahoti *et al.* [52] and Choi *et al.* [62], however, put forward analyses which predicted that the limit strain is insensitive to r for negative minor strain. Choi *et al.* indicates, instead, that an increase in r simply extends the range of the FLD farther to the left (increasingly negative minor strain).

In terms of the punch test strain distribution, increasing r tends to suppress the

deformation over the punch, moving the localization away from the pole (the top of the dome) and decreasing the limiting dome height (refer to Figure 1.24) [63].

1.7.2 Thickness

In general, the forming limit is considered to increase with increasing sheet thickness [6, 17]. This is consistent with the observation that a material with a higher initial thickness will undergo more plastic flow in the region of the localization before fracture occurs [11].

1.7.3 Friction

Although not generally considered to be a material parameter, it is important to include a discussion of the effect of friction on the forming limit. Lubrication or variations in frictional coefficient do not have an effect on the forming limit diagram directly. The influence of μ is solely on the strain state in the material and does not change the actual limit strain for a given strain state. In formability studies, however, it can have a significant effect. As mentioned in Section 1.5.1, Hecker [17] found that increasing the lubrication in a formability test decreases the sensitivity of the material to small changes in the material properties.

In a punch test, a higher frictional coefficient will have the effect of stabilizing the deformation over the punch and driving the localization farther from the pole. As a result, a lower pole strain is expected along with a sharper localization closer to the punch-blank tangency point. A lower μ will tend to smooth out the deformation resulting in a more diffuse localization and a larger minor strain, either more positive for stretching or more negative for drawing.

1.8 Application of FEM to the Simulation of Sheet Forming

The purpose of this section is to outline some of the previous finite element investigations which have been carried out in this field with particular attention to the simulations of the punch test. A detailed outline of the finite element code and model used in this investigation is contained in Chapter 3.

In recent years, there have been a large number of papers and other publications discussing various components of FEM techniques and their application to sheet forming simulations. A significant number have dealt with element formulations, material models, and the accurate description of frictional contact forces and other boundary conditions. The focus of this research, however, is the application of FEM to the prediction of the strain distribution and fracture in sheet metal forming. In view of this, the review of the literature will focus on applications of FEM techniques to forming processes and the comparison of the results with experimental predictions.

1.8.1 Trends in the FEM Simulation of Metal Forming Operations

This area of research is advancing rapidly, so much so that reviews of the state of the research are out of date as soon as they are published [68]. Such a review by Wagoner and Zhou [68] comparing the field as it was in 1989 to the field just before publication of their paper in 1992 gives some idea of the advances made over a period of only three years. The interest level in sheet forming analysis was high in 1989 but no accurate and reliable methods were demonstrated to simulate general forming problems. Most simulations were limited to simple geometries and used membrane elements and static implicit formulations [10, 12]. Other areas of importance such as CAD/CAM compatibility which are critical to the practical application of such simulations in terms of die try out and refinement, were not available either. Interestingly, the early

study by Honecker and Mattiasson [69] using DYNA3D was cited, not only as one of the very few using the explicit method, but also because of its reasonably accurate simulation of wrinkling. In summation of the state of numerical simulations of sheet forming operations at that time (1989), Wagoner and Zhou [68] conclude that:

- the scatter among programs is great
- the operator-operator scatter is about equal to the program-program scatter
- general-purpose and custom programs exhibit similar scatters
- single-step and similar programs (geometric etc.) lack predictive capability
- contact and frictional conditions cause the most serious simulation problems
- more experienced groups consistently get better results

In contrast, just 3 years later, Wagoner and Zhou found that many of the earlier problems plaguing the field had been solved to a greater or lesser degree to the point that industrial applications of the FEM technique were being made [71, 70]. A greater use of explicit formulations with shell elements was observed. In addition, scatter among various solutions of the VDI Benchmark [68] problem was significantly reduced even though the problem was more complex than those upon which the previous conclusions were based.

1.8.2 Major Issues

Time Integration Scheme

The use of dynamic explicit formulations over implicit FEM models in the field of sheet forming has really only occurred recently [68]. Explicit time integration is based on direct integration of the equations of motion using the central difference scheme which avoids the need to form and solve a large set of stiffness equations. Implicit formulations, on the other hand, utilize large stiffness matrices and require the solution of a set of simultaneous equations.

Both methods are capable of simulating forming processes but the implicit method has inherent problems which make the explicit method much more attractive. Some of these are outlined in detail by Rebelo *et al.* [72] and by Honecker and Mattiasson [69] who noted that implicit codes have difficulty dealing with intermittent contact conditions and wrinkling which is a buckling instability.

Also of interest with larger analyses is the non-proportional increase in run time and storage capacity required by an implicit solver. In contrast, dynamic explicit model run times tend to increase in proportion to the model size. The benefit is even greater when the applicability of explicit programs to parallel processing is considered. The general conclusion of both studies was that explicit formulations offer significant advantages over implicit solutions in terms of flexibility and run time especially for large, more complex problems [32, 69, 72]. For smaller problems, such as parametric studies on simple geometries, implicit solutions can offer some advantages in terms of efficiency and accuracy [69].

Boundary Conditions

The area that is least well understood but which has the most significant effect on any formability study is the frictional boundary condition.

An example of a study of the effect of friction has been performed by Knibloe and Wagoner [12] who undertook an FEM simulation of a hemispherical punch-stretching experiment. The experimental portion of the work was based on measured strain distributions and the FEM predictions were compared to these distributions in an effort to investigate the anisotropic coefficient, M , of Hill's non-quadratic yield function and the friction coefficient which were observed to have opposite effects on the strain distribution. The computation model was based on a membrane formulation of a rigid viscoplastic theory with the yield criterion based on Hill's non-quadratic yield criterion with normal anisotropy. Contact forces were determined using a contact algorithm and Coulomb friction. In their attempts to obtain reasonable results for

comparison with experimental measurements, Knibloe and Wagoner found that it was necessary to fit the frictional coefficient such that the predicted pole strains were comparable to measured pole strains. The use of Hill's non-quadratic yield function with M coefficients fit to experimental material characterisation data, was found to result in a lower peak radial strain, a higher strain at the pole, and a peak location closer to the pole [12]. These trends are characteristic of a lower frictional coefficient. To fit the experimentally measured strain distributions, Knibloe and Wagoner had to increase the friction coefficient to compensate for the effect of M . μ values as high as 0.64 were required to match the pole strains. Significantly better results were obtained using Hill's quadratic yield function (i.e. $M = 2.0$) with frictional coefficients between 0.094 and 0.351 giving acceptable predictions of the strain distribution.

Similar results were obtained by Wang and Wenner [63] using an elastic-plastic analysis of hemispherical punch forming. They found that it was not possible to accurately predict the strain distribution at a given punch depth, using a single value of μ . As a result, a μ value was chosen such that the pole strain is accurately predicted for one punch depth.

Burford *et al.* [10] provide further evidence to support the importance of μ on numerical predictions in a parameter sensitivity study by demonstrating that the limiting dome height and predicted strain distributions showed the greatest sensitivity to μ .

Despite the importance of friction in determining the deformation characteristics, the lack of an accurate model of frictional effects makes the use of realistic frictional coefficients difficult. It is a common approach, as noted above, to fit the frictional coefficient used to give an accurate prediction of experimentally obtained data. One might expect that such an approach would be impractical as it would not allow the simulation of different geometries without the need to repeat the fitting procedure. Results presented by Mattiasson *et al.* [32], however, provide some evidence that frictional coefficients from a relatively simple geometry can be applied to other more

complex problems. The study was aimed at assessing the applicability of a commercial explicit FEM code to metal forming operations. The code used incorporated Hill's quadratic yield criterion with the Coulomb formulation for friction and was applied to the stretch forming and deep drawing of a cylindrical cup and a rectangular pan. As with the studies noted above, close agreement between measured and predicted punch force for the stretching and drawing operations was achieved by fitting the predicted pole strain to the experimental results using the μ coefficient. Of particular interest however, was the observation that the coefficients of friction determined using the simple geometry provided accurate predictions of punch force for the more complex rectangular box. It is unclear, however, if similar success was achieved in predicting the strain distributions in both parts.

Constitutive Models

The constitutive equation incorporated in a numerical model is critical to the results which will be obtained. One parameter which is particularly critical in sheet metal forming is the state of anisotropy in the material. The most commonly used anisotropic yield function, as noted in Section 1.6.6 above, is Hill's quadratic yield criterion [10, 12, 32, 73, 74]. Also used, but less frequently, is Hill's non-quadratic function [10, 75].

In a sensitivity study carried out by Burford *et al.* [10], the effect of r was found to be closely related to the yield function used. The FEM model used was based on a 3-dimensional non-linear rigid viscoplastic formulation with triangular membrane elements. Both Hosford's and Hill's non-quadratic yield criterion were implemented, both of which can be degenerated to Hill's 1948 criterion and to the von Mises criterion with the appropriate parameter settings. Friction was predicted using the Coulomb formulation with a single value of μ .

The effect of r predicted by the FEM model was yield theory dependent. According to Burford *et al.*, Hill's quadratic yield criterion predicts a significant dependence of

the limiting dome height at failure (LDH_f) on r which is consistent with the analytical results mentioned above [49, 53, 55]. It is generally agreed, however, that r has less effect on the FLD than was predicted [17]. Hosford's yield criterion, was shown to give better agreement with experimental measurements, predicting a significantly lower sensitivity to variations in the r value chosen. Over all, r was shown to have a smaller effect on the predicted strain distributions than the other parameter in the study including μ and n .

Despite the reservations expressed by a number of authors with the general agreement of the quadratic theory to experimental trends of r , it seems that the quadratic yield theory does provide reasonable results for numerical studies involving steel where r is on the order of 1.5 to 2.0. Although a more general constitutive model such as Hill's non-quadratic yield function does give better agreement with experimental stress-strain data, their application in FEM analyses has been limited. The reason for this could be the added complexity introduced by the additional parameters involved, such as M in Hill's non-quadratic yield criterion.

1.9 Current Research

Drawing on the observations and conclusions of the publications reviewed, it is clear that there is the need for further study of the FEM simulation of metal forming operations. There are many aspects of the field which are not well understood such as the implementation of a more realistic frictional boundary conditions and the correct modelling of materials exhibiting low anisotropy, most notably aluminium.

In the chapters to follow, the present course of research will be outlined. The goal of the current research was to investigate the application of FEM to the prediction of strain in sheet metal forming operations. The experimental investigation, outlined in Chapter 2, was used to provide measured strain distributions with which to verify the numerical predictions. The numerical study was based on a commercial 3-D non-linear FEM code, LS-DYNA3D. The choice of an explicit code over an implicit code

was based on the advantages inherent with the use of explicit time integration (e.g. efficiency, lower run time for large analyses, and the efficiency of the implementation of contact algorithms). A FEM model of the limiting dome height test used in the experimental investigation was constructed as described in Chapter 3. A series of analyses were run to investigate predicted strain distributions for three materials, two steels and one aluminum. Parametric studies of the effect of r predicted by Hill's anisotropic yield criteria and μ , the Coloumb frictional coefficient, on the strain distribution were also carried out. Results of the experimental and numerical investigations are presented in Chapter 4. Finally, conclusions for the research are given in Chapter 5.

Chapter 2

Experimental Programme

Limiting dome height (LDH) experiments were performed on a variety of materials exhibiting varying degrees of planar anisotropy. These experiments were used to assess and validate a numerical simulation of the forming process. During the first part of the investigation, forming limit curves for the four materials were produced. From the FLDs, three specimen geometries were chosen giving a variation in strain condition at failure representative of the three forming conditions (i.e. biaxial stretching, plane strain, and drawing). These three geometries were then used as the basis for a series of interrupted dome tests. By performing a series of tests that were interrupted at varying punch depths, it was possible to determine the strain distribution as a function of punch depth, material characteristics, and strain condition. The development of the localization during the course of the test could then be visualized.

2.1 Material Selection

Four materials were chosen for the experimental investigation. The main property of interest was the level of anisotropy present in each material. Three commercial steels were supplied by the Metal Forming Technology Laboratories (MTL) of CANMET, an Interstitial Free (IF) steel and two Aluminum Killed Drawing Quality (AKDQ or simply AK) steels. The IF steel, designated IF, and one of the AK steels, designated

AK1, were supplied to CANMET by Dofasco Inc. The other AK steel (designated AK2) was a lighter grade commercial sheet steel bought from a local supplier. As mentioned previously, commercial drawing quality steel tends to have quite a high r value, usually on the order of 1.8-2.5. To provide a contrast, a relatively low r value material, aluminum, was also used in the experimental investigation. Two sheets of X544.0 aluminum sheet (designated AL) were provided by the ALCAN Kingston Research and Development Laboratories, Kingston, Ontario.

The sheets of the IF and AK1 steel supplied by Dofasco and the aluminum sheets were all cut from the same coil thereby ensuring a level of uniformity between individual sheets. It was impossible to ascertain if all of the sheets of the commercial AK steel, bought from a local supplier (material AK2), was also cut from the same coil. As a result, the material characterization (FLD blanks and tensile test specimens) and the interrupted dome height tests were carried out using different sheets of the AK2 material. In this way all specimens for a given series of tests were assured to be uniform.

2.2 Material Characterization

The chemical compositions of the four materials used in the investigation are given in Tables 2.1 and 2.2. Of particular interest in terms of the composition of the steels is the extremely low carbon content of the IF steel compared to that of the two AK steels, AK1 and AK2. This low carbon level reduces the possible nucleation points for void growth significantly improving its plastic properties in terms of ductile fracture.

The mechanical properties of each material were determined using tensile tests. Three tensile test specimens for each of the 0°, 45°, and 90° directions, relative to the rolling direction (RD), were cut from the as-received sheets (refer to Figure 2.1). The samples were pulled in uniaxial tension in a MTS servo-hydraulic 100 kN (22 Kips) testing machine with an Instron 8500 controller. This work was carried out by the mechanical testing laboratory of CANMET and followed ASTM E517 [76] and

E646 [77].

The mechanical properties for the 0°, 45°, and 90° specimens are summarized in Table 2.3. The values given represent averages from the three tests for each orientation. The L, D, and T designations refer to tensile properties along the rolling direction (RD), inclined at 45° to the RD, and transverse to the RD respectively. The mean values for each material are presented in Table 2.4 and represent an equivalent anisotropic material exhibiting planar isotropy. Note the values for $\bar{r}$ were calculated using the weighted average discussed previously (Equation 1.12). True stress true strain curves for the materials are shown in Figures 2.2 through 2.5 and were derived from the load-extension data recorded during the tensile tests.

2.3 LDH Specimen Preparation

2.3.1 Blank Cutting

Blanks were cut from the sheets of the various materials using a powered shear and trimmed to exact dimensions with a hand operated shear. The dimensional tolerance on the blanks was held to less than 1 mm overall. A 50 – 75 mm margin was removed from all edges of the sheet so that non-uniformities due to edge effects of rolling were eliminated. For consistency, all blanks were cut with the major dimension in the rolling direction. Although it is understood that, as a result of planar anisotropy, the formability of a partially clamped blank will vary depending on the direction of the principal axis of anisotropy, cutting the blank parallel to the rolling direction is a common practice for the prediction of the FLD. This practice was adopted for all test specimens.

Areas with severe corrosion were cut away from the sheet to provide a consistent surface finish. This consistent finish is critical for repeatability of test results as corrosion causes a variation in the frictional boundary conditions between the plate and the punch. Surface condition is also critical for the etching procedure as outlined

in Section 2.3.2. Thickness measurements were made on each blank to ensure that no large variations existed throughout each sheet. Randomizing the specimens has been suggested as a method of removing any variations in material properties along the length of the coil which may lead to variations in the FLD [34]. Because of the relatively small amount of material used in the present investigation, it was felt that such a process was not warranted.

2.3.2 Surface Preparation

Strain measurements were made using the circle grid analysis first introduced by Keeler [78]. A regular circle grid pattern was electrochemically etched on the surface of the blanks before testing (see Figure 2.6). The circle diameter used for most specimens was 2.54 mm (0.10 inches) as is common practice in dome test strain measurements [10, 12, 19, 79]. The only exception was on the uniaxial test specimens of the IF, AK1, and AK2 materials where grids 1.27 mm (0.05 in) in diameter were used. The smaller grid size is usually required to capture the strain next to the localization because of the relatively small area of the localization and the high strain gradient. Best results, in terms of grid sharpness and dimensional precision, are achieved using a photochemical etch procedure but this technique was not available at the MTL laboratories during this research. An electrochemical etching system from Chem-Etch and Stamp of Canada Ltd. [80] was used. Considerable effort was made to revise the etching procedure to improve the quality of the resulting grids. The procedure is material dependent as sheet thickness, conductivity, surface finish, and any protective coating can affect the quality of the etched grids. The results for the steel samples were quite good with fairly uniform, crisp grids being produced.

The condition of the surface of the specimen to be etched is a critical factor in the finished result. Any surface corrosion, surface treatment, corrosion protection (e.g., protective oil or galvanizing), even oil from finger prints, will prevent the etchant from being in full contact with the metal. To this end, a careful surface preparation

routine was followed. For the steel, this consisted of removing the protective oil coating and any loose debris or corrosion using an acetone bath in an ultrasonic degreaser. Care was taken during subsequent handling of the specimens to ensure that no contaminants, water, or finger prints were deposited on the sheets before the etching procedure.

The surface condition of the as-received aluminum was very good. Any oil on the aluminum was picked up from the shear used to cut the blanks and a methanol soaked cloth was sufficient to clean the surface before etching. To develop an acceptable etching procedure for aluminum, some blanks were etched using the manufacturers suggested practices (a combination of aluminum etchant or steel etchant and either AC or DC current). These trials indicated that the oxide layer on the aluminum presented a significant resistance to the etching process. A nitric acid bath (50% HNO_3) and a sodium peroxide bath (5% NaOH) were used to temporarily remove this oxide layer and pre-etch the aluminum surface so as to provide a dull surface to contrast the etched grids. The relatively high oxidation rate of aluminum made it necessary to etch the specimens soon after the initial surface treatment in order to achieve the best results. The results of the etching procedure were not as good as on the steel blanks but were sufficient to allow measurement of the strain to a reasonable degree of accuracy (refer Section 2.6 below). The grids were not complete and only the inner and outer edges were etched leaving a hollow donut shape.

2.4 Experimental Setup

The experimental formability test used in this investigation is similar to that proposed by Hecker [17] and Ghosh [19]. A hemispherical punch, 101.6 mm (4 inches) in diameter is pressed through a rectangular blank. Draw-in is prevented by clamping the periphery of the blank in a draw bead. All tests were performed on the limiting height dome test apparatus at CANMET (refer to Figures 2.7, 2.8, and 2.9). A schematic of the data acquisition system is shown in Figure 2.10.

The limiting dome height press consist of a 922.3 kN (207 350 lbs) press with interchangeable clamping dies and punch. For this study, a hemispherical tool steel punch, 101.6 mm (4 inches) in diameter, was used. Punch pressure is applied by a 304.8 mm (12 inch) bore main cylinder and two 101.6 mm (4 inch) bore auxiliary cylinders. Hydraulic pressure is provided by an on-board hydraulic pump which provides a maximum supply pressure of 10.34 MPa (1500 psi) to each cylinder giving a maximum applied load of 754.6 kN (169 650 lbs.) for the main cylinder and 83.8 kN (18 850 lbs.) for each of the auxiliary cylinders. Control of the velocity of the punch is achieved using manually adjusted electronic flow and pressure controllers. Simultaneous adjustment of both controllers allows for punch speeds between 0.5 mm/s (0.02 in/s) and 1.0 mm/s (0.04 in/s).

The clamping dies used for this investigation are of the same dimensions as those used by Hecker [17] and Ghosh [19], shown previously in Figure 1.9. Blank holder force is applied by a single 127.0 mm (5 inch) bore cylinder supplied by an off-board Enerpac Model PEM3055A1 hydraulic pump. The maximum supply pressure is approximately 69.0 MPa (10 000 psi) giving a maximum clamping force of 889.6 kN (200 000 lbs), well in excess of the suggested minimum blank holder forces discussed in Section 1.5.2.

Clamping pressure, punch load, and punch displacement were continually monitored during each test. The blank holder force is derived from the supply pressure of the Enerpac using an Omega Model PX-310-10KGV pressure transducer mounted on the hydraulic supply line of the clamping cylinder. The resolution of the measurement is 1788 kN (402 lbs.) or 0.2% of full scale. Note that the resolutions or system resolutions quoted account for the accuracy of the measuring device and the resolution of the data acquisition system used (12-bit A/D conversion).

Punch load was measured using a custom made load cell mounted between the punch and the punch mounting plate. The load cell was calibrated by the National Aeronautical Establishment of the National Research Council of Canada and is rated

at 896 kN (200 000 lbs.) with a system resolution of 2815 kN (633 lbs.).

Punch displacement was measured with a Schaevitz Engineering Model 3000DC-D linear variable differential transformer (LVDT) which was mounted on the press frame. The core of the LVDT was attached to the punch mounting plate. The range of the LVDT is ± 76.2 mm (± 3 inches) with a system resolution of 0.076 mm (0.003 in).

The signal conditioning component of the data acquisition system is made up of an Omega 901 Power Supply, a 911 General I/O Module, a 941 Strain Module, and a 991 CPU Module. Signals from the pressure transducer and LVDT are fed into the AC 911 General I/O Module while the signal from the punch load cell is fed into the 941 Strain I/O Module. The output from the signal processor is routed to an 286 IBM compatible computer where it is recorded by a data acquisition program called Notebook. The current configuration uses 4 channels (one for time and one for each of the signals) and samples at a rate of 10 Hz while a graphics display of the data is being presented on the screen in near real time. Notebook converts the voltage signals to real values (i.e. pressures, forces and displacements) for display. The raw voltage signals are also displayed during the run on digital displays (refer to Figure 2.10).

2.5 Specimen Test Procedure

Prior to testing, the tooling was thoroughly cleaned using methanol and a clean absorbent cloth to remove any debris and oil. Alignment of the blank in the die was achieved with the aid of two spring loaded pins set into the lower clamping die. Butting the edges of the blank against the pins and centring the blank relative to the two edges of the die, as shown in Figure 2.11, ensured that the blank was centred in the press. This step was critical to the repeatability of the test as any misalignment causes a change in the deformation of the blank and therefore the strain path and strain state at failure. The 203.0 mm (8 inch) wide blanks had to be centred in both directions as the pins are only 191 mm (7.5 inches) apart. Careful measurement using

a scale allowed larger blanks to be centred.

With the blank centred, the clamping die was closed and the clamping pressure was increased to between 24.1 to 27.6 MPa (3500 and 4000 psi). It was noted during the course of the preliminary tests that the clamping pressure decayed during the run. The reduction in the clamping pressure over the course of the test was usually on the order of 1.0 MPa (150 psi) but the applied pressure was sufficient such that the clamping pressure at failure was still well in excess of the suggested minimum to prevent any draw-in (see Section 1.5.2). With the specimen fully clamped, the punch was forced through the blank. Punch head velocities on the order of 0.5 mm/s (0.02 in/s) were used for all tests. The low punch velocity provided quasi-static deformation, eliminating rate effects. The low velocity also aided in the determination of the failure point which can occur quite suddenly. Clamping pressure, punch depth, and punch load were continuously monitored and recorded during the test.

Tests were terminated either when the desired punch depth was reached or when a load drop was observed in punch force. Ideally, tests to failure should be stopped just as localization or necking occurs in the specimen. In practice this is very difficult to achieve as the load drop can be quite sudden. The relatively low resolution of the load cell and chatter in the frictional contact forces at the punch-blank interface can obscure the load drop. The size of the fracture was used as an indication of the success of the test. If the blank deformed significantly after failure (i.e., a large crack or tear was observed in the failed blank), the test was determined to be invalid as the strain distribution in and around the failure would have changed during the post-fracture deformation. In this case, the test was repeated. The termination point of the repeated test was based on the punch depth and/or punch load at failure determined from the load-displacement history of the unsuccessful test as shown in Figure 2.12.

2.6 Grid Measurement

Grid measurements were made using a low powered monoscope with a graduated reticle. The middle of the etched circle was chosen as the reference for the measurements (see Figure 2.13). The graduated reticle has divisions of 0.05 mm. For the steel specimens, it was possible to estimate the grid dimensions to within 0.025 mm (or half a division) but because of the variations in the quality of the deformed grids, measurements were taken to the nearest 0.05 mm, giving an absolute error of ± 0.05 mm or $\pm 2\%$ strain and a relative error on the smallest grid measurement of approximately $\pm 2.6\%$ on a measurement of 1.90 mm. For the aluminum, the grid measurement was significantly more difficult because of the poorer quality of the grids. The etched lines were more prone to being rubbed off during deformation. As well, the shiny surface of the aluminum made the viewing of the grids difficult. The absolute error on a grid measurement was estimated to be ± 0.1 mm or $\pm 4\%$ strain and the maximum relative error was estimated to be approximately $\pm 4.1\%$ on a measurement of 2.45 mm.

2.7 Determination of the Forming Limit Diagram

Specimen widths of 203.2, 152.4, 139.7, 127.0, 120.7, 114.3, 101.6, 76.2, 50.8, and 25.4 mm (8, 6, 5, 4.75, 4.5, 4, 3, 2, and 1 inch) were used. In addition to the 203.2 mm (8 inch) dry specimens, 203.2 mm (8 inch) blanks with various lubricants were also tested to allow an evaluation of the formability on the extreme right (tending towards equibiaxial tension) of the FLD. A lubricated 152.4 mm (6 inch) specimen of AK2 material was also tested but the limit minor strain was approximately the same as that for the dry 203.2 mm (8 inch) blank. Rectangular blanks of less than 101.6 mm (4 inches) in width tore at the draw bead before a localization occurred under the punch. This was due to the relatively sharp draw bead radii and large draw bead depth which contribute to a severe stress concentration at this location and a large

shear band which forms in the blank during deformation. As a result, a machined 'notched' specimen geometry (Figure 2.14) was used to obtain formability data for the extreme left of the FLD. These 'notched' blanks give a full 101.6 mm (4 inch) width at the draw bead, preventing tearing, while tapering to a 25.4, 50.8, or 76.2 mm (1, 2, or 3 inch) section where the localization occurs at a large negative minor strain. Minor strains of the order -11 to -25% were possible using these configurations. Table 2.5 gives a summary of the blank sizes used in this part of investigation as well as the strain conditions (i.e. minor limit strain) achieved for each material.

Three blanks were tested for each geometry to demonstrate repeatability. The only exceptions were the lubricated specimens where repeatability was nearly impossible. Measurement of the grids was carried out as outlined in Section 2.6. As discussed in Section 1.2, the FLD is a plot of the strain state in the sheet adjacent to the fracture. Only grids immediately adjacent to the localization were measured. Grids broken by the failure or a significant localization were usually not measured as they give no useful information due to the large strain gradient. Any measured grids situated across a localization are indicated as **Failed** on the FLDs (refer to Figure 2.15 for a closeup view of the grids surrounding a localization). Grids where a small neck was observed or where a neck was initiating were labelled **Necked** for the purposes of the FLD. **Safe** grids were those right next to the tear or neck where no localization was observed. The forming limit curve (FLC) is drawn through the upper band of the safe points and the lower band of the necked points.

The FLC produced for the three steels are shown in Figures 2.16, 2.17, and 2.18 (IF, AK1, and AK2 materials, respectively). The limited amount of aluminum available was not sufficient to prepare the number of blanks required to produce a FLD. As result only FLDs for the three steel sheets could be produced.

The large scatter of safe points can be attributed to the somewhat coarse distribution of grids. The grid nearest to the failure may actually be some distance away from the localization depending on the path of the fracture. The size of the grid pattern

is critical to the accuracy of the measurements of the FLD and involves a trade-off. A fine mesh will better capture the localization and will allow a measurement of the strain very close to the fracture or neck; however the relatively small gauge length results in a large error in the strain measurements. A more accurate measurement of the deformation of the grid is possible with a coarser pattern but this sacrifices the accuracy of the strain measurement because of the relatively high strain gradient in the area of the localization. A coarser grid pattern will lead to underestimating the FLC as the strain measurement will be an averaged over some distance from the localization.

At this point in the investigation, the IF material was removed from consideration. The similarity in failure strains between the IF and AK1 material, as presented by the respective FLDs (Figures 2.16 and 2.17), and the similarity in averaged material properties including $\bar{r}$ (refer to Table 2.4) lead to the conclusion that the measurement of the strain distributions for both materials would give little additional information at a cost of a significant effort in experimental measurement and numerical modelling. The AK1 material was chosen for the remainder of the investigation because of the extreme range in planar anisotropy, with r varying from 1.674 to 3.007 (Table 2.3).

2.8 Interrupted Dome Tests

Interrupted dome tests formed the core of the current investigation as they allowed the measurement of the development of the localization in the sheet as it deforms. These strain distributions were then used as the basis for a comparative study of the numerical simulation of the forming process. Three representative geometries were chosen from those tested for the production of the FLD. The 50.8 mm (2 inch) machined specimen ($\epsilon_2 \approx -10\%$), 127.0 mm (5 inch) specimen ($\epsilon_2 \approx 0\%$), and dry 203.2 mm (8 inch) specimen ($\epsilon_2 \approx +10\%$) were chosen to give strain states representative of the three modes of forming. A series of six blanks were cut for each material and blank size. The sample preparation outlined in Section 2.3 was followed

as was the testing procedure given in Section 2.5.

A series of dome heights were chosen based on the limiting dome heights observed in the tests for the FLD. Small dome height increments (5 mm and 2.5 mm) were used to capture the development of a localization which, as will be seen in the presentation of the experimental results, can occur over a very small change in punch depth. Blanks were then tested up to each of the dome heights shown in Table 2.6. An example of such a series of test is shown in Figure 2.19 for the 127.0 mm (5 inch) blanks of the AK1 material. Principal strain measurements were then taken along a strip of grids lying approximately at the line of symmetry in the rolling direction. The results of these measurements will be presented in terms of principal strain versus radial distance from the pole, or axis, of the dome (see Figure 2.20).

The number of measurements required to completely characterize the strain distribution in the entire dome is prohibitive. As a result, the measurements were concentrated along the rolling direction or length of the blank. In addition to the 203.2 mm AK1 blanks tested dry, a lubricated AK1 blank was also measured to investigate the effect of lubrication on the strain distribution.

Chapter 3

Numerical Model

Numerical predictions of the strain distribution were made using the LS-DYNA3D finite element code due to Halquist [81]. The analysis runs were performed on an SGI Crimson 100 MIP workstation.

LS-DYNA3D employs the central difference method of explicit time integration. This method greatly reduces the storage requirements compared to the implicit method where a large global system of equations are formed and solved simultaneously. Another benefit of using the explicit formulation is the efficiency with which intermittent contact surfaces or slide lines can be implemented. LS-DYNA3D is particularly strong in this respect and a significant effort has been put into the implementation of efficient and reliable contact algorithms [69, 82].

The benefits associated with an explicit formulation are somewhat offset by the stability condition which limits the maximum step size. The maximum time step size corresponds to the period associated with the highest natural frequency of the system according to the Courant criterion [81]. This stable time step is approximated as being equal to the time for a stress wave to traverse some characteristic length of the smallest element (e.g. the shortest edge or element diagonal). Typically, explicit formulation step sizes are 100 – 1000 times smaller than those in an implicit solution [69]. In general, sheet formability tests are quasi-static with punch velocities on the order of 0.1 to 10 mm/s. In order to achieve acceptable run times, however, explicit

models are run at artificially increased velocities, typically 2-3 orders of magnitude larger than those observed in the actual process. The limitation on time step imposed by the geometric size of the elements also has implications on any mesh refinements. Improvements in the accuracy of the results through the use of a finer mesh come at a cost of reduced time step size and increased computation time. This trade-off must be considered in any mesh optimization and convergence study.

3.1 Finite Element Mesh

3.1.1 Mesh Description

The numerical model of the dome test was created using the PATRAN MCAE package. Details of the press geometry were provided by CANMET and the geometric representation of the tooling was created from technical drawings of the press. Four-node quadrilateral shell elements were used for all components of the model. The tooling was assumed to be rigid and the tool surfaces were modelled using rigid surface elements. Only one quarter of the geometry was modelled (see Figures 3.1, 3.2a, and 3.2b) with symmetry being enforced through appropriate boundary conditions (refer to Section 3.5). The narrower width of the 50.8 mm blank allowed for a further reduction in the amount of tooling included in the model. As shown in Figure 3.3, only one sixth of the clamping dies was modelled for this geometry.

The deformable blank was modelled using Belytschko-Lin-Tsay [83, 84] four-node quadrilateral shell elements. Two constitutive formulations were considered in the study: an isotropic elasto-plastic material and a through-thickness anisotropic elasto-plastic material. These formulations provided a basis for an investigation of the effects of anisotropy predicted by Hill's quadratic yield function, the yield function incorporated in the through-thickness anisotropic material model.

Table 3.1 gives a comparison of the relative model sizes for the three blank geometries considered. As mentioned above, the size and geometry of the 50.8 mm

machined specimen allowed a reduction in the tooling required, reducing the number of rigid elements by almost third. Despite the difference in physical size between the 203.2 mm and 127.0 mm specimens, a corresponding reduction in the number of elements needed to model the 127.0 mm blank was not possible. Initial runs of the 127.0 mm model indicated that severe deformation occurred in the blank as it passes through the edge of the drawbead (as indicated on Figure 3.4) due to a combination of bending and drawing. The geometry of this area required careful meshing with relatively small elements to capture the curvature during deformation. The smaller time step size associated with the small element size in the refined area of the 127.0 mm model also resulted in larger run times for the 127.0 mm blank over the 203.2 mm blank (approximately 10 hours difference in run time).

The majority of the elements in the model were used to represent the rigid tooling. The reduction in size of the portion of tooling required for the 50.8 mm machined specimen (refer to Table 3.1) resulted in significant savings in run time. The relative simplicity of the shape of the specimen and the less complex mode of deformation observed in the 50.8 mm sample also allowed for a slightly coarser mesh and hence a larger time step.

3.1.2 Model Refinement

The major area of mesh refinement was the draw bead. Although the focus of this investigation was on the limiting dome height test where no drawing of the material is permitted, the accurate simulation of the drawbead action was considered to be critical to modelling of the punch. 'Locking' of the material in the drawbead and between the clamping dies is easy to simulate. A coarse mesh, for example, will effectively prevent the material from drawing through the drawbead as the elements are not small enough to move around the draw bead radii. Alternately, a rigid boundary condition could be imposed on the blank nodes at the position of the draw bead [12]. Neither of these, however, accurately model the real punch operation. The

127.0 mm and 50.8 mm blanks, for example, exhibit significant drawing at the outer edge of the drawbead (see Figure 3.5). Even the 203.2 mm blank can show some asymmetric deformation which will lead to sliding or movement of the material along the drawbead. This asymmetries can result from in-plane anisotropy of the blank, for example.

Geometric locking of the mesh was observed during runs of the coarse mesh. The mesh locking was the result of an inaccurate discretization of the draw bead which prevented the blank mesh from deforming around the draw bead radii correctly. As a result, the mesh in the area of the draw bead in both the blank and dies was refined to better capture the sharp draw bead radii.

The major model refinements were carried out on the 203.2 mm model. The 50.8 mm and 127.0 mm models were then assembled based on the experience gained on the 203.2 mm blank. As discussed above, the major mesh refinement involved the accurate modelling of the drawbead. Other investigations aimed at improving the model involved the contact stiffness (Section 3.6.2), the applied loads (Section 3.8.1), and the frictional boundary condition (Section 3.8.2). The final meshes adopted for the investigation for the 50.8 mm, 127.0 mm, and 203.2 mm blanks are shown in Figures 3.6, 3.7, and 3.8 respectively.

3.2 Element Description

3.2.1 Choice of Element Formulation

The choice of shell elements over brick elements was made largely on the basis of efficiency. Brick or hex elements are computationally very expensive because of the need to carry out a full three-dimensional constitutive evaluation at each integration point. The assumption of a plane stress condition in shell elements halves the state variables and computations required [83]. Shell formulations commonly use through-thickness integration points to account for bending with results that are comparable

to full three-dimensional formulations. For thin shell problems, shell elements can be used with a greater efficiency at little if any loss in accuracy.

The choice of element formulation for this investigation was made on the basis of element efficiency and accuracy. A comparative study of a number of shell element formulations applicable to sheet metal forming operations was carried out recently by Galbraith and Halquist [85]. In particular, the cost-benefit relationship of the seven four-node shell elements currently implemented in LS-DYNA3D, the commercial explicit dynamic three-dimensional finite element code used in this investigation, were studied. The elements fall into three broader categories; membrane shells, full integration shells (using 2x2 Gauss point quadrature), and reduced integration shells (using single point quadrature).

Based upon reasonable computation time and an accuracy comparable to the more expensive full integration formulations, Galbraith and Halquist [85] found that reduced integration shells are a reasonable choice for most sheet forming problems. A summary of the results of their study is given in Table 3.2. Although membrane shells are extremely efficient, the lack of bending terms limits their use to first approximations. Full integration formulations tend to give very accurate results but at the expense of higher computational cost. This computational expense comes largely from the need to evaluate the constitutive relations at each of the four Gauss points. As mentioned above, reduced integration shells are much more efficient as they require a single evaluation of the constitute equations at each through-thickness integration point. The saving does come at a small expense, however, as the use of a single point of quadrature permits the presence of zero energy deformation modes called hourglassing (discussed in more detail in Section 3.3.2 below). In most cases, however, the implementation of simple hourglassing control routines effectively eliminates the presence of these undesirable deformation modes while adding only slightly to the overall run time.

Results for the three reduced integration formulations implemented in DYNA3D

were comparable. Based on computation time, the Belytschko-Tsay shell is the most efficient and Galbraith and Halquist [85] indicate that, as it is the default formulation in DYNA3D, it may be more robust than the other comparable formulations. For most forming problems, it may therefore be concluded that, of the formulations implemented in LS-DYNA3D, the Belytschko-Tsay formulation provides the best compromise between accuracy and efficiency.

Through-thickness variations (e.g. bending) are captured in shell elements using through-thickness integration points. The constitutive relations are evaluated at each through-thickness integration point. Trapezoidal integration is then used to integrate the stresses over the thickness of the element giving the resultant forces and moments. Typically three to seven integration points are used. Improved accuracy can be obtained using more integration points but at the expense of run time. For this study, five through-thickness integration points were used.

Tooling deformation was not taken into account in this study. The tooling was modelled using rigid shell elements. As a result, only the surface geometry of the dies and punch needed to be included in the model of the dome test.

3.3 The Belytschko-Lin-Tsay Four Node Quadrilateral Shell Element

As outlined in the previous section, the Belytschko-Tsay [83, 84] four-node quadrilateral element possess the characteristics required for a study of metal forming, namely simplicity, efficiency, and an accuracy which is comparable with more complex elements. The particular strength of the Belytschko-Lin-Tsay bilinear four-node quadrilateral element, namely efficiency, lies in it's use of a co-rotational coordinate system. The reference surface of the quadrilateral shell element is defined by the mid-surface containing the four corner nodes. A co-rotational local coordinate system greatly simplifies the element formulation by embedding the local element spatial

coordinate system in the element at the reference surface. Rigid body rotations are automatically accounted for thereby satisfying the frame invariance of the constitutive relations [86] without the need to change the constitutive routines [83]. This eliminates the need to evaluate the computationally expensive Jaumann stress rate rotation [85].

A complete description of the element formulation and its implementation in LS-DYNA3D can be found in the original papers by Belytschko, Tsay, and Lin [83, 84] and the LS-DYNA3D Theoretical Manual [81]

3.3.1 Thickness Update Option

An important feature of any finite element routine which is to be applied to a metal forming problem is the implementation of a thickness update. Thickness updates are particularly crucial in predicting the effect of sheet thinning on the strain distribution. LS-DYNA3D uses a procedure developed by Hughes and Carnoy [87]. Essentially, the mean through- thickness strain component is added to the nodal fibre lengths to obtain the updated shell thickness. The importance of this option in conjunction with the thickness change consideration in the contact algorithm was outlined by Mattiasson *et al.* [32]. In particular, the need to accurately capture thickness changes and their effect on contact forces around sharp radii and in the region of flanges in a drawing problem was demonstrated. Thickness variations in the flange area are important as they can result in intermittent contact (refer to the discussion of friction in Section 1.5.1).

3.3.2 Hourglass Control

The use of a reduced integration and selective/reduced integration (e.g. a single integration point in the plane of the element) allows for the presence of modes of deformation which do not cause stress at the central integration point [83, 88]. The first three displacement modes shown in Figure 3.9 are the rigid body modes

and are not detrimental to the solution. The presence of one of the kinematic modes, however, can result in a spatial oscillation of the solution or a singularity which prevents successful solution especially if the pattern of these modes is similar to the mesh design [84, 88]. These modes are commonly called hourglass or keystoneing modes due to the characteristic shape of the kinematic mode shown in Figure 3.9.

The difficulty in controlling hourglassing comes in the implementation of a control routine which will not influence or be influenced by the rigid body modes or deformation. Recent developments in the field have provided a relatively efficient and robust hourglass control as is the case with the Flanagan and Belytschko routine [84, 88] implemented in LS-DYNA3D [81]. The other attractive feature is the hourglass control is orthogonal to the normal deformation modes and, as a result, has a relatively small effect on the solution [83].

For the purposes of this investigation, the default stiffness damping form of hourglass control was used. Some hourglassing was observed in initial runs of the model but these were attributed to the interface stiffness and hourglassing effects were removed by appropriate corrections to the interface stiffness scaling factor (refer to Section 3.6.2). No hourglassing was observed during the remainder of the study. As a result, no effort was made to assess the effects of the other hourglass control methods available in LS-DYNA3D on the accuracy of the solution.

3.4 Constitutive Model

As outlined in Section 2.2, the mechanical properties of the materials were measured using uniaxial tensile tests. These material properties were then used in the material models incorporated in the numerical model. Two constitutive models were used in the analyses. The first, an isotropic elasto-plastic material, uses an implementation of the von Mises yield function (see Section 1.6.2) for plane stress. The purpose of this isotropic model was to provide a basis of comparison to assess the effect of the $\bar{\epsilon}$ value on the strain distribution.

The second constitutive model was an implementation of Hill's quadratic yield surface for plane stress (see Section 1.6.4). This anisotropic elastic-plastic material model is similar to the isotropic model discussed previously with the exception of the yield function. The model allowed the incorporation of anisotropy effects characterized by the $\bar{r}$ parameter derived from the series of material tensile tests outlined in Section 2.2. Table 3.3 shows the material parameters incorporated in the material models.

In both models, the plastic hardening of the material is treated as linear. The hardening is characterised by the tangent modulus, E_T , where E_T is equal to the slope of the linearized hardening curve of the material as shown in Figure 3.10. The value of E_T and the corresponding σ'_Y were approximated by fitting linear elastic and linear hardening curves to the material stress-strain curves of the AK1, AK2, and AL materials, as shown in Figures 2.3, 2.4, and 2.5 respectively. The representation of the material response by a yield point, σ'_Y , and a constant hardening modulus, E_T , is reasonable for large strains although the actual yield will be below σ'_Y .

3.5 Boundary Conditions

3.5.1 Displacement Boundary Conditions

The movement of the rigid tooling was controlled by displacement boundary conditions. The lower die was clamped fully by enforcing zero displacement and rotations at all nodes. The upper die and punch were constrained in all directions except the y direction and were allowed to displace in this direction. The symmetry of the blank along the x and z symmetry planes was enforced with appropriate displacement and rotational boundary conditions.

3.5.2 Applied Loads

Control of the various applied loads is carried out in LS-DYNA3D through the use of load-time histories commonly referred to as load curves. Each load curve consists of a series of time-value coordinates which are used to define a piece-wise linear variation of the corresponding load (e.g. pressure or prescribed velocity). Since the magnitudes of the punch and clamp velocities have a direct effect on the duration of the run, optimization of the respective load curves is of great importance to the overall efficiency of the model. During this investigation, the punch velocity load curve had the most significant effect on the run time because of the long punch stroke (i.e. on the order of 35 mm).

Artificially high tooling velocities were required to keep run times within reasonable bounds because of the small time step which is characteristic of an explicit solution. Strain rate hardening effects are not incorporated in the constitutive relations used in this study. As a result, the only rate effects that had to be avoided were hourglassing and dynamic or inertial responses. It is possible to control some of the effects through effective hourglass control and alteration of the contact stiffness but with high tooling velocities dynamic stress waves can occur in the flexible blank. An assessment of the effect of high tooling velocities was made during initial runs of model. Both the clamping action and the punch movement were modelled using applied constant velocities. Optimization of the clamping velocity and punch velocity was achieved through a series of runs. It was found that a clamp velocity of 1 m/s and a punch velocity of 4 m/s did not initiate a dynamic response in the blank and still gave a reasonable run time.

To provide a more realistic simulation of the clamping process and clamping pressure, the clamping motion of the upper die was modelled using an applied pressure. Initial runs with various pressure load curves resulted in extremely high die velocities which in turn induced a large dynamic response in the blank. The large velocities were the result of the low inertia of the upper clamping die, a result of using thin shells.

To overcome this problem, a simpler approach was adopted. A two stage clamping procedure using an applied velocity followed by a clamping pressure was implemented. The upper die was closed using an applied velocity of 1 m/s. The velocity boundary condition was then removed and replaced by a ramped pressure (see Figure 3.11a). A gradually increasing pressure completes the motion of closing the die. Once the die is fully closed the pressure was rapidly ramped to a final clamping pressure of 6.9 MPa (1000 psi). This pressure is lower than that used experimentally but it was sufficient to fully clamp the blank, restricting the draw-in to the acceptable levels outline in Section 1.5.2.

The application of a constant punch velocity allowed specific states, corresponding to the experimental interrupted dome heights, to be analyzed. The example shown in Figure 3.11a is the series of load curves used for the 203.2 mm specimens. Because of the lower rigidity of the narrower 50.8 mm and 127.0 mm blanks, it was found that it was necessary to close the upper die more slowly and to apply the clamping pressure more gradually in order to avoid a dynamic response (refer to Table 3.4). The final termination time for the run was determined by the limiting punch depth found experimentally. Different punch depths were used for the steels and aluminum reflecting the lower experimentally measured limiting dome heights of the aluminum specimens.

3.6 Contact Modelling

3.6.1 Contact Surfaces

Each model consists of several distinct meshes which come into contact during the course of the run. As no element connectivity exists between the various parts, special logic or contact algorithms are used to prevent parts from passing through one another.

The contact algorithm implemented in LS-DYNA3D is of the penalty stiffness

form. Interfaces are defined by listing all the elements forming the two contact surfaces, one side being designated the master surface and the other, the slave surface. The terms master and slave originated in the kinematic constraint and distributed parameter methods [81] where the slave surface nodes are constrained to move along the master surface. Under the penalty stiffness method, however, the surface designations are interchangeable and have no effect on the final solution.

The contact search routine loops through each node of the slave surface and checks for penetration of the node through an element face on the master surface. Under the penalty stiffness method, penalty functions are introduced which act as interface springs between penetrating nodes and the contact surface. The stiffness of the interface spring is scaled to be a function of the bulk modulus of the master segment (i.e. the element through which penetration has been detected). The magnitude of the restoring force is then determined by the stiffness of this interface spring, a user defined interface stiffness scale factor, and the depth of penetration.

The spring remains in effect as long as penetration is detected. When the node is no longer determined to be penetrating, the spring is removed and no contact forces are added. This search routine is then repeated to check for penetration of the master surface nodes on the slave surface segments. One advantage of the penalty stiffness method is that energy is conserved.

The penalty stiffness method is more resistant to the initiation of hourglassing modes than other contact algorithms. In cases where the stiffness of the interface seems to excite hourglassing modes, it is possible to 'soften' the contact by scaling down the penalty stiffness. Shell elements, in particular, are prone to the initiation of hourglassing by the contact stiffness because of the low through-thickness stiffness.

Recent improvements to the surface contact algorithm have been driven by the analysis of sheet metal forming operations [81, 82]. In particular, contact search routines have been modified to be less sensitive to problems caused by large deformations and poor element aspect ratio which can interfere with the logic routines in

the penetration search. Even so, it is important to minimise the chance of severely deformed elements occurring during the deformation. One way to do this is to mesh the blank in such a way that the elements follow the expected flow lines of the deforming material. It is also important to match the element sizes in the areas of contact between the rigid tooling and deformable blank. This practice was followed during the meshing of the model especially in the area of the punch-to-blank contact where incorrect penetration searches and errors in the interface stiffness and hence interface force would cause problems with the prediction of the frictional interface forces.

Three contact surfaces were used in the model; one for the lower die to lower blank surface contact and one each for the punch and upper die to upper blank surface contact, respectively. The division of the upper contact surface into two distinct master-slave pairs allowed the punch load-displacement data to be extracted from the runs.

3.6.2 Interface Stiffness

The first runs of the model showed large oscillations initiating in the blank soon after the punch started moving. These oscillations, characteristic of hourglassing, destroyed the solution within a few time steps. Subsequent investigation traced the initiation of hourglassing to the contact stiffness. As was mentioned above in the discussion of contact surfaces, the penalty stiffness method incorporates a stiffness scaling factor which can be used to stiffen or soften the contact. By default the stiffness scale factor is set to 0.1, scaling the interface stiffness to one tenth of the stiffness of the element normal to the contact interface. If the penalty stiffness is too high, the movement of one contact surface against another can cause the interface force to be large enough to introduce perturbations which excite hourglassing. When the interface stiffness was scaled back by a factor of 10 (i.e. to 0.01), no hourglassing was observed.

3.7 Frictional Boundary Conditions

The interface frictional boundary condition implemented in LS-DYNA3D is based on the Coulomb formulation,

$$F = \mu |f_n| \quad (3.1)$$

where F is the frictional force, μ is the Coulomb coefficient of friction, and f_n is the surface contact force determined by the contact algorithm.

If the interface frictional shear force is larger than the shear carrying capability of the material, the frictional force is set to the limiting tangential shear stress,

$$f = \min(f_{Coulomb}, \kappa A_{master}) \quad (3.2)$$

where κ is the viscous coefficient and A_{master} is the area of the master segment.

As was discussed in some detail in Section 1.5.1, the Coulomb formulation for the frictional forces at the tool-blank interface is a highly simplified description of the complex interaction of surface characteristics at the interface. It does, however, provide a starting point for an investigation of the frictional force and its influence on the forming operation, strain distribution, and forming forces.

Following the review of the literature (Section 1.5.1), experimental friction coefficients ranging from 0.05 to 0.2 were quoted for dry steel-steel contact. An even larger range has been used in other numerical solutions, with coefficients from 0.2 through 0.8 being used to fit strain results to experimental measurements. This approach was rejected in the current study and more realistic frictional coefficients were used. μ values ranging from 0.08 to 0.26 gave varying results depending on the material and blank size. The results of a brief parametric study of friction coefficient are presented in Section 4.3.2.

3.8 Mesh Optimization Studies

3.8.1 Load Curve Optimization

A significant effort was made to increase the velocity of the clamping die and punch and the rate of application of the clamping pressure in order to reduce the overall run times. The limiting factor was found to be the excitation of a dynamic response in the blank. A rather crude attempt was made at ramping the clamping pressure in order to avoid sudden ‘impact’ loading of the blank but step load functions were used for both the clamping and punch velocities mainly for simplicity. Run times of between 120 and 196 hours, were initially felt to be tolerable. Over the course of the study, however, the length of the run was seen to restrict the scope of any parametric study to be undertaken. In addition, the long runs time precluded any form of mesh convergence study.

To alleviate this problem, sinusoidal load curves were adopted to describe the punch and clamp velocity time histories following the approach used by Honecker and Mattiasson [69] and Galbraith and Halquist [85]. The clamp and punch velocities were initially set to zero and gradually accelerated to very high values and then decelerated back to zero at the end of the clamp and punch strokes respectively. In this manner, the sharp transitions associated with the step velocity profile were eliminated while minimizing the total solution time.

Trial calculations of the implementation of the sinusoidal load curves were carried out using the 127.0 mm and 203.2 mm sample geometries for the AK1 steel. In all cases, the anisotropic material model was used with a friction coefficient of 0.18. Trials were made using three levels of peak clamp and punch velocities and associated load curve durations. The clamping pressure was applied as a sinusoidally increasing pressure with a ramp duration of 0.5 ms followed by a constant clamping pressure of 6.9 MPa (refer to Table 3.5 for a summary of the trials carried out). Figure 3.11b shows an example of the load curve history for a punch depth of 36 mm using the

sinusoidal method and provides a comparison with the step function load curves (Figure 3.11a) for the same punch depth.

Variations in clamping velocity were not observed to have a significant effect on the strain distribution in either the 127.0 mm or 203.2 mm models although a small dynamic effect was observed in the run with the clamping velocity period of 0.25 ms. As a result, a clamping period of 1.0 ms was chosen for the remaining runs.

Punch speed was observed to have a more significant effect. A large inertial effect was observed in the runs with the punch velocity period of 0.25 ms. Results for 1 ms and 4 ms periods were very similar. The small differences in predicted strain between the 1 ms and 4 ms period runs was not deemed to be large enough to warrant the additional run time associated with the 4 ms period and the 1 ms period was adopted. Note that the peak punch velocity varies with the final punch depth. For the maximum punch depths observed experimentally (i.e. 36 to 38 mm), the peak punch velocity is on the order of 60 m/s.

Some spring back was observed during the clamping phase of the runs. This was thought to be due to the release of the constraint on the movement of the upper die after the velocity boundary condition was removed. The spring back is then removed by the application of the clamping pressure which completes the closure of the upper die. In an effort to overcome the spring back, a trial was carried out in which the clamping pressure starts at a non-zero value (0.1 MPa) resulting in a stepped pressure application followed by a sinusoidal ramp up to the maximum clamping pressure of 6.9 MPa. The step application of the pressure solved the spring back problem and did not induce any variation in the predicted strain distribution over the non- stepped run.

Figure 3.12 shows a comparison of the major and minor strain distributions predicted using step function load curves and, for the same geometry, material, frictional coefficient, and punch depth, using sinusoidal load curves. The step function run required approximately 120 hours to run while the sinusoidal load curve run

required approximately 23 hours. The conclusion of the series of sinusoidal load curve calculations was that runs performed in 2.5 ms (1.0 ms for the clamp closure, 0.5 ms for the blank holder pressure application, and 1.0 ms for the punch stroke) gave results comparable with the original 10 – 15 ms runs using step function load application with a saving of more than 4 days of run time. Based on these results, the sinusoidal load curve method was adopted.

3.8.2 Initial Mesh Investigation of Frictional Forces

Initial calculations using a coarse mesh were made with a value of $\mu = 0.25$. The resulting strain distributions showed a diffuse neck at a significantly lower strain, for a given depth, than was measured experimentally. This diffuse neck is characteristic of a low friction coefficient (i.e. a well lubricated blank). The under-prediction of the frictional contact forces was consistent with published results which were based on a high frictional coefficient (ie. on the order of 0.5 – 0.6) assumed in the model [12]. These high μ values were required to achieve close agreement of measured and predicted strain distributions (refer to Section 1.8.2).

Friction coefficients for a given material, lubricant, and forming condition are derived from simulative tests (refer to Section 1.5.1). It is difficult or impractical to implement a realistic analytical prediction of frictional force in an FEA model (see Section 1.5.1). The error introduced by the simple representation of the frictional forces is compounded by an imprecise description of the contact surfaces resulting from the discretization of the contact surface geometry. Element size effects can be quite significant in terms of geometric contact especially when one is considering curvilinear surfaces. This observation was reinforced by results from the refined mesh, shown previously in Figure 3.8, in which the density of elements in the punch and the corresponding area in the blank was increased. A significant improvement in the accuracy of the predicted strain distribution was observed using realistic friction coefficients ($\mu = 0.13 - 0.18$). The prediction of the magnitude and location of the

peak strain was significantly better with the refined mesh and lower μ coefficient.

3.8.3 Convergence Check

The substantial decrease in run time achieved by adopting the sinusoidal load curve method allowed a more detailed mesh to be run to check for convergence.

The configuration chosen to examine mesh convergence was the 203.2 mm blank of the AK1 material. Neither the load curves nor the frictional coefficient ($\mu = 0.18$) were changed. The blank and tooling meshes were refined to include 21951 elements, 9492 rigid shells modelling the tooling and 12459 representing the blank (see Figure 3.13). The run time for this model was approximately 60 hours.

Figure 3.14 shows a comparison of predicted major and minor Green's strain for the FEM mesh used in the numerical study (3072 elements) and the refined mesh (9492 elements). The slightly lower deformation predicted over the punch (ie. between the pole and the localization) may be the result of a more accurate prediction of the frictional contact due to the better discretization of the contact surfaces. The results for the refined mesh do not, however, indicate a significant improvement over the strain distributions predicted by the 3072 element mesh and it is unlikely that a less drastic refinement of the mesh used for the study would yield any measurable benefit at the cost of a significant increase in computation time.

Chapter 4

Discussion of Numerical and Experimental Results

This chapter presents results from experimental measurements and FEM predictions of strain distribution. The experimentally observed trends are presented first in terms of major and minor principal Green's strain distributions. The FEM predictions are then outlined and compared to the measured distributions. Finally, parametric studies of the effects of r and μ on strain distribution are presented.

4.1 Experimental Results

4.1.1 Comparison of Material Formabilities

Material formability is commonly measured using the Forming Limit Diagram (FLD). As outlined in Section 2.7, FLDs for the three sheet steels were produced early in the investigation to assess the relative formabilities of the materials used in this study. Figure 4.1 shows the three Forming Limit Curves (FLCs) from Figures 2.16, 2.17, and 2.18 superimposed on one graph. Of particular interest is the similarity between the FLCs of the IF and AK1 materials. Both have LDH_0 limit strains of approximately 50%. In addition, the measured limit strains in both biaxial stretching ($\epsilon_2 > 0$) and drawing ($\epsilon_2 < 0$) are virtually identical. Based solely on the FLCs, one could conclude

that the performance of both materials would be similar.

The AK2 material, on the other hand, exhibits slightly different formability. The LDH_0 limit strain is slightly higher for this material at 55%. The FLC also indicates better formability in drawing for this material with a lower formability in stretching as compared to the IF and AK1 materials.

The difference between the AK2 FLC and the IF and AK1 FLC's can be attributed to two competing mechanisms. The upward shift of the curve is a result of the higher initial sheet thickness of the AK2 material ($t = 0.908$ mm) compared to the IF ($t = 0.664$ mm) and AK1 ($t = 0.656$ mm) sheets. Another factor which affects the FLD is the strain hardening coefficient, n . The lower n of the AK2 sheet ($n = 0.2253$) would tend to lower the FLC compared to the IF and AK1 materials, both of which have higher hardenability ($n = 0.2871$ and $n = 0.2487$, respectively). This would tend to offset the increased formability due to higher initial material thickness. Note that a similar trend would be expected between the IF and AK1 materials as the IF material has a significantly higher hardenability than the AK1 material while the initial sheet thicknesses are similar. The lower yield stress of the IF steel ($\sigma_Y = 142.1$ MPa) compared to the AK1 material ($\sigma_Y = 165.7$ MPa) however, would tend to lower the FLC offsetting the effect of the higher n .

As noted earlier, the amount of AL544.0 material available was not sufficient to produce the number of blanks required to generate an FLD. For comparison, an FLC for 1050-0 aluminum from Kohara *et al.* [89] has been included in Figure 4.1. Note the generally lower formability of aluminum at all levels of minor strain.

4.1.2 Experimentally Measured Strain Distributions

The IF material was dropped from the investigation of strain distribution because of the similarity in formability and in $\bar{r}$ compared to the AK1 material. The AK1 material was chosen over the IF steel because of the larger planar anisotropy observed in tensile tests of the AK1 material.

As detailed previously in Section 2.8, a series of interrupted dome tests were performed on 50.8 mm, 127.0 mm, and 203.2 mm wide blanks to measure the developing strain distributions in the AK1, AK2, and AL blanks as they deform. Measurements of major and minor Green's strain along a series of grids lying approximately on the symmetry plane of the blank were carried out using the circle grid technique. Plotting the measured strains against lateral position relative to the pole of the dome (Figure 2.20) gives a strain distribution along the symmetry plane of the dome.

4.1.3 Sheet AK1

Measured major and minor principal strain distributions for the AK1 material are presented in Figures 4.2, 4.3, and 4.4 for the 203.2 mm, 127.0 mm, and 50.8 mm wide blanks, respectively. Each curve corresponds to measured strain distribution tests interrupted at the dome height indicated.

Figure 4.2 clearly shows the development of a localization in the 203.2 mm blank of the AK1 material as reflected by the sharp strain peaks. The development of a diffuse localization is seen early in the deformation at a punch depth of 20.0 mm. This localization seems to be driven by the frictional contact forces as the peak strain occurs near the tangency point (TP) between the blank and punch. Outside the contact region, the strain decreases towards the drawbead where there is little deformation. As the deformation proceeds, the localization becomes more defined. The frictional contact force constrains the material over the punch and little deformation occurs in the region near the pole. The position of the localization is driven by the frictional contact and occurs slightly behind the TP where the frictional contact forces prevent the material from deforming.

Measurements of minor strain show considerably more scatter than the measurements of major strain. This scatter is a result of higher relative error in the measurement of small strains. As a result, it is difficult to make detailed interpretations of trends although qualitative observations of the trends and relative magnitudes of the

minor strain distributions are possible. The magnitude of the minor strain does not seem to vary over the contact area. As the punch depth increases, the magnitude of the minor strain does not seem to change although the extent of material straining increases. Outside the contact region, the minor strain decreases as the distance from the pole increases. Near the drawbead, the material experiences a plane strain state which results from the lateral constraint provided by the drawbead.

As shown in Figure 4.3, the major strain distribution for the 127.0 mm blank of AK1 is very similar to the strain distribution of the 203.2 mm blank shown in Figure 4.2. The peak strain at failure is slightly lower at approximately 50% which is consistent with the lower formability expected at or near plane strain. The main difference between the 127.0 mm and 203.2 mm blanks is in the measured minor strain. The 127.0 mm blank shows little if any indications of lateral straining over the entire blank during the deformation. The frictional contact forces prevent any stretching or drawing in the material throughout the deformation resulting in approximately plane strain deformation.

The 50.8 mm blank shows significant differences in strain distribution compared to results for the 127.0 mm and 203.2 mm blanks. These differences are largely the result of the different frictional contact forces experienced by the narrower blank. The initial straining in the blank is fairly uniform in the material in contact with the punch. Because of the tapered geometry of the blank, the contact area of the punch and blank is small. As a result, the frictional forces will also be small, allowing material to draw-in over the punch. Only in the area of the pole of the punch is the frictional force sufficient to limit deformation. As shown in Figure 4.4, there is significant deformation at the pole of the punch between punch depths of 25.0 mm and 35. mm. This is in contrast to the 127.0 mm and 203.2 mm geometries where deformation over the punch was constrained very early in the punch stroke. At a depth of approximately 35 mm, the frictional forces over the punch overwhelm the draw-in of the material preventing any further deformation. A sharp localization then

forms near the punch.

The minor strain distribution indicates a small amount of drawing over the punch with minor strains of approximately -10% . As with the 127.0 mm and 203.2 mm geometries, the extent of the lateral deformation is controlled by the frictional contact force. Outside the contact region, the amount of deformation decreases with increasing distance from the pole.

4.1.4 Sheet AK2

Measured strain distributions for the 203.2 mm blank of the AK2 material are shown in Figure 4.5. The results show trends similar to those discussed for the 203.2 mm wide blank of AK1. The developing localization occurs at a similar location and has a similar peak strain as would be expected from the similarity in material properties and formability predicted by the respective FLDs.

The results for the 127.0 mm wide blank (Figure 4.6) are also similar to those obtained for the 127.0 mm AK1 blank. The peak major strain is slightly higher reflecting the thicker material but the location of the localization and the magnitude and general trend of the minor strain distribution are the same, within experimental error, as was measured in the AK1 material.

Figure 4.7 shows the strain distributions for the 50.8 mm wide blank. As with the same blank geometry of AK1 steel, the major strain distribution over the punch is fairly uniform although, in contrast with the AK1 material, the minor strain distribution seems to indicate more drawing. This may be a result of the higher r of the AK1 material ($\bar{r}_{AK1} = 2.21$) compared to that of the AK2 sheet ($r_{AK2} = 1.93$). Midway through the deformation, at a punch depth of 30 mm, a diffuse localization develops towards the TP. Although subsequent distributions (i.e. those at 35.1 mm and 37.1 mm) seem to show a uniform distribution of strain, the final distribution at fracture again shows a localization away from the pole of the punch. The position of the fracture is still some distance in from the TP but it is farther from the pole than

was observed in the AK1 material.

4.1.5 Sheet AL

Trends in the measured strain distributions for the AL material are harder to identify because of the scatter caused by the lower grid quality and higher relative error on a lower strain measurement.

Figure 4.8 shows the results for the 203.2 mm blank size. Again, the trends measured are similar to those observed in the AK1 material. As can be expected from the lower formability of the aluminum, the peak major strain at failure is significantly lower than that of either of the steels but the general trends are similar. The frictional constraint seems to be stronger in the aluminum as the localization occurs outside of the punch to blank contact. The minor strain is lower than was measured for the steels with a peak minor strain of approximately 10% although the scatter makes the identification of such trends difficult.

Similar observations can be made concerning the results for the 127.0 mm blank size which are presented in Figure 4.9. The localization develops outside the TP with the strain distribution at fracture being characterised by an extremely sharp localization at plane strain. Scatter in the minor strain results is very high although the deformation in the region of the punch contact seems to have taken place under plane strain. Outside the dome, the deformation is characterised by lateral drawing with a minor strain of approximately -5%. The plane strain state under the dome and the lateral draw-in outside the dome seem to develop early in the deformation and do not change over the course of the dome formation indicating that most of the deformation occurs outside the contact area.

The 50.8 mm blank exhibits the same trends as the 50.8 mm AK1 blank only at a lower level of strain (Figure 4.10). Deformation under the punch takes place until a relatively low punch depth (less than 17.9 mm) is reached. At a punch depth of approximately 19.8 mm, the frictional contact forces constrain any further

deformation of the material under the punch, driving a localization to occur at the TP. This localization develops rapidly after a punch depth of 22.1 mm to a final fracture at a major strain of approximately 32%. The deformation takes place under plane strain as little if any lateral deformation is shown to be present in the minor strain distribution.

4.2 Numerical Results

For the purposes of comparison with predicted strain distributions, only the strains on one side of the punch were examined. Experimental results showed that the deformation up to fracture was relatively symmetric across the pole. As a result, no information is lost by considering only one side of the experimental results. For consistency of approach, the side of the distribution containing the localization was chosen for all comparisons.

Prior to comparing the predicted and measure distributions, it is useful to first examine the numerical results since they show better the overall distribution of strain within the sheet.

4.2.1 Predicted Strain Distribution

The predicted strain distribution development for the 203.2 mm wide blank of AK1 is shown in Figures 4.11 through 4.16. The predictions are from an analysis run using Hill's quadratic yield surface with $\bar{r} = 2.210$ and a Coulomb frictional coefficient of $\mu = 0.18$. Figures 4.11, 4.12, and 4.13 shows the prediction distribution of major Green's strain for punch depths of 0 mm, 19 mm, and 35.9 mm, respectively. The areas of peak strain at a punch depth of 0 mm are limited to the drawhead area. At a punch depth of 19 mm, the distribution of Green's strain under the punch begins to develop. The peak strain occurs at some distance from the pole, a result of the frictional constraint at the punch to blank interface. As the deformation continues,

the frictional constraint continues to drive the localization away from the pole while restricting further deformation under the punch. At a punch depth of 35.9 mm (Figure 4.13), the development of a localization near the punch to blank tangent point is clearly shown. These trends are more clearly seen in a plot of major Green's strain versus distance from the pole as shown in Figure 4.17.

The predicted minor principal Green's strain distribution for the same punch depths discussed above are shown in Figures 4.14, 4.15, and 4.16. Here we see the development of the minor strain distribution early in the deformation at a punch depth of 19.0 mm. The majority of the lateral stretching takes place under the punch as is evidenced by the tensile strain in this region. Farther from the pole, the minor strain decreases as the lateral constraint of the drawbeads resists the lateral stretching providing a strain state close to plane strain near the drawbead (approximately 50 mm from the pole). The same trend is observed at a punch depth of 35.9 mm (Figure 4.16). The minor strain under the punch has increased although it is still fairly uniform. Again, these trends are clearly demonstrated in a plot of minor Green's strain versus distance from the pole (Figure 4.17).

Figures 4.18, 4.19, and 4.20 show the predicted sheet thickness for punch depths of 0 mm, 19.0 mm, and 35.9 mm. The predicted thickness along the symmetry plane of the dome is also plotted in Figure 4.21. The distributions are included to demonstrate the capability of the model to capture the variations in sheet thickness, including the localization. In the region of tensile strain, the model correctly predicts thinning of the material. Although no experimental measurements of sheet thickness were made during this investigation, sheet thickness can be used as another measure of the deformation of the material. The use of sheet thickness as a gauge of material deformation is advantageous for large scale parts where ultrasonic measurement techniques provide an inexpensive and quick measurement technique in contrast to the relatively cumbersome grid analysis technique.

Also to be noted in the results presented in Figures 4.11 through 4.16 is the

relatively small variation in the predicted strain distribution along the circumferential direction within the 203.2 mm wide blank. This is consistent with the transverse anisotropic constitutive model (i.e. planar isotropic) adopted in the investigation. Figure 4.22 shows the major and minor principal Green's strain distributions for a strip of elements along the line of symmetry of the model and a strip of elements running along the diagonal of the blank. The general trends of both distributions are virtually identical with the diagonally measured distribution being slightly smoother, with a more pronounced localization.

Fringe plots of predicted major and minor Green's strain and thickness variation for the 127.0 mm and 50.8 mm blank geometries (again $r = 2.210$ and $\mu = 0.18$) are shown in Figures 4.23 through 4.25 and Figures 4.26 through 4.28, respectively. Results for the 127.0 mm blank show a sharp localization characterised by a high strain gradient in the region of the peak strain. The predicted location of the localization is close to the punch to blank tangent point, again, characteristic of the frictional constraint of the punch to blank interface. Note also the prediction of a shear band which develops at the corner of the drawbead. This shear band contributes to the severe deformation at the edge of the blank (refer to Figure 3.4) which caused problems with the initial mesh design for this geometry. Also of interest is the predicted thickening of the blank material at the outer edge of the specimen as shown in Figure 4.25. This thickening is consistent with the first stages of wrinkling or buckling which was observed in the experiments for this geometry (visible in Figure 2.15).

The localization in the 50.8 mm blank occurs closer to the pole than the blank to punch tangent point (Figure 4.26). This prediction is consistent with experiments in which where the failure occurred near the pole, a result of the lower friction constraint due to the smaller punch to blank contact area. The lower frictional constraint also allows the material to draw-in over the entire dome giving the negative minor strains shown in Figure 4.27.

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

4.2.2 Comparison Between Predictions of Numerical and Experimental Strain Distributions

Comparison of the predicted and measured strain distributions was made based on the numerical and measured strain distributions described above. As noted previously, the results of a circle grid analysis are in-plane Green's strains. The output of LS-DYNA3D is in the form of a global upper and lower surface infinitesimal strain tensors for each element. This global strain tensor is of little use for comparisons with the experimentally measured data. To obtain consistency with experimental results requires principal Green's strain measured in local element coordinates (i.e. in-plane). As a result, a post-processing routine was written to transform the strain components into in-plane principal values referenced to a known direction.

AK1 Material

Figure 4.29 compares the predicted and measured major and minor strain distributions for the 203.2 mm wide AK1 blank. Results for analyses using the isotropic and anisotropic ($\bar{r} = 2.21$) constitutive models are shown separately in each figure. The predicted strain distribution for the 203.0 mm AK1 blank is greatly improved through the use of Hill's anisotropic yield function. The isotropic material model predicts diffuse major and minor strain distributions with significant deformation over the punch. This prediction is contrary to the measured distributions which show low deformation in the region of punch to blank contact. The pole strain develops very early in the deformation and does not change over the course of the punch stroke. The same is true of the minor strain which does not change significantly as the punch depth increases. More importantly, the sharp localization which develops at the edge of the blank to punch contact is captured accurately using the anisotropic yield model. Figure 4.29 shows good agreement between the predicted and the observed localization behaviour over the entire history of the deformation. The predicted strain distributions for all punch depths shown in Figure 4.29 are very close to the

experimental results. In addition, the small changes in minor strain are predicted as is the overall magnitude which is well within the experimental measurement error.

Similar agreement is shown in Figure 4.30 for the 127.0 mm blank. The isotropic material model tends to over predict the deformation slightly in the contact region. The location and magnitude of the peak strain, as well as the general form of the localization, is very similar to the measured trends.

Results for the 50.8 mm specimen (Figure 4.31) seem to indicate one of the weaknesses of Hill's quadratic yield function. The predicted trends in major strain are similar for the isotropic and anisotropic models. The deformation is predicted to occur uniformly over the punch with a slight increase in peak strain towards the TP. Numerically, this diffuse deformation is predicted to continue right up to the punch depth at which a localization was measured experimentally. One possible explanation for the significant difference between observed trends and predicted strain distributions could be the under-prediction of the frictional contact forces. Friction plays a significant role in the localization formation for this blank size. A lower frictional force would tend to allow the material to draw-in, resulting in a more diffuse deformation, a more negative minor strain, and hence, a higher amount of deformation before fracture. These trends are clearly shown in the major and minor strain distributions of the anisotropic model.

The diffuse deformation may also be a result of the effect of r on the yield surface at negative minor strains. There is evidence (refer to Section 1.7.1) that r has little effect on the formability at negative minor strains. Hill's yield surface, however, predicts an increase in formability for higher r at negative minor strain. As a result, one would expect Hill's yield surface to predict a more uniform distribution for negative minor strain. Again, this trend can be seen in the results with an increase in r (e.g. from the isotropic to the anisotropic model) resulting in a more uniform strain distribution.

Using the observed trends and results for the AK1 material, upon which most of the runs were based, numerical models of the other two materials were created.

Measured frictional coefficients for AK2 steel were in the range of 0.15 to 0.147 for a relatively poor lubricant [90]. For a non-lubricated specimen, 0.18 does not seem an unreasonable estimate. $\mu = 0.18$ has also been indicated to be a reasonable choice for the aluminum [91]

AK2 Material

The predicted trends observed in the model of the AK1 203.2 mm blank were also observed in the AK2 material. The isotropic material model predicted a more uniform strain distribution than was observed experimentally (Figure 4.32). Results for the anisotropic material model were much better (Figure 4.32). The correlation with the experimentally measured distribution was not as good as was observed for the AK1 material but the results are still fairly good with the location and magnitude of the predicted localization being very close to that measured experimentally. Minor strain results were within experimental accuracy.

Results for the 127.0 mm wide blank are presented in Figure 4.33. The predicted location of the localization is very close to that observed experimentally. The over-prediction of the strain in the region of the localization may be a result of the lower hardenability of the AK2 material.

Results for the 50.8 mm blank show much better agreement with the measured distributions. The isotropic material prediction of major strain is reasonably close to that measured (refer to Figure 4.34) however, the numerical model predicts significantly more lateral draw-in (e.g. larger negative minor strain) than was measured. The anisotropic model predicts a relatively diffuse localization with a high strain gradient on either side. This 'plateau' develops early in the deformation and grows as the blank is deformed. The model does not, however, predict the development of a sharper localization, at least not in the range of punch depths used experimentally.

Sheet AL - Aluminum

Figures 4.35, 4.36, and 4.37 present comparisons of the measured and predicted strain distributions for the 203.2 mm, 127.0 mm, and 50.8 mm AL blanks. In all cases, the predictions of both the isotropic and anisotropic material models were similar. The implementation of an anisotropic constitutive model did not change the major strain distribution significantly for any of the strain states investigated for this material. In general, the predicted deformation is fairly diffuse with no significant localization occurring in the 203.2 mm model. This trend is in contrast to the measured distributions which clearly show the development of a localization.

Results for the 127.0 mm blank capture the developing strain distribution out of the contact region very well but, under the punch, significantly more deformation was predicted than is measured. Minor strain predictions for this geometry are also quite different from those measured. Experimentally, the majority of the deformation takes place under plane strain conditions. The predicted minor distribution, however, displays a significant amount of stretching over the punch.

Results for the 50.8 mm blank gave reasonable predictions of strain. The frictional constraint of the material at the pole of the punch, characterised by a low deformation, is accurately captured by the model. This result is in contrast with the more diffuse distribution of strain predicted by the numerical model of the 127.0 mm wide blank.

The results for the AL material are consistent with previous studies [38, 50, 53, 54] on the use of Hill's quadratic yield function to predict flow in low r value materials. It is well established that the quadratic yield function does not accurately capture the stress-strain characteristic of materials with r values of less than 1.0. The poor agreement between predicted and measured strain distributions for the 203.2 mm and 127.0 mm wide blanks clearly show this problem. The improved agreement for the 50.8 mm wide blank is due to the lower minor strain for this narrow specimen. Near plane strain, r is observed to have little or no effect on the formability of a material as discussed in Section 1.6.4. As a result, the predicted deformation should

be reasonably independent of the r value at plane strain.

4.2.3 Effects of Small Variations in r

The characterization of the anisotropy in a material exhibiting planar anisotropy using a single $\bar{r}$ value is of questionable accuracy. As was discussed in Section 1.6.3, comparisons between the predicted and measured uniaxial and biaxial hardening characteristics of various materials have indicated that the use of an averaged r value is not always appropriate. This observation seems reasonable when the specimen is cut in such a way that the properties in the rolling direction, for example, would tend to dominate the deformation. Such is the case for the 50.8 mm blank and, to some degree, the 127.0 mm blank as well. It is also true that small variations in r do not significantly affect the limit strains. One would also expect only small variations in the predicted strain distributions with small variations in r . To investigate this hypothesis, analyses were carried out using the r_0 values (i.e. r measured in the rolling direction) in addition to the calculations using $\bar{r}$ and $r = 1.0$ for the two narrower geometries.

Results from these runs indicated that the small variations in r did not significantly change the predicted strain distribution. Figure 4.38 shows a comparison of the predicted strain distributions using $\bar{r} = 2.210$ and $\bar{r} = r_0 = 2.48$ for the 50.8 mm and 127.0 mm blank of the AK1 material. The location and magnitude of the peak strain at the depths shown are not influenced by the slight change in r . As was indicated by Knibloe and Wagoner [12], there is evidence that a numerical model incorporating Hill's anisotropic yield surface is less sensitive to small variations in r than to variations in other parameters such as μ and n . In addition, at or near plane strain, the influence of r has been shown to be insignificant and there is evidence that r has little effect on the limit strain at negative minor strains [52, 62]. As the results described above are for the 50.8 mm and 127.0 mm blanks, both of which show minor strains on the order of 0% to -5% strain, such small variations in r would not be

expected to influence the predicted strain distributions. Similar results were found for the 50.8 mm and 127.0 mm geometries of the AK2 and AL materials.

If, on the other hand, a larger variation between $\bar{r}$ and the r value characteristic of the specimen geometry and orientation with respect to the RD were to be observed, such a difference in r may have an effect on the predicted strain distributions. This may also be true for higher minor strains. A large Δr was present in the AK1 material although experimental dome tests were not performed on the narrower specimens in orientations other than along the RD.

4.3 Parametric Studies

To examine the effect of the friction coefficient, μ , and the anisotropy, r , on the predicted strain distributions, a parametric study was carried out in which the two parameters, r and μ , were varied independently.

4.3.1 Effect of Large Variations in r

The effect of large variations in r were studied using numerical model of the 203.0 mm wide blank of the AK1 material. Values of $r = 1.67, 2.49$, and 3.00 were chosen to coincide with values of the anisotropy observed in the AK1 sheet along the RD, transverse to the RD, and at 45° to the RD. Together with calculations using $\bar{r} = 2.21$ and those for the isotropic constitutive model ($r = 1.0$), they provide five different strain distributions. As shown in Figure 4.39, the variation of the predicted strain distribution with r is significant. The higher the value of r , the more pronounced the localization and the farther the localization is from the pole. The minor strain distribution also changes with r . The peak minor strain decreases with increasing r while the position follows the same trend as the location of the peak major strain. Hence, a higher r tends to favour the initiation of localization at a lower minor strain resulting in a less favourable deformation (i.e. closer to plane strain on the FLD with

a corresponding lower forming limit strain). These observations are consistent with the predicted effect of r on the limit strain described in Section 1.7.1.

The correlation between a high r value and sharper localization may also be presented in terms of the thickness distribution. The positive correlation between r and the rate of formation of a localization is clearly demonstrated in Figure 4.10 which shows thickness plotted against distance from the pole for the range of r values considered in this study. At a low r value, the material exhibits a uniform deformation characterised by the relatively smooth thickness distribution. As r increases, the material becomes more susceptible to thinning (r is the ratio of thickness strain to transverse strain) and a sharp variation in thickness is observed at the localization.

Experimental measurements did not provide data for comparison with these results. Even though measurements were taken along the 45° and 90° directions, relative to the RD, no significant variation in strain distribution was seen in both the lubricated and unlubricated specimens.

4.3.2 Effect of Frictional Variations

The effect of the frictional coefficient was investigated using the model of the AK1 material. Predicted strain distributions for μ values of 0.08, 0.13, 0.18, and 0.26 are shown in Figures 4.41, 4.42, and 4.43 for the 203.2 mm, 127.0 mm, and 50.8 mm wide blanks, respectively. The numerical results shown are for punch depths corresponding to the dome heights of the failed test specimens of the 203.2 mm, 127.0 mm, and 50.8 mm wide AK1 blanks. Results of the respective necked interrupted dome tests are also shown for comparison.

Increasing the frictional contact force of the blank and punch suppresses the deformation of the material in contact with the punch. This effect is shown in Figure 4.41 which presents the predicted effect of μ on the strain distribution for the 203.0 mm blank. Increasing μ suppresses the deformation under the punch as is shown by the reduced major and minor strain in the area of the pole. The same

trend is predicted in the 127.0 mm blank although the effects of increasing μ are more significant (refer to Figure 4.42). The increased sensitivity to μ is most likely a result of the lower minor strain which tends towards a plane strain condition for high μ values.

As shown in Figure 4.43, an increase in μ from 0.08 to 0.18 results in a more uniform major strain distribution with a lower peak strain in the 50.8 mm blank. At lower frictional forces, the notched shape of the 50.8 mm blank localizes the deformation to the material at the pole of the punch. As μ is increased, the frictional contact forces at the pole of the punch increase, resisting the deformation of the material at the pole of the punch where the highest contact forces exist. The strain distribution becomes more uniform through the material as a result of these two competing mechanisms. As the frictional force increases, the deformation begins to localize again as frictional forces over more of the punch reach a level such that the friction makes a significant contribution to resisting deformation of material in contact with the punch. At some value of μ (between 0.13 and 0.18 for this example), the frictional contact forces at the punch drive a localization at the edge of the punch to blank contact. This is clearly shown in Figure 4.43.

Chapter 5

Conclusion

Comparisons between measured and predicted strain distributions support the use of a commercial FEM code in modelling sheet metal forming processes. Successful simulation of the processes will rely heavily upon obtaining accurate material and frictional data and on the use of appropriate constitutive models. Results for the steel materials indicate good agreement between predicted and measured strain. A parametric study of the effect of r has demonstrated the sensitivity of the numerical results to the material parameters.

Variations in frictional boundary conditions are also seen to have a significant effect on the predicted strain distributions. It is generally agreed that a higher frictional coefficient constrains the deformation of the material in the region of punch to blank contact. The sensitivity to changes in μ seems to increase as the strain state in the material approaches plane strain. This behaviour is consistent with the observation that a material is more prone to localization at or near plane strain (e.g. LDH_0 on the FLD).

The model clearly shows the difficulties associated with modelling materials with anisotropy characterised by an r value less than 1.0. The numerical predictions were consistent with other published results which indicate that Hill's quadratic yield criterion does not accurately model the flow characteristics of materials with very low r values.

5.1 Future Work

This investigation has provided a starting point for a more detailed investigation of the application of FEM to sheet metal forming operation. The resulting models have been validated against experimental data for steel but the results for aluminum indicate the need for further study of constitutive relations applicable to low r value materials.

Non-quadratic yield criteria have been proposed which give significantly better predictions of the response of materials with $r \leq 1$ as discussed by Burford *et al.* [10]. Other material parameters which need to be investigated are the hardening coefficient, n , and the strain rate sensitivity, m . In aluminum, these parameters are particularly important because of the competing mechanisms of strain rate softening and strain hardening. The present study focussed on the effects of r and μ and did not investigate strain hardening effects to any degree. It is clear from other sensitivity analyses [10] that n plays a significant role in determining the strain distribution in a formed part and more detailed investigations of forming should incorporate a more accurate prediction of the hardening characteristics of a material. Strain rate effects are more difficult to incorporate into the analysis mainly due to the limited amount of data available.

The final area of concern for future work is an investigation of in-plane anisotropy. The use of the $\bar{r}$ parameter to characterize the anisotropy of materials, particularly those which exhibit significant in-plane variations in r , has been questioned by a number of researchers [42, 43, 47, 59]. Results of this study show reasonable agreement with measured strains can be achieved using the simplifying assumption of $\bar{r}$ however this may not be the case for more complex geometries.

In the material characterization stage of this investigation, the AK1 and IF materials were seen to possess similar averaged material properties. As a result, the IF steel was not considered in the validation of the numerical models. If in-plane anisotropic effects are to be examined, however, the two materials would provide

an excellent basis for comparison based on the wide difference in-plane anisotropic characteristics and the similarity in most other respects. This similarity would allow for isolation of the in-plane anisotropy effects from other material property related trends.

References

- [1] Keeler, S.P., "Sheet Metal Stamping Technology-Need for Fundamental Understanding," *Mechanics of Sheet Metal Forming - Material Behaviour and Deformation Analysis*, D.P. Koistinen and N. Wang. (eds.), Plenum Press, New York, 1978, pp. 3-17.
- [2] Jalinier, J.M., Barata Da Rocha, A, Barlat, F., Chakari, L., and Rio., G., "Modelling of Sheet Metal Forming," *Advanced Technology of Plasticity*, Vol. 1, 1984, pp. 741-746.
- [3] Yoshida, K., "Trends in Forming Technology for Car body in Japan," *Advanced Technology of Plasticity* , Vol. 1, 1984, pp. 729-734.
- [4] Yoshida, K. and Miyauchi, K., "Experimental Studies of Material Behaviours as Related to Sheet Metal Forming," *Mechanics of Sheet Metal Forming - Material Behaviour and Deformation Analysis*, D.P. Koistinen and N. Wang. (eds.), Plenum Press, New York, 1978, pp. 19-49.
- [5] Saran, M.J., Schedin, E., Samuelsson, A., Melander, A., and Gustafsson, C., "Numerical and Experimental Investigation of Deep Drawing of Metal Sheets," *Transactions of the ASME*, Vol. 112, Aug. 1990, pp. 272-285.
- [6] Siekirk, J.F., "Process Variable Effects on Sheet Metal Quality," *Journal of Applied Metal Working*, Vol. 4, No. 3, July 1986, pp. 262-269.

- [7] Liu, S.-D. and Karima, M., "A One Step Finite Element Approach for Product Design of Sheet Metal Stampings," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A A. Balkema, Rotterdam, 1992, pp. 497-502.
- [8] Wilson, D.V., "Representation of Material Behaviour in Finite Element Methods of Modelling Sheet Forming Processes," *International Materials Reviews*, Vol. 35, No. 6, 1990, pp. 329-348.
- [9] Marciniak, Z., "Assessment of Material Formability," *Advanced Techniques of Plasticity*, Vol. 1, 1984, pp. 685-694.
- [10] Burford, D.A., Narasimhan, K., and Wagoner, R.H., "A Theoretical Sensitivity Analysis for Full-Dome Formability Tests: Parameter Study for n , m , r , and μ ," *Metallurgical Transactions A*, Vol. 22A, 1991, pp. 1775-1788.
- [11] Wang, Y., Bayoumi, A.E., and Zbib, H.M., "An Experimental/Analytical Approach to Study Formability Using a Punch Test," *Journal of Material Shaping Technology*, Vol. 9, 1991, pp. 143-152.
- [12] Knibloe, J.R. and Wagoner, R.H., "Experimental Investigation and Finite Element Modelling of Hemispherical Stretched Steel Sheet," *Metallurgical Transactions A*, Vol. 20A, 1989, pp. 1509-1521.
- [13] Swift, H.W., "The Mechanism of a Simple Deep-Drawing Operation," *Sheet Metal Industries*, Vol. 31, 1954, pp. 817-828.
- [14] Kaftanoglu, B. and Alexander, J.M., "An Investigation of the Erichsen Test," *Journal of the Institute of Metals*, Vol. 90, 1962, pp. 457-470.
- [15] "E643 - Standard Test Method for Ball Punch Deformation of Metallic Sheet Material," *Annual Book of ASTM Standards*, American Society of Testing and Materials, Philadelphia, Vol. 3.01, 1992, pp. 663-666.

- [16] Yokai, M. and Alexander, J.M., "A Further Investigation of the Erichsen Test," *Sheet Metal Industries*, Vol. 44, 1967, pp. 466-475.
- [17] Hecker, S.S., "A Cup Test for Assessing Stretchability," *Metals Engineering Quarterly*, Vol. 14, No. 4, 1974, pp. 30-36.
- [18] Schey, J.A., *Tribology in Metalworking - Friction, Lubrication and Wear*, American Society for Metals, Ohio, 1983.
- [19] Ghosh, A.K., "The Effect of Lateral Draw-In on Stretch Formability," *Metals Engineering Quarterly*, Vol. 15, No. 3, pp. 53-64, 1975.
- [20] Danckert, J. and Wanheim, T., "The Use of a Square Grid as an Alternative to a Circle Grid in the Determination of Strains," *Journal of Metal Working Technology*, Vol. 3, 1979, pp. 5-15.
- [21] Sowerby, R., Chu, E., and Duncan, J.L., "Determination of Large Strains in Metalforming," *Journal of Strain Analysis*, Vol. 17, No. 2, 1982, pp. 95-101.
- [22] Keeler, S.P., "Circle Grid System - A Valuable Aid for Evaluating Sheet Metal Formability," SAE Paper No. 680092, 1968.
- [23] Goodwin, G.M., "Application of Strain Analysis to Sheet Metal Forming Problems in the Press Shop," Presented at SAE Automotive Engineering Congress, Detroit, January 1968, SAE Paper 680093.
- [24] Karima, M., Chandrasekaran, N., and Tse, W., "Process Signatures in Metal Stamping: Basic Concepts," *Journal of Materials Shaping Technology*, Vol. 7, No. 3, 1982, pp. 169-183.
- [25] Story, J.M., "Variables Affecting Dome Test Results," *Journal of Applied Metalworking*, Vol. 2, No. 2, 1982, pp. 119-125.

- [26] Nine, H.D., "Drawbead Forces in Sheet Metal Forming," *Mechanics of Sheet Metal Forming - Material Behaviour and Deformation Analysis*, D.P. Koistinen and N. Wang. (eds.), Plenum Press, New York, 1978, pp. 179-207.
- [27] Wilson, W.R.D., "Friction and Lubrication in Sheet Metal Forming," *Mechanics of Sheet Metal Forming - Material Behaviour and Deformation Analysis*, D.P. Koistinen and N. Wang. (eds.), Plenum Press, New York, 1978, pp. 157-174.
- [28] Meier, B.T., Reissner, J., and Bloeck, M., "Effect of the Surface Quality of the Workpiece Material on the Frictional Behaviour During Sheet Forming," *Proceedings of the 16th Biennial Congress of the International Deep Drawing Research Group*, Borlänge, Sweden, June 11-13, 1990, pp. 43-52.
- [29] Wright, R.N., and Schurman, T., "Die Friction Variations in Simulated Sheet Metal Forming," *Proceedings of the 16th Biennial Congress of the International Deep Drawing Research Group*, Borlänge, Sweden, June 11-13, 1990, pp. 113-121.
- [30] Maisch, R.P., Meier, B., Kubli, W., and Reissner, J., "Considering Friction Effects in the FE-Simulation of Forming Processes," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 163-168.
- [31] Bochmann, E. and Doege, E., "Friction as a Critical Phenomenon in the Simulation of Metal Forming," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 415-420.
- [32] Mattiasson, K. Bernspång, L., Samuelsson, A., Hännman, T., Schedin, E., and Melander, A., "Evaluation of a Dynamic Approach Using Explicit Integration in 3-D Sheet Forming Simulation," *Numiform 92: Numerical Methods in Industrial*

- Forming Processes*, Chenot, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 55-67.
- [33] Baaijens, F., Brekelmans, W., Veldpaus, F.E., and Starman, F.J.M., "A Constitutive Equation for Frictional Phenomena Including History Dependency," *Numiform 89: Numerical Methods in Industrial Forming Processes*, Thompson, E.G., Wood, R.D., Zienkiewicz, O.C., and Samuelsson, A. (eds.), A.A. Balkema, Rotterdam, 1989, pp. 91-96.
- [34] Schey, J.A. and Fisher, R.F., "Effect of Clamp Force in Limiting Dome Height (LDH) Test," *Journal of Materials Shaping Technology*, Vol. 9, No. 2, 1991, pp. 85-88.
- [35] Semiatin, S.L. and Jonas, J.J., *Formability and Workability of Metals: Plastic Instability and Flow Localization*, American Society for Metals, Ohio, Chapters 1, 2, 5, and 6, 1984.
- [36] Hill, R., "On Discontinuous Plastic States, with Special Reference to Localization Necking in Thin Sheets," *Journal of Mechanics and Physics of Solids*, Vol. 1, 1952, pp. 19-30.
- [37] Benham, P.P. and Crawford, R.J., *Mechanics of Engineering Materials*, John Wiley and Sons, New York, 1987.
- [38] Pearce, R., "Some Aspects of Anisotropic Plasticity on Sheet Metals," *International Journal of Mechanical Science*, Vol. 10, 1968, pp. 995-1005.
- [39] Taylor, G.I., "Plastic Strain in Metals," *Journal of the Institute of Metals*, Vol. 62, 1938, pp. 307-324.
- [40] Hill, R., *The Mathematical Theory of Plasticity*, Oxford University Press, Oxford, 1986.

- [41] Chin, G., Hosford, W., and Mendorf, D., "Accommodation of Constrained Deformation in f.c.c. Metals by Slip and Twinning," *Proceedings of the Royal Society A*, Vol. 309, 1969, pp. 433–456.
- [42] Bramley, A.N., and Mellor, P.B., "Plastic Flow in Stabilized Sheet Steel," *International Journal of Mechanical Science*, Vol. 8, 1966, pp. 101–114.
- [43] Kobayashi, S., Caddell, R.M., and Hosford, W.F., "Examination of Hill's Latest Yield Criterion using Experimental Data for Various Anisotropic Sheet Metals," *International Journal of Mechanical Science*, Vol. 27, No.7/8, 1985 pp. 509–517.
- [44] Horta, R.M., Roberts, W.T., and Wilson, D.V., "Effects of Plastic Anisotropy on the Biaxial Stretching Performance of Low-Carbon Steels," *International Journal of Mechanical Science*, Vol. 12, 1970, pp. 231–243.
- [45] Wang, N., Hall, F.R., Pillinger, I., Hartley, P., Sturgess, C.F.N., Van Bael, A., Winters, J., Van Houtte, P., and Aernoudt, E., "Finite-Element Prediction of Texture Evolution in Material Forming," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 193–198.
- [46] Hill, "A Theory of the Yielding and Plastic Flow of Anisotropic Metals," *Proceedings of the Royal Society A*, Vol. 193, 1948, pp. 281–297.
- [47] Dillamore, I.L., "The Relevance of Tensile Properties to Sheet Formability," *Journal of Physics D: Applied Physics*, Vol. 7, 1974, pp. 979–991.
- [48] Hill, R., "The Theory of Plane Plastic Strain for Anisotropic Materials," *Proceedings of the Royal Society A*, Vol. 193, 1949, pp. 428–437.
- [49] Marciniak, Z. and Kuczyński, K., "Limit Strains in the Processes of Stretch Forming Sheet Metal," *International Journal of Mechanical Science*, Vol. 9, 1967, pp 609–620.

- [50] Chan, K.S., "Effects of Plastic Anisotropy and Yield Surface Shape on Sheet Metal Stretchability," *Metallurgical Transactions A*, Vol. 16A, 1985, pp. 629–639.
- [51] Chan, K.S., Koss, D.A., and Ghosh, A.K., "Localized Necking of Sheet at Negative Minor Strains," *Metallurgical Transactions A*, Vol. 15A, 1984, pp. 323–329.
- [52] Lahoti, G.D., Nagpal, V., and Altan, T., "Prediction of Limit Strains in Biaxial Stretching of Sheets Using Marciniak's Approach," *Advanced Technology of Plasticity*, Vol.1, 1984, pp. 712–718.
- [53] Sowerby, R. and Duncan, J.L., "Failure in Sheet Metal in Biaxial Tension," *International Journal of Mechanical Science*, Vol. 13, 1971, pp. 217–229.
- [54] Venter, R., Johnson, W., and De Malherbe, M.C., "The Limit Strains of Inhomogeneous Sheet Metal in Biaxial Tension," *International Journal of Mechanical Science*, Vol. 13, 1971, pp. 209–308.
- [55] Parmar, A. and Mellor, P.B., "Predictions of Limit Strains in Sheet Metal using a more General Yield Criterion," *International Journal of Mechanical Science*, Vol. 20, 1978, pp. 385–391.
- [56] Marciniak, Z., Kuczyński, K., and Pokora, T., "Influence of the Plastic Properties of a Material on the Forming Limit Diagram for Sheet Metal in Tension," *International Journal of Mechanical Science*, Vol. 15, 1973, pp. 789–805.
- [57] Jonas, S.E. and Gillis, P.P., "A Generalized Quadratic Flow Law for Sheet Metals," *Metallurgical Transactions A* Vol. 15A, 1984, pp. 129–132.
- [58] Hosford, W.F., 7th North American Metalworking Conference, SME, Dearborn, MI, 1979, pp. 191–197.

- [59] Lequeu, P.H. and Jonas, J.J., "Modeling of the Plastic Anisotropy of Textured Sheet," *Metallurgical Transactions A*, Vol. 19A, 1988, pp. 105-120.
- [60] Stören, S. and Rice, J.R., "Localized Necking in Thin Sheets," *Journal of Mechanical Science*, Vol. 23, 1975, pp. 421-441.
- [61] Graf, A. and Hosford, W.F., "Calculations of Forming Limit Diagrams," *Metallurgical Transactions A*, Vol. 21A, 1990, pp. 87-90.
- [62] Choi, W., Gillis, P.P., and Jones, S.E., "Calculation of the Forming Limit Diagram," *Metallurgical Transactions A*, Vol. 20A, 1989, pp. 1975-1987.
- [63] Wang, N.-M., and Wenner, M.L., "Elastic-Viscoplastic Analyses of Simple Stretch Forming Problems," *Mechanics of Sheet Metal Forming - Material Behaviour and Deformation Analysis*, D.P. Koistinen and N. Wang. (eds.), Plenum Press, New York, 1978, pp. 367-398.
- [64] Melander, A. and Schedin, E., "On the Forming Limit in Sheet metal Forming," *Advanced Technology of Plasticity*, Vol. 1, 1984, pp. 635-639.
- [65] Ohsawa, H., Yamazaki, S., and Nishimura, H., "The Role of Strain Hardening and Strain Rate Sensitivity in Uniaxial Tension and Hemispherical Punch Stretching," *Advanced Technology of Plasticity*, Vol.1, 1984, pp. 640-645.
- [66] Nie, Q.Q. and Lee, D., "The Effect of Rate Sensitivity on History Dependent Forming Limits of Anisotropic Sheet Metals," *Journal of Material Shaping Technology*, Vol. 9, 1991, pp. 233-240.
- [67] Jonas, S.E. and Gillis, P.P., "An Analysis of Biaxial Stretching of a Flat Sheet," *Metallurgical Transactions A*, Vol. 15A, 1984, pp. 133-138.
- [68] Wagoner, R.H., and Zhou, D., "Analyzing Sheet Forming Operations - Recent Numerical and Experimental Advances," *Numiform 92: Numerical Methods*

- in *Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 123-132.
- [69] Honecker, A. and Mattiasson, K., "Finite Element Procedures for 3D Sheet Forming Simulation," *Numiform 89: Numerical Methods in Industrial Forming Processes*, Thompson, E.G., Wood, R.D., Zienkiewicz, O.C., and Samuelsson, A. (eds.), A.A. Balkema, Rotterdam, 1989, pp. 457-463.
- [70] Anerlene, F. and Babeau, J.-L., "OPTRIS and FRITURE: Industrial Tools to Modelize Sheet Metal Forming," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 397-402.
- [71] Wang, M.-M. and Somaratna, N., "Numerical Simulation of Industrial Sheet Forming Processes," *Numiform 89: Numerical Methods in Industrial Forming Processes*, Thompson, E.G., Wood, R.D., Zienkiewicz, O.C., and Samuelsson, A. (eds.), A.A. Balkema, Rotterdam, 1989, pp. 75-84.
- [72] Rebelo, N., Nagtegaal, J.C., Taylor, L.M., and Passman, R., "Comparison of Implicit and Explicit Finite Element Methods in the Simulation of Metal Forming Processes," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 99-108.
- [73] Kawka, M., and Makinouchi, A., "Finite Element Simulation of Sheet Metal Forming Processes by Simultaneous Use of Membrane, Shell, and Solid Elements," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 491-496.

- [74] Narasimhan, K. and Wagoner, R., "Finite Element Modeling Simulation of In-Plane Forming Limit Diagrams of Sheets Containing Finite Defects," *Metallurgical Transactions A*, Vol. 22A, 1991, pp. 2655–2665.
- [75] Hambrecht, J., Choudhry, S., Lee, J., and Wagoner, R., "Numerical Study of Two-Dimensional Sheet Forming Processes using Bending, Membrane, and Solid Finite Element Models," *Numiform 92: Numerical Methods in Industrial Forming Processes*, Chenat, J.-L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 451–456.
- [76] "E517 - Standard Test Method for Plastic Strain Ratio r for Sheet Metal," *Annual Book of ASTM Standards*, American Society of Testing and Materials, Philadelphia, Vol. 3.01, 1992, pp. 580–585.
- [77] "E646 - Standard Test Method for Tensile Strain-Hardening Exponents of Metallic Sheet Materials," *Annual Book of ASTM Standards*, American Society of Testing and Materials, Philadelphia, Vol. 3.01, 1992, pp. 667–673.
- [78] Keeler, S.P., "Determination of Forming Limit in Automotive Stampings," Presented at SAE Mid-Year Meeting, Chicago, May 1965, SAE paper 650535
- [79] Marciniak, Z. and Duncan, J., *Mechanics of Sheet Metal Forming*, Hodder and Stoughton, London, England 1992.
- [80] Chem-Etch of Canada Ltd., 5715 McAdam Road, Mississauga, Ontario, L4Z 1N9, Tel. (416) 890-4935.
- [81] Halquist, J.O., *LS-DYNA3D Theoretical Manual*, Livermore Software Technology Corporation, Livermore, California, USA, June 1991.
- [82] Schweizerhof, K., Halquist, J., and Wainscott, B., "Improvements in Contact Algorithms for the Simulation of Thin Sheet Metal Forming with Explicit Finite Element Programs," *Numiform 92: Numerical Methods in Industrial*

- Forming Processes*, Chenat, J. L., Wood, R.D., and Zienkiewicz, O.C. (eds.), A.A. Balkema, Rotterdam, 1992, pp. 535-542.
- [83] Belytschko, T., Lin, J.I., and Tsay, C., "Explicit Algorithms for the Non-Linear Dynamics of Shells," *Computer Methods in Applied Mechanics and Engineering*, Vol. 42, 1984, pp. 225-251.
- [84] Belytschko, T. and Tsay, C., "A Stabilization Procedure for the Quadrilateral Plate Element with One-Point Quadrature," *International Journal for Numerical Methods in Engineering*, Vol. 19, 1983, pp. 405-419.
- [85] Galbraith, P.C. and Halquist, J.O., "Shell Element Formulations in LS-DYNA3D: Their Use in modelling Sheet Metal Forming," in progress.
- [86] Engelmann, B.E., Whirley, R.G., and Goudreau, G.L., "A Simple Shell Element Formulation for Large-Scale Elastoplastic Analysis," *Analytical and Computational Models of Shells*, Noor, A.K., Belytschko, T., and Simo, J.C. (eds.), CED-Vol.3, ASME, New York, New York, 1989.
- [87] Hughes, T.J.R and Carnoy, E., "Nonlinear Finite Element Shell Formulation Accounting for Large Membrane Strains," AMD-Vol. 48, ASME, 1981, pp. 193-208.
- [88] Flanagan, D.P. and Belytschko, T., "A Uniform Strain Hexahedron and Quadrilateral with Orthogonal Hourglass Control," *International Journal for Numerical Methods in Engineering*, Vol. 17, 1981, pp. 679-706.
- [89] Kohara, S., Katsuta, M., Aoki, K., and Susuki, K., "Forming Limit Curves for Aluminum, Copper and their Alloys," *Advanced Technology of Plasticity*, Vol. 1, 1984, pp. 765-768.
- [90] Martin, P., Williams, K., and Worswick, M., "Deep Drawability of Stainless Steel," *Proceedings of the International Conference on Strip Casting, Hot and*

Cold Deformation of Stainless Steel, Ryan, N.D., Brown, A.J., and McQueen, H.J. (eds.), Quebec City, Quebec, August, 1993 pp. 195-210.

- [91] P. Galbraith, ALCAN Kingston Research Laboratories, Kingston, Ontario, May, 1993 (private communication).
- [92] Roizard, X., von Stebut, J., Paintendre, B., and Venumiere, L., "The Influence of Contact Temperature of Metal Transfer and Galling in Strip Drawing," *Proceedings of the 16th Biennial Congress of the International Deep Drawing Research Group*, Borlänge, Sweden, June 11-13, 1990, pp. 161-171
- [93] *Marks Standard Engineering Handbook*, Avallone, E. and Baumeister, T. (eds.), Ninth Edition, McGraw Hill, New York, USA.

Table 1.1: Summary of frictional coefficients quoted in published numerical and analytical studies of sheet metal forming

Author	Material	Coefficient of Friction μ	Lubrication Condition	Notes
Swift [13] Wright	Steel	0.06 0.128	Graphite None	Theoretical calculations compared to experimental dome test (Source of values unknown)
and Schurman [29]	AKDQ HDGS	0.129-0.138 0.074-0.080	G M Montgomery 4284 Lubricant (5:1 Dilutant)	Draw bead and binder radius simulator - dynamic coefficient
Wright and Schurman [29]	AKDQ 0.4 μ m 0.8 μ m 1.6 μ m 2.0 μ m	0.12 0.12 0.15 0.18	Unspecified	Draw bead with surface finish variations (sand blasted) - dynamic coefficient
Roisard et al [92]	Un-coated 25°C steel 60°C 100°C Zn coated 25°C steel 60°C 100°C	0.15-0.17 0.18-0.18 0.21-0.23 0.19-0.22 0.21-0.31 0.27-0.33	Fuchs no 5012 S oil air dried for 20 hours	Strip drawing with temperature effect - repeated passes dynamic coefficient steel unspecified
Burford et al [10]	Steel	0.20	Unspecified	Coulomb friction in a rigid viscoplastic FEM with triangular constant strain elements (Reason for choice unspecified)
Wang et al [45]	1010 Cold rolled steel and	0.22 0.10	Unspecified Unspecified	Analytical analysis of punch test using membrane theory (Assumed values)
Wilson [8]	Steel	0.15	Unspecified	Press forming a rectangular pane using a elastic-plastic membrane FEM formulation with normal anisotropy and Coulomb friction (Assumed value)
	2036-T4 alloy	0.05-0.25	Unspecified	Stretch forming over a square punch using a rigid viscoplastic FEM formulation with Coulomb friction (Assumed values for comparison)
Saran et al [5]	DDQS 70/30 Brass HSS Stainless Steel Aluminum	0.22 0.08 0.15 0.14 0.06	Unspecified	Continuum mechanics based mathematical and computational model using Hill's quadratic yield function and Coulomb model for friction. (Assumed values giving relatively good agreement)
Galbraith and Halquist [85]	Aluminum	0.162	Unspecified	Shell formulation comparison within LS-DYNA3D for an elastic-plastic material with isotropic hardening and Coulomb friction
Mattinson et al [32]	Deep Drawing Steel High Strength Steel Stainless Steel Brass Aluminum	0.40 0.12 0.08 0.18 0.03	Unspecified	Explicit FEM analysis (DYNA3D) using Coulomb friction of the deep drawing of a cylindrical cup (μ fit to experimental punch force versus displacements plots)
	Deep Drawing Steel Stainless Steel Brass	0.12 0.40 0.20	Unspecified	The same analysis as above but for stretching. (μ fit to experimental pole strains)
Wang and Somaratna [71]	Steel	0.10 0.15	Rust protective oil Drawing compound	FEM analysis of an industrial sheet forming process using Hill's quadratic yield criterion with Coulomb friction (μ estimated from a comparison of numerical and experimental measurements of punch and binder force)

Table 1.2: Experimentally determined coefficients of friction [18].

Sheet Material	Lubrication	Strain	μ
Al-killed steel	Dry (rough punch)	0.06	0.6
	Grease	0.04	0.42
		0.09	0.29
		0.13	0.24
	Dry (smooth punch)	...	0.2
	Mill oil	...	0.13
	PTFE spray + PE sheet	...	0.12
Stainless steel (409)	Dry	...	0.14
	Mill Oil	...	0.13
	PTFE spray + PE sheet	...	0.09
Soft copper	Grease	0.1	0.32
	PTFE film	0.16	0.135
		0.33	0.07
Soft Brass (70/30)	Grease	0.1	0.31
	Dry	...	0.36
	PTFE + PE film	...	0.29
Soft aluminum 3003-0 Al	Grease	0.1	0.31
	Dry	...	0.22
	PTFE + PE film	...	0.07
2024-T4 Al	Dry	...	0.31
	E.P. oil	...	0.19
	PTFE + PE film	...	0.17
	Polymer coat + oil	...	0.045

Table 2.1: Chemical composition of the sheet steels.

Sheet	Type	Chemical Composition (weight %)						
		Si	Mn	Al	Nb	Ti	S	C
IF	IF	0.016	0.14	0.049	0.011	0.061	0.062	0.005
AK1	AKDQ	0.014	0.24	0.046	< 0.010	< 0.005	0.018	0.050
AK2	AKDQ	0.015	0.17	0.044	< 0.010	< 0.005	0.076	0.040

Table 2.2: Chemical composition of the aluminum sheet.

Sheet	Type	Chemical Composition (weight %)				
		Si	Cu	Mg	Zn	Fe
AL	X544.0	< 0.070	< 0.030	3.5	< 0.030	0.16
		Ti	Mn	Ni	Cr	
		< 0.010	0.32	< 0.010	< 0.007	

Table 2.3: Uniaxial tensile test data. The L, D, and T designations refer to the tensile properties along the rolling direction (RD), inclined at 45° to the RD, and transverse to the RD respectively.

Spec.	Yield Stress (0.2%) (MPa)	Ultimate Tensile Stress (MPa)	n Value (4%)	r Value (15%)	Unif. Elong. (%)	Elong. (%)
IF-D	139.3	317.3	0.2871	2.489	29.12	42.00
IF-L	140.6	332.9	0.2891	1.632	27.89	39.82
IF-T	146.4	333.5	0.2847	2.480	26.91	41.38
AK1-D	168.3	326.1	0.2453	1.674	25.98	36.24
AK1-L	162.2	314.8	0.2556	2.485	28.01	40.89
AK1-T	166.7	313.6	0.2453	3.007	24.83	38.00
AK2-D	171.6	296.0	0.2232	1.516	23.10	42.51
AK2-L	159.0	284.0	0.2297	2.173	28.60	44.12
AK2-T	164.2	278.2	0.2230	2.517	25.08	44.28
AL-D	114.1	240.7	0.2589	0.446	—	22.60
AL-L	116.1	241.6	0.2679	0.733	—	19.60
AL-T	117.0	244.9	0.2591	1.010	—	19.20

Table 2.4: Summary of material property data. Tensile test data except where noted.

Spec.	Average Sheet Thickness (mm)	Mean Yield Stress (0.2%) (MPa)	Mean Ultimate Tensile Stress (MPa)	Average n Value (4%)	$\bar{r}$ Value (15%)	E (MPa)	ν
IF	0.664	142.1	327.9	0.2871	2.273	208000 [†]	0.29 [†]
AK1	0.656	165.7	318.2	0.2487	2.210	208000 [†]	0.29 [†]
AK2	0.908	163.9	286.3	0.2253	1.931	208000 [†]	0.29 [†]
AL	1.602	115.7	242.6	0.2623	0.659	73010	0.33 [†]

[†] Handbook values [93]

Table 2.5: Typical limit minor strains for various blank sizes

Blank Size (mm)	Material			
	IF	AK1	AK2	AL
Tensile Spec.	-27%	-30%	-28%	N/A
203.2 x 25.4 (Mach.)	-23%	-26%	-26%	N/A
203.2 x 50.8 (Mach.)	-11%	-14%	-15%	-2%
203.2 x 76.2 (Mach.)	-9%	-12%	-10%	N/A
203.2 x 88.9	N/A	N/A	FDB	N/A
203.2 x 95.3	N/A	N/A	FDB	N/A
203.2 x 101.6	-11%	-12%	-8%	N/A
203.2 x 114.3	-7%	-5%	-5%	N/A
203.2 x 120.7	-5%	-5%	N/A	N/A
203.2 x 127.0	-5%	-4%	0%	0%
203.2 x 139.7	0%	0%	N/A	N/A
203.2 x 152.4	5%	6%	8%	N/A
203.2 x 152.4 Lub1	N/A	N/A	12%	N/A
203.2 x 203.2	11%	11%	12%	9%
203.2 x 203.2 Lub2	11%	N/A	N/A	N/A
203.2 x 203.2 Lub3	N/A	N/A	23%	N/A
203.2 x 203.2 Lub4	27%	N/A	24%	N/A
203.2 x 203.2 Lub5	N/A	16%	36%	N/A
203.2 x 203.2 Lub6	35%	24%	N/A	N/A

N/A Blank size not tested for this material

FDB Failed at the draw bead

Lub1 Protective oil air dried for 30 minutes

Lub2 Vaseline only

Lub3 Blank not cleaned (dried protective oil)

Lub4 Vaseline and two 101.6 mm polyethalene disks

Lub5 Vaseline and 101.6 neoprene disk

Lub6 Vaseline, 101.6 neoprene disk, and 101.6 polyethalene disk

Table 2.6: Interrupted dome test dome heights. Specimens denoted failed exhibited large necks or failures and were not measured.

	Blank Size (mm)	Dome Height						
		Blank Identification Number						
		11	12	13	14	15	16	17
AK1	50.8	25.0 mm	30.1 mm	35.1 mm	37.2 mm	39.0 mm	Failed	N/A
	127.0	19.8 mm	25.4 mm	30.1 mm	32.7 mm	35.2 mm	Failed	36.4 mm
	203.2	20.0 mm	25.0 mm	30.1 mm	32.3 mm	34.9 mm	N/A	N/A
AK2	50.8	25.0 mm	30.1 mm	35.1 mm	37.1 mm	Failed	38.4 mm	N/A
	127.0	20.1 mm	25.0 mm	30.1 mm	32.6 mm	35.2 mm	Failed	35.5
	203.2	20.0 mm	25.0 mm	30.1 mm	32.5 mm	34.8 mm	Failed	35.1
AL	50.8	24.5 mm	17.9 mm	19.8 mm	22.1 mm	23.9 mm	Failed	N/A
	127.0	15.1 mm	20.1 mm	22.6 mm	Failed	24.2 mm	17.6 mm	N/A
	203.2	17.0 mm	15.2 mm	19.9 mm	22.6 mm	25.1 mm	Failed	N/A

Table 3.1: Outline of analysis mesh sizes used in the investigation.

Blank Geometry	Rigid Shell Elements	B-L-T Shell Elements	Total
50.8 mm (mach.)	2760	2862	5622
127.0 mm	4140	2784	6924
203.2 mm	4140	3072	7212
203.2 mm (fine)	12459	9492	21951

Table 3.2: Comparison of shell element formulations implemented in LS-DYNA3D. Run times are for the Numisheet'93 Benchmark problem [85]. Note that the cpu times were quoted for analyses run on a single processor of an SGI 4D480 workstation.

Element Type	Integration Point Layout Used	Total CPU Time (hrs.)	% of CPU Time for Element Solution	Number of Time Steps
Hughes-Liu	1 x 7 through thick.	4.92	82.7	4971
Belytschko-Tsay	1 x 7 through thick.	3.23	85.5	4970
Belytschko-Tsay Membrane	1 x 7 through thick.	2.20	50.8	5231
S/R Hughes-Liu	2 x 2 x 7 through thick.	21.00	96.6	4977
S/R co-rotational Hughes-Liu	2 x 2 x 7 through thick.	13.50	94.6	4978
Englemann-Whirley	1 x 7 through thick.	3.39	72.5	4982
Fully integrated Bely.-Tsay Membrane	2 x 2 x 1 through thick.	2.63	62.5	5090

Table 3.3: Summary of material property data for the numerical material models.

Material	Sheet Thickness (mm)	Density (g/mm ³) $\times 10^{-3}$	Yield Stress σ_Y (MPa)	Hardening Modulus E_T (MPa)	$\bar{r}$ Value	Young's Modulus E (MPa)	Poisson's Ratio ν
AK1	0.656	7.887 [†]	295.0	450.0	2.210	208000 [†]	0.29 [†]
AK2	0.908	7.887 [†]	270.0	340.0	1.931	208000 [†]	0.29 [†]
AL	1.602	2.643 [†]	195.0	500.0	0.659	73010	0.33 [†]

[†] Handbook values [93]

Table 3.4: Breakdown of model run time showing the three stages of load application for the step load curves.

	Blank Size (mm)					
	Steels			Aluminum		
	50.8	127.0	203.2	50.8	127.0	203.2
Clamp Closure	9.54 ms	9.54 ms	4.77 ms	9.54 ms	9.54 ms	4.77 ms
Pressure Application	1.46 ms	1.46 ms	1.21 ms	1.46 ms	1.46 ms	1.21 ms
Punch Stroke	10.00 ms	9.00 ms	9.00 ms	6.50 ms	6.25 ms	7.00 ms
Total Time	21.00 ms	20.00 ms	15.00 ms	17.50 ms	17.25 ms	13.00 ms
Punch Depth	40 mm	36 mm	36 mm	26 mm	25 mm	28 mm

Table 3.5: Summary of the sinusoidal load curve trials.

Run Title	Clamping Velocity Duration (ms)	Clamping Pressure Application (ms)	Punch Velocity Duration (ms)
d250-c-0.25ms.37	0.25 [†]	0.5	1.00
d250-c-4ms.37	4.00	0.5	1.00
d250-p-0.25ms.37	1.00	0.5	0.25 [‡]
d250-p-1ms.37	1.00	0.5	1.00
d250-p-4ms.37	1.00	0.5	4.00
J250-cpl-step.37	1.00	0.5*	1.00
d280-c-0.25ms.37	0.25 [†]	0.5	1.00
d280-c-4ms.37	4.00	0.5	1.00
d280-p-0.25ms.37	1.00	0.5	0.25 [‡]
d280-p-1ms.37	1.00	0.5	1.00
d280-p-4ms.37	1.00	0.5	4.00

† Small dynamic response observed in the blank

‡ Large inertial effect in the blank

* Initial step in pressure followed by a sinusoidal ramp

Figure 1.1: Example of the time requirements for the various stages of a car body development and production tooling cycle [3].

Figure 1.2: The position of formability in the die-face design cycle [3].

Figure 1.3: Factors affecting the material behaviour in press forming [4].

Figure 1.4: Limit strain loci for simple sheet forming operations [77].

Figure 1.5: The representation of the main deformation modes in strain space and their relation to the position of the forming limit diagram (due to Karima *et al.* [24]).

Figure 1.6: Comparison of the failure strain predicted at the failure (Goodwin curve) and adjacent to the failure (Keeler curve) [53].

Figure 1.7: Schematic of the Swift drawing test [13].

Figure 1.8: Schematic of the Erichsen punch test [15].

Figure 1.9: Schematic of the punch and drawbead arrangement of the Hecker and Ghosh punch test (due to Hecker [17]).

Figure 1.10: Effect of the lubrication condition on the major strain distribution of a limiting dome height test [18]. The pole refers to the top of the spherical indenter

Figure 1.11: Schematic of a draw bead simulator used to assess the frictional characteristic of a material being drawn through a set of draw beads [26].

Figure 1.12: Elements of a metalworking lubrication system [18].

(a)

(b)

Figure 1.13: Variation of draw-in and punch depth with specimen width for a clamping force of (a) 125 kN and (b) 250 kN [34].

Figure 1.14: Variation of draw-in with clamping force showing the minimum required clamping force for a maximum draw-in of 0.25 mm [34].

Figure 1.15: The predicted effect of r value on the yield locus. a represents the stress ratios of common loading states [38].

Figure 1.16: Experimentally measured variation of r with the orientation to the rolling direction. Curves represent four unspecified steels. [42].

Figure 1.17: Experimental and analytically predicted work hardening characteristics of a steel with $\bar{r} = 1.48$ using Hill's quadratic yield criterion. (1-3) experimental curves for simple tension at 0° , 45° , and 90° to the rolling direction, (4) experimental biaxial test, (5) theoretical curves based on $\bar{r} = 1.48$, (6) theoretical curve based on 90° tensile curve and on an assumption of loading along the anisotropic axes, and (7) theoretical curve based on 0° tensile curve and on an assumption of loading along the anisotropic axes [42].

Figure 1.18: Uniaxial and biaxial stress-strain curves for commercial-purity aluminum in the annealed ($\bar{r} = 0.68$) and hard rolled ($\bar{r} = 0.43$) conditions [38].

Figure 1.19: Uniaxial and biaxial stress-strain curves for annealed copper-containing steel ($\bar{r} = 1.62$) [38].

Figure 1.20: Predicted effect of r on the FLD according to Marciniak and Kuczński theory [55].

Figure 1.21: Qualitative non-mathematical interpretation of the Marciniak and Kuczyński predictions using the predicted effect of the r value on the yield locus. (a) isotropic material ($r = 1.0$) and (b) anisotropic material ($r \geq 1$) due to Sowerby and Duncan [53].

Figure 1.22: Effect of the exponent, M , of Hill's non-quadratic yield function on the yield locus [55].

Figure 1.23: Predicted influence of the exponent, M , of Hill's non- quadratic yield criterion on the right hand side of the FLD [55].

Figure 1.24: Effect of anisotropy, r , on the strain distribution in a hemispherical dome test specimen [63].

Figure 1.25: Effect of the strain hardening exponent, n , on the FLD [77].

Figure 2.1: Orientation of tensile test specimens relative to the sheet rolling direction.

Figure 2.2: Experimental true stress versus true strain curves for sheet IF (Dofasco IF steel) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.

Figure 2.3: Experimental true stress versus true strain curves for sheet AK1 (Dofasco AK steel) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.

Figure 2.4: Experimental true stress versus true strain curves for sheet AK2 (commercial grade AK steel) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.

Figure 2.5: Experimental true stress versus true strain curves for sheet AL (experimental aluminum X544.0 supplied by Alcan) showing a hardening modulus, E_T , and corresponding yield stress, σ'_Y fit to the curve.

Figure 2.6: Close up of a deformed 203.2 mm specimen (AK1) showing the circle grid pattern etched on the blanks.

Figure 2.7: Limiting dome height press facility at CANMET.

Figure 2.8: View of the clamping die and punch arrangement.

Figure 2.9: Schematic view of the limiting dome height press facility.

Figure 2.10: Schematic view of the limiting dome height test data acquisition setup.

Figure 2.11: Schematic showing the positioning of the blank in the punch test setup using the location pins.

Figure 2.12: Example of a load-displacement plot of a blank deformed past necking indicating the approximate punch load and displacement at failure.

Figure 2.13: Schematic of a deformed circle grid showing the grid measurement technique.

Figure 2.14: Geometry of the narrow, notched specimens used for the 25.4 mm, 50.8 mm, and 76.2 mm blank sizes. The 50.8 mm specimen geometry is shown above.

Figure 2.15: Photograph of a failed 127.0 mm AK1 blank showing the neck and the surrounding grids.

Figure 2.16: Experimentally measured forming limit curve for sheet IF.

Figure 2.17: Experimentally measured forming limit curve for sheet AK1.

Figure 2.18: Experimentally measured forming limit curve for sheet AK2.

Figure 2.19: Example of a series of interrupted dome test specimens (127.0 mm blanks of sheet AK1). Counterclockwise from the lower left, the depths are 19.8 mm, 25.4 mm, 30.2 mm, 32.7 mm, 35.2 mm, and 36.4 mm.

Figure 2.20: Schematic cross section of the deformed blank showing the pole of the dome, the blank to punch tangency point (TP) and the coordinate frame for dome height (y) and strain measurement position (x).

Figure 3.1: Exploded view of the numerical model of the 127.0 mm wide blank.

Figure 3.2: Schematic of models showing the symmetry used for the (a) 127.0 mm and (b) 203.2 mm blank geometries.

Figure 3.3: Schematic showing the reduced tooling required to model the 50.8 mm blank geometry.

Figure 3.4: Plot of the deformed 127.0 mm wide blank indicating the region of severe deformation which combines stretching, bending, and shear.

(a)

(b)

Figure 3.5: Indications of drawing parallel to the draw bead in the 127.0 mm wide blank of AK1 material (a) observed experimentally and (b) predicted numerically.

Figure 3.6: FEM mesh plot of the 50.8 mm wide blank model (2862 elements).

Figure 3.7: FEM mesh plot of the 127.0 mm wide blank model (2784 elements).

Figure 3.8: FEM mesh plot of the 203.2 mm wide blank model (3072 elements).

Figure 3.9: Quadrilateral shell element displacement modes (a-c) and hourglass mode (d) [88].

Figure 3.10: Example of the elastic-plastic material model with isotropic hardening represented by the hardening modulus, E_T , fit to actual true stress versus true strain curve.

Figure 3.11: Graphical representation of the stages of the modelled process and the load histories used. (a) step function load curves and (b) sinusoidal load curve functions. (Both graphs represent the load curves used for the 203.2 mm wide steel models)

Figure 3.12: Comparison of predicted strain distribution using (a) step function load curves and (b) sinusoidal load curves.

Figure 3.13: Refined mesh of the 203.2 mm wide blank used in the convergence check (9492 elements).

Figure 3.14: Comparison of predicted strain distributions for varying mesh densities.

Figure 4.1: Comparison of measured forming limit curves for the AK1 and AK2 steels with published results for Al 1050-0 from Kohara *et al.* [87].

Figure 4.2: Measured major and minor principal Green's strain for the 203.2 mm wide blank size of AK1.

Figure 4.3: Measured major and minor principal Green's strain for the 127.0 mm wide blank size of AK1.

Figure 4.4: Measured major and minor principal Green's strain for the 50.8 mm wide blank size of AK1.

Figure 4.5: Measured major and minor principal Green's strain for the 203.2 mm wide blank size of AK2.

Figure 4.6: Measured major and minor principal Green's strain for the 127.0 mm wide blank size of AK2.

Figure 4.7: Measured major and minor principal Green's strain for the 50.8 mm wide blank size of AK2.

Figure 4.8: Measured major and minor principal Green's strain for the 203.2 mm wide blank size of AL.

Figure 4.9: Measured major and minor principal Green's strain for the 127.0 mm wide blank size of AL.

3 of/de 3

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Figure 4.10: Measured major and minor principal Green's strain for the 50.8 mm wide blank size of AL.

Figure 4.11: Fringe plot of predicted major principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 0 mm.

Figure 4.12: Fringe plot of predicted major principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 19.0 mm

Figure 4.13: Fringe plot of predicted major principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$). at a punch depth of 35.9 mm

Figure 4.14: Fringe plot of predicted minor principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 0 mm.

Figure 4.15: Fringe plot of predicted minor principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 19.0 mm.

Figure 4.16: Fringe plot of predicted minor principal Green's strain for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$), at a punch depth of 35.9 mm.

Figure 4.17: Plot of predicted major and minor principal Green's strain versus position relative to the pole of the dome ($\bar{r} = 2.21$ and $\mu = 0.18$) for the 203.2 mm wide AK1 blank.

Figure 4.18: Fringe plot of predicted thickness distribution for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 0 mm.

Figure 4.19: Fringe plot of predicted thickness distribution for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 19.0 mm.

Figure 4.20: Fringe plot of predicted thickness distribution for the 203.2 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 35.9 mm.

Figure 4.21: Plot of predicted thickness distribution versus position relative to the pole of the dome ($\bar{r} = 2.21$ and $\mu = 0.18$) for the 203.2 mm wide AK1 blank.

Figure 4.22: Plot of predicted major and minor principal Green's strain versus position relative to the pole showing the variation in the predicted strain distribution across the sheet.

Figure 4.23: Fringe plot of the predicted major principal Green's strain for the 127.0 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 37.1 mm.

Figure 4.24: Fringe plot of the predicted minor principal Green's strain for the 127.0 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 37.1 mm.

Figure 4.25: Fringe plot of predicted thickness distribution for the 127.0 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 37.1 mm.

Figure 4.26: Fringe plot of the predicted major principal Green's strain for the 50.8 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 38.0 mm.

Figure 4.27: Fringe plot of the predicted minor principal Green's strain for the 50.8 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 38.0 mm.

Figure 4.28: Fringe plot of predicted thickness distribution for the 50.8 mm wide blank of AK1 ($\bar{r} = 2.21$ and $\mu = 0.18$) at a punch depth of 38.0 mm.

Figure 4.29: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 203.2 mm wide AK1 blank (F denotes failed test specimen).

Figure 4.30: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 127.0 mm wide AK1 blank (F denotes failed test specimen).

Figure 4.31: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 50.8 mm wide AK1 blank (F denotes failed test specimen).

Figure 4.32: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 203.2 mm wide AK2 blank (F denotes failed test specimen).

Figure 4.33: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for 127.0 mm wide AK2 blank (F denotes failed test specimen).

Figure 4.34: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for 50.8 mm wide AK2 blank (F denotes failed test specimen).

Figure 4.35: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for the 203.2 mm wide AL blank (F denotes failed test specimen).

Figure 4.36: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for 127.0 mm wide AL blank (F denotes failed test specimen).

Figure 4.37: Comparison of the measured and predicted strain distributions for (a) an isotropic material and (b) a through-thickness anisotropic material according to Hill's quadratic yield criterion for 50.8 mm wide AL blank (F denotes failed test specimen).

Figure 4.38: Predicted effect of small variation in r on the strain distribution. AK1 material (a) 50.8 mm blank and (b) 127.0 mm blank sizes. FEM results are for punch depths of 38.0 mm and 37.1 mm for the 50.8 mm blank and 127.0 mm blank sizes respectively.

Figure 4.39: Predicted effect of large variations in r on the strain distribution of the 203.2 mm wide AK1 blank (FEM results are for a punch depth of 34.4 mm).

Figure 4.40: Predicted effect of variations in r on the material thickness distribution for the 203.2 mm wide blank of AK1 material.

Figure 4.41: Predicted effect of variations in μ for the 203.0 mm wide blank of AK1 material (FEM results for a punch depth of 34.4 mm).

Figure 4.42: Predicted effect of variations in μ for the 127.0 mm wide blank of AK1 material (FEM results are for a punch depth of 37.1 mm).

Figure 4.43: Predicted effect of variations in μ for the 50.8 mm wide blank of AK1 material (FEM results are for a punch depth of 38.0 mm).

END

'21-07-94

FIN