Interpolation of Experimental Error Statistics for Mission Design of GPS Partial Attitude Determination

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Abstract

Attitude determination is a critical component of many navigation applications, including missions utilizing autonomous as well as piloted platforms. While Global Navigation Satellite Systems (GNSS) can be used to indirectly compute attitude angles, the cost of precise receivers and designing the antenna separation for a GNSS attitude system can pose significant design challenges. There is a need for a direct means of choosing the receiver quality as well as the necessary antenna separation distance, particularly for partial attitude determination systems using only 2 receivers.

This research presents an exhaustive simulation method for designing a GNSS pitch and heading determination system. A simulated environment, validated experimentally with 2 GNSS antennas, offset from each other along a front to rear platform axis, is used in conjunction with a fixed satellite geometry in order to generate pseudorange and carrier phase measurements from satellites to receiver antennas. Antenna separation as well as receiver quality are varied, with error statistics collected and tabulated for each variation. Experiments with real receivers are conducted in a rural environment as well as an urban environment to validate positioning algorithms and simulations. Raw Global Positioning System (GPS) data is collected in Receiver Independent Exchange Format (RINEX), then processed in post. Low cost GNSS viability for partial attitude determination is discussed based on tabulated simulation data and experimental data, especially in cases where dual frequency receivers and the carrier phase observable are not available. A method of directly computing minimum antenna separation or minimum receiver quality using interpolation of collected simulation data is proposed.

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1 Introduction

1.1 Motivation

One major design consideration in mission planning for navigation is that it is highly desirable to minimize the budget for sensors while maximizing the robustness, accuracy and precision of the solutions that the sensors yield. One aspect of navigation is measurement of the platform position. Global Navigation Satellite Systems (GNSS) allow for determination of position in a global frame at any location on earth which has a line of sight to at least four satellites. Use of a second antenna on the same platform also allows the GNSS to double as a determiner of partial attitude. Specifically, a front to rear antenna setup allows for calculation of pitch and compass heading. In order to make a partial attitude determination system, an integrated Inertial Navigation System (INS) can be employed, which allows for the third attitude parameter, roll, to be computed along with the partial attitude. It also tends to offer higher precision attitude computation than a pseudorange-only GPS attitude solution with short antenna separation. Robustness of partial attitude determination can be increased by incorporating INS or using a high precision GPS system which incorporates the high precision carrier phase observables, or by increasing the separation between antennas. In the case of INS, it incurs an additional cost on top of positioning costs and it has a bias drift which increases over time due to the Inertial Measurement Unit (IMU) [1]. The use of high precision GPS requires a high grade of receiver, or a base station receiver at minimum so that the carrier phase can be utilized. Increasing antenna separation is only viable insofar as the geometry of the platform allows it.

Therefore, it is pertinent to quantify the viability of a low-cost GPS partial attitude / positioning system for navigation in order to aid in mission design.

1.2 Problem Statement

One difficulty in designing a GPS partial attitude determination system is that using high quality sensors incurs costs which will not meet some budgets. The use of a GPS base station or dual frequency receiver on the platform, for instance, allows for a variety of strategies for solving the integer ambiguity problem of the carrier phase , which allows for high precision positioning[2]. That translates to higher precision of pitch and heading computations using GPS antenna positions. However, this incurs additional costs. A real time kinematic (RTK) GPS system with dual frequency capability currently ranges in price from approximately \$3000 CDN to \$20,000 CDN. On the other hand, a single frequency, single point GPS system costs much less (approximately \$100 CDN for receiver boards with antennas, \$500 CDN for a system with a built-in interface). The main challenge of using the less expensive systems is that it is much more difficult to readily access the carrier phase. In fact, some systems such as the Huawei P20 do not support access to carrier phase measurement with raw data.

Therefore, it is important to determine what limitations a GPS partial attitude system may have when only pseudoranges are used. While it is well known that increasing antenna separation increases the precision of the pitch and heading computation, this effect has not been quantified in a thorough way to the author's knowledge. When designing a partial attitude system for navigation, it would be helpful to have tabulated statistics with varying antenna separation as well as varying pseudorange noise. This allows a designer to solve the problem of

how far 2 antennas must be separated to meet their specifications, or the reverse problem of what maximum receiver noise standard deviation is required for a limited fixed separation. This thesis is intended to address both problems.

1.3 Previous Work

Use of satellite positioning systems for attitude determination is not a new concept. A great deal of research has been done regarding all steps which need to be taken in the attitude determination process using GPS. This section will discuss this research in 3 categories: GNSS error mitigation, use of the carrier phase observable and attitude determination strategies. Error mitigation and use of the carrier phase are important because computed positions from GPS solutions are used directly to compute attitude, while review of attitude determination strategies is important in order to evaluate available options in a low-cost system.

1.3.1 GNSS Error Mitigation

Mitigation of GPS errors entails isolating and attempting to remove the effects of ranging errors including atmospheric delays, multipath delay, receiver and satellite clock errors, satellite orbital errors and receiver noise. One of the simplest errors to mitigate is atmospheric bias.

Atmospheric delays generally come from two sources: the ionosphere and the troposphere. The ionosphere refracts a transiting signal due to ionized particles, while the troposphere has a refractive effect due to temperature and pressure changes. The most readily available way to mitigate these errors for a single point solution is to use models for the atmosphere. In order to model the troposphere, one could rely on the model developed by J. Saastamoinen[3] or utilize the Hopfield model[4]. Both models rely on weather information including pressure and temperature in conjunction with estimated geometry between satellites

and the receiver antenna, which means that they can be utilized for any solution as long as weather parameters are available. A study by C. Satirapod and P. Chalermwattanachai[5] compared the effectiveness of these techniques and concluded that both methods yield very similar pseudorange positioning accuracy and that both yield optimal results for baseline positioning. Therefore, they recommended use of either method. Another study by Wielgosz et al.[6] compared the Hopfield model with more robust techniques which make use of reference networks, European Position Determination Systems (EUPOS) or precise point positioning based techniques. While these techniques were more effective in troposphere mitigation than the Hopfield model, the authors concluded that the Hopfield model still yielded results which were suitable for many positioning applications.

The other atmospheric delay for GPS pseudoranges is ionospheric delay. The most readily available method for modelling the ionosphere delay is the Total Electron Content (TEC) model outlined by the International GPS Service (IGS) Interface Control Document[7]. This is an important method because it only requires a measurement of the TEC in a particular geographic area at the time of acquisition, which can be acquired from a TEC map provided by various organizations. The method was evaluated by Jakowski et al.[8], who noted that the technique's ease of use and simplicity make it well-suited for use in near-real-time applications. Another method of ionosphere mitigation is using augmentation systems such as Wide Area Augmentation Systems (WAAS). This was examined by Paul Cannon[9], who suggested that while forecast based systems utilizing ground instruments could be improved by increasing the number of available ground stations for redundancy, these techniques simply will not be sufficient for some applications. A very effective way of mitigating ionosphere delay is by using dual frequency techniques, which use the dependency of ionosphere effects on signal frequency

to make a linear combination of measurements from two signals which eliminates bias. Some methods using multiple frequencies were evaluated by P. Bolla and K. Borre[10], who noted that while first and second order ionosphere effects can be mitigated by dual frequency techniques, such receivers can consume considerable battery power. It is also important to note that dual frequency receivers are generally more expensive than single frequency receivers, making them a poor option for the purposes of this thesis.

Another error source which can induce significant biases is multipath error. Most methods for minimizing the impacts of multipath require improvements to antenna and receiver technology or planning missions in multipath-free environments. One such method examined by Jones et al[11] uses a pulse aperture correlator with narrow lane characteristics. The authors noted that while choke-ring ground planes can be used to optimize antenna gain patterns and site selection can also reduce multipath as well as noise effects, these are not necessarily practical for kinematic applications. The additional bulk of this equipment makes it impractical for many moving platforms. They concluded that the narrow correlation function in the receiver causes rejection of multipath signals at more than 1 chip length while having negligible influence from multipath in the 0.1 to 0.9 chip length range. Further, G. Seepersad and S. Bisnath[12] illustrated that modelling multipath temporal variability in order to mitigate it in static applications can improve pseudorange precision and decrease convergence time in precise point positioning applications. This temporal variability is a characteristic of most GNSS errors, not only multipath. M. Olynik[13] studied these effects in detail, concluding that using precise orbits, precise clocks, dual frequency receivers and a troposphere correction yields the best possible solution in a 10-50 second time interval. The narrow correlator incurs additional cost and the temporal variability modelling is restricted to static applications, which means that they cannot

be used for this research, which will be dealing with low cost solutions which can be applied to kinematic applications. This suggests that noise and multipath would be expected as major factors to the effectiveness of a low-cost GPS attitude solution.

1.3.2 Carrier Phase Observables

One ranging error which is difficult to eliminate from code pseudoranges is random noise. The second option for range measurement is the carrier phase range measurement. This is by far the less noisy of the two measurements, so there is a great deal of motivation to use the carrier phase when high precision positioning is required. However, the integer ambiguity problem prevents use of the measurement without isolating the ambiguities first. In particular, the carrier phase precisely measures a fractional number of partial cycles of the carrier signal. While the effects from noise on this measurement are minimal, there is an integer number of complete cycles which separates this measurement from the actual range from the satellite to the receiver antenna phase center. This is illustrated in Figure 4, found in section 2.3.2.3. This problem has been a significant topic for research. Techniques for ambiguity resolution can generally be placed in three different categories: geometric searches, dual frequency solutions and triple differencing.

Geometric searches are currently the best available ambiguity resolution option for applications where the antenna is not stationary over time and dual frequency measurements are not available. Many authors have investigated the use of such techniques, such as G. Teunissen[14] who developed the Least-Squares Ambiguity Decorrelation Adjustment (LAMBDA) technique, which uses a transformation based on the estimated variance-covariance matrix of the unknown coordinates to transform and decorrelate the double difference float ambiguities. This technique has been adopted and studied by many authors as well, such as D. Cox and J. Brading[15], who applied the method to a Kalman filtered spacecraft navigation solution. They computed a Kalman filtered state vector and its corresponding variance-covariance matrix at each epoch which were used as input to the LAMBDA algorithm, finding

that the combination of a Kalman filter with LAMBDA for navigation yielded somewhat robust integer ambiguity selection success as well as reduced sensitivity to selection of incorrect integer combinations. The method was also used in Junjie Liu's thesis[16] on implementation and analysis of ambiguity resolution techniques in single and multiple reference station scenarios. Using the FLYKIN+ software package (which was developed using a C++ navigation lab library belonging to the University of Calgary Geomatics Engineering Department), he evaluated 8 different ambiguity resolution strategies, finding that double differenced ionosphere delay directly impacts the success of most strategies and that stochastic ionospheric modelling strategies generally performed better than ionosphere-free strategies in adverse ionosphere conditions.

While ionosphere-free solutions require use of a dual frequency receiver which generally imposes an increased cost to the GPS budget for a mission, they can nonetheless be very successful tools for ambiguity resolution. It has been analyzed in detail, such as the case with G. Blewitt[17], who applied the ionosphere free phase combination solution to baselines up to 2000km apart and based on results, hypothesized that bias optimization was superior to bias fixing of the ionosphere free solution over some sparse networks, but could not conclusively say so with the available data to the study. It is also worth noting that this technique was using more than a single receiver for positioning, which is an additional cost for the mission. On the other hand, Collins et al[18] used a decoupled clock model as an alternative to the differential solution requirement. The authors noted that the ionosphere-free model cannot explicitly solve for the ambiguities because the correlation of the phase and code biases with the clock biases interferes with the isolation of the ambiguity. However, it was noted that by decoupling the code and phase oscillator parameters, the ambiguity can be more easily isolated and ambiguities can be

successfully fixed to integer values. There is also an algorithm used by Bertiger et al[19] which works by first defining integer ambiguities in both L1 and L2 frequencies, then widelanes are computed and subsequently narrow lanes from a Kalman filter smoothed ionosphere free solution. The authors found that using IGS orbit products in conjunction with their modifications to the method, the precision in measured baselines could be significantly improved. Laurichesse et al[20] produced experimental results using undifferenced dual frequency measurements yielding positioning precision of less than 1cm, which does support use of both frequencies when available, even if a base receiver is not available.

Another method of resolving the integer ambiguity problem is using a triple difference of 2 double differences across 2 subsequent epochs. While this can be done by simply using the triple difference directly as an observable and assuming cancellation of the ambiguity term, this can be dangerous if cycle slip detection is not used such as the technique presented by Deng et al[21]. Some authors, such as L. Zhenkun and Huang Shunji[22] have done research on actually using the triple difference as an aid to solve for the ambiguity as opposed to just using the triple difference directly. In any case, the triple difference can only be utilized if the antenna coordinates remain constant over time, so that limitation makes it unsuitable for this thesis.

Other authors have done some useful evaluations of ambiguity resolution methods. D. Kim and R. Langley[23] noted that the focus research on ambiguity resolution has largely changed from computational efficiency to actual performance of various algorithms and asserted that many modern resolution techniques have sufficient computational efficiency for a wide variety of applications. Geng et al[24] compared fractional cycle bias determination with integer-recovery clock determination in the context of ambiguity-fixed position estimates, concluding that both approaches yield positioning precision and accuracy that are approximately equivalent

due to having identical design matrices. For the purposes of this thesis, triple differencing and dual frequency techniques will not be used due to the assumed stationary geometry and availability of higher cost equipment, respectively. For any carrier phase ranges, a simple linear geometric ambiguity search will be used with the same search criteria that LAMBDA uses. This will ensure equivalent selection accuracy in a small region and does not require additional complexity for the ambiguity selection, which is not the primary focus of this thesis in any case. To attempt to fix ambiguities in simulation, the ratio test proposed in [25] will be implemented, which compares the smallest sum square of residuals combination with the combination producing the second smallest sum square of residuals.

1.3.3 GPS for Pitch and Heading Determination

Because the use of GNSS for attitude determination is contingent on access to robust positioning solutions, much of the research on the concept involves some degree of hardware sophistication and multi-sensor data fusion. An example of this is the multifrequency fusion of the GPS system with the Chinese Beidou system by Nadarajah et al[26]. In their paper they demonstrated the benefit of adding additional satellites to the positioning solution, which reduces dilution of precision and improved the heading computations. They also recommended that the theoretical framework for such a system be expanded in the future. Yang et al[27] proposed a method of fusing low-cost inertial and magnetic field sensors with GNSS to compute attitude. After using their fusion algorithm to smooth double difference carrier phase-based attitude measurements, they noted a significant decrease in the standard error of their pitch and yaw calculations compared to using raw measurements alone, which is to be expected. Xu et al[28] fused GPS, GLONASS and BEIDOU constellation satellites at high data rates in 3 Trimble R9 receivers to measure the pitch, roll and yaw as applied to a deformation analysis and used an

IMU as a reference sensor to analyze the errors. They noted that, separately, the GPS solutions yielded the best results, followed by GLONASS and then Beidou having the noisiest solutions. They also interestingly noted that in some dynamic cases, the fusion of all 3 systems can perform slightly worse than the GPS only solution, suggesting that the other signals may not have been weighted optimally in their algorithm.

There has also been significant research on GPS-based attitude determination without high grade equipment being available. For example, Wang et al[29] developed a way of introducing constraints to the LAMBDA ambiguity resolution algorithm[14] in order to reduce ambiguity fixing time, including expansion of search space, as a way of using GPS to precisely compute attitudes in adverse error conditions. They found that introducing their constraints reduced the standard deviations of their pitch and heading computations to approximately one fifth that of the unconstrained solution. It is worth noting that a base station would still be required in order to obtain a solution sufficiently close to make the LAMBDA algorithm feasible with short ambiguity search periods. Chang et al[30], also utilizing a double difference measurement model, presented a vectorization approach to computing first estimates for the position and Euler angles in a GNSS attitude network. They showed that using this analytical approach can optimize the initial estimates for the iterative least squares estimation process, requiring fewer iterations while improving the precision of the solution. Sun et al[31] proposed a method of using GNSS multi-antenna baseline separations in conjunction with a Monte Carlo sampling strategy in order to establish a search space for the LAMBDA ambiguity resolution algorithm without the need of code pseudorange measurements to aid the algorithm. They found that, in a setup of 3 antennas with a baseline distance of approximately 0.5 m or less, the simultaneous computation of quaternion attitude as well as double difference ambiguities was

nearly 100 percent successful. This was especially true when 7 or more satellites were available, and computation times for their algorithm ranged between 0.6 s and 4.3 s.

To the author's knowledge, the most relevant research to that of this thesis is the work done by Jiun Han Keong[32] and Chaochao Wang[33]. Keong's thesis comprised the fusion of GPS with GLONASS signals in a non-dedicated, dual-antenna, external oscillator system with a single-difference measurement model for determining pitch and heading. It was determined that, while the code measurement noise from GLONASS was twice that of GPS, the carrier phase of each had approximately 1.1 mm standard deviation of random ranging error due to noise. Keong also noted that, for attitude determined using the carrier phase, multipath was a far more dominant error source than noise. It was also found that, while using the GLONASS system in conjunction with GPS increased attitude determination availability, but it decreased accuracy of the attitude solution due to the additional noise of the GLONASS signals. Keong recommended investigation of the effects of temperature on the error budget, expansion of the testing platform into additional environments such as urban canyons, testing with additional receiver types as well as using a double difference measurement model. The effects of temperature, additional receiver types and the double difference model are implicit in the simulations used in this thesis.

Wang's thesis focused on developing an attitude determination system at low cost. 3 CMC Allstar receivers were used in conjunction with 3 Murata rate gyros in order to measure the angular velocity of the platform. It was noted that, in absence of multipath, the low-cost system achieved similar accuracy to that of the high-performance Novatel OEM4 receivers integrated in a similar system. It was also asserted that the antenna had a significant effect on the attitude determination success due to multipath and its correlation with the antenna gain pattern. Interestingly, it was noted that ambiguity resolution success was severely degraded for the low-

cost system due to increased noise and susceptibility to cycle slip occurrences. The author recommended increased data rates to solve this problem. The gyros were found to add significant benefits to the system, especially due to the Kalman filter coasting ability for the state vector during GPS outage periods. Some notable recommendations that Wang made were integration of doppler measurements for use in cycle slip detection as well as investigating the effects of increased inter-antenna separation which would theoretically improve the attitude calculation performance. A thorough simulation analysis using varied inter-antenna separation distances is a major focus of this thesis.

1.4 Thesis Objectives

This work seeks to develop a method of solving two GPS partial attitude determination design problems. The method uses a collection of GPS error statistics with interpolation to solve for a minimum antenna separation at a given GPS receiver quality (specifically the noise characteristics), or to solve for maximum receiver noise specification given a specified GPS antenna separation length.

Experimental GPS control point surveys with a low-cost, wide correlator, L1-signal-only Emlid Reach M+ (hereon referred to as test receiver 1), a Huawei P20 cell phone with pseudorange only capability (hereon referred to as test receiver 2) and a VectorNav VN-300 wide correlator, L1 only integrated system (hereon referred to as test receiver 3) are to be conducted to validate positioning algorithms as well as to validate the partial attitude simulations.

The simulation results provide a potential design tool for navigation mission planning. Specifically, a designer could use the simulation results to design for a particular design criteria by fixing one of either the pseudorange noise or antenna separation, then interpolating the other parameter from the simulation results.

1.5 Contributions

The contributions of this work to the field of positioning and navigation are as follows:

- A novel partial attitude simulation method using a fixed satellite constellation availability with pseudorange noise and GNSS antenna separation as input parameters
- Experimental control point surveys to validate simulated GPS data
- Novel quantification of the effects of antenna separation and noise on pitch and heading computation.
- Proposal of a mission design technique using interpolation from simulated data

1.6 Organization

The remainder of this work is organized into the following parts:

- Part 2: Background on coordinate systems, GPS positioning and partial attitude
- Part 3: Methodology for least squares estimation, satellite orbit propagation, positioning methods, Kalman filtering and partial attitude computation
- Part 4: Simulation for GPS positioning, validation of positioning algorithms by experimental survey, and partial attitude simulation explanation
- Part 5: Partial attitude control point survey, discussion and comparison with simulated results
- Part 6: Concludes the research, discusses the results and recommends future work for expansion of the concept.

2 Background Information

2.1 Coordinate Systems

2.1.1 Earth-Centered, Earth-Fixed (ECEF) Coordinates

ECEF Coordinates use the center of the earth as the origin, with the Z axis being collinear with the earth rotation axis and positive being in the north direction. The X axis lies on the equatorial plane and is aligned with the prime meridian in Greenwich, UK. The Y axis completes a left-handed coordinate system, as illustrated in Figure 1:

Figure 1: Illustration of the Earth-Centered, Earth-Fixed (ECEF) Coordinate System

As shown in Figure 1, coordinates in this system are not referenced to the earth surface. This makes the system a good coordinate system to use when determining GPS receiver/antenna coordinates in terms of satellite coordinates and is therefore used in this research for GPS positioning computations.

2.1.2 Geographic Coordinates

Like ECEF coordinates, geographic coordinates use the earth's equator and prime meridian as references. However, geographic coordinates are polar in nature. The 3 coordinates associated are latitude, longitude and height, where latitude is an angle subtended by the point in question, the earth's center and the closest point on the equator which lies on the same meridian as the point in question. The height is collinear with the gravity vector and is measured using sea level as a reference datum. This sea level is modelled by an ellipsoid, as illustrated below:

Figure 2: Geographic Polar Coordinate References (Left), Geographic Height Above Sea Level (Right)

As can be observed in Figure 2, the reference ellipsoid does not use a constant mean sea level because this would approximate the earth as a sphere, which is not as accurate a description as an ellipsoid. This research will use the WGS-84 ellipsoid as a reference ellipsoid for all purposes.

2.1.3 Local Vertical Compass Reference Frames

A local vertical reference frame uses a nearby reference point as an origin and the Z axis is collinear with the gravity vector at the reference point and is referred to as the ‘vertical’ axis. The remaining 2 axis’ lie on the horizontal plane at the reference point, with a ‘north’ axis being tangent to the local meridian at the reference point, positive to the north, and an ‘east’ axis being tangent to the local parallel at the reference point, positive to the east. This is illustrated below:

Figure 3: Illustration of a Local Vertical Compass-Based Reference Frame

Local vertical frames such as the one illustrated in Figure 3 have a distinct advantage over projected coordinate systems such as Mercator projections in that they allow for a good navigation frame (from one point on the earth’s surface to the next) without suffering the same

scaling distortions that projections do. This research will use local vertical frames extensively for error analysis as well as navigation and mapping.

2.2 GNSS Operating Principles

Global Navigation Satellite Systems (GNSS) is a time-of-flight positioning system. A constellation of satellites is maintained in varying orbits and each satellite constantly transduces a signal which can be received via antenna and interpreted by a receiver at the antenna location.

The fundamental GNSS measurement is the time taken for the signal to travel from the satellite to the receiver antenna. The message in the satellite signal contains a time of emission, measured by a precise atomic clock in a specific reference time system. The receiver records the time of arrival for the signal in its own time system which is offset from the satellite time reference. Because all electromagnetic radiation travels at the speed of light, the time-of-flight can be used to calculate a range between the satellite and receiver[2]:

$$P_{range} = c\Delta t \quad (1)$$

Where c is the speed of light and Δt is the travel time of the signal. If the position of the satellite is known, then this measurement yields a sphere of possible locations of the receiver in the GNSS coordinate frame:

$$P_{range}^2 = (X_{satellite} - X_{receiver})^2 + (Y_{satellite} - Y_{receiver})^2 + (Z_{satellite} - Z_{receiver})^2 \quad (2)$$

If 3 of these ranges are simultaneously obtained from 3 different satellites with sufficiently optimized geometry relative to the receiver, then 3 spheres can be constructed. These

spheres will all intersect at two points in space, one of which is the location of the receiver and the other only represents a geometric constraint of the three spheres. With additional ranges, a unique solution can be computed and the confidence in that solution can be improved. The intersection is illustrated below:

Figure 4: Intersection of three spheres representing ranges from satellite to receiver antenna. A unique position solution exists when more than three satellites are in line of sight[34].

However, computing this position is made more complicated by the fact that the measured time-of-flight is subject to delays and noise which induce uncertainty to the measurement. In addition, there are errors associated with the satellite position and the antenna

setup by the user. The noise and biases must be modelled or avoided if an accurate and precise solution is to be computed.

2.3 GNSS Error Sources

2.3.1 Pseudorange Error

Generally, code pseudoranges are subject to errors in signal propagation as well as errors due to the receiver hardware quality. These include atmospheric errors, clock errors, multipath and noise.

2.3.1.1 Receiver and Satellite Clock Error

The satellite records a time of propagation while the receiver records a time of receiving the signal. However, the satellite and receiver have a non-zero error in recording these times. Part of this is measurement error, while part of it is manifest from the fact that the receiver and satellite each have their own offset from the system's reference time. This is due to imperfect synchronization of the clocks to the standard GPS reference time. Since both recorded times are used in the measurement of propagation time, it is useful to simply model this error as a single aggregate error due to the offsets and measurements in both instruments. The offset can be very significant. In fact, it is typically so large that it needs to be modelled as a separate unknown when computing a position[2].

2.3.1.2 Ionosphere delay

The Ionosphere extends from an elevation of approximately 50 km until greater than 1000 km. It exists due to ultraviolet radiation from the sun and contains ionized gas molecules. These ionized particles will delay a GNSS signal by refracting it, and the amount of refraction depends on the frequency of the signal. The total ranging bias due to delay by the ionosphere can

range from 1 m to about 100 m, making it the most significant delay caused by the earth's atmosphere[13].

Since this error is caused by solar radiation, it is subject to temporal characteristics. It varies according to an 11-year solar cycle as well as daily variation due to the earth's rotation.

The ionosphere delay is also subject to spatial variability. It is most significant at the earth's poles and the geomagnetic equator. It is also typically more significant during winter time. The relative location of the satellite is also significant, as the delay is about 3 times greater for a satellite near the horizon compared to one at zenith. This is because the signal has a greater travel distance through the atmosphere and the refraction angle is greater.

The delay can be compensated by modelling according to the total electron content (TEC) at the time of recording, or with techniques using dual-frequency receivers to remove first-order effects.

2.3.1.3 Troposphere Delay

The troposphere is the layer of the atmosphere closest to the earth's surface. For GNSS purposes, the troposphere is the non-dispersive part of the atmosphere, which is to say that the delay of a GNSS signal is not correlated with the frequency of the signal. This extends from the earth's surface to approximately 60 km elevation.

Like the ionosphere, the troposphere delays a GNSS signal by refracting it. However, the troposphere has a much smaller effect than the delay caused by the ionosphere. The delay typically ranges between 2.3 m (at zenith) and 9.3 m at a 75 degree zenith angle[13].

Simple models which do not require dual frequency data can be used to estimate the delay and compensate it to within about 20 cm of the actual effect. Therefore This delay is not a

problem for approximate positioning, though it can pose an obstacle to precise positioning applications, particularly when estimating height.

The delay is divided into a dry (hydrostatic) and wet component. The wet component is more difficult to model than the dry component. One way to mitigate the troposphere delay is via differential positioning with short baselines at similar heights.

There is some temporal variability in the troposphere delay, particularly in areas of high humidity and unstable weather.

2.3.1.4 Multipath Error

Multipath delay occurs when a GNSS signal reflects off of a surface before reaching the antenna, hence delaying the signal by the additional propagation time. GNSS multipath is particularly prone to occur in environments where there are smooth surfaces for reflection. In particular, smooth metallic surfaces, polished glass or water are all conducive to multipath effects[35].

GNSS receivers are designed with small chip length indicating higher noise and multipath mitigation properties, usually increasing the cost of the receiver. The delay from multipath can reach magnitudes of about half of a code chip length, which for the C/A code of a wide correlator receiver is 150 m. This is very large, so it can be a limiting factor in positioning, especially for C/A code only, wide correlator receivers.

Multipath has a very high spatial variability, and it decorrelates very rapidly when the position changes. Two identical receiver/antenna combinations recording at the same time will yield very different results in an urban skyscraper environment near polished windows compared to an open sky environment with no reflective surfaces nearby.

Multipath will also have temporal variability for a static position. Specifically, it will vary periodically according to the satellite orbital period. Unlike atmospheric errors, multipath is not generally mitigated by differential positioning. In fact, using differential techniques can amplify the effects.

2.3.1.5 Code Noise

In addition to biases due to signal delays during propagation, the pseudorange is also subject to random, gaussian distributed noise. This noise is a receiver-based error, not an atmospheric or environmental delay.

In general, higher quality receivers tend to have more options for limiting noise. The lowest quality receivers are generally wide-correlator, single-frequency receivers which can only access the C/A code, not the P code.

The C/A code has significantly higher noise characteristics than the P code, so it is generally desirable to have access to the P code. That is generally a determining factor for the price of a receiver.

The noise on a receiver can be as large as about 3 m for low-quality receivers, down to the dm level for high-quality receivers[12].

Random noise is difficult to compensate with a model, but it can be modelled by connecting 2 receivers to the same antenna and recording data, since that allows for isolation of all other error sources. Once that is modelled, a standard deviation of the noise can be used in stochastic models for least squares adjustments.

2.3.2 Carrier Phase Error

2.3.2.1 Multipath

Like the pseudorange, the carrier phase is also subject to multipath delays. However, due to the fact that the carrier phase measures fractional cycles rather than an entire range, the multipath delay is limited to a quarter wavelength in principle (though it is typically no more than a tenth of a wavelength in practice). Because this delay is much smaller than multipath effects on pseudoranges, this gives the carrier phase a significant advantage in precision positioning applications when multipath conditions are not favorable[24].

2.3.2.2 Noise

The carrier phase is also subject to gaussian random receiver noise. However, this effect is very small for the carrier phase (less than 1 mm typically)[36]. This makes the carrier phase essential for high-precision applications.

2.3.2.3 Integer Ambiguity

The carrier phase measurement is a measurement of the phase angle of the carrier signal on time of receipt. This angle corresponds to the fraction of a full cycle of the signal from reference to point and time of receipt. This is illustrated in the figure below:

CARRIER-PHASE MEASUREMENTS

Figure 5: Explanation of the GNSS carrier phase measurement and integer ambiguity offset[37]

Because what the carrier phase measurement yields is a fractional number of cycles plus an arbitrary integer, there is an unknown remaining distance between the signal and satellite constituting an ambiguous integer number of cycles:

Figure 6: Illustration of Carrier Phase Ambiguity

It can be observed from the above figure that, as long as no cycle slips occur, the integer ambiguity will remain constant over time despite movement of satellites and the antenna. This is accounted for by the integer portion of the measurement itself. Therefore, the ambiguity can be fixed at one value and then used in the carrier phase measurement model to construct a precise pseudorange[23].

However, resolving the ambiguity is not simple, and in fact most techniques for ambiguity resolution require triple-differences or dual frequency capabilities for the receiver. This study will employ a technique which does not require triple-differencing or dual frequency observations in order to eliminate the need for additional costs and equipment. Namely, it will utilize an integer search-based technique.

2.3.3 Orbital Errors

Orbital errors are not errors which affect the range measurement, but instead they refer to the fact that at any given time, a satellite's position cannot be known with certainty. Along with the observations, a satellite also broadcasts orbital characteristics in the navigation message. However, there are two reasons that this does not perfectly indicate a satellite's motion at the time of signal transduction.

First, the orbital elements broadcast in the navigation message are not perfectly accurate. They are observed with limited precision.

Second, the broadcast orbits are only transmitted for a single time before logging of observations began. Therefore, they must be propagated in order to obtain orbital elements at the time of signal transduction. There is a loss of precision associated with the nonlinear propagation algorithm used to do this which depends on the step size as well as how many perturbations are accounted for in the computation.

One way to mitigate this error is to use precise International GPS Service (IGS) orbits[7] instead of the broadcast orbits, though this can only be done in post-processing and not real-time applications since the IGS orbits are only updated periodically.

2.3.4 Antenna Offset Error

This error refers to the antenna being set up in a position other than the one which is recorded. This is usually caused by either human error, non-GNSS related measurement errors or physical limitations of the setup itself.

For instance, when using a GNSS to measure the position of a point where coordinates are already known, it might be the case that the antenna center was not placed vertically on top of the point with sufficient precision, which would manifest in an apparent horizontal error bias relative to the known position. This is independent of any error in the sensor itself and is caused by the user.

The vertical offset of the antenna relative to the point in question may also be difficult to measure due to the need of levels to ensure vertical measurement as well as errors inherent to the measuring tape or laser used to measure the distance. Errors in this will manifest in an apparent vertical error bias in the solution. This is caused by a combination of lack of user skill and limitation of the levelling and distance measurement instruments used.

This error is especially important to mitigate for when a control point is being used as a base station for differential positioning. The technique works on the assumption that the antenna position of the base is known, so deviations from the correct setup will adversely affect the position solution for the rover antenna.

It is also very important to avoid setup errors for any GNSS antennas that are being used to compute attitude. The attitude of the vehicle is aligned to driving, upward and lateral directions, so deviations from those vectors in setup will induce a bias to the angles computed.

2.4 Data Formats

In application and validation of the various techniques used in this thesis, data was collected using multiple instruments including GPS receivers, and an Inertial Measurement Unit. In order to fuse data from each of these sensors, the raw data must be collected and then processed separately. This section will discuss the form that this data takes as well as what processing needs to be done before the data can be used.

2.4.1 GNSS Data

Raw GNSS data can take a number of different forms, but for the purposes of this thesis, the Receiver Independent Exchange Format (RINEX) will be used, precisely because the receiver used to collect data will not affect a software's ability to read and store raw data (IGS 2015). The data comes in 2 separate files: The navigation message and the observation files containing epoch-by-epoch measurements from each visible satellite.

2.4.1.1 GPS RINEX Navigation Message

The navigation message contains information about satellite orbits for discrete timestamps which are not concurrent with the observation file timestamps. An example of a GPS navigation message file is depicted in figure 5:

```
1 2.11 N: GPS NAV DATA RINEX VERSION / TYPE
2 teqc 2018Dec12 UNAVCO Archive Ops 20190105 00:02:29UTC PGM / RUN BY / DATE
3 Solaris x86 5.10|AMD64|cc SC5.8 -xarch=amd64|+=|+= COMMENT
4 7.4506D-09 -1.4901D-08 -5.9605D-08 1.1921D-07 ION ALPHA
5 9.0112D+04 -6.5536D+04 -1.3107D+05 4.5875D+05 ION BETA
6 1.862645149231D-09 1.776356839400D-15 61440 2035 DELTA-UTC: A0,A1,T,W
7 END OF HEADER
8 11 19 1 4 0 0 0.0-6.801565177739D-04 5.002220859751D-12 0.000000000000D+00
9 7.700000000000D+01-1.265000000000D+02 5.358080328410D-09-2.533745914082D+00
10 -6.373971700668D-06 1.662355300505D-02 6.578862667084D-06 5.153682119370D+03
11 4.320000000000D+05 1.657754182816D-07-7.913106360528D-01 1.583248376846D-07
12 9.080422048598D-01 2.212187500000D+02 1.852558703969D+00-8.229985669080D-09
13 -2.271523189487D-10 1.000000000000D+00 2.034000000000D+03 0.000000000000D+00
14 2.000000000000D+00 0.000000000000D+00-1.210719347000D-08 7.700000000000D+01
15 4.305120000000D+05 4.000000000000D+00
16 3 19 1 4 0 0 0.0 1.747305504978D-04 2.728484105319D-12 0.000000000000D+00
17 9.500000000000D+01 1.809375000000D+01 4.524117019062D-09-2.945343849279D+00
18 9.834766387939D-07 1.873593893833D-03 7.649883627892D-06 5.153602052689D+03
19 4.320000000000D+05 0.000000000000D+00 6.922204607321D-01 3.539025783539D-08
20 9.621591858150D-01 2.323750000000D+02 6.925635632980D-01-7.899614764949D-09
21 5.896674191577D-10 1.000000000000D+00 2.034000000000D+03 0.000000000000D+00
22 2.000000000000D+00 0.000000000000D+00 2.328306436539D-09 9.500000000000D+01
23 4.248180000000D+05 4.000000000000D+00
24 7 19 1 4 0 0 0.0 5.830219015479D-05-7.048583938740D-12 0.000000000000D+00
25 2.400000000000D+01 6.325000000000D+01 4.336966366343D-09-2.486825422146D+00
26 3.015622496605D-06 1.212974800728D-02 1.119449734688D-05 5.153626543045D+03
27 4.320000000000D+05 1.210719347000D-07 2.802020430872D+00-2.086162567139D-07
28 9.568857582013D-01 1.619687500000D+02-2.479351672115D+00-7.646747089138D-09
29 -2.146517982518D-10 1.000000000000D+00 2.034000000000D+03 0.000000000000D+00
30 2.000000000000D+00 0.000000000000D+00-1.117587089539D-08 2.400000000000D+01
31 4.248180000000D+05 4.000000000000D+00
32 28 19 1 4 0 0 0.0 7.608016021550D-04 1.136868377216D-12 0.000000000000D+00
33 6.100000000000D+01 1.035312500000D+02 4.262320399896D-09 1.184539330669D+00
34 4.997476935387D-06 1.952434494160D-02 3.937631845474D-06 5.153696651459D+03
35 4.320000000000D+05-1.899898052216D-07-2.361480967035D+00-4.637986421585D-07
```

Figure 7: RINEX Navigation Message Example

In general, this file will begin with a header, and the header always finishes with the phrase “END OF HEADER”. Beyond this, the file begins to list navigation information for each satellite at a given time. It gives values in this order[7]:

Table 1: Information Provided in RINEX GPS Navigation Messages

Value	Units
Satellite PRN Number	
Time of Clock	
Year (last 2 digits)	
Month	1 through 12
Day	
Hour	Hours
Minute	Minutes
Second	Seconds
SV Clock Bias	Seconds
SV Clock Drift	$\frac{\text{Seconds}}{\text{second}}$
SV Clock Drift Rate	$\frac{\text{Seconds}}{\text{Second}^2}$
IODC Issue of Data, Ephemeris	
Crs	meters
Δn	$\frac{\text{radians}}{\text{second}}$
M_0	radians
Cuc	radians
Eccentricity e	
Cus	radians
$\sqrt{A}$	$\sqrt{\text{m}}$
Time of Ephemeris	Seconds of GPS week
Cic	radians
Ω	radians
CIS	radians
i_0	radians
Crc	m
ω	radians
$\dot{\Omega}$	$\frac{\text{radians}}{\text{second}}$
$\dot{i}$	$\frac{\text{radians}}{\text{second}}$
Codes on L2 Channel	
GPS Week Number	
L2 P Data Flag	
SV Accuracy	m
SV Health	
TGD	Seconds
Issue of Data, Clock	
Transmission Time of Message	Seconds of GPS Week
Fit Interval	Hours

The navigation message entries will always come in the same order, so reading of the data files can be written assuming that. Since the classical orbital elements are given at discrete times, they cannot be used directly to calculate satellite positions at time of observation. Instead they must be propagated.

2.4.1.2 GNSS RINEX Observation file

The mixed RINEX observation file contains all of the logged raw data from every visible satellite, from every satellite system that the receiver is capable of transponding signals from, at each epoch. An example of a RINEX mixed observation file is depicted below:

```

1      3.03      OBSERVATION DATA      M: Mixed      RINEX VERSION / TYPE
2      RTKCONV 2.4.3 Emlid      20180820 043054 UTC      PGM / RUN BY / DATE
3      log: C:\Users\Mike\Documents\School\MASc\Thesis\GNSS\raw_2      COMMENT
4      format: RINEX      COMMENT
5
6      MARKER NAME
7      MARKER NUMBER
8      MARKER TYPE
9      OBSERVER / AGENCY
10     REC # / TYPE / VERS
11     ANT # / TYPE
12     APPROX POSITION XYZ
13     ANTENNA: DELTA H/E/N
14     G 31 C1C L1C D1C S1C C1P L1P D1P S1P C1W L1W D1W S1W C1Y      SYS / # / OBS TYPES
15     L1Y D1Y S1Y C1M L1M D1M S1M L1N D1N S1N C1S L1S D1S      SYS / # / OBS TYPES
16     S1S C1L L1L D1L S1L      SYS / # / OBS TYPES
17     R 8 C1C L1C D1C S1C C1P L1P D1P S1P      SYS / # / OBS TYPES
18     E 20 C1C L1C D1C S1C C1A L1A D1A S1A C1B L1B D1B S1B C1X      SYS / # / OBS TYPES
19     L1X D1X S1X C1Z L1Z D1Z S1Z      SYS / # / OBS TYPES
20     J 20 C1C L1C D1C S1C C1S L1S D1S S1S C1L L1L D1L S1L C1X      SYS / # / OBS TYPES
21     L1X D1X S1X C1Z L1Z D1Z S1Z      SYS / # / OBS TYPES
22     S 4 C1C L1C D1C S1C      SYS / # / OBS TYPES
23     C 12 C1I L1I D1I S1I C1Q L1Q D1Q S1Q C1X L1X D1X S1X      SYS / # / OBS TYPES
24     2018 8 15 18 0 20.5940000 GPS      TIME OF FIRST OBS
25     2018 8 15 18 6 34.9940000 GPS      TIME OF LAST OBS
26     G      SYS / PHASE SHIFT
27     R      SYS / PHASE SHIFT
28     E      SYS / PHASE SHIFT
29     J      SYS / PHASE SHIFT
30     S      SYS / PHASE SHIFT
31     C      SYS / PHASE SHIFT
32     0      GLONASS SLOT / FRQ #
33     C1C 0.000 C1P 0.000 C2C 0.000 C2P 0.000      GLONASS COD/PHS/BIS
34     > 2018 8 15 18 0 20.5940000 0 14      END OF HEADER
35     G 6 18710782.363 1 98325802.387 1 -1564.943 44.000
36     G19 19250097.222 1 101159959.459 1 -2412.008 42.000
37     G17 20913268.314 1 109900037.44311 -3048.196 43.000

```

Figure 8: A Logged RINEX Observation File

Just as is the case for navigation message files, a RINEX observation file will always begin with a header which is terminated by the phrase “END OF HEADER”. After this, the

observation records begin. Each record begins with a line of values in the following order for the associated epoch[7]:

Table 2: GNSS RINEX Mixed Observation Record Epoch Information

Value	Units
Year	
Month	01-12
Day	01-31
Hour	00-23
Minute	00-59
Second	00-59
Event Flag	0 or 1
Number of Satellites	Total from all systems

After each epoch record, the measurements for each satellite are listed. The format of each individual satellite observation record is dependent on what system the satellite belongs to, and each system's format is contained in the header of the file.

3 Methodology

3.1 GNSS Pseudorange and Carrier Phase Corrections

3.1.1 Pseudorange Corrections

Because delays to GNSS pseudoranges can be quite substantial, the raw pseudoranges from logged RINEX files must be corrected before they can be used in a least squares adjustment.

3.1.1.1 Troposphere Correction

To correct for the troposphere delay, the Saastamoinen model [3] was implemented. Based on literature, this model tends to yield the greatest baseline accuracy possible and is statistically comparable to the Hopfield model. The Saastamoinen model computes the troposphere correction as follows:

$$\delta_{trop} = \frac{0.002277}{\cos(z)} \left(P + \left(\frac{1255}{T} + 0.05 \right) pp - B \tan^2(z) \right) + \delta R \quad (3)$$

$$\text{where } pp = R_h e^{-37.2465 + 0.213166T - 0.000256908T^2}$$

$$T(K) = T(\text{Celsius}) + 273.16$$

$$P = P_0 \left(1 - 0.000226 (H - H_0) \right)^{5.225}$$

$$R_h = \rho e^{-0.0006396 (H - H_0)}$$

Where z is the zenith angle of the satellite and δR is a parameter depending on z which is interpolated from an array of calibrated values documented in [3]. The model requires some input parameters, so the following parameters were used, including some standard atmospheric values:

Table 3: Saastamoinen Model Input Values

Variable	Value
Reference Elevation H_0 (m)	0
Reference Pressure P_0 (mbar)	1013.25
Temperature T (K)	265
Reference Humidity R_h	0.5

3.1.1.2 Ionosphere Correction

To correct for the ionosphere delay, the International GPS Service (IGS) ionosphere model will be used. This is because dual frequency methods are not applicable to this study since only single frequency receivers are being utilized. The IGS model uses a mapped value of the Total Electron Content in the ionosphere on a regional basis. In order to compute an ionosphere correction, a mapping function must first be computed based on the satellite geometry:

$$F_{map} = \frac{1}{\sqrt{1 - \left(\frac{6371}{6371 - H_{shell}} \cos(\alpha_{Elevation}) \right)^2}} \quad (4)$$

Where H_{shell} is an estimate of the outer shell height of the ionosphere and $\alpha_{Elevation}$ is the elevation angle of the satellite for which the pseudorange is being corrected. With the mapping function computed, now the ionosphere correction can be computed:

$$\delta_{iono} = F_{map}(TEC)(0.163) \quad (5)$$

Where 0.163 is a constant calibrated to the L1 signal and TEC is a value for the total electron content in the region. Typically, this can be obtained from freely available maps which are updated frequently, such as the following from Trimble GNSS:

Figure 9: TEC Map from Trimble GNSS

Reading from these maps allows for interpolation by eye in order to acquire an approximate value of the TEC content for the data collection period.

3.1.1.3 Relativistic Effects Correction

For all GPS related purposes an earth centered earth fixed (ECEF) inertial coordinate system at rest is assumed during computations. This necessitates taking into consideration relativistic effects such as path range effects and the Earth's rotation itself. Path range effects have a maximum effect on the pseudoranges of only about 2 cm. For a wide correlator receiver this is much smaller than the receiver noise and has little chance of affecting ambiguity resolution, so it is not compensated for. The effects of the earth's rotation, however, can be up to

30 m which would have a substantial effect on positioning solutions. It will therefore be corrected as follows:

$$\delta p = \frac{(r_r - r_s)(\vec{v}_r)}{c} \quad (6)$$

$$\vec{v}_r = \omega_e r_r + \vec{v}_k \quad (7)$$

Where δp is the position correction due to the earth's rotation, r_r is the receiver position vector, r_s is the satellite position vector, ω_e is the rotational velocity of the earth and $\vec{v}_k$ is the velocity of the receiver relative to the earth surface[2].

3.1.2 Carrier Phase Ambiguity Resolution

Carrier phase measurements cannot be used without first resolving their ambiguities. Techniques which are frequently used involve dual frequency or triple-differences as the observation model. However, this is not viable if only one frequency is available, and triple-differences are less viable for mobile applications than static ones, so neither of those concepts will be used. Instead, a vector-based search technique will be implemented.

First, a solution must be computed using pseudoranges. Refer to section 2.2 for pseudorange positioning techniques. Since this solution represents the current best estimate of the antenna position, a best estimate for each 3D geometric range from satellite to receiver can be calculated:

$$\begin{aligned} range_{geo} \\ = \sqrt{(X_{est} - X_{satellite})^2 + (Y_{est} - Y_{satellite})^2 + (Z_{est} - Z_{satellite})^2} \end{aligned} \quad (8)$$

Next, this range can be used to calculate a float ambiguity:

$$N_{float} = \frac{range_{geo} - \phi_{cycles} * \lambda_{L1}}{\lambda_{L1}} \quad (9)$$

Where ϕ_{cycles} is the fractional number of cycles measured and λ_{L1} is the wavelength of the L1 signal. Since it is known that the ambiguity must be an integer, this float ambiguity is then rounded. Now, a search area needs to be defined. In order to be thorough and maximize the chances of finding the correct integer for each satellite, the search will be done by adding an integer to the float ambiguity and then testing it.

The number of integers to test will determine the required processing time as well as the likelihood of finding the correct integer. For the purposes of this study, the integers -2, -1, 0, 1 and 2 are tested. Since there are 5 integers to be tested and m satellites contributing measurements, there are 5^m possible combinations of integer ambiguities to be tested. In order to organize these combinations, a $5^m \times m$ matrix of combinations is built:

$$Combinations_{5^m \times m} \begin{bmatrix} -2 & -2 & \dots & \dots & -2 & -2 \\ -2 & -2 & \dots & \dots & -2 & -1 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 2 & 2 & \dots & \dots & 2 & 1 \\ 2 & 2 & \dots & \dots & 2 & 2 \end{bmatrix} \quad (10)$$

Next, every combination is tested, one at a time, in a least squares adjustment. The integer choice is used with the float ambiguity to construct a carrier phase-based range measurement:

$$\Phi_{measured} = \left(N_{float(rounded)} + N_{tested} + \phi_{cycles} \right) \lambda_{L1} \quad (11)$$

Where $\Phi_{measured}$ is the measured range using the total number of cycles calculated and N_{tested} is the tested integer taken from the matrix of cycle combinations. These measured ranges are used to build the least squares measurement vector:

$$l_{observed_{mx1}} = \begin{bmatrix} \Phi_{measured_1} \\ \Phi_{measured_2} \\ \dots \\ \Phi_{measured_m} \end{bmatrix} \quad (12)$$

This is used to compute $\hat{\delta}$ in the same process used in section 2.2. This can be done using the single point model or the differential model. Afterward, the vector of residuals $\hat{v}$ is computed using the stochastic model techniques from section 2.3. One way to select the optimal combination of integer ambiguities is to choose the combination yielding the smallest sum square of the residuals:

$$Total = \Sigma v_i^2 \quad (13)$$

Once the optimal combination is chosen, a full iterative least squares adjustment can be done using the measurements derived from the optimal combination of integer ambiguities.

It is important to note that if the solution computed from the pseudoranges is not sufficiently close to the true coordinates of the antenna, the true geometric range will not fall within the search region and the ambiguity selected will be biased by a minimum of one full wavelength. This is especially important when using low cost, wide correlator receivers with single frequency and access only to the P code, since achieving high accuracy is much less likely with those receivers than with high quality equipment.

3.2 Satellite Orbit Propagation

In order to use a GNSS satellite position at any given epoch, an ephemeral set of classical orbital elements at a previous time must be propagated to the time of observation. These orbital elements must be acquired from the navigation message for real time use, while more precise

orbits can be obtained from IGS products later on for post-processing. Some navigation files contain sets of orbital elements at multiple different times for the same satellite, giving an option for which record to use for propagation. Since this is a nonlinear process, there is loss of precision in propagation when the step size for propagation is not infinitely small. For this reason, the ephemeris set which is temporally closest to the observation times is used. First, the difference in time between the observation epoch and the ephemeris record is calculated:

$$\Delta t_{eph} = t_{obs} - t_{eph} \quad (14)$$

Where t_{obs} is the time of observation in seconds and t_{eph} is the time of ephemeris in seconds. Next, the rate of change of the mean anomaly is computed:

$$\dot{MA} = \sqrt{\frac{\mu_{earth}}{a_{semimajor}^3}} + \Delta n \quad (15)$$

Where μ_{earth} is the gravitational constant (3.986005×10^{14}), $a_{semimajor}$ is the semimajor axis from the navigation message and Δn is the difference in mean motion. Now the mean anomaly at the time of observation can be calculated:

$$MA = MA_0 + \dot{MA}\Delta t_{eph} \quad (16)$$

Where MA_0 is the initial mean anomaly from the navigation message. Next, the eccentric anomaly is set equal to the mean anomaly. Then, an adjustment to the eccentric anomaly is calculated iteratively:

$$d_{1temp} = 1 - e_{eccentricity} \cos(EA) \quad (17)$$

$$\delta EA = \frac{MA - EA + e_{eccentricity} \sin(EA)}{d_{1temp}} \quad (18)$$

$$EA_{new} = EA + \delta EA \quad (19)$$

Where $e_{eccentricity}$ is the orbital eccentricity from the navigation message and EA is the eccentric anomaly. The new eccentric anomaly is used again for processes 16-18 and the cycle is repeated until the value of δEA falls below a desired threshold. Now the rate of change of eccentric anomaly and an intermediate value are calculated:

$$\dot{EA} = \frac{\dot{MA}}{d_{1temp}} \quad (20)$$

$$E_{ein} = -4.442807633 \times 10^{-10} e_{eccentricity} \sqrt{a_{semimajor}} \sin(EA) \quad (21)$$

Next, a satellite clock correction is calculated:

$$\Delta cor_{satclock} = sv_{clockbias} + \Delta t_{eph}(sv_{clockdrift} + \Delta t_{eph}sv_{clockdriftrate}) + E_{ein} - TGD \quad (22)$$

Where $sv_{clockbias}$, $sv_{clockdrift}$, $sv_{clockdriftrate}$ and TGD (inter-signal correction term) are all taken from the navigation message. Now the satellite clock drift can be calculated:

$$clockdrift = sv_{clockdrift} + 2\Delta t_{eph}sv_{clockdriftrate} \quad (23)$$

Next, the argument of latitude and its rate of change are computed:

$$a_{lat} = \tan^{-1} \frac{(1 - e_{eccentricity}^2) \sin(EA)}{\cos(EA) - e_{eccentricity}} + \omega \quad (24)$$

$$\dot{a}_{lat} = \frac{(1 - e_{eccentricity}^2) \dot{EA}}{d_{1temp}} \quad (25)$$

Where ω is the argument of perigee from the navigation message. Now corrections to these 2 terms are computed:

$$a_{lat_{corr}} = a_{lat} + C_{harm_{a_{lat}s}} \sin(2a_{lat}) + C_{harm_{a_{lat}c}} \cos(2a_{lat}) \quad (26)$$

$$\dot{a}_{lat_{corr}} = \dot{a}_{lat} \left(1 + 2(C_{harm_{a_{lat}^c}} \cos(2a_{lat}) - C_{harm_{a_{lat}^s}} \sin(2a_{lat})) \right) \quad (27)$$

Where $C_{harm_{a_{lat}^c}}$ is the amplitude of the cosine harmonic term, taken from the navigation message, $C_{harm_{a_{lat}^s}}$ is the amplitude of the sine harmonic term, taken from the navigation message, and $a_{lat_{corr}}$ and $\dot{a}_{lat_{corr}}$ are the corrected values for the argument of latitude and its derivative, respectively. Next the orbital radius and its derivative are calculated:

$$R_{orbit} = a_{semimajor} d_{1temp} + C_{harm_{R_{orbit}^c}} \cos(2a_{lat}) + C_{harm_{R_{orbit}^s}} \sin(2a_{lat}) \quad (28)$$

$$\dot{R}_{orbit} = a_{semimajor} e_{eccentricity} \sin(EA) \dot{EA} + 2\dot{a}_{lat} (C_{harm_{R_{orbit}^c}} \cos(2a_{lat}) - C_{harm_{R_{orbit}^s}} \sin(2a_{lat})) \quad (29)$$

Where $C_{harm_{R_{orbit}^c}}$ is the orbital radius cosine term amplitude and $C_{harm_{R_{orbit}^s}}$ is the orbital radius sine term amplitude. Now the satellite orbital inclination and its derivative are computed:

$$inc_{orbit} = inc_0 + \dot{I} \Delta t_{eph} + C_{harm_{inc_{orbit}^c}} \cos(2a_{lat}) + C_{harm_{inc_{orbit}^s}} \sin(2a_{lat}) \quad (30)$$

$$\dot{inc}_{orbit} = \dot{I} + 2\dot{a}_{lat} (C_{harm_{inc_{orbit}^c}} \cos(2a_{lat}) - C_{harm_{inc_{orbit}^s}} \sin(2a_{lat})) \quad (31)$$

Where $C_{harm_{inc_{orbit}^c}}$ is the orbital radius cosine term amplitude and $C_{harm_{inc_{orbit}^s}}$ is the orbital radius sine term amplitude. Now the 2D position of the satellite as it lies in the orbital plane, as well as its partial rates of change, are calculated:

$$x_{orbit_{plane}} = R_{orbit} \cos(a_{lat_{corr}}) \quad (32)$$

$$y_{orbit_{plane}} = R_{orbit} \sin(a_{lat_{corr}}) \quad (33)$$

$$\dot{x}_{orbit_plane} = \dot{R}_{orbit} \cos(a_{lat_corr}) - y_{orbit_plane} \dot{a}_{lat_corr} \quad (34)$$

$$\dot{y}_{orbit_plane} = \dot{R}_{orbit} \sin(a_{lat_corr}) + x_{orbit_plane} \dot{a}_{lat_corr} \quad (35)$$

These coordinates and rates are only with respect to the orbital plane, so they will need to be transformed in order to obtain 3D coordinates in the earth-centered, earth-fixed frame. First, right ascension of the ascending node and its rate of change, adjusted for the earth rotation rate, need to be computed:

$$\dot{\omega}_{adj} = \dot{\Omega}_{long} - \omega_{earth} \quad (36)$$

$$\omega_{adj} = \Omega_{long} + \Delta t_{eph} \dot{\omega}_{adj} - \omega_{earth} t_{eph} \quad (37)$$

Where Ω_{long} and $\dot{\Omega}_{long}$ are the longitude of the ascending node and its derivative, respectively and ω_{earth} is the earth rotation rate. Now the satellite's 3D coordinates can be calculated:

$$X_{sat} = x_{orbit_plane} \cos(\omega_{adj}) - y_{orbit_plane} \cos(inc_{orbit}) \sin(\omega_{adj}) \quad (38)$$

$$Y_{sat} = x_{orbit_plane} \sin(\omega_{adj}) - y_{orbit_plane} \cos(inc_{orbit}) \cos(\omega_{adj}) \quad (39)$$

$$Z_{sat} = y_{orbit_plane} \sin(inc_{orbit}) \quad (40)$$

Finally, an intermediate value as well as the satellite velocities can be computed:

$$d_{2temp} = \dot{y}_{orbit_plane} \cos(inc_{orbit}) - \dot{y}_{orbit_plane} \sin(inc_{orbit}) \dot{inc}_{orbit} \quad (41)$$

$$\dot{X}_{sat} = -\dot{\omega}_{adj} Y_{sat} + \dot{x}_{orbit_plane} \cos(\omega_{adj}) - d_{2temp} \sin(\omega_{adj}) \quad (42)$$

$$\dot{Y}_{sat} = \dot{\omega}_{adj} X_{sat} + \dot{x}_{orbit_plane} \sin(\omega_{adj}) + d_{2temp} \cos(\omega_{adj}) \quad (43)$$

$$\dot{Z}_{sat} = y_{orbit_plane} \cos(inc_{orbit}) \dot{inc}_{orbit} + \dot{y}_{orbit_plane} \sin(inc_{orbit}) \quad (44)$$

Note that the coordinates and velocities for each satellite obtained from this process will be less accurate than the IGS produced values which can be obtained after all raw data is collected.

3.3 Parametric Least Squares Adjustments

3.3.1 Least Squares Adjustment in General

In general, a least squares adjustment uses all available observations simultaneously to compute the most probable value for related unknowns in the presence of error and at any redundancy at or beyond the minimum to avoid singularities. This is done with a goal of minimizing the sum square value of all residuals between observed and modelled values. This algorithm will be utilized for most nonlinear, time-independent computations because it is widely used in geomatics networks, it is simple to implement, and it is a rigorous method with strong potential for precise results.

First, a mathematical and stochastic model must be selected. The mathematical could either be conditional (in which unknowns are given constraints in terms of observed values) or parametric (in which observations are constrained by the related unknowns). In this case, a parametric model is used because the pseudorange observable can easily be isolated without other measurements appearing in the equation. The derivatives of each observation with respect to the unknowns can be inserted into a design matrix as follows:

$$A_{design_{m \times n}} = \begin{bmatrix} \frac{d_{l_1}}{d_{x_1}} & \frac{d_{l_1}}{d_{x_2}} & \dots & \frac{d_{l_1}}{d_{x_m}} \\ \frac{d_{l_2}}{d_{x_1}} & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \frac{d_{l_n}}{d_{x_1}} & \dots & \dots & \frac{d_{l_n}}{d_{x_m}} \end{bmatrix} \quad (45)$$

Since the derivatives of the observation equations in a nonlinear relationship will contain the estimations of the unknowns, this matrix will need to be updated after an adjustment is made.

Using the same observation equation, we can populate a vector of modelled observations using our current estimate of the unknowns:

$$l_{estimated_{nx1}} = \begin{bmatrix} l_1(x) \\ l_2(x) \\ \dots \\ l_n(x) \end{bmatrix} \quad (46)$$

For use with the stochastic model, it is also necessary to define the variance-covariance matrix of the observations, using the variances of each measurement to populate the diagonals:

$$C_{l_{nxn}} = \begin{bmatrix} \sigma_{l_1}^2 & 0 & 0 & 0 \\ 0 & \sigma_{l_2}^2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_{l_n}^2 \end{bmatrix} \quad (47)$$

The least squares procedure operates based on minimizing the misclosure:

$$w_{misclosure_{nx1}} = l_{observed} - l_{estimated} \quad (48)$$

Where $l_{observed}$ is a vector containing all of the actual observed measurements corresponding to the observation equations used to construct $l_{estimated}$. In the case of an unweighted solution where no stochastic model is selected, constructing these matrices and vectors using initial estimates of the unknowns gives enough information to compute an adjustment to the unknowns as follows:

$$\hat{\delta}_{mx1} = N^{-1} A_{design}^T w_{misclosure} \quad (49)$$

Where

$$N = A_{design}^T A_{design} \quad (50)$$

Now the adjustment $\hat{\delta}$ can be used to adjust the current estimate of the unknowns:

$$\hat{x}_{mx1} = x_{0_{mx1}} + \hat{\delta}_{mx1} \quad (51)$$

Where $\hat{x}$ is the updated estimate of the unknowns and x_0 is the estimate of the unknowns used to compute $\hat{\delta}$. The new estimate of unknowns is used to compute the next iteration of A_{design} , $l_{estimated}$ and $\hat{\delta}$. It is important to note that if the model is nonlinear, more than one iteration of the algorithm may be required. In fact, the procedure will be repeated iteratively until the magnitude of $\hat{\delta}$ falls below a desired threshold.

Since many geometric mathematical models are non-linear, it is important to ensure that the initial estimates of the unknown values are sufficiently close to the optimal values before beginning the least-squares procedure. In addition, care must be taken to ensure that all measurements are done with biases eliminated or modeled as much as possible. If either of these conditions are not met, then the procedure may diverge from the optimal solution.

3.3.2 Weighted Parametric Least Squares Adjustment

It is often useful to use a stochastic model to assist in model optimization. The stochastic model is built by defining a weight matrix:

$$P_{weight_{n \times n}} = \begin{bmatrix} weight_1 & 0 & 0 & 0 \\ 0 & weight_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & weight_n \end{bmatrix} \quad (52)$$

Where each weight corresponds to a measurement used in the adjustment. There are many ways to decide these weights, but as a general rule the goal is to give more weight to measurements which are trusted and less weight to measurements with less confidence. One way to select weights is using the standard deviations of each measurement. In that case, the weight matrix can be simply defined as:

$$P_{weight} = C_l^{-1} \quad (53)$$

Now, this weight matrix can be utilized to augment the solution for computing the correction:

$$\hat{\delta}_{mx1} = N^{-1} A_{design}^T P_{weight} W_{misclosure} \quad (54)$$

Where

$$N = A_{design}^T P_{weight} A_{design} \quad (55)$$

If weights are well-selected and the measurements are sufficiently free of bias, then this should yield a solution which more accurately reflects the physical reality of the estimated values. From the equations, it becomes apparent that when no stochastic model is selected, a model is implicitly imposed on the model where the weight matrix is simply equal to an identity matrix.

3.4 Least Squares Mathematical and Stochastic Models

3.4.1 Global Navigation Satellite Systems Positioning Models

3.4.1.1 Single Point Positioning

In general, GNSS pseudorange measurements are modelled using the pseudorange equation:

$$P_{range} = c\Delta t \quad (56)$$

$$\begin{aligned} & \text{pseudorange} = \\ & P_{range} - c\delta T + \delta_{troposphere} + \delta_{ionosphere} + \delta_{multipath} + \text{noise} \end{aligned} \quad (57)$$

Where $\rho_{pseudorange}$ is the value directly extrapolated from the receiver's time of flight measurement from satellite to antenna, P_{range} is the actual geometric range between the satellite and antenna, $c\delta T$ is the delay caused by the difference between the satellite clock offset and receiver clock offset, $\delta_{troposphere}$ is the error due to the troposphere delay, $\delta_{ionosphere}$ is the error due to ionosphere delay, $\delta_{multipath}$ is the multipath error and noise is the gaussian white noise of the signal.

While the least squares procedure is intended to compensate for the white noise and the multipath is highly spatially dependent requiring static testing to model its periodic behaviour, the atmospheric errors are very substantial and need to be modelled to mitigate their effects. Therefore, The vector of observables will be:

$$l_{obs} = \begin{bmatrix} \rho_{pseudorange}^{satellite\ 1-antenna} \\ \rho_{pseudorange}^{satellite\ 2-antenna} \\ \dots \\ \rho_{pseudorange}^{satellite\ n-antenna} \end{bmatrix} \quad (58)$$

The clock delay can be very significant as well. Since there is not a similar way to model this delay (it is unknown before the adjustment), it will be one of the unknown values. Therefore the vector of unknowns will be:

$$\hat{x}_{single\ point} = \begin{bmatrix} X_{WGS84_{antenna}} \\ Y_{WGS84_{antenna}} \\ Z_{WGS84_{antenna}} \\ c\delta T \end{bmatrix} \quad (59)$$

Where $X_{WGS84_{antenna}}$, $Y_{WGS84_{antenna}}$ and $Z_{WGS84_{antenna}}$ are the coordinates of the antenna in the earth-centered, earth-fixed coordinate frame using the WGS84 datum. The observation equation relating the unknowns with the observations is as follows:

$$\begin{aligned} \rho_{pseudorange} &= l(x) \\ &= \sqrt{(X_{antenna} - X_{satellite})^2 + (Y_{antenna} - Y_{satellite})^2 + (Z_{antenna} - Z_{satellite})^2} \\ &\quad - c\delta T + \delta troposphere + \delta ionosphere \end{aligned} \quad (60)$$

This is used to construct the vector of estimated observables:

$$\begin{aligned} l_{estimated_{nx1}} &= \begin{bmatrix} l(\text{antenna position, satellite 1 position, clock error}) \\ l(\text{antenna position, satellite 2 position, clock error}) \\ \dots \\ l(\text{antenna position, satellite n position, clock error}) \end{bmatrix} \end{aligned} \quad (61)$$

Due to the requirement of computing the clock error, it is important to note that a minimum of four satellites must be used to compute a unique solution, because there are 4 unknown quantities (X, Y and Z coordinates plus a clock offset) and each satellite provides one pseudorange measurement. When the derivative is computed of each pseudorange with respect to each unknown, the following is the result:

$$\frac{d\rho_i}{dX_{antenna}} = \frac{X_{antenna} - X_{satellite_i}}{\rho_i} \quad (62)$$

$$\frac{d\rho_i}{dY_{antenna}} = \frac{Y_{antenna} - Y_{satellite_i}}{\rho_i} \quad (63)$$

$$\frac{d\rho_i}{dZ_{antenna}} = \frac{Z_{antenna} - Z_{satellite_i}}{\rho_i} \quad (64)$$

$$\frac{d\rho_i}{d(c\delta T)} = -1 \quad (65)$$

These derivatives are used to construct the design matrix:

$$A_{design} = \begin{bmatrix} \frac{d\rho_1}{dX_{antenna}} & \frac{d\rho_1}{dY_{antenna}} & \frac{d\rho_1}{dZ_{antenna}} & -1 \\ \frac{d\rho_2}{dX_{antenna}} & \frac{d\rho_2}{dY_{antenna}} & \frac{d\rho_2}{dZ_{antenna}} & -1 \\ \dots & \dots & \dots & \dots \\ \frac{d\rho_n}{dX_{antenna}} & \frac{d\rho_n}{dY_{antenna}} & \frac{d\rho_n}{dZ_{antenna}} & -1 \end{bmatrix} \quad (66)$$

It is important to note that the only way that the single point measurement model can compensate for pseudorange bias in the least squares procedure is with optimal satellite geometry. This is why atmospheric models must be modelled before they are used as observables.

In addition, while this is the least robust measurement model in terms of precision, it is also the least demanding model in terms of hardware requirements. Since it only requires access to a single antenna and receiver, it will still be used when a base station is not available.

Where the carrier phase is used instead of the pseudorange, the same procedure is used as with pseudoranges, but the carrier phase equation will be used as the fundamental parametric equation:

$$P_{range} - c\delta T + \delta_{troposphere} - \delta_{ionosphere} + \delta_{multipath} + noise + \lambda N = \phi_{carrier} \quad (67)$$

The primary differences here are that the equation contains an additional term λN , which refers to the integer ambiguity term scaled by the wavelength of the carrier signal and that the ionosphere term is negative rather than positive. The design matrix is identical to the pseudorange design matrix, while the $l_{estimated}$ vector is constructed by subtracting the ionosphere term and adding the scaled ambiguity term.

3.4.1.2 Between Receiver Single Difference

The between receiver single difference is modelled as follows:

$$\Delta\rho = \Delta P - c\Delta T + \Delta troposphere + \Delta ionosphere + \Delta multipath + \Delta noise \quad (68)$$

Where ΔP is the geometric difference in ranges between the satellite and each receiver, $\Delta troposphere$ is the residual troposphere error after differencing, $\Delta ionosphere$ is the residual ionosphere error after differencing, $\Delta multipath$ is the differential multipath delay and $\Delta noise$ is the measurement noise.

Since $\Delta troposphere$ and $\Delta ionosphere$ are atmospheric errors, the difference effectively mitigates those errors, especially for short baselines between the base and rover. This is because both signals will experience similar atmospheric delays when they travel through similar regions geometrically. Therefore, we can use this model to create differences from corrected ranges directly:

$$l_{obs\,differentialGNSS} = \begin{bmatrix} \rho_{rx1-sat1} - \rho_{rx2-sat1} \\ \rho_{rx1-sat2} - \rho_{rx2-sat2} \\ \dots \\ \rho_{rx1-satn} - \rho_{rx2-satn} \end{bmatrix} \quad (69)$$

The act of differencing eliminates the satellite clock error but not receiver clock.

Therefore, the vector of unknowns is the same as in single point positioning:

$$\hat{x}_{differential} = \hat{x}_{single\,point} = \begin{bmatrix} X_{WGS84\,antenna} \\ Y_{WGS84\,antenna} \\ Z_{WGS84\,antenna} \\ c\delta T \end{bmatrix} \quad (70)$$

Where $WGS84$ is the reference frame and *antenna* indicates the antenna location on the platform. The observation equation for the differential solution is as follows:

$$\begin{aligned} \Delta\rho_{used} &= l(x) \\ &= \sqrt{(X_{antenna1} - X_{satellite})^2 + (Y_{antenna1} - Y_{satellite})^2 + (Z_{antenna1} - Z_{satellite})^2} \\ &\quad - \sqrt{(X_{antenna2} - X_{satellite})^2 + (Y_{antenna2} - Y_{satellite})^2 + (Z_{antenna2} - Z_{satellite})^2} \end{aligned} \quad (71)$$

This is used to construct the vector of estimated observations:

$$l_{estimated_{nx1}} = \begin{bmatrix} l(\text{antenna position1, antenna position2, satellite 1 position, clock error}) \\ l(\text{antenna position1, antenna position2, satellite 2 position, clock error}) \\ \dots \\ l(\text{antenna position1, antenna position2, satellite n position, clock error}) \end{bmatrix} \quad (72)$$

Next, the derivatives of the observation equation with respect to each unknown is computed for each observation:

$$\frac{d\Delta\rho_i}{dX_{rover}} = \frac{X_{rover} - X_{satellite_i}}{\rho_i} \quad (73)$$

$$\frac{d\Delta\rho_i}{dY_{rover}} = \frac{Y_{rover} - Y_{satellite_i}}{\rho_i} \quad (74)$$

$$\frac{d\Delta\rho_i}{dZ_{rover}} = \frac{Z_{rover} - Z_{satellite_i}}{\rho_i} \quad (75)$$

$$\frac{d\rho_i}{d(c\Delta T)} = -1 \quad (76)$$

Where *rover* indicates that it is the non-base antenna location (at the platform). The derivatives are used to construct the design matrix:

$$A_{design} = \begin{bmatrix} \frac{d\Delta\rho_1}{dX_{rover}} & \frac{d\Delta\rho_1}{dY_{rover}} & \frac{d\Delta\rho_1}{dZ_{rover}} & -1 \\ \frac{d\Delta\rho_2}{dX_{rover}} & \frac{d\Delta\rho_2}{dY_{rover}} & \frac{d\Delta\rho_2}{dZ_{rover}} & -1 \\ \dots & \dots & \dots & \dots \\ \frac{d\Delta\rho_n}{dX_{rover}} & \frac{d\Delta\rho_n}{dY_{rover}} & \frac{d\Delta\rho_n}{dZ_{rover}} & -1 \end{bmatrix} \quad (77)$$

It is interesting to note that the derivatives with respect to the unknowns are identical in this model compared to the single point model. This is because the term containing the base coordinates has a zero-derivative due to all variables being constant in that term.

It is also interesting to note that the property of mitigating atmospheric error makes the differential model preferable in almost all situations where a base is available. However, since

the atmospheric error mitigation degrades at longer baselines and the differential model does not mitigate multipath errors, a single point model may be preferable to the differential one in some extreme situations, such as long baseline from the base or high multipath environments.

As is the case for single point positioning, the carrier phase merely uses a different equation in single differencing than the pseudorange solution does:

$$\Delta\phi = \Delta P - c\Delta T + \Delta troposphere + \Delta ionosphere + \Delta multipath + \Delta noise + \Delta\lambda N \quad (78)$$

3.4.1.3 Double Differencing

The double difference is a difference between satellites of 2 between receiver single differences, and is modelled as follows:

$$\Delta\nabla\rho = \Delta\nabla P + \Delta\nabla troposphere + \Delta\nabla ionosphere + \Delta\nabla multipath + \Delta\nabla noise \quad (79)$$

Where $\Delta\nabla\rho$ is the measured, between receiver, between satellite double difference, $\Delta\nabla P$ is the true double difference based on the 2 satellite coordinates as well as the base and rover receiver coordinates, $\Delta\nabla troposphere$ and $\Delta\nabla ionosphere$ are the residual atmospheric biases after differencing, $\Delta\nabla multipath$ is the multipath bias and $\Delta\nabla noise$ is the double difference noise.

The same mitigative properties are true for double differences as are true for a between receiver single difference. In fact, for short baselines it eliminates even more of the atmospheric biases than the single difference does. It also has the property of completely removing the receiver and satellite clock biases. However, it does not remove multipath effects and noise effects can be significantly amplified compared to the single difference and single point models.

In order to create a measurement vector with this model, a satellite must first be selected as a reference satellite, the single difference from which all other single differences are subtracted. In order to minimize the influence of the reference error on the whole solution, it is prudent to select a satellite which is expected to have the lowest bias effects. For the purposes of this study, the satellite which has the greatest weight in the stochastic model is selected. To be clear, that is the satellite which has the highest elevation angle relative to the receivers. Figure 8 below illustrates all 3 models along with the setup and how they combine together to form double differences:

Figure 10: GNSS Double Difference Model and Setup

Once the reference satellite is selected, the vector of observed double differences can be constructed from all available between receiver single differences:

$$l_{obsDoubleDifference_{(n-1) \times 1}} = \begin{bmatrix} \Delta\rho_{reference} - \Delta\rho_{sat1} \\ \Delta\rho_{reference} - \Delta\rho_{sat2} \\ \dots \\ \Delta\rho_{reference} - \Delta\rho_{satn} \end{bmatrix} \quad (80)$$

It is important to note that, although there are only 3 unknowns to be solved in this model, there is also 1 less observation available for redundancy. The vector of unknowns for the double difference model is as follows:

$$\hat{x}_{double\ difference} = \begin{bmatrix} X_{WGS84_{rover}} \\ Y_{WGS84_{rover}} \\ Z_{WGS84_{rover}} \end{bmatrix} \quad (81)$$

The observation equation for the double difference model is as follows:

$$l(x) = (range_{reference-rover} - range_{reference-base}) - (range_{differencer-rover} - range_{differencer-base}) \quad (82)$$

Where all ranges are constructed using the coordinates of the satellites, known base coordinates and estimates of the rover coordinates. These double differences are used to construct the vector of estimated measurements:

$$l_{estimated_{(n-1) \times 1}} = \begin{bmatrix} l(reference\ position, differencer_1\ position, rover\ position, base\ position) \\ l(reference\ position, differencer_2\ position, rover\ position, base\ position) \\ \dots \\ l(reference\ position, differencer_n\ position, rover\ position, base\ position) \end{bmatrix} \quad (83)$$

Now the derivatives are computed with respect to each unknown:

$$\frac{d\Delta\nabla\rho_i}{dX_{rover}} = \frac{X_{rover} - X_{reference}}{\rho_{rover-reference}} - \frac{X_{rover} - X_{differencer_i}}{\rho_{rover-differencer}} \quad (84)$$

$$\frac{d\Delta\nabla\rho_i}{dY_{rover}} = \frac{Y_{rover} - Y_{reference}}{\rho_{rover-reference}} - \frac{Y_{rover} - Y_{differencer_i}}{\rho_{rover-differencer}} \quad (85)$$

$$\frac{d\Delta\nabla\rho_i}{dZ_{rover}} = \frac{Z_{rover} - Z_{reference}}{\rho_{rover-reference}} - \frac{Z_{rover} - Z_{differencer_i}}{\rho_{rover-differencer}} \quad (86)$$

The derivatives are used to construct the design matrix:

$$A_{design(n-1) \times 3} = \begin{bmatrix} \frac{d\Delta\nabla\rho_1}{dX_{rover}} & \frac{d\Delta\nabla\rho_1}{dY_{rover}} & \frac{d\Delta\nabla\rho_1}{dZ_{rover}} \\ \frac{d\Delta\nabla\rho_2}{dX_{rover}} & \frac{d\Delta\nabla\rho_2}{dY_{rover}} & \frac{d\Delta\nabla\rho_2}{dZ_{rover}} \\ \vdots & \vdots & \vdots \\ \frac{d\Delta\nabla\rho_n}{dX_{rover}} & \frac{d\Delta\nabla\rho_n}{dY_{rover}} & \frac{d\Delta\nabla\rho_n}{dZ_{rover}} \end{bmatrix} \quad (87)$$

It is interesting to note that, unlike the single difference model, the double difference derivatives are not identical to the single point model derivatives.

The double difference essentially amplifies the mitigative properties of the single difference. Since this can be quite effective at mitigating atmospheric errors at short baselines, this model is quite commonly used. However, multipath and noise effects could still render the single point model preferable in some situations since the double difference can amplify both errors.

As is the case with other measurement models, the carrier phase simply uses a different parametric equation than pseudoranges in double differencing:

$$\Delta\nabla\rho = \Delta\nabla P + \Delta\nabla troposphere + \Delta\nabla ionosphere + \Delta\nabla multipath + \Delta\nabla noise + \Delta\nabla\lambda N \quad (88)$$

As can be seen, the carrier phase can greatly benefit from differential positioning models because it is still subject to atmospheric error. Differential carrier phase solutions can mitigate bias while retaining the robust noise characteristics of the carrier phase.

3.4.1.4 GNSS Stochastic Models

Typically, the least squares stochastic model is one which is used to construct the weight matrix. One very common model uses the variance-covariance matrix of the measurements:

$$Cl_{n \times n} = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_n^2 \end{bmatrix} \quad (89)$$

Where σ_i^2 is the expected variance of the individual measurement. This value is typically taken from a manufacturer's specification or a value observed by experiment. In the case of a GNSS pseudorange, this refers to the noise variance, which will be the same for each pseudorange. This means that using this matrix will not impact the solution compared to using no stochastic model, since all measurements would be weighted equally regardless. Therefore, this stochastic model will not be used for the purpose of this study in the GNSS context.

One other method of weighting observations is to use the elevation angle of the satellite. When a satellite is directly overhead at an elevation angle of 90 degrees, it has optimal propagation geometry since it has the smallest possible distance to cover and the least refractive effects due to the atmosphere. At the horizon, satellites have the poorest propagation geometry, since the signal takes the most curved path possible, through the longest possible stretch of atmosphere, before reaching the antenna.

For this reason, it can be useful to use the elevation angle in the stochastic model:

$$weight_i = \cos(90 - \alpha_{elevation_i}) \quad (90)$$

Where $\alpha_{elevation_i}$ is the elevation angle of each satellite. This ensures that satellites with poor geometry are given smaller weights relative to optimal satellites in the solution. These weights are used to build the weight matrix:

$$P_{weight_{n \times n}} = \begin{bmatrix} weight_1 & 0 & 0 & 0 \\ 0 & weight_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & weight_n \end{bmatrix} \quad (91)$$

This weight matrix is used to augment the solution as follows:

$$N = A^T P A \quad (92)$$

$$\hat{\delta} = N^{-1} A_{design}^T P_{weight} w_{misclosure} \quad (93)$$

$$C_{\hat{x}} = N^{-1} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} & \sigma_{x\delta T} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{yz} & \sigma_{y\delta T} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z^2 & \sigma_{z\delta T} \\ \sigma_{x\delta T} & \sigma_{y\delta T} & \sigma_{z\delta T} & \sigma_{\delta T}^2 \end{bmatrix} \quad (94)$$

Where $C_{\hat{x}}$ is the variance-covariance matrix of the unknowns. This provides a measure of estimated precision in the calculated coordinates based on the estimated measurement noise. It does not indicate accuracy since the least squares procedure does not take bias into account and the measurement noise is not perfectly known. Accuracy can only be determined by comparing computed coordinates with known values (if available).

3.5 Conditional Least-Squares

Also discussed in [38] is the conditional least-squares model. The difference between this model and the parametric model is that this model uses unknown parameters in terms of measured quantities. This can be convenient for error propagation when this derivative is less complex than the parametric model. In general, the design matrix for the conditional model is:

$$B_{design_{m \times n}} = \begin{bmatrix} \frac{dx_1}{dl_1} & \dots & \frac{dx_1}{dl_n} \\ \dots & \dots & \dots \\ \frac{dx_m}{dl_1} & \dots & \frac{dx_m}{dl_n} \end{bmatrix} \quad (95)$$

In the conditional model, the measurement covariance matrix and weight matrix are the same as in the parametric model. In order to propagate the uncertainty, the inverse of the normal matrix is computed:

$$N_{m \times m} = BPB^T \quad (96)$$

$$C_x = N^{-1} \quad (97)$$

For the purposes of this research, this relationship will only be used to propagate uncertainty, and only in some select cases where it is convenient to do so. The reason for using this method of uncertainty propagation is that it preserves information about covariance between different observables and unknowns.

3.6 Pitch and Heading Determination Using Dual

Antenna GNSS

3.6.1 Computation of Pitch and Heading

3.6.1.1 Definition of Heading

For the purposes of this research, a heading will be defined as the direction of platform orientation, given as the clockwise angle between the geodetic north to the unit vector of platform orientation, projected onto the horizontal plane. This is illustrated below:

Figure 11: Definition of the Heading Angle on the North / East Horizontal Plane

Figure 9 illustrates the meaning of the heading angle as it relates to a moving car, but the heading holds the same definition regardless of the platform. There simply needs to be a reference defining the platform forward axis. An angle which holds a similar meaning to the heading angle is a bearing between 2 points, as illustrated below:

Figure 12: Definition of a Bearing Between 2 Points as Projected on the North / East Horizontal Plane

The bearing in Figure 10 and the heading in Figure 6 can each take on a positive value between 0 and 360 degrees for the purposes of this research. The heading angle is an angle critical to navigation because navigating pertains traversing from one location to an objective, and projected coordinate systems are very useful for terrestrial applications. The heading defines the direction of travel, so any adjustments to trajectories will need to take this angle into account.

3.6.1.2 Definition of Pitch

For the purposes of this research, pitch will be defined as the inclination of the platform plane, given as the counter-clockwise angle between the horizontal plane (perpendicular to the gravity vector) and the platform frontal axis. The pitch will be a rotation about the platform lateral axis by definition when using a zero-roll assumption. This is illustrated in Figure 11:

Figure 13: Definition of Pitch Relative to the Horizontal Plane and Vertical Axis

The pitch angle uses the horizontal plane as a reference, which does not have a fixed direction. Therefore, unlike the heading which uses a fixed north axis as a reference, pitch can take a value between -90 and 90 degrees. Pitch is an angle that could have safety implications for many applications, including marine and land navigation. High pitch angle indicates a sharp incline, which can compromise safety in some adverse weather conditions and on some surfaces.

3.6.1.3 Computation and Error Propagation of Pitch and Heading Using GPS

Suppose that the unit vector representing the platform direction of travel is defined by 2 points. If the coordinates of those points are known in a local vertical frame. The heading and pitch can then both be computed by triangle geometry:

$$\Delta E = E_{front} - E_{rear} \quad (98)$$

$$\Delta N = N_{front} - N_{rear} \quad (99)$$

$$\Delta U = U_{front} - U_{rear} \quad (100)$$

$$\theta_{pitch} = \tan^{-1} \frac{\Delta U}{\sqrt{\Delta E^2 + \Delta N^2}} \quad (101)$$

$$\alpha_{heading} = \tan^{-1} \frac{\Delta E}{\Delta N} \quad (102)$$

Where E_{front} and E_{rear} are easting coordinates of the front and rear point, N_{front} and N_{rear} are northing coordinates of the front and rear point and U_{front} and U_{rear} are vertical coordinates of the front and rear point in a local vertical reference frame. It is important to note that the arctangent in (134) is a quadrant-dependent function. Therefore, the sign of the numerator and denominator must both be taken into account as follows:

$$\text{if } \Delta E > 0 \text{ and } \Delta N > 0: \quad 0 < \alpha < 90 \quad (103)$$

$$\text{if } \Delta E > 0 \text{ and } \Delta N < 0: \quad 90 < \alpha < 180 \quad (104)$$

$$\text{if } \Delta E < 0 \text{ and } \Delta N < 0: \quad 180 < \alpha < 270 \quad (105)$$

$$\text{if } \Delta E < 0 \text{ and } \Delta N > 0: \quad 270 < \alpha < 360 \quad (106)$$

For the purposes of this research, the coordinates computed for the first epoch of the rear GPS antenna will be used for the reference point in the coordinate transformation from the earth-centered, earth-fixed (ECEF) coordinate frame to the local vertical frame. In order to propagate the uncertainty from the computed ECEF coordinates to the local vertical equivalents, the parametric least-squares method will be used. The estimated covariance matrixes of the

coordinates computed using GPS will be used to construct a new measurement covariance matrix:

$$C_{l_{transform}} = \begin{bmatrix} C_{x_{GPS}1,1} & C_{x_{GPS}1,2} & C_{x_{GPS}1,3} \\ C_{x_{GPS}2,1} & C_{x_{GPS}2,2} & C_{x_{GPS}2,3} \\ C_{x_{GPS}3,1} & C_{x_{GPS}3,2} & C_{x_{GPS}3,3} \end{bmatrix} \quad (107)$$

As in (132), only the positional elements are extracted from the covariance matrix of unknowns from the least-squares position solution, whether or not a combined clock offset was solved for in the solution. This is because only the geometry is being rotated in the transformation, not a pseudorange bias. The weight matrix for the transformation will be the inverse of this matrix. As for the design matrix, the rotation matrix from the ECEF frame to the local vertical frame can conveniently be used as the design matrix in a conditional least squares model instead of a parametric model. The rotation matrix will be a combination of two rotations, depending on the computed geodetic latitude and longitude of the GPS antenna:

$$R_{\lambda} = \begin{bmatrix} \cos(\lambda_{long}) & -\sin(\lambda_{long}) & 0 \\ \sin(\lambda_{long}) & \cos(\lambda_{long}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (108)$$

$$R_{\phi} = \begin{bmatrix} \cos(\phi_{lat}) & 0 & -\sin(\phi_{lat}) \\ 0 & 1 & 0 \\ \sin(\phi_{lat}) & 0 & \cos(\phi_{lat}) \end{bmatrix} \quad (109)$$

$$R_{transform} = R_{\phi} R_{\lambda} \quad (110)$$

Where λ_{long} is the geodetic longitude and ϕ_{lat} is the geodetic latitude. With a weight matrix and rotation matrix computed, the uncertainty can be propagated using the conditional model:

$$C_{x_{transform}} = (R_{transform} C_{l_{transform}}^{-1} R_{transform}^T)^{-1} \quad (111)$$

This will represent a covariance matrix of computed coordinates in a local vertical frame. If this uncertainty is propagated for both antenna solutions, each individual covariance matrix can be used to build a new measurement covariance matrix for the pitch and heading computation:

$$C l_{GPS\ Attitude} = \begin{bmatrix} C_{xrear\ 1,1} & C_{xrear\ 1,2} & C_{xrear\ 1,3} & 0 & 0 & 0 \\ C_{xrear\ 2,1} & C_{xrear\ 2,2} & C_{xrear\ 2,3} & 0 & 0 & 0 \\ C_{xrear\ 3,1} & C_{xrear\ 3,2} & C_{xrear\ 3,3} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{xfront\ 1,1} & C_{xfront\ 1,2} & C_{xfront\ 1,3} \\ 0 & 0 & 0 & C_{xfront\ 2,1} & C_{xfront\ 2,2} & C_{xfront\ 2,3} \\ 0 & 0 & 0 & C_{xfront\ 3,1} & C_{xfront\ 3,2} & C_{xfront\ 3,3} \end{bmatrix} \quad (112)$$

It is worth noting that, while some covariance information is retained through propagation, this matrix assumes that the front and rear GPS antenna are uncorrelated. This is likely not true for this research, because the front and rear antenna share a common receiver.

With 2 antennas being utilized, (133) and (134) yield a unique solution, eliminating the need for a least-squares solution for the unknowns. However, the least-squares stochastic model is still useful for propagating the uncertainty from the previous step. Therefore, parametric equations and their partial derivatives are computed:

$$U_{front} = \tan(\theta_{pitch}) \sqrt{\Delta E^2 + \Delta N^2} + U_{rear} \quad (113)$$

$$U_{rear} = -\tan(\theta_{pitch}) \sqrt{\Delta E^2 + \Delta N^2} - U_{front} \quad (114)$$

$$E_{front} = \sqrt{\frac{\Delta U^2}{\tan^2 \theta_{pitch}} - \Delta N^2} + E_{rear} \quad (115)$$

$$E_{rear} = -\sqrt{\frac{\Delta U^2}{\tan^2 \theta_{pitch}} - \Delta N^2} - E_{front} \quad (116)$$

$$N_{front} = \sqrt{\frac{\Delta U^2}{\tan^2 \theta_{pitch}} - \Delta E^2} + N_{rear} \quad (117)$$

$$N_{rear} = -\sqrt{\frac{\Delta U^2}{\tan^2 \theta_{pitch}} - \Delta E^2} - N_{front} \quad (118)$$

$$E_{front} = \Delta N \tan(\alpha_{heading}) + E_{rear} \quad (119)$$

$$E_{rear} = -\Delta N \tan(\alpha_{heading}) - E_{front} \quad (120)$$

$$N_{front} = \frac{\Delta E}{\tan(\alpha_{heading})} + N_{rear} \quad (121)$$

$$N_{rear} = -\frac{\Delta E}{\tan(\alpha_{heading})} - N_{front} \quad (122)$$

$$\frac{dU_{front}}{d\theta_{pitch}} = \sec^2 \theta_{pitch} \sqrt{\Delta E^2 + \Delta N^2} \quad (123)$$

$$\frac{dU_{rear}}{d\theta_{pitch}} = -\frac{dU_{front}}{d\theta_{pitch}} \quad (124)$$

$$\frac{dE_{front}}{d\theta_{pitch}} = \frac{-\cot(\theta_{pitch}) \csc^2 \theta_{pitch} \Delta U^2}{\sqrt{\frac{\Delta U^2}{\tan^2 \theta_{pitch}} - \Delta N^2}} \quad (125)$$

$$\frac{dE_{rear}}{d\theta_{pitch}} = -\frac{dE_{front}}{d\theta_{pitch}} \quad (126)$$

$$\frac{dN_{front}}{d\theta_{pitch}} = \frac{-\cot(\theta_{pitch}) \csc^2 \theta_{pitch} \Delta U^2}{\sqrt{\frac{\Delta U^2}{\tan^2 \theta_{pitch}} - \Delta E^2}} \quad (127)$$

$$\frac{dN_{rear}}{d\theta_{pitch}} = -\frac{dN_{front}}{d\theta_{pitch}} \quad (128)$$

$$\frac{dE_{front}}{d\alpha_{heading}} = \sec^2(\alpha_{heading}) \Delta N \quad (129)$$

$$\frac{dE_{rear}}{d\alpha_{heading}} = -\frac{dE_{front}}{d\alpha_{heading}} \quad (130)$$

$$\frac{dN_{front}}{d\alpha_{heading}} = -\csc^2(\alpha_{heading}) \Delta E \quad (131)$$

$$\frac{dN_{rear}}{d\alpha_{heading}} = -\frac{dN_{front}}{d\alpha_{heading}} \quad (132)$$

Where computed coordinates and headings will be used to substitute into the derivatives.

These derivatives can now be used to fill the design matrix:

$$A_{pitch,heading} = \begin{bmatrix} \frac{dE_{rear}}{d\theta_{pitch}} & \frac{dE_{rear}}{d\alpha_{heading}} \\ \frac{dN_{rear}}{d\theta_{pitch}} & \frac{dN_{rear}}{d\alpha_{heading}} \\ \frac{dU_{rear}}{d\theta_{pitch}} & 0 \\ \frac{dE_{front}}{d\theta_{pitch}} & \frac{dE_{front}}{d\alpha_{heading}} \\ \frac{dN_{front}}{d\theta_{pitch}} & \frac{dN_{front}}{d\alpha_{heading}} \\ \frac{dU_{front}}{d\theta_{pitch}} & 0 \end{bmatrix} \quad (133)$$

With a covariance matrix and design matrix constructed, the uncertainty can be propagated:

$$C_{x_{pitch,heading}} = (A_{pitch,heading}^T C l_{GPS_{attitude}}^{-1} A_{pitch,heading})^{-1} \quad (134)$$

3.6.1.4 Cropping and Linear Interpolation of GPS Data Sets for an Undedicated Pitch and Heading Determination System

Computation of pitch and heading requires simultaneous positions from 2 separate GPS antennas. However, the physical system used in this research is an undedicated system in which 1 receiver has a different data rate and is not synced with the other receiver. The receivers also begin recording data at different times. Therefore, before pitch and heading can be computed in post processing, data must be cropped to give both datasets the same duration, then data from the set with the lower data rate must be interpolated to fill a position at all times where a position is available in the set with the higher data rate.

Because the lower data rate set is to be interpolated, the longer dataset will be cropped to be equal in duration to the shorter dataset such that:

- The first and last epochs of each dataset share the same GPS time stamp if possible, or if not possible
- (If the high data rate set is longer) The first time stamp in the high data rate set which is later than the first time stamp in the low data rate set becomes the new first time stamp in both sets. In addition, the last time stamp in the high data rate set which is earlier than the last time stamp in the low data rate set becomes the new last time stamp in both sets.
- (If the low data rate set is longer) The first time stamp in the high data rate set becomes the new first time stamp in both sets. In addition, the last time stamp in the high data rate set becomes the new last time stamp in both sets.

Once the data has been cropped to the appropriate size, the time stamps from the high data rate set will be used in conjunction the time stamps and positions in the low data rate set in order to interpolate a data set with the same data rate as the high data rate set:

$$x_{new}(t) = \frac{t - t_1}{t_2 - t_1} (x_2 - x_1) \quad (135)$$

$$y_{new}(t) = \frac{t - t_1}{t_2 - t_1} (y_2 - y_1) \quad (136)$$

$$z_{new}(t) = \frac{t - t_1}{t_2 - t_1} (z_2 - z_1) \quad (137)$$

Where t_1 is the nearest GPS time stamp in the low rate data set preceding time t in the high data rate set, t_2 is the nearest GPS time stamp in the low data rate set after time t and $x_{new}(t)$, x_1 and x_2 are the interpolated coordinate, preceding coordinate in the low rate set and next coordinate in the low rate set, respectively. This interpolation yields 2 GPS positioning datasets which can be used directly to compute pitch and heading.

3.6.2 Static Kalman Filter

For the purposes of exploring how the results for pitch and heading computation in a static setup can be improved, a simple Kalman filter will be applied to computed angles from raw data in post processing. The algorithm is derived in [39] and it is a recommended technique for fusion of different sensors [40] as is the case for the undedicated GPS pitch and heading system implemented in this thesis. In order to utilize the recursive Kalman filter algorithm, some key matrices must be defined:

$$x_k = \begin{bmatrix} \theta_{pitch_k} \\ \theta_{heading_k} \end{bmatrix} \text{ (state vector at time } k) \quad (138)$$

$$\Phi_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ (state transition matrix)} \quad (139)$$

$$z_k = \begin{bmatrix} \theta_{pitch_{k_{measured}}} \\ \theta_{heading_{k_{measured}}} \end{bmatrix} \text{ (vector measurement update)} \quad (140)$$

$$H_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ (connection between the measurements and state vector)} \quad (141)$$

$$P_k = \begin{bmatrix} \sigma_{\theta_{pitch}}^2 & 0 \\ 0 & \sigma_{\theta_{heading}}^2 \end{bmatrix} \text{ (covariance matrix of the measurements)} \quad (142)$$

$$R_k = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0005 \end{bmatrix} \text{ (process noise matrix)} \quad (143)$$

Where the process noise matrix will be kept constant for this implementation. For $\sigma_{\theta_{pitch}}^2$ and $\sigma_{\theta_{heading}}^2$, the diagonals of the estimated covariance matrix of the pitch and heading from GPS error propagation will be used. Only the diagonals are used because this implementation is assuming no correlation between pitch and heading. While this may not reflect reality, the uncorrelated model produced superior results compared to the correlated model in practice. $\theta_{pitch_{k_{measured}}}$ and $\theta_{heading_{k_{measured}}}$ are the computed pitch and roll values at time k using raw interpolated GPS coordinates from the front and rear GPS receiver. Process noise was tuned until optimal results were obtained, yielding the values in Equation 142. A trial/error method was used to tune the process noise in this case. Values were iteratively chosen until the pitch and heading errors were minimized.

In general, Kalman filtering is an iterative process. In the first epoch, the state vector is equal to the measurement update because no time has passed for propagation. In all proceeding steps, the state vector is a result of the filter cycle. First, Kalman gain K_k is calculated:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad (144)$$

Where P_k^- is either the propagated covariance matrix from the previous epoch's filter (if using Kalman propagation for the noise) or the covariance matrix propagated from all measurements at time k (if using the least squares covariance matrix at each time instead). The Kalman gain is used to fuse the predicted state and measured state:

$$x_k = x_k^- + K_k(z_k - H_k x_k^-) \quad (145)$$

Where x_k^- is the predicted state based on the system dynamics and previous state. Next, if Kalman gain is used to propagate covariance, 2 steps will be undertaken:

$$P_k = (I - K_k H_k) P_k^- \quad (146)$$

$$P_{k+1} = \Phi_k P_k \Phi_k^T + P_k \quad (147)$$

Where P_k^- is the previously propagated covariance matrix. If the alternative least squares adaptation is used (as in this thesis), P_k^- will simply be updated with a computed covariance matrix propagated from measurement geometry in the next epoch. Finally, the prediction for the next epoch is calculated:

$$x_{k+1}^- = \Phi_k x_k \quad (148)$$

This procedure is repeated for all available measurement updates. The iterative cycle is illustrated in the flow chart below:

Figure 14: Kalman Filter Iterative Cycle Flow Chart with Standard as well as Adapted Covariance Update Strategies

3.7 Error and Uncertainty Metrics

In order to assess the validity of a measured or computed result, metrics for the accuracy and precision are necessary. Precision refers to the repeatability of a result from measured quantities, while accuracy refers to the difference between observed results and true quantities if they are known.

3.7.1 Estimated Standard Deviation

In the case of a GPS single point position solution, the covariance matrix is as in (110), while it is the result of (133) in the case of the pitch and heading. The diagonals of the covariance matrix represent the estimated variance of the corresponding variance. Therefore, the

estimated standard deviation used to construct error ellipses as well as reported in simulation and surveys for pitch and heading is the square root of the variance:

$$\sigma_{est_i} = \sqrt{\sigma_{est_i}^2} = \sqrt{C_x(i, i)} \quad (149)$$

This is also known as dilution of precision (DOP) in the GNSS industry. It is perhaps a better term than standard deviation, because while it often does not reflect observed standard deviation, it does give a relative measure of precision with respect to geometry and provides a basis for construction of a confidence ellipse.

3.7.2 Confidence Ellipses

An error ellipse, otherwise called a confidence ellipse, will be used in this research to determine a region where 95% of positioning results are expected to lie if the coordinates are projected onto the horizontal plane at the true location of the antenna. In order to construct this ellipse, the components defining the ellipse must be defined. The procedure outlined in [38] will be used to construct the ellipses. First, the semi-eastings and semi-northings lengths are defined by the estimated standard deviations in the eastings and northings directions, respectively. The larger of the 2 values will represent the semimajor axis of the ellipse with approximately 68% confidence. In order to draw an ellipse at a different confidence level, the semi-eastings and semi-northings must be scaled:

$$a_E = S_0 \sqrt{Cx(1,1)} \quad (150)$$

$$a_N = S_0 \sqrt{Cx(2,2)} \quad (151)$$

Where S_0 is the scale factor (in this case a factor of 2.447 is used for the 95% confidence ellipse). Due to correlation between the eastings and northings, a rotation about the vertical axis is computed:

$$\tan(2\theta_{ellipse}) = \frac{2C_x(1,2)}{C_x(2,2) - C_x(1,1)} \quad (152)$$

Where $\theta_{ellipse}$ is the rotation angle about the vertical axis. The equation of the unrotated ellipse is defined by:

$$\frac{(E - E_0)^2}{a_E^2} + \frac{(N - N_0)^2}{a_N^2} = 1 \quad (153)$$

Where E_0 and N_0 are the true eastings and northings of the antenna. The ellipse is generated by entering a selection of eastings from $E_0 - a_E$ to $E_0 + a_E$, taking both the positive square root and negative square root solutions for N. The coordinates are then rotated by $\theta_{ellipse}$ about the vertical axis. The rotated ellipse coordinates in the north and east directions are used for the confidence ellipse plots in this research.

3.7.3 Root Mean Square (RMS) Error

The RMS Error is used to give a measure of true accuracy of a solution on average. In order to compute this error, the true value must be known. In general, it can be defined as follows:

$$RMS = \sqrt{\frac{\sum_{i=1}^n (observed - true)^2}{n}} \quad (154)$$

Where n is the number of samples. The advantage that the RMS error has as a statistic is that it can aid in quantifying the effects of biases which are not random noise effects. While this

is used to quantify errors on individual variables, it is also useful to have a broader positioning accuracy metric for computed GPS positions. One such metric used in this thesis is the 2DRMS:

$$2DRMS = \sqrt{\frac{\sum_{i=1}^n ((E_{observed} - E_{true})^2 + (N_{observed} - N_{true})^2)}{n}} \quad (155)$$

Where E denotes eastings and N denotes northings. This research uses 2DRMS to compare horizontal and vertical accuracy of GPS positioning solutions.

3.7.4 Standard Deviation

The standard deviation is a measure of precision of a data set. That is, it is the quantity, sharing units of the data set, such that approximately 68% of all data points in a normally distributed set fall within the mean plus or minus one standard deviation. It is computed as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (observed - mean)^2}{n - 1}} \quad (156)$$

Where n is the number of samples. The advantage of the standard deviation is that it gives a better measure of noise effects than the RMS error does. This can be adversely affected by temporal variability of measurement biases, however. It will be used as the primary measure of observed precision in this research.

4 Simulations and GPS Positioning Validation

4.1 Positioning Simulation

Real GPS data is susceptible to multipath bias which is difficult to model. Therefore, a MATLAB simulation will be done to evaluate computation methods under a variety of environmental conditions and based on the experimental control marker survey done in Didsbury, Alberta in which all errors will be controlled as an input parameter. The location was selected for its open sky environment as well as for easy access to precisely surveyed control monuments, each having coordinates and revision history available to the public.

4.1.1 Simulated Measurements:

In order to simulate an L1 pseudorange measurement, first the true geometric range is computed between the antenna location and satellite location. In this case, the known ASCM 764522 location is used as the antenna location and the satellites will take on identical geometry to the visible satellites in the first epoch of the actual logged data from the control marker survey in Didsbury.

4.1.2 Simulated Results:

Using satellite and marker geometry identical to that of the first epoch of the Didsbury ASCM survey, collections of results were simulated in a variety of bias, noise and differential baseline conditions. All positioning data collections are contained and discussed in this section.

4.1.2.1 Collection A: 0.1 ppm Bias, 10 cm Noise Standard Deviation, 100 m

Differential Baseline

Figure 15: Horizontal and Vertical Errors for Simulation Set A

Figure 16: Horizontal 95 Percent Confidence Ellipses for Simulation Set A

Table 4: Mean Error Statistics for Set A

Technique	3DRMS Error (m)	Vertical Error (m)	Error Standard deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	0.728	0.647	0.156	-
Single Point: Carrier Phase	0.615	0.527	0.653	0.1486
Single Difference: Pseudoranges	0.411	-0.010	0.227	-
Single Difference: Carrier Phase	0.250	0.019	0.480	0.020
Double Difference: Pseudoranges	0.527	-0.004	0.608	-
Double Difference: Carrier Phase	0.486	0.052	0.290	0.1814

From Figure 13, it is apparent that the atmospheric biases have offset the vertical error whereas the differential solutions have a vertical mean closer to the true value, which is expected. It is also interesting to note that the differential solutions are noisier than the single point solution. This is also expected, since the noise from 2 receivers' signals will sometimes meet destructively but sometimes constructively in the difference.

From Figure 14, it is apparent that the true position falls within the confidence ellipse of the single point and single difference solutions are nearly circular while the double difference solution ellipse is elongated. This is expected since errors are highly correlated with the position of the reference satellite.

From table 4 it is interesting to see that the single point solution has some advantages over the differential solution, particularly with regards to noise. This is because the error properties of Collection A are very optimistic. 10 cm pseudorange noise is what a high-precision P-code can yield if the equipment is high-quality, and only 0.1 ppm of unmitigated atmospheric error is very difficult to achieve. Therefore, this set will provide a baseline for the ideal situation for positioning.

4.1.2.2 Collection B: 0.1 ppm Bias, 1 m Noise Standard Deviation, 100 m Differential Baseline

Figure 17: Horizontal and Vertical Errors for Simulation Set B

Collection B yields some interesting artifacts and trends which separate it from Collection A. Regarding Figure 15, it is interesting that while noise has increased, the positioning solutions are not significantly offset from each other. There is a noticeable difference in the 3D positioning variation, however. A very extreme spike in positioning error is observed for the double difference solution near epoch 40. This shows how strongly a constructive difference in signal noise can affect the solution.

Figure 18: Horizontal 95 Percent Confidence Ellipses for Simulation Set B

Table 5: Mean Error Statistics for Set B

Technique	3DRMS Error (m)	Vertical Error (m)	Error Standard deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	3.233	0.591	1.646	-
Single Point: Carrier Phase	3.231	0.571	1.640	1.23
Single Difference: Pseudoranges	4.780	-0.287	2.242	-
Single Difference: Carrier Phase	4.827	-0.336	2.304	1.63
Double Difference: Pseudoranges	5.032	-0.117	2.732	-
Double Difference: Carrier Phase	5.031	-0.111	2.746	0.26

Table 5 shows some interesting differences with Collection A. For instance, the correct integer ambiguity is not found as often, and since Collection B uses very optimistic atmospheric biases, there is only very marginal benefit overall to using the carrier phase, and in fact the pseudoranges performed better than the carrier phase when using the single difference measurement model.

One of the most significant statistics here which changed is the standard deviation of the error, being greater than 1.5 m in all cases. This is to be expected, since the control variable which was changed for this set is the measurement noise standard deviation.

4.1.2.3 Collection C: 0.1 ppm Bias, 3 m Noise Standard Deviation, 100 m Differential

Baseline

Figure 19: Horizontal and Vertical Errors for Simulation Set C

Figure 20: Horizontal 95 Percent Confidence Ellipses for Simulation Set C

Table 6: Mean Error Statistics for Set C

Technique	3DRMS Error (m)	Vertical Error (m)	Error Standard deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	9.543	0.516	5.252	-
Single Point: Carrier Phase	9.542	0.463	5.299	-0.60
Single Difference: Pseudoranges	12.480	0.556	6.837	-
Single Difference: Carrier Phase	12.537	0.644	6.800	1.29
Double Difference: Pseudoranges	16.236	-0.388	8.269	-
Double Difference: Carrier Phase	16.263	-0.330	8.249	4.87

From Figure 17, a very pronounced difference is observed between differential and single point solutions is observed, though the mean position calculated for each is very similar. Collection C very clearly illustrates the amplified effect that differential measurement models have on noise.

Since the noise is so significant in this set, the pseudorange solution falls well outside of the linear search regions for the carrier phase ambiguities quite often. This results in discrete variations of carrier phase error in both directions, despite the carrier phase noise being modelled at only 1mm. This is to be expected. The pseudorange error is almost entirely due to noise in this set, so solutions are expected to fall anywhere inside of a 3D confidence ellipsoid defined by the 3m measurement noise, with the mean being close to the true position.

Benefits of the carrier phase are only marginal in collection C, in fact the benefits are mostly random and inconsistent. However, there does seem to be a reasonable improvement to vertical positioning accuracy using the carrier phase in the single point model, possibly because that model has the least vulnerability to noise of the 3 models.

4.1.2.4 Collection D: 5 ppm Bias, 10 cm Noise Standard Deviation, 100 m

Differential Baseline

Figure 21: Horizontal and Vertical Errors for Simulation Set D

Figure 22: Horizontal 95 Percent Confidence Ellipses for Simulation Set D

Table 7: Mean Error Statistics for Set D

Technique	3DRMS Error (m)	Vertical Error (m)	Error Standard deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	33.158	32.809	0.158	-
Single Point: Carrier Phase	33.065	32.720	0.374	-20.48
Single Difference: Pseudoranges	0.418	-0.005	0.192	-
Single Difference: Carrier Phase	0.264	-0.013	0.464	0.31
Double Difference: Pseudoranges	0.592	0.017	0.302	-
Double Difference: Carrier Phase	0.626	-0.006	0.649	0.34

It is evident from looking at figures 19 and 20 that single point positioning solutions are strongly affected by the pseudorange biases. This is expected since the model has no way of mitigating residual atmospheric delays. Since the base to rover baseline is only 100 m, the differential solutions do effectively mitigate the pseudorange biases, hence the solutions are far more accurate on average in the vertical and horizontal directions. It is possible that sufficiently optimal satellite geometry would improve the 2D positioning accuracy for single point solutions, but this would not significantly aid the vertical error, since this would require line of sight to satellites which are obscured by the earth itself.

Collection D yields results which show use of the carrier phase improving the positioning accuracy with all 3 measurement models. This includes the single point model, which had a significant average error in estimating the integer ambiguity, so this is a good sign that use of carrier phase observables can be helpful in the presence of atmospheric biases.

4.1.2.5 Collection E: 5 ppm Bias, 1 m noise Standard Deviation, 100 m Differential

Baseline

Figure 23: Horizontal and Vertical Errors for Simulation Set E

Figure 24: Horizontal 95 Percent Confidence Ellipses for Simulation Set E

Collection E yields results which do not support the use of carrier phase measurements in this scenario. The only method which yielded a better global error statistic using the carrier phase was the double difference solution, which had a slightly better error standard deviation. Even with that method, though, Table 8 shows that the other error statistics did not benefit from use of the carrier phase. It is possible that expanding the search region would improve this, but using this integer combination search, any extra integers would increase processing time exponentially.

Table 8: Mean Error Statistics for Set E

Technique	3DRMS Error (m)	Vertical Error (m)	Standard Deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	33.220	32.760	1.790	-
Single Point: Carrier Phase	33.262	32.800	1.858	-19.76
Single Difference: Pseudoranges	4.628	-0.318	2.521	-
Single Difference: Carrier Phase	4.656	-.273	2.529	-2.05
Double Difference: Pseudoranges	5.549	0.118	2.902	-
Double Difference: Carrier Phase	5.593	0.156	2.917	1.32

4.1.2.6 Collection F: 5 ppm Bias, 3 m Noise Standard Deviation, 100 m Differential

Baseline

Figure 25: Horizontal and Vertical Errors for Simulation Set F

Figure 26: Horizontal 95 Percent Confidence Ellipses for Simulation Set F

Table 9: Mean Error Statistics for Set F

Technique	3DRMS Error (m)	Vertical Error (m)	Standard Deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	34.438	32.662	4.708	-
Single Point: Carrier Phase	34.381	32.610	4.792	-20.74
Single Difference: Pseudoranges	14.538	-0.692	8.267	-
Single Difference: Carrier Phase	14.550	-0.626	8.229	-3.62
Double Difference: Pseudoranges	16.028	0.488	8.754	-
Double Difference: Carrier Phase	16.024	0.431	8.798	1.50

Interestingly, Collection F yields results which reduced the mean error overall as well as the vertical error for every measurement model when the carrier phase is used, which is a stark contrast to the data from Collection E. This may suggest that the success rate for selection of optimal ambiguities depends on the geometry of the estimated pseudorange solution relative to both the true position and the satellites broadcasting measurements.

Since differential measurement models are being used here, it would also be interesting to examine results when the rover-base baseline distance is varied.

4.1.2.7 Collection G: 5 ppm Bias, 1 m Noise Standard Deviation, 1 km Differential

Baseline

Figure 27: Horizontal and Vertical Errors for Simulation Set F

Figure 28: Horizontal 95 Percent Confidence Ellipses for Simulation Set G

Table 10: Mean Error Statistics for Set G

Technique	3DRMS Error (m)	Vertical Error (m)	Standard Deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	32.220	32.760	1.790	-
Single Point: Carrier Phase	32.262	32.799	1.858	-19.76
Single Difference: Pseudoranges	4.283	-0.016	2.184	-
Single Difference: Carrier Phase	4.345	0.055	2.185	2.52
Double Difference: Pseudoranges	5.580	0.075	2.833	-
Double Difference: Carrier Phase	5.622	0.083	2.185	1.03

Collection G is yielding very similar results to those of Collection E, especially in the case of the single point solutions. This is expected, since the rover is simply shifted from its position in Collection E. This does, however, increase the error statistics for the differential solutions, particularly in the horizontal direction. This is because the separation of the baselines reduces the ability of the model to mitigate atmospheric errors and is reflected in the RMS error statistics in Table 10.

It is important to note that the altitudes in both sets are assumed to be identical, which is potentially more ideal for the differential models depending on the satellite geometry, which

could be reducing the effect of increasing the baseline. A set of data will also be produced varying the altitude difference between the rover and base antennas.

4.1.2.8 Collection H: 5 ppm Bias, 1 m Noise Standard Deviation, 1 km Differential

Baseline With 5m Vertical Offset

Figure 29: Horizontal and Vertical Errors for Simulation Set H

Figure 30: Horizontal 95 Percent Confidence Ellipses for Simulation Set H

Table 11: Mean Error Statistics for Set H

Technique	3DRMS Error (m)	Vertical Error (m)	Error Standard deviation (m)	Average Ambiguity Error
Single Point: Pseudoranges	33.450	32.954	1.824	-
Single Point: Carrier Phase	33.476	32.977	1.897	-19.71
Single Difference: Pseudoranges	4.682	0.189	2.440	-
Single Difference: Carrier Phase	4.749	0.191	2.514	3.56
Double Difference: Pseudoranges	4.845	0.088	2.579	-
Double Difference: Carrier Phase	4.841	0.097	2.597	-0.02

From Table 11, it is apparent that the impact of the carrier phase on datasets with set E biases and noise was not strongly affected by the ground geometry of the receivers. Despite 1 km

of separation as well as 5m of vertical offset, no processing method had any significant benefit using the epoch-by-epoch integer ambiguity carrier phase measurements.

It is interesting to note that the double difference was more effective in mitigating the effect of the vertical offset on the vertical error than the single difference was.

4.1.2.9 Positioning Simulation Summary

When observing the results from simulation collections A through H, some general patterns are evident. First, the pseudorange noise effect is proportional to the positioning error standard deviation. From the measurement model and from error propagation, this is expected. It is also reflected in the size of the confidence ellipse produced under different noise conditions. Larger confidence ellipses indicate more noise in the positioning solution.

Second, the amount of atmospheric bias added to the range to generate the pseudorange is proportional to the positioning bias, especially in the single point measurement model. This is expected, because the single point model has no inherent mechanism to isolate unmitigated atmospheric bias, while differential models do so with the differencing between base and rover ranges.

Another pattern is that the baseline distance between base and rover antennas is proportional to the positioning bias (especially in the vertical direction). This is also expected, because as the baseline distance increases, so does the difference between the base-to-satellite range versus the rover-to-satellite range. Hence, the bias is not cancelled as effectively by the differencing model when the baseline distance is large.

4.2 Didsbury Control Marker Survey

To validate the single point positioning algorithms for pseudorange-only as well as carrier phase solutions, an Alberta Survey Control Marker (ASCM) was located in Didsbury Alberta which has the following characteristics:

Table 12: ASCM 764522 Information

Latitude (DMS)	52° 40'02.81212" North
Longitude (DMS)	114° 08'35.55293" West
Elevation (m)	1041.004
Type	ASC cap on 5 cm square heavy wall steel pipe

The cap in Table 12 is situated in a relatively ideal location outside of the rural town, minimizing the possibility of multipath effects:

Figure 31: Location of ASCM 764522 Outside of Didsbury, Alberta. Brass Cap (Left) With Precise Coordinates Used for Antenna Placement. Conditions are Ideal for Positioning in Open Sky (Right).

As can be observed from Figure 30, the location of the ASCM has minimal multipath potential. The iron post east of the cap and the wooden stakes may cause some minimal

multipath effects, but it is a superior setup to an urban setting when considering multipath and access to optimal satellite geometry.

In order to conduct an experimental survey of the marker, test receiver 1 was utilized. The antenna was centered on top of the cap as accurately as possible and left in a static position to log data via the Reachview Android application.

Data was logged in the RINEX format and processed using the C++ software developed for the purposes of this research, with and without the use of the L1 carrier phase observable. The same data was also processed using RTKPost software using the Saastamoinen troposphere model and the broadcast ionosphere model for comparison. Results are as follows:

Figure 32: True Directional Error for Processed Rural ASCM Survey Data When Compared to Archived ASC Coordinates

Table 13: Mean Error Statistics for Rural ASCM Survey Data Processing For Each Processing Method

Processing Model	3DRMS Error (m)	Vertical Error (m)	Standard Deviation (m)
Pseudoranges	12.907	-12.174	0.991
Carrier Phase	12.921	-12.176	1.011
RTKPost Software Solution	19.898	-19.635	0.833

There are several interesting trends which immediately differentiate this real processed data from the processed simulated data. One such trend is that there is clear temporal variability in the error bias which was not observed in the simulations. One major reason for this is that satellite geometry was kept constant in simulations in order to isolate white noise effects while the satellite geometry changes over time in real data, illustrated in Figure 32:

Figure 33: Skyplot for Visualizing the Satellite Headings and Elevation Angles Relative to the Antenna Over the Data Collection Period for Didsbury ASCM Survey

In fact, the effect of changing satellite geometry is clearly evident in the overall trend curves in all directions in Figure 32. This makes it very difficult to directly measure the effect of white noise in this data. The standard deviations computed in Table 13 cannot be directly interpreted as the standard deviation of the white noise in the solution. In the absence of more information, this forces the user to rely only on the dilution of precision values from propagating an assumed pseudorange noise through the least squares solution. The standard deviation of the error in Table 13 suggests that test receiver 1's noise is consistent with a pseudorange noise of between 10 cm and 1 m.

4.3 Carleton University Control Point Survey

In order to assess test receiver 3's positioning capabilities, raw RINEX data was collected with the rear antenna placed on top of a control point in Ottawa, Ontario which was surveyed in aggregate by the Carleton University Civil Engineering Department. Computed GPS positions from raw data were compared with coordinates provided by the Civil Engineering Department which were quoted as having a maximum estimated 6cm of 3DRMS error.

Table 14: Properties of Local Control Point ID 2003 From Civil Engineering Local Carleton Campus Survey

Latitude (DMS)	45° 23'02.55181" North
Longitude (DMS)	75° 41'53.11418" West
Elevation (m)	64.393
Type	Bronze cap embedded in walking path

Figure 34: Attitude Survey Control Point Locations. Rear Antenna Environment (Top Left), Rear Antenna Control Point in Concrete (Top Right), Front Antenna Environment (Bottom Left), Front Antenna Control Point in Concrete (Bottom Right)

Figure 34 shows the location of the control point on which this experimental survey was conducted. The location of this control point has significantly greater multipath potential than the control point used in the rural Didsbury survey. There are tall permanent structures and numerous reflective surfaces nearby such as windows and smooth metal surfaces, which have a high potential for multipath, especially when paired with a wide correlator receiver, which is one attribute of test receiver 3.

In order to conduct a survey of the control point, test receiver 3 was used, which is an integrated system with an IMU. The rear antenna/receiver was used in conjunction with a round

steel plate to act as a ground plane. The antenna was centered on top of the cap as accurately as possible and left in a static position to log raw data via the VectorNav Control Center application.

Data was logged in a RINEX format and processed using the C++ software developed for the purposes of this research, with the pseudorange-only single point model. The same data was also processed with the VN-300 real time filtered result for comparison. Results are as follows:

Figure 35 True Directional Error for Processed Carleton Campus Control Point Survey Data When Compared to Civil Engineering Department Control Coordinates

Table 15: Mean Error Statistics for Carleton University Survey Data Processing For Each Processing Method

Processing Model	3DRMS Error (m)	Vertical Error (m)	Standard Deviation (m)
Pseudoranges (Local Software)	32.096	-32.007	4.053
VN-300 Solution	31.862	-31.819	2.302

As can be observed in Table 15 and Figure 33, there is much more temporal variability of coordinates computed in the Carleton experimental survey when compared with values in Figure 31 and Table 13. While there does appear to be more noise in these coordinates, it is not clear how much of the standard deviation can be accounted for by temporal variability in biases, especially multipath. The mean vertical error is much greater for the test receiver 3 survey than with the rural Alberta test receiver 1 survey, which suggests that the receiver was subject to more bias in general. Part of this can be accounted for by the lower elevation of the Carleton location, but models are used to mitigate those biases, so multipath is likely playing a significant factor in the error. The satellite geometry was in fact better in this survey compared to the rural survey, as seen in Figure 34:

Figure 36: Skyplot for Visualizing the Satellite Headings and Elevation Angles Relative to the Antenna Over the Data Collection Period Carleton University Survey on February 1st, 2019

While there is an absence of satellite availability northwest of the antenna setup in Figure 34 (likely because they are obstructed by a nearby building), the solution has 1 more available satellite than the rural survey did, which should improve positioning precision. This further supports that the bias is being affected by multipath.

While the error standard deviation in Table 15 is consistent with simulation data with pseudorange noise between 1 m and 3 m, the noise profile of the receiver is likely on the lower end, closer to 1 m.

4.4 Pitch and Heading Simulation

4.4.1 Simulation Results

Using satellite and marker geometry identical to that of the first epoch of the Didsbury ASCM experimental survey, sets of results were simulated in a variety of bias, noise and differential baseline conditions. In addition, an antenna is added in a location with a predetermined offset from the first rover to act as a frontal rover for partial attitude determination. For the pitch and attitude simulations, the base offset from the rear rover is fixed at 10 km and pseudorange bias is fixed at 3 ppm. All sets are found in Appendix A through C, with antenna separation being varied between 1 m and 90 m, while receiver pseudorange noise being varied between 10 cm and 3 m standard deviation. Set A-1 is displayed in this section in order to explain the dataset layouts.

Set A-1: 1 m Dual Antenna Separation

Figure 37: Computed and Measured Headings for Simulation Set A-1

Figure 38: Computed and Measured Pitch for Simulation Set A-1

The first 2 figures presented for each dataset are computed and measured values for pitch and heading for each method and model, over the course of the 100-sample set. It is important to note that the true simulated value of heading is 45 degrees, while the true pitch is 0 degrees. It is also important to note that this does not represent a temporal analysis. The satellite geometry is constant and therefore all variation is due to noise.

Figure 39: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-1

The next figure presented for each set is a pair of 95% confidence ellipses plotted in the horizontal plane. The red ellipse is centered on the true coordinates of the rear GPS antenna, while the green ellipse is centered on the true coordinates of the front antenna. The ellipse represents a horizontal region where 95% of all computed points would be expected to fall inside of if the propagated, expected standard deviations reflect the true standard deviation.

Table 16: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	15.769	12.779	9.117
Single Point GPS (Carrier Phase)	33.264	24.640	0.050
Single Difference GPS (Pseudoranges)	22.561	17.098	10.739
Single Difference GPS (Carrier Phase)	27.603	17.057	0.038
Double Difference GPS (Pseudoranges)	23.283	20.526	8.490
Double Difference GPS (Carrier Phase)	30.602	22.521	0.061
IMU Measurement	0.007	-	0.006

Table 17: Mean Global Error Statistics of Unfused Heading for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	19.880	12.825	53.769
Single Point GPS (Carrier Phase)	62.022	28.296	0.163
Single Difference GPS (Pseudoranges)	47.757	22.831	35.088
Single Difference GPS (Carrier Phase)	56.885	20.697	0.068
Double Difference GPS (Pseudoranges)	42.724	19.831	28.739
Double Difference GPS (Carrier Phase)	59.971	21.806	0.146
IMU Measurement	0.320	-	0.269

Table 18: Mean Global Error Statistics of GPS/IMU Fused Pitch for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.092	0.033	0.194
Single Point GPS (Carrier Phase)	6.742	4.346	0.024
Single Difference GPS (Pseudoranges)	2.316	0.266	0.194
Single Difference GPS (Carrier Phase)	6.337	3.498	0.018
Double Difference GPS (Pseudoranges)	0.049	0.028	0.198
Double Difference GPS (Carrier Phase)	7.714	4.151	0.029
IMU Measurement	0.007	-	0.006

Table 19: Mean Global Error Statistics of GPS/IMU Fused Heading for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.149	1.123	3.663
Single Point GPS (Carrier Phase)	54.142	21.719	0.077
Single Difference GPS (Pseudoranges)	9.489	2.301	3.828
Single Difference GPS (Carrier Phase)	49.911	17.033	0.034
Double Difference GPS (Pseudoranges)	2.036	0.971	4.181
Double Difference GPS (Carrier Phase)	45.847	17.767	0.072
IMU Measurement	0.320	-	0.269

Next, each data set lists 4 tables of global mean error statistics for the heading and pitch. The first pair of tables lists statistics for the GPS only solutions, while the second pair lists the statistics for the solutions which fused the GPS with IMU solutions.

Table 20: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	55	59	60	64	79	86	91	95
Single Point GPS (Carrier Phase)	65	65	65	65	65	75	79	85
Single Difference GPS (Pseudoranges)	57	59	60	66	71	75	77	81
Single Difference GPS (Carrier Phase)	77	77	77	77	81	88	91	92
Double Difference GPS (Pseudoranges)	45	46	48	56	67	74	83	87
Double Difference GPS (Carrier Phase)	74	74	74	74	75	81	85	90
IMU Measured	100	100	100	100	100	100	100	100

Table 21: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	45	46	51	56	71	81	85	89
Single Point GPS (Carrier Phase)	58	58	66	66	71	71	78	81
Single Difference GPS (Pseudoranges)	58	60	61	64	68	73	75	77
Single Difference GPS (Carrier Phase)	70	70	71	75	82	82	82	82
Double Difference GPS (Pseudoranges)	53	53	54	56	65	74	77	80
Double Difference GPS (Carrier Phase)	71	72	72	74	79	79	79	79
IMU Measured	91	100	100	100	100	100	100	100

Table 22: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	66	80	94	96	96	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	77	77	84	93	95	95	99	99
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	74	74	83	97	97	97	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 23: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	70	82	91	96	100	100	100	100
Single Point GPS (Carrier Phase)	58	59	66	70	72	77	87	88
Single Difference GPS (Pseudoranges)	79	86	90	94	97	97	98	98
Single Difference GPS (Carrier Phase)	70	71	71	81	82	87	89	89
Double Difference GPS (Pseudoranges)	74	83	92	96	99	100	100	100
Double Difference GPS (Carrier Phase)	70	71	72	77	80	80	82	85
IMU Measured	91	100	100	100	100	100	100	100

Next, each data set lists 4 tables of specification success rates for the heading and pitch.

The first pair of tables contains success rates for the GPS only solutions, while the second pair

lists success rates for the solutions which fused the GPS with IMU solutions. Rates are listed for all specification categories.

Table 24: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-1

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.858	0.107
Single Difference GPS (Pseudoranges)	0.937	0.146
Double Difference GPS (Pseudoranges)	0.573	0.046

Finally, each dataset presents a table of correlation coefficients between the computed pitch and heading. These are taken from the off diagonals of the covariance matrix of the unknowns.

4.5 Mission Design Using Simulation Results

The simulation results collected can be used to solve design problems for GPS partial attitude determination systems. The 2 design parameters which can be solved for are one of either antenna separation required, or the pseudorange noise requirement for the receiver.

4.5.1 Example when Solving for Pseudorange Noise

Problem: Suppose that a navigation platform allows for a maximum 20 m of GPS antenna separation. What is the maximum allowable pseudorange noise on the receiver if 80% of the pitch measurements need to fall within a range of plus or minus 2 degrees of error?

Solution: Each collected set has statistics tabulated for 10 m and 30 m separations.

Therefore, endpoint values will be chosen from those sets in series' which have statistics bounding the 80% required specification. The following summarizes the values and sets used:

Table 25: Statistics from Selected Data Sets for Interpolation of Pseudorange Noise

Simulation Set	Antenna Separation (m)	Pseudorange Noise (m)	Success Rate of Meeting ± 2 Degree Specification (%)
B-4	10	1	64
B-5	30	1	87
A-4	10	0.1	93
A-5	30	0.1	100

First, interpolations will be done to compute 2-degree specification success rates for each series at 20m separation:

$$\text{Series A Rate (20m)} = 93 + \frac{(100 - 93)(20 - 10)}{(30 - 10)} \quad (157)$$

$$= 96.5\%$$

$$\text{Series B Rate (20m)} = 64 + \frac{(87 - 64)(20 - 10)}{(30 - 10)} \quad (158)$$

$$= 75.5\%$$

Now, a final interpolation is done to find a maximum pseudorange noise to meet an 80% specification:

$$\text{pseudorange noise (80\% success)} = 0.1 + \frac{(80 - 75.5)(1 - 0.1)}{(96.5 - 75.5)} \quad (159)$$

$$= 0.19\text{m}$$

This suggests that at most 20 cm are allowable in meeting the required specifications. For the designer, this will indicate that a high-quality receiver with access to the precise p code is a minimum requirement here.

4.5.2 Example when Solving for Antenna Separation

Problem: Suppose that 2 receivers with 2 m standard deviation of random error due to pseudorange noise are available for use in developing a low-cost partial attitude system. What is the minimum antenna separation required if the heading measurements must meet a required maximum of 2.5 degrees standard deviation?

Solution: Data series' are available with pseudorange noise of 1 m and 3 m. The same strategy will be used as in the first problem, finding sets in Series B and Series C which encompass the required standard deviation. These sets are summarized below:

Table 26: Statistics from Selected Data Sets for Interpolation of Antenna Separation

Simulation Set	Antenna Separation (m)	Pseudorange Noise (m)	Heading Standard Deviation (Degrees)
B-4	10	1	6.45
B-5	30	1	1.98
C-6	50	3	2.91
C-7	70	3	1.63

First, interpolations are done to compute the antenna separation at which each series meets the design requirement:

$$Separation (B) = 10 + \frac{(2.5 - 6.45)(30 - 10)}{(1.98 - 6.45)} \quad (160)$$

$$= 27.673\text{m}$$

$$Separation (C) = 50 + \frac{(2.5 - 2.91)(70 - 50)}{(1.63 - 2.91)} \quad (161)$$

$$= 56.406\text{m}$$

Now the interpolated separations are used to interpolate the required separation with 2 m of pseudorange noise:

$$\begin{aligned}
 \text{Separation (2m Noise)} &= 27.673\text{m} + \frac{(2 - 1)(56.406 - 27.673)}{(3 - 1)} & (162) \\
 &= 42.040\text{m}
 \end{aligned}$$

This indicates that at least 42 m of separation is required to meet the 2.5 degree standard deviation specification. Most passenger aircraft meet this specification by their length. In fact, some aircraft meet this requirement with their wingspan, such as the Boeing 747. This means such an aircraft could double as a roll determiner as well as meeting this heading requirement given this low-cost GPS specification.

5 GPS Pitch and Heading Experimental Control

Point Survey

In order to validate the simulated results, an experimental survey was conducted at the Carleton University campus using 2 different GPS receivers of which the antennas were placed atop 2 control points previously surveyed by the Civil Engineering Department to high precision. This provides a large advantage for validation, as the known coordinates of the control points can be used to directly assess positioning accuracy as well as providing a true value of the pitch and heading defined by the 2 points.

5.1 Control Points

The 2 control points used for the survey are annotated in Figure 38:

Figure 40: Locations of Control Points Used for Pitch and Heading Survey on Carleton University Campus

As can be seen in Figure 115, the location of the 2 control points have a somewhat open view of the sky, but also have the potential for multipath hazards. Control point 2003 more likely will suffer from multipath due to window reflections as well as obscuring of satellites by the buildings themselves. Control point 2005 is situated very close to the roadside, and therefore the parked cars which were present at the time of the survey likely were the largest multipath contributors. This is shown in Figure 41:

Figure 41: Attitude Experiment Location. Top Left: Rear GPS Environment. Top Right: Rear Control Point. Bottom Left: Front GPS Environment. Bottom Right: Front GPS Control Point

The coordinates of the control points are listed below:

Table 27: Geographic Coordinates of Control Points in Attitude Survey, WGS84 Used as Ellipsoidal Datum for Height Reference

Control Point ID	Latitude			Longitude			Height (m)
	Degrees	Minutes	Seconds	Degrees	Minutes	Seconds	
2003	45	23	2.552	75	41	53.114	64.393
2005	45	23	4.891	75	41	55.567	64.158

The control markers for both points were a bronze cap engraved with the point ID and embedded in the concrete walking paths at their locations. Test receiver 2 was placed on top of control point 2003 and the Geo++ RINEX Logger application was used to log raw GPS raw RINEX data captured by the internal GPS receiver of the cell phone. Unlike test receiver 1, test receiver 2 does not have the capacity to log carrier phase measurements, so the carrier phase will not be used in the pitch and heading analysis for this survey. The test receiver 1 antenna was placed on top of control point 2005, using the ReachView application to log raw RINEX data for it, just as was done in the Didsbury survey.

5.2 Positioning Results

In order to assess the true positioning error for each receiver during this survey, the computed coordinates by each receiver were compared directly with the known coordinates of the markers by transforming computed coordinates into local vertical frames using their marker coordinates as reference points:

Figure 42: True Error of Computed GPS Coordinates for the Front and Rear GPS Receivers Using Known Marker Locations as a Reference Position

As can be observed in Figure 117, test receiver 1 was collecting data for a longer period than receiver 2 was. It is also important to note that the cell phone collected data at a rate of 1 Hz while receiver 1 collected data at a rate of 5 Hz. Therefore, in order to use computed positions for pitch and heading computation, the cell phone data will be interpolated in order to obtain positions at times synced to receiver 1 data. It is also very interesting to note the noticeable difference in data quality between the two receivers. Receiver 2 was subject to a significantly higher amount of noise and likely multipath as well. Some of that can be explained by the

location, but more of this disparity is likely explainable by the lower receiver quality in the cell phone. Global error statistics for each receiver are summarized below:

Table 28: Global Mean Positioning Error Statistics for Each Receiver Across Carleton Pitch and Heading Survey

Receiver	3DRMS Error (m)	Vertical Error (m)	Standard Deviation (m)
Test Receiver 1	34.095	-33.890	5.873
Test Receiver 2	41.871	-35.372	24.010

Table 28 shows that, while the receiver 1 and receiver 2 yielded somewhat comparable mean true errors, the receiver 2 data was far noisier than that of receiver 1. This may have implications in the comparison with simulation data, because all simulated data used an assumption that both receivers used to compute attitude have the same noise characteristics. Some of the trends in the data may be accounted for by changing satellite geometry, which is illustrated below:

Figure 43: Sky Plot of Trajectories of Available GPS Satellites During Carleton Pitch and Heading Survey

While not each of the satellites shown in Figure 118 are available to both receivers for the duration of the survey, it is still interesting to note that neither receiver had a line of sight to a GPS satellite directly north of it in the sky, which may affect the quality of positioning solutions in the north direction. In addition, solutions which have different available satellite geometry between satellites could induce some biases to the pitch and heading computations. This is because the 2 positions used for the angle computation would have slightly different biases in the solution caused by a disparity of satellite geometry. In an open sky environment this would not be a problem.

5.3 Pitch and Heading Results

To directly assess the accuracy of the pitch and heading results, coordinates were computed for control point 2005 relative to control point 2003 in a local vertical frame. Because control point 2003 represents the origin of this system, the northings, eastings and vertical coordinates of control point 2005 can be said to represent the change in eastings, northings and vertical of the front and rear points on a platform for orientation computation, with control point 2005 representing the front and control point 2003 representing the rear. This is also very convenient because the baseline distance between control points is 89.776 m. This very close to the 90m baseline, which was the longest baseline used in simulations, making a direct comparison more feasible. Positioning results laid out in the horizontal plane are shown below:

Figure 44: Positioning Results in Horizontal Plane for Test Receivers 1 and 2 in Carleton Pitch and Heading Survey

As can be seen in Figure 41, while the computed confidence ellipses somewhat effectively encompass the computed coordinates, they do not contain all of them. This is consistent with the simulation results, where 95% confidence ellipses were typically optimistic compared to the observed values. This was true even when pseudorange noise was forced to conform to a known standard deviation in simulation, so it is not likely caused to an optimistic choice of pseudorange noise in these results. Time series graphs of the pitch and heading errors are as follows:

Figure 45: True Pitch Error of Undedicated GPS Pitch and Roll System Using Test Receivers 1 and 2 in Carleton Survey. Raw Solutions Using Interpolated Data as Well as Kalman Filtered Results are Shown

Figure 46: True Heading Error of Undedicated GPS Pitch and Roll System Using Test Receivers 1 and 2 in Carleton Survey. Raw Solutions Using Interpolated Data as Well as Kalman Filtered Results are Shown

There are some interesting trends to be observed in Figures 42 and 43. First, it appears that the interpolation of the cell phone data has had a false smoothing effect on both the pitch and heading results. While this does not improve the solution where receiver 1 and receiver 2 positions are chronologically synced naturally, it does appear to induce solutions which are clustered closer to the true value than the random data points alone. It is also interesting that the pitch had larger peaks in error than the heading did. This is possibly due to the large variation in vertical error between and within datasets, which could be caused in part by multipath effects. The vertical coordinates are not used in the heading computation, so the heading is not affected by that. Global error statistics are summarized below:

Table 29: Global Mean Error Statistics of Undedicated GPS Pitch and Heading System During Carleton Survey

Method		Observed Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Raw Interpolation	Pitch	26.991	0.647	13.881
	Heading	5.567	1.446	4.287
Kalman Filter	Pitch	3.609	-	0.714
	Heading	4.449	-	3.539

Table 30: Portion of Observations Obtaining Desired Specifications for Each Method and Metric During Carleton Survey

Metric / Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Pitch (Interpolation)	47	48	49	56	68	77	85	90
Heading (Interpolation)	47	50	59	78	94	99	100	100
Pitch (Kalman Filter)	88	96	98	98	98	98	98	98
Heading (Kalman Filter)	45	54	63	87	96	99	100	100

Using simulation results with a 90 m inter-receiver baseline as a basis for comparison, it is interesting to note that the results in Table 29 most closely resemble simulated dataset C-6 in magnitude, while the fact that heading was more successful in reaching specifications than pitch

is more in keeping with simulation dataset C-8. Dataset C-6 had a baseline separation of only 50 m compared to this dataset having approximately 90 m of separation. This suggests that more error is present in the survey than would be expected from a 3 m pseudorange noise solution with noise alone. The standard deviation of the receiver 2 data solutions is much higher than any simulated solutions having 3 m of pseudorange noise, which suggests that receiver 2 likely has a combination of more pseudorange noise as well as more multipath at play here. It is likely that simulations modelled with noise greater than 3 m would yield pitch and heading computation error statistics comparable to those found in this survey.

One important thing to extrapolate from these results is that, for a given platform which has at least 90 m of front to rear separation, a GPS pitch and heading determination system could be constructed at low cost and still achieve desired specifications in some applications. In fact, this system benefitted significantly from a Kalman filter of the raw data. This is especially true for the pitch. The only specification category which the filtered pitch struggled to achieve was $\pm 0.5^\circ$. Depending on the application, this could be very acceptable. This includes nautical applications where the pitch and heading are important for navigation purposes.

5.4 Effects of Antenna Separation on Pitch and Heading

Accuracy

Heading and pitch are angular quantities confined to a 180-degree range and 360-degree range, respectively. This means that the standard deviation is not a sufficient metric for assessing the effectiveness of a GPS attitude system. This is especially true at short antenna separation distances, based on the simulated results in Appendix A through C. The estimated standard deviations of heading exceed more than 360 degrees when noise is high and antenna separation

is small. This does not make sense due to the cyclical nature of the angular quantities, because that standard deviation could never be observed in reality. It is geometrically understandable why this happens, because the general trend yields increasing partial attitude noise with decreasing antenna separation and increasing pseudorange noise. Suppose that two receivers with equal pseudorange noise are utilized in a pitch and heading system. Also suppose that a confidence ellipse can be drawn which contains 100% of possible computed positions for each receiver. The following figure illustrates the nature of the pitch and heading computation as it varies with antenna separation:

Figure 47: True Headings vs Extreme Error Computed Headings, Receiver Noise Properties Fixed at a Constant with Varying Antenna Separation. Long separation (Top), Medium Separation (Middle), Short Separation (Bottom)

As can be observed in Figure 44, far enough separation between antennas minimizes the maximum error in computed attitude. This is true regardless of how much noise a receiver has in the position solution. A noisier receiver simply requires a longer separation to achieve the same heading and pitch computation precision. On the other hand, at close distances, there is a region in which solutions for the two receiver positions overlap (magenta). This is a critical region in which solutions for heading could be in any quadrant. In order to avoid this problem, the separation must be increased or a higher quality receiver must be used with less pseudorange noise. The simulations produced in this thesis will aid in solving this design problem.

6 Summary and Recommendations

This section will reiterate some of the motivation of this work as well as summarize the findings and contributions of the author's research to the field of positioning and navigation. Subsequently, it will lay out possibilities for future expansion of the thesis.

6.1 Thesis Summary

Positioning and navigation are vital components of any moving platform system, whether they are autonomous or manually driven. Small platforms such as autonomous rovers or autonomous vehicles, or large platforms such as offshore seismic or cargo vessels, or passenger aircraft, all rely on sensors to aid in this task. As with any design problem, there is always also a need to minimize cost while still meeting specifications. Such is the case for GNSS sensors, particularly if they are used for attitude or partial attitude determination. While the current status quo accepts use of the carrier phase observable as a necessity to have sufficient precision for most missions, the author's knowledge the feasibility of low-cost receivers, especially using pseudoranges only, has not been quantified in a way that is easily accessible to designers.

This thesis approached the design problem using an exhaustive simulation approach, validating those simulations using control point surveys with low-cost sensors. First a positioning simulation was conducted by generating biased pseudorange and carrier phase measurements from satellites with known positions to receivers with known positions. The effects of atmospheric errors, multipath and noise were quantified using 3 different positioning models: single point positioning, between receiver single differences and double differences. Differential solutions were shown to mitigate atmospheric biases, while not mitigating multipath effects and increasing noise effects relative to single point solutions.

The positioning simulation was validated by conducting an experimental control point survey in Didsbury, Alberta on an Alberta Survey Control Marker (ASCM) with known coordinates. Data was collected using test receiver 1, and positions were computed in single point mode using pseudoranges as well as using an integer search to optimize carrier phase ambiguities. The carrier phase was found to not be helpful to the position calculation due to the search space being much too small to encompass the bias caused by atmospheric effects as well as due to only one frequency of observations being available on the low-cost receiver. The author's positioning methods were compared to results obtained by processing the same raw data with RTK Post software and while the author yielded higher error in the horizontal, the result was much closer to the true coordinate in the horizontal overall. Temporal variability of satellite geometry was observed to cause trends in the positioning results which were not simply caused by random noise. While the standard deviation was used to estimate a noise category for the receiver, more study would be required if the receiver noise characteristics are to be determined with precision. This is true for the VN-300 system (test receiver 3) as well as the Huawei P20 (test receiver 2) which were also utilized in this research.

The simulation framework was expanded into a thorough pitch and heading simulation. Raw measurements were generated just as was done with the positioning simulation, but 2 rover antennas with known positions were utilized instead of 1. With the known pitch and heading of the system, a simulated partial attitude measurement was generated by adding gaussian noise with the standard deviation observed with the VN-300 recording pitch and heading in a static environment for approximately 10 minutes. The simulation computed pitch and heading using GPS coordinates only, IMU only and also a GPS/IMU fused solution using a weighted average approach with the estimated standard deviation of the GPS solution and the known standard

deviation of the IMU. Interestingly the GPS seldom improved the IMU solution in fusion, but this might be due to lack of access to a Kalman filter because the simulation only repeated measurements with static satellite geometry rather than using temporally variable constellations. It is also important to note that single point solutions were found to perform better than differential solutions due to cancellation of bias effects and reduced noise by comparison. This may be counterintuitive to many designers, because in general, using more sensors is correlated with higher accuracy and precision.

The partial attitude simulation was consisted of data sets and data series'. A set determines the dual antenna separation distance, while a series determines the pseudorange noise (and by extension the receiver quality) of the data set. Pseudorange noise was varied at 10 cm, 1 m and 3 m to yield 3 series', and antenna separation was varied from 1 m up to 90 m for a total of 8 data sets per series. A double interpolation method was proposed which could aid a mission designer in receiver selection or in setting out the necessary antenna separation distance for receivers with known noise characteristics. This was found to give a novel method of directly quantifying the necessary mission design criteria rather than reverting to higher cost receivers, differential solutions to access the carrier phase or buying an IMU because of known noise characteristics. This has potential to save costs as well as make mission designing an easier task on the whole.

The partial attitude simulation was validated with a control point survey on Carleton University campus in Ottawa, Ontario. Test receivers 1 and 2 were placed on 2 control points with known coordinates. Receiver 2 had a much noisier position solution than the receiver 1, which makes direct comparison to the simulation difficult. However, several findings from the survey validated the simulations. For instance, the control points were 90 m apart, as was the

case with the 8th simulation set. Just as was observed in simulation, the data collected yielded pitch and heading with very little bias due to cancellation of atmospheric effects, which supports the earlier contention that single point solutions are preferable to differential solutions. In addition, many angular specification categories were met with high success rates due to the significant separation of the antennas. While this does not indicate that low-cost undedicated systems are feasible for small platforms, it does hold implications for larger platforms. For example, many marine seismic vessels meet this 90 m length description as a platform, so this suggests that a low-cost solution for pitch and heading could be considered by a ship designer, assuming a safety factor is used to consider possible multipath effects from the water surface. A simple static Kalman filter was also shown to drastically improve the partial attitude solution when propagated covariance information from the least squares solution was utilized.

It is also interesting to note that Chaochao Wang [33] did not factor in a specific attitude precision into his design as a constraint, while this research was directed toward solving the problem of maximizing quality and minimizing cost quantitatively. Instead, Wang decided to integrate gyroscopic sensors onto the design platform in order to achieve a relative improvement to the attitude solution while sacrificing cost. In the case of JiunHang Keong's [32] research, the relationship between inter-antenna distance and heading accuracy is discussed qualitatively but is not directly factored into the design parameters. Instead, the farthest possible distance allowable while maintaining the platform heading parallel to GNSS vector heading was used for a combination of accuracy maximization and computation convenience. To the author's knowledge, the interpolation technique presented in this thesis is currently the only quantitative technique which directly addresses the precision and accuracy design constraint problem.

6.2 Future Recommendations

While the author has made some novel contributions to the field of partial attitude determination, some steps could be taken to expand on the research:

- The LAMBDA ambiguity resolution technique could be implemented in order to increase viability of the carrier phase. This would likely allow for ambiguity resolution when a base station is utilized
- The simulation could be expanded to include three-receiver platforms. This would allow for computation of all orientation angles, accurately representing a full attitude determination system and would add complexity to the angle computations.
- The simulation could be expanded to a temporal analysis with changing satellite geometry. While the author predicts that the direct effects of satellite geometry would be minimal due to the close proximity of antennas to each other, there could be multipath implications due to its dependence on satellite geometry
- A different fusion technique could be used to improve the result from the simple weighted average used between the IMU and GPS solutions in simulation. This technique rarely improved the IMU-only solution; therefore, it would be worth testing a Kalman filter in a temporal analysis as an alternative
- A pair of receivers could be tested to determine the pseudorange noise properties and allow for a direct validation of the simulation. This would require connecting two receivers of the same quality to the same antenna, then logging simultaneous data. The difference in measured pseudoranges could be used to compute a standard deviation over the data set.

- A dedicated system of two GPS receivers sharing an oscillator could be built where the budget allows. This would allow for partial attitude computation without interpolating or extrapolating data from 1 receiver.

7 Conclusion

To conclude, the author has successfully developed a novel, quantitative method for low-cost, GNSS partial attitude mission design using interpolation of simulation data. Key findings are as follows:

- Pseudorange noise is proportional to pitch and heading noise but inversely proportional to success rates of meeting specifications. This can be used as a design requirement and minimum receiver quality (maximum noise) can be computed by interpolating the author's simulation results.
- Antenna separation is inversely proportional to pitch and heading noise but proportional to success rates of meeting specifications. This can be used to interpolate minimum antenna separation for a system with known receiver noise.
- Single point positioning solutions are preferable to differential solutions when being used to compute attitude, unless the differential solution is used to optimize the search space for carrier phase ambiguity resolution.
- If high noise receivers have a sufficiently small antenna separation (approximately 3 m for a system with 1m receiver noise standard deviation 10m for a system with 3 m receiver noise standard deviation), there is a potential for heading solutions in any quadrant, which should be mitigated against strongly as it could have adverse effects on navigation.
- Kalman filtering can significantly improve the solutions from GPS-only partial attitude systems when the least squares covariance matrix is used to construct the measurement noise.

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Appendix A: 10 cm Pseudorange Noise Standard

Deviation Pitch and Heading Simulation Results

Set A-1: 1 m Dual Antenna Separation

Figure 48: Computed and Measured Headings for Simulation Set A-1

Figure 49: Computed and Measured Pitch for Simulation Set A-1

Figure 50: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-1

Table 31: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	15.769	12.779	9.117
Single Point GPS (Carrier Phase)	33.264	24.640	0.050
Single Difference GPS (Pseudoranges)	22.561	17.098	10.739
Single Difference GPS (Carrier Phase)	27.603	17.057	0.038
Double Difference GPS (Pseudoranges)	23.283	20.526	8.490
Double Difference GPS (Carrier Phase)	30.602	22.521	0.061
IMU Measurement	0.007	-	0.006

Table 32: Mean Global Error Statistics of Unfused Heading for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	19.880	12.825	53.769
Single Point GPS (Carrier Phase)	62.022	28.296	0.163
Single Difference GPS (Pseudoranges)	47.757	22.831	35.088
Single Difference GPS (Carrier Phase)	56.885	20.697	0.068
Double Difference GPS (Pseudoranges)	42.724	19.831	28.739
Double Difference GPS (Carrier Phase)	59.971	21.806	0.146
IMU Measurement	0.320	-	0.269

Table 33: Mean Global Error Statistics of GPS/IMU Fused Pitch for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.092	0.033	0.194
Single Point GPS (Carrier Phase)	6.742	4.346	0.024
Single Difference GPS (Pseudoranges)	2.316	0.266	0.194
Single Difference GPS (Carrier Phase)	6.337	3.498	0.018
Double Difference GPS (Pseudoranges)	0.049	0.028	0.198
Double Difference GPS (Carrier Phase)	7.714	4.151	0.029
IMU Measurement	0.007	-	0.006

Table 34: Mean Global Error Statistics of GPS/IMU Fused Heading for Simulation Set A-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.149	1.123	3.663
Single Point GPS (Carrier Phase)	54.142	21.719	0.077
Single Difference GPS (Pseudoranges)	9.489	2.301	3.828
Single Difference GPS (Carrier Phase)	49.911	17.033	0.034
Double Difference GPS (Pseudoranges)	2.036	0.971	4.181
Double Difference GPS (Carrier Phase)	45.847	17.767	0.072
IMU Measurement	0.320	-	0.269

Table 35: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	55	59	60	64	79	86	91	95
Single Point GPS (Carrier Phase)	65	65	65	65	65	75	79	85
Single Difference GPS (Pseudoranges)	57	59	60	66	71	75	77	81
Single Difference GPS (Carrier Phase)	77	77	77	77	81	88	91	92
Double Difference GPS (Pseudoranges)	45	46	48	56	67	74	83	87
Double Difference GPS (Carrier Phase)	74	74	74	74	75	81	85	90
IMU Measured	100	100	100	100	100	100	100	100

Table 36: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	45	46	51	56	71	81	85	89
Single Point GPS (Carrier Phase)	58	58	66	66	71	71	78	81
Single Difference GPS (Pseudoranges)	58	60	61	64	68	73	75	77
Single Difference GPS (Carrier Phase)	70	70	71	75	82	82	82	82
Double Difference GPS (Pseudoranges)	53	53	54	56	65	74	77	80
Double Difference GPS (Carrier Phase)	71	72	72	74	79	79	79	79
IMU Measured	91	100	100	100	100	100	100	100

Table 37: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	66	80	94	96	96	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	77	77	84	93	95	95	99	99
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	74	74	83	97	97	97	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 38: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	70	82	91	96	100	100	100	100
Single Point GPS (Carrier Phase)	58	59	66	70	72	77	87	88
Single Difference GPS (Pseudoranges)	79	86	90	94	97	97	98	98
Single Difference GPS (Carrier Phase)	70	71	71	81	82	87	89	89
Double Difference GPS (Pseudoranges)	74	83	92	96	99	100	100	100
Double Difference GPS (Carrier Phase)	70	71	72	77	80	80	82	85
IMU Measured	91	100	100	100	100	100	100	100

Table 39: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-1

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.858	0.107
Single Difference GPS (Pseudoranges)	0.937	0.146
Double Difference GPS (Pseudoranges)	0.573	0.046

Set A-2: 3 m Dual Antenna Separation

Figure 51: Computed and Measured Headings for Simulation Set A-2

Figure 52: Computed and Measured Pitch for Simulation Set A-2

Figure 53: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-2

Table 40: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	4.881	2.547	4.017
Single Point GPS (Carrier Phase)	13.424	0.012	9.816
Single Difference GPS (Pseudoranges)	7.115	2.476	5.611
Single Difference GPS (Carrier Phase)	11.071	0.012	6.506
Double Difference GPS (Pseudoranges)	8.269	1.82	6.893
Double Difference GPS (Carrier Phase)	12.179	0.022	7.567
IMU Measurement	0.007	-	0.005

Table 41: Mean Global Error Statistics of Unfused Heading for Simulation Set A-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	4.164	2.43	3.427
Single Point GPS (Carrier Phase)	9.022	0.01	6.601
Single Difference GPS (Pseudoranges)	6.941	2.869	4.893
Single Difference GPS (Carrier Phase)	10.51	0.01	5.265
Double Difference GPS (Pseudoranges)	7.275	1.95	5.339
Double Difference GPS (Carrier Phase)	9.088	0.026	5.902
IMU Measurement	0.301	-	0.245

Table 42: Mean Global Error Statistics of Fused Pitch for Simulation Set A-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.14	0.192	0.036
Single Point GPS (Carrier Phase)	5.133	0.006	4.255
Single Difference GPS (Pseudoranges)	0.222	0.188	0.057
Single Difference GPS (Carrier Phase)	4.383	0.006	2.8
Double Difference GPS (Pseudoranges)	0.217	0.179	0.091
Double Difference GPS (Carrier Phase)	3.46	0.011	2.181
IMU Measurement	0.007	-	0.005

Table 43: Mean Global Error Statistics of Fused Heading for Simulation Set A-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.052	0.69	0.723
Single Point GPS (Carrier Phase)	8.716	0.005	6.385
Single Difference GPS (Pseudoranges)	1.893	0.885	1.123
Single Difference GPS (Carrier Phase)	9.652	0.005	4.941
Double Difference GPS (Pseudoranges)	3.013	0.68	1.799
Double Difference GPS (Carrier Phase)	8.192	0.013	5.368
IMU Measurement	0.301	-	0.245

Table 44: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	54	58	62	82	99	100	100	100
Single Point GPS (Carrier Phase)	69	69	69	73	86	90	94	95
Single Difference GPS (Pseudoranges)	56	60	66	80	93	98	99	100
Single Difference GPS (Carrier Phase)	85	85	85	86	91	91	93	95
Double Difference GPS (Pseudoranges)	62	64	69	80	93	99	100	100
Double Difference GPS (Carrier Phase)	79	79	79	80	86	89	93	98
IMU Measured	100	100	100	100	100	100	100	100

Table 45: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	51	57	62	81	98	100	100	100
Single Point GPS (Carrier Phase)	58	58	58	66	89	96	99	100
Single Difference GPS (Pseudoranges)	47	54	60	74	89	95	98	99
Single Difference GPS (Carrier Phase)	77	77	77	86	86	89	96	96
Double Difference GPS (Pseudoranges)	50	53	61	71	89	95	96	98
Double Difference GPS (Carrier Phase)	67	67	69	77	82	86	100	100
IMU Measured	96	100	100	100	100	100	100	100

Table 46: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	69	69	75	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	85	85	88	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	79	79	87	99	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 47: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	72	88	95	100	100	100	100	100
Single Point GPS (Carrier Phase)	58	58	58	66	89	99	99	100
Single Difference GPS (Pseudoranges)	70	80	87	97	100	100	100	100
Single Difference GPS (Carrier Phase)	77	77	77	86	86	91	96	96
Double Difference GPS (Pseudoranges)	64	70	82	93	99	99	100	100
Double Difference GPS (Carrier Phase)	67	67	69	77	82	91	100	100
IMU Measured	96	100	100	100	100	100	100	100

Table 48: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-2

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.839865	0.291616
Single Difference GPS (Pseudoranges)	0.834911	0.304303
Double Difference GPS (Pseudoranges)	0.779955	0.307236

Set A-3: 5 m Dual Antenna Separation

Figure 54: Computed and Measured Headings for Simulation Set A-3

Figure 55: Computed and Measured Pitch for Simulation Set A-3

Figure 56: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-3

Table 49: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	3.01	1.539	2.474
Single Point GPS (Carrier Phase)	7.364	0.006	5.549
Single Difference GPS (Pseudoranges)	3.794	1.55	2.892
Single Difference GPS (Carrier Phase)	6.192	0.006	3.856
Double Difference GPS (Pseudoranges)	5.075	1.243	3.974
Double Difference GPS (Carrier Phase)	7.555	0.013	4.821
IMU Measurement	0.007	-	0.006

Table 50: Mean Global Error Statistics of Unfused Heading for Simulation Set A-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.97	1.303	2.292
Single Point GPS (Carrier Phase)	4.386	0.004	3.233
Single Difference GPS (Pseudoranges)	3.155	1.416	2.559
Single Difference GPS (Carrier Phase)	3.981	0.004	2.332
Double Difference GPS (Pseudoranges)	3.506	1.137	2.852
Double Difference GPS (Carrier Phase)	4.914	0.014	3.111
IMU Measurement	0.306	-	0.245

Table 51: Mean Global Error Statistics of Fused Pitch for Simulation Set A-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.026	0.192	0.017
Single Point GPS (Carrier Phase)	4.45	0.003	3.298
Single Difference GPS (Pseudoranges)	0.06	0.19	0.023
Single Difference GPS (Carrier Phase)	3.638	0.003	2.297
Double Difference GPS (Pseudoranges)	0.154	0.183	0.051
Double Difference GPS (Carrier Phase)	2.973	0.007	1.789
IMU Measurement	0.007	-	0.006

Table 52: Mean Global Error Statistics of Fused Heading for Simulation Set A-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.685	0.365	0.536
Single Point GPS (Carrier Phase)	4.317	0.002	3.187
Single Difference GPS (Pseudoranges)	1.033	0.397	0.696
Single Difference GPS (Carrier Phase)	3.915	0.002	2.296
Double Difference GPS (Pseudoranges)	1.403	0.4	0.917
Double Difference GPS (Carrier Phase)	4.694	0.007	2.974
IMU Measurement	0.306	-	0.245

Table 53: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	58	64	71	98	100	100	100	100
Single Point GPS (Carrier Phase)	65	65	65	84	92	100	100	100
Single Difference GPS (Pseudoranges)	58	62	71	91	98	100	100	100
Single Difference GPS (Carrier Phase)	81	81	81	87	91	100	100	100
Double Difference GPS (Pseudoranges)	51	56	66	82	96	100	100	100
Double Difference GPS (Carrier Phase)	67	67	67	77	87	99	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 54: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	50	63	73	95	99	100	100	100
Single Point GPS (Carrier Phase)	54	54	63	89	99	100	100	100
Single Difference GPS (Pseudoranges)	57	60	79	94	100	100	100	100
Single Difference GPS (Carrier Phase)	80	81	87	89	99	100	100	100
Double Difference GPS (Pseudoranges)	63	65	74	94	100	100	100	100
Double Difference GPS (Carrier Phase)	77	78	84	88	98	99	100	100
IMU Measured	97	100	100	100	100	100	100	100

Table 55: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	65	76	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	81	81	87	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	67	71	83	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 56: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	70	90	98	100	100	100	100	100
Single Point GPS (Carrier Phase)	54	55	63	89	99	100	100	100
Single Difference GPS (Pseudoranges)	78	92	96	100	100	100	100	100
Single Difference GPS (Carrier Phase)	80	81	87	89	100	100	100	100
Double Difference GPS (Pseudoranges)	76	86	92	99	100	100	100	100
Double Difference GPS (Carrier Phase)	77	78	84	88	98	99	100	100
IMU Measured	97	100	100	100	100	100	100	100

Table 57: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-3

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.839865	0.291616
Single Difference GPS (Pseudoranges)	0.834911	0.304303
Double Difference GPS (Pseudoranges)	0.779955	0.307236

Set A-4: 10 m Dual Antenna Separation

Figure 57: Computed and Measured Headings for Simulation Set A-4

Figure 58: Computed and Measured Pitch for Simulation Set A-4

Figure 59: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-4

Table 58: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.46	0.777	1.185
Single Point GPS (Carrier Phase)	3.63	0.003	2.518
Single Difference GPS (Pseudoranges)	2.048	0.799	1.644
Single Difference GPS (Carrier Phase)	3.156	0.003	1.647
Double Difference GPS (Pseudoranges)	2.141	0.622	1.691
Double Difference GPS (Carrier Phase)	3.391	0.006	2.037
IMU Measurement	0.007	-	0.006

Table 59: Mean Global Error Statistics of Unfused Heading for Simulation Set A-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.212	0.624	0.962
Single Point GPS (Carrier Phase)	2.252	0.002	1.606
Single Difference GPS (Pseudoranges)	1.648	0.684	1.376
Single Difference GPS (Carrier Phase)	2.058	0.002	1.118
Double Difference GPS (Pseudoranges)	1.778	0.477	1.433
Double Difference GPS (Carrier Phase)	2.483	0.005	1.608
IMU Measurement	0.329	-	0.253

Table 60: Mean Global Error Statistics of Fused Pitch for Simulation Set A-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.015	0.177	0.012
Single Point GPS (Carrier Phase)	2.709	0.001	1.836
Single Difference GPS (Pseudoranges)	0.02	0.178	0.016
Single Difference GPS (Carrier Phase)	2.129	0.001	1.152
Double Difference GPS (Pseudoranges)	0.027	0.167	0.021
Double Difference GPS (Carrier Phase)	2.043	0.003	1.155
IMU Measurement	0.007	-	0.006

Table 61: Mean Global Error Statistics of Fused Heading for Simulation Set A-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.477	0.198	0.371
Single Point GPS (Carrier Phase)	2.24	0.001	1.596
Single Difference GPS (Pseudoranges)	0.635	0.215	0.487
Single Difference GPS (Carrier Phase)	2.042	0.001	1.11
Double Difference GPS (Pseudoranges)	0.801	0.179	0.657
Double Difference GPS (Carrier Phase)	2.443	0.003	1.584
IMU Measurement	0.329	-	0.253

Table 62: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	66	76	93	100	100	100	100	100
Single Point GPS (Carrier Phase)	69	69	71	93	100	100	100	100
Single Difference GPS (Pseudoranges)	59	67	81	99	100	100	100	100
Single Difference GPS (Carrier Phase)	79	79	83	96	98	100	100	100
Double Difference GPS (Pseudoranges)	63	74	84	97	100	100	100	100
Double Difference GPS (Carrier Phase)	79	79	81	89	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 63: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	66	78	94	100	100	100	100	100
Single Point GPS (Carrier Phase)	66	69	87	98	100	100	100	100
Single Difference GPS (Pseudoranges)	72	79	93	100	100	100	100	100
Single Difference GPS (Carrier Phase)	85	93	93	100	100	100	100	100
Double Difference GPS (Pseudoranges)	58	67	81	100	100	100	100	100
Double Difference GPS (Carrier Phase)	76	80	83	100	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 64: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	69	69	86	98	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	79	79	89	98	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	79	81	87	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 65: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	81	99	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	66	69	87	98	100	100	100	100
Single Difference GPS (Pseudoranges)	85	96	99	100	100	100	100	100
Single Difference GPS (Carrier Phase)	85	93	93	100	100	100	100	100
Double Difference GPS (Pseudoranges)	68	84	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	76	80	83	100	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 66: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-4

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.877546	0.678435
Single Difference GPS (Pseudoranges)	0.881098	0.623469
Double Difference GPS (Pseudoranges)	0.795612	0.557494

Set A-5: 30 m Dual Antenna Separation

Figure 60: Computed and Measured Headings for Simulation Set A-5

Figure 61: Computed and Measured Pitch for Simulation Set A-5

Figure 62: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-5

Table 67: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.467	0.264	0.372
Single Point GPS (Carrier Phase)	1.23	0.001	0.874
Single Difference GPS (Pseudoranges)	0.666	0.262	0.54
Single Difference GPS (Carrier Phase)	1.159	0.001	0.679
Double Difference GPS (Pseudoranges)	0.797	0.212	0.642
Double Difference GPS (Carrier Phase)	1.13	0.002	0.706
IMU Measurement	0.007	-	0.006

Table 68: Mean Global Error Statistics of Unfused Heading for Simulation Set A-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.401	0.214	0.315
Single Point GPS (Carrier Phase)	0.863	0.001	0.606
Single Difference GPS (Pseudoranges)	0.569	0.211	0.449
Single Difference GPS (Carrier Phase)	0.714	0.001	0.407
Double Difference GPS (Pseudoranges)	0.622	0.161	0.515
Double Difference GPS (Carrier Phase)	0.748	0.002	0.486
IMU Measurement	0.299	-	0.242

Table 69: Mean Global Error Statistics of Fused Pitch for Simulation Set A-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.013	0.11	0.01
Single Point GPS (Carrier Phase)	1.099	0	0.775
Single Difference GPS (Pseudoranges)	0.019	0.11	0.015
Single Difference GPS (Carrier Phase)	1.024	0	0.6
Double Difference GPS (Pseudoranges)	0.027	0.094	0.022
Double Difference GPS (Carrier Phase)	0.874	0.001	0.544
IMU Measurement	0.007	-	0.006

Table 70: Mean Global Error Statistics of Fused Heading for Simulation Set A-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.267	0.094	0.213
Single Point GPS (Carrier Phase)	0.861	0	0.605
Single Difference GPS (Pseudoranges)	0.362	0.093	0.285
Single Difference GPS (Carrier Phase)	0.713	0	0.406
Double Difference GPS (Pseudoranges)	0.427	0.075	0.351
Double Difference GPS (Carrier Phase)	0.744	0.001	0.483
IMU Measurement	0.299	-	0.242

Table 71: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	86	99	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	64	90	95	100	100	100	100	100
Single Difference GPS (Pseudoranges)	80	96	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	80	86	92	100	100	100	100	100
Double Difference GPS (Pseudoranges)	71	90	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	73	84	92	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 72: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	90	99	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	78	85	97	100	100	100	100	100
Single Difference GPS (Pseudoranges)	84	97	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	85	85	99	100	100	100	100	100
Double Difference GPS (Pseudoranges)	73	94	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	81	85	99	100	100	100	100	100
IMU Measured	94	100	100	100	100	100	100	100

Table 73: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	64	91	99	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	83	86	98	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	81	86	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 74: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	97	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	78	85	97	100	100	100	100	100
Single Difference GPS (Pseudoranges)	89	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	85	85	99	100	100	100	100	100
Double Difference GPS (Pseudoranges)	85	98	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	81	85	99	100	100	100	100	100
IMU Measured	94	100	100	100	100	100	100	100

Table 75: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-5

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.873332	0.834883
Single Difference GPS (Pseudoranges)	0.866496	0.818398
Double Difference GPS (Pseudoranges)	0.803797	0.772162

Set A-6: 50 m Dual Antenna Separation

Figure 63: Computed and Measured Headings for Simulation Set A-6

Figure 64: Computed and Measured Pitch for Simulation Set A-6

Figure 65: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-6

Table 76: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.28	0.159	0.233
Single Point GPS (Carrier Phase)	0.742	0.001	0.526
Single Difference GPS (Pseudoranges)	0.446	0.158	0.362
Single Difference GPS (Carrier Phase)	0.793	0.001	0.487
Double Difference GPS (Pseudoranges)	0.502	0.126	0.418
Double Difference GPS (Carrier Phase)	0.67	0.001	0.397
IMU Measurement	0.007	-	0.006

Table 77: Mean Global Error Statistics of Unfused Heading for Simulation Set A-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.26	0.129	0.207
Single Point GPS (Carrier Phase)	0.498	0	0.363
Single Difference GPS (Pseudoranges)	0.387	0.127	0.31
Single Difference GPS (Carrier Phase)	0.55	0	0.348
Double Difference GPS (Pseudoranges)	0.355	0.094	0.279
Double Difference GPS (Carrier Phase)	0.447	0.001	0.294
IMU Measurement	0.305	-	0.239

Table 78: Mean Global Error Statistics of Fused Pitch for Simulation Set A-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.015	0.074	0.012
Single Point GPS (Carrier Phase)	0.687	0	0.487
Single Difference GPS (Pseudoranges)	0.02	0.074	0.016
Single Difference GPS (Carrier Phase)	0.735	0	0.451
Double Difference GPS (Pseudoranges)	0.029	0.06	0.023
Double Difference GPS (Carrier Phase)	0.572	0.001	0.338
IMU Measurement	0.007	-	0.006

Table 79: Mean Global Error Statistics of Fused Heading for Simulation Set A-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.203	0.061	0.167
Single Point GPS (Carrier Phase)	0.497	0	0.363
Single Difference GPS (Pseudoranges)	0.297	0.06	0.244
Single Difference GPS (Carrier Phase)	0.549	0	0.348
Double Difference GPS (Pseudoranges)	0.274	0.046	0.21
Double Difference GPS (Carrier Phase)	0.445	0	0.293
IMU Measurement	0.305	-	0.239

Table 80: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	97	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	83	94	99	100	100	100	100	100
Single Difference GPS (Pseudoranges)	88	99	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	87	90	99	100	100	100	100	100
Double Difference GPS (Pseudoranges)	88	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	87	94	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 81: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	97	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	95	99	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	90	99	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	84	97	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	89	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	82	100	100	100	100	100	100	100
IMU Measured	97	100	100	100	100	100	100	100

Table 82: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	97	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	83	94	99	100	100	100	100	100
Single Difference GPS (Pseudoranges)	88	99	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	87	90	99	100	100	100	100	100
Double Difference GPS (Pseudoranges)	88	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	87	94	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 83: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	86	94	99	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	88	90	99	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	91	95	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 84: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-6

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.86429	0.846041
Single Difference GPS (Pseudoranges)	0.867223	0.849816
Double Difference GPS (Pseudoranges)	0.784954	0.769789

Set A-7: 70 m Dual Antenna Separation

Figure 66: Computed and Measured Headings for Simulation Set A-7

Figure 67: Computed and Measured Pitch for Simulation Set A-7

Figure 68: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-7

Table 85: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.203	0.113	0.166
Single Point GPS (Carrier Phase)	0.48	0	0.34
Single Difference GPS (Pseudoranges)	0.302	0.113	0.241
Single Difference GPS (Carrier Phase)	0.51	0	0.304
Double Difference GPS (Pseudoranges)	0.323	0.09	0.243
Double Difference GPS (Carrier Phase)	0.54	0.001	0.323
IMU Measurement	0.007	-	0.006

Table 86: Mean Global Error Statistics of Unfused Heading for Simulation Set A-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.16	0.091	0.127
Single Point GPS (Carrier Phase)	0.367	0	0.25
Single Difference GPS (Pseudoranges)	0.258	0.091	0.208
Single Difference GPS (Carrier Phase)	0.309	0	0.19
Double Difference GPS (Pseudoranges)	0.251	0.068	0.197
Double Difference GPS (Carrier Phase)	0.315	0.001	0.204
IMU Measurement	0.342	-	0.263

Table 87: Mean Global Error Statistics of Fused Pitch for Simulation Set A-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.013	0.055	0.011
Single Point GPS (Carrier Phase)	0.454	0	0.322
Single Difference GPS (Pseudoranges)	0.019	0.055	0.015
Single Difference GPS (Carrier Phase)	0.482	0	0.288
Double Difference GPS (Pseudoranges)	0.025	0.044	0.019
Double Difference GPS (Carrier Phase)	0.48	0	0.286
IMU Measurement	0.007	-	0.006

Table 88: Mean Global Error Statistics of Fused Heading for Simulation Set A-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.157	0.044	0.125
Single Point GPS (Carrier Phase)	0.367	0	0.25
Single Difference GPS (Pseudoranges)	0.2	0.044	0.161
Single Difference GPS (Carrier Phase)	0.308	0	0.19
Double Difference GPS (Pseudoranges)	0.221	0.033	0.175
Double Difference GPS (Carrier Phase)	0.314	0	0.203
IMU Measurement	0.342	-	0.263

Table 89: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	90	98	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	97	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	86	98	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	95	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	87	98	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 90: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	92	100	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	98	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	92	100	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	98	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	94	100	100	100	100	100	100	100
IMU Measured	99	100	100	100	100	100	100	100

Table 91: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	90	98	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	86	98	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	87	98	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 92: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	92	100	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	92	100	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	99	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	94	100	100	100	100	100	100	100
IMU Measured	99	100	100	100	100	100	100	100

Table 93: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-7

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.869998	0.861423
Single Difference GPS (Pseudoranges)	0.874294	0.866835
Double Difference GPS (Pseudoranges)	0.786869	0.779073

Set A-8: 90 m Dual Antenna Separation

Figure 69: Computed and Measured Headings for Simulation Set A-8

Figure 70: Computed and Measured Pitch for Simulation Set A-8

Figure 71: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set A-8

Table 94: Mean Global Error Statistics of Unfused Pitch for Simulation Set A-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.187	0.088	0.15
Single Point GPS (Carrier Phase)	0.395	0	0.275
Single Difference GPS (Pseudoranges)	0.196	0.088	0.16
Single Difference GPS (Carrier Phase)	0.379	0	0.207
Double Difference GPS (Pseudoranges)	0.254	0.07	0.201
Double Difference GPS (Carrier Phase)	0.392	0.001	0.243
IMU Measurement	0.007	-	0.005

Table 95: Mean Global Error Statistics of Unfused Heading for Simulation Set A-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.124	0.071	0.1
Single Point GPS (Carrier Phase)	0.233	0	0.165
Single Difference GPS (Pseudoranges)	0.197	0.071	0.155
Single Difference GPS (Carrier Phase)	0.215	0	0.116
Double Difference GPS (Pseudoranges)	0.212	0.053	0.165
Double Difference GPS (Carrier Phase)	0.235	0.001	0.141
IMU Measurement	0.326	-	0.258

Table 96: Mean Global Error Statistics of Fused Pitch for Simulation Set A-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.015	0.043	0.012
Single Point GPS (Carrier Phase)	0.379	0	0.263
Single Difference GPS (Pseudoranges)	0.016	0.043	0.012
Single Difference GPS (Carrier Phase)	0.362	0	0.198
Double Difference GPS (Pseudoranges)	0.023	0.035	0.018
Double Difference GPS (Carrier Phase)	0.357	0	0.221
IMU Measurement	0.007	-	0.005

Table 97: Mean Global Error Statistics of Fused Heading for Simulation Set A-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.116	0.035	0.092
Single Point GPS (Carrier Phase)	0.233	0	0.165
Single Difference GPS (Pseudoranges)	0.177	0.035	0.141
Single Difference GPS (Carrier Phase)	0.215	0	0.116
Double Difference GPS (Pseudoranges)	0.188	0.026	0.148
Double Difference GPS (Carrier Phase)	0.234	0	0.141
IMU Measurement	0.326	-	0.258

Table 98: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set A-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	91	97	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	86	99	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	99	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	89	100	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 99: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set A-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	96	100	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	99	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	96	100	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	97	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	97	100	100	100	100	100	100	100
IMU Measured	97	100	100	100	100	100	100	100

Table 100: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set A-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	92	99	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	89	99	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	93	100	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 101: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set A-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	96	100	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	99	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	96	100	100	100	100	100	100	100
Double Difference GPS (Pseudoranges)	98	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	97	100	100	100	100	100	100	100
IMU Measured	97	100	100	100	100	100	100	100

Table 102: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set A-8

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.869504	0.86444
Single Difference GPS (Pseudoranges)	0.873157	0.86824
Double Difference GPS (Pseudoranges)	0.78564	0.780952

Appendix B: 1 m Pseudorange Noise Standard

Deviation Pitch and Heading Simulation Results

Set B-1: 1 m Dual Antenna Separation

Figure 72: Computed and Measured Headings for Simulation Set B-1

Figure 73: Computed and Measured Pitch for Simulation Set B-1

Figure 74: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-1

Table 103: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	34.603	121.283	28.382
Single Point GPS (Carrier Phase)	32.445	0.156	25.695
Single Difference GPS (Pseudoranges)	37.475	88.682	31.4
Single Difference GPS (Carrier Phase)	36.746	0.102	31.41
Double Difference GPS (Pseudoranges)	36.496	63.396	28.925
Double Difference GPS (Carrier Phase)	36.134	0.126	29.682
IMU Measurement	0.007	-	0.006

Table 104: Mean Global Error Statistics of Unfused Heading for Simulation Set B-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	94.459	531.372	68.048
Single Point GPS (Carrier Phase)	100.135	0.437	68.413
Single Difference GPS (Pseudoranges)	83.104	338.806	68.928
Single Difference GPS (Carrier Phase)	89.304	0.298	68.102
Double Difference GPS (Pseudoranges)	104.337	419.144	66.08
Double Difference GPS (Carrier Phase)	103.95	2.527	66.694
IMU Measurement	0.305	-	0.244

Table 105: Mean Global Error Statistics of Fused Pitch for Simulation Set B-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.05	0.199	0.015
Single Point GPS (Carrier Phase)	4.354	0.058	2.631
Single Difference GPS (Pseudoranges)	0.985	0.198	0.109
Single Difference GPS (Carrier Phase)	7.104	0.044	4.17
Double Difference GPS (Pseudoranges)	0.021	0.2	0.013
Double Difference GPS (Carrier Phase)	7.46	0.051	3.774
IMU Measurement	0.007	-	0.006

Table 106: Mean Global Error Statistics of Fused Heading for Simulation Set B-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.837	8.187	0.552
Single Point GPS (Carrier Phase)	66.438	0.186	51.033
Single Difference GPS (Pseudoranges)	5.049	8.229	1.014
Single Difference GPS (Carrier Phase)	68.512	0.134	49.084
Double Difference GPS (Pseudoranges)	1.761	8.177	0.654
Double Difference GPS (Carrier Phase)	59.456	0.355	39.253
IMU Measurement	0.305	-	0.244

Table 107: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	59	59	60	62	66	73	77	79
Single Point GPS (Carrier Phase)	60	60	60	63	71	76	76	79
Single Difference GPS (Pseudoranges)	59	60	62	66	70	71	76	78
Single Difference GPS (Carrier Phase)	63	63	63	64	67	74	78	81
Double Difference GPS (Pseudoranges)	56	59	60	64	68	78	81	84
Double Difference GPS (Carrier Phase)	58	58	59	61	68	76	79	83
IMU Measured	100	100	100	100	100	100	100	100

Table 108: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	46	46	46	49	52	54	57	60
Single Point GPS (Carrier Phase)	50	50	52	52	57	57	60	60
Single Difference GPS (Pseudoranges)	46	46	47	52	54	56	58	59
Single Difference GPS (Carrier Phase)	46	46	46	48	52	52	56	58
Double Difference GPS (Pseudoranges)	58	58	59	59	62	65	66	68
Double Difference GPS (Carrier Phase)	58	58	59	60	60	61	64	66
IMU Measured	95	100	100	100	100	100	100	100

Table 109: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	70	86	91	97	99	100	100	100
Single Difference GPS (Pseudoranges)	99	99	99	99	100	100	100	100
Single Difference GPS (Carrier Phase)	68	76	86	91	97	98	98	98
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	69	76	88	96	98	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 110: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	85	95	99	100	100	100	100	100
Single Point GPS (Carrier Phase)	39	39	40	48	51	55	58	58
Single Difference GPS (Pseudoranges)	87	98	98	99	99	99	99	99
Single Difference GPS (Carrier Phase)	38	38	38	45	50	54	57	57
Double Difference GPS (Pseudoranges)	75	94	98	99	99	99	100	100
Double Difference GPS (Carrier Phase)	48	50	51	52	58	61	62	63
IMU Measured	95	100	100	100	100	100	100	100

Table 111: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-1

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.080141	0.000005
Single Difference GPS (Pseudoranges)	-0.93829	-0.00998
Double Difference GPS (Pseudoranges)	0.981793	0.00015

Set B-2: 3 m Dual Antenna Separation

Figure 75: Computed and Measured Headings for Simulation Set B-2

Figure 76: Computed and Measured Pitch for Simulation Set B-2

Figure 77: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-2

Table 112: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	23.486	81.661	17.29
Single Point GPS (Carrier Phase)	29.048	0.094	23.031
Single Difference GPS (Pseudoranges)	27.737	67.485	21.473
Single Difference GPS (Carrier Phase)	30.238	0.071	23.913
Double Difference GPS (Pseudoranges)	31.053	51.166	26.163
Double Difference GPS (Carrier Phase)	31.336	0.094	25.91
IMU Measurement	0.008	-	0.006

Table 113: Mean Global Error Statistics of Unfused Heading for Simulation Set B-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	48.586	307.582	26.949
Single Point GPS (Carrier Phase)	48.17	0.902	27.213
Single Difference GPS (Pseudoranges)	49.4	254.488	30.983
Single Difference GPS (Carrier Phase)	57.415	0.336	31.994
Double Difference GPS (Pseudoranges)	59.328	289.133	37.515
Double Difference GPS (Carrier Phase)	65.969	0.427	37.35
IMU Measurement	0.29	-	0.227

Table 114: Mean Global Error Statistics of Fused Pitch for Simulation Set B-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.036	0.199	0.011
Single Point GPS (Carrier Phase)	3.872	0.042	2.59
Single Difference GPS (Pseudoranges)	0.027	0.199	0.013
Single Difference GPS (Carrier Phase)	4.733	0.032	3.762
Double Difference GPS (Pseudoranges)	0.056	0.197	0.018
Double Difference GPS (Carrier Phase)	4.172	0.043	2.686
IMU Measurement	0.008	-	0.006

Table 115: Mean Global Error Statistics of Fused Heading for Simulation Set B-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.085	7.141	0.66
Single Point GPS (Carrier Phase)	36.851	0.188	16.656
Single Difference GPS (Pseudoranges)	1.907	6.87	0.646
Single Difference GPS (Carrier Phase)	40.769	0.151	23.42
Double Difference GPS (Pseudoranges)	4.257	7.051	1.208
Double Difference GPS (Carrier Phase)	33.634	0.194	20.362
IMU Measurement	0.29	-	0.227

Table 116: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	51	54	59	63	75	79	81	86
Single Point GPS (Carrier Phase)	52	52	53	55	64	68	80	84
Single Difference GPS (Pseudoranges)	52	52	55	62	68	72	76	79
Single Difference GPS (Carrier Phase)	51	51	55	57	65	69	73	76
Double Difference GPS (Pseudoranges)	70	71	73	76	82	87	89	91
Double Difference GPS (Carrier Phase)	67	68	70	73	79	82	87	92
IMU Measured	100	100	100	100	100	100	100	100

Table 117: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	43	43	47	56	60	64	71	74
Single Point GPS (Carrier Phase)	44	44	46	52	61	63	73	73
Single Difference GPS (Pseudoranges)	52	53	54	60	65	68	71	74
Single Difference GPS (Carrier Phase)	51	52	53	56	66	66	71	73
Double Difference GPS (Pseudoranges)	49	50	51	55	60	63	66	69
Double Difference GPS (Carrier Phase)	49	49	52	57	60	62	66	69
IMU Measured	98	100	100	100	100	100	100	100

Table 118: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	54	79	92	97	98	99	99	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	57	70	86	94	99	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	78	86	92	98	98	99	99	99
IMU Measured	100	100	100	100	100	100	100	100

Table 119: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	89	94	97	100	100	100	100	100
Single Point GPS (Carrier Phase)	45	47	48	57	73	83	87	90
Single Difference GPS (Pseudoranges)	92	96	98	100	100	100	100	100
Single Difference GPS (Carrier Phase)	50	52	53	60	70	77	82	84
Double Difference GPS (Pseudoranges)	91	94	95	97	97	99	99	99
Double Difference GPS (Carrier Phase)	48	50	54	58	67	73	79	81
IMU Measured	98	100	100	100	100	100	100	100

Table 120: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-2

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.870646	0.010325
Single Difference GPS (Pseudoranges)	0.748203	0.000944
Double Difference GPS (Pseudoranges)	0.819718	0.016298

Set B-3: 5 m Dual Antenna Separation

Figure 78: Computed and Measured Headings for Simulation Set B-3

Figure 79: Computed and Measured Pitch for Simulation Set B-3

Figure 80: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-3

Table 121: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	15.844	42.771	12.058
Single Point GPS (Carrier Phase)	16.467	0.044	12.563
Single Difference GPS (Pseudoranges)	22.756	43.824	17.798
Single Difference GPS (Carrier Phase)	25.581	0.054	20.098
Double Difference GPS (Pseudoranges)	28.967	57.048	21.966
Double Difference GPS (Carrier Phase)	29.548	0.111	22.877
IMU Measurement	0.008	-	0.006

Table 122: Mean Global Error Statistics of Unfused Heading for Simulation Set B-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	18.776	95.456	11.997
Single Point GPS (Carrier Phase)	19.408	0.067	12.766
Single Difference GPS (Pseudoranges)	36.572	116.748	21.849
Single Difference GPS (Carrier Phase)	36.94	0.097	22.508
Double Difference GPS (Pseudoranges)	40.825	179.035	27.003
Double Difference GPS (Carrier Phase)	50.691	0.354	28.424
IMU Measurement	0.351	-	0.277

Table 123: Mean Global Error Statistics of Fused Pitch for Simulation Set B-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.013	0.2	0.009
Single Point GPS (Carrier Phase)	3.291	0.021	2.708
Single Difference GPS (Pseudoranges)	0.072	0.199	0.017
Single Difference GPS (Carrier Phase)	4.907	0.025	3.801
Double Difference GPS (Pseudoranges)	1.455	0.198	0.157
Double Difference GPS (Carrier Phase)	5.16	0.045	2.938
IMU Measurement	0.008	-	0.006

Table 124: Mean Global Error Statistics of Fused Heading for Simulation Set B-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	3.584	5.469	0.954
Single Point GPS (Carrier Phase)	13.924	0.033	9.924
Single Difference GPS (Pseudoranges)	2.214	6.111	0.894
Single Difference GPS (Carrier Phase)	29.077	0.045	17.246
Double Difference GPS (Pseudoranges)	5.512	6.568	1.209
Double Difference GPS (Carrier Phase)	29.718	0.157	15.133
IMU Measurement	0.351	-	0.277

Table 125: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	58	58	61	65	76	88	93	94
Single Point GPS (Carrier Phase)	57	61	61	67	77	86	92	93
Single Difference GPS (Pseudoranges)	50	50	50	55	69	73	81	84
Single Difference GPS (Carrier Phase)	52	52	54	58	67	74	78	83
Double Difference GPS (Pseudoranges)	54	57	58	62	67	74	80	82
Double Difference GPS (Carrier Phase)	53	53	55	62	67	75	81	84
IMU Measured	100	100	100	100	100	100	100	100

Table 126: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	53	56	56	62	74	81	84	88
Single Point GPS (Carrier Phase)	50	50	56	65	76	81	84	84
Single Difference GPS (Pseudoranges)	49	52	55	60	67	74	79	82
Single Difference GPS (Carrier Phase)	48	49	52	59	69	73	78	81
Double Difference GPS (Pseudoranges)	49	49	51	54	59	67	71	74
Double Difference GPS (Carrier Phase)	50	51	53	58	62	65	70	74
IMU Measured	93	99	100	100	100	100	100	100

Table 127: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	62	77	98	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	57	66	86	94	98	99	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	67	77	92	97	99	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 128: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	82	93	95	98	99	99	99	99
Single Point GPS (Carrier Phase)	50	53	56	67	78	87	90	93
Single Difference GPS (Pseudoranges)	80	92	94	98	99	100	100	100
Single Difference GPS (Carrier Phase)	48	50	52	59	74	81	85	89
Double Difference GPS (Pseudoranges)	82	91	97	99	99	99	99	99
Double Difference GPS (Carrier Phase)	50	51	54	60	73	79	86	88
IMU Measured	93	99	100	100	100	100	100	100

Table 129: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-3

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.21306	0.001037
Single Difference GPS (Pseudoranges)	0.945381	0.050411
Double Difference GPS (Pseudoranges)	-0.91592	-0.02784

Set B-4: 10 m Dual Antenna Separation

Figure 81: Computed and Measured Headings for Simulation Set B-4

Figure 82: Computed and Measured Pitch for Simulation Set B-4

Figure 83: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-4

Table 130: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	7.083	19.072	5.376
Single Point GPS (Carrier Phase)	8.144	0.018	6.007
Single Difference GPS (Pseudoranges)	10.461	17.47	8.288
Single Difference GPS (Carrier Phase)	10.753	0.016	8.799
Double Difference GPS (Pseudoranges)	13.366	17.167	10.322
Double Difference GPS (Carrier Phase)	13.266	0.033	10.497
IMU Measurement	0.007	-	0.006

Table 131: Mean Global Error Statistics of Unfused Heading for Simulation Set B-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	6.45	21.464	4.957
Single Point GPS (Carrier Phase)	7.372	0.015	5.877
Single Difference GPS (Pseudoranges)	9.91	27.881	7.3
Single Difference GPS (Carrier Phase)	10.055	0.021	7.436
Double Difference GPS (Pseudoranges)	12.247	26.039	8.498
Double Difference GPS (Carrier Phase)	12.374	0.049	8.636
IMU Measurement	0.278	-	0.228

Table 132: Mean Global Error Statistics of Fused Pitch for Simulation Set B-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.084	0.197	0.02
Single Point GPS (Carrier Phase)	3.027	0.009	2.072
Single Difference GPS (Pseudoranges)	0.109	0.197	0.023
Single Difference GPS (Carrier Phase)	4.306	0.008	3.754
Double Difference GPS (Pseudoranges)	0.018	0.2	0.011
Double Difference GPS (Carrier Phase)	3.094	0.016	2.427
IMU Measurement	0.007	-	0.006

Table 133: Mean Global Error Statistics of Fused Heading for Simulation Set B-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.246	3.898	0.517
Single Point GPS (Carrier Phase)	6.94	0.008	5.561
Single Difference GPS (Pseudoranges)	1.364	3.531	0.714
Single Difference GPS (Carrier Phase)	8.566	0.01	6.724
Double Difference GPS (Pseudoranges)	1.912	4.268	0.852
Double Difference GPS (Carrier Phase)	10.611	0.025	7.445
IMU Measurement	0.278	-	0.228

Table 134: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	54	56	64	78	88	97	99	100
Single Point GPS (Carrier Phase)	59	60	63	75	87	95	98	99
Single Difference GPS (Pseudoranges)	60	62	64	75	91	95	97	98
Single Difference GPS (Carrier Phase)	59	59	61	73	87	97	97	98
Double Difference GPS (Pseudoranges)	56	57	60	71	78	88	95	98
Double Difference GPS (Carrier Phase)	56	57	57	68	81	88	95	99
IMU Measured	100	100	100	100	100	100	100	100

Table 135: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	51	53	57	78	90	95	99	100
Single Point GPS (Carrier Phase)	54	54	56	72	90	95	98	100
Single Difference GPS (Pseudoranges)	51	52	56	75	85	93	94	97
Single Difference GPS (Carrier Phase)	54	56	59	73	84	90	94	96
Double Difference GPS (Pseudoranges)	55	58	61	72	81	86	92	96
Double Difference GPS (Carrier Phase)	54	56	61	72	81	85	92	95
IMU Measured	99	100	100	100	100	100	100	100

Table 136: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	62	69	92	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	61	66	94	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	61	74	91	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 137: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	96	97	98	99	100	100	100	100
Single Point GPS (Carrier Phase)	54	54	58	73	91	96	98	100
Single Difference GPS (Pseudoranges)	82	87	94	100	100	100	100	100
Single Difference GPS (Carrier Phase)	54	56	61	76	85	92	98	99
Double Difference GPS (Pseudoranges)	85	91	94	98	99	100	100	100
Double Difference GPS (Carrier Phase)	54	57	61	72	83	90	94	98
IMU Measured	99	100	100	100	100	100	100	100

Table 138: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-4

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.775919	0.012938
Single Difference GPS (Pseudoranges)	0.910818	0.129777
Double Difference GPS (Pseudoranges)	-0.66525	-0.02701

Set B-5: 30 m Dual Antenna Separation

Figure 84: Computed and Measured Headings for Simulation Set B-5

Figure 85: Computed and Measured Pitch for Simulation Set B-5

Figure 86: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-5

Table 139: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.158	6.313	1.806
Single Point GPS (Carrier Phase)	2.621	0.004	2.181
Single Difference GPS (Pseudoranges)	3.506	6.75	2.736
Single Difference GPS (Carrier Phase)	3.81	0.005	2.992
Double Difference GPS (Pseudoranges)	3.836	5.085	3.108
Double Difference GPS (Carrier Phase)	3.923	0.01	3.194
IMU Measurement	0.006	-	0.005

Table 140: Mean Global Error Statistics of Unfused Heading for Simulation Set B-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.978	5.088	1.638
Single Point GPS (Carrier Phase)	2.144	0.003	1.752
Single Difference GPS (Pseudoranges)	3.088	6.22	2.446
Single Difference GPS (Carrier Phase)	3.181	0.003	2.584
Double Difference GPS (Pseudoranges)	2.973	4.278	2.387
Double Difference GPS (Carrier Phase)	3.08	0.009	2.44
IMU Measurement	0.343	-	0.287

Table 141: Mean Global Error Statistics of Fused Pitch for Simulation Set B-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.007	0.2	0.006
Single Point GPS (Carrier Phase)	1.598	0.002	1.356
Single Difference GPS (Pseudoranges)	0.008	0.2	0.006
Single Difference GPS (Carrier Phase)	2.174	0.003	1.719
Double Difference GPS (Pseudoranges)	0.02	0.199	0.01
Double Difference GPS (Carrier Phase)	2.076	0.005	1.496
IMU Measurement	0.006	-	0.005

Table 142: Mean Global Error Statistics of Fused Heading for Simulation Set B-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.358	1.266	0.299
Single Point GPS (Carrier Phase)	2.126	0.001	1.737
Single Difference GPS (Pseudoranges)	0.366	1.588	0.289
Single Difference GPS (Carrier Phase)	3.143	0.002	2.556
Double Difference GPS (Pseudoranges)	0.638	1.353	0.426
Double Difference GPS (Carrier Phase)	2.993	0.004	2.373
IMU Measurement	0.343	-	0.287

Table 143: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	68	73	87	100	100	100	100	100
Single Point GPS (Carrier Phase)	68	74	82	96	100	100	100	100
Single Difference GPS (Pseudoranges)	56	63	69	88	99	100	100	100
Single Difference GPS (Carrier Phase)	57	60	68	89	98	100	100	100
Double Difference GPS (Pseudoranges)	53	57	64	91	100	100	100	100
Double Difference GPS (Carrier Phase)	52	56	71	94	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 144: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	59	65	80	100	100	100	100	100
Single Point GPS (Carrier Phase)	56	62	79	100	100	100	100	100
Single Difference GPS (Pseudoranges)	64	67	75	96	100	100	100	100
Single Difference GPS (Carrier Phase)	61	66	77	95	100	100	100	100
Double Difference GPS (Pseudoranges)	55	60	75	95	100	100	100	100
Double Difference GPS (Carrier Phase)	55	63	71	94	100	100	100	100
IMU Measured	91	99	100	100	100	100	100	100

Table 145: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	71	82	94	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	59	67	80	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	61	74	95	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 146: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	89	99	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	56	63	80	100	100	100	100	100
Single Difference GPS (Pseudoranges)	90	99	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	61	67	77	95	100	100	100	100
Double Difference GPS (Pseudoranges)	84	93	99	100	100	100	100	100
Double Difference GPS (Carrier Phase)	55	63	71	94	100	100	100	100
IMU Measured	91	99	100	100	100	100	100	100

Table 147: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-5

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.88543	0.136893
Single Difference GPS (Pseudoranges)	0.770276	0.057302
Double Difference GPS (Pseudoranges)	0.800147	0.401223

Set B-6: 50 m Dual Antenna Separation

Figure 87: Computed and Measured Headings for Simulation Set B-6

Figure 88: Computed and Measured Pitch for Simulation Set B-6

Figure 89: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-6

Table 148: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.234	3.995	0.981
Single Point GPS (Carrier Phase)	1.461	0.003	1.149
Single Difference GPS (Pseudoranges)	2.144	3.897	1.757
Single Difference GPS (Carrier Phase)	2.338	0.003	1.9
Double Difference GPS (Pseudoranges)	2.396	3.1	1.893
Double Difference GPS (Carrier Phase)	2.551	0.006	2.028
IMU Measurement	0.007	-	0.005

Table 149: Mean Global Error Statistics of Unfused Heading for Simulation Set B-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.332	3.286	0.986
Single Point GPS (Carrier Phase)	1.306	0.002	0.964
Single Difference GPS (Pseudoranges)	1.825	3.245	1.446
Single Difference GPS (Carrier Phase)	1.785	0.002	1.426
Double Difference GPS (Pseudoranges)	2.01	2.423	1.545
Double Difference GPS (Carrier Phase)	2.057	0.005	1.604
IMU Measurement	0.3	-	0.24

Table 150: Mean Global Error Statistics of Fused Pitch for Simulation Set B-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.007	0.199	0.005
Single Point GPS (Carrier Phase)	1.033	0.001	0.816
Single Difference GPS (Pseudoranges)	0.008	0.199	0.007
Single Difference GPS (Carrier Phase)	1.735	0.001	1.383
Double Difference GPS (Pseudoranges)	0.009	0.199	0.007
Double Difference GPS (Carrier Phase)	1.391	0.003	1.093
IMU Measurement	0.007	-	0.005

Table 151: Mean Global Error Statistics of Fused Heading for Simulation Set B-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.294	0.826	0.233
Single Point GPS (Carrier Phase)	1.299	0.001	0.959
Single Difference GPS (Pseudoranges)	0.333	0.82	0.263
Single Difference GPS (Carrier Phase)	1.774	0.001	1.418
Double Difference GPS (Pseudoranges)	0.387	0.766	0.317
Double Difference GPS (Carrier Phase)	2.027	0.002	1.58
IMU Measurement	0.3	-	0.24

Table 152: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	65	79	94	100	100	100	100	100
Single Point GPS (Carrier Phase)	66	77	92	100	100	100	100	100
Single Difference GPS (Pseudoranges)	60	65	82	100	100	100	100	100
Single Difference GPS (Carrier Phase)	57	68	80	100	100	100	100	100
Double Difference GPS (Pseudoranges)	61	68	80	96	100	100	100	100
Double Difference GPS (Carrier Phase)	62	69	77	97	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 153: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	67	82	93	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	79	92	100	100	100	100	100
Single Difference GPS (Pseudoranges)	63	75	87	99	100	100	100	100
Single Difference GPS (Carrier Phase)	63	77	88	99	100	100	100	100
Double Difference GPS (Pseudoranges)	58	68	89	97	100	100	100	100
Double Difference GPS (Carrier Phase)	60	71	87	97	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 154: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	69	82	97	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	62	74	93	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	69	78	96	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 155: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	95	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	79	93	100	100	100	100	100
Single Difference GPS (Pseudoranges)	92	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	63	77	88	99	100	100	100	100
Double Difference GPS (Pseudoranges)	88	99	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	60	71	87	97	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 156: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-6

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.879492	0.205581
Single Difference GPS (Pseudoranges)	0.849702	0.14371
Double Difference GPS (Pseudoranges)	0.774141	0.172438

Set B-7: 70 m Dual Antenna Separation

Figure 90: Computed and Measured Headings for Simulation Set B-7

Figure 91: Computed and Measured Pitch for Simulation Set B-7

Figure 92: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-7

Table 157: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.88	2.849	0.699
Single Point GPS (Carrier Phase)	1.041	0.002	0.819
Single Difference GPS (Pseudoranges)	1.534	2.8	1.257
Single Difference GPS (Carrier Phase)	1.674	0.002	1.359
Double Difference GPS (Pseudoranges)	1.716	2.236	1.355
Double Difference GPS (Carrier Phase)	1.828	0.004	1.451
IMU Measurement	0.007	-	0.005

Table 158: Mean Global Error Statistics of Unfused Heading for Simulation Set B-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.947	2.323	0.702
Single Point GPS (Carrier Phase)	0.929	0.001	0.686
Single Difference GPS (Pseudoranges)	1.304	2.29	1.035
Single Difference GPS (Carrier Phase)	1.276	0.001	1.021
Double Difference GPS (Pseudoranges)	1.434	1.715	1.104
Double Difference GPS (Carrier Phase)	1.467	0.003	1.146
IMU Measurement	0.3	-	0.24

Table 159: Mean Global Error Statistics of Fused Pitch for Simulation Set B-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.007	0.199	0.005
Single Point GPS (Carrier Phase)	0.804	0.001	0.634
Single Difference GPS (Pseudoranges)	0.008	0.198	0.007
Single Difference GPS (Carrier Phase)	1.327	0.001	1.067
Double Difference GPS (Pseudoranges)	0.008	0.197	0.007
Double Difference GPS (Carrier Phase)	1.121	0.002	0.888
IMU Measurement	0.007	-	0.005

Table 160: Mean Global Error Statistics of Fused Heading for Simulation Set B-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.283	0.591	0.223
Single Point GPS (Carrier Phase)	0.926	0.001	0.684
Single Difference GPS (Pseudoranges)	0.322	0.584	0.255
Single Difference GPS (Carrier Phase)	1.271	0.001	1.017
Double Difference GPS (Pseudoranges)	0.352	0.545	0.289
Double Difference GPS (Carrier Phase)	1.451	0.002	1.134
IMU Measurement	0.3	-	0.24

Table 161: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	69	85	98	100	100	100	100	100
Single Point GPS (Carrier Phase)	68	82	95	100	100	100	100	100
Single Difference GPS (Pseudoranges)	61	72	93	100	100	100	100	100
Single Difference GPS (Carrier Phase)	62	74	90	100	100	100	100	100
Double Difference GPS (Pseudoranges)	65	73	85	100	100	100	100	100
Double Difference GPS (Carrier Phase)	64	73	85	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 162: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	75	86	96	100	100	100	100	100
Single Point GPS (Carrier Phase)	69	87	98	100	100	100	100	100
Single Difference GPS (Pseudoranges)	66	84	96	100	100	100	100	100
Single Difference GPS (Carrier Phase)	67	84	98	100	100	100	100	100
Double Difference GPS (Pseudoranges)	64	79	92	100	100	100	100	100
Double Difference GPS (Carrier Phase)	65	76	92	100	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 163: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	75	91	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	66	77	96	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	71	80	97	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 164: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	96	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	69	87	98	100	100	100	100	100
Single Difference GPS (Pseudoranges)	92	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	68	84	98	100	100	100	100	100
Double Difference GPS (Pseudoranges)	91	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	65	76	92	100	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 165: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-7

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.876989	0.26835
Single Difference GPS (Pseudoranges)	0.855939	0.209688
Double Difference GPS (Pseudoranges)	0.778497	0.229756

Set B-8: 90 m Dual Antenna Separation

Figure 93: Computed and Measured Headings for Simulation Set B-8

Figure 94: Computed and Measured Pitch for Simulation Set B-8

Figure 95: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set B-8

Table 166: Mean Global Error Statistics of Unfused Pitch for Simulation Set B-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.79	2.217	0.634
Single Point GPS (Carrier Phase)	0.885	0.002	0.73
Single Difference GPS (Pseudoranges)	1.213	2.217	0.968
Single Difference GPS (Carrier Phase)	1.278	0.002	1.023
Double Difference GPS (Pseudoranges)	1.193	1.784	0.989
Double Difference GPS (Carrier Phase)	1.225	0.004	0.987
IMU Measurement	0.007	-	0.006

Table 167: Mean Global Error Statistics of Unfused Heading for Simulation Set B-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.74	1.808	0.594
Single Point GPS (Carrier Phase)	0.8	0.001	0.634
Single Difference GPS (Pseudoranges)	0.927	1.827	0.782
Single Difference GPS (Carrier Phase)	0.972	0.001	0.826
Double Difference GPS (Pseudoranges)	1.032	1.377	0.808
Double Difference GPS (Carrier Phase)	1.053	0.003	0.799
IMU Measurement	0.293	-	0.237

Table 168: Mean Global Error Statistics of Fused Pitch for Simulation Set B-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.008	0.197	0.006
Single Point GPS (Carrier Phase)	0.721	0.001	0.594
Single Difference GPS (Pseudoranges)	0.008	0.197	0.006
Single Difference GPS (Carrier Phase)	1.033	0.001	0.829
Double Difference GPS (Pseudoranges)	0.008	0.195	0.007
Double Difference GPS (Carrier Phase)	0.816	0.002	0.654
IMU Measurement	0.007	-	0.006

Table 169: Mean Global Error Statistics of Fused Heading for Simulation Set B-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.278	0.469	0.225
Single Point GPS (Carrier Phase)	0.798	0	0.632
Single Difference GPS (Pseudoranges)	0.277	0.475	0.229
Single Difference GPS (Carrier Phase)	0.969	0	0.824
Double Difference GPS (Pseudoranges)	0.32	0.444	0.258
Double Difference GPS (Carrier Phase)	1.045	0.001	0.793
IMU Measurement	0.293	-	0.237

Table 170: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set B-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	73	93	99	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	88	98	100	100	100	100	100
Single Difference GPS (Pseudoranges)	64	80	97	100	100	100	100	100
Single Difference GPS (Carrier Phase)	62	80	96	100	100	100	100	100
Double Difference GPS (Pseudoranges)	60	76	96	100	100	100	100	100
Double Difference GPS (Carrier Phase)	58	71	95	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 171: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set B-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	78	94	98	100	100	100	100	100
Single Point GPS (Carrier Phase)	81	94	99	100	100	100	100	100
Single Difference GPS (Pseudoranges)	69	83	97	100	100	100	100	100
Single Difference GPS (Carrier Phase)	64	81	97	100	100	100	100	100
Double Difference GPS (Pseudoranges)	63	81	97	100	100	100	100	100
Double Difference GPS (Carrier Phase)	68	81	94	100	100	100	100	100
IMU Measured	92	100	100	100	100	100	100	100

Table 172: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set B-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	75	91	100	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	72	87	99	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	65	90	100	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 173: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set B-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	92	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	81	94	99	100	100	100	100	100
Single Difference GPS (Pseudoranges)	94	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	64	81	97	100	100	100	100	100
Double Difference GPS (Pseudoranges)	92	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	68	81	94	100	100	100	100	100
IMU Measured	92	100	100	100	100	100	100	100

Table 174: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set B-8

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.863665	0.28378
Single Difference GPS (Pseudoranges)	0.876758	0.3122
Double Difference GPS (Pseudoranges)	0.776088	0.238242

Appendix C: 3 m Pseudorange Noise Standard

Deviation Pitch and Heading Simulation Results

Set C-1 1 m Dual Antenna Separation

Figure 96: Computed and Measured Headings for Simulation Set C-1

Figure 97: Computed and Measured Headings for Simulation Set C-1

Figure 98: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-1

Table 175: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	34.368	317.315	27.65
Single Point GPS (Carrier Phase)	35.132	0.163	29.164
Single Difference GPS (Pseudoranges)	32.066	191.936	26.756
Single Difference GPS (Carrier Phase)	31.344	0.111	25.78
Double Difference GPS (Pseudoranges)	37.743	116.399	31.284
Double Difference GPS (Carrier Phase)	37.634	0.135	31.608
IMU Measurement	0.006	-	0.005

Table 176: Mean Global Error Statistics of Unfused Heading for Simulation Set C-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	96.937	2166.759	73.757
Single Point GPS (Carrier Phase)	93.557	0.573	73.619
Single Difference GPS (Pseudoranges)	95.091	959.116	84.036
Single Difference GPS (Carrier Phase)	97.803	0.352	84.817
Double Difference GPS (Pseudoranges)	102.56	581.813	76.231
Double Difference GPS (Carrier Phase)	104.158	0.587	76.725
IMU Measurement	0.34	-	0.264

Table 177: Mean Global Error Statistics of Fused Pitch for Simulation Set C-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.023	0.2	0.008
Single Point GPS (Carrier Phase)	3.506	0.059	2.449
Single Difference GPS (Pseudoranges)	0.025	0.2	0.009
Single Difference GPS (Carrier Phase)	5.641	0.044	3.392
Double Difference GPS (Pseudoranges)	0.013	0.2	0.009
Double Difference GPS (Carrier Phase)	9.263	0.052	4.428
IMU Measurement	0.006	-	0.005

Table 178: Mean Global Error Statistics of Fused Heading for Simulation Set C-1

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.989	8.513	0.729
Single Point GPS (Carrier Phase)	66.968	0.203	47.943
Single Difference GPS (Pseudoranges)	0.823	8.644	0.417
Single Difference GPS (Carrier Phase)	78.956	0.142	65.154
Double Difference GPS (Pseudoranges)	8.182	8.481	1.347
Double Difference GPS (Carrier Phase)	59.943	0.245	48.695
IMU Measurement	0.34	-	0.264

Table 179: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	55	57	60	64	70	74	80	83
Single Point GPS (Carrier Phase)	58	58	61	63	70	77	80	83
Single Difference GPS (Pseudoranges)	58	58	58	64	72	77	82	87
Single Difference GPS (Carrier Phase)	56	57	57	60	66	76	81	86
Double Difference GPS (Pseudoranges)	61	64	64	67	71	76	80	83
Double Difference GPS (Carrier Phase)	65	65	65	66	69	74	80	82
IMU Measured	100	100	100	100	100	100	100	100

Table 180: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	48	48	49	51	54	57	58	59
Single Point GPS (Carrier Phase)	49	50	52	54	55	56	57	60
Single Difference GPS (Pseudoranges)	41	41	42	42	42	44	47	50
Single Difference GPS (Carrier Phase)	43	43	43	44	45	46	49	51
Double Difference GPS (Pseudoranges)	53	53	54	56	60	60	60	60
Double Difference GPS (Carrier Phase)	54	54	54	56	59	59	60	61
IMU Measured	95	100	100	100	100	100	100	100

Table 181: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	73	84	92	98	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	63	80	89	94	99	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	69	79	85	93	96	97	99	99
IMU Measured	100	100	100	100	100	100	100	100

Table 182: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-1

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	89	96	96	98	98	99	99	99
Single Point GPS (Carrier Phase)	41	42	44	48	53	57	60	60
Single Difference GPS (Pseudoranges)	88	96	99	100	100	100	100	100
Single Difference GPS (Carrier Phase)	38	38	40	42	45	49	51	52
Double Difference GPS (Pseudoranges)	81	94	97	98	98	98	99	99
Double Difference GPS (Carrier Phase)	40	40	41	47	51	51	52	52
IMU Measured	95	100	100	100	100	100	100	100

Table 183: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-1

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.995489	0.001466
Single Difference GPS (Pseudoranges)	0.950991	0.002704
Double Difference GPS (Pseudoranges)	0.916941	0.003229

Set C-2 3 m Dual Antenna Separation

Figure 99: Computed and Measured Headings for Simulation Set C-2

Figure 100: Computed and Measured Pitch for Simulation Set C-2

Figure 101: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-2

Table 184: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	29.784	234.755	24.357
Single Point GPS (Carrier Phase)	29.828	0.125	25.185
Single Difference GPS (Pseudoranges)	33.737	164.225	27.97
Single Difference GPS (Carrier Phase)	34.755	0.113	29.312
Double Difference GPS (Pseudoranges)	37.695	182.592	30.948
Double Difference GPS (Carrier Phase)	38.317	0.159	31.498
IMU Measurement	0.007	-	0.005

Table 185: Mean Global Error Statistics of Unfused Heading for Simulation Set C-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	65.73	1780.105	41.604
Single Point GPS (Carrier Phase)	64.01	2.032	41.466
Single Difference GPS (Pseudoranges)	82.914	1066.18	49.601
Single Difference GPS (Carrier Phase)	83.239	1.977	49.891
Double Difference GPS (Pseudoranges)	85.076	1734.718	65.706
Double Difference GPS (Carrier Phase)	88.017	0.807	65.345
IMU Measurement	0.289	-	0.229

Table 186: Mean Global Error Statistics of Fused Pitch for Simulation Set C-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.007	0.2	0.006
Single Point GPS (Carrier Phase)	3.158	0.048	2.479
Single Difference GPS (Pseudoranges)	0.02	0.2	0.009
Single Difference GPS (Carrier Phase)	6.199	0.044	3.475
Double Difference GPS (Pseudoranges)	0.034	0.199	0.015
Double Difference GPS (Carrier Phase)	9.625	0.056	4.75
IMU Measurement	0.007	-	0.005

Table 187: Mean Global Error Statistics of Fused Heading for Simulation Set C-2

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.545	8.179	0.316
Single Point GPS (Carrier Phase)	39.345	0.326	25.653
Single Difference GPS (Pseudoranges)	2.203	8.485	0.592
Single Difference GPS (Carrier Phase)	63.207	0.275	37.751
Double Difference GPS (Pseudoranges)	2.466	8.392	0.646
Double Difference GPS (Carrier Phase)	45.794	0.314	32.907
IMU Measurement	0.289	-	0.229

Table 188: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	66	66	67	72	78	81	84	85
Single Point GPS (Carrier Phase)	65	65	68	70	77	80	82	87
Single Difference GPS (Pseudoranges)	62	62	63	67	71	77	81	88
Single Difference GPS (Carrier Phase)	60	61	63	64	69	78	80	83
Double Difference GPS (Pseudoranges)	55	55	58	60	66	70	78	81
Double Difference GPS (Carrier Phase)	54	54	57	61	65	71	78	79
IMU Measured	100	100	100	100	100	100	100	100

Table 189: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	51	51	52	53	56	61	65	66
Single Point GPS (Carrier Phase)	48	48	49	53	55	60	62	64
Single Difference GPS (Pseudoranges)	51	52	53	56	62	62	64	65
Single Difference GPS (Carrier Phase)	53	53	53	57	60	63	65	67
Double Difference GPS (Pseudoranges)	47	47	47	48	50	52	55	57
Double Difference GPS (Carrier Phase)	46	47	47	49	52	53	56	58
IMU Measured	96	100	100	100	100	100	100	100

Table 190: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	75	83	91	97	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	67	82	91	96	98	99	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	67	79	88	96	96	96	96	97
IMU Measured	100	100	100	100	100	100	100	100

Table 191: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-2

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	91	97	98	100	100	100	100	100
Single Point GPS (Carrier Phase)	46	47	48	56	65	71	75	77
Single Difference GPS (Pseudoranges)	90	97	99	100	100	100	100	100
Single Difference GPS (Carrier Phase)	51	52	53	60	69	73	74	75
Double Difference GPS (Pseudoranges)	89	97	99	100	100	100	100	100
Double Difference GPS (Carrier Phase)	37	38	40	46	55	61	65	69
IMU Measured	96	100	100	100	100	100	100	100

Table 192: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-2

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	-0.83668	-0.00099
Single Difference GPS (Pseudoranges)	-0.92067	-0.00024
Double Difference GPS (Pseudoranges)	-0.32977	-1.7×10^{-5}

Set C-3 5 m Dual Antenna Separation

Figure 102: Computed and Measured Headings for Simulation Set C-3

Figure 103: Computed and Measured Pitch for Simulation Set C-3

Figure 104: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-3

Table 193: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	26.931	223.215	21.626
Single Point GPS (Carrier Phase)	27.681	0.129	22.343
Single Difference GPS (Pseudoranges)	29.725	159.458	23.437
Single Difference GPS (Carrier Phase)	29.075	0.103	23.445
Double Difference GPS (Pseudoranges)	36.4	103.456	27.756
Double Difference GPS (Carrier Phase)	36.192	0.108	27.871
IMU Measurement	0.007	-	0.006

Table 194: Mean Global Error Statistics of Unfused Heading for Simulation Set C-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	48.704	870.824	32.053
Single Point GPS (Carrier Phase)	56.576	0.343	33.057
Single Difference GPS (Pseudoranges)	82.843	594.777	48.459
Single Difference GPS (Carrier Phase)	81.585	0.202	48.019
Double Difference GPS (Pseudoranges)	80.516	547.863	40.76
Double Difference GPS (Carrier Phase)	77.333	0.435	40.52
IMU Measurement	0.316	-	0.267

Table 195: Mean Global Error Statistics of Fused Pitch for Simulation Set C-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.011	0.2	0.007
Single Point GPS (Carrier Phase)	3.276	0.049	2.35
Single Difference GPS (Pseudoranges)	0.018	0.2	0.009
Single Difference GPS (Carrier Phase)	5.25	0.043	3.373
Double Difference GPS (Pseudoranges)	0.026	0.2	0.011
Double Difference GPS (Carrier Phase)	10.853	0.046	4.683
IMU Measurement	0.007	-	0.006

Table 196: Mean Global Error Statistics of Fused Heading for Simulation Set C-3

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.91	7.903	0.645
Single Point GPS (Carrier Phase)	34.519	0.142	19.706
Single Difference GPS (Pseudoranges)	5.395	7.918	0.987
Single Difference GPS (Carrier Phase)	63.32	0.089	33.647
Double Difference GPS (Pseudoranges)	1.436	7.993	0.659
Double Difference GPS (Carrier Phase)	44.332	0.195	27.166
IMU Measurement	0.316	-	0.267

Table 197: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	46	48	50	53	60	70	76	82
Single Point GPS (Carrier Phase)	46	47	49	52	57	67	76	80
Single Difference GPS (Pseudoranges)	54	55	61	62	63	67	75	78
Single Difference GPS (Carrier Phase)	55	56	57	60	62	66	74	80
Double Difference GPS (Pseudoranges)	53	53	54	60	67	69	73	77
Double Difference GPS (Carrier Phase)	49	49	53	60	69	70	75	79
IMU Measured	100	100	100	100	100	100	100	100

Table 198: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	58	59	61	64	68	71	73	74
Single Point GPS (Carrier Phase)	57	57	59	60	67	68	71	71
Single Difference GPS (Pseudoranges)	58	58	59	62	66	71	73	74
Single Difference GPS (Carrier Phase)	58	58	59	61	64	69	73	73
Double Difference GPS (Pseudoranges)	51	52	52	56	60	65	67	73
Double Difference GPS (Carrier Phase)	53	54	56	58	60	67	69	72
IMU Measured	94	100	100	100	100	100	100	100

Table 199: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	61	78	94	98	99	99	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	62	81	91	94	97	98	99	99
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	67	76	89	95	97	98	98	98
IMU Measured	100	100	100	100	100	100	100	100

Table 200: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-3

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	93	98	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	56	57	59	63	70	77	79	85
Single Difference GPS (Pseudoranges)	92	97	99	99	99	99	99	99
Single Difference GPS (Carrier Phase)	52	52	53	55	65	70	73	74
Double Difference GPS (Pseudoranges)	84	91	95	98	100	100	100	100
Double Difference GPS (Carrier Phase)	48	50	52	58	65	69	74	77
IMU Measured	94	100	100	100	100	100	100	100

Table 201: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-3

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	-0.4922	-8.8×10^{-5}
Single Difference GPS (Pseudoranges)	-0.60894	-9.4×10^{-5}
Double Difference GPS (Pseudoranges)	0.822186	0.00113

Set C-4 10 m Dual Antenna Separation

Figure 105: Computed and Measured Headings for Simulation Set C-4

Figure 106: Computed and Measured Pitch for Simulation Set C-4

Figure 107: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-4

Table 202: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	16.345	89.422	12.72
Single Point GPS (Carrier Phase)	16.598	0.048	12.615
Single Difference GPS (Pseudoranges)	22.023	81.832	17.422
Single Difference GPS (Carrier Phase)	22.286	0.052	17.699
Double Difference GPS (Pseudoranges)	27.019	95.913	21.088
Double Difference GPS (Carrier Phase)	28.275	0.09	21.95
IMU Measurement	0.007	-	0.005

Table 203: Mean Global Error Statistics of Unfused Heading for Simulation Set C-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	16.952	230.015	13.286
Single Point GPS (Carrier Phase)	16.68	0.126	13.175
Single Difference GPS (Pseudoranges)	29.232	838.467	20.336
Single Difference GPS (Carrier Phase)	28.991	0.867	20.34
Double Difference GPS (Pseudoranges)	44.332	248.361	26.771
Double Difference GPS (Carrier Phase)	43.974	0.225	26.889
IMU Measurement	0.315	-	0.252

Table 204: Mean Global Error Statistics of Fused Pitch for Simulation Set C-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.007	0.201	0.006
Single Point GPS (Carrier Phase)	3.901	0.023	2.578
Single Difference GPS (Pseudoranges)	0.279	0.198	0.035
Single Difference GPS (Carrier Phase)	4.112	0.024	3.38
Double Difference GPS (Pseudoranges)	0.018	0.2	0.008
Double Difference GPS (Carrier Phase)	2.744	0.042	1.996
IMU Measurement	0.007	-	0.005

Table 205: Mean Global Error Statistics of Fused Heading for Simulation Set C-4

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.652	6.958	0.38
Single Point GPS (Carrier Phase)	13.212	0.059	10.381
Single Difference GPS (Pseudoranges)	5.123	6.373	1.21
Single Difference GPS (Carrier Phase)	18.57	0.221	13.449
Double Difference GPS (Pseudoranges)	1.536	7.811	0.563
Double Difference GPS (Carrier Phase)	25.235	0.107	15.388
IMU Measurement	0.315	-	0.252

Table 206: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	52	55	56	60	75	82	88	93
Single Point GPS (Carrier Phase)	51	51	54	61	77	85	91	95
Single Difference GPS (Pseudoranges)	55	56	57	62	68	74	77	80
Single Difference GPS (Carrier Phase)	54	54	54	57	67	72	78	83
Double Difference GPS (Pseudoranges)	50	52	53	62	70	77	82	87
Double Difference GPS (Carrier Phase)	48	50	51	62	71	75	81	86
IMU Measured	100	100	100	100	100	100	100	100

Table 207: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-4

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	40	41	45	48	64	75	84	89
Single Point GPS (Carrier Phase)	43	43	43	48	63	76	86	90
Single Difference GPS (Pseudoranges)	58	60	61	63	69	74	77	79
Single Difference GPS (Carrier Phase)	57	57	59	67	71	75	77	79
Double Difference GPS (Pseudoranges)	51	51	52	57	65	74	77	81
Double Difference GPS (Carrier Phase)	51	52	53	56	65	73	76	79
IMU Measured	92	100	100	100	100	100	100	100

Table 208: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	58	76	99	99	99	99	99	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	56	66	90	97	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	60	75	91	97	99	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 209: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	85	98	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	43	43	43	51	70	86	91	94
Single Difference GPS (Pseudoranges)	85	96	96	97	98	99	99	99
Single Difference GPS (Carrier Phase)	58	58	61	70	78	84	87	89
Double Difference GPS (Pseudoranges)	81	95	97	99	99	100	100	100
Double Difference GPS (Carrier Phase)	51	52	54	61	73	80	82	87
IMU Measured	92	100	100	100	100	100	100	100

Table 210: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-4

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.785031	0.000681
Single Difference GPS (Pseudoranges)	0.945713	0.4263
Double Difference GPS (Pseudoranges)	-0.92185	-0.00346

Set C-5 30 m Dual Antenna Separation

Figure 108: Computed and Measured Headings for Simulation Set C-5

Figure 109: Computed and Measured Pitch for Simulation Set C-5

Figure 110: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-5

Table 211: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	4.691	25.361	3.786
Single Point GPS (Carrier Phase)	4.849	0.01	3.834
Single Difference GPS (Pseudoranges)	7.441	23.58	5.87
Single Difference GPS (Carrier Phase)	7.591	0.01	5.939
Double Difference GPS (Pseudoranges)	8.398	18.793	6.91
Double Difference GPS (Carrier Phase)	8.739	0.019	7.173
IMU Measurement	0.007	-	0.005

Table 212: Mean Global Error Statistics of Unfused Heading for Simulation Set C-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	4.247	23.916	3.281
Single Point GPS (Carrier Phase)	4.413	0.007	3.439
Single Difference GPS (Pseudoranges)	7.089	27.081	5.033
Single Difference GPS (Carrier Phase)	7.155	0.011	5.032
Double Difference GPS (Pseudoranges)	7.3	19.968	5.652
Double Difference GPS (Carrier Phase)	7.277	0.02	5.647
IMU Measurement	0.312	-	0.255

Table 213: Mean Global Error Statistics of Fused Pitch for Simulation Set C-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.017	0.2	0.007
Single Point GPS (Carrier Phase)	2.65	0.005	1.877
Single Difference GPS (Pseudoranges)	0.027	0.199	0.009
Single Difference GPS (Carrier Phase)	3.719	0.005	2.947
Double Difference GPS (Pseudoranges)	0.029	0.198	0.013
Double Difference GPS (Carrier Phase)	3.141	0.009	2.595
IMU Measurement	0.007	-	0.005

Table 214: Mean Global Error Statistics of Fused Heading for Simulation Set C-5

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.477	4.412	0.288
Single Point GPS (Carrier Phase)	4.29	0.004	3.351
Single Difference GPS (Pseudoranges)	0.739	3.921	0.408
Single Difference GPS (Carrier Phase)	6.32	0.005	4.703
Double Difference GPS (Pseudoranges)	1.492	3.82	0.751
Double Difference GPS (Carrier Phase)	6.582	0.01	5.23
IMU Measurement	0.312	-	0.255

Table 215: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	56	60	64	84	99	100	100	100
Single Point GPS (Carrier Phase)	59	60	69	85	98	100	100	100
Single Difference GPS (Pseudoranges)	61	65	72	81	92	98	99	100
Single Difference GPS (Carrier Phase)	59	63	67	81	89	99	99	99
Double Difference GPS (Pseudoranges)	61	64	69	81	93	99	100	100
Double Difference GPS (Carrier Phase)	65	65	70	81	92	99	99	100
IMU Measured	100	100	100	100	100	100	100	100

Table 216: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	54	62	66	82	96	100	100	100
Single Point GPS (Carrier Phase)	51	56	65	83	96	100	100	100
Single Difference GPS (Pseudoranges)	50	57	60	73	88	95	98	99
Single Difference GPS (Carrier Phase)	49	52	62	76	86	94	98	99
Double Difference GPS (Pseudoranges)	53	55	61	70	89	95	97	99
Double Difference GPS (Carrier Phase)	48	55	59	71	87	95	97	99
IMU Measured	94	100	100	100	100	100	100	100

Table 217: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	61	72	91	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	63	72	86	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	70	81	94	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 218: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-5

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	93	99	99	100	100	100	100	100
Single Point GPS (Carrier Phase)	51	56	65	83	98	100	100	100
Single Difference GPS (Pseudoranges)	87	95	98	100	100	100	100	100
Single Difference GPS (Carrier Phase)	49	52	62	76	87	96	99	100
Double Difference GPS (Pseudoranges)	81	89	93	97	100	100	100	100
Double Difference GPS (Carrier Phase)	48	55	60	73	88	96	99	100
IMU Measured	94	100	100	100	100	100	100	100

Table 219: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-5

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.914066	0.147612
Single Difference GPS (Pseudoranges)	-0.90226	-0.14099
Double Difference GPS (Pseudoranges)	-0.85897	-0.09175

Set C-6 50 m Dual Antenna Separation

Figure 111: Computed and Measured Headings for Simulation Set C-6

Figure 112: Computed and Measured Pitch for Simulation Set C-6

Figure 113: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-6

Table 220: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	3.001	15.258	2.5
Single Point GPS (Carrier Phase)	3.011	0.006	2.49
Single Difference GPS (Pseudoranges)	3.722	15.697	2.86
Single Difference GPS (Carrier Phase)	3.783	0.006	2.963
Double Difference GPS (Pseudoranges)	5.312	12.479	4.219
Double Difference GPS (Carrier Phase)	5.452	0.012	4.267
IMU Measurement	0.008	-	0.006

Table 221: Mean Global Error Statistics of Unfused Heading for Simulation Set C-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.909	12.756	2.244
Single Point GPS (Carrier Phase)	2.912	0.004	2.234
Single Difference GPS (Pseudoranges)	3.046	14.389	2.482
Single Difference GPS (Carrier Phase)	3.092	0.004	2.51
Double Difference GPS (Pseudoranges)	3.633	11.547	2.789
Double Difference GPS (Carrier Phase)	3.673	0.012	2.881
IMU Measurement	0.304	-	0.239

Table 222: Mean Global Error Statistics of Fused Pitch for Simulation Set C-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.008	0.2	0.006
Single Point GPS (Carrier Phase)	1.856	0.003	1.467
Single Difference GPS (Pseudoranges)	0.01	0.2	0.007
Single Difference GPS (Carrier Phase)	2.264	0.003	1.696
Double Difference GPS (Pseudoranges)	0.025	0.199	0.011
Double Difference GPS (Carrier Phase)	2.768	0.006	1.868
IMU Measurement	0.008	-	0.006

Table 223: Mean Global Error Statistics of Fused Heading for Simulation Set C-6

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.318	2.966	0.253
Single Point GPS (Carrier Phase)	2.869	0.002	2.206
Single Difference GPS (Pseudoranges)	0.402	3.206	0.29
Single Difference GPS (Carrier Phase)	3.06	0.002	2.483
Double Difference GPS (Pseudoranges)	0.859	3.192	0.409
Double Difference GPS (Carrier Phase)	3.545	0.006	2.781
IMU Measurement	0.304	-	0.239

Table 224: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	60	65	72	98	100	100	100	100
Single Point GPS (Carrier Phase)	58	68	73	97	100	100	100	100
Single Difference GPS (Pseudoranges)	56	59	67	93	98	100	100	100
Single Difference GPS (Carrier Phase)	54	57	70	91	98	100	100	100
Double Difference GPS (Pseudoranges)	50	55	64	79	96	100	100	100
Double Difference GPS (Carrier Phase)	51	53	64	80	96	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 225: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	48	62	72	94	99	100	100	100
Single Point GPS (Carrier Phase)	51	60	69	93	99	100	100	100
Single Difference GPS (Pseudoranges)	58	65	80	95	100	100	100	100
Single Difference GPS (Carrier Phase)	58	65	76	96	100	100	100	100
Double Difference GPS (Pseudoranges)	59	63	72	93	99	100	100	100
Double Difference GPS (Carrier Phase)	56	60	73	93	99	100	100	100
IMU Measured	97	100	100	100	100	100	100	100

Table 226: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	68	74	95	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	56	70	90	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	57	69	92	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 227: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-6

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	96	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	52	60	69	93	99	100	100	100
Single Difference GPS (Pseudoranges)	94	98	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	58	65	77	96	100	100	100	100
Double Difference GPS (Pseudoranges)	87	94	98	99	100	100	100	100
Double Difference GPS (Carrier Phase)	56	60	73	93	99	100	100	100
IMU Measured	97	100	100	100	100	100	100	100

Table 228: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-6

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.887863	0.059522
Single Difference GPS (Pseudoranges)	0.901028	0.08013
Double Difference GPS (Pseudoranges)	0.748957	0.029468

Set C-7 70 m Dual Antenna Separation

Figure 114: Computed and Measured Headings for Simulation Set C-7

Figure 115: Computed and Measured Pitch for Simulation Set C-7

Figure 116: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-7

Table 229: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	2.113	11.093	1.701
Single Point GPS (Carrier Phase)	2.202	0.004	1.76
Single Difference GPS (Pseudoranges)	2.875	11.265	2.335
Single Difference GPS (Carrier Phase)	2.896	0.004	2.345
Double Difference GPS (Pseudoranges)	2.87	8.581	2.361
Double Difference GPS (Carrier Phase)	2.97	0.009	2.397
IMU Measurement	0.007	-	0.006

Table 230: Mean Global Error Statistics of Unfused Heading for Simulation Set C-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.632	9.1	1.33
Single Point GPS (Carrier Phase)	1.694	0.003	1.364
Single Difference GPS (Pseudoranges)	2.345	9.897	1.905
Single Difference GPS (Carrier Phase)	2.345	0.003	1.915
Double Difference GPS (Pseudoranges)	2.598	6.653	2.075
Double Difference GPS (Carrier Phase)	2.582	0.007	2.074
IMU Measurement	0.327	-	0.257

Table 231: Mean Global Error Statistics of Fused Pitch for Simulation Set C-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.007	0.2	0.006
Single Point GPS (Carrier Phase)	1.469	0.002	1.142
Single Difference GPS (Pseudoranges)	0.007	0.2	0.006
Single Difference GPS (Carrier Phase)	1.869	0.002	1.487
Double Difference GPS (Pseudoranges)	0.008	0.2	0.006
Double Difference GPS (Carrier Phase)	1.493	0.004	1.15
IMU Measurement	0.007	-	0.006

Table 232: Mean Global Error Statistics of Fused Heading for Simulation Set C-7

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.321	2.19	0.255
Single Point GPS (Carrier Phase)	1.68	0.001	1.353
Single Difference GPS (Pseudoranges)	0.347	2.355	0.272
Single Difference GPS (Carrier Phase)	2.326	0.001	1.899
Double Difference GPS (Pseudoranges)	0.377	1.996	0.299
Double Difference GPS (Carrier Phase)	2.525	0.003	2.03
IMU Measurement	0.327	-	0.257

Table 233: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	63	66	88	99	100	100	100	100
Single Point GPS (Carrier Phase)	62	65	83	99	100	100	100	100
Single Difference GPS (Pseudoranges)	59	63	74	93	100	100	100	100
Single Difference GPS (Carrier Phase)	58	62	72	95	100	100	100	100
Double Difference GPS (Pseudoranges)	58	70	83	98	100	100	100	100
Double Difference GPS (Carrier Phase)	60	65	80	97	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 234: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	62	71	85	100	100	100	100	100
Single Point GPS (Carrier Phase)	62	71	86	100	100	100	100	100
Single Difference GPS (Pseudoranges)	59	70	83	98	100	100	100	100
Single Difference GPS (Carrier Phase)	62	71	80	98	100	100	100	100
Double Difference GPS (Pseudoranges)	61	67	75	96	100	100	100	100
Double Difference GPS (Carrier Phase)	59	68	75	97	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 235: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	78	95	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	60	69	84	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	69	84	97	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 236: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-7

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	96	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	62	71	86	100	100	100	100	100
Single Difference GPS (Pseudoranges)	94	99	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	62	71	80	98	100	100	100	100
Double Difference GPS (Pseudoranges)	92	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	59	68	75	97	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 237: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-7

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.882005	0.070036
Single Difference GPS (Pseudoranges)	0.854762	0.050123
Double Difference GPS (Pseudoranges)	0.799174	0.072105

Set C-8 90 m Dual Antenna Separation

Figure 117: Computed and Measured Headings for Simulation Set C-8

Figure 118: Computed and Measured Pitch for Simulation Set C-8

Figure 119: Propagated 95% Confidence Ellipses for Front and Rear GPS Antennas in Simulated Data Set C-8

Table 238: Mean Global Error Statistics of Unfused Pitch for Simulation Set C-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.542	8.795	1.23
Single Point GPS (Carrier Phase)	1.58	0.003	1.288
Single Difference GPS (Pseudoranges)	2.261	8.6	1.804
Single Difference GPS (Carrier Phase)	2.177	0.003	1.738
Double Difference GPS (Pseudoranges)	2.766	7.102	2.139
Double Difference GPS (Carrier Phase)	2.779	0.007	2.132
IMU Measurement	0.007	-	0.005

Table 239: Mean Global Error Statistics of Unfused Heading for Simulation Set C-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	1.504	7.291	1.178
Single Point GPS (Carrier Phase)	1.564	0.002	1.195
Single Difference GPS (Pseudoranges)	1.909	7.126	1.517
Single Difference GPS (Carrier Phase)	1.936	0.002	1.544
Double Difference GPS (Pseudoranges)	2.054	5.781	1.68
Double Difference GPS (Carrier Phase)	2.104	0.006	1.726
IMU Measurement	0.301	-	0.24

Table 240: Mean Global Error Statistics of Fused Pitch for Simulation Set C-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.006	0.2	0.005
Single Point GPS (Carrier Phase)	1.099	0.002	0.892
Single Difference GPS (Pseudoranges)	0.007	0.2	0.006
Single Difference GPS (Carrier Phase)	1.515	0.002	1.214
Double Difference GPS (Pseudoranges)	0.008	0.2	0.006
Double Difference GPS (Carrier Phase)	1.584	0.004	1.109
IMU Measurement	0.007	-	0.005

Table 241: Mean Global Error Statistics of Fused Heading for Simulation Set C-8

Method	Measured Standard Deviation (degrees)	Estimated Standard Deviation (degrees)	Mean RMS Error (degrees)
Single Point GPS (Pseudoranges)	0.294	1.776	0.234
Single Point GPS (Carrier Phase)	1.554	0.001	1.187
Single Difference GPS (Pseudoranges)	0.299	1.738	0.239
Single Difference GPS (Carrier Phase)	1.924	0.001	1.534
Double Difference GPS (Pseudoranges)	0.491	1.786	0.295
Double Difference GPS (Carrier Phase)	2.071	0.003	1.697
IMU Measurement	0.301	-	0.24

Table 242: Success Rates of Reaching Angular Specifications of Unfused Pitch for Simulation Data Set C-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	60	75	89	100	100	100	100	100
Single Point GPS (Carrier Phase)	59	69	91	100	100	100	100	100
Single Difference GPS (Pseudoranges)	65	72	82	98	100	100	100	100
Single Difference GPS (Carrier Phase)	62	73	84	98	100	100	100	100
Double Difference GPS (Pseudoranges)	50	60	75	96	100	100	100	100
Double Difference GPS (Carrier Phase)	50	60	75	95	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 243: Success Rates of Reaching Angular Specifications of Unfused Heading for Simulation Data Set C-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	68	80	91	100	100	100	100	100
Single Point GPS (Carrier Phase)	71	77	90	100	100	100	100	100
Single Difference GPS (Pseudoranges)	64	72	90	98	100	100	100	100
Single Difference GPS (Carrier Phase)	62	73	87	98	100	100	100	100
Double Difference GPS (Pseudoranges)	54	62	77	98	100	100	100	100
Double Difference GPS (Carrier Phase)	55	63	80	98	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 244: Success Rates of Reaching Angular Specifications of Fused Pitch for Simulation Data Set C-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	65	83	98	100	100	100	100	100
Single Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	70	80	94	100	100	100	100	100
Double Difference GPS (Pseudoranges)	100	100	100	100	100	100	100	100
Double Difference GPS (Carrier Phase)	62	79	94	100	100	100	100	100
IMU Measured	100	100	100	100	100	100	100	100

Table 245: Success Rates of Reaching Angular Specifications of Fused Heading for Simulation Data Set C-8

Method	Portion of Trials in Which Target Specification was reached (%)							
	$\pm 0.5^\circ$	$\pm 1^\circ$	$\pm 2^\circ$	$\pm 5^\circ$	$\pm 10^\circ$	$\pm 15^\circ$	$\pm 20^\circ$	$\pm 25^\circ$
Single Point GPS (Pseudoranges)	94	100	100	100	100	100	100	100
Single Point GPS (Carrier Phase)	71	78	90	100	100	100	100	100
Single Difference GPS (Pseudoranges)	94	100	100	100	100	100	100	100
Single Difference GPS (Carrier Phase)	62	74	87	98	100	100	100	100
Double Difference GPS (Pseudoranges)	89	98	99	100	100	100	100	100
Double Difference GPS (Carrier Phase)	55	63	81	98	100	100	100	100
IMU Measured	95	100	100	100	100	100	100	100

Table 246: Pitch / Heading Correlation Coefficients for Each Computation Method in Simulation Data Set C-8

Method	Pitch / Heading Correlation Coefficient	
	Unfused	Fused
Single Point GPS (Pseudoranges)	0.89194	0.121567
Single Difference GPS (Pseudoranges)	0.81833	0.049476
Double Difference GPS (Pseudoranges)	0.815788	0.096819

Appendix D: Summary of Global Trends Across all Partial Attitude Simulation Data Sets

The highest performing GPS attitude method across all simulations was the single point pseudorange solution. Therefore, this section will examine error and success metrics for the pseudorange single point method across each simulation series.

Series A Summary

Figure 120: Global Error and Noise Trends for Pseudorange Solutions Across All Series A Simulation Data Sets

Figure 121: Series A Success Levels in Meeting Angular Specifications Between 1/2 Degree and 5 Degrees

Figure 122: Series A Success Levels in Meeting Angular Specifications Between 10 Degrees and 25 Degrees

Series B Summary

Figure 123: Global Error and Noise Trends for Pseudorange Solutions Across All Series B Simulation Data Sets

Figure 124: Series B Success Levels in Meeting Angular Specifications Between 1/2 Degree and 5 Degrees

Figure 125: Series B Success Levels in Meeting Angular Specifications Between 10 Degrees and 25 Degrees

Series C Summary

Figure 126: Global Error and Noise Trends for Pseudorange Solutions Across All Series C Simulation Data Sets

Figure 127: Series C Success Levels in Meeting Angular Specifications Between 1/2 Degree and 5 Degrees

Figure 128: Series C Success Levels in Meeting Angular Specifications Between 10 Degrees and 25 Degrees