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STRUCTURAL AND AERODYNAMIC STABILITY ANALYSIS OF LONG-SPAN CABLE-STAYED BRIDGES

by

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A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

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Department of Civil and Environmental Engineering

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**STRUCTURAL AND AERODYNAMIC STABILITY
ANALYSIS OF LONG-SPAN CABLE-STAYED BRIDGES**

submitted by

SHAOHONG, CHENG, B.S., M.S.

in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

External Examiner

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December 1999

Abstract

The increasing span length of the modern cable-supported bridges makes the aerodynamic stability a major concern for this type of structure. Flutter, which is a self-excited aerodynamic instability phenomenon, has become one of the governing criterion in the design of the long-span bridges.

The determination of the dynamic characteristics of the bridge is an essential first step in the flutter analysis. The common practice is to use the 3D equivalent beam finite elements to develop the analytical model. Some assumptions on the behaviour of the bridge response must be made in order to simulate the mass and stiffness properties of a complex bridge girder by the equivalent beam, which may be over-simplified and thus have a significant impact on the accuracy of the results. Further, the state-of-the-art 3D flutter analysis models for bridges based on the equivalent beam finite elements cannot account the spatial distribution effects over the bridge deck cross-section. A major component of the present research is to study the application of the finite strip method in the dynamic characteristics and flutter analysis of the long-span cable-stayed bridges. The mass and stiffness properties of the bridge girder can be accurately represented by minimum modelling effort. The spatial distribution effects of the aerodynamic forces can be taken into account by distributing the load on the individual finite strip elements.

Under the dynamic loads, the vibrational behaviour of the cable-stayed bridges involves the motion of the bridge girder, pylon(s) as well as the stay cables. The transverse vibration of the cables, which can be significant in some cases, are usually neglected in most of the research. The impact of the cable vibration effect on the dynamic characteristics and the flutter behaviour of the long-span cable-

stayed bridges having different combinations of structural layout and aerodynamic properties is studied in the current research.

The unique properties of the CFRP make it an ideal substituent cable material of the traditional steel to be used in the long-span cable-supported structure. Very few research has been conducted to study the bridge response by using CFRP cables. In the present study, the impact of using CFRP cable on the dynamic characteristics and the flutter response of the cable-stayed bridges having different combinations of structural layout and aerodynamic properties are carried out.

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List of Symbols

A	the cross-sectional area
A_i^*, H_i^*, P_i^*	the flutter derivatives of the bridge deck cross-section, $i = 1, 2 \dots 6$
A_m, B_m	the coefficients for defining the B_3 spline function $\Phi_m(y)$
A_{CFRP}	the cross-sectional area of the CFRP cable
A_{steel}	the cross-sectional area of the steel cable
$\{A\}$	the self-excited aerodynamic force vector
$[A_d]$	the global aerodynamic damping matrix
$[A_d]_c$	the aerodynamic damping matrix of the cable element
$[A_d]^g$	the generalized aerodynamic damping matrix
$[A_s]$	the global aerodynamic stiffness matrix
$[A_s]_c$	the aerodynamic stiffness matrix of the cable element
$[A_s]^g$	the generalized aerodynamic stiffness matrix
$\{A_i\}, \{A_j\}$	the lumped aerodynamic force vectors of node i and j
B	the width of the bridge deck
$[B]$	the strain-displacement relationship matrix
C_D	the static drag force coefficient
C_L	the static lift force coefficient
$C(k)$	the Theodorsen circular function
C_m, D_m	the coefficients for defining the B_3 spline function $\Phi_m(y)$

D_p	the drag force
$[D]$	the global damping matrix
$[D]^g$	the generalized damping matrix
$[D_E]$	the elastic matrix
$[D_E]_p$	the elastic matrix of the plane stress strip
$[D_E]_b$	the elastic matrix of the bending plate strip
E	the Young's modulus
E_{CFRP}	the Young's modulus of the CFRP cable
E_c	the elastic modulus of concrete
E_{eq}	the Ernst equivalent modulus
E_s	the elastic modulus of steel
E_{steel}	the Young's modulus of the steel cable
$ F $	the flutter determinant
$F(k)$	the real part of the Theodorson circular function
$F_{D_{se}}$	the self-excited drag force component
$F_{D_{st}}$	the static drag force component
$F_{L_{se}}$	the self-excited lift force component
$F_{L_{st}}$	the static lift force component
G	the shear modulus
$G(k)$	the imaginary part of the Theodorson circular function

H_{1c}^*, H_{5c}^*	the flutter derivatives of the stay cable
I_M	the mass moment of inertia
$I_{\bar{y}}$	the moment of inertia about $\bar{Y}$ axis
$I_{\bar{z}}$	the moment of inertia about $\bar{Z}$ axis
J	the moment of inertia about $\tilde{X}$ axis
J_d	the free torsional moment of inertia
J_ω	the principal sectorial moment of inertia
K	the reduced frequency, $K = B\omega/U$
K_c	the static stretching stiffness of the cable
K_e	the elastic stiffness of the cable
K_g	the geometric stiffness of the cable
$K(\omega)$	the dynamic cable stiffness
$[K]$	the global stiffness matrix
$[K]^g$	the generalized stiffness matrix
$[K]_s^e$	the stiffness matrix of the eight-node sub-element in the global coordinate system
$[\bar{K}]_c$	the stiffness matrix of the stay cable element in the local coordinate system
$[\bar{K}]_s^e$	the stiffness matrix of the eight-node sub-element in the local coordinate system

L	the chord length of the cable
L_h	the lift force
M_α	the aerodynamic moment
$[M]$	the global mass matrix
$[M]^g$	the generalized mass matrix
$[M]_s^e$	the mass matrix of the eight-node sub-element in the global coordinate system
$[\bar{M}]_c$	the total mass matrix of the cable element in the local coordinate system
$[\bar{M}]_c^t$	the local mass matrix of the cable element related to the transverse motion of the cable
$[\bar{M}]_s^e$	the mass matrix of the eight-node sub-element in the local coordinate system
$[N]$	the shape function matrix
N_{im}	the vertical displacement shape function of the spline finite strip, $i = 1, 2, 3, 4$
$[N_l]$	the shape function matrix of displacement l , $l = \bar{u}, \bar{v}, \bar{w}$
P	the complex value function, $P = K(\delta + i)$
P_{1c}^*, P_{5c}^*	the flutter derivatives of the stay cable
$P_{i,1}, P_{i,2}$	the starting values in flutter iteration

$\{P\}_Y$	the equivalent nodal force of the three-node cable element
$\vec{R}, \vec{S}, \vec{T}$	the unit vectors of X , Y and Z axes
$\{R\}$	the response amplitude vector
S	the total number of finite strips or the tension in the string
S_i	the Strouhal Number of the i^{th} mode
T	the mean tension in the cable
$[T]$	the coordinate transformation matrix
$[T_3]$	the displacement transformation matrix of the transition cable element
U	the wind velocity
U_{10}	the annual extreme wind velocity for a 100-year return period
U_{cur}	the current wind velocity
U_D	the design wind velocity
U_f	the critical flutter wind velocity
U_{high}	the highest wind velocity
U_{inc}	the increment of the wind velocity
U_{low}	the lowest wind velocity
U_p	the critical flutter wind velocity of an equivalent thin flat plate
U_r	the reduced wind velocity or relative wind velocity
W	the cable weight per unit length
$\bar{X}, \bar{Y}, \bar{Z}$	the axes in the local coordinate system

$X, \dot{X}, \ddot{X}$	the displacement, velocity and acceleration vector
Y	the generalized coordinate vector
$Y_n(x)$	the mode shape of the n_{th} transverse vibration mode of a horizontal fixed-fixed string
Z_e, X_e, M_e	the lumped aerodynamic lift force, drag force and moment at the elasticity center of the deck cross-section
a_0, a_1	the proportionality constants of the Rayleigh damping matrix
a_{yn}, a_{zn}	the generalized coordinates corresponding to the n^{th} sine shape function term
b	the half width of the thin flat plate or width of the finite strip
d	the diameter of the cable
f_b	the frequency of the lowest symmetrical heave mode
f_t	the frequency of the lowest symmetrical torsional mode
f_n	the n^{th} modal frequency of the transverse motion of a horizontal of a horizontal fixed-fixed string
$h, \dot{h}$	the vertical displacement, velocity of the bridge girder
k	the reduced frequency, $k = b\omega/U$
l	the length of the element or a horizontal string
l_I	the width of the I^{th} strip
l_h	the horizontal projection of an inclined cable

l_{ij}	the width of the strip formed by two nodal lines i and j
$p, \dot{p}$	the lateral displacement, velocity of the bridge girder
$\{p\}_m^F$	the equivalent knot load at the cross-section m due to the aerodynamic force F , $F = L_H, D_p, M_\alpha$
r	the total number of longitudinal sections on a nodal line or the radius of gyration of the deck cross-section
$\vec{r}, \vec{s}, \vec{t}$	the unit vectors of $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes
t	the thickness of the strip or the time variable
$\bar{u}, \bar{v}, \bar{w}$	displacements along the $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes
$\bar{u}_{km}, \bar{v}_{km}$	displacements parameter of nodal line k at knot m , $k = 1, 2$
$\bar{w}_{km}, \bar{\theta}_{km}$	displacements parameter of nodal line k at knot m , $k = 1, 2$
$\bar{x}$	the local coordinates of a point in the $\bar{X}$ direction
$\bar{x}_M, \bar{y}_M$	the coordinates of an arbitrary point M within the strip
$\dot{y}, \dot{z}$	the velocities in the cross-wind and along-wind direction
y_i	coordinates of the $(i + 1)^{th}$ knot, $i = m - 2, m - 1, m, m + 1, m + 2$
$\Phi_{km}^l(y)$	the unequal section length B_3 spline function centered at $y = y_m$, $k = 1, 2; l = u, v, w, \theta$
$[\Phi]$	the mode shape matrix
$\alpha, \dot{\alpha}$	the rotational displacement, velocity of the bridge girder
α_e	the effective wind attack angle

α_m	the coefficients used for interpolating an arbitrary function by B_3 spline function
$\bar{\alpha}, \bar{\beta}, \bar{\gamma}$	rotations around the $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes
β	the angle between the X and $\bar{X}$ axes or Z and $\bar{Z}$ axes
$\bar{\chi}_x, \bar{\chi}_y$	the curvatures of the strip element in the local coordinate system
$\bar{\chi}_{xy}$	the twists of the strip element in the local coordinate system
δ	the logarithmic decrement of the response
$\{\delta\}$	the displacement parameter vector in the global coordinate system
$\{\bar{\delta}\}$	the displacement parameter vector in the local coordinate system
ϵ	the frequency ratio between the torsional and heave modes
$\bar{\epsilon}_x, \bar{\epsilon}_y, \bar{\nu}_{xy}$	the strains of the strip element in the local coordinate system
$\{\bar{\epsilon}\}$	the strain matrix in the local coordinate system
γ	the weight per unit volume
γ_1	the height modification factor
γ_2	the horizontal length modification factor
η	the shape modification factor
$[\varphi]$	the initial shape vector matrix
$[\lambda]$	the coordinate transformation matrix
μ	the mass ratio between the bridge deck and the surrounding air or mass of a string per unit span

ν	the Poisson's ratio
ρ	the mass density or air mass density
σ	the stress induced by the self-weight of the cable
$[\sigma]_{CFRP}$	the ultimate stress of the CFRP material
$[\sigma]_{steel}$	the ultimate stress of the steel material
$\{\bar{\sigma}\}$	the stress matrix in the local coordinate system
ω	the circular frequency of response
ω_0	the fundamental frequency of a taut wire
ω_f	the critical circular flutter frequency
ω_n	the n_{th} circular frequency of the transverse motion of a horizontal fixed-fixed string
ξ, η, ζ	nondimensional parameters
$\{\xi\}$	the generalized coordinate vector

Chapter 1

Introduction

1.1 Background of Cable-Stayed Bridges

The concept of the cable-stayed bridge, namely, to support a beam by the stays from a pylon, can be dated back to the 1600s, when a Venetian engineer named Verantius built a bridge with several diagonal chain stays in Venice (Hopkins, 1970; Kavanagh, 1973). Some pedestrian cable-stayed bridges were built during the period of the 17th to the 19th century. The stayed bridge might have become a common form of construction at the time had it not been for the bad publicity that followed the failure of several bridges, including one over the Tweed River near Ryburgh-Abbey, England (Leonhardt and Zellner, 1970), and another one across the Saale River near Nienburg, Germany (Feige, 1967). The main reason of the failure of these bridges is the lack of adequate theoretical tools and techniques for the analysis of the structural behaviour and the lack of suitable high-strength materials for the fabrication of the cable stays. In addition to the collapse of the early cable-stayed

bridges in Britian, the French engineer Claude Louis Navier (1823) identified some of the shortcomings of the early cable-stayed bridges at the time, which ruled out the acceptance and further development of this type of bridges. Navier considered the cable-stayed bridges have no economic and aesthetic advantages over the suspension bridges. His opinions had such a wide influence on the engineering community that the idea of cable-stayed bridges was in fact abandoned in the next one hundred and twenty-five years.

The German engineer F.Dischinger (1949) rediscovered the stayed bridge, while designing a suspension bridge across the Elbe River near Hamburg in 1938. He recognized that the inclined cables of the early cable-stayed bridges were never subject to any initial tension, thus cables started to perform properly only after considerable deformations of the whole structure. In earlier development, this behaviour led to the misconception that this type of bridge was unacceptably flexible and consequently unsafe.

During World War II, approximately 15,000 bridges were destroyed in Germany. The demand to rebuild these bridges was urgent. The requirements of efficient use of materials and speedy construction made cable-stayed bridges the most economical design for the replacements. The first modern cable-stayed bridge, the Strömsund Bridge designed by F.Dischinger, was completed in Sweden in 1955. The design and construction of this bridge represent the beginning of a new era of modern cable-stayed bridges.

The rapid growth of modern cable-stayed bridges throughout the world afterwards is due to the many advances in bridge engineering leading towards better understanding of the behaviour and performance , and recognizing the advantages of this type of bridges in terms of economy, ease of fabrication and construction,

aesthetics and the different possibilities in structural arrangements, etc.. Nowadays, the cable-stayed bridge has been recognized as a very efficient and competitive design for bridges of span ranging from 200m to 800m. For a span length between 200m and 400m, the reinforced concrete girder design for the longitudinal bridge member is generally considered more economical. For a span length between 400m to 600m, the composite deck cross-section can be considered, whereas for 600m to 800m, the steel box girder or composite deck design is preferable.

The recent trend of longer span length is quite remarkable in the design of cable-stayed bridges. The Normandy Bridge (Virlogeux, 1988) which crosses the Seine River in France has a main span of 856m. It was completed in 1994. The Tatara Bridge (Honshu-shikoku Bridge Authority, 1989; Technical Comm. Wind Resistant Design of Honshu-shikoku Bridges, 1990) in Japan, which is part of the Onomichi-Imabari-route of the Honshu-Shikoku Project, has a main span of 890m. It was completed recently in May, 1999. Bridges of such long span length used to be the domain of the suspension bridge system. More recently, Kindmann (1988), Leonhardt (1980), Gimsing (1987) and some others have proposed the construction of even longer span cable-stayed bridges in the range of 1100m to 3000m.

Results from wind tunnel experimental studies and computer simulations have indicated that because of the very long span length, modern cable-stayed bridges have lower stiffness and mechanical damping properties than other cable-stayed bridges of typical span length, which results in a more flexible structural system. Thus, the sensitivity of the long-span cable-stayed bridge behaviour to the dynamic loads, such as earthquake and wind, is of even greater concern to the bridge engineers and designers. The wind stability behaviour and performance are often one of the governing criteria in the design of modern long-span cable-stayed bridges.

1.2 Review of Studies on Bridge Flutter Phenomenon

1.2.1 Wind-induced Vibration

A bridge structure immersed in an air flow field is subjected to the surface stresses induced by that flow. Due to the effect of turbulence in the atmospheric boundary layer, the velocity, direction and spatial distribution of the wind load are random and non-steady (time-dependent). For a typical highway bridge, the shape of the deck cross-section is generally unstreamlined. When the air flow passes around the bridge structure, vortices and separation of the flow might occur, which results in the complicated aerodynamic forces. If the bridge structure is stiff, the relative wind forces acting on it will not be affected by the responses of the bridge. For flexible structures such as long-span cable-suspended bridges, the wind-induced motions of the structures may lead to changes in the boundary conditions, which will not only affect the wind force but in turn influence the responses of the structures as well.

Aeroelasticity is the discipline concerning the study of the interaction problem between the structural motions and the aerodynamic forces (Simiu and Scanlan, 1986). If the interaction leads to an increase in the magnitude of the oscillatory motions and/or results in a divergent behaviour, an aeroelastic instability is said to have occurred. The aeroelastic instability is caused by the aerodynamic forces induced on the structure as a consequence of its motion. Such aerodynamic forces are called self-excited forces. The aerodynamic behaviour of a structure depends on the interaction between the aerodynamic forces defined by the aeroelasticity model, and the inertial, the damping and the deformation properties due to the dynamic characteristics of the structures, as shown in Figure 1.1 (Dowell *et al*, 1978).

For wind design of structures, Smeaton, who is considered as the “first civil engineer” to apply the practical knowledge of wind in design applications, presented a “Table of Wind Force” to the Royal Society of London in 1759 (Smeaton, 1759). In the early studies of wind responses of structures, the wind actions were approximated as the static forces caused by the wind pressure. It was not until the collapse of the Tacoma Narrows Bridge under a wind condition of only moderate intensity at 19m/s on November 7, 1940, did the engineers realize the importance of the dynamic effects of the wind loads on the structures.

In the past 50 years, the mechanisms of various types of wind-induced vibration in bridges have been studied by structural engineers and aerodynamicists. The dynamic responses to the wind load on bridge structures are the combined effects of the transient actions of the natural wind, the dynamic characteristics of the structure, and the interactions between the wind load and the structural motions. The aerodynamic phenomena of the bridge structures can be classified into four types based on the properties and characteristics of the induced vibration. These are flutter, galloping, buffeting and vortex-induced vibration. Flutter and galloping are self-excited vibrations with divergent amplitude. They are caused by the mean air flow of the wind. Buffeting is a kind of limited-amplitude, forced vibration resulting from the fluctuating component of the wind velocity. Vortex-induced vibration has the characteristics of both of these two types of vibrational behaviour. The classification and nature of the wind-induced vibrations are presented in Figure 1.2.

1.2.2 Bridge Flutter Behaviour

The term “flutter” is originally derived from the aeronautical engineering where it is used to describe an aeroelastic instability phenomenon which involves the coupling of the torsional motion and vertical bending motion of the airfoils. Flutter is a self-excited oscillatory instability phenomenon. It is induced by the interaction between the inertial forces, the damping forces, the elastic forces and the self-excited aerodynamic forces acting on the structures. When a slender bridge structure is subjected to wind load of a certain critical wind velocity, instead of dissipating energy, the aerodynamic forces start to feed energy into the oscillating bridge structural system, which makes the bridge vibrate in a divergent, destructive manner.

According to Scanlan (1987), bridge flutter can be classified into two types based on their different driving mechanisms as follows:

Stiffness-driven Flutter

The first type of flutter behaviour is also known as classical flutter or binary flutter. This type of flutter behaviour is driven by the stiffness of the air flow. For bridges, it is most likely to occur with a streamlined flat deck cross-section. The flutter behaviour observed in the airfoils belongs to this category. The single degree-of-freedom galloping and torsional flutter can hardly occur in bridges with streamlined deck cross-sections. However, when the vertical bending and the torsional motions are coupled by a particular difference in the phase response of this type of bridge, the classical coupled flutter can occur under a certain critical wind velocity. The aerodynamic stiffness effect induced by a high speed air flow may alter the dynamic characteristics of the structure in such a way that the frequency of the

bending mode is increased, while that of the torsional mode is decreased. Thus at a critical point, the two vibration modes are coupled and cause the bridge to vibrate in a divergent manner. The critical wind velocity for this type of flutter is relatively high.

In the early studies of wind responses of bridges, it was thought that the frequency ratio between the torsional and the vertical bending modes plays an important role in the stiffness-driven type of flutter behaviour. But results from recent analytical studies and wind tunnel tests have shown that it is not the frequency ratio, but the torsional frequency that is the predominant factor affecting the stiffness-driven flutter behaviour in bridges. The relationship between the critical wind velocity and the torsional vibration mode frequency is approximately linear. Another important factor which has been found to have a significant effect on the flutter phenomenon is the mass ratio between the mass of the bridge structure and that of the surrounding air. The variation of this dimensionless parameter has a considerable impact on the critical wind velocity of the coupled flutter.

Damping-driven Flutter

The second type of flutter behaviour is also known as the separated-flow torsional flutter or single-degree-of-freedom torsional flutter. It often occurs when the bridge structure has a blunt deck cross-section, and relates directly to the flow separation and vortex-shedding of the deck. The effective damping of the structure, i.e. the combined damping effects of the mechanical damping and the aerodynamic damping, varies with the wind velocity. The increase of the wind velocity may

lead the effective damping of the bridge to change from positive to negative when the negative damping effect of the air flow overcomes the positive damping of the structure. Under such conditions, the bridge continuously absorbs energy from the surrounding flow field, resulting in the increase of the oscillatory amplitude of the structural motion which then finally initiates flutter. The frequency of this type of flutter behaviour is close to that of the torsional mode.

Typically, the natural wind contains a turbulent component. The wind load usually acts on the bridge structure at an attack angle. This increases the vertical projection of the streamlined deck cross-section, which thus decreases the flat ratio between the horizontal and vertical projections of the deck and magnifies the blunt effect of the deck cross-section. Thus, even in the case of a bridge with streamlined deck cross-section, the damping-driven flutter phenomenon can still be an important factor for consideration in design. This phenomenon is similar to the observed behaviour of stall flutter when an airfoil is subjected to wind load under a large attack angle. Considering the aerodynamic properties, the behaviour of a streamlined deck cross-section subjected to wind at an attack angle are modified towards that of an unstreamlined cross-section. Under this condition, two separate flutter mechanisms are involved in the wind responses of the bridge deck. The actual type or mode of the complicated flutter response must be determined through careful analysis and more research is needed in this area. It can be generally concluded that with the wind load at an attack angle, the bridge deck cross-section, whether streamlined or not, may have similar aerodynamic properties in nature as that of a blunt deck cross-section.

The critical flutter wind velocity depends on a number of parameters related to the structural and aerodynamic properties of the bridge. The former includes

the stiffness, the damping and the dynamic characteristics of the bridge structure, whereas the latter concerns the geometric shape of the deck cross-section. It is the combining effects of these two groups of factors that determines the flutter behaviour in the bridge structure.

The accurate prediction of the critical flutter wind velocity U_f is an important task in the design of modern long-span bridges. In the wind-resistant design, the critical flutter wind velocity of the bridge should be higher than the maximum possible wind velocity with a low probability of occurrence at the bridge site in order to ensure the safe performance of the structure. This includes consideration for the bridge under different construction stages as well as the completed bridge.

1.2.3 Bridge Flutter Analysis

The vigorous study of the mechanism and the critical condition of flutter began after the failure of the Tacoma Narrows Bridge in 1940. The ensuing investigations into the phenomenon involved in the bridge failure have precipitated many advances in the wind design of bridges. The research results clearly demonstrated the need to consider the aerodynamic effects more accurately in the design of long-span bridges. There are mainly three different approaches for flutter analysis: the classical analytical approach, the direct experimental approach, and the semi-analytical and semi-experimental approach. Generally, the classical analytical approach is only suitable for the case of an ideally streamlined cross-section. Since the bridge decks are generally blunt, the latter two methods are usually adopted in the bridge flutter analysis.

Classical Analytical Approach

In the development of the classical analytical approach, the classical flutter problems of airfoils and thin flat plates were often used as examples. Theodorsen (1935) first derived a complete set of analytical expressions for the aerodynamic forces acting on an sinusoidally oscillating thin flat plate with infinitesimally small amplitude, as shown in Figure 1.3. These expressions are based on the potential energy theory, which assumes that there is no separation between the flow and the solid surface of the body. They have the form as follows

$$L_h = -2\pi\rho bU^2 \left\{ C(k) \left[\alpha + \frac{\dot{h}}{U} \right] + [1 + C(k)] \frac{b}{2} \frac{\dot{\alpha}}{U} \right\} \quad (1.1)$$

$$M_\alpha = \pi\rho b^2U^2 \left\{ C(k) \left[\alpha + \frac{\dot{h}}{U} \right] + [1 - C(k)] \frac{b}{2} \frac{\dot{\alpha}}{U} \right\} \quad (1.2)$$

where L_h is the lift force; M_α is the aerodynamic moment; ρ is the air mass density; b is the half width of the thin flat plate; U is the wind velocity; k is the reduced frequency defined as $k = \omega \cdot b/U$; ω is the circular frequency of response; and $C(k) = F(k) + iG(k)$ is the Theodorsen circulation function (Theodorsen, 1934) defined by Bessel or Hankel functions.

As a part of the studies on the wind-resistant design of the second Tacoma Narrows Bridge, Fr. Bleich (1948-1949) conducted a series of research to apply the classical airfoil flutter theory based on the Theodorsen circulation function to the analysis of suspension bridges. He modelled the bridge deck as a thin flat plate and developed a procedure to calculate the critical wind velocity of the suspension bridges. He also tried to modify the aerodynamic forces.

Klöppel and Thiele (1967) extended the analytical procedure developed by Bleich. They presented the nomograms for the flutter solution of different thin flat

plates in terms of a number of non-dimensional parameters. These parameters are: the frequency ratio ϵ between the torsional and vertical bending modes; the mass ratio μ between the bridge deck and the surrounding air; the ratio between the radius of gyration and the half width of the deck cross-section; and the damping ratio of the torsional and the vertical bending modes. In order to apply the nomograms to calculate the critical flutter wind velocity of the suspension bridges, the shape modification factor η corresponding to various types of deck cross-section were introduced since the contours of the real bridge decks were not ideally streamlined as the thin flat plate. In a typical flutter analysis, the solution of an equivalent thin flat plate with the same non-dimensional parameters as that of the bridge deck is first determined by using the nomograms. The shape modification factor of the deck is then measured in the wind tunnel test of the bridge deck cross-section model. The critical wind velocity of the bridge is then obtained as follows

$$U_f = \eta \cdot U_p \quad (1.3)$$

where U_f is the critical wind velocity for the bridge, η is the shape modification factor, and U_p is the critical wind velocity of an equivalent thin flat plate obtained from the nomograms.

Van der put (1976) simplified the flutter analysis procedure by neglecting the influence of the variation in the mechanical damping of the bridge and considering that the relationship between the reduced wind velocity $U/(b\omega)$ and the frequency ratio ϵ as approximately linear. Through regression analysis, he proposed a practical formula for the calculation of the critical wind velocity for the classical coupled flutter of thin flat plates.

Two simple similar formulas were proposed by Selberg (1961) and Rocard (1963)

to apply the classical airfoil flutter theory on the analysis of suspension bridges.

The arguments on the rationality of applying the classical approach in bridge flutter analysis mainly concentrate on two aspects. First, it concerns the separation of the air flow surrounding the bridge deck. Since the real bridge deck is not ideally streamlined as in the case of a thin flat plate, the aeroelastic effects induced by the surrounding air flow are directly related to the flow separation and vortex-shedding phenomena. The second argument is related to the effect due to multi-mode participation. The classical flutter theory considers only the contributions from the lowest bending and torsional modes in the flutter behaviour of the bridge. It assumes that the coupled flutter is the dominant mode of response in the flutter behaviour of the bridge. Consequently, the influences caused by the other modes were not considered in the analysis. Research results have shown that although this assumption may be acceptable for suspension bridges, it may not be applicable to the flutter analysis of the more complex cable-stayed bridges.

Direct Experimental Approach

Since the flow field surrounding a bridge deck is complex, and no suitable analytical expression has been found to date to model the aerodynamic phenomena, the direct experimental approach seems to be the most effective way to study bridge flutter.

In general, there are three types of experimental models, i.e. section model, taut strip model and 3D full bridge model.

The section model test is the most popular among the three due to its convenience in design and fabrication, and relatively low cost. Another important advantage is

that a section model can represent the geometry of the bridge deck cross-sections in detail. Thus, a section model is suitable for use in the sensitivity analysis of checking the effects due to changes in geometric configuration and the adoption of aerodynamic devices. However, a section model cannot simulate the actual boundary conditions of the whole structure and it cannot take into account the multi-mode participation effect. The effects of wind turbulence is typically neglected in this approach. The section model test is usually carried out to measure the static wind force coefficients and the aerodynamic parameters such as the flutter derivatives and the aerodynamic admittance needed in the analytical study of the bridge-wind interaction problem. In general, the critical flutter wind velocity obtained from the section model test is lower than the actual value.

The full bridge model test has obviously the distinct advantage of taking into account the influence of the structural characteristics and the wind turbulence in three-dimensions. It is the most accurate method for predicting the flutter behaviour of the bridge structures. An important example of the application of this type of model is that the wind behaviour of a bridge structure under different construction stages can only be tested by using the 3D model. However, it is noted that the design and fabrication of the 3D model is much more complex and expensive than a section model. Further, testing of a 3D model requires the use of larger wind tunnel facilities. Because of the higher costs and the other requirements, the prototype model tests are carried out for large and important bridge projects, in which cases the results obtained from the analytical models or section models are often checked by those obtained from the 3D model tests.

The taut strip model, which was first introduced by Davenport (1972), could be considered as a semi-experimental, three-dimensional section model. It fills the gap

between the section model and the prototype model. The main idea is to simulate the geometric shape and mass distribution of the bridge deck with the consistent scaling factor as that used in the simulation of the natural wind. The simulation of the main cable behaviour, which is often difficult with large scaling, is not included in the model. Thus, the non-linearity effect due to the gravitational acceleration of the bridge structure is ignored. The stiffness is provided by the stretched wires between the anchors. It is less expensive and much easier than the prototype model in terms of design and fabrication, yet a taut strip model can predict the elastic deformations of the structure under wind loads with sufficient accuracy (Davenport *et al*, 1992).

Semi-Experimental Semi-Analytical Approach

Considering that flutter response in a bridge is significantly different from that of an airfoil, Scanlan (1967) suggested to modify the classical coupled flutter theory for bridges by introducing the flutter derivatives H_1^* , H_2^* , H_3^* and A_1^* , A_2^* , A_3^* . The determination of these aerodynamic motion-related coefficients for bridge decks with purely sinusoidal motions under smooth flow has been experimentally carried out since the 1960s (Ukeguchi *et al*, 1966; Scanlan and Sabzevari, 1967; Sabzevari and Scanlan, 1968). The aerodynamic forces acting on a bluff bridge deck per unit span are expressed as follows

$$L_h = \frac{1}{2}\rho U^2(2B) \left[KH_1^*(K)\frac{\dot{h}}{U} + KH_2^*(K)\frac{B\dot{\alpha}}{U} + K^2H_3^*(K)\alpha \right] \quad (1.4)$$

$$M_\alpha = \frac{1}{2}\rho U^2(2B^2) \left[KA_1^*(K)\frac{\dot{h}}{U} + KA_2^*(K)\frac{B\dot{\alpha}}{U} + K^2A_3^*(K)\alpha \right] \quad (1.5)$$

Scanlan and Tomko (1971) also proposed the experimental procedure to measure the flutter derivatives. The separate-flow flutter theory, which successfully explains the failure of Tacoma Bridge in 1940, was formally presented by Scanlan (1978). According to the theory, the collapse of the Tacoma Bridge was caused by the separated-flow torsional flutter, or sometimes also known as the single degree-of-freedom torsional flutter. The theory can predict the flutter behaviour of the suspension bridges with unstreamlined deck cross-section. But for the case of the cable-stayed bridges, the effect of the inclined cables causes the lateral and torsional vibration motions of the bridge girder to be coupled together, which leads to the difficulties in determining the dominant torsional modes as the participating modes in the flutter response analysis.

In a recent study, Scanlan (1990) considered also the effect of the sway motion in the flutter behaviour of the cable-stayed bridges. By including the sway motion in the flutter analysis model, Scanlan extended the two-dimensional flutter analysis procedure to account the full three-dimensional effects. In Scanlan's procedure, the coupling terms between the individual degrees-of-freedom of the bridge deck model are taken into account. The three-dimensional coupled aerodynamic forces are expressed in terms of eighteen flutter derivatives as follows (Jones *et al*, 1994; Jain *et al*, 1996A; Jain *et al*, 1996B)

$$L_h = \frac{1}{2}\rho U^2(2B) \left[KH_1^*(K)\frac{\dot{h}}{U} + KH_2^*(K)\frac{B\dot{\alpha}}{U} + K^2H_3^*(K)\alpha + K^2H_4^*(K)\frac{h}{B} \right. \\ \left. + KH_5^*(K)\frac{\dot{p}}{U} + K^2H_6^*(K)\frac{p}{B} \right] \quad (1.6)$$

$$D_p = \frac{1}{2}\rho U^2(2B) \left[KP_1^*(K)\frac{\dot{p}}{U} + KP_2^*(K)\frac{B\dot{\alpha}}{U} + K^2P_3^*(K)\alpha + K^2P_4^*(K)\frac{p}{B} \right. \\ \left. + KP_5^*(K)\frac{\dot{h}}{U} + K^2P_6^*(K)\frac{h}{B} \right] \quad (1.7)$$

$$M_\alpha = \frac{1}{2}\rho U^2(2B^2) \left[KA_1^*(K)\frac{\dot{h}}{U} + KA_2^*(K)\frac{B\dot{\alpha}}{U} + K^2A_3^*(K)\alpha + K^2A_4^*(K)\frac{h}{B} \right. \\ \left. + KA_5^*(K)\frac{\dot{p}}{U} + K^2A_6^*(K)\frac{p}{B} \right] \quad (1.8)$$

With the development of the finite element method and the computer modelling techniques, the common practice in the three-dimensional flutter analysis of bridge structures nowadays is based on the combination of the finite element analytical model and the experimentally obtained data from the wind tunnel tests.

Xie and Xiang (1985) considered the bridge structure and the air flow as a complete system. They developed the *State-Space* method. Agar (1989) presented the *Modal Technique* which formulates the flutter equation in the form of an order $2M$ eigenvalue problem. The *M-S* (Multi-mode Searching) method proposed by Chen (1994), which is based on the Modal Technique, further reduces the equation into an M -order complex eigenvalue problem. Namini *et al* (1992) presented a general computational algorithm, which is called the *PK-F* method, to solve the flutter equation of motion. The critical flutter point, the pre-flutter and the post-flutter response can be determined through iterations.

Comparison between different approaches

Comparing the above three different approaches which can be adopted in the study of the flutter behaviour of bridges, it can be concluded that

1. All approaches use the critical wind velocity as an index to evaluate the aerodynamic stability behaviour of the bridge structures.
2. There are some limitations in the first two approaches. The classical theoretical approach is only suitable for the case of a bridge with ideally streamlined deck cross-section. The complex and costly prototype model tests should be done in the direct experimental approach to ensure the accuracy of the results. The semi-experimental, semi-analytical approach is more general in that it can be applied in the flutter analysis of different types of bridges, such as suspension bridges and cable-stayed bridges, and for bridges with either streamlined or unstreamlined deck cross-section.
3. Among the available methods in the semi-experimental, semi-analytical approach, the *PK-F* method is the most general and versatile one in that it can consider many different scenarios, such as the flutter behaviour of a bridge during the construction stages, and it allows the use of different sets of flutter derivatives corresponding to different part of the bridge girder in the case that the variation of the geometric contour of the bridge deck is significant.

1.3 Numerical Models of Bridge Analysis

The numerical models used for the analysis of bridge structures include the equivalent beam finite element model, the conventional full 3D finite element model and the finite strip model.

The equivalent beam finite element model is extensively used in the static and dynamic response analysis of bridge structures. However, in order to simulate the

mass and stiffness properties of a complex bridge girder by an equivalent beam, some assumptions on the behaviour of the bridge response must be made, which may be over-simplified and thus sometimes may have a significant impact on the accuracy of the results.

In a conventional full 3D finite element model, the bridge structure is modelled by 3D solid and/or shell elements. It can model the bridge structure in detail and provide accurate results. But the high costs of the large amount of computer storage and *CPU* time are the disadvantages of this model.

The finite strip method, first proposed by Cheung (1968) for the analysis of simply supported bridge deck, has been found to be a powerful, efficient and versatile analysis tool for the analysis of bridge structures which have complicated geometric layout and different material properties. Since then, application of the method has been extended to the modelling and analysis of other more complex structures and bridges under various loading conditions, both static and dynamic. In comparison to the original finite strip method, the recently developed spline finite strip method (Cheung *et al*, 1982) is more flexible, which can be used to model structures with more complex and different types of boundary condition. It is more efficient in analyzing multispan structures and structures subjected to concentrated forces. Because of its accuracy and efficiency, the spline finite strip method has been successfully applied in the static, dynamic, stability and postbuckling analysis of box girder bridges (Cheung *et al*, 1987; Mizusawa, 1988; Zhu and Cheung, 1989; Cheung *et al*, 1996).

In the finite strip method, the bridge girder is divided into a number of flat shell finite strips in the longitudinal direction. Compared to the equivalent beam finite element model, the mass and stiffness properties of the girder can be accurately

modelled with minimal input data preparation. On the other hand, the finite strip model has a significantly less number of degrees-of-freedom than the full three-dimensional finite element model, and thus it is computationally more efficient.

The stay cable is one of the major structural members of the cable-stayed bridge. It plays an important role in the structural performance and appearance of the bridge. The proper interpolation of the cable behaviour is very important in the dynamic analysis of the cable-stayed bridges. In most of the research, the stay cable is usually modelled as the single-truss element. Thus, the transverse motion of the stay cables as well as the interaction between the motion of the cables and the other parts of the bridge superstructure cannot be considered in the analysis.

The multi-link model, of which each cable is divided into a number of links, has been adopted in some recent studies (Abdel-Ghaffar *et al*, 1991; Tuladhar *et al*, 1995; Jones *et al*, 1997). Results show that the transverse motion of the cables may affect the dynamic response of the cable-stayed bridges and should be considered in the dynamic analysis.

1.4 Objectives and Scope

The semi-experimental, semi-analytical flutter analysis approach is a numerical procedure for determining the critical flutter wind velocity, with the utilization of the aerodynamic information provided by the wind tunnel tests. It employs the flutter derivatives measured in the wind tunnel tests in the flutter analysis of the bridges. The development of computer technology and advances in computer-aided analysis and modelling techniques have made it possible to model complex bridge structures in detail. The responses of bridges under various load conditions can be accurately

determined.

In aerodynamic studies of bridges, it is important to recognize that the flow field of the wind action on a bridge girder is spatially distributed. The finite strip method can take this into account by allocating the spatially distributed wind loads to different strips in the model representing the bridge girder, as opposed to lumping all the aerodynamic forces at the center of elasticity of the deck cross-section in the case of an equivalent beam finite element model. Further, the mass and stiffness properties of a complex bridge deck cross-section can be accurately represented by the finite strip model with minimal modelling effort. These advantages of the finite strip method motivate the present research to develop a 3D flutter analysis procedure based on the finite strip method, which will provide bridge engineers and designers an alternative convenient and accurate way to evaluate the aerodynamic stability behaviour of bridges. The application of the finite strip method has been, for the first time here, extended to the wind engineering area.

The importance of the cable vibrations to the bridge responses is an interesting topic in bridge aerodynamics. However, most of the research in this field focuses on the cable dynamics such as the rain-induced cable vibration of the cable-stayed bridges. Recent studies (Abdel-Ghaffar *et al*, 1991; Tuladhar *et al*, 1995; Jones *et al*, 1997) have shown that the interaction between the vibrating cables and the bridge structures may affect the dynamic behaviour of the bridge. A new cable model, which can properly represent the behaviour of the stay cable in the oscillating bridge system, and is also computationally more efficient than the multi-link model proposed by Abdel-Ghaffar *et al* (1991), will be developed in the current study.

In the previous flutter studies of the cable-stayed bridges, only the self-excited aerodynamic forces acting on the bridge girder have been considered. With the de-

velopment of multi-stay cable-stayed bridges, the influence of the self-excited forces acting on the stay cables on the wind stability behaviour of the whole bridge structure should be examined.

The CFRP (Carbon Fibre Reinforced Polymer) material has higher tensile strength and lower mass density in comparison to steel. It has been used as the substituent material for the stay cables in some short-span cable-stayed bridges. With the development of modern long-span and super long-span cable-stayed bridges, of which the minimization of the weight of the design structure is an important design criterion, the CFRP is considered as the most ideal material for the cables. It is important to evaluate the impact of replacing the traditional steel stay cables with the light weight CFRP cables on the dynamic characteristics and aerodynamic stability of long-span cable-stayed bridges.

The objectives and scope of the current study are summarized as follows

1. Development of an analytical model for determining the dynamic characteristics of cable-stayed bridges. The model is based on the spline finite strip method. Comparisons on the efficiency and accuracy of different analytical models, including the 3D equivalent beam finite element model, the conventional full 3D finite element model and the proposed spline finite strip model, will be made. The advantages and disadvantages of those analytical models will be discussed.
2. Development of an aerodynamic force model for the three-dimensional flutter analysis of long-span cable-stayed bridges based on the spline finite strip method, which can take into account the spatial distribution of the aerodynamic forces. The *PK-F* method is employed as a computational algorithm for

seeking the critical point of flutter. A numerical example of the flutter analysis of a simply supported thin flat plate is presented to verify the reliability and accuracy of the proposed model and method.

3. Verifying the proposed structural and flutter analysis model by employing it to study the dynamic characteristics and flutter behaviour of a real long-span cable-stayed bridge — the Nanpu Bridge. Compare the results with those obtained from experiments and other analytical models or methods. Discuss the potential of the proposed finite strip model and method.
4. Study of the impact of the cable vibration behaviour on the dynamic characteristics and flutter response of the cable-stayed bridges. Development of a three-node cable element, which can not only represent properly the motion of the cable in the oscillating bridge structure, but also is computationally efficient. Studying the influence of the aerodynamic forces acting on the stay cables to the flutter behaviour of the cable-stayed bridges.
5. Study of the impact of using CFRP material as the substituent material for the cable stays on the dynamic characteristics and aerodynamic stability properties of the cable-stayed bridges.

Altogether four computer programs, two for the dynamic analysis of bridge structures, and two for the post-processing of analysis results, have been developed in the current study. The computer program *SFSFA* has been developed for the finite strip based 3D flutter analysis of bridges. The element library includes the flat shell finite strip element, the three-dimensional beam element and the transition cable element derived in the present study. The program *3NFEFA* has been developed

for the dynamic analysis of bridge structures considering the cable vibration effect. The element library includes the three-dimensional beam element, the single-truss element and the three-node cable element developed in the current research. Both of the programs are written in FORTRAN. Each of them contains two modules, i.e. the dynamic characteristic analysis module and the flutter analysis module. The programs *DRAW* and *DRAW_3N* are written in Visual Basic. They are developed for the post-processing of the analysis results obtained from the program *SFSFA* or *MFEFA*. The mechanical model as well as the mode shapes of the structure can be shown graphically in three dimensions.

1.5 Layout of the Thesis

A general introduction of the cable-stayed bridge and review of studies on bridge flutter phenomenon and numerical models employed in the bridge analysis are contained in the present chapter. The objectives and scopes of the current study are presented.

A spline finite strip analytical model of the cable-stayed bridges is developed in Chapter 2. The main girder of the bridge is modelled by the flat shell spline finite strips; the pylons are modelled by the 3D beam finite elements, while the cables are modelled by the new transition elements developed in the present study. The penalty function is adopted for the modelling of the boundary conditions. The resulting eigenvalue problem is solved by the subspace iteration method. The advantages and disadvantages of the different mathematical models which can be used to model long-span bridges, such as the equivalent beam finite element model, the full 3D finite element model and the spline finite strip model, are discussed by presenting

two numerical examples.

In Chapter 3, the aerodynamic force model of the bridge girder is developed based on the spline finite strip method. The aerodynamic stiffness and damping matrices are derived by allocating the aerodynamic forces on each nodal line of the finite strip model representing the bridge girder. The spatial distribution of the aerodynamic forces can thus be considered. The flutter equation of motion is established according to the dynamic equilibrium condition of the bridge structure. The *PK-F* method is employed as a computational algorithm for seeking the critical point of flutter. A numerical example of the flutter analysis of a simply supported thin flat plate is presented to verify the reliability and accuracy of the proposed model and method.

The case study of a real long-span cable-stayed bridge—the Nanpu Bridge, is presented in Chapter 4. The dynamic characteristics and the flutter response of the bridge are analyzed by using the proposed finite strip structural and aerodynamic force model. The dynamic analysis results are then compared with those obtained from the wind tunnel tests and other analytical models or methods. The potential of the future application of the proposed finite strip model is also discussed.

Chapter 5 deals with the impact of cable vibration behaviour on the dynamic response of the cable-stayed bridges. A three-node cable model is developed, which can properly represent the behaviour of the stay cables in the oscillating bridge system. The models of the aerodynamic loads acting on the cables and the bridge deck are derived for the flutter analysis. The case study of the Nanpu Bridge is conducted to analyze the cable vibration effect on the dynamic responses of a real long-span cable-stayed bridge. Eight model bridges are being analyzed as the parametric study to fully understand the impact of cable vibration effect on the dynamic responses

of the cable-stayed bridges with different combinations of structural layout.

In Chapter 6, the history of the cable materials, the properties of the CFRP material and the state-of-the-art application of the CFRP material are first reviewed. The case study of the Nanpu Bridge by using the CFRP cables is carried out to study the impact of CFRP cables on the dynamic response of the bridge. A more comprehensive study of the CFRP cable effect has been conducted by using eight model bridges with different combinations of structural layout.

The summary and conclusions of the present study, and some suggestions for the future work are presented in Chapter 7.

Figure 1.1: Aeroelasticity collar triangle

Figure 1.2: Wind-induced vibration

Figure 1.3: Aerodynamic forces on a thin flat plate

Chapter 2

Structural Analysis of Cable-Stayed Bridges

2.1 Introduction

The main components of a cable-stayed bridge are the girders, cables and pylons. The girder resists bending moments and axial forces from the vertical dead and live loads carried by the bridge deck and the lateral loads induced by wind and earthquakes. The cables can only sustain axial load. The pylon resists the very large axial forces, and sometimes also bending moments, transmitted from the stay cables.

In the structural layout of a cable-stayed bridge, the inclined cables provide elastic supports to the main girder system. With the latest development of increasing long bridge span length in modern cable-stayed bridges, the cable systems have

changed from just a few pairs of stay cables used in the early design to the multi-stay cable system that are widely adopted today. When resisting the external loads, the early cable-stayed bridge systems behave as multi-span, elastically supported continuous beams, whereas the modern multi-stay cable systems behave as quasi-truss systems.

The dynamic characteristics of cable-stayed bridges are very complicated. Due to the presence of the inclined cables, the swaying and torsional motions are strongly coupled together. Since determining the dynamic characteristics is an essential first step for calculating the dynamic responses and the flutter behaviour of a bridge system, it is very important to have an accurate analytical model, which includes sufficient details of the dynamic behaviour of the bridge, in order to ensure satisfactory results.

The spline finite strip method was proposed by Cheung *et al* (1982) for the analysis of rectangular plates. The use of the ' B_3 spline function' as the interpolation function in the longitudinal direction was introduced as an improvement to the original formulation of the conventional finite strip method. The accuracy and efficiency of the method have been proven through applications of the method to the analysis of skew and arbitrary shaped plates (Tham *et al*, 1986; Cheung *et al*, 1988), arbitrary curved slab bridges (Cheung *et al*, 1993), continuous haunched bridges (Cheung and Li, 1990; Cheung *et al*, 1992) and other applications in vibration (Chen and Chong, 1984), stability (Cheung *et al*, 1987) and postbuckling studies (Chen and Chong, 1985; Chong and Chen, 1986; Lau and Hancock, 1986; Zhu and Cheung, 1989).

In this chapter, the spline finite strip method is adopted to develop a mathematical model for the dynamic analysis of the cable-stayed bridges.

2.2 B_3 Spline Function

The name 'spline' originally comes from the tool employed by the draughtsman, which is a small flexible wooden strip for drawing continuous smooth curve segment by segment.

Spline functions are well-established mathematical tool. There are a number of different classes of spline functions commonly used as interpolation functions for field problems in engineering applications. The advantage of the spline functions is that the assemblage of the spline functions used as the interpolation functions can satisfy any required continuity or discontinuity conditions.

In typical formulation, the B_3 spline function with equal section length is chosen to represent the variations of the displacements in the longitudinal direction of the analysis model. In the spline finite strip method, the support points and the locations of the concentrated load are taken as the longitudinal knots. Thus, in the case of the cables in a cable-stayed bridge, which may have unequal spacing, the unequal length spline functions must be adopted. For greater flexibility, the B_3 spline function with unequal section length is chosen to represent the displacement field of the bridge girder in the present study. The function is a cubic polynomial with continuity up to the second derivative.

Assuming an arbitrary function $g(y)$ over the interval $a \leq y \leq b$, the entire interval is divided into r sections of unequal length by knots y_m , with $0 \leq m \leq r$, $y_0 = a$ and $y_r = b$ for the interpolation of the function by spline functions.

The unequal section length B_3 spline function $\Phi_m(y)$ with the center at $y = y_m$ is defined as follows

$$\Phi_m(y) = \begin{cases} A_m(y - y_{m-2})^3 & y_{m-2} \leq y \leq y_{m-1} \\ A_m(y - y_{m-2})^3 + C_m(y - y_{m-1})^3 & y_{m-1} \leq y \leq y_m \\ B_m(y_{m+2} - y)^3 + D_m(y_{m+1} - y)^3 & y_m \leq y \leq y_{m+1} \\ B_m(y_{m+2} - y)^3 & y_{m+1} \leq y \leq y_{m+2} \\ 0 & \text{otherwise} \end{cases} \quad (2.1)$$

where

$$A_m = [(y_{m-1} - y_{m-2})(y_m - y_{m-2})(y_{m+1} - y_{m-2})]^{-1} \quad (2.2a)$$

$$B_m = [(y_{m+2} - y_{m+1})(y_{m+2} - y_m)(y_{m+2} - y_{m-1})]^{-1} \quad (2.2b)$$

$$C_m = -(y_{m+2} - y_{m-2}) [(y_{m-1} - y_{m-2})(y_m - y_{m-1})(y_{m+1} - y_{m-1})(y_{m+2} - y_{m-1})]^{-1} \quad (2.2c)$$

$$D_m = -(y_{m+2} - y_{m-2}) [(y_{m+2} - y_{m+1})(y_{m+1} - y_m)(y_{m+1} - y_{m-1})(y_{m+1} - y_{m-2})]^{-1} \quad (2.2d)$$

and y_i (with $i = m-2, m-1, m, m+1$ and $m+2$) are the coordinates of the $(i+1)^{th}$ knot.

As an example, the variation of $\Phi_m(y)$ and its first and second derivatives, with $y_{m-2} = 0$, $y_{m-1} = 5$, $y_m = 6$, $y_{m+1} = 8$ and $y_{m+2} = 10$ are plotted in Figure 2.1. The values of the functions at the knots are also given in Table 2.1. For comparison purposes, the variations of the equal section length B_3 spline function and its

derivatives over the same interval are also plotted in Figure 2.1. Although the curves related to the unequal section length spline functions are un-symmetric, while those of the equal section length spline are symmetric, both functions are twice continuously differentiable over the interval. The second derivatives of both cases are linear in terms of y .

Considering the interpolation of an arbitrary function $g(y)$ by local B_3 splines $\Phi_m(y)$, the interval of interest is divided into sections separated by knots. The interpolating function in each section is represented by the combination of a total of four splines centered around the point, two on each side. Because of the involvement of two spline functions on each side, the interpolation of the function at points on the two end sections between knots 0 and 1 and between knots $(r - 1)$ and r thus requires the spline functions centered at two additional knots y_{-1} and y_{r+1} outside the interpolation interval. Therefore, there are a total of $(r + 3)$ section knots. The series of spline functions has the form of

$$g(y) = \sum_{m=-1}^{r+1} \alpha_m \Phi_m(y) \quad (2.3)$$

where $\alpha_m (m = -1, 0, 1, \dots, r + 1)$ are the coefficients to be determined from the following set of linear equations

$$g'(y_0) = \alpha_{-1} \Phi'_{-1}(y_0) + \alpha_0 \Phi'_0(y_0) + \alpha_1 \Phi'_1(y_0) \quad (2.4a)$$

$$\begin{aligned} g(y_m) &= \alpha_{m-2} \Phi_{m-2}(y_m) + \alpha_{m-1} \Phi_{m-1}(y_m) + \alpha_m \Phi_m(y_m) \\ &\quad + \alpha_{m+1} \Phi_{m+1}(y_m) + \alpha_{m+2} \Phi_{m+2}(y_m) \\ \text{for } 0 \leq m \leq r \end{aligned} \quad (2.4b)$$

$$g'(y_r) = \alpha_{r-1} \Phi'_{r-1}(y_r) + \alpha_r \Phi'_r(y_r) + \alpha_{r+1} \Phi'_{r+1}(y_r) \quad (2.4c)$$

2.3 Structural Model of Bridge Deck System

2.3.1 Displacement Function

In the formulation here, a rectangular bridge deck system consisting of bridge deck slab on girder is modelled by the finite strip method. Since the bridge deck system resists both bending and in-plane loading, it is modelled as a flat shell finite strip, as shown in Figure 2.2, by combining the behaviour of a bending plate strip and that of a plane stress strip.

Defining $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ respectively as the transverse, longitudinal and vertical direction of the individual strip element, the corresponding displacements $\bar{u}$, $\bar{v}$ and $\bar{w}$ within the finite strip are expressed as follows

$$\bar{u} = \sum_{m=-1}^{r+1} [(1 - \bar{x}/b)\Phi_{1m}^u(y)\bar{u}_{1m} + (\bar{x}/b)\Phi_{2m}^u\bar{u}_{2m}] \quad (2.5a)$$

$$\bar{v} = \sum_{m=-1}^{r+1} [(1 - \bar{x}/b)\Phi_{1m}^v(y)\bar{v}_{1m} + (\bar{x}/b)\Phi_{2m}^v\bar{v}_{2m}] \quad (2.5b)$$

$$\bar{w} = \sum_{m=-1}^{r+1} [N_{1m}\bar{w}_{1m} + N_{2m}\bar{\theta}_{1m} + N_{3m}\bar{w}_{2m} + N_{4m}\bar{\theta}_{2m}] \quad (2.5c)$$

where $\bar{x}$ is the coordinate in the $\bar{X}$ direction, b is the width of the strip, r is the total number of longitudinal sections on a nodal line, $\bar{u}_{km}, \bar{v}_{km}, \bar{w}_{km}$ and $\bar{\theta}_{km}$ are the displacement parameters of nodal line k , with $k = 1, 2$. As shown in Eqs.2.5(a)-(c), the variation of the inplane displacements $\bar{u}$ and $\bar{v}$ are modelled by the linear shape functions in the transverse direction of the strip and by the unequal section length B_3 spline functions in the longitudinal direction. For the vertical displacement $\bar{w}$,

the cubic Hermitian polynomial variations in the transverse direction is modelled by the beam displacement shapes, and in the longitudinal direction by the B_3 spline functions as follows

$$N_{1m} = [1 - 3(\bar{x}/b)^2 + 2(\bar{x}/b)^3] \Phi_{1m}^w(y) \quad (2.6a)$$

$$N_{2m} = \bar{x} [1 - 2(\bar{x}/b) + (\bar{x}/b)^2] \Phi_{1m}^\theta(y) \quad (2.6b)$$

$$N_{3m} = [3(\bar{x}/b)^2 - 2(\bar{x}/b)^3] \Phi_{2m}^w(y) \quad (2.6c)$$

$$N_{4m} = \bar{x} [(\bar{x}/b)^2 - (\bar{x}/b)] \Phi_{2m}^\theta(y) \quad (2.6d)$$

In the above equations, $\Phi_{km}^l(y)$ (with $k = 1, 2$; $l = w, \theta$) is the unequal section length B_3 spline function centered at $y = y_m$, as defined by Equation (2.1).

For a B_3 spline function, $\Phi_m(y)$ is non-zero over the interval $[y_{m-2}, y_{m+2}]$. Thus, the displacement at an arbitrary point $M(\bar{x}_M, \bar{y}_M)$ on the strip between nodal line e and nodal line f of a rectangular plate (with $\bar{x}_e \leq \bar{x}_M \leq \bar{x}_f$ and $\bar{y}_h \leq \bar{y}_M \leq \bar{y}_{h+1}$, where $-1 \leq h \leq r+1$) is related to the spline functions defined by eight knots in its proximity, i.e. $e_{h-1}, e_h, e_{h+1}, e_{h+2}, f_{h-1}, f_h, f_{h+1}, f_{h+2}$, as shown in Figure 2.3(a).

The stiffness and mass matrices of the eight-node sub-element are derived here. The global matrices can be obtained by following the standard assemblage procedure of the finite element method. In the derivation, the nodes of the sub-element are numbered as shown in Figure 2.3(b).

Thus, according to Equation (2.5), the displacements $\bar{u}, \bar{v}, \bar{w}$ at an arbitrary point (x, y) can be expressed as follows

$$\begin{aligned} \bar{u} = & (1 - \bar{x}/b)\Phi_1(y)\bar{u}_1 + \bar{x}/b\Phi_2(y)\bar{u}_2 + (1 - \bar{x}/b)\Phi_3(y)\bar{u}_3 + \bar{x}/b\Phi_4(y)\bar{u}_4 + \\ & (1 - \bar{x}/b)\Phi_5(y)\bar{u}_5 + \bar{x}/b\Phi_6(y)\bar{u}_6 + (1 - \bar{x}/b)\Phi_7(y)\bar{u}_7 + \bar{x}/b\Phi_8(y)\bar{u}_8 \end{aligned} \quad (2.7a)$$

$$\begin{aligned} \bar{v} = & (1 - \bar{x}/b)\Phi_1(y)\bar{v}_1 + \bar{x}/b\Phi_2(y)\bar{v}_2 + (1 - \bar{x}/b)\Phi_3(y)\bar{v}_3 + \bar{x}/b\Phi_4(y)\bar{v}_4 + \\ & (1 - \bar{x}/b)\Phi_5(y)\bar{v}_5 + \bar{x}/b\Phi_6(y)\bar{v}_6 + (1 - \bar{x}/b)\Phi_7(y)\bar{v}_7 + \bar{x}/b\Phi_8(y)\bar{v}_8 \end{aligned} \quad (2.7b)$$

$$\begin{aligned} \bar{w} = & N_1(x)\Phi_1(y)\bar{w}_1 + N_2(x)\Phi_1(y)\bar{\theta}_1 + N_3(x)\Phi_2(y)\bar{w}_2 + N_4(x)\Phi_2(y)\bar{\theta}_2 + \\ & N_1(x)\Phi_3(y)\bar{w}_3 + N_2(x)\Phi_3(y)\bar{\theta}_3 + N_3(x)\Phi_4(y)\bar{w}_4 + N_4(x)\Phi_4(y)\bar{\theta}_4 + \\ & N_1(x)\Phi_5(y)\bar{w}_5 + N_2(x)\Phi_5(y)\bar{\theta}_5 + N_3(x)\Phi_6(y)\bar{w}_6 + N_4(x)\Phi_6(y)\bar{\theta}_6 + \\ & N_1(x)\Phi_7(y)\bar{w}_7 + N_2(x)\Phi_7(y)\bar{\theta}_7 + N_3(x)\Phi_8(y)\bar{w}_8 + N_4(x)\Phi_8(y)\bar{\theta}_8 \end{aligned} \quad (2.7c)$$

Equation (2.7) can be written in a more compact form as follows

$$\{\bar{f}\} = [\bar{u} \quad \bar{v} \quad \bar{w}]^T = [N] \cdot \{\bar{\delta}\}^e \quad (2.8)$$

where

$$\{\bar{\delta}\}^e = \{\bar{u}_1, \bar{v}_1, \bar{w}_1, \bar{\theta}_1, \bar{u}_2, \bar{v}_2, \bar{w}_2, \bar{\theta}_2, \dots, \bar{u}_8, \bar{v}_8, \bar{w}_8, \bar{\theta}_8\}^T \quad (2.9)$$

is the nodal displacement parameter vector of the eight-node sub-element; $[N]$ is the shape function matrix, which has the form

$$[N] = \{[N_1], [N_2], [N_3], \dots, [N_8]\}^T \quad (2.10)$$

where

$$[N]_i = \begin{cases} \begin{bmatrix} (1 - \bar{x}/b)\Phi_i & 0 & 0 & 0 \\ 0 & (1 - \bar{x}/b)\Phi_i & 0 & 0 \\ 0 & 0 & N_1\Phi_i & N_2\Phi_i \end{bmatrix}, & \text{for } i = 1, 3, 5, 7 \\ \begin{bmatrix} \bar{x}/b\Phi_i & 0 & 0 & 0 \\ 0 & \bar{x}/b\Phi_i & 0 & 0 \\ 0 & 0 & N_3\Phi_i & N_4\Phi_i \end{bmatrix}, & \text{for } i = 2, 4, 6, 8 \end{cases} \quad (2.11)$$

2.3.2 Strains

The strains, curvatures and twists are determined as follows

$$\bar{\epsilon}_x = \frac{\partial \bar{u}}{\partial \bar{x}} \quad (2.12a)$$

$$\bar{\epsilon}_y = \frac{\partial \bar{v}}{\partial \bar{y}} \quad (2.12b)$$

$$\bar{\nu}_{xy} = \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}} \quad (2.12c)$$

$$\bar{\chi}_x = -\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} \quad (2.12d)$$

$$\bar{\chi}_y = -\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} \quad (2.12e)$$

$$\bar{\chi}_{xy} = 2\frac{\partial^2 \bar{w}}{\partial \bar{x} \partial \bar{y}} \quad (2.12f)$$

Writing Equation (2.12) in matrix form

$$\begin{aligned} \{\bar{\epsilon}\} &= [\bar{\epsilon}_x, \bar{\epsilon}_y, \bar{\nu}_{xy}, \bar{\chi}_x, \bar{\chi}_y, \bar{\chi}_{xy}]^T \\ &= [B]\{\bar{\delta}\}^e \end{aligned} \quad (2.13)$$

where $[B]$ is the strain-displacement relationship matrix which can be expressed as follows

$$[B] = [[B]_{12}, [B]_{34}, [B]_{56}, [B]_{78}] \quad (2.14)$$

in which

$$[B]_{ij} = \begin{bmatrix} -\Phi_i/b & 0 & (1 - \bar{x}/b)\Phi_i' & 0 & 0 & 0 \\ 0 & (1 - \bar{x}/b)\Phi_i' & -\Phi_i/b & 0 & 0 & 0 \\ 0 & 0 & 0 & -N_1''\Phi_i & -N_1\Phi_i'' & 2N_1'\Phi_i' \\ 0 & 0 & 0 & -N_2''\Phi_i & -N_2\Phi_i'' & 2N_2'\Phi_i' \\ \Phi_j/b & 0 & \bar{x}/b\Phi_j' & 0 & 0 & 0 \\ 0 & \bar{x}/b\Phi_j' & \Phi_j/b & 0 & 0 & 0 \\ 0 & 0 & 0 & -N_3''\Phi_j & -N_3\Phi_j'' & 2N_3'\Phi_j' \\ 0 & 0 & 0 & -N_4''\Phi_j & -N_4\Phi_j'' & 2N_4'\Phi_j' \end{bmatrix}^T \quad (2.15)$$

with $i = 1, 3, 5, 7$ and $j = 2, 4, 6, 8$.

2.3.3 Stresses

The relationship between stresses and strains can be expressed as follows

$$\{\bar{\sigma}\} = [D_E]\{\bar{\epsilon}\} \quad (2.16)$$

where

$$\{\bar{\sigma}\} = \{ \bar{N}_x, \bar{N}_y, \bar{N}_{xy}, \bar{M}_x, \bar{M}_y, \bar{M}_{xy} \}^T \quad (2.17)$$

and $[D_E]$ is the elastic matrix. For an isotropic folded plate,

$$[D_E] = \begin{bmatrix} [D_E]_p & 0 \\ 0 & [D_E]_b \end{bmatrix} \quad (2.18)$$

in which the elastic matrix $[D_E]_p$ for the plane stress strip is

$$[D_E]_p = \frac{Et}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (2.19)$$

and the elastic matrix $[D_E]_b$ for the bending plate strip is

$$[D_E]_b = \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (2.20)$$

where E is the Young's modulus, t is the thickness of the strip, and ν is the Poisson's ratio.

2.3.4 Derivation of Sub-element Matrices

Stiffness Matrix

The strain energy of the eight-node sub-element is

$$U = \frac{1}{2} \int_A \{\bar{\epsilon}\}^T \{\bar{\sigma}\} dA \quad (2.21)$$

Substituting Equation (2.13) and Equation (2.16) into Equation (2.21)

$$\begin{aligned} U &= \frac{1}{2} \{\bar{\delta}\}^e{}^T \left(\int_A [B]^T [D_E] [B] dA \right) \{\bar{\delta}\}^e \\ &= \frac{1}{2} \{\bar{\delta}\}^e{}^T [\bar{K}]_s^e \{\bar{\delta}\}^e \end{aligned} \quad (2.22)$$

in which

$$[\bar{K}]_s^e = \int_A [B]^T [D_E] [B] dA \quad (2.23)$$

is the stiffness matrix of the eight-node sub-element in the local coordinate system.

Mass Matrix

The sub-element mass matrix derived according to the minimum potential energy principle, has the form as follows

$$[\tilde{M}]_s^e = \rho t \int_0^b \int_0^h [N]^T [N] dx dy \quad (2.24)$$

where ρ is the mass density, t is the thickness of the strip, and $[N]$ is the shape function matrix as given in Equation (2.10).

2.3.5 Coordinate Transformation

The sub-element stiffness and mass matrices are derived in the local coordinate system, of which the $\bar{X}$ and $\bar{Y}$ axes coincide with the mid-surface of the strip. Since the bridge deck system is divided into a number of strips in the analysis model, some of the strips will meet at an angle as in the case of a box girder bridge. To obtain the global stiffness and mass matrices of the sub-elements, all the matrices are transformed into global coordinate system before assembling.

Defining X , Y , and Z as the global coordinate axes, and $\bar{X}$, $\bar{Y}$, and $\bar{Z}$ as the local coordinate axes, with Y and $\bar{Y}$ coincide with each other, as shown in Figure 2.4, the displacement transformation relationships between these two coordinate systems can be expressed as follows

$$\{\bar{\delta}\} = [T]\{\delta\} \quad (2.25)$$

where $\{\bar{\delta}\}$ and $\{\delta\}$ are respectively the displacement parameter vectors in the local and global coordinate systems; $[T]$ is the coordinate transformation matrix defined as follows

$$[T] = \begin{bmatrix} [t] & 0 \\ 0 & [t] \end{bmatrix} \quad (2.26)$$

with

$$[t] = \begin{bmatrix} \cos \beta & 0 & \sin \beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \beta & 0 & \cos \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.27)$$

in which β is the angle between the X and $\bar{X}$ axes or Z and $\bar{Z}$ axes.

The sub-element stiffness and mass matrices in the global coordinate system, denoted by $[K]_s^e$ and $[M]_s^e$ respectively, can be obtained as follows

$$[K]_s^e = [T]^T [\bar{K}]_s^e [T] \quad (2.28)$$

$$[M]_s^e = [T]^T [\bar{M}]_s^e [T] \quad (2.29)$$

2.4 Structural Model of Pylon

2.4.1 Displacement Function

In the model, the pylons of the cable-stayed bridge are discretized into a number of 3D beam elements. Defining $\bar{X}$, $\bar{Y}$, $\bar{Z}$ as the axes of the local coordinate system of

the beam element, with $\bar{X}$ axis coincides with the beam axis, as shown in Figure 2.5.

The 3D beam element has six degrees-of-freedom at each end of the element

$$\{\bar{\delta}\} = [\bar{u}, \bar{v}, \bar{w}, \bar{\alpha}, \bar{\beta}, \bar{\gamma}]^T \quad (2.30)$$

where $\bar{u}, \bar{v}, \bar{w}$ — displacements along the $\bar{X}, \bar{Y}, \bar{Z}$ axes;

$\bar{\alpha}, \bar{\beta}, \bar{\gamma}$ — rotations around the $\bar{X}, \bar{Y}, \bar{Z}$ axes.

The nodal displacement vector of the beam element is therefore

$$\{\bar{\delta}\}^e = [\{\bar{\delta}\}_i, \{\bar{\delta}\}_j]^T \quad (2.31)$$

or

$$\{\bar{\delta}\}^e = [\bar{u}_i, \bar{v}_i, \bar{w}_i, \bar{\alpha}_i, \bar{\beta}_i, \bar{\gamma}_i, \bar{u}_j, \bar{v}_j, \bar{w}_j, \bar{\alpha}_j, \bar{\beta}_j, \bar{\gamma}_j]^T \quad (2.32)$$

The linear displacement $\bar{u}, \bar{v}, \bar{w}$ at an arbitrary point $(\bar{x}, \bar{y}, \bar{z})$ within the beam element can be expressed as follows

$$[\bar{u}, \bar{v}, \bar{w}]^T = [N] \cdot \{\bar{\delta}\}^e \quad (2.33)$$

where $[N]$ is the shape function matrix of the beam element as follows

$$[N]^T = \begin{bmatrix} 1 - \xi & 0 & 0 \\ 6(\xi - \xi^2)\eta & 1 - 3\xi^2 + 2\xi^3 & 0 \\ 6(\xi - \xi^2)\zeta & 0 & 1 - 3\xi^2 + 2\xi^3 \\ 0 & -(1 - \xi)l\zeta & -(1 - \xi)l\eta \\ (1 - 4\xi + 3\xi^2)l\zeta & 0 & -(\xi - 2\xi^2 + \xi^3)l \\ -(1 - 4\xi + 3\xi^2)l\eta & (\xi - 2\xi^2 + \xi^3)l & 0 \\ \xi & 0 & 0 \\ -6(\xi - \xi^2)\eta & 3\xi^2 - 2\xi^3 & 0 \\ -6(\xi - \xi^2)\zeta & 0 & 3\xi^2 - 2\xi^3 \\ 0 & -l\xi\zeta & -l\xi\eta \\ (-2\xi + 3\xi^2)l\zeta & 0 & (\xi^2 - \xi^3)l \\ (2\xi - 3\xi^2)l\eta & (-\xi^2 + \xi^3)l & 0 \end{bmatrix} \quad (2.34)$$

where l — length of the beam element;

ξ, η, ζ — nondimensional parameters with $\xi = \bar{x}/l, \eta = \bar{y}/l, \zeta = \bar{z}/l$,
respectively.

2.4.2 Member Stiffness and Mass matrices

By applying the virtual displacement theorem, the member stiffness matrix of the beam element in the local coordinate system can be derived as follows

$$[\tilde{K}]_p^e = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & 12\frac{EI_x}{l^3} & 0 & 0 & 0 & 0 & 0 & -12\frac{EI_x}{l^3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 12\frac{EI_y}{l^3} & 0 & 0 & 0 & 0 & 0 & -12\frac{EI_y}{l^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 \\ 0 & 0 & -6\frac{EI_y}{l^2} & 0 & 4\frac{EI_y}{l} & 0 & 0 & 0 & 6\frac{EI_y}{l^2} & 0 & -2\frac{EI_y}{l} & 0 \\ 0 & 6\frac{EI_x}{l^2} & 0 & 0 & 0 & 4\frac{EI_x}{l} & 0 & 0 & 0 & 0 & 0 & -6\frac{EI_x}{l^2} \\ -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & -12\frac{EI_x}{l^3} & 0 & 0 & 0 & -6\frac{EI_x}{l^2} & 0 & 12\frac{EI_x}{l^3} & 0 & 0 & 0 & 0 \\ 0 & 0 & -12\frac{EI_y}{l^3} & 0 & 6\frac{EI_y}{l^2} & 0 & 0 & 0 & 12\frac{EI_y}{l^3} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 \\ 0 & 0 & -6\frac{EI_y}{l^2} & 0 & 2\frac{EI_y}{l} & 0 & 0 & 0 & 6\frac{EI_y}{l^2} & 0 & 4\frac{EI_y}{l} & 0 \\ 0 & 6\frac{EI_x}{l^2} & 0 & 0 & 0 & 2\frac{EI_x}{l} & 0 & -6\frac{EI_x}{l^2} & 0 & 0 & 0 & 4\frac{EI_x}{l} \end{bmatrix} \quad (2.35)$$

where E — Young's modulus;

G — shear modulus;

l — length of the beam element;

A — cross-sectional area of the beam element;

$I_{\bar{y}}$ — bending moment of inertia of the cross-section about $\bar{Y}$ axis;

$I_{\bar{z}}$ — bending moment of inertia of the cross-section about $\bar{Z}$ axis;

J — rotational moment of inertia of the cross-section about $\bar{X}$ axis.

In the current study, the lumped mass matrix is used, which has the form

$$[\bar{M}]_p^e = \begin{bmatrix} \frac{1}{2}\rho Al & & & & & & & & & & & \\ 0 & \frac{1}{2}\rho Al & & & & & & & & & & \\ 0 & 0 & \frac{1}{2}\rho Al & & & & & & & & & \\ 0 & 0 & 0 & I_{x_i} & & & & & & & & \\ 0 & 0 & 0 & 0 & I_{y_i} & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & I_{z_i} & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\rho Al & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\rho Al & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\rho Al & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{x_i} & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{y_i} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{z_i} \end{bmatrix} \quad (2.36)$$

where ρ — mass density;

$I_{x_i}, I_{y_i}, I_{z_i}, I_{x_j}, I_{y_j}, I_{z_j}$ — rotational inertia of the mass around $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes on the i^{th} and j^{th} end of the element;

2.4.3 Coordinate Transformation

The member stiffness and mass matrices of the 3D beam element are transformed from the local coordinate system to the global coordinate system by the vector cross

product approach. The vectors $\vec{r}$, $\vec{s}$, $\vec{t}$ are respectively the unit vectors in the positive directions of the local axes $\bar{X}$, $\bar{Y}$, $\bar{Z}$, and $\vec{R}$, $\vec{S}$, $\vec{T}$ are the unit vectors in the positive directions of the global axes X , Y , Z , as shown in Figure 2.6. For a beam element ij , node i is taken as the starting point in the definition of the direction of the element, and the $\bar{X}$ -axis is along the longitudinal direction of the beam. The two other axes $\bar{Y}$ and $\bar{Z}$ lie on the principal axes of inertia of the beam cross-section. The vectors $\vec{r}$, $\vec{s}$ and $\vec{t}$ can be determined as follows

$$\vec{r} = i\vec{j} = A_1\vec{R} + A_2\vec{S} + A_3\vec{T} \quad (2.37)$$

in which

$$A_1 = x_j - x_i, \quad A_2 = y_j - y_i, \quad A_3 = z_j - z_i \quad (2.38)$$

Selecting an arbitrary point k on the plane determined by the $\bar{X}$ and $\bar{Y}$ -axes, and denoting a vector $i\vec{k}$ as $\vec{g}$

$$\vec{g} = i\vec{k} = B_1\vec{R} + B_2\vec{S} + B_3\vec{T} \quad (2.39)$$

in which

$$B_1 = x_k - x_i, \quad B_2 = y_k - y_i, \quad B_3 = z_k - z_i \quad (2.40)$$

Thus

$$\vec{t} = \vec{r} \times \vec{g} = C_1\vec{R} + C_2\vec{S} + C_3\vec{T} \quad (2.41)$$

i.e.

$$\vec{t} = \begin{vmatrix} \vec{R} & \vec{S} & \vec{T} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} \quad (2.42)$$

and

$$C_1 = A_2B_3 - A_3B_2 \quad (2.43a)$$

$$C_2 = A_3B_1 - A_1B_3 \quad (2.43b)$$

$$C_3 = A_1B_2 - A_2B_1 \quad (2.43c)$$

Finally

$$\vec{s} = \vec{t} \times \vec{r} = U_1\vec{R} + U_2\vec{S} + U_3\vec{T} \quad (2.44)$$

i.e.

$$\vec{s} = \begin{vmatrix} \vec{R} & \vec{S} & \vec{T} \\ C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \end{vmatrix} \quad (2.45)$$

and

$$\begin{aligned} U_1 &= A_3C_2 - A_2C_3 \\ &= A_3(A_3B_1 - A_1B_3) - A_2(A_1B_2 - A_2B_1) \\ &= (A_1^2 + A_2^2 + A_3^2)B_1 - (A_1B_1 + A_2B_2 + A_3B_3)A_1 \\ &= AA * B_1 - AB * A_1 \end{aligned} \quad (2.46a)$$

Similarly

$$U_2 = AA * B_2 - AB * A_2 \quad (2.46b)$$

$$U_3 = AA * B_3 - AB * A_3 \quad (2.46c)$$

in which

$$AA = A_1^2 + A_2^2 + A_3^2, \quad AB = A_1B_1 + A_2B_2 + A_3B_3 \quad (2.47)$$

Denoting $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ respectively as the unit vectors of $\vec{r}$, $\vec{s}$ and $\vec{t}$ gives

$$\vec{s}_1 = T_{11}\vec{R} + T_{12}\vec{S} + T_{13}\vec{T} \quad (2.48a)$$

$$\vec{s}_2 = T_{21}\vec{R} + T_{22}\vec{S} + T_{23}\vec{T} \quad (2.48b)$$

$$\vec{s}_3 = T_{31}\vec{R} + T_{32}\vec{S} + T_{33}\vec{T} \quad (2.48c)$$

where

$$T_{11} = \frac{A_1}{DL}, \quad T_{12} = \frac{A_2}{DL}, \quad T_{13} = \frac{A_3}{DL}, \quad DL = \sqrt{A_1^2 + A_2^2 + A_3^2} \quad (2.49)$$

$$T_{21} = \frac{U_1}{UU}, \quad T_{22} = \frac{U_2}{UU}, \quad T_{23} = \frac{U_3}{UU}, \quad UU = \sqrt{U_1^2 + U_2^2 + U_3^2} \quad (2.50)$$

$$T_{31} = T_{12} \cdot T_{23} - T_{13} \cdot T_{22} \quad (2.51)$$

$$T_{32} = T_{13} \cdot T_{21} - T_{11} \cdot T_{23} \quad (2.52)$$

$$T_{33} = T_{11} \cdot T_{22} - T_{12} \cdot T_{21} \quad (2.53)$$

Writing in matrix form gives

$$\begin{Bmatrix} \vec{s}_1 \\ \vec{s}_2 \\ \vec{s}_3 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{Bmatrix} \vec{R} \\ \vec{S} \\ \vec{T} \end{Bmatrix} \quad (2.54)$$

i.e.

$$\{\vec{G}\} = [T]\{\vec{M}\} \quad (2.55)$$

where

$$\{\vec{G}\} = [\vec{s}_1 \quad \vec{s}_2 \quad \vec{s}_3]^T, \quad \{\vec{M}\} = [\vec{R} \quad \vec{S} \quad \vec{T}]^T \quad (2.56)$$

According to virtual displacement principle, the member stiffness and mass matrices of the 3D beam element in the global coordinate system can be obtained as follows

$$[K]_p^e = [\lambda]^T [\bar{K}]_p^e [\lambda] \quad (2.57)$$

$$[M]_p^e = [\lambda]^T [\bar{M}]_p^e [\lambda] \quad (2.58)$$

where $[K]_p^e$ — member stiffness matrix in the global coordinate system;
 $[M]_p^e$ — member mass matrix in the global coordinate system;
 $[\bar{K}]_p^e$ — member stiffness matrix in the local coordinate system;
 $[\bar{M}]_p^e$ — member mass matrix in the local coordinate system;
 $[\lambda]$ — coordinate transformation matrix,

$$[\lambda] = \begin{bmatrix} [T] & & & \\ & [T] & & \\ & & [T] & \\ & & & [T] \end{bmatrix} \quad (2.59)$$

2.5 Structural Model of Cable

2.5.1 Displacement Function

In the analysis of cable-stayed bridges, the main girder is modelled by the flat shell spline finite strips, and the pylon is modelled by the 3D beam elements. The cable, with one end connecting to the pylon and the other connecting to the girder, can be modelled by a special transition element, as shown in Figure 2.7. One end of the cable element has the same degrees-of-freedom as the flat shell spline finite strip, while the other end has the same degrees-of-freedom as the 3D beam element.

The displacement function of a cable I shown in Figure 2.8 is derived here. In the formulation, one of the cable element end i is connected to the main girder. It coincides with a knot on a nodal line of the spline finite strip model of the bridge

deck. The nodal displacements are related to the displacement parameters of the two adjacent knots $i - 1$, $i + 1$ on the same nodal line and itself i as follows

$$\bar{u}_i = \Phi_i(y_{i-1})\bar{u}'_{i-1} + \Phi_i(y_i)\bar{u}'_i + \Phi_i(y_{i+1})\bar{u}'_{i+1} \quad (2.60a)$$

$$\bar{v}_i = \Phi_i(y_{i-1})\bar{v}'_{i-1} + \Phi_i(y_i)\bar{v}'_i + \Phi_i(y_{i+1})\bar{v}'_{i+1} \quad (2.60b)$$

$$\bar{w}_i = \Phi_i(y_{i-1})\bar{w}'_{i-1} + \Phi_i(y_i)\bar{w}'_i + \Phi_i(y_{i+1})\bar{w}'_{i+1} \quad (2.60c)$$

where $\Phi_i(y_k)$ — B_3 spline functions centered at $y = y_i$ ($k = i - 1, i, i + 1$);

$\bar{u}'_k, \bar{v}'_k, \bar{w}'_k$ — nodal displacement parameters of knot k in the local coordinate system ($k = i - 1, i, i + 1$);

$\bar{u}_i, \bar{v}_i, \bar{w}_i$ — nodal displacements of knot i in the local coordinate system.

Writing Equation (2.60) in matrix form gives

$$\begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \bar{w}_i \end{Bmatrix} = \begin{bmatrix} \Phi_i(y_{i-1}) & 0 & 0 \\ 0 & \Phi_i(y_{i-1}) & 0 \\ 0 & 0 & \Phi_i(y_{i-1}) \\ 0 & 0 & 0 \\ \Phi_i(y_i) & 0 & 0 \\ 0 & \Phi_i(y_i) & 0 \\ 0 & 0 & \Phi_i(y_i) \\ 0 & 0 & 0 \\ \Phi_i(y_{i+1}) & 0 & 0 \\ 0 & \Phi_i(y_{i+1}) & 0 \\ 0 & 0 & \Phi_i(y_{i+1}) \\ 0 & 0 & 0 \end{bmatrix}^T \cdot \begin{Bmatrix} \{\bar{\delta}'_{i-1}\} \\ \{\bar{\delta}'_i\} \\ \{\bar{\delta}'_{i+1}\} \end{Bmatrix} \quad (2.61)$$

where $\{\bar{\delta}'_k\} = [\bar{u}'_k \ \bar{v}'_k \ \bar{w}'_k \ \bar{\theta}'_k]$ is the displacement parameter vector of knot k in the local coordinate system, with $k = i - 1, i, i + 1$.

Denoting

$$[T_2] = \begin{bmatrix} \Phi_i(y_{i-1}) & 0 & 0 \\ 0 & \Phi_i(y_{i-1}) & 0 \\ 0 & 0 & \Phi_i(y_{i-1}) \\ 0 & 0 & 0 \\ \Phi_i(y_i) & 0 & 0 \\ 0 & \Phi_i(y_i) & 0 \\ 0 & 0 & \Phi_i(y_i) \\ 0 & 0 & 0 \\ \Phi_i(y_{i+1}) & 0 & 0 \\ 0 & \Phi_i(y_{i+1}) & 0 \\ 0 & 0 & \Phi_i(y_{i+1}) \\ 0 & 0 & 0 \end{bmatrix}^T \quad (2.62)$$

Equation (2.61) can be expressed as follows

$$\begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \bar{w}_i \end{Bmatrix} = [T_2] \cdot \begin{Bmatrix} \{\bar{\delta}'_{i-1}\} \\ \{\bar{\delta}'_i\} \\ \{\bar{\delta}'_{i+1}\} \end{Bmatrix} \quad (2.63)$$

Thus, for the I^{th} cable, the nodal displacement in the local coordinate system can

be expressed as follows

$$\begin{Bmatrix} \bar{u}_i \\ \bar{v}_i \\ \bar{w}_i \\ \bar{u}_j \\ \bar{v}_j \\ \bar{w}_j \end{Bmatrix} = \begin{bmatrix} [T_2]_{3 \times 9} & 0 \\ 0 & [I]_{3 \times 3} \end{bmatrix} \cdot \begin{Bmatrix} \{\bar{\delta}'_{i-1}\} \\ \{\bar{\delta}'_i\} \\ \{\bar{\delta}'_{i+1}\} \\ \{\bar{\delta}_j\} \end{Bmatrix} \quad (2.64)$$

or in matrix form

$$\{\bar{\delta}\}^e = [T_3] \cdot \{\bar{\delta}'\}^e \quad (2.65)$$

in which

$$\{\bar{\delta}\}^e = [\bar{u}_i, \bar{v}_i, \bar{w}_i, \bar{u}_j, \bar{v}_j, \bar{w}_j]^T \quad (2.66)$$

is the nodal displacement vector;

$$[T_3] = \begin{bmatrix} [T_2]_{3 \times 9} & 0 \\ 0 & [I]_{3 \times 3} \end{bmatrix} \quad (2.67)$$

is the displacement transformation matrix;

$$\{\bar{\delta}'\}^e = [\{\bar{\delta}'_{i-1}\}^T, \{\bar{\delta}'_i\}^T, \{\bar{\delta}'_{i+1}\}^T, \{\bar{\delta}_j\}^T]^T \quad (2.68)$$

is the nodal displacement parameter vector.

2.5.2 Member Stiffness and Mass Matrices

Stiffness Matrix

The member stiffness matrix of cable I in the local coordinate system is

$$[\bar{K}]_c^e = \frac{EA}{l} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.69)$$

where E — Young's modulus;

A — cross-sectional area of the cable;

l — length of the cable.

The sagging effect due to the self-weight of the cable can be approximately taken into account by reducing the value of the elastic Young's modulus. This is accomplished by using the Ernst equivalent modulus (Walter *et al*, 1988), which has the form as follows

$$E_{eq} = \frac{E}{1 + \frac{(\gamma l_h)^2 E}{12\sigma^3}} \quad (2.70)$$

where E — elastic Young's modulus;

γ — weight per unit volume;

l_h — horizontal distance between two ends of the cable;

σ — stress induced by the self-weight of the cable.

The value of the equivalent modulus relates to the self-weight and the tension in cable. It combines the effects of both material and geometric deformation.

Denoting $\{\bar{S}\}^e$ and $\{\bar{S}'\}^e$ as the nodal force vectors corresponding to the nodal displacement vector $\{\bar{\delta}\}^e$ and the nodal displacement parameter vector $\{\bar{\delta}'\}^e$, respectively. By applying the principle of virtual work, the member stiffness matrix of the cable element in terms of $\{\bar{\delta}'\}^e$ can be derived as follows

$$\{\bar{\delta}\}^{eT} \{\bar{S}\} = \{\bar{\delta}'\}^{eT} \{\bar{S}'\} \quad (2.71)$$

Substituting Equation (2.65) into Equation (2.71) yields

$$([T_3]\{\bar{\delta}'\}^e)^T \{\bar{S}\} = \{\bar{\delta}'\}^{eT} \{\bar{S}'\} \quad (2.72)$$

or

$$\{\bar{\delta}'\}^{eT} [T_3]^T \{\bar{S}\} = \{\bar{\delta}'\}^{eT} \{\bar{S}'\} \quad (2.73)$$

Since

$$\{\bar{S}'\} = [T_3]^T \{\bar{S}\} \quad (2.74)$$

and

$$\{\bar{S}\} = [\bar{K}]_c^e \{\bar{\delta}\}^e \quad (2.75)$$

thus

$$\begin{aligned}
 \{\bar{S}'\} &= [T_3]^T [\bar{K}]_c^e \{\bar{\delta}\}^e \\
 &= [T_3]^T [\bar{K}]_c^e [T_3] \{\bar{\delta}'\}^e \\
 &= [\bar{K}']_c^e \{\bar{\delta}'\}^e
 \end{aligned} \tag{2.76}$$

Substituting Equation (2.74) and Equation (2.75) into Equation (2.73) yields

$$[\bar{K}']_c^e = [T_3]^T [\bar{K}]_c^e [T_3] \tag{2.77}$$

where $[\bar{K}']_c^e$ is the member stiffness matrix of the transition cable element expressed in the form of the nodal displacement parameter vector $\{\bar{\delta}'\}^e$.

Mass Matrix

In the formulation, the lumped mass matrix for which the simple lumping procedure of lumping one half of the total element mass to each end of the element is adopted to form the member mass matrix of the cable element. Thus, the member mass matrix of the cable element in the local coordinate system can be expressed as follows

$$[\bar{M}]_c^e = \frac{1}{2} \rho A l \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.78}$$

Similarly, by applying the virtual displacement principle, the member mass matrix $[\bar{M}]_c^e$ in terms of the nodal displacement parameter vector is

$$[\bar{M}]_c^e = [T_3]^T [\bar{M}]_c^e [T_3] \quad (2.79)$$

Consequently, the member stiffness matrix $[K']_c^e$ and mass matrix $[M']_c^e$ of the cable element in the global coordinate system can be obtained as follows

$$\begin{aligned} [K']_c^e &= [\lambda]^T [\bar{K}]_c^e [\lambda] \\ &= [\lambda]^T [T_3]^T [\bar{K}]_c^e [T_3] [\lambda] \\ &= ([T_3][\lambda])^T [\bar{K}]_c^e ([T_3][\lambda]) \end{aligned} \quad (2.80)$$

and

$$\begin{aligned} [M']_c^e &= [\lambda]^T [\bar{M}]_c^e [\lambda] \\ &= [\lambda]^T [T_3]^T [\bar{M}]_c^e [T_3] [\lambda] \\ &= ([T_3][\lambda])^T [\bar{M}]_c^e ([T_3][\lambda]) \end{aligned} \quad (2.81)$$

2.6 Boundary Conditions

Various methods can be applied to impose the boundary conditions in the finite strip method. Among these the scheme of modifying the local splines at and immediately adjacent to the boundary point is commonly used (Tham *et al*, 1986; Li *et al*, 1986). In the present study, a more convenient approach of the penalty function method is adopted to impose the boundary conditions at the end supports and at any intermediate supporting points of the strips. In the formulation, the degrees-of-freedom are restrained by appropriate stiff springs.

Considering the boundary conditions of fixed, hinged and roller supports at a nodal line, some of the displacements and rotations are constrained to be zero. For the case where the vertical displacement component w at a point (x, y) is zero, i.e.

$$w(x, y) = \sum_{m=-1}^{r+1} N_m(x, y) \bar{w}_m = 0 \quad (2.82)$$

where $N_m(x, y)$ and $\bar{w}_m$ are the displacement shape functions and the displacement parameters, respectively. A fictitious spring with a sufficiently large stiffness as represented by the large penalty parameter β in the Z direction is introduced at the point. The potential energy of the fictitious spring can be written as follows

$$U = \frac{1}{2} \beta w^2 = \frac{1}{2} \beta \sum_m \sum_n N_m(x, y) N_n(x, y) \bar{w}_m \bar{w}_n \quad (2.83)$$

Through minimization of the potential energy, the stiffness matrix of the spring can be obtained as follows

$$[K_{mn}] = [\beta N_m(x, y) N_n(x, y)] \quad (2.84)$$

If the first derivative of the virtual deflection, for example $\frac{\partial w}{\partial y}$, is constrained to be zero at the point (x, y)

$$\frac{\partial w(x, y)}{\partial y} = \sum_{m=-1}^{r+1} \frac{\partial N_m(x, y)}{\partial y} \bar{w}_m = 0 \quad (2.85)$$

the stiffness matrix of the fictitious spring can be derived in a similar way, which is expressed as follows

$$[K_{mn}] = \left[\beta \frac{\partial N_m(x, y)}{\partial y} \frac{\partial N_n(x, y)}{\partial y} \right] \quad (2.86)$$

Similar fictitious springs and penalty terms can be introduced to enforce the boundary conditions in other translational directions as well as in the rotations. The

final equilibrium matrix equations of the entire structure, including modelling of the boundary conditions by the penalty functions, can then be obtained by assembling the stiffness matrices of all the structural members and the fictitious springs.

2.7 Eigenvalue problem

2.7.1 Free Vibration Equation

The natural vibration frequencies of the bridge structure are determined by solving the equations of motion of a free undamped system, which can be expressed in the matrix form as follows

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\} \quad (2.87)$$

where $[M]$ is the global mass matrix; $[K]$ is the global stiffness matrix; $\{X\}$ and $\{\ddot{X}\}$ are the displacement vector and the corresponding acceleration vector.

The displacement vector $\{X\}$ can be expressed in terms of the generalized coordinate vector as follows

$$\{X\} = [\Phi]\{Y\} \quad (2.88)$$

where $[\Phi]$ is the mode shape matrix, and $\{Y\}$ is the generalized coordinate vector.

Substituting Equation (2.88) into Equation (2.87), and assuming harmonic motions, the free vibration equation can be rearranged as follows

$$[K][\Phi] = [M][\Phi][\Lambda] \quad (2.89)$$

where $[\Lambda] = \text{diag}(\omega^2)$, and ω is the circular frequency. The eigenvalues of the above equation are the natural frequencies of the structure, and the corresponding eigenvectors represent the vibration mode shapes.

2.7.2 The Subspace Iteration Method

The subspace iteration method developed by Bathe (1971) is one of the most efficient schemes for solving large eigenproblems. It is aimed at the solution of only the first few lowest eigenvalues and the corresponding eigenvectors which are mostly required in the dynamic analysis of large systems. The subspace iteration combines the fundamental concepts of the Rayleigh-Ritz method and the inverse iteration, which significantly reduces the order of the initial eigenproblem.

In order to obtain the set of the lowest p modes with sufficient accuracy, it is desirable to start the iteration with a larger number of q trial vectors. Studies have found that $q = \min\{2p, p + 8\}$ is generally an effective value. Expressing the displacement vector $\{\phi\}$ in the form of a linear combination of the q eigenvectors as follows

$$\begin{aligned}\{\phi\} &= a_1\{\phi_1\} + a_2\{\phi_2\} + \dots + a_q\{\phi_q\} \\ &= [\varphi]\{A\}\end{aligned}\tag{2.90}$$

where $[\varphi]$ — the lowest q mode shape matrix, $[\varphi] = [\{\phi_1\}, \{\phi_2\}, \dots, \{\phi_q\}]$;
 $\{A\}$ — column vector of the unknown amplitude of the lowest mode shapes. Then the circular frequency ω can be obtained according to the Rayleigh principle

$$\omega^2 = \frac{\{\phi\}^T[K]\{\phi\}}{\{\phi\}^T[M]\{\phi\}}\tag{2.91}$$

Substituting Equation (2.90) into Equation (2.91) yields

$$\omega^2 = \frac{\{A\}^T[\varphi]^T[K][\varphi]\{A\}}{\{A\}^T[\varphi]^T[M][\varphi]\{A\}}\tag{2.92}$$

Denoting

$$[K]^g = [\varphi]^T [K] [\varphi] \quad (2.93)$$

$$[M]^g = [\varphi]^T [M] [\varphi] \quad (2.94)$$

then

$$\omega^2 = \frac{\{A\}^T [K]^g \{A\}}{\{A\}^T [M]^g \{A\}} \quad (2.95)$$

Since ω^2 is at the minimum for the first mode, therefore the variation of ω^2 with respect to $\{A\}$ for the lowest mode must satisfy

$$\frac{\partial \omega^2}{\partial \{A\}} = 0 \quad (2.96)$$

Substituting Equation (2.95) into Equation (2.96) yields

$$[K]^g \{A\} = \omega^2 [M]^g \{A\} \quad (2.97)$$

The solution $\{A\}$ obtained from the above equation can give a better approximation of the required eigenvectors $[\Phi]$ in Equation (2.89). This is what is called the Rayleigh-Ritz method. The disadvantage of the method lies in the fact that the results depends on the assumption of the initial shape vector matrix $[\varphi]$.

In order to improve the accuracy of the solution, the obtained $\{\phi\}$ is substituted into the right hand side of Equation (2.89). A new set of approximation eigenvectors $\{\bar{\phi}\}$ can be obtained by inverse iteration. $\{\bar{\phi}\}$ has stronger component of the lower order mode shapes than $\{\phi\}$. This $\{\bar{\phi}\}$ is used to replace $\{\phi\}$ in Equation (2.90), and the Rayleigh-Ritz method is applied again. These steps repeat until convergence is achieved. This algorithm is called the subspace iteration.

The main procedures of the subspace iteration method can be briefly summarized as follows

$$[K][\bar{\varphi}_k] = [M][\varphi_{k-1}] \quad (2.98a)$$

$$[K]^g = [\bar{\varphi}_k]^T [K][\bar{\varphi}_k] \quad (2.98b)$$

$$[M]^g = [\bar{\varphi}_k]^T [M][\bar{\varphi}_k] \quad (2.98c)$$

$$[K]^g[A_k] = [M]^g[A_k][\Omega_k^2] \quad (2.98d)$$

$$[\varphi_k] = [\bar{\varphi}_k][A_k] \quad (2.98e)$$

2.8 Numerical Examples

To verify the validity and reliability of the proposed finite strip analytical model for cable-stayed bridges, numerical examples of the cable-stayed bridges are presented in this section. Among the examples presented, one is a single pylon cable-stayed bridge with asymmetric spans, the other is a double pylon, symmetric three-span cable-stayed bridge. These two examples represent the typical geometric layout of modern cable-stayed bridges. For comparison purpose, the example bridges are also analyzed by using the equivalent beam finite element approach and the full three-dimensional finite elements.

2.8.1 Example 1: Single Pylon Cable-stayed Bridge

A single pylon cable-stayed bridge is shown in Figure 2.9. The lengths of the two asymmetric spans are $60.0m$ and $120.0m$. The width and depth of the bridge deck are $15m$ and $1m$, respectively. The Young's modulus is $3.5 \times 10^4 MPa$, and the Poisson's ratio is $1/6$. The pylon has an H shape, with the bending rigidity of $EI = 7.46 \times 10^5 MN - m^2$. The Young's modulus of the cables is $2.0 \times 10^5 MPa$, and the specific weight is $0.078 MN/m^3$. The area of the cross-section of the cables are $A_1 = A_4 = 0.004m^2$, $A_2 = A_3 = 0.002m^2$, with the subscript i of A_i being the numbering sequence of the cables given in Figure 2.9.

In the spline finite strip model, the main girder is divided into two equal width strips by three nodal lines, as shown in Figure 2.10(a). Each nodal line is further divided into ten sections by the unevenly spaced knots in the longitudinal direction. The pylon is modelled by the three-dimensional beam elements, as shown in Figure 2.10(b). The cables are modelled by the cable elements derived in Section 2.5.

The main girder is assumed simply supported at both ends. At the intersection of the pylon and girder, the lateral and vertical displacements of the girder are restrained to that of the pylon. The spline finite strip model of the bridge is shown in Figure 2.11.

The natural frequencies of the bridge are determined in a modal analysis using a computer program *SFSFA* developed in this study. The main functions of this program will be described in more detail later in Section 3.8. The natural frequencies of the first ten modes of the bridge are listed in Table 2.2. The corresponding mode shapes are shown in Figure 2.12.

For the purpose of comparison, the example bridge is also analyzed by two other

models: the equivalent beam finite element model and the full 3D finite element model. In the equivalent beam model, the main girder of the bridge is divided into ten sections modelled by the equivalent beam finite elements in the longitudinal direction, whereas in the full 3D finite element model, the bridge deck is modelled by 3D flat shell finite elements with the mesh size of 5×90 . In both of the models, the pylon is modelled by the 3D beam elements, and the cables by the 3D truss elements. The self-weight effect of the cable is taken into account by using the equivalent modulus of elasticity. These two models are shown in Figure 2.13 and Figure 2.14, respectively.

SAP5 and *COSMOS* are two popular commercial softwares for structural analysis. *SAP5* (A Structural Analysis Program for Static and Dynamic Response of Linear Systems) was developed by Bathe *et al* from the University of California, Berkeley in 1970. It is one of the most widely used commercial program for structural analysis, with applications in the fields of mechanics, aerospace, transportation, hydraulics and other disciplines of engineering. *COSMOS* is developed by Structural Research and Analysis Corporation(SRAC). It is a complete, modular, self-contained finite element program for solving linear and nonlinear, static and dynamic structural problems, and problems in the fields of heat transfer, fluid mechanics, electromagnetics and structural optimization. These two programs are employed in the dynamic analysis of the example bridge for the models of the equivalent beam finite element model and the full 3D finite element model. The results obtained by using these models are also presented in Table 2.2.

It can be noted from the data in the table that all three sets of results agree well with each other, both in terms of the ordering and frequencies of the natural modes. Especially for the bending modes of the pylon, the results from the three

models are almost the same. This is because that in all three models, the pylon is modelled by the 3D beam elements. For the modes related to the main girder, results obtained from the equivalent beam model is slightly higher, whereas those from the full 3D finite element model is slightly lower. The reason is that in the equivalent beam model, the girder is divided into elements only in the longitudinal direction, whereas for the spline and full 3D model, the main girder is discretized both in the longitudinal and transverse directions, resulting the discretized models more flexible than the equivalent beam model. Based on the principles in the formulation of the finite element method, the lower the obtained frequencies, the more accurate is the result. Thus, according to the results presented in Table 2.2, the full 3D model gives the most accurate results. Compared with the results given by the equivalent beam model, the results given by the spline model is closer to those given by the full 3D model.

The total number of degrees-of-freedom and the computer times required for each model are listed in Table 2.3. The spline finite strip model has the fewest number of degrees-of-freedom, comparing to the other two models. The most time consuming part of the analysis is to calculate and assemble the stiffness and mass matrices, and to perform the subspace iteration. In the spline finite strip formulation, the main girder is discretized into a number of eight-node sub-elements. The integration involved in the formulation of the stiffness and mass matrices of these sub-elements are calculated by Gaussian integration. In the equivalent beam model, the main girder is divided into equivalent beam elements, for which the stiffness and mass matrices are directly calculated according to Equation (2.35) and Equation (2.36). Although the equivalent beam model has more degrees-of-freedom than the spline model, the time required for computing the mass and stiffness matrices is

much less, which results in the use of less total computer time in the solution of the problem. The full 3D model has the larger number of degrees-of-freedom, which obviously requires the most computer time among the three models.

However, if considering the input data preparation, much more effort is required for the equivalent beam model, since the geometric properties of the deck cross-section need to be calculated in advance. This part of the data preparation is always very time consuming.

Thus, it is concluded that for the analysis of the example bridge, the spline finite strip model can achieve the same level of accuracy in the results with minimal data preparation effort, the fewest number of degrees-of-freedom and the least computer time.

2.8.2 Example 2: Double Pylon Cable-stayed Bridge

A double pylon concrete box girder cable-stayed bridge is shown in Figure 2.15(a). The span arrangement is symmetric, with a center span of $225m$ and two side spans of $100m$ each. The cross-section of the main girder is shown in Figure 2.15(b). The Young's modulus for the girder and pylons is $3.5 \times 10^4 MPa$, and the Poisson's ratio is $1/6$. There are eight pairs of cables laying out from each pylon in a semi-fan configuration. The two cable planes are parallel to each other. The cross-sectional area of the cables are $A_1 = A_2 = A_7 = A_8 = 0.004m^2$, $A_3 = A_4 = A_5 = A_6 = 0.002m^2$, with the subscript i of A_i being the numbering order of the cables given in Figure 2.15(a). The girder is simply supported at the two ends. At the intersections of the pylons with the girders, the lateral and vertical displacements are constrained to those of the pylons.

The cross-section of the box girder is divided into six strips by six nodal lines, as shown in Figure 2.15(c). In the longitudinal direction, each nodal line is divided into 17 sections by the knots. These knots are located along the girder at the cable support points and the intersections of the pylons and the girder. The spline finite strip model of the bridge is shown in Figure 2.16.

The results of the natural frequency analysis are given in Table 2.4, and the mode shapes are given in Figure 2.17.

Similarly, the example bridge is also discretized respectively by the equivalent beam finite elements and the full 3D finite elements for comparison. These two numerical models are shown in Figure 2.17 and Figure 2.18. The equivalent beam model is analyzed by *SAP5*, and the full 3D model is analyzed by *COSMOS*. Results obtained from these two models are also given in Table 2.4. The total degrees-of-freedom and the computer time required for each model are listed in Table 2.5.

From Table 2.4, it can be noted that the match in terms of the modal order and the natural vibration frequencies of those three sets of results are obvious. Again, the obtained natural frequencies of the girder modes is slightly higher in the case of equivalent beam model, and slightly lower in the full 3D model. The reason is the same as stated in example 1.

Since in this example, the bridge deck has a box cross-section, which is more complicated than the case of the slab girder in example 1, more spline finite strips is required for the modelling purpose.

It can be seen from Table 2.5 that the equivalent beam model has the minimum number of degrees-of-freedom, and costs the least computer time. But if considering the input data preparation, the spline model can save much more time and effort in that for the girder, only the dimensions and material properties need to be input.

It is not necessary to calculate the geometric properties of the deck cross-section, as what should be done in advance in the case of the equivalent beam model. If the main girder has a more complicated or compound deck cross-section, even more effort is required to model it as an equivalent beam.

The full 3D finite element model can definitely provide the most accurate and detailed model, but the cost of computer storage and time is too expensive.

It can also be concluded from this example that for achieving the results in the same level of accuracy, the total modelling efforts and time required in the spline finite strip model is the minimal.

2.9 Summary and Conclusions

In this chapter, the application of the spline finite strip method has been extended to the modal analysis of the cable-stayed bridges. The main girder of the bridge is modelled by flat shell spline finite strips, the pylons by the 3D beam elements, and the cables by the new special transition elements which connect the spline element and the beam element at two ends, respectively. The penalty function is used for the modelling of the boundary conditions. The eigenvalue problem is solved by the subspace iteration method. The advantages and disadvantages of the different analytical models of the bridge structure, i.e. the equivalent beam finite element model, the full 3D finite element model and the spline finite strip model are discussed by using two numerical examples. The following conclusions can be drawn from the studies presented in this chapter:

1. The spline finite strip analytical model developed in the current study is a

reliable and accurate alternative to the finite element approach for the analysis of bridge structures.

2. Compared to the equivalent beam finite element model and the full 3D finite element model, the proposed spline finite strip model can achieve the same level of accuracy in the results with minimal data preparation and modelling effort.
3. The proposed finite strip model can represent the bridge girder in more detail than the equivalent beam model, and requires much less computer storage and time than the full 3D model.

Table 2.1: Values of Unequal Section Length B_3 Spline Function and Its Derivatives at Knots

Function	y_{m-2}	y_{m-1}	y_m	y_{m+1}	y_{m+2}
$\Phi_m(y)$	0	0.5208	0.7677	0.2000	0
$\Phi'_m(y)$	0	0.3125	0.0050	-0.3000	0
$\Phi''_m(y)$	0	0.1500	-0.6167	0.3000	0

Table 2.2: Natural Frequencies and Mode Shapes of Bridge Example 1

Mode No.	Frequency (Hz)			Mode Shape
	Proposed Finite Strip	Equivalent Beam Finite Element	Full 3D Finite Element	
1	0.2491	0.2511	0.2475	Heave(L)
2	0.5639	0.5706	0.5594	Heave(S,L)
3	0.7422	0.7509	0.7375	Heave(S,L)
4	1.2001	1.2245	1.1884	Heave(S,L)
5	1.2390	1.2360	1.2352	Bending(P)
6	1.4320	1.4108	1.4289	Torsion(L)
7	1.6641	1.6631	1.6635	Bending(P)
8	1.7995	1.7943	1.7986	Bending(P) + Torsion(L)
9	1.9094	1.9667	1.9000	Heave(S,L)
10	2.1629	2.1621	2.1619	Bending(P)

Mode Designation: L—Long span; S—Short span; P—Pylon.

Table 2.3: Computer Storage and Time of Different Models in Example 1

Model	Total DOF's	Computer Time (s)
Proposed Finite Strip Model	252	7
Equivalent Beam Model	300	2
Full 3D Finite Element Model	3456	61

Table 2.4: Natural Frequencies and Mode Shapes of Bridge Example 2

Mode No.	Frequency (Hz)			Mode Shape
	Proposed Finite Strip	Equivalent Beam Finite Element	Full 3D Finite Element	
1	0.3595	0.3604	0.3580	Sym. Heave(Girder)
2	0.4930	0.4972	0.4877	Asy. Heave(Girder)
3	0.6844	0.6913	0.6758	Sym. Heave(Girder)
4	0.7336	0.7417	0.7189	Sym. Sway(Girder)
5	0.7466	0.7585	0.7351	Asy. Heave(Girder)
6	0.9177	0.9410	0.8920	Sym. Heave(Girder)
7	1.1451	1.1730	1.1223	Sym. Torsion(Girder)
8	1.1563	1.1600	1.1424	Asy. Bending(Pylon)
9	1.2317	1.2376	1.2201	Sym. Bending(Pylon)
10	1.2358	1.2330	1.2273	Sym. Bending(Pylon)

Table 2.5: Computer Storage and Time of Different Models in Example 2

Model	Total DOF's	Computer Time (s)
Proposed Finite Strip Model	756	78
Equivalent Beam Model	600	16
Full 3D Finite Element Model	7512	642

Figure 2.1: Unequal section length B_3 spline function and its derivatives

Figure 2.2: Flat shell spline finite strip model

Figure 2.3: Structural model of girder (a) Mesh of the girder (b) Eight-node sub-element

Figure 2.4: Coordinate transformation of spline finite strip

(a)

(b)

Figure 2.5: Three-dimensional beam element in local and global coordinate systems (a) Nodal displacements in the local coordinate system (b) Nodal displacements in the global coordinate system

Figure 2.6: Coordinate transformation of three-dimensional beam element.

Figure 2.7: Cable element

(a)

(b)

Figure 2.8: Degrees-of-freedom of cable element (a) In the local coordinate system
(b) In the global coordinate system

Figure 2.9: Single-pylon cable-stayed bridge

Figure 2.10: Model of example 1 (a) Mesh of the deck cross-section (b) Mesh of the pylon

Figure 2.11: Spline finite strip model of example 1

Figure 2.12: Mode shapes of example 1

Figure 2.13: Equivalent beam finite element model of example 1

Figure 2.14: Full 3D finite element model of example 1

(a)

(b)

(c)

Figure 2.15: Double-pylon cable-stayed bridge (a) Structure layout (b) Deck cross-section (c) mesh of the deck cross-section

Figure 2.16: Spline finite strip model of example 2

Figure 2.17: Mode shapes of example 2

Figure 2.18: Equivalent beam finite element model of example 2

Figure 2.19: Full 3D finite element model of example 2

Chapter 3

Flutter Analysis of Cable-stayed Bridges

3.1 Introduction

Flutter is a self-excited aerodynamic instability phenomenon. At a certain critical wind velocity, the aerodynamic forces act to feed energy into the oscillating system, which makes the bridge vibrate in a divergent destructive manner. Bridge flutter is a complicated phenomenon which results from the interactions between the inertia force, the damping force, the elastic force and the self-excited aerodynamic forces. In wind design of the bridges, the critical wind velocity is often used as an indicator of the aerodynamic stability property of the bridge system. Previous research in bridge flutter has focused on the understanding of the mechanism of the flutter instability phenomenon and the determination of the critical wind condition at which flutter

will occur. The latter of which is an important step in the design of modern long-span bridges.

In this chapter, the modelling of the self-excited aerodynamic forces is presented. It is based on the spline finite strip method. The aerodynamic matrices are derived, which represent the external load from the wind in the flutter equation of motion. The flutter equation is then solved by the computational algorithm known as the *PK-F* method. A flutter analysis computer module is developed and implemented in the computer program *SFSFA*. The numerical example of a thin flat plate is presented to demonstrate the reliability of the proposed model.

3.2 Definition of Aeroelastic Forces

3.2.1 Notations and Assumptions

When a three-dimensional structure is immersed in a flow field, the action of the flow field on the structure can be represented in general by three force components and three moment components, as shown in Figure 3.1. They are denoted as follows

D : Drag force

S : Side force

L : Lift force

M_R : Rolling moment

M_P : Pitching moment

M_Y : Yawing moment

In practice, it is often unnecessary to consider all six components. Two-dimensional mathematical models can be developed and used effectively in the practical analysis

and design of typical bridge structures.

“Strip Theory” Assumption

Since a bridge structure has a much larger dimension in the longitudinal direction than the other two directions, and the wind-induced responses of interest is under the condition when the wind load acts perpendicular to the longitudinal axis of the bridge, the aerodynamic stability of the bridge can be studied with the behaviour of a unit strip cross-section of the bridge deck. The “Strip Theory” assumption (Dowell *et al*, 1978) was originally introduced for the analysis of large aspect ratio airfoils. It is applied to the wind response analysis and design of flexible long-span bridges. In the “Strip Theory”, it is assumed that the aerodynamic forces acting on the whole bridge deck structure can be obtained by the integration of the aerodynamic forces acting on a unit cross-section of the bridge deck and modelled by the two-dimensional aerodynamic theory. This simplifies the analysis that instead of modelling the entire structure, only a 2D cross-section strip of the bridge deck and the associated aerodynamic forces need to be considered. In the strip model, only the two force and one moment components of the aerodynamic force actions need to be considered, i.e. the lift force, the drag force and the pitching moment, as shown in Figure 3.2. In the following derivation, these are denoted by L_h , D_p and M_α , respectively.

Linearization Assumption

It is generally considered that if there exists separation between the airflow and the structure, the aerodynamic forces become non-linear functions of the wind attack angle α_e . The present research concerns the determination of the critical state which initiates the divergent vibration of the bridge structure. Thus, the current study focuses on the behaviour of the incipient small amplitude vibration of long-span bridges. Recent wind tunnel experimental results (Bridge research Group of Tongji University, 1980 & 1982; Structural Theory Research Institute of Tongji University, 1983) have shown that the relationship between the aerodynamic forces and the attack angle for a bridge structure which vibrates with a small amplitude near the zero attack angle is approximately linear even the bridge deck cross-section has a very bluff configuration. It is reasonable to assume a linear relationship between the aerodynamic forces and the wind attack angle in the derivation and modelling of the wind load in the current study.

Constant Attack Angle Assumption

It is generally accepted that a change in the attack angle of the approaching flow will result in a change in the aerodynamic forces acting on the structure. But this change can also be considered as the result from the alteration of the effective aerodynamic configuration of the bridge deck cross-section. In other words, a streamlined deck cross-section subjected to wind at a non-zero wind attack angle will behave similarly in terms of aerodynamics as an unstreamlined one with a zero attack angle. Thus in the wind stability study of bridges, it can be assumed that the attack angle is a constant value and the aerodynamic behaviour of the bridge

can be obtained from an equivalent aerodynamic configuration.

3.2.2 Self-excited Force Model for Flutter Analysis

Because of the complexity and the large variation in the configuration and cross-sectional layout of bridge structures, there are still not sufficient field data today to accurately express the spatial variation of the air flow field surrounding the bridge deck. Although in recent years, advances have been made in this regard through numerical studies (Mendes and Branco, 1994; Larsen and Walther, 1996; Larsen, 1997), models used today for analysis of the wind-induced responses of bridge structures still require to be calibrated by information and data obtained from the wind tunnel tests.

In the derivation here, a bridge structure with a representative bridge deck cross-section as shown in Figure 3.2 is immersed in the wind flow field perpendicular to the longitudinal axis of the bridge at a mean wind speed of U . The “Strip Theory” assumption is applied. If the bridge oscillates with an angular frequency ω , the aerodynamic loads acting on the deck cross-section may be separated into two parts, i.e. the aeroelastic component, which is motion dependent, and the buffeting component, which is motion independent. Further, the linerization assumption and the constant attack angle assumption are adopted in the derivation of the wind load model.

The linearized self-excited lift force L_h , drag force D_p and aerodynamic moment M_a , as shown in Figure 3.2, are determined from the flutter derivatives obtained by wind tunnel tests of the section model of the bridge. According to the Scanlan’s model (Jones *et al*, 1994; Jain *et al*, 1996A; Jain *et al*, 1996B), the aerodynamic forces

per unit span length are related to the heave, sway and torsional displacements of the bridge deck, as denoted respectively by h , p , α , and the associated velocity $\dot{h}$, $\dot{p}$, $\dot{\alpha}$ at the center of elasticity of the deck cross-section, and the experimentally obtained flutter derivatives as follows

$$L_h = \frac{1}{2}\rho U^2(2B) \left[KH_1^*(K)\frac{\dot{h}}{U} + KH_2^*(K)\frac{B\dot{\alpha}}{U} + K^2H_3^*(K)\alpha + K^2H_4^*(K)\frac{h}{B} + KH_5^*(K)\frac{\dot{p}}{U} + K^2H_6^*(K)\frac{p}{B} \right] \quad (3.1a)$$

$$D_p = \frac{1}{2}\rho U^2(2B) \left[KP_1^*(K)\frac{\dot{p}}{U} + KP_2^*(K)\frac{B\dot{\alpha}}{U} + K^2P_3^*(K)\alpha + K^2P_4^*(K)\frac{p}{B} + KP_5^*(K)\frac{\dot{h}}{U} + K^2P_6^*(K)\frac{h}{B} \right] \quad (3.1b)$$

$$M_\alpha = \frac{1}{2}\rho U^2(2B^2) \left[KA_1^*(K)\frac{\dot{h}}{U} + KA_2^*(K)\frac{B\dot{\alpha}}{U} + K^2A_3^*(K)\alpha + K^2A_4^*(K)\frac{h}{B} + KA_5^*(K)\frac{\dot{p}}{U} + K^2A_6^*(K)\frac{p}{B} \right] \quad (3.1c)$$

where ρ — the air mass density;
 B — the bridge deck width;
 U — the wind velocity;
 K — the reduced frequency with $K = B\omega/U$;
 ω — the circular frequency of response.

H_i^*, A_i^*, P_i^* — the flutter derivatives, $i = 1, 2, \dots, 6$;

Note that h , p , α in Eq.(3.1) are equivalent to w , u , θ in the formulation of the spline finite strip model discussed earlier in Chapter 2. Since the coefficients KH_i^* ,

KP_i^* , KA_i^* ($i = 1 \dots 6$) represent the gradients of the aerodynamic forces, and Equation (3.1) holds only for pure sinusoidal motion, H_i^* , P_i^* , A_i^* ($i = 1 \dots 6$) are known as the flutter derivatives. They are the functions of the reduced frequency and depend on the geometrical configuration of the bridge deck cross-section and the characteristics of the approaching flow. The flutter derivatives can be presented in graphical form as functions of the reduced wind velocity U_r , which is defined as follows

$$U_r = \frac{2\pi}{K} = \frac{U}{nB}, \quad n = \frac{\omega}{2\pi} \quad (3.2)$$

The terms H_1^* , H_4^* in Equation (3.1) represent the contributions of the heave motions to the lift force, whereas P_1^* and P_4^* represent the contributions of the sway motions to the drag force, and A_2^* and A_3^* represent the contributions of the torsional motions to the aerodynamic moment. The terms H_1^* , H_4^* , P_1^* , P_4^* , A_2^* and A_3^* are called the direct-derivatives. The other flutter derivatives are the coupling terms which represent the aerodynamic forces along one particular degree-of-freedom due to the motion in another degree-of-freedom. For example, A_1^* and A_4^* relate the pitching moment induced by the vertical motion. Therefore, the terms H_2^* , H_3^* , H_5^* , H_6^* , P_2^* , P_3^* , P_5^* , P_6^* , A_1^* , A_4^* , A_5^* and A_6^* are called the cross-derivatives.

Alternative expressions of the aerodynamic forces other than Equation (3.1) are used at ONERA in France (Szechenyi, 1973), where the flutter derivatives are known as the Küssner coefficients: $\{K_1^a, K_2^a, K_1^b, K_2^b\}$ and $\{m_1^a, m_2^a, m_1^b, m_2^b\}$.

The aerodynamic forces given by the models which involve the flutter derivatives or the Küssner coefficients have the “mixed time-frequency form”. They are convenient for application in frequency domain analysis. For time-domain analysis, the time-domain expressions of the aerodynamic forces are required. Scanlan (1974)

proposed to employ the indicial response functions to represent the variations of the lift force and the aerodynamic moment due to a step change in the wind attack angle and the velocity of the vertical motion. The indicial response functions and the corresponding flutter derivatives are Fourier transformation pair.

Another aerodynamic force model proposed by Y.K.Lin (1978, 1979) employs impulse functions for the stochastic stability analysis of bridge structures in a turbulent flow field. The impulse functions also correspond to the flutter derivatives in the Scanlan's model.

A unified convention for a standard representation of the flutter derivatives was recently proposed by Zasso (1996). This new convention has the advantages of allowing an easier and direct quantitative comparison of the performances of different deck cross-sections.

In the flutter analysis, the frequency-domain approach may be considered as more convenient and practical than the other approaches (Scanlan,1993). The reason is that the wind spectra and the parameters which characterize the aerodynamic effects such as flutter derivatives and aerodynamic admittances are all functions in the natural frequency-domain.

The time-frequency mixed form of the aerodynamic forces for 2D flutter analysis has been widely used since it was first presented by Scanlan (1967). In subsequent years, the expression has been extended to apply in the case of full 3D flutter analysis which not only includes the sway motion (Scanlan,1990), but also accounts for the coupled terms between different degrees-of-freedom as well (Jones *et al*, 1994; Jain *et al*, 1996A; Jain *et al*, 1996B). In addition to the classical flutter problem, the improved aerodynamic force model can also be used to analyze the separated-flow flutter problem. This model, as given in Equation (3.1), is adopted in the current

research.

The flutter derivatives in the expressions are determined by the section model wind tunnel tests, usually under the smooth flow conditions. The successful extraction of flutter derivatives under turbulent flow have been reported by a number of authors (Scanlan and Lin, 1978; Huston, 1986; Kumarasena, 1989; Sarkar *et al*, 1991; Sarkar, 1992; Davenport *et al*, 1992). It has been observed (Scanlan and Lin, 1978; Sarkar, 1992) that flutter derivatives obtained under turbulence conditions do not have much difference from those obtained under smooth flow conditions.

3.3 Allocation of the Aerodynamic Forces

In Scanlan's model, as given by Equation (3.1), the self-excited aerodynamic forces are related to the displacements and velocities at the center of elasticity of the deck cross-section. When applying the spline finite strip method to model the bridge structure, the main girder is divided into a number of strips. In order to account for the spatial distribution effects in the flutter response analysis of the bridges, it is assumed that the aerodynamic forces are distributed to all the nodal lines of the finite strip model representing the bridge girder.

In the spline finite strip model of the bridge girder as shown in Figure 3.3, each nodal line is divided into a number of sections in the longitudinal direction by knots. Considering the section M between knots $(m - 1)$ and m , the aerodynamic loads acting on the bridge cross-sections $(m - 1)$ and m can be obtained by lumping half of the respective forces to each end, as shown in Figure 3.4 to Figure 3.6.

At cross-section $(m - 1)$,

$$Z_{e_{m-1}} = \frac{h_m}{2} \cdot L_h(u_{e_{m-1}}, w_{e_{m-1}}, \theta_{e_{m-1}}, \dot{u}_{e_{m-1}}, \dot{w}_{e_{m-1}}, \dot{\theta}_{e_{m-1}}) \quad (3.3)$$

$$X_{e_{m-1}} = \frac{h_m}{2} \cdot D_p(u_{e_{m-1}}, w_{e_{m-1}}, \theta_{e_{m-1}}, \dot{u}_{e_{m-1}}, \dot{w}_{e_{m-1}}, \dot{\theta}_{e_{m-1}}) \quad (3.4)$$

$$M_{e_{m-1}} = \frac{h_m}{2} \cdot M_\alpha(u_{e_{m-1}}, w_{e_{m-1}}, \theta_{e_{m-1}}, \dot{u}_{e_{m-1}}, \dot{w}_{e_{m-1}}, \dot{\theta}_{e_{m-1}}) \quad (3.5)$$

At cross-section m ,

$$Z_e = \frac{h_m}{2} \cdot L_h(u_e, w_e, \theta_e, \dot{u}_e, \dot{w}_e, \dot{\theta}_e) \quad (3.6)$$

$$X_e = \frac{h_m}{2} \cdot D_p(u_e, w_e, \theta_e, \dot{u}_e, \dot{w}_e, \dot{\theta}_e) \quad (3.7)$$

$$M_e = \frac{h_m}{2} \cdot M_\alpha(u_e, w_e, \theta_e, \dot{u}_e, \dot{w}_e, \dot{\theta}_e) \quad (3.8)$$

where h_m — the length of the bridge span between knots $(m - 1)$ and m ;
 Z_e, X_e, M_e — the lumped aerodynamic lift force, drag force and moment
at the center of elasticity of the bridge deck cross-section,
 u_e, w_e, θ_e — the transverse, vertical displacements and rotation about an
axis parallel to the longitudinal direction of the bridge
deck cross-section;
 $\dot{u}_e, \dot{w}_e, \dot{\theta}_e$ — the associated velocities;
 L_h, D_p, M_α — the aerodynamic forces as given by Equation (3.1).

The following derivation applies to the bridge cross-section $(m-1)$. The subscript $(m - 1)$ is dropped for convenience purposes.

3.4 Displacement Relationship

Although the finite strip method developed here can consider the spatial distribution effects of the aerodynamic forces by applying different aerodynamic loads at the knots of the structural model, accurate field and experimental data of such distributed aerodynamic loads are not presently available for the purpose of research here. Consequently, to simulate the distribution effects, the aerodynamic force model given by Scanlan at the center of elasticity of the bridge cross-section are distributed to the nodal lines of the finite strip model. The distributions are based on the rigid body motion relationships between the displacements and velocities at the nodal lines and those at the center of elasticity of the deck cross-section.

Assume the deck cross-section undergoes a rigid-body motion. Select the centroid o of the cross-section as the base point. The displacement of point o in the X and Z direction are u_o and w_o , respectively. The rotation of the cross-section is θ . The displacements of an arbitrary point $p(x_p, z_p)$ of the cross-section, as shown in Figure 3.7, can be expressed as follows

$$\begin{cases} u_p = u_o - r_{op} \cdot \theta \sin\phi = u_o - (z_p - z_o) \cdot \theta = u_o - \bar{z}_p \cdot \theta \\ w_p = w_o + r_{op} \cdot \theta \cos\phi = w_o + (x_p - x_o) \cdot \theta = w_o + \bar{x}_p \cdot \theta \\ \theta_p = \theta \end{cases} \quad (3.9)$$

Assume point e is the center of elasticity of the deck cross-section, with its coordinates being (x_e, z_e) . Thus, the displacement of point e can be expressed as follows

$$\begin{cases} u_e = u_o - \bar{z}_e \cdot \theta \\ w_e = w_o + \bar{x}_e \cdot \theta \\ \theta_e = \theta \end{cases} \quad (3.10)$$

or

$$\begin{Bmatrix} u_e \\ w_e \\ \theta_e \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\bar{z}_e \\ 0 & 1 & \bar{x}_e \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_o \\ w_o \\ \theta \end{Bmatrix} \quad (3.11)$$

where

$$\bar{x}_e = x_e - x_o \quad (3.12)$$

$$\bar{z}_e = z_e - z_o \quad (3.13)$$

For a symmetric single-cell box girder or slab girder cross-section, the centroid o and the elasticity center e coincide with each other, i.e.

$$\begin{cases} u_e = u_o \\ w_e = w_o \\ \theta_e = \theta \end{cases} \quad (3.14)$$

whereas for an open type deck cross-section such as ' π ' configuration, $\bar{x}_e \neq 0$, $\bar{z}_e \neq 0$.

The translational displacements at the centroid of the deck cross-section can be expressed in the form of the nodal displacements as follows

$$u_o = \sum_{i=1}^r \frac{\sum_j l_{ij}}{2 \sum_{I=1}^S l_I} u_i \quad (3.15a)$$

$$w_o = \sum_{i=1}^r \frac{\sum_j l_{ij}}{2 \sum_{I=1}^S l_I} w_i \quad (3.15b)$$

where r — total number of nodal lines;
 S — total number of finite strips;
 l_I — width of the I^{th} strip;
 l_{ij} — width of the strip formed by two nodal lines i and j ;
 u_i, w_i, θ_i — displacement parameters of nodal line i .

Expand the right hand side of Equation (3.15a), and write it in the matrix form gives

$$\begin{aligned} u_o &= \frac{\sum_j l_{1j}}{2 \sum_{I=1}^S l_I} u_1 + \frac{\sum_j l_{2j}}{2 \sum_{I=1}^S l_I} u_2 + \cdots + \frac{\sum_j l_{rj}}{2 \sum_{I=1}^S l_I} u_r \\ &= \begin{bmatrix} a_1 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & \cdots & a_r & 0 & 0 & 0 \end{bmatrix} \{\delta\} \\ &= [N_u] \{\delta\} \end{aligned} \quad (3.16)$$

where $[N_u]$ — shape function matrix of displacement u ,

$\{\delta\}$ — the displacement parameter vector,

$$\{\delta\} = \left\{ u_1 \quad v_1 \quad w_1 \quad \theta_1 \quad u_2 \quad v_2 \quad w_2 \quad \theta_2 \quad \cdots \quad u_r \quad v_r \quad w_r \quad \theta_r \right\}; \quad (3.17)$$

$$[N_u] = \begin{bmatrix} a_1 & 0 & 0 & 0 & a_2 & 0 & 0 & 0 & \cdots & a_r & 0 & 0 & 0 \end{bmatrix} \quad (3.18)$$

$$a_i = \frac{\sum_j l_{ij}}{2 \sum_{l=1}^s l_l}, \quad i = 1, 2, 3, \dots, r \quad (3.19)$$

Similarly, by expanding the right hand side of Equation (3.15b) gives

$$w_o = [N_w]\{\delta\} \quad (3.20)$$

where $[N_w]$ is the shape function matrix of the vertical displacement w ,

$$[N_w] = \begin{bmatrix} 0 & 0 & a_1 & 0 & 0 & 0 & a_2 & 0 & \cdots & 0 & 0 & a_r & 0 \end{bmatrix} \quad (3.21)$$

The majority of the bridge structures have symmetric deck cross-section. Based on rigid body motion assumption, the rotation θ of the deck cross-section can be determined from the relative displacements in the Z direction of a pair of nodal lines symmetric about the deck center. For the finite strip model with q pairs of symmetric nodal lines, q sets of rotation $\theta_i (i = 1, 2, 3, \dots, q)$ can be determined. The rotation θ of the deck cross-section is then taken as the average of these rotation angles. For the case of a single-cell box girder with five pairs of symmetric nodal lines, as shown in Figure 3.8, the displacement shape function $[N_\theta]$ has the form as follows

$$[N_\theta] = \frac{1}{5} \begin{bmatrix} 0 & 0 & b_1 & 0 & 0 & 0 & b_2 & 0 & \cdots & 0 & 0 & b_{10} & 0 \end{bmatrix} \quad (3.22)$$

where $b_1 = -b_5 = -\frac{1}{s_1}$, $b_2 = -b_4 = -\frac{1}{s_2}$, $b_3 = b_9 = 0$, $b_6 = -b_7 = -\frac{1}{s_3}$, $b_8 = -b_{10} = -\frac{1}{s_4}$.

Thus

$$\begin{Bmatrix} u_o \\ w_o \\ \theta_o \end{Bmatrix} = \begin{Bmatrix} [N_u] \\ [N_w] \\ [N_\theta] \end{Bmatrix} \cdot \{\delta\} \quad (3.23)$$

The form and size of the displacement shape function matrices $[N_u]$, $[N_w]$ and $[N_\theta]$ depend on the geometry and discretization of the deck cross-section.

Substituting Equation (3.23) into Equation (3.11) gives the relationship between the displacement at the center of elasticity of the deck cross-section and those at the nodal lines, which has the form as follows

$$\begin{Bmatrix} u_e \\ w_e \\ \theta_e \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -\bar{z}_e \\ 0 & 1 & -\bar{x}_e \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} [N_u] \\ [N_w] \\ [N_\theta] \end{Bmatrix} \cdot \{\delta\} \quad (3.24)$$

or

$$u_e = ([N_u] - \bar{z}_e[N_\theta]) \cdot \{\delta\} \quad (3.25a)$$

$$w_e = ([N_w] + \bar{x}_e[N_\theta]) \cdot \{\delta\} \quad (3.25b)$$

$$\theta_e = [N_\theta] \cdot \{\delta\} \quad (3.25c)$$

3.5 Aerodynamic Matrices

3.5.1 Effects due to Lift Force L_h

As shown in Figure 3.4(b), the virtual work done by $Z_{e_{m-1}}$ through the virtual displacement $\bar{w}_{e_{m-1}}$ can be written as follows

$$W = -Z_{e_{m-1}} \bar{w}_{e_{m-1}} = -\{\delta\}^T \{p\}_{m-1}^{L_h} \quad (3.26)$$

where $\{p\}_{m-1}^{L_h}$ — the equivalent knot load at the end cross-section $(m-1)$ due to the lift force L_h acting on the section M ,

$$\{p\}_{m-1}^{L_h} = \left(\frac{1}{2} \rho U^2 \right) \left(\frac{1}{U} [A_d]_{M,m-1}^{L_h} \{\dot{\delta}\} + [A_s]_{M,m-1}^{L_h} \{\delta\} \right) \quad (3.27)$$

$[A_d]_{M,m-1}^{L_h}, [A_s]_{M,m-1}^{L_h}$ — the components of the aerodynamic damping and stiffness matrices induced by L_h at the end cross-section $(m-1)$,

$$\begin{aligned} [A_d]_{M,m-1}^{L_h} = & (KBh_m) [H_1^*([N_w]^T + \bar{x}_e[N_\theta]^T)([N_w] + \bar{x}_e[N_\theta]) + BH_2^*([N_w]^T + \bar{x}_e[N_\theta]^T)[N_\theta] \\ & + H_5^*([N_w]^T + \bar{x}_e[N_\theta]^T)([N_u] - \bar{z}_e[N_\theta])] \end{aligned} \quad (3.28)$$

$$\begin{aligned} [A_s]_{M,m-1}^{L_h} = & (K^2 h_m) [BH_3^*([N_w]^T + \bar{x}_e[N_\theta]^T)[N_\theta] + H_4^*([N_w]^T + \bar{x}_e[N_\theta]^T)([N_w] + \bar{x}_e[N_\theta]) \\ & + H_6^*([N_w]^T + \bar{x}_e[N_\theta]^T)([N_u] - \bar{z}_e[N_\theta])] \end{aligned} \quad (3.29)$$

3.5.2 Effects due to Drag Force D_p and Aerodynamic Moment M_α

The components of the aerodynamic damping and stiffness matrices $[A_d]_{M,m-1}^{D_p}$ and $[A_s]_{M,m-1}^{D_p}$ induced by the drag force D_p , and $[A_d]_{M,m-1}^{M_\alpha}$ and $[A_s]_{M,m-1}^{M_\alpha}$ induced by

the aerodynamic moment M_α , can be obtained by following similar derivations, and referring to Figure 3.5(b) and Figure 3.6(b), as follows

$$\begin{aligned} [A_d]_{M,m-1}^{D_p} = & (KBh_m) [P_1^*([N_u]^T - \bar{z}_e[N_\theta]^T)([N_u] - \bar{z}_e[N_\theta]) + BP_2^*([N_u]^T - \bar{z}_e[N_\theta]^T)[N_\theta] \\ & + P_5^*([N_u]^T - \bar{z}_e[N_\theta]^T)([N_w] + \bar{x}_e[N_\theta])] \end{aligned} \quad (3.30)$$

$$\begin{aligned} [A_s]_{M,m-1}^{D_p} = & (K^2h_m) [BP_3^*([N_u]^T - \bar{z}_e[N_\theta]^T)[N_\theta] + P_4^*([N_u]^T - \bar{z}_e[N_\theta]^T)([N_u] - \bar{z}_e[N_\theta]) \\ & + P_6^*([N_u]^T - \bar{z}_e[N_\theta]^T)([N_w] + \bar{x}_e[N_\theta])] \end{aligned} \quad (3.31)$$

$$\begin{aligned} [A_d]_{M,m-1}^{M_\alpha} = & (KB^2h_m) [A_1^*[N_\theta]^T([N_w] + \bar{x}_e[N_\theta]) + BA_2^*[N_\theta]^T[N_\theta] + A_5^*[N_\theta]^T([N_u] - \bar{z}_e[N_\theta])] \end{aligned} \quad (3.32)$$

$$\begin{aligned} [A_s]_{M,m-1}^{M_\alpha} = & (K^2Bh_m) [BA_3^*[N_\theta]^T[N_\theta] + A_4^*[N_\theta]^T([N_w] + \bar{x}_e[N_\theta]) + A_6^*[N_\theta]^T([N_u] - \bar{z}_e[N_\theta])] \end{aligned} \quad (3.33)$$

3.5.3 Aerodynamic Matrices $[A_d]_{M,m-1}^e$ and $[A_s]_{M,m-1}^e$

The aerodynamic damping and stiffness matrices $[A_d]_{M,m-1}^e$ and $[A_s]_{M,m-1}^e$ at the end cross-section $(m-1)$ of section M can finally be obtained by summing all the effects due to the lift force, drag force and aerodynamic moment as follows

$$[A_d]_{M,m-1}^e = [A_d]_{M,m-1}^{L_h} + [A_d]_{M,m-1}^{D_p} + [A_d]_{M,m-1}^{M_\alpha} \quad (3.34)$$

$$[A_s]_{M,m-1}^e = [A_s]_{M,m-1}^{L_h} + [A_s]_{M,m-1}^{D_p} + [A_s]_{M,m-1}^{M_\alpha} \quad (3.35)$$

The size of the aerodynamic matrices $[A_d]_{M,m-1}^e$ and $[A_s]_{M,m-1}^e$ depends on the mesh size of the deck cross-section. Since the aeroelastic forces are non-conservative,

the aerodynamic matrices are nonsymmetric and vary with the wind velocity and response frequency. The global aerodynamic matrices $[A_d]$ and $[A_s]$ for the spline finite strip model can be obtained by following the same assembling procedures as in the standard finite element method.

3.6 Flutter Equation of Motion

A three-dimensional self-excited vibration equation of motion for the finite strip model can be derived based on the dynamic equilibrium condition as follows

$$[M]\{\ddot{X}\} + [D]\{\dot{X}\} + [K]\{X\} = \{A\} \quad (3.36)$$

where $[M]$ — the global mass matrix;

$[D]$ — the global damping matrix;

$[K]$ — the global stiffness matrix;

$\{X\}$ — the displacement vector;

$\{A\}$ — the self-excited aerodynamic force acting on the bridge structure,

which can be separated into two parts and expressed as follows

$$\{A\} = \frac{1}{2}\rho U^2 \left([A_s]\{X\} + \frac{1}{U}[A_d]\{\dot{X}\} \right) \quad (3.37)$$

in which $[A_s]$ and $[A_d]$ are respectively the global aerodynamic stiffness and damping matrices.

Substituting Equation (3.37) into (3.36) yields

$$[M]\{\ddot{X}\} + [D]\{\dot{X}\} + [K]\{X\} = \frac{1}{2}\rho U^2 \left([A_s]\{X\} + \frac{1}{U}[A_d]\{\dot{X}\} \right) \quad (3.38)$$

As indicated by Equation (3.38), the flutter response results from the interactions between the inertial force, the damping force, the elastic force and the self-excited aerodynamic forces.

Following the mode-superposition method, the responses at all the degrees-of-freedom can be discretized into the product of the spatial and time-dependent components, i.e. natural mode shapes and the generalized coordinates, as follows

$$\{X\} = [\phi]\{\xi\} \quad (3.39)$$

where $[\phi]$ — the mode shape matrix of size $n \times m$, with the i^{th} column corresponding to the i^{th} mode shape;

$\{\xi\}$ — the generalized coordinate vector of size m , with the i^{th} row corresponding to the i^{th} generalized coordinate;

n — the total number of degrees-of-freedoms;

m — the number of modes considered in the flutter analysis.

The mode shape matrix $[\phi]$ is formed by any modes susceptible to participate in the flutter motion. There is no restriction on the number of the involving modes.

Substituting Equation (3.39) into (3.38) and premultiplying with the transpose of $[\phi]$ yields

$$[M]^g \{\ddot{\xi}\} + [D]^g \{\dot{\xi}\} + [K]^g \{\xi\} = \frac{1}{2} \rho U^2 \left([A_s]^g \{\xi\} + \frac{1}{U} [A_d]^g \{\dot{\xi}\} \right) \quad (3.40)$$

where $[M]^g = [\phi]^T[M][\phi]$ is the generalized mass matrix;

$[D]^g = [\phi]^T[D][\phi]$ is the generalized damping matrix;

$[K]^g = [\phi]^T[K][\phi]$ is the generalized stiffness matrix;

$[A_s]^g = [\phi]^T[A_s][\phi]$ is the generalized aerodynamic stiffness matrix;

$[A_d]^g = [\phi]^T[A_d][\phi]$ is the generalized aerodynamic damping matrix.

Assume the global damping matrix has the form of Rayleigh damping, i.e.

$$[D] = a_0[M] + a_1[K] \quad (3.41)$$

where a_0 and a_1 are the proportionality constants. With Rayleigh damping, the mode shapes are orthogonal with respect to the mass and stiffness matrices $[M]$ and $[K]$. Consequently, $[M]^g$, $[D]^g$ and $[K]^g$ in Equation (3.40) are diagonal matrices. However, the generalized aerodynamic matrices $[A_s]^g$ and $[A_d]^g$ are non-diagonal. The flutter response is the coupled response between different modes. Equation (3.40) is called the flutter equation of motion.

Since the flutter response is generally a harmonic motion, it is assumed that the generalized coordinates of the response have the damped, harmonic form as follows

$$\{\xi\} = \{R\} \exp(\mu t) \quad (3.42)$$

where $\{R\}$ is the response amplitude vector; $\mu = (\delta + i)\omega$; δ is the logarithmic decrement of the response; and ω is the circular frequency of the response.

Using the relationship between the circular frequency ω and the reduced frequency K , which is $K = B\omega/U$, Equation (3.42) can be rewritten as follows

$$\begin{aligned} \{\xi\} &= \{R\} \cdot \exp \left[\frac{U}{B} K (\delta + i) t \right] \\ &= \{R\} \cdot \exp \left(\frac{U}{B} P t \right) \end{aligned} \quad (3.43)$$

where P is a complex value function defined as follows

$$P = K(\delta + i) \quad (3.44)$$

Substituting Equation (3.43) into (3.40) gives

$$\left[[M]^g \left(\frac{U}{B} \right)^2 P^2 + [D]^g \left(\frac{U}{B} \right) P + [K]^g - \frac{1}{2} \rho U^2 \left([A_s]^g + \frac{1}{B} [A_D]^g P \right) \right] \cdot \{R\} \exp \left(\frac{U}{B} P t \right) = \{0\} \quad (3.45)$$

Because at the critical point of flutter, the total effective damping of the structure is zero and the response is of an undamped type, the factor P for the generalized aerodynamic damping matrix $[A_d]^g$ should satisfy the condition such that $\delta = 0$, or $K(\delta + i) = iK$. Thus, the final form of the flutter equation of motion can be expressed as follows

$$\left[[M]^g \left(\frac{U}{B} \right)^2 P^2 + [D]^g \left(\frac{U}{B} \right) P + [K]^g - \frac{1}{2} \rho U^2 \left([A_s]^g + \frac{1}{B} [A_D]^g iK \right) \right] \cdot \{R\} \exp \left(\frac{U}{B} P t \right) = \{0\} \quad (3.46)$$

Since in Equation (3.46), the response amplitude vector $\{R\}$ cannot be zero, and the exponential term $\exp(\frac{U}{B} P t)$ will never be zero, the determinant of the remaining part must be zero for a nontrivial solution. This determinant is termed as flutter determinant F , which has the form as follows

$$F = \left| [M]^g \left(\frac{U}{B} \right)^2 P^2 + [D]^g \left(\frac{U}{B} \right) P + [K]^g - \frac{1}{2} \rho U^2 \left([A_s]^g + \frac{1}{B} [A_D]^g iK \right) \right| \quad (3.47)$$

The flutter determinant F is the function of the complex variable P , and P itself is the function of reduced frequency K . To obtain the actual response, the problem now becomes to select an appropriate value of P , which under a given wind velocity U and reduced frequency K , renders the determinant to be zero. The corresponding

circular frequency ω of structural response and the logarithmic decrement δ can be obtained as follows

$$\omega = \frac{U}{B} \text{Im}(P) \quad (3.48)$$

$$\delta = \frac{\text{Re}(P)}{\text{Im}(P)} \quad (3.49)$$

where $\text{Re}(P)$ and $\text{Im}(P)$ denotes the real and imaginary part of the complex value P , respectively.

According to the sign of the logarithmic decrement, the nature of the response can be classified into three types, i.e.

1. $\delta < 0$, which indicates that the amplitude of the response decreases, the structure is under stable condition;
2. $\delta = 0$, which indicates that the amplitude of the response does not change, the structure is under neutrally stable condition;
3. $\delta > 0$, which indicates that the amplitude of the response increases, the structure is under unstable condition.

The critical flutter point corresponding to the neutrally stable condition gives the flutter wind velocity U_f and the flutter frequency ω_f .

3.7 *PK-F* Method

The *PK-F* method, which is modified and improved form the *P-K* method (Hassig, 1971) used in the aircraft industry, is adopted as a computational algorithm to

solve flutter equation of motion in the present study. By means of this general numerical procedure, not only the structural response at the critical flutter point can be determined, but the pre- and post-flutter responses can be obtained as well. Also, more complicated cases such as the flutter responses during different construction stages or the responses of bridge structures having variable cross-section can be analyzed.

According to *PK-F* method, the main procedures for finding the critical wind velocity are as follows

1. Select the range of wind velocity for flutter analysis, from U_{low} to U_{high} , with an increment U_{inc} .
2. Based on the natural frequency analysis, select the vibration modes which may participate and contribute to the flutter motion. Assume the number of the selected modes is m , and calculate two starting values $P_{i,1}$ and $P_{i,2}$ for each of the selected modes, where the subscript i corresponds to the i^{th} mode.
3. Initialize the current wind velocity by setting it equal to U_{low} .
4. Increase the current wind velocity U_{cur} by U_{inc} .
5. Repeat the following procedure for each selected mode i , where $i = 1, 2, 3, \dots, m$.
6. Under the current wind velocity U_{cur} and response frequency ω , calculate the reduced frequency K_1 and K_2 , where $K_1 = Im(P_{i,1})$, $K_2 = Im(P_{i,2})$.
7. Calculate the generalized aerodynamic matrices corresponding to K_1 and K_2 , i.e. $[A_s(K_1)]^g$, $[A_d(K_1)]^g$, $[A_s(K_2)]^g$ and $[A_d(K_2)]^g$.

8. According to Equation (3.47), form the flutter determinants $F_1(K_1)$ and $F_2(K_2)$ corresponding to the reduced frequencies K_1 and K_2 and calculate their values.
9. Comparing the value of flutter determinant $F_2(K_2)$ with the tolerance. If convergence has not achieved, the next iteration value $P_{i,3}$ is assumed to be

$$P_{i,3} = \frac{P_{i,2}F_1 - P_{i,1}F_2}{F_1 - F_2} \quad (3.50)$$

10. Repeat steps 6 to 9, until $F_2(K_2)$ converges.
11. When the flutter determinant F_2 is lower than tolerance, the true structural response corresponding to the U_{cur} and the mode i is found, which is denoted as $P_{i,c}$. The logarithmic decrement δ and response frequency ω can thus be computed according to Equation (3.48) and (3.49). Depending on the sign of δ , the nature of the response can be determined.
12. For all the selected m modes, repeat steps 5 to 11.
13. Select two starting values $P_{i,1}$ and $P_{i,2}$ based on $P_{i,c}$ for each mode for the next wind velocity considered in the analysis.
14. For the considered range of wind velocity, repeat steps 4 to 13.

Several points should be considered in the implementation of the *PK-F* method to perform flutter analysis for bridge structures:

- (1) The iteration procedure is based on the search of the critical point where the structural behaviour becomes divergent. The starting values $P_{i,1}$ and $P_{i,2}$ in step 2 is determined from a nearly undamped condition, i.e. $\delta \approx 0$. Physically, it can be interpreted as that at the starting point, the aerodynamic forces

acting on the structure are generated by extremely low wind velocities. Since there are no aerodynamic forces at zero wind velocity, and thus cannot initiate the numerical iterative procedure, U_{low} is chosen a small value close but not exactly zero. The recommended starting values for P are

$$P_{i,1} = -0.01S_i + iS_i \quad (3.51)$$

$$P_{i,2} = iS_i \quad (3.52)$$

where $S_i = (\omega_i B)/U_{low}$ is the Strouhal Number of the i^{th} mode.

- (2) There is no strict restriction for U_{high} . It can be chosen according to the requirement of the analysis, but should not be higher than the meteorologically possible values at the bridge site.
- (3) In step 12, the starting values $P_{i,1}$ and $P_{i,2}$ for the next wind velocity is determined from $P_{i,c}$ under the current wind velocity as follows

$$P_{i,1} = P_{i,c} \frac{U_{cur}}{U_{cur} + U_{inc}} \quad (3.53)$$

$$P_{i,2} = 1.01P_{i,c} \frac{U_{cur}}{U_{cur} + U_{inc}} \quad (3.54)$$

To ensure the proportional relations given by Equation (3.53) and (3.54) to be correct, the incremental wind velocity U_{inc} should be small such that the prorating of the response will not be affected by the preflutter results.

- (4) The generalized aerodynamic matrices $[A_s]^g$ and $[A_d]^g$ are functions of the reduced frequency K . Since the experimental data obtained from the wind

tunnel tests may not cover the entire range of the reduced frequency considered in analysis, linear interpolation and extrapolation are thus employed in the calculation of $[A_s]^g$ and $[A_d]^g$. Further, if the bridge has variable cross-section along the span length, different sets of flutter derivatives corresponding to the representative cross-section should be used to account for the 3D effects of the structure and the surrounding flow field.

- (5) In step 9, $P_{i,3}$ is updated by linear extrapolation, as shown in Figure 3.9, which is numerically stable for low-order determinants (Bathe, 1982).
- (6) In step 11, if the sign of δ changes from negative to positive, it indicates that flutter has occurred. A refined search for the critical wind velocity U_f around U_{cur} should be carried out by using a smaller incremental value U_{inc} in the evaluation of the update for the U_{cur} . For each level of refined search, take $U_{inc}^j = 0.5U_{inc}^{j-1}$, where j represents the current iteration loop, and $(j - 1)$ represents the previous iteration loop. This refinement continues until the absolute value of the logarithmic decrement or U_{inc} is less than or equal to the tolerance, where the tolerance should be a small number.

3.8 Computer Interpolation

A computer program *SFSFA* (Spline Finite Strip-Based Flutter Analysis Program) has been developed based on the derivations presented in Chapter 2 and Chapter 3. It is written in Fortran language.

The main functions of *SFSFA* are summarized as follows

1. Perform natural frequency analysis of the bridge structure. Three types of ele-

ments, i.e. spline finite strip flat shell element, 3D beam element and stay cable element have been developed and implemented in the program. The eigenvalue problem is solved by subspace iteration. The computed mode shapes are stored in a file for later use.

2. A post-processing module *DRAW* has been developed to graphically illustrate the frequency analysis results. The module is developed in the windows environment using Visual Basic. The mechanical model of the structure and the mode shapes can be shown graphically. The module can greatly assist in the evaluation of the frequency analysis results, in visualizing the deformation of each mode shape, and thus in choosing the necessary modes for flutter analysis.
3. Perform flutter analysis. By iterating on the flutter determinant, the oscillating response over the predefined range of wind velocity can be obtained, and the critical flutter wind velocity U_f can be determined.

The general flow chart of *SFSFA* is shown in Figure 3.10. The program is composed of two modules, i.e. the natural frequency analysis module and the flutter analysis module. These two modules are linked via an intermediate file.

Information for the natural frequency analysis such as geometric dimensions and material properties of the structure, numbering sequences of the mathematical model, etc. are provided in Data file 1. This part is an essential prerequisite for flutter analysis. Execution of module 1 generates two result files, i.e. *fn.plt* and *fn.out*, and one intermediate file *fn.vt*. The post-processing program *DRAW* will use the data stored in *fn.plt* to plot the mechanical model and mode shapes.

In the flutter analysis, only the aerodynamic forces acting on the bridge girder are considered, while those on the pylon(s) and the cables are neglected. Thus only the mode shape components of the nodes on the girder are required. They are stored in the file *fn.vt* for calculating the aerodynamic forces.

The wind flow field and the aeroelastic data are stored in Data file 2. Result file 3 stores the flutter analysis results, which include the critical wind velocity U_f , the flutter frequency ω_f , and the logarithmic decrement and frequency of the oscillating response corresponding to each iterating wind velocity.

The advantage of this program lies in that it can give estimates on the critical flutter wind velocity U_f of the bridge structure even in the initial design stage. The program can also be used to evaluate the variation of the critical wind velocity during different construction stages. The analysis results provide reliable data for evaluating the aerodynamic stability property of bridge structures.

The flow charts for the natural frequency analysis module and flutter analysis module are given in Figure 3.11 and Figure 3.12, respectively.

3.9 Numerical Example: Simply Supported Thin Flat Plate

A numerical example for the flutter analysis of a simply supported thin flat plate is presented to illustrate the reliability and accuracy of the proposed method.

3.9.1 Description of the Problem

The span length of a simply supported plate representing a bridge deck is 300m. The width and depth of the deck cross-section are 40m and 1m, respectively, as shown in Figure 3.13. It is assumed that the modulus of elasticity $E = 3.5 \times 10^4 MPa$, the mass density $\rho = 2.5 \times 10^3 Kg/m^3$ and the poisson's ratio $1/6$.

The plate is divided into two strips of equal width in the X direction and ten equal length sections in the Y direction.

3.9.2 Flutter Analysis

From the frequency analysis, the frequency of the first symmetrical heave mode is determined to be $f_b = 0.01887 Hz$, and that of the first symmetrical torsional mode is $f_t = 0.1943 Hz$. These two modes are selected for the flutter analysis.

For a plate with uniform thin cross-section, the Theodorson aerodynamic force can be used to calculate the flutter derivatives. In practice, the Theodorson aerodynamic force can be represented approximately by the R.T.Jones's expression as follows

$$C(k) = 1 - \frac{0.165}{1 - \frac{0.0455}{k}i} - \frac{0.335}{1 - \frac{0.3}{k}i} \quad (3.55)$$

or

$$C(k) = F(k) + iG(k) \quad (3.56)$$

where

$$F(k) = 1 - \frac{0.165}{1 + \frac{0.0455}{k}} - \frac{0.335}{1 + \frac{0.3}{k}} \quad (3.57)$$

$$G(k) = 1 - \frac{\frac{0.165 \times 0.0455}{k}}{1 + \left(\frac{0.0455}{k}\right)^2} - \frac{\frac{0.335 \times 0.3}{k}}{1 + \left(\frac{0.3}{k}\right)^2} \quad (3.58)$$

The flutter derivatives of a thin flat plate cross-section can thus be expressed as follows

$$\begin{aligned} H_1^*(K) &= -\frac{2\pi}{K} F(K/2) & A_1^*(K) &= \frac{\pi}{2K} F(K/2) \\ H_2^*(K) &= -\frac{\pi}{K} \left(\frac{1+F(K/2)}{2} + \frac{G(K/2)}{K} \right) & A_2^*(K) &= \frac{\pi}{K} \left(\frac{F(K/2)-1}{8} + \frac{G(K/2)}{2K} \right) \\ H_3^*(K) &= -\frac{\pi}{K^2} \left(2F(K/2) - \frac{KG(K/2)}{2} \right) & A_3^*(K) &= \frac{\pi}{K^2} \left(\frac{K^2}{64} + \frac{F(K/2)}{2} - \frac{KG(K/2)}{8} \right) \\ H_4^*(K) &= \frac{\pi}{2} \left(1 + \frac{4G(K/2)}{K} \right) & A_4^*(K) &= -\frac{\pi}{2K} G(K/2) \end{aligned} \quad (3.59)$$

The critical flutter wind velocity of the plate is determined to be $123.5m/s$, and the associated flutter frequency is $0.1045Hz$. Figures 3.14 and Figure 3.15 give the $f - U$ curve and $\delta - U$ curve, which describe respectively the variation of structural response frequency and logarithmic decrement with respect to the wind velocity. Figure 3.15 shows that when the wind velocity reaches $123.5m/s$, the logarithmic decrement of the fundamental torsional mode becomes zero, which indicates that flutter occurs. Since the two curves representing the frequency of the torsion and heave motions shown in Figure 3.14 converge to each other as the wind velocity approaches the critical flutter velocity, this implies the flutter response is a bending-torsion coupled classical flutter.

3.9.3 Comparison with Other Methods

The numerical results obtained by the spline finite strip method are compared with that obtained from the *Van der put* (Van der put, 1976) formula, which is commonly

used for calculating the critical flutter wind velocity of the thin flat plate. The *Van der put* formula has the form

$$U_f = \left[1 + (\epsilon - 0.5) \sqrt{(r/b) \times 0.72\mu} \right] \times \omega_b \times b \quad (3.60)$$

where U_f is the critical flutter velocity; ϵ is the frequency ratio between fundamental torsional and bending modes; r is the radius of gyration of the deck cross-section; b is the half deck width; μ is the mass ratio between the bridge deck and the surrounding air, and ω_b is the circular frequency of the first bending mode. For the example here, the calculated theoretical flutter wind velocity is $U_f = 121.9m/s$. The numerical results by the spline finite strip method agrees with the analytical result. The difference is only 1.3%.

The thin flat plate example has also been analyzed by using a three-dimensional finite element-based flutter analysis program *FEFA* (Cheng, 1994), which has been successfully used in the flutter analysis of many long-span bridges. The calculated critical flutter wind velocity by the finite element model using the same mesh size in the longitudinal direction of the finite strip model is $125.4m/s$ and the flutter frequency is $0.1034Hz$. The finite strip results agree well with the analytical and finite element results.

3.9.4 Parametric Study

The results obtained from a parametric study to determine the influence of the mesh size in both the transverse and the longitudinal directions on the critical flutter wind velocity and frequency are presented in Table 3.1 and Table 3.2.

The results show that the number of sections considered in the Y direction of the analysis model has a significant effect on the critical flutter wind velocity. The

critical flutter velocity for the case of a 1×4 mesh is $108.50m/s$, while that for a 1×16 mesh is $125.38m/s$, where there is a 15.6% difference. According to the spline finite strip model, the magnitude of the aerodynamic forces allocated to the knot is proportional to the sectional length. Thus with fewer and a longer sectional length, larger forces are allocated to the knots, consequently more wind energy is input into the structure through these knots. This initiates the occurrence of flutter earlier. Increasing the number of sections in the Y direction, the critical flutter wind velocity also increases, while the flutter frequency decreases. The results converge in the case of the length of the section less than or equal to $1/16$ of the span length.

In the current example, the number of finite strips do not seem to have significant effect on the critical flutter wind velocity. The reason is that since there is no sufficient field data to date to describe the actual spatial variation of the air flow field surrounding the bridge deck, the aerodynamic model developed in the current study is based on the equivalent allocation of Scanlan's aerodynamic forces to each nodal line of the finite strip model. With the future availability of the spatially varied field data, the formulation proposed in the present study can better demonstrate the impact of taking into account the spatial distribution of the aerodynamic loads on the critical flutter wind velocity.

3.10 Summary and Conclusions

In this chapter, the self-excited aerodynamic force model is developed by using the spline finite strip method. The spatial distribution of the wind load can be taken into account by allocating the aerodynamic forces on each nodal line of the finite strip model representing the bridge girder. The *PK-F* method is employed

as a computational algorithm for seeking the critical point of flutter. A computer program *SFSFA* for the flutter analysis of long-span bridges is developed according to the principles and derivations in Chapter 2 and Chapter 3. The numerical example of the flutter analysis of a simply supported thin flat plate verifies the reliability and accuracy of the proposed model and method. The following conclusions can be drawn from the results obtained in the current study:

1. A 3D flutter analysis procedure based on the finite strip method is developed. The proposed finite-strip-based aerodynamic force model for the bridge girder can easily take into account the spatial distribution effects of the wind load. It provides the bridge engineers and designers a convenient alternative way to evaluate the aerodynamic stability behaviour of the bridges, especially in the early design stage.
2. The reliability and accuracy of the proposed aerodynamic model are verified via a numerical example of the simply supported thin flat plate.
3. The flutter response of a simply supported thin flat plate is a bending-torsion coupled classical flutter, of which the frequencies of the heave and torsional motions converge to each other as the wind velocity approaches the critical flutter wind velocity U_f .
4. In the spline finite strip analysis, the span length of the bridge should be divided into sufficient number of sections in order to achieve satisfactory results.

Table 3.1: Critical flutter wind velocity of simple support plate (m/s)

No. of strips	No. of nodal lines	No. of sections in the Y Direction					
		4	8	10	12	16	20
1	2	108.50	121.63	123.19	124.13	125.38	125.38
2	3	108.81	121.94	123.50	124.75	125.38	125.38
4	5	108.81	122.25	123.50	124.75	125.38	125.38
8	9	108.81	122.25	123.50	124.75	125.38	125.38

Table 3.2: Critical flutter frequency of simple support plate (Hz)

No. of strips	No. of nodal lines	No. of sections in the Y Direction					
		4	8	10	12	16	20
1	2	0.1056	0.1047	0.1048	0.1046	0.1045	0.1046
2	3	0.1053	0.1045	0.1045	0.1042	0.1044	0.1045
4	5	0.1053	0.1044	0.1045	0.1043	0.1044	0.1045
8	9	0.1053	0.1044	0.1045	0.1043	0.1044	0.1045

Figure 3.1: Wind force components on a three-dimensional structure

Figure 3.2: Wind force components on the deck cross-section

Figure 3.3: Spline finite strip model of bridge girder

Figure 3.4: Equivalent knot load due to lift force (a) Lift force acting on section M
 (b) Equivalent knot load due to lift force

Figure 3.5: Equivalent knot load due to drag force (a) Drag force acting on section M (b) Equivalent knot load due to drag force

Figure 3.6: Equivalent knot load due to aerodynamic moment (a) Aerodynamic moment acting on section M (b) Equivalent knot load due to aerodynamic moment

Figure 3.7: Rigid body motion of a cross-section

Figure 3.8: Displacement shape function $[N_\theta]$ of a single-cell box girder

Figure 3.9: Linear extrapolation of the root

Figure 3.10: Flow chart of program SFSFA

Figure 3.11: Flow chart of dynamic characteristic analysis module

Figure 3.12: Flow chart of flutter analysis module

Figure 3.13: Simple support thin flat plate

Figure 3.14: $f - U$ relationship of simple support thin flat plate

Figure 3.15: $\delta - U$ relationship of simple support thin flat plate

Chapter 4

Case Study – Free Vibration and Flutter Analysis of Long-Span Cable-Stayed Bridge

To further demonstrate the capability and accuracy of the proposed method, the free vibration and flutter analysis of the Nanpu Bridge, a long-span cable-stayed bridge completed in 1992, is presented in this chapter. The validity and reliability of the proposed model are discussed. The finite strip method results are compared with the finite element results, as well as with the experimental results obtained from the wind tunnel tests of the prototype model . The advantages and disadvantages of the different modelling schemes, and the potential of the proposed method are discussed.

4.1 Bridge Description

The Nanpu Bridge spans across the Nanpu River in Shanghai, China, which connects the eastern and western part of the city. It is an important civil engineering project in the sense that it not only improves the transportation network in Shanghai, but also facilitates the economic development of the city. The structural form of this cable-stayed bridge is a double-pylon, double-cable-plane, longitudinally floating system.

The structural configuration of half of the bridge is shown in Figure 4.1. The main bridge has five spans: a 423m center span between the two pylons, and two additional spans which include an auxiliary span of 94.5m and an anchorage span of 76.5m on each side of the center span. The total span length is 765.0m. The length and height of the center span are determined in accordance to the navigational and clearance requirements.

The configuration of the prestressed concrete pylon is shown in Figure 4.2. It has an *H*-shape. The height of the pylon is 154m. The pile group foundation under each pylon consists 98 steel pipe piles. Each pile is 0.914m in diameter and 51.1m long.

The two stayed cable planes are parallel to each other. There are 22 pairs of stayed cables laying out from each side of the pylon in a semi-fan configuration. Two vertical cables hanging from the pylon support the girder and isolate the bridge deck from the pylon in the longitudinal direction. The spacing between two adjacent points of attachment of the cables on the girder is typically 9m, except near the two anchorage piers where it is 4.5m.

The main girder of the bridge has a composite cross-section, as shown in Fig-

ure 4.3. It comprises of a prestressed concrete slab supported by two *I*-shaped steel beams. The spacing between the two stiffened beams is 24.55m. The height of the main girder is 2.25m. The total width of the bridge deck is 30.35m. The total width of the traffic lanes is 25.0m. The width of the pedestrian lane on each side of the bridge is 2.675m.

The Nanpu Bridge is a flexible long-span structure. The geometric configuration of the deck cross-section is of an open type. The clearance under the center span varies from 50 to 60m. Since the bridge is located in a typhoon prone area, the wind-resistant stability and safety of the bridge during different construction stages and after its opening to traffic are important factors which required careful consideration in its design.

4.1.1 Boundary Conditions

Nanpu Bridge has a longitudinally floating deck system. The lateral motions of the pylons and that of the main girder at the pylon connections, as well as those at the anchorage piers, are coupled together by lateral restraining blocks. On the secondary spans, the vertical and torsional motion of the bridge deck are restrained at the supports of the auxiliary and anchorage piers by pull rods, while longitudinal displacements and rotations about the transverse axis are allowed by the bridge bearings. The supports at the bottom of the pylons are assumed to be fixed.

4.1.2 Modelling of the Bridge

In the numerical model of the Nanpu Bridge as shown in Figure 4.4, the bridge deck is modelled by three-dimensional spline finite strips, the twin pylons by three-

dimensional beam elements, and the stay cables by the special transition elements which take into account the sagging effect of the cables.

In developing the model, a coordinate system is chosen such that the bridge deck is parallel to the XY -plane of the coordinate system with the Y -axis along the longitudinal direction of the bridge. The Z -axis is perpendicular to the bridge deck surface.

The main girder is modelled as spline finite strip flat shell elements. Since the main girder of the Nanpu Bridge has composite deck cross-section, the concrete deck surface is divided into three strips, and the two stiffening steel beams are modelled by two vertical strips. One of the advantages of the finite strip method here as compared to the equivalent beam finite element model is that the composite deck cross-section can be modelled more accurately without the need of transformation. The material properties of concrete and steel are applied respectively to the strips modelling the deck and the I -shaped steel beams. For the concrete, the modulus of elasticity is $E_c = 3.3 \times 10^4 MPa$ and the Poisson's ratio is $1/6$. For the steel, the modulus of elasticity is $E_s = 2.1 \times 10^5 MPa$, the Poisson's ratio is $1/3$. The mesh and numbering of the deck cross-section is shown in Figure 4.5. The composite deck is modelled by a total of five spline finite strips. In the model, each nodal line is divided into 94 segments separated by the cable support points, the supports of the piers and the pylon locations.

The pylons are modelled by 3D beam finite elements. The shaft of the pylons has variable dimensions. The geometric properties of the representative shaft cross-sections are presented in Table 4.1. Sixty-nine 3D beam elements are used to model each pylon.

The cable is modelled by special cable elements derived in Section 2.5. The cross-

sectional area of the cables are presented in Table 4.2. The numbering sequence of the cables starts from the one closest to the pylon. The cable hanging vertically from the pylon to support the girder is labelled as number 0. The cables in the side span are denoted with the symbol 'S', while for those in the center span are denoted with the symbol 'C'. The cable element has the same degrees-of-freedom at one end as the spline finite strip of the deck and at the other end as the beam finite element of the pylon. The influence on the axial stiffness of the cable due to the sag caused by its self weight is accounted for in the analysis by an effective elastic modulus given by the Ernst formula (Walther *et al*, 1988).

Two rigid horizontal bars are used to simulate the action of the lateral restraining blocks at the intersections of the main girder and the pylons in restraining the lateral motions of the girder to that of the pylon. At the auxiliary pier supports, the vertical motion and rotation about the longitudinal axis of the bridge are restrained, whereas at the supports of the anchorage piers, all displacements are restrained except the longitudinal displacement and rotation about the transverse axis.

The numerical model has a total of 754 nodes and 3084 active degrees-of-freedom.

4.2 Free Vibration Analysis

The dynamic characteristics of a cable-stayed bridge, including its natural vibration frequencies, vibration mode shapes, mechanical damping properties etc., are important factors which can significantly affect its wind-resistant stability behaviour and performance. The self-excited oscillatory motion of a long-span cable-stayed bridge is most significantly related to the lowest heave and torsional modes of the bridge, and its energy dissipation behaviour which is quantified by the mechanical damping

ratios of the structure. The characteristics of and relationship between the heave and torsional modes depend on the configuration and layout of the structural system, the boundary conditions, the geometries of the bridge deck cross-section and the type of cable arrangement.

The *Natural Frequency Analysis* module of the program *SFSFA* is employed to perform the free vibration analysis of the Nanpu Bridge. The first ten natural vibration modes are calculated. Table 4.3 presents the first ten natural vibration frequencies and nature of the vibration modes of the Nanpu Bridge. Some typical mode shapes are shown in Figure 4.6.

The frequency of the fundamental mode is $0.1294Hz$, which is an anti-symmetric floating mode in the longitudinal direction. This type of vibration mode is expected for the floating deck system. Of particular concern to the wind stability of cable-stayed bridges are the heave and torsional motions of the bridge deck, which in the case of the Nanpu Bridge are the second and the fifth vibration modes, with the frequencies of $0.3309Hz$ and $0.4995Hz$ respectively. The mode shapes of these two modes are similar to each other and have the same symmetric nature. The frequency ratio of these two modes is $\epsilon = f_{t1}/f_{b1} = 1.51$. This ratio is considered as low, which is a disadvantage factor to the wind stability behaviour of the bridge. It is noted that as the frequencies of the heave mode and the torsional mode become closer, the possibility of interaction or coupling between these two motions is higher.

4.3 Flutter Analysis

4.3.1 Design Wind Speed

Annual extreme wind speed

For the design of the Nanpu Bridge, measured meteorological statistical data at the bridge site are not available. The annual extreme wind speed at the site required for the determination of the design wind speed of the bridge is obtained from the meteorological weather station located near the bridge site or the National Wind Pressure Distribution Map of China, whichever gives the higher value.

The annual extreme wind speed is determined as the maximum 10-minute mean wind speed measured by anemometers installed 10 meters above the ground surface at the site. In the wind-resistant design of the Nanpu Bridge, the return period of the extreme wind speed is taken as 100 years. Samples of the statistical data measured at the Longhua Meteorological Weather Station of the Shanghai Meteorological Bureau are shown in Table 4.4. Assuming that the probability density function of the extreme wind speed has a Type *I* maximum or Gumbel distribution (Gumbel, 1954), the annual extreme wind speed corresponding to a return period of 100 years at the bridge site is calculated to be 30.0 m/s. According to the National Wind Pressure Distribution Map, the fundamental wind pressure of Shanghai is 800 Pa, the magnitude of the extreme wind velocity is estimated to be 32.0 m/s. Selecting the higher of the two, the annual extreme wind speed for a 100-year return period at the site of the Nanpu Bridge is thus taken as 32.0 m/s.

Design wind speed

The design wind speed is determined using the following formula (Honshu-Shikoku Bridge Authority, 1976) with $U_{10} = 32.0m/s$.

$$U_D = \gamma_1 \cdot \gamma_2 \cdot U_{10} \quad (4.1)$$

where γ_1 — the height modification factor.

γ_2 — the horizontal length modification factor.

U_{10} — the annual extreme wind speed for a 100-year return period.

Considering $h = 50m$, $\gamma_1 = 1.26$, whereas for $L = 423m$, $\gamma_2 = 1.18$ (Honshu-Shikoku Bridge Authority, 1976). Therefore, the design wind speed U_D is calculated to be $47.6m/s$.

To account for the uncertainties in the various estimates of the design and other factors not considered here, the critical flutter wind velocity in design is required typically to be at least 1.2 times greater than the design wind speed over the range of the wind attack angle from -3° to $+3^\circ$ in order to ensure that the critical flutter phenomenon in the bridge can be avoided. Thus, in the case of the Nanpu Bridge, the critical flutter wind velocity of the structure should be greater than $57.1m/s$ for the self-excited vibration.

4.3.2 Critical Flutter Wind Velocity

According to the test results obtained by Xiang (1993) from the section model wind tunnel tests, the bridge deck cross-section manifests the most unfavorable aerodynamic stability property when the wind attack angle is $+3^\circ$. In the current study, this critical case is analyzed to validate the proposed method. The numerical

results are correlated with the wind tunnel test data. The flutter derivatives of the representative deck cross-section of the Nanpu Bridge at the wind attack angle of $+3^\circ$, as measured by the Wind Tunnel Testing Division of State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China, are shown in Figure 4.7. The flutter derivatives are plotted against the reduced wind velocity in Figure 4.7 with the wind velocity normalized by the response frequency and width of the bridge deck.

The *Flutter Analysis* module of the program *SFSFA* is employed to analyze the flutter behaviour of the Nanpu Bridge. The lowest symmetric and anti-symmetric heave and torsional modes are commonly found to be the contributing motions in the flutter response. They are the 2nd, 4th, 5th and 9th modes in the case of the Nanpu Bridge. For flutter analysis, these modes are selected to represent the wind response motions of the bridge. The structural damping ratios of the bridge are taken as 0.01 and 0.02 in two separate analyses. The critical flutter wind velocity results obtained for the two cases are presented in Table 4.5. The finite strip method results are compared with the experimental results obtained from prototype model wind tunnel tests (Xiang, 1993; Cheng, 1994), as well as with the finite element results by the State-Space method (Xiang, 1993; Cheng, 1994). The finite element model employed here has 501 nodes, 180 three-dimensional truss elements and 448 three dimensional beam elements.

Results given in Table 4.5 show that the critical flutter wind velocity of the Nanpu Bridge is higher than the critical design value of 57.1m/s for the self-excited vibration. Thus, the design of the Nanpu Bridge is considered safe in its resistance against flutter.

Comparing the results with the wind tunnel test data, the spline finite strip

model has an error of only a 1.5%. It is noted that the proposed method gives more accurate results than the finite element model, when compared to the wind tunnel test results. The results also show that the finite strip method can accurately model the stiffness property and behaviour of the deck, whereas the finite element model gives stiffer results as demonstrated by the higher estimate of the critical flutter velocity than the test data.

The variation of the vibration frequencies with the wind velocity for the heave and torsional modes are plotted in Figure 4.8. The results in the figure show that the frequencies of the torsional modes decrease with higher wind velocity, while the frequencies of the heave modes generally stay constant. The decrease in the frequencies of the torsional modes is due to the contributing effect of the aerodynamic stiffness, as given in Equation (3.38). The relationship of the logarithmic decreament with the wind velocity is presented in Figure 4.9. It is clearly shown in Figure 4.9 that when the wind velocity reaches $67.3m/s$, the logarithmic decrement of the lowest symmetric torsional mode changes from negative to positive. This wind velocity is defined as the critical flutter wind velocity, which represents the initiation of the flutter behaviour. For wind velocity higher than the critical flutter velocity, the effective damping becomes negative because the negative damping associated with the aerodynamic forces is greater than the mechanical damping of the structure. Thus, the bridge will vibrate in a divergent manner. As shown in Figure 4.8, the frequency of the lowest symmetric torsional mode decreases from about $0.5Hz$ to $0.46Hz$ at the critical flutter point and drops rapidly beyond that point. The frequency of the anti-symmetric torsional mode also decreases by 4.5% at the critical point. This is due to the contribution of the flutter derivative A_3^* , which represents the torsional aerodynamic stiffness, in softening the structure. The increase of A_3^* with higher

wind velocity, as shown in Figure 4.7, results in the increase of the aerodynamic stiffness given in Equation (3.38). Consequently, the effective stiffness of combining the structural stiffness and the aerodynamic stiffness in Equation (3.38) is reduced with a higher A_3^* .

The analytical results by the proposed spline finite strip method correlate well with the experimental wind tunnel test results, which indicate that the flutter of the Nanpu Bridge is caused by the divergence behaviour of the lowest symmetric torsional mode. The flutter behaviour is single-degree-of-freedom torsional flutter, as can be expected from the positive nature of the flutter derivative A_2^* .

4.4 Comparison and Discussion

The typical approach for bridge flutter analysis employs the three-dimensional equivalent beam finite elements to model the bridge structure. This approach assumes that the self-excited aerodynamic forces can be related to the motions at the centre of elasticity of the deck cross-section.

In order to make comparisons between the different analytical models, i.e. the 3D equivalent beam finite element model and the spline finite strip model, the free-vibration and flutter analysis of the Nanpu Bridge by the equivalent beam models are also presented.

4.4.1 Finite Element Models

Three types of widely adopted equivalent beam finite element model are illustrated in Figure 4.10.

In the single-spine model as shown in Figure 4.10(a), it is assumed that the mass (translational and rotational) and the stiffness (vertical and lateral bending rigidity and torsional rigidity) of the girder system are concentrated on the axis through the shear center of the deck cross-section. The stay cables are connected to the spine through rigid links or via master-slave relationships. If the rigid link connection is adopted, the total degrees-of-freedom as well as the number of elements in the numerical model may be increased considerably. The stiffness of the transverse beam as well as the warping torsional stiffness of the main girder cannot be properly represented by this model. Thus, the single-spine model is more suitable for the modelling of the single-cell or multi-cell box girder of which the girder cross-section has a relatively high free torsional stiffness.

The double-beam model shown in Figure 4.10(b) consists of two beams which are connected by the transverse beams. The cross-sectional area and the vertical bending stiffness of the two beams are taken as half of those of the original bridge girder, while the lateral bending stiffness is the equivalent stiffness calculated by assuming that the transverse deflection of the girder at the center span should be kept the same as that of the original girder system. The actual stiffness of the transverse beam is included in the model. The number of nodes and elements in this type of model is still relatively small compared to a full three-dimensional finite element model. The two separate beams in the model can approximately account some of the warping torsional stiffness effect. However, when the structure deforms in the transverse direction, this model behaves like a shear truss system, whereas the actual structural behaviour is basically of the flexural type. Thus, the lateral bending stiffness of the girder cannot be properly modelled. In the cable-stayed bridges, due to the existence of the inclined stay cables, the lateral and torsional

motions of the bridge girder are usually strongly coupled. The accuracy of the fundamental torsional frequency, which is an important factor in the wind stability analysis, can be significantly affected by the accuracy in the modelling of the stiffness in the transverse direction.

Compared to the double-beam model, the triple-beam model shown in Figure 4.10(c) can achieve better accuracy by appropriately allocating the mass and stiffness of the girder among the central and the two side beams. It is most suitable for modelling the open type bridge cross-section, which has relatively low free torsional stiffness. The contribution of the warping torsional stiffness is taken into account.

When modelling a bridge by the equivalent beam finite elements, careful considerations should be made in advance for the selection of the model type.

The Nanpu Bridge is separately modelled by using the single-spine model and the triple-beam model for comparison.

The single-spine model has 501 nodes, 180 three-dimensional truss elements and 262 three-dimensional beam elements; while the triple-beam model also has 501 nodes, 180 three-dimensional truss elements, but 448 three-dimensional beam elements. The boundary conditions are taken the same as those in the spline finite strip model.

The natural vibration frequency analysis and the flutter analysis are performed by using a three-dimensional finite element-based flutter analysis program *FEFA* (Cheng,1994), which has been successfully applied to evaluate the wind stability property of many long-span bridges.

The results of free-vibration analysis are given in Table 4.6. It can be seen that for the frequencies of floating, heave and sway modes, there are no significant

differences between the results obtained from the two different finite element models. But when considering the torsional modes, the frequencies obtained from the triple-beam model is 17.7% higher than that obtained from the single-spine model for the fundamental symmetric torsional mode, and 37.2% higher for the fundamental anti-symmetric torsional mode. This significant discrepancy is due to the way the structure is modelled. The deck cross-section of the Nanpu Bridge is a composite deck and has an open π -type configuration. For this type of cross-section, the free torsional stiffness is relatively small. The warping torsional stiffness should be included when calculating the torsional stiffness of the deck cross-section. Further, the two cable planes of the bridge are parallel to each other, which indicates that the torsional stiffness of the whole structure is mainly provided by the main girder. Studies (Fan,1996) have shown that in the case that the deck cross-section has relatively high free torsional stiffness, or the double cable planes are inclined which can thus provide higher torsional stiffness for the whole structure, the impact caused by the warping torsional stiffness is relatively small and can be neglected. For lower order modes, the difference is less than 2%, and for higher order modes less than 3%. Thus, the single-spine model can be used with no significant sacrifice in the accuracy of the results.

Since the lowest torsional mode is the most important mode in wind stability analysis of the bridges, typically for those having unstreamlined deck cross-section, such as the Nanpu Bridge, the single-spine model would give lower torsional frequency and thus lower critical flutter wind velocity U_f , which is conservative.

Again, the *PK-F* method is employed as the computational algorithm to solve the flutter problem. The flutter analysis results obtained from these two models are listed in Table 4.7.

It can be seen from Table 4.7 that for the Nanpu Bridge, the impact of the warping torsional stiffness can be considerable. If it is not accounted for, for the case of damping ratio $\xi = 0.01$, the critical wind velocity is 55.3m/s , which is lower than the critical design value of 57.1m/s . If the warping torsional stiffness is considered, the critical flutter wind velocity increases by 18.4% to 65.5m/s , and thus is higher than the requirement of the bridge design.

Results obtained from these two finite element models also show that the nature of the flutter for Nanpu Bridge is the single-degree-of-freedom torsional flutter. This conclusion is the same as that drawn from the analysis by the spline finite strip model and the wind tunnel tests.

4.4.2 Discussion

Based on the case study of the Nanpu Bridge, comparison has been made between the spline finite strip model and the 3D equivalent beam finite element models. The advantages and disadvantages of these different modelling schemes are discussed.

Structural Model

The first important step in using a finite element model is to determine which type of model, i.e. the single-spine, the double-beam or the triple-beam model, is appropriate for the modelling of the given bridge. In comparison, the spline finite strip model is simple as there is no need to select the model type.

A significant difference between the spline finite strip model and the equivalent beam model is in the way the bridge deck cross-section is modelled. To simulate the mass and stiffness properties of a complex bridge girder by an equivalent beam,

some assumptions and simplifications on the behaviour of the bridge response have to be made. An important advantage of the proposed spline finite strip method in flutter analysis as compared to the equivalent beam finite element model is that the torsional properties and behaviour of the bridge system can be modelled more accurately and more conveniently. In the finite strip approach, the bridge deck cross-section is divided into strips which can accurately represent different parts of the deck made of different materials, such as concrete deck and steel girder. More importantly, the modelling of the cross-section by finite strips accurately accounts for both the free and warping torsional stiffness. As the torsional behaviour is a significant factor in the wind stability of long-span cable-stayed bridges, the proposed finite strip method obviously has an advantage over the finite element model. As in the case of the Nanpu Bridge, the comparison of using the single-spine model and the triple-beam model has shown that accurate modelling of the warping torsion of the bridge deck cross-section can affect the results of the lowest symmetric torsional mode by 17.7%, and for the lowest anti-symmetric torsional mode by 37.2%. This consequently has a considerable impact on the flutter analysis results. In addition to the advantage of higher accuracy in modelling the torsional behaviour, the spline finite strip model requires less data preparation.

In the modelling of the complex deck cross-section, the finite strip method divides the deck cross-section into finite number of strips. Each strip can represent the geometry and material properties of the detailed cross-section. In comparison to the equivalent beam approach in which the bridge girder is modelled by a single beam element using appropriate parameters, the degree of modelling accuracy depends on the complexity of the deck cross-section. Although the finite strip method has a larger number of degrees-of-freedom than the equivalent beam approach, the finite

strip method has the flexibility and the capability to model the complex deck cross-section to a higher degree of accuracy as often required in detailed wind stability analysis of long-span flexible bridges.

Self-Excited Aerodynamic Force Model

The spline finite strip model has the obvious advantage over the equivalent beam model in that it can take into account the spatial distribution of the aerodynamic forces. With the future advances in the computational fluid mechanics when such a model is available to describe the surrounding air flow field of the bridge structure, this advantage in the formulation of the spline finite strip method can be fully utilized. The interaction between the structure and the wind can be accurately modelled as aerodynamic forces on the nodal lines of the finite strip method distributed throughout the deck cross-section. On the other hand, bridges of nowadays tend to be longer, more slender and lighter in weight. The adoption of the streamlined deck cross-section is common in the design of the long-span bridges due to its high performance in the aerodynamic stability. This type of deck cross-section can be conveniently modelled by the spline finite strip method with minimal data preparation efforts. Thus, the proposed finite strip model has the potential in its future application to the dynamic analysis of long-span bridges.

4.5 Summary and Conclusions

In this chapter, the case study of a real long-span cable-stayed bridge—the Nanpu Bridge, is presented. The dynamic characteristics and the flutter response of the bridge are analyzed by employing the proposed spline finite strip structural and

aerodynamic force model. The validity and reliability of the proposed models have been demonstrated. The advantages and disadvantages of the different numerical models and the potential of the proposed finite strip models are discussed. The following conclusions can be drawn from the results presented in the current study:

1. The critical flutter wind velocity of the Nanpu Bridge is higher than the checking wind velocity for the self-excited vibration. The bridge can safely resist the occurrence of flutter.
2. Due to the unstreamlined property of the bridge deck cross-section, the flutter behaviour of the Nanpu Bridge is single-degree-of-freedom torsional flutter. The occurrence of the flutter is caused by the divergent behaviour of the lowest symmetrical torsional mode.
3. The spline finite strip model can accurately and conveniently represent the mass and stiffness properties of the bridge girder, especially in the cases that the bridge girder has an open type or composite cross-section.
4. For modelling large structures, the 3D equivalent beam finite element models have fewer degrees-of-freedom than the spline model and require less computer time and storage. However, the mass and stiffness properties of the structure can be accurately represented by the spline finite strip model with minimal effort in data preparation.
5. The single-spine model cannot properly represent the stiffness of the transverse beam and the warping torsional stiffness of the main girder. In some cases where the warping torsion of the bridge girder should be considered, the adoption of this type of model yields conservative results.

6. The triple-beam finite element model can consider the warping torsion of the deck cross-section and is most suitable to be employed when the bridge girder has an open type deck cross-section.
7. The spline finite strip model has the obvious advantages over the equivalent beam model in that it can take into account the spatial distribution of the aerodynamic forces. This advantage can be fully utilized and demonstrated with the future advances in the computational fluid mechanics. The proposed finite strip model has the potential in its future application to the dynamic analysis of the long-span bridges.

Table 4.1: Geometric properties of Pylon shaft of Nanpu Bridge

Range	$A(m^2)$	$J_d(m^4)$	$I_1(m^4)$	$I_2(m^4)$
0 – 8m	20.1962	137.4668	187.0820	58.2095
8 – 16m	18.7621	121.3928	153.5258	52.7201
16 – 25m	17.4016	106.4310	124.8312	47.6040
25 – 33m	17.9624	101.5258	114.7114	45.6260
33 – 41m	16.6602	86.6762	91.6743	40.6021
> 41m	14.8300	79.0259	78.7961	38.1945
Upper Crossbeam	19.0666	218.6422	133.0895	164.8195
Lower Crossbeam	18.6200	201.9411	118.6263	158.8893

Table 4.2: Cross-Sectional Area of Cables of Nanpu Bridge

Type	Numbering Sequence	Cross-Sectional Area (cm^2)
1	0, S1, S8, S9, S21, S22, C1, C21, C22	102.0
2	S10	81.0
3	S2, S7, S13 – S20, C2, C7, C13 – C20	72.0
4	S6, S11, S12, C6, C11, C12	63.0
5	S5, C5, C8, C9, C10	58.0
6	S3, S4, C3, C4	49.0

Table 4.3: Natural Frequency of Nanpu Bridge (Spline Finite Strip Model)(Hz)

Mode No.	Frequency (Hz)	Mode Shape	
1	0.1294	Floating	Anti-Sym.
2	0.3309	Heave(girder)	Sym.
3	0.3786	Sway (girder)	Sym.
4	0.4030	Heave(girder)	Anti-Sym.
5	0.4995	Torsion(girder)	Sym.
6	0.5138	Bending(pylon)	Anti-Sym.
7	0.5579	Bending(pylon)	Sym.
8	0.5581	Heave(girder)	Sym.
9	0.6097	Torsion(girder)	Anti-Sym.
10	0.6268	Heave(girder)	Anti-Sym.

Table 4.4: (1956 – 1988) Annual Extreme Wind Speed in Shanghai (m/s)

No.	YY/MM/DD	Annual Extreme Wind Speed (m/s)	Wind Direction
1	1956.08.02	30.0	E
2	1957.06.05	14.7	WNW
3	1958.01.15	12.5	NW
4	1959.01.16	16.7	WNW
5	1960.06.10	15.7	NE
6	1961.10.04	20.0	NE
7	1962.08.06	15.3	ESE
8	1963.09.12	18.3	NE
9	1964.04.03	14.0	NW
10	1965.01.10	14.3	WNW
11	1966.03.15	15.7	WNW
12	1967.01.15	13.3	WNW
13	1968.01.14	13.3	NW
14	1969.08.11	12.5	SSE
15	1970.04.17	12.2	W
16	1971.07.27	12.3	SSE
17	1972.08.17	12.3	E

Table 4.4: (Continue) (1956 – 1988) Annual Extreme Wind Speed in Shanghai
(*m/s*)

No.	YY/MM/DD	Annual Extreme Wind Speed (<i>m/s</i>)	Wind Direction
18	1973.12.21	11.0	NW
19	1974.08.20	12.0	ENE
20	1975.08.12	14.0	E
21	1976.04.22	11.3	NNW
22	1977.09.11	14.0	S
23	1978.06.12	15.7	ENE
24	1979.08.24	13.7	ENE
25	1980.08.25	13.7	WNW
26	1981	12.7	
27	1982	12.3	
28	1983	12.3	
29	1984	11.3	
30	1985	11.3	
31	1986	12.3	
32	1987	10.7	
33	1988	10.0	

Table 4.5: Critical Flutter Wind Velocity of Nanpu Bridge (m/s)

Damping Ratio	Proposed Finite Strip	Finite Element	Wind Tunnel Tests
0.01	62.9	67.0	—
0.02	67.3	71.0	66.4

Table 4.6: Natural Frequency of Nanpu Bridge (Finite Element Model)(Hz)

Mode No.	Single-spine Model			Triple-beam Model		
	Freq.(Hz)	Mode Shape		Freq.(Hz)	Mode Shape	
1	0.1214	Floating	Anti-Sym.	0.1216	Floating	Anti-Sym.
2	0.3431	Heave(girder)	Sym.	0.3433	Heave(girder)	Sym.
3	0.3667	Sway (girder)	Sym.	0.3687	Sway (girder)	Sym.
4	0.4240	Heave(girder)	Anti-Sym.	0.4255	Heave(girder)	Anti-Sym.
5	0.4288	Torsion(girder)	Sym.	0.4992	Torsion(girder)	Sym.
6	0.4450	Torsion(girder)	Anti-Sym.	0.5069	Bending(pylon)	Anti-Sym.
7	0.5071	Bending(pylon)	Anti-Sym.	0.5392	Bending(pylon)	Sym.
8	0.5334	Bending(pylon)	Sym.	0.6106	Torsion(girder)	Anti-Sym.
9	0.5663	Torsion(girder) ^a		0.6337	Heave(girder)	Sym.
10	0.5663	Torsion(girder) ^a		0.7336	Heave(girder)	Anti-Sym.

^a Local torsional behaviour of the girder near the pylons where no cables are attached.

Table 4.7: Comparison of Critical Flutter Wind Velocity of Nanpu Bridge (m/s)

Mechanical Damping Ratio	Single-spine F.E. Model	Triple-beam F.E. Model
0.01	55.3	65.5
0.02	59.2	70.1

Figure 4.1: Structural layout of Nanpu Bridge(half)

Figure 4.2: Pylon of Nanpu Bridge

Figure 4.3: Deck cross-section of Nanpu Bridge

Figure 4.4: Spline finite strip model of Nanpu Bridge

Figure 4.5: Mesh of the deck cross-section

MODE NUMBER 1 FREQUENCY=0.129377E+00 HZ

Figure 4.6: (a) Mode shapes of Nanpu Bridge

MODE NUMBER 2 FREQUENCY= 0.330939E+00 HZ

Figure 4.6: (b) Mode shapes of Nanpu Bridge

MODE NUMBER 3 FREQUENCY= 0.378630E+00 HZ

Figure 4.6: (c) Mode shapes of Nanpu Bridge

MODE NUMBER 4 FREQUENCY=0.403042E+00 HZ

Figure 4.6: (d) Mode shapes of Nanpu Bridge

MODE NUMBER 5 FREQUENCY= 0.499476E+00 HZ

Figure 4.6: (e) Mode shapes of Nanpu Bridge

MODE NUMBER 6 FREQUENCY= 0.513829E+00 HZ

Figure 4.6: (f) Mode shapes of Nanpu Bridge

MODE NUMBER 7 FREQUENCY=0.557944E+00 HZ

Figure 4.6: (g) Mode shapes of Nanpu Bridge

MODE NUMBER 8 FREQUENCY= 0.558078E+00 HZ

Figure 4.6: (h) Mode shapes of Nanpu Bridge

MODE NUMBER 9 FREQUENCY= 0.609657E+00 HZ

Figure 4.6: (i) Mode shapes of Nanpu Bridge

MODE NUMBER 10 FREQUENCY= 0.626803E+00 HZ

Figure 4.6: (j) Mode shapes of Nanpu Bridge

Figure 4.7: Flutter derivatives of Nanpu Bridge

Figure 4.8: $f - U$ relationship of Nanpu Bridge

Figure 4.9: $\delta - U$ relationship of Nanpu Bridge

(a)

(b)

(c)

Figure 4.10: Finite element models for dynamic analysis of bridges (a) Single-spine model (b) Double-beam model (c) Triple-beam model

Chapter 5

Effects of Cable Vibration on the Dynamic Responses of Cable-stayed Bridges

5.1 Introduction

In recent years, the trend in bridge construction indicates that the design and construction of cable-stayed bridges are increasing rapidly all over the world, both in number and in the length of the main span. The dynamic behaviour of this type of long span, flexible structure induced by various dynamic loads, such as wind, earthquake and traffic, is three-dimensional and complex. The dynamic responses of a cable-stayed bridge under typical dynamic load conditions involve motions of the deck and the pylons, as well as that of the stay cables. With the development of

computer technology and the finite element method, the computer-aided simulation of numerical model has become a major tool which the engineers can now use to predict the responses of the complex bridge structures. The accuracy and reliability of the analysis results from the numerical model on the bridge behaviour depend on many factors, among which the accurate modelling of the structural properties and behaviour of the bridge deck, the pylons and the stay cables are particularly important.

The stay cables, which support the bridge deck and transmit the dead and live loads carried by the deck to the pylons, play an important role in the dynamic behaviour of the cable-stayed bridges. Under the conditions of heavy traffic load or unfavorable weather conditions with wind accompanied by rain, the vibration amplitude of the stay cables becomes large. The significance of the rain-wind vibration problem of the stay cables was first reported in Japan (Hikami, 1986), when the Meiko-Nishi cable-stayed bridge was under construction. During the unfavorable weather conditions of wind accompanied with rain approximately in the longitudinal direction of the bridge axis, the vibration of the stay cables was observed with an amplitude of approximately 25cm.

In the last several decades, the dynamic behaviour of cable-stayed bridges due to wind, earthquakes and traffic loads have attracted increasing attention by civil engineers. In most of the previous analytical studies, the stay cables are modeled by truss elements. The sagging effect due to the self-weight of the cable is taken into account by using an equivalent Young's modulus. This approach, of which each individual stay cable is represented by only one truss element, is a common practice in the analysis of both static and dynamic problems of cable-stayed bridges (Fleming and Egeseli, 1980; Parvez and Wieland, 1987; Wethyavivorn, 1987; Abdel-Ghaffar

and Nazmy, 1991; Nazmy and Abdel-Ghaffar, 1992). For the static or quasi-static analysis, previous researches have shown that the single-truss element model can give accurate results. However, in the case of the dynamic responses, the truss model of the stay cable cannot account for the lateral vibration motions of the cable. The mode shapes of the structure therefore include only the vibrations of the deck and the pylons. The interaction between the vibration behaviour of the cables and the other parts of the bridge superstructure, as well as the vibration of the stay cable itself, are thus ignored.

In most cable-stayed bridges, the fundamental frequencies of the high strength steel cables lie in the frequency range of the first several basic modes of the bridge structure. Thus, under certain dynamic load conditions, the vibration of the stay cables and the interactions between the vibration behaviour of the cables and that of the deck and the pylons can become significant. Some research has been carried out to improve the modelling scheme of the stay cables. Causevic and Screckovic (1987) suggested using linear springs and lumped masses to represent the stay cables in the analysis model. A more convenient approach of using the multi-link model, which discretizes each stay cable into several elements, has been adopted by several researches in analyzing the dynamic behaviour of the cable-stayed bridges due to wind and seismic loads (Baron and Lien, 1973; Yiu, 1982; Tuladhar and Brotton, 1987). However, results obtained from these studies have not been compared in a systematic way with those obtained from the single-truss element approach. In some recent studies, comparisons have been made between the results obtained from the single-truss model and the multi-link model. However, the impact of cable vibration effect on the dynamic responses of the cable-stayed bridges is still not clear.

By discretization of the stay cable into multiple elements in the analysis model,

additional vibration modes involving primarily the vibration motions of the stay cables and new cable-superstructure coupled modes are identified. In the seismic analysis, it has been found that the vibration behaviour of the cables significantly influences the participation factors in the earthquake response (Abdel-Ghaffar and Khalifa, 1991; Tuladhar *et al*, 1995). In the buffeting analysis of the Sunshine Skyway Bridge (Jones *et al*, 1997), comparisons of the results indicate that the single-truss element approach overestimates the buffeting response in the side spans of the bridge. The importance of including the effects of the stay cable vibration in the dynamic response analysis of cable-stayed bridges has been demonstrated and emphasized by a number of researchers. In recent years, the problem of the cable vibration behaviour has gained increasing attention in the field of bridge aerodynamics (Hikami and Shiraishi, 1983; Yamaguchi, 1990; Knisely, 1990; Wardlaw, 1991; Miyata, 1991; Matsumoto *et al*, 1992; Zasso, 1992; Dielen, 1993; Kusakabe *et al*, 1995; Kobayashi *et al*, 1995; Matsumoto *et al*, 1995; Bossdogianni and Olivari, 1996; Jones *et al*, 1997). However, the majority of the research on cable vibration focus on the mechanisms and countermeasures of the cable motion itself, e.g. the wind-induced cable vibration, the rain-induced cable vibration and the combined wind-rain-induced cable vibration. The effect of the cable vibration on the aerodynamic response of the entire bridge structure has rarely been studied.

Flutter is a self-excited aerodynamic instability phenomenon. Under a certain critical flutter wind velocity, the divergent behaviour of the bridge may cause the failure of the structure. The aerodynamic stability property has become one of the governing criteria in the design of modern long-span cable-supported bridges. In the common practice of flutter analysis of long-span cable-stayed bridges, the stay cable is usually modelled by the single-truss element. With the trend of increasingly

longer span cable-stayed bridges, the length of the stay cables becomes longer. The vibration of the cables and their interaction with the motion of the bridge superstructure become more significant, and thus can influence the dynamic response of the bridge. In view of the sensitivity of long-span cable-stayed bridges to aerodynamic instability, it is therefore essential to consider the effects and impact of the cable vibration on the flutter response behaviour of cable-stayed bridges.

Although the multi-link approach has been proven to be accurate in the analysis of modern multi-stay cable-stayed bridges, the discretization of each individual stay cable into several elements significantly increases the degrees-of-freedom of the analytical model. In this chapter, a new and more convenient cable model of using three-node cable elements is developed. It can accurately model the vibrational characteristics of the stay cables with fewer degrees-of-freedom than the multi-link model. The interaction behaviour between the cables, the pylons and the bridge deck are considered by introducing the dynamic stiffness effect of the cable. The accuracy and reliability of this new cable model are demonstrated by a numerical example. In the case study of the Nanpu Bridge, the cable vibration effects on the dynamic characteristics and the flutter response behaviour of the Nanpu Bridge are analyzed. To further study and demonstrate the effect of the cable vibration on the dynamic behaviour of the cable-stayed bridges, eight model bridges of medium to long span length and different cable arrangements, i.e. the parallel cable-plane type and the inclined cable-plane type, have been analyzed. A comprehensive parametric study has been carried out to quantify the impact of cable vibration behaviour on the dynamic characteristics and the flutter response of the cable-stayed bridges having different span length, different cable arrangement type and different aerodynamic property of the deck cross-section. The aerodynamic loads acting on the stay

cables are included in the flutter analysis. The effect of the aerodynamic loads on the stay cables on the flutter response of the bridge is also discussed.

By careful consideration in the derivation of the numerical model and accurately accounting for the mass and stiffness properties in the computer model of the bridge, the equivalent beam finite element method can give accurate result for most bridges with minimum requirement on the computer storage and time. In this chapter, the study of the effect of the cable vibration on the dynamic behaviour of long-span cable-stayed bridges is carried out, the triple-beam finite element method is employed to model the behaviour of the bridge deck superstructure.

5.2 Structural Model of the Stay Cable

5.2.1 Displacement Function

5.2.1.1 Transverse Motion of the Stay Cable

Transverse vibration of a string fixed at both ends

A string with length l is shown in Figure 5.1(a). The X-axis coincides with the longitudinal axis of the string, whereas the Y-axis is perpendicular to the string. The string is fixed at both ends. The mass per unit length of the string is μ . With small amplitude vibration in the transverse direction, it is assumed that the tension S in the string remains constant when the string vibrates.

Considering an arbitrary infinitesimal small segment of the string shown in Figure 5.1(b), the equilibrium at time t is maintained by the transverse component of

the tension S and the inertial force of the string which can be expressed as follows

$$S \left(\frac{\partial y}{\partial x} + \frac{\partial y^2}{\partial x^2} dx \right) - S \frac{\partial y}{\partial x} - \mu dx \frac{\partial y^2}{\partial t^2} = 0 \quad (5.1)$$

or

$$S \frac{\partial y^2}{\partial x^2} - \mu \frac{\partial y^2}{\partial t^2} = 0 \quad (5.2)$$

Denoting $C = \sqrt{S/\mu}$ and substituting it into Equation (5.2) yields

$$\frac{\partial y^2}{\partial t^2} - C^2 \frac{\partial y^2}{\partial x^2} = 0 \quad (5.3)$$

Equation (5.3) is similar in form to the equation of wave propagation, where C is equivalent to the wave propagation velocity $\lambda = \sqrt{E/\rho}$, with E and ρ being respectively the elastic modulus and the mass density of the propagation medium.

By separating the variables in the linear equation, and assuming the vibration is a harmonic motion, the solution of Equation (5.3) can be expressed in the form as follows

$$y(x, t) = Y(x)(A \sin \omega t + B \cos \omega t) \quad (5.4)$$

where ω is the circular frequency.

By substituting Equation (5.4) into Equation (5.3), and introducing the boundary conditions

$$Y(0) = 0 \quad (5.5a)$$

$$Y(l) = 0 \quad (5.5b)$$

the transverse vibration frequency for a horizontal string fixed at both ends can be obtained as follows

$$\omega_n = \frac{Cn\pi}{l} = \frac{n\pi}{l} \sqrt{S/\mu} \quad (n = 1, 2, 3, \dots) \quad (5.6a)$$

or

$$f_n = \frac{n}{2l} \sqrt{S/\mu} \quad (5.6b)$$

and the corresponding mode shape is

$$Y_n(x) = \frac{\omega l}{n\pi} \sin \frac{n\pi x}{l} \quad (5.7)$$

Transverse displacement function of the stay cable

The transverse displacements at any arbitrary point on a stay cable can be decomposed into two parts. The first part is the linear displacements related to the nodal displacements at the two ends of the cable connected to the girder and the pylon. The second part can be treated as displacements due to the transverse vibration of a cable fixed at both ends. Since the mode shapes of the transverse vibration of a fixed-fixed cable are the sine functions given by Equation (5.7), the second part of the displacements can be expressed as the summation of the sine series using the mode superposition method. Thus, the displacements in the transverse direction at an arbitrary point on a stay cable, as shown in Figure 5.2, can be expressed in the form as follows

$$\bar{v}(\bar{x}) = \frac{\bar{x}}{L} \bar{v}_i + \left(1 - \frac{\bar{x}}{L}\right) \bar{v}_j + \sum_n a_{\bar{v}_n} \sin \frac{n\pi \bar{x}}{L} \quad (5.8)$$

$$\bar{w}(\bar{x}) = \frac{\bar{x}}{L} \bar{w}_i + \left(1 - \frac{\bar{x}}{L}\right) \bar{w}_j + \sum_n a_{\bar{z}_n} \sin \frac{n\pi \bar{x}}{L} \quad (5.9)$$

where L — the length of the cable;

$\bar{v}, \bar{w}$ — displacements along the $\bar{Y}, \bar{Z}$ axes in the local coordinate system;

$\bar{v}_i, \bar{w}_i, \bar{v}_j, \bar{w}_j$ — nodal displacements of nodes i and j along the $\bar{Y}, \bar{Z}$ axes in the local coordinate system;

$a_{\bar{y}_n}, a_{\bar{z}_n}$ — generalized coordinates corresponding to the n^{th} sine shape function terms.

The number of terms to be taken in the harmonic summation of Equations (5.8) and (5.9) is determined by the accuracy requirement of the analysis.

5.2.1.2 Longitudinal Motion of the Stay Cable

Static cable stiffness

The extensional behaviour of the stay cable is another important component of its dynamic motion. A typical inclined stay cable with the slight sagging due to its self-weight is shown in Figure 5.3. In practice, the nonlinear static stretching stiffness K_c of the cable is composed of the elastic stiffness K_e due to the elastic stretch of the cable, and the geometrical stiffness K_g due to the change in the cable geometry. It has the form as follows

$$\frac{1}{K_c} = \frac{1}{K_e} + \frac{1}{K_g} \quad (5.10)$$

or

$$K_c = \frac{K_e K_g}{K_e + K_g} \quad (5.11)$$

where K_c — the nonlinear static stretching stiffness;

K_e — the elastic stiffness $K_e = \frac{EA}{L}$;

K_g — the geometrical stiffness.

E — elastic Young's modulus;

A — cross-sectional area of the cable;

L — the chord length of the cable;

L_h — the length of the horizontal projection of the cable;

T — the mean tension in the cable;

W — the weight per unit length.

If the sag of the cable is assumed to have a parabolic shape, the geometric stiffness

$$K_g = \frac{12T^3}{W^2L_h^3}.$$

When the bridge is subjected to dynamic loads, the bridge superstructure vibrates both in and out of the plane of the structure. The stay cables resist the movements of the deck and the pylon at the two end supports. The dynamic behaviour of the stay cable is affected not only by the vibration of the cable itself, but also by the motions of the deck and the pylon. The dynamic interaction between the stay cable and the other parts of the superstructure should be considered in the dynamic analysis of the cable-stayed bridge. Further, the longitudinal and the transverse motions in the plane of the stay cable are dynamically coupled. If the tension of the cable changes, the sag in the cable will change which in turn will affect the transverse acceleration and the inertia load in the cable. The geometric nonlinearity of the stay cable induced by the dynamic coupling between the motions

in the transverse and the longitudinal direction is considered in the model by the dynamic cable stiffness.

Dynamic cable stiffness

The dynamic extensional cable stiffness at frequency ω of the structure has been derived by Davenport (1994), which has the form as follows

$$K(\omega) = \frac{K_e \left[1 - \left(\frac{\omega}{\omega_0} \right)^2 \right]}{1 + \left(\frac{K_e}{K_g} \right) - \left(\frac{\omega}{\omega_0} \right)^2} \quad (5.12)$$

where $\omega_0 = \frac{\pi}{L} \sqrt{\frac{T_g}{W}}$ is the fundamental frequency of the “taut wire”.

The variation of the dynamic cable stiffness is plotted in Figure 5.4. It is clearly shown in the figure that when the response frequency of the structure is low ($\omega \rightarrow 0$), the dynamic extensional stiffness of the cable approaches the nonlinear static stretching stiffness K_e . When the structural frequency is high ($\omega \gg \omega_0$), the cable stiffness approaches the elastic stiffness K_e . Resonance occurs when the transverse and the extensional motions of the cable are coupled. At that time, the structural frequency ω satisfies the following condition

$$\omega = \omega_1 = \omega_0 \sqrt{1 + \frac{K_e}{K_g}} \quad (5.13)$$

The shaded area shown in Figure 5.4 between the frequencies ω_0 and ω_1 represents the region of the unstable behaviour of the stay cable, i.e. the effective stiffness of the cable is negative within the frequency range of the shaded region. The significance of the negative stiffness is that instead of resisting the elongation of the cable, it tends to increase the deformation. Studies on the massive guy cables (Davenport and

Steels, 1965) have shown that the dynamic stiffness effect had a significant impact on the dynamic behaviour of guy cable structures. At certain frequencies when the effective stiffness becomes negative, a dangerous situation occurs that instead of applying a restoring force to the mast, the guy cable tends to pull it over. In the case of the cable-stayed bridges, this phenomenon has not been studied in the evaluation of the influence of the dynamic stiffness of the stay cables on the behaviour of the cable-stayed bridges. It is of practical importance to know how significant is the effect of the dynamic stiffness as compared to the static stiffness, and the possibility of when the effective stiffness of the cable becomes negative.

In order to more accurately account for the cable behaviour in the dynamic response of the cable-stayed bridges, the formulation of the dynamic cable stiffness is included in the cable model employed in the current study. The impact of the dynamic stiffness is investigated in detail in the case study of the Nanpu Bridge.

Longitudinal displacement function of the stay cable

A spring-mass-spring system as shown in Figure 5.5(a) is adopted in the dynamic modelling of the guyed cables by Davenport (Davenport and Steels, 1965; Davenport, 1986). The end connection of the guy on the ground is assumed to be anchored. In the vibration of the cable-stayed bridge, both ends of the stay cable move in response to the motions of the deck and the pylon. Considering the relative displacement at the two end supports of the cable with the displacement at the cable-pylon connection taken as the reference base, a similar spring-mass-spring model is developed for the derivation of the dynamic cable stiffness. Appropriate constraints are applied to the motion of the lumped mass of the cable when consid-

ering the longitudinal motion of the stay cable. This model considers the dynamic interaction between the motion of the stay cable and that of the other parts of the bridge superstructure.

The finite element model representing the dynamic behaviour of the stay cable in the longitudinal direction is shown in Figure 5.5(b). For convenience in the derivation of the longitudinal displacement shape function of the cable, the cable mass which participates in the translational motion in the longitudinal direction is assumed to be lumped at the center point m of the cable model. The stiffness of the springs at the two sides of the mass are K_g and K_e , respectively. The absolute longitudinal displacements at points i , m and j are respectively $\bar{u}_i$, $\bar{u}_m$ and $\bar{u}_j$. From the assumption that point j , which represents the cable support point at the pylon, is anchored, the relative displacements of point m and i with respect to the point j , which are $(\bar{u}_m - \bar{u}_j)$ and $(\bar{u}_i - \bar{u}_j)$ respectively, are used in the derivation. The in-plane displacement $\bar{u}(\bar{x})$ of an arbitrary point $\bar{x}$ on the cable relative to the displacement of the cable support point j at the pylon can be expressed as follows

$$\bar{u}(\bar{x}) = \begin{cases} \frac{2\bar{x}}{L}(\bar{u}_m - \bar{u}_j), & 0 \leq \bar{x} \leq \frac{L}{2} \\ \frac{2(L-\bar{x})}{L}(\bar{u}_m - \bar{u}_j) + \frac{2\bar{x}-L}{L}(\bar{u}_i - \bar{u}_j), & \frac{L}{2} < \bar{x} \leq L \end{cases} \quad (5.14)$$

5.2.2 Stiffness Matrix of the Stay Cable Element

Under the dynamic loads, the motion of the stay cable involves the translational movement in the transverse direction and extension of the cable in the longitudinal direction. For the derivation here, the first three terms in the harmonic summation in Equations (5.8) and (5.9) are taken to express the transverse displacements $\bar{v}$ and $\bar{w}$ along the $\bar{Y}$ and $\bar{Z}$ axes. By referring to Figure 5.6 and Section 5.2.1, the displacements at an arbitrary point within the cable element can be expressed in

the matrix form as follows

$$\{\bar{\delta}\} = \begin{Bmatrix} [N_u] \\ [N_v] \\ [N_w] \end{Bmatrix} \cdot \{\bar{\delta}\}^e \quad (5.15)$$

where $\{\bar{\delta}\}$ is the displacement vector,

$$\{\bar{\delta}\} = \begin{bmatrix} \bar{u} & \bar{v} & \bar{w} \end{bmatrix}^T \quad (5.16)$$

$\{\bar{\delta}\}^e$ is the displacement parameter vector,

$$\{\bar{\delta}\}^e = \begin{bmatrix} \bar{u}_i & \bar{v}_i & \bar{w}_i & \bar{u}_j & \bar{v}_j & \bar{w}_j & \bar{u}_m & a_{y_1} & a_{x_1} & a_{y_2} & a_{x_2} & a_{y_3} & a_{x_3} \end{bmatrix}^T \quad (5.17)$$

$[N_u], [N_v], [N_w]$ are the shape function matrices for displacements $\bar{u}, \bar{v}, \bar{w}$, respectively

$$[N_u] = \begin{cases} \begin{bmatrix} 0 & 0 & 0 & -\frac{2x}{L} & 0 & 0 & \frac{2x}{L} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, & 0 \leq \bar{x} \leq \frac{L}{2} \\ \begin{bmatrix} \frac{2x-L}{L} & 0 & 0 & -1 & 0 & 0 & \frac{2(L-x)}{L} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, & \frac{L}{2} < \bar{x} \leq L \end{cases} \quad (5.18)$$

$$[N_v] = \begin{bmatrix} 0 & (1 - \frac{x}{L}) & 0 & 0 & \frac{x}{L} & 0 & 0 & \sin \frac{\pi x}{L} & 0 & \sin \frac{2\pi x}{L} & 0 & \sin \frac{3\pi x}{L} & 0 \end{bmatrix} \quad (5.19)$$

$$[N_w] = \begin{bmatrix} 0 & 0 & (1 - \frac{x}{L}) & 0 & 0 & \frac{x}{L} & 0 & 0 & \sin \frac{\pi x}{L} & 0 & \sin \frac{2\pi x}{L} & 0 & \sin \frac{3\pi x}{L} \end{bmatrix} \quad (5.20)$$

Under small amplitude vibration, it is assumed that the tension T in the cable remains constant. The strain energy of the cable due to elongation and change in geometry can be expressed as follows

$$U = \frac{1}{2} \int_0^{\frac{L}{2}} E_1 A \left(\frac{d\bar{u}}{d\bar{x}} \right)^2 d\bar{x} + \frac{1}{2} \int_{\frac{L}{2}}^L E_2 A \left(\frac{d\bar{u}}{d\bar{x}} \right)^2 d\bar{x} + T \cdot \frac{1}{2} \int_0^L \left[\frac{1}{2} \left(\frac{d\bar{v}}{d\bar{x}} \right)^2 + \frac{1}{2} \left(\frac{d\bar{w}}{d\bar{x}} \right)^2 \right] d\bar{x} \quad (5.21)$$

By substituting Equations (5.15) and (5.16), Equation (5.21) can be simplified as follows

$$U = \frac{1}{2} \{\bar{\delta}\}^e T [\bar{K}]_e \{\bar{\delta}\}^e \quad (5.22)$$

where $[\bar{K}]_e$ is the stiffness matrix of the stay cable element in the local coordinate system of the size 13×13 .

The non-zero terms in the lower-triangular part of the cable element stiffness matrix are given as follows

$$\begin{aligned} K_{11} &= -K_{71} = K_e \\ K_{22} &= K_{33} = K_{55} = K_{66} = -K_{52} = -K_{63} = T/L \\ K_{44} &= -K_{74} = K_g \\ K_{77} &= K_e + K_g \\ K_{88} &= K_{99} = \frac{\pi^2}{2L} \cdot T \\ K_{10,10} &= K_{11,11} = \frac{2\pi^2}{L} \cdot T \\ K_{12,12} &= K_{13,13} = \frac{9\pi^2}{2L} \cdot T \end{aligned} \quad (5.23)$$

where K_e is the elastic stiffness, $K_e = \frac{EA}{L}$; K_g is the geometric stiffness, $k_g = \frac{12T^3}{W^2 L^3}$.

5.2.3 Mass Matrix of the Stay Cable Element

The dynamic behaviour of the stay cable involves the motions of the cable in both the transverse and the longitudinal directions. For the modelling of the longitudinal motion as discussed earlier in section 5.2.1, the translational mass related to the extensional behaviour of the cable is assumed to be lumped at the center point m .

The mass related to the transverse vibration are determined from the kinetic energy of the transverse motion, which can be expressed as follows

$$T = \frac{1}{2} \int_0^L (\rho AL)(\dot{v}^2 + \dot{w}^2) d\bar{x} \quad (5.24)$$

where ρ is the mass density. Substituting Equations (5.15) and (5.16) into Equation (5.24) gives

$$T = \frac{1}{2} \left\{ \dot{\delta} \right\}^e{}^T [\bar{M}]_c^t \left\{ \dot{\delta} \right\}^e \quad (5.25)$$

$$[\bar{M}]_c^t = \rho AL \int_0^L \left([N_v]^T [N_v] + [N_w]^T [N_w] \right) d\bar{x} \quad (5.26)$$

where $[\bar{M}]_c^t$ is the local mass matrix of the stay cable element related to the transverse motion.

The total mass matrix $[\bar{M}]_c$ of the cable element can be obtained by combining

the contributions to the mass effect from all the movement components as follows

$$[\bar{M}]_c = \rho AL \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{\pi} & 0 & -\frac{1}{2\pi} & 0 & \frac{1}{3\pi} & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{\pi} & 0 & -\frac{1}{2\pi} & 0 & \frac{1}{3\pi} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{\pi} & 0 & -\frac{1}{2\pi} & 0 & \frac{1}{3\pi} & 0 \\ 0 & 0 & \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & 0 & \frac{1}{\pi} & 0 & -\frac{1}{2\pi} & 0 & \frac{1}{3\pi} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\pi} & 0 & 0 & \frac{1}{\pi} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\pi} & 0 & 0 & \frac{1}{\pi} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2\pi} & 0 & 0 & -\frac{1}{2\pi} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2\pi} & 0 & 0 & -\frac{1}{2\pi} & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3\pi} & 0 & 0 & \frac{1}{3\pi} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3\pi} & 0 & 0 & \frac{1}{3\pi} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \quad (5.27)$$

5.2.4 Numerical Example

Introduction

To verify the validity and reliability of the proposed three-node cable model, the numerical example of a three-span cable-stayed bridge is presented in this section. Modal analysis of the bridge is carried out. This example bridge has been used by Tuladhar *et al* (1995) to study the effect of cable vibration on the modal analysis

results and seismic response of the cable-stayed bridge. In the previous study, the stay cables are modeled by the multi-link elements. In the current study, slight modifications on the bridge layout has been made due to the lack of detailed structural information. The variation in the bridge deck elevation has been neglected. The dead load of the bridge is calculated based on the unit weight of the bridge deck provided in the reference.

In the present research, the emphasis is placed on the comparison of the different modelling schemes of the transverse motion of the cable, and the influence of the cable vibration effect on the modal analysis results. The dynamic cable stiffness effect is not considered here in the current numerical example.

The geometry and details of the mesh of the example bridge are shown in Figure 5.7. The bridge has a symmetric layout with a main span of $371m$ and two side spans of $159m$ each. The bridge deck is steel, while the pylons are concrete. For the mesh shown in Figure 5.7(b), the cables are modelled by the proposed three-node cable elements. The properties of the bridge deck, pylon and cables are presented in Table 5.1 and Table 5.2.

Three different analytical models are employed in the comparison study. The bridge deck and pylons in all three models are modelled by three-dimensional beam elements. The stay cables are respectively modelled by the proposed three-node cable elements, the multi-link elements and the single-truss elements in separate analytical models. The tension forces in the cables are obtained from the dead load condition. The motion of the bridge is assumed to be restricted in the plane of the structure, i.e. only three degrees-of-freedom (two translations and one rotation) are considered at each mode of the numerical model of the bridge.

The modal analysis of the example bridge by the single-truss cable model and

the multi-link model are carried out by using the computer program *FEFA* (Cheng, 1994); while that by the three-node cable model is analyzed by the computer program *3NFEFA* developed in the current study.

Modal analysis results and comparison

Comparison of the modal analysis results of the example bridge obtained from the proposed three-node cable element model and the single-truss cable model are presented in Table 5.3. The first twenty natural modes of the single-truss model and the corresponding modes in the three-node model are given in the table. It has been found that concerning the basic modes of the bridge itself, results obtained from the two different models are close, with the discrepancy attributed to the cable vibration effect. The transverse vibrational behaviour of the stay cables can be accurately predicted by the proposed three-node cable model. The pure cable vibration modes are identified. These local cable vibration modes show up as additional vibration modes independent of the structural modes with frequencies between those of the natural modes of the bridge structure itself.

A post-processing program *DRAW-3N* has been developed to display in three-dimensions the modal analysis results from the program *3NFEFA*.

Shown in Figure 5.8 are the mode shapes of some typical corresponding modes obtained from the three-node cable model and the single-truss cable model. The impact on the mode shapes due to the vibrational behaviour of the cables are clearly demonstrated. The mode shapes obtained from the three-node cable model include the motions not only of the deck and the pylons, but also of the stay cables as well.

In the case of the single-truss cable model, only the motions of the deck and the pylons are considered. The cables in the mode shapes by the single-truss model remain straight as the vibrational behaviour of the cables are ignored. Figure 5.9 shows the mode shapes of some pure cable vibration modes obtained by the three-node cable model. In these modes, only the vibration of the cables are developed, without any significant movement or deformation in the other parts of the bridge superstructure.

The modal frequencies obtained from the proposed three-node cable model and the single-truss cable model are plotted against the modal order in Figure 5.10. The flat regions on the curve of the three-node cable model result indicate the pure cable modes. They show up between the basic modes of the bridge itself.

The natural frequencies and the associated mode shapes of the transverse motion of a string fixed at both ends have been derived in Section 5.2.1. The frequency of the n^{th} mode, as given by Equation (5.6), is $f_n = \frac{n}{2l} \sqrt{\frac{S}{\mu}}$ ($n = 1, 2, 3, \dots$). Although in the case of the cable-stayed bridge, the two ends of the stay cable may move with the deck and the pylon(s) under dynamic loads, Equation (5.6) can still be used with reasonable accuracy as a reference for the estimates of the transverse motion frequencies of the stay cables. The first three modal frequencies of the stay cables in the example bridge, obtained from both the analytical solutions expressed by Equation (5.6) and the modal analysis results using the three-node cable element, are presented in Table 5.4. Good agreement is observed for the first three natural frequencies of all the cables.

The frequency range of the first ten modes of the example bridge without consideration of the vibration of the cables and the fundamental frequencies of the longest and shortest cables are presented in Table 5.5. It is observed that in the case of

the example bridge, which has a steel deck, the fundamental frequencies of the stay cables between $0.4844Hz$ to $1.1913Hz$ are within the range of the first ten modal frequencies of the bridge, which is between $0.3493Hz$ and $1.9673Hz$. This overlap in the frequency range illustrates the importance of the cable vibration behaviour in the dynamic analysis of the example cable-stayed bridge. Depending on the external load conditions, the impact of cable vibration effects on the bridge response can be significant. Comparisons between the cable frequencies and the first few frequencies of the bridge itself with respect to the concrete deck, the steel deck and the composite deck have been made by Tuladhar *et al* (1995). It has been found that for the steel or composite deck bridges, the cable frequencies generally overlap the first few frequencies of the bridge itself, whereas for the concrete deck bridges, the cable frequencies are typically higher and are less likely to fall in the range of the first few bridge frequencies. The higher cable frequencies are due to greater tension in the stay cables, which results from the heavier superstructure of the concrete deck, as compared to the steel or the composite deck.

The results given in Table 5.3 and Table 5.4 verify the validity and accuracy of the proposed three-node cable model in modelling the transverse vibrational behaviour of the stay cables, and demonstrate the influence of the cable vibration effect on the modal properties of the cable-stayed bridge. To further demonstrate the efficiency of the proposed model, the example bridge is also analyzed by using the multi-link elements to represent the cables. Each cable is divided respectively into two links, four links and eight links in three separate analyses. The fundamental cable frequencies obtained from the analytical expression given by Equation (5.6), the three-node model and the multi-link models are presented in Table 5.6. Results show that for the example here, each cable should be divided into at least eight

links to achieve accurate results. The cable vibration can be modelled accurately by the eight-link model. The first twenty-two natural frequencies obtained from the three-node model and the eight-link model, including the bridge frequencies and all the cable fundamental frequencies, are presented in Table 5.7. The agreement between these two sets of results is excellent. The total number of degrees-of-freedom of the three-node cable model is 171, whereas that of the eight-link model is 387. For the modern multi-stay cable-stayed bridges, there can be more than a hundred stay cables in one bridge. If the multi-link element is employed to model the stay cables, the number of degrees-of-freedom of the analytical model is therefore significantly higher than that of the proposed cable model. It not only requires more effort in data preparation, but also increases drastically the cost in computer time and storage.

Conclusion

In this section, the modal analysis of a three-span steel deck cable-stayed bridge is presented. The stay cables in the bridge are modeled separately by the proposed three-node cable element, the single-truss element and the multi-link element. It is found that the fundamental cable frequencies fall in the range of the first few frequencies of the bridge itself. Numerous pure cable vibration modes have been identified by using either the three-node cable model or the multi-link model. However, the effects of these modes in the dynamic response are ignored in the single-truss model. Considering the effects of the cable vibration behaviour, the modal frequencies and mode shapes of the bridge are affected due to the interaction between the cable

motions and that of the deck and the pylons. The proposed three-node cable element model can accurately describe the transverse motion of the stay cables, and is computationally more efficient than the multi-link model.

5.3 Model of Aerodynamic Forces Acting on the Stay Cable

5.3.1 Wind Load on the Stay Cable

The wind exerts both static and dynamic loads on the stay cables. These wind effects can be mainly classified as follows

- (i) The static drag force. For certain types of cables which has unsymmetric cross-section, the static lift force is also induced. The drag and lift force coefficients vary with the Renolads Number.
- (ii) The random gust force which induces oscillations with random amplitude.
- (iii) The wind load associated with the vortex excitation, which is due to the vortex shedding phenomenon.
- (iv) The wind load associated with the galloping excitation, which is due to the instability caused by lift and drag forces.

All these wind-induced forces are primarily associated with the drag force. The gust force, the vortex excitation and the galloping excitation are not considered in the current study.

Assuming that the wind blows normal to the stay cable with a mean velocity U , as shown in Figure 5.11(a), the cable oscillates in the cross-wind direction Y and the along-wind direction Z with velocities $\dot{y}$ and $\dot{z}$, respectively ($|\dot{y}| \ll U, |\dot{z}| \ll U$). The effective wind attack angle α_e , as shown in Figure 5.11(b), is expressed as follows

$$\alpha_e = \frac{-\dot{y}}{U - \dot{z}} \quad (5.28)$$

and the relative wind velocity U_r is

$$U_r = \sqrt{(U - \dot{z})^2 + (-\dot{y})^2} \quad (5.29)$$

The static wind forces acting on the stationary cable, as shown in Figure 5.12(a), are

$$F_{L, st} = \frac{1}{2} \rho U^2 d C_L \quad (5.30)$$

$$F_{D, st} = \frac{1}{2} \rho U^2 d C_D \quad (5.31)$$

where $F_{L, st}$ — Static lift force;

$F_{D, st}$ — Static drag force;

d — Diameter of the cable;

U — Mean wind speed;

ρ — Air mass density;

C_L — Static lift force coefficient;

C_D — Static drag force coefficient.

If the cable oscillates in both the cross-wind as well as the along-wind directions, as shown in Figure 5.12(b), the wind load acting on the oscillating cable can be obtained by modifying Equations (5.30) and (5.31) as follows

Lift force

$$F_L = \frac{1}{2}\rho U_r^2 d \left(C_L + \frac{dC_L}{d\alpha} \alpha_e \right) \quad (5.32)$$

Substituting Equations (5.28) and (5.29) into (5.32) yields

$$\begin{aligned} F_L &= \frac{1}{2}\rho d [(U - \dot{z})^2 + (-\dot{y})^2] \left[C_L + \frac{dC_L}{d\alpha} \left(\frac{-\dot{y}}{U - \dot{z}} \right) \right] \\ &= \frac{1}{2}\rho U^2 d C_L + \frac{1}{2}\rho U^2 d \left(-2C_L \frac{\dot{z}}{U} - \frac{dC_L}{d\alpha} \frac{\dot{y}}{U} \right) \\ &= F_{L_{st}} + F_{L_{se}} \end{aligned} \quad (5.33)$$

where $F_{L_{st}}$ — Static lift force component as defined by Equation (5.30);

$F_{L_{se}}$ — Self-excited lift force component in the form of

$$F_{L_{se}} = \frac{1}{2}\rho U^2 d \left(-2C_L \frac{\dot{z}}{U} - \frac{dC_L}{d\alpha} \frac{\dot{y}}{U} \right) \quad (5.34)$$

Drag force

The expression of the drag force can be obtained by following the similar derivation procedure, which can be expressed as follows

$$\begin{aligned} F_L &= \frac{1}{2}\rho U_r^2 d \left(C_D + \frac{dC_D}{d\alpha} \alpha_e \right) \\ &= F_{D_{st}} + F_{D_{se}} \end{aligned} \quad (5.35)$$

where $F_{D_{st}}$ — Static drag force component as defined by Equation (5.31);

$F_{D_{se}}$ — Self-excited drag force component in the form of

$$F_{D_{se}} = \frac{1}{2}\rho U^2 d \left(-2C_D \frac{\dot{z}}{U} - \frac{dC_D}{d\alpha} \frac{\dot{y}}{U} \right) \quad (5.36)$$

The projection of F_L and F_D in the along-wind direction Z is then

$$F_Z = F_L \sin \alpha_e + F_D \cos \alpha_e \quad (5.37)$$

Consider the case of the incipient(small) motion, i.e. the condition in the vicinity of $\dot{y} = 0$ and $\dot{z} = 0$, the effective wind attack angle is then

$$\alpha_e = \frac{-\dot{y}}{U - \dot{z}} \approx 0 \quad (5.38a)$$

$$\sin \alpha_e \approx \alpha_e \approx -\frac{\dot{y}}{U} \quad (5.38b)$$

$$\cos \alpha_e \approx 1 \quad (5.38c)$$

Thus Equation (5.37) can be simplified as follows

$$F_Z = F_L \alpha_e + F_D \quad (5.39)$$

Substituting Equations (5.33), (5.35) and (5.38) into Equation (5.39) yields

$$\begin{aligned} F_Z &= \frac{1}{2} \rho U^2 d \left[\left(C_L - 2C_L \frac{\dot{z}}{U} - \frac{dC_L}{d\alpha} \frac{\dot{y}}{U} \right) \left(-\frac{\dot{y}}{U} \right) + \left(C_D - 2C_D \frac{\dot{z}}{U} - \frac{dC_D}{d\alpha} \frac{\dot{y}}{U} \right) \right] \\ &\approx \frac{1}{2} \rho U^2 d \left[C_D - 2C_D \frac{\dot{z}}{U} - \left(C_L + \frac{dC_D}{d\alpha} \right) \frac{\dot{y}}{U} \right] \\ &= F_{Z_{st}} + F_{Z_{se}} \end{aligned} \quad (5.40)$$

where $F_{Z_{st}}$ — Static component of F_Z ,

$$F_{Z_{st}} = \frac{1}{2} \rho U^2 d C_D \quad (5.41)$$

$F_{Z_{se}}$ — Self-excited component of F_Z ,

$$F_{Z_{se}} = \frac{1}{2} \rho U^2 d \left[-2C_D \frac{\dot{z}}{U} - \left(C_L + \frac{dC_D}{d\alpha} \right) \frac{\dot{y}}{U} \right] \quad (5.42)$$

Similarly, the projection of F_L and F_D in the cross-wind direction Y can be obtained as follows

$$\begin{aligned}
 F_Y &= F_L \cos \alpha_e - F_D \sin \alpha_e \\
 &\approx F_L - F_D \alpha_e \\
 &\approx \frac{1}{2} \rho U^2 d \left[C_L - 2C_L \frac{\dot{z}}{U} + \left(C_D - \frac{dC_L}{d\alpha} \right) \frac{\dot{y}}{U} \right] \\
 &= F_{Y_{st}} + F_{Y_{se}}
 \end{aligned} \tag{5.43}$$

where $F_{Y_{st}}$ — Static component of F_Y ,

$$F_{Y_{st}} = \frac{1}{2} \rho U^2 d C_L \tag{5.44}$$

$F_{Y_{se}}$ — Self-excited component of F_Y ,

$$F_{Y_{se}} = \frac{1}{2} \rho U^2 d \left[-2C_L \frac{\dot{z}}{U} + \left(C_D - \frac{dC_L}{d\alpha} \right) \frac{\dot{y}}{U} \right] \tag{5.45}$$

For the flutter analysis, only the self-excited components $F_{Y_{se}}$ and $F_{Z_{se}}$ of the cross-wind and the along-wind forces F_Y and F_Z acting on the stay cables are considered.

Comparing Equations (5.42), (5.45), which represents the self-excited aerodynamic forces acting on the stay cable, with the Scanlan expression for the aerodynamic force given by Equation (3.1), and also notice that $\dot{y}$ and $\dot{z}$ in Equations (5.42) and (5.45) are equivalent to $\dot{h}$ and $\dot{p}$ respectively in Equation (3.1), the expressions of $F_{Y_{se}}$ and $F_{Z_{se}}$ can be rewritten in the form as follows

$$F_{Y_{se}} = \frac{1}{2} \rho U^2 d \left[K H_{1c}^* \frac{\dot{y}}{U} + K H_{5c}^* \frac{\dot{z}}{U} \right] \tag{5.46}$$

$$F_{Z_{se}} = \frac{1}{2} \rho U^2 d \left[K P_{1c}^* \frac{\dot{z}}{U} + K P_{5c}^* \frac{\dot{y}}{U} \right] \tag{5.47}$$

where K — reduced frequency, $K = \frac{d\omega}{U}$;

$H_{1c}^*, H_{5c}^*, P_{1c}^*, P_{5c}^*$ — Flutter derivatives of the stay cable,

$$H_{1c}^* = \frac{1}{K} \left(C_D - \frac{dC_L}{d\alpha} \right) \quad (5.48a)$$

$$H_{5c}^* = -\frac{2}{K} C_L \quad (5.48b)$$

$$P_{1c}^* = -\frac{2}{K} C_D \quad (5.48c)$$

$$P_{5c}^* = -\frac{1}{K} \left(C_L + \frac{dC_D}{d\alpha} \right) \quad (5.48d)$$

5.3.2 Aerodynamic Matrices of the Stay Cable Element

Equivalent nodal load

The equivalent nodal load of the three-node stay cable element due to the self-excited aerodynamic forces $F_{Y_{se}}$ and $F_{Z_{se}}$ can be derived by applying the virtual work principle. The virtual work done by $F_{Y_{se}}$ through the virtual displacement $\bar{y}$ can be written as follows

$$\begin{aligned} W &= - \int_L F_{Y_{se}}(x) \bar{y}(x) dx \\ &= - \int_0^L \frac{1}{2} \rho U^2 d \left(K H_{1c}^* \frac{\dot{y}}{U} + K H_{5c}^* \frac{\dot{z}}{U} \right) \bar{y} dx \\ &= - \{\bar{\delta}\}^e \{P\}_Y \end{aligned} \quad (5.49)$$

where $\{\bar{\delta}\}^e$ — displacement parameter vector of the three-node cable element;
 $\{P\}_Y$ — equivalent nodal force of the three-node cable element caused by
the self-excited force $F_{Y_{se}}$.

$$\begin{aligned}\{P\}_Y &= \frac{1}{2}\rho U^2 d \int_0^L \frac{1}{U} \left(K H_{1c}^* [N_v]^T [N_v] + K H_{5c}^* [N_v]^T [N_w] \right) \{\dot{\bar{\delta}}\}^e dx \\ &= \left(\frac{1}{2}\rho U^2 \right) \frac{1}{U} [A_d]_c^Y \{\dot{\bar{\delta}}\}^e\end{aligned}\quad (5.50)$$

where $[N_v], [N_w]$ — displacement shape function matrices defined by
Equations (5.19) and (5.20);

$\{\dot{\bar{\delta}}\}^e$ — associated velocity vector of $\{\bar{\delta}\}^e$;

$[A_d]_c^Y$ — component of the cable element aerodynamic damping
matrix $[A_d]_c$ due to the effect of $F_{Y_{se}}$.

$$\begin{aligned}
[A_d]_c^Y &= d \int_0^L \left(KH_{1c}^* \cdot [N_v]^T [N_v] + KH_{5c}^* \cdot [N_v]^T [N_w] \right) dx \\
&= KdL \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{H_{1c}^*}{3} & \frac{H_{5c}^*}{3} & 0 & \frac{H_{1c}^*}{6} & \frac{H_{5c}^*}{6} & 0 & \frac{H_{1c}^*}{\pi} & \frac{H_{5c}^*}{\pi} & \frac{-H_{1c}^*}{2\pi} & \frac{-H_{5c}^*}{2\pi} & \frac{H_{1c}^*}{3\pi} & \frac{H_{5c}^*}{3\pi} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{H_{1c}^*}{6} & \frac{H_{5c}^*}{6} & 0 & \frac{H_{1c}^*}{3} & \frac{H_{5c}^*}{3} & 0 & \frac{H_{1c}^*}{\pi} & \frac{H_{5c}^*}{\pi} & \frac{-H_{1c}^*}{2\pi} & \frac{-H_{5c}^*}{2\pi} & \frac{H_{1c}^*}{3\pi} & \frac{H_{5c}^*}{3\pi} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{H_{1c}^*}{\pi} & \frac{H_{5c}^*}{\pi} & 0 & \frac{H_{1c}^*}{\pi} & \frac{H_{5c}^*}{\pi} & 0 & \frac{H_{1c}^*}{2} & \frac{H_{5c}^*}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{-H_{1c}^*}{2\pi} & \frac{-H_{5c}^*}{2\pi} & 0 & \frac{-H_{1c}^*}{2\pi} & \frac{-H_{5c}^*}{2\pi} & 0 & 0 & 0 & \frac{H_{1c}^*}{2} & \frac{H_{5c}^*}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{H_{1c}^*}{3\pi} & \frac{H_{5c}^*}{3\pi} & 0 & \frac{H_{1c}^*}{3\pi} & \frac{H_{5c}^*}{3\pi} & 0 & 0 & 0 & 0 & 0 & \frac{H_{1c}^*}{2} & \frac{H_{5c}^*}{2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{aligned}
\tag{5.51}$$

The aerodynamic damping matrix component $[A_d]_c^Z$ of the cable element caused by the self-excited force $F_{Z_{se}}$ can be derived in a similar way. It can be expressed

as follows

$$\begin{aligned}
 [A_d]_c^Z &= d \int_0^L \left(K P_{1c}^* [N_w]^T [N_w] + K P_{5c}^* [N_w]^T [N_v] \right) dx \\
 &= KdL \begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{P_{5c}^*}{3} & \frac{P_{1c}^*}{3} & 0 & \frac{P_{5c}^*}{6} & \frac{P_{1c}^*}{6} & 0 & \frac{P_{5c}^*}{\pi} & \frac{P_{1c}^*}{\pi} & \frac{-P_{5c}^*}{2\pi} & \frac{-P_{1c}^*}{2\pi} & \frac{P_{5c}^*}{3\pi} & \frac{P_{1c}^*}{3\pi} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{P_{5c}^*}{6} & \frac{P_{1c}^*}{6} & 0 & \frac{P_{5c}^*}{3} & \frac{P_{1c}^*}{3} & 0 & \frac{P_{5c}^*}{\pi} & \frac{P_{1c}^*}{\pi} & \frac{-P_{5c}^*}{2\pi} & \frac{-P_{1c}^*}{2\pi} & \frac{P_{5c}^*}{3\pi} & \frac{P_{1c}^*}{3\pi} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{P_{5c}^*}{\pi} & \frac{P_{1c}^*}{\pi} & 0 & \frac{P_{5c}^*}{\pi} & \frac{P_{1c}^*}{\pi} & 0 & \frac{P_{5c}^*}{2} & \frac{P_{1c}^*}{2} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{-P_{5c}^*}{2\pi} & \frac{-P_{1c}^*}{2\pi} & 0 & \frac{-P_{5c}^*}{2\pi} & \frac{-P_{1c}^*}{2\pi} & 0 & 0 & 0 & \frac{P_{5c}^*}{2} & \frac{P_{1c}^*}{2} & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{P_{5c}^*}{3\pi} & \frac{P_{1c}^*}{3\pi} & 0 & \frac{P_{5c}^*}{3\pi} & \frac{P_{1c}^*}{3\pi} & 0 & 0 & 0 & 0 & 0 & \frac{P_{5c}^*}{2} & \frac{P_{1c}^*}{2}
 \end{bmatrix}
 \end{aligned}
 \tag{5.52}$$

The aerodynamic damping matrix $[A_d]_c$ of the three-node cable element can be obtained by combining the effects of both $F_{Y_{se}}$ and $F_{Z_{se}}$, i.e.

$$[A_d]_c = [A_d]_c^Y + [A_d]_c^Z \tag{5.53}$$

In the case that the object has a symmetric cross-section such as in the case of

the round stay cable, the following conditions apply

$$C_L \approx 0 \quad (5.54a)$$

$$\frac{dC_L}{d\alpha} \approx 0 \quad (5.54b)$$

$$\frac{dC_D}{d\alpha} \approx 0 \quad (5.54c)$$

Thus, the expressions of the stay cable flutter derivatives can be simplified as follows

$$H_{1c}^* = \frac{1}{K} \left(C_D - \frac{dC_L}{d\alpha} \right) \approx \frac{1}{K} C_D \quad (5.55a)$$

$$H_{5c}^* = -\frac{2}{K} C_L \approx 0 \quad (5.55b)$$

$$P_{1c}^* = -\frac{2}{K} C_D \quad (5.55c)$$

$$P_{5c}^* = -\frac{1}{K} \left(C_L - \frac{dC_D}{d\alpha} \right) \approx 0 \quad (5.55d)$$

It is clear from Equation (5.55) that the flutter derivatives of the stay cable only relates to the drag coefficient C_D . For the cable having a circular cross-section, C_D is usually taken as 0.7. This value is adopted in the current study. The final form of the aerodynamic damping matrix for a cable having a symmetric cross-section can

be written as follows

$$\begin{aligned}
 [A_d]_c &= [A_d]_c^Y + [A_d]_c^Z \\
 &= KdL \begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{H_{1c}^*}{3} & 0 & 0 & \frac{H_{1c}^*}{6} & 0 & 0 & \frac{H_{1c}^*}{\pi} & 0 & -\frac{H_{1c}^*}{2\pi} & 0 & \frac{H_{1c}^*}{3\pi} & 0 \\
 0 & 0 & \frac{P_{1c}^*}{3} & 0 & 0 & \frac{P_{1c}^*}{6} & 0 & 0 & \frac{P_{1c}^*}{\pi} & 0 & -\frac{P_{1c}^*}{2\pi} & 0 & \frac{P_{1c}^*}{3\pi} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{H_{1c}^*}{6} & 0 & 0 & \frac{H_{1c}^*}{3} & 0 & 0 & \frac{H_{1c}^*}{\pi} & 0 & -\frac{H_{1c}^*}{2\pi} & 0 & \frac{H_{1c}^*}{3\pi} & 0 \\
 0 & 0 & \frac{P_{1c}^*}{6} & 0 & 0 & \frac{P_{1c}^*}{3} & 0 & 0 & \frac{P_{1c}^*}{\pi} & 0 & -\frac{P_{1c}^*}{2\pi} & 0 & \frac{P_{1c}^*}{3\pi} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{H_{1c}^*}{\pi} & 0 & 0 & \frac{H_{1c}^*}{\pi} & 0 & 0 & \frac{H_{1c}^*}{2} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \frac{P_{1c}^*}{\pi} & 0 & 0 & \frac{P_{1c}^*}{\pi} & 0 & 0 & \frac{P_{1c}^*}{2} & 0 & 0 & 0 & 0 \\
 0 & -\frac{H_{1c}^*}{2\pi} & 0 & 0 & -\frac{H_{1c}^*}{2\pi} & 0 & 0 & 0 & 0 & \frac{H_{1c}^*}{2} & 0 & 0 & 0 \\
 0 & 0 & -\frac{P_{1c}^*}{2\pi} & 0 & 0 & -\frac{P_{1c}^*}{2\pi} & 0 & 0 & 0 & 0 & \frac{P_{1c}^*}{2} & 0 & 0 \\
 0 & \frac{H_{1c}^*}{3\pi} & 0 & 0 & \frac{H_{1c}^*}{3\pi} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{H_{1c}^*}{2} & 0 \\
 0 & 0 & \frac{P_{1c}^*}{3\pi} & 0 & 0 & \frac{P_{1c}^*}{3\pi} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{P_{1c}^*}{2}
 \end{bmatrix}
 \end{aligned}
 \tag{5.56}$$

5.4 Model of Aerodynamic Forces Acting on the Girder

For the dynamic analysis in this and the following chapter, the bridge girder is discretized into three-dimensional beam elements. If the girder is modelled by suffi-

cient number of beam elements, the aerodynamic forces acting on each element can be approximated by the uniformly distributed loads. The self-excited aerodynamic forces acting on the girder per unit span are given by Equation (3.1). In order to represent the uniformly distributed load by the nodal forces at the two ends of the beam element, a simple lumping procedure is adopted. It is assumed that one-half of the distributed load is lumped on each end of the 3D beam element, as shown in Figure 5.13.

For an arbitrary 3D beam element ij with nodes i and j at its two ends, the aerodynamic lumping effects on both ends can be decomposed into the aerodynamic stiffness component as well as the aerodynamic damping component as follows

$$\begin{aligned}\{A\}^e &= [\{A_i\}, \{A_j\}]^T \\ &= \frac{1}{2}\rho U^2 \left([A_s]^e \{\delta\}^e + \frac{1}{U} [A_d]^e \{\dot{\delta}\}^e \right)\end{aligned}\quad (5.57)$$

where $\{A_i\}, \{A_j\}$ — lumped aerodynamic force vectors for node i and j ,

$$\{A_l\} = \begin{bmatrix} F_{Xl} & F_{Yl} & F_{Zl} & M_{Xl} & M_{Yl} & M_{Zl} \end{bmatrix}^T, \quad (l = i, j);$$

$\{\delta\}^e$ — nodal displacement vector of element ij , $\{\delta\}^e = [\{\delta_i\}, \{\delta_j\}]^T$,

$$\{\delta_l\} = \begin{bmatrix} u_l & v_l & w_l & \alpha_l & \beta_l & \gamma_l \end{bmatrix}, \quad (l = i, j);$$

$\{\dot{\delta}\}^e$ — associated nodal velocity vector of element ij ;

$[A_s]^e$ — aerodynamic stiffness matrix of element ij , with size of 12×12 ;

$[A_d]^e$ — aerodynamic damping matrix of element ij , with size of 12×12 .

Assuming that the length of the element ij is L , and the aerodynamic lift force, drag force and moment per unit span acting on the girder are respectively L_h , D_p and M_a , the lumped nodal forces on nodes i and j induced by the aerodynamic

forces distributed within the element can be calculated. Defining

$$\begin{aligned}\{A\}^e &= [\{A_i\}, \{A_j\}]^T \\ &= \begin{bmatrix} F_{Xi} & F_{Yi} & F_{Zi} & M_{Xi} & M_{Yi} & M_{Zi} & F_{Xj} & F_{Yj} & F_{Zj} & M_{Xj} & M_{Yj} & M_{Zj} \end{bmatrix}\end{aligned}\quad (5.58)$$

yields

$$F_{Xi} = F_{Xj} = M_{Yi} = M_{Yj} = M_{Zi} = M_{Zj} = 0 \quad (5.59a)$$

$$\begin{aligned}F_{Yi} = F_{Yj} &= \frac{1}{2}L_h L \\ &= \frac{1}{2}\rho U^2 \left(BLKH_1^* \frac{\dot{h}}{U} + B^2 LKH_2^* \frac{\dot{\alpha}}{U} + BLK^2 H_3^* \alpha + LK^2 H_4^* h + BLKH_5^* \frac{\dot{p}}{U} \right. \\ &\quad \left. + LK^2 H_6^* p \right)\end{aligned}\quad (5.59b)$$

$$\begin{aligned}F_{Zi} = F_{Zj} &= \frac{1}{2}D_p L \\ &= \frac{1}{2}\rho U^2 \left(BLKP_1^* \frac{\dot{h}}{U} + B^2 LKP_2^* \frac{\dot{\alpha}}{U} + BLK^2 P_3^* \alpha + LK^2 P_4^* h + BLKP_5^* \frac{\dot{p}}{U} \right. \\ &\quad \left. + LK^2 P_6^* p \right)\end{aligned}\quad (5.59c)$$

$$\begin{aligned}M_{Xi} = M_{Xj} &= \frac{1}{2}M_\alpha L \\ &= \frac{1}{2}\rho U^2 \left(B^2 LKA_1^* \frac{\dot{h}}{U} + B^3 LKA_2^* \frac{\dot{\alpha}}{U} + B^2 LK^2 A_3^* \alpha + BLK^2 A_4^* h \right. \\ &\quad \left. + B^2 LKA_5^* \frac{\dot{p}}{U} + BLK^2 A_6^* p \right)\end{aligned}\quad (5.59d)$$

Substituting Equations (5.58) and (5.59) into Equation (5.57), the aerodynamic stiffness matrix $[A_s]^e$ and the aerodynamic damping matrix $[A_d]^e$ of the 3D beam element can be obtained. These are sparse matrices. The non-zero terms in $[A_s]^e$ are

$$A_s(2, 2) = A_s(8, 8) = K^2 L H_4^* \quad (5.60a)$$

$$A_s(2, 3) = A_s(8, 9) = K^2 L H_6^* \quad (5.60b)$$

$$A_s(2, 4) = A_s(8, 10) = -K^2 B L H_3^* \quad (5.60c)$$

$$A_s(3, 2) = A_s(9, 8) = K^2 L P_6^* \quad (5.60d)$$

$$A_s(3, 3) = A_s(9, 9) = K^2 L P_4^* \quad (5.60e)$$

$$A_s(3, 4) = A_s(9, 10) = -K^2 B L P_3^* \quad (5.60f)$$

$$A_s(4, 2) = A_s(10, 8) = -K^2 B L A_4^* \quad (5.60g)$$

$$A_s(4, 3) = A_s(10, 9) = -K^2 B L A_6^* \quad (5.60h)$$

$$A_s(4, 4) = A_s(10, 10) = K^2 B^2 L A_3^* \quad (5.60i)$$

and the non-zero terms in $[A_d]^e$ are

$$A_d(2, 2) = A_d(8, 8) = K B L H_1^* \quad (5.61a)$$

$$A_d(2, 3) = A_d(8, 9) = KBLH_5^* \quad (5.61b)$$

$$A_d(2, 4) = A_d(8, 10) = -KB^2LH_2^* \quad (5.61c)$$

$$A_d(3, 2) = A_d(9, 8) = KBLP_5^* \quad (5.61d)$$

$$A_d(3, 3) = A_d(9, 9) = KBLP_1^* \quad (5.61e)$$

$$A_d(3, 4) = A_d(9, 10) = -KB^2LP_2^* \quad (5.61f)$$

$$A_d(4, 2) = A_d(10, 8) = -KB^2LA_1^* \quad (5.61g)$$

$$A_d(4, 3) = A_d(10, 9) = -KB^2LA_5^* \quad (5.61h)$$

$$A_d(4, 4) = A_d(10, 10) = KB^3LA_2^* \quad (5.61i)$$

Because of the non-conservative nature of the aeroelastic forces, the aerodynamic matrices $[A_d]^e$ and $[A_s]^e$ are non-symmetric matrices. Since flutter derivatives are functions of the reduced frequency K , $[A_d]^e$ and $[A_s]^e$ may vary with the wind velocity and the response frequency. Different sets of flutter derivatives corresponding to the representative varying deck cross-section should be used in the flutter analysis if the variation of the geometric contour of the deck cross-section is significant.

5.5 Case Study: Nanpu Bridge

5.5.1 Effect of Cable Vibration on the Dynamic Characteristics of the Nanpu Bridge

The impact of cable vibration effect on the dynamic characteristics and the flutter response of the Nanpu Bridge is studied in this section. The influence of the dynamic cable stiffness on the extensional behaviour of the stay cables is investigated by comparing the behaviour of the longest and the shortest stay cables in the bridge. The dynamic analysis of other long-span cable-stayed bridges are also presented to study the cable vibration effect in more detail. The parameters considered in the configuration include an inclined cable-plane arrangement and inverted- Y shape pylons, as compared to the parallel cable-plane arrangement and H shape pylons in the Nanpu Bridge.

Modeling of the bridge

In the common application of the finite element approach in bridge analysis, the main girder of the bridge can be modeled by the single-spine, the double-beam and the triple-beam approach, as discussed in Section 4.4.1. The triple-beam model, which can take into account the warping torsional stiffness of the bridge deck cross-section, is adopted in this section to model the main girder of the Nanpu Bridge.

The coordinate system of the numerical model is chosen such that the bridge deck is parallel to the XY -plane of the coordinate system, with Y -axis along the longitudinal direction of the bridge. The Z -axis is perpendicular to the bridge deck surface.

In the triple-beam model, the properties of the bridge girder are represented by a central beam located at the bridge central axis and two side beams located at the intersection of the cable planes and the bridge girder, as shown in Figure 5.14. The two side beams are connected with the central beam by a rigid link or by a master-slave relationship. The geometric properties of these beams are defined as follows

A_i — area;

I_{z_i} — lateral bending moment of inertia;

I_{x_i} — vertical bending moment of inertia;

J_{d_i} — free torsional moment of inertia

where $i = 1$ represents the central beam, and $i = 2$ represents the side beam. The mass and stiffness of the bridge girder are allocated among the central and the two side beams according to the following rules:

(a) Equivalence of the area and the lateral bending stiffness

The area and the lateral bending moment of inertia of the girder are assumed to be lumped on the central beam, i.e.

$$A_1 = A \quad (5.62a)$$

$$A_2 = 0 \quad (5.62b)$$

$$I_{z_1} = I_z \quad (5.62c)$$

$$I_{z_2} = 0 \quad (5.62d)$$

(b) Equivalence of the vertical bending stiffness

The vertical bending moment of inertia of the girder is distributed to three beams by

$$I_{x_1} + 2I_{x_2} = I_x \quad (5.63)$$

(c) Equivalence of the warping torsional stiffness

Assume the torsional motion of the girder cross-section is a rigid motion. Thus, the warping torsional stiffness of the girder is provided by the vertical bending stiffness of the two side beams as

$$2I_{x_2}b^2 = J_\omega \quad (5.64)$$

where J_ω is the principal sectorial moment of inertia of girder warping torsion, and b is the horizontal distance between the side and the central beam.

Substitute Equation (5.64) into Equation (5.63) gives

$$I_{x_1} = I_x - J_\omega/b^2 \quad (5.65)$$

(d) Equivalence of the free torsional stiffness

$$J_{d_1} + 2J_{d_2} = J_d \quad (5.66)$$

(e) Equivalence of the Mass

The mass of the girder can either be assumed to be lumped only on the central beam, or to be allocated to all three beams as follows

$$M = M_1 + 2M_2 \quad (5.67a)$$

$$I_M = 2M_2b^2 \quad (5.67b)$$

where I_M is the mass moment of inertia of the girder.

The main girder of the Nanpu Bridge has a composite deck cross-section comprised of a prestressed concrete slab and two I -shaped strengthening steel beams. The whole cross-section is first transformed to an equivalent cross-section with the uniform material of steel. The mass and stiffness properties of this equivalent steel girder cross-section are modelled by a central beam and two side beams in the triple-beam finite element model. Each of these three beams are then further divided into ninety-four 3D beam elements in the longitudinal direction separated by the cable support points, the supports of the piers and the pylon locations.

The two H -shaped pylons are modeled by 3D beam finite elements. The geometric properties of the representative pylon shaft cross-sections are listed in Table 4.1. Each pylon is modeled by sixty-nine 3D beam elements.

The stay cables are modeled by the proposed three-node cable elements. The transverse motion of the cables are considered. For the extensional motion, the dynamic cable stiffness is included. The numbering sequence of the cables are the same as in Chapter 4, which starts from the one closest to the pylon. The cable hanging vertically from the pylon to support the girder is labelled as number 0. The cables in the side spans are denoted by the symbol "S", whereas for those in the center span are denoted by "C". For example, "C10" represents the 10th cable in the center span counting from the pylon, and "S10" represents the 10th cable in the side span counting from the pylon.

The boundary conditions are the same as described in Chapter 4.

This numerical model of the Nanpu Bridge has a total of 646 nodes and 2670

active degrees-of-freedom.

Dynamic characteristic analysis

Two different numerical models are employed to study the cable vibration effect on the dynamic characteristics of the Nanpu Bridge. In these two models, the modelling schemes for the main girder and the pylons are the same, i.e. the main girder is modelled by the triple-beam model, and the pylons are modelled by the 3D beam elements. The difference of the two models exists in the way the behaviour of the stay cables is represented. In the first model, the stay cables are modelled by single-truss elements, whereas in the second model, the cables are modelled by the proposed three-node cable elements. For convenience, these two models are referred respectively as the “single-truss cable model” and the “three-node cable model”. The modal analysis results obtained from these two models are presented in Table 5.8.

The results show that by considering the cable vibration in the dynamic behaviour of the example bridge, additional pure cable vibration modes as well as new cable-deck coupled modes are identified in the modal analysis. These modes appear in between the basic modes of the bridge itself. The new cable-deck coupled modes involve the transverse motion of some stay cables in the cable plane and the torsional behaviour of the main girder in the center span. The cable motions in these modes are the dominant component in the deformation shape of the modes. These coupled motions as well as the pure cable vibration modes cannot be identified by the single-truss model. Comparing the two sets of results, it is noted that

the natural frequencies of the corresponding basic bridge modes are close to each other, with those obtained from the three-node cable model somewhat lower. This is because the transverse vibrational behaviour of the stay cables are considered in the three-node cable model. The oscillations of the cables increase the vibration mass of the bridge system. With the same stiffness but greater mass, the calculated natural frequencies are therefore lower.

Figure 5.15 shows the influence of the cable vibration effect on the dynamic characteristics of the Nanpu Bridge in terms of modal order. The frequencies of the first 20 modes obtained from the single-truss cable model and the first 200 modes obtained from the three-node cable model are plotted against the mode number. It can be observed that in the same frequency range of $0 - 0.8\text{Hz}$, there are 14 modes extracted from the single-truss cable model, whereas more than 200 modes are identified by the three-node cable model. Thus, by considering the cable vibration behaviour in the modal analysis, 186 new modes have been extracted in the frequency range below 0.8Hz . For the curve corresponding to the results obtained from the three-node cable model in Figure 5.15, many flat regions in the curve can be identified. These flat regions represent the local vibration modes of the stay cables.

The mode shapes of some typical basic bridge modes obtained by using the three-node cable model are shown in Figure 5.16. The interaction of the cable vibration behaviour with the motion of the main girder and the pylons are clearly demonstrated. Some pure cable vibration modes are shown in Figure 5.17. They include the in-plane and out-of-plane vibration of the cables at the center and the side spans. Some of the new cable-deck coupled modes are shown in Figure 5.18. In these modes, the dominant in-plane motions of some of the long cables are coupled

with the torsional motion of the main girder at the center span. This is due to the open π shape deck cross-section of the Nanpu Bridge, the torsional stiffness of the main girder is relatively low, and thus the torsional behaviour of the main girder is easily excited.

The frequencies of some of the long cables in the center and the side spans of the Nanpu Bridge obtained from the modal analysis are compared with the analytical results using Equation (5.6). The results of the comparison are presented in Table 5.9. Altogether ten stay cables, five in the center span and another five in the side spans, are studied. Fairly good agreement between these two sets of results is observed. The minor difference between the analytical and the modal analysis results is due to the difference in the boundary conditions. The analytical solution is based on the fixed-fixed end boundary condition, whereas in the modal analysis, the two ends of the cable move with the deck and the pylons.

According to the analytical solution, the fundamental frequency of the longest cable in the Nanpu Bridge is $0.4762Hz$, whereas that of the shortest is $1.5794Hz$. The cable frequencies fall overlap with the frequency range between mode 5 to 45 of the bridge structure, i.e. between $0.4255Hz$ and $1.607Hz$. For dynamic motions of the bridge in this frequency range, the vibration of the cables and their interaction with the motions of the deck and the pylons can be significant, which can severely affect the dynamic response of the bridge.

Dynamic cable stiffness

The dynamic cable stiffness accounts for the interaction effect between the mo-

tion of the cables and that of the other parts of the bridge superstructure. It influences the extensional behaviour of the cable by modifying the longitudinal stiffness. Assuming there is no tension loss due to corrosion or fatigue problems of the cable and the cable tensions are obtained under the dead load condition, the dynamic cable stiffness of the longest and the shortest cables of the Nanpu Bridge are determined for the cases of bridge natural frequencies at $0.1Hz$, $0.2Hz$, $0.5Hz$ and $1.0Hz$. The results are presented in Table 5.10.

Consider the case of the longest cable, no significant change in the extensional stiffness of the cable is found. If the dynamic interaction between the cable and the other parts of the superstructure is neglected, the extensional cable stiffness obtained is $0.9862 \times 10^7 N/m$. For the dynamic stiffness corresponding to the bridge structural frequency of $0.5Hz$, the extensional cable stiffness is $0.9861 \times 10^7 N/m$. Comparing to the previous result, there is only a difference of 0.01%. For the case of the extensional stiffness of the cable approaches the critical state of zero, the corresponding bridge frequency for the longest cable is $7.76Hz$, and for the shortest cable is $105.6Hz$. These frequencies are obviously beyond the bridge frequency range that are of most concern in the dynamic response analysis of bridges due to wind. In the case of the shortest cable, the dynamic stiffness effect is negligible. Thus, if there is no tension loss due to the corrosion or fatigue problem, because of the existence of the high pre-tension and relatively lower mass of the cables, the impact of the dynamic stiffness on the extensional behaviour of the stay cables in the cable-stayed bridges is not as significant as that pointed out by Davenport and Steels (1965) for the guy cables, which typically have a lower tension and a greater mass. However, in reality the stay cables are usually subjected to deterioration due to the corrosion and fatigue problems. This may affect the dynamic stiffness behaviour of the stay

cables.

There are a number of reported cases that the cable replacement are required because of the corrosion and fatigue problems in the cables. These two phenomena happen simultaneously and reinforce each other. The tension in the cable can drop significantly due to the damage to the wires inside the stay cable. Under such conditions, the dynamic cable stiffness may have a significant impact on the longitudinal behaviour of the cable.

To study the effect of the tension loss on the dynamic behaviour of the stay cables, the dynamic stiffness of the longest and the shortest cables of the Nanpu Bridge are investigated for the cases of tension in the cable taken as $0.75T_{DL}$, $0.50T_{DL}$ and $0.25T_{DL}$ respectively, where T_{DL} is the initial cable tension caused by the dead load. The results are shown in Table 5.11

If the remaining cable tension is $0.50T_{DL}$, when the bridge vibrates with a frequency of $0.2Hz$, the extensional stiffness of the longest cable considering the dynamic interaction may drop by 15.18% to a value of $0.8365 \times 10^7 N/m$. From the data presented in Table 5.11, it can be noticed that the value of the dynamic cable stiffness depends more on the tension in the cable than on the external exciting frequency of the bridge motion. A negative value of the extensional stiffness of the longest cable occurs under the condition when the remaining tension force is $0.25T_{DL}$ and the oscillating frequency of the bridge itself is $1.0Hz$. Physically, this negative stiffness indicates that instead of resisting the elongation of the cable, the deformation tends to increase. The cable then is under an unstable condition. No such negative stiffness has been found in the case of the shortest cable for the same combination of the cable tensions and the bridge frequencies. The effect of the dynamic interaction between the cables and the other parts of the superstructure on

the extensional stiffness of the shortest cable is much less than that on the longest cable.

In the dynamic analysis of the cable-stayed bridges, if the cable is significantly corroded and fatigue are considered, the tension in the cable may reduce significantly. The dynamic stiffness should then be considered, especially for the long cables in the bridge. The dynamic responses of the bridge can be significantly affected. Since the problems of the cable corrosion and fatigue are not included in the current study, the dynamic stiffness effect of the cables can be neglected.

5.5.2 Effect of Cable Vibration on the Flutter Response of the Nanpu Bridge

The natural frequency analysis results used in the flutter analysis of the bridge are extracted by using the three-node cable model. Altogether four natural modes, i.e. the lowest symmetric and anti-symmetric heave and torsional modes are selected as the participating modes in representing the flutter response of the bridge. The flutter derivatives of the deck cross-section of the Nanpu Bridge are presented in Figure 4.7. The $PK-F$ method, as described in Section 3.7, is employed here as the computational algorithm for seeking the critical flutter point of the bridge structure. By considering the cable vibration behaviour and also including the aerodynamic forces acting on the stay cables, the critical flutter wind velocity of the Nanpu Bridge is determined to be $65.49m/s$ for the case of the structural damping ratio taken as 0.01. There is a 4.16% increase when compared to the result of $62.88m/s$ obtained from the single-truss cable model, of which neither the transverse motion of the stay cables nor the aerodynamic loads acting on the cables are considered in the

analysis. Similar results have been obtained by Mendes and Semião (1999) in the aerodynamic stability study of the Vasco da Gama Bridge.

To further investigate in more detail the separate contributions from the cable vibration effect and the aerodynamic loads acting on the stay cables on the flutter stability of the Nanpu bridge, the critical flutter wind velocities for four load cases of different combinations are analyzed. The results are presented in Table 5.12. If the aerodynamic loads on the stay cables are neglected in the flutter analysis, the cable vibration effect increases the critical flutter wind velocity U_f by 1.12%, from 62.88m/s to 63.58m/s. If the aerodynamic loads on the stay cables are considered, the cable vibration effect increases the U_f by 3.92%, from 63.02m/s to 65.49m/s. On the other hand, if the cable vibration behaviour is ignored, the consideration of the aerodynamic loads results in a 0.23% increase of the U_f from 62.88m/s to 63.02m/s, whereas in the case that the cable vibration is considered, the inclusion of the aerodynamic loads leads to a 3.0% increase of the U_f from 63.58m/s to 65.49m/s.. Thus, the effect induced by the cable vibration behaviour plays a more important role than that caused by the cable aerodynamic forces.

The influence of these two effects can be considered as additional damping to the bridge structure. Flutter is a self-excited aerodynamic instability phenomenon. The oscillating bridge structure absorbs energy from the surrounding air flow mainly through the motion of the bridge girder. The vibrations of the stay cables dissipate energy from the bridge system, and thus helps to stabilize the structure. The aerodynamic loads acting on the stay cables are mainly in the form of the drag force, which also contributes to the stabilization of the structure.

The impact of cable vibration effect on the flutter response of the Nanpu Bridge in terms of the relationship between the logarithmic decrement of the heave and

torsional modes with respect to the wind velocity is shown in Figure 5.19. There are two sets of curves in the figure, one corresponding to the case that the cable vibration effect and the aerodynamic loads on the cables are ignored, whereas in the other case both effects are considered. The results show that the behaviour of the two cases are similar. It is observed that in the behaviour of the symmetric torsional mode, the intersection of the logarithmic decrement curve with the horizontal line representing zero logarithmic decrement occurs at a higher wind velocity if the transverse motion of the cables is considered. This delay in the intersection corresponds to a higher critical flutter wind velocity of the bridge and thus demonstrates the stabilization effect of the cable vibration phenomenon..

The cable vibration effects on the relationship of response frequencies of the participating modes and the wind velocity are shown in Figure 5.20. Again, two sets of curves are presented for comparison purposes. The trend of variation in the response frequencies with respect to the wind velocity is found to be similar for the two sets of curves. The frequencies of the heave modes have only minor changes. The two curves representing the frequency variation of the symmetric torsional modes drop noticeably in the vicinity of the critical flutter point. When the wind velocity increases, the frequencies of the anti-symmetric torsional mode in both cases decrease. For all the participating modes, the frequencies obtained in the case of considering cable vibration and cable aerodynamic forces are always found to be slightly higher than those obtained in the case of ignoring these effects.

5.5.3 Effect of Cable Vibration on the Dynamic Response of Long-span Bridges

5.5.3.1 Description of the Example Bridge

The Nanpu Bridge has a parallel cable-plane arrangement and two *H*-shaped pylons. The cable vibration effect influences the modal analysis results in terms of the natural frequencies, the mode shapes and the modal order. Further, it helps to increase the critical flutter wind velocity of the bridge. To study the impact of cable vibration behaviour on the dynamic responses of the cable-stayed bridges having different structural layout, the dynamic analysis of other long-span bridge structures modified from the configuration of the Nanpu Bridge are carried out.

For the example bridge considered here, the span length, the cross-section of the main girder, the cross-sectional area of the stay cables and the boundary conditions are taken as same as the Nanpu Bridge. The shape of the pylons is changed to an inverted-*Y* shape, as shown in Figure 5.21. The pylon shafts under the lower cross-beam remain unchanged from the Nanpu Bridge. The upper cross-beam is removed. The upper ends of the pylon shafts between the upper and the lower cross-beams are connected at the center point of the upper cross-beam. The two pylon shafts beyond the upper cross-beam are merged as one. Consequently, the parallel cable-plane arrangement of the Nanpu Bridge is changed to the inclined cable-plane arrangement in the example bridge here.

In the numerical model of the example bridge, the composite main girder is represented by the triple-beam model, the stay cables are modelled by the three-node cable elements and the pylons by the 3D beam elements. The numerical model,

as shown in Figure 5.22, has a total of 646 nodes and 2754 active degrees-of-freedom.

5.5.3.2 Dynamic Characteristic Analysis

Two sets of modal analysis results, of which the stay cables are modeled respectively by the single-truss element and the three-node element, are presented in Table 5.13. Again, numerous pure cable vibration modes are extracted by using the three-node cable model. Corresponding to the first ten basic bridge modes obtained from the single-truss cable model, there are 154 modes obtained from the three-node cable model in the same frequency range, i.e. 144 new pure cable modes are identified by using the three-node cable model. No cable-deck coupled mode is identified in this case. For the modal frequencies listed in the table, the torsional frequencies are increased when considering the influence of the cable vibration behaviour, whereas for the other basic bridge modes, the modal frequencies are slightly decreased. This is different from the case of the Nanpu Bridge, of which the bridge natural frequencies, including the torsional modes, all decrease due to the influence of the transverse motion of the cables.

The impact of cable vibration effect on the modal order of the example bridge is shown in Figure 5.23. The flat regions in the curve of the three-node cable model result represent the local pure cable vibration modes.

Some basic bridge mode shapes are presented in Figure 5.24. They include the motions of the girder, the pylons and the cables. Some typical pure cable vibration modes are given in Figure 5.25. The range of the fundamental frequencies of the cables for the analyzed bridge is from $0.4270Hz$ for the longest cable to $1.5355Hz$ for the shortest cable. It coincides with the frequency range of the first 40 bridge

structural vibration modes.

Comparing the results presented in Table 5.8 with those in Table 5.13, it can be observed that by changing the pylon shape from H to the inverted- Y , the frequency of the floating mode has a minor decrease, whereas those for the heave modes are slightly increased. A 3.94% increase in the frequency of the sway mode is observed. For the torsional modes, the impact is much more significant. The frequency of the lowest symmetric torsional mode has been increased by 31.99% from $0.4951Hz$ to $0.6535Hz$, whereas that of the lowest anti-symmetric torsional mode has been increased by 20.15% from $0.6073Hz$ to $0.7297Hz$. This is because the torsional rigidity of the entire bridge system is mainly attributed to the contributions from the resistance of the deck, the cables as well as the pylons. For the torsional behaviour of the bridge, the two sides of the bridge deck move up and down, the effect of the motion of the deck are transmitted to the pylons by the cables. The upper part of the pylon shafts provide the anchorage support to the stay cables. In the case of the Nanpu Bridge with H shape pylons, the two upper pylon shafts are not connected and thus oscillate in opposite directions during the torsional motion of the girder. In this mode of behaviour, the pylons do not contribute much to the torsional rigidity of the whole system. However, in the case of the modified bridge, the two upper pylon shafts are combined together. This has significantly higher torsional resistance than the previous case, and thus can suppress more effectively the torsional motion of the girder. The torsional frequencies of the bridge system increase significantly due to the contribution to the torsional rigidity of the system by the inverted- Y shape pylons.

5.5.3.3 Flutter Analysis

In the flutter analysis of the modified bridge, the adopted flutter derivatives of the main girder are the same as those of the Nanpu Bridge, as presented in Figure 4.7. The aerodynamic loads acting on the stay cables are calculated by taking the drag coefficient $C_D = 0.7$. The lowest symmetric and anti-symmetric heave and torsional modes are selected for consideration in the flutter analysis. The critical flutter wind velocity of the modified bridge under four different modelling cases are presented in Table 5.14. It can be seen that considering the cable vibration behaviour and also the aerodynamic loads on the cables, the obtained critical flutter wind velocity U_f is determined as 86.59m/s , which is 6.49% higher than the result of 81.31m/s obtained by ignoring these effects.

Compared to the increase of 4.15% in the case of the Nanpu Bridge, the stabilizing effects contributed by the cable vibration and the aerodynamic loads on the cables are more significant in the case of the modified bridge. This is due to the differences in the structural layout of these two cable-stayed bridges in the cable arrangement and the shape of the pylons. Because of the higher torsional rigidity of the modified bridge, the structure is more capable of resisting the occurrence of the aerodynamic instability. The consideration of cable vibrations in the analysis modal allows the input energy from the wind action to be also dissipated from the oscillating cables, which helps even more to stabilize the structure. Again, the results in Table 5.14 indicates that the stabilizing effect resulting from the cable vibration behaviour is more significant than that by considering the aerodynamic loads on the stay cables.

The impact of cable vibration effect on the flutter response of the modified

bridge are shown in Figure 5.26 and Figure 5.27. Figure 5.26 shows the impact of the cable vibration on the logarithmic decrement of the structural response, whereas Figure 5.27 shows the impact on the response frequency. Both the symmetric and anti-symmetric torsional and vertical vibration modes are considered. The results demonstrate the significance of the cable vibration effect, and the impact of the cable aerodynamic loads on the flutter behaviour of the long-span bridge. Figure 5.26 shows the variation of the logarithmic decrement of the participating modes with respect to the wind velocity. The behaviour of the torsional and heave vibration modes with and without considering the cable vibration effect are generally found to be similar. For the heave modes, the results of the two cases are essentially the same. For the torsional modes, the results show some differences, especially for the case of the symmetric torsional mode. When the cable vibration effect is considered, the critical flutter wind velocity increases. The divergence occurs later in the case of considering cable vibration effect, as indicated by the delay in the intersection of the curve with the horizontal line representing the neutral state of the structure. For the influence on response frequencies as presented in Figure 5.27, if cable vibration is considered, the frequencies of the heave modes are slightly lower than those obtained by ignoring the cable vibration. For the torsional modes, behaviour is opposite. Ignoring the effect of cable vibration yields slightly higher torsional frequency results. The variation of frequencies of the heave modes hardly changes with the wind velocity in both the symmetric and anti-symmetric cases. In comparison, the frequencies of the torsional modes drop slightly when the wind velocity approaches the critical flutter point.

Figure 5.28 and Figure 5.29 show the impact of structural layout on the flutter response of a cable-stayed bridge. The relationships between the logarithmic decre-

ment of the participating modes and the wind velocity, as well as those between the response frequencies of the modes and the wind velocity are plotted respectively in these two figures for the Nanpu Bridge and the modified bridge. It can be observed from Figure 5.28 that the responses of the heave modes for both cases decay at about the same rate, whereas for the torsional modes, the response of the modified bridge decay faster than that in the Nanpu Bridge. The critical flutter state occurs at a significantly higher wind velocity for the case of the modified bridge. This is due to the higher torsional rigidity of the modified bridge provided by the inverted-*Y* shape pylons and the inclined cable-plane arrangement. This type of structural layout improves the torsional behaviour of the structure, and thus helps to enhance the aerodynamic stability of the bridge. This type of structural layout is thus recommended for the long-span cable-stayed bridges, which are slender, flexible and vulnerable to flutter. The discrepancies between the torsional frequencies of these two bridges, as shown in Figure 5.29, further demonstrate the effect of adopting the inverted-*Y* shape pylon and the inclined cable-plane arrangement on the improvement of the bridge torsional behaviour. No significant effect has been found in the behaviour of the heave modes caused by the change in the structural layout.

In order to further understand the effect of the aerodynamic loads acting on the stay cables on the flutter behaviour of the cable-stayed bridges, and the sensitivity of the critical wind velocity to the variation of the aerodynamic loads on the stay cables, a parametric study of the drag coefficient C_D , which represents the aerodynamic loads on the stay cables, is carried out. The range of variation of C_D has been taken from -1.4 to 1.4 . In reality, the drag coefficient C_D is typically a positive value. The negative numbers included in the parametric study is for the purpose of studying the trend of the critical flutter wind velocity variation induced by the change of

the aerodynamic loads on the cable. Physically, these negative values mean that instead of having the aerodynamic loads on cables help to stabilize the structure, these loads have a destabilizing effect.

The deck cross-section of the Nanpu Bridge is unstreamlined. It has an unfavourable aerodynamic stability property. To assess the significance of this design parameter, the effect of drag coefficient variation on the bridges having a streamlined deck cross-section is also investigated. In these cases, the flutter derivatives of the main girder of the Nanpu Bridge and the modified bridge are replaced by those obtained from a streamlined flat steel box girder of the Tiger Gate Bridge in China. The uncoupled flutter derivatives H_1^* , A_2^* and A_3^* of this streamlined flat box cross-section are measured in the wind tunnel of Tongji University in China. They are plotted in Figure 5.30 against the reduced wind velocity U/fB . The other flutter derivatives are assumed to be zero in the flutter analysis.

Four curves in terms of the critical flutter wind velocity against the variation of the drag coefficient are plotted in Figure 5.31. Each curve represents the analysis results obtained from the bridge having one particular type of structural layout and aerodynamic property of the deck cross-section. The symbol “NPSH” represents the case of the Nanpu Bridge, whereas “NPSY” represents the case of the modified bridge. For each bridge, the original unstreamlined deck cross-section and the assumed streamlined deck cross-section are considered in the analysis. It is shown in the figure that for the same structural layout, the impact of varying the drag coefficient is more significant in the case of the streamlined deck cross-section than that with an unstreamlined one. For the decks with the same aerodynamic property, the impact of varying the drag coefficient is found to be more significant in the inclined cable-plane arrangement case of the modified bridge than in the par-

allel cable-plane arrangement case of the Nanpu Bridge. This indicates that if the structure itself has a better aerodynamic stability performance, the consideration of the aerodynamic loads on the stay cables may affect the flutter behaviour of the bridge more significantly in terms of the critical flutter wind velocity. The effect can be either destabilizing or stabilizing. The percentage of change in the critical flutter wind velocities with respect to the drag coefficient C_D for the four analyzed bridges are shown in Figure 5.32. The phenomena described above are presented in a non-dimensional form. For the extreme case of the modified Nanpu Bridge with the streamlined deck cross-section, the variation of U_f ranges from -13.40% which corresponds to $C_D = -1.4$, to 13.15% which corresponds to $C_D = 1.4$, in reference to the case of $C_D = 0.0$ as the base line.

5.6 Parametric Study

5.6.1 Definition of the Model Bridges

In Section 5.5, the cable vibration effect on the dynamic characteristics and the flutter response of the Nanpu Bridge and the example bridges modified from the Nanpu Bridge have been analyzed. To better understand the cable vibration behaviour on the dynamic responses of modern cable-stayed bridges, parametric studies are carried out in this section. The varying parameters include the structural properties of the bridge superstructure, the layout as well as the aerodynamic aspects of the bridge. They are listed as follows:

Structural category:

1. Span length
2. Cable arrangement
3. Shape of the pylon
4. Modelling of the stay cable behaviour

Aerodynamic category:

1. Aerodynamic properties of the deck cross-section
2. Aerodynamic loads acting on the stay cables

Four different center span lengths are considered, which are 300m, 423m(exact span length of the Nanpu Bridge), 500m and 600m. They cover the range of the span length of modern medium- to long-span cable-stayed bridges. Two cases of typical cable arrangement in modern cable-stayed bridges are considered, i.e. the parallel cable-plane arrangement and the inclined cable-plane arrangement. Correspondingly, the associated pylon shapes are taken as the *H* shape as in the Nanpu Bridge, and the inverted-*Y* shape as in the modified bridge. The combinations of these structural parameters yields eight different cable-stayed bridges, as listed in Table 5.15. The names of these bridges are being assigned according to the following rules:

1. The first letter "B" represents "Bridge".
2. Following, numerical number represents the span length of the bridge.

3. The letter “S” indicates the material of the stay cables are made of the conventional high-strength steel. This is for the purpose to distinguish from other parametric bridges analyzed in Chapter 6, of which the carbon fiber reinforced polymer is used as the alternative material for the stay cables.
4. The last letter represents the shape of the pylon and the type of cable arrangement, with the letter “H” representing the H shape pylon and parallel cable-plane arrangement, and the letter “Y” representing the inverted- Y shape pylon and the inclined cable-plane arrangement.
5. For a span length of 423m, the Nanpu Bridge with H shape pylons is denoted by the symbol “NPSH”, whereas that with the inverted- Y shape pylons is denoted by the symbol “NPSY” to represent the modified bridge.

In each of the eight bridges analyzed, two cases of with or without the consideration of the cable vibration behaviour are considered. In the flutter analysis, the cases considered for the deck cross-section of each of the analyzed bridges include both unstreamlined as in the Nanpu Bridge and streamlined section of a flat box girder. For the case that the cable vibration effect is ignored, the aerodynamic loads acting on the stay cables are also neglected. Otherwise, they are included in the flutter analysis by taking the drag coefficient $C_D = 0.7$.

The structural dimensions of the bridges with span lengths of 300m, 500m and 600m are modified proportionally from the original Nanpu Bridge, as presented in Table 5.16. It is assumed that for these bridges, the properties of the main girder are taken the same as that of the original Nanpu Bridge. The cross-sectional areas of the cables in these bridges are presented in Table 5.17. These cable areas are determined in accordance with the common design practice of cable-stayed bridges

by assuming that the vertical component of the cable tension should be able to resist the dead load and live load (taken as 20% – 30% of the dead load) between the two adjacent cable support points on the girder.

5.6.2 Impact of Cable Vibration Effect on the Dynamic Characteristics of Cable-stayed Bridges

The dynamic characteristics of the eight model bridges as discussed in Section 5.6.1 are presented in this section. The cable vibration effects on the behaviour of modern cable-stayed bridges of different structural layout are studied.

5.6.2.1 Variation of Span Length

The impact of span length variation on the natural frequencies of cable-stayed bridges with parallel cable-plane and the inclined cable-plane are shown in Figure 5.33 and Figure 5.34, respectively. The frequencies of six typical low order modes are plotted against the span length variation. They are the fundamental floating mode(floating), the lowest symmetric sway mode(1-S-L), the lowest symmetric heave(1-S-V) and anti-symmetric heave(1-A-V) modes, and the lowest symmetric torsional(1-S-T) and anti-symmetric torsional(1-A-T) modes. Two curves are drawn for each mode, with the solid line representing the results of neglecting the cable vibration behaviour and the dotted line representing the results of considering the cable vibration behaviour.

In all the cases analyzed, the frequencies drop more rapidly when the span length changes from a medium span of 300m to a long span of 423m. Changing from span

length $423m$ to $600m$, the changes in frequencies are more gentle. It is observed that the variation in the span length has a significant impact on the sway behaviour of the bridge girder. For the case of the parallel cable-plane arrangement, the frequency of the sway mode drops from $0.7909Hz$ for the case of $300m$ center span to $0.1937Hz$ for the case of $600m$ center span. When the center span length is doubled, the sway frequency is reduced by 75.5%. The reduction percentage of the sway mode in the case of the inclined cable-plane arrangement is found to be almost the same.

It can be seen from Figure 5.33 and Figure 5.34 that the natural frequencies of the floating, sway and heave modes are hardly affected by the cable vibration behaviour, whereas in the case of considering also the transverse motion of the cables, the frequencies of the torsional modes are slightly changed. No obvious relationship is identified between the cable vibration effect on the change of the modal frequencies and the variation of the span length. The impact of the transverse motion of the cables on the dynamic characteristics of the cable-stayed bridges is mainly reflected by the change in modal order, the appearances of pure cable modes and the new cable-deck coupled modes, as well as the alteration in the mode shapes of the basic bridge modes.

5.6.2.2 Cable Arrangement Type

Comparing Figure 5.33 with Figure 5.34, the frequencies of the floating, sway and heave modes have not been significantly affected by the change in the cable arrangement type and the pylon shape. However, the frequencies of the torsional modes have been significantly increased when the type of cable arrangement is changed from the parallel cable-plane to the inclined cable-plane. For the lowest symmetric

torsional mode, a 32.8% increase is observed for the span length of 300m, and a 37.58% increase is observed for the case of span length 600m. This increase in torsional frequencies is due to the fact that the inclined cable-plane arrangement type provides a bridge system that is torsionally more rigid. Since the lowest symmetric torsional mode is of most concern in the flutter analysis of cable-stayed bridges, and the critical flutter wind velocity depends significantly on the frequency of this mode, the increase in the lowest symmetric torsional frequency may help to improve the flutter behaviour of the bridge. The inclined cable-plane arrangement is recommended in the design of the long-span cable-stayed bridges to enhance the wind-resistant stability properties of the structure.

The cable vibration behaviour hardly affects the frequencies of the floating, sway and heave modes in both type of the cable arrangements. By taking into account the transverse cable motions, the obtained frequencies of these modes are slightly lower. For the torsional modes, the same trend of decrease in the frequency values due to the cable vibration effect is observed when the bridge has a parallel cable-plane arrangement, whereas the torsional frequencies are found to have slight increases in the case when the inclined cable-plane arrangement is adopted for the bridge.

5.6.3 Impact of Cable Vibration Effect on the Flutter Analysis of Cable-stayed Bridges

The following bridge parameters are chosen in this section to analyze the influence of cable vibration behaviour on the flutter response of the cable-stayed bridges:

- (a) Type of cable arrangement

- (b) Aerodynamic properties of the deck cross-section
- (c) Aerodynamic loads on the stay cables
- (d) Variation of the span length

5.6.3.1 Cable Arrangement Type

The influence of cable vibration on the critical flutter wind velocities of the model bridges with either the parallel cable-plane or the inclined cable-plane are shown in Figure 5.35. This set of flutter analysis results are obtained by choosing four participating modes which are of most concern in the bridge flutter analysis, i.e. the lowest symmetric and anti-symmetric heave and torsional modes. It is assumed that the deck cross-section of these bridges are unstreamlined. The flutter derivatives of the deck cross-section in the Nanpu Bridge are adopted. As shown by the results in Figure 5.35, the critical flutter wind velocity U_f decreases with increasing in the span length. This is because when the span length increases, the torsional frequency decreases as the bridge structure becomes more slender, flexible and prone to flutter.

In both types of cable arrangement, the cable vibration behaviour may help to stabilize the bridge structure. The obtained critical flutter wind velocity is observed to be always higher when the transverse cable motion is considered. The vibrations of the cables help to dissipate additional energy from the bridge system and thus help to stabilize the bridge system. The impact of the cable vibration effect is observed to be more significant for the type of inclined cable-plane arrangement than for the parallel cable-plane arrangement. This is because a bridge with a parallel cable-plane arrangement has a lower torsional stiffness and thus the critical flutter wind velocity of the bridge is lower. The contribution from the cable motion in stabilizing

the bridge may work as an off-set for the potential destabilizing effect. For a bridge with inclined cable-plane arrangement, the structure itself typically has a higher torsional rigidity than that of parallel cable-plane arrangement, the wind-resistant stability behaviour is thus more favourable. The energy dissipation due to the cable vibration tend to enhance the stability performance of this type of cable-stayed structure. Thus, the cable vibration behaviour has more significant effect on the critical flutter wind velocity for the cable-stayed bridges having inclined cable-plane arrangement. This has been demonstrated by the discrepancies between the two sets of curves representing different types of cable arrangement.

5.6.3.2 Aerodynamic Properties of the Deck Cross-section

In order to study the cable vibration effect on bridges with deck cross-sections of different aerodynamic properties, the flutter analysis of eight model bridges are conducted assuming that the bridge deck has the aerodynamic property of the streamlined cross-section. The flutter derivatives of a streamlined flat box deck cross-section, as presented in Figure 5.30, are adopted. The critical flutter wind velocities of all the model bridges are plotted in Figure 5.36. For each bridge, the results obtained by considering or neglecting the cable vibration behaviour are presented.

Similar phenomena in terms of the relationship between the critical flutter wind velocity and the span length variation, as well as the cable vibration effect and the cable arrangement type have been found as presented in Figure 5.35. By comparing these two figures, it is noticed that the cable vibration behaviour has a more significant impact on the critical flutter wind velocity of the bridges having a streamlined deck cross-section than those having an unstreamlined one. It can thus be concluded

that if the cable-stayed bridge itself has favourable aerodynamic stability properties such as high torsional rigidity provided by the inclined cable-plane arrangement or streamlined deck cross-section, the stabilizing effect comes from the cable vibrations may lead to additional improvement to the flutter behaviour of the bridges.

The percentage change in the critical flutter wind velocities due to the cable vibration effect of the model bridges having different type of cable arrangement and different aerodynamic properties of the deck cross-section are shown in Figure 5.37. The impact of cable vibration behaviour is demonstrated more directly in a non-dimensional form. It can be seen in the figure that for the same span length, if the aerodynamic properties of the deck cross-sections are the same, the critical flutter wind velocity of the bridge having a inclined cable-plane arrangement may increase more than that of the bridge having a parallel cable-plane arrangement. This increase is induced by the cable vibration effect. On the other hand, if the bridges have the same span length and the same type of cable arrangement, the stabilizing effect contributed by the cable motion is more significant in the case of a streamlined deck cross-section than that in an unstreamlined one. The increase of the critical wind velocity is found to be greater in the combination case of the inclined cable-plane arrangement and the streamlined deck cross-section than that of the parallel cable-plane arrangement and the unstreamlined deck cross-section. The extreme case of which the cable vibration behaviour has the least significant effect on increasing the critical flutter wind velocity is the case of the parallel cable-plane arrangement combined with the unstreamlined deck cross-section, whereas the most significant effect is observed in the extreme combination case of the inclined cable-plane arrangement with the streamlined deck cross-section.

5.6.3.3 Aerodynamic Loads on the Stay Cable

The importance of including the aerodynamic loads on the stay cables in the flutter analysis of the cable-stayed bridges are studied in this section. The drag coefficient C_D , which represents the aerodynamic loads acting on the cables, are assumed to vary from -1.4 to 1.4 . The hypothetical negative value of C_D physically means that instead of helping to stabilize the structure, the aerodynamic loads on the cables have a destabilizing effect to the bridge system.

The cable-stayed bridges having four different combinations of cable arrangement type and aerodynamic property of the deck cross-section are being studied. For each type of the combination, flutter analysis for bridges of span length $300m$, $423m$, $500m$ and $600m$ are conducted, with the value of the drag coefficient taken as -1.40 , -1.05 , -0.70 , -0.35 , 0.00 , 0.35 , 0.70 , 1.05 and 1.40 , respectively. The analysis results are being plotted in Figure 5.38 to Figure 5.41, with each figure corresponding to one combination case. In each figure, four curves representing the critical flutter wind velocity of four bridges having span lengths varying from $300m$ to $600m$ are plotted against the variation of the drag coefficient. In all cases, the relationship between the change in the bridge critical flutter wind velocity and the drag coefficient variation is found to be approximately linear. The inclusion of the aerodynamic loads on the cables in the flutter analysis may help to stabilize the structure, as indicated by the ascending trend of the curves in the figures.

5.6.3.4 Variation of Span Length

Span length is an important parameter in bridge design. The relationship between the bridge span length and the cable vibration effect on the flutter response is

discussed in this section.

As shown in Figure 5.35 and Figure 5.36, for any selected set of curves representing the critical flutter wind velocity U_f of the cable-stayed bridges, which has certain type of combination in terms of the cable arrangement type and the aerodynamic property of the deck cross-section, the discrepancy between the two curves shows the increase of U_f due to the cable vibration effect. The amount of the discrepancy increases with respect to the increase in the span length, i.e. when the bridge has longer span length, the transverse motion of the cable may contribute more to the stabilization of the structure. This relationship is clearly indicated in Figure 5.37 in a non-dimensional form. For the extreme combination case of the inclined cable-plane arrangement and streamlined deck cross-section, by considering the cable vibration behaviour in the flutter analysis, the critical flutter wind velocity increases by 5.91% for the bridge with 300m center span length and by 18.70% for the bridge with 600m center span length. If the center span of the bridge is longer, the length of the stay cable is also longer. Thus, the transverse motion of the cable may be more significant, which helps to dissipate more energy from the bridge system. Consequently, the cable vibration behaviour has a more significant impact on improving the aerodynamic stability property of the bridge with longer span length.

The effect of including the cable aerodynamic loads in the flutter analysis on the critical flutter wind velocity of the cable-stayed bridges with different combinations of cable arrangement type and aerodynamic property of the deck cross-section has been shown in Figure 5.38 to Figure 5.41. It is indicated by the slope of the curves representing the critical flutter wind velocity of bridges having different span length that the consideration of the aerodynamic loads on the cables in the flutter analysis

has a more significant impact for bridges having longer span length. Since the aerodynamic loads on the cables are functions of the cable length and the velocities of the transverse cable motion, the transverse motions of the cables are more significant because the cables are longer in a bridge of longer span length. These result in the greater aerodynamic loads on the cables and thus affects more on the flutter behaviour of the bridge.

To have a more clear picture of the effect of the cable aerodynamic loads on the bridge flutter response, the percentage change of the critical flutter wind velocities of the cable-stayed bridges having different combinations of the span length, cable arrangement type and aerodynamic property of the deck cross-section are plotted in Figure 5.42 to Figure 5.45, by taking the case of drag coefficient $C_D = 0.0$ as the base. The results obtained are compared to those presented in Figure 5.38 to Figure 5.41. The effect of the cable aerodynamic loads is demonstrated in a non-dimensional form. In all four figures, the level of influence due to the consideration of cable aerodynamic loads on the flutter response of the bridges having different span length can be deduced by the slope of the curves. The impact increases with the increase in span length in all cases.

5.7 Summary and Conclusion

The effect of cable vibration behaviour on the dynamic characteristics and the flutter response of the cable-stayed bridges is studied in this chapter. The structural model of the stay cable is first developed. The proposed three-node cable model can represent accurately not only the transverse motion of the cable, but the interaction between the motion of the cables and the other parts of the superstructure

by introducing the dynamic stiffness in the longitudinal direction. The models of the aerodynamic loads acting on the cables and the bridge deck are derived for use in the flutter analysis. The case study of the Nanpu Bridge is conducted to analyze the cable vibration effect on the dynamic responses of a long-span cable-stayed bridge. Eight model bridges are analyzed in a parametric study to fully understand the impact of cable vibration effect on the dynamic responses of the cable-stayed bridges of different span length, cable arrangement type and aerodynamic property of the deck cross-section. The following conclusions can be drawn from the results obtained in this chapter:

1. The fundamental frequencies of the conventional high strength steel cables in the modern cable-stayed bridges fall in the frequency range of the first several basic bridge modes. The dynamic response of the bridge is influenced by the motion of the cables. The cable vibration behaviour should be considered in the dynamic analysis of the modern long-span cable-stayed bridges.
2. The proposed three-node cable model can accurately represent the transverse motion of the stay cable. It is computationally more efficient than the multi-link approach, especially in the analysis of the modern multi-stay cable-stayed bridges.
3. By considering the cable vibration behaviour, the natural frequencies, mode shapes and modal order of the cable-stayed bridges are altered. A large number of pure cable vibration modes as well as new cable-deck coupled modes are identified. These modes cannot be predicted if the cable is modeled by the single-truss element.

4. The dynamic cable stiffness relates essentially to the value of the cable tension. If the tension drops significantly due to the fatigue and corrosion in the cable, the extensional stiffness of some of the longest cables may become negative and thus influence the dynamic behaviour of the structure. If there is no such significant tension loss, the dynamic cable stiffness does not have pronounced impact on the longitudinal behaviour of the cables.
5. The case study of the Nanpu Bridge shows that the cable vibration behaviour helps to improve the aerodynamic stability of the structure. The critical flutter wind velocity of the bridge is increased by 4.16% due to the consideration of cable motions and the aerodynamic loads on the stay cables.
6. The inclined cable-plane arrangement provides higher torsional rigidity for the bridge system than the parallel cable-plane arrangement. The critical flutter wind velocity is found to be higher in bridges having this type of cable arrangement.
7. For medium to long-span cable-stayed bridges, the frequencies of the floating, sway and heave modes are hardly affected by the cable motion, while the torsional frequencies are slightly influenced. The cable motions tend to decrease the modal frequencies, except for the torsional modes of the bridges having inclined cable-plane arrangement.
8. The stabilizing effect contributed by the transverse motion of the cables is more significant for bridges having favourable aerodynamic stability properties, such as the high torsional rigidity of the bridge system resulting from the inclined cable-plane arrangement, inverted-Y shape pylons, and the deck cross-section

of a streamlined contour.

9. The cable aerodynamic loads may help to increase the critical flutter wind velocity of the bridge. The effect is more significant if the aerodynamic stability properties of the bridge are good.
10. The impact of cable vibration behaviour on the flutter response of the cable-stayed bridges increases with respect to the span length. It contributes to the stabilization of the structure.
11. The cable vibration behaviour and cable aerodynamic loads may have a significant impact on the flutter response of the cable-stayed bridges with long span length. These effects should be considered in the flutter analysis of this type of bridges.

Table 5.1: Properties of the Example Bridge

Item	$A(m^2)$	$I(m^4)$	$E(Mpa)$	Unit weight (KN/m^3)
Deck	1.20	2.58	200000	110.00
Pylon1	14.20	30.00	31600	24.00
Pylon2	35.80	40.00	31600	24.00

Table 5.2: Cable Area of the Example Bridge

Cable No.	Area(m^2)
1	0.060
2	0.040
3	0.028
4	0.028
5	0.040
6	0.050

* The cable elastic modulus $E = 165000Mpa$; unit weight $w = 92.5KN/m^3$.

Table 5.3: Comparison of Three-Node Cable Model and Single-Truss Cable Model
Natural Frequencies and Mode Shapes of the Example Bridge

Proposed Three-Node Cable Element		Single-Truss Cable Element		Mode Shape
Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	
1	0.3481	1	0.3493	Heave(girder, Sym.)
2	0.4615	2	0.4660	Heave(girder, Anti-Sym.)
3-6	0.4844-0.5148			Pure cable motion
7	0.5892	3	0.5872	Heave(girder, Anti-Sym.)
8-10	0.7526-0.8131			Pure cable motion
11	0.8206	4	0.8196	Heave(girder, Sym.)
12	0.8289			Pure cable motion
13	0.9240	5	0.9139	Heave(girder, Anti-Sym.)
14-17	0.9677-1.0250			Pure cable motion
18	1.1017	6	1.0959	Heave(girder, Sym.)
19-22	1.1342-1.1913			Pure cable motion
23	1.3365	7	1.3328	Heave(girder, Anti-Sym.) + Bending(pylon)
24,25	1.4515-1.4559			Pure cable motion
26	1.4956	8	1.4975	Heave(girder, Anti-Sym.)
27-32	1.5226-1.6439			Pure cable motion

Table 5.3: (Continue) Comparison of Three-Node Cable Model and Single-Truss Cable Model Natural Frequencies and Mode Shapes of the Example Bridge

Proposed Three-Node Cable Element		Single-Truss Cable Element		Mode Shape
Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	
33	1.8409	9	1.8309	Heave(girder, Sym.)
34	1.9757	10	1.9673	Heave(girder, Anti-Sym.)
35	2.0741	11	2.0596	Bending(pylon)
36	2.2427	12	2.2466	Heave(girder, Sym.)
37-44	2.2718-2.4678			Pure cable motion
45	2.6979	13	2.6770	Bending(pylon)
46	2.8248	14	2.7972	Heave(girder, Anti-Sym.)
47	3.2054	15	3.1924	Bending(pylon)
48,49	3.4004-3.4217			Pure cable motion
50	3.4382	16	3.4382	Heave(girder, Sym.)
51,52	3.5556-3.5736			Pure cable motion
53	3.6115	17	3.5807	Bending(pylon)
54	3.7316	18	3.7210	Heave(girder, Anti-Sym.)
55	3.9831	19	3.9702	Heave(girder, Sym.)
56	4.1688	20	4.1490	Bending(pylon)

Table 5.4: First Three Modal Frequencies of Stay Cables in the Example Bridge

Cable No.	f_1 (Hz)		f_2 (Hz)		f_3 (Hz)	
	Analytical	Three-node	Analytical	Three-node	Analytical	Three-node
1	0.4838	0.4844 (3)	0.9676	0.9677 (14)	1.4515	1.4514 (24)
2	0.7601	0.7526 (8) 0.7588 (9)	1.5202	1.5226 (27)	2.2803	2.2833 (38)
3	1.1332	1.1342 (19)	2.2663	2.2718 (37)	3.3995	3.4004 (48)
4	1.1913	1.1913 (22)	2.3826	2.3866 (42)	3.5739	3.5736 (52)
5	0.8218	0.8206 (11)	1.6435	1.6439 (32)	2.4653	2.4618 (43) 2.4678 (44)
6	0.5117	0.5091 (5) 0.5147 (6)	1.0234	1.0240 (16)	1.5351	1.5372 (29) 1.5454 (30)
7	0.4853	0.4888 (4)	0.9706	0.9707 (15)	1.4559	1.4558 (25)
8	0.7622	0.7526 (3) 0.7588 (9)	1.5244	1.5259 (28)	2.2865	2.2860 (39) 2.2915 (40)
9	1.1402	1.1410 (20)	2.2803	2.2860 (39) 2.2915 (40)	3.4205	3.4217 (49)
10	1.1849	1.1847 (21)	2.3697	2.3719 (41)	3.5546	3.5556 (51)
11	0.8212	0.8131 (10) 0.8289 (12)	1.6423	1.6417 (31)	2.4635	2.4618 (43) 2.4678 (44)
12	0.5122	0.5091 (5) 0.5147 (6)	1.0243	1.0250 (17)	1.5365	1.5372 (29) 1.5454 (30)

Note: Numbers in the brackets indicate the modal order.

Table 5.5: Comparison of Bridge Frequencies Versus Cable Frequencies

Bridge Frequencies (Hz)		Cable Frequencies (Hz)	
First Mode	Tenth Mode	Longest Cable	Shortest Cable
0.3493	1.9673	0.4844	1.1913

Table 5.6: Comparison of the Fundamental Frequencies of the Stay Cables in the
Example Bridge

Cable No.	Analytical (Hz)	Three-Node Cable Element (Hz)	Multi-link Element (Hz)		
			2 links	4 links	8 links
1	0.4838	0.4844	0.4348-0.4364	0.4633-0.4810	0.4813-0.4876
2	0.7601	0.7526-0.7588	0.6827-0.6860	0.7364	0.7496
3	1.1332	1.1342	1.0208	1.1013-1.1083	1.1277
4	1.1913	1.1913	1.0653-1.0714	1.1623	1.1842
5	0.8218	0.8206	0.7391-0.7415	0.8013	0.8163
6	0.5117	0.5091-0.5147	0.4589	0.4975-0.5029	0.5067-0.5125
7	0.4853	0.4888	0.4348-0.4364	0.4633-0.4810	0.4813-0.4876
8	0.7622	0.7526-0.7588	0.6827-0.6860	0.7408	0.7546
9	1.1402	1.1410	1.0208-1.0267	1.1013-1.1176	1.1347
10	1.1849	1.1847	1.0653-1.0714	1.1554	1.1775
11	0.8212	0.8131-0.8289	0.7391-0.7415	0.7991	0.8139
12	0.5122	0.5091-0.5147	0.4623	0.4975-0.5029	0.5067-0.5125

Table 5.7: Comparison of Three-Node Cable Model and Multi-link Cable Model
Modal Analysis Results

Mode No.	Natural Frequency (Hz)		Mode Shape
	Proposed Three-Node Cable Element	Multi-link Element (8 links per cable)	
1	0.3481	0.3465	Heave(girder, Sym.)
2	0.4615	0.4584	Heave(girder, Anti-Sym.)
3-6	0.4844-0.5148	0.4813-0.5125	Pure cable motion
7	0.5892	0.5895	Heave(girder, Anti-Sym.)
8-10	0.7526-0.8131	0.7496-0.8139	Pure cable motion
11	0.8206	0.8259	Heave(girder, Sym.)
12	0.8289	0.8306	Pure cable motion
13	0.9240	0.9204	Heave(girder, Anti-Sym.)
14-17	0.9677-1.0250	0.9530-0.9997	Pure cable motion
18	1.1017	1.1110	Heave(girder, Sym.)
19-22	1.1342-1.1913	1.1277-1.1842	Pure cable motion

Table 5.8: Impact of Cable Vibration Behaviour on the Dynamic Characteristics of Nanpu Bridge

Single-Truss Cable Element		Proposed Three-Node Cable Element		Mode Shape
Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	
1	0.1216	1	0.1216	Floating(girder, Anti-Sym.)
2	0.3433	2	0.3422	Heave(girder, Sym.)
3	0.3687	3	0.3679	Sway(girder, Sym.)
4	0.4255	4	0.4246	Heave(girder, Anti-Sym.)
		5	0.4696	New mode *
		6-12	0.4758-0.4767	Pure cable motion (#C22)
		13	0.4829	New mode *
		14-20	0.4890-0.4901	Pure cable motion (#C21)
5	0.4992	21	0.4951	Torsion(girder, Sym.)
6	0.5069	22	0.5042	Bending(pylon, Anti-Sym.)
		23-30	0.5256-0.5262	Pure cable motion (#S22)
7	0.5392	31	0.5376	Bending(pylon, Sym.)
		32-38	0.5407-0.5419	Pure cable motion (#C20)
		39	0.5448	New mode *
		40-47	0.5464-0.5474	Pure cable motion (#S21)
		48	0.5537	New mode *
		49-54	0.5571-0.5577	Pure cable motion (#C19)
		55	0.5599	New mode *
		56-63	0.6048-0.6058	Pure cable motion (#S20)
8	0.6106	64	0.6073	Torsion(girder, Anti-Sym.)

* New mode: Dominant cable vibration coupled with torsion of main girder at the center span.

Table 5.9: Fundamental Frequencies of Some Longest Stay Cables in the Nanpu Bridge

Cable No.	Fundamental Frequency (Hz)	
	Analytical	Proposed Three-node Cable Element
C22	0.4762	0.4758 - 0.4767
C21	0.4894	0.4890 - 0.4901
C20	0.5418	0.5407 - 0.5419
C19	0.5593	0.5571 - 0.5577
C18	0.6202	0.6171 - 0.6203
S22	0.5278	0.5256 - 0.5262
S21	0.5495	0.5464 - 0.5474
S20	0.6078	0.6048 - 0.6058
S19	0.6102	0.6105 - 0.6120
S18	0.6574	0.6587 - 0.6599

Table 5.10: Effect of Dynamic Stiffness on the Extensional Stiffness of the Longest and Shortest Cables in the Nanpu bridge

Item		Longest Cable	Shortest Cable
^a $K_e(N/m)$		1.0119×10^7	3.0943×10^7
^b $K_g(N/m)$		3.8815×10^8	2.3517×10^{10}
^c $K_0(N/m)$		0.9862×10^7	3.0902×10^7
^d $K(\omega)$ (N/m)	$\omega = 0.1 \times 2\pi$	0.9862×10^7	3.0902×10^7
	$\omega = 0.2 \times 2\pi$	0.9862×10^7	3.0902×10^7
	$\omega = 0.5 \times 2\pi$	0.9861×10^7	3.0902×10^7
	$\omega = 1.0 \times 2\pi$	0.9812×10^7	3.0902×10^7

$$^a K_e = \frac{EA}{L}, \quad ^b K_g = \frac{12T^3}{(\gamma A)^2 L_h^3}, \quad ^c K_0 = \frac{K_e K_g}{K_e + K_g}, \quad ^d K(\omega) = \frac{K_e(K_g - m\omega^2)}{K_e + (K_g - m\omega^2)},$$

Table 5.11: Impact of Cable Tension Variation on the Dynamic Cable Stiffness

Tension (N)	ω (rad/s)	Dynamic Cable Stiffness $K(\omega)$ (N/m)	
		Longest Cable	Shortest Cable
$0.75T_{DL}$	$0.1 \times 2\pi$	0.9530×10^7	3.0847×10^7
	$0.2 \times 2\pi$	0.9529×10^7	3.0847×10^7
	$0.5 \times 2\pi$	0.9524×10^7	3.0847×10^7
	$1.0 \times 2\pi$	0.9507×10^7	3.0847×10^7
$0.50T_{DL}$	$0.1 \times 2\pi$	0.8370×10^7	3.0620×10^7
	$0.2 \times 2\pi$	0.8365×10^7	3.0620×10^7
	$0.5 \times 2\pi$	0.8323×10^7	3.0620×10^7
	$1.0 \times 2\pi$	0.8156×10^7	3.0620×10^7
$0.25T_{DL}$	$0.1 \times 2\pi$	0.3766×10^7	2.8538×10^7
	$0.2 \times 2\pi$	0.3689×10^7	2.8538×10^7
	$0.5 \times 2\pi$	0.3092×10^7	2.8535×10^7
	$1.0 \times 2\pi$	-0.3964×10^7	2.8525×10^7

Table 5.12: Impact of Cable Vibration Effect and the Aerodynamic Loads Acting on Cables on the Critical Flutter Wind Velocity of Nanpu Bridge (m/s)

Load Case		Cable Vibration Effect	
		Consider	Not Consider
Aerodynamic Loads	Consider	65.49	63.02
	Not Consider	63.58	62.88

**Table 5.13: Impact of Cable Vibration Behaviour on the Dynamic Characteristics
of the Modified Nanpu Bridge**

Single-Truss Cable Element		Proposed Three-Node Cable Element		Mode Shape
Mode No.	Frequency (Hz)	Mode No.	Frequency (Hz)	
1	0.1208	1	0.1208	Floating(girder, Anti-Sym.)
2	0.3450	2	0.3435	Heave(girder, Sym.)
3	0.3863	3	0.3824	Sway(girder, Sym.)
4	0.4270	4	0.4248	Heave(girder, Anti-Sym.)
		5-92	0.4250-0.5763	Pure cable motion
5	0.5825	93	0.5802	Bending(pylon, Anti-Sym.)
		94-101	0.5852-0.5863	Pure cable motion
6	0.5902	102	0.5887	Bending(pylon, Sym.)
		103-126	0.6055-0.6304	Pure cable motion
7	0.6322	127	0.6371	Heave(girder, Sym.)
8	0.6463	128	0.6535	Torsion(girder, Sym.)
		129-144	0.6591-0.6828	Pure cable motion
9	0.6895	145	0.6895	Bending(Anchorage pier, Sym.)
		146-153	0.6973-0.7034	Pure cable motion
10	0.7218	154	0.7297	Torsion(girder, Anti-Sym.)

Table 5.14: Impact of Cable Vibration Effect and the aerodynamic Loads Acting on Cables on the Critical Flutter Wind Velocity of the Modified Nanpu Bridge (m/s)

Load Case		Cable Vibration Effect	
		Consider	Not Consider
Aerodynamic Loads	Consider	86.59	82.20
	Not Consider	83.10	81.31

Table 5.15: Name of the Model Bridges Used in the Parametric Study of Cable
Vibration Effect

Span Length (m)	Pylon Shape	
	H	Inverted- Y
300	B300SH	B300SY
423	NPSH	NPSY
500	B500SH	B500SY
600	B600SH	B600SY

Table 5.16: Structural Dimensions of the Model Bridges

Bridge	Center Span (m)	Side Span (m)	Pylon Height (m)
B300SH, B300SY	300	121.27	109.22
NPSH, NPSY	423	171.00	154.00
B500SH, B500SY	500	202.16	182.03
B600SH, B600SY	600	242.54	218.44

Table 5.17: Cross-sectional Areas of Cables in the Model Bridges

Type	Numbering Sequence	Cross-sectional Area (cm^2)			
1	0, S1, S8, S9, S21, S22 C1, C21, C22	73.0	102.0	121.0	145.0
2	S10	58.0	81.0	96.0	115.0
3	S2, S7, S13 - S20 C2, C7, C13 - C20	51.0	72.0	85.0	102.0
4	S6, S11, S12 C6, C11, C12	45.0	63.0	75.0	90.0
5	S5 C5, C8, C9, C10	41.0	58.0	69.0	83.0
6	S3, S4 C3, C4	35.0	49.0	58.0	70.0

(a)

(b)

Figure 5.1: Transverse vibration of a horizontal string (a) A horizontal string fixed at both ends (b) Equilibrium of an infinitesimal small segment of the string

Figure 5.2: Transverse vibration of the stay cable

Figure 5.3: Layout of a typical inclined stay cable

Figure 5.4: Dynamic cable stretching stiffness

(b)

Figure 5.5: Dynamic behaviour of the stay cable in the longitudinal direction

(a) Equivalent spring-mass-spring system (b) Finite element model

Figure 5.6: Displacement parameters of the three-node stay cable model

Figure 5.7: Geometry and mesh details of the example bridge (a) Geometry of the example bridge (b) Mesh details of the example bridge (c) Connection details

Figure 5.8: (a) Some typical mode shapes of the example bridge (a) Single-truss cable model (b) Proposed three-node cable model

Figure 5.8: (b) Some typical mode shapes of the example bridge (a) Single-truss cable model (b) Proposed three-node cable model

Figure 5.8: (c) Some typical mode shapes of the example bridge (a) Single-truss cable model (b) Proposed three-node cable model

Figure 5.9: Some typical pure cable modes of the example bridge

Figure 5.10: Impact of cable vibration effect on the dynamic characteristics of the example bridge

(a)

(b)

Figure 5.11: Motion of cable in the wind (a) Direction of mean wind and cable motion (b) Effective attack angle

Figure 5.12: Wind load acting on the cable (a) Wind load on the stationary cable
(b) Wind load on the oscillating cable

Figure 5.13: Lumping effects of the distributed aerodynamic forces (a) Lift force
(b) Drag force (c) Aerodynamic moment

Figure 5.14: Triple-beam finite element model of the bridge girder

Figure 5.15: Impact of cable vibration effect on the dynamic characteristics of the Nanpu Bridge

Mechanical Model

Figure 5.16: (a) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 1 FREQUENCY=0.121588 HZ

Figure 5.16: (b) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 2 FREQUENCY=0.342221 HZ

Figure 5.16: (c) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 3 FREQUENCY=0.367946 HZ

Figure 5.16: (d) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 4 FREQUENCY=0.424588 HZ

Figure 5.16: (e) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 21 FREQUENCY=0.495140 HZ

Figure 5.16: (f) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 22 FREQUENCY= 0.504233 HZ

Figure 5.16: (g) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 31 FREQUENCY=0.537643 HZ

Figure 5.16: (h) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 64 FREQUENCY= 0.607344 HZ

Figure 5.16: (i) Mode shapes of the Nanpu Bridge (Three-node cable model)

MODE NUMBER 6 FREQUENCY=0.475750 HZ

Figure 5.17: (a) Pure cable vibration modes of the Nanpu Bridge

MODE NUMBER 7 FREQUENCY= 0.475983 HZ

Figure 5.17: (b) Pure cable vibration modes of the Nanpu Bridge

MODE NUMBER 40 FREQUENCY=0.546377 HZ

Figure 5.17: (c) Pure cable vibration modes of the Nanpu Bridge

MODE NUMBER 42 FREQUENCY= 0.546984 HZ

Figure 5.17: (d) Pure cable vibration modes of the Nanpu Bridge

MODE NUMBER 5 FREQUENCY= 0.469573 HZ

Figure 5.18: (a) New cable-deck coupled modes of the Nanpu Bridge

MODE NUMBER 13 FREQUENCY=0.482913 HZ

Figure 5.18: (b) New cable-deck coupled modes of the Nanpu Bridge

MODE NUMBER 48 FREQUENCY=0.553718 HZ

Figure 5.18: (c) New cable-deck coupled modes of the Nanpu Bridge

Figure 5.19: Impact of cable vibration on the flutter response of the Nanpu Bridge

$\delta - U$ relationship

Figure 5.20: Impact of cable vibration on the flutter response of the Nanpu Bridge

$f - U$ relationship

Figure 5.21: Modification of the pylon shape in the modified Nanpu Bridge (a) Pylon of the original Nanpu Bridge (b) Pylon of the modified Nanpu Bridge

Mechanical Model

Figure 5.22: Numerical model of the modified Nanpu Bridge

Figure 5.23: Impact of cable vibration effect on the dynamic characteristics of the modified Nanpu Bridge (inclined cable-plane)

MODE NUMBER 1 FREQUENCY=0.120830 HZ

Figure 5.24: (a) Mode shapes of the modified Nanpu Bridge

MODE NUMBER 2 FREQUENCY=0.343549 HZ

Figure 5.24: (b) Mode shapes of the modified Nanpu Bridge

MODE NUMBER 3 FREQUENCY=0.382369 HZ

Figure 5.24: (c) Mode shapes of the modified Nanpu Bridge

MODE NUMBER 4 FREQUENCY= 0.424779 HZ

Figure 5.24: (d) Mode shapes of the modified Nanpu Bridge

MODE NUMBER 128 FREQUENCY= 0.653504 HZ

Figure 5.24: (e) Mode shapes of the modified Nanpu Bridge

MODE NUMBER 154 FREQUENCY= 0.729718 HZ

Figure 5.24: (f) Mode shapes of the modified Nanpu Bridge

MODE NUMBER 7 FREQUENCY= 0.426267 HZ

Figure 5.25: (a) Pure cable vibration modes of the modified Nanpu Birdge

MODE NUMBER 8 FREQUENCY= 0.426374 HZ

Figure 5.25: (b) Pure cable vibration modes of the modified Nanpu Birdge

MODE NUMBER 39 FREQUENCY=0.526421 HZ

Figure 5.25: (c) Pure cable vibration modes of the modified Nanpu Birdge

MODE NUMBER 58 FREQUENCY= 0.532404 HZ

Figure 5.25: (d) Pure cable vibration modes of the modified Nanpu Birdge

Figure 5.26: Impact of cable vibration on the flutter response of the modified Nanpu Bridge (inclined cable-plane) $\delta - U$ relationship

Figure 5.27: Impact of cable vibration on the flutter response of the modified Nanpu Bridge (inclined cable-plane) $f - U$ relationship

Figure 5.28: Impact of cable arrangement on the flutter response of the Nanpu

Bridge $\delta - U$ relationship

Figure 5.29: Impact of cable arrangement on the flutter response of the Nanpu Bridge $f - U$ relationship

Figure 5.30: Flutter derivatives of a streamlined flat box deck cross-section

Figure 5.31: Impact of varying drag coefficients on the critical flutter wind velocity of the original and modified Nanpu Bridge

Figure 5.32: Percentage change of critical flutter wind velocity of original and modified Nanpu Bridge due to the variation of drag coefficient

Figure 5.33: Impact of span length variation on the natural frequency of parallel cable-plane cable-stayed bridges

Figure 5.34: Impact of span length variation on the natural frequency of inclined cable-plane cable-stayed bridges

Figure 5.35: Impact of cable vibration effect on the critical flutter wind velocity of cable-stayed bridges (unstreamlined deck cross-section)

Figure 5.36: Impact of cable vibration effect on the critical flutter wind velocity of cable-stayed bridges (streamlined deck cross-section)

Figure 5.37: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the impact of cable vibration effect

Figure 5.38: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (parallel cable-plane, unstreamlined deck cross-section)

Figure 5.39: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (inclined cable-plane, unstreamlined deck cross-section)

Figure 5.40: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (parallel cable-plane, streamlined deck cross-section)

Figure 5.41: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (inclined cable-plane, streamlined deck cross-section)

Figure 5.42: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (parallel cable-plane, unstreamlined deck cross-section)

Figure 5.43: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (inclined cable-plane, unstreamlined deck cross-section)

Figure 5.44: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (parallel cable-plane, streamlined deck cross-section)

Figure 5.45: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (inclined cable-plane, streamlined deck cross-section)

Chapter 6

Dynamic Behaviour of Cable-stayed Bridges Using Carbon Fiber Reinforced Polymer Cables

6.1 Introduction

History of cable material

The stay cables are important structural elements in a cable-stayed bridge. They have significant influences on the structural performance and appearance of the bridge. The development of the cable-stayed bridge benefits greatly from the

progress in the development of the stay cable material.

The use of cable in a structural application can be dated back to the time before recorded history. Cables made of tangled grape vines were used to cross small rivers or large ravines. Suspension bridges made of hemp rope and iron chains were built in China. The adoption of the copper cables in an ancient city of Ninevah near Babylon about 685 B.C. was recorded (Scalzi *et al*, 1969).

The stay cables in cable-stayed bridges are subjected to tension induced by the dead and live loads. The tensile strength of the stay cables is one of the most important mechanical properties of the cable-stayed bridge. The first great application of the cables in the present form was in the 240m suspension bridge across the gorge below Niagara Falls. It was designed by John A. Roebling. The same structural system was applied by Roebling to the Brooklyn Bridge completed in 1882. The tensile strength of the cable wires was only $1,100 \text{ N/mm}^2$ for a 4.2mm diameter galvanized wire. Later, the tensile strength of the cables used in the Williamsburg Bridge completed in 1903 reached $1,470 \text{ N/mm}^2$. In early bridge construction, the cables were made of ungalvanized steel wire to minimize the tensile strength loss due to hot dip galvanizing. The galvanized wires were used in the Manhattan Bridge completed in 1909 with a tensile strength of $1,470 \text{ N/mm}^2$. In 1932, the George Washington Bridge was built. The tensile strength of the wires was upgraded to $1,520 \text{ N/mm}^2$. In the next fifty years, the tensile strength of the bridge wires remained at about the same level. At present, the designed wire strength for the Tatara Bridge, which is the longest cable-stayed bridge today, is about $1,570 \text{ N/mm}^2$. The Akashi Bridge in Japan, which is the longest suspension bridge in the world, has a center span of 2km and two side spans of 1km each. It has been designed with a tensile strength of approximately $1,800 \text{ N/mm}^2$ in its main cables. The progress of the tensile strength

of the bridge wires is shown chronologically in Figure 6.1.

FRP material

The common types of cables used in cable-stayed bridges are mainly the wire ropes, the parallel wire strands (PWS), the parallel wire cables (PWC), and the parallel stranded cables, typically made of steel. Due to their important role in the performance of the cable-stayed bridges, the following requirements of the stay cables should be satisfied (Ohashi, 1991):

- (1) High tensile strength
- (2) High and stable Young's modulus
- (3) Compact cross-section
- (4) High fatigue resistance
- (5) Ease in corrosion protection
- (6) Ease in handling and installation, and
- (7) Low in cost

However, for the traditional steel cable, corrosion and fatigue are two major problems in bridges with high traffic volume or bridges located in corrosion environment. Often, one leads to the other. Corrosion and fatigue problems in stay cables progress almost simultaneously. When the cable vibrates frequently with the bridge, the relative movements between the individual wires damage the corrosion-resistant

surface of the wires through friction, resulting in corrosion damage at these areas. Consequently the fatigue strength drops significantly, which causes the premature rupture of the wires inside the cable. The cost of replacement and maintenance of the steel cable is high.

On the structural performance, the steel cable is relatively heavy, resulting in significant sagging effect due to the self-weight of the steel cable, thus reducing the effective stiffness of the cable and making it behave “softer” under service load.

The recent progress in the development of the advanced composite materials (ACM), which began in the 1940's, offers the alternative of using new materials having improved characteristics as a substituent to the traditional cable material of steel in long-span cable-stayed bridge construction.

The early application of the fiber reinforced plastic (FRP) composites were mainly in the areas of military, aerospace, automobile and sports industries. Due to many of the advantages of the composite materials such as high strength, light weight, good fatigue strength, excellent corrosion resistance ability and non-magnetic properties, they are widely used nowadays in civilian constructions. They can be applied as individual structural members in structural or non-structural systems as well as prestressing systems. Some examples of application are listed as follows:

- The stiffening ribs of the storage tanks
- Poles for highway lighting
- Prestressing systems for concrete
- Stay cable system for the cable-stayed bridges
- Bridge deck

- Guy cables
- Grid systems

The fibers used to produce the fiber reinforced plastic materials include several types of glass, aramid, graphite, boron, silicon carbide and alumina. A wide range of structural properties such as the strength and elastic modulus are produced by these fibers.

The application of FRP materials to the bridge system began in the 1970's. Since then, FRP pedestrian bridges have been built for research purposes. A 4.9m glass fiber reinforced plastic (GFRP) pedestrian bridge was built in 1976 in the USA (Gregory, 1971; McCormick, 1985). The Chongqing cable-stayed pedestrian bridge which has a GFRP box girder (Bruce, 1989), the Beijing-Miyun road bridge constructed in 1982 (Shu, 1983) and the Guanyingqiao bridge completed in 1988 (Bruce, 1989) are all in China. In Germany, the Lünen'sche Gasse experimental bridge was built in 1980 using GFRP strands for the first time (Mufti *et al*, 1991; Meier, 1991; Bedard, 1992). The Adolf Kiepert Bridge, which has two spans of 27.61m and 22.98m, is built more recently in Berlin (Bedard, 1992). In addition, there are several important GFRP highway bridges built in recent years. The roadway of the Ulenbergstrasse Bridge completed in 1986 is the first application of the GFRP prestressing tendons (Ballinger, 1990; Meier, 1991; Mufti *et al*, 1991; Bedard, 1992). The carbon fiber reinforced plastic (CFRP) materials have also been used in bridge construction, such as the Ludwigshafen Bridge in Germany and the Shinmyabashi Bridge in Japan (Zoch *et al*, 1991).

In other cases, the aramid fibers and aramid fiber reinforced plastics have been used for the prestressing, bracing and cable-staying of the structural members, par-

ticularly for the anchorage of oil drill platforms (Burgoyne, 1987; Gerritse, 1990).

CFRP Cable

As mentioned earlier, among the different types of FRP materials of glass, aramid and carbon fibers being considered for possible application in bridge structures, the carbon fiber is identified to have the greatest potential based on its favourable properties as follows:

- (a) High tensile strength
- (b) High elastic modulus
- (c) Low mass density
- (d) Excellent corrosion-resistant ability
- (e) Excellent fatigue-resistant ability
- (f) Low order thermal expansion coefficients

Due to the fact that the CFRP has the highest modulus-density ratio among the available structural materials, it is most suitable for application as the structural members where maximum stiffness and light weight are required, such as the case of cables in long-span cable-stayed and suspension bridges.

The high strength-mass density ratio of CFRP makes it possible to design and construct cable supported bridges of greatly increased spans (Meier, 1987; Kim and Meier, 1991; Saul, 1991). Its high stiffness-mass density ratio reduces the longitudinal deformation of the cables under loads and the sagging effect due to

the self-weight. The outstanding fatigue resistant ability of CFRP satisfies the typical loading conditions of the cable-stayed bridge consisting of large cyclic load amplitudes. Test results (Meier, 1991) show that CFRP can endure at least three times higher in the amplitude of stress and a higher mean stress than the steel material for 2×10^6 cycles without damage. The excellent corrosion resistance ability of CFRP makes it more economical for maintenance as compared to steel. Large structures exposed to severe environment may be subjected to large thermal fluctuations due to the change of climate conditions. This can significantly affect the performance of the structure. The CFRP material, with its low axial thermal expansion coefficient can minimize this temperature related effect.

Carbon fibres are produced by pyrolysis of pitch or polyacrylonitrile (PAN) precursors (Chung, 1994). They are composed of at least 92% by weight of carbon. The polymer matrix serves as a binder and protective coating for the fibres. It allows the applied loads to be transferred to the fibers by shear at the matrix-fibre interface (Neale and Labossiere, 1988). The excellent adhesion between the fibers and the matrix makes CFRP a reliable material. In the case that a single fiber breaks, the shear stress can be transferred to the unbroken fibers a few millimeters away through the matrix. The full load is then taken up by the unbroken fibers. This unique mechanism minimizes the chance of cable failure.

Compared to the traditional cable material of high strength steel, the CFRP has the properties of about 2 times higher in tensile strength, about 80% of the elastic modulus and only 20% of the mass density of steel.

The CFRP cables are manufactured as the assemblies of pultruded wires. The design approach for cables made of unidirectional CFRP wires are similar to that of the conventional steel cables. Some principal aspects that are of most concerned

in the design of the CFRP cable are

- Allow minimal strength loss of the wires in a bundle as compared to single wire.
- Prevent friction between wires.
- Shield against decay due to environmental factors such as erosion and UV radiation.
- Compact cross-section to minimize the aerodynamic drag.
- Convenient to handle.

The strength requirements can be met to a great extent by using the parallel arrangement of the wires. Although the twisting or weaving method to produce cable made it easier for handling, these methods may result in lateral stresses within the loaded cable, and the associated loss in cable stiffness and strength. The typical properties of the T700 CFRP wire are given in Table 6.1 (Miere, 1992).

The CFRP materials are generally unidirectional material. The outstanding mechanical properties of the CFRP materials as discussed are only valid in the longitudinal direction. The properties in the transverse direction including the inter-laminar shear are relatively poor. The development of an efficient anchorage system to achieve the full tensile strength of the CFRP wire becomes a main challenge. Research on this key issue, which is a decisive factor for the wide acceptance of the CFRP cable, has been solved at EMPA (Swiss Federal Laboratories for Materials Testing and Research). The anchorage system developed by Meier (1996) is an effective cone-shaped resin-cast termination. A gradient casting material, also known

as load transfer media (LTM) is developed to fill the space between the metallic cone of the termination and the CFRP wires.

The first application of CFRP as cables in a real cable-stayed bridge is in a road bridge which crosses several railway tracks in Winterthur, Switzerland in April, 1996. As shown in Figure 6.2, among the total of twenty-four cables, two are made of CFRP, while the rest are the conventional steel cables.

The feasibility of using CFRP cables in long-span cable supported bridges attracts increasing attentions from civil engineers. The slender and flexible characteristics of the long-span cable-supported bridge make it sensitive to the dynamic loads. The adoption of CFRP cables, which has different properties from the conventional steel cables, may significantly affect the dynamic response of the bridges. Khalifa (1992) studied the effect of CFRP cables on the seismic response of the cable-stayed bridge. He indicated that the modal participation factors can be significantly influenced.

Very little research on the impact of using CFRP cables in cable-stayed bridges has been conducted. In the current chapter, the effects of using CFRP cables on the dynamic characteristics and flutter response of the cable-stayed bridges are studied. The case study of the Nanpu Bridge is first presented. The effects of using CFRP cables in the cable-stayed bridges having different structural layout and different aerodynamic properties are discussed in a parametric study.

6.2 Dynamic Behaviour of the Nanpu Bridge Using CFRP Cable

6.2.1 Dynamic Characteristic Analysis

In order to study the impact of using CFRP cables on the dynamic behaviour of the Nanpu Bridge, the conventional steel cables of the bridge are substituted by CFRP cables. The mechanical properties and the mass density of the CFRP cables employed in the study are taken the same as those listed in Table 6.1.

Determination of the cross-sectional area of the CFRP cable

The cross-sectional area of the CFRP cables depends on the strength or the stiffness requirement of the bridge, whichever is the controlling factor.

- Case 1: Equivalent cable strength

$$[\sigma]_{steel}A_{steel} = [\sigma]_{CFRP}A_{CFRP} \quad (6.1)$$

or

$$A_{CFRP} = 0.5A_{steel} \quad (6.2)$$

- Case 2: Equivalent cable stiffness

$$E_{steel}A_{steel} = E_{CFRP}A_{CFRP} \quad (6.3)$$

or

$$A_{CFRP} = 1.212A_{steel} \quad (6.4)$$

The cross-sectional areas of the CFRP cables determined respectively by the relationships of Equation (6.2) and Equation (6.4) are used in two separate analyses. In both cases, the stresses in the cables are lower than the ultimate stress $[\sigma]_{CFRP}$ of the CFRP material. However, results show that in case 1, the deflection at the mid-span reaches $4.084m$, which is almost two times of that in the Nanpu Bridge and thus does not satisfy the stiffness requirement. In case 2, a mid-span deflection of $1.942m$ is obtained, which is smaller than that of the Nanpu Bridge and thus meet the stiffness requirement. From the results, the stiffness requirement is the controlling factor for the determination of the cross-sectional areas of the CFRP cables in the case study of the Nanpu Bridge. The cross-sectional areas of the CFRP cables are taken as 1.212 times of that of the corresponding steel cables.

Numerical model

Two types of numerical models are used. In both models, the pylons are modelled as 3D beam elements; the main girder is represented by the triple-beam model with each of the three beams being further discretized into ninety-four 3D beam elements separated by the cable support points, the supports of the piers and the pylon locations in the longitudinal direction; the stay cables are respectively modelled as the single-truss elements and the three-node cable elements proposed in Chapter 5 in the two numerical models. These two numerical models are referred to as “single-truss cable model” and “three-node cable model” in the following discussion of this chapter. To isolate the impact of using different cable materials on the dynamic responses of cable-stayed bridges, the single-truss cable model, which ignores the

cable vibration effect, is adopted in some cases.

6.2.1.1 Natural Frequencies of the Nanpu Bridge Using CFRP Cables

The single-truss cable model is first adopted to study the effect of using CFRP cables on the basic bridge vibration modes. The natural frequencies of the first 10 modes of the Nanpu Bridge, using either the CFRP cables or the conventional steel cables, are presented in Table 6.2. Results show that there is no changes in the modal order of the basic bridge vibration modes and in the mode shapes by substituting the steel cables with the CFRP cables. The modal frequencies for the case of CFRP cables are higher than those of the Nanpu Bridge using the conventional steel cables.

Since the CFRP cables in the case study are designed according to the principles of equivalent cable stiffness given by Equation (6.3) and Equation (6.4), the mass density of the CFRP material is about 20% of that of the conventional steel, the resulting total mass of the CFRP cable is significantly less, about 24% of that of the steel cable. With the same stiffness and much lower mass attached to the whole system, the vibration modes of the bridge thus have higher frequencies. The adoption of the CFRP cables in the Nanpu Bridge has the effect of stiffening the structure by increasing its natural frequencies.

6.2.1.2 Vibration Effect of CFRP Cable

The vibrational characteristics of the CFRP cables, as well as the cable vibration effect on the dynamic characteristics of the Nanpu Bridge using CFRP cables, are

studied in this section.

The natural frequencies of the first 15 modes of the Nanpu Bridge using CFRP cables, either considering or neglecting the cable vibration behaviour, are presented in Table 6.3. As shown in the table, the modal frequencies of the basic bridge modes are not affected by the motion of the cables. For the first 15 modes given in the table, the modal orders of the two sets of results are the same, no pure cable modes or new cable-deck coupled modes have been extracted between the frequency range of $0.1240Hz$ to $0.8403Hz$ of the first 15 basic bridge modes of the Nanpu Bridge by the use of CFRP cables.

The modal frequencies of the Nanpu Bridge using the CFRP cables are plotted in Figure 6.3 against the mode number. Two curves, with one representing the results of considering the cable motion, and the other of neglecting the cable motion effect, are presented. It can be seen from the figure that the flat regions representing the pure motion of the CFRP cables do not appear until the frequency reaches higher than $0.9Hz$.

Compared to the results given in Table 5.8, which shows the impact of cable vibration behaviour on the dynamic characteristics of the Nanpu Bridge using the conventional steel cable, within the frequency range of $0.1216Hz$ to $0.6106Hz$ of the first 8 lowest bridge modes, 56 extra modes, including 51 pure cable modes and 5 new cable-deck coupled modes have been identified. The mode shapes of the basic bridge modes, as shown in Figure 5.16, have been found to be significantly affected by the cable motions.

In order to compare the vibrational behaviour of the cables in the Nanpu Bridge made of different materials, the fundamental frequencies of ten longest cables in the bridge presented in Table 6.4 are studied. Among the ten cables, five are in the

center span with labels from “C22” to “C18”, another five are in the side span with labels from “S22” to “S18”. The fundamental frequencies of the vertical (in-plane) and swing (out-of-plane) modes of these steel and CFRP cables, obtained from either the modal analysis by using the three-node cable model or the analytical solution according to Equation (5.6), are listed in the table. The agreement between the modal analysis results and the analytical solutions are fairly good, which demonstrates again the validity and reliability of the three-node cable model proposed in chapter 5. The frequency of the CFRP cable is found to be almost twice that of the corresponding steel cable. The fundamental frequency of the cable depends on its length, tension and mass per unit length. For the corresponding CFRP and steel cable, there is no change in the cable length, the cable tension is affected slightly due to the change in the cable mass. The most significant change exists in the cable mass density. As discussed earlier in this chapter, the mass of the CFRP cable is about 24% of that of the corresponding steel cable. Thus, if the cable is made of CFRP instead of steel, the fundamental frequency of the cable may be almost doubled. The frequency range of the CFRP cables is beyond the range of the first few basic bridge modes, such as the lowest heave and torsional modes that are most concerned in the flutter analysis. The motions of the CFRP cables thus do not show pronounced influence on the first few basic bridge modes, in terms of either the modal frequencies or the mode shapes.

Shown in Figure 6.4 are the relationships between the modal frequencies and the modal order of the Nanpu Bridge, using either the CFRP cables or the steel cables. Comparison between the two curves show that the vibration of the steel cables developed much earlier than the CFRP cables. They appear between the first few basic bridge modes. Due to the overlap of the frequency ranges between

the steel cables and the basic bridge modes, the vibrational behaviour of the steel cables may have a more significant impact on the dynamic characteristics of the cable-stayed bridge than that of the CFRP cables. The influence of the CFRP cable motions appears in higher modes. Besides the pure cable modes, some new modes, including the swing motion of a group of cables and the cable-deck coupled motion are developed. Some of the selected new modes are shown in Figure 6.5.

6.2.2 Flutter Analysis

6.2.2.1 Flutter Response of the Nanpu Bridge Using CFRP Cables

To isolate the influence of using different cable materials on the flutter response of the Nanpu Bridge, the cable vibration effect is first neglected in the flutter analysis. The critical flutter wind velocity of the Nanpu Bridge with the CFRP cables is 64.75m/s , as presented in Table 6.5. It is increased by 2.97%, when compared to the result of 62.88m/s using the conventional steel cable. This is because that by using the CFRP cables, the modal frequencies, including that of the lowest symmetric torsional mode, are increased. The torsional rigidity of the whole system has been improved. For the bridge having an unstreamlined deck cross-section such as the Nanpu Bridge, flutter is usually induced by the divergence behaviour of the lowest symmetric torsional mode. Thus, the enhancement of the bridge torsional rigidity helps to increase the critical flutter wind velocity of the bridge structure, as is the case here.

The flutter response of the Nanpu Bridge of both cases with the CFRP cables

and the conventional steel cables are presented in Figure 6.6 and Figure 6.7. Shown in Figure 6.6 are the relationships between the logarithmic decrement of the participating modes and the wind velocity. For the cases of using different cable materials, similar trends in the change of the oscillating amplitudes of the corresponding modes are observed. The critical flutter wind velocity of the bridge using the CFRP cables is slightly higher than that using the steel cables, as indicated by the slight shift to the right of intersection of the curve representing the logarithmic decrement of the lowest symmetric torsional mode and the horizontal line representing the neutral state of the structure. The variations of the frequencies of the participating modes with respect to the wind velocity are plotted in Figure 6.7. It is found that for the Nanpu Bridge using the CFRP cables or the conventional steel cables, behaviour between the response frequencies and the wind velocity are similar. The response frequencies of the bridge using CFRP cables are always higher.

The adoption of the CFRP cables in the cable-stayed bridges shows some improvement in the aerodynamic stability of the structure.

6.2.2.2 CFRP Cable Vibration Effect

As discussed in Chapter 5, the cable vibration effect increases the critical flutter wind velocity of the cable-stayed bridge. Due to its light weight and high strength, the CFRP cables have different vibrational characteristics from that of the steel cables. The influence of considering the motions of CFRP cables on the flutter response of the cable-stayed bridge needs to be studied. It is conducted in this section.

Considering the cable vibration effect, the critical flutter wind velocity of the

Nanpu Bridge with the CFRP cables is presented in Table 6.5. It is indicated by the obtained result that in the case of the Nanpu Bridge, which has a center span of $423m$, the CFRP cable motions do not affect the aerodynamic stability of the bridge. This is because for the Nanpu Bridge using the CFRP cables, the fundamental frequency of the longest cable is $0.9213Hz$, which is far beyond the frequency range of the heave and torsional modes chosen for the flutter analysis. The modal frequencies and mode shapes of these modes are not affected by the transverse motions of the CFRP cables. In these modes, the oscillation amplitudes of the CFRP cables are too small to be of significance. The energy dissipation due to the vibrational behaviour of the CFRP cables are thus much less significant than the case of the conventional steel cable. The magnitude of the cable aerodynamic loads depends on the motions of the cables. If the motions are very small, the generated self-excited aerodynamic loads on the cable are small, and thus do not contribute significantly to the enhancement of the flutter performance of the bridge. For the case of the steel cables, the critical flutter wind velocity of the bridge can be increased by 4.15% if the cable motions are considered.

It is very important to note here that in the traditional finite-element based flutter analysis approach, the cable vibration effect is neglected by using the single-truss element to model the stay cable, and only the self-excited aerodynamic loads acting on the bridge deck are considered. Using this traditional approach, the adoption of the CFRP cables in the cable-stayed bridge shows some help in the increase of the critical flutter wind velocity of the bridge. But in reality, when the bridge oscillates under the dynamic loads, vibrations in the cables also occur. Further, the cable motions will produce the self-excited aerodynamic loads on the cables. Thus, the single-truss cable model cannot properly represent the dynamic behaviour of the

stay cables in cable-stayed bridges.

By employing the three-node cable model to consider the vibrational behaviour of the cables, and also includes the aerodynamic loads on the cables, the critical flutter wind velocity obtained for the case of the CFRP cables is lower than that of the steel cables. This is due to the more significant oscillatory behaviour of the steel cable which helps to dissipate more energy from the system, and the larger magnitude of the aerodynamic loads helps more on the stabilization of the structure. Thus, in reality, the adoption of the CFRP cables in the Nanpu bridge does not have an advantage in the flutter behaviour.

Since cable motions can significantly influence the dynamic response of a cable-stayed bridge, especially when the conventional steel cables are used, the cable motions should be considered in the dynamic analysis of the cable-stayed bridges. The motions of the cables cannot be appropriately modelled by using the single-truss cable element. The proposed three-node cable model is more accurate and efficient approach to model the vibrational behaviour of the cables.

The impact of the cable vibration behaviour on the flutter response of the Nanpu Bridge with the CFRP cables are shown in Figure 6.8 and Figure 6.9. The relationships between the logarithmic decrement of the participating modes and the variation of the response frequencies are plotted respectively in these two figures with respect to the wind velocities. The overlap of the corresponding curves proves again that by considering the vibrational behaviour of the CFRP cable, there is slight change in the flutter behaviour of the Nanpu bridge.

6.2.3 Dynamic Behaviour of the Modified Nanpu Bridge Using CFRP Cable

For the purpose of studying the impact of using CFRP cable on the dynamic response of the cable-stayed bridge having different structural layout from the Nanpu bridge, the modified bridge, which has inclined cable-plane arrangement and inverted-Y shaped pylons as described in Section 5.5.3, is analyzed here by using the CFRP cables. The cross-sectional area of the CFRP cables are determined according to Equation (6.4).

6.2.3.1 Dynamic Characteristic Analysis

The natural frequencies of the first ten modes of the modified bridge, either by using the CFRP cables or the conventional steel cables are presented in Table 6.6. The cable vibration effect is neglected here to focus more on studying the influence induced by the different cable materials. Results indicate that by using the CFRP cables, the modal frequencies of the corresponding modes are increased by 0.6% – 6.6%, except for the ninth mode, which is the local bending of the anchorage pier. The reason of this “stiffening” effect caused by using the CFRP cable is the same as explained in Section 6.2.1.1. The modal order as well as the mode shapes of the first few modes of the bridge are found here not to be affected by the use of different cable materials.

The effect of the CFRP cable vibration on the dynamic characteristics of the modified Nanpu Bridge is presented in Table 6.7. For the first few modal frequencies listed in the table, there is no significant difference between the two sets of results.

Considering the case of the fundamental frequency of the longest CFRP cable given in Table 6.8, the frequency of 0.91Hz is beyond the frequency range of the first few modes that are usually used in the dynamic analysis. As observed from the mode shapes, the cable motions occur at frequencies beyond that of the first five basic bridge modes. For the rest of the modes, the cable motions become more and more significant when the bridge frequency increases.

Figure 6.10 shows the impact of CFRP cable vibration on the modal frequencies and the modal order of the modified bridge. No difference between the two curves has been observed for the first few modes. The flat regions representing the local vibrations of the CFRP cables appear much later than the case of using the conventional steel cable. This has been clearly demonstrated in Figure 6.11, where by considering the cable vibration behaviour, the modal frequencies obtained by using both the CFRP cables and the conventional steel cables are plotted against the modal order. Again, it is concluded that the motions of the steel cables have a more significant impact on the dynamic characteristics of the cable-stayed bridges than that of the CFRP cables. The fundamental frequencies of the ten longest cables in the modified Nanpu Bridge, using either CFRP or steel material, are presented in Table 6.8. Due to its light weight and high strength, the frequency of the CFRP cable is found to be nearly doubled as that of the corresponding steel cable.

6.2.3.2 Flutter Analysis

Effect due to different cable material

Ignoring the cable vibration effect, the critical flutter wind velocity of the modified Nanpu Bridge with CFRP cables is obtained to be 86.31m/s . The flutter

analysis results of the bridge obtained from four combinations in terms of different cable material and different interpolation of the cable behaviour are listed in Table 6.9. If the transverse motion of the cable is neglected in the analysis, it is found that by using the CFRP cables, the critical flutter wind velocity of the modified bridge has been increased by 6.15% to 86.31m/s , as compared to the result of 81.31m/s obtained by using the steel cables. This is because of the “stiffening” effect of the CFRP cables to the bridge system. The enhancement to the torsional rigidity of the structure yields higher aerodynamic stability property of the bridge. By comparing to the corresponding results of 64.75m/s and 62.88m/s of the Nanpu Bridge given in Table 6.5, it can be seen that the adoption of the CFRP cables helps more to increase the critical flutter wind velocity of the cable-stayed bridge having inclined cable-plane arrangement and inverted- Y shape pylons than that in the case of the parallel cable-plane arrangement and H shape pylons, provided that the other structural informations are the same.

Neglecting the motions of the cables, the flutter response of the modified bridge, using either the CFRP cables or the conventional steel cables, are shown in Figure 6.12 and Figure 6.13. As indicated by the curves plotted in these two figures, the variations of the decaying rate of the oscillating amplitudes, as well as the response frequencies of the participating modes with respect to the wind velocity are found to have the same trend for the bridges using different materials of the stay cables. For the curves representing the logarithmic decrement of the lowest symmetric torsional mode shown in Figure 6.12, the “stabilizing” effect induced by using the CFRP cables is demonstrated by the delay in the intersection of the curve with the horizontal line representing the neutral state of the structure. The “stiffening” effect contributed by the CFRP cables is indicated in Figure 6.13 by the differences

in frequencies between the corresponding two curves representing the same mode.

Effect of cable vibration behaviour

By referring to the flutter analysis results presented in Table 6.9, the influence of the CFRP cable vibration on the critical flutter wind velocity of the modified bridge is very small, 86.63m/s as compared to 86.31m/s , the increase is only 0.36%. In reality, this order of difference in the critical flutter wind velocity is negligible.

The effect of CFRP cable vibration on the flutter response of the modified bridge is shown in Figure 6.14 and Figure 6.15. The variation of the logarithmic decrement and the response frequency of the participating modes are respectively plotted against the wind velocity in these two figures. The corresponding curves obtained by either considering or neglecting the transverse motion of the cable are nearly the same, which indicates that the vibrational behaviour of the CFRP cables does not affect the flutter response of the modified bridge. This is the same conclusion obtained in Section 6.2.2.2 for the Nanpu bridge using the CFRP cables.

Listed also in Table 6.9 are the critical flutter wind velocities obtained by using the steel cables, either neglect or consider the motions of the cables with the value of 81.31m/s and 86.59m/s , respectively. A more significant impact due to the cable vibration on the aerodynamic stability of the bridge has been found in the case of using the conventional steel cables. The increase in the critical flutter wind velocity is 6.49%. The insignificant impact of considering the cable motions of the CFRP cables on the flutter response of the modified bridge can be explained similarly by the reasons given in Section 6.2.2.2.

Effect of the variation of cable aerodynamic loads

As indicated by the study results in Section 5.5.3.3, for both of the original and the modified Nanpu Bridge using steel cables, the variation of the cable aerodynamic loads may affect the flutter behaviour of the cable-stayed bridges. Previous results show that the motion of the CFRP cable and its effect on the response of the bridge system are different from the case of the steel cable. As the magnitudes of the cable aerodynamic loads depend on the motion of the cables, it is therefore necessary to study the effect of the variation of the aerodynamic loads acting on the stay cables on the flutter behaviour of the cable-stayed bridge using CFRP cables.

For convenience in the following discussion, the Nanpu Bridge using CFRP cables is referred as “NPFH”, while the modified bridge using CFRP cables is referred as “NPFY”.

Shown in Figure 6.16 are the impact of the variation of cable aerodynamic loads on the flutter response of the bridges NPFH and NPFY. The bottom two curves representing the results obtained under the condition that the bridge deck is assumed to have the same unstreamlined geometric contour as the Nanpu Bridge. Corresponding to the variation range of the drag coefficient C_D from -1.4 to 1.4 , the critical flutter wind velocities of the bridges NPFH and NPFY increase very slowly. This is because that the fundamental frequencies of the CFRP cables are beyond the frequency range of the heave and the torsional modes chosen for flutter analysis. The motions of the cables are not excited in these modes. The magnitude of the self-excited aerodynamic loads on the cables relates directly to the velocities of the cable motion. For small amplitude of CFRP cable motions, the associated cable aerodynamic loads are small, which have minor effect on the flutter behaviour of the bridges. Thus, the variation of the small cable aerodynamic loads may not

have significant influence on the critical flutter wind velocity of the cable-stayed bridges.

Effect of different aerodynamic property of the deck cross-section

The impact of different aerodynamic property of the deck cross-section on the critical flutter wind velocity of the cable-stayed bridges NPFH and NPFY are presented in Table 6.10. The results for the cases of the unstreamlined deck cross-section in the table are obtained from the same deck cross-section of the Nanpu Bridge, whereas for cases of the streamlined deck cross-section are obtained from the flat box cross-section used in Section 5.5.3.3. As shown by the results, the critical flutter wind velocities of the bridges NPFH and NPFY are almost triple for the case of the streamlined flat box cross-section as compared to that of the Nanpu Birdge. The critical flutter wind velocities of the bridges are more significantly affected by the transverse motion of the cables if the bridge girder has a streamlined deck cross-section. This stabilization effect attributed to the oscillation of the CFRP cables are found to be more pronounced in the case of bridge NPFY, which has an inclined cable-plane arrangement and inverted-Y shape pylons. Similar behaviour is observed in the cases of bridges using steel cables that the transverse motions of the stay cables have a more significant impact on the flutter behaviour of the cable-stayed bridges with favourable aerodynamic stability properties such as high torsional rigidity of the structural system and a streamlined deck cross-section.

The upper two curves shown in Figure 6.16 represents the variations of the critical flutter wind velocities of the bridges NPFH and NPFY with respect to the

drag coefficient C_D , for the case of a streamlined deck cross-section. The effect of varying the drag coefficient on the critical flutter wind velocity is found to be more significant in the case of a bridge having a streamlined deck cross-section, as can be noted from the different slopes of the four curves in the figure. The relationship between the variations of the drag coefficient and the corresponding critical flutter wind velocity is found to be almost linear. The percentage change of the critical flutter wind velocity of the bridges NPFH and NPFY due to variation of the drag coefficient is plotted in Figure 6.17. The influence of the drag coefficient variation on the flutter response behaviour of bridges NPFH and NPFY, with streamlined and unstreamlined deck cross-sections, is shown in a non-dimensional form. The most significant change in the behaviour has been noticed in the case of the bridge NPFY using streamlined deck cross-section, where the combination of the inclined cable-plane arrangement, the inverted-Y shape pylons and the streamlined deck cross-section makes it to have the most favourable aerodynamic stability property among the group of the four different cases considered.

6.3 Parametric Study

6.3.1 Definition of the Model Bridges

To better understand the effect of using CFRP cables on the dynamic response of the cable-stayed bridges, eight model bridges, with the same structural information as given in Section 5.6.1, are analyzed in this section. The conventional steel cables in the bridges are replaced by the CFRP cables according to the relationship given by Equation (6.4), i.e. $A_{CFRP} = 1.212A_{steel}$. The notations adopted to identify

the analyzed bridges are the same as before, as explained in Section 5.6.1. Since the model bridges in this section use CFRP cables, so the letter “S”, which represents steel cables in Chapter 5, are replaced by “F” to represent the CFRP cables considered in this chapter. The cases of the modal bridges analyzed are listed in Table 6.11.

6.3.2 Impact of Using CFRP Cables on the Dynamic Characteristics of the Cable-stayed Bridges

6.3.2.1 Effect of Different Cable Materials

The influences of using CFRP cables as substituents to the conventional steel cables on the dynamic characteristics of the model bridges B300FH, NPFH, B500FH and B600FH are analyzed. These four bridges, with the range of span length from 300m to 600m, all have parallel cable-plane arrangement and *H*-shaped pylons. The natural frequencies of six typical low order modes in these bridges, i.e. the floating, lowest sway, heave (Sym. and Anti-Sym.) and torsion (Sym. and Anti-Sym.), are plotted against the span length in Figure 6.18.

For comparison purpose, frequencies of these six modes in model bridges B300SH, NPSH, B500SH and B600SH with the conventional steel cables, as discussed in Chapter 5, are also plotted in the figure. For the same mode, the modal frequency of the bridge using the CFRP cables is found to be always higher than that using the conventional steel cables. The replacement of the conventional steel cables by the CFRP cables manifests a “stiffening” effect to the structural system. Among the six typical low order modes, the floating mode is not affected by the replacement

of the stay cable material. The influence on the sway mode is also small. The modal frequencies of the heave and torsional (1-S-V, 1-A-V, 1-S-T, 1-A-T) modes are increased by 2% to 6% when CFRP cables are used. The increase in the modal frequencies is attributed to the properties of the CFRP material as compared to that of the steel. The light weight CFRP material reduces the total mass of the system, its high strength helps to increase the rigidity. With the reduced mass and increased rigidity, the natural frequencies of the structural system becomes higher.

6.3.2.2 Effect of Different Cable Arrangement Type

The influence of using the CFRP cables on the dynamic characteristics of the model bridges B300FY, NPFY, B500FY and B600FY are shown in Figure 6.19. These four model bridges all have the inclined cable-plane arrangement and the inverted-Y shape pylons. In the figure, the dotted lines represent the modal analysis results of the model bridges using the CFRP cables, whereas the solid lines represent the results of the model bridges using the steel cables, i.e. B300SY, NPSY, B500SY and B600SY, as described in Section 5.6.1. As indicated by the differences between the corresponding curves, the frequencies of the floating and sway modes are slightly increased by using the CFRP cables. The dotted line and the solid line which represent respectively the CFRP cable results and the steel cable results are nearly the same for the two modes considered. The most significant increase is found in the case of the frequency of the lowest symmetric torsional mode(1-S-T) by 3.17% for the bridge B300FY, and by 9.3% for the bridge B600FY.

Figure 6.20 shows the comparison of the “stiffening” effect resulted from using the CFRP cables in cable-stayed bridges with parallel cable-plane arrangement and

inclined cable-plane arrangement. The percentage changes in the modal frequencies are plotted in the figure using the steel cable results as the reference base. The effect of using CFRP cables on the floating, sway, heave and the lowest anti-symmetric torsional modes for the two different types of cable-stayed bridges are found to be almost the same. For the case of the lowest symmetric torsional mode, the adoption of CFRP cables shows the most significant stiffening effect when the bridge has an inclined cable-plane arrangement.

6.3.2.3 Effect of Cable Vibration Behaviour

The effect of cable vibration behaviour on the dynamic characteristics of the cable-stayed bridges having parallel cable-plane arrangement, *H* shape pylons and inclined cable-plane arrangement, inverted-*Y* shape pylons are shown respectively in Figure 6.21 and Figure 6.22. Small difference in the behaviour is noticed in the lowest anti-symmetric heave mode (1-A-V) and the lowest torsional modes (1-S-T, 1-A-T) for bridges having inclined cable-plane arrangement as shown in Figure 6.22, where the frequencies of these modes are found to be slightly decreased due to the motion of the cable, especially when the bridge has a longer span length. Since the fundamental frequencies of the CFRP cables are relatively high as compared to the conventional steel cables, they are usually beyond the frequency range of the first few basic bridge modes that are often chosen for the dynamic analysis. Thus, the motions of the cables do not affect the modal properties of these modes. In the cable-stayed bridges having inclined cable-plane arrangement and inverted-*Y* shape pylons, the torsional frequencies are significantly increased due to the enhanced torsional rigidity of the structure. These frequencies are closer to the fundamental

frequencies of the CFRP cables and thus may be more easily affected. The cable vibrations have the effect similar to attaching mass to the system, which decreases the modal frequencies of the bridge.

6.3.2.4 Effect of Span Length Variation

In this section, discussion is focused on the effect of variation of the span length on the dynamic characteristics of the cable-stayed bridges with CFRP cables.

From the results shown in Figure 6.18 and Figure 6.19, it is obvious the modal frequencies of bridges decrease with respect to the increase in the span length. The change in the frequency of the floating mode is the smallest, whereas the most significant change is observed in the sway mode(1-S-L). The trend of frequency change with respect to the variation in the span length is found to be the same in both cases of the steel cables and the CFRP cables. As can be noted from the non-dimensional relationship presented in Figure 6.20, the stiffening effect caused by the adoption of the CFRP cables increases with the increase in the span length, i.e. the longer the span length of the bridge, the more pronounce is the stiffening effect.

As shown by the results in Figure 6.21 and Figure 6.22, the motions of the CFRP cables have no effect on the modal frequencies of the cable-stayed bridges having span length between 300m to 600m considered in the current study. No apparent relationship is observed between the CFRP cable vibration effect and the span length for the model bridges considered in the present study.

6.3.3 Impact of Using CFRP Cables on the Flutter Response of the Cable-stayed Bridges

6.3.3.1 Effect of Different Cable Arrangement Type

The critical flutter wind velocity of all eight model bridges, i.e. B300FH, NPFH, B500FH, B600FH, B300FY, NPFY, B500FY and B600FY, obtained under both conditions of either neglecting or considering the cable vibration effect are plotted in Figure 6.23. The aerodynamic property of the deck cross-section of these bridges are assumed to be unstreamlined as the Nanpu Bridge. The flutter derivatives corresponding to the original Nanpu Bridge deck cross-section, as given by Figure 4.7, are adopted to determine the critical flutter point. Four natural modes, i.e. the lowest symmetric and anti-symmetric heave and torsional modes are chosen in the flutter analysis of each bridge. The cable vibration effect on the critical flutter wind velocities of the cable-stayed bridges using CFRP cables will be discussed later in Section 6.3.3.3.

As shown in Figure 6.23, if the bridges have the same span length but different type of cable arrangement, the bridge with the inclined cable-plane arrangement always yields higher critical flutter wind velocity than that with the parallel cable-plane arrangement. This is because that the inclined cable-plane arrangement may provide higher torsional stiffness to the whole structural system. For cable-stayed bridge having unstreamlined deck cross-section, flutter is usually induced by the divergence behaviour of the lowest symmetric torsional mode. If the torsional rigidity of the bridge is enhanced, the divergence of the torsional mode may show up later, which results in the higher critical flutter wind velocity.

The increase of the bridge critical flutter wind velocity induced by the adoption of the inclined cable-plane arrangement as compared to the parallel cable-plane arrangement for the cable-stayed bridges using either the conventional steel cables or the CFRP cables is shown in Figure 6.24. The difference between the critical flutter wind velocity of the model bridges having the same span length, the same cable material, the same aerodynamic property of the deck cross-section, but different cable arrangement type are plotted in a dimensionless form against the span length by taking the results of the case of the parallel cable-plane bridge as the reference base. As indicated by the second and the fourth curves counting from the top in the figure, which represents bridges having unstreamlined deck cross-section, the type of cable arrangement has a more significant effect on the critical flutter wind velocity if the CFRP cables are employed in the cable-stayed bridges instead of the conventional steel cables.

6.3.3.2 Effect of Different Aerodynamic Property of the Deck Cross-section

Assume that the deck cross-section of all eight model bridges have the same aerodynamic property as a streamlined flat box. The effect of the CFRP cables on the flutter behaviour of the cable-stayed bridges having streamlined deck cross-section is discussed. Using the flutter derivatives of the streamlined flat box cross-section given by Figure 5.30, the critical flutter wind velocities of the eight model bridges using the CFRP cables are plotted in Figure 6.25. Results show that by using the streamlined deck cross-section, the critical flutter wind velocity of the model bridges are drastically increased. The cable vibration effect will be discussed later in Sec-

tion 6.3.3.3. The difference between the second and the fourth curves counting from the top in the figure demonstrates the effect of the cable arrangement type on the critical flutter wind velocity of the model bridges if the deck cross-section of the bridges are assumed to be streamlined.

This difference is plotted in a non-dimensional form in Figure 6.24, which is the first curve at the top. For comparison purpose, the effect of the cable arrangement on the critical flutter wind velocity of the cable-stayed bridges having same streamlined deck cross-section, but using the conventional steel cables are also plotted in Figure 6.24, which is the third curve counting from the top.

By comparing the four curves shown in Figure 6.24, it can be seen that if the same cable material is used, either the conventional steel or the CFRP, the critical flutter wind velocities of the cable-stayed bridges having streamlined deck cross-section are increased more than those having unstreamlined deck cross-section. Further, if the aerodynamic properties of the deck cross-sections are the same, the impact of cable arrangement type on the flutter behaviour is found to be more significant if the CFRP cables are employed in the cable-stayed bridges instead of the conventional steel cables.

6.3.3.3 Effect of Cable Vibration Behaviour

The effect of cable vibrations on the critical flutter wind velocities of the cable-stayed bridges using the CFRP cables are shown respectively in Figure 6.23 and Figure 6.25 for bridges having either the unstreamlined or the streamlined cross-sections.

In Figure 6.23, the corresponding curves representing the critical flutter wind velocities of the bridges having unstreamlined deck cross-section by either considering

or neglecting the motions of the CFRP cables are found to be nearly the same. This indicates that by using the CFRP cables, the cable motions do not significantly affect the flutter behaviour of the cable-stayed bridges having unstreamlined deck cross-section. The oscillations of the CFRP cables are found to have a slight beneficial effect on the aerodynamic stability of the cable-stayed bridges having streamlined deck cross-section, as noted in Figure 6.25. This is shown by the small differences between the corresponding flutter wind velocity curves obtained by neglecting and considering the CFRP cable motions.

The effect of the CFRP cable vibration on the flutter behaviour of the cable-stayed bridges are presented in a dimensionless form in Figure 6.26. When compared to Figure 5.37, which demonstrates the steel cable vibration effect on the critical flutter wind velocity of the cable-stayed bridges, similar phenomena can be observed that the motions of the cables have a more significant impact on the flutter response of the cable-stayed bridges having favourable aerodynamic stability properties such as higher structural torsional rigidity contributed by the inclined cable-plane arrangement and streamlined deck cross-section. If two bridges with the same span length have the same type of cable arrangement, the critical flutter wind velocity may be increased more because of the cable motion in the bridge having streamlined deck cross-section. On the other hand, if two bridges with the same span length have the same deck cross-section, the stabilizing effect contributed by the cable motions is found to be more significant in the bridge having inclined cable-plane arrangement.

The influence of the cable vibration on the flutter behaviour of the cable-stayed bridges is found to be much more significant in the case of the steel cables than that in the case of the CFRP cables. Since the oscillation of the cables helps to dissipate energy from the system, the more violent the motions of the cables, the more energy

of the bridge system may be dissipated. Consequently, the more stabilizing effect due to the cable motions may be demonstrated. Because the fundamental frequencies of the steel cables are much lower than those of the CFRP cables and fall in the frequency range of the first few modes that are usually used in the dynamic analysis, the modal characteristics of the first few basic bridge modes are more affected if steel cables are employed. The mode shapes of the first few basic bridge modes involve significant cable motions in the case of the steel cable, whereas for the CFRP cables, this is not the case. The more significant motions of the steel cables, as compared to that of the CFRP cables, contribute more to the stabilizing effect of the bridge.

6.3.3.4 Effect of Drag Coefficient Variation

The self-excited aerodynamic loads acting on the stay cables are functions of the drag coefficient and cable motions. The variation of the drag coefficient may change the magnitude of the aerodynamic loads and thus affect the flutter behaviour of the bridge structure. The effect of drag coefficient variation on the critical flutter wind velocity of the Nanpu Bridge (NPFH) and the modified bridge (NPFY) using CFRP cables have been discussed early in Section 6.2.3. A more comprehensive study covering range of the bridge span length from 300m to 600m is conducted in this section.

The eight model bridges described in Section 6.3.1, i.e. B300FH, NPFH, B500FH, B600FH, B300FY, NPFY, B500FY and B600FY, are used in the current study. The critical flutter wind velocity of each bridge are calculated by taking the drag coefficient C_D respectively as -1.40 , -1.05 , -0.70 , -0.35 , 0.00 , 0.35 , 0.70 , 1.05 and 1.40 . The influences of the drag coefficient variation on the critical flutter wind velocities

of the cable-stayed bridges having different span length, different type of cable arrangement as well as different aerodynamic property of the deck cross-section are shown in Figures 6.27 through 6.30.

In each figure, the critical flutter wind velocity of the cable-stayed bridges having the same structural layout, the same aerodynamic property of the deck cross-section and different span length ranging from 300m to 600m are plotted against the variation of the drag coefficient. Each figure contains four curves. Every individual curve represents the flutter analysis results of a particular bridge. In all the cases, the flutter response of the bridge is found to be slightly affected by the variation of the cable aerodynamic loads, as indicated by the gentle slope of the curves in these figures.

Shown in Figures 6.31 through 6.34 are the percentage changes of the critical flutter wind velocity of the cable-stayed bridges having different combinations of span length, cable arrangement type and deck aerodynamic property. They correspond respectively to Figures 6.27 through 6.30. Taking the critical flutter wind velocity obtained under the condition of $C_D = 0.0$ as the reference base, the influence of the drag coefficient variation on the flutter behaviour of the cable-stayed bridges using CFRP cables is demonstrated in a dimensionless form.

Compare Figure 6.31 with Figure 6.33, and Figure 6.32 with Figure 6.34, it can be seen that if the bridges have the same type of cable arrangement, the variation of the drag coefficient may have a more significant impact on the flutter response of the bridge having streamlined deck cross-section. On the other hand, if comparisons are made between bridges having same aerodynamic property of the deck cross-section, such as compare Figure 6.31 with Figure 6.32, or Figure 6.33 with Figure 6.34, the influence on the bridge flutter response due to the drag coefficient

variation is found to be more significant if the bridge has the inclined cable-plane arrangement. Thus, similar conclusions as those obtained in the case of the cable-stayed bridges using steel cables discussed in Section 5.6.3.3 can be drawn here. The cable aerodynamic loads contribute to the stabilization of the bridge structure. The inclusion of aerodynamic loads on the stay cables in the flutter analysis may help to increase the critical flutter wind velocity of the cable-stayed bridge. The stabilizing effect induced by the cable aerodynamic loads is more distinct when the structure itself has favourable aerodynamic stability property, such as the relatively higher structural torsional rigidity provided by the inclined cable-plane arrangement or/and streamlined deck cross-section.

When compared to the figures from Figure 5.38 to Figure 5.45, which shows the drag coefficient variation effect on the cable-stayed bridges using steel cables, it can be observed that the flutter response of the cable-stayed bridges may be affected more significantly by the drag coefficient variation when the conventional steel cables are used instead of the CFRP cables.

Since the magnitudes of the aerodynamic loads acting on the cables are proportional to the velocities of the cable motions, the aerodynamic loads acting on the steel cables may have larger magnitudes than those acting on the CFRP cables. The inclusion of the cable aerodynamic loads in the flutter analysis may lead to an increase of the critical flutter wind velocity of the bridge. The larger the magnitudes of the cable aerodynamic loads, the more increase of the critical flutter wind velocity will be. Thus, the cable aerodynamic loads may contribute more to the improvement of the flutter behaviour if the steel cables are used instead of the CFRP cables.

6.3.3.5 Effect of Different Cable Materials

The adoption of the light weight, high strength CFRP cables may affect the dynamic performance of the cable-stayed bridges. The influence induced by different cable materials on the flutter behaviour of the cable-stayed bridges are analyzed in this section. Flutter analysis of sixteen model bridges are conducted, with eight using steel cables as described in Section 5.6.1 and another eight using CFRP cable as described in Section 6.3.1.

These sixteen model bridges are divided into four groups according to the different combinations of structural layout and aerodynamic property of the deck cross-section. Flutter analysis results of each group of the bridges, using either the conventional steel cables or the CFRP cables, are presented in Figures 6.35 through 6.38. For each case, the critical flutter wind velocity obtained under conditions of either neglecting or considering the cable vibration behaviour are plotted in the figures against the bridge span length. Results show that for bridges having the same span length, the same cable arrangement type and the same aerodynamic property of the deck cross-section, if the transverse motion of the cables is not considered, the CFRP-cable-system may yield higher critical flutter wind velocity than that of the steel-cable-system. This is because that the replacement of the conventional steel cables with the light weight, high strength CFRP cables helps to enhance the rigidity of the structure, particularly for the torsional motion. The torsional stiffness of the system is an important factor in the determination of the aerodynamic stability of the bridge structure. The higher the structural torsional stiffness, the more stable the structure will be. However, it is very important to note here that if the cable vibration effect is considered, the more pronounced motions of the steel cables

and larger magnitudes of the aerodynamic loads acting on the steel cables result in the steel-cable-bridge-system having a better aerodynamic performance than the CFRP-cable-bridge-system, as demonstrated by the results shown in Figures 6.35 to 6.38. In each group of the bridges, the highest critical flutter wind velocity is always achieved when the steel cables are employed and the motions of the steel cable are considered in the analysis.

The impact of different cable materials on the flutter behaviour of the cable-stayed bridges are shown more clearly by Figure 6.39 and Figure 6.40 in a non-dimensional form. In Figure 6.39, the effect of the different cable materials is studied under the condition that the cable motions are not considered in the analysis. The percentage change of the bridge critical flutter wind velocity due to the adoption of different cable materials are plotted against the variation of span length by taking the flutter analysis result of the steel-cable-bridge-system as the base. It can be observed from the figure that if cable motions are neglected in the analysis, using CFRP cables in the bridge instead of the conventional steel cables may yield higher critical flutter wind velocity, as demonstrated by the positive ordinate of the points on the curves. The impact induced by the different cable material on the flutter behaviour of the bridge is found to be more significant when the bridges have inclined cable-plane arrangement type or/and unstreamlined deck cross-section.

Curves plotted in Figure 6.40 also shows the impact of different cable materials on the critical flutter wind velocity of cable-stayed bridges. However, in this case, the motions of the cables are considered. Similar to the results shown in Figure 6.39, the percentage change of the bridge critical flutter wind velocities are plotted against the span length variation by taking the results of the steel-cable-system as the reference base. The negative ordinates of the points on the curves indicate that by considering

oscillations of the cables, the steel-cable-system is aerodynamically more stable than the CFRP-cable-system. If bridges having same span length and same aerodynamic property of the deck cross-section are considered, the cable material effect is found to be more significant when the bridge has inclined cable-plane arrangement, as indicated by comparing the first and the second, or the third and the fourth curves counting from the top of the figure.

If the model bridges have the same span length and the same type of cable arrangement, by considering the cable motion in the analysis, the effect of different cable materials may have a more significant impact on the flutter response of the bridge having streamlined deck cross-section. This can be obtained by comparing the first and the third, or the second and the fourth curves counting from the top of the figure.

In the traditional finite-element-based flutter analysis approaches, the transverse motions of the cables as well as the cable aerodynamic loads are neglected in the flutter analysis. By following such approaches, the CFRP-cable-system manifests better aerodynamic stability performance than the conventional steel-cable-system. In reality, the stay cables tend to oscillate with the deck and pylon(s) under the dynamic loadings. The transverse motions of the stay cables generate the self-excited aerodynamic loads on the cables. If the real situation is modelled by considering the cable vibration behaviour and cable aerodynamic loads, the adoption of the CFRP cables may not manifest advantages on the aerodynamic stability of the cable-stayed bridges, as compared to the conventional steel-cable-bridge-system. Thus, the accurate modelling of the bridge behaviour and the load conditions are essential, or the totally different conclusions will be drawn.

6.3.3.6 Effect of Span Length Variation

From Section 6.3.3.1 to Section 6.3.3.5, the impact of using CFRP cables on the flutter response of the cable-stayed bridges have been conducted. Eight model bridges with span length ranging from 300m to 600m are used in the study. It is found that the critical flutter wind velocity of the bridge decreases due to the increase of the span length. This is because that the longer span length results in more slender and flexible structure, which is prone to flutter.

By referring to the parametric studies carried out in the previous sections as well as the associated figures from Figure 6.23 to Figure 6.40, it can be seen that the significance of the impact on the flutter response of the CFRP cable-stayed bridges due to the different type of cable arrangement, different aerodynamic property of the deck cross-section, cable vibration behaviour, drag coefficient variation and different cable materials increases with respect to the increase of the span length. The longer the bridge span length, the more influence will be found in the flutter response of the CFRP cable-stayed bridges due to the variation of various parameters. This conclusion is the same as that obtained from the parametric studies of the cable-stayed bridges using steel cables in Chapter 5.

6.4 Summary and Conclusion

The carbon fiber reinforced polymer (CFRP) material has the unique properties of high tensile strength, high elastic modulus, excellent corrosion resistance ability, outstanding fatigue performance, low mass density and low thermal expansion. These distinctive characteristics of the CFRP material made it an ideal substitute to

the traditional steel to be used for the cables in the cable supported structures, such as the cable-stayed bridge. Due to the light weight and high strength of the CFRP, there is the potential for its application in the long-span cable supported bridges in the near future.

The history of the stay cable materials, the properties of the CFRP material and the state-of-the-art application of the CFRP material are first reviewed in this chapter. Then, the case study of the Nanpu Bridge by using the CFRP cables is carried out to study the impact of CFRP cables on the dynamic response of the bridge. A more comprehensive study of the effects of using CFRP cables has been conducted by using eight model bridges with different combinations of span length and structural layout. The following conclusions can be drawn from the study conducted in this chapter:

1. The adoption of the CFRP cables in the cable-stayed bridges may have a stiffening effect on the bridge system by increasing the modal frequencies.
2. Compared to the conventional steel cables, the fundamental frequencies of the CFRP cables are relatively high due to its light weight and high strength. They may be beyond the frequency range of the first few basic bridge modes that are usually used in the dynamic analysis.
3. The motions of the CFRP cables do not affect the modal characteristics of the first few basic bridge modes. By considering the motions of the cables, the modal order does not change, the modal frequencies are slightly decreased and the vibrations of the cables do not develop in these modes. The effect of cable motions on the dynamic characteristics of the cable-stayed bridges using

CFRP cables is not obvious unless the frequency of the basic bridge mode is close to the fundamental frequency of the CFRP cables.

4. Compared to the cable-stayed bridges using steel cables, if the cable motions as well as the cable aerodynamic loads are neglected in the analysis, the adoption of the CFRP cables manifests stabilization effect by achieving higher critical flutter wind velocity. However, if the motions of the cables and the cable aerodynamic loads are considered in the analysis, as is what happens in reality, the steel-cable-bridge-system is found to have a better aerodynamic stability performance than the CFRP-cable-bridge-system. The accurate modelling of the structural behaviour and load conditions are essential for the proper determination of the structural responses.
5. The effect induced by the different cable arrangement type on the flutter response of the cable-stayed bridges using CFRP cables is found to be more significant than that in the case of the cable-stayed bridges using steel cables. Further, in the CFRP-cable-system, this effect is more significant if the bridge has a streamlined deck cross-section instead of an unstreamlined one.
6. The influence of the variation of the cable aerodynamic loads on the flutter response of the cable-stayed bridges using CFRP cables is much less significant than that of the cable-stayed bridges using steel cables. The relationships between the varied drag coefficient and the associated critical flutter wind velocities of the cable-stayed bridges using CFRP cables are found to be almost linear.
7. The increase of the bridge span length yields a more slender and flexible struc-

ture. The critical flutter wind velocity of the bridge may decrease with respect to the increment of the span length. The impact due to the different cable arrangement type, different aerodynamic properties of the deck cross-section, cable vibration behaviour, variation of the cable aerodynamic loads, different cable materials on the flutter response of the cable-stayed bridges using CFRP cables are found to be more and more significant with the increasing of the span length.

Table 6.1: Properties of T700 (LY556/HY917/DY070) CFRP Wires

Tensile strength	$\sim 3300N/mm^2$
Load capacity of $\phi 6mm$ wire	85 – 90000N
Elastic modulus	$\sim 165000N/mm^2$
Density	$1.56g/cm^3$
Fiber content	$\sim 68\%$
Thermal expansion	$0.2 \times 10^{-6}m/m/^{\circ}C$ (longitudinal) $35.0 \times 10^{-6}m/m/^{\circ}C$ (transverse)

Table 6.2: Comparison of Natural Frequencies of the Nanpu Bridge by Using Cables of Different Materials (Not Consider Cable Vibration Effect)

Mode No.	Natural Frequency (Hz)		Mode shape
	Steel cable	CFRP cable	
1	0.1216	0.1240	Floating(girder, Anti-Sym.)
2	0.3433	0.3573	Heave(girder, Sym.)
3	0.3687	0.3819	Sway(girder, Sym.)
4	0.4255	0.4362	Heave(girder, Anti-Sym.)
5	0.4992	0.5171	Torsion(girder, Sym.)
6	0.5069	0.5206	Bending(pylon, Anti-Sym.)
7	0.5392	0.5517	Bending(pylon, Sym.)
8	0.6106	0.6248	Torsion(girder, Anti-Sym.)
9	0.6365	0.6513	Heave(girder, Sym.)
10	0.6895	0.6895	Bending(Anchorage pier, Sym.)

Table 6.3: Impact of Cable Vibration Behaviour on the Dynamic Characteristics of the Nanpu Bridge by Using CFRP Cable

Mode No.	Natural Frequency (Hz)		Mode shape
	Single-truss cable model	Proposed three-node cable model	
1	0.1240	0.1240	Floating(girder, Anti-Sym.)
2	0.3573	0.3572	Heave(girder, Sym.)
3	0.3819	0.3818	Sway(girder, Sym.)
4	0.4362	0.4362	Heave(girder, Anti-Sym.)
5	0.5171	0.5168	Torsion(girder, Sym.)
6	0.5206	0.5203	Bending(Pylon, Anti-Sym.)
7	0.5517	0.5515	Bending(Pylon, Sym.)
8	0.6248	0.6246	Torsion(girder, Anti-Sym.)
9	0.6513	0.6508	Heave(girder, Sym.)
10	0.6895	0.6895	Bending(Anchorage pier, Sym.)
11	0.7502	0.7497	Heave(girder, Anti-Sym.)
12	0.7706	0.7701	Heave(girder, Sym.)
13	0.7861	0.7853	Sway(girder, Anti-Sym.)
14	0.7940	0.7936	Heave(girder, Anti-Sym.)
15	0.8403	0.8358	Torsion(girder, Sym.)

Table 6.4: Comparison of the fundamental frequencies of the stay cables using different materials in the Nanpu Bridge

Cable No.	First vertical mode (Hz)				First swing mode (Hz)			
	Steel cable		CFRP cable		Steel cable		CFRP cable	
	FEM	Analytical	FEM	Analytical	FEM	Analytical	FEM	Analytical
C22	0.4758	0.4762	0.9213	0.9213	0.4760	0.4762	0.9204	0.9213
C21	0.4891	0.4894	0.9389	0.9391	0.4890	0.4894	0.9383	0.9391
C20	0.5407	0.5418	1.0118	1.0138	0.5410	0.5418	1.0129	1.0138
C19	0.5572	0.5593	1.0963	1.0964	0.5571	0.5593	1.0958	1.0964
C18	0.6171	0.6202	1.1776	1.1776	0.6177	0.6202	1.1765	1.1776
S22	0.5261	0.5278	1.0792	1.0867	0.5256	0.5278	1.0777	1.0867
S21	0.5470	0.5495	1.0945	1.0921	0.5464	0.5495	1.0940	1.0921
S20	0.6057	0.6078	1.0963	1.0965	0.6048	0.6078	1.0958	1.0965
S19	0.6119	0.6102	1.1097	1.1107	0.6105	0.6102	1.1092	1.1107
S18	0.6596	0.6574	1.1223	1.1275	0.6587	0.6574	1.1261	1.1275

Table 6.5: Impact of Using CFRP Cables on the Critical Flutter Wind Velocity of the Nanpu Bridge (m/s)

Cable material	Transverse cable motion and cable aerodynamic loads	
	Not Consider	Consider
CFRP	64.75	64.75
Steel	62.88	65.49

Table 6.6: Comparison of Natural Frequencies of the Modified Nanpu Bridge by Using Cables of Different Materials (Not Consider Cable Vibration Effect)

Mode No.	Natural Frequency (Hz)		Mode shape
	Steel cable	CFRP cable	
1	0.1208	0.1228	Floating(girder, Anti-Sym.)
2	0.3450	0.3569	Heave(girder, Sym.)
3	0.3863	0.3913	Sway(girder, Sym.)
4	0.4270	0.4363	Heave(girder, Anti-Sym.)
5	0.5825	0.5917	Bending(pylon, Anti-Sym.)
6	0.5902	0.6036	Bending(pylon, Sym.)
7	0.6322	0.6523	Heave(girder, Sym.)
8	0.6463	0.6892	Torsion(girder, Sym.)
9	0.6895	0.6895	Bending(Anchorage pier, Sym.)
10	0.7218	0.7451	Torsion(girder, Anti-Sym.)

Table 6.7: Impact of Cable Vibration Behaviour on the Dynamic Characteristics of the Modified Nanpu Bridge by Using CFRP Cable

Mode No.	Natural Frequency (Hz)		Mode shape
	Single-truss cable model	Proposed three-node cable model	
1	0.1228	0.1228	Floating(girder, Sym.)
2	0.3569	0.3569	Heave(girder, Sym.)
3	0.3913	0.3913	Sway(girder, Sym.)
4	0.4363	0.4363	Heave(girder, Anti-Sym.)
5	0.5917	0.5917	Bending(Pylon, Anti-Sym.)
6	0.6036	0.6036	Bending(Pylon, Sym.)
7	0.6523	0.6518	Heave(girder, Sym.)
8	0.6892	0.6873	Torsion(girder, Sym.)
9	0.6895	0.6895	Bending(Anchorage pier, Sym.)
10	0.7451	0.7438	Torsion(girder, Anti-Sym.)
11	0.7508	0.7503	Heave(girder, Anti-Sym.)
12	0.7683	0.7679	Heave(girder, Sym.)
13	0.7930	0.7926	Heave(girder, Anti-Sym.)
14	0.7964	0.7957	Sway + Torsion(girder, Anti-Sym.)
15	0.8467	0.8443	Sway + Torsion(girder, Sym.)

Table 6.8: Comparison of the fundamental frequencies of the stay cables using different materials in the Modified Nanpu Bridge

Cable No.	First vertical mode (Hz)				First swing mode (Hz)			
	Steel cable		CFRP cable		Steel cable		CFRP cable	
	FEM	Analytical	FEM	Analytical	FEM	Analytical	FEM	Analytical
C22	0.4250	0.4270	0.9099	0.9205	0.4259	0.4270	0.9193	0.9205
C21	0.4355	0.4373	0.9370	0.9387	0.4362	0.4373	0.9374	0.9387
C20	0.4793	0.4812	1.0121	1.0142	0.4799	0.4812	1.0133	1.0142
C19	0.5280	0.5321	1.0964	1.0975	0.5291	0.5321	1.0958	1.0975
C18	0.5741	0.5792	1.1723	1.1792	0.5748	0.5792	1.1769	1.1792
S22	0.5227	0.5260	1.0902	1.1009	0.5215	0.5260	1.0911	1.1009
S21	0.5264	0.5283	1.1018	1.1056	0.5258	0.5283	1.1023	1.1056
S20	0.5315	0.5353	1.1041	1.1087	0.5312	0.5353	1.1048	1.1087
S19	0.5424	0.5458	1.1179	1.1215	0.5420	0.5458	1.1181	1.1215
S18	0.5544	0.5538	1.1340	1.1369	0.5541	0.5538	1.1342	1.1369

Table 6.9: Impact of Using CFRP Cables on the Critical Flutter Wind Velocity of the Modified Nanpu Bridge (m/s)

Cable material	Transverse cable motion	
	Not Consider	Consider
CFRP	86.31	86.63
Steel	81.31	86.59

Table 6.10: Impact of Different Aerodynamic Property of the Deck Cross-section on the Critical Flutter Wind Velocity of the Original and the Modified Nanpu Bridge Using CFRP Cables (m/s)

Deck cross-section		Unstreamlined		Streamlined	
Cable motion		Neglect	Consider	Neglect	Consider
Bridge name	NPFH	64.75	64.75	185.69	186.63
	NPFY	86.31	86.63	245.38	248.50

Table 6.11: Name of the Model Bridges Used in the Parametric Study of CFRP Cable Effect

Span Length (m)	Pylon Shape	
	H	Inverted- Y
300	B300FH	B300FY
423	NPFH	NPFY
500	B500FH	B500FY
600	B600FH	B600FY

Figure 6.1: Historical change of bridge wire tensile strength

(a)

(b)

Figure 6.2: Application of the CFRP cables in a real cable-stayed bridge (a) View of the bridge from west to east (b) Bridge in service

Figure 6.3: Impact of cable vibration effect on the dynamic characteristics of the Nanpu Bridge (CFRP cable)

Figure 6.4: Impact of using CFRP cable and considering cable vibration effect on the dynamic characteristics of the Nanpu Bridge

MODE NUMBER 15 FREQUENCY= 0.856082 HZ

Figure 6.5: (a) Some selected higher order new modes of the Nanpu Bridge using CFRP cable

MODE NUMBER 16 FREQUENCY= 0.857910 HZ

Figure 6.5: (b) Some selected higher order new modes of the Nanpu Bridge using CFRP cable

MODE NUMBER 17 FREQUENCY= 0.904466 HZ

Figure 6.5: (c) Some selected higher order new modes of the Nanpu Bridge using CFRP cable

MODE NUMBER 18 FREQUENCY= 0.906511 HZ

Figure 6.5: (d) Some selected higher order new modes of the Nanpu Bridge using CFRP cable

Figure 6.6: Impact of using CFRP cable on the flutter response of the Nanpu

Bridge $\delta - U$ relationship

Figure 6.7: Impact of using CFRP cable on the flutter response of the Nanpu Bridge $f - U$ relationship

Figure 6.8: Impact of cable vibration on the flutter response of the Nanpu Bridge
(CFRP cable) $\delta - U$ relationship

Figure 6.9: Impact of cable vibration on the flutter response of the Nanpu Bridge
(CFRP cable) $f - U$ relationship

Figure 6.10: Impact of cable vibration effect on the dynamic characteristics of the modified Nanpu Bridge (CFRP cable, inclined cable-plane)

Figure 6.11: Impact of using CFRP cable and considering cable vibration effect on the dynamic characteristics of the modified Nanpu Bridge (inclined cable-plane)

Figure 6.12: Impact of using CFRP cable on the flutter response of the modified Nanpu Bridge (inclined cable-plane) $\delta - U$ relationship

Figure 6.13: Impact of using CFRP cable on the flutter response of the modified Nanpu Bridge (inclined cable-plane) $f - U$ relationship

Figure 6.14: Impact of cable vibration on the flutter response of the modified Nanpu Bridge (CFRP cable, inclined cable-plane) $\delta - U$ relationship

Figure 6.15: Impact of cable vibration on the flutter response of the modified Nanpu Bridge (CFRP cable, inclined cable-plane) $f - U$ relationship

Figure 6.16: Impact of varying drag coefficient on the critical flutter wind velocity of the original and modified Nanpu Bridge (CFRP cable)

Figure 6.17: Percentage change of critical flutter wind velocity of the original and modified Nanpu Bridge due to the variation of drag coefficient (CFRP cable)

Figure 6.18: Impact of using CFRP cable on the natural frequency of parallel cable-plane cable-stayed bridges

Figure 6.19: Impact of using CFRP cable on the natural frequency of inclined cable-plane cable-stayed bridges

Figure 6.20: Comparison of stiffening effect induced by the CFRP cables on the cable-stayed bridges having different structural layout

Figure 6.21: Impact of span length variation on the natural frequency of the parallel cable-plane cable-stayed bridges (CFRP cable)

Figure 6.22: Impact of span length variation on the natural frequency of the inclined cable-plane cable-stayed bridges (CFRP cable)

Figure 6.23: Impact of cable arrangement type and cable vibration effect on the critical flutter wind velocity of cable-stayed bridges (CFRP cable, unstreamlined deck cross-section)

Figure 6.24: Percentage change of the critical flutter wind velocity of the cable-stayed bridges induced by the different cable arrangement type

Figure 6.25: Impact of cable arrangement type and cable vibration effect on the critical flutter wind velocity of cable-stayed bridges (CFRP cable, streamlined deck cross-section)

Figure 6.26: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the impact of cable vibration effect (CFRP cable)

Figure 6.27: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (CFRP cable, parallel cable-plane, unstreamlined deck cross-section)

Figure 6.28: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (CFRP cable, inclined cable-plane, unstreamlined deck cross-section)

Figure 6.29: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (CFRP cable, parallel cable-plane, streamlined deck cross-section)

Figure 6.30: Impact of drag coefficient variation on the critical flutter wind velocity of cable-stayed bridges (CFRP cable, inclined cable-plane, streamlined deck cross-section)

Figure 6.31: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (CFRP cable, parallel cable-plane, unstreamlined deck cross-section)

Figure 6.32: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (CFRP cable, inclined cable-plane, unstreamlined deck cross-section)

Figure 6.33: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (CFRP cable, parallel cable-plane, streamlined deck cross-section)

Figure 6.34: Percentage change of critical flutter wind velocity of cable-stayed bridges due to the variation of drag coefficient (CFRP cable, inclined cable-plane, streamlined deck cross-section)

Figure 6.35: Impact of using CFRP cable on the critical flutter wind velocity of cable-stayed bridges (parallel cable-plane, unstreamlined deck cross-section)

Figure 6.36: Impact of using CFRP cable on the critical flutter wind velocity of cable-stayed bridges (inclined cable-plane, unstreamlined deck cross-section)

Figure 6.37: Impact of using CFRP cable on the critical flutter wind velocity of cable-stayed bridges (parallel cable-plane, streamlined deck cross-section)

Figure 6.38: Impact of using CFRP cable on the critical flutter wind velocity of cable-stayed bridges (inclined cable-plane, streamlined deck cross-section)

Figure 6.39: Impact of using CFRP cable on the percentage change of the critical flutter wind velocity of cable-stayed bridges (not consider cable vibration effect)

Figure 6.40: Impact of using CFRP cable on the percentage change of the critical flutter wind velocity of cable-stayed bridges (consider cable vibration effect)

Chapter 7

Summary and Conclusions

7.1 Summary of the Research

The primary objective of the present research is to study the structural and aerodynamic behaviour of the long-span cable-stayed bridges. The finite strip-based 3D structural and aerodynamic force models for the cable-stayed bridges have been developed. The impact of the cable vibration behaviour and the use of CFRP cables on the dynamic responses of the cable-stayed bridges have been analyzed. Altogether four computer programs have been developed for the dynamic analysis and post-processing of the results.

A spline finite strip analytical model of the cable-stayed bridges is developed. The main girder of the bridge is modeled by the flat shell spline finite strips. The pylons are modeled by the 3D beam finite elements. The cables are modeled by the new transition elements developed in the present study, which connect the spline finite strip elements modelling the bridge deck with the 3D beam finite elements

modelling the pylon at the two ends. The penalty function is adopted for the modelling of the boundary conditions. The resulting eigenvalue problem is solved by the subspace iteration method. The advantages and disadvantages of the different mathematical models which can be used to model the long-span bridges, such as the equivalent beam finite element model, the full 3D finite element model and the spline finite strip model, are discussed by presenting two numerical examples.

The aerodynamic force model of the bridge girder is developed by using the spline finite strip method. The mixed time-frequency domain expressions for the aerodynamic forces by Scanlan are adopted as the basis for establishing the aerodynamic model. The aerodynamic stiffness and damping matrices are derived by allocating the aerodynamic forces on each nodal line of the finite strip model representing the bridge girder. The spatial distribution of the aerodynamic forces can thus be considered. The flutter equation of motion is established according to the dynamic equilibrium condition of the bridge structure. The *PK-F* method is employed as a computational algorithm for seeking the critical point of flutter. Numerical example of the flutter analysis of a simply supported thin flat plate is presented to verify the reliability and accuracy of the proposed model and method.

The case study of a real long-span cable-stayed bridge—the Nanpu Bridge, is presented. The dynamic characteristics and the flutter response of the bridge are analyzed by using the proposed finite strip structural and aerodynamic force model. The dynamic analysis results are then compared with those obtained from the wind tunnel tests and other analytical models or methods. The potential of the future application of the proposed finite strip model is also discussed.

Studies of the cable vibration impact on the dynamic response of the cable-stayed bridges are carried out. A three-node cable model is developed, which can

properly represent the behaviour of the stay cables in the oscillating bridge system. The models of the aerodynamic loads acting on the cables and the bridge deck are derived for the flutter analysis. The case study of the Nanpu Bridge is conducted to analyze the cable vibration effect on the dynamic responses of a real long-span cable-stayed bridge. Eight model bridges are analyzed in the parametric study to fully understand the impact of cable vibration effect on the dynamic responses of the cable-stayed bridges with different combinations of structural layout.

The history of the cable materials, the properties of the CFRP material and the state-of-the-art application of the CFRP material are reviewed. The case study of the Nanpu Bridge by using the CFRP cables is carried out to study the impact of CFRP cables on the dynamic characteristics and flutter response of the bridge. A more comprehensive study of the CFRP cable effect on the dynamic response of the cable-stayed bridges has been conducted by using eight model bridges with different combinations of structural layout.

7.2 Conclusions of the Present Study

The main conclusions drawn from the studies undertaken in the present research are listed in the sequence in which they appear in the thesis:

Spline finite strip-based structural analysis

1. The spline finite strip analytical model developed in the current study is a reliable and accurate alternative to the finite element approach for the analysis of bridge structures.

2. Compared to the equivalent beam finite element model and the full 3D finite element model, the proposed spline finite strip model can achieve the same level of accuracy in the results with minimal data preparation and modelling effort.
3. The proposed finite strip model can represent the bridge girder in more detail than the equivalent beam model, and requires much less computer storage and time than the full 3D model.

Spline finite strip-based flutter analysis

1. A 3D flutter analysis procedure based on the finite strip method is developed. The proposed finite-strip-based aerodynamic force model for the bridge girder can easily take into account the spatial distribution effects of the wind load. It provides the bridge engineers and designers a convenient alternative way to evaluate the aerodynamic stability behaviour of the bridges, especially in the early design stage.
2. The reliability and accuracy of the proposed aerodynamic model are verified via a numerical example of the simply supported thin flat plate.
3. The flutter response of a simply supported thin flat plate is a bending-torsion coupled classical flutter, of which the frequencies of the heave and torsional motions converge to each other as the wind velocity approaches the critical flutter wind velocity U_f .
4. In the spline finite strip analysis, the span length of the bridge should be divided into sufficient number of sections in order to achieve satisfactory results.

Case study of the Nanpu Bridge

1. The critical flutter wind velocity of the Nanpu Bridge is higher than the checking wind velocity for the self-excited vibration. The bridge can safely resist the occurrence of flutter.
2. Due to the unstreamlined property of the bridge deck cross-section, the flutter behaviour of the Nanpu Bridge is single-degree-of-freedom torsional flutter. The occurrence of the flutter is caused by the divergence behaviour of the lowest symmetrical torsional mode.
3. The spline finite strip model can accurately and conveniently represent the mass and stiffness properties of the bridge girder, especially in the cases that the bridge girder has an open type or composite cross-section.
4. For modelling large structures, the 3D equivalent beam finite element models have less number of degrees-of-freedom than the spline model and requires less computer time and storage. However, the mass and stiffness properties of the structure can be accurately represented by the spline finite strip model with minimal effort in data preparation.
5. The single-spine model cannot properly represent the stiffness of the transverse beam and the warping torsional stiffness of the main girder. In some cases where the warping torsion of the bridge girder should be considered, the adoption of this type of model may yield conservative results.
6. The triple-beam finite element model can consider the warping torsion of the deck cross-section and is most suitable to be employed when the bridge girder has an open type deck cross-section.

7. The spline finite strip model has the obvious advantages over the equivalent beam model in that it can take into account the spatial distribution of the aerodynamic forces. This advantage can be fully utilized and demonstrated with the future advances in the computational fluid mechanics. The proposed finite strip model has the potential in its future application to the dynamic analysis of the long-span bridges.

Impact of cable vibration behaviour

1. The fundamental frequencies of the conventional high strength steel cables in the modern cable-stayed bridges fall in the frequency range of the first several basic bridge modes. The dynamic response of the bridge is influenced by the motion of the cables. The cable vibration behaviour should be considered in the dynamic analysis of the modern long-span cable-stayed bridges.
2. The proposed three-node cable model can accurately represent the transverse motion of the stay cable. It is computationally more efficient than the multi-link approach, especially in the analysis of the modern multi-stay cable-stayed bridges.
3. By considering the cable vibration behaviour, the natural frequencies, mode shapes and modal order of the cable-stayed bridges are altered. A large number of pure cable vibration modes as well as new cable-deck coupled modes are identified. These modes cannot be predicted if the cable is modeled by the single-truss element.

4. The dynamic cable stiffness relates essentially to the value of the cable tension. If the tension drops significantly due to the fatigue and corrosion in the cable, the extensional stiffness of some longest cables may become negative and thus influence the dynamic behaviour of the structure. If there is no such significant tension loss, the dynamic cable stiffness does not have pronounced impact on the longitudinal behaviour of the cables.
5. The case study of the Nanpu Bridge shows that the cable vibration behaviour helps to improve the aerodynamic stability of the structure. The critical flutter wind velocity of the bridge is increased by 4.16% due to the consideration of cable motions and the aerodynamic loads on the stay cables.
6. The inclined cable-plane arrangement provides higher torsional rigidity for the bridge system than the parallel cable-plane arrangement. The critical flutter wind velocity is found to be higher in bridges having this type of cable arrangement.
7. For medium to long-span cable-stayed bridges, the frequencies of the floating, sway and heave modes are hardly affected by the cable motion, while the torsional frequencies are slightly influenced. The cable motions tend to decrease the modal frequencies, except for the torsional modes of the bridges having inclined cable-plane arrangement.
8. The stabilizing effect contributed by the transverse motion of the cables is more significant for bridges having favourable aerodynamic stability properties, such as the high torsional rigidity of the bridge system resulting from the inclined cable-plane arrangement, inverted-Y shape pylons, and the deck cross-section

of a streamlined contour.

9. The cable aerodynamic loads may help to increase the critical flutter wind velocity of the bridge. The effect is more significant if the aerodynamic stability property of the bridge is good.
10. The impact of cable vibration behaviour on the flutter response of the cable-stayed bridges increases with respect to the span length. It contributes to the stabilization of the structure.
11. The cable vibration behaviour and cable aerodynamic loads may have a significant impact on the flutter response of the cable-stayed bridges with long span length. These effects should be considered in the flutter analysis of this type of bridge.

Impact of the adoption of CFRP cables

1. The adoption of the CFRP cables in the cable-stayed bridges may have a stiffening effect on the bridge system by increasing the modal frequencies.
2. Compared to the conventional steel cables, the fundamental frequencies of the CFRP cables are relatively high due to its light weight and high strength. They may be beyond the frequency range of the first few basic bridge modes that are usually used in the dynamic analysis.
3. The motions of the CFRP cables do not affect the modal characteristics of the first few basic bridge modes. By considering the motions of the cables, the modal order does not change, the modal frequencies are slightly decreased

and the vibrations of the cables do not develop in these modes. The effect of cable motions on the dynamic characteristics of the cable-stayed bridges using CFRP cables is not obvious unless the frequency of the basic bridge mode is close to the fundamental frequency of the CFRP cables.

4. Compared to the cable-stayed bridges using steel cables, if the cable motions as well as the cable aerodynamic loads are neglected in the analysis, the adoption of the CFRP cables manifests stabilization effect by achieving higher critical flutter wind velocity. However, if the motions of the cables and the cable aerodynamic loads are considered in the analysis, as is what happens in reality, the steel-cable-bridge-system is found to have a better aerodynamic stability performance than the CFRP-cable-bridge-system. The accurate modelling of the structural behaviour and load conditions are essential for the proper determination of the structural responses.
5. The effect induced by the different cable arrangement type on the flutter response of the cable-stayed bridges using CFRP cables is found to be more significant than that in the case of the cable-stayed bridges using steel cables. Further, in the CFRP-cable-system, this effect is more significant if the bridge has a streamlined deck cross-section instead of an unstreamlined one.
6. The influence of the variation of the cable aerodynamic loads on the flutter response of the cable-stayed bridges using CFRP cables is much less significant than that of the cable-stayed bridges using steel cables. The relationships between the varied drag coefficient and the associated critical flutter wind velocities of the cable-stayed bridges using CFRP cables are found to be almost linear.

7. The increase of the bridge span length yields a more slender and flexible structure. The critical flutter wind velocity of the bridge may decrease with respect to the increment of the span length. The impact due to the different cable arrangement type, different aerodynamic properties of the deck cross-section, cable vibration behaviour, variation of the cable aerodynamic loads, different cable materials on the flutter response of the cable-stayed bridges using CFRP cables are found to be more and more significant with the increases of the span length.

7.3 Recommendations for Future Work

Following are some recommendations for the future work:

1. The measurement of the spatially distributed self-excited aerodynamic forces around a bridge deck cross-section would be useful and help to fully utilize and demonstrate the advantages of the finite strip model proposed in the current study.
2. The application of the proposed finite strip model may be extended to the analysis of buffeting response of the bridge structure.
3. The impact of cable vibration effect on the dynamic response of super-long-span cable-stayed bridges needs to be studied.
4. The present study of the CFRP cable effect could be extended to the dynamic response analysis of super-long-span cable-stayed bridges.

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