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**THERMAL STRESSES IN
INTERSECTING MASONRY WALLS - A FINITE ELEMENT
STUDY**

by

Ahmad Habibur Rahman, B.Sc.Eng.(Civil), M.Sc. Eng.(Civil)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Department of Civil Engineering

Carleton University

Ottawa, Ontario, Canada

July 3, 1986

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The undersigned hereby recommends to
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acceptance of the thesis;

Thermal Stresses in Intersecting Masonry Walls - A Finite Element Study

submitted by

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in partial fulfilment of the requirements for the degree of

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August, 1986

Dedicated to the memory of my late father

ABSTRACT

In typical buildings of loadbearing and shear wall construction, interior walls are rigidly connected to exterior walls. The exterior wall is subjected to the diurnal and seasonal variations in the ambient air temperature and to solar radiation. Consequently, its temperature fluctuates between daily and annual extremes. On the other hand, the interior wall undergoes little variation in temperature. The ensuing temperature differential between the walls induces differential movement, and hence stresses, in the rigidly connected walls. This study essentially investigates these thermal stresses and other effects in intersecting walls of concrete masonry buildings.

The extreme temperatures to which the exterior concrete masonry loadbearing wall of a typical building is subjected under typical Canadian climatic conditions were determined using both steady state and transient thermal analysis techniques. All climatological factors including solar radiation were considered. It was found that steady state analysis significantly overestimates the extreme temperature particularly in summer. The temperature profiles along the intersecting interior wall corresponding to the extreme temperature conditions in the exterior wall were found to be close to stepped profiles.

A linear elastic finite element computer programme using plane stress elements and a special substructuring technique was developed to compute the thermal stresses efficiently. Stress analyses for critical combinations of thermal and gravity loads using this programme indicated some cracking and compressive overstressing particularly in the intersecting walls.

Since thermal loads are likely to be somewhat relieved by cracking and compressive overstressing beyond the elastic limit, a nonlinear analysis was performed to determine the thermal effects in a more realistic manner. Orthotropy of concrete block masonry was considered by using an analytical model. The one-way floor system consisting of open-web steel joists and reinforced concrete slab was modelled in a special manner for efficiency. The results of the nonlinear analysis substantiated the key findings of the linear elastic analysis.

The above stress analyses indicate that the exterior wall may undergo some compressive overstressing and extensive cracking under summertime and wintertime thermal loads, respectively. The interior wall may also crack in summer. Such thermal cracks may reduce the lateral rigidity of the walls in addition to creating serviceability problems.

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LIST OF SYMBOLS

A	area transverse to heat flow
a_{bm}	fraction of cross sectional area of block material per unit length of wall
a_g	fraction of cross sectional area of grout per unit length of wall
c	specific heat of material
E_t	tangent modulus of elasticity
E_{bm}	initial tangent modulus of block material
E_c	modulus of elasticity of concrete
E_e	elastic modulus of steel
E_m	modulus of elasticity of masonry
E_g	initial tangent modulus of grout
E_o	initial tangent modulus of elasticity
E_p	strain-hardening modulus of steel
E_s	secant modulus of elasticity at peak stress
E_1	modulus of elasticity in direction 1
E_2	modulus of elasticity in direction 2
f	moisture correction factor
f_y	yield strength of steel
f_c	compressive strength of concrete
f_m	compressive strength of masonry
f_t	uniaxial tensile strength of concrete or masonry
G	shear modulus of isotropic material
G_{orth}	shear modulus of orthotropic material
G'	a constant related to the shear modulus of orthotropic material
h_c	convective heat transfer coefficient
h_r	radiative heat transfer coefficient
k	coefficient of thermal conductivity
k_{bl}	coefficient of thermal conductivity of block
k_{br}	coefficient of thermal conductivity of brick

k_{dbl}	coefficient of thermal conductivity of block in oven-dry condition
k_{dbr}	coefficient of thermal conductivity of brick in oven-dry condition
k_{fm}	coefficient of thermal conductivity between nodes l and m
L_g	spacing of grouted cores
l	distance between two points (Chapter 2) number of load cases (Chapter 4)
N	number of storeys in building
n	number of finite element nodes (Chapter 4) number of crosswebs in 1000 mm length of wall (Chapter 6)
q	rate of heat flow
R	thermal resistance of material of given thickness
r	reduction factor to account for discontinuity of block webs
T	temperature
T_f	final temperature
T_i	initial temperature
$T_{i,\tau}$	temperature at node i at time τ
$T_{i,\tau+\Delta\tau}$	temperature at node i after interval of time $\Delta\tau$
t_c	thickness of hollow cores in concrete block
t_{fs}	thickness of the face-shell of concrete blocks
t_g	thickness of grouted cores (equal to t_c)
t_i	inside temperature
t_o	outside temperature
t_w	thickness of webs of concrete blocks
w	moisture content
α	solar absorptivity or fraction of block cores grouted (Chapter 2) coefficient of thermal expansion (elsewhere)
β	shear retention factor
ϵ	strain
ϵ_{cu}	strain at ultimate strength of concrete or masonry
ϵ'_{cu}	ultimate strain in compression of concrete or masonry

ϵ_i	biaxial strain in direction i , $i = 1, 2$
ϵ_{iu}	equivalent uniaxial strain in direction i , $i = 1, 2$
ϵ_{su}	ultimate strain of steel
ϵ_T	initial thermal strain
ϵ'_{tu}	ultimate strain in tension of concrete or masonry
ν	fraction solid of block masonry (Chapter 2) Poisson's ratio (elsewhere)
ρ	mass density of material
σ	stress
σ_o	peak stress in compression
ψ	ratio of net block to gross block area

Matrix Notations:

[D]	displacement matrix of basic finite element model (Chapter 4) constitutive relationship matrix (Chapter 6)
[d]	matrix of boundary displacements common to the crosswall
[d]'	primary displacement matrix
[d]''	secondary displacement matrix
[K]	link stiffness matrix
[P]	link load matrix
[Q]	load matrix of basic finite element model
[S]	stiffness matrix of basic finite element model
{K}	link stiffness vector
{P}	link load vector
[Δ]	actual displacement matrix of substructure model

Subscripts:

ew	exterior wall
ww	web wall, i.e. crosswall
cw	corridor wall
fs	floor slab
iso	isotropic
orth	orthotropic

1.1 The Problem

In typical buildings of shear wall construction, interior crosswalls intersect exterior walls and corridor walls. In reinforced concrete buildings, these intersecting walls are built monolithically with reinforcement running across the intersection so as to make a rigid connection. In masonry buildings, such walls are bonded usually by interlocking of the masonry units and some reinforcement or connectors placed across the intersection.

Structurally, such rigid connections of intersecting walls serve mainly two important purposes, viz., to provide mutual lateral support to the walls so that their load bearing capacity with respect to elastic buckling is increased, and to increase the section modulus of the crosswall in shear wall action. While it may not be necessary in all cases to rigidly connect the walls together from these structural considerations, it is desirable from serviceability considerations in order to prevent the development of unsightly cracks or distress in floor slabs and wall finishes, which may ensue because of separation or sliding at the unbonded interface unless special detailing is provided.

However, while the structural performance of the walls are improved and the problem of separation or sliding at the interface is solved, another problem is created. Since relative deformation of one wall with respect to the other is restrained, some stresses are likely to develop in the walls. Several factors have been identified which may induce such relative deformation of the walls at their intersection. Two of these factors selected for the purpose of this study are discussed in the following paragraphs.

The exterior wall is exposed to the natural environment. It is therefore subjected to the seasonal and daily variations in the ambient air temperature as well as to solar radiation in the daytime. Consequently, its temperature fluctuates between daily as well as annual extremes. But the interior crosswall, typically being in a controlled environment, undergoes little variation in temperature. This creates a temperature differential across the intersection which varies in magnitude and sense, both diurnally and annually, but attains maximum absolute values in the temperature peaks of summer and winter. Evidently, this temperature differential will induce relative deformation, and hence stresses, in the rigidly connected walls. In summer, such stresses are compressive in the exterior walls and tensile in the crosswall. In winter, this stress distribution is reversed. Additionally, shear stresses develop along the intersection.

Another factor that may tend to deform the walls differently is differential gravity loading of

the walls. In North American practice, particularly in masonry buildings, floors are typically of one-way construction. Although in concrete masonry walls some adjustment of the axial stiffness of walls is possible through variation of grout spacing, the stiffnesses of the intersecting walls are hardly proportional to their respective gravity loads. This is mainly because in most situations minimum grout spacing governs. Consequently, the wall which carries the floor loads will tend to deform more than the other wall. But since they are rigidly connected, some load will be transferred to the unloaded wall giving rise to some extra compressive stress in that wall. Again, shear stresses will also develop along the intersection.

The compressive stresses developed due to the above two causes may substantially reduce the margin of safety against compression failure of building walls. The implications of tensile thermal stresses are even more serious in that they may lead to cracking of the walls. Such cracks, in addition to causing serviceability problems, may decrease the lateral stiffnesses of the walls. Of further concern is the fact that the thermal stresses are cyclic in nature due to fluctuations of the ambient temperature stated earlier. Stiffness degradation due to cracking is likely to be aggravated through loss of aggregate interlock along the cracks by the cyclic thermal loads.

Neither concrete nor masonry design practice currently in vogue considers the above stresses in intersecting walls of buildings. It appears that problems in buildings emanating from differential movement including that caused by differential temperature are generally well known. Nevertheless, little study or research has been done in the past to investigate the associated stresses and their implications, particularly in intersecting walls.

The overall purpose of this study is to estimate the stresses caused by the factors discussed above, to determine the influence of some selected parameters on these stresses, and to discuss the implications of these stresses on the structural and serviceability behaviour of the walls. While such stresses could also occur in reinforced concrete buildings of shear wall or loadbearing wall construction, this study will be restricted to buildings of loadbearing concrete masonry wall construction.

1.2 Literature Survey

1.2.1 General

Although some problems emanating from differential thermal movement in buildings have been known for 10, 20 and even 30 years, very little research has been done so far to analyse the associated stresses in intersecting walls. Some research has been carried out on thermal movements and associated stresses in tall buildings of frame structure. But a vast amount of work has been done on thermal stresses in nuclear reactors and bridges of both reinforced concrete

and composite construction. Many of these works were studied and were found helpful for this study. Some of these are briefly covered in this section and reference to many others will be made in subsequent chapters.

The author did not find any work on the investigation of stresses in intersecting walls due to differential gravity loads, such as caused by one-way floors. This is presumably because, as will be shown later, these stresses alone are of relatively insignificant magnitude although substantial amounts of load is transferred from the loaded to the unloaded walls.

1.2.2 Masonry Buildings

Problems in masonry walls due to thermal movement has been known at least since the early nineteen sixties. Latta and Garden [59]¹ reported in 1962 that the range in building envelope temperature could be more than 110 °C whereas the internal surface would be subjected to a much smaller range. Consequently, they inferred that a solid wall would tend to deflect in and out and a multilayered wall would have one layer sliding over the other. In 1964, Hutcheon [46] reported that the range in temperature of the exterior wall of a building could be as high as 83 °C, and since the interior structure remains at a nearly uniform temperature, the differential thermal movement thus created could lead to cracking in portions of both the exterior walls and crosswalls.

Differential thermal movement is the principal factor that causes distress and often potentially hazardous failures in masonry veneers and claddings. Causes and mechanisms of these failures and ways to prevent them have been discussed by Plewes [70,71], Fricke et al [31], Kuchar [55], and Suter and Hall [86]. Recently, Fenton [28] derived an analytical means to predict the differential movement arising between a masonry veneer and its supporting frame.

But only recently, in 1980, apparently the first study on the analysis of stresses due to differential thermal movement in masonry walls was reported by Glanville and Kobak [37]. They used a simplified analytical procedure to compute thermal stresses in a crosswall of a ten storey building. Part of this wall is protruding out of the building envelope to support balconies. In terms of thermal loads and resulting stresses, this problem is analogous to the one studied in this report, the principal difference being that in reference 37 the thermal load is generated in the protruding part of the crosswall, whereas in this study it is generated in the exterior wall intersecting with the crosswall.

However, while the work in reference 37 demonstrates that a potential exists for cracking and overloading of the masonry walls, the procedure used is limited to a linear elastic precracking range. Even in that range, some of the computed stresses are only approximate, because the stiffening effect of the floor slabs has not been considered. Also, the assumption of linearly

¹ Numbers in square brackets indicate references listed at the end of this report

distributed restraining forces along the height of the building for the laterally restrained case is likely to be erroneous. Further inaccuracy in the stresses arises, particularly in the upper levels of the ten storey building or at any level of shorter buildings, due to the neglect of deep beam effects.

1.2.3 Concrete Buildings

Until the 1940s, structures in tall buildings were typically concealed behind a facade within a temperature controlled environment. Following World War II, architects and building designers began to expose the exterior structural elements principally to reduce building weight and cost, although the increasing popularity of the new aesthetics of expressing the structure was another factor [20]. Since the exterior part of the structure was now subjected to the variation of temperature, problems arising from differential thermal movements inevitably followed.

In the 1950s, the effects of temperature along with other factors which change the volume of concrete such as creep and shrinkage were considered only as an academic subject. The building designers rarely considered them quantitatively in their designs. As a consequence, some tall buildings built in the 1960s developed partition distress as well as structural overstress leading to cracks in some cases [29].

Between the years 1965 and 1968, Fintel and Khan [30, 53, 54] established a simple engineering procedure for the solution of architectural as well as structural problems of column exposure in tall buildings. The procedure consists of determination of a realistic design temperature and the corresponding average temperatures of the exposed columns based on steady-state heat transfer as a first step. The length change and the bowing stresses in the exposed columns and the load transferred to the interior structure are then computed. Finally the structural design is revised to include the thermal effects and appropriate architectural and structural detailing is provided to accommodate the thermal movements. The procedure, meant for the routine design of tall buildings, appears reasonable. However, for the summer design temperature, effect of solar radiation apparently has not been considered. Depending on the colour, texture and density of the concrete, solar radiation may substantially elevate the summer temperature.

But no evidence of any work on thermal stresses and cracking in intersecting walls of concrete buildings have been found by the author. It appears that in reinforced concrete construction, shear walls located within the building but intersecting with an exterior wall are not used very often, although stair walls and elevator cores are sometimes placed on the periphery of the building.

1.2.4 Concrete and Composite Bridges

Like building envelopes, bridge structures are exposed to variations of temperature. The

main types of deformation caused by these temperature changes are longitudinal expansion and contraction, and vertical deflection. The longitudinal deformation is caused by changes in average temperature and the vertical deflection results from a vertical temperature gradient across the cross section of the bridge. This vertical temperature gradient, which is markedly nonlinear at web flange intersections of box girders or multicellular bridges, gives rise to thermal stresses in a manner analogous to that in intersecting walls of buildings. Dissimilar thermal coefficients of expansion is an additional factor in composite steel girder concrete deck bridges to cause such stresses.

A large number of works on the thermal stresses in bridges of reinforced concrete or composite construction has been published. In 1961, a theoretical study of thermal stresses in a composite steel and concrete section was published by Zuk [96]. An elastic theory was developed to compute the stresses and strains caused by various linear thermal gradient patterns. Longitudinal stresses in the steel beam as high as 166 MPa were computed by this theory. Liu and Zuk [62] extended this work of Zuk to prestressed concrete sections. For a temperature difference of 14 °C through the prestressed slab and beam, calculated stresses in the slab in the direction of the beam were significant and varied from 6.9 MPa in tension to 5.5 MPa in compression.

Wah and Kirksey [92] reported in 1969 results of theoretical as well as experimental and field investigations. Measured thermal strains from an existing simple span bridge indicated thermal tensile stresses up to 10 MPa at the top and 6.6 MPa at the bottom of the concrete slab. Creep effects were found to have a significant influence on the thermal stresses.

Berwanger and Symko [7] reported in 1975 results of a finite element analysis of temperature and associated thermal stresses in the cross sectional plane of a composite bridge of steel girder and concrete deck. Based on their analysis, they reported, for example, that due to a uniform temperature drop from the zero-stress datum temperature of 21 °C to -40 °C in winter, tensile stress in the steel beam and compressive stress in the concrete deck could be as high as 41.5 MPa and 2.6 MPa, respectively. They assumed steady state boundary conditions for the temperature distribution and inferred that this condition is rarely reached in practice and stresses under transient conditions would be greater than shown by their analysis.

Hunt and Cooke [45] in 1975 used one-dimensional heat flow analysis to determine temperature distributions across bridge cross sections. Results of temperature distribution based on this analysis showed excellent agreement with experimental temperature measurements published earlier by Priestley [73]. Maximum tensile and compressive stresses of 1 MPa and 4 MPa, respectively, were indicated by this analysis in a concrete box girder bridge of simple span.

In the period between 1976 and 1981, Emanuel [25] and his co-workers developed a theoretical procedure for determining stresses and strains in composite steel and concrete

bridges resulting from thermal loading. They also derived realistic thermal loading and temperature distributions from 20 years of recorded weather data. Subsequent experimental studies conducted by them on a model composite two-span bridge verified that the theoretical procedure developed earlier was adequate for a reasonable prediction of the behaviour of composite girder bridge structures under thermal loading.

In 1983, Dilger et al [22] attributed the collapse of a composite steel box girder bridge in western Canada to temperature induced stresses. They concluded that under the worst combination of environmental conditions and thermal properties of the materials, temperature differentials up to 70 °C may develop in such bridges. They also developed a computer programme to predict the temperature distribution in the cross section of a composite steel box girder bridge. Applied to two typical cases, this programme indicated that compressive stresses up to 100 MPa could develop in the steel web and tensile stresses up to 2.9 MPa could develop in the concrete slab.

A study similar to the above, but for concrete bridges of arbitrary geometry, was published by Elbadry and Ghali [24] in 1983. They performed a series of transient heat flow analyses by the finite element method to study the influence of various parameters on the thermal response of concrete bridges of various cross section types. They concluded that temperature distribution over the cross section is generally non-planar and results in longitudinal stresses even in a simply supported bridge. For a daily ambient temperature range of 40 °C in summer, the maximum stresses in a simple span concrete box girder bridge were computed as 2.4 MPa and 2.0 MPa in compression and tension, respectively.

Berwanger [5] considered the transient nature of heat flow in bridge cross sections in 1983. Again using the finite element technique, he computed the temperature and stress distributions in a model continuous bridge of composite construction. As the top surface of the bridge was cooled by rapid placement of ice from a temperature of 25.8 °C, which was uniform throughout the bridge, to 2.6 °C, the computed maximum stresses reached about 7.8 MPa and 6.8 MPa in the longitudinal and transverse directions of the concrete deck slab, respectively. Experimental measurements showed that the finite element predictions of the temperatures in the cross sectional plane were accurate.

In all the above works, stresses due to thermal loads were computed on the assumption of linear elastic behaviour of the materials. Most of the reported maximum tensile stresses in concrete exceed the tensile strength of concrete and therefore the concrete in the corresponding cases is cracked. Cracking may substantially relieve the thermal loads generated. Therefore the actual stresses could be significantly lower than reported. Thurston et al [90] reported that for a thermally loaded model continuous bridge of reinforced concrete T-beam construction the reduction in moments caused by cracking was more than 50 percent compared

to uncracked moments. A more realistic analysis of the thermal stresses can only be done considering cracking and other nonlinear behaviour of concrete.

1.2.5 Nuclear Reactor Vessels

Reinforced and prestressed concrete containment vessels in nuclear power plants are subjected to much higher differential temperatures than buildings. A large number of works have been published on the analysis of concrete reactor vessels for thermal loads. Some of these were reviewed and are discussed below.

Katayama et al [52] presented a study on the thermal stresses in the outer shielding of a nuclear reactor due to solar radiation and ambient temperature. They developed computer programmes to determine the non-axisymmetric solar radiation, the resulting temperature distribution based on 3-dimensional heat conduction, and the thermal stresses in the outer shield. However, although they reported that cracking would occur on the inside surface of the structure, cracking was not considered in their analysis of thermal stresses.

Buchert and Ballard [13] reported that thermal stress analysis of concrete containment vessels without considering cracking may lead to overly conservative designs in most cases and unconservative designs in some cases. They discussed two techniques used for the determination of stresses in cracked container vessels. One of these techniques is based on the classical shell theory in which the axial and flexural rigidities are calculated on the basis of different effective thicknesses to allow for cracking. The other is layered shell analysis by the finite element method in which a solution is obtained iteratively by changing the constitutive relationship in each cycle to reflect the cracking. The latter technique has in fact been used in this study to determine the thermal stresses considering cracking and other material nonlinearities of the materials.

McGeorge and Swec [63] developed a two-step procedure to analyse cracked concrete structures efficiently. A linear analysis is first carried out on the entire structure to obtain the gross response of the structure; a nonlinear postcracking analysis is then performed on selected portions of the structure where the primary analysis has indicated cracking. Compatibility of behaviour between the linear and nonlinear analyses is achieved by iterating through a number of analyses. The results of such an analysis of an actual containment structure carried out by the authors correlated well with experimental results.

ACI Committee 349 [2] presented a design-oriented approach for considering thermal loads on reinforced concrete structures, particularly nuclear power plant structures. Two types of structures, frames and axisymmetric shells, were covered. A number of graphs were presented to compute the thermal moments in cracked frame members and shells.

1.2.6 Other Structures

Larrabee et al [58] analysed a hyperbolic cooling tower for thermal loads caused by two

phenomena: rising hot air heating the inside surface while the outside surface temperature is lower, such as in winter, and uneven solar heating of the tower. They used the finite element method for a linear elastic stress analysis of the cooling tower under these thermal loads. On the basis of the results, they concluded that the concrete shell would crack. Also, they inferred that while the thermal stresses are relieved by cracking, the performance of the structure under other loadings such as earthquakes may be adversely affected.

Kar [50] discussed some inconsistencies in the provisions made by the ACI about thermal load analysis of structures. In order to avoid the usual iterative approach for the computation of thermal loads and stresses, he proposed a method for reinforced concrete frame members based on an 'average effective member stiffness' lying between the stiffnesses corresponding to cracked and uncracked sections. However, no application was presented to demonstrate the validity, accuracy, or merits of the method.

Besio et al [8] proposed a similar approach to compute the thermal loads in simple concrete members after cracking. The basis of this approach is to multiply the elastically determined thermal load or moment by a factor which is governed by the magnitude of other loads as well as temperature. Using a piecewise linear elastic analysis code, they computed this factor for several simple frame members under various combinations of thermal and other loads. The factor was found to be as low as 0.3 for a member where bending was prevalent.

A simplified analysis technique considering cracking, shrinkage and creep for computing hoop stresses in heated reinforced concrete cylinders was proposed by Haydl and Sherbourne [41]. The shell of the cylinder is divided into a number of layers in the radial direction. For each layer a stress is assumed and the strain is calculated as the sum of thermal, creep and shrinkage strains. The average strain over these layers is then calculated. The stress in each layer is calculated using the modulus of elasticity and the difference between the layer strain and the average strain. If stress in any layer exceeds the tensile strength, the strain in that layer is assumed as zero and the average strain is revised excluding that layer. Finally, comparison is made between the assumed and computed stresses and the whole process is repeated until they match. This process is similar in essence to the iterative procedure in many finite element programmes for nonlinear analysis.

Heller [42] studied the thermal stresses in a large diameter concrete cylinder encased in steel considering the diurnal and annual cycles of ambient temperature. Using a mathematical approach, he showed that the tangential stresses at the interface of the casing and the cylinder and at the centre of the cylinder are out of phase. Also, unlike the temperature cycle which attenuates as it penetrates the cylinder, stresses at the centre remain high.

1.3 Objectives and Scope

The primary objectives of the present study are the following:

1. To determine the extreme temperatures in the intersecting exterior and interior walls of a typical loadbearing concrete masonry building.
2. To estimate the stresses in intersecting walls due to differential temperatures in the walls and differential gravity loads on the walls.
3. To determine the influence of some selected parameters on the stresses caused by the differential temperature.
4. To discuss the implications of the above stresses on the serviceability and structural behaviour of the walls.

Although, as indicated earlier, stresses of comparable magnitude and bearing similar implications could develop due to temperature differentials in intersecting walls of both concrete and masonry buildings, this study will be restricted to buildings of concrete masonry loadbearing wall construction. However, modified by using appropriate material properties, the results of the linear elastic finite element analysis and the parametric study could readily be applied to concrete and brick masonry buildings of similar layout.

Interior and exterior walls of various lengths can form intersections at various locations of the building layout. Boundary conditions for the mathematical model of such sets of intersecting walls can be all too numerous. This study will be restricted to a wall layout of interior crosswalls intersecting with an exterior wall at one end and with an interior corridor wall at the other. Such wall layouts are commonly found in apartment and hotel buildings; a typical floor plan of such a building is shown in Fig. 1.1. These buildings, rarely taller than 14 stories, constitute a substantial fraction of all loadbearing masonry buildings in North America.

The floor systems of the above buildings in North America are rarely of two-way construction. Most floors and roofs are one-way, either reinforced concrete slabs on steel joists, or prestressed concrete planks [87]. The analysis in this study will be restricted to one-way floor systems of reinforced concrete slab and steel joist construction. However, as will be shown later, the results could also be applied to other one-way floor systems without any significant error.

The climatological factors which govern the temperatures in a building, and thus the thermal stresses, are highly variable around the world; so are the materials used and practices of masonry construction. This study principally deals with a Canadian climate and North American masonry practices. However, the key findings of this work, as will be shown later, are also applicable to masonry buildings in other regions of the world.

Fig. 1.1 Floor Plan of Typical Multistorey Building of Loadbearing Masonry Wall Construction

The thermal insulation in the exterior wall can be placed on either the inside face or the outside face of the wall. The range of temperature fluctuation to which the loadbearing wall is subjected depends on the position of the insulation. This study will be restricted to the case where insulation is placed on the inside face, because in most existing buildings insulation is placed on that face of the exterior loadbearing wall [87]. Also, such a position of the insulation causes the largest range of temperature change to occur in the loadbearing wall.

In determining the thermal stresses, the differential temperatures in the walls only due to the environmental factors are considered in this study. Differential temperatures caused by fire or any other heating and cooling source in the building are excluded.

From a theoretical point of view, the thermal stress analysis problem at hand can be treated as a thermoelastic problem up to the point of cracking. The temperature changes and associated deformations are small; the basic thermoelastic equations are therefore linear with respect to stress, strain, and temperature. Also, it can be assumed that the heat produced within the domain under analysis by the small deformations is insignificant. Therefore, the basic equations of heat conduction and stresses and strains are uncoupled. Finally, the process of temperature change within the domain is slow; therefore, no inertia effects enter the solution process.

However, treatment of the problem in this study by the classical theory of thermoelasticity appeared to be complex, if not impossible, apart from the obvious limitation of its applicability only up to the onset of cracking and other inelastic behaviour of the materials. The complexity arises mainly because of coupling of floor slabs with walls in resisting the thermal loads generated. Besides, no general solution can be developed which is applicable for different geometries of the walls and slabs and different sets of possible boundary conditions.

The finite element technique appeared to be a better alternative for this problem. Once the finite element model of the problem is prepared, variation of different parameters to study their influence on the stresses and deformations poses no additional problem. Besides, to carry the analysis into the nonlinear postcracking range, the finite element method appears to be the only analytical tool readily available at hand.

1.4 The Thesis Layout

To achieve the objectives outlined in the previous section, the present research work proceeded in the following steps:

1. Determination of the range of temperatures to which the intersecting exterior and interior walls of typical buildings are subjected under typical Canadian conditions.
2. Development of an efficient computer programme for the linear elastic finite element

analysis of buildings particularly under differential thermal and gravity loads.

3. Linear elastic analysis of a typical building to determine the stresses under differential thermal and gravity loads.
4. Adaptation of an available nonlinear finite element programme for the nonlinear postcracking analysis of the same building to determine the stresses and cracking in a more realistic manner.
5. Study of some parameters which influence the thermal stresses in buildings

Chapter 2 of this work deals with the thermal analysis. Climatic conditions all over Canada were studied to select a locality which typically represents the extreme temperature conditions. The effects of solar and long wave radiation were considered. Both steady state and transient analyses were carried out to determine the extreme temperatures in typical exterior walls and the temperature profile along the interior crosswall.

Chapter 3 deals with the preliminaries of stress analysis. The stress analysis problem is defined, the relevant properties of the materials in the range of applicable temperatures are discussed, and the need to develop a linear finite element analysis programme is explained.

In Chapter 4, the development of a computer programme for the linear elastic finite element analysis is described. Based on certain assumptions, the 3-D problem is reduced to a 2-D one and a special substructuring technique is used so that the large structure under study can be analysed efficiently.

Using the programme developed in Chapter 4, a typical 10-storey building of concrete block masonry loadbearing wall construction is analysed under differential thermal and gravity loads. This analysis is presented in Chapter 5.

Since the linear analysis indicated cracking and compressive overstressing in masonry walls and roof slabs, a nonlinear analysis was deemed necessary to determine the stresses in a more realistic manner. An available computer programme for the nonlinear finite element analysis of reinforced and prestressed concrete structures is adapted primarily to model the orthotropic behaviour of concrete block masonry. Chapters 6 and 7 deal with the nonlinear analysis of the present problem using this adapted programme.

In Chapter 8, the influence of some selected parameters on the thermal stresses in typical buildings is studied. The linear elastic programme is used for this purpose.

Finally, in Chapter 9, a summary of the present work and its conclusions are presented. Recommendations of some remedial measures and for further work in this area are also provided.

2

Thermal Analysis

2.1 General

In order to determine the thermal stresses by the finite element method, it is first necessary to determine the temperature distribution in the domain under analysis. The base temperature at which stresses are zero is also required to be known.

The temperature distribution in the exterior wall of a building is governed by several factors. First of all, it is transient, changing every moment principally because of fluctuations in the environmental conditions. On the other hand, temperature variation in the interior of a building with respect to time is hardly significant. This is because the temperature inside a building is controlled within a narrow range suitable for human habitation. In the North American climate, this control is exercised through insulation of the building envelope and additionally, in most areas, heating the interior in winter and cooling in summer. For the purpose of this study, the temperature of the interior air will be assumed invariant with time.

The objective of this study is to focus on the stresses and associated distresses of maximum severity in a typical case. Therefore, the immediate objective in this chapter on thermal analysis is to determine the temperature distributions in the walls under the maximum differential of temperature between the intersecting walls. Since exterior wall temperatures are typically higher in summer and lower in winter than the interior temperature, two maximum differentials, one in summer and the other in winter having opposite deforming tendencies, are in fact sought. This requires the maximum and minimum temperatures in the exterior wall to be determined. Although the interior room temperature is assumed constant, the interior wall may have different temperatures particularly near the intersection with the exterior wall. Therefore, temperature distribution in the interior wall corresponding to the maximum and minimum temperature in the exterior wall will also have to be determined. First, the factors that govern the exterior wall temperature will be reviewed briefly.

2.2 Factors Governing Exterior Wall Temperature

In typical North American practice the exterior walls of a masonry building are of the so-called cavity wall type. The loadbearing concrete masonry wall is covered on the outside usually by a brick masonry cladding leaving a cavity in between. The cladding is used mainly to

enhance the appearance of the building and to protect the loadbearing wall from moisture penetration. However, single wythe loadbearing walls with no cladding are also sometimes used as exterior walls [87]. Typical cross sections of these exterior walls are shown in Fig. 2.1. Analysis was carried out in this study for both types of walls.

(a) Cavity Wall

(b) Single Wythe Wall

Fig. 2.1 Typical Exterior Wall Sections

The temperature at any point in a single or multilayered wall is governed by the following factors:

1. Amount of total incident thermal radiation
2. Absorptivity and emissivity of the surface
3. Ambient air temperature
4. Wind
5. Conductivity of the wall elements
6. Thermal mass of the wall elements

These factors are briefly discussed in the following subsections.

2.2.1 Radiation

The radiation incident on an exterior wall surface is comprised of solar radiation, atmospheric radiation, and radiation reflected from surrounding objects such as other buildings. The first one is high temperature short wave radiation and the latter two are low temperature long wave radiations.

Solar Radiation- Of all the factors that influence the temperature in an exterior wall, solar radiation is by far the most influential. The amount of radiation incident on a surface at right angles to the sun rays and just outside the earth's atmosphere, known as extraterrestrial solar radiation, is fairly constant. It varies only about 3.5% about the mean value of 1.37 kW/m^2 due to the changing earth to sun distance as the earth follows an elliptical orbit around the sun.

Only a fraction of the extraterrestrial solar radiation reaches terrestrial surfaces, like building walls, as direct solar radiation. The rest is diffused and absorbed by scattering by the air molecules, by scattering and absorption by the aerosol particles suspended in the air, and by absorption by the molecules of ozone and water vapour present in the atmosphere. The diffused part, still short wave radiation, reaching the earth's surface is known as diffuse solar radiation.

The total solar radiation reaching a surface on earth, which is comprised of both direct and diffuse solar radiation, is a function of several factors such as time of day, time of year, latitude, orientation of the surface, and atmospheric conditions like cloud cover, dust and water vapour content of the atmosphere. Since the earth is revolving about its axis, the obliqueness with which the direct and diffuse rays strike a surface is constantly changing. This introduces the transience in the heat flow analysis mentioned earlier. However, this transience has a periodicity, since the earth makes a complete revolution around its axis every 24 hours and is therefore predictable. On the other hand, atmospheric conditions, which are also constantly changing are almost unpredictable.

Long Wave Radiation- Although the radiation received by a surface on the earth is

predominantly solar radiation, long wave radiation should be taken into account for an accurate calculation of temperatures. The part of solar radiation which is absorbed by aerosol particles, ozone and water vapour, is reradiated by the atmosphere at long wave lengths. This is sometimes known as atmospheric radiation. The marked effect of this radiation on temperatures at the earth's surface is felt in heavily overcast nights when temperatures remain above normal and in clear cloudless nights when temperatures are cooler. Long wave radiation could also emanate from terrestrial objects like the ground, buildings, and paved surfaces, when they are radiating heat absorbed by them after being exposed to the sun or when just reflecting other long wave radiations. Long wave radiation exchange also takes place between two surfaces within a wall cross section separated by an air space and at the interior surface of the wall.

More detailed discussions on radiation are available in references 1, 17, 48, 51, and 67

2.2.2 Absorptivity and Emissivity

All real bodies have a certain capacity to absorb and emit radiation. The amount of heat a body can gain by absorbing radiation is governed by its capacity to absorb radiation known as absorptivity. Likewise, the amount of heat a body can lose through radiation is determined by its capacity to emit radiation known as emissivity.

In general, absorptivity and emissivity are dependent upon the direction in which radiation is taking place, the spectral distribution, i.e. the wave length distribution of radiation, the temperature of the surface which is absorbing or emitting radiation, and the composition and structure, or in other words, texture of the surface. However, there is no relationship, consistent for all materials, between these factors and absorptivity and emissivity. Some surfaces have higher absorptivities for short wave radiation while some have the opposite. There are still others whose emissivity and absorptivity do not vary with wave length of radiation. These surfaces are said to be grey. Masonry surfaces exhibit a moderate variation of absorptivities with wave length, having slightly lower values for solar radiation than for long wave radiations. In the temperature range pertinent to this study, all surfaces including masonry emit radiation only in long waves. The emissivity and absorptivity for long wave radiation exchange are practically the same. In fact, according to Kirchhoff's law, under a state of thermal equilibrium, the emissivity and absorptivity of a surface are the same.

Of particular interest in this study is the solar absorptivity, i.e. the absorptivity for solar radiation, of masonry surfaces because heat gained by a wall through absorption of solar radiation at its exterior surface is largely responsible for the elevation of the wall temperature. It will be shown later that the temperature at the exterior surface of walls is quite sensitive to the variation of solar absorptivity values. Solar absorptivity has been found to be largely influenced by the colour of the surface.

The heat gained or lost by a wall through long wave radiation exchange with the environment is governed by the absorptivity and emissivity for long wave radiation. Although long wave radiation exchange at a wall surface occurs in daytime together with absorption of solar radiation, and is the only radiation exchange occurring at night, its influence on the wall temperature is relatively small.

The available data on solar absorptivity of common building materials including masonry is rather sparse. The importance of solar radiation was first realised by airconditioning and heating engineers in calculating the cooling loads and heating requirements for buildings. The effect of incident solar radiation is typically included through the use of sol-air temperature which requires specifying the solar absorptivity of the element. But only a very limited selection of solar absorptivity values based on a crude classification of colour, such as "light" or "dark", is typically available in standard design information for making heat transfer calculations.

2.2.3 Ambient Air Temperature and Wind

Since the exterior wall is in constant contact with outside air, the ambient air temperature is an important determinant of the exterior wall temperature. The ambient air temperature at any particular place is a part of the overall climate of that area and is therefore governed by geographic features like, latitude, longitude, and altitude. It varies both daily and annually because of variation of solar radiation described earlier. But since unpredictable environmental factors like atmospheric conditions and wind also play a role, it is difficult to make any estimate on an analytical basis. However, from the temperature measurement data accumulated over several years, as is done by weather bureaus, it is possible to predict the trends on a statistical basis.

An important mode of heat transfer at the surfaces of a building wall is convection. This convective heat transfer is possible at an exterior surface in still air, i.e. when there is no wind, by the motion of air masses due to density differences caused by the temperature differences. But the amount of heat transferred by this natural convection is rather small. Convection induced or forced by wind can dramatically increase the heat transfer depending upon the characteristics of wind flow.

Theoretically there is a relationship between wind velocity and convective heat transfer. One such relationship is given by Hutcheon and Handegord [48], which is the same as given by ASHRAE but modified for the metric system of units. However, although it is possible to know wind velocities in a certain area from the climatological information published by weather bureaus, those can not be readily translated into the wind velocities prevailing at a particular wall surface in a particular building located in that area. The direction of the wind flow, ambient temperature and humidity, characteristics of buildings and other obstacles in the path of wind around the building of interest are only a few factors which influence the air velocity prevailing at a wall surface. Further

complications arise due to the fluctuation of the wind velocity and change in the direction of wind flow.

2.2.4 Conductivity and Thermal Mass

Conduction is the predominant mode of heat transfer from one face of the wall to the other across the thickness of the wall. In fact, it is the only mode of heat transfer across solid elements of the wall. Fortunately, unlike most other factors that influence the temperature in a wall, conductivities and conductances of the wall elements can be established with reasonable accuracy.

The basic relationship for heat flow by conduction is given by Fourier's law as

$$dq/d\theta = -kA dt/dx \quad (2.1)$$

where $dq/d\theta$ = rate of heat flow, q , per unit time, θ
 k = coefficient of thermal conductivity
 A = area transverse to heat flow
 dt/dx = temperature gradient

In metric units, the coefficient of thermal conductivity, k , can be defined as the amount of heat flow in watts through a cross sectional area of 1 m^2 across a thickness of 1 m for a temperature differential of 1°K . Conductivity of a material of definite thickness is known as conductance.

It is evident from Eq. 2.1 that the higher the conductivity, the higher is the heat flow rate when two different temperatures are maintained at two ends of the heat flow path. In a situation where a fixed temperature is maintained only at the upstream end of the heat flow, an element of high conductivity will attain a higher temperature at the other end than an element of low conductivity. Thermal conductivity is principally a function of density, temperature, and moisture content. However, in the range of temperature relevant to this study, masonry shows little variation of thermal conductivity with temperature.

Another important property of materials that influence heat flow through them is the heat storage capacity, sometimes referred to as thermal mass or capacity insulation. Some materials, like masonry, can store a considerable amount of heat. Metals, on the other hand, have a low thermal mass or heat storage capacity. Thermal mass is a function of specific heat, mass, and density of the material. Specific heat in metric units is the amount of heat in joules required to elevate the temperature of 1 kg of material by 1°K . A material of high thermal mass will take longer to reach equilibrium temperatures, i.e. steady state conditions, when maintained under a fixed temperature gradient than a material of low thermal mass.

2.3 The Thermal Analysis Problem

It has been already mentioned that the thermal analysis problem at hand is one of determination of the maximum and minimum temperatures in the exterior wall and the corresponding temperature distributions in the interior wall that intersects with the exterior wall. A typical intersection of exterior and interior walls is shown in Fig. 2.2. To illustrate the heat flow problem, consider the temperature outside the building is higher than that inside the building, as is typical in a summer day. Heat therefore will flow from outside to the inside across the exterior wall. Rigorously speaking, heat will flow in all three dimensions, across the wall as well as along the wall in both vertical and horizontal directions. The heat flow in the plane of the wall may be caused by uneven shading against solar radiation, the presence of intersecting walls acting as heat sinks, and air movements. But it is very difficult to consider these effects in a general way. Shading, for example, is not necessarily an integral feature of the building. It is mostly dependent on the position and height of other buildings around it.

For the purpose of this study, the building is assumed unshaded at all times. Air movements will induce negligible heat flow in the plane of the wall, and heat flow in the plane of the wall around intersections with other walls is highly localised. Therefore, for all intents and purposes, the heat flow can be treated essentially as one-dimensional across the exterior wall. Since the interior wall is in physical contact with the exterior wall, some heat will flow into the interior wall and eventually dissipate through the wall faces in contact with the inside air. The possible heat flow pattern is shown in Fig. 2.2.

The solution of the temperature analysis problem was achieved in two stages. First, the maximum and minimum temperature in the exterior wall were determined. This was done using both steady state and transient heat flow analyses as dealt with in the next three sections. The steady state analysis was done at the primary stage of this work to obtain a primary and quick impression about the maximum and minimum exterior wall temperatures. Although the transient nature of the heat flow was appreciated at the very outset, the extent of crudeness of the steady state analysis, particularly for maximum temperature, was realised only after the transient analysis. Next, steady state heat flow analysis was used to determine the temperature distribution in the interior crosswall with the temperature in the middle plane of the exterior wall used as input temperature at the intersection of the interior wall. This had to be done since the transient thermal analysis programme that was used for the exterior wall can model the one-dimensional heat flow across a wall only; it does not have the capability to include the intersecting interior crosswall in which the heat flow is in fact two-dimensional. However, as will be shown later in Sec. 2.7, the temperature profile along the interior crosswall determined in this manner is accurate enough for the purpose of this study.

Fig. 2.2 Heat flow Pattern around a Typical Intersection of Walls

2.4 Parameters of Heat Flow Analysis

As discussed in Sec. 2.2, heat flow through an exterior wall is governed by climatological factors as well as by thermal properties of the materials involved. This section deals with the data on these parameters of heat flow used or assumed for the purpose of this study; the data is primarily based on the Canadian climate and Canadian masonry construction materials and practices.

2.4.1 Climatological Factors

Although Canada's vast territory spans between the latitudes of 42° and 83° , more than 90% of the population lives below the 60th parallel. All the major cities are located below the 50th parallel. The absolute maximum and minimum temperatures in these cities, as recorded up to 1980 and reported by Environment Canada [27], are presented in Table 2.1. Some of these records span a period of more than 90 years while some cover less than 30 years. All of the temperatures were recorded in airport locations; extreme temperatures at city centres tend to be a few degrees higher. The maximum temperatures range between 30.6°C and 43.3°C , and the minimum temperatures range between -15.6°C and -50°C . The highest temperature ever observed in Canada was 46°C in Gleichen, Alberta on July 28, 1903; the lowest, -63°C , was observed at Snag, Yukon Territory, on February 3, 1947 [79].

Table 2.1 Extreme Temperatures in Major Canadian Cities

Location	Temperatures, °C		No. of years of record	Location	Temperatures, °C		No. of years of record
	Max	Min			Max	Min	
Newfoundland				S. S. Marie	36.1	- 38.9	36
St. John's	30.6	- 23.3	39	Sudbury	38.3	- 37.8	27
Nova Scotia				Timmins	38.9	- 45.6	26
Halifax	34.5	- 26.1	22	Toronto	38.3	- 31.1	43
P. E. Island				Trenton	38.9	- 32.2	46
Charlottetown	34.4	- 28.1	38	Windsor	38.3	- 26.1	41
New Brunswick				Manitoba			
St. John	34.5	- 36.7	35	Brandon	37.8	- 45.6	40
Fredericton	37.2	- 37.2	30	Winnipeg	40.6	- 45.0	43
Moncton	37.2	- 32.2	41	Saskatchewan			
Quebec				Prince Albert	37.8	- 50.0	39
Montreal	37.8	- 37.8	40	Regina	43.3	- 50.0	87
Quebec	35.6	- 36.1	38	Saskatoon	40.0	- 47.8	40
Sherbrooke	33.3	- 40.0	19	Alberta			
Ontario				Calgary	36.1	- 45.0	97
Hamilton	35.6	- 27.8	22	Edmonton	34.4	- 48.3	44
Kingston	33.3	- 32.8	51	Medicine Hat	42.2	- 46.1	98
London	36.7	- 31.7	41	British Columbia			
Ottawa	37.8	- 36.1	43	Vancouver	33.3	- 17.8	44
				Victoria	36.1	- 15.6	41

The objective of this study is to determine thermal stresses and the extent of associated distress, like cracking, that could occur under extreme temperatures of typical magnitudes. Both the high and low extreme temperatures of the Ottawa location are very close to the means of all the corresponding values presented in Table 2.1. The climatological data of Ottawa will therefore be used in this study for the thermal analysis as being typically representative of the Canadian climate.

Ambient Temperature- For the transient heat flow analysis, the ambient temperature variation over a 24-hour period is important. Fig. 2.3 shows two hourly temperature plots over

24-hour periods for a typical summer and winter day in Ottawa. With the break of dawn, around 5:00 in the morning, the ambient temperature starts to rise and reaches a peak around 4:00 in the afternoon. The temperature then starts to drop and reaches the lowest of the day in the early hours of the following morning.

Fig. 2.3 Typical Daily Ambient Temperature Variations in Ottawa

The transient analysis computer programme used in this work creates an idealised daily temperature history assuming that the daily minimum and maximum temperatures occur at 5.00 a.m. and 4:00 p.m., respectively, and requires the magnitudes of the daily maximum and minimum temperatures to be input. For summer, the extreme high temperature of 37.8 °C for Ottawa, as presented in Table 2.1, was used as the maximum daily temperature; similarly, for winter, the extreme low temperature of -36.1 °C was used as the minimum daily temperature. To determine the minimum daily temperature in summer and the maximum daily temperature in winter, the differences between the maximum and minimum temperatures on the hottest and the coldest days of each of the last ten years, as obtained from reference 26, were calculated. The means of these ten differences, 13.9 °C and 9.7 °C, for the hottest and the coldest days, respectively, were then calculated as shown in Table 2.2. The minimum daily temperature in summer and the maximum daily temperature in winter were then obtained as follows;

$$\text{Minimum daily temperature in summer} = 37.8 - 13.9 = 23.9^{\circ} \text{C}$$

$$\text{Maximum daily temperature in winter} = -36.1 + 9.7 = -26.4^{\circ} \text{C}$$

Table 2.2 Differences Between the Daily Maximum and Minimum Temperatures

SUMMER				WINTER			
Date	Max	Min	Diff	Date	Max	Min	Diff
01.08.75	35.4	21.4	14.0	20.01.75	-20.9	-29.3	8.4
05.07.76	32.6	16.5	16.1	23.01.76	-23.3	-31.7	8.4
20.07.77	33.6	20.5	13.1	17.01.77	-20.8	-27.9	7.1
20.07.78	33.4	21.0	12.4	04.02.78	-14.8	-24.3	9.5
14.07.79	32.8	19.1	13.7	11.02.79	-21.2	-29.3	8.1
25.06.80	32.4	17.7	14.7	25.12.80	-22.9	-31.6	8.7
08.07.81	34.1	19.9	14.2	04.01.81	-21.1	-35.2	14.1
18.07.82	34.2	20.1	14.1	10.01.82	-16.0	-30.7	14.7
22.06.83	33.3	18.9	14.4	20.12.83	-20.5	-27.8	7.3
09.06.84	32.6	20.0	12.6	16.01.84	-18.5	-29.4	10.9

 $\sum \text{diff} = 139.3$ $\sum \text{diff} = 97.2$

Mean difference = 13.9 °C

Mean difference = 9.7 °C

Wind- The average wind speed in Ottawa is 3.25 m/sec (11.8 kph) in July and 4.45 m/sec (16.2 kph) in January [26]. These wind speeds were measured at the airport located amidst open surroundings. Wind speeds near building walls in the city are likely to be lower in general. Besides, the hottest record temperatures are also accompanied by very light winds. Considering these facts, the high temperature analysis was performed with a nominal wind speed of 1 m/sec; for low temperature analysis, 2 m/sec was assumed.

Solar Radiation- It was mentioned in Sec. 2.2 that the intensity and duration of solar radiation received by a surface over a 24-hour period depends upon the location on earth and the orientation of the surface. The transient analysis programme requires the input on location in terms of latitude, longitude, and standard time meridian which, for Ottawa, are 45°19', 75°40', and 75°, respectively. Except northfacing walls, all other walls are exposed to solar radiation of varying intensity and duration depending upon the orientation. It is therefore obvious that for the minimum temperature analysis, a northfacing wall will govern. But because of the complex interplay of thermal properties of the materials and geometry of the wall cross section together with the hourly variation of solar radiation and ambient temperature, it is difficult to envisage which orientation of the wall will create a maximum temperature in the loadbearing element of the wall. A series of transient heat flow analyses was therefore done with the orientation of wall as a parametric variable. Since the sun is always to the south of Ottawa, walls oriented with outward normals in the northern quadrants do not receive any solar radiation; hence, they are excluded from this parametric study. Fig. 2.4 shows the results of these analyses. The maximum

temperatures have been normalised with respect to those for an orientation angle of 90° and plotted against angle of orientation. It is observed that while maximum surface temperatures are reached in a westfacing wall ($\gamma = 270^\circ$), maximum temperatures in the loadbearing block walls are reached in walls oriented approximately west-southwest ($\gamma = 245^\circ$). It is also interesting to note that although a southfacing wall ($\gamma = 180^\circ$) receives solar radiation of the longest duration, it is subjected to minimum temperatures in all cases. This is so because the angle of incidence of solar radiation on a southfacing wall is very oblique in summer.

Fig. 2.4 Relationship of Maximum Temperature with Wall Orientation

Solar radiation received by a wall also depends upon the atmospheric conditions. It was assumed that the sky is clear for both maximum and minimum temperature analyses. The average total number of hours of sunshine received by Ottawa is about 277 in July, 59% of that possible, the highest July record being 339 in 1982 [26]. A clearness factor of 1.05 was assumed.

2.4.2 Thermal Properties of Materials

Absorptivity and Emissivity- Solar radiation absorbed by a wall is the principal determinant of the maximum temperature in the wall. Reliable values of solar absorptivity for masonry surfaces are difficult to obtain. BIA [12] classifies glazed and unglazed brick masonry surfaces into three colour groups viz., light, medium, and dark. The absorptivities of these groups are given as 0.3 and less, 0.3 to 0.5, and 0.7 and over, respectively. GEC [35] provides specific values of emissivities at different wave lengths of radiation including solar. Since terrestrial bodies cannot emit at wave lengths of solar radiation under ordinary circumstances, the solar emissivities

are probably meant to be the solar absorptivities in reality. Twenty different values of this so-called solar emissivity, depending upon colour and material composition and ranging between 0.45 and 0.90, have been given for bricks. The solar emissivity of uncoloured concrete tiles has been given as 0.65. Kasten [51] gives only long wave absorptivity of brick wall surfaces as 0.94. ASHRAE [1] gives solar absorptivities of 0.65 to 0.80 for red brick and concrete, 0.50 to 0.70 for bricks of medium colour, and 0.30 to 0.50 for light coloured bricks. Emissivities of all these surfaces in the range of temperature relevant to this study vary between 0.85 and 0.95 according to this report. Garden [33] provides solar absorptivities on the basis of colour only, ranging from 0.45 for white to 0.90 for black surfaces. Reagan and Acklam [76] developed a probe and employed it to measure solar reflectivities of common building materials. Solar absorptivities can be obtained from their data by deducting solar reflectivities from unity. Accordingly, solar absorptivity of concrete masonry surfaces range between 0.55 and 0.79, 0.61 being the value for plain or grey surfaces. Solar absorptivities of walls made of bricks vary between 0.49 and 0.72 depending on a colour coding scheme used by the authors to designate the bricks. These absorptivity values appear to be slightly lower than reported by others.

A series of heat flow analyses was carried out to determine the sensitivity of temperatures in a wall section with respect to solar absorptivities of the exterior surface. The results of these analyses are shown in Fig. 2.5 in which normalised temperatures are plotted against solar absorptivity. The temperatures have been normalised with respect to those with solar absorptivities of 0.85 and 0.75 for the cavity wall and the single wythe wall, respectively.

Fig. 2.5 Relationship of Maximum Temperature with Solar Absorptivity

It can be seen that surface temperatures are fairly sensitive to changes in solar absorptivity. The mean temperatures in the block wall are less sensitive. An underestimation of solar absorptivity by about 12% introduces an error of only about 3% in the block mean temperature for the cavity wall; similar sensitivity is also observed for the single wythe wall. For this work, solar absorptivities of brick and block surfaces were assumed as 0.85 and 0.75, respectively.

Conductivity and Thermal Mass- Fenton [28] conducted a literature survey of thermal characteristics of the materials typically used in masonry wall construction. Based on this survey, he used the following formulas for thermal conductivities of clay bricks and concrete blocks which are also used in this study:

$$k_{br} = f k_{dbr} \quad (2.2)$$

$$k_{bl} = \{ v (f - 1.0) + 1.0 \} k_{dbl} \quad (2.3)$$

$$\text{where, } k_{dbr} = (\rho / 2505.46)^{6.642} + 0.25917 \times 10^{-3} \rho - 0.0504 \quad (2.4)$$

$$k_{dbl} = 0.6078 \times 10^{-6} \rho^2 - 1.31 \times 10^{-3} \rho + 0.7702 \quad (2.5)$$

$$f = 1 + 0.3157 w^{0.6} - 0.01567 w \quad (2.6)$$

$$v = 1.0 - (1.0 - \alpha)(1.0 - \psi) \quad (2.7)$$

In the above equations,

k_{br} , k_{bl} = thermal conductivity of brick and block, respectively, W/m °C

k_{dbr} , k_{dbl} = same as above, but in oven-dry condition, W/m °C

ρ = density of the material, kg/m³ (500 < ρ < 2600)

f = moisture correction factor

w = moisture content in percent by volume

v = fraction solid of block masonry

α = fraction of block cores grouted

ψ = ratio of net block to gross block area

Densities of brick and block were assumed as 2100 kg/m³ and 2000 kg/m³, respectively. As suggested in Eq. 2.6, moisture content of brick masonry affects its conductivity rather dramatically; an increase in moisture content from 0% to 8% by volume almost doubles the conductivity. A similar moisture correction factor suggested by Grím [38] indicates a weaker

influence, thermal conductivity doubling at a moisture content of 20% by volume. Moisture content of an exterior wall is highly variable, depending on the moisture absorption capacity of the materials, severity of exposure to rain, relative humidity, flashing details, and position of insulation. It is difficult to make a rational judgment on the moisture content considering all these factors. However, moisture content also increases the specific heat of bricks and blocks since water has a higher specific heat than dry masonry materials. The changes in conductivity and specific heat due to any change in moisture content of the outer skin have opposing effects on the maximum temperature reached at any point inside the wall, the net effect being insignificant. This was verified by a series of three analyses with moisture content varying from 3 to 12 percent in the brick cladding and from 5 to 10 percent in the block wall; this showed almost no change in the maximum block wall temperatures. Average water absorption of clay bricks and concrete blocks commonly used are 14% and 10% by weight, respectively [85]. Expressed in percent by volume, these are 30% and 20%, respectively. Grimm [38] assumes a moisture content of 12 to 20 percent by volume for brick walls after a rainy period. For the cavity wall, moisture contents of 12% for the brick cladding and 5% for the interior block wythe were assumed; for the single wythe block wall, 10% was assumed.

Specific heat of bricks varies between 0.84 to 0.96 kJ/kg °C and that of hollow concrete blocks of normal weight is about 0.92 kJ/kg °C [28]. A value of 0.90 kJ/kg °C was assumed for both bricks and blocks in this study.

A variety of materials is used as insulation and wall finish in building construction. Thermal properties of many of these materials are listed by Fenton [28]. The insulation in this study was assumed as 50 mm thick styrofoam having a specific heat of 1.21 kJ/kg °C and thermal conductivity given by

$$k = 0.1686 \times 10^{-4} (T) - 0.0066 \quad (2.8)$$

where T = temperature in °K

A 12 mm thick gypsum board was assumed as wall finish. Specific heat and thermal conductivity of this material are 1.09 kJ/kg °C and 0.136 W/m °C, respectively.

2.4.3 Summary of Thermal Analysis Data

A summary of all the data used or assumed for the thermal analysis is given below:

Location:	Ottawa
Ambient Temperature:	Summer 38 °C max 24 °C min

Winter -26 °C max -36 °C min

Wind: 1 m/sec in summer, 2 m/sec in winter

Solar Radiation Data:

- Latitude = $45^{\circ} 19' N$
- Longitude = $75^{\circ} 40' W$
- Standard Time Meridian = 75°
- Orientation of wall = west-southwest (245°) in summer
= north (0°) in winter
- Solar absorptivity, brick = 0.85
block = 0.75
- Emissivity, brick and block = 0.90
- Clearness factor = 1.05

Thermal Properties:	Thermal conductivity,	brick = as given by Eq. 2.2 block = as given by Eq. 2.3 insulation = as given by Eq. 2.8 wall finish = $0.136 \text{ W / m } ^\circ\text{C}$
	Density,	brick = 2100 kg/m^3 block = 2000 kg/m^3
	Moisture content,	brick = 12% by vol. block = 5% by vol. for cavity wall = 10% by vol. for single wythe
	Specific heat,	brick = $0.90 \text{ kJ/kg } ^\circ\text{C}$ block = $0.90 \text{ kJ/kg } ^\circ\text{C}$ insulation = $1.21 \text{ kJ/kg } ^\circ\text{C}$ wall finish = $1.09 \text{ kJ/kg } ^\circ\text{C}$

2.5 Exterior Wall temperatures - Steady State Analysis

Until the early seventies, cooling and heating requirements for buildings were calculated on the basis of steady state conditions. A steady state condition is said to be reached when the heat flow rate does not change with time. If two points in a wall have temperatures T_1 and T_2 , then the steady state heat flow rate, q , can be derived directly from Eq. 2.1 as

$$q = kA(T_1 - T_2)/l' \quad (2.9)$$

where k = coefficient of thermal conductivity

l = distance between the points

A = cross sectional area perpendicular to heat flow

Eq. 2.9 can be rewritten as

$$q/A = (1/R)(T_1 - T_2) \quad (2.10)$$

where, $R = l/k$, thermal resistance of the material of thickness l

Heat transmission through a unit area of a composite wall section is given by [48]

$$q/A = U(t_i - t_o) \quad (2.11)$$

where, t_i = inside temperature

t_o = outside temperature

U = overall coefficient of heat transmission

The total thermal resistance of the composite wall section, R_{tot} , is given by

$$R_{tot} = 1/U = R_1 + R_2 + R_3 + \dots + R_n \quad (2.12)$$

where, $R_1 \dots R_n$ = individual resistances of the elements

Thermal resistances of solids are easily derived from their conductances. But thermal resistances of air films at the inside and outside surfaces of and any air space in the wall have to include the effect of radiation and convection. This is done by deriving the thermal resistance of air films and air spaces from

$$R_{air} = 1/(h_c + h_r) \quad (2.13)$$

where, h_c = convective heat transfer coefficient

h_r = radiative heat transfer coefficient

More detailed discussions about these coefficients and their derivations are available in Chapter 8 of reference 48.

The inside air temperature, t_i , is assumed as 20 °C in this work, which is the standard temperature maintained for optimum comfort in both winter and summer. To calculate the maximum summertime temperature in the exterior wall, the effect of solar radiation must be reflected in the outside temperature, t_o . This is done by using the sol-air temperature concept. Sol-air temperature is the fictitious air temperature which will cause the same amount of heat flow by surface conductance alone as that caused by radiation, convection, and conduction at the surface of the wall. More details on sol-air temperature are given in Appendix A. To calculate the minimum temperature in the exterior wall, t_o was assumed equal to the minimum ambient temperature occurring in winter.

The calculations of maximum and minimum temperatures for both types of wall cross sections shown in Fig. 2.1 are presented in tabular form in Tables 2.3 through 2.6. Calculations for the conductances of air films and air spaces are shown in Appendix B. Figs. 2.6 and 2.7 show

the temperature profiles for both types of walls as calculated in the abovementioned tables. It is observed that the maximum mean temperatures in the loadbearing block wall are about 68 °C and 73 °C for the cavity wall and the single wythe wall, respectively; the minimum mean temperatures in the same walls are about -25 °C and -31 °C, respectively. Also, the maximum exterior surface temperature for both walls is about 75 °C. These temperatures will be compared with those obtained from transient analysis later.

Table 2.3 Steady State Maximum Temperatures in Cavity Wall

Element	Conductance $W/m^2^{\circ}C$	Resistance $m^2^{\circ}C/W$	Temperature Drop $^{\circ}C$	Temperature $^{\circ}C$
Outside air film	18.14	0.055	1.7	76.7
Brick cladding	14.67	0.068	2.1	75.0
Air space	10.16	0.098	3.1	72.9
Block	7.60	0.132	4.2	69.8
Insulation	0.80	1.250	39.3	65.6
Wall finish	12.50	0.080	2.5	26.3
Inside film	8.22	0.122	3.8	23.8

Total resistance = 1.805

Table 2.4 Steady State Maximum Temperatures in Single Wythe Wall

Element	Conductance $W/m^2^{\circ}C$	Resistance $m^2^{\circ}C/W$	Temperature Drop $^{\circ}C$	Temperature $^{\circ}C$
Outside air film	17.82	0.056	1.9	76.7
Block	7.60	0.132	4.6	74.8
Insulation	0.80	1.250	43.2	70.2
Wall finish	12.50	0.080	2.8	27.0
Inside film	8.22	0.122	4.3	24.2

Total resistance = 1.640

Table 2.5 Steady State Minimum Temperatures in Cavity Wall

Element	Conductance $\text{W/m}^2\text{°C}$	Resistance $\text{m}^2\text{°C/W}$	Temperature Rise °C	Temperature °C
Outside air film	20.12	0.050	1.5	-36.0
Brick cladding	14.67	0.068	2.0	-33.5
Air space	6.14	0.163	4.8	-26.7
Block	7.60	0.132	3.9	-22.8
Insulation	0.80	1.250	36.9	14.1
Wall finish	12.50	0.080	2.4	16.5
Inside air film	7.96	0.126	3.6	20.0

Total resistance = 1.869

Table 2.6 Steady State Minimum Temperatures in Single Wythe Wall

Element	Conductance $\text{W/m}^2\text{°C}$	Resistance $\text{m}^2\text{°C/W}$	Temperature Rise °C	Temperature °C
Outside air film	20.12	0.050	1.7	-36.0
Block	7.60	0.132	4.4	-33.3
Insulation	0.80	1.250	42.0	-28.9
Wall finish	12.50	0.080	2.7	13.1
Inside air film	8.22	0.126	4.1	15.8

Total resistance = 1.638

2.6 Transient Heat Flow Analysis

It was explained in Sec. 2.2 that important determinants of the exterior wall temperature, like solar radiation, are constantly changing. Therefore, a steady state condition is never reached in the heat flow through the wall. Strictly speaking, even a steady-periodic condition is not reached because the variations of atmospheric conditions and wind are aperiodic, although

variation of extraterrestrial solar radiation is periodic.

(a) Cavity Wall

(b) Single Wythe Wall

Fig. 2.6 Steady State Maximum Temperature Profiles in Wall Sections

Past research conducted by BIA [11] indicates that the actual heat transfer through masonry walls may be up to 20% less than calculated by steady state methods. NBS [64] states that heat flows calculated on the basis of steady state conditions were 29% to 69% greater than those measured under dynamic conditions for masonry walls. Therefore, a transient heat flow

analysis is necessary to determine the temperatures in the exterior wall with reasonable accuracy.

Fig. 2.7 Steady State Minimum Temperature Profiles in Wall Sections

Under typical summer conditions, the exterior wall begins to get warmed early in the morning as the ambient temperature begins to rise due to sunrise. For some length of time during the day, depending on its orientation, the wall is also exposed to some solar radiation. During the

course of the day, the wall transmits some heat into the interior, but also stores some heat because of its thermal mass described earlier. After reaching a peak at some instant during the day, the temperature at the exterior surface begins to fall. After the sunset, the wall begins to lose heat stored in it mainly by long wave radiation into the cooler evening air, and this continues through the night until the temperature again begins to rise the next day at dawn, completing a 24-hour cycle in the heat flow. Due to thermal mass, a certain time has elapsed between the occurrence of peak temperature at the exterior surface of the wall and that at any point in the wall cross section. The further the point from the exterior surface, the longer is the time elapsed. An attenuation of the peak temperature also occurs with increasing distance from the surface into the wall due to the thermal resistance of the wall.

A 24-hour heat flow cycle similar to the above also occurs in winter. However, the heat flow is almost always from inside to the outside as opposed to the direction of heat flow switching between day and night in summer. Since a northfacing wall in Ottawa receives no solar radiation and since the daily variation of ambient air temperature in winter is small, the heat flow is fairly close to steady state during the early hours of the morning when the wall reaches the lowest temperature. This will be evident from the results of thermal analysis presented later

2.6.1 Analysis Technique

The tool that was used in this study to determine the maximum and minimum temperatures in the exterior wall is a computer programme recently developed by Fenton [28]. The programme uses the finite difference technique to solve the differential equation of heat flow which is based on the basic equation of heat equilibrium given, for cases where no internal heat is generated, by

$$\text{HEAT STORED} = \text{HEAT IN} - \text{HEAT OUT} \quad (2.14)$$

The governing differential equation for heat flow in one dimension only by conduction is

$$\rho c A \frac{\delta T}{\delta \tau} = \frac{\delta}{\delta x} \left(k A \frac{\delta T}{\delta x} \right) \quad (2.15)$$

where ρ = mass density of the material
 τ = time
 c = specific heat
 x = distance along heat flow path
 k = coefficient of thermal conductivity
 A = area of cross section perpendicular to heat flow
 T = temperature of the material

To illustrate the technique used by the programme, consider the simple example of a uniform bar in which heat is flowing from one end to the other. The bar may be modelled as a series of small nodal elements linked together as shown in Fig. 2.8. For a typical interior node, say node 2, where heat transfer is taking place only by conduction, the heat equilibrium equation applied during the small interval of time $\Delta\tau$ reduces to

$$\rho A \Delta x c (T_{2, \tau + \Delta\tau} - T_{2, \tau}) = k_{12} A \frac{T_{1, \tau} - T_{2, \tau}}{\Delta x} \Delta\tau - k_{23} A \frac{T_{2, \tau} - T_{3, \tau}}{\Delta x} \Delta\tau \quad (2.16)$$

where $T_{i, \tau}$ = Temperature at node i at time τ

$T_{i, \tau + \Delta\tau}$ = Temperature at node i after the interval $\Delta\tau$

k_{lm} = coefficient of thermal conductivity between nodes l and m

The future temperature, $T_{2, \tau + \Delta\tau}$, may therefore be expressed as

$$T_{2, \tau + \Delta\tau} = \frac{\Delta\tau}{\rho c (\Delta x)^2} \{ k_{12} (T_{1, \tau} - T_{2, \tau}) - k_{23} (T_{2, \tau} - T_{3, \tau}) \} \quad (2.17)$$

Fig. 2.8 Finite Difference Model of Uniform Bar

This indicates that the future temperature at node 2 after the interval $\Delta\tau$ can be determined if the present temperatures at node 2 and the adjacent nodes are known. The same argument applies for any other node as well. Now suppose the temperature at node 1 is raised slowly starting at an instant of time when all the nodal temperatures are known, and the problem is to determine the nodal temperatures at definite time intervals. The programme subdivides each of these time intervals into a number of subintervals and applies the increase in temperature in node 1 for the first subinterval. It then uses Eq. 2.17 to calculate the temperature at node 2, then at node 3, and so on until all the nodal temperatures are obtained for the first subinterval of time. The process is repeated for the subsequent subintervals. When all the subintervals for the first interval of time have been exhausted, the programme outputs the nodal temperatures. Then it proceeds to the next time interval and uses the same procedure all over again.

The temperatures at different points in a wall cross section are obtained in the same manner. Each different element in the wall is subdivided into a number of nodal elements so as to form a series of these elements but having different thermal properties. Also radiation and convection are involved at the solid-air interfaces. But these pose no additional difficulty to the procedure described for the bar example.

However, a problem arises about the initial temperatures to be assigned to the nodes at the start of the analysis. The temperatures at the nodes at a particular instant are determined by the previous history of heat flow, and are therefore unknown. Fenton [28] assumes that a steady state condition is reached just before dawn. He therefore starts the transient analysis at dawn with the initial temperatures given by steady state analysis. However, his own results indicate that a steady state condition is in fact not reached at dawn in summer. Had it been so, there would be no discrepancy between the initial temperatures and the temperatures at the end of the 24-hour analysis. Therefore, the nodal temperatures indicated by such analysis are only approximate.

An improvement in the results can be achieved by simply repeating the 24-hour analysis a number of times until the final temperatures from two consecutive cycles are practically the same. The nodal temperatures given by the final 24-hour cycle can be considered as those closest to the true temperatures. If this technique is used, the initial temperatures could be any arbitrary values. The only difference it would make is in the number of iterations required to reach the convergence in the final temperatures. Temperatures at dawn given by steady state analysis are good initial values in this respect. Evidently, wall temperatures obtained by this iterative approach would most closely represent the real temperatures reached by the wall when more or less the same environmental conditions repeat for a few consecutive days. Such a condition is indeed often associated with heat or cold waves that cause record high or low temperatures, respectively. This iterative approach will be used in this study to determine the maximum and minimum temperatures in the wall. It was found that after 5 iterations the differences remaining between the initial and final temperatures become negligible.

2.6.2 Results

The variations of temperature at different locations in the walls over a 24-hour period in summer and winter, as obtained from transient heat flow analysis, are shown in Figs. 2.9 and 2.10, respectively. The maximum and minimum temperatures reached in the same locations along with the times of the day at which these occur are presented in Table 2.7. It is observed from Figs. 2.9 and 2.10 that a considerable attenuation of the amplitudes of the daily cycles of temperature occur as the heat wave reaches the block wall. Also, there is a considerable time lag between the occurrence of maximum or minimum temperatures at the exterior surface and those at the block wall, about 5 hours for the cavity wall and 2 hours for the single wythe wall.

Fig. 2.9 Daily Temperature Variations in Summer by Transient Analysis

Fig. 2.10 Daily Temperature Variations in Winter by Transient Analysis

Table 2.7 Maximum and Minimum Temperatures by Transient Analysis

Type of wall	Temperature, °C, and hour of occurrence			
	Maximum		Minimum	
	Ext. surface	Block mean	Ext. surface	Block mean
Cavity	64.6 17:00	41.3 22:00	-30.3 5:00	-18.7 10:00
Single Wythe	57.6 17:00	48.5 19:00	-28.8 5:00	-25.9 7:00

Figs. 2.11 and 2.12 show the transient state temperatures in the wall cross sections at such times of the day when maximum or minimum temperatures occur in the exterior surface and in the block wall. The steady state temperature profiles shown earlier in Figs. 2.6 and 2.7 are also shown in these figures for the purpose of comparison. It is evident that steady state analysis significantly overestimates the maximum mean temperatures in the block wall, by about 27 °C for the cavity wall and about 25 °C for the single wythe wall. Thus, with respect to the constant interior temperature of 20 °C, the steady state analysis overestimates the maximum summertime temperature differentials in a cavity wall and a single wythe wall by about 65% and 51%, respectively. The spreads between the minimum temperatures obtained from the steady state and transient analyses are much smaller, but still substantial, about 7 degrees for the cavity wall and 6 degrees for the single wythe wall. In other words, the maximum temperature differentials in winter, with respect to the constant temperature of 20 °C, are overestimated by about 18% and 13% for the cavity wall and the single wythe wall, respectively. Fenton [28] used steady state analysis to determine the minimum wintertime temperature in a cavity wall similar to the one studied in this work; the results obtained thereby are therefore only approximate.

2.7 Temperatures in Interior Wall

It was mentioned earlier that the heat flow around the intersection of exterior and interior walls is two-dimensional (Fig. 2.2). However, a two-dimensional transient analysis of heat flow is rather complex, and is not warranted as will be shown later.

The objective is to determine the temperature distributions along the interior cross wall when the mean temperature in the exterior loadbearing block wall is maximum and minimum. The problem, as depicted in Fig. 2.13, is analogous to a fin of uniform cross section protruding out of a body maintained at a constant temperature. It is assumed that the temperature distribution is uniform along the thickness of the interior wall. Since temperature, t_i , is maintained on both sides of the interior wall all along its length, it can be assumed that at a distance l from O , the point of intersection, the temperature in the wall is equal to t_i . In other words, there is no further heat flow into or out of the wall beyond the point, P .

Since under steady state conditions no heat is stored, application of Eq. 2.14 to the heat flow into and out of the prism of length Δx and unit thickness yields

$$q_1 = q_2 + 2 q_3 \quad (2.18)$$

Using Eqs. 2.11 and 2.12,

(a) Cavity Wall

(b) Single Wythe Wall

LEGEND:

- At the time of day when surface temperature is maximum
- - - At the time of day when mean temperature in block is maximum
- Steady State

Fig. 2.11 Transient State Temperatures in Exterior Wall Cross Sections in Summer

(a) Cavity Wall

(b) Single Wythe Wall

LEGEND:

- Transient state - at the time of day when exterior surface temperature is minimum
- Transient state - at the time of day when mean temperature in block is minimum
- Steady state

Fig. 2.12 Transient State Temperatures in Exterior-Wall Cross Sections in Winter

Fig. 2.13 Analytical model of 2-D Heat Flow in Interior Wall

$$q_1 = -kb \left(\frac{dt}{dx} \right)_x \quad q_2 = -kb \left(\frac{dt}{dx} \right)_{x+\Delta x} \quad q_3 = \frac{1}{R_{wf} + R_{air}} (t - t_r) \Delta x \quad (2.19)$$

where k = thermal conductivity of concrete blocks
 R_{wf} = thermal resistance of wall finish
 R_{air} = thermal resistance of air film

But neglecting higher order terms of Taylor's expansion,

$$\left(\frac{dt}{dx} \right)_{x+\Delta x} = \left(\frac{dt}{dx} \right)_x + \left(\frac{d^2t}{dx^2} \right) \Delta x \quad (2.20)$$

Substituting Eqs. 2.19 and 2.20 in Eq. 2.18, simplifying, and rearranging,

$$\frac{d^2t}{dx^2} - \frac{2}{kb} \frac{1}{R_{wf} + R_{air}} (t - t_r) = 0 \quad (2.21)$$

Let

$$t = t - t_r \quad \text{and} \quad (2.22)$$

$$m = \sqrt{\frac{2}{kb} \frac{1}{R_{wf} + R_{air}}} \quad (2.23)$$

Therefore, Eq. 2.21 can be rewritten as

$$\frac{d^2 t}{dx^2} - m^2 t = 0 \quad (2.24)$$

The general solution to the above equation is

$$t = A e^{-mx} + B e^{mx} \quad (2.25)$$

The following are the boundary conditions:

$$\begin{aligned} \text{at } x = 0, \quad t = t_e \quad \text{i.e. } t' = t_e - t_r \text{ and} \\ \text{at } x = l, \quad t = t_r \quad \text{i.e. } t' = 0 \end{aligned} \quad (2.26)$$

Using the above boundary conditions in Eq. 2.25, A and B can be solved as,

$$A = - \frac{e^{2ml}}{1 - e^{2ml}} (t_e - t_r) \quad \text{and} \quad B = \frac{(t_e - t_r)}{1 - e^{2ml}} \quad (2.27)$$

Substituting Eqs. 2.22 and 2.27 in Eq. 2.25 and rearranging, the following expression is obtained:

$$\frac{t - t_r}{t_e - t_r} = \frac{1}{1 - e^{2ml}} (e^{mx} - e^{2ml} e^{-mx}) \quad (2.28)$$

The value of m can now be found by using appropriate values of conductivity of the materials. From a rigorous point of view, the radiative heat transfer coefficient, h_r , of the inside air film is variable from summer to winter (see Appendix B); it is also variable along the length of the interior wall since the temperature differential varies. However, the overall influence of these variations is insignificant. Hence a constant value of $t_i = 8.10 \text{ W/m}^2 \text{ K}$, which is an average of the winter and summer values computed in Appendix B, was used. Thermal conductivity of concrete blocks as calculated using Eq. 2.3 is about $1.1 \text{ W/m}^2 \text{ K}$, and that of the wall finish is about $0.137 \text{ W/m}^2 \text{ K}$.

Using the above conductivities in Eq. 2.23, m was computed as 6.75. Let the distance l where t becomes equal to t_r be 5 m, which will be verified later. Eq. 2.28 can now be used to plot

the interior wall temperature profile along its length with $t_r = 20^\circ\text{C}$ and known values of the exterior wall temperature, t_e . Four such plots, two for the maximum (summertime) and two for the minimum (wintertime) temperatures in the exterior wall as determined by the transient analysis earlier, are shown in Fig. 2.14. It can be seen that the temperature drops or rises very rapidly along the interior wall at points away from the intersection and, for all practical purposes, can be assumed to reach the room temperature of 20°C within a distance of 0.5 m from the intersection. One can easily visualise that under transient conditions these temperature profiles would be even steeper.

Fig. 2.14 Temperature Profiles along the Length of Interior Wall

From a study of the sensitivity of computed stresses to the temperature profile along the interior wall, very small increases in the key stresses were observed when the temperature profile was assumed as a stepped function from the exterior wall to the interior wall compared to the stresses obtained on the basis of the profile determined by the steady state analysis. Therefore, any refinement of the temperature profile by transient analysis would contribute even lesser changes in the stresses computed later using the steady state profile. Hence, a transient analysis to determine the temperature profile along the interior wall was not necessary. For the linear elastic analysis presented in Chapter 5 a stepped temperature profile was assumed; this means

that a uniform temperature of 20°C was assumed throughout the interior wall while only at the outer end the temperature was assumed equal to that in the exterior wall. However, the profiles given in Fig. 2.14 were used in the nonlinear stress analysis for greater accuracy.

2.8 Temperatures in Roof Slab

Like the exterior wall, the roof is also exposed to ambient temperature and solar radiation. Because of its orientation, the total insolation on a roof is greater than on a wall. However, there are several factors which tend to reduce the range of temperature variation the structural slab of the roof typically undergoes. First, the roof construction today is almost always 'inverted' which means that the insulation is usually placed on top of the slab. Secondly, the roof membrane is usually of light colour which absorbs less solar radiation. Steady state analysis of typical roofs comprised of 50 mm stone ballast and 64 mm rigid insulation on top of 64 mm reinforced concrete slabs indicates an average maximum temperature of 35°C and an average minimum temperature of 10°C in the reinforced concrete slab. It is evident that under transient conditions these extremes would be reduced, i.e. closer to the datum temperature of 20°C . Finally, the extreme temperatures in the roof slab do not coincide with those in the exterior wall in terms of time of occurrence; at the time when the temperature in the exterior loadbearing wall is a maximum, the temperature in the roof slab is likely to be lower than the maximum. Considering these facts, the temperature in the roof slab was assumed to be the same as in other floor slabs which is the datum temperature of 20°C .

3 Stress Analysis Preliminaries

3.1 General

The overall problem of intersecting walls in typical concrete masonry loadbearing buildings under investigation in this work was introduced in Chapter 1. In order to achieve the objectives as set forth in that chapter, a large amount of stress analysis work must be performed. In this chapter the stress analysis problem is formulated. The methods available and the one selected for the purpose of stress analysis are treated. Also discussed are the properties of the materials relevant to the linear analysis and the basic assumptions in formulating the problem. The constitutive relationships of the materials required for the nonlinear analysis will be treated in Chapter 6.

3.2 The Stress Analysis Problem

Two situations in which the intersecting exterior and interior walls of a building may undergo relative deformations were identified in Chapter 1. It was mentioned that since relative deformation is not possible at the intersection due to a rigid connection, stresses would inevitably be induced in the walls. Stresses and deformations that could develop in the walls due to a rise in the exterior wall temperature are illustrated in Fig. 3.1. Stresses and deformations somewhat similar to these which could occur under differential gravity loading of the intersecting walls are shown in Fig. 3.2.

Only the intersection area has been focussed upon in the above illustrations. It is obvious that the stresses and deformations will be governed by the boundary conditions applied at the other ends of the walls. In a general case, no defined boundary conditions exist at these locations and therefore, from an ideal point of view, the entire structure has to be analysed. Fortunately in reality, some symmetry exists about one or more axes of the building which reduces the domain of analysis. The building plan that was used for analysis in this work has one such geometrical axis of symmetry as shown in Fig. 3.3. Such a plan was chosen for this study to keep the computational efforts within manageable proportions; in spite of this, the computational requirements were very large as will become apparent later.

Advantage of this symmetry can only be taken if the loading is also symmetrical about the axis. The gravity loadings for which these buildings are normally designed are evidently symmetrical. The thermal loading, however, is not truly symmetrical since the exterior walls on the two sides of the building will generally experience different temperatures. For the cavity type of

exterior wall, the temperature in the eastfacing exterior wall is 39.6°C corresponding to the maximum summertime temperature of 41.3°C in the westfacing exterior wall; for the single wythe type of exterior wall, these temperatures are 42.7°C and 48.5°C , respectively. These differences are small, and even smaller differences between such temperatures exist in the case of wintertime minimum temperature loading. Therefore, for all practical purposes, it can be assumed that loading symmetry also exists for thermal loading about the geometrical axis of symmetry shown in Fig. 3.3. Hence, only one half of the building will be analysed.

Fig. 3.1 Possible Stresses in Intersecting Walls due to a Rise in the Temperature of the Exterior Wall

Further reduction of the domain of analysis can be effected depending upon conditions prevailing in the longitudinal direction of the building. It is assumed that the same bay width and wall arrangement are repeated along the length of the building. It is also assumed that the building has a length to height ratio greater than unity. For such buildings, it can be reasonably assumed that no deformations are possible in the longitudinal direction. Therefore, only one bay, in fact half a bay since symmetry also exists about the crosswall in each bay, needs to be modelled analytically for the purpose of stress analysis.

Fig. 3.2 Possible Stresses in Intersecting Walls due to Differential Gravity Loads

The resulting domain of analysis is shown as a shaded area in Fig. 3.3. Appropriate boundary conditions can now be applied to the structure within this domain. The boundary conditions are as shown in Fig. 3.4. It has been assumed that both vertical and horizontal degrees of freedom are restrained at the foundation level. While some horizontal movement may be possible at that level in reality, the resulting effects on the stresses will be only localised; the key stresses which occur at the upper levels of the building will remain virtually unaffected.

From Figs. 3.1 and 3.2 it is evident that the walls are loaded in plane under both loading conditions. The gravity load on the floor slab is of course out of plane, but stresses in the slab due to the gravity loads are of no interest in the linear elastic stress analysis as explained in the next section. Since the crosswall lies on a plane of symmetry, it cannot undergo any out-of-plane deformation; in other words, a pure plane stress condition exists in that wall. Under the loads considered in this study, the exterior wall, the corridor wall, and the floor slabs can theoretically deform out of plane. But since the walls and slabs are thin, their in-plane rigidities are far greater than their out-of-plane rigidities. This means that the loads are carried by them predominantly by

membrane action. The flexural stresses in the walls and slabs were therefore disregarded in this study. These flexural stresses would have only a localised influence in the vicinity of wall to wall or wall to slab connections which are usually rigid. The assumption of plane stress conditions in walls and floor slabs means they can be modelled by plane stress finite elements.

Fig. 3.3 Part Plan of Building showing Axes of Symmetry and Domain of Analysis

3.3 Loading Cases

The stress analysis was performed for two loading cases, one emanating from differential temperature in the intersecting walls, and the other from differential gravity loading of the intersecting walls. The latter is caused by one way slabs carrying the floor loads onto either of the walls.

Since the one-way floor slabs can span in either of the two directions as indicated in Fig 3.2, the question arises as to which orientation will produce the most critical combination of stresses caused by the gravity and thermal loads. Careful examination of Figs. 3.1 and 3.2 will reveal that such critical combinations are different for different types of stresses. For example,

maximum compressive stress in the exterior wall is likely to be created under a combination of summertime thermal loading and the floor loads being carried by the exterior and corridor walls; whereas, maximum tensile stress in the same wall would be created under a combination of wintertime thermal loading and the floor loads being carried by the crosswalls.

Fig. 3.4 Boundary Conditions of the Domain of Analysis

Stress analysis should be carried out for all such combinations. In the linear elastic analysis part of this work, stress analysis was performed for each of the following loading cases:

1. Thermal loads under summertime maximum temperatures
2. Thermal loads under wintertime minimum temperatures
3. Gravity Loads, Case 1 - floor loads on the longitudinal walls
4. Gravity Loads, Case 2 - floor loads on the crosswalls

The key stresses were then identified and possible combinations of the stresses from the above cases were tried to study the effect on the key stresses. The most significant combination was then selected for the nonlinear stress analysis.

One might wonder why lateral loads have not been included. The objective of the linear elastic stress analysis is to throw light on the additional stresses that could occur due to thermal loads and differential gravity loads which are not considered in conventional design procedures.

Lateral load is considered in normal design practice. However, the implications of the stresses and any cracking due to thermal loads on the lateral load carrying capacity of the building will be discussed. For the same reason, it was deemed unnecessary to compute the flexural stresses in the floor slabs caused by the gravity loads. Hence the floor loads were directly applied onto the walls on which the slabs bear.

3.4 Properties of Materials

While a variety of materials are used in typical building construction, the structural materials relevant to this study are concrete block masonry, concrete, and reinforcing steel. Although brick masonry is used as a cladding in cavity wall construction, its mechanical properties are irrelevant since it does not participate in the load carrying mechanism. For the purpose of stress analysis, the objective is to use such material properties which are representative of typical buildings. The properties selected or assumed for each of the above materials are discussed in the following subsections.

3.4.1 Concrete Block Masonry

The following properties of concrete block masonry are of particular interest in this study:

1. Coefficient of thermal expansion
2. Compressive strength
3. Tensile strength
4. Modulus of elasticity

Coefficient of thermal expansion- Although this property is generally treated as a thermal property, it is more appropriately a thermomechanical property in that the thermal deformations can also produce stresses. It is treated here since it is more contextual in stress analysis than in thermal analysis.

Concrete block masonry is a composite material made of concrete blocks, mortar, and sometimes also grout. Each of these primary constituents is made more or less from the same basic ingredients such as cement, aggregate and water. The coefficient of thermal expansion of block masonry is therefore likely to be close to that of the individual constituents. Information available on the coefficient of thermal expansion of block masonry is rather scanty. Jessop [49] reports a wide range of the coefficient, 6×10^{-6} to 13×10^{-6} per $^{\circ}\text{C}$, for concrete blocks. According to PCA [72], the coefficients are 7.7×10^{-6} per $^{\circ}\text{C}$ for lightweight and 9.4×10^{-6} per $^{\circ}\text{C}$ for normal weight concrete masonry. BIA [10] gives a series of values of the coefficient

according to the type of aggregate used. For lightweight aggregates such as pumice, and expanded slag, the coefficient varies between 5.6×10^{-6} per $^{\circ}\text{C}$ and 8.3×10^{-6} per $^{\circ}\text{C}$; for dense aggregates it is given as 9.4×10^{-6} per $^{\circ}\text{C}$.

None of these sources treat the thermal expansion coefficient of concrete masonry in depth. A large body of literature dealing with the thermal coefficient of a similar material, concrete, indicate that the coefficient is influenced by several factors. Values of the coefficient for concrete published by different sources vary fairly widely, one of the reasons being that no common conditions of all the parameters exist among various research reports. It is very likely that the same factors also influence the thermal coefficient of expansion of concrete block masonry, and there may be more. For example, the coefficient for concrete block masonry may be direction-dependent.

In the absence of any specific research information on the parameters peculiar to masonry and their influence on the thermal expansion coefficient, a value for the coefficient must be selected from the available sources. Considering normal weight concrete blocks made of dense aggregates such as gravel and sand, a value of 10×10^{-6} per $^{\circ}\text{C}$ appears reasonable. A constant value of the coefficient is satisfactory in the range of temperature relevant to this study. According to Weigler and Fischer [93], the influence of temperature on the thermal expansion coefficient of concrete is negligible at temperatures below 100°C . Tests conducted by Berwanger [6] on concrete slabs also indicate that there is almost no change in the thermal expansion coefficient of concrete in the temperature range of -35°C to 25°C .

Compressive strength- For concrete block masonry, which is usually hollow, compressive strength is customarily expressed on the basis of net cross sectional area. The area of grouting, if any, is also included. Compressive strength of the assemblage is a function of the compressive strengths of individual constituents viz., concrete block, mortar, and grout. Other factors like workmanship and thickness of the mortar joints also have a major influence.

Concrete block masonry is an orthotropic material in that its strength is direction-dependent. Experimental results reported by Hamid and Drysdale [40] show about the same strengths in the principal directions for both grouted and ungrouted block masonry. But loaded at skew angles, both types indicate a lower strength, the ungrouted specimens showing a much greater drop in strength than the grouted specimens. However, orthotropy was not considered in this study for the purpose of linear elastic stress analysis. It was considered in a special way in the nonlinear analysis which will be explained later. Compressive strength of block masonry usually implies its strength perpendicular to the bed joints.

A wide range of compressive strengths of concrete masonry is used in the design of buildings, the most common range being 10 to 12 MPa. Since strength requirement in the upper

floors is less, often two different strengths are used in the upper and the lower levels of multistorey buildings. In this work, a constant strength of 12 MPa was assumed over the entire height of the building. This strength is typically achieved by using concrete blocks of 20 MPa compressive strength and Type S (1:1.5:4.5) mortar.

No information is available on the relationship between compressive strength of concrete block masonry and temperature. A large number of studies have been carried out on the effects of elevated temperature on the strength of concrete. Schellenbach and Schimmelpfennig [82] reported that the compressive strength of concrete generally decreases with an increase in temperature but a great scattering is indicated in the available results due to the multiplicity of relationships between compressive strength and other factors. However, in the range of temperature relevant to this study, any decrease in compressive strength with increasing temperature as shown by the average trend is negligible. Lankard et al [57] reviewed a number of these studies. The trend of the results shown by these studies indicates about a 6% decrease in the compressive strength at a temperature of 50 °C as compared to the strength at 20 °C when free moisture is present during heating. A 3% increase in strength is indicated in the same temperature range when the specimen is restrained during heating. Since both of these conditions are more or less present in the concrete block masonry walls under study, it appears that the net change in the compressive strength of masonry in the present range of temperature will be negligible. A constant compressive strength of 12 MPa was used for the entire range of temperature, -20 °C to 42 °C, in this study.

Tensile strength- Unlike compressive strength, tensile strengths of ungrouted block masonry, as determined from splitting tests, are considerably different along the two principal axes [23]. The tensile strength perpendicular to bed joints is about half of that parallel to bed joints. However, grouted masonry shows about the same tensile strength in both principal directions although somewhat higher strength is indicated at skew angles.

Tensile strength perpendicular to bed joints is little influenced by the strength of the blocks; tensile strength of grout and mortar to block bond strengths are the most influential factors in that direction. On the other hand, block strength has a significant influence on the tensile strength parallel to bed joints since the split occurs across the blocks, whereas grout strength has almost no influence. Test results presented by Drysdale et al [23] show that, depending upon percent grouting and strength of grout and mortar, tensile strength of concrete block masonry may vary from 0.25 to 1.0 MPa in the direction perpendicular to bed joints; tensile strength parallel to bed joints was found to vary only slightly from 0.7 to 0.8 MPa. Compressive strength of the masonry used in these tests, as derived from the materials data, was about 12.0 MPa.

For the purpose of linear elastic analysis in this work, a uniform tensile strength of 0.5 MPa in the direction normal to bed joints was assumed. This is deemed to be the best representative of the tensile strength of concrete block masonry, which is partially grouted with medium strength grout, in the direction perpendicular to the bed joints. In the direction parallel to bed joints, a tensile strength of 0.7 MPa was assumed. In diagonal directions, an intermediate value of 0.6 MPa was assumed.

Modulus of elasticity- This is another key property of concrete block masonry in this study in that thermal stress developed under elastic conditions is directly proportional to the modulus. Like concrete, it is related to the compressive strength of masonry. The most widely used relation is the one given in the CSA code on masonry design and construction [15] which is given below:

$$E_m = 1000 f_m' \quad (3.1)$$

where E_m = modulus of elasticity of masonry in compression, MPa

f_m' = compressive strength of masonry, MPa

Sturgeon [88] made an extensive review of the available information on modulus of elasticity of concrete block masonry. He concludes that Eq. 3.1 possibly overestimates the modulus by about 15%. Results presented by Sahlin [81] however indicate that the same equation rather gives a conservative but realistic estimate of the modulus. Because of its wide acceptance, Eq. 3.1 was used in this study; hence, for the 12 MPa masonry, the modulus of elasticity is 12 000 MPa.

Berwanger [6] reports that an increase occurs in the modulus of concrete with decreasing temperature below the freezing point of water. The increase was explained as being caused by the increased cohesive force between the particles in concrete due to the expansive force produced by the freezing of free moisture. Hence, the amount of increase in the modulus at a particular below-freezing temperature depends on the amount of free moisture present. Since no quantitative relation between the moisture content and the increase in the modulus is available, particularly for concrete block masonry, this phenomenon can not be considered in this study. Results shown by Berwanger indicate only a slight increase in modulus for old concrete at below-normal temperatures. As reported by Schellenbach and Schimmelpfennig [82], the modulus of elasticity of concrete decreases with increasing temperature but only by about 7%, for an increase of temperature from 20 °C to 50 °C.

3.4.2 Concrete and Steel

Concrete typically used in construction of the particular type of floor and roof slabs considered in this study has a compressive strength of 20 MPa. For normal density concrete, the CSA code for the design of concrete structures [14] provides the following formula for the modulus of elasticity of concrete:

$$E_c = 5000 (f'_c)^{0.5} \quad (3.2)$$

where E_c = modulus of elasticity of concrete, MPa

f'_c = compressive strength of concrete, MPa

With f'_c equal to 20 MPa, the modulus of elasticity of concrete is about 22 360 MPa.

Intermediate grade steel with a yield strength of 400 Mpa is the most frequently used steel for reinforcing in masonry building construction. The modulus of elasticity of steel is almost constant, about 200 000 MPa. The thermal coefficient of expansion of steel is about 11.5×10^{-6} per °C.

The mechanical properties of all the materials relevant to this study are summarised in Table 3.1.

Table 3.1 Mechanical Properties of Materials

Material	Compressive Strength, MPa	Tensile Strength, MPa	Modulus of Elasticity, MPa	Thermal Coefficient of Expansion, per °C
Concrete Block Masonry	12.0	0.5 $\perp$ 0.7	12 000	0.00001
Concrete	20.0	2.0	22 360	n.a
Steel	400.0	400.0	200 000	0.0000115

$\perp$ perpendicular to bed joints; || parallel to bed joints

3.5 Method of Analysis

It was decided at the outset that the finite element method of analysis would be used for the stress analysis in this work. A brief period of time was spent, however, on the possibility of using classical methods. It was realised that the overall problem would be too complex to be

treated by classical methods. Coull and Magd [19] analysed a wide flanged shear wall structure under lateral loads by classical methods using some simplifying assumptions about the stress distribution. However, the interaction of the floor slabs with the walls in carrying the applied loads was not considered in their study. It was also decided to undertake first a linear elastic stress analysis in order to estimate the magnitude and general distribution of stresses. Based on the results of this linear elastic stress analysis, it could be decided later if a nonlinear stress analysis was necessary. Eventually, a nonlinear stress analysis was indeed deemed necessary and was thus carried out following the linear elastic stress analysis.

In performing a finite element stress analysis using the available computing facilities, the author had the following options:

1. Use the finite element stress analysis software available on campus.
2. Look for suitable finite element stress analysis software available commercially and at other academic institutions, select and procure one, install in the campus computer, and use.
3. Develop a finite element stress analysis programme specially suited to the problem at hand and use.

The first option was tried first. It was soon realised that no suitable software existed on campus to solve the problem. The structure to be analysed has elements in all three mutually perpendicular planes in a three-dimensional space. In order to model the structure by plane stress elements, either 2-D elements generally oriented in space, i.e. having three translational degrees of freedom at each node are required, or the 3-D problem has to be reduced to a 2-D problem using some conceptual simplification.

Bowes and Russel [9] explained how a T-section can be reduced to a coplanar system of plates joined by short bar elements, as illustrated in Fig. 3.5. Since floor slabs were not included in the stress analysis in the beginning, the wall having an I section could be reduced to a coplanar system of three plates in this manner. A problem arose with the memory capacity of the computer; with a finite element mesh of reasonable fineness and using an available programme SP44B [9], it was found that buildings no higher than 4 stories could be analysed by this technique. Above all, as eventually the floor slabs had to be included, the technique can not be extended to include the floor slabs since, when the slabs are folded, they overlap the wall and thus can no longer be modelled separately. This approach was therefore abandoned.

Another finite element stress analysis programme, ASGARD, which later became available on the campus computer, was also tried. Being a general purpose programme for stress analysis and heat flow analysis, both linear and nonlinear, this programme required a substantial overhead in memory, reducing the memory available for data all the more. Besides, the output was voluminous. To obtain satisfactory accuracy, it was estimated that about 5000 degrees of

freedom were required to model a 10-storey building. With the limitations of memory, execution time, and the use of other peripherals of the campus computer, it was impossible to analyse a problem of this size using ASGARD.

Fig. 3.5 Reduction of 3-D T-Beam Problem to a 2-D problem

The second option, one of procuring a suitable programme from elsewhere did not appear to be a viable one. In addition to the time, effort, and financial resources required for the procurement, and the adaptation and installation of the programme in the campus computer, a job fraught with many uncertainties, limitations of the campus computer would still be a hindrance. The most serious limitation of the available computer was its core memory available to a FORTRAN user, only 256 k words, limited by the hardware. While limitations of computing resources and use of peripherals were manageable to some extent, this memory limitation was insurmountable.

Therefore, a finite element stress analysis computer programme, suitable for the problem at hand and workable within the constraints of the available computer facilities, was required to be developed. Since the building has distinct components such as exterior wall, interior crosswall, corridor wall, and floor slabs, and since all these are repeated in the vertical direction a number of times, it is ideally suited to substructure analysis. A finite element programme for linear elastic stress analysis was developed using substructuring and a special technique to reduce the 3-D problem to a 2-D one. The next chapter deals with this substructure analysis.

4

**A Special Substructure
Finite Element Analysis**

4.1 General

The finite element method of analysis is a modern computational tool which is very powerful and versatile. Originally making its debut in stress analysis, it is now widely used in major engineering disciplines such as thermal analysis, fluid mechanics and electromagnetics. Today's fast and powerful computers have popularised the method obviating many expensive and time-consuming experimental methods.

The wide acceptance of the method is primarily due to its ability to analyse complex problems. However, powerful as they may be, computers have practical limits of memory and computation time. In applying the method to large structures such as buildings, ships, and airplanes, the number of finite elements required for a solution of satisfactory accuracy is so large as to very often exceed these limits.

The stress analysis problem in this work is one of such relatively large magnitude. With the available computational facilities, both hardware and software, it was impossible to solve the problem described in Section 3.2. This necessitated the development of a stress analysis programme. A special substructuring technique was introduced which not only made the solution of the problem possible but also increased the computational efficiency. This chapter deals primarily with the development of this special substructure finite element analysis procedure.

4.2 The Substructuring Technique

To overcome the computational difficulties associated with solving a problem of a large number of degrees of freedom, efforts have been made primarily in two areas. One is to improve the algorithms for solutions of large systems of equations. Today many equation solving routines are available which can solve large sets of equations taking advantage of the sparseness of stiffness matrices and using out-of-core memory, which is virtually unlimited, in conjunction with core memory. As a price for the reduction of core memory requirement, however, the computation time increases since accessing out-of-core memory is very slow. The other is to use the technique of what is known as substructuring. The essence of this technique, discussed in further detail in the next section, is to divide the equation solving process into more than one

stage so that the number of unknowns to be solved at any particular stage can be accommodated in the available core memory. While it may be practically impossible to solve some large structures without substructuring, it may in fact reduce the computational cost of solving others which are also solvable without substructuring. The substructure method of analysis is specially advantageous for solving structural units built up from different types of members like plates, membranes, shells, beams, and solids. Each of these, treated as substructures, can be handled individually with highly specialised subprogrammes or programmes by different groups of analysts. In analysing structures having repetitive units, substructuring can reduce the cost of analysis substantially.

4.2.1 Theory of Substructuring

Substructuring essentially consists of dividing the structure to be analysed into a number of substructures which are, in turn, further subdivided into a number of finite elements. The stiffness and the load matrices of each of the substructures are derived from those of the finite elements in the usual manner. These matrices are then condensed, i.e. reduced, so that only those degrees of freedom pertaining to the boundaries of the substructure are retained. The structure can now be treated as an assemblage of the substructures, which are sometimes also known as superelements, and the global stiffness and load matrices of the structure can be assembled from the corresponding condensed matrices of the superelements. The unknown degrees of freedom, which belong only to the substructure boundaries, are then solved. The known boundary degrees of freedom are then used as boundary conditions for each of the substructures and the interior degrees of freedom which were previously condensed out are solved. Computation of stress and strain then follows in the routine manner.

What is described above is single level substructuring. The same basic procedure can be repeated for multiple level substructuring, where substructures of level 1 are built up of basic finite elements and those of higher levels are built up of the next lower level substructures. In this manner, structures of practically any size can be solved by finite element methods within the constraints of available computational resources.

4.2.2 Static Condensation

A structure composed of m substructures is shown in Fig. 4.1. For the j th substructure, the well known force equilibrium equations are

$$\begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix}^j \begin{Bmatrix} U_i \\ U_b \end{Bmatrix}^j = \begin{Bmatrix} P_i \\ P_b \end{Bmatrix}^j \quad (4.1)$$

Fig. 4.1 Division of a Structure into Substructures

In the above matrix equations, U_b and U_i are the vectors of displacements occurring at the boundary nodes and the internal nodes, respectively. The corresponding load vectors are denoted by P_b and P_i . The superscript j indicates that all these matrices pertain to the j th substructure and j varies from 1 to m . The displacements of the internal nodes can be eliminated in the manner described below.

If the first matrix equation in Eq. 4.1 is written out and solved for U_i , the following is obtained where j has been omitted for clarity:

$$U_i = K_{ii}^{-1} (P_i - K_{ib} U_b) \quad (4.2)$$

The second matrix equation can be written out as

$$K_{bi} U_i + K_{bb} U_b = P_b \quad (4.3)$$

Eliminating U_i from the above equation with the help of Eq. 4.2 and rearranging, the following equation is obtained:

$$(K_{bb} - K_{bi} K_{ii}^{-1} K_{ib}) U_b = (P_b - K_{bi} K_{ii}^{-1} P_i) \quad (4.4)$$

Introducing the superscript j again, the above equation can also be written as

$$K_E^j U_b^j = P_E^j \quad (4.5)$$

$$\text{where } K_E^j = K_{bb} - K_{bi} K_{ii}^{-1} K_{ib} \quad (4.6)$$

$$P_E^j = P_b - K_{bi} K_{ii}^{-1} P_i \quad (4.7)$$

K_E^j and P_E^j are the stiffness matrix and the load matrix, respectively, for the j th substructure obtained by eliminating the internal degrees of freedom. The process by which these matrices have been obtained and summarised by Eqs. 4.2 through 4.7 is known as static condensation.

The matrix inversion in Eqs. 4.6 and 4.7 is not usually done explicitly. The products $K_{ii}^{-1}K_{ib}$, denoted as L_i , and $K_{ii}^{-1}P_i$, denoted as Q_i , can be obtained by solving

$$\begin{bmatrix} K_{ii} \end{bmatrix} \begin{Bmatrix} L_i \\ Q_i \end{Bmatrix} = \begin{Bmatrix} K_{ib} \\ P_i \end{Bmatrix} \quad (4.8)$$

Matrices K_E^j and P_E^j can also be obtained by reducing the stiffness matrix of Eq. 4.1 to an upper triangular form by Gaussian elimination but stopping the elimination when the first row of the rectangular matrix $[K_{bf}^j \ K_{bb}^j]$ has been reduced. The matrix now located where K_{bb} originally was is K_E^j and the vector located where P_b originally was is P_E^j . Both of these approaches are computationally more efficient than direct inversion of K_{ii} .

4.2.3 Previous Work

The concept of dividing a structure into parts to carry out analysis existed in the form of matrix partitioning even before the advent of computers. Research interest in substructuring surfaced during the early sixties. Przemieniecki [74] introduced the theory of substructuring and discussed its advantages in solving large structures, such as aircraft, in 1963. He predicted that even with the advent of larger and faster computers, substructure analysis would not be wholly superseded by an overall analysis carried out on the complete structure.

While the theory of substructuring introduced by Przemieniecki remains the same, many variations and improved schemes have been proposed. Rosen and Rubinstein [78] reported that computational advantage may be achieved over the standard method of substructuring, which they called 'full release' method, by using a 'partial release' technique. Their method utilises the substructure transformation inherent in the Cholesky decomposition method. Furike [32] developed a computerised multiple level substructuring analysis method using partitioned block data sets in the Cholesky decomposition for both forward and backward substitution to reduce the I/O and CPU times. He concluded that large band active column matrices and sparse matrices with wide band can virtually be eliminated by substructuring. Utku [91] proposed a scheme he called 'systematic substructuring' wherein the computer itself is instructed to partition the problem such that it can be solved with the available core memory with a minimum number of partitions. Thus it is different from many schemes in which the analyst has to select the partitions or from those where the problem is automatically partitioned but not necessarily with the minimum number of partitions which is imperative for economical solutions. Petersson and Popov [69] developed a procedure to reduce the number of equations in different substructure levels

without disturbing the economic storage mode which sometimes happens because of the requirement to follow some particular node numbering scheme. They also presented a generalised Gaussian elimination scheme for the solution of systems of substructure equations which, according to them, would be very useful in solutions of mixed finite element models and moderately ill-conditioned problems.

Another improved scheme of substructuring particularly suitable for structures with large expanse in one direction, such as multistorey frames and towers, has been proposed by Gurujee and Deshpande [39]. Unlike many methods including Rubinstein's [78] wherein the internal nodes are numbered first for the convenience of reducing the stiffness matrix, they numbered the nodes starting with one boundary, continuing with interior nodes for minimum band width, and finishing with the other boundary. By means of two examples they demonstrated that using their scheme, less memory, and much fewer number of operations especially for structures having many identical substructures, were required as compared to many other schemes. A number of substructuring aspects such as multilevel substructuring algorithms, substructuring in elastoplastic problems, and use of hypermatrix and other sparse matrix schemes were considered by Noor et al [66] with a view to assessing and improving the efficiency of substructuring techniques and exploiting new computer hardware achievements. They confirmed the importance of proper selection of size, topology, and node numbering of individual substructures. They suggested nested dissection schemes as guidelines regarding the numbering of nodes to reduce arithmetic operations and disk storage requirement. They also suggested the use of constrained substructuring technique, the essence of which is to condense out some of the boundary degrees of freedom kinematically coupled to the remaining boundary degrees of freedom in addition to the interior ones. The kinematic coupling mentioned above is done by using some displacement interpolation scheme along the boundaries. The accuracy of the analysis therefore depends on this interpolation scheme.

Computational advantages of substructure analysis have been demonstrated and verified by many authors. Arora and Govil [4] presented a general method of optimal design of structures by using substructuring. They showed that up to 9% saving in computing time could be achieved even for a small scale problem of a 25-member transmission tower. However, the saving in computing time was much more significant, up to 66%, in solving a 200 member plane truss. The memory requirement was also less, by about 15%, for the Jattér problem. Hrabok and Hosain [43] used substructuring to analyse stub-girders, which are composite girders made up of a steel beam and a concrete slab connected by a series of short steel rolled sections called 'stubs'. They showed that the storage requirements for the substructure analysis were 50% of those for a conventional analysis. Since the girder has a number of repetitive units, substructuring also reduced the computational effort and execution time. For structures having many identical units,

Gurujee and Deshpande [39] concluded that substructuring would be much more efficient than sparse matrix techniques such as wave front. Williams [94] showed that the substructure method is equivalent to the sparse matrix method if rows are not interchanged; thus the sparse matrix method can never be more efficient if a suitable choice of substructure is made. But when the structure is comprised of identical substructures, the method of substructuring has definite advantage over the best sparse matrix method, he concluded. Anand and Shaw [3] used substructuring in conjunction with a mesh refinement scheme in elastic-plastic finite element analysis. They also reported considerable savings in memory requirements and computational efforts for their analysis. An excellent study of computational advantages of substructuring has been reported by Noor et al [66]. They investigated the influence of several parameters on the requirement of computational resources. For a problem of 6534 degrees of freedom, single level substructuring required 23% less and four level substructuring required 46% less memory compared to direct analysis. The effectiveness of substructuring as a means of exploiting some features of minicomputers, used widely nowadays in structural analysis, and the virtual memory of large computers like CDC-STAR 100 was also verified by them.

4.3 Conventional Substructuring

The structure to be analysed in this study is ideally suited to substructuring in that it has distinct components such as exterior wall, interior crosswall, corridor wall, and the floor slabs, which are merely repeated in the vertical direction a number of times equal to the number of storeys. Superiority of substructuring over sparse matrix techniques in reducing computational cost for the analysis of such structures has been substantiated by many researchers as discussed in the previous section.

The walls and slabs are lying in three mutually perpendicular planes forming a three dimensional structure. How the standard technique of substructuring could be applied to the problem will be examined first. Each of the walls and slabs can be treated as a substructure as shown in Fig. 4.2. In order to model these substructures one could use plane stress elements having all the three translational degrees of freedom since the nodes common to more than one substructure can deform in all the three directions. However, while assembling the global stiffness matrix for each of the substructures as well as the entire structure would be straightforward and simple, the degrees of freedom perpendicular to the plane of each substructure, except at the common boundary nodes, would be wasted. Alternatively, one could use plane stress elements having two translational degrees of freedom at each node to model the substructures. But a special routine would have to be developed to assemble the global stiffness matrix for the entire structure bearing in mind that at some boundaries there are three

degrees of freedom per node while at others there are only two. Supposing such an assembling procedure is not difficult to formulate; the size of the stiffness matrix of the substructure assemblage, i.e. the global stiffness matrix of the entire structure, will be examined.

Fig. 4.2 A Substructure Representation of a Typical Storey

Assume tentatively that in order to obtain a solution of satisfactory accuracy, the substructures are to be divided into the finite element meshes shown in Fig. 4.3. After the elimination of the internal degrees of freedom from each substructure, the resulting assemblage of the substructures is shown in Fig. 4.4. It can be easily verified that for a building higher than 3 stories, the nodes must be numbered in the sequence as shown in that figure in order to keep the bandwidth of the global stiffness matrix to a minimum. It can be seen that the largest differences between the lowest and highest node numbers occur in the substructures 2 and 3 in the second level and upwards. Since some nodes have three degrees of freedom, while others have two, it is however not so obvious which substructure governs the bandwidth. The following calculation shows that the largest half bandwidth is 218 and occurs in substructure no. 2.

Largest half bandwidth:

Substructure no. 2

$$\text{Highest degree of freedom} = (117 - 36) \times 2 + 36 \times 3 = 270$$

$$\text{Lowest degree of freedom} = (48 - 5) \times 2 + 5 \times 3 - 2 = 99 \quad \text{Difference} = 171$$

Fig. 4.3 Basic Finite Element Meshes for Substructures

Substructure no. 3

$$\text{Highest degree of freedom} = (123 - 42) \times 2 + 42 \times 3 = 288$$

$$\text{Lowest degree of freedom} = (58 - 15) \times 2 + 15 \times 3 - 2 = 129 \quad \text{Difference} = 159$$

$$\text{Therefore, largest semi bandwidth} = 171 + 1 = 172$$

Fig. 4.4 A 3-Storey Assemblage of Substructures

The total number of degrees of freedom will now be calculated. Assuming that a 15-storey building is to be analysed

$$\text{Total number of nodes} = 80 + (80 - 21) \times 14 = 906$$

$$\text{Number of nodes with 3 dof's} = 21 \times 15 = 315$$

$$\text{Number of nodes with 2 dof's} = 906 - 315 = 591$$

$$\text{Therefore, total number of dof's} = 315 \times 3 + 591 \times 2 = 2127$$

The size of the global stiffness matrix of the substructure assemblage would therefore be 172×2127 , requiring about 357 k words of memory. It would be impossible to accommodate this matrix given that the overall limit of core memory in the on-campus CP-6 computer system is only

256 k words.

However, the memory requirement could be reduced. Many of the degrees of freedom are constrained and therefore need not enter the solution process. In other words, the constrained degrees of freedom could be excluded from the stiffness matrix, and only those degrees of freedom could be numbered sequentially which are unknown. Revised calculations on this basis yield a half bandwidth of 146 and total number of degrees of freedom of 1425. Therefore, the revised size of the global stiffness matrix is now 146×1425 , which requires 203 k words of memory. But considering storage space required for other arrays and variables, the code, and system overhead, it would still be impossible to accommodate a matrix of this size in the available computer.

As a means to further reduce the core memory requirement, consider the prospects of introducing another level of substructuring. Suppose that each storey of the building, comprised of three walls and a floor slab, be considered in its entirety as a primary substructure and each of the walls and the floor slab in each storey are considered as secondary substructures. One such probable primary substructure is shown in Fig. 4.5 with its boundaries along which it is connected to other primary substructures highlighted.

Fig. 4.5 A Possible Primary Substructure Comprising of Secondary Substructures

According to conventional substructuring, all the degrees of freedom except those on these interface boundaries should be condensed out. But since some degrees of freedom along each of the edges and ridges besides those boundaries are also constrained, they can not be

condensed out. The unrestrained degrees of freedom on these edges and ridges can of course be condensed out but a careful examination would reveal that a high price in terms of computing is to be paid in order to do this. The degrees of freedom have to be numbered so that the unrestrained ones, i.e. the ones to be condensed out, are numbered either in the beginning or in the end. This would evidently result in a highly dispersed matrix of order 181×181 for the primary substructures. Condensation of such a matrix would be inefficient. Moreover, the global stiffness matrix of the entire structure obtained by assembling the primary substructure stiffness matrices still requires a substantial, although manageable, amount of memory. The number of unknown degrees of freedom at each interface boundary, as shown in Fig. 4.5, is 50. It follows that the size of the global stiffness matrix would be 100×800 requiring about 78 k words of memory.

In the following section is presented what, the author believes, is a better alternative to the multilevel substructuring scheme described above.

4.4 An Alternative Substructure Analysis

4.4.1 Basic Technique

In this approach, substructuring is still used, but in a special way. The basic concept will be presented first, the procedural details will follow.

Two assumptions are to be made at the outset; their justifications and implications will be discussed later. The assumptions, illustrated in Fig. 4.6, are the following:

1. The floor slabs are connected only to the crosswalls; no connection exists between the slabs and the exterior and corridor walls.
2. The partial restraint provided by the lintel beam at the top corner of the exterior wall at each floor in the horizontal direction can be replaced by a full restraint.

An exploded view of a 3-storey building is shown in Fig. 4.7, three stories being selected only for the purpose of illustration. The walls, full three stories high, and each floor slab are treated as individual structural units. These units are connected to one another, but one important point to be observed is that none of them is connected to two different units. This is an outcome of the first assumption stated above. Looking at the connections between the structural units (Fig. 4.7), the following observations are made:

- i) Only the vertical (z) degrees of freedom of each neighbouring unit interact along the exterior wall to crosswall intersection.
- ii) Only the vertical (z) degrees of freedom of each neighbouring unit interact along the

corridor wall to crosswall intersection.

- iii) Only the horizontal (y) degrees of freedom of each neighbouring unit interact along each floor slab to crosswall intersection.

Fig. 4.6 Illustration of Assumptions for Substructure Analysis

Fig. 4.7 Exploded View of a 3-Storey Building

Let each of the structural units now be divided into some substructures which are in turn further subdivided into some finite element meshes such that the nodes at the interfaces of substructures are conformable. Let there be n such number of nodes on the common boundaries of the exterior wall and the crosswall. Dealing with the substructure model of the exterior wall only, let a unit vertical displacement be given to the m th node while the rest of the n nodes are restrained in the vertical direction and all other restraints shown in Fig. 4.7 are applied to that wall. Let $\{K\}^m$ be the vector of forces of order n necessary to cause this displacement and to restrain the other nodes in the vertical direction. Repeating the above process for the rest of the n nodes, a total of n such force vectors can be obtained. Let these vectors be combined into a matrix, of order $n \times n$, denoted by $[K]_{ew}$. Because this matrix will be used to link the exterior wall to the crosswall, let us call it the *link stiffness matrix* of the exterior wall. Tentatively, assume that $[P]_{ew}$ is a matrix, of order $n \times l$ and called the *link load matrix* of the exterior wall, whose columns $\{P\}_{ew}^1, \{P\}_{ew}^2, \dots, \{P\}_{ew}^l$ represent the loads that are equivalent to the applied loads on the entire exterior wall for l number of load cases in the same common vertical degrees of freedom mentioned above. How $[P]_{ew}$ can be obtained will be discussed later. Also, let $[d]_{ew}$ be the matrix of primary displacements in all the degrees of freedom of the substructure model of the exterior wall due to the l number of load cases, when the degrees of freedom common to the web wall are also restrained in addition to the applied restraints.

Similar link stiffness matrices, link load matrices, and primary displacement matrices, $[K]_{cw}$, $[P]_{cw}$ and $[d]_{cw}$ for the corridor wall, and $[K]_{fs}$, $[P]_{fs}$ and $[d]_{fs}$ for a typical floor slab, can be obtained in the same manner. Now let $[k]_{ww}$ be the stiffness matrix of the web wall or crosswall having, at least, the degrees of freedom which are common to all the other units connected to it, and those which exclusively belong to itself but are unknown. Also let $[p]_{ww}$ be the matrix of loads corresponding to these degrees of freedom representing all the loads applied on the web wall. If the matrices $[K]_{ew}$, $[K]_{cw}$ and $[K]_{fs}$ are added to $[k]_{ww}$, as shown schematically in Fig. 4.8, the resulting matrix, $[K]_{ww}$ is the stiffness matrix of the crosswall which now incorporates the connections of the crosswall to the other units. Note that in Fig. 4.8, the degrees of freedom common to the other units have been numbered in the beginning for ease of illustration of matrix superposition only. In a similar fashion, the resulting matrix of loads, $[\Pi]_{ww}$, obtained by superposing $[P]_{ew}$, $[P]_{cw}$ and $[P]_{fs}$ on $[p]_{ww}$ represent all the loads which are applied on, and transferred from the other units to, the crosswall. The resulting system of equations is given below:

$$[K]_{ww} [\Delta]_{ww} = [\Pi]_{ww} \quad (4.9)$$

Fig. 4.8 Schematic of Formation of K_{ww} and Π_{ww}

When the above equation is solved, the displacements $[\Delta]_{ww}$, having / number of vectors, will be the actual displacements in the crosswall when the entire structure deforms as an integral unit under the / number of applied load cases.

The displacements $[d]_{ew}$, $[d]_{cw}$, and $[d]_{fs}$, which are also common to the exterior wall, corridor wall, and the floor slabs, respectively, can be extracted from $[\Delta]_{ww}$ and applied to the corresponding structural unit. Displacements in these units in the remaining degrees of freedom due to these applied displacements are determined. Let $[d]_{ew}^*$, $[d]_{cw}^*$ and $[d]_{fs}^*$ be the matrices of these secondary displacements in the substructure models which includes all the degrees of freedom. The actual displacements $[\Delta]_{ew}$, $[\Delta]_{cw}$ and $[\Delta]_{fs}$ can therefore be obtained by summing up the primary displacements $[d]$ and the secondary displacements $[d]^*$ of the respective structural unit.

The rest of the process is routine. Displacements at the boundaries of each substructure are extracted from the displacement matrices $[\Delta]$ and applied to the basic finite element model of the respective substructures together with the applied loads, if any, at the internal degrees of

freedom. Displacements in these internal degrees of freedom are then solved and finally the stresses in the finite elements determined.

The essence of the present technique has been presented above. The feature that distinguishes it from the conventional substructuring is that the global displacements of the three-dimensional problem can be obtained without solving a three-dimensional assemblage. Unlike substructures in the usual sense, the exterior wall, crosswall, corridor wall, and the floor slabs, called structural units in this technique, remain physically separated throughout the solution process.

4.4.2 Procedure

The analysis of the problem at hand by the present technique is carried out according to the following stepwise procedure:

1. Consider each of the exterior wall, crosswall, corridor wall, and each floor slab as independent structural units. Consider each storey of each wall as a substructure or superelement of the corresponding wall. Therefore each wall has N substructures where N is the number of stories.
2. Divide a typical substructure of each of the walls and a typical floor slab into a basic finite element mesh of constant strain triangles so that nodes at the common boundaries of each neighbouring units are conformable.
3. Generate the banded stiffness matrix $[S]$ and the load matrix $[P]$ for each of these finite element models.
4. Condense out all the degrees of freedom to reduce each of the above matrices to the ones having the boundary degrees of freedom only.
5. Assemble the banded global stiffness matrix $[k]$ and the global load matrix $[p]$ for each of the N -storey walls with the reduced substructure stiffness matrix and load matrix of the corresponding wall.
6. Derive the link stiffness matrices $[K]_{ew}$, $[K]_{cw}$ and $[K]_{fs}$ and the link load matrices $[P]_{ew}$, $[P]_{cw}$ and $[P]_{fs}$ (see Subsection 4.3.1). Also, derive the primary displacement matrices $[d]_{ew}$ and $[d]_{cw}$.
7. Add the matrices $[K]_{ew}$, $[K]_{cw}$ and $[K]_{fs}$ to $[k]_{ww}$ in the corresponding positions shown schematically in Fig. 4.8 to obtain $[K]_{ww}$. Similarly, add the matrices $[P]_{ew}$, $[P]_{cw}$ and N number of $[P]_{fs}$ to $[p]_{ww}$ to obtain $[P]_{ww}$.
8. Solve the resulting system of equations, Eq. 4.9, for the displacements $[\Delta]_{ww}$ of the substructure model of the crosswall.

9. Extract the common boundary displacements $[d]_{ew}$ and $[d]_{cw}$ from $[\Delta]_{ww}$. Apply these displacements to the corresponding substructure models of the walls and obtain the secondary displacements $[d]''_{ew}$ and $[d]''_{cw}$ for all the degrees of freedom in those substructure models. Add the displacements $[d]''$ to $[d]'$ to obtain $[\Delta]$ for each wall.
10. Extract the displacements occurring at the boundaries of each substructure of each wall from $[\Delta]_{ww}$, $[\Delta]_{ew}$ and $[\Delta]_{cw}$. Also extract the N number of displacement matrices, $[d]_{fs}$, from $[\Delta]_{ww}$.
11. Apply the above displacements, and any external loads applied at the interior degrees of freedom, to the corresponding basic finite element models and obtain the displacements in the basic finite element nodes of each substructure of each wall, and each floor slab, by solving the general equation of the form

$$[S][D] = [Q] \quad (4.10)$$

12. Determine the stresses in the finite elements and convert them into nodal stresses by expressing the stresses at each node as a weighted average of the stresses in the elements meeting at that node.

The details of how the important steps were executed will now be discussed.

Generation and Condensation of Basic FE Stiffness Load Matrices (Steps 2-4):

Constant strain triangular (CST) plane stress elements were used for the linear elastic analysis. The main reason for choosing CST elements is the availability of a stress analysis routine using these elements from which several subroutines could be readily used. Besides, results from preliminary analyses indicated high stress gradients in the walls around the intersection requiring some mesh grading in that area for which triangular elements are better suited than quadrilateral elements.

Several basic finite element meshes of varying fineness were tried for each wall. The meshes shown in Fig. 4.9 were found to yield results of satisfactory accuracy; therefore, they were used in this analysis. Note that the nodes in each neighbouring mesh at their interfaces are conformable in number and location.

The node numbering schemes to number the degrees of freedom to be condensed out at the beginning and to obtain a minimum possible bandwidth are also shown in Fig. 4.9. Note that since no condensation was done on the basic stiffness matrix of the slab mesh, the node numbering shown is to obtain the minimum bandwidth only.

The banded finite element stiffness matrix, $[S]$, of each of the structural units generated from the above meshes was condensed by partial triangulation using Gaussian elimination. This

Fig. 4.9 Node Numbering Schemes of Basic Finite Element Meshes

was done by the same routine used for solution of the systems of equations but stopping the elimination process and exiting from the routine when the row containing the last degree of freedom to be condensed out had just been processed.

Note that the band widths of the smaller walls suggest that a symmetrical storage mode would require less memory to store the stiffness matrices. Alternatively, a skyline storage mode could be used for the banded matrix which contains many consecutive null elements. But since the memory requirement was already quite low, only 11 k words for the largest matrix, such refinements were not made. Besides, generation and condensation of the basic stiffness matrices was done by a totally separate programme and the condensed matrices were stored in the disk for later use by another programme performing the rest of the analysis, again and again, for the parametric studies. This means that condensation was a one-time process for all the walls in most of the analyses except for the exterior wall in those analyses where its width was varied as a parameter.

Derivation of Link Stiffness Matrices and Link Load Matrices (Steps 5 and 6):

The node numbering scheme for the assemblage of substructures that constitute a typical wall is shown in Fig. 4.10. Note that this scheme renders minimum semi-bandwidths. The link stiffness matrices $[K]_{ew}$, $[K]_{cw}$ and $[K]_{fs}$, and the link load matrices $[P]_{ew}$, $[P]_{cw}$ and $[P]_{fs}$ were derived by solving the system of equations shown schematically in Fig. 4.11. The square matrix in that figure is the global substructure stiffness matrix, $[k]_{ew}$ or $[k]_{cw}$, for the exterior wall or corridor wall, respectively, and the basic finite element stiffness matrix, $[S]_{fs}$, for the typical floor slab. The method, often referred to as unit displacement method, uses the solution scheme introduced by Payne and Irons [68]. Suppose, $[K]_{ew}$ and $[P]_{ew}$ are to be derived. Let a, b, c , etc. be the degrees of freedom of the exterior wall which are also common to the crosswall and α, β, χ , etc. be its degrees of freedom having prescribed zero displacements. All the diagonal elements in the stiffness matrix corresponding to the above degrees of freedom are given a large value, 10^{20} . On the right hand side, m fictitious load vectors are generated; each load vector has only one non-zero element, 10^{20} , corresponding to one common degree of freedom, m being the number of such common degrees of freedom. Besides these fictitious load vectors, there may be l number of actual load vectors for l load cases on the right hand side. When the equations are solved for the $(m + l)$ number of displacement vectors, where all the displacements are treated as unknowns, the first m solution vectors will have zero displacements corresponding to the degrees of freedom α, β, χ , etc. and a unit displacement corresponding to one of a, b, c , etc. Using these displacements, the forces at the common degrees of freedom can be calculated and thus $[K]_{ew}$ can be constructed. Using the remaining solution vectors of displacements, which will

also have zero displacements corresponding to the degrees of freedom α , β , χ , etc., nodal loads at the common nodes can be calculated to which any load directly applied at these nodes must be added in order to get $[P]_{ew}$.

The same procedure is repeated for the corridor wall and the floor slabs. Note that $[K]_{fs}$ and $[P]_{fs}$ are to be derived for only one slab since all floor slabs are identical.

Alternatively, the link stiffness and load matrices could be derived by static condensation, a procedure described in the previous step. However, since the degrees of freedom which are also common to the web wall and have prescribed restraints would have to be numbered in the end in this procedure, it would destroy the banded nature of the stiffness matrices $[k]_{ew}$, $[k]_{cw}$ and $[S]_{fs}$ giving rise to highly dispersed matrices. Consequently, a much larger memory would be required. For example, the stiffness matrix of a 10-storey corridor wall requires only (48×354) or 17 k words of memory in the banded form whereas it requires (177×355) or 62 k words in the symmetric storage mode. Besides, computation time required by the condensation method would be longer unless operations on null elements are avoided by using sparse matrix techniques.

Fig. 4.10 Node Numbering Scheme in a Typical Structural Unit for Minimum Band Width

Solution of Displacements in the Substructure Model of the Crosswall (Step 8):

The Payne and Irons scheme discussed above was used to solve the displacements in the substructure model of the crosswall. Therefore, partitioning of the global stiffness matrix, $[K]_{ww}$.

was not necessary, and hence the same simple assembly routine could be used for all the walls in Step 5. Since only the degrees of freedom at the base, which constitute only 5% of all the degrees of freedom in a 10-storey wall, have known (zero) displacements, partitioning would not effect any appreciable saving in computation anyway.

Solution of Displacements in the Substructure Models of the Exterior and Corridor Walls (Step 9):

In this step, the exterior wall and the corridor wall have known displacements corresponding to the vertical degrees of freedom at their boundaries common to the web wall. These displacements have been derived in the previous step. Additionally, there are other degrees of freedom having known (zero) displacements as imposed boundary conditions. Therefore the total number of known degrees of freedom is substantial in this step. But again, partitioning was not done since this would require rearranging the numbering system of the degrees of freedom resulting in a dispersed matrix requiring a large amount of memory. The same Payne and Irons scheme and Gaussian elimination were again used here to solve the displacements.

Solution of Displacements in the Finite Element Models (Step 11):

In the finite element models of the walls, the nodes had to be numbered as shown in Fig. 4.9 primarily to number at the beginning the degrees of freedom to be condensed out. These numbering systems do not yield the absolute minimum possible bandwidths. In this step, therefore, the nodes in the basic finite element meshes of the walls were renumbered to obtain the minimum possible bandwidth.

Since all the substructures in a particular wall have the same finite element composition, the finite element stiffness matrices for all of these substructures are the same. For the same reason, all floor slabs also have the same finite element stiffness matrix. Therefore, instead of solving the displacements in each substructure of any wall or each floor slab individually, they could be solved in one single operation for each of the structural units by solving Eq. 4.10 where both $[D]$ and $[Q]$ have N sets of vectors, N being the number of stories, and each set has l number of vectors, l being the number of load cases. Obviously, Eq. 4.10 was solved four times, once for each of the structural units. The Payne and Irons scheme together with Gaussian elimination were used once again for the solutions.

4.4.3 Key Assumptions

The two assumptions stated in Subsection 4.3.1 are key to the segregated substructure analysis technique described above. The reasonableness of these assumptions will now be discussed.

Fig. 4.11 Schematic of the Derivation of Link Stiffness and Load Matrices

The first assumption was that no connection exists between the floor slabs and the exterior wall or the corridor wall. In actual construction, however, physical connection exists between the floor slabs and all the walls. But under the loads dealt with in this study, such as thermal and gravity loads, and the imposed boundary conditions (see Fig. 4.7), there is little tendency of any relative deformation or sliding along the common degrees of freedom at the interfaces of the floor slabs with the exterior wall and the corridor wall. Therefore, it is unlikely that ignoring the above physical connections will cause any significant errors in the key stresses. Any discrepancy in stresses and deformations will be on the conservative side because the analytical model based on this assumption is more flexible than the real structure.

The second assumption, that the partial restraint provided by the lintel beam at the top corner of the exterior wall at each floor can be replaced by a full restraint, is also reasonable insofar as the overall stresses and deformations are concerned. This was verified by doing two analyses, one with a full restraint and the other with no restraint at the above position. While the stress distribution in the exterior wall was somewhat different locally around that restraint, the key stresses in all the structural units were almost identical in those analyses.

Further verifications of the validity of these assumptions are provided in Sec. 4.5 by comparing the results of a finite element analysis under these assumptions with those derived from another analysis in which such assumptions are not made.

4.5 Computer Programmes

Computer implementation of the procedure of the substructure analysis of multistorey thin-walled buildings described in the previous section was carried out in a manner so as to maximise computing efficiency for the large volume of analyses required in this study. As mentioned before, the substructure stiffness and load matrices of all the walls were generated by a separate programme, called SUBMAT. The link stiffness and the link load matrices of the typical floor slab were also generated by a separate programme, called SLABLINK. Note that since the typical slab was not divided into substructures, no condensation was necessary. Also since the slabs are connected only to the web wall, the link stiffness and load matrices of the typical floor slab were added by SLABLINK to the corresponding substructure matrices of the web wall generated by SUBMAT. The substructure stiffness and load matrices of the web wall thus modified, and the substructure and load matrices of the remaining walls generated by SUBMAT were stored in disk for later use by the programme, SUFEA, for the rest of the analysis. A post-processor programme, called PLOTTER, was also developed to plot the deformations and stresses in all the structural units.

The above disposition of the computer analysis eliminated wasteful repetition of such

2

1.0

1.1

1.25

1.4

1.6

1.8

2.0

2.2

2.5

2.8

3.2

3.6

4.0

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STANDARD REFERENCE MATERIAL 1010a
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time-consuming steps, as generation of substructure stiffness matrices, in the parametric study involving a large number of analyses in which the wall dimensions and the basic finite element meshes were not changed. Such disposition also minimised the cost of developing the programme since a countless number of runs were required to develop the programme. However, all the programmes can be combined into one integral unit with very little effort which will perhaps be easier to handle as a 'black box' by design office users.

All the programmes are written in FORTRAN. Two versions of the analysis programmes compatible with the CP-6 and the APOLLO computer systems are available. The plotting routine is only available with the CP-6 version. A flow diagram showing the sequence of the key functions performed by the finite element analysis programmes is given in Fig. 4.12. A brief description of each of the programmes is given in the following subsections.

4.5.1 Programme SUBMAT

This programme generates the substructure stiffness and load matrices for all the walls. The basic input data required for the programme are the following:

1. Elastic properties of the materials
2. Number of boundary nodes
3. Coordinates of all finite element nodes
4. Topology, thickness, and temperature (if different from 20 °C) of finite elements
5. Loads (maximum 5 cases)

A simple mesh generation programme was developed to generate the mesh data with minimum possible bandwidths. Since the degrees of freedom that are to be condensed out have to be numbered in the beginning for SUBMAT, the nodes were renumbered manually in the sequence shown in Fig. 4.9. With the help of another small programme the element topology derived from the mesh generation programme was reconstructed using the modified node numbers.

Note that any in-plane load applied to the internal nodes only were considered in this programme. Loads applied to the peripheral nodes were treated directly by the programme SUFEA.

Each of the walls were treated separately by SUBMAT. This was done to facilitate data handling by minimising the volume of data and thus to reduce possible wastage of computer time caused by data errors.

4.5.2 Programme SLABLINK

This programme generates the link stiffness matrix and the link load matrix of the typical

Fig. 4.12 Schema of Execution of Computer Programmes

floor slab and adds them to the substructure stiffness and load matrices of the web wall derived from SUBMAT. The basic input data for this programme are the following:

1. Elastic properties of the slab material
2. Degrees of freedom common to the web wall
3. Coordinates of all finite element nodes in the slab mesh
4. Topology, thickness, and temperature (if different from 20 °C) of finite elements

Since no condensation was done on the basic finite element stiffness matrix of the typical slab, the mesh data generated by the mesh generation programme was directly used by this programme. Note that no loads except thermal loads are considered in slabs. The only applied load on slabs is out of plane and need not be considered in the elastic analysis for reasons stated in Sec. 3.3. However, these slab loads, as carried over onto the walls, are of course considered in the walls.

4.5.3 Programme SUFEA

This programme deals with the major part of the analysis. It executes the steps 5 through 12 of the procedure described in Subsection 4.3.2, except deriving the link stiffness and link load matrices of the typical floor slab and adding them to the substructure stiffness matrix and load matrix of the web wall which are done by SLABLINK.

To minimise human error, the data to be input manually was reduced by automating data generation as far as possible. For example, to form the global load vector of gravity loads for the substructure models, the only input required is the intensity of the dead load and live load on the floor slabs. Similarly, although not used in this study, the programme can generate the global load vectors for earthquake loads from the input of the values of the earthquake parameters A, S, K, I, and F only. The applied boundary restraints are also generated from the mere input of a boundary condition code for each of the structural units.

The basic input data required by the programme are the following:

Manually input data:

1. Boundary condition codes (1 of 4 for each unit)
2. Load case codes (up to 5)
3. Number of stories
4. Earthquake load parameters (A, S, K, I, and F)
5. Elastic property factors (1 for each structural unit)

Data generated by other programmes:

1. Substructure stiffness and load matrices of all walls

2. Nodal coordinates and element topology of basic finite element mesh of all units

To minimise the core memory requirement, maximum possible utilisation of the EQUIVALENCE statement has been made in the programme. Also, several temporary disk files are opened to store stiffness and load matrices at different stages of the programme. Thus at no stage, the stiffness matrix and the right hand side matrix of more than one unit reside in the core at a time.

Other features of the programme include the option of stopping or continuing after the displacements in the substructure models of the walls have been derived. User option can also be exercised to limit the stress analysis in the finite element model of any one or more desired structural units. The programme outputs the nodal stresses and displacements of the finite element models in the increasing sequence of storeys on a formatted file to make hardcopies of the results. It also outputs the nodal displacements of the substructure models and nodal principal stresses and shearing stresses of the finite element models on an unformatted file for post-processing by the plotting programme described below.

4.5.4 Programme PLOTTER

This programme plots the nodal displacements of the substructure models of the walls. The applied boundary restraints are indicated on these plots. The programme also plots the principal stresses and the shearing stress in the x-y system of axes at the finite element nodal points on separate diagrams for each individual structural unit. The shearing stress distribution at the major interfaces of units can also be plotted. Individual scale factors to suit each of the above plots can be chosen by the user; the programme prints the scale factor selected on the corresponding plot.

The programme uses some of the CP-6 library routines belonging to the graphics package called CALPLOT.

4.5.5 Core Memory Requirement

The computer core memory required by the three finite element analysis programmes described above are different. The maximum core memory is required by the programme SUFEA. Core memory required by the data segment of this programme depends on the size of the problem to be analysed and can be calculated using the following formula:

$$\begin{aligned} DATMEM = & MAXFEL \times 11 + MAXFEN \times 22 + MAXFESM + MAXSSM + MAXSM_{su} \\ & + MAXRHS + MAXLOD_{su} + SUMNODW \times 2 + SUMPNOD \times (NS + 2) + NCN \times 4 \\ & + 428 \end{aligned}$$

where $DATMEM$ = core memory required by the data segment

- MAXFEL** = maximum number of finite elements in a single substructure or slab
- MAXFEN** = maximum number of nodes in the finite element mesh of a single substructure or slab
- MAXFESM** = size of the largest finite element stiffness matrix in banded form
- MAXSSM** = size of the largest substructure stiffness matrix in symmetrical storage mode
- MAXSM_{su}** = size of the largest 'structure' stiffness matrix of the substructure model of any structural unit
- MAXRHS** = size of the largest right hand side matrix
- MAXLOD_{su}** = size of the largest load matrix of any structural unit
- SUMNOPW** = sum of the number of nodes in the substructure models of all walls
- SUMPNOD** = sum of the number of peripheral nodes in all substructures
- NS** = number of stories in the building
- NCN** = number of the common nodes at the interfaces of the crosswall with the exterior wall and the corridor wall

The memory requirements for the data segment as a whole and also for the main components of data are shown in Table 4.1 for different sizes of the problem modelled by the basic finite element meshes shown in Fig. 4.9.

Core memory required by the code depends on the machine on which the programme is run and also the compiler options used. On the Honeywell CP-6 mainframe computer, the programme SUFEA requires about 25 k words; on the APOLLO DN320 using floating point hardware, it requires about 18 k words.

Table 4.1 Memory Requirements of the Substructure Analysis Programme

No. of stories	Total No. of DOF's	Memory requirement in k (1024) words				
		Total	MAXFESM	MAXSSM	MAXSM _{su}	MAXRHS
2	788	17604	3024	2080	6784	1148
5	1910	29964	3024	2080	14848	4784
10	3780	61668	3024	2080	28488	21948
20	7520	153468	3024	2080	55168	84668

• for one load case

4.6 Comparative Analysis

In order to verify the validity of the assumptions stated in Subsection 4.3.1 on which the present substructure analysis technique is based and the assumption of plane stress conditions in the walls and slabs under the loadings dealt with in this study, comparative analyses of the present problem were done using the computer programmes developed and another programme, FELARC [36]. The computational advantage of the computer programme based on the present substructuring technique was also assessed from these comparative analyses. Only thermal load in a 2-storey building was considered for this comparative study; it was assumed that the temperature in the exterior wall was 20 °C higher than that in the rest of the structure.

The computer programme FELARC was developed elsewhere for the nonlinear finite element analysis of reinforced and prestressed concrete structures and has been used in the later part of this work for the nonlinear analysis of the present problem. A detailed description of the programme is given in Chapter 6. FELARC can also be used for linear elastic analysis by applying only a certain fraction of the the input loads so that the stresses developed are very low, and then dividing the stresses and deformations output by the programme by the same fraction to obtain actual stresses and deformations under the intended loads. The actual analysis by FELARC was thus done for a 2 °C rise in the exterior wall temperature; the results were therefore multiplied by a factor of 10 to obtain the stresses and deformations for a 20 °C rise in the exterior wall temperature.

For the analysis by FELARC, all the walls and floor slabs were modelled by quadrilateral thin shell elements with six degrees of freedom at each node, three for in-plane and the other three for bending deformations. The elements in the exterior wall, corridor wall, and the floor slabs were divided into 3 layers to appraise the bending stresses developed in those units. The lintel beam was modelled as an axial member by a bar element of appropriate cross sectional area. Since the lintel beam is also subjected to the same rise in temperature as the exterior wall, an initial stress equivalent to the thermal stress was applied to it. The finite element mesh used for the analysis by FELARC is shown in Fig. 4.13; the basic finite element meshes used by the author's programmes are shown in Fig. 4.9. All other data pertinent to the analysis are listed below:

Building Dimensions:

Length of crosswall, $l_{ww} = 7.5 \text{ m}$

Length of corridor wall, $l_{cw} = 7.5 \text{ m}$

Length of exterior wall, $l_{ew} = 2.5 \text{ m}$

Width of corridor, $b_{cd} = 1.5 \text{ m}$

Height of storey, $h = 2.76 \text{ m}$

NOTES:
 Arrows indicate ascending order of node numbers.
 The structural units have been shown segregated for ease of illustration. Node numbers are the same at common edges.

Fig. 4.13 Finite Element Mesh Used for Analysis by FELARC

Material Properties:

Modulus of elasticity of wall material = 10 000 MPa

Modulus of elasticity of slab material = 20 000 MPa

Thermal coefficient of expansion of wall material = 0.00001 per °C

4.6.1 Accuracy of Results

Deformed shapes of the crosswall as obtained by plotting the nodal displacements output by both programmes are depicted in Fig. 4.14. It is observed that these displacements are practically the same. Such good agreement was achieved despite the neglect of bending and the connection between the floor slabs and the longitudinal walls in the analysis by the author's programmes.

Fig. 4.14 Comparison of Deformations in Crosswall - Author's vs. FELARC

The stress distributions along a vertical line in the crosswall 625 mm from its exterior end, shown in Fig. 4.15, indicate overall good agreements for all the stresses, but particularly the normal stress in the vertical direction which is the most significant stress. It is believed that use of a finer mesh with the programme FELARC would indicate an even better agreement.

Fig. 4.15 Comparison of Stresses in Crosswall - Author's vs. FELARC

Deformed shapes of the exterior wall are compared in Fig. 4.16. Again, good agreement is observed for the key displacements. It is also observed that the influence of replacing the lintel beam by a fixed support in the analysis by the author's programmes is localised and affects only the horizontal displacements along the free edge with any significance. But these horizontal displacements are only trivial, in any case, in that these are mostly unrestrained thermal deformations. This is further clarified in the plots of the stress distributions along the midline of the exterior wall shown in Fig. 4.17. It is seen that significant magnitudes of the normal stress in

the x-direction are localised at the floor and foundation levels only; for the rest of the wall, this stress is not so significant. Note that overall good agreement is again indicated in the vertical normal stress distributions. The wavering displayed by the normal stress in the y-direction given by FELARC would presumably be less prominent had a finer mesh been used.

Fig. 4.16 Comparison of Deformations in Exterior Wall - Author's vs. FELARC

Good agreement between the results of the two analyses is also indicated by the deformations of the exterior edge of the roof slab shown in Fig. 4.18. The distributions of stresses in the slab along a line 625 mm from the line of intersection with the crosswall, shown in Fig. 4.19, are also in good agreement, except that the magnitudes of these stresses given by FELARC are mostly somewhat higher. Since the stress gradient in the slab is high in both directions particularly in the vicinity of the exterior edge, it is again believed that the mesh used

with FELARC is too coarse or at least not well graded and therefore the stresses are slightly overestimated by FELARC. The higher tensile or lower compressive stresses in the y-direction indicated by the author's programmes are likely caused by the additional flexibility of the slab due to the exclusion of the lintel beam and any connection between the exterior wall and the slab.

Fig. 4.17 Comparison of Stresses in Exterior Wall - Author's vs. FELARC

The stresses in the corridor wall and the first floor slab are trivial and therefore are not compared. All stresses in the exterior wall and the roof slab discussed above which pertain to the programme FELARC are at the middle layer of the 3-layered elements and therefore represent the average stresses in those elements. Stresses caused in those elements due to bending only can be computed using the stresses at the outer layer of those elements. The highest bending stress in the slab thus computed is only 0.35 MPa in the direction normal to the crosswall at the corner between the exterior wall and the crosswall. Also, bending stresses of such an order of magnitude are highly localised in the area of the slab around the exterior edge and the exterior

end of the crosswall. Obviously, these bending stresses can not be determined by the programmes developed in this work since plane stress elements are used to model all parts of the structure.

Fig. 4.18 Comparison of Deformations in Roof Slab - Author's vs. FELARC

Fig. 4.19 Comparison of Stresses in Roof Slab - Author's vs. FELARC

From the above discussion, it can be inferred that except for being deficient in indicating the localised bending stresses in the floor slabs, the present substructure analysis using plane stress elements can yield results for the thermal stress analysis which are as good as those obtained from a direct finite element analysis using shell elements. In fact, any loading that is resisted by the structure predominantly by membrane action or is applied in the plane of the walls can be adequately treated by the programmes developed in this study. Most loadings on buildings can be grouped into forces causing local bending in certain parts of the structure, such as gravity loads on floor slabs, and those in the plane of the walls, such as floor load reactions

4.6.2 Computational Advantage

To assess the computational advantage offered by the computer programmes in this work, a comparison of the computer resource requirements was made between the present programmes and FELARC. For this purpose, both FELARC and the present programmes were run on the same machine, an APOLLO DN320 microcomputer, to analyse the same problem for which the results have been compared in the previous subsection. Table 4.2 shows the results of this comparison. The CPU time taken by the author's programme is obtained by summing up the CPU times taken by all the three finite element analysis programmes viz., SUBMAT, SLABLINK and SUFEA. The greater part of this CPU time is taken up by the programme SUFEA; for the 2-storey analysis, for example, SUFEA takes 130 seconds out of the total of 152 seconds. The core memory shown for the author's programme is the maximum core memory required by any of the above programmes, usually SUFEA. The 5-storey analysis by FELARC was not run in reality since it would take a long time to execute; however, it is estimated that the CPU time required would be between 75 and 100 minutes.

Table 4.2. Comparison of Computational Requirements - Author's vs FELARC

No. of Stories	Programme	Total No. of DOFs	No. of Unknown DOFs	Core Memory Required, k words			CPU Time min - sec
				Data	Code	Total	
2	Author's	788	684	20	18	38	02 52
	FELARC	714	446	75	50	119	25 27
5	Author's	1910	1710	33	18	51	06 13
	FELARC	1686	1073	175	50	225	.

• This analysis was not run. Estimated CPU time is between 75 and 100 minutes

The computational efficiency of the programmes developed in this work as compared to the programme FELARC is clearly evident from Table 4.2. The latter is a direct finite element analysis programme in that it does not use any substructuring. Also, as mentioned before, shell elements with 24 degrees of freedom were used in that programme to model the structure to include the effects of bending. Therefore, the comparison shown in Table 4.2 together with the observations made on the accuracy of the results in the previous subsection indicate that building structures under usual loadings such as dealt with in this work can be analysed without any significant deficiency at a much smaller computational cost using the present substructuring technique and simple CST plane stress elements as compared to using direct finite element analysis and shell type of elements.

4.7 Scope and Limitations

The substructure finite element analysis computer programmes discussed above have been developed specially to analyse building structures of loadbearing wall construction. The following limitations apply to the programmes with respect to the analysis concept on which they are based:

1. The programmes are developed for linear elastic analysis; therefore, the stresses and deformations computed by the programmes are accurate or valid as long as the stresses do not exceed the proportional limit or the ultimate strength of the material whichever occurs earlier.
2. All loads must be applied in the plane of the walls and slabs; loads that are applied out of plane to any part of the structure must be treated separately.
3. The wall layout and the boundary displacement conditions should be such that no significant bending occurs in any wall due to the applied loads.

Apart from the above theoretical limitations certain other limitations also apply to the programmes with respect to their present status which can be surmounted with varying degrees of difficulty by modifying the programmes in an appropriate manner. These limitations are given below:

1. The wall layout in the building plan should be symmetrical with respect to both axes such as is shown in Fig. 3.3.
2. The walls and slabs must intersect orthogonally.
3. The longitudinal (exterior and corridor) walls should be symmetrical with respect to the crosswall.

4. The number of walls intersecting with the crosswall should be less than or equal to two.
5. The number of load cases should be less than or equal to five, and the loadings are limited to one or more of the following:
 - i) Earthquake loads
 - ii) Thermal loads
 - iii) Floor loads on the longitudinal walls
 - iv) Floor loads on the crosswalls
 - v) Any other loads (data input manually) applied at finite element nodal points

Within the constraints of the basic assumptions, the programmes can also be used to analyse many thin-walled structures such as box girders and multiple span folded plate roof systems.

5 Linear Elastic Stress Analysis

5.1 General

One of the primary objectives of this study is to determine the stresses that may develop in the intersecting walls of typical buildings of loadbearing masonry wall construction under differential thermal and gravity loads. This chapter deals with the linear elastic analysis carried out using the substructure finite element analysis programme discussed in the previous chapter. The principal assumptions on which the linear elastic analysis is based were discussed in Chapters 3 and 4, and are also summarised below:

1. The materials constituting the structure are homogeneous, isotropic, and linearly elastic.
2. The walls and floor slabs constituting the structure are thin.
3. The loadings are such that they are resisted by the structure predominantly through membrane action.
4. Symmetry of loading and geometry exists about the planes indicated in Fig. 3.3.
5. The floor slabs are free to slide over the exterior and corridor walls but are rigidly connected to the crosswall.

The early parts of this chapter deal with the deformations and stresses that could develop in a typical 10-storey building under differential thermal and gravity loads applied separately. Later, the thermal and gravity loads are combined in such combinations as to produce the worst conditions.

5.2 A Typical Analysis

5.2.1 General Description and Data

A part plan of the building analysed showing a typical bay which is repeated along the length of the building is given in Fig. 3.3. The general dimensions of the building are listed in the summary of data below. The floor slabs of such buildings, mostly of one-way composite construction, have a typical thickness of 64 mm. This thickness was slightly increased to 67.5 mm to account for the reinforcement present in the slab. For the linear elastic analysis, the contribution of the joists to the in-plane stiffnesses of the slabs was ignored. Also, possible

reduction of the slab thickness due to flexural precracking was neglected.

The thickness of the concrete masonry blocks used to construct the walls of a 10-storey building is typically 190 mm. However, the effective thickness of the walls, defined as the transformed area of all solids, viz., block material, grout and reinforcement in the cross section of the wall per unit length of the wall, is smaller and may vary from wall to wall. For this typical analysis, all walls have been assumed to be of the same effective thickness. It has been assumed that the walls are grouted at a nominal spacing of 1200 mm on centres. The total reinforcement has been assumed to be 0.2 percent of the gross cross sectional area, which is the minimum reinforcement required by the CSA code [15] for reinforced masonry loadbearing and shear walls. This reinforcement was assumed to be distributed equally in the horizontal and vertical directions. Note that grouting increases the effective thickness of the wall only in the vertical direction, thus introducing a certain degree of orthotropy in the walls. This orthotropy was ignored for this linear elastic stress analysis and will be considered later in the nonlinear stress analysis.

Because of lower stresses in the upper floors, concrete blocks of lower strengths are frequently used in the upper few floors. Again, this distinction was ignored and is believed to have no significant influence on the key stresses. Also, although sometimes provided, no lintel beam in itself was considered over the window openings. However, the equivalent¹ effect of a lintel beam on the exterior wall was simulated by restraining the latter in the horizontal direction at each floor level. Door openings in the corridor wall and the crosswall and other small openings were disregarded.

The loading cases for which the linear elastic stress analyses were carried out were given in Section 3.3. The mean maximum and minimum temperatures that could arise in the exterior loadbearing walls in summer and winter, respectively, were determined in Chapter 2 and were summarised in Table 2.7. Also, it was shown in Section 2.7 that the temperature can be assumed as 20 °C throughout the crosswall in both seasons. The thermal stresses will be computed for only those temperature differentials that could arise when the exterior wall is of a cavity type.

The base temperature at which no thermal stress develops in the building is now required to be known. If the changes in temperature including the almost periodic diurnal fluctuations during the construction period are disregarded, the temperature at which thermal stresses are zero can be assumed to be equal to the average temperature during construction. However, as long as this average temperature during construction is deemed uniform for all intersecting walls, any arbitrary temperature can be assumed as the base temperature. Since in both seasons the temperature in the crosswall is assumed to be 20 °C, the base temperature was also assumed to be 20 °C to reduce the computation related to thermal loads.

¹ This equivalence is justified in Subsection 4.6.1.

All dead and live loads were considered in the analysis for gravity loads. Live load reduction was applied as stipulated in the NBC code [65].

All pertinent data for the linear elastic stress analysis are summarised below:

Type of Building:	Residential		
Number of Stories:	10		
Length of Crosswall, l_{ww}	7.5 m		
Length of Corridor wall, l_{cw}	7.5 m		(See Fig. 3.3.)
Length of Exterior Wall, l_{ew}	2.5 m		
Storey Height:	2.76 m		
Wall Thickness,	nominal	190 mm	
	effective	84 mm	
Slab Thickness,	nominal	64 mm	
	effective	67.5 mm	
Temperatures:	Exterior wall	41.3 °C in summer,	-18.7 °C in winter
	Crosswall	20 °C in summer,	20 °C in winter
	Corridor wall	20 °C in summer,	20 °C in winter
	All floor slabs	20 °C in summer,	20 °C in winter
Gravity Loads:	Dead load on floors	= 2.4 kN per sq. m	
	Live load on floors	= 2.0 kN per sq. m	
	Unit weight of 190 mm block masonry	= 2.8 kN per sq. m	
Material Properties:	Modulus of Elasticity,	block masonry	= 12 000 MPa
		reinforced concrete	= 22 360 MPa
	Poisson's Ratio,	block masonry	= 0.15
		reinforced concrete	= 0.15
Coeff. of thermal expansion of block masonry = 10×10^{-6} per °C			

5.2.2 Results

The deformations in the crosswall and the exterior wall under different cases of loading are depicted in Figs. 5.1 through 5.4. The deformations in the corridor wall are not presented because those deformations are insignificant compared to the deformations in the other walls particularly under thermal loads. Note that different scales have been used for different loading cases in order to accentuate the nodal displacements adequately.

The principal stresses in the crosswall, exterior wall, and the floor slabs at the uppermost two levels are plotted in Figs. 5.5 through 5.16. The corridor wall and the remaining floor slabs are excluded because the stresses in those parts are either relatively insignificant or irrelevant to this study. The length and direction of the spikelike lines represent the magnitude and direction, respectively, of the principal stresses in these plots. The maximum tensile and compressive stresses are identified with an enlarged symbol of the proper type and their magnitudes written on the top of the plots. It should be noted again that different scales have been used in different plots to represent the magnitude of these stresses. Also, Principal Stress 1 is that principal stress which subtends an angle less than 45° with the horizontal axis.

The distributions of shearing stresses at the interface of the exterior wall and the crosswall in the x-y coordinate system for various loading cases are presented in Figs. 5.17 and 5.18.

5.3 Discussion

The deformed shapes of the crosswall and the exterior wall presented in Figs. 5.1 through 5.4 are useful in visualising the overall deformation tendencies of the walls under the respective loading conditions. These deformations agree well with the ones one would intuitively derive.

From a preliminary examination of the plots of principal stresses presented in Figs. 5.5 through 5.16, the following observations can be made:

Summertime Thermal Stresses:

1. The crosswall is subjected to a thermal stress field which is predominantly tensile (Fig. 5.5). These tensile stresses are in a predominantly vertical direction and are significant in the upper floors of the wall near the intersection with the exterior wall with a maximum magnitude of 2.71 MPa at the top exterior corner.
2. The exterior wall is subjected to a state of biaxial compression (Fig. 5.9). In a more or less vertical direction, these compressive stresses vary from about 1 MPa at the top increasing gradually to about 2.5 MPa at the bottom of the wall. A higher compressive stress of about 3 MPa but only localised at the floor levels occurs in a predominantly horizontal direction. The maximum compressive stress of 5.1 MPa occurs at the foundation. Also, a small tensile stress occurs near the top of the wall at its free edge.
3. The roof slab is subjected to a thermal stress field which is predominantly compressive (Fig. 5.13). These compressive stresses are highest along the intersection with the crosswall and have a highest magnitude of about 1.04 MPa at the corner where the exterior wall and the crosswall meet.

Wintertime Thermal Stresses:

4. The crosswall is subjected to a thermal stress field which is predominantly

compressive (Fig. 5.6). High compressive stresses, up to 4.93 MPa, occur in the upper floors near the intersection with the exterior wall. These high compressive stresses are accompanied by significant tensile stresses in the perpendicular direction particularly in the uppermost floor. The highest magnitude of these tensile stresses is 1.57 MPa.

5. The exterior wall is in a state of biaxial tension (Fig. 5.10). Tensile stresses, varying from 2.0 MPa at the top to about 4.5 MPa at the bottom of the wall, occur in a predominantly vertical direction. Also, in a more or less horizontal direction but localised at the floor levels, tensile stresses of 5.5 MPa occur in the wall. A maximum tensile stress of about 9.28 MPa, but highly localised, occurs at the foundation level.
6. The roof slab is subjected to significant magnitudes of tensile stresses particularly along the line of intersection with the crosswall (Fig. 5.14). These tensile stresses vary from about 1.25 MPa at the corridor end to about 1.90 MPa at the exterior end.

Stresses Due to Gravity Loads - Case 1:

7. Although no slab load is directly applied on the crosswall, a significant amount of floor load is transferred from the exterior and corridor walls to the crosswall. This is evident directly from the magnitude and distribution of shear stresses at the interface of the exterior wall and the crosswall (Fig. 5.18) and indirectly from the near equality of the magnitudes of compressive stresses at the bases of those walls (Figs 5.7 and 5.11). Since self-weight accounts for only about 0.9 MPa of these compressive stresses, the net stress transferred from the exterior wall to the crosswall at the base is about 1.7 MPa.

Stresses Due to Gravity Loads - Case 2:

8. Similar to Case 1, a significant load transfer also occurs under the gravity loads of Case 2. Again, deducting the stress of 0.9 MPa due to self weight, it can be seen from Fig. 5.12 that the net stress transferred from the loaded crosswall to the exterior wall at the base is about 1.6 MPa.

As is evident from a comparison of the interfacial shear stresses given in Figs. 5.17 and 5.18, the shear transfer under differential gravity loads is much smaller than that under differential thermal loads.

The thermal stresses plotted in Figs. 5.5, 5.6, 5.9, 5.10, 5.13, 5.14, and 5.17 are due to thermal loads only. In other words, these plots indicate the thermal stresses that could develop in addition to the stresses due to other loads, such as gravity loads. Thus, for example, it can be said that additional compressive stresses up to 4.93 MPa may occur in the upper part of the crosswall under summertime thermal loading. However, the magnitudes and directions of principal stresses

are functions of both normal and shear stresses present in the x-y system of axes. Therefore, the principal stresses caused only by the thermal loads can not be directly superimposed on those caused by the gravity loads. Moreover, any potential of cracking can only be determined on the basis of net tensile stresses in the structure. In order to ascertain the implications of thermal stresses in a realistic manner, the stresses in the x-y coordinate system occurring under thermal loads and gravity loads must be combined and the principal stresses determined from these combined stresses.

The combinations of the thermal and gravity loads which produce worst conditions with respect to stresses are to be determined now. A close examination of the stresses plotted in Figs. 5.5 through 5.18 indicate that there is no unique combination that is worst for all the key stresses in all the components of the building. For example, on the basis of similarity of sign of the interfacial shearing stresses plotted in Figs. 5.17 and 5.18, it appears that the summer thermal loads must be combined with the gravity loads of Case 2 to obtain the maximum tensile stress near the exterior edge of the crosswall whereas the winter thermal loads must be combined with the gravity loads of Case 1 to obtain the maximum compressive stresses in the same location. But again, while the combination of summer thermal loads and gravity loads of Case 1 enhances the thermal tensile stresses at the exterior edge of the crosswall, it reduces the thermal tensile stresses near the interior edge of the same wall in the upper floors somewhat more than does the other possible combination of summer thermal loads with the gravity loads of Case 1 (Fig. 5.5, 5.7 and 5.8). Nevertheless, since the tensile and compressive stresses near the exterior edge of the crosswall are the most significant ones and since significant stresses in other components such as exterior wall and roof slab are also affected in the same manner, the following combinations of the thermal and gravity loads are considered most critical:

Combination I : Summertime Thermal Loads + Gravity Loads, Case 2

Combination II : Wintertime Thermal Loads + Gravity Loads, Case 1

It should be noted that since the stress distributions in the walls under the gravity loads of either case are more or less the same in the lower floors, no significant difference in the combined stresses in that area would arise if the summertime thermal loads were combined with the gravity loads of Case 1 instead of Case 2 or if wintertime thermal loads were combined with gravity loads of Case 2 instead of Case 1.

The combined principal stresses in the various components of the building under the above combinations I and II are plotted in Figs. 5.19 through 5.24. The implications of the thermal loads can now be discussed on the basis of these stresses.

5.3.1 Cracking under Thermal Loads

It can be seen in Fig. 5.19 that moderate to high tensile stresses occur in the upper floors of the crosswall near its exterior edge. These tensile stresses are inclined at an angle of about 45° to 55° with the horizontal, are highest at the top exterior corner, and decrease in both inward and downward directions of the wall. It was stated in Chapter 3 that concrete block masonry has a tensile strength of 0.6 MPa in a diagonal direction. Therefore, under the Load Combination I, cracks are expected to occur in a generally diagonal direction in the vicinity of the exterior edge of the crosswall particularly in the upper two floors. The possible adverse effects of these cracks will be discussed in Chapter 6.

Some tensile stresses are also indicated in the same location under the Load Combination II but in directions perpendicular to the tensile stresses occurring under Load Combination I (Fig. 5.20). Although these tensile stresses are smaller in magnitude, it should be noted that the other principal stresses are compressive and of moderate to high magnitude. This implies that the tensile strength under this tension-compression biaxial stress condition would be even smaller than 0.6 MPa. While a rigorous assessment of cracking on the basis of such refined tensile strength is deferred until the nonlinear postcracking stress analysis in Chapter 6, it is sufficient to infer, at this stage that cracks oriented at angles of about 135° with the positive x-axis are expected to occur in the upper exterior corner of the crosswall under the Load Combination II.

Fig. 5.22 indicates that under the Load Combination II, tensile stresses much higher than the uniaxial tensile strength of 0.5 MPa occur in a predominantly vertical direction along the entire height of the exterior wall. Therefore, horizontal cracks are expected to occur in the exterior wall along its entire height under that load combination. In addition, some vertical cracks are also expected to occur as indicated by the high tensile stresses in a more or less horizontal direction at the floor levels.

It is seen in Fig. 5.24 that under the Load Combination II, the roof slab is subjected to tensile stresses up to 2.18 MPa. The highest tensile stress occurs at an angle of about 45° with the positive x-axis at the exterior corner of the slab and gradually decreases in both directions. On the basis of a tensile strength of concrete of 2.0 MPa, a few cracks may therefore occur in the slab in the vicinity of its exterior corner with the crosswall at angles of about 135° with the positive x-axis. It should be noted that the stresses in the slabs have been computed on the basis of uncracked sections. It is quite likely that the slabs are precracked due to flexural tensile stresses caused by transverse loads and therefore the thermal cracks are likely to be more extensive.

5.3.2 Overstressing in Compression under Thermal Loads

Fig. 5.20 indicates that the crosswall is subjected to some localised high compression at the top exterior corner under the combination of wintertime thermal loads with gravity loads of

Case 1. These compressive stresses near the exterior edge in the uppermost floor are well beyond the allowable axial compressive stress of $0.225f'_m$ or 2.7 MPa as stipulated in the CSA code [15].

Fig. 5.21 indicates that the exterior wall is subjected to high compressive stresses along most of its height under the combination of summertime thermal loads and gravity loads of Case 2. These compressive stresses are generally in the vertical direction and vary from about 2.0 MPa at Level 8 to about 7.0 MPa at the base. Therefore, a high degree of compressive overstressing also occurs in the exterior wall particularly in the lower floors under the Load Combination II.

From Fig. 5.23, it appears that some compressive stresses occur in the roof slab under the Load Combination I. However, these compressive stresses are not so significant as compared to the compressive strength of concrete of 20 MPa.

5.4 Limitations of Linear Elastic Analysis

From the above discussion, two significant effects of thermal loads on a building are evident. Firstly, cracking may occur in some parts of the exterior wall, crosswall, and roof slab. Secondly, particularly in the exterior wall, overstressing in compression beyond the linear elastic limit may occur.

Both of these effects are likely to cause a redistribution of stresses so that the stresses indicated by the linear elastic analysis are altered. For example, it was indicated by Fig. 5.21 that cracks would develop in the exterior wall due to wintertime thermal loads. As soon as one or more of these cracks develop, the thermal load causing the cracks is somewhat relieved. Consequently, the tendency to develop further cracks is also reduced and the cracking in that wall may not be as extensive as indicated by the distribution of tensile stresses in Fig. 5.21. For the same reason, the compressive stresses indicated at the top corner of the crosswall under wintertime thermal loads (Fig. 5.20) may not be as high.

While the locations where cracking would initiate can be predicted by linear elastic stress analysis with reasonable accuracy, the extent of cracking and the magnitude of compressive stresses determined by such an analysis are unlikely to be accurate. The actual extent of cracking and magnitudes of compressive stresses can be determined either experimentally or analytically by a nonlinear postcracking analysis by the finite element method.

The next chapter deals with a nonlinear finite element analysis of the above problem.

FIG. 5.1 CROSSWALL DEFORMATION - THERMAL LOADS

LEGEND:

 Δ Restraint

FIG. 5.2 CROSSWALL DEFORMATION - GRAVITY LOADS

FIG. 5.3 EXTERIOR WALL DEFORMATION - THERMAL LOADS

LEGEND:

△ Restraint

CASE 1

DEFLECTION SCALE: 2.000 mm

CASE 2

DEFLECTION SCALE: 2.000 mm

FIG. 5.4 EXTERIOR WALL DEFORMATION - GRAVITY LOADS

LEGEND:

—◆— Tension
—■— Compression

Maximum Tension = 2.71 MPa

106

Maximum Compression = .86 MPa

Enlarged symbol indicates a maximum

FIG. 5.5 PRINCIPAL STRESSES IN CROSSWALL - SUMMER THERMAL LOADS

FIG. 5.6 PRINCIPAL STRESSES IN CROSSWALL - WINTER THERMAL LOADS

LEGEND:

Maximum Tension = .27 MPa

—●— Tension

—■— Compression

Enlarged symbol indicates a maximum

Maximum Compression = 2.61 MPa

108

FIG. 5.7 PRINCIPAL STRESSES IN CROSSWALL - GRAVITY LOADS, CASE 1

LEGEND:

Maximum Tension = .33 MPa

—●— Tension

—●— Compression

Enlarged symbol indicates a maximum

Maximum Compression = 2.54 MPa

109

FIG. 5.8 PRINCIPAL STRESSES IN CROSSWALL - GRAVITY LOADS, CASE 2

LEGEND:

—●— Tension

Maximum Tension = .34 MPa

—■— Compression

Maximum Compression = 5.10 MPa

Enlarged symbol indicates a maximum

110

FIG. 5.9 PRINCIPAL STRESSES IN EXTERIOR WALL - SUMMER THERMAL LOADS

LEGEND:

—●— Tension

Maximum Tension = 9.28 MPa

—■— Compression

Maximum Compression = .63 MPa

Enlarged symbol indicates a maximum

111

FIG. 5.10 PRINCIPAL STRESSES IN EXTERIOR WALL - WINTER THERMAL LOADS

LEGEND:

112

Maximum Tension = .48 MPa

—●— Tension

—■— Compression

Enlarged symbol indicates a maximum

Maximum Compression = 3.04 MPa

FIG. 5.11 PRINCIPAL STRESSES IN EXTERIOR WALL - GRAVITY LOADS, CASE 1

LEGEND:

Maximum Tension = .04 MPa

—●— Tension

—■— Compression

Enlarged symbol indicates a maximum

Maximum Compression = 2.54 MPa

113

FIG. 5.12 PRINCIPAL STRESSES IN EXTERIOR WALL - GRAVITY LOADS; CASE 2

LEGEND:

- ◆— Tension
 —■— Compression
 Enlarged symbol indicates a maximum

Maximum Tension = .24 MPa

Maximum Compression = 1.04 MPa

LEVEL 10

LEVEL 9

STRESS SCALE: .500 MPa

FIG. 5/13. PRINCIPAL STRESSES IN SLABS - SUMMER THERMAL LOADS

LEGEND:

- ◆— Tension
- Compression
- Enlarged symbol indicates a maximum

FIG. 5.14 PRINCIPAL STRESSES IN SLABS - WINTER THERMAL LOADS

LEGEND:

- Tension
- Compression
- Enlarged symbol indicates a maximum

FIG. 5.15 PRINCIPAL STRESSES IN SLABS - GRAVITY LOADS, CASE 1

LEGEND:

- Tension
- Compression
- Enlarged symbol indicates a maximum

Maximum Tension = .32 MPa

Maximum Compression = .10 MPa

LEVEL 10

LEVEL 9

STRESS SCALE: .100 MPa

FIG. 5.16 PRINCIPAL STRESSES IN SLABS - GRAVITY LOADS, CASE 2

FIG. 5.17 SHEAR STRESSES AT EXTERIOR INTERSECTION - THERMAL LOADS

FIG. 5.18 SHEAR STRESSES AT EXTERIOR INTERSECTION - GRAVITY LOADS

LEGEND:

—●— Tension
 —x— Compression

Enlarged symbol
 indicates a maximum

Maximum Tension = 2.76 MPa

Maximum Compression = 2.12 MPa

FIG. 5.19 PRINCIPAL STRESSES IN CROSSWALL, SUMMER THERMAL+GRAVITY2

LEGEND:

Maximum Tension = 1.82 MPa

—●— Tension
 —■— Compression

Enlarged symbol
 indicates a maximum

Maximum Compression = -5.32 MPa

FIG. 5.20. PRINCIPAL STRESSES IN CROSSWALL, WINTER THERMAL+GRAVITY

FIG. 5.21 PRINCIPAL STRESSES IN EXT. WALL, SUMMER THERMAL+GRAVITY2

Maximum Tension = 7.56 MPa

—●— Tension
 —■— Compression

Maximum Compression = 1.15 MPa

Enlarged symbol
 indicates a maximum

STRESS SCALE: 2.000 MPa

FIG. 5.22 PRINCIPAL STRESSES IN EXT. WALL; WINTER THERMAL+GRAVITY1

LEGEND:

—◆— Tension

—■— Compression

Enlarged symbol
indicates a maximum

Maximum Tension = .25 MPa

Maximum Compression = 1.12 MPa

LEVEL 10

LEVEL 9

STRESS SCALE: .500 MPa

FIG. 5.23 PRINCIPAL STRESSES IN SLABS, SUMMER THERMAL+GRAVITY2

LEGEND:

- Tension
- Compression
- Enlarged symbol indicates a maximum

FIG. 5.24 PRINCIPAL STRESSES IN SLABS, WINTER THERMAL+GRAVITY

6 Adaptation of a Nonlinear Stress Analysis Programme

6.1 General

Some limitations of the linear elastic stress analysis were stated in the previous chapter. It was mentioned that the stresses predicted by linear elastic analysis were unlikely to be accurate. The objective of the present nonlinear analysis is to predict the thermal stresses and other effects of thermal loads in a more realistic manner.

Nonlinearity in structural behaviour may emanate from two sources viz., nonlinear stress-strain relationships known as material nonlinearity and nonlinear strain-displacement relationships known as geometric nonlinearity. Since both strains and displacements in the structure in this study are small under the applied loads, geometric nonlinearity can be neglected. Therefore, by the term 'nonlinearity' in this study, only material nonlinearity is implied.

In a structure made of reinforced concrete and masonry, material nonlinearity may be introduced by such phenomena as nonlinear constitutive relationships, cracking and crushing of masonry and concrete, yielding of steel, bond slip, shear friction, and dowel action. Still other factors such as creep and shrinkage, which are time-dependent, may also be involved. In this study, however, the time-dependent effects were not considered. Also, it was assumed that a perfect bond exists between steel and masonry or steel and concrete. This latter assumption was unavoidable because the nonlinear programme available for this study does not consider bond slip.

In the earlier part of this chapter a brief introduction of the nonlinear computer programme used in this study is presented. In the later part, adaptation of the programme to model the anisotropic behaviour of concrete block masonry and the one-way floor systems in typical masonry buildings is presented.

6.2 The Nonlinear Analysis Computer Programme

The computer programme used in this study for nonlinear stress analysis, known as FELARC (Finite Element Layered Analysis of Reinforced Concrete), was developed by Ghoneim [36]. The programme can trace the nonlinear behaviour of reinforced and prestressed concrete structures subjected to monotonic or cyclic loading. In addition to the nonlinear constitutive

relationships of concrete and steel, time-dependent effects such as creep and shrinkage are also accounted for. A variety of reinforced and prestressed concrete structural elements such as beams, slabs, folded plates, shells, and box girder bridges can be analysed by the programme.

The theoretical aspects of the programme which are most relevant to this study are briefly treated in the following subsections. A more comprehensive treatment is available in reference 36.

6.2.1 Material Modelling

The materials involved in the present analysis are concrete block masonry, concrete and steel. However, in its original state, the programme can handle only concrete and steel. While the adaptation of the programme to include concrete block masonry will be treated later, the modelling of the stress-strain behaviour of concrete and steel as used by the programme is discussed here.

The stress-strain curve for concrete under uniaxial compression is typically as shown in Fig 6.1. The programme adopts the following two equations to represent the curve:

For the segment ab,

$$\sigma = \frac{E_0 \epsilon}{1 + \left(\frac{E_0}{E_s} - 2\right) \frac{\epsilon}{\epsilon_{cu}} + \left(\frac{\epsilon}{\epsilon_{cu}}\right)^2} \quad (6.1)$$

and for the segment bc,

$$\sigma = \sigma_0 \left(\frac{\epsilon}{\epsilon_{cu}}\right)^{1 - \epsilon/\epsilon_{cu}} \quad (6.2)$$

where E_0 = initial tangent modulus of elasticity
 E_s = secant modulus of elasticity at peak stress
 σ = stress
 σ_0 = compressive strength
 ϵ = strain
 ϵ_{cu} = strain at peak stress

Equation 6.1 is the widely accepted relationship suggested by Saenz [80]; Equation 6.2 was proposed by Smith and Young [84].

In uniaxial tension, a linear relationship between stress and strain with the same modulus E_0 is assumed.

Fig. 6.1 Stress-Strain Curve for Concrete in Uniaxial Compression and Tension

Darwin and Pecknold [21] report that under a biaxial state of stress concrete exhibits strongly stress-induced orthotropy. This means that unless the ratio of stresses in the two axes, γ , is equal to 1, the initial tangent moduli, for example, in the two directions are different. This is reported to be caused by the confinement of microcracking in addition to Poisson's effect. The programme considers this stress-induced orthotropy using the concept of equivalent uniaxial strain introduced by Darwin and Pecknold [21, 18]. The same relationships given by Equations 6.1 and 6.2 for uniaxial compression are used except that ϵ is replaced by the equivalent uniaxial strain, ϵ_{iu} . The latter, in the two principal directions $i = 1$ and 2 , are related to the biaxial strains ϵ_1 and ϵ_2 by:

$$\begin{aligned}\epsilon_{1u} &= \frac{1}{1 - \nu\alpha} \epsilon_1 \\ \epsilon_{2u} &= \frac{1}{1 - \nu\alpha} \epsilon_2\end{aligned}\tag{6.3}$$

where ϵ_{iu} = equivalent uniaxial strain in direction i , $i = 1, 2$
 ϵ_i = biaxial strain in direction i , $i = 1, 2$

- ν = Poisson's ratio
 α = stress ratio, σ_1/σ_2
 σ_i = biaxial stress in direction i , $i = 1, 2$.

In order to determine the point of maximum stress in Fig. 6.1 under a biaxial state of stress, the programme uses the modified biaxial failure envelope proposed by Kupfer and Gerstle [56] which is shown in Fig. 6.2. The ratio γ of the current principal stresses σ_1 and σ_2 are determined and used in the envelope to determine concrete strengths σ_{ic} in the principal directions. For the principal direction in which the stress is compressive, the strain ϵ_{ic} corresponding to the peak stress is assumed by the programme as given by different researchers for different biaxial stress states. With these known values of σ_{ic} and ϵ_{ic} , and the value of the ultimate strain ϵ_u as input (usually 0.0004), the programme defines new stress-strain curves using the equations 6.1 and 6.2 for each of the principal directions. The tangent moduli of elasticity E_1 and E_2 in the principal directions are then determined and used in the constitutive matrix $[D]$ for an orthotropic material given by:

$$[D] = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1E_2} & 0 \\ \nu\sqrt{E_1E_2} & E_2 & 0 \\ 0 & 0 & \frac{1}{4}(E_1 + E_2 - 2\nu\sqrt{E_1E_2}) \end{bmatrix} \quad (6.4),$$

where ν = Poisson's ratio.

The expression for the shear modulus in the above equation, suggested by Darwin and Pecknold [21], is direction-independent.

The criterion assumed for concrete cracking in this programme is that cracking occurs perpendicular to a principal stress whenever that stress exceeds the uniaxial tensile strength of concrete f_t' . Once a crack forms, say when $\sigma_1 > f_t'$, the principal directions are locked and a crack in the other direction can only form if $\sigma_2 > f_t'$. After cracks have formed in both directions, all the coefficients of the constitutive matrix become zero except that the shear modulus may still retain some value, βG , because of the dowel action of reinforcement and aggregate interlock across the cracks. Ghoneim [36] reported on the basis of Lin's [61] observations that the value of this shear

retention factor β has little effect on the results. Razaqpur [75], who subsequently used FELARC, reported that this may not be always true.

Fig. 6.2 Biaxial Failure Envelope of Concrete

The tension carried by uncracked portions of concrete between cracks transferred from steel by bond action, referred to as tension-stiffening, is also accounted for in FELARC. This is accomplished by introducing a linear descending branch to the stress-strain relationship of concrete in tension as shown in Fig. 6.3. To avoid numerical difficulty often arising with negative stiffnesses, the descending branch is actually modelled in the analysis by a stepped function. This means that while the cracked concrete can carry some tension, its stiffness is zero as indicated by the horizontal lines in Fig. 6.3.

A linear stress-strain relationship identical in tension and compression is assumed for steel. The relationship as shown in Fig. 6.4 is defined by four parameters: the yield strength, f_y , the elastic modulus, E_s , the strain-hardening modulus, E_p , and the ultimate strain, ϵ_{su} .

Fig. 6.3 Stress-Strain Relationship of Concrete in Tension showing Tension Stiffening

Fig. 6.4 Stress-Strain Relationship of Steel

6.2.2. Finite Element Modelling

The programme can model various structures by three types of elements viz , (1) quadrilateral thin shell element, (2) one-dimensional axial element and (3) spring element. The structure in this study was modelled by the quadrilateral thin shell element.

The quadrilateral thin shell element in FELARC is obtained by combining a quadrilateral in-plane element, QLC3, developed by Sisodia [83] and a quadrilateral plate bending element, QBE, developed by Zienkiewicz and Cheung [95]. This shell element has in all 24 degrees of freedom as shown in Fig. 6.5 of which 12 are in-plane and the remaining 12 are out-of-plane.

Fig. 6.5 The Quadrilateral Thin Shell Element

The element is divided into a number of concrete layers in the thickness direction. Each layer of distributed steel reinforcement is represented as a 'smeared' layer of steel having an equivalent thickness to represent the same area of steel per unit length as provided by the discrete reinforcing bars. This layered representation of the element is shown in Fig. 6.6. In deriving the stiffness matrix of the element any integration in the z -direction is transformed into a summation over the concrete and steel layers. The purpose of this multilayered approach is to allow the stiffness of the individual layers to vary according to the state of stress present in them.

6.2.3 Thermal Loading

The programme treats thermal effects as initial strains. From the input values of the initial and final temperatures and the coefficient of thermal expansion, it computes the initial strain from the following:

$$\epsilon_T = \alpha (T_f - T_i) \quad (6.5)$$

where ϵ_T = initial thermal strain

α = coefficient of thermal expansion

T_f = final temperature

T_i = initial temperature

Fig. 6.6 Layered Composition of Thin Shell Element

This initial strain is used to compute the equivalent nodal forces using the constitutive relationships corresponding to the states of stress at the time of applying the thermal load. After these equivalent nodal forces are computed, they are treated exactly as externally applied loads except that in the first iteration of any increment of these loads stresses are calculated based on the net strain increment given by

$$\Delta \epsilon_{\text{net}} = \Delta \epsilon_m - \Delta \epsilon_T \quad (6.6)$$

where $\Delta \epsilon_m$ = strain increment due to the applied equivalent load increment
 $\Delta \epsilon_T$ = initial thermal strain increment

6.2.4. The Analysis Technique

The programme uses an incremental iterative tangent stiffness procedure, schematically shown in Fig. 6.7, to solve the nonlinear problem. The procedure can be summarised in the following steps:

1. The load is divided into a number (including one) of increments
2. After a certain increment of load is applied, the incremental nodal displacements are calculated using the tangent stiffness evaluated at the end of the previous load increment.
3. The incremental strains and stresses are calculated and added to the total strains and stresses at the end of the previous load increment.

4. The resulting current stresses corresponding to the resulting current strains are only approximate since a tangent stiffness has been used. The actual current stresses corresponding to the current total strains are therefore determined from the stress-strain relationships corresponding to the current states of stress.
5. The differences between the approximate and actual stresses are calculated to compute the unbalanced nodal forces. The tangent stiffnesses corresponding to the current strains are also calculated.
6. The unbalanced nodal forces are then applied and again the incremental displacements due to these loads are calculated using the tangent stiffnesses determined in step 5.
7. Steps 3 through 6 are repeated until the unbalanced forces or the incremental displacements are smaller than some specified tolerable limits or a preset number of iterations are completed. Instead of the unbalanced forces or incremental displacements becoming smaller, i.e., converging, divergence may occur indicating a collapse of the structure. In such case the solution process is stopped whenever a specified divergence criterion is exceeded.
8. Steps 2 through 7 are repeated until all the load increments are exhausted.

Fig. 5.7 Incremental-Iterative Solution Scheme Using Tangent Stiffness

6.3 Orthotropy of Concrete Block Masonry

Concrete block masonry is evidently orthotropic: the units have different sectional areas in the vertical and horizontal directions and the grout cores run only in the vertical direction. For a realistic stress analysis, particularly under thermal loads, this orthotropy must be considered; for, unlike other loads, thermal loads are functions of the stiffnesses of the structure. However, FELARC did not have any provision to deal with such external orthotropy. Such provisions therefore had to be introduced in the programme for this study.

The layout of the concrete block masonry wall in plan which falls within the domain of analysis (Fig. 3.3) is shown in Fig. 6.8. Due to symmetry about the middle plane of the crosswall BC, it is evident that this wall cannot undergo any bending and its effective area of cross section is derived from a half of its thickness; however, it is still orthotropic.

The exterior and corridor walls were modelled by a special three layered orthotropic element (Type 1) and the crosswall was modelled by a special single layered orthotropic element (Type 2). In deriving these orthotropic formulations, the objective was to use the basic analytical procedure of FELARC as much as possible so that modifications of FELARC were minimal.

Fig. 6.8 Layout of Concrete Block Masonry Wall

6.3.1 Orthotropic Element of Type 1

A blown-up view of the part of a typical masonry wall is shown in Fig. 6.9(b). This view is obtained if one imagines two vertical planes of separation as indicated in Fig. 6.9(a).

The masonry wall can therefore be imagined as a 3-layered sandwich panel, with the two outer skins represented by the face shell material of the block and the head and bed mortar joints while the core is represented by the webs of the blocks and the grout columns. Since the mortar and the face shells in the outer skins have similar basic composition and mechanical properties, and since mortar joints constitute a relatively small part of the skin, it can be reasonably assumed that the outer skins are isotropic having the same mechanical properties as those of the block material. On the other hand, as is evident from Fig. 6.9, the inner core is highly orthotropic because the grout columns and the block webs extend only in the vertical direction. Therefore, the inner layer must be represented by an orthotropic formulation. The resulting Type 1 orthotropic element is schematically shown in Fig. 6.10.

The constitutive relationship of an orthotropic material in a 2-D plane stress condition is given by the following:

$$[D]_{\text{orth}} = \frac{1}{1 - \nu_{12}\nu_{21}} \begin{bmatrix} E_1 & \nu_{12}E_1 & 0 \\ \nu_{21}E_2 & E_2 & 0 \\ 0 & 0 & G \end{bmatrix} \quad (6.7)$$

where E_1 and E_2 = tangent moduli of elasticity in principal directions 1 and 2, respectively
 ν_{12} and ν_{21} = Poisson's ratios
 G = a constant related to the shear modulus

The programme can treat multilayered elements but considers all layers as "externally" isotropic. The constitutive matrix is given by the following:

$$[D]_{\text{iso}} = \frac{1}{1 - \nu^2} \begin{bmatrix} E & \nu E & 0 \\ \nu E & E & 0 \\ 0 & 0 & \frac{E}{2}(1 - \nu) \end{bmatrix} \quad (6.8)$$

where E = Tangent modulus of elasticity
 ν = Poisson's ratio

Fig. 6.9 Blown-up View of Concrete Block Masonry Wall

Fig. 6.10 Orthotropic Element of Type 1

The easiest way to represent the external orthotropy¹ for the inner layer of Fig. 6.10 is to modify the isotropic constitutive relationship for that layer built in FELARC to the one given in Equation 6.7. This can be done by multiplying the elements of the constitutive matrix $[D]$ by appropriate factors as shown below:

$$[D]_{\text{orth}} = \begin{bmatrix} C_1 d_{11} & C_2 d_{12} & 0 \\ C_2 d_{21} & C_3 d_{22} & 0 \\ 0 & 0 & C_4 d_{33} \end{bmatrix} \quad (6.9)$$

$$\text{where } d_{11} = d_{22} = E/(1 - \nu^2)$$

$$d_{21} = d_{12} = \nu E/(1 - \nu^2)$$

$$d_{33} = E/2(1 + \nu)$$

The problem now reduces to finding appropriate values of E_1 , E_2 , ν_{12} , ν_{21} and G' so that the multiplication factors C_1 through C_4 can be determined. Evidently, E_1 , E_2 , ν_{12} , ν_{21} and G' are the equivalent properties of a fictitious homogeneous but orthotropic layer which can duplicate the actual stress-strain relationship of the heterogeneous inner layer shown in Fig. 6.9.

It is observed that in the vertical direction, the inner layer (Fig. 6.9) derives its axial stiffness from two sources: the grout columns and the crosswebs of the blocks. It is assumed that the

¹ This is distinguished from the stress-induced orthotropy of concrete discussed in Subsection 6.2.1

crosswebs are not bedded in any mortar joint except those adjacent to the grout columns. This assumption is based on the common practice of block masonry construction in which the blocks are only face-shell bedded.

The initial tangent stiffness of the fictitious layer in the vertical direction, E_v , can be expressed by the following:

$$E_v = a_{bm} E_{bm} + a_g E_g \quad (6.10)$$

where E_{bm} and E_g = initial tangent modulus of block material and grout
 a_{bm} and a_g = factors representing the fraction of cross sectional areas of block material and grout, respectively, in a unit length of wall.

The factors a_{bm} and a_g are functions of the block dimensions and number of cores grouted per unit length of the wall. With reference to Fig. 6.11 these factors are given by the following formulas:

$$a_{bm} = (1/1000) r n t_w \quad (6.11)$$

$$a_g = t_g / L_g$$

where t_w = thickness of the crosswebs of blocks, mm
 n = number of crosswebs in 1000 mm length of wall
 r = reduction factor described below
 t_g = thickness of grouted cores, mm (Fig. 6.11)
 L_g = spacing of grouted cores, mm

Fig. 6.11 Horizontal Cross Section of the Inner Layer of Type 1 Element

The reduction factor r is to account for the discontinuity of some of the webs. A value of 0.5 appears reasonable for typical walls assuming that 50 percent of the webs are discontinuous.

Note that even those webs which are not connected by bed joints partly contribute to the vertical stiffness since they are monolithic with the face shells. Although this contribution can be theoretically determined, such a refined assessment of the reduction factor appears unnecessary since the vertical stiffness of the full wall section is not so sensitive to this factor.

In the horizontal direction, except when all cores are grouted which is rare, there is nothing in the inner layer to resist any displacement. This implies that the stiffness in that direction is zero. However, since the grout columns are also in contact with the face shells they contribute somewhat to the overall horizontal stiffness of the wall section through shear transfer. While it is theoretically possible to quantify this contribution, it appears to be insignificant for walls of typical construction. A number of test analyses revealed that the stresses change by very insignificant amounts as E_h is varied between $0.02 E_v$ and $0.1 E_v$. A value of $0.02 E_v$ was selected for E_h .

Since the Poisson's ratios of grout and block concrete are about the same, it is reasonable to assume that ν_{hv} is equal to ν . Since $\nu_{vh} E_v$ is equal to $\nu_{hv} E_h$, it follows that $\nu_{vh} = 0.02 \nu$.

It remains to determine the value of G' . Unlike isotropic materials, the shear modulus for orthotropic materials is an additional unknown and therefore must be determined experimentally. Analytical expressions for the torsional rigidities of orthogonal stiffened plates have been proposed by many researchers such as Huffington [44]. Szilard [89] reports that the shear modulus of an orthotropic material can be approximately expressed as

$$G_{orth} = \frac{\sqrt{E_1 E_2}}{2(1 + \sqrt{\nu_{12} \nu_{21}})} \quad (6.12)$$

While the above approximate relation may be reasonable for stiffened plates with comparable values of E_1 and E_2 , its applicability to the fictitious homogeneous layer in the present study where E_1 and E_2 differ widely is open to question. However, in the absence of any experimental data or any analytical formula for an analogous condition, Equation 6.12 was used in this study. Accordingly, taking $E_h = 0.02 E_v$, $\nu_{hv} = 0.15$, and $\nu_{vh} = 0.02 \nu$, and the vertical and horizontal directions as principal directions 1 and 2, respectively, G_{orth} can be computed from the formula below:

$$G_{orth} = \frac{0.0707 E_v}{1 + 0.141 \nu} \quad (6.13)$$

But since $G' = G_{orth} (1 - \nu_{vh} \nu_{hv})$

$$G' = \frac{0.0707 E_v (1 - 0.02 v^2)}{1 + 0.141 v} \quad (6.14)$$

Taking $v = 0.15$ the multiplication factors C_1 through C_4 can now be determined from the following approximate formulas:

$$\begin{aligned} C_1 &= \frac{E_h}{E} \frac{1 - v^2}{(1 - 0.02 v^2)} \approx 0.0196 \frac{E_v}{E} \\ C_2 &= \frac{v_{hv}}{v} \frac{E_h}{E} \frac{1 - v^2}{(1 - 0.02 v^2)} \approx 0.0196 \frac{E_v}{E} \\ C_3 &= \frac{E_v}{E} \frac{1 - v^2}{(1 - 0.02 v^2)} \approx 0.978 \frac{E_v}{E} \\ C_4 &= \frac{0.0707 (1 - 0.02 v^2) E_v}{(1 + 0.141 v) E/2(1 + v)} \approx 0.159 \frac{E_v}{E} \end{aligned} \quad (6.15)$$

6.3.2 Orthotropic Element of Type 2

Unlike the orthotropic element of Type 1 which must represent the flexural behaviour of a masonry wall, the Type 2 orthotropic element is required to represent only the in-plane behaviour of the wall it models. This permits the use of a single orthotropic layer of any arbitrary thickness provided suitable material constitutive relationships are used.

It was assumed that the thickness of the Type 2 orthotropic element is equal to that of the face shell of concrete blocks; the rest of this orthotropic formulation is similar to that for Type 1. Suitable multiplication factors C_1 through C_4 were calculated to modify the isotropic constitutive relationship of the block material to an orthotropic one. The same formula as given in Eq 6.10 was used for E_v except that a_{bm} and a_g are given by the following formulas:

$$\begin{aligned} a_{bm} &= 1 + \frac{1}{2000} r n t_w \frac{t_c}{t_{fs}} \\ a_g &= \frac{t_g t_c}{2 L_g t_{fs}} \end{aligned} \quad (6.16)$$

where a_{bm} and a_g = fractions of the areas of block mortar and grout in 1000 mm length of wall

- t_w = thickness of crosswebs of blocks, mm
- t_c = thickness of cores, mm (Fig. 6.12)
- t_g = length of grout cores, mm (Fig. 6.12)
- t_{fs} = thickness of face shell of blocks, mm
- L_g = spacing of grouted cores, mm
- n = number of crosswebs in 1000 mm of wall
- r = reduction factor

Again a value of 0.5 was selected for the reduction factor r . In the horizontal direction, stiffness is derived from the face shell and also from the grout columns through shear transfer mechanism. However, the contribution of the latter is negligible compared with that of the former. Therefore, E_h was assumed equal to E . Using Eq. 6.11 for G_{orth} and taking $v_{hv} = v = 0.15$ the multiplication constants C_1 through C_4 for type 2 orthotropic elements can thus be computed using the following approximate formulas:

$$C_1 = C_2 \approx \frac{0.9775}{1 - 0.0225 R_E}$$

$$C_3 \approx \frac{1}{R_E} \frac{0.9775}{1 - 0.0225 R_E} \quad (6.17)$$

$$C_4 \approx \frac{1.15 - 0.1725 \sqrt{R_E}}{\sqrt{R_E}}$$

where $R_E = E/E_v$

E = isotropic initial tangent modulus of block concrete

E_v = equivalent initial tangent modulus in the vertical direction

6.4 One-Way Floors

The modelling of the one-way floor slabs by the available elements in FELARC posed a serious problem. The open-web steel joists (Fig. 6.13) could be modelled by the uniaxial element and the floor slab by shell elements. However, this would force one to use a definite mesh of shell elements for slabs and walls to conform to the nodal points of the trussed joists. This would entail a large number of elements, much larger than would be practical to deal with. To surmount this problem, a special approach similar to substructuring was used. A suitable mesh of rectangular shell elements was selected to model the floor slabs; the only restriction was that nodal points of the mesh were to align with the joists, a condition which was not difficult to accommodate.

Fig. 6.12 Orthotropic Element of Type 2

Then the following two assumptions were made:

1. The top chord of the open-web steel joists which is encased in the slab does not contribute significantly to the axial and bending stiffnesses of the deck slab. This is reasonable because the top chord of typical open-web steel joists has a negligible cross sectional area compared with the deck slab.
2. The open-web joists behave structurally as trusses.

The first assumption implies that the joists contribute stiffness to the shell elements of the slabs only in the vertical (z) degree of freedom. The essence of the approach was to compute these stiffness contributions of the joists and add these stiffnesses directly to the global stiffness matrix generated by FELARC at the proper locations.

The stiffness contributions of the joists were computed by a truss analysis programme. With the two ends simply supported, a unit displacement in the vertical direction was applied at, say node 2 (Fig. 6.13), while nodes 3 through 6 were vertically restrained. The forces necessary to cause this unit vertical displacement at node 2 and to restrain the other nodes vertically were then calculated. The process was repeated for the other nodes. Note in Fig. 6.13 that a slight error is involved in the case of nodes 2, 4 and 6 since displacements or restraints are actually applicable to points on the top chord corresponding to the nodal points in the slab whereas the

nodes 2, 4 and 6 belong to the bottom chord. This was unavoidable since displacements in a truss can be applied or restrained only at nodal points.

Fig. 6.13 One-Way Floor System

6.5 Adaptation of FELARC

Some modifications of FELARC were necessary to implement in it the above procedures to treat orthotropy of masonry and one-way floor systems. The programme was modified to read the multiplication factors C_1 through C_4 for each type of orthotropic element used in the analysis as input data. These factors were then passed to the subroutine which generates the element stiffness matrices. The subroutine was modified to multiply the appropriate elements of the current constitutive matrix by those factors when it generates the stiffness matrix of the Type 2 orthotropic element or the inner layer of the Type 1 orthotropic element.

Constitutive matrices were modified only when calculating element stiffness matrices; for calculating stresses, the unmodified current constitutive matrices were used. Thus, the computed stress in the vertical direction for the inner layer of the Type 1 orthotropic element is the actual stress in the grout columns and the crosswebs of blocks, not just an average stress over the fictitious cross sectional area of that layer which also contains hollows. Similarly, the vertical stress thus calculated in a Type 2 orthotropic element represents the actual stress over the areas of the face shell, grout columns and crosswebs of blocks. But in the horizontal direction of only the inner layer of the Type 1 orthotropic element, the computed stress is fictitious because no continuous stress resisting element exists in that layer in the horizontal direction. However, this stress is inconsequential for two reasons: the horizontal stresses in most parts of the masonry walls are trivial under the loads dealt with in this study, and any cracking assumed by the programme on the basis of an overestimation of this stress reflects the actual condition of discontinuity only.

To implement the procedure for the one-way floors, the programme was modified to read the stiffnesses of the joist computed by a separate programme as input data. Appropriate modifications were made to the subroutine which assembles the global stiffness matrix so that these joist stiffnesses are added to the global stiffness matrices in the appropriate degrees of freedom.

6.6 Assumptions and Limitations

The above special approaches to deal with orthotropy of masonry and one-way floors, and also the nonlinear analysis in general, are based on some assumptions. These assumptions are summarized below:

1. The outer skins (Fig. 6.9) of concrete block masonry walls comprising of the face shells of blocks and mortar joints can be treated as homogeneous and isotropic on a macroscopic scale.
2. The modified constitutive relations of the Type 2 orthotropic element and the inner layer of the Type 1 orthotropic element are governed by the biaxial states of stress in the same manner as the constitutive relation of concrete.
3. The material constants governing the constitutive relations and thermal properties of block material (microconcrete), mortar and grout are close enough to be represented by a single set of constants.
4. The open-web steel joists are not stressed to or beyond yielding so that their stiffnesses remain constant in the entire range of loadings dealt with in this study.

Clearly, stresses emanating from the dissimilarity of mechanical properties of the block material and the jointing mortar are disregarded because of the first assumption. Such stresses in fact precipitate failure in brick masonry under axial compression; but in concrete block masonry, particularly in its outer skins, such stresses are not likely to be high because of a much smaller difference in material properties than in brick masonry.

While it is difficult to justify or discuss the implications of the second assumption, the second assumption is unavoidable unless the general approach of FELARC with respect to constitutive relationships is extensively modified. However, since orthotropy is restricted to only the inner layer of the Type 1 orthotropic element and the degree of orthotropy is not high in either type of element, and also since horizontal stresses are generally small, it is believed that this assumption will not introduce any significant error.

The third assumption is reasonable with respect to the materials typically used in concrete block masonry construction. The fourth assumption was verified after the analyses by comparing the deflections of the joists obtained from the analyses with those causing yielding.

6.7 Some Computational Difficulties

In carrying out the nonlinear analysis of the structure under study using the programme FELARC, several problems were encountered. As discussed previously in the case of linear elastic analysis, the structure is large and therefore large amounts of computer memory and time are required to analyse it. With respect to core memory requirement alone, the Honeywell CP6 mainframe computer system could not be used to run FELARC for the analysis of even a four-storey building. The core memory available in the APOLLO microcomputer system was larger than in CP6, yet the maximum number of stories of the building that could be analysed was eight. Higher buildings could be analysed using virtual memory management at the expense of a virtually impractical amount of time. It was estimated that the nonlinear analysis of even a eight-storey building would engage an APOLLO computer for hundreds of hours.

An FPS-164 attached processor was also available which used the APOLLO as host computer. The core memory available on this machine allowed a seven-storey building to be analysed requiring a CPU time of about 6 hours. Since the user load on this machine was light, it could be used for such an analysis. However, at the time of writing this thesis, this machine was still extremely unreliable in terms of continuity of functioning because of both hardware and software problems. Nevertheless, the intermittent though unpredictable availability of this machine over a prolonged period of time made several runs of the programme FELARC possible. These nonlinear analyses are presented in the next chapter.

7 Nonlinear Stress Analysis

7.1 General

In the previous chapter, an available computer programme for nonlinear stress analysis by the finite element method was adapted to the problem dealt with in the present study. In this chapter, this programme is applied to the combined gravity and thermal stress analysis of the same masonry building analysed by the linear elastic programme in Chapter 5. The outcome of the nonlinear analysis is compared with that of the linear elastic analysis and the implications of the thermal stresses and cracking on the behaviour of the building are discussed.

7.2 The Finite Element Model

The mesh of quadrilateral shell elements for a typical storey used for the nonlinear analysis is shown in Fig. 7.1. As indicated in the figure, the lintel beam was also modelled by shell elements instead of a bar element as was done in Chapter 4. The top and bottom reinforcement in the beam was represented by two uniaxial elements of steel.

A five-storied version of the building, a part plan of which is shown in Fig. 3.3, was actually designed to determine the grout spacing and reinforcements in different walls. All loads including lateral loads were considered. The required grout spacing in the exterior wall was found to be 600 mm while the minimum grout spacing of 1200 mm governed for the crosswall and the corridor wall. The minimum reinforcement of 0.2 percent of the gross cross sectional area specified by the code [15] was used. This reinforcement in each wall was distributed equally in the horizontal and vertical directions and was represented by smeared steel layers of appropriate thicknesses in the middle plane of the elements. The reinforcement in the floor slabs was also represented by two smeared steel layers at the middle plane of the slab equivalent to the reinforcement actually provided.

Since stresses in all walls and slabs are higher in the vicinity of intersection as indicated by the linear elastic analysis, it was desirable to select a graded mesh so that sizes of elements near intersections were smaller. However, this would complicate the derivation of the joist stiffnesses discussed in Sec. 6.4; therefore, a uniform mesh was selected.

The kinematic boundary conditions used for the nonlinear analysis are the same as used for linear analysis; these were shown in Fig. 3.4.

Fig. 7.1 Finite Element Mesh for a Typical Storey for Analysis by FELARC

Some of the key information about the finite element mesh is given below.

Total number of shell elements = 265

Total number of nodes = 291

Total number of unknown dof's = 1068

Number of layers in slab = 8

7.3 Material Properties and Other Data

The uniaxial constitutive relationship of concrete block masonry as a composite material is likely to be considerably different from that of concrete; particularly, unlike that of concrete, it is direction dependent. However, because of the particular way masonry has been modelled in this study, the constitutive relationship curve for masonry elements can be assumed to have the same shape as that of concrete. The likeness is justifiable if one considers the basic materials constituting masonry, viz., block material, mortar and grout as microconcrete.

The sets of constants defining the shapes of the constitutive relationships were however assumed to be different for concrete masonry and concrete. The values of these sets of constants used in this study are listed below. Some of the values such as ϵ_u , ϵ_{cu} etc. for concrete are readily available in concrete literature; others such as β and ϵ_{tu} which are peculiar to FELARC were chosen on the basis of judgment. A low value of β for masonry as compared to concrete was chosen to reflect the low reinforcement ratio and smaller aggregate size of masonry. However, several values of β ranging from 0.1 to 0.8 were tried and the results were found to be insensitive to the value of β which corroborates Lin's [61] observation as reported by Ghoneim [36]. Also, although the assumed tensile strength of masonry is only 30 percent of that of concrete, ϵ_{tu} of masonry which reflects tension stiffening was assumed the same as that of concrete; this was done to recognize the larger portions of masonry between cracks owing to thinly distributed reinforcement.

Properties of Concrete and Concrete Block Masonry:

Property	Concrete	Masonry
Ultimate compressive strength, f_c' or f_m' , MPa	20.0	12.0
Ultimate tensile strength, f_t' , MPa	2.0	0.6
Strain at ultimate compressive strength, ϵ_{cu}	0.0018	0.0015

Ultimate strain in compression, ϵ_{cu}	0.0035	0.0035
Ultimate strain in tension, ϵ_{tu}	0.001	0.001
Initial tangent modulus, E_c or E_m , MPa	22 600	12 000
Poisson's ratio, ν	0.15	0.15
Cracked shear constant, β	0.5	0.2
Coefficient of thermal expansion, α , per $^{\circ}\text{C}$	0.000014	0.00001

Properties of Steel:

Modulus of elasticity, E_s , MPa	200 000
Strain-hardening modulus, E_p , MPa	2000
Yield strength, F_y , MPa	400
Ultimate strain, ϵ_{su}	0.03
Coefficient of thermal expansion, α , per $^{\circ}\text{C}$	0.0000117

To keep execution time (see Sec. 6.7) within reasonable limits, the number of stories in the building was limited to five for the nonlinear analyses. Also, a small number of load steps was used in these analyses. Several test analyses of one and five-storey buildings using different number of load steps indicated that the results are insensitive to the number of load steps. All the gravity loads were applied in one load step; this is deemed satisfactory since the stresses due to gravity loads are quite low in most parts of the structure. The thermal loads were applied in four load steps so that the nonlinear load-deformation behaviour could be traced reasonably well.

To achieve greater accuracy, the actual temperature profiles along the interior crosswall as obtained from the thermal analysis (Sec. 2.7, Fig. 2.14) were used for the nonlinear analysis instead of the stepped profiles used in the linear analysis. Also, since the exterior wall is represented by a 3-layered shell element, the across-the-wall variation of temperature could also be taken into account by assigning different temperatures in the layers. The absolute difference between the temperatures at the exterior and interior surfaces of the exterior wall is about 3.5°C in both summer and winter (Figs. 2.11 and 2.12).

Two combinations of the gravity and thermal loads were found to be critical in Chapter 5. To repeat both combinations for the nonlinear analyses, both orientations of the one-way floor system would have to be considered. This would require a duplication of the efforts necessary to model the one-way floor system in the manner described Sec. 6.4. To reduce computational work, only the one orientation of the floor joists which is parallel to the crosswalls and gives rise to gravity loads of Case 2 was considered. Also, only the cavity type of exterior wall was analysed.

A displacement criterion was used for the convergence or divergence in the iterative solution process. A tolerance factor of 0.01, i.e., one percent, was used as a convergence criterion; for divergence, a factor of 200 was used.

7.4 Analysis for Summertime Thermal Loads

The programme FELARC outputs the nodal displacements and rotations; it also outputs the principal stresses and directions at selected Gauss points in addition to the stresses in the local coordinates. Further, it outputs a condition code for each of the Gauss points for which stresses are output to indicate whether the material has cracked, crushed, or yielded.

The vertical deflection at the top of the intersection between the crosswall and the exterior wall is plotted against the increments of the thermal loads in Fig. 7.2. Although slight, the nonlinearity of the load-deflection behaviour is clearly perceptible in this figure

Fig. 7.2 Load vs. Deflection at top of Wall Intersection from Nonlinear Analysis

The cracks that would occur in the crosswall of a five-storey building under summertime thermal loads as indicated by the nonlinear analysis are depicted in Fig. 7.3. The short lines in this figure drawn at the Gauss points of the elements indicate the approximate direction of cracking

The principal stresses and expected cracks as obtained from a linear elastic analysis of a five-storey building are shown in Fig. 7.4. From a comparison of Figs. 7.3 and 7.4 it is apparent that the cracks indicated by the linear analysis are substantiated by the nonlinear analysis. Additionally, however, the nonlinear analysis indicates a greater penetration of the cracked region particularly in the downward direction of the wall.

LEGEND:

- cracked zone
finite element boundaries
cracks

Fig. 7.3 Cracking in Crosswall under Summertime Thermal Loads indicated by Nonlinear Analysis

Fig. 7.4 Principal Stresses and Cracking in Crosswall under Summertime Thermal Loads Indicated by Linear Analysis

Although a 10-storey building could not be analysed by the nonlinear programme due to computational constraints, it is not difficult to predict intuitively the cracking in the crosswall of taller buildings from the results for the five-storey building. A comparison of Figs. 5.19 and 7.4 indicates that the zones of cracking predicted by the linear analysis are almost identical for a five and a 10-storey building with respect to the orientation of cracks and the location as well as the horizontal and vertical penetration of the cracked zone. The reason why cracks do not penetrate to a greater depth with the increase in the number of stories is as follows: the thermal strain in the exterior wall and thus the relative strain between the exterior wall and the crosswall does not change with the height of the building; therefore, the zone of cracking penetrates only up to a constant depth from the top of the building where the counteracting compressive stresses due to the overlying gravity loads reduce the thermal tensile stresses below the cracking stress. For the same reason, the zone and pattern of cracking that would be predicted by nonlinear analysis for any number of stories greater than four is expected to be similar to that indicated for a five-storey building in Fig. 7.3.

The variations of the average vertical compressive stress with increasing load at two locations of the exterior wall are plotted in Fig. 7.5. Nonlinearity in both of these plots are noticeable; however, the curve for the 5th floor location indicates a more pronounced nonlinearity than that for the ground floor location. This is because a higher degree of relief occurs in the 5th floor location due to cracking of the crosswall in the upper floors.

The relief that occurs in these stresses due to cracking and other nonlinear effects can now be calculated. From the output of the nonlinear analysis, it was observed that after the application of the first increment of thermal load no element cracked and compressive stresses were low. Therefore, the load-stress relationship during this increment of load can be reasonably assumed to be linear. The linear stress after the entire thermal load is applied can be extrapolated from an extension of this linear relationship as shown in Fig. 7.5. The difference between this stress and the corresponding nonlinear stress is a measure of the relaxation that occurs due to cracking other effects. Such differences for the 5th floor and the ground floor location are 0.21 MPa and 0.13 MPa, respectively. Expressed in percentage of the total linear thermal stress, this relaxation is about 27 percent for the 5th floor location; however, for the ground floor location where the stress is highest, it is only about 7 percent.

The vertical normal stresses in the two outer layers of the three-layered exterior wall along a vertical line 625 mm away from the intersection are plotted in Fig. 7.6. The local discontinuities at the floor levels in these stress distributions are caused by the bending moments and vertical forces transferred from the floors to the wall. It is seen that the maximum vertical compressive stress occurring at the base of the wall under the combined gravity and thermal loads is about 3.4 MPa. The vertical compressive stress at the same location due to the gravity loads only is about

1.5 MPa, as obtained from the output. Therefore, an additional compressive stress of about 1.9 MPa develops in the exterior wall of a five-storey building due to the summertime thermal loads. Note that the linear elastic analysis indicated an additional stress of about 2.1 MPa in the same location.

Fig. 7.5 Load vs. Vertical Compressive Stress in Exterior Wall from Nonlinear Analysis

From a parametric study using the linear analysis programme presented in the next chapter, it was found that the above stress for a 10-storey building could be 20 percent higher. Since the zone of cracking in the crosswall penetrates to a constant depth from the top, the relief that occurs in the vertical compressive stress at the base of the exterior wall due to cracking of the

crosswall is expected to be less in a 10-storey building than in a five-storey building. Therefore, the additional vertical compressive stress due to summertime thermal loads at the base of the exterior wall in a 10-storey building would be at least 20 percent higher than that in a five-storey building, i.e., about 2.3 MPa.

Fig. 7.6 Vertical Normal Stress Distribution along Height in Exterior Wall under Gravity plus Summertime Thermal Loads - Nonlinear Analysis

7.5 Analysis for Wintertime Thermal Loads

Unlike the analysis for summertime thermal loads, that for the wintertime thermal loads could not be carried out successfully. In spite of several attempts with various corrective measures, the programme stopped the solution process because of alleged large unbalanced forces or displacements after the first increment of the thermal loads was applied, indicating a collapse of the structure. The first increment of the wintertime thermal load, which is a quarter of the total thermal load, is sufficient to cause cracking of masonry in the exterior wall. But since the walls are reinforced in both directions, they are likely to retain sufficient stiffness after cracking to prevent a collapse. Also, after cracks are formed in the exterior wall, the thermal loads are likely to be relieved substantially. It was therefore expected that the programme would exit normally after completing the analysis for all thermal load steps.

A considerable amount of time and effort was spent to detect the cause of such an unexpected exit of the programme. Several analyses were carried out, mostly on one-storey buildings to reduce computation time, with various measures to correct suspected problems. The number of thermal load steps was increased to 16 from four. This refined load-step analysis revealed no useful information except that the programme quit at such a load step when some elements began to crack. A higher amount of reinforcement, about twice as much as actual, was also tried. To prevent any possible flexural failure of the exterior wall in the vertical direction, the vertical reinforcement was distributed equally in the outer layers although this contradicts the practicability of placing this reinforcement. Finally, a large factor of 2000 was used as the divergence criterion. However, none of these measures succeeded in making the programme continue the solution process beyond the first increment of thermal load.

The output, taken after each iteration, clearly indicates that the exterior masonry wall cracks. However, this observation from the nonlinear analysis is of little additional value since this had been earlier established by the results of the linear analysis. It was most desirable from the nonlinear analysis to know how much relief occurs in the high compressive stress indicated by the linear analysis at the upper floors of the crosswall. Also, it was of interest to know whether the roof slab actually cracks in winter as indicated by the linear analysis.

The analytical attempt to come up with quantitative measures of these stresses and cracks having failed, a qualitative estimate can be obtained intuitively. As the temperature starts going down in winter, tension develops in the exterior wall. At a certain temperature, this tension exceeds the tensile strength of masonry along most of the height of that wall; thus cracks develop, principally in the horizontal direction as the analyses suggest. After these cracks develop, the axial stiffness of the wall is likely to be drastically reduced and hence the thermal loads are likely to be relieved substantially; for, the only sources of the stiffness that remain are

the nominal amount of reinforcement and the tension-stiffening effect of masonry between cracks. Therefore, while the exterior wall itself is likely to crack extensively along most of its height, it is unlikely that any significant unbalanced thermal load is left to cause any significant compression in the crosswall or cracking in the roof slab.

7.6 Adequacy of Linear Elastic Analysis

The linear elastic analysis discussed in Chapters 5 and 6 was based on several assumptions. Nonlinearity and orthotropy of materials were ignored; bending was disregarded so that the walls and slabs could be modelled by plane stress elements; the temperature profile along the crosswall was assumed as a stepped function; also, the analysis was carried out using a substructuring technique based on certain simplifying assumptions (see Subsection 4.4.3).

On the other hand, the nonlinear stress analysis presented above was carried out simulating actual material, structural, and loading conditions as closely as possible. Material nonlinearity, including cracking, was considered; orthotropic behaviour of concrete block masonry was reflected by a theoretical model; the actual temperature profile along the crosswall as suggested by a thermal analysis was used; and the analysis was done by the direct finite element method which involved no simplifying assumption as to the interconnection of the structural members. However, as a rule to which the analysis of the present problem poses no exception, the nonlinear analysis is more complex, more time-consuming, and hence more expensive, than linear analysis. Therefore, it is useful to examine the adequacy of linear analysis for the present problem.

It has already been demonstrated in Chapter 4 that the assumptions on which the substructure analysis was based and the assumption of negligible bending in the walls and slabs introduce no significant error in the key stresses and displacements. Any deficiency of the linear elastic analysis due to the combined effect of these assumptions together with the neglect of orthotropy of concrete block masonry can also be determined from the results of the nonlinear analyses presented in Sec. 7.4. The average additional compressive stress in the exterior wall at the ground floor location (Fig. 7.6) due to summertime thermal loads as computed on the basis of linear extrapolation is about 1.9 MPa; the linear elastic analysis indicates about 2.0 MPa. This indicates that in spite of the above assumptions, the linear analysis can predict the key stresses with good accuracy. Also, the nonlinear analysis closely substantiates the orientation and zone of cracking predicted by the linear analysis as already observed in Sec. 7.4. The relaxation that occurs in the maximum compressive stress in the exterior wall due to cracking of the crosswall and other nonlinear effects is only 7 percent. Hence, it can be concluded that the linear analysis can predict the key thermal stresses and cracking with a reasonable degree of accuracy that is

adequate for the present problem.

7.7 Adverse Thermal Effects on Buildings

From the linear and nonlinear stress analysis presented above, it has been established that in typical buildings of loadbearing masonry wall construction in which interior crosswalls intersect exterior walls, the crosswall may undergo some cracking at the upper floors and the exterior wall may develop an additional compressive stress of about 2.3 MPa under summertime thermal loads. In winter, the exterior wall may develop horizontal cracks distributed along the entire height of the building. These analyses were carried out for the cavity type of exterior wall only; for single wythe exterior walls which are subjected to higher temperature differentials in both seasons, these thermal cracking and overstressing effects are obviously more severe. The possible adverse effects of these thermal cracks and stresses on the serviceability and structural performances of typical buildings will now be examined.

The maximum compressive stress of 2.3 MPa that occurs due to thermal load alone in the exterior wall of a 10-storey building is about 20 percent of the ultimate strength of masonry of 12.0 MPa. The code [15] allows an axial compressive stress of 22.5 percent of the ultimate strength for reinforced concrete block masonry construction. Therefore, the additional thermal compressive stress reduces the factor of safety built into the code provisions by about 50 percent, from about 4.5 to about 2.35.

The thermal cracks appear to have more serious implications. At best, these cracks are likely to create serviceability problems; at worst, they may pose as safety-related problems. The serviceability problems created by moisture penetration and air leakage through cracks and other permeable paths in an exterior wall are well known [34, 47, 60, 77]. The thermal cracks in the exterior wall provide additional paths to those commonly existing in unit masonry construction. In winter, when the thermal cracks are open widest, moisture in the exfiltrating air is likely to condense and freeze in the exterior wall unless adequate vapour barriers are provided between the interior and the wall. In early spring or late fall, moisture or water penetrating into the cracks from the outside may also freeze. The alternate freeze-thaw action over a prolonged period can lead to the deterioration of masonry. Moreover, air leakage promoted by these cracks increases heating and cooling expenses.

The thermal cracks may also adversely affect the capacity of the walls to resist loads, particularly lateral loads. Castellani and Chesi [16] investigated the effects of cracking, both numerically and experimentally, on two categories of reinforced concrete structures: shear walls of buildings and cylindrical containment structures. On the basis of their numerical model, they reported that a single vertical crack in a shear wall may reduce its lateral rigidity by about 10 to 20

percent depending upon the crack width. Their experimental investigation showed that the lateral rigidities of horizontally cracked concrete cylinders could be as low as 70 percent of the uncracked cylinders. They also reported from a limited set of tests that a further reduction of the rigidity by about 10 percent may occur due to cycling of the lateral loads. In cracked masonry walls also, similar losses of rigidity are likely to occur. Rather, since masonry walls are lightly reinforced and the aggregate interlocking effect in masonry construction is weaker, these losses are likely to be higher. Because of widely distributed reinforcement, the spacing of thermal cracks may be large resulting in large crack widths. Above all, the thermal load is cyclic; temperature in the exterior wall fluctuates over a 24-hour period between daily extremes which very often reverses the temperature differential between the exterior and the intersecting interior crosswall. A progressive degradation of the rigidity of masonry walls is therefore expected due to this cycling effect. Since the crosswall cracks only partially, it may not lose rigidity substantially. However, the exterior wall is likely to lose lateral rigidity by a considerable amount since horizontal cracks all across the thickness and width, and distributed all along its height, occur in that wall. Thus, in buildings or parts of buildings in which such walls are designed to resist lateral loads, the implication of the cracks evidently is significant.

Although, for the purpose of this study, the wintertime temperature differentials in typical Canadian climatic conditions were considered, it should be noted that only about a quarter of this temperature differential is required to form cracks in the exterior wall. Therefore, such cracks, but presumably of lesser widths, are likely to form in buildings located in regions of milder climates around the world. Needless to mention that the summertime temperatures in many parts of the world are much higher than in Canada. Also, no cladding is used in masonry building construction in many of those regions, particularly in underdeveloped countries. The margin of safety against compression failure of the exterior wall in such buildings is likely to be reduced by an even greater magnitude.

8

Parametric Study

8.1 General

In the previous chapters, it has been established that buildings of loadbearing masonry wall construction in which interior crosswalls are rigidly connected to exterior walls may be adversely affected by thermal loads either through cracking of walls and roof slabs, or through compressive overstressing of the walls, or both. In order to take these thermal effects into design consideration, it is necessary to know the factors which govern, and also how they govern, the thermal stresses. Some of these factors have been identified and their relationships with the thermal stresses have been studied. This chapter deals with this study.

8.2 Thermal Stress Parameters

The following parameters, which are in no way exhaustive, were selected on an intuitive basis for this parametric study:

1. Number of stories, i.e. height of building
2. Width of exterior wall
3. Relative strength of crosswall
4. Relative area of cross section of crosswall
5. Area of cross section of floor slabs

The structural designer may have little choice over the first parameter, i.e. the number of stories in the building, which is mainly dictated by architectural considerations. Although the second parameter, width of exterior wall, is dictated by the requirements of windows and other openings and other architectural considerations, it may be possible to adjust it to some extent. The structural designer has much greater control over the last three factors.

The term 'relative' as used in parameters 3 and 4 needs some clarification. What is meant in parameter 3 is the strength of masonry in a crosswall as compared to the strength of masonry in the exterior wall. In other words, the ratio of the strength of masonry in a crosswall to that in the exterior wall has been implied. Similarly, in parameter 4, the ratio of the cross sectional area per unit length of the crosswall to that of the exterior wall has been suggested. The reason behind selecting relative strength or relative cross sectional area will now be explained. Unlike other

loads, such as gravity and wind loads, thermal loads in a structure are geared to the elastic properties, such as the modulus of elasticity, of the material. Since the modulus of elasticity of masonry is directly proportional to its strength (see Eq. 3.1), it is obvious that within the elastic limit any increase in strength is totally offset by an increase in thermal stresses. Hence, a general increase in the strength of masonry of all the walls is of no importance to the designer as a measure to control the thermal stresses. A similar argument also holds true for the area of cross section. Only a relative change in the strength or cross sectional area of intersecting walls can bring about some relief of some of the thermal stresses.

In each of the next five subsections the relationships between some key thermal stresses and the above parameters as determined from several series of analyses are presented.

8.2.1 Number of Stories

In order to ascertain the relationship between the number of stories in a building and the key thermal stresses, a series of analyses was carried out for thermal loads only with the number of stories varying as 1, 3, 5, 7, and 10. The same building layout as shown in Fig. 3.3 was used except that the exterior flange width was 2.1 m instead of 2.5 m. Other pertinent data are as given in Subsection 5.2.1 with the following exceptions:

Effective wall thickness (all walls) = 190 mm

Modulus of elasticity of block masonry = 10 000 MPa

Modulus of elasticity of reinforced concrete = 20 000 MPa

An arbitrary temperature differential of 40 °C between the exterior wall and the interior crosswall was used in these analyses.

The results of these analyses are presented in Fig. 8.1 wherein the key thermal stresses in the crosswall and the exterior wall have been plotted against the number of stories. These key stresses, which are the maximums of the respective kind, have been normalised with respect to the corresponding stresses in a one-storey building. The locations of these key stresses have been indicated in the inset which remain the same as the number of stories is varied. Also listed as reference stresses are the magnitudes of the key stresses in a one-storey building.

It is observed that the most significant stress in the crosswall, σ_{yy} , remains unchanged with the increase in number of stories. This is explained by the fact that at the top of the building neither the relative thermal deformation between the intersecting walls nor the stiffness of these walls in the vertical direction change with the change in number of stories. But the maximum normal stress in the horizontal direction, σ_{xx} , in the crosswall increases at a decreasing rate with number of stories and levels off at seven stories at a magnitude which is about 47 percent higher

than that in a one-storey building.

The normalised vertical stress, σ_{yy} , in the exterior wall that has been plotted in Fig. 8.1 is not a maximum. The maximum σ_{yy} occurs at the foundation and is obviously influenced by the degree of restraint in the horizontal direction at the base. To eliminate this influence, σ_{yy} at Level 1 which is close to that at the foundation in magnitude has been selected as the reference stress. It is observed that this stress varies significantly with number of stories particularly for buildings up to seven stories. For a building higher than 10 stories, the increase in this stress with the increase in number of stories is insignificant as indicated by the trend of the curve.

Fig. 8.1 Relationships Between Key Thermal Stresses and Number of Stories in the Building

8.2.2 Width of Exterior Wall

A series of analyses was carried out to evaluate the effect of varying widths of exterior wall on the key thermal stresses. Again, the same building layout as given in Fig. 3.3 was used except that the width of exterior wall was varied as 2.1, 3.0 and 4.2 meters. Although ten would be a more desirable number of stories since it would be more representative of typical buildings, the number of stories for this series of analyses was chosen as five in order to reduce the volume of computation. Nevertheless, it is believed that the trend shown by the present analyses would

apply equally well to any number of stories within the common range for such buildings. Other data for the analyses are the same as given for parameter 1.

Fig. 8.2 shows the results of these analyses. The key stresses have been normalised with respect to the stresses corresponding to an exterior wall width of 2.1 m. The locations of the key stresses are shown in the inset. The reference stresses shown are those for an exterior wall width of 2.1 m.

Fig. 8.2 Relationships Between Key Thermal Stresses and Width of Exterior Wall

Again, it is observed that the maximum vertical normal stress, σ_{yy} , which occurs at the top exterior corner of the crosswall, remains unaffected by the variation in the width of exterior wall. However, σ_{yy} at Level 1 increases almost linearly with the increase in the width of exterior wall. This is explained as follows: the thermal shear force transferred at the top of the crosswall from the exterior wall does not depend on the width of exterior wall since the latter is completely free to deform in the vertical direction at that level except at the node common to the crosswall; but at points below, such as at Level 1, the sum total of the thermal shear forces transferred to the crosswall above that point is governed by the width of exterior wall.

The maximum horizontal normal stress, σ_{xx} , in the crosswall increases more or less linearly with the increase in the width of exterior wall. The rate of this increase is significant; for a 100 percent increase in the width of exterior wall, σ_{xx} increases by about 50 percent. All other factors remaining constant, this stress is a function of the total shear force transferred to the crosswall from the exterior wall. Since the latter is, in turn, a function of the width of exterior wall, the above observed relationship was qualitatively predictable.

The most significant stress, σ_{yy} , at Level 1 of the exterior wall is observed to decrease slightly with the increase in the width of exterior wall. As the width of exterior wall is increased a larger amount of vertical shear is transferred to the crosswall and consequently a larger deformation in the vertical direction at the interface can occur. This, in turn, relieves the vertical thermal stresses in the exterior wall by a higher amount and thus explains the above observation.

8.2.3 Relative Strength of Masonry in Crosswall

For the study of this parameter, a series of three analyses was carried out. The strength of masonry in the crosswall was varied as 10.0, 12.5 and 15.0 MPa, and all other conditions were the same as for parameter 2. The results of these analyses are plotted in Fig. 8.3. The stresses have been normalised with respect to the corresponding stresses for f'_m equal to 10.0 MPa.

These plots indicate that the horizontal normal stress, σ_{xx} , at the roof level of the crosswall increases in about the same proportion as the increase in the ratio of the masonry strength of crosswall to that of exterior wall. The maximum vertical normal stress, σ_{yy} , also increases with the above strength ratio but at a lower rate. Since a stronger crosswall also means a stiffer crosswall, it will attract more thermal shear force from the exterior wall. Therefore, it is no surprise that stresses in the crosswall will also be higher. Fig. 8.3 further indicates that σ_{yy} in the exterior wall also increases by about 11 percent for a 50 percent increase in the strength of crosswall. This increase is explained by the fact that less vertical deformation at the intersection and hence less relaxation of the thermal stresses in the exterior wall is possible as the crosswall becomes stiffer.

8.2.4 Relative Area of Cross Section of Crosswall

In this series of analyses only the area of cross section of the crosswall per unit length of wall was varied by varying the effective thickness of the wall. In actual construction, the effective thickness of the walls can be varied by varying any or all of the following: number of grouted cores, percentage of reinforcement, and gross or net thickness of blocks. Three ratios of crosswall to exterior wall thickness were tried in this study.

The normalised key stresses are plotted against the ratio of cross sectional area in Fig. 8.4. An increase in the area of cross section of the crosswall makes the wall stiffer and thus causes a greater amount of thermal shear force to be transferred to the crosswall from the exterior wall. However, this increased thermal load on the crosswall is more than offset by the increased cross sectional area and hence the stresses in the crosswall are expected to decrease. This is indeed indicated in Fig. 8.4. The maximum vertical stress, σ_{yy} , in the crosswall decreases by about 42 percent and the maximum horizontal stress, σ_{xx} , in the same wall decreases by about 32 percent for a 50 percent increase in the cross sectional area of the crosswall. Since relative stiffness of the crosswall has the same linear relationship with the strength of masonry and the area of cross section, it is no surprise that σ_{yy} in the exterior wall has the same quantitative relationship with the area ratio (Fig. 8.4) as it has with the strength ratio (Fig. 8.3).

Fig. 8.3 Relationships Between Key Thermal Stresses and Relative Strength of Crosswall

8.2.5 Area of Cross Section of Floor Slabs

A variety of floor types is used in buildings of loadbearing wall construction. The net thickness of the floor slabs, usually reinforced or prestressed concrete, varies from 64 mm for a

one-way system to 200 mm for a two-way system. In order to determine the influence of the area of cross section of floor slabs on the thermal stresses in the walls and slabs, a series of analyses was carried out with slab thickness varying from zero to 200 mm keeping the elastic properties of the slab constant. Since the horizontal normal stress, σ_{xx} , in the crosswall at the roof level, which is likely to be influenced most by slab thickness, varies significantly with height of building up to 10 stories, the number of stories for this series of analyses was selected as ten. Other data are the same as those for parameter 1.

Fig. 8.4 Relationships Between Key Thermal Stresses and Relative Cross Sectional Area of Crosswall

The results of these analyses are presented in Fig. 8.5 where in the most significant stress, σ_{xx} , in both crosswall and roof slab and the maximum σ_{yy} in the crosswall are plotted against slab thickness. The stresses have been normalised with respect to the corresponding ones for a slab thickness of 67.5 mm, the transformed thickness of a reinforced concrete slab of 64 mm nominal thickness.

The trend of the curve for σ_{xx} in the crosswall indicates that this stress decreases significantly with the increase in slab thickness. For example, it decreases by about 50 percent as the slab thickness is increased from 67.5 mm to 150 mm. A similar trend is indicated by the maximum σ_{xx} in the roof slab.

It is also observed that the maximum σ_{yy} in the crosswall remains unchanged as the slab thickness varies. A similar effect was observed for σ_{yy} in the exterior wall. Since slab thickness does not affect the vertical stiffnesses of the structure and affects the horizontal stiffnesses only locally at the floor levels, these observations were predictable.

Fig. 8.5 Relationships Between Key Thermal Stresses and Thickness of Floor Slabs

8.3 Discussion

In the range of the building materials, type of construction, and architectural aspects common in practice, most of the parameters dealt with above are believed to be uncoupled. This implies that the combined effect of varying two parameters simultaneously can be obtained by superimposing the individual effects of the parameters. For example, if the width of exterior wall is increased from 2.1 m to 4.2 m and the strength of masonry in the crosswall is increased from 10.0

MPa to 12.5 MPa, then it can be said that the combined effect on the maximum horizontal normal stress, σ_{xx} , at the roof level of the crosswall is an increase by $(1.5 \times 1.27 - 1.00) \times 100$ or 90 percent. This is because the increases in the above stress caused individually by the above parameters are 50 percent and 27 percent, respectively.

The prospects of using the above parameters to control the thermal effects on a building will now be discussed. It has already been mentioned that the structural designer is not likely to have much control over the number of stories in the building because of the overriding importance of other factors. However, it is useful to know that in buildings taller than 10 stories, all the key thermal stresses are virtually unaffected by the increase in number of stories while some of these stresses increase significantly with the increase in number of stories up to 10 stories.

The parameter which appears most promising as a means to control the thermal effects is the width of exterior wall. For a zero width of exterior wall, i.e. when there is no exterior wall rigidly connected to the interior crosswall, there is no thermal load and hence no thermal stress in the building. Therefore, eliminating a return at the exterior end of the crosswall altogether would be the best solution. If required for reasons such as stiffening the crosswall, keeping the width of exterior wall as small as possible would be the second best solution.

Since increasing the strength of the crosswall relative to that in the exterior wall effects a less than proportionate increase in the most significant stress, σ_{yy} , in the crosswall (Fig. 8.3), it will reduce cracking in summer and compressive overstressing in winter in the crosswall. However, σ_{xx} in the crosswall increases slightly more than proportionately, and σ_{yy} in the exterior wall, which is already severe, increases further. Depending upon the relative importance of the above stresses, this parameter may therefore be used in a rather limited way to control the thermal effects.

Similarly, increasing the area of cross section of the crosswall significantly reduces both σ_{xx} and σ_{yy} in the crosswall; however, it also increases the already critical stress σ_{yy} in the exterior wall (Fig. 8.4). Therefore, increasing the area of cross section of the crosswall will have the same limitation as the strength of masonry in the crosswall as a means to control thermal effects.

Although increasing the slab thickness has virtually no effect on σ_{yy} in the walls, it significantly reduces σ_{xx} in both the crosswall and roof slab (Fig. 8.5). Therefore, where these stresses need to be controlled, use of thicker floor slabs appears to be a good solution.

9 Summary and Conclusions

9.1 General

In the introductory chapter of this thesis, the potential of some distress in buildings of loadbearing wall construction due to differential movements between walls rigidly bonded at their intersection was described. Two factors which may induce the relative deformation at intersections of the exterior and interior walls were identified. These are differential temperature and differential gravity load.

A review of the available literature showed that little work had been done to investigate the above potential of distress in intersecting exterior and interior walls. Such intersections of exterior and interior walls are commonly present in buildings of loadbearing masonry wall construction and therefore the need to investigate quantitatively this potential of distress in masonry buildings was established. This work was undertaken with the objectives to estimate the stresses in intersecting masonry walls caused particularly by differential temperature, to identify and study the parameters influencing these stresses, and to examine the implications of these stresses on the structural and serviceability behaviour of the walls.

A summary of the important phases of this research work and important conclusions drawn thereby are given in the following sections.

9.2 Thermal Analysis

To estimate the stresses due to differential temperature, the ranges of temperature to which the intersecting walls are subjected were required to be known. In the absence of such information, thermal analyses were carried out to determine the extreme temperatures in the walls. These thermal analyses, presented in Chapter 2, were performed for both cavity and single wythe type of exterior walls on the basis of both steady state and transient heat flow conditions. All climatological factors were considered and data on such factors and material properties were used as available in the literature, supplemented by studies of those parameters whose influences on the wall temperatures were not clearly or readily known.

The following conclusions were derived from the thermal analyses:

1. At latitudes such as of Ottawa, maximum mean temperatures in the loadbearing

concrete block wall for both cavity and single wythe walls are reached for wall orientations approximately West-Southwest; however, maximum surface temperatures in both types of walls are reached in westfacing walls.

2. The maximum mean temperatures in the loadbearing concrete block wall are relatively insensitive to the solar absorptivity of the exterior surface. For example, if solar absorptivity of brick masonry for a cavity type of exterior wall is decreased from 0.85 to 0.70, the maximum mean temperature in the block wall decreases by only about 5 percent.
3. Steady state heat flow analysis significantly overestimates the summertime maximum mean temperatures in the loadbearing block wall of both cavity and single wythe type of exterior walls. For a typical building located in Ottawa, Canada, this overestimation is by about 27 °C for the cavity wall and 25 °C for the single wythe wall, as compared to temperatures indicated by transient analysis. Steady state analysis underestimates wintertime minimum mean temperatures in the same walls, but by relatively modest amounts, 7 °C for the cavity wall and 5 °C for the single wythe wall.
4. In a typical building located in Ottawa, summertime maximum mean temperatures in the loadbearing concrete block wall may be as high as 41.3 °C in a brick-clad exterior wall, and 48.5 °C in an unclad single wythe exterior wall; the wintertime minimum mean temperatures in the same walls may be as low as -18.7 °C and -25.9 °C, respectively.
5. The summertime maximum temperature in the exterior surface may be as high as 64 °C in a brick-clad exterior wall and 57 °C in a single wythe unclad concrete block wall; the wintertime minimum temperatures at those surfaces may be as low as -30 °C and -28 °C, respectively.
6. The temperature profile along the horizontal axis of an interior crosswall intersecting with an exterior wall can be assumed as a stepped function for the purpose of stress analysis.

9.3 Stress Analysis

To determine the stresses, the finite element method appeared to be the best available analytical tool. However, no suitable computer programme was readily available to solve the large-size problem using the computing facilities available at the time of undertaking this study. A linear elastic finite element stress analysis programme was therefore developed using a special substructuring technique and constant strain triangular plane stress elements. With respect to computational efficiency without any significant loss of accuracy for the present problem, this programme was demonstrated to be far superior to direct finite element programmes such as the

one used for the nonlinear analysis in the later part of this study.

Using the above programme, stress analyses of a typical 10-storey building were carried out for differential gravity and thermal loads. Since a one-way floor system is common in practice and since the floors can span along any principal direction of the building, both orientations of the floor joists were considered. Also, since the temperature differential between the exterior and interior walls that develops in summer is reversed in winter, both summertime and wintertime thermal loads were considered. The gravity and thermal loads were then combined to produce the worst conditions. These analyses revealed that under summertime thermal loading, the interior crosswall may develop some diagonal cracks near the intersection with the exterior wall at the upper floors while the exterior wall may undergo some compressive overstressing. The analyses also indicated that under wintertime thermal loading, the exterior wall may crack horizontally along almost its entire height while the crosswall may be subjected to some compressive overstressing at the upper floors; also, the roof slab may develop some cracks.

But cracking and compressive overstressing beyond the elastic limit are expected to relieve the thermal stresses to some extent. Therefore, the extent of cracking and the magnitudes of compressive stresses predicted by linear elastic analysis are likely to be inaccurate. Considering this limitation of linear elastic analysis, a nonlinear postcracking analysis by the finite element method was also carried out. An available nonlinear analysis programme written to analyse reinforced and prestressed concrete structures was modified to implement an analytically developed orthotropic model of concrete block masonry. The one-way floor system was also modelled by a special technique similar to substructuring to reduce the computational load and thus solve the structure in an efficient manner. The nonlinear analysis substantiated the key findings of the linear analysis. It was observed that the effects of environmental thermal loads on typical buildings can be determined by the linear analysis with an accuracy that is acceptable for all practical purposes.

Based on the above linear and nonlinear analyses, the following conclusions were derived:

1. Under typical Canadian climatic conditions in summer, and in buildings having interior walls intersecting with exterior walls, the interior wall may develop some cracks in a more or less diagonal direction near the intersection with the exterior wall in the upper three or four floors.
2. Under the same conditions as above, the loadbearing concrete block wall of an exterior brick-clad cavity type of wall may develop an additional compressive stress of about 2.3 MPa in the vertical direction; such a stress in an unclad exterior concrete block wall is likely to be about 3.0 MPa. This additional compressive stress reduces the

factor of safety against compression failure substantially, by about 46 percent in the cavity wall and about 53 percent in the single wythe wall.

3. In winter, the exterior wall is likely to develop horizontal cracks extended all across the width and thickness and distributed all along the height of the wall.
4. A linear elastic analysis using plane stress elements can adequately predict the thermal cracking and overstressing effects for the present problem.

The possible adverse effects of thermal cracks on buildings were examined. These cracks were found to have the potential to give rise to serviceability problems, such as moisture wetting and degradation by freeze-thaw action, and safety-related problems, such as loss of lateral rigidity.

9.4 Parametric Study

Having substantiated the outcome of the linear elastic analysis by the nonlinear analysis, the linear analysis programme was again used to determine how the thermal stresses are influenced by certain parameters. The parameters considered are, number of stories, i.e., height of building, width of exterior wall, strength and cross sectional area of the crosswall relative to the exterior wall, and cross sectional area of floor slabs. The most important conclusions are listed below:

1. The key thermal stresses which signify the overall effects of environmental thermal loads on typical buildings remain unchanged with the increase in number of stories above 10 stories. The thermal vertical stress in the exterior wall increases significantly with the number of stories up to 10 stories although the vertical stress in the crosswall, which causes cracks in summer, remains unchanged.
2. The width of the exterior wall is the key factor in that the wall in itself is the source of the thermal load when rigidly bonded with the interior wall. The wider the exterior wall, the deeper the cracked zone penetrates in the interior wall in summer; however, the concomitant compressive stress in the exterior wall itself becomes slightly smaller.
3. Using a higher strength masonry or a thicker cross section for the interior crosswall reduces the possibility of cracking or overstressing in the interior crosswall, but increases the vertical thermal stress in the exterior wall. However, these effects are moderate only; for example, a 50 percent increase in the relative strength of crosswall reduces the possibility of cracking in that wall by about 15 percent and increases the vertical normal stress in the exterior wall by about 11 percent.

9.5 Recommendations

9.5.1 Remedial

The potential problems created by temperature differentials between rigidly connected exterior and interior walls of typical loadbearing masonry buildings have been addressed in this work. Quantitative measures of these problems have been derived. Two remedies, which are somewhat obvious, are suggested.

The exterior wall rigidly connected to the interior wall is in itself the source of the thermal load. Therefore, elimination of a return at the end of the interior wall altogether appears to be the best solution, provided it is permissible by other constraints, such as stiffening of the wall. This is possible by shifting the openings in the exterior wall so that they are contiguous with the exterior crosswall.

Another alternative is to place the insulation outside the loadbearing block wall, in the cavity between the cladding and the loadbearing wall. This will significantly reduce the temperature differential between the exterior and interior walls. In fact, such a placement of the insulation, suggested by Hutcheon [46], reportedly also reduces the possibility of degradation caused by wetting and wet-freezing. However, in the single wythe type of exterior wall, it is impractical to place the insulation outside. Moreover, the placement of the insulation in the cavity of the cavity type of wall dramatically increases the temperature differential between the cladding and the back-up wall. The differential movement which may ensue may lead to serious problems in the cladding [28, 31, 55, 70, 71, 86] unless special detailing is provided.

9.5.2 Future Work

This work was limited to the consideration of thermal load as applied one time for a short duration. In other words, cycling of the thermal loads and time-dependent properties of masonry, such as creep, were not considered. However, creep is not likely to introduce any significant relaxation in the thermal stresses, since it does not take more than a few weeks for the temperature differential to reach its extreme from a neutral level. Nevertheless, creep and cycling of the thermal loads may be included in future works to further investigate thermal problems in buildings. Also, shrinkage and moisture-induced deformations may be coupled with thermal loads for a more comprehensive study.

The analytical approach used in this study to model the orthotropic behaviour of concrete block masonry walls needs to be verified experimentally.

The possible loss of lateral rigidity of concrete block masonry walls because of cracks caused by thermal loads such as indicated in this study should also be investigated.

Appendix A

Sol-Air Temperature

Sol-air temperature can be considered as the temperature reached by a surface irradiated by solar radiation and which is not losing heat by conduction. In other words, it is the fictitious air temperature which will cause the same amount of heat flow into an irradiated surface by surface conduction as would be caused by the radiation. The surface conductance, h_o , is a combined coefficient equal to the sum of convection coefficient h_c and radiation coefficient h_r of the air film:

Fig. A.1 Heat Flow at an Irradiated Surface

Heat balance equation for the above surface can be written as

$$q/A = \alpha I_t - h_c(t_s - t_o) - \epsilon \sigma T_s^4 + \epsilon I_l \quad (A.1)$$

where, q/A = heat flow rate across the surface by conduction, W/m^2

T_s = absolute temperature of the surface, $^{\circ}K$

t_s = surface temperature, $^{\circ}C$

t_o = air temperature, $^{\circ}C$

I_t = total solar radiation incident on the surface, W/m^2

I_l = total long wave radiation incident on the surface, W/m^2

h_c = convective heat transfer coefficient, $W/m^2 \text{ } ^{\circ}C$

α = solar absorptivity of the surface

ϵ = emissivity of the surface

σ = Stefan-Boltzman constant

Also, $q/A = h_o (SAT - t_s)$ as per definition of sol-air temperature.

3

OF/DE

3

1.0

1.1

1.25

1.4

1.6

1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0

2.8

2.5

3.2

2.2

3.6

2.0

4.0

1.8

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

where, SAT = sol-air temperature

$$h_o = h_c + h_r \quad (A.2)$$

Therefore,

$$h_o (SAT - t_s) = \{ \alpha I_t - \epsilon (\sigma T_s^4 - I_l) \} - h_c (t_s - t_o) \quad (A.3)$$

which can also be written as,

$$h_o (SAT - t_s) = \alpha I_t + \{ \epsilon \sigma (T_o^4 - T_s^4) + \epsilon (I_l - \sigma T_o^4) \} + h_c (t_o - t_s) \quad (A.4)$$

h_r is obtained from the following equation which governs the radiation exchange between a blackbody and its surroundings;

$$h_r (t_o - t_s) = \epsilon \sigma (T_o^4 - T_s^4) \quad (A.5)$$

Using the above relationship in Eq. A.4 and substituting $(I_l - \sigma T_o^4)$ by ΔR , we get

$$h_o (SAT - t_s) = \alpha I_t + (h_r + h_c) (t_o - t_s) - \epsilon \Delta R$$

Again, using Eq. A.2 in the above relationship, we get

$$h_o (SAT - t_s) = \alpha I_t + h_o (t_o - t_s) - \epsilon \Delta R$$

Dividing throughout by h_o , and reducing, we get

$$SAT = t_o + (\alpha I_t - \epsilon \Delta R) / h_o \quad (A.6)$$

Sol-air temperatures based on the above equation with $\alpha = 0.9$ and for Ottawa area (Latitude = 45° N) are listed in Table 9.7 of reference 27.

Appendix B

B.1 Conductance of Air Films

Conductance of air films, f , is given by

$$f = h_c + h_r \quad (B.1)$$

where, h_c = convective heat transfer coefficient

h_r = radiative heat transfer coefficient

Outside Air Film:

From Table 8.7 of Reference 27,

$$h_c = 5.62 + 3.91V \quad V < 5 \text{ m/sec} \quad (B.2)$$

With wind velocity, V equal to 1 m/sec in summer and 2 m/sec in winter,

$$h_c = 5.62 + 3.91 \times 1 = 9.53 \text{ W/m}^2 \text{ } ^\circ\text{K} \quad \text{in summer}$$

$$h_c = 5.62 + 3.91 \times 2 = 17.35 \text{ W/m}^2 \text{ } ^\circ\text{K} \quad \text{in winter}$$

In summer, ambient air temperature = $38^\circ\text{C} = 311^\circ\text{K}$

From Table 9.7 of Reference 27, after modifying for solar absorptivity, $\alpha = 0.85$ for brick masonry and $\alpha = 0.75$ for block masonry,

$$\begin{aligned} (\text{Sol-air temperature} - \text{ambient air temperature}) &= 38.7^\circ\text{C} \text{ for brick masonry} \\ &= 34.2^\circ\text{C} \text{ for block masonry} \end{aligned}$$

Assuming a temperature drop of 3°C across the air film,

$$\text{Approximate brick surface temperature} = 273 + (38.0 + 38.7) - 3.0 = 346.7^\circ\text{K}$$

$$\text{Approximate block surface temperature} = 273 + (38.0 + 34.2) - 3.0 = 342.2^\circ\text{K}$$

In winter, ambient air temperature = $-36^\circ\text{C} = 237^\circ\text{K}$

Again assuming a temperature rise of 3°C across the air film,

Approximate surface temperature for

$$\text{both brick and block masonry} = 273 + (-36.0) + 3.0 = 240^\circ\text{K}$$

Radiative heat transfer coefficients can now be determined using Eq. 8.10 of Reference

27 which is given as

$$h_r = \epsilon \sigma (T_s^2 + T_a^2) (T_s + T_a) \quad (B.3)$$

where, ϵ = emissivity of the material

σ = Stefan-Boltzmann constant, $5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \cdot \text{K}^4)$

T_a = ambient air temperature, $^{\circ}\text{K}$

T_s = surface temperature, $^{\circ}\text{K}$

Therefore,

In summer

$$\begin{aligned} h_r &= 0.9 \times 5.67 \times 10^{-8} \{ (349.7)^2 + (346.7)^2 \} \{ 349.7 + 346.7 \} \\ &= 8.61 \text{ W}/(\text{m}^2 \cdot \text{K}) \text{ for brick} \end{aligned}$$

$$\begin{aligned} h_r &= 0.9 \times 5.67 \times 10^{-8} \{ (345.2)^2 + (342.2)^2 \} \{ 345.2 + 342.2 \} \\ &= 8.29 \text{ W}/(\text{m}^2 \cdot \text{K}) \text{ for block} \end{aligned}$$

In winter

$$\begin{aligned} h_r &= 0.9 \times 5.67 \times 10^{-8} \{ (240.0)^2 + (237.0)^2 \} \{ 240.0 + 237.0 \} \\ &= 2.77 \text{ W}/(\text{m}^2 \cdot \text{K}) \text{ for both brick and block} \end{aligned}$$

Conductances for the outside air film, f_o , are therefore computed as follows:

In summer

$$f_o = 9.53 + 8.61 = 18.14 \text{ W}/(\text{m}^2 \cdot \text{K}) \text{ for brick}$$

$$f_o = 9.53 + 8.29 = 17.82 \text{ W}/(\text{m}^2 \cdot \text{K}) \text{ for block}$$

In winter

$$f_o = 17.35 + 2.77 = 20.12 \text{ W}/(\text{m}^2 \cdot \text{K}) \text{ for both brick and block}$$

Inside Air Film:

From Table 8.7 of Reference 27, the convective heat transfer coefficient, h_c , at an indoor wall surface is given by

$$h_c = 1.87 (\Delta t)^{0.32} H^{-0.05} \quad (B.4)$$

where, Δt = temperature drop across the air film

H = height of the wall

Assuming $\Delta t = 5^{\circ}\text{C}$ and a 3 m-high wall,

$$h_c = 1.87 (5.0)^{0.32} \times 3^{-0.05} = 2.96 \text{ W}/(\text{m}^2 \cdot \text{K})$$

The inside air temperature is 20 °C or 293 °K. With the assumed temperature drop of 5 °C across the air film, the inside surface temperature is 298 °K in summer and 288 °K in winter.

Therefore, using Eq. B.3,

$$h_r = 0.9 \times 5.67 \times 10^{-8} \{ (298.0)^2 + (293.0)^2 \} \{ 298.0 + 293.0 \}$$

$$= 5.26 \text{ W/(m}^2\text{.K)} \quad \text{in summer}$$

$$\text{and } h_r = 0.9 \times 5.67 \times 10^{-8} \{ (293.0)^2 + (288.0)^2 \} \{ 293.0 + 288.0 \}$$

$$= 5.00 \text{ W/(m}^2\text{.K)} \quad \text{in winter}$$

Therefore,

$$f_1 = 2.96 + 5.26 = 8.22 \text{ W/(m}^2\text{.K)} \quad \text{in summer}$$

$$\text{and } f_1 = 2.96 + 5.00 = 7.96 \text{ W/(m}^2\text{.K)} \quad \text{in winter}$$

B.2 Conductance of Air Space

For radiation exchange between two parallel planes, the effective emissivity factor, F_e , is given by

$$F_e = \frac{1}{1/\epsilon_1 + 1/\epsilon_2 - 1} \quad (\text{B.5})$$

where, ϵ_1 and ϵ_2 are the emissivities of the two surfaces.

With $\epsilon_1 = \epsilon_2 = 0.90$, $F_e = 0.82$

Assuming,

$T_1 = 343 \text{ °K (70 °C)}$ and $T_2 = 336 \text{ °K (63 °C)}$ in summer

and $T_1 = 241 \text{ °K (-32 °C)}$ and $T_2 = 248 \text{ °K (-25 °C)}$ in winter

$$h_r = 0.82 \times 5.67 \times 10^{-8} \{ (343.0)^2 + (336.0)^2 \} \{ 343.0 + 336.0 \}$$

$$= 7.28 \text{ W/(m}^2\text{.K)} \quad \text{in summer}$$

$$\text{and } h_r = 0.82 \times 5.67 \times 10^{-8} \{ (248.0)^2 + (241.0)^2 \} \{ 248.0 + 241.0 \}$$

$$= 2.72 \text{ W/(m}^2\text{.K)} \quad \text{in winter}$$

Based upon 96 separate tests carried out for a range of conditions, convection coefficients for air spaces are shown in Fig. 8.3 of Reference 27. From that figure,

$$(h_c)_0 = 0.08$$

where, l = thickness of the air space = 25 mm

Therefore,

$$(h_c)_{10} = 0.08 / 0.025 = 3.2$$

Summer mean temperature = $(70 + 63) / 2 = 66.5^\circ\text{C}$

Winter mean temperature = $(-32 - 25) / 2 = -28.5^\circ\text{C}$

Convective heat transfer coefficient, h_c , of air spaces is given by Eq. 8.13 of Reference 27 which is

$$(h_c)_t = (h_c)_{10} \{ 1 - 0.0018 (t - 10) \} \quad (\text{B.6})$$

Therefore,

$$h_c = 3.2 \{ 1 - 0.0018(66.5 - 10.0) \} = 2.88 \text{ in summer}$$

$$h_c = 3.2 \{ 1 - 0.0018(-28.5 - 10.0) \} = 3.42 \text{ in winter}$$

Finally,

$$f_{as} = 2.88 + 7.28 = 10.16 \text{ W/(m}^2\text{.K)} \text{ in summer}$$

$$f_{as} = 3.42 + 2.72 = 6.14 \text{ W/(m}^2\text{.K)} \text{ in winter}$$

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