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Effects of Stacking Sequence on the Impact Damage Resistance of Composite Laminates

by

Edgar Fuoss

B. A. Sc. (Engineering Physics)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of Master of Engineering
in Mechanical Engineering

Department of Mechanical and Aerospace Engineering
The Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario, Canada

December 1996

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Impact Damage Resistance of Composite Laminates**

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Thesis Supervisor

Chair, Department of Mechanical and Aerospace Engineering

Carleton University
17 January 1997

Abstract

The stacking sequence of a composite laminate is an important design parameter which affects the strength, stiffness, and impact damage resistance of the material. Changes in stacking sequence may be classified by three separate parameters: ply grouping, the interface angle between stacked plies, and the ply orientation relative to the boundary supports of the material. Each parameter will affect the damage resistance differently, in a manner which is difficult to predict. The effect of each parameter on damage resistance is studied using a static finite element analysis. From the findings of this analysis, guidelines are proposed to improve the damage resistance. A laminate ranking method, based on the bending stiffness of a laminate, is also proposed to evaluate the damage resistance of different stacking sequences. The proposed laminate ranking method combined with the damage resistance guidelines should provide designers with a valuable tool suitable for preliminary design analysis.

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Nomenclature

$[A]$	Extensional stiffness matrix
$[B]$	Coupling matrix between the strains and curvatures
$[D]$	Bending stiffness matrix
$[\bar{Q}]^k$	Rotated elastic modulus matrix at ply k
D_a	Scaling factor for Choi/Chang criterion (Equation 2-5)
D_b, D_t	Bending stiffness coefficients for the bottom and top laminae groups comprising an interface (Equation 2-8)
D_{mn}	Coefficients of the bending stiffness matrix ($m=1, 2, \dots, 6; n=1, 2, \dots, 6$)
DR	Damage resistance parameter
E_1	Longitudinal modulus
E_2	Transverse modulus
G_{12}	In-plane shear modulus
G_{13}, G_{23}	Out-of-plane shear modulus
G_I, G_{II}, G_{III}	Mode I, II, III fracture toughnesses
$G_{Ic}, G_{IIc}, G_{IIIc}$	Mode I, II, III critical strain energy release rates
M	Bending moment
Q_{mn}^k	Elastic modulus coefficients at ply k ($m=1, 2, \dots, 6; n=1, 2, \dots, 6$)
$\bar{Q}_{mn}^k$	Rotated elastic modulus coefficients at ply k ($m=1, 2, \dots, 6; n=1, 2, \dots, 6$)
R, S	Transverse shear strength in directions 4 and 5 respectively (see Figure 2-2)
T	In-plane shear strength

X_T, X_C	Fibre tensile and compression strength, respectively
Y_T, Y_C	In-plane matrix tensile and compression strength, respectively
Z_T, Z_C	Out-of-plane matrix tensile and compression strength, respectively
Δa	Crack growth rate
$a(\alpha)$	Damage radius parameter
a, b	Constants (Equation 2-8); Plate dimensions (Chapter 5)
c	Distance through-thickness from the neutral axis to the bottom face
n	Ply number (Table 2-1)
q_{mn}	Load function (Equation 5-17)
r	Radius
u_n	Normal displacement (see Figure 3-3)
u_z	Transverse displacement (see Figure 3-3)
w	Parameter proportional to the out-of-plane transverse displacement of the composite laminate
x_b, y_b, z_b	Coordinates in the bending coordinate system
x_p, y_p, z_p	Coordinates in the principal coordinate system
z	Distance through-thickness measured from the neutral axis
α	Offset in-plane angle of bending coordinate system to principal coordinate system
ϵ_b	Maximum bending strain of equivalent beam section at angle α
ϵ_{cr}	Critical ply strain
κ_x	Beam curvature
$\nu_{12}, \nu_{13}, \nu_{23}$	Poisson's Ratio
σ_{xx}	Global in-plane longitudinal stress

σ_{yy}	Global in-plane transverse stress
σ_{zz}	Global out-of-plane transverse stress
σ_{xy}	Global in-plane shear stress
σ_{xz}, σ_{yz}	Global transverse shear stresses
σ_1	Local in-plane longitudinal stress
σ_2	Local in-plane transverse stress
σ_3	Local out-of-plane transverse stress
σ_4, σ_5	Local out-of-plane transverse shear stresses
σ_6	Local in-plane shear stress
θ	Ply angle with respect to a fixed coordinate axis

Chapter 1

Introduction

Advanced fibre reinforced composite materials have played an important role in the development of a wide range of light-weight structures. Today, composite materials can be found in everything from the latest commercial aircraft such as the Boeing 777 to common sporting equipment such as bicycle frames and tennis racquets. The wide acceptance of composites is due to a number of desirable properties which these materials have over traditional engineering materials such as steel and aluminum. Composites have excellent stiffness and strength properties with respect to weight. The stiffness and strength properties may be tailored to create more efficient structures for specific applications. During the manufacturing process, the material may be formed into a variety of complex shapes including tubes, angled stiffeners, and curved panels.

In spite of these advantages, composites do have significant limitations. The limitations include material degradation due to environmental factors such as temperature and humidity. Manufacturing and material costs are higher than conventional materials. In addition, composites are very susceptible to impact damage. Impact from foreign objects may produce internal damage which extends far beyond the localised surface

damage created at the point of impact. The internal damage is often barely visible, yet can seriously degrade the load-carrying capability of the material. Avery and Grande (1990) have reported that impact damage could result in up to a 50% reduction in the compressive strength.

Considerable research has been performed to address the limitations of impact damage (Abrate, 1991). The research areas include understanding the damage propagation modes, improving inspection methods to detect impact damage, determining the strength degradation due to impact damage, and predicting the amount of impact damage for a given load. The research areas may be classified into two categories: *damage resistance* and *damage tolerance* (Lagace *et al.*, 1993). Damage resistance is the ability of a material to resist damage resulting from an impact from another body. Damage resistance is assessed by quantifying the amount of damage for a fixed set of impact conditions. Damage tolerance is a measure of the structural performance of the material in the presence of damage. Damage tolerance is used as a design requirement for structures containing composite materials. Design requirements specified by the Federal Aviation Regulations (FAR 25.571-1, 1991) state that materials containing barely visible impact damage (BVID) must be able to withstand ultimate flight loads under extreme environmental conditions.

The ability to predict the impact damage resistance would be of great benefit to engineers and others using composite materials. Such a method would allow more

efficient structures to be designed. However, the prediction of damage resistance has proved to be a challenging task due to the complexity of damage which may occur, both internally and externally. Damage may propagate through several mechanisms, including fibre breakage, micro-buckling of laminates, delamination between adjacent laminae, and cracking of the bonding matrix. The propagation of damage is a progressive event. Once damage occurs, the stress state in the immediate vicinity of the damage regions is altered, affecting the propagation of subsequent damage. Comparisons of damage resistance have also proved to be difficult due to a number of parameters which affect the damage state. These parameters include supporting boundary conditions, method of loading, strength and stiffness of the material, and stacking sequence of plies within the laminate. By using standardised impact testing procedures, comparisons can be made to assess specific parameters such as material performance or stacking sequence by eliminating the effects of boundary supports or load conditions.

The objective of this thesis is to analyse the effects of laminate stacking sequence on the damage resistance in carbon fibre reinforced composite laminates. The stacking sequence represents an important design parameter, affecting both the strength properties and the damage resistance of the material. To meet the objective, the following tasks were carried out:

- Development a finite element model suitable for calculating the internal stress state of a laminate subjected to a transverse point load.
- Performance of a parametric study of various stacking sequence configurations by comparing the internal stress properties and the predicted damage area.
- Proposal of design guidelines based on the trends observed in the parametric study.
- Determination of a method of ranking the damage resistance capabilities for different stacking sequence configurations.

This thesis contains the results of the analysis performed for this study. The structure of the thesis is as follows. Chapter 2 contains a literature review of research performed to date in the area of composite materials. Chapter 3 contains a description of the finite element model used for stress calculations. The results of the finite element analysis are presented in Chapter 4. A description of a proposed laminate ranking method for damage resistance is given in Chapter 5. Conclusions and recommendations for future research are given in Chapter 6.

Chapter 2

Review of Impact Damage Resistance in Composite Materials

2.1 Introduction

This chapter gives an introduction to the study of impact damage resistance in composite materials. Definitions and conventions which are used in this thesis are presented first followed by a review of research performed to date in this field. The study of impact damage resistance may be classified into three areas: damage characterisation, parametric studies, and prediction techniques. A comprehensive review of each of the three areas is given in this chapter. Impacts studied for this thesis involved low-velocity/high mass impacts on coupon specimens; therefore, research reviewed in this chapter will focus on this type of impact.

2.2 Definitions and Conventions

2.2.1 Basic Definitions

Throughout this thesis, the following definitions are used. A plane of unidirectional fibres within a bonding matrix is referred to as a *lamina*, *layer*, or *ply*. A

laminate is two or more unidirectional laminae stacked together at various orientations. *Lamina* or *ply orientation* refers to the orientation of the fibres within the lamina. The laminae may be stacked at various orientations depending on the properties required. *Layup* refers to the composition of the laminate including material, number and relative orientation of each lamina. The exact orientation of each lamina within the laminate is defined by the *stacking sequence*. Several common stacking sequences exist. *Cross-ply* laminates contain laminae stacked at 90° with respect to each other. *Angle-ply* laminates contain laminae which are oriented at $+\theta$ and $-\theta$ to each other. *Quasi-isotropic* laminates are a special class of laminates in which the in-plane stiffnesses are identical in all directions, but the coupling and bending stiffnesses are not. Examples of such laminates include $[0/60/-60]_s$ and $[0/45/90/-45]_s$.

Several definitions are used to describe the properties of a composite laminate. *The impact face* is the outside surface of the laminate which makes contact with an impacting body. *The back face* is the outside surface opposite the impact face. *Ply group* refers to a group of adjacent plies within the laminate oriented in the same fibre orientation. *Lamina interface* is defined as the plane between two adjacent plies or ply groups which contain a change in ply orientation. The *interface angle* is defined as the angle between the ply orientations of the two plies or ply groups which comprise an interface.

2.2.2 Coordinate Systems

Two coordinate systems are used for this thesis to reference a laminated composite material: global and local. Both coordinate systems used are illustrated in Figure 2-1. The global coordinate system references the specimen as a whole, using three orthogonal axes labelled x , y , and z . The origin is placed at the specimen centre at the back face, with the z -axis pointing normal to the specimen plane. The local coordinate system is used to reference an individual lamina. This system, designated by axes 1, 2, and 3, is oriented such that axis 3 points in the same direction as the z -direction, but axes 1 and 2 are rotated in the specimen plane by angle θ to correspond with the lamina fibre orientation. Axis 1 is parallel to the fibre direction, while axis 2 is normal to fibre direction, as shown in Figure 2-1. The impact load was oriented to cause displacement of the specimen in the negative z -direction.

The lamina numbering convention starts with the lamina at the back face and sequentially increases towards the impact face. Likewise, the lamina interface numbering convention starts with the interface furthest from the impact face and increases sequentially towards the interface closest to the impact face.

2.2.3 Description of Stress State

A laminated composite material under load develops a complicated three-dimensional stress state. The stress state at an arbitrary point within the laminate may be

described using the two coordinate systems defined in Section 2.2.2, as illustrated in

Figure 2-2. The stresses in the global coordinate system are defined as follows:

σ_{xx} , σ_{yy} are the in-plane normal stresses

σ_{zz} is the transverse normal stress

σ_{xy} is the in-plane shear stress

σ_{xz} , σ_{yz} are the transverse shear stresses

The local stresses are defined as follows:

σ_1 is the in-plane longitudinal fibre stress

σ_2 is the in-plane transverse stress

σ_3 is the out-of-plane transverse stress

σ_4 , σ_5 are the out-of-plane transverse shear stresses

σ_6 is the in-plane shear stress

The global stresses may be transformed into the local stresses by the standard rotation matrix (Ochoa and Reddy, 1992):

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & 2mn \\ n^2 & m^2 & 0 & 0 & 0 & -2mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -mn & mn & 0 & 0 & 0 & (m^2 - n^2) \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} \quad (2-1)$$

where $m = \cos \theta$ and $n = \sin \theta$

Figure 2-1: Global and local coordinate systems for a laminated plate

Figure 2-2: Global and local stress state

2.3 Damage Characterisation

2.3.1 Characterisation Methods

An assessment of impact damage resistance begins with a selection of an appropriate experimental impact test. A number of tests have been developed to examine various aspects of impact damage. The tests can be classified into two categories based on the rate of loading: low-velocity and high-velocity (Cantwell and Morton, 1991). A low-velocity impact represents an impact by a large mass object (> 4 kg) at velocities below 10 m/s, such as a dropped tool. A high-velocity impact represents an impact by a low mass ($\ll 4$ kg) object at speeds above 10 m/s, such as runway debris. The distinction between low and high velocity impact is not fixed and is classified based on the structural response of the material. All impacts performed for this study are classified as low-velocity. A number of tests exist to simulate low-velocity impact loading, including Charpy and Izod pendulums, drop weight impact towers, and hydraulic machines. Of these, the drop weight tower test is the most common. This test is capable of closely modelling actual impact conditions and testing a variety of geometric configurations. However, comparisons of damage characteristics resulting from drop weight impact tests are hampered due to a lack of accepted testing standards.

The size and geometric configuration of the test specimen may vary based on the parameters that are to be studied. Large scale structures such as stiffened panels are used when the global impact response of the component is to be measured. These types of

tests are costly and time-consuming to perform. For comparisons of material response, small coupon specimens are used. Coupons are easy to standardise and allow a basis for comparison. To relate data obtained from coupon specimens to large scale structures, scaling laws are employed (Abrate, 1991). The impactor shape may also vary to include configurations such as sharp points and blunt cylinders. Generally, a spherical or hemispherical head is used (Cantwell and Morton, 1991).

A wide range of techniques, both destructive and non-destructive, are available to characterise the damage state (Vietinghoff, 1994). Non-destructive techniques such as X-radiography and through-transmission ultrasonics provide a topographical survey of the internal damage region. More detailed information of the through-thickness damage is provided by a pulse-echo time-of-flight ultrasonic scan. This type of scan will allow identification of the topographical geometry and the depth-wise location of internal delaminations. However, this type of scan is limited since the ultrasonic signal is not able to penetrate beyond the first layer of damage. For a true picture of the internal damage, destructive techniques are required. Dye-penetrant enhanced fractography reveals the through-thickness distribution of the damage at a given cross-section within the material. For a full analysis of the internal damage, more costly de-ply techniques are utilised.

2.3.2 Characteristic Damage State

The impact damage state for a variety of laminates has been extensively characterised to define what is termed the Characteristic Damage State (CDS) (Gosse and Mori, 1988). When a laminate is subjected to a transverse impact, damage will propagate by three different mechanisms: matrix cracking, delamination, and fibre breakage (Agarwal and Broutman, 1990). Idealised representations of the three damage mechanisms are found in Figures 2-3 and 2-4. The damage modes generally occur in the order as listed with increasing impact energy. (Choi *et al.*, 1991; Joshi and Sun, 1987). For low velocity/high mass impacts, damage tends to initiate internally at the bottom interface and propagate towards the impact face (Abrate, 1991). Delaminations between interfaces are interconnected by matrix cracks. The largest delamination damage generally occurs at the back face and progressively becomes smaller toward the impact face.

When the impact load is sufficient to create stresses which exceed the fibre strength of the material, fibre breakage will occur both on the impact face and most notably on the back face of the specimen. Vietinghoff (1994) has extensively observed and documented fibre breakage phenomena. At the impact face, fibre breakage results from the micro-buckling of the top ply under in-plane compressive forces due to plate bending and the transverse compressive forces resulting from the impactor. Tensile forces due to bending will create fibre breakage at the back face with noticeable outward deformation. The outward deformation is created from internal plies being crushed and

deformed underneath the impact point. This process will continue with increasing energy until complete penetration of the specimen occurs.

These damage mechanisms will create visible damage on both the impact face and back face of the laminate, as shown in Figure 2-4. At the impact face, plastic deformation at the impact point occurs in the form of a semi-spherical indent (Cantwell and Morton, 1989). Fibre breakage occurs in the form of a single crack extending from both sides the indented region. At the back face, fibre breakage appears together with visible matrix cracks. The matrix cracks are aligned with the fibre orientation of the bottom ply.

The formation of matrix cracking and delamination has been observed by Choi and Chang (1992) to be related in the following manner. Damage initiates when the energy from an impact exceeds a threshold value. Below this threshold, no damage was observed to occur. Damage initiates in the form of matrix cracking. The location of the initial matrix cracks is dependent on the stacking sequence of the laminate. The cracks are oriented in directions parallel to the fibre direction of a damaged ply. As the load is increased, delamination will occur at the locations of the initial matrix cracks. The formation of delamination will, in turn, initiate new micro-cracks. Due to the coupling between matrix cracking and delamination, the location of the initial matrix cracks strongly influences the propagation of delamination. Choi and Chang classified two types of matrix cracks: shear cracks within the laminate and bending cracks at the back face of the laminate.

Figure 2-3: Typical matrix cracking and delamination damage in a composite laminate

Figure 2-4: Typical damage exhibited on the faces of a coupon specimen

The delamination at an interface has been reported in literature to occur in a wide variety of shapes, depending on the stacking sequence. The common characteristics of a delamination are illustrated in Figure 2-5. The delamination appears as an elongated ellipse with a section which narrows at the impact point. The elongated portion of the delamination is oriented in a direction which is closely aligned with the fibre direction of the bottom ply comprising the interface. For this thesis, two measurements are used to characterise the delamination shape. The delamination length is defined as the size of the delamination along the fibre direction of the bottom ply. The delamination width is defined as the size of the delamination along a direction normal to the fibre direction.

Gosse and Mori (1988) and Vietinghoff (1994) have observed that the elongated section of the delaminations were shaped as two circular wedges for quasi-isotropic laminates. The size of the wedges depended on the angular difference of fibre orientations between the two adjacent plies comprising the interface. The apex of each elongated section was slightly offset from the fibre direction of the bottom ply. Lesser and Filippov (1991) and Liu (1988) reported that delamination area for cross-ply layups resembled the shape of a peanut. Hitchen and Kemp (1995) analysed several orthotropic layups and found in addition to the shapes described above, the delaminations appeared as cigar-shaped having a large elongated section with a narrow width, circular-shaped, and elliptical-shaped.

As previously described, delaminations between plies are connected by matrix cracks. For quasi-isotropic layups, Gosse and Mori (1988) and Vietinghoff (1994) have observed that the damage pattern through-thickness resembles a circular staircase. The pattern is illustrated in Figure 2-6 for the quasi-isotropic layup $[-45/0/45/90]_{3S}$. The ply numbering scheme used in Figure 2-6 is consistent with the convention used for this thesis, as described in Section 2.2.2. The circular staircase pattern may be repeated several times through-thickness based on the stacking sequence. When viewing the damage using ultrasonic C-Scan methods, the total damage region appeared as circular or elliptical. Gosse and Mori, and Vietinghoff have determined from both fractography and ultrasonic C-Scanning, that the delamination damage was greatest at the back face, and progressively decreased toward the impact face. Therefore, the delamination damage through-thickness is contained within a three-dimensional conical volume.

Figure 2-5: Typical delamination damage at an interface within a composite laminate

Figure 2-6: Plan view of delamination damage through the last 12 plies of a $[-45/0/45/90]_{3S}$ laminate (Gosse and Mori, 1988). Impact occurs at ply 24.

2.4 Parameters affecting Impact Damage

Damage within a laminate is affected by several structural, material, and impact parameters. Each parameter will have a different effect on the damage resistance. To isolate and compare the performance of a specific parameter, fixed impact test conditions are used. This will allow the effect of specimen geometry and supporting boundary conditions on damage resistance to be neglected. The most significant parameters are material, loading rate, stacking sequence, and structural response. Each parameter is discussed in detail below.

2.4.1 Material

Initiation and extension of damage is dependent on the material properties of the laminate. Matrix properties affect matrix cracking and delamination while fibre properties affect fibre breakage (Abrate, 1991). Altering the properties of the matrix or fibres will change the damage resistance of the composite material. Vietinghoff (1994) found that the toughened matrix of Toray T800H/3900-2 will have a far greater damage resistance capability as compared with the brittle resin of Hercules AS4/3501-6. Srinivasan *et al.* (1991) also found that damage resistance is a strong function of the matrix system.

Researchers have attempted to correlate the initiation and extension of damage for different materials through two different approaches: stress analysis and fracture mechanics. Stress-based analysis uses the tensile, compressive, and shear strength of the material to determine the existence of failure. Similarly, analysis based on fracture mechanics uses the fracture strength of the material to determine the existence of failure. Choi and Chang (1992) have used stress-based analysis to predict reasonably the initiation of matrix cracking and delamination. However, other researchers including Wang and Vu-Khanh (1994) and Qian *et al.* (1990), have found that stress-based analysis is inadequate to predict the growth of delaminations. Instead, delamination growth was proposed to be governed by the fracture toughness of the material. Liu and Chang (1994) found that damage size within T800H/3900-2 laminates was significantly smaller than for T300/976 laminates under common impact conditions. Both materials have similar strength and stiffness properties, but the T800H/3900-2 system has vastly superior fracture toughness properties.

Fracture modes of a material are classified into three types: mode I - opening, mode II - in-plane shear, and mode III - anti-plane shear (Gibson, 1994). The growth rate in each of the modes is governed by a fracture toughness parameter G . Based on experiments, Liu and Chang (1994) determined that delamination initiation is governed by G_I , while damage growth is governed by G_{II} and G_{III} . Many tests have been developed to measure each parameter individually or several parameters at once (Jones *et al.*, 1988). However, difficulties exist in isolating and measuring a single fracture mode parameter.

Even greater difficulties exist in determining the fracture properties of realistic composite structures using analytical or numerical means. These areas are still the subject of active research.

Changes in material properties affect damage in a manner which is complex and difficult to predict. A thorough analysis would use a combined approach, using the strength of materials to predict damage initiation and fracture mechanics to predict damage growth. However, prediction of damage in realistic structures using a fracture mechanics approach is not yet possible. This is due to the difficulty in determining individual fracture parameters and complexity of the analytical calculations. Future research in the area of fracture mechanics will hopefully address this limitation. In the interim, researchers have used the strength of material approach to predict both the initiation and propagation of damage.

2.4.2 Loading Rate

The structural response of the specimen and the resulting damage state depend on the loading rate. The response may be classified into two categories, as described previously in Section 2.3.1: low-velocity and high-velocity. Cantwell and Morton (1989) described the differences in structural response between the categories. Low-velocity impacts produce a global response in the specimen, where the energy is absorbed by the entire structure. In contrast, high-velocity impacts produce a response which is localised around the impact region. Morita *et al.* (1995) studied damage from both types of

loadings and found that each loading produced similar damage mechanisms at a given impact energy. However, the resulting damage area was significantly higher for high-velocity impacts.

At sufficiently low rates of loading, the resulting structural response approaches a static response. Swanson (1992) compared the contact force, normal strains behind the impact point, and peak interlaminar shear stress from dynamic and static loading. Swanson found that dynamic effects become important when the period of impact is less than 3 times the fundamental vibration period of the structure. Assessing the vibrational response of low-velocity impacts, Kwon and Sankar (1991) and Olsson (1992) have found that the impact period is much larger than the fundamental vibration period of the plate, producing a response which may be treated as quasi-static. Treating the low-velocity impact event as quasi-static allows the use of simpler static analysis to determine the properties of the system.

Comparative studies between static indentation and low-velocity impact loading have been performed. Kwon and Sankar (1991) have assessed the damage in both quasi-isotropic and cross-ply Hercules AS4/3501-6 laminates due to both types of loading. Each type of load created similar damage, with static loading creating a slightly larger damage area. However, a clear trend is difficult to establish due to the scatter in the measured data. Sjöblom, Hartness, and Cordell (1988) compared the static and dynamic load-deflection responses of $[0/45/-45/90]_{ns}$ Hercules AS4/3502 specimens, where

$n = 2, 4$. Good agreement was found, with the dynamic response containing large oscillations due to vibration. An examination of the damaged specimens revealed that static and impact loading produced virtually the same damage mechanisms, including similar patterns of matrix cracking and delamination. Based on the observed similarities of the damage state between static and dynamic loading, Kaczmarek and Maison (1994), and Lagace *et al.* (1993) proposed that static indentation testing could be used to characterise impact damage resistance. Static testing has several advantages over impact testing. For example, it has a lower cost, is easier to conduct, and is easier to standardise.

Despite the similarities, differences do exist between static and low-velocity impact loading. Lagace *et al.* (1993) found that for the same peak contact force, the topographical damage area is more extensive under impact loading. A contradictory result was found by Lee and Zahuta (1991). Comparing contact force, energy absorption, and damage area, Lee and Zahuta observed that, while the response is similar, static loading gave slightly greater damage areas with respect to peak contact force and absorbed energy. The disagreement of the above researchers on the effects of loading rate is possibly linked to the fracture properties of the material tested. Sohn and Hu (1995) compared the fracture properties under static and dynamic loading for Toray T300 and Hercules MI1510 carbon fibre composites. Noticeable differences existed between the two materials in the response of fracture strength G_I and G_{II} with respect to crack growth Δa . Examining the fracture surfaces of MI1510 in detail using scanning electron microscopy, Sohn and Hu found that interfacial bond was stronger under dynamic

loading conditions, which would lead to a improved damage resistance under impact loading.

The differences between static and dynamic loading has not been adequately examined in literature published to date. Further research is required to establish the strain rate sensitivity which exists for different material systems. Therefore, caution must be exercised when modelling low-velocity impacts using static analysis.

2.4.3 Stacking Sequence

The manner in which stacking sequence is linked to the damage resistance of a laminate is still not clearly understood. This is due, in part, to the difficulties of determining the damage propagation mechanisms which exist for a particular layup. In attempts to determine a law governing the effects of stacking sequence on damage resistance, researchers have focused on specific aspects of stacking sequence. These areas include studying the effects of ply grouping and the difference in fibre orientation between adjacent ply groups on damage resistance.

Liu (1988) examined changes in the relative angle of fibre orientations between adjacent plies for $[0_s/\theta_s/0_s]$ laminates for various ply angles θ . Liu found that the delamination area will increase significantly with increasing θ . Strait *et al.* (1992) compared the energy of impact at different stages of the impact event and the peak contact force for various woven, cross-ply and quasi-isotropic layups. Strait found that quasi-isotropic

layups contained the highest damage resistance capability. However, a clear trend of performance was difficult to establish due to the vast differences in layups tested.

Hitchen and Kemp (1995) performed a more systematic study of stacking sequence involving plies oriented at 0° , 45° , -45° , and 90° . These ply orientations are commonly used in industry when constructing laminated composites (Vosteen and Hadcock, 1994). The effects of ply grouping and ply orientation placement were studied by comparing the damage initiation energy of impacted specimens. Hitchen and Kemp found that the damage initiation energy increased by placing the $\pm 45^\circ$ plies in the surface layers and by increasing the number of interfaces within the laminate. Finn, He, and Springer (1993) studied the above effects as well as the effect of increasing the number of plies in the back ply group. Laminates with $[0_n/90_{(8-n)}]_s$ layups were impacted for various values of n . Finn *et al.* found that increasing the back face ply group also increased the delamination size. Ply grouping increased the stresses which were aligned in the fibre direction, promoting delamination growth.

While several researchers have investigated various aspects of stacking sequence, not one has developed a simple method of predicting the damage resistance for an arbitrary layup. Changes in stacking sequence produce changes in the internal stress state which are complex and difficult to model. Therefore, the effects of stacking sequence are difficult to establish without extensive experimental testing or detailed numerical modelling.

2.4.4 Structural Response

A number of parameters are available from a drop weight impact test which measures the global response of the system. These include structural response parameters such as peak contact force, impact kinetic energy, maximum absorbed energy, and impact velocity; and visual damage parameters such as indentation depth and visual surface damage area. Considerable efforts have been made by researchers to determine if any of these parameters are directly linked to the damage state of the laminate. Such a parameter, if found, would be very useful for prediction purposes.

The use of visual damage parameters is attractive since they are compatible with inspection procedures currently used in the aerospace industry. New non-destructive technologies are emerging to allow easier identification of damaged areas. Unfortunately, damage within composite materials also occurs internally. The internal damage is often much larger than the externally visible damage. A simple visual inspection is insufficient to gage the true damage state of the material. An additional complication is that the indent depth of the damage region has been observed to reduce with time. As a result, the usefulness of a visual parameter has been very limited.

Peak contact force has been identified by Lagace *et al.* (1993) and Sjöblom, Hartness, and Cordell (1988) as the most suitable structural parameter for assessing damage resistance. A strong correlation was found between peak contact force and topographical damage area for fixed impact and material conditions. With peak contact

force, comparisons of damage resistance capabilities may be made between different materials and layups. However, peak contact force may not be used to predict damage within an arbitrary composite specimen. For fixed initial impact conditions, Straznicky *et al.* (1995) have determined that peak contact force will vary based on material, loading rate, and stacking sequence in a manner such that no noticeable trends are apparent. A combined parameter which accounts for structural geometry, layup, and material would be required for prediction of damage resistance.

2.5 Failure Theories

A number of theories has been proposed to predict the initiation of failure and its progression. The formulation of these theories was based on a particular parameter such as stress, total strain energy, or fracture toughness. Stress forms the basis for the majority of the theories. The theories may be categorised into two stages of the impact event: initiation and post-failure. Initiation theories, as the name implies, determine the locations of the onset of damage within the laminate. Once damage has occurred, the laminate may still be capable of carrying load, with the damaged regions exhibiting a reduction in stiffness and strength. Thus, post-failure theories are used in progressive damage modelling to determine the amount of strength reduction at each modelled time increment during dynamic loading.

Various initiation failure theories exist to predict the intralaminar and interlaminar damage. Intralaminar damage includes fibre fracture and matrix cracking. Interlaminar

damage refers to the damage between laminae, namely delamination. The theories are applied in the form of a failure criteria. Parameters such as stress are checked against pre-determined allowed values. If these values are exceeded, failure is assumed to occur. The most common damage initiation failure criteria are described below.

2.5.1 Stress-based Failure Criteria

Stress-based failure criteria may be classified into two categories: independent and interactive. Failure modes for an independent failure criterion are separate and distinct with no interaction between the modes. On the other hand, failure modes in an interactive failure criterion are combined with each other to form a failure parameter. A survey of both types of failure criteria is listed in Table 2-1. The following nomenclature is used in Table 2-1: (X_T , Y_T , Z_T) are the tensile strengths in the (1, 2, 3) directions, (R , S , T) are the shear strengths in the (4, 5, 6) directions, (X_C , Y_C , Z_C) are the compressive strengths in the (1, 2, 3) directions, respectively.

In the maximum stress criterion (Ochoa and Reddy, 1992), failure is assumed to occur if any of the six stress components exceed the maximum tensile or compressive strengths. A similar criterion exists for strains. This criterion is simple and allows for immediate identification of the failure mode. Good agreement with experimental results is obtained when uniaxial loading is applied (Gibson, 1994). However, since failure is often the result of a combination of stress components, the maximum stress criterion tends to give conservative predictions of damage.

For loading applied along multiple axes, interactive based criteria such as the Tsai-Wu criterion (Ochoa and Reddy, 1992) are more appropriate. These criteria for composite materials have evolved from failure theories for isotropic materials such as von Mises criterion. The terms in Tsai-Wu criterion, given in tensor notation in Table 2-1, combine to form an elliptical failure surface. Unlike the maximum stress criteria, the Tsai-Wu criterion is more complex and identification of the damage modes is difficult.

The Hashin (1980) criteria are similar to the Tsai-Wu criterion, but allow for the identification of the damage modes. Hashin identified some of the limitations of using the Tsai-Wu criterion, and proposed a separate failure criterion for four types of failure modes: fibre tension, fibre compression, matrix tension, and matrix compression. Each criterion is based on a summation of terms containing a squared ratio of stresses to strengths. The Hashin criteria apply to damage modes based on unidirectional laminates, not multi-directional damage modes such as matrix cracking between layers and delamination.

Choi and Chang (1992) proposed a set of interactive failure criteria to predict matrix cracking and delamination in multi-directional laminates. The criteria are similar in form to the Hashin criteria and to other previously proposed interactive failure criteria, including that of Brewer and Lagace (1988). Matrix cracking is initiated by the in-plane matrix stress σ_2 and the transverse shear stress σ_4 . The matrix cracking criterion is applied at each layer n , where stresses σ_2 and σ_4 are the averaged stresses within the

layer. Delamination is initiated by the transverse shear stress σ_4 in the top layer $n+1$ and by the in-plane matrix stress σ_2 and transverse shear stress σ_5 in the bottom layer n of the n th interface. The delamination criterion is applied at each interface, where the stresses σ_2 , σ_4 , and σ_5 are given as the averaged stresses in the respective layers. A scaling factor D_a is used to correlate predictions with experimental results. Choi and Chang applied their criteria to predict damage in several experimentally impacted laminates and found good agreement.

2.5.2 Other Failure Criteria

Alternative failure criteria have been developed, often in response to limitations of stress-based failure criteria. Finn and Springer (1993) developed a failure criterion to predict the size and shape of delaminations based on strain energy. The strain energy density was calculated from stresses determined by Finn and Springer to promote delamination. Failure occurred when the strain energy density exceeded the energy Γ required to delaminate a surface of unit area. Determining the delamination energy Γ is difficult, and is often assumed to be the mode I fracture strength G_{Ic} .

Table 2-1: Stress-Based Failure Criteria

Maximum Stress:	
Fibre Tension	$\sigma_1 \geq X_T \quad (\sigma_1 > 0) \quad (2-2)$
Fibre Compression	$ \sigma_1 \geq X_C \quad (\sigma_1 < 0)$
Matrix Tension	$\sigma_2 \geq Y_T \quad \sigma_3 \geq Z_T \quad (\sigma_2, \sigma_3 > 0)$
Matrix Compression	$ \sigma_2 \geq Y_T \quad \sigma_3 \geq Z_T \quad (\sigma_2, \sigma_3 < 0)$
Matrix Shear	$ \sigma_4 \geq R \quad \sigma_5 \geq S \quad \sigma_6 \geq T$
Tsai-Wu:	
	$F_i \sigma_i + F_{ij} \sigma_i \sigma_j \geq 1 \quad (i, j = 1, 2, \dots, 6) \quad (2-3)$
	$F_1 = \frac{1}{X_T} - \frac{1}{X_C} \quad F_2 = \frac{1}{Y_T} - \frac{1}{Y_C} \quad F_3 = \frac{1}{Z_T} - \frac{1}{Z_C}$ $F_{11} = \frac{1}{X_T X_C} \quad F_{22} = \frac{1}{Y_T Y_C} \quad F_{33} = \frac{1}{Z_T Z_C}$ $F_{44} = \frac{1}{R^2} \quad F_{55} = \frac{1}{S^2} \quad F_{66} = \frac{1}{T^2}$ $F_{12} = -\frac{1}{2\sqrt{X_T X_C Y_T Y_C}} \quad F_{13} = -\frac{1}{2\sqrt{X_T X_C Z_T Z_C}} \quad F_{23} = -\frac{1}{2\sqrt{Y_T Y_C Z_T Z_C}}$
Hashin:	
Fibre Tension	$\left(\frac{\sigma_1}{X_T}\right)^2 + \frac{\sigma_6^2 + \sigma_5^2}{T^2} \geq 1 \quad (2-4)$
Fibre Compression	$\sigma_1 \geq -X_C$
Matrix Tension	$\frac{(\sigma_2 + \sigma_3)^2}{Y_T^2} + \frac{(\sigma_4^2 + \sigma_5^2 + \sigma_6^2 - \sigma_2 \sigma_3)}{T^2} \geq 1$
Matrix Compression	$\frac{1}{Y_C} \left[\left(\frac{Y_C}{2T}\right)^2 - 1 \right] (\sigma_2 + \sigma_3) + \frac{(\sigma_2 + \sigma_3)^2}{4T^2} + \frac{(\sigma_4^2 + \sigma_5^2 + \sigma_6^2 - \sigma_2 \sigma_3)}{T^2} \geq 1$
Choi and Chang:	
Matrix cracking	$\left(\frac{\sigma_2}{Y}\right)_n^2 + \left(\frac{\sigma_4}{R}\right)_n^2 \geq 1 \quad \begin{cases} Y = Y_T & \text{if } \sigma_2 \geq 0 \\ Y = Y_C & \text{if } \sigma_2 < 0 \end{cases} \quad (2-5)$
Delamination	$D_a \left[\left(\frac{\sigma_4}{R}\right)_{n+1}^2 + \left(\frac{\sigma_5}{S}\right)_n^2 + \left(\frac{\sigma_2}{Y}\right)_n^2 \right] \geq 1 \quad \begin{cases} Y = Y_T & \text{if } \sigma_2 \geq 0 \\ Y = Y_C & \text{if } \sigma_2 < 0 \end{cases}$

It has been suggested that the extension of delamination is governed by the fracture properties of the material rather than the stress state, as previously described in Section 2.4.1. One approach, applied by Wang and Vu-Khanh (1994), uses the strain energy release rate of the material. Prediction of final extent of delamination requires the knowledge of a delamination arrest toughness, which Wang and Vu-Khanh determined through finite element modelling and experiments to be close to the mode II fracture strength G_{IIc} .

For predicting damage from mixed-mode fractures, an interactive failure criterion is often used (Jones *et al.*, 1988). The criterion is expressed as:

$$\left(\frac{G_I}{G_{Ic}}\right)^a + \left(\frac{G_{II}}{G_{IIc}}\right)^b \geq 1 \quad (2-6)$$

where G_{Ic} and G_{IIc} are critical strain energy release rates, and a and b are constants. This criterion has been used by Liu and Chang (1994) to determine the matrix cracking effect on delamination growth. The strains rates are determined numerically using a crack closure technique (Rybicki and Konninen, 1977). The variables a and b were determined to give a best fit to experimental data.

2.6 Damage Prediction

Under static or impact loading, a complicated three-dimensional stress state which may lead to matrix or fibre damage develops within a laminated composite. The internal stress distribution is dependent on many factors as described above, including stacking sequence and ply thickness. Once damage occurs, the internal stress state will be altered, affecting future damage as the load is increased. This complicated stress state and the progressive nature of damage propagation are factors which need to be addressed when attempting to predict impact damage.

2.6.1 Empirical Methods

Several researchers have attempted to apply simple empirical methods to predict damage. These approaches attempt to model the key characteristics of the final damage state without calculating the internal stress state or progressive damage propagation. Clark (1989) proposed an empirical delamination model based on a two-ply configuration. Under load, the curvatures of each individual ply within the laminate will create regions of tensile forces promoting delamination and compressive forces suppressing delamination. Clark predicted that the major axis of the characteristic “peanut-shaped” delamination will be oriented along the fibre direction of the lower ply within an interface. This model makes no attempt to determine the size or exact shape of the delaminated region, thus is only useful for visualisation purposes.

Liu (1988) also studied the delamination damage present within two-ply specimens. Liu proposed that shape of the delamination is dependant on the bending stiffness mismatch between the two plies, represented by the coefficient M:

$$M = \frac{D_{11}(\theta_b) - D_{11}(\theta_t)}{D_{11}(0^\circ) - D_{11}(90^\circ)} \quad (2-7)$$

where $D_{11}(\theta)$ is the bending stiffness coefficient at angle θ , θ_b and θ_t are the fibre angles of the bottom and top lamina respectively. The mismatch coefficient M was used to predict the size and shape of $[0_4/\theta_4]$ graphite/epoxy specimens at various ply angles θ . While this approach could give general predictions of delamination damage for two-ply specimens, extending this hypothesis to predict damage in multi-ply specimens was found to be inadequate for several reasons, as reported by Fuoss, Straznicky, and Poon (1994). For multi-ply laminates, computation of the top and bottom stiffness coefficients D_{11} at a given interface would now involve summing the stiffness of each lamina above and below the delamination respectively. Since the interface is no longer limited to the midplane of the laminate, the difference of bending coefficients may be skewed to give erroneous results when calculating coefficient M.

An alternative approach was attempted by Fuoss *et al.* by first assuming the top and bottom plies comprising the interface form a two-ply laminate, for which M could be calculated using Equation (2-7). Then, the coefficient M was scaled at each interface i using:

$$M_L^{[i]} = a \left(\frac{D_b^{[i]}}{D_t^{[i]}} \right)^b M^{[i]} \quad (2-8)$$

where D_b , D_t are the stiffness coefficients for the bottom and top laminae groups respectively, and a , b are constants. This approach would avoid the skewing effects associated with a direct extension of the mismatch method as described earlier. A similar stiffness ratio was used by Wu and Springer (1988) to give reasonable predictions of delamination size. On comparison to impact damage within 24-ply quasi-isotropic specimens, Fuoss *et al.* found that coefficient M , given by Equation (2-8), did not accurately predict the delamination size through-thickness. Based on the above analysis, bending stiffness was found to be insufficient to predict impact damage. A combination of several parameters may be required to accurately model damage within a laminated plate.

Morita *et al.* (1995) also addressed another limitation of Liu's mismatch coefficient M . For the case of unidirectional laminates, the difference in in-plane stiffness equals zero. However, the calculated value of M for unidirectional laminates does not equal zero in all cases. This situation arises from the fact that M is calculated solely on the difference of bending stiffnesses between plies. Morita *et al.* identified this limitation and proposed a new parameter β which accounts for the differences of in-plane and bending stiffness:

$$\beta = \frac{1}{2\pi} \int_0^{2\pi} \frac{(\Delta Q_{11}(\theta) \cdot T(\theta))_{\max}}{D_{11}(\theta)} d\theta \quad (2-9)$$

where $\Delta Q_{11}(\theta)$ is the difference in the in-plane stiffness between the adjacent laminae in direction θ , $T(\theta)$ is distance through-thickness from the neutral surface to the interface, and D_{11} is the bending stiffness coefficient of the entire laminate. Parameter β is a measure of the maximum bending stress discontinuity, given as an integrated quantity with respect to θ . Morita *et al.* compared parameter β to the topographical impact damage area of APC-2/AS4 specimens with various stacking sequences and found reasonable linear correlation. Further testing is required to verify this method, as only a few specimens were tested for this study.

2.6.2 Analytical Methods

While empirical methods are useful for visualising the characteristics of impact damage, they fall short of being a useful tool for damage prediction. Therefore, attempts have been made to formulate analytical mathematical models which determine the response and internal stress state in the material. These stresses can then ultimately be used to predict damage.

Several researchers have developed analytical models to predict the dynamic response of the specimen. Dobyns (1981) used plate equations from Whitney and Pagano (1970) to determine the deflection and interlaminar stresses within simply-supported orthotropic plates. Christoforou and Swanson (1991) used a Fourier series expansion with Laplace transform techniques to determine the impact load history for simply-

supported plates. More complicated methods were used by Matsushashi *et al.* (1993) to account for the non-linear effects of membrane stiffening.

Bogdanovich and Friedrich (1994) developed a rigorous mathematical model to determine the displacement history of the structure. The maximum stress and a second-order tensor-interactive failure criteria were applied to determine the initiation of damage. A ply-by-ply progressive failure algorithm proposed by Bogdanovich *et al.* was then applied to determine the final damage state. However, the calculations are complex and the analysis was limited to simple configurations such as unidirectional or cross-ply layups.

2.6.3 Numerical methods

The majority of the current research is focused on using numerical methods, such as the finite element method, to determine the stress or energy state of the material under load. This approach has the advantage of being able to handle a variety of stacking sequences and loading types. Choi and Chang (1992) employed the use of a dynamic finite element model with an appropriate contact law to determine the stress state. A damage model was developed based on experimental observations to predict damage based on the calculated stress state. This model is semi-empirical as it does not account for the progressive nature of the damage propagation or the interaction of delaminations between plies. In spite of these limitations, reasonable agreement was found between experimental and numerical calculated damage within quasi-isotropic specimens.

More detailed modelling of the impact problem was performed by Majeed (1995). A dynamic finite element model with contact analysis and a progressive failure criterion was utilised to simulate the impact of a quasi-isotropic coupon specimen. The specimen was discretized with four-noded Belytschko-Tsay quadrilateral shell elements. The use of quadrilateral elements allows large reductions in computational time. However, these shell elements do not allow the calculation of through-thickness normal stresses and poorly model the transverse shear stresses. As a result, internal delaminations were not modelled. Solid hexagonal elements were used to discretize the supporting base and impact apparatus. Majeed applied the Chang-Chang failure criteria (1987) with a modified post-failure stress degradation based on Humphreys (1981) to account for the progressive growth of damage due to matrix cracking, crushing, and fibre breakage.

With this comprehensive composite damage model, Majeed was able to accurately simulate low energy impacts resulting in an elastic response with no damage. The calculated impact force, energy, and strain histories were all in good agreement with experimental values. Simulations of high energy impacts, sufficient to create back-face fibre damage, were not as promising. With the simulation of damage, the model became very computationally intensive and suffered from mesh dependency. The calculated impact response was found to be sensitive to both the failure threshold and the fibre post-failure behaviour. With modifications to the post-failure criteria, Majeed was able to reasonably predict back-face fibre breakage, but greatly overpredicted the extent of matrix cracking by 119%.

Other researchers have attempted damage modelling using dynamic finite element analysis with varying degrees of success. Finn and Springer (1993) analysed and identified the stresses which promoted delamination and matrix cracking. A failure criterion based on the total strain energy of the system was then used to predict damage. The model employs the use of 3D finite elements. However, to reduce computational effort, several plies through-thickness were modelled within a single element. This created a smearing effect on the calculation of stresses of individual plies within the element.

Given the computing power available, designs of new composite structures are not yet feasible using the dynamic finite element models as presented above. Thus, other approaches have been attempted. Hong, Choi, and Kim (1994) took advantage of quasi-static characteristics of the low-velocity impact response by using a static FE model to calculate the desired stresses. The peak contact force was first determined from a new analytical model of the contact force history (Choi and Hong, 1994). Then, the peak force was applied as a static point load in the FE model. Nine-noded 2D shell elements with a higher-order shear deformation formulation were used to model the laminated plate. Failure was determined using a modified Choi and Chang (1992) failure criterion. Hong *et al.* used this model to reasonably predict damage in cross-ply laminates; therefore, this model provides a promising approach to determine damage in specimens with general layups.

2.7 Summary

Impact damage in composite materials is a complex event which involves several damage mechanisms including matrix cracking, delamination, and fibre breakage. The damage may occur both internally and externally. For quasi-isotropic layups, the damage state has been extensively characterised and occurs in a manner defined as the Characteristic Damage State. Exact prediction of impact damage is difficult due to the large number of parameters which may affect the damage state. Such parameters include changes in material, layup, and boundary conditions. To study the effect of a single parameter, fixed test conditions are used.

Two approaches have been used to predict damage: strength of materials and fracture mechanics. Strength of materials is suitable to predict the damage initiation, but was found to be limited in predicting damage growth. Fracture mechanics is a more suitable approach for damage growth prediction. However, it is more complex and has so far been able to predict damage only in simple layup configurations such as cross-ply. As a result, the strength of materials approach, despite its limitations, is used to predict both damage initiation and growth.

Several methods have been used to predict damage based on the strength of materials approach. The most common method found in literature is the application of the finite element (FE) method combined with a failure criterion. The FE method is used

to calculate the internal stress state of the material under load. The stresses are checked using a failure criterion to determine the existence of damage. The impact load has been modelled using either static or dynamic analysis. Due to the large computational expense of the FE method, efforts have been made to develop a simple parameter which governs the amount of impact damage.

Future research is currently directed in several areas. New methods are being examined to improve the speed and accuracy of the FE calculations, including the development of more efficient element formulations. Research is continuing to allow fracture mechanics based analysis to predict damage in more common layup configurations. Also, the patterns and trends of impact damage are being closely examined with hopes to determine a simplified prediction method.

Chapter 3

Model Development

3.1 Introduction

As reported in Chapter 2, altering the stacking sequence of a laminate will have a significant effect on the damage resistance. The effects of stacking sequence on damage resistance were analysed in this thesis by modelling the internal stress state associated with a given stacking sequence. The analysis was performed in the following manner. A standardised test procedure was adopted to isolate the effects of stacking sequence from other parameters such as boundary supports and load conditions. A detailed finite element (FE) model was developed to simulate the stress state within a transversely loaded laminate. Results from the FE model were compared with experimental data to establish trends associated with changes in stacking sequence.

This chapter presents the details of the FE model in three parts. First, a description of the experimental test procedure adopted for this thesis is given. Next, an overview of the FE model is presented. Finally, details of the verification checks performed to establish the accuracy of the results are given. The results from the numerical model, including comparisons to experimental data, are presented in Chapter 4.

3.2 Experimental Test Procedure

The numerical model used for this study simulates a drop-weight impact test outlined in Boeing Specification BSS 7260 (Boeing, 1988). The Boeing specification is a standard used for impact and subsequent compression testing of composite laminates. The specification was adopted for this study to allow comparisons with other experimental data available in literature. Unless otherwise noted, experimental data published in this thesis were previously obtained by Vietinghoff (1994). A brief description of the test specimens and experimental procedure is given in this section. A full description of the experimental procedures may be found in the work published by Vietinghoff.

3.2.1 Coupon Specimens

The experimental coupon specimens are flat panels with planar dimensions of 152.4 mm x 101.6 mm (6.0 in x 4.0 in). The coupon specimens are constructed from a fibre/matrix prepreg roll. The prepreps are cut into layers which form the individual plies of the laminate. The plies are oriented and stacked by hand, with the aid of templates, to give the desired layup. The prepreg panel was cured in an autoclave, in accordance with the manufacturer's specifications. The cured panel was then sectioned into six specimens, with each specimen machined to the correct dimensions. Each coupon specimen was checked using ultrasonic C-scanning to insure no manufacturing flaws were present.

Two different material systems were tested by Vietinghoff: Toray T800H/3900-2 and Hercules AS4/3501-6. The Toray T800H/3900-2 is newer composite material with increased toughness to give improved mode I and mode II fracture strengths. Hercules AS4/3501-6 is an older material system with a brittle matrix, which has a poorer damage resistance capability as compared with T800H/3900-2. The in-plane material properties of both material systems are found in Table 3-1 (Gaudert *et al.*, 1993; Poon *et al.*, 1991).

3.2.2 Test Apparatus

Impact tests were performed using the Dynatup 8200 instrumented drop weight impact system. The system was configured to comply with Boeing BSS 7260 specification for class 1 impacts. The test apparatus, illustrated in Figure 3-1, consisted of three main components: impactor, supporting base, and data acquisition system (not shown). The impactor tup was comprised of a 15.9 mm (5/8 in) hemispherical head made of hardened steel with an attached load cell to measure the impact load. The total mass of the tup assembly was 5.44 kg. The tup was mounted on a drop tower assembly which restricted the motion of the tup to directions normal to the plane of the specimen. The drop tower was located to allow the tup to make contact at the specimen's centre. A latching mechanism was mounted on the drop tower to prevent repeated impacts.

The aluminum/plywood supporting base with a rectangular cut-out supports the specimen at its outer boundaries. The specimen was positioned over the rectangular cut-out by three locating pins. Four rubber-tipped clamps, providing light clamping pressure.

Table 3-1: In-Plane Material Properties (Gaudert *et al.*, 1993; Poon *et al.*, 1991).

	T800H/3900-2	AS4/3501-6
Longitudinal Modulus, E_1 (GPa)	152	140
Transverse Modulus, E_2 (GPa)	8.07	8.2
Shear Modulus, G_{12} (GPa)	4.14	6.2
Poisson's Ratio, ν_{12}	0.35	0.30
Longitudinal Tensile Strength, X_T (MPa)	2772	2144.3
Longitudinal Compressive Strength, X_C (MPa)	1480	824.6
Transverse Tensile Strength, Y_T (MPa)	79.3	46.2
Transverse Compressive Strength, Y_C (MPa)	231.0	172.4
Shear Strength, R (MPa)	132.8	110.3
Ply Thickness, h_o (mm)	0.19	0.14

were used to support the specimen during impact to prevent rebounding. This configuration approximates simply-supported boundary conditions (Avery *et al.*, 1990).

Data measurements were obtained using a personal computer equipped with a GRC 730-I instrumentation package. The system monitors the load during the impact event at a sampling rate of 1024 data points within a time interval of 10 ms. The initial impact velocity at the point of contact was measured using an infrared detector. The detector also triggers the start of load sampling. The data were stored on hard disk for future analysis. The system software is capable of determining the energy history, displacement history, and total energy absorbed during impact from the load history. The software can also determine the peak contact load, absorbed energy, and maximum impactor displacement.

Figure 3-1: Experimental apparatus

3.3 Model Methodology

A comprehensive model was developed to determine the internal stress state of a composite specimen and the subsequent damage which occurs under transverse impact loading. To accurately account for the complexity of the impact event, as previously described in Chapter 1, the finite element (FE) method was employed. The FE method allows the modelling of realistic geometric configurations including lamination stacking sequence and boundary conditions. A failure criterion was applied using the FE data to determine the extent of the predicted damage.

Numerical modelling was performed by the NISA II finite element software program (EMRC 1994a,b). The NISA II program includes the DISPLAY III pre- and post-processor, an interactive graphical interface for model development and subsequent data analysis. The NISA II code is capable of calculating a variety of parameters including stress, strain, and displacement. All computations were performed using a Silicon Graphics IRIX Challenge L/2 workstation, at the Institute for Aerospace Research, National Research Council Canada.

A static load model was used to determine the internal stresses. The use of a static model to analyse a dynamic impact problem is appropriate for the low-velocity, high-mass impact loading used for this study, as previously discussed in Section 2.4.2. For this class of impacts, the contact duration of the impactor is much larger than the

fundamental vibration period of the plate. Several researchers, including Kwon and Sankar (1991) and Jil and Sun (1993), have proposed that this type of response could be modelled statically. The primary advantage of the static model is sharp reduction of computational time. This allows the opportunity to perform a more detailed stress analysis, including the determination of the transverse interlaminar shear stresses, as compared with dynamic modelling. A static analysis also allows a quicker turn-around time for model changes and data post-processing. A verification check was performed to insure that the modelled test problem may be classified as quasi-static. Details of the verification check may be found in Section 3.4.1.

3.3.1 Model Formulation

The experimental coupon specimens were modelled using the NISA type 4 solid hexahedron finite elements. The hexahedron elements are suitable for modelling the full three-dimensional stress state, including interlaminar shear stresses σ_{xz} and σ_{yz} . To reduce computational time, only first order elements with a linear interpolation scheme were used. The NISA type 4 element also contains additional shape functions to improve the element behaviour in bending. Each element contains 8 nodes, with each node containing three degrees of freedom: u_x , u_y , and u_z . The element formulation is based on the classical linear elasticity theory using an orthotropic material model. Each element models a single orthotropic lamina through-thickness. Perfect bonding is assumed to exist between laminae. For the composite materials modelled in this study, only the in-

plane properties were available. The out-of-plane properties were obtained by assuming the composite is transversely isotropic in the 2-3 plane. This assumption is reasonable since the material behaviour transverse to the fibre direction is similar for both the in-plane direction 2 and out-of-plane direction 3 (Agarwal and Broutman, 1990). As a result, the following relationships were assumed:

$$E_2 = E_3 \quad G_{12} = G_{13} \quad \nu_{12} = \nu_{13} = \nu_{23} \quad G_{23} = \frac{E_2}{2(1 + \nu_{23})} \quad (3-1)$$

The out-of-plane shear strengths were also assumed to be equal to the in-plane shear strength:

$$R = S = T \quad (3-2)$$

where R, S, and, T are the shear strengths in the 4, 5, and 6 directions. The in-plane properties listed in Table 3-1 together with the out-of-plane properties given by Equations (3-1) and (3-2) form the complete set of material properties used by the numerical model.

The in-plane element discretization of the FE model is illustrated in Figure 3-2. The entire specimen was modelled, as symmetry did not exist for the majority of layups examined for this study. The model contained four levels of mesh refinement towards the specimen's centre, giving a total of 300 in-plane elements. The mesh-refined areas improve the computational accuracy since regions with changing stress gradients exist near the point of loading. The in-plane aspect ratio of the elements was unity for all elements except for elements used to transition from coarse to fine meshed regions. This minimised the interpolation errors due to element distortion. Each lamina within the

coupon specimen was modelled using a minimum of three elements through-thickness. For a 24-ply laminate, a total of 21600 elements were used to model the plate. The size of this FE model was the maximum size which the Silicon Graphics IRIX Challenge L/2 workstation, used for calculations, was capable of handling in terms of hard disk space.

The boundary supports of the experimental test jig were idealised as simply-supported at each of the specimen edges, as shown in Figure 3-3. The simply-supported conditions were imposed by constraining, to zero, displacements the normal to the edge, u_n , and transverse displacements, u_z , at the midplane nodes of each edge. A reduced specimen size of 127 mm $\times$ 76.2 mm (5 in $\times$ 3 in) was modelled to correspond with the dimensions of the rectangular cut-out present within the supporting base. These idealised support conditions allow a large reduction of computational time by allowing fewer elements to be used when modelling the specimen. This simplification will increase the error in the stresses calculated near the plate boundaries, due to the differences between the modelled and actual bending of the plate. However, around the point of impact away from the plate boundaries, the modelled support conditions have a minimal effect on calculated stresses. These support conditions are consistent with the works of other researchers, including Finn and Springer (1993), and Ochoa and Reddy (1992).

Figure 3-2: Element discretization of coupon specimen

Figure 3-3: Simply-supported boundary conditions for numerical model

The impact load is idealised as a point force at the top centre node of the model. The static load corresponds with the peak contact force of the impact event. For comparisons purposes, a force of 7.5 kN was applied. This load was chosen to create damage at each interface, while keeping the extent of the damage within the mesh refined regions as much as possible for Toray T800H/3900-2 specimens. This improves the accuracy of the computed stresses within the projected damage regions. The 7.5 kN applied load corresponds approximately to an 12 J impact. Experimental data used in this chapter were from specimens subjected to a 15 J impact, with a peak contact force of 8.5 kN.

A set of constitutive equations which govern the material response are formulated by NISA II using the principle of minimum potential energy to satisfy the applied load and boundary conditions. The displacement and subsequent stress solutions were found by solving the set of constitutive equations using a wavefront technique. A typical solution run required approximately 700 MB of storage space and 75 minutes of processing time. The output of a run includes the displacement, stress, and strain energy at each node. All quantities from a NISA II run are referenced to the global coordinate axis of the laminate (see Figure 2-1).

3.3.2 Damage Prediction

Prediction of damage within the specimen was made by applying the Choi and Chang (1992) failure criterion, given by Equation (2-5) (see Section 2.5.1). The criterion uses the stress state as determined by the FE model to predict the initiation of matrix cracking and delamination. Back-face fibre breakage and indentation damage underneath the impactor are not considered in this criterion. The criterion also does not account for material degradation or the interaction of delaminations and matrix cracking between plies during damage propagation. In spite of these limitations, the Choi/Chang criterion gave good predictions of the damage state within impacted specimens. Therefore, while the criterion does not model all the mechanisms involved in damage propagation, the stress state before damage initiation has been found to indicate the trends of the final damage state.

The criterion is applied at each interface using stresses which are averaged within the ply or ply group adjacent to the interface. To determine an appropriate stress averaging scheme, the through-thickness stresses at a selected point were examined for a Toray T800H/3900-2 specimen with a quasi-isotropic layup of $[-45_3/0_3/45_3/90_3]_s$. A 7.5 kN point load was modelled at the specimen centre. The stresses from this specimen were considered typical for the layups examined in this study. A survey of stresses with respect to thickness z are found in Figures 3-4 through 3-6. The interface locations are indicated by the vertical lines. The point $z = 0$ corresponds with the back face of the

laminate. Stresses σ_{xx} , σ_{yy} , and σ_{xy} were found to be discontinuous at each interface. For these stresses, the values calculated exactly at an interface indicate the average stress between the two adjacent laminae. Including this value when averaging stresses within a ply group will skew the result. Therefore, stress values calculated exactly at the interfaces were ignored for stress averaging. Stresses σ_{zz} , σ_{yz} , and σ_{xz} were found to be continuous across each interface. Thus all points including the stress values at the interfaces were used when stress averaging within a ply group.

Before the failure criterion was applied, the global stresses were transformed into the local lamina coordinate system using the standard rotation matrix given in Equation (2-1). A custom-made program was created in FORTRAN to read the global stress results from NISA II, transform global stresses into local stresses, apply a failure criterion, and finally output the data in a DISPLAY III readable results file. The remaining post-processing was performed in DISPLAY III to sample data and plot results. The FORTRAN program is available from the Department of Mechanical and Aerospace Engineering, Carleton University, upon request.

Figure 3-4: In-plane stresses σ_{xx} and σ_{yy} at point (1.58 mm, 1.58 mm, z) for $[-45_3/0_3/45_3/90_3]_S$, load 7.5 kN

Figure 3-5: Transverse normal stress σ_{zz} and in-plane shear stress σ_{xy} at point (1.58 mm, 1.58 mm, z) for $[-45_3/0_3/45_3/90_3]_S$, load 7.5 kN

Figure 3-6: Transverse shear stresses σ_{xz} and σ_{yz} at point (1.58 mm, 1.58 mm, z) for $[-45_3/0_3/45_3/90_3]_S$, load 7.5 kN

3.4 Model Verification

Several verification tests were performed to determine the quality of numerical results. The tests are designed to evaluate the performance of the elements used for modelling and the errors associated with modelling the geometry of the coupon specimen.

3.4.1 Quasi-Static Approximation

The quasi-static modelling of the impact response is a fundamental approximation which is used for this study. A calculation check was performed to insure that the dynamic effects of the system could be neglected. From a comparison of stresses under both dynamic and static loading, Swanson (1992) reported that the quasi-static approximation is valid when the frequency of the impact ω_i is less than 1/3 of the natural frequency ω_n of the structure. The impact frequency ω_i was determined experimentally from the measured force history during the impact event:

$$\omega_i = \frac{1}{2T_i} \quad (3-3)$$

where T_i is the contact period. For this verification check, ω_i was determined for a Toray T800H/3900-2 quasi-isotropic specimen with stacking sequence $[-45/0/45/90]_{3s}$. This specimen is typical of specimens examined in this study. The measured period T_i for this specimen is 4.44 ms, and the resulting frequency ω_i was calculated to be:

$$\omega_i = \frac{1}{2 \cdot 4.44 \text{ ms}} = 113 \text{ Hz} \quad (3-4)$$

The natural frequency ω_n was calculated using the NISA II eigenvalue solver. The coupon specimen was simulated as a 127 mm by 76.2 mm (5 in by 3 in) plate with simply-supported boundary conditions. The reduced specimen size was used to give better correlation with the actual experimental support conditions. The plate was modelled using 20×12 NISA type 32 composite shell elements. The mathematical formulation of the shell elements include effects due to membrane stretching, bending, and transverse shear (EMRC 1994b). The density was adjusted to simulate the actual mass of the specimen using the reduced modelled volume. In-plane translations and rotations about the normal to the plate were constrained at every node. Transverse displacements were constrained at the edges.

The first four calculated vibrations modes of the coupon specimen are illustrated in Figure 3-7. The natural frequency ω_n was determined to be 2234 Hz; a frequency which is almost 20 times greater than the actual impact frequency. This satisfies the quasi-static criterion, allowing dynamic vibrational effects to be neglected for this study.

Figure 3-7: Vibration modes of a 24-ply quasi-isotropic coupon specimen

3.4.2 Element Performance

The performance of the NISA type 4 finite elements was tested by solving a test problem with a known analytical solution. The NISA type 4 elements are 8-node solid elements using linear interpolation. The numerical accuracy was then assessed by comparing the numerical solution against the analytical solution. The test problem modelled the transverse loading of a cross-ply simply-supported rectangular plate. This problem was previously solved by Pagano (1970a). Material and geometry data for the problem are given in Table 3-2. The notation used to reference the plate is given in Figure 3-8. The applied load q_o at the top surface of the plate was specified as:

$$q_o(x, y) = \sigma \sin\left(\frac{a}{\pi} x\right) \sin\left(\frac{b}{\pi} y\right) \quad (3-5)$$

where constant $\sigma = -1$. The imposed boundary conditions were taken as:

$$\begin{aligned} \text{At } x = 0, a: \sigma_{xx} = u_y = u_z &= 0 \\ \text{At } y = 0, b: \sigma_{yy} = u_x = u_z &= 0 \end{aligned} \quad (3-6)$$

A 3-ply laminate was modelled with a stacking sequence of [0/90/0]. Each lamina has a thickness of 1 in.

A sampling of the numerically-generated stress state is found in Figures 3-9 through 3-11. The following normalisation constants were used when reporting results:

$$(\bar{\sigma}_{xx}, \bar{\sigma}_{yy}) = \frac{1}{\sigma S^2} (\sigma_{xx}, \sigma_{yy}) \quad \bar{\sigma}_{xz} = \frac{1}{\sigma S} \sigma_{xz} \quad S = \frac{a}{h} \quad \bar{z} = \frac{z}{h} \quad (3-7)$$

On comparison with stress plots published by Pagano, the agreement is excellent. A comparison of stresses sampled at selected points for both analytical and numerical solutions are found in Table 3-3. The largest errors occurred at the edges of the plate. This is due to the reduced number of points available for stress averaging. The deviation between the exact and FE solution appears greater in Figure 3-11. This is due to the smaller scale used for $\bar{\sigma}_{xy}$. The error of approximately ± 0.02 for stress $\bar{\sigma}_{xy}$ is consistent with the calculation errors for the other stresses. Stresses sampled within the laminate, namely $\bar{\sigma}_x$, agree very well with the analytical solution. The results verify the FE model formulation and solution generated by the NISA II code.

Table 3-2: Material and Geometry Data for Rectangular Plate Test Problem

Parameter	Value
Longitudinal Modulus, E_1 (psi)	25×10^6
Transverse Modulus, E_2 (psi)	1.0×10^6
Shear Modulus, $G_{12} = G_{13}$ (psi)	0.5×10^6
Shear Modulus, G_{23} (psi)	0.2×10^6
Poisson's Ratio, ν_{12}	0.25
Length, a (in)	12
Width, b (in)	12
Thickness, h (in)	3

Figure 3-8: Plate notation

Figure 3-9: In-plane stress distribution of plate

Figure 3-10: Transverse shear stress distribution of plate

Figure 3-11: In-plane shear stress distribution of plate

Table 3-3: Analytical and Numerical Stress Solutions for Plate Bending Problem

	Analytical Solution	NISA II
$\bar{\sigma}_{xx}(\frac{a}{2}, \frac{b}{2}, \frac{1}{2})$	0.801	0.768
$\bar{\sigma}_{xx}(\frac{a}{2}, \frac{b}{2}, -\frac{1}{2})$	-0.755	-0.728
$\bar{\sigma}_{xz}(0, \frac{b}{2}, 0)$	0.256	0.251
$\bar{\sigma}_{xz}(0, \frac{b}{2}, 0.27)$	0.282	0.280
$\bar{\sigma}_{xy}(0, 0, \frac{1}{2})$	-0.0511	-0.0490
$\bar{\sigma}_{xy}(0, 0, -\frac{1}{2})$	0.0505	0.0483

3.4.3 Geometric Modelling

Specimens modelled in this study have in-plane dimensions which are much larger than the thickness. As a result, elements used to model this geometry will have poor length-to-thickness aspect ratios. Normal guidelines suggest using finite elements with aspect ratios of 3 or less for stress solutions (EMRC 1994b). Exceeding this ratio may result in an ill-conditioned stiffness matrix, which in turn produces high round-off errors in the solution. Ill-conditioning is a particular problem in regions which contain a transition from a coarse to fine mesh. For this study, elements used to model regions of high stress contain length-to-thickness aspect ratios as high as 50. The high aspect ratio is an obvious cause for concern when modelling the structural response of the specimen.

A parametric study of a test problem with a known exact solution was performed to determine the effect of using high aspect ratio elements. The test problem chosen for this study is the cylindrical bending of a composite beam under a sinusoidal load. This problem was solved previously by Pagano (1970b). The beam contains two laminae of equal thickness with a layup of [15/-15]. This type of layup will also test the accuracy of the NISA II software code to calculate shear coupling effects which exist between the layers. Material and geometry data for the problem is given in Table 3-4. The beam notation is illustrated in Figure 3-12. Translations normal to the plate are constrained at two opposing edges of the beam, while the remaining edges are free. A sinusoidal load is placed the top face of the beam, equivalent to:

$$q(x) = \sin\left(\frac{\pi x}{10}\right) \quad (3-8)$$

A relatively coarse mesh of 8 x 4 elements was used to model the in-plane dimensions. Several runs were performed, with each run increasing the number of elements through the thickness while holding the number of in-plane elements fixed. Stresses were sampled along the midpoint of the width of the beam to avoid stress concentrations which are present at the edges.

The transverse shear stress $\bar{\sigma}_{xz}$ at point $x = 0$ and the direct in-plane stress $\bar{\sigma}_{xx}$ at point $x = 5$ were calculated and compared for each run, as shown in Figure 3-13 and Figure 3-14 respectively. The reported quantities were normalised using Equation (3-7). The number of elements used through-thickness is indicated in the legend of each figure. All stresses are reported in the global coordinate system. Both plots show convergence of the solution at 32 elements and greater, through-thickness, and give excellent agreement when compared with the exact solution as reported by Pagano (1970b). The highest aspect ratio modelled is 640, using 512 elements through-thickness. Even at this aspect ratio, the solution shows no signs of ill-conditioning. From these results, accurate solutions of the internal stress state may still be obtained when using a high element aspect ratio through-thickness for this problem.

Table 3-4: Material and Geometry Data for Beam Bending Test Problem

Parameter	Value
Longitudinal Modulus, E_1 (psi)	25×10^6
Transverse Modulus, E_2 (psi)	1.0×10^6
Shear Modulus, $G_{12} = G_{13}$ (psi)	0.5×10^6
Shear Modulus, G_{23} (psi)	0.2×10^6
Poisson's Ratio, ν_{12}	0.25
Length, l (in)	10
Width, b (in)	10
Thickness, h (in)	1

Figure 3-12: Beam notation

Figure 3-13: Transverse shear stress distribution of beam

Figure 3-14: In-plane stress distribution of beam

3.4.4 Convergence

The solutions obtained from a finite element analysis are dependent on the mesh size of the model. By continually increasing the number of elements within the mesh, the stress solution normally converges to a particular value. The convergence of the numerical model used in this study was performed through an iterative process of mesh refinement. The mesh was refined at the centre of the specimen. Stresses calculated at the specimen's centre will have the largest errors, due to the rapidly changing stress gradients which are present in this region. For this study, a Toray T800H/3900-2 specimen with layup $[-45/0/45/90]_{3s}$ was modelled using three different element discretizations, illustrated in Figure 3-15. The mesh of model CS5-LA is the same mesh used for simulation models of this study. Each successive model contains the same nodes of the previous model, but doubles the number of in-plane elements at the specimen's centre. Keeping the nodes of previous models in the refined model allows direct comparisons of the stress solution to be made from model-to-model. The number of elements through-thickness remained unchanged and is the same for all three models. Model CS5-LA, which contains the most refined mesh, represents the largest model size which the computer system was capable of solving due to limitations in hard disk storage space.

The test program, given in Table 3-5, details the stress sampled, location, and models used for stress calculation. Stresses were sampled through-thickness at the given

in-plane locations and are reported in the global coordinate system. Simulations 1 and 2 are sampled at point common to all three models, while simulations 3 and 4 are sampled at a point which is common only to models CS4-LA and CS5-LA. To evaluate the effect of mesh refinement, the relative error between an unrefined model n and the refined model $n+1$ was calculated as:

$$\Delta\sigma = |\sigma_{n+1} - \sigma_n| \quad (3-9)$$

The stresses and relative errors between levels of mesh refinement for each simulation are given in Figures 3-16 through 3-19. A point load of 7.5 kN was applied in all cases.

In all cases, successive mesh refinements gave relatively similar stress values below the midplane ($z < 2.29$ mm). Above the midplane, greater differences existed between the various mesh refinement models, with no clear indications of a converging trend. The different responses above and below the midplane is a result of sampling stresses relatively close to the point of loading. The high gradient stress field due to loading created wide variations in the stresses calculated above the midplane, in turn increasing the solution error. However, the region of greatest interest is below the midplane, as this region contains the largest damage within the entire specimen. Therefore, the focus of this convergence study was twofold: 1) to determine the level of convergence below the midplane, 2) to insure that the large calculation errors near the point of loading do not significantly influence the results at other points within the specimen.

Table 3-5: Test Program for Convergence Study

Simulation Number	Stress Sampled	In-Plane Location (mm, mm)	Models used for Calculations
1	σ_{xx}	(3.18,0)	CS3-LA, CS4-LA, CS5-LA
2	σ_{yy}	(0,3.18)	CS3-LA, CS4-LA, CS5-LA
3	σ_{xz}	(1.59,1.59)	CS4-LA, CS5-LA
4	σ_{xy}	(1.59,1.59)	CS4-LA, CS5-LA

Model CS3-LA

Model CS4-LA

Model CS5-LA

Figure 3-15: Element discretization of convergence study models

Figure 3-16: Longitudinal stresses σ_{xx} and relative errors $|\Delta\sigma_{xx}|$ through thickness z sampled at point (3.18 mm, 0 mm)

Figure 3-17: Longitudinal stresses σ_{yy} and relative errors $|\Delta\sigma_{yy}|$ through-thickness z sampled at point (0 mm, 3.18 mm)

Figure 3-18: Longitudinal stresses σ_{xz} and relative errors $|\Delta\sigma_{xz}|$ between models CS4-LA and CS5-LA through-thickness z at point (1.59 mm, 1.59 mm)

Figure 3-19: Longitudinal stresses σ_{xy} and relative errors $|\Delta\sigma_{xy}|$ between models CS4-LA and CS5-LA through-thickness z at point (1.59 mm, 1.59 mm)

For the first level of mesh refinement between models CS3-LA and CS4-LA, the difference in solution is primarily below ± 60 MPa for stresses σ_{xx} and σ_{yy} located below the midplane, with the exception of a few points containing errors around ± 100 MPa. The difference is significantly reduced with the second level of mesh refinement between models CS4-LA and CS5-LA. For each simulation case, the difference is below ± 10 MPa except for a few points which are below ± 20 MPa. The stress difference between models CS4-LA and CS5-LA represent the bound of calculation error for simulation models used in the study. Additional errors will exist between the actual and calculated stresses due to approximations made in the formulation of the FE model. The approximations include the assumption of perfect bonding between layers and simply-supported boundary conditions.

To better visualise the effect of mesh refinement, delamination damage was determined using the Choi/Chang failure criterion at interfaces 1, 5, and 11 below the midplane. A comparison of results for the three test models is found in Figure 3-20. For interfaces 5 and 11, increasing the number of elements at the specimen centre will increase the predicted delamination area, but does not appreciably increase the delamination length as defined in Figure 2-5. In contrast, increasing the number of elements at interface 1 reduced the overall size of the predicted delaminations. Regions which did not undergo a mesh refinement had no change in the predicted delamination shape.

The areas of the predicted delaminations shapes were determined from Figure 3-20, and are compared in Figure 3-21. A larger change occurred with the first level of mesh refinement. The area change for a given mesh refinement varies from interface-to-interface since the mesh spacing was not common for all projected delamination shapes. A fair comparison of changes may be made for interface 11, since the delamination area for each model is contained almost fully within the meshed refined region. The first mesh refinement produced a 228% increase in damage area, while the second mesh refinement gave a 28%. Therefore, a rapid convergence trend is indicated with each level of mesh refinement.

Based on the limited runs performed for this study, a trend of convergence was observed for stresses below the midplane. However, convergence was not reached due to the limitations of the computer system. The most refined mesh produced a clear and well-defined shape of the predicted delamination area. This is sufficient to allow comparative studies to be made using this mesh. The numerical accuracy of this refined mesh was determined by comparing the numerical solutions to experimental results, and is reported in Chapter 4.

Figure 3-20: Predicted delamination areas at selected interfaces

Figure 3-21: Comparison of delamination areas at selected interfaces

Chapter 4

Results

4.1 Introduction

The effects of stacking sequence on the damage resistance were analysed in this thesis through a parametric study of three parameters: interface mismatch angle, ply orientation relative to the boundary supports, and ply grouping. Each parameter will have a different and unique effect on the damage resistance. Using the finite element (FE) model, as previously described in Chapter 3, calculations were performed to determine the internal stress state and to predict damage for several stacking sequences under common loading conditions. By systematically altering each parameter, the changes in predicted damage area were compared to give assessment of how the damage resistance was affected. This chapter presents the results of the FE calculations and the findings of the parametric study. Section 4.2 gives an overview of the results of the FE model for selected layups on a ply-by-ply basis. Comparisons between experimental and analytical results are found in Section 4.3. A complete description of the parametric study and the findings are given in Section 4.4. The FE model was also used to assess the damage resistance of quasi-isotropic layups as well as typical layups used in industry. The results

of this study is found in Section 4.5. Lastly, recommendations to improve the damage resistance of a composite laminate are proposed in Section 4.6.

The layups analysed in this chapter contain 24 plies and are stacked in a manner which is symmetric about the midplane. All plies are constructed using the Toray T800H/3900-2 material system, except where noted. Various layups were modelled, including common layups used in industry and special layups used specifically for the parametric study. Where possible, the layups contain plies which are oriented at angles 0° , 45° , -45° and 90° to maintain consistency with ply orientations which are commonly used in industry. Each laminate was subjected to a point load of 7.5 kN, unless noted otherwise. This load was chosen to give delamination damage at each interface within a laminate, but to keep the extent of the damage within the finely meshed region at the centre of the model. This load represents an impact of approximately 12 J. Experimental tests referenced in this chapter represent impacts ranging from 15 J to 20 J.

The damage resistance of a layup was assessed by determining the delamination damage at specific interfaces below the midplane and the topographical delamination area using the Choi/Chang delamination criterion (see Section 3.3.2). The topographical delamination area is the overall projected area of all delaminations within a laminate from a plan view. It has been used by Vietinghoff (1994) and Dost *et al.* (1991) to compare performances of various experimentally impacted specimens. When more detailed

comparisons are required, the delamination damage will be assessed at individual interfaces.

The numerical data presented in this chapter is presented in the plan view of each specimen, showing the impact face. The coordinate system used for ply orientation is shown in Figure 2-1.

4.2 Initial Survey of Numerical Results

Delamination damage is a complex mechanism which may occur at multiple interfaces within a laminate. The delaminations at each interface will vary based on location within the laminate and the fibre orientation of plies for a fixed impact conditions. Based on experimental data of delamination damage, as discussed previously in Section 2.3.2, the following general trends are expected:

- The delaminations may vary in shape from elliptical to "peanut-shaped".
- The elongated section of the delamination is oriented in fibre direction of the bottom layer comprising the interface, with the apex of the delamination slightly offset from the fibre direction.
- The delaminations decrease in size the closer the interface is located to the impact face.

To determine if the FE model is simulating these effects correctly, the delamination damage at individual interfaces was calculated for a series of quasi-isotropic layups listed in Table 4-1. For each layup, a constant angular difference between the fibre orientations at adjacent layers was maintained throughout the laminate. The angular difference is specified by the interface angle given in Table 4-1.

A plot of the predicted delamination areas at the first 12 interfaces is given in Figure 4-1. All delamination regions, indicated by the black outlines, are drawn to a common scale as indicated. The delaminations are tabulated starting at the interface closest to the back face, designated as interface 1. Examining the delamination contours in Figure 4-1 reveals that all three observations listed above were modelled. The largest damage occurs at interface 1, and successively becomes smaller the further the interface is located from the back face. Delaminations above the 6th interface are relatively the same size for each layup. The major axis of each delamination is aligned with the fibre direction of the bottom ply comprising the interface, as expected. The shapes of the delaminations range from elliptical to "peanut-shaped", all consistent with shapes observed experimentally by Vietinghoff (1994).

Significant differences in delamination damage existed between the various layups modelled in Figure 4-1. This is due to the combined effects of varying the angular difference in fibre orientation between layers and varying the orientation of plies relative

to the boundaries of the test specimens modelled. The effects of each parameter will be discussed later in this chapter.

As discussed above, the topographical delamination area was chosen to assess the damage resistance of a laminate. To determine how the predicted delaminations at each interface contributed to the topographical area, the delaminations at the first four interfaces of layup LA were superimposed over each other, as shown in Figure 4-2. The outside boundary of all delamination contours represents the predicted topographical damage area. Due to the rapid decrease in delamination size away from the back face, only delaminations at the first 3 or 4 interfaces contribute to the predicted topographical delamination area. Delaminations at the remaining interfaces were smaller than the delaminations at the first 4 interfaces, and therefore did not contribute to the total combined area. This will still give a reasonable representation of the damage characteristics within a composite, as the delaminations at the back face are the most critical, and will fail first when the specimen is loaded in compression.

Table 4-1: Layups Analysed for Interface Mismatch Angle Study

Layup Code	Stacking Sequence	Interface Angle
LF	[-75/-60/-45/-30/-15/0/15/30/45/60/75/90] _S	15°
LE	[-60/-30/0/30/60/90] _{2S}	30°
LA	[-45/0/45/90] _{3S}	45°
LG	[-30/30/90] _{4S}	60°
LH	[-15/60/-45/30/-75/0/75/-30/45/-60/15/90] _S	75°
LI	[0/90] _{6S}	90°

Figure 4-1: Predicted delamination damage at the first 12 interfaces for quasi-isotropic layups listed in Table 4-1, load 7.5 kN

Figure 4-1 continued

Figure 4-1 continued

Figure 4-2: Contributions of delaminations at the first 4 interfaces to the total topographical area for layup LA

A survey of the stresses for layup LA used in the Choi/Chang delamination criterion (σ_2 , σ_4 , and σ_5) are given in Figures 4-3 through 4-5. Each figure gives stress contour plots for interfaces 1 and 4, with all values reported in MPa. The stresses were calculated using an applied load of 8.53 kN, representing an impact energy of 15 J. The black regions in the centre indicate stresses which have exceeded the strength of the material. Internal stresses for other layups follow a similar trend as the ones shown in Figures 4-3 through 4-5.

At interface 1, the interface closest to the back face, the primary stress which contributes to predicted delamination failure is σ_2 . The interlaminar shear stresses σ_4 and σ_5 show minimal contribution. This is expected since the in-plane transverse stress σ_2 results from plate bending, and is greatest at the two face surfaces. The shear stresses σ_4 and σ_5 are expected to be close to zero at the face surfaces and greatest at the midplane. This trend is seen at interface 4, where a sharp drop occurs in the tensile stress σ_2 and an increase in the shear stresses σ_4 and σ_5 are observed. The predicted damage at interface 4 is due to the combined interaction of these interlaminar stresses. The contour shape of the shear stresses σ_4 and σ_5 at interface 4 appears as two elliptical shaped regions, which closely resemble the damage areas which have been observed from experimental specimens.

Interface 1

Interface 4

Figure 4-3: Contour plot of stress σ_2 at interfaces 1 and 4, $[-45/0/45/90]_{3S}$, load 8.5 kN

Figure 4-4: Contour plots of stress σ_4 at interfaces 1 and 4, $[-45/0/45/90]_{3s}$, load 8.5 kN

Figure 4-5: Contour plots of stress σ_5 at interfaces 1 and 4, $[-45/0/45/90]_{3s}$, load 8.5 kN

4.3 Comparison of Numerical and Experimental Results

The accuracy of the FE model was assessed by comparing the predicted results from the FE model to the total topographical damage areas of experimentally impacted specimens. The applied load in the FE model was set to correspond with the peak contact force measured during the impact event. The test layups varied in material, stacking sequence, and peak contact load. A comparison of results is presented in Figure 4-6. The black region represents the topographical damage area of the experimental specimen as determined by ultrasonic C-scanning. The grey outline represents the predicted damage area using the Choi/Chang delamination criterion.

The shape of experimental damage area for each test layup is approximately circular, with the exception of test layups 5 and 6. Test layups 5 and 6 each contain a central circular core region similar to the other layups, but with two elongated regions which extend out from the core. The elongated regions are oriented in the direction of the first ply, and represent damage at the first interface, as determined from C-Scan images published by Vietinghoff (1994). The elongation region for test layup 5 was found to give a small increase to the core region, whereas the elongation region for test layup 6 gave a noticeable increase in the total topographical area.

From a comparison of numerical and experimental results, the length of the predicted damage areas was found to correspond closely with the actual core damage areas for each layup. However, the width of the predicted damage areas was found to correlate poorly with the experimental results for all test cases. Predictions for test layups 5 and 6 are noticeably different than the other test layups. In both cases, the elongated damage which extends from the core region was significantly under predicted.

The difference between experimental and numerical results for test layups 5 and 6 are most likely attributed to the fracture mechanisms which cause delamination growth. The fracture mechanisms for test layups 5 and 6 promoted large delamination growth, primarily along the fibre direction of the bottom ply of the interface. This is due to unique stress concentrations which existed in the bottom plies as a result of either ply grouping or small changes in the ply orientation between layers. The other layups contained a more distributed stress concentration, causing fracture growth along both the length and width of the delamination. Since fracture growth is not modelled using a strength of material approach, true delamination damage within layups containing ply grouping or small changes in ply orientation between layers is not predicted.

** Experimental tests for this layup performed by author

Figure 4-6: Comparison of finite element predictions against experimental results

In all cases, a closer examination of the delaminations areas at individual plies reveals the discrepancies between experimental and numerical results. The accuracy of damage prediction at an interface was found to decrease the closer the interface was located to the impact face. As an illustration, the predicted delamination areas at interfaces 1 and 4 of test layup 1 were superimposed over a topographical C-Scan of an experimental specimen with impact damage, as shown in Figure 4-7. The C-Scan projects damage at the back face, with colour contours indicating the depth of the delaminations. The light blue region indicates delamination at interface 4. Delaminations at interfaces 1 through 3 were not detected by the C-Scan, but have been shown to exist from a fractograph indicating the through-thickness damage and a picture of the back face damage, as published by Vietinghoff (1994). The delamination length at interface 1 is expected to be similar in length to the largest delamination shown in Figure 4-7. The predicted delamination length at interface 1 slightly under predicts the diameter of the damaged region. Predicted damage at interface 4 indicates the general shape and orientation of the delaminated area. However, the delamination length is significantly underestimated by the FE model. This under prediction of delamination at interfaces away from the bottom interface would cause the discrepancies which are seen in Figure 4-6.

Test Layup 1: [-45/0/45/90]_{3S}, Specimen #: 663-1, Impact Energy: 15.5 J, Peak Load: 8.67 kN

Figure 4-7: Comparison of predicted delaminations at interfaces 1 and 4 against C-Scan of experimental damage (actual delamination of interface 1 is not shown)

The under prediction of the interlaminar shear stresses is due to a number of factors: progressive damage, inaccurate interlaminar shear strengths used in calculations, and the use of static analysis for stress calculations. Progressive damage, cited as the primary cause for the under prediction, refers to the changes to the stress state within a specimen after damage has occurred. The altered stress state will affect the formation of future damage as the load is increased. The FE model used for this study does not account for progressive damage formation during loading. The formation of damage such as matrix cracking or delamination will also affect the fracture modes within the specimen. As reported in Section 2.3.2, coupling exists between matrix cracking and delamination. A large matrix crack in a ply may lead to a delamination in similar length in an adjacent ply. This type of damage propagation is not modelled using a stress-based approach and is better predicted by using a fracture mechanics approach.

The interlaminar shear strengths of the laminate used in the Choi/Chang criterion were unknown, and were assumed to be the in-plane shear strength of a unidirectional composite. Measurement of the interlaminar shear strength presents great difficulties. Existing experimental methods are not able to distinctly isolate the shear strength component from other in-plane tensile strengths. In addition, the shear strength depends on the thickness and stacking sequence of the laminate (Chang and Chen, 1987). Therefore, the strength value used in the failure criterion is possibly inaccurate.

The interlaminar shear stresses may also be sensitive to dynamic loading, an effect which is not modelled by static analysis. From line-loading experiments, Choi, Wu, and Chang (1991) found that the interlaminar shear stresses were larger near the loading point under dynamic loading as compared with static loading. A similar result could be expected for point loading. The material properties of the matrix resin could also be sensitive to the loading rate. However, as shown in Section 3.4.1, the impacts studied for this thesis may be classified as quasi-static such that dynamic effects do not contribute significantly to the specimen response. Therefore, this factor is not expected to be a major cause of the under prediction of the damage area.

The FE model has been shown to give reasonable predictions of the shape and orientation of delamination damage at each interface for various materials and stacking sequences. The model is in good agreement with experimental results when predicting delamination size at the first interface, but under predicts delaminations at interfaces away from the back face. As a result, the FE model is not capable of accurately predicting the actual delamination damage. However, it is capable of indicating the relative performance of various stacking sequences and materials provided the relative damage mechanisms for each modelled specimen does not change during failure progression. Therefore, the FE model is considered suitable for a study of parameters affecting stacking sequence, as discussed in Section 4.4.

4.4 Effects of Layup Parameters

4.4.1 Interface Angle

The interface angle is defined as the angular difference in fibre orientation between two layers which comprise an interface. Each layer may contain one or more plies oriented in a common direction. Several researchers including Liu (1988), Finn, He, and Springer (1993), and Straznicky *et al.* (1995), have previously observed that the interface angle significantly affected the delamination damage within a laminate. For this thesis, a systematic study of the effects of interface angle on delamination damage was performed on two types of stacking sequences. The first type maintains a constant interface angle at each interface. The second type uses two or more interface angles between layers to stack the plies. Altering the interface angle for each stacking sequence type will have a different effect on damage resistance, as discussed below.

4.4.1.1 Layups containing a constant interface angle

Six layups of stacking sequence $[0_4/\theta_4/0_4]_s$ were analysed, with angle θ varying in 15° increments, as listed in Table 4-2. The stacking sequence was designed to orient the length of the delamination along 0° direction. Each layup was subjected to a reduced point load of 4.5 kN, to keep the extent of the predicted delamination within the finely meshed regions of the model, where possible.

Table 4-2: Layups Analysed for Interface Angle Study

Layup Code	Stacking Sequence	Interface Angle
SA	$[0_4/15_4/0_4]_S$	15°
SB	$[0_4/30_4/0_4]_S$	30°
SC	$[0_4/45_4/0_4]_S$	45°
SD	$[0_4/60_4/0_4]_S$	60°
SE	$[0_4/75_4/0_4]_S$	75°
SF	$[0_4/90_4/0_4]_S$	90°

A comparison of damage areas at interface 1 for all layups are found in Figure 4-8. The results clearly indicate that increasing the interface angle will decrease the predicted delamination area. This is in marked contrast with the bending stiffness mismatch theory proposed by Liu (1988) which related delamination size to the difference in bending stiffness between adjacent plies comprising an interface. Using Liu's mismatch coefficient M given by Equation (2-7) (Section 2.6.1), the largest delamination size is predicted for layup SF containing an interface angle of 90°; the smallest delamination size is predicted for layup SA containing an interface angle of 15°. This is completely opposite to the FE results given in Figure 4-8. However, the FE results are consistent with the results published by Vietinghoff (1994). Vietinghoff tested several quasi-isotropic specimens using three different interface angles. The topographical delamination area was found to increase as the interface angle was decreased. The results are also consistent with a similar experimental and numerical study performed by Finn He. and Springer (1993).

To determine how the predicted delaminations change between each layup, the delamination length and width, as defined in Figure 2-5, were measured for each layup and are plotted in Figure 4-9. Both the delamination length and width are found to decrease as the interface angle is increased. The length experiences a far greater change as compared to the width between interface angles 15° and 90° . The length changed 26 mm while the width changed 8 mm.

The change in delamination area may be attributed to the change in bending stiffness that occurs between different stacking sequences. From classical bending theory, the bending stress is inversely proportional to the bending stiffness of the structure. Therefore, an increase in the stiffness will decrease the bending stresses for a given applied moment. The stiffness within a ply is maximum when the ply orientation is perpendicular to the plane of bending. Conversely, the stiffness is a minimum when the ply orientation is parallel to plane of bending. Stacking plies in a common direction will act to increase the stiffness in the fibre direction while decreasing the stiffness in a direction transverse to the fibres. This will create regions of high and low bending stress concentrations within the laminate plane. The optimum stacking sequence is one which contains a uniform bending stiffness in all directions. For such a layup, the bending stresses are also approximately uniform in all regions.

Examining the layups in Table 4-2, the optimum configuration is layup SF, with an interface angle of 90° . For this layup, the bending stiffness is more uniformly

distributed as compared with the other layups, since the amount of ply grouping in a single direction is minimised. The worst configuration is layup SA, with an interface angle of 15° . For this layup, the plies are stacked in relatively similar orientations. This will produce high bending stresses transverse to the fibre direction of the bottom ply group, creating a larger damage area.

Figure 4-8: Delamination area at interface 1 vs. interface angle, load 4.5 kN

Figure 4-9: Delamination length and width vs. interface angle, load 4.5 kN

4.4.1.2 Layups containing multiple interface angles

The effect of using multiple interface angles to stack plies was examined through comparisons of two sets of layups, as listed in Table 4-3. Test case 1 examined the effect of using three different interface angles to stack plies. Test case 2 examined the effect of switching layers 4 and 5 in layup LE to give two different interface angles throughout the layup. Each test case compared the layup containing multiple interface angles to a similar layup maintaining a single interface angle between each layer. The applied load for all layups was 7.5 kN.

Table 4-3: Layups Containing Multiple Interface Angles

Layup Code	Stacking Sequence	Interface Angles used
Test Case 1		
LA	$[-45/0/45/90]_{3S}$	45°: All interfaces
MB	$[-45/0/30/45/90/-60]_{2S}$	45°: Interfaces 1, 4, 7, 10, 13, 16, 19, 22 30°: Interfaces 2, 5, 8, 11, 12, 15, 18, 21 15°: Interfaces 3, 6, 9, 14, 17, 20
Test Case 2		
LE	$[-60/-30/0/30/60/90]_{2S}$	30°: All interfaces
ME	$[-60/-30/0/60/30/90]_{2S}$	30°: Interfaces 1, 2, 4, 6, 7, 8, 10, 13, 15, 16, 17, 19, 21, 22 60°: Interfaces 3, 5, 9, 11, 12, 14, 18, 20

For test case 1, the topographical delamination area as well as delamination areas at first 4 interfaces were assessed for both layups LA and MB. A comparison of results is found in Figure 4-10. Stacking plies at multiple interface angles in layup MB was found to increase the delamination size at each of the 4 interfaces over layup LA. A different trend in the delamination sizes from interface-to-interface is also seen. For layup LA, the

delamination size decreases at each higher interface. For layup MB, the delamination size initially increases at interface 2, then successively decreases at interfaces 3 and 4. From the results given in Section 4.4.1.1, decreasing the interface angle at an interface is expected to increase the delamination area. This occurs with interfaces 2 and 3. However, the delamination area at interfaces 1 and 4, which contains no change in the interface angle, also increases. Therefore using multiple interface angles to stack plies will affect delamination damage at every interface. For this test case, the overall effect of using multiple interface angles reduces the damage resistance of the layup, as seen in a comparison of topographical delamination area in Figure 4-10. This effect can also be seen in Figure 4-6 for the experimental test layups 1 and 4. Test layups 1 and 4 have the same stacking sequences as layups LA and MB, respectively, and both test layups were impacted at relatively the same impact energy. The damage area for the multi-interface angle specimen (test layup 4) was found to be greater.

A similar comparison of delamination areas was made for test case 2. Delamination damage was examined at interfaces 3, 4, and 5, for both layups LE and ME. Interfaces 3 and 5 of layup ME are stacked with an interface angle of 60° , while interface 4 is stacked at 30° . Layup LE maintains a constant interface angle of 30° at each interface. A comparison of delamination areas including the topographical area is found in Figure 4-11. Two very different damage trends exist between the two layups. For layup LE, the delamination size decreased at each higher interface, in a similar fashion to

layup LA. For layup ME, the delamination size dramatically decreased at interface 4, and increased again at interface 5. This trend indicated that delaminations were greater at interfaces which are stacked at an angle of 60° as compared with interfaces stacked at 30° . This is contrary to expectations based on findings from Section 4.4.1.1. Larger interface angles should contain smaller delaminations. The topographical areas for both layups were found to be the same, as the changes in the delamination area occurred at interfaces away from the back face.

Comparing interfaces 3 and 5, which contain a change in interface angle, also reveals some interesting results. Interface 3 showed a slight reduction of 3 mm^2 in delamination area when changing the interface angle from 30° to 60° . However, the trend is reversed for interface 5, where changing the interface angle from 30° to 60° resulted in an increase of 9 mm^2 . Again, this is contrary to findings in Section 4.4.1.1. An increase in interface angle should decrease delamination area.

The results from the two test cases revealed that the extent of delamination damage depended on the particular stacking sequence being used. Using multiple interface angles to stack plies affected the delamination size at each interface differently. The delamination size was found not to be directly associated with its corresponding interface angle, as different trends in damage were observed for a given angle. For the two cases analysed, using multiple interface angles to stack plies was found to increase the damage area as compared to stacking plies with a constant interface angle.

Figure 4-10: Comparison of delamination areas between layups LA and MB, test case 1, load 7.5 kN

Figure 4-11: Comparison of delamination areas between layups LE and ME, test case 2, load 7.5 kN

4.4.2 Ply Orientation

The orientation of a ply relative to the boundaries of a coupon specimen will alter the bending properties of the specimen. This will affect both the internal stresses and predicted delaminations. The effects of ply orientation were examined for this study using the layups listed in Table 4-4. Layups LA, GB-GF are all quasi-isotropic layups with a constant interface angle of 45° maintained throughout each layup. Layup LA is the base layup used for comparisons. Layups GB through GD, are each similar to layup LA except that the entire stacking sequence is successively rotated by 45° . Layups GE and GF use a reverse stacking sequence as compared to layup LA, stacking plies clockwise instead of counter-clockwise. Three cross-ply layups, GG, GH, and LI, were also considered in this study. Each of the three cross-ply layups start the stacking sequence with a different ply orientation.

Altering the orientation of the plies with respect to the plate boundaries was found to result in a large variation of topographical delamination areas, as shown in Figure 4-12. The best performance for the quasi-isotropic layups is given by layup GD at 58 mm^2 . Layup GD starts the stacking sequence with a 90° degree ply, aligning the delamination at the first interface along the shortest in-plane dimension of the plate. In contrast, the worst performance for the quasi-isotropic layups occurs with layup GB at 90 mm^2 . In this case, the first ply is oriented at 0° , aligning the delamination with the largest in-plane

dimension of the plate. For the cross-ply layups, Layup GG, with plies oriented at -45° and 45° , performed significantly better than the layup LI, with plies oriented at 0° and 90° .

Quasi-isotropic layups LA, GC, GE, and GF, with the first ply oriented at either 45° or -45° , resulted in delamination areas which are between layups GD and GB. Layups with the second ply oriented at 90° , namely GC and GE, performed better than layups LA and GF with the second ply oriented at 0° . Layups GE and GF, using a clockwise stacking sequence, were found to give to the same damage areas as compared with the counter-clockwise stacking sequences GC and LA, respectively. This is expected since clockwise and counter-clockwise stacking sequences have identical bending stiffnesses. The only difference in the two stacking methods is the orientation of the delamination shape.

Table 4-4: Layups Analysed for Geometric Orientation Study

Layup Code	Stacking Sequence
LA	$[-45/0/45/90]_{3S}$
GB	$[0/45/90/-45]_{3S}$
GC	$[45/90/-45/0]_{3S}$
GD	$[90/-45/0/45]_{3S}$
GE	$[-45/90/45/0]_{3S}$
GF	$[45/0/-45/90]_{3S}$
GG	$[-45/45]_{6S}$
GH	$[90/0]_{6S}$
LI	$[0/90]_{6S}$

Figure 4-12: Comparison of topographical damage areas for geometric orientation study. load 7.5 kN

The cause of the variations observed above may be explained by examining the transverse displacements of the plate. A typical distribution of the predicted transverse displacement is illustrated in Figure 4-13 for the first ply of a $[-45/0/45/90]_{3S}$ layup. The displacements are indicated as negative quantities. The greatest displacement will occur at the centre of plate due to the applied load. Away from the centre, the transverse displacement will decrease. The greatest reduction will occur along the shortest dimension of the plate, in this case the width. Along the length of the plate, the transverse displacements decrease more gradually with respect to distance away from the centre. The amount of transverse displacement at any given location will be largely affected by the dimensions of the plate.

The internal stress distribution will resemble the distribution of transverse displacements which are exhibited in Figure 4-13, since the stresses which create delaminations are a function of transverse displacement. The high stress gradient which exists along the width will act to constrain the size of the delamination. Therefore, delaminations which are oriented along the width of the plate will be smaller as compared to delaminations which are oriented along the length. The trend may be seen in a plot of delaminations areas at interface 1 for layups GB, GC, and GD, given in Figure 4-14. As the orientation of the delamination is rotated from the length to the width (from 0° to 90°), the area decreases.

Figure 4-13: Predicted transverse displacement distribution for layup LA, load 7.5 kN (dimensions are in mm)

Figure 4-14: Predicted delamination damage at interface 1 for layups containing an interface angle of 45°

The performance of the cross-ply layups is also explained in a similar fashion. For layup LI, an equal number of plies are oriented in the 0° direction as the 90° direction. As a result, interfaces with the bottom ply oriented at 0° will exhibit large delaminations, while interfaces with bottom ply oriented at 90° will exhibit small delaminations. For layup GG, each interface will exhibit relatively the same delamination size due to common geometric orientation of the plies relative to the boundaries. This will reduce the size of the topographical area. Layup GH, with a starting ply orientation of 90° , will have a relatively small delamination at interface 1, but will have a much larger delamination at interface 2, since the second ply is oriented in the 0° direction. As the delamination at interface 2 is larger than the delaminations for layup GG, the topographical area for layup GH also will be larger than GG, but still significantly less than layup LI.

The results of this study indicated that the ply orientation relative to the boundaries of the impacted specimen will affect delamination damage. The manner in which the delamination is affected will depend on the geometry of the specimen. This is an important point to consider when applying results from one configuration to another, such as applying results obtained from coupon tests to full scale structures. For the specimen geometry used in this thesis, placing the first ply at 90° for layups with an interface angle of 45° was found to give lowest damage area. For cross ply layups, the optimum configuration was $\pm 45^\circ$.

4.4.3 Ply Grouping

The effect of grouping plies together was examined for the layups listed in Table 4-5. Each layup contains an equal number of plies oriented in each of the following fibre orientations: 0° , 45° , -45° , and 90° . A constant interface angle of 45° was maintained throughout each layup. Layup LA, a quasi-isotropic layup with no ply groupings, was used as the base layup for comparisons. Layups PB, PC, and PD have ply groupings of 1 and 2 plies arranged in different combinations. Layup PE contains equal ply groupings of 3 plies thick.

A comparison of the predicted topographical damage areas, given in Figure 4-15, revealed that ply grouping reduces the damage resistance in a laminate. The base layup LA with no ply grouping contained the lowest damage at 78 mm^2 . As the amount of ply grouping increased, the damage area increased dramatically. The damage area for layup PE was double the area of Layup LA at 158 mm^2 . The location of the ply grouping also has an effect on the damage resistance. A larger damage area will result when the ply grouping occurs near the impact and back faces of the laminate, as shown for Layup PB. An improvement of 8 mm^2 is made when the ply grouping occurs at the midplane. The optimum configuration is achieved by uniformly dispersing the ply grouping through the laminate, as done for Layup PD. Layup PD contained the least damage at 83 mm^2 of the three layups contain 2 ply thickness groupings.

Stacking plies of the same fibre orientation together will increase the stress concentration at the adjacent interfaces, due to the increased bending stiffness within that ply group. This increase in stress concentration will, in turn, create larger delaminations. Ply grouping will also reduce the number of interfaces available for delamination, since delaminations can occur only at interfaces which contain a difference in fibre orientation between the adjoining plies. Since delamination acts to absorb energy from an impact, reducing the number of locations available for delamination will increase delamination size at the remaining interfaces.

Table 4-5: Layups Analysed for Ply Grouping Study

Layup Code	Stacking Sequence
LA	$[-45/0/45/90]_{3S}$
PB	$[-45_2/0_2/45_2/90_2/-45/0/45/90]_S$
PC	$[-45/0/45/90/-45_2/0_2/45_2/90_2]_S$
PD	$[-45_2/0/45/90_2/-45/0_2/45_2/90]_S$
PE	$[-45_3/0_3/45_3/90_3]_S$

Figure 4-15: Comparison of topographical delamination areas for ply grouping study. load 7.5 kN

4.5 Damage Resistance of General Layups

In the previous section, the effects of three different stacking sequence parameters were studied. Each parameter was studied individually by analysing test layups which isolated the effects of the single parameter on the damage resistance. In this section, the damage resistance performance is assessed for layups ranging from quasi-isotropic to general orthotropic. For these layups, all three parameters are altered at once, allowing the study of their combined effects on the damage resistance. Several layups examined in this study are based on layups which are used in industry or layups previously examined by other researchers.

4.5.1 Quasi-Isotropic Layups

The performance of various quasi-isotropic layups was assessed for damage resistance capability. The layups examined for this study are listed in Table 4-1. The predicted topographical damage area was determined for each layup and plotted against interface angle in Figure 4-16. Layups with interface angles of 45° and 60° gave the best performance with an area around 75 mm². Layups with interface angles of 15° and 90° gave the worst performance with areas 121 mm² and 115 mm² respectively.

The effect of the various layups on delamination sizes at individual interfaces is seen in Figure 4-1. Layup LF, with an interface angle of 15°, had the largest

delaminations at the first four interfaces. As the interface angle was increased, the delamination length tended to decrease while the width increased. The overall effect reduced the delamination size. The optimum interface angle was between 45° and 60° , where large reductions in delamination size were seen when compared to layup LF. The stresses within layups LA and LG were more evenly distributed around the plate, resulting in smaller delaminations. As the interface angle was increased past 60° , both the width and length of the delaminations increased. For layup LI, a significant increase in delamination area was seen, particularly in interfaces 1 and 3.

The observed results may be explained using the findings obtained from the parametric study performed in Section 4.4.1. For layups with a small interface angle, the damage at each interface was larger due to increased bending stiffness of the laminate. The increase in stiffness resulted from plies being stacked at similar orientations. The optimum configuration was attained by using an interface angle of 45° or 60° . For these layups, the plies were stacked in a manner that gave similar stiffness properties in all directions. For layups with an interface angle above 60° , the ply directions for every second layer were again stacked in similar orientations. This increased the stiffness along particular fibre directions, while reducing the stiffness in other directions. The increase in stiffness also increased the damage area as compared to Layup LA. However, the increase in damage area was still smaller than the damage area predicted for layups with small interface angles.

Figure 4-16: Predicted delamination area vs. interface angle for quasi-isotropic layups listed in Table 4-1

4.5.2 Orthotropic Layups

In this section, other orthotropic layups were examined which did not contain in-plane isotropic characteristics. These types of layups often contain a disproportionate number of plies oriented in a given direction or contain several interface angles at different interfaces throughout the laminate. In practice, orthotropic layups are used to tailor the strength and stiffness properties of a laminate for a desired application. When designing for in-plane strength, several permutations exist to stack plies to give the desired strength properties, but each permutation will have different impact damage resistance capabilities. To examine the effect of different stacking sequences on damage resistance, several layups were studied as listed in Table 4-6. The orientation of the plies within each layup is restricted to directions 0° , 45° , -45° , and 90° . Results from the quasi-isotropic layup LA are repeated here for comparison purposes. Each layup contains the same number of plies in each of the four possible orientations, with the exception of layup MC. A comparison of topographical damage areas for each layup is found in Figure 4-17.

Layup MC was designed to study both the effects of placing $\pm 45^\circ$ plies at the surface layers and ply grouping. The use of $\pm 45^\circ$ plies is designed to improve impact damage resistance at surface layers, while ply grouping the remaining laminae is designed to reduce fabrication costs. The stacking sequence is similar to common

laminates used in aircraft structures, previously studied by Ambur *et al.* (1995). This particular layup results in a noticeably smaller damage area of 57 mm^2 as compared with the base layup LA at 78 mm^2 . Ply grouping the 0° and 90° laminae did not appreciably increase the damage area since the ply grouping was located away from the surface layers.

Layup MD is a laminate which was examined previously by Dost *et al.* (1991). The stacking sequence was designed to resist impact damage using a strength of material approach at individual matrix cracks which connect delaminations between layers. The layup showed an improvement over the base layup LA with a damage area of 66 mm^2 . However, the damage area was higher than layup MC. This is due to the large number of plies at 90° and -45° orientations stacked near the surface layers. This acts to increase the bending stiffness, and subsequently the stresses, in these orientations.

Another variation of a stacking sequence using ± 45 plies at the surface layers is layup MF. This layup is very similar to the base layup LA with a set of 4 plies which are repeated through the laminate. Two interface angles, 90° and 45° , are used in the stacking sequence. The damage area which is predicted by the FE model for layup MF is virtually identical to layup MC. Changing the interface angle when stacking the surface plies will create a slightly higher damage area for layup MF. Otherwise, both layups can be seen to have comparable damage resistance capabilities.

Layup MG maintains a constant interface angle of 45° throughout the layup, but stacks the 90° plies closer to the outer surfaces and the 0° plies closer to the midplane. This orients the delamination along the width of the plate at the surface plies in attempt to reduce delamination size. From the results given in Figure 4-17, this layup was not effective in improving damage resistance. The damage area for layup MG at 81 mm^2 is slightly higher than the base layup LA. Again this is due to stacking plies at the same orientation close together. For layup MG, the 90° plies are stacking in close proximity to each other at the back face, increasing both the stiffness and stress at this orientation.

Table 4-6: Stacking Sequences Analysed for Orthotropic Layup Study

Layup Code	Stacking Sequence	Effect Studied
LA	$[-45/0/45/90]_{3S}$	Base Layup
MC	$[\pm 45_2/90_2/\pm 45_2/0_2]_S$	Placing ± 45 plies at the surface layers, multiple interface angles, ply grouping
MD	$[45/(90/-45)_3/(0/45)_2/0]_S$	Placing 90° plies near the surface layers, ply grouping sets of laminae
MF	$[\pm 45/90/0]_{3S}$	Placing ± 45 plies at the surface layers, multiple interface angles
MG	$[90/45/90/-45/90/45/0/-45/0/45/0/-45]_S$	Placing 90° plies near the surface layers

Figure 4-17: Comparison of predicted delamination areas for various orthotropic layups

4.6 Summary

The damage resistance capabilities of various types of layups were examined using the FE model. Damage resistance of a layup was assessed by comparing its topographical delamination damage area to the base layup $[-45/0/45/90]_{3S}$. From a comparative analysis of topographical damage within laminates generated both numerically and experimentally, the FE model was found to be suitable for performing comparative studies of damage resistance. The FE model gave reasonable predictions of delamination damage at interfaces near the back face, but under predicted the delaminations for the remaining interfaces. The discrepancy between the FE predictions and the experimental results is primarily due to the lack of progressive damage modelling. As a result, the FE model was not capable of predicting the actual damage area.

Three parameters affecting stacking sequence were studied: interface angle, geometric orientation of plies, and ply grouping. Each parameter significantly affected the damage resistance. In general, stacking plies to give uniform stiffness properties in all in-plane directions improved damage resistance. Ply grouping or stacking plies in similar orientations increased both the stiffness and damage in the stacked orientation, thus reducing damage resistance. With careful attention to the geometry and boundary

supports of the specimen, the plies may be stacked in a manner to increase the damage resistance.

From the analysis performed in this chapter, the following guidelines are proposed to improve the impact damage resistance of composite plates:

- Avoid ply grouping laminae or stacking laminae in similar orientations.
- Avoid stacking laminae at interface angles below 45° .
- Give attention to the orientation of laminae relative to the plate boundaries.

For rectangular plates, the following guidelines are suggested:

- For stacking sequences maintaining an interface angle of 45° , start the stacking sequence with the first lamina oriented along width (the shortest dimension) of the plate.
- For stacking sequences maintaining an interface angle of 90° , start the stacking sequence with the first lamina oriented at 45° or -45° to the width of the plate.

Chapter 5

Laminate Ranking Method for Damage Resistance

5.1 Introduction

The ability to predict the damage resistance of composite materials would greatly assist designers in developing structures capable of withstanding impacts from foreign objects. As reported in Chapter 2, this has proved to be a challenging task, due to the many factors which may affect the damage state. To predict impact damage, the finite element method is often used due to its ability of modelling the dynamic response of the specimen and the progressive damage propagation. However, this type of prediction method has found only limited usage in the preliminary design stage due to the model complexity and the time required for analysis.

A simpler approach would be to use a parameter which is directly related to the damage state. With this parameter, laminates could be ranked for damage resistance. Predictions of the damage area could then be made by correlating the ranking parameter to baseline experimental data. The assessment of this parameter would be faster as compared to the required computational time when using detailed analytical or numerical

modelling. Therefore, using a parameter to predict damage would be suitable for preliminary design analysis. Ideally, this damage resistance parameter should include the effects of material, stacking sequence, and thickness on the damage state.

Previous attempts to find a damage resistance parameter have met with only limited success. The most successful attempt was made by Lagace *et al.* (1993), who linked the damage state to the peak contact force of the impact event; a finding which has been echoed by Straznicky *et al.* (1995). The use of peak contact force as a damage resistance parameter is illustrated in Figure 5-1, where peak contact force is plotted against the topographical damage area for four different stacking sequences. All data plotted in Figure 5-1 was obtained from Vietinghoff (1994). For each layup, a near linear relationship existed between peak contact force and damage area until a threshold point was reached. Specimens impacted at energies above the threshold point showed a further increase in the measured damage area, but no further increase in the peak contact force. The threshold point is described in greater detail in Section 5.4. The correlation between peak contact force and damage area was unique for each stacking sequence, material, and thickness. Therefore, peak contact force may be used to rank only laminates with a common stacking sequence, material, and thickness, impacted at an energy below the threshold point.

The three stacking sequence parameters examined in Chapter 4, ply grouping, ply orientation, and interface angle, were each found to significantly affect the damage

resistance. Nevertheless, the results of this parametric study did not reveal any identifiable trends which directly link damage resistance to any of these parameters.

This chapter examines the use of a damage resistance parameter based on the bending strain to rank laminates for damage resistance. Bending strain provides a convenient method of ranking laminates with respect to differences in material, thickness, and stacking sequence, as it is a function of all three variables. For this thesis, laminate rankings are made with respect to changes in stacking sequence only. However, methods of ranking laminates with respect to changes with material and thickness are also briefly discussed. A review of the appropriate lamination theory is presented first in Section 5.2. The derivation and proposal of the damage resistance parameter are then presented in Section 5.3. An evaluation of the proposed damage resistance parameter is given in Section 5.4, followed by a discussion of the results in Section 5.5.

Figure 5-1: Laminate ranking using peak contact force (data from Vietinghoff, 1994)

5.2 Lamination Theory

From classical lamination theory (Gibson, 1994), the strains $\{\epsilon\}$ and curvatures $\{\kappa\}$ in the laminate are related to the applied forces $\{N\}$ and moments $\{M\}$ by:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix} \quad (5-1)$$

where $[A]$ is the extensional stiffness matrix, $[D]$ is the bending stiffness matrix, and $[B]$ is the coupling matrix between the strains and curvatures. The classical laminate theory assumes that each ply is in a state of plane stress. Perfect bonding is assumed to exist between plies with no slippage, and small deflections are assumed to occur such that lines drawn normal to each ply remain normal after bending.

Matrices $[A]$, $[B]$, and $[D]$ are defined as follows:

$$[A] = \sum_{k=1}^N [\bar{Q}]^k (z_k - z_{k-1}) \quad (5-2)$$

$$[B] = \frac{1}{2} \sum_{k=1}^N [\bar{Q}]^k (z_k^2 - z_{k-1}^2) \quad (5-3)$$

$$[D] = \frac{1}{3} \sum_{k=1}^N [\bar{Q}]^k (z_k^3 - z_{k-1}^3) \quad (5-4)$$

where k is the ply number, N is the total number of plies, and z_k is the distance from the laminate mid-plane to ply k . Variables k , N , and z are illustrated in Figure 5-2.

Matrix $[\bar{Q}]^k$ is the rotated elastic modulus matrix at ply k , relating the stresses and strains by:

$$\{\sigma\}^k = [\bar{Q}]^k \{\epsilon\}^k \quad (5-5)$$

Matrix $[\bar{Q}]^k$ was transformed from local laminate coordinate system to the direction of the calculated stresses and strains by the standard rotation matrix, given in equation (2-1).

Each stiffness matrix given in equation (5-1) is symmetric and has a dimension of 3 by 3. For matrix $[D]$, the terms are defined as:

$$[D] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \quad (5-6)$$

The stiffness component along and transverse to the fibre direction are primarily governed by coefficients D_{11} and D_{22} . The stiffness due to in-plane shear is given by coefficients D_{66} . Coupling between stiffnesses along and transverse to the fibre direction is given by coefficient D_{12} . Coupling between tensile and shear stiffnesses is given by coefficients D_{16} and D_{26} . Coefficients for the other stiffness matrices $[A]$ and $[B]$ follow in a similar fashion. For layups symmetric about the mid-plane, the coupling matrix $[B]$ is zero as are coefficients D_{16} and D_{26} for the bending stiffness matrix $[D]$ and coefficients A_{16} and A_{26} for the extensional stiffness matrix $[A]$.

Figure 5-2: Laminate nomenclature

5.3 Proposal of a Damage Resistance Parameter

A number of methods have been proposed in literature to predict impact damage using an empirical approach, as described in Section 2.6.1. While each method predicted some of the key characteristics of impact damage, none were able to predict damage for a variety of stacking sequences. The bending mismatch parameter proposed by Liu (1988), as given in equation (2-7), gave reasonable predictions for two ply specimens, but was found to give inaccurate predictions for multi-ply laminates. Morita *et al.* (1995) proposed a modified version of the bending mismatch parameter, as given by equation (2-9), correcting for some of the limitations experienced when using equation (2-7). The form of this modified parameter β closely resembles the bending stress of a laminated beam, which is integrated with respect to angle θ . Parameter β mainly measures the maximum in-plane longitudinal stresses σ_1 of each layer, since the greatest beam bending stresses occur in the fibre direction. However, Choi and Chang (1992) have found that delamination is caused by the transverse in-plane stresses σ_2 , not the σ_1 stresses which parameter β measures. As a result, the proposed parameter β will lead to erroneous predictions of damage.

A new damage resistance parameter is proposed for this thesis, addressing the limitations experienced by previously proposed parameters. The damage resistance parameter is based on the bending strain in the laminate. The bending strain gives an indirect measure of the damage susceptibility of a laminate, as regions with high strain

will contain greater amounts of damage. To assist in the derivation of the damage resistance parameter, two coordinate systems are introduced. A *principal coordinate system* is defined with axes x_p , y_p , and z_p , aligned in the same directions as the global coordinate system defined in Section 2.2.2, except with the origin placed at $(0,0,t/2)$ in global system coordinates, where t is the thickness of the laminate. The *bending coordinate system* is defined using three axes x_b , y_b , and z_b , having an origin located also at $(0,0,t/2)$ in global system coordinates, with the z_b axis aligned in the same direction as axis z_p of the principal coordinate system. Axes x_b and y_b are rotated about the z_b axis by an angle α to axes x_p and y_p of the principal coordinate system respectively. The *plane of bending* is defined to be a plane defined by the y_b and z_b axes of the bending coordinate system at $x_b = 0$. A *moment vector* is defined with a base at the origin and tip at point $(0,-1,0)$ in the bending coordinate system. All moments M used in the calculations are referenced with respect to this moment vector. The orientation of each ply always remains fixed with respect to the principal axis. However, the orientation of each ply with respect to bending coordinate system may change based on a change in angle α . The two coordinate systems and plane of bending are defined in Figure 5-3.

Figure 5-3: Definition of coordinate systems and plate nomenclature

A damage resistance parameter of the following form is proposed:

$$\boxed{DR = \int_{\alpha=0^{\circ}}^{360^{\circ}} \int a(\alpha) \partial a \partial \alpha} \quad (5-7)$$

Parameter $a(\alpha)$ is a measure of the topographical damage radius measured from the plate centre at angle α . A measure of the total damage area is then obtained by integrating parameter $a(\alpha)$ with respect to a and angle α to give the damage resistance parameter DR. Parameter DR is not intended to directly predict damage, but rather to give a value that is related to the total topographical damage. A laminate with a high DR value indicates a high susceptibility to impact damage. Therefore to improve damage resistance, parameter DR is to be minimised.

Bending strain was used as a measure of the damage radius $a(\alpha)$. Computing the bending strain using a full analytical treatment of the plate bending problem is formidable for even the most basic stacking sequence configurations. Therefore, an approximate method was used instead by examining an equivalent one-dimensional beam bending problem. At any desired in-plane direction α , the plate was treated as a beam of unit width with a constant applied moment, containing the same ply orientations as the original plate with respect to the desired calculation direction x_b . Stated in different terms, the cross-sections of both the plate and the beam formed by the plane of bending, as illustrated in Figure 5-3, will have the same layup distribution. The strains were then calculated by classical beam bending theory for the equivalent beam cross-section of unit

width. The in-plane strains may be calculated at different orientations of α in a similar manner to give a two-dimensional profile of the in-plane strains.

Approximating the plate problem as a beam introduces a few errors. The first error is that the beam strain calculations do not account for the geometrical shape of the specimen. The geometrical shape greatly influences the value of the calculated strains; thus, ignoring this effect may lead to large errors. This was compensated by including an out-of-plane displacement parameter in the calculation of parameter $a(\alpha)$. The beam calculations used a constant moment of unity at all angles of α . However, the actual applied bending moments would not be constant with respect to angle α , due to the twisting moments that are also present within the plate. Ignoring the twisting moments will lead to errors of approximately 10% to 20% in the strain calculations. The error is largely dependent on the stacking sequence and geometry of the laminate. Using a stacking sequence containing uniform in-plane stiffness properties with respect to angle α will greatly reduce the error, since the applied moments at any given point are relatively the same for all angles of α . No compensation for this error was made in the formulation of parameter $a(\alpha)$, as the amount of error was not considered to be critical.

Based on the above approach, a damage radius parameter $a(\alpha)$ is proposed as:

$$a(\alpha) = \frac{w(r, \alpha) \cdot \epsilon_b(\alpha)}{\epsilon_{cr}} \quad (5-8)$$

where:

ε_b : Maximum bending strain of equivalent beam section at angle α

w : Parameter proportional to the out-of-plane transverse displacement

ε_{cr} : Critical ply strain

Parameter ε_b is the primary measure of the damage radius, accounting for effects of stacking sequence. The displacement parameter w , specified in terms of radius r and angle α , accounts for the effects of the plate geometry and support conditions on the damage area. Radius r , as illustrated in Figure 5-3, is chosen to give best profile of the displacement. The value of r is arbitrary, but should be located away from both the plate boundaries and the point of loading. A value of $r = 0.1b$ is suggested for this thesis, where b is the smallest dimension of the specimen. The radius r must remain constant for all angles of α . The critical ply strain ε_{cr} is included in parameter $a(\alpha)$ as a method of accounting for different material strengths when ranking laminates. Here, the critical ply strain is defined as the maximum permissible strain which the specimen is allowed to sustain. This could be the limit strain or ultimate strain of the material depending on the requirements of the structure. Parameters ε_b , w , and ε_{cr} are described fully in Sections 5.3.1 through 5.3.3, respectively.

The proposed damage resistance parameter is valid for moderately thick laminates whose response to transverse loading is primarily bending. Thin laminates which respond to transverse loading primarily by in-plane membrane forces are not covered by this parameter. The damage resistance parameter is related to internal damage only: delaminations and matrix cracking. Back face damage such as ply blow-out, fibre breakage, and matrix cracking are not predicted. Equations derived in this thesis are valid for mid-plane symmetrical laminates which conform to the Boeing Specification BSS 7260 (Boeing, 1988), as discussed in Section 3.2.2. Equations for other specimen and test configurations may be derived by following the methodology presented in Sections 5.3.1 and 5.3.2.

5.3.1 Bending strain parameter ϵ_b

Bending strain was chosen as the basis to assess the extent of impact damage, allowing for easier identification of the critical areas within the laminate as compared with using bending stress. Using a strain-based approach, a ply is assumed to fail when a critical amount of strain is reached, regardless of the ply orientation. Therefore, the maximum bending strain can be used to identify the critical locations of the laminate. Using a stress-based approach, identification of critical locations is more difficult as the failure stress depends on the ply orientation. Since the stress distribution through-thickness is not continuous, a stress-based failure criterion must be applied for each ply to determine failure.

From classical beam bending theory, the bending strain in the x_b direction for a beam of unit width is defined as:

$$\varepsilon_{x_b} = -z\kappa_x \quad (5-9)$$

where κ_x is the beam curvature and z is the distance through-thickness measured from the neutral axis. Using equation (5-1), κ_x may be expressed in terms of an applied moment M :

$$\kappa_x = \frac{M}{D_{11}} \quad (5-10)$$

Equation (5-10) is valid only for mid-plane symmetrical laminates such that $[B]=0$ and $D_{16}=D_{26}=0$. Placing equation (5-10) into (5-9) gives:

$$\varepsilon_{x_b} = -\frac{Mz}{D_{11}} \quad (5-11)$$

As previously noted in Section 4.2, the primary stresses/strains which create delaminations at interfaces located near the back face are the in-plane transverse stresses σ_2 and strains ε_2 . The σ_2 stresses and the ε_2 strains have been observed to form a delamination damage area, with the major axis closely aligned with the fibre direction of the lower ply of the interface, as illustrated in Figure 2-5. Relating these observations to equation (5-11), the strains promoting delamination damage along the major axis are oriented along the y_b axis for plies whose fibres are aligned in the x_b direction. To insure

the major axis of the predicted delamination is aligned with the fibre direction, the bending strain parameter is proposed as:

$$\varepsilon_b(\alpha) = -\frac{c}{D_{22}(\alpha)} \quad (5-12)$$

where c is the distance through-thickness from the neutral axis to the bottom face.

Equation (5-12) represents the maximum bending strain per unit width in the y_b direction for a unit moment. The applied moment is assumed to be constant with respect to angle α , and therefore is not included in damage area parameter. For a given value of α , parameter ε_b will be greatest for plies whose fibres are oriented along the x_b direction. When polar-plotting parameter ε_b versus α for a given laminate, the resulting contour will resemble a topographical damage contour for the laminate.

It is useful to note that the usage of either D_{11} or D_{22} in equation (5-12) will give the same results when calculating the damage resistance parameter DR, given in equation (5-7). This fact is due to the close relationship between both bending coefficients:

$$D_{11}(\alpha) = D_{22}(\alpha + 90^\circ) \quad (5-13)$$

The bending stiffness coefficient D_{22} in equation (5-12) is defined by equation (5-4), where coefficient $\bar{Q}_{22}^k$ is defined as a function of ply orientation θ with respect to axis x_b :

$$\bar{Q}_{22}^k(\theta) = Q_{11}^k \sin^4 \theta + Q_{22}^k \cos^4 \theta + 2(Q_{12}^k + 2Q_{66}^k) \sin^2 \theta \cos^2 \theta \quad (5-14)$$

Q_{ij}^k are the reduced in-plane elastic modulus coefficients at ply k :

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}} & Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}} \\ Q_{12} &= \frac{\nu_{12}E_1}{1 - \nu_{12}\nu_{21}} & Q_{66} &= G_{12} \end{aligned} \quad (5-15)$$

Angle θ can be redefined in terms of angle α using the following relationship:

$$\theta = \phi - \alpha \quad (5-16)$$

where angle ϕ is defined to be the ply orientation with respect to the principal coordinate axis x_p . Angle ϕ is constant for each ply regardless of angle α .

5.3.2 Displacement parameter w

The displacement parameter w is included in the calculation of the damage radius parameter $a(\alpha)$ to account for the effects of specimen geometry and support conditions on the damage shape. A different analytical or numerical expression for displacement w will exist for each unique geometric and support configuration. For this thesis, a rectangular specimen was modelled with simply supported edges, as described in Section 3.2. For these conditions, the displacement was approximated using classical plate theory (Ashton and Whitney, 1970). Using a Navier series solution, the displacement is given as:

$$w = \frac{1}{\pi^4} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{D_{11} \left(\frac{m}{a}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{mn}{ab}\right)^2 + D_{66} \left(\frac{n}{b}\right)^4} \quad (5-17)$$

where q_{mn} is the load function, a and b are the length and width of the plate. A unit point load was assumed at the plate centre $(a/2, b/2)$, giving an expression for q_{mn} as:

$$q_{mn} = \frac{4}{ab} \sin \frac{m\pi}{2} \sin \frac{n\pi}{2} \quad (5-18)$$

Equation (5-17) was derived for a mid-plane symmetrical laminate which satisfies the conditions $[B]=0$ and $D_{16}=D_{26}=0$. Hygrothermal effects and effects of transverse shear deformation were neglected, and static loading was assumed. Analytical displacement equations for other geometry and support conditions may be derived as outlined by Ashton and Whitney (1970).

5.3.3 Critical strain parameter ϵ_{cr}

Composite materials vary in both stiffness and strength. Therefore when ranking laminates of different materials, both the effects of stiffness and strength must be taken into account. The stiffness changes are accounted for by the bending strain parameter ϵ_b , given by equation (5-12). An extra parameter must be included in equation (5-8) to account for the differences in strength. The critical strain parameter ϵ_{cr} is proposed to be used as a normalising parameter for material strength. The critical strain represents the maximum permissible bending strain which the specimen is allowed to sustain. For the

materials examined in this thesis, in-plane strains transverse to the fibre direction are considered to be the most critical strains in promoting damage. The critical strain was defined to be:

$$\epsilon_{cr} = \frac{Y_T}{E_2} \quad (5-19)$$

where Y_T is the transverse tensile strength and E_2 is the transverse modulus.

A limit strain or ultimate strain of the material may be used to define the critical strain parameter ϵ_{cr} , depending on the requirements of the structure. However, the definition of the critical strain must remain consistent for all materials examined. The inclusion of the parameter ϵ_{cr} in the definition of damage radius parameter $a(\alpha)$ is a first attempt to correlate the damage resistance of specimens composed of different materials. Other material related properties, such as fracture toughness, were not considered in the formulation of the damage resistance parameter.

5.4 Evaluation of the Laminate Ranking Method

An evaluation was made of the proposed damage resistance parameter to rank laminates for damage resistance. The method was evaluated by comparing the ranked laminate results to experimental data published by Vietinghoff (1994) and Dost *et al.* (1991). Also, laminate rankings made using the damage resistance parameter were compared against predicted FE model results to determine the differences between each method. All specimens examined in this chapter conform with Boeing Specification BSS 7260. The reduced specimen size of 127 mm by 76.2 mm (5 in by 3 in), corresponding to the locations of the hinged supports, was used in the calculations of the damage resistance parameter. Variables used in the damage resistance parameter calculations are given in Table 5-1. The critical strains given in Table 5-1 were calculated using equation (5-19). A FORTRAN program was created to calculate the damage resistance parameter DR. The program is available from the Department of Mechanical and Aerospace Engineering, Carleton University upon request.

To illustrate the calculation procedure of the damage resistance parameter, the parameter $a(\alpha)$, given by equation (5-8), was plotted against angle α for a few selected layups, as shown in Figure 5-4. A unique contour was observed for each layup, varying in both size and shape. Angles which contain the largest values of $a(\alpha)$ are expected to receive the greatest damage. The damage resistance parameter DR, which represents the total area outlined by $a(\alpha)$, is then expected to be related to the total topographical

damage area. The calculated DR values are indicated for each layup in Figure 5-4. For layups LA and LD, the maximum value of $a(\alpha)$ occurs around $\alpha = -25^\circ$, an angle which lies in between the first two ply orientations of -45° and 0° . For layup LF, the maximum value of $a(\alpha)$ occurs around $\alpha = -45^\circ$. Comparing the DR contours to the results from the FE model reveals a number of similarities. The DR contours have a similar shape and orientation as those predicted FEM damage areas, as shown in Figure 4-6 for test layups 1, 6, and 5 corresponding to layups LA, LD, and LF, respectively. The predicted DR values also indicate a similar trend as the FE model; both layups LD and LF have significantly larger DR values than LA. This trend is seen for the FE model in Figure 4-6, where the damage areas of test layups 5 and 6 are larger than test layup 1. Also, the DR contours reveal a similar limitation as the FE model; the interlaminar stresses are not accounted for in the proposed DR parameter, causing an underprediction of the damage at ply orientations away from the back face.

Table 5-1: Parameters Used in the Calculations of the Damage Resistance Parameter

Parameter	Value
General Properties	
Length of plate, a (mm)	127
Width of plate, b (mm)	76.2
Radius for displacement calculation, r (mm)	7.62
Number of terms in fourier series for displacement calculation, n	30
Material Properties	
T800H/3900-2 (Gaudert <i>et al.</i> , 1993; Poon <i>et al.</i> , 1991)	
Longitudinal Modulus, E_1 (GPa)	152
Transverse Modulus, E_2 (GPa)	8.07
Shear Modulus, G_{12} (GPa)	4.14
Poisson's Ratio, ν_{12}	0.35
Critical Strain, ε_{cr}	0.0098
IM7/8551-7 (Dow and Smith, 1988)	
Longitudinal Modulus, E_1 (GPa)	139
Transverse Modulus, E_2 (GPa)	9.38
Shear Modulus, G_{12} (GPa)	4.50
Poisson's Ratio, ν_{12}	0.33
Critical Strain, ε_{cr}	0.0079

Figure 5-4: Polar plot of parameter $a(\alpha)$ for selected layups

The correlation of the damage resistance parameter DR to experimental data was determined for the specimens listed in Table 5-2. Each specimen was impacted at approximately the same energy, ranging from 15.5 to 19.2 J. Although the range of impact energies is small, the differences in the impact energy will have an effect on the damage size. To account for this effect, the measured damage area was normalised by the impact energy when plotted against parameter DR. For each specimen, the damage resistance parameter DR was calculated, and then normalised by the calculated DR value for layup LA. A plot of the damage resistance parameter DR against the measured damage area is shown in Figure 5-5. A best-fit line was drawn for each material and the R^2 value, a measure of linear correlation, is indicated beside each line. A R^2 value of 1 indicates a linear correlation, while a R^2 coefficient of 0 indicates no linear correlation.

A strong linear correlation was observed for both materials. The correlation was higher for the Toray T800H/3900-2 specimens, due in part to a more accurate assessment of the topographical damage area. The damage areas measured by Dost *et al.* (1991) were reported as a damage diameter, giving an approximation of the actual damage, whereas the damage areas reported by Vietinghoff (1994) were a measurement of the actual damage. Only two specimens, BA and BF, were observed to deviate from the linear trend. The measured damage area for specimen BA was observed to be unusually low as compared with the measured damage areas of the other specimens, and therefore is considered to be an outlier point. Specimen BF, however, indicates a limitation of the

damage resistance parameter. The stacking sequences for both BE and BF are similar except for the switch of the -30° and -60° plies, as listed in Table 5-2. The effect of switching the -30° and -60° plies produces only a minimal stiffness change, as shown by the similar DR values in Figure 5-5. The large increase in damage is hypothesised to be caused by the change in interlaminar stresses at the affected interfaces; an effect which is not modelled by the damage resistance parameter. Also, switching the -30° and -60° plies varies the constant stacking angle of 30° , apparently reducing the damage resistance. This observation was also seen with specimens LA and MB.

As a comparison of the laminate ranking using the FE model and the damage resistance parameter, the FE damage predictions were correlated with the Vietinghoff specimens in Table 5-2, and are plotted in Figure 5-6. The FE data were normalised with respect to predicted damage area of specimen LA, and the measured damage areas were normalised with respect to impact energy. Like the damage resistance parameter, the correlation between the FE results and the measured damage area was linear. The FE results gave a slightly better correlation of the experimental data, with a R^2 value of 0.997 compared to 0.991 for the parametric results.

Table 5-2: Experimental Data Used to Evaluate the Damage Resistance Parameter

Layup Code	Stacking Sequence	Impact Energy (J)	Damage Area (mm ²)
Data from Vietinghoff (1994)			
LA	[-45/0/45/90] _{3S}	15.5	247
LD	[-45 ₃ /0 ₃ /45 ₃ /90 ₃] _S	15.7	1020
LE	[-60/-30/0/30/60/90] _{2S}	15.6	275
LF	[-75/-60/-45/-30/-15/0/15/30/45/60/75/90] _S	15.6	727
MB**	[-45/0/30/45/90/-60] _{2S}	19.2	332
** Test performed by author			
Data from Dost <i>et al.</i> (1991) (Data reported originally as a damage diameter)			
BA	[45/90/-45/0] _{3S}	16.3	115
BB	[45/(90/-45) ₃ /(0/45) ₂ /0] _S	16.3	585
BC	[45/(0/45) ₃ /(90/45) ₂ /0] _S	16.3	638
BD	[45 ₃ /90 ₃ /-45 ₃ /0 ₃] _S	16.3	1320
BE	[30/60/90/-60/-30/0] _{2S}	17.6	423
BF	[30/60/90/-30/-60/0] _{2S}	19.0	638

Figure 5-5: Damage resistance parameter versus measured topographical damage area, Impact energy 15-19J

Figure 5-6: Predicted FEM damage area versus measured damage area, impact energy 15-19J

The ranking of specimens impacted at different energies was also evaluated for layups LA, LD, LE, and LF. As previously mentioned in Section 5.1, peak contact force provided a convenient method of correlating impact damage for specimens with a common stacking sequence, material, and thickness. To correlate specimens with different stacking sequences, the peak contact force was multiplied by the damage resistance parameter to provide a ranking parameter. Using the experimental data previously plotted in Figure 5-1, the peak contact force multiplied by the damage resistance parameter was plotted against the measured damage area, as shown in Figure 5-7. As in previous plots, the damage resistance parameter DR was normalised with respect to the calculated DR parameter for layup LA.

Figure 5-7 indicated two trends for each layup. Specimens impacted at low energies below a threshold point have a linear correlation between the ranking parameter and damage area. Specimens impacted at energies above the threshold point diverged away from the linear trend, showing a further increase in the measured damage area, but only a minimal increase in the ranking parameter. The threshold point for each layup is indicated in Figure 5-7 by the intersection of the main trend line with the vertical threshold line. The threshold energy represented the transition of the damage propagation mechanisms, based on observations made by Vietinghoff (1994). For specimens impacted at energies below the threshold point, the damage propagated primarily by internal damage, delamination and matrix cracking, with a minimal amount of back face

damage. For specimens impacted at energies above the threshold point, the damage propagated through back face damage, ply blow-out, matrix cracking of the bottom ply, and fibre breakage, with a minimal increase to the internal damage. The threshold point was unique for each layup plotted in Figure 5-7.

A linear correlation of data for all four layups was seen for specimens impacted below the threshold points. The data contains some degree of scatter resulting from experimental error in measurement and variability in manufacturing. However, a best-fit line for data below the threshold points contained a R^2 value of 0.953, indicating a high degree of linearity. Damage measured from specimens impacted at energies above the threshold energy are not accounted for by the damage resistance parameter.

To compare the performance of the FE model to the damage resistance parameter, the experimental data of Figure 5-1 were also ranked using FEM damage predictions. Predictions of damage area were made for each layup subjected to a point load of 7.5 kN. The predicted areas were first normalised by the predicted damage for layup LA, then multiplied by peak contact force to give a ranking parameter. The peak contact force * predicted FEM damage was plotted against the measured damage area, as shown in Figure 5-8. As in Figure 5-7, a linear trend was seen for all layups for specimens subjected impacts at energies below the threshold points. Laminate ranking using the FEM predicted damage gave a slightly better correlation to the experimental data, with a R^2 value of 0.970 as compared to 0.953 when using the damage resistance parameter.

Figure 5-7: Laminate ranking of specimens impacted at various energies using the damage resistance parameter (threshold point indicated by intersection of the main trend line with vertical threshold line for each layup)

Figure 5-8: Laminate ranking of specimens impacted at various energies using predicted FEM damage (threshold point indicated by intersection of the main trend line with vertical threshold line for each layup)

To evaluate the differences between the damage resistance parameter and the FE model, the two methods were correlated against each other for a series of stacking sequences listed in Table 5-3. All stacking sequences in Table 5-3 contain plies oriented at 0°, 45°, -45°, and 90°. The D-series layups contain plies stacked randomly to test the parametric ranking method. The L-series layups are quasi-isotropic layups. The M-series layups are general orthotropic layups used in industry, previously examined in Section 4.5.2. The P-series layups test the ply grouping effect, previously examined in Section 4.4.3. A plot of the damage resistance parameter, normalised with respect to layup LA, against the predicted FEM damage is found in Figure 5-9. A good linear trend was found between the damage resistance parameter and the predicted FEM damage, as indicated by the R^2 value of 0.923. However, a fair amount of scatter was observed between the two parameters. This would indicate that although the damage resistance parameter predicts similar trends as the FE model, the two predictions methods are not equivalent.

The differences between the two methods were more pronounced when comparing the quasi-isotropic layups listed in Table 4-1, as shown in Figure 5-10. Each ranked specimen was stacked with a constant interface angle, as noted in Figure 5-10. For layups with an interface angle less than or equal to 60°, the DR parameter correlated well with the predicted FEM damage area. For these layups, an increase in the DR parameter corresponded with an increase in the predicted FEM damage in relatively the same proportions. As the interface angle increased, however, the calculated DR value

deviated greatly from the predicted FEM damage, as noted with layups LH and LI. An increase in the predicted damage did not correspond with an appropriate increase in the calculated DR value for these layups. The cause of deviation between the two methods is examined in Section 5.5.

Table 5-3: Specimen Data used to Compare Rankings Using the Damage Resistance Parameter DR and Predicted FEM Damage

Layup Code	Stacking Sequence	Damage Resistance Parameter DR [~]	Predicted FEM Damage (mm ²)
LA	[-45/0/45/90] _{3S}	1.00	78
LD	[-45 ₃ /0 ₃ /45 ₃ /90 ₃] _S	2.80	158
DA	[-45 ₃ /0/-45/45/0/-45/0/90/-45/0] _S	4.71	210
DB	[(-45/0) ₂ /-45 ₃ /0/90/0/45/-45] _S	7.55	495
DC	[-45/0 ₃ /45/0/90 ₂ /45/0/-45/45] _S	2.06	164
DE	[-45/0/90/0/-45 ₃ /90/0 ₂ /45/-45] _S	2.16	170
DF	[-45/0/90 ₂ /45/-45/0/-45/0/45/-45 ₂] _S	1.08	82
DG	[-45/0/90 ₃ /0 ₆ /-45] _S	1.14	99
MC	[±45 ₂ /90 ₂ /±45 ₂ /0 ₂] _S	1.05	57
MD	[45/(90/-45) ₃ /(0/45) ₂ /0] _S	1.37	66
MG	[90/45/90/-45/90/45/0/-45/0/45/0/-45] _S	1.63	81
PB	[-45 ₂ /0 ₂ /45 ₂ /90 ₂ /-45/0/45/90] _S	1.35	94
PC	[-45/0/45/90/-45 ₂ /0 ₂ /45 ₂ /90 ₂] _S	1.13	86
PD	[-45 ₂ /0/45/90 ₂ /-45/0 ₂ /45 ₂ /90] _S	1.30	83
[~] Damage resistance parameters were normalised with respect to calculated DR value for Layup LA

Figure 5-9: Comparison of damage resistance parameter against predicted FEM damage for layups listed in Table 5-3

Figure 5-10: Comparison of damage resistance parameter against predicted FEM damage for layups listed in Table 4-1

5.5 Discussion

As noted in Section 5.4, the damage resistance parameter was capable of ranking laminates with respect to damage resistance. The rankings correlated reasonably well when compared to the available experimental data, providing a linear correlation between the damage resistance parameter DR and measured damage area as demonstrated in Figures 5-5 and 5-7. These results show great promise for the use of the damage resistance parameter as a means of impact damage prediction. By correlating the damage resistance parameter to baseline experimental data, predictions of impact damage may be made without the need of sophisticated and time-consuming finite element analysis. Table 5-4 shows clearly the advantage of using a damage resistance parameter by comparing the computational time of various prediction methods. The most sophisticated model, created by Majeed (1995), required one week of computational time to analyse a single layup. The damage resistance parameter showed a tremendous improvement in performance over both the Majeed model and the FE model presented in this thesis, requiring a mere 3.4 seconds.

Table 5-4: Comparison of Damage Prediction Methods

Model	Calculation Time (approximate)
Dynamic FE Model with contact analysis and progressive failure modelling (Majeed, 1995)	1 week
Static FE Model with volume elements	2 hours
Damage resistance parameter	3.4 seconds
Computational time was measured for the analysis of a single layup configuration. All times were measured using a Silicon Graphics Challenge L/2 Workstation. Times do not include pre- and post- processing of data.	

The damage resistance parameter, with such an enormous speed advantage, is clearly suited for preliminary design analysis. When initially designing a composite structure, the damage resistance parameter can provide quick estimates of the impact damage resistance for various stacking sequences being considered. By creating an optimising routine, it is conceivable that a program could be developed to find an optimised stacking sequence for damage resistance based on the required number of plies at the various orientations. After finding an "optimised" stacking sequence, a more accurate assessment of the impact damage resistance may be made by either experimental or numerical means. The results of the method could also be used as an input for damage tolerance studies, such as compression-after-impact analysis (Vietinghoff, 1994). Damage tolerance studies require the amount of damage present within the laminate to determine the strength degradation. Using this approach in the preliminary design stage will undoubtedly lower both the costs and time of development.

While the results of using the damage resistance parameter were generally positive, it is by no means a verification of the method. Further experimental testing would be required to determine the limitations and strengths of the method. The evaluation performed in Section 5.4 has indicated some weaknesses in the damage resistance parameter. Analysing a wide variety of stacking sequences has revealed a noticeable degree of scatter between the predicted FEM damage and the damage resistance parameter as shown in Figure 5-9. With the absence of actual experimental data for these specimens, it is difficult to estimate the error of prediction when using the

damage resistance parameter. A particular weakness was found with layups containing stacked plies with interface angles greater than 60° . The damage resistance parameter significantly underestimated the damage for these types of specimens.

The cause of the scatter is due to a number of reasons. The damage resistance parameter, like the FE model, was shown to underpredict damage which is created from internal interlaminar stresses. Stacking sequence changes which cause a change to the interlaminar stresses but not to the overall bending stiffness of the laminate lead to significant errors when ranked by the damage resistance parameter. However, the largest problem with the damage resistance parameter is with the method of assessing damage. The FE model predicts damage by first calculating the transverse in-plane stresses σ_2 and the interlaminar stresses σ_4 and σ_5 at each interface, and then applying a failure criterion. In contrast, the damage resistance parameter does not calculate the equivalent strain distribution at each individual interface, but rather the maximum ϵ_y strain of an entire laminate cross-section. The main contribution to the damage resistance parameter will be the transverse in-plane strains ϵ_2 of the ply or plies oriented closest to angle of calculation α . Since the ϵ_2 strain is a principal strain which causes delamination damage, the damage resistance parameter will be related to the impact damage in most cases. When several plies are closely oriented with the angle of calculation α , the damage resistance parameter will primarily be a measure of the ϵ_2 strains at these α -oriented plies. When the plies are not closely aligned with angle α , the damage resistance parameter will be a measure of both the ϵ_1 and ϵ_2 strains. For layups with a small interface angle, one or more plies will

be always oriented at a similar angle as the angle α , giving a better measure of the ϵ_2 strains which create damage. For layups with a large interface angle, the plies are not as likely to be oriented with the angle α , causing the damage resistance parameter to be a combined measure of ϵ_1 and ϵ_2 strains at any layer. This will lead to erroneous predictions of the damage area.

The damage resistance parameter was primarily evaluated to rank laminates with respect to changes in stacking sequence. A brief examination of ranking laminates with different materials, as shown in Figure 5-5, was shown not be correlated by the damage resistance parameter. Although the damage resistance parameter accounts for the structural stiffness of the material, the final damage is largely a function of the material's fracture toughness properties; an effect not predicted by a strength of materials approach. A ranking parameter would need to account for the fracture toughness properties, in order to rank laminates on the basis of material.

The proposed damage resistance parameter, in its current form as given by equation (5-7), does not account for changes in thickness. A change in thickness will create an associated change in the bending strain. However by integrating the calculated beam strain, as is done in equation (5-7), the changes in strain are effectively squared, biasing the results when making comparisons to laminates with a different thickness. Therefore to include thickness as a basis for ranking laminates, the damage resistance parameter must be modified in a way as not to bias thickness changes when integrating

the calculated beam strain. Also more experimental data are required to determine the effect of altering thickness on the impact damage.

The proposed damage resistance parameter is a first attempt of correlating impact damage with respect to changes in stacking sequence. As noted above, several modifications are required to improve the accuracy and robustness of the method. These modifications, when implemented, will undoubtedly alter the original proposed form as given in equation (5-7). However, the concept of using a damage resistance parameter to predict impact damage has proved to be feasible, in spite of the simplifying assumptions made, including the use of static analysis and the lack of progressive damage modelling. With further research, the damage resistance parameter could provide impact damage predictions for a wide range of layups at a fraction of the time and cost of damage prediction methods used in industry today.

5.6 Summary

A parameter based on the bending stiffness of a laminate was proposed in this thesis as a method of ranking the damage resistance of laminates with respect to changes in stacking sequence. The method was evaluated by comparing the ranked laminate results to existing experimental data of impacted specimens. The results were generally positive, as the calculated damage resistance parameter was found to have a high linear correlation with the measured damage areas. When the parameter is combined with the peak contact force of the impact event, rankings can also be made for specimens impacted at different energies. In addition, the damage resistance parameter was shown to have superior performance in computing time over the finite element method, making the damage resistance parameter a suitable candidate for use in the preliminary design stage. Several limitations were noted with the method, including the inaccurate ranking of laminates containing interface angles greater than 60° . As a result, future research is required to address these limitations and extend the method to allow damage resistance ranking with respect to material and thickness changes. With these modifications, the damage resistance parameter should provide a viable alternative to the sophisticated and time-consuming finite element method when making impact damage predictions.

Chapter 6

Conclusions

6.1 Conclusions

This thesis analysed the effects of stacking sequence on the damage resistance in carbon fibre reinforced composite laminates. Based on the research performed for this thesis, the following conclusions are drawn:

1. Damage within composite laminates due to transverse loading is a complex phenomenon involving multiple damage mechanisms and progressive damage propagation.
2. The use of a static finite element model using a strength of material formulation was found to give reasonable predictions of delamination damage for the low-velocity impacts considered. The main limitation of the model was the lack of progressive damage modelling. This caused an underprediction of the interlaminar shear stresses.
3. Changes to the stacking sequence of a laminate will cause significant changes in the damage resistance capability.

4. Three stacking sequence parameters were identified to affect damage resistance: ply grouping, interface angle between laminae, and laminae orientation. Each parameter will have a different effect on the damage resistance.
5. The bending stiffness of the laminate is a parameter that is strongly linked to the damage resistance of the laminate. Regions of the laminate containing a high bending stiffness were found to have a greater amount of impact damage.
6. A damage resistance parameter based on the bending stiffness of the laminate and the peak contact force of the impact event was formulated and was found to give good predictions of damage resistance for layups containing interface angles of 60° and below.
7. Bending stiffness is not exclusively related to damage, as other parameters such as fracture toughness and thickness will also affect the damage state. Therefore, the use of bending stiffness alone will not give estimates on the damage resistance of a particular stacking sequence.

6.2 Future Research

The following is proposed for future research in the field of impact damage resistance in composite materials:

1. Improve the accuracy of the finite element model. This will include the development of more efficient elements which will reduce computational effort and allow more detailed stress modelling.
2. Develop an improved laminate failure theory which accounts for fracture growth of delamination damage.
3. Perform more experimental tests to evaluate the accuracy of the proposed laminate ranking method. Layups examined in the parametric study presented in this thesis are suggested as the basis for an experimental test program.
4. Improve the proposed laminate ranking method to account for damage created by interlaminar stresses and to account for stacking sequences containing interface angles of 60° or greater.
5. Expand the proposed laminate ranking method by modelling the effects of fracture growth to account for the changes in material properties and by modelling changes in the structural response due to changes in laminate thickness.
6. Further investigate the effects of changes in specimen geometry and boundary supports on the damage resistance in the material.

6.3 Summary of Contributions

This thesis has made the following contributions to the general knowledge in the field of impact damage resistance in composite materials:

1. Three parameters affecting stacking sequence were identified: ply grouping, interface angle between laminae, and laminae orientation. The effect of each parameter on damage resistance was extensively analysed using the finite element method.
2. A set of guidelines was proposed to improve the impact damage in composite plates, based on trends observed from finite element modelling and the experimental findings of other researchers.
3. A laminate ranking method was proposed to evaluate the impact damage resistance of different stacking sequences.

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