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**A STUDY
OF THE EFFECTS
OF THERMAL PARAMETER VARIATIONS
ON ESTIMATED GEOTHERMS
FOR THE KAPUSKASING STRUCTURAL ZONE**

by

KATARZYNA TYRCHA, Dipl. Geophys. (Cracov.).

A thesis submitted to the Faculty of Graduate Studies
and Research in partial fulfillment of the requirements
for the degree of Master of Science.

Department of Earth Sciences

The Ottawa-Carleton Geoscience Centre

Ottawa, Ontario

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The undersigned recommend to the Faculty of Graduate Studies
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ON ESTIMATED GEOTHERMS FOR THE KAPUSKASING
STRUCTURAL ZONE"

submitted by

Katarzyna Tyrcha, Dipl. Geophys.

in partial fulfillment of the requirements for
the degree of Master of Science

Thesis Supervisor

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ABSTRACT

The Kapuskasing Structural Zone (KSZ) is an example of lower continental crust which has been brought to the surface along a major thrust fault. This study is concerned with the present thermal state of the crust in the KSZ. The variability and accuracy of thermal parameter determinations are reviewed and assessed, for Precambrian shields in general, and the KSZ in particular. The variability of heat production can be very high, because of the spatial distribution of heat producing elements and the increase of the uncertainty at greater depth. Adjustments to measured heat flow are necessary to compensate for such factors as climatic, topographic, water flow, and erosion effects. Having estimated thermal parameters, temperature-depth distributions are calculated by solving the heat conduction equation with appropriate boundary conditions. Solutions for several models are derived using both an analytical and a finite difference approach. An optimum heat generation-depth model is selected by a trial and error procedure and combined with compiled rock thermal properties and surface heat flow measurements to generate characteristic one-dimensional geotherms for the region. Temperature distribution is a function of variations in surface heat flow, heat production properties with depth, and thermal conductivity. Comparison of different models lead to the conclusion that in the KSZ surface heat flow (corrected for glaciation) of $38 - 55 \text{ mW m}^{-2}$, heat production that decreases exponentially in each unit, and a Moho heat flow of $20 \pm 5 \text{ mW m}^{-2}$ satisfy all the available constraints.

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INTRODUCTION

The solution of the heat conduction equation has a number of very interesting applications in the Earth Sciences. Thermal aspects of geological processes play a central role in tectonophysics. The physical properties of the crust and upper mantle (seismic velocities and attenuation, density, electrical conductivity, and viscosity) are all dependent on the temperature and/or the ratio of the temperature to the melting temperature.

A variety of methods, exact, approximate, and purely numerical, are available for the solution of the heat conduction equation. The choice of a particular method depends on the nature of the problem under consideration.

Finite difference methods (which have been used in this study) are a particularly powerful technique for solving heat transfer problems. The basis of the finite difference method is the approximation of the partial derivatives at a given point by finite differences, thus reducing the problem to the solution of a large number of coupled algebraic equations for temperature. The temperature is calculated at the nodal points of a grid covering the region under investigation.

Whenever there is a temperature gradient in a system, heat will flow, and therefore temperature distribution is essential to the study of heat transfer in a given region. There are three distinct modes of heat transfer: conduction, convection, and radiation. In the Earth, the temperature distribution is controlled by the combined effects of these three modes of heat transfer. However, for simplicity, one mode of heat transfer such as conduction may be considered separately where heat transfer

by convection and radiation is negligible. This study was concerned solely with conductive heat transfer. Although heat transport by convection of hydrothermal fluids may be important locally, it is implicitly assumed here to be negligible on a large scale. This assumption may have to be revised if model calculations do not match observations.

Terrestrial heat flow, which is the outflow of heat from the Earth's interior, is affected by deep-seated tectonic processes. For this reason a knowledge of the surface heat flow is an important tool in understanding the nature and long-term evolution of the lithosphere. Local inhomogeneities in the spatial distribution of the observed heat flow are indicators of heat flow anomalies at depth or reflect lateral and vertical inhomogeneities in the distribution of heat-producing elements. The complexity of heat transfer in the Earth usually necessitates a multi-dimensional solution to the heat-conduction equation. Since this equation can be solved in an analytical form only in few single cases, no practical advancement in solution techniques was achieved before the widespread use of numerical methods.

Recently, much progress has been made in the understanding of the thermal state of the lithosphere and in developing qualitative models of dynamic lithospheric processes. Future advancements depend on the development of solutions for more rigorous, quantitative models.

In this thesis, a geothermal model for the crust in the Kapuskasing Structural Zone (KSZ) is proposed. It is assumed that the surface heat flow is in equilibrium with heat flowing into the crust at its base plus the heat generated by radioactive decay of unstable isotopes within it. The following topics are covered

in the thesis. In Chapter I the heat conduction fundamentals are presented. Review of some solutions of the heat transfer equation and their application to geothermics are covered in Chapter II. In Chapter III the thermal parameters of crustal rocks are discussed. The geology and the thermal regime of the KSZ are presented in Chapter IV. The results of model calculations are discussed in Chapter V.

1. HEAT CONDUCTION FUNDAMENTALS

As mentioned in the introduction, this thesis is concerned with calculating the distribution of temperature with depth across the Kapuskasing Structural Zone and examining the geological consequences of this distribution. Although heat may be transferred by conduction, convection and radiation, this study is concerned with conductive heat transfer only, as this is the main form of heat transfer in the lithosphere. In this chapter some of the fundamentals of heat conduction are presented; methods for solving the differential equation of heat conduction are outlined and a detailed discussion of the finite-difference technique is presented. The latter technique is used to solve the steady-state heat conduction equation in this study. A more detailed treatment can be found in Carslaw & Jaeger (1959).

1.1 The differential equation of heat conduction

Heat conduction theory is concerned with the distribution of temperature within solids as a function of position and time. Heat conduction is governed by Fourier's Law, which states that

$$\mathbf{q}(\mathbf{r},t) = -k \nabla T(\mathbf{r},t) \quad (1.1)$$

where $\mathbf{r}$ is position and t time. The temperature gradient $\nabla T(\mathbf{r},t)$ is a vector normal to the isothermal surfaces. The heat flow vector $\mathbf{q}$ represents heat flow per unit time per unit area across the isothermal surface in the direction of decreasing temperature. The parameter k is thermal conductivity, with dimensions

(W m⁻¹ K⁻¹).

The energy balance equation for a small volume V is :

$$\begin{array}{lcl} \text{rate of heat} & \text{rate of heat} & \text{rate of} \\ \text{entering through} & \text{generation} & \text{storage of} \\ \text{the bounding} & \text{within V} & \text{energy in V} \\ \text{surfaces of V} & & \end{array} =$$

(1.2)

By means of Green's identity:

$$\int \mathbf{q} \cdot \mathbf{n} \, dS = \int \nabla \cdot \mathbf{q} \, dV \quad (1.3)$$

Equation (1.2) becomes

$$-\int \nabla \cdot \mathbf{q} \, dV + \int A(\mathbf{r}, t) dV = \int \rho c \, \partial T(\mathbf{r}, t) / \partial t \, dV \quad (1.4)$$

where S is the surface area of the volume V, $\mathbf{n}$ is the outward normal unit vector to the surface element dS, $\mathbf{q}$ is the heat flow vector at dS, $A(\mathbf{r}, t)$ is internal heat generation per unit time and volume, ρ is density (kg m⁻³), and c is specific heat at constant pressure (J kg⁻¹ K⁻¹).

From equation (1.4) we obtain (since the equation is valid for any volume, no matter how small)

$$-\nabla \cdot \mathbf{q}(\mathbf{r}, t) + A(\mathbf{r}, t) = \rho c \, \partial T(\mathbf{r}, t) / \partial t \quad (1.5)$$

Using equations (1.1) and (1.5) the differential equation of heat conduction for a

stationary, homogenous, isotropic solid with internal heat generation becomes:

$$\nabla [k \nabla T(r,t)] + A(r,t) = \rho c \partial T(r,t) / \partial t \quad (1.6)$$

When the thermal conductivity is assumed to be constant (i.e. independent of position and temperature), equation (1.6) simplifies to:

$$\nabla^2 T(r,t) + 1/k A(r,t) = 1/\kappa \partial T(r,t) / \partial t \quad (1.7)$$

where

$$\kappa = k/\rho c \text{ (m}^2 \text{ s}^{-1}\text{)} \quad (1.8)$$

is the thermal diffusivity.

There are three particular cases of the differential equation of heat conduction:

(a) Steady-state (Laplace equation)

In this case there is no heat generation and no change in temperature with time, i.e.,

$$\nabla^2 T(r,t) = 0 \quad (1.9)$$

(b) Diffusion equation

In this case there is no heat generation, but the problem is time dependent,

$$\nabla^2 T(r,t) = 1/\kappa \partial T(r,t)/\partial t \quad (1.10)$$

(c) Poisson equation

In this case the problem is steady-state, but with non-zero heat generating function

$$\nabla^2 T(r,t) = -1/k A(r,t) \quad (1.11)$$

In the more general case, in which there is heat advection (i.e. the material moves with velocity v) equation (1.7) takes the form (in Cartesian coordinates)

$$-\partial/\partial x_i \left(k(\partial T/\partial x_i) \right) + \rho c v_i \partial T/\partial x_i = A - \rho c \partial T/\partial t \quad (1.12)$$

The advective term $\rho c v_i \partial T/\partial x_i$ accounts for the flow of heat associated with mass transport.

1.2 Initial and boundary conditions

The differential equation of heat conduction has numerous solutions unless a set of initial and boundary conditions is prescribed.

The **initial** conditions specify the temperature distribution in the medium at some instant in time, usually at the beginning of some heating or cooling process; the **boundary** conditions specify the temperature or the heat flow at the boundaries of the region. Boundary conditions fall into a number of categories:

(a) The temperature distribution T_0 at the boundary may be prescribed (boundary condition of the first kind). A boundary at a constant temperature $T = T_0$ satisfies this condition.

(b) The heat flow distribution may be prescribed (boundary condition of the second kind).

(c) If there is convective heat exchange with an environment at a prescribed temperature, then the boundary condition is called a boundary condition of the third kind.

1.3 Methods of solution of the heat conduction equation

A considerable amount of work in the literature has been devoted to the development of exact, approximate, and numerical methods for solving heat conduction problems. The following is a brief discussion of some of the more common methods (for more details, see Carslaw & Jaeger, 1959).

1.3.1 Separation of variables

This is a technique for solution of a certain class of linear partial differential equations. Let us consider the case of a homogenous boundary-value problem for which there is no internal heat generation. The partial differential equations can be separated into space- and time- dependent functions as follows:

$$T(r,t) = F(r) G(t) \quad (1.13)$$

where $F(r)$ and $G(t)$ are respectively space and time dependent functions. The application of this method to the solution of linear, homogeneous boundary-value problems of heat conduction is rather straightforward. That is, the partial differential equation of heat conduction in the n independent variables is separated into n ordinary differential equations, and in this separation process $n-1$ arbitrary separation constants are introduced. The resulting ordinary differential equations $F(r)$, $G(t)$ are readily solved and the complete solution is constructed by the linear superposition of all separated solutions. The unknown coefficients associated with the superposition are then determined.

The method is inconvenient if the problem is nonhomogeneous either due to the presence of the heat generation term in the differential equation or due to the nonhomogeneity of boundary conditions or both. Multidimensional steady-state heat conduction problems with no heat generation can be solved using this technique if only one of the boundary conditions is nonhomogeneous. Problems involving more than one nonhomogeneous boundary conditions may be split up into simpler problems each containing only one nonhomogeneous boundary condition (Özisik, 1977).

1.3.2 Laplace Transform Technique

This has been widely used for the solution of transient problems, if the partial derivative with respect to the time variable is removed from the differential equation by the application of the Laplace transform. Generally, the mathematical form of the problem is given by

$$F(s) = \int f(t)e^{-st}dt \quad (\text{Trajdos-Wróbel, 1970}) \quad (1.14)$$

where s is the transform variable. The function $F(s)$ is the Laplace's transform of the function $f(t)$. For example, in the 1-D, time-dependent case, the heat-conduction problem is reduced to an ordinary differential equation in the space variable for the transform of the temperature function

$$d^2T/dx^2 - (s/\kappa)T = (1/\kappa) f(x) \quad (1.15)$$

where $T(x,s)$ denotes the Laplace's transform of the temperature

$$T(x,s) = \int e^{-st}T(x,t)dt \quad (1.16)$$

and $f(x)$ is the initial temperature. When the ordinary differential equation is solved and the resulting transform is inverted, the desired solution for the temperature distribution is obtained (Hill & Dewynne, 1987).

1.3.3 Approximate analytic methods

The above described methods are suitable for the solution of conduction problems in bodies having simple geometries such as slabs, rectangles, cylinders, spheres, etc. Heat conduction problems in more complicated geometries cannot be solved analytically. Therefore, numerous approximate analytic methods have been applied for the analysis of steady and transient heat conduction problems in bodies

having more complicated geometries or for situations when the differential equation or the boundary condition is nonlinear. However, the disadvantage of these method is that the accuracy of an approximate analysis cannot be assessed unless the results are compared with the exact solution. The approximate methods include techniques such as the Rayleigh-Ritz method, the Galerkin method, and the integral method (Özisik, 1980).

1.3.4 Numerical methods

For heat conduction problems involving complicated boundary conditions, irregular boundaries and variable thermal properties, the analytical solutions become impossible. Approximate solutions are usually sought, though they may not provide the desired degree of accuracy. Numerical methods of solution are useful for such situations.

Numerical methods used to solve heat conduction equation problems include the Monte Carlo method, the finite difference method and the finite element method.

Of the various numerical methods we consider only finite differences, which has been widely used to solve the heat transfer problem. The basic concept in this technique is the approximation of partial derivatives at a given point by finite differences, thus converting the partial equation to a large number of coupled algebraic equations.

1.4 A finite difference representation of the heat conduction problem

In this study the finite difference method has been used to solve the heat conduction equation. The following is a brief discussion of the method (Croft & Lilley, 1970; Özisik, 1977, 1980; Reeves, 1988).

The finite difference method approximates the derivative of a function at a given point to a Taylor series expansion of the function about that point. For example, the first order derivative $f'(x)$ of a function $f(x)$ can be expressed in a finite difference term as follows:

$$f'(x) = [f(x + h) - f(x - h)]/2h + O(h^2) \quad (1.17)$$

where h is the spacing between two contiguous points. $O(h^2)$ indicates that the error involved is of the order of h^2 . This is known as a **central differences** approach since x lies in a central position between $(x + h)$ and $(x - h)$ (see Fig.1.1).

Letting $x = ih$, $x + h = (i + 1)h$, $x - h = (i - 1)h$, then $f(x) = f_i$, $f(x + h) = f_{i+1}$, $f(x - h) = f_{i-1}$, the first derivative is:

$$f'_i = (f_{i+1} - f_{i-1})/2h + O(h^2) \quad (1.18)$$

and the second derivative:

$$f''_i = (f_{i-1} - 2f_i + f_{i+1})/h + O(h^2) \quad (1.19)$$

For example, the 2-D steady-state heat conduction equation with internal heat generation, which has the form

$$\partial^2 T / \partial x^2 + \partial^2 T / \partial y^2 + (1/k) A(x,y) = 0 \quad \text{in region R} \quad (1.20)$$

may be expressed in a finite difference form using the above approach. First, a net of rectangular mesh points with spacing Δx , Δy in the x- and y-coordinate direction respectively is constructed (see Fig. 1.2). Assuming $\Delta x = \Delta y = 1$ then the finite difference approximation to equation (1.17) becomes:

$$(T_{i-1,j} + T_{i+1,j} + T_{i,j-1} + T_{i,j+1} - 4T_{i,j}) + A/k = 0 \quad (1.21)$$

This reduces the partial differential equation to a set of simultaneous algebraic equations which can be solved for the unknown temperature. These equations can be written in matrix form. The matrix is banded, i.e. the nonzero elements lie in a band on either side of the main diagonal. E.g. the matrix equation for 7 - node 1-D, time independent model has the form:

$$\begin{bmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} A' - T_s \\ A' \\ A' \\ A' \\ A' + q_6 \end{bmatrix} \quad (1.22)$$

Figure 1.1. Nomenclature for a Taylor series representation (from Özisik, 1980)

Figure 1.2 Rectangular network of mesh (Δx , Δy) (from Özisik, 1980)

where $A' = (\Delta r)^2 \cdot A/k$, Δr is the grid spacing, T_s is the surface temperature, q_b is the basal heat flow. The general form of equation (1.22) is

$$D T = A \quad (1.23)$$

where T and A are column vectors and D is a diagonally banded matrix.

Simultaneous algebraic equations of this form can be solved by direct methods that make use of the Gauss elimination or Gauss-Jordan elimination procedure, or by iterative methods such as the Gauss-Siedel iteration procedure (Reeves, 1988). The advantage of the iterative method is that it does not require large computer storage space. In this method a first approximation is used to calculate a second approximation, and so on; the iteration is terminated when the desired convergence criterion is satisfied. One difficulty with the Gauss-Siedel method is that its convergence is relatively slow.

A Taylor series expansion involves a truncation error (or discretization error), as a result of the fact that the first- or the second- order derivative is replaced by the finite-difference approximation. Also there is a roundoff error, since numerical calculations are carried out only to a finite number of decimal places at each step in the calculation. Ideally, if it were possible to carry out finite-difference calculations with infinitesimally small steps and to perform the calculations to an infinite number of decimal places, the resulting solution would be exact.

In the time-dependent heat conduction problems, the stability and convergence consideration plays an important role. The stability (with respect to

size of time step Δt and grid spacing Δr) and **convergence** (which indicates that refining the space-time grid makes the approximate computed solution approach the exact solution) of the solution which we calculate are generally interrelated. Convergence means that as the step size tends to zero, the finite difference solution of the differential equation tends to the exact solution at each location considered. The stability of finite difference equation depends on the step size (both time Δt and space Δr). For a stable solution $n = \kappa \Delta t / (\Delta r)^2$ should be ≤ 0.5 (Barakat & Clark, 1966). When $n \leq 0.5$, small perturbations in the initial temperature field diminish rather than grow on successive iterations and the final numerical solution approaches the exact solution (Reeves, 1988).

The time-dependent heat conduction equation can be expressed in a finite difference form. We can use either an implicit or explicit approach to do so. In the **explicit** approach the unknown temperature T at a time step $(n + 1)$ is directly determined from the known temperatures T_{j-1} , T_j , and T_{j+1} at a previous time (n) since

$$T_j^{n+1} = rT_{j-1}^n + (1 - 2r)T_j^n + rT_{j+1}^n \quad (1.24)$$

$$\text{for } j = 1, 2, 3, \dots, N-1; n = 0, 1, 2, \dots$$

where $r = \kappa \Delta t / (\Delta x)^2$. However, the disadvantage of this method is a restriction imposed on the parameter r :

$$\tau = \kappa \Delta t / (\Delta x)^2 < 1/2 \quad (1.25)$$

If $\tau > 1/2$ then the explicit solution is unstable.

In the implicit approach to determine unknown temperature T_j^{n+1} , for $j = 1, 2, \dots, N-1$ at the $(n + 1)$ th time step, a set of simultaneous algebraic equations have to be solved. The main advantage of this method is that it is stable for all sizes of time step Δt , thus there is no size restriction on Δt . The only size restriction on Δt is due to the consideration of the truncation error. A modified implicit method known as the Crank-Nicolson method has been used in this study (Reeves, 1988). The partial derivative with respect to the space variable (i.e. $\partial^2 T / \partial x^2$) is represented in finite difference form by taking the arithmetic average of the explicit and the implicit form e.g.,

$$\begin{aligned} (T_j^{n+1} - T_j^n) / \Delta t = (\kappa/2) \{ [T_j^{n+1} - 2T_j^{n+1} + T_{j+1}^{n+1}] / (\Delta x)^2 \\ + [T_{j-1}^n - 2T_j^n + T_{j+1}^n] / (\Delta x)^2 \} \end{aligned} \quad (1.26)$$

The advantage of this method is that for given values of the space and time steps (Δx and Δt respectively), the resulting solution involves less truncation error due to Δt than either the explicit or implicit forms. The method however involves additional computation.

In this study the finite difference scheme was implemented using the Pascal programming language. A generalized matrix solution algorithm is implemented in the Turbo Pascal procedure MatSolve and GSSolve (Reeves, 1988) which can solve

problems of the general form expressed by the steady-state equation (1.20). MatSolve uses Gauss, GaussPivot, Thomas, and GaussSiedel procedures. They are supported by the library MatLib which includes a large number of vector/matrix procedures. The Gauss elimination is provided in the procedure Gauss. GaussPivot is an alternative solver which allows to help control numerical errors. Using the Thomas procedure the problem simplifies reducing the amount of computation and required storage. GSSolve implements the Gauss-Siedel iteration algorithm in a such a way that enables one to handle much larger models than MatSolve (which allow only about 30 mesh points). A complete listing of the program is given in Appendix 1.1.

2. REVIEW OF SOME SOLUTIONS OF THE HEAT TRANSFER EQUATION AND THEIR APPLICATION TO GEOTHERMICS

Most tectonic processes produce or are responses to either variations in the mode of heat transfer in the Earth or to variations in the temperature-depth distribution. To understand these processes, the temperature-depth distribution in a given region must be determined. Much of the mathematical theory required to do so is discussed in Carslaw & Jaeger (1959) while Turcotte & Schubert (1982) examine some applications to geological/geophysical problems.

Any temperature-depth model which we derive must be in agreement with observation. In particular, models must agree with any measured surface heat flow values and an explanation must be sought for any discrepancies. Often, however, surface heat flow values quoted in the literature are not consistent, i.e. only some values will have been corrected for topographic, climatic and other effects.

This chapter reviews the corrections implemented for the measurements of thermal parameters, and considers some geodynamic models connected with the heat transfer equation.

The important factors influencing measured surface heat flow are oscillation of surface temperature, groundwater circulation, topographic effects, presence of a water-body, erosion/sedimentation, and past climatic changes. The mode of heat transfer depends on the geological/geophysical properties of the region. For example, in a tectonically young region with active plutonism the effects of convection due to circulating hydrothermal fluids will be important. In cratonic areas, where there is a closer approach to steady-state, it is usually sufficient to consider

conductive effects on the temperature distribution.

2.1 Perturbations of the temperature gradient caused by the oscillations of surface temperature

In general, the transient oscillations of the surface temperature (diurnal, annual, etc.) can be represented by Fourier series in which the higher harmonics die out much more rapidly with depth than the lower harmonics. An adequate equation describing these oscillations is obtained from the 1-D, time- dependent solution to the heat conduction equation (Jaeger, 1965)

$$T = T_0 \exp(-k_w z) \cos(\omega t - k_w z) \quad (2.1)$$

where T_0 is the temperature at $t = 0$, $\omega = 2\pi/P$, P is the period, $k_w = (\omega/2\kappa)^{1/2}$, and the wavelength λ is $\lambda = 2\pi/k_w = (4\pi\kappa P)^{1/2}$.

The diurnal oscillation can be neglected at depths greater than 1 m, the annual at depths greater than 20 m.

2.2 Past climatic changes

The surface temperature of the Earth has not remained constant in time. In particular, many regions have been subjected to Pleistocene glaciation. The thermal effects of such glaciation are preserved in the geotherm. A single ice age may perturb the geothermal gradient by as much as 15°C/km at shallow depths (Jaeger, 1965). As reviewed by Jaeger (1965), the effect can be serious when compared with

the undisturbed gradient, typically about 30 mK/m. Gough (1963) (see review by Jaeger, 1965) and Čermák and Jessop (1971) studied the temperature in a number of relatively deep boreholes in which climatic temperature variations over time periods up to a million years were considered to have been significant. In this case it was necessary to estimate the climatic changes and to compare temperatures thus calculated with observed temperature-depth curves. More details about the influence of the past climatic changes on the resulting geotherms will be discussed in the Chapter IV concerning the Kapuskasing Structural Zone (KSZ).

2.3 Topographic effects

In this case the Laplace transformation is used. It must satisfy the conditions $T \rightarrow \beta z$ as $z \rightarrow \infty$ (β is the temperature gradient, z is the depth), $T = T_s(x,y)$ at the surface, where $z = -h(x,y)$, $h(x,y)$ being the height above the sea level. The temperature is usually assumed to vary linearly with height, that is

$$T_s(x,y) = T_s - \beta h(x,y) \quad (2.2)$$

where T_s is constant, and β is a temperature gradient, usually assumed to be constant (10 mK/m). Both Carslaw & Jaeger (1959) and Turcotte & Schubert (1982) solved the problem for topographic correction assuming that there are lateral variations in surface temperature T_s . The surface temperature is assumed to be a periodic function of x (horizontal distance), given by

$$T_s = T_o + T \cos(2\pi x/\lambda) \quad (2.3)$$

Using the method of separation of variables, the solution for the temperature distribution in the continental crust in the x, z coordinate system with periodic topography is (Jaeger, 1965)

$$T = T_o + q_m z/k + (A_s b^2/k)(1 - e^{-z/b}) + \\ + (\beta - q_m/k - A_s b/k) h_o \cos(2\pi x/\lambda) e^{-2\pi z/\lambda} \quad (2.4)$$

where q_m is mantle heat flow, b is a length scale for the decrease in heat production with depth, A_s is surface heat production, k is thermal conductivity, and β is the vertical temperature gradient. For instance in a mountain range with assumed wavelength of 100 km and amplitude of 1.2 km, for the measured surface heat flow $q_s = 46 \text{ mW m}^{-2}$ and $k = 2.5 \text{ W m}^{-1} \text{ K}^{-1}$, the topographic correction is - 2.2 mW m^{-2} . The values of the topographic correction are affected by the changes in surface conditions (sea or lakes) and variations in thermal conductivity.

2.4 Groundwater circulation

The movement of underground water may greatly affect measured geothermal gradients, particularly those obtained in shallow boreholes. Bullard & Niblett (1951) give the solution for the temperature distribution in 1-D (see review by Jaeger, 1965); Danis & Quintard (1989) discuss the application of the 2-D solution, with

extention to the 3-D numerical models incorporating both thermal conductive and forced convective heat transfer to aquifer problems in the Vosges region of France. It was found that variations from the mean heat flow value were due to both downwards water flow and to a thermal spring beneath the sedimentary cover. The application of Darcy's law together with the related boundary conditions was the starting point for the thermal problem. The solution is possible and reliable if hydrogeological and thermal conditions are resonably well-known.

Large variations in heat flow within fractured zones, which are not associated with comparable changes in thermal conductivity, suggest active hydrothermal circulation (Drury, 1989). Lewis (1975) found large variations in the heat flow values in a small area (3 km²) of the shield near Lake Dufault (Churchill Province). The variation in the measured heat flow was much larger than possible errors in measurements. Also it was shown that variations in heat production and thermal conductivity are unlikely to cause the observed differences in heat flow. The large deviations from the straight line in the geotherms were noticed. The variation in the heat flow values (in the range 25 - 39 mW m⁻²) was explained by the water flow along thrust faults and fractures.

2.5 Erosion, sedimentation, and uplift

If material is being eroded from the surface at a rate v , then the effect on the geotherm is equivalent to material below the actual surface moving toward the surface at this speed. Similarly, if material is added by sedimentation, the material below can be regarded as moving away from the surface. These problems require the

solution of the heat conduction equation for a moving medium. In the case of movement in the vertical direction only this has the following form

$$\kappa \partial^2 T / \partial z^2 = \partial T / \partial t + v(\partial T / \partial z) \quad (2.5)$$

The solution of equation (2.5) is (Carslaw & Jaeger, 1959)

$$T = T_0 + \beta(z - vt) + 1/2(\beta + \Delta T/v)\{(z + vt) \cdot e^{v^2/4\kappa} \operatorname{erf}[(z + vt)/2(\kappa t)^{1/2}] + (vt - z) \cdot \operatorname{erf}[(x - vt)/2(\kappa t)^{1/2}]\} \quad (2.6)$$

where the sedimentation or erosion process begins at time $t = 0$ when the initial temperature is $T_0 + \beta z$, the surface $z = 0$ is subsequently maintained at temperature $T_0 + \Delta T t$ (where ΔT is the rate of temperature change with time t), and $\operatorname{erf}(\zeta)$ is the error function. The above solution covers all the cases referred to above, and it is readily extended to incorporate internal heat production in the medium. Fast sedimentation will reduce significantly the measured heat flow (see review by Carslaw & Jaeger, 1959). The evaluation of sedimentation effects on heat flow depends on the compaction history of the material. Due to compaction, the sediment velocity decreases and the diffusivity increases with depth. For example, in the Gulf of Oman sedimentary basin, forming the part of the Arabian plate the estimated sedimentation rate is between 250-400 m Ma⁻¹, which may depress the heat flux up to 35% (Hutchison *et al.*, 1981).

2.6 Thermal state of the lithosphere

The factors determining the thermal regime of the continental lithosphere are (1) the heat flow from the asthenosphere (2) the vertical and lateral variations of both thermal conductivity and radiogenic heat production (3) tectonic history (4) climatic history and the shallow hydrothermal regime (Morgan & Sass, 1984).

A negative correlation between heat flow and tectonic age of the continental lithosphere has been observed. However, the data show wide dispersion. From several reviews of continental heat flow (Sclater *et al.*, 1980; Vitorello & Pollack, 1980; Pollack, 1982) two widely accepted generalizations have emerged: (1) heat flow generally decreases with increasing age, where "age" refers to either the last significant tectonic and/or magmatic event or the last regional event involving sufficient heating of crustal rocks (2) the thermal histories of continental and oceanic lithosphere can both be explained by the same cooling model. Continental heat flow as a function of age is shown in Fig.2.1(a),(b) (Vitorello & Pollack, 1980). In Fig.2.1(a) the age is assigned on the basis of the most recent major tectonic or regional magmatic (tectonothermal) event. In Fig.2.1(b) (after Chapman & Furlong, 1977) the age is assigned on the basis of the basement age at each heat site. Rao *et al.* (1982) have pointed out that it is not possible strictly to establish a correlation in any statistical way, because the data are not normally distributed. In the evaluation of the decrease of continental surface heat flow with increasing age the contribution of crustal radioactivity to the continental heat flow and the amount of the heat flow escaping from the underlying mantle should be taken into account. Those are determined partly by the distribution of heat production with depth. The

Figure 2.1. The continental heat flow as a function of (a) tectonothermal age (b) crustal age (from Vitorello & Pollack, 1980). C denotes Cenozoic, M-Mesozoic, LPa-late Paleozoic, EPa-early Paleozoic, LPr-late Proterozoic, EPr-early Proterozoic, A-Archean

empirical relationship $q_c = q_r + bA_c$, where q_c is the surface heat flow, q_r is the reduced heat flow (interpreted as the contribution to the observed heat flow coming from the sources below the upper crust), and bA_c is the contribution of the radiogenic heat sources within the upper crust, is often used (for details see equations 3.2, 3.3, and 3.4, Chapter III). The continental heat flow, on the average, can be interpreted in terms of three main contributions as follows (Fig. 2.2, after Vitorello & Pollack, 1980): (i) about 40% is contributed by radiogenic heat sources in the upper crust; (ii) about 20% comes from cooling of heated continental lithosphere or erosional effects or both; (iii) up to about 40% is interpreted as a uniform heat flow contributed by radiogenic heat sources in the continental lithosphere below the enriched upper crust and by upward flow from the mantle below the lithosphere. There are considerable uncertainties and these will be discussed in Chapter III and IV.

2.6.1 Thickness and geotherms of stable lithosphere

Investigations have shown a trend of increasing lithospheric thickness with general decrease in continental heat flow. Seismic studies of lithospheric thickness appear to be in agreement with surface heat flow values and continental geotherms (see Fig.2.3 for some examples of continental geotherms). The concept of a single stable continental geotherm is invalid in lithosphere with lateral variations of heat production. Lateral variations in the heat production and in the thermal conductivity will result in the lateral temperature and possibly thickness variations in the lithosphere in a single, stable province, and potentially large variations between

Figure 2.2. Secular decrease of continental heat flow with age and its three principal components: component I is radiogenic heat from the crust, component II is heat from a transient thermal perturbation associated with tectogenesis, and component III is background heat flow from deeper sources. The data points are from Chapman & Furlong (1977). C denotes Cenozoic; M-Mesozoic; LPr-late Paleozoic; EPr-early Paleozoic; LPr-Late Proterozoic; EPr-early Proterozoic and A-Archean. Points are plotted at the mean age of the respective age ranges. Double bars represent the standard error of the mean; single bars the standard deviation. The upper curve corresponds to a visually fitted curve through the data points; the middle curve corresponds to 60% of the upper curve; the lower curve represents the background heat flow from deeper sources (from Vitorello & Pollack, 1980).

Figure 2.3. Lithosphere thickness as a function of crustal radioactivity. Crustal thickness = 30 km. Lithosphere thickness estimated on the basis of the intersection of the geotherm with the BDS (Basalt Dry Solidus). (a) step distribution; $b = 10$ km, $q_s = q^* = 27 \text{ mW m}^{-2}$. (b) exponential distribution; $b = 15$ km (dashed curves), $q_s = q^* = 27 \text{ mW m}^{-2}$. Assumed thermal conductivity for the crust and upper mantle were 2.5 and $3.4 \text{ W m}^{-1} \text{ K}^{-1}$ respectively, A_s is surface heat flow (from Morgan & Sass, 1984).

different stable provinces (Morgan & Sass, 1984). Pollack & Chapman (1977) constructed geotherms and calculated lithospheric thickness on the assumptions that crustal heat production contributes a constant fraction to the surface heat flow and that surface heat flow can be predicted from the age of the last tectonothermal activity. According to Morgan & Sass (1984) these assumptions are not valid on a regional scale and thus the geotherms and continental lithospheric thickness maps generated under these assumptions must be treated with caution.

2.6.2 Temperature and the rheology of the crust

Surface heat flow and temperature distribution affect the rheology of the crust and upper mantle. The influence of temperature on the viscosity (i.e. strength) of rocks in the plastic regime is well known (Ranalli, 1987; Ranalli & Murphy, 1987). The steady-state rheological equation can be written in generalized form

$$\dot{\epsilon} = F(\sigma)\exp[-(E + pV_a)/k_bT] = F(\sigma)\exp(-M T_m/T) \quad (2.7)$$

where $F(\sigma)$ is a function primarily dependent on stress, E and V_a are the activation energy and activation volume for creep, k_b is Boltzmann's constant, M is a material constant, T_m is the melting temperature, and T and p are the ambient temperature and pressure. Therefore, for a given stress level, variations in homologous temperature T/T_m correspond directly to variations in viscosity. The viscosity-depth relationship for selected geotherms corresponding to different heat flow provinces has been calculated (Ranalli, 1987) inserting laboratory data on rheological

properties of crustal rocks into Weertman's general creep equation (where strain rate $\dot{\epsilon}$ is expressed as a function of stress σ in a power law). Under most geothermal conditions, calculations indicate a low-viscosity lower crust.

2.7 The effect of intrusions

The cooling of an igneous intrusion is a relatively rapid phenomenon over geological time scales. In its simplest form, the problem is solved for the 1-D, time dependent heat conduction equation, $\partial T/\partial t = \kappa \partial^2 T/\partial z^2$. An example of the influence of the present or recent plutonic activity on surface heat flow is given for the Okinawa Trough region by Yamano *et al.*, 1989. The age of the intrusive body is estimated from the amplitude and horizontal extent of the associated surface heat flow anomaly. If we assume an intrusion of infinite length and rectangular cross-section, where $2a$ is the width, h is the thickness, and d is the depth to the top surface, the surface heat flow anomaly is (Yamano *et al.*, 1989)

$$q = q_0 \frac{2d}{\pi^{1/2} \ell} E_0(x, \ell, a) \{ \exp(-d^2/\ell^2) - \exp[-d^2/\ell^2(1 + h/d)^2] \} \quad (2.8)$$

where $E_0(x, \ell, a) = 1/2 \{ \text{erf}[(a/\ell)(1 - x/a)] + \text{erf}[(a/\ell)(1 + x/a)] \}$, $\ell = (4\kappa t)^{1/2}$, $q_0 = (kT_0)/d$, and T_0 is the initial temperature inside the block. The normalized heat flow anomaly directly above the centre of the intrusion, q_c/q_0 can be constructed (q_c is heat flow at $x = 0$) and the maximum value of ℓ which can produce the observed

heat flow distribution can be determined. The maximum value of ℓ is about 6 km assuming that the heat flow anomaly at $x = 10$ km should be negligible as compared with that at $x = 0$. The obtained maximum value of q_e/q_0 is 0.48 assuming $h/d = 10$. The maximum depth and age of the intrusive body, assuming conductive cooling, is 3.5 km and $3 \cdot 10^5$ yrs respectively. When advective heat transfer is taken into account, the age must be much younger.

2.8 Thermal consequences of tectonic activity

The thermal structure of the continental lithosphere can be modified in a number of ways during tectonism (Morgan & Sass, 1984). Lithospheric thermal perturbations can take a variety of forms. Extensional tectonic regimes and hot spots cause anomalously high temperatures in the lithosphere and lithospheric thinning. Convergent regimes cause a depression of isotherms and lithospheric thickening associated with the downward movement of material, but are associated with high temperatures at shallow depths in island arcs arising from melting in the subducting slab. Strike-slip regimes are associated with local, frictional heating on the fault planes. Within these broad generalizations, the details of individual thermal perturbations may be complex, and details of the disturbed lithospheric geotherm cannot be predicted simply.

2.9 Thermal structure of subduction complexes

The evolution of subduction complexes (accretionary prisms) may be an important phase in the development of continents.

The thermal structure in the subduction complex is determined by the subduction rate, the amount of the heat flow from its base, and the radioactivity in the sediments (Wang & Shi, 1984). Cloos (1989) analyzed the temperature distribution in subduction complexes on the basis of the example of the Franciscan complex of California. Assuming that the movement of sediments in a subduction complex may be approximated by assuming a 2-D steady-state laminar flow of an incompressible viscous fluid of uniform viscosity, a finite element method was applied to solve the advection equation. This is a 2-D case of the heat conduction equation, discussed in the Chapter I (see equation 1.11) Thermal conductivity is treated as a tensor because the model is anisotropic.

A quantitative analysis of the thermal structure in the interiors of the subduction complexes is valuable because it allows quantitative prediction of the distribution of metamorphic grades within these complexes. Quantitative thermal modelling can also reveal the relative significance of the various processes in shaping the evolution of subduction complexes. The study of the thermal structure of subduction complexes is related to the study of the dynamics of sediments inside the complexes. Important parameters to consider are variations of viscosity with depth, angle of subduction, and convective cooling. The viscosity is a strong function of temperature, and the relationship between strain rate and the deviatoric stress may be nonlinear; these factors may significantly alter the results of the assumed model. In general, results of these studies reveal that thermal modeling and prediction of the distribution of metamorphic grades and ages of sediments in subduction complexes can provide powerful tests on hypotheses on the evolution of these

complexes.

2.10 Cooling of the oceanic lithosphere

The oceanic lithosphere can be treated as a thermal boundary layer that cools as it moves away from the ridge. The problem is illustrated schematically in Fig.2.4. To determine the temperature distribution, the equation for a moving medium has to be solved. The solution has the form (see e.g. Turcotte & Scubert, 1982)

$$(T - T_m)/(T_s - T_m) = \text{erf}[z/2(\kappa x/v_r)^{1/2}] \quad (2.9)$$

where T_m is the temperature at the ridge crest ($x = 0$) and at the base of the plate. It can be proven that equation 2.9 leads to a distribution of surface heat flow with age (or distance from the ridge) as

$$q = kT_s/(\pi\kappa t_s)^{1/2} = kT_s(v_r/\pi\kappa x)^{1/2} \quad (2.10)$$

where k is the thermal conductivity, κ is thermal diffusivity, T_s is the temperature of the asthenosphere, t_s is the age of the lithosphere, x is the distance from the spreading axis, and v_r is the spreading rate.

Similarly, the thickness of oceanic lithosphere depends on age as

$$H = 2.32(\kappa t_s)^{1/2} \quad (2.11)$$

Figure 2.4. Scheme for the cooling oceanic lithosphere (from Turcotte & Schubert, 1982).

where H is thickness (km), and t_a is in Ma. For oceanic crust less than 50 Ma old the observed heat flow measurements are consistently less than theoretical calculations suggest they should be. This is attributed to convective cooling of the young, hot ocean crust by hydrothermal circulation of seawater. The latter phenomenon becomes less important as the age of the oceanic lithosphere increases.

Equation 2.10, with the application of the principle of isostasy, may be used to predict the ocean topography. Assuming isostatic equilibrium,

$$\rho_m(h_w + H) = \rho_w h_w + \int_0^H \rho dz \quad (2.12)$$

where ρ is the density of the lithosphere, ρ_m is the density of the mantle, h_w is the depth of the ocean floor, and H is the thickness of the lithosphere. The thermally-induced density change in the mantle can be written in the form

$$\rho - \rho_m = \alpha \rho_m (T_m - T) \quad (2.13)$$

where α is the volume coefficient of thermal expansion. Combining equations 2.10, 2.12, and 2.13 we obtain

$$h_w = 2\alpha \rho_m (T_m - T_0) (\kappa x / \pi v_r)^{1/2} / (\rho_m - \rho_w) \quad (2.14)$$

which predicts that the average depth of the ocean floor increases as the square root of the age or distance from the ridge axis. These and other relationships are reviewed in Turcotte & Schubert (1982).

3. THERMAL PARAMETERS

In this chapter the fundamental thermal parameters and their influence on the solution of the heat conduction equation in the crust are discussed. More details about these parameters with reference to the Kapuskasing Structural Zone will be given in Chapter IV.

As can be seen from the general form of the heat conduction equation, four thermal parameters determine the temperature distribution in the crust. They are: heat production (per unit volume) (A , [$\mu\text{W m}^{-3}$]), thermal conductivity (k , [$\text{W m}^{-1} \text{K}^{-1}$]), density (ρ , [kg m^{-3}]) and specific heat (c , [$\text{kJ kg}^{-1} \text{K}^{-1}$]). Another parameter, namely thermal diffusivity κ , defined as $\kappa = k/\rho c$ ($\text{m}^2 \text{s}^{-1}$), combines the last three. The most important parameters determining the temperature distribution within any crustal region are the heat production, thermal conductivity, and thermal diffusivity.

3.1 Heat production and its correlation with heat flow

Heat production is the heat generated by the radioactive decay of unstable isotopes of thorium (Th), uranium (U), and potassium (K). The heat production of a rock with a given concentration of these elements can be calculated from the formula (Birch, 1954)

$$A = 0.01\rho (9.52 U + 2.56 \text{Th} + 3.48 \text{K}) \quad (3.1)$$

where U and Th are concentrations expressed in ppm, K in weight per cent, and ρ

is density in kg m^{-3} .

In general, radiogenic heat production is higher in rocks of granitic composition than in mafic rocks, and it is most significant in most continental upper crustal rocks (Morgan & Sass, 1984). There may be however, different depth parameters controlling the distribution of U, Th and K in the crust and different distributions at different depths (Lachenbruch & Bunker, 1971). Processes affecting the radiogenic element distribution include primary differentiation of the crust, magmatic and metamorphic fluid circulation, and meteoric water circulation. Studies of uranium in metamorphic rocks indicate that it is essentially immobile up to amphibolite grade, but mobile due to migrating fluid phases during granulite metamorphism. Thorium is mobilized at a similar metamorphic grade (Jaupart *et al.*, 1981).

The average heat production value of surface rocks in continents is about $2 \mu\text{W m}^{-3}$ (Ashwal *et al.*, 1987). If we consider a uniform section of crust approximately 30 km thick with this heat production value and ignore heat flow from the mantle, the resulting surface heat flow is 60 mW m^{-2} , i.e., about equal to the average continental heat flow. As it is impossible that the continental mantle heat flow is zero, and since there is no general correlation between surface heat flow and crustal thickness, heat production must decrease with depth.

Consequently, certain assumptions must be made about the variation of heat production with depth. For instance one might assume that the heat generation A has different but constant values corresponding to different rock types, so that variations of A with depth are represented by a step function. In the step model of

Roy *et al.* (1968), the heat production is assumed to remain constant from the surface to a depth given by the parameter b , the thickness of the enriched near-surface layer. Another model assumes an exponential decrease of heat production with depth (Lachenbruch, 1970). Such a model may be expressed as follows:

$$A(z) = A_s \exp (-z/b) \quad (3.2)$$

where A_s is the surface heat production ($\mu\text{W m}^{-3}$), $A(z)$ is the heat production at a depth z , and b is the depth parameter.

The observed values of A and b are functions of the initial concentrations of the isotopes, the decay constant and the exponential depth constant (Jessop & Lewis, 1978). The best way to investigate the distribution of heat production with depth is to measure it directly in samples from vertical cross-sections of the crust exposed at the surface by tectonism. Such sections have been measured for example in the Vredefort Structure in South Africa (Nicolaysen *et al.*, 1981), in the granitic rocks in the Pennine basement in eastern Alps (Hawkesworth, 1974), in the Sudbury Structure of the Archean granodioritic rocks in the Superior Province of Ontario (Schneider *et al.*, 1987), and in the Kapuskasing Structural Zone in the Canadian Shield (Percival & Card, 1983). The problem of the distribution of the heat producing elements in the KSZ will be discussed in detail in Chapter IV.

Profiles across the Vredefort structure, where the granite core has been overturned, exposing to view a thick section of 15 km through the Archean crystalline crust, provide an opportunity for detailed studies of the distribution of U,

Th, and K with depth in the upper and middle crust (see Fig.3.1 after Nicolaysen *et al.*, 1981). The radiogenic isotopes follow neither a simple nor the same distribution with depth. Potassium does not vary significantly with depth, but uranium and thorium data can be approximated by exponential curves. Interesting results have been obtained in Austria (Alpine Pennine basement). Here an exposed vertical section through the crust allows the exponential model for the dependence of heat production with depth to be tested. The heat production for the four units of the structure taken separately fit a straight line (uniform decrease with depth) equally well as they do an exponential curve (Hart *et al.*, 1981).

Birch *et al.* (1968) and Lachenbruch (1968) discovered that the surface heat flow (q_s) is correlated with the radioactivity of the exposed rock (A_s). A linear relationship exists of the type

$$q_s = q_r + b A_s \quad (3.3)$$

where b is slope of the linear trend (with the dimensions of length) and q_r represents the reduced heat flow at the base of the enriched radiogenic zone. When plotting surface heat flow versus surface heat production, the points cluster along an approximately linear trend. An example is given in Fig.3.2 (after Vigneresse *et al.*, 1989). This correlation lead to the definition of "heat flow provinces" by Roy *et al.* (1972). This term denotes a region with the same value of reduced heat flow (q_r), in recognition of the fact that this parameter is of greater geophysical significance than the distribution of heat producing elements in the upper crust.

Figure 3.1. Heat production distribution of U, Th. and K with depth in the Vredefort structure (from Nicolaysen *et al*, 1981).

Figure 3.2. Linear relationship between heat flow and heat production with confidence intervals. The size of the symbols reflects error bounds (from Vignerresse *et al.*, 1989).

A variety of models for the decrease of heat production with depths satisfy the linear relationship found by Birch *et al.* (1968). The step-function (Roy *et al.*, 1968), the linear and the exponential (Lachenbruch, 1970) models are mathematically the simplest. However, only an exponential dependence of heat production with depth enables the relationship between heat flow and the heat production to remain valid after differential erosion (Lachenbruch, 1970).

The linear relationship between q_r and q_s allows the establishment of a new empiricism relating reduced heat flow (q_r) to mean surface heat flow (q_s) in a heat-flow province. Pollack & Chapman (1977) suggested that, on the average,

$$q_r = 0.6 q_s \quad (3.4)$$

that is, within continental regions with mean heat flow q_s , approximately 40% of the mean flux arises from near-surface radiogenic sources, and 60% comes from greater depths. The reduced heat flow q_r is made up of a radiogenic contribution from the lower crust and upper mantle, and a deeper contribution which arises from convectively delivered heat and/or viscous dissipation in the asthenosphere, and which enters the lithosphere at its base. Also, a transient (cooling) component may be present for a few hundred Ma after a tectonothermal event (Vitorello & Pollack, 1980).

All the above discussed relations are found to be of doubtful validity (see discussion below). In the Soviet Kola Peninsula, for example, a linear relationship between surface heat flow and the heat production exists (see review by Jolivet *et*

al., 1989). The heat production, however, does not decrease exponentially with depth, but rather increases from a value of $1.26 \mu\text{W m}^{-3}$ at surface to $2.1 \mu\text{W m}^{-3}$ at 10 km depth. The correlation between heat flow and heat production has not been observed in the French Massif Central, though it was confirmed in Southern England (as reviewed by Jolivet *et al.*, 1989). Data from Brittany (Western France) do not fit a strict linear relationship either. The wide scatter results from measurements in sedimentary or metasedimentary rocks, which have very low heat production levels; hence A_r values sometimes are not representative of the underlying heat production (Jolivet *et al.*, 1989). Results from a variety of tectonic provinces are summarized in Tab.3.1 (a), (b).

The relationship between the surface heat flow and reduced heat flow is not always valid (Pujol & Fountain, 1985; Drury, 1989). The surface heat flow/reduced heat flow relationship for continents was reexamined statistically by Pujol & Fountain (1985). The relationship $q_r = 0.6 q_s$, suggests that the upper crustal radiogenic contribution to heat flow is coupled to changes in q_r . For stable regions the slope of equation (3.3) is small and intercept is large. Inclusion of data from regions which have been tectonically active in the Cenozoic gives results closer to relation (3.4). The average upper crustal radiogenic component has a large standard deviation (14 mW m^{-2}), suggesting that variation in q_r for a given heat flow province in stable regions is largely controlled by upper crustal radiogenic variations. Because q_r and q_s exhibit large errors and are correlated, a standard least squares regression is not ideal to find a relationship between them.

A computer simulation method can be used to fit a straight line to correlated

Tab. 3.1 (a)

Mean heat flow, reduced heat flow, and characteristic depth of heat source distribution for several heat flow provinces (from compilation of Pollack and Chapman, 1977)

Province	Mean heat flow ± std deviation (mW m⁻²)	Reduced heat flow ± uncer tainty (mW m⁻²)	Characteristic depth (km)
Basin and Range	92 ± 13	59 ± 8	9.4
Central Australia	77 ± 10	27 ± 6	11.1
Zambia	67 ± 7	40 ± 6	
Eastern U.S.A.	57 ± 17	33 ± 4	7.5
Canadian shield	39 ± 7	28 ± 6	9.8
Western Australia	39 ± 8	26 ± 8	4.5
Sierra Nevada	39 ± 12	17 ± 2	10.1
Baltic shield	36 ± 8	22 ± 6	8.5
Niger	20 ± 8	11 ± 8	8

Tab. 3.1 (b)

Reduced heat flow and characteristic depth for several shield heat-flow provinces and theoretical models (from Fountain *et al.*, 1987)

Province	Reduced heat flow (mW m ⁻²)	Characte ristic depth (km)
Brazilian Coastal Shield	28±7	13.1
Baltic Shield	22±6	8.5
Wester Australia	26±8	4.5
Indian Shield (Prot)	38±2	14.8
Indian Shield (Arch)	33±1	7.5
Superior	21±1	14.4
Superior (1-D model)*	31.7	7.8
Superior (2-D model)**	31.9	8.3
Superior (2-D model)***	22.3	7.8

* values obtained by Fountain *et al.* (1987) in the Pikwitonei-Sachigo Subprovince (Superior Province) in the 1-D model assuming $q_m = 20 \text{ mW m}^{-2}$.

** values obtained by Fountain *et al.* (1987) in the Pikwitonei-Sachigo Subprovince (Superior Province) in the 2-D model assuming $q_m = 20 \text{ mW m}^{-2}$.

*** values obtained by Fountain *et al.* (1987) in the Pikwitonei-Sachigo Subprovince (Superior Province) in the 2-D model assuming $q_m = 10 \text{ mW m}^{-2}$.

variables with errors (Pujol *et al.*, 1985). The method includes the effect of errors and the correlation of variables. The results of the simulation procedure show that the 60% partition arises from tectonic influences and does not imply any fundamental coupling between the upper crust and lower crust-upper mantle heat production. Upper crustal heat sources are primarily responsible for wide global variations in mean heat flow for stable tectonic regions.

Due to the uncertainty as to the petrology of lower crust, some uncertainties exist in the evaluation of lower crustal heat production values. However, it is evident that the lower crust is less endowed with heat-producing isotopes than the surface enriched zone. Often, a uniform lower-crustal heat production value independent of the variation in upper-crustal enrichment is assumed (for example, Chapman, 1986). The range of the heat production values in different rock types is given in Tab.3.2 (after Wollenberg & Smith, 1987).

3.2 Correlation of heat production and seismic velocity

Much new information on the lower crust has become available as a result of the development and widespread application of refraction and reflection profiling and advances in quantifying metamorphic conditions. Compressional seismic velocities in the lower crust are usually between 6.5 and 7.3 km s⁻¹ and above 8 km s⁻¹ beneath the Moho (Bolland *et al.*, 1988, 1989). Metamorphic conditions are bracketed in the ranges 650-850 °C, 700-1400 MPa (Percival & Berry, 1987). Generally, the distinction between the upper and lower crust is based on (1) bulk chemical difference between a more felsic upper crust and more mafic lower crust,

Tab. 3.2

Radiogenic heat production ($\mu\text{W m}^{-3}$) (from Wollenberg and Smith, 1987)

Rock Class	Mean	S.D.	Range	no. of samples
Acid Extrusives	4.1	2.7	1.10 - 16.0	124
Acid Intrusives	4.8	2.8	0.70 - 24.0	569
Intermediate Extrusives	1.9	0.9	0.60 - 5.3	57
Intermediate Intrusives	2.6	1.6	0.10 - 12.0	271
Basic Extrusives	0.7	0.4	0.04 - 2.4	77
Basic Intrusives	0.7	0.8	0.04 - 4.1	109
Ultrabasic	0.3	0.2	0.04 - 1.1	24
Alkali Feldspathoidal Intermediate Extrusives	20.0	8.1	4.50 - 33.0	35
Alkali Feldspathoidal Intermediate Intrusives	13.0	12.0	0.90 - 98.0	61
Alkalic Basic Extrusives	2.2	2.0	0.80 - 9.8	27
Alkalic Basic Intrusives	2.0	1.4	0.40 - 4.5	8
Chemical Sedimentary Rocks*	1.4	2.3	0.04 - 11.0	38
Carbonates	0.6	0.4	0.04 - 1.6	32
Detrital Sedimentary Rocks**	3.0	3.5	0.10 - 39.0	298
Clay	1.9	1.0	0.50 - 1.0	29
Shale	4.1	1.4	0.90 - 27.0	126
Sandstone and Conglomerate	2.3	2.6	0.10 - 35.0	143
Metamorphosed Igneous Rocks	3.1	4.1	0.30 - 45.0	138
Metamorphosed Sedimentary Rocks	2.5	1.7	0.10 - 17.0	207

* Includes carbonates

** Includes clay, shale, sandstone and conglomerate

and/or (2) difference in metamorphic grade. Additionally, there is a difference in mechanical behaviour between the stronger, elastic and brittle upper crust and the softer lower crust which may deform by plastic flow. The lower boundary of the lower crust is determined by the Mohorovicic discontinuity, present almost everywhere. The boundary between the upper and lower crust (Conrad discontinuity), on the contrary, is often poorly defined or absent.

The continental crust is characterized by lateral and vertical heterogeneities of varying scale. The large-scale layering in the continental crust may be metamorphic as well as compositional (see Fig.3.3 after Fountain & Salisbury, 1981 for an example). The lower crust consists of granulite facies rocks of mafic to intermediate composition. Mafic-ultramafic bodies and anorthosites also occur in the deeper levels of some of the sections. The intermediate levels consist predominantly of amphibolite facies and range from silicic to intermediate in composition. Tonalitic to granodioritic gneisses and migmatites are common in most sections. Metasedimentary and metavolcanic rocks are also common at this level. The upper crust is characterized by low-grade metamorphosed sedimentary rocks, volcanic rocks, and their unmetamorphosed equivalents. The evolution of the crust is complex and is a result of continuous evolution by recycling and metamorphism through time in a variety of tectonic environments. The main feature of the continental crust is its variability. General relations between the heat production and depth, therefore, are of very limited value.

A different approach to the heat distribution problem is presented in papers by Allis (1979), Rybach (1979), Rybach & Buntebarth (1984), Čermák (1989) and

Figure 3.3. Generalized cross-section of the continental crust (from Fountain & Salisbury, 1981).

Čermák & Rybach (1989). It is based on an empirical correlation between heat production and seismic P-wave velocity. Rybach & Buntebarth (1984) proposed for Precambrian rocks the following relationship

$$\ln A = 12.6 - 2.17 V_p \quad (3.6)$$

and for Phanerozoic rocks:

$$\begin{aligned} \ln A &= 16.5 - 2.74 V_p && \text{at 50 MPa} \\ \ln A &= 13.7 - 2.17 V_p && \text{at 100 MPa} \\ \ln A &= 12.4 - 1.93 V_p && \text{at 200 MPa} \end{aligned} \quad (3.7)$$

where $\ln$ is natural logarithm, A is heat production ($\mu\text{W m}^{-3}$), and V_p is P-wave velocity (km s^{-1}), measured at room temperature. To render in-situ and laboratory data comparable, a correction factor is introduced

$$V_p(20,100) = V_p(T,p) [1 + B/V_p] \quad (3.8)$$

where T , p are temperature (in $^{\circ}\text{C}$) and pressure (in MPa) respectively, $B = (\partial V_p / \partial T) \Delta T + (\partial V_p / \partial p) \Delta p$, $\Delta T = 20 - T$, $\Delta p = 100 - p$ (Čermák & Bodri, 1986). The relation between heat production and seismic velocity is shown in Fig.3.4.

The A - V_p relation is not confirmed by data obtained by Fountain (1986) and Kern & Siegesmund (1989). The lack of confirmation is attributed to the fact that

Figure 3.4. Radiogenic heat production versus seismic velocity. The ellipses correspond to Rybach & Buntebarth's (1984) laboratory formula; heavy lines correspond to the relation (3.7) after application of corrections for pressure and temperature and are applicable for "in situ" conditions (from Čermák, 1989).

the seismic velocities are mainly determined by the physical properties of the major minerals, whereas heat production in the rocks results from the radiogenic components, which are largely concentrated in accessory minerals (apatite, monazite, sphene, zircon). There is no apparent correlation between heat production and shear velocity. For geochemical reasons, radioactive elements are generally enriched in acidic rocks and depleted in rocks of basaltic composition. This may be the reason why the proposed relationship between heat production and compressional wave velocity of Rybach (1979) holds in a general way for igneous rocks, for which the original relationship was derived. A linear trend is apparent, if only granites, basalts, norites and peridotites are considered. Because the constitution of the continental crust is dominated by metamorphic rocks with very complex histories, Kern & Siegesmund (1989) conclude that the heat production-velocity relationship reported in the literature cannot be used to predict heat production from seismic velocity for most crustal lithologies. However, Rybach & Buntebarth (1986) pointed out that the $A(V_p)$ relationship is of stochastic nature. They also emphasized the necessity to understand the relation within a petrophysical systematics of the interrelation V_p - A - ρ .

From the above discussion, we must conclude that, although the distribution of heat production with depth is an important factor controlling the temperature distribution in the crust, it is not clear how radiogenic elements are distributed throughout the lithosphere, and all models are subject to large uncertainties. Generally, the step or the exponential models may be applied with success to one dimensional problems. For some regions the combination of two models may be

more appropriate. For example in the southern part of the Rhinegraben (Vosges, France), an exponential model was chosen for the upper crust and a constant value of heat production assumed for the lower crust (Royer & Danis, 1989). Although a linear relationship between surface heat flow and heat production may exist, it has not been observed in all regions and it should be treated with caution. Moreover, models of $A(z)$ totally disregard lateral variations, which may play an important role in the geothermal modelling in the crust.

3.3 Thermal conductivity

Thermal conductivity is another parameter of considerable importance in the solution of steady state heat transfer problems. It is controlled by chemical and mineralogical composition, texture, grain size, and to a lesser extent by temperature and pressure effects. Earth materials in the lithosphere may have a wide range of thermal conductivity values at surface temperatures, for example from $< 1 \text{ W m}^{-1} \text{ K}^{-1}$ for unconsolidated sediments to $> 6 \text{ W m}^{-1} \text{ K}^{-1}$ for halite and pure quartzite. Variations within a given rock type may be due to the effects of low conductivity fluids filling pore spaces. Large variations may result from the presence of high conductivity monomineralic rocks (for example graphite). The values of the thermal conductivity for some different materials are given in Table 3.3.

Some aspects of thermal conductivity including its variation with depth and its effect on the temperature gradient will be described below. The data for thermal conductivity concerning the KSZ will be discussed in Chapter IV.

Thermal conductivity (k) for most rocks under crustal conditions varies

Tab. 3.3

Specific heats and thermal conductivities for various elements and materials under standard conditions (from Özisik, 1977)

Material	specific heat c	thermal conductivity k
	$\text{kJ kg}^{-1} \text{K}^{-1}$	$\text{W m}^{-1} \text{K}^{-1}$
Aluminium	0.90	190
Copper	0.39	390
Gold	0.13	310
Iron	0.45	78
Lead	0.13	37
Nickel	0.44	89
Silver	0.23	418
Steel	0.49	46
Tin	0.23	63
Granite	0.88	2.50
Graphite	0.72	45
Ice	2.10	2.20
Limestone	0.92	1.70
Sandstone	0.96	2.50
Silicon	0.71	84
Sulphur	0.70	0.28
Water	4.20	0.60

inversely with temperature and directly with pressure (or depth) according to the relation (Clark, 1966)

$$k = k_0 (1 + c_0 z) / (1 + b_0 T) \quad (3.10)$$

where k_0 is conductivity measured at 0°C and room pressure; T is in degrees Celsius; b_0 and c_0 are constants; z is depth in km. The values for b_0 , c_0 , and k are approximately

	b_0	c_0	k
upper crust	$1.5 \cdot 10^{-3} \text{ K}^{-1}$	$1.5 \cdot 10^{-3} \text{ km}^{-1}$	$3.0 \text{ W m}^{-1} \text{ K}^{-1}$
lower crust	$1.0 \cdot 10^{-4} \text{ K}^{-1}$	$1.5 \cdot 10^{-3} \text{ km}^{-1}$	$2.6 \text{ W m}^{-1} \text{ K}^{-1}$

The above refers to lattice thermal conductivity. The total conductivity is the sum of conductivities caused by lattice vibrations k_l and by the transfer of heat by radiation k_r . At $T < 1000 \text{ K}$ the thermal conductivity is due almost entirely to lattice vibrations, and the radiative conductivity is negligible. Radiative heat transfer, however, becomes important at mantle temperatures, and as a result total thermal conductivity below the lithosphere tends to increase with increasing temperature. Limiting our considerations to the crust we can assume that the thermal conductivity decreases with increasing temperature approximately as (Clark, 1966)

$$k \approx k_0 / (1 + b_0 T) \quad (3.11)$$

since the influence of depth (pressure) is negligible relative to that of temperature.

Assuming the upper crust to be dominated by rocks of granitic composition, equation (3.11) can be written as

$$k = 3/(1 + 0.0008 T) \quad (\check{\text{C}}ermák, 1982) \quad (3.12)$$

where T is in $^{\circ}\text{C}$. A lower conductivity k , causes higher temperatures at all depths, but within the existing values of conductivity in the crust ($1.5\text{-}2.5 \text{ W m}^{-1} \text{ K}^{-1}$), this effect is usually smaller than the level of uncertainty in measuring both heat production and surface heat flow. In the model of the lower crust used by $\check{\text{C}}ermák$ (1982), who assumed that it is composed of "basaltic" rocks, the thermal conductivity for the temperature range $200 - 600 \text{ }^{\circ}\text{C}$ does not significantly depend on the temperature and the value for k can be assumed to be constant, about $2.0 \text{ W m}^{-1} \text{ K}^{-1}$. A compilation of experimentally determined and estimated thermal conductivity values by different authors for crust and upper mantle temperatures is presented in Fig.3.5 ($\check{\text{C}}ermák, 1982$).

On the basis of laboratory measurements, Royer & Danis (1989) consider a value of $k = (2.0\text{-}3.0) \text{ W m}^{-1} \text{ K}^{-1}$ as appropriate for the granitic upper crust. $\check{\text{C}}ermák$ & Rybach (1989) use a value $k = 2.6 \text{ W m}^{-1} \text{ K}^{-1}$ as appropriate for rocks of lower crustal composition.

Representative values of thermal conductivity for various rock types are given in Tab.3.4 (a), (b). Crystalline rocks which constitute a significant volume of the crust and upper mantle have a range in mean thermal conductivities at temperatures up to 1000 K between 2.0 and $3.5 \text{ W m}^{-1} \text{ K}^{-1}$. For analytical convenience, most

Figure 3.5. Thermal conductivity in the crust and the upper mantle as a function of temperature (k_T). k_L - thermal conductivity caused by lattice vibrations, k_R - thermal conductivity caused by transfer of heat by radiation (from Čermák, 1982).

Tab. 3.4

Values of the thermal conductivity (k) for various rocks types
(a) from Čermak and Rybach (1982)

rock type	k (W m ⁻¹ K ⁻¹)
granite	3.00
diorite	2.91
granodiorite	2.65
gabbro	2.63
amphibolite	2.46
gneiss	2.44
granulite	2.40 - 2.90

(b) from Jones *et al.* (1989)

Limestone	2.4 ± 0.9
Dolomite	3.1 ± 1.4
Igneous, metamorphic and meteorite	2.9 ± 0.4
Shale	1.4 ± 0.4
Sandstone	3.1 ± 1.3
Siltstone	3.2 ± 1.3
Marlstone	3.0 ± 1.1
Chert	1.4 ± 0.5
Coal	0.2 ± 0.1
Conglomerate	3.2 ± 1.8
Salt	5.7 ± 1.0
Miscellaneous	2.4 ± 1.5

researchers adopt constant conductivities of about $2.5 \text{ W m}^{-1} \text{ K}^{-1}$ for the crust and $3.0 - 3.5 \text{ W m}^{-1} \text{ K}^{-1}$ for the mantle when modelling lithospheric geotherms (Morgan & Sass, 1984).

Chapman (1986) summarizes the results of different authors and assumes values for the thermal conductivities of the upper and lower crust taking into account "in situ" conditions. For the coldest thermal regime, corresponding to a surface heat flow of 40 mW m^{-2} , the base of the upper crust attains a temperature of approximately of 200°C , which lowers k from a surface value of $3.0 \text{ W m}^{-1} \text{ K}^{-1}$ to $2.4 \text{ W m}^{-1} \text{ K}^{-1}$. For $q = 90 \text{ mW m}^{-2}$, k varies from $3.0 \text{ W m}^{-1} \text{ K}^{-1}$ to $1.7 \text{ W m}^{-1} \text{ K}^{-1}$. In the lower crust the temperature and pressure effects tend to cancel each other. For $q = 40 \text{ mW m}^{-2}$, k increases from 2.61 to $2.64 \text{ W m}^{-1} \text{ K}^{-1}$.

The thermal conductivity in anisotropic materials becomes a second-rank tensor k_{ij} . The general anisotropic case includes the isotropic case, if $k_{ij} = 0$ ($i \neq j$) and $k_{ij} = k$ ($i = j$), that is, all non-diagonal terms of the tensor vanish and the non-zero diagonal ones are equal. Often there is one principal conductivity direction k_{11} , perpendicular to bedding or to the foliation and $k_{22} = k_{33}$, parallel to it. In anisotropic rocks, the component of heat flux in one direction is affected by all three components of the temperature gradient.

Local variations in thermal conductivity have an effect on the geothermal gradient. These effects can be studied numerically or analytically. Carslaw & Jaeger (1959) give numerical solutions for some of the problems which arise from such local effects, many of which can be treated similarly to electric flow problems. In the calculation of geotherms the emphasis is placed on understanding the sensitivity to

measurable properties (Chapman, 1986). Computed geotherms are most sensitive to surface heat flow or reduced heat flow, and uncertainties in these quantities will limit the accuracy of temperature estimations for lower crust and lithosphere. Otherwise, the sensitivity to thermal parameters such as A and k , and to crustal thickness, depends on the depth and on whether the region is characterized by high or low heat flow. For example, an underestimation of upper crustal thermal conductivity produces an overestimation of temperature in the lower crust and upper mantle. The temperature dependence of thermal conductivity is not negligible for high heat flow geotherms. In the case of thicker lower crust the mantle heat flow value is less, thus lower than actual temperatures are predicted for greater depths. Upper mantle thermal parameters are relatively insensitive parameters for lower crustal temperature calculations. In the upper crust, the decrease in thermal conductivity at high temperatures observed in felsic rocks tends to counteract the decrease in heat flow which results from the depth decrease of crustal radioactivity and therefore a constant gradient can usually be assumed. For depths in excess of 10 km, the temperature estimate is less certain, depending both on the model of heat transfer adopted (with or without some fluid circulation) and on the accuracy of the thermal conductivity and heat production values for the lower crust and uppermost mantle. The uncertainty of temperature estimates becomes progressively greater with increasing depth. A summary of parameter sensitivity is given in Tab.3.5 (after Chapman, 1986).

Tab. 3.5

Parameter sensitivities for geotherm calculations (from Chapman, 1986)

Parameter	Value	Pertur bation (%)	T(%) at 50 km	
			shield	rift
heat flow q_s	40 mW m ⁻² (shield)	+10%	+13%	-
	80 mW m ⁻² (rift)	+10%	-	+11%
heat production:				
depth parameter b	8 km	+25%	+6%	+4%
lower crust A	0.45 μ W m ⁻³	+20%	- 5%	- 1%
thermal conductivity				
upper crust k	3.0 W m ⁻¹ K ⁻¹	+20%	- 8%	- 8%
lower crust k	2.6 W m ⁻¹ K ⁻¹	+20%	- 5%	- 1%
Moho depth	35 km	+20%	- 2%	+1%
Conrad depth	16 km	+25%	+1%	+3%

3.4 Thermal diffusivity

Thermal diffusivity is the thermophysical rock property required for solution of the transient heat conduction equation. The bulk thermal diffusivity of rock is a function of the diffusivities, amount and distribution of its constituents. As was mentioned at the beginning of this chapter, thermal diffusivity is related to thermal conductivity k , density ρ and specific heat c according to the relation

$$\kappa = k/\rho c \quad (3.15)$$

Within any rock unit there may be both random and systematic variations, depending on (1) chemical composition, (2) crystal or particle size, (3) orientation, (4) nature of crystal boundaries, and (5) cooling history if the rock is igneous and/or metamorphic. For crystalline rocks, the range of diffusivity values is between 0.6 and 1.9 mm² s⁻¹, the lower end being appropriate for basic rocks and the upper end for quartz-bearing acidic rocks. The uncertainty in measurements of thermal diffusivity is about 3 to 5% (Drury, 1987a). The scatter in diffusivity is significantly more than that of thermal conductivity. This is explained by the fact that the diffusivity of water is typically less than 10% of the diffusivity of most common minerals, whereas water conductivity is 25 - 30% of the conductivity of the minerals. Therefore, for a sample of uniform mineralogy in which porosity varies, a greater variation of diffusivity than of conductivity is expected. Quartz has the highest diffusivity of the commonly occurring major minerals. However, there is usually no linear correlation between diffusivity and either quartz content (κ is related to quartz through geometrical

relationship) or porosity for the samples. Data for thermal diffusivity are presented in Tab.3.6.

A high value of κ could result either from a high value of thermal conductivity, which would indicate a rapid energy-transfer rate, or from a low value of the specific heat c . A low value of the specific heat implies that less of the energy moving through the material is absorbed and used to raise the temperature of the material; thus more energy would be available for further transfer.

Density is a well-known property of rocks and is usually uniform within a given petrological type within about 3%. Allowing for an increase in density with depth, the mean density of the continental crust is usually estimated to be in the range 2750 - 2900 kg m⁻³ (see for example Bott, 1982).

Variations in the values of thermal parameters are discussed by Drury (1987) on the basis of measurements made on samples from the Canadian Shield. Measurements of the specific heat of crystalline rocks were generally higher than previously published measurements for similar rocks and minerals. The range 700-800 J kg⁻¹ K⁻¹ is typical of most common minerals. However, for some rocks, such as basalts, the measured value is much larger than that of the constituent minerals (see Tab.3.7). This can be explained in terms of relatively high water content since water has a significantly high specific heat than that of most minerals. So even a small amount of water in a rock can substantially increase the specific heat of the sample. For example, the value of specific heat for dry basalt is 900 J kg⁻¹ K⁻¹ but for a water saturated sample it is 1020 J kg⁻¹ K⁻¹ when the mean porosity of the basalt samples equal 3.6% (Drury, 1987a).

Tab. 3.6
Thermal diffusivity according to the compilation of Drury (1987a)

Rock type	old range	new range
	(mm ² s ⁻¹)	(mm ² s ⁻¹)
granite	0.5 - 1.51	0.9 - 1.87
gneiss	1.13 - 1.41	0.87 - 1.18
granodiorite	0.50 - 0.91	0.56 - 1.87
basalts	0.64 - 0.68*/0.45 - 1.05	0.56 - 0.93

"old range" refers to compilation of Drury (1984)

"new range" refers to the data presented by Drury (1987a)

* data from Kappelmayer & Haenel (1974)

Tab. 3.7

Thermophysical data for various rock types (mean $\pm$ standard deviation). Conductivity, diffusivity and density were measured individually on each sample, and specific heat was calculated from those parameters (from Drury, 1987a)

Rock type	Conductivity	Diffusivity	Density	Specific heat	No. of samples
	(W m ⁻¹ K ⁻¹)	(mm ² s ⁻¹)	(Mg m ⁻³)	(J kg ⁻¹ K ⁻¹)	
Granite	3.62 $\pm$ 0.30	1.34 $\pm$ 0.21	2.63 $\pm$ 0.03	1060 $\pm$ 192	171
Granodiorite	3.35 $\pm$ 0.35	1.21 $\pm$ 0.25	2.65 $\pm$ 0.03	1070 $\pm$ 187	59
Gneiss	2.29 $\pm$ 0.21	0.87 $\pm$ 0.14	2.79 $\pm$ 0.09	796 $\pm$ 467	22
Gabbro	2.34 $\pm$ 0.39	0.83 $\pm$ 0.22	3.05 $\pm$ 0.08	947 $\pm$ 100	6
Basalt	1.93 $\pm$ 0.19	0.69 $\pm$ 0.08	2.83 $\pm$ 0.17	1010 $\pm$ 129	47

Difficulties in modelling the thermal diffusivity are similar to those encountered in modelling conductivity (both quantities are tensors in anisotropic materials). Most commonly used models do not take into account factors such as the anisotropy of diffusivity, or grain boundary effects, including porosity. Thermophysical parameter values as compiled by Drury (1987a) are shown in Tab.3.7.

3.5 Coefficient of volume expansion

Considering the thermal parameters of the heat transfer equation the coefficient of volume expansion of the rock α (K^{-1}) should be mentioned, since it affects density. For an isotropic material, α is expressed by (Turcotte & Schubert, 1982)

$$\alpha = 1/V(\partial V/\partial T)_p = -1/\rho(\partial \rho/\partial T)_p \quad (3.16)$$

where V is volume, T temperature, p pressure, and ρ is density. For crystalline rocks α lies in the range $1.5-3.3 \cdot 10^{-5} K^{-1}$. For the upper mantle just below the Moho, it is about $3.0 \cdot 10^{-5} K^{-1}$; at the base of the mantle the value of α is about $1.0 \cdot 10^{-5} K^{-1}$ (see e.g. Stacey, 1977).

4. THE KAPUSKASING STRUCTURAL ZONE AND THE THERMAL REGIME OF PRECAMBRIAN SHIELDS

4.1 Geological and geophysical evidence

The Kapuskasing Structural Zone (KSZ) is part of the Superior Province, the oldest subdivision of the Canadian Shield and the largest unit of exposed Archean rocks. Isotopic age determinations throughout the Superior Province suggest that the Kenoran orogeny (2500 Ma) is the most significant tectonometamorphic event to have affected these rocks. Some of the age determinations (see review by Percival & Card, 1983) reveal significantly older dates, up to 3550 Ma in some regions. The Superior Province is thought to have stabilized by early Proterozoic time.

A positive gravity anomaly known as the Kapuskasing gravity high (first observed by Garland, 1950, see review by Percival & McGrath, 1986) is one of the major features of the gravity map of northern Ontario. The anomaly belt is at least 1000 km long and extends from James Bay in the NE to Chapleau in the SW, where it gradually disappears (see Fig.4.1). Parallel to its axis, but displaced some 40 km to the east is an aeromagnetic high. A NE trending thrust fault is situated parallel to the anomaly to the SE (see review by Percival & McGrath, 1986).

The Superior Province is composed of the Pontiac, Abitibi, and Opatika Belts to the east and the Wawa and Quetico Belts to the west of the gravity anomaly (Fig.4.1). The principal rock types in the belts include granitic rocks, migmatites, metasediments, metavolcanics, mafic and ultra-mafic intrusive and syenite (see review by Forsyth *et al.*, 1983). The Michipicoten and Abitibi belts consist of

Figure 4.1. (a) Location of the Kapuskasing Uplift (KU) with respect to major features of the Canadian Shield. KS - Kenyon structure, WRF - Winisk River Fault.

(b) Regional geology of the central Superior Province showing major tectonic features associated with the Kapuskasing Zone, a composite structure made up of the Chapleau (CB), Groundhog River (GRB), and Fraserdale-Moosonee (FMB) blocks. The Val Rita Block (VRB) is bounded by the Lepage (LF), Foxville (FF), Kneras (KF), and Saganash Lake (SLF) faults. Additional structures indicated include the Bad River (BRF) and Winisk River (WRF) faults, and Ivanhoe Lake cataclastic zone (ILCZ) (from Percival & McGrath, 1986).

metavolcanic and metasedimentary suites, while heterogeneous tonalitic gneisses and granodiorites characterize the Wawa domal gneiss terrane. Mafic paragneiss and anorthosite are typical in the Kapuskasing zone. Depending on the predominance of a rock type, belts are classified as "Greenstone Belts" (volcanic-rich) or "Metasedimentary Belts" (metasediment/gneiss-rich). The Pontiac, Quetico, and Opatica Belts are metasedimentary gneiss belts whereas the Wawa and Abitibi Belts are greenstone belts (Forsyth *et al.*, 1983). These belts are of the order of 50-300 km in width and generally display greenschist to amphibolite facies metamorphism.

Different hypotheses have been proposed to account for the origin of the Kapuskasing Structural Zone. First it was treated as a thinned upper crustal layer (Garland, 1950). Later it was assumed to be a graben associated with a mid-Proterozoic rifting event (Innes *et al.*, 1967). McGlynn (1970), interpreted the KSZ as a horst structure composed of numerous fault slices. It has also been proposed that the KSZ is a Proterozoic suture (Wilson, 1968), a failed arm of a late Proterozoic rift (Burke & Dewey, 1973), and a sinistral transcurrent shear zone (Watson, 1980). Gibb (1983) suggested several different components of movement at different times (see review by Percival & McGrath, 1986). The new accepted hypothesis of thrusting has been proposed by Percival & Card (1983). The KSZ is now thought to be an oblique cross-section through some 25 km of crust, which has been tectonically transported to the surface along a west-dipping major thrust fault in the early Proterozoic (2250-1950 Ma) (Percival & McGrath, 1986).

In the literature the terms "Kapuskasing zone" and "Kapuskasing uplift" are used. The former refers to the linear NNE trending belt of geophysical anomalies

and granulites. The latter is the region transitional between greenschist and granulite facies metamorphic zones, interpreted as a nearly complete, oblique crustal cross section through an Archean Greenstone Belt (Percival & McGrath, 1986).

The KSZ is approximately 120 km wide (Chapleau Block) and characterized by a west-to-east increase in metamorphic grade. Detailed geologic mapping and geophysical investigations have revealed the presence of a major boundary separating high-grade rocks of the Archean Wawa Belt (Wawa Domal Gneiss Terrane) and the KSZ to the west from the low-grade Archean rocks of the Abitibi Belt (Abitibi Greenstone Belt) to the east. The rocks to the west of this boundary are characterized by an easterly increase in metamorphic grade from greenschist facies in the Michipicoten Belt west of the Wawa Belt to granulite facies in the KSZ (Percival & McGrath, 1986). The boundary between the Kapuskasing zone and the Abitibi Belt is delineated by a narrow zone of fault-related rocks (mylonites, blastomylonites, and pseudotachylites; Percival *et al.*, 1983; Percival & Card, 1983). This zone, known as the Ivanhoe Lake Cataclastic Zone (ILCZ) extends for several hundred of kilometres with an approximately northeast-southwest strike. It is characterized by a positive linear magnetic anomaly, and coincides with the through of a paired high (west)/low (east) Bouguer gravity anomaly (Percival *et al.*, 1983). The ILCZ is a major structural break. The analysis of seismic refraction data across the KSZ supports the geological model of an intra-cratonic thrust (Boland *et al.*, 1988; Bolland & Ellis, 1989). The upper crustal P-wave velocities increase progressively from 5.9 km s⁻¹ over the Wawa Domal Gneiss in the west to 6.5 km s⁻¹ over the southern KSZ, and then decrease abruptly back to 6.25 km s⁻¹ to the

east over the Abitibi Greenstone Belt (Fig.4.2). The roof of material with a velocity of 6.6-6.7 km s⁻¹ is situated at a depth of about 20 km beneath the Wawa and Abitibi belts, but only at 4 km under the KSZ.

Seismic reflection profiles (Cook, 1985), recorded in the Kapuskasing Structural Zone near Foleyet, Ontario, reveal reflections with travel times of at least 5.0 s. An event dipping west from about 1.7 to 3.4 s probably corresponds to the Ivanhoe Lake Cataclastic Zone. The true dip on this boundary is calculated to be about 35-38° west-northwest and is consistent with interpretation of this zone as a west-dipping, crustal-scale thrust fault (Percival *et al.*, 1983).

A single, 500 km long thrust fault, of which the Ivanhoe Lake fault is part, is postulated to have carried the granulites to the surface and to underlie the entire KSZ (Cook, 1985). There are along-strike normal faults of variable displacement on the western side of the KSZ (Percival & McGrath, 1986). They limit (from the south to north) the Chapleau Block (CB), the Groundhog River Block (GRB), and Fraserdale Moosonee Block (FMB) (see Fig.4.1 (b)). The three fault bounded tectonic blocks are composed of granulite to upper amphibolite facies rocks. The CB is dominantly orthogneiss in character. Seismic refraction studies (see review by Percival & McGrath, 1986) indicate a depression of the Moho beneath the Chapleau block relative to the more northerly blocks. Uplift of the CB from a crustal position beneath the Conrad discontinuity is suggested by high average density (Percival, 1986) and high seismic velocities (Cook, 1985). GRB contains a heterogeneous sequence of tonalitic orthogneiss, mafic gneiss and rare paragneiss. GRB is separated from FMB to the north by a fault-bounded wedge of tonalitic gneiss of amphibolite

Figure 4.2. Geological cross-section (a) and isovelocity profile (b) of the KSZ (from Boland *et al.*, 1988).

facies, the Val Rita Block. The FMB consists mainly of metasedimentary granulites. The Saganash Lake Fault (SLF) separates the Kapuskasing granulites from amphibolite facies tonalitic gneiss and granite of the Wawa Belt to the west of the CB.

Major northeast-striking faults that bound the Kapuskasing zone on the west have been geologically & geophysically investigated. Bouguer anomalies have been modelled to determine fault dips and geobarometry of garnet-orthopyroxene-plagioclase-quartz assemblages has been used to estimate vertical displacements (Percival & McGrath, 1986). Granulites in the Kapuskasing zone record pressures of 0.6 to 1 GPa whereas those in the Quetico Belt to the west indicate metamorphic pressures of only 0.4-0.6 GPa. Individual calibrations of the barometer yield consistent pressure differences of 0.2-0.3 GPa, suggesting 7-10 km of west-side-down movement on the faults. These major normal faults which are subparallel to the northwest dipping thrust fault (ILCZ) on the eastern side, dip 60° - 70° NW and probably formed as collapse structures in response to a crustal thickening episode which occurred during the preceding compressional uplift stage (early Proterozoic). Easterly increasing metamorphic pressure over a 120 km wide zone suggest oblique exposure of a 20 km thick crustal layer (Percival, 1983). The temperature is assumed to have been greater than 700°C in all the granulites (Percival, 1983).

The KSZ has the potential to provide valuable information on the nature of the continental crust to depths corresponding to the highest observed metamorphic grade (about 25 km; Cook, 1985).

4.2 Thermal regime of the Superior Province and the Kapuskasing Structural Zone

Modelling the present thermal regime of the KSZ, as a part of the Superior Province in the Canadian Shield, requires several preliminary steps. The input parameters are corrected values of surface heat flow (q_s), the heat production distribution as a function of depth ($A(z)$), and thermal conductivity (k), density (ρ) and specific heat (c) which together determine the thermal diffusivity.

4.2.1 Thermal regime of the Precambrian Shield

Both the heat flow distribution within cratons and the differences between craton and mobile belts in regard to temperature-depth gradients have been studied extensively (for example, Pollack, 1986; Ballard & Pollack, 1987; Fountain *et al.*, 1987).

There is an obvious paradox when the early thermal regime of the stabilized Archean cratons is considered. There is evidence that Archean temperatures, both in the crust and cratonic upper mantle, were not different from those of the present day (Pollack, 1986), although the total heat loss from the Earth must have been substantially higher in the Archean than at present, since heat production from the decay of radioisotopes in the Earth was probably 2-3 times the present-day rate. The paradox may be explained by assuming that the extra heat in the Archean was lost not through the cratons, but through the surrounding Archean oceans (Ballard & Pollack, 1987). Since mantle convection is a more efficient heat transfer mechanism than conduction (responsible for heat transfer in the craton), it is expected that some asthenosphere heat flow would be advected away from the cratonic root and

subsequently escape to the surface through non-cratonic terrains (Ballard & Pollack, 1987). Thus the heat flow into the base of the craton was low enough that the thermal conditions within the craton in the Archean may have been very similar to what they are today. Another mechanisms for higher heat loss in the Archean is more vigorous mantle convection and sea-floor spreading in that time.

The mean surface heat flow in the interior of Archean cratons is typically about $40 \pm 8 \text{ mW m}^{-2}$, while that in surrounding Proterozoic and younger terrains is considerably higher (about $60 \pm 12 \text{ mW m}^{-2}$ on the average, Ballard & Pollack, 1987). The values of heat flow measured in a number of different cratons including the Superior Province are presented in Tab.4.1 (Drury, 1988). It can be seen that the heat flow values in the Superior Province range between 37 and 54 mW m^{-2} with mean value equal $43 \pm 8.6 \text{ mW m}^{-2}$ (Ballard & Pollack, 1987). The differences between heat flow in craton and in surrounding mobile belts may be attributed to two separate causes: firstly, the difference in crustal heat production between the two terrains; secondly, the existence of a "lithospheric root" beneath the Archean cratons. This root serves to divert heat away from the craton into the thinner surrounding lithosphere. Ballard & Pollack (1987) suggest that the combination of differences in crustal heat production and lithospheric thickness can explain the contrast in observed surface heat flow values. The cratonic root beneath the Superior Province may extend to a depth of 400 km (Ballard & Pollack, 1987), whereas in southern Africa it extends to depths of 200-300 km. The mean heat production in the cratonic crust is between $0.45 \text{ } \mu\text{W m}^{-3}$ and $0.90 \text{ } \mu\text{W m}^{-3}$ and the contrast in crustal heat production between the Archean cratons and younger surrounding terrains (mobile

Tab. 4.1

Comparison of heat flow in different tectonic areas (from Drury, 1988).

Area	Heat flow (mW m ⁻²)		
	mean	range	number of observations
Superior craton (exposed)	43	37-54	16
Wyoming craton	64	50-88	12
Churchill craton*	80	70-90	-
South Keewatin*	65	60-70	-
Exposed PMB*	46	23-59	5
Buried PMB*	50	45-55	-

* Estimated from the heat-flow map of Drury (1988), where PMB is the Proterozoic Mobile Belt.

belts) is usually less than about $0.35 \mu\text{W m}^{-2}$. Ballard & Pollack (1987) have shown that it is possible for a root of the inferred thickness range to divert enough heat to account for 50-100% of the observed contrast in surface heat flow, the remaining being attributed to differences in crustal heat production between the craton and the surrounding younger terrains.

4.2.2 Glacial Correction to measured heat flow

The need for corrections in the measured heat flow values arises from neglecting the effects of climatic variations in drillholes. Other measurement errors, resulting from groundwater movements and topographic effects, have been mentioned in Chapter III.

There are considerable difficulties in inferring a comprehensive picture of past climatic variations. Some attempts have been made to correlate climatic variations with variations in geophysical phenomena such as magnetic intensity change (Wollin *et al.*, 1973), and astronomical cycles (see review by Beck, 1977).

Disturbances due to climatic history may range from zero to significant values of the observed heat flow. The magnitude of the effect and the depth range over which it is significant depend on the time period and amplitude of the surface temperature variations, as well as latitude and thermal conductivity. The problem has been discussed by many authors, for example by Birch (1948), Beck (1977, 1982), Beck & Shen (1989) and in particular for KSZ by Čermák & Jessop (1971).

The final retreat of the Wisconsin ice sheet is the most significant event affecting the disturbance in the Canadian Shield. In regions which have been subject

to Pleistocene glaciations, such as the KSZ, the effect of varying surface temperatures on the measured underground temperature must be calculated and eliminated in order that the corrected heat flow values accurately represent the internal thermal regime.

Depending on the model used to apply the glacial correction, different values for the heat flow are obtained. Čermák & Jessop (1971), for example, assumed a 4 K temperature rise since the Wisconsin Glaciation by taking the average ground temperature to be close to the melting point of ice under the appropriate pressure at the base of ice cover. Probably some of the most important information on patterns of temperature variations in the past has come from a study of the $^{18}\text{O}/^{16}\text{O}$ oxygen isotope abundance ratio in various materials. Beck (1977) proposed a 12 K temperature rise, based particularly on $^{18}\text{O}/^{16}\text{O}$ ratio variations in ice cores from Greenland and the actual observed temperature at the base of the Greenland ice shield. Nielsen & Beck (1989), on the basis of a comparative analysis, take the temperature rise since the end of glaciation to be 8 K with standard deviation of 4 K. The difference between assuming a temperature rise of 4 or 12 K, in terms of paleoclimatic corrections to terrestrial heat flow amounts to 10 mW m^{-2} .

The particular model of climatic history adopted for the Superior Province is based on data from Prest (1957), and Benett *et al.* (1967) (see review by Čermák & Jessop, 1971). To correct the measured value, a knowledge of the time intervals when the surface temperature differed from its present value, and the amounts of these deviations, is necessary. A step function model of the surface temperature is chosen based on the times of advance and retreat of ice sheets. The estimates of the

ground temperature below the ice cover during the glacial age range from 0°C to -4°C (Crain, 1967, see review by Čermák & Jessop, 1971). The detailed ice base temperature reflected the pre-glacial environment and depended on the thickness of the ice, its movement, etc.. The thickness of the ice cover preserved the heat being lost from the surface, so the temperature at the base increased slowly. The estimated surface temperatures during late interglacials suggest that KSZ has been free of any significant permafrost and the temperature of the ice-ground boundary can be assumed to be close to the melting point of ice during most of the glacial times. Because the thickness of the Wisconsin ice cover exceeded 3 km (thereby shielding the ground from surface temperature disturbances), the ice base temperature of -1.5°C was adopted to represent the ground temperature during the entire ice age. The surface temperature during interglacials periods is usually equivalent to the present-day surface temperature.

The climatic corrections are based on the difference between the assumed temperatures in the past and the extrapolated surface temperature from the straight part of the geotherm. The correction for the effect of the glaciations can be as much as a 30% increase of the measured heat-flow value (Čermák & Jessop, 1971). The magnitude of the correction depends on the thermal diffusivity of the rocks. In their calculation of the climatic corrections Čermák & Jessop (1971) assumed diffusivities of $1.3 \cdot 10^{-8}$ and $1.4 \cdot 10^{-8} \text{ m}^2 \text{ s}^{-1}$. The change in temperature gradient in the KSZ it is calculated to be 1.9-1.4 °C/km in depth interval 300 - 700 m using the model of Čermák & Jessop (1971). The application of the glaciation correction in the Superior Province changes the values of surface heat flow (q_s) from $33 \pm 6 \text{ mW m}^{-2}$ to 40

mW m^{-2} , reduced heat flow (q_r) from 21 to 28 mW m^{-2} and characteristic depth from $14.4 \pm 0.5 \text{ km}$ to $13.6 \pm 0.6 \text{ km}$ (Jessop & Lewis, 1978).

4.2.3 Thermal parameters

(i) Heat generation

Measurements of heat generation from Superior Province and KSZ samples are presented in this section. Tab.4.2 shows heat generation values from the Superior Province as given by Jessop & Lewis (1978). The heat generation values are generally less than $3 \mu\text{W m}^{-3}$ (for the Kapuskasing Structural Zone the average value is $0.46 \pm 0.08 \mu\text{W m}^{-3}$). These values are lower than those from other Archean rocks. The heat production measurements confirm the low values of heat production measured in other shields and their lateral variations.

Recent ideas on the evolution of the crust have led to a 2-layer model of heat generation in the crust: an upper layer of variable heat generation, and a lower layer of uniformly low heat generation (Drury, 1989). The higher and more variable heat generation of the upper layer is consistent with ideas concerning removal of heat-producing elements from the lower crust during amphibolite to granulite metamorphism and their redistribution into the upper crust (Drury, 1989). The granulite nature of the lower crust suggests a large-scale migration of CO_2 -rich fluids through the lower crust, with transfer of heat-producing isotopes and H_2O to higher crustal levels during the late Archean and early Proterozoic, occasionally accompanied by partial melting. However, the redistribution of heat-producing elements in such a process would have only a small effect on crustal geotherm, so

Tab. 4.2
Heat production values (from Jessop and Lewis, 1978)

Site	n	*	Heat production ($\mu\text{W m}^{-3}$)
Winnipeg	13	B	1.42 ± 0.50
Jackfish	15	B	1.01 ± 0.28
	51	P	1.41 ± 0.60
Otoskwin River	12	B	0.18 ± 0.08
Minchin Lake	17	B	0.86 ± 0.53
	116	P	2.10 ± 1.30
English River			
north of collar	41	P	2.01 ± 0.89
south of and near collar	41	P	3.70 ± 2.30
	8	S	2.70 ± 1.80
	17	B	3.10 ± 2.20
Nielsen Island	13	B	1.25 ± 0.17
Lake Dufault	4	S	0.55 ± 0.09
Sudbury	11	B	1.43 ± 0.98
northwest of Windy Lake	3	S	3.40 ± 1.70
west of Wanapitei Lake	5	S	2.60 ± 1.30
Merrill Island	6	B	0.07 ± 0.03
Red Lake	4	\$, B	0.83 ± 0.20
Manitouwadge	7	S	0.94 ± 0.61
	20	B	1.30 ± 1.10
	19	B	1.51 ± 0.77
Cochrane		1	1.30 ± 0.20
Kapuskasing	1		0.46 ± 0.08
Hearst	1		1.80 ± 0.30
Echo L.	9	2	0.10 ± 0.07
Snow L.	19	2	0.29 ± 0.09
Thompson	10	2	1.04 ± 0.85
Lynn L.	4	2	0.06 ± 0.01
Fox L.	34	2	0.92 ± 0.11

* B = borehole samples; S = surface samples; P = portable gamma-ray spectrometer measurements on outcrops; \$ = composite samples; 1 = Cermak and Jessop (1971) 2 = data from Churchill Province (from Drury, 1985).

n - number of samples

2

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that partitioning of heat production was not a major factor in the crustal stabilisation (Drury, 1989).

Detailed data on heat production (A), thermal conductivity (k), and surface heat flow values (q_s) from the Pikwitonei-Sachigo (Manitoba) region of the Superior Province are presented by Fountain *et al.* (1987). The heat production, thermal conductivity and density data for the Pikwitonei-Sachigo Subprovince samples are shown in Tab.4.3.

A detailed study of heat production from the KSZ was undertaken by Ashwal *et al.* (1987). Fifty eight samples representative of the main lithologies exposed in a 100 km transect were measured. Samples were taken from the Michipicoten (Wawa) greenstone belt, through a domal gneiss terrane of amphibolite grade, to the granulite belt of the KSZ (Fig.4.3). Heat flow measurements from three sites located in the KSZ yield values of 51, 33 and 43 mW m⁻² respectively (corrected for glaciation effect), with mean site heat production values of 1.8, 0.5 and 1.3 μ W m⁻³, respectively. The location of the sites is shown in Fig.4.3. Values of heat production measured in the KSZ samples are given in Tab.4.4. Mean heat production values for these terrains, based on surface distribution of major rock types, are shown in Tab.4.5. From the restored profile (profile A - B, see Fig.4.3) we see that the heat production data are consistent with the hypothesis that granitoid intrusives in the Wawa domal gneisses terrane are enriched in radiogenic elements relative to the lower parts of the section. Among the silicic plutonic rocks (tonalites, granites, and gneisses), the relatively large variation in heat production correlates well with abundances of accessory minerals. The relatively high weighed mean heat

4.3. (a) Generalized geological map of the central Superior Province showing location of study area. Borehole-sites 1, 2, and 3 are Hearst, Kapuskasing, and Cochrane respectively. (b) Geological map of the Wawa-Foley areas showing sample locations. A-B represents line of section to which sample locations were projected in constructing depth profile by Ashwal *et al.*, 1987). C-D represents line of section in this study (from Ashwal *et al.*, 1987).

Tab. 4.3
Heat production and thermal-conductivity data (from Fountain *et al.*, 1987)

Sam ple	Loca tion	Lithology	ρ (kg m ⁻³)	A (μ W m ⁻³)	k (W m ⁻¹ K ⁻¹)	
Felsic gneisses						
P1	PR	Enderbite gneiss		2663	0.26	n.d.
P2	PR	Enderbite gneiss		2638	0.28	2.57
P6	PR	Enderbite gneiss		2685	0.21	2.85
P7	NL	Enderbite gneiss		2723	0.34	2.45
P11	CL	Tonalite gneiss		2674	0.38	2.56
P13	CL	Tonalite gneiss		2654	0.26	2.89
P25	OL	Tonalite orthogneiss		2734	0.75	n.d.
P37	CL	Enderbite gneiss		2688	0.52	2.51
P48	GL	Tonalite gneiss		2663	0.70	2.64
P50	NR	Enderbite gneiss		2693	0.53	3.20
P52	PL	Enderbite gneiss		2667	0.85*	2.95
P60	OL	Tonalite gneiss		2723	0.71	2.95
P61	OL	Tonalite gneiss		2708	0.41	2.95
P64	CI	Enderbite gneiss		2738	1.37	2.92
Mafic gneisses						
P5	NL	Norite gneiss		3269	0.15	4.16
P9	CL	Hornblende gneiss		3103	0.29	2.12
P12	CL	Mafic gneiss (inclusion)		2837	1.51	1.93
P14	CL	Amphibolite		3034	0.09	1.82
P31	NL	Norite gneiss		3086	0.10	2.30
P51	NR	Norite gneiss		3034	0.13	2.12
P62	BR	Mafic gneiss		2957	0.09	2.35
Anorthosite						
P36	CL	Anorthosite		2713	0.12	2.58
Metasedimentary rocks						
P8	NL	gar-pyx-mag gneiss		3599	0.06	n.d.
P17	UL	bi-plag gneiss		2745	1.35	2.48
P18	UL	bi-plag-sill gneiss		2711	1.23	3.00
P23	BL	Metagraywacke		2702	1.08	3.07
P24	BL	Metagraywacke		2854	1.37	1.88
P26	OL	Metagraywacke		2801	1.00	n.d.
P27	OL	Metaconglomerate		2849	2.81	2.52
P33	NL	Quartzite		2651	2.70*	3.12
P53	BL	bi-plag gneiss		2827	0.71	2.22

Metavolcanic and intrusive rocks					
P19	UL	Metabasalt	3076	0.05	2.01
P20	UL	Metabasalt	3069	0.23	2.44
P22	BL	Metatuff	2902	4.96	2.10
P28	OL	Metabasalt	3019	0.02	3.04
P29	OL	Metagabbro	3025	0.10	3.02
P38	UL	Metagabbro	3113	0.25	2.71
P39	UL	Metabasalt	2996	0.12	2.42
P40	UL	Metagabbro	3023	0.24	2.42
P42	UL	Hornblende gneiss	2797	0.65	2.60
P44	OL	Metagabbro	3041	0.22	3.39
P45	OL	Metabasalt	3041	0.11	2.94
P46	OL	Metabasalt	2840	0.11	2.28
P47	OL	Meta-andesite (basaltic)	2941	0.36	3.17
P55	BL	hb-bi-plag gneiss	2793	1.34	2.13
P56	BL	hb-plag gneiss (tuff breccia)	2851	1.78	1.98
Silicic intrusive rocks					
P16	UL	Bear Lake granite	2628	1.67	3.39
P21	BL	Bear Lake granite	2680	3.33	n.d.
P41	UL	Leucogranite	2659	1.45	2.93
P43	OL	Granodiorite	2744	0.87	2.70
P54	BL	Quartz diorite	2640	0.96	2.55
P57	OL	Granodiorite	2695	0.49	2.74
Trondhjemites and garnet-bearing gneisses					
P10	CL	Trondhjemite gneiss	2613	1.30	3.22
P15	CL	Trondhjemite gneiss	2682	0.44	2.30
P32	NL	gar-bi gneiss	2812	4.22*	3.22
P34	CL	gar trond. gneiss	2714	0.76	2.92
P35	CL	gar trond. gneiss	2681	3.76*	2.64
P49	NR	gar-bi gneiss	2693	0.63	3.06
Molson dike					
P3	PR	Diabase	2999	0.19	2.83
Mylonites					
P58	OL	Metadacite	2817	0.35	3.24
P59	OL	Amphibolitic schist	2967	0.59	1.65

Notes: BL, Bear Lake; BR, Burntwood River; CL, Cauchon Lake; GL, Goulet Lake; NL, Natawahunan Lake; NR, Nelson River; OL, Oxford Lake; PL, Pikwitonei Lake; PR, Pikwitonei River; UL, Utik Lake; bi, biotite; gar, garnet; hb, hornblende; mag, magnetite; plag, plagioclase; pyx, pyroxene; sill, sillimanite; trond., trondhjemite; n.d., not determined. U concentration estimated using Th/U ratio of 4. Heat production estimated with this

value of U.

Tab. 4.4

Heat production in Kapuskasing samples (from Ashwal *et al.*, 1987)

Sample No.	Description	Metamorphic grade	Heat production ($\mu\text{W m}^{-3}$)
1	Leucogranite, Oswald L. Pluton, E of ILCZ	"granitic"	6.09 $\pm$ 0.14 5.89 $\pm$ 0.16 avg. = 5.99 $\pm$ 0.15
2	Flaser tonalite, Abitibi Belt	amphibolite	0.57 $\pm$ 0.06
3	Paragneiss, graphite-bearing	granulite	0.66 $\pm$ 0.05
4	Mafic volcanic, Abitibi belt, E of ILCZ	amphibolite	0.12 $\pm$ 0.04
5	Mafic gneiss	granulite	0.36 $\pm$ 0.06
6A	Mafic tonalite gneiss, garnet-rich	granulite	0.24 $\pm$ 0.05
6B	Mafic tonalite gneiss, garnet-free	granulite	0.59 $\pm$ 0.04
7	Paragneiss	granulite	1.02 $\pm$ 0.05
7A	Mafic gneiss	granulite	0.11 $\pm$ 0.03
8	Mafic gneiss with leucosome	granulite	0.48 $\pm$ 0.04
9	Banded gneiss	granulite	0.34 $\pm$ 0.05
10	Xenolithic tonalite gneiss	amphibolite	0.35 $\pm$ 0.04
11	Diorite-monzonite, Floranna Complex	"granitic"	1.03 $\pm$ 0.08
12	Mafic granulite gneiss, garnet bearing	granulite	0.09 $\pm$ 0.03
13	Xenolithic tonalite	amphibolite	0.52 $\pm$ 0.05 0.53 $\pm$ 0.05 avg. = 0.53 $\pm$ 0.05
14	Tonalite gneiss	amphibolite	1.70 $\pm$ 0.07 1.74 $\pm$ 0.08 avg. = 1.72 $\pm$ 0.08
15A	Granite	"granitic"	4.51 $\pm$ 0.15
15B	Tonalite gneiss (inclusion in 15A)	amphibolite	0.20 $\pm$ 0.05
16	Paragneiss (minus leucosome)	amphibolite	1.21 $\pm$ 0.07
17	Leucosome of paragneiss	"granitic"	2.20 $\pm$ 0.08

			2.18±0.08
		avg. =	2.19±0.08
18	Tonalite gneiss	amphibolite	0.87±0.06
19	Xenolithic tonalite	amphibolite	0.23±0.05
20	Chaplin Lake granodiorite	"granitic"	0.74±0.05
21	Mafic xenolith from xenolithic tonalite	amphibolite	0.21±0.03
22	Tonalite gneiss (layered)	amphibolite	0.40±0.05
23	Massive granite	"granitic"	1.57±0.08
			1.58±0.08
		avg. =	1.58±0.08
24	Xenolithic tonalite	amphibolite	1.51±0.06
			1.41±0.06
		avg. =	1.46±0.06
25	Flat banded gneiss	amphibolite	0.43±0.05
26	Pegmatite, sphenebearing	"granitic"	3.59±0.12
27	Tonalite gneiss	amphibolite	0.68±0.05
28	Xenolithic tonalite	amphibolite	0.44±0.05
29	Tonalite gneiss	amphibolite	0.50±0.05
30	Tonalite gneiss with crosscutting granite dike	amphibolite	2.27±0.08
31A	Granitic gneiss (cataclastic)	"granitic"	2.73±0.08
31B	Tonalite gneiss	amphibolite	6.99±0.17
			6.78±0.14
		avg. =	6.89±0.16
32	Mafic gneiss	amphibolite	0.08±0.04
32A	Tonalite gneiss	amphibolite	0.23±0.05
33	Mafic volcanic	amphibolite	0.08±0.03
34	Massive granite	"granitic"	0.43±0.05
			0.42±0.05
		avg. =	0.43±0.05
35	Mafic volcanic	greenschist	0.05±0.03
36	Lineated tonalite (mafic sample + felsic sample)	amphibolite	0.71±0.06
			0.73±0.05
		avg. =	0.72±0.06
37	Massive granodiorite	"granitic"	0.56±0.05
38	Tonalite	greenschist	1.20±0.06
			1.14±0.06
		avg. =	1.17±0.06
39	Pillowed basalt, Sir James Mine	greenschist	0.09±0.03
40	Felsic volcanic Sir James Mine	greenschist	2.77±0.10

41	Felsic volcanic	greenschist	2.20±0.08
42A	Felsic volcanic, McLeod Mine	greenschist	1.59±0.06
			1.57±0.06
		avg. =	1.58±0.06
42B	Iron formation, McLeod Mine	greenschist	0.03±0.02
43	Dore conglomerate	greenschist	1.37±0.08
44	Mafic tonalite to diorite	granulite	0.90±0.06
45	Paragneiss	granulite	0.11±0.05
46	Flaser diorite to mafic tonalite	granulite	0.31±0.05
47	Mafic tonalite to diorite	granulite	0.87±0.09
48	Paragneiss	granulite	0.91±0.07
49	Paragneiss	granulite	0.71±0.06
50	Leucogranite assoc. with aplite and pegmatite	"granitic"	0.58±0.12
51	Foliated mafic tonalite	amphibolite	0.76±0.08
52	Unfoliated granodiorite	"granitic"	1.70±0.18
53	Diorite-monzonite	"granitic"	0.90±0.09
54	Tonalite	greenschist	0.39±0.05
55	Foliated granodiorite	amphibolite	0.69±0.06
56	Anorthosite	granulite	0.04±0.03
57	Anorthosite	granulite	0.06±0.03
58	Anorthosite	granulite	0.04±0.03

Tab. 4.5

Average heat production values of main lithologic units in the Kapuskasing crustal cross section subdivided by metamorphic grade (from Ashwal *et al.*, 1987)

Unit	Heat Production $\mu\text{W m}^{-3}$
Michipicoten greenstone belt	
Iron Formation (3%)	0.03
Conglomerate (12%)	1.37
Granitoid stocks (20%)	0.73
Volcanics (65%):	
IREE-enriched basalt (3%)	0.50
HREE-depleted dacite (1%)	0.77
Rhyolite MV 1 (3%)	2.01
LREE-depleted basalt (40%)	0.10
Rhyolite FV 1 (10%)	1.65
Dacite (3%)	1.08
Rhyolite FV 3 (5%)	1.67
Weighted mean	0.72
Amphibolite terrane	
Paragneiss (5%)	1.21
Mafic gneiss (8%)	0.12
Granites (24%)	2.85
Tonalites (63%)	0.84
Weighted mean	1.37
Granulite terrane	
Tonalite (40%)	0.54
Paragneiss and mafic gneiss (43%)	0.49
Anorthosite (17%)	0.05
Weighted mean	0.44

production of the domal gneiss terrane can be accounted for by the presence of a large proportion of late-stage Th-, U-, and K- rich granitoid intrusives. It has been suggested by Ashwal *et al.* (1987) that these may have been derived from the underlying Kapuskasing granulite terrane, leaving it slightly depleted in heat producing elements. Transport of Th, U, and K, therefore, could have taken place in silicate melts rather than in aqueous or carbonic metamorphic fluids. The pre-metamorphic radioactivity profile for the KSZ crustal section is uniformly low to depths of 25 km, with a mean heat production value less than $1 \mu\text{W m}^{-3}$. These results are completely different from the hypotheses by other authors, where the low values of heat production in some Archean terrains are explained as a result of the removal of upper crustal enriched zones by erosion. The results of Ashwal *et al.* (1987) are distinctly different from measured profiles in more silicic terrains, which show decreasing heat production with depth. This implies fundamental differences in crustal radioactivity distribution between granitic and more mafic terrains.

(ii) Spatial variations of heat production

Crustal heat production is characterized by lateral and vertical variations.

Three plausible models of radioactivity for the crust below the surface enriched layer are shown in Fig.4.4 (see review by Rao & Jessop, 1975). Model 1 is in accordance with the idea that high-grade metamorphic rocks become increasingly abundant with depth. Model 2 is the same as model 1 except that it has a transition layer 5 km thick. Model 3 takes into account the presence of the Conrad discontinuity and its probable implication as a heat generation boundary. The temperatures generated in model 3 are intermediate between those of models 1 and

Figure 4.4. Models of heat production distribution in the crust (from Rao & Jessop, 1975).

2. A single crustal conductivity value of $2.5 \text{ W m}^{-1} \text{ K}^{-1}$ or two constant values of $2.94 \text{ W m}^{-1} \text{ K}^{-1}$ for upper crust and $2.1 \text{ W m}^{-1} \text{ K}^{-1}$ for lower crust were adopted in the analysis of Rao & Jessop (1975). The assumption of a conductivity layering as opposed to a single uniform conductivity does not significantly alter the Moho temperature, but it does alter the shape of the temperature profile, depending on where the discontinuity is placed.

Detailed modelling of the distribution of the heat generation with depth in the Superior Province, in the region of Sachigo and Pikwitonei Subprovinces, has been done by Fountain *et al.* (1987). Various possibilities for the heat production distribution with depth are shown in Fig.4.5. Thermal parameters used in the models are shown in Tab.4.6. In general, most heat production models that take into account lateral variations in the $A(z)$ dependence show that the $q_s - A_s$ linear correlation may arise spuriously and therefore any interpretation of the correlation must be treated with caution (see next subsection).

(iii) The relationship between surface heat flow and surface heat production

One of the most important problems in the study of the thermal regime is the investigation of the surface heat flow-surface heat production relation (discussed in detail in Chapter III). The initial work of Birch *et al.* (1968) and Lachenbruch (1968), was restricted to plutonic rocks, but subsequently measurements were made in metamorphic terrains (Jessop & Lewis, 1978) and metavolcanics and metasediments of Precambrian greenstone belts (Drury, 1985). The results of these studies for the Superior Province are summarized below.

Figure 4.5. Theoretical models of the variation of heat production with depth in the Archean shield based on (a) exponential model; (b) model of Allis (1979), using empirical correlation between P-wave velocity and heat production (GS is greenstone terrain, GB is granitoid batholith terrain, and GT is gneiss terrain); (c) weighted average of heat production for every 2.5 km depth increment from the model presented by Fountain *et al.*, 1987) in the Pikwitonei and Sachigo Subprovinces. Horizontal bars indicate the standard deviation.

Tab. 4.6

Heat-production and thermal conductivity values used in heat-flow models* (from Fountain *et al.*, 1987)

Rock unit	Heat production	Conductivity
	($\mu\text{W m}^{-3}$)	($\text{W m}^{-1} \text{K}^{-1}$)
Background	0.02	3.20
Felsic granulites	0.40	2.77
Mafic granulite and amphibolite facies	0.15	2.11
Amphibolite-facies felsic gneisses	0.70	2.64
Greenschist-facies felsic gneisses	0.70	2.64
Felsic plutons	1.50	3.16
Metabasalts	0.15	2.50
Metasedimentary rocks	1.20	2.60

* The models are presented in Fig.4.6 (b).

On the basis of heat flow and heat production measurements made during the period 1968-70 in three boreholes in the Superior Province of the Canadian Shield, (borehole Hearst, Kapuskasing and Cochrane) the linear relationship was confirmed, yielding values of $b = 13.5 \text{ km}$ and $q_s = 27 \text{ mW m}^{-2}$ (Cermák & Jessop, 1971). Rao & Jessop (1975) in their discussion of general thermal characteristics of shields, also support the idea of a linear relationship between surface heat flow and surface heat production in the Superior Province. The results of the above relationship for the Superior Province as well as other tectonic provinces are presented in Fig.4.6.

By 1978 there were twenty one heat flow values (see Tab.4.7 after Jessop & Lewis, 1978) available for the Superior Province of the Canadian Shield. These heat flow values were all corrected for effects related to the Pleistocene glaciations and for topography. The average value is 40 mW m^{-2} with a standard deviation of 8 mW m^{-2} (in particular for the KSZ it is $33 \pm 6 \text{ mW m}^{-2}$).

The results of Jessop & Lewis (1978) confirmed that a linear relation between heat flow and surface heat generation exists in the Canadian Shield. A reduced heat flow of $20\text{-}21 \text{ mW m}^{-2}$ (uncorrected for glaciation) and a characteristic depth of 14 km (which lies within the range $5\text{-}14 \text{ km}$, proposed by different authors, see Tab.3.2, Chapter 3) have been proposed to satisfy this relationship. Both q_s and b are different from other heat flow provinces. Since it is inferred that a large thickness of the crust was eroded away and that the original heat production in Archean times was much larger than the present value, a linear relation equivalent to those found in younger heat flow provinces is not expected (Jessop & Lewis, 1978). The characteristic depth of 14 km is greater than values inferred for most other areas.

Figure 4.6. Heat flow - heat generation relation.
 (a) in different tectonic provinces. In the Canadian Shield line I corresponds to the gneissic terrains (data point 4 corresponds to the measurements in Adirondack, North America); data points 5 in line II corresponds to measurements in a granitic pluton and data point 6 to the anorthosite plutons (from Rao & Jessop, 1975).

(b) as calculated from one and two-dimensional heat flow models in the Pikwitonei-Sachigo Subprovince: top - 1-D model, middle - 2-D model with $q_r = 20 \text{ mW m}^{-2}$, bottom - 2-D model with $q_r = 10 \text{ mW m}^{-2}$ (from Fountain *et al.*, 1987).

Tab. 4.7

Heat flow data for Canadian Precambrian Shield (from Jessop and Lewis, 1978 and Drury, 1985)

Name	Depth	Temp.	k	heat flow		Stand
	Interval	grad.	average	(mW m ⁻²)		devia-
	(m)	(mK m ⁻¹)	(W m ⁻¹ K ⁻¹)	measu-	corre-	tion
				red	cted	
Jackfish	150- 610	14.2	2.91	35	41	0.3
Otoskwin River	200- 606	9.1	2.69	20	25	0.6
Minchin Lake	150- 605	14.4	3.00	35	42	0.3
English River	150- 612	13.5	3.02	34	40	0.2
Sudbury Basin						
Moose Lake-a	200-1840	14.5	3.14	41	45	0.2
Onaping	50- 990	16.9	3.13	45	51	0.2
Moose Lake-b	150-1680	16.2	2.85	42	46	0.4
average value				43	47	
Merrill Island	100- 742	9.1	3.40	23	30	0.4
Red Lake	80- 311	11.9	4.09	37	44	0.6
Manitouwadge						
269	80- 884	11.8	3.33	30	35	0.5
S-158	300-1279	13.1	3.04	34	38	0.3
273	150- 869	11.7	3.28	30	36	0.4
274	170- 892	11.8	3.27	32	38	0.3
309	200- 714	11.6	3.46	33	39	0.9
306	100- 629	11.9	3.15	27	33	0.6
average value				31	37	
Launay	150- 366	10.7	3.66	31	40	0.5
Kirkland Lake				42	(43)	
Malartic				29	(35)	
Larder Lake				37	(44)	
Timmins				31	(36)	
Elliot Lake						
(7 holes)				50	(60)	
Cochrane				39	43	
Kapuskasing				27	33	
Hearst				46	52	
Winnipeg				29	38	
Nielsen Island				25	26	
Experimental Lakes						

(11 sites)					49	
Lake Dufault						
(70 holes)				36	42	
Rumpel Lake	550-1450	4.82	6.51	31	39	0.3
Echo Lake	150- 275	13.0	2.73	35	39	0.3
	325- 465	12.4	2.89	33	37	0.3
Snow Lake	210- 353	11.1	3.73	31	44	5
	120- 185	17.0	2.95	48	52	0.8
Thompson	300- 866	17.5	3.38	57	59	0.2
Lynn Lake	200- 589	7.51	2.53	18	21	0.5
	200- 494	7.06	3.10	22	25	0.4
Fox Lake	150- 383	11.9	3.53	41	44	0.5
	150- 378	12.5	2.80	34	36	3

Figures in brackets represent approximate corrections for Pleistocene climatic disturbance.

It confirms the observations of several authors that stable areas have low surface heat flow.

Fountain *et al.* (1987), on the basis of the models presented for the distribution of heat production with depth (see Fig.4.5) and measurements of both surface heat production and thermal conductivity (see Tab.4.6), constructed models for the relationships between surface heat flow and surface heat generation (see Fig.4.6 (b)). Both 1-D and 2-D models were used. In the 2-D model it was noted that the value of b increased in response to increases in the value of q_s , a phenomenon not observed in the case of the 1-D model. Furlong & Chapman (1987) interpreted results presented by Fountain *et al.* (1987), and concluded that the observed linear relationship $q_s - A_s$ is a consequence of horizontal crustal heterogeneity. They included 2-D and 3-D effects in their heat transfer models in an attempt to estimate middle and lower crustal temperature and heat flux and to explain the observed heat flow-heat production relationship. Their results can be summarized as follows: (1) in a compositionally heterogeneous crust temperatures may vary by $> 100^\circ\text{C}$ at mid-crustal levels over horizontal distances of 50 - 75 km, even in areas where mantle heat flux is constant; (2) even if no systematic decrease of heat production with depth exists, the relationship between $q_s - A_s$ is approximately linear; (3) reduced heat flow (q_r) is the average heat flow at depth b and typically is significantly less than the surface heat flow (q_s); (4) the characteristic depth increases with increasing q_s ; (5) estimates of both q_r and b are dependent on the location of sampling sites. In addition, their results show that many regions of high reduced heat flux need not be regions of high mantle heat flux and may be simply regions with a relatively

radiogenic middle crust.

These results indicate that the apparent linear heat flow-heat production relationship does not necessarily relate to the distribution of heat production with depth. Crustal thickness may be a more important parameter controlling crustal heat production than the slope of the linear relationship. Heat flow at the Moho in the KSZ is estimated to be about 20 mW m^{-2} taking into account the measured value of surface heat flow (33 mW m^{-2}), heat production ($0.44 \mu\text{W m}^{-3}$) and thickness of the crust (30 km).

An important factor in the above consideration is the uncertainty in measured heat flux. Drury (1987) estimated an uncertainty of at least 10% in a heat flow measurement and of 20-25% in heat production. The concept of heat flow province, e.g. a region in which the surface heat flow and surface heat production are linearly related because of some special distribution in the crust of heat-producing elements, and in which reduced heat flow is constant, is of doubtful significance (Drury, 1987b).

Taking the above considerations into account a thermal model for a cross-section through the KSZ was constructed. It will be discussed in detail in the next chapter.

5. TEMPERATURE CALCULATIONS

5.1. Procedure

In order to determine the present temperature distribution (assumed steady-state) in the KSZ, six vertical profiles were chosen (see Table 5.1 and Fig.5.1; they are discussed in detail in section 5.2). Two methods are used to calculate temperatures. The first method is a 1-D finite difference technique, which requires as input a heat production-depth distribution, a surface temperature, and an assumed basal heat flow. Nodal spacing is 2 km in all cases. A Pascal computer program was used to carry out the finite difference calculations. The required form of the input data for this program is illustrated in Appendix 5.1. The program generates a graphical output showing temperature as a function of depth. The program, called "Geotherm" was written by Reeves (1988) in Turbo Pascal (Version 4.0) for the IBM-PC. It uses a direct matrix solution technique employing Thomas' algorithm (the method of storing and manipulating tridiagonal matrices so that only the non-zero terms are saved and operated upon) for the 1-D solution of the steady-state heat conduction equation. Since in the present case that material is non-homogenous, internal conditions of continuity were assumed in all calculations. The original program was modified by prescribing conditions of continuity of temperature and heat flux at the boundaries between materials with differing thermal properties. Thermal conductivity is considered temperature-independent.

In addition to the finite difference method, an analytical solution was also used. This method assumes a steady-state heat conduction in a medium consisting

of homogenous, isotropic layers each with a characteristic heat generation constant A_i and thermal conductivity k_i . The calculated thickness for each unit in the profiles (Tab.5.1) was used for layer thickness. The analytical solution for $T(z)$ was obtained using a dBase III-plus computer program developed by the author (see Appendix 5.2). The solution for each layer can be written as follows:

$$T_i(z) = T_{i-1} + (q_{i-1}/k_i)z - (A_i/2k_i) z^2 \quad (5.1)$$

where T_{i-1} and q_{i-1} are the temperature and heat flow at the top of the model respectively; the vertical axis z is positive downwards and with origin at the top of each layer. If a layer has a thickness Δz_i ,

$$T_i = T_{i-1} + (q_{i-1}/k_i)\Delta z_i - A_i \cdot \Delta z_i^2 / 2k_i \quad (5.2)$$

$$q_i = q_{i-1} - A_i \Delta z_i \quad (5.3)$$

describe the temperature and heat flow respectively at the top of underlying layer, equal to the values solved for the bottom of the previous layer. An example of the results obtained using this program is shown in Appendix 5.3. Tab.5.2 shows the Moho temperatures calculated for each profile using this method. When the analytical and finite difference solutions are compared, the temperature difference are seen not to exceed 50 K at the base of the model.

The implicit assumption in any 1-D temperature-depth solution is that heat

Tab. 5.1
Thickness of various units at selected profiles

Profile Number	Total crustal thickness (km)	Crustal units (from surface to Moho)	thickness of unit (km)
1	36	1	6
		3	11
		2	19
2	38	3	13
		2	25
3	41	2	20
		2	21
4	39	2	7
		3	14
		2	18
5	37	3	20
		2	17
6	36	1	3
		3	16
		2	17

unit 1 - Michipicoten greenstone belt and Abitibi belt (metavolcanics)
unit 2 - Kapuskasing granulites
unit 3 - Wawa domal gneiss

Tab. 5.2

Moho temperature (T_M) and Moho heat flow (q_M) as functions of thermal conductivity (k), average heat production of each unit (A_{avg}) and surface heat flow (q_s) (from analytical solution).

profile	unit	k ($W\ m^{-1}\ K^{-1}$)	A_{avg} ($\mu W\ m^{-3}$)	q_s ($mW\ m^{-2}$)	q_M ($mW\ m^{-2}$)	T_M (K)
1	1	2.60	0.72	48	21.51	768
	3	2.44	1.37			
	2	2.44	0.44			
2	3	2.44	1.37	49	21.09	784
	2	2.44	0.44			
3	2	2.44	0.44	38	20.18	787
4	1	2.60	0.72	50	19.12	801
	3	2.44	1.37			
	2	2.44	0.44			
5	3	2.44	1.37	55	21.02	809
	2	2.44	0.44			
6	1	2.60	0.72	51	20.70	777
	3	2.44	1.37			
	2	2.44	0.44			

flows only vertically. This generalization yields accurate results in regions where isotherms are horizontal and also gives a good approximation in areas where isotherms have only gentle slopes. A fully 2-D solution would be required for areas where there are significant lateral variations in heat flow. Therefore by calculating initially the 1-D solution for each of the selected profiles in KSZ and comparing the results we can roughly determine the geometry of the isotherms. If their slopes are not large, a 2-D solution would introduce only minor modifications (see also Fountain *et al.*, 1987 and Baumann, 1988).

5.2. Thermal parameters for the KSZ

A structural cross-section of the KSZ is shown in Fig.5.1 (Cook, 1985). The location of the cross-section is shown in Fig.4.3. From west to east the cross-section traverses the Michipicoten Belt (MB), Wawa Belt (WA), Kapuskasing Structural Zone (KSZ), and the Abitibi Belt (AB).¹

The four crustal geological units are shown on the cross-section. They are the Michipicoten and Abitibi Belts (both consisting of metasediments), the Kapuskasing granulites, and Wawa domal gneiss terrane. Mean thermal parameters have been estimated; thermal conductivity k , mean value of heat production A , and density ρ . The specific heat has been taken as $c = 0.8 \text{ kJ kg}^{-1} \text{ K}^{-1}$ (typical value for most common minerals), and surface temperature as $T_s = 300 \text{ K}$ (see Tab.5.3). Radioactive heat generation appears to be a most important parameter in crustal

¹A revised cross-section of the KSZ based on newly acquired Lithoprobe reflection data has been published by Percival *et al.* (1989). It does not differ greatly from the one used here.

Figure 5.1. Cross-section of the KSZ

Tab. 5.3

Mean model parameters for KSZ units

unit	thermal conductivity	mean heat production	density	specific heat
	k (W m ⁻¹ K ⁻¹)	A_{avg} (μW m ⁻³)	ρ (Mg m ⁻³)	c (kJ kg ⁻¹ K ⁻¹)
1	2.60	0.72	2.90	0.8
2	2.44	0.44	2.82	0.8
3	2.44	1.37	2.70	0.8

unit 1 - Michipicoten greenstone belt and Abitibi belt (metavolcanics)

unit 2 - Kapuskasing granulites

unit 3 - Wawa domal gneiss

temperature calculations. In estimating the mean heat production (A), for each geological unit, the mean values of heat production as measured by Ashwal *et al.* (1987) were used. Various models of heat production vs depth were used in the temperature calculations. These models will be discussed and assessed later in this chapter.

The thermal conductivity (k) was estimated in a number of ways. Direct measurements on comparable rocks cited in the literature (e.g. Clark, 1966; Chapman, 1986; Fountain *et al.*, 1987), and discussed in Chapter III and IV were used where possible. Additionally, thermal conductivities measured in the Pikwitonei-Sachigo region (Superior Province), where the rock types are similar to those of the KSZ, were also taken into account. Laboratory data for thermal conductivity (average $3.0 \text{ W m}^{-1} \text{ K}^{-1}$ for the upper crust and $2.6 \text{ W m}^{-1} \text{ K}^{-1}$ for the lower crust), corrected for the pressure, give the value $2.37 \text{ W m}^{-1} \text{ K}^{-1}$ (decreasing with depth) and $2.64 \text{ W m}^{-1} \text{ K}^{-1}$ (increasing with depth) for the upper and lower crust respectively (according to the relation 3.10, also Chapman, 1986; see Chapter III).

On the basis of these considerations the chosen average values for the thermal conductivity in this study are $2.60 \text{ W m}^{-1} \text{ K}^{-1}$ for the metasediments (Michipicoten greenstone belt and Abitibi belt, unit 1), $2.44 \text{ W m}^{-1} \text{ K}^{-1}$ for the Kapuskasing granulites (unit 2), and $2.44 \text{ W m}^{-1} \text{ K}^{-1}$ in the geological unit 3 (Wawa domal gneiss). Conductivity measurements are usually accurate to $\pm 5\%$. However, a $\pm 20\%$ variation in the magnitude of k has only a small effect on the calculated temperature-depth profile (Chapman, 1986) and often a constant value of k throughout the crust is assumed (see for example Jessop & Lewis, 1978, Čermák,

1982).

In estimating the specific heat c ($\text{J kg}^{-1} \text{K}^{-1}$) and volume expansion coefficient α (K^{-1}) the values selected lay within the range quoted for comparable rocks in the literature (for example, Clark, 1966; Bott 1982, Drury, 1987a; see Chapter III).

Six vertical temperature profiles were selected in the study area from the west to east (denoted by numbers 1 - 6 respectively, see Fig.5.1). The thickness of the geological units beneath each profile was estimated from the cross-section of Cook (1985) shown in Fig.5.1 (see Tab.5.1). For each profile the temperature distribution with depth was calculated using both a finite difference technique and an analytical method, as detailed in Section 5.1.

The selected thermal parameters presented in Tab.5.3 are the basis of the geothermal model constructed for the KSZ. Surface heat flow, where available, forms an additional piece of input. The observed surface heat flow q_s was resolved into two components; one which is generated by heat sources above the Moho (the crustal contribution) and another related to heat flowing upwards from below the Moho (and which forms a lower boundary condition in the numerical solution). Three measurements of surface heat flow from the Superior Province in the KSZ are available. They are: 51, 33 and 43 mW m^{-2} (corrected for glaciation; their location is shown in Fig.4.3). The three corresponding surface heat production values are 1.8, 0.5 and 1.3 $\mu\text{W m}^{-3}$ respectively. Only one of these falls within the KSZ proper (33 mW m^{-2} , corresponding approximately with "virtual" surface heat flow, profile 3). "Virtual" surface heat flow at each profile was obtained by calculating the contribution of crustal heat production to the surface heat flow and assuming an

approximately constant value for the Moho heat flow. At profile 3, when q_b has been measured, "virtual" heat flow is approximately equal to the measured heat flow.

The magnitude of the glaciation correction is subject to debate (see Chapter III). Čermák & Jessop (1978) proposed a correction of + 4 K to the temperature for the glacial effect. The same value is used by Ashwal *et al.* (1987). However Nielsen & Beck (1989) suggest that a higher correction factor (+ 8 K) is necessary to compensate for the temperature rise at the end of the Wisconsin glaciation period. This higher correction factor gives higher surface heat flow values. For example in the case of a measured value of 43 mW m⁻² (Cochrane borehole) the value for higher glacial correction becomes 50 mW m⁻². Similarly these results suggest that the quoted value of 33 mW m⁻² for the KSZ is probably an underestimation. If a measured surface heat flow of 33 mW m⁻² is assumed to be representative of the whole cross-section within the study area, calculations yield negative estimates for the Moho heat flow in some cases. Therefore, the thermal parameters (primarily the surface heat flow value) must be reassessed. Estimated value for the surface heat flow used in profile 3 is 38 mW m⁻². The heat flow values estimated for other profiles are shown in the Tab.5.4. These revised values imply that the larger glacial correction, as suggested by Nielsen & Beck, is probably more realistic.

An important factor determining the thermal regime in the region is the value of the basal heat flow (the value of the heat flow at the Moho). In the KSZ, this parameter may be assumed to be constant, as the KSZ is well within the Superior Province. In Precambrian shields the Moho heat flow is usually estimated to be 20

Tab. 5.4

Surface heat flow (q_s) at the location of the six profiles estimated on the basis of (i) an assumed constant basal heat flow (q_m) of 20 mW m^{-2} ; (ii) a variable crustal heat production (q_{cr}) as computed from average heat productions in the various units.

No. of profile	q_{cr} (mW m^{-2})	q_s (mW m^{-2})
1	28.0	48
2	29.0	49
3	18.0	38
4	30.0	50
5	34.0	55
6	32.0	51

$\pm 4 \text{ mW m}^{-2}$ (Čermák & Jessop, 1971; Ashwal *et al.*, 1987). As discussed in Chapter III, another important factor which affects the thermal regime of an area is the model of the heat production distribution with depth together with the value of the depth parameter b (if applicable) and the $q_v - A_v$ relationship in the terrain under investigation. The most recent studies of the $q_v - A_v$ relationship (Drury, 1989) do not confirm the hypothesis that an exponential distribution of the heat production with depth is the only model capable of yielding the observed linear relationship; this relationship may be achieved using a variety of heat production-depth distributions. These will be considered in the next section.

5.3. Selection of the optimum heat production-depth distribution model

The selection of mean thermal parameters for each unit is not sufficient to characterise the heat production in the KSZ. Variations of heat production with depth other than those related to petrology must also be taken into account.

In order to check whether heat production varies systematically with depth within a given unit, the actual sample locations (as measured by Ashwal *et al.*, 1987) have been projected from a geological cross-section to a pseudo depth-profile, by restoring the exposed layers to their original horizontal position. The actual heat production values used were taken from Ashwal *et al.* (1987). The projection of these measurements as pseudo-depth profiles was carried out for profiles 1, 2, and 3 (see Fig 5.2a to 5.2.c). These figures illustrate the large scatter of heat production with depth. For example values range from 0.5 to 3.5 $\mu\text{W m}^{-3}$ in the Wawa domal gneiss terrane. This terrane is characterized by a higher mean heat production value of 1.37

Figure 5.2(a). Projection of the heat production measurements to depth carried out for profile 1

Figure 5.2(b). Projection of the heat production measurements to depth carried out for profile 2

Figure 5.2(c). Projection of the heat production measurements to depth carried out for profile 3

$\mu\text{W m}^{-3}$ (Ashwal *et al.*, 1987) than the other units. However, despite the large scatter, a general decrease in heat production with depth is observed on all three profiles.

As part of this study a wide variety of heat production-depth models were used. Clearly, the same model must yield "acceptable" results (i.e. the resulting temperature at the Moho should be in the range 600-800 K; this range is typical for Precambrian shields - see for example Morgan & Sass, 1984) for all the profiles. The crustal contribution to the surface heat flow should give reasonable values with respect to the measured value of surface heat flow q_s , which is estimated to be in the range 38 - 55 mW m^{-2} for the selected six profiles in the study area if the higher (and probably more realistic) glacial correction is required.

The highest measured values of heat production do not always occur at the surface on the restored profiles. This is a consequence of the structure of the KSZ, where units are overthrust one above the other. Consequently, it is impossible to find a single model expressing the variation of heat production with depth $A(z)$ which yields reasonable results for all profiles in the study area. It is therefore necessary to consider different $A(z)$ models for each unit. The measured value of the mean heat production within each geological unit is used as a primary constraint in choosing an appropriate $A(z)$ model. The observed general decrease in heat production with depth in each unit (Fig.5.2) is modelled by a (linearly or exponentially) decreasing function of depth. The actual function chosen is that which yields a mean heat production value within each unit equivalent to the measured value. (The mean value of heat production within Kapuskasing granulites, Wawa domal gneiss, Abitibi and Michipicotan Belts units is $0.44 \mu\text{Wm}^{-3}$, $1.37 \mu\text{Wm}^{-3}$ and

0.72 μWm^{-3} respectively, see Tab.5.3). Another constraint is that the estimated Moho temperature must be in the range 700 ± 100 K.

A number of different models describing the heat production-depth distribution in the study area are used in the calculations: (1) A model with constant mean heat production for each unit gives unrealistic (negative) values for the Moho heat flow at some of the profiles, (i.e. it exceeds crustal contribution to the surface heat flow); moreover, the Moho temperature calculated from this model is higher than values obtained through the application of other models (i.e. it exceeds 800 K). (2) A model, where $A(z)$ was assumed to vary according to a single decreasing linear function throughout the whole crust, generated equally unacceptable results (i.e. it underestimated the crustal contribution to the surface heat flow requiring unacceptably high Moho heat flow values). (3) A model in which $A(z)$ varies linearly with depth separately in different units yields Moho temperatures much lower than in other cases. (4) Similarly, a combination of constant heat production in the top part of the profiles and linear decrease with depth in the lower part of the profiles results in Moho temperatures which are too low (~ 500 K). (5) A model with an exponentially decreasing heat production-depth function (common to the whole crust) yields estimates of the crustal heat production contribution to the surface heat flow which are very low and consequently unacceptably high estimates of Moho heat flow values along some of the profiles. (6) Acceptable results are obtained using a model with a combination of a constant heat production value for the upper units and an exponential decreasing function calculated for the lower units. (7) A model with exponentially decreasing heat production function separate for each unit, when

applied to all the selected profiles in the region, yields the most geophysically reasonable results in terms of Moho temperature and heat flow.

Although some models may fit some profiles individually, the purpose of the work was to find a model of the heat production distribution with depth that would fit all selected profiles. Results from some models are presented in Tab.5.5. The Moho temperature for the preferred model (exponential distribution of heat producing elements in each unit) is in the range 750-808 K, which fits well estimates in other Precambrian regions (see Chapter III).

Models of the heat production distribution with depth used for the KSZ structure can be compared with other models presented in the literature (see Chapters III and IV). A geothermally most similar area is the Sachigo-Pikwitonei Subprovince (Superior Province). The models of Fountain *et al.* (1987) consist of a two-layered crust (see Fig.4.5). For example, in variant Fig.4.5 (b), the upper crust has been characterized by three different terrains: greenstone, gneiss and granitoid were assumed to have three distinct $A(z)$ distributions (for greenstone increasing heat production with depth and for gneiss and granitoid decreasing heat production with depth), while in the lower crust an exponential decrease of heat production with depth was assumed.

Generally, the KSZ also represents a type of two-layer crust, in the sense that the top part is relatively more heterogenous and the lower part is more homogenous with respect to heat producing elements. The concept of two-layer crust is discussed in detail by Drury (1989).

As discussed in section 3.2, seismic refraction data can be used as a constraint

Tab. 5.5

Heat production-depth distributions and their effect on Moho temperature (T_m), surface heat flow (q_s), reduced heat flow ($q_r = 0.6 q_s$) and basal heat flow (q_m) (from finite difference algorithm).

profile	q_s (mW m ⁻²)	q_r (mW m ⁻²)	q_m (mW m ⁻²)	T_m (K)
p1*	56.85	31.36	20.56	750
p1s	52.39	30.90	20.19	721
p2*	53.40	35.80	20.52	808
p2cl	40.51	24.07	20.08	619
p2ce	53.30	36.87	20.52	817
p2s	46.00	33.60	20.01	793
p3*	38.72	31.63	20.46	771
p31	37.12	26.40	20.14	713
p3s		37.71	21.20	827
p3se		38.84	20.14	692
p4*	42.35	33.39	20.45	769
p4c	50.59	39.58	20.87	830
p5*	55.35	33.29	20.56	773
p5c	52.46	36.80	20.89	793
p5cl	48.70	31.80	20.14	736
p6*	49.80	33.35	20.56	750
p6c	34.47	23.81	20.91	802

p1, p2, p3, p4, p5, p6 - exponential decrease of heat production elements with depth, separately in each unit.

p31 - single exponentially decreasing heat production with depth ($b = 14$ km).

p1s, p2s, p3s - heat production-depth distribution as determined by measured surface heat generation, projected on pseudo-depth sections.

p3c, p4c, p5c - constant heat generation in each layer, equal to the mean of the measured heat production values for that layers.

p2cl, p5cl - combination of constant heat production distribution in the upper layer and linear heat production distribution in the lower layer.

p3se - seismological model of the heat production distribution with depth

p2ce - combination of constant heat production in the upper layer and exponential heat production in the lower layer

*** - the best model**

in constructing heat production-depth models, by using the conversion of seismic velocities into heat production values proposed empirically by Rybach & Buntebarth (1982). These formulae allow heat production to be evaluated without any a priori model describing the depth distribution of radiogenic elements in the continental crust. The validity of the method has been criticised by some authors (see discussion in Chapter III). An example of the method for the Kapuskasing gneisses (profile 3, 3se) has been presented in Fig.5.3 and results included in Tab.5.5. In this case the results obtained were reasonable, although they do not prove the correctness of the method.

The present work suggests that the most reasonable model for heat generation with depth in Precambrian crust (at least in the KSZ and surrounding regions in the Superior Province), is an approximately exponential decrease of heat production with depth, separate for each geological unit. Such a model takes into account the complicated nature of the KSZ, for example, the fact that layers are overthrust and so higher heat production values may be found in lower crustal units. Diagrams which illustrate these various models of heat production distributions with depth for the KSZ are enclosed in Appendix 5.1.

5.4. Effects of variations in thermal parameters on estimated geotherms

In each of the above models the value of the depth parameter was set equal to 14 km for each unit. Obviously, no physical meaning can be assigned to this value, which is simply the result of fitting an exponential distribution to available data. A value of 14 km, however is widely quoted in the literature (see for example Jessop

Figure 5.3 An example of the heat production distribution model calculated from the seismic refraction data (for profile 3).

& Lewis, 1978), where the thermal regimes of the Canadian Shield in general, or Superior Province in particular, are discussed.

The b -value for Precambrian shields (see discussion in Section 4.2) is usually between 7 ± 1 and 14 ± 1 km. In order to investigate the effect of a varying b on the calculated geotherms, additional calculations have been performed. In the first case (model A) the depth parameter was taken to be 7 km, with an exponential heat production-depth distribution model separate for each geological unit, which gave an average heat production value in the unit equal to that measured by Ashwal *et al.* (1987). In model B the heat production-depth distribution was assumed to be the same as in the model A, but with the value of the depth parameter 14 km (see previous section). In model C the heat production-depth distribution model used was the same as described above for model B with the following modification. Where the evidence indicates that only the deeper parts of some units are present along one of the selected profiles, then the average heat production value of that unit is chosen to be somewhat less than that measured when the entire unit (including the upper radiogenically enriched part) is present. The geotherms obtained from these three different models have been presented in the Appendix 5.1(a) (model A), Appendix 5.1(b) (model B) and Appendix 5.1(c) (model C).

5.5 Discussion

Fig.5.4 shows variations in q_r and q_m along the six profiles calculated for $b = 14$ km (model C). Usually, reduced heat flow is taken as $q_r = 0.6 q_m$. However, the validity of this empirical relation has been questioned by some workers: a

Figure 5.4. Variations in q_s and q_m along the six profiles (positions marked by squares along the cross-section).

proportionality coefficient of 0.7 or 0.8 may be just as appropriate (Drury, 1989). Also the value for the depth parameter b may vary. Fountain *et al.* (1987), for example, using both a 1-D model and a 2-D model to solve the heat conduction equation in the Pikwitonei-Sachigo region (Superior Province) obtained $b = 7.8$ km (2-D model) and $b = 8.3$ km (1-D model). Rao & Jessop (1975) present two curves for the Superior Province, one with $b = 12.4$ km and another one with $b = 7.1$ km. Both were determined using the relationship $q_s = q_r + bA_s$. A regression analysis of q_s and A_s values for the study region yields $b = 6.7 \pm 2.3$ km, and a ratio $q_r/q_s = 0.85$ (see Fig.5.5). However, the fit is very poor and the validity of the relation is in doubt. Usually the difference in the temperature at the Moho using $b = 7$ km and 14 km is not large (≤ 60 K). Tab.5.6 summarizes the effects of variations in b . Obviously, other things being equal, the lower the value of b , the lower the Moho temperature.

The calculated temperature distribution in the study area for $b = 7$ km and $b = 14$ km are shown in Fig.5.6 (model A), 5.7 (model B) and 5.8 (model C), respectively. The distribution is only pseudo-two-dimensional, as it was obtained by contouring 1-D solutions by the program Lotus Freelanes. It can be seen that the effect of the parameter b is more significant (see the differences in models A and B) than that of using two different variants of the exponential heat production distribution (models B and C). Model A is characterized by very gently sloping isotherms, while in models B and C reasonably flat-lying lower crust isotherms develop a more pronounced relief beneath the KSZ. Generally, the isotherms across the KSZ are approximately flat-lying, however, there is a shallowing of isotherms (in

Regression Output:

Constant	40.17017
Std Err of Y Est	4.503241
R Squared	0.684904
No. of Observations	6
Degrees of Freedom	4
X Coefficient(s)	6.720136
Std Err of Coef.	2.279053

Figure 5.5. Regression analysis of q_s on A_s . q_s - estimated surface heat flow values for the six profiles of the cross-section; A_s - measured values of heat production (from Ashwal *et al.*, 1987)

Figure 5.6. Temperature distribution for the model A

Figure 5.7. Temperature distribution for the model B

Figure 5.8. Temperature distribution for the model C

Tab. 5.6

Variations in calculated Moho temperature with changes in depth parameter b (see text for description of model A, B and C).

Profile	Model	b (km)	T _m (K)
1	A	7	760
	B	14	749
	C	14	758
2	A	7	741
	B	14	763
	C	14	808
3	A	7	736
	B	14	725
	C	14	771
4	A	7	787
	B	14	805
	C	14	769
5	A	7	730
	B	14	758
	C	14	773
6	A	7	754
	B	14	778
	C	14	758

the lower crust) approximately beneath the ILCZ. Because of the low slope of the isotherms, a 1-D solution to the heat conduction equation is acceptable as a first approximation. However, with an increase in our knowledge of the thermal properties of the crust and with a more detailed picture of surface heat flow, a fully 2-D solution (which would yield a more precise determination of the temperature in the Earth's crust) may become justified.

It is important to evaluate the uncertainties in the estimated values of the thermal parameters. Density (ρ) and thermal conductivity (k) are measured with a 5% accuracy (Drury, 1985; Chapman, 1986). The great variability of heat flow values (sometimes exceeded 50%, see for example review by Lewis, 1975), may be attributed to climatic, topographic, groundwater circulation, and erosion effects. Taking the above factors very carefully into considerations, an accuracy of 5-10% in a heat flow measurements can be assumed. The accuracy of heat production measurements (A) should be of 10%, but for average heat production the reasonable uncertainty is taken to be in the range 20-25%. The variability of heat production values can be very high (the ratio of standard deviation to mean can be in range 9-74%; Drury, 1987b). Exact models of the heat production - depth distribution do not exist and the uncertainty of the estimated heat - production values increases at greater depth. Tab.5.7 shows the variations in Moho temperature when both the above parameters are altered within these accuracy limits. Clearly, the calculated Moho temperature is more sensitive to inaccuracies in surface heat flow than to inaccuracies in heat production. These tests were carried out assuming constant heat production in each layer and using the analytical solution provided by Chapman

Tab.5.7

Effect of a $\pm 10\%$ and $\pm 20\%$ change in the estimated values of average heat production (A) and surface heat flow (q_s) on the calculated Moho temperature (T_M) and Moho heat flow (q_M) for profile 3

	A $\mu\text{W m}^{-3}$	q_s mW m^{-2}	q_M mW m^{-2}	T_M K
$q_s = 38 \text{ mW m}^{-2}$, A = $\pm 10\%$				
	0.44	38	20.4	789
	0.40	38	22.0	802
	0.48	38	18.0	775
$q_s = -10\%$, A = $0.44 \mu\text{W m}^{-3}$ and A = $\pm 10\%$				
	0.44	34	16.4	721
	0.40	34	18.0	735
	0.48	34	14.8	708
$q_s = +10\%$, A = $0.44 \mu\text{W m}^{-3}$ and A = $\pm 10\%$				
	0.44	42	24.4	856
	0.40	42	26.0	869
	0.48	42	22.8	842
$q_s = 38 \text{ mW m}^{-2}$, A = $\pm 20\%$				
	0.36	38	30.1	816
	0.52	38	17.2	761
$q_s = -10\%$, A = $\pm 20\%$				
	0.36	34	19.6	748
	0.52	34	13.2	694
$q_s = +10\%$, A = $\pm 20\%$				
	0.36	42	24.0	987
	0.52	42	20.9	827
$q_s = +20\%$, A = estimated value and A = $\pm 10\%$				
	0.44	46	28.4	923
	0.40	46	30.0	936
	0.48	46	26.8	909

$q_s = -20\%$, $A = \text{estimated value and } A = \pm 10\%$

0.44	30	12.4	654
0.40	30	14.0	668
0.48	30	10.6	639

$q_s = +20\%$, $A = \pm 20\%$

0.36	46	31.6	950
0.52	46	25.2	896

$q_s = -20\%$, $A = \pm 20\%$

0.36	30	15.8	681
0.52	30	19.2	627

$q_s = \pm 5\%$, $A = \text{estimated value}$

0.44	36	18.4	755
0.44	40	22.4	822

Tab. 5.8

The influence of variations in basal heat flow (q_m) on Moho temperature (T_m), surface heat flow (q_s) and reduced heat flow (q_r) (from finite difference algorithm).

profile	q_m (mW m ⁻²)	q_s (mW m ⁻²)	q_r (mW m ⁻²)	T_m (K)
p1	20	56.85	31.36	750
p1q	18	54.75	29.40	721
p1q1	15	51.66	28.90	677
p1q2	10	43.63	21.56	603
p2	20	53.40	35.80	808
p2s	20	45.99	33.59	793
p2sq	18	44.08	31.68	762
p2sq1	15	41.21	28.80	716
p2q2	10	36.42	24.02	540
p3	20	38.72	31.63	771
p3q	18	36.72	29.64	738
p3q1	15	33.72	26.63	689
p3q2	10	28.74	21.60	607
p4	20	42.35	33.39	769
p4q	18	40.34	31.47	738
p4q1	15	37.40	28.61	692
p4q2	10	32.35	23.81	616
p5	20	55.35	33.29	773
p5q	18	53.46	31.38	744
p5q1	15	50.57	28.51	701
p5q2	10	45.77	23.71	629
p6	20	49.80	33.35	758
p6q	18	47.75	31.42	729
p6q1	15	44.65	28.56	687
p6q2	10	39.52	23.76	614

p1 profile 1 with heat flow with basal heat flow equal 20 mW m⁻²
q basal heat flow equal 18 mW m⁻²
q1 basal heat flow equal 15 mW m⁻²

q2 basal heat flow equal 10 mW m^{-2}
ps profile with surface heat production projected on pseudo-depth section

(1986) (eq.5.1). A $\pm 10\%$ change in surface heat flow results in a temperature difference at the Moho of ± 130 K. In the case of a $\pm 5\%$ change in surface heat flow the Moho temperature varies by about ± 80 K. A change of $\pm 10\%$ in heat production results in a variation of the Moho temperature of ± 30 K.

A change of $\pm 20\%$ in thermal conductivity (k) hardly affects the geotherm (Chapman, 1986). Variations in thermal conductivity are small in the KSZ and any temperature dependence is practically negligible.

Another factor which may affect temperature distribution in the crust is groundwater flow. However, its importance cannot be estimated unless the hydrogeological situation is at least roughly known. A purely conductive heat transfer solution may not be sufficient in areas where large - scale groundwater circulation is important.

Surface heat flow, reduced heat flow, and Moho temperature calculated as a function of the Moho heat flow are shown in Tab.5.8. Three values 10, 15, and 20 mW m^{-2} for the Moho heat flow were used. A variation in the Moho heat flow of 5 mW m^{-2} (from 20 to 15 mW m^{-2}) results in a variation of the Moho temperature of 80 K.

In conclusion, for a relatively undisturbed area such as the KSZ (where, however, surface heat flow is not known in detail), the uncertainty in estimated geotherms can be qualitatively assessed to be in the 10 - 20% range.

CONCLUSION

Heat flow studies of continental areas show that as a rule the abundance of heat-producing elements decreases with increasing depth. Fountain & Salisbury (1981) suggested that measured values of heat production in rocks from the overthrust block in the KSZ should decrease toward the suture. This has been confirmed by this study, by projecting measured heat production values to depth along a line of section constructed perpendicular to the ILCZ (see section 5.3 and Fig. 5.2).

Fountain & Salisbury (1981) also point out that the most pervasive layering in the crust is one of metamorphic grade, and not necessarily composition. Lithologic variations in the crust are controlled by the superposition of increasing grades of metamorphism with depth on changes in rock composition, which can be both vertical and lateral. In the KSZ, it appears that composition of the crust is of primary importance and metamorphic grade plays a less important role in the temperature-depth distribution. The uppermost part of the crust is very heterogeneous. The lower crust is relatively more homogeneous. The two-layer heat-generation model of the crust is consistent with crustal models derived from several types of geophysical and geological evidence, both direct and indirect (Drury, 1989). Such two layer models explain the apparent constancy of the parameter b across terrains of different age and tectonic history.

The results of this study on the distribution of temperature and heat flow at depth in the KSZ can be summarized as follows:

- (1) Crustal heat production is the main factor contributing to variations in the

surface heat flow, since the Moho heat flow may be assumed to be constant in the KSZ and surrounding regions.

(2) The model of heat production-depth distribution considered most "reasonable" in this study is that with an exponentially decreasing heat production with depth separate for each geological unit. Other models, with an exponentially decreasing heat production in the top part of the crust and either constant, exponential or linearly decreasing heat production in the lower crust, also gave satisfactory results. However, crustal models with a single constant or linearly decreasing distribution of heat production with depth did not give realistic results.

(3) Both b values of 7 and 14 km generated reasonable geotherms and realistic Moho temperatures. However, each of the three heat production-depth distribution models (section 5.3) used produced somewhat different results. Isotherms with very gentle relief were obtained using $b = 7$ km. For $b = 14$ km, isotherms with a sharper relief were obtained. In particular, there appears to be a temperature anomaly in the lower crust in the vicinity of the ILCZ. The depth parameter b obtained from a regression analysis of surface heat flow vs surface heat production is 6.7 ± 2.3 km. This is similar to the values of 8.3 km (1-D solution) and 7.8 km (2-D solution) obtained by Fountain *et al.* (1987).

(4) The average value of q_s (using one measured value at profile 3 and others obtained assuming only crustal heat production contribution to the surface heat flow) in the KSZ cross-section is 47 mW m^{-2} . This value is much higher than values obtained by other workers (see chapter IV, for example Čermák & Jesso^v₁, 1971 or Ashwal *et al.*, 1987). In the heat flow measurements obtained by these authors the

calculated glacial correction factor was probably too small (Jessop & Lewis 1978; Nielsen & Beck, 1989, see Tab.4.8).

(5) The Moho heat flow beneath the KSZ is $20 \pm 4 \text{ mW m}^{-2}$ which is taken as indicative for the whole Superior Province

(6) The Moho temperatures beneath the KSZ vary from 700 to 800 K. This is in accordance with other studies by Cermák & Jessop (1971) and Ashwal *et al.* (1987).

(7) Testing the sensitivity of the calculated geotherms to variations in the thermal parameters showed that a $\pm 10\%$ variation in surface heat flow results in a $\pm 130 \text{ K}$ temperature difference at the Moho. A $\pm 20\%$ change in the average value of heat production changes the Moho temperature by $\pm 30 \text{ K}$. Variations in thermal conductivity across the region were small and the temperature variations resulting from them were negligible.

(8) The number of measurements of thermal parameters, especially of surface heat flow, in the KSZ is too small to map accurately the temperature-depth distribution. Only one value of surface heat flow for the KSZ is available. This value (33 mW m^{-2} for the smaller glacial corrections, Jessop & Lewis, 1978; 40 mW m^{-2} for the larger correction, Nielsen & Beck, 1989) corresponds geographically to the position of profile 3, where heat production values are also very low.

(9) Knowledge of the surface heat flow is an extremely important constraint on the calculated geotherm in any region. The other thermal parameters play a less significant role. Where surface heat flow measurements are not available and appropriate values have to be assumed, the errors involved most likely exceed the

temperature differences between a 1-D and a 2-D solution.

(10) The differences between a 1-D and 2-D solution are probably not very significant in the case of the KSZ (a similar conclusion was reached by Fountain *et al.*, 1987). Lateral temperature variations are small and the error caused by the assumption that heat flows only vertically is not substantial.

SYMBOLS

	Symbol	Units
Activation volume	V_a	m^3
Activation energy	E_a	$J \text{ mole}^{-1}$
Age	a	Ma
Bedding angle	θ	
Boltzman's constant	k_B	$J \text{ K}^{-1}$
Convection heat-transfer coefficient	h_i	$W \text{ m}^{-2}$
Correction factor	B	$km \text{ s}^{-1}$
Density	ρ	$kg \text{ m}^{-3}$
Depth parameter	b	km
Depth of the ocean floor	h_w	km
Depth	z	km
Edge factor	E_e	
Error function	$\text{erf}(\zeta)$	
Exponential time factor	λ_i	
Grid spacing	Δr	km
Heat generation per unit time and volume	A	$\mu \text{ Wm}^{-3}$
Heat flow	q	$mW \text{ m}^{-2}$
Melting temperature	T_m	K, $^{\circ}\text{C}$
Moho heat flow	q_M	$mW \text{ m}^{-2}$
Number of decay constants	n_d	
Outward normal unit vector to the surface element	$\mathbf{n}$	
Period	P	
Position	$\mathbf{r}$	m
Pressure	p	MPa
P-wave velocity	V_p	$km \text{ s}^{-1}$
Reduced heat flow	q_r	$mW \text{ m}^{-2}$
Specific heat	c	$J \text{ kg}^{-1} \text{ K}^{-1}$
Strain rate	ϵ	s^{-1}
Surface	S	m^2
Surface element	dS	m^2
Surface heat flow	q_s	$mW \text{ m}^{-2}$
Temperature	T	K, $^{\circ}\text{C}$
Temperature gradient	β	$K \text{ km}^{-1}, ^{\circ}\text{C km}^{-1}$
Thermal conductivity	k	$W \text{ m}^{-1} \text{ K}^{-1}$
Thermal expansion coefficient	α	K^{-1}
Thermal diffusivity	κ	$m^2 \text{ s}^{-1}$
Thickness of the lithosphere	H	km
Time	t	s
Time step	Δt	s

Volume
Wavelength

V
 λ

m³
m

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APPENDIX 1.1

T-1
H-5
100

137

Program GEOTHERM;

(\$M 49152,0,655360)

(\$N-)

(\$R-)

(\$D-)

(\$S-)

(\$L-)

Uses Globals,HPGL,Graph,MatLib,LinSolve,Strings,Files,Win,IOStuff,GWin,GIOStuff,
Crt;

Const

g = 9.807; DaysPerYear = 365.25; SecsPerDay = 86400;
LogonWindow = 1; DialogueWindow = 2; EntryWindow = 3;
MiniHelpWindow = 4; WarningWindow = 5; HelpWindow = 6;
ULGraphWindow = 7; LLGraphWindow = 8; URGraphWindow = 9;
LRGraphWindow = 10; FullGraphWindow = 11; FLGraphWindow = 12;
MLGraphWindow = 13; MRGraphWindow = 14; FRGraphWindow = 15;
LHGraphWindow = 16; RHGraphWindow = 17;
MaxChanges = 10; DWCol = 28; EWCol = 25;
ProgName = 'GEOTHERM : Version 1.1 Date : 05/05/88'; DefaultFile = '\$\$\$';
Temperature = 'K'; Pressure = 'GPa'; Density = 'Mg/m3'; Time = 'Ma';
Distance = 'km'; Conductivity = 'W/m/K'; HeatCapacity = 'kJ/kg/K';
HeatProduction = 'uW/m3'; ExpansionCoeff = '/K'; Item = 'integer';
LatentHeat = 'kJ/kg'; Gradient = 'MPa/K'; Rate = 'km/Ma'; HeatFlow = 'mW/m2';
PlotRaw : boolean = true; PlotProcessed : boolean = false;
ShowRaw : boolean = true; ShowProcessed : boolean = true;
InitSteady : boolean = false; ModelType : boolean = true;
Steady : boolean = true; Transient : boolean = false;
Direct : boolean = true; Indirect : boolean = false;

Type

Transition = Record pT0,dpdT,LH,dRo:real; end;
pTdata = Array[1..MaxChanges] of Transition;

Var

KappaProfile,LithoProfile,RoProfile,AProfile,TaProfile,TcProfile : GraphDefinition;
PhaseChange : pTdata;
TempString,FileName : String;
Dz,AccumTime,ActivePeriod,ErosionRate,SedRate,StepLength,Acceleration : Real;
Field,Dec,Nodes,NumChanges,NumSteps,TotSteps,i : integer;
Depth,Litho,Ta,Alpha,T,Ro,k,Cp,A,Kappa : RVector;
RFileVar : text;

Procedure ClearKeyBuffer;

var Ch : char;

Begin

While keypressed do Ch:=ReadKey;

End;

Procedure DefineWindows;

Const

None = 0; Single = 1; Double = 2;

begin

WinSet(LogonWindow, 1, 1,45, 6,Double,Black,White);

WinSet(DialogueWindow, 1, 6,45,25,Single,Black,White);

WinSet(EntryWindow, 46, 1,80,25,Single,White,Black);

```

WinSet(MiniHelpWindow, 3, 7, 43, 24, Double, White, Black);
WinSet(WarningWindow, 48, 22, 78, 23, None, Black, White);
WinSet(HelpWindow, 1, 6, 80, 25, Double, Black, White);
GWinSet(FullGraphWindow, 1, 1, 80, 24, Single, Black, White);
GWinSet(ULGraphWindow, 1, 1, 40, 12, Single, Black, White);
GWinSet(LLGraphWindow, 1, 13, 40, 24, Single, Black, White);
GWinSet(URGraphWindow, 41, 1, 80, 12, Single, Black, White);
GWinSet(LRGraphWindow, 41, 13, 80, 24, Single, Black, White);
GWinSet(FLGraphWindow, 1, 1, 20, 24, None, Black, White);
GWinSet(MLGraphWindow, 21, 1, 40, 24, None, Black, White);
GWinSet(MRGraphWindow, 41, 1, 60, 24, None, Black, White);
GWinSet(FRGraphWindow, 61, 1, 80, 24, None, Black, White);
GWinSet(LHGraphWindow, 1, 1, 40, 24, None, Black, White);
GWinSet(RHGraphWindow, 41, 1, 80, 24, None, Black, White);
end;

```

```

Procedure SignOn;

```

```

Begin
  ClrScr;
  OpenTextWindow(True, 'LogonWindow');
  Writeln;
  Writeln('    GEOTHERM 1D heat flow');
  Writeln('    Finite-difference model');
  Write('    (c) Malcolm Reeves, 1988');
End;

```

```

Procedure SignOff;

```

```

Begin
  ClrScr;
  OpenTextWindow(True, 'LogonWindow');
  Writeln;
  Writeln(' GEOTHERM is a product of GEMLAB');
  Writeln('More GEMLAB software may be downloaded');
  Write('from the CCUGGS BBS at (306) 955-3078');
End;

```

```

Procedure SetUniformVector(Var V:RVector; N:integer);

```

```

Var
  i : integer;
Begin
  for i := 0 to N+1 do V[i] := V[0];
End;

```

```

Procedure GetFileVector(Var V:RVector; N:integer);

```

```

Var
  i : integer;
Begin
  for i := 0 to N+1 do Read(DataFile, V[i]);
End;

```

```

Procedure MakeDiffusivityVector(Var Kappa:RVector);

```

```

var
  i : integer;
Begin
  for i := 0 to Nodes+1 do Kappa[i] := k[i] / (Ro[i] * Cp[i]);
End;

```

```
Procedure MakeLithostaticVector(Var Litho:RVector);
```

```
var
```

```
  i : integer;
```

```
Begin
```

```
  Litho[0] := 0;
```

```
  for i := 1 to Nodes+1 do
```

```
    Litho[i] := Litho[i-1] + 1e-3*g*Ro[i]*Dz;
```

```
End;
```

```
Procedure MakeDepthVector(Var Depth:RVector);
```

```
var
```

```
  i : integer;
```

```
Begin
```

```
  Depth[0] := 0;
```

```
  for i := 1 to Nodes+1 do
```

```
    Depth[i] := Depth[i-1] - Dz;
```

```
End;
```

```
Procedure MakeAdiabaticVector(Var Ta:RVector);
```

```
var
```

```
  i : integer;
```

```
  temp : real;
```

```
Begin
```

```
  if T[0] = 0 then for i := 0 to Nodes+1 do T[i] := T[i] + 273;
```

```
  Ta[0] := T[0];
```

```
  for i := 1 to Nodes do
```

```
    begin
```

```
      temp := Alpha[i-1]*Dz*g/(2*Cp[i-1]);
```

```
      Ta[i] := Ta[i-1]*(1+temp)/(1-temp);
```

```
    end;
```

```
End;
```

```
Procedure SetVectors;
```

```
Begin
```

```
  SetUniformVector(k,Nodes);
```

```
  SetUniformVector(Ro,Nodes);
```

```
  SetUniformVector(Cp,Nodes);
```

```
  SetUniformVector(Alpha,Nodes);
```

```
  SetUniformVector(A,Nodes);
```

```
  SetUniformVector(T,Nodes);
```

```
End;
```

```
Procedure GetVectors;
```

```
Const
```

```
  ok = 'vector read ok';
```

```
Begin
```

```
  ClrScr;
```

```
  Read(DataFile,Nodes,Dz);
```

```
  Writeln(Centre(WINWidth,'GEOTHERM FILE INPUT UTILITY'));
```

```
  Writeln; Writeln('Vector length.....',Nodes:3); Writeln;
```

```
  if Nodes>MaxPoints-1 then
```

```
    begin
```

```
      Writeln('Too many nodes for GEOTHERM'); Halt;
```

```
    end;
```

```
  Writeln; Writeln('Mesh spacing.....',Dz:4:1,' km'); Writeln;
```

```

GetFileVector(k,Nodes);  Writeln('Conductivity.....',ok);
GetFileVector(Ro,Nodes); Writeln('Density.....',ok);
GetFileVector(Cp,Nodes); Writeln('Specific heat.....',ok);
GetFileVector(Alpha,Nodes); Writeln('Expansion coefficient..',ok);
GetFileVector(A,Nodes);  Writeln('Heat production.....',ok);
GetFileVector(T,Nodes);  Writeln('Initial temperature....',ok);
End;

Procedure GetClapeyronInteractive;
var
  Count :integer;
Begin
  OpenTextWindow(True,'CLAPEYRON CURVES',MiniHelpWindow);
  Writeln; Writeln;
  Writeln('A linear approximation of the p-T');
  Writeln('"Clapeyron" curve for transition');
  Writeln('is required by GEOTHERM along with');
  Writeln('associated latent heat and density');
  Writeln('changes.');
```

TextPause;

```

WinWipe(1,1);
Writeln('Zero changes will ABORT this data');
Writeln('entry section. GEOTHERM can handle');
Writeln('up to 10 phase transitions.');
```

Writeln;

```

NumChanges := Kound(UserData('Number of changes',Item,EWCol));
if NumChanges<=0 then begin CloseTextWindow(NoPause); Exit; end;
Count := 0;
Repeat
  Count := Count + 1;
  WinWipe(1,1);
  Write('PHASE TRANSITION'); Writeln(Count:3,' of ',NumChanges:3);
  Writeln;
  With PhaseChange[Count] do
  begin
    pT0 := UserData('Clapeyron Intercept',Pressure,EWCol);
    dpdT := UserData('Clapeyron Gradient',Gradient,EWCol)/1000;
    LH := UserData('Latent Heat',LatentHeat,EWCol);
    dRo := UserData('Density change',Density,EWCol);
  end;
Until (Count=NumChanges);
CloseTextWindow(WithPause);
End;

Procedure GetActiveData;
var
  Count :integer;
Begin
  OpenTextWindow(True,'DEPOSITION/EROSION',MiniHelpWindow);
  Writeln; Writeln;
  Writeln('An exponentially varying erosion');
  Writeln('or sedimentation rate can be spec-');
  Writeln('ified by a peak rate and a "half-');
  Writeln('life" for the period of activity.');
```

Writeln;

```

if Query('Deposition or Erosion','DE',EWCol) then
```

```

    SedRate := UserData('Peak deposition rate',Rate,EWCol)
  else
    ErosionRate := -UserData('Peak erosion rate',Rate,EWCol);
    ActivePeriod := UserData('Half-life for activity',Time,EWCol);
    CloseTextWindow(WithPause);
  End;

```

```

Procedure GetPhaseChanges;
var
  i : integer;
Begin
  Read(DataFile,NumChanges);
  for i := 1 to NumChanges do
    With PhaseChange[i] do
      Readln(DataFile,pT0,dpdT,LH,dRo);
    End;
  End;

```

```

Procedure GetClapeyron;
Begin
  FileName:="";
  GetFile(FileName,DefaultFile);
  GetPhaseChanges;
  Close(DataFile)
End;

```

```

Procedure GetInteractiveData;
Var
  Count : integer;
  Uniform : boolean;
Begin
  Field := 5; Dec := 2; Count := -1; Uniform := false;
  Repeat
    Nodes := Round(UserData('Number of nodes',Item,DWCol));
  Until (Nodes<30);
  Dz := UserData('Model mesh spacing',Distance,DWCol);
  if Query('All nodes the same','YN',DWCol) then Uniform := true;
  Repeat
    Count := Count + 1;
    WinWipe(1,1);
    Write('NODAL DATA SPECIFICATION');
    if Uniform then Writeln else Writeln(Count:3,' of ',Nodes+1:3);
    Writeln;
    k[Count] := UserData('Thermal conductivity',Conductivity,DWCol);
    Ro[Count] := UserData('Rock density',Density,DWCol);
    Cp[Count] := UserData('Heat capacity',HeatCapacity,DWCol);
    Alpha[Count] := UserData('Linear expansion coeff.',ExpansionCoeff,DWCol);
    A[Count] := UserData('Heat production',HeatProduction,DWCol);
    T[Count] := UserData('Initial temperature',Temperature,DWCol);
  Until (Count=Nodes+1) or Uniform;
  if Uniform then SetVectors;
  A[Nodes+1] := UserData('Basal heat flow',HeatFlow,DWCol);
End;

```

```

Procedure GetFileData;
Begin
  Filename := "";

```



```

GetFile(FileName,DefaultFile);
GetVectors;
Close(DataFile)
End;

Procedure PhaseTransitions(Var Ta,Litho,Ro,Cp:RVector; PhaseChange:pTdata);
var
  i,j : integer;
Begin
  for j := 1 to NumChanges do
    With PhaseChange[j] do
      begin
        for i := 1 to Nodes do
          begin
            if (Litho[i] > pT0+Ta[i]*dpdT/1000) then
              begin
                Ro[i] := Ro[i] + dRo;
                Ta[i] := Ta[i] + LH / Cp[i];
              end;
            end;
          end;
        end;
      MakeLithostaticVector(Litho);
    End;
  End;

Procedure SetUpModel;
Begin
  OpenTextWindow(True,'PROBLEM SPECIFICATION',DialogueWindow);
  Writeln; Writeln;
  Writeln(Centre(WINWidth,"H" for help and information)); Writeln;
  NumSteps := 0; StepLength := 500; Acceleration := 1;
  ErosionRate := 0; SedRate := 0; ActivePeriod := 0;
  if Query('Steady-state or transient','TS',DWCol) then
    begin
      ModelType := Transient;
      NumSteps := Round(UserData('Number of time steps',Item,DWCol));
      StepLength := UserData('Length of step',Time,DWCol);
      Acceleration := UserData('Time step acceleration','',DWCol);
      if Query('Erosion or deposition','YN',DWCol) then GetActiveData;
    end;
  NumChanges := 0;
  if Query('Any phase changes','YN',DWCol) then
    if Query('File or keyboard input','FK',DWCol) then
      GetClapeyron
    else
      GetClapeyronInteractive;
  Writeln; Writeln(Centre(WINWidth,'MAIN DATA INPUT')); Writeln;
  if Query('File or keyboard input','FK',DWCol) then
    GetFileData
  else
    GetInteractiveData;
  MakeAdiabaticVector(Ta);
  MakeLithostaticVector(Litho);
  MakeDepthVector(Depth);
  if SedRate<>0 then TcProfile.Ymin := Depth[Nodes] - SedRate * ActivePeriod;
  if NumChanges>0 then PhaseTransitions(Ta,Litho,Ro,Cp,PhaseChange);
  MakeDiffusivityVector(Kappa);

```

```

TextPause;
WinWipe(1,1);
if (ModelType=Transient) then
  if Query('Find initial steady-state','YN',DWCol) then InitSteady := True;
  if Query('Plot computed geotherms','YN',DWCol) then PlotProcessed := True;
  if Query('Plot raw data profiles','NY',DWCol) then PlotRaw := False;
  if Query('Show raw data','NY',DWCol) then ShowRaw := False;
  if Query('Show initial geotherms','NY',DWCol) then ShowProcessed := False;
End;

```

```

Procedure DisplayVector(VectorName:String; V:RVector; N:integer);

```

```

Var

```

```

  Row,Count : integer;

```

```

Begin

```

```

  Count := 0;

```

```

  OpenTextWindow(True,VectorName,EntryWindow);

```

```

  Repeat

```

```

    if (Count mod 16)=0 then

```

```

      begin

```

```

        ClrScr;

```

```

        Writeln(Centre(WINWidth,VectorName)); Writeln;

```

```

        Writeln('  Depth      Value');

```

```

        Row := WhereY - 1;

```

```

      end;

```

```

      Count := Count + 1; Row:=Row + 1;

```

```

      Writeln(Depth[Count]:9:0,V[Count]:14:3);

```

```

      if ((Count mod 16)=0) and (Count<>N) then TextPause;

```

```

      ClearKeyBuffer;

```

```

    Until (Count=N);

```

```

  CloseTextWindow(WithPause);

```

```

End;

```

```

Procedure XYPlot(Scaling:boolean; UserPen,GWindow:integer; XVal,YVal:RVector;
XYGraph:GraphDefinition);

```

```

Var

```

```

  ix,iy : Vector;

```

```

  Count : integer;

```

```

Begin

```

```

  With XYGraph do

```

```

    begin

```

```

      if Scaling then

```

```

        begin

```

```

          OpenGraphicsWindow(True,Title,GWindow);

```

```

          FindLimits(YVal,Nodes,Ymin,Ymax);

```

```

          ForceLimits(Ymin,Ymax); TickY := (Ymax - Ymin)/NTickY;

```

```

          FindLimits(XVal,Nodes,Xmin,Xmax);

```

```

          ForceLimits(Xmin,Xmax); TickX := (Xmax - Xmin)/NTickX;

```

```

          HPSetColor(Pen2); DrawAxes(XYGraph,XYWin,PWin);

```

```

        end;

```

```

        ScaleGraphicsWindow(Horizontal,PWin,XVal,Nodes,ix);

```

```

        ScaleGraphicsWindow(Vertical,PWin,YVal,Nodes,iy);

```

```

        HPSetColor(UserPen); HPMoveTo(ix[1],iy[1]);

```

```

        for Count := 2 to Nodes do HPLineTo(ix[Count],iy[Count]);

```

```

      end;

```

```

End;

```

Procedure SetUpProfiles;

Begin

With KappaProfile do

begin

Box := True; Title := 'THERMAL DIFFUSIVITY';

XTitle := 'Diffusivity (mm²/s)';

NTickX := 2; FieldX := 5; DecX := 2; TickX := 0.1; Xmin := 0; Xmax := 2;

YTitle := 'Depth (km)';

NTickY := 10; FieldY := 4; DecY := 0; TickY := 10; Ymin := 0; Ymax := 0;

end;

With LithoProfile do

begin

Box := True; Title := 'PRESSURE';

XTitle := 'Pressure (GPa)';

NTickX := 2; FieldX := 5; DecX := 0; TickX := 10; Xmin := 0; Xmax := 0;

YTitle := 'Depth (km)';

NTickY := 10; FieldY := 4; DecY := 0; TickY := 10; Ymin := 0; Ymax := 0;

end;

With RoProfile do

begin

Box := True; Title := 'DENSITY';

XTitle := 'Density (Mg/m³)';

NTickX := 2; FieldX := 5; DecX := 2; TickX := 0.1; Xmin := 2; Xmax := 0;

YTitle := 'Depth (km)';

NTickY := 10; FieldY := 4; DecY := 0; TickY := 10; Ymin := 0; Ymax := 0;

end;

With AProfile do

begin

Box := True; Title := 'HEAT PRODUCTION';

XTitle := 'Production (uW/m³)';

NTickX := 2; FieldX := 5; DecX := 0; TickX := 10; Xmin := 0; Xmax := 0;

YTitle := 'Depth (km)';

NTickY := 10; FieldY := 4; DecY := 0; TickY := 10; Ymin := 0; Ymax := 0;

end;

With TaProfile do

begin

Box := True; Title := 'CONVECTION GEOTHERM';

XTitle := 'Temperature (K)';

NTickX := 8; FieldX := 5; DecX := 0; TickX := 10; Xmin := 0; Xmax := 0;

YTitle := 'Depth (km)';

NTickY := 10; FieldY := 4; DecY := 0; TickY := 10; Ymin := 0; Ymax := 0;

end;

With TcProfile do

begin

Box := True; Title := 'CONDUCTION GEOTHERM';

XTitle := 'Temperature (K)';

NTickX := 8; FieldX := 5; DecX := 0; TickX := 10; Xmin := 0; Xmax := 2000;

YTitle := 'Depth (km)';

NTickY := 10; FieldY := 4; DecY := 0; TickY := 10; Ymin := 0; Ymax := 0;

end;

End;

Procedure BeginGraphics;

Begin

LinkDrivers;

```

HPInit("");
GWinInit;
End;

```

```

Procedure EndGraphics;
Begin
  GWinExit;
  HPClose;
  CloseGraph;
  RestoreCrtMode;
End;

```

```

Procedure NumericalSolution(ModelType:boolean; Var T:RVector; A,Kappa,Ro,Cp,k:RVector;
Dz:real;
NRows,NCols:integer);
var
  Coeff : Matrix;
  Itol, Omega : Real;
  Order, Diag, Count : integer;

```

```

Procedure Form1D(ModelType:boolean; Var Coeff:Matrix; Var A,T:RVector;
Kappa,Ro,Cp,k:RVector;
Dz:real;
NRows:integer);
{
  Set up coefficient matrix for 1D implicit finite-difference scheme
}
var
  N,Row,Col,Test : integer;
  Dt,C0,CN,K0,KN,value : real;

```

```

Procedure CPotModify(Row:Integer; Cval:Real);
begin
  A[Row] := A[Row]-Cval*Kappa[Row]; {Modify for constant potential boundary}
end;

```

```

Procedure GradModify(Row:Integer; Kval:Real);
begin
  {Modify for constant gradient boundary}
  Coeff[Row,Row] := Coeff[Row,Row]+ Kappa[Row];
  A[Row] := A[Row]+Kval*Dz/(Ro[Row]*Cp[Row]);
end;

```

```

Begin
  N := NRows; Dt := StepLength * DaysPerYear * SecsPerDay * 1e-6;
  K0 := A[0]; KN := -A[N+1]; {Gradient boundary term}
  C0 := T[0]; CN := T[N+1]; {Constant potential value}
  for Row := 1 to N do
    begin
      A[Row] := -Sqr(Dz)*A[Row]/(Ro[Row]*Cp[Row]);
      if (ModelType=Transient) then A[Row] := A[Row]-T[Row]*Sqr(Dz)/Dt;
    end;
  for Row := 1 to N do
    begin
      for Col := 1 to N do
        begin

```

```

    value := 0; Test := Row-Col;
    if (Test=0) then (it's a diagonal coefficient)
    begin
        value := - 2 * Kappa[Col];
        if (ModelType=Transient) then value := value - Sqr(Dz)/Dt;
    end;
    if Abs(Test)=1 then value := Kappa[Col]; (it's an adjacent row coefficient)
)
    Coeff[Row,Col] := value;
end;
(Modify for boundary conditions)
if Row = 1 then CPotModify(Row,C0);
if Row = N then GradModify(Row,KN);
end;
End;

Begin
    MatLibInit;
    Order := NRows*NCols; Diag := 2*NCols+1; Itol := 1e-3; Omega := 1;
    Form1D(ModelType,Coeff,A,T,Kappa,Ro,Cp,k,Dz,NRows);
    MatSolve(Direct,Coeff,A,k,T,Order,Diag,Count,Itol,Omega,Dz);
End;

Procedure WriteFullGraphInfo;
Begin
    Str(NumSteps:5,TempString);
    HPOutTextXY(45*BitsPerChar,4*LinesPerChar,'Number of steps      = '+TempString);
    Str(StepLength:5:2,TempString);
    HPOutTextXY(45*BitsPerChar,5*LinesPerChar,'Initial step size (Ma) = '+TempString);
    Str(Acceleration:5:2,TempString);
    HPOutTextXY(45*BitsPerChar,6*LinesPerChar,'Acceleration factor   = '+TempString);
    if SedRate<>0 then
    begin
        Str(SedRate:5:2,TempString);
        HPOutTextXY(45*BitsPerChar,7*LinesPerChar,'Deposition rate (km/Ma) = '+TempString);

        Str(ActivePeriod:5:2,TempString);
        HPOutTextXY(45*BitsPerChar,8*LinesPerChar,'Deposition period (Ma) = '+TempString);
    end;
    if ErosionRate<>0 then
    begin
        Str(ErosionRate:5:2,TempString);
        HPOutTextXY(45*BitsPerChar,7*LinesPerChar,'Erosion rate (km/Ma)   = '+TempString);

        Str(ActivePeriod:5:2,TempString);
        HPOutTextXY(45*BitsPerChar,8*LinesPerChar,'Erosion period (Ma)   = '+TempString);
    end;
End;

Procedure ModifyDepth(ChangeRate:Real);
var
    temp : Real;
    Count : integer;
Begin
    if ChangeRate<>0 then

```

```

begin
  temp := ChangeRate * StepLength * Exp(-AccumTime/ActivePeriod)/Nodes;
  Dz := Dz + temp;
end;
End;

Begin
  Assign(RFilVar,'trial.dat');
  ReWrite(RFilVar);
  DefineWindows;
  Signon; SetUpProfiles; SetUpModel;
  if ShowRaw then
    begin
      DisplayVector(KappaProfile.XTitle,Kappa,Nodes);
      DisplayVector(LithoProfile.XTitle,Litho,Nodes);
      DisplayVector(RoProfile.XTitle,Ro,Nodes);
      DisplayVector(AProfile.XTitle,A,Nodes);
    end;
  if InitSteady then
    begin
      NumericalSolution(Steady,T,A,Kappa,Ro,Cp,k,Dz,Nodes,1);
      Writeln(RFilVar,'These are the temps. for the initial steady-state.');
```

```

    begin
      for i := 1 to Nodes do
        Writeln(RFilVar,T[i]:8:2);
      end;
    end
  else
    NumericalSolution(ModelType,T,A,Kappa,Ro,Cp,k,Dz,Nodes,1);
  if ShowProcessed then
    begin
      DisplayVector(TaProfile.XTitle,Ta,Nodes);
      DisplayVector(TcProfile.XTitle,T,Nodes);
    end;
    CloseTextWindow(WithPause);

  BeginGraphics;
  HPSetColor(Pen3);
  if PlotRaw then
    begin
      XYPlot(True,Pen3,FLGraphWindow,Kappa,Depth,KappaProfile);
      XYPlot(True,Pen3,MLGraphWindow,Litho,Depth,LithoProfile);
      XYPlot(True,Pen3,MRGraphWindow,Ro,Depth,RoProfile);
      XYPlot(True,Pen3,FRGraphWindow,A,Depth,AProfile);
      CloseGraphicsWindow(WithPause);
    end;
  if PlotProcessed then
    begin
      HPClearDevice;
      if (ModelType=Steady) then
        begin
          Writeln(RFilVar,'This is a steady-state solution.');
```

```

          begin
            for i := 1 to Nodes do
              Writeln(RFilVar,T[i]:8:2);
            end;
          end;
        end;
      end;
    end;
  end;
end;

```

```

    XYPlot(True, Pen3, LHGraphWindow, Ta, Depth, TaProfile);
    XYPlot(True, Pen3, RHGraphWindow, T, Depth, TcProfile);
    XYPlot(False, Pen3, RHGraphWindow, Ta, Depth, TcProfile)
end
else
begin
    XYPlot(True, Pen3, FullGraphWindow, T, Depth, TcProfile);
    WriteFullGraphInfo;
    TotSteps := NumSteps;
    NumSteps := Pred(NumSteps);
    Writeln(RFilVar, 'The total number of time-steps is ', TotSteps);
    AccumTime := 0;
    Repeat
        NumSteps := Pred(NumSteps);
        AccumTime := AccumTime + StepLength;
        ModifyDepth(SedRate);
        ModifyDepth(ErosionRate);
        MakeDepthVector(Depth);
        NumericalSolution(Transient, T, A, Kappa, Ro, Cp, k, Dz, Nodes, I);
        XYPlot(False, Pen4, FullGraphWindow, T, Depth, TcProfile);
        StepLength := StepLength * Acceleration;
        Writeln(RFilVar, 'The current step number is ', TotSteps - NumSteps);
    begin
        for i := 1 to Nodes do
            Writeln(RFilVar, T[i]:8:2);
        end;
    Until NumSteps=0;
end;
CloseGraphicsWindow(WithPause);
end;
Close(RFilVar);
EndGraphics;
Signoff;
End.
^Z

```

Appendix 5.1 (a)

Output of Pascal program for the six profiles from the KSZ cross-section (model A): $b = 7$ km; A - exponential heat production distribution with depth separate for each geological unit with an average heat production value equal to that measured by Ashwal *et al.* (1987).

THERMAL DIFFUSIVITY

Diffusivity (m2/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m3)

HEAT PRODUCTION

Production (uW/m3)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

1. Equilibrium steady-state solution.

200.0

201.00

202.00

211.04

212.00

213.00

214.00

215.00

216.00

217.00

218.00

219.00

220.00

221.00

222.00

223.00

224.00

225.00

226.00

227.00

228.00

229.00

230.00

231.00

232.00

233.00

234.00

235.00

236.00

237.00

238.00

239.00

240.00

241.00

242.00

243.00

244.00

245.00

246.00

247.00

248.00

249.00

250.00

251.00

252.00

253.00

254.00

255.00

256.00

257.00

258.00

259.00

260.00

261.00

262.00

263.00

264.00

265.00

266.00

267.00

268.00

269.00

270.00

271.00

272.00

273.00

274.00

275.00

276.00

277.00

278.00

279.00

280.00

281.00

282.00

283.00

284.00

 

THEMAL DIFFUSIVITY

PRESSURE

DENSITY

HEAT PRODUCTION

Diffusivity (mm²/s)

Pressure (GPa)

Density (Mg/m³)

Production (μW/m³)

CONVECTION GEOTHERM

CONDUCTION GEOTHERM

Temperature (K)

Temperature (K)

THERMAL DIFFUSIVITY

Diffusivity (m²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (uW/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

this is a steady-state solution.

330.87
359.61
386.74
412.67
437.70
462.07
485.90
509.00
532.57
555.54
576.80
596.70
615.30
634.10
651.95
669.20
686.20
703.00
719.66
736.15

10

1.44	2.44	3.44	4.44	5.44	6.44	7.44	8.44	9.44	10.44	11.44	12.44	13.44	14.44	15.44	16.44	17.44	18.44	19.44	20.44	21.44	22.44	23.44	24.44	25.44	26.44	27.44	28.44	29.44	30.44	31.44	32.44	33.44	34.44	35.44	36.44	37.44	38.44	39.44	40.44	41.44	42.44	43.44	44.44	45.44	46.44	47.44	48.44	49.44	50.44	51.44	52.44	53.44	54.44	55.44	56.44	57.44	58.44	59.44	60.44	61.44	62.44	63.44	64.44	65.44	66.44	67.44	68.44	69.44	70.44	71.44	72.44	73.44	74.44	75.44	76.44	77.44	78.44	79.44	80.44	81.44	82.44	83.44	84.44	85.44	86.44	87.44	88.44	89.44	90.44	91.44	92.44	93.44	94.44	95.44	96.44	97.44	98.44	99.44	100.44	101.44	102.44	103.44	104.44	105.44	106.44	107.44	108.44	109.44	110.44	111.44	112.44	113.44	114.44	115.44	116.44	117.44	118.44	119.44	120.44	121.44	122.44	123.44	124.44	125.44	126.44	127.44	128.44	129.44	130.44	131.44	132.44	133.44	134.44	135.44	136.44	137.44	138.44	139.44	140.44	141.44	142.44	143.44	144.44	145.44	146.44	147.44	148.44	149.44	150.44	151.44	152.44	153.44	154.44	155.44	156.44	157.44	158.44	159.44	160.44	161.44	162.44	163.44	164.44	165.44	166.44	167.44	168.44	169.44	170.44	171.44	172.44	173.44	174.44	175.44	176.44	177.44	178.44	179.44	180.44	181.44	182.44	183.44	184.44	185.44	186.44	187.44	188.44	189.44	190.44	191.44	192.44	193.44	194.44	195.44	196.44	197.44	198.44	199.44	200.44	201.44	202.44	203.44	204.44	205.44	206.44	207.44	208.44	209.44	210.44	211.44	212.44	213.44	214.44	215.44	216.44	217.44	218.44	219.44	220.44	221.44	222.44	223.44	224.44	225.44	226.44	227.44	228.44	229.44	230.44	231.44	232.44	233.44	234.44	235.44	236.44	237.44	238.44	239.44	240.44	241.44	242.44	243.44	244.44	245.44	246.44	247.44	248.44	249.44	250.44	251.44	252.44	253.44	254.44	255.44	256.44	257.44	258.44	259.44	260.44	261.44	262.44	263.44	264.44	265.44	266.44	267.44	268.44	269.44	270.44	271.44	272.44	273.44	274.44	275.44	276.44	277.44	278.44	279.44	280.44	281.44	282.44	283.44	284.44	285.44	286.44	287.44	288.44	289.44	290.44	291.44	292.44	293.44	294.44	295.44	296.44	297.44	298.44	299.44	300.44	301.44	302.44	303.44	304.44	305.44	306.44	307.44	308.44	309.44	310.44	311.44	312.44	313.44	314.44	315.44	316.44	317.44	318.44	319.44	320.44	321.44	322.44	323.44	324.44	325.44	326.44	327.44	328.44	329.44	330.44	331.44	332.44	333.44	334.44	335.44	336.44	337.44	338.44	339.44	340.44	341.44	342.44	343.44	344.44	345.44	346.44	347.44	348.44	349.44	350.44	351.44	352.44	353.44	354.44	355.44	356.44	357.44	358.44	359.44	360.44	361.44	362.44	363.44	364.44	365.44	366.44	367.44	368.44	369.44	370.44	371.44	372.44	373.44	374.44	375.44	376.44	377.44	378.44	379.44	380.44	381.44
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THERMAL DIFFUSIVITY

Diffusivity (m²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (μW/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

THERMAL DIFFUSIVITY

Diffusivity (m²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

FLAT PRODUCTION

Production (uM/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

this is a steady-state solution.

339.67

377.98

415.28

449.98

481.84

510.42

536.53

560.79

583.65

E05.4E

626.42

E46. 24

666. 27

ER5.14

707.21

720.67

737.67

754.02

32

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.3 0.8 0.8 0.8

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4-6 5a-6 5a-6 5a-6 5a-6 5a-6 5a-6

.88 .88 .66 .50 2.67 2.00 1.51 1.13 .85 .64 .48 .36 .86

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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Appendix 5.1 (b)

Output of Pascal program for the six profiles from the KSZ cross-section (model B): $b = 14$ km; A - exponential heat production distribution with depth separate for each geological unit with an average heat production value equal to that measured by Ashwal *et al.* (1987).

THERMAL DIFFUSIVITY

PRESSURE

DENSITY

HEAT PRODUCTION

Diffusivity (mm²/s)

Pressure (GPa)

Density (Mg/m³)Production (μW/m³)

CONVECTION GEOTHERM

CONDUCTION GEOTHERM

Temperature (K)

Temperature (K)

his is a steady-state solution.

331.97
361.74
389.61
415.82
440.61
464.15
486.61
508.13
528.85
548.87
568.28
587.16
605.59
623.62
641.31
658.70
675.84
692.74
709.44
725.98

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.82 2.82 2.82 2.82 2.82 2.82 2.82 2.82 2.82 2.82 2.82 2.82 2.82

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.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8

.8 0.8 0.8 0.8 0.8 0.8

e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6

e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6

.34 1.34 1.16 1.01 0.76 0.66 0.57 0.49 0.43 0.37 0.32 0.28

.24 0.21 0.18 0.16 0.14 0.12 0.10 0.09 20.00

00 300 300 300 300 300 300 300 300 300 300 300 300 300

00 300 300 300 300 300

THERMAL DIFFUSIVITY

Diffusivity (m²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (uW/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

[illegible]

THERMAL DIFFUSIVITY

PRESSURE

DENSITY

HEAT PRODUCTION

Diffusivity (mm²/s)

Pressure (GPa)

Density (Mg/m³)

Production (uW/m³)

CONVECTION GEOTHERM

CONDUCTION GEOTHERM

Appendix 5.1 (c)

The output of Pascal program for the six profiles from the KSZ cross-section (model C): $b = 14$ km; A - exponential heat production distribution with depth separate for each geological unit.

T4S
M
110
100
100
100

1. The first part of the document is a list of names and addresses of the members of the committee. The names are listed in alphabetical order, and the addresses are given below each name. The list includes the names of the members of the committee, the names of the members of the sub-committee, and the names of the members of the advisory committee. The addresses are given in the following order: the address of the member of the committee, the address of the member of the sub-committee, and the address of the member of the advisory committee.

2. The second part of the document is a list of the names and addresses of the members of the committee. The names are listed in alphabetical order, and the addresses are given below each name. The list includes the names of the members of the committee, the names of the members of the sub-committee, and the names of the members of the advisory committee. The addresses are given in the following order: the address of the member of the committee, the address of the member of the sub-committee, and the address of the member of the advisory committee.

THERMAL DIFFUSIVITY

diffusivity (mm²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (kg/m³)

HEAT PRODUCTION

production (W/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

TABLE 1

1. Name of the person or organization
2. Address
3. City
4. State
5. Zip
6. Telephone
7. Fax
8. E-mail
9. Website
10. Other

1. Name of the person or organization
2. Address
3. City
4. State
5. Zip
6. Telephone
7. Fax
8. E-mail
9. Website
10. Other

THERMAL DIFFUSIVITY

Diffusivity (mm²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (uM/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

THERMAL DIFFUSIVITY

Diffusivity (mm²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (uW/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

0.12

0.44 2.14 2.44 2.14 2.44 2.44 2.44 2.44 2.44 2.14 2.44 2.14 2.14

0.44 2.44 2.14 2.44 2.44 2.14 2.44 2.44

0.82 2.82 2.82 2.82 2.82 2.70 2.7 2.70 2.70 2.70 2.70 2.70 2.70

[illegible]

state solution.

a b c

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HEAT DIFFUSIVITY

PRESSURE

DENSITY

HEAT PRODUCTION

Diffusivity (m²/s)

Pressure (GPa)

Density (Mg/m³)

Production (uW/m³)

CONVECTION GEOTHERM

CONDUCTION GEOTHERM

THERMAL DIFFUSIVITY

Diffusivity (m²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (uW/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

this is a steady-state solution.

238.71
375.74
411.13
445.08
476.32
506.20
537.54
559.10
589.12
605.81
627.36
647.60
668.08
687.55
706.22
724.21
741.59
758.44

2

[illegible]

THERMAL DIFFUSIVITY

PRESSURE

DENSITY

HEAT PRODUCTION

Diffusivity (nm²/s)

Pressure (GPa)

Density (Mg/m³)

Production (uW/m³)

CONVECTION GEOTHERM

CONDUCTION GEOTHERM

340.30
377.12
419.60
443.41
470.02
504.48
530.31
550.30
570.37
593.42
612.30
636.52
653.70
670.75
687.33
704.63
721.37

[illegible]

THERMAL DIFFUSIVITY

Diffusivity (mm²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (μW/m³)

CONVECTION GEOTHERM

Temperature (K)

CONDUCTION GEOTHERM

Temperature (K)

his is a solid, stable conclusion.

377.75
374.50
404.00
411.00
411.00
500.40
570.90
557.00
500.00
600.00
620.00
650.00
700.50
700.71
720.00
741.70
750.00
770.00
780.00

10

[illegible]

THERMAL DIFFUSIVITY

Diffusivity (mm²/s)

PRESSURE

Pressure (GPa)

DENSITY

Density (Mg/m³)

HEAT PRODUCTION

Production (μW/m³)

CONVECTION GEOTHERM

CONDUCTION GEOTHERM

for a steady-state solution.

512.00
540.46
567.26
594.40
621.00
647.80
674.26
700.57
727.00
753.64
780.36
807.14
833.06
859.16
885.27
911.50
937.45
963.13
988.67
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1.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8 0.8
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1.0 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6 5e-6
1.00 1.00 0.71 0.50 0.25 1.00 1.00 0.70 0.25 0.40 0.60 1.10 0.71
1.50 0.75 1.00 1.00 0.70 0.25 0.40 0.60 20.00

APPENDIX 5.2

TH

to

T

...

* Program . . . : GEOT.PRG
* Author . . . : KATARZYNA TYRCHA
* Date : 01-25-89

```
set talk off
use deep
public row, choice, T, Tt1, Tt2, Tt3, Tt, qs, qs1, qs2, qs3, k,
k1, k2, k3, A, A1, A2, A3, z, zd, za, zb, zc, choice1, Tb, Tbl,
qm, prof
row = 3
@ 0,0 clear
store 0 to choice
do while choice < 5
  clear
  @ 5,30 say "MODELS"
  @ 7,19 say "Choose a selection shown below: "
  @ 9,25 say "1 Model: 1 interval"
  @ 10,25 say "2 Model: 2 intervals"
  @ 11,25 say "3 Model: 3 intervals"
  @ 13,25 say "5 Exit program"
  store 0 to choice
  store 0 to T
  store 0 to Tt
  store 0 to Tt1
  store 0 to Tt2
  store 0 to Tt3
  store 0 to Tb
  store 0 to Tbl
  store 0 to qm
  store 0 to qs
  store 0 to qs1
  store 0 to qs2
  store 0 to qs3
  store 0 to k
  store 0 to k1
  store 0 to k2
  store 0 to k3
  store 0 to A
  store 0 to A1
  store 0 to A2
  store 0 to A3
  store 0 to za
  store 0 to zb
  store 0 to zc
  store 0 to zd
  store " " to prof
  @ 15,20 say "Enter selection: "GET choice picture "9"
  READ
  DO CASE
    CASE choice = 1
      @ 0,0 clear
      @ 6,20 say "ENTER NAME OF PROFILE: "GET prof picture "XXXX"
      @ 10,25 say "Enter z(km): "GET za picture "99"
      @ 12,5 say "Enter Tt: "GET Tt picture "999.99"
```

```
@ 12,27 say "Enter qs:"Get qs picture "99.99"  
@ 12,50 say "Enter k: "GET k picture "99.99"  
@ 14,25 say "Enter A: "GET A picture "99.99"  
store " " to choice1  
@ 18,15 say "DIRECT THE OUTPUT TO THE PRINTER? [Y/N] "GET  
choice1  
  READ  
  DO CASE  
    CASE upper(choice1) = "Y"  
      set device to print  
      do geot11  
      set device to screen  
    CASE upper(choice1) = "N"  
      do geot1  
      wait  
  ENDCASE choice1  
  CASE choice = 2  
    do geot12  
  CASE choice = 3  
    do geot13  
  CASE choice = 5  
    do exit  
  OTHERWISE  
    * Display error when user presses wrong key *  
    @ 0,0 clear  
    @ 15,25 say "ERROR, Please choose 1, 2, 3 or 5"  
    wait  
  ENDCASE choice  
enddo  
clear all  
set talk on  
RETURN
```

^Z

* Program GEOT1.PRG

```

@ 0,0 clear
@ 0,25 say "Model 1 T = f(z)"
@ 2,2 say "z(km)"
@ 2,10 say "T(K)"
@ 2,20 say "z(km)"
@ 2,28 say "T(K)"
@ 2,38 say "z(km)"
@ 2,46 say "T(K)"
@ 2,56 say "z(km)"
@ 2,64 say "T(K)"
row = 3
store 0 to z
go top
DO CASE
  CASE za <= 12
    do while z <= za
      z = z + 1
      T = Tt + (qs/k) * z - (A/(2*k)) * (z ** 2)
      row = row + 1
      @ row,3 say z picture "99"
      @ row,10 say T picture "9999.99"
      skip
    ENDDO
    qsl = ((Tt + (qs/k)*za - (A/(2*k))*(za**2)) - (Tt + (qs/k) *
(z-1) - (A/(2*k))*((z-1)**2)))*k
    @ 18,28 say "qsl ="
    @ 18,34 say qsl picture "99.99"
    RETURN
  clear
  CASE za <= 25
    store 0 to z
    go top
    do while z <= 12
      z = z + 1
      T = Tt + (qs/k)*z - (A/(2*k))*(z**2)
      row = row + 1
      @ row,3 say z picture "99"
      @ row,10 say T picture "9999.99"
      skip
    enddo
    row = 3
    store 0 to z
    go 14
    do while z <= za
      z = z + 1
      T = Tt + (qs/k)*z - (A/(2*k)) * (z ** 2)
      row = row + 1
      @ row,21 say z picture "99"
      @ row,28 say T picture "9999.99"
      skip
    enddo
    qsl = ((Tt + (qs/k)*za-(A/(2*k))*(za**2)) - (Tt + (qs/k) *
(z-1) - (A/(2*k))*((z-1)**2)))*k

```

```

@ 18,28 say "qsl ="
@ 18,34 say qsl picture "99.99"
RETURN
clear
CASE za <= 38
store 0 to z
go top
do while z <= 12
  z = z + 1
   $T = Tt + (qs/k) * z - (A/(2*k)) * (z**2)$ 
  row = row + 1
  @ row,3 say z picture "99"
  @ row,10 say T picture "9999.99"
  skip
enddo
row = 3
store 0 to z
go 14
do while z <= 25
  z = z + 1
   $T = Tt + (qs/k) * z - (A/(2*k)) * (z**2)$ 
  row = row + 1
  @ row,21 say z picture "99"
  @ row,28 say T picture "9999.99"
  skip
enddo
row = 3
store 0 to z
go 27
do while z <= za
  z = z + 1
   $T = Tt + (qs/k) * z - (A/(2*k)) * (z**2)$ 
  row = row + 1
  @ row,39 say z picture "99"
  @ row,46 say T picture "9999.99"
  skip
enddo
qsl = ((Tt + (qs/k)*za-(A/(2*k))*(za**2)) - (Tt + (qs/k) *
(za-1)-(A/(2*k))*((za-1)**2)))*k
@ 18,28 say "qsl ="
@ 18,34 say qsl picture "99.99"
RETURN
clear
CASE za <= 45
store 0 to z
go top
row = 3
DO WHILE z <= 12
  z = z + 1
   $T = Tt + (qs/k) * z - (A/(2*k)) * (z**2)$ 
  row = row + 1
  @ row,3 say z picture "99"
  @ row,10 say T picture "9999.99"
  skip
enddo
row = 3

```

```

store 0 to z
go 14
do while z <= 25
  z = z + 1
  T = Tt + (qs/k)*z - (A/(2*k))*(z**2)
  row = row + 1
  @ row,21 say z picture "99"
  @ row,28 say T picture "9999.99"
  skip
enddo
row = 3
store 0 to z
go 27
do while z <= 38
  z = z + 1
  T = Tt + (qs/k)*z - (A/(2*k))*(z**2)
  row = row + 1
  @ row,39 say z picture "99"
  @ row,46 say T picture "9999.99"
  skip
enddo
row = 3
store 0 to z
go 40
do while z <= za
  z = z + 1
  T = Tt + (qs/k)*z - (A/(2*k))*(z**2)
  row = row + 1
  @ row,57 say z picture "99"
  @ row,64 say T picture "9999.99"
  skip
enddo
qsl = ((Tt+(qs/k)*za-(A/(2*k))*(za**2)) - (Tt+(qs/k) * (za-1)
- (A/(2*k))*((za-1)**2)))*k
@ 18,28 say "qsl ="
@ 18,34 say qsl picture "99.99"
ENDCASE za
RETURN

```

^Z

* Program GEOT11.PRG

```

@ 0,0 clear
@ 0,28 say "PROFILE: "
@ 0,37 say prof picture "XXXX"
@ 2,25 say "T = f(Tb, A, qs, k, z)"
@ 4,20 say "z(km)"
@ 4,46 say "T(K)"
row = 6
store 0 to z
go top
do while z <= za
  z = z + 1
  T = Tt + (qs/k) * z - (A/(2*k)) * (z ** 2)
  row = row + 1
  @ row,21 say z picture "99"
  @ row,45 say T picture "9999.99"
  skip
enddo
qsl = ((Tt +
(qs/k)*za-(A/(2*k))*(za**2))-(Tt+(qs/k)*(za-1)-(A/(2*k))*((za-1)**
2)))*k
@ row+2,13 say "*****"
@ row+3,13 say "*"
@ row+3,58 say "*"
@ row+4,13 say "*"
@ row+4,16 say "k = "
@ row+4,20 say k picture "99.99"
@ row+4,30 say "qs = "
@ row+4,35 say qs picture "99.99"
@ row+4,44 say "A = "
@ row+4,49 say A picture "99.99"
@ row+4,58 say "*"
@ row+5,13 say "*"
@ row+5,58 say "*"
@ row+6,13 say "*****"
@ row+8,30 say "qm = "
@ row+8,34 say qsl picture "99.99"
RETURN
clear

```

* Program GEOT12.PRG

```
@ 0,0 clear
@ 2,20 say "ENTER NAME OF PROFILE: "GET prof picture "XXXX"
@ 5,20 say "Enter z(km) for 1st interval:"GET za picture "99"
@ 6,20 say "Enter z(km) for 2nd interval:"GET zb picture "99"
@ 10,15 say "Enter Tt:"GET Tt picture "999.99"
@ 10,40 say "Enter qs:"GET qs picture "99.99"
@ 12,15 say "Enter k:"GET k picture "99.99"
@ 12,40 say "Enter k1:"GET k1 picture "99.99"
@ 14,15 say "Enter A:"GET A picture "99.99"
@ 14,40 say "Enter A1:"GET A1 picture "99.99"
READ
set device to print
do geot12a
set device to screen
```

^Z

* Program GEOT12A.PRG

```

@ 0,0 clear
row = 6
@ 0,30 say "PROFILE: "
@ 0,39 say prof picture "XXXX"
@ 2,25 say "T = f(Tb, A, qs, k, z)"
@ 4,20 say "z(km)"
@ 4,40 say "T(K)"
store 0 to z
go top
do while z <= za
  z = z + 1
  T = Tt + (qs/k) * z - (A/(2*k)) * (z ** 2)
  row = row + 1
  @ row,21 say z picture "99"
  @ row,39 say T picture "9999.99"
  skip
enddo
qsl = ((Tt + (qs/k)*za - (A/(2*k))*(za**2)) - (Tt + (qs/k)*(za-1)
- (A/(2*k))*((za-1)**2)))*k
Tt1 = Tt + (qs/k)*za-(A/(2*k))*(za**2)
store 0 to z
go 2
do while z <= zb
  z = z + 1
  T = Tt1 + (qsl/k1)*z - (A1/(2*k1))*(z**2)
  row = row + 1
  @ row,21 say z picture "99"
  @ row,39 say T picture "9999.99"
  skip
enddo
qm = ((Tt1 + (qsl/k1)*zb - (A1/(2*k1))*(zb**2)) - (Tt1
+ (qsl/k1)*(zb-1) - (A1/(2*k1))*((zb-1)**2)))*k1
@ row +2,15 say "*****"
@ row +3,15 say "*"
@ row +3,50 say "*"
@ row+4,15 say "*"
@ row +4,27 say "qs = "
@ row +4,32 say qs picture "99.99"
@ row +4,50 say "*"
@ row +5,15 say "*"
@ row +5,50 say "*"
@ row +6,15 say "*"
@ row +6,20 say "k = "
@ row +6,24 say k picture "99.99"
@ row +6,36 say "k1 = "
@ row +6,41 say k1 picture "99.99"
@ row +6,50 say "*"
@ row +7,15 say "*"
@ row +7,50 say "*"
@ row +8,15 say "*"
@ row +8,20 say "A = "
@ row +8,24 say A picture "99.99"
@ row +8,36 say "A1 = "
@ row +8,41 say A1 picture "99.99"

```

```
@ row +8,50 say "*"
@ row +9,15 say "*"
@ row +9,50 say "*"
@ row +10,15 say "*****"
@ row+12,28 say "Tb ="
@ row+12,34 say Ttl picture "9999.99"
@ row+14,28 say "qsl ="
@ row+14,34 say qsl picture "99.99"
@ row+16,28 say "qm ="
@ row+16,34 say qm picture "99.99"
RETURN
clear
```

^Z

*** Program GEOT13.PRG**

```
@ 0,0 clear
@ 2,20 say "ENTER NAME OF PROFILE: "GET prof picture "XXXX"
@ 5,20 say "Enter z(km) for 1st interval:"GET za picture "99"
@ 6,20 say "Enter z(km) for 2nd interval:"GET zb picture "99"
@ 7,20 say "Enter z(km) for 3rd interval:"GET zc picture "99"
@ 10,15 say "Enter Tt:"GET Tt picture "999.99"
@ 10,40 say "Enter qs:"GET qs picture "99 99"
@ 12,5 say "Enter k:"GET k picture "99.99"
@ 12,30 say "Enter k1:"GET k1 picture "99.99"
@ 12,50 say "Enter k2:"GET k2 picture "99.99"
@ 14,5 say "Enter A:"GET A picture "99.99"
@ 14,30 say "Enter A1:"GET A1 picture "99.99"
@ 14,50 say "Enter A2:"GET A2 picture "99.99"
READ
set device to print
do geot13a
set device to screen
```

^Z

* Program GEOT13A.PRG

```

@ 0,0 clear
row = 6
@ 0,28 say "PROFILE: "
@ 0,37 say prof picture "XXXX"
@ 2,25 say "T = f(Tb, A, qs, k, z)"
@ 4,20 say "z(km)"
@ 4,40 say "T(K)"
store 0 to z
go top
do while z <= za
  z = z + 1
  T = Tt + (qs/k) * z - (A/(2*k)) * (z ** 2)
  row = row + 1
  @ row,21 say z picture "99"
  @ row,39 say T picture "9999.99"
  skip
enddo
qs1 = ((Tt + (qs/k)*za - (A/(2*k))*(za**2)) - (Tt + (qs/k)*(za-1)
- (A/(2*k))*((za-1)**2)))*k
Tt1 = Tt + (qs/k)*za-(A/(2*k))*(za**2)
store 0 to z
go 2
do while z <= zb
  z = z + 1
  T = Tt1 + (qs1/k1)*z - (A1/(2*k1))*(z**2)
  row = row + 1
  @ row,21 say z picture "99"
  @ row,39 say T picture "9999.99"
  skip
enddo
qs2 = ((Tt1+(qs1/k1)*zb - (A1/(2*k1))*(zb**2)) - (Tt1 +
(qs1/k1)*(zb-1)-(A1/(2*k1))*((zb-1)**2)))*k1
Tt2 = Tt1 + (qs1/k1)*zb-(A1/(2*k1))*(zb**2)
store 0 to z
go 2
do while z <= zc
  z = z + 1
  T = Tt2 + (qs2/k2)*z - (A2/(2*k2))*(z**2)
  row = row + 1
  @ row,21 say z picture "99"
  @ row,39 say T picture "9999.99"
  skip
enddo
qm = ((Tt2 + (qs2/k2)*zc - (A2/(2*k2))*(zc**2)) - (Tt2
+ (qs2/k2)*(zc-1) - (A2/(2*k2))*((zc-1)**2)))*k2
@ row +2,10 say "*****"
@ row +3,10 say "*"
@ row +3,55 say "*"
@ row+4,10 say "*"
@ row +4,27 say "qs = "
@ row +4,32 say qs picture "99.99"
@ row +4,55 say "*"
@ row +5,10 say "*"
@ row +5,55 say "*"

```

```

@ row +6,10 say "*"
@ row +6,13 say "k = "
@ row +6,17 say k picture "99.99"
@ row +6,27 say "k1 = "
@ row +6,32 say k1 picture "99.99"
@ row +6,41 say "k2 = "
@ row +6,46 say k2 picture "99.99"
@ row +6,55 say "*"
@ row +7,10 say "*"
@ row +7,55 say "*"
@ row +8,10 say "*"
@ row +8,13 say "A = "
@ row +8,17 say A picture "99.99"
@ row +8,27 say "A1 = "
@ row +8,32 say A1 picture "99.99"
@ row +8,41 say "A2 = "
@ row +8,46 say A2 picture "99.99"
@ row +8,55 say "*"
@ row +9,10 say "*"
@ row +9,55 say "*"
@ row +10,10 say "*****"
@ row+12,28 say "Tb ="
@ row+12,34 say Tt1 picture "9999.99"
@ row+14,28 say "Tb1 ="
@ row+14,34 say Tt2 picture "9999.99"
@ row+16,28 say "qs1 ="
@ row+16,34 say qs1 picture "99.99"
@ row+18,28 say "qs2 ="
@ row+18,34 say qs2 picture "99.99"
@ row+20,28 say "qm ="
@ row+20,34 say qm picture "99.99"
RETURN
clear

```

^Z

APPENDIX 5.2

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Program GHOT12A.PRG

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Appendix 5.3

An example of output from dBase III plus program (analytical solution of the heat conduction equation)

THE UNIVERSITY OF CHICAGO

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