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**SEMI-CYLINDRICAL SURFACE GRADIENT COILS.
A NOVEL APPROACH TO NMR GRADIENT COIL DESIGN**

by

Patrick L. Rapley, H.B.Sc., M.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Doctor of Philosophy

Department of Physics

Carleton University
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A NOVEL APPROACH TO NMR GRADIENT COIL DESIGN

submitted by

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in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

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Thesis Supervisor

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Abstract

A novel NMR gradient coil design has been introduced that provides relatively fast switching, strong gradient fields. The design, called Semi-Cylindrical Surface Gradient Coils (SCSGC), is based on a geometry with the current density confined to the surface of a half cylinder. A new method for the design of gradient coils has been developed as computer software. The numerical optimization algorithm used is a hybrid between the *downhill simplex* and *simulated annealing* techniques. The software optimizes gradient coils, and provides all necessary performance specifications.

A three direction SCSGC has been designed and constructed to fit inside of a 40 cm bore, 4.7T NMR system. The coil's performance has been measured and agrees with specifications calculated using the design software. The constructed x, y, and z gradient coils can attain gradient strengths of 6.1, 4.3, and 5.6 G/cm respectively at 150 amperes of current. In all cases, the effective ramp time to these values is less than 140 μ s. The gradient field is linear to within 5% (peak to peak) over cuboid volumes of greater than 3 cm $\times$ 3 cm $\times$ 3 cm while the usable volume for imaging is found to be greater than 6 cm $\times$ 6 cm $\times$ 10 cm for the x, y, and z directions respectively. The coil is found to be appropriate for Echo Planar Imaging, as well as diffusion studies.

A variation of the SCSGC providing a larger effective imaging volume (at the cost of reduced coil efficiency) has been investigated. The design incorporates movable current loops providing variability in the position of the imaging volume. A gradient coil set based on this modified SCSGC principle has been designed. The result is a gradient coil allowing human whole body access, yet capable of producing gradient fields appropriate for Echo Planar Imaging.

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This work is the result of six years of my life's attention. There are many people without whom this accomplishment would not have been possible. First I would like to thank JKS for his patience, support and encouragement. Thanks to Piotr and Andrej for many insightful discussions. I also acknowledge everyone at IBD for their assistance and fellowship. I offer special thanks to Slawek for easing the pain of the arduous task of coil wrapping.

I dedicate this thesis to Jane Ann, Mom and Dad.

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Chapter 1: General NMR theory

1.1 Introduction

Gradient coils are an essential part of a magnetic resonance imaging (MRI) system. The presence of a gradient in the z-component of the magnetic field while acquiring a nuclear magnetic resonance (NMR) signal provides spatial information about the spins producing the signal. In recent years, imaging methods have been developed that require gradient coil performance exceeding that provided by conventional designs. Specifically, many of the newer imaging methods require gradient coils capable of attaining a large gradient amplitude in a relatively short period of time.

This thesis is a study of gradient coils. A new method of gradient coil design is described, followed by the introduction of two novel gradient coil geometries.

Chapter 1 presents an overview of general NMR theory. It is intended to provide the reader with the necessary tools to understand the remainder of the thesis. Specifically, it is meant to provide an understanding of the link between gradient coils and the NMR signal.

The specifics of gradient coil theory are introduced in chapter 2. Conventional design methods and geometries are presented. The advantages and disadvantages of the various methods and designs are also discussed.

Chapter 3 describes a new method of gradient coil design developed for this work. The method introduced is an optimization technique called T-simplex, a hybrid of the downhill simplex method with the method of simulated annealing.

Although used in other disciplines, T-simplex has not been previously implemented in coil design for NMR.

This thesis introduces a new geometry (Semi-Cylindrical Surface Gradient Coil) of gradient coils which is shown to provide relatively fast switching, strong gradient coils. Chapter 4 discusses the general principles of the novel coil design. A prototype coil designed and built for this thesis is introduced and evaluated for performance.

Throughout this thesis, gradient coil design will be seen to be a compromise between several performance attributes. The Semi-Cylindrical Surface Gradient Coil is found to provide a relatively small usable volume for imaging compared to conventional full cylinder coil geometries. Chapters 5 and 6 study a variation on the Semi-Cylindrical Surface Gradient Coil featuring a movable imaging region, thus improving the shortcoming of small imaging volume.

1.2 Theory

1.2.1 Preliminary concepts

Nuclei have been shown to possess intrinsic spin and an associated magnetic moment¹. The nuclear spin is known to behave like angular momentum in the way it couples with a magnetic field². Designate the nuclear spin angular momentum as $\underline{S}$ and the nuclear magnetic moment as $\underline{\mu}$. The direction of the magnetic moment vector will precess when placed in an external magnetic field. It is this precession that will be seen to lead to the NMR signal.

Classically, the magnetic dipole moment ($\underline{m}$) of a spinning particle (velocity $\underline{v}$) of mass M and charge q is given by

$$\underline{m} = \frac{1}{2} \int_V \rho_c(\underline{r}) (\underline{r} \times \underline{v}) dV \quad , \quad (1.1)$$

while the angular momentum ($\underline{L}$) is given by

$$\underline{L} = \int_V \rho_m(\underline{r}) (\underline{r} \times \underline{v}) dV \quad . \quad (1.2)$$

Here, the integral is taken over the three dimensional volume enclosing the particle. Note that the 1/2 factor in equation (1.1) is not the result of classical theory, but is included to conform with quantum mechanics. If the charge density (ρ_c) and mass density (ρ_m) are constant over the particle (that is no $\underline{r}$ dependence), equations (1.1) and (1.2) can be simplified to

$$\underline{m} = \frac{q}{2} \underline{I} \quad (1.3)$$

and

$$\underline{L} = M \underline{I} \quad , \quad (1.4)$$

where the vector integral $\underline{I}$ is given by

$$\underline{I} = \int_V (\underline{r} \times \underline{v}) dV \quad . \quad (1.5)$$

Therefore, using equations (1.3) and (1.4) the magnetic moment is found to be related to the angular momentum by

$$\underline{m} = \Gamma \underline{L} \quad , \quad (1.6)$$

where the proportionality constant is given by

$$\Gamma = \frac{q}{2M} \quad . \quad (1.7)$$

In the case of the particle being a nucleon, equation (1.6) is written as

$$\underline{\mu} = \gamma \underline{S} \quad , \quad (1.8)$$

where $\underline{\mu}$ is the nuclear magnetic dipole moment, $\underline{S}$ is the spin angular momentum of the nucleon, and γ is the gyromagnetic ratio.

The equations describing the precession of the magnetic moment vector can be derived using figure 1.1. A magnetic dipole in an external magnetic field $\underline{B}$ will feel a torque $\underline{N}$ given by

$$\underline{N} = \underline{\mu} \times \underline{B} = \frac{\partial \underline{S}}{\partial t} \quad . \quad (1.9)$$

Figure 1.1 Illustration of precession of magnetic moment vector.

Figure 1.2 Parameter definition for Faraday's law.

Therefore, the time derivative of the angular momentum is given by

$$\frac{\partial \underline{S}}{\partial t} = \underline{S} \times \gamma \underline{B} = -SB\gamma \sin\theta \hat{\Phi} \quad , \quad (1.10)$$

where the direction was determined from figure 1.1 using the right hand rule for the vector cross product. In general, the time derivative of any vector $\underline{S}$ can be written as

$$\frac{\partial \underline{S}}{\partial t} = \underline{S}' = S' \hat{r} + S\Phi' \sin\theta \hat{\Phi} + S\theta' \hat{\theta} \quad (1.11)$$

in terms of spherical coordinates. In the case of precession, θ and S are constant in time, and therefore the only non-zero term in equation (1.11) is the Φ component. That is equation (1.11) is reduced to

$$\frac{\partial \underline{S}}{\partial t} = S\Phi' \sin\theta \hat{\Phi} \quad . \quad (1.12)$$

Referring to figure 1.1, it is clear that the time derivative of the angle Φ is the magnitude of angular frequency of the precession of the vector $\underline{S}$. This can be written as

$$\omega = \frac{\partial \Phi}{\partial t} = \Phi' \quad . \quad (1.13)$$

Thus equations (1.10) and (1.12) can be equated to determine the relationship between the precessional angular frequency (ω) and the magnetic field strength ($\underline{B}$):

$$\underline{\omega} = -\gamma \underline{B} \quad . \quad (1.14)$$

Note that the direction of ω is opposite that of the magnetic field ($\underline{B}$). The frequency of precession is called the *Larmor* precessional frequency, and is given by

$$f = -\frac{\gamma}{2\pi} B \quad . \quad (1.15)$$

Equation (1.15) is of fundamental importance in NMR, defining the precession frequency of the various nuclear spins in a magnetic field. To determine the gyromagnetic ratio (γ), in general a complicated quantum electrodynamics problem must be solved. However γ can be determined to a first approximation by equation (1.7), if the nucleon is taken to be a simple sphere of homogeneous charge and mass. In such a case, γ is found to be proportional to the nucleon charge, and inversely proportional to the mass. For example, the gyromagnetic ratio for the proton (^1H) is

$$\frac{\gamma}{2\pi} = 42.57 \frac{\text{MHz}}{\text{T}} \quad (1.16)$$

while for phosphorus ^{31}P it is

$$\frac{\gamma}{2\pi} = 17.23 \frac{\text{MHz}}{\text{T}} \quad (1.17)$$

It is easy to show both classically³ and quantum mechanically^{12,45} that the energy of a nucleon with spin in the presence of an external magnetic field ($\underline{B}$) is greater when anti-aligned with the field than when aligned. Define the aligned and anti-alignment spin states as $1/2$ and $-1/2$ (or spin-up $\uparrow$, spin-down $\downarrow$) respectively, with corresponding energy values of

$$\begin{aligned} E_{+\frac{1}{2}} &= -\hbar\gamma B_0/2 \\ E_{-\frac{1}{2}} &= \hbar\gamma B_0/2 \end{aligned} \quad (1.18)$$

where $\hbar$ is Planck's constant. In the absence of an external magnetic field, the spin-up and spin-down states have the same energy (i.e. they are degenerate), and at thermal equilibrium, the two states are equally populated. To understand the magnetic properties of bulk matter, consider matter as an assemblage of magnetic dipoles.

Magnetization $\underline{M}$ is defined as the magnetic dipole moment per unit volume of matter, and is a macroscopic characteristic of magnetic properties of an object. It can be shown that in the presence of an external magnetic field, two opposing energy processes determine the degree of magnetization. Firstly, thermal energy causes the spins to tumble under the influence of Brownian motion⁶, tending to distribute the spins into random orientation (i.e. no net magnetism). Secondly, the presence of an external $\underline{B}$ field tends to align a fraction of the spins with the field. The second process is a consequence of the difference in energy between the spin $\uparrow$ and spin $\downarrow$ states shown in equation (1.18). The natural tendency is toward the state of lowest energy, which is achieved when some (albeit a very small) degree of alignment is present. The resultant process due to the two competing mechanisms is known as the *spin-lattice* interaction. When an external $\underline{B}$ field is applied, the spin-lattice interactions cause an increase in $\underline{M}$ from zero to an equilibrium value ($\underline{M}_0$) determined by statistical thermodynamics⁷. The magnetization increases according to

$$\underline{M}(t) = \underline{M}_0 (1 - e^{-\frac{t}{T_1}}) \quad , \quad (1.19)$$

where T_1 is the so-called longitudinal relaxation time. From equation (1.19) T_1 is seen to represent the time taken to achieve a magnetization of $0.63M_0$. For high water content samples, the value of T_1 typically ranges from $0.1 \rightarrow 10$ s. To determine the degree of alignment (or M_0) consider the following results from statistical thermodynamics. Define the density of nuclei in the spin up and spin down states as $n_\uparrow$ and $n_\downarrow$ respectively.

In a spin system the ratio between the number of $\uparrow$ and $\downarrow$ is found to be given by

$$\frac{n_{\downarrow}}{n_{\uparrow}} = e^{\frac{-\gamma \hbar B_0}{kT}} \quad (1.20)$$

where k is Boltzmann's constant, and T is the temperature of the system. The ratio has a value of 1 at $B_0=0$ and approaches 0 as $B_0 \rightarrow \infty$. The difference in spin population is derived from equation (1.20) to be

$$n_{\uparrow} - n_{\downarrow} = n \tanh\left(\frac{\gamma \hbar B_0}{2kT}\right) \quad (1.21)$$

where $n=n_{\uparrow}+n_{\downarrow}$ is the total number of spins. An example shows the poor sensitivity associated with NMR. For $T = 310$ K (body temperature), and a magnet field strength of 15 Tesla ($\sim$ largest field used for NMR), the fraction of excess spins aligned with the field is (using equation (1.21)) found to be

$$\frac{(n_{\uparrow} - n_{\downarrow})}{n} = 4.9 \times 10^{-7} \quad (1.22)$$

Note that for most cases in NMR, $\gamma \hbar B_0 \ll kT$, so the tanh function of equation (1.21) can be replaced by its argument, and the fraction of excess spin-up nuclei can be written as

$$\frac{n_{\uparrow} - n_{\downarrow}}{n} = \frac{\gamma \hbar B_0}{2kT} \quad (1.23)$$

Now, by solving the eigenvalue problem for the Q.M. spin angular momentum operator analogous to $\underline{L}$ of classical mechanics in equation (1.8), the magnetic dipole moment for a single spin can be found to be

$$\mu_{\text{single spin}}^{\uparrow} = \frac{1}{2} \gamma \hbar \quad (1.24)$$

for the spin-up state,

and

$$\mu_{\text{single spin}}^{\downarrow} = -\frac{1}{2}\gamma\hbar \quad (1.25)$$

for the spin-down state. Therefore the bulk nuclear magnetization is given by

$$\begin{aligned} M_0 &= (n_{\uparrow} - n_{\downarrow})\frac{1}{2}\gamma\hbar \\ &= \gamma^2 \hbar^2 B_0 \frac{n}{4kT} \end{aligned} \quad (1.26)$$

where equation (1.23) is used to get the second approximation. Clearly for cases where the condition $\gamma\hbar B_0 \ll kT$ is satisfied, the magnetization is proportional to the magnetic field. The materials studied in NMR tend to be of the *Linear Isotropic Homogeneous (LIH)* type whose constituents are classified by a magnetization that is linearly related to the applied external field as in equation (1.26). For LIH materials, the relationship between magnetization and applied magnetic field can be generalized as

$$\underline{M} = \frac{\chi_m}{(1 + \chi_m)} \underline{B} \quad (1.27)$$

where $\underline{B}$ is the external field, and χ_m is the magnetic susceptibility which is a characteristic of the material.

1.2.2 Excitation and detection

In order to understand the origin of the NMR signal, some further electromagnetic theory is needed. One of the fundamental features of electromagnetism is the connection between electric fields and magnetic fields. In one direction, Ampere's law³ and the Biot-Savart Law (discussed in detail in chapter 2) tell us that electric current produced by an

electric field induces a magnetic field. Inversely, Faraday's law of induction tells us that a magnetic field that changes in time induces an electric field (and therefore potential for current). In integral form, Faraday's law is written

$$\oint_C \underline{E} \cdot d\underline{s} = - \frac{d}{dt} \int_S \underline{B} \cdot d\underline{a} \quad (1.28)$$

Figure 1.2 illustrates the use of this equation. The area S is any surface enclosed by the curve C . The equation specifies that if there is a change (in time) in the magnetic flux through the surface S , an electric field will be produced along the path C . If C is a loop of wire, any change (in time) in either the $\underline{B}$ field (through S) or the area of S will cause an induced voltage in the wire. As a result, current will flow if the wire is a complete circuit.

B_0 field and the equilibrium position

The NMR experiment begins with a strong magnetic field $\underline{B}_0$ applied in the z direction, causing a small degree of magnetization $\underline{M}_0$ (as discussed in section 1.2.1: equation (1.26)) in the z direction. In NMR, the $\underline{B}_0$ field, or *main* field, is always present, and should be as spatially homogeneous as possible. The reason for this will be described shortly. Figure 1.3a illustrates this equilibrium (relaxed) state of the spin system.

RF pulse (excitation)

The excitation consists of temporarily applying a second magnetic field ($\underline{B}_1$), known as the RF or radio frequency pulse, that causes the spin to rotate from the z -axis onto the xy (transverse)-plane (figure 1.3b). The RF pulse requires application of a magnetic field $\underline{B}_1$ that is perpendicular to the z -axis. The B_1 field is generated by an RF-coil which must be designed to provide the required nuclear magnetic dipole nutation.

Figure 1.3 Schematic of spin dynamics for NMR.

The simplest example to demonstrate an RF pulse is shown in figure 1.3b. The insert figure shows an RF-coil system consisting of two orthogonal loops of wire[†]. Consider a sinusoidally varying current applied to both loops, with a phase offset of 90° between them. With the Biot Savart law it is possible to show that the magnetic field ($\underline{B}_1$) at the coordinate system origin will be given by

$$\begin{aligned} B_{1x}(t) &= B_1 \cos(\omega_0 t) \\ B_{1y}(t) &= B_1 \sin(\omega_0 t) \end{aligned} \quad (1.29)$$

This is equivalent to the equation representing a vector undergoing circular motion with angular frequency ω_0 . To see the effect of such an RF pulse on a nucleus with its dipole moment initially aligned in the z direction, consider a reference frame ($X_{rot}, Y_{rot}, Z_{rot}$) that is rotating about the z axis at the Larmor frequency (ω_0). Now consider a nucleus with a magnetic dipole moment that is not aligned with $\underline{B}_0$. In the laboratory frame of reference, this nucleus will precess about $\underline{B}_0$ at the Larmor frequency. In the rotating frame, however, this spin is observed to be stationary. Since the dipole moment is not precessing, the $\underline{B}_0$ field must be zero strength in this rotating frame. Therefore the only magnetic field present in the rotating frame is $\underline{B}_1$, which has a constant magnitude. The net result is that the magnetization will precess about $\underline{B}_1$ (x-axis) with frequency $\omega_1 = -\gamma B_1$. The angle of rotation (or the pulse angle) of the net magnetization is then given by

$$\theta_{pulse} = \omega_1 t_{pulse} \quad (1.30)$$

where t_{pulse} is the length of the RF pulse.

[†] Two orthogonal loops have been chosen to avoid complex analysis in this discussion. In practical terms, the simplest RF-coil configuration is a single loop with axis perpendicular to the xy-plane.

So, for the simple NMR sequence shown in figure 1.3 the RF power (which determines B_1) and t_{pulse} are chosen so that the angle of nutation is $\theta_{\text{pulse}} = \pi/2$ radians.

Detection

With the net spin magnetization aligned with the xy-plane, and the RF power off, the spins precess about the B_0 field, and can be detected by a single loop coil as shown in figure 1.3c. The figure shows the magnetic field and spin magnetization configuration for the detection phase of the basic NMR sequence. The detection is made possible via Faraday's law of induction. In the case of a single closed loop of wire, Faraday's law can be simplified to

$$V_{\text{ind}} = - \frac{d\Phi}{dt} \quad , \quad (1.31)$$

where V_{ind} is the induced voltage, and Φ is the magnetic flux given by (see figure 1.4)

$$\Phi \propto \mathbf{M}_{\text{tran}} \cdot \mathbf{A} \cdot \cos(\theta) \quad . \quad (1.32)$$

Here, A is the area of the wire loop, and $\mathbf{M}_{\text{tran}}$ is the transverse component of the magnetization. θ is the angle between $\mathbf{M}_{\text{tran}}$ and the loop axis. Immediately after the RF pulse $\mathbf{M}_{\text{tran}} = \mathbf{M}_0$, but $\mathbf{M}_{\text{tran}}$ decays to zero due to T_2 relaxation. Because of precession, $\theta = \pi/2 - \omega_0 t$, and the induced voltage can be written, using equation (1.31) and (1.32), as

$$V_{\text{ind}} \propto \omega_0 \mathbf{M}_{\text{tran}} \cdot \mathbf{A} \cdot \cos(\omega_0 t) \quad . \quad (1.33)$$

The RF detection circuit must be designed with a bandwidth large enough so that it can detect the frequency offsets of all the nuclear spins present. The RF voltage across the tuned circuit is sensed by a high impedance, low noise pre-amplifier and sent to the console where it is further amplified, de-modulated, and processed.

Figure 1.4 Illustration of parameters for signal detection example (single loop coil).

Figure 1.5 Example of T_2 relaxation.

The details of tuned circuits for NMR detection are beyond the scope of this thesis and will not be discussed further here.

Free induction decay (signal)

To understand the origin of the NMR signal, one must consider the distribution in precessional frequency of nuclear spins making up the sample. Because of the additive nature of B fields, the total signal is made up of the superposition of the signal from each nuclear spin. Inhomogeneity in the B_0 field as well as chemical shift (discussed in the spectroscopy section) leads to a spread of Larmor frequencies throughout the sample. To account for the distribution of frequencies, the concept of isochromats is introduced. In the discrete isochromat model, nuclear spins are grouped in bins according to their frequency. For example, assume that there is a total of N nuclear spins in the sample. Also assume that the spread of spin frequency (ω) is such that

$$\omega_1 \leq \omega \leq \omega_2 \quad . \quad (1.34)$$

The frequency distribution is then segregated into m discrete bins or isochromats labelled with the index i where $i=0,1,2,\dots,m-1$. Define the number of spins in and the frequency of the i th isochromat as n_i and ω_i respectively. The following relationships define the system:

$$N = \sum_{i=0}^{m-1} n_i \quad (1.35)$$

$$\omega = \omega_i + i\Delta\omega \quad . \quad (1.36)$$

where

$$\Delta\omega = \frac{\omega_2 - \omega_1}{m-1} \quad (1.37)$$

Equation (1.26) determines the equilibrium magnetization due to the i th isochromat to be

$$M_{0,i} = n_i \cdot \frac{\gamma^2 \hbar^2 B_0}{4kT} \quad (1.38)$$

The NMR signal known as the free induction decay (FID) is made from a digitized sampling of the voltage induced in the receiver coil. Assume an ideal RF pulse that uniformly rotates all spin isochromats 90° to flip them onto the transverse plane. From equation (1.33) the signal from the i th isochromat is found to be

$$S_i(t) \propto M_{\text{trans},i} \cdot \sin(\omega_i t) \quad (1.39)$$

and the FID signal due to the complete (all isochromats) sample is simply

$$S(t) = \sum_{i=0}^{m-1} S_i(t) \quad (1.40)$$

At the end of the ideal RF pulse $M_{\text{trans},i} = M_{0,i}$. Gradually, spin-lattice relaxation returns the spins from the transverse plane back to the z -aligned state. The z component of M grows exponentially with time as in equation (1.19). It is easy to show that if the magnitude of M is assumed constant, the transverse component decays exponentially as

$$M_{\text{trans},i} = M_{0,i} e^{-\frac{t}{T_1}} \quad (\text{spin-lattice relaxation only}) \quad (1.41)$$

In reality, the magnitude of M is found to decay as a result of a second natural relaxation process designated as spin-spin or transverse relaxation.

The thermal tumbling that provides the mechanism for longitudinal relaxation leads to local fluctuations in the $\underline{B}$ field. On average, the fluctuations will produce a component of $\underline{B}$ in the transverse as well as longitudinal direction. The fluctuating field produced by a given spin will interact with other spins in its vicinity. Both the longitudinal and transverse components of the randomly fluctuating fields (in space and time) lead to a spatial randomizing of the phase and amplitude of the transverse magnetization. The resultant reduction in transverse magnetization is called the natural spin-spin relaxation. For example, assume all the spins to be aligned (same phase) in the xy-plane immediately following the RF pulse. Figure 1.5 shows the spin orientation at an arbitrary time following the RF pulse. In the figure, each arrow represents the magnetic moment of a single nucleus in a fictional seven spin system. In this simplified description, each nucleus in the sample sees a slightly different (randomly chosen) effective $\underline{B}$ in which to precess. At time $t=0$, all the nuclei are aligned with the y-axis. If spin-spin relaxation interactions are not present, all nuclei will precess to the same angle as shown in the figure. In the presence of spin-spin effects, the nuclear magnetic moments (precessing at slightly different, randomly distributed frequencies) will spread out in the x-y plane. The effect of spin-spin interactions is to cause a dephasing (or fanning out) of the transverse magnetization. The dephasing is found to lead to exponential (to first approximations) decay in the transverse magnetization of the form

$$M_{\text{trans},t} = M_{0,t} e^{-\frac{t}{T_2}} \quad (\text{spin-spin relaxation only}) \quad (1.42)$$

T_2 is the natural spin-spin relaxation time constant, and is a property of the NMR sample.

Additional transverse dephasing occurs due to B_0 inhomogeneity and magnetic susceptibility effects. The relaxation time constants are specified as T_{2m} and T_{2ms} for the magnet inhomogeneity and magnetic susceptibility respectively. The resulting effective transverse relaxation time constant T_2 is given by

$$\frac{1}{T_2} = \frac{1}{T_{2n}} + \frac{1}{T_{2m}} + \frac{1}{T_{2ms}} \quad (1.43)$$

The net result of the combined relaxation mechanisms is to cause the transverse magnetization to decay as

$$M_{\text{trans}} = M_{0,i} e^{-\frac{t}{T_2}} e^{-\frac{t}{T_1}} \quad (1.44)$$

Thus the signal including relaxation can be written as

$$S(t) \propto \sum_{i=0}^{m-1} n_i e^{-\frac{t}{T_2}} e^{-\frac{t}{T_1}} \cdot \sin(\omega_i t) \quad (1.45)$$

where equation (1.38) has been used to replace $M_{0,i}$ with n_i . The general form of the FID given by equation (1.45) is illustrated in figure 1.6. The signal shown is made up of an arbitrary six isochromats to generalize the example.

Note that one further cause of signal decay is diffusion. The effect of diffusion on the signal will be discussed in the image contrast section of 1.2.5.

Phase sensitive quadrature detection (discussed in section 1.2.6) is usually used in NMR, and the resulting signal is complex having the form

$$S(t) \propto \int n(\omega) e^{-i2\pi\omega t} d\omega \quad (1.46)$$

where i is the square root of -1.

Figure 1.6 Schematic example of free induction decay.

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Figure 1.7 Schematic example of time-frequency space correspondence (FT).

Here, relaxation has been neglected, and the signal is taken as non-discrete as indicated by the integral replacing the summation of equation (1.45).

1.2.3 Frequency space analysis

If, during the signal acquisition period, the sample contains spins with a distribution of frequencies, the FID will contain a superposition of sinusoidal signals as seen in equation (1.45) or (1.46). The frequency distribution $n(\omega)$ can be evaluated by applying the mathematical tool known as the Fourier Transform (FT). Ignoring the exponential relaxation factors, the discrete FT of $S(t)$ in equation (1.45) will determine the $i=0,1,2,\dots,m-1$ values of n_i . If n_i is plotted as a function of i (or ω_i), we obtain a spectrum of the number of spins at each frequency between ω_1 and ω_2 . Note that the n_i values are not absolute measures of the number of spins, but are scaled relative to the maximum n_i value. For an example of the frequency space analysis, consider a spin system that contains only two different isochromats ($m=2$). Assume that $n_1=2n_2$, and that $\omega_1=(1/2)\omega_2$. Neglecting relaxation, and assuming an infinite signal acquisition time, the signal from such a system is illustrated in figure 1.7a. If this signal undergoes a FT, the resulting spectrum will have the form of figure 1.7b. The effect of relaxation in the time domain is that the signal amplitude is multiplied by an exponentially decreasing value. Without going into the mathematical details, the effect of this relaxation in frequency space is a broadening of the spectral width. An example of line broadening is shown in figure 1.8, where the spectrum of the isochromat system in figure 1.7 is shown under the influence of the natural spin-spin interactions. The magnitude of line broadening ($\Delta\omega_{1/2}$) can be measured as the width of the peak at 1/2 the maximum peak amplitude.

Figure 1.8 Broadening effect of T_2 relaxation on spectral lines. Figure 1.9 Line broadening due to B_z gradient.

As can be seen, the spectral width is related to the spin-spin relaxation time constant as

$$\Delta\omega_1 \propto \frac{1}{T_{2n}}. \quad (1.47)$$

The other relaxation mechanisms of equation (1.43) will contribute cumulatively to the broadening effect. An additional spectral line broadening results from the finite length of the signal acquisition time. However, the acquisition time is usually long compared to T_2 and T_1 thus rendering its broadening effect negligible compared with relaxation.

The above time $\Leftrightarrow$ frequency correspondence provided by the Fourier Transform will be seen to be of paramount importance in both NMR spectroscopy and imaging.

1.2.4 Spectroscopy

Although the focus of this thesis is on imaging, spectroscopic methods were used to map the B_z fields produced by the gradient coils. Therefore, a brief introduction of spectroscopy is in order.

Orbital electrons produce small perturbations in the magnetic field at the site of the nuclear spins. The magnitude and direction of the perturbations are dependent on the specific electronic configuration in the vicinity of the nucleus. For example, protons in pure water experience a different effective magnetic field than do protons in the biologically prevalent CH_2 (chemical formula for two hydrogen atoms bound to one carbon atom). The spins at different effective B_z fields will precess at different Larmor frequencies, and can thus be distinguished from one another in frequency space (spectrum). This so-called *chemical shift* in frequency enables the determination of relative quantities of spins in the various chemical environments. For a detailed analysis of NMR spectroscopy, the reader is referred to Slichter⁵ and Abragam⁶.

In this thesis, ^1H spectroscopy is performed on a small homogeneous water sample in order to measure the magnetic field produced by a gradient coil. Assuming the sample to be pure water, there should be no chemical shift differentiation present. Therefore, in a homogeneous B_z field (i.e. no gradients), the spectrum should consist of a single peak with a line-width determined only by the relaxation line-broadening effects. However, the presence of field gradients (due to B_0 inhomogeneity, and/or the applied gradient) over the dimensions of the sample, produces a frequency spread resulting in further broadening of the spectral line. Figure 1.9 shows an example of the broadening effect for a spherical sample in the presence of a linear gradient in the x direction. Notice that the line-width is proportional to both the gradient strength and the physical width of the sample. The position of the centre of the spectral peak is determined by the strength of B_z relative to the resonance frequency of the RF coil (i.e. the spectrometer frequency).

1.2.5 Imaging

The goal of imaging is to obtain visually discernible contrast due to differences in the physical qualities measured. In the simplest NMR imaging, contrast in the density of protons provides a map of water concentration. For example, white matter in the brain is ~84% water by weight, while the grey matter is ~70% water by weight⁹. Therefore, a method that can provide spatial separation of spins at different positions, as well as sufficient intensity contrast between the water concentrations of interest is required.

As previously discussed, if the B_z field varies in space, the angular precession frequency will differ for spins at different B_z values. In NMR imaging, a strong gradient (of known $B_z(x,y,z)$ distribution) is applied that causes spins at different locations to precess at different frequencies. A FT of the signal obtained in the presence of such a gradient field will determine the density of spins precessing at each of the frequencies present in the sample. Knowing the $B_z(x,y,z)$ distribution allows a mapping of precession frequency into a spatial position. The result is a mapping of spin density as a function of spatial position. The most practical choice of gradient field is a linear one specified by

$$B_z(x,y,z) = B_0 + G_x x \quad (1.48)$$

in the case where spatial separation in the x direction is required. In the case of a linear gradient, the gradient strength (G_x) is constant over the imaging region. B_z field gradients are discussed in detail in chapter 2. For now assume perfectly linear gradients as defined by equation (1.48). When a linear x -gradient is applied to the object in figure 1.10a the resulting spectrum is shown in figure 1.10b. For a given x value (say x_1) spins at all values of y precess at the same frequency (ω_1). The resulting spectral amplitude $n(\omega_1)$ is the sum of all the spin densities at positions $x=x_1$, $y_{min} \leq y \leq y_{max}$. This is called the projection of $n(\omega_1)$ along the y direction. In the case of continuous time and frequency space, the density at ω_1 is given by the line integral

$$n(\omega_1) = \int_{y_{min}}^{y_{max}} n(x_1, y) dy \quad (1.49)$$

In order to also get spatial separation in the y direction (i.e. 2-dimensional image), a second orthogonal gradient (y -direction) must be applied subsequently.

Figure 1.10 Effect of linear gradient on an annular 2 dimensional sample (from Morris 1986).

Figure 1.11 Illustration of Aliasing.

In order to describe 2-dimensional (2-D) imaging, some MR imaging parameters need to be defined. The FID signal is stored as a discrete sampling of the continuous current induced in the RF receiver coil. Define the time between sampled points as the dwell time (DT). A FT of a discrete signal will result in a discrete spectrum. If there are N sample points in the time domain, there will be N sample points in the frequency domain. A 2-D image is made up of a discrete sampling of the NMR signal obtained during specific FT imaging sequences. Define the number of sample points in the x and y directions as NP_x and NP_y respectively ($N = NP_x \times NP_y$). Also, define the range in the x and y direction to be covered in the image as FOV_x (field of view) and FOV_y respectively. Sampling theory dictates that the FT of a discretely sampled signal will give frequency information over a finite band of frequencies. Define the sweep-width (SW) as the range of the frequency space that is covered by the FT. SW is related to DT by

$$SW = \frac{1}{DT} \quad , \quad (1.50)$$

so that shorter DT leads to greater SW. If the FID contains spins precessing at frequencies outside of the SW, *aliasing* will occur. Aliasing is the folding back (see figure 1.11) of all $n(\omega)$ for

$$|\omega| \geq \frac{SW}{2} \quad . \quad (1.51)$$

Aliasing causes artifacts in the image, and should be minimized by choosing appropriate DT, gradient strengths and low pass filter width for the specified FOV. The choice of DT and gradient strength is discussed in section 2.2.2, while the effect of a low pass filter is discussed in the MRI hardware section of this chapter.

As mentioned, an MR image is made up from a discrete sampling of the frequency space spectrum acquired using various sequences of RF pulses and gradients. The so-called *k-space* formalism^{10,11,12,13} has been introduced to facilitate the description of these pulse and gradient imaging sequences. The *k-space* model will now be described in one dimension (*x* direction). In the presence of a gradient field, the frequency of spin precession is a function of position. For a linear gradient in the *x* direction the functional correspondence ($\omega \Leftrightarrow x$) can be written

$$\omega(x,t) = -\gamma G_x(t)x \quad , \quad (1.52)$$

where G_x is constant in space. Note that the time-and-space invariant quantity γB_0 has been dropped from the expression. In NMR, the detection sequence includes the subtraction of a Larmor frequency alternating current from the received signal. This effectively transforms the signal into the rotating reference frame previously introduced, and removes the γB_0 term as in equation (1.52). According to equation (1.52), the phase of the complex exponential at time *t* and frequency ω (or *x*) will be given as

$$\phi(x,t) = -\gamma x \int_0^t G_x(t') dt' \quad , \quad (1.53)$$

and using equation (1.46) the signal can be written as

$$S(t) \propto \int_{\text{sample}} n(x) e^{-i\phi(x,t)} dx \quad . \quad (1.54)$$

Since the phase is linearly dependent on *x*, the maximum and minimum phase accumulation will correspond to the maximum and minimum values of *x* ($x_{\max}$ and $x_{\min}$) in the FOV_{*x*}:

$$x_{\max}^{\min} = \mp \frac{\text{FOV}_x}{2} \quad . \quad (1.55)$$

Define the x component of the k-space vector corresponding to an x direction gradient as

$$k_x(t) = \gamma \int_0^t G_x(t') dt' \quad , \quad (1.56)$$

where the x subscript on k denotes the k-space direction, not an x dependency. Notice that k is directly related to the time dependent gradient strength. Now, to prevent aliasing, the maximum and minimum phase accumulated by a spin between any two sample points must be π and $-\pi$ respectively. Therefore

$$k_x(DT)x_{\max} = \pi \quad , \quad (1.57)$$

or

$$k_x(DT) = \frac{2\pi}{FOV_x} \quad . \quad (1.58)$$

The k-space value at each point along the discrete acquisition is given by

$$k_x(i) = i \cdot \frac{2\pi}{FOV_x} \quad i=1,2,\dots, NP_x \quad . \quad (1.59)$$

Remembering that i represents increments in time, equation (1.59) determines the k-space trajectory (in time) that must be taken in order to satisfy the imaging conditions. This k-space path is shown for a 1 dimensional (k_x) image in figure 1.12. Clearly, to encode frequency information over a certain range (i.e. FOV) a corresponding range of k-space must be covered. The 2 dimensional case is a direct analogy. To obtain a 2 dimensional image with FOV_x and FOV_y , the corresponding range in 2 dimensional k-space (k_x and k_y) must be covered (see figure 1.13). A gradient sequence must therefore be chosen such that the required k-space coverage (specified by FOV_x, FOV_y) is provided. One possible trajectory in k-space is shown in figure 1.13.

Figure 1.12 One-dimensional example of k-space.

Figure 1.13 K-space trajectory for spin echo imaging sequence

This trajectory can be implemented using the *spin-echo* imaging sequence whose pulse and gradient sequence is shown in figure 1.14. Assume the slice, phase, and read gradients of figure 1.14 to be in the z , x , and y directions respectively. Notice that the 90 degree pulse is applied in the presence of a z -gradient. Ideally, this selective pulse excites spins uniformly over a defined band of frequency. In the presence of a z -gradient, the band of frequency corresponds to a band of z positions. Therefore, the combination of selective pulse and gradient is to excite spins only in a selected slice in the z direction. The position and thickness of the slice are controlled by the frequency offset and shape of the rf-pulse shape and gradient amplitude respectively. Although slice selection is very important in imaging, it is not the focus of this thesis, and thus will not be discussed further. The reader is referred to references 14, 15, and 16 for in-depth coverage of RF selective excitation. The 180 degree pulse applied at time $TE/2$ reverses the transverse dephasing (except for the dephasing due to the natural spin-spin interactions), so that the effects of T_{2m} and T_{2ms} (see equation (1.43)) are eliminated at the centre of the read gradient. This refocusing π pulse produces a so-called echo of the FID that has long since decayed to minimal amplitude following the $\pi/2$ pulse. The negative lobe on the slice select and read gradients acts to refocus the loss of phase coherence due to the gradients (equation (1.53)). In the case of the slice gradient, the accumulated phase (due to slice gradient) should be zero after the gradients. The negative lobe in the read gradient leads to zero accumulated phase (due to read gradient) at the centre of the acquisition. Assume for the moment the phase and read directions are x and y respectively.

Figure 1.14 Pulse sequence for spin-echo imaging.

Neglecting the effect of the phase gradient, one step through the sequence in figure 1.14 corresponds to one of the horizontal sweeps in k_x direction of figure 1.13. The effect of a phase encode gradient (x direction) is to move the position of the k_x sweep up or down in the k_x direction. In order to cover the required 2-dimensional region of phase space, the whole sequence in figure 1.14 must be repeated NP_x times with different values of phase gradient for each phase encoding step. The phase encoding gradient in figure 1.14 is shown to be ramped from a negative value to a positive value. As an illustration consider the first step through the cycle, where the phase gradient is at its minimum value. Just after the slice selective $\pi/2$ pulse the position in k-space is at the origin (position A on figure 1.14 and 1.13). The negative phase gradient in the x direction causes a move in the negative k_x direction to point B. The negative lobe of the read (y) gradient produces a move to C. At this point, the digitizer is activated, and the acquisition begins. The positive read gradient sweeps k-space along the k_y direction from C to D to E.

Image contrast

Thus far the contrast in an MR image has been seen to come from differences in densities of nuclear spins. There are however other contrast mechanisms available. Important to this thesis are T_2 , T_1 , and diffusion contrast methods. Other MRI contrast methods including Magnetization Transfer^{17,18}, flow^{19,20} and contrast enhancing agents² will not be discussed here. The relative contribution to the overall contrast of each mechanism is dependent on the imaging sequence used.

As has been shown, spin-spin and spin-lattice relaxation cause an exponentially decreasing amplitude of the NMR signal. Therefore, equal densities of spin nuclei that have different T_1 or T_2 will produce different signal intensities. The difference will increase with time according to equation (1.45). For example, consider the case where two groups of nuclear spins (labelled as spin groups 1 and 2) have spin-spin relaxation constants $T_{2n,1}$ and $T_{2n,2}$ respectively. At time t_1 after the $\pi/2$ RF pulse the ratio in signal amplitude between contributions from group 1 and 2 (assume equal spin density and T_1) will be

$$\frac{A_1(t_1)}{A_2(t_1)} = e^{t_1 \left(\frac{T_{2n,1} - T_{2n,2}}{T_{2n,1} T_{2n,2}} \right)}, \quad (1.60)$$

where A_1 and A_2 are the signal amplitudes.

Appropriate choices of imaging sequence and acquisition parameters can produce signals with contrast due to spin-spin and/or spin-lattice relaxation effects. Note that due to the random nature of the natural T_1 and T_2 processes, their contrast effects will always be present to some degree in an image. The imaging sequence can only affect the degree to which T_1 and T_2 are manifested relative to the other contrast mechanisms. A good overview of the relaxation contrast in imaging is given by Morris².

The effects of diffusion on the NMR signal were presented in 1954 by Carr and Purcell²¹. Simply stated, in the presence of any B_z field inhomogeneity, the signal will be attenuated (beyond what is predicted by T_1 and T_2) due to the random diffusion process. Because a given spin is always in random motion, its position (and thus its Larmor frequency due to a field gradient) varies in time.

The net result is that each different spin (even if in the same isochromat) will acquire a different phase than its neighbouring spins. The phase distribution will be random due to the random nature of the motion. This dephasing due to diffusion causes an attenuation in signal (from an elemental volume) amplitude given by²²

$$\frac{A(\underline{r},t)}{A(\underline{r},0)} = e^{-\gamma^2 D \omega \int_0^t \left[\int_0^{t'} G(\omega, \tau) d\tau \right]^2 dt'} \quad (1.61)$$

$D(\underline{r})$ is the diffusion coefficient and is dependent on the material. Notice that the diffusion attenuation effect is dependent on the time integral of the square of the k-space trajectory given by equation (1.56). Therefore, larger gradients cause greater diffusion effects.

1.2.6 NMR hardware

As should already be clear, three types of magnetic field generating coils are used in MR imaging. Ideally, the B_0 coil produces a time invariant, homogeneous field over the sample region. The RF-coil produces excitation of spin nuclei. The gradient coils produce spatial variations in the B_z field and can be used to encode the signal. These magnetic field producing coils are only a part (albeit a major part) of the hardware producing the final spectrum or image. This section summarizes the hardware used in obtaining an NMR signal. The hardware controlling the B_0 field will not be discussed, as it is not the concentration of this thesis.

RF transmitter

The RF transmitter circuitry produces the electrical current in the RF coil that causes excitation of the nuclear spin system. The specifics of the electronics are beyond the scope of this thesis. Figure 1.15 gives an overview of the generation of RF pulses.

Figure 1.15 General NMR transmitter process.

'The electronics are complicated. See Chen and Hoult 1989.

Figure 1.16 Flow chart of general NMR receiver process.

In general, both phase and amplitude modulation are desired in the current signal sent to the RF coil. For a detailed analysis of the electronic circuitry required, the reader is referred to Chen and Hoult²¹.

Signal receiver

An overview of the receiver sequence is given in figure 1.16. The receiver must amplify only the signal of interest, while adding a minimum of noise. After the signal has been amplified by the pre-amp stage, it is split into two parts using a quadrature phase sensitive detector. The complex signal is then amplified, filtered (to remove signal from outside the FOV) and converted to a digital form for analysis or storage in the computer.

Figure 1.17 gives a block diagram of quadrature phase-sensitive detection. Two signals (the real and imaginary parts of a complex signal) are generated - an input signal, and a reference signal. In each case the original signal is combined with a reference signal using a phase-sensitive detector (PSD). The reference signal used for signal 1 is given a 90 degree phase difference from that used for signal 2. Assume the original signal (voltage or current) is sinusoidal in time of the form

$$V_i = \sin(2\pi f_i t + \phi_i) \quad (1.62)$$

The reference signals entering PSD1 and PSD2 can be written in a similar fashion as

$$V_{r1} = \sin(2\pi f_{r1} t + \phi_{r1}) \quad (1.63)$$

and

$$V_{r2} = \sin(2\pi f_{r2} t + \phi_{r2}) \quad (1.64)$$

Figure 1.17 Schematic of quadrature phase sensitive detection (from Chen and Hoult 1989).

Figure 1.18 Voltage ramping.

A phase-sensitive detector generates an output signal resulting from the multiplication of the two input signals. Therefore, assuming $\phi_i = \phi_{r1} = \pi/2$, the output signal from PSD1 will be

$$\begin{aligned} S_1 &= \cos(2\pi f_i t) \cdot \cos(2\pi f_{r1} t) \\ &= \frac{1}{2} \cos[2\pi t(f_i + f_{r1})] \\ &\quad + \frac{1}{2} \cos[2\pi t(f_i - f_{r1})] \end{aligned} \quad (1.65)$$

where the trigonometric identities for $\cos(a+b)$ and $\cos(a-b)$ have been used. Now if the high frequency component (term 1) is filtered out using a low-pass filter, the final signal (S_{1f}) can be seen to be oscillating at angular frequency given by

$$\Delta\omega_1 = \omega_i - \omega_{r1} \quad (1.66)$$

If ω_{r1} is chosen to be the Larmor frequency ω_0 , the final signal is seen to be represented in terms of the rotating frame of reference. Now, due to the fact that the cosine function is even, it is impossible to distinguish between a positive and negative $\Delta\omega$. That is, the signal S_{1f} does not differentiate between nuclei precessing in the clockwise versus the counter-clockwise directions. This problem is removed via the addition of a second PSD whose reference signal is at a $\pi/2$ phase difference from the input signal (ie: $\phi_{r2} = \pi, \phi_i = \pi/2$). Using trigonometric identities, the signal out of PSD2 is given by

$$\begin{aligned} S_2 &= -\frac{1}{2} \sin[2\pi t(f_i + f_{r2})] \\ &\quad + \frac{1}{2} \sin[2\pi t(f_i - f_{r2})] \end{aligned} \quad (1.67)$$

Now a low-pass filter removes the high frequency first term, leaving only the sinusoidal term with frequency $\Delta\omega_2$. Assuming both reference frequencies are identical, $\Delta\omega_1 = \Delta\omega_2 = \Delta\omega$, and the two signals from quadrature PSD are

$$S_{1f} = \frac{1}{2} \cos(\Delta\omega t) \quad . \quad (1.68)$$

and

$$S_{2f} = \frac{1}{2} \sin(\Delta\omega t) \quad . \quad (1.69)$$

Now, if $\Delta\omega < 0$:

$$S_{1f} S_{2f} < 0 \quad ,$$

while if $\Delta\omega > 0$:

$$S_{1f} S_{2f} > 0.$$

Comparison of the sign of the two signals in quadrature PSD enables the differentiation of nuclear spins rotating in opposite directions.

Gradient control

The gradient controller sends a low voltage signal to the gradient amplifiers, which step-up (amplify) the voltage and send a current signal to the coil. As a consequence of Faraday's law of induction, gradient coils in general have a self-inductance as well as a mutual inductance with the main B_0 coil. The result is that the current in the coil does not follow the simple Ohm's law relationship with the applied voltage³:

$$I = \frac{V}{R} \quad . \quad (1.70)$$

In the case of a step voltage (curve A in figure 1.18) the analogous relationship in the presence of an inductive impedance is given in equation 2.13.

As is described in section 2.2.3 this inductive impedance will give rise to an over-shoot and oscillation in the current supplied to the coil. In general, the size of the over-shoot and oscillations will be dependent on (among other things to be discussed in section 2.2.3) the rate of change of voltage in time during the switching of voltage values. The greater the rate of voltage change, the greater the over-shoot effect.

To minimize the overshoot, the gradient controller sends a ramped voltage signal to the coil (curves B and C in figure 1.18). The steeper the slope of the ramp, the worse the overshoot problem will be.

Chapter 2: Gradient coil design theory

2.1 Introduction

As discussed in chapter 1, in order to acquire an image a gradient in the z component of the magnetic field must be applied. The z component of the magnetic field can be written as

$$\begin{aligned}
 B_z(x,y,z) = & B_0(x,y,z) + \frac{\partial B_z(x,y,z)}{\partial x} x + \frac{\partial B_z(x,y,z)}{\partial y} y + \frac{\partial B_z(x,y,z)}{\partial z} z \\
 & + \frac{\partial^2 B_z(x,y,z)}{2 \partial x^2} x^2 + \frac{\partial^2 B_z(x,y,z)}{2 \partial y^2} y^2 + \frac{\partial^2 B_z(x,y,z)}{2 \partial z^2} z^2 \\
 & + \text{second order cross terms} + \text{Higher Order Terms} .
 \end{aligned} \tag{2.1}$$

Ideally, B_0 is generated by the main coil and is both time and spatially invariant, while the linear and higher order terms are generated by separate gradient coils. The goal of a gradient coil is to modify the linear and/or higher order terms of equation (2.1) to values required for specific imaging sequences. Most imaging experiments, such as the *Fourier transform* (FT) reconstruction methods discussed in chapter 1 require linear (ie. first order terms only) gradients. On the other hand, *field focused nuclear magnetic resonance* (F.o.n.a.r)²⁴ and *topical magnet resonance* (t.m.r)²⁵ imaging methods require high order gradients. Some applications require very strong gradients (echo planar imaging (EPI) and diffusion imaging), while others must have gradients that can be switched on and off quickly (EPI). Still other applications have constraints on the size of the volume over which the gradient must be well defined.

In general the design of a gradient coil system must account for the following considerations:

- 1) *field shape (i.e. linearity).*
- 2) *gradient strength.*
- 3) *switching time.*
- and 4) *physical shape / size.*

This chapter starts with an examination of these considerations in accordance with their relevance to this thesis. This discussion is followed by some general gradient coil design theory, and the chapter concludes with an investigation of some existing design methods.

Chapters 3, 4, 5 and 6 present a novel gradient coil system which has been optimized for strong, fast switching linear gradient fields.

2.2 Gradient coil design considerations

2.2.1 Field shape (linearity)

As shown in Chapter 1, gradients in the B_z field are required in imaging in order to produce a distribution of spin frequencies. This spatial distribution of frequency can then allow for a one-to-one mapping between frequency and position. A spin at any point (x,y,z) is in a field determined by equation (2.1). If there are any linear or higher order terms in the B_z field, then a spin at point (x',y',z') will not necessarily be found in the same field as the spin at point (x,y,z) . Therefore spins at different spatial locations will be precessing at different frequencies.

A Fourier transform of the signal acquired in the presence of such a field gradient will result in a spectrum, plotting the relative number of spins versus frequency (which is directly related to spatial location).

Two criteria must be met in order that the signal obtained in the presence of a general $B_z(x,y,z)$ can be decoded into an image. Firstly, the distribution of B_z must be known so that the frequency space (FT of signal) information can be mapped into a spatial distribution. Secondly, the function $B_z(x,y,z)$ must be single valued within the region of interest. In this general case, the mapping from frequency space to an image depends on the B_z distribution, and can be arbitrarily complex. It is important, therefore, to select a B_z distribution that not only complies with the above two necessary conditions, but one that also simplifies the frequency $\Rightarrow$ image mapping/reconstruction.

The Fourier transform offers a fast and simple tool to map the frequency $\Rightarrow$ image transformation. But, for the FT to be directly applicable, the frequency must vary linearly along a spatial direction. Therefore the gradient must contain only the linear terms in equation (2.1), and all of the higher order terms must vanish.

Fourier methods are by far the most common image reconstruction techniques, and therefore coils that produce linear (as linear as practically possible) gradients are most common and useful. Projection Reconstruction methods, where a Radon transformation is applied², also require linear gradients.

Echo Planar Imaging is the concentration of this thesis, and should therefore be given careful attention. EPI uses Fourier transformations to map frequency space. As shown above, a linear gradient will simplify the reconstruction of an EPI signal.

The Semi-Cylindrical Surface Gradient Coil (SCSGC) presented in this thesis, is designed with EPI in mind, and therefore has gradient linearity as one of the main design considerations. In fact, as will be seen, the primary parameter of optimization in the design algorithm is the Volume of Linearity (VOL) or the volume over which the field is linear (to within a specified tolerance).

2.2.2 Gradient strength

Any imaging sequence must apply gradients in such a way that a certain region of *k-space* (see chapter 1) is covered, corresponding to coverage of a certain volume of physical space. The gradient strength in a given direction, along with the *sweep-width* of the spectrometer determine the size of *k-space* (frequency) coverage in the gradient direction. A general two dimensional FT imaging experiment is a good example. Assume the following imaging parameters (see figure 2.1):

NP_x, NP_y - resolution in the x and y directions respectively

x_{min}, x_{max} - minimum and maximum values of x to be imaged

(i.e. $FOV_x = x_{max} - x_{min}$)

$G_x = \delta B_z / \delta x$ - gradient strength in the x direction

$\Delta f, \Delta \omega$ - sweepwidth or bandwidth of spectrometer in Hz and rad/s respectively.

Label nuclear spins at x_{min} and x_{max} as spins 1 and spins 2 respectively. For spins 1 and 2 to be at the edge of the FOV of the image, they must be precessing at frequencies $\Delta f/2$ and $-\Delta f/2$ respectively.

Figure 2.1 Definition of parameters for gradient strength example.

Figure 2.2 Current as a function of time (for an LR circuit) after instantaneous application of a voltage V .

But spins 1 and spins 2 are known to be rotating at angular frequencies

$$\omega_{1,2} = -\gamma \left(B_0 + \frac{\delta B_z}{\delta x} x_{\min, \max} \right) \quad (2.2)$$

and the difference between the frequencies of spins 1 and 2 is

$$\begin{aligned} \Delta\omega_{21} = \omega_2 - \omega_1 &= -\gamma \frac{\delta B_z}{\delta x} (x_{\max} - x_{\min}) \\ &= -\gamma G_x \text{FOV}_x \end{aligned} \quad (2.3)$$

Using the equation

$$\Delta f = \frac{\Delta\omega}{2\pi} \quad (2.4)$$

one can obtain the relationship between the read gradient strength and the sweepwidth as

$$G_x = -\frac{2\pi}{\gamma} \frac{\Delta f}{\text{FOV}_x} \quad (2.5)$$

So, clearly, the gradient strength is proportional to the sweepwidth, and inversely proportional to the field of view.

The strength of the phase encoding gradient (in the y direction for this example) is found to be dependent on the imaging method used, as well as the length of the phase encode gradient pulse. Assume $\text{FOV}_x = \text{FOV}_y$. In the case of conventional *EPI*, the phase gradient need only be

$$G_y = \frac{G_x}{\text{NP}_y} \quad (2.6)$$

if the duration of each phase encode gradient is equal to that of the read gradient.

For *spin echo* imaging methods, the phase encode gradient strength (duration same as the read gradient) must be increased in each successive step, with the value ranging from

$$-\frac{G_x}{2} \Leftrightarrow \frac{G_x}{2}, \text{ in steps of } \frac{G_x}{NP_y}. \text{ As a numerical example select a}$$

$FOV_x = FOV_y = 4.0 \text{ cm}$, and assume that the digitizer samples the signal at $40 \mu\text{s}$ intervals (i.e. dwell time $DT = 40 \times 10^{-6} \text{ s}$). Sampling theory²⁰ determines that the useful range in frequency space (sweep width $SW = \Delta f$) is determined by the dwell time according to

$$\Delta f = \frac{1}{DT} = \frac{1}{40 \mu\text{s}} = 25 \text{ kHz} \quad (2.9)$$

Therefore the read gradient must be

$$\begin{aligned} G_x &= \frac{2\pi 25 \times 10^3 \text{ s}^{-1}}{\gamma \cdot 0.04 \text{ m}} \\ &= \frac{1}{42.573 \times 10^6 \frac{\text{Hz}}{\text{Tesla}}} \times 6.25 \times 10^5 \frac{\text{Hz}}{\text{m}} \\ &= 1.468 \times 10^{-2} \frac{\text{Tesla}}{\text{m}} \\ &= 1.468 \frac{\text{Gauss}}{\text{cm}} \end{aligned} \quad (2.10)$$

This gradient strength is easily obtained from most designs of gradient coil systems, and no special coil system need be designed. A problem relating to this example arises when EPI is considered.

As will be seen, an EPI signal is formed during application of a gradient sequence following a single 90° RF pulse. As the system is guided through its *k-space* trajectory, T_2 relaxation causes a continuous de-phasing of the spins, resulting in a loss of signal[†]. Also as a result of the *single shot*[†] nature of the signal acquisition, T_{2m} (see the General NMR Theory section in chapter 1) adds to the dephasing problem, as well as causing intensity errors in the image²⁷. To minimize the effect of these problems, it is important in EPI to make the signal acquisition time period as short as possible. For a given image resolution the only way to facilitate this is to make the dwell time as short as possible. A dwell time of 10 μ s results in a sweepwidth of $\Delta f = 100$ kHz, making the read gradient strength for the above example

$$G_x = 5.872 \frac{\text{Gauss}}{\text{m}} \quad (2.9)$$

In contrast, spin echo imaging methods require a 90° RF pulse for each phase encoding step in the sequence, allowing far less time for spin dephasing before each acquisition. A dwell time of 40 μ s is found to be acceptable.

In summary, the gradient strength is inversely dependent on the FOV in the gradient direction, and directly on the dwell time chosen for the digitizer. Spin echo methods have modest constraints on gradient strength, and need only standard gradient coils. EPI becomes less sensitive to spin relaxation/inhomogeneity effects the shorter the dwell time used. Shorter dwell times require stronger gradients.

[†]*Single shot* refers to the fact that only one 90° RF pulse is used for the whole signal acquisition.

Therefore, EPI may require design and construction of coils that can produce relatively stronger gradients (at least in the read direction).

2.2.3 Gradient switching time

In some sequences, it is important to be able to switch the gradient field on and off very quickly. In this work, the switching time of a gradient coil is defined as the time it takes to go from zero gradient strength to either 90% of maximum, or 90% of minimum gradient strength. The reason for the seemingly arbitrary choice of 90% will become clear shortly.

Any circuit or coil has an electrical impedance associated with it. In general, impedance consists of a resistive (R), capacitive ($1/i\omega C$), and inductive ($i\omega L$) component. R , C , and L are the resistance, capacitance, and inductance respectively. The square root of -1 is designated by i . For most gradient coils, capacitance can be neglected, and the circuit can be considered as an *RL circuit*[†]. Consider a circuit with neither an inductive component nor a capacitive component: Here, an applied voltage (V) would cause a practically instantaneous current (I) to flow through the circuit. In this case, the current is simply given by Ohm's law³ as

$$I = \frac{V}{R} \quad (2.10)$$

In the presence of an inductive component, the gradient coil circuits will not respond instantaneously to an applied voltage.

[†] An *RL circuit* has a resistance (R) and inductance (L) in series.

Instead, it can be shown that³ when a constant voltage V is applied, the current I approaches the Ohm's law value of V/R as

$$I(t) = \frac{V}{R} (1 - e^{-\frac{Rt}{L}}) \quad (2.11)$$

Clearly, as $L \rightarrow 0$ the equation approaches Ohm's law. The general shape of this relationship is shown in Figure 2.2 where current I is plotted as a function of time t . In this graph, it is assumed that the voltage V is applied at $t=0$. As can be seen, as $t \rightarrow \infty$ the current approaches V/R . Because the current will never actually reach the V/R value, it is necessary to define a fraction of V/R to characterize the circuit. The convention in electronics is to choose the point where

$$t = \tau = \frac{L}{R} \quad (2.12)$$

Here, τ is known as the time constant of the circuit. At τ , the current is, by equation (2.11),

$$I(\tau) = 0.632 \times \frac{V}{R} \quad (2.13)$$

which is 63% of the maximum (or Ohm's Law) value.

To define the switching time of a gradient coil, further consideration must be taken, accounting for the hardware controlling the gradient coils. As discussed in chapter 1, when a sequence calls for a gradient value to be changed, the gradient controller sends a ramped signal in time (see Figure 1.18 in chapter 1) to the amplifier which in turn sends the required current to the coil.

For example, if the sequence calls for a current I corresponding to a voltage V , the voltage V is not turned on in one single step, but is increased from 0 to a value greater than V in a number of steps specified by the gradient controller. A voltage value greater than V is applied by the amplifier in order that the current growth be operating in the high slope region of figure 2.2, thereby facilitating faster current increase. While improving the switching time, this method unfortunately increases the problem of *overshoot* (illustrated as the oscillations in figure 2.3). Overshoot is a direct consequence of the inductance which causes a time lag between the applied voltage, and the current. In general, three factors determine the shape of the current versus time curve for a gradient coil:

- a) the time constant of the coil,
- b) the shape of the voltage versus time relationship, and
- c) the maximum voltage applied by the amplifier.

Most gradient controllers have a few preset ramp options, which utilize predetermined voltage versus time shapes. Therefore, the time constant of the coil will determine which ramp setting can be used. In general, the shorter the time constant, the shorter the ramp time setting that can be used.

2.2.4 Efficiency

In specifying gradient coil performance, a user will demand that the coil supply a particular gradient strength (G/cm) in a particular rise time (μ s). In general this aspect of coil performance is characterized by a coil's ability to reach higher gradient strengths in shorter periods of time.

Figure 2.3 Characteristic current versus time relationship for a gradient coil.

Figure 2.4 Current configuration for the Maxwell Pair axial gradient coil (gradient in z direction).

For the comparison of different gradient coil geometries in this thesis, efficiency is defined as

$$\text{efficiency} = \frac{(\text{gradient strength at a specified current})^2}{\text{coil inductance}} \quad (2.14)$$

Efficiency defined in this way is independent of the number of turns and is a property of the coil geometry only.

2.2.5 Physical shape

Another consideration that must be taken into account when designing a gradient coil system is physical size and shape of the coil. The coil must be designed to allow access to the patient, or phantom. As will be seen, a spherical gradient system would make for a very linear field calculated from relatively simple methods. However, because of its closed nature, it is not possible to move the samples in and out. Also, smaller coils are better for faster switching times, and higher gradient strengths; but the coils must be large enough to allow the sample to fit into the volume where the gradient is linear.

2.3 Gradient coil design theory

2.3.1 General principles

There are many methods of designing and building gradient coil systems for NMR, some of which will be described in the following sections. First, a brief tour of electromagnetic theory as it applies to coil design will be presented.

Ideally, the goal of coil design is to be able to specify the desired B-field, and from this, calculate the current configuration needed to achieve it. However, due to constraints of finite size of the coil, and limitations on the physical realization of the ideal current configuration, the desired field can never be obtained exactly.

Also, the calculation of fields are in general non-trivial, rendering them complicated and time consuming. Therefore, many design methods use approximations, and make use of simple symmetrical constraints (i.e. axial symmetry), limiting the attainable field.

Electromagnetic theory tells us that if the current density (distribution of current) is known in all space, the magnetic field can be calculated at all points of space. This can be accomplished by applying the Biot-Savart Law³

$$\underline{dB} = \frac{\mu_0 I \underline{dl} \times \underline{r}}{4\pi r^3} \quad , \quad (2.15)$$

where $\underline{dB}$ is the magnetic field vector at point $\underline{r}$, and $\underline{dl}$ is an element of current. The field at $\underline{r}$ can then be determined by integration over the current distribution. This equation can easily be changed into its component form depending on the coordinate system in use. For example if cylindrical coordinates are used (cylindrical geometry to facilitate patient/specimen access), the z-component (axial) of the $\underline{dB}$ vector is given by the complicated elliptical *Legendre* integrals². So, even simple cases with cylindrical symmetry require solutions involving non-trivial elliptical integrals for analytical results.

Although the Biot-Savart Law offers a mathematically complicated tool to calculate B fields, it tells us little about the building blocks needed for gradient coil optimization. What is wanted is an approach that gives insight into how various symmetries, or configurations of current can be combined to form the required field. The following analysis using Laplace's equation offers such insight.

One of the results of electromagnetism, is that the magnetic field B in free space can be written as the gradient of a magnetic scalar potential V_m

$$\underline{B} = -\nabla V_m \quad . \quad (2.16)$$

This scalar is found to satisfy Laplace's equation:

$$\nabla^2 V_m = 0. \quad (2.17)$$

The solution to Laplace's differential equation is well known² in terms of *spherical harmonics* as

$$V_m = \sum_{lm} A_{lm} r^l Y_l^m(\theta, \phi) \quad . \quad (2.18)$$

where r , θ and ϕ are the spherical coordinates, and the *spherical harmonics* $Y_l^m(\theta, \phi)$ are given by:

$$Y_l^m(\theta, \phi) = (-1)^m \left[\frac{2l+1(1-m)!}{4\pi (1+m)!} \right]^{\frac{1}{2}} e^{im\phi} P_l^m(\cos \phi) \quad . \quad (2.18)$$

In this equation $P_l^m(\cos \phi)$ are the associated *Legendre* polynomials²⁸. The spherical

harmonics (SH) for all l and m form an orthonormal set. This has two implications:

- 1) As suggested in equation (2.18), any scalar potential field can be formed by a linear combination of spherical harmonics.
- 2) Each of the SH terms is mutually independent of the others, meaning that one cannot be formed by combinations of the others.

The consequence of this second point is that if one can design coils that will generate specific harmonics, a required field can be built up one coil (or harmonic) at a time. If V_m is known from equation (2.18), $\underline{B}$ can be determined through equation (2.16). In NMR the axial (z) component of the magnetic field is of primary interest. B_z is determined by equation (2.16) to be²

$$B_z(r, \theta) = \sum_{l=0}^{\infty} \frac{r^l}{l!} \frac{\partial^l B_z(0)}{\partial z^l} P_l(\cos \theta) \quad (2.19)$$

This is a Taylor series expansion of $B_z(z)$ evaluated at $z = 0$. Axial symmetry has been assumed to exclude any ϕ variation. If the derivatives of the axial component of B_z are known at the field origin, one can calculate the field at any point using equation (2.19). This result provides a set of building blocks whereby approximations to desired fields can be achieved by using loops of current as elements. Design methods based on equation (2.19) will be referred to as 'loop element' techniques. As an example, consider a B_z field made up of sets of current loop elements, all oriented parallel to one another. Because the system has axial symmetry (gradient coil loop axis parallel to B_z), $B_z(z)$ for a single loop at points on the z axis can be written simply as²⁹

$$B_z(z) = \frac{\mu I}{2a(1 + \frac{z^2}{a^2})^{\frac{3}{2}}} \quad (2.20)$$

where a is the diameter of the loop. The derivatives with respect to z for the single loop can then be easily evaluated from equation (2.20).

The m th order derivative is defined as

$$B^{(m)} = \frac{d^m B_z(z)}{dz^m} \quad (2.21)$$

When more than one loop is present, the contribution from each loop on any given order of derivative is superposed. That is the m th order derivative (re z) of the B_z field is the sum over all current loops

$$B_{\text{tot}}^{(m)} = \sum_k B_k^{(m)} \quad (2.22)$$

In the case where the current distribution is symmetric about the plane perpendicular to the z -axis, and in the centre between the coils, all of the odd-order terms are zero. Conversely, when anti-symmetry exists about this central plane, all of the even-order terms are zero. The task in designing coils with the Legendre polynomials (equation (2.19)) is to choose appropriate combinations of loop diameters, positions and current such that the undesired orders of equation (2.22) are summed up to zero. For example an ideal z gradient coil is formed when there is anti-symmetry cancelling all even order terms, as well as loop combinations rendering all but the first order term to zero. Although such an ideal gradient coil is not possible due to a finite number of loops, a good approximation can be made with as few as two loops of current. The two loop Maxwell Pair³⁰ configuration is shown in figure 2.4 and discussed in section 2.3.2.

As mentioned, this 'loop element' method is based on a simplification due to axial symmetry of the current. Different measures must be taken for the transverse (G_x and G_y) gradient directions, as they do not have the required symmetry.

Note that designs need only be made for one of the transverse directions, since the other direction can be obtained by a $\pi/2$ rotation about the z axis. Zupancic¹¹ formulated a method for generating the transverse (G_x, G_y) as well as axial (G_z) gradients using four infinite straight wires oriented in two planes perpendicular to the z axis. The configuration and parameters are defined in figure 2.5. Using the Biot-Savart law, the transverse and axial gradients are given respectively by¹²

$$\frac{\partial B_z}{\partial y} = \frac{\mu_0 \pi I}{6g^2}, \quad (2.23)$$

when the wire configuration is as in figure 2.5a, and

$$\frac{\partial B_z}{\partial z} = \frac{\sqrt{5} \mu_0 \pi I}{24 g^2} \quad (2.24)$$

for figure 2.5b. To obtain these results, the B_z field was expanded into a geometric power series which contains only odd terms (z^1, z^3, z^5 , etc). For small z and y all terms of order greater than three can be neglected. For linear gradients, the third order term (z^3) must also equal zero. This is the case when the geometrical parameters a and g are found in the ratios³²

$$\frac{\pi a}{2g} = \cosh^{-1} \sqrt{\frac{3}{2}}, \quad \text{and} \quad \frac{\pi a}{2g} = \cosh^{-1} \sqrt{6} \quad (2.25)$$

for figures 2.5a and 2.5b respectively. Gradient coil designs based on parallel straight lines of current will be referred to as 'line element' methods.

In addition to the above analytical solutions for determining the B_z field, the Biot-Savart Law integrals can be evaluated using numerical integration algorithms.

Figure 2.5 Wire configuration for four line element gradient system.

Figure 2.6 Most common geometries for gradient coils.

Gradient Coil Shapes

Such numerical methods lend themselves well to 'numerical optimization' of the B_z field.

In general, numerical optimization involves the following algorithm:

- 1/ Define an initial current configuration.
- 2/ Calculate the B_z field.
- 3/ Compare with the desired B_z field.
- 4/ Change the current configuration.
- 5/ Go back to 2/.

After step 3, the algorithm must determine whether the calculated and desired field match well enough to stop the cycle. Also, step 4 is usually not just a random change of current configuration, but a change in a direction favouring the desired field. There are many possible algorithms of numerical optimization, most of which are beyond the scope of this thesis. The reader is referred to **Numerical Recipes in C**³³ for an in depth coverage of the topic of numerical optimization. An example of using numerical optimization to design gradient coils is described by Crozier³⁴, where 'simulated annealing' is used to minimize a combination of gradient coil properties. The coil optimization algorithm used in this thesis is a hybrid combining the 'simplex' and 'simulated annealing' methods and is described in chapter 3.

Although the Biot-Savart Law gives a tool to calculate the field due to any current distribution, it is really the inverse problem that (ideally[†]) must be solved in coil design. That is, given a desired field, can the necessary current distribution be determined?

[†] The difficulty occurs when the ideal current density is approximated by the constructed current density.

In 1986, R. Turner³⁵ presented such a method of analytically calculating the required current configuration to produce a specified field. The so called 'target field approach' uses a Fourier-Bessel expansion of the magnetic field to effectively invert the Biot-Savart Law. The Fourier-Bessel inversion is applicable only to current distributions that are constrained to the surface of the cylinder. Turner has since extended the inversion method to current confined to a planar surface by making use of Lagrange multipliers³⁶. Further extensions using Green's functions make inversion possible for Bi-planar³⁷ and elliptical³⁸ current configurations.

Another interesting inverse method of designing coils is presented by V.R. Portugal'skii³⁹. The method is based on representing the current density as a trigonometric polynomial, whose coefficients are determined for the smallest standard deviation of the difference between the created field and the desired field. The method can be used for current configurations confined to a surface of any given shape.

Most existing gradient coils for NMR were designed using either direct numerical evaluation of fields using the Biot-Savart law, or the analytical solutions provided by the Legendre Polynomials and spherical harmonics. Recently, coil designs based on the 'inverse approach' introduced by Turner³⁵ have become common. The best method to use depends on the specific design criterion involved. The next section gives an overview of some existing gradient coil designs.

2.3.2 Existing design geometries

Most existing gradient coil designs consist of current confined to the surface of a cylinder. Two other planar geometries have been recently proposed. These three common gradient coil shapes illustrated in figure 2.6, will now be discussed in detail.

Cylindrical geometries

The simplest way to produce a useably linear gradient is with the Maxwell pair described by Tanner³⁰. The Maxwell pair is illustrated in figure 2.4. The field produced by such a loop pair can be calculated using equation (2.20). The most uniform gradient possible with two loops is 5th order (3rd order term = 0). Setting the 3rd order equal to zero in equation (2.22) the coil dimensions (a and d) can be solved for using equations (2.20) and (2.21). The result is that the Maxwell pair configuration gets the best linearity at

$$d = a \sqrt{3} \quad , \quad (2.26)$$

where a is the loop diameter, and d is the loop spacing.

Improvement to the linearity can be achieved by adding more Maxwell pair loops. In general, the more loop pairs, the higher the order of linearity that can be achieved. A four coil (2 pairs) system is discussed by Maltempo et. al.⁴⁰. Quine et. al.⁴¹ offer a model for optimizing coil dimensions in gradient systems with any number of coil pairs.

As mentioned, this technique of combining loops is valuable only when designing gradients in the axial (z) direction of the main B_0 magnet.

The transverse gradients can be produced using the four line element method³² already discussed.

An improved transverse gradient set based on a modification to the four line element method consists of loops of current on the surface of a cylinder as illustrated in figure 2.7 for an x gradient coil⁴². These are termed Golay type gradient coils and consist of two pairs of saddle coils with current oriented as shown in the figure 2.7. By adding another two pairs of saddle coils (dotted elements) the modified Golay coils will have improved linearity. Optimized Golay gradient coils are linear to within 3% over a spherical volume of radius $0.31a$, while optimized modified Golay coils have a sphere of linearity of $0.36a$ (a is the radius of the saddle coil).

Line element coils

In deriving the expression for the gradient strength in equations (2.23) and (2.24), the line elements were assumed to be infinite in length. Of course in application, the current lines must be of finite length, and must be completed with return current paths. Three practical return current configurations for transverse gradient coils have been suggested and are shown in figure 2.8a, b, and c. In the configuration in a⁴³, the return current is placed at the same angular orientation as the main current. This results in no introduction of third order gradients thus maintaining the linearity present for the case with no return current. The strength of the gradient (for a given current) is however reduced with this configuration. An alternative path for the return current is shown in figure 2.8b⁴¹, where all four current loops are oriented in parallel planes. In this configuration, the wires running parallel to the z axis do not contribute to the B_z field.

Figure 2.7 Current configuration for the Golay (solid) and Modified Golay (dash) gradient coils (gradient direction = x).

Figure 2.8 Return paths for the four parallel straight wire gradient system (gradient direction = y).

a) equal angle

b) parallel to z -axis

c) Anderson type flat gradient coils

Therefore if the length d in the figure 2.8b is made long enough, the current running out of the page will be a large distance from the useful region of the coil, and will thus contribute negligibly to the B_z field. To accomplish this, the coil will need to be large. This will be seen to lead to a high inductive impedance therefore requiring long switching times. The third configuration for the return current is shown in figure 2.8c⁴⁴⁻⁴⁶. This shape, known as the Anderson type flat gradient coil system, does not have as high a degree of linearity as in a and b, but does have the advantage of requiring far less space along the magnet axis. The flat plates do, however, limit access to the imaging region.

As shown in figure 2.5, the four line element configuration can also be used to produce linear gradient fields in the axial (z) direction. One design based on this so called Quadrupole coil is shown to be suitable for diffusion measurements⁴⁷ (radius of coil is 1 cm). Recently, a modified quadrupole gradient set has been designed for high resolution imaging⁴⁸. These gradient coils will be discussed in the results section of chapter 4, where a comparison will be made with the SCSGC designed for this thesis.

Planar surface gradient coils

Recently, with the advent of fast imaging techniques such as EPI (requires fast switching time), diffusion imaging and high resolution imaging (both require strong gradients), planar surface gradient coils have been designed^{49,50,51,52}. In general, surface gradient coils provide strong gradients, with a low inductance, but with a small volume of linearity. For example, Cho and Yi⁵² report a gradient set with gradient strength >6 G/cm in a 'useful region' of 20% of the coil dimension, and with inductances $\sim 1500 \mu\text{H}$.

As will be seen, the useful region is only for imaging planes with slice selection in the y direction, where y is perpendicular to the coil surface. One problem with planar surface gradient coils is connected with the coil used for generating a gradient in the direction perpendicular to the surface. Consider a planar gradient coil with the current constrained to the xz plane. Using finite wire segments, an optimized y gradient coil is of the general configuration shown in figure 2.9a⁵². Figure 2.9b gives a plot of B_z as a function of y (or the distance from the coil surface). As should be expected the field strength decreases with distance from the surface. In a certain region (designated region of interest ROI in the figure 2.9b), the gradient has linearity usable for imaging. However, at the centre of the ROI, the B_z value is some finite positive offset value (B_A). For this type of current configuration, the stronger the gradient strength, the larger the offset. The result of such an offset, is that the image will be shifted in the direction of the gradient with the offset. If the offset is large enough, the entire signal will be shifted outside of the FOV of the spectrometer, and the image will be lost. This offset can be compensated for in the following ways:

- 1/ The frequency of the spectrometer can be shifted to centre the FOV at the desired offset. Problem: All other directions of gradient coil used in the imaging sequence will be shifted in the reverse direction.
- 2/ The filter width of the spectrometer can be increased to include the frequencies corresponding to the offset gradient which will be aliased into the FOV. Problem: Information from outside the FOV in all directions will be aliased into the image.

Figure 2.9a Current configuration for planar surface y-gradient coil.

Figure 2.9b B_z field versus distance (y) from surface for planar surface gradient.

3/ A second coil with current in the opposite sense of coil A in figure 2.9a can be added to the gradient coil design. Problem: Gradient strength is reduced.

4/ No correction is made, and the y-gradient coil is used for the slice gradient only. Problem: Limited imaging plane orientation.

The third option, and that taken by Yi and Cho⁵⁰, is shown as loop B in figure 2.9a. The plot of B_z as a function of y is shown in figure 2.9b. As is seen, this correction loop (B) can be configured such that the B_z value at the centre (B_B) of the ROI is -1 times the B_z value produced by wires A ($B_A = -B_B$). The net B_z at the centre of the ROI is therefore zero as required. The correction of the B_z offset will be further discussed in the discussion section of chapter 3. Another gradient coil design with the offset correction current in a displaced plane is described by Jasinski et. al.⁵³. This hexagonal surface gradient coil was designed to be fastened directly to an *in vivo* heart to remove motional errors in localized MR spectroscopy. A gradient coil designed and built using the option 4 to deal with the $B_z=0$ offset is presented in reference 50, 52. This coil design will be used for comparison with the SCSGC designed and built for this thesis.

Biplanar gradient coils

Martens et. al.³⁷ combine the work of Turner³⁵ and Roemer⁴⁹ to optimize current confined to two parallel planes. The results will be discussed in the discussion section of chapter 3 where they are used for comparison with the SCSGC.

Others

A literature search was done using the *Medline* and *Inspec* data bases, using MRI gradient coil design (and all pertinent combinations of these three words) as search fields. Two other gradient coil shapes were found. A Triangular Pyramidal gradient coil was proposed by Sobol and Blicharski⁵⁴. The results were compared with Tanner's two loop coil³⁰. The two coils had similar performance except that the triangular coil had a 50% larger linear region, and a 10% weaker gradient strength. Finally, a *z*-direction gradient coil with wire wound on an elliptical surface was optimized and presented by Petropolis et al.³⁸. The results show an improvement over cylindrical geometry with regards to both size of VOL and gradient efficiency.

Chapter 3: Numerical optimization of gradient coils

3.1 Introduction

Numerical optimization is any computing method that determines the configuration of a system that results in the *best* value of a given system dependent. The word *best* in this statement refers to the desired system configuration. In mathematical terms, optimization is an attempt to determine the maximum or minimum of a given function. Many algorithms exist for one dimensional and multidimensional optimization, and the best choice for a given situation depends on the specifics of the problem. For the optimization of SCSGCs in this work, a hybrid of two multidimensional methods has been used. This combination of the *simplex* and *simulated annealing* algorithms is a unique solution to the problem of gradient coil optimization. This chapter gives an outline of the optimization theory, as well as its application to SCSGC design.

3.2 Simplex minimization

Assume you are given a function f that depends on N independent variables. The goal of an N -dimensional minimization is to obtain the N values of these independent parameters (ip) which result in the minimum possible value of f . The term optimization parameter, or OP, will often be used to refer to f in this thesis. Nelder and Mead⁵⁵ developed the *downhill simplex method* which offers a multidimensional optimization that requires only function evaluations as opposed to most other methods which require the calculation of derivatives.

A simplex is a geometrical entity consisting of $N+1$ points, and all the lines joining the points. For example, in the case of $N=2$ independent variables, the simplex is a triangle. To begin the optimization, an initial simplex consisting of $N+1$ points must be chosen (each point of the simplex has N *ip*'s specifying its location). The initial simplex is most simply defined by a single point P_0 in the N -dimensional *ip*-space in conjunction with N values of e_i . The e_i values represent a characteristic length scale for each of the i independent parameters. The full starting simplex (S) is then defined from P_0 and the N e_i values according to the matrix equation shown in equation 3.1 (next page). In this matrix representation, each point of the initial simplex is specified by a single row with N entries corresponding to the value of the *ips*. The matrix on the right side of the equation represents the initial simplex which is made up of $N+1$ rows each corresponding to the $N+1$ points of the starting simplex. The left hand side shows how the initial simplex is made up from the matrix representation of P_0 and e_i . Here, the matrix e_i is represented by a 2-dimensional matrix with the first row containing all zero elements and the e_i values completing the diagonal. With the initial simplex specified, the algorithm then takes the simplex through a series of steps, where the points are moved in one of the four possible courses illustrated in figure 3.1. Eventually, all of the points converge to a minimum of the function f .

The simplex algorithm is summarized in the flow chart in figure 3.2. The cycle of simplex metamorphosis is terminated when either f is below some pre-specified value $ftol$, or all the $N+1$ points in the simplex are at the same position (with distance tol of each other).

Equation 3.1

$$\mathbf{P}_0$$

N+1 rows
N cols
Each row is a single point
in ip space

$$\mathbf{e}_i$$

N+1 rows
N cols
Diagonal Elements
are characteristic size
scale of each ip

$$\mathbf{S}$$

N+1 rows
N cols
Matrix representation
of initial simplex

$$\begin{bmatrix} P_{0,1} & P_{0,2} & P_{0,3} & \dots & P_{0,N} \\ P_{1,1} & P_{1,2} & P_{1,3} & \dots & P_{1,N} \\ P_{2,1} & P_{2,2} & P_{2,3} & \dots & P_{2,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{N,1} & P_{N,2} & P_{N,3} & \dots & P_{N,N} \\ P_{N+1,1} & P_{N+1,2} & P_{N+1,3} & \dots & P_{N+1,N} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ \mathbf{e}_1 & 0 & 0 & \dots & 0 \\ 0 & \mathbf{e}_2 & 0 & \dots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & \mathbf{e}_N \end{bmatrix} = \begin{bmatrix} P_{1,1} & P_{1,2} & P_{1,3} & \dots & P_{1,N} \\ P_{2,1} & P_{2,2} & P_{2,3} & \dots & P_{2,N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P_{N,1} & P_{N,2} & P_{N,3} & \dots & P_{N,N} \\ P_{N+1,1} & P_{N+1,2} & P_{N+1,3} & \dots & P_{N+1,N} \end{bmatrix}$$

The first row in the simplex is the same as the starting point $\mathbf{P}_0$

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Figure 3.1 Possible moves of simplex are: a) reflection of high point, b) reflection and expansion, c) contraction from high, or d) contraction along all dimensions toward low.

Figure 3.2 Flow chart of simplex algorithm.

In general, the downhill simplex method is time consuming compared to most of the other multidimensional minimization techniques. This is because it requires a greater number of function evaluations to converge to the minimum point. However, the simplex method is less sensitive to the choice of initial point for starting the optimization, and is often the best method if calculation time is not a constraint. Consider the above problem with N independent parameters. A given set of the N *ips* results in a value of the OP. Somewhere in the N -dimensional space of *ip* there is a point for which the OP is a minimum. This is the optimized configuration that is being searched for. For most multidimensional minimization algorithms, the algorithm determines the direction of a move in *ip* space that results in the smallest value of OP. The optimization proceeds along this path of steepest descent until no further downward movement is possible. A simple analogy for the $N=2$ case is instructive. Consider the optimization problem of finding the lowest point within a given area of land. In this case, the OP is the height and the 2 *ips* are the coordinates of latitude and longitude. The algorithm determines the direction where the slope is the most negative, and takes a step of predetermined length in that direction. The step is repeated until the bottom of the valley has been reached. It is simple to see that the initial point of the algorithm is critical to finding the global minimum. Consider the case of figure 3.3a which is a slice along one of the *ip* directions. If the initial point is chosen to be at the point A_i in figure 3.3a, a *steepest descent* type minimization will culminate at the point A_f . Point A_f is at the bottom of a valley (i.e. a *local* minimum), but it is not the lowest point in the range of *ip*#1.

Figure 3.3a Illustration of importance of initial point for conventional optimization methods.

Figure 3.3b Illustration of importance of initial point for optimization using the simplex method.

If however, the point B_1 is specified as the initial point, the minimization will lead to the true *global* minimum at B_1 . This example is easily expanded to any number of *ip* dimensions. In contrast, the simplex method starts with a group of initial points that covers a range of the *ip*-space. Thus, if the characteristic length for each of the *ips* is chosen wisely, the range is more likely to encompass the global minimum. A 1-dimensional case illustrating the advantage of simplex is given in figure 3.3b. The initial point $P_{0,1}$ is located in a valley containing only a local minimum. However, the range of the simplex including the point at $P_{0,1} + e_1$ covers the region including the valley with the global minimum. Thus, the simplex algorithm with initial points chosen as in figure 3.3b should lead to the global minimum while the conventional minimization algorithm with the same initial point (figure 3.3a) will lead only to a local minimum. It should be noted that even with the simplex method there is still a significant dependence on the choice of initial points. All that can be said is that for a given initial point (and the e_i values for simplex), there is a greater probability of finding a global minimum with simplex than with methods relying on a single initial point.

3.3 Method of Simulated Annealing

Simulated Annealing (SA) is a minimization technique that can significantly improve the probability of finding the global minimum independent of the choice of starting point. The method introduced in 1953 by Metropolis et. al.⁵⁶ has recently become a popular tool of optimization in a wide range of disciplines including coil design for NMR. Bellina et. al.⁵⁷ and Ishyama et. al.⁵⁸ describe using SA for main field (B_0) coil design. Gradient coil design by SA has been reported by Crozier et al.^{59,60}.

A literature search with MEDLINE and INSPEC revealed only these four articles on "coil design" using "simulated annealing".

The method of simulated annealing is based on an analogy with the naturally occurring process of the cooling of metals. The process, called annealing, is also analogous with the crystallization of some liquids. As the liquid is cooled in both of these thermodynamic processes, the atoms attempt to align and configure themselves into a state of minimum energy. It seems as though nature has a way of seeking out the atomic configuration resulting in the minimum energy state. It should be noted that if the liquid is cooled too quickly, the system may end up in a state with an elevated energy. This is analogous to having found a local minimum only, and such a state has been given the adjective "quenched". By definition, annealing is the *slow* cooling of a liquid where sufficient time is provided for the atoms to redistribute as energy is removed from the system.

With the exception of SA, all methods of minimization follow the most direct downhill route determined from the starting point. For example in the simplex method, any move that is accepted, must lead to an overall state with a lower value of the OP (energy in the cooling analogy). In the accepted model of natural annealing, and in simulated annealing optimization, the cooling or minimization algorithm does not follow this *quick*, direct downhill path. Instead, for any given step, there is a finite probability for the system to accept a move that results in a higher value of OP or energy state.

In the thermodynamic model the probability of the system existing in a state characterized by energy E is given by the Boltzmann distribution

$$P(E) \propto e^{-\frac{E}{kT}} \quad (3.2)$$

T is the temperature of the system, and k is Boltzmann's constant relating temperature to energy. Any system containing states governed by the Boltzmann distribution will move between states both in the direction of increasing and decreasing energy. Equation (3.2) shows that the lower the temperature the less probable a given uphill (increasing energy) move will be.

The so-called Metropolis⁵⁶ algorithm for SA uses this annealing analogy for the numerical minimization of an OP. In its original form, the OP will make a change with the probability

$$p = e^{-\frac{(OP_{new} - OP_{old})}{kT}} \quad (3.3)$$

Notice that the OP must be transformed into units of energy in order to maintain dimensional consistency. In general, the SA algorithm must contain the following elements:

1. A concise representation of the system. This system configuration is determined by the set of independent parameters.
2. An objective function (same as OP above) whose single scalar value is to be minimized. The OP is a function of the N values of the ip , and can be used to quantify compromises between conflicting objectives.

2

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3. An algorithm to rearrange the system configuration, consisting of random changes in the *ips*.
4. A control parameter T and an *annealing schedule* that determines the path taken by T in going from a high to a low temperature.

The specific implementation of the SA algorithm is dependent on the application. For the case of gradient coil design used in this thesis, the elements of the SA method are defined in section 3.5.

3.4 Simulated Annealing combined with Simplex minimization

Implementation of the SA method to cases of multi-dimensional *ip*-space has been found to be difficult³³. The problems involve planning a generator of random moves in the *ip* configuration (element 3 above). Most schemes presented in the literature^{61,62,63,64} appear to produce too many uphill moves, resulting in inefficiencies in computing time.

Numerical Recipes in C³³ suggests a method that combines SA with the downhill simplex method. This is the so-called T-Simplex[†] method. In the work described in this thesis, it has been found to give good results for gradient coil design. No reference to use of the T-Simplex method for optimization of gradient coils has been found in an extensive search of the literature using both the MEDLINE and INSPEC databases.

The complete algorithm for simplex based SA is given in Numerical Recipes in C³³, and will not be repeated here. An overview is given in figure 3.4.

[†] The 'T' refers to the temperature (or SA) dependence of the method.

Figure 3.4 Flow chart of overview of T-simplex algorithm.

At the core of the algorithm is a simplex method that is modified in two basic ways:

1/ The annealing is simulated by adding a thermal fluctuation to the OP value every time any of the *ips* is changed, and the result is accepted. This is accomplished by adding to the value of OP a random number that is weighted by the temperature T. For example, if a movement of an *ip* is accepted, then the OP corresponding to the moved simplex point is given a random thermal fluctuation defined by

$$OP^{new} = OP^{old} \pm T \log(\text{random}) \quad , \quad (3.4)$$

where T is the annealing temperature, and *random* is a random number between 0 and 1. The sign (+ or -) of the temperature term is determined by an independent second randomization with a 50% chance of each sign. Therefore a move that would surely lead to a smaller value of OP in the absence of thermal variations may lead to a larger value of OP when T has a finite value. The inverse case is also possible, providing the necessary 'uphill' moves for simulated annealing. The points in the simplex algorithm where random fluctuations are added are indicated by the shaded areas of figure 3.4.

2/ The simplex algorithm shown in figure 3.4 is repeated N_T times where N_T is number of steps in the SA temperature reduction schedule. On each repetition, the characteristic length scale parameters e_i are scaled according to the temperature T of the SA. The functional dependence of the e_i values on T affects the efficiency of the optimization, and must therefore be chosen carefully.

In this work, the effective length scale (e_i') used for each simplex repetition was calculated as

$$e_i' = e_i T \quad (3.5)$$

when $T \neq 0$ and

$$e_i' = e_i \quad (3.6)$$

when $T=0$. It follows that at the start of the T-simplex algorithm, when T is large, the size of the simplex is correspondingly large. For the last repetition of the simplex, when $T=0$, the values of e_i' used for the simplex definition are just those that would be defined for the standard simplex method (i.e. e_i).

As mentioned, the standard simplex method, with random thermal fluctuations, is repeated N_T times, with decreasing temperature on each pass. The schedule by which T is reduced must be selected according to the specifics of the optimization problem. The schedule must be chosen to provide cooling that is sufficiently slow in order to avoid quenching of the system. Of the three annealing schedules proposed in Numerical Recipes in C, the following was found to be best for the optimization in this thesis:

Starting at T_0 , reduce the temperature after each simplex step according to

$$T = T_0 \left(1 - \frac{n}{N_T}\right)^\alpha, \quad (3.7)$$

where n is the number of simplex steps thus far, and α determines the relative amount of time spent at higher versus lower temperatures.

Clearly larger values of α lead to more time being spent at lower temperatures

Although this T-Simplex method provides a way of optimizing for a global minimum, its efficiency is dependent on the user's choice of optimization parameters. When the optimization software written for this work is run, the user is prompted for values of:

- T_0 initial temperature,
- T_f final temperature,
- N_T number of simplex steps,
- F_{max} maximum number of function evaluations per
 simplex step,
- e_i characteristic scale for each ip ,
- β overall size of simplex, and
- α power in temperature annealing schedule.

Experience from repeated use of the software leads to an intuitive understanding of the use of these parameters. The parameter β was introduced to simplify the use of the software. The e_i values can be chosen with emphasis on the relative differences between the size scale of each of the ips . The β value is chosen as a common overall scale factor by which each of the e_i values are multiplied by prior to each simplex move. Thus once the e_i values have been set, the optimization is controlled and modified by specification of the single parameter β .

3.5 T-Simplex SCSGC optimization

This section describes the general implementation of the T-Simplex algorithm for the design of SCSGCs. First the manifestation of the physical parameters to the *ips* and *OP* will be discussed, followed by a study of the effectiveness of the optimization method for SCSGC design.

3.5.1 Description of independent parameters for T-Simplex optimization of SCSGC

The current configuration of a coil is represented by nodes specifying the corners of the current loops. The nodes are defined by parameters that are described in detail in section 4.2.2. For the optimization described in this thesis, the position of the node points is allowed to be varied in the azimuthal and *z* directions. The current through the loop defined by each node is also given the freedom to be varied. Thus for each node there are 3 degrees of freedom, or 3 *ips*. The choice of the number of nodes used to define a current configuration is guided by both the time of optimization, as well as construction constraints. In general, more nodes leads to both more complicated optimization and construction stages. For this work, the number of nodes ranged from ~10 to ~30.

In some cases the number of *ips* was reduced by grouping several nodes together into *node_groups* that were each defined by 6 or fewer independent parameters. Consider the example shown in figure 3.5 which gives the node points (in one quadrant) of a possible initial configuration for a *y*-direction gradient coil. As shown in chapter 4, configurations are most efficiently represented by specifying node points in one quadrant only. To simplify the diagram, neither the current in the *z* direction, nor the return current for loop completion are shown.

Figure 3.5 Example of node_group configuration for a y-gradient coil optimization (one quadrant only).

Table 3.1 Definition of node_group parameters.

node_group	ip	# of ip	start node	end node
1	strt ϕ fin ϕ strt z fin z strt I fin I	6	1	5
2	ϕ z I	3	6	6
3	same as group 1	6	7	9
4	same as group 1	6	10	12

If each node is treated separately in optimization, there would be $12 \times 3 = 36$ *ips* to be varied. In contrast by forming *node_groups* defined in table 3.1 (shown by the dashed lines in figure 3.5), the number of *ips* is reduced to 21. In this simplification, *node_groups* that contain more than one node are specified by a starting and finishing value for each of z , ϕ , and I . The z , ϕ , and I values for the intermediate nodes are calculated as the evenly spaced values between the two end-points. The reduction in the number of *ips* can be even more significant in cases that have a larger number of nodes to begin with.

Many other methods of grouping nodes are possible. Each grouping scheme must be accompanied by its own C-code that initialises and configures the general optimization software. One such alternative grouping scheme has been used for gradient coil design in chapter 6 of this thesis. In chapter 6 a gradient coil is designed that requires movement (variation of z , ϕ , and I) of a single group of current while all other nodes in a previously optimized coil are maintained stationary. The bold dashed box in figure 3.5 shows the necessary node grouping for this situation. Nodes 1 to 6 are not allowed to vary, while the one *node_group* containing nodes 7 to 12 is given freedom in the z , ϕ , and I parameters. The resultant optimization requires only 3 *ips*.

3.5.2 Description of optimization parameters for T-Simplex optimization of SCSGC.

The optimization parameter (OP) is in general a function combining several gradient coil properties. The contribution from a given property can be weighted according to the significance of that property in the desired coil design.

In the software for SCSGC design used in this thesis, it is possible to optimize for the following coil properties:

- 1/ maximum size of VOL,
- 2/ maximum gradient strength,
- 3/ minimum coil inductance, and
- 4/ y-position of the VOL (for y-direction gradient coil only).

For example, given a set of *ips*, the OP could be defined as

$$\text{OP} = -C_1 N_y + C_2 \frac{\text{Inductance}}{G_{\text{strength}}} + C_3 (y_{\text{pos}} - y_{\text{des}})^2, \quad (3.8)$$

where the C_i are the adjustable weighting factors, N_y is the number of points of linearity in the y-direction (with a pre-defined x and z direction linearity width), y_{pos} is the y position of the centre of the VOL, and y_{des} is the desired y-position of the centre of the VOL. The efficiency of the coil (ratio of inductance to gradient strength) is also included in the determination of the OP.

3.5.3 Effectiveness of T-Simplex for SCSGC design

In designing the software for the optimization of the SCSGC, three methods were considered including Simplex, simulated annealing, and T-Simplex. The algorithms were implemented, evaluated and compared with reference to SCSGC design. The results indicate that T-Simplex provides the fastest convergence to the 'best' coil configuration. This section describes a specific case study illustrating the effectiveness of the T-Simplex method in the design of SCSGC.

The section concludes with a brief comparison between T-Simplex, Simplex, and simulated annealing.

Figure 3.6 gives an example of the convergence of a y gradient coil using the T-simplex method. The OP was specified according to equation (3.8) with the weighting factors $C_1=1$, $C_2=0$, and $C_3=1$. The width of the VOL in the x and z direction was specified as 4.0 cm in both directions. The radius of the coil was constrained to 24 cm, and the node group configuration was specified as in figure 3.5. The desired y-direction centre of the VOL was specified to be at y position 28 (y positions have values ranging from 0 at the centre of the cylinder to 60 at the coil radius). Because of the large computational time needed to calculate the inductance for a given configuration, the efficiency has been ignored in this example (i.e. $C_2=0$). The 21 *ips* of the initial simplex point were specified by using past experience with y-SCSGC design. This initial point P_0 resulted in an initial OP value of -19. The optimization parameters were set as $T_0=30$, $T_{final}=0$, # of function evaluations per temperature step=100, # of temperature steps=6, simplex size $\beta=2$, power in cooling schedule $\alpha=2$. Figure 3.6a shows a plot of optimization parameter versus the number of function evaluations. Both the best (lowest OP) and worst (highest OP) simplex point are tracked, and are seen to converge at approximately 450 function calculations. The OP scale in Figure 3.6b has been chosen to emphasize the convergence of the best simplex point only. The result of the optimization was to produce a configuration with an OP of -36 corresponding to a VOL ranging 37 points in the y direction, and centred at $y=27$ (see equation (3.8)).

Figure 3.6a Example of T-simplex convergence.

Figure 3.6b Compressed OP scale plot of convergence for the best simplex point.

From these plots, some general observations can be made that characterize the T-Simplex optimization of SCSGCs:

- The start of each new temperature step is marked by a large spike in the worst simplex point curve
- The height of the spike decreases with each new temperature step
- Both the best and worst simplex points contain thermal fluctuations, and
- The height of the fluctuations in both best and worst simplex point curves decreases with each new temperature step.

Further insight into the T-Simplex algorithm is gained by observing the coverage of independent parameter space during the convergence process. This would be impossible to visualize in the above situation consisting of 21 independent parameters. It is possible to simplify the optimization problem to two *ips* by grouping the nodes 7 through 12 in figure 3.5 together, and allowing only variation of current as well as movement of the whole group in the azimuthal angle direction. This type of optimization will be used in chapter 6 when moveable VOLs are considered. The idea is, that given an optimized coil with a VOL centred at a specific point in the y-direction, the VOL can be shifted in the y-direction by simply varying the current and/or position of the above specified node group. For example, starting with the current configuration that resulted from the optimization in figure 3.6 the following 2-dimensional T-simplex case can be studied. First consider the final *ip* configuration left by the figure 3.6 optimization, which has VOL of 37 points in the y direction, and is centred at point 27.

Now use T-simplex with variation in current and position of the above node group only, to find the optimal configuration with a y-centre at $y=35$. The results are shown in figure 3.7 where the OP (determined from the worst simplex point) is plotted as a function of the azimuthal angle and the relative current through the node group. The optimization parameters were $T_0=50$, $T_{\text{final}}=0$, # function evaluations per temperature=100, # of temperature steps=10, $\beta=0.5$, and power in cooling schedule $\alpha=1$. Only the configurations that resulted in a move of the worst point are shown on the plot. In this case, the coverage of the 2-dimensional ip space is seen to be populated along the diagonal of the plane. This is a result that is specific to this example, and should not be considered a general property of ip space coverage for T-simplex. It is simply a manifestation of the fact that certain values of current and azimuthal angle give OP values larger than the thermal fluctuations can compensate for. Larger temperatures in the optimization will give increased coverage of ip space, but will result in a less efficient convergence due to too many 'uphill' moves.

To use T-simplex in an efficient manner requires some insight into how the controllable input parameters affect the convergence and final results of the optimization. Careful selection of input parameters can mean the difference between finding the global or local minimum. Assume the user has configured the problem with reasonable values of ϵ_i (which should be determined by trial and error), the most significant input parameters to be considered are simplex size (β), initial annealing temperature (T_0), and the power in the cooling schedule (α).

Figure 3.7 Coverage of independent parameter space for a simple case of two dimensional T-simplex for SCSGC.

The effect of changing the simplex size is demonstrated by the graphs in figure 3.8, which all take the same input configuration as that used in figure 3.6. The simplex size can be thought of as controlling the size of the steps taken in attempting to find moves of the simplex. Larger β values result in larger attempted moves of the simplex. As β is increased from 0.5 to 4.0, the size of the spikes, for a given temperature, can be seen to increase. Notice however that the fluctuations for a given temperature remains unchanged as the value of β is varied. In effect, increasing the size of β increases the overall range in ip space that will be 'looked at'. Note further that the final value of OP reached by the optimization varies from -22 to -36 depending on the value of β chosen.

In contrast, increasing the initial temperature has no effect on the height of the spikes, but increases the size of the thermal fluctuations at a given temperature. Figure 3.9 illustrates the effect of increasing T_0 from 15 to 60. Note that the value of β has also been decreased such that the product of T_0 and β remains constant. This must be done in order to isolate the effects of the temperature from the simplex size, as the size of the attempted moves in the simplex algorithm have been multiplied by T_0 . The OP curves in figure 3.9 clearly demonstrate an increase in thermal fluctuations as the temperature is increased. In fact the thermal fluctuations can be quantified by applying a polynomial regression of order 2 (remember that the temperature is decreasing by a polynomial of order 2). When this is done, the standard deviation of the fit gives a good indication of the fluctuations. The results are 1.7, 2.5, and 3.6, for the standard deviation of the polynomial fit for the $T=15$, 30, and 60 cases respectively.

Figure 3.8 Illustration of the effect of simplex size (β) on optimization.

a) $\beta = 0.5$

b) $\beta = 1.0$

c) $\beta = 2.0$

d) $\beta = 4.0$

•Initial Temperature	30
•Final Temperature	0
•# of function evaluations per temperature	100
•# of temperature steps	6
•power in temperature schedule	2

Figure 3.9 Illustration of the effect of initial simulated annealing temperature (T_0) on optimization of SCSGC. $T_{final} = 0$, # of func. evals. per temp. = 100, # of temp. steps = 6, power in cooling schedule = 2.

a) $T = 15, \beta = 4$

b) $T = 30, \beta = 2$

c) $T = 60, \beta = 1$

Most significant is that the final value of OP obtained is also dependent on the initial temperature of the simplex, with the best result (OP=-36) obtained for $T_0=30$ case.

The power in the cooling schedule (α in equation (3.7)) can be seen to affect the convergence as illustrated in figure 3.10. As expected, the size of the spikes is seen to reduce in a linear and quadratic fashion for the $\alpha=1$ and $\alpha=2$ case respectively. At the start of the optimization, both schemes cover the same *ip* space. However, as the cooling proceeds, the $\alpha=1$ case covers more *ip* space, while the $\alpha=2$ case is more sensitive to the local *ip* contour. In this case, the final result is the same (OP=-36), although the convergence for the $\alpha=1$ case occurs more quickly at ≈ 250 function calculations (≈ 425 function calculations for $\alpha=2$). The optimal value of α is dependent on the specifics of the *ip* space, the starting simplex point, as well as the number of function evaluations at each temperature. In general the best value of α should be determined by trial and error, guided by any available knowledge of the *ip* surface. For a given number of function evaluations per temperature, lower values of α explore a larger area of *ip* space, while higher α values are more efficient in areas of more contoured *ip* surfaces.

The number of function calculations per temperature step, and the number of temperature steps, must be determined by experiment. Experience has shown that it is better to require too many calculations per step than too few, as the main expense is in computation time rather than the resultant optimized OP. The same rule holds for the number of temperature steps.

Figure 3.10 Illustration of the effect of the power in the cooling schedule (α) on T-simplex optimization of SCSGC. $T_0 = 30$, $T_{\text{final}} = 0$, # of func. evals. per temp. step = 100, # of temp. steps = 6.

a) $\alpha = 2$

b) $\alpha = 1$

3.5.4 Comparison of T-simplex with other optimization methods

When simplex alone was applied to the optimization of SCSGCs, the result was found to be heavily dependent on the size of the simplex. Even in the case of optimal simplex size, several restarts were needed in order to get the same level of minimization as with T-simplex. Often, simplex failed to match the results of T-simplex even after repeated restarts leading to the same final OP. Because of its 'always go downhill' bias, simplex is intrinsically limited to find a local minimum (albeit more global than a steepest descent type algorithm).

Simulated annealing has been suggested as a method of gradient coil design by Crozier et. al^{59,60}. Simulated annealing has been implemented as an option for optimization in the software written for this thesis. When SA optimization is compared to T-Simplex for the design of SCSGCs, two general patterns appear. First, the simulated annealing algorithm is found to be far more dependent on the choice of optimization parameters. The SA algorithm had to be repeated several times with variations of the input parameters before the optimization would converge to as low a value as that obtained using T-simplex. In the case of T-simplex, a generic set of input parameters was found that result in 'global' optimization for a wide range of problems. Secondly, simulated annealing consistently requires a considerably larger number of function evaluations than T-Simplex to converge to the same minimum OP. As an example consider the OP convergence plots depicted in figure 3.11. The optimization problem is identical to that shown in figure 3.6, however the results of SA and T-Simplex are shown on the same graph for comparison.

Figure 3.11 A comparison between simulated annealing and T-Simplex for SCSGC optimization.

The following optimization parameters were used:

Simulated Annealing

- $T_0 = 10$
- $T_{\text{final}} = 0$
- # of function calculations per temperature = 600
- # of temperature steps = 10
- power in cooling schedule = 2.

T-simplex

- $T_0 = 30$
- $T_{\text{final}} = 0$
- # of function calculations per temperature = 100
- # of temperature steps = 6
- simplex size (β) = 2
- power in cooling schedule = 2.

For the T-simplex case, only the convergence of the best simplex point is shown to prevent clutter on the graph. The input parameters used in both methods are given on the figure. Note that eight different combinations of input parameters were tried for the SA case before achieving convergence to $OP = -36$ as obtained with the T-simplex. The improvement in calculation cost in going from SA to T-simplex is clearly demonstrated by observing convergence in ≈ 600 versus over 4000 function evaluations respectively. These shortcomings of SA have been discussed by Press and Teukolsky⁶⁵, where it is suggested that it is problematic to select a generator of random configuration changes that leads to efficient optimization. It is difficult to select a random change generator that doesn't almost always propose an uphill move when a local downhill move is possible. The modified simplex algorithm seems to offer an effective random move generator, as confirmed by the relatively fast optimization convergence shown in figure 3.11.

Chapter 4: A Novel design of gradient coils

4.1 Introduction

This chapter describes a novel gradient coil design, based on a hybridization of the cylindrical and surface-planar gradient coils discussed in chapter 2. The resulting design, named *Semi-Cylindrical Surface Gradient Coil* (SCSGC), offers a compromise between the fast switching but small volume of linearity (VOL) of the planar coils and the slower, larger VOL of the cylindrical designs.

The chapter begins with a description of the method used for the design of the SCSGC, including an overview of the algorithm of the computer program written to assist in the design. The usefulness of the method is then demonstrated by examining an optimized xyz SCSGC, designed and built for a 40 cm diameter bore NMR magnet.

4.2 Description of *Semi-Cylindrical Surface Gradient Coil* design

4.2.1 General considerations

The SCSGC design is based on distributing current onto the surface of an approximate semi-cylinder as illustrated in Figure 4.1. This figure also defines the coordinate system used in this discussion. The ideal shape of the current distribution was determined by computer optimization of the following parameters:

- a) Size and linearity of VOL,
- b) Gradient strength,
- c) Inductance.

In the case of y-gradient coils, a fourth optimization parameter was included that controlled the strength of the B_z field at the centre of the VOL.

Figure 4.1 Schematic of general SCSGC geometry and coordinate definition.

Figure 4.2a Definition of Quadrant Conventions.

Figure 4.2b Quadrant symmetry definition, and some examples of typical loop patterns.

This $B_z = 0$ offset, which was discussed in chapter 2 as a problem with planar surface gradient coils, can be controlled in the SCSGC design.

The overall goal in SCSGC design is to maximize the size of the VOL and the gradient strength, while minimizing the inductance. This was accomplished by first using trial and error (guided by an understanding of the fields produced by straight current elements and the symmetries needed to produce gradients in the three orthogonal directions) to determine a *starting point* for optimization. Next a *simplex* type algorithm was used to optimize the *starting point* current configuration for any, or all, of the above optimization parameters.

4.2.2 Method used for *Semi-Cylindrical Surface Gradient Coil* design

Defining a current configuration

In this work, the current distributions were obtained by wrapping loops of copper wire into grooves on the surface of semi-cylindrical formers. The design software represents these loops (and in fact the whole current configuration) by *node* points at the corners of the loops. The node points are characterized by several node parameters which will be discussed shortly in an example.

In order to simplify the number of nodes that must be specified when defining a current configuration, the surface has been separated into quadrants. The quadrant definition is illustrated in figure 4.2a. The software user need only specify node parameters for quadrant 1, and the program calculates the nodes for the other quadrants using the symmetry parameter associated with the node. The quadrant symmetries for gradients in the three directions are given in figure 4.2b.

A quadrant designated as positive (+ve) symmetry will have the current flow in the same sense (clockwise or counter-clockwise) as in quadrant 1. A negative (-ve) designation requires current flow in the opposite sense to that in quadrant 1. With all the nodes established in quadrant 1, the software then calculates all of the current elements present. Note that the current elements oriented along the z-axis need not be considered since they do not contribute to the z component of the magnetic field (see the Biot-Savart Law in Chapter 2).

Figure 4.3 gives an example of the node definition for a y-type gradient coil. As can be seen, nodes are found at the corners of current loops, and are defined by the following parameters:

$\rho...$ radial component in a cylindrical coordinate system

$\phi...$ azimuthal angle in a cylindrical coordinate system

$z...$ z component in a cylindrical coordinate system

$I...$ current passing through loop with sign convention:

+ve...current flows into node

-ve...current flows away from node

node symmetry... the *node symmetry* specifies how the node point is to be reflected into quadrants 2, 3, and 4.

The software follows certain rules for each reflection depending on the *symmetry type* and *node type*.

The node *symmetry* can have one of three values:

0...x type gradient

1...y type gradient

2...z type gradient.**node type...** there are three node types that specify

association with other nodes. When two nodes are associated together, each node

is said to be the *mate* of the other:

0...no mate: (e.g. node 1 in figure 4.3) implies association with only a non-specified reflection (about $\phi=90$) node.

1...primary with mate: (e.g. node 3) implies association with another node

(specified in its *node mate* parameter. In any pair of node mates, the

current is specified only for one of the nodes. This mate is known as *primary*.

2...secondary with mate: (e.g. node 5) implies association with another node; current specified with other mate.

node mate... specifies which node is mate. Node mate is defined only if *node type* = 1 or 2.

The node points specified as such fully define any current distribution (made of line elements). The node points can be saved in a file and/or used for further calculations (i.e. B_z , gradient, inductance, current configuration optimization).

Figure 4.3 Illustration of node definition.

Node #	Azimuthal angle ϕ (degrees)	Z position (cm)	Current (amperes)	Node type	Node symmetry	Node mate
1	75	2	-150	0	1	NA
2	75	14	+150	0	1	NA
3	25	6	+150	1	1	5
4	25	15	-150	1	1	6
5	45	6	-150	2	1	3
6	45	15	+150	2	1	4

B_z field calculations

All B_z calculations are done using the Biot-Savart Law. A numerical approximation is applied to integrate B_z over all the current elements in the defined current configuration. The software allows for a discrete B_z field to be calculated at grid points in Cartesian space. Either a line of points or a plane of points of arbitrary orientation can be calculated with full control over the resolution (or spacing) of the grid. The fields calculated as such can be stored in files and viewed graphically, and/or used in the coil optimization software to calculate gradient strength, size of VOL, etc.

Gradient field calculations

The gradient field associated with the above B_z field can be calculated for line or planar grids. Again, the gradient field can be stored in a file and viewed graphically, and/or used for subsequent calculations in coil optimization.

Volume of linearity calculations

The volume of linearity is defined as the volume over which the gradient strength does not vary by more than *tolerance*, a predefined value (usually set to 5%). A program function, known as *get_pen()*, makes the VOL calculation. The algorithm for *get_pen()* is given in figure 4.4a. The result is that for a given width in the x and z directions (pre-specified in software options), *get_pen()* calculates the y position of the largest region (largest # of y points) where the gradient value does not differ by more than *tolerance*.

Inductance calculations

The inductance of a given current configuration can be calculated using a numerical algorithm developed by the author specifically for current defined by the *node* convention. The inductance calculation is based on determining the *magnetic flux* Φ through each loop of the coil using a numerical evaluation of the integral¹

$$\Phi = \int_s \underline{B} \cdot d\underline{a} \quad , \quad (4.1)$$

where s is the surface contained by the loop, and $d\underline{a}$ is the vector element of area. To simplify having to evaluate all three components of the dot product, the radial component of the $\underline{B}$ field is calculated using the *Biot-Savart Law*. The surface chosen for the integral in equation (4.1) then becomes the cylindrical surface bound by the current loop, and the problem is reduced to evaluating only one component of the dot product. Once the net flux of the coil circuit has been evaluated, the inductance (L) can be determined by the equation³

$$L = \frac{\Phi}{I} \quad , \quad (4.2)$$

where I is the current assumed for the $\underline{B}$ field calculations used to determine Φ .

The inductance as calculated can be used as an optimization parameter in the coil optimization algorithm. The inductance calculation is computer time intensive, and was not used as an optimization parameter in this study. The inductance was, however, always checked after the maximization of gradient strength and size of VOL.

Figure 4.4a Algorithm of function `get_pen( )` that calculates the size of the volume of linearity (VOL).

Calculate gradient value for points on a line along y direction ($x=z=0.0$).

Function `pen( )`.

- Determine the largest region (# of y points)
where $(\text{grad}_{\text{max}} - \text{grad}_{\text{min}}) \leq \text{tolerance}$ (tolerance=5% usually).
- returns
 - start (`strt`) and end (`fin`) points (y-dir) of largest region.
 - spacing of grid in the y direction.
 - minimum and maximum values of gradient in the largest region.

Calculate gradient values for points in the volume defined by

- $x_{\text{wid}} \leq x \leq x_{\text{wid}}$,
- $\text{strt}-2 \leq y \leq \text{fin}+2$,
- $z_{\text{wid}} \leq z \leq z_{\text{wid}}$.

where x_{wid} and z_{wid} are the desired widths of the VOL in the x and z directions respectively. x_{wid} and z_{wid} are pre-specified in software.

Scanning from `strt-2` to `fin+2` **determine the largest region** (# of y points) where all points within $|x| \leq x_{\text{wid}}$ and $|z| \leq z_{\text{wid}}$ satisfy

$$\left| \frac{\text{gradient}_{\text{max}} - \text{gradient}_{\text{min}}}{\text{average gradient}} \right| \leq \text{tolerance} \quad .$$

Function returns:

- `strt` $\Leftrightarrow$ y value of the start of the VOL,
- `max_cnt` $\Leftrightarrow$ # of y direction points in the VOL,
- `bzcenter` $\Leftrightarrow$ value of B_z at the center of the VOL,
- average gradient $\Leftrightarrow$ average value of the gradient throughout the VOL.

Node group definition.

- define # of independent parameters (ip) per node in each of the node groups.
- define start and finish node # for each of the node groups.

Allocate memory for optimization arrays.

- opt_par $\Rightarrow$ node group definitions and initial current configuration.
- con $\Rightarrow$ physical constraints for the ip.
- pre $\Rightarrow$ precision (# of decimal places) for the ip.
- p $\Rightarrow$ initial array of ip to pass to Tsimplex algorithm.

Define initial node values.

- con,pre,opt_par (node group definitions).
- get initial current configuration from:
 - a) pre defined node configuration.
 - or
 - b) a node config. previously returned from Tsimplex.

Define parameters for the Tsimplex optimization.

- initial / final temperature (T) values
- power in the cooling schedule
(ie. path for T to follow from T_{max} to T_{min})
- # of passes and maximum # of function evaluations per pass
- characteristic size of simplex

Optimization Algorithm.

- Minimizes the result of min_func()^{*} using the Tsimplex algorithm.

Free allocated memory.

^{*} See figure 4.4c for algorithm of the function that calculates the OP (min_func()).

Figure 4.4d Algorithm for function `min_func( )` that calculates the OP.

Note: an array (`sim_pt`) of independent parameters is passed to `min_func( )`.

round off `sim_pt` to predefined precision `pre`.

`sim_pt` $\Rightarrow$ `node_pt`

transform `sim_pt` into form usable by B_z and Gradient calculation software.

Check `node_pt` for constraint violations.

-calculates a penalty to add to the minimization variable based on

$$\text{penalty} = \left[\frac{|\text{tv} - \text{violation constraints}|}{|\text{violation constraints} - \text{nonviolation constraints}|} \right]^{\text{power}}$$

where power is controllable in software.

Function `get_pen( )` (see figure 4.4d for algorithm of `get_pen( )`).

For a given width in the z and x directions, this function calculates the y position of the largest region (largest # of y points) where the gradient value does not differ by more than tolerance (ie. tolerance=5%) .

The function returns the number of points in the y direction (`max_cnt`) as a non-penalized OP.

Penalize for B_z at center of VOL $\neq 0$.

calculates a value called `bz_pen` based on:

$$\text{num} = \left| \frac{\text{bz}_{\text{center}}}{\text{average gradient}} \right|$$

$$\text{if } (\text{num} \leq \text{bz}_{\text{tol}} \text{ then } \text{bz}_{\text{pen}} = 0.0)$$

$$\text{else } \text{bz}_{\text{pen}} = 100 \times (\text{num} - \text{bz}_{\text{tol}})$$

Calculate penalized OP.

$$\text{OP} = -\text{max_cnt} + \text{bz}_{\text{pen}} + \text{penalty}$$

Return OP.

If the inductance was too high, the starting current configuration was changed in such a way as to reduce the inductance (i.e. reduce the number of current elements).

Optimization software

Figure 4.4b is an overview of the steps performed in determining the optimal current configuration.

The optimization of the current is based on a variation of the multidimensional downhill simplex method. The dimension (N) of the optimization is determined by the number of independent parameters (ip) that can be changed resulting in different values of the optimization parameter (OP). The ips in the coil optimization software are the position (p , ϕ , and z) and the current (I) associated with each node. The current is constrained to a set radius p , thus reducing the number of independent variables by $1/4$. Therefore, if there are 5 nodes, the optimization parameter is dependent on $3 \times 5 = 15$ independent variables (i.e. $N=15$).

The first step in determining the optimum current configuration is to obtain initial values for the N independent variables. This is done based on an understanding of how simple straight wire segments can form crude gradient fields. Trial and error (using the B_z and gradient software to map fields) assists in finding a reasonable starting point.

Figures 4.4a, 4.4c, and 4.4d give an overview of the optimization software algorithm. As can be seen from the darkly shaded box in figure 4.4c, the core of the optimization is the T-Simplex method which minimizes the result of a function *min_func()*. At the core of the function *min_func()* is the function *get_pen()* discussed above, and summarized in figure 4.4a.

The optimization parameter returned by *min_func()* is equivalent to -1 times the size of the VOL with some penalties added on to it. These penalties are due to violations of the specified physical constraints as well as B_z at the centre of the VOL being different from zero. Note also that if the gradient strength at the centre of the VOL is lower than a pre-defined value (G_{min}), an arbitrarily assigned large value of OP is returned. This will cause the simplex algorithm to favour current configurations with gradient strength greater than G_{min} . Tsimplex acts to minimize OP (penalties included), which basically results in a maximized size of VOL.

The T-Simplex algorithm is a slight variation on the well known simplex method combined with the recently described simulated annealing method. Both of these techniques are discussed in detail in chapter 3. Basically the simplex algorithm can be described as a stretching and contracting of an N-dimensional object (called a *simplex*) until a local minimum (or maximum) in the optimization parameter is found. The 'T' in the name refers to 'temperature' or 'sensitivity' of the optimization and is controllable in software. Starting with a high value of T and working to a low value allows one to be more confident of finding a *global*¹ minimum (or maximum).

¹In general the N-dimensional space is a complicated surface with many *local* maxima and minima. There will be one position in the space with the lowest value. This point is called the global minimum.

4.3 An optimized design for an xyz SCSGC

4.3.1 Introduction

As a test of the optimization software and algorithm, a SCSGC was designed and built for a 40 cm NMR magnet. The goal was to design a three directional gradient coil system capable of generating gradient fields for EPI. Specifically, the design criteria were set out as follows:

- Gradient strength > 4 G/cm at 150 A current,
- Gradient switching time < 100 μ s corresponding to inductances < 160 μ H,
- VOL > 3 cm $\times$ 3 cm $\times$ 3 cm.

The design generated by the optimization algorithm exceeded all of these specifications. Furthermore, the resultant constructed coil performed in accordance with the specifications generated by the software.

The method of construction involves wrapping wire in grooves along the inner (y-coil), and outer (x, z-coils) surfaces of a cylindrical former. The architecture for the optimized coil is therefore presented by showing the blueprints for the groove patterns accompanied by tables specifying the number of wire loops in each groove.

Section 4.3.2 presents the design blueprints as well as any other information needed to construct the SCSGC set. Section 4.3.3 shows the calculated specifications of the optimized coil, while section 4.3.4 presents calculated field plots from various orientations, and shows the size and location of the VOL corresponding to a tolerance of 5.0%.

Section 4.3.5 presents images obtained using a computer simulation of a spin echo imaging technique, and the calculated gradient fields corresponding to the optimized SCSGC design.

4.3.2 Blueprints and construction information for the optimized design

Blueprints

Figure 4.1b shows the orientation of the view from which the groove blueprints are given. The view given is one looking down from the z,x (or ϕ) plane along the y axis of figure 4.1a. Now, imagine unrolling the cylindrical surface to become a planar surface. The grooves are then mapped as a function of z and ϕ , where ϕ now represents the transformed (unrolled) version of the azimuthal angle.

Figure 4.2a shows the quadrant definition from the view of figure 4.1b. The groove blueprints give the groove positions only in the first (shaded) quadrant. The groove positions for the other three quadrants are exact reflections about the x and z axis. As discussed above, the sense, or direction, of the wire wrapping (i.e. current direction) although arbitrarily chosen in the first quadrant, must follow strict symmetry and anti-symmetry rules for the other three quadrants. The symmetry definitions for the wrapping sense of the x , y , and z gradient coils are given in figure 4.2b. In each case, the sense in quadrant one is defined as positive. The sense for the other three quadrants is then designated either as positive (same sense as quadrant one) or negative (opposite sense as quadrant one).

For example, in the case of the x-gradient, if current for a particular loop in the first quadrant has an azimuthal segment of current flowing to the left, then it will be flowing to the right, to the right, and to the left respectively in the second, third, and fourth quadrants.

Figure 4.5 shows a cross-section of the coil and defines the dimensions of the inner and outer radii of the semi-cylinder. Note that although the bore of the B_0 magnet is specified to be 40 cm in diameter, the factory installed gradient set limits the usable space to 30 cm in diameter. The SCSGC was therefore designed to fit tightly inside the 30 cm space using screw tighteners to facilitate a snug fit.

Figure 4.6 gives an enlarged view of the area of the cross-section circled in figure 4.5. This enlarged view demonstrates that the y-gradient coil is wound on the inner surface, while the x and z gradient coils are wound on the outer surface. The respective groove depth for each coil surface is also given. The z coil must be wound on the inner layer of the outer surface before the x coil is wound on the outer layer of the outer surface. As can be seen on both cross-sectional views, the inner surface (y-grad.) consists of two surfaces of differing radius. The smaller radius surface (shown in black) is for the $B_0 = 0$ correction current. In order to save on material, this correction surface need not be built to span the whole surface in the z direction, but can be constructed as smaller radius islands on the larger radius surface. The necessary size and position of the correction surface island is shown as the dark surface in figure 4.8.

Figure 4.5: SCSGC Crossection to Define Radial Dimensions.

Figure 4.6 Expanded region of SCSGC crossection.

Figures 4.7, 4.8, and 4.9 are the actual blueprints for the first quadrant grooves for the x, y, and z gradient coils respectively. The horizontal grooves are designated by numbered circles while the vertical grooves are designated by numbered squares. Tables 4.1, 4.2, and 4.3 give detailed numerical values of position and width of the grooves. In the case of the horizontal grooves, the tables specify:

- ▶ the start and finish azimuthal angle value
- ▶ the z position of the centre of the groove
- ▶ the width of the groove
- ▶ the depth of the groove,

and

- ▶ the number of wires to be wrapped in the groove.

The vertical grooves are referenced to vertical guidelines or "vertical markers" (dashed lines) on the figures. The tables specify the distance from these vertical markers to the edge of the groove. A positive value means that the edge of the groove is at a greater angle than the vertical marker, while a negative value specifies the edge at a lower angle than the marker. As can be seen in figure 4.7, some of the vertical grooves and their associated vertical markers (see groove A) do not have distance specifiers. It is not necessary to specify distances for such grooves because they are exact reflections of other grooves and groove markers. The groove marker at angle -5° and the groove marker at 35° are example of such reflection pairs. The distance value specified at A would be exactly negative the position value specified for vertical groove reference 8. All such vertical marker pairs are designated in the tables.

Figure 4.7 X-Gradient Coil Groove Blueprint.

@ designates grooves required to bridge the quadrants.

Table 4.1a X-Gradient Blueprint Information (Horizontal Grooves).

Groove #	start angle/ end angle (degrees)	Z position (cm)	groove width/ groove depth (cm)	# of wires ¹
1	43.6 → 81.4	2.5	0.2 / 0.4	1
2	45.0 → 80.0	4.9	0.2 / 0.4	1
3	46.4 → 78.6	7.3	0.2 / 0.4	1
4	-10.7 → 40.7	1.5	0.2 / 0.4	2
5	-8.6 → 38.6	3.8	0.4 / 0.4	3
6	-5.5 → 35.0	6.1	0.6 / 0.4	5
7	-0.7 → 30.7	8.4	0.6 / 0.4	5
8	46.4 → 78.6	18	0.6 / 0.4	-3
9	-0.7 → 30.7	18	1.8 / 0.4	-9

¹ positive means left to right wrapping sense, negative means right to left.

Table 4.1b X-Gradient Blueprint Information (Vertical grooves)

Groove #	1	2	3	4	5	6	7	8	9
Distance ¹ (degree)	0.5	-0.5	1.5	-1.5	-3.5	-1.5	1.5	4.5	-3.5
Groove depth (cm)	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

¹ Distance is measured from the vertical marker to the edge of the groove.

Vertical marker line reflection pairs:

- ▶ -5.0° and 35.0°
- ▶ 45.0° and 80.0°

Figure 4.8 Y-Gradient coil groove blueprint.

@ designates grooves required to bridge the quadrants.

Table 4.2a Y-Gradient Blueprint Information (Horizontal Grooves).

Groove #	start angle/ end angle (degrees)	Z position (cm)	groove width/ groove depth (cm)	# of wires ¹
1	35.0 → 90.0	1.1	0.2 / 0.4	0.5 [†]
2	35.0 → 90.0	3.5	0.2 / 0.4	0.5 [†]
3	36.0 → 90.0	5.9	0.2 / 0.4	0.5 [†]
4	36.0 → 90.0	8.3	0.2 / 0.4	0.5 [†]
5	-15.0 → 25.0	4.6 → 5.6 [*]	0.4 / 0.8	-7
6	-13.0 → 23.0	7.6 → 8.6 [*]	0.4 / 0.8	-8
7	36.0 → 90.0	14.2	0.4 / 0.4	-2
8	-13.0 → 23.0	14.4	0.8 / 0.8	15

¹ positive means left to right wrapping sense, negative means right to left.

[†] must make a parallel junction so that current from one wire is split into two.

^{*} start → finish values of z are given.

Table 4.2b Y-Gradient Blueprint Information (Vertical grooves)

Groove #	1	2	3	4	5	6
Distance ¹ (degree)	0.0	1.0	-1.0	0.0	2.0	-2.0
Groove depth (cm)	0.4	0.4	0.4	0.8	0.8	0.8

¹ Distance is measured from the vertical marker to the edge of the groove.

Vertical marker line reflection pairs:

► -15.0° and 25.0°

Figure 4.9 Z-Gradient coil groove blueprint.

Table 4.3a Z-Gradient Blueprint Information (Horizontal Grooves).

Groove #	start angle/ end angle (degrees)	Z position (cm)	groove width/ groove depth (cm)	# of wires ¹
1	41.0 → 90.0	9.1	0.4 / 0.9	3
2	39.0 → 90.0	11.3	0.4 / 0.9	3
3	37.0 → 90.0	13.5	0.4 / 0.9	3
4	4.5 → 23.5	3.1	0.6 / 0.9	5
5	1.5 → 26.5	5.9	0.6 / 0.9	6
6	-1.5 → 29.5	8.7	0.6 / 0.9	7

¹ positive means left to right wrapping sense, negative means right to left.

Table 4.3b Z-Gradient Blueprint Information (Vertical grooves)

Groove #	1	2	3	4	5	6	7	8
Distance ¹ (degree)	2.0	0.0	-2.0	3.0	0.0	-3.0	-6.0	-4.0
Groove depth (cm)	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9

¹ Distance is measured from the vertical marker to the edge of the groove.

Vertical marker line reflection pairs:

► 1.5° and 26.5°

The depth of all grooves can be found in the legend of the blue-print diagrams, as well as in the accompanying tables.

One important point must be mentioned in reference to the x-gradient coil. Figure 4.7 has three islands designated with black fill. These parts must be made separately from the rest of the former so that they can be fastened on after the wrapping of the x-gradient coil is completed. If this step is not followed, it is difficult if not impossible to wrap the z-coil whose grooves are below the x grooves on the outer surface of the former.

A final note about the x and y groove blueprints is that each contains grooves marked with the @ symbol. These grooves are included to bridge the gap between the quadrants.

Construction considerations

The material used for the semi-cylindrical former as well as the method of producing the grooves is left to the manufacturer. Note the following recommendations:

- There are strong Lorentz³ forces and torques on the structure. The material must not be brittle. Also, it is important to construct the coil system so that it can be securely fastened to the NMR magnet bore.

- The material must be non-magnetic.

For our coil, the wire former was constructed using a method known as *stereo lithography*[†]. In this method a *laser* is used to harden a liquid epoxy into any three dimensional shape desired.

[†]Stereo Lithography work was done by Institute for Advanced Manufacturing Technology, National Research Council Canada, Ottawa, K1A 0R6.

The resultant former gave grooves accurate to within 0.2 mm of the specified design, which is within the allowed tolerance of 0.5 mm for wire wrapping. During testing of the coil, forces from the fast switching caused some cracking and breakage of the former which seemed to become more brittle with the passage of time[†]. The former was immediately reinforced with fiberglass and epoxy, and no further damage has occurred.

To fasten the wire into the grooves, the wires were first glued into position with a fastener that bonds in ~10-20 seconds. Then, after each direction of coil was constructed and tested, the remaining space in the grooves was filled with epoxy.

There are many possible ways of wrapping the wire so that the net magnetic field is as desired. The exact route that is chosen is left to the wrapper. Several important considerations are necessary.

- ▶ The design is for 14 gauge (AWG) varnished copper wire.
- ▶ Each direction of gradient coil should be wrapped with one single length of wire to avoid post wrapping breakage in the current line. The fewer soldered joints the better. In the case of the y-gradient, there are four unavoidable solder points, due to the need for the current dividers (i.e. parallel circuit splits- see table 4.2). Be sure these solder points are strong before filling over with epoxy. The length of wire needed in each case must be calculated after the wrapper has chosen the exact route the wire will take.

[†]According to the Institute for Advanced Manufacturing Technology group, recent advancements in materials for stereo lithography have eliminated this problem.

A rough guide for the lengths needed is approximately 30, 25, and 23 meters for the x, y, and z directions respectively.

- The number of wires that must go through each groove is given in the tables. It is best to consider each horizontal groove, and determine the current loops associated with it. One must then choose a route such that the necessary loops are completed exactly.

- All of the quadrant jumping grooves marked with @ in the blueprints must have a net current of zero flowing through them. That means that there must be an equal number of wires wrapped in each direction along them.

- Using the above two considerations, it is possible to plan a route such that all of the necessary current loops are exactly completed (that is fully closed loops), while all the quadrant joining current paths have no net current in them. It is highly recommended that this route planning stage be carried out on paper, and that the result is double checked for these considerations.

- For grooves with more than one wire through them, one must determine a packing order. The design has allowed for 2 mm width per wire. The packing order must be such that when the wrapping is finished, the grooves are filled from the bottom of the groove up. An example best illustrates this. Horizontal groove 6 in figure 4.8 is specified to have 7 wires in it. The groove is made to be 6 mm (three wire widths) wide. Therefore when the wrapping is finished, there should be three wires on the bottom layer, three wires in the second layer, and one wire on the top layer. Note, it is sometimes convenient to start a new layer before finishing the present one.

This is acceptable, as long as the end result has the bottom heavy packing result.

- It is helpful to use small wooden sticks to bend the wire, and hold the wire in place while the glue hardens. The sticks can be made from doweling, and can have their ends carved into helpful tools.

- Care should be taken not to scratch the insulating varnish on the wire, else a short in the coil circuit could result.

4.3.3 Calculated coil performance specifications

The resistance, inductance, size of VOL and gradient strength of the optimized 30 cm bore SCSGC have been calculated using the above described software. The results are summarized in table 4.4. As can be seen all the specifications meet or exceed the design criteria discussed in section 4.3.1. Note that a current of 150 amperes was chosen in the calculations because this is the maximum current delivered by our gradient amplifier system. The 14 gauge wire chosen should be able to handle twice this current (300 A) for the short duty cycles used in most NMR gradient sequences.

4.3.4 Calculated B_z fields

Further insight into performance is gained by examining the B_z fields produced by each of the three directions of the optimized coil. Figures 4.10, 4.11 and 4.12 show contour plots of the B_z field for the x, y, and z gradient directions respectively. Plotted on the same graphs is the boundary of the region where the gradient does not differ by more than 5%. The B_z contours are represented by the thin lines in the figures, while the 5% difference lines are thicker. For each gradient direction, three plane orientations are given.

Table 4.4 Calculated specifications of the designed and constructed X, Y, and Z SCSGC.

Gradient Direction	Resistance³ ($\pm 0.02\Omega$)	Inductance ($\mu H \pm 20\%$)	Size of VOL¹ ($\pm 0.1\text{cm}$)	Gradient strength² ($\pm 0.2\text{G/cm}$)
X	0.26	140	x=4.0 y=3.0 z=4.0	6.1
Y	0.17	120	x=3.0 y=3.0 z=3.0	4.1
Z	0.18	110	x=3.0 y=3.0 z=3.0	5.4

¹ The volume of linearity (VOL) is the volume over which the gradient strength does not vary by more than 5.0%. The VOL is given by its x,y, and z directions in cm.

² At current of 150 Amperes .

³ Assumes 14 gauge (AWG) wire and 40° Celsius .

Figure 4.10 Contour plots of B_z (gauss) in the xy , yz , and zx planes for the x -gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

Figure 4.11 Contour plots of B_z (gauss) in the xy , yz , and xz planes for the y -gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

Figure 4.12 Contour plots of B_z (gauss) in the xy, yz, and zx planes for the z-gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

Each plane orientation, in turn, is given at three separate values of displacement, resulting in a total of nine plots per gradient direction.

To calculate the % difference lines, the gradient field is calculated as described in section 4.2.2. The gradient strength is found to vary smoothly in all directions. The % difference is then simply calculated at each point on the grid as

$$\%diff = \left| \frac{\text{gradient value} - \text{average gradient}}{\text{average gradient}} \right|$$

where *average gradient* is the average value of gradient over the volume defined in figure 4.4a. This region is slightly different from the VOL calculated in the optimization software of figure 4.4. The difference is illustrated by looking at the results for the optimized y-gradient coil. Table 4.4 shows the y-gradient as having a VOL of 3.0 cm × 3.0 cm × 3.0 cm for the x, y, and z directions respectively. Examination of figure 4.11 shows that nowhere within a volume of 4.0 cm × 3.5 cm × 3.5 cm is the % difference greater than 5%. The same observation can be made of the other two gradient directions as well.

4.3.5 Simulated images using calculated fields

Software has been written for this thesis that simulates NMR signal acquisition in the presence of various pulse/gradient sequences. Gradient fields for imaging simulation can be chosen as linear, or defined from calculated fields due to a specified node configuration. Phantom geometries can be programmed in as needed. Image reconstruction is sequence dependent, and has been programmed for each of the imaging methods available.

Figure 4.13 is a simulated image of a 6 cm $\times$ 6 cm grid phantom. The imaging method is *spin echo* (see chapter 1) with a 6 cm FOV and 64 $\times$ 64 pixel resolution. Relaxation effects have been ignored. Figure 4.14 shows the results of the same simulation using the gradient values calculated from the B_z fields in figures 4.10 to 4.12. Simulated images from the three principal orthogonal planes with zero offset are shown. The read and phase encode direction has been toggled for each of the plane orientations.

4.4 Performance of the constructed optimized SCSGC set

4.4.1 Introduction

An xyz SCSGC was built using the method discussed in section 4.3. Figure 4.15 is a photograph of the finished constructed coil set. The coil was constructed for two reasons:

- 1) A coil was needed for the 40 cm magnet capable of generating gradient fields for EPI imaging sequences.

- 2) Verification of the coil design and optimization software was necessary.

As will be seen, the actual performance of the constructed coil agrees well with the specifications calculated in software. The performance will be evaluated by comparing the B_z field calculated in software to fields measured by experiment. Simulated images will then be compared to actual images acquired using the constructed coil set. Finally, rise time measurements will be presented that determine the switching speed of the constructed coils.

Figure 4.13 Simulated images of a grid and circular phantom with linear gradients. FOV=6.0 cm Resolution=64x64.

Figure 4.14 Simulated images of a grid phantom using gradient fields calculated for the 30 cm SCSGC. FOV=6.0 cm Resolution=64 x 64.

Figure 4.15 Photograph of constructed 30 cm SCSGC.

4.4.2 Measured B_z fields

The B_z field was measured using the following two independent methods:

- 1) Gaussmeter.
- 2) NMR spectrometer.

The results will be presented separately.

Gaussmeter measurements

Method

A *Lakeshore 410 Gaussmeter* was used to make measurements of the z component of the magnetic field due to the x , y , and z SCSGC at various grid points in space. For each direction of the coil, B_z was measured at 6×5 grid points on the xy plane respectively. For each coil, four such xy plane grids were measured, corresponding to four different positions of z .

A photograph of the apparatus is shown in figure 4.16.

The probe of the gaussmeter consists of a Hall probe which measures the component of the magnetic field perpendicular to its surface. The probe was oriented to face the z direction, and held in position by a stereotactic mount. Once fastened into the mount, the probe position was controllable with a precision of 0.1 mm. The probe was capable of measuring magnetic field strength to ± 0.2 gauss.

The coil was connected to a Techron model 7570 amplifier capable of delivering ≈ 20 amperes of direct current to the coil. The amplifier and connected cable are not shown in the photograph.

Figure 4.16 Photograph showing gaussmeter and stereotactic holder with coil in position

The position of the centre of the coil is needed in order to make absolute the coordinates determined from the stereotactic mount's vernier scales. This is needed to enable a one-to-one mapping between the calculated and measured fields. To determine the centre of the coil (the x , y , z points where $B_z=0$ for each of the coil directions), the following procedure was used:

1) Estimate the approximate centre (x , y , and z) of the coil.

2) To find the x centre:

►Measure a line of B_z points in the x direction, starting from well onto one side of the estimated centre of x , and scanning to well onto the other side. The y , and z values of the line were chosen as the estimated value of the centre.

►Plot a graph of B_z as a function of x .

►Get the centre from the $B_z=0$ point on the graph.

3) Repeat for the y , and z directions.

Results

x,y,z -centre

Figure 4.17 shows the plots used to determine the stereotactic mount vernier scale reading corresponding to the centre of the coil. At the side of each plot is the result from a linear regression of the data. In all three directions the correlation is excellent, and the resulting coordinates are:

$$x = 5.6 \pm 0.1 \text{ cm}$$

$$y = 3.6 \pm 0.1 \text{ cm}$$

$$z = 5.0 \pm 0.1 \text{ cm.}$$

Figure 4.17a B_z vs x to determine x center.

$$B_z = -3.72 + 0.67 x$$

$$\text{correlation} = 0.99971$$

$$\therefore B_z = 0 \text{ at } x = 5.6 \pm 0.1 \text{ cm}$$

Figure 4.17b B_z vs y to determine y center.

$$B_z = -1.64 + 0.45 y$$

$$\text{correlation} = 0.99873$$

$$\therefore B_z = 0 \text{ at } y = 3.6 \pm 0.1 \text{ cm}$$

Figure 4.17c B_z vs z to determine z center.

$$B_z = -2.89 + 0.58 z$$

$$\text{correlation} = 0.99833$$

$$\therefore B_z = 0 \text{ at } z = 5.0 \pm 0.1 \text{ cm}$$

These coordinates mark the position of the centre of the VOL in terms of the vernier scale readings on the stereotactic mount. This information determines the mapping between vernier scale and calculation coordinates as:

stereotactic $\Leftrightarrow$ calculation

$$x = 5.6 \text{ cm} \Leftrightarrow x = 0.0$$

$$y = 3.6 \text{ cm} \Leftrightarrow y = y_{\text{centre of VOL}}$$

$$z = 5.0 \text{ cm} \Leftrightarrow z = 0.0$$

where the y center of VOL is determined in the optimization function `get_pen()` (see figure 4.4a).

B_z field plots

The above mapping is used to overlay the B_z field measured with the gaussmeter, with the B_z field calculated in software. Maps of the xy plane field were obtained at four different positions of z, for each direction of gradient. The results for the x direction are shown in Figures 4.18 a, b, c, and d which give the plots for the z=0.0 cm, z=2.0 cm, z=4.0 cm, and z=6.0 cm xy planes respectively. The left side of each illustration shows a three dimensional wire plot of the measured (black) and calculated (white) B_z fields as a function of the x and y position. On the right of each figure, a grey-scale map of the difference

$$B_z \text{ difference} = \text{measured } B_z - \text{calculated } B_z$$

between the measured and calculated B_z field is given. The uncertainty in the measured B_z is 0.2 gauss, and the uncertainty in the calculated is taken to be negligible, so the corresponding uncertainty in the difference is 0.2 gauss.

Figure 4.18 Calculated and measured (gauss-meter) B_z as a function of x and y for the x-gradient coil. On the right is a grey scale map of measured - calculated B_z .

a) $z=0.0$ cm

b) $z=2.0$ cm

Figure 4.18 (con't) Calculated and measured (gauss-meter) B_z as a function of x and y for the x-gradient coil. On the right is a grey scale map of measured - calculated B_z .

c) $z=4.0\text{cm}$

d) $z=6.0\text{cm}$

The grey scale maps show any position with B_z difference > 0.2 G as black, while positions with B_z difference < -0.2 G are designated as white. All values within the experimental uncertainty are scaled with various tones of grey. Figure 4.19 and figure 4.20 give the results for the y and z gradient directions respectively.

Spectrometer measurements

The B_z field can be indirectly mapped using proton spectroscopy. The general principle of spectroscopy has already been discussed in chapter 1. As was shown, under the right experimental conditions, a homogeneous sample (containing nuclei all with the same resonance frequency) can lead to a frequency spectrum with a single well defined peak. The position of the peak is determined by the frequency at which the nuclei are oscillating, which in turn is determined by the B_z field at their location. To accurately map the B_z field inside a spectrometer, an rf probe and a sample must be constructed that have the following properties:

- 1) The physical size of the sample and coil must be as small as possible to optimize the resolution of the measurement.
- 2) The sample and coil must be large enough to afford a reasonable signal to noise ratio.
- 3) The sample coil combination must result in a spectral peak with a small line width to facilitate accurate measurements of peak position and hence frequency.

Clearly, properties 1 and 2 conflict with each other, and a compromise must be found

Figure 4.19 Calculated and measured (gauss-meter) B_z as a function of x and y for the y -gradient coil. On the right is a grey scale map of measured - calculated B_z .

Figure 4.19 (con't) Calculated and measured (gauss-meter) B_z as a function of x and y for the y -gradient coil. On the right is a grey scale map of measured - calculated B_z .

Figure 4.20 Calculated and measured (gauss-meter) B_z as a function of x and y for the z -gradient coil. On the right is a grey scale map of measured - calculated B_z .

Figure 4.20 (con't) Calculated and measured (gauss-meter) B_z as a function of x and y for the z -gradient coil. On the right is a grey scale map of measured - calculated B_z .

c) $z=4.0$ cm

d) $z=6.0$ cm

The mapping of the B_z field due to a gradient coil has some additional technical problems which are not present for B_0 coils. The presence of a gradient causes the field over even a small sample to vary over the dimensions of the sample. This results in an intrinsic broadening of the spectral line (stronger gradient $\Rightarrow$ broader spectrum). The broader the spectrum, the greater the uncertainty in the position of the frequency peak. However, in order to resolve the shifts in frequency at different positions, a stronger gradient is required (i.e. larger gradient $\Rightarrow$ larger gradient shifts). A compromise is required to find a gradient strength just strong enough to resolve the required separation in frequency. Such a gradient strength will give the minimum allowed uncertainty in the frequency measurement.

Method

The experimental setup is shown in figures 4.21a and b.

The rf coil/sample combination was designed and built[†] specifically for mapping the field inside of the 3T whole body magnet at NRC. A photograph of the coil and sample is given in figure 4.21b. The sample is a spherical glass container with water sealed inside. The coil consists of a copper wire wrapped around the sample in a solenoid-like form. The spherical sample container has an inner diameter of 4.0 mm. The solenoid coil is preceded by a 1 pF capacitor in series with a parallel circuit of a pair of 6.8 pF capacitors. The spectral shape and linewidth were found to be highly dependent on the shape of the wire leads to the coil. Trial and error (bending the leads) determined the optimal configuration for minimum linewidth.

[†]The coil/sample were designed and built by Zenon Starcuk at the NRC lab.

Figure 4.21a Schematic illustration of the stereo-tactic mount use for the 3 T whole body magnet.

Figure 4.21b Photograph showing field mapping probe and sample

The SCSGC was placed into the bore of the 3T magnet, and fastened into place with duct tape. The coils were connected to the existing gradient amplifier system used for the gradient set installed in the 3T magnet. The amplifier system (for each direction of coil) consisting of 4 Techtron model 7700 amplifiers was connected as a parallel pair of 2 amplifiers in series. The net result is an amplifier system capable of generating 300 V peak voltage, and a peak current of 360 A. Note: This amplifier system has more than enough power to cause extensive damage to the 40 cm SCSGC set. Therefore, care was taken to ensure that only very small gradient signals were used.

The sample was held in position by a stereotactic (s.t.) holder designed specifically for the Magnex[†] 3T magnet. The s.t. holder is shown schematically in figure 4.21a. The holder offers three degrees of freedom for the position of the sample:

- a sliding mechanism in the Z direction,
- a sliding mechanism in the radial direction,
- and a rotating mechanism in the azimuthal direction.

Both of the sliding mechanisms for the z and radial directions offer infinitesimal positioning abilities. The azimuthal position can be selected with 15 degree increments. The centre of the azimuthal rotation is the centre of the 3T magnet bore. Because of this, and the small dimensions of the 40 cm SCSGC set, only one plane (yz) of B_z mapping was physically possible.

[†]Magnex Manufacturing Inc. Abingdon, U.K. (MRI magnet manufacturers)

The spectrometer acquisition parameters were set as follows:

Sweep-width	50 kHz.
# points in acquisition	4096 (real + imaginary).
Spectrometer frequency	127.751 MHz.
Gradient ramp time	512 μ s.

The spectrometer pulse sequence used can be seen in figure 4.22a. The gradients can be switched on independently in the software. To select an appropriate gradient strength for resolution of the B_z field, the strength was increased from zero until the necessary spectral separation between neighbouring points along the z direction, was observed in the experimental protocol to be described next. The gradient strength used was measured to be 622 Hz/cm. For the 3T magnet this corresponds to a gradient strength of 0.05 G/cm. Figure 4.22b gives an example of the spectra used for the gradient field mapping. The linewidth of the spectrum can be measured as the width at half maximum to be 60 Hz.

Software has been written[†] that runs the spectrometer acquisition and processing in an automated protocol. In this protocol, the above pulse sequence is repeated with a specified period, for a specified number of FID acquisitions. The software then takes each of the obtained FIDs, Fourier transforms to spectra, and calculates the positions of the peak. The peak position in Hz for each of the acquisitions is then stored sequentially in a file.

[†]Zenon Starcuk NRC.

Figure 4.22b Example spectrum for measurement of B_z field. $x=y=0.0\text{ cm}$, $z=7.0\text{ cm}$, z -gradient on. Spectral position scale -- 1 point = 48 Hz.

For the SCSGC B_z spectroscopy field measurements, the xyz centre of the coil in terms of the 3T s.t.-holder was needed to allow superposition with the calculated field. In the case of the z-direction, the procedure already described for *centering* in the gaussmeter method section was used. Physical constraints dictated that the x position was fixed to one location, while the y centre was beyond the reach of the probe on the s.t.-holder. The centres for the x and y direction were calculated after the acquisition of the spectroscopy field measurements. These calculations will be shown in the results section.

With the z centre determined, the spectrometer automated protocol was set to acquire 11 sequential FIDs allowing enough time between each acquisition to physically move the s.t.-holder in the z direction. For each scan group of 11 acquisitions, the z position was varied from 0.0 to 10.0 cm at increments of 1.0 cm. Without changing the y position, this step (11 points on a z line) was repeated for the x, y, and z gradients turned on, as well as with no gradients. The y position was then changed, and the z line acquisitions were repeated. The y position was varied from the maximum extension of the s.t.-holder (sty_{max}) to $sty_{max}+4.0$ cm at increments of 0.5 cm. The resulting grid of B_z points is clearly 9 by 11 in the y and z directions respectively.

It should be noted that the spectra were also measured for the case of no gradient turned on. This was to eliminate the field variations caused by the inhomogeneity in the main 3T B_z field. For this study, only the field due to the gradient coil is of interest, and the background field can be subtracted away.

For each point on the grid, the frequency value obtained with no gradient present is subtracted from the frequency value with the gradient on. Because the fields are independent, the result is the frequency due to the gradient coil alone.

Results

Figure 4.23 shows an example of the spectra from one acquisition group, for the z gradient turned on. Table 4.5 quantifies the peak positions for the five peaks chosen. Both the peak position measured from figure 4.23 and the values calculated in software are included for comparison. The average gradient strength can be calculated to be 622 Hz/cm.

The centre of the SCSGC in terms of the 3T s.t.-holder must be determined in order to be able to superimpose the spectroscopically measured B_z and the calculated B_z . In fact the centre was needed before calculations of B_z could be carried out.

Z-centre

The z -centre of SCSGC is the z point where $G_z = 0.0$ when the z -gradient is on. This was determined by setting the number of acquisitions in the automated spectroscopy protocol to 17, and scanning in the z direction with the gradient turned on to a positive value. The procedure was repeated for the same z range but with a negative value of gradient. The z point where the two curves cross must be the centre of the z -coil. The result is shown in figure 4.24 where the frequency is plotted against the z position. The two curves meet at the point 8.5 cm. This point was marked on the s.t.-holder as the $z = 0.0$ position.

Figure 4.23 Superimposed series of spectra each corresponding to a different position of z .

Table 4.5 Comparison of peak positions determined from software versus graphical method.

Peak label	spectral position	centre position (1028) - spectral position	peak position determined from graph (Hz)	peak position as calculated in software (Hz)	difference (Hz)
z_{max}	870	154	3758	3834	76
$z_{max} + 2cm$	924	100	2440	2562	122
$z_{max} + 4cm$	975	49	1196	1300	104
$z_{max} + 6cm$	1030	-6	-146	28	173
$z_{max} + 8cm$	1074	-50	-1220	-1041	179

Figure 4.24 Plot of frequency vs z position to determine center of z-coil.

Table 4.6a Segment of frequency data for y-gradient coil. Shaded part used in calculations. All frequencies in Hz.

	$y=sty_{max}$	+ 0.5cm	+ 1.0cm	+ 1.5cm
$z=0.0$	427	658	852	1039
1.0cm	394	674	817	1052
2.0cm	421	698	840	1081
3.0cm	444	734	863	1115

Table 4.6b Segment of frequency data for x-gradient coil. Shaded part used in calculations. All frequencies in Hz.

	$y=sty_{max}$	+ 0.5cm	+ 1.0cm	+ 1.5cm
$z=0.0$	725	849	778	789
1.0cm	690	861	788	804
2.0cm	736	865	791	804
3.0cm	733	855	781	789

Allowing for the possible variation (due to uncertainty in the frequency) in the position of both lines in figure 4.24, the uncertainty in the position of the centre is 0.25 cm.

Y-centre

To determine the y centre of the SCSGC, some calculations must be made using the spectroscopic data obtained in the presence of a y-gradient. Table 4.6a shows a 4 by 4 segment of the frequency data obtained from the y-gradient spectroscopy experiment. The upper left box gives the frequency obtained (427 Hz) when the s.t.-holder was at the position $y = sty_{max}$, $z = 0.0$. In order to obtain the value of sty_{max} , the following assumption was made:

Assume that the gradient is linear over the region to be considered for the calculations.

To validate this assumption, only the shaded region was used in the calculation that follows. This region can be considered to have linear gradients because the optimization software calculates a 5% VOL with $z < 4.0$ cm and $y < 3.0$ cm. Therefore the gradient is linear in the z direction at least out to 2.0 cm, to where the shaded region is extended. Remember the VOL is also reflected about the $z=0$ point (ie. from -2.0 cm to +2.0 cm). The y region of assumed linearity extends to $sty_{max}+0.5$ cm. This value was chosen by measuring with a ruler (graduated in mm) the distance from the surface of the SCSGC to the position of the sample/coil. Using this measurement of 6.0 cm and knowledge (from optimization software) that the centre of the y coil should be at 5.0 cm from the surface, it was *estimated* that $sty_{max}=1.0$ cm relative to the centre. Therefore the shaded region was chosen to extend to 1.5 cm from the centre according to the 3.0 cm y VOL.

Now, the average frequency at sty_{max} is 414 Hz (at $y=sty_{max}$, $f=414$ Hz). The average change in frequency between $y=sty_{max}$ and $y=sty_{max}+0.5$ cm is 211 Hz. Therefore a distance of 0.5 cm corresponds to a frequency change of 211 Hz ($\Delta y=0.5$ cm, $\Delta f=211$ Hz). So to calculate the value of sty_{max} , one must evaluate the following ratio equation:

$$\frac{0.5 \text{ cm}}{sty_{max}} = \frac{211 \text{ Hz}}{414 \text{ Hz}} \quad (4.3)$$

$$\therefore sty_{max} = 0.98 \text{ cm} = 1.0 \pm 0.5 \text{ cm} .$$

X-centre

A similar method was used to calculate the x position (x_{shift}) of the yz plane. Table 4.6b gives the frequency values for the same region as for the y-gradient. From optimization software, we know that the strengths of the x and y gradients are related by

$$\frac{G_x}{G_y} = \frac{6.1}{3.9} = 1.564 \quad (4.4)$$

Also, we know from the above y-centre calculations that the absolute strength of the y gradient was $G_y=422$ Hz/cm. Therefore $G_x = 1.564$ and $G_y = 560.1$ Hz/cm. From table 4.6b, the average frequency of the plane is found to be 738 Hz.

This frequency corresponds to a shift in the x direction of

$$x_{\text{shift}} = \frac{764 \text{ Hz}}{660 \text{ Hz/cm}} = 1.1 \pm 0.5 \text{ cm} \quad (4.5)$$

B_z field plots

The above values for the positions of x , y , and z are used to overlay the B_z field measured with the spectrometer, with the B_z field calculated in software. Maps of the yz plane ($x=1.2 \text{ cm}$) were obtained, for each direction of gradient. The results are shown in figures 4.25, 4.26, and 4.27 which give the plots for the x , y , and z gradient directions respectively. At the top of each illustration is a three dimensional wire plot of the spectroscopically measured (black) and calculated (white) B_z fields as a function of the x and y position. Note that the B_z values are presented as frequencies (in Hertz) relative to a frequency of zero for $B_z = 3.0\text{T}$. Below the wire map, a grey-scale map of the difference

$$B_z \text{ difference} = \text{measured frequency} - \text{calculated frequency}$$

between the measured and calculated B_z field is given. The uncertainty in the values of frequency from spectral analysis has been determined to be $\pm 20 \text{ Hz}$. However, further uncertainties arise due to error in alignment of the *st-mount* relative to the SCSGC. The effective uncertainty in the measured B_z has been taken to be $\pm 100 \text{ Hz}$, and the uncertainty in the calculated is taken to be negligible, so the corresponding uncertainty in the difference is $\pm 100 \text{ Hz}$. This 100 Hz includes the 20 Hz on the spectral value and the combined uncertainties of the values of the x , y , and z positions

Figure 4.25 Calculated and measured (spectroscopy) B_z as a function of y and z for the x-gradient coil. Below is a grey scale map of measured - calculated B_z .

Figure 4.26 Calculated and measured (spectroscopy) B_z as a function of y and z for the y -gradient coil. Below is a grey scale map of measured - calculated B_z .

Figure 4.27 Calculated and measured (spectroscopy) B_z as a function of y and z for the z -gradient coil. Below is a grey scale map of measured - calculated B_z .

The grey scale maps show any position with B_z difference > 100 Hz as black, while positions with B_z difference < -100 Hz are designated as white. All values within the experimental uncertainty are displayed with various tones of grey.

4.4.3 Images using constructed coil

With the SCSGC fastened inside the bore of the NRC's 40 cm 4.7T NMR magnet, images were obtained to qualitatively evaluate the linearity of the gradient field. For this purpose, a cylindrical bottle containing water was used as a phantom. The inner diameter of the water sample was previously measured (using the permanently installed, calibrated cylindrical gradient set) to be 4.9 cm. The spin echo imaging sequence described in chapter 1 was used with the following parameters:

slice direction:	z ,
slice thickness (z direction):	4.0 mm,
FOV (x and y):	6 cm $\times$ 6 cm,
resolution:	64 $\times$ 64 , and
phase encode direction:	x .

A saddle-type rf-coil with an inner diameter of 7 cm was used to excite and receive signal from the phantom. Morris² suggests that such a saddle coil produces a uniform rf-field over radial positions not in excess of $0.7a$ where a is the radius of the coil cylinder.

Figure 4.28a and b give the resulting experimental images as well as images generated using the NMR simulation software written for this thesis. Relaxation effects have been neglected in the simulated images. Images acquired at six different z offset values ranging from $z = 0.0$ cm to $z = 6.0$ cm are compared

Figure 4.28a Actual and simulated x,y slice images of a cylindrical homogeneous water phantom. Slice thickness = 4 mm, FOV = 6.0 cm, resolution = 64 x 64. Phase gradient in x direction.

Figure 4.28b Actual and simulated x,y slice images of a cylindrical homogeneous water phantom. Slice thickness = 4 mm. FOV = 6.0 cm, resolution = 64×64 . Phase gradient in x direction.

Due to the nature of the geometry of the gradient and rf coils, it was not possible to position the cylindrical phantom in the y-centre of the gradient coil. This is apparent in all of the actual (experimental) images in the figures. Further examination shows the y-centre offset worsens as the z offset increases. This gradual y-shifting is also due to physical geometrical constraints and is a result of the rf-coil not being completely horizontal. The simulated images have all been centred in both the x and y directions.

4.4.4 Measured rise time and specifications

The rise time measurements were made on the 4.7T 40 cm spectrometer. To determine a quantitative value for rise time, the output of the gradient amplifier was monitored by an oscilloscope. At a specified time during the trace cycle, the amplifier was switched on from zero to a chosen value between 0 and 28 in arbitrary units (28 corresponds to the maximum gradient strength allowed by the controller). The gradient controller has three ramp-switch settings, which determine the voltage signal that is applied to the gradient coil during the gradient rise time. The switches, labelled 1,2,3, attempt to ramp the gradients to the specified value in 512 μ s, 128 μ s, and 64 μ s respectively. However, the actual ramp time is of course dependent on the properties of the coil. The specific path followed by the voltage versus time curve on approach to the desired voltage is dependent on the coil as well. For this reason the measured rise time was arbitrarily defined as the time taken for the voltage to reach 90% of the specified gradient value. In this fashion, rise time measurements were made with the x, y, and z directions for both the SCSGC and the permanently installed cylindrical gradient set.

For each coil, 5 different gradient strengths, between 5 and 28 (arbitrary units), were evaluated.

The results of the rise time measurements can be seen in figures 4.29a, b, and c for the x, y, and z directions respectively. The rise time values for the installed gradients are shown as solid symbols while the SCSGC values have hollow symbols. Note that only the ramp-switch 2 and 3 results are shown because all the coils handled the slowest setting (ramp-switch 1) with no problems (i.e. all coils, and all voltages gave rise times of $470 \pm 10 \mu\text{s}$).

The measured and calculated performance specifications of the SCSGC are summarized in tables 4.7a, b, and c for the x, y, and z directions respectively. For comparison, the tables also show the corresponding measured specifications of the permanently installed cylindrical gradient set.

4.5 Discussion of the constructed 40 cm SCSGC

4.5.1 Introduction

The results of the constructed SCSGC gradient coil system will now be discussed. The design resulting from optimization gave gradient fields linear to within 5% over an xyz volume of at least $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$, with strength $> 4.0 \text{ G/cm}$ in all directions. The switching time (to 90% of maximum or $\approx 140 \text{ A}$) was found to be $< 160 \mu\text{s}$ for all directions.

Table 4.7a Summary of specifications for the x gradient.

	X SCSGC			Installed gradient X coil Measured
	Measured	Calculated	% diff.	
Resistance (Ω)	0.25 ± 0.01	0.26 ± 0.05	4.0	0.38 ± 0.05
Inductance (μH)	152 ± 5	140 ± 25	7.9	225 ± 10
Strength (G/cm)	6.1 ± 0.3	6.1 ± 0.2	0.0	4.2 ± 0.2
Rise time (μs)	165 ± 10	not calculated		270 ± 10
size of VOL (xyz cm ³)	not measured ¹	4×3×4		not measured ²

¹ Although this parameter was not calculated specifically, the measured and calculated B_z field match well. Hence the VOL should correspond.

² The VOL was not measured, but it is known to be considerably larger than that of the SCSGC.

Figure 4.29a Rise time (0 → 90% of gradient value) vs gradient strength for the installed and SCSGC x gradients.

Table 4.7b Summary of specifications for the y gradient.

	Y SCSGC			Installed gradient Y coil Measured
	Measured	Calculated	% diff.	
Resistance (Ω)	0.18 ± 0.01	0.20 ± 0.02	11.0	0.38 ± 0.05
Inductance (μH)	112 ± 5	$120 \pm 20\%$	7.1	218 ± 10
Strength (G/cm)	4.3 ± 0.3	4.1 ± 0.2	4.8	4.2 ± 0.2
Rise time (μs)	140 ± 10	not calculated		240 ± 10
size of VOL (xyz cm ³)	not measured ¹	3x3x3		not measured ²

¹ Although this parameter was not calculated specifically, the measured and calculated B₁ field match well. Hence the VOL should correspond.

² The VOL was not measured, but it is known to be considerably larger than that of the SCSGC.

Figure 4.29b Rise time (0 → 90% of gradient value) vs gradient strength for the installed and SCSGC y gradients.

Table 4.7c Summary of specifications for the z gradient.

	Z SCSGC			Installed gradient Z coil
	Measured	Calculated	% diff.	Measured
Resistance (Ω)	0.18 ± 0.01	0.18 ± 0.02	0.0	0.36 ± 0.05
Inductance (μH)	105 ± 5	$110 \pm 20\%$	4.7	160 ± 10
Strength (G/cm)	5.6 ± 0.3	5.4 ± 0.2	3.7	3.7 ± 0.2
Rise time (μs)	140 ± 10	not calculated		240 ± 10
size of VOL (xyz cm^3)	not measured ¹	$3 \times 3 \times 3$		not measured ²

¹ Although this parameter was not calculated specifically, the measured and calculated B_z fields match well. Hence the VOL should correspond.

² The VOL was not measured, but it is known to be considerably larger than that of the SCSGC.

Figure 4.29c Rise time (0 $\rightarrow$ 90% of gradient value) vs gradient strength for the installed and SCSGC z gradients.

4.5.2 Calculated B_x field contour plots

For the x gradient, the calculated B_x contour plots shown in figure 4.10, indicate a gradient in the x direction. For example, figure 4.10a shows apparently parallel contour lines running in the y direction. The B_x field is therefore increasing in the x direction, resulting in an x gradient field. Further examination shows that the field is increasing by ≈ 6 G/cm near the $x = 0$ cm, $y = 0.7$ cm centre of the VOL. In each plot of figure 4.10, the region of 5% linearity is plotted as the bold line. By taking some simple measurements of distance based on these difference lines, the 5% VOL can be determined from figure 4.10 to be a box at least 5 cm $\times$ 5 cm $\times$ 5 cm. This is larger than the x -gradient VOL of 4 cm $\times$ 3 cm $\times$ 4 cm calculated in the optimization software, and is reported in table 4.4. The reason for this difference is that the calculation used for table 4.4 determined the volume over which the gradient does not vary by more than 5% while the 5% line in figure 4.10 gives the region where the gradient does not differ from the value at the centre of the VOL by more than 5%. The former calculation results in a tighter constraint on the field linearity, and was therefore used in all reported values of VOL. However, the VOL determined from the contour plots seems to represent a region where the linearity is more than adequate for Fourier transform imaging. This is confirmed qualitatively by the simulated images in figures 4.14 and figure 4.28 as well as the actual images of figure 4.28, which all show usable image quality in as large a region as $FOV_x = FOV_y = 5.0$ cm. A quantitative measure of the effect of the non-linearity of the SCSGC can be determined by considering the resulting shifting of pixels on the grid phantom simulated images.

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
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This pixel shifting manifests itself as curvature of the ideally straight lines on the grid phantoms of figure 4.14. For all images, the worst shift is seen to be 4 pixels which corresponds to 4.6% relative to the 64 pixel width.

A similar examination of figure 4.11 (y-gradient) shows contour lines running parallel to the zx -plane, thus designating a B_y gradient in y direction. The gradient strength appears to be ≈ 4 G/cm at the centre of the VOL, which is in agreement with the calculated value given in table 4.4. Measurements from figure 4.11 show a 5% VOL of $4 \text{ cm} \times 5 \text{ cm} \times 4 \text{ cm}$, which as expected is larger than the calculated VOL of $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$ reported in table 4.4.

The contour lines in figure 4.12 run parallel to the xy -plane suggesting the presence of a B_z gradient in the z direction as is expected. The field line density indicates a gradient strength of greater than 4 G/cm for the z -gradient coil. The 5% difference lines indicate an xyz VOL of $5 \text{ cm} \times 3 \text{ cm} \times 5 \text{ cm}$ compared with the $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$ VOL calculated in the optimization software.

4.5.3 Calculated and measured field comparison

Figures 4.18, 4.19, and 4.20 show direct comparisons of the measured (gauss meter) and calculated B_z fields for the x , y , and z gradient directions respectively. Considering an uncertainty of 0.2 G in the measured values, the 3d wire-frame plots result in a good match over most of the volume studied. The B_z difference plots on the right of each figure give a more quantitative comparison of the measured and calculated fields. Any point (on the xy plane) with a B_z difference greater than the 0.2 G uncertainty is plotted as black, while points with difference less than -0.2 G are plotted as white.

The intermediate values are scaled in levels of grey. The x gradient (figure 4.18) is seen to agree within experimental uncertainty for all values of y in an x region out to 3.5 cm. The results for the y-gradient show similar regions of disagreement, with the exception of the $z = 6$ cm plane (figure 4.19d) which has $\text{diff} > 0.2$ G at $y = -4.0$ cm, $x = 3.0$ cm, where B_z differs by ~ 0.3 G. The z-gradient results indicate agreement for all x and y except for the $y = 4$ cm lines for the $z = 4.0$ cm and 6.0 cm planes (figures 4.20c and d).

Figures 4.25, 4.26, and 4.27 give comparisons of the calculated and spectroscopically measured B_z field for the x, y, and z gradient coils respectively. In general the 3d wire plots indicate (qualitatively) good agreement in the shape and strength of the field. The B_z difference plots at the bottom of each figure indicate an excellent match for all values of y and z, and all gradient directions. The only exception is the band of y points with $z = 8$ cm $\rightarrow$ 10 cm for the y-gradient in figure 4.26, where the difference is greater than the uncertainty of 100 Hz.

There are two explanations for the results outside the uncertainty width. First, any calculation at a point that is near the surface of the coil has the possibility of error due to the finite physical dimensions of the wire wrapping. In the calculation software, all of the current is assumed to be concentrated in an infinitesimal width of wire. In construction, this same current may be obtained by wrapping several layers of finite diameter wire. It follows that near such wire, the calculated field may vary significantly from the actual field generated. Note that this is not a problem when designing the SCSGC because the calculated VOL (which is the only region of interest) can be tested with the lines of current placed at various positions.

These position offsets can be chosen such that they will more than account for the changes due to finite wire wrapping. Secondly, the precise orientation of the plane (xy and yz for the gaussmeter and spectrometer respectively) during measurement cannot be determined. The SCSGC and the stereotactic mount were aligned by eye, in an attempt to match the planes of the mount and coils. The size of this effect would depend on both the degree of misalignment, and the strength of the B_z gradient in the direction perpendicular to the plane of measurement.

Although the measured and calculated B_z fields exhibit some differences greater than the experimental uncertainty, it is important to note that there are no discrepancies within the calculated VOL. In fact, in all cases, there are no discrepancies within a volume of $2X_{VOL} \times 2Y_{VOL} \times 2Z_{VOL}$, where X_{VOL} is the x dimension of the VOL calculated in the optimization software. The conclusion from this is that the B_z (i.e. VOL) and gradient calculations within and near to the VOL can be accepted as correct.

4.5.4 Actual versus simulated images

Ignoring the shift in position of centre, the experimental images of figure 4.28 show good qualitative agreement with their simulated counterparts. A quantitative comparison is made difficult by the presence of a shift in x and y position of the phantom in the actual images. The lack of symmetry about the x-centre of the experimental images is a direct result of the x-position shift.

Ideally, an image of a homogeneous water phantom should exhibit constant intensity throughout the phantom area. Nonlinearity of gradients will result in inhomogeneous distribution of intensity.

Therefore, a study of the variation of image intensity can offer a quantitative evaluation of gradient linearity. The raw data file representing the simulated images of figure 4.28 have been found to contain intensity variations of less than 5% relative to the central pixel. This result considers pixels from the centre out to and including a 3 pixel distance from the edge of the phantom. The edge of the phantom is arbitrarily defined as the pixel where the intensity has dropped to less than 50% of the value at centre. Using the same criterion, the actual images of figure 4.28 for $z = 0$ cm, 1 cm and 2 cm have been studied to show less than 10% variation. The larger spread of intensity is a result of the darker bands that can be seen on the edges of the phantom in the x direction. These darker bands are likely to be due to inhomogeneity in the B_1 field produced by the RF coil. As mentioned in section 4.4.3, the homogeneous region of a saddle type RF coil is considered to extend to 0.7 of the coil radius. The actual images at larger z values show considerable random intensity variation that can be attributed to noise introduced by the RF coil.

A further observation that can be made from the images is the gradual compression of the phantom in the x direction as the z position is increased. From the images, the width of the phantom in the x direction can be measured as 4.9 cm and 4.6 cm for the $z = 0.0$ cm and $z = 6.0$ cm planes respectively. This 6.1% compression must correspond to an equal decrease in the x gradient strength. This shift can be predicted from the calculated x -coil B_z field that is plotted in figure 4.10.

The zx plane plots of figure 4.10 g,h, and i all show that in the region from $x = 0$ to 3 cm (the region covered in the actual and simulated images), the spacing of lines of equal B_z (iso-lines) increases with increase in z . Examination of the raw data used to generate figure 4.10 quantifies the increase in spacing (decrease in gradient strength) to be 6.0% from $z = 0$ to $z = 6.0$ cm. This agrees with the compression seen in the images.

4.5.5 Specifications

Table 4.7a, b, and c summarize the measured and calculated specifications for the x , y , and z gradient coils respectively. The uncertainties in the measured values of resistance and inductance were determined from the LCR meter scale. The uncertainty in the measured gradient strength results from the sensitivity of the distance measurement tool in the image display/analysis software. The uncertainty in rise time results from the oscilloscope's scale and precision. The uncertainty values quoted for the calculated results were determined by convergence investigations with the optimization software.

The calculated resistances agree with the measured (ohm-meter) values to well within the experimental uncertainties. The gradient strengths calculated in the optimization software agree with the measured values to within the uncertainties shown. The gradient strength was measured on the 4.7T 40 cm spectrometer, by imaging a cylindrical water phantom of known diameter. The diameter was then measured from the image, and using the known current applied, the gradient strength could be calculated. The inductance measured on an LCR meter (BK Precision model 875A) also agreed with the values calculated in software.

The calculated inductances have an uncertainty of $\pm 20\%$. This uncertainty could be reduced at the cost of computing time. The 20% value comes from sectioning the surface into 200 by 200 areas, each of which requires 5000 integral steps to calculate the magnetic flux. The computing time for such a calculation was 7 hours on a *Silicon Graphics IRIS* work station.

The rise time has been determined by measuring (on an oscilloscope) the time taken to reach 90% of the specified gradient strength. The rise time to reach 90% of the maximum gradient strength (150 A) is shown for the x, y, and z gradient coils as 165 μs , 140 μs , and 140 μs respectively. It is valuable to note from the rise time plots (figures 4.29a, b, and c) that for all gradient directions, the SCSGC has rise time less than 80 μs for gradient strengths up to 75% of the maximum allowed gradient strength. It is not possible to calculate the rise time because it is dependent on hardware such as the gradient controller. It is, however, possible to calculate a characteristic time constant τ (discussed in chapter 2) based on the electrical impedance of the coil. Using the measured values of L and R, the time constant $\tau = L/R$ is found to be 608 μs , 622 μs and 583 μs for the x, y, and z SCSGC coils respectively. The rise time to a given gradient strength can then be estimated (neglecting the effect of the ramped signal) using equation (2.11). The maximum voltage that can be applied with the *Techron* amplifier is 250 V.

The Ohm's law current attainable is

$$I = \frac{V}{R} = \frac{250 \text{ V}}{0.25 \text{ } \Omega} = 1000 \text{ A} \quad . \quad (4.7)$$

for the x gradient coil. The time to reach 150 A is determined by equation (2.11) to be

$$150 = 1000 (1 - e^{-\frac{t}{608 \text{ } \mu\text{s}}}) \quad . \quad (4.8)$$

Solving for t , the time to reach 150 amps for the SCSGC x-gradient coil with 250 V applied in *one step* is $t = 99 \text{ } \mu\text{s}$. Calculations based on similar *one-step* voltage applications result in rise time values of 71 μs and 67 μs for the y and z directions respectively. However, as mentioned in chapter 2, the gradient controller sends a ramped voltage signal to the amplifier in an attempt to reduce overshoot (discussed in chapter 2). In the case of rampswitch #3 on the Bruker[†] gradient control unit, the effect of ramping the voltage was to produce a rise time (to 150 A) of 165 μs using the x SCSGC. For each of the SCSGC directions, the overshoot was measured (on the oscilloscope) to be less than 5% of the specified voltage (1.85 volts in the case of the 37.5 volts required to generate 150 A of current) for all gradient strengths. Furthermore, the overshoot oscillations faded to less than 1% of the specified voltage in an additional 20 μs . These measurements of overshoot were taken with the pre-optimized gradient pre-emphasis unit turned on.

The calculated size of the VOL cannot be confirmed directly. However, it has already been shown that the B_z and gradient field calculations are accurate to within the

[†]Bruker is an MR manufacturer in Germany.

Therefore, the VOL calculations, based on these calculated gradient fields, can be assumed to be as accurate as the algorithm used to specify the volume. As previously mentioned, the VOL calculation algorithm tends to under evaluate the size of the VOL. Therefore, the calculated size of VOL can be taken as a minimum result, and the gradient coil can be expected to have an effective VOL at least as large as the calculated one.

4.5.6 Design objectives

The design objectives for the 40 cm SCSGC presented in section 4.3.1 have been liberally met. The goal was to design a gradient coil capable of generating the strong and fast switching gradient fields needed for EPI. The following is a summary of the design objectives, and their significance to imaging.

VOL

The criterion specified for the size of the imaging, was to design a gradient set with a VOL at least $3\text{ cm} \times 3\text{ cm} \times 3\text{ cm}$. Even using the tightest rules for calculating the VOL, all directions of the SCSGC exceed this size. Using the average of the x,y,z VOL values measured from the field plots, the VOLs represent 18%, 20%, and 16% (x,y, and z directions) of the characteristic coil dimension. The characteristic coil dimension represents the size of the coil, and was taken to be the diameter of the coil.

Gradient strength

The objective gradient strength was specified to be greater than 4 G/cm at 150 A. Tables 4.7a, b, and c give values of 6.1 G/cm, 4.3 G/cm, and 5.6 G/cm for the x, y, and z SCSGC directions respectively. The value of 4.0 G/cm was chosen to enable images with a minimum field of view (FOV) of ~6 cm in all directions.

To calculate the gradient strength needed to obtain a given FOV, equation (2.5) was used

$$\frac{23.5 \text{ G/cm}}{\text{FOV}} = \text{Gradient Strength} \quad (4.9)$$

The value of 23.5 G/cm is obtained by assuming a sweep-width (SW) of 100 kHz, which corresponds to a dwell-time (DT) of 10 μ s. This represents the minimum dwell time[†] available with the digitizer in use on the 40 cm 4.7T spectrometer. Using the measured gradient strength for the SCSGC, the minimum FOV available at a current of 150 A is 3.9 cm, 5.5 cm, and 4.3 cm for the x, y, and z directions respectively. The x and z minimum FOVs are smaller than the VOL determined from the B₁ contour plots for the SCSGC. The minimum FOV available in the y direction is only slightly larger than the VOL provided by the y coil. It should be noted that the SCSGC was designed to handle up to 300 A of current which effectively halves the minimum FOV attainable with the coil. Also note that the minimum FOV scales inversely with the DT of the acquisition. Therefore imaging sequences that are not sensitive to a long DT (such as any spin echo sequence) can have considerably smaller FOV than those specified above.

Rise time

The objective was to design coils with rise times less than 100 μ s. According to the rise time measurement, gradient strengths measuring 75% of the maximum strength (at 150A) can be achieved in less than 80 μ s for all directions with acceptable levels of overshoot.

[†] A small DT is desired for EPI to minimize field inhomogeneity errors.

EPI

According to the specifications, EPI sequences will be possible with the following limitations. With a rise time of 100 μ s allowed for the ramping of the oscillating read gradient, the minimum FOVs will be:

$$DT=10\mu s \quad FOV_{min}^x = 4.5 \text{ cm}$$

$$FOV_{min}^y = 6.3 \text{ cm}$$

$$FOV_{min}^z = 5.0 \text{ cm.}$$

Here, a value of 80% gradient is used, which is the approximate percentage of maximum at 100 μ s rise time in figures 4.29a, b, and c. Smaller minimum FOV can be obtained if either the ramp time is increased (allowing for larger gradient values), or the DT is increased.

Diffusion imaging

Diffusion imaging refers to images where contrast is caused as a result of differences in the diffusion coefficient between protons in different environments. Diffusion imaging was briefly discussed in chapter 1. As was shown, in order to obtain diffusion information, a strong diffusion gradient must be applied. The strength of gradient needed to give contrast was shown to depend on:

- The difference in diffusion coefficient, between the protons being resolved.
- The desired level of contrast.
- The strength of the diffusion gradient.
- The length of time the diffusion gradient is applied.

Therefore, the shorter the diffusion gradient pulse, the stronger the gradient must be to obtain a given level of diffusion contrast.

Diffusion imaging using a spin echo sequence involves applying a diffusion gradient G_D between the 90 and 180 degree pulses, followed by a second diffusion gradient between the 180 degree pulse and the echo (or acquisition). Shorter times of diffusion gradient pulse improve the SNR, as well as reducing the competing T_2 relaxation effect (T_2 for water in tissue ranges from 35 ms to 400 ms). For example, spin echo diffusion imaging has been achieved on NRC's 30 cm 4.7T spectrometer using diffusion gradients 7 G/cm in strength, and 10 ms in duration. The echo time (TE) of the sequence was 50 ms.

Turner and Le Bihan⁶⁶ show that diffusion imaging is possible in an EPI sequence as well. Again the diffusion gradient pulses are located first between the $\pi/2$ and π pulses and secondly between the π pulse and the start of the acquisition. Turner and Le Bihan found that to achieve enough diffusion attenuation to be accurately measured, gradients of 4 G/cm in strength, and 20 ms in duration were required.

Measurement of the diffusion coefficient is also possible with the spin-echo method originally presented by Stejskal and Tanner⁶⁷ in 1965. Li and Sotak⁶⁸ vary the method slightly to form their multiple spin-echo method for measuring the diffusion coefficient. These methods in general require gradient strengths as large as possible, however Li and Sotak use diffusion gradients with ~4 G/cm strength and ~10 ms duration.

The constructed SCSGC set can produce gradient fields with strength in excess of 12 G/cm (x-direction, 300 A). Clearly, the SCSGC set will produce fields sufficiently strong to implement any of the above diffusion methods.

4.5.7 Comparison with other coil configurations

To further evaluate the performance of the SCSGC, a comparison will now be made with 3 different geometries of gradient coils reported previously. First the planar surface gradient coil of Cho and Yi⁵² will be compared, followed by the biplanar design of Martens et al.³⁷, and finally a quadrupole gradient set recently presented by O'Dell et al.⁴⁸. All gradient strengths have been scaled to a current of 150 A.

Planar surface gradient coil.

For a planar coil with rectangular dimensions 12.3 cm $\times$ 25 cm the useful imaging volume of 5 cm $\times$ 5 cm $\times$ 5 cm is specified. However, this VOL cannot be directly compared with the SCSGC because the planar x, y, z gradients do not provide linearity in the y direction. The coil is designed to be used for imaging in only the zx plane (slice selection in the y direction). For all gradient directions, the gradient strength is found to decrease with the distance from the coil surface. The x and y gradient strength varies from $\sim 9 \rightarrow 42$ G/cm over the useful y range, while the stronger z gradient varies from 22.5 $\rightarrow$ 80 G/cm. The inductance is specified as 1800 μ H, 1600 μ H, and 1200 μ H for the planar x, y, and z gradient coils respectively. The resistance is shown to be 1.4 Ω , 1.9 Ω , and 1.5 Ω for the planar x, y, and z gradient coils respectively. To scale these results for comparison with the SCSGC, it should be noted that to achieve a two-fold gain in the gradient strength, twice the number of wires must be wrapped.

The flux used to determine inductance has a four-fold increase. $2 \times$ inductance for the doubling in the area of the loops, and approximately a two-fold increase for the increase in B_z at each point in the loop. Therefore, if the number of loops in the SCSGC design is multiplied by $2 \times 2 = 4$, the inductances will be multiplied by approximately $4 \times 4 = 16$, and the resistances will be multiplied by 4. The resultant SCSGC coils would have inductances of $\sim 2200 \mu\text{H}$, $1900 \mu\text{H}$, and $1700 \mu\text{H}$, and resistances of 1.0Ω , 0.8Ω , and 0.7Ω for the x , y , and z directions respectively. The gradient strength of such coils would be $G_x = 24.5 \text{ G/cm}$, $G_y = 17.2 \text{ G/cm}$, and $G_z = 22.5 \text{ G/cm}$. A comparison of the efficiency (as calculated by equation (2.14)) of the scaled SCSGC and the planar coil is summarized in table 4.8. The x and y gradient directions for the SCSGC fall into the centre of the range of efficiency determined for the planar coil. The z SCSGC is below the efficiency range calculated for the planar z coil. The SCSGC is seen, therefore, to offer a gradient set with approximately the same usable VOL, gradient strength, and inductance as the optimized planar surface gradient set. However, a significant advantage of the SCSGC is that it provides linearity in the y directions making it useful for imaging in any plane.

Yi and Cho⁵⁰ have described another planar surface gradient coil set that uses an offset plane of current (in the opposite sense to the main gradient coil current). This method of removing the $B_z = 0$ offset has been described in section 2.3.2 of this thesis. The coils were constructed on a plate measuring $22 \text{ cm} \times 40 \text{ cm}$. The results specify a VOL (over which the gradients vary from 5 G/cm to 10 G/cm) of $5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$. A rise time of $200 \mu\text{s}$ is indicated.

Table 4.8 Efficiency comparison between SCSGC and planar surface gradient coil systems.

	(SCSGC effic.)/ (planar effic.)	SCSGC efficiency ¹ ((G/cm) ² /μH)	Planar coil efficiency ((G/cm) ² /μH)
x	5.4 → 0.28	0.27	0.05 → 0.98
y	3.2 → 0.15	0.16	0.05 → 1.10
z	0.71 → 0.06	0.30	0.42 → 5.30

¹The number of wires has been multiplied by 4 to produce gradient strength in centre of planar coil strength range.

Table 4.9 Efficiency comparison between SCSGC and Biplanar gradient coil systems.

	(SCSGC effic.)/ (Biplanar effic.)	SCSGC efficiency ((G/cm) ² /μH)	Biplanar coil efficiency ((G/cm) ² /μH)
x	45	0.27	0.006
y	10	0.16	0.016
z	10	0.30	0.030

Table 4.10 Efficiency comparison between SCSGC and modified quadrupole gradient coil systems.

	(SCSGC effic.)/ (Quadrupole effic.)	SCSGC efficiency ((G/cm) ² /μH)	Quadrupole coil efficiency ((G/cm) ² /μH)
x	0.9	0.28	0.31
y	1.0	0.16	0.16
z	0.9	0.30	0.32

Clearly the allowed gradient inhomogeneity (apparently greater than 50% difference over the VOL) used in specifying the VOL for these planar gradients is much worse than that specified for the SCSGC (5%).

Comparison between figure 2.7b (planar surface coils) and figure 4.30 (SCSGC) illustrates a fundamental shortcoming associated with the planar coils that is improved with the SCSGC. This shortcoming has to do with the correction of the $B_z = 0$ offset. Figure 2.7b shows that for planar surface gradients, current A and current B can be configured in such a way that the B_z value at the centre of the ROI is zero. The $B_z = 0$ offset correction in the SCSGC design is provided by 'correction current' applied (in the opposite sense to the main gradient current) at azimuthal angles near to or less than zero. Although both geometries provide a means of B_z offset correction, the SCSGC method is found to be more efficient. This can be seen by observing that for the planar gradient system, coil A has a B_z field that increases in value (becomes less negative) as y increases, while the correction coil B has a B_z field that decrease in value (becomes less positive) as y increases. When the fields due to A and B are added together, the result is a smaller slope (or gradient) than that for either of the coils in isolation. The case for the SCSGC is opposite of that for the planar gradient coils. Figure 4.30 illustrates this 'constructive' superposition of gradients associated with SCSGCs. As can be seen, the B_z field due to the main coil (triangular points) decreases (becomes more negative) with distance from the coil surface. Similarly, the B_z field due to the correction current (oval points) decreases (becomes less positive) with the distance from the surface.

Figure 4.30 B_z versus distance (y) from surface for a SCSGC ($x=0, z=0$)

The resultant field in the presence of both the main coil and the correction coil is a gradient strength that is larger than either of the coils in isolation. Thus a $B_z = 0$ offset correction current for the SCSGC increases the gradient strength while correction coils for the planar gradient systems decrease the gradient strength.

Biplanar gradient coils

Martens et al.³⁷ used variational methods to minimize the inductance in a biplanar gradient coil set. As will be seen, this optimized coil offers improved linearity over the single planar coils described above. Also, for a comparable diameter the biplanar coils achieve higher gradient strength with shorter rise times than those of cylindrical coil designs. The optimized biplanar coils consist of two parallel planes of current approximately $z = 120$ to 140 cm length and $x = 40$ to 50 cm width. The separation between the planes is 44 cm. The only measure of linearity available is that for the $y = 0$ plane, within a circular slice of radius 12.5 cm, the B_z field is linear to within 20% . All three coils generate gradients of ~ 2 G/cm at 150 A, and the inductances have been determined to be $646 \mu\text{H}$, $254 \mu\text{H}$, and $133 \mu\text{H}$ for the x , y , and z directions respectively. This planar coil with its 40 cm space allows more access than the 30 cm diameter of the SCSGC. However, the efficiency of the SCSGC is seen to be much better than that of biplanar coils (see table 4.9). Taking measurements from field plots for the SCSGC, a spherical VOL with 20% linearity can be determined. The gradient is linear to 20% over at least a sphere of radius 6 cm for each of the gradient directions. Thus the VOL of the SCSGC is only about half that of the biplanar coil. Clearly, the SCSGC offers higher gradient strength with lower inductance, while the biplanar coil provides a larger VOL.

Optimized Modified Quadrupole Gradient Set

W. O'Dell et al.⁴⁸ presented a three directional gradient coil based on the four line element gradient system discussed in chapter 2. The linearity was reported as the percent of spatial distortion measured using images taken from a grid phantom. For a cylindrical VOL measuring 8 cm in diameter and 8 cm in length, the images indicate less than 2% spatial distortion. A comparison of coil efficiency is summarized in table 4.10. The quadrupole coils have been scaled to 30 cm diameter for a meaningful comparison. The x and z SCSGCs are only slightly less efficient than their quadrupole counterparts. The y SCSGC is equally efficient as the quadrupole y gradient coil. One further consideration should be discussed. The quadrupole gradient set has poor patient/sample accessibility. The coil must be removed from the magnet in order to insert the sample. Furthermore, even in large scale versions of the quadrupole coil, the sample is limited in length along the z-axis. This prevents use of this modified quadrupole coil with long thin objects (such as the human body).

Chapter 5: Improvement on the SCSGC providing larger VOL.

5.1 Introduction

In chapter 4 a novel design of gradient coils was presented that provides relatively strong/fast switching gradient fields. As shown in the discussion, the main limitation of the SCSGC is a generally smaller VOL than for coils based on full cylindrical geometry. A detailed study of the y-direction SCSGC has led to the following observation:

the y-position of the VOL is dependent on the position and strength of the $B_z=0$ correction current (i.e. grooves 5 and 6 in figure 4.8).

This observation suggests a way of effectively increasing the size of the VOL in the y direction. Consider the following example. Construct the y gradient coil of figure 4.8 in such a way that the currents in grooves 5 and 6 are independent from the rest of the coil. In such a modified SCSGC the currents and positions of the wires in grooves 5 and 6 can be varied independently of those in the other grooves. Figure 5.1 shows a schematic representation of the above observation. In such a coil, the usable VOL can be moved in the y direction effectively allowing imaging of a larger volume than with stationary equatorial current. In the discussion that follows, the current group that is altered (in magnitude and/or position) will be referred to as the *equatorial current set*, because of its general proximity to the $\phi = 0^\circ$ azimuthal angle.

This relationship between the equatorial current and VOL position was observed for the x and z directions as well. The movable equatorial current in these cases are grooves 4, 5, 6, 7 for the x-coil (figure 4.7) and grooves 4, 5, 6 for the z-coil (figure 4.9).

Figure 5.1 Schematic illustration of the effect of equatorial current variation.

This chapter presents an investigation into the effect of the magnitude and position of the movable equatorial current on the position and size of the VOL. All three gradient directions will be considered.

In chapter 6, an optimized whole body[†] x, y, z gradient system is presented that allows for the movement of the VOL.

5.2 Results: movement of the VOL

Using the optimization software designed for this thesis it was possible to explore the relationships between the equatorial current and the size and position of the VOL. Figures 5.2, 5.3, and 5.4 show the effect of varying current strength and the azimuthal position of the equatorial current for the constructed 30 cm x, y, and z coil system presented in chapter 4. Part a of the figures shows the effect of varying the current in the equatorial coil set. In part b of figures 5.2, 5.3, and 5.4 the azimuthal position as well as the current of the equatorial coil set has been varied. In this case, the current for each different position has been optimized separately. The optimized current magnitude and corresponding gradient strength are included in part b of the figures. All the results have been optimized for maximum size of VOL under the constraints of the specified position or current of the equatorial coil. The VOL was determined as the length in the y-direction over which the gradient linearity varies by no more than 5%. The size of the VOL in the x and z direction was specified to be 3.0 cm for all calculations. Note the difference in scale between figures a and b for all three directions of gradient. To discuss these results, each gradient direction will be considered separately.

[†] Inner diameter of 50 cm to allow access to most patients.

Figure 5.2a Effect of varying equatorial current strength in an x-SCSGC.

Figure 5.2b Effect of varying equatorial current azimuthal position in an x-SCSGC.

Figure 5.3a Effect of varying equatorial current strength in a y-SCSGC.

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Figure 5.3b Effect of varying equatorial current azimuthal position in a y-SCSGC.

Figure 5.4b Effect of varying equatorial current azimuthal position in a z-SCSGC.

5.2.1 X-gradient

The following results are based on varying the strength and / or position of current grooves 4, 5, 6, and 7 of the x gradient coil of figure 4.7. In all cases the position and current of loops 1, 2, and 3 remains as specified in figure 4.7 and table 4.1. The relative current in each loop (in units of # of wires) is seen to be 1=2, 3, 5, and 5 in loops 4, 5, 6, and 7 respectively.

For this study, the current in the equatorial set will be specified by the number of wires in loop 4. The current in loops 5, 6, and 7 is then determined by the ratio of current relative to loop 4 in figure 4.7. For example if the loop set is designated as having a relative current $I = 3$, the current will be 3, 4.5, 7.5, and 7.5 for grooves 4, 5, 6, and 7 respectively.

When investigating the effect of the position of the movable groove set, the z position has been held constant for the sake of simplicity. The z degree of freedom has only a small effect on these results, and will be reintroduced in the optimization of the whole body coil in chapter 6. The position of the groove set is designated by the azimuthal position at the start (lowest ϕ) of the groove set. For example, the position of groove set (grooves 4, 5, 6, and 7) in figure 4.7 which ranges from -5° to 35° is designated by $\phi = -5^\circ$. Therefore a coil with groove set position designated as $\phi = -15^\circ$ will have all grooves (4, 5, 6, and 7) shifted by -10° relative to figure 4.7.

The results of varying the current are shown in figure 5.2a, which plots the position and size of the VOL as a function of the relative current.

The centre of the VOL is indicated with the diamond point, while the VOL range is given by the line. Neglecting the result for a relative current $I = 1.5$ (which has a VOL size of only 1 cm), a change in the current by a factor of 1.75 leads to a movement of the centre of the VOL by 2.5 cm. It was found that increasing the relative current beyond $I = 3.5$ results in a VOL size < 2 cm.

When the azimuthal position is varied in conjunction with the optimized current, the VOL changes as illustrated in figure 5.1. The position and size of the VOL are plotted as a function of azimuthal angle. The current resulting in the maximum VOL size as well as the gradient strength are given beside each VOL line. The results show that by varying the position of the equatorial current from 5° to -35° the VOL centre can be moved by slightly more than 5 cm, which is of the order of the size of the VOL.

5.2.2 Y-gradient

Figure 4.8 shows the configuration for the y-direction of the optimized SCSGC. Grooves 5 and 6 represent the $B_z = 0$ correction (or equatorial) current. In the case of the y-direction gradient, the centre of the VOL is defined as the position where $B_z = 0$.

Varying the current in the equatorial component of the y-direction gradient coil affects the position of the VOL as shown in figure 5.3a. A increase in current of 2 loops is seen to lead to less than a 1.5 cm shift in the centre of the VOL.

The effect of varying the azimuthal position of the correction current in conjunction with optimizing the current for maximum size of VOL is far more significant.

The results obtained from altering the azimuthal angle from 15° to -20° are shown in figure 5.3b. The centre is seen to be movable by ~ 3.5 cm which is on the order of the size of the VOL.

5.2.3 Z-gradient

Grooves 4, 5, and 6 represent the equatorial current set for the constructed z -gradient coil in figure 4.9. In the case of the z -direction gradient, the current variation is again shown to lead to an insignificant change in VOL. Figure 5.4a indicates movement in the centre of the VOL of ~ 1.5 cm as a result of a 2 loop increase in the current.

Figure 5.4b shows the results of movement of the equatorial current in the case of the z -direction gradient coil. Excluding the result for start azimuth $\phi = 29^\circ$ (which has y direction VOL size of only 2 cm), a shift of $\sim 40^\circ$ in the position of the current leads to ~ 3.0 cm variation in the position of the centre of the VOL.

5.3 Discussion: movement of VOL

In order that a shift in the VOL position improve the SCSGC design, the shift should be at least nearly as large as the size of the VOL provided by the optimized coil without shift capabilities. In the case of the constructed gradient set used for this investigation, a VOL of at least $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$ was achieved for each gradient direction. Therefore, a VOL that is movable by 3 cm in the y direction would offer a 2 fold increase in the usable VOL. It should be noted that each gradient direction in an x , y , z coil system must have a spatially coincidental VOL in order that all gradient directions be usable for a single imaging sequence.

Thus optimizing a coil system is not just a matter of independently finding the best x, y, and z configuration. Instead the final optimized coil system must provide the largest usable VOL common to all directions of gradient coils. As discussed in chapters 2 and 3, coil inductance and gradient strength must be considered in conjunction with VOL size, thus further complicating the optimization procedure. It is therefore not possible to generalize the effectiveness of the variation of equatorial current without considering the specific design criteria for a specific gradient set. A whole body gradient coil applying the movable equatorial current technology has been designed and is described in detail in chapter 6. The present discussion describes only the direct effects of equatorial coil variation on the independent gradient directions, and does not attempt to unify the design characteristics of a 3-directional movable VOL coil system.

Before discussing the results, the relationship between VOL, and the imaging FOV will be presented. The VOL movement investigated involves shifts in the y-direction only. This leads to an important difference in design considerations for the x and z gradient coils compared with the y gradient coil. In imaging, the centre of the FOV in a given direction is determined by the position (in that direction) where $B_z = 0$. The centre position indicated in figure 5.3 for the y gradient direction is also the y-position where $B_y = 0$. In this y-gradient case, the range of the VOL is determined as the maximum range (with the usual linearity constraints) centred at the $B_z = 0$ y-position. In the x and z (figures 5.2 and 5.4) cases, however, B_z is zero everywhere along the y-axis due to symmetry. The VOL range is calculated, and the centre is just the middle point of the range. Most 2 dimensional imaging methods produce images with a square FOV.

Therefore the maximum allowed FOV is determined by the smallest VOL direction for both gradient directions used in the imaging slice. Put another way, the usable VOL for imaging is the logical AND operation applied to the individual VOLs of the X, Y, and Z gradient coils. For example consider 2 dimensional imaging (xy-plane) in the presence of x and y gradient fields with VOLs defined in figure 5.5. The x-gradient field is seen to have a VOL of 3 cm $\times$ 8 cm centred at $x = 0$, $y = 0$. The y-gradient has a VOL of 4 cm $\times$ 3 cm and is centred at $x = 0$ (as necessary by symmetry) and $y = 1.5$ cm. The maximum usable VOL for imaging is the largest square common to both gradients, and is depicted as the shaded region in figure 5.5. Note that the y-centre of the VOL for the x gradient has no bearing on the resulting image FOV. The important criterion for the x-gradient coil is that the y-range of the VOL cover the same range as indicated by the y-gradient coil. The same reasoning controls the z-gradient VOL for a yz-plane image. Summarizing, only the y-range of VOL is relevant in the x and z-gradient cases, while the range and centre of range need to be considered for the y-gradient case.

5.3.1 Variation of current

With the exception of the x direction, the effect of varying only the current strength is found to produce insignificant shifts in the position of the VOL. Current variations in the y and z directions lead to y-shifts in the VOL of less than 2 cm. Figure 5.2a indicates a shift of ~ 3 cm is possible for the x-gradient by variation of current only. It should be noted, however, that the range of the VOL for the relative current $I = 2.5$ case covers y-space from $y = 3.75$ cm $\leftrightarrow$ $y = 8.25$ cm.

Figure 5.5 Example of VOL coincidence.

Figure 5.6 Schematic of proposed rail and clamp system for movable coil.

In comparison, the VOL ranges for relative current values $I = 3.50$ and $I = 1.75$ are $y = 3.25 \text{ cm} \leftrightarrow y = 6.00 \text{ cm}$ and $y = 5.75 \leftrightarrow y = 8.25 \text{ cm}$ respectively. The total VOL range covered by this current varying system is therefore $y = 3.25 \text{ cm} \leftrightarrow y = 8.25 \text{ cm}$, which is only a 10% improvement over the single current $I = 2.5$ coil. Clearly, no significant gain in size of VOL is produced by allowing for variation of the equatorial current strength alone.

5.3.2 Variation of azimuthal position

In all directions, the greatest range in y-position of the VOL is obtained when allowing for variation of the position of the equatorial current in conjunction with an optimized current change. Figure 5.2b shows that for the x-gradient coil, a net VOL range from $y = 0 \text{ cm} \leftrightarrow y = 9 \text{ cm}$ can be provided by varying the start azimuthal angle from -35° to 5° . Notice, however, that the range for the $\phi = -15^\circ$ configuration provides a VOL range $y = 1 \text{ cm} \leftrightarrow y = 8 \text{ cm}$ losing only 22% compared to the net movable range. Because only range is important (and not centre) for the x-direction, there may be little advantage in using a movable VOL system for this x-gradient case. The specific design considerations such as inductance, gradient strength, and compatibility with y and z coils must be addressed to evaluate the advantages gained by movable VOL.

Variation of equatorial current position in the z-gradient case is seen to provide a shift in the centre of the VOL of $\sim 3.0 \text{ cm}$ in the y direction. Variation of azimuth from $-25^\circ \leftrightarrow 15^\circ$ results in a net VOL range from $y = 2 \text{ cm} \leftrightarrow y = 8.5 \text{ cm}$. This range produces a VOL that is more than double that of the constructed z-gradient coil.

It should be noted that the number of current loops increases from 5 to 8, while the gradient strength remains the same (to within 10%) as the start azimuth is decreased. The resultant decrease in gradient coil efficiency ($G_z^2/\text{inductance}$) must be considered when choosing coil design.

The results for the y-gradient coil present a significant advantage over the previously constructed y-coil. Because of the limitation imposed by the position of $B_z = 0$, and the size of the VOL in x and z direction, the only way to gain y-range in the VOL for a y gradient coil is by a shift in the y-centre. Figure 5.3b shows that a variation of azimuth from $-20^\circ \leftrightarrow 15^\circ$ results in a shift of VOL centre of $y = 5 \text{ cm} \leftrightarrow y = 8 \text{ cm}$. Considering the VOL has x and z ranges of $3 \text{ cm} \times 3 \text{ cm}$, such a shift will lead to a doubling in the size of the usable VOL. The FOV usable for imaging will remain 3 cm, but the volume over which the coil will be effective is increased.

5.3.3 General discussion

In the above, only the azimuthal angle was given the freedom to move. In optimizing the whole body coil of chapter 6, the azimuth and z value are allowed to move. The addition of the z degree of freedom will be seen to slightly enlarge the size of the VOL compared to adjusting the azimuth and current only.

The modification to the basic SCSGC design providing a movable VOL position is an advantage that is not possible with the planar surface gradient coil systems described in chapter 2. A SCSGC with a movable VOL keeps all of the advantages (fast switching, strong gradient) associated with a surface type gradient.

In addition, the modification results in larger effective VOL, approaching the advantage of the fully cylindrical type coils.

It should be noted that when the equatorial current is moved to low azimuthal angles, the total coil covers nearly all of the azimuthal dimension of the surface. In these configurations, the SCSGC actually resembles the modified Golay type gradient coils presented in chapter 2. In Golay coils, the centre of the VOL is at the centre of the cylinder, while for the SCSGC the y centre is offset towards the main current loops at $\phi = 90$. In effect, this results in the average distance from current to the VOL being reduced in the modified SCSGC compared to the Golay coils. This leads to an improvement in the efficiency of the SCSGC. Of course this is at a cost to the size of the VOL.

Problems

The implementation of a movable VOL has been shown to require both movement and variation in strength of the equatorial current. The current variations must occur only in the equatorial portion of the coil, while the current in the main loops (grooves 1, 2, 3, and 4 in figure 4.8) must remain constant. Clearly this is most easily implemented by providing separate amplifiers for the main and equatorial current loops. It follows that the gradient controller must be capable of providing independent signals to the gradient amplifiers, a property not installed in most NMR systems. This limitation should be considered before building and installing such a movable VOL gradient system.

The non-trivial construction of a coil system that has movable parts has not been attempted at the time of writing. The movable portion of the coil system must maintain structural strength as well as an electrical connection. One possible way to accomplish these criteria is to use a conducting rail, onto which the movable coil is fastened with conducting clamps (see figure 5.6). The rail is set into the main coil housing structure, and follows a straight line ($d\phi/dz = \text{constant}$) path between the necessary azimuth and z points. The exact engineering of the conducting clamp mechanism will need to be worked out. Further clamp/rail systems will be required for structural integrity, although only one will be electrically conducting. The number of clamp/rail attachment points is left to the builder.

One further problem associated with this proposed movable VOL system is that to preform a move, the imaging experiment must come to a halt while the user removes the patient/sample, and physically moves four (one for each quadrant) coils each of which will have any number of clamps.

Chapter 6: A modified SCSGC design for a whole body system

6.1 Introduction

Gradient coils in whole body imaging systems tend to be large in order to allow patient access, as well as a large imaging volume. As has been shown in chapter 2, large coils will generally produce slower switching and weaker gradients relative to their smaller coil counterparts. This chapter presents an x, y, z gradient coil system large enough to permit human access, while providing gradient fields strong enough and fast enough to permit Echo Planar Imaging. The design criteria set for the optimization of the coil were specified as:

minimum VOL size: $\geq 8 \text{ cm} \times 8 \text{ cm} \times 8 \text{ cm}$

gradient strength: $\geq 1.8 \text{ G/cm}$

switching time: $\leq 300 \mu\text{s}$

patient access: $\approx 0.5 \text{ m diameter}$.

In addition, the goal was to make use of the concept presented in chapter 5 thus allowing for movement of the y-position of the VOL. As will be seen, the resulting coil design meets all of the specified design criteria. The y-direction gradient coil uses a movable equatorial current design, increasing the effective VOL in the y direction by approximately 50%. The optimized x and z coils were found to provide VOL ranges covering sufficient range to avoid the need for movable parts.

6.2 Optimization details

In all cases, the T-Simplex algorithm was used in the same fashion as that described in chapters 3 and 4. The initial simplex was determined by up-scaling the 30 cm SCSGCs optimized in chapter 4. The z and azimuthal positions of the current elements were increased in proportion to the increases in the radii of the coils. The radii were chosen to allow an inner access separation of 45 to 50 cm (see figure 6.1a), which should be sufficient for the average hip to penetrate. This somewhat arbitrary choice of coil size, was chosen only to demonstrate the possibilities of the SCSGC for whole body imaging systems. The optimization process can be repeated for any desired access size.

In general, with the initial simplex chosen, the T-simplex algorithm was applied several times, with an increase in the x and z VOL size on each restart. The OP was calculated as the negative of the y -direction size of the VOL. Most often the optimization parameters used were:

- T_0 30,
- T_{final} 0,
- # of function calculations
 per temperature step 100,
- # of temperature steps 6,
- simplex size (β) 2, and
- power in cooling schedule . . 2.

When the restart produced no further improvement in the OP, the algorithm repeated with several variations of the optimization parameters to ensure a global minimum.

Figure 6.1a Cross section of whole body SCSGC to define radial dimensions.

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Figure 6.1b Expanded region from a).

The y-gradient was optimized first, with no constraint on the position of the centre of the VOL or the number of current elements eligible for movement. The resultant design has been called the *central* configuration. The optimization was then changed to allow only three degrees of freedom (3 *ips*): namely, the z and azimuthal coordinates as well as the current through the moveable portion of the coil as discussed in chapter 5. In addition, a penalty for the position of the centre of the VOL was included in the optimization as defined in equation (3.8). Under the above constraints, the T-simplex algorithm was applied repeatedly with a stepwise decrease in the desired centre of the VOL on each pass. The y-position of the VOL was decreased from the centre of the optimized central configuration to the point where the optimized size of the VOL became too small to be practical for imaging. The so called *far* configuration of the y gradient coil was then chosen as that configuration giving a usable VOL centred farthest away from the coil surface ($\phi = 90^\circ$). The process was then repeated with increases in the y-position of the VOL on each restart of the T-simplex. The resultant design with the optimized VOL closest to the coil surface has been called the *near* configuration.

The x and z gradient coils were optimized with no penalty for the position of the y-centre of the VOL. The resulting configurations were found to provide VOLs with sufficient range in the y direction to remove the need of a movable VOL.

6.3 Current configuration

The current configurations reported for the whole body coil will use the same coordinate system and quadrant and symmetry conventions as used in chapter 4 for the description of the 30 cm gradient system (see figures 4.1b, 4.2a, and 4.2b).

A cross-section of the jig for the whole-body coil is given in figure 6.1a. Figure 6.1b gives an expanded view illustrating the approximate range of groove depths. As with the smaller constructed SCSGC of chapter 4, the whole-body design has the y-gradient on the inner surface (with movable $B_z = 0$ correction current) and the x and z coils on the outer surface. Notice that the jig is a complete cylinder to facilitate the azimuthal migration of the y-gradients movable current former. As can be seen, the clearance inside the coil is 45 cm minimum and 50 cm maximum diameter.

The groove paths for the optimized whole body modified-SCSGC are defined in figures 6.2 for the x coil, 6.3, 6.4, and 6.5 for the y coil, and 6.6 for the z coil. Only quadrant 1 (figure 4.2a) grooves are shown in the illustrations because the groove paths for the other three quadrants are easily determined from symmetry (figure 4.2b). The design is specified for construction with American wire gauge (AWG) 12-gauge copper wire corresponding to 2.05 mm diameter. Experience from the 30 cm SCSGC suggests that to facilitate wrapping, the width of the constructed groove should be at least 10% larger than the actual width of the wire(s) in the groove. This suggests that the grooves be made with 2.5 mm per width of wire. For example, any groove designated as being n wires wide will have width = $n \times 2.5$ mm.

The x-direction gradient case of figure 6.2 will be used to illustrate the use of these groove maps. Figure 6.2a shows the schematic representation of the position of the current loops. Again, as in the case of the groove maps for the 30 cm coil set, the horizontal axis can be interpreted as the cylinder (azimuth ϕ) having been rolled out onto the x-axis. Each loop is labelled by a number (1-8) for reference.

Figure 6.2a Schematic drawing of current configuration for quadrant 1 of the whole body x-direction SCSGC.

Figure 6.2b Quantitative x-gradient loop parameters.

Figure 6.2b continued

Note: These loops must be reflected about both the z and x -axis to define the full current configuration. Effective width of 1 wire is 2.5mm.

Figure 6.3 Overview of current configuration for the whole body y-SCSGC with movable $B_z=0$ correction current.

Figure 6.4a Schematic drawing of current configuration for quadrant 1 of the whole body y-direction SCSGC (non movable portion).

Figure 6.4b Quantitative loop parameters for the stationary y-gradient set.

Figure 6.4b continued

quadrant 2 coordinates

Note: must reflect these loops about the x-axis to get full current configuration. Effective width of 1 wire is 2.5mm.

Figure 6.5a Schematic drawing of the current configuration for quadrant 1 of the whole body y-direction SCSGC (movable current group).

Note: These loops must be reflected about both the z and x -axis to define the full current configuration. Effective width of 1 wire is 2.5mm

Table 6.1 Transformation of the moving to the stationary coordinate system ($z' \leftrightarrow z$, $\phi' \leftrightarrow \phi$).

near	centre	far
$z = z' + 0.13 \text{ m}$	$z = z' + 0.115 \text{ m}$	$z = z' + 0.09 \text{ m}$
$\phi = \phi' + 27^\circ$	$\phi = \phi' - 7^\circ$	$\phi = \phi' - 13^\circ$

Figure 6.6a Schematic drawing of current configuration for quadrant 1 of the whole body z-direction SCSGC.

Figure 6.6b Quantitative loop parameters for the whole body z-gradient coil design.

Figure 6.6b continued

Note: Loops 1,2, and 3 require no reflection to specify the complete current configuration. Loops 4,5,6,and 7 require a reflection about the z-axis.

Effective width of 1 wire is 2.5mm

5 current split in half via a parallel circuit.

Figure 6.2b specifies the details for all of the loops labelled in figure 6.2a. Each loop is fully specified by a table providing the ϕ and z coordinates of its four corners moving clockwise from the bottom right. The depth, width, and radial position of the groove for each loop are shown schematically beside the coordinate table. This schematic side-bar also specifies the number of wires and relative sense of wrapping for each loop. For example, loop #4 is specified as having a continuous groove running from $\phi = -38^\circ$, $z = 13.0$ cm in the lower right corner of figure 6.2a across to $\phi = 40^\circ$, $z = 14.8$ cm, straight up along z to $\phi = 40^\circ$, $z = 38.75$ cm, back across the azimuth to $\phi = -38^\circ$, $z = 38.75$ cm, and finally straight back down z to close the loop at the starting point. The side-bar shows that the loop is made of a groove of width = 2 wires = 2×2.5 mm = 5.0 mm and depth = 325.0 mm - (320.0 mm + 1 wire) = 7.5 mm. Also, there are two wires on the first layer, and 1 on the second layer. All wires for loop 4 are wrapped in a clockwise direction in figure 6.2a. Note that all grooves for the x-direction gradient coil are set into the outer ($r = 0.325$ m) surface.

A schematic overview of the y-direction gradient coil optimized design is given in figure 6.3. The main gradient coil is shown enclosed in a white dashed box, while the movable $B_z = 0$ correction current is encased in a black dashed box. As displayed, the coil is in its configuration where the movable current is nearest to the -90° azimuthal angle. Details of the groove specification for the stationary main part of the y-gradient coil are given in figure 6.4. Figure 6.4a gives an expanded view of the schematic description of the loops, while figure 6.4b gives the quantitative details.

The same convention has been used as in the x-direction. Note that the grooves are to be cut out from the 24.5 cm radius inner surface.

Figure 6.5 defines the groove specification for the movable correction current of the y-gradient coil. Again, the loop schematic and quantitative details are given in parts a and b of the figure respectively. The specified grooves are set into the inner surface of the correction current former which has a radius of 22.5 cm. Because of the spatial variability of this correction current, all points are specified in terms of a movable coordinate system (ϕ' and z'). To convert to the stationary coordinate system (ϕ and z in figures 6.3 and 6.4), the position of the origin of the movable coordinate system can be given in terms of its stationary coordinates. Optimization resulted in a y-coil that could be used in three configurations (i.e. three position and current values for the correction current). The position of the origin of the movable coordinate system for the three useful configurations is given in table 6.1 (page 235). To simplify further references to the various configurations, they have been named according to the relative position of each VOL. Specifically, the optimized configurations will be shown to have VOL centres at y distances of 9.5 cm, 13.0 cm, and 17.5 cm from the inner (24.5 cm radius) surface of the coil former. These configurations will be referred to as *near*, *centre*, and *far* respectively.

Figure 6.6 specifies the current loops resulting in optimization of the z-direction whole body gradient coil. Seven loops have been optimized as illustrated in figure 6.6a. The details of loop position, and groove width depth etc. are given in figure 6.6b.

The grooves for the z-direction coil are set into the 32.5 cm outer radius surface of the coil former.

Some general points about the overall coil design should be discussed:

- Firstly, nothing has been mentioned about the construction of the above defined whole-body modified-SCSGC set. The details will be left up to the builder. In the x and z gradient wire maps (figures 6.2a and 6.6a), none of the grooves for joining the current loop groups or for connecting to the amplifiers have been specified. For example, in figure 6.2, loops 1, 2, and 3 make up one group, while loops 4, 5, 6, 7, and 8 make up another. There must exist a groove to join these two loop groups, as well as grooves that join the quadrants. The builder must consider the position and size of these joining grooves before beginning construction. It is recommended that the construction guidelines discussed in chapter 4 be reviewed before initiating construction.

- Secondly, the reader is reminded that for the 30 cm SCSGC construction, some parts of the x-gradient jig outer surface had to be glued on after the z-coil had been wrapped. This was due to a conflict between the positions of the current for the two coil directions (x and z) both wrapped in grooves set in the outer surface. A careful comparison between figure 6.2 and 6.6 shows that for the whole body SCSGC, this problem has been alleviated. The radial position of the wires in the x and z coils can be shown to be offset resulting in no conflicts. The comparison in the z and ϕ direction is facilitated by figure 6.7 which displays the x gradient loop configuration superimposed above the z coil. There exists no point where the presence of a z groove affects the wrapping of the x coil in the x grooves, and vice versa.

Figure 6.7 X gradient coil configuration (black) superimposed on the Z configuration (grey) for the modified whole body SCSGC.

6.4 Whole body SCSGC performance

6.4.1 Specifications

The designed whole-body y-gradient coil has three operating configurations. Figure 6.8 illustrates the usable FOV position and size, as determined by calculation, for the three configurations. The z dimension is not shown for illustrative simplicity. However, the size of the VOL in the z direction is greater than the x direction at all points. The y-centre of the FOV for the centre, near, and far coil positions is seen to be 13.0 cm, 9.5 cm, and 17.5 cm respectively. The net y direction range of the usable FOV is 15.5 cm. The central configuration VOL has an x and z direction width of 8 cm minimum while the near and far configurations span at least 7 cm in these directions.

The detailed specifications of the whole body modified SCSGC are presented in table 6.2, where parts a, b, and c give the specifications for the centre, near, and far configurations respectively. The resistance, inductance, size and position of the centre of the VOL as well as the gradient strength have been calculated using the software described in chapters 3 and 4. The resistance calculations are based on the length of wire, the diameter of 12 gauge wire, and the resistivity of copper. The inductance calculation parameters used were:

ϕ resolution:	201 points
z resolution:	201 points
# of increments in	
B field calculations:	1000.

These values result in an error on the inductance calculation of $\pm 20\%$.

Figure 6.8 Usable FOV for the 3 configurations of the whole body y-gradient coil.

Table 6.2a Specifications for the **central** configuration of the whole body modified-SCSGC.

	Resistance ($\pm 0.02\Omega$)	Inductance ($\mu\text{H} \pm 20\%$)	Size of VOL ¹ ($\pm 0.1\text{cm}$)	Gradient strength ² ($\pm 0.05\text{G/cm}$)
X	0.96	161	10.0×12.0×10.0	0.77
Y	0.15 , 0.50	55 , 300	8.0×8.5×8.0	1.37 (@ 100A , 135A)
Z	0.50	168	10.0×12.0×10.0	0.90

Table 6.2b Specifications for the **near** configuration of the whole body modified-SCSGC.

	Resistance ($\pm 0.02\Omega$)	Inductance ($\mu\text{H} \pm 20\%$)	Size of VOL ¹ ($\pm 0.1\text{cm}$)	Gradient strength ² ($\pm 0.05\text{G/cm}$)
X	0.96	161	7.0×7.0×7.0	0.77
Y	0.15 , 0.45	60 , 280	7.0×7.5×7.0	1.60 (@ 100A , 145A)
Z	0.50	168	7.0×7.0×7.0	0.91

Table 6.2c Specifications for the **far** configuration of the whole body modified-SCSGC.

	Resistance ($\pm 0.02\Omega$)	Inductance ($\mu\text{H} \pm 20\%$)	Size of VOL ¹ ($\pm 0.1\text{cm}$)	Gradient strength ² ($\pm 0.05\text{G/cm}$)
X	0.96	161	8.0×8.0×8.0	0.78
Y	0.15 , 0.45	70 , 325	7.0×7.5×7.0	1.01 (@ 100A , 125A)
Z	0.50	168	7.0×8.5×7.0	0.88

¹ Size of VOL is given as x × y × z region where gradient does not differ by more than 5%.² X and z gradient strength are calculated at 100A. Y current is as specified in brackets as (@stationary coil current, equatorial current) .

In the case of the y-gradient, the current in the $B_z = 0$ correction coil must be controllable independently of the main coil. Therefore the two coils in the y direction gradient set must be supplied current from different amplifiers. The two coils making up the y-gradient set must be specified by separate values of resistance and inductance, as indicated by the two numbers separated by a comma in the tables. The first value relates to the non-movable coil, while the second specifies the movable $B_z = 0$ correction coil. The inductances for the stationary and movable y-coils were determined by first calculating separately the self inductance of each, followed by inclusion of the mutual inductance between them. The gradient strengths for the x and z directions are all designated as being calculated at a current of 100 A. Because the ratio between the current in the stationary and movable y-coils must be varied for the different configurations, the current must be indicated separately. The gradient strength for the y-direction is specified, followed by, in order, the current for the stationary and the movable coil.

6.4.2 Calculated B_z field

The B_z field has been calculated for the x, y, and z whole-body modified SCSGC. Figure 6.9 shows contour plots of the B_z field for the three plane orientations of the x gradient direction. Each plane is plotted at three offset positions progressing from the centre to the edge of the calculated VOL. The region where the gradient is linear to $\leq 5\%$ (relative to the gradient at the centre of the VOL) is plotted as the bold lines in each figure. The B_z and 5% gradient contour plots for the centre, near, and far configurations of the y coil are given in figure 6.10, 6.11, and 6.12 respectively.

Figure 6.9 Contour plots of B_z (gauss) in the xy , yz , and zx planes for the x -gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

————— 5% diff line ————— B_z contours (gauss)

a) $z=0.0\text{cm}$

b) $z=2.0\text{cm}$

c) $z=4.0\text{cm}$

d) $x=0.0\text{cm}$

e) $x=2.0\text{cm}$

f) $x=4.0\text{cm}$

g) $y=16.0\text{cm}$

h) $y=12.5\text{cm}$

i) $y=9.5\text{cm}$

Figure 6.10 Contour plots of B_z (gauss) in the xy, yz, and zx planes for the centre y-gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

Figure 6.11 Contour plots of B_z (gauss) in the xy, yz, and zx planes for the near y-gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

Figure 6.12 Contour plots of B_z (gauss) in the xy , yz , and zx planes for the far y -gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

The fields resulting from the z-direction gradient coil are shown in figure 6.13. The results for the x and z gradient directions are obtained using a current of 100 A in the coils. The current in the stationary main coil for the y direction was taken to be 100 A for all three configurations. In the case of the movable $B_z = 0$ correction y-coil, the current was chosen for the calculations to be 135 A, 145 A, and 125 A for the centre, near, and far configurations respectively.

6.4.3 Simulated images

Using the B_z fields plotted in figures 6.9 to 6.13, simulated images of a grid phantom have been produced by the NMR simulation software written for this work. Figure 6.14 gives the resulting images in the xy, yz and zx planes for the FOV corresponding to the central coil configuration. The imaging parameters were set as:

- spin echo pulse sequence,
- $\text{FOV} = 10 \text{ cm} \times 10 \text{ cm}$,
- resolution = 64×64 .

The slice thickness was assumed to be infinite. For comparison, a simulated image of the grid phantom under linear gradient conditions was given in figure 4.13. Note that both possible orientations of the phase and read gradient direction are shown side by side for each plane. Figures 6.15 and 6.16 present the simulated images corresponding to the near and far FOV configurations respectively. The raw image data consists of natural numbers between 0 and 255 representing a range of image intensity.

Figure 6.13 Contour plots of B_z (gauss) in the xy , yz , and zx planes for the z -gradient coil. Each orientation is plotted at 3 positions along the perpendicular line.

Figure 6.14 Simulated images of a grid phantom using gradient fields calculated for the whole body SCSGC (central configuration). FOV=10 cm Resolution=64 x 64.

Figure 6.15 Simulated images of a grid phantom using gradient fields calculated for the whole body SCSGC (near configuration). FOV=10 cm Resolution=64 x 64.

Figure 6.16 Simulated images of a grid phantom using gradient fields calculated for the whole body SCSGC (far configuration). FOV=10 cm Resolution=64 x 64.

Picture elements (pixels) represented by data value 0 will appear black on the image, while pixels with data value 255 will be white. Pixels with intermediate data values will be scaled with appropriate levels of grey.

6.5 Discussion of the whole body modified-SCSGC performance

As mentioned in the discussion of the 30 cm SCSGC in chapter 4, the VOL as calculated by the optimization software is smaller than the VOL determined by examination of the 5% gradient non-linearity lines seen in the B_z contour plots. This can be confirmed for the whole-body coil results by comparing figures 6.9 through 6.13 to table 6.2. For example, the z-gradient in figure 6.13 shows 5% linearity out to at least 5 cm in the x and z directions along a range in the y direction that spans from $y = 2.5$ cm to $y = 22.0$ cm. In the central region, the z coil is seen to provide linearity with 13 cm and 21 cm width in the x and z directions respectively. By comparison, table 6.2 specifies the largest z coil VOL of 10.0 cm $\times$ 12.0 cm $\times$ 10.0 cm for the central coil region. Similar *inflated* VOLs can be seen for the x and y gradient coils if figures 6.9, 6.10, 6.11, and 6.12 are examined.

Additional considerations must be made when the practical imaging volume is discussed. First note that in the case of the y-gradient direction, the centre of the image FOV must coincide with the point where $B_z = 0$. Furthermore, the FOV in both orthogonal directions of an imaging plane must be equal. This fact was used for the schematic representation of the FOVs in figure 6.8. As can be seen the FOV for each of the three configurations is given as the largest square centred at $x = 0$ and $y = y_{centre}$ (y_{centre} is the point where $B_z = 0$).

The practical consequence of this is that in most cases, if the FOV is increased beyond the size defined in figure 6.8 only one of the gradient directions will exceed its 5% linearity region. Often the resultant image will be acceptable if the phase versus read gradient orientation is chosen carefully.

The simulated images of figures 6.14 to 6.16 help to demonstrate these points. The linearity of the lines in the grid phantom can be used as a measure of the linearity of the gradient fields. In all cases the gradients can be seen qualitatively to produce a useful image out to the 10 cm FOV. Note that the spurious vertical line in the z-y plane of figure 6.15 is caused by aliasing due to an increase in the gradient strength from the central value. This FOV of 10 cm is larger than the FOV values determined from the field calculations and shown in figure 6.8. However, the images contain useful information out to the full 10 cm FOV even in the case of the near and far configurations which both have a calculated FOV of $7 \text{ cm} \times 7 \text{ cm}$. This increase in the usable VOL is due not only to above mentioned *inflated* VOL, but also to the fact that the 5% tolerance used throughout this thesis tends to under-define the VOL that is useful for imaging.

A gradient strength of 1.8 G/cm was specified as the target in the goals of the optimization. This value was determined as the strength required for an EPI imaging sequence with $\text{FOV} = 8 \text{ cm}$, 64×64 pixel resolution, and a digitizer dwell time of $5 \mu\text{s}$. Using table 6.2 as a guide, the current required through the x gradient to attain 1.8 G/cm is calculated to be 233 amperes. The 12 gauge wire recommended for the blueprints of the whole-body SCSGC can handle more than double this current for the duty times used in imaging sequences.

The required current to produce 1.8 G/cm in the moveable central, near, and far y-gradient coil configurations is 177, 163, and 221 amperes respectively. Similarly the stationary y coil requires 132, 113, and 176 amperes. For the z coil 204 amperes of current is necessary for 1.8 G/cm gradient strength.

These large current values are attainable with the gradient amplifier system available to drive the existing *factory installed* gradient system for the 1 m whole body magnet at the Institute for Biodiagnostics at the National Research Council of Canada. For each direction of gradient coil, the system consists of 4 *Techron* model 7700 amplifiers: two parallel groups of two series interconnected amplifiers. The resultant amplifier group is capable of producing 300 volts across the load and 300 amperes through it.

Using the specifications of this existing amplifier system, the ramp time for the SCSGCs to acquire a specified current can be calculated using equation (2.11). The rise time to 1.8 G/cm has been determined to be 231 μ s for the x gradient coil. The y-coil will reach 1.8 G/cm in 211 μ s, 175 μ s, and 292 μ s for the centre, near, and far current configurations. The z coil is the fastest switching gradient coil of the SCSGC set, attaining 1.8 G/cm in 140 μ s. In all cases, the SCSGC can reach 1.8 G/cm within the 300 μ s design criterion.

Table 6.3 gives a comparison of ramp time (to 180 amperes) between the designed whole body SCSGC and the *factory installed* gradient coil set. The calculations for the installed set are based on measured values of inductance and resistance. The installed coil values have been scaled from 70 cm diameter to 50 cm diameter.

Table 6.3 Comparison between scaled whole body factory installed gradient coil set and the whole body SCSGC.

	Factory Installed Coils		Whole Body SCSGC	
	Ramp time ¹ (μ s)	Efficiency ((G/cm) ² / μ H) $\times 100$	Ramp time ¹ (μ s)	Efficiency ((G/cm) ² / μ H) $\times 100$
X	330	2.6	143	3.7
Y-centre	330	2.6	214	3.4
Y-near	330	2.6	196	3.9
Y-far	330	2.6	228	2.5
Z	250	3.2	120	4.8

¹ Time taken to 180 amperes.

As can be seen switching time is lower using the SCSGC for all directions of field gradient.

As previously described, the ratio of the square of the gradient strength (at a specified current) to the inductance is a measure of the efficiency of a gradient coil. In table 6.3 the efficiencies of the x, y, and z installed coil set are compared to those of the SCSGC. Again, the installed coil set has been scaled in diameter from 70 cm to 50 cm. Except for the far y-gradient configuration, the coil efficiency of the SCSGC is greater than that of the scaled installed coils. The far-gradient configuration provides efficiency closely matching the installed y-gradient coil. It is interesting to note a loss in efficiency as one goes from the near to far configurations of the y SCSGC.

As seen in figure 6.8, the size of the VOL of the SCSGC has effectively been doubled by the moveable VOL. However, the VOL sizes quoted in table 6.2 are still smaller than that of the installed coils whose specified VOL is scaled to a 28 cm sphere of 5% linearity. The gain in switching time and coil efficiency of the SCSGC is significant, and will permit imaging requiring faster, stronger coils. Clearly if the main criterion of a coil design is size of VOL, a more conventional coil design is in order. However if switching time and/or gradient strength are the main considerations, the SCSGC will give the necessary results.

A comment is required regarding to the name *semi-cylindrical surface gradient coil*. In a design with moveable current corrections, the blueprints show that the required former is a cylindrical surface rather than the approximately semi-cylindrical design when no VOL movement is required.

The name SCSGC has been maintained for the moveable current design because it evolved from the original design that was truly semi-cylindrical.

The results presented in this chapter are meant only as an example application of the movable VOL. The resultant coil design was optimized under specific conditions, and will not be the best coil for all needs. For example if the range of VOL movement were reduced, it should be possible to optimize a gradient set with a more efficient y coil than that described in this chapter.

Chapter 7: Conclusions

7.1 Summary

Imaging sequences in NMR are often limited by the technology of the hardware available. This is particularly evident in the case of gradient coil designs. With the ever-increasing demands set on gradient coil performance, specific designs have become necessary for each application. The SCSGC and movable VOL SCSGC introduced in this thesis offer alternatives to existing gradient coil concepts and designs. The optimization software described in this thesis provides a tool for designing gradient coils with preset performance specifications.

An SCSGC prototype was designed and constructed for use in a 30 cm cylindrical magnet bore. The design was tested in the 40 cm 4.7 T magnet at the Institute for Biodiagnostics at Canada's National Research Council. The results were used to confirm the accuracy of the gradient coil design software. All of the calculated specifications were found to agree with those measured using the constructed SCSGC set, thus validating further use of the optimization software. The prototype also provides evidence of the attributes of the SCSGC. Comparison with other gradient coil geometries suggests that the SCSGC is an excellent option when gradient strength and switching speed are of primary importance.

The main shortcoming of the SCSGC was seen to be a relatively small VOL compared with coils based on full cylindrical geometry. Gradient coils with moveable current loops of geometry similar to those of the SCSGC were found to improve this

small VOL limitation. A gradient coil design employing these movable current elements was optimized for use in a human whole-body NMR system. The resultant coil design indicates improvements in switching speed and efficiency (gradient strength to inductance ratio) over conventional gradient coils with the same access size.

In general, the best choice of gradient coil geometry depends on the significance assigned to the various performance characteristics. The main characteristics that must be considered are size of the VOL, gradient strength, inductance (switching time), and accessibility to the VOL. Table 7.1 shows a general guide to the pros and cons of the different coil geometries discussed in this thesis. The table ranks the various coil geometries from best to worst for each of the main gradient coil characteristics. The gradient strength and inductance are characterized together in the table as efficiency.

Table 7.1 Overview of coil geometry attributes.

	efficiency	size of VOL	patient access
best	PI	CC	CC
↓	SCSGC	Quad	SCSGC, Bipl, PI
↓	Quad, Bipl	SCSGC, Bipl	
↓			
worst	CC	PI	Quad

- PI ↔ planar surface gradient coils. Note: PI coils suffer from $B_z \neq 0$ at coil centre for the y gradient direction.
- Quad ↔ quadrupole gradient coils.
- Bipl ↔ biplanar gradient coil set.
- CC ↔ conventional full cylinder gradient set.

Planar surface coils are described by Chow and Yi⁵², quadrupole coils by O'Dell et al.⁴⁸, while Martens et al.³⁷ give a good analysis of biplanar surface gradient coils. The

conventional full-cylinder gradient set characteristics are obtained from the installed gradient sets at NRC. These are general results, and should only be used as a guideline. For a more detailed analysis, the reader is referred to the discussion in chapter 4 of this thesis.

The T-Simplex optimization method was shown to be more efficient than either simplex or simulated annealing techniques for the design of gradient coils. For productive use of the T-Simplex algorithm, the optimization variables must be chosen wisely. An intensive study of the optimization variables was carried out, and the results summarized in this thesis. This information can be used both to solidify one's understanding of the T-Simplex process, and to provide the software user with a more informed choice of optimization variables.

7.2 Future Work

- The constructed SCSGC set allows NMR sequences not possible with the installed gradient coils in the NRC's 40 cm 4.7T magnet. The coil provides *Echo Planar Imaging* and improved *Diffusion* imaging options for this NMR system. The size of the coil will provide access to small pigs, dogs and cats. Many new experiments are now possible.

- The whole-body SCSGC design presented in chapter 6 can be constructed to provide EPI options for the whole-body 3T magnet at the NRC. This will enable a wide range of imaging studies to be performed on human volunteers.

- The gradient coil design software written for this thesis can be used to design further coils for specific needs as they arise.

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