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Design of Concrete Shear Wall Buildings for Earthquake Induced Torsion

by

Soheil Yavari, B.Sc.

A thesis submitted to the
Faculty of Graduate Studies and Research in partial fulfillment
of the requirements for the degree of

Master of Applied Science

Department of Civil and Environmental Engineering
Faculty of Engineering
Carleton University
Ottawa, Ontario, Canada
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Design of Concrete Shear Wall Buildings for Earthquake Induced Torsion

Submitted by
Soheil Yavari

in partial fulfillment of the requirements for
the degree of Master of Applied Science

Chair, Department of Civil and Environmental
Engineering

Thesis Supervisor

Carleton University
Ottawa, Ontario, Canada
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Abstract

Observation of damage after earthquakes has confirmed that torsional vibration of buildings, induced by lateral seismic ground motion, is one of the contributing factors to the damage and subsequent collapse of structures. Many researchers have therefore studied the elastic and inelastic torsional response of building models. Recent studies have shown that the conventional method of design of concrete shear wall structures in which stiffness is considered as being independent of strength needs to be revised. Both analytical and experimental investigations have revealed that stiffness of reinforced concrete elements is significantly influenced by the amount of reinforcing steel, the axial load and the arrangement of reinforcement and hence by the strength of the wall.

The interrelation between the strength and stiffness of concrete shear wall structures and an amended method of design for torsion is presented in this work. Inelastic analyses of building models, carried out in the presented study, show that buildings designed according to the proposed method exhibit peak ductility demands that are equal or less than those for the associated torsionally balanced buildings, indicating that the proposed method is rational and adequate.

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List of Important Notations

a	plan dimension of the building parallel to the direction of earthquake
A_g	gross area of section
A_{st}	total steel area
a/v	ratio of peak ground acceleration expressed in terms of g to the peak ground velocity expressed in the units of m/s
b	plan dimension of the building perpendicular to the direction of earthquake
C	damping matrix
CM	centre of mass
CR	centre of rigidity
CV	centre of strength
D_I	distance between the centre of rigidity and the centre of strength

E_c	modulus of elasticity of concrete
e_R	static eccentricity
e_d	effective dynamic eccentricity
e_{d1}, e_{d2}	design eccentricities
F_{y1}, F_{y2}, F_{y3}	yield strengths of resisting planes 1, 2, 3, respectively
F_t	Top floor force
f'_c	standard concrete compressive strength
f_y	steel yield strength
g	ground acceleration
h_w	total height of wall
h_e	effective height of wall
I_g	gross moment of inertia
I_{eff}	effective moment of inertia
k_1, k_2, k_3	stiffness of planes 1, 2, 3, respectively
K	stiffness matrix
K_{tot}	total stiffness of structure

K_{θ}	rotational stiffness of a storey about the centre of mass
l_w	wall length
M	mass matrix
M	applied moment to the structure
M_y	first yield moment of the section
M_n	nominal flexural strength
M_u	ultimate moment of the section
m	mass of the floor
N	axial load on shear wall
Plr_f	plastic hinge rotation of the flexible edge
Plr_s	plastic hinge rotation of the stiff edge
r_{Plrf}	ratio of plastic hinge rotation of the flexible edge
r_{Plrs}	ratio of plastic hinge rotation of the stiff edge
R	strength reduction factor
r	radius of gyration of the floor about the centre of mass

$r_{\delta f}$	mean of ratio of displacements for the flexible edge, obtained for a set of earthquakes
$r_{\delta s}$	mean of ratio of displacements for the stiff edge, obtained for a set of earthquakes
$r_{\mu f}$	mean of ratio of ductilities for the flexible edge, obtained for a set of earthquakes
$r_{\mu s}$	mean of ratio of ductilities for the stiff edge, obtained for a set of earthquakes
$SDOF$	single degree of freedom
S_a	spectral acceleration corresponding to frequency
T	translational period of structure
V_e	total elastic strength of resisting planes
V_0	base shear in the torsionally balanced building
α_2, α_3	ratio of length of the second and third walls to the length of first wall, respectively
β_2, β_3	ratio of strength of the second and third walls to the strength of first wall, respectively

δ_u	maximum displacement
δ_{uf}	maximum displacement of the flexible edge
δ_{us}	maximum displacement of the stiff edge
η	ratio of the effective moment of inertia to the gross moment of inertia
$\dot{\varphi}_y$	first yield curvature
φ_y	yield curvature
φ_u	curvature at failure
γ_2, γ_3	ratio of stiffness of the second and third walls to the stiffness of first wall, respectively
μ	displacement ductility
μ_b	displacement ductility in the torsionally balanced building
μ_f	displacement ductility at the flexible edge of the torsionally unbalanced building
μ_s	displacement ductility at the stiff edge of the torsionally unbalanced building
Ω_R	frequencies ratio

ρ	steel ratio
w_i	i^{th} floor weight
ω_y	uncoupled translational frequency
ω_θ	uncoupled rotational frequency
ω_1, ω_2	first two coupled frequencies
ξ	damping ratio

Chapter 1

Introduction

1.1 General

In quite a number of building structures, the centre of rigidity does not coincide with the centre of mass. Therefore, in case of an earthquake, lateral base motion induces torsional vibration in the structure. Field observations of earthquake damage have shown numerous examples of structural failure due to such motion. Damage due to torsional motion was observed during the 1971 San Fernando earthquake (Hart et al, 1975), the 1985 Mexico earthquake (Rosenbleuth and Meli, 1986; Esteve, 1987), the 1985 Loma Prieta earthquake (Mitchell et al, 1990), the 1990 Philippines earthquake (Booth et al, 1991), the 1994 Northridge earthquake (Mitchell et al, 1995), the 1995 Kobe earthquake (Mitchell et al, 1996), the 1998 Turkey earthquake and the 1999 Taiwan earthquake. These events have led to a renewed interest in the study of torsional response of building structures.

Many researchers have studied the coupled lateral-torsional motions of both single and multi-storey building structures and have investigated both elastic and inelastic behaviour of building models. The torsional behaviour of buildings, especially multi-storey building, is not easy to determine since a large number of parameters affect this

behaviour. As a consequence, the results of various studies are not always consistent. Although results obtained by different researchers give a general understanding of the nature of torsional response and the relevant parameters, they are applicable only to the models studied and the particular assumptions made in each of those.

Currently, most seismic code provisions for design against torsion are based on well-established principles of elastic structural behaviour. Several of the recent theoretical studies, which have critically examined the appropriateness of established code rules, have attempted to modify the existing design eccentricity parameters to achieve a better agreement with the findings of the dynamic analyses of non-linear structural models.

The results of such studies have led to the refinement of code provisions. However, the design provisions for seismic torsion continue to be based on the specification of design eccentricities. The seismic forces are applied through points located a distance from the centre of mass that is equal to the design eccentricities. The design forces in the individual elements are based on elastic analysis and are proportional to the stiffness of the elements. The resisting elements are then designed to have a strength that is equal to or greater than the calculated design force.

Recent studies (Paulay, 1998,1999 and Priestley and Kowalsky, 1998) have shown that for reinforced concrete structures, in particular shear wall structures, strength is an independent parameter that can be assigned arbitrarily. Also, once the strength has been assigned, the stiffness is automatically determined. In other words, contrary to the

traditional belief, stiffness is not determined only by the geometry of the element, but is also dependent on the strength.

The objective of the present study is to investigate the effects of variation of the fundamental quantities like stiffness and strength on the design of shear wall-type structures for seismic torsion. Moreover, inelastic torsional responses of single and multi-storey building models are studied to identify important quantities and to make design recommendations based on the results obtained. The validity and limitations of the recommendations made are established by studying the response of a series of building models to a significant number of earthquake records.

1.2 Causes of torsional motion

The dynamic forces that act on a structure during an earthquake are related to the inertia and act through centres of mass. These inertia forces are resisted by elastic forces in the lateral load resisting elements whose resultants pass through centre of strengths (CV). If the resisting elements and their assigned strengths in a building are so distributed that the centres of rigidity and the centres of strengths do not coincide with the centres of mass, lateral seismic forces cause torsional motion in the structure. Torsional motion induced in the structure by these eccentricities is referred to as *natural torsion*.

Torsional motion may also result due to a variety of factors other than known asymmetry. One such factor is asymmetry that may exist in a normally symmetric structure because of uncertainty in the evaluation of centre of mass and stiffness. For example, the actual distribution of mass may be different from the one assumed in the computations. The

estimation of the stiffness of resisting elements may be inaccurate because of the lack of precise data on the modulus of elasticity, extent of cracking in concrete etc. Another factor is the torsion of structure caused by ground rotation about a vertical axis. Torsional motion due to above two reasons is referred to as *accidental torsion*.

1.3 Literature review

Earthquake response of linear elastic structures whose centres of mass are eccentric with respect to the centres of rigidity and strengths has been studied by several researches. Lumped mass model has been used in many early studies (Ayre, 1956; Housner and Outinen, 1958; Medearis, 1966; Rosenbluth and Elorduy, 1969). Other studies have used continuous parameter models (Berg, 1962; Gibson et al, 1972). Based on the numerical results for specific models used, several studies indicate a reduction in base shear as a result of coupling (Bustamante and Rosenbluth, 1960; Rosenbluth and Elorduy, 1969; Jhaveri, 1968).

On the basis of a study of the elastic response of torsionally coupled building, for idealized flat and hyperbolic spectra, Kan and Chopra (1976) concluded that lateral torsional coupling induces torsion, amplifying the static torque but generally reducing the base shear. When the excitation was applied in one direction, they derived a relation between the base shear and torque in a torsionally coupled system and the corresponding quantities in the associated uncoupled system.

Kan and Chopra showed that the results obtained from single storey models were also applicable to a *special class* of multi-storey buildings. In such buildings the centres of

mass and stiffness lie approximately on two vertical lines and the ratio of the storey stiffness in two orthogonal directions in the floor plan is about the same for all storeys. It should be noted that the initial studies of Kan and Chopra (1976) were based on elastic response, which is not affected by the location of the centre of strength. Observations similar to those of Kan and Chopra have given rise to torsional provisions in building codes for the multi-storey buildings of special class, based on the results obtained for single storey elastic models. The application of code specified eccentricity concept to the multi-storey buildings of a general type, however, has not been easy because there is no generally accepted definition of centres of rigidity for multi-storey buildings. For a general multi-storey building, centre of rigidity has been defined in many different ways by researchers (Poole, 1977; Humar 1984; Smith and Vezina, 1985; Cheung and Tso, 1986; Riddell and Vasquez, 1984) and there is no universally accepted definition.

Some later studies (Dempsey and Tso, 1982; Tso 1983) showed that the maximum displacements on the edges of a building model provide a more realistic assessment of torsional effects for design purposes as compared to maximum torsional moments. This gave rise to the concept of effective edge eccentricity, e_e . The effective edge eccentricity was defined as the distance measured from the centre of rigidity at which the static shear should act in order to give rise to the same edge displacement as that obtained by using dynamic analysis.

Accidental torsion created due to uncertainties in stiffness and mass and due to ground rotation has been studied by several researchers (Awad and Humar, 1984; Pekau and Guimond, 1990). De La Llera and Chopra (1992, 1994a) studied the response of

nominally symmetric plan building models using a probability distribution of structural element stiffness and concluded that the additional displacement due to stiffness uncertainty is less than 10 and 5 percent for reinforced concrete and steel structures respectively. De La Llera and Chopra (1994a) studied the torsional response of symmetric buildings for a derived set of rotational excitations and concluded that the additional response due to ground rotation about a vertical axis is less than 5% for torsionally stiff edge.

Although the effects of torsional coupling on the linear elastic response of buildings are now well understood, these results are not directly applicable to the calculation of earthquake design forces for a majority of the buildings because they are usually designed to deform significantly beyond the yield limit during ground motions. To understand inelastic torsional response, a number of studies on the eccentric single mass system have been carried out. Kan and Chopra (1981) showed that the torsional coupling effects in the inelastic system are similar to those in the elastic system. Using a single element model in their studies, they concluded that the response in the inelastic range is affected by torsional coupling generally to a lesser degree than the elastic response and after initial yielding the system responds primarily in translation, behaving more like a single degree of freedom system.

Using a three-element model, Tso and Sadek (1985) demonstrated that:

1. Torsionally coupled systems do not respond primarily in translation when they are excited well into the inelastic range, on the contrary, significant torsional motion exists when peak ductility demand is reached.

2. An increase by a factor of two in the ductility demand of edge elements is not uncommon for systems with large eccentricity as compared to symmetrical plan systems.
3. Ductility demand is not sensitive to the ratio of uncoupled torsional to lateral frequencies.

Further studies considered additional ductility demands and additional deformations at the edge frames of a building, as the inelastic torsional response parameters. Tso and Ying (1990) demonstrated that the distribution of strengths among elements has a strong influence on the inelastic seismic response of structures.

To clarify the discrepancies in the assessment of ductility demands of edge elements, Tso and Zhu (1992a and 1992b) studied a wide range of three-element single storey models. They examined the strength distribution of lateral load resisting elements in a torsionally unbalanced system, based on codified torsional provisions. It was shown that the element strength can be expressed conveniently as the element strength of a similar but torsionally balanced system multiplied by a strength factor. This strength factor depends upon three system parameters, namely, the location of the element relative to the centre of rigidity, torsional stiffness, and eccentricity of the structure. Although this method has been shown to work well, it requires explicit determination of the centre of rigidity.

The ultimate goal of all the studies performed on single storey inelastic model has been to generate globally acceptable design guidelines for asymmetric single as well as multi-storey structures that undergo significant inelastic deformations during a major earthquake. Using generally accepted concepts for specifying the strength distribution in

single storey models, several studies have extrapolated the knowledge available to multi-storey inelastic models. Many researchers have designed a multi-storey building according to the torsional provisions of several codes (that are primarily based on the results of single storey models) and investigated the validity of such provisions by means of a three dimensional dynamic analysis (De La Llera and Chopra, 1992; Sedarat and Gupta, 1992).

Yoon and Smith (1995) suggested a method to predict the degree of translational-torsional coupling of mixed-bent-type multi-storey building structures (i.e. wall-frame structures) subjected to dynamic loading.

Some recent studies have concentrated on improving the torsional provisions of building codes for the design of general multi-storey buildings. De Stefano (1993) studied a model having additional lateral load resisting elements in the direction perpendicular to the ground motions. However, these transverse elements were assumed to remain elastic throughout the analysis and the results obtained were therefore not relevant to actual buildings. A subsequent study (Wong and Tso, 1995) evaluated the seismic torsional provisions of NBCC 1990, by considering single mass structural model with bi-directional earthquake motion and inelastic transverse elements. It was recommended that for considering torsional effects in such cases, the edge elements should be designed for the worst case of (1) 100% x-directional effects + 30% y-directional effects, and (2) 100% y-directional effects + 30% x-directional effects.

Inelastic behaviour of asymmetric multi-storey buildings has been studied by De La Llera and Chopra (1995). They have investigated six different structural characteristics and

their effects on the torsional response of buildings. The six quantities are strength of orthogonal resisting planes, stiffness asymmetry, strength asymmetry, plan wise distribution of strength, number of resisting planes, and intensity of ground motion component in the orthogonal direction. De La Llera and Chopra concluded that the two most effective means of increasing the torsional capacity of the system are; introducing resisting planes in the orthogonal direction, and modifying the stiffness and strength distribution to localize yielding in selected resisting planes.

Humar and Kumar (1998 I, 1998 II) have studied the elastic and inelastic torsional response of both single and multi-storey building models subjected to earthquake motion. One conclusion of their studies is that the ratio of the uncoupled torsional frequency to the uncoupled translational frequency of the building is one of the most important parameter among those governing torsional response. They have suggested some modifications to the design provisions, which give design forces closer to the results obtained from a dynamic analysis.

Dusicka, et al. (2000) have investigated the significance of strength characteristics in inelastic torsional seismic response and have concluded that the stiffness as well as strength distribution has a significant impact on the overall system torsional response and both should be considered in the analysis of torsional susceptible inelastic structures.

Using a two elements model, Myslimaj and Tso (2001) have examined different distributions of strength and stiffness to minimize the torsional effects and the results have indicated that locating the centre of strength and the centre of rigidity on the opposite sides of centre of mass minimizes the rotation of the diaphragm.

Several studies have been carried on the relationship between the yield curvature of a reinforced concrete wall and the steel ratio, reinforcement arrangement and axial load ratios (Paulay and Priestley 1993; Paulay 1997; Priestley and Kowalsky 1998). Based on these studies, Paulay (1998, 1999) has suggested that the lateral stiffness distribution is no longer defined prior to the assignment of strengths to the elements. Consequently the stiffness eccentricity, e_R , cannot be computed readily and the strength eccentricity defined as the distance from centre of mass to the centre of strength, e_V , should be taken into account as well as e_R .

1.4 Issues related to the design of concrete shear wall structures subjected to seismic torsion

Most of the studies referred to in the previous section as well as the torsion design provisions of seismic codes are based on the implicit assumption that the stiffnesses of the resisting elements are related only to their geometric dimensions and can therefore be determined once these dimensions have been specified. A knowledge of the stiffness properties permits the determination of the centres of rigidity and hence the eccentricity between the centres of mass and the centres of rigidity. The seismic torsion forces in turn depend on these eccentricities. Code provisions relate the design strengths to the torsion induced forces calculated from the determined eccentricities which have been amplified for dynamic effect. The stiffness of the element being designed appears in the calculation of these forces.

As stated earlier, Priestley and Kowalsky (1998), as well as Paulay (1998,1999), have pointed out that for concrete structures, particularly shear wall structures, the stiffness is not dependent on the geometry, alone. In fact, strength, rather than stiffness, is the independent parameter and the latter is automatically determined once a strength has been assigned to an element with a well-defined geometry.

It is apparent that for shear wall structures of concrete, the traditional method of design for seismic torsion must be revised. One of the issues to be addressed is the interrelationship between the geometry, the yield curvature, the yield strength and the stiffness. Another issue is to devise a design procedure that recognizes the interdependence of strength and stiffness. An additional topic for study is the impact of the relative locations in plan of the centres of rigidity, the centres of strength and the centres of mass.

1.5 Statement of the problem

As described in the previous sections, several researchers have studied various aspects of the torsional response of buildings. During the course of such studies, many issues have been identified, some of which are yet to be settled. A number of issues related to the seismic torsional response of buildings can be identified. A few of them are listed below. The present work addresses some of these issues, as outlined in the next section.

1. What is the relationship between the stiffness of an earthquake load-resisting element and its strength and, as a result, what is the relationship between the locations of the centre of rigidity and the centre of strength of a building.

2. For a reinforced concrete structure, to what extent the reinforcement and axial load may affect the lateral stiffness of an element? Are the code provisions adequate for estimation of stiffness of earthquake-load resisting elements?
3. What is the nature of displacement and ductility demands in the edge elements of a single-storey building model, designed according to current code provisions?
4. What design strategy is applicable to ductile buildings in which the element stiffness varies with variation in strength?
5. Can existing design provisions of building codes be applied to the torsional design of structure in which the element stiffness depends on the strength of the element?
6. Can the knowledge obtained from single storey models be applied to the design of multi-storey models? What are the limitations?

1.6 Objectives and scope

The main objective of this study is to provide a better understanding of the torsional behaviour of concrete shear wall buildings and to recommend design strategies that are both rational and simple to implement. It is clear from the literature review that the results obtained by several researchers are mostly based on the traditional method in which the element stiffness is assumed to be an independent parameter related only to the geometry of the element. However, as pointed out recently by several researchers, this assumption needs to be re-examined.

As a part of ongoing effort to better understand the torsional behaviour of buildings, this study addresses some of the issues listed in the previous section by analysing a wide

range of reinforced concrete shear-wall type single storey and multi-storey building models.

The scope of the work carried out in this study is as follows:

1. Study the moment-curvature relationship for different type of reinforced concrete shear walls and investigate the effects of steel ratio, reinforcement arrangement and axial load ratio on the stiffness of a shear wall.
2. Study the interdependence of element strength and stiffness, taking yield displacement as a parameter that depends mainly on the geometry.
3. Based on the above study investigate the best locations for the centres of rigidity and strengths in relation to the centre of mass.
4. Suggest a strategy for designing shear wall type structures, based on the above observations.
5. Design single storey inelastic models according to the procedure developed as above and investigate the adequacy of the existing code provisions in relation to the new procedure.
6. Study the inelastic behaviour of multi-storey asymmetric building of special class designed according to the suggested procedure.

Chapter 2

Relation Between Strength and Stiffness for Concrete Shear Walls

2.1 Introduction

A fundamental basis for the traditional method of design for earthquake-induced torsion is the assumption that the gross section size of elements can be used to find the related stiffnesses, and the strengths can then be assigned in proportion to these stiffnesses. This is to some extent true for a homogeneous material such as structural steel. Research has shown that in designing reinforced concrete structures, other factors must be considered, as well. These include axial load on the structure, the amount of steel and its distribution and the cracking of concrete under tension. Concrete cracking reduces the section moment of inertia and hence the element stiffness, while increases in the steel reinforcement and axial compressive load, both, increase the stiffness. Effect of some of these parameters has been incorporated in the design codes. For example, the Canadian Standard Association, CSA A23.3-94, for the design of reinforced concrete structures under earthquake-induced loads recommends an effective moment of inertia, I_{eff} , equal to $0.7I_g$ for columns and walls (Notes on Clauses 21.2.2.1 and 21.2.2.2). Paulay and

Priestley (1992) have suggested an effective moment of inertia for columns in the range of $0.4I_g$ and $0.8I_g$, depending on axial load. In a similar manner, they recommend a value between $0.3I_g$ to $0.5I_g$ for walls.

However, the approach of using stiffness in allocating strength to members causes significant errors even when the effective stiffness is taken as a fraction of the gross stiffness to account for cracking. In the current design practice, the stiffness is assumed as a fundamental property so that the yield curvature of section becomes directly proportional to yield moment. However, in reality, yield curvature does not change appreciably even as the yield moment varies. This is illustrated in Fig. 2.1 where the current design approximations are shown in Fig. 2.1(a) while the more realistic conditions are shown in Fig. 2.1(b).

Evaluation of the stiffnesses of cantilever shear walls and their relation to the wall strengths is considered in detail in this chapter. It may be noted that in recent years, Paulay and Priestley (1993-2000) have carried out several studies in this area. In the present work, their results are reviewed and further expanded.

2.2 Moment-curvature relationship

The flexural stiffness of an element depends on its geometry as well as on the moment-curvature relationship for a cross-section of the element. Frequently, the shear wall cross-section is a rectangle as shown in Figs. 2.2(a) and 2.3(a). Each of these two cross-sections has a length, l_w , and thickness, b . In Fig. 2.2(a) the reinforcing steel is placed in two layers and distributed uniformly along the length. The first and last sets of bar are placed

at a distance, d_c from the nearest concrete face. The remaining bars are placed at an even spacing, s . In Fig. 2.3(a), a portion of the steel is concentrated at a distance, d_b , while the remaining is evenly distributed.

Several types of cross-section shapes other than rectangle may be used for a wall. Examples are a T-shape, an L-shape, an I-shape or a channel-shape cross-section. In the present work, a channel-shape section is analyzed to examine the behaviour of a non-rectangular section. If the non-rectangular wall consists of long flanges, the designer must decide how much of the width of such wide flanges should be considered to be effective. Code provisions have suggested different effective widths of compression and tension flanges. Paulay and Priestley (1992) have suggested an approximation for the effective width in wide-flanged structural walls. This is based on the assumption that vertical forces due to shear stresses introduced by the web of the wall into the tension flange, spread out at a slope of 1:2. Accordingly, the effective width of the tension flange is

$$b_{eff} = 0.5h_w + b_w \leq b \quad (2.1a)$$

where b_w is the thickness of wall web and h_w is the height of the wall.

The flexural strength of a wall section with the flange in compression is insensitive to the assumed effective width. It should be noted, however that after significant tension yield excursion in the flange, the compression contact area becomes rather small after load reversal with outer bars toward the tips of the flange still in tensile strain. Considering this effect, Paulay and Priestley suggest a flange effective width in compression given by

$$b_{eff} = 0.15h_w + b_w \leq b \quad (2.1b)$$

Wall section, strains and relevant stresses for the studied channel-shape section are shown in Fig. 2.4.

A representative moment-curvature relationship for the cross-sections in Figs. 2.2(a), 2.3(a) and 2.4(a) is shown in Fig. 2.5, and is derived on the assumption that in bending, plane sections remain plane. This assumption is valid except in the case of squat shear wall. However, shear walls are most often used to resist earthquake forces in relatively tall structures, in which the shear walls are slender and the assumption of plane section is therefore valid. The strain distribution across the cross-section is as shown in Figs. 2.2(b), 2.3(b) and 2.4(b). The strain is zero at the neutral axis located at distance, c , from the compressive face. The concrete is in tension below the neutral axis. As long as the tensile strain in concrete does not exceed the tensile strain at cracking, this concrete carries the resulting tensile stress, which is approximately proportional to the strain. The section can be considered as being homogeneous at this stage and for the symmetrical distribution of steel, neutral axis is at mid-depth of the section. The flexural rigidity of the section is $E_c I_t$, where E_c is the modulus of elasticity of concrete and I_t is the moment of inertia of the transformed section. The slope of moment-curvature curve is equal to $E_c I_t$ as long as the concrete is not cracked. The curvature of the section, which is equal to the included angle in the strain diagram, is given by

$$\phi = \frac{M}{E_c I_t} \quad (2.2)$$

where M is the applied moment.

When the applied moment reaches a value M_{cr} , the concrete below the neutral axis is cracked and is assumed not to carry any stress. The flexural rigidity of the section and hence the slope of the moment-curvature curve decrease simultaneously at this stage as shown in Fig. 2.5.

With increasing applied moment, the strains in concrete and reinforcing steel increase until the steel in the layer that is farthest from the neutral axis starts to yield in tension. The curvature of the section at first yield is denoted by ϕ'_y and the corresponding moment by M_y . It is easily shown that ϕ'_y is given by

$$\phi'_y = \frac{\epsilon_y}{(d - c)} \quad (2.3)$$

Where ϵ_y is the strain at tensile yielding of steel and d is the distance from the compression face of concrete to the extreme layer of steel.

The slope of the moment-curvature diagram decreases progressively beyond first yield as additional layers of steel start to yield. Ultimately, failure of section takes place either through fracture of steel layers or crushing of concrete in compression. The failure moment is denoted by M_u and the curvature at failure by ϕ_u .

In many researches by Paulay, it has been shown that for the purpose of analyzing a reinforced concrete structure for its dynamic response to earthquake motion, it is adequate to represent the moment-curvature relationship for individual sections by an

idealized bilinear relationship, shown by dashed line in Fig. 2.5. The first branch of the curve is obtained by drawing a straight line from the origin passing through the point (M_y, ϕ'_y) and terminating at point (M_n, ϕ_y) . A number of alternative methods have been proposed for defining the point (M_n, ϕ_y) . In the present work, the definition proposed by Priestley et al. (1998) is used. In this definition, point (M_n, ϕ_y) is selected so that a horizontal straight line through (M_n, ϕ_y) intersects the moment-curvature curve at $5\phi_y$.

Based on extensive analyses of bridge columns, Priestley et al. (1996) have shown that the yield curvature is an almost invariable parameter of a concrete column section and can be considered as a fundamental section property. In other words the yield curvature does not vary substantially with either the steel ratio or the applied axial load and is approximately given by

$$\phi_y = \frac{K_1 \varepsilon_y}{D} \quad (2.4)$$

where D is the section depth of the column and K_1 is a constant, which will be explained later. It is interesting to compare Eq. (2.4) with the exact expression for a section of homogeneous material

$$\phi_y = \frac{2\varepsilon_y}{D} \quad (2.5)$$

Studies by Priestley et al. (1996) have shown that for a section of concrete column with an axial load ratio in the range $0 \leq N/f'_c A_g \leq 0.4$ and longitudinal steel ratio of $0.01 \leq \rho = A_{st}/A_g \leq 0.04$, the yield curvature, ϕ_y , is given by

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2.3 Description of shear wall model and material properties

The moment-curvature relationships for a rectangular shear wall are derived for a wall length, l_w , varying from 1.5 m to 5 m and wall widths, b , of 200, 250 and 300 mm. The base model is a shear wall with the length of 3 m and 200 mm width. The standard concrete compressive strength is taken as $f'_c=30$ MPa and the steel yield stress as $f_y=400$ MPa. The compressive stress induced by axial load is varied between 0.0 and 3.0 MPa corresponding to axial load ratio, $N/f'_c A_g$, varying between 0.0 and 0.10. The range of assumed axial load ratio covers the common value of axial loads for multi-storey buildings up to 12 stories in height. The longitudinal steel ratio is varied between 0.25% and 3%. For the wall section with concentrated steel, Fig. 2.3(a), 0.5% of steel is taken as being distributed along the wall length and the remaining are symmetrically concentrated at two end of section with the distance from the concrete face, d_c , is taken as being equal to 100 mm and d_b is assumed as $0.04l_w$.

In deriving the moment-curvature relationships for a channel-shape wall section, dimensions of the cross-section are assumed to be $l_w=3$ m, $b=1.5$ m and $t=300$ mm. The longitudinal steel ratio for the web is taken as 0.5% of $l_w t$ and steel ratios in flanges are varied between 0.5% and 3% of bt .

In addition to the geometric and material properties, determination of the moment-curvature relationship requires that the stress-strain relationship for both the concrete and steel material be defined. An idealized stress-strain curve that follows the Hognestad's model is used for concrete and is shown in Fig. 2.6. The effect of confinement on the stress-strain relationship is ignored, because confinement of concrete is not very practical

for shear walls. The stress-strain curve for steel is shown in Fig. 2.7. The effect of strain hardening in steel is included, however, the stress-strain curve beyond the onset of strain hardening is represented as a straight line as shown in Fig. 2.7.

A special purpose computer program was developed to carry out the analyses for the determination of moment-curvature relationship. The program provides both the complete moment-curvature curve and the idealized form shown in Fig. 2.5. The later provides the yield curvature, ϕ_y , as well as the nominal yield moment, M_n .

2.4 Results of the analysis

Analytical results are presented here for the variation of yield curvature with axial load and reinforcement ratio. The yield curvature is represented in the following dimensionless format,

$$K_I = \phi_y \frac{l_w}{\epsilon_y} \quad (2.8)$$

The variation of K_I with the steel ratio and axial load is presented in Fig. 2.8 for a rectangular wall. It is observed that the dimensionless yield curvature K_I is comparatively insensitive to the axial load. The curvature does vary with the steel ratio, increasing with an increase in that ratio. However, the variation is not very large. If the minimum steel ratio of 0.25% is excluded, the yield curvature for a wall with distributed reinforcement can be expressed as

$$\phi_y L_w = 2.1 \epsilon_y \pm 15\% \quad (2.9)$$

For a wall with some of the steel concentrated at ends and the rest uniformly distributed, the minimum steel ratio is taken as 0.5%. In fact, at this minimum, all of the steel is uniformly distributed. For larger steel ratios, 0.5% is assumed to be uniformly distributed along the wall and the remaining assumed to be concentrated at the ends as explained earlier.

The yield curvature for such a distribution of steel reinforcement can be expressed as,

$$\phi_y L_w = 1.85 \epsilon_y \pm 10\% \quad (2.10)$$

In fact, in most cases, the variation is less than 10%.

The slope of the initial line of the idealized bilinear moment-curvature can be taken as the effective flexural rigidity, $E_c I_{eff}$, to be used in the analytical study of dynamic response.

The effective flexural rigidity is then given by

$$E_c I_{eff} = \frac{M_y}{\phi'_y} = \frac{M_n}{\phi_y} \quad (2.11)$$

The effective flexural rigidity depends on both the axial load ratio $N/f_c A_g$ and the longitudinal steel ratio ρ . The variation of the ratio I_{eff}/I_g , where I_g is the gross moment of inertia given by $I_g = bl_w^3/12$, with the axial load and longitudinal steel ratio is presented in Fig. 2.9. For distributed steel, I_{eff}/I_g varies from 0.07 to 0.5 as the steel ratio varies from 0.25% to 3%. The ratio I_{eff}/I_g also varies with the axial load, but the variation is not very significant. For shear walls with part of steel concentrated at the ends the ratio I_{eff}/I_g varies from 0.12 to 0.85.

The curves shown in Figs. 2.8 and 2.9 are for steel with a yield strength of 400 MPa. Analytical results were also obtained for steel yield strength of 300 MPa and are presented in Figs. 2.10 and 2.11. Figure 2.10 shows the variation of K_I while Fig. 2.11 shows the variation of I_{eff}/I_g .

The variations of K_I and I_{eff}/I_g for the channel-shape section are graphically represented in Figs. 2.12(a) and 2.12(b). Although Fig. 2.12(a) shows a different value of K_I for channel-shape wall than the one for a rectangular section, it still demonstrates that K_I remains practically constant for this type of walls. The ratio of I_{eff}/I_g shown in Fig. 2.12(b) illustrates that the suggested value of $I_{eff}=0.7I_g$ is too high and should be revised.

The implication of presenting the foregoing analytical results in dimensionless form is that they would apply to walls of arbitrary geometrical dimensions. In order to verify this, analyses are repeated for rectangular walls with different widths and lengths. Some of the results are presented in Figs. 2.13 to 2.15. These figures present comparisons for K_I and I_{eff}/I_g . The results support the validity of the observation that the dimensionless forms of the analytical results for yield curvature and effective moment of inertia apply for the tested wall widths and lengths used in the present investigation.

The analytical results presented in Fig. 2.8 show that for practical purposes, the yield curvature can be taken as being independent of the reinforcement and axial load ratios, but is inversely proportional to the wall length. A consequence of this is that the yield deflection is also independent of the axial load and steel ratios, but varies inversely as the wall length.

For a cantilever wall, the yield deflection is given by

$$\delta_y = \frac{F_y}{K_{eff}} \quad (2.12)$$

where the effective stiffness, K_{eff} , is obtained from

$$K_{eff} = \frac{3EI_{eff}}{h_e^3} \quad (2.13)$$

where h_e is the effective wall height. h_e will be different based on the lateral load distribution. For a single storey building with lateral load applied to the roof, h_e would be equal to the total wall height, h_w .

Substitution of Eq. (2.13) in Eq. (2.12) gives

$$\delta_y = \frac{F_y h_e^3}{3EI_{eff}} \quad (2.14)$$

The moment corresponding to yielding at the base of a cantilever wall is obtained from

$$M_n = F_y h_e \quad (2.15)$$

From Eq. (2.11), Eq. (2.14) and Eq. (2.15) the formula for yield deflection is obtained as,

$$\delta_y = \varphi_y \frac{h_e^2}{3} \quad (2.16)$$

Substitution from Eq. (2.8) gives

$$\delta_y = \left(\frac{K_l \epsilon_y}{l_w} \right) \frac{h_c^2}{3} \quad (2.17)$$

where K_l is obtained from graphs shown in Figs. 2.8 and 2.10 or from Eq. (2.9) and (2.10).

Eq. (2.17) implies that it is not necessary to know the wall strength to calculate its yield displacement, knowledge of wall length and height and steel strain at yield is sufficient.

Another interesting result obtained from Fig. 2.8 and Eq. (2.17) is that in a building with a number of walls of different lengths oriented in the same direction, it will be impossible for all walls to have the same yield displacement because yield displacement is inversely proportional to the length of wall. This means that the presumption made in the current design that allocating strengths to walls in proportion to their stiffnesses will lead to simultaneous yielding of walls and hence a uniform ductility demand is impossible to achieve.

The strength of an individual wall could be assigned arbitrarily within the range dictated by the range of admissible steel ratio. The stiffness of the wall is then obtained upon dividing the strength by the yield displacement derived from Eq. (2.17). This subject will be further discussed in Chapter 4.

2.5 Summary and conclusion

Analytical results are presented here for the moment-curvature relationships for a variety of shear walls. It is shown that the stiffness of a wall is not dependent just on the

geometry of the wall section, but depends on the strength of the wall and the yield curvature.

The following conclusion can be drawn from the work presented here.

1. From the analyses of the moment-curvature relationship of walls, it is found that the yield curvature is insensitive to the axial load ratio as well as the steel reinforcement ratios, but depends only on the steel strain at yield and the length of wall. Even the type of distribution of steel has only a slight effect on the yield curvature.
2. Since yield curvature (and hence the yield displacement) is independent of strength and is dependent on wall length, it is not possible to design a structure having different wall sizes to ensure that all walls will yield simultaneously.
3. The effect of longitudinal reinforcement ratios is an important parameter that should be considered in design. Thus, the flexural rigidity of a wall may change from 7% of $E_c I_g$ for a 0.25% steel ratio to 60% of $E_c I_g$ for a 3% steel ratio.

(a)

(b)

Figure 2.1: Moment-curvature relationship for different stiffness distributions
 (a) constant stiffness (design assumption) (b) realistic condition (constant yield curvature)

Figure 2.2: Wall section with distributed steel (a) wall cross-section (b) strain at section (c) stress at section

Figure 2.3: Wall section with end concentrated steel (a) wall cross section (b) strain at section (c) stress at section

Figure 2.4: Channel-shape wall properties (a) wall section (b) strain at section (c) stress distribution

Figure 2.5: Moment-curvature relationship for a rectangular concrete wall

Figure 2.6: Idealized stress-strain curve for concrete in uniaxial compression

Figure 2.7: Realistic and tri-linear stress-strain relationship for reinforcing steel

(a)

(b)

Figure 2.8: Variation of dimensionless yield curvature of a rectangular wall with axial load and steel ratio for $f_y = 400$ MPa (a) distributed reinforcement (b) end concentrated reinforcement

(a)

(b)

Figure 2.9: Ratio of effective moment of inertia to gross moment of inertia of a rectangular wall for $f_y=400$ MPa (a) distributed reinforcement (b) end concentrated reinforcement

(a)

(b)

Figure 2.10: Variation of dimensionless yield curvature of a rectangular wall with axial load and steel ratio for $f_y = 300$ MPa (a) distributed reinforcement (b) end concentrated reinforcement

(a)

(b)

Figure 2.11: Ratio of effective moment of inertia to gross moment of inertia of a rectangular for $f_y=300$ MPa (a) distributed reinforcement (b) end concentrated reinforcement

(a)

(b)

Figure 2.12: Parameters for channel-shape shear wall (a) variation of dimensionless yield curvature with axial load and steel ratio for $f_y=400$ MPa (b) ratio of effective moment of inertia to gross moment of inertia

Figure 2.13: Comparison of yield curvatures of rectangular walls for different wall lengths and widths for $f_y=400$ MPa (a) distributed reinforcement $\rho=0.5\%$ and 2% (b) end concentrated reinforcement $\rho=0.5\%$

(b)

Figure 2.14: Comparison of yield curvatures of rectangular walls for different wall lengths and widths, end concentrated reinforcement $\rho=2\%$

Figure 2.15: Comparison of the ratios of l_{eff}/l_g for rectangular walls of different wall lengths and widths, $\rho=0.5\%$ and $\rho=2\%$ (a) distributed reinforcement (b) end concentrated reinforcement

Chapter 3

Effects of Strength Distribution on Torsional Response of Asymmetric Structures with Shear Walls

3.1 Introduction

Torsional vibration of an asymmetric building during earthquakes may cause severe damage to the structure. Torsion increases the displacement of the flexible side while decreasing the displacement of the stiff side of the structure. To account for the additional demand placed on the lateral resisting system by torsion, special design provisions are specified in building codes. The torsional design provisions are mainly based on elastic response of the structure. In reality, most structures are designed to be strained into the inelastic range during a design earthquake. Several researches have carried out studies on torsional inelastic response to verify the adequacy of current code provisions through numerical modeling (Chandler et al. 1997, Tso and Thu 1992, Wong and Tso 1994 and Humar and Kumar 1998 I, 1998 II).

Research by Humar and Kumar has shown that the two parameters that have the greatest impact on torsional seismic response are

1. The eccentricity between the centre of mass and the centre of rigidity
2. The torsional stiffness about the centre of rigidity

Most code guidelines for minimizing the effects of torsional vibrations are based on the traditional perception that element strengths are dependent on the stiffnesses. As discussed in the previous chapter, stiffness of a reinforced concrete resisting element is not a fundamental parameter. Studies have pointed out that for reinforced concrete structures, stiffness is in fact a strength-dependent parameter. This implies that the stiffnesses of the resisting elements cannot be defined prior to the assignment of their strengths. Therefore, computing the stiffness eccentricity is not as easy as implied in the traditional method, in which the centre of rigidity is fixed when the element geometries have been selected. In the new design context, it is the yield displacement distribution and not the stiffness distribution of the structure that is known prior to the assignment of strengths. With this information in hand, the task is to distribute the strength to each element in plan in a way that torsional response will be minimized. Once strengths and yield displacements distributions are specified, the stiffness distribution is fully determined. The objective of this chapter is to develop guidelines on how strength must be distributed so that the torsional response is minimized.

3.2 Description of the problem and modeling

In order to provide a physical understanding of the problem, consider the simple single storey structure shown in plan view in Fig. 3.1. The lateral load resisting elements consist of cantilever shear walls oriented along the x and y directions. The mass, m , of the structure is concentrated at the floor level and is uniformly distributed so that the centre of mass, CM, is located at the centroid of the plan. The shear walls oriented along the x -axis are located symmetrically about the CM. However the walls along the y -axis are

located arbitrarily so that the centre of rigidity, CR, is offset from the centre of mass and the distance between the CR and CM is denoted by e_R and is given by

$$e_R = \frac{\sum K_{yi} x_i}{\sum K_{yi}} \quad (3.1)$$

where K_{yi} is the stiffness of the i^{th} wall along the y -axis and x_i its distance from CM.

The torsional stiffness of the structure about CR is given by

$$K_{\theta R} = \sum_{i=1}^N K_{yi} (x_i - e_R)^2 + \sum_{i=1}^M K_{xi} y_i^2 \quad (3.2)$$

Where K_{xi} is the stiffness of the i^{th} wall along the x -axis and y_i its distance from CM, N is the number of walls oriented along the y -axis and M the number of walls along x -axis.

It is useful to define two frequency terms, an uncoupled lateral frequency, ω_y , and an uncoupled torsional frequency, ω_θ , given by

$$\omega_y = \sqrt{\frac{K_y}{m}} \quad (3.3a)$$

$$\omega_\theta = \sqrt{\frac{K_{\theta R}}{mr^2}} \quad (3.3b)$$

where $K_y = \sum K_{yi}$ is the total lateral stiffness in y direction and r is the radius of gyration about CM. Ratio $\Omega_R = \omega_\theta / \omega_y$ is referred to as the frequency ratio and provides a measure of the torsional stiffness about CR. A structure represented by Fig. 3.1 is often referred to

as a torsionally unbalanced structure. For an earthquake excitation in the y direction, it undergoes both a translation in the y direction and a rotation about CR. For a given ground motion, the response of the structure in the y direction is governed by ω_y , e_R and Ω_R (Humar and Kumar 1998 I). In the inelastic range, a number of additional parameters control the response. In each case, however, e_R and Ω_R are considered as the most important parameters.

In the model studied here, the length and width of plan are taken as $b=36$ m and $a=18$ m, respectively. The lengths of walls No. 1, 2 and 3 are assumed to be l_w , $\alpha_2 l_w$ and $\alpha_3 l_w$ (Fig. 3.2a). In most cases, α_2 and α_3 are taken to be greater than unity in the present study. This implies that the left edge is the flexible edge while the right edge is the stiff edge of the structure. The nominal strengths of the left, middle and right walls are respectively taken as f , $\beta_2 f$ and $\beta_3 f$ (Fig. 3.2b). The location of centre of strength, CV, varies with changes in β_2 and β_3 . The CV could be located on the left side or the right side of CM or coincide with it. Taking the force-displacement relation of the elements to be elasto-plastic, the stiffness of wall No.1 is given by $K=f/\delta_y$, where δ_y is the yield displacement of the first wall which can be obtained from Eq. (2.16). In the same manner, the stiffnesses of the second and third walls can be expressed as

$$K_2 = \frac{f_2}{\delta_{y2}} = \frac{\beta_2 f}{\delta_{y2}} \quad (3.4a)$$

$$K_3 = \frac{f_3}{\delta_{y3}} = \frac{\beta_3 f}{\delta_{y3}} \quad (3.4b)$$

As described in the previous chapter, the yield displacement is inversely proportional to the length of wall, so that

$$\delta_{y2} \propto \frac{1}{l_{w2}} \quad (3.5a)$$

$$\delta_{y3} \propto \frac{1}{l_{w3}} \quad (3.5b)$$

Substitution of $l_{w2} = \alpha_2 l_w$ and $l_{w3} = \alpha_3 l_w$ gives

$$\delta_{y2} \propto \frac{1}{\alpha_2 l_w}$$

$$\delta_{y3} \propto \frac{1}{\alpha_3 l_w}$$

and consequently,

$$\delta_{y2} = \frac{\delta_y}{\alpha_2} \quad (3.6a)$$

$$\delta_{y3} = \frac{\delta_y}{\alpha_3} \quad (3.6b)$$

From Eqs. (3.4) and (3.6), stiffnesses are obtained as

$$K_2 = \alpha_2 \beta_2 \frac{f}{\delta_y} = \gamma_2 K \quad (3.7a)$$

$$K_3 = \alpha_3 \beta_3 \frac{f}{\delta_y} = \gamma_3 K \quad (3.7b)$$

The stiffness distribution is shown in Fig. 3.2c

Using the foregoing definitions, the stiffness eccentricity, e_R , and strength eccentricity, e_V , are obtained as

$$e_R = \left(\frac{\alpha_3 \beta_3 - 1}{1 + \alpha_2 \beta_2 + \alpha_3 \beta_3} \right) \frac{b}{2} \quad (3.8a)$$

$$e_V = \left(\frac{\beta_3 - 1}{1 + \beta_2 + \beta_3} \right) \frac{b}{2} \quad (3.8b)$$

Finally, the distance between the centre of rigidity, CR, and the centre of strength, CV, can be expressed as

$$D_I = \left(\frac{\alpha_3 \beta_3 - 1}{1 + \alpha_2 \beta_2 + \alpha_3 \beta_3} - \frac{\beta_3 - 1}{1 + \beta_2 + \beta_3} \right) \frac{b}{2} \quad (3.9)$$

Different values of α and β have been chosen to examine the dependency of D_I on these parameters. Parameter α_2 is varied from 0.6 to 2, α_3 from 1 to 3 and β_2 and β_3 from 0 to 3. Even within these wide ranges of strength distribution and geometry, for a given set of length ratios, the distance between CR and CV does not change much with β_2 and β_3 for values of these parameters higher than 1. This means that D_I is reasonably insensitive to the strength distribution in this range. Figs. 3.3a and 3.3b show two cases, one for $\alpha_2=0.8$ and the other for $\alpha_2=1.3$. Parameter α_3 is varied from 1.3 to 2. In the first case, β_2 is equal

to 0.8 while in the second case, β_2 is equal to 1.5. In both cases, β_3 is changing from 0 to 3. The effect of variation of β_2 on D_I has been shown in Figs. 3.4a and 3.4b. Again, it is observed that D_I does not vary much with the variation of β_2 .

The fact that D_I is not very sensitive to a variation in β gives the freedom to assign different strength ratios to the walls in order to investigate the best locations for CR and CV with respect to CM so as to obtain the minimum rotational response.

3.3 Analysis procedure

As mentioned in the pervious section, for a given structure geometry, the distance between CR and CV does not vary noticeably with variation in the strength distribution. This can be attributed to the fact that, stiffness is dependent on the strength. Consequently, the centre of rigidity moves along with the centre of strength and the distance between these two points remains almost constant.

There are four different possible positions of CR and CV:

1. CR coincides with CM, but CV could have different positions (Model No.1)
2. CR and CV are on the opposite sides of CM (Model No.2)
3. CV coincides with CM, but CR could have different positions (Model No.3)
4. CR and CV are located on the same side of CM (Model No.4)

These four models are graphically shown in Fig. 3.5. Geometry of structures is the same for the four models, but distribution of strengths is chosen in such a manner as to achieve the desirable locations for CR and CV.

For the purpose of an analytical study we assume that the length of first wall is $l_w=1$ m and take α_2 for the second wall as 2. The length of the third wall is obtained as 2.3, 2.5, 2.8, and 3 m corresponding to the values of α_3 equal to 2.3, 2.5, 2.8 and 3, respectively. For each of these four cases, three different values are selected for the ratio β_2 . The selected values are 0.5, 1.0 and 1.5. For each β_2 , the ratio of strength of the third wall to the first one, β_3 , is taken as $1/\alpha_3$, 0.5, 1.0 and 1.5. When β_3 is selected as equal to $1/\alpha_3$, the first and the third walls have equal stiffnesses, as a result CR coincides with CM. For the others values of β_3 , CR is not coincident with CM. In each set of four models, the four different values for β_3 generate the four models with different location of CV and CR as explained earlier. A constant lateral period of 0.6 s has been taken for all analyses. Because of the given yield displacement distribution according to Eq. (2.17), it is not possible to construct all models to have the same total strength and also the same total lateral stiffness. Since they will be subjected to dynamic excitation, it is considered more appropriate that they should have the same lateral period. Consequently, the total strength is varied slightly between each set of four models to achieve this objective. Tables 3.1 to 3.4 give the details for each set of four models. Yield displacements, stiffnesses and strengths for the three walls and the total strength, stiffness and yield displacement of the system, which is obtained from $f_{y\text{tot}}/K_{\text{tot}}$, all have been shown in these tables.

Table 3.1: Properties of the structure in case No.1 with $\alpha_2=2.0$ and $\alpha_3=2.3$

Model No.	β_2	β_3	δ_{y1} (m)	δ_{y2} (m)	δ_{y3} (m)	f_{y1} (kN)	f_{y2} (kN)	f_{y3} (kN)	$f_{y\text{tot}}$ (kN)	K_1 (kN/m)	K_2 (kN/m)	K_3 (kN/m)	K_{tot} (kN/m)	δ_y (m)	e_v (m)	e_R (m)	D (m)
1	0.5	0.435	0.035	0.0175	0.0152	665.28	332.64	289.25	1287.18	19008.1	19008.1	19008.1	57024.4	0.023	-5.26	0.00	5.26
2	0.5	0.500	0.035	0.0175	0.0152	633.60	316.80	316.80	1267.21	18103.0	18103.0	20818.4	57024.4	0.022	-4.50	0.86	5.36
3	0.5	1.000	0.035	0.0175	0.0152	464.15	232.08	464.15	1160.38	13261.5	13261.5	30501.4	57024.4	0.020	0.00	5.44	5.44
4	0.5	1.500	0.035	0.0175	0.0152	366.21	183.11	549.32	1098.63	10463.2	10463.2	36098.0	57024.4	0.019	3.00	8.09	5.09
1	1.0	0.435	0.035	0.0175	0.0152	498.96	498.96	216.94	1214.87	14256.1	28512.2	14256.1	57024.4	0.021	-4.18	0.00	4.18
2	1.0	0.500	0.035	0.0175	0.0152	480.93	480.93	240.46	1202.32	13740.8	27481.6	15801.9	57024.4	0.021	-3.60	0.65	4.25
3	1.0	1.000	0.035	0.0175	0.0152	376.58	376.58	376.58	1129.73	10759.3	21518.6	24746.4	57024.4	0.020	0.00	4.42	4.42
4	1.0	1.500	0.035	0.0175	0.0152	309.43	309.43	464.15	1083.02	8841.0	17682.0	30501.4	57024.4	0.019	2.57	6.84	4.27
1	1.5	0.435	0.035	0.0175	0.0152	399.17	598.76	173.55	1171.48	11404.9	34214.6	11404.9	57024.4	0.021	-3.47	0.00	3.47
2	1.5	0.500	0.035	0.0175	0.0152	387.54	581.32	193.77	1162.63	11072.7	33218.1	12733.6	57024.4	0.020	-3.00	0.52	3.52
3	1.5	1.000	0.035	0.0175	0.0152	316.80	475.20	316.80	1108.81	9051.5	27154.5	20818.4	57024.4	0.019	0.00	3.71	3.71
4	1.5	1.500	0.035	0.0175	0.0152	267.90	401.85	401.85	1071.60	7654.3	22962.8	26407.3	57024.4	0.019	2.25	5.92	3.67

Table 3.2: Properties of the structure in case No.2 with $\alpha_2=2.0$ and $\alpha_3=2.5$

Model No.	β_2	β_3	δ_{y1} (m)	δ_{y2} (m)	δ_{y3} (m)	f_{y1} (kN)	f_{y2} (kN)	f_{y3} (kN)	f_{tot} (kN)	K_1 (kN/m)	K_2 (kN/m)	K_3 (kN/m)	K_{tot} (kN/m)	δ_y (m)	e_v (m)	e_R (m)	D (m)
1	0.5	0.400	0.035	0.0175	0.0140	665.28	332.64	266.11	1264.04	19008.1	19008.1	19008.1	57024.4	0.022	-5.68	0.00	5.68
2	0.5	0.500	0.035	0.0175	0.0140	614.11	307.05	307.05	1228.22	17546.0	17546.0	21932.5	57024.4	0.022	-4.50	1.38	5.88
3	0.5	1.000	0.035	0.0175	0.0140	443.52	221.76	443.52	1108.81	12672.1	12672.1	31680.2	57024.4	0.019	0.00	6.00	6.00
4	0.5	1.500	0.035	0.0175	0.0140	347.10	173.55	520.66	1041.31	9917.3	9917.3	37189.8	57024.4	0.018	3.00	8.61	5.61
1	1.0	0.400	0.035	0.0175	0.0140	498.96	498.96	199.59	1197.51	14256.1	28512.2	14256.1	57024.4	0.021	-4.50	0.00	4.50
2	1.0	0.500	0.035	0.0175	0.0140	469.61	469.61	234.81	1174.03	13417.5	26835.0	16771.9	57024.4	0.021	-3.60	1.06	4.66
3	1.0	1.000	0.035	0.0175	0.0140	362.88	362.88	362.88	1088.65	10368.1	20736.1	25920.2	57024.4	0.019	0.00	4.91	4.91
4	1.0	1.500	0.035	0.0175	0.0140	295.68	295.68	443.52	1034.89	8448.1	16896.1	31680.2	57024.4	0.018	2.57	7.33	4.76
1	1.5	0.400	0.035	0.0175	0.0140	399.17	598.76	159.67	1157.59	11404.9	34214.6	11404.9	57024.4	0.020	-3.72	0.00	3.72
2	1.5	0.500	0.035	0.0175	0.0140	380.16	570.24	190.08	1140.49	10861.8	32585.4	13577.2	57024.4	0.020	-3.00	0.86	3.86
3	1.5	1.000	0.035	0.0175	0.0140	307.05	460.58	307.05	1074.69	8773.0	26318.9	21932.5	57024.4	0.019	0.00	4.15	4.15
4	1.5	1.500	0.035	0.0175	0.0140	257.53	386.29	386.29	1030.12	7358.0	22074.0	27592.4	57024.4	0.018	2.25	6.39	4.14

Table 3.3: Properties of the structure in case No. 3 with $\alpha_2=2.0$ and $\alpha_3=2.8$

Model No.	β_2	β_3	δ_{y1} (m)	δ_{y2} (m)	δ_{y3} (m)	f_{y1} (kN)	f_{y2} (kN)	f_{y3} (kN)	$f_{y\text{tot}}$ (kN)	K_1 (kN/m)	K_2 (kN/m)	K_3 (kN/m)	K_{tot} (kN/m)	δ_y (m)	e_v (m)	e_R (m)	D (m)
1	0.5	0.357	0.035	0.0175	0.0125	665.3	332.6	237.6	1235.5	19008.1	19008.1	19008.1	57024.4	0.022	-6.23	0.00	6.23
2	0.5	0.500	0.035	0.0175	0.0125	587.0	293.5	293.5	1174.0	16771.9	16771.9	23480.6	57024.4	0.021	-4.50	2.12	6.62
3	0.5	1.000	0.035	0.0175	0.0125	415.8	207.9	415.8	1039.5	11880.1	11880.1	33264.2	57024.4	0.018	0.00	6.75	6.75
4	0.5	1.500	0.035	0.0175	0.0125	321.9	161.0	482.9	965.7	9197.5	9197.5	38629.4	57024.4	0.017	3.00	9.29	6.29
1	1.0	0.357	0.035	0.0175	0.0125	499.0	499.0	178.2	1176.1	14256.1	28512.2	14256.1	57024.4	0.021	-4.91	0.00	4.91
2	1.0	0.500	0.035	0.0175	0.0125	453.6	453.6	226.8	1134.0	12960.1	25920.2	18144.1	57024.4	0.02	-3.60	1.64	5.24
3	1.0	1.000	0.035	0.0175	0.0125	344.1	344.1	344.1	1032.3	9831.8	19663.6	27529.0	57024.4	0.018	0.00	5.59	5.59
4	1.0	1.500	0.035	0.0175	0.0125	277.2	277.2	415.8	970.2	7920.1	15840.1	33264.2	57024.4	0.017	2.57	8.00	5.43
1	1.5	0.357	0.035	0.0175	0.0125	399.2	598.8	142.6	1140.5	11404.9	34214.6	11404.9	57024.4	0.02	-4.05	0.00	4.05
2	1.5	0.500	0.035	0.0175	0.0125	369.6	554.4	184.8	1108.8	10560.1	31680.2	14784.1	57024.4	0.019	-3.00	1.33	4.33
3	1.5	1.000	0.035	0.0175	0.0125	293.5	440.3	293.5	1027.3	8385.9	25157.8	23480.6	57024.4	0.018	0.00	4.76	4.76
4	1.5	1.500	0.035	0.0175	0.0125	243.4	365.1	365.1	973.6	6954.2	20862.6	29207.6	57024.4	0.017	2.25	7.02	4.77

Table 3.4: Properties of the structure in case No.4 with $\alpha_2=2.0$ and $\alpha_3=3.0$

Model No.	β_2	β_3	δ_{y1} (m)	δ_{y2} (m)	δ_{y3} (m)	f_{y1} (KN)	f_{y2} (KN)	f_{y3} (KN)	$f_{y\text{tot}}$ (KN)	K_1 (KN/m)	K_2 (KN/m)	K_3 (KN/m)	K_{tot} (KN/m)	δ_y (m)	e_v (m)	e_R (m)	D (m)
1	0.5	0.333	0.035	0.0175	0.0117	665.28	332.64	221.76	1219.69	19008.1	19008.1	19008.1	57024.4	0.021	-6.55	0.00	6.55
2	0.5	0.500	0.035	0.0175	0.0117	570.24	285.12	285.12	1140.49	16292.7	16292.7	24439.0	57024.4	0.020	-4.50	2.57	7.07
3	0.5	1.000	0.035	0.0175	0.0117	399.17	199.59	399.17	997.93	11404.9	11404.9	34214.6	57024.4	0.017	0.00	7.20	7.20
4	0.5	1.500	0.035	0.0175	0.0117	307.05	153.53	460.58	921.16	8773.0	8773.0	39478.4	57024.4	0.016	3.00	9.69	6.69
1	1.0	0.333	0.035	0.0175	0.0117	498.96	498.96	166.32	1164.25	14256.1	28512.2	14256.1	57024.4	0.020	-5.14	0.00	5.14
2	1.0	0.500	0.035	0.0175	0.0117	443.52	443.52	221.76	1108.81	12672.1	25344.2	19008.1	57024.4	0.019	-3.60	2.00	5.60
3	1.0	1.000	0.035	0.0175	0.0117	332.64	332.64	332.64	997.93	9504.1	19008.1	28512.2	57024.4	0.017	0.00	6.00	6.00
4	1.0	1.500	0.035	0.0175	0.0117	266.11	266.11	399.17	931.40	7603.3	15206.5	34214.6	57024.4	0.016	2.57	8.40	5.83
1	1.5	0.333	0.035	0.0175	0.0117	399.17	598.76	133.06	1130.98	11404.9	34214.6	11404.9	57024.4	0.020	-4.24	0.00	4.24
2	1.5	0.500	0.035	0.0175	0.0117	362.88	544.32	181.44	1088.65	10368.1	31104.2	15552.1	57024.4	0.019	-3.00	1.64	4.64
3	1.5	1.000	0.035	0.0175	0.0117	285.12	427.68	285.12	997.93	8146.3	24439.0	24439.0	57024.4	0.017	0.00	5.14	5.14
4	1.5	1.500	0.035	0.0175	0.0117	234.81	352.21	352.21	939.23	6708.8	20126.3	30189.4	57024.4	0.016	2.25	7.41	5.16

The models described in Tables 3.1 to 3.4 are analyzed for their response to the NS component of the 1940 El Centro earthquake record scaled to a peak ground acceleration of 0.5g and acting in the y direction of the model. The force-displacement relationship for the shear walls resisting the earthquakes is assumed to be bilinear as shown in Fig. 3.6. The slope of the initial branch is assumed to be equal to k_i . The slope of the second branch is equal to $0.05k_i$, and unloading is parallel to the initial branch.

Computer program DRAIN-TABS (Guendelman and Powell, 1977) developed at University of California, Berkeley, is used for carrying out the analyses. The program performs a step-by-step time history numerical analysis using the average acceleration method of integration. The time step employed in the analyses is 0.001 s. Also, a Rayleigh type damping has been used with the damping values adjusted to 5% in each of the two coupled modes involving translation in the y direction and rotation about CR.

3.4 Discussion on the results of analyses

Analyses of the models with a period $T=0.6$ s show that all of the analyzed systems are excited into the inelastic range under the described ground motion. Consequently, ductility demand is one of the most important parameters in designing such structures. The ductility demand, μ , is calculated by comparing the maximum displacement, δ_u , to the yield displacement, δ_y , as

$$\mu = \frac{\delta_u}{\delta_y}$$

Results of analyses are summarized in Tables 3.5 to 3.8. These tables show the rotations of the floor as well as the ductility demands on the flexible edge, μ_f , on the middle plane, μ_2 , and ductility demands on the stiff edge, μ_s . In addition, these tables show the maximum displacements experienced by flexible edge, δ_{uf} , middle plane, δ_{u2} , and stiff edge, δ_{us} . The results for flexible and stiff edge are also presented in a graphically form in Figs. 3.9 to 3.12. Maximum diaphragm rotations in four models are shown in Figs 3.7 and 3.8.

Table 3.5: Maximum displacements, ductility demands and rotation in case No.1

Model No.	β_2	β_3	δ_{uf} (m)	δ_{u2} (m)	δ_{us} (m)	μ_f	μ_2	μ_s	Rotation
1	0.5	0.435	0.069	0.059	0.099	1.97	3.37	6.51	0.00272
2	0.5	0.500	0.073	0.064	0.088	2.09	3.66	5.78	0.00268
3	0.5	1.000	0.125	0.073	0.042	3.57	4.17	2.76	0.00296
4	0.5	1.500	0.134	0.083	0.035	3.83	4.74	2.30	0.00314
1	1.0	0.435	0.076	0.066	0.096	2.17	3.77	6.31	0.00272
2	1.0	0.500	0.076	0.069	0.086	2.17	3.94	5.65	0.00218
3	1.0	1.000	0.127	0.075	0.049	3.63	4.29	3.22	0.00342
4	1.0	1.500	0.143	0.077	0.043	4.09	4.40	2.83	0.00434
1	1.5	0.435	0.077	0.072	0.09	2.20	4.11	5.91	0.00211
2	1.5	0.500	0.078	0.073	0.083	2.23	4.17	5.45	0.00165
3	1.5	1.000	0.121	0.075	0.048	3.46	4.29	3.15	0.00329
4	1.5	1.500	0.141	0.074	0.049	4.03	4.23	3.22	0.00446

Table 3.6: Maximum displacements, ductility demands and rotation in case No.2

Model No.	β_2	β_3	δ_{ur} (m)	δ_{u2} (m)	δ_{u5} (m)	μ_r	μ_2	μ_5	Rotation
1	0.5	0.400	0.069	0.058	0.101	1.97	3.31	7.21	0.00287
2	0.5	0.500	0.074	0.065	0.084	2.11	3.71	6.00	0.00265
3	0.5	1.000	0.123	0.077	0.04	3.51	4.40	2.86	0.00311
4	0.5	1.500	0.128	0.079	0.032	3.66	4.51	2.29	0.00347
1	1.0	0.400	0.076	0.064	0.095	2.17	3.66	6.79	0.00282
2	1.0	0.500	0.077	0.071	0.083	2.20	4.06	5.93	0.00195
3	1.0	1.000	0.127	0.074	0.048	3.63	4.23	3.43	0.00352
4	1.0	1.500	0.134	0.073	0.042	3.83	4.17	3.00	0.00443
1	1.5	0.400	0.078	0.071	0.089	2.23	4.06	6.36	0.00232
2	1.5	0.500	0.078	0.074	0.079	2.23	4.23	5.64	0.00153
3	1.5	1.000	0.122	0.075	0.048	3.49	4.29	3.43	0.00339
4	1.5	1.500	0.136	0.071	0.048	3.89	4.06	3.43	0.00463

Table 3.7: Maximum displacements, ductility demands and rotation in case No.3

Model No.	β_2	β_3	δ_{ur} (m)	δ_{u2} (m)	δ_{u5} (m)	μ_r	μ_2	μ_5	Rotation
1	0.5	0.357	0.069	0.058	0.104	1.97	3.31	8.32	0.00304
2	0.5	0.500	0.075	0.065	0.079	2.14	3.71	6.32	0.00240
3	0.5	1.000	0.137	0.084	0.039	3.92	4.80	3.12	0.00350
4	0.5	1.500	0.144	0.067	0.034	4.11	3.83	2.72	0.00385
1	1.0	0.357	0.077	0.063	0.096	2.20	3.60	7.68	0.00293
2	1.0	0.500	0.077	0.072	0.076	2.20	4.11	6.08	0.00170
3	1.0	1.000	0.137	0.077	0.048	3.91	4.40	3.84	0.00369
4	1.0	1.500	0.140	0.065	0.041	4.00	3.71	3.28	0.00456
1	1.5	0.357	0.079	0.07	0.088	2.26	4.00	7.04	0.00255
2	1.5	0.500	0.078	0.073	0.072	2.23	4.17	5.76	0.00134
3	1.5	1.000	0.124	0.075	0.047	3.54	4.29	3.76	0.00344
4	1.5	1.500	0.142	0.065	0.047	4.06	3.71	3.76	0.00480

Table 3.8: Maximum displacements, ductility demands and rotation in case No.4

Model No.	β_2	β_3	δ_{ur} (m)	δ_{u2} (m)	δ_{us} (m)	μ_r	μ_2	μ_s	Rotation
1	0.5	0.333	0.069	0.058	0.105	1.97	3.31	9.00	0.00314
2	0.5	0.500	0.076	0.067	0.075	2.17	3.83	6.43	0.00218
3	0.5	1.000	0.142	0.082	0.039	4.06	4.69	3.34	0.00342
4	0.5	1.500	0.143	0.068	0.035	4.09	3.89	3.00	0.00410
1	1.0	0.333	0.077	0.062	0.096	2.20	3.54	8.23	0.00299
2	1.0	0.500	0.078	0.072	0.072	2.23	4.11	6.17	0.00161
3	1.0	1.000	0.142	0.082	0.047	4.06	4.69	4.03	0.00377
4	1.0	1.500	0.143	0.064	0.041	4.09	3.66	3.51	0.00462
1	1.5	0.333	0.079	0.068	0.087	2.26	3.89	7.46	0.00270
2	1.5	0.500	0.079	0.074	0.072	2.26	4.23	6.17	0.00122
3	1.5	1.000	0.129	0.077	0.047	3.69	4.40	4.03	0.00351
4	1.5	1.500	0.148	0.066	0.047	4.23	3.77	4.03	0.00496

The results presented in graphs and tables show that for all cases the model with CR and CV on the opposite side of CM, Model No.2, has the minimum value of rotation. This indicates that if the objective is to minimize the rotational response, one should assign the strengths to walls in such a manner as to get the centre of rigidity on one side and the centre of strength on the other side of centre of mass. However, rotation is not the only parameter that should be taken into account. As mentioned earlier, in design, ductility demand on walls is another important parameter. Results of analyses show that among the four models, model 4 has the largest displacement at the flexible edge while model 1 has the largest displacement at the stiff edge. Although model 4 has the largest absolute value of edge displacement among all the models, this model does not necessarily have

the highest ductility demand. Calculation of ductility demands for each element of the four models demonstrates that in all the cases model 1 has the largest ductility demand. To understand why model 1 has the largest ductility demand while model 4 has the largest edge displacement, it should be noted that yield displacements are not equal for the two models, because the wall lengths are not equal, and as discussed in the previous chapter, yield displacement is inversely proportional to the length of the wall. The yield displacement of the largest wall is less than that of other walls and this could make the ductility demand on this wall to be the largest among all ductility demands. This implies that a strength distribution that minimizes the displacement or rotation does not necessarily lead to minimum ductility demands. It follows that both the displacements and ductility demands should be considered and examined in any seismic response study of asymmetric system.

The peak displacement at each edge results from a combination of translation and rotational responses of the system. During strong ground motions, the system goes through a number of different states. Initially, all elements are in the elastic state. In this state, the stiffness distribution of the elements has a great influence on the response. Due to torsional response, elements at flexible edge may yield while the others remain in the elastic range. Once yielding has taken place, substantial torque is generated in the system. This is the elastic/plastic state. Finally, all the elements become yielded and the system goes to the fully plastic state. Because of the characteristics of seismic excitation, some elements will experience unloading before others and re-enter into the elastic state when the ground motion reverses its direction. It should be noted that sometimes the system would not go through the three states in the order described. Occasionally, some elements

do not go to the inelastic state and may return to elastic state after the system has experienced the elastic/plastic state. The interaction of the elastic and inelastic contributions to the overall torsional response makes the torque trace very different from the edge displacement traces. Using a two element models, Myslimaj and Tso (2001) found that when CR and CV were located on the opposite sides of CM, the torsional response was the smallest. On the other hand, when CR and CV were both on the same side of CM, very large torque magnitudes resulted. The slabs that experience large torque show large rotation responses. The last columns of the Tables 3.5 to 3.8 exhibit the peak rotation of the four different models. All the results show that model 4 with CR and CV on the same side of CM has the largest rotation. Neither model 3 with zero strength eccentricity nor model 1 with zero stiffness eccentricity has the smallest rotational response. Model 2 with CR and CV located on the opposite sides of CM is the only model that gives the smallest rotational response.

3.5 Summary and conclusion

In the present work, using time history dynamic analysis, the best location of the centres of rigidity and strength with respect to the centre of mass for single storey systems has been investigated. Up to 48 models have been examined and the following conclusions can be drawn from these analyses

1. Systems that are perfectly balanced elastically can still have a significant torsional response due to strength irregularity. Yet, strength eccentricity alone is not the only parameter that contributes to the torsional response, because systems with no strength eccentricity but having stiffness eccentricity also show significant

torsional response. It is found that both the stiffness as well as the strength eccentricities contribute to the torsional response of ductile structures.

2. Distance between the centre of rigidity and the centre of strength is quite insensitive to the strength distribution. Yield displacement, which is dependent on the elements geometry (length of elements), is the most important parameter that controls this distance.
3. Since yield displacement depends only on the geometry of the wall, stiffness of a wall depends on the strength assigned to it. Consequently, the strength distribution controls the location of both the rigidity and strength centres in relation to CM. Taking into consideration the insensitivity of the distance between CV and CR, it is apparent that a small strength eccentricity would result in a correspondingly large stiffness eccentricity and vice versa.
4. It is shown that when CR and CV are on the opposite sides of CM the rotational response is a minimum. However, this does not necessarily give the least ductility demand or the least edge displacement. In addition, assigning of strengths to the elements to achieve such a location of CR and CV is not always possible due to the structural and non-structural limitations in design.

Figure 3.1: Plan view of a single storey building

(a)

(b)

(c)

Figure 3.2: (a) wall length distribution (b) strength distribution (c) stiffness distribution

(a)

(b)

Figure 3.3: Dependency of distance between CR and CV on length and strength ratios

(a) $\alpha_2=0.8, \beta_2=0.8$ (b) $\alpha_2=1.3, \beta_2=1.5$

(a)

(b)

Figure 3.4: Dependency of distance between CR and CV on length and strength ratios

(a) $\alpha_3=0.8, \beta_3=0.8$ (b) $\alpha_3=1.3, \beta_3=1.5$

Figure 3.5: Different locations of centre of mass, CM, centre of rigidity, CR, and centre of strength, CV, for each of four models

Figure 3.6: Force-displacement relationship for the i^{th} plane (Bilinear Model)

(a)

(b)

Figure 3.7: Comparison of rotations among four models having different strength distribution (a) $\alpha_2=2, \alpha_3=2.3$ (b) $\alpha_2=2, \alpha_3=2.5$

(a)

(b)

Figure 3.8: Comparison of rotations among four models having different strength distribution (a) $\alpha_2=2, \alpha_3=2.8$ (b) $\alpha_2=2, \alpha_3=3.0$

(a)

(b)

Figure 3.9: Comparison of flexible edge ductility among four models for different strength ratios (a) $\alpha_2=2, \alpha_3=2.3$ (b) $\alpha_2=2, \alpha_3=2.5$

Figure 3.10: Comparison of flexible edge ductility among four models for different strength ratios (a) $\alpha_2=2, \alpha_3=2.8$ (b) $\alpha_2=2, \alpha_3=3.0$

(a)

(b)

Figure 3.11: Comparison of stiff edge ductility among four models for different strength ratios (a) $\alpha_2=2, \alpha_3=2.3$ (b) $\alpha_2=2, \alpha_3=2.5$

(a)

(b)

Figure 3.12: Comparison of stiff edge ductility among four models for different strength ratios (a) $\alpha_2=2, \alpha_3=2.8$ (b) $\alpha_2=2, \alpha_3=3.0$

Chapter 4

Design of Shear Wall Structures for Seismic Torsion

4.1 Introduction

As described in the previous chapter, torsional vibration may cause significant additional forces in the lateral load resisting planes of an asymmetric structure that remains elastic during an earthquake. The majority of structures, however, are designed for only a fraction of the calculated elastic forces and are expected to undergo significant inelastic deformations during a major earthquake. For such structures, torsional motion caused by natural and accidental eccentricities leads to additional displacement and ductility demands in the lateral loads resisting elements at the edges. In asymmetric buildings, inelastic hysteresis behaviour due to yielding, unloading and reloading of structure as well as strength and stiffness deterioration in reinforced concrete elements, change the relative stiffness of the structural elements and alter the vibration period. Element stiffness changes, in turn, shift the centre of rigidity of the building and affect the torsionally coupled response of inelastic structures, causing a behaviour that is different from that of elastic system.

A number of researchers have studied the inelastic torsional response of single storey buildings and have drawn important conclusions regarding the susceptibility of certain load resisting elements to excessive additional displacement and ductility demands as compared to that in the corresponding torsionally balanced system. Some of these studies on inelastic response have focused on the existing torsional design provisions of various seismic codes.

Recently, Humar and Kumar (1998 I and 1998 II) proposed guidelines for designing asymmetric buildings. Although these guidelines are based on the elastic torsional response, they are also applicable to building structures that are expected to become inelastic. The formulas relate the design strengths of the structure elements to the stiffness of such elements.

The objective of the work presented here is to review Humar and Kumar's torsional provisions and to evaluate whether these design guidelines, based on traditional concept of strength being related to stiffness, are also applicable to cases where the strength is an independent quantity and the stiffness is automatically determined once a strength has been assigned.

4.2 Torsional code provisions

Traditionally, the codes for design against earthquake-induced torsion in buildings have specified the use of a static analysis procedure. In this procedure the design inertia forces are applied through a point that is eccentric with respect to the centre of mass. The design eccentricity e_d measured from the centre of rigidity, is defined as the ratio of design

torque to the base shear. Calibration with the results obtained from linear dynamic analyses have led to prescription of two separate design eccentricity expressions; one suggesting an amplification in static eccentricity, applicable to lateral load-resisting elements or planes on the flexible side of the building, and the other suggesting a reduction in static eccentricity, applicable to elements on the stiff side of the building. These expressions are the origin and basis of the torsional design provisions in modern building codes.

The design eccentricity formulas given in building codes can be written in the following form

$$e_{d1} = \alpha \cdot e_R + \beta \cdot b \quad (4.1)$$

$$e_{d2} = \gamma \cdot e_R - \beta \cdot b \quad (4.2)$$

Where e_{d1} and e_{d2} are design eccentricities, e_R is the distance between the centre of mass and the centre of rigidity, b is plan dimension of the building perpendicular to the direction of earthquake and α , β and γ are coefficients, that have different values in different building codes.

Codes require that to determine the design load for an individual lateral load-resisting element at a floor, the storey shear should be applied at distance e_{d1} or e_{d2} , whichever produces the higher design force, from the centre of rigidity. Usually, e_{d1} governs the design of elements on the flexible side of the building while e_{d2} governs the design of elements on the stiff side.

The first term in the expressions for design eccentricity represents natural torsion, while the second term is supposed to represent the accidental torsion. Factors α and γ are applied to static eccentricity e_R , to take into account the effects of dynamics torque amplification. Accidental torsion, which can be assessed only in an indirect manner, is taken as a fraction of plan dimension b . NBCC (1995) suggested the following expressions,

$$e_{d1} = 1.5e_R + 0.1b \quad (4.3)$$

$$e_{d2} = 0.5e_R - 0.1b \quad (4.4)$$

Humar and Kumar (1998 I) suggest the following expressions,

$$e_{d1} = e_R + 0.1b \quad (4.5)$$

$$e_{d2} = e_R - 0.1b \quad \text{For } \Omega_R \geq 1 \quad (4.6a)$$

$$e_{d2} = -0.1b \quad \text{For } \Omega_R < 1 \quad (4.6b)$$

Eqs. (4.5) and (4.6a) are also being considered for the new version of NBCC. The proposals for NBCC do not include Eq. (4.6b). Instead, they would limit the use of equivalent static method to structures with $\Omega_R \geq 1$, although in an indirect manner.

It has been shown in previous studies that a three-plane system is adequate for representing the inelastic behaviour of most asymmetric buildings with a plan view similar to Fig. 3.1, including those having more than three planes (De La Llera and

Chopra 1994). It has been shown that the presence of x direction planes always improves the performance of torsionally balanced buildings. On the other hand, the contribution from such planes cannot be relied upon because they may have yielded under the action of a simultaneous ground motion in the x direction (Goel and Chopra 1990; De La Llera and Chopra 1994). For these reasons, the three-plane model shown in Fig. 4.1 is used in the present study. In this model, there are three planes in the y direction. The plan aspect ratio a/b is assumed to be 0.5, where a , is the dimension parallel to the direction of earthquake. The force-displacement relationship for each resisting plane is assumed to be that shown in Fig 4.2. The relationship is represented by two straight lines in which the slope of the second line is taken as being 5% of the first. Unloading branch is assumed to be parallel to that initial line.

For purpose of comparison, an associated torsionally balanced model is defined as a single degree of freedom system, (SDOF), which has the same total stiffness, K_y , and mass, m , as the torsionally unbalanced model. In fact, the response of this system represents the response of a single storey structure whose CM coincides with the CR as well as the CV so that the structure responds only in translation.

In the elastic range, the displacement of the flexible edge, δ_f , is given by

$$\delta_f = \frac{V_0}{K_y} + \frac{V_0 e_{d1}}{K_{\theta R}} \left(e_R + \frac{b}{2} \right) \quad (4.7)$$

where K_y is the total lateral stiffness of the structure and $K_{\theta R}$ is the total rotational stiffness. V_0 is the design base shear for the associated torsionally balanced building. The

design base shear, V_0 , is taken as being equal to V_e/R where V_e is the total elastic base shear induced in the balanced building by the design earthquake and R is a force modification factor. All of the above-referred parameters are shown in Fig. 4.3. Factor R starts from 1.0, at a period near to zero, increases with period in the short period range and then becomes constant and almost equal to the ductility ratio, μ (Bertero et al. 1991). R is also dependent on the a/v ratio of the record (ratio of peak acceleration to peak velocity of ground motion) and the ductility capacity.

The design strength, defined as the lateral load at which first yield may take place at a critical section of the wall, is given by $f_1 = k_1 \delta_f$. Thus

$$f_1 = V_0 k_1 \left[\frac{1}{K_y} + \frac{e_{d1} \left(e_R + \frac{b}{2} \right)}{K_{\theta R}} \right] \quad (4.8)$$

As stated earlier, the ratio of uncoupled frequencies, Ω_R , is given by

$$\Omega_R = \frac{\omega_\theta}{\omega_y} = \sqrt{\frac{K_{\theta R}}{K_y r^2}} \quad (4.9)$$

where r is the radius of gyration about CM. For a uniform distribution of mass in plan, r can be expressed as

$$r^2 = \frac{a^2 + b^2}{12} \quad (4.10)$$

Substitution of $K_{\theta R}$ from Eq. (4.9) and $\bar{e} = e_R/b$ in Eq. (4.8) gives

$$f_1 = V_0 \frac{k_1}{K_y} \left[1 + \frac{1}{\Omega^2_R} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (\bar{e} + 0.5) \right] \quad (4.11)$$

In a similar manner, the design strengths of second and third walls can be obtained as

$$f_2 = V_0 \frac{k_2}{K_y} \left[1 + \frac{1}{\Omega^2_R} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} \bar{e} \right] \quad (4.12)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega^2_R} \left(\frac{b}{r} \right)^2 \frac{e_{d2}}{b} (0.5 - \bar{e}) \right] \quad (4.13a)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega^2_R} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (0.5 - \bar{e}) \right] \quad (4.13b)$$

In determining f_3 , the larger of the absolute values obtained from Eqs. (4.13a) and (4.13b) should be used. The research by Humar and Kumar (1998 II) has shown that in a system that is torsionally very flexible, the rotational component of displacement may exceed the translational component. Consequently, Eq. (4.13b) may govern the strength of stiff edge wall and should be taken into account for designing.

The strengths of resisting planes in the corresponding torsionally balanced system are given by

$$f_{10} = V_0 \frac{k_1}{K_y} \quad (4.14a)$$

$$f_{20} = V_0 \frac{k_2}{K_y} \quad (4.14b)$$

$$f_{30} = V_0 \frac{k_3}{K_y} \quad (4.14c)$$

Also,

$$V_0 = f_{10} + f_{20} + f_{30} \quad (4.15)$$

In the traditional method of design, Eqs. (4.11) to (4.13) are used to assign the strengths of the walls. Through extensive inelastic response studies, Humar and Kumar (1998 II) have shown that use of Eqs. (4.11) to (4.13) along with Eqs. (4.5) and (4.6) lead to satisfactory design.

4.3 Proposed design procedure for shear wall structures

The design procedure outlined in the pervious section would be valid provided the stiffness values used in Eqs. (4.11) to (4.13) were correct. However, as described in Chapter 2, the stiffness values themselves are dependent on the strengths. Thus, strengths also appear on the right hand side of Eqs. (4.11) to (4.13). Consequently, these equations cannot be easily solved to obtain the strengths, and an iterative procedure must be used.

Consider a system with plan view similar to Fig. 4.1. Assuming the lengths and widths of walls have been determined from architectural consideration, the first step in designing against earthquake is to choose a reinforcement distribution; either evenly distributed or end concentrated reinforcement. In the next step a trial value of steel ratio is taken and from graphs shown in Figs. 2.8 and 2.9 the parameter K_I and ratio of effective moment of inertia to gross moment of inertia, $\eta = I_{eff}/I_g$, are obtained.

The next step in design is to assign strengths to the walls. It has been shown in Chapter 2 and in a series of studies by Paulay (1998-2000) that, because yield displacements are independent of the strength of components, strengths may be assigned arbitrarily. Limitations on this arbitrariness stem from rational engineering practice in providing the chosen strength and the consideration of the effects of this choice on the seismic response of the system. As described in the previous chapter, assignment of strengths to elements affects the locations of centre of rigidity and centre of strength, which in turn affects the torsional response of ductile structural systems. Locating CV and CR on the opposite sides of CM, as depicted in the earlier chapter, gives the least rotation but not the least ductility demand on the structure.

For a cantilever shear wall, the yield strength of wall is

$$P_y = \frac{M_y}{h_e} \quad (4.16)$$

where M_y is the yield moment and h_e is the height of wall. As mentioned in Chapter 2, the yield moment of a section can be written as

$$M_y = \phi_y \cdot E_c \cdot I_{eff} \quad (4.17)$$

where ϕ_y is the yield curvature, E is the modulus of elasticity of concrete and I_{eff} is the effective moment of inertia of the wall section.

Substitution of the Eq. (4.17) in Eq. (4.16) gives

$$P_y = \frac{\varphi_y \cdot E_c \cdot I_{eff}}{h_e} \quad (4.18)$$

According to Eq. (2.8), yield curvature is given by

$$\varphi_y = \frac{K_1 \cdot \varepsilon_y}{l_w}$$

Therefore,

$$P_y = \frac{K_1 \cdot \varepsilon_y \cdot E_c \cdot I_{eff}}{h_e \cdot l_w} \quad (4.19)$$

Defining $\eta = I_{eff}/I_g$, and substituting this ratio and $I_g = bl_w^3/12$ in Eq. (4.19) gives,

$$P_y = \frac{\eta \cdot K_1 \cdot \varepsilon_y \cdot E_c \cdot b \cdot l_w^2}{12 \cdot h_e} \quad (4.20)$$

As described in Chapter 2, the yield displacement of a wall of given dimensions can be expressed as

$$\delta_y = \left(\frac{K_1 \varepsilon_y}{l_w} \right) \frac{h_e^2}{3} \quad (4.21)$$

The stiffness of the wall is now obtained from

$$K = \frac{P_y}{\delta_y} \quad (4.22)$$

The iterative procedure may begin by assuming values for the steel ratios in the walls. The strengths and stiffnesses of individual walls are obtained from Eqs. (4.20) to (4.22). This in turn allows the determination of the total strength and stiffness, the uncoupled translational period, T , the frequency ratio, Ω_R , and all other important parameters of the structure.

The design provisions given by Eqs. (4.11) to (4.13) are now used to calculate the required strengths. If these calculated strengths are close to the strengths determined earlier from Eq. (4.20), the procedure has converged. Otherwise, new trial values of the steel ratios should be chosen. In order to minimize the number of trial and error cycles, one can employ Eq. (4.20) inversely, using the f_y obtained from last try to calculate η ,

$$\eta = \frac{12 \cdot P_y \cdot h_e}{K_l \cdot \epsilon_y \cdot E_c \cdot b \cdot l_w^2} \quad (4.23)$$

As shown in Chapter 2, the value for K_l can be taken as 2.1 for evenly distributed reinforcement, or 1.85 for end concentrated steel bars. Choice of these values of K_l would involve errors that are less than 15%, which is acceptable for a trial step.

For the given axial load ratio and the new value of η , the required steel ratio can be obtained from graphs similar to those shown in Fig. 2.9. A new iteration is now carried out with this new trial value of the reinforcement ratio.

To illustrate the design procedure a numerical example is presented here. For the plan view of Fig. 4.1, suppose that the length of the first wall is 3 m. α_2 is 0.9 and α_3 is 1.5. The total mass is 520 tonnes. Try a steel ratio of 1.0 % for the first wall. Trial steel ratios for the second and third walls are taken as

$$\rho_2 = \frac{l_w}{l_{w2}} \rho \quad , \quad \rho_3 = \frac{l_w}{l_{w3}} \rho$$

For the sake of simplicity axial loads on walls are not taken into account in this example. Using the procedure described in Chapter 2, exact values of η and K_I are calculated for each wall and are shown in Table 4.1. Assigned strengths, P_y , as well as, yield displacements, δ_y , and stiffnesses, k , can respectively be obtained by using Eqs. (4.20), (4.21) and (4.22).

The stiffness values determined as described in the previous paragraph are used to determine the uncoupled period. This allows the evaluation of the elastic base shear, V_e , from the provided design spectrum. Using a selected value of R , the design shear V_0 , is obtained from $V_0 = V_e/R$. Substitution of the stiffnesses and V_0 in Eqs. (4.11) to (4.13) gives the design strengths for the walls. These design strengths are denoted by F_y and have also been entered in Table 4.1. The process is assumed to have converged if the values of F_y are close to the values of P_y .

Table 4.1: Properties of three walls for the first iteration

Wall No.	l_w (m)	ρ (%)	η	K_1	δ_y (m)	P_y (kN)	k (kN/m)	F_y (kN)
1	3.0	1.0	0.204	2.15	0.0120	648.61	54276	625.38
2	2.7	1.1	0.221	2.18	0.0134	576.05	42886	304.27
3	4.5	0.7	0.153	2.07	0.0077	1052.1	137096	648.68
Σ						2276.7	234256	1.219V ₀

In the present case, the elastic response spectrum corresponding to NS component of El Centro earthquake is used and R is taken as equal to 4. Uncoupled period of the structure is easily calculated from the formula

$$T = 2\pi \sqrt{\frac{m}{K_y}} \quad (4.24)$$

The period of this system works out to $T=0.296$ s and the frequency ratio is calculated as $\Omega_R=1.298$.

In the case presented here, comparison between P_y and F_y for the three walls shows that steel ratios assumed for wall No.2 and 3 are overestimates and should be revised. Using Eq. (4.23) to assist in the selection of the revised steel ratios we find that the minimum steel ratio value of 0.25% should be tried for these two walls. Properties calculated according to new steel ratios are shown in Table 4.2.

Table 4.2: Properties of the system, second iteration

Wall No.	l_w (m)	ρ (%)	η	K_1	δ_y (m)	P_y (KN)	k (KN/m)	F_y (KN)
1	3.0	1.00	0.2040	2.150	0.0120	648.61	54276	589.31
2	2.7	0.25	0.0630	1.850	0.0114	139.61	12225	107.15
3	4.5	0.25	0.0634	1.855	0.0061	391.33	56959	590.39
Σ						1179.5	123461	1.20V ₀

This time for the second wall, the provided strength is close to that required. However, the strength of the third wall has dropped below the code prescribed value and the steel ratio for this wall should be increased. In addition, the total stiffness of the structure has decreased and hence, the uncoupled period of the system has increased. For the second iteration, we obtain $T=0.408$ s. We assume $\rho_3=0.45\%$ which gives $P_y=690.85$ and $k_3=94781$. Results for this final iteration are shown in Table 4.3.

Table 4.3: Properties of the system, final iteration

Wall No.	l_w (m)	ρ (%)	η	K_1	δ_y (m)	P_y (KN)	k (KN/m)	F_y (KN)
1	3.0	1.00	0.2040	2.150	0.0120	648.61	54277	646.10
2	2.7	0.25	0.0630	1.850	0.0114	139.61	12225	99.76
3	4.5	0.45	0.1055	1.968	0.0073	690.85	94781	651.91
Σ						1479.1	161284	1.20V ₀

It will be noted that the provided strength for the second wall is somewhat higher than that required. However the lower limit for longitudinal steel ratio presented in many codes is 0.25%. In the presented example, steel ratio corresponding to required strength would be less than this limit, therefore the minimum code value is adopted for design.

4.4 Validity of the suggested procedure for design against torsion

Based on studies by Humar and Kumar (1998 I and 1998 II), it should be expected that the design procedure described in the previous paragraph would lead to a structure that satisfies the requirements that the ductility demands in the torsionally unbalanced system be less than the demands in the associated torsionally balanced system. To further verify this, a number of single story building models having different ratios of the third wall length to the first wall length, α_3 , and consequently different eccentricities and different frequency ratios, Ω_R , are analyzed for their inelastic responses to two sets of 12 and 15 recorded motions. The models are designed according to the procedure described in previous section in a manner such that the provided strengths of walls are reasonably close to and higher than those given by design expressions, Eqs. (4.11) to (4.13).

Structural details of the models

A single storey shear-type building model having three resisting planes in the y direction, as shown in Fig. 4.1, is selected. The mass of the building floor, m , is taken as 520 t, mass moment of inertia, mr^2 , as 70,200 t-m², aspect ratio, a/b , as 0.5, floor width, b , as 36 m and storey height, h_w , as 5 m. The force-displacement ratio is assumed to be similar to that shown in Fig 4.2 with the strain hardening ratio taken as 5%. Damping is assumed to

be 5% in each mode of vibration. The ratio of the third wall length to the first wall length, α_3 , is taken as 1.0, 1.25, 1.50, 1.75, 2.0, 2.25 and 2.5. The reason for choosing 2.5 as the upper limit is that beyond this limit, e_R/b , the ratio of eccentricity, e_R , to the width of diaphragm, b , becomes greater than 0.3, which is not very practical.

Earthquake motions

The building models are subjected to two sets of 12 and 15 earthquake records, details of which are given in Tables 4.4 and 4.5. The records are selected from McMaster University's database (Naumoski et al, 1988). The first set of records has a wide range of peak ground acceleration to peak ground velocity ratios (a/v). The records in the second set have a peak ground acceleration to peak ground velocity (a/v) ratio in the intermediate range, which varies from 0.8 to 1.2. An elastic response spectrum is obtained for each of the records in the two sets of earthquakes after the records have been scaled to a peak ground acceleration of 0.5g. Elastic response spectra for the two sets of 12 and 15 earthquake records are shown in Figs 4.4 and 4.5, respectively. In each figure, the mean value spectrum is shown by bold line.

Table 4.4: Description and peak ground motion parameters for earthquake records, set No.1

No.	Earthquake	Date	Magn	Site	Epic Dist (Km)	Comp	Max Acc (cm/s²)	Max Vel (cm/s)	A/V
1	Nahanni Aftershock Canada	23/12/85	6.9	Iverson Northwest Territories	7	280	1319.1	45.06	2.98
2	Nahanni Aftershock Canada	23/12/85	6.9	Battlement Creek, N.W. Territories	21	360	190.2	3.43	5.65
3	Miyagi Prefecture Japan	12/6/79	6.3	Ofunato Harbor Jetty	103	E41S	222.1	14.10	1.61
4	ADAK USA	2/5/71	6.8	ADAK Naval Base	69	West	182.8	8.00	2.33
5	ADAK USA	2/5/71	6.8	ADAK Naval Base (Hand dig)	69	N90E	183.7	6.35	2.95
6	ADAK USA	2/5/71	6.8	ADAK Naval Base	69	N90E	203.7	6.28	3.31

Table 4.4: (contd.) Description and peak ground motion parameters for earthquake records, set No.1

No.	Earthquake	Date	Magn	Site	Epic Dist (Km)	Comp	Max Acc (cm/s ²)	Max Vel (cm/s)	A/V
7	Sitka USA	30/7/72	7.6	Sitka Magnetic Observatory	48	West	91.3	9.32	1.00
8	Sitka USA	30/7/72	7.6	Sitka Magnetic Observatory	48	N90E	89.4	6.58	1.36
9	San Fernando California	9/2/71	6.5	Lake Huges Arrays 4	28	S69E	168.2	5.75	2.98
10	Coyote Lake USA	6/8/79	5.8	Gilory Array 6 San Ysidro	10	230	409.0	43.80	0.95
11	Loma Prieta California	18/10/89	7.1	Appel Arr 9 Crystal spr. reservoir	62	137	115.1	17.13	0.68
12	Loma Prieta California	18/10/89	7.1	Calaveras Cherry flat reservoir (left abut.)	42	360	78.2	8.73	0.91

Table 4.5: Description and peak ground motion parameters for earthquake records, set No.2

No.	Earthquake	Date	Magn	Site	Epic Dist (Km)	Comp	Max Acc (cm/s ²)	Max Vel (cm/s)	A/V
1	Imperial Valley California	18/5/40	6.6	El Centro	8	S00E	0.348	0.334	1.04
2	Kern County California	21/7/52	7.6	Taft Lincoln School Tunnel	56	S69E	0.179	0.177	1.01
3	Kern County California	21/7/52	7.6	Taft Lincoln School Tunnel	56	N21E	0.156	0.157	0.99
4	Borrego Mtn. California	8/4/68	6.5	San Onofre SCE power plant	122	N57W	0.046	0.042	1.10
5	Borrego Mtn. California	8/4/68	6.5	San Onofre SCE power plant	122	N33E	0.041	0.037	1.11
6	San Fernando California	9/2/71	6.4	3838 Lankershim BLvd, LA	24	S90W	0.150	0.149	1.01
7	San Fernando California	9/2/71	6.4	Hollywood Storage P.Lot, LA	35	N90E	0.211	0.211	1.00
8	San Fernando California	9/2/71	6.4	3407 6 th St. LA	39	N90E	0.165	0.166	0.99

Table 4.5: (contd.) Description and peak ground motion parameters for earthquake records, set No.2

No.	Earthquake	Date	Magn	Site	Epic Dist (Km)	Comp	Max Acc (cm/s²)	Max Vel (cm/s)	A/V
9	San Fernando California	9/2/71	6.4	Griffith Park Obsv, LA	31	S00W	0.180	0.205	0.88
10	San Fernando California	9/2/71	6.4	234 Figueroa St., LA	41	N37E	0.199	0.167	1.19
11	Nearest coast of Honshu Japan	16/11/74	6.1	Kishima Harbour Works	38	N00E	0.070	0.072	0.97
12	Nearest coast of Honshu Japan	2/8/71	7.0	Kushiro Central Wharf	196	N90E	0.078	0.068	1.15
13	Monte Negro Yugoslavia	15/4/79	7.0	Albatros Hotel Uicing	17	N00E	0.171	0.194	0.88
14	Mexico Earthquake	19/9/85	8.1	El Suchil Guerrero Array	230	S00E	0.105	0.116	0.91
15	Mexico Earthquake	19/9/85	8.1	La Villita Guerrero Array	44	N90E	0.123	0.105	1.17

Details of analysis

For the design of the lateral load resisting elements, each of the original acceleration records is scaled to a peak ground acceleration of 0.5g and an elastic response spectrum for each scaled earthquake is obtained.

When the trial values of steel ratio, the strengths and stiffnesses of the walls and hence the period of the structure have been determined, the required total elastic strength of elements in y-direction, V_e , is obtained from the response spectrum for a selected earthquake. The building model is designed for 25% of the elastic force so that, $V_0 = V_e/4$. The required yield strengths are now determined from Eqs. (4.11) to (4.13). The total strength from these equations is greater than V_0 . If the trial strengths are lower than the required strengths, iteration is repeated.

When the process has converged, a dynamic time history analysis is carried out on building model obtained from final design. Since the design strength is significantly lower than the strength obtained from elastic spectrum, all the lateral resisting elements undergo considerable inelastic deformation. If the results of analyses show that the ductility demands are lower than those produced in the associated torsionally balanced building by the same earthquake, the design is satisfactory.

Accidental torsion

Accidental torsion in structures is caused by uncertainty in the estimation of mass and stiffness. Inaccuracies in estimating the mass and stiffness affect the assessment of the response of both torsionally unbalanced and torsionally balanced structures. However, the

primary interest here is to calculate the response of torsionally unbalanced structure relative to the response of corresponding torsionally balanced structure. The difference between the two responses is dependent mainly on the distribution of stiffness and mass rather than on their absolute values. Therefore, in assessing the accidental torsion, we will focus on the uncertainty in estimating the distribution of mass and stiffness.

A change in the distribution of mass and stiffness will affect both the static eccentricity and the frequency ratio Ω_R . The variability in the distribution of mass or stiffness is a random parameter and must be treated by statistical means. De la Llera and Chopra (1994) have carried out an exhaustive study of mass and stiffness uncertainties and their effect on response. They obtained the increase in edge displacements due to the various sources contributing to accidental torsion, including uncertainty in the distribution of mass and stiffness and ground rotational motion. They concluded that the dynamic response magnification obtained by increasing or decreasing the eccentricity by $0.05b$ provides a reasonable estimate of the contribution from all sources of accidental torsion.

The design provisions given by Eqs. (4.11) to (4.13) take into account the accidental eccentricity. Consequently, in evaluating the dynamic responses of structures designed according to these equations, the effects of accidental eccentricity must be included. In the present work, to account for these effects, the analyses are in fact carried out on building models that have been modified by moving the centre of mass, CM, $\pm 0.05b$ along the x-axis. The higher of the responses of two alternative models is considered for each element. For comparison with the displacement and ductility demands in the associated torsionally balanced model the latter is also analyzed.

The maximum ductility demand in a resisting plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand in the associated torsionally balanced model subjected to the same earthquakes is denoted by μ_b . The ratio of ductility demands, μ_u/μ_b , provides a measure of the effect of torsional motion. A mean value of the ductility ratios, obtained for a set of earthquakes is denoted by r_μ .

Computer program DRAIN-TABS (Guendelman and Powell, 1977) has been used for carrying out the inelastic analyses. The program performs a step-by-step time history numerical analysis using the average acceleration method of integration. The time step employed in the analyses is 0.001 s. In addition, a Rayleigh type damping has been used with the damping values adjusted to 5% in each of the two coupled modes involving translation in the y direction and rotation about CR.

Results of analyses

Dynamic analyses for response to the 12 and 15 scaled earthquake records are carried out on the torsionally balanced, TB, as well as torsionally unbalanced, TUB, models with different ratios of α_3 , following the procedure described earlier. Results for models are presented in Table 4.6 and 4.7. As stated earlier, the effect of accidental torsion is accounted for in the dynamic analyses by shifting the mass centre by a distance of $\pm 0.05b$ from the geometric centre, and taking the larger of the ductility values determined from the analysis of the two TUB models so obtained.

Table 4.6: Mean values of displacements and ductility demands for 12 earthquake

$\alpha_3=l_w3/l_w$	$r_{\delta} = \frac{\delta_{us}}{\delta_{ub}}$	$r_{\delta} = \frac{\delta_{uf}}{\delta_{ub}}$	$r_{\mu} = \frac{\mu_{us}}{\mu_b}$	$r_{\mu} = \frac{\mu_{uf}}{\mu_b}$
1.00	1.110	1.107	0.813	0.811
1.25	1.006	1.396	0.851	0.884
1.50	0.868	1.395	0.796	0.782
1.75	0.714	1.604	0.664	0.767
2.00	0.773	1.644	0.749	0.718
2.25	0.720	1.692	0.662	0.618
2.50	0.651	1.663	0.728	0.604

Table 4.7: Mean values of displacements and ductility demands for 15 earthquake

$\alpha_3=l_w3/l_w$	$r_{\delta} = \frac{\delta_{us}}{\delta_{ub}}$	$r_{\delta} = \frac{\delta_{uf}}{\delta_{ub}}$	$r_{\mu} = \frac{\mu_{us}}{\mu_b}$	$r_{\mu} = \frac{\mu_{uf}}{\mu_b}$
1.00	1.180	1.179	0.899	0.898
1.25	0.973	1.315	0.822	0.856
1.50	0.879	1.404	0.844	0.823
1.75	0.624	1.411	0.617	0.705
2.00	0.577	1.567	0.612	0.717
2.25	0.492	1.588	0.491	0.641
2.50	0.480	1.658	0.546	0.615

An important criterion in the seismic design is that the ductility demand in any resisting plane should not exceed the ductility capacity. Since the force reduction factor R has been assumed to be approximately equal to the ductility capacity μ_t , it implies that the ductility demand should not, in this case, exceed 4. Now it is known that selection $R=\mu_t$ which is based on the assumptions of equal total displacements in the elastic and inelastic models in a single-degree-of-freedom systems, does not always ensure that the ductility demand μ is equal to the target ductility μ_t . The purpose of torsional provisions is to ensure that the ductility demand imposed on asymmetric systems is no greater than that imposed on the corresponding balanced system. The design requirement can therefore be stated in an alternative form: the ratio of TUB model response to TB model response, where the TUB model response has been calculated by including the effect of accidental eccentricity, should not exceed 1. Average ratios r_{μ_s} and r_{μ_f} for the stiff and flexible edge planes respectively are shown in columns 4 and 5 of Tables 4.6 and 4.7 and plotted in Figs. 4.8 and 4.9 as a function of α_3 . It is observed that the ratios of r_{μ_s} and r_{μ_f} are always less than 1.0. The standard deviation for each graph has also been calculated to show the dispersion of results, and the values of the average plus the standard deviation and the average minus the standard deviation are also shown in Figs. 4.6 to 4.9. It will be noted that even the average plus the standard deviation values of r_{μ_s} and r_{μ_f} are generally lower than 1. In all cases the displacement ratio, r_{δ} , is greater than 1, implying that the displacement of the flexible edge plane in the unbalanced building model is greater than the lateral displacement of the associated balanced building model.

4.5 Summary and conclusions

In the present work, a procedure for the design of asymmetric shear wall structures resisting earthquake forces is presented. The procedure is iterative and is based on the fact that stiffness and strength are interrelated. According to the proposed method a value of steel ratio is assumed for each shear wall and this allows the determination of effective moment of inertia, I_{eff} , as well as the strengths of resisting elements. These strengths are compared to those obtained from suggested design equations. If the former are close but higher than the latter, the process has converged, otherwise the values of steel ratios are revised and the procedure repeated.

The suggested design provisions for strengths of elements required to resist earthquake-induced translational and rotational displacements are reviewed in this work. The new design expressions are based on elastic analysis, but the applicability of these expressions for inelastic design has been established by previous research (Humar and Kumar 1998 II). Studies in this chapter examine the applicability of such provisions to the design of buildings designed by the method proposed in the chapter.

Analytical studies are carried out on single storey building models that are symmetrical about the x -axis but are asymmetrical about the y -axis. The lateral resistance in the y direction is provided by three planes, which also provide the entire torsional resistance. The torsionally balanced building models are designed to have $\frac{1}{4}$ th of the design strength obtained from the elastic spectrum of an earthquake selected out of two sets of 12 and 15 scaled earthquake records. The strengths for resisting planes in torsionally unbalanced building models are derived according to the suggested design method. Inelastic time

history analyses are carried out on all building models for the sets of earthquake records. Mean values of the ratio of ductility demand in planes of an unbalanced building to that in the associated balanced building, both subjected to the same earthquake, are obtained to assess the applicability of the suggested design provisions to the structures whose properties are determined as described in the present chapter.

Results show that the use of the suggested torsion design expressions leads to ductility demands in the flexible and stiff edge planes that are smaller than the corresponding values in the torsionally balanced model.

Figure 4.1: Plan view of a three plane single storey building model

Figure 4.2: Force-displacement relationship for the i^{th} plane (Bilinear Model)

Figure 4.3: Base shear-displacement relationship for elastic and inelastic stages

Figure 4.4: Elastic response spectra for 12 selected ground motions having a wide range of (a/v) ratio. The records have been scaled to a peak ground acceleration of 0.5g. Mean value is shown by bold line

Figure 4.5: Elastic response spectra for 15 selected ground motions with an (a/v) ratio in the intermediate range. The records have been scaled to a peak ground acceleration of 0.5g. Mean value is shown by bold line

(a)

(b)

Figure 4.6: Ratio of maximum displacement in a torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes
 (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 4.7: Ratio of maximum displacement in a torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 15 earthquakes
 (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 4.8: Ratio of ductility demand in a torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 4.9: Ratio of ductility demand in a torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 15 earthquakes (a) ratio for stiff edge (b) ratio for flexible edge

Chapter 5

Implementation of the Proposed Design Procedure for Unbalanced Multi-Storey Buildings

5.1 Introduction

The inelastic response of multi-storey buildings is very complex. The characteristics of the building as well as characteristics of the earthquake, including the frequency content, the duration and the nature of time history of ground acceleration affect the inelastic response. As a consequence of variation in one or more of the earthquake characteristics, significantly different responses may be excited by two different ground motions even with similar frequency content, similar maximum amplitude and similar duration. It is therefore difficult to draw generalized conclusions from the results of a set of analyses. Nevertheless, response studies on simplified models of buildings can provide a useful insight into inelastic seismic behaviour of building structures and the effect of torsional motion.

Response analyses have been carried out in the study presented here on a series of torsionally unbalanced and associated balanced multi-storey building models for twelve

different earthquake records. The applicability of proposed method stated in the previous chapter for design of multi-storey buildings of special class is studied.

5.2 Example of multi-storey building

The results obtained from the analyses of single storey inelastic models indicate that the proposed method, which employs design expressions based on the results of elastic analysis, can be successfully applied to the design of single storey inelastic buildings. The design procedure has been verified earlier and it has been shown that torsionally unbalanced single storey buildings designed according to the proposed design method exhibit almost no additional ductility demand beyond that in the associated balanced models. This means that the design procedure and expressions are satisfactory for the design of single storey structures.

A multi-storey building model is employed in the present study to examine the applicability of the proposed design procedure to this type of building structures. It is apparent that a more extensive study on a number of models with different number of floors and different characteristics should be carried out to generalize the design method for all cases.

Building description

The model studied represents a 5-storey shear wall type building with a plan view similar to Fig. 4.1. As stated earlier, a special class of building, in which the centres of mass and rigidity at different floors lie on two vertical lines, is studied to illustrate the design

method. For the sake of simplicity, the dimensions and steel reinforcement ratio of walls are assumed to be constant along the height of the walls. This leads to equal stiffnesses for all the storeys. Mass of each floor of the building is taken as 200 t , mass moment of inertia as $27,000 \text{ t.m}^2$, aspect ratio as 0.5, storey height as 4 m , and floor width b as 36 m . Each lateral load resisting plane is assumed to have a bilinear force-displacement relationship similar to the one shown in Fig. 4.2. A set of building models with five ratios of the length of third wall to the length of first wall, $\alpha_3=1, 1.25, 1.5, 1.75$ and 2.0 are considered. The length of the first wall, L_1 , is assumed to be equal to 4 m and the width of all the 3 walls is taken as 300 mm .

Earthquake motions

Twelve earthquake motions, as used in the study of single storey models in Chapter 4, are employed here, too. Details of these earthquakes are given in Table 4.4. In the present study, earthquake Nos. 9 and 11 are replaced by two ground motions; El Centro, 1940; and San Fernando, 1971 each of which is known to cause significant incursion into the inelastic range. Descriptions of these earthquakes are presented in Table 6.1. It should be noted that the two earthquakes have different peak ground acceleration to peak ground velocity, a/v , ratios. El Centro earthquake has an a/v ratio in the intermediate range and San Fernando earthquake has an a/v ratio in the low to very low range. Similar to earthquakes used in previous chapter, each of these two earthquakes is scaled so that its peak ground acceleration value is $0.5g$.

Table 5.1: Characteristics of the selected earthquake records

No.	Earthquake	Date	Magn	Site	Epic Dist (Km)	Comp	Max Acc. A(g)	Max Vel. V(m/s)	A/V
1	Imperial Valley California	18 May 1940	6.6	El Centro	8	S00E	0.348	0.334	1.04
2	San Fernando California	9 Feb 1971	6.4	4680 Wiltshire Blvd, LA	38	N75W	0.084	0.209	0.4

Analysis steps

The set of multi-storey building models described in previous paragraphs are studied here for both elastic and inelastic behaviour. The study includes design of the building and both static and dynamic response analyses. The steps involved in the analysis are summarised as below:

1. A trial value for reinforcement ratio of each wall is assumed and by using graphs similar to those shown in Chapter 2, the ratio $\eta = I_{eff}/I_g$ and nominal flexural strength of the shear walls, M_n , are determined.
2. The multi-storey building is analysed for an earthquake motion with floor rotations restrained. A Rayleigh damping matrix is used in this analysis.

To determine the damping parameters, α and β , that are mass proportional and stiffness proportional damping coefficients, respectively, the first two uncoupled circular frequencies, ω_1 and ω_2 , are used. The Rayleigh damping matrix is a linear combination of the mass and the stiffness matrices, so that

$$C = \alpha M + \beta K \quad (5.1)$$

Using Eq. (5.1), the damping ratio for any two modes, say i^{th} and j^{th} , can be specified as follows:

$$\varphi_i^T C \varphi_i = 2 \xi_i \omega_i = \alpha + \beta \omega_i^2 \quad (5.2a)$$

$$\varphi_j^T C \varphi_j = 2 \xi_j \omega_j = \alpha + \beta \omega_j^2 \quad (5.2b)$$

where φ is the shape function matrix of the structure.

Assuming damping ratio, $\xi_1 = \xi_2 = 5\%$, and using Eq. (5.2), the parameters α and β can be expressed as

$$\beta = \frac{0.1\omega_2 - 0.1\omega_1}{\omega_2^2 - \omega_1^2} \quad (5.3)$$

$$\alpha = 0.1\omega_1 - \beta\omega_1^2 \quad (5.4)$$

The elastic dynamic analysis of the building with trial properties referred to step 1 provides the elastic shear, V_e , of the structure corresponding to the selected earthquake. Dividing the elastic shear by reduction factor, assumed to

be 4, gives the design base shear of the structure. The floor forces can now be obtained from

$$F_i = (V_0 - F_t) \frac{w_i h_i}{\sum_{i=1}^N w_i h_i} \quad (5.5)$$

where F_i is the floor force at storey level i , N is the total number of floors and w_i and h_i are weight and height of i^{th} floor, respectively. F_t is the top floor force as defined in NBCC and is given by

$$F_t = \begin{cases} 0 & T_1 \leq 0.7 \\ 0.07T_1 V_0 & 0.7 < T_1 < 3.6 \\ 0.25V_0 & T_1 \geq 3.6 \end{cases} \quad (5.6)$$

where T_1 is the fundamental period of the structure.

It should be noted that in routine design, the elastic base shear, V_e , may be obtained from an idealized design response spectrum using an estimated value of fundamental natural period.

3. The dynamic floor forces obtained from the previous step are then applied statically to the same building. As pointed out in the previous chapter, in the proposals for the new version of NBCC, two design eccentricities, ed_1 and ed_2 , are specified for the design of earthquake resisting elements on the flexible and stiff edge of the building, respectively.

$$ed_1 = e_R + 0.1b \quad (5.7)$$

$$ed_2 = e_R - 0.1b \quad (5.8)$$

Equation (5.8) is applicable for the cases where $\Omega_R > 1$.

4. Moments at the bottom of the walls obtained from step 3 are compared to the nominal flexural strengths obtained from step 1. If the latter are close to and slightly larger than the former, the design process has converged, if not, the procedure should be repeated starting from the first step.

Computer program SUPER-ETABS (Maison and Nuess, 1983) is used to carry out the analysis to calculate the mode shapes and frequencies required in step 2, while computer program DRAIN-TABS (Guenelman and Powell, 1977) developed at University of California, Berkeley, is employed for carrying out the static and dynamic analyses in elastic and inelastic studies.

To investigate the inelastic behaviour of the model designed according to the design procedure outlined in the previous paragraphs, and to examine the applicability of the proposed design method to multi-storey buildings, a torsionally balanced model is defined first. This model may consist of 3 identical walls whose total stiffness is equal to the total stiffness of walls in the unbalanced model. The total moment capacity of the walls for balanced model is taken as being equal to the total static base moment induced by the design base shear, V_0 , distributed across the height according to Eqs. (5.5) and (5.6). In the balanced model centres of mass and stiffness coincide, so that a lateral earthquake motion does not induce a rotation in the structure. Both the torsionally unbalanced and the associated torsionally balanced models are analysed for their

response to the selected earthquakes. Studies by Humar and Kumar (1998 I, 1998 II) show that a portion of design eccentricity equal to $0.05b$ in Eq. (5.7) and $-0.05b$ in Eq. (5.8) can be assumed to account for accidental torsion; the portion of design eccentricity that accounts for natural torsion thus works out to be $e+0.05b$ and $e-0.05b$, respectively for the flexible and stiff edges of the building. To consider the accidental torsion in dynamic analysis, the centre of mass of the unbalanced building model is shifted to left (toward the flexible edge) by a distance $0.05b$ to find the maximum effect on the flexible edge. In a similar manner, to find the maximum effect on the stiff edge, CM of the unbalanced model is shifted to the right (toward the stiff edge) by a distance $0.05b$.

In a manner similar to studies described in the previous chapter, the displacement ratio δ_s/δ_f is calculated for all storeys. This ratio provides a measure of the effect of torsional motion. It should be noted that in a torsionally balanced model, all planes have the same displacement in each floor.

The variation of the ratio of δ_s/δ_f for stiff and flexible edges with $\alpha_3=L_1/L_3$ is shown in Figs. 5.1 and 5.2 for El Centro and San Fernando earthquakes. Figure 5.1 demonstrates the displacement ratios for the first floor while Fig. 5.2 shows the displacement ratios for the fifth floor. The plastic hinge rotation at the base of the shear walls is a critical parameter and would be used to provide a measure of the ratio of ductility demands in torsionally unbalanced building and the associated torsionally balanced models. If the ratio of plastic hinge rotations is less than or equal to 1, the design procedure and the design provisions meant to account for torsion can be considered as being adequate. The

ratio of plastic hinge rotation of the torsionally unbalanced model to the one related to the associated balanced model is shown in Fig. 5.3.

Results of analyses

The final designed properties of a model subjected to El Centro earthquake are shown in Table 5.2. The results of analysis including the displacement ratios and plastic hinge rotation ratios of torsionally unbalanced to associated torsionally balanced model are given in Table 5.3. It should be mentioned that the plastic hinge rotation ratios presented in Table 5.3 are calculated at the base of the shear walls.

Table 5.2: Properties of three walls for the final iteration. $\alpha_3=1.25$

Wall No.	l_w (m)	t (m)	ρ (%)	η	K_I	M_n (KN.m)	M_{req} (KN.m)
1	4.0	0.30	0.70	0.153	2.070	6232.91	5619.50
2	3.6	0.30	0.25	0.063	1.853	1878.16	1248.00
3	5.0	0.30	0.45	0.106	1.968	6401.08	5655.40

Table 5.3: Maximum displacement ratios and plastic hinge ratios of stiff and flexible edges. $\alpha_3=1.25$

Floor No.	$r_{\delta} = \frac{\delta_{us}}{\delta_{ub}}$	$r_{\delta} = \frac{\delta_{uf}}{\delta_{ub}}$	$r_{Plr_s} = \frac{Plr_s}{Plr_b}$	$r_{Plr_f} = \frac{Plr_f}{Plr_b}$
1	0.6818	1.2727	0.4847	0.9569
2	0.6462	1.1692	-	-
3	0.6875	1.2143	-	-
4	0.7160	1.2469	-	-
5	0.7406	1.2877	-	-

Figures 5.1a for the first floor and 5.2a for the fifth floor show that the stiff edge displacement in the unbalanced model is, in most cases, less than that in the balanced building while the flexible edge displacements in the unbalanced building are higher than those in the balanced building. Figure 5.3a demonstrates that the stiff edge plastic hinge rotation in unbalanced model is always less than those in the associated balanced building. As observed from Fig. 5.3b, the plastic hinge rotation in the flexible edge plane of unbalanced model is also in general smaller than that in the balanced model. However, in some cases the opposite is true and the plastic hinge rotation in the unbalanced model is actually larger than that in the balanced model. Nevertheless, the difference between the two is not large and may be considered as being acceptable for the purpose of design.

To complete the study, the same model is subjected to the twelve earthquakes described earlier and the mean value of displacement ratios as well as the mean values of plastic hinge rotation ratios are shown in Figs. 5.4 to 5.6. Calculation of the standard deviation

of the results shows the dispersion of results. The mean values of ratios of plastic hinge rotations for both flexible and stiff edge are less than 1, which demonstrates the adequacy of design method and expressions. However, in some cases, specially for flexible edge, the mean plus a standard deviation of the plastic hinge rotation ratio is greater than 1.

5.3 Summary and conclusion

In the previous chapter, a method of designing single storey buildings for seismic torsion, in which the stiffness is assumed to be dependent on strength, was proposed. Results of a study on the inelastic response of models, designed according to the proposed method, to many ground motions were presented. The study on single storey buildings is extended to multi-storey building models. Five shear wall type buildings of *special class*, five storeys in height, are designed according to the suggested method for two different scaled earthquake records. The ratios of maximum displacements in the unbalanced and balanced buildings as well as the plastic hinge rotation ratios are obtained. As expected, the response behaviour of multi-storey building is seen to be quite complex. However, the ductility demand in the unbalanced buildings is found to be comparable to that in the associated balanced models.

The study is extended to cover 12 different scaled earthquakes having different range of a/v ratio. Results show that in most of the cases studied here, the suggested design procedure leads to a safe design. The graphs illustrate that in all cases the, stiff edge plastic hinge rotations in the unbalanced models are significantly less than those in the associated balanced models. The plastic hinge rotations of flexible edge in the unbalanced models are generally less than those in the associated balanced models,

except in a few cases. Even when the plastic hinge rotation in unbalanced model is larger than that in the associated balanced model, the difference is not large and may be acceptable for the purpose of design.

(a)

(b)

Figure 5.1: Ratio of maximum displacement in a torsionally unbalanced building model to that in the associated torsionally balanced building model for first floor (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 5.2: Ratio of maximum displacement in a torsionally unbalanced building model to that in the associated torsionally balanced building model for the fifth floor (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 5.3: Ratio of plastic hinge rotation at the base of a shear wall in a torsionally unbalanced building model to that in the associated torsionally balanced building model
 (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 5.4: Ratio of maximum displacement in a torsionally unbalanced building model to that in the associated torsionally balanced building model for first floor; mean value for 12 earthquakes (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 5.5: Ratio of maximum displacement in a torsionally unbalanced building model to that in the associated torsionally balanced building model for the fifth floor; mean value for 12 earthquakes (a) ratio for stiff edge (b) ratio for flexible edge

(a)

(b)

Figure 5.6: Ratio of plastic hinge rotation at the base of shear wall in a torsionally unbalanced building model to that in the associated torsionally balanced building model; mean value for 12 earthquakes (a) ratio for stiff edge (b) ratio for flexible edge

Chapter 6

Summary and Conclusion

6.1 Summary

Damage studies of recent earthquakes have shown that torsional vibration induced by earthquake motion is one of the sources of damage to structures. It is therefore imperative that sufficient torsional resistance be provided in building systems. Many studies on the elastic and inelastic torsional behaviour of single and multi-storey building models have been carried out, but due to the complexity of problem, conclusions drawn from such investigations have not always been consistent. A recent observation, based on study of concrete shear walls that stiffness varies with a variation in strength requires adjustments in the traditional method of designing structures for torsion. The objective of the work presented in this thesis is to study this issue and to develop a method for the design of shear wall structures to resist torsion.

The present study consists of the following main parts:

1. Review of the causes of torsion, of issues related to torsion design of concrete shear wall structures, of the literature on torsional response, and the recent

observations related to the stiffness of concrete shear walls with a view to identifying the unresolved issues in torsion design.

2. Study of the moment-curvature relationship for shear wall type structures and investigation of the effects of governing parameters on stiffnesses of walls.
3. Study of the relationship between the locations of centres of mass, stiffness and strength and investigation of the effects of the location of such centres on the torsional response of a structure.
4. Development of a new method for the torsion design of structures based on observations in step 2 and 3 and study of the applicability of the design method through inelastic analyses of torsionally unbalanced single storey building models.
5. Inelastic analysis of torsionally unbalanced multi-storey building models of special class, and assessment of the applicability of torsional design method referred to above to the design of multi-storey buildings.

In Chapter 2, study of different shapes of shear walls with different steel ratios, different steel arrangements and different axial loads have been carried out to investigate the effects of these parameters on the stiffness of shear walls. It has been reported in previous studies that stiffness of a resisting shear wall element is not a fundamental parameter but varies with the variation in strength. The yield curvature and consequently the yield displacement of an element are determined by the geometry of the element. As a result, once a strength is assigned to the section of an element, the stiffness of the element is automatically determined. To confirm the validity of these observations, many shear wall models are analyzed. The graphs obtained for the ratio of effective stiffness to gross

stiffness show that the constant moment of inertia of $0.7I_g$ recommended by Canadian Standard Association, CSA A23.3-94, for the design of reinforced concrete shear walls under earthquake-induced loads is in most cases higher than the realistic value of effective moment of inertia.

In Chapter 3, analytical studies are presented for torsional response of single storey building models. The objective of these studies is to investigate the optimum locations of the centres of rigidity and strength in relation to the centre of mass, to minimize the rotation of the structure. Four different models with different strengths and stiffnesses are selected so as to achieve the four possible relative locations of the centres of rigidity and strength in plan with reference to the centre of mass. It is observed that when centre of rigidity and centre of strength are located on the opposite sides of the centre of mass, rotation of the structure would be minimum. However, the presented studies also show that the least rotation of the structure plan does not lead to minimum ductility demand. This result stems from the fact that the yield displacements of resisting elements in the group are different. Consequently, while a reduction in the rotation of the structure will reduce the maximum displacement of the flexible edge, the ratio of maximum displacement to yield displacement, which is denoted as ductility demand of the structure, may not be reduced.

In Chapter 4, a study is presented on a proposed method for the design of shear wall structures for seismic torsion. Traditionally, the code provisions for design against earthquake-induced torsion in buildings have specified the use of a static analysis procedure. In this procedure, the design inertia forces are applied through a point that is

eccentric with respect to the centre of mass. Most code provisions consider two design eccentricities, one for the design of flexible edge and the other for the design of the stiff edge of the structure. Revisions to design eccentricities provided in NBCC have recently been proposed. These revisions are based on studies by Humar and Kumar (1998 I, 1998 II). The objective is to investigate the validity of these design eccentricities in the design of shear wall structures in which the stiffness is not an independent parameter but is dependent on strength.

A new design method is proposed that takes the recent definition of stiffness into account. In this method, the yield displacement of a wall element is determined by using the graphs shown in Chapter 2 and the stiffness is obtained by dividing the yield strength by the yield displacement. The validity of this method is verified through inelastic analyses of a number of single storey models for two sets of 12 and 15 different earthquakes.

In Chapter 5, a five-storey building is designed according to the suggested method. The objective is to study whether the proposed design method can be applied to the design of special class of multi-storey buildings in which the centres of mass and rigidity lie on two vertical lines. Different torsionally unbalanced and associated torsionally balanced models are analyzed to investigate the inelastic torsional behaviour of buildings designed according to the new procedure. In most cases, the proposed method and design eccentricities are found to be valid for the design of special class of multi-storey buildings

6.2 Conclusions

The following important conclusions can be drawn from the work presented here

1. The yield curvature of a concrete shear wall section is insensitive to the axial load ratio as well as the steel reinforcement ratios, but depends only on the steel strain at yield and the length of wall. Even the type of distribution of steel has only a slight effect on the yield curvature.
2. Dependency of yield displacement on wall length implies that it is not possible to design a structure having different wall sizes to ensure that all walls will yield simultaneously.
3. The longitudinal reinforcement ratio is an important parameter that may considerably change the stiffness of elements and should be considered in design.
4. Systems that are perfectly balanced elastically can still have a significant torsional response due to strength irregularity. Both the stiffness as well as the strength eccentricities contribute to the torsional response of structures.
5. Distance between the centre of rigidity and the centre of strength is quite insensitive to the strength distribution. Yield displacement, which is dependent on the elements geometry (length of elements), is the most important parameter controlling this distance.
6. The strength distribution controls the location of both the rigidity and strength centres in relation to CM. Taking into consideration the insensitivity of the

distance between CV and CR, it is apparent that a small strength eccentricity would result in a correspondingly large stiffness eccentricity and vice versa.

7. When CR and CV are on the opposite sides of CM, the rotation of the plan is minimum. However, this does not necessarily give the least ductility demand or the least edge displacement.
8. The torsion design provisions proposed for the next version of NBCC are valid even when the stiffness is dependent on strength.
9. The design procedure based on strength dependent stiffness is an iterative procedure.
10. The design procedure based on studies of single storey building models can be applied to the design of special class of multi-storey building in which the centres of mass and stiffness lie on two vertical lines.

6.3 Recommendations for future works

A detailed study of properties governing stiffness and strength of shear walls as well as the inelastic torsional response of shear wall type single and multi-storey building models has been presented in this thesis. The adequacy of torsional design provisions proposed for the next version of NBCC for building models that are expected to go into inelastic range during an earthquake are verified by means of a study of ductility demand in the edge elements of torsionally unbalanced models. The following recommendations for additional work on torsional response emerge from the present work.

1. In the present work, only models that are mono-symmetric in plan are studied. However, in reality the buildings are likely to have an eccentric distribution of

mass or stiffness about both orthogonal horizontal axes. Therefore, the torsional behaviour of models with bi-axial eccentricities is required to be investigated.

Some issues that should be addressed in this area are listed as follows:

- (a) What is the effect of eccentricity in one direction on the torsional behaviour of a single storey system, when the structure is subjected to earthquake motion in the perpendicular direction?
 - (b) What changes would be required to make the proposed method of design applicable for more general models?
 - (c) What is the inelastic torsional behaviour of bi-axially asymmetric single storey models subjected to ground motion in one direction? How can peak ductility demands of edges of such models be predicted?
2. The present work deals with multi-storey models that belong to a special class of building. Is it possible to develop a simple design procedure for general type of multi-storey building?
3. The presented study for the effective stiffness of walls is limited to single walls. What is the nature of effective stiffness in coupled shear walls?
4. What parameters govern the stiffness of reinforced concrete frame structures, and do the torsion design provisions need any adjustment for such structures?

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