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EVALUATION OF MAGNITUDE RECURRENCE PARAMETERS BASED ON A GLOBAL EARTHQUAKE CATALOG

by

Mladen Jorgic, Dipl. Eng. Physics (Zagreb)

A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of requirements for the
degree of Master of Science

Department of Earth Sciences
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December, 2000

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Chair, Department of Earth Sciences

Thesis Co-Supervisor

Thesis Co-Supervisor

Carleton University,
January 2001

The size distribution of earthquakes is one of the most important factors to be determined in seismic hazard assessment and in developing a physical theory for earthquake generation. In this study, we examine global earthquake recurrence statistics to define the significance of observed deviations from the Gutenberg-Richter magnitude-frequency relation. We determine the parameter values for the Gutenberg-Richter distribution using the Flinn-Engdahl's regionalization of global seismicity and the Harvard Centroid Moment Tensor (CMT) catalog for shallow earthquakes. Deviation from the linear trend (in log-linear plot) of the Gutenberg-Richter cumulative distribution is approximated by the gamma law; i.e., a version of the Gutenberg-Richter cumulative distribution with an exponential taper towards the larger magnitudes. The maximum magnitude in a given tectonic region can be evaluated by using statistical procedures, except for midoceanic ridges. There is no significant differences in b values for subduction zones, collision zones and intracontinental regions ($b=0.98\pm0.02$), while midoceanic ridges show a large b value ($b=1.42\pm0.05$).

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CHAPTER 1

Introduction

The relationship between earthquake size and frequency of occurrence has interested many geophysicists, starting with its first discussions by Ishimoto and Iida (1939) and Gutenberg and Richter (1944). The distribution of earthquake sizes is usually invoked as a first confirmation for practically all models of seismicity. For historical reasons the most commonly used measure of earthquake size is magnitude, as determined from a variety of different magnitude scales. Magnitude is generally a simplified description of physical parameters such as scalar moment (Hanks and Kanamori, 1979), radiated energy (Kanamori, 1977) or wave amplitude (Richter, 1935). The earthquake size distribution is usually described by the Gutenberg-Richter magnitude-frequency law (1954)

$$\log n(m) = a' - b'm \quad (1.1)$$

where $n(m)$ is the number of earthquakes for equal-magnitude classes $m \pm \delta m$, or

$$\log N(m) = a - bm \quad (1.2)$$

where $N(m)$ is the cumulative number of earthquakes with magnitude larger than m . The parameters a' and a are linked by

$$a = a' - \log(b' \ln 10) \quad (1.3)$$

since

$$N(m) = \int_m^{\infty} n(m) dm$$

The equality of parameters $b = b'$ applies only if the integration is made within the limits $[m, -\infty]$ (Båth, 1978; Karnik, 1993). The advantage of using the cumulative distribution (1.2) of earthquakes is that it smoothes the scatter of data. Clearly, b is a statistic measuring the proportion of large and small earthquakes in the group (sample); for a given number of small earthquakes, a large b value implies that large events are very rare. Thus b values have important implications for the evaluation of seismic hazard.

A few authors have attempted to relate the value of b to the physical processes which generate earthquakes. One approach is to assume that the number of earthquakes of a given size is proportional to the area or volume available for faulting and thus that the b value is closely related to the dimensionality of this volume (Kanamori and Anderson, 1975; Aki, 1981; Lomnitz, 1985). Another approach is to attempt to derive a relation between the stress or stress drop at the source and the b value (Scholz, 1968, 1992; Wyss, 1973; Hanks, 1979); the general result is that higher stresses correspond to lower b values.

Earthquakes worldwide are generally observed to follow the Gutenberg-Richter relation, but there is controversy over whether there are deviations in the large magnitude range. One of the proposed prominent deviations is a “bulge” of the $\log N(m)$ curve (Figure 1.1 B) in the range of higher magnitudes which is associated with abnormally high incremental frequencies (Duda and Rashidi, 1972; Purcaru, 1975; Båth, 1981; Singh et al., 1983; Main and Burton, 1984; Pacheco and Sykes, 1992, 1993; Scholtz, 1998; Sykes 1999). The bulge effect results in underestimation of the observed frequencies of large events when frequencies of large earthquakes are estimated by extrapolation of the Gutenberg-Richter relation.

Similar underestimations have been observed when instrumentally determined magnitude statistics are compared with the number of large events inferred from paleoseismological or geological data (Schwartz and Coppersmith, 1984; Qian et al., 1991).

On the other hand, world earthquake data show that at higher magnitude values the log $N(m)$ curve departs from a straight line and trends toward the vertical in the vicinity of $m \approx 7.5-9.0$ (Chinnery and North, 1975; Bloom and Erdmann, 1980; Pacheco et al., 1992). In this case the extrapolation of smaller magnitude values overestimates the number of large events. Figure 1.1 shows typical shapes of the cumulative distribution of earthquakes.

Several explanations have been proposed to interpret the bulge effect. One is the so-called mainshock-aftershock hypothesis (Báth, 1981), assuming that the bulge is a consequence of the difference between the magnitudes of main shocks m_1 and of their largest aftershocks m_2 ("Báth's law", $m_1 - m_2 = 1.2$). Báth regarded it as a typical phenomenon in high-seismicity regions. The characteristic earthquake model (see Figure 1.1 D) (Schwartz et al., 1981; Wesnousky et al., 1983; Schwartz and Coppersmith, 1984; Davison and Scholz, 1985) constitutes an alternative, i.e., the nature of earthquakes originating from the same fault shows distinct differences from the collective nature of earthquakes. The magnitude-frequency distribution of earthquakes occurring in a specific region is well represented by the Gutenberg-Richter relation (equation 1.1 or 1.2). However, the magnitudes of earthquakes originating from a single fault do not follow this relation and this model assumes that a fault generates earthquakes of a characteristic size. The latter model is found to be consistent with observations in some cases. According to

Pacheco et al. (1992) a break in self-similarity from small to large earthquakes occurs at a point where the dimension of the event equals the downdip width of the seismogenic layer. In a specified region the long return period of larger magnitude earthquakes, with respect to the period of time covered by the instrumental data, and the existence of a maximum possible magnitude for the region may provide plausible explanations for the deviation from the straight line and trend toward the vertical of world and regional earthquake data.

The objectives of this thesis are as follows:

1. The primary objective is to examine global earthquake recurrence statistics, and draw conclusions regarding the significance of observed deviations from the Gutenberg-Richter relation.
2. The secondary objective is to make statistical estimates of maximum possible magnitude in different seismic regions if warranted.

Chapter 2 describes the database used in the study for selected seismic regions. The earthquake catalog is attached in Appendix 1 (see disc #1) in which I have added additionally the Flinn-Engdhal code for each seismic region so that the data can easily be accessed.

Chapter 3 gives the modification of Gutenberg – Richter relation by introducing an additional parameter called the “maximum moment or maximum magnitude”.

Chapter 4 presents the method of analysis. The statistical procedure of determining the b value is discussed. The statistical evaluation of maximum possible earthquakes is described.

Chapter 5 presents the results. For each seismic region and for worldwide data, the b value and maximum magnitude are calculated and tabulated. The incremental, cumulative, G-R and gamma distributions are also displayed in graphical form.

Chapter 6 presents the conclusion and discusses the results of this study in comparison with those of previous studies.

Figure 1.1 Schematic diagram of the cumulative distribution of earthquakes, showing three types of deviation from the G-R relation (A): concave type (B), convex type (C) and distribution for a "characteristic earthquake" model (D).

CHAPTER 2

Data

2.1. Seismic data

For this study, the Harvard Centroid Moment Tensor (CMT) catalog compiled by the Harvard group (Dziewonski et al., 1999) has been used. The catalog contains 15539 solutions of moment magnitude $M \geq 5.0$ in the period from January 1, 1977 to December 31, 1998, 11905 of which are shallow earthquakes (depth limits of hypocenters 0-70 km). To test for possible nonuniformity of the catalog because of the time bias in the data (Bloom and Erdmann, 1979; Ogata and Katsura, 1993; Kagan 1997). I compared numbers of earthquakes with different moment magnitude thresholds in one year time intervals throughout the catalog's time span (Figure 2.1). The curves for $M \geq 5.0$, $M \geq 5.2$, $M \geq 5.4$ have an obvious temporal linear trend with the slopes of about 32, 15, 7 (earthquakes per year) respectively. The number of events for $M \geq 5.6$ is uniform in time with slope of the linear trend of only 3 earthquakes per year. On the basis of this analysis the CMT catalog of shallow earthquakes is essentially complete with threshold moment magnitude $M_t = 5.6$.

2.2. Seismic regions

To analyze global seismicity it is necessary to select seismic regions. In this thesis, the Flinn - Engdahl Regionalisation Scheme (Flinn et al., 1974; Young et al., 1995) is used, which is the standard list of region names and boundaries, known as the F-E Code, used by seismologists to specify and identify regions of the Earth. The scheme was first proposed by Flinn and Engdahl (1965) who described the criteria for defining and naming regions. The regionalisation proposed by Flinn and Engdahl (1965) is a subdivision of 50 of the Gutenberg and Richter regions (1954) (regions 51 and 30 being merged), so forming a mosaic within the larger region. Initially, there were a total of 729 subdivisions. Some adjustment of the large region boundaries also had to be made. Flinn and Engdahl called the large regions "seismic regions" (Figure 2.2) and the subdivisions forming the mosaic "geographical regions". The 50 seismic regions are listed in Table 5.1. They are classified into five broad categories (classes) according to the tectonic style prevalent in each zone:

- Subduction zones,
- Collision zones,
- Intracontinental Regions,
- Midoceanic ridges,
- Others.

Figure 2.1 Time distribution of earthquake numbers: $M \geq 5.0$, $M \geq 5.2$, $M \geq 5.4$, $M \geq 5.6$. Average number of events is shown by straight horizontal line.

Figure 2.2 Worldwide Flinn-Engdhal scheme.

CHAPTER 3

Modification of G-R relation

The model of the Gutenberg-Richter relation (equation 1.1 or 1.2) is generally considered satisfactory for small and moderate earthquakes, while for the largest events, some authors actually suggest a formal smoothing of equation (1.1) (Wyss, 1973; Purcaru 1975; Knopoff and Kagan, 1977; Lomnitz-Adler and Lomnitz, 1978, 1979; Utsu 1979; Makjanić 1978, 1980;), truncated in the high magnitudes range, with the high magnitude distribution tail showing a decay stronger than 10^{-bm} . This problem is generally solved by introducing an additional parameter called the "maximum moment or maximum magnitude" (M_{0max} or m_{max}). Anderson and Luco (1983) propose truncation at M_{0max} of either the cumulative distribution, or its density (the first derivative), or the second derivative of the distribution. Main and Barton (1984) and Kagan (1993, 1994, 1997) use the gamma distribution, a version of the Gutenberg-Richter distribution with an exponential roll-off at large moments, to describe earthquake size distribution.

The gamma distribution has a probability density

$$\Phi(M) = C^{-1} M_0^{-1-B} \exp(-M_0/M_{0max}) \quad M_{0t} \leq M_0 < \infty \quad (3.1)$$

where C is a normalizing coefficient

$$C = M_{0max}^{-B} \Gamma(-B, M_{0t}/M_{0max}) \quad (3.2)$$

$\Gamma(x, y)$ is the incomplete gamma function, M_{ot} is the lower moment threshold, M_{omax} is a maximum moment parameter which can be considered an approximate estimate of maximum moment, and $B=(2/3)b$.

For $M_{omax} \gg M_{ot}$ the coefficient C in equation (3.2) is (Bateman & Erdelyi 1953)

$$C \approx M_{ot}^{-B} \exp(M_{ot}/M_{omax})/B \quad (3.3)$$

and therefore equation (3.1) becomes

$$\Phi(M_0) = B M_{ot}^B M_0^{-1-B} \exp[(M_0 - M_{ot})/M_{omax}] \quad M_{ot} \leq M_0 < \infty \quad (3.4)$$

The scalar seismic moment M_0 is used as an indicator of earthquake size (Hanks and Kanamori, 1979).

The relation between scalar seismic moment M_0 and moment magnitude M is

$$M = (2/3) \log M_0 - 10.7 \quad (3.5)$$

where M_0 is in $\text{dyn}\cdot\text{cm}$ (dyne centimeters).

Using the rule of transformation of variables in the probability density function

$$f(M) = \Phi[M_0(M)] \cdot dM_0/dM \quad (3.6)$$

the probability density function of the gamma distribution as a function of a moment magnitude is

$$f(M) = \beta \exp[-\beta(M - M_t)] \exp\{\exp[3/2 \ln(10) \cdot (M - M_{max}) - 3/2 \ln(10) \cdot (M_t - M_{max})]\} \quad (3.7)$$

where $\beta = b \ln(10)$.

According to equation (3.7) the cumulative magnitude-frequency distribution becomes

$$\log N(M) = a - bM - 10^{2/3(M-M_{\max})} \quad (3.8)$$

which differs from the G-R relationship (equation 1.2) by decreasing more rapidly as the value $M_{\max}$ is approached.

CHAPTER 4

Method of Analysis

4.1. Determination of b value

The incremental $n(m)$, and cumulative $N(m)$ frequencies for most F-E seismic zones have been examined. Common procedures for estimating the parameter b are the least squares method and the method of maximum likelihood. Many authors have extensively discussed the problem of selecting the preferred method of calculating the b values of the G-R relation. Knopoff and Kagan (1977) suggested that methods which utilize all available data give superior estimates of the parameters of seismicity. Karnik (1995) and Båth (1987), recommend both methods but use the least square method in their analysis. Ranalli (1969) and Papadopoulos and Kijko (1991) used and compared both methods in their analysis. Pacheco and Sykes (1992) used the least squares method (1992), but Frolich and Davis (1993) used only maximum likelihood method.

Others, however, believe that the maximum likelihood is superior to linear regression in its handling of the largest magnitudes (Aki 1965; Bender 1983). The b value estimated by Aki (1965) is

$$b = \log(e) / (m_{av} - m_t) \quad (4.1)$$

and by Bender (1983) is

$$q / (1 - q) - nq^k / (1 - q^k) = (m_{av} - m_t) / \delta m \quad (4.2)$$

where $q = \exp[-\ln(10) \cdot b \delta m]$, m_{av} is the mean value of all magnitudes within the stated range, k is number of discrete bins of width δm between a threshold magnitude m_t and a maximum magnitude. Equation (4.2) is a transcendent equation and its solution for the b value can only be obtained numerically.

Error estimates for b were given by both Aki (1965) and Utsu (1966). Aki (1965) uses the central limit theorem to arrive at a Gaussian distribution of b around its maximum likelihood estimate b_0 with a standard deviation

$$\sigma = b_0 n^{-1/2} \quad (4.3)$$

where n is the total number of earthquakes. This should not be used for small n . Also, Aki (1965) tabulates values for $n \geq 50$. Utsu's (1966) chi-square distribution with variance σ^2 which is described by equation (4.3) is used for small numbers of earthquakes.

In this thesis, both least square and maximum likelihood methods (equation 4.2) are used for the calculation of b values and their standard deviations.

4.2. Statistical Evaluation of Maximum Possible Earthquakes

The problem of evaluating the maximum possible earthquake for a particular seismic region has always occupied one of the central places in seismology. This is attributable, on the one hand, to the fact that the maximum possible magnitude m_{max} is one of the parameters of the earthquake recurrence law, and on the other hand, to the fact that the possible destructive consequences of earthquakes in a particular region are quite dependent on m_{max} . If we know the magnitude distribution law, we may be able to use recurrence statistics to estimate m_{max} . It should be clear that neglecting the uncertainty

resulting from the selection of an incorrect model of magnitude distribution can lead to significantly biased estimates of $m_{\max}$.

In this thesis we followed the convention in which a capital letter is used for a random variable, and the same letter, in lower case, represents the values which it may assume.

For the classical Gutenberg-Richter relation (equation 1.1), which is a frequency-magnitude relation that is unbounded from above (i.e., $m_{\max} \rightarrow \infty$), the probability density function (PDF) and cumulative density function (CDF) $f_M(m) \equiv f_M(m|\infty)$ and $F_M(m) \equiv F_M(m|\infty)$ are continuous and equal to (Aki, 1965)

$$f_M(m) = \begin{cases} \beta \exp[-\beta(m-m_t)], & \text{for } m \geq m_t, \\ 0, & \text{for } m < m_t, \end{cases} \quad (4.4)$$

$$F_M(m) = \begin{cases} 1 - \beta \exp[-\beta(m-m_t)], & \text{for } m \geq m_t, \\ 0, & \text{for } m < m_t, \end{cases} \quad (4.5)$$

The respective PDF and CDF of earthquake magnitude, which are bounded from above by $m_{\max}$, are (Page, 1968; Cosentino et al., 1977)

$$f_M(m|m_{\max}) = \begin{cases} \frac{\beta \exp[-\beta(m-m_t)]}{1 - \exp[-\beta(m_{\max}-m_t)]}, & \text{for } m_t \leq m \leq m_{\max} \\ 0, & \text{for } m < m_t, m > m_{\max} \end{cases} \quad (4.6)$$

$$F_M(m|m_{\max}) = \begin{cases} 0, & \text{for } m < m_t, \\ \frac{1 - \beta \exp[-\beta(m-m_t)]}{1 - \exp[-\beta(m_{\max}-m_t)]}, & \text{for } m_t \leq m \leq m_{\max}, \\ 1, & \text{for } m > m_{\max}. \end{cases} \quad (4.7)$$

Where $\beta = b \ln(10)$, and b is the parameter of the Gutenberg-Richter relation (1.1). Both distribution functions are obtained as a result of truncation of unbounded distributions (4.4) and (4.5) at the point $m_{\max}$, that is

$$f_M(m|m_{\max}) = \begin{cases} \frac{f_M(m)}{F_M(m_{\max})}, & \text{for } m_1 \leq m \leq m_{\max} \\ 0, & \text{for } m_1 > m > m_{\max} \end{cases} \quad (4.8)$$

$$F_M(m|m_{\max}) = \begin{cases} 0, & \text{for } m < m_1, \\ \frac{F_M(m)}{F_M(m_{\max})}, & \text{for } m_1 \leq m \leq m_{\max}, \\ 1, & \text{for } m > m_{\max}. \end{cases} \quad (4.9)$$

where

$$F_M(m) = \int_{m_1}^m f_M(\xi) d\xi \quad (4.10)$$

The estimator derived here (Kijko and Graham, 1998) is based on the "order statistics" of earthquake magnitudes $M_1 \leq M_2 \leq \dots \leq M_n$, where the unordered M_i are independent random values and identically distributed according to CDF $F_M(m|m_{\max})$ and n is the total number of earthquakes. In addition it is assumed that $F_M(m|m_{\max})$ belongs to the class of functions which allow a Taylor series expansion about the point $m_{\max}$. We observe that after the transformation $Y_1 = F_M(M_1|m_{\max})$, $Y_2 = F_M(M_2|m_{\max})$, ..., $Y_n = F_M(M_n|m_{\max})$, the values $Y_1, \dots, Y_n$ form an ordered data set $Y_1 \leq Y_2 \leq \dots \leq Y_n$, and all of them are uniformly distributed, that is

$$F_M(m|m_{\max}) = \begin{cases} 0, & \text{for } y < 0, \\ y, & \text{for } 0 \leq y \leq 1, \\ 1, & \text{for } y > 1. \end{cases} \quad (4.11)$$

The CDF of the largest among $(Y_1 \leq Y_2 \leq \dots \leq Y_n)$, that is Y_n , is

$$\begin{aligned} F_{Y_n} &\equiv \Pr[Y_n \leq y] \\ &= \Pr[Y_1 \leq y, Y_2 \leq y, \dots, Y_n \leq y] \\ &= [F_y(y)]^n \\ &= y^n \end{aligned} \quad (4.12)$$

The PDF of Y_n is given by

$$f_{Y_n}(y) \equiv \frac{dF_{Y_n}(y)}{dy} = \begin{cases} 0, & \text{for } y < 0, \\ ny^{n-1}, & \text{for } 0 \leq y \leq 1, \\ 1, & \text{for } y > 1. \end{cases} \quad (4.13)$$

and its expected value is

$$E(Y_n) = \int_0^1 \xi f_{Y_n}(\xi) d\xi = n \int_0^1 \xi^n d\xi = \frac{n}{n+1} \quad (4.14)$$

One possibility for obtaining the estimator $\hat{m}_{\max}$ is to introduce the condition

$$E(Y_n) = y_n \quad (4.15)$$

where y_n is calculated from the relation $F_M(m_{\max}^{obs} | \hat{m}_{\max})$, and $m_{\max}^{obs}$ is the largest

observed magnitude. Bearing in mind that $E(Y_n) = \frac{n}{n+1}$ we obtain the equation

$$F_M(m_{\max}^{obs} | \hat{m}_{\max}) = \frac{n}{n+1} \quad (4.16)$$

Thus the estimator of $m_{\max}$ becomes a function of the known observations $m_{\max}^{obs}$ and n , and is obtained as a root of equation (4.16). The above result is valid for any of the magnitude CDF $F_M(m|m_{\max})$, and does not require the fulfilment of the truncation

condition (equations 4.8-4.10). It can be shown that the estimator of the parameter $m_{\max}$ as defined above, belongs to the class of so-called “moment estimators” (Kendall and Stuart, 1967).

One of the simplest ways to assess the properties of the above estimator is to use the Taylor series expansion of an inverse of CDF $F_M^{-1}(Y_n|m_{\max})$. The expansion of the function $F_M^{-1}(Y_n|m_{\max})$ in a Taylor Series about the point $Y_n=1$ yields

$$M_n = F_M^{-1}(Y_n|m_{\max}) - \frac{dF_M^{-1}(Y_n, m_{\max})}{dY_n} \Big|_{Y_n=1} (1-Y_n) + \dots \quad (4.17)$$

Since $F_M^{-1}(Y_n|m_{\max}) = m_{\max}$ and $E(1-Y_n) = 1 - \frac{n}{n+1} = \frac{1}{n+1}$

$$\frac{dF_M^{-1}(Y_n, m_{\max})}{dY_n} \Big|_{Y_n=1} = \frac{1}{\frac{dF_M(m, m_{\max})}{dm} \Big|_{m_{\max}}} = \frac{1}{f_M(m_{\max}|m_{\max})}. \quad (4.18)$$

For n large, $E(1-Y_n) \simeq 1/n$, and $f_M(m_{\max}|m_{\max}) \simeq f_M(m_{\max}^{\text{obs}}|m_{\max}^{\text{obs}})$. After replacing $1-Y_n$ by its expected value, viz. $1/n$, the estimator (4.16) takes the simple form

$$\hat{m}_{\max} = m_{\max}^{\text{obs}} - \frac{1}{n \cdot f_M(m_{\max}^{\text{obs}}|m_{\max}^{\text{obs}})} \quad (4.19)$$

The end-point estimator (4.19) was first derived by Tate (1959). It was used by Kendall and Stuart (1967). Pisarenko (1991, 1996), Kijko and Graham (1998) applied it into the estimation of maximum regional magnitude $m_{\max}$.

It is easy to extend this approach and assess the approximate variance of the estimator (4.19). By applying the relation between the derivative of a continuous, strictly monotonic function and its inverse function, the approximate variance of $\hat{m}_{\max}$ can be written as

$$\sigma^2(\hat{m}_{\max}) = \left[\frac{dF_M(m_{\max}^{\text{obs}} | m_{\max})}{dm_{\max}} \right]^{-2} \Big|_{m_{\max} = \hat{m}_{\max}} \cdot \sigma^2(\hat{y}_n). \quad (4.20)$$

Following equations (4.13) and (4.14) we obtain

$$E(Y_n^2) = \int_0^1 \xi^2 f_{Y_n}(\xi) d\xi = n \int_0^1 \xi^{n+1} d\xi = \frac{n}{n+2} \quad (4.21)$$

and

$$\sigma^2(Y_n) = E(Y_n^2) - [E(Y_n)]^2 = \frac{n}{(n+2)(n+1)^2} \quad (4.22)$$

We also obtain

$$\frac{dF_M(m_{\max}^{\text{obs}} | m_{\max})}{dm_{\max}} = -F_M(m_{\max}^{\text{obs}} | \hat{m}_{\max}) \cdot f_M(m_{\max} | m_{\max}) \quad (4.23)$$

and after replacing $[F_M(m_{\max}^{\text{obs}} | \hat{m}_{\max})]^2$ by its expected value $E(Y_n^2)$, for large n , we obtain

$$\sigma^2(\hat{m}_{\max}) = \frac{1}{n^2 \cdot f_M^2(m_{\max}^{\text{obs}} | m_{\max}^{\text{obs}})} \quad (4.24)$$

The above equation describes the variance of $\hat{m}_{\max}$, estimated according to equation (4.19).

4.3. Parameter Estimation for the Frequency-Magnitude Relation

We assume that we have a complete data set of n earthquakes with threshold magnitude m_t and larger, $m_1, m_2, \dots, m_n$. These are considered as random samples from a population whose magnitude distribution is represented by the Gutenberg-Richter magnitude-frequency law with probability density functions represented by equations

(4.4), (4.7), or a gamma probability density function (equation 3.7). The parameters in the probability density functions give the values which maximize the log-likelihood function

$$\ln L = \sum_{i=1}^n \ln f(m_i) \quad m_i \geq m_t \quad (4.25)$$

and it is possible to compute AIC (Akaike information criterion. Akaike, 1974)

$$AIC = -2 \cdot \ln L_m + 2v \quad (4.26)$$

where L_m is the maximum of L and v is the number of parameters in the probability density function $f(m)$. Based on the AIC values that are calculated for different probability density distributions for a given data set, we can judge which is the most suitable distribution for the data set. The one which provides the smallest AIC is the best, though a difference of less than about 2 in AIC is considered insignificant.

4.4. Significance Test of the Difference in b Values Between Two Groups

When we obtain fairly different values of b , say b_1 and b_2 ($b_1 > b_2$) for two groups of earthquakes, the problem is to decide whether this difference is statistically significant or not. A test can be performed by using the AIC (Utsu, 1999). The null hypothesis is that the two groups have the same b value (b for combined group is b_0) and the alternative one is that the b values are different. The AIC for the former and latter hypothesis is denoted by AIC_0 and AIC_{12} . The numbers of earthquakes in each group are n_1 and n_2 . The parameter $\beta = b \ln(10)$ in the probability density function (equation 4.4), calculated by equation (4.1), maximizes the log-likelihood function (equation 4.25). The AIC for the former and latter hypothesis are

$$AIC_0 = -2(n_1 + n_2) \ln[b_0 \ln(10)] + 2(n_1 + n_2) + 2 \quad (4.27)$$

$$AIC_{12} = AIC_1 + AIC_2 = -2n_1 \ln[b_1 \ln(10)] + 2n_1 - 2n_2 \ln[b_2 \ln(10)] + 2n_2 + 4 \quad (4.28)$$

and

$$\delta AIC = AIC_0 - AIC_{12} = -(n_1 + n_2) \ln(n_1 + n_2) + 2n_1 \ln(n_1 + n_2 b_1 / b_2) + 2n_1 \ln(n_1 b_2 / b_1 + n_1) - 2 \quad (4.29)$$

If AIC_{12} is significantly smaller than AIC_0 , we can reject the null hypothesis and accept that the two groups have different b values. Usually the difference in AIC is considered significant if δAIC exceeds about 2. If $\delta AIC > 5$ the difference is highly significant.

4.5. The Statistical Homogeneity of the b Values

The homogeneity of the b values can be tested by using a statistical test based on a modification of Bartlett's statistics (Pearson and Hartley, 1966). Utsu (1971) and Kagan (1977) have shown that for the Gutenberg-Richter distribution (1.1), the reciprocal of the estimate of b is distributed as a χ^2 variable with $2n$ degrees of freedom. For estimates b_i in k regions with numbers of earthquakes n_i in each, it is necessary to calculate the value

$$S = 2 \sum_{i=1}^k n_i \ln b_i - 2n \ln \bar{b} \quad (4.30)$$

where

$$n = \sum_{i=1}^k n_i \quad (4.31)$$

$$\bar{b} = n / \sum_{i=1}^k \frac{n_i}{b_i} \quad (4.32)$$

If all of the b_i belong to the same population with the universal value of b , S should be approximately distributed as a χ^2 variable with $k-1$ degrees of freedom. The significance

level α obtained from tables of χ^2 indicates whether we should reject the null hypothesis that all regions share the same value of the b parameter. For example, $\alpha = 5\%$ means that the difference of b estimates is not statistically significant at the 5% level, that is, because of random fluctuations, a similar distribution of b is expected in one of 20 region categories.

In the following chapter, we determine the b values and standard deviations for cumulative and incremental distributions. The statistical procedures for testing of the difference in the b values between two groups and for homogeneity of the b values are used. Also, the maximum possible magnitude is determined for all regions.

CHAPTER 5

Results

To determine b values for worldwide, broad tectonic zones, and F-E seismic zones different approaches are employed. The Harvard Centroid Moment Tensor (CMT) catalog of shallow earthquakes from 1/1/1977 to 31/12/1998 with threshold moment magnitude $M_t = 5.6$ is divided into five catalogs for the following categories: subduction zones, collision zones, intracontinental regions, midoceanic ridges, and others. It is further subdivided into fifty F-E Seismic Zones (see Table 5.1)

The cumulative distribution is used. The earthquakes close to $M_{\max}^{\text{obs}}$, that is, those in the interval $[M_c, M_{\max}^{\text{obs}}]$ where the magnitude-frequency plot becomes nonlinear, are excluded from consideration (Karnik, 1993; Kagan, 1997; Triep and Sykes, 1999). This means that the deviations from linearity begin at a value M_c which depends on the largest observed $M_{\max}^{\text{obs}}$ and on the distribution type. The distribution in the magnitude range $[M_t, M_c]$ is approximated by the Gutenberg-Richter distribution, and in the magnitude range $[M_c, M_{\max}^{\text{obs}}]$ it is approximated by the gamma distribution (Figures 5.1-5.6).

To obtain estimates of b values and their standard deviations σ , the least square (LS) method and the maximum likelihood (ML) procedure (Bender, 1983) are used. The methods give the same b values but different values for standard deviation. The values for $b \pm \sigma_{\text{LS}}$ and $b \pm \sigma_{\text{ML}}$ are listed in Table 5.1 for all regions and region categories having a number of events $n \geq 7$.

The b values for the five categories were also calculated for the incremental distribution in the magnitude range $[M_t, M_{\max}^{\text{obs}}]$. The differences of the b values between incremental and cumulative distribution are insignificant except for midoceanic ridges.

The parameter $M_{\max}$ in the gamma distribution is obtained by the proposed method for evaluating maximum magnitude $M_{\max}$ (see equation 4.19). It is a convenient and simple statistical method, which uses only earthquakes catalogs. Of course, it follows that the inaccuracy of the estimate of $M_{\max}$ depends on whether the statistical model is complete and correct. The values of $M_{\max}$ with their standard deviations σ_M (see equation 4.24) are shown in Table 5.1.

The Akaike information criterion (AIC) (see section 4.4) is used as a significance test of the difference in b values. The results are shown in Table 5.2, 5.3, 5.4, 5.5, 5.6 and 5.7.

The Bartlett's Test (see section 4.5) is used for statistical homogeneity of the b value and the values of k , b , S , and α are shown in Table 5.8.

5.1 Subduction Zones

Both cumulative magnitude-frequency distributions $N(M)$ and incremental distribution $n(M)$ (Figure 5.1) have a convex shape. The corner magnitude is $M_c=7.3$, $b=1.0\pm0.01$ and $M_{\max} = 8.4\pm0.1$ (where the b value is determined by maximum likelihood using all data in subduction regions). The gamma distribution is a very good approximation to $N(M)$ for $M>7.3$. The b value for incremental distribution is $b=1.01\pm0.01$. Table 5.2 shows the result of the test on the difference in b values among

the eighteen seismic regions in the subduction zone category. There are 11% highly significant differences in b value, 14% significant and 75% not significant. It is seen that most significant and highly significant differences are related to the small b values in regions 5 (Mexico-Guatemala) and 8 (Andean S. America). Barlett's Test accepts the null hypothesis that all regions share the value $b=1.0\pm0.01$ ($\alpha=50\%$), where α is the percentage of regions expected to have a similar distribution of b . Note that this b value is determined from the reciprocals of b values of all subduction zone regions (equation 4.32), which is a χ^2 distributed variable.

5.2 Collision Zones

The cumulative magnitude-frequency distributions $N(M)$ and incremental distribution $n(M)$ (Figure 5.2) have a convex shape. The corner magnitude is $M_c=7.1$. $b=1.04 \pm 0.03$ and $M_{\max}=7.7 \pm 0.1$. The gamma distribution estimates $N(M)$ very well for $M>7.1$. The b value for incremental distribution is $b=0.98\pm0.06$. Table 5.3 shows the result of the test of the difference in b value among the ten seismic regions in the collision zone category. Almost all b values are not significantly different. Barlett's test accepts the null hypothesis that all regions share the same value $b=1.04 \pm 0.03$ ($\alpha=64\%$).

5.3 Intracontinental Regions

A flat region for $6.4 \leq M \leq 7.2$ in the incremental frequency plot $n(M)$ produces a bulge effect for the same magnitude interval in the cumulative frequency plot $N(M)$

(Figure 5.3) . The linear trend of the G-R distribution is well expressed up to the corner magnitude $M_c = 6.6$ and beyond, with $b = 0.83 \pm 0.04$. The b value for incremental distribution is $b = 0.71 \pm 0.14$. The maximum magnitude is $M_{max} = 7.5 \pm 0.2$. The gamma distribution is not a good estimation for $N(M)$ for $M > 6.6$. Table 5.4 shows the result of the test on the difference in b values among the ten seismic regions in the intracontinental regions category. There is only a 17% significant difference in b value, related to region 42 (N.E. Asia-Greenland). Barlett's test accepts the null hypothesis that all regions share the value $b = 0.83 \pm 0.04$ ($\alpha = 18\%$)

5.4 Midoceanic ridges

A flat region for $M > 7.1$ in the incremental frequency plot $n(M)$ produces a sharp bulge effect for $7.1 < M < 8.1$ in the cumulative frequency plot $N(M)$ (Figure 5.4). The linear trend of the G-R distribution is well expressed to the corner magnitude $M_c = 7.1$ with very large $b = 1.46 \pm 0.05$. The b value for incremental distribution is $b = 1.08 \pm 0.08$. The maximum magnitude M_{max} cannot be determined by the method employed in this study. This can be explained by the large b value and fewer numbers of earthquakes. For the gamma distribution the historically maximum magnitude $M_{max} = 8.4$ is used (Pacheco et al. 1992). The gamma distribution is not a good estimation of $N(M)$ for $M > 7.0$. Table 5.5 shows the result of the test on the difference in the b value among the twenty one seismic regions in the midoceanic ridges category. There are 17% highly significant differences in the b value, 14% significant and 69% are not significant. Most of the differences are related to the large b values in regions 44 (Galapagos), and 43 (Macquarie

Loop). Barlett's test rejects the null hypothesis that all regions share the value

$$b=1.46\pm0.05. (\alpha=0.4\%)$$

5.5 Others

The category "Other" is defined as the seismic regions with mixed style of deformation. A flat region for $M>7.4$ in the incremental frequency plot $n(M)$ produces a very weak bulge effect for $7.4<M<7.9$ in the cumulative frequency plot $N(M)$ (Figure 5.5). The linear trend of the G-R distribution is well expressed to the corner magnitude $M_c=7.2$, with $b=0.97\pm0.01$. The b value for incremental distribution is $b=0.89\pm0.04$. The maximum magnitude is $M_{\max}=8.2\pm0.2$. The gamma distribution estimates well $N(M)$ for magnitude $M>7.2$. Table 5.6 shows the result of the test of the difference in b value among the twenty-one regions in the others category. There are 19% highly significant differences in the b value. It is seen that they are related to the large b values in regions 17 (Caroline Island-Guam), and 10 (S. Antilles). Barlett's test rejects the null hypothesis that all regions share the value $b=0.97\pm0.01$ ($\alpha=0.5\%$). However, this rejection is not caused by the different b estimates in any particular region, since we obtain $\alpha=8.3\%$ if we remove only one region (44 Galapagos) from the set.

5.6 Worldwide

The cumulative magnitude-frequency distributions $N(M)$ has a convex shape with a corner magnitude $M_c=7.8$, $b=1.06\pm0.02$, $M_{\max}=8.4\pm0.1$ (Figure 5.6). The gamma distribution very well estimates $N(M)$ for $M>7.8$. The incremental magnitude-frequency

distribution $n(\mathbf{M})$ has a linear trend from threshold magnitude to the maximum observed magnitude $\mathbf{M}_{\max}^{\text{obs}}=8.3$ with the slope $b=1.06 \pm 0.04$ i.e., the Gutenberg-Richter distribution very well estimates the incremental distribution $n(\mathbf{M})$ in whole magnitude range. Table 5.7 shows the result of the test of the difference in b value among the five broad worldwide categories: subduction zones, collision zones, intracontinental regions, midocenic ridges, and others. It rejects only midocenic ridges.

If we consider subduction zones, collision zones and intracontinental region, according to the results of Barlett's test (Table 5.8), as a single region with continental boundaries, the calculated b value in the magnitude range $5.6 \leq \mathbf{M} \leq 7.4$ is 0.98 ± 0.01 . The b value for incremental distribution is $b=0.99 \pm 0.04$. Barlett's test accepts the null hypothesis that continental regions share the value $b=0.98 \pm 0.01$ ($\alpha=9.5\%$).

Figure 5.1 Magnitude-frequency plots for Subduction Zones-total. Circles, full line and dashed line denote 22-year normalized (p.a.) frequencies, gamma distribution and G-R distribution, respectively, for the cumulative distribution. Squares denote the 22-year normalized (p.a.) frequencies for the incremental distribution.

Figure 5.1a. As in Figure 5.1 for Subduction Zones 1, 5, 6, and 7.

Figure 5.1b As in Figure 5.1 for Subduction Zones 8, 12, 13, and 14.

Figure 5.1c As in Figure 5.1 for Subduction Zones 15, 16, 18, and 19.

Figure 5.1d As in Figure 5.1 for Subduction Zones 20, 21, 22, and 23.

Figure 5.1e As in Figure 4 for Subduction Zones 24 and 26.

Figure 5.2 Magnitude-frequency plots for Collision Zones-total. Circles, full line and dashed line denote 22-year normalized (p.a.) frequencies, gamma distribution and G-R distribution, respectively, for the cumulative distribution. Squares denote the 22-year normalized (p.a.) frequencies for the incremental distribution.

Figure 5.2a As in Figure 5.2 for Collision Zones 25, 26, 27, and 28.

Figure 5.2b As in Figure 5.2 for Collision Zones 29, 30, 31, and 41.

Figure 5.2c As in Figure 5.2 for Collision Zones 47 and 48.

Intracontinental Regions-total

Figure 5.3 Magnitude-frequency plots for Intracontinental Regions-total. Circles, full line and dashed line denote 22-year normalized (p.a.) frequencies, gamma distribution and G-R distribution, respectively, for the cumulative distribution. Squares denote the 22-year normalized (p.a.) frequencies for the incremental distribution.

Figure 5.3a As in Figure 5.3 for Intracontinental Regions 34, 37, 38 and 42.

Figure 5.4 Magnitude-frequency plots for Midoceanic Ridges-total. Circles, full line and dashed line denote 22-year normalized (p.a.) frequencies, gamma distribution and G-R distribution, respectively, for the cumulative distribution. Squares denote the 22-year normalized (p.a.) frequencies for the incremental distribution.

Figure 5.4a As in Figure 5.4 for Ridges 4, 32, 33 and 40.

Figure 5.4b As in Figure 5.4 for Ridges 44 and 45.

Others-total

Figure 5.5 Magnitude-frequency plots for Others-total. Circles, full line and dashed line denote 22-year normalized (p.a.) frequencies, gamma distribution and G-R distribution, respectively, for the cumulative distribution. Squares denote the 22-year normalized (p.a.) frequencies for the incremental distribution.

Figure 5.5a As in Figure 5.5 for Others 2, 3, 9 and 10.

Figure 5.5b As in Figure 5.5 for Others 11, 17 and 39.

Figure 5.6 Magnitude-frequency plots for worldwide. Circles, full line and dashed line denote 22-year normalized (p.a.) frequencies, gamma distribution and G-R distribution, respectively, for the cumulative distribution. Squares denote the 22-year normalized (p.a.) frequencies for the incremental distribution.

Table 5.1 List of seismic zones with F-E code, number of earthquakes n , b values with corresponding standard deviations $\pm\sigma_{LS}$ and $\pm\sigma_{ML}$, corner magnitude M_c , maximum magnitude M_{max} with corresponding standard deviation and maximum observed magnitude M_{max}^{obs} .

FE Code	Name <i>Subduction Zones</i>	n	b	$\pm\sigma_{LS}$	$\pm\sigma_{ML}$	M_c	M_{max}	$\pm\sigma$	M_{max}^{obs}
1	Alaska Aleutina Arc	243	0.93	0.02	0.06	6.9	8.2	0.3	7.9
5	Mexico-Guatemala	160	0.81	0.02	0.06	7.5	8.3	0.3	8.0
6	Central America	152	1.12	0.04	0.09	6.6	8.0	0.4	7.6
7	Caribbean Loop	49	1.14	0.06	0.16	6.2	7.5	0.4	7.1
8	Andean S. America	247	0.82	0.01	0.05	6.6	8.3	0.2	8.1
12	Kermadec-Tonga-Samoa	470	1.14	0.02	0.05	6.9	8.4	0.4	8.0
13	Fiji Island	80	1.41	0.09	0.16	6.8	7.0	0.2	7.0
14	New Hebrides Island	361	0.96	0.02	0.05	6.3	7.8	0.1	7.7
15	Bismarck-Solomon Island	377	0.97	0.01	0.05	6.9	7.8	0.1	7.7
16	New Guinea	208	1.14	0.02	0.08	7.1	9.9	1.7	8.2
18	Guam-Japan	80	0.93	0.06	0.10	7.3	8.2	0.5	7.7
19	Japan-Kamchatka	394	0.98	0.03	0.05	6.3	8.8	0.5	8.3
20	S.E. Japan-Ryukyu Island	60	1.13	0.05	0.15	7.1	7.4	0.3	7.1
21	Taiwan	80	1.06	0.06	0.12	6.3	7.8	0.4	7.4
22	Philippines	252	0.87	0.02	0.05	6.4	7.8	0.1	7.7
23	Borneo-Celebes	270	1.02	0.01	0.06	6.8	8.2	0.3	7.9
24	Sunda Arc	217	0.98	0.04	0.07	6.7	9.2	0.9	8.3
46	Andaman Island-Sumatra	47	1.35	0.08	0.20	6.7	7.3	0.4	6.9
Subduction Zones-total:3747		1.00	0.01	0.02	7.3	8.4	0.1	7.3	
<i>Collision Zones</i>									
25	Burma-S.E. Asia	31	1.03	0.04	0.18	6.1	7.4	0.4	7.0
26	India-Tibet-Szechwan-Yunan	48	0.92	0.04	0.13	6.5	8.0	0.5	7.5
27	S. Sinkiang-Kansu	44	0.91	0.03	0.14	6.0	7.1	0.2	6.9
28	Alma-Ata-Baikar	14	0.91	0.08	0.24	6.1	7.4	0.5	6.9
29	W. Asia	79	0.89	0.06	0.10	6.2	7.6	0.2	7.4
30	Middle East-Crimea-Balkans	73	0.84	0.08	0.10	6.1	7.1	0.1	7.0
31	W. Mediterranean	36	1.31	0.10	0.22	6.1	7.9	0.9	7.1
41	E. Asia	11	1.27	0.16	0.38	6.0	8.8	1.9	7.0
47	Baluchistan	19	1.22	0.18	0.28	6.1	8.3	1.3	7.1
48	Hindu Kush. Pamir	18	0.76	0.06	0.18	6.0	7.7	0.5	7.2
Collision Zones-total: 373		1.04	0.03	0.05	7.1	7.6	0.1	7.5	

Table 1(continued)

FE Code	Name <i>Intracontinental Regions</i>	n	b	$\pm\sigma_{LS}$	$\pm\sigma_{ML}$	M_c	M_{max}	$\pm\sigma$	M_{max}^{obs}
34	E. N. America	7	0.73	0.04	0.28	5.8	8.7	1.5	7.3
35	E. S.America	0							
36	N.W. Europe	0							
37	Africa	52	1.08	0.06	0.15	6.6	7.6	0.4	7.2
38	Australia	11	0.87	0.09	0.26	6.0	6.9	0.3	6.4
42	N.E. Asia-Greenland	9	0.47	0.1	0.16	6.0	6.9	0.2	6.3
49	N. Asia	0							
50	Antarctica	0							
Intracontinental Regions-total:		79	0.85	0.04	0.09	6.6	7.5	0.2	7.3
<i>Midoceans Ridges</i>									
4	Baja Calif.-Gulf Calif.	23	1.34	0.11	0.28	6.2	6.9	0.3	6.6
32	Atlantic Ocean	213	1.21	0.04	0.08	7.0	7.2	0.1	7.1
33	Indian Ocean	181	1.35	0.06	0.10	6.8	8.9	1.2	7.7
40	Arctic	22	1.36	0.16	0.29	6.1	7.1	0.5	6.7
43	S.E. and Antarctic Pacific	206	1.61	0.1	0.11	6.5	6.9	0.1	6.8
44	Galapagos	49	2.26	0.07	0.32	6.2	6.6	0.3	6.4
45	Macquarie Loop	92	1.54	0.1	0.16	6.5	29.8	21.8	8.1
Midoceans Ridges-total:		786	1.46	0.05	0.06	7.1	10.9	2.8	8.1
<i>Others</i>									
2	E. Alaska-Vancouver I.	22	1.05	0.02	0.22	6.3	9.3	1.9	8.0
3	California-Nevada	70	0.82	0.02	0.10	6.7	7.5	0.2	7.3
9	Extreme S. America	25	1.12	0.05	0.22	6.3	6.9	0.3	6.7
10	S. Antilles	130	1.16	0.04	0.10	6.4	7.4	0.2	7.2
11	New Zealand	76	0.73	0.02	0.08	7.2	8.4	0.4	8.0
17	Caroline Island-Guam	43	1.65	0.09	0.25	6.0	6.7	0.2	6.5
39	Pacific Basin	17	1.11	0.05	0.27	6.0	6.9	0.3	6.6
Others-total:		383	0.97	0.01	0.05	7.2	8.2	0.2	8.0
<i>Worldwide</i>		5368	1.06	0.02	0.01	7.8	8.4	0.1	8.3

Table 5.3

As in Table 5.2 for

Collision Zones

Name and

F-E Code

[illegible]

Table 5.4

As in Table 5.2 for

Intracontinental Regions

Name and

F-E Code

E. N. America	34
Africa	× 37
Australia	× × 38
N.E. Asia-Greenland	× ⊗ × 42

Table 5.5
As in Table 5.2 for

Midocean Ridges

*Name and
F-E Code*

Baja Calif.-Gulf Calif.	4
Atlantic Ocean	× 32
Indian Ocean	× × 33
Arctic	× × × 40
S.E. and Antarctic Pacific	× · × × 43
Galapagos	· · · ⊗ ⊗ 44
Macquarie Loop	× × × × × ⊗ 45

Table 5.6
As in Table 5.2 for

Others

*Name and
F-E Code*

E. Alaska-Vancouver I.	2
California-Nevada	× 3
Extreme S. America	× × 9
S. Antilles	× · × 10
New Zealand	× × × · 11
Caroline Island-Guam	× · × × · 17
Pacific Basin	× × × × × × 39

Table 5.7

As in Table 5.2 for

WORLDWIDE**Subduction Zones**× **Collision Zones**× × **Intracontinental Regions*** * * **Midoceanic Ridges**× × × * **Others**Table 5.8 *Barlett's Test for b Value Homogeneity*

Regions Class	k	$\bar{b}$	S	$\alpha\%$
Subduction Zones	18	0.99	18.2	50
Colision Zones	10	1.02	8.1	64
Intracon. Regions	4	0.85	6.7	18
Midoceanic Ridges	7	1.42	19.2	0.4
Others	7	0.98	18.5	0.5
Continental Classes	32	0.98	44.1	9.5

CHAPTER 6

Conclusions

The purpose of this thesis was to define the significance of observed deviations from the Gutenberg-Richter relation the global earthquake recurrence statistics.

The worldwide Harvard Centroid Moment Tensor (CMT) shallow earthquake catalog has been used. The data sample the period from January 1, 1977 to December 31, 1998, and are free of time bias i.e., the catalog with the threshold moment magnitude $M_t=5.6$ is homogeneous (see section 2.1) and it is accepted as a database.

The Flinn-Engdal Regionalisation Scheme (Flinn et al., 1974; Young et al. 1995) based on the seismic zones defined by Gutenberg and Richter (1954) is used in this thesis (see Figure 2.2).

The b values and their standard deviations were calculated for each seismic region, category and worldwide (Table 5.1). The calculated standard deviations, using the LS method, are significantly smaller than the calculated standard deviations using the ML method. However, the differences in values of standard deviation decrease with increasing number of earthquakes.

The differences in b value between cumulative and incremental distributions in the magnitude range $M_t \leq M \leq M_c$ are negligible. This agrees with Riznichenko's (1964) proof that, if the summation for cumulative distribution starts at a value of $m_{\max}^{\text{obs}}$, cumulative and incremental distribution become parallel only after three summation

steps. The difference in the slopes is of the order of 5% or less. In the CMT catalog the above proof is in agreement with the data in the magnitude range $M_t \leq M \leq M_c$ i.e. the differences between $M_{\max}^{\text{obs}}$ and M_c have values between 1.2 and 0.5 except for midoceanic ridges.

The difference between the b value for the cumulative distribution in the magnitude range $M_t \leq M \leq M_c$ and that for the incremental distribution in the magnitude range $M_t \leq M \leq M_{\max}^{\text{obs}}$ for worldwide data is negligible i.e. the difference is in the range of standard deviation (0.03). Pacheco and Sykes (1992) calculated the b value of CMT catalog (for the period 1977-1990) in the magnitude range $6.0 \leq M \leq 7.5$ and $7.6 \leq M \leq 8.0$ for the cumulative distribution and obtained $b=1.07$ and $b=1.60$ respectively, but that is only a statistical artifact. Pacheco and Sykes (1992) did not take into consideration the incremental distribution.

Statistical analysis (Table 5.2-5.8) of the magnitude distribution indicates that two separate populations of earthquakes have significantly different statistical properties: midoceanic ridges events and earthquakes on the continents and their boundaries. For continental regions (we consider subduction zones, collision zones and intracontinental region as a single continental region with continental boundaries) the b value is 0.98 ± 0.01 . For ridge events the b value is 1.46 ± 0.05 .

Similar results from global analysis of b values were obtained by Kagan (1991,1997), Frolich and Davis (1993), Okal and Romanowicz (1994) and Godano and Pingue (2000) using the CMT catalog (shallow earthquakes) with different threshold magnitudes and different time periods. The significant differences are only in the values of standard deviations of b values.

The statistical straightforward procedure for estimating the maximum regional magnitude $M_{\max}$ was used to determine a parameter of the cumulative gamma distribution. It is assumed that both the analytical form and the parameters of the distribution function of earthquake magnitude are known. The largest observed magnitude $M_{\max}^{\text{obs}}$ plays a crucial role in the procedure (equation 4.19 and Table 5.1). For ridge events the b value is 1.46 ± 0.05 and the maximum magnitude cannot be determined by the methods employed in this thesis. It is possible that only the exponentially decaying part of the cumulative distribution is observed for ridge events (Figure 5.4, 5.4a, 5.4b). This would explain the large estimate of b value (1.46). However, the maximum magnitude should depend on the size of the volume available for brittle fracture (Sholz, 1990). Thus, the moment magnitude $M=8.1$ of historically observed events in the midoceanic ridges (Appendix 1) seems plausible as a maximum magnitude.

The gamma distribution describes well the behavior of earthquakes worldwide in the magnitude range $M_c \leq M \leq M_{\max}$, except in midoceanic ridges.

The results can be summarized as follows:

- The falling off of cumulative frequency at large magnitudes is not observed in the incremental distribution.
- Regional differences in the Gutenberg-Richter relation for worldwide shallow earthquakes, on the base of CMT catalog, were seen only in the midoceanic ridges.
- The parameter b of the Gutenberg –Richter relation has a universal value 0.98 ± 0.01 for continents and their boundaries.

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