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SOIL-STRUCTURE INTERACTION IN DISCONTINUOUS SHEAR ZONES

by

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B.Sc.(Hons.), Queen's University, Kingston, Ontario

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A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

in

Engineering

Department of Civil Engineering

Carleton University

Ottawa, Ontario, Canada

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A.P.S. Selvadurai

Carleton University

April, 1989

dedicated to my dear parents

Abstract

Three dimensional non-linear finite element techniques were utilized to investigate the class of soil-structure interaction problems involving embedded structural elements subjected to discontinuous shear displacements. The Drucker-Prager and the modified Cam-Clay constitutive laws were employed to simulate elasto-plastic behaviour of geotechnical materials. Simple experimental modelling techniques were incorporated to provide the means for the understanding of the effects of dislocation movements on embedded structural elements. Strength and deformation characteristics of a sand-bentonite buffer mixture were determined by conventional geotechnical laboratory tests. These material parameters were later used for the implementation of numerical techniques to solve the container-buffer-rockmass interaction problem within the context of the Canadian nuclear waste disposal endeavours. Parametric studies and comparisons with existing numerical and analytical results were made for typical two dimensional problems. The developed numerical techniques were used to investigate three dimensional problems involving buried pipelines and laterally loaded piles.

Keywords

soil-structure interaction, three dimensional, finite element analysis, soil non - linearity, constitutive modelling, scaled model experiments, buffer material, nuclear waste disposal, buried pipelines, laterally loaded piles.

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List of Symbols

SYMBOL	DESCRIPTION
c	cohesion
c_p	post peak cohesion
D	average diameter
d	depth of embedment
E	Young's modulus
E_b	Young's modulus of buffer
E_c	Young's modulus of container
E_p	Young's modulus of pipe or pile
E_s	Young's modulus of soil
E_{strip}	Young's modulus of strip
E_{tube}	Young's modulus of tube
E_{ij}	deviatoric strain tensor
e_0	current void ratio
F	yield function
G	shear modulus
G_i	interface shear modulus
H	length of waste disposal vault
h	distance from top of buffer
I_p	moment of inertia of pipe
K	bulk modulus
L	length of strip or tube
i	distance from bottom of strip or tube
M	slope of critical state line on q-p plot
M_{max}	maximum bending moment
m_{opt}	optimum moisture content

N_i	shape function value at node i
P	applied concentrated load
P_v, P_h	vertical and horizontal concentrated load
p	mean hydrostatic stress
p_h, p_r	hoop and radial stress
S_{ij}	deviatoric stress tensor
q	deviatoric shear stress
R^*	relative stiffness parameter
r	radius of cavity
t	thickness of lining (or shell)
t_i	thickness of shell at node i
u^c	radial displacement at crown
u^s	radial displacement at springline
$[B]$	strain-displacement matrix (global axes)
$[B']$	strain-displacement matrix (local axes)
$[D]$	elastic constitutive matrix (global axes)
$[D']$	elastic constitutive matrix (local axes)
$[D^{ep}]$	elastic-plastic constitutive matrix
$[d]$	generic displacement vector
$[J]$	Jacobian matrix
$[J]^{-1}$	inverse of Jacobian matrix
$[K]$	stiffness matrix
$[K_o]$	original element stiffness matrix
$[K_r]$	reduced element stiffness matrix
$[M_o]$	moment vector
$[N]$	shape functions matrix
$[P_o]$	translational load vector
$[P_o^o]$	original load vector
$[P_o^r]$	reduced load vector
$[T]$	transformation matrix

$[U_0^0]$	original displacement vector
$[u]$	continuous displacement vector
α, k	material constants of Drucker-Prager model
$\gamma_{d(max)}$	maximum dry unit weight
Δ	induced displacement
Δ_0	maximum induced displacement
Δ_p	induced plastic displacement
Δ_{max}	maximum induced plastic displacement
ϵ_{ij}	strain tensor
ϵ_{zz}	longitudinal strain
$d\epsilon_{ij}^p$	incremental plastic strain vector
κ	slope of isotropic recompression line
λ	slope of virgin compression line
ν	Poisson's ratio
ν_p	Poisson's ratio of pipe or pile
ν_s	Poisson's ratio of soil
ϕ	friction angle
ϕ_p	post peak friction angle
σ_{ij}	stress tensor
σ_{rr}	radial stress
σ_{zz}	longitudinal stress
$\sigma_{\theta\theta}$	tangential stress
dv^p	incremental plastic volumetric strain
$[\delta_s]$	translational displacement vector
$[\delta_s^r]$	reduced displacement vector
$[\epsilon]$	strain vector (global axes)
$[\epsilon']$	strain vector (local axes)
$[\sigma]$	stress vector (global axes)
$[\sigma']$	stress vector (local axes)
$[\theta_s]$	rotational displacement vector

Chapter 1

Introduction

1.1 General

The category of problems which deals with the interaction between embedded foundations and structures such as piles, casings, pipelines, tunnels, reinforcements in soil retaining structures and the soil media constitutes an important aspect of geotechnical engineering. In such problems, the combined response of the soil-structure system is governed by the individual mechanical responses of the structural element and the constitutive responses of the soil and the soil-structure interface. Recent interests in the behaviour of structural elements, located in either discontinuously deforming soil media or soil regions subjected to intense shear deformations, arise from applications of soil-structure interaction analyses to geotechnical problems encountered in energy resources development and transportation, nuclear waste management, slope stabilization by dowels and earth reinforcements by flexible strips.

Pipelines are extensively used for the transportation of recovered energy resources such as heated oil, natural gas and slurried coal (Ariman, 1970 ; Pickell, 1983; Selvadurai and Lee, 1981). Such pipelines traverse long distances where variability in soil and environmental conditions can threaten their integrity. For example, soil-pipe interaction of an adverse nature can be induced in soil regions which are subjected to ground swelling by frost heave, soil consolidation and ground movements due to external loading, soil creep or excavation (Esparzo, 1985; Ladanyi, 1984; Nixon et al., 1983; Selvadurai, 1985a). The occurrence of active faults along the route of a pipeline can initiate discontinuous movements along its length (Fig. 1.1). An evaluation of the flexural stresses that are induced at the transition zone is therefore important to the assessment of the structural integrity of the pipeline.

Figure 1.1 : Buried Pipeline Subjected to Fault Displacement

Pipelines buried at shallow depths can be subjected to surface loadings. The prediction of the induced flexural stresses and deformations are therefore necessary to provide design guidelines which include the adequate depth of burial.

The response of borehole casings which are subjected to lateral soil displacement is of interest to both offshore and on-shore energy resources exploration. Drilling or production casings embedded within caisson-retained islands in the Beaufort Sea are being frequently damaged by lateral ice loadings exerted on the outer steel caisson (Byrne et al., 1984). Significant stresses in production well casings can also be developed as a result of either ground compaction or ground consolidation due to the withdrawal of energy resources such as oil and natural gas. Deflections, shear forces and bending moments induced in these borehole casings should be accurately determined such that adequate remedial measures or modifications of the overall design may be accomplished.

The development of containment technologies for the long term storage and disposal of irradiated fuel is an area of research which is of crucial importance to the nuclear energy industry. The Fuel Waste Technology Branch of Atomic Energy of Canada Limited (AECL) is the lead agency in the development of the necessary containment technologies (Hancox, 1986). The Canadian proposals for the disposal scheme specifically concentrates on the deep burial (approximately at 1000m in depth) of the waste in specially constructed vaults in plutonic rocks of the Canadian Shield. The engineered multi-barrier concept constitutes a key feature in the waste disposal effort. Current proposals for the disposal facility involve four principal components, which include the vault system, the waste container, the backfill material and the buffer material. The functional requirements of the various components of the system are fully documented in the literature (see e.g. Selvadurai, 1988a,b). The buffer material which fills the space between the waste container and the borehole walls is a mixture of bentonitic clay and sand which should fulfill certain functional criteria. It should act as a barrier to suppress the detrimental effects of the corrosive water in the host rock and enhance the life of the waste

container; serve as a geochemical filter for the sorption of radionuclides released either by accidental damage or natural breakdown of the waste containers; provide the mechanical strength to support the waste containers; isolate the containers from detrimental rockmass movements; act as a medium for conducting the radiogenic heat and it should be capable of swelling under fluid influx to seal gaps, cavities etc., that may be developed by moisture depletion during the heat and moisture transfer processes.

The plasticity of the buffer region is an important aspect particularly in relation to the isolation of the waste container from detrimental rockmass movements. Such rock movements can take place either due to the thermal loading of the repository area induced by the stored waste, removal of ground water from the repository area, earthquake effects or by stress relief due to the creation of the repository. Estimates indicate that rockmass movements due to thermal loadings can be of the order of 2 to 5 cm. Such displacements can occur in a discontinuous fashion when fractures are generated and/or reactivated by the thermal loadings. In the particular instance when a fracture plane intersects the borehole, a dislocation type rock movement can prematurely threaten the integrity of the waste container (Fig.1.2). Furthermore, the experimental investigations conducted in connection with heat and moisture migration in the buffer in the absence of moisture influx (Selvadurai, 1988a,b) indicates that loss of moisture in the buffer region leads to a severe loss of its 'plasticity' properties. Therefore, a discontinuous movement that can take place under conditions with no moisture influx into the buffer region but moisture loss due to heating, can present an adverse environment for the waste container. Since it is not practically feasible to stabilize the rock surfaces to prevent such unpredictable occurrence of fracture, the interaction behaviour between the buffer and the container must be investigated for possible movements along fracture planes.

Soil reinforcement techniques have been used extensively. Since its first introduction, the reinforced earth technique, which utilizes flexible reinforcing strips,

Figure.1.2 : Possible Orientation of Discontinuity Across Nuclear Waste Disposal Vault

provides an economical and efficient solution to soil retaining problems (Ingold, 1982). In addition, the notion of soil nailing has been used to describe a technique of reinforcing in-situ soil by stiff bars or piles (Sommer, 1979; Juran et al., 1983). This technique has been widely used in conjunction with soil retaining structures and for slope stabilization. In a nailed soil wall, the grouted bars are able to sustain tensile forces, shear forces and bending moments. This type of soil-reinforcement interaction depends on factors such as the relative rigidity of the reinforcements, their orientation with respect to a potential failure surface, the mechanical properties of the soil and of the reinforcements etc. Gudehus and Schwarz (1985) reported that dowels (piles) had been successfully applied to stabilize a creeping slope. It is not uncommon to encounter a situation in which a pile or a casing is located in a layered soil system whereby the upper stratum is displaced laterally with respect to the stiffer lower stratum. The movement may be triggered by factors such as earthquake effects or soil creep. In a similar fashion, a foundation pile embedded in a layered soil system can be subjected to lateral loading (Fig.1.3). Thus it is important to investigate the interaction behaviour between the reinforcing element and the surrounding soil.

The focus of this research work initiates from the importance of these soil-structure interaction problems to geotechnical engineering. An examination of the geotechnical literature indicates that these categories of soil-structure interaction problems have received limited analytical and experimental treatment. Most theoretical treatments utilize idealized soil models such as Winkler type spring models to simplify what is otherwise a very complex soil-structure interaction problem. Experimental results are limited in scope and comparisons are usually made with either the Winkler type model or two dimensional plane strain finite element analyses. Full scale field tests are often too site specific and only empirical relationships are derived from the test results.

The actual mechanisms of the interaction including the effects of separation and the propagation of plastic zones have not been investigated. Also, three dimensional

Figure 1.3 : Pile Subjected to Lateral Load in a Layered Soil System

effects are generally ignored. In view of these observations, a research programme was established to investigate the interaction behaviour between the soil media and the structural elements subjected to dislocation movements and lateral loadings. It is hoped that the results of the research can contribute to the understanding of the relevant geotechnical influences on structural elements which are located in discontinuous shear zones.

1.2 Literature Review

In the geotechnical literature, the publications which deal with the problems under investigation can be broadly classified into two categories, namely (i) theoretical, (ii) experimental and field studies.

1.2.1 Theoretical Studies

In order to examine the soil-structure interaction problems, it is necessary to model the mechanical response of the soil. For instance, in the case of a buried pipeline problem, the mechanical response of the soil is influenced by factors including the soil type, the variability of the soil properties along the route, the condition of the soil-pipeline interface etc. In view of these complexities, researchers employ simplified models to account for the soil behaviour. One such form of idealization is to consider the soil mass surrounding the buried structure as an elastic region. The elastic model serves as a useful approximation for the mathematical analysis of several complex soil-pipeline interaction problems. The analytical schemes that may be used for the analysis can be broadly classified into analytical methods and numerical methods such as the finite and boundary element methods. Instead of solving complex three dimensional equations, a simplified modelling of soil-structure interaction may be pursued. This method concentrates on the development of a mathematical relationship for the load versus the displacement.

The most common method used in these simplified analytical analyses is the Winkler type foundation model which treats the soil as a series of unconnected linear elastic springs to which the structural element is attached. The orientation

of the springs depends on the particular nature of the problem. In the case where an axially loaded pile is subjected to lateral ground displacement effects, the springs are incorporated both vertically and horizontally. The differential equation governing the flexural behaviour of the structural element is given by (see e.g. Hetenyi, 1946; Selvadurai, 1979)

$$EI \frac{d^4 y}{dx^4} + ky = P(x) \quad (1.1)$$

where

EI = flexural rigidity of the structural element.

y = deflection.

k = modulus of subgrade reaction.

$P(x)$ = external load. (i.e. $P(x) = 0$ for no external load)

This equation can be solved analytically or numerically using finite difference, finite element or boundary element techniques in order to determine bending moments, shear forces and deflections. The non-linearity of the soil response could be accounted for by an elasto-plastic Winkler model (Madhav et al, 1971), a hyperbolic Winkler model (Soldatos and Selvadurai, 1985) or the more popular 'p-y' approach (Reese, 1977) developed for piles. In the method proposed by Reese (1977), finite difference techniques were used to solve the governing differential equation which also included the input of axial load. A series of so called 'p-y' curves were the input for various points along the pile and an iterative procedure was used to obtain compatible values of p and y at all points. The major disadvantages of the Winkler type formulation are that there is a lack of continuity in the response of the soil and that ambiguity exists in the description of the value of k .

Selvadurai (1985a) considered a Winkler type foundation for a soil-pipeline interaction problem as shown in Fig.1.4. The assumptions were that (i) the pipe was of infinite length, (ii) the relative ground displacement was the only known quantity, (iii) the modulus of subgrade reaction, k , remained constant along the

pipeline, (iv) the pipe underwent rigid body movement (i.e. no stresses developed) away from the transition zone. The following analytical results were obtained.

$$y_R(x) = \Delta \left[1 - \frac{e^{-\lambda x}}{2} \cos(\lambda x) \right] \quad (1.2)$$

$$y_L(x) = \frac{\Delta}{2} e^{\lambda x} \cos(\lambda x) \quad (1.3)$$

$$M_R(x) = EI \Delta \lambda^2 e^{-\lambda x} \sin(\lambda x) \quad (1.4)$$

$$M_L(x) = EI \Delta \lambda^2 e^{\lambda x} \sin(\lambda x) \quad (1.5)$$

where

$$\lambda^4 = \frac{k}{4EI}$$

$y_R(x)$ = deflection for $x \geq 0$.

$y_L(x)$ = deflection for $x \leq 0$.

$M_R(x)$ = bending moment for $x \geq 0$.

$M_L(x)$ = bending moment for $x \leq 0$.

The accuracy of the above quantities is dependent upon the accuracy with which the value of k can be estimated for the buried soil-pipeline system. In general, the estimation of k will depend upon (i) the flexural rigidity of the pipeline, (ii) the depth of location of the pipeline, (iii) the elastic parameters E_s and ν_s of the soil surrounding the pipeline and (iv) the type of loading on the pipeline. In the situation where $E_s/E_p = 0.01$ (E_p is the Young's modulus of the pipe), the Winkler type response closely models the continuum response. The parameter k essentially depends on the elastic constants E_s and ν_s of the soil medium surrounding the pipeline.

Selvadurai (1985a) presented the following approximate relationships. For near surface pipelines

$$k = \frac{0.65E}{(1 - \nu^2)} \quad (1.6a)$$

and for deeply buried pipelines

$$k = \frac{6E(1 + \nu)}{(1 - \nu)(3 - 4\nu)} \quad (1.6b)$$

Figure 1.4 : Pipeline Traversing a Ground Subsidence Zone

which could be employed to define the value of k in terms of the continuum type linear elastic properties E_s and ν_s of the soil.

Finite element and boundary element techniques have been applied extensively for the analysis of soil-foundation interaction problems (see e.g. Banerjee and Davies, 1978; Desai and Christian, 1977 etc.) Selvadurai (1985b) implemented a combined finite element-boundary element formulation to study soil-pipeline interaction during fault movement (Fig.1.1). The assumptions were that (i) the pipeline was modelled as a structural element which experienced only flexure during the interaction, (ii) the flexural response was modelled by a conventional elastic beam element, (iii) the elastic soil medium surrounding the pipe was represented by boundary elements. Numerical analyses showed the influence of the flexibility of the pipe on its displacement and the manner in which the maximum bending moment varied with the relative rigidity of the pipe.

Nixon et al. (1983) considered a problem involving the traversing of a pipeline from frozen to unfrozen ground or shallow permafrost which would result in a differential movement due to frost heave. The soil was considered to be either an elastic continuum or a non-linear viscous continuum and the frost heave was dependent on the applied pressure. The pipe was assumed to be a completely passive structural element with infinite length. Again these authors carried out finite element analyses which were restricted to a two dimensional model of the problem.

The Swedish concept of nuclear waste disposal is largely similar to that of the Canadian concept. The major difference is that the Swedish method uses bentonitic clay alone as the buffer material while the Canadian proposal advocates the use of a sand-bentonite mixture. Borgesson (1986) investigated the effects of rock displacement across a radioactive waste deposition hole by means of the finite element method. Linear elastic three dimensional finite element analyses were carried out to simulate a full scale deposition hole subjected to shear displacement. The radius of the container was set at 0.35m, the radius of the deposition hole at 0.70m and

the height of the container at 4.7m. A shear deformation of 0.01m was simulated by letting the upper part of the boundary move horizontally. Elastic parameters corresponding to that of copper were used for the container. The results showed that the highest stresses induced in the container were those along the longitudinal direction. High stress concentrations were also observed at the surface of the container.

Furthermore, non-linear two dimensional finite element analyses were carried out to investigate the influence of the non-linear behaviour of copper and bentonite. The non-linear behaviour was modelled by a simple bi-linear stress-strain curve. The maximum stress obtained in the non-linear two dimensional calculation was only one third that of the corresponding value from the elastic three dimensional calculation. However, the maximum stress obtained from a linear elastic two dimensional analysis exceeded that of the corresponding value in a non-linear calculation. The inconsistencies in the above mentioned results seem to suggest that more reliable results can only be reached by means of non-linear three dimensional analyses.

Budhu and Davies (1987) presented the results of a boundary element analysis of single laterally loaded piles embedded in cohesionless soils by taking soil yielding into consideration. The analysis provided an independent alternative to the well known p-y method. The analysis required only conventional soil data, namely the angle of internal friction and a parameter characterizing the rate of increase of Young's modulus with depth. Results of parametric studies showed clearly that soil yielding greatly increased the displacements, rotations and bending moments of laterally loaded piles. The applicability of these results to design practice was evidenced by good agreement between the predictions and published case histories.

Byrne et al. (1984) described a modification of the original Winkler type formulation to take into account the so called 'free-field' soil displacements. The responses of piles and casings embedded within caisson-retained islands in the Beaufort Sea subjected to horizontal ice loadings were investigated. The results indicated that

flexible casings essentially tracked with the soil and were subjected to relatively low shear forces and bending moments, whereas stiff casings deflected as a rigid body and experienced large shear forces and bending moments.

Alternatively, the soil could be treated as an ideal elastic continuum on which the structure, considered as a beam, was rested. The model accounted for the continuous nature of the soil (Poulos, 1971; Poulos, 1972). The non-linear yielding of the soil was taken into account by an iterative solution procedure. However, at the regions where the soil had not yielded (N.B. soil usually yields right from the start of loading), the displacements were still governed by the elastic theory. Soil-pile separation was considered by a further iteration process until no resulting net tensile stress existed in the soil surrounding the pile. The major disadvantages of this approach are that the frictional force at the interface is assumed to be zero and that there is difficulty in determining the relevant soil moduli.

1.2.2 Experimental and Field Studies

The last section provides an overview of the analytical and numerical treatments of the various soil-structure interaction problems. However, the validity and accuracy of such theoretical modelling can be established only by recourse to either model or full scale experimentation. It is not unusual to conduct suitable laboratory experiments prior to considering full scale field studies. In most cases, field tests are both time consuming and expensive. Nevertheless, it has been realized that laboratory experimentation can never substitute for full scale prototype field test. This section provides an overview of the documented experimental and field studies that are relevant to the present scope of investigation.

Williams (1986) discussed a joint programme between Canada and France with regard to the investigation of frost heave effects on a long distance buried pipeline. In a controlled-environment laboratory in France, a 16m length of pipe 27.3cm in diameter was buried in $250m^3$ of soil. In this particular experiment, the soil was made 'uniform' in all respects except that there would be two types of soils

expected to heave at different rates under a known, steady and cold temperature. Since the soils and the temperatures were thoroughly known, it was possible to find relationships between the rate and amount of frost heave and the characteristics of the soils. It was known that the differential heaving of the soils constituted the bending of the pipe. Although this project was expected to provide valuable information as to (i) how much differential heave could the pipeline tolerate and (ii) how often would the design limits be exceeded, necessitating expensive remedial measures and so on, the project was supposed to last five years bearing a cost of over a million dollars. It is clear that engineers and scientists working in the pipeline industry cannot depend entirely on this type of exhaustive experiments. Cheaper and simpler mathematical formulations should be utilized to provide the necessary information for such problems.

Ladanyi et al. (1984) presented the results of observations made during the first phase of operation of a large-scale model of a chilled pipeline buried in two different types of soils. This task was a part of the joint Canada-France project discussed earlier to study freezing around a refrigerated pipeline in a controlled environment facility in Caen, France. Along the length of the pipe, half of the pit contained a slightly frost susceptible sand while the other half contained a very frost susceptible silty soil. The pipe was refrigerated to -2°C , and the ambient chamber temperature was held slightly below zero. The pipe and the two soils were heavily instrumented to monitor thermal and moisture regime, ground heave, pipe deformations and displacements. The experimental results led to the conclusion that for a buried gas pipeline, subjected to freezing temperatures, the simulation of the mechanical interaction between the soil and the pipe presented a major difficulty, i.e. the evaluation of the resistance of the medium. It is difficult to adapt the present knowledge of frozen soil mechanics to a heterogeneous medium whose properties vary with time. A finite element analysis is needed to model this complex problem. In spite of the favourable comparisons with the results obtained from the tests in Caen, the model experiments presented in this paper, which simplifies the problem to the extreme, represents only a first step towards a general solution.

Trautmann et al (1985a, 1985b) reported the results of some plane strain experiments on (i) the uplift force - displacement responses and (ii) the lateral force-displacement responses of buried pipes. Variables such as soil density, depth of burial of pipe and pipe roughness etc. were investigated. A distinct shear plane was not introduced in the experimental set-up although analytical treatments of such cases were mentioned in the literature review. The experimental results were compared with the results obtained from existing theoretical and empirical approaches and design procedures were presented in each of the cases considered.

Borgesson et al. (1986) reported a series of model tests involving discontinuity across a deposition borehole in the Swedish nuclear waste disposal research programme. The model was scaled 1 to 10 to a prototype hole. The hole was filled with a container made of solid copper surrounded by a highly compacted water saturated bentonite. Before shear, the swelling pressure was measured by transducers in order to monitor the water uptake process. During shear, pressure, strain, force and deformation were measured at various points. Extensive samplings after shear were made through which the density, water content, degree of saturation and the effect of shear on the bentonite and the container could be studied. However, the model tests failed to address the long standing problem of scale effects. In addition, in-situ compaction of the bentonite in the prototype deposition hole could produce strength and deformability characteristics which differ from those observed in the model tests.

Juran et al (1983) reported research results including both theoretical and experimental studies on the mechanism of interaction between the soil and the reinforcing elements during direct shearing of a nailed soil mass. The finite element technique was used to analyse the direct shear tests on samples of a silty soil reinforced by bars placed perpendicularly to the failure surface. This two dimensional analysis replaced a row of bars by an equivalent plate considering plane strain conditions. The plate and the soil were assumed to satisfy the von Mises and the Mohr-Coulomb failure criteria respectively. The interfaces were assumed to be

smooth and separation was not considered. Friction elements were used to simulate the potential failure surface in the soil between the lower and upper parts of the shear box. The analysis was able to demonstrate the mobilization of an apparent cohesion in the nailed soil mass. It also investigated the relative influences of different parameters such as the normal stress at the failure surface and the rigidity of the reinforcing elements on the efficiency of the reinforcement, on the behaviour of the nailed soil mass and on the mobilization of its overall shearing resistance.

Gudehus and Schwarz (1985) reported that creeping slopes in stiff clay could be stabilized by using dowels (piles) made of concrete or steel. The dowels transmitted the stabilizing force from the underlying stratum to the creeping soil. The lateral load on the dowel was assumed to increase linearly with the displacement up to a maximum value. The design of the stabilization scheme was carried out assuming the dowels as elastic beams with a constant coefficient of lateral subgrade reaction.

Gassler et al. (1981) dealt with some mechanical aspects of soil nailing at nearly vertical cuts in cohesionless soils. Several large scale field tests were described and various failure mechanisms were analysed. Earth pressures and displacement data were used to establish some design guidelines in standard cases.

1.3 Scope of the Research

Current approaches had been reviewed in the previous sections. The Winkler type formulation and its modifications were extensively used in theoretical considerations. Finite element analyses were concentrated on the two dimensional plane strain formulation. Three dimensional effects of the problems were largely limited to linear elastic analysis. Other aspects of the problem such as the non-linear characteristics of soils and the occurrence of separation along the soil-structure interface were treated only superficially. The use of constitutive models developed for soils was scarce. Features such as the influences of a propagating plastic zone and the orientation of the plane of discontinuity were rarely considered in the existing literature.

In the present doctoral thesis research programme, an attempt is made to model, both numerically and experimentally, the problems in which a structural element located at a discontinuous soil region is subjected to differential lateral displacements. The research project can be divided into the following three phases : (i) constitutive modelling of the soil media, (ii) experimental modelling of shear displacement problems and (iii) implementation of numerical models to solve soil-structure interaction problems.

1.3.1 Constitutive Modelling

The accurate characterization of the mechanical behaviour of the soil medium is a necessary first step in any attempt to solve complex soil-structure interaction problems, such as those involving embedded structural elements considered in the present study. A soil can be modelled using an appropriate constitutive model. A constitutive model is a mathematical model that describes the behaviour of a given material. Many of these models are based on fundamental concepts such as the theory of elasticity and the theory of plasticity. In plasticity models, both the yield criterion and the post-yield behaviour must be considered. Classical constitutive models that are still in use include von Mises, Tresca, Mohr-Coulomb and Drucker-Prager. In order to take into account other features such as volume change, strain hardening and softening behaviour, there is an increase in the use of sophisticated plasticity models such as the Critical State and Cap models. In such models, progressive yielding of the soil is defined by the continuous expansion of the so called hardening caps. The variable moduli model (quasilinear model) and the linear elastic model are also available.

The selection process of a suitable constitutive model for a given soil can consist of the following steps (Desai and Siriwardane, 1984).

- 1) mathematical formulation.
- 2) identification of significant parameters.
- 3) determination of parameters from appropriate soil tests.

- 4) verification with respect to laboratory test data under stress paths other than those used for parameter determination.
- 5) application to the solving of boundary value problems.

In the soil characterization programme, standard soil mechanics laboratory tests such as the direct shear and the triaxial test are carried out on the selected soils under consideration. Shear strength parameters such as cohesion, angle of internal friction and deformability characteristics such as the modulus of elasticity and the Poisson's ratio can be determined by the triaxial compression tests. The direct shear test provides data for an alternative stress path. In the present research, results of direct shear tests are compared with results obtained from triaxial tests. In some cases where the constitutive parameters cannot be directly determined, an estimation based on other information such as published 'typical' values can be made.

1.3.2 Numerical Implementation

As mentioned before, a majority of problems in geotechnical engineering, including those within the present scope of investigation, are three dimensional in nature. Behaviour of soil is usually highly non-linear and poses a difficult problem for the analyst. This can be handled by using idealized mathematical models and efficient computational procedures. In this research programme, the finite element code SSTIN-3D constitutes the starting point. This code was developed under the direction of Professor C.S. Desai and it is capable of examining three dimensional non-linear soil-structure interaction problems (Siriwardane and Desai, 1983; Siriwardane, 1980). A general shell element based on the isoparametric concept has been chosen to model the structural element in this thesis. Relevant computer codes are developed such that this element can be implemented in conjunction with the already existing 20-node 'brick' element.

The problems of structural elements located in discontinuously deforming soil regions involve geometric, material and interface non-linearities. In the present

study, only non-linearities due to material behaviour and interface conditions are considered. Material non-linearity is due to several factors such as the initial state of stress; changes in the physical state at the rupture zone and interfaces as defined by alterations of density, moisture content or void ratio; possible dilatant effects of dense and overconsolidated soils; stress path effects; hardening and softening behaviour etc.

From the procedures outlined previously, an appropriate constitutive law may be chosen to model a particular soil. The structural element may be modelled by the linear elastic law or the von Mises criterion which was first developed for metals. 'Thin' interface elements (Desai et al., 1984) are used to model the interaction behaviour between the structural element and the soil.

1.3.3 Experimental Modelling

In order to model experimentally the soil-structure interaction problems, a reduced scale model was designed and fabricated. This model is essentially a rigid cylinder with a horizontal discontinuity at mid-section. To provide meaningful results for comparisons with numerical stress analysis results, restrictions and assumptions of the model apparatus are considered. The influence of side friction of the model container are taken into account.

Displacement tests are carried out in conjunction with instrumented model structural elements which include (i) hollow pipes to model piles, pipes or casings, (ii) metal strips to provide qualitative information on the flexural behaviour. Concrete sand is selected as the soil for testing primarily due to its wide range of particle size distribution which can contribute to the attainment of higher density after compaction. Details of the model design and the experiments can be found in Chapter 5.

1.3.4 Objectives

The main objective of this doctoral thesis research programme is to investigate both experimentally and numerically the soil-structure interaction problems

involving structural elements subjected to discontinuous shearing displacements and lateral loads. The numerical analyses are carried out primarily through the implementation of non-linear three dimensional finite element techniques. The method used is different from current approaches as discussed above.

- 1) Verification of the finite element code by means of comparisons with existing analytical and numerical solutions as well as parametric studies of typical problems.
- 2) Soil properties characterization using conventional geotechnical laboratory tests in order to determine the constitutive parameters to be used in conjunction with the constitutive models.
- 3) Investigation of the flexural responses of the instrumented structural elements subjected to lateral shear displacements by means of reduced scale model tests.
- 4) Application of the developed non-linear three dimensional finite element techniques to analyse various soil-structure interaction problems which include (i) buried pipelines subjected to external displacements and loadings, (ii) nuclear waste disposal vaults subjected to lateral rock dislocation movements and (iii) piles in homogeneous and layered soil strata under lateral loadings.

1.4 Summaries of Chapters

Chapter 2 deals with the fundamental theories of solid mechanics and the finite element implementation procedures on which this research is based. This chapter starts with the basic definitions of the stress and strain invariants. Then the formulations of the constitutive models, namely Drucker-Prager and Critical State models, are discussed. This is followed by the description of the general shell element, the 20-node continuum element and the interface element used in the finite element procedures. The chapter concludes with the discussion of the coupling between the shell and the solid element.

Chapter 3 concentrates on the verification of the validity of the computer pro-

gramme for solving the soil-structure interaction problems under investigation. The reinforced hole-in-infinite-plate problem subjected to 'far field' loading is solved by the conventional Airy's stress function approach and the finite element approach after which comparisons are made. Parametric studies are then conducted on several two dimensional problems such as a shallow buried tube subjected to uniform overburden pressure and a deeply buried tube subjected to unequal vertical and horizontal concentrated loads. A three dimensional example of a buried pipeline is also investigated.

Chapter 4 discusses a series of geotechnical laboratory tests on the buffer material chosen for use in the Canadian Nuclear Waste Disposal Programme. Grain size distribution analyses, compaction tests, direct shear and triaxial tests are conducted on the sand-bentonite mixture. Constitutive parameters are then determined from these test results for use in the finite element analysis of the container-buffer-rockmass interaction problem.

Chapter 5 focusses on the laboratory simulation of an embedded structural element subjected to discontinuous shearing. A 'split cylinder' apparatus is specially designed for this investigation. The shear experiments are conducted in conjunction with instrumented aluminium strip and tube. The experimental results are compared with numerical results obtained from three dimensional finite element modelling of the experiments.

The next three chapters concentrate on three dimensional linear and non-linear finite element analyses on several important soil-structure interaction problems involving buried pipelines, nuclear waste disposal vault and laterally loaded piles. Chapter 6 deals with long distance buried pipelines subjected to discontinuous fault movement and surface loadings. Chapter 7 focusses on the problem concerning a transverse rock dislocation movement across a waste disposal borehole. Laboratory determined and estimated material parameters are used. Chapter 8 discusses a series of parametric studies on a laterally loaded single pile embedded in homogeneous and layered soils. In all three chapters, the effects of non-linearity and the

relative stiffness between the structure and the soil are emphasized. The induced stresses and deformations on the embedded structural elements are also considered.

Finally, Chapter 9 provides a summary of the numerical and experimental results and a number of conclusions drawn from the research programme.

Chapter 2

Basic Theories and Numerical Formulations

2.1 General

In this thesis, finite element techniques are employed to examine the various classes of soil-structure interaction problems. The non-linear constitutive behaviour of the soil medium is modelled via the Drucker-Prager and Critical State models. The structural elements are assumed to exhibit a linear elastic response. In the finite element adaptation, the isoparametric solid elements which represent the soil medium are coupled with shell elements which model the structural elements. The continuity between the soil and the shell is assumed to be in fully bonded conditions. In certain situations, the interface is modelled via special interface elements having elastic and plastic responses. This chapter presents a brief overview of the mathematical formulations associated with the elastic and elastic-plastic modelling and gives a description of the finite element procedures which lead to the coupling of the shell element and the non-linear soil continuum element.

2.2 Basic Definitions

The behaviour of the soil media and structural elements encountered in this thesis is based on conventional solid mechanics principles. It is therefore essential to define basic quantities such as stress, strain and their respective invariants as applicable to the implementation of the constitutive relationships. In order to maintain clarity, all the definitions are expressed by means of the indicial notation.

2.2.1 Definition of Stress

The state of stress in a continuum can be expressed in terms of the Cauchy stress σ_{ij} which represents the tractions on various planes, measured per unit area of the plane under consideration. When the deformation experienced by the medium is considered to be small, the tractions can be measured either in relation to areas in the deformed or the undeformed body. In classical theories of elasticity and plasticity where the small strain considerations are implicit, the Cauchy stress is referred to the undeformed domain. The sign convention adopted in the thesis treats compressive stresses as positive. In the rectangular Cartesian coordinate system, σ_{ij} takes the form

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad (2.1)$$

In the development of constitutive theories, it is convenient to employ measures of σ_{ij} which are independent of the frame of reference (see e.g. Fung, 1965). These stress invariants of σ_{ij} are defined as follows :

$$J_1 = \sigma_{ii} \quad (2.2a)$$

$$J_2 = \frac{1}{2} \sigma_{ij} \sigma_{ji} \quad (2.2b)$$

$$J_3 = \frac{1}{3} \sigma_{ik} \sigma_{kj} \sigma_{ji} \quad (2.2c)$$

It is also convenient to introduce a deviatoric stress tensor which is defined as

$$S_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \quad (2.3)$$

where δ_{ij} = Kronecker delta function.

Similarly, the invariants corresponding to the S_{ij} are defined by

$$J_{1D} = 0.0 \quad (2.4a)$$

$$J_{2D} = \frac{1}{2} S_{ij} S_{ij} \quad (2.4b)$$

$$J_{3D} = \frac{1}{3} S_{ik} S_{kj} S_{ji} \quad (2.4c)$$

The invariants J_1 and J_{2D} are widely used in the formulation of advanced constitutive models. It is therefore advantageous to express these quantities in terms of principal stresses. In this case,

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad (2.5a)$$

$$J_{2D} = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2] \quad (2.5b)$$

In engineering practice, a quantity called the *octahedral shear stress*, τ_{oct} , is often used. This shear stress is equal to the magnitude of the projection of the stress vector on an *octahedral plane* which makes equal angles with the three principal directions. In other words, this plane is perpendicular to the space diagonal in the three dimensional principal stress space. The second invariant of the deviatoric stress tensor is related to τ_{oct} by the expression

$$J_{2D} = \frac{3}{2} \tau_{oct}^2 \quad (2.6)$$

2.2.2 Definition of Strain

Consider a continuum region in space which is deformed in such a way that its points undergo displacements. With respect to the rectangular Cartesian coordinate system, consider a particle in the undeformed body having coordinates x_i (i

$= x, y, z$). After deformation the particle will occupy a new position having coordinates $(x_i + u_i)$ where u_i are the displacement components. Assuming that u_i are differentiable functions of the coordinates and attention is restricted to sufficiently small strains so that terms in the deformation gradients higher than the first order are neglected. The deformation gradients consist of normal and shear strains. Normal strain is the change in length of a line element with respect to the undeformed length. Shear strain is the change in the angle of an originally right angled corner. The sign convention is similar to the one adopted for stresses where compressive strains are treated as positive. In the rectangular Cartesian coordinate system, the state of strain at a point is given by

$$[\epsilon_{ij}] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \quad (2.7)$$

As in the case of stresses, three strain invariants are defined as follows :

$$I_1 = \epsilon_{ii} \quad (2.8a)$$

$$I_2 = \frac{1}{2} \epsilon_{ij} \epsilon_{ji} \quad (2.8b)$$

$$I_3 = \frac{1}{3} \epsilon_{ik} \epsilon_{kj} \epsilon_{ji} \quad (2.8c)$$

Similarly, a deviatoric strain tensor can be defined as

$$E_{ij} = \epsilon_{ij} - \frac{1}{3} \epsilon_{kk} \delta_{ij} \quad (2.9)$$

The invariants corresponding to E_{ij} are defined by

$$I_{1D} = 0.0 \quad (2.10a)$$

$$I_{2D} = \frac{1}{2} E_{ij} E_{ji} \quad (2.10b)$$

$$I_{3D} = \frac{1}{3} E_{ik} E_{ki} E_{ji} \quad (2.10c)$$

2.3 Constitutive Relationships

The determination of the response of soil media to external loads is of fundamental importance to the analysis of soil-structure interaction problems. Such evaluation can be made only from a knowledge of the complete stress-strain characteristics of the soil. The mathematical description of the mechanical properties of the soil are referred to as the constitutive relationship of the soil. Owing to the variety of soils and soil conditions that can be encountered in engineering practice, it seems unlikely that generalized stress-strain relations will be developed to fulfill the requirements of every type of soil behaviour, especially with respect to the analysis of interaction between the soil and the embedded structural element.

The inherent complexities in the behaviour of real soils have led to the development of many idealized soil models. The classical theories of elasticity and plasticity are two such idealizations commonly employed in the analysis of problems in geomechanics. Any such model of soil response or material behaviour provides a useful description of certain features of earth materials under limited conditions of operation. Idealized soil models prove to be particularly useful in the analysis of soil-structure interaction problems. The relevant choice of an idealized behaviour of the soil of a soil-structure interaction problem depends on a variety of factors including the type of soil and soil conditions, the type of structure and the nature of external loading. In addition, consideration should also be given to more practical aspects of the problem such as the method of construction, the purpose and life span of the structure as well as economical considerations. Within the context of the present research programme, it is decided that the Drucker-Prager model and

the Critical State (modified Cam-Clay) model can be adopted for the representation of the stress-strain behaviour of soils. The following subsections focus on the description of the two chosen constitutive relationships.

2.3.1 Elastic Behaviour

As discussed in fundamental solid mechanics, an external load causes stresses and strains in a body. If the body returns to its original state of stress or strain, it is considered to have deformed elastically. The type of behaviour may be described by the generalized Hooke's Law expressed as

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad (2.11)$$

where C_{ijkl} is a fourth order tensor and is also known as the elasticity tensor, or the constitutive tensor for an elastic body.

In order to facilitate numerical computations such as the finite element analysis, eq.(2.11) may be written in matrix notation as

$$[\sigma] = [D][\epsilon] \quad (2.12)$$

where $[\sigma]$ and $[\epsilon]$ are the stress and strain vectors respectively expressed in the form as follows.

$$[\sigma]^T = [\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{xy} \quad \sigma_{yz} \quad \sigma_{zx}] \quad (2.13a)$$

$$[\epsilon]^T = [\epsilon_{xx} \quad \epsilon_{yy} \quad \epsilon_{zz} \quad \epsilon_{xy} \quad \epsilon_{yz} \quad \epsilon_{zx}] \quad (2.13b)$$

For an isotropic material, the stress-strain matrix is given by

$$[\mathbf{D}] = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & \text{sym.} & & \frac{1-2\nu}{(1-\nu)} & 0 & 0 \\ & & & & \frac{1-2\nu}{(1-\nu)} & 0 \\ & & & & & \frac{1-2\nu}{(1-\nu)} \end{bmatrix} \quad (2.14)$$

where E is the modulus of elasticity (Young's modulus) and ν is Poisson's ratio. Further details regarding the above relationships may be found in Timoshenko and Goodier (1970).

2.3.2 Plastic Behaviour

The developments of the theories of plasticity and their applications are extensively documented in the engineering literature (Davis, 1969; Prager, 1959; Hill, 1950 etc.). This section, however, attempts to provide an overview of the important aspects of plasticity as pertinent to the scope of investigation involving soil-structure interaction problems. As described in the previous section, elastic materials undergo recoverable deformations; i.e. the loaded body returns to its initial configuration when the load is removed. If the body exhibits some degree of time independent irrecoverable deformation upon load removal, it is considered to have behaved in a plastic manner.

Consider a material under a uniaxial state of stress. When the material is stressed beyond the point A (Fig.2.1) and unloaded, the non-linear elastic material will recover the original configuration by tracing back the original stress path; while the plastic material will follow a different path, as shown, resulting in permanent plastic deformation, OC. Some material exhibits an ideally plastic behaviour in which a limiting yield stress, σ_y , exists at which the strains are indeterminate (Fig.2.2). For all stresses below such yield stress, a linear (or non-linear) elastic relationship is assumed. A further refinement of this model is one that involves hardening/softening material in which the yield stress depends on some hardening

Figure 2.1 : Stress-Strain Curves for Non-Linear Elastic and Plastic Materials (schematic representation)

Figure 2.2 : Stress-Strain Curves for Elastic-Perfectly Plastic Material (schematic representation)

Figure 2.3 : Stress-Strain Curve for Elastic-Plastic (Strain Hardening) Material (schematic representation)

**Figure 2.4 : Stress-Strain Curve for Elasto-Plastic Material
(schematic representation)**

parameter, such as the plastic strain (Fig.2.3). Furthermore, Fig.2.4 illustrates the case where both elastic and plastic deformations occur as the material is stressed from O to A. This is known as elasto-plastic behaviour. Many common geotechnical materials behave in an elasto-plastic manner right from the start of loading without undergoing the phase of purely elastic response. Since the reloading path does not follow the original loading path, the strains are dependent on the history of stress applications when plastic deformations occur. It should be emphasized that time effects are neglected in the theory of plasticity (Drucker et al., 1955).

During incremental stress changes of infinitesimal magnitudes, changes of strain are assumed to be divisible into elastic and plastic parts. Thus

$$d\epsilon_{ij} = d\epsilon_{ij}^e + d\epsilon_{ij}^p \quad (2.15)$$

After some matrix and algebraic manipulations (Zienkiewicz, 1979), the elasto-plastic stress-strain relation which determine the stress changes in terms of imposed strain changes gives rise to

$$[d\sigma] = [D^{ep}] [d\epsilon] \quad (2.16)$$

where $[D^{ep}]$ takes the place of the elasticity matrix in incremental analysis. It can be shown that (see e.g. Zienkiewicz et al., 1969; Yamada et al., 1968)

$$[D^{ep}] = [D] - [D] \left(\frac{\partial Q}{\partial \sigma} \right) \left(\frac{\partial F}{\partial \sigma} \right)^T [D] \left[A + \left(\frac{\partial F}{\partial \sigma} \right)^T [D] \left(\frac{\partial Q}{\partial \sigma} \right) \right]^{-1} \quad (2.17)$$

where Q is the *plastic potential*, F is the *yield function* and A is a parameter relating to hardening behaviour which will be discussed in later sections. In the case of ideal plasticity, A is equal to zero.

The theory of plasticity consists of the following :

- 1) Yield Criterion and
- 2) Post Yield Behaviour.

2.3.2.1 Yield Criterion

A yield criterion can be described as the limit of elastic deformation defined by a combination of states of stress. However, under the three dimensional states of stress, the yield criterion is most appropriately represented by an idealized mathematical formulation. This definition has to be established based on experimental observations. The influence of the hydrostatic stress on the plastic deformation has been found to be negligible for homogeneous materials. This assumption leads to the fact that the yield criterion depends only on the state of the deviatoric stress. Therefore the yield criterion can be expressed as

$$F(J_{2D}, J_{3D}) = 0 \quad (2.18)$$

where J_{2D} and J_{3D} are the invariants of the deviatoric stress tensors defined previously.

2.3.2.2 Post Yield Behaviour

Plastic Flow Behaviour

When the state of stress in a body reaches the yield criterion, it will undergo plastic deformation. This plastic flow behaviour is defined through a flow rule which is fundamental to the development of plastic stress-strain relationships. In the theory of plasticity, the plastic flow is defined through a flow rule by assuming the existence of a plastic potential function to which the incremental plastic strain vectors are normal. The normality rule can be written as

$$d\epsilon_{ij}^p = d\lambda \frac{\partial F}{\partial \sigma_{ij}} \quad (2.19)$$

where $d\lambda$ is a positive scalar factor of proportionality and F is the yield function.

In a similar fashion, the plastic strain increment can also be defined by

$$d\epsilon_{ij}^p = d\lambda \frac{\partial Q}{\partial \sigma_{ij}} \quad (2.20)$$

where Q is the plastic potential function.

In the particular case where the plastic potential function Q coincides with the yield function F , the flow rule is called an *associative flow rule*. When this relation is not satisfied, the flow rule is considered to be *non-associative*.

Hardening Behaviour

Due to plastic flow, work hardening or strain hardening occurs in many materials including earth materials. One hypothesis assumes that hardening depends only on the plastic work done and is independent of the strain path. This implies that the resistance to further yielding depends only on the total plastic work done on the material. This is known as the *work hardening hypothesis*. The second hypothesis assumes that plastic strain is a measure of hardening and is known as the *strain hardening hypothesis* (see e.g. Drucker, 1950).

It is recognized that many soils experience plastic deformation almost from the start of a loading process (Desai and Siriwardane, 1984). During a particular loading path, the soil undergoes a process of continuous yielding until it reaches the conventional failure state defined by the yield function. It is possible to represent the foregoing behaviour by a series of yield surfaces prior to reaching the failure or the final yield state. During the successive yielding processes, the soil generally hardens and exhibits strain or work hardening behaviour. For such a soil, when a stress point moves beyond the current yield surface, a new yield surface is established. These surfaces that define the hardening behaviour are referred to as *hardening caps*. For instance, the modified Cam-Clay model (Roscoe and Burland, 1968) and the Cap model (DiMaggio and Sandler, 1971) are formulated according to the assumption that successive hardening caps are formed as the soil yields.

In the case where a soil specimen is subjected to a conventional triaxial stress condition, Drucker, Gibson and Henkel (1955) proposed that the yield surface should intersect the J_1 -axis as shown in Fig.2.5a in order to account for this phenomenon. A soil specimen which is subjected to a hydrostatic state of stress tends to deform elastically when unloaded. Until the stress state reaches point A upon reloading,

Figure 2.5a : Hardening Behaviour of Soil and Rock Media - Yield Surface Intersecting J_1 Axis

Figure 2.5b : Hardening Behaviour of Soil and Rock Media - Behaviour Under Hydrostatic Loading and Unloading

the soil continues to deform elastically (Fig.2.5b). It should be noted that the behaviour under a hydrostatic state of stress involves working hardening and not perfect plasticity. In the case of work hardening materials, a new yield surface is established when the stress point moves outside the existing yield surface. But the actual yield surface (or cap) may not necessarily be circular. Several constitutive models based on the concept of yield caps have been developed. One of these, the Critical State Model, will be discussed in the following sections.

2.4 Constitutive Laws for Soils

It is known that the strength of most soils is dependent on the hydrostatic stress. Under fully or partially drained conditions, the strength of the soil increases with mean pressure and exhibit frictional characteristics. The undrained behaviour of a saturated soil such as clay is, however, similar to that of metals. In the context of our scope of investigation of various soil-structure interaction problems, it is decided that the Drucker-Prager model and the Critical State (modified Cam-Clay) model can be adopted for the representation of the stress-strain behaviour of soils. One of the advantages for selecting these two soil models is that the input parameters can be readily determined from triaxial, direct shear and consolidation tests.

2.4.1 Drucker-Prager Model

The Drucker-Prager model has been used quite successfully to model the elastic-plast' behaviour of soils which exhibit both cohesive and frictional responses. The yield criterion characterizing the Drucker-Prager model takes the form

$$F = \sqrt{J_{2D}} - \alpha J_1 - k \quad (2.21)$$

where α and k are material parameters; J_1 is the first invariant of the stress tensor and J_{2D} is the second invariant of the deviatoric stress tensor (Drucker and Prager, 1952).

Equation 2.21 represents a straight line on a J_1 vs $\sqrt{J_2 D}$ plot (Fig.2.6). In the three dimensional principal stress space, this criterion represents a right circular cone. The projection on the π -plane is a circle. Moreover, the associative flow rule dictates that the incremental plastic strain vector, $d\epsilon_{ij}^p$, is always perpendicular to the yield surface.

When α is equal to zero, the yield surface reduces to the von Mises criterion which is suitable for representing a purely cohesive (constant strength) material. The values of α and k are dependent on the angle of internal friction, ϕ , and the cohesion, c . For the plane strain case, α and k can be expressed as

$$\alpha = \frac{\tan \phi}{\sqrt{(9 + 12 \tan^2 \phi)}} \quad (2.22a)$$

$$k = \frac{3c}{\sqrt{(9 + 12 \tan^2 \phi)}} \quad (2.22b)$$

For the axi-symmetric case, α and k can be expressed as

$$\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)} \quad (2.23a)$$

$$k = \frac{6c \cos \phi}{\sqrt{3}(3 - \sin \phi)} \quad (2.23b)$$

The required material parameters for the implementation of this model are the four basic constants E , ν , c and ϕ , all of which may be determined by standard geotechnical laboratory tests such as the triaxial test. Further details of the relevant finite element implementation procedures may be found in Desai and Siriwardane (1984).

As shown in Fig.2.6, the incremental plastic strain vector has a negative volumetric component which indicates volume increase or dilation at failure. Experimental data on normally consolidated clays and loose sands indicates that only compressive deformations or decrease in volume occur during shear (Desai and

Figure 2.6 : Drucker-Prager Yield Criterion

Siriwardane, 1984). These observations are thus in conflict with the predictions from the Drucker-Prager model. The possible reasons for this discrepancy are that the normality rule may not be valid and (ii) the Drucker-Prager yield criterion may not be applicable to these materials. If the normality rule is neglected, a situation will exist such that the stresses satisfy the yield criterion while the strain states are forced to satisfy conditions such as no volume change (Drucker, 1950). The shortcomings of the Drucker-Prager model are further discussed in Desai and Christian (1977), Gyebi and Dasgupta (1987).

2.4.2 Critical State Model

2.4.2.1 Critical State Concept

The critical state concept due to Roscoe, Schofield and Wroth (1958) relates the effective stresses and the corresponding specific volume, v , of a clay during shearing under drained or undrained conditions. It thus unifies the characteristics of shear strength and deformation. Extensive experiments involving conventional triaxial compression tests were conducted by Roscoe and his co-workers on reconstituted cylindrical samples of kaolin. This concept represents an idealization of the observed patterns of behaviour of the saturated remoulded clays, but it is assumed to be applicable to undisturbed clays. It was demonstrated that a characteristic surface, the *state boundary surface*, exists which limits all possible states of the clay and that all effective stress paths reach or approach a line, the *critical state line*, on that surface which defines the state at which the clay will continue to yield at constant volume under constant effective stresses. The concept can be extended to include general three dimensional states of stress.

2.4.2.2 Modified Cam-Clay Model

Based on the original critical state concept, Roscoe and Burland (1968) proposed the modified Cam-Clay model which is based on the isotropic hardening concept. For the axisymmetric triaxial condition where $\sigma_2 = \sigma_3$, the relevant quantities are defined as follows.

$$p = \frac{\sigma_1 + 2\sigma_3}{3} = \frac{J_1}{3} \quad (2.24a)$$

$$q = \sigma_1 - \sigma_3 = \sqrt{3J_2D} \quad (2.24b)$$

$$dv = d\epsilon_1 + 2d\epsilon_3 \quad (2.24c)$$

$$d\epsilon = \frac{2}{3}(d\epsilon_1 - d\epsilon_3) \quad (2.24d)$$

where p is the mean hydrostatic pressure, q is the deviatoric shear stress, dv is the incremental volumetric strain, $d\epsilon$ is the incremental shear strain and σ_1, σ_3 are the major and minor principal effective stresses respectively.

When a saturated soil sample is sheared, it passes through progressive states of yielding before reaching a state of collapse. Thus the stress path passes through several yield surfaces (hardening caps) involving plastic deformations. The yielding continues to occur until the material reaches a critical state at which the void ratio remains constant during subsequent deformations. This is a state in which no volume change takes place during shear and can be considered as the ultimate state of the material. The points corresponding to various ultimate states form a straight line called the critical state line on the *Cambridge* $q - p$ stress space. The attainment of this state is found to be independent of the orientation of the stress paths concerned. Results from a series of drained cylindrical triaxial tests on the same soil have shown that the ultimate states (or failure points) lie on the critical void ratio line observed in the undrained behaviour. The critical state line and the isotropic volume-pressure line are parallel to each other on the $e - \ln p$ plot (Fig.2.7).

2.4.2.3 Material Parameters

Some of the important parameters used in the development and implementation of the cam-clay models are discussed in this section. The slope of the critical state line as shown in Fig.2.9 is denoted by the material parameter M where

Figure 2.7 : Void Ratio - $\ln p$ Relationship for the Critical State Model

$$M = \frac{q}{p} \quad (2.25)$$

Figure 2.8 is a $e - \ln p$ plot depicting isotropic loading and unloading behaviour of a soil. The parameter λ is the slope of the virgin compression line while κ is the slope of the unloading-reloading line. The values of λ and κ can be related to the conventional compression index, C_c , and the swelling index, C_s , as

$$\lambda = \frac{C_c}{2.303} \quad (2.26a)$$

$$\kappa = \frac{C_s}{2.303} \quad (2.26b)$$

In addition to the above three basic constants, the following parameters associated with the current state are also required to describe the constitutive behaviour within the context of the modified Cam-Clay model.

e_0 = current void ratio

p_0 = hardening parameter or maximum past isotropic stress

p, q = current state of stress (as defined earlier).

Moreover, the elastic properties such as the bulk and shear moduli, denoted by K and G respectively, can be related to the above quantities as follows.

$$K = \frac{p(1 + e_0)}{\kappa} \quad (2.27a)$$

$$G = \frac{3}{2} \frac{p(1 + e_0)}{\kappa} \frac{(1 - 2\nu)}{(1 + \nu)} \quad (2.27b)$$

where ν is Poisson's ratio. It is clear that both moduli are functions of the current mean pressure, p , and the current void ratio, e_0 .

2.4.2.4 Plastic Stress-Strain Behaviour

In Fig.2.8, the vertical distance AD shows the plastic component and DE shows

Figure 2.8 : Isotropic Loading and Unloading Behaviour for the Critical State Model

Figure 2.9 : Yield Locus for the Critical State Model in the q - p Plane

the elastic component of the change in volume of a soil subjected to an isotropic stress state. The plastic volumetric strains, dv^p , can therefore be evaluated as

$$dv^p = \frac{(\lambda_c - \kappa)}{(1 + e_0)} \frac{dp_0}{p_0} \quad (2.28)$$

where the superscript 'p' denotes plastic changes. In the stress-strain theory based on the critical state concept, it is assumed that there is no recoverable energy associated with shear distortions, i.e. $d\epsilon^e = 0$ where the superscript 'e' denotes elastic deformations. In other words, the relationship

$$d\epsilon = d\epsilon^p \quad (2.29)$$

holds true at all times.

According to the normality rule discussed in previous sections, the incremental plastic strain vector is normal to the yield surface at any point (Fig.2.9). In the q-p diagram as shown, the yield surface has the shape of an ellipse. This condition can be expressed as

$$\frac{d\epsilon^p}{dv^p} = \frac{dq}{dp} \quad (2.30)$$

As described in Roscoe and Burland (1968), the ratio of shear and volumetric plastic strains can be defined in terms of p and q by assuming a dissipated energy function. By integrating eq.(2.30), the yield function of the modified Cam-Clay model can be obtained as

$$F = q^2 - M^2 p_0 p - M^2 p^2 = 0 \quad (2.31)$$

In the three dimensional stress space, this equation represents an ellipsoid.

2.5 Finite Element Method of Analysis

2.5.1 General

Conventional analytical methods of stress analysis usually involve the solution of the governing differential equations or integral equations by direct means. Such direct methods can be used effectively in situations where the geometry of the domain is simple and regular, the constitutive relationships are linear, and the configuration has symmetry conditions. However, engineering problems, such as the soil-structure interaction problems to be dealt with in this research programme, are complicated in nature. The limitations of these analytical methods and the recent rapid advancement of computer technology have led to the development of numerical techniques for stress analysis. Computational methods such as the finite difference scheme, the finite element method and the boundary element method and their combinations have been used extensively in recent years for the solution of static and dynamic stress analysis problems involving linear and non-linear material properties. In this section, attention is focussed on the important aspects of the finite element method which is used extensively in this thesis. Further details of this technique can be found in references such as Cook (1974); Zienkiewicz (1979); Bathe (1982) and Desai and Abel (1972) etc.

The finite element method involves the approximation of the values of various unknown quantities at a discrete number of points within a body. The process of selecting a certain number of points in the body is called *discretization*. One way to discretize a body is to divide into an equivalent system of smaller bodies or units. This forms the basis of the finite element method where a body is represented by an assemblage of subdivisions called *finite elements*. The elements are assumed to be connected at the joints which are called *nodes*. In solid mechanics applications, if the unknown function is the displacement field in the body, simple mathematical functions can be chosen to approximate the variation of the actual displacements over each finite element. The assumed functions are called

displacement functions. The unknown amplitudes of the displacement functions at the nodes are called *generic displacements*. In essence, the major objective of the finite element technique as applied to solid mechanics problems is to determine the generic displacements which will approximate the actual displacement field in the body.

The basic step of the finite element technique is to generate the equilibrium equations for each element. The equilibrium equations for the entire body are then obtained by assembling the equations for the individual elements. In solid mechanics, formulation of the finite element method is commonly used. The displacement field in the body is related to the nodal displacement by

$$[u] = [N][d] \quad (2.32)$$

where

$[u]$ = continuous displacement function

$[N]$ = shape function

$[d]$ = generic displacement vector

The shape function must satisfy the following condition.

$$N_i(\xi_j, \eta_j) = \delta_{ij} \quad (2.33a)$$

and

$$\sum_{i=1}^n N_i(\xi, \eta) = 1 \quad (2.33b)$$

where n is the total number of joints in the element; ξ and η are curvilinear local coordinates unique to the element which can be represented in terms of the Cartesian coordinate system; δ_{ij} is Kronecker's delta function.

The strain-displacement equation can be written in matrix form as

$$[\epsilon] = [\nabla][u] \quad (2.34)$$

where

$[\nabla]$ = matrix containing the differential operators relating displacements to strains.

It follows that

$$[\epsilon] = [D][N][d] = [B][d] \quad (2.35)$$

where

$[B]$ = strain-displacement relationship

Provided that the material obeys Hooke's Law, stresses can be expressed as

$$[\sigma] = [D][\epsilon] \quad (2.36)$$

where

$[\sigma]$ = vector containing stress components

$[D]$ = material property matrix

The element stiffness matrix can be obtained by the following relationship.

$$[K] = \int_V [B]^T [D] [B] dv \quad (2.37)$$

where

$[K]$ = element stiffness matrix

dv = incremental volume of the element

The unknown quantities that need to be determined are the generic displacements $[d]$.

2.5.2 Isoparametric Formulation

Isoparametric elements have been used with great success to examine problems dealing with two and three dimensional elasticity, plates and general shells (Zienkiewicz, 1979). The expression *isoparametric* originates from the fact that the same interpolation functions are used to define both the element geometry and the displacements within the element. Once such element has been implemented, it is relatively straight forward to extend the formulation to other types of isoparametric elements. Linear elements have straight sides, but quadratic and higher order elements may have either straight or curved sides, which make them very useful for the modelling of curved boundaries.

The coordinates of isoparametric elements employ a so called 'natural' coordinate system $r - s - t$ (three dimensional case) which is defined by the element geometry and not by the element orientation in the global $x - y - z$ coordinate system. The systems are related by means of appropriate transformations. Coordinates of a three dimensional element are scaled in a way that the sides are defined by $r = +1$, $r = -1$, $s = +1$, $s = -1$, $t = +1$ and $t = -1$.

2.5.3 Three Dimensional Solid 'Brick' Element

Since most geomechanics problems are three dimensional in nature, it is necessary to employ solid elements to model the soil behaviour (Bathe, 1982; Zienkiewicz, 1979; Desai and Abel, 1972). The computer code SSTIN-3D was used as a starting point for non-linear three dimensional finite element analysis in this thesis. After careful consideration of the nature of the problems to be investigated in this research programme, the 20-node element is selected as the primary element.

The basic procedure in the isoparametric finite element formulation is to express the element coordinates and displacements in the form of interpolations using the natural coordinate system of the element.

For a 20-node three dimensional element (Fig.2.10), the coordinate interpola-

Figure 2.10 : 20-node Three Dimensional Continuum (Brick) Element

tions are given by

$$x = \sum_{i=1}^{20} N_i x_i \quad (2.38a)$$

$$y = \sum_{i=1}^{20} N_i y_i \quad (2.38b)$$

$$z = \sum_{i=1}^{20} N_i z_i \quad (2.38c)$$

where x , y and z are the coordinates at any point of the element, and x_i , y_i and z_i , $i = 1, 2, \dots, 20$ are the coordinates of the twenty element nodes. The interpolation functions N_i 's are defined in the natural coordinate system of the element, which has variables r , s and t each varying from -1 to +1.

The element displacements are interpolated in a similar way as follows.

$$u = \sum_{i=1}^{20} N_i u_i \quad (2.39a)$$

$$v = \sum_{i=1}^{20} N_i v_i \quad (2.39b)$$

$$w = \sum_{i=1}^{20} N_i w_i \quad (2.39c)$$

where u , v and w are the local element displacements at any point of the element and u_i , v_i and w_i , $i=1, 2, \dots, 20$ are the corresponding element displacements at its nodes.

Furthermore, eqs.(2.39a) to (2.39c) can be written as

$$[u] = [N][a] \quad (2.40)$$

where

$$[u] = [u, v, w]$$

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & \dots & N_{20} & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & \dots & 0 & N_{20} & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & \dots & 0 & 0 & N_{20} \end{bmatrix}$$

$$[a]^T = [u_1, v_1, w_1, u_2, v_2, w_2 \dots u_{20}, v_{20}, w_{20}]$$

2.5.3.1 Strain-Displacement Relationship

For a full three dimensional analysis, six strain components are sufficient. The strain matrix can be defined as

$$[\epsilon] = \begin{bmatrix} \epsilon_{zz} \\ \epsilon_{yy} \\ \epsilon_{xx} \\ \epsilon_{zy} \\ \epsilon_{yx} \\ \epsilon_{zx} \end{bmatrix} \quad (2.41)$$

By means of appropriate manipulations, the vector of strain components can be expressed in terms of nodal displacements as

$$[\epsilon] = [B][a] \quad (2.42)$$

where $[B]$ is the strain-displacement relationship. After partitioning the $[B]$ matrix, eq.(2.42) can be written as

$$[\epsilon] = [[B]_1 \quad [B]_2 \quad \dots \quad [B]_{20}] [a] \quad (2.43)$$

A typical submatrix $[B]_i$ takes the form

$$[B] = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial z} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad (2.44)$$

In order to evaluate the $[B]$ matrix, it is necessary to establish a transformation relationship from global to local coordinates. By the usual rules of partial differentiation, it can be shown that

$$\begin{bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \end{bmatrix} = [J] \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix} \quad (2.45a)$$

$$[J] = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t} \end{bmatrix} \quad (2.45b)$$

where $[J]$ is the Jacobian matrix. In order to find the global derivatives, the following relationship is used.

$$\begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \end{bmatrix} \quad (2.46a)$$

$$[J^*] = \begin{bmatrix} J_{11}^* & J_{12}^* & J_{13}^* \\ J_{21}^* & J_{22}^* & J_{23}^* \\ J_{31}^* & J_{32}^* & J_{33}^* \end{bmatrix} \quad (2.46b)$$

where $[J]^{-1}$ is the inverse of $[J]$.

2.5.3.2 Stress-Strain Relationships

For an isotropic material, the stress-strain matrix for a three dimensional solid element is given by eq.(2.14).

2.5.4 General Shell Element

2.5.4.1 General

In order to model the flexural behaviour of the structural elements encountered in the soil-structure interaction problems in this research programme, it is necessary to employ a general shell element that must satisfy at least the following : (i) compatible with the existing SSTIN-3D code which employs variable node isoparametric elements and (ii) meet the considerable demands for correctness of response and versatility of behaviour.

Certain difficulties may be encountered if the three dimensional solid element is used as a shell element. Firstly, the existence of three degrees of freedom at each node can lead to large stiffness coefficients for relative displacements along the shell thickness direction. When the shell thickness becomes small, compared with other element dimensions, numerical ill-conditioning may arise. Secondly, the high number of degrees of freedom across the shell thickness leads to the unnecessary consumption of computing time.

If the geometric complexities of doubly curved elements based on general shell theory are to be avoided, a three dimensional solid element may be specialized to achieve the desired purposes. In other words, the geometry and displacement interpolation concepts that are used in the isoparametric continuum elements can also be employed in the evaluation of shell element matrices. In the shell element formulation, the displacements in the global x , y , z directions are interpolated in terms of mid-surface nodal displacements and rotations. Since a higher degree of interpolation is used for the geometry than for the displacements, the shell element thus formed may be termed *superparametric* element.

2.5.4.2 Geometric Formulation

Consider a 20-node (60 dof's) solid element as shown in Fig.2.10. Referring to the local coordinate axes, $t = -1$ and $t = +1$ correspond to the bottom and the top of the element respectively. If the lines joining the two surfaces are constrained to be

straight; that is, linear interpolation in the thickness direction, the element may be defined in terms of the shape functions of the 8-node two dimensional isoparametric element (Bathe, 1982; Zienkiewicz, 1979).

It should be noted that the local coordinate direction 'n' is not truly 'normal' to the mid-surface. A vector $\mathbf{v}_{3i}$ which is normal to the middle surface and spans the thickness t_i of the shell node i can be defined in terms of direction cosines as

$$\mathbf{v}_{3i} = \begin{bmatrix} l_{3i} \\ m_{3i} \\ n_{3i} \end{bmatrix} t_i \quad (2.47)$$

where l_{3i} , m_{3i} and n_{3i} are direction cosines of the vector $\mathbf{v}_{3i}$ with the three global coordinate directions x , y , z respectively. The coordinates of the shell element may be expressed as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \sum_{i=1}^8 N_i \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \sum_{i=1}^8 N_i \frac{n_i}{2} \mathbf{v}_{3i} \quad (2.48)$$

2.5.4.3 Displacement Formulation

In addition to the displacements in the directions of the global axes $x - y - z$, there exist two rotations α_i and β_i , about the two local tangential axes r and s respectively (Fig.2.11) at each node. Displacements at any point in terms of element nodal displacements are given by

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{i=1}^8 N_i \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} + \sum_{i=1}^8 N_i \frac{t_i}{2} [\mu] \begin{bmatrix} \alpha_i \\ \beta_i \end{bmatrix} \quad (2.49)$$

where

$$[\mu] = \begin{bmatrix} -l_{2i} & l_{1i} \\ -m_{2i} & m_{1i} \\ -n_{2i} & n_{1i} \end{bmatrix} \quad (2.50)$$

Figure 2.11 : Super-parametric General Shell Element

and l_{1i} , l_{2i} etc. are direction cosines of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. The vectors $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ form an orthogonal set of vectors.

In essence, the 40 dof shell element is formed by the appropriate reduction of the 60 dof solid element. As implicitly stated in the above equations, the strain normal to the mid-surface is neglected, and the lines originally normal to the mid-surface are constrained to remain straight during deformation. Since these lines need not remain normal to the mid-surface, the element is able to model transverse shear deformation. Furthermore, since the isoparametric interpolation is employed, the geometry of curved shell surfaces can be represented to a high degree of accuracy.

2.5.4.4 Shape Functions

The shape functions of the general shell element in their explicit forms are as follows.

$$N_1 = \frac{1}{4}(1-r)(1-s)(-r-s-1) \quad (2.51a)$$

$$N_2 = \frac{1}{4}(1+r)(1-s)(r-s-1) \quad (2.51b)$$

$$N_3 = \frac{1}{4}(1+r)(1+s)(r+s-1) \quad (2.51c)$$

$$N_4 = \frac{1}{4}(1-r)(1+s)(-r+s-1) \quad (2.51d)$$

$$N_5 = \frac{1}{2}(1-s)(1-r^2) \quad (2.51e)$$

$$N_6 = \frac{1}{2}(1+r)(1-s^2) \quad (2.51f)$$

$$N_7 = \frac{1}{2}(1+s)(1-r^2) \quad (2.51g)$$

$$N_8 = \frac{1}{2}(1 - r)(1 - s^2) \quad (2.51h)$$

where the numbers 1 to 8 represent the shell element node numbers as shown in Fig.2.11.

2.5.4.5 Stress-Strain Relationship

With the strain in the t (thickness) direction neglected, the strain components of interest are given in the local coordinate directions as

$$[\epsilon'] = \begin{bmatrix} \epsilon_{x'} \\ \epsilon_{y'} \\ \epsilon_{x'y'} \\ \epsilon_{x'z'} \\ \epsilon_{y'z'} \end{bmatrix} \quad (2.52)$$

It should be noted that none of these directions necessarily coincide with the local coordinate axes $r - s - t$.

The stresses corresponding to these strains are defined by the usual stress-strain relationships as follows.

$$[\sigma'] = [D'][\epsilon'] \quad (2.53a)$$

where

$$[\sigma'] = \begin{bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \sigma_{x'y'} \\ \sigma_{x'z'} \\ \sigma_{y'z'} \end{bmatrix} \quad (2.53b)$$

The reduced elasticity matrix $[D']$ is defined with respect to the local axes and is capable of incorporating any anisotropic properties. This matrix is given by

$$[D'] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ & 1 & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 \\ & & & \frac{1-\nu}{k} & 0 \\ & & & & \frac{1-\nu}{k} \end{bmatrix} \quad (2.54)$$

It may be seen from the displacement definition that the shear distribution is approximately constant across the thickness whereas in reality the shear distribution is taken as approximately parabolic. Thus the factor k included in the last two shear terms is taken as 1.2 in order to improve the shear displacement approximation (Cook, 1974; Zienkiewicz, 1979). In addition, the form of $[D']$ ensures that there is no stress in the thickness direction and that plane stress conditions exist in the $x'-y'$ plane.

2.5.4.6 Strain-Displacement Relationship

The 3 x 3 Jacobian matrix required for this element is given by eq.(2.46b) where

$$\frac{\partial x}{\partial r} = \sum_{i=1}^8 \frac{\partial N_i}{\partial r} x_i + \sum_{i=1}^8 \frac{\partial N_i}{\partial r} t_i \frac{t_i}{2} l_{3i} \quad (2.55a)$$

$$\frac{\partial x}{\partial s} = \sum_{i=1}^8 \frac{\partial N_i}{\partial s} x_i + \sum_{i=1}^8 \frac{\partial N_i}{\partial s} t_i \frac{t_i}{2} l_{3i} \quad (2.55b)$$

$$\frac{\partial x}{\partial t} = \sum_{i=1}^8 N_i \frac{t_i}{2} l_{3i} \quad (2.55c)$$

and so on.

As in the conventional isoparametric formulation, the inverse of $[J]$ is given by eq.(2.46b).

The strain-displacement relationship at a given point i is given by

$$[B_i] = \begin{bmatrix} a_i & 0 & 0 & d_i l_{2i} & d_i l_{1i} \\ 0 & b_i & 0 & e_i m_{2i} & e_i m_{1i} \\ 0 & 0 & c_i & g_i n_{2i} & g_i n_{1i} \\ b_i & a_i & 0 & -e_i l_{2i} - d_i m_{2i} & e_i l_{1i} + d_i m_{1i} \\ 0 & c_i & b_i & -g_i m_{2i} - e_i n_{2i} & g_i m_{1i} + e_i n_{1i} \\ c_i & 0 & a_i & -d_i n_{2i} - g_i l_{2i} & d_i n_{1i} + g_i l_{1i} \end{bmatrix} \quad (2.56)$$

where

$$a_i = J_{11}^* \frac{\partial N_i}{\partial r} + J_{12}^* \frac{\partial N_i}{\partial s}$$

$$b_i = J_{21}^* \frac{\partial N_i}{\partial r} + J_{22}^* \frac{\partial N_i}{\partial s}$$

$$c_i = J_{31}^* \frac{\partial N_i}{\partial r} + J_{32}^* \frac{\partial N_i}{\partial s}$$

$$d_i = \frac{t_i}{2} (a_i t + J_{13}^* N_i)$$

$$e_i = \frac{t_i}{2} (b_i t + J_{23}^* N_i)$$

$$g_i = \frac{t_i}{2} (c_i t + J_{33}^* N_i)$$

2.5.4.7 Matrix Transformation

Since matrix $[B]$ yields the strain vector $[\epsilon]$ in the global x - y - z directions rather than strains $[\epsilon']$ in the x' - y' - z' coordinates, coordinate transformation must be applied to convert $[D']$ to the matrix $[D]$ in the global direction. This can be accomplished by using a 6 x 6 transformation matrix $[T]$ and its transpose $[T]^T$. Thus

$$[D] = [T]^T [D'] [T] \quad (2.57)$$

The elements of the matrix $[T]$ are composed of the direction cosines of the x' - y' - z' coordinate axes measured from the global x - y - z coordinate directions as follows.

$$[T] = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & l_1 m_1 & m_1 n_1 & n_1 l_1 \\ l_2^2 & m_2^2 & n_2^2 & l_2 m_2 & m_2 n_2 & n_2 l_2 \\ l_3^2 & m_3^2 & n_3^2 & l_3 m_3 & m_3 n_3 & n_3 l_3 \\ 2l_1 l_2 & 2m_1 m_2 & 2n_1 n_2 & l_1 m_2 + l_2 m_1 & m_1 n_2 + m_2 n_1 & n_1 l_2 + n_2 l_1 \\ 2l_2 l_3 & 2m_2 m_3 & 2n_2 n_3 & l_2 m_3 + l_3 m_2 & m_2 n_3 + m_3 n_2 & n_2 l_3 + n_3 l_2 \\ 2l_3 l_1 & 2m_3 m_1 & 2n_3 n_1 & l_3 m_1 + l_1 m_3 & m_3 n_1 + m_1 n_3 & n_3 l_1 + n_1 l_3 \end{bmatrix} \quad (2.58)$$

where $l_i, m_i, n_i, i=1,2,3$ are direction cosines of the local r - s - t coordinate axes measured in the global x - y - z coordinate directions (see Bathe, 1982).

In general, a separate transformation is required at each integration point as the x' - y' - z' directions change from one point to another. Consequently, different sets of direction cosines are needed and may be established by first finding vectors v_1, v_2 and v_3 at each integration point. This may be achieved by the following sequence of calculations.

$$e_1 = J_{1norm} \quad (2.59a)$$

$$e_3 = (J_1 \times J_2)_{norm} \quad (2.59b)$$

$$e_2 = e_3 \times e_1 \quad (2.59c)$$

where J_{1norm} denotes the first row of the Jacobian matrix normalized to unit length; while e_1, e_2 and e_3 are unit vectors corresponding to the local x' - y' - z' directions

respectively. Further details of these procedures can be found in Bathe (1982) and Zienkiewicz (1979).

2.6 Interface Element

The interface element used in this thesis is basically a very thin isoparametric element with a constitutive matrix similar to that of a solid continuum element. The constitutive relationship is modified to account for relative slip behaviour by introducing an independent shear modulus (see Desai et al., 1984). The thickness of the interface element is considered to have a very small value. Since this interface element is based on the isoparametric concept, its nodal connectivity is similar to that of any other solid element. The linear constitutive relationship for the three dimensional interface element is given by

$$[D]^{int} = C_1 \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ & 1 & \frac{\nu}{1-\nu} & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & G_{int}/C_1 & 0 & 0 \\ \text{sym.} & & & & G_{int}/C_1 & 0 \\ & & & & & G_{int}/C_1 \end{bmatrix} \quad (2.60)$$

where

$$C_1 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

and G_{int} = shear modulus for the interface element.

The value of this modulus does not necessarily be consistent with the relationships with E and ν within the scope of conventional theory of elasticity. The value of G_{int} is either determined from laboratory direct shear experiments or estimated from other methods such as parametric studies. In this thesis, G_{int} is taken as a very low value relative to the other terms of the constitutive matrix.

2.7 Coupling

As mentioned earlier, incompatibility exists between the shell element and the regular 20-node brick element. Each node in the general shell element has three orthogonal translational degrees-of-freedom and two rotational degrees-of-freedom; while each node in the solid element has only three translational degrees-of-freedom. It is therefore necessary to reduce the number of degrees-of-freedom in each shell element node in such a way that the two different elements can be coupled together to facilitate an overall finite element solution for the system.

The method of *static condensation* has been adopted to deal with this situation. This approach involves the elimination of the terms associated with the two rotational degrees of freedom at each of the eight nodes during the formation of the element stiffness matrix. As a result, the reduced element stiffness matrix has a size of 24 x 24. It should be noted that the effects of rotation are taken care of by the resulting translational degrees of freedom after all the matrix manipulations have been accomplished.

The relevant matrices and vectors are defined as follows :

$[P_s^o]$ = original load vector

$[K_s^o]$ = original element stiffness matrix

$[U_s^o]$ = original displacement vector

$[P_s]$ = translational load vector

$[M_s]$ = moment vector

$[\delta_s]$ = translational displacement vector

$[\theta_s]$ = rotational displacement vector

$[P_s^r]$ = reduced load vector

$[K_s^r]$ = reduced element stiffness matrix

$[\delta_s^r]$ = reduced displacement vector

The shell element stiffness matrix takes the form

$$[\mathbf{P}_s^o] = [\mathbf{K}_s^o][\mathbf{U}_s^o] \quad (2.61)$$

where

$$[\mathbf{P}_s^o] = \begin{bmatrix} [\mathbf{P}_s] \\ [\mathbf{M}_s] \end{bmatrix}$$

$$[\mathbf{U}_s^o] = \begin{bmatrix} [\delta_s] \\ [\theta_s] \end{bmatrix}$$

The stiffness matrix may be partitioned as

$$\begin{bmatrix} [\mathbf{P}_s] \\ [\mathbf{M}_s] \end{bmatrix} = \begin{bmatrix} [\mathbf{K}_{11}^o] & [\mathbf{K}_{12}^o] \\ [\mathbf{K}_{21}^o] & [\mathbf{K}_{22}^o] \end{bmatrix} \begin{bmatrix} [\delta_s] \\ [\theta_s] \end{bmatrix} \quad (2.62)$$

As a result of the matrix manipulations associated with static condensation, the reduced translational load vector may take the form

$$[\mathbf{P}_s^r] = \left[[\mathbf{K}_{11}^o] - [\mathbf{K}_{12}^o][\mathbf{K}_{22}^o]^{-1}[\mathbf{K}_{21}^o] \right] [\delta_s^r] \quad (2.63a)$$

or

$$[\mathbf{P}_s^r] = [\mathbf{K}_s^r][\delta_s^r] \quad (2.63b)$$

where the reduced shell element stiffness matrix is defined with respect to translational degrees of freedom only.

Chapter 3

Program Verification and Parametric Studies

3.1 General

This chapter presents the solution of a series of soil-structure interaction problems which will verify the accuracy and capabilities of the finite element procedures described in Chapter 2. This is attempted by examining certain specific two-dimensional soil-structure interaction problems involving circular cavities, both lined and unlined, embedded in linear elastic or non-linear elastic-plastic soil media. In these two-dimensional treatments, the capabilities of the finite element technique are demonstrated by means of parametric studies. The performance of the general shell element used is fully examined. The plane strain condition is assumed for all of the two-dimensional analyses.

The soil-structure interaction problems investigated in this thesis are three-dimensional in nature. There are, however, only a very limited number of three-dimensional solutions for these problems available in the literature. A large body of work focusses on the two-dimensional idealization of various problems (see e.g. Brown et al., 1983; Sakurai and Abe, 1979; Kennedy and Lindberg, 1978; Ladanyi, 1974 etc.). The verification undertaken in this chapter is focussed on some aspects of the two-dimensional problems involving embedded cylinders. It should be noted that the results obtained from these two dimensional analyses are not meant to provide complete verification of the corresponding three-dimensional analyses. These solutions should instead be considered as an illustration of the extended capabilities of the developed finite element procedures which are based primarily on three-dimensional formulations. Comparisons of numerical predictions will, however, be

made with a linear elastic three-dimensional finite element analysis which involves a buried pipeline adjacent to a trench excavation subjected to surface loadings.

3.2 Deeply Buried Cylinder - Plane Strain

3.2.1 General

The first problem examined in this chapter relates to the classical elasticity problem of a cylindrical cavity which is located in an elastic solid of infinite extent. The loading corresponds to either uniform biaxial or radially symmetric loading at large distances from the cavity. This configuration is similar to the practical engineering problem where a circular tunnel of radius, a , is situated within a homogeneous, isotropic initially elastic rockmass subjected to a hydrostatic stress field, p_0 (Fig. 3.1). It can be shown that this axisymmetric tunnel problem can be simulated by an equivalent configuration as shown in Fig. 3.2 if the pressures are applied at a distance far enough from the tunnel (say $b/a = 10$). In addition, the tunnel face can also be subjected to a radial internal support pressure. If the tunnel is surrounded by a layer of material with different material properties such as that in the case of a tunnel reinforced by concrete lining, the entire nature of the problem is changed.

A large number of solutions have been obtained for the unlined cavity problem. Brown et al. (1983) provided a summary of the solutions available in the literature. It is clear that the Mohr-Coulomb yield criterion is the predominant yield function used. Three main stress-strain models are used in most of these previous solutions, namely (i) elastic-plastic, (ii) elastic-brittle plastic and (iii) elastic-strain softening. There are, however, a very limited number of solutions dealing with lined cavities although some solutions attempted to simulate the lining by applying an internal radial pressure. Table 3.1 attempts to summarize some of the publications that involve this buried tunnel problem.

	Unlined Cavity	Lined Cavity	Features
Elastic Analytical	Timoshenko and Goodier (1970)	-	Airy stress function approach
	-	Pang (1988)	Airy stress function approach
Elastic Numerical	Reyes and Deera (1966)	-	Two dimensional linear elastic finite element analysis
	-	Sakurai and Yamamoto (1976)	Visco-elastic finite element analysis
Non-linear Analytical	Lindberg and Kennedy (1978)	-	Elastic-plastic analytical method (Mohr-Coulomb yield criterion)
	-	Ladanyi (1974)	Closed-form analytical solution
Non-linear Numerical	Sakurai and Abe (1979)	-	Two dimensional elastic-plastic finite element analysis (Mohr-Coulomb yield criterion)
	-	Giorda and Jurina (1979)	Two dimensional elasto-plastic finite element analysis

Table 3.1 : Summary of Selected References Involving Deeply Buried Lined and Unlined Cavities in Soil or Rock Media

2

MicroD

Figure 3.1 : Unlined Cavity Subjected to Axi-symmetric Free-Field Stress

Figure 3.2 : Unlined Cavity Subjected to Uniform Biaxial Free-Field Stress

3.2.2 Finite Element Approach

In order to assess the performance of the non-linear Drucker-Prager soil model and the capabilities of the general shell element used within the context of the isoparametric finite element formulations, the above mentioned reinforced hole-in-infinite-plate problem is discretized as shown in Fig.3.3. The relative dimensions are chosen such that the original free field circumferential stress can be adequately simulated by applying the same stress intensity to the horizontal and vertical edges. The reinforcement of the hole is modelled by general shell elements while the surrounding solid material is modelled by 20-node continuum elements. Since this is a two dimensional problem, the plane strain situation is simulated by constraining all displacements in the out-of-plane, z , direction. The entire mesh consists of 287 nodes with 32 solid elements and 4 shell elements. This mesh is considered to be fine enough to provide accurate results.

3.3 Unlined Cavities

3.3.1 Elastic Solutions

The classical solution for the problem where a hole is embedded in a homogeneous, isotropic elastic medium subjected to uniform free field stresses is readily available from well known sources such as Timoshenko and Goodier (1970) and will not be elaborated here. Reyes and Deere (1966) presented results of elasto-plastic finite element analyses of underground openings in rock media. This treatment of the problem was extended to cover cavities of a variety of shapes.

3.3.2 Non-linear Solutions

It has long been recognized that the calculations of rock displacements and stresses surrounding tunnel openings are essential to the understanding of tunnel support mechanisms. The resulting ground support interaction diagrams may also aid in the selection and dimensioning of tunnel support elements. Brown et al. (1983) proposed two new analytical solutions to the simple axisymmetric plane

strain tunnel problem (Fig.3.2). The surrounding rockmass was initially subjected to a hydrostatic in-situ stress field. Both solutions utilized a non-linear strength criterion for the rockmass which is dependent on rock type and rockmass quality. The first was a closed-form solution that used an elastic-brittle-residual plastic stress-strain model with dilation occurring at a constant rate. The second solution used a trilinear elastic-strain softening-residual plastic stress-strain model with dilation occurring at a variable rate. Due to algebraic complexities, this second solution was not closed-form, but it was presented as a stepwise sequence of calculations.

Kennedy and Lindberg (1978) reported the investigation of the effect of the nonlinearity of the Mohr-Coulomb yield function in the problem of the elastic-plastic closure of a cylindrical cavity in rock subjected to an axisymmetric load. The paper focussed on the diametral closure of the cavity. The analysis gave rise to a closed form solution with rock yielding via the use of a piecewise linear approximation of a nonlinear Mohr-Coulomb type yield function, in which internal friction decreased with an increase of the mean normal stress. Strains were calculated by the adoption of the associative flow rule applied over the entire plastic zone. In the analyses, the combination of the internal and the free field pressures were applied in such a way that the internal pressure reached its maximum value before the occurrence of rock yielding.

Numerical results involving the variation of the radial closure of the cavity with respect to the increasing free field stress were presented. The case where full dilatancy took effect were selected for the purpose of comparison. The finite element mesh shown in Fig.3.3 was used. The Drucker-Prager yield criterion in conjunction with an associative flow rule was chosen to simulate nonlinear rock behaviour. The elastic constants used were as follows : Young's modulus of weak rock (tuff), $E_r = 6.9 \times 10^3$ MPa, Poisson's ratio, $\nu = 0.49$. In order to provide the upper and lower bound approximations of the nonlinear Mohr-Coulomb type failure envelope, two sets of soil material properties were employed where (i) cohesion, $c_1 = 8.0$ MPa, $\phi_1 = 16.7^\circ$ and (ii) $c_2 = 14.0$ MPa, $\phi_2 = 1.67^\circ$. Figure 3.4 shows the curves

obtained from the various sources. Kennedy and Lindberg (1978) had explained the discrepancy resulting from the nonlinear Mohr-Coulomb failure envelope and the linear approximation of it. It is clear that the curves from the finite element analyses do provide a bounded region within which the linear approximation curve from the analytical treatment is situated.

Sakurai and Abe (1979) proposed a method evaluating the closures of an underground opening and the pressures acting on the lining. The method was based on the displacements measured at a test tunnel so that it must be used in conjunction with field measurements. The medium surrounding the test tunnel was assumed to be linear elastic, although the mechanical behaviour of the real media was more complex; for instance, elasto-plastic behaviour. In this method, back analyses were conducted by considering the assumed elastic constants and the displacements measured at the test tunnel. When this hypothetical initial stresses was assumed for the elastic medium, the mechanical behaviour of the underground opening could be simulated by an elastic analysis. The proposed method was demonstrated by means of finite element computations via the use of the Mohr-Coulomb yield criterion. The non-linear elasto-plastic analysis was used in conjunction with 8-node isoparametric plane strain elements. For simplicity of calculation, it was assumed that the tunnel was located at a large depth underneath the horizontal ground surface such that the problems could be simulated as an infinite plate with a hole under biaxial compressive loading.

Based on the non-linear elasto-plastic analysis discussed above, numerical results from the finite element techniques used in this thesis were produced. Figure 3.5 shows a set of curves relating the initial free field stress and the radial displacements of the cavity. The material characteristics used in the analyses were as follows : (i) $E = 10,000$ MPa, $\nu = 0.3$ and 0.45 ; $c = 5$ MPa, $\phi = 30^\circ$. The initial stresses were $p = 35$ MPa and $q = 17.5$ MPa for the vertical and horizontal components respectively. The results obtained from the two different numerical analyses exhibit reasonable agreement with each other. In addition, Table 3.2 shows the compar-

	Sakurai and Abe (1979)	Pang
u^c/a	0.008	0.058
u^s/a	0.027	0.024

Table 3.2 : Unlined Cavity Subjected to Biaxial Free-Field
Stress - Comparison with Sakurai and Abe (1979)

Figure 3.4 : Radial Closure of Unlined Cavity - Comparison with Kennedy and Lindberg (1978)

Figure 3.5 : Radial Closure of Unlined Cavity - Comparison with Sakurai and Abe (1979)

isons between the maximum radial displacements for the case where $\nu = 0.45$, $p = 50$ MPa and $q = 25$ MPa. It is apparent that the two non-linear finite element predictions agreed with each other for radial displacements at the springline. The discrepancies in the displacements at the crown may be attributed to the difference in the yield criteria used.

3.4 Lined Cavities

3.4.1 Elastic Solutions

This section focusses on the examination of the buried cavity problem except that lining is considered around the opening. As a first step, it is convenient to solve this radially symmetric problem by considering linear elastic responses for both the continuum and the shell. The Airy stress function approach is selected to solve the problem.

3.4.1.1 Airy Stress Function Approach

In this section, a solution to the lined cavity problem using the Airy stress function is presented. Figure 3.6 illustrates the case where a hole surrounded by a shell within a medium, with an inner radius 'a' and an outer radius 'b' units is subjected to an axi-symmetric free field stress of p_0 . The shell has a Young's modulus of E_1 and a Poisson's ratio of ν_1 while the surrounding material has corresponding values of E_2 and ν_2 respectively. The bi-harmonic equation in its general form may be expressed by

$$\nabla^2 \nabla^2 \Phi^{(i)}(r) = 0 \quad (3.1a)$$

where $i = 1, 2$

$$\nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \quad (3.1b)$$

Figure 3.6 : Lined Cavity Subjected to Axi-symmetric Free-Field Stress

The radial and tangential stress components, $\sigma_{rr}^{(i)}$ and $\sigma_{\theta\theta}^{(i)}$, can be expressed in the following forms.

$$\sigma_{rr}^{(i)} = \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} \quad (3.2a)$$

$$\sigma_{\theta\theta}^{(i)} = \frac{\partial^2 \Phi}{\partial r^2} \quad (3.2b)$$

The general solution of the governing differential equation takes the form

$$\Phi^{(i)}(r) = A \ln r + B r^2 \ln r + C r^2 + D \quad (3.3)$$

where A, B, C and D are constants.

From compatibility considerations it can be shown that $B = D = 0$ for the plane strain situation (Timoshenko and Goodier, 1970). Hence the reduced form of the Airy stress function is given by

$$\Phi(r) = A \ln r + C r^2 \quad (3.4)$$

The radial stress may then be expressed as

$$\sigma_{rr}^{(i)}(r) = \frac{A^{(i)}}{r^2} + 2C^{(i)} \quad (3.5)$$

and likewise the tangential stress is given by

$$\sigma_{\theta\theta}^{(i)}(r) = -\frac{A^{(i)}}{r^2} + 2C^{(i)} \quad (3.6)$$

From Fig.3.6, several boundary conditions may be imposed as follows.

$$\sigma_{rr}^{(2)}(\infty) = p_0 \quad (3.7a)$$

$$u_r^{(1)}(b) = u_r^{(2)}(b) \quad (3.7b)$$

$$\sigma_{rr}^{(1)}(b) = \sigma_{rr}^{(2)}(b) \quad (3.7c)$$

$$\sigma_{rr}^{(1)}(a) = 0 \quad (3.7d)$$

where u_r represents the radial displacement with the superscripts '1' and '2' referring to the shell and the surrounding medium respectively.

Since the Poisson's ratio is considered to be an insensitive parameter, let $\nu_1 = \nu_2 = \nu$. Consider the soil-structure interface where $r = b$ and a series of algebraic manipulations, the following expressions are obtained.

$$\sigma_{\theta\theta} = -A_1 \left(\frac{1}{b^2} + \frac{1}{a^2} \right) \quad (3.8)$$

$$\sigma_{rr} = \frac{A_2}{b^2} + p_0 \quad (3.9)$$

where

$$A_1 = \frac{a^2}{a^2 - b^2} (A_2 + b^2 p_0) \quad (3.10a)$$

$$A_2 = \frac{(-\alpha - \beta - \gamma)}{(-\alpha + \beta - \gamma)} b^2 p_0 \quad (3.10b)$$

and

$$\alpha = \frac{(1 - \nu^2)}{E_2} \quad (3.11a)$$

$$\beta = \frac{(1 - \nu^2)}{E_1} \frac{(a^2 + b^2)}{(a^2 - b^2)} \quad (3.11b)$$

$$\gamma = \nu(1 + \nu) \left(\frac{1}{E_2} - \frac{1}{E_1} \right) \quad (3.11c)$$

The results obtained from these derived equations are then compared with some numerical results obtained from the finite element code used in this thesis. Table 3.3 illustrates how the radial and hoop stresses vary with the ratios D/t and E_1/E_2 , where D is the average diameter of the hole, t is the thickness of the shell, E_1 and E_2 are the Young's moduli of the shell and the solid respectively. The range of D/t ratios varies from 25 (large thickness) to 200 (medium thickness); and the E_1/E_2 ratios range from 50 to 3000. These values are chosen such that a wide variety of soil-structure interaction problems within our scope of investigation are addressed. The hoop stresses are obtained from stresses calculated at integration points within the shell elements. The radial stresses are obtained from integration points within the solid elements closest to the soil-shell interface.

Moreover, it can be observed that the stress values obtained from the two methods are closest to each other when the D/t ratio is small. Larger discrepancies are observed when this ratio becomes large, i.e. a medium to thin shell situation. But for all practical purposes within our scope of investigation, the correlation between the results obtained from the finite element technique and the theoretical solution is considered satisfactory.

3.4.2 Non-linear Solutions

Sakurai and Yamamoto (1976) proposed an analytical approach for the direct determination of the maximum earth pressure acting on the lining of a tunnel constructed at a large depth underneath the ground surface. A time dependent analysis was not required to obtain the maximum pressure. Techniques similar to that of an elastic analysis were employed although the medium around the tunnel was assumed to have a viscoelastic behaviour. Finite element analyses were conducted to solve geometrically more complicated problems such as elliptical and horseshoe-shaped tunnels. The ground medium was assumed to be homogeneous and isotropic. The effects of the relative rigidity of the linings were discussed.

A closed-form solution for the determination of the true ground pressure acting on tunnel supports in the rockmass was presented in Ladanyi (1974). Although

		Hoop stresses		Radial stresses	
D/t	E ₁ / E ₂	p_h^{SF}/p_0	p_h^{FE}/p_0	p_r^{SF}/p_0	p_r^{FE}/p_0
25	3000	16.8	17.4	1.40	1.36
50	3000	34.1	36.3	1.39	1.36
100	3000	68.5	75.8	1.38	1.35
200	3000	136.0	146.7	1.37	1.35
25	500	16.6	16.4	1.37	1.33
50	500	33.2	32.6	1.35	1.33
100	500	63.9	65.2	1.29	1.32
200	500	122.4	122.3	1.23	1.31
25	50	14.5	14.8	1.20	1.20
50	50	25.7	28.0	1.05	1.13
100	50	49.5	50.9	1.00	1.05
200	50	99.5	91.2	1.00	0.96

Table 3.3 : Comparison of Hoop and Radial Stresses Obtained from Airy Stress Function and Finite Element Approaches

only valid for a simple tunnel geometry (e.g. circular) and field stress conditions, the solution offered the advantage of taking into consideration a number of significant characteristics of the rockmass, such as its linear or non-linear failure envelope, its loss of strength after failure associated with dilation, as well as its strength decrease with time. The solution was able to produce a reasonable first estimate of the true ground pressure, in particular at high field stresses and in creep-sensitive rock materials.

3.5 Deeply Buried Cylinder Subjected to Concentrated Loads - Plane Strain

In the last section, the deeply buried cylinder problem was dealt with by applying distributed loads. This section focusses on the investigation of the responses of the shell around the hole when the plate is subjected to concentrated loads applied as shown in Fig.3.7. The horizontal load can assume a different value from the vertical load. This configuration depicts a typical practical situation where a 'deeply' buried tube is subjected to a load applied vertically above the crown. The horizontal load may be considered as the resultant of the induced lateral stress on the tube. The value of the latter depends on the K_0 value of the soil.

3.5.1 Finite Element Analysis

The finite element mesh used in this section is the same one used in the last section. Since this is also a plane strain situation, all out-of-plane nodes in the 'z' direction are restrained from movement. The concentrated loads are applied as shown in Fig.3.7. Two cases where $P_1 = P_3$ ($P_v = P_h$) and $P_1 = 2P_3$ ($P_v = 2P_h$) are considered.

Figure 3.8 illustrates the variation of the maximum hoop stress $\sigma_{\theta\theta}/P_1$ with the modular ratio ratio E_p/E_s . Also, D/t ratios of 100 and 200 corresponding to shells of medium thickness were used. It is apparent that the maximum hoop stress only increased with the modular ratio when the latter was not exceeding 500. All the curves are more or less horizontal for E_p/E_s values greater than 1000.

Figure 3.7 : Lined Cavity Subjected to Concentrated Loads

Figure 3.8 : Lined Cavity Subjected to Concentrated Loads -
Maximum Hoop Stress vs. Modular Ratio

Figure 3.9 illustrates the circumferential distribution of hoop stresses around the shell. The highest stresses were concentrated at the immediate vicinity where the point loads were applied. Remarkably uneven distribution was observed for the two cases where $P_1 = 2P_3$ ($P_v = 2P_h$). Similarly, radial soil pressure was plotted against the angle measured from the vertical (Fig.3.10). Substantially higher pressures were exerted on the crown of the tube when $P_1 = 2P_2$ ($P_v = 2P_h$).

The above example illustrates to a certain extent how the shell element behaves in relation to the surrounding medium when the system is subjected to external concentrated loads. The analyses performed were by no means complete but they had served the purpose of illustrating how the shell hoop stresses and the soil radial pressures varied with the change in values of certain important variables.

3.6 Shallowly Buried Cylinders Subjected to Uniform Surcharge Loads - Plane Strain

3.6.1 General

The problem of shallow buried pipelines (Fig.3.11) subjected to uniform overburden pressure has been studied extensively. Duns and Butterfield (1968) compared the results obtained from three different approaches, namely : (i) Airy's stress function, (ii) Savin's analytical solution and (iii) two dimensional finite element solution. It should be noted that in the numerical scheme, straight line bar elements connected end to end were used to model the tube. Similar procedures to those used in the present research programme were employed in that paper to eliminate the rotational degrees-of-freedom at the soil-structure interface. Tube deflections and hoop stresses were the primary focus of that investigation. The main purpose of this section is to model the same soil-structure interaction problem by means of the proposed finite element techniques outlined in Chapter 2.

3.6.2 Finite Element Discretization

The shallow buried pipeline problem is discretized as shown in Fig.3.12. There

Figure 3.9 : Lined Cavity Subjected to Concentrated Loads
Circumferential Distribution of Hoop Stress

Figure 3.10 : Lined Cavity Subjected to Concentrated Loads
Circumferential Distribution of Soil Pressure

are a total of 363 nodes in the mesh with 40 solid elements used to model the soil and 8 shell elements used to model the tube. The boundary conditions are as shown in Fig.3.12. Since the length of the pipe along its centroidal axis is assumed to be large in comparison with its diameter, this problem could be analysed by adopting plane strain assumptions. This situation is achieved by restricting movement in the out-of-plane, or z , direction in the mesh.

In order to illustrate the influence of non-linearity of the surrounding soil, the Drucker-Prager failure criterion was used in conjunction with an associative flow rule to simulate the soil behaviour. The cases where the soil was 'purely' cohesive and 'purely' frictional were being analysed and the results compared with those obtained from linear elastic analyses. The soil-pipe interface was assumed to be in a fully bonded condition. The pipe was assumed to behave in a linear elastic manner throughout the analyses.

3.6.3 Numerical Results

Figures 3.13 and 3.14 illustrate the manner in which the vertical diameter of the pipe was closing when subjected to increasing overburden pressure. Different ratios of E_s/E_p and E_s/c were used. A purely cohesive material was modelled by assigning $\phi = 0$ and a purely friction material was assigned a value of $c = 0$. From the figures, it is apparent that a less stiff soil would lead to higher values of closure since the soil above the crown of the pipe could readily yield and thus losing the required capacity to sustain excess external pressure. Moreover, a purely frictional material would seem to be able to sustain more pressure thus reducing the diametral closure induced in the pipe.

The variations of the hoop stresses with the applied overburden pressure are plotted in Figs.3.15 and 3.16. Again the effect of non-linearity of the soil had rather significant effects on the hoop stresses induced in the pipe. In all cases, the purely elastic and the purely cohesive soil properties tended to form the upper and the lower bounds of the results while the results with purely frictional properties always lie in between the extremes. However, these trends could be altered if the values of

Figure 3.11 : Shallowly Buried Pipe Subjected to Uniform Overburden Pressure

Figure 3.12 : Finite Element Discretization of Shallow Buried Pipe

Figure 3.13 : Shallowly Buried Pipe - Overburden Pressure vs. Radial Closure ($E_p / E_s = 5000$)

Figure 3.14 : Shallowly Buried Pipe - Overburden Pressure vs. Radial Closure ($E_p / E_s = 50$)

Figure 3.15 : Shallowly Buried Pipe - Overburden Pressure vs. Hoop Stress ($E_p / E_s = 5000$)

Figure 3.16 : Shallowly Buried Pipe - Overburden Pressure vs. Hoop Stress ($E_p / E_s = 50$)

ϕ and E_s/c are changed.

3.7 Buried Cylinder Subjected to Surcharge Loading - Plane Strain

3.7.1 General

As discussed in the previous section, the importance of thorough research concerning the analysis and design of buried flexible pipes should not be overlooked. Fourie and Beer (1986) presented a series of finite element results on the simulation of original laboratory testing on full scale (1m diameter) pipes at the Transport and Road Research Laboratory in the United Kingdom and a corresponding set of centrifuge tests conducted at Cambridge University.

The finite element mesh used in that study corresponded to all the dimensions of the laboratory test facility. The 1m diameter pipe was buried in a medium dense sand with 0.25m cover. The test tank was 3m wide and there was thus a minimum of 1m between the steel pipe and the tank side wall. Surcharge loads were applied at ground surface right above the crown of the pipe through 0.6m wide plates. Since only loads which were symmetrical about the pipe centreline had been considered, only one half of the configuration needed to be discretized. Joint elements which allowed slippage between the soil and the pipe (Beer 1985) were introduced at the interface. The soil was represented by 20-noded isoparametric 'brick' elements and the steel pipe by three dimensional shell elements similar to those used in the present research programme. It should be noted, however, that the details of the coupling between the two elements were not described in the paper. Assuming plane strain conditions, all the results presented in the paper were obtained from plane strain analyses achieved by restraining the longitudinal displacement of a 1m long pipe.

3.7.2 Finite Element Modelling

The same configuration was discretized by a different mesh as shown in Fig.3.12. Similar 20-noded brick elements and 8-noded shell elements were used for this

purpose. There were a total of 363 nodes with 40 brick elements and 8 shell elements. All the dimensions corresponded to the original testing configuration. Plane strain conditions were simulated by restraining out-of-plane displacements for all the nodes. The boundary conditions were such that all the nodes were free to move in the vertical direction except for the base where all the nodes were fixed.

3.7.3 Numerical Results

3.7.3.1 Effects of Poisson's Ratio

Tables 3.4a, 3.4b and 3.4c show the comparison among the numerical results obtained from the two finite element procedures discussed previously. The first series of analyses assumed linear elastic behaviour for both the steel pipe and the sand. The Young's modulus for the pipe was 207×10^6 kPa. A base value of 42,000 kPa was used as the Young's modulus of the soil. The shell thickness was fixed at 6mm and the applied surcharge load was 99.2 kPa. Table 3.4a shows the variations of the maximum hoop stresses with changing Poisson's ratios obtained from the two different finite element calculations. The influence of the Poisson's ratio value ranging from 0.2 to 0.4 was very slight. The two sets of values agreed remarkably well with each other as shown.

3.7.3.2 Effects of Soil Modulus

The results of the influence of the Young's modulus on the maximum hoop stress is shown in Table 3.4b. Three different values of 4,200 kPa, 42,000 kPa and 420,000 kPa were used for the analyses. It was expected that the use of such wide range (two orders of magnitude) of moduli values should have a greater influence on induced stresses than the effect created by the variation of the Poisson's ratio. However, the results from Hourie and Beer (1986) indicated virtually no change in the maximum hoop stress while the values obtained from the present research indicated only very slight changes. A constant Poisson's ratio of 0.30 was used for all the analyses. Most of the changes in hoop stresses occurred at the crown region whereas the maximum hoop stress was located near the shoulder of the pipe.

3.7.3.3 Effects of Soil-Pipe Interface Condition

It has long been recognized that the representation of the buried pipe problem by purely elastic material properties is inadequate and some allowance for slippage at the interface between the pipe and the soil is necessary. Table 3.4c shows the effects of varying the interface condition on the maximum hoop stresses. Results are shown for the two series of analysis each having three separate interface conditions. The analysis assuming linear elastic material properties is compared with the analysis in which the shear modulus of the joint is equal to one-third the shear modulus of the sand (all other properties remained the same). The third comparison is made between assigning elasto-plastic strength parameters of $c = 0$, $\phi = 30^\circ$ and assigning a joint shear modulus of 1.0×10^{-5} . Details of the joint element used in the Fourie and Beer (1986) paper can be found in Beer (1985) while the formulation of the interface element used in the present research can be found in Chapter 2. The introduction of the joint element, regardless of properties, can be seen to influence the hoop stress significantly. All the analyses gave higher maximum hoop stress for the pipe.

It has been demonstrated in this section that the finite element formulation employed in the present research programme gives satisfactory results when compared with those obtained from a similar formulation using a different discretization pattern. Thus these results further reinforce the suitability of this type of numerical technique in solving plane strain problems concerning buried structures in soils. More complex soil-structure interaction problems of a three dimensional nature are to be dealt with in the following chapters.

3.8 Comparison with Linear Elastic Three-Dimensional Finite Element Results

As discussed previously, three dimensional finite element analysis results for buried structures are scarce in the engineering literature. Some papers available (e.g. Katzenbach and Breth, 1981; Nath, 1983) are not suitable for comparison in

Effects of Poisson's ratio $E_s = 42 \text{ MPa}$		
	Pang	Fourie and Beer (1986)
ν	$P_{h(\max)}$ (MPa)	$P_{h(\max)}$ (MPa)
0.20	2.46	2.35
0.30	2.50	2.55
0.40	2.57	2.70

Table 3.4a : Effects of Poisson's Ratio on Maximum Hoop stress - Comparison with Fourie and Beer (1986)

Effects of soil modulus ($\nu = 0.30$)		
	Pang	Fourie and Beer (1986)
E_s (MPa)	$P_{h(\max)}$ (MPa)	$P_{h(\max)}$ (MPa)
4.2	3.02	2.55
42.0	2.50	2.55
420.0	1.67	2.55

Table 3.4b : Effects of Soil Modulus on Maximum Hoop stress - Comparison with Fourie and Beer (1986)

Effects of interface conditions		
	Pang	Fourie and Beer (1986)
Interface	$P_{h(\max)}$ (MPa)	$P_{h(\max)}$ (MPa)
$G_i = G_s$	2.50	2.15
$G_i = 1/3 G_s$	2.93	2.35
$G_i \ll G_s$	2.97	2.55

Table 3.4c : Effects of Interface Conditions on Maximum Hoop Stress - Comparison with Fourie and Beer (1986)

view of the scope of the present research. O'Rourke and Ahmed (1985) summarizes the results of linear elastic three-dimensional finite element analyses to evaluate the strains and displacements sustained by cast iron pipelines parallel to shallow trench construction. The analytical results compare well with field measurements. The most severe strains are caused by surface construction loads. The bending strains are closely related to the soil modulus and are not influenced significantly by the pipe diameter for the range of sizes considered. Due to the various limitations of the available computing facilities, it is not possible to have the same number of elements in the mesh as that reported in the paper. In the present analysis, there are 307 nodes with 44 solid elements while a total of 16 shell elements are used to simulate the pipeline. Figure 3.17 shows the comparison between the results obtained from both finite element analyses for two different relative stiffness of the soil-pipe system. It is seen that there are differences in the values and the locations of the maximum tensile and compressive strains. These discrepancies may be attributed to the following : (i) the finite element mesh used in the present analysis is not as fine as the one used in the analysis presented in the paper and (ii) there may be a difference in the method of simulation of the pipe between the two approaches.

Figure 3.17 : Buried Pipeline Adjacent to Shallow Trench - Longitudinal Strain Distribution Along Pipe Bottom Chord

Chapter 4

Geomechanical Properties of a Sand and a Sand-Bentonite Mixture

4.1 General

The determination of constitutive parameters for the soil media is an important phase in the numerical analysis of the various soil-structure interaction problems. In this thesis, the constitutive models used in connection with the numerical implementations focus on elastic-plastic models which are based on the Drucker-Prager and the Critical State yield criteria. It is essential to determine the numerical values of the constitutive parameters governing the modelling of the deformability and strength characteristics of the soil. The objective of the experimental programme reported in this chapter is to obtain the material parameters of the Critical State model used to simulate the mechanical behaviour of the sand-bentonite mixture under conditions of quick or undrained loading. This mixture is proposed to be used as a buffer material in the Canadian nuclear waste disposal endeavours (Selvadurai, 1988a,b; Lopez, 1984). Such information on material properties will be utilized in the theoretical examination of the container-buffer-rockmass interaction during possible discontinuous movements along rock fracture planes intersecting the disposal vault borehole (see Chapter 7). The theoretical predictions are based on non-linear finite element simulation of the problem. The use of conventional triaxial compression tests and direct shear tests is necessary in order to evaluate the elastic constants and the strength parameters. Grain size distribution analyses and compaction tests are also essential for providing valuable information on soil composition and density. In the ensuing sections, the discussion will focus on the evaluation of the constitutive parameters which govern the behaviour of the buffer material.

4.2 Buffer Mixture

Clay based buffer materials have been proposed for use in nuclear waste disposal endeavours (Pusch et al., 1985; Radhakrishna, 1984; Selvadurai, 1988a,b). In such a disposal activity involving deep rock repositories the buffer material is used to isolate the waste container from a borehole which houses containers of radioactive wastes (Fig.4.1). Within the context of the Canadian Nuclear Waste Disposal programme, the buffer material is a mixture of bentonitic clay and quartz sand. The mix composition of the buffer is organized in such a way that the functional requirements of mechanical strength, permeability, swelling capabilities and heat conduction are simultaneously satisfied. This buffer material is intended to serve as a barrier for the chemical absorption of the radionuclides which may be released either by accidental damage or the natural breakdown of the container. In the borehole emplacement scheme shown in Fig.4.1, the buffer fills the annular space between the rockmass and the container. During the expected lifetime of the disposal vault, a variety of loading configurations may result in distress to the waste container. One such situation could involve the stressing of the container due to movements at rock joints or fractures which intersect the borehole. Rock fractures could develop at the vault region either due to stress relief as a result of the creation of the vault system, due to the thermal gradients induced by the radioactive waste or due to earthquake activity. Furthermore, in view of the time scales involved in the corresponding waste disposal situation, it is clearly not possible to effectively stabilize (grouting or rockbolts) the entire rockmass in the vicinity of the repository in order to prevent the occurrence of fractures. Consequently the buffer material should possess sufficient *plasticity* to safeguard the container from detrimental effects of rock movements at rock fracture zones. A critical situation could involve the discontinuous movement at a fracture plane which can intersect a disposal vault borehole (Fig.4.2).

To achieve many of the desirable attributes of the buffer, it becomes necessary to compact the buffer material, with a predominantly sodium bentonite clay

Figure 4.1 : Typical Cross-Section Through the Waste Disposal Vault - TEC-2 Container Configuration (after Wardrop et al.,1985).

Figure 4.2 : Possible Orientation of Plane of Discontinuity with the Waste Disposal Vault

mineral, to very high densities (up to 2.0 Mg/m^3). Such high densities can be attained by pre-compacting the buffer prior to emplacement in the vault (Pusch, 1985). Engineering studies suggest that for the Canadian concept, owing to the large number of containers needed for the disposal of radioactive waste and the potential use of clay-sand mixtures, in-situ compaction of the buffer may be a more appropriate alternative (Wardrop, 1985).

The dry density of pure clays which are compacted in-situ can be as low as 1.3 Mg/m^3 ; however, by mixing sand with the clay, the attainable dry density can be increased. The addition of sand can also decrease the creep potential, increase the thermal conductivity, decrease the hydraulic conductivity, and improve the strength properties of the mixture. For these reasons, a mixture of 50 per cent sand and 50 per cent clay by mass appears to optimize the physical properties of the buffer (Dixon et al., 1985).

4.3 Components of the Buffer

The principal components of the buffer mixture are a graded silica sand (Indusmin) and sodium bentonite (Avonseal); both recommended and supplied by the Atomic Energy of Canada Limited (AECL). The two components are to be mixed in equal proportions by weight and compacted to its maximum dry density in order to serve all the required purposes. Relevant properties such as swelling potential, hydraulic conductivity and creep behaviour are being investigated and reported elsewhere (Dixon and Woodcock, 1986). The following sections concentrate on the evaluation of the strength and deformability characteristics of the buffer mixture and its components pertinent to the numerical investigations on the effects of rock joint movement on the container. The results of the laboratory testing programme are to be presented in the following sequence : (i) grain size distribution analysis, (ii) compaction tests (standard and modified Proctor), (iii) direct shear tests, (iv) triaxial compression tests and (v) evaluation of material parameters that may be used in incremental non-linear three dimensional finite element analyses of a

container-buffer-rockmass interaction problem.

4.3.1 Sodium Bentonite

The clay used in the buffer mixture is a sodium bentonite, sold commercially as 'Avonseal', produced by Avonlea Mineral Industries at Regina, Saskatchewan. The natural material comes from the Bearpaw Formation of the Upper Cretaceous Age in southern Saskatchewan. The material is a whitish grey powdered clay obtained by controlled, dried (less than 120° C) sand grinding processes on the natural soil (Quigley, 1984; Dixon and Gray, 1985). The water content of the commercially available material is about 6% to 8%. The bentonite consists predominantly of sodium montmorillonite (79%); among other minerals are illite, quartz, feldspar, gypsum and carbonate. The bentonite is predominantly a clay (montmorillonite) material with a substantial proportion of silt. The bentonite has a liquid limit of 250% and a plasticity index of 200%. The material has a high affinity for water; a property typical of the smectite mineral present. According to the Unified Soil Classification System, the sodium bentonite is a highly plastic clay with a group symbol of CH.

4.3.2 Silica Sand

The particle size distribution was selected using the Talbot-Richart formula (Dixon and Woodcock, 1986) to facilitate the attainment of a high density in the compacted buffer. It should be noted that a high density is desirable in nuclear waste containment endeavour since it reduces the hydraulic conductivity and increases the absorption of radionuclides by the buffer material.

As discussed earlier, the sand used in the mixture is intended to increase the strength characteristics of the buffer without greatly reducing the swelling properties of the sodium bentonite. The particle size distribution of the sand shown in Fig. 4.3 was specified to yield high maximum compacted density (Yong et al, 1986). Table 4.1 shows the proportions of the buffer constituents.

4.3.3 Compacted Buffer Mixture

Dixon et al. (1985) reported experimental results indicating that the average saturation of the specimens was only about 97%. This phenomenon was also observed in the test results which will be presented in the following sections. The incomplete saturation was in part due to the viscous nature of the adsorbed water layers around the clay particles which required a high input of compactive energy to achieve full saturation. In addition, due to the confining nature of the compaction mould, it is foreseeable that air bubbles could be trapped during specimen preparation and could not be driven out easily. They were most likely forced to dissolve into the pore water during the compaction process. Upon relief or removal of the compaction pressure some of the dissolved air could be released from solution, causing incomplete saturation of the specimens.

4.4 Specimen Preparation

Batches of samples of the quartz sand were prepared according to the proportions as shown in Table 4.1. Grain size distribution analyses were performed according to the procedures set out in ASTM D421-58 and D422-63. Figure 4.3 shows the grain size curves for the sand and the bentonite. According to the Unified Soil Classification System, the sand is classified as a well-graded sand with a group symbol of SW.

The buffer mixture was prepared by mixing the sand and the clay in equal proportions by weight with the aid of a mechanical mixer. The grain size distribution of the mixture is also shown in Fig.4.3. Also shown are the grain size distribution curves of the mixtures with various proportions of sand and clay. The 50%- 50% sand-bentonite mixture was found to contain approximately 50% of sand, 20% of silt and 30% of clay particles. The well-graded nature of this soil would indicate that the mixture can be compacted to a high dry density. According to the Unified Soil Classification System, the material is considered to be a low plasticity silt or

PROPERTIES OF BUFFER CONSTITUENTS		
Grain Size	Sodium Bentonite	Quartz Sand
Clay (<0.002mm)	60	0
Silt (0.002 to 0.06mm)	39	5
Sand (0.06 to 0.2mm)	1	19
(0.2 to 1mm)	0	58
(1 to 2mm)	0	16
(>2mm)	0	2
Unified Classification	CH	SW
Liquid Limit (%)	250	-
Plastic Limit (%)	49	NP

Table 4.1 : Properties of Buffer Constituents

Figure 4.3 : Grain Size Distribution of Buffer Mixture - Different Proportions by Weight

clay with a group symbol of ML or CL.

The test specimens used in the triaxial tests were prepared from cylindrical specimens of the compacted 50%-50% sand-bentonite mixture. The preparation procedures adopted were similar to the modified Proctor tests discussed earlier. All the specimens were compacted to the pre-determined dry density of 1.73 Mg/m^3 at a water content of 18.0% in a split mould 4-inch diameter by 8-inch long such that sample disturbance could be minimized during sample extraction. Since the split mould was slightly longer than the conventional compaction mould, some modifications as to the number of compacting layers and the corresponding blow counts had to be made such that the applied compactive effort (energy per unit volume of compacted material) remained the same as that specified in the ASTM standards.

De-aired water was used throughout the testing programme. Prior to mixing, the silica sand and the sodium benonite were oven-dried for 24 hours at 110°C (ASTM,1980). After cooling to room temperature, the required amount of water and silica sand were added to a bowl attached to a mechanical mixer. Bentonite was than added slowly in several batches to the sand-water mixture. Constant stirring was necessary in order to ensure that the particles were thoroughly mixed. The mixture was then stored in an air-tight container for 2 days before compaction in the split mould to allow for the attainment of uniform moisture distribution.

4.5 Test Details and Results

The following sections focus on the discussion of the test procedures and results of the compaction tests, direct shear tests and triaxial tests conducted on samples of the sand-bentonite mixture.

4.5.1 Compaction Tests

The dry density - moisture content relationships for the mixture and the sand are shown in Fig.4.4. Both the standard and the modified Proctor tests were

Figure 4.4 : Compaction Characteristics of Buffer Mixture

conducted on the 50%-50% sand-bentonite mixture. This is in contrast to the small scale tests reported by Dixon et al. (1984). Also shown in the figure are two points representing the maximum dry densities : (i) a second series of modified Proctor test points obtained at Carleton University, and (ii) quoted as the reference 'wet' maximum dry density in AECL documents (Dixon and Gray, 1985). The maximum dry densities obtained from modified Proctor tests range from 1.71 Mg/m^3 to 1.76 Mg/m^3 while the corresponding optimum water contents varied between 18% and 19%. A well defined curve with a distinct 'peak' was observed for the modified Proctor test conducted in a 4-inch diameter compaction mould. The dry densities were relatively insensitive to changing water contents on the 'dry' side of the optimum value for the standard Proctor tests. The maximum dry density in this case was found to be 1.42 Mg/m^3 with a corresponding optimum water content of 30%. The insensitivity can be attributed to the fact that the clay mineral had a high affinity for water. The applied compaction energy was insufficient to overcome the high shearing resistance of the attached water molecules; that is, the increase in the amount of water did not serve the purpose of 'lubricating' the particles in order to make it easier for them to reorient into a denser configuration (Lambe and Whitman, 1979).

The large difference in the applied compactive energy between the standard and modified Proctor tests can be attributed to the large difference in the maximum dry densities. As for the small scale tests, a similar trend was also observed for the water contents below optimum. Dixon et al. (1984) have observed that there was a high correlation between the data points obtained from the small scale and the modified Proctor tests. However, the compactive effort (energy input) and the type of test (dynamic, kneading, static) was not specified. The volume of soil used in the small scale test was only 25 cm^3 as compared to the 940 cm^3 used in the Proctor tests. If dynamic tests were conducted, the ratio of the diameter of the hammer to the thickness of each soil layer would have been an important factor influencing the effectiveness of compaction. If kneading compaction was employed, the dry density and its corresponding water content would have been proportional to the duration

of application of the pressure.

4.5.2 Direct Shear Tests

Quick direct shear tests were conducted on the sand-bentonite mixture at two different maximum dry densities corresponding to the standard and the modified Proctor tests. Figure 4.5a shows the stress vs displacement relationships for the samples at a maximum dry density of 1.76 Mg/m^3 with a moisture content of 18.1%. The applied normal stresses were chosen to be 20 kPa, 40 kPa, 100 kPa and 200 kPa respectively. These curves clearly illustrate the dense and brittle nature of the mixture due to the presence of pronounced peak strengths and the subsequent strain softening behaviour. Small amount of dilation at the beginning of each test was also apparent as shown in the curves relating vertical and horizontal displacement (Fig.4.5b). The relatively small volume increase for such a dense material might be attributed to the relatively large normal stresses that were applied to the samples. When the peak strengths were plotted on a graph of shear stress vs normal stress (Fig.4.6), the points were sufficiently collinear to define the Mohr-Coulomb failure criterion with an angle of internal friction $\phi = 30^\circ$ and an apparent cohesion $c = 280 \text{ kPa}$. A corresponding post peak strength failure envelope can be characterized by the following parameters; $\phi_p = 30^\circ$, $c_p = 10 \text{ kPa}$. Within the context of the investigation concerning transverse rock joint movements across the waste disposal vault, the post peak strength values may seem to be more appropriate for use as input parameters in the finite element analyses due to the fact that at the plane of discontinuity within the buffer material, strain softening is expected to occur as the lateral displacement increases.

A similar series of tests were also performed on samples compacted to a maximum dry density of 1.40 Mg/m^3 at an optimum moisture content of 32%. The strength parameter values were similar to those obtained by the standard Proctor tests. Figures 4.7a, 4.7b and 4.8 summarize the results from this series of direct shear tests. A strain softening behaviour is also observed in these tests. The reduction in the shear stresses was, however, much less abrupt than that which was

Figure 4.5a : Direct Shear Test - Shear Stress vs Horizontal Displacement
(Modified Proctor Compaction)

Figure 4.5b : Direct Shear Test - Vertical vs Horizontal Displacement
(Modified Proctor Compaction)

Figure 4.6 : Direct Shear Test - Shear Stress vs Normal Stress
(Modified Proctor Compaction)

Figure 4.7a · Direct Shear Test - Shear Stress vs Horizontal Displacement
(Standard Proctor Compaction)

Figure 4.7b : Direct Shear Test - Vertical vs Horizontal Displacement
(Standard Proctor Compaction)

Figure 4.8 : Direct Shear Test - Shear Stress vs Normal Stress
(Standard Proctor Compaction)

observed previously. This is particularly relevant when the applied normal stresses were in the range 100 kPa to 200 kPa. This material is expected to behave in an elastic-plastic manner at high normal stresses. Moreover, it should be noted that the Mohr-Coulomb failure envelope is pronouncely non-linear as shown in Fig.4.8. These curves seem to suggest that as the normal (confining) stress increases, the buffer mixture at this particular density displays an apparent behaviour which varies from a frictional response to a purely cohesive response. A friction angle of 10° and an apparent cohesion of 70 kPa were measured from the 'peak' failure envelope while the corresponding post peak envelope revealed a lower c_p value of 50 kPa. These observations seem to suggest that a high confining stress is necessary to induce the desirable plastic behaviour of the buffer material. Hence the surcharge to be imposed by the backfill above the buffer material is expected to assume an important role in enhancing the performance of the buffer as it relates to the deformability and plastic-type strength characteristics.

Two other sets of direct shear tests were performed separately on saturated sand and sand which was compacted according to the modified Proctor procedures (Figs.4.9 and 4.10). All the stress-displacement curves exhibit distinct peak strengths and subsequent strain-softening behaviour. The corresponding Mohr-Coulomb strength envelopes give rise to a peak friction angle of 40° and a residual value of 32° . These results illustrate a well known fact that compaction does not substantially alter the density of sand, thus further justifying the mixing of sand with the sodium bentonite to form a material with a much wider range of grain size distribution such that a higher maximum dry density may be attained.

4.5.3 Triaxial Tests

Quick undrained triaxial compression tests were conducted on the compacted buffer mixture in order to obtain the stress-strain and shear strength parameters. In these triaxial investigations no attempt was made to measure the pore fluid pressures in the compacted, partially saturated buffer. One justification for the choice of a quick (undrained) triaxial test is the very low permeabilities which are

Figure 4.9a : Direct Shear Test - Shear Stress vs Horizontal Displacement (Compacted Sand)

Figure 4.9b : Direct Shear Test - Shear Stress vs Normal Stress (Compacted Sand)

Figure 4.10a : Direct Shear Test - Shear Stress vs Horizontal Displacement (Saturated Sand)

Figure 4.10b : Direct Shear Test - Shear Stress vs Normal Stress (Saturated Sand)

associated with the compacted buffer material. Furthermore, in the context of the envisaged application, namely the problem of interaction between the waste container and the buffer during discontinuous rock movement, it is plausible to assume that the rate of such discontinuous movement is such that the buffer region essentially responds in an undrained fashion. It may also be noted that in an actual deep repository, the buffer region may be fully saturated due to the presence of high fluid pressures of the order of 10 MPa.

Figure 4.11 illustrates the stress-strain curves for samples of the buffer material sheared under confining pressures of 0 kPa (unconfined), 100 kPa, 200 kPa, 300 kPa, 500 kPa and 1000 kPa. These tests were conducted at a strain rate of 0.635%/min. The test results indicate the characteristic 'brittle' behaviour of the compacted buffer material especially in the low range (0 - 100kPa) of confining stresses. The very distinct failure planes were oriented approximately 50° to 52° with the σ_3 direction. With increasing cell pressure, the samples of the compacted buffer displayed traits of strain-hardening behaviour. The initial tangent modulus E was found to be approximately 140 MPa. From the results of the investigations, i.e. both direct shear and triaxial tests, it would appear that the buffer material will exhibit plasticity when the confining stress is higher than a certain critical value. These findings seemed to agree qualitatively with similar conclusions drawn from triaxial testing of the buffer material at lower dry densities conducted by Graham et al (1986); Sun (1986) and Wan (1987).

The peak strength values obtained from the triaxial tests are used to plot Figs.4.12 to 4.14. In Fig.4.12, $q = (\sigma_1 - \sigma_3)/2$ and $p = (\sigma_1 + \sigma_3)/2$, the test points seemed to be very well represented by the conventional K_f line as inferred by the Mohr-Coulomb failure criterion. The cohesion, c_u , was found to be approximately 600 kPa and the angle of internal friction, ϕ_u , was estimated at 10°. The latter values agreed with the observation that the failure planes made an angle of about 50° with the horizontal; i.e. the theoretical prediction for the orientation of the failure plane is $\theta = (45 + \phi/2)$. Also shown in the figure are the data points

Figure 4.11 : Triaxial Test - Deviator Stress vs Axial Strain
 (Modified Proctor Compaction)

Figure 4.12 : Direct Shear and Triaxial Tests on Buffer Material
 q vs p plot (Mohr-Coulomb Yield Criterion)

Figure 4.13 : Triaxial Test on Buffer Material - $\sqrt{J_{2D}}$ vs J_1
 (Drucker-Prager Yield Criterion)

Figure 4.14 : Triaxial Test on Buffer Material - $\sqrt{3J_2D}$ vs $J_1/3$
(Critical State Yield Criterion)

corresponding to the direct shear tests conducted on the buffer mixture compacted to the same maximum dry density. These values were obtained by assuming that the samples were experiencing a state of pure shear. An apparent bilinear trend of the K_f line is observed. Furthermore, in order to facilitate the use of these test results in determining the parameters used for the purpose of defining the Drucker-Prager and the Critical State (modified Cam-clay) constitutive models, the same set of data points were plotted again on the $\sqrt{J_{2D}}$ vs J_1 and the $\sqrt{3J_{2D}}$ vs $J_1/3$ stress spaces; these plots are shown in Figs.4.13 and 4.14 respectively. The parameters for the two constitutive laws discussed above are defined as follows.

Drucker-Prager (plane strain case) :

$$\alpha = \frac{\tan \phi}{\sqrt{(9 + 12 \tan^2 \phi)}} \quad (4.1)$$

$$k = \frac{3c}{\sqrt{(9 + 12 \tan^2 \phi)}} \quad (4.2)$$

Drucker-Prager (axi-symmetric case) :

$$\alpha = \frac{2 \sin \phi}{\sqrt{3}(3 - \sin \phi)} \quad (4.3)$$

$$k = \frac{6c \cos \phi}{\sqrt{3}(3 - \sin \phi)} \quad (4.4)$$

Critical State (axi-symmetric case) :

$$M = \frac{6 \sin \phi}{(3 - \sin \phi)} \quad (4.5)$$

Figure 4.15 shows a typical pressure-volume change relationship for a hydrostatic test conducted on the buffer mixture. Volumetric strains were determined by considering the three principal stresses. The bulk modulus, K , estimated from these results corresponds to a value of 385 MPa. Considering the experimental values for E and K determined from the triaxial tests and assuming isotropic elastic

Figure 4.15 : Hydrostatic Test on Buffer Material - Cell Pressure vs Volumetric Strain

behaviour in the linear range, it can be shown that the Poisson's ratio is approximately 0.45.

4.6 Summary

Tables 4.2, 4.3 and 4.4 provide a summary of the average values of the measured and derived quantities relating to the deformability properties and strength characteristics of the buffer mixture. From the results of the compaction tests, it is found that the compacted sand-bentonite mixture is able to achieve the desired dry density. The modified Proctor compaction procedures are able to provide a higher dry density and a lower optimum moisture content than similar quantities obtained from the standard Proctor procedures.

It may be observed from the direct shear tests that the buffer mixture is an extremely brittle material. However, the extent of strain softening can be reduced by applying higher confining stresses. Dilatant behaviour is demonstrated in the tests where the buffer is compacted according to the modified Proctor procedures.

The triaxial test results provide valuable information on the deformability properties of the buffer mixture at relatively high confining stresses. Although the strength characteristics of the buffer mixture obtained separately from direct shear and triaxial tests cannot be directly compared with each other due to the difference in confining stresses, the bilinear trend of the K_f line shown on the $q - p$ plot does confirm the dilatant behaviour discussed previously. The typical hydrostatic test results display the characteristic features of the behaviour of the buffer mixture under isotropic stress conditions.

COMPACTION TESTS		
Compaction Method	$\gamma_{d(max)}$ (Mg/m^3)	m_{opt} (%)
Standard Proctor	1.42	30
Modified Proctor	1.73	18

Table 4.2 : Compaction Test Results

DIRECT SHEAR TESTS				
	Peak Strength		Post Peak Strength	
Compaction Tests	c (kPa)	ϕ (deg)	c_p (kPa)	ϕ_p (deg)
Standard Proctor	70	10	50	10
Modified Proctor	280	30	10	30

Table 4.3 : Direct Shear Test Results

TRIAXIAL TESTS				
Elastic		Drucker-Prager		Critical State
E (MPa)	ν	α	k (MPa)	M
140	0.45	0.07	0.74	0.4

Table 4.4 : Triaxial Test Results

Chapter 5

Modelling of Structural Element Subjected to Discontinuous Shearing

5.1 General

In engineering practice, one approach to investigate the soil-structure interaction problems, such as those involving buried pipelines, nuclear waste disposal vaults and foundation piles, is to conduct full-scale prototype field tests. However, as discussed previously, this approach is either too expensive or too exhaustive in both human and time resources. The other feasible alternatives which may be considered are (i) to conduct reduced scale laboratory experiments and (ii) to simulate the problems by means of numerical methods (finite element and boundary element methods etc.). In view of all these situations, it is decided that laboratory experiments are necessary to simulate the soil-structure interaction problems encountered in this thesis.

This chapter focusses on the discussion of the experimental simulation of the flexural responses of a structural element embedded in a soil medium subjected to transverse shear dislocation movement. Some of these experimental results will be compared with computational results from finite element simulation of the experiment. The purposes for conducting these experiments are two fold. Firstly, qualitative information of the responses of the structural element can be obtained. Secondly, further verification of the capabilities of the developed finite element procedures may be achieved.

5.2 Experimental Design

The apparatus used in these series of reduced scale model experiments was specially designed at the Geotechnical Research Laboratories and fabricated at the Mechanical Engineering Workshops at Carleton University. Figures 5.1 and 5.2 show the elevation and plan of the apparatus respectively. It was originally designed to model a borehole within the the context of the Canadian nuclear waste disposal programme. A horizontal discontinuity at mid-section of the tube was used to simulate a fracture plane intersecting at right angle to the borehole. However, other soil-structure interaction problems having a similar configuration can also be investigated.

The borehole portion of the apparatus was fabricated from a mild steel pipe. Two pieces of box sections were used to reinforce the connection of the split cylinder to the reaction support which was provided by a 1m long I-beam. Steel plates were employed to model the plane of discontinuity and to provide base support to the whole apparatus. As noted earlier, the main feature is the assembly of four semi-circular halves to form the 1m long rigid cylinder. The cylinder has a wall thickness of 19mm and internal diameter of 245mm. A total of twelve 12.5mm diameter bolts were used to ensure perfect contact of the two halves during the course of an experiment. The reasons for designing the split cylinder and reinforcing it with such large diameter bolts are as follows. First of all, the original purpose of the apparatus was to investigate the behaviour of the model container embedded in a sand-bentonite buffer mixture compacted to a very high density (see Chapter 4). Due to the anticipated strength of the buffer, it was necessary to provide adequate reinforcement to the split cylinder. Secondly, after each experiment was completed, the cylinder could be opened to facilitate speedy and thorough emptying of the soil and the instrumented structural element. Owing to the change in the planning of the research programme, the apparatus was used instead to conduct experiments involving structural elements embedded in compacted sand; even though these

Figure 5.1 : Elevation View of 'Split Cylinder' Apparatus

Figure 5.2 : Plan View of 'Split Cylinder' Apparatus

experiments might not necessarily require the type of heavy reinforcements associated with this apparatus.

The lateral displacement is induced by a 2-ton capacity worm gear actuator installed in the space between the cylinder and the I-beam. The latter is used to provide the reaction force necessary to generate motion. In the case where a continuously strain-controlled experiment is to be conducted, the actuator can be connected to a motor. Displacements of up to 25mm are permitted. A load cell is also installed in between the actuator and the cylinder in order to monitor the load-displacement response of the system. Surcharge loadings are applied by placing dead weights on the top surface of the soil.

5.3 Experimental Programme

Prior to the examination of the flexural responses of the embedded structural elements in soil, shearing experiments were conducted on the empty cylinder in order to determine the friction between the two sliding steel plates at mid-section of the cylinder. The frictional force used in subsequent calculations were then determined by averaging the values obtained from these preliminary experiments. The accuracy of the DCDT—'s (Direct Current Displacement Transducers) were also examined by comparing their readings with the prescribed distances travelled by the shaft of the manually operated worm gear actuator.

5.3.1 Concrete Sand

Moist concrete sand was used as the soil medium for the entire testing programme. Particle size distribution analysis results were reported by Gnanendran (1988). The effective diameter, D_{10} , was found to be 0.27mm, the coefficient of uniformity, C_u , was calculated to be 3.0 and the coefficient of curvature, C_c , was found to be 0.95. By means of the Unified Soil Classification system, the soil was classified as a poorly graded, gravelly sand with little or no fines (group symbol of SP). Conventional direct shear tests were conducted on the sand (Gnanendran,

1988) at an average moisture content of approximately 4.5% and a bulk unit weight of 17.6 kN/m^3 . The shear strength parameters were found to be $c = 5.0 \text{ kPa}$ and $\phi = 40^\circ$.

The moisture content of the concrete sand was maintained between 4% and 5% during the reduced scale model tests. In order to achieve this consistent moisture content, the sand was oven-dried for 24 hours. Prior to each experiment the sand was cooled and mixed with a pre-determined amount of water which would result in a final mixture having the desired moisture content.

5.3.2 Instrumented Structural Elements

Experiments were carried out in the apparatus using moist concrete sand (5% moisture content) in conjunction with an aluminium strip and an aluminium tube both instrumented with strain gauges as shown in Figs. 5.3a and 5.3b. The strip is relatively flexible in bending and has a length of 1100mm, a width of 50mm and a thickness of 5mm. The tube has an outside diameter of 25mm, a wall thickness of 2mm and a length of 1100mm. Longitudinal strain measurements were conducted on only one side of the strip by means of strain gauges arranged as shown. The strain gauges were installed on the tube in such a way that measurements could be taken on both the 'passive' and the 'active' sides. The strain readings were originally measured in microstrains.

5.3.3 Data Acquisition System

All the wires from the strain gauges were connected to a Hewlett Packard HP86B data acquisition system. This system consisted of a multi-channel digital strain gauge reader, a 350mm (14in.) monitor on which screen display of experimental readings was possible and a hard disc drive (Figs. 5.4a and 5.4b). A data acquisition software package was also available. All the relevant data associated with an experiment could be recorded on a hard disc while hard copies could also be produced from the attached dot matrix printer.

Figure 5.3a : Schematic Diagram of Strain Gauge Arrangements on Aluminium Strip

Figure 5.3b : Schematic Diagram of Strain Gauge Arrangements on Aluminium Tube

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Figure 5.4a : Multi-Channel Strain Gauge Reader

Figure 5.4b : Monitor and Disc Drive of Data Acquisition System

5.3.4 Installation of Structural Element and Sand Compaction

The manual compaction of the buffer mixture was carried out by using the modified Proctor compacting equipment, i.e. a 10-lb hammer (see Bowles, 1978). The compactive effort was determined so that the specific energy applied to the sand (i.e., the energy input per unit compacted volume) compacted in the cylinder was identical to that which would be applied to the sand in a standard compaction mould. The sand was compacted in layers of approximately 10cm thick with 25 blows for each layer. This process was continued until the sand level had reached the top of the cylinder. The moisture content was kept constant at approximately 5%. The average bulk density of the compacted sand was found to be approximately $16.6kN/m^3$.

The installation of the structural element in the sand was a major concern in the experimental set-up phase. Since the structural element had to maintain its position at the middle of the split cylinder, it was decided that clamps could be used to hold the strip or tube in position. This procedure was particularly useful during sand compaction. In spite of the inconvenience created by the presence of the structural element, the sand could still be compacted to the desired density. Particular attention was paid to ensure adequate contact at the soil-structure interface. Tremendous care was required such that none of the strain gauges and their components were disturbed during compaction. The leads from the strain gauges were glued onto the surface of the structural element (strip or tube). The upper exposed surface of the borehole was covered with a clear polyethylene sheet to minimize premature loss of moisture from the sand surface prior to the commencement of an experiment. A step by step illustration of the entire experimental set-up process is shown in Fig.5.5.

5.4 Discontinuous Shear Test on Sand

Direct shear tests without reinforcements were conducted on concrete sand with

Step 1 : Instrumented structural element held in place by clamp

Step 2 : Sand compacted in layers inside cylinder

Step 3 : Interior of cylinder filled with sand

Step 4 : Clamp removed and ready for testing

Figure 5.5 : Procedures Showing Structural Element Installation and Sand Compaction

a moisture content of approximately 5%. The entire cylinder was filled by compacting the sand placed in 5 equal layers with 50 blows for each layer. The density was found to be around 16.6 kN/m^3 . Friction calibration was carried out before the placing of sand in each test such that the force required to shear along the two machined surfaces at mid-height of the cylinder could be determined. As mentioned before, all loading and displacement output were displayed on the monitor screen and recorded on hard disc by means of the data acquisition system (Fig.5.4). Displacement increments of 0.8 mm each were applied by manually turning the handle attached to the actuator.

Figure 5.6 shows the several load-displacement relationships corresponding to surcharge load values of 0, 5 kPa and 10 kPa. The surcharge was applied by the placing of dead weights on the surface of the sand. Two tests were performed for each surcharge value. From Fig.5.6, it is apparent that the sand was behaving elastically up to the 'yield' stress beyond which non-linear behaviour commenced. Plastic yielding continued until no more load could be taken by the sand. At this stage, the curves became horizontal indicating that a plastic state had been reached. The difference in yield stresses due to different surcharge values was small. These series of tests proved that the sand did behave in a non-linear manner and that surface surcharge did have an effect on both the yield and the ultimate loads.

5.5 Discontinuous Shear Tests with Embedded Strip

In the next stage of the testing programme, similar shear tests on the same sand with an embedded aluminium strip were conducted. In addition to all the testing procedures described in the previous sections, an extra DCDT was installed to measure the displacement of the strip relative to the induced displacement. The strain gauges and their wirings had obstructed sand compaction during the set-up stage of the experiment. Tremendous care was taken during the compaction of the sand in order to ensure full contact along the soil-strip interface before the test

Figure 5.6 : Stress-Displacement Relationship of Discontinuous Shear Test on Sand

commenced. After the soil was compacted and the required surcharge imposed, the strain gauges were 'zeroed' with the aid of a data acquisition computer programme used in conjunction with the HP 86B.

Figure 5.7 shows the load-displacement relationships with respect to the three surcharge values. Bi-linear relationships are observed for all the curves. No apparent 'ultimate' state is observed for the soil-strip system. It is apparent that the embedded strip did provide a reinforcing effect thus increasing the lateral load carrying capacity of the system. It is interesting to note that the presence of the strip did not significantly alter the value of the 'yield' stress as compared with the unreinforced case. Moreover, the curves in Fig.5.7 seems to suggest that a simple bi-linear constitutive model could well be adequate when this problem is analysed using numerical methods.

5.6 Numerical Simulation of Discontinuous Shear Tests with Strip

As shown in Figs.5.6 and 5.7, the soil was behaving in a linear elastic manner up to the yield stress beyond which a non-linear behaviour commenced. The Drucker-Prager yield criterion was employed for the purpose of constitutive modelling. The entire experimental set-up was discretized by using solid 'brick' elements and general shell elements. Figure 5.8 is a schematic illustration of the finite element mesh. The solid elements were used to model the soil and the cylinder while the shell element acting as a plate was used to model the aluminium strip. There were a total of 364 nodes and 54 solid elements, 42 of which were used to model the soil medium while the rest were used to simulate a 'rigid' cylinder. Boundary conditions were imposed in such a way that the outer edge of the half below the discontinuity was fixed. The upper half was allowed to move freely in the horizontal, x , direction. A total of 7 general shell elements were used in the form of plates to model the aluminium strip.

For the purpose of finite element simulation of the strip, the general shell

Figure 5.7 : Stress-Displacement Relationship of Discontinuous Shear Test with Strip

Figure 5.8 : Finite Element Discretization of Discontinuous Shear Test with Strip

element was used in the form of a 'thick' plate. It was realized that the transformations necessary for its implementation within this context were unnecessary. However, the shell element was still chosen for convenience. The following simplifications for the implementation of the shell element as a plate may be pursued as follows (Zienkiewicz, 1979).

- (i) the thickness, t , of the plate can be aligned with any one of the global x , y and z directions.
- (ii) the unit vectors, v_1 , v_2 and v_3 can be taken in the directions of x , y and z respectively.
- (iii) it is no longer necessary to transform stress and strain components to a local system of axes r , s and t since global definitions can be used throughout.

A fully bonded soil-strip interface was assumed in order to simulate a 'rough' surface. It should be noted that the strain gauges and wirings were expected to contribute to the overall 'roughness'. Appropriate magnitudes of displacements were induced by means of the application of distributed loads in the x direction.

For all the analyses, the shell elements assumed the material properties of aluminium, i.e. a Young's modulus of 77,000 MPa and a Poisson's ratio of 0.35. Both elastic and elastic-plastic behaviour of the soil were considered. It should be noted that the sand was in fact a ' $c - \phi$ ' soil probably due to the presence of moisture. Soil anisotropy was not considered in the numerical analyses. As discussed earlier, Gnanendran (1988) conducted conventional direct shear tests on the concrete sand. A cohesion value of 50 kPa and a friction angle of 40° were determined at a bulk density of 17.6 kN/m^3 . In view of the margin of error of the experimental results that might exist due to compaction and moisture distribution effects, a selected range of values for c and ϕ were used in the analyses.

Figures 5.9, 5.10 and 5.11 show the finite element results as compared with the experimental results for the cases of 0, 5 kPa and 10 kPa surcharge respectively. In order to provide an 'upper' bound and a 'lower' bound to the experimental results, several analyses were performed with different values of soil modulus, E_s . It was

Figure 5.9 : Discontinuous Shear with Embedded Strip - Longitudinal Strain vs Induced Displacement (no surcharge)

Figure 5.10 : Discontinuous Shear with Embedded Strip - Longitudinal Strain vs Induced Displacement (5 kPa surcharge)

Figure 5.11 : Discontinuous Shear with Embedded Strip - Longitudinal Strain vs Induced Displacement (10 kPa surcharge)

Figure 5.12 : Discontinuous Shear with Embedded Strip - Longitudinal Strain Distribution Along Strip (no surcharge)

Figure 5.13 : Discontinuous Shear with Embedded Strip - Longitudinal Strain Distribution Along Strip (5 kPa surcharge)

Figure 5.14 : Discontinuous Shear with Embedded Strip - Longitudinal Strain Distribution Along Strip (10 kPa surcharge)

found that E_s values of 100 MPa and 125 MPa gave rise to the limits as shown. These values correspond to modular ratios, E_{strip}/E_s , of 770 and 620 respectively. The elastic analyses were sufficient to model the experiments even though local yielding of the soil adjacent to the plane of discontinuity was expected. The numerical results were not particularly sensitive to the applied surcharge. Figures 5.12 to 5.14 illustrate the comparisons between finite element results and experimental results regarding the distribution of longitudinal strains along the strip. The strength parameters of the non-linear analyses were chosen based on results reported by Gnanendran (1988). A cohesion of $c = 50$ kPa and ϕ values of 30° and 35° were selected. The range of friction angle was chosen to reflect the lower density of 16.6 kN/mm^3 used in the experiments under consideration. It should be noted that the difference in the friction angle did not alter significantly the strains induced on the strip due to discontinuous shearing.

5.7 Discontinuous Shear Tests with Embedded Tube

In order to provide further insight in the understanding of this class of soil-structure interaction problems, a similar series of direct shear tests was conducted in conjunction with an aluminium tube instrumented with strain gauges. Nine strain gauges were mounted on each side (passive and active) of the tube coplanar with the plane of symmetry (Fig.5.3). Aspects such as the application of load and the measurement of strains by means of the available data acquisition system were the same as those discussed for the previous experiments.

Figure 5.15 shows the load-displacement relationships with respect to the three surcharge values of 0, 5 kPa and 10 kPa. All the curves exhibit slightly non-linear trends at larger relative displacements. However, the bi-linear behaviour observed in the strip experiments is not significant here. As in the cases for the strip, no ultimate state can be observed in spite of the abrupt relative displacements. The tube had, however, shown to have provided a reinforcing effect to the entire system.

5.8 Numerical Simulation of Discontinuous Shear Tests with Tube

In this series of analyses, the cylinder-soil-tube system was discretized as shown in Fig.5.16. There were a total of 376 nodes and 54 solid elements, 36 of which were used to model the soil medium while the rest were used to simulate a 'rigid' cylinder. A total of 18 shell elements were used to model the aluminium tube. Due to the presence of strain gauges and their wirings, it was decided that the soil-tube interface could be considered as a completely bonded surface at which relative slippage was not allowed. Appropriate boundary conditions were applied to restrict the horizontal and transverse (x and y directions) movement of the lower half while displacements in the horizontal direction was allowed for the upper portion. Vertical displacements were allowed for all the elements except for the bottom of the mesh at $z = 0$. The aluminium tube was assumed to behave in an elastic manner throughout the analyses. A Young's modulus value of $77,000 \text{ N/mm}^2$ and a Poisson's ratio of 0.35 were used for the shell elements. The soil properties were the same as those used in the simulation of the 'strip' experiments.

Figures 5.17, 5.18 and 5.19 show the comparisons between elastic finite element results and experimental results for the variation of longitudinal strain, ϵ_{xx} , with the induced displacements. The numerical results were able to predict reasonably well the trend of the curves. Figures 5.20 to 5.22 show the results for the longitudinal distribution of ϵ_{xx} along the tube. Here the finite element solutions illustrate a much more abrupt change in signs for ϵ_{xx} at the region of intense shearing. Only the two extreme bounds in these analyses are shown in the diagrams. Better correlation between experimental and numerical results is possible if the Young's modulus of the soil is adjusted appropriately.

5.9 Summary

Numerical analyses results have shown that the experimental curves can

Figure 5.15 : Stress-Displacement Relationship of Discontinuous Shear Test with Tube

Figure 5.16 : Finite Element Discretization of Discontinuous Shear Test with Tube

Figure 5.17 : Discontinuous Shear with Embedded Tube - Longitudinal Strain vs Induced Displacement (no surcharge)

Figure 5.18 : Discontinuous Shear with Embedded Tube - Longitudinal Strain vs Induced Displacement (5 kPa surcharge)

Figure 5.19 : Discontinuous Shear with Embedded Tube - Longitudinal Strain vs Induced Displacement (10 kPa surcharge)

Figure 5.20 : Discontinuous Shear with Embedded Tube - Longitudinal Strain Distribution Along Tube (no surcharge)

Figure 5.21 : Discontinuous Shear with Embedded Tube - Longitudinal Strain Distribution Along Tube (5 kPa surcharge)

Figure 5.22 : Discontinuous Shear with Embedded Tube - Longitudinal Strain Distribution Along Tube (10 kPa surcharge)

theoretically be 'duplicated' within acceptable margins of error by altering relevant material parameters through the use of three dimensional finite element techniques. It has already been mentioned that the experimental determination of these parameters could only be treated as starting points. The soil modulus E_s is the most unpredictable quantity in all these experiments due to its sensitivity to the degree of compaction and the distribution of moisture among other factors. Nevertheless, the so called 'upper bound' and 'lower bound' solutions are still informative as long as the experimental curve is always lying in between these two limits.

Chapter 6

Mechanics of Soil-Pipeline Interaction

6.1 General

Buried pipelines form a safe and economical means for the bulk transportation of energy resources such as heated oil, natural gas and slurried coal (see Ariman et al., 1979; Jeyapalan, 1985; Pickell, 1983). In many parts of the world, long distance pipelines are used for the transportation of both onshore and offshore oil and natural gas. The interaction behaviour between the pipeline and the surrounding soil constitutes an important aspect in the analysis and design of buried pipelines. Such interactions can be induced by a variety of effects including thermal and pressure induced expansion of the pipeline, external loads imposed by backfill, berm construction and traffic; and loadings of a geotechnical origin such as ground subsidence, fault movement, frost heave, thaw settlement, relief due to adjacent excavations, landslides etc.

In the context of soil-pipeline interactions, the mechanical response of soils is influenced by a number of factors including the differences in properties between the natural soil and the compacted backfill, cohesive or granular nature of the backfill or in-situ soil, the density and void ratio of the soil, the groundwater conditions and the rate of application of external loads. These factors contribute to the complex material characteristics which are both time and stress dependent. Furthermore, the variability of the soils along the length of a pipeline is such that it is extremely difficult to simulate such soil-structure interaction behaviour (see e.g. Desai and Siriwardane, 1984; Selvadurai, 1987). For this reason, simplified models are considered in order to accommodate reasonably accurately the essential or dominant aspects of soil behaviour. The time-dependent mechanical behaviour of soil can be modelled by linear and non-linear spring elements which interact with the pipeline

in a discrete or continuous fashion. The major advantage of such an approach is that simplified analytical and numerical solutions to soil-pipeline interaction problems can be obtained with relative ease (see e.g. Pickell, 1983; Selvadurai and Lee, 1981, 1982; Selvadurai, 1985b). The primary limitation of these models is the difficulty that is encountered in obtaining accurately and uniquely the parameters characterizing these models.

In view of the shortcomings of the theoretical treatments discussed above, it is essential to seek alternative approaches to deal with this class of soil-structure interaction problems. Such alternatives include scaled model tests in the laboratory and full scale prototype experiments in the field. It is not unusual to conduct reduced scale model tests prior to full scale field studies. The following paragraphs describe some of the more recent laboratory and field investigations involving buried pipelines subjected to discontinuous shearing.

Trautmann et al. (1985a, 1985b) reported the results of some plane strain experiments on (i) the uplift force-displacement responses and (ii) the lateral force-displacement responses of buried pipes. In the first case, the experimental study deals in particular with the influence of soil density and depth of burial. The results compared well with several published models for medium and dense sand. In the second case, the effects of pipe depth, soil density, pipe diameter and pipe roughness were considered. Pipe surface roughness was found to have little effect on the overall response. Soil density had a large influence on displacements required to mobilize the maximum force. Simplified approaches for design purposes were introduced for both cases. These series of test results had provided valuable information on the behaviour of model pipelines subjected to transverse shear loadings.

Williams (1986) discussed a joint programme between Canada and France with regard to the investigation of frost heave effects on a long distance buried pipeline. In a controlled-environment laboratory in France, a 16m length of pipe 27.3cm in diameter was buried in $250m^3$ of soil. In this particular experiment, the soil was made 'uniform' in all respects except that there would be two types of soils

expected to heave at different rates under a known, steady and cold temperature. Since the soils and the temperatures were thoroughly known, it was possible to find relationships between the rate and amount of frost heave and the characteristics of the soils. It was known that the differential heaving of the soils constituted the bending of the pipe. This project was expected to provide valuable information as to (i) how much differential heave could the pipeline tolerate and (ii) how often would the design limits be exceeded.

Ladanyi et al. (1984) presented the results of observations made during the first phase of operation of a large-scale model of a chilled pipeline buried in two different types of soils. This task was a part of the joint Canada-France project discussed earlier to study freezing around a refrigerated pipeline in a controlled environment facility in Caen, France. Along the length of the pipe, half of the pit contained a slightly frost susceptible sand while the other half contained a very frost susceptible silty soil. The pipe was refrigerated to -2°C , and the ambient chamber temperature was held slightly below zero. The pipe and the two soils were heavily instrumented to monitor thermal and moisture regime, ground heave, pipe deformations and displacements. The experimental results led to the conclusion that for a buried gas pipeline, subjected to freezing temperatures, the simulation of the mechanical interaction between the soil and the pipe presented a major difficulty, i.e. the evaluation of the resistance of the medium. It is difficult to adapt the present knowledge of frozen soil mechanics to a heterogeneous medium whose properties vary with time.

In spite of the information that can be extracted from these experimental and field studies, it is still beneficial to employ more sophisticated numerical techniques such that more variables can be dealt with at the same time. The advancement of computer hardware in recent years certainly contributes to this cause. Due to the complexities of the problems as discussed above, it is conceivable that non-linear finite element techniques are necessary in order to provide further insight to the mechanics of this type of soil-pipeline interaction problems. These analyses

should take into account such variables as soil density, strength and deformability characteristics of the soil and in-situ stress conditions etc.

6.2 Deeply Buried Pipeline Subjected to Discontinuous Fault Movement

6.2.1 Problem Definition

In order to examine the soil-pipeline interaction problem related to ground subsidence or fault movement, a number of facets of the problem need to be considered. These include (i) the modelling of the mechanical response of the soil surrounding the pipeline, (ii) the modelling of the mechanical behaviour of the pipeline, (iii) the modelling of the mechanical behaviour of the soil-pipeline interface and (iv) the geometry or orientation of the pipeline in relation to the abrupt displacement feature. In addition, the constitutive responses of the soil at the discontinuous shear zones need to be given careful consideration. The reason is that it is not clear whether the extent of the non-linear behaviour is confined only to the shear zones. In the case where the non-linear zone propagates outside the shear zone, it is important to investigate the extent of the propagation and the resulting effects on the buried pipeline.

The objective of this section is to consider an idealized model of the soil-pipeline system with a view to establishing primarily the influence of soil non-linearity on the flexural behaviour of the pipeline. The flexural interaction is induced by a discontinuous ground displacement, Δ , at the base of the soil layer containing the pipeline (Fig.6.1). The entire soil-pipeline system is assumed to be underlain by bedrock which is considered to have a high relative rigidity. The chosen model attempts to simulate a situation that can occur at a ground subsidence zone or an active fault zone. The induced relative ground movement results in the generation of stresses and strains in the pipeline without the application of any direct external load. The discontinuous displacement is assumed to occur over a wide region and it is also assumed that the pipeline is perpendicular to the fault plane.

6.2.2 Finite Element Modelling

In view of the complexities associated with this type of soil-structure interaction problems, the numerical analysis is achieved via the non-linear three dimensional finite element techniques discussed in Chapter 2. This analysis considers the pipeline as a cylindrical shell with a purely elastic response. The surrounding soil mass is modelled both as a linear elastic solid and an ideal elastic-plastic solid. The latter behaviour is modelled by employing the Drucker-Prager yield criterion used in conjunction with an associative flow rule. According to Trautmann et al., (1985b) the 'roughness' of the soil-pipe interface has very little influence on the behaviour of the pipeline subjected to transverse discontinuous shearing. It is also assumed to be buried at a large enough depth such that perfect contact between the pipe and the soil is consistently sustained partially due to the effect of the soil overburden pressure. These assumptions thus justify a completely bonded interface throughout any analysis.

For illustrative purposes and in order to minimize the use of computational resources, attention is restricted to the idealized soil-pipeline interaction problem associated with a finite region (Fig.6.2). The corresponding finite element discretization is as shown in Fig.6.3. There are 54 solid 'brick' elements each of which has 20 nodes. Of these solid elements, 40 are assigned soil material properties while the rest are designated as 'rigid' elements which are located at the bottom of the mesh and used to induce the abrupt ground displacement. There are 20 superparametric shell elements used to model the pipeline behaviour. The boundary conditions shown in Fig.6.3 indicate that upward movement is permitted. For the solid element, the optimum choice of 27 Gaussian integration points are used; i.e., a $3 \times 3 \times 3$ configuration with respect to the three local axes; a $3 \times 3 \times 2$ configuration (with respect to the r - s - t local coordinate system) for a total of 18 integration points is selected for the shell element with 2 integration points in the shell 'thickness' direction.

**Geometry Of Soil-Pipeline System
With Discontinuity**

Figure 6.1 : Soil-Pipeline Interaction in a Fault Zone

3

MicroD

Figure 6.2 : The Idealized Soil-Pipeline System

Section A-A Plane of Asymmetry

Figure 6.3 : Finite Element Discretization of the Idealized
Soil-Pipeline System

Furthermore, it should be noted that a plane of asymmetry exists at section A-A (Fig.6.3). All the in-plane displacements with respect to the global x and y directions are prescribed to be zero. Owing to the fact that asymmetric conditions are associated with deformation, only half of the soil-pipeline system needs to be discretized. The dimensions in both the x and the y directions are chosen such that the effects of imposed boundary conditions can be minimized. The ratios of x/D and y/D are 9.0 and 7.5 respectively. The dimension in the longitudinal direction allows for the simulation of ground movement along a relatively long stretch of the pipeline. The abrupt displacement is induced by the application of upward pressures from the bottom of the rigid base. It should, however, be emphasized that the load is not being applied directly onto the pipeline. This finite element mesh configuration is chosen after considering the trade-off between computational accuracy and reasonable computing time (CPU) within the capacities of the Carleton University mainframe computing system (a Honeywell Level 66 Computer).

6.2.3 Numerical Results

In this section, results of engineering interest regarding interaction between a buried pipeline and the surrounding soil are presented. The focus of the numerical analyses is on the effects of soil non-linearity with respect to the induced bending moments and hoop stresses of the pipe. The relationship between the stresses in the pipe and the relative stiffness of the soil-pipe system is emphasized. For illustrative purposes, a relative stiffness parameter, R^* , is defined. This parameter relates the bending stiffness of the pipeline to the elastic property and geometry of the soil as follows.

$$R^* = \frac{E_p I_p}{E_s D^4} \quad (6.1)$$

where E_p = Young's modulus of pipe, I_p = moment of inertia of pipe cross-section, E_s = Young's modulus of soil, D = average diameter of pipe.

A typical E_s value for dense sand of 70 MPa is used (Bowles, 1977). The

thickness of the pipe is chosen to give a D/t ratio of 200 (thin-walled pipe). For the pipe itself, values of 1.5×10^7 psi, 5.0×10^6 psi and 5.0×10^5 psi are used for E_p . These values approximately correspond to typical values quoted in the literature for cast iron, concrete and PVC pipes respectively. The above parameters give rise to corresponding R^* values of 15.7, 1.57 and 0.157. For the purpose of non-linear analyses, two combinations of input parameters are used. Firstly, a Poisson's ratio of 0.48 and an apparent cohesion of 70 kPa giving a E_s/c ratio of 1000 are used to simulate a purely cohesive soil. On the other hand, a Poisson's ratio of 0.30 and a friction angle of 30° are used to model a purely frictional soil. The soil-pipeline interaction is induced by a translational relative displacement, Δ , perpendicular to the pipe in the absence of any external force.

6.2.3.1 Bending Moments in the Pipe

Figures 6.4 to 6.6 illustrate the manner in which the maximum bending moment in the pipe varies with the induced displacement. The deviation from the purely elastic behaviour seems to be very insignificant. However, it should be noted that the moment-displacement relationship begins to deviate from the linear trend as R^* decreases (Fig.6.6). It is also evident that the linear elastic solution does provide a 'conservative' estimate for this type of problem. Hence the development of plastic zones is an advantage as far as the structural integrity of the pipeline is concerned. Figures 6.7 to 6.9 illustrate the distribution of the bending moment along the pipeline. It is also clear that for all values of R^* considered, the maximum moment is located close to the transition zone (plane of asymmetry). The reduction in the moment is abrupt in all cases. Figure 6.10 shows the variation of the maximum bending moment with the relative stiffness. The general trend is that the elastic solution seems to lie in between the purely frictional and purely cohesive cases for a wide range of R^* .

6.2.3.2 Hoop Stresses in the Pipe

Figures 6.11 to 6.13 show the circumferential variation of the hoop stresses from the crown to the invert of the pipe at the location of maximum flexural responses.

Figure 6.4 : Maximum Bending Moment vs. Induced Ground Displacement ($R^* = 15.7$)

Figure 6.5 : Maximum Bending Moment vs. Induced Ground Displacement ($R^* = 1.57$)

Figure 6.6 : Maximum Bending Moment vs. Induced Ground Displacement ($R^* = 0.157$)

Figure 6.7 : Pipeline in Fault Zone - Bending Moment Distribution Along Pipeline ($R^* = 15.7$)

Figure 6.8 : Pipeline in Fault Zone - Bending Moment Distribution Along Pipeline ($R^* = 1.57$)

Figure 6.9 : Pipeline in Fault Zone - Bending Moment Distribution Along Pipeline ($R^* = 0.157$)

Figure 6.10 : Pipeline in Fault Zone - Maximum Bending Moment vs. Logarithm of Relative Stiffness

6.3.2 Finite Element Modelling

The numerical treatment of the above mentioned problem is achieved via non-linear three dimensional finite element scheme described previously. The finite element discretization of the region is as shown in Fig.6.16. Owing to the symmetry of the problem about the x-y and the y-z planes, it is sufficient to discretize only one quarter of the problem domain. The pipeline is considered as a cylindrical shell with a purely elastic behaviour. The surrounding soil is treated as an elastic-plastic solid which can be represented by the Drucker-Prager constitutive model. The relative dimensions of the longitudinal and the transverse directions are chosen such that the possible effects of the boundary constraints can be minimized.

A total of 45 isoparametric 'brick' elements are used to model the soil and 20 super-parametric type shell elements are used to model the pipe. The boundary conditions are such that all the sides of the finite element mesh are restrained from lateral movement. The rest of the nodes are free to displace in the vertical, x, direction except for the bottom face where the nodes are fixed in all directions. For the purposes of illustration, the numerical computation is concentrated on the determination of the influences of the depth of embedment, d , and the relative stiffness, R^* , of the soil-pile system. A E_s/c ratio of 1000 and a D/t ratio of 100 are used for all the cases considered.

6.3.3 Numerical Results

Figures 6.17 to 6.20 illustrate the manner in which the flexural deflections of the pipeline are influenced by its depth of embedment. Four different d/D ratios of 1, 2 and 3 respectively are used. It is clear that the most shallowly buried pipeline ($d/D = 1$) experiences the largest deflections. Moreover, the value of R^* also imposes a very significant influence on the magnitudes of deflection. An R^* of 15.7 probably depicts the situation of a 'rigid' pipe in which only translational displacements are experienced by the pipeline when the surface load is applied. In all cases, the non-linearity of the soil tends to increase the deflection of the pipe

Figure 6.15 : Soil-Structure Interaction of Pipelines
Subjected to Surface Loading

Figure 6.16 : Finite Element Discretization of Pipeline
Subjected to Surface Loading

Figure 6.17 : Pipeline Under Surface Loading - Vertical Displacement Profile Along Length of Pipeline ($R^* = 15.7$)

Figure 6.18 : Pipeline Under Surface Loading - Vertical Displacement Profile Along Length of Pipeline ($R^* = 1.57$)

Figure 6.19 : Pipeline Under Surface Loading - Vertical Displacement Profile Along Length of Pipeline ($R^* = 0.157$)

Figure 6.20 : Pipeline Under Surface Loading - Vertical Displacement Profile Along Length of Pipeline ($R^* = 0.0157$)

especially within the immediate vicinity below the loading surface.

Figures 6.21 to 6.23 show the variation of the induced maximum bending moment with the applied surface load. It appears that the effect of soil non-linearity is significant. Three different R^* values of 15.7, 1.57, 0.157 are used. In addition, the effects of the depth of embedment deserves particular attention. It is apparent that when d/D reaches 2 or more, the maximum bending moment remains constant. This observation may prove to be very informative with respect to the selection of a 'critical depth' at which a pipeline should be buried. Figure 6.24 illustrates the manner in which the maximum bending moment varies with the logarithm of R^* . The depth of embedment seems to have very limited influence. Figure 6.25 shows the bending moment distribution along the pipe at $R^* = 15.7$. It is observed that the maximum value occurs at the location immediately below the loading surface. As in the case of the pipeline located in a fault zone, the effect of the distresses imposed on the pipeline is very much localized.

Figure 6.21 : Pipeline Under Surface Loading - Maximum Bending Moment vs. Applied Surface Load ($R^* = 15.7$)

Figure 6.22 : Pipeline Under Surface Loading - Maximum Bending Moment vs. Applied Surface Load ($R^* = 1.57$)

Figure 6.23 : Pipeline Under Surface Loading - Maximum Bending Moment vs. Applied Surface Load ($R^* = 0.157$)

Figure 6.24 : Pipeline Under Surface Loading - Maximum Bending Moment vs. Logarithm of Relative Stiffness

Figure 6.25 : Pipeline Under Surface Loading - Bending Moment Distribution Along Pipeline ($R^* = 15.7$)

Chapter 7

Mechanics of Container-Buffer-Rockmass Interaction

7.1 General

The development of containment technologies for the long term storage and disposal of irradiated fuel is an area of research which is of crucial importance to the nuclear energy industry. The Fuel Waste Technology Branch of Atomic Energy of Canada Limited (AECL) is the lead agency in the development of the necessary containment technologies (Hancox,1986). The Canadian proposals for the disposal scheme specifically concentrates on the deep burial (approximately at 1000m in depth) of the waste in specially constructed vaults in plutonic rocks of the Canadian Shield. The engineered multi-barrier concept constitutes a key feature in the waste disposal effort. Current proposals for the disposal facility involves four principal components, which include, the vault system, the waste container, the backfill material and the buffer material. The functional requirements of the various components of the system are fully documented in the literature (see e.g. Selvadurai,1988a,b). The buffer material which fills the space between the waste container and the borehole walls is a mixture of bentonitic clay and sand which should fulfill certain functional criteria. It should act as a barrier to suppress the detrimental effects of the corrosive water in the host rock and enhance the life of the waste container; serve as a geochemical filter for the sorption of radionuclides released either by accidental damage or natural breakdown of the waste containers; provide the mechanical strength to support the waste containers; isolate the containers from detrimental rockmass movements; act as a medium for conducting the radiogenic heat and it should be capable of swelling under fluid influx

Figure 6.11 : Pipeline in Fault Zone - Pipe Hoop Stress Distribution from Crown to Invert of Pipe ($R^* = 15.7$)

Figure 6.12 : Pipeline in Fault Zone - Pipe Hoop Stress Distribution from Crown to Invert of Pipe ($R^* = 1.57$)

to seal gaps, cavities etc., that may be developed by moisture depletion during the heat and moisture transfer processes.

This particular research focusses on the investigation of the manner in which the buffer is capable of isolating the waste container from detrimental effects of rockmass movements. The 'plastic' behaviour of the buffer region is an important aspect particularly in relation to the isolation of the waste container from detrimental rockmass movements. Such rock movements can take place either due to the thermal loading of the repository area induced by the stored waste, removal of ground water from the repository area, earthquake effects or by stress relief due to the creation of the repository. Estimates indicate that rockmass movements due to thermal loadings can be of the order of 2 to 5 cm. Such displacements can occur in a discontinuous fashion when fractures are generated and/or reactivated by the thermal loadings. In the particular instance when a fracture intersects the borehole, a dislocation type rock movement can prematurely threaten the integrity of the waste container. Furthermore, the experimental investigations conducted in connection with heat and moisture migration in the buffer in the absence of moisture influx (Selvadurai, 1988a,b) indicates that loss of moisture in the buffer region leads to a severe loss of its 'plastic' properties. Therefore, a discontinuous movement that can take place under conditions with no moisture influx into the buffer region but moisture loss due to heating, can present an adverse environment for the waste container. The theoretical modelling of the effects of such discontinuous rock movements on the behaviour of the buffer-container system should take into consideration a number of complex material and interface phenomena. These include (i) the mathematical representation of the changes in the deformability and failure (fracture/yield) characteristics of the buffer associated with the moisture depletion, (ii) investigation of the extent of separation and fracture that is associated with the moisture depletion process, (iii) non-linear processes that exist at the buffer-container and buffer-rockmass interfaces, (iv) changes in the geometry of the buffer due to heat induced shrinkage processes, (v) elastic and plastic deformations of the container which accounts for large deflections and large strains and (vi)

Figure 6.13 : Pipeline in Fault Zone - Pipe Hoop Stress Distribution from Crown to Invert of Pipe ($R^* = 0.157$)

Figure 6.14 : Pipeline in Fault Zone - Maximum Hoop Stress vs. Logarithm of Relative Stiffness

The variations between linear and non-linear analyses are again small but yet distinguishable. It is clear that the hoop stresses change from tensile to compressive. This change is abrupt and both peak stresses are located close to the neutral axis of the pipe cross-section. Figure 6.14 illustrates the variations of the maximum hoop stresses with the logarithm of R^* . In spite of the general decreasing trend of the maximum moment with increasing R^* , it appears that there is a certain critical value of R^* below which the general trend no longer prevails.

6.3 Pipeline Subjected to Surface Loads

External loadings such as berm construction and surface traffic loads have long been recognized as serious threats to the integrity of buried pipelines. In many cases, the pipelines are buried at a depth of no more than two or three times the pipe diameter. It has already been discussed in Chapter 3 that concentrated surface loads imposed by stationary construction vehicles have very significant influence on the induced stresses in buried pipelines situated close to shallow trenches. It is therefore necessary to undertake a more in depth examination of the problem of surface loadings on buried pipelines with special emphasis on the effects of soil non-linearity.

6.3.1 Problem Definition

In order to have a clearer understanding of the distresses caused by surface loads on a shallowly buried pipeline, the problem of a pipeline which is embedded in a soil stratum of finite thickness is investigated. For the purposes of illustration, the lateral extent of the soil region is considered to be finite. A pipeline of average diameter, D , and thickness, t , is embedded in a soil stratum at a depth, d . The interface between the pipeline and the soil mass is assumed to be perfectly bonded. The relative stiffness parameter, R^* , as defined in Equation(6.1) is again utilized. The surface of the soil medium is subjected to a uniformly distributed load of intensity, p , and dimensions, $D \times D$. The geometry of the problem is summarized in Fig.6.15.

analysis. Since the rockmass is modelled as essentially a rigid region, a horizontally directed distributed load is applied to induce the relative displacement. The resistance to the movement along the plane of discontinuity is not considered since the attention is focussed on the effects imposed by the magnitude of the induced relative displacement.

7.2.2 Buffer Region

In these investigations the buffer material considered is a mixture of bentonitic clay and quartz sand in a 1 to 1 ratio by weight. The mix composition of the buffer is organized in such a way that the functional requirements of mechanical strength, permeability, swelling capabilities and heat conduction are simultaneously satisfied. The mechanical behaviour of the buffer can be influenced by a variety of factors including grain size distribution, soil micro-structure, stress history, moisture content and degree of saturation. These factors contribute to soils which display marked non-linear and irreversible time-dependent phenomena. It is evident that any attempt to solve the container- buffer-rockmass interaction problem by taking into account all such characteristics is a complex task. It is therefore prudent to idealize the mechanical behaviour of the buffer mass in order that progress in the analytical treatment of these soil-structure interaction problems can be made within reasonable computational efforts. In these studies the critical state model (or the modified Cam-clay model) is chosen to model the time independent mechanical behaviour of the buffer material. Details of the formulation and numerical implementation of this model are given by Roscoe and Burland (1968), Siriwardane and Desai (1983).

In the numerical implementation of such models, the accuracy largely depends on the accuracy with which the input parameters can be defined via laboratory and field tests. With regard to the constants used in the constitutive model based on the Critical State model, it is required to determine the shear strength parameters and the stress-strain properties of the soils. Both conventional triaxial compression tests and direct shear tests can be employed to determine the relevant mechanical parameters. Chapter 4 had discussed the results of a series of geotechnical labora-

tory tests on the compacted buffer material. Graham et al. (1986), Sun (1986) and Wan (1987) have also reported geotechnical laboratory test results for the buffer mixture. The results of these investigations form the basis for estimating the mechanical parameters that will be used in the numerical studies. It may be noted that changes in initial dry density, void ratio and residual compaction stresses can lead to variations in the estimated parameters.

For the modified Cam-clay model, seven basic parameters need to be specified; these include M , λ , κ , p_0 , e_0 and the basic elastic parameters E and ν . The data is obtained from two separate sources: (i) laboratory tests on buffer mixture compacted to a maximum dry density of 1.76 Mg/m^3 at an optimum moisture content of 18%, (ii) similar tests on buffer mixture compacted to a dry density of 1.51 Mg/m^3 at a moisture content of 28%. As reported in Chapter 4, for the denser buffer material, the Young's modulus of the buffer is found to be approximately 140 MPa with a Poisson's ratio of 0.45. From the plot of $\sqrt{3J_{2D}}$ vs $J_1/3$ (Fig.4.14), M is estimated to be 0.4. The initial void ratio is found to exert a relatively little influence. From basic phase relationships, e_0 is found to have a value of 1.0. The initial stress in the buffer, p_0 , is assumed to have a value of 100 kPa as a first approximation of the stresses due to compaction. The quantity κ can be estimated from the following relationship.

$$K = \frac{p_0(1 + e_0)}{\kappa} \quad (7.1)$$

where K is the bulk modulus obtained from the hydrostatic test results reported in Chapter 4. The values of 0.0025 and 0.0005 are used for λ and κ respectively. For the buffer material with lower compacted density (standard Proctor), the Young's modulus is determined to be 50 MPa (average value) and the Poisson's ratio is set equal to 0.48. Since there is a scarcity of relevant data at the 'critical state', M is taken as 1.0. Similarly p_0 is assigned a value of 100 kPa. From the results reported by Graham et al (1986), an e_0 value of 2.0 and a λ value of 0.50 are used. Since the swelling procedure is not performed during the isotropic consolidation tests, the

κ value of 0.15 is also estimated from equation (1). A summary of the material parameters employed to model the two densities of the buffer region is given in Table 7.1.

7.2.3 Interface Conditions

There are two interfaces associated with the problem; namely the buffer - container interface and the buffer-rockmass interface. Two extreme conditions of the buffer-rockmass interface are considered, namely perfectly 'rough' and perfectly 'smooth'. The rough case is simulated by constraining the relative movement between the buffer and the rockmass in the vertical, or 'z' direction. The smooth case is simulated by allowing vertical displacements at the boundaries. The latter configuration allows the surcharge to take effect over the entire buffer region. It should also be noted that the analysis focusses on a single cell of the vault configuration and that the influence of neighbouring cells is neglected. During the container-buffer-rockmass interaction, the cylinder would experience a rigid body translation. The boundary between the container and the buffer could exhibit a variety of interface characteristics ranging from completely smooth to completely bonded conditions. In the present investigation, the interface is assumed to be in a fully bonded condition.

7.2.4 Waste Container

One container option under study is the supported-shell metal matrix concept (Nuttall, 1984). This concept involves the packing of the used fuel bundles into a thin, corrosion-resistant shell and casting the remaining space with a low melting point metal or alloy. Structural support for the shell is provided by the resulting metal matrix. Several materials have been identified for fabrication of the corrosion-resistant shell, including commercially available pure and low alloy titanium, high nickel-based alloy such as Inconel 625 and pure copper. A conceptual design has also been produced for a copper shell structurally supported container (Ko and Hosaluk, 1987). The ensuing finite element analyses focusses on the performance

of the copper cylindrical shell. According to the conceptual design, the prototype metal-matrix container has a relatively thick shell of more than 25 mm in order to provide adequate mechanical strength and corrosion allowance. In the present analyses, the waste container is modelled as a cylindrical shell with an average thickness of 25 mm. Material properties for copper are used for all the analyses; a Young's modulus of 99,000 MPa and a Poisson's ratio of 0.35. The shell is assumed to behave as a linear elastic material throughout the dislocation type movement of the system.

7.3 Finite Element Discretization

The main objective of this chapter is to investigate the buffer - container - rockmass interaction problems due to distress caused by dislocation type movements along fracture planes intersecting nuclear waste disposal vault boreholes (Fig.7.1). Due to symmetry along the x-z plane, only one half of the borehole needs to be discretized. The effect of the backfill is modelled as a surcharge at the upper surface of the buffer. A schematic diagram of the idealized disposal vault borehole is shown in Fig.7.2. Figure 7.3 shows the finite element mesh used in the analyses. There are 48 solid 'brick' elements and 26 shell elements resulting in 380 nodes. The boundary conditions are such that the half of the borehole below the plane of discontinuity is restrained from lateral movement. The upper half only is allowed to move horizontally in the 'x' direction by the application of horizontal loadings applied to the solid elements that are used to simulate the rockmass surrounding the borehole. The plane of discontinuity is assumed to intersect at (i) the mid-section level of the borehole and (ii) the mid-section level of the container. Only the horizontal orientation is considered in these analyses. The container and the buffer region are modelled by superparametric Ahmad-type general shell elements and 20-node isoparametric brick elements respectively. One of the major advantages of this shell element is its capability to model curved and irregular boundaries owing to the presence of a midside variable node at each edge of the element. Details of

Figure 7.1 : Possible orientation of plane of discontinuity to disposal vault borehole

Figure 7.2 : Schematic representation of an idealized disposal vault borehole

Figure 7.3 : Finite element discretization of disposal vault borehole

the numerical techniques have been presented earlier and are also given in Zienkiewicz (1979) and Selvadurai and Pang (1988).

7.4 Numerical Results

In the three-dimensional finite element analyses, the results of linear elastic analyses of the corresponding buffer-container interaction problems are used as a basis for comparison. As discussed previously, the modified Cam-clay constitutive model is used to simulate the buffer behaviour. The analyses emphasize on the evaluation of critical stresses in the container and the soil pressure at the container-buffer interface. These stresses are the longitudinal stresses (σ_{zz}) and the hoop stresses ($\sigma_{\theta\theta}$) in the container as well as the radial soil stresses (σ_{rr}) around the cylinder. Table 7.1 summarizes the material parameters used in the numerical analyses.

7.4.1 Container Stresses

Figures 7.4 and 7.5 illustrate the variations in the maximum stress along the axial direction of the container with the magnitude of the displacement of the surrounding rockmass when the discontinuity is located across the mid-section level of the borehole ($h/H = 0.5$). The stresses are normalized with respect to E_c , the Young's modulus of the container (copper), and the displacement is normalized with t , the thickness of the container. The value of D/t is held fixed at 24, this corresponds to a relatively thick shell. The modular ratio E_c/E_b , where E_b is the Young's modulus of the buffer region, is set equal to 2000 and 700. The non-linear constitutive model is implemented by using 6 equal load increments with 2 iterations each increment. A surcharge of 100 kPa is applied on the upper surface of the buffer to simulate the effects of a 5m height of backfill material. Figures 7.6 to 7.7 show similar sets of curves for an h/H ratio of 0.65 (mid-container level).

In general, distinct non-linearities are observed in the curves from the analyses which incorporate non-linear effects in the buffer behaviour. The results show that a stiffer buffer (lower E_c/E_b) would experience higher longitudinal stresses.

MATERIAL PARAMETERS								
Problem	Elastic		Critical State				Surcharge	
Category	$E(\text{MPa})$	ν	M	λ	κ	e_0	$p_0(\text{MPa})$	(MPa)
A	140	0.45	0.4	0.0025*	0.0005*	1.0	0.1*	-
B	140	0.45	0.4	0.0025*	0.0005*	1.0	0.1*	0.1
C	50	0.48	1.0*	0.50	0.15*	3.0	0.1*	-
D	50	0.48	1.0*	0.50	0.15*	3.0	0.1*	0.1

* estimated

Table 7.1 : Summary of Material Parameters

Figure 7.4 : Maximum longitudinal stress vs displacement
 $(E_c/E_b = 2000) (h/H = 0.50)$

Figure 7.5 : Maximum longitudinal stress vs displacement
 $(E_c/E_b = 700) (h/H = 0.50)$

Figure 7.6 : Maximum longitudinal stress vs displacement
($E_c/E_b = 2000$) ($h/H = 0.65$)

Figure 7.7 : Maximum longitudinal stress vs displacement
($E_c/E_b = 700$) ($h/H = 0.65$)

Moreover, the surcharge load tends to reduce the maximum stresses induced in the container. It is clear that the backfill contributes to the structural integrity of the container during the dislocation type rock movement. In general, the increase in stresses predicted by the non-linear analyses becomes significant as the induced displacements are increased. Thus for this particular type of container loading, the linear elastic solution does not give rise to conservative estimates for the container stresses. Figures 7.8 and 7.9 indicate that for the situations modelled ($h/H=0.5$), and for the range of displacements considered, the maximum hoop stress in the container is relatively uninfluenced by the modular ratio E_c/E_b . Similar trends are observed from the curves for the case where h/H has a value of 0.65 (Figs. 7.10 to 7.11). It is evident, however, that the stress magnitudes are higher. Also for a given displacement, the hoop stresses have values which are substantially higher than the longitudinal stresses. From the point of view of the structural design of the container, the hoop stress is thus a more critical measure.

Figures 7.12 and 7.13 illustrate the variation in the maximum longitudinal stress along the length of the container for the $h/H=0.5$ case. These compressive stresses reach a peak at the level of the discontinuity. It is also shown that stress distributions predicted from the analyses which utilize non-linear modelling of soil behaviour are substantially higher than those calculated by assuming linear elastic behaviour of the soil. In all cases, the variation of stress around the region of the discontinuity is very abrupt. It can be seen from Figs. 7.14 to 7.15 that the trends of the stress distribution are similar for the $h/H=0.65$ situation except for the fact that the peak stresses are located at a different position. Figures 7.16 to 7.19 show the circumferential distribution of hoop stresses at levels where the corresponding maximum values occur. In all cases, the stresses change from compressive to tensile to compressive. Also, the tensile stresses are larger than the compressive stresses. These stress distributions point to the fact that the container shell undergoes contra-flexural bending along the circumferential direction during the discontinuous rock movement.

Figure 7.8 : Maximum hoop stress vs displacement
 $(E_c/E_b = 2000) (h/H = 0.50)$

Figure 7.9 : Maximum hoop stress vs displacement
 $(E_c/E_b = 700) (h/H = 0.50)$

development of fracture and separation zones at the various interfaces due to the discontinuous rock deformation. A theoretical modelling (analytical/numerical) of the buffer-container interaction which takes into account all the non-linear thermo-mechanical processes outlined above presents a problem of considerable complexity.

The objective of this chapter is to provide an elementary treatment of this rather complex buffer-container interaction problem with a view to providing an assessment of the possible influence of the discontinuous rock movement on the stresses generated in the container shell. The chapter specifically examines the interaction of a container shell and the buffer which possesses elastic-plastic material properties. A non-linear finite element technique is used to assess the manner in which the location of the fracture planes, surcharge loads at the borehole surface, variable interfaces at the buffer-container and buffer-rock surfaces and non-linear material characteristics of the buffer material influences the stresses induced in the container shell by discontinuous rock movement.

As mentioned previously, the Swedish concept of nuclear fuel waste disposal is remarkably similar to that of the Canadians. Borgesson (1986) reported the investigation of the consequences of rock displacement across a deposition hole by some model tests. The model was scaled 1 : 10 to a prototype deposition hole. It was filled with a container made of solid copper surrounded by highly compacted, water saturated MX-80 bentonite. Before shear the swelling pressure was measured by six transducers in order to follow the water uptake process. During shear, pressure, strain, force and deformation were measured in altogether 18 points. The shearing was made at different rates in the various tests.

Extensive sampling after shear was conducted through which the density, water content, degree of saturation, homogenization and the effect of shear on the buffer and the container could be studied. The results from these experiments were compared to those of several numerical computation procedures including the (i) linear elastic three dimensional finite element calculations, (ii) non-linear two dimensional finite element calculations and (iii) three dimensional boundary element

calculations. These comparisons results in the conclusion that three dimensional finite element analyses using non-linear material properties is necessary to simulate the actual shearing process. The report also recommends that it is necessary to combine a mathematical model with experimental work in order to further understand the behaviour of the entire soil-structure interaction system.

7.2 Physical Modelling

In this chapter attention is restricted on the problem of an axisymmetric single borehole configuration. The geometries of the borehole and the container are selected to conform approximately to those proposed for the Canadian concepts (Wardrop et al. 1985). The action of the backfill is represented purely as a surcharge which acts at the surface of the buffer region. The fracture plane in the rockmass, along which the discontinuous rock movement takes place, is oriented horizontally (Fig.7.1). For purposes of illustration, the location of the discontinuity is assumed to be at the midway point of the borehole. The discontinuous deformation corresponds to a differential displacement Δ which induces a state of symmetry about the x-z plane. In the modelling, the region below the discontinuity is held stationary and the upper half is allowed to displace in the horizontal direction. It is assumed that the fracture plane offers no resistance to the movement. In the ensuing, the modelling of the various components of the idealization of the buffer-container-rockmass configuration will be briefly discussed. The descriptions are divided into four subsections in order to account for the rockmass, buffer, interfaces and the container.

7.2.1 Rockmass

The stiffness and strength characteristics of the rock are expected to be significantly higher than that of the buffer region. In the numerical computations, the Young's modulus of the rock is set equal to 10^9 MPa and the Poisson's ratio is set equal to 0.48. The rockmass is assumed to behave elastically throughout the

Figure 7.12 : Longitudinal stress distribution along container
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.50$)

Figure 7.13 : Longitudinal stress distribution along container
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.50$)

Figure 7.14 : Longitudinal stress distribution along container
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.65$)

Figure 7.15 : Longitudinal stress distribution along container
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.65$)

Figure 7.16 : Circumferential hoop stress distribution
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.50$)

Figure 7.17 : Circumferential hoop stress distribution
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.50$)

Figure 7.18 : Circumferential hoop stress distribution
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.65$)

Figure 7.19 : Circumferential hoop stress distribution
 $(\Delta/D = 6.0 \times 10^{-3})$ ($h/H = 0.65$)

7.4.2 Buffer Stresses

Figures 7.20 and 7.21 illustrate the longitudinal distribution of maximum soil pressure exerted in the container-buffer interface. Two major features could be highlighted from the plots : (i) separation is detected at the region of the interface above the plane of discontinuity (existence of 'tensile' stresses in the buffer), (ii) the maximum radial soil pressure does not coincide with the location of the discontinuity. The extent of the zone of separation is expected as in the 'tube' experiments discussed in Chapter 5. The plots also showed that the highest stresses existed in the cases where $E_c/E_b = 700$ with the presence of the surcharge. In contrast, the analyses involving a linear elastic soil behaviour show remarkably uniform soil pressure distribution along the length of the container. Again, the stresses from elastic analyses are relatively smaller than that predicted from non-linear analyses.

The circumferential soil pressure distributions at the level of maximum stresses are plotted as in Figs. 7.22 and 7.23. The distribution patterns are more or less as expected with stresses on the 'passive' side of the container substantially higher than that on the 'active' side. Signs of apparent separation zones are also located at the buffer-container interface at the lower (stationary) portion of the system.

Figure 7.20 : Soil Pressure Distribution Along Container ($h/H = 0.50$)

Figure 7.21 : Soil Pressure Distribution Along Container ($h/H = 0.65$)

Figure 7.22 : Circumferential Soil Pressure Distribution Around Container ($h/H = 0.50$)

Figure 7.23 : Circumferential Soil Pressure Distribution Around Container ($h/H = 0.65$)

Chapter 8

Soil-Pile Interaction in Homogeneous and Layered Soils

8.1 General

Piled foundations are frequently required to carry inclined loads which result from the combined action of the dead load of the structure and the horizontal loads from wind, water or earth pressure on the structure. When the horizontal component of the load on the pile is small in relation to the vertical load, the structure can be supported safely by vertical piles. Thus special provision is not usually made in piled foundations to buildings for the horizontal loading resulting from wind pressure. However, in the cases where piles are frequently subjected to lateral forces and moments, such as those piles in wharves and jetties carrying the impact forces of berthing ships and wave action; in offshore structures subjected to wind and wave action; in pile-supported earth retaining structures; in transmission tower foundations; and in earthquake areas such as Japan and the west coast of continental North America, the horizontal load is relatively large such that vertical piles cannot generally be relied on to withstand these forces.

8.2 Pile Subjected to Lateral Loading

When a horizontal load is applied to the head of a vertical pile which is free to move in any lateral direction, the load is initially carried by the soil close to the ground surface. Since the soil compresses elastically, there is some transfer of load to the soil at a greater depth. When the horizontal load is increased, the soil yields plastically and the load transfer extends to greater depths. A pile with low relative stiffness to the soil will rotate and passive resistance will develop at the toe on the

same face at which the load is applied in addition to the passive resistance of the soil near the ground surface on the opposite face. Failure occurs by rotation when the passive resistance at the head and toe are exceeded. The failure mechanism of a pile with high relative stiffness is different since the cumulative passive resistance at the lower part of the pile is very high. Hence the pile cannot rotate and failure occurs by fracture of the pile at the point of maximum bending moment (Tomlinson, 1975).

The head of a laterally loaded pile may displace through an appreciable distance before failure in rotation or fracture of the pile takes place, such that the movement of the structure carried by the piles may exceed tolerable limits. Therefore, in addition to ensuring that there is an adequate safety factor against failure under the lateral load, the deflection of the pile should also be examined.

The dominant factor in the interaction of the pile and the soil is the relative stiffness of the pile. The stiffness influences the degree of deflection and determines whether the failure mechanism is one of rotation, translation or failure in bending. The type of loading, whether sustained or time dependent, influences the extent of yielding of the soil. External influences such as removal of soil from around the pile by scour, or shrinkage of soil away from the pile surface, affect the passive resistance of the soil against the upper part of the pile.

The main objective of this chapter is to utilize the developed non-linear three dimensional finite element techniques for the analysis of the soil-pile interaction problem. Particular attention is paid on the evaluation of the influences of the non-linear behaviour of the soil on the deflection and induced bending moment responses of the pile. The analyses also provide an illustration of the importance of the relative stiffness of the pile to the soil.

8.3 Current Methods of Analysis

For the analysis and design of the lateral resistance of piles, it is pertinent to investigate its load-deflection behaviour (Poulos and Davis, 1980). During the pro-

cess of designing pile foundations to resist lateral loads, the major design criterion is the maximum deflection of the piles. Apart from the various empirical approaches that are already widely used, theoretical and numerical approaches for predicting lateral movements have been developed. Two of the more popular methods are as follows.

- (1) The subgrade reaction approach in which the continuous nature of the soil medium is ignored and the pile reaction at a point is simply related to the deflection at that point.
- (2) The elastic approach which assumes the soil to be an ideal elastic continuum.

8.3.1 The Subgrade Reaction Approach

The subgrade reaction (Winkler-type) model of soil behaviour characterizes the soil as a series of independent linear elastic or non-linear springs (Poulos and Davis, 1980). Despite the fact that this approach is widely employed in practice due to its simplicity and its ability to take into account, in a very approximate fashion, the factors such as non-linearity, variation of soil stiffness with depth and layering of the soil profile. In addition, considerable amount of experience has been gained in applying the theory to the solution of practical problems and a number of empirical correlations are available for determining the spring modulus. However, there are many disadvantages associated with the subgrade reaction method. Two of the major disadvantages are that there (i) is a lack of continuity of the soil model, and (ii) exists a dependence of the spring modulus on the size of the foundation.

In the Winkler soil model, the pressure p and deflection δ at a point are assumed to be related through a modulus of subgrade reaction, which for horizontal loading, is denoted as k_h . Thus,

$$p = k_h \delta \quad (8.1)$$

The pile is usually assumed to act as a thin strip whose behaviour is governed by the beam equation. As in the simple theory of beam bending, the effect of axial

load in the pile is ignored. The governing equation for the deflection of a laterally loaded pile is given by

$$E_p I_p \frac{d^4 \delta}{dz^4} + k_h \delta = 0 \quad (8.2)$$

where $E_p I_p$ = bending stiffness of the pile.

The solution to eq.(8.2) may be obtained either analytically or numerically. Analytical solutions are only available in a convenient form for the case of constant k_h along the pile. For other distributions of k_h , solutions can be obtained by the numerical finite difference method (Palmer and Thompson, 1948). By using this method, any variation of k_h with depth can be considered (Poulos and Davis, 1980).

8.3.2 The Elastic Continuum Approach

Theoretically, the modelling of the soil as an elastic continuum is more appropriate due to the continuous nature of the soil. Modifications to the model are possible in order to take into account the non-linear behaviour of the soil, for the varying soil modulus with depth and for layered systems. One advantage of the elastic model is that it enables consistent analysis of the immediate movements and the final movements. The major disadvantage to the application of this method in practice is again the difficulty of determining the appropriate soil moduli. The problems involving soil-pile interaction under lateral loading conditions is a comprehensive three dimensional problem in continuum mechanics. The elasto-plastic behaviour of the soil medium further complicates the attempt to achieve an exact solution to the problem. It is, however, convenient to employ numerical techniques such as the finite element and the boundary element methods for the purpose of solving this soil-pile interaction problem. Some attempts have been made to deal with this problem by means of two dimensional finite element techniques (e.g. Rowe and Poulos, 1979). Baguelin and Frank (1979) used the finite element method of analysis to obtain a degenerate linear elastic solution.

Banerjee and Davies (1978) employed a special finite element technique which

is capable of dealing with general three dimensional loading for axi-symmetric geometries under elastic conditions. Desai and Appel (1976) used the general three dimensional finite element approach with allowance for axial slippage via the use of joint elements. More recently, the method of substructuring is utilized in conjunction with three dimensional finite element procedures to deal with the laterally loaded pile problem.

Selvadurai and Rajapakse (1986) examined the problems related to the axial lateral and rotational loadings of a vertical rigid cylindrical inclusion which is embedded in bonded contact at the boundary of an isotropic elastic half-space. Comparisons were made between results from numerical schemes and equivalent results derived via analytical techniques which focussed on the solution of the governing integral equation and other approximate solution schemes. Spillers and Stoll (1964) considered the problem of lateral loading of an elastic pile which was embedded in an elastic half-space. The pile was treated as a line inclusion obeying the Bernoulli-Euler beam theory.

8.3.3 Other Methods of Analysis

Many of the methods discussed above had assumed the plain strain conditions which was inappropriate for modelling the three dimensional laterally loaded pile problem. The three dimensional analyses presented had neglected the elasto-plastic nature of the soil.

Muqtadir and Desai (1986) employed a non-linear three dimensional finite element procedure to analyse the response of pile group foundations subjected to lateral loading. An interface (joint) element which could take into account the interaction effects of relative slip and debonding was used throughout the soil-pile interface. The objective of this study was to analyse the soil-structure interaction and load-deformation behaviour of pile groups. The circular hollow pile sections were approximated as equivalent rectangular solid sections. This approximation was adopted due to its simplicity and to avoid a complicated and expensive mesh.

This analysis adopted the hyperbolic type non-linear model to represent soil behaviour (Duncan and Chang, 1970). The methodology was an improvement to the various analytical and numerical techniques discussed previously.

8.4 Single Pile in Homogeneous Soil Subjected to Lateral Load

In order to rectify some of the inconsistencies and shortcomings that exist in the presently available methods of analysis for the soil-pile interaction problem, an approach which utilizes the general shell element and non-linear soil constitutive models in conjunction with three-dimensional finite element procedures is used. Details of these formulations may be found in Chapter 2. Figure 8.1 shows a schematic diagram of the problem under consideration. The hollow circular single pile which is fitted with a rigid pile cap is embedded in a homogeneous soil medium of finite extent. The pile is subjected to a horizontal load applied to the pile cap as shown. The curved surface of the pile is modelled by the general shell elements with a flat base plate at the pile tip and a solid pile cap at the pile head. The soil medium is modelled by solid 'brick' elements. The non-linear behaviour of the soil is modelled by an elastic-plastic response. Special conditions at the soil-pile interface are considered wherever appropriate.

8.4.1 Finite Element Discretization

Figure 8.2 shows a schematic diagram of the finite element mesh of a single circular pile (with pile cap) embedded in a cohesive soil with a thin layer of solid elements between the pile and the soil. This thin layer of elements can incorporate special interface conditions. There are a total of 391 nodes with 56 solid elements and 15 shell elements. The interface element possesses the capabilities of allowing for the modelling of relative slip between the pile and the surrounding soil (see Chapter 2). Owing to the symmetry of the geometry and the loading, only half of the soil-pile system is discretized. The boundary conditions are such that all the

Figure 8.1 : Laterally Loaded Pile in Homogeneous Soil

Figure 7.10 : Maximum hoop stress vs displacement
($E_c/E_b = 2000$) ($h/H = 0.65$)

Figure 7.11 : Maximum hoop stress vs displacement
($E_c/E_b = 700$) ($h/H = 0.65$)

Figure 8.2 : Finite Element Discretization of Pile in Homogeneous Soil Subjected to Lateral Load

nodes are free to displace in the vertical direction except for those at the base of the mesh. All the outside vertical faces of the mesh are restrained in the out-of-plane direction. At the plane of symmetry, the degrees-of-freedom in the out-of-plane direction are fixed. For all the analyses, the pile and the cap are assumed to behave in a linear elastic manner. As in the cases for pipelines (Chapter 6), a relative stiffness parameter, R^* , is defined as follows.

$$R^* = \frac{E_p t}{E_s D} \quad (8.3)$$

where E_p = Young's modulus of pile; E_s = Young's modulus of soil; t = thickness of pile cross-section and D = mean diameter of pile.

For the purpose numerical studies, four different values of R^* were used. The pile modulus, E_p , was chosen to assume values of 2.0×10^5 MPa, 1.0×10^5 MPa, 2.0×10^4 MPa and 2.0×10^3 MPa while E_s was kept constant throughout. These values were expected to cover the entire range of materials normally used for piles (Tomlinson, 1980). A Poisson's ratio, ν_p , of 0.30 was used for all the analyses. The Drucker-Prager yield criterion was employed in conjunction with an associative flow rule to model the constitutive behaviour of the soil. The Young's modulus of the soil, E_s , was taken at 50 MPa. This was considered as a typical value for a medium to stiff clay. The Poisson's ratio for the soil was also chosen to be 0.30. The friction angle, ϕ , was assigned a zero value and the E_s/c ratios of 2000, 1000 and 500 were selected. These values essentially covered the range from soft to stiff clays (Selvadurai, 1979). The interface elements had elastic properties identical to that of the soil elements except that the shear modulus, G , assumed a very low value of 1.0×10^{-5} MPa in the tangential directions at the soil-pile interface. This low G value allowed for the possible relative slip to occur between the pile and the soil. The horizontal load was essentially a concentrated load applied to the rigid pile cap along the horizontal direction as shown in Fig.8.1. For the linear elastic analyses, this load was applied in one increment. For the non-linear analyses, the same lateral load was applied in 8 equal increments with 3 iterations for each increment.

8.4.2 Numerical Results

8.4.2.1 Load-Displacement Behaviour

The load-deformation behaviour of the pile cap (Fig.8.1) are as shown in Figs.8.3 to 8.5. The effects of soil properties represented by the E_s/c ratios are illustrated for three different values of relative stiffness, R^* . The curves indicate that the pile experiences larger deformations as E_s/c increases. It is also shown that the induced displacement is inversely proportional to R^* . The influence of the special interface properties on the load-displacement relationships is found to be very insignificant.

Figures 8.6 to 8.8 illustrate the variation in the plastic deformation with incremental loading. The increase in R^* plays an important role in the increasing magnitude of the plastic displacements. It is apparent that the soil experiences progressive yielding. Furthermore, Fig.8.9 shows the variation of the maximum plastic displacement with the logarithm of R^* . Within this context, the plastic displacement, Δ_p , is defined as the magnitude of the deviation of the non-linear load-displacement curve from the corresponding linear elastic relationship under the same load conditions. This observation seems to indicate that the plastic deformation tends to increase as the value of R^* increases.

8.4.2.2 Load-Bending Moment Relationship

Figures 8.10 to 8.12 illustrate the influence of the applied load on the maximum induced bending moment. The curves show that the maximum moment increases significantly with the increasing load. The deviation from the corresponding elastic relationship is very significant for relatively low R^* values. Moreover, it is observed that a weaker soil with a correspondingly higher E_s/c ratio allows the pile to bend more under the same applied load thus resulting in a higher maximum bending moment. Also shown in the figures are the curves resulting from the inclusion of interface elements. It appears that there is a noticeable increase in the induced bending moment in all cases. The difference is, however, very small when compared

Figure 8.3 : Single Pile in Homogeneous Soil -
Lateral Load vs Total Displacement ($R^* = 15.7$)

Figure 8.4 : Single Pile in Homogeneous Soil -
Lateral Load vs Total Displacement ($R^* = 1.57$)

Figure 8.5 : Single Pile in Homogeneous Soil -
Lateral Load vs Total Displacement ($R^* = 0.157$)

Figure 8.6 : Single Pile in Homogeneous Soil -
Lateral Load vs Plastic Displacement ($E_s/c = 500$)

Figure 8.7 : Single Pile in Homogeneous Soil -
Lateral Load vs Plastic Displacement ($E_s/c = 1000$)

Figure 8.8 : Single Pile in Homogeneous Soil -
Lateral Load vs Plastic Displacement ($E_s/c = 2000$)

Figure 8.9 : Single Pile in Homogeneous Soil -
Plastic Displacement vs Logarithm of Relative Stiffness

Figure 8.10 : Single Pile in Homogeneous Soil
Lateral Load vs Maximum Bending Moment ($R^* = 15.7$)

Figure 8.11 : Single Pile in Homogeneous Soil
Lateral Load vs Maximum Bending Moment ($R^* = 1.57$)

Figure 8.12 : Single Pile in Homogeneous Soil
Lateral Load vs Maximum Bending Moment ($R^* = 0.157$)

with the difference between an elastic-plastic and a purely elastic case without the incorporation of any special interface condition.

The variation of the maximum moment with R^* is shown in Fig.8.13. A trend of convergence is again observed for these curves. The decrease in the normalized maximum bending moment is significant with the increase in R^* . Figure 8.14 shows the typical pattern of the variation of the bending moment from the pile tip to the pile cap. There is a change of sign in the bending moment as shown in the figure. The maximum value is located close to the pile cap.

8.4.2.3 Soil Pressure Distribution

Figures 8.15 and 8.16 illustrate the distribution of soil pressure along the pile on the passive side of the lateral movement as shown in the diagrams. The incorporation of non-linear soil properties leads to the increase of the lateral earth pressure for cases of low R^* values and the decrease in the same pressure for cases with high R^* values. The distribution of the soil pressure is more or less uniform for the upper half of the pile. For the lower portion of the pile, the pressure variation is more noticeable. This variation is substantially diminished as the E_s/c ratio decreases (stronger soil). The elastic analyses exhibit remarkably uniform distributions. Furthermore, it is also observed that the sensitivity of the soil pressure to R^* can be significantly reduced as the soil yields along the length of the pile.

8.5 Soil Nailing

Soil nailing is a technique of in-situ soil reinforcement by bars which are generally used for slope stabilization and for retaining of excavations. In spite of numerous examples of applications, the design methods used are still rather empirical in nature. This situation is due to the fact that the mechanism of soil-bar interaction in a nailed soil is not clearly understood. The mechanism depends primarily upon the type of application and the characteristics of the reinforcing elements.

In a nailed-soil retaining structure, the soil-bar interaction is essentially governed by the friction mobilized at the interfaces. Hence, this structure is comparable

Figure 8.13 : Single Pile in Homogeneous Soil
Maximum Moment vs Logarithm of Relative Stiffness

Figure 8.14 : Single Pile in Homogeneous Soil
Bending Moment Distribution Along Pile Length

Figure 8.15 : Single Pile in Homogeneous Soil
Soil Pressure Distribution Along Pile Length

Figure 8.16 : Single Pile in Homogeneous Soil
Soil Pressure Distribution Along Pile Length

to the common reinforced earth wall. The major differences between a reinforced earth wall and a nailed soil wall are as follows : (i) the construction process and (ii) the relative rigidity of the reinforcing elements. In a reinforced earth structure, the reinforcing elements are flexible and can therefore withstand only tensile forces, whereas in a nailed-soil structure the reinforcing bars are relatively rigid and consequently they can withstand both tensile forces and bending moments (Juran et al., 1983).

For slope stabilization, the bars are generally placed perpendicular to the potential sliding surface. If the bars were extremely rigid, they could be considered as piles subjected to a lateral earth thrust on both sides of the sliding surface and would have to withstand mainly bending moments. However, in practice, the bars are relatively flexible and consequently both tensile forces and bending moments can be mobilized (Gudehus and Schwarz, 1985).

In a creeping slope, the soil experiences time dependent movement in a downhill direction. Typical creep velocities range from 0.1 mm to 5 cm per month (Gudehus and Sshwarz, 1985). In the usual cases where the ground water table is located near the surface, the creen velocity decreases and sometimes the creep nearly stops with the lowering of the ground water table. Slight disturbances such as small cuts or fills can induce the movement of a slope. Structures situated on a creeping slope will be influenced by this movement. A flexible structure, for instance, an asphalt road, a pipeline or a cable, will be unable to resist the movement and will be damaged in due course. In the case of a rigid foundation such as an anchored diaphragm wall or a deeply founded abutment, the soil movement will be slowed down until the build-up of earth pressure due to soil creep destroys the structure. In recent years, many structures have had to be abandoned because slope movement was ignored in the original design. In order to facilitate the building of structures on or near creeping slopes, it is necessary for the geotechnical engineers to provide safe and economical solutions in the foundation design of such structures.

Reinforcement by inserting piles increases the mechanical strength of a slope. If

the prevailing loading of the piles is transverse shearing, this method is called dowelling. Soil dowelling is a rather old invention and its application had been proved to be successful in recent years. In order to provide a safe and economical design method for such dowells, it is necessary for engineers to understand thoroughly the mechanisms involved. The effects of soil non-linearity is among the major aspects under investigation.

8.6 Lateral Load on a Single Pile in Layered Soil

In order to provide further insight into the understanding of soil nailing and the mechanisms of dowelling, an idealized model of the problem of restricting soil movement by rigid piles is considered. This involves the task of approximating lateral soil pressure acting on the pile as a horizontal concentrated load being applied to the rigid pile cap. Figure 8.17 shows the laterally loaded pile in a double-layer soil system. The following sections focus on the numerical analysis of this problem.

The lower soil layer is considered to be stiffer than the upper one. The pile is taken to be relatively rigid with a length to diameter ratio of 10. The configuration is used to simulate the situation where a short, rigid pile is socketed onto a stiffer lower layer of soil. This configuration is common, for instance, in the cases where piles are used for dowelling purposes; or where a foundation is to be built on piles that are designed to carry the load to the lower, stiffer soil medium.

8.6.1 Finite Element Modelling

Figure 8.18 is a diagram showing the finite element mesh that is used for the analysis. This mesh has essentially the same number of nodes and elements as the one shown in Fig.8.2. The boundary conditions are also applied in the same manner as in the case of homogeneous soil. In order to model a pile of high relative stiffness socketed into a lower stiffer layer of soil, the dimensions of the mesh are arranged in such a way that the boundary between the two soil layers is located in

Figure 8.17 : Laterally Loaded Pile in Layered Soil

Figure 8.18 : Finite Element Discretization of Pile in Layered Soil Subjected to Lateral Load

the mid-section level of the pile (Fig.8.18). The Young's modulus of the lower layer is taken to be ten times that of the upper layer. The softer layer of soil is assumed to behave in an elastic-plastic manner. The same relative stiffness parameter, R^* , as defined in eq.(8.3) is used. In this case, the E_s in eq.(8.3) refers to the Young's modulus of the softer soil layer. For illustrative purposes, four different values of R^* , 15.7, 7.85, 1.57, 0.157, are used. A Poisson's ratio of 0.3 is chosen for all soil materials. The pile and the cap is again assumed to behave in a linear elastic manner.

The Drucker-Prager yield criterion is again employed in conjunction with an associative flow rule to model the behaviour of the soil in the upper, softer layer. The ratio of E_s/c of 1000 is used for all the analyses. Three different cases are considered in the finite element modelling, namely (i) elastic behaviour for all materials, (ii) elastic pile and elastic soil with non-linear interface properties and (iii) elastic pile with non-linear interface and non-linear upper soil layer. The horizontal load is applied in the form of a concentrated load on the pile cap in the 'x' direction. For case (i), the load is applied in one increment. For cases (ii) and (iii), the same lateral load is applied in 10 equal increments with 3 iterations for each increment.

8.6.2 Numerical Results

The load-deformation curves of the pile cap (in Fig.8.18) are shown in Figs. 8.19 to 8.22. It is clear that the lateral displacement increases as R^* decreases under the same loading condition. In all cases, the effect of non-linearity is significant. However the difference between a system with only a non-linear interface and a system with a non-linear upper soil layer is relatively small. Figure 8.23 shows the change of the maximum lateral displacement with the change in the logarithm of R^* . It is observed that the maximum displacement becomes increasingly sensitive to the value of R^* as the latter increases. Similar trends are observed for the variation of the induced maximum bending moment with the applied load increments (Figs. 8.24 to 8.27). It is also observed that the maximum moment increases substantially as the R^* value decreases. In addition, it may be noted, from Fig.8.28, that the

Figure 8.19 : Single Pile in Layered Soil -

Lateral Load vs Displacement ($R^* = 15.7$)

Figure 8.20 : Single Pile in Layered Soil -

Lateral Load vs Displacement ($R^* = 7.85$)

Figure 8.21 : Single Pile in Layered Soil -
Lateral Load vs Displacement ($R^* = 1.57$)

Figure 8.22 : Single Pile in Layered Soil -
Lateral Load vs Displacement ($R^* = 0.157$)

Figure 8.23 : Single Pile in Layered Soil -
Maximum Displacement vs Logarithm of Relative Stiffness

effects of non-linearity is far more significant to the bending moment than for the pile cap displacement. Figures 8.29 to 8.30 illustrates the variation in the bending moment along the length of the pile. There is an apparent change in signs in the induced moment. It also appears that there are two distinct peaks in the distribution. The difference becomes clearer as R^* decreases.

The soil pressure distribution on the passive side of the pile movement for different values of R^* are shown in Figs. 8.31 to 8.34. In all cases, the influence of the non-linearity of the upper soil layer is significant. The abrupt change in magnitude of the soil pressure is attributed to the change in soil stiffness between the two soil layers. But the elastic cases seem to produce higher soil stresses within the lower soil layer.

Figure 8.24 : Single Pile in Layered Soil -
Maximum Bending Moment vs Displacement ($R^* = 15.7$)

Figure 8.25 : Single Pile in Layered Soil -
Maximum Bending Moment vs Displacement ($R^* = 7.85$)

Figure 8.26 : Single Pile in Layered Soil -
Maximum Bending Moment vs Displacement ($R^* = 1.57$)

Figure 8.27 : Single Pile in Layered Soil -
Maximum Bending Moment vs Displacement ($R^* = 0.157$)

Figure 8.28 : Single Pile in Layered Soil -
Maximum Moment vs Logarithm of Relative Stiffness

Figure 8.29 : Single Pile in Layered Soil -
Moment Distribution Along Pile (Drucker-Prager Interface)

Figure 8.30 : Single Pile in Layered Soil -
Moment Distribution Along Pile (Drucker-Prager)

Figure 8.31 : Single Pile in Layered Soil -
Soil Pressure Distribution Along Pile ($R^* = 15.7$)

Figure 8.32 : Single Pile in Layered Soil -
Soil Pressure Distribution Along Pile ($R^* = 7.85$)

Figure 8.33 : Single Pile in Layered Soil -
Soil Pressure Distribution Along Pile ($R' = 1.57$)

Figure 8.34 : Single Pile in Layered Soil -
Soil Pressure Distribution Along Pile ($R^* = 0.157$)

Chapter 9

Summary and Conclusions

In this thesis, attention is focussed on the experimental and numerical modelling of soil-structure interaction problems involving discontinuous shearing. The effects of soil non-linearity and relative stiffness of the system are emphasized. Extensive literature review has revealed that existing analytical and numerical treatments of these types of problems are largely restricted to two dimensional elastic analyses. The limited number of non-linear analyses concentrates on the use of non-linear spring models and hyperbolic type stress-strain models. The documented laboratory and field investigations are either too site specific or have provided very limited information. This research programme is thus initiated to provide further insight into this class of soil-structure interaction problems. A substantial portion of the research is focussed on the linear and non-linear three dimensional finite element modelling of various problems of engineering interest. The numerical analyses is complimented by experimental modelling involving scaled model tests as well as geotechnical laboratory tests for material property characterization.

The finite element technique used in this research is primarily based on the isoparametric concept. The 20-node solid continuum element is used to model the soil or rock medium. Each node has three translational degrees-of-freedom with respect to the three global coordinate directions. All the embedded structures encountered in these problems are simulated by a super-parametric type general shell element. This element has 8 nodes each of which has three translational and two rotational degrees-of-freedom. The two elements are coupled together by means of the *static condensation* method whereby the two rotations in each shell node are eliminated after some matrix manipulations.

The non-linear behaviour of the soil and rock media is simulated by either

the Drucker-Prager or the Critical State (Modified Cam-Clay) constitutive models. In particular, the modified Cam-Clay model is used to simulate the mechanical behaviour of the buffer material used in the Canadian nuclear waste disposal endeavours. One of the major advantages of these two constitutive laws is that only the elastic constants, the strength and consolidation parameters are required for numerical implementation. All these parameters can be determined from conventional geotechnical laboratory procedures such as the triaxial, direct shear and consolidation tests.

Prior to the investigation of the full three dimensional soil-structure interaction problems, the accuracy and capabilities of the finite element procedures are verified by examining some two dimensional plain strain problems. These problems involve circular cavities, both lined and unlined, embedded in linear elastic or non-linear elastic-plastic soil media. Both shallowly buried and deeply buried cavities are considered. One example of linear elastic three dimensional analysis involving a pipeline buried near a shallow trench is also examined. Most of the numerical predictions obtained from the present finite element procedures compare favourably with closed form solutions and published numerical results.

The experimental modelling undertaken in this research involves the simulation of the flexural responses of a structural element embedded in a soil medium subjected to transverse shear dislocation movement. The experimental results are compared with numerical results obtained from the finite element analyses of the experiment. The apparatus is specifically designed to serve the anticipated testing purposes. Moist concrete sand is used as the soil medium for the tests. The structural elements are an aluminium strip and an aluminium tube instrumented with strain gauges and connected to a computerized data acquisition system.

The experimental load-displacement relationships exhibit a bi-linear trend. It is apparent that the embedded structural element did provide a reinforcing effect for the system. Elastic finite element predictions are sufficient for numerical simulation of the experiments. Favourable comparisons for the bending moment distributions

are possible. It is observed that the numerical predictions are able to provide 'upper bound' and 'lower bound' solutions such that the experimental curves lie in between these two limits.

The last three chapters of the thesis focus on the application of non-linear three dimensional finite element techniques to analyse various problems of interest to the geotechnical engineer. These problems include (i) the effects of fault movement on a deeply buried pipeline, (ii) the effects of surface loading on a pipeline buried at shallow depths, (iii) mechanics of container-buffer-rockmass interaction due to rock dislocation movement in a proposed Canadian nuclear waste disposal vault, (iv) soil-pile interaction in homogenous and layered soil media.

For the deeply buried pipeline problem, an idealized model of the system is considered. A discontinuous ground displacement at the base of the soil medium is induced and the resulting structural behaviour of the pipe is analysed. The pipeline is modelled as a cylindrical shell with a purely elastic response. The surrounding soil is modelled as an ideal elastic-plastic solid. The soil behaviour is simulated by employing the Drucker-Prager yield criterion used in conjunction with an associative flow rule. For illustrative purposes, a relative stiffness parameter, R^* , is introduced in order to relate the bending stiffness of the pipe to the elastic property and geometry of the soil. From the numerical results, it is observed that the bending moment versus induced displacement relationship begins to deviate from the linear trend as R^* decreases. The distribution of bending moment along the pipeline is such that the maximum value is located close to the shear zone. An examination of the circumferential variation of the hoop stresses of the pipe indicates that there is a relatively abrupt change of signs.

The idealized problem of a pipeline embedded in a soil stratum subjected to surface loading is also investigated. A uniformly distributed load of constant intensity is applied. The finite element modelling is very similar to the one used in the previous investigation. Numerical results show that the pipeline buried at the shallowest depth experiences the largest deflection. The influence of R^* on the

magnitudes of deflection is also significant. It is also shown that the non-linear behaviour of the soil substantially increases the maximum bending moment. The bending moment distribution along the pipeline shows that the peak value occurs at the location immediately below the loading surface.

The thesis is then focussed on the investigation of the manner in which the sand-clay buffer material can function in order to protect the waste container from detrimental effects of abrupt and discontinuous rockmass movement. This complicated configuration is idealized such that the induced stresses in the container can be predicted. In the finite element analyses, the container is considered as a shell structure with a linear elastic behaviour. The modified Cam-Clay model is employed to simulate the behaviour of the buffer solid. The relevant shear strength parameters and stress-strain properties are extracted either from the geotechnical laboratory tests reported in this thesis or from experimental results published elsewhere. The required tests for material characterization include the triaxial, direct shear and consolidation tests.

Numerical results show that the container experiences higher stresses when buffer non-linearity is incorporated. A stiffer container relative to the surrounding buffer will experience higher longitudinal stresses. The surcharge load applied on top of the buffer tends to reduce container stresses. For the same relative displacement, the hoop stresses have a higher magnitude than the corresponding longitudinal stresses. It is also shown that the container shell undergoes contra-flexural bending along the circumferential direction during the discontinuous movement. With respect to the buffer pressure exerted at the container-buffer interface, the stresses predicted from taking into account the elasto-plastic behaviour of the buffer are much higher than those predicted from considering only the linear elastic response.

The emphasis of the research is finally focussed on the soil-pile interaction problems involving lateral loading. The relative stiffness of the soil-pile system influences the magnitude of deflection and determines whether the failure mechanism is one of

rotation, translation or failure in bending. The first problem investigated is the one involving a hollow, circular single pile fitted with a rigid pile cap which is embedded in a homogeneous soil medium of finite extent. The pile is subjected to a horizontal load applied to the pile cap. The curved surface of the pile is modelled by shell elements with linear elastic responses. The soil is simulated by solid elements whose properties are governed by the Drucker-Prager yield criterion. Thin 'interface' elements are used in some cases to simulate the possible relative slip between the pile and the soil. A relative stiffness parameter, R^* , similar to the one used in the pipeline analyses is used.

Load-deformation curves of the pile cap indicate that the pile experiences larger deformations as the ratio E_p/c increases. It is observed that the pile cap displacement is inversely proportional to R^* . As R^* increases, the magnitudes of the plastic displacement increases. The magnitudes and distribution of soil pressures are substantially altered when soil non-linearity is incorporated.

A similar idealized problem involving the lateral loading of a pile embedded in a two-layer soil strata is then investigated. The lower layer is taken to be ten times as stiff as the upper layer. The finite element simulation is similar to that of the previous problem except that an elastic-plastic interface element is used in some cases. It is observed that the lateral displacement increases as R^* decreases for the same loading conditions. There is an apparent change of signs of the induced bending moment along the pile length. Abrupt change in soil pressure is also observed at the interface of the two soil layers.

The major contributions of this thesis may be summarized as follows :

- (1) extension of the scope of knowledge with regard to the soil-structure interaction problems involving embedded structural elements subjected to discontinuous shear displacements,
- (2) utilization of non-linear three dimensional finite element techniques, in conjunction with established constitutive models, to solve the above mentioned problems,

- (3) incorporation of simple experimental modelling techniques for the understanding of the mechanism of soil-structure interaction behaviour,
- (4) illustration of the importance of soil material property characterization and the implementation of laboratory testing results in subsequent numerical analyses; one such example involves the investigation of the problem of container-buffer-rockmass interaction in a nuclear waste disposal vault,
- (5) verification of the developed finite element techniques and demonstration of the versatility of the method by solving a number of typical two dimensional soil-structure interaction problems,
- (6) application of the developed finite element techniques to the investigation of three dimensional geotechnical problems such as those involving buried pipelines and laterally loaded piles.

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