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AS PREDICTED BY 5 MATHEMATICAL MODELS

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A COMPARISON OF THE STEADY STATE CORNERING
RESPONSE OF A 5-AXLE TRACTOR-SEMITRAILER AS
PREDICTED BY FIVE MATHEMATICAL MODELS

BY

J. Graham Thomas B. Eng.

A thesis submitted to the Faculty of
Graduate Studies and Research in partial
fulfillment of the requirements for the degree of
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5-Axle Tractor-Semitrailer as Predicted by Five Mathematical Models"
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ABSTRACT

Directional response of tractor-semitrailers, particularly as it relates to vehicle safety is of considerable interest to vehicle designers and operators.

Considerable effort has been expended in the field of vehicle simulation by mathematical modelling, due to the cost of running experimental test program. A variety of models ranging from very simple to very complex are currently used to study vehicle behaviour. A review of the literature revealed that there has been no systematic comparison of the different models.

This work compares the response of a five-axle tractor semitrailer in a steady state turn as predicted by four existing vehicle models. The investigation showed that one of the more widely used models was not suitable for simulating constant speed cornering manoeuvres under certain conditions. A new model was developed by the author to fill the gap left by the unsatisfactory one.

FOREWORD

All data, experimental results and computer simulation results in this thesis are presented in imperial units. This was done to maintain consistency with reference 8 for clarity to future researchers.

I would like to thank my supervisor, Dr. J. Y. Wong and his associate, Dr. M. El Gindy, for all the support and guidance they provided during the course of this work. My wife, Diana, deserves special credit for putting up with the many late nights that went into this thesis. Thanks to my friend, Patricia Leger, for typing the entire text.

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NOMENCLATURE

a	- distance from tractor cg to front axle
a'	- distance from tractor sprung mass cg to front axle
$[A]$	- rotation matrix in γ
b	- distance from tractor cg to tractor leading tandem axle
b'	- distance from tractor sprung mass cg to tractor leading tandem axle.
b_1	- distance from tractor cg to tractor trailing tandem axle
$[B]$	- rotation matrix in ϕ and θ
c	- distance from tractor leading to trailing tandem axles
C_1	- total cornering stiffness of tires on axle 1
C_2	- total cornering stiffness of tires on axle 2
C_3	- total cornering stiffness of tires on axle 3
C_4	- total cornering stiffness of tires on axle 4
C_5	- total cornering stiffness of tires on axle 5
CA	- cornering stiffness of a tire
CA'	- modified cornering stiffness of a tire
cg	- centre of gravity
CS	- longitudinal stiffness of a tire
d	- distance from tractor cg to fifth wheel
d'	- distance from tractor sprung mass cg to fifth wheel
$d\mathbf{i}/dt$	- derivative of unit vector with time
$d\mathbf{j}/dt$	- derivative of unit vector with time
DOF	- degree of freedom
e	- distance from fifth wheel to trailer cg
e'	- distance from fifth wheel to trailer sprung mass cg

NOMENCLATURE

f_1	- lateral force on axle 1
f_2	- lateral force on axle 2
f_3	- lateral force on axle 3
f_4	- lateral force on axle 4
f_5	- lateral force on axle 5
FA	- tire pavement characterization parameter
F_{iL}	- load transfer force on the left wheelset of the ith axle
f_{iL}	- force vector acting on the left wheel set of the ith axle
f_{in1}	- inertial force vector acting on the tractor cg
f_{in2}	- inertial force vector acting on the trailer cg
F_{iR}	- load transfer force on the right wheelset of the ith axle
f_{iR}	- force vector acting on the right wheelset of the ith axle
ft	- feet
ft/s	- feet per second
FX	- longitudinal force acting on a tire
FY	- lateral force acting on a tire
h	- distance from trailer cg to trailer leading tandem axle
h_1	- height of tractor cg above the ground
h_2	- height of trailer cg above the ground
h_3	- height of tractor cg above the fifth wheel
h'_3	- height of tractor sprung mass cg above fifth wheel
h_4	- height of the fifth wheel above the ground
h_5	- height of the trailer cg above the fifth wheel
h'_5	- height of the trailer sprung mass cg above the fifth wheel
HPCG	- subroutine implementing Hamming's Predictor - Corrector algorithm for integrating differential equations
I_1	- yaw moment of inertia of the tractor

NOMENCLATURE

I_2	- yaw moment of inertia of the trailer
$\vec{i}$	- unit vector along x axis
$\vec{j}$	- unit vector along y axis
KF	- cornering stiffness modification constant
k_i	- suspension stiffness of a wheelset on axle i
l	- distance between trailer tandem axles
l_1	- distance from the fifth wheel to trailer leading tandem axle.
l_2	- distance from the fifth wheel to the trailer trailing tandem axle.
l_3	- distance from the fifth wheel to the trailer tandem axle centre.
lb	- pound force
m_1	- tractor mass
m'_1	- tractor sprung mass
m_2	- trailer mass
m'_2	- trailer sprung mass
N	- normal force
r_{cg1}	- position vector, fifth wheel to tractor cg
r_{cg2}	- position vector, fifth wheel to trailer cg
r_{iL}	- position vector, fifth wheel to left wheelset of the ith axle.
r_{iR}	- position vector, fifth wheel to right wheelset of the ith axle.
RR	- rolling radius

NOMENCLATURE

s	- longitudinal tire slip
T_1	- half track of axle 1
T_2	- half track of axles 2 and 3
T_3	- half track of axles 4 and 5
Δt	- time increment
U	- tractor longitudinal velocity
$\dot{U}$	- tractor longitudinal acceleration
V	- trailer lateral velocity
$\dot{V}$	- trailer lateral acceleration
v_s	- wheel slip velocity
X	- longitudinal hitch force
Y	- lateral hitch force
Z	- vertical hitch force
z_{iL}	- vertical displacement of left wheelset on the i th axle.
z_{iR}	- vertical displacement of right wheelset on the i th axle.
α	- tire side slip angle
$\bar{\alpha}$	- tire stiffness modification constant
δ	- steer angle
λ	- longitudinal co-ordinate in tire contact patch
μ	- tire-road coefficient of friction
μ_0	- tire characterization parameter
ω	- yaw rate
$\dot{\omega}$	- yaw acceleration
ϕ	- tractor pitch angle
ψ	- articulation angle
θ	- tractor roll angle

CHAPTER 1 INTRODUCTION

Over the last 25 years there has been ever increasing interest in the handling characteristics of articulated road vehicles, particularly as it relates to vehicle safety. Three aspects of handling may be identified as being of primary interest; directional response, directional stability and roll stability. For each of the categories listed, three manoeuvres are considered to describe the full range of vehicle operation; steering, braking and combined braking and steering.

Directional response is a self descriptive term. Generally it describes the vehicle response to steering and/or braking inputs.

Directional stability is the tendency for the vehicle to remain on a given path. A directionally stable vehicle will tend to regain its original path if disturbed by an external force (e.g. a wind gust).

Directionally unstable vehicles exhibit a divergent response when disturbed by either an external force or a steering input.

The tendency for a vehicle to roll over during extreme steering and/or braking manoeuvres is a measure of its roll stability. This aspect of vehicle handling is especially important for vehicles with a high centre of gravity (c.g.)

Due to the prohibitive cost of running experimental test programs to examine all aspects of handling of all classes and configurations of vehicles, considerable work has been done in the field of mathematical modelling of articulated vehicles. For purposes of this work, a mathematical model is defined as the representation of a dynamic system by simultaneous equations.

Reference (15) eloquently describes a model as "a mathematical abstraction for representing behaviour of some physical system. It is a homomorphism of the system structure into the numerical relational - structure."

Before introducing the mathematical representations of the tractor-semitrailer its physical attributes will be described.

1.1 PHYSICAL ATTRIBUTES OF THE TRACTOR-SEMITRAILER

The tractor and semitrailer bodies physically comprise most of the vehicle (figure 1). ~~They~~ are referred to as the "sprung" masses because they are supported by the suspension springs. The tractor sprung mass may be considered to be a rigid body. As such, it has the usual 6 rigid body degrees of freedom to describe its orientation in space (3 translational and 3 rotational).

Connection of the tractor and semitrailer sprung masses is through the so-called fifth wheel. The commonly used conventional and inverted fifth wheel allows the semitrailer to yaw and pitch relative to tractor but constrains it to roll with the tractor along the tractor's longitudinal axis. Kinematically this means that only two additional degrees of freedom (DOF) are required to fully describe the trailer orientation. The less popular king pin type fifth wheel only allows the semitrailer to yaw relative to the tractor. Thus, it requires only one additional DOF to describe trailer orientation.

Figure 1 - The Tractor-Semitrailer

Thus, eight degrees of freedom fully describe the tractor and semitrailer orientation (with conventional or inverted fifth wheel). Axles are referred to as unsprung masses because they are directly in contact with the ground through the tires. An axle may be considered to have 2 DOF. One degree is up and down motion and the other is a side to side roll motion. Because of the nature of the suspension, axles yaw with the sprung mass.

With the exception of air resistance all external forces are applied through the tires. Modelling of tire forces will be discussed in Section 2.2.

Now consider the three most common types of models used to represent tractor-semitrailer dynamics. These being the 4 DOF, 8 DOF and complex models.

1.2 MATHEMATICAL REPRESENTATION OF THE TRACTOR-SEMITRAILER

This section briefly introduces the general types of models used in current research. In the section following this, an overview of the literature is presented to demonstrate how these models are being used.

1.2.1 THE 4 DOF MODEL

This is the simplest representation of an articulated vehicle. It only considers motion in the road plane. Roll (side to side motion), pitch (end to end rocking) and heave (vertical translation) are ignored.

As a result of these assumptions the sprung and unsprung masses and

inertias are lumped together. Degrees of freedom that remain after the assumptions are:

- (1) Longitudinal translation of the tractor
- (2) Lateral translation of the tractor
- (3) Yaw motion of the tractor
- (4) Yaw motion of the trailer

Unless special assumptions are made, this model cannot consider the effect of redistributing the tire loads as a result of cornering or braking accelerations. This effect is referred to as load transfer. It is most dramatically demonstrated in the case of rollover during extreme cornering manoeuvres. Section 2.3.3 discusses the methods used to calculate load transfer with the 4 DOF model.

1.2.2 THE 8 DOF MODEL

This model allows for all eight of the rigid body degrees of freedom associated with the tractor and semitrailer sprung masses. It is not a full representation as the axle masses and yaw inertias are lumped with the sprung masses. The 8 degrees of freedom include:

- (1) Tractor longitudinal motion
- (2) Tractor lateral motion
- (3) Tractor vertical motion
- (4) Tractor yaw
- (5) Tractor pitch
- (6) Tractor roll
- (7) Trailer pitch
- (8) Trailer yaw

Since the pitch and roll degrees of freedom are included, load transfer may be modelled as a function of suspension deflection from the static position.

1.2.3 COMPLEX MODELS

This general classification includes all models with more than 8 degrees of freedom. Additional degrees of freedom are used to model axle motion, wheel spin, steering compliance and frame flex.

1.3 LITERATURE SURVEY

There has been a considerable amount of work published on theoretical and experimental studies of the handling and performance of articulated vehicles. This section highlights some of the analytical work that has been done. It concludes with some observations on the continuity of this work.

In 1963 Hales investigated the directional stability of an articulated vehicle using a linearized 4 DOF model (1)*. This early work concluded that jackknifing (tractor yaw divergence) was associated with locking of the rear tractor wheels. Ellis reached the same conclusions that year using the nonlinear 4 DOF model in his analysis (2).

* The numbers in brackets refer to references in the Bibliography.

Jindra (3) studied the response of various tractor-trailer combinations to steering inputs with a linearized model similar to the 4 DOF model. This mathematical representation was an extension of the original model that allowed additional degrees to account for a pintle hitch and a second full trailer. The investigation concluded that the model was a useful tool for studying the effect of changing vehicle parameters.

In 1968 Mikulcik presented the first mathematical model of the articulated vehicle that offered a full rigid body representation of the two sprung masses (4). His paper in 1971 (5) compares the linearized and nonlinear 8 DOF model results for braking and steering manouvres. The nonlinear model predicted divergent vehicle behaviour in many cases where the linearized model predicted stable behaviour. Mikulcik concluded that one must be wary of the results predicted by linear models.

Tobler (6) reformulated the 8 DOF model in 1974 with a numerical formulation technique based on (4x4) transformation matrices. This technique allows complex dynamics problems to be computer programmed directly without writing out the analytical equations. His work showed that Mikulcik's 8 DOF nonlinear model contained some errors which produced the divergent behaviour mentioned. Thus, Mikulcik's conclusions may not be valid. With this technique Tobler also formulated a 20 DOF model that included axle degrees of freedom.

Leucht introduced the method of quasistatic load transfer with his paper in 1970 (7). The method uses the 4 DOF model to describe the vehicle dynamics. Rather than using suspension travel as a result of roll, pitch and heave to describe load transfers, the method uses a static summation of moments.

The model was used to investigate the braking in a turn manoeuvre.

In 1973 Bernard et al presented a sophisticated model for simulating trucks and articulated vehicles (8). It was based on Tobler's work. The report includes comparisons with validation tests for steady-state cornering and for the braking in a turn manoeuvre.

This is one of the most complete and general models available for vehicle simulation.

Citing the need for a simple general purpose program to predict articulated vehicle performance, Bernard et al (9) developed a program based on Leuchts quasistatic model. It employed the 4 DOF model with a quasistatic representation for load transfer. Since its introduction in 1975, this model has seen a lot of application. For instance, Billing and Visentin (10) used it in their investigation of 5 jackknife control devices.

The complex model from reference (5) has also seen a lot of use. It was the basis of a lengthy investigation by Bernard et al (11) into yaw instability during pure cornering manoeuvres.

The advent of the more complex models has not spelled the end of the 4 DOF model. In 1978 Mallikarjunarao and Fancher used a linearized extension of this model (12) to analyse the directional response of double tankers. Ervin and MacAdam (13) used a similar model in conjunction with a complete rigid body representation to study the effect of lateral acceleration amplification in vehicles with multiple articulation points. The linear model was used to determine frequency response to sinusoidal steering inputs while the complete model was used for transient analysis.

As may be gathered from the preceding discussion, there have been many publications dealing with the theoretical aspects of articulated vehicle modelling. There have been significantly less publications on experimental work and even fewer comparing analytic results to experimental results. Table 1, taken from Vlks' (14) review of the literature, summarizes the relative amounts of work done in each area.

Table 1 - Number of Publications on the Handling Performance of
Tractor-Semitrailers

	Theory	Experiment	Comparison
Directional Performance	62	15	10
Roll Dynamics	25	14	8
Braking Performance	31	22	5
Combined Steering &	31	12	6

Braking

The significance of Table 1 lies in the small number of comparison studies done relative to theoretical or experimental studies. In the literature review, only one comparison of different models was found. Tobler compared Mikulcik's rigid body model with two of his own. No comparison of all classes of models was found.

1.4 PURPOSE OF THIS WORK

It is apparent from the literature review of section 1.3 that there have been wide ranging investigations into different aspects of articulated vehicle handling. This work has been carried out with the aid of many different versions of the three classes of vehicle models described in Section 1.2. As was pointed out, these investigations have been primarily theoretical with relatively little work done comparing experiment to theory. Even more significant is the fact that no work has been uncovered which compared different mathematical models to each other and experiment.

The importance of such a comparison is obvious if one considers the nature of models of increasing complexity. Generally a more sophisticated model requires more input parameters and greater computational effort for solution to a manoeuvre. The work involved in obtaining input parameters for very sophisticated models is quite significant as it involves disassembling the vehicle and using elaborate test rigs (8).

With this in mind, it becomes clear that the use of the simplest model for accurate representation of a manoeuvre can result in a considerable saving in effort. If the appropriate model is to be chosen for an analysis, there must be some basis of comparison.

This thesis attempts to establish the foundation for a basis of comparison between existing mathematical representations of the tractor-semitrailer. It makes the first step in a systematic comparison of existing models by examination of the steady state.

Specifically, this thesis compares the steady state cornering response of a five axle tractor semitrailer as predicted by five different models to experimental results. One of the models was developed by the author. With this basis, research may be continued in the area of transient response.

Chapter 2 begins with a general discussion of time domain modelling as it applies to the vehicle models. Since the external forces on a vehicle in a steady state turn are applied through the tires (ignoring air resistance) the tire model is presented next.

The five vehicle models representing the five axle tractor-semitrailer are then outlined. They are as follows:

- (1) Classical 4 DOF Model (time domain model)
- (2) Linearized 3 DOF Model (steady state solution)
- (3) 4 DOF Model with Quasi-Static Load Transfer (time domain model)
- (4) 28 DOF Model (time domain model)
- (5) Linearized 6 DOF Model (steady state solution)

Model 1 is the oldest mathematical representation of the articulated vehicle. It has been used in various forms by a number of researchers to investigate directional response and stability for all types of manoeuvres (3,4,12,14 and 18).

Model 2 is a steady state solution to the linearized model 1 for the steady state cornering manoeuvre (17).

Model 3, as implemented in the program TBS (9), based on Leuchts equations (7) has been used extensively to study all aspects of vehicle dynamics. It is essentially model 1 with a quasistatic scheme for approximating load transfer. Unfortunately this model has serious drawbacks as will be pointed out.

Model 4 is a full rigid body representation of the articulated vehicle. It is one of the most complete representations of the 5-axle tractor-semitrailer in existence.

The Linearized 6 DOF Model was formulated by the author to demonstrate a more rigorous method to approximate load transfer with a quasistatic approach. It offers a steady state solution to the cornering manoeuvre.

Next, results from field tests run by the University of Michigan Transportation Research Institute (UMTRI nee HSRI) are described. In these tests, steady state turning performance was measured experimentally for a five-axle tractor-semitrailer with an empty and a fully loaded trailer (8).

Steady state cornering results from the five models are then compared with the experimental results. The two responses compared are the steady state yaw velocity and the lateral acceleration of the tractor. These variables will be plotted as a function of front wheel steer angle. For this manoeuvre the only other response variable that could have been compared was the articulation angle between the tractor and the semitrailer. Since this information was not presented as part of the experimental results in Reference 8, it was not considered in the comparison.

The thesis concludes with a relative assessment of the five models and suggestions for further research.

Chapter 2 THE MATHEMATICAL MODELS

2.1 TIME DOMAIN MODELLING

The models discussed in sections 2.2.1, 2.2.3 and 2.2.4 are classed as time domain models. They all represent the vehicle by systems of nonlinear differential equations. In each case, the equations must be given initial conditions and then integrated. The complexity of the systems of equations precludes obtaining a solution by analytical integration. Thus, in each case, the equations must be integrated numerically.

Since each numerical integration scheme has its own order of accuracy and time step size related stability characteristics, it was deemed necessary to use the same integration scheme for each model. This guaranteed the same order of accuracy for the three models. The scheme used was the Hamming's Predictor-Corrector Method as employed in the program HPCG (16).

A fourth order Runge-Kutta algorithm is used to start the integration. Error control is effected by doubling or halving the time step size to achieve a user specified weighted error norm.

To check the accuracy of HPCG, it was used to integrate the function $f(x)=\cos x$ twice. The routine was used to evaluate the double integral at $x=\pi$. The function was chosen because of its continuously varying behaviour and because its integrals are easily evaluated.

$$\iint \cos x dx dx = -\cos x$$

Obviously the double integral evaluated at $x=\pi$ yields the result $-\cos x = 1.0$. To find the answer numerically, HPCG was given the initial conditions:

$$x = 0.0$$

$$\int \cos x dx = 0.0$$

$$\left(\int \int \cos x dx dx = -1.0 \right.$$

Two abscissa increments were tried in the integration, $x=\pi/2$ and

$x=\pi/4$. These rather large increments were used to investigate the effectiveness of the error checking capability claimed by HPCG.

Allowable error value of 0.1 was used in this test. Effect of varying allowable error was checked for $x=\pi/4$. Errors of 0.01 and 0.001 were used in this test. The results of these tests are presented in Appendix I.

All three time domain models presented problems insofar as achieving steady state was concerned. In general, the longitudinal acceleration was a complicated function of the forces on the vehicles' tires and the kinematic variables. As such it was very difficult to calculate the tractive effort required at the drive axles to maintain constant forward velocity. Thus, in a steady turning manoeuvre, the forward velocity was continuously decreasing in all three of the time domain models. When UMTRI discovered this problem (8) while comparing the 28 DOF model results to field tests, they decided to start the simulation at a higher speed than the test was carried out at. When the simulation slowed to the test speed, the variables of interest were recorded. It was assumed that the transient phase of the response was complete at this point and the results could be considered to be "quasisteady state".

To circumvent the same problem when obtaining results with the models in sections 2.2.1 and 2.2.3, the same approach was taken. The simulations were started at the same speeds as in reference 8 (44 ft/s). To study the effect of initial forward speed on the output variables, two higher initial speeds (48 ft/s and 52 ft/s) were run in the simulation for the 4 DOF model described in section 2.2.1

2.2 THE TIRE MODEL

Since this work deals only with the steady state cornering characteristics of a five axle tractor-semitrailer, only the cornering properties of tires are of direct interest. However, for the sake of completeness this section will present a full mathematical model of the tire describing both tractive and cornering performance. The model presented is the Dugoff model. It was selected because Reference 8 showed that it did a good job of modelling the tire properties of the vehicle tested in that report. The outline of the model presented is taken from Reference (8).

For the most general case, consider a steerable wheel such as the one in Figure 2. The tire centre has a lateral and a longitudinal component of velocity with respect to a body axes system fixed to the end of the axle. These velocity components are:

$$u_i = U - \omega T_j \text{ (longitudinal velocity of the } i\text{th tire)} \quad (2.1)$$

$$v_i = V + \omega L_j \text{ (lateral velocity of the } i\text{th tire)} \quad (2.2)$$

where

Figure 2 - Force Components in the Tire Axis System

ω - is the vehicle yaw rate

U - is the vehicle longitudinal velocity

V - is the vehicle lateral velocity

T_j - is the half track of the j th axle

L_j - is the longitudinal distance from the cg to the j th axle

Investigation of the tire-road interaction has shown that the lateral force on a tire is related to the angle between the tire direction of travel and the wheel plane of the tire. This is commonly known as the slip angle. In the case of the steerable wheel, the slip angle of the i th wheel is:

$$\alpha_i = \tan^{-1} (v_i/u_i) - \delta \quad (2.3)$$

where δ is the steer angle (Figure 2). For a free rolling tire at small slip angles (less than 3 degrees) the lateral force may be represented as:

$$F_Y = CA \alpha_i \quad (2.4)$$

where CA is the absolute value of the slope of the lateral force verses slip angle curve evaluated at $\alpha_i = 0$ and $s = 0$. This is the linear Dugoff model.

Parameter s is referred to as the longitudinal slip or simply the slip. It is expressed as:

$$s = 1 - \frac{RR \Omega}{u_w} \quad (2.5)$$

where:

$$u_w = u \cos \delta + v \sin \delta \quad (2.6)$$

is the longitudinal velocity of the tire.

Further, RR is the effective rolling radius of the tire and Ω is the spin velocity of the tire. Slip results from either braking or driving torque on the wheel. Studies have shown that for the case of no slip angle, longitudinal force is related to small values of slip ($s = 0.09$) by:

$$F_X = -CS \cdot s \quad (2.7)$$

Parameter CS is the absolute value of the slope of the longitudinal force versus slip curve evaluated at zero slip.

In the general case, both side slip and longitudinal slip may be present. For this case the relations are more complicated and have been defined as follows:

$$F_X = -CS \cdot s \cdot f(\lambda) / (1 - s) \quad (2.8)$$

$$F_Y = -CA \cdot d \cdot f(\lambda) / (1 - s) \quad (2.9)$$

where

$$\lambda = \frac{1}{2} \mu N (1 - s) / [(CS \cdot s)^2 + (CA \alpha)^2]^{1/2} \quad (2.10)$$

$$f(\lambda) = (2 - \lambda) \lambda \quad \text{for } \lambda < 1 \quad (2.11)$$

$$f(\lambda) = 1 \quad \text{for } \lambda \geq 1 \quad (2.12)$$

$$\mu = \mu_0 (1 - FA \cdot VS) \quad (2.13)$$

$$VS = uw (s^2 + \alpha^2)^{1/2} \quad (2.14)$$

with the following parameter definitions:

μ_0 and FA - characteristic parameters for specific tire-pavement combinations

μ - coefficient of friction between the tire and the road,

VS - effective sliding velocity of the tire

N - normal force exerted by the tire on the road

λ - longitudinal co-ordinate describing where slip between the tire and the road begins in the tire-road contact patch.

Thus far, the model presented correlates kinematic variables with tire performance through consideration of the most up-to-date tire physics. This correlation has been demonstrated to work quite well at small slips and slip angles. To extend the range of the model validity, Reference (8) introduces a curve fitting technique which worked well for the tires considered in that report. The cornering stiffness coefficient, CA, was modified depending on slip angle as follows:

$$CA' = CA (1 - KF |\alpha|) \text{ for } |\alpha| < \bar{\alpha} \quad (2.15)$$

$$CA' = CA (1 - KF \bar{\alpha}) \text{ for } |\alpha| \geq \bar{\alpha} \quad (2.16)$$

with $KF = 1.7$

$$\bar{\alpha} = 0.15707 \text{ radians}$$

CA' is then used in place of CA in equations 2.8, 2.9, and 2.10. Figure 3 shows a plot of lateral force versus side slip angle using the Dugoff tire model with curve fitting for a 10 x 20 F tire at 85 psi. The 4 DOF model in section 2.2.1 was implemented with both the linear and the nonlinear Dugoff model. The nature of the 3 DOF model in section 2.2.2 required that the linear Dugoff model be used. TBS, the program implementing the 4 DOF model with quasistatic load transfer (section 2.2.3) represents lateral tire force as a polynomial function of the tire side slip angle. Since the vehicle dynamics were dominant in this model, the tire routine was not changed. This fact is discussed further in section 4.3. As with the 4 DOF model, the 6 DOF model in section 2.2.5 was implemented with both the linear and nonlinear Dugoff model.

Figure 3

Lateral force vs. sideslip angle.

FA = 0, $\mu_0 = .85$, C_{α} fromTable 2, KF = 1.2, $\bar{\alpha} = 9$

The 28 DOF model outlined in section 2.2.4 uses a table lookup scheme for properties. Since the Dugoff model very closely matched the tabulated tire response, it was felt that consistency was maintained with the table lookup approach.

2.3 THE VEHICLE MODELS

2.3.1 CLASSICAL 4 DOF MODEL

This mathematical representation of the tractor-semitrailer first appeared in the literature in about 1963 (1,2).

The model is classified as a 2-D representation since it only considers motion in the horizontal plane. No vertical, pitch or roll motion is considered. A body fixed or path variable co-ordinate set attached to the tractor cg as shown in Figure 4 is used to describe vehicle motion.

The degrees of freedom are:

- (1) tractor longitudinal velocity (U)
- (2) tractor lateral velocity (V)
- (3) tractor yaw rate (ω)
- (4) trailer yaw rate (ω')

The translational and angular velocity of the tractor may be written as:

$$\begin{aligned} \text{VEL} &= U\bar{i} + V\bar{j} \\ \bar{\Omega} &= \omega\bar{k} \end{aligned} \quad 2.17$$

Write the acceleration as the derivative of the velocity

$$\overline{\text{accel}} = \dot{U}\bar{i} + \dot{V}\bar{j} + U\frac{d\bar{i}}{dt} + V\frac{d\bar{j}}{dt} \quad 2.18$$

Figure 4 - 4 DOF Representation of the 5 Axle Tractor-Semitrailer

To find the time derivatives of the unit vectors, consider Figure 5 showing the co-ordinate system change from time equals t to time equals $t + \Delta t$. Note that the translational components have been subtracted out.

The difference between $\bar{I}$ and $\bar{I}'$ caused by rotation θ is $\Delta\bar{I}$. If $\Delta\bar{I}$ is very small corresponding to very small Δt , it may be written as:

$$\Delta\bar{I} = 1 \cdot \theta \bar{J} \quad 2.19$$

The direction $\bar{J}$ is apparent from the diagram. Write θ as a function of Δt .

$$\theta = \omega \Delta t + 1/2 \dot{\omega} \Delta t^2 + \dots \quad 2.20$$

Now combine 2.19 and 2.20 to find

$$d\bar{I}/dt = \lim_{\Delta t \rightarrow 0} \frac{(\omega \Delta t + 1/2 \dot{\omega} \Delta t^2 + \dots)}{\Delta t} = \omega \bar{J} \quad 2.21$$

Similarly, write $d\bar{J}/dt$ as:

$$d\bar{J}/dt = -\omega \bar{I} \quad 2.22$$

Finally, rewrite the acceleration:

$$\text{accel} = (\dot{U} - V\omega) \bar{I} + (\dot{V} + U\omega) \bar{J} \quad 2.23$$

With this expression for acceleration, two equations for translation may be written using the Newton formulation:

$$m_1 (\dot{U} - V\omega) = X - f_1 \sin \delta \quad 2.24$$

$$m_1 (\dot{V} + U\omega) = Y + f_3 + f_2 + f_1 \cos \delta \quad 2.25$$

A third equation may be obtained by summing moments about the tractor cg:

$$I_1 \dot{\omega} = a f_1 - b f_2 - (b + c) f_3 - Yd \quad 2.26$$

The trailer may be analysed using the (U', V') co-ordinate set.

With the same analysis used for the tractor, the three trailer equations are:

Figure 5 - Rotating Unit Vectors

$$m_2(\dot{U}' - V'\omega') = -X \cos\psi + Y \sin\psi \quad 2.27$$

$$m_2(\dot{V}' + U'\omega') = -X \sin\psi - Y \cos\psi + f_4 + f_5 \quad 2.28$$

$$I_2\dot{\omega}' = -e(Y \cos\psi + X \sin\psi) - hf_4 - (1+h)f_5 \quad 2.29$$

To ensure compatibility, write hitch velocities with respect to (U,V) and (U', V') and equate:

$$U' = U \cos\psi - V \sin\psi + d\dot{\psi} \sin\psi \quad 2.30$$

$$V' + e\omega' = U \sin\psi + V \cos\psi - d\dot{\psi} \cos\psi \quad 2.31$$

Differentiating 2.30 and 2.31 yields expressions for $\dot{U}'$ and $\dot{V}'$

$$\dot{U}' = \dot{U} \cos\psi - U \dot{\psi} \sin\psi - \dot{V} \sin\psi - V \dot{\psi} \cos\psi + d\dot{\psi} \sin\psi + d\omega' \cos\psi \quad 2.32$$

$$\dot{V}' = \dot{U} \sin\psi + U \dot{\psi} \cos\psi + \dot{V} \cos\psi - V \dot{\psi} \sin\psi - d\dot{\psi} \cos\psi + d\omega' \sin\psi - e\dot{\omega}' \quad 2.33$$

Now equations 2.30, 2.31, 2.32 and 2.33 may be substituted in 2.27 and 2.28 to eliminate U' , V' , $\dot{U}'$ and $\dot{V}'$. If equations 2.24 to 2.29 are combined to eliminate hitch forces X and Y, the following four equations are obtained:

$$\begin{aligned} (m_1 + m_2)\dot{U} - (m_1 + m_2)V\omega + m_2 d\omega^2 - m_2 e \dot{\omega}' \sin\psi + m_2 e \omega'^2 \cos\psi \\ = (f_4 + f_5) \sin\psi - f_1 \sin\delta \end{aligned} \quad 2.34$$

$$\begin{aligned} (m_1 + m_2)\dot{V} + (m_1 + m_2)U\omega - m_2 d\dot{\omega} - m_2 e \dot{\omega}' \cos\psi - e \omega'^2 m_2 \sin\psi \\ = f_3 + f_2 + f_1 \cos\delta + (f_4 + f_5) \cos\psi \end{aligned} \quad 2.35$$

$$\begin{aligned} I_1 \dot{\omega} + m_1 e \dot{V}, + m_1 d U \omega = (a + d) f_1 \cos\delta - (b - d) f_2 + \\ + (d - b - c) f_3 \end{aligned} \quad 2.36$$

$$\begin{aligned} I_2 \dot{\omega}' + m_1 e V \cos\psi + m_1 e U \omega \cos\psi + m_1 e \dot{U} \sin\psi - m_1 e V \omega \sin\psi \\ = e(f_3 + f_2 + f_1 \cos\delta) \cos\psi - h f_4 - (1 + h) f_5 - e f_1 \sin\delta \sin\psi \end{aligned} \quad 2.37$$

These four differential equations comprise the 4 DOF model. They are integrated numerically using the Hamming's Predictor-Corrector method as implemented in the routine HPCG (16).

This is the simplest of the time domain models considered. Since roll, pitch and vertical displacement are not considered, load transfer due to lateral or longitudinal accelerations cannot be accounted for.

Since tire cornering stiffness is a function of normal load, the lack of load transfer will be of significance during extreme manoeuvres.

This is discussed further in Section 2.3.3.

To find the lateral forces f_1, f_2, f_3, f_4 , and f_5 write the slip angles as lateral velocity over longitudinal velocity:

$$\alpha_1 = \delta - (V + \omega)/U \quad 2.38$$

$$\alpha_2 = - (V - b\omega)/U \quad 2.39$$

$$\alpha_3 = - (V - b\omega)/U \quad 2.40$$

$$\alpha_4 = - (V' - h\omega)/U \quad 2.41$$

$$\alpha_5 = - (V' - (h+1)\omega)/U \quad 2.42$$

These slip angles may now be used with tire stiffnesses to find lateral forces as described in Section 2.1. Notice that the slip angle was calculated for an equivalent wheel located on the U axis. Since there is no load transfer in this model (i.e. tire cornering stiffness is constant), the vehicle is considered to be laterally telescoped and the average tire slip angle may be used. Another result of this "telescoping" is that average steer angle at the front wheels may be used as steering input.

The 4 DOF model was implemented in a program written by the author. All vehicle physical data as well as the initial conditions were input data to the program.

Prescribed steer angle was applied as a ramp function over a half second time interval to match test conditions. Data was generated for the full range of steer angles found in the experimental results for the cases of both loaded and empty trailers. This was done with the linear tire model of equation 2.4 and with the nonlinear tire model of equations 2.15 and 2.16. Data was also generated for the full trailer with both tire models for various initial speeds. The reason for this test was explained more fully in Section 2.1.

2.3.2 THE LINEAR 3 DOF MODEL

This model is a closed form, steady state solution to the constant turn manoeuvre obtained by linearizing the 4 DOF model (17). In the steady state there are no angular or translational accelerations.

Longitudinal velocity is constant. In addition, the following linearizing assumptions are made:

(1) small angle assumptions $\cos \psi = 1, \cos \delta = 1$

$\sin \psi = \psi, \sin \delta \approx 0$

(2) variables are small therefore products of variables are ignored.

(3) $U = U'$

Each of these assumptions may be substantiated by programming the full 4 DOF equations and examining the output variables. If the preceding steady state assumptions are substituted in equations 2.24 through 2.29, the following four equations result:

$$m_1 U \omega = Y + f_3 + f_2 + f_1 \quad 2.43$$

$$0 \approx a f_1 - b f_2 - (b+c) f_3 - Y d \quad 2.44$$

$$m_2 U \dot{\omega} = -Y + f_4 + f_5 \quad 2.45$$

$$0 = -eY - hf_4 - (1 + h)f_5 \quad 2.46$$

To eliminate the lateral forces from these equations, the linear Dugoff model is used to represent tire forces (equation 2.4). Substitute equations 2.30 and 2.31 in 2.41 and 2.42, then use equations 23.1 through 24.2 to define the slip angle in equation.

$$f_1 = -C_1(\delta - (V + a\omega)/U) \quad 2.47$$

$$f_2 = -C_2(V - b\omega)/U \quad 2.48$$

$$f_3 = -C_3(V - b_1\omega)/U \quad 2.49$$

$$f_4 = -C_4(\gamma + (V - (d + l_1)\omega)/U) \quad 2.50$$

$$f_5 = -C_5(\gamma + (V - (d + l_2)\omega)/U) \quad 2.51$$

Use equation 2.43 to eliminate the lateral hitch force Y from equations 2.44, 2.45, and 2.46. Substitute for lateral forces with the equations above. The result is the set of three equations shown in Figure 6.

The variables in this system are lateral velocity, yaw rate and articulation angle. Input parameters are the tractor trailer physical data, longitudinal velocity and steer angle.

Solution of this set of equations requires only the solution of the 3x3 matrix in Figure 6, a considerable savings of effort over the time integration required for the 4 DOF model. As with the 4 DOF model, load transfer from one side of an axle to another or fore and aft is not taken into account with the Linear 3 DOF model.

$$\begin{bmatrix} ((m_1+m_2)U^2+C_1a-C_2b-C_3b_1-C_4(d+1)_1-C_5(d+1)_2) \\ (em_2U^2-(e+1)_1)(d+1)_1C_4-(e+1)_2(d+1)_2C_5 \\ (dm_1U^2+a(a+d)C_1-b(d-b)C_2-b_1(d-b_1)C_3) \end{bmatrix}
 \begin{bmatrix} (C_1+C_2+C_3+C_4+C_5) \\ ((e+1)_1)C_4+(e+1)_2C_5 \\ ((a+d)C_1+(d-b)C_2+(d-b_1)C_3) \end{bmatrix}
 \begin{bmatrix} U(C_4+C_5) \\ ((e+1)_1)C_4+(e+1)_2C_5)U \\ 0 \end{bmatrix}
 =
 \begin{bmatrix} \omega \\ \sim \\ v \end{bmatrix}
 \begin{bmatrix} C_1\delta U \\ 0 \\ U(a+d)\delta C_1 \end{bmatrix}$$

Figure 6 - The Linear 3 DOF Equations

2.3.3 4 DOF MODEL WITH QUASISTATIC LOAD TRANSFER

One of the shortcomings of the classical 4 DOF model is its inability to account for load transfers between tire sets on an axle or between axles on a vehicle due to longitudinal and/or lateral accelerations. The significance of load transfer may be appreciated by considering the extreme case. At high lateral accelerations (i.e. greater than 0.3g) the load transfer can be great enough to completely shift all of the weight to the outside wheels causing the vehicle to roll over. Lateral accelerations below this level are still significant because of the nonlinear variation of tire cornering stiffness with normal load. Figure 7 depicts the variations in linearized cornering stiffness with normal load for the tractor and semi-trailer tires of the vehicle analysed in this report. (Table 2 summarizes the data).

True representation of the load transfer is obviously obtainable if full rigid body dynamics are considered. To avoid the complexities of this representation of load transfer, Leucht formulated the 4 DOF model with quasistatic load transfer (7).

In this model, the tractor and semitrailer are allowed to roll independantly. Load transfer resulting from this roll is calculated by equating the moment of the inertial force with the load transfer couple and the moment of the hitch force. For example, consider the front view of the tractor shown in Figure 8.

Table 2.- Linearized Tire Cornering Stiffness Properties

 $(\alpha < 3^\circ)$

Cornering Stiffness for
Tires on Axles 1, 4 & 5

NORMAL	CA
LOAD (lb)	(lb/rad)

1400	12260
2800	20860
4200	26760
5430	29970
6700	31520
8100	31970
9200	31920

Cornering Stiffness for
Tires on Axles 2 & 3

NORMAL	CA
LOAD (lb)	(lb/rad)

1400	11170
2800	19600
4200	25730
5430	29570
6700	31290
8100	31060
9200	31860

FIGURE 7 LINEARIZED TIRE CORNERING STIFFNESS PROPERTIES ($\alpha < 3$)

Figure 8 - Front View of a Tractor in a Turn

Solving for the load transfer force F as described.

$$h_1 m_1 a = h_4 Y - 2 T_1 F \quad 2.52$$

$$F = \frac{h_4 Y - h_1 m_1 a}{2 T_1} \quad 2.53$$

Note that only the load transfer portion of the tire normal forces are considered in the analysis and that the weight is neglected completely. This is because the static normal load on the tires and the weight acting at the cg cancel. The load transfer, F , may be apportioned between the front and rear axles with a ratio of front suspension roll stiffness to total vehicle roll stiffness. A similar analysis may be performed for the trailer to find its lateral load transfers. Finally a similar analysis is performed on both the tractor and trailer in the longitudinal directions to find longitudinal load transfers.

The two load transfer components at each wheel (lateral and longitudinal) may then be added to the static normal load to find the net normal load. The physical properties of each tire may then be calculated as a function of the normal load.

Note that this calculation scheme considers the "static" case at each time step and thus does not account for roll or pitch inertias.

Further, no allowance is made for a longitudinal hitch moment as would exist in real vehicle. These facts will be discussed further in Chapters 4 and 5.

The computer program TBS was written by the University of Michigan Transportation Research Institute (UMTRI) to implement this algorithm (9). Reference 9 gives a more thorough explanation of the model. TBS may be obtained by writing UMTRI.

As mentioned in section 2.2, the tire model in TBS was inconsistent with the those used in the rest of the simulations. As will be seen in Chapter 4 the vehicle dynamics were the dominant factor in determining vehicle response (i.e. the vehicle rolled over at small steer angles and the response was divergent). To compare the variation of total tire stiffness per axle with steer angle, it was felt that the differences in the tire models may be important. To remedy this problem, the 3 DOF model was modified to use the quasistatic load transfer scheme in conjunction with the linear tire model of equation 2.4. Total tire stiffnesses per axle were found based on quasistatic load transfer with the same tire property interpolation scheme used in the 6 DOF model (section 2.3.5).

Since the 3 DOF model is a steady state solution, an iterative scheme similar to the one in Figure 13 was used to find values of the response variables. As will be seen in Chapter 4, the linear 3 DOF model results were virtually identical to those from the 4 DOF model. Since the 4 DOF representation is the heart of TBS, this approach for finding the variation of per axle tire stiffness with steer angle was thought to be reasonable.

2.3.4 THE 28 DOF MODEL

This model is one of the more sophisticated mathematical representations of the tractor-semitrailer in existence. It provides a full rigid body representation of the articulated vehicle. Derivation of the model is quite time consuming and complicated and will not be given here.

Tobler presents a good discussion of the mathematics of the formulation in his PhD thesis (6). Reference 8 offers a good discussion of the physical considerations required in the implementation of the model.

Degrees of freedom incorporated in the model include:

1. 3 translational for the tractor (3 DOF)
2. 3 rotational for the tractor (3 DOF)
3. 2 rotational for the trailer (2 DOF)
4. 1 vertical translation for each axle (5 DOF)
5. 1 rotation (roll plane) for each axle (5 DOF)
6. 1 spin degree for each tire set on (10 DOF)

either side of an axle.

In addition to discussing attributes of the model, Reference 8 presents the experimental results used as the standard of comparison in this thesis. The results from the 28 DOF model were also taken from this report.

2.3.5 A PROPOSED LINEAR 6 DOF MODEL

Of the four models presented so far, two attempt to take load transfer effects into account. The 28 DOF model of section 2.3.4 uses a mathematically rigorous method to simulate suspension travel. Thus load transfers may be computed as the change in force from the equilibrium transmitted by the suspension.

The quasistatic model of section 2.3.3 tries to approximate the load transfer by performing a "quasistatic" summation of moments. If the technique worked, it would offer a simple calculation scheme for considering load transfer effects.

As will be pointed out in Chapters 4 and 5, this technique has serious drawbacks.

In an attempt to fill the need for a model with a simple yet reasonably accurate load transfer scheme, the author formulated the linearized 6 DOF model. It differs from the quasistatic model of section 2.3.3 in that it attempts to account for load transfer by considering the steady state position of the vehicle body relative to the ground. The formulation of this steady state solution is more complicated than the model of section 2.3.3. However, it requires only two additional input parameters and the solution scheme is ultimately no more complicated. Probably the main improvement of the 6 DOF model over the 4 DOF with quasistatic load transfer is that the longitudinal hitch moment has been included in the former.

As in the linearized 3 DOF model of section 2.3.2, degrees of freedom for this model include lateral velocity, yaw rate and articulation angle. In addition, three displacement variables are introduced to describe steady state vehicle position relative to the ground. These displacements are:

- (1) ϕ tractor pitch angle
- (2) θ tractor roll angle
- (3) z vertical translation of the fifth wheel

These new displacements are measured from the coordinate set UVW at the tractor center of gravity (figure 9). Due to the kinematic constraints of the hitch, the tractor and semitrailer must roll together along the tractor's longitudinal axis. The semitrailer pitch DOF was not modelled.

Experimenting with different displacements proved that neglecting trailer pitch and including vertical translation gave the best results. As such, trailer pitch was neglected and general vertical translation z included. By neglecting trailer pitch as an independent DOF, the hitch is being modelled as a king pin. As such, the model is not kinematically consistent with the real vehicle. However, allowing the hitch to transmit a roll moment is an improvement over the quasistatic model of Section 2.3.3 as it included neither trailer pitch or hitch moment. Consider Figure 9 depicting the freebody diagram of the tractor semitrailer. Using standard Newtonian formulation, write the translation equations for the tractor with respect to the UVW axes set. Accelerations are determined by the methods in section 2.2.1.

$$m_1 V\omega = X \quad (a)$$

$$m_1 U\omega = f_1 + f_2 + f_3 + Y \quad (b) \quad 2.54$$

$$0 = F_{1L} + F_{1R} + F_{2L} + F_{2R} + F_{3L} + F_{3R} + Z \quad (c)$$

Notice that in equation 2.54(c), the summation in the W direction, there are no terms to account for weight or static tire loads as these cancel. The F and F terms are forces due to load transfer.

Write the semitrailer translation equations:

$$m_2 V'\omega = -X\cos\gamma - Y\sin\gamma \quad (a)$$

$$m_2 U'\omega = f_4 + f_5 - Y\cos\gamma + X\sin\gamma \quad (b) \quad 2.55$$

$$0 = F_{4L} + F_{4R} + F_{5L} + F_{5R} - Z \quad (c)$$

A final set of equations may be obtained by summing moments of all forces in the system. Moments for the entire vehicle will summed about the XYZ axes (Figure 9)

To facilitate this summation, write the co-ordinates of both centres of gravity and all tire-road contact points (refer to Figure 10 for dimensions).

Figure 9 - Freebody Diagrams of the Tractor and the Semitrailer

Figure 10 - Dimensions of the Tractor-Semitrailer

For the tractor:

$$r_{1L} = (a+d, -T_1, h_4) \quad (a)$$

$$r_{1R} = (a+d, T_1, h_4) \quad (b)$$

$$r_{2L} = (d-b, -T_2, h_4) \quad (c) \quad 2.56$$

$$r_{2R} = (d-b, T_2, h_4) \quad (d)$$

$$r_{3L} = (d-b_1, -T_2, h_4) \quad (e)$$

$$r_{3R} = (d-b_1, T_2, h_4) \quad (f)$$

$$r_{cg1} = (d, 0, -h_3) \quad (g)$$

For the trailer, first write the co-ordinates with respect to $x''y''z''$ shown in Figure 9.

$$r'_{4L} = (-l_1, -T_3, h_4) \quad (a)$$

$$r'_{4R} = (-l_1, T_3, h_4) \quad (b)$$

$$r'_{5L} = (-l_2, -T_3, h_4) \quad (c) \quad 2.57$$

$$r'_{5R} = (-l_2, T_3, h_4) \quad (d)$$

$$r'_{cg2} = (-e, 0, -h_5) \quad (e)$$

To write the co-ordinates in 2.57 with respect to XYZ operate on them with rotation matrix [A].

$$\begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = [A] \quad 2.58$$

The transformed trailer co-ordinates are:

$$r_{4L} = (-l_1 \cos\psi - T_3 \sin\psi, l_1 \sin\psi - T_3 \cos\psi, h_4) \quad (a)$$

$$r_{4R} = (-l_1 \cos\psi + T_3 \sin\psi, l_1 \sin\psi + T_3 \cos\psi, h_4) \quad (b)$$

$$r_{5L} = (-l_2 \cos\psi - T_3 \sin\psi, l_2 \sin\psi - T_3 \cos\psi, h_4) \quad (c)$$

$$r_{5R} = (-l_2 \cos\psi + T_3 \sin\psi, l_2 \sin\psi + T_3 \cos\psi, h_4) \quad (d)$$

$$r_{cg2} = (-e \cos\psi, e \sin\psi, -h_5) \quad (e) \quad 2.59$$

Now write the tractor inertial force and tire forces in vector form:

$$f_{1L} = (0, f_1/2, F_{1L}) \quad (a)$$

$$f_{1R} = (0, f_1/2, F_{1R}) \quad (b)$$

$$f_{2L} = (0, f_2/2, F_{2L}) \quad (c)$$

$$f_{2R} = (0, f_2/2, F_{2R}) \quad (d)$$

$$f_{3L} = (0, f_3/2, F_{3L}) \quad (e)$$

$$f_{3R} = (0, f_3/2, F_{3R}) \quad (f)$$

$$f_{in1} = (-m_1 V \omega, m_1 U \omega, 0) \quad (g) \quad 2.60$$

Write the trailer forces in vector form with respect to $x'' y'' z''$.

$$f'_{4L} = (0, f_4/2, F_{4L}) \quad (a)$$

$$f'_{4R} = (0, f_4/2, F_{4R}) \quad (b)$$

$$f'_{5L} = (0, f_5/2, F_{5L}) \quad (c)$$

$$f'_{5R} = (0, f_5/2, F_{5R}) \quad (d)$$

$$f'_{in2} = (-m_2 V' \omega, m_2 U' \omega, 0) \quad (e) \quad 2.61$$

Operate on these forces with matrix [A] to transform them to X Y Z co-ordinates:

$$f_{4L} = (f_4 \sin \gamma / 2, f_4 \cos \gamma / 2, F_{4L}) \quad (a)$$

$$f_{4R} = (f_4 \sin \gamma / 2, f_4 \cos \gamma / 2, F_{4R}) \quad (b)$$

$$f_{5L} = (f_5 \sin \gamma / 2, f_5 \cos \gamma / 2, F_{5L}) \quad (c)$$

$$f_{5R} = (f_5 \sin \gamma / 2, f_5 \cos \gamma / 2, F_{5R}) \quad (d)$$

$$f_{in2} = (-m_2 \omega (V' \cos \gamma - U' \sin \gamma), m_2 \omega (V' \sin \gamma + U' \cos \gamma), 0) \quad (e) \quad 2.71$$

The moment summation may now be written as:

$$r_{cg1} \times f_{in1} + r_{cg2} \times f_{in2} = \sum_{i=1}^5 (r_{iL} \times f_{iL} + r_{iR} \times f_{iR}) \quad 2.81$$

If the summation in 2.81 is performed and the results separated into 4 equations representing the three co-ordinate directions, the following equations are obtained. The equations in the z direction for the

tractor and the semitrailer have been written separately. This was done because there is no moment coupling along this axis.

$$h_5 m_2 (V' \omega \sin \psi + U' \omega \cos \psi) + h_3 m_1 U \omega = T_1 (F_{1R} - F_{1L}) + T_2 (F_{2R} - F_{2L} + F_{3R} - F_{3L}) + T_3 \cos \psi (F_{4R} - F_{4L} + F_{5R} - F_{5L}) \quad (a) \quad 2.82$$

(Moments about the X axis)

$$h_3 m_1 V \omega - h_5 m_2 (U' \omega \sin \psi - V' \omega \cos \psi) = -(a+b) (F_{1L} + F_{1R}) - (d-b) (F_{2L} + F_{2R}) - (d-b_1) (F_{3L} + F_{3R}) + l_1 \cos \psi (F_{4L} + F_{4R}) + l_2 \cos \psi (F_{5L} + F_{5R}) + T_3 \sin \psi (F_{4L} - F_{4R} + F_{5L} - F_{5R}) + h_4 (f_4 + f_5) \sin \psi \quad (b)$$

(Moments about the Y axis)

$$m_1 dU \omega = (a+d) f_1 + (d-b) f_2 + (d-b_1) f_3 \quad (c)$$

tractor)

$$m_2 eU \omega = l_1 f_4 + l_2 f_5 \quad (d)$$

(Moments about Z axis semitrailer)

If equations 2.54, 2.55 and 2.82 are to be solved as a steady state problem, some linearizing assumptions must be made. Assume the following:

- (1) Products of variables are small and can be ignored.
- (2) Make small angle assumptions $\cos \psi = 1$ and $\sin \psi = 0$
- (3) let $U' = U$

Further, let the tire forces, f_i , be defined by equations 2.47 through 2.51. With these assumptions equations 2.54(a) and 2.55(a) vanish. Equations 2.54(b) and 2.55(b) may be combined to eliminate Y. Write this new equation with 2.82(c) and 2.82(d):

$$(m_1 + m_2) U \omega = f_1 + f_2 + f_3 + f_4 + f_5 \quad (a)$$

$$m_1 dU \omega = (a+d) f_1 + (d-b) f_2 + (d-b_1) f_3 \quad (b)$$

$$m_2 eU \omega = l_1 f_4 + l_2 f_5 \quad (c) \quad 2.83$$

These are the same equations as 2.43 through 2.46. Substituting for the tire forces produces the system of linear equations shown in Figure 6. Notice that this system of equations is in fact the Linear 3 DOF model.

Thus, ω , V and γ may be found by solution of this system and used as constants in equations 2.54(c), 2.55(c), 2.82(a) and 2.82(b).

Equations 2.30 and 2.31 may be used to evaluate U' and V' to reduce the number of approximations in the remaining equations. Before solving the remaining equations, expressions for the load transfer terms (F_{iL} and F_{iR} , $i = 1, 5$) must be found. If the position of the vehicle relative to the ground in a steady state turn can be found, then load transfer forces can be calculated. Load transfer forces may be written as the product of change in vertical height from the static position with suspension spring constant. To define the change in height due to accelerations, introduce the co-ordinates ϕ , θ , and z as previously mentioned.

Define axes set xyz to have a common origin with UW . The x - y plane is parallel to the ground. Tractor orientation may now be described relative to the xyz axes. First, let UW and xyz be completely aligned (Figure 11a). Roll UW through θ radians about the x axis. (Figure 11b). Now pitch UW ϕ radians about the y axis in its new position (Figure 11c) to arrive at the steady state tractor orientation. These two rotations are expressed mathematically by transformation matrix $[B]$ in 13.4.

$$[B] = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ -\sin\phi\cos\theta & \cos\theta & \sin\phi\sin\theta \\ -\cos\phi\sin\theta & -\sin\theta & \cos\phi\cos\theta \end{bmatrix} \quad (a)$$

$$[B] = \begin{bmatrix} 1 & 0 & \phi \\ 0 & 1 & \theta \\ -\phi & \theta & 1 \end{bmatrix} \quad (b)$$

2.84

Equation 2.84(b) is the small angle approximation of the transformation matrix.

Figure 11 - Coordinates Describing Steady State Tractor Orientation

Due to the nature of the tandem axle suspension geometry, both axles carry the same amount of load. Since this is true the tandem axle sets may be modelled as a single equivalent axle acting midway between the two axle positions. With this approximation in mind, redefine the tire-road contact points in the static position with respect to axes set xyz.

$$\begin{aligned} 1L &= (a, -T_1, h_1) & 2'L &= (-b_3, -T_2, h_1) \\ 1R &= (a, T_1, h_1) & 2'R &= (-b_3, T_2, h_1) \end{aligned} \quad 2.85$$

where

2'L and 2'R refer to the left and right wheel sets of the tractor equivalent axle. $(b_3 = (b+b_1)/2)$

Operating on the co-ordinates in 2.85 with [B] transforms them to the UW system. If this is done and h_1 is subtracted from the W co-ordinate and vertical translation z added to it, the remaining terms in W express the suspension travel. The suspension extensions for each of the tractor wheels are:

$$\begin{aligned} Z_{1L} &= -a\phi + T_1\theta + z \quad (a) & Z_{2L} &= b_3\phi + T_2\theta + z \quad (c) \\ Z_{1R} &= -a\phi - T_1\theta + z \quad (b) & Z_{2R} &= b_3\phi - T_2\theta + z \quad (d) \end{aligned} \quad 2.86$$

To find the suspension travel for the semi-trailer, a similar approach is used. First the co-ordinates of the equivalent single axle are redefined with respect to XYZ.

$$\begin{aligned} 3'L &= (-l_3 \cos \gamma - T_3 \sin \gamma, l_3 \sin \gamma - T_3 \cos \gamma, h_4) \quad (a) \\ 3'R &= (-l_3 \cos \gamma + T_3 \sin \gamma, l_3 \sin \gamma + T_3 \cos \gamma, h_4) \quad (b) \end{aligned} \quad 2.87$$

Next, the origin is shifted to the xyz set by subtracting d from the X co-ordinate and h_3 from the Z co-ordinate. Final semi-trailer position is found by applying transformation [B] to the co-ordinates. After subtracting the original z co-ordinate and adding the general translation z , the suspension travel is written as:

$$Z_{3L} = (l_3 \cos \gamma + T_3 \sin \gamma + d) \phi - (l_3 \sin \gamma - T_3 \cos \gamma) \theta + z \quad (a) \quad 2.88$$

$$Z_{3R} = (l_3 \cos \gamma - T_3 \sin \gamma + d) \phi - (l_3 \sin \gamma + T_3 \cos \gamma) \theta + z \quad (b)$$

Load transfer forces may now be written as the product of suspension extension and suspension spring constant.

$$F_{1L} = k_1(-a\phi + T_1\theta + z) \quad (a)$$

$$F_{1R} = k_1(-a\phi - T_1\theta + z) \quad (b)$$

$$F_{2L} = k_2(b_3\phi + T_2\theta + z)/2 \quad (c)$$

$$F_{2R} = k_2(b_3\phi - T_2\theta + z)/2 \quad (d)$$

$$F_{3L} = F_{2L} \quad (e)$$

$$F_{3R} = F_{2R} \quad (f) \quad 2.89$$

$$F_{4L} = k_3((l_3 \cos \gamma + T_3 \sin \gamma + d) \phi - (l_3 \sin \gamma - T_3 \cos \gamma) \theta + z)/2 \quad (g)$$

$$F_{4R} = k_3((l_3 \cos \gamma - T_3 \sin \gamma + d) \phi - (l_3 \sin \gamma + T_3 \cos \gamma) \theta + z)/2 \quad (h)$$

$$F_{5L} = F_{4L} \quad (i)$$

$$F_{5R} = F_{4R} \quad (j)$$

The expressions for load transfer forces in equations 2.89 may be substituted in equations 2.54(c), 2.55(c), 2.82(a) and 2.82(b) to yield four equations in the variables ϕ , θ , z and Z . Before proceeding to solve these equations, consider the physical nature of equations 2.82(a) and 2.82(b). They are the summation of moments about the X and Y axes. In the preceding formulation they are used to solve for the pitch, roll and heave of the vehicle relative to the ground.

In a turning manoeuver the axles of the vehicle remain essentially parallel to the ground. As such, it is the sprung masses that are pitching, rolling and heaving. Including the unsprung masses in the formulation for the displacement variables would thus give an inaccurate picture of the load transfer. To allow for this, the primed (') version of the tractor semi-trailer parameters are substituted in equations 2.82(a) and 2.82(b). These represent the parameters for the sprung masses as shown in Figure 10. If the vertical hitch force Z is eliminated the resulting system of equations is shown in Figure 12. If the equations in Figure 12 are solved, the values of ϕ , θ and z may be substituted in equations 2.89 to calculate the load transfer at each wheel set. Doing this simply predicts the load transfer for the 3 DOF model. However, if the calculated load transfer is used to modify the tire cornering stiffness of each wheel set (see the variation of cornering stiffness with normal load Figure 7), vehicle response may be recalculated. This recalculation procedure may be repeated until the response variables converge. Figure 13 shows the algorithm diagrammatically.

The program RBLT (Rigid Body Load Transfer) implements the 6 DOF model. Appendix II contains a listing of the program. The program was run with both the linear and the nonlinear tire models as mentioned in section 2.2. Section 4.5 presents and discusses the results from the 6 DOF simulation.

$$\begin{bmatrix}
 2(-a'k_1 + b'_3k_2 + (l_3 \cos \psi + d')k_3) & -2k_3l_3 \sin \psi & 2(k_1 + k_2 + k_3) & 0 \\
 2k_3(l_3 \sin \psi (l_3 \cos \psi + d') - T_3^2 \cos \psi \sin \psi) & -2(k_1 T_1^2 + k_2 T_2^2 + k_3 T_3^2 \cos^2 \psi + k_3 l_3^2 \sin^2 \psi) & 2l_3 \sin \psi k_3 & h'_5 m'_2 (V' \omega \sin \psi + U' \omega \cos \psi) + h'_3 m'_1 U' \omega + h_4 (f_1 + f_2 + f_3 + (f_4 + f_5) \cos \psi) \\
 -2(-a'(a' + b')k_1 + b'_3(d' - b'_3)) & 2k_3(-l_3^2 \cos \psi \sin \psi + T_3^2 \cos \psi \sin \psi) & -2(k_1(a' + b') + k_2(d' - b'_3) - l_3 \cos \psi k_3) & h'_3 m'_1 V' \omega - h'_5 m'_2 (U' \omega \sin \psi - V' \omega \cos \psi) - h_4 (f_4 + f_5) \sin \psi
 \end{bmatrix}$$

Figure 12 - The 6 DOF Displacement Equations

Figure 13 - Flow Chart for the 6 DOF Model

CHAPTER 3 EXPERIMENTAL RESULTS

The experimental results used in this thesis were obtained by UMTRI and are extensively documented in Reference 8. For completeness, this section will highlight the significant features of the tests. Appendix I tabulates the tractor-semitrailer instrumentation used in the tests. Figures 14 and 15 summarize the important vehicle data for the empty and loaded trailer cases.

The tractor was a White 4x6, 46000 lb (20900 kg) GVW, Cab over engine on a 142 inch (3.60m) wheelbase with tandem drive axles. The trailer was a Fruehauf 40 ft. (12.2 m) with tandem axles and a 34000 lb. (15400 kg) gross load capacity.

Testing was conducted on the skid pad at the Bendix Automotive Development Centre (BADC). Skid pad surface was high coefficient Jennite.

To maintain the correct steer angle during cornering, a steering stop was mounted on the steering column. Initially, the vehicle was driven in a straight line. A "ramp steer" was applied by the driver until the steering wheel hit the preset stop. Speed was maintained by the driver until the vehicle achieved steady state conditions. All variables were recorded automatically (see Appendix II for instrumentation).

Tests were run with the trailer empty and with a 46800 lb. (21200 kg) payload of containerized gravel. Speeds were 39 ft/s (11.9 m/s) with the loaded trailer and 40 ft/s (12.2 m/s) with the empty trailer.

Tire data used in all the simulations were obtained from the UMTRI Flat Bed Tester detailed in Reference 18 (Table 2).

TRACTOR

a = 67"	a' = 35.9"	T1 = 40"
b = 47.8"	h1 = 52.5"	T2 = 36"
c = 54.4"	h4 = 48"	k1 = 1018 lb/in
d = 75"		k2 = 6590 lb/in
m1 = 14970 lb	m'1 = 9245 lb	

Tractor Yaw Moment of Inertia = 9917 slug ft**2

1

SEMITRAILER

e = 265"	e' = 222"	T3 = 36"
l = 49.3"	h2 = .69"	k3 = 7880 lb/in
h = 76"		
m2 = 11160 lb	m'2 = 8120 lb	

Trailer Yaw Moment of Inertia = 50417 slug ft**2

NORMAL LOADS ON EACH AXLE

AXLE	LOAD (lbs)
1	8550
2	5043
3	5043
4	3747
5	3747

Figure 14 - Data for the Empty Trailer

TRACTOR

a = 67"	a' = 35.9"	T1 = 40"
b = 47.8"	h1 = 52.5"	T2 = 36"
c = 54.4"	h4 = 48"	k1 = 1018 lb/in
d = 75"		k2 = 6590 lb/in
m1 = 14970 lb	m'1 = 9245 lb	

Tractor Yaw Moment of Inertia = 9917 slug ft**2

SEMITRAILER

e = 218"	e' = 188.5"	T3 = 36"
l = 49.3"	h2 = 68.3"	k3 = 7880 lb/in
h = 123"		
m2 = 57960 lb	m'2 = 54920 lb	

Trailer Yaw Moment of Inertia = 318130 slug ft**2

NORMAL LOADS ON EACH AXLE

AXLE	LOAD (lbs)
1	10306
2	15705
3	15705
4	15607
5	15607

Figure 15 - Data for the Loaded Trailer

CHAPTER 4 RESULTS AND DISCUSSION

This chapter presents and discusses the results of the cornering simulations from all the mathematical models. A comparison of all the models to the experimental results may be found at the end of the chapter in Tables 17 and 18 and Figures 32 to 35.

Progressing from the simplest to the most complicated, the results of the models are analysed. As the discussion proceeds, results from investigations into various aspects of the models behaviour are presented. The chapter concludes with a discussion of the new linear 6 DOF model proposed by the author.

4.1 THE LINEAR 3 DOF MODEL

Results from the 3 DOF model are tabulated in Tables 17 and 18 and shown graphically in Figures 32 through 35 along with the results from all the other models. As might be expected from consideration of the name of this model, response was a linear function of the steer angle. Figures 32 and 33 show that the 3 DOF model gave reasonable results for the empty trailer at small steer angles (less than 4 degrees). Error was approximately 50% for extreme steer angles. The results of the simulation for the full trailer were less satisfactory. Error was fairly consistent at about 100% throughout the range of steer angles simulated. With this level of error, the simulation could at best be labelled a good order of magnitude estimate of response. What is significant about these results is that they are virtually identical to those from the 4 DOF model.

Since the 3 DOF model does not require yaw moments of inertia, gathering input data for it is very easy compared to the 4 DOF model. Supplementing this advantage of the 3 DOF representation is its simple solution scheme. Since it is a steady state solution requiring only the solution of a 3x3 matrix for the response at a steer angle, computation time is reduced by a factor of 1000 (approximately).

4.2 THE 4 DOF MODEL

The results of the 4 DOF simulation with the linear and nonlinear tire models are listed in Table 3 and Table 4 along with the corresponding experimental results. Figures 16 through 19 depict this information graphically. From Figures 16 and 17, it is apparent that the nonlinear tire model offered better approximation of the experimental results for the empty trailer case. However, Figures 18 and 19 show that the opposite was true for the loaded trailer. Here the linear tire model offered the better representation. In either case, both tire models yielded results which were higher than the experimental values. Obviously it is possible to select either tire model as being the superior representation depending on the selection criteria. If the better tire model were to be chosen on the basis of average absolute error, Table 5 may be useful. For the comparisons in this work the linear tire model was used.

TABLE 3 RESULTS - 4 DOF MODEL - EMPTY TRAILER

3(a) YAW RATE (deg/sec)

steer angle (deg)	exper results	4 dof linear tires	4 dof nonlinear tires
0.00	0.00	0.00	0.00
4.20	9.30	12.14	12.03
4.60	10.50	13.30	13.16
5.96	13.20	17.21	16.96
6.66	14.10	19.25	18.91
7.86	15.90	22.72	22.19
8.91	17.90	25.76	25.00
9.17	17.70	26.50	25.68
10.37	19.80	29.98	28.78
10.43	19.80	30.14	28.92

3(b) LATERAL ACCELERATION (ft/sec**2)

steer angle (deg)	exper results	4 dof linear tires	4 dof nonlinear tires
0.00	0.00	0.00	0.00
4.20	6.50	8.50	8.42
4.60	7.70	9.31	9.22
5.96	9.60	12.08	11.91
6.66	10.30	13.52	13.31
7.86	11.90	16.00	15.69
8.91	12.80	18.19	17.77
9.17	13.20	18.73	18.28
10.37	14.80	21.25	20.62
10.43	14.50	21.37	20.73

TABLE 4 RESULTS - 4 DOF MODEL - LOADED TRAILER

4(a) YAW RATE (deg/sec)

steer angle (deg)	exper results	4 dof linear tires	4 dof nonlinear tires
0.00	0.00	0.00	0.00
4.59	7.60	14.20	14.52
5.40	9.70	16.72	17.23
8.30	14.30	25.81	28.29
8.81	15.00	27.42	29.78
9.15	15.40	28.50	30.62
10.06	15.90	31.41	32.77
11.31	17.10	35.45	37.05
11.72	17.80	36.80	38.55

4(b) LATERAL ACCELERATION (ft/sec**2)

steer angle (deg)	exper results	4 dof linear tires	4 dof nonlinear tires
0.00	0.00	0.00	0.00
4.59	5.50	9.71	9.62
5.40	6.70	11.46	11.87
8.30	9.60	17.89	20.08
8.81	10.00	19.04	21.11
9.15	10.90	19.82	21.74
10.06	11.50	21.93	23.47
11.31	12.20	24.89	26.54
11.72	12.90	25.88	27.59

FIGURE 16 4 DOF MODEL - YAW RESPONSE - EMPTY TRAILER

FIGURE 17 4 DOF MODEL - LATERAL ACCELERATION - EMPTY TRAILER

FIGURE 18 4 DOF MODEL - YAW RESPONSE - LOADED TRAILER

FIGURE 19 4 DOF MODEL - LATERAL ACCELERATION - LOADED TRAILER

Table 5 - Average Absolute Error of 4 DOF Simulations

	Average Absolute Error (%)	
	Empty	Loaded
	Trailer	Trailer
No Cornering Stiffness Mod.	40	90
Cornering Stiffness Mod.	37	100

As mentioned in section 2.1, all time domain simulations had to be started at an initial velocity higher than the experimental velocity because of the difficulty in predicting the tractive effort necessary to maintain constant forward velocity. Thus, time domain simulations were begun at 44 ft/s (30 mph). Yaw velocity and lateral acceleration were recorded when the simulations had slowed to 39 ft/s for the loaded trailer and 40 ft/s for the empty trailer. Tables 6 through 9 list the results of simulations started at 48 ft/s and 52 ft/s for loaded and empty trailers with both tire models. As mentioned in section 2.1, this was done to investigate the effect of different initial velocities on the response variables. It is apparent from Tables 6 and 7 that initial velocity had no effect on the response of the empty trailer. Tables 8 and 9 indicate that this was not the case for the loaded trailer. Figures 20 through 23 graphically depict this information. It is obvious that increased initial velocity began to have an effect on the response as the steer angle increased. For both the linear and nonlinear tire, response magnitude increased with increasing initial velocity. This effect was relatively small for the linear tire model compared to that of the nonlinear tire model.

TABLE 6 VARYING INITIAL VELOCITY - EMPTY TRAILER - 4 DOF
LINEAR TIRES

6(a) YAW RATE (deg/sec)

steer angle (deg)	Uinitial 44 ft/sec	Uinitial 48 ft/sec	Uinitial 52 ft/sec
0.00	0.00	0.00	0.00
4.20	12.14	12.14	12.14
4.60	13.30	13.30	13.30
5.96	17.21	17.21	17.21
6.66	19.25	19.25	19.25
7.86	22.72	22.72	22.72
8.91	25.76	25.75	25.75
9.17	26.50	26.50	26.51
10.37	29.98	29.98	29.98
10.43	30.14	30.14	30.13

6(b) LATERAL ACCELERATION (ft/sec**2)

steer angle (deg)	Uinitial 44 ft/sec	Uinitial 48 ft/sec	Uinitial 52 ft/sec
0.00	0.00	0.00	0.00
4.20	8.50	8.50	8.50
4.60	9.31	9.31	9.31
5.96	12.08	12.08	12.08
6.66	13.52	13.53	13.52
7.86	16.00	16.00	16.00
8.91	18.19	18.19	18.18
9.17	18.73	18.73	18.74
10.37	21.25	21.28	21.27
10.43	21.37	21.39	21.38

TABLE 7 VARYING INITIAL VELOCITY - EMPTY TRAILER - 4 DOF
NONLINEAR TIRES

7(a) YAW RATE (deg/sec)

steer angle (deg)	Uinitial 44 ft/sec	Uinitial 48 ft/sec	Uinitial 52 ft/sec
0.00	0.00	0.00	0.00
4.20	12.03	12.03	12.03
4.60	13.16	13.16	13.16
5.96	16.96	16.96	16.96
6.66	18.91	18.91	18.91
7.86	22.19	22.19	22.19
8.91	25.00	24.99	24.99
9.17	25.68	25.68	25.68
10.37	28.78	28.76	28.76
10.43	28.92	28.90	28.89

7(b) LATERAL ACCELERATION (ft/sec**2)

steer angle (deg)	Uinitial 44 ft/sec	Uinitial 48 ft/sec	Uinitial 52 ft/sec
0.00	0.00	0.00	0.00
4.20	8.42	8.43	8.42
4.60	9.22	9.22	9.22
5.96	11.91	11.92	11.92
6.66	13.31	13.32	13.32
7.86	15.69	15.69	15.69
8.91	17.77	17.77	17.77
9.17	18.28	18.29	18.29
10.37	20.62	20.65	20.65
10.43	20.73	20.77	20.76

FIGURE 21 LOADED TRAILER LATERAL ACCELERATION AT VARIOUS
INITIAL VELOCITIES - NONLINEAR TIRES

TABLE 9 VARYING INITIAL VELOCITY - LOADED TRAILER - 4 DOF
NONLINEAR TIRES

9(a) YAW RATE (deg/sec)

steer angle (deg)	Uinitial 44 ft/sec	Uinitial 48 ft/sec	Uinitial 52 ft/sec
0.00	0.00	0.00	0.00
4.59	14.52	14.52	14.52
5.40	17.23	17.23	17.23
8.30	28.29	28.42	28.47
8.81	29.78	29.82	29.84
9.15	30.62	30.66	30.66
10.06	32.77	33.23	33.64
11.31	37.05	38.06	38.85
11.72	38.55	39.69	40.72

&

9(b) LATERAL ACCELERATION (ft/sec**2)

steer angle (deg)	Uinitial 44 ft/sec	Uinitial 48 ft/sec	Uinitial 52 ft/sec
0.00	0.00	0.00	0.00
4.59	9.62	9.96	9.96
5.40	11.87	11.88	11.87
8.30	20.08	20.25	20.34
8.81	21.11	21.35	21.57
9.15	21.74	22.11	22.46
10.06	23.47	24.48	25.27
11.31	26.54	28.32	29.69
11.72	27.59	29.54	31.37

This assumption in the formulation of the model is kinematically inconsistent with the real world. It was these results and this belief that led the author to formulate the more rigorous linear 6 DOF model. Even with these shortcomings, the model may still be of use. It was originally formulated to study vehicle dynamics during braking manoeuvres (7) and has been shown to give reasonable results under these conditions (9,10). If the braked vehicle has a very small articulation angle and very low lateral acceleration, longitudinal hitch moments will not be significant and the quasistatic method may be valid. It is on this basis the model has not been dismissed as inadequate. The recommendation made is that this model should not be used to simulate manoeuvres involving steering inputs alone.

4.4 THE 28 DOF MODEL

As mentioned in section 2.2.4, the results for this model were obtained from Reference 8. They are tabulated along with the results of all of the other models in Tables 17 and 18 and plotted in Figures 32 through 35.

As might be expected, the 28 DOF model gave the best overall prediction of response for both vehicle configurations. It is "expected" because generally, the most complete models tend to give the best results (assuming that the model is properly formulated and the input parameters are correct).

FIGURE 21 LOADED TRAILER LATERAL ACCELERATION AT VARIOUS
INITIAL VELOCITIES - NONLINEAR TIRES

TABLE 11 RESULTS - 6 DOF MODEL - LOADED TRAILER

11(a) YAW RESPONSE (deg/sec)

steer angle (deg)	linear tires	nonlinear tires
0.00	0.00	0.00
4.59	13.83	13.83
5.40	16.00	16.07
8.30	23.17	22.47
8.81	23.41	23.07
9.15	24.29	23.43
10.06	25.80	25.04
11.31	25.36	25.41
11.72	25.70	27.87

11(b) LATERAL ACCELERATION (ft/sec**2)

steer angle (deg)	linear tires	nonlinear tires
0.00	0.00	0.00
4.59	9.42	9.42
5.40	10.89	10.94
8.30	15.77	15.29
8.81	15.94	15.71
9.15	16.54	15.95
10.06	17.56	17.04
11.31	17.26	17.30
11.72	17.49	18.97

FIGURE 23 LOADED TRAILER LATERAL ACCELERATION AT VARIOUS
INITIAL VELOCITIES - LINEAR TIRES

For example, contrast the extreme cases of initial velocity equals 44 ft/s versus 52 ft/s for yaw response at 11.72 degrees of steer angle for the loaded trailer. The nonlinear tire model shows a 5.6% spread versus a 0.60% spread for the linear model. These results indicate that care must be exercised with nonlinear models when prescribing initial conditions.

Insofar as the accuracy of the 4 DOF model is concerned, the discussion in section 4.1 holds since the predicted response is virtually identical to that from the 3 DOF model.

4.3 THE 4 DOF MODEL WITH QUASISTATIC LOAD TRANSFER

The results from this simulation are listed along with those from the other models in Tables 17 and 18. Figures 32 through 35 show this information graphically. Although this model predicted the response of the vehicle with an empty trailer (Figures 32 and 33) very well for up to 6 degrees of steer, results began to diverge after this point. The simulation predicted that the trailer wheels lifted off the ground at approximately 8.9 degrees of steer.

Simulation results of the loaded trailer (Figures 34 and 35) showed that the response was the same as for the 3 DOF model up to about 3 degrees of steer. At this steer angle, the results began to diverge culminating in trailer wheel lift off by 6.5 degrees of steer.

For both trailer loading conditions, the important trend to observe was that the response variables diverged before trailer wheel lift off was predicted. The author believes that the diverging response predicted by this model results from the fact that roll moment across the fifth wheel has not been considered.

This assumption in the formulation of the model is kinematically inconsistent with the real world. It was these results and this belief that led the author to formulate the more rigorous linear 6 DOF model. Even with these shortcomings, the model may still be of use. It was originally formulated to study vehicle dynamics during braking manoeuvres (7) and has been shown to give reasonable results under these conditions (9,10). If the braked vehicle has a very small articulation angle and very low lateral acceleration, longitudinal hitch moments will not be significant and the quasistatic method may be valid. It is on this basis the model has not been dismissed as inadequate. The recommendation made is that this model should not be used to simulate manoeuvres involving steering inputs alone.

4.4 THE 28 DOF MODEL

As mentioned in section 2.2.4, the results for this model were obtained from Reference 8. They are tabulated along with the results of all of the other models in Tables 17 and 18 and plotted in Figures 32 through 35.

As might be expected, the 28 DOF model gave the best overall prediction of response for both vehicle configurations. It is "expected" because generally, the most complete models tend to give the best results (assuming that the model is properly formulated and the input parameters are correct).

Figures 32 through 35 show that response was tracked very well but in all cases an offset error existed. The authors of Reference 8 were at a loss to explain this discrepancy. It was felt by the author that the error may partially be due to the fact that the simulation was started at a velocity approximately 10% higher than the desired value.

Response was then recorded when the vehicle had slowed to the desired speed. The tests of the 4 DOF model at varying initial velocities were made to investigate this hypothesis. It was found that for the loaded trailer with nonlinear tires, increasing the initial velocity increased the response level. However, this effect diminished at smaller steer angles. The relatively constant offset error would tend to cast doubt on this effect being the source of the discrepancy. Nevertheless, the steady state response of a nonlinear system is a function of the initial conditions. Considering this and the fact that the 28 DOF model is a highly nonlinear system of equations, it is still possible that a portion of the offset error results from the simulation method. Even with this discrepancy, the 28 DOF results were consistently better. What must be kept in mind when deciding if this model should be used is the cost of obtaining the input data. To get good data for the program, the tractor-semitrailer must be partially disassembled. Components must be weighed, measured and tested using a variety of apparatus. After all this effort, error in response variables may be as much as 15% for an empty trailer and 50% for a full trailer for the vehicle in a steady state turning manoeuvre. An analysis of the transient response of all the models might demonstrate the advantages of this model.

4.5 THE PROPOSED LINEAR 6 DOF MODEL

Due to the high cost of acquiring input data for sophisticated models that do a reasonable job of simulating load transfer effects, the need for a good approximate model has been identified (9,10). The 4 DOF model with quasistatic load transfer has been presented as being such a model (7,9). However, the discussion in section 4.3 has shown that this model is not a good representation of the five axle tractor semitrailer in a steady state cornering manoeuvre. Further, it was postulated that the reason for this was that no provision was made to account for a roll moment being transmitted across the fifth wheel. To demonstrate a more kinematically consistent technique for quasistatically approximating load transfer, the author formulated the linear 6 DOF model. This section presents and discusses the results from this model.

As with the 4 DOF model, this simulation was run with both the linear and nonlinear tire models. The results of these simulations are listed in Table 10 and 11 and are shown graphically in Figures 24 through 27. Examination of Figures 24 and 25 clearly shows that the nonlinear tire model offered better results for the empty trailer. The two tire models produced virtually identical results at small steer angles (i.e. less than 4.2° of steer). As steer angle increased, results of the two models began to diverge. At the maximum steer angle simulated (10.43°) the linear tire model predicted results 7.2% higher than the nonlinear model.

TABLE 10 RESULTS - 6 DOF MODEL - EMPTY TRAILER

10(a) YAW RATE (deg/sec)

steer		
angle	linear	nonlinear
(deg)	tires	tires
0.00	0.00	0.00
4.20	11.98	11.76
4.60	13.07	12.81
5.96	16.69	16.23
6.66	18.46	17.88
7.86	21.42	20.51
8.91	23.87	22.63
9.17	24.53	23.11
10.37	27.06	25.26
10.43	27.17	25.34

10(b) LATERAL ACCELERATION (ft/sec**2)

steer		
angle	linear	nonlinear
(deg)	tires	tires
0.00	0.00	0.00
4.20	8.36	8.21
4.60	9.13	8.95
5.96	11.65	11.33
6.66	12.89	12.48
7.86	14.95	14.32
8.91	16.67	15.80
9.17	17.13	16.13
10.37	18.89	17.63
10.43	18.97	17.69

TABLE 11 RESULTS - 6 DOF MODEL - LOADED TRAILER

11(a) YAW RESPONSE (deg/sec)

steer angle (deg)	linear tires	nonlinear tires
0.00	0.00	0.00
4.59	13.83	13.83
5.40	16.00	16.07
8.30	23.17	22.47
8.81	23.41	23.07
9.15	24.29	23.43
10.06	25.80	25.04
11.31	25.36	25.41
11.72	25.70	27.87

11(b) LATERAL ACCELERATION (ft/sec**2)

steer angle (deg)	linear tires	nonlinear tires
0.00	0.00	0.00
4.59	9.42	9.42
5.40	10.89	10.94
8.30	15.77	15.29
8.81	15.94	15.71
9.15	16.54	15.95
10.06	17.56	17.04
11.31	17.26	17.30
11.72	17.49	18.97

FIGURE 24 6 DOF MODEL YAW RESPONSE - EMPTY TRAILER

FIGURE 25 6 DOF MODEL LATERAL ACCELERATION - EMPTY TRAILER

FIGURE 26 6 DOF MODEL YAW RESPONSE - LOADED TRAILER

FIGURE 27 6 DOF MODEL LATERAL ACCELERATION - LOADED TRAILER

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For the loaded trailer, response with linear and nonlinear tires was virtually identical at steer angles less than 5 degrees (Figures 26 and 27). Above 5 degrees of steer, the nonlinear tires gave slightly better results except in the extreme case. Notice that at larger steer angles the simulation response curve is not smooth. It tends to be somewhat erratic. This apparent scatter in the response is a result of the solution technique. As mentioned in Section 2.3.5, the solution scheme for the 6 DOF model is iterative. In order to check for convergence, the error norm of the response variables is compared to a user supplied allowable error (see Appendix II for details of the algorithm). The empty trailer response converged at an 8.1% error. The loaded trailer converged at an allowable error of 14%. In either case, if the allowable error less than these values was used, the mid-range steer angles would not converge. It should be noted that the response did not diverge either. Rather, successive iterations occurred within a range defined by the allowable error for which convergence occurred. In other words, for the loaded trailer, the data points exhibiting an erratic pattern are single points lying in a convergence range. It is the existence of this range that made the response for 11.72 degrees of steer for the nonlinear tires higher than for the linear tires. Adjusting the allowable error to be higher would have yielded another response in the aforementioned range. On this basis, the nonlinear tires were chosen as the better tire model for the 6 DOF vehicle model. For both loading cases, the nonlinear tires yielded better results.

The linearized 6 DOF model predicted the same response as the 4 DOF model at relatively small steer angles (i.e. 4 degrees and less) for both the loaded and empty trailer cases (Figures 32 through 35). For both vehicle configurations, response level fell below that of the 4 DOF model as steer angle increased. This trend made it better than any of the three simple models discussed so far (the 28 DOF results were the best). The importance of this finding is that these superior results were obtained with substantially less computational effort than those from the two 4 DOF time domain models.

Since the two 4 DOF models and 6 DOF model are essentially the same models with different representations for tire cornering stiffness, this information was tabulated and plotted. As mentioned in section 2.3.3, the linear 3 DOF model was modified to iteratively solve for cornering response with the quasistatic load transfer technique. This was done to ensure that compatible tire models were being compared. The results of this investigation are listed in Tables 12 and 13 and plotted in Figures 28 and 29.

Figure 32 shows that for the empty trailer the quasistatic model did a good job of simulating vehicle response up to 6 degrees of steer. At this point the response began to diverge culminating in wheel lift off. Figure 28 shows that this divergence trend is paralleled with a sharp increase in tire stiffnesses on axles 2 and 3. Notice that the two simulations exhibit markedly different total axle stiffness characteristics and thus have different mechanisms responsible for simulation results.

TABLE 12 VARIATION OF PER AXLE TIRE STIFFNESS WITH STEER ANGLE

12(a) Empty Trailer - 6 DOF Model

STEER ANGLE (degrees)	C1 (lb/rad)	C2 and C3 (lb/rad)	C4 and C5 (lb/rad)
0.00	53911	40236	32813
4.20	48353	37576	23374
4.80	47713	37383	23163
5.96	45480	36856	22493
6.66	44294	36657	22492
7.86	42228	36485	23681
8.91	40473	36419	24593
9.17	40012	36446	24721
10.39	38021	36429	25709
10.43	37895	36446	25732

12(b) Loaded Trailer - 6 DOF Model

STEER ANGLE (degrees)	C1 (lb/rad)	C2 and C3 (lb/rad)	C4 and C5 (lb/rad)
0.00	58494	98125	102012
4.60	47695	83906	65763
5.40	44684	80204	62942
8.30	33462	67467	52327
8.81	31691	66203	51104
9.15	30295	64742	50189
10.06	28870	63476	49279
11.31	23431	56158	46834
11.72	25348	57725	46850

TABLE 13 VARIATION OF PER AXLE TIRE STIFFNESS WITH STEER ANGLE

13(c) Empty Trailer - 4 DOF with Quasistatic Load Transfer

STEER ANGLE (degrees)	C1 (lb/rad)	C2 and C3 (lb/rad)	C4 and C5 (lb/rad)
0.00	53911	40236	32813
4.20	53360	37259	32813
4.80	53272	36961	32813
5.96	52928	35761	32381
6.66	52790	36606	32102
7.86	52456	38411	31728
8.91	52131	39727	31411
9.17	52052	40082	31305
10.39	51708	41494	30978
10.43	51708	41484	31046

13(d) Loaded Trailer - 4 DOF with Quasistatic Load Transfer

STEER ANGLE (degrees)	C1 (lb/rad)	C2 and C3 (lb/rad)	C4 and C5 (lb/rad)
0.00	58494	98125	102012
4.60	45298	62896	62947
5.40	43087	62896	62947

FIGURE 28 CORNERING STIFFNESS vs STEER ANGLE - EMPTY TRAILER

FIGURE 29 CORNERING STIFFNESS vs STEER ANGLE - LOADED TRAILER

The most predominant characteristic of the 6 DOF model is the steady decrease in cornering stiffness of the front axle. This would effectively decrease the lateral force due to steer angle and lower the response variables (witness Figure 32). The same trend is demonstrated for the loaded trailer case (Figure 29). The reason for the divergence of the quasistatic model in this case is the substantial decrease in the cornering stiffnesses of axles 2 and 3. Reducing the stiffness in these axles reduces the lateral force available to resist turning. This explanation is given credibility by the fact that the stiffness of the other axles for both models are very nearly the same.

To investigate the mechanisms that cause the change in per axle tire stiffness on the 6 DOF model, Table 14 tabulates the normal loads on each axle during the manoeuvring. Figure 30 shows this information graphically for the empty trailer. Notice that the decrease in cornering stiffness on axle 1 corresponds to a decrease in net normal load on that axle. Figure 30 shows the same trend for the loaded trailer.

Table 15 further breaks this loading data down into loads carried by each wheelset of the empty trailer. These data show that the trailer wheels lift off the ground at 6.66 degrees of steer. Table 16 shows the same results for the loaded trailer at 11.31 degrees of steer. Notice in all cases that the load transfer trends are shifting weight from the inside wheelsets to the outside wheelsets.

TABLE 14 NORMAL LOADS ON EACH AXLE - 6 DOF MODEL

14(a) Empty Trailer

STEER ANGLE (degrees)	AXLE 1 (lbs)	AXLE 2&3 (lbs)	AXLE 4&5 (lbs)
0.00	8550	5043	3747
4.20	8236	5198	3749
4.60	8179	5226	3750
5.96	7963	5331	3753
6.66	7841	5390	3754
7.86	7611	5505	3754
8.91	7417	5601	3756
9.17	7396	5616	3751
10.37	7139	5739	3756
10.43	7131	5745	3754

14(b) Loaded Trailer

STEER ANGLE (degrees)	AXLE 1 (lbs)	AXLE 2&3 (lbs)	AXLE 4&5 (lbs)
0.00	10306	15705	15607
4.60	8944	16116	15877
5.40	8449	16263	15978
8.30	7024	16595	16358
8.81	6789	16691	16380
9.15	6629	16720	16430
10.06	6504	16759	16454
11.31	5463	16984	16749
11.72	5806	16949	16613

FIGURE 30 - VARIATION OF NORMAL LOADS WITH STEER ANGLE
EMPTY TRAILER

TABLE 15 NORMAL LOADS ON EACH WHEEL SET - 6DOF MODEL

15(a) Empty Trailer - Left Side Wheel Sets

Steer Angle (deg)	Axle 1 (lbs)	Axle 2&3 (lbs)	Axle 4&5 (lbs)
0.00	4275	2522	1874
4.20	4484	3665	3161
4.60	4486	3770	3273
5.96	4477	4109	3632
6.66	4461	4273	3804
7.86	4416	4532	4068
8.91	4370	4730	4270
9.17	4368	4762	4299
10.37	4292	4976	4518
10.43	4289	4983	4523

15(b) Empty Trailer - Right Side Wheel Sets

Steer Angle (deg)	Axle 1 (lbs)	Axle 2&3 (lbs)	Axle 4&5 (lbs)
0.00	4275	2522	1874
4.20	3752	1533	588
4.60	3693	1456	477
5.96	3486	1222	120
6.66	3379	1118	-49
7.86	3195	973	-313
8.91	3047	871	-514
9.17	3028	854	-548
10.37	2847	763	-762
10.43	2841	762	-769

TABLE 15 NORMAL LOADS ON EACH WHEEL SET - 6DOF MODEL

15(a) Empty Trailer - Left Side Wheel Sets

Steer Angle (deg)	Axle 1 (lbs)	Axle 2&3 (lbs)	Axle 4&5 (lbs)
0.00	4275	2522	1874
4.20	4484	3665	3161
4.60	4486	3770	3273
5.96	4477	4109	3632
6.66	4461	4273	3804
7.86	4416	4532	4068
8.91	4370	4730	4270
9.17	4368	4762	4299
10.37	4292	4976	4518
10.43	4289	4983	4523

15(b) Empty Trailer - Right Side Wheel Sets

Steer Angle (deg)	Axle 1 (lbs)	Axle 2&3 (lbs)	Axle 4&5 (lbs)
0.00	4275	2522	1874
4.20	3752	1533	588
4.60	3693	1456	477
5.96	3486	1222	120
6.66	3379	1118	-49
7.86	3195	973	-313
8.91	3047	871	-514
9.17	3028	854	-548
10.37	2847	763	-762
10.43	2841	762	-769

TABLE 16 NORMAL LOADS ON EACH WHEEL SET - 6DOF MODEL

16(a) Loaded Trailer - Left Side Wheel Sets

Steer Angle (deg)	Axle 1 (lbs)	Axle 2&3 (lbs)	Axle 4&5 (lbs)
0.00	5153	7853	7804
4.59	5781	11876	12558
5.40	5742	12554	13366
8.30	5548	14233	15481
8.81	5483	14434	15702
9.15	5448	14582	15902
10.06	5428	14725	16074
11.31	5152	15549	17188
11.72	5215	15216	16709

16(b) Loaded Trailer - Right Side Wheel Sets

Steer Angle (deg)	Axle 1 (lbs)	Axle 2&3 (lbs)	Axle 4&5 (lbs)
0.00	5153	7853	7804
4.59	3162	4240	3319
5.40	2709	3708	2612
8.30	1476	2363	877
8.81	1306	2257	678
9.15	1180	2138	529
10.06	1076	2034	381
11.31	311	1435	-439
11.72	591	1733	-96

The results of the simulation for the normal loads carried by each wheelset (Tables 17 and 18) show that although macroscopic results (i.e. response) from this model were good, the microscopic results (i.e. Tables 15 and 16) were not so good. According to this information, the semitrailer wheels lift off the ground at 6.6 degrees of steer for the empty trailer and at 11.3 degrees of steer for the loaded trailer. This could be due to the fact that trailer pitch relative to the tractor has not been modelled. As was pointed out in section 2.3.5 this degree of freedom could not be modelled without rendering the system of equations insoluble. Including trailer pitch relative to the tractor introduces a fourth displacement variable to be solved for. As shown in section 2.3.5 there are only three equations available for displacements, hence only 3 displacements may be found. Although wheel lift off was predicted by the 6 DOF model, the response results were good. In contrast, the 4 DOF model with quasistatic load transfer predicts wheel lift off in conjunction with diverging results. Thus the 6 DOF model may be used to predict the yaw and lateral acceleration response of the tractor-semitrailer. Results from this model will be the best obtainable from any of the simpler models available at the time of writing. However, the model should not be used in its present form to predict rollover thresholds. Since wheel lift off was predicted, the program should be modified to terminate when this occurs if it is to be used commercially. Perhaps if a way to include the pitch angle of the trailer relative to the tractor is found this flaw may be corrected.

TABLE 17 RESULTS - EMPTY TRAILER

17(a) Yaw Rate (deg/sec)

Steer Angle (deg)	Exper. Results	28 DOF Results	Linear 3 DOF Results	QS Load Transfer (TBS)	4 DOF Results	6 DOF Results
0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.20	9.30	11.00	12.15	9.40	12.14	11.76
4.60	10.50	11.10	13.30	10.30	13.30	12.81
5.96	13.20	15.00	17.24	13.40	17.21	16.23
6.66	14.10	16.10	19.26	15.20	19.25	17.88
7.86	15.90	18.20	22.73	19.00	22.72	20.51
8.91	17.90	20.00	25.77		25.76	22.63
9.17	17.70	20.70	26.52		26.50	23.11
10.37	19.80	21.80	29.99		29.98	25.26
10.43	19.80	21.80	30.16		30.14	25.34

17(b) Lateral Acceleration (ft/sec**2)

Steer Angle (deg)	Exper. Results	28 DOF Results	Linear 3 DOF Results	QS Load Transfer (TBS)	4 DOF Results	6 DOF Results
0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.20	6.50	7.90	8.48	6.60	8.50	8.21
4.60	7.70	8.00	9.29	7.20	9.31	8.95
5.96	9.60	10.70	12.03	9.40	12.08	11.33
6.66	10.30	11.40	13.45	10.70	13.52	12.48
7.86	11.90	12.90	15.87	13.50	16.00	14.32
8.91	12.80	14.30	17.99		18.19	15.80
9.17	13.20	14.60	18.51		18.73	16.13
10.37	14.80	15.80	20.94		21.25	17.63
10.43	14.50	15.00	21.06		21.37	17.69

FIGURE 32 YAW RATE vs STEER ANGLE - EMPTY TRAILER

FIGURE 33 LATERAL ACCELERATION vs STEER ANGLE - EMPTY TRAILER

TABLE 18 RESULTS LOADED TRAILER

18(a) Yaw Rate (deg/sec)

Steer Angle (deg)	Exper. Results	28 DOF Results	Linear 3 DOF Results	QS Load Transfer (TBS)	4 DOF Results	6 DOF Results
0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.59	7.60	12.30	14.17	14.50	14.20	13.83
5.40	9.70	14.20	16.67	18.40	16.72	16.07
8.30	14.30	19.90	25.63		25.81	22.47
8.81	15.00	21.70	27.20		27.42	23.07
9.15	15.40	22.00	28.25		28.50	23.43
10.06	15.90	22.40	31.06		31.41	25.04
11.31	17.10	22.70	34.92		35.45	25.41
11.72	17.80	23.00	36.19		36.80	27.87

18(b) Lateral Acceleration (ft/sec**2)

Steer Angle (deg)	Exper. Results	28 DOF Results	Linear 3 DOF Results	QS Load Transfer (TBS)	4 DOF Results	6 DOF Results
0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.59	5.50	8.40	9.65	10.00	9.71	9.42
5.40	6.70	9.50	11.35	13.10	11.46	10.94
8.30	9.60	13.50	17.44		17.89	15.29
8.81	10.00	13.50	18.52		19.04	15.71
9.15	10.90	14.00	19.23		19.82	15.95
10.06	11.50	15.30	21.14		21.93	17.04
11.31	12.20	16.20	23.77		24.89	17.30
11.72	12.90	16.60	24.63		25.88	18.97

FIGURE 34 YAW RATE vs STEER ANGLE - LOADED TRAILER

FIGURE 35 LATERAL ACCELERATION vs STEER ANGLE - LOADED TRAILER

CHAPTER 5 CONCLUSION

In this thesis the steady state cornering response of a 5 axle tractor-semitrailer predicted by five mathematical models of varying complexity was examined. Responses from the models were compared with each other and with experimental results from Reference (8). Models included both time domain and steady state solutions.

Investigation has shown that the linear tire model given by equation 2.4 was the suitable for use with the 4 DOF model. Equation 2.9 (the nonlinear tire model) was appropriate for the 6 DOF model.

Comparison of the linear 3 DOF and 4 DOF representations showed that these two models predicted virtually identical results. Thus, the 3 DOF model should be used if steady state cornering results are all that are desired. Use of this model precludes the need to measure yaw inertias. Computation time is also substantially less with this model. Both models gave reasonable results for the empty trailer at small steer angles (less than 4 degrees of steer). Error at extreme steer was in the order of 50%. The models gave an order of magnitude approximation for the loaded trailer over the whole range of steer angles (error was 100%). This was probably because the loaded vehicle had a large trailer mass with a high centre of gravity causing load transfer effects to be more significant.

The 4 DOF model with quasistatic load transfer gave the best prediction of all the models for the empty trailer at steer angles of less than 6 degrees. Beyond this the results diverged with trailer wheel lift off predicted at less than 9 degrees of steer. This model gave the worst results for the loaded trailer.

Table I - Integration Results for $x=\pi/2$ and Error=0.1

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHLF
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.7853981	-.7071068	-.7007803	.7071068	.7110446	1
1.5707963	-.0000001	.0064419	1.0000000	1.0027796	1
2.3561944	.7071067	.7221377	.7071069	.7110446	1
3.1415925	1.0000000	1.0165942	.0000002	.0000001	1
STOP					

Table II- Integration Results for $x=\pi/4$ and Error=0.1

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHLF
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.3926991	-.9238795	-.9237612	.3826834	.3828957	1
.7853982	-.7071068	-.7069554	.7071068	.7072018	1
1.1780971	-.3826835	-.3823559	.9238795	.9241646	1
1.5707961	-.0000002	.0001915	1.0000000	1.0002232	1
1.9634951	.3826832	.3831117	.9238796	.9240567	1
2.3561941	.7071065	.7077203	.7071070	.7072035	1
3.1415923	1.0000000	1.0084128	.0000004	-.0035551	0
STOP					

Table III- Integration Results for $x=\pi/4$ and Error=0.01

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHLF
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.3926991	-.9238795	-.9237612	.3826834	.3828957	1
.7853982	-.7071068	-.7069554	.7071068	.7072018	1
1.1780971	-.3826835	-.3823559	.9238795	.9241646	1
1.5707961	-.0000002	.0001915	1.0000000	1.0002232	1
1.9634951	.3826832	.3831117	.9238796	.9240567	1
2.3561941	.7071065	.7077203	.7071070	.7072035	1
2.7488931	.9238794	.9246976	.3826838	.3827253	1
3.1415921	1.0000000	1.0009019	.0000005	.0000012	1
STOP					

A major shortcoming of the model was that it, like the 4 DOF model with quasistatic load transfer, predicted trailer wheel lift off at moderate steer angles. Unlike its counterpart, the 6 DOF model results did not diverge. Thus, although this model will provide the best results with minimal computation, it should not be used in its present form for rollover analysis. It could be useful for studying rollover if relative trailer pitch could be included as a degree of freedom. Since the results did not diverge, perhaps this load transfer approach could be implemented in a time domain simulation.

In conclusion it should be stressed that this work has examined the steady state response of 5 vehicle models. For a complete comparison of model types, work must be continued in the area of transient analysis.

Table IV - Integration Results for $x = \pi/4$ and Error=0.001

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHL.F
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.3926991	-.9238795	-.9237612	.3826834	.3828957	1
.7853982	-.7071068	-.7069554	.7071068	.7072018	1
1.1780971	-.3826835	-.3823559	.9238795	.9241646	1
1.3744465	-.1950906	-.1947807	.9807852	.9810316	2
1.5707960	-.0000003	.0003604	1.0000000	1.0002948	2
1.7671455	.1950900	.1955070	.9807853	.9810712	2
1.9634950	.3826831	.3831699	.9238797	.9241632	2
2.1598444	.5555698	.5561195	.8314699	.8317515	2
2.3561939	.7071064	.7077114	.7071072	.7073881	2
2.5525434	.8314693	.8321301	.5555708	.5558511	2
2.7488929	.9238793	.9245944	.3826840	.3829638	2
2.9452424	.9807851	.9815550	.1950910	.1953702	2
3.1415919	1.0000000	1.0008241	.0000007	.0002796	2
STOP					

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APPENDIX I - INTEGRATION ROUTINE TEST

To validate the integration routine, it was used to evaluate the double integral of $\cos x$ at $x = \pi$ given the initial conditions

$$x = 0.0$$

$$\int \cos x dx = 0.0$$

$$\iint \cos x dx dx = -1.0$$

Table I and II give the results for the test of HPCG with $x = \pi/2$ and $x = \pi/4$ with an error value of 0.1. HPCG found the integral to within an accuracy of 1.55% with 4 increments of the abscissa (Table I). It is worth noting that this routine uses a fourth order Runge-Kutta algorithm to generate four points to start the integration. As such only the last x increment was calculated with the predictor-corrector in Table I. For the second case, $x = \pi/4$, half of the eight intermediate points were calculated by the predictor-corrector. Final error was 0.84%. The integers in the column labelled IHLF indicate the number of times the abscissa increment was halved. It is always halved initially to generate the Runge-Kutta starting values. Notice that in the $x = \pi/4$ test, the increment was returned to its initial value of $\pi/4$ for the last interval. This is the same increment size as the program used in the $\pi/2$ test for the last interval. Thus, given two different abscissa increment sizes and the same allowable error, the routine found the same increment size for the final interval. The smaller initial increments in the $\pi/4$ test resulted in its final answer being more accurate.

Table III lists the results of the integration for $x = \pi/4$ an error of 0.01. Decreasing allowable error by an order of magnitude improved the accuracy of the answer by an order of magnitude.

Table I - Integration Results for $x=\pi/2$ and Error=0.1

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHLF
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.7853981	-.7071068	-.7007803	.7071068	.7110446	1
1.5707963	-.0000001	.0064419	1.0000000	1.0023796	1
2.3561944	.7071067	.7221377	.7071069	.7110446	1
3.1415923	1.0000000	1.0165942	.0000002	.0000001	1
STOP					

Table II- Integration Results for $x=\pi/4$ and Error=0.1

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHLF
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.3926991	-.9238795	-.9237612	.3826834	.3828957	1
.7853982	-.7071068	-.7069554	.7071068	.7072018	1
1.1780971	-.3826835	-.3823559	.9238795	.9241646	1
1.5707961	-.0000002	.0001915	1.0000000	1.0002232	1
1.9634951	.3826832	.3831117	.9238796	.9240567	1
2.3561941	.7071065	.7077203	.7071070	.7072035	1
3.1415923	1.0000000	1.0084128	.0000004	-.0035551	0
STOP					

Table III- Integration Results for $x=\pi/4$ and Error=0.01

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHLF
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.3926991	-.9238795	-.9237612	.3826834	.3828957	1
.7853982	-.7071068	-.7069554	.7071068	.7072018	1
1.1780971	-.3826835	-.3823559	.9238795	.9241646	1
1.5707961	-.0000002	.0001915	1.0000000	1.0002232	1
1.9634951	.3826832	.3831117	.9238796	.9240567	1
2.3561941	.7071065	.7077203	.7071070	.7072035	1
2.7488931	.9238794	.9246976	.3826838	.3827253	1
3.1415921	1.0000000	1.0009019	.0000005	.0000012	1
STOP					

Table IV - Integration Results for $x = \pi/4$ and Error=0.001

X	-COS(X)	CALCULATED	SIN(X)	CALCULATED	IHL.F
.0000000	-1.0000000	-1.0000000	.0000000	.0000000	0
.3926991	-.9238795	-.9237612	.3826834	.3828957	1
.7853982	-.7071068	-.7069554	.7071068	.7072018	1
1.1780971	-.3826835	-.3823559	.9238795	.9241646	1
1.3744465	-.1950906	-.1947807	.9807852	.9810316	2
1.5707960	-.0000003	.0003604	1.0000000	1.0002948	2
1.7671455	.1950900	.1955070	.9807853	.9810712	2
1.9634950	.3826831	.3831699	.9238797	.9241632	2
2.1598444	.5555698	.5561195	.8314699	.8317515	2
2.3561939	.7071064	.7077114	.7071072	.7073881	2
2.5525434	.8314693	.8321301	.5555708	.5558511	2
2.7488929	.9238793	.9245944	.3826840	.3829638	2
2.9452424	.9807851	.9815550	.1950910	.1933702	2
3.1415919	1.0000000	1.0008241	.0000007	.0002796	2
STOP					

Further reducing the allowable error by an order of magnitude had a less significant effect (Table IV). Error was 0.082% versus the previous 0.090%.

This test demonstrates that HPCG can give good results with large increments of the abscissa when integrating a continuously varying function. It also demonstrates the routines' ability to vary abscissa increment size based on allowable error supplied by the user.

APPENDIX II
VEHICLE SPECIFICATIONS AND INSTRUMENTATION
(From Reference 8)

Vehicle Specifications, Tractor

Model	4x6, 46,000 lb gvwr, 142-in. wheelbase, COE (sleeper type)	
Engine	V-8, 335	
Transmission	5 speed forward, 1 reverse, 2 speed auxiliary spicer	
Rear Axle	34,000 with 4.11 ratio	
Steering Gear	28:1 constant ratio, lock to lock	
Wheels	Cast spoke	
Brakes	Front—dual chamber wedge type	Rear—dual chamber wedge type
Special equip.	limiting and quick release valve	relay valve and quick release valve
Air chamber	type 12	Type 12
Wedge angle	12°	12°
Size	15 x 4	15 x 7
Linings	RM-MR-417A	RM-MA-417A
Parking-emer.	---	single wedge, spring actuated, 4 rear wheels
Axles	12,000 lb	34,000 lb
Suspension	leaf spring	4 spring
Tires	highway tread, tube type	deep lug, tube type
Size	10.00-20	10.00-20
Load Range	F	F
Axle Weights		
Bobtail	8100 lb	6800 lb
Total Vehicle C.G.		
Position,	67 inches aft of front axle	
Bobtail	40 inches above ground level	

Trailer Specifications

Model	40 ft, van type, 2 axle, semitrailer
Suspension	4 spring (3 leaf)
Axles	34,000 lb
Brakes	S-cam, leading-trailing
Air chambers	type 30
Slack adjusters	6-inch length
Size	16-1/2 x 7
Linings	SAE friction code "EE"
Tires	highway tread, tube type
Size	10.00 x 20
Load range	F

Loading Conditions for the Articulated Vehicle

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Loading Condition	Static Axle Load (lb)			
	Front	Rear	Trailer	Total
Empty	8,900	10,500	7,800	27,200
Loaded	10,500	32,000	31,800	74,300

Loading Condition	C.G. Position			
	Tractor		Trailer	
	Aft of front axle (in.)	Height (in.)	Aft of Kingpin (in.)	Height (in.)
Empty	67	40	265	56
Loaded	67	40	218	66

Instrumentation, Tractor-Trailer Combination

Variable	Instrumentation
Left front steering angle, δ_L	Markite, Type 3595 Potentiometer, 5 K ohms
Right front steering angle, δ_R	Markite, Type 3595 Potentiometer, 5 K ohms
Steering wheel angle, δ_s	Amphenol Model 2101B Potentiometer, 10 K ohms
Brake line pressure at foot valve, P_f	CEC Type 4-327 Strain Gauge Pressure Transducer
Brake line pressure at front axle, P_1	Dynisco Model APT136 Strain Gauge Pressure Transducer
Brake line pressure at tractor rear axle, P_2	Dynisco Model APT136 Strain Gauge Pressure Transducer
Brake line pressure at trailer rear axle, P_3	Dynisco Model APT136 Strain Gauge Pressure Transducer
Tractor pitch, θ , roll, ϕ , Longitudinal acceleration, A_x , Lateral acceleration, A_y	Hymphry Inc., Stabilized Platform Unit SA07-0114-1
Yaw rate, $\dot{\psi}$, of tractor	Daystrom Pacific rate Gyro Model R59B90-1
Articulation angle between tractor and trailer, $\psi - \psi_1$	Beckman Helipot Mod 3301, 1 K
Wheel rotation/lock-up for each of ten wheels, LU_{1-10}	Erwell Bicycle Generators for go/no-go indication
Vehicle velocity, V_x	Tracktest Fifth Wheel
Brake lining temperature for each of ten wheels, T_{1-10}	Serve-Rite, Iron-Constantan Thermocouple
Recorders: Two Honeywell Visicorders, Model 2206, 14 Channel, light beam oscillograph	

APPENDIX III - LISTING OF RBLT (6DOF MODEL).

DICTIONARY OF VARIABLES

A	- Distance from front axle to tractor cg
A11	- Matrix coefficient for calculating response variables
A12	- " " " " "
A13	- " " " " "
A21	- " " " " "
A22	- " " " " "
A31	- " " " " "
A32	- " " " " "
A33	- " " " " "
ANS	- Array for storing simulation results
AP	- Distance from front axle to tractor sprung mass cg
B	- distance from tractor cg to leading tandem axle
B1	- distance from tractor cg to trailing tandem axle
B3P	- distance from tractor sprung mass cg to tandem axle center
BP	- distance from tractor sprung mass center to leading tandem axle
C	- distance between tractor tandem axles
C1	- cornering stiffness of tires on axle 1
C2	- cornering stiffness of tires on axle 2
C3	- " " " " " 3
C4	- " " " " " 4
C5	- " " " " " 5
CL	- cornering stiffness of left wheel set on an axle (temporary variable)
CM1	- array of normal loads vs tire stiffness axles 1 4 and 5
CM2	- array of normal loads vs tire stiffness axles 2 and 3
CR	- cornering stiffness of right wheel set on an axle (temporary variable)
D	- distance from cg of tractor to fifth wheel
DELTA	- vector of steering angles
DET	- determinant of equations in Figure 6
DP	- distance from tractor sprung mass to fifth wheel
E	- distance from fifth wheel to trailer cg
EP	- " " " " " " sprung mass cg
ERROR	- allowable error in response variables between iterations
ERRTEST	- actual error in response variables between iterations
F1L	- load transfer left side axle 1
F1R	- " " right " " 1

F2L	-	"	"	left	"	"	2 and 3
F2R	-	"	"	right	"	"	"
F3L	-	"	"	left	"	"	4 and 5
F3R	-	"	"	right	"	"	"
FTOT	-	sum of loads on all wheel sets					
H	-	distance from trailer cg to trailer leading tandem axle					
H1	-	height of tractor sprung mass cg above ground					
H2	-	height of trailer sprung mass cg above ground					
H3	-	"	"	tractor	"	"	fifth wheel
H4	-	height of fifth wheel above ground					
H5	-	height of trailer sprung mass above fifth wheel					

note: variable definitions for H1, H2, H3 and H5 are opposite to the convention in Figure 10 for the sake of programming brevity.

I	-	counter
I1	-	counter
ISTEER	-	number of steering angles to be input
ITER	-	number of iterations allowed
ITR	-	vector of number of iterations used
K1	-	1/2 stiffness of front suspension including the contribution from the tires
K2	-	1/2 stiffness of the tractor tandem suspension including contribution from the tires
K3	-	1/2 stiffness of the trailer tandem suspension including contribution from the tires
L	-	distance from trailer leading to trailing tandem axles
L1	-	distance from fifth wheel to trailer leading tandem axle
L2	-	distance from fifth wheel to trailer trailing tandem axle
L3	-	distance from fifth wheel to trailer trailing tandem axle
M1	-	mass of tractor
M1P	-	mass of tractor sprung mass
M2	-	mass of trailer
M2P	-	mass of trailer sprung mass
N	-	vector of static normal loads on the axles
NL	-	array of normal effective normal loads for each steer angle
NTEMP	-	temporary variable for calculating tire cornering stiffness
OMEGA	-	current calculated yaw rate
OMEGOLD	-	last calculated yaw rate
PI	-	3.14...
PSI	-	current calculated articulation angle
PSIOLD	-	last calculated articulation angle
T1	-	half track of axle 1
T2	-	half track of axles 2 and 3

T3	- half track of axles 4 and 5
U	- forward velocity of the simulation
V	- current calculated lateral velocity
V1	- RHS of the equations in figure 6
V2	- " " " " " "
VOLD	- last calculated lateral velocity

PROGRAM RBLT

```

C This program finds the steady state cornering response for a
C 5-axle tractor semi-trailer in a steady state turn
C
      REAL A,A11,A12,A13,A21,A22,A31,A32,A33,B,B1,C1,C2,C3,C4,C5,
1  D,DELTA(15),DET,E,L,L1,L2,M1,M2,OMEGA,PSI,PI,U,V,V1,V2
2  ,H1,H2,H3,H4,H5,T1,T2,T3,K1,K2,K3,CM1(7,2),CM2(7,2),M1P,
3  M2P,AP,BP,B3P,DP,L3,N(3),ERROR,ERRTEST,F1L,F1R,F2L,F2R,
4  F3L,F3R,FTOT,OMEGOLD,VOLD,PSIOLD,NL(15,3,2),ANS(15,5),CL,CR
5  ,C,H,NTEMP,EP
      INTEGER I,ISTEER,ITER,ITR(15),I1
      COMMON/DATA/ AP,BP,B3P,DP,H3,H4,H5,L3,T1,T2,T3,M1P,M2P,U,
1  VOLD,PSIOLD,OMEGOLD,K1,K2,K3,F1L,F1R,F2L,F2R,F3L,F3R,C1,C2,
2  C5,DELTA,A,B1,B,D,EP,L1,L2
      OPEN(5,FILE='INPUT')
      OPEN(6,FILE='OUTP',STATUS='NEW')

C
C INPUT THE TRACTOR SEMITRAILER PARAMETERS
C UNITS :  LENGTH inches
C           MASS    lbs
C           SPRING FORCE  lbs/inch
C           VELOCITY  ft/sec
C           STEER ANGLE  degrees
C           NORMAL LOAD  lbs
C           CORNERING STIFFNESS  lbs/rad
C
C IN THIS PROGRAM THE VARIABLES H1, H2, H3 AND H5 REPRESENT THE
C VARIABLES H1', H2', H3' AND H5' IN FIGURE 10. THEY ARE WRITTEN
C AS THEY ARE FOR BREVITY IN PROGRAMMING.
C
      READ(5,1000) A,B
      READ(5,1000) C,D
      READ(5,1000) E,L
      READ(5,1000) H,H1
      READ(5,1000) H2,H4
      READ(5,1000) T1,T2
      READ(5,1000) T3,K1
      READ(5,1000) K2,K3
      READ(5,1000) M1,M2
      READ(5,1000) N(1),N(2)
      READ(5,1000) N(3)
      READ(5,1000) ((CM1(I,J),J=1,2),I=1,7)
      READ(5,1000) ((CM2(I,J),J=1,2),I=1,7)
      READ(5,1000) M1P,M2P
      READ(5,1000) EP
      READ(5,1000) AP,ERROR
      READ(5,1010) ITER
      READ(5,1000) U
      READ(5,1010) ISTEER
      DO 10 I=1,ISTEER
10      READ(5,1000) DELTA(I)
      WRITE(6,2010)
      WRITE(6,2015)
C

```


C CALCULATE SPRUNG MASS VARIABLES

C

BP=A+B-AP
B3P=BP+C/2.
DP=A+D-AP

C

C WRITE THE INPUT VARIABLES

C

WRITE(6,2000) ' A = ',A, ' B = ',B
WRITE(6,2000) ' C = ',C, ' D = ',D
WRITE(6,2000) ' E = ',E, ' L = ',L
WRITE(6,2000) ' H = ',H
WRITE(6,2000) ' M1 = ',M1, ' M2 = ',M2
WRITE(6,2000) ' N1 = ',N(1), ' N2 = ',N(2)
WRITE(6,2000) ' N3 = ',N(3)
WRITE(6,2000) ' U = ',U
WRITE(6,2000) ' AP = ',AP
WRITE(6,2000) ' H1 = ',H1, ' H2 = ',H2
WRITE(6,2000) ' H4 = ',H4, ' T1 = ',T1
WRITE(6,2000) ' T2 = ',T2, ' T3 = ',T3
WRITE(6,2000) ' K1 = ',K1, ' K2 = ',K2
WRITE(6,2000) ' K3 = ',K3
WRITE(6,2000) ' M1P = ',M1P, ' M2P = ',M2P
WRITE(6,2000) (' CM1 ',CM1(I,1), ' ',CM1(I,2),I=1,7)
WRITE(6,2000) (' CM2 ',CM2(I,1), ' ',CM2(I,2),I=1,7)
WRITE(6,2000) (' ',DELTA(I),I=1,ISTEER)

2000 FORMAT(A,F10.2,A,F10.2)

2010 FORMAT(1X, ' STEADY STATE SOLUTION TO THE LINEARIZED 6DOF',
1 ' EQUATIONS')

2015 FORMAT(/ ' UNITS: LENGTH - inches'/11X, 'MASS - pounds'/
1 11X, 'TIRE STIFFNESS - pounds/rad')
PI=ACOS(-1.)/180.

C

C CONVERT ALL VARIABLES TO slugs, feet, lbs and radians

C

A=A/12.
B=B/12.
C=C/12.
D=D/12.
E=E/12.
H=H/12.
L=L/12.
B1=B+C
L1=H+E
L2=L1+L
L3=(L1+L2)/2.
AP=AP/12.
BP=BP/12.
B3P=B3P/12.
DP=DP/12.
T1=T1/12.
T2=T2/12.
T3=T3/12.
H1=H1/12.
H2=H2/12.

```

      H4=H4/12.
      H3=H1-H4
      H5=H2-H4
      M1=M1/32.2
      M2=M2/32.2
      M1P=M1P/32.2
      M2P=M2P/32.2
      K1=K1*12.
      K2=K2*12.
      K3=K3*12.
      DO 20 I=1,ISTEER
20    DELTA(I)=DELTA(I)*PI
      WRITE(6,2020)
2020  FORMAT(/' STEER ANGLE   C1          C2 AND C3   C4 AND C5'
1, ' NET WEIGHT')
      DO 4000 I=1,ISTEER
      IF (I.EQ.1) THEN
C
C  INITIALIZE RESPONSE VARIABLES TO ZERO IF THIS IS THE
C  FIRST STEER ANGLE
C
      VOLD=0.0
      PSIOLD=0.0
      OMEGOLD=0.0
      ELSE
C
C  FIRST ITERATION BUT NOT FIRST STEER ANGLE SO SET RESPONSE
C  VARIABLES TO THOSE FOR THE PREVIOUS STEER ANGLE
C
      OMEGOLD=ANS(I-1,1)*PI
      VOLD=ANS(I-1,2)
      PSIOLD=ANS(I-1,3)*PI
      ENDIF
      I1=0
C
C  BEGIN ITERATIONS
C
3000  I1=I1+1
C
C  CALL ROUTINE TO CALCULATE LOAD TRANSFERS BASED ON CURRENT ESTIMATE
C  OF RESPONSE VARIABLES
C
      CALL TRANSFER(I)
C
C  CALCULATE EFFECTIVE NORMAL LOAD ON EACH WHEEL AND FIND TOTAL
C  TIRE CORNERING STIFFNESS FOR EACH AXLE
C
C  LEFT SIDE AXLE 1
      NTEMP=N(1)/2.-F1L
      FTOT=NTEMP
      ALPH=DELTA(I)-(VOLD+A*OMEGOLD)/U
      CALL TSTIFF(ALPH,NTEMP,1,CL,CM1)
C  RIGHT SIDE AXLE 1
      NTEMP=N(1)/2.-F1R
      FTOT=FTOT+NTEMP

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```

      CALL TSTIFF(ALPH,NTEMP,1,CR,CM1)
      C1=CL+CR
C   LEFT SIDE AXLES 2 AND 3
      NTEMP=N(2)/8.-F2L/2.
      FTOT=FTOT+NTEMP*4.
      ALPH=-(VQLD-(B+C/2.)*OMEGOLD)/U
      CALL TSTIFF(ALPH,NTEMP,2,CL,CM2)
C   RIGHT SIDE AXLES 2 AND 3
      NTEMP=N(2)/8.-F2R/2.
      FTOT=FTOT+NTEMP*4.
      CALL TSTIFF(ALPH,NTEMP,2,CR,CM2)
      C2=CL+CR
      C3=C2
C   LEFT SIDE AXLES 4 AND 5
      NTEMP=N(3)/8.-F3L/2.
      FTOT=FTOT+NTEMP*4.
      ALPH=PSIOLD-(VOLD-L3*OMEGOLD)/U
      CALL TSTIFF(ALPH,NTEMP,2,CL,CM1)
C   RIGHT SIDE AXLES 4 AND 5
      NTEMP=N(3)/8.-F3R/2.
      FTOT=FTOT+NTEMP*4.
      CALL TSTIFF(ALPH,NTEMP,2,CR,CM1)
      C4=CL+CR
      C5=C4
C
C   CALCULATE THE RESPONSE VARIABLES FOR THE CURRENT ESTIMATES OF
C   CORNERING STIFFNESS
C   SOLVE EQUATIONS IN FIGURE 6
C
      A11=(M1+M2)*U*U+C1*A-C2*B-C3*B1-C4*(D+L1)-C5*(D+L2)
      A12=C1+C2+C3+C4+C5
      A13=U*(C4+C5)
      A21=D*M1*U*U+A*(A+D)*C1-B*(D-B)*C2-B1*(D-B1)*C3
      A22=(A+D)*C1+(D-B)*C2+(D-B1)*C3
      A31=E*M2*U*U-L1*(D+L1)*C4-L2*(D+L2)*C5
      A32=L1*C4+L2*C5
      A33=A32*U
      DET=A11*A22*A33+A13*A21*A32-A13*A22*A31-A12*A21*A33
      V1=U*C1*DELTA(I)
      V2=U*(A+D)*DELTA(I)*C1
      OMEGA=(V1*A22*A33+A13*V2*A32-A12*V2*A33)/DET
      V=(A11*V2*A33-A13*V2*A31-V1*A21*A33)/DET
      PSI=(A12*V2*A31+V1*A21*A32-V1*A22*A31-A11*V2*A32)/DET
1000  FORMAT(2F10.0)
1010  FORMAT(I2)
C
C   CHECK IF RESPONSE HAS CONVERGED. CALCULATE THE NORM OF THE ERROR
C   VECTOR AND COMPARE IT TO THE ALLOWABLE ERROR
C   IF RESPONSE VARIABLES ARE ZERO ( CORRESPONDING TO ZERO STEER
C   ANGLE ) SET ERRTEST TO A LARGE VALUE AND SKIP ERROR NORM
C   CALCULATION. THIS IS DONE BECAUSE THE ERROR TEST WOULD
C   INVOLVE A DIVIDE BY ZERO IN THE CASE OF ZERO RESPONSE VARIABLES
C
      ERRTEST=ERROR+1.
C

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```

      IF (OMEGA.EQ.0.0) GOTO 22
      ERRTEST=SQRT(((OMEGA-OMEGOLD)/OMEGA)**2+((V-VOLD)/V)**2+
1((PSI-PSIOLD)/PSI)**2)
22      IF (ERRTEST.GT.ERROR) THEN
          OMEGOLD=OMEGA
          VOLD=V
          PSIOLD=PSI
          ENDIF
          IF (I1.GT.ITER) GOTO 3010
          IF (ERRTEST.GT.ERROR) GOTO 3000
C
C  EITHER THE SOLUTION HAS CONVERGED OR MAXIMUM ITERATIONS HAVE BEEN
C  REACHED. IN EITHER CASE STORE THE RESULTS FOR OUTPUT LATER.
C
3010      NL(I,1,1)=N(1)/2.-F1L
          NL(I,1,2)=N(1)/2.-F1R
          NL(I,2,1)=N(2)/4.-F2L
          NL(I,2,2)=N(2)/4.-F2R
          NL(I,3,1)=N(3)/4.-F3L
          NL(I,3,2)=N(3)/4.-F3R
          ANS(I,1)=OMEGA/PI
          ANS(I,2)=V
          ANS(I,3)=PSI/PI
          ANS(I,4)=U*OMEGA
          ANS(I,5)=ERRTEST
          ITR(I)=I1
C
C  OUTPUT THE TOTAL STIFFNESS PER AXLE
C
4000      WRITE(6,9010) DELTA(I)/PI,C1,C2,C4,FTOT
9000      FORMAT(1X,'STEER ANGLE      C1      C2 AND C3      C4 AND C5'
1, '      WEIGHT')
9010      FORMAT(1X,F8.2,4F10.0)
C
C  OUTPUT THE REST OF THE RESULTS
C
          WRITE(6,9020)
          WRITE(6,9030) (DELTA(I)/PI, (ANS(I,J),J=1,5), ITR(I), I=1, ISTEER)
          WRITE(6,9040)
          WRITE(6,9050) (DELTA(I)/PI, (NL(I,J,1),J=1,3), I=1, ISTEER)
          WRITE(6,9060)
          WRITE(6,9050) (DELTA(I)/PI, (NL(I,J,2),J=1,3), I=1, ISTEER)
          WRITE(6,9070)
          WRITE(6,9050) (DELTA(I)/PI, (NL(I,J,1)+NL(I,J,2),J=1,3), I=1,
1 ISTEER)
9020      FORMAT(/'  DELTA      OMEGA      V      PSI      LAT ACCL',
1'  ERROR  ITERATIONS')
9030      FORMAT(1X,2F8.2,2F8.4,F8.3,F8.6,I2)
9040      FORMAT(/'NORMAL LOADS ON EACH WHEEL SET'/'STEER ANGLE
1, 'AXLE 1 LEFT  AXLES 2 AND 3 LEFT  AXLES 4 AND 5 LEFT')
9050      FORMAT((1X,F10.2,3(3X,F10.0)))
9060      FORMAT(/'STEER ANGLE  AXLE 1 RIGHT AXLES 2 + 3 RIGHT',
1'  AXLES 4 + 5 RIGHT')
9070      FORMAT(/'NORMAL LOADS ON EACH AXLE'/'STEER ANGLE ',
1'  AXLE 1      AXLES 2 AND 3      AXLES 4 AND 5')

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        STOP
        END
        SUBROUTINE TSTIFF(ALPHI,NI,I,CI,CM1)
C
C THIS SUBROUTINE FINDS TIRE STIFFNESS BASED ON THE EFFECTIVE
C NORMAL LOAD. STIFFNESS VALUES ARE INTERPOLATED FROM A USER
C DEFINED TABLE
C
        REAL ALPHI,CI,CM1(7,2),N,NI
        INTEGER I,J
        N=NI
C
C IF NORMAL LOAD ON THE WHEEL SET IS LESS THAN ZERO SET
C CORNERING STIFFNESS TO ZERO
C
        IF(N.LE.0.0) THEN
            CI=0.0
            RETURN
        ENDIF
C
C IF THE NORMAL LOAD IS GREATER THAN THE MAXIMUM NORMAL LOAD
C IN THE ARRAY CM1 OR CM2 SET THE CORNERING STIFFNESS TO THE
C MAXIMUM VALUE
C
        IF(N.GE.CM1(7,1)) THEN
            CI=CM1(7,2)
            CALL CIMOD(CI,ALPHI,I)
            RETURN
        ENDIF
C
C NORMAL LOAD IS IN THE RANGE OF THE ARRAYS CM1 OR CM2 SO
C INTERPOLATE TO FIND CORNERING STIFFNESS
C
        IF(N.LE.CM1(1,1)) THEN
            CI=N*CM1(1,2)/CM1(1,1)
            CALL CIMOD(CI,ALPHI,I)
            RETURN
        ENDIF
        J=2
1000    IF(N.LE.CM1(J,1)) THEN
            CI=(CM1(J,2)-CM1((J-1),2))*(N-CM1((J-1),1))/(CM1(J,1)-CM1((J-1),1))+CM1((J-1),2)
            CALL CIMOD(CI,ALPHI,I)
            RETURN
        ENDIF
        J=J+1
        IF (J.LE.7) GOTO 1000
        WRITE(6,9000) 'TIRE ROUTINE FAILED LOAD = ',N
9000    FORMAT(1X,A,F10.0)
        STOP
        END
        SUBROUTINE CIMOD(C,ALPH,I)
C
C THIS ROUTINE MODIFIES TIRE STIFFNESS FOR AXLES WITH 2
C WHEELS PER SIDE. THE NONLINEAR TIRE MODEL MAY BE USED

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C BY REMOVING THE COMMENT INDICATORS FROM THE LOGICAL
C IF LOOP
      REAL ALPHA,C
      IF (I.NE.1) C=C*2.
      IF (ABS(ALPH).LT.0.15707) THEN
      C=C*(1.-1.7*ABS(ALPH))
      ELSE
      C=C*0.732981
      ENDIF
      RETURN
      END
C*****
C
C THIS SUBROUTINE CALCULATES VEHICLE ORIENTATION AND LOAD TRANSFER
C BY SOLVING THE SYSTEM OF EQUATIONS IN FIGURE 12
C
C*****
      SUBROUTINE TRANSFER(I)
C
C IN THIS SUBROUTINE THE PRIMED AND UNPRIMED VARIABLES HAVE BEEN
C SWITCHED. IN OTHER WORDS THE VARIABLE "A" FROM THE MAIN
C PROGRAM IS "AP" IN THIS SUBROUTINE AND VICE-VERSA
C THIS WAS DONE TO KEEP THE EQUATION LENGTHS UNDER CONTROL
C
      REAL A,B,B3,D,DP,H3,H4,H5,L3,T1,T2,T3,M1P,M2P,U,V,PSI,
1 OMEGA,K1,K2,K3,F1L,F1R,F2L,F2R,F3L,F3R,C1,C2,C5,DELTA(15)
2 ,A11,A12,A13,A21,A22,A23,A31,A32,A33,V1,V2,CS,SN,UP,VP,
3 DET,PHI,THETA,Z,F1,F2,F3,F4,F5,E,AP,B1,BP,L1,L2,VT
      INTEGER I
      COMMON /DATA/ A,B,B3,D,H3,H4,H5,L3,T1,T2,T3,M1P,M2P,U,VT,PSI,
1 OMEGA,K1,K2,K3,F1L,F1R,F2L,F2R,F3L,F3R,C1,C2,C5,DELTA,
2 AP,B1,BP,DP,E,L1,L2
C
C CALCULATE CONSTANTS
C
      IF (OMEGA.EQ.0.0) GOTO 1000
      CS=COS(PSI)
      SN=SIN(PSI)
C
C CALCULATE LATERAL VELOCITY OF THE TRACTOR SPRUNG MASS
C
      V=VT+(AP-A)*OMEGA
C
C CALCULATE THE LATERAL AND LONGITUDINAL VELOCITIES OF
C THE TRAILER SPRUNG MASS
C
      UP=U*CS-V*SN+(DP+AP-A)*OMEGA*SN
      VP=U*SN+V*CS-(DP+AP-A)*OMEGA*CS-E*OMEGA
C
C CALCULATE THE LATERAL FORCES ON EACH AXLE
C
      F1=C1*(DELTA(1)-(VT+AP*OMEGA)/U)
      F2=-C2*(VT-BP*OMEGA)/U
      F3=-C2*(VT-B1*OMEGA)/U
      F4=-C5*(PSI+(VT-(DP+L1)*OMEGA)/U)

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      F5=-C5*(PSI+(VT-(DF+L2)*OMEGA)/U)
C
C  DEFINE MATRIX COEFFICIENTS AND SOLVE FOR DISPLACEMENTS
C
      A11=2.*K3*SN*(L3*(L3*CS+D)-T3**2.*CS)
      A12=-2.*(K1*T1**2.+K2*T2**2.+K3*(L3**2.*SN*SN+T3**2.*
1 CS*CS))
      A13=2.*L3*SN*K3
      V1=H5*M2P*OMEGA*(VP*SN+UP*CS)+H3*M1P*U*OMEGA+H4*(F1+F2+
1 F3+(F4+F5)*CS)
      A21=-2.*(-A*(A+B)*K1+B3*(D-B3)*K2-L3*CS*K3*(L3*CS+D)-
1 T3**2.*SN*SN*K3)
      A22=2.*K3*SN*CS*(T3**2.-L3**2.)
      A23=-2.*(K1*(A+B)+K2*(D-B3)-L3*CS*K3)
      V2=OMEGA*(H3*M1P*V-H5*M2P*(UP*SN-VP*CS))-H4*(F4+F5)*SN
      A31=2.*(-A*K1+B3*K2+(L3*CS+D)*K3)
      A32=-2.*K3*L3*SN
      A33=2.*(K1+K2+K3)
      DET=A11*A22*A33+A12*A23*A31+A13*A21*A32-A12*A21*A33-
1 A11*A23*A32-A13*A22*A31
      PHI=(V1*A22*A33+A13*V2*A32-A12*V2*A33-V1*A23*A32)/DET
      THETA=(A11*V2*A33+V1*A23*A31-V1*A21*A33-A13*V2*A31)/DET
      Z=(A12*V2*A31+V1*A21*A32-A11*V2*A32-V1*A22*A31)/DET
C
C  CALCULATE THE LOAD TRANSFER FORCES AND RETURN THEM TO THE
C  MAIN PROGRAM
C
      F1L=K1*(-A*PHI+T1*THETA+Z)
      F1R=F1L-2.*T1*THETA*K1
      F2L=K2*(B3*PHI+T2*THETA+Z)/2.
      F2R=F2L-K2*T2*THETA
      F3L=K3*((L3*CS+T3*SN+D)*PHI-(L3*SN-T3*CS)*THETA+Z)/2.
      F3R=F3L-K3*T3*(SN*PHI+CS*THETA)
      RETURN
C
C  FIRST ITERATION OF FIRST STEER ANGLE SET LOAD TRANSFER FORCES
C  TO ZERO
C
1000  F1L=0.0
      F1R=0.0
      F2L=0.0
      F2R=0.0
      F3L=0.0
      F3R=0.0
      RETURN
      END

```

INPUT DATA FOR EMPTY TRAILER CASE

STEADY STATE SOLUTION TO THE LINEARIZED 4DOF EQUATIONS

UNITS: LENGTH - inches
 MASS - pounds
 TIRE STIFFNESS - pounds/rad

A =	67.00	B =	47.80
C =	54.40	D =	75.00
E =	265.00	L =	49.30
H =	76.00		
M1 =	14970.00	M2 =	11160.00
N1 =	8550.00	N2 =	10086.00
N3 =	7494.00		
U =	40.00		
AP =	35.90		
H1 =	52.50	H2 =	69.00
H4 =	48.00	T1 =	40.00
T2 =	36.00	T3 =	36.00
K1 =	1018.00	K2 =	6596.00
K3 =	7880.00		
M1P =	9245.00	M2P =	8120.00
CM1	1400.00		12260.00
CM1	2800.00		20860.00
CM1	4200.00		26760.00
CM1	5430.00		29970.00
CM1	6700.00		31520.00
CM1	8100.00		31970.00
CM1	9200.00		31920.00
CM2	1400.00		11170.00
CM2	2800.00		19600.00
CM2	4200.00		25730.00
CM2	5430.00		29570.00
CM2	6700.00		31290.00
CM2	8100.00		31060.00
CM2	9200.00		31860.00

INPUT DATA FOR THE LOADED TRAILER CASE

STEADY STATE SOLUTION TO THE LINEARIZED 4DOF EQUATIONS

UNITS: LENGTH - inches
 MASS - pounds
 TIRE STIFFNESS - pounds/rad

A = 67.00 B = 47.80
 C = 54.40 D = 75.00
 E = 218.00 L = 49.30
 H = 123.00
 M1 = 14970.00 M2 = 57960.00
 N1 = 10306.00 N2 = 31410.00
 N3 = 31214.00
 U = 39.00
 AP = 35.90
 H1 = 52.50 H2 = 68.30
 H4 = 48.00 T1 = 40.00
 T2 = 36.00 T3 = 36.00
 K1 = 1018.00 K2 = 6596.00
 K3 = 7880.00
 M1P = 9245.00 M2P = 54920.00

CM1	1400.00	12260.00
CM1	2800.00	20860.00
CM1	4200.00	26760.00
CM1	5430.00	29970.00
CM1	6700.00	31520.00
CM1	8100.00	31970.00
CM1	9200.00	31920.00

CM2	1400.00	11170.00
CM2	2800.00	19600.00
CM2	4200.00	25730.00
CM2	5430.00	29570.00
CM2	6700.00	31290.00
CM2	8100.00	31060.00
CM2	9200.00	31860.00

END

2 1 0 2 8 6

FIN