Analysis of the Nonlinear Optical Loop Mirror with Feedback and Birefringent Fibers in the Loop

by,

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Abstract

The behavior of the nonlinear optical loop mirror with feedback with high birefringence fiber in the loop is examined. It was found that the use of a birefringent fiber in the loop will affect the output dynamics of the system. The output bistable and chaotic behaviour of the power is shown to rely on the polarization state of the input beam. The output polarization of the system is shown to also exhibit bistable and chaotic behavior for a fixed input polarization state. The use of a high birefringence fiber in the loop simplifies the propagation equations of the system and offers a measure of control of the nonlinear optical loop mirror with feedback, allowing the output bistable and chaotic behavior to be modified by adding a polarization controller to the input of the device.

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Chapter 1

Introduction

Nonlinear effects in optical materials were first observed in the 19th century by a reverend with a masters degree in divinity, working at the Free Church Training College in Glasgow, Scotland [1]. Reverend John Kerr had an enthusiasm for the physical and natural sciences, helping found the first physical laboratory at the university of Glasgow. At around the age of fifty, Kerr discovered that an electric field applied in a plane perpendicular to the propagation of a beam of light would change the refractive index seen by the polarized light in that plane. This electric field applied along one polarization axis created a difference in refractive index between polarizations, inducing birefringence. This effect is known as the Kerr electro-optic effect, with the change in refractive index being dependent on the square of the applied electric field. The discovery of lasers allowed the use of light's own electric field to induce this change in the propagating media, opening the discoveries of many phenomena in nonlinear optics, including optical bistability and optical chaos seen in this thesis.

Optical resonators can exhibit bistable, multi-stable and chaotic behavior. Figure 1.1 shows the power output of an optical resonator as the input power is increased and then

Figure 1.1: Bifurcation diagram showing the bistable region, multi-stable and chaotic regions of an optical resonator, in this case a nonlinear optical loop mirror with feedback.

decreased into the system. At lower powers, there is a bistable region, and at higher powers the output becomes multi-stable and chaotic. The ability of the optical resonator to operate in these different regions makes it a useful device for many different applications.

Bistable and chaotic phenomena are found in many different aspects of nature. For example, for the general public who live near beaches, bistability has been discovered in the dynamic modelling of tilted sandpiles [2] where the authors “address the twin phenomena of bistability and hysteresis at the angle of repose of a sandpile, θ_r ; this is the maximum inclination of the free surface of a stationary pile.” where bistable behavior occurs around this angle. For chaotic phenomena, circuits have been made for uses such as recreating the Lorenz equations which were developed for modelling the weather. This bistable and chaotic behavior in the field of nonlinear fiber optics developed only within the last thirty

years.

The bistable region of the optical resonator has applications in all optical computing. When operating in the bistable region, the optical resonator performs the optical equivalent of a semiconductor flip-flop [3]. The resonator is seen to have two stable states, one high transmission level, and one low transmission level. The resonator will reach either state depending on the intensity of the light passing through the system. The use of flip-flops in semiconductor electronics is seen in devices such as static memory and hardware registers. In the field of all optical computing, the optical bistability of a nonlinear optical resonator would be an important core component for optical memory and logic operations.

The chaotic region of the optical resonator will generate seemingly random output for a fixed input power, but in fact the system is deterministic. The “common” definition of chaos tends to be misleading, with Thomas Leiber pointing out that although there are different definitions of chaos in literature, that “there is no general definition of deterministic chaos applicable to the majority of interesting cases” [4]. Yale University uses the rather cliché (but still amusing) idea of examining the Webster’s definition of the word chaos [5]. The dictionary states that chaos derives from the Greek (as all English words are derived) word for “abyss”, and the definition is given as

1. An empty, immeasurable space; a yawning chasm. [Archaic]
2. The confused, unorganized condition or mass of matter before the creation of distinct and orderly forms.
3. Any confused or disordered collection or state of things; a confused mixture; confusion; disorder.

This definition equating chaos to randomness, is of course quite wrong when considering chaotic behavior in mathematics. With the popular definition of chaos being generally incorrect in the eyes of the general public, mathematicians solved this problem by adding an extra term, giving the origin of “deterministic chaos”, an oxymoron for the uninformed, however offering clarity for the mathematically inclined. Leiber notes that only in the special case of the iteration of a function, there is agreement on the definition of the mathematical properties of deterministic chaos, given as:

1. sensitive dependence on the initial conditions
2. dynamical mixing in state space
3. periodic points lying dense in state space

Leiber also mentions that “while the chaos industry is still expanding, the interpretational practice, or meaning variance, of the term, chaos, varies a lot, and some scientists are even quite unhappy about the very notion of deterministic chaos itself which was accidentally introduced by Li and Yorke (1975).” For the purposes of this thesis, the definition of chaos will be the mathematical one offered from Leiber shown above [4], and not anything related to what is found in Webster’s dictionary.

One benefit of using a system exhibiting deterministic chaos is for encryption [6]. The idea is that a simple optical or electronic system with only a few degrees of freedom can give a complex dynamic output which then can be used for secure communications. An input signal would have a chaotic system added to it, and after transmission it would be decrypted by subtracting an identical synchronized chaotic output, as shown in figure 1.2. As a chaotic output has a sensitive dependence on input conditions, small variations in

Figure 1.2: Example of a simple chaotic encryption system

optical resonator design can generate quite different chaotic output. This another important reason why the analysis of the optical resonator with birefringence and polarization effects are important, as these parameters will be shown to have an effect on the output of the device.

The mid-1970s showed the development of Fabry-Perot cavities in mediums having a Kerr-type response, which exhibited bistable behavior [7, 8]. The mathematics required in the analysis of this system was considerably complex, and a mathematically simpler optical system consisting of a ring cavity with a Kerr medium was proposed to allow the system to be treated analytically [9, 10]. This bulk cavity ring (BCR) resonator is shown as figure 1.3.

The 1970s also saw the development of manufacturing techniques that enabled lower and lower transmission losses in fiber optic cables. The availability of these fibers opened up research in the use of silica fibers as optical devices as well as transmission media. As the fiber itself was discovered to be a medium with a Kerr-type response, it exhibits nonlinear behaviors and the field of nonlinear fiber optics was born [11]. The fiber optic cables could be used to create a ring resonator system similar to the BCR, using the fiber itself as the Kerr-type media. A ring resonator structure is shown as figure 1.4, where a

straight piece of fiber is coupled to a fiber loop.

In 1993, Shi introduced a new type of optical resonator circuit, the nonlinear optical loop mirror (NOLM) with feedback applied to the loop [12]. The NOLM is a Sagnac interferometer which operates in the nonlinear regime [13]. Shi discovered that applying feedback to the NOLM will give rise to an optical system which displays bistable, multi-stable, and chaotic output as power is increased in the system, similar to the behavior found in the BCR and the fiber ring resonator, although the system operates quite differently from those other resonators [14]. The NOLM with feedback is shown in figure 1.5.

Early analysis of the bistable behavior of these devices was performed using a technique known as the graphical method, introduced by Felber and Marburger [15, 16]. In 1979, Ikeda applied an iterative method and linear stability analysis to the examination of the BCR, and discovered the dynamical behavior of period doubling and eventual chaotic output [17]. However, while the graphical method was useful in examining the bistable nature of these devices, Steele et al. later showed it was insufficient for examining the unstable regions of the device [14]. They show that one technique for examining the bistable and chaotic nature of optical resonators is to use an iterative technique which produces bifurcation diagrams. A sample of a bifurcation diagram from an optical resonator is shown as figure 1.1. The input of this system is a quasi-CW Gaussian pulse, with the pulse width several orders of magnitude larger than the period length of the cavity. In the figure, the bistable region is generated with a low transmission on the rising edge of the quasi-CW pulse, and a higher transmission on the trailing edge of the pulse. The multi-stable and chaotic regions occur at higher power levels in the input.

Figure 1.3: The bulk cavity ring (BCR) with a nonlinear dielectric medium.

Figure 1.4: The fiber ring resonator.

Figure 1.5: The nonlinear optical loop mirror with feedback.

Earlier analysis of the NOLM with feedback [12, 18] showed that the NOLM with feedback is sensitive to parameters such as coupler splitting ratio, fiber lengths and fiber type, which are all fixed on the initial design of the NOLM with feedback. This thesis will show that the inclusion of polarization effects is an important parameter in the complete analysis of the NOLM with feedback, as it was found that the input polarization state has an effect on the behavior of the NOLM with feedback when birefringent fiber is used in the loop. Careful choice of the birefringence of the fiber will allow a variable of control over the output behavior of the NOLM with feedback with the use of a polarization controller on the input. As all fibers have some degree of birefringence, the characterization of this polarization effect is important for systems using a NOLM with feedback.

This polarization dependence in nonlinear fiber optic devices has become a topic of growing interest. Recently, analysis of the NOLM without feedback, often used in devices such as all optical switching, optical pulse compression and filtering [13], has included polarization effects [19–23]. However these devices do not behave as optical resonators since the NOLM without feedback only passes light once around the loop, and as well these devices tend to work with short pulses which simplifies the propagation equations. Korneev has recently examined the effects of polarization in optical resonators [24] looking at the ring resonator structure, although Steele et al. showed in previous optical resonator comparisons without birefringence effects [14], that the NOLM with feedback exhibits a different dynamics than the ring resonator structures. Including polarization effects in the NOLM with feedback type of optical resonator is important to compliment Korneev's work, to get a more complete picture on how optical resonators behave with birefringent properties.

This thesis examines the polarization behavior of the NOLM with feedback with birefringent fiber in the loop. It will be shown that the polarization state of the input beam, a controllable parameter, has an effect on both the power output, and output polarization state of the NOLM with feedback. The location and behavior of bistable and chaotic regions in the output power of the NOLM with feedback is found to depend on the input polarization state. As well, it is discovered that the output polarization state of the NOLM with feedback is found to exhibit bistable and chaotic behavior. Both of these factors are important when using this system for applications in optical systems that depend on specific bistable or chaotic behavior.

Chapter 2

Background Work

This section outlines the derivations of the nonlinear propagation equations in a fiber, in the order of complexity. First the relation between the propagating electric field and the induced polarization of the fiber is developed. Then, the propagation behavior of an unpolarized single beam is modelled, followed by the interaction of two unpolarized propagating beams. Then, birefringence is introduced to the fiber and the single and dual beam propagations are redeveloped in terms of orthogonally polarized beams. The final section introduces the nonlinear fiber optic loop mirror (NOLM) with feedback, outlining previous research in the subject.

2.1 Maxwell's Equations

Here, the equations relating the electric field and the linear and nonlinear induced polarizations in optical fibers are described, using previous work [11]. This is used for the development of beam propagation in nonlinear dispersive media using the slowly varying envelope approximation. It is appropriate to start off with the classic Maxwell's equations

below, in SI units [25]

$$\nabla \times \mathbf{E} = -\frac{\delta \mathbf{B}}{\delta t} \quad (2.1)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\delta \mathbf{D}}{\delta t} \quad (2.2)$$

$$\nabla \cdot \mathbf{D} = \rho_f \quad (2.3)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.4)$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field vectors, and $\mathbf{D}$ and $\mathbf{B}$ are electric and magnetic flux densities. The density of electric charge ρ_f and the current density $\mathbf{J}$ both are zero, as glass fibers do not have a free charge [11]. The electric and magnetic fields are related to the flux densities by the well known relations [25]

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad (2.5)$$

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} \quad (2.6)$$

where $\mathbf{P}$ and $\mathbf{M}$ are the induced electric and magnetic polarizations. In an optical fiber which is nonmagnetic, $\mathbf{M}$ is zero.

The wave equation describing light propagation in waveguides is generated from these equations and is given in [11] by taking the curl of 2.1 and using 2.2, 2.3 and 2.4 to write the wave equation in terms of $\mathbf{E}$ and $\mathbf{P}$. This wave equation is shown as

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2} \frac{\delta^2 \mathbf{E}}{\delta t^2} - \mu_0 \frac{\delta^2 \mathbf{P}}{\delta t^2} \quad (2.7)$$

The evaluation of the induced polarization is a complicated procedure and requires a

quantum mechanical approach, however if the optical frequency is far from the medium resonance [26–29], as is the case with optical fibres and telecommunications wavelengths, the total induced polarization by electric dipoles will satisfy [11]

$$\mathbf{P} = \epsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{E}\mathbf{E} + \chi^{(3)} : \mathbf{E}\mathbf{E}\mathbf{E} + \dots \right) \quad (2.8)$$

For silica glass, as it is a centro-symmetric molecule, $\chi^{(2)}$ vanishes and we consider only the third order nonlinear effects governed by $\chi^{(3)}$. Thus, the induced polarization can be broken into a linear and nonlinear part, given as

$$\mathbf{P}(\mathbf{r}, t) = \mathbf{P}_L(\mathbf{r}, t) + \mathbf{P}_{NL}(\mathbf{r}, t) \quad (2.9)$$

where, assuming an instantaneous response of the silica molecules in the neglecting of the Raman response (which is the molecular vibration contribution to $\chi^{(3)}$), the nonlinear relationship of $\mathbf{P}_{NL}$ is given as

$$\mathbf{P}_{NL}(\mathbf{r}, t) = \epsilon_0 \chi^{(3)} : \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) \mathbf{E}(\mathbf{r}, t) \quad (2.10)$$

where this is valid for pulse widths $> 1\text{ps}$ [11]. This is well satisfied for the case of a CW beam used in the NOLM with feedback.

2.2 Beam Propagation Equations

The study of nonlinear optical loop mirrors (NOLMs) requires equations describing the propagation of the optical beam inside of a fiber. The wave equation shown in 2.7 is the basis of these beam propagations. To rewrite this wave equation, we need the relationship

shown in 2.9, and the identity given below as

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E} \quad (2.11)$$

with the assumption that the index of refraction is independent of spatial coordinates for a single mode, step index fiber to obtain this relation [11]. Using these two formulas, the wave equation can be rewritten as

$$\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\delta^2 \mathbf{E}}{\delta t^2} = \mu_0 \frac{\delta^2 \mathbf{P}_L}{\delta t^2} + \mu_0 \frac{\delta^2 \mathbf{P}_{NL}}{\delta t^2} \quad (2.12)$$

and using 2.9 and 2.10, the beam propagation equations can be developed.

2.2.1 Single Beam Propagation with No Polarization

The most simple propagation equation in a fiber is with a single beam without birefringence in the fiber. Following the work shown in Chapter 2 of [11], several assumptions must be made to solve the wave equation 2.12. The nonlinear induced polarization $\mathbf{P}_{NL}$ is treated as a small perturbation of $\mathbf{P}_L$, because typical changes of silica refractive index in fibers is of the order 10^{-6} . For the unpolarized fiber, the second assumption is that the beam maintains its polarization as it propagates, such that a scalar approach will be sufficient. This will change as birefringence is introduced in the fiber as shown in section 2.2.3. The last assumption is one of a quasi monochromatic field, where the ratio of the frequency spread about the center frequency $\Delta\omega/\omega_0 \ll 1$, which for telecommunications wavelengths is valid for sub-picosecond and wider pulses. This leads to the slowly varying envelope approximation shown below of the electric and induced polarization

components

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2} \hat{x} [E(\mathbf{r}, t) \exp(-i\omega_0 t) + c.c.] \quad (2.13)$$

$$\mathbf{P}_L(\mathbf{r}, t) = \frac{1}{2} \hat{x} [P_L(\mathbf{r}, t) \exp(-i\omega_0 t) + c.c.] \quad (2.14)$$

$$\mathbf{P}_{NL}(\mathbf{r}, t) = \frac{1}{2} \hat{x} [P_{NL}(\mathbf{r}, t) \exp(-i\omega_0 t) + c.c.] \quad (2.15)$$

The nonlinear component of the induced polarization is of concern when developing the beam propagation equation. The relation between the nonlinear polarization and the electric field is given in 2.10, and when the electric field slowly varying envelope approximation 2.13 is substituted, the below result for $\mathbf{P}_{NL}$ is found, containing both a term at ω_0 and a term at the third harmonic frequency of $3\omega_0$.

$$\begin{aligned} \mathbf{P}_{NL} = \epsilon_0 \chi^{(3)}: & \left[\frac{3}{8} \hat{x} (E(\mathbf{r}, t) E(\mathbf{r}, t) \bar{E}(\mathbf{r}, t)) \exp(-i\omega_0 t) + \right. \\ & \left. \frac{1}{8} \hat{x} (E(\mathbf{r}, t) E(\mathbf{r}, t) \bar{E}(\mathbf{r}, t)) \exp(-3i\omega_0 t) + c.c. \right] \end{aligned} \quad (2.16)$$

The third harmonic term requires phase matching and for optical fibers is neglected [11], therefore the above equation can be simplified somewhat, with the aid of the identity of $z \cdot z^* = |z|^2$, giving

$$\mathbf{P}_{NL} = \epsilon_0 \chi^{(3)}: \left[\frac{3}{8} \hat{x} |E(r, t)|^2 E(r, t) \exp(-i\omega_0 t) + c.c. \right] \quad (2.17)$$

This can be also shown by using 2.15 to rewrite in terms of the linear and nonlinear dielectric constant as shown below

$$P_{NL}(\mathbf{r}, t) \approx \epsilon_0 \epsilon_{NL} E(\mathbf{r}, t) \quad (2.18)$$

where the nonlinear dielectric constant is given as

$$\epsilon_{NL} = \frac{3}{4} \chi_{xxxx}^{(3)} |E(r, t)|^2 \quad (2.19)$$

which is related to the intensity of the electric field. With these results, the process of solving the wave equation can occur, by the substitution of 2.13, 2.14 and 2.15 into 2.12. The solution of this is easier in the frequency domain, which is difficult because of the nonlinearity from the intensity dependence of ϵ_{NL} , however as shown in [30, 31] one can treat the nonlinearity as a constant during the propagation equation derivation because of the slowly varying envelope approximation. Following the work from [11], the Fourier transform of

$$\tilde{E}(r, \omega - \omega_0) = \int_{-\infty}^{\infty} E(r, t) \exp[i(\omega - \omega_0)t] dt \quad (2.20)$$

solves the Helmholtz equation given as

$$\nabla^2 \tilde{E} + \epsilon(\omega) k_0^2 \tilde{E} = 0 \quad (2.21)$$

where k_0 is the wavenumber given as ω/c and the dielectric constant as a function of ω is given as

$$\epsilon(\omega) = 1 + \chi_{xx}^{(1)}(\omega) + \epsilon_{NL} \quad (2.22)$$

which is a complex entity. The dielectric constant in general can be expressed in terms of a refractive index and an absorption coefficient as shown

$$\epsilon = \left(n + \frac{i\alpha c}{2\omega} \right)^2 \quad (2.23)$$

where in this case, since the refractive index and absorption coefficient have an intensity dependence because of the nonlinearity, they are rewritten as

$$\tilde{n} = n + n_2|E|^2 \quad (2.24)$$

$$\tilde{\alpha} = \alpha + \alpha_2|E|^2 \quad (2.25)$$

and substituted into the expression of the dielectric constant as $\epsilon = (n + i\alpha c/2\omega)^2$. The terms n_2 and α_2 are defined as

$$n_2 = \frac{3}{8n} \text{Re}(\chi_{xxxx}^{(3)}) \quad (2.26)$$

$$\alpha_2 = \frac{3\omega_0}{4nc} \text{Im}(\chi_{xxxx}^{(3)}) \quad (2.27)$$

Note that the nonlinear term of α_2 is often neglected in silica fibers as the effect is quite small. Values of n_2 for single mode fibers are typically on the order of $2.3 - 2.6 \times 10^{-20} \text{m}^2/\text{W}$, when working around wavelengths of $1.55 \mu\text{m}$ [11]. The Helmholtz equation 2.21 can be solved when using the separation of variables method if the propagating field is written in the form

$$\tilde{E}(r, \omega - \omega_0) = F(x, y) \tilde{A}(z, \omega - \omega_0) \exp(i\beta_0 z) \quad (2.28)$$

which will give rise to the solutions in terms of the beam cross section (x,y) and the propagation (z) as

$$\frac{\delta^2 F}{\delta x^2} + \frac{\delta^2 F}{\delta y^2} + \left(\epsilon(\omega) k_0^2 - \tilde{\beta}^2 \right) F = 0 \quad (2.29)$$

$$2i\beta_0 \frac{\delta \tilde{A}}{\delta z} + (\tilde{\beta}^2 - \beta_0^2) \tilde{A} = 0 \quad (2.30)$$

where for this thesis the effect of the modal distribution $F(x, y)$ is not important to the effects on the NOLM with feedback. For the propagating beam along the z-axis in 2.30, the substitution of $\tilde{\beta}^2 - \beta_0^2 \approx 2\beta_0(\tilde{\beta} - \beta_0)$ gives

$$\frac{\delta \tilde{A}}{\delta z} - i(\tilde{\beta} - \beta_0) \tilde{A} = 0 \quad (2.31)$$

where an expression is required for the wave number $\tilde{\beta}$ which can be generated from first order perturbation theory. Using this, we can express $\tilde{\beta}$ as a function of

$$\tilde{\beta}(\omega) = \beta(\omega) + \Delta\beta \quad (2.32)$$

which can be substituted into 2.31. Since there is no exact solution of the wave number function $\beta(\omega)$, it can be approximated with a Taylor series expansion around the carrier frequency ω_0 , as given by

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2 + \frac{1}{6}(\omega - \omega_0)^3\beta_3 + \dots \quad (2.33)$$

$$\beta_m = \left(\frac{\delta^m \beta}{\delta \omega^m} \right)_{\omega=\omega_0}$$

where it is only necessary to keep up to the square of the term as the higher order terms are small for the criteria of a small spectral width $\Delta\omega/\omega_0 \ll 1$, which is the quasi-monochromatic assumption made earlier. This gives an expression for the beam propaga-

tion in the frequency domain of

$$\frac{\delta \tilde{A}}{\delta z} - i \left(\beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2}(\omega - \omega_0)^2\beta_2 + \Delta\beta \right) \tilde{A} = 0 \quad (2.34)$$

and then it is convenient to Fourier transform this back to the time domain, where the expressions of $(\omega - \omega_0)^m$ get replaced by the differential operator of $\delta^m/\delta t^m$, and this gives the expression

$$\frac{\delta A}{\delta z} = -\beta_1 \frac{\delta A}{\delta t} - i \frac{\beta_2}{2} \frac{\delta^2 A}{\delta t^2} + i \Delta\beta A \quad (2.35)$$

where now an expression is needed for the $\Delta\beta$ term. This requires the modal distribution $F(x, y)$ from the separation of variables expression of 2.28, and involves the integration of this modal field as shown with

$$\Delta\beta = k_0 \frac{\int \int_{-\infty}^{\infty} \Delta n |F(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy} \quad (2.36)$$

and Δn is given as a small perturbation as $\Delta n = n_2 |E|^2 + \frac{i\tilde{\alpha}}{2k_0}$ which is taken by using the perturbation theory from 2.23, 2.24 and 2.25. With this, $\Delta\beta$ evaluates to

$$\Delta\beta = \frac{-\alpha}{2} + i \frac{n_2 \omega_0}{c A_{eff}} |A|^2 \quad (2.37)$$

and the common substitution is made consisting of the nonlinear parameter $\gamma = \frac{n_2 \omega_0}{c A_{eff}}$, where A_{eff} is the effective core area of the fiber. This gives the final expression for the propagation of the beam through a fiber as

$$\frac{\delta A}{\delta z} = -\beta_1 \frac{\delta A}{\delta t} - i \frac{\beta_2}{2} \frac{\delta^2 A}{\delta t^2} + \frac{-\alpha}{2} A + i \gamma |A|^2 A \quad (2.38)$$

or for a CW beam which has no change in envelope with respect to time, which is used in the NOLM with feedback this expression can be simplified to become

$$\frac{\delta A}{\delta z} = \frac{-\alpha}{2}A + i\gamma|A|^2A \quad (2.39)$$

The significance of this is that the beam's phase changes with respect to the intensity of itself, shown by the $i\gamma|A|^2A$ term. This is known as the self phase modulation of the beam. Similar propagation equations but with different phase changes come up when a second beam propagates with the first beam, or when the beam is broken down into polarization components in a birefringent fiber. This is known as cross phase modulation (XPM), and will be shown next.

2.2.2 Two Beam Propagation with No Polarization

The operation of a NOLM with feedback (as will be explained in detail in section 2.3.2) requires a phase expression for the interaction of two counter-propagating CW beams in a fiber. The two beams interact with each other through the fiber nonlinearity, which will be shown here. Each beam will see a nonlinear phase generated from its own intensity (SPM), and the intensity of the other beam, known as cross phase modulation (XPM). The procedure for developing this beam propagation expression is similar to section 2.2.1, but with a new expression for $E(\mathbf{r}, t)$ given as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2}\hat{x} [E_1 \exp(-i\omega_0 t) + E_2 \exp(-i\omega_0 t) + c.c.] \quad (2.40)$$

This expression can then be substituted into the induced polarization expression from

2.10, and expanded to generate a rather complicated expression for P_{NL} which consists of terms at ω_1 and ω_2 , and four wave mixing terms at $(2\omega_1 - \omega_2)$ and $(2\omega_2 - \omega_1)$. This is summarized in [11], however a brief look at the math is shown here. The terms involving the frequency ω_1 are collected as

$$\epsilon_0 \chi^{(3)} \tilde{x} \left(\frac{3}{8} E_1 E_1 \bar{E}_1 + \frac{3}{4} E_1 E_2 \bar{E}_2 + \text{four wave mixing terms} \dots \right) \exp(-i\omega_1 t) \quad (2.41)$$

which can be factored and simplified using the identity $z \cdot \bar{z} = |z|^2$ to bring about the expression for the contribution of ω_1 to the nonlinear induced polarization as

$$\frac{3}{8} \epsilon_0 \chi^{(3)} \tilde{x} (|E_1|^2 + 2|E_2|^2 + \text{four wave mixing terms} \dots) E_1 \exp(-i\omega_1 t) \quad (2.42)$$

where the four wave mixing terms are neglected assuming phase matching does not occur. If the induced polarization is written in terms similar to 2.18 with respect to frequency ω_j ($j = 1, 2$), we have

$$P_{NL}(\omega_j) = \epsilon_0 \epsilon_j^{NL} E_j \quad (2.43)$$

which is combined with the linear induced polarization P_L to give the expression

$$P(\omega_j) = \epsilon_0 \epsilon_j E_j \quad (2.44)$$

with the net dielectric constant $\epsilon_j = \epsilon_j^L + \epsilon_j^{NL} = (n_j^L + \Delta n_j)^2 \approx n_j^2 + 2n_j \Delta n_j$, where Δn_j is the change caused by the third order nonlinearity. This is related to 2.42 from the

similar procedure in deriving ϵ_{NL} from 2.19, giving the expression for Δn and ϵ^{NL} as shown for a given wavelength as

$$\Delta n_j \approx \frac{\epsilon_j^{NL}}{2n_j} \approx n_2 (|E_j|^2 + 2|E_{3-j}|^2) \quad (2.45)$$

with n_2 defined in 2.26. From the example given using frequency ω_1 in 2.42, Δn_1 is given as $n_2 (|E_1|^2 + 2|E_2|^2)$. Using this expression containing n_2 , the derivation of the beam propagation equations can be derived similar to section 2.2.1, with the expansion of the $\Delta\beta$ in the formula 2.36 using the Δn from 2.45. This technique will be similar for the derivation of the polarization components, with different values for the change in refractive index, based on the different ways the beams and polarization components interact. For two identical beams of the same wavelength, as is the case for the NOLM with feedback (the more general case is shown in [11]), and using the separated variable assumption relating $E(r, t) = F(x, y)A(z, t)\exp(i\beta_0 z)$, the coupled propagation equations become for each beam

$$\begin{aligned} \frac{\delta A_1}{\delta z} &= -\beta_1 \frac{\delta A_1}{\delta t} - i \frac{\beta_2}{2} \frac{\delta^2 A_1}{\delta t^2} + \frac{-\alpha}{2} A_1 + i\gamma (|A_1|^2 + 2|A_2|^2) A_1 \\ \frac{\delta A_2}{\delta z} &= -\beta_1 \frac{\delta A_2}{\delta t} - i \frac{\beta_2}{2} \frac{\delta^2 A_2}{\delta t^2} + \frac{-\alpha}{2} A_2 + i\gamma (|A_2|^2 + 2|A_1|^2) A_2 \end{aligned} \quad (2.46)$$

and again the simplification can be made removing the time derivatives in the case of a CW beam, to get the two beam propagation equations

$$\begin{aligned} \frac{\delta A_1}{\delta z} &= \frac{-\alpha}{2} A_1 + i\gamma (|A_1|^2 + 2|A_2|^2) A_1 \\ \frac{\delta A_2}{\delta z} &= \frac{-\alpha}{2} A_2 + i\gamma (|A_2|^2 + 2|A_1|^2) A_2 \end{aligned} \quad (2.47)$$

which are the equations used in the work of Shi, Steele et al. [12, 14, 18, 32] for propagation through a NOLM with feedback without consideration of polarization effects. The similarity between the one and two beam propagation equations is evident when comparing equations 2.39 and 2.47, where the two beam case has the familiar self phase modulation term, and in addition has a cross phase modulation term which has twice the nonlinear phase effect.

2.2.3 One Beam Propagation with Polarization

Two orthogonally polarized modes are supported in a fiber, with the degenerate case being an ideal fiber which has identical refractive indexes for each mode (no birefringence). The propagation formulas of section 2.2.1 and 2.2.2 are examples of this ideal fiber assumption, which is sufficient for many cases. However most fibers in reality have some small degree of birefringence and different refractive indices n_x and n_y because of imperfections in the fibre core shape and because of stresses in the fiber. For non-polarization maintaining fibers, this is generally of a random nature, the degree of birefringence, and the orientation of the polarization axis changing over ~ 10 m lengths of fiber [11]. However some fibers are specifically made to have polarization maintaining properties (PM fibers) for use in optical systems which require little change in polarization state as light propagates down the fiber. As the intensity of a beam was shown to change the refractive index of a fiber in the previous sections, in a birefringent fiber there will be changes in the refractive indexes for each polarization mode, which induces a nonlinear birefringence of the fiber, depending on the beam intensity, and polarization orientation.

With a birefringent fiber, the fiber behaves as a long waveplate. For a higher refractive

index along the x-axis, $n_x > n_y$, the x- and y-axis are labeled the slow and fast axis respectively. Excluding nonlinear effects, (using a low power beam), as a beam of light launched at an angle to the x- or y- axis propagates down the fiber, it will rotate through polarization states from linear, to elliptical, and eventually back to the linear state if the waveplate is perfect. This periodicity happens over a length known as the beat length L_B , which is related to the birefringence by $L_B = \lambda/|n_x - n_y|$. The relation of the beat length to the overall length of the fiber is one of the ways to define the difference between a low and high birefringent fiber, which is important in further development of the beam propagation equations.

In the case of a single beam which has two orthogonal polarization modes, each mode will interact with itself, and that of its orthogonal mode in a form of XPM. To express the electric field in terms of the orthogonal modes, it can be written as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2} (\hat{x}E_x + \hat{y}E_y) \exp(-i\omega_0 t) + c.c. \quad (2.48)$$

where E_x and E_y are the complex amplitudes of each orthogonal mode in an x-y coordinate system. The procedure of substituting 2.48 into the expression for the induced polarization in equation 2.10 is similar to that of sections 2.2.1 and 2.2.2, however some changes to the induced polarization formulas must be incurred, as the third order susceptibility $\chi^{(3)}$ must be expanded. From references [11, 29], the susceptibility is shown to have three elements which are independent of each other, which is

$$\chi_{xxxx}^{(3)} = \chi_{xxyy}^{(3)} + \chi_{xyxy}^{(3)} + \chi_{xyyx}^{(3)} \quad (2.49)$$

Substituting 2.48 into 2.10, and using the susceptibilities from 2.49, the nonlinear

induced polarization can be written in terms of x- and y-polarization components, as shown

$$\mathbf{P}_{NL}(\mathbf{r}, t) = \frac{1}{2} \left(\hat{x}P_x + y\hat{P}_y \right) \exp(-i\omega_0 t) + c.c. \quad (2.50)$$

where P_x and P_y are given as

$$\begin{aligned} P_x &= \frac{3\epsilon_0}{4} \left((\chi_{xxxy}^{(3)} E_x E_x E_x^* + \chi_{xyxy}^{(3)} E_x E_x E_x^* + \chi_{xyyx}^{(3)} E_x E_x E_x^*) + \right. \\ &\quad \left. (\chi_{xxxy}^{(3)} E_x E_y E_y^* + \chi_{xyxy}^{(3)} E_y E_x E_y^* + \chi_{xyyx}^{(3)} E_y E_y E_x^*) \right) \\ P_y &= \frac{3\epsilon_0}{4} \left((\chi_{xyxy}^{(3)} E_y E_y E_y^* + \chi_{xyxy}^{(3)} E_y E_y E_y^* + \chi_{xyyx}^{(3)} E_y E_y E_y^*) + \right. \\ &\quad \left. (\chi_{xxxy}^{(3)} E_y E_x E_x^* + \chi_{xyxy}^{(3)} E_x E_y E_x^* + \chi_{xyyx}^{(3)} E_x E_x E_y^*) \right) \end{aligned} \quad (2.51)$$

If the third order susceptibilities are assumed to be approximately equal, then 2.51 can be simplified as

$$\begin{aligned} P_x &= \frac{3\epsilon_0}{4} \chi_{xxxx}^{(3)} \left[\left(|E_x|^2 + \frac{2}{3} |E_y|^2 \right) E_x + \frac{1}{3} (E_x^* E_y) E_y \right] \\ P_y &= \frac{3\epsilon_0}{4} \chi_{xxxx}^{(3)} \left[\left(|E_y|^2 + \frac{2}{3} |E_x|^2 \right) E_y + \frac{1}{3} (E_y^* E_x) E_x \right] \end{aligned} \quad (2.52)$$

From the P_x equation, the first term in the brackets which is proportional to E_x gives the nonlinear contribution to Δn_x . The second term is a degenerate four-wave mixing term, where the importance will depend on the amount of phase matching, which is related to the birefringence and beat length of the fiber. This importance will be shown as the propagation equations are developed. For the nonlinear contribution to Δn_x , the result of $P_x^{NL} = \epsilon_0 \epsilon_x^{NL} E_x$ with $\epsilon_x = \epsilon_x^L + \epsilon_x^{NL} = (n_x^L + \Delta n_x)^2$, the nonlinear change of refractive

index can be written as

$$\begin{aligned}\Delta n_x &= n_2 \left(|E_x|^2 + \frac{2}{3} |E_y|^2 \right) \\ \Delta n_y &= n_2 \left(|E_y|^2 + \frac{2}{3} |E_x|^2 \right)\end{aligned}\quad (2.53)$$

where from this result, one can see a SPM and XPM term, with the effect of the orthogonal polarization having an XPM factor of 2/3 compared to that of the SPM for equal intensity polarizations. With this result, the beam propagation equations can be solved, with a slightly modified version of the separation of variables approximation from 2.28 shown as

$$E_x(r, \omega - \omega_0) = F(x, y) A_x(z, \omega - \omega_0) \exp(i\beta_{0x} z) \quad (2.54)$$

and following the procedures from section 2.2.1, the beam propagation equations without time derivatives for the CW beams can be shown to be

$$\begin{aligned}\frac{\delta A_x}{\delta z} &= -\frac{\alpha}{2} A_x + i\gamma \left(|A_x|^2 + \frac{2}{3} |A_y|^2 \right) A_x + i\frac{\gamma}{3} A_x^* A_y^2 \exp(-2i\Delta\beta z) \\ \frac{\delta A_y}{\delta z} &= -\frac{\alpha}{2} A_y + i\gamma \left(|A_y|^2 + \frac{2}{3} |A_x|^2 \right) A_y + i\frac{\gamma}{3} A_y^* A_x^2 \exp(-2i\Delta\beta z)\end{aligned}\quad (2.55)$$

with γ being the nonlinear coefficient given as $\gamma = \frac{2\pi n_2}{\lambda A_{eff}}$, with A_{eff} being the effective core area of the fiber. The $\Delta\beta$ in 2.55 is different from the one in 2.32, with this being related to the birefringence from the different refractive indexes. It is given as $\beta_{0x} - \beta_{0y} = (2\pi/\lambda) B_m = 2\pi/L_B$, where L_B is the beat length of the fiber defined at the beginning of this section. The propagation equations contain two important terms. The

first is the nonlinear phase part containing the SPM and XPM terms that each polarization contributes. The last part involving the exponential with the $\Delta\beta$ term is a result of a degenerate four-wave mixing term. For $L \gg L_B$, there are many periodical changes over the length of the fiber, which is the case for a high birefringent fiber of L_B on the order of centimeters and L on the order of meters and kilometers. With these many periodical changes, the last terms involving the exponential will average out to zero [11], and the propagation equations for a high birefringent fiber are shown as

$$\begin{aligned}\frac{\delta A_x}{\delta z} &= -\frac{\alpha}{2}A_x + i\gamma \left(|A_x|^2 + \frac{2}{3}|A_y|^2 \right) A_x \\ \frac{\delta A_y}{\delta z} &= -\frac{\alpha}{2}A_y + i\gamma \left(|A_y|^2 + \frac{2}{3}|A_x|^2 \right) A_y\end{aligned}\tag{2.56}$$

2.2.4 Two Beam Propagation with Polarization

The final derivation of equations, and the most important for the analysis of the NOLM with feedback, is the situation where two beams propagate in a fiber which is birefringent. It can be inferred that each beam's polarization modes will see a self phase modulation effect, and cross phase modulation effects between itself and its orthogonal polarization mode and the modes of the other beam. Indeed, it will be shown that the final propagation equations will be a combination of the effects of equations 2.47 and 2.56. As well, it will be shown that there will be four wave mixing terms in the propagation equations, similar in form to 2.55. This analysis is quite complicated, however the basics of the derivations will be shown here, with the final solution taken from [33] adapted for counter-propagating beams.

The first step in the derivation is the basic expression for the total electric field in

the fiber. Similar to previous expressions, the E-field will be now be represented as a combination each beam, and of the polarization components of each beam. It can be written as

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{2} \hat{x} \left[\frac{1}{2} (\hat{x} E_{1x} + \hat{y} E_{1y}) \exp(-i\omega_0 t) + \frac{1}{2} (\hat{x} E_{2x} + \hat{y} E_{2y}) \exp(-i\omega_0 t) + c.c. \right] \quad (2.57)$$

The derivation in [33] uses the separation of variables approximation similar to 2.28, more generalized to cover two beams of different wavelengths. This leads to the expression given as

$$\begin{aligned} E(x, y, z, t) = & \hat{x} (F_{1x}(x, y) A_{1x}(z, t) \exp[i(\beta_{1x} z - \omega_1 t)] + \\ & F_{2x}(x, y) A_{2x}(z, t) \exp[i(\beta_{2x} z - \omega_2 t)]) + \\ & \hat{y} (F_{1y}(x, y) A_{1y}(z, t) \exp[i(\beta_{1y} z - \omega_1 t)] + \\ & F_{2y}(x, y) A_{2y}(z, t) \exp[i(\beta_{2y} z - \omega_2 t)]) \end{aligned} \quad (2.58)$$

Using a similar process as shown in previous sections, this expression can be substituted into the induced polarization equation found in 2.10, and the propagation equation becomes in it's general form

$$\begin{aligned} i \left(\frac{\delta E_{lm}}{\delta z} + \beta'_{lm} \frac{\delta E_{lm}}{\delta t} \right) - \frac{1}{2} \beta''_{lm} \frac{\delta^2 E_{lm}}{\delta t^2} + \gamma_{lm} \left(|E_{lm}|^2 + 2|E_{pm}|^2 + \frac{2}{3}|E_{lq}|^2 + \frac{2}{3}|E_{pq}|^2 \right) E_{lm} = \\ - \frac{1}{3} \gamma_{lm} (E_{lq}^2 E_{lm}^* \exp[(2i(\beta_{lq} - \beta_{lm})z)] + \end{aligned}$$

$$\begin{aligned}
& 2E_{lq}E_{pq}E_{pm}^* \exp \{i [(\beta_{lq} - \beta_{lm}) + (\beta_{pq} - \beta_{pm})] z\} \\
& - \frac{2}{3} \gamma_{lm} E_{lq} E_{pm} E_{pq}^* \exp \{i [(\beta_{lq} - \beta_{lm}) - (\beta_{pq} - \beta_{pm})] z\}
\end{aligned} \tag{2.59}$$

where we have

$$l, m = 1, 2 \quad p = 3 - l \quad q = 3 - m$$

Here, some simplifications can be made because both beams have the same wavelength, and the time derivatives can also be neglected due to the CW nature of the input to the NOLM with feedback. As well, as the beams propagate in the same medium, they both see the same birefringence values and therefore the β terms can be simplified and combined into $\Delta\beta$ values. As the beams are counter propagating the directions of the propagation are accounted for in the $\exp(i\beta z)$ terms, and in the conjugations of the beams. The final simplification comes with noting that the nonlinear coefficients are approximately the same between polarization states, and are identical the same polarization states of each beam, giving $\gamma_{1x} = \gamma_{2x} \approx \gamma_{1y} \approx \gamma_{2y}$. Therefore the coupled DE equations can be written in a simplified way as shown (with $P = 2/3$ for linearly polarized beams):

$$\begin{aligned}
& i \frac{\delta E_{1x}}{\delta z} + \gamma \left(|E_{1x}|^2 + 2|E_{2x}|^2 + \frac{2}{3}|E_{1y}|^2 + \frac{2}{3}|E_{2y}|^2 \right) E_{1x} = \\
& - \frac{1}{3} \gamma (E_{1y}^2 E_{1x}^* \exp[-2i\Delta\beta z] + 2E_{1y} E_{2y}^* E_{2x}) - \frac{2}{3} \gamma E_{1y} E_{2x}^* E_{2y} \exp\{-2i\Delta\beta z\} \\
& i \frac{\delta E_{2x}}{\delta z} + \gamma \left(|E_{2x}|^2 + 2|E_{1x}|^2 + \frac{2}{3}|E_{2y}|^2 + \frac{2}{3}|E_{1y}|^2 \right) E_{2x} =
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3}\gamma (E_{2y}^2 E_{2x}^* \exp[(2i\Delta\beta z)] + 2E_{2y} E_{1y}^* E_{1x}) - \frac{2}{3}\gamma E_{2y} E_{1x}^* E_{1y} \exp\{2i\Delta\beta z\} \\
& i\frac{\delta E_{1y}}{\delta z} + \gamma \left(|E_{1y}|^2 + 2|E_{2y}|^2 + \frac{2}{3}|E_{1x}|^2 + \frac{2}{3}|E_{2x}|^2 \right) E_{1y} = \\
& -\frac{1}{3}\gamma (E_{1x}^2 E_{1y}^* \exp[(2i\Delta\beta z)] + 2E_{1x} E_{2x}^* E_{2y}) - \frac{2}{3}\gamma E_{1x} E_{2y}^* E_{2x} \exp\{2i\Delta\beta z\} \\
& i\frac{\delta E_{2y}}{\delta z} + \gamma \left(|E_{2y}|^2 + 2|E_{1y}|^2 + \frac{2}{3}|E_{2x}|^2 + \frac{2}{3}|E_{1x}|^2 \right) E_{2y} = \\
& -\frac{1}{3}\gamma (E_{2x}^2 E_{2y}^* \exp[(-2i\Delta\beta z)] + 2E_{2x} E_{1x}^* E_{1y}) - \frac{2}{3}\gamma E_{2x} E_{1y}^* E_{1x} \exp\{-2i\Delta\beta z\} \quad (2.60)
\end{aligned}$$

These relatively long equations are used to describe two beam propagation in an arbitrarily birefringent fiber. The expressions relating the XPM and SPM components of the beam intensities to the nonlinear phase of a particular beam polarization are readily visible, as a combination of the two beam XPM factor of 2 from section 2.2.2, and the polarization XPM factor of 2/3 from section 2.2.3. The last terms are the four wave mixing terms. As in section 2.2.3, for $L \gg L_B$, there are many periodical changes over the length of the fiber, and with these many periodical changes, the last terms involving the exponential will average out to zero. Thus for a high birefringence fiber, these equations

can further be simplified to be

$$\begin{aligned}
\frac{\delta E_{1x}}{\delta z} &= i\gamma \left(|E_{1x}|^2 + 2|E_{2x}|^2 + \frac{2}{3}|E_{1y}|^2 + \frac{2}{3}|E_{2y}|^2 \right) E_{1x} \\
\frac{\delta E_{2x}}{\delta z} &= i\gamma \left(|E_{2x}|^2 + 2|E_{1x}|^2 + \frac{2}{3}|E_{2y}|^2 + \frac{2}{3}|E_{1y}|^2 \right) E_{2x} \\
\frac{\delta E_{1y}}{\delta z} &= i\gamma \left(|E_{1y}|^2 + 2|E_{2y}|^2 + \frac{2}{3}|E_{1x}|^2 + \frac{2}{3}|E_{2x}|^2 \right) E_{1y} \\
\frac{\delta E_{2y}}{\delta z} &= i\gamma \left(|E_{2y}|^2 + 2|E_{1y}|^2 + \frac{2}{3}|E_{2x}|^2 + \frac{2}{3}|E_{1x}|^2 \right) E_{2y}
\end{aligned} \tag{2.61}$$

With these equations, the NOLM with feedback in a birefringent fiber can be analyzed using numerical solving techniques. The next sections and chapters will describe how these equations are used in the unpolarized, and polarized versions of the NOLM with feedback.

2.3 Nonlinear Optical Loop Mirror

The nonlinear optical loop mirror (NOLM) is the nonlinear application of a Sagnac interferometer. The fiber Sagnac interferometer, invented in 1976 by Vali et al. [34], is shown in as figure 2.1. The Sagnac interferometer has many uses such as applications for wavelength conversion, optical switching and fiber laser mode locking [13].

2.3.1 Nonlinear Optical Loop Mirror without Feedback

Figure 2.1: Nonlinear optical loop mirror, an application of a Sagnac interferometer

The behavior of the NOLM is rather straightforward. Light enters the input of the NOLM with amplitude E_{in} . The coupler K_1 splits the power of the input field into the ratio $K_1 : (1 - K_1)$, as shown by the two counter-propagating fields E_3 and E_4 given as

$$\begin{aligned} E_3 &= K_1^{1/2} E_{in} + i(1 - K_1)^{1/2} E_2 \\ E_4 &= K_1^{1/2} E_2 + i(1 - K_1)^{1/2} E_{in} \end{aligned} \quad (2.62)$$

where the field E_2 is usually zero bringing the simplified version of the fields as

$$\begin{aligned} E_3 &= K_1^{1/2} E_{in} \\ E_4 &= i(1 - K_1)^{1/2} E_{in} \end{aligned} \quad (2.63)$$

For low powers, each field E_3 and E_4 will propagate the same distance, but opposite directions, around the loop. They both see the same linear phase from propagation, and before recombination at the coupler, are known as E'_3 and E'_4 , defined for low powers as

$$E'_3 = E_3 \exp(-i\phi_L)$$

$$E'_4 = E_4 \exp(-i\phi_L) \quad (2.64)$$

and thus, at the coupler, the output field E_{out} can be calculated as

$$E_{out} = K_1^{1/2} E'_3 + (1 - K_1)^{1/2} E'_4 \quad (2.65)$$

With a 50:50 coupler splitting ratio, all of the power in the NOLM will be reflected back to the input ($E_{out} = 0$). For a slightly unbalanced splitting ratio of 45:55 for example, the output at low powers is minimal, with the majority of the power still being reflected back into the input. However, with higher powers, and thus the nonlinear effect of the fiber being taken into account, there is an additional phase seen by each counter-propagating beam E_3 and E_4 , generated from the equations 2.47 derived in section 2.2.2. When including these nonlinear effects, the phase of the counter propagating beams can be rewritten as shown from [18] as

$$\begin{aligned} E'_3 &= E_3 \exp[-i(\phi_L + \phi_c)] \\ E'_4 &= E_4 \exp[-i(\phi_L + \phi_{cc})] \end{aligned} \quad (2.66)$$

where the phases ϕ_c and ϕ_{cc} are the nonlinear phase contributions generated from the propagation equations from 2.47, shown below in equation 2.67, derived by Shi [12].

$$\begin{aligned} \phi_c &= \frac{2\pi n_2 L}{\lambda A_{eff}} (2 - K_1) P_1 \\ \phi_{cc} &= \frac{2\pi n_2 L}{\lambda A_{eff}} (1 + K_1) P_1 \end{aligned} \quad (2.67)$$

As these phases are dependent on the power P_1 entering the NOLM, the slightly unbalanced coupler will generate different total phases of the beams at E'_3 and E'_4 , depending on the input power. The result of this is, as power is increased into the input of the NOLM, the output power will oscillate, as opposed to linearly increase as in the case of operating the NOLM at low powers. Both of these effects are shown as figure 2.2, in a NOLM with 500 m of single mode fiber with a nonlinear coefficient value of $\gamma = 1.2 \times 10^{-3}$, with a linear phase contribution of $\phi_L = 0$ for simplicity, although this value can be between 0 and 2π .

Figure 2.2: NOLM output, illustrating the effect of including the nonlinear phase contribution

2.3.2 Nonlinear Optical Loop Mirror with Feedback

When feedback is applied to the NOLM, it behaves as an optical resonator, and it can be used to generate bistable and chaotic output [12, 14, 18, 32]. These previous papers on the NOLM with feedback were done with the assumption of no polarization effects in the NOLM with feedback in the fiber, using the equations found in section 2.2.2. The NOLM

The diagram shows a fibre loop with two main paths. On the left, a path labeled "UNIDIRECTIONAL FEEDBACK" contains a square block with an upward arrow. This path has nodes E_6 and E_8 . On the right, a path labeled "FIBRE LOOP" contains nodes E_4 and E_3 . The paths are connected at two central nodes, E_1 and E_2 . Input signals E_{IN} and E_{IN}' enter from the top. Feedback signals E_3' and E_4' exit from the top. Gain blocks K_1 and K_2 are shown with feedback loops $(1-K_1)$ and $(1-K_2)$ respectively. The diagram illustrates the unidirectional feedback mechanism in a fibre loop.

Analysis of the instability and bistability of the NOLM with feedback is contained thoroughly in [18], with an example of the NOLM with feedback output reproduced here in 2.4, at $\lambda = 1.55\mu m$, using parameters from [18] of $K_1 = 0.25$, $K_2 = K_3 = 0.8$, $n_2 = 3.2 \times 10^{-20}$ and $A_{eff} = 30\mu m^2$. The power input is incrementally increased after every cavity period pass around the NOLM, with the input power following the Gaussian shaped quasi-CW input shown in the bottom picture of 2.4.

Figure 2.4: Figures showing power output of the NOLM with feedback vs the input (top), and the power output of the NOLM vs cavity period passes (bottom). The input power to the NOLM is shown as the Gaussian shaped curve on the bottom graph.

By adjusting the NOLM with feedback coupler values, as well as the type and length of the fiber used in the NOLM, different outputs can be generated. The NOLM output is sensitive to the length and type of optical fiber, the coupler ratios, and the linear phase contributions in the coupler and isolator portions of the system [18]. The next two chapters examines the change of the NOLM with feedback behavior when using a birefringent fiber, and adding input polarization state as an additional parameter of control.

Chapter 3

High Birefringence Fiber

3.1 Introduction

All fibers typically have some form of birefringent properties because of imperfections in the manufacturing process, various stresses induced on the fiber, and other factors. For birefringent fibers, B_m is the degree of birefringence given by $B_m = |n_x - n_y|$, and this birefringence amount and orientation generally changes over the length of the fiber ($\sim 10\text{m}$) unless specially constructed fibers are used [11]. Polarization maintaining (PM) fibers are designed to have a linear birefringence built into them which is much larger than the naturally occurring birefringence, and therefore have a nearly constant B_m throughout the fiber. For a high birefringence PM fiber, the birefringence is typically on the order of $B_m \sim 10^{-4}$, and the beat length defined as $L_B = \lambda/B_m$ and is usually around $L_B \sim 1\text{ cm}$ for a high birefringent fiber [11], giving an indication of the strength of the waveplate inherent in the fiber. Because there are many beat lengths in a fiber in a NOLM (where the fiber loop is on the order of meters or kilometers), the approximations in section 2.2.4 can be used and the propagation equations for high birefringence fibers are applicable.

This chapter covers the use of high birefringence fiber in the NOLM, and the NOLM with feedback, showing the effects on the output of the systems when changing the input polarization state.

3.1.1 The Poincare Sphere

The polarization of the light in a fiber can be represented as Stokes vectors on a Poincare sphere. The Poincare sphere is given as figure 3.1. For the output polarization modes of the NOLM and the NOLM with feedback in Cartesian coordinates, one can represent them in an alternate fashion in terms of the Stokes vectors using the following relations

$$S_0 = |E_{out_X}|^2 + |E_{out_Y}|^2$$

$$S_1 = |E_{out_X}|^2 - |E_{out_Y}|^2$$

$$S_2 = 2Re \{ E_{out_X}^* E_{out_Y} \}$$

$$S_3 = 2Im \{ E_{out_X}^* E_{out_Y} \} \quad (3.1)$$

The S1 axis represents the linear polarization state corresponding to the horizontal and vertical polarizations. A value of +1 on the normalized sphere represents horizontal polarization in the Cartesian coordinate system, and a value of -1 represents a vertically polarized beam. The S2 axis represents a 45 degree polarization state, with a value of +1 on the S2 axis being equal to +45 degree linear polarization, and -1 being equal to

Figure 3.1: The Poincare Sphere, showing the polarization orientation for each axis.

-45 degrees. The S3 axis represents the circular polarizations, with a +1 value on this axis being a right-hand circularly polarized beam, and a -1 value being a left-hand circularly polarized beam. The Poincare sphere can be alternately viewed as a set of three planes, corresponding to the three axis represented on the sphere. This representation of the output polarization state of the NOLM and the NOLM with feedback is used in the subsequent sections.

Figure 3.2: The Nonlinear Optical Loop Mirror with high birefringence fiber in the loop

3.2 Nonlinear Optical Loop Mirror with High Birefringence Fiber

3.2.1 Overview of the Nonlinear Optical Loop Mirror

Recent research has shown that the input polarization of the light and the twist of the fiber can have an important effect on the performance of the nonlinear optical loop mirror (NOLM). There has been research examining polarization effects on the NOLM [19, 20, 35], and these papers typically use a pulse input and neglect the effects of cross phase modulation (XPM) of the counter propagating beams in the NOLM by assuming very short pulses. Reference [36] analyzes SPM and XPM in a NOLM, although with a pulsed input instead of the CW input used here. This section looks briefly at the behavior of the NOLM with highly linearly birefringent fiber in the loop, considering the SPM and XPM effects of each counter-propagating CW beam. With this understanding, feedback can be applied to the NOLM and its behavior as an optical resonator is examined in section 3.3.

Figure 2.1 illustrates the NOLM, shown again as figure 3.2 for clarity. The NOLM behavior is described for unpolarized CW light in section 2.3.1 and in [13]. The input

CW beam in this analysis is considered to be made up of two orthogonal linear polarized beams, although this can be done with any orthogonal basis. With a NOLM with birefringent fiber in the loop, the coupler equations are modified, giving

$$\begin{aligned} E_{3x} &= K_1^{\frac{1}{2}} E_{1x} + i(1 - K_1)^{\frac{1}{2}} E_{2x} \\ E_{3y} &= K_1^{\frac{1}{2}} E_{1y} + i(1 - K_1)^{\frac{1}{2}} E_{2y} \end{aligned} \quad (3.2)$$

for the clockwise beam, and

$$\begin{aligned} E_{4x} &= K_1^{\frac{1}{2}} E_{2x} + i(1 - K_1)^{\frac{1}{2}} E_{1x} \\ E_{4y} &= K_1^{\frac{1}{2}} E_{2y} + i(1 - K_1)^{\frac{1}{2}} E_{1y} \end{aligned} \quad (3.3)$$

for the counter-clockwise beam.

The two beams propagate around the loop using the propagation equations found in 2.2.4, which are repeated here as 3.4, with the variable names changed to reflect the beams at each port 3 and 4.

$$\begin{aligned} \frac{\delta E_{3x}}{\delta z} &= i\gamma \left(|E_{3x}|^2 + 2|E_{4x}|^2 + \frac{2}{3}|E_{3y}|^2 + \frac{2}{3}|E_{4y}|^2 \right) E_{3x} \\ \frac{\delta E_{4x}}{\delta z} &= i\gamma \left(|E_{4x}|^2 + 2|E_{3x}|^2 + \frac{2}{3}|E_{4y}|^2 + \frac{2}{3}|E_{3y}|^2 \right) E_{4x} \\ \frac{\delta E_{3y}}{\delta z} &= i\gamma \left(|E_{3y}|^2 + 2|E_{4y}|^2 + \frac{2}{3}|E_{3x}|^2 + \frac{2}{3}|E_{4x}|^2 \right) E_{3y} \\ \frac{\delta E_{4y}}{\delta z} &= i\gamma \left(|E_{4y}|^2 + 2|E_{3y}|^2 + \frac{2}{3}|E_{4x}|^2 + \frac{2}{3}|E_{3x}|^2 \right) E_{4y} \end{aligned} \quad (3.4)$$

These propagation equations can be solved using Matlab ODE45 solver as they are

propagated around the loop, however they can be integrated easily assuming a constant electric field amplitude, to give a solution of the form shown in equation 3.5 which results in much quicker computational times.

$$\begin{aligned}
 E'_{3x} &= E_{3x} \exp \left(i\gamma L \left(|E_{3x}|^2 + 2|E_{4x}|^2 + \frac{2}{3}|E_{3y}|^2 + \frac{2}{3}|E_{4y}|^2 \right) \right) \\
 E'_{4x} &= E_{4x} \exp \left(i\gamma L \left(|E_{4x}|^2 + 2|E_{3x}|^2 + \frac{2}{3}|E_{4y}|^2 + \frac{2}{3}|E_{3y}|^2 \right) \right) \\
 E'_{3y} &= E_{3y} \exp \left(i\gamma L \left(|E_{3y}|^2 + 2|E_{4y}|^2 + \frac{2}{3}|E_{3x}|^2 + \frac{2}{3}|E_{4x}|^2 \right) \right) \\
 E'_{4y} &= E_{4y} \exp \left(i\gamma L \left(|E_{4y}|^2 + 2|E_{3y}|^2 + \frac{2}{3}|E_{4x}|^2 + \frac{2}{3}|E_{3x}|^2 \right) \right)
 \end{aligned} \tag{3.5}$$

Similar to section 2.3.1 and to the unpolarized NOLM with feedback analysis by Lynch et. al [18], the nonlinear phase ϕ accumulated by the clockwise and counter-clockwise propagating beams beam can be written from these equations. The power seen at port 3 and port 4 of the coupler is given as

$$P_3 = K \cdot P_{IN}$$

$$P_4 = (1 - K) \cdot P_{IN} \tag{3.6}$$

where P_{IN} is the input power in the NOLM. Looking at the powers P_3 and P_4 , they can be written in terms of the E fields as shown below.

$$P_3 = |E_3|^2$$

$$P_4 = |E_4|^2 \tag{3.7}$$

With the beams E_3 and E_4 being rewritten in terms of their orthogonally polarized modes as shown below.

$$\begin{aligned} E_{3x} &= E_3 \cos(\theta) & E_{3y} &= E_3 \sin(\theta) \\ E_{4x} &= E_4 \cos(\theta) & E_{4y} &= E_4 \sin(\theta) \end{aligned} \quad (3.8)$$

Using the value for the phase from the integrated propagation equations given as the general solution $\exp(i\phi L)$, the phase can be then expressed as

$$\phi_{3x} = \gamma \left(|E_{3x}|^2 + 2|E_{4x}|^2 + \frac{2}{3}|E_{3y}|^2 + \frac{2}{3}|E_{4y}|^2 \right) \quad (3.9)$$

for the E_{3x} case. Using the identities found in 3.6, 3.7 and 3.8 the phases can be written in terms of the input power and the coupler K value as shown below.

$$\begin{aligned} \phi_{3x} &= \gamma \left[\frac{2}{3}P_{IN} + \left(\frac{4}{3} - K \right) P_{IN} \cos^2 \theta \right] \\ \phi_{3y} &= \gamma \left[\frac{2}{3}P_{IN} + \left(\frac{4}{3} - K \right) P_{IN} \sin^2 \theta \right] \\ \phi_{4x} &= \gamma \left[\frac{2}{3}P_{IN} + \left(\frac{1}{3} + K \right) P_{IN} \cos^2 \theta \right] \\ \phi_{4y} &= \gamma \left[\frac{2}{3}P_{IN} + \left(\frac{1}{3} + K \right) P_{IN} \sin^2 \theta \right] \end{aligned} \quad (3.10)$$

These equations can be used for a linear stability analysis much as they were used by Steele, Lynch et al. [14, 18] in the analysis of the unpolarized NOLM with feedback, how-

ever this is beyond the scope of this thesis. Applying a horizontal or vertical polarization to these equations ($\theta = 0, 90^\circ$) will yield the same degenerate-case phase expressions for the clockwise and counter-clockwise values in section 2.3.1 and in previous work by Steele et al. [18].

After the beams have propagated around the loop and acquired their nonlinear phases, they are recombined in the coupler using similar coupler equations as in 3.2 and 3.3. The output power of the system is given as $P_{out} = |E_{out_x}|^2 + |E_{out_y}|^2$.

3.2.2 Output Power Behavior

As the CW light propagates around the NOLM, each beam's phase relies on both its own polarization components, and the polarization components of the counter-propagating beam. If the beam is polarized along either the fast or slow axis, the equations degenerate into the scalar forms of XPM seen in section 2.2.2. The power output behavior of the NOLM changes when the input polarization angle is rotated, due to the power distribution in each polarization mode affecting the nonlinear phase at the output of the loop. This is seen when examining the propagation equations in 3.4. An example of the NOLM behavior with different polarization input angles is numerically simulated in figure 3.3, with input polarizations from 0 to 45 degrees in 15 degree increments.

As the polarization rotates between 45 to 90 degrees, the NOLM performance will behave in a symmetrical fashion to 3.3, with the NOLM output at 90 degrees being identical to the output at 0 degrees. This behavior allows the output of the NOLM to be tuned by rotating the input polarization angle between 0 and 45 degrees from either the vertical or horizontal state. Note that adding a phase difference between each polarization

Figure 3.3: NOLM output with varying input polarization. Minimum and maximum transmission power increases as polarization angle is rotated through to 45 degrees from either axis.

mode, equivalent to creating an elliptical polarization state, will not affect this output as the propagation equations in 3.4 rely on the intensity (modulus-square) of the beam. This output behavior that depends on the input polarization angle seen by the NOLM is of importance when adding feedback to the NOLM, when bistable and chaotic tunability can be important. This is examined in section 3.3.

3.2.3 Output Polarization Behavior

In a straight lossless piece of fiber for a single beam input E_1 polarized at 45 degrees with respect to the optical fiber axis (equal powers in both polarization states), the nonlinear phase seen by each propagating polarization mode is the same because the propagation equations derived in section 2.2.3 shown below rewritten slightly, will be identical.

$$\begin{aligned}\frac{\delta E_{1x}}{\delta z} &= i\gamma \left(|E_{1x}|^2 + \frac{2}{3}|E_{1y}|^2 \right) E_{1x} \\ \frac{\delta E_{1y}}{\delta z} &= i\gamma \left(|E_{1y}|^2 + \frac{2}{3}|E_{1x}|^2 \right) E_{1y}\end{aligned}\tag{3.11}$$

This is shown below with all beam polarization modes having the same power $E_{1x} = E_{1y} = E$.

$$\begin{aligned}\frac{\delta E_{1x}}{\delta z} &= i\gamma \left(\frac{5}{3}|E|^2 \right) E_{1x} \\ \frac{\delta E_{1y}}{\delta z} &= i\gamma \left(\frac{5}{3}|E|^2 \right) E_{1y}\end{aligned}\tag{3.12}$$

Therefore at the output of the fiber, as E_x and E_y have the same intensity and thus

Figure 3.4: Beam polarization at output E'_4 , just before recombination at the coupler. Input polarization is at 44.35 degrees, with no PDL in the system.

see the same nonlinear phase change, there will not be any rotation in polarization at the output, for any input power. For a single beam polarized at an angle between $0 < \theta < 45$ or $45 < \theta < 90$, the power between the E_x and E_y fields is unbalanced, and thus the polarization modes will end up with a different, intensity-dependent, nonlinear phase between them, as seen by the nature of equation 3.11. As the phase difference increases, the combined beam E_1 , as it propagates along the fiber will rotate from a linear polarization to an elliptical state, and back to linear, corresponding to a circle on the S2/S3 plane of the Poincare sphere whose size depends on the input polarization angle. When two beams propagate in a fiber, the situation is similar, as each beam will rotate polarization states depending on the intensity of its own polarization modes, and the modes of the other beam. This is illustrated in figure 3.4 for a 1 km NOLM and a coupler splitting ratio of 45:55, and an input of slightly less than 45 degrees. The state of polarization for the propagated beam E_4 , just before re-coupling into the output, at port E'_4 is shown as input power to the NOLM is increased.

As the coupler is unbalanced with more power (55% of the input) being seen in beam

Figure 3.5: Comparison of the polarization states at E'_4 and E'_3 before recombination at the coupler.

E_4 than beam E_3 , the polarization state of the beam at E'_4 , will be different than for the beam at the output E'_3 . This is shown in figure 3.5, illustrating that as power is increased, the beam E_4 has rotated further around the Poincare sphere than the beam E_3 .

The effect of this is that upon recombination in the coupler, the output polarization state of the beam E_{out} will spiral over the Poincare sphere as the power in the NOLM is increased. This means that the output polarization state changes depending on the input power, and the input polarization angle. This can have an adverse effect on any polarization sensitive devices after the NOLM. The power output and 2D stokes vectors are shown in figure 3.6, and a picture of the polarization rotation on a Poincare sphere is shown in figure 3.7.

Figure 3.6: Polarization and Power output of the NOLM. Power output is shown in (a), and the planes of the Poincare sphere are shown in (b).

Figure 3.7: Polarization Rotation on the Poincare Sphere.

3.3 High Birefringence Fiber in NOLM with feedback

With the understanding of the behavior of the NOLM in section 3.2, the behavior of the NOLM with feedback applied can be more easily understood. It will be shown that since, in the previous section, the NOLM output power and polarization was found to depend on the input polarization, a similar situation will be seen when feedback is applied to the NOLM. For interest of comparison, setting input polarization angles to linear horizontal or vertical will give the unpolarized, degenerate case of the propagation equations used in previous work [12, 14, 18, 32]. This section examines the output power and state of polarization when using highly linear birefringent fiber in a NOLM with feedback with a quasi-CW input.

3.3.1 Overview of the Nonlinear Optical Loop Mirror with Feedback

The configuration of the NOLM with feedback is shown in Figure 3.8. The behavior of the NOLM portion of the device is the same as outlined in section 3.2.1, but with an isolator coupled between the input and output with couplers K_2 and K_3 , providing feedback to the system.

The couplers behave in the same fashion as in the NOLM without feedback, shown in equations 3.2 and 3.3. As light is fed back into the input, careful tuning of the couplers K_2 and K_3 will allow a system exhibiting bistable, multi-stable and chaotic behavior. The description of the unpolarized NOLM with feedback is shown in section 2.3.2, however when high birefringence fibers are used in the loop, the propagation equations from 3.4 are used instead of the unpolarized forms.

Figure 3.8: Optical loop mirror with feedback schematic. The electric field at position j is E_j and $K_m : (1 - K_m)$ indicates the power coupling ratio at coupler m .

3.3.2 Output Power Behavior

As seen from the behavior of the NOLM with high birefringence fiber in section 3.2.2, the input polarization angle will give a significant effect on the output power and polarization state of the beam. Examining the two extreme cases of polarization inputs, namely 0 and 45 degree input polarizations from either the x- or y- axis, the behavior of the NOLM with feedback is examined. Using values of a 80 m fiber, and coupler splitting ratios of $K_1 = 0.15$ and $K_2 = K_3 = 0.85$ consistent with earlier NOLM with feedback analysis [18, 32], and with a modified $n_2 = 3.4 \times 10^{-20} \text{ m}^2/\text{W}$ and $A_{eff} = 41 \text{ } \mu\text{m}^2$ for a high birefringence fiber [37], the polarization angle was rotated and the results numerically simulated in Matlab. Note that power levels used in this simulation are high, however this can be reduced significantly by using longer fibers, but in the interest of comparison with earlier work [12, 14, 18], the shorter fiber lengths were used. Fig. 3.9 shows the bistable regions for input beams launched at 0 degree and 45 degree polarization angles. The 45 degree polarization orientation requires a power of approximately 20.61 Wm^{-2} to enter the bistable region, compared to the 15.63 Wm^{-2} for the beam polarized along the fast or

Figure 3.9: Output of the NOLM with feedback when the input polarization is at 0 and 45 degrees to the fast (or slow) axis. Solid line is the 0 degree, and dashed line is 45 degree polarization angle.

slow axis. This is due to the phase behavior from 3.4 showing that as the power is divided within the polarizations, the overall nonlinear phase reduces. Other angles between 0 and 45 degrees tune this bistable region, allowing a control of desired switching power over a certain range of input powers.

Fig. 3.10 shows the output power levels for each cavity period pass of the NOLM with feedback. It is shown that the chaotic area, and the clipping effects on the power output are significantly different for the 0 and 45 degree polarization input angles. Examining the bistable, multi-stable and beginnings of the chaotic region for both the 0 and 45 degree polarization states in figure 3.11, one can see that although the bistable region is pushed to higher powers for the 45 degree polarization, it exhibits chaotic behavior at approximately the same input power, and the chaotic output amplitude is much more

pronounced with the 45 degree input. As well for the beam polarized at 45 degrees in the input, the pitchfork bifurcation spans a greater output power range as compared to the 0 degree input polarization. This shows that although the polarization angle at 0 degrees will generate a lower bistable switching power than at 45 degrees, the chaotic output can be greater by using the 45 degree input. This behaviour is only for this particular design of NOLM with feedback, designed to illustrate the potential differences that can be seen in the output by a polarization rotation. Much more detailed analysis on the dynamics of the NOLM with feedback with polarization would need to be performed to develop ways to design in the tunability of the system.

Figure 3.10: Plot of power against number of cavity period passes (iterations). Input power is the Gaussian shape, output power is the inside curve. The top figure shows the 0 degree polarization power output, and the bottom figure shows the 45 degree polarization power output.

With a different NOLM with feedback coupler selection of $K_1 = 0.25$ and $K_2 = K_3 = 0.85$, and an increase in power, the behavior of the NOLM with feedback is shown in figure 3.12. Here, a return to stability is seen for the 0 degree polarization case around 33.6 W, with a bifurcation occurring shortly after. As well for the 0 degree polarization input, the chaotic region from the input power range of 21.7 W to 33.6 W, and the chaotic region from 37.2 W onward, both are limited in scope to approximately a power level between 10 and 22 W. With the input polarization angle of 45 degrees, the return to stability is no longer seen at the same input powers for both the increasing and decreasing edge of the pulse, and also the power limiting or clipping effect of the NOLM with feedback is no longer seen. Note that in this NOLM with feedback design, the chaotic region is pushed out to higher powers for the 45 degree input compared to the 0 degree input. For the case in figure 3.11 the chaotic region develops at slightly lower powers for the 45 degree input compared to the 0 degree input.

With the appropriate CW input levels to select the region to work in the NOLM with feedback, rotating the polarization with a polarization controller can allow tuning of the bistable, unstable, and chaotic output, a feature not present in previous NOLM with feedback designs [12, 14, 18, 32].

3.3.3 Polarization Dependent Loss in NOLM Coupler

With the polarization angle having an effect on the output of the NOLM with feedback, it is important to examine the effect of polarization dependent loss (PDL) in the NOLM coupler. When the coupler is assumed to have a loss associated to one linear polarization state, there are two different situations examined here, the same PDL existing in both

Figure 3.11: Plot of the input versus output power for the 0 degree polarization input (top) and 45 degree input polarization (bottom), illustrating the bistable, multi-stable and chaotic outputs of each.

Figure 3.12: NOLM with feedback with $K_1 = 0.25$, and $K_2 = K_3 = 0.85$. The 0 degree input polarization (top) shows a return to stability and output power limiting by the system. The 45 degree input polarization (bottom) shows no power limiting.

Figure 3.13: PDL in the y-component of a propagating E-field. The angle θ of the field E depends on the amount of PDL.

output arms of the coupler, or a difference in PDL between one output arm of the coupler and the other output arm. As the NOLM with feedback has a dependence on the input polarization and input power, the PDL in the NOLM coupler can have a significant effect. A schematic showing the PDL attenuation on the y-component of a propagating electric field is shown as figure 3.13.

In the case of the same PDL existing in both output arms of the coupler, the coupler equations can be written as shown,

$$E_{3x} = K_1^{\frac{1}{2}} \cdot E_{1x}$$

$$E_{3y} = K_1^{\frac{1}{2}} \cdot \alpha^{\frac{1}{2}} \cdot E_{1y} \quad (3.13)$$

$$E_{4x} = i(1 - K_1)^{\frac{1}{2}} \cdot E_{1x}$$

$$E_{4y} = i(1 - K_1)^{\frac{1}{2}} \cdot \alpha^{\frac{1}{2}} \cdot E_{1y} \quad (3.14)$$

with α as the power attenuation caused by the PDL, given as $\alpha = 10^{-PDL_{dB}/10}$. For example, a device with 0.2 dB of PDL would have an α equal to 0.995, meaning 95.5% of the light in that polarization state is transmitted through the coupler. The PDL affects the polarization of the beams entering the NOLM as shown in figure 3.14 for a y-axis polarization loss. As the beam is attenuated in one polarization state, the overall power entering the NOLM is reduced, and the effective angle of the polarization is reduced.

Figure 3.14: The effect of PDL on the angle and amplitude of the input beam. Top picture has no PDL, bottom picture has 0.5dB of PDL applied to both output fibers of the 55:45 coupler. Note the slight attenuation and angle change when PDL is applied to the system.

The effective polarization angle as a function of the coupler PDL is shown as the left figure in 3.15, assuming an input polarization angle of 45 degrees and a y-axis PDL.

Figure 3.15: Polarization angle and intensity versus coupler PDL of the light at E_3 with an input polarization of 45 degrees. PDL is applied to the y-axis for these graphs.

Because of the power loss, the overall power entering the NOLM is reduced, as is shown in the right figure of 3.15. This effect is dependent on the input polarization angle, with beams polarized closer to the axis with PDL showing a greater overall effect.

The PDL effect on the polarization propagation equations of 2.61 can be easily visualized, as the nonlinear phase seen in the NOLM is directly related to the intensity of the polarized components. The output of the NOLM will depend on the amount of PDL, and the input polarization angle. An example of a NOLM with feedback bistable output with both 0 and 0.5 dB PDL with an input angle of 45 degrees is shown as 3.16. The bistable area can be seen to be slightly smaller, and located at higher input powers for a coupler with PDL due to the power attenuation and slight polarization angle rotation.

With PDL in only one output arm of the coupler, only one polarization of the four will see a polarization dependent loss. This is a degenerate case of two different PDLs being seen in each coupler arm. This PDL is implemented by removing one of the alpha terms in 3.13 or 3.14. The modified coupler equations for a PDL at the output of one arm of the coupler at E_4 are shown as

Figure 3.16: NOLM with feedback and 45 degree input polarization, with and without PDL in both coupler arms.

$$E_{3x} = K_1^{0.5} \cdot E_{1x}$$

$$E_{3y} = K_1^{0.5} \cdot E_{1y}$$

$$E_{4x} = i(1 - K_1)^{0.5} \cdot E_{1x}$$

$$E_{4y} = i(1 - K_1)^{0.5} \cdot \alpha^{0.5} \cdot E_{1y} \quad (3.15)$$

The output of the NOLM with feedback with PDL in only one arm of the coupler is seen in figure 3.17 to behave in between the behaviours of the no PDL, and both coupler arm PDL situations. An input polarization angle close to the axis exhibiting PDL was used to accentuate the PDL effect.

Figure 3.17: NOLM with PDL behavior, with input polarization angle 10 degrees from the axis with PDL.

3.3.4 Output Polarization Behavior

The NOLM with feedback can give a widely varying polarization output based on a fixed input polarization angle. In the purely nonlinear propagation of a beam in a straight piece of fiber assuming a high birefringence, the output polarization is seen to rotate about the S_3 axis of the Poincare sphere as power is increased [11]. However, for each counter-propagating beam in a NOLM the intensities are different, so at recombination at coupler K_1 , the beams are recombined each having rotated a different amount around the Poincare sphere. Without feedback, this results in the output polarization spiraling around the Poincare sphere as shown figure 3.7 in section 3.2.3. When feedback is introduced, the system becomes much more complex, and the polarization begins to exhibit instabilities and chaos of the system as it rotates around the Poincare sphere.

For the case of a NOLM with feedback operating in the bistable region, the resulting output power and polarization of the system is shown as figure 3.18. The NOLM was designed with a 8 km fiber, with $K_1 = 0.25$, and $K_2 = K_3 = 0.85$, a nonlinear coefficient

Figure 3.18: NOLM with feedback operating in the bistable region, with polarization angle of 40 degrees. Left graph is the output power of the device, and the right graph shows polarization vector projected on the S1/S2 plane of the Poincare sphere.

$\gamma = 3.4 \times 10^{-20} \text{m}^2/\text{W}$ and an effective area for high birefringence fiber of $A_{eff} = 41 \mu\text{m}^2$ [37], with an quasi-CW input Gaussian pulse of 250 mW. Figure 3.18 shows the polarization evolution of the output of the graph following a hysteresis at the same time as the output power, where the polarization does not follow the same trajectory on the leading and trailing edges of the Gaussian pulse.

As the power is increased in the NOLM, the output will go unstable and lead to chaotic power output. Figure 3.19 shows the output polarization evolution on the S2/S3 plane of the Poincare sphere for an input beam close to 45 degrees. This figure shows that the polarization output is nearly circular around the S2/S3 plane, showing that the combination of the two unbalanced beams at coupler K_1 results in an output that generally rotates from linear to circular. The picture on the right in figure 3.19 shows the Stokes vectors connected, showing how the polarization jumps from different orientations on the S2/S3 plane in the chaotic region as power is increased. This rapid polarization scrambling is

Figure 3.19: Polarization output of the NOLM with feedback showing the points (left), and the points connected illustrating the chaotic output of the polarization (right)

important if devices using the output of a NOLM with feedback are polarization sensitive.

The NOLM with feedback can be used to generate a chaotic output by inputting a constant CW input of high enough power to keep the NOLM with feedback in the unstable region. An example of an 8km NOLM with $K_1 = 0.25$ and $K_2 = K_3 = 0.8$ and a constant input of 10 W is shown as figure 3.20. With the chaotic power output, there is a chaotic polarization output. However the chaotic polarization output relies on the input polarization angle, and does not cover the entire Poincare sphere. The general behavior of the output polarization is that it lies between bands on the Poincare sphere. The Poincare sphere with circular rings showing polarization bands is shown as figure 3.21. This is generated by taking a linear polarized beam between 0 and 45 degrees (between the S1 and S2 axis) and then retarding the phase of one of the beams incrementally. For a 45 degree beam, this will rotate the polarization from linear polarization (S2) to circular polarization (S3) and back to linear. This, on the S2/S3 plane is traced out as a circle with a diameter from -1 to 1. For polarizations not originating on the S2 axis, circles are traced

Figure 3.20: NOLM with feedback in chaotic region with constant 10 watt input, showing the chaotic output

out as well, but with smaller diameters. This is shown on the Poincare sphere as figure 3.21.

The output for various input linear polarizations is shown on the planes of the Poincare sphere with a 44 degree input angle as figure 3.22, and a 5 degree input angle as figure 3.23. The polarization chaos falls between boundaries of polarization output, rotating around with phase values between the polarizations between zero and 2π , but falling within circular bands. Figure 3.24 compares the 5 degree input polarization on the S2/S3 plane, with two linear beams polarized at 2 and 9 degrees and rotated around the S1 axis. The chaotic polarization output of the 5 degree input is seen to fall between these two regions and nowhere else on the Poincare sphere.

The output polarization states of the NOLM with feedback which is driven well into the chaotic region can produce some interesting patterns as the polarization evolves. Fig-

Figure 3.21: The Poincaré sphere, showing various input polarization angles rotated around the S1 axis

Figure 3.22: Polarization Output of a NOLM with feedback in the chaotic region. Input angle is 44 degrees.

Figure 3.23: Polarization Output of a NOLM with feedback in the chaotic region. Input angle is 5 degrees.

Figure 3.24: The S2/S3 plane comparison of the chaotic output from the 5 degree input polarization, and the elliptical rotation of a 2 and 9 degree input polarization beam. Note the chaotic points fall between these circles.

Figure 3.25: The S2 / S3 plane of the Poincare sphere, with $K1 = 0.15$

ures 3.25, 3.26 and 3.27 show the polarization output of the S2 and S3 plane of the Poincare sphere as the coupler $K1$ is set to 0.15, 0.2 and 0.25, while keeping the other couplers $K2$ and $K3$ at 0.85. The rest of the system is fixed at a length of 80 m and a nonlinear coefficient $\gamma = 3.4 \times 10^{-20} \text{m}^2/\text{W}$ and an effective area for high birefringence fiber of $A_{eff} = 41 \mu\text{m}^2$.

The polarization output of the NOLM with feedback follows similar trends to its power output. The polarization output exhibits bistable and chaotic behavior for a fixed

Figure 3.26: The S_2 / S_3 plane of the Poincaré sphere, with $K_1 = 0.2$

Figure 3.27: The S_2 / S_3 plane of the Poincare sphere, with $K_1 = 0.25$

polarization input, which is important when using the NOLM with feedback with other optical systems. Changes in the output polarization can affect PDL sensitive devices in optical networks, and therefore the analysis of the polarization output of the NOLM with feedback is important when designing the optical resonator for use in optical circuits.

Chapter 4

Low Birefringence

4.1 Introduction

All fibers exhibit some form of birefringent properties, whether they are controlled as in the case of the PM fiber, or natural birefringences induced by stresses that vary randomly. As discussed in section 3.1, the degree of birefringence given by $B_m = |n_x - n_y|$ generally changes over the length of the fiber ($\sim 10\text{m}$) unless specially constructed fibers are used [11]. For a low birefringence fiber, the birefringence is typically on the order of $B_m \sim 10^{-7}$ [38], and the beat length defined as $L_B = \lambda/B_m$ and is of the same order as the length of the fiber. With non polarization maintaining fibers, this value of B_m and L_B change along the fiber length. Previous research has been done with polarization evolution in twisted birefringent fibers [39], and with nonlinear optical loop mirrors using low birefringent twisted fiber in the loop [19–23, 40] in both the linear and nonlinear regimes, however no work has been done with the nonlinear optical loop mirror with feedback. As well, this earlier work neglects the effects of XPM in the loop, by assuming short input pulses or by working in the linear region of the device. With the quasi-CW input of the

NOLM with feedback, it is necessary to include both the SPM and XPM effects in the analysis using low birefringence fiber.

As the beat lengths in a low birefringence fiber are of the order of the fiber length, the cross coupling terms found in 4.1 cannot be neglected. This opens up another parameter space in the analysis of the NOLM with feedback; that of the ratio of the beat length to the fiber length, otherwise known as the normalized fiber length $Z_N = L/L_B$. Previous research on the NOLM has looked at normalized fiber lengths of $Z_N = 0.5$ and 1.5 [20], and this approach is taken here, with normalized fiber lengths close to unity examined in section 4.2.2. One model for a low birefringence fiber in a NOLM is to take into account the randomly varying birefringence over the length of the fiber by approximating the fiber loop with small segments of fiber with random birefringence and orientation [39], however this is beyond the scope of this thesis and is left as future work.

Most of the research of the NOLM with pulse input has been done using twisted fibers [19, 20]. These use propagation equations with the coupled differential equations with circularly polarized modes to enable a circular birefringence to be additionally incorporated to the equations induced by the fiber twist. However as a pulse input is generally used, the propagation equations used are much easier to write in terms of circular polarization and therefore fiber twist can be incorporated. In the case of the NOLM with feedback requiring complicated coupled propagation equations, adding a twist is not a trivial task and is left as future research. At any rate, a direct comparison between the high birefringent PM fiber of chapter 3 and a similar, low birefringent fiber are advantageous. The dynamics of only one additional parameter space of the normalized fiber length Z_N complicates the NOLM with feedback output, and therefore this chapter will

4.2 Nonlinear Optical Loop Mirror with Low Birefringence Fiber

The configuration of the NOLM with feedback is identical to that seen in chapter 3 with high birefringence fiber, however the fiber loop is constructed with a fiber with lower birefringence. The NOLM with feedback is reproduced here as figure 4.1 for clarity. The presence of additional terms in the propagation equations will modify the beams at port 3 and 4 of the coupler as shown in the propagation equations rewritten as shown in 4.1.

$$\begin{aligned}
& i \frac{\delta E_{3x}}{\delta z} + \gamma \left(|E_{3x}|^2 + 2|E_{4x}|^2 + \frac{2}{3}|E_{3y}|^2 + \frac{2}{3}|E_{4y}|^2 \right) E_{3x} = \\
& -(1-P)\chi \left(E_{3y}^2 E_{3x}^* \exp[(-2i\Delta\beta z)] + 2E_{3y}E_{4y}^* E_{4x} \right) - P\chi E_{3y}E_{4x}^* E_{4y} \exp\{-2i\Delta\beta z\} \\
& i \frac{\delta E_{4x}}{\delta z} + \gamma \left(|E_{4x}|^2 + 2|E_{3x}|^2 + \frac{2}{3}|E_{4y}|^2 + \frac{2}{3}|E_{3y}|^2 \right) E_{4x} = \\
& -(1-P)\chi \left(E_{4y}^2 E_{4x}^* \exp[(2i\Delta\beta z)] + 2E_{4y}E_{3y}^* E_{3x} \right) - P\chi E_{4y}E_{3x}^* E_{3y} \exp\{2i\Delta\beta z\} \\
& i \frac{\delta E_{3y}}{\delta z} + \gamma \left(|E_{3y}|^2 + 2|E_{4y}|^2 + \frac{2}{3}|E_{3x}|^2 + \frac{2}{3}|E_{4x}|^2 \right) E_{3y} = \\
& -(1-P)\chi \left(E_{3x}^2 E_{3y}^* \exp[(2i\Delta\beta z)] + 2E_{3x}E_{4x}^* E_{4y} \right) - P\chi E_{3x}E_{4y}^* E_{4x} \exp\{2i\Delta\beta z\} \\
& i \frac{\delta E_{4y}}{\delta z} + \gamma \left(|E_{4y}|^2 + 2|E_{3y}|^2 + \frac{2}{3}|E_{4x}|^2 + \frac{2}{3}|E_{3x}|^2 \right) E_{4y} = \\
& -(1-P)\chi \left(E_{4x}^2 E_{4y}^* \exp[(-2i\Delta\beta z)] + 2E_{4x}E_{3x}^* E_{3y} \right) - P\chi E_{4x}E_{3y}^* E_{3x} \exp\{-2i\Delta\beta z\}
\end{aligned} \tag{4.1}$$

These propagation equations will degenerate into the forms for unpolarized light as used in previous papers [12, 14, 18, 32], when horizontal or vertical input polarizations are used. When using input linear polarization angles other than 0 or 90 degrees, the additional cross coupling effects change the dynamics of the loop as will be seen in the power and polarization output of the low birefringence NOLM with feedback.

Figure 4.2: Bistable region of the NOLM with feedback, with low birefringence fiber. Normalized fiber lengths of 0.5, 1 and 1.5 are simulated.

4.2.2 Output Power Behavior

For a fixed length of 80 m in the fiber loop, the beat length L_B of the fiber will change depending on the choice of normalized fiber lengths used in the simulation, from the relation $Z_N = L/L_B$. The effect of the normalized fiber lengths Z_N of 0.5, 1 and 1.5 ($L_B = 160, 80$, and 40m) on the output is shown as figure 4.2, with an input polarization of 45 degrees which has equal power in both polarization modes. When Z_N is varied, the beat length L_B changes, which affects the birefringence through the relation $L_B = \lambda/B_m$, where B_m was given earlier as the degree of birefringence. This degree of birefringence affects the cross coupling terms in equation 4.1, which gives rise to the different bistable outputs in figure 4.2.

A comparison of the use of low birefringence fiber to that of the high birefringence fiber is shown in figure 4.3. This shows the bistable regions when an input polarization angle of 45 degrees into a NOLM with feedback with both low and high birefringence fiber is used. An input of 0 degree polarization is also included for comparison. For this simulation, coupler values of $K_1 = 0.15$ and $K_2 = K_3 = 0.85$, a normalized fiber length $Z_N = 0.5$, effective fiber core areas $A_{eff} = 87 \mu\text{m}$ and a nonlinear parameter value of $n_2 = 2.5 \times 10^{-20}$ were used for consistency in the comparison, making the low and high birefringence fibers identical except for the degree of birefringence in the fiber, with the low birefringence model having a normalized fiber length Z_N of 0.5. The low birefringence fiber is seen to have a smaller tunability range over the 0 to 45 degree input range.

For a fiber with high birefringence, the output behavior of the NOLM with feedback is symmetrical about the 45 degree input polarization because of the symmetrical behavior found in equations 2.61. This means that the output of the NOLM with feedback would be identical for a beam polarized (for example) 10 degrees from either the x- or y-polarization axis. The addition of the cross coupling terms for a low birefringence fiber makes the phase seen by each modes no longer symmetrical about the 45 degree input angle. Therefore the bistable region can be tuned to the maximum range for an input polarization angle of approximately 63 degrees as simulated in figure 4.4. Continued rotation of the polarization angle to 90 degrees (vertical polarization) will return the system to the same as a 0 degree input, although the system will return to this behavior in a smaller input angle change. Figure 4.4 shows the bistable regions for a 0, 45, and 63 degree input polarization input illustrating that the linear 45 degree input no longer gives

Figure 4.3: Comparison of the bistable regions between low and high birefringent fibers with the same fiber core size and n_2 values.

Figure 4.4: Bistable regions showing the maximum tuning range of the NOLM with feedback using low birefringence fiber

the maximum tuning range.

Changing the polarization to an elliptical state also has an effect on the output of the NOLM with feedback when including the cross coupling terms for a low birefringence fiber. It can be seen by examining the equations in 4.1 that the cross coupling terms no longer depend on the intensity (square of the modulus) of the beam, but that changing the phase between the polarization modes will also affect the output of the NOLM with feedback. Figure 4.5 shows the effect of setting the input linear polarization to the maximum linear tuning range of 63 degrees, and then adding ellipticity to the beam by applying a fixed retardance to the x-component of the beam. A detailed analysis of the phase behavior of the propagation equations 4.1 is required for a particular NOLM with feedback

Figure 4.5: NOLM with feedback output with low birefringence fiber when setting the input linear polarization angle at 63 degrees, and then applying a retardance to the beam.

design to determine the maximum amount of tuning range available, and this is left as future work.

4.2.3 Output Polarization Behavior

The NOLM with feedback with low birefringence has a varying output polarization behavior for a fixed polarization input, similar to the NOLM with feedback using a high birefringence fiber in the loop. For a NOLM with feedback with varying normalized fiber lengths of $Z_N = 0.5, 1$, and 1.5 , the output polarization changes. This can be viewed in an alternate fashion by plotting the power in each of the polarization modes, being $|E_{out_X}|^2$ and $|E_{out_Y}|^2$, where the total power output of the NOLM is the sum of the power in each

mode. Figures 4.6 and 4.7 show the output powers in each polarization mode for an 80m fiber with coupler values of $K_1 = 0.15$ and $K_2 = K_3 = 0.85$, effective fiber core area $A_{eff} = 87 \mu\text{m}^2$ and a nonlinear parameter value of $n_2 = 2.5 \times 10^{-20}$, and a varying normalized fiber length of $Z_N = 0.5, 1$, and 1.5 , and also for a high birefringence fiber with $Z_N \gg 1$.

The NOLM with feedback with low birefringence operating in the output power bistable region will also show a bistability in the polarization output. The trajectory on the Poincare sphere will follow a different polarization path depending on if the quasi-CW pulse is increasing or decreasing in power. Figure 4.8 shows the power output and polarization output of the low birefringence NOLM with feedback.

Increasing the power in the NOLM into the chaotic region becomes difficult for Matlab to solve the coupled differential equations, which is not a problem with the integrated solutions of the high birefringence equations for chapter 3. However it can be shown by increasing the input power of the quasi-CW gaussian pulse, that as the output of the NOLM period doubles towards chaos, the polarization state will oscillate rapidly between states in the output similar to the behavior seen in chapter 3. Figure 4.9 shows the power output of the system as the power is increased into the chaotic region of the NOLM with feedback. Figures 4.10, 4.11, and 4.12 show the output polarization of the low birefringence NOLM with feedback as planes of the poincare sphere. The top pictures in each figure show the points connected, showing the quickly varying polarization state between the trajectories shown in the bottom pictures.

The use of a low birefringence fiber in the NOLM with feedback adds extra parameters to the system, with the additional cross coupling terms in the propagation equations

Figure 4.6: Power in each polarization mode for $Z_n = 0.5$ (top) and $Z_n = 1.0$ (bottom) for an input polarization angle of 45 degrees.

Figure 4.7: Power in each polarization mode for $Z_n = 1.5$ (top), and for a high birefringent fiber (bottom) which has identical powers in both modes with an input polarization angle of 45 degrees.

Figure 4.8: Output showing the bistable power and polarization region of the NOLM with feedback with low birefringence fiber in the loop.

Figure 4.9: Power output of the NOLM with feedback with low birefringence fiber, showing bistability, and period doubling leading to chaos.

Figure 4.10: Output polarization of the NOLM with feedback with low birefringence fiber showing the S_1 / S_2 plane of the poicare sphere. Top picture shows the polarization states connected, and the bottom picture shows the polarization trajectories.

Figure 4.11: Output polarization of the NOLM with feedback with low birefringence fiber showing the $S1 / S3$ plane of the poincare sphere. Top picture shows the polarization states connected, and the bottom picture shows the polarization trajectories.

Figure 4.12: Output polarization of the NOLM with feedback with low birefringence fiber showing the S_2 / S_3 plane of the poicare sphere. Top picture shows the polarization states connected, and the bottom picture shows the polarization trajectories.

relying on the birefringence of the system. These cross coupling terms also mean that other polarization states besides linear ones will cause changes in the dynamical behavior of the NOLM with feedback. With the extra parameter space opened up with a low birefringence fiber, much more detailed analysis and numerical simulations would be needed to explore the dynamic behavior of the system, however this chapter has shown some of the possible outputs that can be realized by using a low birefringent fiber, to try to give some insight into the system behavior.

Chapter 5

Conclusion

This thesis has explored the analysis of both the power output and polarization output of a NOLM with feedback with birefringent fiber in the loop for the first time. Important findings were that the output power behavior of the NOLM with feedback depends on the input polarization angle, which allows for a controllable parameter of the NOLM. As well, it was discovered that the output polarization of the NOLM with feedback will exhibit polarization bistability and chaos, which is important when designing a birefringent NOLM in an optical network, as other devices might be polarization sensitive.

The use of a highly birefringent fiber in the loop offers several advantages over using a low birefringent or randomly birefringent fiber. With the large birefringence value, the propagation equations used to model the system become simplified, and the device can be modelled with one additional parameter space opening up to the design, that of the input polarization angle. This offers a tuning ability not seen in the earlier NOLM with feedback research which did not take polarization into account. The simplified propagation equations allow for easier modelling and more control over the design space of the device, which is not the case when using a low birefringence fiber in the loop. The work with the

low birefringence fiber shows that the cross coupling terms in the propagation equations also have an effect on the output of the device. This adds additional parameter space to the design such as the fiber beat length, which is related to the birefringence of the fiber. As well, the simulation of this system is more difficult to perform, as the NOLM behavior in the chaotic region is quite computationally intensive.

In the practical situation of construction of a NOLM with feedback, the fibers used are susceptible to stresses and thermal variations, and therefore fibers of a low birefringence value tend to be random in nature, and this is a hard feature to control. Techniques such as measurement of the beat length of a fiber [40] can be used to help select the appropriate fiber for the design, however there is no guarantee that additional stresses or bends will not change this value. The use of a highly birefringent PM fiber in the loop, with a birefringence value well above that of low or randomly birefringent fiber tends to not see the random birefringence-inducing stresses, and offers a constant birefringence throughout, making the practical construction of the device potentially much simpler along with the modelling of the device.

Future work would be to more completely analyze the NOLM with feedback with birefringent fiber in the loop, by performing a linear stability analysis of the system. The high birefringence propagation equations, although more complex than the unpolarized forms, can still be examined using a linear stability analysis because the nonlinear phases seen by the modes can be expressed analytically similar to the work done in [18]. Applying a fiber twist to the high birefringence fiber might not be practical as it is generally a stress induced fiber and circular birefringence will most likely not have an impact, but a more complete analysis of a twisted low birefringence NOLM with feedback would be a

good addition to previous NOLM research in the field. Finally, the ideal future work with the NOLM with feedback would be to obtain good experimental results, the lack of which being the motivation for this thesis.

Appendix A

Matlab Code

```

%*****
% NOLM with Feedback - Master Simulation File          Clark Merchant
%                                                    Jan 1, 2004
%
% This file is the core simulation file that is used to model the different
% polarization forms of the NOLM with feedback. It has been built from the
% many separate Matlab models previously written. The file is designed to
% model the different forms of NOLM with feedback by simply changing the
% parameters below.

close all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% FILE PARAMETERS:
%
% The only parameters one should want to set

FiberTypeFlag = 'LoBi'          % HiBiInt (use integrated prop eqns), HiBi, LoBi, NoBi
FeedbackFlag = true;            % apply feedback true/false
PulseStyleFlag = 'gaussian';    % gaussian, linear
PulseMinPower = sqrt(120);      % applies to linear ramp only
PulseMaxPower = sqrt(120);      % E-field amplitude = sqrt(power)
PDL1 = 0.0;                    % PDL in dB for straight-thru coupler (K)
PDL2 = 0.0;                    % PDL for cross-over part of coupler i(1-K)
PDLK2 = 0.0;
PDLK3 = 0.0;

PolAngle = 45;                 % Input Polarization Angle
WPRetardance = 0;              % waveplate retardance (to input elliptical beams)

K1 = 0.25;                      % coupler splitting ratios;
K2 = 0.85;
K3 = 0.85;

L = 80;                         % Fiber Length
lambda = 1.55*10^-6;           % 1550 nm
c = 299792458;                 % speed of light

PDL1 = 10^(-PDL1/10);
PDL2 = 10^(-PDL2/10);
PDLK2 = 10^(-PDLK2/10);
PDLK3 = 10^(-PDLK3/10);

if strcmp(FiberTypeFlag, 'NoBi') % Behaves like the unpolarized Steele et al. formulas
    n2=3.2*10^-20;
    Aeff=3*10^-11;
    PolAngle = 0;
    WPRetardance = 0;

elseif strcmp(FiberTypeFlag, 'HiBiInt') | strcmp(FiberTypeFlag, 'HiBi')
    % Use the n2/Aeff that's calculated from a hi-bi fibre's n2 measurement
    % from J. Opt. A: Pure Appl. Opt. 2 (2000) 314-318.
    n2=3.4*10^-20;
    Aeff=4.1*10^-11;
    Aeff = 8.7*10^-11;          % Fiber Effective Area (rough calc)
    n2 = 2.5*10^-20;

elseif strcmp(FiberTypeFlag, 'LoBi')
    Aeff = 8.7*10^-11;          % Fiber Effective Area (rough calc)
    n2 = 2.5*10^-20;           % Standard value for n2
    n = 1.4682;                % Corning SMF-28 @1550
    dn=10^-7;                  % birefringence - Taken from Optics Letters vol 20 no. 24 (Dec
15, 1995) Wai, Menyuk
    Lb = lambda / dn;           % beat length calc
    Zn = L / Lb;               % Normalized fiber length

    % here solve for dn if we want to choose our own normalized fiber
    % length.
    Zn = 0.5
    Lb = L / Zn
    dn = lambda / Lb
    % End Recalc Here.

delta_beta = 2*pi/Lb;          %Calculation of delta-beta for our LoBi CDE function

```

```

end

% Calculate our gamma here based on our fiber type
omega = 2*pi*c/lambda;
gamma = n2*omega/(c*Aeff);
% Set up our fiberlength array - for the propagation DEs
fiberlength=[0 L]; % 0 to XXm in length

% Defining our input power

if strcmp(PulseStyleFlag, 'linear')
    EInAmplitude = linspace(PulseMinPower, PulseMaxPower, 1000);
elseif strcmp(PulseStyleFlag, 'gaussian')
    EInAmplitude = linspace(0, 1000, 2000);
    EInAmplitude=PulseMaxPower*exp(-(EInAmplitude./200)-2.5).^2);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Start the Simulation Loop here

close all;
stdout = 1;
fprintf(stdout,'\n\nNOLM with or Without Feedback - Master File\n');
fprintf(stdout,'\nBy Clark Merchant - Revised Feb 2, 2004\n');
fprintf(stdout,'Filename: NOLM_Master_Simulation_File.m\n');

%Allocate space for our output matrices
EOut = zeros(2, length(EInAmplitude));
PowerOuts = zeros(1, length(EInAmplitude));

%Zero our output and our isolator feedback before we start
E6 = [0 0];
E5 = [0 0];

WP = [exp(i*WPRetardance) 0; 0 exp(-i*WPRetardance)];
for idx=1:length(EInAmplitude)

    theta =PolAngle*pi/180; %Angle in radians of the polarization

    %Set our input "AInput" to come in at a particular
    % polarization angle wrt the fiber.
    Ein = [EInAmplitude(idx)*cos(theta) EInAmplitude(idx)*sin(theta)];
    % Add in any waveplate retardance for ellipticity
    Ein = transpose(WP*transpose(Ein));

    if FeedbackFlag == true
        %E1 is the combination of E6 (Isolator feedback) + Ein
        E1 = [(K3^0.5)*Ein(1)+(i*(1-K3)^0.5)*E6(1)
        (K3^0.5)*Ein(2)+(sqrt(PDLK3)*(i*(1-K3)^0.5)*E6(2))];
    else
        E1 = Ein;
    end

    E3 = [(K1^0.5)*E1(1) sqrt(PDL1)*(K1^0.5)*E1(2)];
    E4 = [(i*(1-K1)^0.5)*E1(1) sqrt(PDL2)*(i*(1-K1)^0.5)*E1(2)] ;

    % The initial conditions for the Integrated DE function. This is done
    % this way to be similar to the non-integrated coupled DEs, just so
    % we can drop-in either function.
    %[E1x E2x E1y E2y]
    CDEInitial = [E3(1) E4(1) E3(2) E4(2)];

    if strcmp(FiberTypeFlag,'HiBiInt') | strcmp(FiberTypeFlag, 'NoBi')

```

```

% For No Birefringence, we set the pol angle to zero, and the
% integrated propagation equations degenerate into the forms
% used for the unpolarized light from Steele et al.
w3 = CDE_Pol_HiBi_Int(fiberlength, CDEInitial, gamma, gamma, 0, 0);

% Here we have propagated around the loop, and we now need to combine
% the outputs through the coupler, to get our net output.
%Note = E3' is equal to E3_ for the notation
E4=[w3(2) w3(4)];
E3_=[w3(1) w3(3)];
elseif strcmp(FiberTypeFlag,'HiBi')

OPTIONS = [];
OPTIONS = odeset('Refine',6,'RelTol',1e-4,'AbsTol',1e-7);
[z3 w3] = ode45(@CDE_Pol_HiBi, fiberlength, CDEInitial, OPTIONS, gamma, gamma,
0, 0);

% Here we have propagated around the loop, and we now need to combine
% the outputs through the coupler, to get our net output.
%Note = E3' is equal to E3_ for the notation
E4=[w3(length(w3),2) w3(length(w3),4)];
E3_=[w3(length(w3),1) w3(length(w3),3)];

elseif strcmp(FiberTypeFlag,'LoBi')

OPTIONS = [];
OPTIONS = odeset('Refine',1,'RelTol',1e-8,'AbsTol',1e-10);
[z3 w3] = ode45(@CDE_Pol_LoBi, fiberlength, CDEInitial, OPTIONS, gamma,
delta_beta);

% Here we have propagated around the loop, and we now need to combine
% the outputs through the coupler, to get our net output.
%Note = E3' is equal to E3_ for the notation
E4=[w3(length(w3),2) w3(length(w3),4)];
E3_=[w3(length(w3),1) w3(length(w3),3)];

end

if FeedbackFlag == true

E5 = [(K1^0.5)* E3_(1) + (i*(1-K1)^0.5)* E4_(1)) (sqrt(PDL1)*(K1^0.5)* E3_(2)
+ (sqrt(PDL2)*(i*(1-K1)^0.5)* E4_(2)))];
Eout = [(K2^0.5)*E5(1) K2^0.5*E5(2)]; % Only one input to E5
E6 = [(i*(1-K2)^0.5)*E5(1) sqrt(PDLK2)*(i*(1-K2)^0.5)*E5(2)];

else
Eout = [(K1^0.5)* E3_(1) + (i*(1-K1)^0.5)* E4_(1)) (sqrt(PDL1)*(K1^0.5)*
E3_(2) + (sqrt(PDL2)*(i*(1-K1)^0.5)* E4_(2)))];
end

EOut(1,idx) = Eout(1);
EOut(2, idx) = Eout(2);

end

PowerOuts = abs(EOut(1,:)).^2 + abs(EOut(2,:)).^2;
Pin = abs(EInAmplitude).^2;

PinFirstHalf = zeros(length(EInAmplitude)/2+1);
POutFirstHalf = zeros(length(EInAmplitude)/2+1);
PinSecHalf = zeros(length(EInAmplitude)/2+1);
POutSecHalf = zeros(length(EInAmplitude)/2+1);

for index=1:length(EInAmplitude)/2
PinFirstHalf(index)=abs(EInAmplitude(index)).^2;
POutFirstHalf(index)=PowerOuts(index);
end

for index=(length(EInAmplitude)/2):length(EInAmplitude)
PinSecHalf(index)=abs(EInAmplitude(index)).^2;
POutSecHalf(index)=PowerOuts(index);
end

```

```

figure(2);
plot (PInFirstHalf, POutFirstHalf, 'r.', PInSecHalf, POutSecHalf, 'b. ');
title('Output Power vs Input Power');
xlabel('Input Power (W)');
ylabel('Output Power (W)');
axis square;

tilefigs([2 3], 5);
pause;

close all;
Ap = abs(EOut(1,:)).^2 - abs(EOut(2,:)).^2;
Bp = 2*real(conj(EOut(1,:)).*EOut(2,:));
Cp = 2*imag(conj(EOut(1,:)).*EOut(2,:));

A = Ap; B=Bp; C=Cp;

ABCNorm = sqrt(abs(Ap).^2+abs(Bp).^2+abs(Cp).^2);
A = Ap./ABCNorm;
B = Bp./ABCNorm;
C = Cp./ABCNorm;

save ('c:\A.txt', 'A', '-ascii');
save ('c:\B.txt', 'B', '-ascii');
save ('c:\C.txt', 'C', '-ascii');

%figure(4);

% title('Output Stokes Vectors (Normalized)');
% %plot(A,B,'r+',B,C,'go',A,C,'b*');
% plot(A,B,'k+',B,C,'k:',A,C,'kx');
% legend('A vs B plane','B vs C plane','A vs C plane');
% axis square;
% axis([-1 1 -1 1]);

figure(1);
plot(A,B,'k:');
title('S2 vs S1 Plane');
xlabel('S1');
ylabel('S2');
axis square;
axis([-1 1 -1 1]);

figure(2);
plot(B,C,'k:');
title('S3 vs S2 Plane');
xlabel('S2');
ylabel('S3');
axis square;
axis([-1 1 -1 1]);

figure(3);
plot(A,C,'k:');
title('S3 vs S1 Plane');
xlabel('S1');
ylabel('S3');
axis square;
axis([-1 1 -1 1]);

A1=A(1:length(A)/2);
A2=A(((length(A)/2)+1):length(A));

B1=B(1:length(B)/2);
B2=B(((length(B)/2)+1):length(B));

C1=C(1:length(C)/2);
C2=C(((length(C)/2)+1):length(C));

figure(4);

```

```

%plot(A,B,'r+',B,C,'go',A,C,'b*');
plot(A1,B1,'r.',A2,B2,'b.');
```

```

title('S2 vs S1 Plane');
xlabel('S1');
ylabel('S2');

%legend('A vs B increasing','A vs B decreasing');
axis square;
axis([-1 1 -1 1]);

figure(5);

%plot(A,B,'r+',B,C,'go',A,C,'b*');
plot(A1,C1,'r.',A2,C2,'b.');
```

```

title('S3 vs S1 Plane');
xlabel('S1');
ylabel('S3');

%legend('A vs C increasing','A vs C decreasing');
axis square;
axis([-1 1 -1 1]);

figure(6);

%plot(A,B,'r+',B,C,'go',A,C,'b*');
plot(B1,C1,'r.',B2,C2,'b.');
```

```

title('S3 vs S2 Plane');
xlabel('S2');
ylabel('S3');

%legend('B vs C increasing','B vs C decreasing');
axis square;
axis([-1 1 -1 1]);

tilefigs([2 3], 5);
pause;
close all;
```

```

%*****
% CDE_Pol_HiBi_Int.m
%
% This function contains the coupled integrated equations (four
% equations) which calculates the SPM and XPM of two beams propagating in a
% fiber. These formulas are integrated from the formulas commented out
% below. We ignore the attenuation for now.

% Formulas taken directly from Applications in Nonlinear Fiber Optics, by
% G.P. Agrawal, chapter 7, pg 265 (7.1.19 and 7.1.20)

function f=CDE_Pol_HiBi_Int(z, w, gamma1, gamma2, alpha1, alpha2)

Alx=w(1);
Aly=w(3);
A2x=w(2);
A2y=w(4);
L=z(length(z));

% [ Alx - Beam 1 X-Polarization ;
%   A2x - Beam 2 X Polarization ;
%   Aly - Beam 1 Y-Polarization ;
%   A2y - Beam 2 Y-Polarization ];

% f=[ - alpha1/2*Alx + i*gamma1*(abs(Alx)^2 + 2*abs(A2x)^2 + (2/3)*abs(Aly)^2 +
(2/3)*abs(A2y)^2)*Alx ;
%     - alpha2/2*A2x + i*gamma2*(abs(A2x)^2 + 2*abs(Alx)^2 + (2/3)*abs(Aly)^2 +
(2/3)*abs(A2y)^2)*A2x ;
%     - alpha1/2*Aly + i*gamma1*(abs(Aly)^2 + 2*abs(A2y)^2 + (2/3)*abs(Alx)^2 +
(2/3)*abs(A2x)^2)*Aly ;
%     - alpha1/2*A2y + i*gamma2*(abs(A2y)^2 + 2*abs(Aly)^2 + (2/3)*abs(Alx)^2 +
(2/3)*abs(A2x)^2)*A2y ];

% Returns a set of four single results here (at the end of the fiber), as opposed to the
% above solves
% throughout each z-position of the fiber

f= [ Alx*exp(i*gamma1*L*(abs(Alx)^2 + 2*abs(A2x)^2 + (2/3)*abs(Aly)^2 +
(2/3)*abs(A2y)^2));
    A2x*exp(i*gamma2*L*(abs(A2x)^2 + 2*abs(Alx)^2 + (2/3)*abs(Aly)^2 +
(2/3)*abs(A2y)^2));
    Aly*exp(i*gamma1*L*(abs(Aly)^2 + 2*abs(A2y)^2 + (2/3)*abs(Alx)^2 +
(2/3)*abs(A2x)^2));
    A2y*exp(i*gamma2*L*(abs(A2y)^2 + 2*abs(Aly)^2 + (2/3)*abs(Alx)^2 +
(2/3)*abs(A2x)^2))];

```



```

%*****
% CDE_Pol_LoBi.m
%
% This function contains the coupled differential equations (four
% equations) which calculates the SPM and XPM of two beams propagating in a
% fiber. These coupled differential equations include the polarization
% states of each beam (compared to earlier scalar CDEs used here). Time-derivative
% amplitude variance is also neglected (since we are quasi-CW and slowly varying)
%
% Formulas taken from Kmar et al., OSA B, Vol 11, No. 5, May 1994 p 80
%
% Note :We don't use an alpha (attenuation) in this formula, although it
% could be included as in the high birefringence case. Also, we assume the
% same gamma (nonlinear coeff) since our wavelengths are the same.

function f=CDE_Pol_LoBi(z, w, gamma, delta_beta)

Elx=w(1);
Ely=w(3);
E2x=w(2);
E2y=w(4);

% [ Elx - Beam 1 X-Polarization ;
%   E2x - Beam 2 X Polarization ;
%   Ely - Beam 1 Y-Polarization ;
%   E2y - Beam 2 Y-Polarization ];

f=[ i*gamma*(abs(Elx)^2 + 2*abs(E2x)^2 + (2/3)*abs(Ely)^2 + (2/3)*abs(E2y)^2)*Elx +
i*(1/3)*gamma*((Ely^2)*conjElx)*exp(-2*i*delta_beta*z) + 2*Ely*conjE2y)*E2x) +
i*(2/3)*gamma*Ely*conjE2x)*conjE2y)*exp(-2*i*delta_beta*z) ;
i*gamma*(abs(E2x)^2 + 2*abs(Elx)^2 + (2/3)*abs(E2y)^2 + (2/3)*abs(Ely)^2)*E2x +
i*(1/3)*gamma*((E2y^2)*conjE2x)*exp(2*i*delta_beta*z) + 2*E2y*conjEly)*Elx) +
i*(2/3)*gamma*E2y*conjElx)*Ely*exp(2*i*delta_beta*z) ;
i*gamma*(abs(Ely)^2 + 2*abs(E2y)^2 + (2/3)*abs(Elx)^2 + (2/3)*abs(E2x)^2)*Ely +
i*(1/3)*gamma*((Elx^2)*conjEly)*exp(2*i*delta_beta*z) + 2*Elx*conjE2x)*E2y) +
i*(2/3)*gamma*Elx*conjE2y)*E2x*exp(2*i*delta_beta*z) ;
i*gamma*(abs(E2y)^2 + 2*abs(Ely)^2 + (2/3)*abs(E2x)^2 + (2/3)*abs(Elx)^2)*E2y +
i*(1/3)*gamma*((E2x^2)*conjE2y)*exp(-2*i*delta_beta*z) + 2*E2x*conjElx)*Ely) +
i*(2/3)*gamma*E2x*conjEly)*Elx*exp(-2*i*delta_beta*z) ];

```

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