**Design and Phase Noise Analysis of
CMOS Differential and Quadrature VCOs**

by

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of the requirements for the degree of

Master of Applied Science

In

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Submitted by Sundus Kubba, B.Eng. in partial fulfillment of the
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Thesis supervisor

Chair, Department of Electronics

Carleton University
June, 2002

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Abstract

The design of monolithic oscillators in standard CMOS process has been the subject of several recent papers, especially after the Local Area Networks technologies, namely Bluetooth gained popularity.

In this research two circuits were built. The first is a 2.25GHz differential voltage-controlled oscillator, designed and built in 0.18 μ CMOS technology. The core circuit consumes 1.8mW of power. Its phase noise is -93dBc/Hz @100KHz offset from a 2.25 GHz carrier frequency. The second circuit is a 2.4GHz quadrature differential voltage-controlled oscillator, designed and built in 0.18 μ CMOS technology. The second circuit was built to provide signals 90° apart that are used in receivers, along with other circuits, for image rejection.

In addition, a study of how phase noise appears around the carrier signal was performed. The sources of noise were broken down and analyzed separately and then added together. These analyses can be used as a tool to predict phase noise. The calculated values were compared with measured results and they agreed to within a few dB.

Table Of Contents

Acknowledgements	iii
Abstract	iv
List of Figures	ix
List of Abbreviations	xi
Chapter 1 Introduction	1
1.1 Motivation	1
1.2 Thesis Overview	1
1.3 Thesis Outline	2
Chapter 2 Transceivers	3
2.1 Frequency Stages in the Receiver Side	3
2.2 The Image Frequency Problem	4
2.3 Receiver Topologies	5
2.3.1 Heterodyne Receiver	5
2.3.2 Double Conversion Receiver	6
2.3.3 Homodyne Receiver	7
2.3.4 Image Reject Receiver	8
2.4 Future Trends	11
Chapter 3 Oscillator Circuit Background	12
3.1 Oscillator Circuit Overview	12
3.2 Performance parameters	12
3.3 Operation of LC Oscillators	13
3.4 LC Oscillator Circuit Topologies	14
3.4.1 Colpitts and Hartley oscillators	14
3.4.2 Negative-gm Oscillator	15
3.4.3 Complementary Negative-gm oscillators	20

3.5	Phase Noise in LC Oscillators.....	23
3.5.1	Noise Sources	23
3.5.1.1	Thermal (White) Noise	23
3.5.1.2	Flicker Noise.....	24
3.5.2	Phase Noise Mechanisms	24
3.5.2.1	Q of the Tank.....	25
3.5.2.2	Amplitude Modulated Noise	28
3.5.2.3	Frequency Modulated Noise.....	32
3.5.2.4	Oscillation Amplitude vs. Phase Noise	35
3.5.3	Calculation of Combined Phase Noise	35
3.6	Quadrature VCO Circuit.....	37
3.6.1	Motivation	37
3.6.2	Methods of Generating Quadrature	37
3.6.2.1	RC phase-shift circuit	37
3.6.2.2	RC polyphase Network.....	38
3.6.2.3	Dividing the Double Frequency	39
3.6.2.4	The Quadrature Oscillator	40
3.6.3	Effect of mismatch.....	42
3.6.4	Phase Noise Analysis.....	45
3.7		46
3.8	Previous Work	47
3.8.1	Single Differential VCO	47
3.8.1.1	Completely Monolithic Oscillators	48
3.8.1.2	With Bond Wire Inductors	49
3.8.2	Quadrature Differential VCO	50
Chapter 4	Design and Layout.....	52
4.1	0.18μ CMOS Technology	52
4.2	Building the VCO Circuit	52
4.2.1	Tank circuit.....	52
4.2.1.1	Inductor.....	52
4.2.1.1.1	Modeling and Simulation	52
4.2.1.1.2	Layout and Optimization	53
4.2.1.1.3	Inductor Values.....	55
4.2.1.2	Varactor	57
4.2.1.2.1	Modeling and Simulation	58
4.2.1.2.2	Varactor Values	58
4.2.1.2.3	Placement and Layout	58
4.2.2	Amplifier circuit	59
4.2.3	Biassing.....	61
4.2.4	Buffers	61
4.2.5	Differential VCO Layout.....	62

4.3	Phase Noise Calculations of the Single Differential Oscillator	63
4.3.1	Q of the tank	63
4.3.2	Low Frequency Flicker Noise	63
4.3.3	Amplitude Modulated Flicker Noise	64
4.3.4	Frequency Modulated Flicker Noise	65
4.3.5	Total Combined Phase Noise	68
4.4	Building the QVCO Circuit	69
4.4.1	Tank Circuit	69
4.4.2	Amplifier Circuit	70
4.4.3	Biasing	70
4.4.4	Buffers	70
4.4.5	Quadrature VCO layout	70
4.5	Phase Noise Calculations of the Quadrature Differential Oscillator	71
4.5.1	Q of the tank	71
4.5.2	Low Frequency Flicker Noise	72
4.5.3	AM Modulated Flicker Noise	72
4.5.4	FM Modulated Flicker Noise	73
4.5.5	Total Combined Phase Noise	73
4.6	Final Chip Layout	75
Chapter 5	Testing and Results	76
5.1	Test Setup	76
5.2	VCO Results	77
5.2.1	Tuning Range	77
5.2.2	Phase Noise	78
5.2.3	Comparison between Calculated and Measured Phase Noise Results	82
5.2.3.1	Phase Noise at a 100kHz offset	82
5.2.3.2	Phase Noise versus Offset Frequency from the Carrier	82
5.3	QVCO Test Results	83
5.3.1	Tuning Range	83
5.3.2	Phase Noise	84
5.3.3	Comparison between Calculated and Measured Phase Noise Results	87
5.3.3.1	Phase Noise at a 100kHz offset	87
5.3.3.2	Phase Noise versus Offset Frequency from the Carrier	87
5.3.4	Phase Shift	88
5.4	Summary	92
Chapter 6	Conclusion	94
6.1	Summary of Work in this Thesis	94

6.2	Thesis Contributions	95
6.3	Future Work	96
	References.....	97
	Appendices	100
	Amplitude Modulated Flicker Noise	100
	Frequency Modulated Flicker Noise	101
	Leeson's Noise	102
	Combined Phase Noise of Differential Oscillator.....	103
	Quadrature Amplitude Modulated Noise	104
	Quadrature Frequency Modulated Noise	105
	Quadrature Lesson Noise	106
	Combined Phase Noise of the Quadrature Differential Oscillator.....	107

List of Figures

Figure 2.1 The image frequency problem	4
Figure 2.2 Heterodyne receivers.....	5
Figure 2.3 Double conversion receivers	7
Figure 2.4 Direct conversion receivers.....	8
Figure 2.5 Hartley image reject receivers.....	9
Figure 2.6 Weaver image reject receivers	10
Figure 3.1 Feedback model	13
Figure 3.2 Colpitts and Hartley oscillators.....	15
Figure 3.3 Negative gm oscillator	16
Figure 3.4 Small signal model of the negative gm oscillator	16
Figure 3.5 Transistor loading to the tank in the negative gm oscillators.....	17
Figure 3.6 Single-ended negative gm oscillators.....	18
Figure 3.7 Possible configurations of the negative gm oscillator.....	20
Figure 3.8 Complementary negative gm oscillators.....	22
Figure 3.9 The current mirror	22
Figure 3.10 Single-ended models with noise current source.....	26
Figure 3.11 Phase noise according to equation 33	28
Figure 3.12 Amplitude modulation of noise with the switching transistors.....	29
Figure 3.13 Representing the oscillator as a mixer	30
Figure 3.14 Amplitude-modulated flicker noise based on equation 38.....	31
Figure 3.15 Spectrum of a frequency-modulated signal.....	32
Figure 3.16 Frequency-modulated flicker noise based on equation 43 and 45	35
Figure 3.17 Combined phase noise calculation	36
Figure 3.18 RC phases –shift circuit	38
Figure 3.19 Obtaining Q and I from the RC phase shifter	38
Figure 3.20 RC poly-phase networks	39
Figure 3.21 Dividing the double frequency.....	40
Figure 3.22 Direct coupling in the quadrature VCO	41
Figure 3.23 Cross coupling in the quadrature VCO	41
Figure 3.24 Direct and cross coupling of the transistors in the quadrature VCO.....	42
Figure 3.25 Model for calculating the effect of amplitude and phase mismatch	42
Figure 3.26 Image rejection vs. phase error and amplitude mismatch (dB).....	44
Figure 3.27 Image rejection vs. phase error and percentage amplitude mismatch.....	45
Figure 3.28 Calculated frequency-modulated flicker noise of the quadrature oscillator ..	46
Figure 3.29 Calculated combined phase noise of the quadrature oscillator	46
Figure 4.1 Inductor model	53
Figure 4.2 Model of two inductors connected in series.....	54
Figure 4.3 Symmetrical square inductor layout.....	55
Figure 4.4 Inductor model with parameter values	56
Figure 4.5 Inductance and Q values of the inductor versus frequency.....	56
Figure 4.6 Cross section of a pn –junction varactor	57
Figure 4.7 Cross section of a MOS varactor	58

Figure 4.8 Varactors placement in the tank circuit.....	59
Figure 4.9 Transistor layout.....	60
Figure 4.10 Buffer circuit	61
Figure 4.11 Single differential VCO layout	62
Figure 4.12 Flicker noise in dBm versus frequency for an NMOS transistor	64
Figure 4.13 Mixer representation of the VCO.....	65
Figure 4.14 different mechanism contributions to phase noise in the single oscillator.....	67
Figure 4.15 Combined phase noise in dBc /Hz versus frequency offset from the carrier	68
Figure 4.16 Combined phase noise in the differential VCO figure showing the limiting thermal noise floor.....	69
Figure 4.17 Quadrature differential VCO layout.....	71
Figure 4.18 Flicker noise in dBm versus frequency for an NMOS transistor	72
Figure 4.19 Different mechanism contribution to phase noise in the quadrature oscillator	73
Figure 4.20 Combined phase noise in dBc /Hz versus frequency offset from the carrier	74
Figure 4.21 Combined phase noise in the quadrature VCO figure showing the limiting thermal noise floor.....	74
Figure 4.22 Layout of the two circuits and placement of bond pads.....	75
Figure 5.1 Single differential VCO test setup	76
Figure 5.2 Output of the signal VCO at 2.3GHz	77
Figure 5.3 Measured frequency vs. control voltage for the single VCO.....	78
Figure 5.4 Calculated K_{VCO} from measurements of the single VCO	78
Figure 5.5 Phase noise of the single differential VCO at 100KHz offset from a 2.25 GHz carrier signal	79
Figure 5.6 Phase noise of the single differential VCO at 100KHz offset from a 2.25 GHz carrier signal	80
Figure 5.7 Phase noise of the single VCO across the tuning range.....	80
Figure 5.8 Measured change in center frequency vs. change in bias current	81
Figure 5.9 Measured rate of change in center frequency vs. changes in bias current	81
Figure 5.10 Measured and calculated combined phase noise.....	82
Figure 5.11 Measured frequency vs. control voltage for the quadrature VCO	83
Figure 5.12 Calculated K_{VCO} from measurements of the quadrature VCO.....	84
Figure 5.13 Phase noise of the quadrature differential VCO at 100KHz offset form a 2.57 GHz.....	85
Figure 5.14 Phase noise of the quadrature across the tuning range.....	85
Figure 5.15 Measured change in center frequency vs. change in bias current	86
Figure 5.16 Measured rate of shift in center frequency vs. change in bias current	86
Figure 5.17 Measured and calculated combined phase noise.....	87
Figure 5.18 Image rejection test configuration with down-conversion technique	88
Figure 5.19 Image rejection test configuration with up-conversion technique	89
Figure 5.20 Test setup for image rejection measurement.....	89
Figure 5.21 Image rejection result 1	91
Figure 5.22 Image rejection result 2	91

List of Abbreviations

AC	Alternating Current
AM	Amplitude Modulation
B	Frequency Bandwidth
BPF	Band Pass Filter
CMOS	Complementary-channel Metal Oxide Semiconductor
DC	Direct Current
FM	Frequency Modulation
F	Noise factor of an amplifier
GSM	Global System for Mobile Communication
g_p	Equivalent parallel conductance in LC tank
g_m	Transistors' transconductance
IF	Intermediate Frequency
IRF	Image Reject Filter
I_n	Noise current
I_b	Bias current
K_{VCO}	Change in Frequency corresponding to change in voltage Hz/V
$K_{VCO, Ib}$	Change in Frequency corresponding to change in current Hz/A
LO	Local Oscillator
LNA	Low Noise Amplifier
MOSFET	Metal Oxide Silicon Field Effect Transistor
NMOS	N-channel Metal Oxide Semiconductor
PMOS	P-channel Metal Oxide Semiconductor
Q	Quality Factor
QVCO	Quadrature Voltage-Controlled Oscillator
QPSK	Quadrature Phase Shift Keying
RF	Radio Frequency
R_p, r_p	Equivalent parallel resistance in LC tank
S_{nv}	Voltage noise spectral density
S_{ni}	Current noise spectral density
VCO	Voltage-Controlled Oscillator
V_n	Noise voltage
V_{gs}	Gate source voltage
V_t	Threshold voltage
W/L	Ratio of the gate width to gate length of a transistor
β	Frequency modulation index
Δf	Change/shift in frequency
f_c	Carrier frequency
f_m	Modulating frequency
k	Boltzmann's constant
$\mathcal{L}$	Phase noise in dBc/Hz
ω_o	Carrier frequency
$\Delta\omega$	Offset frequency from the carrier
ϕ_e	Phase error

Chapter 1 Introduction

1.1 Motivation

The design of monolithic oscillators in standard CMOS process has been the subject of several recent papers, especially after the Local Area Networks technologies, namely Bluetooth gained popularity. Circuits designed for this standard are required to be relatively inexpensive and low-power with somewhat relaxed phase noise and output power level requirements. As a result, CMOS technology is a viable choice for monolithic Bluetooth RF circuits.

Bluetooth receivers designed, with a low IF topology, require image reject receivers. These receivers need quadrature signals and one way to provide such signals is with a quadrature voltage-controlled oscillator.

1.2 Thesis Overview

In this research two circuits were built. The first is a 2.25GHz differential voltage-controlled oscillator, designed and built in 0.18 μ CMOS technology. The core circuit consumes 1.8mW of power. Its phase noise is -93dBc/Hz @100KHz offset from a 2.25 GHz carrier frequency. The tuning range of this VCO is between 1.65GHz to 2.35 GHz, which corresponds to 31% tuning. This single differential oscillator is the basis of the design of the quadrature VCO.

The second circuit is a 2.4GHz quadrature differential voltage-controlled oscillator, designed and built in 0.18 μ CMOS technology. The power consumption of this VCO is 5.4mW for the core circuit, with phase noise of -76dBc/Hz @100KHz offset from a 2.4 GHz carrier frequency. The tuning range of this VCO is from 2.2GHz to 2.55 GHz, which corresponds to 15% tuning.

In addition, a study of how phase noise appears around the carrier was performed. The sources of noise were broken down and analyzed separately. Phase noise values for each of these sources were calculated. For “Leeson’s noise”, the amount of loss in the

tank was estimated and used to calculate the noise value around the carrier. Flicker noise values were estimated from the spice model of the transistors and used to calculate the frequency and amplitude modulated noise. These calculated values were compared with measured results and they agreed fairly well.

1.3 Thesis Outline

Chapter 2 is a detailed look into the receiver front end and different topologies commonly used in its realization. Chapter 3 contains background theory for oscillators, their operation, circuit topologies and phase noise. The quadrature generation of signals using quadrature oscillators is also studied in this chapter. In Chapter 4, details of the implementation of the two circuits, the single differential oscillator and the quadrature differential oscillator in 0.18 μ CMOS technology, are presented. Chapter 5 contains the test results of both of the fabricated circuits. In Chapter 6 analysis of phase noise performance of the two oscillators are presented and compared to measurement results.

Chapter 2 Transceivers

A transceiver is a transmitter-receiver system. It is the front-end of the radio that provides the interface between the antenna and base-band.

The main function of the transmitter is to modulate the base band signal and send it on a carrier at a specified frequency within the spectrum. The transmitted signal has to follow specific regulations regarding its power, bandwidth and harmonics. These regulations correspond to different wireless communication standards that are regulated by the FCC (Federal Communication Commission).

The receiver, in turn, chooses the signal from the spectrum, filters, amplifies and mixes it down to base band, where signal processing takes place.

An oscillator is a device that provides the signal with which the mixer converts the signal from RF to baseband in the receiver, and from baseband to RF in the transmitter. In order for the transceiver to function properly, the local oscillator has to provide a “relatively” clean, large and stable signal at a specified center frequency.

For the purpose of understanding receiver architectures, frequency stages and the image frequency concept are first introduced.

2.1 Frequency Stages in the Receiver Side

The high frequency modulated message is referred to as the Radio Frequency (RF) signal. RF refers to the range of frequencies described by the commercial wireless communication standards for transmitting their signals.

In order to demodulate the received signal, the receiver has to convert the high frequency RF signal into its original baseband form. Signal processing is done in this baseband frequency. The translation of signal from RF to baseband is called down conversion. Down-conversion could be done in one step directly from RF to baseband, or in more than one step, where the RF is first converted to (single or multiple) Intermediate Frequencies (IF) and then to baseband.

Since the RF signal usually consists of several channels, the receiver has to select the wanted channel and filter out the rest. Channel selection is done using a frequency synthesizer, which provides the correct LO signal to get the correct channel, either in the RF or the IF stage depending on the receiver architecture. The rest of the channels are filtered out in IF or baseband using a filter.

2.2 The Image Frequency Problem

For any mixer, two different signals will mix down to the IF stage. The desired RF is one of them. The other is the image, which is any signal or noise occurring at the input of the mixer, which will also mix down to the IF.

The image frequency problem can be understood by examining Figure 2.1¹.

Figure 2.1 The image frequency problem

¹ The graph assumes that LO is on the lower side band of the RF frequency.

Ideally the RF signal, in Figure 2.1.A, mixes with the LO, in Figure 2.1.B, to produce the IF, Figure 2.1.C. Note that $RF - LO = IF$. However, if some information existed at $LO - IF$ (which is the image frequency in this case) then it will also mix with the LO signal and convert down to IF since $LO - (LO - IF) = IF$. At the intermediate frequency stage it would be impossible to distinguish between the down-converted RF signal and the down-converted image signal.

2.3 Receiver Topologies

The receiver is required to perform multiple tasks in order to recover the original transmitted signal. These tasks include:

- Band selection using a Band Pass Filter (BPF)
- Signal Amplification using Low Noise Amplifier (LNA)
- Image Filtering
- Down Conversion from RF to IF or baseband using the mixer.
- Channel selection, and out-of-band channel filtering.

All receivers employ the above functions. However, the main difference between these topologies is the number of IF stages they have. They also might differ in the way they filter out the image frequency.

2.3.1 Heterodyne Receiver

The heterodyne receiver shown in Figure 2.2 uses an adjustable oscillator to mix the desired signal into a fixed-frequency IF.

Figure 2.2 Heterodyne receivers

This is the most widely used topology. In this topology, a band pass filter (BPF) is first used to roughly select the part of the spectrum that is of interest. A Low Noise Amplifier (LNA) then follows this BPF. As the name implies, this LNA is designed to have high gain and minimal noise figure. After the signal has been received and amplified an image-reject filter (IRF) with passband centered at RF is used to remove any information at the image frequency. Now a frequency-synthesizer chooses the desired channel and converts it down to the fixed IF stage with a mixer. Once the signal is at IF, the surrounding unwanted channels, are suppressed using a channel select filter.

Since image-rejection takes place at RF, numerous discrete components are required to build the receiver. In addition, since channel selection (using a synthesizer) occurs at relatively high frequencies, the integration of these components is even more difficult.

From a systems designer's point of view, choosing where to place the IF stage presents a trade off between channel selection and image rejection. The further the IF stage is from DC, the further the image from the RF, and the easier it is to filter that image frequency. At the same time, since channel selection takes place in IF, a choice of a relatively high frequency IF might present a challenge in designing high quality channel select filters. One way to overcome this trade off, is to use the dual IF stage (or double conversion) receiver.

2.3.2 Double Conversion Receiver

In this case the frequency selected for the first IF stage can be chosen large enough to make image filtering easy. Then another mixer can be used to convert the signal down to a second IF stage, where narrow-band filtering (for channel selection) is made easier.

Figure 2.3 Double conversion receivers

However, this receiver still requires high frequency image reject filters, which are not easily integrated. The next two receivers do not need image-reject filters for the image frequency.

2.3.3 Homodyne Receiver

In this receiver the RF signal, with all its in-band potential channels, are amplified by the LNA, then directly converted to DC. For this reason, it is also called the Direct Conversion Receiver. This direct conversion requires that the LO frequency is the same as that of the RF. In this case no image frequency exists since $f_{IF}=0$. After translating the signal band to DC, a channel-select filter is simply a lowpass filter with center frequency at DC. As a result the Q requirements of the filters are relaxed.

This receiver is a good solution for both the image and channel filtering, but it has some disadvantages. A very high frequency synthesizer is still required for channel selection at RF. Another problem is the moderate isolation between the LO port and the inputs of the mixer and the LNA, will still allow the LO to mix with itself resulting in a DC offset, potentially saturating the following stages in the system.

Another disadvantage is seen when frequency and phase-modulated signals are received. In this case the down conversion of this receiver must provide quadrature outputs [1], so as to avoid loss of information. This is because in FM and QPSK the two sides of the spectra carry different information and will have to be separated when trans-

formed to the baseband frequency². Figure 2.4 shows a block diagram of a direct conversion receiver.

Figure 2.4 Direct conversion receivers

2.3.4 Image Reject Receiver

This receiver has an Intermediate Frequency stage (IF), however, it does not need a filter for image rejection. Instead, the incoming RF signal is mixed with two oscillator signals, in-phase and quadrature. The two IF signals are then processed and added together to reconstruct the incoming channel and cancel the image. The two possible implementations of this receiver are shown in Figure 2.5 and Figure 2.6.

The following is a set of derivations that explain the process, assuming ideal quadrature signals with no phase and amplitude mismatch. First, let us define $S(t)$ as the message signal, $IM(t)$ as the image; $LO_I(t)$ and $LO_Q(t)$ as the in-phase and quadrature oscillator signals. Note that $S(t)$ and $IM(t)$ are random signals with a phase reference with respect to the $LO(t)$ signals:

$$S(t) = \cos(\omega_{RF} t) + j \sin(\omega_{RF} t) \quad (1)$$

$$IM(t) = \cos(\omega_{IM} t) + j \sin(\omega_{IM} t) \quad (2)$$

$$LO_I(t) = \cos(\omega_{LO} t) \quad (3)$$

$$LO_Q(t) = \sin(\omega_{LO} t) \quad (4)$$

² [1] This is not required for double-sideband AM signals since it overlaps positive and negative parts of the input spectrum.

After passing through the LNA, the incoming RF signal (message plus image) is mixed with the local oscillator signal. The local oscillator in this case produces both quadrature and in phase signals. The results are shown in equations (5)-(8). Note that the high frequency terms are eliminated using a low-pass filter:

$$I_S(t) = \frac{1}{2} \cos(\omega_{IF}t) + \frac{j}{2} \sin(\omega_{IF}t) \quad (5)$$

$$I_{IM}(t) = \frac{1}{2} \cos(\omega_{IF}t) - \frac{j}{2} \sin(\omega_{IF}t) \quad (6)$$

$$Q_S(t) = -\frac{1}{2} \sin(\omega_{IF}t) + \frac{j}{2} \cos(\omega_{IF}t) \quad (7)$$

$$Q_{IM}(t) = +\frac{1}{2} \sin(\omega_{IF}t) + \frac{j}{2} \cos(\omega_{IF}t) \quad (8)$$

Where $I_S(t)$ and $Q_S(t)$ are the in-phase and quadrature signals of the message signal. $I_{IM}(t)$ and $Q_{IM}(t)$ are the in-phase and quadrature of the image signal.

Now the Q terms can be shifted by 90°, and then added to the In-phase terms. The result is the canceling of the image frequency and reconstruction of the message signal. This technique could be used with either single or double IF stage receiver.

$$S(t) = \cos(\omega_{IF}t) + j \sin(\omega_{IF}t) \quad (9)$$

$$IM(t) = 0 \quad (10)$$

The above corresponds to the receiver shown in Figure 2.5, also known as the Hartley Architecture.

Figure 2.5 Hartley image reject receivers

Another possible implementation of an image-reject receiver is known as the Weaver Architecture [1] shown in Figure 2.6. In this case the phase shifter after the mixer in Figure 2.5 is replaced by another set of mixers to perform basically the same operation; See Figure 2.6.

As explained earlier, after the first quadrature mixing stage and the low-pass filters, the signals are still described by equations (5-8).

The in-phase signals are mixed with the in-phase oscillator signal, while the quadrature signals are then mixed with the 90° shifted oscillator signal. The results are (looking at the low frequency terms only):

$$I_{2S}(t) = \frac{1}{4} \cos(\omega_{IF2}t) + \frac{j}{4} \sin(\omega_{IF2}t) \quad (11)$$

$$I_{2IM}(t) = \frac{1}{4} \cos(\omega_{IF2}t) - \frac{j}{4} \sin(\omega_{IF2}t) \quad (12)$$

$$Q_{2S}(t) = +\frac{1}{4} \sin(\omega_{IF2}t) + \frac{j}{4} \cos(\omega_{IF2}t) \quad (13)$$

$$Q_{2IM}(t) = -\frac{1}{4} \sin(\omega_{IF2}t) + \frac{j}{4} \cos(\omega_{IF2}t) \quad (14)$$

Now adding the last two equations results in:

$$S(t) = \cos(\omega_{IF2}t) + j \sin(\omega_{IF2}t) \quad (15)$$

$$IM(t) = 0 \quad (16)$$

Figure 2.6 Weaver image reject receivers

The image-reject receiver eliminates the need for high frequency image reject filters. However, it requires very good phase and amplitude match of the quadrature signal, which is not always easy to achieve.

2.4 Future Trends

With the fast growth in the wireless area, efforts are focused on building smaller and cheaper devices. The use of CMOS technology is becoming widely popular in the RF world since it is relatively inexpensive. It also allows the RF front end to be integrated with the digital backend circuits on a single chip. With CMOS technology lower power consumption levels are possible, compared to similar bipolar technology, however noise performance might be compromised.

Chapter 3 Oscillator Circuit Background

3.1 Oscillator Circuit Overview

An oscillator provides a sinusoidal signal at a certain center frequency given only DC power. Ideally the signal is a pure sinusoid and its frequency spectrum appears as a single spike. However, noise appears around the oscillator signal widening the spectrum in the frequency domain. This noise is referred to as *phase noise*.

A Voltage-Controlled Oscillator (VCO) is an oscillator that has the capability of operating over a range of frequencies. An input voltage can be used as a control signal to choose the center frequency of the VCO.

3.2 Performance parameters

One of the most important performance parameters in oscillators is noise, referred to as *phase noise*. Phase noise limits immunity against interference from nearby channels [21]. In the receiver, the oscillator converts the desired signal down to the IF stage (or base-band in direct conversion receivers). If the phase noise is high around the carrier, it will also mix down the nearby channels into the IF stage. On the transmitter side, phase noise in the oscillator will cause a portion of the signal to be converted into adjacent channels. The next section will have a more detailed discussion about phase noise.

Another important issue in voltage-controlled oscillators is frequency tuning. Usually the VCO is required to cover a certain frequency range. The frequency of oscillation is dependent on the value of inductors and capacitors (varactors) in the circuit. Since modeling of these elements is still in its early stages (especially in digital CMOS technologies), the prediction of the center frequency and the tuning range will present a challenge. Further shifts in frequency are possible due to parasitic capacitance and inductance in the circuit itself and its load.

An important issue in today's high growth wireless technologies is power consumption. Using CMOS technologies, power supplies as low as 1 volt are desirable. Phase noise on the other hand is usually compromised when using these low power lev-

els. For this reason new circuit design techniques and deeper understanding of phase noise sources became more essential.

3.3 Operation of LC Oscillators

These oscillators have an inductor-capacitor (LC) resonance circuit. This resonance circuit has a certain resonance frequency. Resonance is the frequency at which the energy oscillates between the inductor and the capacitor. The value of this frequency is $1/(2\pi\sqrt{LC})$.

If there were ideal inductors and capacitors, making an oscillator would simply mean putting the two elements together and providing the circuit with some initial triggering. Unfortunately, losses exist in the inductors and capacitors and additional circuitry is required to compensate for these losses.

The additional circuitry that compensates for the losses, in LC oscillators, usually consists of amplifier transistors, which provide feedback gain in the signal path.

Figure 3.1 is a block diagram of a linear feedback system that can be used to represent the oscillator. The transfer function of the system is

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 - H(s)} \quad (17)$$

$H(s)$ is the transfer function of the resonator plus the amplifier. The criteria for oscillation is

- 1) $|H(s_0)| = +1$, the loop gain at resonance should be equal to unity,
- 2) $\angle H(s_0) = 0$, and the phase shift around the loop should be equal to zero degrees [1].

Figure 3.1 Feedback model

A variety of topologies can be used to build LC VCOs depending on how the tank elements and the feedback circuit are connected. Colpitts, Hartley and Negative-gm are all examples of LC Oscillators.

3.4 LC Oscillator Circuit Topologies

3.4.1 Colpitts and Hartley oscillators

Figure 3.2.a shows an example of a common-base Colpitts oscillator. The resonance frequency of the tank is

$$\omega = \frac{1}{\sqrt{L1Ceq}} \quad (18)$$

Where Ceq is the equivalent capacitance in Figure 3.2.a. One important note about the Colpitts oscillator is that the feedback connection from the amplifier circuit to the tank is done at point A in Figure 3.2.a. In the case where the feedback is connected at point B, the tank would see a parallel load from transistor Q1 of the value $1/gm$. Such a load would degrade the Q of the circuit dramatically. However, when using point A instead of B, the impedance seen by the tank will be transformed by a ratio of $(1 + \frac{C1}{C2})^2$, making the parallel resistance seen by the tank higher by a factor of $(1 + \frac{C1}{C2})^2$. It also improves the Q of the tank by this same factor [1].

Figure 3.2 Colpitts and Hartley oscillators

The Hartley oscillator, seen in Figure 3.2.b, uses the same technique for impedance transformation. However, in comparison to the Colpitts oscillator, this resonant circuit substitutes inductors with capacitors and vice versa. Since this circuit requires two inductors, instead of one for Colpitts, it is less attractive for on-chip integration.

3.4.2 Negative-gm Oscillator

Negative-gm is another configuration of LC oscillators, and it is the design of choice in this thesis. These are simple structures and can be built with all NMOS, all PMOS, or a combination of both. One advantage of the negative-gm oscillators is that the transistors provide the LC tank directly with negative transconductance that cancels the losses in the tank. Unlike Colpitts and Hartley oscillators, the loading from these transistors to the tank does not require impedance transformation. The output impedance of transistor M1 is presented to the tank through a buffer, that is the second transistor M2. M2 provides active transformation of the output impedance of M1. M1, in turn, provides active transformation of the output impedance of M2 [1].

One possible configuration of negative-gm oscillator consists of NMOS transistors as the amplifier. The circuit is shown in Figure 3.3 and the small signal model is shown in Figure 3.4.

Figure 3.3 Negative gm oscillator

Figure 3.4 Small signal model of the negative gm oscillator

The R_p in the small signal model includes all the losses from the tank and from the transistors themselves. The $R_p/2$ connected between point A and ground consists of half the losses from the tank elements, the output impedance from transistor M1 and the input impedance from transistor M2. The same applies for the loss $R_p/2$ connected between point B and ground with the respective transistor losses.

In order to determine the gain of this circuit, the feedback can be broken, at point B for example, see Figure 3.4. Now, to sustain the oscillation, a gain of at least 1 is re-

quired between point B1 and B2. Note that a gain of higher than one is needed to start the oscillation and to guarantee it will oscillate at non-zero amplitude. A value of two or three is usually used.

Another way of looking at the circuit is in terms of the negative gm. Each of the transistors should have a negative resistance equivalent to $R_p/2$. This negative resistance comes from the transistor's transconductance. The resistance is regarded as negative because current flows in the opposite direction to that of a positive resistor when the same voltage is applied across them.

Now when closing the loop, there should be no losses in the circuit and the energy keeps oscillating in the LC tank at the resonance frequency.

Figure 3.5 shows the total negative resistance seen by the tank. The gm from each transistor is referred to as Gm and its value should be

$$\frac{R_p}{2} = \frac{1}{G_m} \Rightarrow G_m = \frac{2}{R_p} \quad (19)$$

And for the total resistance seen by the tank Rin

$$R_{in} = \frac{2}{G_m} = R_p \quad (20)$$

Figure 3.5 Transistor loading to the tank in the negative gm oscillators

Another way to represent the circuit is by its single-ended, open-loop model shown in Figure 3.6. In this figure, M1 is the amplifier while M2 is a common source output buffer. In this design, the tank is loaded by the output impedance of M1 multiplied by the AC gain (g_m) of M2. The small signal model is similar to that in Figure 3.4 with the exception that only half the tank circuit is present.

Figure 3.6 Single-ended negative gm oscillators

Before discussing the negative-gm configurations, some points have to be addressed. First, NMOS transistors have higher flicker noise than PMOS transistors. However, NMOS transistors have a higher mobility factor and provide more power per W/L at a given bias. Second, current sources can be one of the main contributors to noise, if not designed and placed correctly. For all the circuits, a common concern for noise is tank quality.

Figure 3.7 shows five possible configurations of negative gm oscillators. Transistors M1 and M2 are the feedback transistors and M3 and M4 are the current source (current mirror) transistors. In the following comparison it was assumed that all M1 transistors have the same dimensions, regardless of their type. It is also the case for M2, M3, and M4.

Circuit (a): This employs NMOS amplifiers with an NMOS current mirror. The main concern in this topology is the tuning of varactors. Some de-coupling capacitors should be used to bias them separately. Another concern is possibly the amplitude modulation of the current source flicker noise by the switching action of M1 and M2 (dis-

cussed further in Section 3.5.2.2). Frequency modulation of flicker noise from the amplifier transistors and from current source is still a big concern.

Circuit (b): This uses NMOS amplifiers with a PMOS current mirror. The main concern is the frequency modulation of the current source flicker noise, which is injected directly into the tank. However, the PMOS transistors in the source have relatively less flicker noise. NMOS transistors also inject $1/f$ noise into the tank. At low frequency, the inductor is a short circuit and this noise appears across both varactors and frequency modulates the oscillator.

Circuit (c): This employs PMOS amplifiers with an NMOS current mirror. The main concern is frequency modulation of the current flicker noise that is injected directly into the tank. Although both NMOS and PMOS transistors are injecting $1/f$ noise into the tank, noise from the current source can have a higher effect because of its smaller size (which leads to higher flicker noise).

Circuit (d): This consists of PMOS amplifiers with a PMOS current mirror. The main concern is the varactor biasing and amplitude modulation of the current source's flicker noise. Frequency modulation is not an issue since the output node is at a fixed DC bias. Compared to circuit (a) the PMOS amplifiers provide less power, but the PMOS current source has less flicker noise.

Circuit (e): This utilizes NMOS amplifiers and no current source. In this case a resistor is used to bias the circuit. Some processes have a variable power supply, in which case there is no need for a resistor. This configuration will have significantly lower phase noise at the carrier frequency since the current source noise is eliminated, assuming that the power supply line is very well de-coupled from other circuits and noise sources and that the resistor value is constant over temperature and process. Note that a resistor can also replace the current source in circuits (b), (c) and (d).

Figure 3.7 Possible configurations of the negative gm oscillator

3.4.3 Complementary Negative-gm oscillators

Figure 3.8 shows a schematic of the circuit with NMOS and PMOS transistors.

Complementary negative gm oscillators are claimed to have better noise performance than all NMOS or all PMOS designs [21]. This is true only if the NMOS and PMOS transistors have the same transconductance at a given bias. This means that half of the negative resistance should come from the NMOS and half from the PMOS transistors.

In the case of matched g_m , the rise and fall time symmetry will result in noise reduction. In other words nodes A or B in Figure 3.8 would not be as sensitive to noise. To understand this, we should examine how the circuit is biased.

First, let us say that the current source is implemented using an NMOS current mirror similar to Figure 3.9. The value of bias current from the current source depends on the size of transistor M5 and V_{gs} (the gate source voltage of the source of transistor M5). The transistor size is constant, but V_{gs} is prone to noise, i.e. flicker noise. The DC voltage at node A is set by the value of the current flowing through that branch and the size of transistors M1 and M3. This DC voltage is approximately $V_{DD} - V_{gs3}$. Now if M1 and M3 were matched for g_m then any noise in the bias current would be canceled. However, if not matched, this noise will appear on node A and will modulate the reactance in the LC tank, which in turn modulates the center frequency.

Also this topology provides more negative resistance for a given bias current than that of the all NMOS or all PMOS configurations. Furthermore, according to [12], the output voltage swing is twice that of all NMOS or all PMOS configuration, where the output voltage is $\frac{4}{\pi} I_{bias} R_p$. While the voltage swing is only $\frac{2}{\pi} I_{bias} R_p$ for the all NMOS or all PMOS one.

This topology was the one chosen for fabrication in the course of this research.

Figure 3.8 Complementary negative gm oscillators

Figure 3.9 The current mirror

3.5 Phase Noise in LC Oscillators

Oscillators are prone to noise. Noise causes the frequency of the oscillator to shift randomly. This frequency deviation of the signal can be viewed as a random variation of the zero crossing points from their ideal position along the time axis.

If we assume that the oscillator signal $S_{LO}(t)$ is given by:

$$S_{LO}(t) = A \cos(\omega_{LO}t) \quad (21)$$

We can include noise and modify this to:

$$S_{LO}(t) = A \cos(\omega_{LO}t + \phi(t)) \quad (22)$$

Where $\phi(t)$ is the noise function that effects the zero crossings of the signal in the time domain.

If we assume that $\phi(t) \ll 1$, then the oscillator's signal can be approximated by:

$$S_{LO}(t) \cong A \cos(\omega_{LO}t) - A \phi(t) \sin(\omega_{LO}t) \quad (23)$$

We can see from equation (23) that the noise function $\phi(t)$ is mixed up to the carrier frequency. This shows that variations in the oscillator's period (phase) translate to a frequency shift of the carrier and vice versa. This explains the name "phase noise".

Two issues will be discussed in the next section: noise sources and phase noise mechanism. Noise sources are the actual physical sources of noise in the oscillator circuit, while phase noise mechanism is the translation of noise from these sources to phase noise around the carrier.

3.5.1 Noise Sources

3.5.1.1 Thermal (White) Noise

Resistors produce thermal noise. This noise is produced by the random fluctuations of the electrons due to thermal agitation. Thermal Noise has a flat spectrum with an rms noise voltage function dependent on the resistor value,

$$V_n(f) = \sqrt{4kTB R} \quad (24)$$

The power density function is given by $V_n^2/R = 4kTB$. The power function is flat across the bandwidth (B) and so is referred to as white noise.

Here, k is Boltzmann's constant, which is equal to 1.38×10^{-23} J/K, and T is the room temperature in Kelvin, that is 290K. This leads to $kT = 4.0 \times 10^{-25}$ J = -174dBm/Hz

In our circuit, losses exist in the tank and in the transistors. The total losses in the circuit can be represented by (R_p) or the equivalent conductance (g_p). Thermal noise from these losses exists over the entire spectrum and is called white noise. Although this noise power density is flat, one should note that thermal noise at DC and at the harmonic frequencies of the oscillator output could be translated to the center frequency, adding to the contribution of thermal noise to phase noise at the fundamental.

For MOS transistors the thermal noise equation is $I_n^2 = 4kT (\frac{2}{3} gm)$. Where gm is the transconductance of the transistor.

3.5.1.2 Flicker Noise

This noise arises from the random trapping of charge at the oxide-silicon interface of MOSFETS [1]. This noise can be modeled as a voltage source in series with the gate

$$V_n^2 = \frac{K}{WLC_{ox}} \frac{1}{f^{af}} \quad (25)$$

K and af are process-dependent constants. W and L are the width and length respectively of the transistor's gate, and C_{ox} is the oxide capacitance per unit area. W and L are the only controllable variables and noise can be reduced by increasing their values (without changing the ratio of gate width to gate length (W/L)). Although this noise diminishes at higher frequencies, this noise can be up-converted to the carrier frequency, contributing to phase noise around the carrier.

3.5.2 Phase Noise Mechanisms

In this section the mechanisms by which noise from circuit sources is translated to phase noise around the carrier will be discussed. These mechanisms are related to the complementary negative-gm oscillator topology.

3.5.2.1 *Q of the Tank*

Losses in the circuit define the white noise power density and thus affect the value of phase noise around the carrier. However, contrary to popular belief, these losses do not define the shape of the filter around the carrier, since the loss is effectively cancelled by the positive feedback in the oscillator. In spite of this, it is still possible to use the Q of the resonator in deriving the expression for phase noise as the following will explain.

A good way to represent the losses in the circuit when we need to predict the noise transfer function is with the quality factor “ Q ” of the tank. All the losses in the circuit can be transformed into a parallel resistance (R_p) across the tank circuit. Q will represent the amount of loss, where infinite Q means zero losses. We could also consider the oscillator a band-pass filter, however, one with infinite Q , which amplifies the noise in its bandwidth. The finite Q in the resonator, due to the losses, is often (incorrectly) considered to be the Q of the oscillator is:

$$Q = \frac{\omega}{B} \quad (26)$$

where B is the 3-dB bandwidth.

Now representing all the circuit losses in the parallel conductance g_p , the resonator 3-dB bandwidth would be:

$$B = \frac{g_p}{C} \quad (27)$$

At the resonance frequency, the energy, picked up from noise, oscillates between the inductor and the capacitor in the tank. At that frequency, the closed loop model of the circuit explains how the losses in the tank are compensated for by the gain of the amplifier, keeping the circuit oscillating forever. A current signal (representing noise) can be injected into the circuit and then the voltage developed on the tank can be calculated.

Reference [4] derives the noise model as below.

A noise current source is placed in the single-ended equivalent of the circuit, as seen in Figure 3.10. This current source S_{ni} represents the overall circuit noise. Now S_{nv} will represent the noise voltage spectral density.

Figure 3.10 Single-ended models with noise current source

We will now derive the relation between this current noise spectral density S_{ni} and the voltage noise spectral density S_{nv} . To do so we will first derive an expression for the impedance of the LC tank of the circuit in Figure 3.10.

The resonance tank has an impedance given by the parallel combination L and C. Taking $\Delta\omega$ as the frequency offset from ω_o .

$$Z(\omega) = \frac{1}{j\omega C} \parallel j\omega L \quad (28)$$

$$Z(\omega_o + \Delta\omega) = \frac{j\omega_o L + j\Delta\omega L}{1 - LC(\omega_o + \Delta\omega)^2} \quad (29)$$

and for $\Delta\omega \ll \omega_o$ we can ignore the $j\Delta\omega L$ and the $(\Delta\omega)^2$ terms, and the result is:

$$|Z(\omega_o \pm \Delta\omega)|^2 \cong \left(\frac{1}{2\Delta\omega C} \right)^2 \quad (30)$$

Therefore,

$$S_{nv} = \frac{S_{ni}}{(2\Delta\omega C)^2} \quad (\text{Where } V=IZ) \quad (31)$$

Substituting equations (26) and (27) in (31) we have,

$$S_{nv} = \frac{1}{4} \frac{S_{ni}}{g_p C} \frac{\omega_o}{Q} \frac{1}{(\Delta\omega)^2} \quad (32)$$

The first contribution to S_{ni} is the thermal noise; that is $4kTg_p$ due to the resistive losses in the circuit. The noise added by the amplifier can be modeled by the noise factor F . The

total noise current density function is $S_{ni} = 4kTg_pF$. Substituting this value in (32) and dividing by the carrier power to get phase noise results in:

$$\mathcal{L}(\Delta\omega) = 10 \log \frac{S_{nv}}{A_o^2 / 2} = 10 \log \frac{2}{A_o^2} \frac{kTF}{C} \frac{\omega_o}{Q} \frac{1}{(\Delta\omega)^2} \quad \text{dBc/Hz} \quad (33)$$

Or:

$$\mathcal{L}(\Delta\omega) = 10 \log \frac{2kTF}{A_o^2 \cdot g_p} \frac{\omega_o^2}{Q^2} \frac{1}{(\Delta\omega)^2} \quad \text{dBc/Hz} \quad (34)$$

$$\text{Since } C = \frac{Q \cdot g_p}{\omega_o} \quad (35)$$

The above equation is known as Leeson's formula. The formula tells us that noise is inversely proportional to the square of the quality factor of the tank and signal amplitude.

Equation 33 also suggests that phase noise increases when adopting a smaller capacitor. Adopting a smaller capacitor means that a larger inductor is required to achieve the same oscillation frequency. This larger inductor means that noise current will go down proportionally with $1/L$, and impedance will go up proportionally with L^2 . Consequently, noise voltage increases relative to L . However, this is only true if the output is voltage limited. In the case where the output is not voltage limited, then, as we have larger inductors, the signal level will go up proportionally with L^2 . Since noise goes up proportionally with L only, this will result in an overall better phase noise performance. [17]

Looking at (33), we could see that the transfer function of noise is a band-pass filter. It has its maximum gain around the resonance frequency of the tank. On each side of this center frequency, the gain rolls off by 20dB (the $1/(\Delta\omega)^2$ term in Leeson's formula). When the Q of the tank is lower, the noise is higher because of higher input noise.

Note that this expression does not account for any non-linearity occurring in the circuit due to clipping. However, this can be seen as Amplitude Modulated noise, which will be discussed next.

Figure 3.11 is a plot of the noise from the tank losses. The numbers used to calculate these values are based on the design presented later, these values can be seen in the Excel sheet attached in the appendix. The figure is shown here to demonstrate equation 33.

Figure 3.11 Phase noise according to equation 33

3.5.2.2 Amplitude Modulated Noise

As discussed earlier, in order for the circuit to continue oscillating, the amplifier has to have a gain that cancels out the losses in the circuit. However, to insure oscillations start up, the open loop gain has to be more than 1. Higher gain results in the circuit operating in the self-limited oscillation amplitude, causing switching in the differential pair [3][4]. The amplifier transistors will act as a commutating mixer.

We can divide the noise that is mixed to f_c into three different categories. First flicker noise from the current source will be up-converted around the carrier f_c through the differential pair. Secondly, white noise around the odd harmonics (B) will be converted down to f_c . Note that mainly odd harmonics exist since the self-limiting mechanism of the oscillator causes the output signal to look more like a square wave, causing the amplitudes of the odd harmonics to increase. Note that even harmonics can exist if the limiting mechanisms for positive and negative swings are not symmetrical, however since the complementary topology is being examined, the assumption of symmetrical swings is made. Finally, white noise around $2f_c$, existing on the common mode node between the two transistors (C), will be down converted to f_c .

Figure 3.12 is an illustration of how the noise is converted to the carrier. (A) represents flicker noise at the gate of the source transistor M3. (B) represents the odd harmonics that exist at the gate of M2, coming from the other differential output M1. (C) represents the $2fc$ noise appearing at the common node point of the two transistor sources. (D) represents the up-converted noise. If we assume that Amplitude Modulation is the only way for flicker noise to get up converted, the output spectrum around the carrier will look like that of transistor M3 with the exception that the flicker noise corner, is lowered. The reason is that white noise from other harmonics is also converted up to the carrier [3].

Figure 3.12 Amplitude modulation of noise with the switching transistors

In order to calculate the flicker noise up conversion we can regard the oscillator as a mixer. The mixer multiplies these low frequency noise sidebands with the carrier and converts them to frequencies around the carrier. We will treat these switching transistors as a mixer to find the transfer function for the noise.

Equation 36 is for the mixer shown in Figure 3.13.a.

$$V_{out+/-} = \frac{1}{2} \frac{V_{IF}(t)}{1/gm_3} \frac{4R_c}{\pi} A_c \cos \omega_{LO} t \quad (36)$$

Figure 3.13 Representing the oscillator as a mixer

where:

R_c is the load

gm_3 is the transconductance of the current source transistor.

V_{IF} is the IF signal voltage

A_c is the amplitude of the carrier

The LO signal = $4/\pi A_c \cos \omega_{LO} t$ (Assuming that the oscillator signal is a square with an amplitude of A_c and ignoring all the harmonics).

Since multiplying $\cos \omega_{LO} t$ by the VIF means shifting the VIF in frequency then:

$$V_{out}(\Delta\omega) = \frac{1}{2} \frac{1}{2} \frac{V_{IF}(\Delta\omega)}{1/gm_3} \frac{4R_c}{\pi} A_c \quad (37)$$

We can use equation (37) to describe the circuit in Figure 3.13.b. as follows:

$$V_{AM\ Noise}(\Delta\omega) = \frac{1}{4} \frac{V_n(\Delta\omega)}{1/gm_3} \frac{1}{gm_5} \frac{4}{\pi} A_c \quad (38)$$

Where:

$$1/gm_5 = R_c \quad (39)$$

gm_5 is the transconductance of transistor M5.

$V_n(\Delta\omega)$ is flicker noise at the gate of the M3 transistor at $\Delta\omega$ frequency offset.

The value of this noise $V_n(\Delta\omega)$ can be estimated from the HSPICE model parameter of the M3 transistor. One technique that can be used to reduce flicker noise modulation is placing signal limiters at the tank.

As for the modulation of the harmonics (B) in Figure 3.12, a value of approximately 3dB can be added to the spectrum to account for this factor [17].

Reference [4] discusses the incorporation of this non-linearity effect in the F factor into Leeson's linear model. However, this is not the only source of noise that is not included in Leeson's formula, frequency modulated noise, discussed in the next section is another one.

Figure 3.14 is a plot of the amplitude-modulated flicker noise. The numbers used to calculate these values are based on the design presented later. The figure is shown here to demonstrate equation 38.

Figure 3.14 Amplitude-modulated flicker noise based on equation 38

3.5.2.3 Frequency Modulated Noise

Frequency modulated noise is a non-linear mechanism in which noise from lower frequencies is transformed around the carrier. The reason is that some noise sources in the circuit affect the capacitance value in the tank, directly or indirectly. The change in the capacitance value causes a shift in the center frequency.

First let us understand frequency modulation. At a given frequency offset from DC, the modulating frequency f_m exists. The modulating signal voltage (V_m) shifts the carrier frequency f_c by Δf . The resultant side band signal power SB at $f_c - f_m$ and $f_c + f_m$ is:

$$SB = 20 \log \beta/2 \quad (40)$$

where:

$$\beta = \frac{\Delta f}{f_m} \quad (41)$$

$$\Delta f = K \cdot V_m \quad (42)$$

β should be < 1 for narrow band frequency modulation.

In (42), K is a constant that describes how much the frequency shifts in Hz for 1 volt change in V_m .

Figure 3.15 Spectrum of a frequency-modulated signal

Two noise sources in the circuit have the potential to be modulated up to the carrier:

1) Noise across the varactor: Any low frequency noise on the control voltage line will be up-converted around the carrier frequency. This noise causes the control voltage to fluctuate across the varactors. Since varactors are voltage-controlled capacitors, their value changes causing a shift in frequency. In other words the noise is frequency modulating the carrier. Also any low frequency noise on the other side of the varactor, for example noise from the cross-coupled transistors will appear directly across the varactors causing the frequency to shift. Note the inductor behaves as a short circuit for noise at low frequency so noise appears equally on both sides so is common-mode noise.

In this case the K constant is equal to the circuit's K_{VCO} since the noise appears as fluctuations on the control voltage. It should be noted that K_{VCO} is the change in frequency with respect to 1V change in voltage (Hz/V). Substituting in equations (40), (41) and (42):

$$\mathcal{L} = 20 \log \frac{K_{VCO} \cdot \Delta V_c}{2 f_m} \text{ dBc/Hz} \quad (43)$$

K_{VCO} is not constant throughout the entire tuning range. For a p-n junction varactor, K_{VCO} usually has a higher value at the lower end of the frequency tuning range and decreases at the higher end. For that reason, the fm-modulated noise is expected to dominate phase noise at these lower frequencies. However, since a certain frequency range is required from the VCO, it would be hard to avoid the frequencies at which the K_{VCO} value is large. De-coupling the control line with a large capacitor can decrease the effect of this noise.

2) Noise on the bias: Any low frequency noise (for example, flicker noise) on the current source bias line there will cause the bias current to fluctuate causing the bias conditions of the amplifier transistors, for example V_{DS} , V_{GS} , to shift as well. This occurs because the DC level of the output (and across the varactor) is dependent on the transistors' bias. This is the case if the circuit is of a complementary topology and where the transconductance of the NMOS and PMOS transistors are not matched. If the bias fluctuates then the varactor capacitance will change, causing frequency modulation. In addition, the change in bias causes the parasitic capacitance of the transistor to change. The combined effect of all these variables can be represented by a factor called $K_{VCO,b}$ where:

$$\Delta f = K_{VCO,lb} * \Delta I_b \quad (44)$$

And

$$f = 20 \log \frac{K_{VCO,lb} \cdot \Delta I_b}{2 f_m} \quad \text{dBc/Hz} \quad (45)$$

$K_{VCO,lb}$ represents the frequency shift in Hz for 1A change in the bias current.

Since the low frequency flicker noise is modeled by a voltage source at the gate of the transistor, we need to translate that into a bias current noise source representation I_n . The following equation was used, where

$$\Delta V_{gs} = \sqrt{V_{bn}^2} \quad (46)$$

And

$$I_{bnoise} = \frac{K_p}{2} \frac{W}{L} ((V_{gs} + \Delta V_{gs}) - V_t)^2 - I_b \quad (47)$$

The value of Δf was calculated by multiplying $K_{VCO,lb}$ by the value of the change of the bias current ΔI_n resulting from the change in gate source voltage $\Delta V_{gs} = V_{bn}$.

Again the $K_{VCO,lb}$ is not linear. This means that the effect of noise is different at different bias current [12]. It also means that there exists a bias value where the $K_{VCO,lb}$, and hence the up-converted noise from the bias line, is minimal.

Unlike the noise on the control line, the designer can avoid the bias points at which the $K_{VCO,lb}$ is high and choose a bias at which the value of $K_{VCO,lb}$ is more linear. Note that this might not be the optimal choice for low-power designs.

Since flicker noise is inversely proportional to the transistor area, the designer can multiply the W & L of the tail transistor by the same factor. By doing so, the flicker noise will be reduced while keeping the W/L ratio constant.

Another source of noise in the bias line is the power supply noise. De-coupling the line will help reduce this effect.

Figure 3.16 is a plot of the frequency-modulated flicker noise. The numbers used to calculate these values are based on the design presented later. The figure is shown here to demonstrate (43) and (45).

Figure 3.16 Frequency-modulated flicker noise based on equation 43 and 45

3.5.2.4 Oscillation Amplitude vs. Phase Noise

Looking at Leeson's formula we can see that noise in general can be reduced by increasing the output power of the oscillator. This is true for a certain range of output levels, where we can observe phase noise decrease relative to the carrier as we increase the power. However, beyond an optimum output level, noise, becomes dominant. As the bias current increases noise performance degrades. This is opposite to what Leeson's linear formula suggests.

Experimental and theoretical results in [4] explain why noise stops decreasing with higher output levels, however it does not describe the reason behind the increase in noise. Another theory suggests that noise degradation in higher-level outputs is due to the increase in $K_{VCO,fb}$, the frequency modulation factor [12].

3.5.3 Calculation of Combined Phase Noise

In order to add all the different noise sources at the output, we can convert the noise power from dBc/Hz to dBm and then add the noise from the three noise mechanisms.

These three mechanisms are the Q of the tank, frequency and amplitude modulation of flicker noise. Equation (48) sums up all phase noise sources.

$$\mathcal{L}(\Delta\omega) = 10 \log \left[\frac{2}{A_o^2} \frac{kTF}{C} \frac{\omega_o}{Q} \frac{1}{(\Delta\omega)^2} + \left(\frac{K_{VCO,lb} \cdot \Delta Ib}{2 \cdot \Delta\omega} \right)^2 + \left(\frac{K_{VCO} \cdot \Delta Vc}{2 \cdot \Delta\omega} \right)^2 + \left(\frac{1}{4} \frac{V_{IF}(\Delta\omega)}{1/gm_3} \frac{4 \cdot R_c}{\pi} \right)^2 \right] \quad (48)$$

Figure 3.17 Combined phase noise calculation

3.6 Quadrature VCO Circuit

3.6.1 Motivation

The Quadrature VCO such as would be used in the image reject receivers as discussed earlier, requires a local oscillator, which generates sine and cosine terms.

3.6.2 Methods of Generating Quadrature

3.6.2.1 RC phase-shift circuit

This is a basic structure for a 90°- phase shifter, see Figure 3.18[18]. The phase shift of this circuit at a given frequency is highly dependent on the components' (resistor and capacitor) values. The transfer function of these circuits can be derived as follows.

In Figure 3.18.a it can be seen that:

$$\frac{V_{o1}}{V_{in1}} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC} \quad (49)$$

Now $\omega = \frac{1}{RC}$ and at $\omega = \omega_0$, if $V_{in1} = 1 \angle 0^\circ$ then $V_{o1} = \frac{1}{\sqrt{2}} \angle 45^\circ$

And if $V_{in1} = 1 \angle 180^\circ$ then $V_{o1} = \frac{1}{\sqrt{2}} \angle -135^\circ$

While in Figure 3.18.b the equation is:

$$\frac{V_{o2}}{V_{in2}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} \quad (50)$$

Now $\omega = \frac{1}{RC}$ and at $\omega = \omega_0$, if $V_{in2} = 1 \angle 0^\circ$ then $V_{o2} = \frac{1}{\sqrt{2}} \angle -45^\circ$

And if $V_{in2} = 1 \angle 180^\circ$ then $V_{o1} = \frac{1}{\sqrt{2}} \angle 135^\circ$

It can be seen that in either case V_{o1} and V_{o2} are 90° apart. The problem with this circuit is its dependence on component values, which may vary widely.

Figure 3.18 RC phases -shift circuit

Figure 3.19 shows the two circuits in Figure 3.18 combined and driven by the same input voltage. The Q outputs and I are 90° apart.

Figure 3.19 Obtaining Q and I from the RC phase shifter

3.6.2.2 RC polyphase Network

Figure 3.20 shows a cascaded RC polyphase network. It may be thought of as a generalized RC network with a four-phase input and output. When driven at the input by a signal and its inverse, the circuit produces four phases at the output [13]. This structure can tolerate mild mismatch in its components and it can provide quadrature outputs with an order of magnitude improvements in phase accuracy over the RC network. Furthermore, the

different stages of the cascaded structure can have different center frequencies, and can provide broader range for its quadrature outputs [14].

According to [34] a 0.3° error is only achievable when matching between RC and CR networks is better than 1% and this is only true for a very small frequency band. They also obtained a 1° phase error by means of extensive tuning and trimming.

The problem with the two previous topologies is that there will be losses of signal power in the resistors and the loading of the phase shifter. This loss can be compensated for by either increasing the input power or adding buffers at the output of the oscillator. Neither solution is desirable since it increases power dissipation. In addition, variations in absolute values of the components, for frequencies further away from the center, result in amplitude mismatch, which starts to dominate the error and affect image rejection.

Figure 3.20 RC poly-phase networks

3.6.2.3 Dividing the Double Frequency

Another technique is to design a VCO to oscillate at double the frequency, and then using positive-edge and negative-edge triggered flip flops, two quadrature outputs can be generated (Figure 3.21). According to [35] the error in the quadrature can be as small as 1° in

a GSM up-conversion receiver. At higher frequencies, however, generating an acceptable oscillator signal at double the required frequency becomes the problem.

Figure 3.21 Dividing the double frequency

3.6.2.4 The Quadrature Oscillator

This circuit consists of two VCOs that are connected, or coupled, in a way that makes them operate in quadrature [9].

Two things are happening in this oscillator. First, injection locking makes the two circuits oscillate at the same frequency (the difference is minimized since the two circuits, and their LC tanks, are made identical). The second is coupling, which is the key to making these two circuits operate in quadrature.

To connect the circuits together, coupling transistors are added. The drain and the source of the coupling transistor will be connected to the drain and source of the core (amplifier) transistor. The gates of the coupling transistors are connected to the other VCO.

The circuits are coupled in two different ways. They are either cross connected or direct-connected. The direct-connection is shown in Figure 3.22. When these two circuits are out of phase (180°) this direct connection between the two circuits will stop the oscillation of the circuits. In this case if node 1 voltage in oscillator A is at 0° then node 1 voltage in oscillator B is also at 0° , so the two oscillators are in phase. Note that the missing connections in Figure 3.22 will be shown in Figure 3.23.

Figure 3.22 Direct coupling in the quadrature VCO

The cross-connection is shown in Figure 3.23. This connection does not allow the two circuits to oscillate when they are in phase (0° apart). Contrary to the direct connection, if node 1 voltage in oscillator A is at 0° , then node 1 voltage in oscillator B has to be at 180° . In Figure 3.23 the gates of VCO B are direct-connected to the drain of VCO A. Note that there is a phase shift of 180° between the gate and the drain of each transistor.

Figure 3.23 Cross coupling in the quadrature VCO

Now if we have a combination of the two connections as in Figure 3.24 the only phase at which the two oscillators can co-exist is at 90° apart. The direct connection suppresses the out of phase co-existence while the cross connection suppresses the in phase co-existence and the quadrature state is the only phase difference at which they can both oscillate. Any drift in the phase away from this 90° would be compensated for by either the direct or cross connections described earlier.

Figure 3.24 Direct and cross coupling of the transistors in the quadrature VCO

3.6.3 Effect of mismatch

Any mismatch in the quadrature signals will lead to imperfect rejection of the image-frequency signal. This mismatch can either be in phase or power level of the quadrature signals.

Figure 3.25 Model for calculating the effect of amplitude and phase mismatch

The mismatch (in degrees or amplitude) can be related to the amount of rejection by the following equations:

$$S(t) = \cos(\omega_{RF} t) + j \sin(\omega_{RF} t) \quad (51)$$

$$IM(t) = \cos(\omega_{IM} t) + j \sin(\omega_{IM} t) \quad (52)$$

$$I_{LO}(t) = \cos(\omega_{LO} t) \quad (53)$$

$$Q_{LO}(t) = (1 + \Delta A) \sin(\omega_{LO} t + \phi_e) \quad (54)$$

Where $S(t)$ is the signal, $IM(t)$ is the image, $I_{LO}(t)$ is the in-phase component of the oscillator signal and $Q_{LO}(t)$ is the quadrature component of the oscillator. ΔA and ϕ_ϵ are the amplitude and phase mismatch respectively.

Now mixing both the signal and image with the local oscillator signals results in the following:

$$I_S(t) = \frac{1}{2} \cos(\omega_{RF}t - \omega_{LO}t) + \frac{1}{2} \cos(\omega_{RF}t + \omega_{LO}t) + \frac{j}{2} \sin(\omega_{RF}t - \omega_{LO}t) + \frac{j}{2} \sin(\omega_{RF}t + \omega_{LO}t) \quad (55)$$

$$I_{IM}(t) = \frac{1}{2} \cos(\omega_{IM}t - \omega_{LO}t) + \frac{1}{2} \cos(\omega_{IM}t + \omega_{LO}t) + \frac{j}{2} \sin(\omega_{IM}t - \omega_{LO}t) + \frac{j}{2} \sin(\omega_{IM}t + \omega_{LO}t) \quad (56)$$

$$Q_S(t) = -\frac{1}{2} (1 + \Delta A) [\sin(\omega_{RF}t - \omega_{LO}t - \phi_\epsilon) + \sin(\omega_{RF}t + \omega_{LO}t + \phi_\epsilon)] \\ + \frac{j}{2} (1 + \Delta A) [\cos(\omega_{RF}t - \omega_{LO}t - \phi_\epsilon) + \cos(\omega_{RF}t + \omega_{LO}t + \phi_\epsilon)] \quad (57)$$

$$Q_{IM}(t) = -\frac{1}{2} (1 + \Delta A) [\sin(\omega_{IM}t - \omega_{LO}t - \phi_\epsilon) + \sin(\omega_{IM}t + \omega_{LO}t + \phi_\epsilon)] \\ + \frac{j}{2} (1 + \Delta A) [\cos(\omega_{IM}t - \omega_{LO}t - \phi_\epsilon) + \cos(\omega_{IM}t + \omega_{LO}t + \phi_\epsilon)] \quad (58)$$

For this calculation we are only interested in the low frequency (IF) component, while the other frequency terms are discarded. Also note that $\omega_{IF} = \omega_{RF} - \omega_{LO} = -(\omega_{IM} - \omega_{LO})$, where these terms are substituted in the above equations.

$$I_S(t) = \frac{1}{2} \cos(\omega_{IF}t) + \frac{j}{2} \sin(\omega_{IF}t) \quad (59)$$

$$I_{IM}(t) = \frac{1}{2} \cos(\omega_{IF}t) - \frac{j}{2} \sin(\omega_{IF}t) \quad (60)$$

$$Q_S(t) = -\frac{1}{2} (1 + \Delta A) \sin(\omega_{IF}t - \phi_\epsilon) + \frac{j}{2} (1 + \Delta A) \cos(\omega_{IF}t - \phi_\epsilon) \quad (61)$$

$$Q_{IM}(t) = +\frac{1}{2} (1 + \Delta A) \sin(\omega_{IF}t - \phi_\epsilon) + \frac{j}{2} (1 + \Delta A) \cos(\omega_{IF}t - \phi_\epsilon) \quad (62)$$

Shifting the Q terms by 90° and adding equations (59) to (61), and (60) to (62) results in:

$$S'(t) = \frac{1}{2} \cos(\omega_{IF}t) + \frac{j}{2} \sin(\omega_{IF}t) - \frac{1}{2} (1 + \Delta A) \cos(\omega_{IF}t - \phi_\epsilon) + \frac{j}{2} (1 + \Delta A) \sin(\omega_{IF}t - \phi_\epsilon) \quad (63)$$

$$\text{IM}'(t) = \frac{1}{2} \cos(\omega_{\text{IF}}t) - \frac{j}{2} \sin(\omega_{\text{IF}}t) + \frac{1}{2} (1+\Delta A) \sin(\omega_{\text{IF}}t - \phi_e) + \frac{j}{2} (1+\Delta A) \cos(\omega_{\text{IF}}t - \phi_e) \quad (64)$$

Substituting $\omega_{\text{IF}}t = 0$ (at $t=0$) and finding the magnitude of the two signals results in:

$$|S|^2 = \left[\frac{1}{2} + \frac{1}{2} (1+\Delta A) \cos(\phi_e) \right]^2 + \left[\frac{1}{2} (1+\Delta A) \sin(\phi_e) \right]^2 \quad (65)$$

$$|IM|^2 = \left[\frac{1}{2} - \frac{1}{2} (1+\Delta A) \cos(\phi_e) \right]^2 + \left[\frac{j}{2} (1+\Delta A) \sin(\phi_e) \right]^2 \quad (66)$$

Now the image rejection ratio can be calculated as:

$$\text{Image Rejection (dB)} = 10 \log \left[\frac{(1 + 2(1+\Delta A) \cos(\phi_e) + (1+\Delta A)^2)}{(1 - 2(1+\Delta A) \cos(\phi_e) + (1+\Delta A)^2)} \right] \quad (67)$$

Figure 3.26 is a plot of the amount of image rejection vs. phase error (in degrees) and amplitude mismatch (in dBs).

Figure 3.27 is a plot of the amount of image rejection vs. phase error (in degrees) and percentage of amplitude mismatch.

Figure 3.26 Image rejection vs. phase error and amplitude mismatch (dB)

Figure 3.27 Image rejection vs. phase error and percentage amplitude mismatch

3.6.4 Phase Noise Analysis

The same phase noise sources and noise generation mechanisms appear in the quadrature oscillator as they do in the single VCO. However, depending on the circuit topology, quadrature VCOs might have higher noise than the single differential oscillator of the same design. One example is the complementary negative gm with NMOS current source. As discussed earlier, if this circuit is not designed carefully the current source can give a significant increase to phase noise. Now in the quadrature version the coupling transistors will not only couple the RF signals of the second VCO, but also the noise from the other circuit. The noise could be a current source that consists of flicker noise and the $2f_c$ noise at the common mode of the two transistors' sources.

Figure 3.28 is a plot of the simulated frequency-modulated flicker noise in the quadrature oscillator design presented in Chapter 4. The increase in the frequency-modulated noise values over the single oscillator can be attributed to the higher $K_{VCO,fb}$ value in the quadrature circuit. Figure 3.29 is the combined phase noise of the same quadrature oscillator, which is mainly dominated by the frequency-modulated flicker noise.

Figure 3.28 Calculated frequency-modulated flicker noise of the quadrature oscillator

Figure 3.29 Calculated combined phase noise of the quadrature oscillator

3.8 Previous Work

Table 1 and Table 2 are some examples of recent publications related to negative-gm oscillators design. The wide range of VCO designs listed below give an overview of how results vary with respect to different topologies and different tank circuits (on chip vs. bond wire inductors and pn-junction vs. MOS varactors).

3.8.1 Single Differential VCO

Source	Inductor Type	Technology	Power	Tuning Range	Frequency	Quality of Tank	Phase Noise
On chip Inductors							
[23]	Three point balanced octagonal	0.25 μ CMOS	18mA x 1.8V	28%	1.8 GHz	8.8	-113.5 dBc/ Hz @ 100kHz
[11]	Symmetrical polygon	0.6 μ BiC-MOS	19mA x 1.8V	1.79-2 GHz 11% 1.1-1.8V	2.0 GHZ	11	* -111.1 dBc/Hz @ 100 KHz
[21]	2 On-chip squares	0.25 μ CMOS	4mA x 1.5V core	10% tuning range	1.8 GHz	7.5	-107 dBc/Hz @100 KHz
[26]	2 On-chip octagonal	0.7 μ CMOS	4mA x 1.5V (core)	250MHz	1.8 GHz	5.7	* -104 dBc / Hz @ 100kHz
[28]	2 On-chip square	0.35 μ CMOS	4mA x 2.5V	26%	2.4 GHz	8.9	-101dBc/Hz @600KHz of 2.6GHz"
[24]	2 On- chip circular	0.25 μ CMOS	12mW (2.5Vx5mA)	250MHz	1.9 GHz	4	"-100dBc/Hz @100KHz"
[22]	2 On- chip octagonal	0.8 μ CMOS	9mA x 2.5V	"15% for 1.5-2.5V"	2.4 GHz	5	"-98dBc /Hz @100 KHz"
[20]	Coupled inductors	25 GHz Bipolar	12mA x 2.7V core+6mA buffers	150MHz 0 to 2.7V	2.0 GHz	4	-102 dBc/Hz @100 KHz
[5]	2 On-chip polygon	20GHz Bipolar	3V	N/A	2.6 GHz	~10	-90dBc/Hz @ 100KHz
[6]	2 squares	0.5 μ 25GHz Bipolar	6.4mA x 3.3V/+6mA buffer	"~1.8-1.9GHz", "w/1-3V"	1.9 GHz		-103dBc@100KHz
Bond Wire Inductors							
[30]	Spiral + bond wire +	0.8 μ CMOS	6.8mW @1.5V &	1-1.125GHz	1.1 GHz	17	-112 dBc/Hz @100 KHz"

	package Leads		12.7mW @ 2.7V	& .925GHz- 1.12GHz			
[29]	Bond wire	0.35 μ CMOS	1mA x?	"1.8- 2.45GHz"	1.9 GHz	14	106.5dBc/Hz @ 100KHz
[25]	Spiral + bond wire	0.35 μ CMOS	6mA x 2V	350MHz "- .9 / -.1" +/-17%"	1.3 GHz	4.5	*-105 dBc/Hz @ 100kHz

Table 1 Examples of negative-gm VCO papers

3.8.1.1 Completely Monolithic Oscillators

*Reference [23] is a PMOS cross coupled VCO with an NMOS current source. It uses a three terminal balanced octagonal inductor. $L=2.875\text{nH}$, 3.7 ohms at 1.8GHz, $Q = 8.8$. It has a shunt capacitor at the gate of the current source. It uses a delay-line method incorporated in a dedicated phase noise measurement method, so it is unclear if noise from amplitude modulation has been taken into account.

Reference [11] is an NMOS cross-coupled VCO with a high-impedance PMOS current source. It refers to the effect of the current source flicker noise on the phase noise performance. Careful design of the current source led to enhanced phase noise performance. PMOS transistors were used for the source since they inherently have less flicker noise. Increasing the area of the PMOS transistor further minimized the noise. A bypass capacitor is added to suppress the $2fc$ signal at the common node to prevent it from being converted down to the center frequency. The inductor Q is 11.

Reference [21] is a complementary negative-gm VCO. For tuning, this circuit has NMOS switches and uses MOS capacitors to fine tune the frequency. At a given level of input power, it suggests that the best phase noise performance is achieved at low supply voltages and higher currents. It also compares all-NMOS VCOs to the complementary devices. Measurement results show that all NMOS circuits have inferior noise performance. One of the reasons behind it is that the complementary design has better rejection for flicker noise up-conversion. This is only true if the gms of the NMOS and PMOS transistors are matched. Q is 7.5.

Reference [26] is an NMOS cross-coupled VCO with a PMOS current source. This circuit has an optimized inductor with Q of 5.7 at 1.8 GHz. It uses p/n junction

varactors as variable capacitors. The phase noise performance of this design is relatively good in spite of the low inductor quality. This could be because it uses the PMOS current source.

Reference [28] is a complementary cross-coupled VCO with an NMOS current source and MOS capacitors. It presents a design optimization process that doesn't allow the circuit to operate in the limited current regime leading to reduced switching in the transistors. The Q of the inductor is 8.9.

Reference [24] is an NMOS cross-coupled VCO with a fixed bias. No current source is used and the circuit is biased with resistors. The inductor Q is 4. Using fixed bias might be one of the reasons behind its relatively good noise performance. The resistor biasing can help eliminate the flicker noise of the transistor current source, which could up-convert to around the carrier.

Reference [22] is an NMOS cross-coupled with a PMOS current source. The VCO uses MOS varactors and two octagonal inductors with a quality factor of 5. This suggests that MOS capacitors have a better quality than pn diodes.

Reference [5] also refers to the modulating factor from the current sources to that tank, $K_{VCO,lb}$. The effect of this factor is more evident at higher biases, since the $K_{VCO,lb}$ value is higher, as it is experimentally proven in this paper. The Q of the tank is ~ 10 .

Reference [20] is a bipolar cross-coupled oscillator with coupled inductors as feedback. Coupling enables higher inductance without an increase in the tank losses.

Reference [6] is a differential common-base Colpitts oscillator. This paper discusses the non-linearity in the varactor. It suggests that at lower frequencies in the tuning range, phase noise degrades due to higher K_{VCO} value at lower frequency. This higher value means that noise modulates the tank with a larger coefficient.

3.8.1.2 With Bond Wire Inductors

Reference [30] is a PMOS cross-coupled VCO with a PMOS current source. Inductors are a combination of spiral, bond wire and packaged lead inductors. Pn junction diodes are used as varactors. The combined inductor Q is 17.

Reference [29] is a complementary cross-coupled VCO with MOS varactors and a combination of bond wire and on chip spiral. Spiral inductors are used to compensate for the spread of the bond wire inductors ($\pm 20\%$). It suggests that the best biasing condition is when the output amplitude is equal to the saturation amplitude. The Q of the tank is 14.

Reference [25] is an NMOS cross-coupled VCO with an NMOS current source. A combination of spiral and bond wire inductors and a MOS varactor. The combined inductor Q is 4.5.

3.8.2 Quadrature Differential VCO

As mentioned earlier, Table 2 contains some examples of recent publications related to quadrature negative-gm oscillators design

Source	Technology	Power	Tuning Range	Center Frequency	Phase Noise	Phase Error
[9]	1 μ CMOS	43mW	860-958MHz	900 MHz	-100dBc/Hz @100 KHz	<1
[33]	0.25 μ CMOS	20mW	1.72-1.99GHz	1.9GHz	~123dBc/Hz@300KHz	<5

Table 2 Examples of negative-gm quadrature VCO papers

Reference [9] is an all NMOS cross-coupled quadrature VCO with a PMOS current source. It also integrates on chip mixers to test the phase error. The inductors used have a deep air pit under them, where the substrate was selectively removed to reduce substrate capacitance. A MOS varactor are used for tuning, this varactor is connected to the rail and is tuned by a control voltage that changes the drain current and so the junction capacitance.

Reference [33] has NMOS and PMOS cross coupled transistors with no current source, and is self biased by the rails. The circuit has symmetrical octagonal inductors and NMOS varactors in the tank. The Q of the inductor is 8.

With regards to the single VCO one notes that high quality tank circuit are not the single most important factor in determining phase noise performance. For example [11]

uses a very high Q inductor ~ 11 , however [23] achieves better noise performance with an $8.8\text{-}Q$ inductor. So more research is needed to further understand phase noise performance of these circuits.

Also the limited research on the quadrature VCOs leaves room to further investigate these circuits.

Chapter 4 Design and Layout

4.1 0.18 μ CMOS Technology

This technology, manufactured by Taiwan Semiconductor Manufacturing Company (TSMC) and supplied through collaboration with PMC-Sierra, is a 0.18-micron, single poly, six metal CMOS process. The recommended nominal supply voltage is 1.8 V. It is possible to use transistors which operate at 3.3 V, but these are non-minimum gate length, so their speed is lower. The 0.18 μ transistor model is HSPICE level 49, which is the Spice equivalent for Berkley's BSIM3 model. Spectre RF models are also available and were used to simulate the circuit [34].

4.2 Building the VCO Circuit

4.2.1 Tank circuit

4.2.1.1 Inductor

The inductor is the key component in determining the performance of a fully integrated VCO. This is because making a high quality inductor, using the conventional available technology, is very difficult.

The losses in the inductor that contribute to this low quality can be attributed to metal losses and substrate losses. Substrate losses include Eddy currents, while metal losses are dependent on the resistivity of the metal as well as skin effect.

Since conventional CMOS technology was used, no replacement material or extra processing steps were available. As a result, the only adjustment that could be made to optimize the quality at the center frequency was to the dimensions of the inductor.

4.2.1.1.1 Modeling and Simulation

The simulation tool ASITIC was used to model the inductor in this design [16]. ASITIC uses an input file (technology file) to describe the process in which the inductor is fabricated. This file contains information about the substrate, metal and vias. For the substrate, resistivity, thickness and permittivity are specified. For the metal, sheet resistivity, metal

thickness and distance from the bottom layer are needed. Given this information, ASITIC performs numerical analysis to calculate the S-parameter values of the 2-port inductor.

Figure 4.1 Inductor model

In this model R is the series resistance of the metal lines. C_{OX} is the parasitic capacitance of the silicon dioxide insulator layer. R_S and C_S are the parasitic resistance and capacitance between the structure and the substrate, while C_{BR} is the interwinding capacitance through the oxide and the substrate [15].

Given the required frequency of operation and value of inductor, ASITIC optimizes the inductor dimensions for highest quality at the given frequency. Then ASITIC will also determine the values of the components in the model of Figure 4.1. This model is not a broadband one, it is calculated using the given center frequency. However, ASITIC has a command that calculates the S-parameter of the 2-port inductor at different frequencies. These S-parameter values can be used to fit the component values of the model in Figure 4.1 using HSPICE to make it a broadband model.

4.2.1.1.2 Layout and Optimization

The inductor layout is a symmetrical square as shown in Figure 4.3. The top-level metal layer (level 6 in 0.18 μ CMOS) is usually used since it has the least amount of loss. For CMOS 0.18 μ the value of metal 6 resistivity is around 0.036 $\Omega/\square$. A lower-level metal is needed to connect the line at the points of overlap.

Only one symmetrical inductor is needed in the LC tank (for the circuit shown in Figure 3.8). In the case of the other designs (Figure 3.7), two inductors are required to make the circuit symmetrical on the differential outputs. The use of a symmetrical inductor saves chip area. Furthermore, it can be shown that the symmetrical layout has a higher Q than that of two inductors connected in series.

Figure 4.2 is a model for two inductors connected in series. It can be seen that in this case there are three nodes at which the inductors have a “lossy” path to ground. In the symmetrical square inductor case there are only two lossy paths to ground.

Figure 4.2 Model of two inductors connected in series

A wider metal line means lower series resistance; it also means smaller inductor values for a given area. Increasing the area to achieve higher inductance will mean more capacitance, leading to a lower self-resonance frequency. As for spacing, the tighter it is, the higher the peak-Q frequency for the same area and number of turns.

Another important factor is area; larger area usually means more substrate losses [6]. In addition, the eddy currents effect would significantly increase if the turns were filled to the center [12]. This is because the magnetic field would be forced to go through the inductor itself, and not through the hollow part, as would be the case if the inductor had fewer turns.

Figure 4.3 Symmetrical square inductor layout

4.2.1.1.3 Inductor Values

Following the procedure described in the previous section, the dimensions of the structure were optimized to create a 5nH inductor with the highest possible quality factor at the center frequency 2.4GHz. The optimized structure is a $250\mu \times 250\mu$ symmetrical square, with 4 turns. The metal width is 11μ and the spacing between the turns is 3μ . The inductor is built with metal 6 (highest level metal with the lowest resistive loss), while the transition segments are built using metal 5. After obtaining the S-parameter values from ASITIC and running the optimizing file of HSPICE to fit the model in Figure 4.1, the values obtained were as follows:

Figure 4.4 Inductor model with parameter values

Using ASITIC the self-resonance frequency and quality factor are given as follows:

$f_{\text{Resonance}} = 7.45\text{GHz}$ $Q = 8.236$ at 2.4GHz . Figure 4.5 shows the inductance and the quality factor of the inductor versus frequency.

Figure 4.5 Inductance and Q values of the inductor versus frequency

4.2.1.2 Varactor

A varactor is a voltage-controlled variable capacitor. It is used in VCOs to vary the center frequency as desired. The quality of the on-chip varactor is usually higher than that of the on-chip inductor. Two ways to create varactors are:

1. A p-n junction which forms a simple diode with a given capacitance value. Changing the reverse bias on the junction varies its capacitance. Applying a larger reverse-bias voltage results in a wider depletion region and hence a smaller capacitance and higher oscillation frequency. The junction could also be slightly forward biased for wider frequency tuning at the lower end of the frequency range.

Figure 4.6 Cross section of a pn-junction varactor

2. MOS Varactors: The basic structure is similar to a MOS transistor. For example, consider the MOS varactor shown in Figure 4.7, which is similar to an n-MOS device except that it is formed in an n-well to reduce parasitics. The structure has the drain and source tied as one terminal and the gate as the other terminal. The operation of this varactor is divided into two modes: accumulation and depletion for which the dominant capacitance is the oxide capacitance and depletion capacitance respectively. At a given voltage the total value of the capacitance is the series combination of these two.

When the applied voltage, between the gate and the drain/source, is above the flatband voltage, the silicon surface is accumulated with electrons and the capacitance seen from the gate is the oxide capacitance. As the applied gate voltage decreases, the silicon surface becomes less accumulated and eventually charge-

free, beyond which the surface goes through depletion. This variation from accumulation to depletion varies the capacitance from a maximum to a minimum [10].

Figure 4.7 Cross section of a MOS varactor

4.2.1.2.1 Modeling and Simulation

The 0.18 μ CMOS technology used for the design has a model for a special pnp transistor. For this reason, in this design, this transistor was used as the basis for a pn-junction diode. The collector of this transistor is the substrate, and so is always grounded. The base is an n-well region and the emitter is p+ diffusion inside the well. Figure 4.6 shows the structure. It also shows that there are two diodes in it, one between the n-well (base) and the p-substrate (collector), and the other is between n-well (base) and the p+ diffusion (emitter). The latter was used in the tank circuit since it has higher quality. The better quality is due to the higher mobility of the p+ region over the p-substrate [6].

4.2.1.2.2 Varactor Values

Each diode, or varactor structure, is created by connecting 38 pnp transistors (each one is 5 μ x 5 μ). In order to have a symmetrical tank, two of these diodes are connected back to back. In the next section, some issues associated with the varactor placement will be discussed.

The total capacitance of these two varactors in the tank is approximately 0.9pF.

4.2.1.2.3 Placement and Layout

Due to the existence of the substrate diode [6], the placement of the varactor should be done carefully. In order to avoid including the lossy diodes in the tank circuit they should be connected as shown in Figure 4.8. In this figure point C is an AC ground since it is connected to the control voltage. Points A and B are the two sides of the LC tank. In this configuration the lossy diodes are grounded and will not degrade the quality of the tank. Since nodes A and B are biased around mid-rail (0.9V) there will not be a difference in the tuning range when placing the varactor this way, as opposed to having them reversed in the tank.

Figure 4.8 Varactors placement in the tank circuit

In order to reduce the resistive losses in this structure, the overall varactor is built with small transistors ($5\mu\text{x}5\mu$) connected in parallel. In this way the capacitance of the junctions is added while the resistance is divided. This can be seen when modeling the varactor as follows

$$Z(\omega) = R + 1/j\omega C. \quad (68)$$

where R is the series resistance of the varactor

Connecting a number of the transistors (N) in parallel would result in an equivalent impedance of

$$Z'(\omega) = R/N + 1/j\omega(NC). \quad (69)$$

Equation (69) shows that connecting a number of varactors in parallel decreases the overall series resistance and increases the quality factor of the capacitor.

4.2.2 Amplifier circuit

A differential topology was used for the VCO with both NMOS and PMOS transistors cross coupled. Figure 3.9 shows the circuit.

The output voltage is at approximately mid-rail (0.9V). A drawback of this topology is the voltage at this point, which is dependent on the bias of the NMOS and PMOS transistors. Now any mismatch in the transconductance will cause this point to swing (with the changing flow of current in the transistor). This swing in the DC voltage modulates the varactor, and degrades phase noise.

Each transistor's layout consisted of multiple long fingers. Since these fingers are connected in parallel, this layout reduces the gate resistance (see Figure 4.8). Furthermore, when connecting the transistor terminals to the metal layers, many contacts were used. This again ensures that no significant resistance is added to the transistors from these contacts.

Each transistor was surrounded by ground connections, where p+ diffusion is connected to substrate.

NMOS transistors were made $40\mu/0.18\mu$ and PMOS were $48\mu/0.18\mu$. Their respective g_m values are 7ms and 4.2ms. The implications for mismatching the g_m values will be discussed later.

Figure 4.9 Transistor layout

4.2.3 Biasing

For the current source, an NMOS current mirror was used. The mirror transistor's size ratio is 1:1. A higher ratio would contribute to phase noise, since the flicker noise in the diode-connected transistor would be multiplied by that ratio.

The transistor size was made to be $20\mu/0.18\mu$. A better design would make the length bigger than the minimum of 0.18μ to increase the output impedance, reduce the short channel effect and reduce flicker noise, since the latter is inversely proportional to the gate area.

4.2.4 Buffers

A source-follower transistor was used as shown in Figure 4.10. The output was directly connected to the gate of the source-follower. The follower was biased with a current mirror. The two current sources, of the two buffers, were connected to the same diode transistor of the mirror with a ratio of 1:3.6. This ratio helps reduce power dissipation.

The buffers are large transistors that have low output impedance. In this design the buffer transistor (M1 in Figure 4.10) was made to be $100\mu/0.18\mu$, which results in an output impedance of around 47 ohms (with a bias current of 1.8mA for each buffer). M2 and M3 are the current source transistors, which are $140\mu/0.18\mu$ and $14\mu/0.5\mu$ respectively.

Figure 4.10 Buffer circuit

4.2.5 Differential VCO Layout

Figure 4.11 shows the final placement of the VCO components. The varactor layout could be optimized further to reduce the occupied chip area.

Figure 4.11 Single differential VCO layout

4.3 Phase Noise Calculations of the Single Differential Oscillator

The contributions of noise from the different mechanisms (tank quality, frequency modulation and amplitude modulation) were calculated and added together, all using Excel spreadsheets. These calculated results were then compared with measured data in Chapter 5.

4.3.1 Q of the tank

Phase noise was calculated according to the filter model of the oscillator using the following equation (Leeson's):

$$\mathcal{L}(\Delta\omega) = 10 \log \frac{2}{A_o^2} \frac{kTF}{C} \frac{\omega_o}{Q} \frac{1}{(\Delta\omega)^2} \text{ dBc/Hz} \quad (70)$$

The values used for the parameters in the above are as follows:

$$kT = 1.38e-23 \times 298$$

$$A_o^2 = 0.126mV^2 (-12dBm)$$

$$\omega_o = 2.25GHz$$

$$Q = 8$$

$$C = 0.9 pF$$

$$F = 3$$

A value of 8 was used for the tank quality (Q), which is a “rounded down” value of the ASITIC simulated inductor Quality factor.

The frequency offset from the carrier ($\Delta\omega$) was swept from 100Hz to 1GHz and phase noise $\mathcal{L}$ was plotted.

The results are plotted in Figure 4.14. Note that phase noise drops at a rate of 20dB/decade as can be predicted from equation (70). At a 100KHz offset, the phase noise is -120dBc/Hz.

4.3.2 Low Frequency Flicker Noise

Flicker noise was plotted for the NMOS current source transistor:

$$V_n^2 = \frac{K}{WLC_{ox}} \frac{1}{f^{af}} \text{ (V}^2 \text{ / Hz)} \quad (71)$$

Where V_n^2 is a voltage noise source at the gate of the source transistor. The values of K , af and C_{ox} were extracted from the HSPICE model. The width and length of the current source transistors in Figure 3.9 are 20μ and 0.18μ respectively. Frequency f was swept between 100Hz and 1GHz.

Figure 4.12 represents the low frequency noise at the gate of the current source transistor.

Figure 4.12 Flicker noise in dBm versus frequency for an NMOS transistor

4.3.3 Amplitude Modulated Flicker Noise

The low frequency flicker noise converts up to the carrier by amplitude modulation since the amplifier transistors act like a commutating mixer.

Equation (72) describes this mechanism, where:

$$V_{AM\ Noise}(\Delta\omega) = \frac{1}{4} \frac{V_n(\Delta\omega)}{1/gm_3} \frac{1}{gm_5} \frac{4}{\pi} A_c \quad (72)$$

Figure 4.13 Mixer representation of the VCO

where:

gm_5 is the transconductance of M5

V_n is the $1/f$ noise voltage at $\Delta\omega$ from DC

gm_3 is the transconductance of the current source transistor.

The results from equation (72) were plotted after calculating the dBm values of $V_{AMNoise}$.

In this case the noise power drops at a rate of -10dBc/Hz , since amplitude modulation drops at $1/f$ rate, as suggested by (72). Results are plotted on Figure 4.14.

4.3.4 Frequency Modulated Flicker Noise

This section is divided into two parts:

1) Noise directly across the varactor:

The main source for this noise is the amplifier transistors. These transistors have thermal and flicker noise that directly modulate the varactors. The noise voltage level between each of the differential outputs and ground is

$$V_n = I_{noise} \cdot R_{eq} \quad (73)$$

where,

$$R_{eq} = \frac{1}{gm} \quad (74)$$

At the output node, the equivalent impedance seen at low frequency, because the inductor is a short, is $1/g_m$ (the PMOS transistor is diode connected).

$$I_{noise} = I_{thermal} + I_{flicker} \quad (75)$$

and,

$$I_{thermal} = \sqrt{4kT \left(\frac{2}{3} g_m \right)} \quad (76)$$

$$I_{flicker} = \left(\sqrt{\frac{K}{WLC_{ox}}} \frac{1}{f^{af}} \right) / R_{eq} \quad (77)$$

$$\text{Now } \Delta f = V_n * K_{VCO} \quad (78)$$

After calculating the Δf , phase noise at an offset ($\Delta\omega=f_m$) from the carrier is then equal to:

$$\mathcal{L} = 20 \log \frac{\Delta f}{2f_m} \quad (79)$$

2) Noise in the bias line:

In order to calculate the value of the frequency-modulated noise, due to bias line noise, we first need to translate the flicker noise voltage fluctuations into current fluctuation. Then these values can be used along with the $K_{VCO,b}$ value measured and plotted in Figure 5.9 to calculate Δf .

The value of noise current is:

$$I_{bnoise} = \left[\frac{K_p}{2} \frac{W}{L} \left((V_{gs} + \Delta V_{gs}) - V_t \right)^2 - I_b \right] \quad (80)$$

where :

$$\Delta V_{gs} = \sqrt{V_{bn}^2} \quad (81)$$

V_{bn}^2 is the value of flicker noise at the gate of the current source transistor at a given offset (f_m) from DC. V_{bn}^2 is calculated and plotted in Figure 4.12 for frequencies from 100Hz

to 10GHz. The value of I_{bnoise} has to be calculated for all the frequencies in this range (the I_n value was calculated for 10 points in each decade).

Now $\Delta f = I_{bnoise} * K_{VCO,fb}$

After calculating the Δf , phase noise at an offset ($\Delta\omega = f_m$) from the carrier is then equal to:

$$\mathcal{L} = 20 \log \frac{\Delta f}{2 f_m} \quad \text{dBc/Hz} \quad (82)$$

Figure 4.14 different mechanism contributions to phase noise in the single oscillator

The results are plotted in Figure 4.14. It can be seen in this figure that frequency-modulated phase noise drops at -30dB/decade at an offset from the carrier. The reason, taking the bias line flicker noise as an example, is that bias current flicker noise (I_{bnoise}) is

proportional to $\frac{1}{f^{0.5}}$, since current noise is calculated as a square root of the flicker noise

power spectral density V_{bn} . Now since Δf is proportional to the current noise spectral

density, it is also proportional to $\frac{1}{f^{0.5}}$. Since $\beta = \frac{\Delta f}{f_m}$ is proportional to $\frac{1}{f^{1.5}}$, when substi-

tuting that into equation (82) we see that phase noise drops at -30 dB/decade. In other words, -10 dB comes from Δf and another -20 dB from $\frac{1}{f_m}$.

4.3.5 Total Combined Phase Noise

Noise power from the three different mechanisms, Q of the tank, frequency modulation of flicker noise and amplitude modulation of flicker noise were. The result is plotted in Figure 4.15.

From Figure 4.15 we can see which noise is more dominant at which offset. For example up to 100 KHz phase noise slopes at a -30 dBc rate, meaning that the frequency modulated flicker noise is more dominant there. Then, between 100 KHz and 1 MHz, phase noise briefly slopes at a rate of -20 dBc/Hz, indicating that noise due to tank losses are more prominent at this frequency. Finally, the slope becomes -10 dBc/Hz in which case the up-converted flicker noise, due to amplitude modulation, is the dominant source. To see these slope values more clearly, refer to the data from the Excel sheet in the appendices.

Figure 4.15 Combined phase noise in dBc /Hz versus frequency offset from the carrier

However, during the measurement, the spectrum analyzer was determined to have a noise floor -140dBm . With respect to the carrier, which is around -15dBm , the noise floor is, where it should be, around -125dBc/Hz . Taking that into consideration, phase noise would look more like Figure 4.16.

Figure 4.16 Combined phase noise in the differential VCO figure showing the limiting thermal noise floor

4.4 Building the QVCO Circuit

In implementing the Quadrature VCO, the same circuit design of the single VCO was used for the individual oscillators. Some modifications were made to the two oscillators to generate quadratures as explained earlier.

4.4.1 Tank Circuit

The inductors in the individual oscillators were left unchanged. However, the varactor size was slightly decreased to compensate for the increase in the parasitic capacitance of the added devices and traces. The total capacitance of the two varactors in the tank is approximately 0.56pF (this reduction was based on simulation results)

4.4.2 Amplifier Circuit

Coupling transistors were added to the oscillators. In this design, the coupling transistors were of the same size as the amplifier transistors. Later investigation revealed that smaller coupling transistors would have been sufficient for quadrature coupling, in which case less input power would have been required.

The coupling transistors' sizes have an effect on the phase noise and image rejection performance of the quadrature oscillator. According to [27] [36] when coupling with a parallel transistor, a 1 to 1 size ratio in the coupling transistors to the amplifier transistor resulted in best image rejection. The implications of this ratio on phase noise performance were not clear. This same reference indicates that a 1 to 2 in a series coupling method has superior phase noise performance when compared to the parallel topology. This 1 to 2 ratio is where the two circuits (parallel and series) have the same image rejection results.

4.4.3 Biasing

The two current sources of the individual VCO were biased with the same current mirror configuration as the previous circuit, however the source transistor size was increased to $45\mu/0.18\mu$ in order to provide the extra current needed for the coupling transistors. In addition, the metal line width was increased in order to handle the extra current.

4.4.4 Buffers

Same-size buffers as the single VCO were used. However, in this circuit there are four buffers, and are all biased with one current mirror of 1:10 ratio.

4.4.5 Quadrature VCO layout

Figure 4.17 shows the placement of the two coupled VCOs. The inductors were placed as far from each other as possible to avoid coupling.

Figure 4.17 Quadrature differential VCO layout

4.5 Phase Noise Calculations of the Quadrature Differential Oscillator

4.5.1 Q of the tank

The same equation was used to calculate this noise as the single differential oscillator. However, the output level and the capacitor values were different. The following values were used in equation (70):

$$kT = 1.38e-23 \times 298$$

$$A_o^2 = 0.02mV^2 (-20dBm)$$

$$\omega_o = 2.4GHz$$

$$Q = 8$$

$$C = 0.56pF$$

$$F = 3$$

Phase noise drops at a 20dB rate as before, Figure 4.19. However, it is higher than the single oscillator. Two reasons for that are lower output power and a smaller capacitor value. Note, we assumed that they both have the same value for the tank quality factor, which is dominated by the quality factor of the inductor.

4.5.2 Low Frequency Flicker Noise

The low frequency flicker noise of the source transistor of the quadrature oscillator is lower than that of the single oscillator. These results for the quadrature circuit were plotted on Figure 4.18. The main reason is that the source transistor is more than twice as big as the single differential circuit, $45\mu/0.18\mu$ as opposed to $20\mu/0.18\mu$ of the single oscillator. Bigger transistors mean less flicker noise. This explains why flicker noise is 3.5dB lower than that of the single VCO.

Figure 4.18 Flicker noise in dBm versus frequency for an NMOS transistor

4.5.3 AM Modulated Flicker Noise

The current source transistor drives more current into the quadrature oscillator than the single VCO. It also has a bigger W/L value, meaning a higher transconductance (g_m). Considering the amplitude modulation equation (72), it can be seen that this leads to a higher amplitude modulation factor. However, since this source transistor is bigger in area than the source transistor in the single oscillator, the low frequency flicker noise is lower. The result is plotted in Figure 4.19.

4.5.4 FM Modulated Flicker Noise

Although the low frequency noise is lower compared to the single oscillator, when converted around the carrier it becomes higher because of the bigger $K_{VCO,fb}$ value. Bigger $K_{VCO,fb}$ (measured bias current modulation of the center frequency) leads to higher noise around the carrier. The result is plotted on Figure 4.19.

Figure 4.19 Different mechanism contribution to phase noise in the quadrature oscillator

4.5.5 Total Combined Phase Noise

Adding the noise from the different mechanisms together results in the total phase noise. The results are plotted on Figure 4.20. The phase noise slopes is -30dB/decade up to 1MHz , after which the slope changes to a lower value indicating a flicker noise corner frequency. Between 1MHz and 10MHz phase noise drops at a 20dB/decade rate, this is where the tank losses are dominant. After 10MHz the amplitude-modulated noise is more prominent and phase noise drops at a 10dB/decade rate.

Figure 4.20 Combined phase noise in dBc /Hz versus frequency offset from the carrier

Again considering the spectrum analyzer's noise floor the phase noise spectrum should look like Figure 4.21. Since the carrier power is -20dBm , the noise floor appears at -120dBc/Hz .

Figure 4.21 Combined phase noise in the quadrature VCO figure showing the limiting thermal noise floor

4.6 Final Chip Layout

When connecting the circuits to the bond pads, many layout issues were taken into consideration. First, the lines carrying the DC bias were made wide enough so they would handle the amount of current designed to flow through them. The current handling capabilities are listed in the design rules, however a typical rule of thumb is that the metal width should be around $1\mu\text{m}$ for 1mA . In addition, for the quadrature VCO attention was paid to make the lines as symmetric as possible; so as to minimize any possible mismatch.

The single and the quadrature VCOs were placed on the same die. The single VCO occupied $(0.4\text{mm} \times 0.6\text{mm})$, while the quadrature VCO was $(0.8\text{mm} \times 1.2\text{mm})$. Some of the buffer bond pads were shared between the two circuits; however, the two circuits are powered separately. Figure 4.22 is a picture of the final circuit layout.

Figure 4.22 Layout of the two circuits and placement of bond pads

Chapter 5 Testing and Results

This chapter contains details of the testing setup for the VCO and the quadrature VCO circuits. It also includes the test results (i.e. frequency of operation, phase noise and frequency tuning range) of both circuits.

5.1 Test Setup

The test setup is shown in Figure 5.1. The chip was mounted on a board for testing. The board has a current-limiting resistor feeding the bias pin on the chip and a potentiometer is connected between this current-limiting resistor and the power supply. The potentiometer is used to adjust the input current.

De-coupling capacitors are placed at the outputs, which are connected to SMA connectors. However since the circuit generates two differential outputs (180 degrees apart), a balun was used to convert these two outputs into one. This balun is placed after the de-coupling capacitors.

The spectrum analyzer was used to measure the output signal. High frequency cables were used to connect the output SMA to the spectrum analyzer.

Figure 5.1 Single differential VCO test setup

5.2 VCO Results

Figure 5.2 Output of the signal VCO at 2.3GHz

When powered up, the oscillator had an output spectrum as shown in Figure 5.2. The chip had a 1.8V power supply and 1mA core circuit current and produced -14.67dBm of output power at 2.3GHz.

5.2.1 Tuning Range

Using a potentiometer placed in the control voltage signal path, this voltage was varied and the output frequency was observed. The measured tuning range of the VCO is plotted in Figure 5.3 and the K_{VCO} is plotted in Figure 5.4. These results were again taken with 1.8V power supply and 1mA core current.

The frequency was from 1.65GHz – 2.35 GHz, that is 600MHz of tuning range. The control voltage was varied from 0.5-1.8 V.

Figure 5.3 Measured frequency vs. control voltage for the single VCO

Figure 5.4 Calculated K_{VCO} from measurements of the single VCO

5.2.2 Phase Noise

Phase noise was measured using software integrated into the Spectrum Analyzer at 100KHz offset from different center frequencies to cover the tuning range. Figure 5.5 shows a plot of these results. The best result was -93dBc/Hz @ 100KHz offset of 2.25GHz carrier, while the result was -85.8dBc/Hz @ 100 KHz offset from a 2.35 GHz

carrier. One thing to note is that in Figures 5.5 and 5.6 the change in noise level is about -30dB between 10KHz to 100KHz. In Figure 5.5 the slope changes at around the 100KHz point to a lower rate, indicating the $1/f$ noise corner. However, in Figure 5.6 the corner between $1/f^3$ noise and $1/f^2$ cannot be seen and the noise continues to fall at $1/f^3$ rate.

Figure 5.5 Phase noise of the single differential VCO at 100KHz offset from a 2.25 GHz carrier signal

Figure 5.6 Phase noise of the single differential VCO at 100KHz offset from a 2.35 GHz carrier signal

Figure 5.7 Phase noise of the single VCO across the tuning range

Another measurement was performed to determine the shift in the center frequency when changing the bias at a fixed control voltage. It was found that changing the bias from 0.8mA to 1.8mA shifts the frequency by about 4.4 MHz. This test shows how noise in the bias line modulates the center frequency. Figure 5.8 contains these results, where we see that at lower currents the slope of the graph is relatively small, Figure 5.9, but it increases as the current goes up. These results are in close agreement with those reported by [12].

Figure 5.8 Measured change in center frequency vs. change in bias current

Figure 5.9 Measured rate of change in center frequency vs. changes in bias current

5.2.3 Comparison between Calculated and Measured Phase Noise Results

Figure 5.10 presents the comparison with two measurement sets, the first is offset from a 2.25GHz carrier and the other is offset from a 2.35GHz carrier. One thing to note is that the measurements at small offsets from the carrier are not very accurate because the oscillator was not locked (i.e. it was free running).

5.2.3.1 Phase Noise at a 100kHz offset

At a 100kHz offset, frequency-modulated flicker noise is the most prominent. This calculation of noise is very sensitive to the $K_{VCO,IB}$, which was measured using a large signal variation in the bias current. This value might not be accurate when dealing with the small signal values of flicker noise.

The measured result was -85.83dBc/Hz from 2.35GHz, while the calculated value was -88.5dBc/Hz from 2.35GHz. These values are very close.

5.2.3.2 Phase Noise versus Offset Frequency from the Carrier

Figure 5.10 is a comparison between calculated and measured phase noise up to 1MHz offset from the center frequency. The two graphs are in good agreement for lower offset numbers. At higher offsets, beyond 100KHz, the measured values seem to have higher noise. The problem could be due to underestimated noise floor calculation.

Figure 5.10 Measured and calculated combined phase noise

5.3 QVCO Test Results

This circuit has four outputs, two for each sub-circuit (since these sub-circuits are differential). Using baluns as we did for the previous VCO, the four outputs are converted to two that are 90° apart.

5.3.1 Tuning Range

This circuit uses a 1.8V power supply and 3mA of core circuit current. A spectrum analyzer verified that the two signals were at the same frequency. Again using the potentiometer, the control voltage was varied and the output frequency was observed. Both outputs were locked to each other throughout the entire range. Figure 5.11 shows the measured center frequency versus control voltage for the circuit with the same bias conditions as above. Figure 5.12 shows the K_{VCO} values versus frequency.

The center frequency varied from 2.2 GHz-2.54 GHz when sweeping the control voltage between 0.65- 1.8V. This is a tuning range of 340 MHz.

Figure 5.11 Measured frequency vs. control voltage for the quadrature VCO

Figure 5.12 Calculated K_{VCO} from measurements of the quadrature VCO

5.3.2 Phase Noise

Phase noise was measured using the Spectrum Analyzer at 100KHz offset for different center frequencies. Figure 5.13 is the spectrum analyzer output screen showing a phase noise of -73.7dBc/Hz @ 100KHz offset. It can be seen from this figure that the noise level between 100KHz and 1MHz drops by 30 dB. The spectrum, for offsets lower than 100KHz, is too noisy to extract meaningful results.

Figure 5.13 Phase noise of the quadrature differential VCO at 100KHz offset from a 2.57 GHz

Figure 5.14 is a plot of the phase noise values at 100KHz offset versus different center frequencies.

Figure 5.14 Phase noise of the quadrature across the tuning range

A measurement was performed of the center frequency shift when changing the bias at a fixed control voltage. It was found that changing the bias from 2mA to 3mA

shifts the frequency by almost 20 MHz. Figure 5.15 shows these results, and Figure 5.16 is the slope of the frequency versus bias current.

Figure 5.15 Measured change in center frequency vs. change in bias current

Figure 5.16 Measured rate of shift in center frequency vs. change in bias current

5.3.3 Comparison between Calculated and Measured Phase Noise Results

5.3.3.1 Phase Noise at a 100kHz offset

Similar to the single VCO, at a 100kHz offset, frequency-modulated flicker noise is the most prominent. This calculation of noise is very sensitive to the $K_{VCO,fb}$, which was measured using a large signal variation in the bias current. This value might not be accurate when dealing with the small signal values of flicker noise.

Measurement was -73.7dBc/Hz from a carrier, and a calculation is -77.83dBc/Hz . The calculation underestimated the noise, again the equations highly depend on the $K_{VCO,fb}$ value.

5.3.3.2 Phase Noise versus Offset Frequency from the Carrier

Measurement: It can be seen in Figure 5.13 that the noise level between 100KHz and 1MHz drops by 30 dB. The spectrum, for offsets lower than 100KHz is too noisy to extract meaningful results.

Calculation: from Figure 5.17 it can be seen that phase noise slopes at a rate of -30dB/decade for frequencies up to about 1MHz, after which it hits the noise floor.

Figure 5.17 Measured and calculated combined phase noise

5.3.4 Phase Shift

To measure the phase difference between the output signals, we require more than just a spectrum analyzer, because all the outputs have the same frequency with different phase shifts. An oscilloscope could have been used to view the outputs and determine their phase shift; unfortunately a 2.4GHz oscilloscope was not available in the Electronics Department at Carleton University. In order to measure the phase shift, a special test setup was created.

The main idea behind the test of quadrature signals [9] is to mix them with another two quadrature signals from a generator in such a way that adding the two mixed signals would cause one side band to be added and the other one (the image) to be subtracted.

The above procedure can be done by down converting the VCO signal, as in Figure 5.18, or up converting the VCO signal, as in Figure 5.19. Testing with down conversion, however, would require an RF signal that contains information at the LO frequency, the wanted sideband frequency, and the image frequency. It would also require a 90° phase-shifter at IF. Testing with up-conversion would only require quadrature signals at IF and no phase shifter. The signal of the VCO under test would be fed to the RF port of the mixer.

Figure 5.18 Image rejection test configuration with down-conversion technique

Figure 5.19 Image rejection test configuration with up-conversion technique

The mixers used are Hewlett Packard silicon bipolar MMIC 5GHz active double balanced mixers. A full specification sheet is provided in the appendix. The mixers work well even with low LO power levels. The data sheet provides a plot for mixer conversion gain versus LO input power, at a -10dBm input RF signal level. According to the plot with a -3dBm of LO power, the conversion gain is around 15dB .

Two mixers were used to up-convert the two-quadrature signals. The VCO signals were fed to the RF port of the mixer. The RF ports were used instead of the LO ports, since the signal power would not be sufficient to switch the mixers. The two LO ports were fed with quadrature signals at 5MHz and -3dBm . The signal generator used is capable of providing signals 90° apart ($\pm 3-5^\circ$) at 5MHz maximum frequency.

Figure 5.20 Test setup for image rejection measurement

Each of the two output signals, from the two mixers, contains information at 2.395, 2.4 and 2.405GHz. When adding the two signals together, with a power combiner, one of the side bands (wanted channel) will be added and the other side band (the image) will be subtracted or suppressed. The difference between the two output signals at 2.395GHz and 2.405 GHz, is the amount of image rejection. If the two signals had no mismatch in amplitude or phase, perfect image rejection would be achieved.

The problem with this setup is that there is a phase shift uncertainty in the power combiner, the signal generator and the mixers. Table 3 summarizes the effect of these errors based on Figure 3.26. Note that we assumed that the two mixers introduce the same phase shift and so will have no effect on image rejection.

Device	Phase Error	Effect on image rejection
Signal generator	3-5 degrees	~25-30dB worse rejection
Power combiner ³	<7 degrees	~32dB worse rejection

Table 3 Effect of phase error, in the testing equipment, on image rejection measurements

Since the setup presumably introduced some “unknown” phase shift, the quadrature signal generator was tuned to maximize the image rejection, which required 83° phase shift. In this case the amount of image rejection was 10dB. It was also noted that the amplitude mismatch of the two LO signals was 2dB. This amount of mismatch, based on Figure 3.26, causes the image rejection to be less than 20dB. This measurement was not a good indication of the actual phase shift between the VCO outputs, since it is hard to distinguish where the limitation in the measurement of image rejection is coming from.

Depending on whether the in-phase signal is mixed with the in-phase or quadrature generator signals, the low, or high, side band would be suppressed. Figure 5.21 shows the low side band being suppressed, where the in-phase LO is mixed with in-phase RF. Figure 5.22 shows the other side band being suppressed when mixing the in-phase LO with the quadrature RF. Note that for these two measurements, suppression of the up-

³ The Combiner used is a mini circuits model # 15542 2FSC-2-10. A 2-way, 0degree and 2 to 10GHz.

per and lower side band, the quadrature signal generator phase shift had to be adjusted to 56° .

Figure 5.21 Image rejection result 1

Figure 5.22 Image rejection result 2

5.4 Summary

Table 4 and Table 5 summarize the results of the two VCOs.

Center frequency	2.25 GHz
Tuning range	1.65-2.35 GHz / 31% tuning
Phase noise	-93dBc/Hz @ 100KHz offset
Power (Core circuit)	1.8mW
Power (Buffers)	6.5mW

Table 4 Summary of the single VCO results

Center frequency	2.4GHz
Tuning range	2.2-2.54 GHz / 15% tuning
Phase noise	-73dBc/Hz @100 KHz offset
Power (Core circuit)	5.4mW
Power (Buffers)	13mW
Phase mismatch / Image rejection	>10° / 10dB

Table 5 Summary of quadrature VCO results

The two circuits were simulated to have the same frequency. However, the parasitics in the single differential VCO were underestimated. Note that the varactor size of the QVCO was decreased to compensate for the increasing parasitics from the coupling lines and the extra coupling transistors.

The tuning range of the single differential oscillator is higher than that of the quadrature VCO and the reason is the higher parasitics of the QVCO that decrease the overall tunable capacitance in the circuit.

Power consumption was designed to be 4 times as high, due to the fact that the QVCO has two circuits and each circuit consumes double the current of the single differential oscillator because of the extra coupling transistors. However, the circuit operated just as well with 3 times the current, that is 3mA.

Phase noise is higher in the quadrature VCO due to the higher frequency-modulation factor of the bias current transistor. The modulation factor is about 3.5 times larger than that in the differential oscillator, which results in 10dB degradation in phase noise. One reason for this high conversion factor could be the coupling from the current

of one VCO to the other VCO circuit through the coupling transistors. One way to reduce this noise is build bigger bias transistors, add de-coupling capacitors or degeneration resistor to reduce flicker noise. Another way to reduce this conversion factor is making the circuit less sensitive to changes in bias current (i.e. match the transconductance and possibly the loading of the NMOS and PMOS transistors).

In comparison to the previous designs listed in section 3.7, the differential VCO has relatively good phase noise taking into consideration the fact that it only consumes 1.8mW of power and produces -93.5dBc/Hz at 100KHz offset from a 2.25GHz carrier. The quadrature VCO, when compared to [33], consumes considerably less power, however, phase noise is traded off.

The quadrature VCO is still considered a good alternative to an oscillator followed by a poly phase filter when considering power consumption. In the poly phase filter case the oscillator will require buffers to drive the input, also the power level in the filter is reduced and extra power from the oscillator is required to compensate that reduction.

Chapter 6 Conclusion

6.1 Summary of Work in this Thesis

A 2.25GHz differential voltage-controlled oscillator in 0.18 μ CMOS technology was designed and built. The power consumption was 1.8mW for the core circuit, and it has a phase noise of -93dBc/Hz @100Khz offset of a 2.25 GHz carrier frequency. The tuning range of this VCO is between 1.65GHz to 2.35 GHz.

A 2.4GHz quadrature differential voltage-controlled oscillator in 0.18 μ CMOS technology was designed and built. The power consumption was 5.4mW for the core circuit, and it has a phase noise of -76dBc/Hz @100Khz offset of a 2.4 GHz carrier frequency. The tuning range of this VCO is between 2.2GHz to 2.55 GHz, which corresponds to 15% tuning.

This circuit is an alternative to a phase shifter in the current designs of image-reject receivers. The image rejection achieved was only 10dB. However, the inability to accurately test the phase difference, and image rejection, was the limiting factor in this measurement.

In addition, a study of how phase noise appears around the carrier was performed. The sources of noise were broken down and analyzed separately. Phase noise values for each of these sources were calculated. For “Leeson’s noise” the amount of loss in the tank was estimated and used to calculate the noise value around the carrier. Flicker noise values were estimated from the spice model of the transistors and used to calculate the frequency and amplitude modulated noise. These calculated values were compared with measured results and they agreed fairly well.

When building a VCO, circuit topology and design, not tank quality, can be the limiting reason in improving phase noise performance. In the case of CMOS VCO designs, this limiting issue can be attributed to flicker noise up-conversion factor, which can be minimized by certain circuit design choices. Some of these design choices were addressed in this thesis, with regard to the complementary negative gm topology. However,

further investigation and verification is required to better understand this conversion factor.

As for the quadrature VCO design, this thesis has shown how a quadrature oscillator can be much noisier than a single differential oscillator. As mentioned earlier, one reason for this high conversion factor is the coupling from the current of one VCO to the other VCO circuit through the coupling transistors. One way to reduce this noise is build bigger bias transistors, so as to reduce flicker noise. Another way to reduce this conversion factor is making the circuit less sensitive to changes in bias current (i.e. match the transconductance and possibly the loading of the NMOS and PMOS transistors).

6.2 Thesis Contributions

- 1) A better understanding of phase noise was developed. This involved a new way to look at phase noise, and resulted on some ability to predict and design for better phase noise performance. It was shown that phase noise is not only dependent on the quality of the tank itself, but on the circuit design to a large degree.
- 2) The transformation of noise into phase noise was calculated from the basic equations.
- 3) It was recognized that flicker noise could be a limiting factor to phase noise performance. A poor circuit design can lead to high up-conversion factors and result in a high flicker noise corner frequency.
- 4) It was shown that it is possible to build RF circuits in a CMOS technology, however, more research should go into making these circuits compliant with wireless LAN standards. An example is Bluetooth at 2.4GHz. Circuits designed for this standard are required to be inexpensive and low power designs with somewhat relaxed phase noise and output power requirements.
- 5) A quadrature oscillator design that can replace conventional phase shifters in image-reject receivers was examined.

6.3 Future Work

- 1) Further verification of the equations and parameters that were used to calculate phase noise is required.
- 2) Design and build a VCO, taking into consideration how the different design parameters affect phase noise performance, special attention should be given to minimizing the flicker noise up-conversion factor
- 3) In order to reduce the up-conversion factor the transconductances of the NMOS and PMOS transistors in the complementary VCO circuit have to be matched.
- 4) The current source has to be carefully designed to minimize flicker noise; some techniques that can be used are building bigger transistors, adding decoupling capacitors and possibly a degeneration resistor.
- 5) In the quadrature VCO, adding series capacitors in the coupling line can reduce noise coupling from one VCO to the other.
- 6) Design a better test setup for the quadrature VCO so as to be able to measure the exact phase difference between the outputs.
- 7) Further investigate the feasibility of using a quadrature voltage-controlled oscillator for image rejection to replace using a single VCO with a phase shifter.

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Appendices

Amplitude Modulated Flicker Noise

K1 PMOS	6.455E-05				
K1 NMOS	8.1E-24	Cox	0.008463382	gm5 (PMOS)	0.131194512
AF	0.8265	W	2.00E-05		
gm3	5.28E-03	L	0.00000018		

Frequency	(Vn)2	Vgs	gm3*vgs	(gm3*vgs/gm5)*1/pi()	AM Phase noise dBm	Frequency	AM noise in dBc
1.0E+02	5.911E-12	0.000002431	1.28E-08	3.11E-08	-99.13	1.0E+02	-111.13
2.0E+02	3.333E-12	1.82565E-06	9.64E-09	2.34E-08	-101.62	2.0E+02	-113.62
3.0E+02	2.384E-12	1.54401E-06	8.15E-09	1.98E-08	-103.08	3.0E+02	-115.08
4.0E+02	1.879E-12	1.37094E-06	7.24E-09	1.76E-08	-104.11	4.0E+02	-116.11
5.0E+02	1.563E-12	1.25017E-06	6.60E-09	1.60E-08	-104.91	5.0E+02	-116.91
6.0E+02	1.344E-12	1.15944E-06	6.12E-09	1.49E-08	-105.56	6.0E+02	-117.56
7.0E+02	1.183E-12	1.08788E-06	5.74E-09	1.39E-08	-106.12	7.0E+02	-118.12
8.0E+02	1.06E-12	1.02948E-06	5.44E-09	1.32E-08	-106.60	8.0E+02	-118.60
9.0E+02	9.615E-13	9.8057E-07	5.18E-09	1.26E-08	-107.02	9.0E+02	-119.02
1.0E+03	8.813E-13	9.38792E-07	4.96E-09	1.20E-08	-107.40	1.0E+03	-119.40
2.0E+03	4.97E-13	7.04967E-07	3.72E-09	9.03E-09	-109.89	2.0E+03	-121.89
3.0E+03	3.555E-13	5.9621E-07	3.15E-09	7.64E-09	-111.34	3.0E+03	-123.34
4.0E+03	2.802E-13	5.29381E-07	2.80E-09	6.78E-09	-112.37	4.0E+03	-124.37
5.0E+03	2.33E-13	4.82747E-07	2.55E-09	6.18E-09	-113.17	5.0E+03	-125.17
6.0E+03	2.004E-13	4.47712E-07	2.36E-09	5.74E-09	-113.83	6.0E+03	-125.83
7.0E+03	1.765E-13	4.2008E-07	2.22E-09	5.38E-09	-114.38	7.0E+03	-126.38
8.0E+03	1.58E-13	3.97528E-07	2.10E-09	5.09E-09	-114.86	8.0E+03	-126.86
9.0E+03	1.434E-13	3.78642E-07	2.00E-09	4.85E-09	-115.28	9.0E+03	-127.28
1.0E+04	1.314E-13	3.62509E-07	1.91E-09	4.64E-09	-115.66	1.0E+04	-127.66
2.0E+04	7.41E-14	2.72219E-07	1.44E-09	3.49E-09	-118.15	2.0E+04	-130.15
3.0E+04	5.3E-14	2.30223E-07	1.22E-09	2.95E-09	-119.61	3.0E+04	-131.61
4.0E+04	4.179E-14	2.04417E-07	1.08E-09	2.62E-09	-120.64	4.0E+04	-132.64
5.0E+04	3.475E-14	1.8641E-07	9.84E-10	2.39E-09	-121.44	5.0E+04	-133.44
6.0E+04	2.999E-14	1.72881E-07	9.13E-10	2.21E-09	-122.09	6.0E+04	-134.09
7.0E+04	2.631E-14	1.62212E-07	8.56E-10	2.08E-09	-122.65	7.0E+04	-134.65
8.0E+04	2.356E-14	1.53503E-07	8.10E-10	1.97E-09	-123.13	8.0E+04	-135.13
9.0E+04	2.138E-14	1.4621E-07	7.72E-10	1.87E-09	-123.55	9.0E+04	-135.55
1.0E+05	1.959E-14	1.39981E-07	7.39E-10	1.79E-09	-123.93	1.0E+05	-135.93
2.0E+05	1.105E-14	1.05116E-07	5.55E-10	1.35E-09	-126.42	2.0E+05	-138.42
3.0E+05	7.903E-15	8.88994E-08	4.69E-10	1.14E-09	-127.87	3.0E+05	-139.87
4.0E+05	6.231E-15	7.89347E-08	4.17E-10	1.01E-09	-128.90	4.0E+05	-140.90
5.0E+05	5.181E-15	7.19813E-08	3.80E-10	9.22E-10	-129.70	5.0E+05	-141.70
6.0E+05	4.457E-15	6.67572E-08	3.52E-10	8.55E-10	-130.36	6.0E+05	-142.36
7.0E+05	3.923E-15	6.26372E-08	3.31E-10	8.02E-10	-130.91	7.0E+05	-142.91
8.0E+05	3.513E-15	5.92744E-08	3.13E-10	7.59E-10	-131.39	8.0E+05	-143.39
9.0E+05	3.188E-15	5.64584E-08	2.98E-10	7.23E-10	-131.81	9.0E+05	-143.81
1.0E+06	2.922E-15	5.40529E-08	2.85E-10	6.92E-10	-132.19	1.0E+06	-144.19
2.0E+06	1.648E-15	4.059E-08	2.14E-10	5.20E-10	-134.68	2.0E+06	-146.68
3.0E+06	1.178E-15	3.4328E-08	1.81E-10	4.40E-10	-136.14	3.0E+06	-148.14
4.0E+06	9.29E-16	3.04802E-08	1.61E-10	3.90E-10	-137.17	4.0E+06	-149.17
5.0E+06	7.726E-16	2.77952E-08	1.47E-10	3.56E-10	-137.97	5.0E+06	-149.97
6.0E+06	6.645E-16	2.57779E-08	1.36E-10	3.30E-10	-138.62	6.0E+06	-150.62
7.0E+06	5.85E-16	2.4187E-08	1.28E-10	3.10E-10	-139.18	7.0E+06	-151.18
8.0E+06	5.239E-16	2.28885E-08	1.21E-10	2.93E-10	-139.66	8.0E+06	-151.66
9.0E+06	4.753E-16	2.18011E-08	1.15E-10	2.79E-10	-140.08	9.0E+06	-152.08
1.0E+07	4.357E-16	2.08723E-08	1.10E-10	2.67E-10	-140.46	1.0E+07	-152.46
2.0E+07	2.457E-16	1.56736E-08	8.28E-11	2.01E-10	-142.95	2.0E+07	-154.95
3.0E+07	1.757E-16	1.32556E-08	7.00E-11	1.70E-10	-144.40	3.0E+07	-156.40
4.0E+07	1.385E-16	1.17698E-08	6.21E-11	1.51E-10	-145.43	4.0E+07	-157.43
5.0E+07	1.152E-16	1.0733E-08	5.67E-11	1.37E-10	-146.23	5.0E+07	-158.23
6.0E+07	9.908E-17	9.95401E-09	5.26E-11	1.28E-10	-146.89	6.0E+07	-158.89
7.0E+07	8.723E-17	9.33969E-09	4.93E-11	1.20E-10	-147.44	7.0E+07	-159.44
8.0E+07	7.812E-17	8.83827E-09	4.67E-11	1.13E-10	-147.92	8.0E+07	-159.92
9.0E+07	7.087E-17	8.41838E-09	4.44E-11	1.08E-10	-148.34	9.0E+07	-160.34
1.0E+08	6.496E-17	8.05971E-09	4.26E-11	1.03E-10	-148.72	1.0E+08	-160.72
2.0E+08	3.663E-17	6.05227E-09	3.20E-11	7.75E-11	-151.21	2.0E+08	-163.21
3.0E+08	2.62E-17	5.11857E-09	2.70E-11	6.56E-11	-152.67	3.0E+08	-164.67
4.0E+08	2.066E-17	4.54483E-09	2.40E-11	5.82E-11	-153.70	4.0E+08	-165.70
5.0E+08	1.718E-17	4.14448E-09	2.19E-11	5.31E-11	-154.50	5.0E+08	-166.50
6.0E+08	1.477E-17	3.84369E-09	2.03E-11	4.92E-11	-155.15	6.0E+08	-167.15
7.0E+08	1.301E-17	3.60647E-09	1.90E-11	4.62E-11	-155.71	7.0E+08	-167.71
8.0E+08	1.165E-17	3.41285E-09	1.80E-11	4.37E-11	-156.19	8.0E+08	-168.19
9.0E+08	1.057E-17	3.25071E-09	1.72E-11	4.16E-11	-156.61	9.0E+08	-168.61
1.0E+09	9.686E-18	3.11221E-09	1.64E-11	3.99E-11	-156.99	1.0E+09	-168.99

Frequency Modulated Flicker Noise

Kf NMOS	8.1E-24	Cox	0.008463382
AF	0.8265	W	2.00E-05
gm	5.28E-03	L	0.00000018
Kp	0.000226		
Vgsn	0.7822163		
Vt	0.5		
KVCOlb	5.00E+09		

Frequency	(Vn) ²	dBm input referre d Vn	I	delta I	Delta Frequency	Beta	lb phase noise dBc	lb noise inverse log	Vc phase noise inverse log	total inverse log	Total in dBc
1.0E+02	5.911E-12	-82.284	1.0000172E-03	1.72E-08	8.61E+01	8.61E-01	-4.32	3.70E-01	4.20E-02	4.12E-01	-3.85
2.0E+02	3.333E-12	-84.772	1.0000129E-03	1.29E-08	6.47E+01	3.23E-01	-12.83	5.22E-02	5.95E-03	5.81E-02	-12.36
3.0E+02	2.384E-12	-86.227	1.0000109E-03	1.09E-08	5.47E+01	1.82E-01	-17.80	1.66E-02	1.90E-03	1.85E-02	-17.33
4.0E+02	1.879E-12	-87.260	1.0000097E-03	9.71E-09	4.86E+01	1.21E-01	-21.34	7.35E-03	8.46E-04	8.20E-03	-20.86
5.0E+02	1.563E-12	-88.061	1.0000089E-03	8.86E-09	4.43E+01	8.86E-02	-24.08	3.91E-03	4.51E-04	4.36E-03	-23.60
6.0E+02	1.344E-12	-88.715	1.0000082E-03	8.21E-09	4.11E+01	6.84E-02	-26.31	2.34E-03	2.70E-04	2.61E-03	-25.84
7.0E+02	1.183E-12	-89.268	1.0000077E-03	7.71E-09	3.85E+01	5.50E-02	-28.21	1.51E-03	1.75E-04	1.69E-03	-27.73
8.0E+02	1.06E-12	-89.748	1.0000073E-03	7.29E-09	3.65E+01	4.56E-02	-29.85	1.04E-03	1.20E-04	1.16E-03	-29.37
9.0E+02	9.615E-13	-90.170	1.0000069E-03	6.95E-09	3.47E+01	3.86E-02	-31.29	7.43E-04	8.65E-05	8.29E-04	-30.81
1.0E+03	8.813E-13	-90.549	1.0000066E-03	6.65E-09	3.32E+01	3.32E-02	-32.59	5.51E-04	6.44E-05	6.16E-04	-32.11
2.0E+03	4.97E-13	-93.037	1.0000050E-03	4.99E-09	2.50E+01	1.25E-02	-41.10	7.77E-05	9.21E-06	8.69E-05	-40.61
3.0E+03	3.555E-13	-94.492	1.0000042E-03	4.22E-09	2.11E+01	7.04E-03	-46.07	2.47E-05	2.96E-06	2.77E-05	-45.58
4.0E+03	2.802E-13	-95.525	1.0000037E-03	3.75E-09	1.87E+01	4.68E-03	-49.61	1.09E-05	1.32E-06	1.23E-05	-49.11
5.0E+03	2.33E-13	-96.326	1.0000034E-03	3.42E-09	1.71E+01	3.42E-03	-52.35	5.83E-06	7.09E-07	6.54E-06	-51.85
6.0E+03	2.004E-13	-96.980	1.0000032E-03	3.17E-09	1.58E+01	2.64E-03	-54.59	3.48E-06	4.26E-07	3.91E-06	-54.08
7.0E+03	1.765E-13	-97.533	1.0000030E-03	2.97E-09	1.49E+01	2.12E-03	-56.48	2.25E-06	2.77E-07	2.53E-06	-55.97
8.0E+03	1.58E-13	-98.013	1.0000028E-03	2.81E-09	1.41E+01	1.76E-03	-58.12	1.54E-06	1.91E-07	1.73E-06	-57.61
9.0E+03	1.434E-13	-98.435	1.0000027E-03	2.68E-09	1.34E+01	1.49E-03	-59.56	1.11E-06	1.38E-07	1.24E-06	-59.05
1.0E+04	1.314E-13	-98.814	1.0000026E-03	2.57E-09	1.28E+01	1.28E-03	-60.86	8.21E-07	1.03E-07	9.23E-07	-60.35
2.0E+04	7.41E-14	-101.302	1.0000019E-03	1.93E-09	9.63E+00	4.81E-04	-69.37	1.16E-07	1.50E-08	1.31E-07	-68.84
3.0E+04	5.3E-14	-102.757	1.0000016E-03	1.63E-09	8.14E+00	2.71E-04	-74.35	3.67E-08	4.90E-09	4.16E-08	-73.81
4.0E+04	4.179E-14	-103.790	1.0000014E-03	1.45E-09	7.23E+00	1.81E-04	-77.88	1.63E-08	2.22E-09	1.85E-08	-77.33
5.0E+04	3.475E-14	-104.591	1.0000013E-03	1.32E-09	6.59E+00	1.32E-04	-80.63	8.66E-09	1.20E-09	9.86E-09	-80.06
6.0E+04	2.989E-14	-105.245	1.0000012E-03	1.22E-09	6.11E+00	1.02E-04	-82.87	5.17E-09	7.30E-10	5.90E-09	-82.29
7.0E+04	2.631E-14	-105.798	1.0000011E-03	1.15E-09	5.73E+00	8.19E-05	-84.76	3.34E-09	4.79E-10	3.82E-09	-84.18
8.0E+04	2.356E-14	-106.278	1.0000011E-03	1.08E-09	5.42E+00	6.78E-05	-86.40	2.29E-09	3.33E-10	2.62E-09	-85.81
9.0E+04	2.138E-14	-106.700	1.0000010E-03	1.03E-09	5.16E+00	5.74E-05	-87.85	1.64E-09	2.42E-10	1.88E-09	-87.25
1.0E+05	1.959E-14	-107.079	1.0000010E-03	9.88E-10	4.94E+00	4.94E-05	-89.14	1.22E-09	1.81E-10	1.40E-09	-88.54
2.0E+05	1.105E-14	-109.567	1.0000007E-03	7.41E-10	3.71E+00	1.85E-05	-97.66	1.71E-10	2.78E-11	1.99E-10	-97.01
3.0E+05	7.903E-15	-111.022	1.0000006E-03	6.26E-10	3.13E+00	1.04E-05	-102.65	5.44E-11	9.40E-12	6.38E-11	-101.95
4.0E+05	6.231E-15	-112.055	1.0000006E-03	5.56E-10	2.78E+00	6.95E-06	-106.18	2.41E-11	4.37E-12	2.84E-11	-105.46
5.0E+05	5.181E-15	-112.856	1.0000005E-03	5.06E-10	2.53E+00	5.06E-06	-108.93	1.28E-11	2.42E-12	1.52E-11	-108.18
6.0E+05	4.457E-15	-113.510	1.0000005E-03	4.69E-10	2.35E+00	3.91E-06	-111.17	7.63E-12	1.50E-12	9.13E-12	-110.39
7.0E+05	3.923E-15	-114.063	1.0000004E-03	4.40E-10	2.20E+00	3.14E-06	-113.07	4.93E-12	1.00E-12	5.93E-12	-112.27
8.0E+05	3.513E-15	-114.543	1.0000004E-03	4.16E-10	2.08E+00	2.60E-06	-114.71	3.38E-12	7.06E-13	4.08E-12	-113.89
9.0E+05	3.188E-15	-114.965	1.0000004E-03	3.96E-10	1.98E+00	2.20E-06	-116.16	2.42E-12	5.19E-13	2.94E-12	-115.32
1.0E+06	2.922E-15	-115.344	1.0000004E-03	3.79E-10	1.90E+00	1.90E-06	-117.46	1.80E-12	3.95E-13	2.19E-12	-116.60
2.0E+06	1.648E-15	-117.832	1.0000003E-03	2.84E-10	1.42E+00	7.10E-07	-126.00	2.51E-13	6.66E-14	3.18E-13	-124.98
3.0E+06	1.178E-15	-119.287	1.0000002E-03	2.40E-10	1.20E+00	3.99E-07	-130.99	7.96E-14	2.39E-14	1.03E-13	-129.85
4.0E+06	9.29E-16	-120.320	1.0000002E-03	2.12E-10	1.06E+00	2.65E-07	-134.54	3.51E-14	1.16E-14	4.68E-14	-133.30
5.0E+06	7.726E-16	-121.121	1.0000002E-03	1.93E-10	9.67E-01	1.93E-07	-137.29	1.86E-14	6.69E-15	2.53E-14	-135.96
6.0E+06	6.645E-16	-121.775	1.0000002E-03	1.79E-10	8.95E-01	1.49E-07	-139.55	1.11E-14	4.27E-15	1.54E-14	-138.13
7.0E+06	5.85E-16	-122.328	1.0000002E-03	1.68E-10	8.39E-01	1.20E-07	-141.45	7.16E-15	2.93E-15	1.01E-14	-139.96
8.0E+06	5.239E-16	-122.808	1.0000002E-03	1.59E-10	7.93E-01	9.91E-08	-143.10	4.90E-15	2.12E-15	7.02E-15	-141.54
9.0E+06	4.753E-16	-123.230	1.0000002E-03	1.51E-10	7.54E-01	8.38E-08	-144.56	3.50E-15	1.59E-15	5.09E-15	-142.93
1.0E+07	4.357E-16	-123.609	1.0000001E-03	1.44E-10	7.21E-01	7.21E-08	-145.86	2.60E-15	1.23E-15	3.83E-15	-144.17
2.0E+07	2.457E-16	-126.097	1.0000001E-03	1.07E-10	5.37E-01	2.69E-08	-154.44	3.60E-16	2.37E-16	5.97E-16	-152.24
3.0E+07	1.757E-16	-127.552	1.0000001E-03	9.03E-11	4.51E-01	1.50E-08	-159.47	1.13E-16	9.19E-17	2.05E-16	-156.89
4.0E+07	1.385E-16	-128.585	1.0000001E-03	7.98E-11	3.99E-01	9.97E-09	-163.05	4.96E-17	4.73E-17	9.69E-17	-160.14
5.0E+07	1.152E-16	-129.386	1.0000001E-03	7.24E-11	3.62E-01	7.24E-09	-165.82	2.62E-17	2.84E-17	5.46E-17	-162.63
6.0E+07	9.908E-17	-130.040	1.0000001E-03	6.69E-11	3.34E-01	5.57E-09	-168.10	1.55E-17	1.88E-17	3.43E-17	-164.65
7.0E+07	8.723E-17	-130.593	1.0000001E-03	6.25E-11	3.13E-01	4.47E-09	-170.02	9.95E-18	1.33E-17	2.32E-17	-166.34
8.0E+07	7.812E-17	-131.073	1.0000001E-03	5.90E-11	2.95E-01	3.69E-09	-171.89	6.78E-18	9.82E-18	1.66E-17	-167.80
9.0E+07	7.087E-17	-131.495	1.0000001E-03	5.60E-11	2.80E-01	3.11E-09	-173.16	4.83E-18	7.54E-18	1.24E-17	-169.08
1.0E+08	6.496E-17	-131.874	1.0000001E-03	5.35E-11	2.67E-01	2.67E-09	-174.48	3.56E-18	5.96E-18	9.53E-18	-170.21
2.0E+08	3.663E-17	-134.362	1.0000000E-03	3.92E-11	1.96E-01	9.81E-10	-183.19	4.80E-19	1.29E-18	1.77E-18	-177.51
3.0E+08	2.62E-17	-135.817	1.0000000E-03	3.26E-11	1.63E-01	5.44E-10	-188.31	1.47E-19	5.36E-19	6.89E-19	-181.65
4.0E+08	2.068E-17	-136.850	1.0000000E-03	2.86E-11	1.43E-01	3.57E-10	-191.97	6.36E-20	2.88E-19	3.52E-19	-184.54
5.0E+08	1.718E-17	-137.651	1.0000000E-03	2.57E-11	1.29E-01	2.57E-10	-194.81	3.30E-20	1.79E-19	2.12E-19	-186.74
6.0E+08	1.477E-17	-138.305	1.0000000E-03	2.36E-11	1.18E-01	1.97E-10	-197.15	1.93E-20	1.21E-19	1.41E-19	-188.52

Leeson's Noise

k	1.38E-23
T	298
A2 (-12dBm)	0.126191469
Center Frequency	2350000000
F	3
Capacitor	9E-10
Q	8

Offset Frequency	Phase Noise	Leesons Phase noise on dBc
1.0E+02	1.3E-05	-48.9
2.0E+02	3.2E-06	-55.0
3.0E+02	1.4E-06	-58.5
4.0E+02	8.0E-07	-61.0
5.0E+02	5.1E-07	-62.9
6.0E+02	3.5E-07	-64.5
7.0E+02	2.6E-07	-65.8
8.0E+02	2.0E-07	-67.0
9.0E+02	1.6E-07	-68.0
1.0E+03	1.3E-07	-68.9
2.0E+03	3.2E-08	-75.0
3.0E+03	1.4E-08	-78.5
4.0E+03	8.0E-09	-81.0
5.0E+03	5.1E-09	-82.9
6.0E+03	3.5E-09	-84.5
7.0E+03	2.6E-09	-85.8
8.0E+03	2.0E-09	-87.0
9.0E+03	1.6E-09	-88.0
1.0E+04	1.3E-09	-88.9
2.0E+04	3.2E-10	-95.0
3.0E+04	1.4E-10	-98.5
4.0E+04	8.0E-11	-101.0
5.0E+04	5.1E-11	-102.9
6.0E+04	3.5E-11	-104.5
7.0E+04	2.6E-11	-105.8
8.0E+04	2.0E-11	-107.0
9.0E+04	1.6E-11	-108.0
1.0E+05	1.3E-11	-108.9
2.0E+05	3.2E-12	-115.0
3.0E+05	1.4E-12	-118.5
4.0E+05	8.0E-13	-121.0
5.0E+05	5.1E-13	-122.9
6.0E+05	3.5E-13	-124.5
7.0E+05	2.6E-13	-125.8
8.0E+05	2.0E-13	-127.0
9.0E+05	1.6E-13	-128.0
1.0E+06	1.3E-13	-128.9
2.0E+06	3.2E-14	-135.0
3.0E+06	1.4E-14	-138.5
4.0E+06	8.0E-15	-141.0
5.0E+06	5.1E-15	-142.9
6.0E+06	3.5E-15	-144.5
7.0E+06	2.6E-15	-145.8
8.0E+06	2.0E-15	-147.0
9.0E+06	1.6E-15	-148.0
1.0E+07	1.3E-15	-148.9
2.0E+07	3.2E-16	-155.0
3.0E+07	1.4E-16	-158.5
4.0E+07	8.0E-17	-161.0
5.0E+07	5.1E-17	-162.9
6.0E+07	3.5E-17	-164.5
7.0E+07	2.6E-17	-165.8
8.0E+07	2.0E-17	-167.0
9.0E+07	1.6E-17	-168.0
1.0E+08	1.3E-17	-168.9
2.0E+08	3.2E-18	-175.0
3.0E+08	1.4E-18	-178.5
4.0E+08	8.0E-19	-181.0
5.0E+08	5.1E-19	-182.9
6.0E+08	3.5E-19	-184.5
7.0E+08	2.6E-19	-185.8
8.0E+08	2.0E-19	-187.0
9.0E+08	1.6E-19	-188.0
1.0E+09	1.3E-19	-188.9

Combined Phase Noise of Differential Oscillator

	Leesons			Frequency modulation			Amplitude modulation			Total		
Frequency	dBc/Hz	dBm/Hz	inverse log	dBc/Hz	dBm/Hz	inverse log	dBm/Hz	inverse log	inverse log	dBm/Hz	dBc/Hz	
1.0E+02	-48.9	-60.9	8.05E-07	-3.85	-15.9	2.60E-02	-99.13	1.22E-10	2.598E-02	-1.6E+01	-3.9	
2.0E+02	-55.0	-67.0	2.01E-07	-12.36	-24.4	3.66E-03	-101.62	6.89E-11	3.663E-03	-2.4E+01	-12.4	
3.0E+02	-58.5	-70.5	8.95E-08	-17.34	-29.3	1.16E-03	-103.08	4.93E-11	1.165E-03	-2.9E+01	-17.3	
4.0E+02	-61.0	-73.0	5.03E-08	-20.87	-32.9	5.17E-04	-104.11	3.88E-11	5.166E-04	-3.3E+01	-20.9	
5.0E+02	-62.9	-74.9	3.22E-08	-23.61	-35.6	2.75E-04	-104.91	3.23E-11	2.750E-04	-3.6E+01	-23.6	
6.0E+02	-64.5	-76.5	2.24E-08	-25.85	-37.8	1.64E-04	-105.56	2.78E-11	1.643E-04	-3.8E+01	-25.8	
7.0E+02	-65.8	-77.8	1.64E-08	-27.74	-39.7	1.06E-04	-106.12	2.45E-11	1.063E-04	-4.0E+01	-27.7	
8.0E+02	-67.0	-79.0	1.26E-08	-29.38	-41.4	7.28E-05	-106.60	2.19E-11	7.286E-05	-4.1E+01	-29.4	
9.0E+02	-68.0	-80.0	9.94E-09	-30.82	-42.8	5.22E-05	-107.02	1.99E-11	5.223E-05	-4.3E+01	-30.8	
1.0E+03	-68.9	-80.9	8.05E-09	-32.11	-44.1	3.88E-05	-107.40	1.82E-11	3.878E-05	-4.4E+01	-32.1	
2.0E+03	-75.0	-87.0	2.01E-09	-40.62	-52.6	5.47E-06	-109.89	1.03E-11	5.471E-06	-5.3E+01	-40.6	
3.0E+03	-78.5	-90.5	8.95E-10	-45.60	-57.6	1.74E-06	-111.34	7.34E-12	1.740E-06	-5.8E+01	-45.6	
4.0E+03	-81.0	-93.0	5.03E-10	-49.13	-61.1	7.72E-07	-112.37	5.79E-12	7.721E-07	-6.1E+01	-49.1	
5.0E+03	-82.9	-94.9	3.22E-10	-51.86	-63.9	4.11E-07	-113.17	4.81E-12	4.111E-07	-6.4E+01	-51.9	
6.0E+03	-84.5	-96.5	2.24E-10	-54.10	-66.1	2.45E-07	-113.83	4.14E-12	2.456E-07	-6.6E+01	-54.1	
7.0E+03	-85.8	-97.8	1.64E-10	-55.99	-68.0	1.59E-07	-114.38	3.65E-12	1.589E-07	-6.8E+01	-56.0	
8.0E+03	-87.0	-99.0	1.26E-10	-57.63	-69.6	1.09E-07	-114.86	3.26E-12	1.090E-07	-7.0E+01	-57.6	
9.0E+03	-88.0	-100.0	9.94E-11	-59.08	-71.1	7.81E-08	-115.28	2.96E-12	7.817E-08	-7.1E+01	-59.1	
1.0E+04	-88.9	-100.9	8.05E-11	-60.37	-72.4	5.80E-08	-115.66	2.72E-12	5.805E-08	-7.2E+01	-60.4	
2.0E+04	-95.0	-107.0	2.01E-11	-68.87	-80.9	8.18E-09	-118.15	1.53E-12	8.206E-09	-8.1E+01	-68.9	
3.0E+04	-98.5	-110.5	8.95E-12	-73.84	-85.8	2.60E-09	-119.61	1.10E-12	2.615E-09	-8.6E+01	-73.8	
4.0E+04	-101.0	-113.0	5.03E-12	-77.37	-89.4	1.16E-09	-120.64	8.63E-13	1.162E-09	-8.9E+01	-77.3	
5.0E+04	-102.9	-114.9	3.22E-12	-80.11	-92.1	6.16E-10	-121.44	7.18E-13	6.197E-10	-9.2E+01	-80.1	
6.0E+04	-104.5	-116.5	2.24E-12	-82.34	-94.3	3.68E-10	-122.09	6.17E-13	3.709E-10	-9.4E+01	-82.3	
7.0E+04	-105.8	-117.8	1.64E-12	-84.23	-96.2	2.38E-10	-122.65	5.44E-13	2.404E-10	-9.6E+01	-84.2	
8.0E+04	-107.0	-119.0	1.26E-12	-85.87	-97.9	1.63E-10	-123.13	4.87E-13	1.652E-10	-9.8E+01	-85.8	
9.0E+04	-108.0	-120.0	9.94E-13	-87.31	-99.3	1.17E-10	-123.55	4.42E-13	1.186E-10	-9.9E+01	-87.3	
1.0E+05	-108.9	-120.9	8.05E-13	-88.60	-100.6	8.71E-11	-123.93	4.05E-13	8.828E-11	-1.0E+02	-88.5	
2.0E+05	-115.0	-127.0	2.01E-13	-97.09	-109.1	1.23E-11	-126.42	2.28E-13	1.276E-11	-1.1E+02	-96.9	
3.0E+05	-118.5	-130.5	8.95E-14	-102.06	-114.1	3.93E-12	-127.87	1.63E-13	4.183E-12	-1.1E+02	-101.8	
4.0E+05	-121.0	-133.0	5.03E-14	-105.58	-117.6	1.75E-12	-128.90	1.29E-13	1.927E-12	-1.2E+02	-105.2	
5.0E+05	-122.9	-134.9	3.22E-14	-108.30	-120.3	9.32E-13	-129.70	1.07E-13	1.072E-12	-1.2E+02	-107.7	
6.0E+05	-124.5	-136.5	2.24E-14	-110.53	-122.5	5.58E-13	-130.36	9.21E-14	6.725E-13	-1.2E+02	-109.7	
7.0E+05	-125.8	-137.8	1.64E-14	-112.42	-124.4	3.62E-13	-130.91	8.11E-14	4.591E-13	-1.2E+02	-111.4	
8.0E+05	-127.0	-139.0	1.26E-14	-114.05	-126.0	2.48E-13	-131.39	7.26E-14	3.335E-13	-1.2E+02	-112.8	
9.0E+05	-128.0	-140.0	9.94E-15	-115.49	-127.5	1.78E-13	-131.81	6.59E-14	2.541E-13	-1.3E+02	-113.9	
1.0E+06	-128.9	-140.9	8.05E-15	-116.77	-128.8	1.33E-13	-132.19	6.04E-14	2.010E-13	-1.3E+02	-115.0	
2.0E+06	-135.0	-147.0	2.01E-15	-125.23	-137.2	1.89E-14	-134.68	3.40E-14	5.499E-14	-1.3E+02	-120.6	
3.0E+06	-138.5	-150.5	8.95E-16	-130.16	-142.2	6.08E-15	-136.14	2.43E-14	3.132E-14	-1.4E+02	-123.0	
4.0E+06	-141.0	-153.0	5.03E-16	-133.65	-145.7	2.72E-15	-137.17	1.92E-14	2.242E-14	-1.4E+02	-124.5	
5.0E+06	-142.9	-154.9	3.22E-16	-136.36	-148.4	1.46E-15	-137.97	1.60E-14	1.774E-14	-1.4E+02	-125.5	
6.0E+06	-144.5	-156.5	2.24E-16	-138.57	-150.6	8.77E-16	-138.62	1.37E-14	1.483E-14	-1.4E+02	-126.3	
7.0E+06	-145.8	-157.8	1.64E-16	-140.43	-152.4	5.71E-16	-139.18	1.21E-14	1.282E-14	-1.4E+02	-126.9	
8.0E+06	-147.0	-159.0	1.26E-16	-142.05	-154.0	3.94E-16	-139.66	1.08E-14	1.134E-14	-1.4E+02	-127.5	
9.0E+06	-148.0	-160.0	9.94E-17	-143.47	-155.5	2.84E-16	-140.08	9.82E-15	1.020E-14	-1.4E+02	-127.9	
1.0E+07	-148.9	-160.9	8.05E-17	-144.74	-156.7	2.12E-16	-140.46	9.00E-15	9.293E-15	-1.4E+02	-128.3	
2.0E+07	-155.0	-167.0	2.01E-17	-153.05	-165.0	3.13E-17	-142.95	5.08E-15	5.127E-15	-1.4E+02	-130.9	
3.0E+07	-158.5	-170.5	8.95E-18	-157.87	-169.9	1.03E-17	-144.40	3.63E-15	3.649E-15	-1.4E+02	-132.4	
4.0E+07	-161.0	-173.0	5.03E-18	-161.27	-173.3	4.71E-18	-145.43	2.86E-15	2.872E-15	-1.5E+02	-133.4	
5.0E+07	-162.9	-174.9	3.22E-18	-163.89	-175.9	2.58E-18	-146.23	2.38E-15	2.386E-15	-1.5E+02	-134.2	
6.0E+07	-164.5	-176.5	2.24E-18	-166.01	-178.0	1.58E-18	-146.89	2.05E-15	2.051E-15	-1.5E+02	-134.9	
7.0E+07	-165.8	-177.8	1.64E-18	-167.80	-179.8	1.05E-18	-147.44	1.80E-15	1.805E-15	-1.5E+02	-135.4	
8.0E+07	-167.0	-179.0	1.26E-18	-169.35	-181.3	7.33E-19	-147.92	1.61E-15	1.616E-15	-1.5E+02	-135.9	
9.0E+07	-168.0	-180.0	9.94E-19	-170.70	-182.7	5.37E-19	-148.34	1.46E-15	1.466E-15	-1.5E+02	-136.3	
1.0E+08	-168.9	-180.9	8.05E-19	-171.91	-183.9	4.06E-19	-148.72	1.34E-15	1.343E-15	-1.5E+02	-136.7	
2.0E+08	-175.0	-187.0	2.01E-19	-179.73	-191.7	6.71E-20	-151.21	7.57E-16	7.571E-16	-1.5E+02	-139.2	
3.0E+08	-178.5	-190.5	8.95E-20	-184.19	-196.2	2.40E-20	-152.67	5.41E-16	5.414E-16	-1.5E+02	-140.7	
4.0E+08	-181.0	-193.0	5.03E-20	-187.30	-199.3	1.18E-20	-153.70	4.27E-16	4.268E-16	-1.5E+02	-141.7	
5.0E+08	-182.9	-194.9	3.22E-20	-189.67	-201.7	6.80E-21	-154.50	3.55E-16	3.549E-16	-1.5E+02	-142.5	
6.0E+08	-184.5	-196.5	2.24E-20	-191.59	-203.6	4.38E-21	-155.15	3.05E-16	3.053E-16	-1.6E+02	-143.2	
7.0E+08	-185.8	-197.8	1.64E-20	-193.19	-205.2	3.02E-21	-155.71	2.69E-16	2.687E-16	-1.6E+02	-143.7	
8.0E+08	-187.0	-199.0	1.26E-20	-194.57	-206.6	2.20E-21	-156.19	2.41E-16	2.407E-16	-1.6E+02	-144.2	
9.0E+08	-188.0	-200.0	9.94E-21	-195.78	-207.8	1.67E-21	-156.61	2.18E-16	2.183E-16	-1.6E+02	-144.6	
1.0E+09	-188.9	-200.9	8.05E-21	-196.85	-208.9	1.30E-21	-156.99	2.00E-16	2.001E-16	-1.6E+02	-145.0	

Quadrature Amplitude Modulated Noise

K1 PMOS	0.000064545				
K1 NMOS	8.1E-24	Cox	0.008463382	gm5 (PMOS)	0.131194512
AF	0.8265	W	4.50E-05		
gm3	1.503330E-02	L	0.00000018		

Frequency	(Vn)2	Vgs	gm3*vgs	(gm3*vgs/gm5)/pi()	AM Noise dBm/Hz	Frequency	Am noise in dBc
1.0E+02	2.62698E-12	1.62079E-06	2.44E-08	5.91E-08	-101.57	1.0E+02	-121.57
2.0E+02	1.48134E-12	1.2171E-06	1.83E-08	4.44E-08	-104.05	2.0E+02	-124.05
3.0E+02	1.05954E-12	1.02934E-06	1.55E-08	3.75E-08	-105.51	3.0E+02	-125.51
4.0E+02	8.35321E-13	9.13959E-07	1.37E-08	3.33E-08	-106.54	4.0E+02	-126.54
5.0E+02	6.94636E-13	8.33448E-07	1.25E-08	3.04E-08	-107.34	5.0E+02	-127.34
6.0E+02	5.97467E-13	7.7296E-07	1.16E-08	2.82E-08	-108.00	6.0E+02	-128.00
7.0E+02	5.25996E-13	7.25256E-07	1.09E-08	2.65E-08	-108.55	7.0E+02	-128.55
8.0E+02	4.71034E-13	6.86319E-07	1.03E-08	2.50E-08	-109.03	8.0E+02	-129.03
9.0E+02	4.27341E-13	6.53713E-07	9.83E-09	2.38E-08	-109.45	9.0E+02	-129.45
1.0E+03	3.91702E-13	6.25861E-07	9.41E-09	2.28E-08	-109.83	1.0E+03	-129.83
2.0E+03	2.20879E-13	4.69978E-07	7.07E-09	1.71E-08	-112.32	2.0E+03	-132.32
3.0E+03	1.57985E-13	3.97473E-07	5.98E-09	1.45E-08	-113.77	3.0E+03	-133.77
4.0E+03	1.24553E-13	3.5292E-07	5.31E-09	1.29E-08	-114.81	4.0E+03	-134.81
5.0E+03	1.03576E-13	3.21832E-07	4.84E-09	1.17E-08	-115.61	5.0E+03	-135.61
6.0E+03	8.90869E-14	2.98474E-07	4.49E-09	1.09E-08	-116.26	6.0E+03	-136.26
7.0E+03	7.84301E-14	2.80054E-07	4.21E-09	1.02E-08	-116.82	7.0E+03	-136.82
8.0E+03	7.02348E-14	2.65018E-07	3.98E-09	9.67E-09	-117.29	8.0E+03	-137.29
9.0E+03	6.37198E-14	2.52428E-07	3.79E-09	9.21E-09	-117.72	9.0E+03	-137.72
1.0E+04	5.84058E-14	2.41673E-07	3.63E-09	8.81E-09	-118.10	1.0E+04	-138.10
2.0E+04	3.29348E-14	1.81479E-07	2.73E-09	6.62E-09	-120.58	2.0E+04	-140.58
3.0E+04	2.35567E-14	1.53482E-07	2.31E-09	5.60E-09	-122.04	3.0E+04	-142.04
4.0E+04	1.85718E-14	1.36278E-07	2.05E-09	4.97E-09	-123.07	4.0E+04	-143.07
5.0E+04	1.54439E-14	1.24274E-07	1.87E-09	4.53E-09	-123.87	5.0E+04	-143.87
6.0E+04	1.32835E-14	1.15254E-07	1.73E-09	4.20E-09	-124.53	6.0E+04	-144.53
7.0E+04	1.16945E-14	1.08141E-07	1.63E-09	3.94E-09	-125.08	7.0E+04	-145.08
8.0E+04	1.04725E-14	1.02335E-07	1.54E-09	3.73E-09	-125.56	8.0E+04	-145.56
9.0E+04	9.50112E-15	9.74737E-08	1.47E-09	3.56E-09	-125.98	9.0E+04	-145.98
1.0E+05	8.70875E-15	9.33207E-08	1.40E-09	3.40E-09	-126.36	1.0E+05	-146.36
2.0E+05	4.91083E-15	7.00773E-08	1.05E-09	2.56E-09	-128.85	2.0E+05	-148.85
3.0E+05	3.51249E-15	5.92663E-08	8.91E-10	2.16E-09	-130.30	3.0E+05	-150.30
4.0E+05	2.76919E-15	5.26231E-08	7.91E-10	1.92E-09	-131.34	4.0E+05	-151.34
5.0E+05	2.30281E-15	4.79876E-08	7.21E-10	1.75E-09	-132.14	5.0E+05	-152.14
6.0E+05	1.98068E-15	4.45048E-08	6.69E-10	1.62E-09	-132.79	6.0E+05	-152.79
7.0E+05	1.74374E-15	4.17582E-08	6.28E-10	1.52E-09	-133.35	7.0E+05	-153.35
8.0E+05	1.56154E-15	3.95163E-08	5.94E-10	1.44E-09	-133.82	8.0E+05	-153.82
9.0E+05	1.41669E-15	3.76389E-08	5.66E-10	1.37E-09	-134.25	9.0E+05	-154.25
1.0E+06	1.29854E-15	3.60353E-08	5.42E-10	1.31E-09	-134.63	1.0E+06	-154.63
2.0E+06	7.32242E-16	2.706E-08	4.07E-10	9.87E-10	-137.11	2.0E+06	-157.11
3.0E+06	5.23739E-16	2.28854E-08	3.44E-10	8.35E-10	-138.57	3.0E+06	-158.57
4.0E+06	4.12908E-16	2.03201E-08	3.05E-10	7.41E-10	-139.60	4.0E+06	-159.60
5.0E+06	3.43366E-16	1.85301E-08	2.79E-10	6.76E-10	-140.40	5.0E+06	-160.40
6.0E+06	2.95334E-16	1.71853E-08	2.58E-10	6.27E-10	-141.06	6.0E+06	-161.06
7.0E+06	2.60005E-16	1.61247E-08	2.42E-10	5.88E-10	-141.61	7.0E+06	-161.61
8.0E+06	2.32837E-16	1.5259E-08	2.29E-10	5.57E-10	-142.09	8.0E+06	-162.09
9.0E+06	2.11239E-16	1.45341E-08	2.18E-10	5.30E-10	-142.51	9.0E+06	-162.51
1.0E+07	1.93623E-16	1.39148E-08	2.09E-10	5.08E-10	-142.89	1.0E+07	-162.89
2.0E+07	1.09183E-16	1.04491E-08	1.57E-10	3.81E-10	-145.38	2.0E+07	-165.38
3.0E+07	7.80936E-17	8.83706E-09	1.33E-10	3.22E-10	-146.83	3.0E+07	-166.83
4.0E+07	6.15678E-17	7.84651E-09	1.18E-10	2.86E-10	-147.87	4.0E+07	-167.87
5.0E+07	5.11985E-17	7.15531E-09	1.08E-10	2.61E-10	-148.67	5.0E+07	-168.67
6.0E+07	4.40366E-17	6.63601E-09	9.98E-11	2.42E-10	-149.32	6.0E+07	-169.32
7.0E+07	3.87688E-17	6.22646E-09	9.36E-11	2.27E-10	-149.88	7.0E+07	-169.88
8.0E+07	3.47178E-17	5.89218E-09	8.86E-11	2.15E-10	-150.35	8.0E+07	-170.35
9.0E+07	3.14974E-17	5.61225E-09	8.44E-11	2.05E-10	-150.78	9.0E+07	-170.78
1.0E+08	2.88706E-17	5.37314E-09	8.08E-11	1.96E-10	-151.16	1.0E+08	-171.16
2.0E+08	1.628E-17	4.03485E-09	6.07E-11	1.47E-10	-153.64	2.0E+08	-173.64
3.0E+08	1.16444E-17	3.41238E-09	5.13E-11	1.24E-10	-155.10	3.0E+08	-175.10
4.0E+08	9.18023E-18	3.02989E-09	4.55E-11	1.11E-10	-156.13	4.0E+08	-176.13
5.0E+08	7.63409E-18	2.76299E-09	4.15E-11	1.01E-10	-156.93	5.0E+08	-176.93
6.0E+08	6.5662E-18	2.56246E-09	3.85E-11	9.35E-11	-157.59	6.0E+08	-177.59
7.0E+08	5.78073E-18	2.40431E-09	3.61E-11	8.77E-11	-158.14	7.0E+08	-178.14
8.0E+08	5.17669E-18	2.27523E-09	3.42E-11	8.30E-11	-158.62	8.0E+08	-178.62
9.0E+08	4.6965E-18	2.16714E-09	3.26E-11	7.90E-11	-159.04	9.0E+08	-179.04
1.0E+09	4.30483E-18	2.07481E-09	3.12E-11	7.57E-11	-159.42	1.0E+09	-179.42

Quadrature Frequency Modulated Noise

K1 NMOS	8.1E-24	Cox	0.008463382
AF	0.8265	W	4.50E-05
gm	0.00E+00	L	0.00000018
Kp	0.000225		
Vgsn	0.766076		
Vt	0.5		
KVCOlb	1.80E+10		

Frequency	(Vn)2	dBm input referred Vn	I	delta I	Delta Frequen cy	Beta	FM phase noise dBc	inverse log of Ib	inverse log of Vc	total inverse log	total in dBc	Frequency
1.0E+02	2.63E-12	-85.8	2.0000244E-03	2.4365962E-08	4.39E+02	4.39E+00	9.82	9.60E+00	1.07E-01	9.60E+00	9.82	1.0E+02
2.0E+02	1.481E-12	-88.3	2.0000183E-03	1.8297114E-08	3.29E+02	1.65E+00	1.31	1.35E+00	1.51E-02	1.35E+00	1.31	2.0E+02
3.0E+02	1.06E-12	-89.7	2.0000155E-03	1.5474361E-08	2.79E+02	9.28E-01	-3.67	4.30E-01	4.83E-03	4.30E-01	-3.67	3.0E+02
4.0E+02	8.353E-13	-90.8	2.0000137E-03	1.3739841E-08	2.47E+02	6.18E-01	-7.20	1.91E-01	2.15E-03	1.91E-01	-7.20	4.0E+02
5.0E+02	6.946E-13	-91.6	2.0000125E-03	1.2529495E-08	2.26E+02	4.51E-01	-9.94	1.01E-01	1.14E-03	1.01E-01	-9.94	5.0E+02
6.0E+02	5.975E-13	-92.2	2.0000116E-03	1.1620154E-08	2.09E+02	3.49E-01	-12.17	6.06E-02	6.85E-04	6.06E-02	-12.17	6.0E+02
7.0E+02	5.26E-13	-92.8	2.0000109E-03	1.0903001E-08	1.96E+02	2.80E-01	-14.07	3.92E-02	4.44E-04	3.92E-02	-14.07	7.0E+02
8.0E+02	4.71E-13	-93.3	2.0000103E-03	1.0317652E-08	1.86E+02	2.32E-01	-15.71	2.69E-02	3.05E-04	2.69E-02	-15.71	8.0E+02
9.0E+02	4.273E-13	-93.7	2.0000098E-03	9.8274774E-09	1.77E+02	1.97E-01	-17.15	1.93E-02	2.19E-04	1.93E-02	-17.15	9.0E+02
1.0E+03	3.917E-13	-94.1	2.0000094E-03	9.4057678E-09	1.69E+02	1.69E-01	-18.44	1.43E-02	1.63E-04	1.43E-02	-18.44	1.0E+03
2.0E+03	2.209E-13	-96.6	2.0000071E-03	7.0653223E-09	1.27E+02	6.36E-02	-26.95	2.02E-03	2.32E-05	2.02E-03	-26.95	2.0E+03
3.0E+03	1.58E-13	-98.0	2.0000060E-03	5.9753343E-09	1.08E+02	3.59E-02	-31.93	6.41E-04	7.42E-06	6.41E-04	-31.93	3.0E+03
4.0E+03	1.246E-13	-99.0	2.0000053E-03	5.3056601E-09	9.55E+01	2.39E-02	-35.46	2.84E-04	3.31E-06	2.84E-04	-35.46	4.0E+03
5.0E+03	1.036E-13	-99.8	2.0000048E-03	4.8381925E-09	8.71E+01	1.74E-02	-38.20	1.51E-04	1.77E-06	1.51E-04	-38.20	5.0E+03
6.0E+03	8.909E-14	-100.5	2.0000045E-03	4.4870559E-09	8.08E+01	1.35E-02	-40.44	9.04E-05	1.06E-06	9.04E-05	-40.44	6.0E+03
7.0E+03	7.843E-14	-101.1	2.0000042E-03	4.2101319E-09	7.58E+01	1.08E-02	-42.33	5.85E-05	6.91E-07	5.85E-05	-42.33	7.0E+03
8.0E+03	7.023E-14	-101.5	2.0000040E-03	3.9841026E-09	7.17E+01	8.96E-03	-43.97	4.01E-05	4.75E-07	4.01E-05	-43.97	8.0E+03
9.0E+03	6.372E-14	-102.0	2.0000038E-03	3.7948248E-09	6.83E+01	7.59E-03	-45.42	2.87E-05	3.42E-07	2.87E-05	-45.42	9.0E+03
1.0E+04	5.841E-14	-102.3	2.0000036E-03	3.6331425E-09	6.54E+01	6.54E-03	-46.71	2.13E-05	2.55E-07	2.13E-05	-46.71	1.0E+04
2.0E+04	3.293E-14	-104.8	2.0000027E-03	2.7282348E-09	4.91E+01	2.46E-03	-55.22	3.01E-06	3.69E-08	3.01E-06	-55.22	2.0E+04
3.0E+04	2.356E-14	-106.3	2.0000023E-03	2.3073422E-09	4.15E+01	1.38E-03	-60.20	9.56E-07	1.19E-08	9.56E-07	-60.20	3.0E+04
4.0E+04	1.857E-14	-107.3	2.0000020E-03	2.0487127E-09	3.69E+01	9.22E-04	-63.73	4.24E-07	5.38E-09	4.24E-07	-63.73	4.0E+04
5.0E+04	1.544E-14	-108.1	2.0000019E-03	1.8882414E-09	3.36E+01	6.73E-04	-66.47	2.26E-07	2.90E-09	2.26E-07	-66.47	5.0E+04
6.0E+04	1.328E-14	-108.8	2.0000017E-03	1.7326519E-09	3.12E+01	5.20E-04	-68.70	1.35E-07	1.75E-09	1.35E-07	-68.70	6.0E+04
7.0E+04	1.169E-14	-109.3	2.0000016E-03	1.6257193E-09	2.93E+01	4.18E-04	-70.60	8.72E-08	1.15E-09	8.72E-08	-70.60	7.0E+04
8.0E+04	1.047E-14	-109.8	2.0000015E-03	1.5384394E-09	2.77E+01	3.46E-04	-72.24	5.98E-08	7.93E-10	5.98E-08	-72.24	8.0E+04
9.0E+04	9.501E-15	-110.2	2.0000015E-03	1.4653508E-09	2.64E+01	2.93E-04	-73.68	4.28E-08	5.73E-10	4.28E-08	-73.68	9.0E+04
1.0E+05	8.709E-15	-110.6	2.0000014E-03	1.4029181E-09	2.53E+01	2.53E-04	-74.97	3.18E-08	4.29E-10	3.18E-08	-74.97	1.0E+05
2.0E+05	4.911E-15	-113.1	2.0000011E-03	1.0534930E-09	1.90E+01	9.48E-05	-83.48	4.48E-09	6.44E-11	4.48E-09	-83.48	2.0E+05
3.0E+05	3.512E-15	-114.5	2.0000009E-03	8.9095764E-10	1.60E+01	5.35E-05	-88.46	1.43E-09	2.14E-11	1.43E-09	-88.46	3.0E+05
4.0E+05	2.769E-15	-115.6	2.0000008E-03	7.9109928E-10	1.42E+01	3.56E-05	-91.99	6.32E-10	9.85E-12	6.32E-10	-91.99	4.0E+05
5.0E+05	2.303E-15	-116.4	2.0000007E-03	7.2141127E-10	1.30E+01	2.60E-05	-94.73	3.36E-10	5.41E-12	3.36E-10	-94.73	5.0E+05
6.0E+05	1.981E-15	-117.0	2.0000007E-03	6.6905415E-10	1.20E+01	2.01E-05	-96.97	2.01E-10	3.32E-12	2.01E-10	-96.97	6.0E+05
7.0E+05	1.744E-15	-117.6	2.0000006E-03	6.2776270E-10	1.13E+01	1.61E-05	-98.86	1.30E-10	2.20E-12	1.30E-10	-98.86	7.0E+05
8.0E+05	1.562E-15	-118.1	2.0000006E-03	5.9406003E-10	1.07E+01	1.34E-05	-100.50	8.91E-11	1.54E-12	8.91E-11	-100.50	8.0E+05
9.0E+05	1.417E-15	-118.5	2.0000006E-03	5.6583727E-10	1.02E+01	1.13E-05	-101.95	6.39E-11	1.13E-12	6.39E-11	-101.95	9.0E+05
1.0E+06	1.299E-15	-118.9	2.0000005E-03	5.4172923E-10	9.75E+00	9.75E-06	-103.24	4.74E-11	8.51E-13	4.74E-11	-103.24	1.0E+06
2.0E+06	7.322E-16	-121.4	2.0000004E-03	4.0680063E-10	7.32E+00	3.69E-06	-111.75	6.69E-12	1.38E-13	6.69E-12	-111.75	2.0E+06
3.0E+06	5.237E-16	-122.8	2.0000003E-03	3.4404234E-10	6.19E+00	2.08E-06	-116.73	2.13E-12	4.83E-14	2.13E-12	-116.73	3.0E+06
4.0E+06	4.129E-16	-123.8	2.0000003E-03	3.0547872E-10	5.50E+00	1.37E-06	-120.26	9.43E-13	2.31E-14	9.43E-13	-120.26	4.0E+06
5.0E+06	3.434E-16	-124.6	2.0000003E-03	2.7856907E-10	5.01E+00	1.00E-06	-123.00	5.02E-13	1.31E-14	5.02E-13	-123.00	5.0E+06
6.0E+06	2.953E-16	-125.3	2.0000003E-03	2.5835166E-10	4.65E+00	7.75E-07	-125.23	3.00E-13	8.24E-15	3.00E-13	-125.23	6.0E+06
7.0E+06	2.6E-16	-125.9	2.0000002E-03	2.4240719E-10	4.36E+00	6.23E-07	-127.13	1.94E-13	5.59E-15	1.94E-13	-127.13	7.0E+06
8.0E+06	2.328E-16	-126.3	2.0000002E-03	2.2939308E-10	4.13E+00	5.16E-07	-128.77	1.33E-13	4.00E-15	1.33E-13	-128.77	8.0E+06
9.0E+06	2.112E-16	-126.8	2.0000002E-03	2.1849502E-10	3.93E+00	4.37E-07	-130.21	9.53E-14	2.98E-15	9.53E-14	-130.21	9.0E+06
1.0E+07	1.936E-16	-127.1	2.0000002E-03	2.0918582E-10	3.77E+00	3.77E-07	-131.50	7.07E-14	2.30E-15	7.07E-14	-131.50	1.0E+07
2.0E+07	1.092E-16	-129.6	2.0000002E-03	1.5708387E-10	2.83E+00	1.41E-07	-140.01	9.97E-15	4.19E-16	9.97E-15	-140.01	2.0E+07
3.0E+07	7.809E-17	-131.1	2.0000001E-03	1.3285010E-10	2.39E+00	7.97E-08	-144.99	3.17E-15	1.58E-16	3.17E-15	-144.99	3.0E+07
4.0E+07	6.157E-17	-132.1	2.0000001E-03	1.1795897E-10	2.12E+00	5.31E-08	-148.52	1.41E-15	7.96E-17	1.41E-15	-148.52	4.0E+07
5.0E+07	5.12E-17	-132.9	2.0000001E-03	1.0756795E-10	1.94E+00	3.87E-08	-151.26	7.48E-16	4.72E-17	7.48E-16	-151.26	5.0E+07
6.0E+07	4.404E-17	-133.6	2.0000001E-03	9.9761103E-11	1.80E+00	2.99E-08	-153.50	4.47E-16	3.08E-17	4.47E-16	-153.50	6.0E+07
7.0E+07	3.877E-17	-134.1	2.0000001E-03	9.3604232E-11	1.68E+00	2.41E-08	-155.39	2.89E-16	2.15E-17	2.89E-16	-155.39	7.0E+07
8.0E+07	3.472E-17	-134.6	2.0000001E-03	8.8578905E-11	1.59E+00	1.99E-08	-157.03	1.98E-16	1.58E-17	1.98E-16	-157.03	8.0E+07
9.0E+07	3.15E-17	-135.0	2.0000001E-03	8.4370675E-11	1.52E+00	1.69E-08	-158.48	1.42E-16	1.21E-17	1.42E-16	-158.48	9.0E+07
1.0E+08	2.887E-17	-135.4	2.0000001E-03	8.0775981E-11	1.45E+00	1.45E-08	-159.77	1.05E-16	9.47E-18	1.05E-16	-159.77	1.0E+08
2.0E+08	1.628E-17	-137.9	2.0000001E-03	6.0657093E-11	1.09E+00	5.46E-09	-168.28	1.49E-17	1.97E-18	1.49E-17	-168.28	2.0E+08
3.0E+08	1.164E-17	-139.3	2.0000001E-03	5.1293953E-11	9.23E-01	3.08E-09	-173.26	4.73E-18	8.01E-19	4.73E-18	-173.26	3.0E+08
4.0E+08	9.18E-18	-140.4	2.0000000E-03	4.5549221E-11	8.20E-01	2.05E-09	-176.79	2.10E-18	4.25E-19	2.10E-18	-176.79	4.0E+08
5.0E+08	7.634E-18	-141.2	2.0000000E-03	4.1536785E-11	7.48E-01	1.50E-09	-179.53	1.12E-18	2.61E-19	1.12E-18	-179.53	5.0E+08
6.0E+08	6.566E-18	-141.8	2.0000000E-03	3.8522214E-11	6.93E-01	1.16E-09	-181.76	6.66E-19	1.76E-19	6.66E-19	-181.76	6.0E+08
7.0E+08	5.781E-18	-142.4	2.0000000E-03	3.6144772E-11	6.51E-01	9.29E-10	-183.66	4.31E-19	1.26E-19	4.31E-19	-183.66	7.0E+08

Quadrature Lesson Noise

k	1.38E-23	capacitor	5.6E-10
T	298	Q	8
A2 (-20dBm)	0.02		
Center Frequency	2442000000		
F	3		
capacitor	5.6E-10		
Q	8		

Offset Frequency	Phase Noise	Phase Noise in dBc/Hz
1.0E+02	1.3E-04	-38.7
2.0E+02	3.4E-05	-44.7
3.0E+02	1.5E-05	-48.3
4.0E+02	8.4E-06	-50.8
5.0E+02	5.4E-06	-52.7
6.0E+02	3.7E-06	-54.3
7.0E+02	2.7E-06	-55.6
8.0E+02	2.1E-06	-56.8
9.0E+02	1.7E-06	-57.8
1.0E+03	1.3E-06	-58.7
2.0E+03	3.4E-07	-64.7
3.0E+03	1.5E-07	-68.3
4.0E+03	8.4E-08	-70.8
5.0E+03	5.4E-08	-72.7
6.0E+03	3.7E-08	-74.3
7.0E+03	2.7E-08	-75.6
8.0E+03	2.1E-08	-76.8
9.0E+03	1.7E-08	-77.8
1.0E+04	1.3E-08	-78.7
2.0E+04	3.4E-09	-84.7
3.0E+04	1.5E-09	-88.3
4.0E+04	8.4E-10	-90.8
5.0E+04	5.4E-10	-92.7
6.0E+04	3.7E-10	-94.3
7.0E+04	2.7E-10	-95.6
8.0E+04	2.1E-10	-96.8
9.0E+04	1.7E-10	-97.8
1.0E+05	1.3E-10	-98.7
2.0E+05	3.4E-11	-104.7
3.0E+05	1.5E-11	-108.3
4.0E+05	8.4E-12	-110.8
5.0E+05	5.4E-12	-112.7
6.0E+05	3.7E-12	-114.3
7.0E+05	2.7E-12	-115.6
8.0E+05	2.1E-12	-116.8
9.0E+05	1.7E-12	-117.8
1.0E+06	1.3E-12	-118.7
2.0E+06	3.4E-13	-124.7
3.0E+06	1.5E-13	-128.3
4.0E+06	8.4E-14	-130.8
5.0E+06	5.4E-14	-132.7
6.0E+06	3.7E-14	-134.3
7.0E+06	2.7E-14	-135.6
8.0E+06	2.1E-14	-136.8
9.0E+06	1.7E-14	-137.8
1.0E+07	1.3E-14	-138.7
2.0E+07	3.4E-15	-144.7
3.0E+07	1.5E-15	-148.3
4.0E+07	8.4E-16	-150.8
5.0E+07	5.4E-16	-152.7
6.0E+07	3.7E-16	-154.3
7.0E+07	2.7E-16	-155.6
8.0E+07	2.1E-16	-156.8
9.0E+07	1.7E-16	-157.8
1.0E+08	1.3E-16	-158.7
2.0E+08	3.4E-17	-164.7
3.0E+08	1.5E-17	-168.3
4.0E+08	8.4E-18	-170.8
5.0E+08	5.4E-18	-172.7
6.0E+08	3.7E-18	-174.3
7.0E+08	2.7E-18	-175.6
8.0E+08	2.1E-18	-176.8

Combined Phase Noise of the Quadrature Differential Oscillator

Leesons		Frequency modulation			Amplitude modulation			Total			
Frequency	dBc/Hz	dBm/Hz	inverse log	dBc/Hz	dBm/Hz	inverse log	dBm/Hz	inverse log	inverse log	dBm/Hz	dBc/Hz
1.0E+02	-35.7	-55.7	2.68E-06	9.82	-10.2	9.60E-02	-101.57	6.97E-11	9.595426617481E-02	-1.0E+01	9.8
2.0E+02	-41.7	-61.7	6.71E-07	1.31	-18.7	1.35E-02	-104.05	3.93E-11	1.352731929500E-02	-1.9E+01	1.3
3.0E+02	-45.3	-65.3	2.98E-07	-3.67	-23.7	4.30E-03	-105.51	2.81E-11	4.300293198362E-03	-2.4E+01	-3.7
4.0E+02	-47.8	-67.8	1.68E-07	-7.20	-27.2	1.91E-03	-106.54	2.22E-11	1.90706958863E-03	-2.7E+01	-7.2
5.0E+02	-49.7	-69.7	1.07E-07	-9.94	-29.9	1.01E-03	-107.34	1.84E-11	1.014981392190E-03	-3.0E+01	-9.9
6.0E+02	-51.3	-71.3	7.45E-08	-12.17	-32.2	6.06E-04	-108.00	1.59E-11	6.062610437851E-04	-3.2E+01	-12.2
7.0E+02	-52.6	-72.6	5.48E-08	-14.07	-34.1	3.92E-04	-108.55	1.40E-11	3.921405412683E-04	-3.4E+01	-14.1
8.0E+02	-53.8	-73.8	4.19E-08	-15.71	-35.7	2.69E-04	-109.03	1.25E-11	2.688651569800E-04	-3.6E+01	-15.7
9.0E+02	-54.8	-74.8	3.31E-08	-17.15	-37.2	1.93E-04	-109.45	1.13E-11	1.927342050327E-04	-3.7E+01	-17.2
1.0E+03	-55.7	-75.7	2.68E-08	-18.44	-38.4	1.43E-04	-109.83	1.04E-11	1.430974855578E-04	-3.8E+01	-18.4
2.0E+03	-61.7	-81.7	6.71E-09	-26.95	-47.0	2.02E-05	-112.32	5.86E-12	2.017592912688E-05	-4.7E+01	-27.0
3.0E+03	-65.3	-85.3	2.98E-09	-31.93	-51.9	6.41E-06	-113.77	4.19E-12	6.414593307737E-06	-5.2E+01	-31.9
4.0E+03	-67.8	-87.8	1.68E-09	-35.46	-55.5	2.84E-06	-114.81	3.31E-12	2.845012112429E-06	-5.5E+01	-35.5
5.0E+03	-69.7	-89.7	1.07E-09	-38.20	-58.2	1.51E-06	-115.61	2.75E-12	1.514328347691E-06	-5.8E+01	-38.2
6.0E+03	-71.3	-91.3	7.45E-10	-40.44	-60.4	9.04E-07	-116.26	2.36E-12	9.046167672633E-07	-6.0E+01	-40.4
7.0E+03	-72.6	-92.6	5.48E-10	-42.33	-62.3	5.85E-07	-116.82	2.08E-12	5.851787364788E-07	-6.2E+01	-42.3
8.0E+03	-73.8	-93.8	4.19E-10	-43.97	-64.0	4.01E-07	-117.29	1.86E-12	4.012565838791E-07	-6.4E+01	-44.0
9.0E+03	-74.8	-94.8	3.31E-10	-45.42	-65.4	2.87E-07	-117.72	1.69E-12	2.876646399323E-07	-6.5E+01	-45.4
1.0E+04	-75.7	-95.7	2.68E-10	-46.71	-66.7	2.13E-07	-118.10	1.55E-12	2.135989048726E-07	-6.7E+01	-46.7
2.0E+04	-81.7	-101.7	6.71E-11	-55.22	-75.2	3.01E-08	-120.58	8.74E-13	3.01417773025E-08	-7.5E+01	-55.2
3.0E+04	-85.3	-105.3	2.98E-11	-60.20	-80.2	9.56E-09	-122.04	6.25E-13	9.590632768916E-09	-8.0E+01	-60.2
4.0E+04	-87.8	-107.8	1.68E-11	-63.73	-83.7	4.24E-09	-123.07	4.93E-13	4.256887504987E-09	-8.4E+01	-63.7
5.0E+04	-89.7	-109.7	1.07E-11	-66.47	-86.5	2.26E-09	-123.87	4.10E-13	2.267517723704E-09	-8.6E+01	-66.4
6.0E+04	-91.3	-111.3	7.45E-12	-68.70	-88.7	1.35E-09	-124.53	3.53E-13	1.35554406861E-09	-8.9E+01	-68.7
7.0E+04	-92.6	-112.6	5.48E-12	-70.60	-90.6	8.72E-10	-125.08	3.10E-13	8.775132441230E-10	-9.1E+01	-70.6
8.0E+04	-93.8	-113.8	4.19E-12	-72.24	-92.2	5.98E-10	-125.56	2.78E-13	6.021470786479E-10	-9.2E+01	-72.2
9.0E+04	-94.8	-114.8	3.31E-12	-73.68	-93.7	4.28E-10	-125.98	2.52E-13	4.319985477175E-10	-9.4E+01	-73.6
1.0E+05	-95.7	-115.7	2.68E-12	-74.97	-95.0	3.18E-10	-126.36	2.31E-13	3.210044800692E-10	-9.5E+01	-74.9
2.0E+05	-101.7	-121.7	6.71E-13	-83.48	-103.5	4.48E-11	-128.85	1.30E-13	4.564359720692E-11	-1.0E+02	-83.4
3.0E+05	-105.3	-125.3	2.98E-13	-88.46	-108.5	1.43E-11	-130.30	9.32E-14	1.464638525683E-11	-1.1E+02	-88.3
4.0E+05	-107.8	-127.8	1.68E-13	-91.99	-112.0	6.32E-12	-131.34	7.35E-14	6.562831100924E-12	-1.1E+02	-91.8
5.0E+05	-109.7	-129.7	1.07E-13	-94.73	-114.7	3.36E-12	-132.14	6.11E-14	3.532895422060E-12	-1.1E+02	-94.5
6.0E+05	-111.3	-131.3	7.45E-14	-96.97	-117.0	2.01E-12	-132.79	5.26E-14	2.136699216036E-12	-1.2E+02	-96.7
7.0E+05	-112.6	-132.6	5.48E-14	-98.86	-118.9	1.30E-12	-133.35	4.63E-14	1.400864252575E-12	-1.2E+02	-98.5
8.0E+05	-113.8	-133.8	4.19E-14	-100.50	-120.5	8.91E-13	-133.82	4.15E-14	9.745619819973E-13	-1.2E+02	-100.1
9.0E+05	-114.8	-134.8	3.31E-14	-101.95	-121.9	6.39E-13	-134.25	3.76E-14	7.095629303688E-13	-1.2E+02	-101.5
1.0E+06	-115.7	-135.7	2.68E-14	-103.24	-123.2	4.74E-13	-134.63	3.45E-14	5.356012749426E-13	-1.2E+02	-102.7
2.0E+06	-121.7	-141.7	6.71E-15	-111.75	-131.7	6.69E-14	-137.11	1.94E-14	9.300945262374E-14	-1.3E+02	-110.3
3.0E+06	-125.3	-145.3	2.98E-15	-116.73	-136.7	2.13E-14	-138.57	1.39E-14	3.813950737602E-14	-1.3E+02	-114.2
4.0E+06	-127.8	-147.8	1.68E-15	-120.26	-140.3	9.43E-15	-139.60	1.10E-14	2.206374261436E-14	-1.4E+02	-116.6
5.0E+06	-129.7	-149.7	1.07E-15	-123.00	-143.0	5.02E-15	-140.40	9.11E-15	1.520458966817E-14	-1.4E+02	-118.2
6.0E+06	-131.3	-151.3	7.45E-16	-125.23	-145.2	3.00E-15	-141.06	7.84E-15	1.158143276816E-14	-1.4E+02	-119.4
7.0E+06	-132.6	-152.6	5.48E-16	-127.13	-147.1	1.94E-15	-141.61	6.90E-15	9.387549181220E-15	-1.4E+02	-120.3
8.0E+06	-133.8	-153.8	4.19E-16	-128.77	-148.8	1.33E-15	-142.09	6.18E-15	7.928714685022E-15	-1.4E+02	-121.0
9.0E+06	-134.8	-154.8	3.31E-16	-130.21	-150.2	9.53E-16	-142.51	5.61E-15	6.891124660353E-15	-1.4E+02	-121.6
1.0E+07	-135.7	-155.7	2.68E-16	-131.50	-151.5	7.07E-16	-142.89	5.14E-15	6.115220336510E-15	-1.4E+02	-122.1
2.0E+07	-141.7	-161.7	6.71E-17	-140.01	-160.0	9.97E-17	-145.38	2.90E-15	3.065014292990E-15	-1.5E+02	-125.1
3.0E+07	-145.3	-165.3	2.98E-17	-144.99	-165.0	3.17E-17	-146.83	2.07E-15	2.134480128408E-15	-1.5E+02	-126.7
4.0E+07	-147.8	-167.8	1.68E-17	-148.52	-168.5	1.41E-17	-147.87	1.63E-15	1.665124378927E-15	-1.5E+02	-127.8
5.0E+07	-149.7	-169.7	1.07E-17	-151.26	-171.3	7.48E-18	-148.67	1.36E-15	1.377262782735E-15	-1.5E+02	-128.6
6.0E+07	-151.3	-171.3	7.45E-18	-153.50	-173.5	4.47E-18	-149.32	1.17E-15	1.180860618978E-15	-1.5E+02	-129.3
7.0E+07	-152.6	-172.6	5.48E-18	-155.39	-175.4	2.89E-18	-149.88	1.03E-15	1.037472484632E-15	-1.5E+02	-129.8
8.0E+07	-153.8	-173.8	4.19E-18	-157.03	-177.0	1.98E-18	-150.35	9.22E-16	9.277474343567E-16	-1.5E+02	-130.3
9.0E+07	-154.8	-174.8	3.31E-18	-158.48	-178.5	1.42E-18	-150.78	8.36E-16	8.408217151333E-16	-1.5E+02	-130.8
1.0E+08	-155.7	-175.7	2.68E-18	-159.77	-179.8	1.05E-18	-151.16	7.66E-16	7.700994532701E-16	-1.5E+02	-131.1
2.0E+08	-161.7	-181.7	6.71E-19	-168.28	-188.3	1.49E-19	-153.64	4.32E-16	4.329673586723E-16	-1.5E+02	-133.6
3.0E+08	-165.3	-185.3	2.98E-19	-173.26	-193.3	4.73E-20	-155.10	3.09E-16	3.094410779860E-16	-1.6E+02	-135.1
4.0E+08	-167.8	-187.8	1.68E-19	-176.79	-196.8	2.10E-20	-156.13	2.44E-16	2.438749284993E-16	-1.6E+02	-136.1
5.0E+08	-169.7	-189.7	1.07E-19	-179.53	-199.5	1.12E-20	-156.93	2.03E-16	2.027630313490E-16	-1.6E+02	-136.9
6.0E+08	-171.3	-191.3	7.45E-20	-181.76	-201.8	6.66E-21	-157.59	1.74E-16	1.743788814812E-16	-1.6E+02	-137.6
7.0E+08	-172.6	-192.6	5.48E-20	-183.66	-203.7	4.31E-21	-158.14	1.53E-16	1.535066666481E-16	-1.6E+02	-138.1
8.0E+08	-173.8	-193.8	4.19E-20	-185.30	-205.3	2.95E-21	-158.62	1.37E-16	1.374584891559E-16	-1.6E+02	-138.6
9.0E+08	-174.8	-194.8	3.31E-20	-186.74	-206.7	2.12E-21	-159.04	1.25E-16	1.247024382078E-16	-1.6E+02	-139.0
1.0E+09	-186.7	-206.7	2.15E-21	-188.03	-208.0	1.57E-21	-159.42	1.14E-16	1.142740871520E-16	-1.6E+02	-139.4