Development and Application of the SLUTH Algorithm in Determining Distance, Depth, Structural Index and Susceptibility of Magnetic Anomalies

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Master of Science, Department of Earth Sciences

By

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Abstract

A useful property of the anomalous magnetic field T associated with a geological body is that it can be generally assumed homogeneous with respect to horizontal x and vertical z displacements. Its horizontal and vertical derivatives are also homogeneous fields. A corollary is that the angle ratio θ of the lateral gradient over the vertical gradient, $\theta = \tan^{-1}(-\partial T / \partial x / \partial T / \partial z)$, is constant along rays emanating outward from a single point approximating the location of the geological source. Hence drawing these ray paths, which connect same θ at different upward continued heights, along a profile can be used to identify the location and depth of the source. Further examination of these ray paths can also determine the structural index of the causative body and, in turn, this allows the susceptibility contrast to be solved for. This is the basis for the Source Location Using Total-field Homogeneity (SLUTH) method.

This thesis describes a computer algorithm which implements the SLUTH algorithm automatically and determines the afore mentioned outputs along magnetic profiles.

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1 Introduction

1.1 Problem Statement and Research Objectives

Magnetic geophysical prospecting methods measure the intensity of magnetic field of the earth. In addition to very slow variations (on the order of 10^5 - 10^6 years) and polarity reversals, which are both phenomena related to the rotation of the liquid outer core of the earth (Bullard and Gellman, 1954), there is also a daily “diurnal” variation, caused by the rotation of the earth in the solar magnetic field (Fowler, 2005). In addition to these temporal variations, there are also significantly smaller lateral variations in the intensity of the field that are caused by subsurface bodies rich in magnetic minerals (Campbell, 2003). Location of these magnetic anomalies is useful in the search of Earth’s resources and is the object of the magnetic method.

There are many different reasons to want to identify magnetic contrasts in the subsurface. In the search for diamonds, it is necessary to map the extent of their host rock, kimberlites. Kimberlites are usually detected by finding circular magnetic anomalies which can either be positive or negative depending on magnetization contrasts with the surrounding rock (Nabighian et al., 2005). In surveying for oil and gas, it is necessary to find the depth of the interface between the basin fill and the crystalline basement. The crystalline basement usually has a much higher magnetic susceptibility than the sedimentary basin beds and thus can be mapped using magnetic surveying. These and many more case histories about the successful use of the magnetic geophysical method in many facets of geological mapping (Nabighian et al., 2005).

The specific type of magnetic data to be discussed throughout this thesis is airborne magnetic or aeromagnetic data. These data are collected by fixed-wing aircraft or helicopter and are recorded by a magnetometer, which measures the magnetic intensity at discrete points in space along a flight profile (Campbell, 2003).

The challenge for the interpreter is to characterize the anomalous subsurface bodies that are the cause or source of the magnetic variations. Unusual or unexplained variations in the magnetic field are termed anomalies. There are many different interpretation techniques currently available for interpreting anomalies (they will be reviewed in Section 2.5), some of which are automated and others which are still too complicated to implement automatically. Some of these interpretation techniques also require assumptions to be made regarding the underlying geology, and there is often insufficient information to justify these assumptions. These restrictions beg the question: is there an interpretation method which can be implemented algorithmically without the need for a priori knowledge of the underlying geology?

The objectives of this thesis are to investigate the potential of the SLUTH (Source Locating Using Total-field Homogeneity) method, introduced by Thurston and Smith (2007), and to create an algorithm written in the programming language C which can interpret entire flight profiles automatically and efficiently.

1.2 Thesis Overview:

Following this introduction, Chapter 2 gives some general background on the magnetic geophysical method and its history. Fundamental concepts on homogeneous equations and the traits which are useful to the SLUTH method are presented. The SLUTH method is compared to other interpretation techniques presently in use.

Chapter 3 outlines the implementation of the SLUTH algorithm. It is a 6 step process:

- Step 1: Inputting data and variables
- Step 2: Creation of the θ matrix
- Step 3: Finding areas of interest
- Step 4: Construction of ray paths from different θ values
- Step 5: Ray path analysis
- Step 6: File output

Chapter 4 covers the application of the SLUTH algorithm to synthetic data. The first section applies the method to three model types: contact, thin sheet and horizontal cylinder. The following section analyzes a thin sheet surveyed from different heights, ranging from 100 m to 1000 m. The final section examines the results when two thin sheets are spaced apart at varying distances along a flight profile. These examples show how the algorithm reacts to different survey and geological parameters.

Chapter 5 discusses the application of the SLUTH algorithm to field data from two different areas from the Abitibi region in northern Ontario and Quebec. The first is a single flight profile from Timmins, Ontario which has been interpreted previously. The second is 43 parallel flight profiles from Chibougamau, Quebec spaced 200 m apart. The results of the second example are presented in planar view and plotted over the original magnetic data.

Chapter 6 formulates conclusions drawn from the research described in the thesis and proposes future work for this project.

2 Theory

2.1 Overview

In this chapter, a brief background of magnetic surveying and data processing will first be presented in Section 2.2. After this, homogeneous functions and their characteristics will be examined in section 2.3 and their extension to synthetic magnetic data will be examined in sections 2.4. Finally, in Sections 2.5 and 2.6, the SLUTH method will be introduced together with an explanation on how it compares to similar interpretation methods.

2.2 Background on airborne magnetics

In the most general sense, the magnetic geophysical exploration method can be defined as measuring the variations of intensity of the Earth's magnetic field over an area in space. These variations are attributed to natural and man-made magnetic sources in the area.

This method dates back to the 19th century where it began with prospectors using vertical compass (dipping needle) to find lodestone and other magnetic rocks. After World War II, the magnetic exploration method grew substantially in popularity as the original fluxgate magnetometer was developed for airborne submarine detection (Reford and Summer, 1964).

The fluxgate magnetometer allowed for many monumental surveys to be conducted, one of which was the magnetic mapping of the sea floor near spreading centers. This survey showed that approximately half of the seafloor igneous rocks had remnant magnetic

orientation opposite to that of today's geomagnetic field. This convinced many scientists of the new model of the earth's magnetic field which allowed for periodic reversals (Vine and Matthews, 1963). The existence of parallel magnetic anomalies away from spreading centres was also a key evidence for plate tectonics.

More sensitive instruments were progressively developed. In the 1950s, the proton precession magnetometer was developed with a sensitivity of 1 nT (Nabighian et al., 2005). This was bettered by two orders of magnitude by the alkaline-vapour magnetometers of the 1970s, to 0.01 nT. Originally the magnetic method was used solely for the detection of iron oxide deposits. It now extends to indirect exploration for minerals (such as copper and gold), hydrocarbons, ground water, geothermal resources and searching for man-made items such as buried pipelines or landmines (Nabighian et al., 2005).

The alkaline-vapour magnetometers are the present day standard for airborne magnetic surveys. Fugro Airborne Surveys (www.fugroairborne.com) of Ottawa, Ontario, the industry sponsor for this project, uses optically pumped alkaline-vapour magnetometers developed by Scintrex Ltd. (www.scintrexltd.com) of Concord, Ontario for their normal operations. In Figure 2-1, a generic alkaline-vapour system is presented. There are four basic parts to this instrument. The most important is the glass vapour cell in the centre of the instrument which is filled with alkali metals such as cesium, potassium or rubidium in gas form (Hrvoic et al., 2008). The metals must be continuously heated to a temperature between 45-55°C. This gas is then exposed to light of a wavelength specific to the alkali

metal being used. According to quantum mechanics, atoms have different levels of energy, which is evident in the radius of rotation of the electrons. Energy levels start at 1 and go up to infinity and are always integers, however, only low levels are stable. Atoms can be excited into a higher energy level by absorbing light of a certain wavelength depending on their current energy level. This light is first created by a lamp, then the specific wavelength, which will excite the electrons, is permitted by the filters, see Figure 2-1 (Hrvoic et al., 2008). The electrons in the glass vapour cell are originally at level 2, which means that the disk is opaque and no light is transferred through. As the light is absorbed by the level 2 electrons, they become excited and rise to level 3. At level 3, the electrons spontaneously decay to either level 1 or level 2, see Figure 2-2. Those which decay to level 1 remain at level 1 and do not absorb any more light. Those which decay to level 2 absorb more light and are once again excited to level 3. This continues until level 1 is fully populated and the disk becomes translucent. The photo measurement at the other end relays that light is being transferred through the magnetometer (Hrvoic et al., 2008).

In order to return the electrons to level 2, radio frequency (RF) depolarization is used. A varying RF is applied and the one which causes level 2 to be repopulated from level 1 is called the Larmor frequency (Hrvoic et al., 2008). Finding the Larmor frequency is the crucial step involved in this magnetometer because this frequency depends on two things: the energy difference between the gas at level 1 and level 2 (which is known) and the magnetic intensity to which the magnetometer is exposed. By measuring the Larmor frequency, the magnetic intensity is easily estimated. Once the vapour cell becomes

opaque again and the light sensor does not receive any light the magnetometer records the RF used and starts the process again (Hrvoic et al., 2008). These readings can be done 1000 times per second.

Being able to record magnetic intensity with a high accuracy meant that positioning errors started to have a greater negative impact on data quality. Slight errors in position called for careful corrections to be applied to the data. The Global Positioning System (GPS) became available in the early 1990s and provided more precise georeferencing. High-resolution magnetic mapping then became possible, and has now become the industry standard.

After the total magnetic intensity (TMI) has been measured at a high resolution for a series of points in space along a flight profile, several corrections have to be made. An exploration survey is not done to measure the magnetic field caused by planet earth as a whole, but more specifically that caused by anomalous magnetic features in the subsurface. Therefore, the first correction is the removal of the earth's internal magnetic field caused by the earth's core. This field is attributed to rotation of liquid iron in the liquid outer core, according to the geodynamo model (Herzenberg, 1958). The intensity of the internal magnetic field of the earth varies with latitude, it can be as small as 30000 nT at the magnetic equator and as large as 60000 nT at the magnetic poles (Schmucker, 1985). Near Ottawa, it is 56000 nT (GSC, 2000). The value at a location is defined by the International Geomagnetic Reference Field (IGRF). The IGRF is updated every five years as the magnetic field of the earth is ever changing.

The second correction aims at compensating for the magnetic field caused by the platform on which the magnetometer is mounted. The influence the aircraft has on the measured field is complex and dependant on aircraft heading, attitude in pitch, roll and yaw, rate of change of attitude, and the local field intensity. This is very difficult to correct for. To lessen the platform's effect the magnetometer is sometimes towed by a long cable or the aircraft is completely rebuilt to reduce its magnetic signature (Nabighian et al., 2005).

The third correction is the external field removal. The external field is the magnetic interference from outer space, mostly caused by solar wind. The variations are found either by using the data from a stationary base station or by using tie-lines. Tie-lines are flight lines flown perpendicular to one another. Where they cross, the difference in the reading defines the variations with time (Whitham and Niblett, 1961). These variations are then corrected for in the data.

The final two corrections which will be discussed here are the lag and heading corrections. The lag correction is used to place the magnetometer at the right position in space. The GPS is giving the position of some point in the plane, but as stated earlier, the magnetometer is often being towed behind by a cable. By measuring the angle and length of the tow cable, the magnetometer's locations can be updated continuously. When a flight is flown along a profile distance, often the aircraft will deviate slightly from a linear flight path. These slight variations are corrected for so that each flight is approximately straight.

Several interpretation techniques require flight data from the same profile but at different heights. One option would be to re-fly each survey at different altitudes, but this would prove to be very expensive. This is why a mathematical method for determining magnetic intensity at different heights using the nominal flight data has been developed; this method is called upward continuation.

Upward continuation can be computed in a number of ways but for the purpose of this thesis the constant elevation method is implemented. This method assumes that the aeromagnetic data was acquired at a constant elevation, i.e. that no variance in altitude occurred during flight. While this is not perfectly true, this method proves to give a great degree of accuracy in results since altitude changes are small and occur slowly. To find the data at the new height (Blakely, 1996) the integral is taken along the x axis as

$$U(x, z_0 - \Delta z) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \left(\frac{1}{r} \frac{\partial U(x', z_0)}{\partial z'} - U(x', z_0) \frac{\partial}{\partial z'} \frac{1}{r} \right) dx' \quad [1]$$

where $r = \sqrt{(x - x')^2 + (z_0 - \Delta z - z')^2}$, x is the profile distance in meters, $U(x)$ is the magnetic intensity due to nearby magnetic bodies, z_0 is the nominal height, prime indicates the integrating variables and where $\Delta z > 0$ is the height difference between the nominal height and height being upward continued to.

Using Green's theorem, this is further simplified to (Blakely, 1996)

$$U(x, z_0 - \Delta z) = \frac{\Delta z}{2\pi} \int_{-\infty}^{\infty} \frac{\partial U(x', z_0)/\partial z}{[(x - x')^2 + \Delta z^2]^{3/2}} dx' \quad [2]$$

where $\Delta z > 0$.

Equation [2] gives the approximate magnetic intensity for any point in space above the nominal flight height. Upward continuation is equivalent to a low pass filter: short-wavelength anomalies are attenuated much more than long-wavelength anomalies. By upward continuing data, small details are lost but the method emphasizes signal from stronger sources at greater depths (Henderson and Zietz, 1949).

Now the airborne magnetic data have been acquired, processed and upward continued. Before the SLUTH interpretation method can be introduced, a few mathematical concepts must be explained.

Figure 2-1: A generic alkaline-vapour magnetometer (Hrvoic et al., 2008).

Figure 2-2 : Quantum mechanics of alkaline-vapour system. Although many more energy levels are theoretically possible, only 1 through 3 are seen in nature. Levels 1 and 2 are both stable but as atoms are excited to level 3 they become unstable and spontaneously decay, i.e. return to a more stable level of energy, level 1 or 2 (Hrvoic et al., 2008) .

2.3 Homogeneous Functions

A function f of n variables for which both $(x_1, \dots, x_n)$ and $(tx_1, \dots, tx_n)$ are in the domain whenever $t > 0$ is homogeneous of degree k if

$$f(tx_1, \dots, tx_n) = t^k f(x_1, \dots, x_n) \quad [3]$$

for all $t > 0$.

If f is a differentiable function of n variables that is homogeneous of degree k , then each of its partial derivatives $f_i' \neq 0$ (for $i = 1, \dots, n$) are homogeneous of degree $k-1$. For the remainder of this thesis, we will assume that $n = 2$ and the two variables are going to be x , distance along the profile, and z , height, both in metres.

Using these facts, a new method to determine depth and position of causative subsurface ore bodies can be constructed. The level curve of a function is described as the contour line between different ranges of values. The slope of the level curve of a function T is given by the formula:

$$dT = \frac{dT/dx}{dT/dz} = \frac{T_x'(x, z)}{T_z'(x, z)} \quad [4]$$

when $T_z'(x, z) \neq 0$. So, the slope of the curve through some point (x_0, z_0) is:

$$dT(x_0, z_0) = \frac{T_x'(x_0, z_0)}{T_z'(x_0, z_0)}.$$

Assume the function T to be homogeneous of some degree, say k . Let $c > 0$ be some scalar and consider the level curve through (cx_0, cz_0) . In this case, the slope of the level curve is:

$$\frac{T'_x(cx_0, cz_0)}{T'_z(cx_0, cz_0)} = \frac{c^{k-1}T'_x(x_0, z_0)}{c^{k-1}T'_z(x_0, z_0)} = \frac{T'_x(x_0, z_0)}{T'_z(x_0, z_0)}.$$

Therefore, the slope at (cx_0, cz_0) is the same as the slope at (x_0, z_0) . For any point (x_0, z_0) , the points (cx_0, cz_0) with $c \geq 0$ define a straight line ray that goes through the origin (when $c = 0$), through (x_0, z_0) (when $c = 1$) and out to infinity for large c (Thurston and Smith, 2007). This implies the following interesting and important corollary: let f be a differentiable function of two variables that is homogeneous of some degree. Then along any given ray from the origin, the slopes of the level curves of f are the same. By connecting same slope values on different level curves a ray path can be constructed. If two or more different ray paths are constructed then their intersection will be the origin. For airborne magnetic data, the origin indicates the location of the causative body (Thurston and Smith, 2007).

As a simple example, it can be easily shown that $f(x, y) = x^{1/4}y^{1/2}$ is a homogeneous equation of degree $3/4$. In Figure 2-3, a contour map of $f(x, y)$ with ray paths and same slopes for each ray path is labelled. The level curves are the contour lines shown by the colours blue through red. The ray paths intersect at the origin as predicted.

In summary, if magnetic data could be modelled as a homogeneous function then the construction of ray paths around anomalies would give the position of the source of these anomalies. In Section 2.4 we will show that some magnetic bodies have fields that behave as homogeneous functions. In these cases, finding subsurface anomalies will involve constructing the ray paths.

Figure 2-3 : The contour plot of the function $f(x, y) = x^{1/4} y^{1/2}$. There are two ray paths which intersect 5 different contour lines, which range from 5 to 30, at the same slopes $m1$ and $m2$, respectively. These ray paths both intersect the origin $(0, 0)$.

2.4 *Three Different Models of Magnetic Bodies*

Throughout this thesis we will consider three types of bodies: a contact, a thin sheet, and a cylinder. In reality, magnetic bodies in the subsurface are not so simple, but these three models will form a basis for getting a rough understanding of the geological context being examined. The mathematic formulae describing the field and the spatial derivatives of the field will be explored and their homogeneity studied. The study of synthetic data derived from these formulae is helpful in understanding the strengths and weaknesses of the SLUTH method.

A contact is when there is a contrast in susceptibility across a planar boundary that is infinite in all directions, but terminates at a horizontal plane. The straight line where the horizontal plane and the other plane (of arbitrary dip) meet is the top edge of the contact. Figure 2-4 (a) shows a contact with a vertical boundary. A thin sheet, also known as a dyke, is a body which is thin in one direction and very long in the two other directions, and is shown in Figure 2-4 (b). In Figure 2-4(b), the thin sheet is dipping to the left. A cylinder is a rock body which is cylindrical in shape, the long axis of which is horizontal, as seen in Figure 2-4 (c). The functions for the total magnetic field intensity (TMI) for a contact (Nabighian, 1972), thin sheet (Reford, 1964) and cylinder (Smith et al., 1998) are respectively,

$$T(x, z) = 2KFb^2 \sin(d) \left[-\tan^{-1}\left(\frac{x}{z}\right) \cos(2I - d - 90) + \ln((x^2 + z^2)^{1/2}) \sin(2I - d - 90) \right] \quad [5]$$

$$T(x, z) = 2KFb^2 w \left(\frac{z \cos(2I - d) - x \sin(2I - d)}{z^2 + x^2} \right) \quad [6]$$

$$T(x, z) = 2KFS \frac{\sin i}{\sin I} \left(\frac{(z^2 - x^2) \cos(2I - 180) + 2xz \sin(2I - 180)}{(x^2 + z^2)^2} \right). \quad [7]$$

Where K is the magnetic susceptibility contrast between the body and the surrounding material, F is the intensity of the earth's regional magnetic field in nanoTeslas, d is the dip of the contact or thin sheet in degrees, x is the distance along the profile in metres from the top edge or centre of the body, z is the depth to the top edge or centre of the body in metres, A is the angle between the profile and magnetic north (declination) in degrees, I is the inclination of the earth's magnetic field from the horizontal in degrees, $i = \tan^{-1}(\tan I * \cos A)$, $b^2 = 1 - \cos^2 i \sin^2 A$, w is the width of the thin sheet in metres, and S is the cross-sectional area of the cylinder in metres squared.

In Figure 2-5 (a) the synthetic TMI for an outcropping thin sheet is shown for 250 m on each side of the top edge and from the ground surface to a height of 500 m.

The horizontal and vertical derivatives of the TMI for a contact, thin sheet and a cylinder are respectively (Smith et al., 2005),

$$\frac{\partial T}{\partial x}(x, z) = 2KFb^2 \sin(d) \left(\frac{(z \cos(2I - d - 90) + x \sin(2I - d - 90))}{x^2 + z^2} \right) \quad [8]$$

$$\frac{\partial T}{\partial z}(x, z) = 2KFb^2 \sin(d) \left(\frac{(x \cos(2I - d - 90) - z \sin(2I - d - 90))}{x^2 + z^2} \right) \quad [9]$$

$$\frac{\partial T}{\partial x}(x, z) = 2KFb^2 w \left(\frac{(x^2 - z^2) \cos(2I - d) - 2xz \sin(2I - d)}{(x^2 + z^2)^2} \right) \quad [10]$$

$$\frac{\partial T}{\partial z}(x, z) = 2KFb^2 w \left(\frac{2xz \cos(2I - d) + (x^2 - z^2) \sin(2I - d)}{(x^2 + z^2)^2} \right) \quad [11]$$

$$\frac{\partial T}{\partial x}(x, z) = 2KFS \frac{\sin i}{\sin I} \left(\frac{(x^3 - 3xz^2) \cos(2I - 180) + (z^3 - 3zx^2) \sin(2I - 180)}{(z^2 + x^2)^3} \right) \quad [12]$$

$$\frac{\partial T}{\partial z}(x, z) = 2KFS \frac{\sin i}{\sin I} \left(\frac{-(z^3 - 3zx^2) \cos(2I - 180) + (x^3 - 3xz^2) \sin(2I - 180)}{(z^2 + x^2)^3} \right). \quad [13]$$

By substituting $x=tx$ and $z=tz$, it is possible to show that the horizontal and vertical derivatives are both homogeneous of degrees -1, -2 and -3 for a contact, thin sheet and cylinder, respectively. The original T functions for the thin sheet [6] and cylinder [7] are homogeneous of degree -1 and -2 respectively which correlate directly to their structural index. The T function for the contact, equation [5], however, is not homogeneous due to the $\ln$ function. This does not pose a problem as the contact's derivatives are homogeneous of degree -1 and a contact's structural index is 0. As an example of the procedure, proof of a thin sheet's homogeneity and degree is shown here,

$$T(tx, tz) = 2kTb^2 w \left(\frac{tz \cos(2I - d) - tx \sin(2I - d)}{(tz)^2 + (tx)^2} \right)$$

$$T(tx, tz) = 2kTb^2 w \left(\frac{t(z \cos(2I - d) - x \sin(2I - d))}{t^2(z^2 + x^2)} \right)$$

$$T(tx, tz) = 2kTb^2 w \left(\frac{(z \cos(2I - d) - x \sin(2I - d))}{t(z^2 + x^2)} \right)$$

$$T(tx, tz) = t^{-1} 2kTb^2 w \left(\frac{(z \cos(2I - d) - x \sin(2I - d))}{(z^2 + x^2)} \right)$$

$$T(tx, tz) = t^{-1} T(x, z).$$

Figure 2-4 : Diagrams of the three model types being examined: (a) contact, (b) thin sheet, and (c) cylinder.

2.5 Homogeneity

For the remainder of this thesis the slope function will be called the m function (Thurston and Smith, 2007),

$$m = -\frac{T_x'(x, z)}{T_z'(x, z)}. \quad [14]$$

In order to construct ray paths it is necessary to show that the m function for each model be homogeneous of degree 0. The three m functions are examined for a contact, thin sheet and cylinder, respectively.

$$m(x, z) = \frac{z \cos(2I - d - 90) + x \sin(2I - d - 90)}{x \cos(2I - d - 90) - z \sin(2I - d - 90)} \quad [15]$$

$$m(x, z) = \frac{(z^2 - x^2) \cos(2I - d) - 2xz \sin(2I - d)}{2xz \cos(2I - d) + (x^2 - z^2) \sin(2I - d)} \quad [16]$$

$$m(x, z) = \frac{(x^3 - 3xz^2) \cos(2I - 180) + (z^3 - 3zx^2) \sin(2I - 180)}{(z^3 - 3zx^2) \cos(2I - 180) - (x^3 - 3xz^2) \sin(2I - 180)} \quad [17]$$

As an example, for the case of the thin sheet, it is shown that the m function is homogeneous of degree 0.

$$\begin{aligned} m(tx, tz) &= \frac{((tz)^2 - (tx)^2) \cos(2I - d) - 2tx tz \sin(2I - d)}{2tx tz \cos(2I - d) + ((tx)^2 - (tz)^2) \sin(2I - d)} \\ m(tx, tz) &= \frac{t^2(z^2 - x^2) \cos(2I - d) - 2t^2 xz \sin(2I - d)}{2t^2 xz \cos(2I - d) + t^2(x^2 - z^2) \sin(2I - d)} \\ m(tx, tz) &= \frac{t^2}{t^2} \left(\frac{2xz \cos(2I - d) + (z^2 - x^2) \sin(2I - d)}{(x^2 - z^2) \cos(2I - d) + 2xz \sin(2I - d)} \right) \\ m(tx, tz) &= m(x, z) \end{aligned}$$

The proof that the contact and cylinder m functions are homogeneous of degree 0 follows along very similar lines. In Figure 2-5 (b) the m plot for the synthetic thin sheet is shown over the same range of offsets and heights as Figure 2.5 (a). It is obvious that all contour lines have the same m values and originate from the causative body.

The homogeneity of the m function means that any ray path connecting same m values caused by an anomaly intersect the body at the source location ($x = z = 0$). This is the theoretical basis for the SLUTH method.

Figure 2-5 : (a) TMI from 0 to 500 m above ground for a thin sheet dipping at 90° which outcrops at 250 m along the profile distance. (b) continuous *m* function for the same area.

2.6 *Comparing Interpretation Techniques*

The SLUTH method is one of many magnetic interpretation techniques that have been proposed. A comparison between some key techniques and the SLUTH method will help to understand its strengths and weaknesses.

2.6.1 Euler

Euler deconvolution, like the SLUTH method, assumes that the magnetic potential field behaves as a homogeneous function. Thompson (1982) showed that the definition of a homogeneous function, equation [3], could be rewritten as

$$(x - x_0) \frac{\partial T}{\partial x} + (z - z_0) \frac{\partial T}{\partial z} = k(B - T) \quad [18]$$

where (x_0, y_0, z_0) is the position of the magnetic causative body in metres, k is the structural index which is equivalent to the degree of homogeneity in section 2.3, T is the total field magnetic intensity and B is the regional value of the total field in nT (Stavrev and Reid, 2007). This method requires that the user inputs the structural index and then applies this equation to a set of different (x, y, z) values in the area nearby the anomaly (Thompson, 1982). The equations for a minimum of three points around the area of interest are created using the known location values, x and z , their respective derivatives, the measured TMI and the known regional field. These two variables, x and z , are solved for using different k and their results are compared to find the structural index which best fits (Thompson, 1982).

As the first technique to explicitly assume that the potential magnetic field behaved homogeneously, the Euler deconvolution method has been the inspiration for other techniques and several improvements have been proposed (Nabighian et al., 2005). However, the method has its shortcomings. Small variations in the regional field and noise cause large inconsistencies in the results. Also, solving for depth requires the user to define the structural index and most solutions come as sets defined by the k chosen and not as a single output. In comparison, the SLUTH method needs no explicit assumption about the structural index. Using its determined structural index, the SLUTH method outputs a unique set of coordinates and susceptibilities for each anomaly interpreted.

2.6.2 Continuous Wavelet Transform

The continuous wavelet transform (CWT) of interpreting a magnetic profile is similar to the SLUTH method in that they are both geometrical analysis techniques and both assume homogeneous sources (Vallée et al., 2004). In both, the horizontal and vertical derivatives are upward continued. With these upward continued points, the CWT then calculates the analytic signal, $\psi(x)$, for each height and uses them to map the magnetic signal waveform to a set of wavelets, $W(b,a)$, where b is the source position and a is the scale of the wavelet. The maxima of these wavelets are then connected linearly and their intersections indicate the depth and location of the anomaly (Sailhac et al., 2009). If only one set of maxima occurs then a best fit is computed by examining the variation of the maxima with depth. The variation in maxima is also used in determining the degree of homogeneity (Sailhac et al., 2009).

The main differences between the CWT and the SLUTH method are the function used in determining intersecting ray paths and the relative simplicity of the SLUTH method. The SLUTH method estimates the m function at different heights and is not restricted to solely connecting a limited number of maxima. It can connect an arbitrary number of similar m values caused by the homogeneous source allowing for an infinite number of ray paths to be constructed. While the CWT and m functions are both homogeneous, the simplicity of the m function allows for a more robust analysis.

2.6.3 Multiridge Geometric Method

The multiridge geometric method (MGM) is similar to both the Euler deconvolution method and the SLUTH method. It constructs a function, F_3 , which takes into account magnetic and spatial variables, and creates ray paths which fall into one of three categories: the zeros for the first order horizontal derivative (R_I), the zeros for the first order vertical derivative (R_{II}) and the null values for the original function F_3 (R_{III}) (Fedi et al., 2009). The SLUTH method and MGM both use ray path intersection points to determine position and depth and also both use variation over height to determine the structural index.

The MGM can be extended to construct an infinite number of ray paths, by taking into account higher-order horizontal and vertical derivatives (Fedi et al., 2009). Taking these higher order derivatives is problematic, however, as each derivative will amplify noise. On the other hand, the SLUTH method can be used to construct an arbitrarily large number of ray paths by reducing the spacing between the m values being used.

3 The SLUTH Algorithm

3.1 Overview

The following is a description of the algorithm which has been designed to interpret aeromagnetic anomalies along a flight profile using the SLUTH method. This algorithm runs in 6 steps: Step 1 is the input of the necessary data; step 2 is the creation of the θ matrix and SLUTH file; step 3 is finding the areas of interest; step 4 in the construction of ray paths; step 5 is the analysis of ray paths and step 6 is the output of data from the analysis, see Figure 3-1.

The algorithm has been coded in C using Microsoft Visual C++ 2008 Express Edition and run on a Compaq laptop with 1.60 GHz processor and 504 MB of RAM. The figures were constructed using MATLAB version 7.6.0.324 (R2008a). A copy of the code is provided on a CD in a pocket at the end of this thesis.

Figure 3-1 : A flow chart of the 6 steps outlined in chapter 3.

3.2 Step 1: Inputting data and variables

The minimum information required to run the algorithm is the line number, profile distance along the flight profile, TMI, the horizontal and vertical derivatives of the magnetic intensity, the nominal height of the magnetic sensor, the spacing between the heights that the data have been upward continued to, the number of upward continuation heights being used, the inclination, declination, regional magnetic intensity and the easting and northing at the beginning of the profile.

In order to implement the SLUTH algorithm using survey data, the user must input two different ASCII data files. The first file is called the magnetic data file. This file contains the data derived while the survey was being conducted. The first column is the line number, the second is the profile distance and the third is the residual magnetic intensity after subtracting the regional magnetic intensity. The next n columns are the horizontal derivative of the magnetic field at survey height and its upward continued values for $n-1$ heights. The following n columns are the vertical derivative at survey height and its upward continued values for the same $n-1$ heights as before. In Figure 3-2 an example of a magnetic data file is shown.

In the second file, the info file, the first row is the total number of heights, n (including the survey height), represented by their derivatives in the magnetic data file, the second is the nominal height of the sensor and the third is the constant spacing between upward continued heights. In this particular implementation of the SLUTH algorithm, it is assumed that the distance between the upward continued heights is constant. The fourth

row is the inclination, the fifth is the declination, the sixth is the local value of the TMI, the seventh is the northing and the eighth is the easting of the flight profile's starting point. There are n remaining rows in the info file. These are switch values where each row represents each height, starting at the nominal height, which are either a 1 or a 0. The code looks through these switch values and corrects the nominal height and spacing depending on which heights are to be used. A 1 means that this height is to be used and a 0 means it is not to be used. Having switch values allows the user to change the heights being used without having to alter the original height and spacing values in the info file. While there is no maximum number of heights that should be used, there should be a minimum of three heights in order for the ray path to be dependable, see Figure 3-3 for an example of an info file.

3.3 Step 2: Creation of the θ matrix

Presently the algorithm has two input files to use. Now it must begin building the data needed to create ray paths which intersect the causative body. The values which create these ray paths are given as m in equation [14].

As a first attempt, a matrix was created where the first three columns were the same as the first three columns of the magnetic data file and the following n columns were m calculated using equation [14] from the upward continued derivatives in the magnetic data file. Only the columns with a switch value set to 1 were used. This matrix was called the m matrix and the number of columns was 3 + the number of switch values set to 1.

There was a problem with using the m function directly. Most values were between 0 and ± 1 but there were also a few outliers with some values larger than 1000, see Figure 3-4 (a). These outliers occurred when the vertical derivative was close to zero. Having this large a range made it difficult to map and create ray paths of similar slopes. So a new function θ , the angle ratio, was created where

$$\theta = \tan^{-1} \left(- \frac{\partial T / \partial x}{\partial T / \partial z} \right), \quad [19]$$

which has a range between $-\pi/2$ and $\pi/2$, see Figure 3-4 (b). The θ matrix has the same data structure as the m matrix. It has the same name as the original input magnetic data file but now has .sluth added to it, see Figure 3-5.

a)

b)

Figure 3-4 : A plot of the (a) m function and (b) θ function along a flight profile 2 km long for magnetic data file test.004. 70 m is the nominal height, and all other heights are upward continued heights.

test.004.sluth - WordPad

File Edit View Insert Format Help

1 2 3 4 5 6 7 8 9 10 11 12 13 14

1.00	15995.86	-89.70	-1.30	-1.29	-1.28	-1.27	-1.26
1.00	16007.91	-90.80	-1.24	-1.24	-1.23	-1.22	-1.21
1.00	16019.91	-92.00	-1.25	-1.21	-1.18	-1.16	-1.15
1.00	16032.33	-93.30	-0.91	-0.94	-0.96	-0.98	-0.99
1.00	16044.33	-94.30	-0.84	-0.85	-0.86	-0.87	-0.89
1.00	16056.38	-95.10	-0.70	-0.75	-0.79	-0.82	-0.84
1.00	16067.98	-95.80	-0.96	-0.92	-0.90	-0.89	-0.89
1.00	16079.98	-96.60	-0.78	-0.81	-0.84	-0.85	-0.86
1.00	16092.03	-97.30	-0.90	-0.88	-0.87	-0.87	-0.87
1.00	16104.07	-98.00	-0.72	-0.78	-0.81	-0.84	-0.86
1.00	16116.11	-98.60	-1.01	-0.98	-0.95	-0.94	-0.94
1.00	16128.11	-99.30	-0.89	-0.95	-0.98	-0.99	-0.99
1.00	16140.15	-100.00	-1.36	-1.26	-1.19	-1.14	-1.10
1.00	16152.20	-101.10	-0.99	-1.01	-1.02	-1.02	-1.02
1.00	16164.23	-102.00	-1.01	-1.00	-0.99	-0.98	-0.98
1.00	16176.23	-103.00	-1.00	-0.99	-0.97	-0.96	-0.95
1.00	16188.28	-104.00	-0.89	-0.90	-0.91	-0.91	-0.90
1.00	16199.90	-105.00	-1.01	-0.97	-0.93	-0.91	-0.89
1.00	16211.88	-106.20	-0.80	-0.80	-0.80	-0.80	-0.79
1.00	16223.88	-107.20	-0.70	-0.70	-0.71	-0.70	-0.70
1.00	16235.93	-108.20	-0.70	-0.68	-0.66	-0.65	-0.64
1.00	16247.92	-109.10	-0.50	-0.52	-0.54	-0.55	-0.55
1.00	16259.92	-109.90	-0.62	-0.58	-0.55	-0.53	-0.52

For Help, press F1

Figure 3-5 : Example of the first five columns of the sluth file, test.004.sluth. The columns are, the line number (1); profile distance (from 15995.860 to 16259.92 in metres); magnetic intensity (from -109.9 to -89.7 in nanoTeslas); the remaining 5 columns are θ values for the nominal height and the first four upward continued heights.

3.4 Step 3: Finding areas of interest

It would be a waste of computer resources for the algorithm to create a ray path for each different θ value along the flight profile. Instead, the algorithm searches through each row of the m matrix and the magnetic data file at the same time in order to find rows of interest. Magnetic data have many different attributes which can be analyzed by interpreters to determine areas of interest. For this algorithm to flag a row of interest, a row must fulfill two criteria: (1) have a spread, e , of θ values over all the heights being considered less than a pre-determined threshold e_T and (2) have an analytic signal, A , greater than a pre-determined threshold A_T .

To find the spread e for each row, the algorithm subtracts the θ_i value at height i from the value at the nominal height, θ_0 . The absolute value of all these differences are summed for all the columns as described by,

$$e = \sum_{i=1}^{n-1} |\theta_0 - \theta_i|. \quad [20]$$

The analytic signal is the absolute value of the square root of the sum of the squares of the horizontal and vertical derivatives of the TMI. To find the analytic signal A , the horizontal and vertical derivatives from the nominal height before upward continuation, $\partial T / \partial x_0$ and $\partial T / \partial z_0$, are used as described by,

$$A = \left| \sqrt{\left(\partial T / \partial x_0 \right)^2 + \left(\partial T / \partial z_0 \right)^2} \right| \quad [21]$$

Deciding on threshold values for e and A varies depends on the survey being examined. The user should try different values and watch the output to see how many rows are being flagged. Once the code runs the user must look back through the output and either loosen or tighten restrictions on these two variables in order to distinguish signal from background noise. A good starting point for our data is letting $e_T=0.4$ and $A_T=0.4$. These threshold values were chosen by trial and error, and lead to stable results. Choosing e_T and A_T could have been made automatic in the code, but this would have left the user not knowing what areas were being neglected. Leaving it as a user task informs the user on how selecting these parameters impacts the results.

Now that a row has been selected it is stored in a new array where the first column is the row number, the second column is the analytic signal and the remaining n columns are either the upward continued horizontal or vertical derivatives, depending on which is larger. These derivatives will be used later for estimating the type of structure that is causing the magnetic anomaly. The small derivatives have a poor signal to noise ratio, so they are not used. Because the horizontal and vertical derivatives are Hilbert transforms (Sundararajan and Srinivas, 1996), when one derivative is small, the other should be large and hence have a good signal to noise ratio. When the magnitude of the horizontal and vertical derivatives are similar their signal to noise should be similar and hence it does not really matter which of the two derivatives is used.

3.5 Step 4: Construction of ray paths from different θ values

The ray paths, as previously stated, are straight lines which originate from the causative magnetic body and insect each upward continued height at the same θ values. To better explain the process of constructing ray paths it will be broken down into sub-steps and Figure 3-6 will be cited regularly to illustrate each step.

Step 4.1:

The algorithm begins with a row of interest. The specific value of the profile distance, x , that corresponds to a row of interest will be the starting point for ray path construction. It then finds out if this x is on a wrap around point. A wrap around point is a point which exists directly before or directly after a discontinuity in the θ data. These points exists near the boundaries of $-\pi/2$ and $\pi/2$. Examples of wrap around points on Figure 3-6 are close to 17,845 (blue line) 18,025 (magenta line), 18,325 (magenta line) and 18,450 (blue line).

If the row of interest is on a wrap around point then it moves two rows in the negative direction, to the left, along the profile distance and this distance now becomes our starting distance, x . The reason that the point of interest is moved away from a wrap around point is so that there will be close neighbours on either side of x to commence ray path construction and to allow step 4.2 to work properly. The algorithm now finds the θ associated with x and stores this as θ_0 .

In Figure 3-6 we can see that x is about 18,200 and $\theta_0 = 0.2$ and that it does not lie on a wrap around point.

Step 4.2:

At x the algorithm finds whether the θ function is increasing or decreasing along this portion of the flight profile. It will use this information in determining how to increment θ_0 as it moves along the flight profile. Depending on whether the function is increasing or decreasing will define whether the θ is increased or decreased while moving along the profile distance.

In Figure 3-6, θ is decreasing around 18,200 m, our distance of interest.

Step 4.3:

In the algorithm, the user inputs the different spacings, $\Delta\theta$, of the θ values which are to be used in determining the different ray paths created and the number of ray path to be created on each side of the source of the magnetic anomaly. The data are first examined in the negative direction from x . The algorithm reads the data along the flight profile in each height (column) of the θ matrix until it reaches the value closest to w , where

$$w = \theta_0 + \Delta\theta. \quad [22]$$

The algorithm then assumes that values in between the two θ values which surround w , can be accurately predicted by linear interpolation. The fractional distance between the two θ values can be used to predict the distance where θ is exactly equal to w . It does this for each row in the θ matrix. While θ does not behave exactly linearly along the profile distance, this is a safe assumption because the sampling rate on magnetic surveys is high enough that θ is nearly linear between sample points.

In Figure 3-6 (bottom panel) $\Delta\theta = 0.5$ rad and the number of ray paths the algorithm is requested to create on each side is 3.

Step 4.4:

The algorithm then takes these distance and height values associated with the θ being examined and treats them as (x,z) values on a 2-D plane. The method of least squares is used to determine a line of best fit for these points and a ray path is created. The θ value is then incremented by $\Delta\theta$ and the linear interpolation process is repeated for each height to obtain another set of (x,z) points and then a new ray path. Figure 3-6 (bottom panel) this line fitting process has been done three times by increasing the θ value by 0.5 and three times by decreasing the θ value by 0.5.

If while incrementing or decrementing by $\Delta\theta$, w approaches a wrap around point the algorithm jumps past this wrap around point and begins again after the discontinuity. This is done by setting two boundary points at 1.4 rad and -1.4 rad. If w exceeds 1.4 rad or drops below -1.4 rad the algorithm subtracts or adds 2.8 rad to w respectively. These values were chosen by trial and error. They allow the data to get close to the boundary points without going beyond a wrap around point.

In Figure 3-6, $\Delta\theta = 0.5$ so the third ray path should have been constructed with $\theta = 1.7$ rad. Since $1.7 > 1.4$ rad the algorithm subtracted 2.8 rad from θ and built a new ray path for $\theta = -1.1$ rad. The reason 2.8 rad was used and not 3.141 rad is that sample points never actually achieved the maximum or minimum θ values. Instead due to discrete

sampling they normally were between 1.4 and 1.5 rad at wrap around points. This process will leave a gap in the ray paths near a wrap around point.

This step is repeated for as many ray paths as the user requests or until the data are outside the zone of influence of the causative body. There are two ways that the algorithm decides that data are outside the zone of influence. The first being that the algorithm has reached the end or beginning of the θ matrix and the second is that it has reached a peak or trough in the θ data. When either occurs, the ray path presently being constructed is abandoned and no more ray paths are attempted for that direction from the row of interest.

Step 4.5:

Now that the ray paths in the negative direction from the row of interest, x , have been constructed, the algorithm checks in the array holding the rows of interest to see if there exists a row of interest in the positive direction within the next three rows in the θ matrix. If it is the case, then the algorithm jumps ahead to it. These clusters of rows are called areas of interest and are examined together as a set. This process prevents the algorithm from treating a large anomaly as many independent anomalies. Also, between close rows of interest θ behaves erratically and gives results which are unreliable. In chapter 4 synthetic examples of two anomalies approaching one another will be studied. These results confirm this fact. In Figure 3-6 there was only one row of interest so this step was skipped.

Step 4.6:

Steps 4.3 to 4.5 are then repeated in the positive direction from x , with θ being decremented by $\Delta\theta$. In Figure 3-6, there are three ray paths constructed on either side of the source of the magnetic anomaly associated with $\theta = -1.1, 1.2, 0.7, -0.3, -0.8$ and -1.2 rad.

Ray paths are intermediate results provided by the algorithm. A file is written with the same name as the magnetic data file but with .lines as the extension where the first column is the z intercept and the second column is the slope of each ray path. In Figure 3-6, the ray path associated with $\theta = -1.3$ has a slope of 1.5 and a z intercept of -27,300 m.

Figure 3-6 : Example of the construction of ray paths. In this example, there exists a thin sheet dipping at 60° at distance 18200 m and at a depth of 0 m. This survey was flown at a height of 150 m and only upward continued once to a height of 300 m. Three ray paths were constructed for each side of the causative body with a spacing $\Delta\theta = 0.5$ starting from the row of interest where $\theta = 0.2$. The ray paths intersect where the thin sheet outcrops along the profile.

3.6 Step 5: Ray Path Analysis

3.6.1 Horizontal distance

At this point, the algorithm has computed a set of linear ray paths which cluster around a common intersection point. The first analysis done about these ray paths is finding the horizontal position of the causative body. The reason this is solved for first is due to the way adjacent magnetic bodies interact with one another. This reason will be expanded on later in Chapter 4.

To find the horizontal distance, first the algorithm removes any ray paths with absolute values of the slopes below some minimum slope and above some maximum slope. These are left as variables for the user. For this thesis, the minimum slope is 0.5 and the maximum slope is 5.85. By doing this, the algorithm omits ray paths which are outliers (slopes <0.5) or too close to the anomaly (slopes >5.85) and only result in vertical ray paths. The algorithm then sorts the remaining ray paths as adjacent pairs and finds where they intersect one another. Adjacent ray paths are used because this reduces the number of outlying intersections. There are many other possible options, for example finding the locations where each ray path intersected every other ray path.

Once all the distance intercepts have been found, the average and standard deviation are computed. Next, 5% of the outliers are removed, allowing the average to be recomputed more robustly. This robust average is the algorithm's first output, the horizontal position.

3.6.2 Depth

Now that the distance has been solved for, depth follows easily. Depth is solved for in a very similar way with one major difference. For distance, we took the intersections of ray paths with one another. For depth, we create a new vertical line which passes through the horizontal distance. Removing those ray paths with slopes too large or too small, an average of ray paths' intersections with this vertical line is found together with the standard deviation. Once again, 5% of the outliers are removed and a robust average of the remaining depths is taken.

This gives us the depth to the critical part of the body depending on its structure. If the body is a contact, the depth is to the top corner of this contact. If the body is a thin sheet, the depth is to the centre of the topmost edge of the sheet. If the body is a cylinder, the depth is to the centre of the cylinder.

Depth is the algorithm's second output and is relative to ground level. A negative number, e.g. $-b$, implies that the critical point on the source is a depth b below surface. While some above-ground anomalies can be man-made and should be examined, all anomalies that are found to exist above the sensor height are omitted.

3.6.3 Structural Index

Earlier, in step 3, when the area of interest was found, we stored the largest of the horizontal and vertical derivatives in an array with the corresponding distance and its row number in the θ matrix. The larger of the horizontal and vertical derivatives was chosen

so that the derivative with the greatest signal to noise ratio would be used in the analysis and hence give a better estimate. These will be used in the following process to determine the structural index. As we showed in section 2.5, the degree of homogeneity tells us which model best represents the causative body. The algorithm now studies how the derivative varies directly above the body by comparing each upward continued point with the derivative of the nominal height, h_0 . It uses the values directly above it because these represent a vertical ray path. If the ray path is vertical this means the distance will not need to be altered for each height as the slope of the ray path will be zero and directly above the causative body. This ray path is also used because at this profile position the signal to noise ratio will be the highest, giving more reliable results. Since we assume that we are directly above the anomaly, $x=0$. If we assume that the horizontal derivative is the largest, then we see that since the TMI, T , is homogeneous, we can combine equations [3] and magnetic equations for magnetic intensity T to give:

$$\frac{\partial T}{\partial x}(0, h_i) = \left(\frac{h_i}{h_0} \right)^a \frac{\partial T}{\partial x}(0, h_0) \quad [23]$$

for all non-negative $i < n$. By taking the logarithm of each side we can solve for the exponent a at each height h_i and take an average.

$$\bar{a} = \frac{\sum_{i=1}^{n-1} \ln \left[\frac{\partial T}{\partial x}(0, h_i) / \frac{\partial T}{\partial x}(0, h_0) \right] / \ln \left[\frac{h_i}{h_0} \right]}{n - 1} \quad [24]$$

Now $\bar{a}$ is the degree of homogeneity of the derivative. In order to find the degree for the original magnetic data we must add one since, as shown earlier in Section 2.5, the degree of a derivative is one less than the degree of the original function. Looking back to

section 2.5 again we can see which structural indices imply which model, see Table 3-1.

The structural index is the third output of the algorithm.

Table 3-1 : Structural index and model associations

Structural Index	Susceptibility Product	Model
greater than 0.7	N/A	Inconclusive
-0.5 to 0.7	Susceptibility-contrast	Contact
-1.5 to -0.5	Susceptibility-thickness	Thin sheet
-2.5 to -1.5	Susceptibility-area	Cylinder
less than -2.5	N/A	Inconclusive

3.6.4 Susceptibility

Smith et al. [2005] summarizes the formulae for finding the susceptibility of three different magnetic anomaly models. The susceptibility of a contact, a thin sheet and a cylinder are, respectively,

$$K = \frac{A\sqrt{(x^2 + h^2)}}{2Fc \sin(d)} \quad [25]$$

$$Kw = \frac{A(x^2 + h^2)}{2Fc} \quad [26]$$

$$KS = \frac{A(x^2 + h^2)^{3/2}}{F(\sin(i)/\sin(I))} \quad [27]$$

where K is the susceptibility-contrast, A is the analytic signal in nanoTesla/metre, x is the horizontal distance in metres, since we are directly above the source $x=0$, h is the height above the anomaly in metres, F is the is the earth's regional magnetic field strength in nanoTeslas, d is the dip of the contact in degrees, w is the width of the thin sheet in metres, S is the cross-sectional area of the cylinder in metres squared,

$c = 1 - \cos^2 i \sin^2 \alpha$, I is the magnetic inclination in degrees, α is the angle between magnetic north and the flight profile in degrees and $\tan I = \tan i / \cos \alpha$.

Since the width and surface area are both unknown, we solve for a susceptibility product; the susceptibility-thickness and susceptibility-area, respectively, see Table 3-1. For simplicity, the dip is assumed to be 90°.

The susceptibility product is the fourth output of the algorithm. The algorithm stores all four outputs and then begins creating ray paths for the next area of interest.

3.7 Step 6: File output

Now that all the rows of interest have been analyzed, the SLUTH algorithm then creates four output files, which hold the characteristics of each anomaly examined. The algorithm creates a .contact file where it assumes every anomaly can be best modeled as a contact. Similarly, it also creates a .thinsheet and a .cylinder file. In these files the susceptibility output is the susceptibility product for that model assuming the causative body is that model. The fourth output file is the .bestfit file. In this file, the algorithm states the model which is implied by the structural index and outputs the susceptibility product which applies to each body.

In all files, the data structure is the same and there is one row per area of interest. Assuming a north-south flight profile, the columns are: easting in metres, northing in metres, northing standard deviation in metres, depth in metres, depth standard deviation

in metres, root mean square of the two standard deviations in metres, structural index and susceptibility, see Figure 3-7. The two standard deviations are summed for plotting later in MATLAB.

Easting	Northing	NorthingSD	Depth	DepthSD	sumSD	SI	Susceptibility
477838.50	16581.21	31.10	-58.12	7.46	38.56	-1.64	0.29360
477838.50	18209.67	5.98	-52.20	13.96	19.94	-1.13	1.51935

Figure 3-7 : Example of the .bestfit output file for magnetic data file test.004 with columns easting, northing, northing standard deviation, depth, depth standard deviation, sum of standard deviations, structural index and susceptibility. In this case, there are two sources of magnetic anomalies which lie at depths of 58.12 m and 52.2 m below surface.

4 Synthetic Data Analysis

4.1 Introduction to synthetic data and why they are used

Testing a new algorithm on field data is challenging as the field data are noisy and the source of the magnetic field is not always entirely understood. It is common practice to test algorithms on synthetic data derived from mathematical equations where the cause is entirely known. This allows the user to simulate any causative body with any susceptibility at any depth and any location. By varying these parameters the user can determine the limitations of the code with and without noise being added to the synthetic data. For this reason, this chapter will examine the code's performance in response to variations in several parameters. Section 4.2 will examine models of three different shapes, Section 4.3 will focus on a thin sheet at varying depths and Section 4.4 will address the issue of lateral resolution.

4.2 Application of SLUTH method algorithm to three model types

The synthetic data are generated using equations [5], [6] and [7] from Chapter 2 to simulate three model types: a contact, thin sheet and cylinder, respectively. Each synthetic flight profile will be assigned a nominal height of 200 m and upward continued to 290 m above the anomaly at increments of 10 m. As noted in Table 4-1, it is assumed that each causative body lies at location 1000 m along the profile and at a depth of 0 m. The contact and thin sheet are assumed to have a dip of 90° , the thin sheet has a thickness of 20 m and the cylinder has a radius of 20 m. The angle between the flight profile and magnetic north (declination) is 0° and the inclination of the earth's magnetic field is 70° .

These parameters are similar to the actual parameters from the survey area which will be examined in Chapter 5.

Three synthetic magnetic data files were created, one for each of the model type. Each of these magnetic data files were then input into the algorithm in order to find rows of interest assuming a maximum spread of $e_T = 0.4$ and a minimum analytic signal of $A_T = 0.4$. A spacing of $\Delta\theta = 0.02$ was used for each of 30 ray paths constructed on either side of the anomaly.

In Figure 4-1, Figure 4-2 and Figure 4-3, the θ plot and ray paths for each model are shown. Each ray path plot shows a very clean and sharp intersection cluster. It is important to note that gaps in the ray path coverage are caused by the buffer zone created around the wrap around points and do not correspond to missing data. In the θ plots, the spread is very small around profile distance 1000 m and consequently this location was chosen as the row of interest.

Table 4-1 : Inputs and outputs for three model types

Model Type	Input			Output		
	Profile Distance (m)	Depth (m)	Dip (°)	Profile Distance (m)	Depth (m)	Structural Index
Contact	1000.0	0.0	90.0	1000.15 ± 1.26	0.13 ± 0.10	0.00
Thin Sheet	1000.0	0.0	90.0	1000.02 ± 0.85	-0.13 ± 0.08	-1.00
Cylinder	1000.0	0.0	N/A	1000.21 ± 5.42	0.17 ± 0.53	-2.00

Table 4-1 shows that the algorithm's outputs were extremely close to the distance and depth used to generate the synthetic data. Also note that the value calculated for the structural index for each model type is accurate to two significant figures.

These results are very encouraging, but they only express one example for each of the three model types.

a)

b)

Figure 4-1 : (a) θ plot of the contact anomaly surveyed at different heights along the flight profile and (b) corresponding ray paths constructed by the SLUTH algorithm.

a)

b)

Figure 4-2 : (a) θ plot of the thin sheet anomaly surveyed at different heights along the flight profile and (b) corresponding ray paths constructed by the SLUTH algorithm.

a)

b)

Figure 4-3 : (a) θ plot of the cylinder anomaly surveyed at different heights along the flight profile and (b) corresponding ray paths constructed by the SLUTH algorithm.

4.3 *Modelling a thin sheet at different depths*

In this section, only the thin sheet model will be examined. The thin sheet is chosen because it was examined thoroughly in Chapter 2, and also because in Section 5.2 the field line being examined was also studied by other groups who tested other interpretation techniques and identified a thin sheet as the source of the field anomaly.

To test how the algorithm behaves when dealing with anomalies at different vertical distances from the sensor, a synthetic outcropping thin sheet will be examined from heights of 100 m, 200 m, 300 m, 400 m, 500 m, 600 m, 700 m, 800 m, 900 m and 1000m. For each case, the inclination will be 70° , the declination will be 0° and the susceptibility-thickness will be 20. The profile length will be from 0 m to 2000 m, but only the range from 750 m to 1250 m will be used in plotting the ray paths. The data will be upward continued at a spacing of 10 m from the nominal height so that data from a total of 10 heights are input into the algorithm.

In Figure 4-4, the θ plots for surveys heights of 100 m, 500 m and 1000 m are shown. From 100 m, (a), to 1000 m, (c), it is clear that the waveform become much broader as the sensor is increasingly higher above the anomaly.

The four outputs of the algorithm are listed in Table 4-2. As expected, as the sensor height increases, the standard deviation of the profile distance increases. The distance values themselves, however, remain very close to the expected value. This, in turn, constrains the vertical line which is used to determine the depth, as discussed in section

3.6.2. Therefore, the depth's standard deviation does not vary greatly as depth increases. The susceptibility thickness is the most sensitive parameter due to the amount of variables which go into its calculation. Small variations in each variable cause large variations in the susceptibility. However, for these thin sheet simulations, the susceptibility-thickness is rather stable, staying close to the input value of 20. For all 10 survey heights, the structural index is constant at -1.00, which corresponds directly to a thin sheet.

Table 4-2 : The four outputs for the 10 different survey heights

Survey Height (m)	Profile Distance (m)	Depth (m)	Structural Index	Susceptibility-Thickness
100	1000 (± 0.5)	-0.2 (± 0.1)	-1.00	20.1
200	1000 (± 0.8)	-0.1 (± 0.1)	-1.00	19.8
300	1000 (± 1.1)	-0.1 (± 0.1)	-1.00	19.7
400	999 (± 1.7)	-0.1 (± 0.1)	-1.00	19.2
500	999 (± 1.9)	0.0 (± 0.3)	-1.00	18.7
600	1000 (± 1.8)	-0.1 (± 0.2)	-1.00	18.2
700	999 (± 4.6)	0.0 (± 0.1)	-1.00	17.6
800	1000 (± 9.4)	-0.3 (± 0.4)	-1.00	16.6
900	998 (± 9.5)	1.7 (± 0.7)	-1.00	15.4
1000	1000 (± 12.1)	-0.2 (± 0.4)	-1.00	18.5

Figure 4-5 (a)-(j) indicates that as the sensor height increases, the ray paths dip at shallower angles. This is because deeper thin sheets cause broader anomalies. In Figure 4-5 (c), when the thin sheet is 300m deep, the manifestation of the first wrap around point is observed to the right where there is a gap in the ray paths. A single ray path shares a common intersection area, but has a slope very different than the other ray paths from the right side of the area of interest. For depths between 300 m to 700 m inclusive, there are similar manifestations of the wrap effect.

While it is not presented here, varying other parameters such as inclination and dip would change the waveform and, as a result, affect the interaction between the wrap around point and ray path construction. Wrap around point interaction does not depend on the depth of the causative body, but on the value of θ for the row of interest, x .

These results demonstrate convincingly the SLUTH algorithm's ability to interpret near-surface magnetic data. At a sensor height of 1000 m, the profile distance's standard deviation estimated by the algorithm is only off from the actual distance by 12 m and the depth's is off by 0.2 m. This section, however, only studied in detail the case of one isolated causative body. In the next section, two adjacent bodies with overlapping anomalies will be examined, which is a more realistic situation often seen in field data.

a)

b)

c)

Figure 4-4 : θ plot for an anomaly (a) 100 m, (b) 500 m and (c) 1000 m below the sensor.

Figure 4-5 : Ray paths constructed by SLUTH algorithm for an outcropping thin sheet (a) 100 m, (b) 200 m, (c) 300 m, (d) 400m, (e) 500 m, (f) 600 m, (g) 700 m, (h) 800 m, (i) 900 m and (j) 1000 m below the sensor at a distance 1000 m along the profile.

4.4 *Modelling two adjacent thin sheets*

If the magnetic fields of two neighbouring bodies are close by one another, which is often the case, the TMI measured by the magnetometer will be the sum of the magnetic intensity of each field at that point in space, which is the same as adding the two signals caused by each independent anomaly.

For this simulation, it is assumed that there exist two outcropping thin sheets dipping at 90° with a susceptibility-thickness of 20. The data are upward continued from a nominal flight height of 200 m for a total of 10 heights at a spacing of 10 m. The regional magnetic field is 50,000 nT, the inclination of the field is 70° and the declination of the field is 0° . The first thin sheet is at profile distance 1000 m and does not move throughout the numerical experiment. The second thin sheet moves away from the first toward the right of the profile at steps of 100 m, starting at 1100 m and ending at 2000 m.

The θ plots for separations of 100 m, 200 m and 600 m are shown in Figure 4-6. In Figure 4-6 (a) it is obvious that the algorithm would pick up only one anomaly along this profile. In Figure 4-6 (b), θ values become mildly erratic between 1000 m and 1200 m. In Figure 4-6 (c) it is clear that there are two defined anomalies along the flight profile.

In Table 4-3, the four outputs for each of the two thin sheets at each spacing are shown. First, it is interesting to note that when the thin sheets are 100 m apart, the algorithm treats them as one area of interest. While this does not reflect what is physically going on, mathematically the two are close enough that even manual interpretation of the

profile would imply only one anomaly. Interestingly, the estimated susceptibility thickness is close to the sum of the two individual values and hence represents a valid interpretation if the bodies are assumed to be one. When the bodies are 200 m apart, we get very close to correct results, but this is due to our choice of parameters for the practical implementation of the SLUTH method in the code and will be explained in the ray path discussion below. After 200 m, the data approach the expected results and the standard deviations decrease.

Table 4-3 : The four outputs for each thin sheet at each spacing with distance and depth given with their standard deviations, SI is the structural index and K is the susceptibility thickness

Spacing (m)	First thin sheet (fixed)				Second thin sheet (moving away)			
	Distance (m)	Depth (m)	SI	K	Distance (m)	Depth (m)	SI	K
100	1050 (± 16)	-65 (± 11)	-1.23	57.4	N/A	N/A	N/A	N/A
200	993 (± 1)	13 (± 0)	-0.76	18.66	1207 (± 34)	15 (± 1)	-0.29	19.4
300	972 (± 72)	47 (± 2)	-0.31	11.25	1302 (± 184)	54 (± 246)	-0.24	10.1
400	966 (± 13)	53 (± 53)	-0.16	10.0	1430 (± 16)	61 (± 59)	-0.32	8.7
500	970 (± 12)	41 (± 43)	0.74	11.65	1529 (± 12)	43 (± 44)	-0.63	11.1
600	972 (± 2)	17 (± 17)	-3.37	15.5	1627 (± 2)	18 (± 17)	-0.81	15.2
700	976 (± 3)	0 (± 2)	-1.78	18.66	1723 (± 4)	0 (± 2)	-0.96	18.4
800	981 (± 4)	-9 (± 7)	-1.63	20.63	1818 (± 4)	-9 (± 6)	-1.12	20.5
900	985 (± 5)	-14 (± 10)	-1.44	21.74	1915 (± 5)	-14 (± 10)	-1.10	21.7
1000	989 (± 4)	-15 (± 10)	-1.30	22.23	2012 (± 4)	-16 (± 10)	-1.16	22.1

To better understand the above results, the ray paths need to be discussed in detail. The ray path figures for all 10 spacings are shown at the end of this section, in Figure 4-7 (a) – (j).

At 100 m, Figure 4-7 (a), the algorithm has grouped the signal from the two causative bodies as one area of interest. All the ray paths which would have been between the two causative bodies have been omitted, which led to some results with standard deviations, which are lower than would be expected, that are in fact very incorrect. This implies that

two sheets within 100 m of one another produce an anomaly that is not distinguishable from the anomaly caused by a single body by this algorithm.

At a spacing of 200 m, Figure 4-7 (b), the algorithm sees the two anomalies as separate rows of interest and finds ray paths independently for each. It is interesting to note that the ray paths between the two causative bodies are nearly vertical, so in turn they are omitted when determining distance and depth since their slopes exceed the maximum slope used. This is why our results for this example have the lowest standard deviations in this experiment for both distance and depth. Since the only place that the ray paths converges is on the negative side for the anomaly at 1000 m and on the positive side for the anomaly at 1200 m, it reads a strong correlation from these clusters. The divergent ray paths are not included in the analysis and so do not contribute detrimentally.

One advantage of the SLUTH method is that in areas of strong interference the diverging ray paths eliminate themselves and do not detract from the results.

From a spacing of 300 m and higher, a pattern begins to emerge in the ray paths. On the positive side of the fixed thin sheet located at 1000 m, the ray paths' intersection clusters are indicating a source above surface. However, as the spacing between the causative bodies increases, the cluster of ray paths' intersections on the positive side moves down and gets closer and closer to the true source location at a depth of 0 m and a profile distance of 1000 m. This is because the thin sheet moving away has gradually a lesser

influence on the positive side of the anomaly associated with the fixed thin sheet located at 1000 m.

At a spacing of 700 m, Figure 4-7 (g), the clusters of ray paths' intersections from the positive side of the thin sheet at 1000 m coincide well with the ray paths from the negative side. As well, the standard deviations are low and it seems that the second body is out of the area of influence of the first body.

At 800 m, Figure 4-7 (h), the ray paths' intersection from the positive side move further in the negative direction and below a depth of 0 m. This is because a magnetic data anomaly does not decrease asymptotically to 0 nT, it first dips below 0 nT then approaches 0 nT from the negative direction.

For spacings of 900 m and 1000 m, Figure 4-7 (i-j), the ray paths' intersections begin increasing toward a depth of 0 m.

Now that the issue of lateral resolution has been studied in detail, users of the algorithm can better interpret results associated with large standard deviations. Possible future work would be to augment the algorithm to look for large standard deviations and relate them to nearby anomalies. Finding out on which side of the anomaly the interference comes from would allow the ray paths on that side to be omitted.

a)

b)

c)

Figure 4-6 : θ plot for a flight profile with one thin sheet outcropping at 1000 m and a second outcropping at (a) 100m, (b) 200m and (c) 600 m.

c) Two Thin Sheets 300 m Apart

d) Two Thin Sheets 400 m Apart

Figure 4-7 : The ray paths created by the SLUTH algorithm for one thin sheet anomaly at 1000 m and another at (a) 1100 m, (b) 1200 m, (c) 1300 m, (d) 1400 m, (e) 1500 m, (f) 1600 m, (g) 1700 m, (h) 1800 m, (i) 1900 m and (j) 2000 m.

4.5 *Summary of synthetic data outcomes*

In synthetic data tests, the SLUTH algorithm led to stable and reliable results. As the depth of the causative body is increased, the estimated depth and distance do stray slightly, but overall results are very acceptable.

Since this algorithm is based on the TMI data behaving as a homogeneous function, having two causative bodies approach one another does cause their ray paths to distort. This effect is greatest on the side of the anomaly closest to the interfering body. By knowing the cause of these interactions, they become easier to recognize and in future work could help in getting more precise results by omitting corrupted ray paths.

In all, these synthetic experiments imply that the algorithm should work well on actual field data. In the following chapter, two survey areas will be analyzed and the outcomes will be compared with the synthetic results from this chapter and interpretations from other groups published in the literature.

5 Application to Field Data

5.1 *Introduction to field data*

Field data in this case refers to data which have been recorded using a magnetometer during an airborne survey. These data are much more challenging to analyze than synthetic data because they are noisy and have been collected in complex geological environments. Many more variables affect field data than synthetic data, such as sensor noise, variations in survey altitude, and man-made magnetic interferences.

In this chapter, two sets of field data will be examined. The first data set is a single flight profile from the Timmins area in Ontario, and the second data set is a group of parallel flight profiles from the Chibougamau area in the Province of Quebec.

5.2 *Investigation of a single flight profile*

The first set of field data which will be examined is a 28,000 m long flight profile from an area located near Timmins in north-eastern Ontario, Canada (OGS, 2000). This region is within the Abitibi Subprovince of the Canadian Precambrian Shield which features many large granitic intrusions and volcanic bands which strike east-west. This N30°W flight profile, perpendicular to the strike, was flown at a nominal height of 70 m. The horizontal and vertical derivatives were then upward continued to 30 other heights at an increment of 2 m. The inclination of this area is 74.8° and the declination is -11.0°. The spacing between the θ values used in constructing ray paths was $\Delta\theta=0.02$ and all heights were used.

In Figure 5-1, the residual TMI is plotted against profile distance. The figure shows that there exist many peaks and troughs along the flight profile. The corresponding θ plot is very crowded and impossible to read, so it is omitted.

A subset of the data shown in Figure 5-1, from a profile distance from 16,000 m to 21,000 m, will be studied in detail because it features an isolated prominent peak at approximately 18,000 m with several smaller peaks in its vicinity. In Figure 5-2, the TMI and θ plots are shown for this area. The θ plot only shows θ values for 70 m, 80 m, 90 m, 100 m, 110 m, 120 m and 130 m so that it is easier to read. Even so, Figure 5-2 (b) is extremely crowded. In Figure 5-2 (c), the algorithm has detected two areas of interest and both have very low spreads.

Table 5-1 : Outputs for two anomalies along flight profile where distance and depth are shown with their standard deviations, and susceptibility ratio depending on model type.

Profile Distance (m)	Depth (m)	Susceptibility Product	Model	Structural Index
16581 (± 31)	-78 (± 7)	0.29	cylinder	-1.13
18209 (± 6)	-72 (± 14)	1.51	thin sheet	-0.33

Let us zoom in the second area of interest listed in Table 5-1 by examining the data from profile distance 17,500 m to 19,000 m. The TMI data, plotted in Figure 5-3 (a), shows a large peak just before 18,200 m. Now looking at the corresponding θ plot in Figure 5-3 (b), it can be seen that there is a strong cross over around 18,200 m. The ray paths for this area of interest shown in Figure 5-3 (c) intersect at a depth of 72.2 m below surface. The algorithm estimates the source of this magnetic anomaly to be a thin sheet at a profile distance of 18209 (± 6) m, at a depth below ground of -72 (± 14) m with a susceptibility-thickness of 1.51 as listed in Table 5-1.

Figure 5-1 : Magnetic data along a 28,000 m long flight profile from Timmins, Ontario.

Closer examination of Figure 5-3(d) shows that, from the left, there is an intersection at a depth just above 70 m and, from the right, there is an intersection just below 70 m. As discussed in Section 4.4, this could be caused by the presence of a weak magnetic source to the right.

Looking at Figure 5-3 (b) we can see a very low spread of θ values around 18,700 m. The algorithm overlooked this anomaly because of the detection thresholds for the minimum spread and the analytic signal we selected for the program.

By lowering the minimum analytic signal to $A_T = 0.2$ and raising the minimum spread to $e_T = 0.6$, the algorithm finds a number of additional anomalies, one of which being a contact at 18,745 (± 34) m at a depth of 20 (± 174) m below surface. The standard deviation of the horizontal distance is very low, but the standard deviation for the depth is very large, so this is not very precise.

The thin sheet at 18,209 m was examined because it has been previously analyzed using other techniques. Salem et al. (2005) used an enhanced local wavenumber method and found the source at a depth of 69 m. Vallée et al. (2004) used the continuous wavelet transform method and found the source at a depth of 75 m. Finally, Thurston and Smith (2007) used a manual version of the SLUTH method and found the anomaly at a profile distance of 18,200 m and a depth of approximately 75 m. Our algorithm leads to results that are consistent with earlier geophysical interpretations. In addition, in the survey area,

the overlying till has been drilled, and borehole data indicate that the bedrock is at or below 45 m (Vallée et al., 2004). This is consistent with our estimated depth of 72 m.

Results for the entire profile line are listed in Table 5-2 and displayed in Figure 5-4. In Figure 5-4, each symbol corresponds to a model type: a star is a contact, a square is a thin sheet and a circle is a cylinder. Susceptibility and standard deviation are represented by the size and shading of the symbol, respectively. A strong correlation between TMI highs and highly susceptible bodies is shown by the lines connecting the bodies to the TMI plot. Also, deeper bodies clearly have much broader anomalies.

If an anomaly is theorized to exist above the sensor, then it is not output to the file, but this is the only criteria which controls which anomalies are output. For some of the data listed in Table 5-2, the standard deviations are incredibly high and unreliable. These could easily be omitted, but have been left in throughout the thesis to show the reader all the outcomes of the algorithm. Deciding which anomalies deserve further investigation is left up to the user.

a)

b)

c)

Figure 5-2 : (a) Magnetic data, (b) θ plot and (c) 120 rays paths constructed for two areas of interest along a 5,000 m long flight profile near Timmins, Ontario. The θ plot shows θ values for 70, 80, 90, 100, 110, 120 and 130 m flight heights.

a)

b)

c)

d)

Figure 5-3 : (a) Magnetic data and (b) θ plot along a 1,500 m flight profile in the Timmins area. The θ plot shows θ values for 70, 80, 90, 100, 110, 120 and 130 m flight heights. (c) and (d) are ray path plots associated with the thin sheet found at 18,209 m.

Table 5-2 : Estimated characteristics of the source bodies detected by the SLUTH algorithm along the Timmins flight line.

Profile Distance (m)	Depth (m)	Susceptibility-Ratio	Structural Index	Model
766 (± 42)	-92 (± 31)	0.0021	0.09	contact
772 (± 33)	-71 (± 22)	0.0017	-0.25	contact
2245 (± 140)	-169 (± 372)	1.1082	-1.32	thin sheet
3348 (± 354)	-110 (± 239)	0.4356	-0.53	thin sheet
3647 (± 33)	-45 (± 17)	0.0019	0.33	contact
4158 (± 6)	-32 (± 2)	0.0069	-0.33	contact
4235 (± 100)	-95 (± 104)	1.7066	-0.66	thin sheet
5020 (± 311)	-375 (± 150)	16.0081	-2.4	cylinder
5877 (± 153)	-228 (± 100)	3.4286	-1.43	thin sheet
6550 (± 119)	56 (± 175)	0.0009	0.51	contact
7816 (± 5)	77 (± 1)	0.0006	0.84	contact
9186 (± 280)	-318 (± 647)	0.0070	0.18	contact
9786 (± 5)	-105 (± 10)	1.8366	-2.06	cylinder
10935 (± 369)	-12 (± 247)	0.0011	0.24	contact
11562 (± 1049)	-36 (± 417)	0.0041	0.38	contact
12028 (± 17)	36 (± 5)	0.0036	0.38	contact
13528 (± 96)	-216 (± 84)	3.5916	-1.64	cylinder
14810 (± 773)	-151 (± 59)	0.7538	-1.32	thin sheet
15605 (± 24)	-131 (± 138)	0.6022	-0.93	thin sheet
16581 (± 31)	-78 (± 7)	0.2936	-1.84	cylinder
18209 (± 5)	-72 (± 13)	1.5194	-1.13	thin sheet
24733 (± 11)	36 (± 4)	0.0005	0.77	contact

Figure 5-4 : (a) TMI versus profile distance from 0 – 25000 m and (b) cross-section displaying the results in a schematic. Lines are drawn connecting anomalies with high susceptibilities to local peaks in the TMI plot.

5.3 *Investigation of a survey area*

As stated before, the SLUTH algorithm only processes single flight profiles and can not import 2D data directly. This does not mean, however, that area data cannot be analyzed by examining many profiles. In this section, parallel flight profiles from the same survey area will be analyzed to see if any patterns arise.

The field data considered are a series of 43 parallel 13,000 m long flight profiles which are spaced approximately 200 m apart. They have been flown over the Chibougamau area in northern Quebec, Canada (Keating et al., 2007). This region is located at the eastern end of the Abitibi Greenstone Belt and is characterized by two volcano-sedimentary cycles of deposition (Vallée et al., 2009). Figure 5-5 shows a geological map of the Chibougamau area. Like the Timmins area, most structures strike east-west. The flight profiles have been flown north-south, perpendicular to the strike of most geological anomalies. The nominal height of the magnetometer was 75 m and the magnetic data were upward continued to 20 more heights at a spacing of 4 m. All heights were used in the SLUTH analysis. The inclination in this area is 74.47° , the declination is -16.5° and the regional magnetic field is 56770 nT. The spacing between the θ values used in constructing ray paths was $\Delta\theta=0.02$ and 30 ray paths were constructed on each side of each row of interest.

In Figure 5-6, the magnetic contour map of the area is shown. There are a few areas of high TMI which create a band across the middle of the map. These correspond to sills and highly susceptible volcanic sequences.

Plotting the outputs from the SLUTH algorithm in map view offered the challenge of finding the best way to display model type, depth, susceptibility and standard deviations at once in an easily interpretable representation. MATLAB was used to display the results. It was first decided that model type be represented by the symbol shape. A star was chosen to represent a contact, a square to represent a thin sheet and a circle to represent a cylinder. In this display the depth is represented by the size and susceptibility represented by the shading of the symbol, see Figure 5-7. The second attempt was to use the same symbols, but have susceptibility represented by the size and the depth represented by the shading of each symbol, see Figure 5-8. Both conveyed the relevant information and are good methods for display. The choice of which display to use will depend on the magnetic data being interpreted.

The display should be able to be rotated to look at cross-sections. For both of the above displays, the depth was presented as an attribute of the symbol, but when looking at the cross-section, depth is obvious and becomes redundant. This created the need for a display where depth was not an attribute of the symbol, see Figure 5-9. In this display, the size represents the susceptibility and the shading represents the standard deviation. Since this display did not show the depth it was chosen to accompany the cross-sectional views presented in Figures 5-10 and 5-11.

Overlaying the output from the SLUTH algorithm over the magnetic map shows a good spatial coincidence between high magnetic readings and bodies detected in the

subsurface, as seen in Figure 5-9. While many of the thin sheets and cylinders coincide with the magnetic highs, there are highly susceptible contacts which seem to be outliers.

Focusing on the area in rectangle 1 in Figure 5-9, Figure 5-10 illustrates the anomalies that have been found along the east-west strip approximately corresponding to northing 5528000 m. In this cross-section, the anomalies are thin sheets and cylinders, and their depths vary smoothly along the profile, which confirms that the outputs of the SLUTH algorithm are stable and consistent. As discussed in Chapter 3, a structural index between -1.5 and -0.5 will be classified as a thin sheet and between -2.5 to -1.5 as a cylinder. Having such lateral variation means that the structural index must have been close to -1.5 and switched between each model type. In Table 5-3 below, the structural indices found along this profile are listed along with depth. For the majority of the estimates, the structural indices range between -0.9 and -1.9, which agrees with the previously stated hypothesis.

Table 5-3 : The structural index (SI) associated with each source body in this subset of the Chibougamau area.

Easting	Northing	Depth	SI	Model
527198	5528325	-189	-1.33	thin sheet
527422	5527956	-37	-0.07	contact
527439	5528228	-68	0.05	contact
527585	5528137	-178	-1.09	thin sheet
527782	5528037	-353	-1.99	cylinder
528026	5527929	-121	-0.75	thin sheet
528206	5527930	-47	-0.18	contact
528374	5527973	-346	-1.69	cylinder
528598	5528006	-291	-1.33	thin sheet
528767	5527989	-190	-0.86	thin sheet
528999	5527770	-91	-0.54	thin sheet
529235	5527796	-81	-0.58	thin sheet
529407	5527821	-103	-0.75	thin sheet
529590	5527818	-117	-0.87	thin sheet
529777	5527886	-103	-0.77	thin sheet

Now focusing on the area in rectangle 2 in Figure 5-9, Figure 5-11 features two independent anomalies striking east-west, slightly offset one from the other. In this area, the model type varies erratically between the three model types. However all detected bodies have approximately the same size, which means a fairly relatively constant susceptibility-product has been estimated for each anomaly. In Table 5-4 below, susceptibility is listed as a susceptibility-contrast assuming each anomaly could be modeled as a contact for easy comparison. Having little variation in properties for different sections of the same two dimensional magnetic anomalies implies that the algorithm is producing stable and consistent results.

Table 5-4 : The susceptibility-contrast associated with each source body in this subset of the Chibougamau area, assuming all are modeled as contacts.

Easting	Northing	Depth	Susceptibility-Ratio
522209	5528881	36	0.00874
522213	5528711	-17	0.02854
522216	5528491	-166	0.06668
522218	5528436	36	0.01206
522373	5528705	27	0.01398
522596	5528546	-167	0.08093
522782	5528590	-155	0.07077
522789	5528911	41	0.00095
522998	5528585	-92	0.05301
523191	5528574	-117	0.06137
523400	5528648	-42	0.04555
523597	5528511	-12	0.04011
523600	5528321	31	0.01903
523815	5528281	-66	0.07567
523992	5528275	-99	0.09586
524193	5528356	-12	0.05169
524379	5528326	-67	0.05164

Figure 5-5 : Bedrock geological map of the Chibougamau area. The rectangle indicates the area which will be investigated using the SLUTH method (Vallée et al., 2009).

Figure 5-6 : Planar view of the magnetic data from Chibougamau.

Figure 5-7 : SLUTH anomalies plotted over the magnetic data from Chibougamau.

Figure 5-8 : SLUTH anomalies plotted over the magnetic data from Chibougamau.

Figure 5-9 : SLUTH anomalies plotted over the magnetic data from Chibougamau.

Figure 5-10 : Zoom view of the area outlined in rectangle 1, in Figure 5.9 Above is the planar view and below is a projection on to a cross-section for an east-west strip approximately corresponding to northing 5528000 m.

Figure 5-11 : Zoom view of the area outlined in rectangle 2, in Figure 5.9 Above is the planar view and below is a projection on to a cross-section for an east-west strip approximately corresponding to northing 5528500 m.

6 Conclusion

The concept of homogeneity assumed in the Euler method has also been applied in the SLUTH method and has provided a solid foundation for the development of this new method of magnetic interpretation. In connecting locations where the slope ratios, θ , are the same, it is possible to construct multiple ray paths. Regardless if the causative body is an isolated contact, thin sheet or horizontal cylinder, these ray paths intersect at the source of the anomaly. The intersection point provides information about both location and depth, and it is also possible to derive the structural index and an estimate of the susceptibility-product. Having these four important output data with no a priori knowledge of the underlying geology is a very useful step in interpreting magnetic data automatically. This is done without the necessity for second or higher order derivatives of the field.

In this thesis, the SLUTH method has been applied to both synthetic and field data. Synthetic data tests have shown that the method could easily discern all three types of bodies, as long as there was no interference. The SLUTH method also worked well for thin sheet synthetic data which simulate surveys flown up to 1000 m above thin sheets with well-defined and strong magnetic anomalies. This is equivalent to the bodies being 1000 m below the sensor. The lateral resolution between thin sheets was shown to be about 200 m when the survey was flown 70 m above the two outcropping thin sheets.

This thesis only examined a few synthetic cases to give an introduction to possible applications of the SLUTH method. Other possible synthetic options which would be of

interest for future work are studying: vertically stacked anomalies, heterogeneous magnetic susceptibilities and other body types such as thick sheets, dipping cylinders, spheres and combination body types. In studying the application to other body types a good first step would be to test the homogeneity of the synthetic equations' horizontal and vertical derivatives. Another option, which was not implemented, would have been to test the algorithm on synthetic data which had noise added to it. Since the algorithm relies on the data behaving as a homogeneous function, noise would lessen the degree to which the field is homogeneous and therefore cause more erratic results.

Application to field data showed that the results correlated well with prior interpretations. The first data analyzed was a flight profile from Timmins, Ontario where there was an anomaly at a location 18,200 m along the profile. This anomaly has been previously investigated using other interpretation techniques, and the results are consistent, all show a thin sheet at a depth close to 70 m. Furthermore, the ground has been drilled close to the anomaly and the overburden found to be 45 m thick. The fact that the source is deeper than 45 m is consistent with the causative body being a basement feature, in this case a thin sheet that almost subcrops. Interestingly, in the same example, some offset was seen in the intersection clusters of the ray paths which could be attributed to a nearby body. The structural index derived using the SLUTH algorithm for this near by body implied it to be a contact. The entire flight profile was input into the algorithm and a cross-section was plotted and compared with the total magnetic intensity (TMI) plot. In general, where the estimated depth was great, the anomaly width was broad; there was

also a link between high amplitudes in the TMI data and large values of the estimated susceptibility contrast.

The second data set analyzed was 43 parallel flight profiles from the Chibougamau area in Québec. Applying the algorithm to these data was a test of extending the method to a larger multi-line data set. Overall, the SLUTH method gave consistent depths along anomalous features that were crossed by multiple lines, although there was some variation between the estimated structural indices. Three different display methods were tested and showed correlations between characteristics of the magnetic data and the output of the SLUTH method. Each display technique has advantages in illustrating certain features. Showing depth as the size of the symbol allows readers to see depth consistencies along magnetic anomalies. In other display techniques, not showing depth as a variation in the symbols size or shading allowed cross-sections to be plotted without having redundant information. The varying symbol size used previously to display the depth information could then be used to show other information.

While the algorithm described in this thesis is a working product, it could still be improved with modifications. For example, in the spatial analysis of synthetic data it was acknowledged that the ray paths intersection clusters will differ on either side of the anomaly depending which side an interfering body lies. If the algorithm were to acknowledge these offsets and determine which are being altered, they could be omitted from calculation of distance and depth, possibly giving more reliable results. It may also

be interesting to examine different intersection clusters when determining depth and distance from the ray path intersections. In the current algorithm, only adjacent ray path intersections are used, but maybe if all intersections were used for finding both distance and depth at the same time would result in better results. Finally, the distance is presently solved for first using adjacent intersections. Another possible avenue may be to find all intersections with the most horizontal (vertical) ray path to find the distance (depth).

While this thesis illustrated that the SLUTH algorithm developed for processing profile data works well, the algorithm does not lend itself to extension to 2D planar (grid) data. One challenge in such an extension could be to use cones instead of ray paths (the former are more complex). A new study of θ variations around causative body would be necessary to make sure that these data were homogeneous and continuous.

This thesis shows the SLUTH method to be a useful tool to algorithmically determine distance, depth, structural index and susceptibility automatically along profile distances.

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