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TORSIONAL RESPONSE OF BUILDINGS DURING EARTHQUAKE

by

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A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy

November 1998

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Civil and Environmental Engineering
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**TORSIONAL RESPONSE OF
BUILDINGS DURING EARTHQUAKE**

submitted by
PRAVEEN KUMAR, B.Eng., M.Eng.

in partial fulfillment of the requirements for
the degree of Doctor of Philosophy

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November 1998

Abstract

Damage reports on recent earthquakes have indicated that torsional motions often cause significant damage to buildings, at times leading to their collapse. Many researchers have studied the elastic and inelastic torsional response of building models. However, the results of these studies have not always been consistent. Inconsistencies also exist in the torsional design provisions of various building codes. The objective of the work presented here is to provide a better understanding of the torsional behaviour of building systems and to develop design recommendations that are both rational and simple to implement.

Analytical studies are carried out on the elastic torsional response of single storey building models for a range of governing parameters. Based on the response results, a new set of design provisions is proposed which gives design forces that are closer to the results obtained from a dynamic analysis. A series of elastic response analyses are carried out on multistorey buildings to show that the results obtained from the analysis of single storey models can be applied to multistorey buildings in which the ratio of uncoupled torsional frequency to the uncoupled translational frequency, defined at the storey level, does not vary appreciably along the height.

Inelastic time history analyses of single storey models, carried out as a part of this work, show that the buildings designed according to the proposed expressions exhibit peak ductility demands that are equal to or less than those for the associated torsionally balanced buildings. Inelastic response studies are also carried out on multistorey models. They indicate that the proposed torsional design expressions can reasonably be used for the design of multistorey buildings that are asymmetric in plan but otherwise fairly

regular.

The effect of orthogonal planes on the inelastic torsional response is studied. The results show that it is the total torsional stiffness and not how it is distributed between parallel and orthogonal planes that controls the response.

A comparison of the proposed expressions with the torsional design provisions of selected building codes is presented. The study shows that the proposed expressions provide an improved, rational and simple method for the design of buildings against torsion.

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Contents

Abstract	iii
Acknowledgements	v
Contents	vi
List of Tables	x
List of Figures	xiii
List of symbols	xxvii
1 Introduction	1
1.1 General	1
1.2 Causes of torsional motion	2
1.3 Code provisions for design against torsion	4
1.4 Literature review	5
1.5 Statement of the problems	18
1.6 Objectives and scope	19
2 Elastic analysis of torsionally unbalanced single storey building	
models	22
2.1 General	22

2.2	Description of the model	23
2.3	Equations of motion	24
2.4	Response spectrum analysis	25
2.5	Proposed new design procedure	33
2.6	Accidental torsion	36
2.7	Mass eccentric system	48
2.8	Response to recorded ground motions	51
2.9	Effect of aspect ratio on torsional response	55
2.10	Summary and conclusions	59
3	Elastic analysis of torsionally unbalanced multistorey building	
	models	96
3.1	General	96
3.2	Examples of multistorey buildings	98
3.3	Summary and conclusions	123
4	Inelastic analysis of torsionally unbalanced single storey building	
	models	170
4.1	General	170
4.2	Description of the model	172
4.3	Provisions for design against torsion	174
4.4	Inelastic response to recorded motions	178
4.5	Mass eccentric systems	185
4.6	Summary and conclusions	191

5	Effect of orthogonal elements on inelastic torsional response of single storey models	211
5.1	General	211
5.2	Inelastic response to recorded motions	213
5.3	Response of short period systems	219
5.4	Period dependence	220
5.5	Collapse mechanism analysis	226
5.6	Summary and conclusions	232
6	Inelastic analysis of torsionally unbalanced multistorey building models	261
6.1	General	261
6.2	Examples of multistorey buildings	262
6.3	Summary and conclusions	272
7	Comparison of torsional provisions in building codes	290
7.1	General	290
7.2	Provisions for design against torsion	291
7.3	Elastic response of single storey models	297
7.4	Inelastic response of single storey models	300
7.5	Summary and conclusions	305
8	Summary and Conclusions	330
8.1	Summary	330
8.2	Conclusions	334

8.3 Recommendations for future work	340
References	342

List of Tables

2.1	Description and Peak Ground Motion Parameters for Earthquake Records	52
2.1	(contd.) Description and Peak Ground Motion Parameters for Earthquake Records	53
3.1	Example 1 of the elastic multistorey building models, calculation of natural frequencies using the Rayleigh's method	102
3.2	Example 1, shear in edge frames, as obtained from dynamic analysis for a flat spectrum	104
3.3	Example 1, shear in edge frames obtained from static analysis	104
3.4	Example 2, calculation of natural frequencies using the Rayleigh's method	106
3.5	Example 2, shear in edge frames, as obtained from dynamic analysis for a flat spectrum	107
3.6	Example 2, shear in edge frames obtained from static analysis	107
3.7	Example 3, shear in edge frames, as obtained from dynamic analysis for a flat spectrum	110
3.8	Example 3, shear in edge frames obtained from static analysis	111
3.9	Example 4, calculation of natural frequencies using the Rayleigh's method	113

3.10 Example 4, shear in edge frames, as obtained from dynamic analysis for a flat spectrum	114
3.11 Example 4, shear in edge frames obtained from static analysis	115
3.12 Example 5, calculation of natural frequencies using the Rayleigh's method	116
3.13 Example 5, shear in edge frames, as obtained from dynamic analysis for a flat spectrum	118
3.14 Example 5, shear in edge frames obtained from static analysis	119
3.15 Storey shears and forces in the L-shaped balanced building, subjected to 1g flat spectrum, with floor rotations restrained	120
3.16 L-shaped building, storey shears at the flexible edge in balanced and un- balanced buildings	121
3.17 L-shaped building, storey shears at the stiff edge in balanced and unbal- anced buildings	121
4.1 Characteristics of building models	179
4.2 Description and Peak Ground Motion Parameters for Earthquake Records	180
4.2 (contd.) Description and Peak Ground Motion Parameters for Earthquake Records	181
5.1 Effect of period on the torsional response of asymmetric building model s	223
5.2 Characteristics and response of the 3-plane torsionally balanced but un- restrained building model	228
5.3 Characteristics and response of the 3-plane torsionally unbalanced and unrestrained building model	229

5.4	Characteristics and response of the 5-plane torsionally balanced and re- strained building model	231
5.5	Characteristics and response of the 5-plane torsionally unbalanced and restrained building model	231
6.1	Characteristics of the selected earthquake records	265
7.1	Torsional design requirements in building codes	293

List of Figures

2.1	Plan view of a single storey building	62
2.2	Spectral Shapes (a) flat spectrum (b) hyperbolic spectrum	63
2.3	Variation of normalised shear with frequency ratio, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum	64
2.4	Variation of eccentricity ratio with frequency ratio, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum	65
2.5	Time history of normalised dynamic shear and dynamic torque	66
2.6	Effective flexible edge eccentricity, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum	67
2.7	Effective stiff edge eccentricity, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum	68
2.8	Normalised flexible edge displacement, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum	69
2.9	Normalised stiff edge displacement, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum	70
2.10	Effective flexible edge eccentricity, including accidental torsion, aspect ra- tio = 1, (a) flat spectrum (b) hyperbolic spectrum	71

2.11 Effective stiff edge eccentricity, including accidental torsion, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum	72
2.12 Translational and torsional response spectra for El-Centro earthquake . .	73
2.13 Flexible edge displacement due to translational and rotational motions of ground, flat spectrum, $\omega_y = 3$ cps, $\tau = 0.05$, (a) $\Omega_R = 1.0$ (b) $\Omega_R = 1.5$.	74
2.14 Stiff edge displacement due to translational and rotational motions of ground, flat spectrum, $\omega_y = 3$ cps, $\tau = 0.05$, (a) $\Omega_R = 1.0$ (b) $\Omega_R =$ 1.5	75
2.15 Effective eccentricities due to combined effect of translational and rota- tional ground motions, flat spectrum, $\omega_y = 3$ cps, $\tau = 0.05$, (a) $\Omega_R = 1.0$ (b) $\Omega_R = 1.5$	76
2.16 Plan view of (a) stiffness eccentric building (b) mass and stiffness eccentric building	77
2.17 Effective stiffness eccentricity, aspect ratio = $1/3$, flat spectrum, (a) $\eta =$ 0.1 (b) $\eta = 0.2$	78
2.17 (c) $\eta = 0.3$	79
2.18 Average response spectrum for a set of 15 earthquake records	80
2.19 Average normalised displacements for recorded ground motions, aspect ratio = 1, $T = 1$ s., (a) flexible edge (b) stiff edge	81
2.20 Average effective eccentricity for recorded ground motions, aspect ratio = 1, $T = 1$ s., (a) flexible edge (b) stiff edge	82
2.21 Effective flexible edge eccentricity, flat spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R =$ 1.0	83

2.21 (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	84
2.22 Effective flexible edge eccentricity, hyperbolic spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	85
2.22 (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	86
2.23 Effective flexible edge eccentricity, (a) flat spectrum (b) hyperbolic spec- trum	87
2.24 Effective stiff edge eccentricity, flat spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	88
2.24 (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	89
2.25 Effective stiff edge eccentricity, hyperbolic spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	90
2.25 (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	91
2.26 Average flexible edge eccentricity for a set of 15 recorded ground motions (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	92
2.26 (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	93
2.27 Average stiff edge eccentricity for a set of 15 recorded ground motions (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	94
2.27 (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	95
3.1 A five storey shear type of building (a) plan view (b) elevation of a typical frame	124
3.2 Normalised interstorey displacement at the flexible edge of a building of example 1, (a) flat spectrum (b) hyperbolic spectrum	125
3.3 Normalised interstorey displacement at the stiff edge of a building of ex- ample 1, (a) flat spectrum (b) hyperbolic spectrum	126

3.4	Comparison of interstorey displacement ratios for a building of example 1, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5	127
3.4	(c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5	128
3.5	Comparison of interstorey displacement ratios for a building of example 1, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5	129
3.5	(c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5	130
3.6	Normalised interstorey displacement at the flexible edge of a building of example 2, (a) flat spectrum (b) hyperbolic spectrum	131
3.7	Normalised interstorey displacement at the stiff edge of a building of example 2, (a) flat spectrum (b) hyperbolic spectrum	132
3.8	Comparison of interstorey displacement ratios for a building of example 2, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5	133
3.8	(c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5	134
3.9	Comparison of interstorey displacement ratios for a building of example 2, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5	135

3.9	(c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5	136
3.10	A ten storey shear type of building (a) plan view (b) elevation of a typical frame	137
3.11	Normalised interstorey displacement at the flexible edge of a building of example 3, (a) flat spectrum (b) hyperbolic spectrum	138
3.12	Normalised interstorey displacement at the stiff edge of a building of ex- ample 3, (a) flat spectrum (b) hyperbolic spectrum	139
3.13	Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5	140
3.13	(c) flexible edge, floors 4, 3, 2 and 1	141
3.13	(d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5	142
3.13	(f) stiff edge, floors 4, 3, 2 and 1	143
3.14	Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5	144
3.14	(c) flexible edge, floors 4, 3, 2 and 1	145
3.14	(d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5	146
3.14	(f) stiff edge, floors 4, 3, 2 and 1	147
3.15	Normalised interstorey displacement at the flexible edge of a building of example 4, (a) flat spectrum (b) hyperbolic spectrum	148

3.16 Normalised interstorey displacement at the stiff edge of a building of example 4, (a) flat spectrum (b) hyperbolic spectrum	149
3.17 Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5	150
3.17 (c) flexible edge, floors 4, 3, 2 and 1	151
3.17 (d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5	152
3.17 (f) stiff edge, floors 4, 3, 2 and 1	153
3.18 Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5	154
3.18 (c) flexible edge, floors 4, 3, 2 and 1	155
3.18 (d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5	156
3.18 (f) stiff edge, floors 4, 3, 2 and 1	157
3.19 Normalised interstorey displacement at the flexible edge of a building of example 5, (a) flat spectrum (b) hyperbolic spectrum	158
3.20 Normalised interstorey displacement at the stiff edge of a building of example 5, (a) flat spectrum (b) hyperbolic spectrum	159

3.21	Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5	160
3.21	(c) flexible edge, floors 4, 3, 2 and 1	161
3.21	(d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5	162
3.21	(f) stiff edge, floors 4, 3, 2 and 1	163
3.22	Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5	164
3.22	(c) flexible edge, floors 4, 3, 2 and 1	165
3.22	(d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5	166
3.22	(f) stiff edge, floors 4, 3, 2 and 1	167
3.23	Five storey L - shaped building of example 6, (a) plan view (b) elevation of frames	168
3.24	Ratio of shears in a five storey, L - shaped, torsionally unbalanced building to those in the associated torsionally balanced building, flat spectrum . .	169
4.1	(a) Plan view of a single storey mono-symmetric building, (b) force-displacement relationship for the i^{th} plane	193
4.2	Three plane building model	194
4.3	Normalised flexible edge plane strength, aspect ratio = 0.5	195
4.4	Normalised stiff edge plane strength, aspect ratio = 0.5	196

4.5	Normalised total strength of planes, aspect ratio = 0.5	197
4.6	Strength eccentricity, aspect ratio = 0.5	198
4.7	Mean elastic spectrum for the set of 16 earthquake records	199
4.8	Ratio of flexible edge displacement in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records	200
4.9	Ratio of stiff edge displacement in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records	201
4.10	Ratio of flexible edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records	202
4.11	Ratio of stiff edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records	203
4.12	Single storey building models (a) stiffness eccentric system (b) mass and stiffness eccentric system: type 1 model (c) mass and stiffness eccentric system: type 2 model	204
4.13	Ratio of ductility demands for plane No. 1 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 1 models, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	205
4.13	(c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	206

4.14	Ratio of ductility demands for plane No. 3 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 1 models, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	207
4.14	(c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$	208
4.15	Ratio of ductility demands for plane No. 1 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 2 models, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$	209
4.15	Ratio of ductility demands for plane No. 3 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 2 models, (c) $\Omega_R = 0.75$ (d) $\Omega_R = 1.0$	210
5.1	Plan view of a 5 plane single storey mono-symmetric building model . . .	235
5.2	Mean elastic spectrum for the set of 12 earthquake records	236
5.3	Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8$, 1.0 (b) $\Omega_R = 1.0$, $\gamma = 0.4$, 0.6 , 1.0 , (c) $\Omega_R = 1.0$, $\gamma = 0.8$, 1.0	237
5.3	(d) $\Omega_R = 1.25$, $\gamma = 0.4$, 0.6 , 1.0 , (e) $\Omega_R = 1.25$, $\gamma = 0.8$, 1.0	238
5.3	(f) $\Omega_R = 1.5$, $\gamma = 0.4$, 0.6 , 1.0 , (g) $\Omega_R = 1.5$, $\gamma = 0.8$, 1.0	239
5.3	(h) $\Omega_R = 1.5$, $\gamma = 0.4$, 0.6 , 1.0 , (i) $\Omega_R = 1.5$, $\gamma = 0.8$, 1.0	240

5.4	Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8$, 1.0	241
5.4	(b) $\Omega_R = 1.0$, $\gamma = 0.4, 0.6, 1.0$, (c) $\Omega_R = 1.0$, $\gamma = 0.8, 1.0$	242
5.4	(d) $\Omega_R = 1.25$, $\gamma = 0.4, 0.6, 1.0$, (e) $\Omega_R = 1.25$, $\gamma = 0.8, 1.0$	243
5.4	(f) $\Omega_R = 1.5$, $\gamma = 0.4, 0.6, 1.0$, (g) $\Omega_R = 1.5$, $\gamma = 0.8, 1.0$	244
5.5	Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8$, 1.0	245
5.5	(b) $\Omega_R = 1.0$, $\gamma = 0.4, 0.6, 1.0$, (c) $\Omega_R = 1.0$, $\gamma = 0.8, 1.0$	246
5.5	(d) $\Omega_R = 1.25$, $\gamma = 0.4, 0.6, 1.0$, (e) $\Omega_R = 1.25$, $\gamma = 0.8, 1.0$	247
5.5	(f) $\Omega_R = 1.5$, $\gamma = 0.4, 0.6, 1.0$, (g) $\Omega_R = 1.5$, $\gamma = 0.8, 1.0$	248
5.6	Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8$, 1.0	249
5.6	(b) $\Omega_R = 1.0$, $\gamma = 0.4, 0.6, 1.0$, (c) $\Omega_R = 1.0$, $\gamma = 0.8, 1.0$	250
5.6	(d) $\Omega_R = 1.25$, $\gamma = 0.4, 0.6, 1.0$, (e) $\Omega_R = 1.25$, $\gamma = 0.8, 1.0$	251
5.6	(f) $\Omega_R = 1.5$, $\gamma = 0.4, 0.6, 1.0$, (g) $\Omega_R = 1.5$, $\gamma = 0.8, 1.0$	252
5.7	Average ductility ratios in 3 and 5 plane models, (a) flexible edge, (b) stiff edge	253
5.8	Average flexible edge ductility ratios in 3 and 5 plane models, (a) $e/b = 0.1$, (b) $e/b = 0.1$	254
5.8	(c) $e/b = 0.2$, (d) $e/b = 0.3$	255

5.9	Average stiff edge ductility ratios in 3 and 5 plane models, (a) $e/b = 0.1$, (b) $e/b = 0.1$	256
5.9	(c) $e/b = 0.2$, (d) $e/b = 0.3$	257
5.10	(a) 3-plane torsionally unrestrained building model, (b) Paulay's displacement pattern, (c) displacement pattern for El-Centro earthquake at the instant when stiff edge displacement is maximum	258
5.11	Displacement pattern for a 3-plane torsionally unrestrained building model at the instant when stiff edge displacement is maximum, El-Centro earthquake, different strengths of the flexible edge plane	259
5.12	(a) 5-plane torsionally restrained building model, (b) Paulay's displacement pattern, (c) displacement pattern for El-Centro earthquake at the instant when stiff edge displacement is maximum	260
6.1	Elastic response spectrum for the selected ground motions, scaled to a peak ground acceleration of $0.3 g$	275
6.2	Ratio of ductility demand in a five storey unbalanced building to that in the associated balanced building, floor 1, El-Centro earthquake, (a) flexible edge (b) stiff edge	276
6.3	Ratio of ductility demand in a five storey unbalanced building to that in the associated balanced building, floor 5, El-Centro earthquake, (a) flexible edge (b) stiff edge	277
6.4	Interstorey ductility at the stiff edge, five storey building of special class, $\Omega_R = 1.0$, El-Centro earthquake	278

6.5	Interstorey ductility at the stiff edge, five storey building of special class, $\Omega_R = 1.0$, San-Francisco earthquake	279
6.6	Interstorey ductility at the stiff edge, five storey building of special class, $\Omega_R = 1.0$, San-Francisco earthquake	280
6.7	Interstorey ductility at the stiff edge, ten storey building of special class, $\Omega_R = 1.0$, San-Francisco earthquake	281
6.8	Interstorey ductility at the stiff edge, ten storey building of special class, $\Omega_R = 1.0$, San-Fernando earthquake	282
6.9	Interstorey ductility at the stiff edge, twenty storey building of special class, $\Omega_R = 1.0$, El-Centro earthquake	283
6.10	Interstorey ductility at the stiff edge, twenty storey building of special class, $\Omega_R = 1.25$, El-Centro earthquake	284
6.11	Interstorey ductility at the flexible edge, five storey building of general type, $\Omega_R = 1.25$, San-Francisco earthquake	285
6.12	Interstorey ductility at the stiff edge, five storey building of general type, $\Omega_R = 1.0$, San-Francisco earthquake	286
6.13	Interstorey ductility at the stiff edge, five storey building of general type, $\Omega_R = 1.25$, San-Francisco earthquake	287
6.14	Interstorey ductility at the stiff edge, ten storey building of general type, $\Omega_R = 0.75$, San-Francisco earthquake	288
6.15	Interstorey ductility at the stiff edge, ten storey building of general type, $\Omega_R = 1.0$, San-Francisco earthquake	289
7.1	Plan view of a single storey mono-symmetric building model	307

7.2	Variation of A_x , specified in Uniform Building Code 1997, aspect ratio = 1	308
7.3	Plan view of a 3-plane single storey building model	309
7.4	Effective flexible edge eccentricity, aspect ratio = 1, (a) flat spectrum, Ω_R = 0.75, (b) hyperbolic spectrum, $\Omega_R = 0.75$	310
7.4	(c) flat spectrum, $\Omega_R = 1.0$, (d) hyperbolic spectrum, $\Omega_R = 1.0$	311
7.4	(e) flat spectrum, $\Omega_R = 1.25$, (f) hyperbolic spectrum, $\Omega_R = 1.25$	312
7.4	(g) flat spectrum, $\Omega_R = 1.50$, (h) hyperbolic spectrum, $\Omega_R = 1.50$	313
7.5	Effective stiff edge eccentricity, aspect ratio = 1, (a) flat spectrum, Ω_R = 0.50, (b) hyperbolic spectrum, $\Omega_R = 0.50$	314
7.5	(c) flat spectrum, $\Omega_R = 0.75$, (d) hyperbolic spectrum, $\Omega_R = 0.75$	315
7.5	(e) flat spectrum, $\Omega_R = 1.0$, (f) hyperbolic spectrum, $\Omega_R = 1.0$	316
7.5	(g) flat spectrum, $\Omega_R = 1.25$, (h) hyperbolic spectrum, $\Omega_R = 1.25$	317
7.6	Normalised flexible edge plane strength, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$	318
7.6	(c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$	319
7.7	Normalised stiff edge plane strength, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$	320
7.7	(c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$	321
7.8	Normalised total strength of planes, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$	322
7.8	(c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$	323
7.9	Strength eccentricity, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$. . .	324
7.9	(c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$	325

7.10	Ratio of flexible edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$	326
7.10	(c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$	327
7.11	Ratio of stiff edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$	328
7.11	(c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$	329

List of Symbols

A_x	amplification factor specified in UBC 1997
D_d	maximum translational displacement of the single storey model, produced by ground translational motion
D_v	maximum translational velocity of the single storey model, produced by ground translational motion
D_a	maximum translational acceleration of the single storey model, produced by ground translational motion
F_i	floor force at level i
K_y	total lateral stiffness of a storey in y-direction
K_θ	rotational stiffness of a storey about centre of mass
$K_{\theta R}$	rotational stiffness of a storey about centre of rigidity
S_{an}	spectral acceleration in mode n
$S_{a\theta}$	spectral acceleration caused by ground rotation
S_{ay}	spectral acceleration corresponding to frequency ω_y
T_i	floor torque at level i
T_n	torque about centre of mass in mode n
$\bar{T}_n$	normalised torque about centre of mass in mode n
$\bar{T}_n^\theta$	normalised torque about centre of mass in mode n , produced by ground rotation
T_{Rn}	torque about centre of rigidity in mode n
$\bar{T}_{Rn}$	normalised torque about centre of rigidity in mode n
$\bar{T}_{Rn}^\theta$	normalised torque about centre of rigidity in mode

	n , produced by ground rotation
V_0	base shear in the torsionally balanced building
V_1, V_2, V_3	yield strengths of resisting planes 1, 2, 3 respectively, in the torsionally unbalanced building
V_{10}, V_{20}, V_{30}	yield strengths of resisting planes 1, 2, 3 respectively, in the torsionally balanced building
V_e	total elastic strength of resisting planes
V_y	total strength of resisting planes in the y-direction
V_n	base shear in mode n
$\bar{V}_n$	normalised base shear in mode n
$\bar{V}_n^\theta$	normalised base shear in mode n , produced by ground rotation
a	plan dimension of the building parallel to the direction of earthquake
a/v	ratio of peak ground acceleration expressed in terms of g to the peak ground velocity expressed in the units of m/s
b	plan dimension of the building perpendicular to the direction of earthquake
c	horizontal shear wave velocity
c_s	true shear wave velocity in the top soil layer
d_d	maximum ground translational displacement
d_v	maximum ground translational velocity
d_a	maximum ground translational acceleration
e	static eccentricity

e_d	effective dynamic eccentricity
e_{d1}, e_{d2}	design eccentricities
e_{dn}	effective dynamic eccentricity in mode n
e_{dn}^θ	effective dynamic eccentricity in mode n , produced by ground rotation
e_f	effective flexible edge eccentricity
e_s	effective stiff edge eccentricity
e_p	strength eccentricity
f_1, f_2, f_3	yield strengths of resisting planes 1, 2, 3 respectively
f_{xi}	yield strength of i^{th} plane in the x-direction
f_{yi}	yield strength of i^{th} plane in the y-direction
k_1, k_2, k_3	stiffness of planes 1, 2, 3 respectively
m	mass of the floor
r	radius of gyration of the floor about centre of mass
$r_{\Delta f}$	ratio of displacements Δ_f/Δ_0
$\bar{r}_{\Delta f}$	mean of ratio of displacements for the flexible edge, obtained for a set of earthquakes
$r_{\Delta s}$	ratio of displacements Δ_s/Δ_0
$\bar{r}_{\Delta s}$	mean of ratio of displacements for the stiff edge, obtained for a set of earthquakes
r_μ	ratio of ductilities μ_u/μ_b
$r_{\mu f}$	ratio of ductilities for the flexible edge
$\bar{r}_{\mu f}$	mean ratio of ductilities for the flexible edge, obtained

	for a set of earthquake records
$r_{\mu s}$	ratio of ductilities for the stiff edge
$\bar{r}_{\mu s}$	mean ratio of ductilities for the stiff edge, obtained for a set of earthquake records
u_{gx}	ground displacement in x-direction
u_{gy}	ground displacement in y-direction
$u_{g\theta}$	ground rotation
u_y	displacement of the floor in y-direction
$\ddot{u}_{gx}$	ground acceleration in x-direction
$\ddot{u}_{gy}$	ground acceleration in y-direction
$\ddot{u}_{g\theta}$	ground rotational acceleration
$\ddot{u}_y$	translational acceleration of the floor in y-direction
u_θ	rotation of the floor about a vertical axis through CM
$\ddot{u}_\theta$	rotational acceleration of the floor about a vertical axis through CM
Δ_0, Δ_b	displacement or interstorey displacement of the torsionally balanced building
Δ_f	displacement or interstorey displacement of the flexible edge in the torsionally unbalanced building
Δ_{fn}	displacement of the flexible edge in mode n
$\bar{\Delta}_f$	normalised displacement of the flexible edge Δ_f/Δ_0
$\bar{\Delta}_{fc}$	normalised displacement of the flexible edge corresponding to proposed design eccentricities
Δ_p	plastic displacement at the centre of mass

Δ_s	displacement or interstorey displacement of the stiff edge in the torsionally unbalanced building
Δ_{sn}	displacement of the stiff edge in mode n
$\bar{\Delta}_s$	normalised displacement of the stiff edge Δ_s/Δ_0
$\bar{\Delta}_{sc}$	normalised displacement of the stiff edge corresponding to proposed design eccentricities
Ω_R	ratio of uncoupled rotational frequency to uncoupled translational frequency
δ_i	floor displacement at level i
ϕ_d	maximum ground rotational displacement
ϕ_v	maximum ground rotational velocity
ϕ_a	maximum ground rotational acceleration
$\phi_{yn}, \phi_{\theta n}$	mass orthonormalised mode shapes of a single storey coupled system
γ	ratio of torsional stiffness of planes parallel to y-axis, to the overall torsional stiffness of the model, $K_{\theta R}$
γ_{12}	cross correlation factor
γ_s	angle of incidence of shear wave
η	normalised mass eccentricity
μ_b	displacement ductility in the torsionally balanced building
μ_f	displacement ductility at the flexible edge of the torsionally unbalanced building
μ_s	displacement ductility at the stiff edge of the torsionally

	unbalanced building
μ_u	displacement ductility in the torsionally unbalanced building
θ_d	maximum translational displacement of the single storey model produced by ground rotational motion
θ_i	floor rotation about CR, at level i
θ_v	maximum translational velocity of the single storey model produced by ground rotational motion
θ_a	maximum translational acceleration of the single storey model produced by ground rotational motion
τ	transit time of the shear wave
ω_y	uncoupled translational frequency
ω_1, ω_2	coupled frequencies of a single storey model
ω_θ	uncoupled rotational frequency
ζ	damping ratio

Chapter 1

Introduction

1.1 General

In many building structures the centres of resistance do not coincide with the centres of mass. As a consequence, lateral base motion during an earthquake gives rise to torsional vibration of the structure. Field observations of earthquake damage have shown numerous examples of structural failure due to such motion. Damages due to torsional motion were observed during 1971 San Fernando earthquake (Hart *et al*, 1975), 1985 Mexico earthquake (Rosenbleuth and Meli, 1986; Esteve, 1987), 1985 Loma Prieta earthquake (Mitchell *et al*, 1990), 1990 Philippines earthquake (Booth *et al*, 1991), 1994 Northridge earthquake (Mitchell *et al*, 1995) and 1995 Kobe earthquake (Mitchell *et al*, 1996). These events have led to a renewed interest in the study of torsional response of building structures.

A large number of researchers have studied the coupled lateral-torsional motion of building structures. These studies include both elastic and inelastic behaviour of several single and multistorey models. Due to the complexity of the problem and a large number of governing parameters, conclusions from various studies have not always been consistent. Often, the results obtained by the researchers are applicable only to the models

studied, the specific parametric values used and the particular assumptions made in a study. However, these studies have created a general understanding of the nature of torsional response and the governing parameters.

The objectives of this study are to investigate the elastic and inelastic torsional response of single and multistorey building models for a range of governing parameters, to identify important parameters and their critical range and to make design recommendations based on the results obtained, that are both rational and simple to implement. The validity and limitations of the recommendations made are evaluated by studying the response of a series of building models to a significant number of different earthquake records.

1.2 Causes of torsional motion

The dynamic forces that act on a structure during an earthquake are related to inertia and act through centres of mass. These inertia forces are resisted by elastic forces in the lateral load resisting elements whose resultants pass through centres of resistance. If the resisting elements in a building are so distributed that the centres of resistance (CR) do not coincide with centres of mass (CM), lateral seismic forces cause torsional motion in the structure. Structures with non-coincident centres of mass and resistance are referred to as asymmetric structures and the torsional motion induced in them is referred to as *natural torsion*.

Torsional motion may also result due to a variety of factors other than known asymmetry. One such factor is the asymmetry that may exist in a nominally symmetric structure because of uncertainty in the evaluation of centres of mass and stiffness. For

example, the actual distribution of mass may be different from the one assumed in the computations. The estimation of the stiffness of resisting elements may be inaccurate because of lack of precise data on the modulus of elasticity, extent of cracking in concrete, inaccuracy in measuring the dimensions etc. Strictly speaking, the torsion resulting from such asymmetry belongs to the category of natural torsion, but its magnitude can not be defined in a deterministic manner and it can only be assessed in a statistical sense. Another factor is the torsion of structure caused by ground rotation about a vertical axis. No measurements are available for ground rotational motion and therefore its effect can be assessed only in an indirect manner. Torsional motion due to above two reasons is referred to as *accidental torsion*.

Researchers have found another mechanism that may cause torsional motion in symmetric as well as asymmetric structures. This mechanism is related to non-linear coupling between lateral and rotational motions. Such coupling is caused by the non-linear force-displacement characteristics of the lateral load resisting planes of the structure. While the lateral motions are excited by lateral ground motions, torsional motions result from coupling with the lateral motions (Tso and Asmis, 1971). In symmetrical structures, the ratio of uncoupled torsional frequency to lateral frequency, Ω_R , is the dominant factor. Torsional response is excited for a critical range of Ω_R which in turn depends upon the force-displacement characteristics of the resisting elements and the characteristics of the ground excitation.

1.3 Code provisions for design against torsion

The existing building codes, e.g. National Building Code of Canada, Uniform Building Code, New Zealand Standard, Mexico Code, etc; take an equivalent static force approach to account for the torsional effects arising from natural and accidental torsion. These codes require that to determine the design load for an individual lateral load resisting element, the storey shear should be applied at a distance e_d (design eccentricity) from the centre of rigidity. A static analysis of the structure for the given storey shears provides the required design forces. The following expressions are given for the design eccentricity:

$$e_{d1} = \alpha e + \beta b \quad (1.1)$$

$$e_{d2} = \gamma e - \beta b \quad (1.2)$$

where e_{d1} and e_{d2} are the design eccentricities to be used in calculating the total shear in resisting elements, e is the distance between the centre of mass and the centre of rigidity, and b is the plan dimension of the building perpendicular to the ground motion. The first term in the expressions represents natural torsion while the second term, which is taken as a fraction of the building dimension, represents accidental torsion. A typical set of values of the factors in Eqs. 1.1 and 1.2 are: $\alpha = 1.5$, $\beta = 0.1$, and $\gamma = 0.5$, all for NBCC. The factors α and γ are applied to e to take into account the dynamic amplifying effect on the static torsional moment. The factor β represents an indirect assessment of accidental torsion. A detailed discussion on the torsional design provisions of all the above mentioned codes is given in chapter eight of this thesis.

The code expressions take into account the dynamic amplification of static torque

but do not address the reduction of base shear that occurs due to torsional coupling. The expressions also do not give adequate importance to torsional stiffness, as expressed in terms of the ratio of uncoupled torsional to lateral frequencies, Ω_R , on the lateral-torsional coupling of a building. Further, the provisions are based primarily on elastic response behaviour. Their applicability to inelastic behaviour needs to be examined.

1.4 Literature review

Earthquake response of linear elastic structures whose centres of mass are eccentric with respect to the centres of resistance has been studied by several researchers. In many early studies a lumped mass model has been used (Ayre, 1956; Housner and Outinen, 1958; Medearis, 1966; Rosenblueth and Elorduy, 1969). Other studies have used continuous parameter models (Berg, 1962; Gibson *et al*, 1972; Hoerner, 1971). These studies have led to several important conclusions. Two of them are: (1) the dynamic torque may exceed considerably the product of horizontal shear and the eccentricity between the centre of mass and resistance, (2) the lateral and torsional motions are strongly coupled if the uncoupled natural frequencies of the building for lateral and torsional motions are close, even for a building having very small eccentricity.

Forced vibration tests (Jennings, Matthiesen and Housner, 1971) corroborate the second conclusion mentioned above. Based on the numerical results for specific models used, several studies indicate a reduction in base shear as a result of coupling (Bustamante and Rosenblueth, 1960; Rosenblueth and Elorduy, 1969; Jhaveri, 1968). However, Hoerner (1971) has reached the opposite conclusion.

On the basis of a study of the elastic response of torsionally coupled building, for

idealised flat and hyperbolic spectra, Kan and Chopra (1976) concluded that lateral-torsional coupling induces torsion, amplifying the static torque but generally reducing the base shear. They also derived a relation between the base shear and torque in a torsionally coupled system to the corresponding quantities in the associated uncoupled system when the excitation was applied in one direction. Dempsey and Irvine (1979) evaluated the dimensionless torque and shear as function of two parameters, viz. frequency ratio and dimensionless eccentricity. Additional studies (Tso and Dempsey, 1980) on asymmetrical single storey elastic models have been carried out to study the effect of modal coupling. In these studies, researchers defined the ratio of dynamic to static torque as dynamic eccentricity e_d , and considered it to be a measure of coupling and hence of the torsional effects. It was found that coupling effects are significant for a range of uncoupled rotational to lateral frequency ratio (0.8 – 1.25). Based on the results of modal coupling effects, some researchers recommended that a value of frequency ratio close to 1, for which modal coupling was maximum, should be avoided in design.

Subsequent studies (Dempsey and Tso, 1982; Tso, 1983) showed that the maximum edge displacements provide a more realistic assessment of torsional effects for design purposes as compared to maximum torsional moments. This gave rise to the concept of effective edge eccentricity, e_e . The effective edge eccentricity was defined as the distance measured from the centre of rigidity at which the static shear should act in order to give rise to the same displacement as that obtained by using dynamic analysis. It was shown (Dempsey and Tso, 1982) that for a building with a given eccentricity, the maximum effective eccentric, e_e , was about 50 – 75% of the dynamic eccentricity, e_d . The explanation given for this difference was that maximum edge displacements from lateral and

torsional responses did not necessarily occur at the same instant of time.

Further studies indicated that coupling also existed between the lateral and torsional responses of symmetric structures. Newmark (1969) examined the response of symmetrical structures excited into torsional motion by the passage of a seismic wave. Torsion resulted because of the phase difference in seismic wave at different points of the ground on which the structure was built. Awad and Humar (1984) derived and used the torsional response spectrum for such wave motion and showed that the effect of ground rotation may be more significant than the effect of dynamic coupling. Tso and Asmis (1971), and Tso (1975) studied the coupled lateral-torsional response of symmetric elastic structures with a non-linear force-displacement relationship for the lateral load resisting elements, subjected to lateral ground motion. They showed that a non-linear coupling existed between the translational and rotational responses. Pekau and Syamal (1981) investigated this subject further and concluded that non-linear coupling can be very significant for unsymmetrical structures.

Accidental torsion created due to uncertainties in stiffness and mass and due to ground rotation has been studied by several researchers (Awad and Humar, 1984; Pekau and Guimond, 1990). De La Llera and Chopra (1992, 1994a) studied the response of nominally symmetric plan building models using a probability distribution of structural element stiffness and concluded that the additional displacement due to stiffness uncertainty is less than 10 and 5 percent for R/C and steel systems, respectively. De La Llera and Chopra (1994a) studied the torsional response of symmetric buildings for a derived set of rotational excitations and concluded that the additional response due to ground rotation about a vertical axis is less than 5% for torsionally stiff systems.

Kan and Chopra (1976) showed that the results obtained from single storey models were also applicable to a *special class* of multistorey buildings. In such buildings the centres of mass and stiffness lie approximately on two vertical lines and the ratio of the storey stiffnesses in two orthogonal directions in the floor plan is about the same for all storeys. Observations similar to those of Kan and Chopra, have given rise to torsional provisions in building codes for the multistorey buildings of special class, based on the results obtained for single storey elastic models. For the design of lateral resisting elements of the multistorey building, the codes specify additional torques at each floor of the building. These torques are obtained by multiplying the code prescribed storey shear and the design eccentricity. As described in Section 1.3, this eccentricity e_d is given as a function of static eccentricity and building dimensions and is assumed to take care of both the dynamic torque amplification and the accidental torsion. Adequacy of these code specified design provisions has been an important topic of study and many models have been studied keeping them in mind (Tso and Meng, 1981; Tso, 1983; Cheung and Tso, 1987; Tinawi and Vachon, 1987; Calderoni *et al* 1995; Wong and Tso, 1995). As a result of these studies, the design provisions of building codes have undergone several amendments with a view to specifying a more rational expression for design eccentricity, e_d .

The application of code specified eccentricity concept to the multistorey buildings of a general type, however, has not been easy because there is no generally accepted definition of the centres of rigidity for multistorey buildings. The confusion originated because the code specified design eccentricity concept is based on the results of single storey models for which centre of rigidity, shear centre and centre of twist are the same.

A multistorey building of special class, fortunately possesses the same property. For a general multistorey building, researchers have defined the centres of rigidity in many different ways (Poole, 1977; Humar, 1984; Smith and Vezina, 1985; Cheung and Tso, 1986; Riddell and Vasquez, 1984) and there is no universally accepted definition. Tso (1990) explained the difficulty involved in different interpretations used by codes and established an equivalence between floor eccentricity which is based on a floor's centre of twist, and storey eccentricity which is based on a storey's shear centre. It was shown that both floor and storey eccentricities are dependent on the distribution of the lateral loads. The application of code provisions requires that the eccentricities be explicitly determined, a process that is not straightforward. The above problem was solved by Goel (1995) who derived analytical techniques that can be used to implement design provisions of building codes without locating the centres of stiffness. This method requires many analysis steps, but is accepted as a viable solution and recommended in the National Building Code of Canada, for the implementation of torsional provisions of the code (part 4, commentary J(66), NBCC 1995).

Although the effects of torsional coupling on the linear elastic response of buildings have been well established, these results are not directly applicable to the calculation of earthquake design forces for a majority of the buildings because they are usually designed to deform significantly beyond the yield limit during intense ground motions. One of the difficulties in investigating the coupled lateral torsional response of asymmetric plan systems in the inelastic range of behaviour has been that many more parameters are required to characterise such systems as compared to the corresponding elastic systems. For a single storey model, some of the additional governing parameters have been

identified as: strength distribution, resistance eccentricity e_r , number of planes parallel to the direction of ground motion, contribution of torsional stiffness from resisting elements perpendicular to ground motion, mass eccentricity, overstrength factor (ratio of the strengths of asymmetric plan and corresponding symmetric plan systems), range of frequency contents of ground motion records employed for inelastic dynamic analysis etc. Most of the previous studies have been devoted to identifying the controlling parameters of the inelastic torsional behaviour. Despite all the efforts, these studies have not been successful in providing a clear relationship between inelastic torsional responses and structural parameters of the eccentric system. Various assumptions made by the researchers in the structural model studied, have restricted the generality of results obtained. Often, researchers have arrived at contradictory conclusions which is apparent from the following review of the previous work.

As a first step towards the understanding of inelastic torsional response, a number of studies on the eccentric single mass system have been carried out. Early studies indicated that the dynamic torque amplification is larger in the inelastic range, as compared to that for the elastic response of the same model. Subsequent studies (Kan and Chopra, 1981; Syamal and Pekau, 1984) contradicted this and showed that the torsional coupling effects in the inelastic system are similar to those in the elastic system. Using a single element model in their studies, Kan and Chopra (1981) concluded that the response in the inelastic range is affected by torsional coupling to generally a lesser degree than the elastic response and after initial yielding the system responds primarily in translation, behaving more like a single degree-of-freedom system. This was well supported by the findings of Irvine and Kountouris (1980) who concluded that: (1) ductility demands in

resisting elements are insensitive to stiffness eccentricity e_s and uncoupled torsional to lateral frequency ratio, and (2) the ductility demand on the worst loaded frame is rarely more than 30% greater than the ductility demand in a similar symmetric plan structure.

Based on the response of four and sixteen element mass eccentric models, Erdik (1975) reached conclusions similar to those obtained in the above two studies.

The validity of the findings of above-mentioned studies were re-examined by Tso and Sadek (1985). Using a three-element model, they demonstrated that: (1) torsionally coupled systems do not respond primarily in translation when they are excited well into the inelastic range, on the contrary, significant torsional motion exists when peak ductility demand is reached; (2) an increase by a factor of two in the ductility demand of edge elements is not uncommon for systems with large eccentricity as compared to symmetrical plan systems; and (3) ductility demand is not sensitive to the ratio of uncoupled torsional to lateral frequencies.

Using a structural model similar to that used by Tso and Sadek, another study (Bozorgnia and Tso, 1986) confirmed the findings cited in the preceding paragraph and showed that the edge displacements, which are affected more by plan asymmetry than the ductility demand, can be more sensitive to the uncoupled torsional to lateral frequency ratio. Palazzo and Fraternali (1988) however concluded that a marked amplification of ductility demand occurs when the uncoupled torsional and lateral frequencies are coincident.

Subsequent studies (Huckelbridge and Lei, 1987; Sadek and Tso, 1989) have suggested resistance eccentricity, e_r to be an alternate measure of asymmetry. It was shown that for a system with given stiffness eccentricity, the inelastic torsional response decreases

with the resistance eccentricity of the system. Bruneau and Mahin (1991) showed that a symmetric structure with asymmetric yield strength distribution in plan is torsionally coupled.

Further studies considered additional ductility demands and additional deformations at the edge frames of a building, as the inelastic torsional response parameters. Tso and Ying (1990) demonstrated that the distribution of strengths among elements has a strong influence on the inelastic seismic response of structures. The elements can be designed ignoring the torsional shear (stiffness proportional model) or including the torsional shear effect based on static equilibrium (static equilibrium model). The former method leads to resistance/strength eccentricity (e_r) equal to stiffness eccentricity (e_s), while the second approach leads to zero resistance eccentricity ($e_r = 0$). Based on the parametric study of a three-element single mass stiffness eccentric model, Tso and Ying (1990) concluded that: (1) resisting elements on the flexible side are most susceptible to torsional effects, (2) the ductility demand, at the flexible edge, is more for a system designed to have $e_r \approx e_s$, as compared to $e_r = 0$ systems, and (3) for $e_r = 0$, flexible edge element shows minimal additional ductility demand while showing an additional displacements of the order of 2 to 3 times of that obtained for a similar but symmetrical structure.

The above results were well corroborated by the conclusions arrived at by Goel and Chopra (1990) who showed that the torsional coupling increases the torsional deformation and decreases the lateral deformation to a lesser degree in systems with $e_r \ll e_s$. They arrived at the following additional conclusions:

1. The inelastic response of structures is influenced significantly by the contribution

to torsional stiffness from the resisting elements perpendicular to the direction of ground motion. The presence of such perpendicular elements tends to reduce effects of coupling, particularly for short period systems. The large ductility demands and edge displacements observed in the studies without these perpendicular elements in the models investigated, are excessive for the design of most buildings.

2. For systems with $e_r \approx e_s$, the number of resisting elements parallel to the direction of ground motion has little influence on the response of system. For systems with $e_r \ll e_s$, the number of resisting elements has little influence on element deformations but significant effect on maximum ductility demands.
3. For strength symmetric systems ($e_r = 0$), the inelastic response of mass eccentric and stiffness eccentric models, with identical elastic parameters (frequency ratio Ω_R , period T , e_s/r , and the damping ratio ζ) and inelastic parameters (e_r , overstrength factor, ductility factor), can be very different, specially for maximum ductility demand.

Realising the importance of mass eccentricity, several studies have investigated inelastic response of mass eccentric systems. In one such study, Gomez *et al* (1987) concluded that excessive ductility demand is imposed on the stiff side element of mass eccentric systems if strength eccentricity is small. They suggested that the ductile eccentric systems should be designed with strength eccentricity e_r not significantly different from their stiffness eccentricity e_s . This was in contrast to the recommendations made by previous researchers for the stiffness eccentric systems, as described earlier. Since mass eccentric and stiffness eccentric systems have different requirements that the strength eccentricity

must satisfy in order to minimise additional ductility demand, a later study (Tso and Zhu, 1992a) argued that a single criterion for strength eccentricity can not be an effective specification for all types of eccentric structures.

Chandler and Duan (1991) investigated the additional ductility demands on the edge elements of a three-element single mass eccentric model. They concluded that for a system with element strengths specified according to code design eccentricity expressions (which result in $e_r \ll e_s$), the element at the stiff edge is critical showing high ductility demands. They observed that the peak ductility demands of the element at flexible edge is always lower than that of corresponding symmetric structures. This is in contrast to the findings of Tso and Ying (1990). It was also pointed out by Chandler and Duan (1991) that the approach of including accidental eccentricity in the code design eccentricity expressions but ignoring uncertainties in the evaluation of centres of mass and stiffness and the ground rotational motion in inelastic dynamic analysis is misleading.

To clarify the discrepancies in the assessment of ductility demands of edge elements, Tso and Zhu (1992a) studied a wide range of three-element single storey models. To avoid the necessity of deciding whether the eccentric system was stiffness eccentric or mass eccentric, they defined a reference system which was not required to be symmetric but only to be torsionally balanced. Using the centre of rigidity as the reference point, they showed that the conclusions reached by Tso and Ying (1990) were limited to the models with stiffness eccentricity less than 0.3 times the width of the building, and the results obtained by Chandler and Duan (1991) were applicable only to the torsionally flexible systems. They also concluded that: (1) systems designed ignoring torsional provisions ($e_r \approx e_s$) will have a large additional ductility demand on elements at the

flexible side; (2) there is always an additional displacement demand on the flexible side of the structure and this demand is insensitive to the form of torsional provisions adopted in design but depends upon stiffness eccentricity, e_s , torsional stiffness of the structure and the distance of flexible edge from the centre of rigidity; and (3) for torsionally flexible systems, designed according to the torsional provisions of codes that allow substantial strength reduction on the element at the stiff side (e.g. UBC 88 and NZS 1984), a substantial additional ductility demand is imposed on the elements at the stiff side.

Zhu and Tso (1992b) examined the strength distribution of lateral load resisting elements in a torsionally unbalanced system, based on codified torsional provisions. It was shown that the element strength can be expressed conveniently as the element strength of a similar but torsionally balanced system multiplied by a strength factor. This strength factor depends upon three system parameters, namely, the location of the element relative to the centre of rigidity, torsional stiffness, and eccentricity of the structure. Although this method has been shown to work well, it requires explicit determination of the centre of rigidity.

The ultimate goal of all the studies performed on single storey inelastic model has been to generate globally acceptable design guidelines for asymmetric single as well as multistorey structures that undergo significant inelastic deformations during a major earthquake. Using the generally accepted concepts for specifying the strength distribution in single storey models, several studies extrapolated the knowledge available to multistorey inelastic models. Many researchers designed a multistorey building according to the torsional provisions of several codes (that are primarily based on the results of single storey models) and investigated the validity of such provisions by means of a three

dimensional dynamic analysis (De La Llera and Chopra, 1992; Sedarat and Gupta, 1992; Calderoni *et al*, 1995). Duan and Chandler (1993) examined the inelastic behaviour of a mono-symmetric, multistorey building of special class, designed according to NBCC 90, UBC 88, Mexico 87, Eurocode EC8 and NZ 92 and concluded that unlike for an elastic system, a single storey model is not sufficient to investigate the torsional effects in an inelastic multistorey model. They also found that the equivalent static design in accordance with the codes underestimates the additional ductility demands due to torsion in certain load resisting elements very significantly.

De La Llera and Chopra (1994a) developed a conceptual framework for understanding the inelastic seismic performance of asymmetric-plan single storey systems. The idea used was to study the effects of plan-asymmetry by considering the base shear and torque response histories of different structural configurations. The researchers showed that at each instant of the response, the base shears V_x , V_y and base torque T , defined a point in the force space, which was bounded by base shear and torque ultimate surface (BST). The BST surface was defined by the set of base shear and torque combinations corresponding to the different collapse mechanisms that could develop in the system. This conceptual framework was extended to understand the inelastic seismic behaviour of multistorey systems and it was found that the asymmetric single and multistorey buildings of special class exhibited similar inelastic behaviour and such behaviour was affected by the same building characteristics. Using storey shear and torque surface (SST), design recommendations were made to control ductility demands in the edge elements. Based on the results obtained, a simplified model consisting of one super element (SE) per building storey, was proposed. It was shown, using SST surfaces, that

the maximum errors in peak responses due to simplifications made by SE model, was less than 20% for most practical systems.

Some recent studies have concentrated on improving the torsional provisions of building codes for the design of general multistorey buildings. De Stefano (1993) studied a model having additional lateral load resisting elements in the direction perpendicular to the ground motions. However, these transverse elements were assumed to remain elastic throughout the analysis and the results obtained were therefore not relevant for actual buildings. A subsequent study (Wong and Tso, 1995) evaluated the seismic torsional provisions of NBCC 1990, by considering single mass structural model with bi-directional earthquake motion and inelastic transverse elements. They recommended that for considering torsional effects in such cases, the edge elements should be designed for the worst case of (1) 100% x-directional effects + 30 % y-directional effects, and (2) 100% y-directional effects + 30 % x-directional effects.

Some studies have been carried out to investigate the torsional effects in setback type buildings (Berg, 1962; Satake and Shibata, 1988; Fukada *et al* 1992; Tso and Shu Yao, 1994; Jain and Annigeri, 1995). One conclusion is that torsional effects in setback buildings need to be considered on storey by storey basis. Under the strength distribution that takes place, the total strength requirements can be reduced as compared to current design values.

Several researchers have studied the effect of torsional coupling in base isolated structures (Lee, 1980; Pan and Kelly, 1983; Eisenberger and Rutenberg, 1986; Takashi, 1988; Nagarajaiah, Reinhorn and Constantinou, 1993a, 1993b). They have found that although the torque generated in base-isolated structures was less than that of the fixed

base structure, the torque amplifications were quite significant. Torsional motion was found to occur in elastomeric isolation systems due to eccentricity in isolation system or the eccentricity in the superstructure, while in sliding isolated structures torsional motions were induced only due to superstructure eccentricity.

1.5 Statement of the problems

As described in the previous section, several researchers have studied various aspects of the torsional response of buildings. During the course of such studies, many issues have been identified, some of which are yet to be settled.

A number of issues related to the seismic torsional response of buildings can be identified. They are listed below. The present work addresses some of these issues as outlined in the next section.

1. What is the importance of torsional stiffness, as represented by the uncoupled frequency ratio Ω_R , in the torsional response and whether $\Omega_R = 1$ is critical and should be avoided?
2. Do existing static design provisions of building codes correctly account for the elastic torsional behaviour for the full practical range of governing parameters?
3. Can the knowledge obtained from single storey elastic models be applied to multi-storey models of a general type? What are the limitations?
4. How does the variation of eccentricity and storey-wise frequency ratio, along the height of the building, affect torsional response?
5. What is the nature of displacement and ductility demands in the edge elements

of a single storey inelastic building model, designed according to static provisions based on elastic studies, and how do they vary with eccentricity and Ω_R ?

6. How does the presence of orthogonal resisting elements affect inelastic torsional behaviour of mono-symmetric single storey models?
7. What is the effect of time period on inelastic torsional response of building models?
8. What is the effect of the type of earthquake selected (qualified by a/v ratio), on inelastic torsional behaviour?
9. What is the nature of ductility demands in torsionally coupled inelastic multistorey building models? How should the existing design procedures be modified to take this into account?

1.6 Objectives and scope

The main objective of this study is to provide a better understanding of the torsional behaviour of buildings and to recommend design provisions that are both rational and simple to implement. It is clear from the literature review that the results obtained by several researchers, specially for the torsional behaviour in the inelastic range, have been contradictory and limited by the assumptions made in the models selected. There are several issues that have not been touched upon and there is a need to do further research.

As a part of the ongoing effort to better understand the torsional behaviour, this study addresses some of the unresolved issues listed in the previous section by studying a wide range of building models that are categorised in four groups: (a) single storey elastic models, (b) multistorey elastic models, (c) single storey inelastic models and (d)

multistorey inelastic models. The elastic models are discussed in Chapter 2 and 3, and the inelastic models are discussed in Chapter 4, 5 and 6. The scope of the work carried out in this study is as follows:

1. Study the elastic torsional response of single storey building models for a range of governing parameters viz. eccentricity, torsional to lateral frequency ratio, aspect ratio of building, translational period of the model, types of earthquake spectrum and recorded ground motions.
2. Study the effect of accidental eccentricity on the elastic torsional response of single storey building models.
3. Study the elastic torsional response of mass eccentric single storey building models, for a range of governing parameters.
4. Based on the above study, investigate whether the existing code specified provisions correctly account for the edge displacements in asymmetric models. Suggest necessary modifications, if any.
5. Study elastic torsional behaviour of multistorey building models of general class for a range of parameters not studied earlier and investigate the applicability of single storey results.
6. Design the single storey inelastic models according to the knowledge obtained from the analysis of single storey elastic models and investigate the adequacy of design provisions.
7. Study how the presence of orthogonal resisting planes affect the torsional behaviour

of single storey inelastic models. Investigate the effect of yielding in orthogonal planes on the inelastic torsional response.

8. Study period dependency of building models on their inelastic torsional behaviour.
9. Study the inelastic behaviour of multistorey asymmetric buildings of special class and general class. Explore how the results obtained can be applied to achieve a rational design.
10. Evaluate some of the existing code provisions on torsional design, in light of the results obtained in this study.

Chapter 2

Elastic analysis of torsionally unbalanced single storey building models

2.1 General

A majority of building structures are expected to become inelastic during a major earthquake. However, certain structures are purposely designed to remain elastic. Besides, even for structures that are expected to deform into the inelastic range during a major earthquake, it is often a design requirement that the structures remain elastic during a moderate earthquake and that their response in such a case be assessed to verify the assumption of elasticity and the requirements of serviceability. Design provisions should therefore address the torsional response of elastic structures. A study of the elastic response also provides an insight into the nature of inelastic behaviour. This is why the elastic behaviour of torsionally unbalanced building structures is of considerable interest. Analytical results are presented here to study the elastic response of single storey building models.

2.2 Description of the model

The study of the torsional response of an idealised single storey building model provides useful insight into the nature of torsional response of a more complicated single storey as well as multistorey system. Hence the single storey model has been studied by several researchers and is re-examined in the present work. Figure 2.1 shows a single storey building model. The building floor is assumed to be infinitely rigid in its own plane and therefore moves as a rigid body. The entire mass of the structure is uniformly distributed at the floor level. The origin of the coordinate axes is at the centre of mass denoted by CM. During an earthquake, the motion is opposed by lateral load resisting elements oriented along the two orthogonal axes. The resisting planes may comprise columns, shear walls, braced frames or a combination thereof. The i th plane parallel to the x axis has a stiffness k_{xi} , while the i th plane in the y direction has stiffness k_{yi} . The distribution of stiffness is symmetrical about the x axis, but is asymmetrical about the y axis. Thus the centre of stiffness lies on the x axis at a distance e from the centre of mass, where e is given by

$$e = \frac{\sum_{i=1}^N k_{yi} x_i}{\sum_{i=1}^N k_{yi}} \quad (2.1)$$

where N is the number of resisting planes in the y direction. For translational motion in the y direction, the resisting elastic forces are proportional to the stiffness of the resisting planes. Hence the centre of resistance coincides with the centre of stiffness, which is therefore designated as CR.

2.3 Equations of motion

For an earthquake ground motion in the y direction, the equations of motion of the system shown in Fig. 2.1 are given by,

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{u}_y \\ r\ddot{u}_\theta \end{bmatrix} + \begin{bmatrix} K_y & \frac{e}{r}K_y \\ \frac{e}{r}K_y & \frac{K_\theta}{r^2} \end{bmatrix} \begin{bmatrix} u_y \\ ru_\theta \end{bmatrix} = -m \begin{bmatrix} \ddot{u}_{gy} \\ 0 \end{bmatrix} \quad (2.2)$$

where m is the mass of the floor, r is the radius of gyration of the floor about CM, u_y is the displacement of the floor in y direction, u_θ is the rotation of the floor about a vertical axis through CM, $K_y = \sum_{i=1}^N k_{yi}$ is the total stiffness in the y direction, K_θ is the torsional stiffness about CM, and $\ddot{u}_{gy}$ is the ground acceleration in the y direction.

The torsional stiffness about the centre of resistance, $K_{\theta R}$, is given by

$$K_{\theta R} = \sum_{i=1}^N k_{yi}(x_i - e)^2 + \sum_{i=1}^M k_{xi}y_i^2 \quad (2.3)$$

where M is the number of resisting planes orthogonal to the direction of excitation, i.e. in the x direction. Rotational stiffness about the centre of mass, K_θ , can be calculated from,

$$K_\theta = K_{\theta R} + K_y e^2 \quad (2.4)$$

Substituting Eq. 2.4 in Eq. 2.2 and dividing by m we get,

$$\begin{bmatrix} \ddot{u}_y \\ r\ddot{u}_\theta \end{bmatrix} + \omega_y^2 \begin{bmatrix} 1 & \frac{e}{r} \\ \frac{e}{r} & \Omega_R^2 + (\frac{e}{r})^2 \end{bmatrix} \begin{bmatrix} u_y \\ ru_\theta \end{bmatrix} = - \begin{bmatrix} \ddot{u}_{gy} \\ 0 \end{bmatrix} \quad (2.5)$$

where $\omega_y = \sqrt{K_y/m}$ is referred to as the uncoupled translational frequency, $\omega_\theta = \sqrt{K_{\theta R}/mr^2}$ is the uncoupled rotational frequency. Frequencies ω_y and ω_θ are respec-

tively the translational and rotational frequencies of an associated torsionally balanced building that has the same structural properties (K_y , $K_{\theta R}$, m and r) as the asymmetrical building, but has coincident centres of mass and resistance. Parameter $\Omega_R = \omega_\theta/\omega_y$ is the frequency ratio and is a measure of torsional stiffness of the building. It is evident from Eq. 2.5 that, for a given ground motion, the response of the single storey building shown in Fig. 2.1 is governed by ω_y , Ω_R , and e/r . The off-diagonal term e/r in Eq. 2.5 indicates coupling between translational and rotational motions.

2.4 Response spectrum analysis

For a response spectrum analysis of the torsionally unbalanced single storey structure, we need to evaluate its frequencies and mode shapes. The two frequencies of the system represented by Eq. 2.5 are given by,

$$\begin{aligned}\frac{\omega_1}{\omega_y} &= \sqrt{A - B} \\ \frac{\omega_2}{\omega_y} &= \sqrt{A + B}\end{aligned}\tag{2.6}$$

where,

$$\begin{aligned}A &= \frac{1}{2} \left[1 + \Omega_R^2 + \left(\frac{e}{r} \right)^2 \right] \\ B &= \sqrt{A^2 - \Omega_R^2}\end{aligned}\tag{2.7}$$

The corresponding mass-orthonormalised mode shapes are given by

$$\begin{bmatrix} \phi_{yn} \\ \phi_{\theta n} \end{bmatrix} = \frac{1}{\sqrt{\left(\frac{e}{r}\right)^2 + \left\{1 - \left(\frac{\omega_n}{\omega_y}\right)^2\right\}^2}} \begin{bmatrix} -\left\{1 - \left(\frac{\omega_n}{\omega_y}\right)^2\right\} \\ \frac{e}{r} \end{bmatrix} \quad n = 1, 2 \quad (2.8)$$

If the acceleration response spectrum for the given earthquake motion and the specified value of damping ratio is available, the maximum response of the structure can be obtained by superposition of the maximum modal responses. Because the two coupled frequencies ω_1 and ω_2 may be close to each other, the commonly used root sum square (RSS) method of superposing the modal responses could lead to erroneous results (Rosenblueth and Elorduy, 1969; Kelly and Sackman, 1980). The complete quadratic combination (CQC) method is therefore used in place of the RSS method. Thus, if R_n represents the maximum response in mode n and R the resultant response obtained from a superposition of two modal responses,

$$R^2 = R_1^2 + R_2^2 + \gamma_{12} R_1 R_2 \quad (2.9)$$

where γ_{12} is the cross-correlation factor given by

$$\gamma_{12} = \frac{8\zeta^2(1 + q_{12})q_{12}^{3/2}}{(1 - q_{12}^2)^2 + 4\zeta^2q_{12}(1 + q_{12})^2} \quad (2.10)$$

$q_{12} = \omega_1/\omega_2$, and ζ is the damping ratio, assumed to be the same for the two modes.

The modal base shear and torque are given by

$$\begin{aligned} V_n &= m\phi_{yn}^2 S_{an} \\ T_n &= mr\phi_{yn}\phi_{\theta n} S_{an} \\ T_{Rn} &= T_n - eV_n \end{aligned} \quad (2.11)$$

where V_n is the base shear in mode n , T_n is the torque about CM in mode n , T_{rn} is the corresponding torque about CR, and S_{an} is the spectral acceleration in mode n .

It is useful to express the base shear and torque in a normalised form. Thus if $V_0 = mS_{ay}$ represents the base shear in the torsionally balanced building, S_{ay} being the spectral acceleration corresponding to the frequency ω_y , we have

$$\bar{V}_n = \frac{V_n}{V_0} = \phi_{yn}^2 \frac{S_{an}}{S_{ay}} \quad (2.12)$$

$$\begin{aligned} \bar{T}_{Rn} &= \frac{T_{Rn}}{V_0 r} = \phi_{yn} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \frac{S_{an}}{S_{ay}} \\ &= \frac{e_{dn}}{r} = \bar{e}_{dn} \end{aligned} \quad (2.13)$$

Parameter e_{dn} can be viewed as an effective eccentricity such that if the shear V_0 were applied at a distance e_{dn} from CR, it would produce a torque about CR equal to T_{rn} .

In Eq. 2.13, eccentricity e_{dn} has been normalised with respect to r . Often, designers prefer to express e_{dn} in terms of the plan dimension b . For a uniform distribution of mass, the radius of gyration r and the plan dimensions are related by the following expression

$$r^2 = \frac{a^2 + b^2}{12} = \frac{b^2}{12} (1 + \alpha^2) \quad (2.14)$$

where $\alpha = a/b$ is the aspect ratio.

The response equations are evaluated here for two different spectral shapes: (1) flat spectrum, and (2) hyperbolic spectrum, both shown in Fig. 2.2. For a flat spectrum, $S_{an}/S_{ay} = 1$, whereas for a hyperbolic spectrum $S_{an}/S_{ay} = \omega_n/\omega_y$.

Effects of torsional coupling

Figures 2.3a and 2.3b show the variation of normalised shear $\bar{V}$ as a function of Ω_R for different values of the static eccentricity. Figure 2.3a is for flat spectrum and Fig. 2.3b for hyperbolic spectrum. In each case, the damping ratio is 0.05. It is evident that $\bar{V}$ is less than 1 in all cases implying that the base shear in a torsionally unbalanced building is less than that in the associated balanced building. This reduction is greater when the uncoupled natural frequencies in translation and rotation are close, i.e. when $\Omega_R = 1$, at which value, a strong coupling is indicated and $\bar{V}$ takes the minimum value.

To demonstrate the effect of torsional coupling, the ratio of the dynamic eccentricity to the static eccentricity, e_d/e , is plotted as a function of Ω_R in Figs. 2.4a and 2.4b. The ratio e_d/e has been referred to as *dynamic torque amplification* by researchers and it has been the basis of design eccentricity expressions specified in various building codes. This approach is however not right, as will be explained shortly. The ratio e_d/e shows a maxima near $\Omega_R = 1$, where torsional to lateral coupling is high.

It is useful to compare the dynamic eccentricity, e_d , with the code specified values of the design eccentricity. In NBCC 1995, the design eccentricities are given by: $e_{d1} = 1.5e + 0.1b$, applicable to the design of elements on the flexible side of the building; and $e_{d2} = 0.5e - 0.1b$, applicable to the design of resisting elements on the stiff side of the building. These expressions are also plotted in Figs. 2.4a and 2.4b. The code design eccentricity does not vary with Ω_R . For a low value of static eccentricity, $e/b = 0.1$, the code design eccentricity is significantly lower than the dynamic eccentricity for a range of values of Ω_R in the vicinity of 1.0. The code provisions therefore underestimate the torque in such cases.

The underestimation of torque does not however mean that the design eccentricity expressions given in the code are unconservative. The forces imposed on the resisting planes depend on both the base shear and dynamic torque. Maximum shear in a resisting element can only be obtained by combining modal values of element shear and not by adding the effects of resultant base shear and resultant torque. This is a very important point to note. It is clear from Figs. 2.3 and 2.4 that the dynamic torque amplification effect is offset by a corresponding base shear reduction. In the vicinity of $\Omega_R = 1$, $\bar{T}$ shows a maxima while $\bar{V}$ shows a minima. It can be shown for both flat as well as hyperbolic spectra (Kan and Chopra, 1976) that the resultant base shear and torque, obtained by modal superposition, are related as:

$$\bar{V}^2 + \bar{T}^2 = 1 \quad (2.15)$$

To stress this point further, the response of a single storey building model similar to the one shown in Fig. 2.1, with $e/b = 0.1$, $\Omega_R = 1$, $T_y = 1$, and aspect ratio = 1, to 1940 El Centro earthquake is examined. The time history of base shear normalised by maximum base shear in the corresponding torsionally balanced model, V_{0max} , and torque about the centre of resistance (CR) normalised by V_{0max} times the mass radius of gyration, are shown in Fig. 2.5. There are two observations to make: (1) there is a time lag between maximum base shear and maximum torque and the two maxima do not occur simultaneously, and (2) effects of base shear and torque are not necessarily additive. From the point of view of design, the quantity of most interest is the maximum shear in a resisting plane. The shear is produced by a combination of the lateral and torsional motions. Since an increased torsional response is accompanied by a reduced

lateral response, conditions leading to amplified torque do not necessarily lead to increase in the shear in the resisting planes.

In view of the foregoing discussions, the dynamic eccentricity, e_d , is of limited practical interest and we need to concentrate on the maximum shears produced in the resisting planes due to a combination of the lateral and torsional responses.

Displacements and effective eccentricities at edge frames

The maximum shears in resisting planes can be obtained by combining the modal values. In the elastic range, the lateral shears in the resisting elements are proportional to the displacements, therefore we may combine the modal displacements rather than the modal shears. The modal displacement of the flexible edge of the building, Δ_{fn} , is given by

$$\Delta_{fn} = \frac{V_n}{K_y} - \frac{T_{Rn}}{K_{\theta R}} \left(e + \frac{b}{2} \right) \quad (2.16)$$

It is useful to normalise Δ_{fn} by the displacement of the torsionally balanced structure, $\Delta_0 = V_0/K_y$ so that

$$\bar{\Delta}_{fn} = \frac{\Delta_{fn}}{\Delta_0} = \bar{V}_n - \frac{T_{Rn}}{V_0 r} \frac{K_y r}{K_{\theta R}} \left(e + \frac{b}{2} \right) \quad (2.17)$$

Substituting Eq. 2.12 and Eq. 2.13 in Eq. 2.17 and noting that $K_{\theta R}/K_y = \Omega_R^2 r^2$,

$$\bar{\Delta}_{fn} = \phi_{yn}^2 \frac{S_{an}}{S_{ay}} - \frac{1}{\Omega_R^2} \phi_{yn} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{e}{r} + \frac{b}{2r} \right) \frac{S_{an}}{S_{ay}} \quad (2.18)$$

The ratio b/r needed in Eq. 2.18 can be obtained from Eq. 2.14 provided the aspect ratio α is given.

The modal displacement at the stiff edge is given by

$$\Delta_{sn} = \frac{V_n}{K_y} + \frac{T_{Rn}}{K_{\theta R}} \left(\frac{b}{2} - e \right) \quad (2.19)$$

so that

$$\bar{\Delta}_{sn} = \frac{\Delta_{sn}}{\Delta_0} = \bar{V}_n + \frac{T_{Rn}}{V_0 r} \frac{K_y r}{K_{\theta R}} \left(\frac{b}{2} - e \right) \quad (2.20)$$

Substitution of Eq. 2.12 and 2.13 in Eq. 2.20 gives

$$\bar{\Delta}_{sn} = \phi_{yn}^2 \frac{S_{an}}{S_{ay}} + \frac{1}{\Omega_R^2} \phi_{yn} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{b}{2r} - \frac{e}{r} \right) \frac{S_{an}}{S_{ay}} \quad (2.21)$$

Modal superposition provides the resultant flexible edge displacement, $\bar{\Delta}_f$, and the stiff edge displacement, $\bar{\Delta}_s$.

We now define an effective eccentricity e_f as the distance from CR at which the application of base shear V_0 would produce a flexible edge displacement Δ_f , and eccentricity e_s as the distance from CR at which the application of V_0 would produce a stiff edge displacement of Δ_s . Thus

$$\Delta_f = \frac{V_0}{K_y} + \frac{V_0 e_f}{K_{\theta R}} \left(e + \frac{b}{2} \right) \quad (2.22)$$

or,

$$\bar{\Delta}_f = 1 + \frac{1}{\Omega_R^2} \frac{e_f}{b} \left(\frac{b}{r} \right)^2 \left(\frac{1}{2} + \frac{e}{b} \right) \quad (2.23)$$

Equation 2.23 gives,

$$\frac{e_f}{b} = (\bar{\Delta}_f - 1) \frac{\Omega_R^2}{\left(\frac{b}{r} \right)^2 \left(0.5 + \frac{e}{b} \right)} \quad (2.24)$$

In a similar manner, it can be shown that,

$$\frac{e_s}{b} = (1 - \bar{\Delta}_s) \frac{\Omega_R^2}{\left(\frac{b}{r} \right)^2 \left(0.5 - \frac{e}{b} \right)} \quad (2.25)$$

The flexible edge effective eccentricity given by Eq. 2.24 is plotted as a function of e/b for four different frequency ratios, $\Omega_R = 0.75, 1.0, 1.25$ and 1.50 in Figs. 2.6a and 2.6b, respectively for flat and hyperbolic response spectra. Also shown is a line representing the NBCC provision, $e_{d1} = 1.5e + 0.1b$. The NBCC expression is clearly quite conservative, particularly for large eccentricity values.

The stiff edge effective eccentricity given by Eq. 2.25 is plotted as a function of e/b for four different values of Ω_R in Figs. 2.7a and 2.7b, respectively for flat and hyperbolic response spectra. Also shown is the NBCC expression, $e_{d2} = 0.5e - 0.1b$. It should be noted that in this case an effective eccentricity value smaller than the code eccentricity implies that the code provision is unconservative. The design eccentricity specified by NBCC is observed to be satisfactory when $\Omega_R \geq 1.0$. For $\Omega = 0.75$, the design eccentricity is unconservative for a range of values of e/b .

The normalised flexible edge displacement $\bar{\Delta}_f$ obtained by modal superposition is plotted as a function of Ω_R in Figs. 2.8a and 2.8b for flat and hyperbolic spectra. In all cases, $\bar{\Delta}_f$ is greater than 1 implying that the flexible edge displacement in the torsionally unbalanced structure is greater than the displacement in the associated torsionally balanced structure. For Ω_R less than 1, there is a steep increase in the displacement.

Similarly the normalised stiff edge displacement $\bar{\Delta}_s$ obtained by modal superposition is plotted as a function of Ω_R in Figs. 2.9a and 2.9b for flat and hyperbolic spectra. The normalised displacement $\bar{\Delta}_s$ is less than 1 for $\Omega_R > 1$. For torsionally flexible systems, i.e. for $\Omega_R < 1$, the stiff edge displacement starts to increase, and for low torsional stiffness, $\bar{\Delta}_s$ can be substantially higher than 1, particularly for a flat spectrum.

Significance of the frequency ratio

The results presented in Figs. 2.8 and 2.9 clearly show that when the frequency ratio Ω_R is less than 1, torsional motion leads to very large displacements. There is a steep increase in the values of $\bar{\Delta}_f$ and $\bar{\Delta}_s$ when Ω_R falls below 1. A small value of Ω_R implies that the torsional stiffness of the structure is small and/or the lateral force resisting planes are located close to the geometric centre of the building. In practice, a design which gives low torsional stiffness should be avoided and efforts should be made to achieve a frequency ratio greater than 1, whenever possible.

For a frequency ratio close to 1, lateral and rotational motions are strongly coupled. At $\Omega_R = 1$, significant resonant torsional vibrations may be excited by translational ground motion. Torque is highly amplified. However, base shear is significantly reduced and the resultant lateral displacements in resisting planes are not necessarily high. In fact, there is no need to avoid a structural design in which Ω_R is close to 1.

2.5 Proposed new design procedure

As explained earlier, effective eccentricities related to the displacements produced at the edge elements, as presented in Figs. 2.6 and 2.7, can be used to assess the adequacy of the design provisions of NBCC. It was observed that code expression for flexible edge, given by Eq. 1.1, is very conservative. For the stiff edge, code provision given by Eq. 1.2 is conservative for $\Omega_R \geq 1$, but leads to unsafe design for $\Omega_R < 1$, for a range of values of e .

Apart from the inadequacies mentioned above, the NBCC design eccentricity expressions include a multiplier on the static eccentricity, which is 1.5 for the flexible edge and

0.5 for the stiff edge. This implies that static eccentricity e and hence the locations of CRs must be determined. This requirement is difficult to comply with all but the simplest of multistorey buildings. This is so, first because there is no clear definition of the centre of rigidity for multistorey structures (Tso, 1990), and second because even when a definition of CR is agreed upon, the determination of such centres requires complicated analytical procedures. Methods have been devised to implement the provisions of NBCC without explicitly determining the locations of CRs, but even these require many analysis steps (Goel and Chopra, 1993).

In view of the considerations outlined in the previous paragraphs, the following design expressions are proposed.

For the flexible side of CR

$$e_{d1} = e + 0.1b \quad (2.26)$$

For the stiff side of CR

$$e_{d2} = e - 0.1b \quad \Omega_R \geq 1 \quad (2.27)$$

$$e_{d2} = -0.1b \quad \Omega_R < 1 \quad (2.28)$$

The design eccentricity given by Eq. 2.26 is also shown in Figs. 2.6a and 2.6b for the purpose of comparison with the effective eccentricities obtained from a modal analysis. It is evident that the proposed expression is satisfactory for all values of e and Ω_R for both flat and hyperbolic spectra.

The design eccentricity given by Eqs. 2.27 and 2.28 are also shown in Figs. 2.7a and

2.7b for comparison with the effective eccentricities determined by modal analysis. It is evident that the proposed expressions provide safe design.

An expression for normalised flexible edge displacement, $\bar{\Delta}_{fc}$ corresponding to the design eccentricity given by Eq. 2.26 can be obtained as follows:

$$\begin{aligned}\Delta_{fc} &= \frac{V_0}{K_y} + \frac{V_0 e_{d1}}{K_{\theta R}} \left(e + \frac{b}{2} \right) \\ \bar{\Delta}_{fc} &= 1 + \frac{1}{\Omega_R^2} \left(\frac{e}{r} + 0.1 \frac{b}{r} \right) \left(\frac{e}{r} + \frac{b}{2r} \right)\end{aligned}\quad (2.29)$$

The expression for the normalised stiff edge displacement, $\bar{\Delta}_{sc}$ corresponding to the design eccentricity given by Eq. 2.27 is derived in a similar manner. Thus,

$$\begin{aligned}\Delta_{sc} &= \frac{V_0}{K_y} - \frac{V_0 e_{d2}}{K_{\theta R}} \left(\frac{b}{2} - e \right) \\ \bar{\Delta}_{sc} &= 1 - \frac{1}{\Omega_R^2} \left(\frac{e}{r} - 0.1 \frac{b}{r} \right) \left(\frac{b}{2r} - \frac{e}{r} \right)\end{aligned}\quad (2.30)$$

The expression corresponding to Eq. 2.28 is

$$\bar{\Delta}_{sc} = 1 + \frac{1}{\Omega_R^2} \left(0.1 \frac{b}{r} \right) \left(\frac{b}{2r} - \frac{e}{r} \right) \quad (2.31)$$

It should be noted that the stiff edge displacement produced by the application of shear V_0 at an eccentricity e_{d1} is given by

$$\bar{\Delta}_{sc} = 1 - \frac{1}{\Omega_R^2} \left(\frac{e}{r} + 0.1 \frac{b}{r} \right) \left(\frac{b}{2r} - \frac{e}{r} \right) \quad (2.32)$$

The second term in Eq. 2.32 represents the normalised displacement produced by a rotation of the building about CR. For a system that is torsionally very flexible, the rotational component of displacement may exceed the translational component; the second

term in Eq. 2.32 is then greater than 1. In such cases, the absolute value of displacement obtained from Eq. 2.32 may exceed that obtained from Eq. 2.30 or Eq. 2.31.

For $\Omega_R \geq 1$, the design stiff edge displacement is given by the larger of absolute values obtained from Eqs. 2.30 and 2.32. For $\Omega_R < 1$, design $\bar{\Delta}_{sc}$ is the larger of the absolute values obtained from Eqs. 2.31 and 2.32. This has been taken into account, in all further analyses.

2.6 Accidental torsion

Torsion caused by uncertainty in mass and stiffness

As stated earlier, accidental torsion in elastic structures is caused by uncertainty in the estimation of mass and stiffness. Inaccuracies in estimating the mass and stiffness affect the assessment of the response of both torsionally unbalanced and torsionally balanced structures. However, our primary interest is to calculate the response of torsionally unbalanced structure relative to the response of corresponding torsionally balanced structure. The difference between the two responses is dependent mainly on the distribution of stiffness and mass rather than on their absolute values. Therefore in assessing the accidental torsion, we will focus on the uncertainty in estimating the distribution of mass and stiffness.

A change in the distribution of mass and stiffness will affect both the static eccentricity and the frequency ratio Ω_R . The variability in the distribution of mass or stiffness is a random parameter and must be treated by statistical means. Recently, De La Llera and Chopra (1994) have carried out an exhaustive study of mass and stiffness uncertainties and their effect on response. They obtained the increase in edge displacements due to the various sources contributing to accidental torsion, including uncertainty in the dis-

tribution of mass and stiffness and ground rotational motion. They concluded that the dynamic response magnification obtained by increasing or decreasing the eccentricity by $0.05b$ provides a reasonable estimate of the contributions from all sources of accidental torsion.

The result of dynamic analyses presented so far did not include the effect of accidental torsion. In order to ascertain whether the design eccentricities given by Eqs. 2.26, 2.27 and 2.28 are adequate even when the effect of accidental torsion is included, dynamic analysis must be repeated with modified static eccentricities as noted in the preceding paragraph, and the revised values of effective eccentricities e_f and e_s so obtained compared with the proposed design values. This is easily achieved by shifting the response curves in Figs. 2.6a and 2.6b by a distance $-0.05e/b$ along the horizontal axis. It is evident from Figs. 2.6a and 2.6b that the effective eccentricity is highest at $\Omega_R = 1.25$. Therefore the revised response including the effect of accidental torsion for $\Omega_R = 1.25$ is presented in Figs. 2.10a and 2.10b, for flat and hyperbolic spectra respectively. It is observed that the design eccentricity given by Eq. 2.26 is still adequate.

Revised stiff edge eccentricities after including the effect of accidental torsion, are obtained by decreasing the eccentricity by $0.05b$. The new response curves are similar to those in Figs. 2.7a and 2.7b but shifted along the horizontal axis to the right by $0.05e/b$. For low values of Ω_R , the proposed eccentricity, $e = -0.1b$, is represented by a horizontal line in Figs. 2.7a and 2.7b. Thus, shifting the curves for $\Omega_R = 0.5$ and 0.75 along the horizontal axis will not alter their position relative to the line $e = -0.1b$. In other words, the proposed expression given by Eq. 2.28 will continue to be adequate even when accidental torsion is included. For higher values of Ω_R , the critical response curve

is the one corresponding to $\Omega_R = 1$. Therefore, the revised response including the effect of torsion is shown only for $\Omega_R = 1$ in Figs. 2.11a and 2.11b. It is observed that the expressions given by Eq. 2.27 are still satisfactory.

Torsion caused by ground rotation

As noted above, the additional eccentricities $\pm 0.05b$ are sufficient to account for all sources of accidental torsion including the torsional motion caused by ground rotation. In fact, the torsional motion due to ground rotation is quite small. This conclusion is based on studies by De La Llera and Chopra (1994) on measured response of several multistorey buildings. It is of interest to determine whether the magnitude of measured response can be explained by an analytical approach. Several previous studies have considered the effect of ground rotation on the response of single and multistorey buildings (Awad and Humar, 1984; Humar, 1984). However, the results of such studies need to be updated to take into account more recent knowledge.

The equations of motion for the single storey building of Fig. 2.1. responding to a ground rotational acceleration $\ddot{u}_{g\theta}$ are given by:

$$\begin{bmatrix} \ddot{u}_y \\ r\ddot{u}_\theta \end{bmatrix} + \omega_y^2 \begin{bmatrix} 1 & \frac{e}{r} \\ \frac{e}{r} & \Omega_R^2 + (\frac{e}{r})^2 \end{bmatrix} = - \begin{bmatrix} 0 \\ r\ddot{u}_{g\theta} \end{bmatrix} \quad (2.33)$$

Using modal superposition we obtain

$$\bar{V}_n^\theta = \frac{V_n^\theta}{V_0} = \phi_{yn} \phi_{\theta n} \frac{r S_{a\theta}(\omega_n, \zeta)}{S_{ay}(\omega_n, \zeta)} \quad (2.34)$$

$$\bar{T}_n^\theta = \frac{T_n^\theta}{V_0 r} = \phi_{\theta n}^2 \frac{r S_{a\theta}(\omega_n, \zeta)}{S_{ay}(\omega_n, \zeta)} \quad (2.35)$$

$$\bar{T}_{Rn}^\theta = \frac{T_{Rn}^\theta}{V_0 r} = \bar{T}_n^\theta - \frac{e}{r} \bar{V}_n^\theta$$

$$\begin{aligned}
&= \left(\phi_{\theta n}^2 - \frac{e}{r} \phi_{yn} \phi_{\theta n} \right) \frac{r S_{a\theta}(\omega_n, \zeta)}{S_{ay}(\omega_y, \zeta)} \\
&= \bar{e}_{dn}^{\theta}
\end{aligned} \tag{2.36}$$

in which V_n^{θ} is the shear in mode n , T_n^{θ} is the torque about the CM, and T_{Rn}^{θ} is the torque about CR, all produced by ground rotation, and $S_{a\theta}(\omega_n, \zeta)$ is the spectral rotational acceleration.

The normalised flexible and stiff edge displacements caused by ground rotation are obtained from expressions similar to those in Eqs. 2.17 and 2.20 respectively except that $\bar{V}_n$ is replaced by $\bar{V}_n^{\theta}$ and T_{Rn} by T_{Rn}^{θ} . The modal values of torsional response are then combined by using the CQC method according to Eqs. 2.9 and 2.10. The building structure is simultaneously subjected to two types of excitations, translational and rotational. Hence the resultant responses obtained for the two types of excitations must be further combined to obtain the final response. A number of studies have shown that the response obtained from two different components of the ground motion are uncorrelated or independent, so that a root sum square (RSS) method can be used to combine them. In one such study, Morgan et al (1983) obtained the response of a coupled system to the translational and derived rotational components of the San Fernando earthquake recorded at PE lot by using a RSS combination. They then compared the RSS responses with values obtained by a time-series analysis and found that the two sets of results were reasonably close. More recently, De La Llera and Chopra (1994) have shown, both analytically and by calculating the correlation coefficients for eight base translation and derived rotation records of the Loma Prieta earthquake, that the ground translational and rotational motions can be considered as being independent. Based on the results of the studies just cited, the RSS combination rule is used in deriving the

results presented here.

The analytical procedure for obtaining the torsional response outlined above requires a knowledge of the response spectrum of torsional ground motion. Measurements of ground rotational motion do not exist, hence ground motion spectra must be obtained in an indirect manner. Estimates of ground rotations have been obtained by assuming that earthquake motions result from travelling shear waves (Newmark, 1969; Scanlan, 1976; Tso and Hsu, 1978). Under the passage of a shear wave, different points under the building foundation are subjected to the same motion but with a phase lag. The phase lag causes a rotational motion of the ground. The ground rotation $u_{g\theta}$ is obtained by the theory of elasticity (Newmark, 1969),

$$u_{g\theta} = \frac{1}{2} \left(\frac{\partial u_{gx}}{\partial y} - \frac{\partial u_{gy}}{\partial x} \right) \quad (2.37)$$

u_{gx} and u_{gy} are the ground translation in the x and y directions, given in their functional form by

$$\begin{aligned} u_{gx} &= f(y - ct) \\ u_{gy} &= g(x - ct) \end{aligned} \quad (2.38)$$

and c is the horizontal shear wave velocity.

In his original work Newmark (1969) assumed that the bi-directional shear waves u_{gx} and u_{gy} were completely correlated and had similar properties so that $g = f$. With these assumptions the ground rotation is given by

$$\begin{aligned}
u_{g\theta} &\approx -\frac{\partial u_{gy}}{\partial x} \\
&= -f'(x - ct)
\end{aligned} \tag{2.39}$$

Later studies have shown that the assumption of complete correlation is far too conservative. The bi-directional waves can, in fact, be treated as independent. If, in addition, they are assumed to have similar properties and equal phase velocities, the expression for ground rotation, Eq. 2.39, should be multiplied by $1/\sqrt{2}$ (Newmark and Rosenblueth, 1971). Using this reasoning and noting that,

$$\dot{u}_{gy} = -cf'(x - ct) \tag{2.40}$$

the following expression is obtained for the ground rotation

$$u_{g\theta} = \frac{1}{\sqrt{2}} \frac{1}{c} \dot{u}_{gy} \tag{2.41}$$

The maximum ground rotational displacement ϕ_d , the maximum rotational velocity ϕ_v , and the maximum rotational acceleration ϕ_a are thus related to the maximum ground translational velocity d_v , the maximum translational acceleration d_a , and the maximum time derivative of acceleration $\dot{d}_a$ by the following expressions

$$\begin{aligned}
\phi_d &= \frac{1}{\sqrt{2}} \frac{d_v}{c} \\
\phi_v &= \frac{1}{\sqrt{2}} \frac{d_a}{c} \\
\phi_a &= \frac{1}{\sqrt{2}} \frac{\dot{d}_a}{c}
\end{aligned} \tag{2.42}$$

On a tripartite logarithmic graph, the response spectrum for translational motion can be approximated by a line of constant displacement D_d , a line of constant velocity D_v , and a line of constant acceleration D_a , where

$$\begin{aligned} D_d &= D d_d \\ D_v &= V d_v \\ D_a &= A d_a \end{aligned} \tag{2.43}$$

and D , V , and A are amplification factors whose values depend on the damping as well as the characteristics of the earthquake motion (Newmark *et al*, 1973).

Because of the relationship given by Eq. 2.43, it can be expected that the rotational response spectrum will also be approximately represented by three straight lines on a tripartite logarithmic graph. In the range of periods where the translational spectrum is represented by a line of constant acceleration (flat spectrum), the rotational spectrum would be represented by a line of constant rotational velocity, and in the range where the translational spectrum is represented by a line of constant velocity (hyperbolic spectrum), the rotational spectrum will be represented by a line of constant rotational displacement. On the torsional spectrum the rotational displacement bound θ_d , the velocity bound θ_v , and the acceleration bound θ_a are given by

$$\begin{aligned} \theta_d &= D' \phi_d \\ \theta_v &= V' \phi_v \\ \theta_a &= A' \phi_a \end{aligned}$$

(2.44)

For a flat translational spectrum along with its associated rotational spectrum,

$$\begin{aligned}\frac{S_{a\theta}(\omega_n, \zeta)}{S_{ay}(\omega_y, \zeta)} &= \frac{\omega_n V' \phi_v}{A d_a} \\ &= \frac{1}{\sqrt{2}} \frac{\omega_n V'}{A c}\end{aligned}\quad (2.45)$$

Substitution of Eq. 2.45 in Eqs. 2.34 and 2.36 yields,

$$\begin{aligned}\bar{V}_n^\theta &= \frac{1}{\sqrt{2}} \frac{V'}{A} \phi_{yn} \phi_{\theta n} \left(\frac{\omega_n}{\omega_y} \right) \omega_y \frac{r}{b} \tau \\ \bar{T}_{Rn}^\theta &= \frac{1}{\sqrt{2}} \frac{V'}{A} \left(\phi_{\theta n}^2 - \frac{e}{r} \phi_{yn} \phi_{\theta n} \right) \left(\frac{\omega_n}{\omega_y} \right) \omega_y \frac{r}{b} \tau\end{aligned}\quad (2.46)$$

where $\tau = b/c$ is the transit time, namely the time it takes a wave to traverse a distance b , the width of the building perpendicular to the earthquake motion.

In a similar manner, for a hyperbolic translational spectrum along with its associated rotational spectrum,

$$\begin{aligned}\bar{V}_n^\theta &= \frac{1}{\sqrt{2}} \frac{D'}{V} \phi_{yn} \phi_{\theta n} \left(\frac{\omega_n}{\omega_y} \right)^2 \omega_y \frac{r}{b} \tau \\ \bar{T}_{Rn}^\theta &= \frac{1}{\sqrt{2}} \frac{D'}{V} \left(\phi_{\theta n}^2 - \frac{e}{r} \phi_{yn} \phi_{\theta n} \right) \left(\frac{\omega_n}{\omega_y} \right)^2 \omega_y \frac{r}{b} \tau\end{aligned}\quad (2.47)$$

For a torsional spectrum associated with a flat translational spectrum the expression for normalised flexible edge displacement becomes

$$\bar{\Delta}_{fn}^\theta = \frac{1}{\sqrt{2}} \frac{V'}{A} \left(\frac{\omega_n}{\omega_y} \right) \omega_y \frac{r}{b} \tau \left[\phi_{yn} \phi_{\theta n} - \frac{1}{\Omega_R^2} \phi_{\theta n} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{e}{r} + \frac{b}{2r} \right) \right] \quad (2.48)$$

The edge displacement expression corresponding to a torsional spectrum associated with a hyperbolic translational spectrum is

$$\bar{\Delta}_{fn}^{\theta} = \frac{1}{\sqrt{2}} \frac{D'}{V} \left(\frac{\omega_n}{\omega_y} \right)^2 \omega_y \frac{r}{b} \tau \left[\phi_{yn} \phi_{\theta n} - \frac{1}{\Omega_R^2} \phi_{\theta n} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{e}{r} + \frac{b}{2r} \right) \right] \quad (2.49)$$

The normalised stiff edge displacements for a flat translational spectrum is given by

$$\bar{\Delta}_{sn}^{\theta} = \frac{1}{\sqrt{2}} \frac{V'}{A} \left(\frac{\omega_n}{\omega_y} \right) \omega_y \frac{r}{b} \tau \left[\phi_{yn} \phi_{\theta n} - \frac{1}{\Omega_R^2} \phi_{\theta n} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{b}{2r} - \frac{e}{r} \right) \right] \quad (2.50)$$

and for a hyperbolic translational spectrum

$$\bar{\Delta}_{sn}^{\theta} = \frac{1}{\sqrt{2}} \frac{D'}{V} \left(\frac{\omega_n}{\omega_y} \right)^2 \omega_y \frac{r}{b} \tau \left[\phi_{yn} \phi_{\theta n} - \frac{1}{\Omega_R^2} \phi_{\theta n} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{b}{2r} - \frac{e}{r} \right) \right] \quad (2.51)$$

Equations 2.46 through 2.51 show that the response to ground rotation increases with both the uncoupled translational frequency ω_y and the transit time τ . The transit time, in turn, depends on the apparent horizontal velocity of the propagation of shear wave at the surface. This apparent horizontal velocity is different from the true propagation velocity in the surface layers and is, in fact, dependent on the angle of incidence of shear waves. Assuming a horizontally layered soil overlying rock, O'Rourke et al (1982) derived the following relationship between the apparent horizontal propagation velocity c , the true shear wave velocity in the top soil layer c_s , and the angle of incidence of the shear wave γ_s

$$c = \frac{c_s}{\sin \gamma_s} \quad (2.52)$$

Obviously, for vertical incident waves ($\gamma_s = 0$), the apparent velocity c is infinite, $\tau = 0$ and rotational ground motion does not exist. O' Rourke et al applied their procedure to estimate c for 17 sites at which earthquake records were obtained during the 1971 San Fernando earthquake. The computed value of c ranged from 540 m/s to 3.3 km/s with a median value of 2.1 km/s. Similar calculations for 19 Imperial Valley earthquake recording sites gave a median apparent propagation velocity of 3.76 km/s. Measured values from instrument arrays in Japan are reported as ranging from 2.5 to 5.3 km/s (O' Rourke et al, 1982).

Assuming a building width of $b = 40$ m and a wave velocity of $c = 2$ km/s, the transit time τ works out to 0.02 sec. In the results that will be presented here, the damping in each mode is assumed to be 5% of critical. For this value of damping, the amplification factor for the translation spectrum can be taken as $D = 2$, $V = 2$, and $A = 3$ (Newmark et al, 1973). Because of the greater number of oscillations in the corresponding rotational components of the ground motion, slightly higher amplification factors are expected for torsional response spectrum. On the basis of values reported in Newmark's work (Newmark, 1969), the following amplification factors are used in this study $D' = 2.2$, $V' = 2.2$, and $A' = 3.3$.

To illustrate the procedure for deriving the rotational spectrum, consider the El-Centro earthquake motion with $d_d = 25.4$ cm, $d_v = 38.1$ cm/s and $d_a = 304.8$ cm/s². The translational spectrum is obtained from the ground motion bounds by using the amplification factors D , V , and A . The spectrum is shown in Fig. 2.12 where it has been normalised with response to the maximum ground velocity d_v .

In order to derive the complete torsional spectrum, we need the value of d_a . Newmark

(1969) has obtained a value of $d_{\dot{a}} = 12700 \text{ cm/s}^3$ for El-Centro. Using this value and Eqs. 2.42 and 2.44, the torsional response spectrum is derived. For comparison with the translational spectrum, the torsional spectrum is expressed in terms of the added motion at the edge of the building. Thus,

$$\begin{aligned}\theta_d \frac{b}{2} &= D' \phi_d \frac{b}{2} = \frac{1}{\sqrt{2}} D' \frac{d_v}{c} \frac{b}{2} \\ &= \frac{1}{2\sqrt{2}} D' d_v \tau\end{aligned}$$

In a similar manner,

$$\begin{aligned}\theta_v \frac{b}{2} &= \frac{1}{2\sqrt{2}} V' d_a \tau \\ \theta_a \frac{b}{2} &= \frac{1}{2\sqrt{2}} A' d_{\dot{a}} \tau\end{aligned}\tag{2.53}$$

The torsional spectrum given by Eq. 2.53 and with $\tau = 0.02 \text{ s}$, is also plotted in Fig. 2.12 after normalising by d_v . Newmark (1969) obtained the torsional spectra assuming fully correlated bi-directional waves and transit times of 0.1 and 0.05 s. Newmark's spectral values, which are from 3.5 to 7 times those derived here, are very conservative.

Recently, De La Llera and Chopra (1994) analysed the records obtained from 30 buildings with stiff foundation systems. Each building was instrumented with at least two accelerometers, so that both the translational and rotational components of acceleration histories could be obtained by simple geometric transformation. De La Llera and Chopra calculated the mean value of the torsional spectrum from these 30 records. The spectral values obtained by them were 1/2 to 1/7 of those obtained by Newmark. This shows that

the Newmark values are very conservative, and confirms the validity of the procedure described here for the derivation of torsional spectra.

Response of single storey building model to Ground Rotation

The single storey building model is analysed for its response to ground motions represented by a flat translational acceleration spectrum and its associated torsional response spectrum. In Fig. 2.12, this is the region between the periods of 0.1 and 0.5 s. The transit time is taken as 0.05 s. In light of the observations made in earlier sections of this report, this value is quite conservative and can be taken as an upper bound. In the analytical results presented here, the uncoupled translational frequency is taken as 3 cps corresponding to a period of 0.33 s. Equations 2.46 through 2.51 show that the torsional component of response increases in proportional to ω_y , the uncoupled translational frequency. A fairly high frequency has been used here to obtain substantial torsional response.

In Figs. 2.13a and b, horizontal flexible edge displacements are plotted as functions of the static eccentricity for two different values of the frequency ratio $\Omega_R = 1$ and 1.5. Both the translational and torsional components of the edge displacement are shown. Also shown is the resultant obtained by taking the root sum square of the two components. For purpose of comparison, the normalised edge displacement obtained from a design eccentricity $e_{d1} = e + 0.1b$ is also shown. It is evident that the torsional component increases the response only slightly and that the suggested design eccentricity is quite conservative even when the effect of ground rotation is taken into account.

Results for normalised stiff edge displacement are presented in Figs. 2.14a and 2.14b. Also shown there are curves corresponding to a design eccentricity of $e_{d2} = e - 0.1b$.

Again, it is found that ground rotation produces only a small increase in response and that the proposed design eccentricity gives safe results.

The normalised edge displacement can be used to calculate the effective eccentricity by using Eqs. 2.24 for flexible edge and Eq. 2.25 for stiff edge. These eccentricities are plotted in Figs. 2.15a and 2.15b along with the design eccentricity values given by Eqs. 2.26, 2.27 and 2.28. The proposed design provisions are conservative in each case. It should be noted that for the stiff edge, an eccentricity that is smaller than the one calculated analytically, leads to safer design.

2.7 Mass eccentric system

In the single storey buildings studied so far, the centre of mass was assumed to be located at the geometric centre of the building plan and the eccentricity was assumed to have been produced by non-uniform distribution of stiffness. This is the case for a majority of real buildings that possess torsional asymmetry. However, it is possible that the torsional asymmetry is produced wholly or partly by non-uniform distribution of mass. Buildings with this type of asymmetry are referred to as mass eccentric buildings.

Figures 2.16a and 2.16b, compare two buildings, one in which asymmetry results entirely from a non-uniform distribution of stiffness and the other in which asymmetry is caused both by a non-uniform distribution of mass and a non-uniform distribution of stiffness. The origin of co-ordinate axis, in each case, is at the centre of mass. In Fig. 2.16b the mass centre is offset a distance ηb from the geometric centre. The eccentricity, that is the distance from the centre of resistance to the centre of mass, is the same for the two buildings. Also, the other parameters that govern the response,

the uncoupled lateral frequency ω_y , the ratio of uncoupled translational and rotational frequencies Ω_R , and the radius of gyration about the centre of mass r , are assumed to be identical. It is evident from Eq. 2.5 that the responses of the two building models to a given earthquake motion, measured in terms of u_y and ru_θ are identical. However, the edge displacements are not the same. In the mass eccentric system, Fig. 2.16b, the flexible edge is at a distance of $b/2 + e - \eta b$ from the centre of resistance, which is smaller than $b/2 + e$, the comparable distance in the stiffness eccentric systems. The torsional displacement of the flexible edge in the mass eccentric system is therefore smaller than that in the stiffness eccentric system. For the flexible edge, the torsional and the lateral responses add to each other to produce the maximum response. Thus the maximum response of the flexible edge in the mass eccentric system will be less than that in the stiffness eccentric system.

The lever arm to the stiff edge in the mass eccentric system is $b/2 - e + \eta b$, which is greater than the comparable value $b/2 - e$ in the stiffness eccentric system. For the stiff edge, the translational and torsional displacements are generally in opposite direction when the maximum resultant displacement is achieved. For torsionally stiff systems the translational response will dominate; hence the net response in the mass eccentric system will be smaller than that in the stiffness eccentric system. On the other hand, for torsionally flexible systems, the torsional response will dominate and the net response in the mass eccentric system may be greater than that in the stiffness eccentric system.

The flexible edge displacement in the mass eccentric system is given by an equation similar to Eq. 2.17

$$\bar{\Delta}_{fn} = \frac{\Delta_{fn}}{\Delta_0} = \bar{V}_n - T_{Rn} \frac{K_y r}{K_{\theta R}} \left(\frac{b}{2} + e - \eta b \right) \quad (2.54)$$

On substituting for $\bar{V}_n$ and $\bar{T}_{Rn}$, Eq. 2.54 reduces to

$$\bar{\Delta}_{fn} = \phi_{yn}^2 \frac{S_{an}}{S_{ay}} - \frac{1}{\Omega_R^2} \phi_{yn} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{b}{2r} + \frac{e}{r} - \eta \frac{b}{r} \right) \frac{S_{an}}{S_{ay}} \quad (2.55)$$

In a similar manner, the stiff edge displacement is obtained from

$$\bar{\Delta}_{sn} = \frac{\Delta_{sn}}{\Delta_0} = \bar{V}_n + \frac{T_{Rn}}{V_0 r} \frac{K_y r}{K_{\theta R}} \left(\frac{b}{2} - e + \eta b \right) \quad (2.56)$$

or

$$\bar{\Delta}_{sn} = \phi_{yn}^2 \frac{S_{an}}{S_{ay}} + \frac{1}{\Omega_R^2} \phi_{yn} \left(\phi_{\theta n} - \frac{e}{r} \phi_{yn} \right) \left(\frac{b}{2r} - \frac{e}{r} + \eta \frac{b}{r} \right) \frac{S_{an}}{S_{ay}} \quad (2.57)$$

After calculating the normalised flexible and stiff edge displacements, the effective flexible and stiff edge eccentricities can be computed from Eqs. 2.24 and 2.25 respectively.

As stated earlier, the flexible edge displacement in a mass eccentric system is less critical than that in the corresponding stiffness eccentric system with identical properties. Attention is therefore focused on the stiff edge displacement. It is found that for the mass eccentric system, the stiff edge displacement increases significantly with a decrease in the aspect ratio. Results are therefore presented for an aspect ratio of 1/3. The effective eccentricity for the stiff edge of a mass eccentric system is plotted as a function of e/b for four different values of Ω_R , 0.75, 1.0, 1.1 and 1.25 and three different values of η , 0.1, 0.2, 0.3 in Fig. 2.17. The proposed design equations are also shown for the purpose of comparison. Again, it should be noted that a code eccentricity that is higher than the calculated eccentricity represents an unsafe condition.

The design provisions corresponding to $\Omega_R = 0.75$, namely Eqs. 2.26 and 2.28 are adequate in the case of $\eta = 0.1$, but may somewhat underestimate the stiff edge displacement for a range of values of e/b when $\eta \geq 0.2$. For $\Omega_R = 1.0$, the proposed provision i.e. Eq. 2.27 underestimates the stiff edge displacement for a range of e/b values, which happens at all the η values. For $\Omega_R = 1.25$, Eq. 2.27 is generally adequate.

It is evident that the stiff edge displacement becomes critical for mass eccentric systems particularly when the mass eccentricity is large and the aspect ratio is low. In practice such combinations are rare; when they do occur, it is recommended that the cut-off value of Ω_R in the proposed design expressions 2.27 and 2.28 should be increased to 1.25.

2.8 Response to recorded ground motions

The results presented in earlier sections of this report were based on ground motions represented by idealised flat and hyperbolic response spectra, shown in Fig. 2.2. The conclusions drawn earlier need to be verified for response to real ground motions. The building model shown in Fig. 2.1 is therefore analysed for its response to a set of recorded earthquake motions. The set comprises 15 earthquake records, details of which are given in Table 2.1. The records are selected from McMaster University's database (Naumoski *et al*, 1988). They have a peak ground acceleration to peak ground velocity (a/v) ratio in the intermediate range, which varies from 0.8 to 1.2. All of the selected records were recorded on rock or stiff soil sites.

An elastic response spectrum is obtained for each of the 15 records normalised by its peak acceleration value. The mean of the calculated response spectra is shown in

Table 2.1: Description and Peak Ground Motion Parameters for Earthquake Records

No	Earthquake	Date	Magn	Site	Epic Dist (km)	Comp	Max Acc A(g)	Max Vel V(m/s)	A/V	Soil Cond
1	Imperial Valley California	May 18 1940	6.6	El Centro	8	S00E	0.348	0.334	1.04	Stiff Soil
2	Kern County California	July 21 1952	7.6	Taft Lincoln School Tunnel	56	S69E	0.179	0.177	1.01	Rock
3	Kern County California	July 21 1952	7.6	Taft Lincoln School Tunnel	56	N21E	0.156	0.157	0.99	Rock
4	Borrego Mtn. California	Apr 8 1968	6.5	San Onofre SCE power plant	122	N57W	0.046	0.042	1.10	Stiff Soil
5	Borrego Mtn. California	Apr 8 1968	6.5	San Onofre SCE power plant	122	N33E	0.041	0.037	1.11	Stiff Soil
6	San Fernando California	Feb 9 1971	6.4	3838 Lankershim Blvd, LA	24	S90W	0.150	0.149	1.01	Rock
7	San Fernando California	Feb 9 1971	6.4	Hollywood Storage P.Lot, LA	35	N90E	0.211	0.211	1.00	Stiff Soil
8	San Fernando California	Feb 9 1971	6.4	3407 6th St. LA	39	N90E	0.165	0.166	0.99	Stiff Soil
9	San Fernando California	Feb 9 1971	6.4	Griffith Park Obsv, LA	31	S00W	0.180	0.205	0.88	Rock
10	San Fernando California	Feb 9 1971	6.4	234 Figueroa St., LA	41	N37E	0.199	0.167	1.19	Stiff Soil

Table 2.1: (contd.) Description and Peak Ground Motion Parameters for Earthquake Records

No	Earthquake	Date	Magn	Site	Epic Dist (km)	Comp	Max Acc A(g)	Max Vel V(m/s)	A/V	Soil Cond
11	Near east coast of Honshu Japan	Nov 16 1974	6.1	Kashima Harbour Works	38	N00E	0.070	0.072	0.97	Stiff Soil
12	Near south coast of Honshu Japan	Aug 2 1971	7.0	Kushiro Central Wharf	196	N90E	0.078	0.068	1.15	Stiff Soil
13	Monte Negro Yugoslavia	Apr 15 1979	7.0	Albatros Hotel Uicing	17	N00E	0.171	0.194	0.88	Rock
14	Mexico Earthquake	Sep 19 1985	8.1	El Suchil Guerrero Array	230	S00E	0.105	0.116	0.91	Rock
15	Mexico Earthquake	Sep 19 1985	8.1	La Villita Guerrero Array	44	N90E	0.123	0.105	1.17	Rock

Fig. 2.18.

Response analyses are carried out on a single storey building model, similar to the one shown in Fig. 2.1, with uncoupled translational period of 1 s and aspect ratio of 1, for six different values of the eccentricity ratio e/b , 0.05, 0.1, 0.15, 0.2, 0.25, 0.3 and five different values of Ω_R , 0.75, 1.0, 1.25, 1.5 and 2.0. In each case, response analysis is repeated for the corresponding torsionally balanced building. Considering that there are 15 earthquake records, the total number of analysis runs works out to be 450 pairs or 900 in all.

For each pair of analyses, the maximum flexible edge displacement ratio $\bar{\Delta}_f$ and the maximum stiff edge displacement ratio $\bar{\Delta}_s$ are evaluated. The mean values (for 15 earthquakes) of $\bar{\Delta}_f$ are plotted against e/b in Fig. 2.19a for different values of Ω_R . The mean values of $\bar{\Delta}_s$ are plotted in Fig. 2.19b, as a function of e/b for different Ω_R values. It is observed that in most cases, the normalised flexible edge displacement is greater than 1 while the normalised stiff edge displacement is less than 1. However, for $\Omega_R = 0.75$, $\bar{\Delta}_s$ may be greater than 1.

The values of effective eccentricities corresponding to the maximum flexible and stiff edge displacements are calculated from Eqs. 2.24 and 2.25. The mean effective flexible edge eccentricity is plotted as a function of e/b in Fig. 2.20a. The proposed design eccentricity, given by Eq. 2.26 is also shown there. It is evident that the proposed design expression provides a conservative estimate of the effective eccentricity for the entire range of values of Ω_R and e/b studied. The effect of accidental torsion can be accounted for by comparing the values of e_f/b with an adjusted Eq. 2.26, i.e. $e_{d1} = e + 0.05b$. This adjusted equation is also plotted in Fig. 2.20a and it shows that the proposed design

provisions are still satisfactory.

The mean effective stiff edge eccentricity is plotted as a function of e/b in Fig. 2.20b. The proposed design eccentricity, given by Eqs. 2.27 and 2.28 are also shown. Again the proposed design eccentricity provides a conservative estimate of the stiff edge displacement. The effect of accidental torsion can be accounted for by using an adjusted Eq. 2.27, i.e. $e_{d2} = e - 0.05b$. This curve is also shown in Fig. 2.20b. The design eccentricity expressions are adequate except for $\Omega_R = 1$ and a range of values of e/b . However, even there, the difference between the effective eccentricity as obtained from dynamic analysis and from Eq. 2.27 is quite small, and the proposed design expressions may still be considered satisfactory.

The results obtained here are consistent with the ones obtained for the idealised response spectra in previous sections.

2.9 Effect of aspect ratio on torsional response

In the previous sections analytical results have been presented to show the effect of several structural parameters governing the torsional response of a single storey building. An important parameter that is radius of gyration r , has not been studied. This parameter gives an idea of the distribution of mass in the structure. Two buildings with identical locations of centres of mass may behave significantly differently, if their mass distribution and hence the radii of gyration are different. Therefore we need to investigate the effect of the variation of r on torsional response. For a building in which the mass is uniformly distributed in plan, the aspect ratio controls the value of r , as shown by Eq. 2.14. In this section, we focus our attention on the effect of aspect ratio on torsional response.

Flexible edge

Using Eqs. 2.24 and 2.18, the effective flexible edge eccentricity, e_f/b , is calculated for a single storey building model similar to the one shown in Fig. 2.1 with $\Omega_R = 0.75$, a series of values of e/b ranging from 0.0 to 0.4, and four different values of aspect ratio 0.0, 0.33, 1.0 and 3.0, all for a flat spectrum. The plots of e_f/b against e/b are shown in Fig. 2.21a. To compare the results obtained with the proposed design expressions including accidental torsion, the plots in Fig. 2.21a have to be shifted to the left by 0.05 and then compared to the plot given by $e/b + 0.1$, or alternatively the plots in Fig. 2.21a can be compared with $e_{d1} = e + 0.05b$. The second option is used and a line showing $e/b + 0.05$ has been plotted in Fig. 2.21a. Similar plots for $\Omega_R = 1.0, 1.25$ and 1.50 are shown in Figs. 2.21b, 2.21c and 2.21d respectively.

Plots of e_f/b versus e/b for different values of Ω_R and a hyperbolic spectrum are shown in Figs. 2.22a through d.

A number of observations can be made from the results presented in Figs. 2.21 and 2.22. First, for low Ω_R values, a low aspect ratio a/b leads to high flexible edge eccentricity. A low aspect ratio is therefore critical in such cases. For high Ω_R values, a high a/b leads to a larger flexible edge eccentricity and hence is therefore critical. To identify the most critical case, e_f/b versus e/b graphs are plotted in Fig. 2.23 for a few low values of Ω_R and low a/b ($= 1/3$) and for a few high values of Ω_R with high a/b ($= 3$). It is evident that buildings with a combination of high Ω_R and high aspect ratio are most critical for flexible edge displacements.

Second, it is observed that the proposed design expressions are somewhat unsafe for the flexible edge in certain cases, specially for buildings with high Ω_R and high a/b , when

subjected to a flat spectrum. However, the differences between the design values and the analytical results are small and the proposed design expression given by Eq. 2.26 may still be considered satisfactory.

For hyperbolic spectrum the proposed design expression is satisfactory for all cases of Ω_R and all aspect ratios.

Stiff edge

Using Eqs. 2.21 and 2.25, effective stiff edge eccentricity e_s/b , is calculated for a single storey building as shown in Fig. 2.1 with $\Omega_R = 0.75$, a range of eccentricity values and four different aspect ratios, for a flat spectrum. Eccentricity e_s/b is plotted as a function of e/b in Fig. 2.24a. Figures 2.24b, 2.24c and 2.24d show similar plots for $\Omega_R = 1.0$, 1.25 and 1.50 respectively. These plots need to be compared with the proposed design expressions, Eqs. 2.27 and 2.28. However, to take the accidental torsion into account, plots in the Fig. 2.24a have to be shifted to the right by $0.05e/b$. Alternatively, plots corresponding to $e_{d2} = e - 0.05b$ instead of Eq. 2.27 may be shown. This has been done in Figs. 2.24 a through d. Plot of Eq. 2.28 remains unaffected. Figures 2.25a through 2.25d are created in a similar manner for a hyperbolic spectrum.

It should be noted that a lower e_s/b value indicates higher stiff edge displacement and is therefore critical. For flat as well as hyperbolic spectrum, it is evident that low a/b values are critical for the entire range of Ω_R values. It should also be noted that effective eccentricity values are very close for aspect ratios of 0.0 and 0.33. Therefore an aspect ratio of 1/3 is a good representative of low a/b cases. For this reason, during this study, whenever the torsional response of elements at the stiff edge becomes critical, a building model with $a/b = 1/3$ is selected and its response is investigated.

Figures 2.24 and 2.25 show that Eq. 2.27 is unsafe for $\Omega_R = 1.0$ and low aspect ratio. It is therefore recommended that the proposed design expressions, Eqs. 2.27 and 2.28 be modified as follows:

for $a/b < 0.5$,

$$e_{d2} = e - 0.1b \quad \Omega_R \geq 1.25$$

$$e_{d2} = -0.1b \quad \Omega_R < 1.25$$

Response to recorded motion

To verify the results obtained from idealised response spectra, effect of aspect ratio on torsional response is investigated using a set of 15 earthquake records as described in Table 2.1. Response analyses are carried out on building models similar to the one shown in Fig. 2.1 with three values of aspect ratios, 0.33, 1.0 and 3.0, four values of Ω_R , 0.75, 1.0, 1.25, 1.50 and a range of static eccentricity values. Only physically possible models are selected. Thus a combination of e , Ω_R and a/b that requires one or more lateral resisting planes to have a negative value of stiffness, has been excluded. For every building model, corresponding torsionally balanced model (obtained by moving CM to CR) is also analysed for the selected earthquake motions.

For each pair of analysis, Δ_f/Δ_0 and Δ_s/Δ_0 are evaluated. A mean of each of these terms is calculated for 15 earthquakes. Finally, using Eqs. 2.24 and 2.25, effective eccentricities e_f and e_s are computed. Flexible edge eccentricity is plotted as a function of e/b for four different Ω_R values in Figs. 2.26a through 2.26d. Similarly, stiff edge eccentricity is plotted in Figs. 2.27a through 2.27d. Proposed design expressions, adjusted for accidental torsion i.e. $e_{d1} = e + 0.05$, $e_{d2} = e - 0.05b$ and $e_{d2} = -0.1b$ are also plotted.

Figures 2.26 and 2.27 are very similar to the corresponding plots obtained for idealised spectra. Therefore, conclusions made for flat and hyperbolic spectra hold good.

2.10 Summary and conclusions

The torsional response induced in asymmetric buildings by earthquake ground motion is an important consideration in the design of such buildings. Analytical results are presented here for the elastic response of single storey building models with non-coincident centres of mass and resistance. Important parameters that govern the torsional response are identified and modified design recommendations are presented. The following important conclusions can be drawn from the work presented here:

1. The torsional response of a single storey asymmetrical building to a specified ground motion is governed by ω_y , Ω_R and e/r .
2. The frequency ratio Ω_R has an important effect on response. For $\Omega_R = 1$, the ratio of dynamic torque to static torque is highly amplified. This fact has provided the basis for a traditional recommendation that designs with $\Omega_R = 1$ should be avoided. However, the displacements and design forces in resisting planes depend on a combination of shear and torque. While torque is amplified, base shear is reduced on account of torsional coupling. Therefore a high dynamic torque amplification does not mean that the design forces are also amplified. Analytical results show that a frequency ratio equal to 1 is not critical and there is no reason to avoid designs with $\Omega_R = 1$.
3. A low value of Ω_R say below 0.8, may lead to a very large increase in displacements of both flexible and stiff edges. Special design considerations may apply in the

design of structures with low values of Ω_R . It is therefore a good design practice to obtain an estimate of Ω_R .

4. The design eccentricity specified by NBCC for determining the strengths of planes on the flexible side of CR is very conservative. The design eccentricity specified by NBCC for stiff edge is conservative for higher values of Ω_R but inadequate for lower values.
5. A new set of design eccentricity expressions is suggested as a replacement for NBCC provisions. Analytical studies on the torsional behaviour of single storey models for a full range of governing parameters, show that the suggested provisions, given by Eqs. 2.26, 2.27 and 2.28 provide a good basis for design. These expressions are simpler to use as there is no multiplier on static eccentricity e which means that the CRs need not be determined explicitly. However, their use does require the evaluation of Ω_R which is an extra step. It should be noted that Ω_R has a significant effect on the torsional response and a good design must avoid low values of Ω_R . A requirement to determine Ω_R will thus promote good design practice.
6. Accidental torsion is always present in buildings. It results from uncertainty in the distribution of mass and stiffness as well as rotational component of ground motion. Previous studies have shown that effect of accidental torsion can be estimated from a pair of dynamic analyses in which the static eccentricity is increased or decreased by $0.05b$. Using such an estimate of accidental torsion, it is shown that the proposed design eccentricity expressions are adequate even when the effect of the accidental torsion is included.

7. For buildings with low aspect ratio, ($a/b < 0.5$), the proposed design expressions for stiff edge may be unsafe. It is recommended that for such cases, the cut-off value of Ω_R as used in Eqs. 2.27 and 2.28 be increased to 1.25.
8. Buildings in which the eccentricity arises partially or fully because of the mass centre being off-set from geometric centre of the building, are called mass eccentric buildings. For such buildings, the stiff edge response becomes critical. For high mass eccentricity and low aspect ratio it is recommended that the value of Ω_R , used in Eqs. 2.27 and 2.28 be increased to 1.25.
9. The conclusions presented above are based on analytical response studies of single storey building models for an earthquake represented by an idealised response spectrum, flat or hyperbolic. The validity of conclusions are checked by subjecting the same building models to a set of 15 ground motion records. The mean values of response parameters like relative displacements and effective edge eccentricities are then investigated and it is found that all of the above conclusions hold good.

Figure 2.2: Spectral Shapes (a) flat spectrum (b) hyperbolic spectrum

(a)

(b)

Figure 2.3: Variation of normalised shear with frequency ratio, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

(a)

(b)

Figure 2.4: Variation of eccentricity ratio with frequency ratio, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.5: Time history of normalised dynamic shear and dynamic torque

(a)

(b)

Figure 2.6: Effective flexible edge eccentricity, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.7: Effective stiff edge eccentricity, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.8: Normalised flexible edge displacement, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.9: Normalised stiff edge displacement, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.10: Effective flexible edge eccentricity, including accidental torsion, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.11: Effective stiff edge eccentricity, including accidental torsion, aspect ratio = 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.12: Translational and torsional response spectra for El-Centro earthquake

Figure 2.13: Flexible edge displacement due to translational and rotational motions of ground, flat spectrum, $\omega_y = 3$ cps, $\tau = 0.05$, (a) $\Omega_R = 1.0$ (b) $\Omega_R = 1.5$

Figure 2.14: Stiff edge displacement due to translational and rotational motions of ground, flat spectrum, $\omega_y = 3$ cps, $\tau = 0.05$, (a) $\Omega_R = 1.0$ (b) $\Omega_R = 1.5$

(a)

(b)

Figure 2.15: Effective eccentricities due to combined effect of translational and rotational ground motions, flat spectrum, $\omega_y = 3$ cps, $\tau = 0.05$, (a) $\Omega_R = 1.0$ (b) $\Omega_R = 1.5$

Figure 2.16: Plan view of (a) stiffness eccentric building (b) mass and stiffness eccentric building

Figure 2.17: Effective stiffness eccentricity, aspect ratio = $1/3$, flat spectrum, (a) $\eta = 0.1$ (b) $\eta = 0.2$

Figure 2.17: Effective stiffness eccentricity, aspect ratio = 1/3, flat spectrum, (c) $\eta = 0.3$

Figure 2.18: Average response spectrum for a set of 15 earthquake records

(a)

(b)

Figure 2.19: Average normalised displacements for recorded ground motions, aspect ratio = 1, $T = 1$ s., (a) flexible edge (b) stiff edge

(a)

(b)

Figure 2.20: Average effective eccentricity for recorded ground motions, aspect ratio = 1, $T = 1$ s., (a) flexible edge (b) stiff edge

Figure 2.21: Effective flexible edge eccentricity, flat spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 2.21: Effective flexible edge eccentricity, flat spectrum, (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

Figure 2.22: Effective flexible edge eccentricity, hyperbolic spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 2.22: Effective flexible edge eccentricity, hyperbolic spectrum, (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

Figure 2.23: Effective flexible edge eccentricity, (a) flat spectrum (b) hyperbolic spectrum

Figure 2.24: Effective stiff edge eccentricity, flat spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 2.24: Effective stiff edge eccentricity, flat spectrum, (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

Figure 2.25: Effective stiff edge eccentricity, hyperbolic spectrum, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 2.25: Effective stiff edge eccentricity, hyperbolic spectrum, (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

Figure 2.26: Average flexible edge eccentricity for a set of 15 recorded ground motions
(a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 2.26: Average flexible edge eccentricity for a set of 15 recorded ground motions
(c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

(a)

(b)

Figure 2.27: Average stiff edge eccentricity for a set of 15 recorded ground motions (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 2.27: Average stiff edge eccentricity for a set of 15 recorded ground motions (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

Chapter 3

Elastic analysis of torsionally unbalanced multistorey building models

3.1 General

The torsional response of a general multistorey building is quite complex. However, it is of interest to note that the response of a *special class* of building, in which centres of mass and centres of stiffness at different floors lie on two vertical lines, can be obtained by analysing an associated torsionally coupled single storey building and a torsionally uncoupled (balanced) multistorey building. As a consequence, the results (effects of torsional coupling) obtained for a single storey building apply also to a multistorey building of such a special class (Kan and Chopra, 1976, De La Llera and Chopra, 1994). It should be noted that for such a building, static eccentricity is the same at all floors and the stiffness matrices of the resisting planes bear a constant ratio to each other. Unfortunately, buildings of this type seldom exist in real life. The chances are that the static eccentricity as well as the torsional stiffness will vary across the floors. It is of interest therefore, to figure out the extent to which the results of single storey models

can be used for a *general* multistorey building.

Estimating frequency ratio by Rayleigh method

The proposed expressions for design eccentricities, Eqs. 2.26, 2.27 and 2.28 do not have a multiplier on the static eccentricity e , which means that the location of CRs need not be explicitly determined. However, use of the proposed design expressions requires the estimation of frequency ratio Ω_R , especially for Eqs. 2.27 and 2.28. For a single storey building this is straightforward. For a multistorey building, the Rayleigh method may be used (De La Llera and Chopra, 1994). To obtain the uncoupled lateral frequency, the building is subjected to a set of floor level forces F_i , which may be taken proportional to the equivalent static forces given by the code. A static analysis for these forces, with the floor rotations restrained, provides the floor displacements δ_i . The building is now subjected to floor torques T_i that are proportional to forces F_i . A static analysis for these torques provides the floor rotations θ_i . The rotations take place about the centres of resistance. The following expressions can now be used to provide good estimates of the uncoupled translational and rotational frequencies:

$$\begin{aligned}\omega_y^2 &= \frac{\sum F_i \delta_i}{\sum m_i \delta_i^2} \\ \omega_\theta^2 &= \frac{\sum T_i \theta_i}{\sum I_{pi} \theta_i^2}\end{aligned}\tag{3.1}$$

where I_{pi} is the polar moment of inertia of the i th floor about its centre of mass. Because rotations θ_i are about the centres of resistance, definition of ω_θ as in Eq. 3.1 is identical to the definition of ω_θ for a single storey building given in Section 2.3.

3.2 Examples of multistorey buildings

Issues involved

Our primary interest is in investigating whether the results obtained from studies on a single storey building model are applicable to a multistorey building of general class in which eccentricity and torsional stiffness vary from floor to floor. For this purpose, analytical studies are carried out on a number of multistorey building models in which eccentricity e and frequency ratio Ω_R are defined from floor to floor. Before presenting the results of the analyses of these models, it is useful to identify the main issues. They are listed below:

1. Is the estimate of Ω_R obtained from the Rayleigh analysis reasonably accurate and does it help in selecting an equivalent single storey building model whose response results can be used to predict the response of the multistorey building model?
2. How well does the edge displacements of the equivalent single storey building predict the interstorey edge displacements of the multistorey building when subjected to the same earthquake motion?
3. Are the proposed design eccentricity expressions, derived for a single storey building, namely Eqs. 2.26, 2.27, 2.28, apply also to a general multistorey building which does not belong the *specialclass* described earlier?

Analysis steps

The set of multistorey building models presented here are investigated keeping the above three issues in mind. The steps involved in the analysis are summarised below:

1. The multistorey building is first analysed for an earthquake motion represented by an idealised spectrum with floor rotations restrained. This provides the maximum value of storey shears from which corresponding floor forces F_i can be obtained. These dynamic floor forces are then applied statically to the same building and storey displacements δ_i are obtained. Now the building is unlocked i.e. floors are allowed to rotate and floor torques equal to $F_i b$ are applied statically to the building. The floor rotations θ_i are obtained. Using the Rayleigh method, Eqs. 3.1, ω_y, ω_θ and thus $\Omega_R = \omega_\theta / \omega_y$ are calculated. Based on this Ω_R , an equivalent single storey model is selected.
2. To assess the effects of torsion, the torsionally coupled multistorey building and a corresponding torsionally balanced building (identical lateral stiffness, mass and Ω_R but CM moved to CR) are subjected to an earthquake motion represented by an idealised spectrum. Shear forces at the flexible and stiff edges in torsionally unbalanced and balanced buildings are obtained. Since the buildings are assumed to be linear elastic, the displacements in the edge frames are proportional to the shears in them. Therefore, the ratio of shear at the flexible edge in unbalanced building to that in a balanced building is equal to the ratio of unbalanced to balanced flexible edge interstorey displacement $\bar{\Delta}_f$. A similar reasoning applies to the stiff edge displacement $\bar{\Delta}_s$. The displacement values obtained for the multistorey building are compared to the corresponding values for an equivalent single storey building.
3. The applicability of Eqs. 2.26, 2.27 and 2.28, to a multistorey building is examined next. As discussed earlier, a portion of design eccentricity equal to $0.05b$ in Eq. 2.26 and $-0.05b$ in Eq. 2.27 can be assumed to account for accidental torsion. The

portion of design eccentricity that accounts for natural torsion thus works out to be $e + 0.05b$ and $e - 0.05b$ respectively for flexible and stiff edges of the building. These design eccentricities are used in a set of static analyses. The dynamic floor forces obtained in analysis step (1) above are applied through points located $0.05b$ to the left (towards the flexible edge) of CM. Shears in the flexible edge frames are recorded. Now the same set of dynamic floor forces are applied through points located either $0.05b$ to the right of CM ($\Omega_R \geq 1$) or at $0.1b$ to the right of CR ($\Omega_R < 1$), depending upon the Ω_R value for that storey. Shears at stiff edges are recorded. Finally floor forces are applied to the corresponding torsionally balanced building and shears at flexible and stiff edge frames are obtained. As discussed earlier, the ratios of unbalanced to balanced shears provide the design interstorey displacement ratios $\bar{\Delta}_f$ and $\bar{\Delta}_s$. Finally the interstorey displacement ratios from dynamic analysis, i.e. step (2) and static analysis, step (3) are compared.

Computer program SUPER-ETABS (Maison and Nuess, 1983) is used to carry out the static analyses described in steps (1) and (3). The same program is used for the response spectrum analyses, described in step (2). The complete quadratic combination (CQC) technique is used in the modal superposition.

The calculation of static design value $\bar{\Delta}_s$, in step (3), for cases in which Ω_R is less than 1, requires some explanation. In these cases, according to Eq. 2.28, the floor forces should be applied at a distance of $0.1b$ to the right of CR. However this can not be done directly because the locations of centres of resistance are not known. An indirect method is therefore used. It requires three sets of analyses: (a) a 3D analysis with the floor forces applied at $0.05b$ to the left of the centres of mass, (b) a 3D analysis with

the floor forces applied at $0.05b$ to the right of the centres of mass, (c) an analysis with floor forces applied through any point on the floor but with floor rotations restrained. Analysis types (a) and (b) are similar to those required for other values of Ω_R and type (c) is identical to the analysis of corresponding balanced building. Thus, no additional analysis is involved in these steps. Let the shear in any storey of the stiff edge frame obtained from the three sets of analyses be denoted as r_a, r_b and r_c . Let the design shear, due to application of floor forces at $0.1b$ to the right of CR, be denoted as r_t , then:

$$r_t = r_c + r_b - r_a \quad (3.2)$$

Example 1

Building description

The building selected for this example is a five storey, shear building as shown in Fig. 3.1. The building has three planes of different stiffness parallel to the y axis and one plane along the x axis. The building is symmetrical about x axis but is asymmetrical about the y axis. For the planes parallel to y axis, the centre of resistance is offset from the centre of mass towards the right and is depicted as CR in Fig. 3.1. Mass of each floor is taken as 400 t , the mass moment of inertia as $86,400 \text{ t} - \text{m}^2$, the floor width b as 36 m and aspect ratio as 1. The height of each storey is taken as 4 m .

For shear type of building, the total lateral stiffness k_y , the stiffness eccentricity e , and the uncoupled torsional to lateral frequency ratio Ω_R can be defined independently for each storey. In the building being considered, the total lateral stiffness of each storey has the same value and this value is so adjusted that the first uncoupled translational period of the building is 1 s .

This building has been selected to demonstrate the effect of eccentricity variation along the height of the building. The stiffness eccentricity e is $0.2b$ for the 1st and 2nd storeys and $0.1b$ for 3rd, 4th and 5th storeys. The frequency ratio Ω_R is constant throughout the height of the building. Five different buildings with frequency ratios of 0.75, 1.0, 1.25, 1.50 and 2.0 are considered.

Details of analysis

Two idealised spectra, one flat with spectral acceleration of $1.0g$, and one hyperbolic, $S_a/g = 1.0/T$, are selected to represent the earthquake motion.

To show the calculations involved in the Rayleigh's method, the results for a building with $\Omega_R = 1$ are presented. Floor forces F_i obtained from a dynamic analysis for flat spectrum, are shown in Table 3.1. Also shown are the storey displacements δ_i produced by a static application of floor forces F_i , with floor rotations restrained, and the rotations θ_i obtained by a static application of torques $T_i = F_i b$.

Table 3.1: Calculation of natural frequencies using the Rayleigh's method

Floor	Max storey shear (kN)	Floor force F_i (kN)	δ_i	θ_i	NBCC floor force
5	5137.9	5137.9	0.3140	0.0523	5788.2
4	9597.6	4459.7	0.2878	0.0479	4630.6
3	13220.8	3623.2	0.2384	0.0397	3472.8
2	15876.2	2655.4	0.1705	0.0284	2315.2
1	17364.4	1488.2	0.0891	0.0184	1157.6

Equations 3.1 give $\omega_y = \omega_\theta = 6.283$. Thus $\Omega_R = 1$ is obtained which is exactly

equal to the selected value of Ω_R for each storey of the building. A similar conclusion is reached for other values of Ω_R as well as for a hyperbolic spectrum. Also shown in the Table 3.1, are the floor forces F_i obtained from the NBCC specified distribution of base shear. Static analyses are performed for floor forces F_i and floor torques $F_i b$, and Eqs. 3.1 are used to calculate the uncoupled frequencies ω_y and ω_θ . It is observed that the value of Ω_R so obtained is the same as that obtained from dynamic floor forces.

Dynamic analysis of the building models are now carried out for ground motions represented by flat and hyperbolic spectra. A complete quadratic combination (CQC) technique is used in modal superposition. Maximum interstorey displacement ratio $\bar{\Delta}_f$ is plotted as a function of Ω_R in Figs. 3.2a and 3.2b for flat and hyperbolic spectra respectively. Figures 3.3a and 3.3b show the variation of $\bar{\Delta}_s$ with Ω_R . In the above figures, also plotted are the curves (solid lines) for two corresponding single storey buildings. It is observed that the variation of interstorey displacement ratio in each storey of a multistorey building closely matches that in a single storey building with identical values of K_y , e and Ω_R , specially for a flat spectrum.

The building models are now analysed for static forces acting at $\pm 0.05b$ from CM, according to the procedures described earlier. The interstorey displacement ratios $\bar{\Delta}_f$ and $\bar{\Delta}_s$ so obtained are compared with the corresponding values found from dynamic analysis. As a representative set of results, Table 3.2 shows shears in edge frames for a building with $\Omega_R = 1$, obtained from the dynamic analysis for a flat spectrum. Table 3.3 shows shears in edge frames obtained from the static analysis, for the same building.

Figures 3.4a and 3.4b show the variation of $\bar{\Delta}_f$ with Ω_R for a flat spectrum, as obtained from dynamic and static analyses. The two sets of values are quite close.

Table 3.2: Shear in edge frames, as obtained from dynamic analysis for a flat spectrum

Floor	Shear in torsionally balanced building (kN)		Shear in torsionally unbalanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge	stiff edge	flexible edge	stiff edge		
5	650.8	1164.6	981.4	929.1	1.510	0.798
4	1215.7	2175.4	1852.9	1714.2	1.524	0.788
3	1674.6	2996.7	2585.6	2328.1	1.544	0.777
2	1693.5	4868.7	3337.4	2936.1	1.982	0.603
1	1852.2	5325.1	3690.0	3194.9	1.992	0.600

Table 3.3: Shear in edge frames obtained from static analysis

Floor	Shear in torsionally balanced building (kN)		Shear in torsionally unbalanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$		
5	650.8	1164.6	1001.7	1024.3	1.539	0.880
4	1215.7	2175.4	1871.6	1913.9	1.540	0.880
3	1674.6	2996.7	2578.9	2637.1	1.540	0.880
2	1693.5	4868.7	3471.0	3553.5	2.050	0.730
1	1852.2	5325.1	3797.0	3887.3	2.050	0.730

Figures 3.4c and 3.4d show the variation of $\bar{\Delta}_s$ with Ω_R for dynamic and static analyses, for a flat spectrum. Similar plots are shown in Figs. 3.5a through d for a hyperbolic spectrum.

It is evident from Figs. 3.4 and 3.5, that the proposed design provisions for single storey buildings, i.e. Eqs. 2.26, 2.27 and 2.28, provide a conservative estimate of edge displacements due to torsion, in the multistorey buildings studied.

Example 2

Building description

The buildings used in this example are similar to those used in the previous example except that the stiffness eccentricity distribution along the height of the building is different. Eccentricity e for 1st, 2nd and 3rd floor is $0.1b$ while for 4th and 5th floor, the eccentricity is $0.2b$. The frequency ratio Ω_R is the same for each storey. Five different buildings with $\Omega_R = 0.75, 1.0, 1.25, 1.50$ and 2.0 are considered.

Details of analysis

The earthquake motion is represented by flat and hyperbolic spectra, as in the previous example.

The Rayleigh method is used to find Ω_R of the building using the procedure described earlier. To show the calculations involved, the results for a building with $\Omega_R = 1$, with floor forces F_i , obtained from the dynamic analysis for a flat spectrum, are shown in Table 3.4.

Equations 3.1 give $\omega_y = 6.283$ and $\omega_\theta = 6.281$. The frequency ratio Ω_R thus works out to be equal to 1, which is same as the selected value for each storey. A similar conclusion is reached for other values of Ω_R as well as for a hyperbolic spectrum.

Table 3.4: Calculation of natural frequencies using Rayleigh's method

Floor	Floor force F_i (kN)	δ_i	θ_i
5	5137.9	0.3140	0.0524
4	4459.7	0.2876	0.0480
3	3623.2	0.2384	0.0397
2	2655.4	0.1705	0.0285
1	1488.2	0.0891	0.0148

A dynamic response spectrum analysis is carried out on the building models selected, for flat and hyperbolic spectra. Interstorey displacement ratio $\bar{\Delta}_f$ is plotted as a function of Ω_R in Figs. 3.6a and 3.6b for flat and hyperbolic spectra respectively. Figures. 3.7a and 3.7b show the variation of $\bar{\Delta}_s$ with Ω_R . Also plotted in these figures, are the curves for two corresponding single storey buildings. The results for single and multistorey buildings are again quite close.

The building models are next analysed for static forces acting at $\pm 0.05b$ from CM. Displacement ratios $\bar{\Delta}_f$ and $\bar{\Delta}_s$ so obtained are compared with the corresponding values found from dynamic analysis. As a representative set of results, Table 3.5 shows shears in edge frames for a building with $\Omega_R = 1$, obtained from the dynamic analysis for a flat spectrum. Table 3.6 shows shears in edge frames obtained from the static analysis, for the same building.

Figures 3.8a and 3.8b show the variation of $\bar{\Delta}_f$ with Ω_R for a flat spectrum, as obtained from dynamic and static analyses. The two set of results are quite close. Figures 3.8c and 3.8d show the variation of $\bar{\Delta}_s$ with Ω_R for dynamic and static analyses, for a flat spectrum. Figures 3.9a through d show similar plots for a hyperbolic spectrum.

It is evident from Figs. 3.8 and 3.9, that the proposed design provisions for single

Table 3.5: Shear in edge frames, as obtained from dynamic analysis for a flat spectrum

Floor	Shear in torsionally unbalanced building (kN)		Shear in torsionally balanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge	stiff edge	flexible edge	stiff edge		
5	1073.356	972.515	548.046	1575.615	1.959	0.617
4	1970.571	842.192	023.749	2943.248	1.925	0.626
3	2495.372	407.383	674.642	2996.709	1.490	0.803
2	2977.508	906.401	010.993	3598.597	1.481	0.808
1	3249.162	184.165	199.496	3935.918	1.477	0.809

Table 3.6: Shear in edge frames obtained from static analysis

Floor	Shear in torsionally unbalanced building (kN)		Shear in torsionally balanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$		
5	1123.486	150.203	548.046	1575.615	2.050	0.730
4	2098.672	148.579	023.749	2943.249	2.050	0.730
3	2578.940	637.111	674.642	2996.709	1.540	0.880
2	3096.918	166.773	010.992	3598.596	1.540	0.880
1	3387.213	463.615	199.496	3935.917	1.540	0.880

storey buildings, i.e. Eqs. 2.26, 2.27 and 2.28, provide a conservative estimate of edge displacements due to torsion in the multistorey buildings selected in this example set. Also obvious is the fact that an alternative distribution of eccentricity across the floors doesn't make much difference in the pattern of torsional response.

Example 3

Building description

The building selected for this example is a ten storey shear building as shown in Fig. 3.10. The building has two planes of equal stiffness parallel to the x axis and three planes of different stiffness parallel to the y axis. The building is symmetrical about x axis but asymmetrical about the y axis. Centre of resistance is offset from the centre of mass towards the right, as shown in the Fig. 3.10. Mass is uniformly distributed at the floor level and is taken as 400 t for each storey. Floor width b is taken as 36 m , aspect ratio as 1 and the height of each storey as 4 m . Total lateral stiffness K_y has the same value for each storey and it is so adjusted that the first uncoupled period for translational in y direction, is 1 s. Damping of 5% is considered.

Similar to the previous examples, this building has also been selected to investigate the effect of eccentricity variation along the height of the building. The eccentricity e is $0.3b$ for 1st, 2nd, 3rd and 4th storeys, $0.2b$ for 5th, 6th and 7th storeys, and $0.1b$ for 8th, 9th and 10th storeys. The frequency ratio Ω_R is the same for each storey. Five different buildings with $\Omega_R = 0.75, 1.0, 1.25, 1.50$ and 2.0 are considered.

Details of analysis

The earthquake motion is represented by flat and hyperbolic spectra, as used in the previous examples.

The Rayleigh method is used to find the frequency ratio Ω_R of the building using the procedures described earlier. The frequency ratio $\Omega_R (= \omega_\theta/\omega_y)$, as obtained from Eq. 3.1 works out to be exactly equal to the selected value for each storey.

A dynamic response spectrum analysis is carried out on the building models selected, using the complete quadratic combination technique for modal superposition. Interstorey displacement ratio $\bar{\Delta}_f$ is plotted as a function of Ω_R in Figs. 3.11a and 3.11b, for flat and hyperbolic spectra respectively, for each of the ten storeys. Figures 3.12a and 3.12b show the variation of $\bar{\Delta}_s$ with Ω_R for flat and hyperbolic spectra respectively. In the above figures, the respective curves for three corresponding single storey buildings with stiffness eccentricities of $0.1b$, $0.2b$ and $0.3b$ are also shown (solid lines).

For the flat spectrum, it is observed that the variation of $\bar{\Delta}_f$ for each storey of the multistorey building matches with that for the corresponding single storey building quite closely, except for low values of Ω_R , where results from single storey models show a localised dip which is smoothened out in the multistorey building.

For the hyperbolic spectrum, only the bottom four floors of the multistorey building show a somewhat close match between the variation of $\bar{\Delta}_f$ with that for a corresponding single storey building. For other floors, the results obtained for single storey models are generally higher than the flexible edge displacements for multistorey building, except in the case of low values of Ω_R . The variation of interstorey displacement ratio $\bar{\Delta}_s$ for each storey of the multistorey building is close to that for the corresponding single storey model.

The building models are now subjected to static forces acting at $\pm 0.05b$ from CM. Interstorey displacement ratios $\bar{\Delta}_f$ and $\bar{\Delta}_s$ so obtained are compared with the corre-

sponding values obtained from dynamic analysis, in Figs. 3.13a through 3.13f for flat spectrum and Figs. 3.14a through 3.14f for hyperbolic spectrum.

As a representative set of results, Tables 3.7 and 3.8 show shears in edge frames obtained from dynamic and static analyses respectively, for a building with $\Omega_R = 1$, and for an earthquake motion represented by flat spectrum.

Table 3.7: Shear in edge frames, as obtained from dynamic analysis for a flat spectrum

Floor	Shear in torsionally balanced building (kN)		Shear in torsionally unbalanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge	stiff edge	flexible edge	stiff edge		
10	230.17	761.33	356.58	582.85	1.549	0.766
9	446.82	1477.93	697.10	1120.08	1.560	0.758
8	646.67	2138.96	1016.75	1601.60	1.572	0.749
7	446.18	4270.42	894.62	2482.56	2.005	0.581
6	533.83	5109.33	1080.14	2942.38	2.023	0.576
5	610.38	5842.02	1245.91	3335.43	2.041	0.571
4	674.88	9351.66	1737.10	4565.83	2.574	0.488
3	726.15	10062.09	1878.71	4900.82	2.587	0.487
2	762.67	10568.13	1978.37	5146.23	2.594	0.487
1	782.43	10841.88	2031.75	5284.70	2.596	0.487

For a flat spectrum, it is evident from Figs. 3.13a through f, that the proposed design expressions are conservative for the flexible edge except for the upper three floors of the torsionally flexible types of buildings ($\Omega_R < 1$), as shown in the Fig. 3.13a. However, the difference between the $\bar{\Delta}_f$ values obtained from dynamic and static analyses are small and the proposed equation Eq. 2.26 may still be considered satisfactory. For the stiff edge, the proposed design expressions are quite conservative.

Table 3.8: Shear in edge frames obtained from static analysis

Floor	Shear in torsionally balanced building (kN)		Shear in torsionally unbalanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$		
10	230.17	761.33	354.46	669.97	1.540	0.880
9	446.82	1477.93	688.10	1300.58	1.540	0.880
8	646.69	2138.95	995.87	1882.28	1.540	0.880
7	446.18	4270.41	914.66	3117.41	2.050	0.730
6	533.83	5109.33	1094.34	3729.82	2.050	0.730
5	610.38	5842.02	1251.27	4264.69	2.050	0.730
4	674.88	9351.67	1808.67	6546.18	2.680	0.700
3	726.15	10062.11	1946.08	7043.49	2.680	0.700
2	762.67	10568.15	2043.95	7397.72	2.680	0.700
1	782.43	10841.90	2096.89	7589.34	2.680	0.700

For a hyperbolic spectra, it is obvious from Figs. 3.14a through f that the proposed design provisions are quite conservative for flexible as well as stiff edges except for a small range of Ω_R (0.8 – 1.1), for which the proposed expressions somewhat underestimate the value of $\bar{\Delta}_s$, at the upper three floors. This is shown in Fig. 3.14d. Again, the difference is small and the proposed equations may still be considered satisfactory.

Example 4

Building description

The building selected for this example is a ten storey shear building as shown in the Fig. 3.10. It has two planes parallel to the x axis and three planes parallel to the y axis. The building is symmetrical about x axis and asymmetrical about y axis. The centre of resistance, CR, is offset from centre of mass, CM, by a distance e . Mass of each floor is taken as 400 t , floor width b as 36 m , aspect ratio as 1 and storey height as 4 m . Total lateral stiffness K_y has same value for each storey and it is so adjusted that the first

uncoupled translational period T_y is 1 s. Damping of 5% is considered.

This building model has been selected to demonstrate the effect of the variation of relative torsional stiffness along the height of the building. The frequency ratio Ω_R is 1.50 for 1st, 2nd, 3rd and 4th storeys, 1.0 for 5th, 6th and 7th storeys, and 0.75 for 8th, 9th and 10th storeys. Static eccentricity e is constant throughout the height of the building. Six different buildings with e/b values of 0.05, 0.1, 0.15, 0.2, 0.25 and 0.3 are considered.

Details of analysis

The earthquake motion is represented by flat and hyperbolic spectra, as used in the previous examples.

The Rayleigh method is used to find the frequency ratio Ω_R of the building using the procedures described earlier. As a representative set of results, calculations for a building with $e = 0.2b$ and for an earthquake motion represented by flat spectrum are presented here. Table 3.9 shows displacements δ_i produced by dynamic floor forces F_i and rotations θ_i produced by floor torques $F_i b$.

Equations 3.1 give $\omega_y = 6.284$ and $\omega_\theta = 7.577$. The frequency ratio Ω_R thus works out to be 1.2, which is close to the mean of Ω_R values for the storeys. Similar results are obtained for buildings with different eccentricities and for earthquake motions represented by a hyperbolic spectrum.

A dynamic response spectrum analysis is carried out on the building models selected. Interstorey displacement ratio $\bar{\Delta}_f$ is plotted as a function of Ω_R in Figs. 3.15a and 3.15b, for flat and hyperbolic spectra respectively. Figures 3.16a and 3.16b show the variation of $\bar{\Delta}_s$ with Ω_R for flat and hyperbolic spectra respectively. These curves are drawn for

Table 3.9: Calculation of natural frequencies using the Rayleigh's method

Floor	Floor force F_i (kN)	δ_i	θ_i
10	5311.610	0.3182	0.0421
9	4999.550	0.3107	0.0399
8	4611.820	0.2961	0.0356
7	4198.300	0.2750	0.0293
6	3756.320	0.2480	0.0248
5	3280.730	0.2156	0.0194
4	2764.360	0.1786	0.0133
3	2197.230	0.1377	0.0102
2	1565.070	0.0937	0.0070
1	846.630	0.0474	0.0036

each of the ten storeys. For the purpose of comparison, curves of $\bar{\Delta}_f$ and $\bar{\Delta}_s$ for three corresponding single storey building models with Ω_R values of 0.75, 1.0 and 1.50, are also shown.

It is evident from Figs. 3.15a and 3.15b that the variation of $\bar{\Delta}_f$ for the 1st, 2nd, 3rd and 4th storeys, matches closely with that for a corresponding single storey building, for flat as well as hyperbolic spectra. For other floors, the results obtained from corresponding single storey models, underestimate the values of interstorey displacement $\bar{\Delta}_f$ for multistorey building, particularly for top three floors for which the difference is large. Figures 3.16a and 3.16b indicate that the variation of normalised displacement $\bar{\Delta}_s$ for a single storey model provides a conservative estimate of the interstorey displacements $\bar{\Delta}_s$ for a multistorey building.

It is also evident from Figs. 3.15 and 3.16 that an associated single storey model with Ω_R equal to the value obtained from the Rayleigh's method for a multistorey building, underestimates interstorey displacement ratios for that multistorey building. For the

building models selected in this example, a corresponding single storey model with $\Omega_R = 1.2$, underestimates $\bar{\Delta}_f$ values for all but 1st, 2nd, 3rd and 4th storeys. It also underestimates $\bar{\Delta}_s$ values for all the storeys.

The building models are now subjected to static forces acting at $\pm 0.05b$ from CM. Interstorey displacement ratios $\bar{\Delta}_f$ and $\bar{\Delta}_s$ so obtained are compared with the corresponding values obtained from dynamic analysis, in Figs. 3.17a through f for flat spectrum and Figs. 3.18a through f for hyperbolic spectrum.

As a representative set of results, Tables 3.10 and 3.11 show shears in edge frames obtained from dynamic and static analyses respectively, for a building with $e = 0.2b$, and for an earthquake motion represented by flat spectrum.

Table 3.10: Shear in edge frames, as obtained from dynamic analysis for a flat spectrum

Floor	Shear in torsionally unbalanced building (kN)		Shear in torsionally balanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge	stiff edge	flexible edge	stiff edge		
10	467.042	991.888	129.467	1191.797	3.607	0.832
9	888.197	1865.724	251.327	2313.573	3.534	0.806
8	1260.058	2588.385	363.736	3348.353	3.464	0.773
7	989.554	2225.014	446.178	4270.415	2.218	0.521
6	1166.021	2644.591	533.828	5109.325	2.184	0.518
5	1311.061	3006.783	610.381	5842.018	2.148	0.515
4	3541.504	5280.929	2603.035	8387.598	1.361	0.630
3	3771.012	5684.592	2800.785	9024.794	1.346	0.630
2	3933.429	5976.821	2941.641	9478.665	1.337	0.631
1	4021.359	6139.820	3017.834	9724.195	1.333	0.631

It is evident from Figs. 3.17a and b that the proposed design eccentricity expression

Table 3.11: Shear in edge frames obtained from static analysis

Floor	Shear in torsionally unbalanced building (kN)		Shear in torsionally balanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$		
10	371.139	619.730	129.467	1191.796	2.867	0.520
9	720.474	1203.051	251.327	2313.573	2.867	0.520
8	1042.717	1741.132	363.736	3348.353	2.867	0.520
7	914.658	3117.416	446.178	4270.414	2.050	0.730
6	1094.340	3729.822	533.828	5109.324	2.050	0.730
5	1251.272	4264.690	610.381	5842.018	2.050	0.730
4	3817.792	7381.084	2603.035	8387.599	1.467	0.880
3	4107.825	7941.817	2800.785	9024.796	1.467	0.880
2	4314.414	8341.225	2941.641	9478.668	1.467	0.880
1	4426.167	8557.289	3017.835	9724.198	1.467	0.880

for the flexible edge of a single storey building, Eq. 2.26, is unsafe for middle and upper storeys of the multistorey building for a flat spectrum. Figures. 3.18a indicates that the proposed expression Eq. 2.26, is also unsafe for the upper storeys for a hyperbolic spectrum. Figures 3.17a, 3.17b and 3.18a indicate that the proposed expression may underestimate the interstorey flexible edge displacements of a multistorey building by a maximum of 20%.

It is obvious from Figs. 3.17d through f and Figs. 3.18d through f, that the proposed design expressions, Eqs. 2.27 and 2.28 are quite conservative for the stiff edge of the multistorey building models selected.

Example 5

Building description

The buildings used in this example are similar to those used in the previous example except that the Ω_R variation along the height of the building is different. Ω_R values for

1st, 2nd, 3rd and 4th storeys is 0.75; for 5th, 6th and 7th storeys is 1.0; and for 8th, 9th and 10th storeys is 1.50. Static eccentricity e is same for each storey and six different buildings with e/b values of 0.05, 0.1, 0.15, 0.2, 0.25 and 0.3 are considered.

Details of analysis

The earthquake motion is represented by flat and hyperbolic spectra, as used in the previous examples.

The Rayleigh method is used to find the frequency ratio Ω_R of the building. To show the calculations involved, results for a building with $e = 0.2b$ and for an earthquake motion represented by flat spectrum are presented. Table 3.12 shows displacements δ_i produced by dynamic floor forces F_i and rotations θ_i produced by floor torques $F_i b$.

Table 3.12: Calculation of natural frequencies using the Rayleigh's method

Floor	Floor force F_i (kN)	δ_i	θ_i
10	5311.610	0.3182	0.0722
9	4999.550	0.3107	0.0716
8	4611.820	0.2961	0.0706
7	4198.300	0.2750	0.0690
6	3756.320	0.2480	0.0644
5	3280.730	0.2156	0.0591
4	2764.360	0.1786	0.0529
3	2197.230	0.1377	0.0408
2	1565.070	0.0937	0.0277
1	846.630	0.0474	0.0140

Using the Eqs 3.1, $\omega_y = 6.284$ and $\omega_\theta = 5.177$ are obtained. The frequency ratio Ω_R thus works out to be 0.814. This value of Ω_R is apparently influenced by the tor-

sional rigidity of lower storeys. Similar results are obtained for buildings with different eccentricities and for a hyperbolic spectrum.

A dynamic response spectrum analysis is carried out on the building models selected. Interstorey displacement ratio $\bar{\Delta}_f$ is plotted as a function of Ω_R in Figs. 3.19a and 3.19b, for flat and hyperbolic spectra respectively. Figures 3.20a and 3.20b show the variation of $\bar{\Delta}_s$ with Ω_R for flat and hyperbolic spectra respectively. These curves are drawn for each of the ten storeys. For the purpose of comparison, curves of $\bar{\Delta}_f$ and $\bar{\Delta}_s$ for three corresponding single storey building models with Ω_R values of 0.75, 1.0 and 1.50, are also shown.

It is evident from Figs. 3.19a and 3.19b that the variation of $\bar{\Delta}_f$ for a single storey building, somewhat underestimates the maximum interstorey displacement for the 1st, 2nd, 3rd and 4th storeys of the multistorey building, for flat as well as hyperbolic spectra. For other floors, single storey results overestimate the interstorey displacement ratios $\bar{\Delta}_f$, for both flat and hyperbolic spectra. For the stiff edge, maximum interstorey displacement ratio $\bar{\Delta}_s$ for the 1st, 2nd, 3rd and 4th storeys of the multistorey building, matches closely with that for a corresponding single storey building, for flat as well as hyperbolic spectra. The results obtained from single storey models, however, underestimate the values of $\bar{\Delta}_s$, for the other storeys of the multistorey building, for both flat and hyperbolic spectra.

The building models are now subjected to static forces acting at $\pm 0.05b$ from CM. Interstorey displacement ratios $\bar{\Delta}_f$ and $\bar{\Delta}_s$ so obtained are compared with the corresponding values obtained from dynamic analysis, in Figs. 3.21a through f for flat spectrum and Figs. 3.22a through f for hyperbolic spectrum.

As a representative set of results, Tables 3.13 and 3.14 show shears in edge frames obtained from dynamic and static analyses respectively, for a building with $e = 0.2b$, and for an earthquake motion represented by flat spectrum.

Table 3.13: Shear in edge frames, as obtained from dynamic analysis for a flat spectrum

Floor	Shear in torsionally unbalanced building (kN)		Shear in torsionally balanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge	stiff edge	flexible edge	stiff edge		
10	503.153	979.174	478.044	1540.371	1.053	0.636
9	984.960	1926.303	928.002	2990.244	1.061	0.644
8	1438.158	2816.617	1343.065	4327.676	1.071	0.651
7	751.655	2949.813	446.178	4270.414	1.685	0.691
6	902.245	3557.580	533.828	5109.325	1.690	0.696
5	1037.453	4095.658	610.381	5842.021	1.700	0.701
4	1887.065	6195.095	704.968	6489.544	2.677	0.955
3	2049.869	6731.490	758.524	6982.548	2.702	0.964
2	2168.679	7103.555	796.671	7333.712	2.722	0.969
1	2235.039	7294.998	817.306	7523.675	2.735	0.970

It is evident from Figs. 3.21 and 3.22 that the proposed design provision for the flexible edge of a single storey building, Eq. 2.26, provides a conservative estimate of the flexible edge displacements in the multistorey building models selected, for flat as well as hyperbolic spectrum. Proposed expression for stiff edge, Eqs. 2.27, however, underestimates the $\bar{\Delta}_s$ values for 5th, 6th and 7th storeys for both flat and hyperbolic spectra.

Table 3.14: Shear in edge frames obtained from static analysis

Floor	Shear in torsionally unbalanced building (kN)		Shear in torsionally balanced building (kN)		$\bar{\Delta}_f$	$\bar{\Delta}_s$
	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$	flexible edge $e + 0.05b$	stiff edge $e - 0.05b$		
10	701.132	1355.526	478.044	1540.371	1.467	0.880
9	1361.072	2631.413	928.002	2990.243	1.467	0.880
8	1969.832	3808.353	1343.065	4327.675	1.467	0.880
7	914.658	3117.414	446.178	4270.413	2.050	0.730
6	1094.340	3729.822	533.828	5109.324	2.050	0.730
5	1251.273	4264.693	610.381	5842.021	2.050	0.730
4	2020.921	3374.542	704.968	6489.545	2.867	0.520
3	2174.449	3630.904	758.524	6982.550	2.867	0.520
2	2283.805	3813.508	796.671	7333.714	2.867	0.520
1	2342.959	3912.285	817.306	7523.677	2.867	0.520

Example 6

Building description

The building selected in this example is an L shaped, five storey model, as shown in the Fig. 3.23. Resistance to lateral forces in the y direction is provided by three steel frames as indicated. The building is asymmetrical about the y axis and the eccentricity varies along the height of the building. A single resisting frame is provided along the x axis and the building is symmetrical about the x axis. The floor mass is same for all the stories and is taken as 774 t . Mass moment of inertia for each storey is $239,480\text{ t} - \text{m}^2$. The details of the lateral resisting frames are shown in the Fig. 3.23.

Details of analysis

The earthquake motion is represented by a flat spectrum with spectral acceleration of $1.0g$.

The Rayleigh method is used to find Ω_R of the building. An arbitrary value is assigned

to base shear V and it is distributed according to the provisions of NBCC, to obtain floor forces F_i . A static analysis for these forces with floors prevented from rotation, provides storey displacements δ_i . A static analysis for floor torques $F_i b$, where building width b is 45 m, results in floor rotations θ_i . The frequency ratio $\Omega_R = \omega_\theta/\omega_y$, obtained from Eq. 3.1 works out to be 1.05.

A dynamic response spectrum analysis is carried out on the building with floor rotations restrained. Table 3.15 shows the storey shears and forces obtained. Also shown in the table are shears in the flexible edge frame which is at the left hand side of the building and stiff edge frame which is at the right hand side of the building.

Table 3.15: Storey shears and forces in the L-shaped balanced building, subjected to 1g flat spectrum, with floor rotations restrained

Floor	Storey shear (kN)	Floor force (kN)	Storey shears (kN)	
			flexible edge	stiff edge
5	10473	10473	3488	3502
4	18961	8488	6304	6356
3	25314	6353	8534	8246
2	29490	4176	9454	10582
1	31987	2497	9350	13290

Response spectrum analysis is now repeated with the floor rotations released. The storey shears for flexible and stiff edge frames obtained from this analysis are shown in the Tables 3.16 and 3.17. Also shown in these tables are the ratio of shears obtained from the analysis of unbalanced building to that of an associated balanced building.

The building is now subjected to static floor forces, as given in Table 3.15, acting

at $\pm 0.05b$ from centre of mass. The shears obtained in the flexible and edge frames are shown in Tables 3.16 and 3.17 respectively. Also shown in these tables are ratios of shears obtained from dynamic analysis of unbalanced building and the static analysis of balanced buildings. The storey shear ratios obtained from dynamic analyses are compared with those obtained from static analyses, in Fig. 3.24.

Table 3.16: Storey shears at the flexible edge in balanced and unbalanced buildings

Floor level (1)	Balanced dynamic (kN) (2)	Unbalanced dynamic (kN) (3)	Ratio (3)/(2) (4)	Unbalanced static (kN) $e + 0.05b$ (5)	Ratio (5)/(2) (6)
5	3488	4640	1.332	4888	1.404
4	6304	8682	1.374	8854	1.404
3	8534	11620	1.362	11786	1.381
2	9454	13598	1.438	13882	1.468
1	9350	14804	1.583	15270	1.637

Table 3.17: Storey shears at the stiff edge in balanced and unbalanced buildings

Floor level (1)	Balanced dynamic (kN) (2)	Unbalanced dynamic (kN) (3)	Ratio (3)/(2) (4)	Unbalanced static (kN) $e - 0.05b$ (5)	Ratio (5)/(2) (6)
5	2502	2834	0.809	3144	0.898
4	6356	5074	0.798	5696	0.896
3	8246	6718	0.815	7594	0.929
2	10582	7974	0.754	9028	0.853
1	13290	8562	0.667	10086	0.759

The results presented in Table 3.12, Table 3.13 and Fig. 3.24 show that the static design provisions, Eqs. 2.26 and 2.27 are quite satisfactory for the building studied.

3.3 Summary and conclusions

Analytical results are presented here for the elastic response of a variety of multistorey building models with non-coincident centres of mass and resistance. It is shown that the results and recommendations derived from the study of single storey models also apply, approximately, to a more general class of multistorey building, provided the ratio of torsional to translational stiffness does not vary significantly along the height of the building. The following important conclusions can be drawn from the work presented here:

1. For multistorey buildings in which the ratio of torsional to lateral stiffness does not vary significantly along the height, Ω_R can be determined with sufficient accuracy by using the Rayleigh method. This requires a pair of static analyses.
2. To investigate the applicability of results obtained from single storey models, a series of multistorey models are studied. Frequency ratio Ω_R is defined storey by storey. It is observed that for the buildings with the same value of Ω_R along the height of the building, the interstorey displacements are similar to those obtained from the corresponding single storey models. The use of Eqs. 2.26, 2.27 and 2.28 provides a conservative estimate of interstorey displacements in such buildings.
3. For a multistorey building in which the frequency ratio Ω_R varies along the height, response values obtained from the analysis of corresponding single storey models do not match the response of multistorey building. Evidently, Eqs. 2.26, 2.27 and 2.28 can not be used in the design of such buildings. Additional study may be required in this area.

(a)

(b)

Figure 3.1: A five storey shear type of building (a) plan view (b) elevation of a typical frame

Figure 3.2: Normalised interstorey displacement at the flexible edge of a building of example 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.3: Normalised interstorey displacement at the stiff edge of a building of example 1, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.4: Comparison of interstorey displacement ratios for a building of example 1, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5

Figure 3.4: Comparison of interstorey displacement ratios for a building of example 1, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5

Figure 3.5: Comparison of interstorey displacement ratios for a building of example 1, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5

Figure 3.5: Comparison of interstorey displacement ratios for a building of example 1, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5

Figure 3.6: Normalised interstorey displacement at the flexible edge of a building of example 2, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.7: Normalised interstorey displacement at the stiff edge of a building of example 2, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.8: Comparison of interstorey displacement ratios for a building of example 2, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5

(c)

(d)

Figure 3.8: Comparison of interstorey displacement ratios for a building of example 2, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5

Figure 3.9: Comparison of interstorey displacement ratios for a building of example 2, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 1 and 2 (b) flexible edge, floors 3, 4 and 5

Figure 3.9: Comparison of interstorey displacement ratios for a building of example 2, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (c) stiff edge, floors 1 and 2 (d) stiff edge, floors 3, 4 and 5

Figure 3.10: A ten storey shear type of building (a) plan view (b) elevation of a typical frame

Figure 3.11: Normalised interstorey displacement at the flexible edge of a building of example 3, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.12: Normalised interstorey displacement at the stiff edge of a building of example 3, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.13: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5

Figure 3.13: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (c) flexible edge, floors 4, 3, 2 and 1

Figure 3.13: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5

Figure 3.13: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (f) stiff edge, floors 4, 3, 2 and 1

Figure 3.14: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5

Figure 3.14: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (c) flexible edge, floors 4, 3, 2 and 1

Figure 3.14: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5

Figure 3.14: Comparison of interstorey displacement ratios for a building of example 3, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (f) stiff edge, floors 4, 3, 2 and 1

Figure 3.15: Normalised interstorey displacement at the flexible edge of a building of example 4, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.16: Normalised interstorey displacement at the stiff edge of a building of example 4, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.17: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5

Figure 3.17: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (c) flexible edge, floors 4, 3, 2 and 1

Figure 3.17: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (d) stiff edge, 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5

Figure 3.17: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (f) stiff edge, floors 4, 3, 2 and 1

Figure 3.18: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5

Figure 3.18: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (c) flexible edge, floors 4, 3, 2 and 1

Figure 3.18: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5

Figure 3.18: Comparison of interstorey displacement ratios for a building of example 4, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (f) stiff edge, floors 4, 3, 2 and 1

Figure 3.19: Normalised interstorey displacement at the flexible edge of a building of example 5, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.20: Normalised interstorey displacement at the stiff edge of a building of example 5, (a) flat spectrum (b) hyperbolic spectrum

Figure 3.21: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5

Figure 3.21: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (c) flexible edge, floors 4, 3, 2 and 1

(d)

(e)

Figure 3.21: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5

Figure 3.21: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, flat spectrum, (f) stiff edge, floors 4, 3, 2 and 1

Figure 3.22: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (a) flexible edge, floors 10, 9 and 8 (b) flexible edge, floors 7, 6 and 5

Figure 3.22: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (c) flexible edge, floors 4, 3, 2 and 1

(d)

(e)

Figure 3.22: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (d) stiff edge, floors 10, 9 and 8 (e) stiff edge, floors 7, 6 and 5

Figure 3.22: Comparison of interstorey displacement ratios for a building of example 5, as obtained from a dynamic analysis and from static design provisions, hyperbolic spectrum, (f) stiff edge, floors 4, 3, 2 and 1

Figure 3.23: Five storey L - shaped building of example 6, (a) plan view (b) elevation of frames

Figure 3.24: Ratio of shears in a five storey, L - shaped, torsionally unbalanced building to those in the associated torsionally balanced building, flat spectrum

Chapter 4

Inelastic analysis of torsionally unbalanced single storey building models

4.1 General

As described in the previous chapter, torsional vibrations may cause significant additional forces in the lateral load resisting planes of an asymmetric structure that remains elastic during an earthquake. A majority of structures however, are designed for only a fraction of their elastic forces and are expected to undergo significant inelastic deformations during a major earthquake. For such structures, torsional motion caused due to natural and accidental eccentricities, leads to additional displacement and ductility demands in the lateral load resisting elements at the edges.

In asymmetric buildings, inelastic hysteresis behaviour due to yielding, unloading and reloading of structural elements as well as stiffness and strength deterioration in reinforced concrete structures, changes the relative stiffness of the structural elements and alters the vibration period. Element stiffness changes, in turn, shift the centres of rigidity of the building and affect the torsionally coupled response of inelastic structures,

triggering a behaviour that is different from that of elastic system.

A number of researchers have studied the inelastic torsional response of single storey buildings and have drawn important conclusions regarding the vulnerability of certain load resisting elements to excessive additional displacement and ductility demands as compared to that in the corresponding torsionally balanced system. Unfortunately the conclusions drawn by various researchers are often contradictory, as described in Section 1.4. This is attributed to the complexity of the problem and a large number of governing parameters. Various assumptions made by the researchers in the structural models studied have limited the applicability of the results obtained, and many questions related to the inelastic torsional behaviour are still unresolved.

Some of the studies on inelastic response have focussed on the existing torsional design provisions of various seismic codes. Here again, the recommendations emerging from such studies are not always consistent, and the implementation of such recommendations has led to considerable variation in the design provisions of different codes. In spite of the difficulties noted above, the research studies have led to a better understanding of the parameters that govern inelastic torsional response. It should therefore be possible now to arrive at more rational guidelines related to design for torsional motion.

The objective of the work presented here is to review the inelastic behaviour of building structures under earthquake induced torsion and to evaluate whether the proposed design guidelines, Eqs. 2.26, 2.27 and 2.28, based on the elastic torsional response, are also applicable when the building structure is expected to become inelastic.

4.2 Description of the model

A number of studies related to the torsional response of inelastic systems have been carried out on the single storey building model shown in Fig. 4.1a. In this model the building floor is assumed to be infinitely stiff in its own plane, the entire mass is distributed at the floor level, and the lateral resisting planes are oriented along the two principal axes of the building. The coordinate axes are taken as being parallel to the principal axes of the building and the origin is located at the centre of mass. In the elastic range, the i th plane parallel to the x axis has a stiffness K_{xi} while the i th plane in the y direction has stiffness K_{yi} . The distribution of stiffness is symmetrical about the x axis but is asymmetrical about the y axis. Thus the centre of stiffness lies on the x axis at a distance e from the centre of mass, given by

$$e = \frac{\sum_{i=1}^N k_{yi} x_i}{\sum_{i=1}^N k_{yi}} \quad (4.1)$$

where N is the number of resisting planes in the y direction and x_i is the distance from CM of the i th plane oriented in the the direction of the y axis.

The elastic force in a resisting plane is proportional to its stiffness. Hence, in the elastic range the centre of resistance coincides with the centre of stiffness and is designated as CR. Each plane is assumed to have a bilinear force-displacement relationship, as shown in Fig 4.1b. The yield strength of the i th plane in the y direction is f_{yi} and that of i th plane in the x direction is f_{xi} . As in the case of stiffness, the distribution of strength is symmetrical about the x axis, but is unsymmetrical about the y axis. The centre of strength lies at a distance e_p from CM, where e_p is given by

$$e_p = \frac{\sum_{i=1}^N f_{yi} x_i}{\sum_{i=1}^N f_{yi}} \quad (4.2)$$

The strength eccentricity e_p described above has also been referred to as resistance eccentricity e_r , in the literature.

In the elastic range, the response of the building shown in Fig. 4.1a to a given ground motion in y direction is governed by ω_y , the uncoupled frequency of translation in y direction, e/r , the ratio of eccentricity e to the mass radius of gyration r , and $\Omega_R = \omega_\theta/\omega_y$, the ratio of uncoupled rotational frequency ω_θ to the uncoupled translational frequency ω_y (Section 2.3).

In the inelastic range, the response of the building shown in Fig. 4.1a is governed by a number of additional parameters. Some of these have been identified as: (1) strength distribution among the planes in x and y directions, (2) strength eccentricity, e_p , (3) number of resisting planes in the y direction, N , (4) contribution of torsional stiffness from the planes in the x direction, (5) strength of planes in x direction, (6) overstrength factor (ratio of $\sum f_{yi}$ for torsionally unbalanced to that in corresponding torsionally balanced model), (7) frequency content of the ground motion used in the dynamic analysis, etc. The effects of some of these parameters have been discussed in detail by De La Llera and Chopra (1994).

It is clear that the method of design employed in determining the strength of lateral resisting planes, plays a major role in the inelastic torsional behaviour of an asymmetric building. Efforts are continuing to come up with design guidelines that ensure good inelastic behaviour and at the same time are simple enough for implementation in routine design.

4.3 Provisions for design against torsion

If the resisting planes in a torsionally unbalanced inelastic system were to have the same strengths as those in the corresponding torsionally balanced systems, torsional motions induced in the former will cause greater displacements and impose greater ductility demands in certain planes. To keep the displacements and ductilities within acceptable limits, it is necessary to increase the yield strengths of the affected planes in the unbalanced system.

It should be noted that for a building designed to remain elastic, torsional motions cause an increase in the lateral forces in resisting planes. The design requirement for the elastic system is to ensure that for each plane the yield strength is greater than the maximum force imposed. On the other hand, in an inelastic system the requirement is to limit the ductility demand. However, the ductility demand is also controlled by varying the yield strength.

Code provisions for design against torsion specify simplified empirical rules for determining the required strength of resisting planes. In general, the design rules are same for elastic and inelastic systems and are expressed in terms of effective design eccentricities. As an example, NBCC requires that the yield strengths be determined by applying the design shear through a point at a distance from CR equal to the design eccentricity and then calculating the forces in resisting planes by means of an elastic analysis. The design eccentricities in NBCC are given by

$$e_{d1} = 1.5e + 0.1b \quad (4.3)$$

$$e_{d2} = 0.5e - 0.1b \quad (4.4)$$

The design shear in each plane is determined by taking the larger of the two values obtained by using Eqs. 4.3 and 4.4. In general, Eq. 4.3 is used for the design of planes on the flexible side of CR, that is, on the same side of CR as the CM. Equation 4.4 is used for the design of planes on the stiff side. However, in some cases Eq. 4.3 yields a larger value for the strength required in stiff edge planes and should be used instead.

It was shown in Section 2.4 that Eq. 4.3 is very conservative. On the other hand, Eq. 4.4 is not adequate for systems with low values of frequency ratio Ω_R . The following alternative expressions for design eccentricities were suggested in Section 2.5

$$e_{d1} = e + 0.1b \quad (4.5)$$

$$e_{d2} = e - 0.1b \quad \Omega_R \geq 1 \quad (4.6)$$

$$e_{d2} = -0.1b \quad \Omega_R < 1 \quad (4.7)$$

It was also suggested in Sections 2.7 and 2.9 that for buildings with low aspect ratio ($a/b < 0.5$) and/or high mass eccentricity ($\eta \geq 0.2$), the cut-off value of Ω_R in Eqs. 4.6 and 4.7 be taken as 1.25.

It has been shown in the previous studies that a three plane system is adequate for representing the inelastic behaviour of most asymmetric buildings with a plan view similar to Fig 4.1a, including those having more than three planes (De La Llera and Chopra, 1994). It has also been shown that the presence of x direction planes always improves the performance of torsionally balanced buildings. On the other hand, the contribution from such planes can not be relied upon because they may have yielded

under the action of a simultaneous ground motion in the x direction (Goel and Chopra, 1990; De La Llera and Chopra, 1994). For the reasons just cited, the three plane model shown in Fig. 4.2 has been used in the present study. In this model, there are three planes in the y direction and one plane along the x axis. The x direction plane does not contribute to the torsional resistance. The slope of the second branch of the force-displacement relationship shown in Fig. 4.1b is taken as 5% of the initial slope. The plan aspect ratio a/b is assumed to be 0.5, where a is the dimension parallel to the direction of earthquake.

Referring to the system shown in Fig. 4.2, the design yield strengths of individual planes are given by

$$f_1 = V_0 \frac{k_1}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (\bar{e} + 0.5) \right] \quad (4.8)$$

$$f_2 = V_0 \frac{k_2}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} \bar{e} \right] \quad (4.9)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d2}}{b} (0.5 - \bar{e}) \right] \quad (4.10)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (0.5 - \bar{e}) \right] \quad (4.11)$$

where $\bar{e} = e/b$. In determining f_3 , the larger of the absolute values obtained from Eqs. 4.10 and 4.11 should be used.

The value of the strength eccentricity, that is the distance of the centre of strength from the centre of mass, is given by

$$e_p = \frac{(f_3 - f_1)b}{2V_y} \quad (4.12)$$

where V_y is the total strength in the y direction, obtained from

$$V_y = f_1 + f_2 + f_3 \quad (4.13)$$

The strengths of resisting planes in the corresponding torsionally balanced system are given by

$$\begin{aligned} f_{10} &= V_0 \frac{k_1}{K_y} \\ f_{20} &= V_0 \frac{k_2}{K_y} \\ f_{30} &= V_0 \frac{k_3}{K_y} \end{aligned} \quad (4.14)$$

Also

$$V_0 = f_{10} + f_{20} + f_{30} \quad (4.15)$$

The normalised strength of flexible edge plane in the torsionally unbalanced system, f_1/f_{10} is plotted in Fig. 4.3 as a function of the eccentricity ratio e/b for several different values of Ω_R . Two sets of curves are shown, one corresponding to the NBCC provision, Eq. 4.3, and the other corresponding to the provision given by Eq. 4.5. It will be noted that not all curves span the entire range of the value of eccentricity. This is because results are presented only for those cases which are physically possible. Thus combinations of e and Ω_R which would require one or more planes to have a negative stiffness are excluded. The required strength in the torsionally unbalanced system is seen to increase with the eccentricity. In all cases, the NBCC provision, Eq. 4.3, requires a strength that is greater than the one obtained from Eq. 4.5.

The normalised strength of the stiff edge plane is plotted in Fig. 4.4 as a function of

e/b for several values of Ω_R . The required strength is seen to decrease as the eccentricity increases. For $\Omega_R < 1$, the proposed eccentricity expressions, Eqs. 4.5 and 4.7, give a strength that is greater than that obtained from NBCC provisions. For $\Omega_R \geq 1$, the strength calculated from Eqs. 4.5 and 4.6 is smaller than that obtained from NBCC provisions.

Figure 4.5 shows the total strength ratio V_y/V_0 plotted as a function of e/b for several values of Ω_R . For $\Omega_R < 1$, both NBCC and the proposed provisions lead to the same total strength, although they are distributed differently. For $\Omega \geq 1$, the strengths specified by NBCC are substantially higher than those required by the suggested provisions, and the difference between the two increases with e/b .

The strength eccentricity e_p/b is plotted in Fig. 4.6 as a function of e/b for several values of Ω_R . The strength eccentricity first increases with e and then starts to decrease. Eccentricity e_p is much smaller than e , except for low values of both Ω_R and e , when e_p may be greater than e . For these cases the stiff edge plane is required to have a comparatively larger strength.

4.4 Inelastic response to recorded motions

In the previous chapter a number of single storey building models having different eccentricity ratios e/b and different frequency ratios Ω_R were analysed for their elastic response to 15 recorded motions. The mean values of the effective eccentricities e_{d1} and e_{d2} obtained from the analytical studies were compared with those given by the proposed design expressions, Eqs. 4.5, 4.6 and 4.7. A similar study is carried out for single storey building models strained into the inelastic range. The models are designed according

to the proposed design expressions and analysed for their response to a set of recorded earthquake motions.

Structural details of the model

A single storey shear type building model having three resisting planes in the y direction and one resisting plane along the x axis, as shown in Fig. 4.2, is selected. The mass of the building floor is taken as $400\ t$, mass moment of inertia as $54,000\ t - m^2$, aspect ratio as 0.5, floor width b as $36\ m$, storey height as $4\ m$ and uncoupled translational period T_y as $1.0\ s$. Damping of 5% and strain hardening ratio of 5% is taken for all planes. The frequency ratio Ω_R and the eccentricity ratio e/b are varied over a range of values as indicated in Table 4.1. Specified values of Ω_R and e/b are achieved by adjusting the values of k_1, k_2 and k_3 , the stiffnesses of the planes. Only physically admissible values of stiffness are accepted.

Table 4.1: Characteristics of building models

Ω_R	e/b						
0.75	0.05	0.10	0.15				
1.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35
1.15	0.05	0.10	0.15	0.20	0.25	0.30	
1.25	0.05	0.10	0.15	0.20	0.25		
1.50	0.05	0.10					

Earthquake motions

The building models are subjected to a set of 16 earthquake records, details of which are given in Table 4.2. The records have a wide range of peak ground acceleration to peak

ground velocity ratios (a/v). An elastic response spectrum is obtained for each of the 16 records, normalised by its peak ground acceleration, for a damping of 5% of critical. The mean of these spectra is shown in Fig. 4.7. All the earthquake records are then scaled to a peak ground acceleration of $0.28g$.

Table 4.2: Description and Peak Ground Motion Parameters for Earthquake Records

No	Earthquake	Date	Magn	Site	Epic Dist (km)	Comp	Max Acc (cm/s^2)	Max Vel (cm/s)	A/V
1	Nahanni Aftershock Canada	12/23/85	6.9	Iverson Northwest Territories	7	280	1319.1	45.06	2.98
2	Nahanni Aftershock Canada	12/23/85	5.4	Iverson Northwest Territories	7	010	224.1	6.78	3.37
3	Nahanni Aftershock Canada	12/23/85	6.9	Battlement Creek, N.W. Territories	21	360	190.2	3.43	5.65
4	Nahanni Aftershock Canada	12/25/85	5.7	Battlement Creek, N.W. Territories	18	360	103.4	1.05	10.04
5	Miyagi Prefecture Japan	6/12/78	6.3	Ofunato Harbor Jetty	103	E41S	222.1	14.10	1.61
6	Michoacan Mexico City	9/19/85	8.1	LA Union	80	N00E	162.8	20.34	0.82
7	Michoacan Mexico City	9/19/85	8.1	La Villita	40	N00E	121.0	16.11	0.77
8	ADAK USA	5/2/71	6.8	ADAK Naval Base	69	West	182.8	8.0	2.33

Table 4.2: (contd.) Description and Peak Ground Motion Parameters for Earthquake Records

No	Earthquake	Date	Magn	Site	Epic Dist (km)	Comp	Max Acc (cm/s^2)	Max Vel (cm/s)	A/V
9	ADAK USA	5/2/71	6.8	ADAK Naval Base (Hand dig)	69	N90E	183.7	6.35	2.95
10	ADAK USA	5/2/71	6.8	ADAK Naval Base	69	N90E	203.7	6.28	3.31
11	Sitka USA	7/30/72	7.6	Sitka Magnetic Observatory	48	West	91.3	9.32	1.0
12	Sitka USA	7/30/72	7.6	Sitka Magnetic Observatory	48	N90E	89.4	6.58	1.36
13	San Ferando California	2/9/71	6.5	Lake Hugues Array 4	28	S69E	168.2	5.75	2.98
14	Coyote Lake USA	8/6/79	5.8	Gilory Array 6 San Ysidro	10	230	409.0	43.80	0.95
15	Loma Prieta California	10/18/89	7.1	Appel Arr 9 Crystal spr. reservoir	62	137	115.1	17.13	0.68
16	Loma Prieta California	10/18/89	7.1	Calaveras Ar. Cherry flat reservoir (Left abut.)	42	360	78.2	8.73	0.91

Details of analysis

In a torsionally unbalanced model, the centre of mass, CM, coincides with the origin of reference axes, and the centre of stiffness, CR, is offset from the origin by a distance e , the eccentricity. An associated torsionally balanced model is achieved by moving CM to the right, to coincide with CR. It is observed that the torsionally balanced models with the same values of Ω_R have identical response to a given earthquake, irrespective of the location of CR. Thus, associated with the torsionally unbalanced models represented by any one row of Table 4.1, there is only one torsionally balanced model.

The total elastic strength V_e of the resisting planes is obtained from the elastic response spectrum of Fig. 4.7 corresponding to the period 1.0 s. The spectral value obtained from Fig. 4.7 is multiplied by 0.28 so that the spectrum is representative of an earthquake with a peak ground acceleration of 0.28 g . The total design strength for the torsionally balanced model is taken as $V_0 = V_e/4$. This total strength is distributed among the individual planes in proportion to their stiffness according to Eq. 4.14. In the torsionally unbalanced model, the strengths of resulting planes are determined according to Eqs. 4.8 through 4.11. The total strength V_y , given by Eq. 4.13, is now greater than V_0 .

Recent studies (De La Llera and Chopra, 1994) have shown that the effect of accidental torsion can reasonably be estimated by increasing or decreasing the eccentricity by $0.05b$. In the present analysis, the following method is adopted to account for the effect of accidental eccentricities. To find the maximum effect on the flexible edge, centre of mass of the unbalanced building model is shifted to the left (towards the flexible edge) by a distance $0.05b$. To find the maximum effect on the stiff edge, CM of the unbalanced model is shifted to the right (towards the stiff edge) by a distance $0.05b$.

All the 46 torsionally unbalanced and 5 associated torsionally balanced models are subjected to the 16 ground acceleration records referred to earlier, each scaled to a peak ground acceleration of 0.28 g . Since the design strength is significantly lower than the strength obtained from mean elastic spectrum, the building models undergo considerable inelastic deformations. The response analyses provide the maximum displacement and maximum ductility demands for each resisting plane in each model.

Computer program DRAIN-TABS (Guendelman and Powell, 1977) developed at University of California, Berkeley, has been used for carrying out the inelastic analyses. The program performs a step-by-step time history numerical analysis using the average acceleration method of integration. The time step employed in the analyses is 0.005 s. Also, a Rayleigh type damping has been used with the damping values adjusted to 5% in each of the two coupled modes involving translation in the y direction and rotation about CR.

The maximum displacements of a plane in a torsionally unbalanced model subjected to a given earthquake is denoted by Δ_u . The maximum displacement of the associated torsionally balanced model subjected to the same earthquake is denoted by Δ_b . Displacement ratio $r_\Delta = \Delta_u/\Delta_b$ provides a measure of the effect of torsional motion. In a similar manner, the maximum ductility demand in a plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand for the associated torsionally balanced model is denoted by μ_b . It should be noted that in a torsionally balanced model all planes have the same displacements and the same ductility demands. The ratio of ductilities $r_\mu = \mu_u/\mu_b$ provides another measure of the effect of torsional motion.

The mean value of displacement ratio, $\bar{r}_{\Delta f}$ for the flexible edge of the building is

plotted as a function of e/b for four different values of Ω_R in Fig. 4.8. In almost all cases, the displacement of the flexible edge in a torsionally unbalanced building is higher than the displacement of an associated torsionally balanced building. Evidently, torsional motion results in a significant increase in the displacement of the flexible edge.

The mean values of displacement ratio, $\bar{r}_{\Delta s}$, for the stiff edge of the building is plotted as a function of e/b in Fig. 4.9. For $\Omega_R \geq 1$, $\bar{r}_{\Delta s}$ is generally less than 1, implying that the displacements of the stiff edge in a torsionally unbalanced building is less than that of the associated torsionally balanced building. For $\Omega_R < 1$, stiff edge displacement in a torsionally unbalanced building is greater than that in the associated torsionally balanced building. This can be explained as follows. For the stiff edge the displacement due to torsion is out of phase with the displacement due to translational motion and the two tend to cancel each other reducing the net displacement. For a torsionally flexible building, the displacement due to torsion is so large that even when it is reduced by translation of the building, the net displacement is still significantly larger than that in the associated torsionally balanced building.

The mean value of the ratio of ductility for the flexible edge of the building $\bar{r}_{\mu f}$ is plotted as a function of e/b in Fig. 4.10. This ratio is less than 1 in all cases, implying that the ductility demand on the flexible edge plane in a torsionally unbalanced building is substantially less than that in the associated torsionally balanced building. It should be noted that while the displacement of the flexible edge in a torsionally unbalanced building is greater, the yield displacement is also greater because the yield strength is greater. The results also demonstrate that the design eccentricity expression given by Eq. 4.5 is quite adequate, and there is no need to use a more conservative NBCC

expressions given by Eq. 4.3.

The mean value of the ratio of ductility for the stiff edge of the building, $\bar{r}_{\mu s}$ is plotted as a function of e/b in Fig. 4.11. For $\Omega_R = 1.25$ and 1.5 the ratio is less than 1, implying that the expression for design eccentricity, Eq. 4.6, is adequate for these cases. The ratio $\bar{r}_{\mu s}$ is less than 1 also for $\Omega_R = 0.75$. For this case the design expression given by Eq. 4.7, which results in the provision of a comparatively larger strength for the stiff edge, is therefore satisfactory. For $\Omega_R = 1$ to which Eq. 4.6 applies, $\bar{r}_{\mu s}$ is greater than 1 for a range of values of e/b implying that torsional motion results in an increase in the ductility demand on the stiff edge. It may be reasoned that the strength provided for the stiff plane in accordance with Eq. 4.6 is not adequate. However, the increase in ductility demand is no more than 15% and may still be considered satisfactory. In fact, a slight increase in Ω_R leads to a much reduced ductility demand. This will be evident from the plots of stiff-edge displacement and ductility ratios shown respectively in Figs. 4.9 and 4.11. These figures include curves corresponding to $\Omega_R = 1.15$, for which both $\bar{r}_{\Delta s}$ and $\bar{r}_{\mu s}$ are less than 1. Also shown there are additional curves for $\Omega_R = 1$ (revised), for which the strength is determined from Eq. 4.7.

4.5 Mass eccentric systems

In the single storey building model studies presented in the previous section, the centre of mass was assumed to be located at the geometric centre of the building plan, and the eccentricity was produced due to a non uniform distribution of stiffness. Such models are referred to as stiffness eccentric models. In this section, analytical results are presented for building models in which torsional asymmetry is produced wholly or partly by a non

uniform distribution of mass. Such building models are referred to as mass eccentric models. A comparison is made of the ductility demands at the edge planes in a mass eccentric model and in its corresponding stiffness eccentric model, with identical values of the elastic response parameters.

Figure 4.12 demonstrates three associated building models with identical values of eccentricity e , uncoupled translational period T , ratio of uncoupled rotational and translational frequencies Ω_R , and the radius of gyration (about CM) r . In the model shown in Fig. 4.12a, the eccentricity is caused entirely by a non uniform distribution of stiffness. Figures 4.12b and 4.12c show building models in which torsional asymmetry is caused both by a non uniform distribution of stiffness and a non uniform distribution of mass. In Fig. 4.12b, the centre of mass is offset to the left of the geometric centre by a distance ηb , while in Fig. 4.12c the centre of mass is offset to the right by a distance ηb . The origin of co-ordinate axes is taken at the centre of mass in all the cases.

As observed in Section 2.7, the dynamic response of plane No. 3 (stiff edge), is critical for the building model shown in Fig. 4.12b, as its elastic displacement is higher than that for the corresponding stiffness eccentric building model shown in Fig. 4.12a. This is because the distance of plane 3 from CR, in the building model of Fig. 4.12b, $(b/2 - e + \eta b)$, is greater than the corresponding value in the model of Fig. 4.12a which is $b/2 - e$. Similarly, the response of plane No. 1, for the building model shown in Fig. 4.12c is critical, as the lever arm for the plane No. 1 is in this case larger than that for the model shown in Fig. 4.12a. Analytical studies, therefore, are carried out on two sets of mass eccentric models; Type 1: centre of mass is offset to the left of geometric centre, and Type 2: centre of mass is offset to the right of geometric centre. It should be noted

that centre of rigidity CR is always to the right of CM. The above two sets encompass all possible cases of mass eccentricities.

Type 1 models

Structural details of the model

A mono-symmetric, single storey model with rigid diaphragm, as shown in Fig. 4.12b, is selected for study. Since the response of stiff edge plane is critical in this case, an aspect ratio that leads to higher stiff edge response, namely 1/3, (refer to Section 2.9) is selected. The mass of the building is taken as 400 t , mass moment of inertia as 48,000 $t - m^2$, uncoupled lateral period as 1.0 s., floor width b as 36 m , and storey height as 4 m . A damping ratio of 5% is assumed. Strain hardening is taken as 5% for all the lateral load resisting planes. A series of building models with a range of eccentricity values ($e/b = 0.05, 0.1, 0.15, 0.2, 0.25, 0.30, 0.35$, and 0.4) and a range of frequency ratios ($\Omega_R = 0.75, 1.0, 1.25$, and 1.50) is considered. Three values of mass eccentricities, $\eta = 0.1, 0.2$ and 0.3 are considered.

Earthquake motions

The set of 16 earthquake records, used in the studies reported in the previous section, are also employed in the present study. The details of the earthquakes are given in Table 4.2. Each earthquake record is normalised with respect to its peak ground acceleration. The mean of the elastic response spectrum, obtained for all 16 normalised earthquakes, for a damping of 5% of critical, is shown in Fig. 4.7. For the purpose of analysis, the earthquake records are scaled so that the peak ground acceleration for each record is $0.28g$.

Details of analysis

The total elastic strength of lateral load resisting elements V_e is obtained from the elastic response spectrum shown in Fig. 4.7, corresponding to a period of 1.0 s. The spectral value is multiplied by 0.28 to match the scaled peak ground acceleration of the design earthquake, $0.28g$. The building model is designed to have a strength that is 25% of elastic strength, i.e. $V_0 = V_e/4$. The yield strengths of individual planes are given by

$$f_1 = V_0 \frac{k_1}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (\bar{e} + 0.5 - \eta) \right] \quad (4.16)$$

$$f_2 = V_0 \frac{k_2}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (\bar{e} - \eta) \right] \quad (4.17)$$

$$f_2 = V_0 \frac{k_2}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d2}}{b} (\bar{e} - \eta) \right] \quad (4.18)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d2}}{b} (0.5 - \bar{e} + \eta) \right] \quad (4.19)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (0.5 - \bar{e} + \eta) \right] \quad (4.20)$$

where $\bar{e} = e/b$. In determining f_2 , the larger of the two values obtained from Eqs. 4.17 and 4.18 is used. It is evident that for $\eta \leq e/b$, Eq. 4.17 will govern while for $\eta > e/b$, Eq. 4.18 will govern the design of plane No. 2. In determining the strength of plane No. 3, the larger of the absolute values obtained from Eqs. 4.19 and 4.20 is used. Since the aspect ratio is low and the stiff edge is likely to be critical, the design eccentricity e_{d2} for building models with $\Omega_R = 1.25$ and 1.50 , is taken to be that given by Eq. 4.6. For building models with $\Omega_R = 0.75$ and 1.0 , the design eccentricity e_{d2} is obtained from Eq. 4.7.

The building models are analysed for their response to the 16 scaled earthquakes. To

account for the effect of accidental torsion, the centre of mass CM is moved $\pm 0.05b$ along the x-axis and higher of the two responses is considered for each element. To assess the effect of torsion, an associated torsionally balanced model, obtained by moving CM to CR, is also analysed.

The ratio of ductility demand in an element in the torsionally unbalanced model to that in the associated torsionally balanced model is denoted by r_μ . The mean value of ductility ratio for plane No. 1, $\bar{r}_{\mu 1}$, obtained for the set of 16 earthquakes, is plotted as a function of e/b in Figs. 4.13a through 4.13d, for $\Omega_R = 0.75, 1.0, 1.25$ and 1.50 respectively. This ratio is less than 1 in all cases, implying that the ductility demand on the plane No. 1, which is on the same side of CR as the CM, in a torsionally unbalanced building is less than that in the corresponding balanced building. It should be noted that the value of $\bar{r}_{\mu 1}$ is more or less the same for all η values, implying that the ductility demand for this plane is insensitive to variation in the mass eccentricity value but depends only on the total eccentricity between CR and CM.

Figures 4.14a through 4.14d show the variation of mean ductility ratio for plane 3, $\bar{r}_{\mu 3}$, as a function of e/b . This ratio is less than 1 for $\Omega_R = 0.75$ and 1.0 . However, it could be significantly higher than 1 for $\Omega_R = 1.25$ and 1.50 , specially for high mass eccentricity η and high static eccentricity e/b values. It is apparent that a combination of low aspect ratio, high mass eccentricity, and high total eccentricity should be avoided.

Type 2 models

Structural details of the model

A mono-symmetric, single storey building model with rigid diaphragm, as shown in Fig. 4.12c, is selected. The centre of mass CM is offset to the right of geometric centre

by a distance ηb . The response of plane No. 1 for this model is likely to be critical because of the larger distance of this plane from CR. On the basis of results obtained from elastic analyses, it can be assumed that an aspect ratio of 1 would lead to higher response. Therefore, in the present studies the aspect ratio is taken as 1. The building models selected, have the same properties as those is Type 1 models. However, only a few combinations of e/b , Ω_R and η values are physically possible.

Details of the analysis

The total elastic strength of lateral load resisting elements V_e is obtained from the elastic response spectrum shown in Fig. 4.7, corresponding to a period of 1.0 s. The spectral value is multiplied by 0.28 to match the scaled peak ground acceleration of design earthquake, $0.28g$. The building model is designed for $1/4$ of its elastic strength, $V_0 = V_e/4$. The yield strength of individual planes are given by

$$f_1 = V_0 \frac{k_1}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (\bar{e} + 0.5 + \eta) \right] \quad (4.21)$$

$$f_2 = V_0 \frac{k_2}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (\bar{e} + \eta) \right] \quad (4.22)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d2}}{b} (0.5 - \bar{e} - \eta) \right] \quad (4.23)$$

$$f_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (0.5 - \bar{e} - \eta) \right] \quad (4.24)$$

For determining the strength of plane No. 3, larger of the absolute values obtained from Eqs. 4.23 and 4.24 is used. The design eccentricity e_{d2} is calculated according to Eq. 4.6 for $\Omega_R = 1.0, 1.25$ and 1.50 . For $\Omega_R = 0.75$, Eq. 4.7 is used.

The building models are subjected to the set of 16 scaled earthquakes described

earlier. To account for the effect of accidental eccentricity, the centre of mass CM is moved by $\pm 0.05b$ along the x-axis, and higher of the responses obtained from the two modified models is considered. To assess the effect of torsion, an associated balanced building model obtained by moving CM to CR is also analysed.

The mean value of ductility ratios in a torsionally unbalanced model to that in associated torsionally balanced model, are plotted as a function of e/b in Figs. 4.15a through 4.15d. The ratios $\bar{r}_{\mu 1}$ and $\bar{r}_{\mu 3}$ for plane No. 1 and plane No. 3 respectively are less than 1 for all cases of η , Ω_R and e/b . This indicates that the ductility demands of edge planes in a torsionally unbalanced building model shown in Fig. 4.12c is less than that in an associated balanced building.

4.6 Summary and conclusions

In the previous chapters, the results of a study on the elastic response of single and multistorey building models subjected to earthquake motions were presented. It was shown that the most important parameter governing the elastic response was the ratio of uncoupled torsional frequency to uncoupled lateral frequency. In a good design, this frequency ratio should be greater than 1. It was also shown that the existing torsional provisions of NBCC are overly conservative for the flexible edge of the building, but may at times be inadequate for the stiff edge, particularly when the uncoupled frequency ratio is less than 1. Based on these observations, new design provisions were developed for the torsional design of elastic buildings. In the present chapter, analytical studies are presented that examine the applicability of such provisions to the design of buildings that are expected to become inelastic during the design earthquake.

Analytical studies are carried out on single storey building models that are symmetrical about the x axis but are asymmetrical about the y axis. The lateral resistance in the y direction is provided by three planes which also provide the entire torsional resistance. The torsionally balanced building models are designed to have 1/4th of the design strength obtained from the mean elastic spectrum of a set of 16 scaled earthquake records. The strengths for the resisting planes in torsionally unbalanced building models is then derived according to the suggested design provisions. Inelastic time history analyses are carried out on all building models for the 16 earthquake records. Mean values of the ratio of ductility demand in planes of an unbalanced buildings to that in the associated balanced building are obtained to assess the applicability of suggested design provisions. It is seen that the design provisions lead to a conservative design for the flexible edge. This also confirms that the NBCC provisions for the flexible edge are overly conservative. The suggested provisions for the design of stiff edge plane are also satisfactory. In addition, they prove the importance of maintaining a high value for the frequency ratio, which should preferably be greater than 1.

It is also shown that for mass eccentric systems the stiff edge plane may have a high ductility demand particularly for low aspect ratio, large mass eccentricity and large total eccentricity. Such combinations should be avoided in design unless a large value of Ω_R (≥ 1.5) can be ensured.

(a)

(b)

Figure 4.1: (a) Plan view of a single storey mono-symmetric building, (b) force-displacement relationship for the i^{th} plane

Figure 4.2: Three plane building model

Figure 4.3: Normalised flexible edge plane strength, aspect ratio = 0.5

Figure 4.4: Normalised stiff edge plane strength, aspect ratio = 0.5

Figure 4.5: Normalised total strength of planes, aspect ratio = 0.5

Figure 4.6: Strength eccentricity, aspect ratio = 0.5

Figure 4.7: Mean elastic spectrum for the set of 16 earthquake records

Figure 4.8: Ratio of flexible edge displacement in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records

Figure 4.9: Ratio of stiff edge displacement in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records

Figure 4.10: Ratio of flexible edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records

Figure 4.11: Ratio of stiff edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records

(a)

(b)

(c)

Figure 4.12: Single storey building models (a) stiffness eccentric system (b) mass and stiffness eccentric system: type 1 model (c) mass and stiffness eccentric system: type 2 model

Figure 4.13: Ratio of ductility demands for plane No. 1 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 1 models, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 4.13: Ratio of ductility demands for plane No. 1 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 1 models, (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

Figure 4.14: Ratio of ductility demands for plane No. 3 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 1 models, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

(c)

(d)

Figure 4.14: Ratio of ductility demands for plane No. 3 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 1 models, (c) $\Omega_R = 1.25$ (d) $\Omega_R = 1.50$

(a)

(b)

Figure 4.15: Ratio of ductility demands for plane No. 1 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 2 models, (a) $\Omega_R = 0.75$ (b) $\Omega_R = 1.0$

Figure 4.15: Ratio of ductility demands for plane No. 3 in a mass eccentric torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 16 earthquakes, type 2 models, (c) $\Omega_R = 0.75$ (d) $\Omega_R = 1.0$

Chapter 5

Effect of orthogonal elements on inelastic torsional response of single storey models

5.1 General

In the analytical studies presented in the previous sections, a three-plane single storey building model as shown in Fig. 4.2, has been used. In such a model, torsional stiffness is provided solely by lateral load resisting elements parallel to the direction of earthquake. However, for a majority of buildings, a part of torsional resistance is contributed by lateral load resisting elements perpendicular to the direction of ground motion, called orthogonal elements. This section addresses the effect of orthogonal elements on inelastic torsional behaviour of building models.

The issue of the effect of orthogonal elements has not been adequately addressed in the previous studies, leading to uncertainties and contradictions about the role played by them in the inelastic torsional response. This is clear from the following review of the previous work.

Goel and Chopra (1990) compared the inelastic torsional response of two single storey

building models with identical torsional stiffnesses but with and without orthogonal elements. They concluded that: (1) peak responses of short period, acceleration sensitive systems are influenced significantly by the contribution to torsional stiffness from orthogonal elements; (2) in the system with orthogonal elements, the flexible side element experiences smaller deformations and stiff side element undergoes larger deformation than in the system without orthogonal elements; and (3) ductility demand for a system with orthogonal elements is smaller at the flexible edge and larger at the stiff edge than in the corresponding system without orthogonal elements. Goel and Chopra's results are based on models that have one specific value of static eccentricity ($e = 0.2$), and one ratio of uncoupled frequencies ($\Omega_R = 1.0$). Further, the results have been obtained from analytical studies in which the ground motion was applied in only one direction and do not indicate whether the orthogonal elements are yielding.

A subsequent study by De La Llera and Chopra (1994) concluded that systems with strong orthogonal elements exhibit more uniform displacement demands on the elements parallel to the direction of earthquake. The results of that study do not, however, provide a complete picture of the effect of orthogonal elements since the total torsional stiffnesses of corresponding systems with and without orthogonal elements, used in the comparison, are not same. Also, there is some question about the applicability of results obtained from single storey inelastic models in which orthogonal elements are assumed to remain elastic during an earthquake, because such elements may in fact be yielding during a real earthquake event with multiple components of motion (Wong and Tso, 1995).

Correnza *et al* studied the response of single storey models with and without orthogonal elements. They also compared the response of models with orthogonal elements

subjected to uni-directional earthquakes to the response of the same models when subjected to bi-directional earthquakes. They concluded that the orthogonal load resisting elements significantly affected the response of torsionally unbalanced systems, particularly in the short and medium period range. Their studies also showed that in carrying out a non-linear dynamic analysis of models with orthogonal elements, it was important to include both horizontal components of the ground motion.

A recent study by Paulay (1996) argues that the ductility demands for systems in which orthogonal elements remain elastic (referred to as torsionally restrained) is very different from that in systems where orthogonal elements undergo yielding (referred to as torsionally unrestrained).

Keeping in view the above observations, the present study addresses the following issues: (1) effect of the variation in torsional stiffness contributed by orthogonal elements while the overall torsional stiffness of the system remains constant, (2) effect of yielding in the orthogonal elements, (3) effect of the uncoupled translational period of the building model studied, and (4) collapse mechanism analysis of torsionally unrestrained systems.

5.2 Inelastic response to recorded motions

Structural details of the models studied

A mono-symmetric, single storey building model with rigid diaphragm, as shown in Fig. 5.1, is selected. The building model has 2 identical planes parallel to the x-axis which are symmetrically placed. There are 3 planes along the y-axis and their stiffness is so distributed that the centre of stiffness lies at a distance e from the centre of mass, along the x-axis. The mass of the building floor is taken as 400 t , mass moment of inertia as 54,000 $t - m^2$, aspect ratio as 0.5, floor width b as 36 m , storey height as 4

m , uncoupled translational period in y-direction as 1.0 s. and uncoupled translational period in x-direction as 0.5 s. A damping ratio of 5% is assumed. A wide range of building models with different combination of e/b and Ω_R values is selected. Results of the analysis of short period, acceleration sensitive systems are presented in a later section.

To study the effect of orthogonal elements a parameter γ is introduced. It is defined as the ratio of the torsional stiffness (about CR) of planes parallel to y-axis to the overall torsional stiffness of the system $K_{\theta R}$. A lower value of γ indicates a higher contribution from orthogonal elements towards the total torsional stiffness of the model.

To address issue (1) described earlier, torsional stiffness of x-planes is varied while keeping the overall torsional stiffness of the building constant. This is achieved by a redistribution of stiffnesses in y-direction planes (keeping ΣK_y same), and by varying the relative position of orthogonal elements with respect to the x-axis. Four values of γ ; 0.4, 0.6, 0.8 and 1.0 are considered.

Earthquake motions

A set of 12 earthquake records each having ground acceleration data in two orthogonal horizontal directions, is selected. This set is similar to the one used earlier, details of which are described in Table 4.2, with the exception that earthquake Nos. 2, 8, 10 and 11 have been omitted. The reason for their omission is that the ratio of peak ground accelerations of minor direction component (one with lower peak ground acceleration) to major direction component for these earthquakes is much lower than the average value for the selected set of 12 records, which is 0.88. In the dynamic time history analysis described in the following paragraphs, the major direction component of each earthquake

is scaled to a peak ground acceleration of $0.3g$, while the minor direction component is scaled to a peak ground acceleration of $0.88 * 0.3g = 0.264g$.

Details of analysis

For the design of the lateral load resisting elements, each of the original acceleration records is normalised with respect to its peak ground acceleration and an average elastic response spectrum for the normalised major direction component of all 12 earthquakes is obtained, as shown in Fig. 5.2. The elastic spectrum is now scaled by 0.3. The total elastic strength of elements in x-direction, V_{ex} , is obtained from the scaled response spectrum corresponding to a period of 0.5 s. Similarly, the total elastic strength of elements in y-direction, V_{ey} , is obtained from the same scaled response spectrum, corresponding to a period of 1.0 s. The building model is designed for 25% of the elastic forces obtained, i.e. $V_x = V_{ex}/4$ and $V_y = V_{ey}/4$. The two x-direction elements are designed for a shear of $V_x/2$ each. The design strengths of resisting elements in the y-direction are obtained from Eqs. 4.8 through 4.11.

A dynamic time history analysis is carried out on building models described, using bi-directional earthquake motions scaled as described under the previous heading of earthquake motions. A 5% strain hardening is assumed for all lateral load resisting elements. Since the design strength is significantly lower than the strength obtained from mean elastic spectrum, all the lateral resisting elements undergo considerable inelastic deformations. To account for the effect of accidental torsion, the analyses are in fact carried out on building models that have been modified by moving the centre of mass $CM \pm 0.05b$ along the x-axis. The higher of the two responses is considered for each element. To investigate additional displacement and ductility demands due to torsional

effects, an associated torsionally balanced model, obtained by moving CM to CR, is also analysed.

To demonstrate the effect of yielding in orthogonal elements on torsional response of the system, the above mentioned set of analyses is repeated with the difference that the strength of orthogonal elements is taken to be very high. This is done to ensure that the orthogonal elements remain elastic.

The maximum ductility demand in a resisting plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand for the associated torsionally balanced model is denoted by μ_b . The ratio of ductilities $r_\mu = \mu_u/\mu_b$ provides a measure of the effect of torsional motion. A mean value of the ductility ratios, obtained for a set of 12 earthquakes, is denoted by $\bar{r}_\mu$.

Computer program DRAIN-TABS (Guendelman and Powell, 1977) has been used for carrying out the inelastic analyses. The program performs a step-by-step time history numerical analysis using the average acceleration method of integration. The time step employed in the analyses is 0.005 s. Also, a Rayleigh type damping has been used with the damping values adjusted to 5% in each of the two coupled modes involving translation in the y direction and rotation about CR.

Results of response analyses

Results are presented first for the building models with the uncoupled period of translation in y-direction equal to 1.0 s.

The mean ductility ratio for the flexible edge element $\bar{r}_{\mu f}$ is plotted as a function of e/b in Fig. 5.3a through 5.3g, for different values of Ω_R and γ . It should be noted that $\gamma = 1$ represents a building model without orthogonal elements. Also plotted in these

graphs are the responses of building models for which orthogonal elements are designed to remain elastic. The value of $\bar{r}_{\mu f}$ is less than 1 in all cases implying that the ductility demand on the flexible edge plane in a torsionally unbalanced building is substantially less than that in the associated balanced building.

The mean value of ductility ratio for the stiff edge element $\bar{r}_{\mu s}$ is plotted against e/b in Figs. 5.4a through 5.4g. Here again, $\gamma = 1$ represents a building model without orthogonal elements and a lower value of γ indicates a higher contribution of orthogonal elements towards total torsional stiffness of the system.

Effect of γ

Figures 5.3a through g indicate that the presence of orthogonal elements reduces the ductility demand at the flexible edge of the building. In general, for a smaller value of γ , i.e. for a higher contribution of orthogonal elements, this effect is higher. However, the reduction in ductility demands on account of the presence of orthogonal elements is quite modest ($< 10\%$) and may be considered insignificant for practical purposes. This observation holds for all values of γ . As would be expected, the reduction is more significant when the orthogonal elements remain elastic. This is dealt with in greater detail later in this section.

Figures 5.4a through g indicate that the presence of orthogonal elements increases the ductility demand at the stiff edge of the building for $\Omega_R = 1$ but reduces this ductility demand for $\Omega_R = 1.25$ and 1.50 . For a lower value of γ , this effect is more pronounced. Here again, the difference in ductility demands for models with different γ values is quite small ($< 10\%$) and may be considered insignificant.

It is interesting to note that when the orthogonal elements are designed to remain

elastic, the ductility demand at stiff edge elements increases further, and in most cases is higher than that in systems in which there are no orthogonal elements. This is dealt with in greater detail in the following paragraphs.

The above results tend to indicate that for a single storey building system in which orthogonal elements as well as parallel elements yield during an earthquake, the ductility demand of an edge element depends more or less on the total torsional stiffness of the building and not on what part of it is contributed by orthogonal elements. The effect of γ on the inelastic torsional response of single storey building models is thus quite small.

Effect of yielding in orthogonal elements

Figures 5.3a through g indicate that the ductility demand at the flexible edge of a building model reduces if the orthogonal elements remain elastic during an earthquake. This is observed to be true for all values of Ω_R and γ . This reduction is quite small ($< 5\%$) for $\Omega_R = 0.75$ and 1.0 , but is a little higher (up to 15%) for $\Omega_R = 1.25$ and 1.50 , specially for large eccentricity values.

Figures 5.4a through g indicate that the ductility demand at the stiff edge of a building model in which orthogonal elements remain elastic during an earthquake, is more or less the same as that for models with yielding orthogonal elements, for $\Omega_R = 0.75$ and 1.0 . For $\Omega_R = 1.25$ and 1.50 , non-yielding orthogonal elements lead to a decreasing ductility demand (up to 5%) for low values of e/b (< 0.1), but an increasing ductility demand (up to 20%) for higher values of e/b . This is observed to be true for all values of γ . Since the stiff edge ductility demand is critical in the inelastic torsional response of single storey models studied, the increase in ductility demand for higher values of Ω_R and e/b , may be considered significant.

The explanation for the decrease in ductility demands at the flexible edge and increase in the ductility demand at the stiff edge elements of a building system, is as follows. The total response of a lateral load resisting element in a single storey building model is a combination of rotational and lateral responses. The rotational motion adds to the lateral displacement at the flexible edge but compensates the lateral displacement at the stiff edge. Except for systems that are torsionally very flexible ($\Omega_R < 0.75$), the total displacement at the stiff edge reduces as a result of torsion. In the scenario when the parallel planes are yielding, if the orthogonal elements remain elastic, they provide a relatively higher torsional resistance, thereby reducing the torsional response of the system. As a consequence, the total response at the the flexible edge reduces while the total response at the stiff edge increases.

5.3 Response of short period systems

In order to establish the effect of γ and the effect of yielding in orthogonal elements for short period systems, a set of analyses similar to the one conducted earlier is repeated. The building models selected have the same properties as described for the models in the previous set with the difference that the uncoupled translational periods are taken as 0.3 s. and 0.5 s. for x and y directions respectively. The set of 12 earthquakes used earlier is employed and the method of design is exactly the same as before. The mean value of ductility ratio at the flexible edge, $\bar{r}_{\mu f}$, is plotted as a function of e/b in Figs. 5.5a through 5.5g, for different values of Ω_R and γ . The mean value of stiff edge ductility ratio, $\bar{r}_{\mu s}$, is plotted against e/b in Figs. 5.6a through 5.6g. Also shown in these figures are the response of similar building models but in which orthogonal elements remain

elastic during ground excitation.

Figures 5.5 and 5.6 indicate a pattern very similar to the one obtained from Figs. 5.3 and 5.4. All the conclusions drawn from the results of previous analyses hold true for short period building models analysed.

5.4 Period dependence

Results presented in the preceding section indicate that the effect of orthogonal elements on the seismic response of asymmetric buildings is similar for short period of 0.5 s and medium period of 1.0 s. This indicates that the effect of orthogonal elements is not dependent on time period of the building model. However, a previous study (Correnza *et al*, 1994) has concluded that the presence of orthogonal elements reduces the ductility demand at the flexible edge, drastically for the short period range systems. This is in contrast to the findings of the present study. Therefore, an investigation of the variation of ductility demands in 3-plane as well as 5-plane models, with respect to time period is conducted in this section. This is done to study the period dependency of the effect of orthogonal elements on inelastic torsional response.

The building models studied are a 3-plane model without orthogonal elements, as shown in Fig. 4.2 and referred to as model A, and a 5-plane model, as shown in Fig. 5.1 and referred to as Model B. The eccentricity is chosen as $0.3b$. For model A, it is assumed that $k_1 = k_2$. These assumptions yield $k_1 = k_2 = 0.1333K_y$, and $k_3 = 0.7333K_y$. For model B, it is assumed that $K_x = K_y$. Further, the torsional stiffness of model B is taken as same as that for model A. This is achieved by reducing the stiffness of planes 1 and 3 by equal amounts and increasing the stiffness of plane 2 appropriately so that K_y remains

unchanged. Model B has the following properties, $k_1 = 0.00833K_y$, $k_2 = 0.38333K_y$, $k_3 = 0.60833K_y$ and $k_4 = k_5 = 0.5K_y$. For the selected values of the stiffnesses, Ω_R for both models A as well as B, turns out to be about 1.1. The load versus lateral displacement relationships for all planes is assumed to be bilinear with a post yield stiffness of 2% of the initial stiffness.

The earthquake motions used in this part of the study are the same 12 earthquakes described earlier. As done earlier, the records are scaled to a peak ground acceleration of $0.3g$, and the average of the elastic spectra for the scaled records and 5% damping is used as the design spectrum. However, for this set of analysis $R = 5$ is chosen so that the design strength is 1/5th of elastic base shear obtained from the design spectrum. The strength of individual planes in the torsionally balanced (TB) and torsionally unbalanced (TUB) models are obtained by procedures described earlier.

In the models described above, the total stiffness K_y is selected to provide a specified period. The period is varied over a range of values.

The modelling details described in the previous paragraphs have been selected to conform to those used in studies by Correnza et al (1992, 1994) and Chandler et al (1994). In their studies, these authors have arrived at conclusions that are contradictory to those obtained from the present study. Comparison with their results is useful in explaining the reasons for this contradictions.

Dynamic analyses for response to the 12 scaled earthquake records are carried out on the TB as well as TUB models A and B, following the procedures described earlier. Results for models A are presented in Table 5.1. The ductility demand in all planes of the TB model is given in column 2 of the table. The demands in the flexible and

stiff edge planes for TUB model, with centre of mass (CM) restrained at the geometric centre, are shown in columns 5 and 6 of Table 5.1, respectively. As described earlier, the design strengths of planes in the TUB models are determined by a procedure that accounts for the effect of accidental eccentricity. The design procedures can be considered adequate only if the response induced by the design earthquake meets the requirements of design even when accidental eccentricity effects are present in the calculated response. In other words, the dynamic analyses must include the effect of accidental eccentricity. As stated earlier, this is achieved by shifting the mass centre by a distance $\pm 0.05b$ from the geometric centre, and taking the larger of the ductility values determined from the analysis of the two TUB models so obtained. The revised ductility values determined as above are shown in columns 7 and 8 of Table 5.1.

An important criterion in the seismic design is that the ductility demand in any resisting plane should not exceed the ductility capacity. Since the force modification factor R has been assumed to be approximately equal to the ductility capacity μ_t , it implies that the ductility demand should not, in this case, exceed 5. Now it is known that selection $R = \mu_t$ which is based on the assumptions of equal total displacements in the elastic and inelastic models in a single-degree-of-freedom systems, does not always ensure that the ductility demand μ is equal to the target ductility μ_t . Consequently, even in the SDOF system, the ductility demand μ may be smaller or larger than the target ductility μ_t . The TB model defined in this study is equivalent to a SDOF system, and as can be seen from the values presented in column 1 of Table 5.1, μ is at times larger than μ_t and at other times smaller than μ_t . The purpose of torsional provisions is to ensure that the ductility demand imposed on asymmetric systems is no greater

Table 5.1: Effect of period on the torsional response of asymmetric building models

Period s.	Ductility TB model μ_b	Ductility TBR model		Ductility TUB model no mass shift		Ductility TUB model mass shifted $\pm 0.05b$		Ductility ratios			
		Flexible edge μ_{brf}	Stiff edge μ_{brs}	Flexible edge μ_{uf1}	Stiff edge μ_{us1}	Flexible edge μ_{uf2}	Stiff edge μ_{us2}	$\bar{\tau}_{\mu1}$ $\frac{\mu_{uf2}}{\mu_b}$	$\bar{\tau}_{\mu2}$ $\frac{\mu_{us2}}{\mu_b}$	$\bar{\tau}_{\mu3}$ $\frac{\mu_{uf1}}{\mu_{brf}}$	$\bar{\tau}_{\mu4}$ $\frac{\mu_{us1}}{\mu_{brs}}$
1	2	3	4	5	6	7	8	9	10	11	12
0.20	10.251	4.005	7.763	5.437	6.589	6.035	9.690	0.628	1.087	1.322	0.958
0.30	8.138	3.857	6.757	3.677	5.905	3.954	7.714	0.546	1.039	0.899	1.053
0.50	6.165	3.323	5.159	3.377	5.966	3.410	7.292	0.602	1.339	1.065	1.224
0.70	6.660	3.425	5.626	3.343	5.479	3.871	6.751	0.624	0.995	1.003	0.992
1.00	5.462	2.879	4.700	2.543	4.529	2.462	5.515	0.508	0.996	0.959	0.956
1.20	4.974	3.382	4.351	2.244	5.197	2.459	5.789	0.500	1.034	0.736	1.146
1.50	5.262	3.067	4.497	2.710	5.827	2.866	6.050	0.553	1.114	0.956	1.243
1.70	4.535	2.622	3.906	2.589	4.841	2.815	5.043	0.621	1.156	1.046	1.287
2.00	4.378	2.680	3.865	2.311	4.386	2.275	4.801	0.532	1.161	0.867	1.230
2.60	4.291	2.852	3.845	1.841	4.163	1.953	4.766	0.516	1.173	0.727	1.240
3.00	4.585	2.885	4.021	1.758	4.567	1.748	4.896	0.438	1.086	0.709	1.196

than that imposed on the corresponding balanced system. The design requirement can therefore be studied in an alternative form: the ratio of TUB model response to TB model response, where the TUB model response has been calculated by including the effect of accidental eccentricity, should not exceed 1. Average ratios $\bar{r}_{\mu f}$ and $\bar{r}_{\mu s}$ for the flexible and stiff edge planes respectively are shown in columns 9 and 10 of Table 5.1, and plotted in Figs. 5.7a and 5.7b as a function of period. It is seen that the ductility ratios do not vary significantly with period. The flexible edge ratio, $\bar{r}_{\mu f}$, is always less than 1.0. The stiff edge ratio, $\bar{r}_{\mu s}$, is slightly in excess of 1.0 which is expected for Ω_R values close to 1.0, based on the results presented earlier.

The procedure described in the preceding paragraph is a logical method of measuring the adequacy of torsional design provisions. Correnza and co-authors (1992, 1994) however specify a different method of evaluating the design procedures. In their method, they do not include the effect of accidental eccentricity in carrying out the dynamic analyses. Instead, they define a revised torsionally balanced reference model (TBR). In this model the strengths of resisting planes are obtained from a static procedure by applying the shear V_0 at $\pm 0.01b$ from CR. The ductility demands in the flexible and stiff edge planes of TBR model are shown in columns 3 and 4 of Table 5.1, respectively. The ductility ratios for flexible edge plane are obtained by dividing the value in column 5 (TUB with no shift of CM) by those in column 3. Similarly, the ductility ratios for stiff edge plane are obtained on dividing column 6 by column 4. These are shown in columns 11 and 12 of Table 5.1 respectively and have also been plotted in Figs. 5.7a and 5.7b. The plots show the flexible edge ductility ratio as being dependent on the period and significantly higher than 1 in the short and medium period range. These conclusions are not correct as can

be seen by reference to the values of ductility demand in the flexible edge plane given in column 7 of Table 5.1. These ductility demands are inclusive of the effect of accidental eccentricity. They are in all cases significantly smaller than the ductility demand in the corresponding TB model. In fact, with the exception of the value corresponding to a period of 0.2 s, they are also less than R , which is 5 in this case. Conclusions to the contrary have been reached only because of the way in which a reference value has been defined.

The stiff edge ductility ratio is seen to be more or less uniform and not dependent on the period, irrespective of what reference model is used.

The ductility ratios for TUB model B are also shown in Figs. 5.7a and b. They are only slightly better but not significantly so in comparison to those for model A. Apparently, torsional behaviour depends strongly on the frequency ratio Ω_R , or the torsional stiffness, and for identical torsional stiffness, the responses of models with and without orthogonal elements are not significantly different.

To investigate the period dependence of the torsional response further, analytical studies are carried out on building models with a range of values of eccentricity (e/b) and Ω_R . The average values of ductility ratios at the flexible edge, $\bar{r}_{\mu f}$, are plotted against time period in Figs. 5.8a through d. The values of $\bar{r}_{\mu s}$ versus time period are shown in Figs. 5.9a through d. It is evident that ductility demands at both flexible and stiff edges can be considered more or less period independent. Figures 5.8 and 5.9 also indicate that the presence of orthogonal elements reduces the ductility demand at the flexible only slightly and increases the ductility demand at the stiff edge, in general. These observations are consistent with the conclusions reached in the previous sections.

5.5 Collapse mechanism analysis

In a recent work, Paulay (1996) has studied the inelastic torsional response of single storey building models. In that study, buildings are classified into two categories: torsionally restrained and torsionally unrestrained. In a torsionally unrestrained building, all elements that resist torsion yield during an earthquake. In a torsionally restrained building, some of the resisting elements (namely orthogonal elements) remain elastic during the ground excitation. One of the conclusions drawn in the above study is that the ductility demand at the stiff edge of a torsionally unrestrained model is much higher than that for a torsionally restrained model. The above conclusion is contradictory to the results obtained in the previous sections. An investigative review of Paulay's work is therefore conducted here.

Consider a three-plane asymmetric building model as shown in Fig. 5.10a. According to Paulay's definition, this building model is torsionally unrestrained for an earthquake motion in the y -direction. The elastic spectral acceleration for the design earthquake corresponding to the uncoupled translation period of the building, causes an elastic shear V_e in the building. The associated torsionally balanced building would therefore be designed to have a total strength of $V = V_e/\mu$, where μ is the allowable ductility. This is based on the assumption usually made that the total displacement induced by the design earthquake in an inelastic building, comprising the yield displacement Δ_y and plastic displacement Δ_p , is equal to the displacement produced in the associated elastic building.

Assume that the strengths of individual planes in the torsionally unbalanced buildings are obtained from Eqs. 4.8 through 4.11 with $e_{d1} = e_{d2} = e$. The corresponding yield

displacements for the three planes are: Δ_{y1} , Δ_{y2} , and Δ_{y3} , as shown in Fig. 5.10b. Paulay reasons that under the design earthquake, the building centre line AB may be assumed to displace first to the position A_1B_1 , as shown in Fig. 5.10b. At this stage, all the planes are yielding and the building is unrestrained against any additional torsion. Then, if the actual strength of the flexible plane, Plane 1, is slightly higher than its design value, the building will pivot about A_1 and displace until the plastic displacement at the centre of mass is Δ_p , given by

$$\Delta_p = (\mu - 1)\Delta_{y2} \quad (5.1)$$

The plastic displacement at the stiff edge will be $2\Delta_p$. Consequently the ductility demand in the stiff plane is given by

$$\mu_3 = 1 + 2(\mu - 1)\frac{\Delta_{y2}}{\Delta_{y3}} \quad (5.2)$$

Paulay argues that μ_3 will thus be substantially higher than the target value μ , and to keep μ_3 below the ductility capacity, the design ductility μ must be selected to be considerably lower than the ductility capacity.

The assumptions made by Paulay are highly conservative. First, the interaction between lateral and torsional responses reduces the base shear below V , so that the plastic displacement at the centre of mass is smaller than Δ_p . Second, contrary to the assumptions made by Paulay, the torsionally unbalanced building does not pivot about A_1 during its dynamic response to the earthquake.

To illustrate the above points, consider a specific example of single storey building model in which $e = 0.1b$, $a/b = 0.5$, $T = 1s$. and the design earthquake is El-Centro. The

resisting planes in the building are assumed to be shear frames, and the diaphragm is considered infinitely rigid. The floor mass is 400 t and mass moment of inertia is $54,000\text{ t-m}^2$. Width of the building perpendicular to the direction of earthquake is taken as 36 m . Strain hardening of 0% is assumed. The El-Centro earthquake produces an elastic shear $V_e = 2025\text{ kN}$, in the building. The ductility capacity is chosen as $\mu = 4$, therefore the design shear is $V = 506\text{ kN}$. For the torsionally balanced building the strengths of resisting planes are proportional to their stiffnesses.

The properties of the torsionally balanced building are shown in Table 5.2. The maximum displacements produced in the building by El-Centro earthquake and the corresponding ductilities are also shown in that table. As would be expected, the displacements and ductilities are identical for all planes. The ductility value is in fact 3.02, which is lower than 4.0. The equal displacement assumption is obviously conservative for this earthquake. However, in assessing the effect of torsion we will use 3.02 as the reference value.

Table 5.2: Characteristics and response of the 3-plane torsionally balanced but unrestrained building model

Plane No	Stiffness kN/m	Yield strength kN	Yield displacement m	Maximum total displacement m	Ductility
1	2026	65.1	0.0321	0.097	3.02
2	8580	275.4	0.0321	0.097	3.02
3	5185	166.5	0.0321	0.097	3.02

The stiffness of the resisting planes in the torsionally unbalanced building are iden-

tical to those in the torsionally balanced building; the strengths derived from Eqs. 4.8 to 4.11 are shown in Table 5.3 along with the yield displacements. The total displacements produced by the El-Centro earthquake are also shown in Table 5.3, along with the corresponding ductilities.

Table 5.3: Characteristics and response of the 3-plane torsionally unbalanced and unrestrained building model

Plane No	Stiffness kN/m	Yield strength kN	Yield displacement m	Maximum total displacement m	Ductility
1	2026	102.5	0.0506	0.121	2.390
2	8580	301.9	0.0352	0.097	2.757
3	5185	102.5	0.0198	0.080	4.044

According to Paulay's Eq. 5.2 and using $\mu = 3.02$, the ductility demand in Plane 3 should be 8.18. The actual value is just 4.04. Obviously, Paulay's estimate is highly conservative. This is easily explained by observing Fig. 5.10c which shows the displaced shape of the building axis at the time when the stiff edge displacement is maximum. It is clear that the plastic displacement at the centre of mass is smaller than that in the balanced building and pivoting does not take place about the yield position of the flexible plane.

It may be argued that pivoting could take place if the actual strength of the flexible plane were higher than the design value. Therefore, the above analysis is repeated with flexible plane strength values 10% and 30% higher than the design strength. The corresponding values of the stiff plane ductility are found to be 4.15 and 4.30 respectively;

still much smaller than that predicted by Paulay. Figure 5.11 shows the displacement of building axis for different levels of flexible edge strength, at an instant when the stiff edge displacement is a maximum. This clearly indicates that contrary to the prediction of Paulay, pivoting does not take place about point A_1 .

The above conclusions were found to be true for other building configurations considered in the present study. The ductility demands at the stiff edge reduce further, both when strain hardening is included and when Eqs. 4.5 through 4.7 are used to calculate the value of design eccentricity.

Figure 5.12a shows a torsionally restrained building in which it is assumed that orthogonal elements i.e. planes 4 and 5 remain elastic when the design earthquake strikes the building along the y-direction. Figure 5.12b shows Paulay's estimate for the displacement along the axis of such a building. The ductility in the stiff plane is now given by

$$\mu_3 = 1 + (\mu - 1) \frac{\Delta_{y2}}{\Delta_{y3}} \quad (5.3)$$

This is significantly less than that in the torsionally unrestrained building. To assess whether torsional restraint due to elastic orthogonal elements would improve the response substantially at the stiff edge, as predicted by Paulay, the analysis in the previous case is repeated for the building shown in Fig. 5.12a. The plan dimensions, the mass, the eccentricity and the frequency ratio are the same as those for the building in Fig. 5.10a. The characteristics of the torsionally balanced buildings as well as its response to El-Centro earthquake are shown in Table 5.4. Similar information for the torsionally unbalanced building is presented in Table 5.5.

Table 5.4: Characteristics and response of the 5-plane torsionally balanced and restrained building model

Plane No	Stiffness kN/m	Yield strength kN	Yield displacement m	Maximum total displacement m	Ductility
1	381.6	12.25	0.0321	0.097	3.02
2	11869.0	381.1	0.0321	0.097	3.02
3	3539.9	113.6	0.0321	0.097	3.02
4	6579.8	Large			
5	6579.8	Large			

Table 5.5: Characteristics and response of the 5-plane torsionally unbalanced and restrained building model

Plane No	Stiffness kN/m	Yield strength kN	Yield displacement m	Maximum total displacement m	Ductility
1	381.6	19.31	0.0506	0.123	2.43
2	11869.0	417.64	0.0352	0.096	2.73
3	3539.9	70.00	0.0198	0.095	4.80
4	6579.8	Large			
5	6579.8	Large			

Paulay's Eq. 5.3 gives a value of 4.6 for the ductility demand in the stiff edge plane of the building (plane No. 3). The value obtained from the analysis is 4.8. It should be noted that this value is higher than that for a corresponding torsionally unrestrained building. The displacement of the building axis at an instant when stiff edge displacement is maximum, is shown in Fig. 5.12c.

In summary, a large number of analytical studies carried out as part of the work on torsionally unrestrained building models, whose results are presented in the previous sections, do not show trends predicted by Paulay's study. The lack of torsional restraint following yielding of resisting planes does not cause any significant problems. It is the overall torsional stiffness and not just the presence of orthogonal elements which is important in assessing the torsional behaviour of inelastic building systems.

The trends observed in the present study also indicate that the results of previous studies, based on building models in which orthogonal elements remain elastic during an earthquake, provide conservative estimates of ductility demands at the stiff edge.

5.6 Summary and conclusions

The torsional behaviour of asymmetric buildings subjected to earthquake motion is strongly influenced by the torsional stiffness as measured by the ratio of uncoupled rotational frequency to the uncoupled translational frequency. The torsional stiffness may arise from the planes parallel to the direction of earthquake, or as in most often the case, is a sum of contributions from resisting elements both parallel and perpendicular to the direction of earthquakes.

To explore the effect of orthogonal elements on inelastic torsional response, analytical

studies have been carried out on 5-plane mono-symmetric single storey building models, as shown in Fig. 5.1. A set of 12 earthquakes each having ground motion data in two orthogonal horizontal directions has been used. The lateral resisting elements of the building oriented in both perpendicular horizontal directions have been designed for one fourth of the average elastic spectra obtained for major direction component of ground motions. The models are now subjected to bi-directional ground motions. All the resisting elements undergo significant inelastic deformations.

The results indicate that presence of orthogonal elements reduces the ductility demand at the flexible edge of the building, though by a small amount ($< 10\%$). On the other hand, the ductility demand in the stiff edge planes may be reduced or increased depending upon the value of frequency ratio Ω_R . In all cases, this reduction or increase is fairly moderate ($< 10\%$). This leads to the conclusion that for single storey building systems, in which all the lateral-load resisting elements yield during an earthquake, the ductility demand of an edge plane depends more or less on the total torsional stiffness of the building and not on what part of it is contributed by orthogonal elements.

Analytical studies mentioned above are repeated with high yield strength values for the orthogonal planes. This is done to ensure that the orthogonal planes remain elastic during the analysis. The results indicate that the ductility demand at the flexible edge of a building model reduces if the orthogonal elements remain elastic during an earthquake. The reduction in ductilities can be upto 15% depending upon the value of frequency ratio Ω_R . It is observed that when the orthogonal elements are designed to remain elastic, the ductility demands for stiff edge elements increase. This increment can be as high as 20%, particularly for large values of e/b .

A study of the period dependence of ductility demands in 3-plane models, as shown in Fig. 4.2 and 5-plane models as shown in Fig. 5.1, is carried out. The results indicate that the influence of orthogonal elements on ductility demands of planes parallel to major direction of ground excitation, as noted in the previous paragraphs, is consistent for all periods.

A study of the collapse mechanism analysis reveals that contrary to the conclusions reached at by Paulay (1996), the ductility demands in torsionally unrestrained models are not very different from those in torsionally restrained models.

Figure 5.1: Plan view of a 5 plane single storey mono-symmetric building model

Figure 5.2: Mean elastic spectrum for the set of 12 earthquake records

Figure 5.3: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8, 1.0$

(b)

(c)

Figure 5.3: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (b) $\Omega_R = 1.0$, $\gamma = 0.4, 0.6, 1.0$ (c) $\Omega_R = 1.0$, $\gamma = 0.8, 1.0$

Figure 5.3: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (d) $\Omega_R = 1.25$, $\gamma = 0.4, 0.6, 1.0$ (e) $\Omega_R = 1.25$, $\gamma = 0.8, 1.0$

Figure 5.3: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (f) $\Omega_R = 1.5$, $\gamma = 0.4, 0.6, 1.0$ (g) $\Omega_R = 1.5$, $\gamma = 0.8, 1.0$

Figure 5.4: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8, 1.0$

Figure 5.4: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (b) $\Omega_R = 1.0$, $\gamma = 0.4, 0.6, 1.0$ (c) $\Omega_R = 1.0$, $\gamma = 0.8, 1.0$

Figure 5.4: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (d) $\Omega_R = 1.25$, $\gamma = 0.4, 0.6, 1.0$ (e) $\Omega_R = 1.25$, $\gamma = 0.8, 1.0$

Figure 5.4: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 1.0$ s., (f) $\Omega_R = 1.5$, $\gamma = 0.4, 0.6, 1.0$ (g) $\Omega_R = 1.5$, $\gamma = 0.8, 1.0$

Figure 5.5: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8, 1.0$

(b)

(c)

Figure 5.5: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (b) $\Omega_R = 1.0$, $\gamma = 0.4, 0.6, 1.0$ (c) $\Omega_R = 1.0$, $\gamma = 0.8, 1.0$

Figure 5.5: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (d) $\Omega_R = 1.25$, $\gamma = 0.4, 0.6, 1.0$ (e) $\Omega_R = 1.25$, $\gamma = 0.8, 1.0$

Figure 5.5: Ratio of flexible edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (f) $\Omega_R = 1.5$, $\gamma = 0.4, 0.6, 1.0$ (g) $\Omega_R = 1.5$, $\gamma = 0.8, 1.0$

Figure 5.6: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (a) $\Omega_R = 0.75$, $\gamma = 0.8, 1.0$

Figure 5.6: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (b) $\Omega_R = 1.0$, $\gamma = 0.4, 0.6, 1.0$ (c) $\Omega_R = 1.0$, $\gamma = 0.8, 1.0$

(d)

(e)

Figure 5.6: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (d) $\Omega_R = 1.25$, $\gamma = 0.4, 0.6, 1.0$ (e) $\Omega_R = 1.25$, $\gamma = 0.8, 1.0$

Figure 5.6: Ratio of stiff edge ductility demand in a 5 plane torsionally unbalanced building model to that in the associated torsionally balanced building model, mean from 12 earthquakes, $T = 0.5$ s., (f) $\Omega_R = 1.5$, $\gamma = 0.4, 0.6, 1.0$ (g) $\Omega_R = 1.5$, $\gamma = 0.8, 1.0$

Figure 5.7: Average ductility ratios in 3 and 5 plane models, (a) flexible edge, (b) stiff edge

Figure 5.8: Average flexible edge ductility ratios in 3 and 5 plane models, (a) $e/b = 0.1$, (b) $e/b = 0.1$

(c)

(d)

Figure 5.8: Average flexible edge ductility ratios in 3 and 5 plane models, (c) $e/b = 0.2$, (d) $e/b = 0.3$

Figure 5.9: Average stiff edge ductility ratios in 3 and 5 plane models, (a) $e/b = 0.1$, (b) $e/b = 0.1$

Figure 5.9: Average stiff edge ductility ratios in 3 and 5 plane models, (c) $e/b = 0.2$, (d) $e/b = 0.3$

Figure 5.10: (a) 3-plane torsionally unrestrained building model, (b) Paulay's displacement pattern, (c) displacement pattern for El-Centro earthquake at the instant when stiff edge displacement is maximum

Figure 5.11: Displacement pattern for a 3-plane torsionally unrestrained building model at the instant when stiff edge displacement is maximum, El-Centro earthquake, different strengths of the flexible edge plane

Figure 5.12: (a) 5-plane torsionally restrained building model, (b) Paulay's displacement pattern, (c) displacement pattern for El-Centro earthquake at the instant when stiff edge displacement is maximum

Chapter 6

Inelastic analysis of torsionally unbalanced multistorey building models

6.1 General

The inelastic response of multistorey buildings is very complex in nature. It is affected not only by characteristics of the building, but also by the characteristics of the earthquake, including the frequency content, the duration, and even the exact nature of the time history of ground acceleration. Thus, two different ground motions even with similar frequency content, similar maximum amplitude, and similar duration may produce significantly different responses. It is therefore difficult to draw generalised conclusions from the results of a set of analyses. Nevertheless, response studies on simplified models of buildings can provide a useful insight into the inelastic seismic behaviour of building structures and the effect of torsional motion. With this in view, response analyses have been carried out in this study on a series of torsionally unbalanced and associated balanced multistorey building models for a set of selected earthquake records.

6.2 Examples of multistorey buildings

Issues involved

The results obtained from the analyses of single storey inelastic models indicate that design expressions based on the results of elastic analyses can be successfully applied to the design of single storey inelastic buildings. Single storey buildings designed according to the proposed design eccentricity expressions showed almost no additional ductility demands due to torsion. It is of interest to know the extent to which the results obtained earlier, can be used for multistorey buildings that are expected to respond in the inelastic range. For this purpose, analytical studies are carried out on a number of multistorey building models. The following are the main issues addressed in this study:

1. Do the proposed design eccentricity expressions derived for a single storey building, namely Eqs. 4.5, 4.6 and 4.7, provide a rational design for multistorey buildings? What improvements, if any, are necessary?
2. What is the nature of ductility demands in a multistorey building? What are the higher mode effects on the additional ductility demands due to torsion?

Analysis procedure

Keeping the above-mentioned issues in mind, a set of multistorey building models are analysed. The models studied are of shear frame type with a plan view similar to Fig. 4.2. The steps involved in the analysis are summarised as below:

The building model is first analysed for its elastic response to the selected ground motion acting in the y direction. In this analysis, the CM is taken as being coincident with the CR. As a result the building undergoes only a translation in the y direction.

This analysis provides the maximum elastic storey shears in the building, V_{ej} , as well as the maximum shears in each storey of the three resisting planes. The elastic shears are scaled by 1/4 to obtain the total design storey strength, V_{0j} , and the design storey strengths in the three planes, f_{ij} , for the torsionally balanced building. Using the storey strengths V_{0j} , the strengths of individual planes in the unbalanced buildings are obtained from Eqs. 4.8 through 4.11.

The balanced and unbalanced building models are now analysed for their response to the selected ground motion. Since the strengths provided are only 1/4 of the maximum elastic shears, the buildings are expected to undergo substantial inelastic deformations. The response quantities obtained from the inelastic analyses include maximum interstorey displacements and maximum interstorey ductilities for resisting planes on the flexible and stiff sides.

Computer program DRAIN-TABS (Guendelman and Powell, 1977) has been used for carrying out the inelastic analyses. The program performs a step-by-step time history numerical analysis using the average acceleration method of integration. Convergence of the results of numerical integration has been studied by selecting several time step values and a time step of 0.002 s. has been selected for the analyses presented here. Also, a Rayleigh type damping matrix has been used with the damping values adjusted to 5% in each of the first two modes of the balanced building.

Example 1

Building description

The models selected in this example represent 5 storey shear frame buildings of *special class*. The plan view is uniform across the height and is similar to the one shown in

Fig. 4.2. The buildings have 3 lateral load resisting planes oriented in the y direction and one resisting plane along the x axis. The planes in y direction have arbitrary stiffnesses so that the centre of stiffness is offset by a distance e from the origin which is located on the central plane. The eccentricity e as well as the storey-wise frequency ratio Ω_R are assumed to be constant throughout the height. The relative values of the total storey stiffnesses across the height of the building are adjusted so that when lateral forces distributed in the form of an inverted triangle are applied to the building, the deflected shape is also an inverted triangle. This and the specified fundamental period of 0.5 s, allows the determination of the absolute values of storey stiffnesses.

Mass of each floor of the building is taken as 198.16 t , mass moment of inertia as 26,752.3 $t - m^2$, aspect ratio as 0.5, storey height as 4 m , and floor width b as 36 m . Each plane is assumed to have a bilinear force-displacement relationship. The second branch of the curve representing strain hardening is assumed to have a slope that is 5% of that of the initial branch. A set of building models with five values of eccentricity, $e/b = 0.05, 0.1, 0.15, 0.2$ and 0.25 and three values of $\Omega_R, 0.75, 1.0, 1.25$ are considered. Models with $\Omega_R = 0.75$ and $e/b > 0.15$ are physically not possible. This leaves us with 13 building models whose responses are investigated.

Earthquake motions

Three earthquake motions are used in the study: El-Centro, 1940; San Francisco, 1957; and San Fernando, 1971. Description of the earthquakes are presented in Table 6.1, and their elastic response spectra are shown in Fig. 6.1. It will be noted that the three earthquakes have very different peak ground acceleration to peak ground velocity, a/v , ratios. El-Centro earthquake has an a/v ratio in the intermediate range, San Francisco

earthquake has an a/v ratio in the high range, and San Fernando earthquake has an a/v ratio in the low to very low range. In the present study, each earthquake is scaled so that its peak ground acceleration value is $0.3g$.

Table 6.1: Characteristics of the selected earthquake records

No	Earthquake	Date	Magn	Site	Epic Dist (km)	Comp	Max Acc A(g)	Max Vel V(m/s)	A/V	Soil Cond
1	Imperial Valley California	May 18 1940	6.6	El Centro	8	S00E	0.348	0.334	1.04	Stiff soil
2	San Francisco California	22 Mar 1957	5.25	State Bldg S. F.	17	S09E	0.085	0.051	1.67	Stiff soil
3	San Fernando California	9 Feb 1971	6.4	4680 Wilshire Blvd, LA	38	N75W	0.084	0.209	0.40	Stiff soil

Details of analysis

A set of building models described earlier, is subjected to three earthquake motions selected. To take the effect of accidental torsion into account, the CM is moved by $\pm 0.05b$, as described earlier in Section 4.4. An associated balanced model is achieved by moving CM to CR. The ratio of interstorey ductility ratio $r_\mu = \mu_u/\mu_b$, is taken as a measure of the effect of torsion. If this ratio is less than 1, the ductility demand in the unbalanced building is less than that in balanced building, and the design provisions meant to account for torsion are adequate.

Figures 6.2a and 6.2b show the ratios of interstorey ductilities for the flexible and stiff

edge planes in the first storey of the buildings, subjected to the El-Centro earthquake. Figures 6.3a and 6.3b show similar results for the fifth storey. For the first storey the flexible edge ductility in the unbalanced building is always less than that in the balanced building; however, the stiff edge ductility is higher for a range of values of e/b , particularly for $\Omega_R = 1$. For the fifth storey, the ductility ratios for the flexible edge in the unbalanced building are higher than those in the balanced building for $\Omega_R = 1.25$, although the maximum difference is only of the order of 10%. On the other hand, the fifth storey ductility ratio for the stiff edge in the torsionally balanced building is significantly higher than that in the balanced building for a range of values of e/b and Ω_R .

It should be noted that in a multistorey building, the distribution of ductility is not uniform across the height, even for a torsionally balanced building. In some storeys, generally the lowest, the ductility demand is higher than the target ductility, while in the other storeys it is lower. Also, the distribution of ductility across the height of the torsionally unbalanced building may be quite different from that in the associated torsionally balanced building. Because of these non-uniform distributions, a ratio of ductilities that is higher than 1 does not necessarily imply that the ductility demand in torsionally unbalanced building has exceeded the target ductility, which in the current study is 4. To demonstrate this, the distribution of stiff edge ductilities across the height for the most critical combination, $e/b = 0.2$ and $\Omega_R = 1.0$, is shown in Fig. 6.4. In this case the ductility demand in the first and second storeys of the unbalanced building is higher than that in the balanced building, and also somewhat higher than the target ductility. In the top storeys the stiff edge ductility demand is less than the target ductility of 4, even though it is higher than the corresponding ductility demand in the balanced

building.

For the sake of brevity, only the results in the form of distribution graphs similar to Fig. 6.4 are shown here. Results have been generated for the whole range of building models but are not included here. Selected results related to cases where the ductility demand in the unbalanced building exceeds target ductility as well as the corresponding ductility demands in the balanced building are included. The number of such cases is quite small, implying that in a vast majority of cases the ductility demand in the unbalanced buildings are quite satisfactory.

For the San Francisco and San Fernando earthquakes, the flexible edge ductility demand in the unbalanced building is mostly less than that in the balanced building. In a few instances, where the demand is higher in the unbalanced building, it is still less than the target ductility of 4. For the stiff edge, the demand in the unbalanced building sometimes exceeds the target value as well as the demand in the balanced building, but such cases are few. Such exceptional cases are presented in Figs. 6.5 and 6.6.

Example 2

Building description

The models selected in this example are 10 storey shear frame buildings of *special class*, very similar to the ones described in the previous example. The plan view is uniform across the height and is similar to the one shown in Fig. 4.2. The fundamental period of the first mode of vibration in the y direction is taken as 1.0. Storey stiffnesses are selected according to the procedure used in the previous example. Other structural properties of the models are also the same as the ones in the previous example. A set of building models with five values of eccentricity, $e/b = 0.05, 0.1, 0.15, 0.2$ and 0.25 and three values

of Ω_R , 0.75, 1.0, 1.25 is considered.

Details of analysis

The earthquake motions are represented by three ground motion records, used in the previous example.

The selected set of building models are subjected to three earthquake motions described. To take the effect of accidental torsion into account, the CM is moved from the central plane by $\pm 0.05b$. An associated balanced model is achieved by moving CM to CR. The ratio of interstorey ductility ratio $r_\mu = \mu_u/\mu_b$, is taken as a measure of the effect of torsion.

Here again the flexible edge ductility demand in the unbalanced building is, in most cases, less than that in the balanced building, and even in cases where it is higher it is still less than the target ductility of 4. This is true of all three earthquakes and all combinations of e/b and Ω_R . Graphs of ductility ratios have been generated but are not included here.

Stiff edge ductility demand in unbalanced building exceeds, in a few cases, both the target ductility and the ductility demand in associated balanced building. All of these cases belong to a frequency ratio Ω_R of 1. Examples of critical cases are shown in Figs. 6.7 and 6.8. In these cases, the ductility demand in the balanced building is also higher than the target ductility.

Example 3

Building description

The models selected in this example are 20 storey shear frame buildings of *special class*, very similar to the ones used in the previous example. The plan view is uniform across

the height and is similar to the one shown in Fig. 4.2. The first uncoupled lateral period of vibration in the y direction is taken as 2.0. Stiffnesses of the storeys are selected according to the procedure described in the Example 1. Mass of each floor is taken as 198.16 t , mass moment of inertia as 26,752.3 $t - m^2$, aspect ratio as 0.5, storey height as 4 m , and floor width b as 36 m . Strain hardening of 5% is considered. A set of building models with five values of eccentricity, $e/b = 0.05, 0.1, 0.15, 0.2$ and 0.25 and three values of Ω_R , 0.75, 1.0, 1.25 are considered.

Details of analysis

The earthquake motions are represented by three ground motion records, used in the previous examples.

The selected set of building models are subjected to three earthquake motions described. To take the effect of accidental torsion into account, the CM is moved by $\pm 0.05b$, as done earlier. An associated torsionally balanced model is achieved by moving CM to CR. The ratio of interstorey ductility ratio $r_\mu = \mu_u/\mu_b$, is taken as a measure of the effect of torsion.

In this case too, the flexible edge ductility demand in the unbalanced building is always satisfactory. For the stiff edge, ductility demand does at times exceed both the target ductility and the demand in the associated balanced building, but such cases are few and occur mostly for $\Omega_R = 1$. Examples of such cases are presented in Figs 6.9 and 6.10.

Example 4

Building description

The models selected in this example represent 5 storey shear frame buildings of a general type. These models are similar to those selected in Example 1, except that the eccentricity ratio e/b varies along the height. The top three stories have an eccentricity ratio of 0.1 and the bottom two stories have an eccentricity ratio of 0.15. The frequency ratio Ω_R which is defined storey by storey, is constant along the height of the building. Mass of each floor is taken as 198.16 t , mass moment of inertia as 26,752.3 $t-m^2$, aspect ratio as 0.5, storey height as 4 m , and floor width b as 36 m . Each plane is assumed to have a bilinear force-displacement relationship as in the previous examples. A set of building models with three values of Ω_R , 0.75, 1.0, and 1.25 are considered.

Details of analysis

The earthquake motions are represented by three ground motion records, used in the previous example.

The selected set of building models are subjected to three earthquake motions described. To take the effect of accidental torsion into account, the CM is moved from the central plane by $\pm 0.05b$. An associated balanced model is achieved by moving CM to CR. The ratio of interstorey ductility ratio $r_\mu = \mu_u/\mu_b$, is taken as a measure of the effect of torsion.

Here again the flexible edge ductility demand in the unbalanced building is, in most cases, less than that in the balanced building, and even in cases where it is higher it is still less than the target ductility of 4. This is true of all three earthquakes and all combinations of e/b and Ω_R . Graphs of ductility ratios are generated but not included

here.

In a few cases, the stiff edge ductility demand in the unbalanced building does exceed both the target ductility and the ductility demand in the associated balanced building. Such critical cases are shown in Figs. 6.11, 6.12 and 6.13. In all of these cases, the ductility demand in the balanced building is also higher than the target ductility.

Example 5

Building description

The models selected in this example represent 10 storey shear frame buildings of a general type. These models are similar to those selected in Example 2, except that the eccentricity ratio e/b varies along the height. The values of eccentricity ratios are selected as 0.05 for top three floors, 0.1 for middle three floors and 0.15 for bottom three floors. The frequency ratio Ω_R which is defined storey by storey, is constant along the height of the building. Mass of each floor is taken as $198.16\ t$, mass moment of inertia as $26,752.3\ t - m^2$. The dimension of building models are same as those for models selected in Example 2. Each plane is assumed to have a bilinear force-displacement relationship as in the previous examples. A set of building models with three values of Ω_R , 0.75, 1.0, and 1.25 is considered.

Earthquake motions

A set of 16 earthquake records, as used in the study of single storey inelastic models in Chapter 4, is employed here. Details of these earthquake are given in Table 4.2. All the earthquake records are scaled to a peak ground acceleration of $0.3g$. An average elastic spectra for the set of 16 scaled earthquake, for a damping of 5% is obtained for the purpose of obtaining design forces.

Details of analysis

The selected models are designed for 25% of the elastic shears obtained from the average spectra for 16 scaled earthquakes records, mentioned earlier. Equations 4.8 through 4.11 are used to design the individual frames.

The selected set of building models is subjected to the set of 16 ground motions. To take the effect of accidental torsion into account, the CM is moved from the central plane by $\pm 0.05b$ and the ductility demand for elements at flexible and stiff edge frames are obtained from the higher of the two responses for these modified unbalanced models. An associated balanced model is achieved by moving CM to CR. The average ratio of interstorey ductilities obtained for the set of 16 earthquakes, $\bar{r}_\mu$, is taken as a measure of the effect of torsion.

Once again the flexible edge ductility demand in the unbalanced building is, in most cases, less than that in the balanced building, and even in cases where it is higher it is still less than the target ductility of 4. This is true for values of Ω_R .

In a few cases, stiff edge ductility demand in the unbalanced building does exceed both the target ductility and the ductility demand in the associated balanced building. Such critical cases are shown in Figs. 6.14 and 6.15. In all of these cases, the ductility demand in the balanced building is also higher than the target ductility.

6.3 Summary and conclusions

In the previous chapter, results of a study on the inelastic response of single storey building models subjected to 16 ground motion records were presented. It was shown that the use of the proposed design expressions leads to ductility demands at the flexible

edge that are smaller than the corresponding values in the torsionally balanced model. In most cases, the ductility demands at the stiff edge planes are also smaller than the corresponding values in the torsionally balanced model. In a few cases when they are larger, the excess is quite small.

Analytical studies are extended to multistorey building models. Three *special class* of shear frame type buildings, 5, 10 and 20 storeys in height and having fundamental periods of 0.5, 1.0, 2.0 s, are designed according to the suggested provisions for three different, appropriately scaled, earthquake records. Static eccentricity e/b and uncoupled frequency ratio Ω_R (defined storey by storey) are kept constant along the height. A number of building models with a range of e/b and Ω_R values are considered. Inelastic time history analyses are carried out on the balanced and unbalanced buildings. The ratios of interstorey ductilities in the unbalanced and balanced buildings as well as the absolute values of ductilities are obtained. As expected, the response behaviour of multistorey building is quite complex. Even in the balanced building, the ductility demand is not uniform along the height, and may be higher than the target ductility in some storeys, while at the same time being substantially lower in others. The behaviour also varies in an unpredictable way from one ground motion to the other. In spite of this, the suggested design provisions generally lead to a conservative design for the flexible edge, and an adequate design for the stiff edge. The conclusions are based on a large number of analytical studies. The data is quite voluminous and is therefore not presented here. Instead, results are shown for only those cases where the ductility demand in the unbalanced building exceeds both the corresponding ductility demand in the balanced building and the target ductility. It is obvious that such cases are quite few.

Analytical studies are then carried out on a general type of 10 storey building having a time period of 1.0 s, designed according to the proposed expressions for the mean spectrum of 16 appropriately scaled earthquakes. The ratio of uncoupled torsional to lateral frequencies, Ω_R , defined storey by storey, is kept constant along the height of the building while the eccentricity e/b varies from storey to storey. Inelastic analyses are carried out on the unbalanced and associated balanced building models for their response to the 16 scaled earthquake records. The mean value of the interstorey ductility demands for 16 earthquakes, is obtained for both the unbalanced and the balanced buildings. It is shown that the proposed design provisions generally lead to a conservative design for flexible edge and an adequate design for the stiff edge frames.

The analytical results also suggest that any refinement of code provisions, that are based on the concept of a design eccentricity, provides no guarantee of a more rational design, and that the simple design provisions suggested here are quite satisfactory for the design of multistorey buildings of a *special class* as well as of a general type which have a fairly regular distribution of the ratio of torsional to lateral stiffness along the height of the building.

Figure 6.1: Elastic response spectrum for the selected ground motions, scaled to a peak ground acceleration of 0.3 g

(a)

(b)

Figure 6.2: Ratio of ductility demand in a five storey unbalanced building to that in the associated balanced building, floor 1, El-Centro earthquake, (a) flexible edge (b) stiff edge

(a)

(b)

Figure 6.3: Ratio of ductility demand in a five storey unbalanced building to that in the associated balanced building, floor 5, El-Centro earthquake, (a) flexible edge (b) stiff edge

Figure 6.4: Interstorey ductility at the stiff edge, five storey building of special class, $\Omega_R = 1.0$, El-Centro earthquake

Figure 6.5: Interstorey ductility at the stiff edge, five storey building of special class, $\Omega_R = 1.0$, San-Francisco earthquake

Figure 6.6: Interstorey ductility at the stiff edge, five storey building of special class, $\Omega_R = 1.0$, San-Francisco earthquake

Figure 6.7: Interstorey ductility at the stiff edge, ten storey building of special class, $\Omega_R = 1.0$, San-Francisco earthquake

Figure 6.8: Interstorey ductility at the stiff edge, ten storey building of special class, $\Omega_R = 1.0$, San-Fernando earthquake

Figure 6.9: Interstorey ductility at the stiff edge, twenty storey building of special class, $\Omega_R = 1.0$, El-Centro earthquake

Figure 6.10: Interstorey ductility at the stiff edge, twenty storey building of special class, $\Omega_R = 1.25$, El-Centro earthquake

Figure 6.11: Interstorey ductility at the flexible edge, five storey building of general type, $\Omega_R = 1.25$, San-Francisco earthquake

Figure 6.12: Interstorey ductility at the stiff edge, five storey building of general type, $\Omega_R = 1.0$, San-Francisco earthquake

Figure 6.13: Interstorey ductility at the stiff edge, five storey building of general type, $\Omega_R = 1.25$, San-Francisco earthquake

Figure 6.14: Interstorey ductility at the stiff edge, ten storey building of general type, $\Omega_R = 0.75$, San-Francisco earthquake

Figure 6.15: Interstorey ductility at the stiff edge, ten storey building of general type, $\Omega_R = 1.0$, San-Francisco earthquake

Chapter 7

Comparison of torsional provisions in building codes

7.1 General

Traditionally, the codes of practice for design against earthquake induced torsion in buildings have specified the use of a static analysis procedure. In this procedure the design inertia forces are applied through a point that is eccentric with respect to the centre of mass. The design eccentricity e_d , measured from the centre of resistance, is defined as the ratio of design torque to the base shear. Calibration with the results obtained from linear dynamic analyses have led to the prescription of two separate design eccentricity expressions: one suggesting an amplification in static eccentricity, applicable to elements on the flexible side of the building; and the other suggesting a reduction in static eccentricity, applicable to elements on the stiff side of the building. These expressions are the origin and basis of the torsional design provisions in modern building codes.

Since the first introduction of the concept of design eccentricity, various researchers have studied the validity of the design provisions of the codes. Results from these studies have not always been consistent. Consequently, while the provisions of many building

codes have undergone several revisions in light of the findings of ongoing research, they do not necessarily account for some of the important considerations. As an example, most codes do not give explicit consideration to the torsional stiffness as measured by the ratio of uncoupled torsional frequency to uncoupled lateral frequency, Ω_R , even though it is the most important parameter governing the torsional response.

In this chapter, the torsional design provisions of selected building codes are reviewed and compared with the design expressions proposed in this thesis. Four building codes have been selected for this study: (1) National Building Code of Canada, NBCC 1995, (2) Uniform Building Code, UBC 1997, (3) New Zealand Standard, NZS 4203-1992, and (4) the Mexico Code 1993. The study consists of three main parts: (1) comparison of design eccentricity expressions in codes with the effective edge eccentricities obtained from response spectrum analysis; (2) comparison of additional strengths required by the building codes, for the edge elements of a torsionally unbalanced building model; and (3) comparison of the ductility demands in the edge elements of an unbalanced model designed according to the torsional provisions of building codes.

7.2 Provisions for design against torsion

The design eccentricity formulae given in building codes can be written in the following form

$$e_{d1} = \alpha e + \beta b \quad (7.1)$$

$$e_{d2} = \gamma e - \beta b \quad (7.2)$$

where e_{d1} and e_{d2} are design eccentricities, e is the distance between the centre of mass

and the centre of rigidity at a storey, b is plan dimension of the building perpendicular to the direction of earthquake and α, β, γ are coefficients, that have different values in different building codes.

Codes require that to determine the design load for an individual lateral load resisting element at a floor, the storey shear should be applied at a distance e_{d1} or e_{d2} , whichever produces the higher design force, from the centre of rigidity. Usually, e_{d1} governs the design of elements on the flexible side of the building, and e_{d2} governs the design of elements at the stiff side. However, for torsionally flexible systems (low Ω_R) where the torsional response may far exceed the lateral response, e_{d1} may produce higher design forces for the elements on the stiff side of the building as well.

The first term in the expressions for design eccentricity represents natural torsion, while the second term is supposed to represent accidental torsion. Factors α and γ are applied to static eccentricity e , to take into account the effects of dynamic torque amplification. Accidental torsion, which can be assessed only in an indirect manner, is taken as a fraction of plan dimension b . The values of these coefficients, for each of the building codes mentioned earlier, are given in Table 7.1. Besides the design eccentricity expressions given by Eqs. 7.1 and 7.2, the building codes have some special requirements as well. These special requirements are also shown in Table 7.1 and discussed in detail in the following paragraphs.

Special requirements

The design provisions proposed in this work, referred to hereafter as the *proposed expressions*, require that when the ratio of uncoupled torsional-to-lateral frequencies Ω_R is less than 1, a revised expression $e_{d2} = -0.1b$ should be used in place of Eq. 7.2. For

Table 7.1: Torsional design requirements in building codes

Building Codes	α	γ	β	Special requirements
Proposed expressions	1.0	1.0 or, 0.0	0.1	for $\Omega_R < 1$, $\gamma = 0.0$
NBCC	1.5	0.5	0.1	
UBC	1.0	1.0	$0.05 A_x$ $1.0 \leq A_x \leq 3.0$	
NZS	1.0	1.0	0.1	Horizontal regularity criterion should be met
Mexico	1.5	0.5	0.1	(1) for $e > 0.1b$ design base shear be increased by 25 % (2) Restriction on e_p

buildings with high mass eccentricity ($\eta \geq 0.2$), and/or low aspect ratio ($a/b \leq 0.5$) the revised expression for e_{d2} should be used whenever Ω_R is less than 1.25. The basis for this suggested increase in the cut-off value of Ω_R is given in detail in Sections 2.7 and 2.9. As would be expected, the revised expression for e_{d2} leads to a significant increase in the design strength of elements on the stiff side of the building.

NBCC 1995 suggests that as an alternative to the use of floor torques equal to the product of floor force and design eccentricity at a floor, applied at each floor, a 3-D dynamic analysis may be carried out to evaluate the effects of torsion. When a dynamic analysis procedure is used, accidental torsion can be accounted for by applying a torque equal to the floor force times $0.1b$, at each floor. The forces produced by these torques should be added to or subtracted from the forces obtained from 3-D dynamic analysis, to obtain the highest design forces in the lateral load resisting elements.

UBC 1997 requires the calculation of a factor A_x to account for torsional irregularity in the building. The factor A_x is applied to the accidental torsion component and is defined as,

$$A_x = \left[\frac{\delta_{max}}{1.2\delta_{av}} \right]^2 \quad (7.3)$$

$$1.0 \leq A_x \leq 3.0$$

where δ_{max} is defined as the maximum storey drift parallel to the direction of earthquake forces, computed including accidental torsion, while δ_{av} is defined as the average of the storey drifts at the two ends of the structure (Table 16-M of UBC 1997). The above definitions do not clearly indicate whether accidental torsion should be considered for the calculation of δ_{av} , and if so, through what point should the shear be applied in such calculations. In the study carried out here, accidental torsion has been ignored in the calculation of δ_{av} . For the 3-plane mono-symmetric building model shown in Fig. 7.1, it can be shown that

$$A_x = \left[\frac{1 + k(\bar{e} + 0.05)(0.5 + \bar{e})}{1.2(1 + k\bar{e}^2)} \right]^2 \quad (7.4)$$

where $\bar{e} = e/b$, and the coefficient k is given by

$$k = \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \quad (7.5)$$

and r is the radius of gyration about centre of mass, which can be expressed in terms of aspect ratio of the building.

It is evident that coefficient A_x is a function of Ω_R , e/b and the aspect ratio of the building. Figure 7.2 shows the variation of A_x with e/b for an aspect ratio of 1.0 and different values of Ω_R .

New Zealand Standard (NZS 4203-1992) allows a two dimensional static lateral method

of analysis only when one of the following horizontal regularity criteria are satisfied: (1) $e/b \leq 0.3$ and eccentricity does not change its sign over the height of the structure; and (2) under the action of equivalent static forces applied at a distance $e_d = e \pm 0.01b$ from CR, the ratio of the horizontal displacements at the ends of axis at any horizontal plane transverse to the direction of forces is in the range of 3/7 to 7/3.

Mexico Code requires that for buildings that do not possess geometrical and structural regularity, the force reduction factor Q , that is applied to elastic forces in order to obtain the design forces ($V_{design} = V_{elastic}/Q$), should be multiplied by 0.8. The requirements of regularity that are relevant to the current study are: (1) $e/b \leq 0.1$; and (2) $a/b \geq 0.4$. This implies that for building models with static eccentricity higher than $0.1b$, or an aspect ratio less than 0.4, the total design strength should be increased by 25%.

In addition to the above, the Mexico Code imposes certain limitations on the value of the strength eccentricity, e_p , to ensure that excessively large torsional moments do not occur after the structure displaces into the inelastic range. The code requires that for $Q \geq 3$, the centroid of the strength of lateral resisting elements should be located on the same side of the point of application of shear force as the centre of twist, and:

$$1. \quad e_p \geq e - 0.2b \quad \text{if } Q = 3$$

$$2. \quad e_p \geq e - 0.1b \quad \text{if } Q > 3$$

For a single storey building system, the centres of twist and rigidity are the same. For a multistorey system, formulae have been given in the code to calculate centres of twist. In the present study, the force reduction factor Q has been selected as equal to 4. The impact of the criterion related to e_p , on the torsional design of resisting elements is

discussed in detail in the following paragraphs.

To explain the impact of restricting the value of strength eccentricity, e_p , consider the 3-plane, mono-symmetric, single storey building model shown in Fig. 7.3. The stiffnesses of the 3 planes shown are k_1 , k_2 , and k_3 , selected so that the centre of rigidity CR is offset from centre of mass CM, by a distance e , called static eccentricity. Assume that the lateral planes 1, 2 and 3 have been designed for shears V_1 , V_2 and V_3 , respectively, where $V_1 + V_2 + V_3 = V_{in}$, is the design base shear. The centre of strength, CS is at a distance e_p from the centre of mass which is also the geometric centroid. Centre of rigidity CR is on the same side of CM as the CS. For $Q = 4$, the Mexico Code requires that, $e_p \geq e - 0.1b$ or, $e - e_p \leq 0.1b$. This means that the centre of strength CS should be no farther than $0.1b$ from CR. If this condition is not met, the strength distribution needs to be revised such that e_p is increased. To achieve this, either the strengths of planes 1 and 2 have to be reduced or the strength of plane 3 has to be increased. Since the former is not desired, the required value of e_p is achieved by increasing the value of V_3 . In other words, restrictions on strength eccentricity limits the amount of strength reduction at the resisting elements on the stiff side of the building. The following steps summarise the design of the building model shown in Fig.7.3,

1. Design the planes 1, 2 and 3 according to torsional provisions of the Mexico Code without considering restrictions on the value of design eccentricity
2. Calculate strength eccentricity, e_p , given by

$$e_p = \frac{(V_3 - V_1)b}{2(V_1 + V_2 + V_3)} \quad (7.6)$$

3. If $e - e_p > 0.1b$ then assume the following value for e_p

$$e_p = e - 0.1b \quad (7.7)$$

and re-evaluate V_3 as

$$V_3 = \frac{2(e_p/b)(V_1 + V_2) + V_1}{(1 - 2(e_p/b))} \quad (7.8)$$

7.3 Elastic response of single storey models

In Chapter 2, the response of a mono-symmetric single storey building model as shown in Fig. 7.1, was obtained for idealised flat and hyperbolic response spectra, by means of a response spectrum analysis. It may be noted that the effective flexible edge eccentricity, e_f , was defined in Section 2.4, as the distance from CR at which the application of base shear V_0 would produce the same flexible edge displacement as the one obtained from response spectrum analysis. Similarly, the effective stiff edge eccentricity, e_s , was defined as the distance from CR at which the application of base shear would produce the same stiff edge displacement as that obtained from response spectrum analysis. The effective edge eccentricities were compared with the design eccentricity expressions of NBCC 1995, as shown in Figs. 2.6 and 2.7. In this section, a similar comparison is made between the effective edge eccentricities obtained from response spectrum analysis, including the effects of accidental torsion, and the design eccentricity expressions given in various building codes. It should be noted that the response spectrum analysis results shown in Figs. 2.6 and 2.7 do not include the effects of accidental torsion. To account for it, the curves shown in Fig. 2.6 are shifted by a distance $-0.05e/b$ along the horizontal

axis. These adjusted curves are plotted in Figs. 7.4. The curves in Fig. 2.7 are shifted to the right by $0.05e/b$ along the horizontal axis to obtain the adjusted curves shown in Figs. 7.5.

Figures 7.4a through h show the variation of e_f with e/b for four different frequency ratios, $\Omega_R = 0.75, 1.0, 1.25$ and 1.50 , aspect ratio of 1, and for flat and hyperbolic spectra. Also plotted in these figures are the design eccentricity expressions e_{d1} , for different building codes. It should be noted that a design eccentricity value that is higher than the effective eccentricity obtained from dynamic analysis, indicates that the design provisions are conservative. It is evident from the Figs. 7.4a through h that the provisions of NBCC and the Mexico code are overly conservative for all values of eccentricities and frequency ratios, for both flat and hyperbolic spectra. Provisions of NZS and the proposed expressions also provide a conservative estimate of flexible edge displacements, except for $\Omega_R = 1.25$ and low values of e/b (<0.2), for a flat spectrum (Fig. 7.4e). However, even in that case the design eccentricity is only slightly lower than that obtained from a response spectrum analysis, and the provisions of NZS and the proposed expressions may be considered adequate. Provisions of UBC are seen to be unconservative for $\Omega_R = 1.0, 1.25$ and 1.50 , for both flat as well as hyperbolic spectrum.

Figures 7.5a through h show the variation of e_s with e/b for four different frequency ratios, $\Omega_R = 0.50, 0.75, 1.0$, and 1.25 , aspect ratio of 1, and for flat and hyperbolic spectra. Also plotted in these figures are code specified effective design eccentricities for the resisting elements at the stiff edge of the building. Usually, code specified design eccentricity e_{d2} governs the design of stiff edge elements and therefore in such codes the effective eccentricity shown in Fig. 7.5 is equal to e_{d2} . However, for torsionally flexible

systems (low Ω_R), e_{d1} may govern the design of stiff side elements. In that case, the effective stiff edge design eccentricity implied in the code provisions can be calculated from

$$\frac{e_s}{b} = \frac{2 \Omega_R^2}{(b/r)^2(0.5 - \bar{e})} - \frac{e_{d1}}{b} \quad (7.9)$$

It should be noted that for the stiff edge elements, a code specified design eccentricity value higher than the effective eccentricity value obtained from response spectrum analysis, implies that the provision of the code is unconservative.

It is evident from Figs. 7.5a and 7.5b that for $\Omega_R = 0.5$, provisions of NZS are unsafe for flat as well as hyperbolic spectrum. Provisions of UBC are seen to be unconservative for a flat spectrum but are safe for a hyperbolic spectrum. Provisions of NBCC and the Mexico code are conservative for flat as well as hyperbolic spectrum provided the more critical of the design eccentricities e_{d1} and e_{d2} is used to calculate the design strength of the stiff edge plane. It should be noted that for a low values of Ω_R , such as 0.50, e_{d1} governs the design of stiff edge elements and Eq. 7.9 is applicable for the calculation of effective design eccentricity for the stiff edge. This has been shown in Fig. 7.5a and 7.5b.

Figures 7.5c and d indicate that for $\Omega_R = 0.75$, all the existing codes (NBCC, UBC, NZS and the Mexico code) underestimate the response at the stiff edge of the building, particularly for large values of eccentricity. Only the proposed expressions provide an adequate measure of the stiff edge displacements.

For $\Omega_R = 1.0$ and a flat spectrum, the proposed expressions as well as the provisions of all codes for the stiff edge design eccentricity are safe. The provisions of NBCC and the Mexico code are, in fact, overly conservative. For $\Omega_R = 1.0$ and a hyperbolic spectrum,

the UBC provisions are unsafe for the entire range of values of e/b . The provisions of NZS as well as the proposed expressions are also unsafe for a range of e/b values. However, the difference between the proposed design eccentricity and spectral analysis values are not large and may be considered acceptable for the purpose of design.

For $\Omega_R \geq 1.25$ all of the code provisions as well as the proposed expressions for stiff edge design eccentricity are safe, and in fact, quite conservative. This is particularly so for the NBCC and the Mexico code provisions.

It should be noted that the special requirements of the Mexico code are not reflected in Figs. 7.4 and 7.5. These special requirements influence the design strengths as will be evident from the inelastic analysis results presented in the following sections.

7.4 Inelastic response of single storey models

Structural details of the models

A single storey shear type building model having three resisting planes in the y direction and one resisting plane along the x axis, as shown in Fig. 7.1, is selected for study. The mass of the building floor is taken as 400 t , mass moment of inertia as 54,000 $t - m^2$, aspect ratio as 0.5, floor width b as 36 m , storey height as 4 m and uncoupled translational period in y -direction, T_y as 1.0 s . Strain hardening ratio of 5% is assumed for all planes and the damping ratio is taken as 5% of critical. The frequency ratio Ω_R and the eccentricity ratio e/b are varied over a range of physically admissible values. Specified values of Ω_R and e/b are achieved by adjusting the values of k_1 , k_2 and k_3 , the stiffnesses of the planes in y -direction. A wide range of building models with eccentricity values $e/b = 0.05, 0.1, 0.15, 0.2, 0.25$ and 0.3 and frequency ratio values $\Omega_R = 0.75, 1.0, 1.25$ and 1.50 are considered.

The yield strength of individual planes are given by

$$V_1 = V_0 \frac{k_1}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (\bar{e} + 0.5) \right] \quad (7.10)$$

$$V_2 = V_0 \frac{k_2}{K_y} \left[1 + \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} \bar{e} \right] \quad (7.11)$$

$$V_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d2}}{b} (0.5 - \bar{e}) \right] \quad (7.12)$$

$$V_3 = V_0 \frac{k_3}{K_y} \left[1 - \frac{1}{\Omega_R^2} \left(\frac{b}{r} \right)^2 \frac{e_{d1}}{b} (0.5 - \bar{e}) \right] \quad (7.13)$$

where $\bar{e} = e/b$, V_0 is the design base shear in the associated balanced (CM moved to CR) building model, and $K_y = k_1 + k_2 + k_3$. In determining the value of V_3 , the larger of the absolute values obtained from Eqs. 7.12 and 7.13 is used.

The strengths of resisting planes in the associated torsionally balanced system are given by

$$\begin{aligned} V_{10} &= V_0 \frac{k_1}{K_y} \\ V_{20} &= V_0 \frac{k_2}{K_y} \\ V_{30} &= V_0 \frac{k_3}{K_y} \end{aligned} \quad (7.14)$$

Also

$$V_0 = V_{10} + V_{20} + V_{30} \quad (7.15)$$

The value of the strength eccentricity, that is the distance of the centre of strength from the centre of mass, can be obtained from Eq. 7.6.

The normalised strength of the flexible edge plane in the torsionally unbalanced system, V_1/V_{10} is plotted against e/b in Figs. 7.6a through d, for four values of frequency ratio $\Omega_R = 0.75, 1.0, 1.25$ and 1.50 , respectively. It will be noted that not all curves span the entire range of the value of eccentricity. This is because results are presented only for those cases which are physically possible. Thus combinations of e and Ω_R which would require one or more planes to have a negative stiffness are excluded. The required strength in the torsionally unbalanced system is seen to increase with the eccentricity. It is evident from Figs. 7.6 that the Mexico Code requires maximum additional flexible edge strength in a torsionally unbalanced model as compared to that in the associated balanced model. For the Mexico code, there is a step increase in the value of V_1/V_{10} at $e/b = 0.1$. This indicates the additional (25%) strength requirements of the structural regularity provisions in the Mexico Code.

The normalised strength of stiff edge plane in the torsionally unbalanced system, V_3/V_{30} is plotted against e/b in Figs. 7.6a through d, for four values of frequency ratio $\Omega_R = 0.75, 1.0, 1.25$ and 1.50 respectively. For $\Omega_R = 0.75$, the proposed expressions require maximum additional strength for the stiff edge elements of the torsionally unbalanced building. However, for $\Omega_R = 1.0, 1.25$ and 1.50 , the Mexico Code requires maximum additional stiff edge strength. In Figs. 7.7b and 7.7c, there is a steep increase in the values of V_3/V_{30} for the Mexico Code, for $e/b \geq 0.15$, which is due to the limitations imposed on strength eccentricity e_p .

Figures 7.8a through d show the variation of total normalised strength V/V_0 as a function of e/b . It is evident that the Mexico Code requires maximum additional total strength in the torsionally unbalanced building systems. Also, in general the NBCC

provisions lead to larger total strength requirements in comparison to the proposed expressions.

Figures 7.9a through d show a plot of e_p/b as a function of e/b . Figures 7.9b and 7.9c show a steep increase in the value of e_p for the Mexico Code for $e/b \geq 0.15$, indicating the effect of a prescribed minimum value of strength eccentricity in the Mexico Code.

Response to recorded ground motions

A set of 16 earthquake records, used in the analytical studies carried out in Chapter 4, is also used here. The details of these earthquakes are given in Table 4.2. An elastic response spectrum is obtained for each of the 16 records, normalised by its peak ground acceleration, and for a damping of 5% of critical. The mean of the calculated response spectra is shown in Fig. 4.6. All the earthquake records are scaled to a peak ground acceleration of $0.28g$, before being used in the analysis described below.

The total elastic strength V_e of the resisting planes in the y-direction is obtained from the mean elastic response spectrum mentioned in the previous paragraph, corresponding to a period of 1.0 s. The spectral value obtained is multiplied by 0.28 so that the spectrum is representative of an earthquake with a peak ground acceleration of $0.28g$. The total design strength for the torsionally balanced model is taken as $V_0 = V_e/4$. This total strength is distributed among the individual planes of balanced building according to Eq. 7.14. The strengths of planes in the unbalanced building are determined from Eqs. 7.10 through 7.13. The strength distribution in an unbalanced models varies from one code to another, as the expressions of design eccentricity vary from code to code. Therefore, five different unbalanced models, designed according to the proposed expressions, NBCC, UBC, NZS and the Mexico Code, are considered. It will be noted that the

associated balanced model is same for all the above codes.

To account for the effects of accidental torsion, the centre of mass CM is moved by $\pm 0.05b$, in each torsionally unbalanced building, to produce two modified unbalanced models corresponding to each set of e/b and Ω_R values. In the analytical results presented here the maximum of the two values obtained from the two modified models is reported. All the modified unbalanced and associated torsionally balanced models are now subjected to the set of 16 scaled earthquake records.

The maximum ductility demand in a plane in any torsionally unbalanced model subjected to a given earthquake is denoted by μ_u while the maximum ductility demand for the associated torsionally balanced model is denoted by μ_b . The ratio of ductilities $r_\mu = \mu_u/\mu_b$ provides a measure of the effect of torsional motion.

The mean value of the ratio of ductilities for the flexible edge, $\bar{r}_{\mu f}$, obtained for the set of 16 earthquakes, is plotted against e/b in Figs. 7.10a through 7.10d, for four values of frequency ratios Ω_R . The value of $\bar{r}_{\mu f}$ is less than 1 for all the codes, in all cases, implying that the flexible edge ductility in a torsionally unbalanced model is less than that in the associated torsionally balanced model. The provisions of the Mexico Code are most conservative of all.

The mean value of the ratio of ductilities for the stiff edge, $\bar{r}_{\mu s}$, obtained for the set of 16 earthquakes, is plotted against e/b in Figs. 7.11a through d, for four values of Ω_R . Figures 7.11 show that $\bar{r}_{\mu s}$ is higher than 1 for NZS for $\Omega_R = 0.75$. The ratio $\bar{r}_{\mu s}$ is also higher than 1 for NZS, the proposed expressions, and UBC for $\Omega_R = 1.0$, particularly for higher values of e/b . However, this difference is small ($< 20\%$) and the provisions of NZS, proposed expressions and UBC may be considered adequate for $\Omega_R = 1.0$.

7.5 Summary and conclusions

Analytical results are presented in this chapter for the elastic and inelastic response of single storey torsionally unbalanced models. The results of elastic studies are compared with the design provisions of different building codes. These results indicate that the provisions of NBCC and the Mexico Code are overly conservative for the design of elements on the flexible side of the building. They also indicate that the provisions of UBC may be unconservative for the flexible edge, for higher values of Ω_R (1.25 and 1.50), particularly for a flat spectrum.

Results of elastic studies indicate that the torsional provisions of all the codes are unconservative for the design of elements on the stiff side of the building, for low values of Ω_R . Only the proposed expressions provide an adequate assessment of the stiff edge responses.

A comparison has been made of the strength distributions recommended in the building codes. In general, the Mexico Code requires highest additional overall strength and UBC requires least overall additional strength, in a torsionally unbalanced building model.

Analytical studies are then carried out for the inelastic response of single storey models designed for one fourth of the elastic base shear obtained from the mean elastic response spectrum for a set of 16 earthquake records. The base shear is distributed in the lateral planes in the y-direction, according to Eqs. 7.10 through 7.13 in the torsionally unbalanced building, and according to Eqs. 7.14 in the associated torsionally balanced building.

The results of inelastic response to recorded motions indicate that the provisions of

all the codes including that implied in the proposed expressions are conservative for the flexible edge, the value of $\bar{r}_{\mu f}$ being less than 1 in all cases. Provisions of NBCC and the Mexico code are most conservative.

The results of analytical studies on inelastic response to recorded motions also indicate that the provisions of certain codes may be unsafe for the elements on the stiff side of the building, in certain situations. The value of $\bar{r}_{\mu s}$ is greater than 1 for NZS for $\Omega_R = 0.75$. The provisions of NBCC, UBC and the Mexico Code are seen to be safe for $\Omega_R = 0.75$, for the entire range of e/b . The New Zealand provisions are however unsafe for a range of values of e/b . For $\Omega_R = 1.0$, $\bar{r}_{\mu s}$ is somewhat more than 1, for the models designed according to the proposed expressions, NZS and UBC, particularly for large values of eccentricity. However, the value of $\bar{r}_{\mu s}$ is only slightly more than 1 and these provisions may be considered adequate for $\Omega_R = 1.0$.

Figure 7.1: Plan view of a single storey mono-symmetric building model

Figure 7.2: Variation of A_x , specified in Uniform Building Code 1997, aspect ratio = 1

Figure 7.3: Plan view of a 3-plane single storey building model

Figure 7.4: Effective flexible edge eccentricity, aspect ratio = 1, (a) flat spectrum, $\Omega_R = 0.75$, (b) hyperbolic spectrum, $\Omega_R = 0.75$

Figure 7.4: Effective flexible edge eccentricity, aspect ratio = 1, (c) flat spectrum, $\Omega_R = 1.0$, (d) hyperbolic spectrum, $\Omega_R = 1.0$

Figure 7.4: Effective flexible edge eccentricity, aspect ratio = 1, (e) flat spectrum, $\Omega_R = 1.25$, (f) hyperbolic spectrum, $\Omega_R = 1.25$

Figure 7.4: Effective flexible edge eccentricity, aspect ratio = 1, (g) flat spectrum, $\Omega_R = 1.50$, (h) hyperbolic spectrum, $\Omega_R = 1.50$

Figure 7.5: Effective stiff edge eccentricity, aspect ratio = 1, (a) flat spectrum, $\Omega_R = 0.50$, (b) hyperbolic spectrum, $\Omega_R = 0.50$

Figure 7.5: Effective stiff edge eccentricity, aspect ratio = 1, (c) flat spectrum, $\Omega_R = 0.75$, (d) hyperbolic spectrum, $\Omega_R = 0.75$

Figure 7.5: Effective stiff edge eccentricity, aspect ratio = 1, (e) flat spectrum, $\Omega_R = 1.0$, (f) hyperbolic spectrum, $\Omega_R = 1.0$

Figure 7.5: Effective stiff edge eccentricity, aspect ratio = 1, (g) flat spectrum, $\Omega_R = 1.25$, (h) hyperbolic spectrum, $\Omega_R = 1.25$

Figure 7.6: Normalised flexible edge plane strength, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$

Figure 7.6: Normalised flexible edge plane strength, aspect ratio = 0.5, (c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$

Figure 7.7: Normalised stiff edge plane strength, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$

Figure 7.7: Normalised stiff edge plane strength, aspect ratio = 0.5, (c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$

Figure 7.8: Normalised total strength of planes, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$

Figure 7.8: Normalised total strength of planes, aspect ratio = 0.5, (c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$

Figure 7.9: Strength eccentricity, aspect ratio = 0.5, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$

Figure 7.9: Strength eccentricity, aspect ratio = 0.5, (c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$

(a)

(b)

Figure 7.10: Ratio of flexible edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$

Figure 7.10: Ratio of flexible edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$

Figure 7.11: Ratio of stiff edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (a) $\Omega_R = 0.75$, (b) $\Omega_R = 1.0$

Figure 7.11: Ratio of stiff edge ductility demand in a torsionally unbalanced building to that in the associated torsionally balanced building, mean from 16 earthquake records, (c) $\Omega_R = 1.25$, (d) $\Omega_R = 1.50$

Chapter 8

Summary and Conclusions

8.1 Summary

The need for providing sufficient torsional resistance in building systems has been emphasised following the damage studies of some recent earthquakes. Various researchers have carried out analytical studies on the elastic and inelastic torsional behaviour of single and multistorey building models. However, due to the complexity of the problem, conclusions drawn from such studies have not always been consistent. The objective of the work presented in this thesis is to provide a better understanding of the torsional behaviour and to develop design guidelines that are both rational and simple to implement.

The present study consists of the following main parts:

1. Review of the causes of torsion, of code provisions for design of torsionally unbalanced building systems, and of the literature on torsional response, with a view to identifying the unresolved issues.
2. Elastic analysis of torsionally unbalanced single storey building models for a range of governing parameters and the development of simplified torsional design provisions.

3. Elastic analysis of torsionally unbalanced multistorey building models of a general type and verification of the applicability of the provisions referred to in 2 above, in the design of such buildings.
4. Inelastic analysis of torsionally unbalanced single storey building models, assessment of displacement and ductility demands, and study of the applicability of design provisions that are based on the results of analytical studies of elastic models.
5. Study of the effect of orthogonal elements on the inelastic torsional response of single storey building models.
6. Inelastic analysis of torsionally unbalanced multistorey building models of *special class* and of general types, and assessment of the applicability of torsional design provisions referred to above.
7. Comparison of torsional design provisions in different building codes.

In Chapter 2, results are presented from analytical response studies on single storey building models, subjected to ground motions represented by idealised flat and hyperbolic spectra as well as to a set of recorded ground motions. Both natural and accidental torsion are considered. The response results are compared to the torsional design provisions of National Building Code of Canada (Canadian Commission, 1995). It is observed that the provisions of NBCC 1995 are overly conservative for the design of lateral resisting elements on the flexible edge. It is also observed that for the design of resisting elements on the stiff side, the provisions of NBCC 1995 are conservative in some cases

and inadequate in other. Based on elastic response results, a set of new provisions is proposed.

Elastic response of single storey models for accidental torsion is also studied. Torsional effects caused by uncertainty in the distribution of mass and stiffness and by ground rotation are assessed. It is observed that the proposed design expressions adequately account for the torsional response of resisting elements on both flexible and stiff edges of the building, even when the effects of accidental torsion are considered.

Studies are presented on the torsional response of mass eccentric single storey building models. The effect of aspect ratio is also studied. It is suggested that for building models with large mass eccentricities and low aspect ratio, the proposed design expressions can still be used but need a slight modification.

In Chapter 3, analytical studies are presented for elastic torsional response of multi-storey building models of a general class. The objective of these studies is to investigate whether the results obtained from the analyses of single storey models could be used to predict the elastic response of such multistorey buildings. A wide range of building models with different values of eccentricity and Ω_R are analysed for their response to idealised flat and hyperbolic spectra. The results indicate that the torsional response results obtained from single storey building models can be applied to multistorey buildings that are asymmetric in plan but in which the storey-wise frequency ratio does not vary appreciably along the height. The analytical results show that for a multistorey building in which frequency ratio varies along the height, the static analysis techniques are not adequate.

In Chapter 4, analytical studies are presented on the inelastic response of single storey

building models subjected to recorded ground motions. Both natural and accidental eccentricities are considered. The objective is to investigate whether the torsional design provisions based on the results of elastic response studies, can be applied to the design of systems that are expected to become inelastic. The results show that the use of the proposed design provisions leads to almost no additional ductility demand at the flexible or the stiff edge elements of unbalanced buildings.

To verify whether the proposed design expressions provide adequate design for building systems in which the static eccentricity is caused by an asymmetric distribution of mass as well of stiffness, a set of building models with several different mass eccentricities and a wide range of e/b and Ω_R values is analysed for the set of 16 earthquake records mentioned earlier. It is suggested that for the design of building models with large mass eccentricities, the proposed expressions should be used with a slight modification.

In chapter 5, analytical studies are presented on the effect of orthogonal elements on the inelastic torsional response of mono-symmetric single storey building models. The study includes an evaluation of the relative torsional stiffness of orthogonal planes and yielding in orthogonal planes, on the inelastic torsional response. In addition, a study is also presented on the effect of time period on the ductility demands of single storey models with and without orthogonal planes. It is seen that it is the total torsional stiffness and not how it is distributed between parallel and orthogonal planes that governs the response.

In Chapter 6, a study is presented on the interstorey ductility demands in torsionally coupled multistorey inelastic building models. The objective is to investigate the nature of ductility demands in the resisting elements at flexible and stiff edges of the multistorey

building, designed according to the proposed expressions. It is observed that the proposed provisions are generally adequate for the design of buildings responding in the inelastic range.

In Chapter 7, the torsional design provisions of 4 selected building codes are compared with the proposed design expressions. It is shown that the proposed expressions provide a simple and rational approach for design against torsion.

8.2 Conclusions

The following important conclusions can be drawn from the work presented here:

1. The elastic torsional response of a single storey asymmetrical building to a specified ground motion is governed by ω_y , Ω_R and e/r .
2. The frequency ratio Ω_R has an important effect on response. For $\Omega_R = 1$, the ratio of dynamic torque to static torque is highly amplified. This fact has provided the basis for a traditional recommendation that designs with $\Omega_R = 1$ should be avoided. However, the displacements and design forces in resisting planes depend on a combination of shear and torque. While the torque is amplified, the base shear is reduced on account of torsional coupling. Therefore a high dynamic torque amplification does not mean that the design forces are also amplified. Analytical results show that a frequency ratio equal to 1 is not critical and there is no reason to avoid designs with $\Omega_R = 1$.
3. A low value of Ω_R , say below 0.8, may lead to a very large increase in the displacements of both flexible and stiff edges. Special design considerations may apply in the design of structures with low values of Ω_R . It is therefore a good design

practice to obtain an estimate of Ω_R . For multistorey buildings in which the ratio of torsional to lateral stiffnesses does not vary appreciably along the height, Ω_R can be determined with sufficient accuracy by using the Rayleigh method. This requires a pair of static analyses.

4. The design eccentricity specified by NBCC for determining the strengths of planes on the flexible side of CR is very conservative. The design eccentricity specified by NBCC for the stiff edge is conservative for higher values of Ω_R but may be inadequate for lower values.
5. A new set of design eccentricity expressions is proposed as a replacement for NBCC provisions. Analytical studies on the elastic torsional behaviour of single storey models for a full range of governing parameters, show that the proposed provisions provide a good basis for design. The proposed expressions are simpler to use as there is no multiplier on static eccentricity e which means that the CRs need not be determined explicitly. However, their use does require the evaluation of Ω_R , which is an extra step. It should be noted that Ω_R has a significant effect on the torsional response and a good design must avoid low values of Ω_R . A requirement to determine Ω_R will thus promote good design practice.
6. Accidental torsion is always present in buildings. It results from uncertainty in the distribution of mass and stiffness as well as from the rotational component of ground motion. Previous studies have shown that the effect of accidental torsion can be estimated from a pair of dynamic analyses in which the static eccentricity is increased or decreased by $0.05b$. Using such an estimate of accidental torsion, it

is shown that the proposed design eccentricity expressions are adequate even when the effect of accidental torsion is included.

7. For buildings with low aspect ratio, ($a/b < 0.5$), the proposed design expressions for stiff edge may be unsafe. It is recommended that for such cases, the cut-off value of Ω_R as used in the design eccentricity related to the stiff edge, be increased to 1.25.
8. Buildings in which the eccentricity arises partially or fully because of the mass centre being off-set from geometric centre of the building, are called mass eccentric buildings. For such buildings, the stiff edge response becomes critical. For high mass eccentricity and low aspect ratio it is recommended that the value of Ω_R , used in the design eccentricity related to the stiff edge, be increased to 1.25.
9. For elastic multistorey buildings in which the frequency ratio Ω_R defined on storey levels, does not vary appreciably along the height, the use of the proposed design expressions provide an adequate design.
10. For a multistorey building in which the frequency ratio Ω_R varies along the height, response values obtained from the analysis of corresponding single storey models do not match the response of multistorey building. Evidently, the proposed expressions can not be used in the design of such buildings. Additional study may be required in this area.
11. Inelastic analyses of a 3-plane mono-symmetric single storey building models designed according to the proposed provisions show that the proposed design provisions lead to a conservative design for the flexible edge plane. It is also seen that

the NBCC provisions for the flexible edge are overly conservative. The proposed provisions for the design of stiff edge plane are also satisfactory. In addition, they prove the importance of maintaining a high value for the frequency ratio, which should preferably be greater than 1.

12. Inelastic time history analyses on single storey building models designed according to the proposed expressions show that the ratio of ductility demands in torsionally unbalanced to that in torsionally balanced models for the resisting elements on the flexible side of the building is always less than 1, and it does not change appreciably with mass eccentricity. However the ratio of ductility demands for the stiff edge elements is significantly higher than 1 for a combination of low aspect ratio, high mass eccentricity and high total eccentricity. Such a combination should be avoided in practice.
13. Inelastic analyses of building models with orthogonal planes indicate that for a single storey building system in which orthogonal elements as well as the parallel elements yield during an earthquake, the ductility demand of an edge element depends more or less on the total torsional stiffness of the building and not on what part of it is contributed by orthogonal elements. The ductility demand at the flexible edge of a building model reduces if the orthogonal elements remain elastic during an earthquake. The reduction in ductilities can be up to 15% depending upon the value of frequency ratio Ω_R . It is also observed that when the orthogonal elements are designed to remain elastic, the ductility demands for stiff edge elements generally increase. This increment can be as high as 20%, particularly for large values of both e/b and Ω_R .

14. Analytical studies on the response of single storey building models, both with and without orthogonal planes, show that the ductility demands do not vary significantly with the elastic period of the model.
15. A study of the collapse mechanism is presented to review the findings of a recent work by Paulay (1996) in which it was concluded that the ductility demands in a torsionally unrestrained model (all lateral resisting elements including orthogonal elements are yielding) is substantially higher than those in a torsionally restrained model (orthogonal elements remain elastic). It is observed that contrary to the predictions of Paulay, torsionally unrestrained models do not show a high ductility demands at the stiff edge. The trends observed also indicate that the results of previous studies, based on building models in which orthogonal elements remain elastic during an earthquake, provide conservative estimates of ductility demands at the stiff edge.
16. Inelastic time history analyses are carried out on various multistorey buildings of *special class* and general class, with a constant value of Ω_R along the height of the building. These building models are designed according to the proposed expressions and subjected to a set of ground motion records. The response behaviour of multistorey building is seen to be quite complex. Even in the balanced building, the ductility demand is not uniform along the height, and may be higher than the target ductility in some storeys, while at the same time being substantially lower in others. The behaviour also varies in an unpredictable way from one ground motion to the other. In spite of this, the proposed design provisions generally lead to a conservative design for the flexible edge, and an adequate design for the stiff

edge. The analytical results also suggest that any refinement of code provisions, that are based on the concept of a design eccentricity, provides no guarantee of a more rational design, and that the simple design provisions proposed here are quite satisfactory for the design of multistorey buildings of a *special class* as well as of a general class which have a fairly regular distribution of the ratio of torsional to lateral stiffness along the height of the building.

17. The results of response spectrum analyses of single storey building models designed according to the provisions of selected building codes (National Building Code of Canada 1995, Uniform Building Code 1997, New Zealand Standards 4203-1992 and the Mexico Code 1993) and the proposed design expressions are compared. It is observed that the provisions of NBCC and the Mexico Code are overly conservative for the design of elements on the flexible side of the building. It is also observed that the provisions of UBC may be unconservative for the flexible edge, for higher values of Ω_R (1.25 and 1.50), particularly for a flat spectrum. The torsional provisions of all the building codes may be unconservative for the design of elements on the stiff side of the building, for certain low values of Ω_R . Only the proposed expressions provide an adequate assessment of the stiff edge responses.
18. A comparison of the strength distributions recommended in the building codes indicates that, in general, the Mexico Code requires the highest additional overall strength and UBC requires the least overall additional strength, in a torsionally unbalanced building model.
19. Analytical studies are carried out on the inelastic response of single storey models

designed according to the torsional provisions of building codes, and subjected to a set of recorded motions. The results indicate that the provisions of all the codes including that of the proposed expressions are conservative for the flexible edge. Provisions of NBCC and the Mexico code are the most conservative.

20. The results of analytical studies on inelastic response to recorded motions also indicate that the provisions of certain codes may be unsafe for the elements on the stiff side of the building, in certain situations. For $\Omega_R = 1.0$, the proposed design expressions, NZS and UBC may be somewhat unconservative for the design of stiff edge elements, particularly for large values of eccentricity. However, the ductility demand in the stiff edge plane is only slightly more than the corresponding value in the associated torsionally balanced model and may be considered as being acceptable.

8.3 Recommendations for future work

A detailed study of the elastic and inelastic torsional response of shear type single and multistorey building models have been presented in this thesis. Based on the results of elastic analyses, a set of design eccentricity expressions have been proposed. The adequacy of these proposed provisions for the design of building models that are expected to go into inelastic range during an earthquake are verified by means of a study of ductility demands in the edge elements of torsionally unbalanced models. The following recommendations for additional work on torsional response emerge from the present work.

1. The work presented here deals with the torsional behaviour of building models that

are mono-symmetric in plan. However, in reality, the buildings are likely to have an eccentric distribution of mass and/or stiffness about two orthogonal horizontal axes. Therefore, there is a need to study the torsional behaviour of models with bi-axial eccentricities e_x and e_y . More research is required to address the issues listed here:

- (a) What is the effect of eccentricity in one direction, on the elastic torsional behaviour of bi-axially asymmetric single storey systems subjected to uni-directional ground motion applied in the perpendicular direction? For example, how does eccentricity e_x influence the variation of effective edge eccentricities with respect to e_y/b ?
 - (b) What changes, if any, would be required in the proposed expressions to take care of the above?
 - (c) What is the nature of inelastic torsional response of bi-axially asymmetric single storey building models, when subjected to uni-directional ground motions? How does the peak ductility demand change for bi-directional ground motion?
2. Is a push-over analysis useful in predicting the torsional behaviour of multistorey building models?
 3. Is it possible to develop a simple design procedure for multistorey buildings in which frequency ratio defined on storey level varies significantly along the height? Is a push-over analysis useful for predicting the torsional behaviour of such buildings?
 4. What is the nature of ductility demand in a torsionally unbalanced multistorey

building with setbacks? What changes, if any, are required in the proposed provisions for the design of such buildings?

5. How can passive damping devices be used to control the torsional response of building systems.

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IMAGE EVALUATION TEST TARGET (QA-3)

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