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MODELLING AND CHARACTERIZATION OF
DOUBLE DIFFUSED BIPOLAR TRANSISTORS

by

Ramesh M. Sirsi

B. Eng. Bangalore University, 1966

M. Eng. Carleton University, 1969

A Thesis submitted to the Faculty of
Graduate Studies in partial fulfilment
of the requirements for the degree of
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ABSTRACT

This thesis examines modelling and characterization of double diffused bipolar transistors with the objective of developing a systematic and accurate, yet as far as possible simple, approach to device design. In order to facilitate the device modelling process, a regional approach has been used, which also permits examination and modelling of different regions of the device independently. The accuracy of the modelling procedures is ascertained by the degree of correlation obtained between computed and experimentally measured parameters.

Using an exponential impurity distribution for the collector-base junction of planar bipolar transistors, methods to characterize junction capacitance and avalanche breakdown behaviour are discussed. Also, an algorithm to evaluate parameters of the exponential model from electrical measurements performed at the terminals of the device is presented. Modelling of forward biased emitter junctions is carried out by an approximate method which takes into account the presence of mobile carriers in the space charge regions of the junction. Completed results from this method show good agreement with results evaluated from rigorous solution of transport equations. Using this approach, for characterization of extremely shallow diffused devices, a generalized Moll-Ross equation is developed and

computed results from this method show good agreement with computer simulated results. From these results, it is concluded that the conventional use of Moll-Ross equation for characterization of the base region would result in error for devices with width ratio larger than 6%.

Using the approximate method to characterize the forward biased emitter junction, a numerical method of evaluating parameters of a double Gaussian impurity distribution model of the based-collector region is presented. Parameters of the model computed from terminal measurements show excellent agreement with those experimentally determined on completed devices.

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LIST OF PRINCIPAL SYMBOLS

A_c	Collector junction planar area
A_e	Emitter junction planar area
a_{2e}	Emitter junction space charge layer semiwidth in the base region
a_{1c}	Collector junction space charge layer semiwidth in the base region
BV_{CBO}	Collector junction breakdown voltage
C_{jc}	Collector junction space charge capacitance
C_{te} and C_{je}	Emitter junction space charge capacitance
$D_n(x)$	Diffusion Coefficient of mobile electrons
$D_p(x)$	Diffusion coefficient of mobile holes
D_1	Diffusion coefficient of impurities during emitter diffusion cycle
D_2	Diffusion coefficient of impurities during base diffusion cycle
d_c	Depletion layer width
$E(R)$	Electric field at distance R from the surface
e_{jn}	Normalized electric field at the metallurgical junction
e_{jx_0}	Normalized electric field at the edge of the transition region
I_c	Collector current

$J_p(x)$	Hole current density
$J_n(x)$	Electron current density
J_c	Collector current density
J_T	Total current density
k	Boltzmann's constant
L	Characteristic length of an exponential model
$N(x)$	Net ionized impurity density
N_D	Donor impurity density
N_A	Acceptor impurity density
N_{s1}	Surface concentration of emitter diffusion profile from a Gaussian model
N_{s2}	Surface concentration of base diffusion profile from a Gaussian model
N_B	Collector background doping density
n_i	Intrinsic carrier concentration
$n(x)$	Electron density
$p(x)$	Hole density
Q_T	Total charge in one half of the space charge region
Q_e	Base majority carrier stored in the emitter space charge region
Q_{bc}	Base majority carrier stored in the neutral base region.

Q_c	Base majority carrier stored in the collector space charge region
q	Electronic charge
R	Normalized distance from the surface in the cylindrical region of the collector junction
r	distance from the surface in the cylindrical region of the collector junction
r_j	Position of the collector junction from the surface in the cylindrical region of the junction
T	Temperature; $^{\circ}\text{K}$
t	Time
$U(y)$	Normalized electrostatic potential
U_n and U_p	Normalized quasi Fermi potentials for electrons and holes
$U_o(y)$	Auxillary electrostatic potential
V_{tc}	Total voltage developed across the space charge region
V_a	Applied Voltage
V_{am_i}	Measured voltage at i^{th} step
V_{ac_i}	Measured voltage at i^{th} step
V_{eb}	Emitter-base applied voltage
V_{cb}	Collector-base applied voltage
W	Neutral base width
W_b	Metallurgical base width
x	Distance from surface of the material

x'	Distance from collector metallurgical junction
x_o	Space charge layer half width
x_{1e} and x_{2e}	Positions of the emitter junction space charge layer edges from the Surface
x_{1c} and x_{2c}	Positions of the collector junction space charge layer edges from the surface
y	Normalized distance
a_o	Gradient of a linearly grader junction
α_n	Ionization coefficient for electrons
α_p	Ionization coefficients for holes
$\rho(x)$	Space charge density
ϵ	Dielectric constant
$\psi(x)$	Electrostatic potential
ϕ_{bo}	Built in potential
ϕ_n and ϕ_p	Quasi Fermi potentials for electrons and holes
δ	Minimum of the mean Square deviation of voltage error
μ_n and μ_p	Mobility of electrons and holes
ξ_b	Voltage drop across the space charge region half width computed from Boltzmann distribution of carriers
ξ_p	Voltage drop across the space charge region half width computed from Poisson's equation

CHAPTER 1

INTRODUCTION

1.1 Background

The last decade has seen the successful development of silicon bipolar transistors with power gain capabilities in the microwave region^(1,2,3,4,52) as the culmination of rapid advancement in semiconductor device technology since the first transistors appeared in 1951⁽⁵⁾. The design and successful fabrication of these devices is predicted on careful manipulation of design parameters so as to achieve useful performance within constraints imposed by the fundamental limitations of device physics. In practice, realization of such devices has been possible by careful tailoring of dopants in order to obtain extremely small geometry devices with the submicron base widths necessary for high frequency operation. Fabrication of these devices was made possible by the development of high resolution lithographic techniques^(6,7,8,9) and complex metallization systems⁽¹⁰⁾ to overcome the problems associated with standard processing technology. Realization of such small geometry devices has led to new concepts in understanding the internal electronics and in the models for representation of their performance characteristics.

In the beginning, rather crude models^(36,46) developed to characterize transistors were found to be adequate and provide simple analy-

tical expressions for device characterization. Elements of the equivalent circuit models⁽¹¹⁾ developed for the analysis of these devices were related to the device structure in a simple manner. With the innovation of diffusion technology⁽¹²⁾, simple models developed earlier were inadequate to characterize devices fabricated by this process. The non-uniform impurity distribution of diffused devices results in complex relationships rendering characterization of the devices difficult. However, this process can be considerably simplified by representing the device structure by a 'physical model' and deriving all the required relationships from this model. Such a model comprises the structural geometry, defined by regions of separation of the metallurgical junction, together with impurity density profile through emitter, base and collector regions. A 'physical model' is a mathematical representation of the device structure, approximating the actual impurity distribution in a device and can be derived by ideal solution of diffusion equations.* It serves as a link between fabrication processes and electrical terminal behaviour of the device via analysis of internal electronic conduction processes.

Through a proper interpretation of the electrical properties measured at the terminals of the device, parameters of a physical

* For devices fabricated by standard diffusion process, the impurity distribution can be approximated by a Gaussian or an Erfc distribution.

model can be evaluated. Methods to evaluate physical parameters have been suggested previously^(17,18,19,20). In addition to the need to determine these parameters for use as a basis of internal analysis in specific cases, an object of carrying out such a parameter evaluation process is to demonstrate the capability of the model as a representation of the device for design purposes, the parameter obtained being checked for correctness by independent means (e.g. deduced junction depths checked by bevelling and staining). The process of physical modelling consists of characterization of a device by means of suitable flow equations derived in terms of an assumed impurity distribution.

The work presented in this thesis is mainly concerned with physical modelling of double diffused transistors. The analysis presented in this work is carried out by assuming a double Gaussian distribution of impurities. Methods developed in the chapters to follow are equally valid for any other impurity distribution. In order to simplify the modelling process, characterization of a device is performed by a regional analysis approach, which enables one to treat the analysis and design of different regions of a device structure independently. Accuracy of such an analysis can be checked by computer simulation of the device.

Methods to determine the impurity distribution by experimental procedures have been presented previously^(16,58). Such procedures involve in sectioning the device. In the 'physical modelling'

approach, the same information can be obtained by determining the model parameters from terminal measurement made on a completed device. This method is attractive for the simple reason that the procedure is non destructive, the device being retained for further study or for application purposes.

Work published previously^(19,20) was applicable to only certain types of device structure. Physical modelling of a graded base transistor was first proposed by Das and Boothroyd⁽¹⁹⁾ by considering a single exponential model for the base region. This model was found to be adequate to characterize devices having a diffused base and alloyed emitter region structure. A further improvement of this model was provided by Boothroyd and Trofimenkoff⁽²⁰⁾, where the base device was modelled by a region of linear impurity grading adjacent to the emitter followed by an exponential distribution of impurities. These analytical approximations were restricted to single diffused structures or, in the case of the linear plus exponential model, to a double diffused case where the retarding field region of linear grading formed a small fraction of the total base width. Thomas and Boothroyd⁽²¹⁾ suggested a double exponential distribution of impurities in the base region and Thomas⁽¹⁷⁾ has discussed in detail methods for evaluating parameters of this model. Although, this model is adequate to characterize commonly used low frequency transistors, it has limitations in its applicability in that the model cannot be employed for characterizing small geometry devices and does not provide complete

information of the diffused layers. In order to obtain complete information on the device structure and also enable one to correlate process control parameters to that of device structure, the model must be capable of representing the entire impurity distribution. In the work presented in this thesis, physical modelling of double diffused transistors is carried out by using a double Gaussian model.

1.2 Objectives and Summary

The primary objective of the work presented in this thesis is to implement a procedure to model and characterize double diffused transistors by Gaussian distributions of impurities obtained from ideal solution of the diffusion equations. Specifically, it was required

1. To establish a theory to characterize the electrical properties of double diffused transistors in terms of a double Gaussian impurity distribution model in such a way as to accommodate devices of very small geometry.
2. Using the theory established above, to develop an algorithm to evaluate parameters of the double Gaussian model from terminal measurements made on completed devices.
3. To carry out experimental verification of the parameters evaluated from the above algorithm.

The first objective permits one to compute electrical characteristics at the terminals of the device by completely specifying the device structure. The theory established also permits one to

characterize high frequency small geometry devices with minimum modification to the basic approach. The second objective is to use the same basic equations developed earlier, in order to evaluate model parameters by making specified electrical measurements at the terminals of a device. This procedure provides information regarding the internal device structure by non-destructive means. The accuracy of such a modelling process is ascertained by the degree of correlation obtained between the computed and experimentally measured parameters, which is the third objective of the work presented in this thesis.

To facilitate the parameter determination process, suitable approximations are made in certain regions of the transistor. In Chapter 2, a method to evaluate parameters of a model for the collector junction of double diffused transistor from capacitance and breakdown voltage measure across the collector junction is presented. In this region, the double Gaussian distribution is approximated by a single exponential model. Analysis of a linearly graded forward biased junction is discussed in Chapter 3. Characterization of such a junction is performed by developing an approximate solution theory in order to linearize Poisson's equation. Using this approximate solution, characterization of small geometry devices is presented in Chapter 4. In Chapter 5, a procedure to evaluate parameters of a double Gaussian model from terminal measurements is developed.

CHAPTER 2

MODELLING AND CHARACTERIZATION OF COLLECTOR REGION OF A DOUBLE DIFFUSED BIPOLAR TRANSISTOR

2.1 Introduction

In order to develop a reliable and systematic approach for assessment and design of a bipolar transistor, the need for a model capable of detailed characterization of the device can be appreciated. The model, which must relate terminal performance to that of the device internal structure, should preferably be as simple as possible yet still afford adequate characterization of the device for the purpose of design and analysis. Using a regional approach to the problem, in this chapter, methods to evaluate the parameters of a model for the base-collector junction region of a diffused bipolar transistor are presented.

2.1.1 Double Diffused Transistor

Throughout this thesis, the devices discussed are assumed to have been fabricated by standard planar diffusion techniques⁽¹²⁾, involving a two-step diffusion process⁽¹³⁾. Without going into details of diffusion mechanisms, the nature of the impurity distribution after completion of a diffusion cycle can be derived from Fick's Laws⁽¹⁴⁾ assuming idealized boundary conditions. The

equations governing these laws are

$$F = -D \frac{\partial N}{\partial x} \quad (\text{in one dimension}) \quad 2.1.1$$

and

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} \quad 2.1.2$$

where

F = flux density

N = impurity density

D = diffusion coefficient

Depending on the nature of the boundary conditions imposed during the diffusion cycle, the solution of the above equations results in either a complementary error function distribution

$$N(x) = N_0 \operatorname{erfc} (x/\sqrt{4Dt}) \quad 2.1.3$$

for impurities diffusing from a constant impurity source of concentration N_0 , or a Gaussian distribution

$$N(x) = \frac{Q}{\sqrt{\pi Dt}} \exp(-x^2/4Dt) \quad 2.1.4$$

for instantaneous diffusion of a source of surface charge Q , pre-deposited on the slice prior to the diffusion step. In the above equations,

D = diffusion coefficient of diffusing impurities

t = diffusion time.

In reality, the impurity distribution does not follow either of the above distributions^(13,15) due to non-linear reactions occurring during diffusion of impurities. It is not possible to include these non-linear effects in the solution of Fick's Laws and retain the mathematical simplicity of the distributions of Equations (2.1.3) and (2.1.4). In fact, for devices fabricated using the two-step diffusion process, it has been shown⁽¹⁶⁾ that the impurity profile is a combination of both Erfc and Gaussian distributions. However, detailed investigation of devices fabricated with the two-step process has shown^(17,18) that a Gaussian distribution of impurities is adequate for a reasonably accurate representation of the device, especially in the base and collector regions of the transistor.

Since most of the devices analyzed in this project were fabricated with the two-step diffusion process, the Gaussian impurity distribution has been assumed throughout the thesis, although both the basic methods and the discussion presented are equally valid for any other distribution.

Since the fabrication of a transistor involves consecutive diffusion of opposite types of impurities into a substrate material, each of the impurity distributions is assumed to follow Equation (2.1.4) independently as shown in Fig. 2.1a. The net impurity

Fig. 2.1a

Fig. 2.1b

distribution is obtained by superimposing the two distributions on the substrate doping density as shown in Fig. 2.1b and represented mathematically by

$$N(x) = N_D - N_A = N_{S1} e^{-x^2/4D_1 t_1} - N_{S2} e^{-x^2/4D_2 t_2} + N_B \quad 2.1.5$$

where

$$N_{S1} = \frac{Q_1}{\sqrt{\pi D_1 t_1}} \quad \text{Emitter N-type diffusion} \quad 2.1.6a$$

$$N_{S2} = \frac{Q_2}{\sqrt{\pi D_2 t_2}} \quad \text{Base P-type diffusion} \quad 2.1.6b$$

are the surface concentrations of the two diffusions.

N_B = substrate doping concentration.

2.2 Representation of Impurities by Exponential Distribution

Earlier attempts in modelling^(19,20,21) devices assumed an exponential distribution of impurities, enabling explicit derivation of relationships between the device structure and the terminal parameters. Detailed analysis of a device⁽¹⁷⁾ represented by a double exponential distribution has shown that the simplified model is adequate for the operating regions of the device, extending from emitter side of the emitter-base space charge region to the

collector contact. This distribution can not, however, be extrapolated back to the surface of the device (i.e. the exponential profile is a good approximation to both the Gaussian and the Erfc distribution when the concentration has fallen below 10^{-3} of its surface value⁽¹⁷⁾). Consequently, parameters depending on the total distribution of impurities cannot be computed using this distribution.

2.3 Single Exponential Representation of Impurities at the Base-Collector Junction of a Transistor

Previous work⁽¹⁷⁾ has shown that for most of the devices the influence of the emitter diffusion on the distribution of impurities near the vicinity of the collector junction can be neglected. This assumption enables one to reduce the complexity of the distribution by neglecting the emitter diffusion in Equation (2.1.5) and the resulting distribution is represented by

$$N(x) = -N_{S2} e^{-x^2/4D_2 t_2} + N_B \quad 2.3.1$$

Using a similar approach, Thomas⁽¹⁷⁾ has suggested that the impurity distribution can also be represented by a single exponential model of the form

$$N(x') = N_D - N_A = -N_B (e^{-x'/L} - 1) \quad 2.3.2$$

and shown in Fig. 2.2a, as a partial representation of the total impurity distribution. The position $x' = x - x_{jc}$ is with respect to the collector metallurgical junction. It has been shown^(17,18) that the single exponential model represented by Equation (2.3.2) is capable of characterizing the depletion region properties of a double diffused transistor at the collector junction with good accuracy.

Due to simplicity of the model in representing the distribution, the related equations that characterize junction properties are much simpler to handle as compared to those derived from Equation (2.3.1). Thus, it is advantageous to use the exponential distribution when applicable to facilitate the model parameter determination process. In the analysis discussed in this chapter and in the work done previously^(18,21), such an approach has been used. The parameters to be determined in order to completely define the model are collector substrate doping density N_B , characteristic length L of the exponential model, and the collector junction depth x_{jc} (r_j in the edge region - see Fig. 2.2b). Methods to evaluate consistent parameters for this model, using capacitance and junction breakdown information obtained from measurements made at the terminals of the device, are presented in this chapter. Results obtained for MESA diode structures from this model have been shown⁽²²⁾ to agree extremely well with the parameters measured on experimental devices.

Fig. 2.2a Exponential impurity distribution model for collector-base junction.

fig. 2.2b Cross-sectional representation of a diffused junction

In practice, impurity diffusion is carried out through windows cut in the oxide on the surface of the silicon slice. A one-dimensional diffusion equation represents a satisfactory means of describing this process, except at the edges of the oxide window. Here, the dopant glass provides a source for impurities which diffuse parallel to the silicon surface as well as at right angles to it. Kennedy and O'Brien⁽²³⁾ have shown the resultant contour is approximately cylindrical, and for the range of devices considered in this thesis it is reasonable to assume the depth of lateral penetration of impurities to be equal to the longitudinal depth.

Detailed computations performed on cylindrical junctions show^(18,24,25,26) that junction breakdown occurs in the cylindrical regions, being sensitively dependent on the radius of curvature of this region which may therefore be inferred from breakdown properties. In the following sections, a plane junction geometry has been assumed for characterizing the junction capacitance properties from which parameters N_B , L are deduced; and a cylindrical junction profile for representing the junction breakdown, which yields the collector junction depth x_{jc} . A single exponential distribution of impurities similar to Equation (2.3.2) has been used for this purpose and is represented by

$$N(r) = - N_B (e^{-(r-r_j)/L} - 1) \quad 2.3.3$$

where $r_j = x_{jc}$ is the collector junction depth in the edge regions as shown in Fig. 2.2b.

2.4 Collector Junction Model

In this section, methods to evaluate N_B and L for an exponential model of the type discussed in Section 2.3 are presented.

For a graded junction, it has been shown⁽³⁶⁾ that the depletion layers have a first order dependence on impurity profile which is reflected in the junction capacitance properties. Proper interpretation of the electrical quantities measured at the terminals of the device can be used to determine the parameters of the model. By comparing the model parameters thus evaluated to those actually measured on the device, one can establish the degree of reliability of the model for junction characterization.

One-dimensional analysis is treated with the following assumptions:

1. Ionization of donors and acceptors is complete at room temperature.
2. The depletion layer is completely void of mobile carriers in equilibrium and reverse bias situations.

2.4.1 Collector Junction Depletion Region Equations

The method for determining the parameters N_B , L of the exponential model is based on the solution of Poisson's equation. The net charge density in the depletion region can be written as

$$\rho = q N(x') \quad 2.4.1$$

From Poisson's equation,

$$\frac{d^2V}{dx'^2} = - \frac{q N(x')}{\epsilon} \quad 2.4.2$$

After substituting Equation (2.3.2) for $N(x')$, integration of Poisson's equation gives the result

$$V_{tc} = \frac{q N_B d_c^2}{\epsilon} \left[\frac{1}{2} \coth \left(\frac{1}{2} \frac{d_c}{L} \right) - \frac{1}{d_c/L} \right] - \frac{kT}{q} \frac{d_c}{L} \quad 2.4.3$$

where V_{tc} is the total voltage developed between the collector and base boundaries of the space charge layer, of width d_c . The value of d_c is known under given voltage bias conditions, being obtained from measured junction capacitance from the relationship

$$C_{jc} = \frac{\epsilon A_c}{d_c} \quad 2.4.4$$

where A_c is the collector junction area (i.e. the area of the plane junction, together with a correction, if necessary, for the edge region).

Assuming that any voltage V_a applied to the collector with respect to the base* is developed entirely across the space charge

* A reverse biased junction, $V_a > 0$, is assumed.

layer, then

$$V_{tc} = V_a + \phi_{bo} \quad 2.4.5$$

where the quantity ϕ_{bo} is called the built-in potential or barrier voltage and is approximately constant.

Using Equations (2.4.3 - 2.4.5), methods of determining the parameters N_B , L of an exponential model from collector junction capacitance measurements have been proposed in earlier work^(20,21). A feature of these methods is the use of just sufficient capacitance data to permit evaluation of N_B and L from specific algebraic relationships derived from Poisson's equation. The steps that were involved in determining the model parameters are briefly reviewed here.

By rearranging terms, Equation (2.4.3) can be written as

$$V_{tc} = \frac{q L^2 N_B}{\epsilon} \zeta_2 \left(\frac{d_c}{L} \right) \quad 2.4.6$$

where

$$\zeta_2 \left(\frac{d_c}{L} \right) = \frac{1 - \exp \left(- \frac{d_c}{L} \right)}{\frac{d_c}{L}} \zeta_1 \left(\frac{d_c}{L} \right)$$

$$\zeta_1 \left(\frac{d_c}{L} \right) = \frac{1}{2} \left(\frac{d_c}{L} \right)^2 \coth \left(\frac{1}{2} \frac{d_c}{L} \right) - \frac{d_c}{L}$$

A straightforward solution of Equation (2.4.6) is not possible because there are two unknown parameters so that before any attempt is made to solve the equation, these terms must be separated. A method to solve this equation with suitable approximations has been suggested by Thomas⁽¹⁷⁾. The procedure involves making use of the capacitance-voltage relationship. In an earlier publication⁽²⁰⁾, where detailed analysis was carried out for a single diffused junction with an exponential grading of impurities, it was shown that

$$\frac{A_c^3 N_B}{L} = \frac{12}{q\epsilon^2 \delta} \zeta_2 \left(\frac{d_c}{L} \right) \quad 2.4.7$$

where

$$\delta = \frac{d(1/C_{tc}^3)}{d V_a}$$

C_{tc} = collector transition region capacitance and δ is the slope of a $1/C_{tc}^3$ vs V_a plot. For low reverse bias voltages, the function $\zeta_2(d_c/L) \rightarrow 1$ as $d_c/L \rightarrow 0$. As a result, it was possible to simplify Equation (2.4.7) by assuming $\zeta_2(d_c/L) \approx 1$ for low reverse bias values.

By knowing C_{tc} at any given voltage V_a , d_c can be computed from Equation (2.4.4). Rearranging terms in Equation (2.4.3), the relationship between total voltage and depletion layer width

can be alternatively written as

$$\frac{V_{tc}}{N_B d_c^3/L} = \frac{q}{\epsilon} \left(\frac{L}{\epsilon} \right)^3 \left\{ \frac{1}{2} \left(\frac{d_c}{L} \right)^2 \coth \left(\frac{1}{2} \frac{d_c}{L} \right) - \frac{d_c}{L} \right\} - \frac{kT}{q} \frac{d_c}{L} \quad 2.4.8$$

$N_B d_c^3/L$ can be computed from Equation (2.4.7) by knowing d_c .

Now Equation (2.4.8) can be solved for only one unknown term L .

In carrying out this procedure, the voltage V_{tc} used in Equation (2.4.8) is the sum of the applied voltage V_a and the built-in potential ϕ_{bo} is shown in Equation (2.4.5). The value for Equation (2.4.5) was previously⁽¹⁷⁾ determined graphically by extrapolating the $1/C_{tc}^3$ vs V_a plot to the horizontal axis, the intercept of the plot being taken as ϕ_{bo} . Since the unknown term L is in an implicit form in Equation (2.4.8), the solution was performed by numerical methods with the aid of a computer.

Using this approach, model parameters N_B and L have been computed^(20,21) for a planar double diffused transistor. The values for N_B thus evaluated indicated an unsatisfactory agreement with the device substrate doping density. It was found that the results obtained from such procedures vary with the choice of capacitance data employed; it is then not clear which out of the possible range of values for N_B and L should be selected for proper characterization of the collector junction. Variation of N_B with the

Fig. 2.3 Dependence of Background doping density N_B on applied voltage V_A across the collector junction

choice of capacitance measured at a particular applied voltage is shown in Fig. 2.3, for a typical device used for investigation in this thesis.

Since junction breakdown is strongly dependent on N_B , the resulting uncertainty in the computed values of N_B leads to uncertainty in the collector junction depth x_{jc} evaluated from the model. Hence, procedures described above were inadequate for the modelling process discussed in this thesis.

Variation of model parameters with the capacitance data stems from the choice of built-in potential ϕ_{bo} that appears in Equation (2.4.5), especially when the capacitance measurement is made at relatively low reverse bias values. In the analysis discussed above, ϕ_{bo} was assumed to be a constant and its value was determined from graphical extrapolation of measured capacitance vs voltage data. However, by considering Boltzmann equilibrium statistics, the built-in potential can be analytically determined from

$$\phi_{bo} = \frac{kT}{q} \ln \left| \frac{N(x'_1)N(x'_2)}{n_1^2} \right| \quad 2.4.9$$

for a space charge extending between x'_1 and x'_2 (Fig. 2.2). While approximately constant, ϕ_{bo} is actually dependent on the applied voltage V_a since x'_1 and x'_2 vary with V_a .

Nuyts and van Overstraeten⁽²⁸⁾ have shown that, owing to the presence of holes and electrons in the depletion region, the value of ϕ_{bo} computed from Equation (2.4.9) for complete depletion is larger than the exact voltage over the same width of the depletion region. The reason for this property can be appreciated as being due to the different distribution of space charge in the complete depletion model and in the actual device. In the model, the space charge density makes a sudden change from full depletion value to zero at the edges of the space charge layer of width d_c , where in the actual device, there is a gradual transition. Thus, for the same amount of space charge in the two cases, the total barrier heights are not the same; the barrier height for the depletion model being smaller than the corresponding voltage barrier between collector and base boundaries of the actual device. Complete simulation of a linearly graded junction⁽²⁹⁾ in equilibrium exhibits this difference. The small voltage discrepancy between the depletion model and the true device can be corrected, approximately, by choosing a value for ϕ_{bo} smaller than that computed from Equation (2.4.9). Exact calculations on a linearly graded junction show⁽²⁸⁾ that ϕ_{bo} evaluated from a linear extrapolation of the computed plot of $1/C_t^3$ vs V_a intercept agree extremely well with that evaluated from actual measurements made on the device the value of ϕ_{bo} thus evaluated being smaller than that evaluated from Equation (2.4.9).

However, for an unsymmetrical junction, e.g. exponential and Gaussian, a linear extrapolation of the $1/C_t^3$ vs V_a plot could lead to considerable error in ϕ_{b0} , since the data points do not lie on a straight line as is expected for a linearly graded junction. This deviation becomes more pronounced the shallower the junction. The effect of this non-linearity on shallow diffused junctions can be understood by considering the function $\xi_2(d_c/L)$ discussed earlier. For the same applied voltage, decreasing the junction depth results in a more pronounced decrease in characteristic length L than in the depletion layer width d_c . Consequently, the ratio d_c/L for the shallow diffused junction will be greater than unity⁽²⁰⁾. Hence, the approximations made in the previous analysis to evaluate the model parameters will no longer be valid.

In order to evaluate consistent model parameters for shallow diffused devices, yet retain the simplicity of the model parameter determination process, a procedure involving computer fitting between the model and measured data is discussed in the next section*.

2.4.2 Determination of Model Parameters by Optimization of Mean Square Voltage Error

The inconsistency observed in model parameters evaluated using a single capacitance value can be eliminated by employing an

* Procedures presented in this chapter were developed independently before the publication of work presented by van Overstraeten and De Man⁽²²⁾.

alternative approach involving fitting the model to a series of measured values rather than to a single data point as in previous analysis. Determination of model parameters using this approach has been published earlier⁽²²⁾ for single diffused mesa diodes. However, the procedure adopted here differs from the previous work in the actual execution of the model fitting process; moreover, the method is extended to the collector junction of a double diffused planar transistor.

For the purposes of modelling, it is assumed for the actual device, with a voltage V_a applied between collector and base terminals, that the relationships of Equations (2.4.3), (2.4.4) and (2.4.5) may be used, with a suitable choice of value for ϕ_{bo} (taken to be a constant). This assumption is based on the knowledge that the capacitance-voltage relationship of a PN junction, for reverse bias, is accurately predicted by depletion theory. In the process to be described for the determination of the model parameters, ϕ_{bo} is treated as an unknown quantity, so that there are three parameters N_B , L and ϕ_{bo} which have to be found. From Equation (2.4.4)

$$d_{c1} = \frac{\epsilon A_c}{C_{jc1}} \quad \text{and} \quad d_{c2} = \frac{\epsilon A_c}{C_{jc2}}$$

where C_{jc1} and C_{jc2} are the collector junction transition capacitances measured at applied voltages V_{a1} and V_{a2} .

Also

$$d_{c_2} = \frac{C_{jc_1}}{C_{jc_2}} d_{c_1} = K_1 d_{c_1} \quad 2.4.10$$

Similarly

$$d_{c_i} = \frac{C_{jc_1}}{C_{jc_i}} d_{c_1} = K_i d_{c_1} \quad 2.4.11$$

where

$$K_i = \frac{C_{jc_1}}{C_{jc_i}}$$

C_{jc_1} = capacitance at any applied voltage V_{a_i}

Normalizing the depletion layer width d_c with respect to L ,
let

$$Z_1 = \frac{d_{c_1}}{L} \quad 2.4.12$$

Then, from Equation (2.4.11),

$$Z_i = K_i Z_1 \quad 2.4.13$$

By rearranging terms in Equation (2.4.3) and substituting the normalized factors, the expression for total voltage can be written as

$$V_{t_i} = \frac{q N_B (K_i d_{c1})^2}{\epsilon} \left\{ \frac{1}{2} \coth \left(\frac{1}{2} K_i Z_1 \right) - \frac{1}{(K_i Z_1)} \right\} - \frac{kT}{q} (K_i Z_1) \quad 2.4.14$$

Substituting

$$Z_2 = \frac{q N_B d_{c1}^2}{\epsilon} \quad 2.4.15$$

in Equation (2.4.14) for total voltage across the junction gives

$$V_{t_i} = Z_2 K_i^2 \left\{ \frac{1}{2} \coth \left(\frac{1}{2} K_i Z_1 \right) - \frac{1}{(K_i Z_1)} \right\} - \frac{kT}{q} (K_i Z_1) \quad 2.4.16$$

and

$$V_{t_i} = \phi_{bo} + V_{a_i} \quad 2.4.17$$

The method adopted for determination of the model parameters is to utilize Equations (2.4.16) and (2.4.17) in order to arrive at the best functional fit to the measured capacitance-voltage data. Substituting the values for d_c from capacitance measurements into Equation (2.4.16), computed values of $V_{ac_i} = V_{\epsilon_i} - \phi_{bo}$ are obtained for specified values of N_B , L and ϕ_{bo} ; these are compared with the corresponding applied voltages and parameter optimization is carried out by minimizing the mean square duration of voltage error, defined by

$$\delta(N_B, L, \phi_{bo}) = \frac{1}{\ell} \sum_{i=1}^{\ell} \left(\frac{V_{am_i} - V_{ac_i}}{V_{am_i}} \right)^2 \quad 2.4.18$$

In this equation, V_{am} and V_{ac} are the measured and computed values respectively of applied voltages V_a and $i=1 \dots \ell$ represents the range of measurements over which the model is made to fit the capacitance-voltage data. The minimization operation can be carried out alternatively, by minimizing mean square capacitance error, as has been done in the previous work⁽²²⁾. However, the former approach has been used in the work presented in this thesis, as the method is more efficient in terms of computational time as compared to the latter method.

2.4.3 Determination of N_B , L and ϕ_{bo} for Actual Devices

Due to complexities involved in optimizing the mean square deviation function with respect to three variables, extensive use of a digital computer has been made to perform this operation. Numerical procedures to evaluate the model parameters of a real device are outlined in this section. These procedures rely on a prior knowledge of the following information obtained from design details and from measurements made on the device.

1. The planar area A_c of the collector junction. If the side wall area is more than 8% of the total area, then appro-

Fig. 2.4 Flow Chart for computing N_B and L
from capacitance data.

appropriate correction has to be applied to the total area.

(For devices examined in this work side wall area was found to be negligible.)

2. Starting value for ϕ_{bo} . This value can be obtained from a linear extrapolation of a measured $1/C_{tc}^3$ vs V_a plot. The intercept of the extrapolation with the horizontal axis defines a starting value for ϕ_{bo} .
3. Collector capacitance C_{tc} measured over a wide range of applied voltage V_a with small intervals of measurement.
4. Initial values for Z_1 and Z_2 as defined by Equations (2.4.12) and (2.4.15). Since Z_1 and Z_2 are both functions of L and N_B , approximate values for these quantities can be estimated by assuming a linearly graded junction and obtaining the slope from capacitance voltage data.

The sequence of computations performed to evaluate model parameters is shown in the flow chart of Fig. 2.4. Computer program developed to perform this operation is presented in Appendix 1. Utilizing the starting value for ϕ_{bo} , δ is minimized using a standard minimization routine⁽³⁰⁾ with respect to N_B and L . The value for ϕ_{bo} is then adjusted in increments until the smallest δ_{min} is achieved. The resulting minimized mean square voltage error and the corresponding model parameter values depend on the range of measured data used in the optimization process. To

Fig. 2.5 Dependence of minimum of the mean square voltage error, $\delta_{\min}$ on the voltage range V_R of measured capacitance data.

illustrate the sensitivity of the minimum mean square voltage error to the choice of the parameter ϕ_{bo} , curves of $\delta(N_B, L, \phi_{bo})$ as determined from Equation (2.4.18) are plotted in Fig. 2.5, versus the voltage range V_R for the data used in the minimization process with ϕ_{bo} as a parameter. It can be observed that the function reaches a minimum only when a fraction of the total voltage range of measured capacitance data is used in the computation and then deviates for smaller or larger voltage ranges.

In applying the method to the parameter determination process, there was found to be an optimum range of applied voltage V_a for the capacitance data utilized for a given device, as illustrated in Fig. 2.5, for one typical transistor sample. Deterioration of closeness of fit between the theory used and the actual measured data is to be expected if the reverse voltage is made sufficiently large in view of the fact that an exponential impurity distribution model is employed to approximate the true double diffused profile. Under high voltage conditions, the space charge layer boundary in the base may be in a region where the exponential model departs appreciably from the device profile. Too small reverse voltage range, on the other hand, tends to increase the significance of the parameter ϕ_{bo} (this now being comparable with V_a) in the fitting of the model, and also to increase the effects of scatter of

experimental data. The procedure followed was to start with initial choice of ϕ_{b0} , and then to adjust ϕ_{b0} iteratively, optimizing the voltage range at each step until an absolute minimum δ value was reached.

The procedure discussed in this section was used for the determination of model parameters for a variety of planar double diffused transistors in an experiment designed as a demonstration of effectiveness of the approach. The transistors, all fabricated simultaneously on the same production line^{*}, fell into groups corresponding to the values of impurity density N_B of the original N-type slices. The substrate was non-epitaxial. The substrate doping density was computed from four-point probe resistivity measurements made on the slice prior to the diffusion of impurities. Table 2.1 lists the parameters measured on the device. The junction depths were measured by bevelling and staining^{**}.

* The author wishes to thank Mr. R. Falconer of Microsystems International Ltd., Ottawa, for fabricating the devices used in this investigation.

** Junction depths were measured by bevelling a sample and staining the bevelled region by Copper Sulphate staining solution to delineate diffused regions. The junctions were assumed to be located at the edge of the stained regions. However, there is uncertainty involved in this assumption, since depletion layers at the emitter and collector junctions have been reported to stain as dark regions⁽⁵³⁾. In order to investigate the influence of the position of the collector junction depth on breakdown voltage, two extreme interpretations of the depth of the stained region were employed as indicated in Table I: the depth listed under " x_i " is on the assumption that stain extends to the metallurgical junction; the depth under " x_{ji} " assumes staining to the edge of the depletion layer boundary in the base region, and is the result of adding to x_i the depletion layer 'semi-width' calculated from the exponential impurity density profile model. Accuracy of the optical system used to measure junction depth is $\pm 0.15\mu$.

Table 2.2 lists the parameters ϕ_{bo} , N_B and L computed from the algorithm discussed earlier. It is noted that the agreement between the N_B value (Table 2.2) obtained from the parameter determination method, and the value from 4-point probe measurement is very close. The perturbation of ϕ_{bo} in minimizing δ_{min} was very small and, in some cases, no change in this value was observed. In general, between 20 to 40 data points have been used to achieve the minimum value for δ . It is also interesting to note that the results obtained give consistently a smaller value for substrate doping density N_B at the edge of the slice than in the centre.

Good correlation obtained between the measured and computed results appears to validate both the choice of the profile model for a planar transistor collector junction, and the method for determining the model parameters by interpretation of the capacitance data. The accuracy of the method employed seems to be sufficient for resistivity profiling across the silicon slice. Further support for the approach used here is to be found in the results of the application of the model in breakdown voltage calculations as discussed in the following sections.

2.5 Collector Junction Breakdown Model

Characterization of the collector junction avalanche breakdown phenomenon has been performed previously^(17,31) assuming a cylindrical step junction geometry for the peripheral region of a

Device	ϕ_{b0} Volts ($1/C^3$ Intercept)	N_B^* cm^{-3}	x_i	x_{jc}^+ μ	x_{if}	BV CBO (Measured) Volts	Remarks
16B #T2	0.45	2.5×10^{15}	3.3		3.6	94.0	Centre of the Slice
16B #T2	0.45	2.4×10^{15}	3.3		3.6	97.0	Edge of the Slice
16B #T4	0.55	1.0×10^{16}	3.0		3.2	57.0	Centre of the Slice
16B #T4	0.53	1.0×10^{16}	3.1		3.3	59.0	Edge of the Slice
16B #T5	0.55	1.0×10^{16}	3.0		3.2	58.0	Centre of the Slice
16B #T10	0.59	2.0×10^{16}	2.7		2.9	44.0	Centre of the Slice

* 4-point probe measurement
+ spherical bevel section and stain

TABLE 2.1. Experimental determination of model parameters
- measured data.

x_i = depth of stained region
 $x_{if} = x_i$ + equilibrium depletion layer semi-width in base region
Accuracy of x_i estimated as $\pm 0.15\mu$ (half a sodium light fringe).

Device	ϕ_{bo} (Corrected) Volts	δ min	Optimal Voltage Range Volts	N_B (Computed) cm^{-3}	L μ	BV [†]		x_j^* (Computed) μ
						V_1	V_2	
16B #T2	0.50	3.40×10^{-5}	0-8	2.53×10^{15}	0.312	94.2	96.6	3.27
16B #T2	0.48	3.75×10^{-5}	0-9	2.33×10^{15}	0.293	96.2	98.7	3.40
16B #T4	0.58	2.03×10^{-4}	0-55	1.11×10^{16}	0.349	56.9	57.4	3.05
16B #T4	0.55	6.50×10^{-5}	0-50	0.99×10^{16}	0.339	59.1	59.5	3.08
16B #T5	0.56	1.15×10^{-4}	0-50	1.07×10^{16}	0.330	57.9	57.9	3.08
16B #T10	0.60	9.00×10^{-5}	0-40	2.25×10^{16}	0.342	43.6	44.0	2.80

* using measured BV_{CBO}

† using values of x_{jc} from sectioning: V_1 corresponds to x_i and V_2 to x_{ii} in Table 2.1

TABLE 2.2. Experimental determination of model parameters
- computed results.

planar junction. However, the accuracy of the methods has been seriously limited because an appreciable fraction of the depletion layer at the onset of breakdown lies on the base side of the junction. This effect is particularly pronounced in shallow diffused junctions. Accurate prediction of the breakdown voltage for single diffused planar diodes has been found possible with the assumption of a Gaussian impurity density model⁽²⁵⁾. In the present work, an exponential profile of impurities of the type shown in Fig. 2.2 and represented by Equation (2.3.3) is used as the basis of breakdown voltage analysis for the collector junction, assuming a cylindrical peripheral region as shown in Fig. 2.2b. The strong dependency of this model on the collector junction depth and the collector doping density is employed to calculate accurately the breakdown voltage of the junction. Alternatively, by specifying the junction breakdown voltage and the impurity profile (i.e. N_B and L), it is possible to compute the junction depth.

2.5.1 Characterization of Junction Breakdown Using An Exponential Distribution of Impurities

Modelling and characterization of junction breakdown using a single exponential model is more appealing than exact models⁽²⁵⁾ for reasons mentioned below.

- a) The model is represented by a simple equation and the related expressions derived for electrostatic potential

and field distribution in the space charge region are easier to handle.

- b) The computation time involved in determining breakdown voltage or junction depth is considerably reduced.
- c) Since this model was used to determine N_B and L from capacitance measurements with good accuracy, the same model could be employed to characterize the breakdown phenomenon with reasonable confidence.

Accurate characterization of the junction is to be expected unless the space charge region in the base is so wide as to cause a major discrepancy between the true double diffused impurity profile and the exponential model. Although the model by itself cannot be used to describe the complete impurity profile in the device, it can be used to generate either a Gaussian or an Erfc distribution which, under approximate conditions, are good representations of the diffused regions in the device.

The distribution of impurities represented by Equation (2.3.3) can alternatively be written in normalized form as

$$N(R) = - N_o (e^{-R\beta} - \lambda) \quad 2.5.1$$

where

$$R = r/r_j$$

$$\beta = r_j/L$$

$$\lambda = N_B/N_o = e^{-\beta}; N_o = N_B e^{\beta}$$

The avalanche breakdown voltage is evaluated in terms of the electric field distribution $E(R)$ in the junction space charge region; $E(R)$ being derived from the solution of Poisson's equation in cylindrical co-ordinates using the complete depletion approximation. In the normalized form, Poisson's equation becomes

$$\frac{1}{r_j R} \frac{d}{dR} [R \cdot E(R)] = \frac{q}{\epsilon} N(R) \quad 2.5.2$$

Solving this equation with one space charge region boundary at $R=R_1$ ($R_1 < 1$), the electric field distribution can be expressed as

$$E(R) = - \frac{q N_B e^\beta r_j}{R \epsilon \beta^2} \left[\rho^{-R_1 \beta} (R_1^{\beta+1}) - \rho^{-R \beta} (R^{\beta+1}) - \frac{\lambda}{2} \beta^2 (R^2 - R_1^2) \right] \quad 2.5.3$$

This equation is valid for $R_1 < R < R_2$, where R_2 is the radius of the deeper boundary in the collector region of the transistor. A relation between R_1 and R_2 can be established from the condition for the electric field to be continuous at the metallurgical junction and can be expressed as

$$\frac{1}{\beta (R_2^2 - R_1^2)} \left[\rho^{-R_1 \beta} \left(R_1 + \frac{1}{\beta} \right) - \rho^{-R_2 \beta} \left(R_2 + \frac{1}{\beta} \right) \right] = \frac{\lambda}{2} \quad 2.5.4$$

The criterion for junction breakdown is set by considering the ionization process induced by electrons and holes in a high electric field region. Due to a difference in the ionization rates of electrons and holes in silicon, a certain ambiguity could arise as to which type of carrier is to be considered. Howard⁽³²⁾ shows that for each source of carriers, the avalanche breakdown is the same as that defined in terms of the multiplication factor M_n , for ionization due to injected electrons alone. On this basis, the breakdown criterion is defined as that voltage at which the avalanche multiplication factor for incident electrons becomes infinite. The condition for onset of breakdown in terms of the ionization coefficient $\alpha(E)$ can be expressed as^(33,34)

$$r_j \int_{R_1}^{R_2} \alpha(E) dR = 1 \quad 2.5.5$$

with

$$\alpha(E) = r_j \alpha_n \exp \int_{R_1}^R \{ -(\alpha_n - \alpha_p) \} dR \quad 2.5.6$$

and the individual electron and hole ionization rates α_n, α_p defined by Chynoweth's⁽³⁵⁾ empirical formula

$$\alpha = A \exp \{-b/|E(R)|\} \quad 2.5.7$$

The coefficients for ionization rates used in this analysis are those determined by van Overstraeten and De Man⁽²²⁾ and are listed in Table 2.3. Equation (2.5.6), which provides a basis for junction breakdown, does not take into account the existence of a threshold region in which the electron is accelerated up to ionization energy. However, as van Overstraeten and De Man⁽²²⁾ point out, this will only be of significance at low voltage and will result in negligible error above 20 volts.

Determination of appropriate values for the depletion region boundaries R_1 and R_2 involves a simultaneous solution of Equations (2.5.4) and (2.5.6) and is carried out iteratively with the aid of a computer.

Having satisfied the two equations, the junction breakdown voltage is then found by integration of the electric field between the depletion layer boundaries as

$$r_j \int_{R_1}^{R_2} E(R) dR = BV_{CBO} \quad . \quad 2.5.8$$

A comparison of the field distribution computed from the single exponential model with that for a Gaussian model is presented in Fig. 2.6. It can be seen that the electric field does not attain a maximum value at the metallurgical junction in either of the two cases in contrast to a plane junction. The difference between the

Fig. 2.6 Distribution of Electric Field at junction breakdown.

Carrier	A cm^{-1}	b V cm^{-1}	m	E V cm^{-1}
Electrons	7.03×10^5	1.231×10^6	1	$1.75 \times 10^5 < E < 6 \times 10^5$
Holes	1.582×10^6	2.036×10^6	1	$1.75 \times 10^5 < E < 4 \times 10^6$
	6.71×10^5	1.693×10^6	1	$4.0 \times 10^5 < E < 6 \times 10^5$

TABLE 2.3. Ionization rate data for silicon
from Van Overstraeten and De Man⁽²²⁾

maximum field in the cylindrical edge region of a graded junction and the field at the metallurgical junction has been pointed out⁽²⁵⁾ to be the factor responsible for a lower breakdown voltage as compared to an abrupt junction. Fig. 2.7 shows a plot of computed breakdown voltage from the exponential model against background doping density N_B with junction radius as a parameter. The figure includes, for purposes of comparison, a Gaussian impurity distribution with a net impurity charge of 10^{15} atoms/cm². Junction gradient has been used as a basis of comparison between the exponential and Gaussian models. Good agreement of computed results from the model with those obtained from a Gaussian model is observed except when the background doping is light (agreement becomes poorer as junction depth is increased). This discrepancy can, in fact, be expected since the doping level on the diffused side of the junction increases more rapidly for the exponential model than for the Gaussian model. Thus, for the exponential model, the depletion layer spreads more into the lightly doped substrate than for the Gaussian model resulting in higher breakdown voltage. However, for shallow junctions fabricated on low resistivity material, the agreement is such that the exponential model can be employed to characterize junction breakdown of real devices. Using the relationships derived above, computed results are presented in the next section for breakdown voltages of the devices described in Section

Fig. 2.7 Variation junction breakdown voltage as a function of collector background doping density.

2.4.3, and a procedure for computing junction depths by non-destructive methods is developed.

2.5.2 Computation of Collector Breakdown Voltage and Collector Junction Depth from Breakdown Voltage Data

The collector breakdown voltage BV_{CBO} was calculated as described in the previous section using values of N_B and L determined from junction capacitance and the collector junction depth x_{jc} obtained by bevelling and staining procedures. For x_{jc} , two extreme interpretations of the depth of the stained region were employed as indicated in Table 2.1; the depth listed under " x_i " is based on the assumption that the stain extends to the metallurgical junction; the depth under " x_{ii} " assumes staining to the edge of the depletion layer boundary in the base region, and is the result of adding to x_i the depletion layer semi-width calculated from exponential impurity density profile model. Table 2.2 lists the computed values of breakdown voltage with the values V_1 and V_2 corresponding to the values x_i and x_{ii} respectively for x_{jc} in Table 2.1. It is seen that V_1 and V_2 are almost identical except for the devices (16B T2) with most lightly doped collector background region. Agreement with measured breakdown voltage values of Table 2.1 is excellent.

There is a possibility, however, for the space charge layer at the collector base junction to extend some distance into the

Fig. 2.8 Flow Chart for computing collector junction depth from measured breakdown voltage.

base region depending on the junction gradient, and may enter a region where the exponential profile departs from true profile. This limitation imposed by the model is observed to be effective only for devices fabricated on lightly doped substrate materials with relatively deep junctions as compared to the devices investigated in this chapter. For the devices presented in Table 2.1, this discrepancy seems to have little effect on computed BV_{CBO} . It was shown by Vincent et al⁽²⁵⁾ for single diffused junctions that, while the maximum field occurs on the diffused side of the junction (here the base), most of the field distribution extends well into substrate material. In a transistor, therefore, most of the avalanche ionization occurs in the maximum field region of the space charge layer, and spreads mainly into the collector region. Thus, ionization effects in the region of the base where the model departs from the true profile are likely to be negligible.

Using the same relationships derived above, it is possible to compute the collector junction depth by a non-destructive method. By specifying N_B and L (which could be computed from capacitance measurements) and using the measured BV_{CBO} as input data, the collector junction depth is computed iteratively by calculating BV_{CBO} for an initial value of x_{jc} and then perturbing x_{jc} until the computed breakdown voltage agrees with measured BV_{CBO} . The sequence of computations performed to execute this operation is shown in the flow chart of Fig. 2.8. A listing of the program used is included

in Appendix 2. x_{jc} values computed from this method are listed in Table 2.2 and show excellent agreement with sectioned junction depths presented in Table 2.1.

The parameters of the exponential model evaluated from the procedures discussed in the previous sections can be used to generate a corresponding Gaussian profile which is a more realistic representation of the impurity distribution in a real device. This approach, in fact, is used in a later chapter to evaluate parameters of a Gaussian model. In the next chapter, an approximate method to characterize forward biased p-n junctions is presented.

CHAPTER 3

MODELLING OF DIFFUSED EMITTER JUNCTION USING AN APPROXIMATE SOLUTION FOR ELECTROSTATIC POTENTIAL DISTRIBUTION IN THE TRANSITION REGION

3.1 Introduction

Operation of a bipolar transistor in its active mode requires the emitter junction to be forward biased and the collector junction reverse biased. Hence, when characterizing the properties of the device associated with the active mode operation, the presence of a forward bias across the emitter junction increases the complexity of the analysis. In Chapter 2, modelling of collector region was accomplished using reverse bias properties of the junction. The process of evaluating model parameters of this region was considerably simplified by assuming the depletion region approximation applied to the space-charge layer, developed across the junction. This assumption eliminates from Poisson's equation the distribution terms for mobile electrons and holes, and thereby reduces a difficult system of differential equations to a single, mathematically tractable differential equation. However, depletion theory contains restrictive features rendering it inapplicable to several important modes of junction operation. In particular, this theory is not strictly applicable to forward biased p-n junctions and reverse

biased junctions operating at large electric current densities (e.g. collector in a transistor).

In an early analysis presented by Shockley⁽³⁶⁾, an estimate was made concerning the influence of mobile electrons and holes on the space-charge layer of a linearly graded junction. It was established in this analysis that near charge neutrality can be obtained within the transition region of a linearly graded junction containing a small impurity atom gradient. On the other hand, for linearly graded junctions with a large impurity gradient, it was shown that the space-charge regions were depleted of mobile carriers. These two conditions represent the extreme operation situations in a p-n junction. Modifications arising from mobile carriers were evaluated by comparing the space-charge electrical capacitance computed from depletion theory with the capacitance of a junction exhibiting near charge neutrality. This approach provides a quantitative measure of the mobile carriers in a space-charge region, the analysis being limited to structures in equilibrium.

The complete behaviour of a junction operating under any bias condition can be obtained from computer simulation of the device structure. The procedure involves rigorous solution of a system of non-linear differential equations describing the dynamics of operation of the device. The solution of these coupled equations can be performed numerically; algorithms to implement this operation have been published by many authors^(27,29,37,39).

Although numerical techniques do not provide explicit equations describing the physical and electrical properties of a device, from a series of computer calculations, it is possible to present, in graphical form, information normally described by mathematical equations. However, simulation data is not readily available and is primarily useful only in the detailed understanding of device operation and not in device design. Device design by a regional approach, as an alternative method, is attractive, simple and widely used but has limitations in its application, particularly to forward biased junctions. To extend this approach to forward biased operation of p-n junctions, the need for improved models can be appreciated.

In order to simplify the analysis, an approximate method to characterize transition regions of a junction has been developed in this Chapter, taking into account presence of electrons and holes in these regions. The notable features of this approach are:

- 1) simplicity of deriving model parameters
- 2) the procedure is independent of biasing condition
- 3) since the boundaries of the transition regions are well defined, the method is suitable for a regional approach to device analysis
- 4) it is ideally suited for modelling the forward biased emitter junction of a transistor.

In this approach, the electrostatic potential distribution in the transition region is modelled by an approximate solution and the parameters of the model are evaluated in terms of the physical structure of the device and the boundary conditions imposed to terminate the transition region. Methods adopted to evaluate the model parameters and the junction characteristics computed from the approximate solution are presented in this Chapter.

Although the approach presented is specifically for a linearly graded junction, the results are applicable to structures for which the actual impurity density profile is reasonably approximated by a linearly graded model. Such a situation arises in the case of the emitter-base junction of a double diffused transistor. Methods developed to characterize a forward biased junction are discussed in the next section.

3.2 Representation of Diffused Emitter Junction by Linear Grading of Impurities

Diffused junctions are a special class of inhomogenous structures that are commonly encountered in microcircuit fabrication. For the class of double diffused devices discussed in Chapter 2, the grading of impurities near the collector junction can be represented by a single Gaussian distribution and at the emitter junction by a distribution resulting from superposing two Gaussian distributions of opposite impurity types. The distribution of net impurities in

the vicinity of both junctions can be closely approximated^(13,31) by exponential impurity gradings on either side of the junctions. Also, single Gaussian or Exponential profiles have been observed⁽⁴⁰⁾ to provide realistic approximations to impurity profiles for diffused p-n junction diode structures.

In shallow diffused high frequency transistors, in order to bring about a compromise between forward current gain and effective base resistance, both emitter and base regions are heavily doped. Consequently, over a short distance from the emitter metallurgical junction, the impurity densities on either side are comparable. Hence, at low values of reverse and forward voltages, the space-charge layers will penetrate in approximately equal amounts on either side of the junction. Also, due to an extremely high gradient of impurities at the junction, typically of the order of 10^{23} atoms/cm⁴, the space-charge layers are confined to very narrow widths. In Section 2.4.1, it was shown for small ratios of d_c/L , L being the characteristic length of the exponential profile and d_c the width of the depletion layer, the exponential profile could be approximated by an equivalent linearly graded junction. Similarly, for small reverse bias and for forward voltages, the emitter junction of the structures considered here can be approximated by a linearly graded junction.

Linear dependency of the impurity profile near the vicinity of the emitter junction can be alternatively illustrated by considering the junction space-charge capacitance variation with respect to

small reverse voltages. For a linearly graded junction, assuming complete depletion to exist in the transition region, the dependency of the transition capacitance on the total voltage across the junction is given by⁽¹³⁾

$$C_{te} = \frac{d Q_e}{d V_j} = A_e \left(\frac{\epsilon_q^2 a_o}{12 V_j} \right)^{1/3} \quad 3.2.1$$

where

Q_e = total charge in one-half of the depletion layer

V_j = total voltage across the junction

A_e = area of the emitter junction

a_o = gradient of the impurities.

Total voltage

$$V_j = V_a + \phi_{bo}$$

where

V_a = applied voltage

ϕ_{bo} = built-in potential

Assuming ϕ_{bo} to be a constant and differentiating Eqn. 3.2.1

with respect to the applied voltage V_a gives

$$\frac{d \left(\frac{1}{C_{te}^3} \right)}{d V_a} = \frac{12}{q \epsilon^2 A_e^3 a_o} \quad 3.2.2$$

The terms on the right-hand side of the above expression are constant. Thus, the inverse cube of junction capacitance varies linearly with applied voltage. Fig. 3.1 shows a plot of measured $1/C_{te}^3$ against applied voltage V_a for a 16B transistor used in the work presented in this thesis, indicating a linear variation of the two quantities confirming that for low voltages the junction may be considered to be linear.

Hence, in the analysis presented in the following chapters, the emitter junction is assumed to be characterized by a linearly graded junction. Because of the symmetry of the junction, the analysis is simplified in that a solution is required for only one side of the space-charge region. Also, in the approach discussed in the next section, the linearly graded junction assumption facilitates obtaining simple boundary conditions which provide continuity of electrostatic potential, electric field and space-charge density.

3.3 Approximate Solution of the Transition Layer Potential Distribution and Related Characteristics of a Linearly Graded Junction

The steady state distributions of electrons, holes and electrostatic potential in a homogenous semiconductor are described by the equations,

$$\frac{d^2\psi(x)}{dx^2} = - \frac{q}{\epsilon} \{p(x) - n(x) + N(x)\} \quad 3.3.1$$

Fig. 3.1.1 Graph of $1/C_{te}^3$ vs. V_{cb}

$$J_p(x) = -q D_p \frac{dp(x)}{dx} - q\mu_p p(x) \frac{d\psi(x)}{dx} \quad 3.3.2$$

$$J_n(x) = q D_n \frac{dn(x)}{dx} - q\mu_n n(x) \frac{d\psi(x)}{dx} \quad 3.3.3$$

$$J_T(x) = J_p(x) + J_n(x) \quad 3.3.4$$

For an accurate assessment of the properties of the different regions of the device, it is essential to solve the above equations simultaneously. Due to the non-linear nature of the equations, the system of equations can be solved only by numerical methods. Several authors^(36,41) were able to solve the system of equations by neglecting the influence of holes and electrons in the transition region (i.e. the depletion approximation), so that this region was characterized by Poisson's Eqn. (3.3.1) alone. However, this assumption is valid only for thermal equilibrium and reverse bias modes of operation of the junction. Morgan and Smits⁽²⁷⁾ were able to obtain a numerical solution for Poisson's equation taking into account the presence of holes and electrons which were related to the electrostatic potential by the Boltzmann factor. Although their investigation was conducted with the aid of a computer, they elimin-

ated many mathematical difficulties by solving coupled differential equations by assuming zero electric current through the transition region even under forward bias. Subsequently, exact solutions performed by Kennedy and O'Brien⁽²⁹⁾ have shown that the zero current assumption does not influence significantly the distributions of electrostatic potential and carrier densities in the transition region. However, these methods are found to be in general complex and not readily applicable to device analysis and design.

A non-linear differential equation, in principle, can be solved by Picard's⁽⁴²⁾ method. This method relies on setting up a recurrence relation for the solution of the differential equation by introducing a guess for the unknown parameters. By solving this differential equation, a first approximation is obtained for the required solution. The approximate solution is next substituted in the original equation and this equation is again solved; a better approximation is thereby obtained for the required solution. In extending the application of Picard's method to the problem considered here, a zero current through the transition region is assumed. Thus, only Poisson's equation needs to be solved to characterize this region. Normalizing electrostatic potential by the Boltzmann voltage, kT/q , and assuming a Boltzmann distribution of carriers for electron and hole densities given by

$$n = n_1 \exp \frac{q}{kT} (\psi - \phi_n)$$

3.3.5a

$$p = n_i \exp \frac{q}{kT} (\phi_p - \psi) \quad 3.3.5b$$

Eqn. 3.3.1 for a linearly graded junction can be expressed as

$$\frac{d^2 u}{dy^2} = - \frac{q^2 x_o n_i}{kT\epsilon} \left\{ \frac{a_o x_o}{n_i} y - 2 \exp\left(\frac{u_p - u_n}{2}\right) \sinh(u) \right\} \quad 3.3.6$$

where the transition layer is bounded by two planes separated by a distance $2x_o$ and

$$u = \frac{q}{kT} \psi \quad 3.3.7$$

$$u_n = \frac{q}{kT} \phi_n \quad 3.3.8$$

$$u_p = \frac{q}{kT} \phi_p \quad 3.3.9$$

$$y = \frac{x}{x_o} \quad 3.3.10$$

Eqn. 3.3.6 is of the form

$$\frac{d^2 u}{dz^2} = C \sinh u + Dz \quad 3.3.11$$

To solve this equation by Picard's method, an initial guess can be introduced for u and by solving the differential equation, a first approximation is obtained. By successive substitution, the initial value of u is improved. In theory, after an infinite number of iterations, an exact solution will be obtained for this differential equation. It is to be emphasized, however, that in solving a differential equation by Picard's technique no general method exists whereby one can establish the error resulting from a finite number of iterations.

The complicated nature of this equation has prevented an extensive application of Picard's algorithm. To cope with this difficulty, Sah⁽³⁸⁾ first simplified the problem by assuming that

$$u = kz$$

3.3.12

Substituting this in Eqn. 3.3.6 results in a linear differential equation of elementary form and thus an analytical solution was readily obtained. A second iteration of this process becomes difficult, if not impossible. For this reason, the solution was limited to a first approximation for the required potential distribution.

For reasons explained above, the approach pursued in determining the approximate solution presented in this Chapter is also restricted to a first approximation but differs from previously published

methods⁽³⁸⁾ in that the approximate solution for the potential distribution and the boundary conditions applied are more realistic.

Simulated results for linearly graded junctions show^(27,28) the resulting distribution of electrostatic potential to be as shown in Fig. 3.2. This can be approximated by a second or a higher order equation. Eqn. 3.3.12 can be recognized to be an unrealistic approximation to the real distribution. In order to preserve simplicity of the analysis, yet provide a reasonable approximation to the real distribution, a parabolic approximation of the form

$$U = Ay + By^2 \quad 3.3.13$$

has been used in the analysis presented in the following sections. The approximate distribution is used to describe the electron and hole densities, so that the Poisson's equation 3.3.6 is linearized. A solution can be derived readily with the use of boundary conditions which are obtained from physical considerations. Due to the nature of the assumed potential distribution, this analysis provides implicit solutions which are easily solved by simple numerical methods. These solutions can be studied in detail to provide physical understanding of the situation within and at the boundary of the transition region where direct physical measurements are not possible.

Fig. 3.2 Relationships between approximate solution and true solution from ref. 27.

3.3.1 Assumption of the Boundary Conditions

The parameters of the approximate solution used to linearize Poisson's equation (3.3.6) are evaluated from boundary conditions obtained from the following assumptions.

Assumption 1

Electron and hole densities within the transition region are assumed to be described by an auxiliary potential function of the form

$$U_0(y) = Ay + By^2 \quad 3.3.14$$

Eqn. 3.3.6 is linearized by substitution of Eqn. 3.3.14 to describe electron and hole densities, and using normalized notations (see list of symbols) Poisson's equation can be expressed as

$$\frac{d^2 u}{dy^2} = - \frac{q^2 x_c^2 n_i}{kT\epsilon} \left(\frac{a_{00} x}{n_i} y - 2 \exp \frac{(u_p - u_n)}{2} \sinh (Ay + By^2) \right) |y| \leq 1 \quad 3.3.15$$

The edges of the transition region at $y=\pm 1$ are defined by the boundary conditions obtained from the following assumptions.

Assumption 2

The boundary space-charge density is

$$\rho(\pm 1) = 0 \quad 3.3.16$$

where

$$\rho(y) = q n_i \left\{ \frac{ax_o}{n_i} y - 2 \exp(\xi_b) \sinh[u_o(y)] \right\} \quad 3.3.17$$

$$u_o(y) = Ay + By^2$$

$$\xi_b = \frac{u_p - u_n}{2} \quad 3.3.18$$

Assumption 3

$u'(\pm 1)$ = the slope of the neutral solution obtained from
Poisson's equation at $y=\pm 1$

This assumption is necessary to avoid a dirac impulse distribution
of space charge at the boundary.

Assumption 4

The normalized electric field of the auxiliary potential at
the metallurgical junction is defined by

$$u'_o(0) - u'(1) = - \frac{Q_T x_o}{\epsilon 0.026} \quad 3.3.19$$

where Q_T is the total integrated space charge within one-half of the
transition region and is given by

$$Q_T = x_o \int_0^1 \rho(y) dy \quad 3.3.20$$

Assumption 5

The final electrostatic potential distribution within the transition with a total charge Q_T is assumed to be described by Poisson's equation (3.3.15) and the distribution is computed directly by integration of this equation. Thus, the externally applied potential is defined by the difference between the barrier potential and the total potential drop across the space-charge layer. Considering only half the magnitude of these potentials, the resulting relationship in normalized notation can be expressed as

$$\xi_p = \ln \left| \frac{a_o x_o}{n_i} \right| - u(\pm 1) \quad 3.3.21$$

where

$$\xi_p = u_a/2$$

$$u_a = \frac{q V_a}{kT}$$

$$V_a = \text{applied voltage}$$

The first assumption was made on the basis of a parabolic potential function being a reasonably good approximation to the true solution as compared to the linear function used previously⁽³⁸⁾. Parameters A and B appearing in the approximate potential function can be recognized to have units of electric field and space-charge

density respectively. Parameter A is assumed to be independent of distance and is a function of electric field at the metallurgical junction while term B is dependent on the position of the edge of the transition region and its value is determined by the boundary conditions imposed.

The second boundary condition states that the charge densities at the edges of and outside the transition region are zero, i.e. electrical neutrality. At the edge and outside the transition region, the distribution of electrostatic potential is defined by the neutral solution, i.e. $d^2u/dy^2 = 0$ $y \gg 1$. Since the auxiliary potential $u_0(y)$ is designated to represent the carrier distribution satisfying the Boltzmann statistical distribution, the corresponding quasi-fermi potential difference ξ_b for non-equilibrium must satisfy the boundary conditions for the potential function $u_0(y)$. Its value is determined from the boundary condition obtained from electrical neutrality conditions at the edge of the transition layer and is given by

$$\frac{ax}{n_i} - 2 \exp(\xi_b) \sinh[u_0(1)] = 0 \quad 3.3.22$$

It should be emphasized that the quasi-fermi potential difference ξ_b evaluated from Eqn. 3.3.22 does not correspond to the externally applied voltage V_a since the potential distribution $u_0(y)$

employed in the above equation is a function of the carrier distribution alone (related through the Boltzmann distribution) as opposed to the potential difference computed from Poisson's equation for an assumed space-charge distribution and given by Eqn. 3.3.21. It is also assumed that quasi-fermi levels for electrons and holes are constant across the transition region. This corresponds to the zero current assumption made earlier and is in accordance with the fact that it has been proven earlier^(36,43) that the quasi-fermi potentials do not change across the transition region even under appreciable current flow. Eqn. 3.3.22 can be solved only by numerical methods due to the implicit nature of the expression. However, under limiting conditions made for low forward bias voltages, it is possible to derive explicit solutions as shown in Appendix 3.

The third assumption establishes a continuity of the auxiliary potential $u_0(y)$ at the edges of the transition region with the neutral solution obtained from Poisson's equation (3.3.6) by equating $d^2u/dy^2 = 0$ at $y=1$. This condition also ensures a non-zero electric field at the edges of the transition region.

Parameter B can be used to satisfy the continuity condition for the auxiliary potential $u_0(y)$ at the edges of the transition region. Although in terms of absolute magnitude, the continuity condition does not deviate appreciably from zero electric field boundary conditions assumed in previously published results^(36,38), the resulting

space-charge distribution at the edges of the transition region obtained from non-zero electric field boundary conditions provides a better approximation to the true solution.

The fourth assumption is the most important difference between the analysis presented here and previously published methods⁽³⁸⁾. This boundary condition defines the magnitude of the derivative of the auxiliary potential $u_0(y)$ at the metallurgical junction. Physically, the electric field generated at the metallurgical junction by the auxiliary potential is constrained to have a magnitude equal to the difference in fields established by the total charge contained in the transition region at $x=0$ and $x=x_0$.

This assumption, coupled with the other assumptions made, is sufficient to specify a space-charge region of width $2x_0$, defined by auxiliary potential $u_0(y)$ which satisfies Poisson and Boltzmann equations at the edges and the middle of the transition region. These conditions permit the function $u_0(y)$ to establish a good first order approximation to the electric field obtained from the true solution.

Restricting the analysis to a first order approximation as postulated earlier, an approximate solution of Poisson's equation can be carried out once the function $u_0(y)$ is generated for a given total charge Q_T and from the boundary conditions discussed above. The problem that now remains to be solved is that of choosing the

appropriate distribution corresponding to a total charge Q_T for any given bias conditions. This operation can be performed from the last boundary condition. Physically, this condition implies that the 'auxiliary potential' $Ay + By^2$ provides distributions for the mobile carrier densities which are good approximations to the exact distributions. By application of Poisson's equation to the resulting space-charge distribution, a better approximation to the parabolic distribution is then obtained. Further, by definition, in any capacitance determination, Poisson's equation must be specified; otherwise, the concept of capacitance is meaningless. An appropriate value of Q_T is chosen to satisfy Eqn. 3.3.21 by an iterative procedure presented in the next section.

3.3.2 Approximate Solution of Poisson's Equation by Numerical Methods

From the boundary conditions considered in the previous section, an approximate solution of Poisson's equation can be obtained by linearizing the non-linear differential equation in terms of the auxiliary potential model used to describe electron and hole densities. The parameters of this model can be evaluated by the simultaneous solution of equations derived from physical considerations.

The use of the parabolic potential distribution given by Eqn. 3.3.14 and the electrical neutrality boundary condition given by Eqn. 3.3.16 results in one of the four important relationships between material properties and junction characteristics, viz.

$$\frac{a_o x_o}{n_i} = 2 \exp(\xi_b) \sinh(A+B) \quad 3.3.23$$

or

$$\xi_b = \ln \frac{a_o x_o}{2n_i \sinh(A+B)} \quad 3.3.24$$

The neutral solution of electrostatic potential for $y \geq 1$ is given by $d^2 u / dy^2 = 0$ and in normalized notations can be expressed as

$$u = \sinh^{-1} \left[\frac{a_o x_o}{2n_i \exp(\xi_b)} y \right] \quad 3.3.25$$

By differentiation of Eqn. 3.3.25 and introducing the third boundary constraint for the auxiliary potential at $y=1$, a second important relationship is derived as follows.

Differentiating Eqn. 3.3.25:

$$\frac{a_o x_o}{n_i} = 2 \exp(\xi_b) \cosh(u) \frac{du}{dy} \quad 3.3.26$$

Substituting $u(y) = Ay + By^2$ at $y=1$

$$A + 2B = \frac{a_o x_o}{2n_i \exp(\xi_b) \cosh(A+B)} \quad 3.3.27$$

From the junction electric field requirements specified by the

fourth assumption, another relationship can be derived. We have

$$u'_o(0) = A \quad 3.3.28$$

From Gauss's theorem,

$$e_{jn} = -A + e_{jx_o} \quad 3.3.29$$

e_{jn} = normalized electric field at the metallurgical junction

e_{jx_o} = normalized electric field at the edge of the transition region

Also, from Poisson's equation,

$$e_{jn} = - \frac{q^2 n_i x_o^2}{kT\epsilon} \int_0^1 \left(\frac{a_o x_o}{n_i} y - 2 \exp(\xi_b) \sinh (Ay+By^2) \right) dy + e_{jx_o} \quad 3.3.30$$

From Eqn. 3.3.20, substituting total charge Q_T for terms appearing on the right-hand side of the above equation, Eqn. 3.3.30 can be expressed as

$$e_{jn} = - \frac{q x_o Q_T}{kT\epsilon} + e_{jx_o} \quad 3.3.31$$

Equating Eqns. 3.3.29 and 3.3.31

$$A = \frac{q x_o Q_T}{kT\epsilon} \quad 3.3.32$$

The parameters of the auxiliary potential can be computed for a given bias voltage by simultaneous solution of Eqns. 3.3.24 through 3.3.32 such that the potential drop across the junction computed from the approximate solution satisfies Eqn. 3.3.21. Due to the implicit nature of the above expressions, the solution of these equations can be performed only by numerical methods. A computer program to perform the numerical operation has been developed in FORTRAN language and a listing of the program is presented in Appendix 4. The sequence of computations performed by the program to evaluate the parameters of the auxiliary potential are shown in Fig. 3.3 and outlined below.

Step 1

Junction gradient ' a_o ', applied voltage V_a and initial values of x_o and Q_T are read in. An estimate of the initial values can be computed from depletion theory.

Step 2

Using the starting values of x_o and Q_T , the parameter A is computed from Eqn. 3.3.32. Choosing initial values for B and ξ_b , Eqns. 3.3.24 and 3.3.27 are next simultaneously solved for B and ξ_b . Initial values for B and ξ_b can be computed from explicit equations

Fig. 3.3 Flow Chart for computing parameters
of the approximate solution model

obtained from assumptions made for low forward and reverse bias voltages. Analysis of this method is presented in Appendix 3.

Step 3

Employing the parameters evaluated above in Eqn. 3.3.17, the total charge in one-half of the depletion layer is computed from Eqn. 3.3.20. The total charge thus computed is compared with Q_T . If the two values do not agree, a new value for x_0 is chosen and steps 2 and 3 are repeated. Iteration is continued until mutually consistent values are obtained before proceeding to the next step.

Step 4

Using the auxiliary potential $u_0(y)$, potential drop $u(1)$ across one-half of the junction is computed from Poisson's equation (3.3.15) and is compared with the corresponding drop computed from Eqn. 3.3.21. If the two values do not agree, a different value for Q_T is assigned and steps 2 to 4 are repeated until mutually agreeing results are obtained.

3.3.3 Calculation of Junction Characteristics

Properties of a p-n junction characterized by the approximate solution method can be computed using the numerical procedures discussed in the previous section for linearly graded junctions ranging from steep to gentle gradients. Results presented in this section have been computed both for equilibrium and non-equilibrium

conditions. The capacitance properties of a junction characterized by this model are treated in a later section.

The equilibrium space-charge distribution within the transition region, computed from the approximate solution developed in this chapter is shown in Figs. 3.4 and 3.5. Along with it, computed results from simulation⁽²⁹⁾, Sah's linear approximation⁽³⁸⁾, and depletion theory are plotted. It is to be emphasized that Kennedy and O'Brien⁽²⁹⁾ arbitrarily terminated the space-charge regions for comparison purposes at locations where electrostatic charge density was one-half of its maximum value on each side of the junction. However, such approximations are not necessary for the other approximate solutions because the termination of the transition region is provided by a well-defined boundary condition. To identify the approximate solution developed here with other solutions, this method hereafter will be referred to as 'Parabolic Approximation'.

Small deviation of the parabolic approximation curve from simulated results could be attributed to the termination of transition region within a finite distance from the junction as opposed to the simulated case where the boundaries are located at the metallurgical contacts of the device. This illustration shows that the complete depletion layer method yields only a crude approximation. Further, Fig. 3.4 also shows that although Sah's⁽³⁸⁾ solution was an improvement over the depletion layer concept, it is also imprecise.

$$a_0 = 1.25 \times 10^{17} \text{ cm}^{-4}$$

- approximate solution
- · — Sah (ref. 38)
- · — Kennedy and O'Brien (ref. 29)
- - - Depletion theory

Fig. 3.4 Space charge density distribution of a linearly graded junction in equilibrium

Fig. 3.5 Space charge density distribution of a linearly graded steep junction in equilibrium.

The parabolic approximation on the other hand provides a better approximation to the simulated results, especially at the edges of the transition region. The slope of the space-charge density distribution at boundary of the transition region can be shown with the appropriate approximation to be equal to zero; a proof of this statement is given in Appendix 3. For junctions with steeper gradients than 10^{15} atoms/cm, the influence of mobile carriers within the transition region becomes negligible and the space-charge region tends to be depleted of mobile carriers. This effect is evidenced by better agreement obtained between the three curves shown in Fig. 3.5.

In Fig. 3.6, computed equilibrium space-charge half-widths from the parabolic approximation are presented as a function of the junction gradient. Also shown in this figure are the corresponding widths computed by the other methods. Although the parabolic approximation yields results with nearly the same magnitude as the simulated results⁽²⁹⁾ it is emphasized that the space-charge layer width from simulated results is an arbitrarily defined parameter and any modification to its definition could substantially change all of the solutions in Fig. 3.6 for the simulated case. However, the purpose of presenting this figure is to illustrate the good agreement obtained by the parabolic approximation with simulated results subject to the same arbitrary limits imposed on simulated results⁽²⁹⁾.

Fig. 3.6. Equilibrium space charge layer width

For a linearly graded junction, the equilibrium built-in voltage ϕ_{b0} is also an arbitrarily defined parameter. This situation arises from an impurity atom gradient within the semiconductor material outside the p-n junction space-charge layer. At thermal equilibrium, this semiconductor material contains sufficient space charge to generate an electric field. The magnitude of this field is determined by the impurity gradient. The drift component imparted to the mobile charge carriers must be equal and opposite to the diffusion component arising from the mobile charge gradient. In the present discussion, the equilibrium voltage is defined as the potential difference between the transition region boundaries. For this reason, the equilibrium potential will increase as the distance over which the voltage drop is increased. In Fig. 3.7, this difference can be observed by comparing the corresponding values evaluated from depletion theory and the parabolic approximation which has a wider transition region. Figs. 3.8 to 3.12 show the electronic charge, electrostatic potential, electric field, electron and hole distribution for equilibrium and various applied voltages for a junction of impurity gradient equal to those typically encountered at the emitter junctions of double diffused transistors.

In Fig. 3.13, the transition region half-width x_0 is plotted as a function of applied voltage. It is interesting to note that x_0 computed from the parabolic approximation decreases with increasing forward voltages in accordance with depletion theory. However, with

Fig. 2.7 Equilibrium diffusion voltage for a silicon pn junction

Fig. 3.8

Fig. 3.9

Figs. 3.8 and 3.9 Graphs of Charge Density and Electric Field distribution as a function of distance for different V_a values

Figs. 3.10, 3.11, and 3.12

Graphs of Electrostatic potential and mobile carrier
distribution as a function of distance.

Fig. 3.10

Fig. 3.11

Fig. 3.12

Fig. 3.13 Graph of space charge layer half width vs. applied voltage.

further increase in forward voltage, the space-charge region begins to spread out. This modification is clearly a result of space-charge neutralization arising from an increased concentration of majority carriers within each side of the junction.

3.3.4 Space Charge Capacitance Properties of Linearly Graded Junctions

Under certain modes of operation, the voltage dependence of stored charge (i.e. ac capacitance) of a p-n junction could become of such importance that the design of the junction is completely governed by this factor. In a transistor, junction capacitances have a first order influence on the high frequency current gain, and the charge stored in the junction capacitance affects the recovery time or the speed with which a transistor diode can be switched. Hence, a model designed to characterize the properties of a p-n junction should be capable of representing the capacitive behaviour of the junction accurately both for design and analysis purposes.

Differential capacitance is defined as the derivative of stored charge with respect to the voltage. The total capacitance of p-n junction diodes, or of devices containing p-n junctions, has been shown⁽²⁷⁾ quantitatively to consist of two components, transition region (or space charge) capacitance and diffusion capacitance. Transition capacitance is associated with differential space charge uncompensated by mobile charge stored within the transition region.

It is the dominant capacitance for reverse and low forward bias voltages. Diffusion capacitance on the other hand is associated with differential storage of mobile charge carriers of one type. It is significant only when the junction is heavily forward biased. Diffusion capacitance depends on the structure of the diode as a whole and is strongly dependent on the frequency and the current levels. Methods have been developed in the past to characterize the junction capacitance of a linearly graded junction for reverse bias by simple depletion theory⁽³⁶⁾ and empirical relationships have been developed⁽⁴⁵⁾ to model a forward biased junction. With modern techniques of analysis, simulation of a p-n junction has been performed by others^(27,28,29,37), analyzed on the basis of transport, continuity and space charge balance equations. Terminal characteristics, including all the capacitive effects, are reproduced by such calculations without the need of classification of capacitance into various categories. On the other hand, for the construction of compact device models, it is very desirable to have available simple, quantitative models of charge storage effects having reasonable accuracy.

Capacitive properties of a p-n junction can be computed from the parabolic approximation for reverse and low forward bias voltages. In this analysis, only the space charge capacitance of the junction has been computed. Being dependent on the stored charge in the space charge layer, the transition region capacitance is strongly dependent

on the junction profile. Transition capacitance of the linearly graded junction can be computed by assuming all the impurities within the material are ionized at room temperature and by considering separately, the capacitances associated with the p and n side of the space charge layer. Each of these capacitances is denoted by $2 C_t$ so as to result in an overall series capacitance of C_t .

With Q_T being the charge associated with one half of the depletion layer and $V_j/2$ the voltage across this half, transition capacitance is given by

$$2 C_t = \frac{dQ}{dV_{j/2}} \quad 3.3.33$$

$$C_t = \frac{dQ}{dV_j} \quad 3.3.34$$

Junction capacitances computed from Eqn. 3.3.34 using the parabolic approximation are shown in Fig. 3.14.

Overall capacitance has been observed to fall off at high current levels⁽⁴⁴⁾. However, the capacitance fall off exhibited by the curves in Fig. 3.14 is not due to this phenomenon. The reduction in capacitance is due to space charge neutralization occurring at increased current levels. In order to compute the total capacitance taking into account the influence of mobile carriers, the geometry of the device has to be considered and the simple approximate solution from a regional approach is no longer possible without modifications to the

Fig. 3.14 Graph of space charge capacitance vs. applied voltage

basic assumptions made. For forward biased junctions where transition capacitance is comparable to diffusion capacitance, the parabolic approximation provides a simple approach to characterize the junction.

In Chapter 4, the parabolic approximation method will be used to characterize the forward biased emitter junction of a double diffused transistor when computing current transport characteristics of the device.

CHAPTER 4

CHARACTERIZATION OF THE ACTIVE BASE REGION OF A DOUBLE DIFFUSED BIPOLAR TRANSISTOR

4.1 Introduction

Since the advent of the junction transistor, considerable effort has been expended in developing technology to fabricate devices with power gain frequency capabilities extending to the microwave region. Realization of such devices has been made possible by a better understanding of the dependency of the device characteristics on the structure of the device, and by accurately controlling the fabrication process in order to obtain desired device geometries. Structure-determined analysis of bipolar transistors has shown^(46,47) the geometry of the base region of a bipolar transistor to be a critical parameter in characterizing the current gain and high frequency capabilities of the device. In fact fabrication of modern high frequency devices was achieved by reducing the metallurgical base to submicron widths and by tailoring the impurity profile to obtain useful current and power gain in the higher frequency ranges. In the beginning, it was necessary to make simplifying assumptions in order to analyze the behaviour of bipolar transistors. In the conventional analysis, the device is sectioned into regions of high electric field (space charge regions) and regions of low electric field (quasi-neutral regions).

A one-dimensional forward current analysis can be carried out by focusing attention on the "active base region" whose boundaries are positions of nominal transition from the quasi-neutral base to the junction space charge region (positions made quite explicitly by the use of complete depletion approximation in the latter). This simplified assumption enables the minority carrier continuity equation to be linearized and an analytical solution can be obtained for the related equations. Analysis based on these simplifying assumptions proved to be more than adequate in modelling low frequency devices studied earlier^(17,21). However with the development of high frequency devices, which are characterized by shallow structures, the base widths have become comparable to space charge region widths, thus throwing doubt upon the concept of 'neutral region boundaries', and necessitating a more detailed analysis. A computer simulation study of such a device, based on rigorous solution of the transport equations carried out recently, has shown⁽⁴⁸⁾ that considerable error could be introduced for shallow structures in characterizing forward current properties by assuming the active base region to be terminated at the edges of the emitter and collector space charge regions. However in the simulation approach, development of compact models to characterize the different regions of the device for both design and analysis purposes becomes difficult and time consuming. In this chapter, a simplified method is presented for the analysis of forward current properties of shallow diffused structures; this is based on a regional

approach and takes into account the excess mobile charge stored in the forward biased base-emitter junction space charge region. This operation is accomplished by developing a generalized Moll-Ross equation similar to the one presented by Gummel⁽⁴⁹⁾, but without the use of the complete depletion approximation in the space charge regions. Results computed for different profiles by the approximate regional approach are compared with those computed from total simulation and conventional analysis of the device.

4.2 Generalized Moll-Ross Equation

The transistor is a charge controlled device. Fundamental to any device of this type is a source of carriers, a flow region through which the carriers must pass, and a sink at which they must be collected. For normal transistor action, the forward biased emitter junction serves as a source by injecting carriers into the base, the base region acts as the flow medium and the reverse biased collector as the sink for mobile carriers. Forward current characteristics and the influence of base doping profile on the performance of the device can be evaluated by solution of the base transport equations which govern the dynamics of mobile carriers in the base region.

In order to ensure high emitter injection efficiency the emitter region of the transistor is normally doped several orders of magnitude higher than the corresponding doping density in the base region, so that most of the current is carried by injected minority carriers in the base. In high frequency devices, the base width is very narrow

as compared to the carrier diffusion length. Hence, the injected carriers can be assumed to be collected at the collector without recombining in the base. Consequently, the base majority carrier current density can be assumed to be equal to zero. When the device is operating under low level injection conditions, the injected minority carrier density can be assumed to be small and the base majority carrier distribution outside the space charge regions remains essentially unchanged. Thus, a quasi-neutral situation exists in the base, exposing a neutral flow region for the minority carriers.

In deriving a relationship between the current density, carrier distribution and base doping profile, the conditions discussed above are assumed to be valid. Although the analysis presented in this section is carried out for a npn device, the method is equally valid for a pnp transistor.

The transport equations for electron and hole current densities in terms of drift and diffusion components in one dimension can be written as

$$J_n = q\mu_n n e + qD_n \frac{dn}{dx} \quad 4.2.1$$

$$J_p = q\mu_p p e - qD_p \frac{dp}{dx} \quad 4.2.2$$

where J_n and J_p are electron and hole current densities, μ and D are the mobility and diffusion coefficients of electrons and holes

(subscripted with n and p) respectively.

Assuming negligible majority carrier current flow in the base region (p-type carriers for an npn transistor), the longitudinal electric field developed in the base can be evaluated by equating Eqn. 4.2.2 to zero. Thus

$$e = \frac{kT}{q} \frac{1}{p} \frac{dp}{dx} \quad 4.2.3$$

where Einstein's relationship $D = \frac{kT}{q} \mu$ has been assumed to be valid in deriving Eqn. 4.2.3.

Substituting Eqn. 4.2.3 into Eqn. 4.2.1, the minority carrier current density in the base can be expressed as

$$J_n = \frac{qD_n}{p} \left[n \frac{dp}{dx} + p \frac{dn}{dx} \right] \quad 4.2.4$$

Integrating the above equation between limits x_1 and x_2

$$\left[pn \right]_{x_1}^{x_2} = \frac{J_n}{q} \int_{x_1}^{x_2} \frac{p(x)}{D_n(x)} dx \quad 4.2.5$$

$$p(x_2)n(x_2) - p(x_1)n(x_1) = \frac{J_n}{q} \int_{x_1}^{x_2} \frac{p(x)}{D_n(x)} dx \quad 4.2.6$$

Eqn. 4.2.6 is the basic form of generalized Moll-Ross relationship. Limits x_1 and x_2 are the emitter and collector contacts of the device. However, these limits can be arbitrarily chosen at any point

within the device so long as the basic assumptions made hold good. In this analysis, these limits are chosen from physical considerations so as to enclose an area where the majority carrier density $p(x)$ appearing on the right hand side of Eqn. 4.2.6 has a significant contribution to the integrand.

From Boltzmann statistics, electron and hole densities can be expressed as

$$n = n_i \exp \frac{q}{kT} [\psi(x) - \phi_n(x)] \quad 4.2.7.a$$

$$p = n_i \exp \frac{q}{kT} [\phi_p(x) - \psi(x)] \quad 4.2.7.b$$

where

ψ - electrostatic potential distribution

ϕ_n and ϕ_p - electron and hole quasi fermi potentials respectively.

The pn product of the two distributions results in

$$pn = n_i^2 \exp \frac{q}{kT} [\phi_p(x) - \phi_n(x)] \quad 4.2.8$$

Thus the pn products appearing on the left hand side of Eqn. 4.2.6 represent a disturbance from equilibrium by an amount proportional to the quasi fermi potential difference. Since the base majority carrier current flow is assumed zero here, any variation of the hole quasi fermi potential in the base can be neglected. For low

current levels, the quasi fermi potentials are assumed to be constant throughout the space charge regions. Using the hole quasi fermi potential as a reference level, Eqn. 4.2.6 can be expressed as

$$J_n = \frac{q n_i^2 \left\{ e^{-\frac{q\phi_n(x_2)}{kT}} - e^{-\frac{q\phi_n(x_1)}{kT}} \right\}}{\int_{x_1}^{x_2} \frac{p(x)}{D_n(x)} dx} \quad 4.2.9$$

Eqn. 4.2.9 establishes a relationship between the current density J_n , base majority carrier distribution $p(x)$ and the potential drops across the emitter-base and collector-base regions.

Emitter-base and collector-base voltages are defined as

$$V_{eb} = \phi_{pb} - \phi_n(x_1) \quad 4.2.10.a$$

$$V_{cb} = \phi_{pb} - \phi_n(x_2) \quad 4.2.10.b$$

where

ϕ_{pb} is the hole quasi fermi level assumed to be constant in the base region. These voltages differ from terminal voltages by ohmic drops. However, under low level injection conditions, the ohmic drops can be assumed to be small. Replacing $D_n(x)$ by an equivalent diffusion coefficient D_{nb} , Eqn. 4.2.9 can be expressed in a charge control form

as

$$I_c = K \frac{e^{\frac{qV_{eb}}{kT}} - e^{\frac{qV_{cb}}{kT}}}{Q_B} \quad 4.2.11$$

where

$$K = q^2 n_i^2 A_e^2 D_{nb} \quad 4.2.12$$

$$Q_B = qA_e \int_{x_1}^{x_2} p(x) dx \quad 4.2.13$$

$I_c = -J_n A_e$ has been used assuming collector current to be mainly composed of injected minority carriers.

In Eqn. 4.2.11, Q_B represents the total base majority carrier charge and depends on the bias voltages and the doping profile through Eqn. 4.2.13. It is to be noted that, in computing the total charge in the base, as limits x_1 and x_2 are varied the denominator of Eqn. 4.2.9 varies; thus $\phi_n(x_1)$, $\phi_n(x_2)$ must vary to keep J_n constant. It seems reasonable to choose x_1 sufficiently into the space charge region for the denominator to become less position dependent as x_1 is taken further towards the emitter, where $\phi_n(x_1)$ is also expected to be a constant. It is in the interpretation of the total charge Q_B , that the analysis of shallow diffused structures differs from the conventional method. A regional modified approach developed to analyze shallow diffused structures is presented in the next section.

4.3 Characterization of the Base Region of a Shallow Diffused Transistor

Development of a systematic approach for the design of a device can be achieved efficiently and economically by representing the structure in terms of a physical model whose parameters are correlated with the processing parameters of the device geometry. Electrical performance of the device can then be evaluated by characterizing the physical model. Conversely, the model should be capable of characterizing the device geometry from electrical properties at the terminals of the device. A successful computer aided modelling approach has been demonstrated previously^(17,18) for large base bipolar transistors. Eqn. 4.2.11, which relates the terminal performance of the device to the base geometry, represents an important relationship and is used in the modelling approach to characterize the base region. However, for high frequency devices, characterized by shallow diffused small geometry base regions, the conventional approach (which assumes the neutral base region boundaries are located at the edges of the space charge regions determined from the depletion approximation) to analysis of this region will result in the deviation of computed results from experimental data.

The total charge Q_B appearing in Eqn. 4.2.11 and defined by Eqn. 4.2.13, can be computed from a rigorous analysis by extending the limits x_1 and x_2 to the emitter and collector contacts. This computation requires the use of a device simulation approach, thus increasing

the complexity of the analysis. Alternatively, a regional approach can be pursued by terminating integral limits in Eqn. 4.2.13 at boundaries established from physical considerations.

The total charge Q_B in the base region is a function of the hole density distribution. Under low level injection conditions, the hole density in the quasi-neutral base region can be assumed to be equal to the ionized impurity density and is given by

$$p(x) = N(x) \quad 4.3.1$$

where

$N(x)$ is the ionized impurity density and Eqn. 4.3.1 is valid in region II shown in Fig. 4.1. Regions I and II correspond to the space charge regions developed across the emitter and the collector junctions.

The total base charge Q_B can be regarded to be composed of the emitter and the collector capacitive components Q_e and Q_c and forward and reverse current controlled base charge Q_{bc} . Since the majority carrier current flow in the base is assumed to be small, the hole density in the neutral emitter and collector bulk regions can be neglected. The total charge in the base can be expressed as

$$Q_B = Q_e + Q_{bc} + Q_c \quad 4.3.2$$

Fig. 4.1 Regional representation of impurity distribution in a double diffused transistor.

The capacitive components originate from mobile holes stored in the space charge regions I and II of Fig. 4.1. Each of the components of the base charge Q_b can be computed by defining the boundaries of the three regions and a model to compute the hole density distribution in each region. These components can be expressed as

$$Q_e = qA_e \int_{x_{1e}}^{x_{2e}} p(x) dx - \text{Region I} \quad 4.3.3$$

$$Q_{bc} = qA_e \int_{x_{2e}}^{x_{1c}} p(x) dx - \text{Region II} \quad 4.3.4$$

$$Q_c = qA_e \int_{x_{1c}}^{x_{2c}} p(x) dx - \text{Region III} \quad 4.3.5$$

In the normal mode of operation, the capacitive component Q_c of the reverse biased junction has been shown⁽⁵⁰⁾ to be much less than Q_e , so that Q_{bc} differs from Q_b by the amount of charge stored in the emitter junction space charge layer. For a reverse biased collector junction, it is straight forward to define the boundary of the space charge region in the base. However, for a forward biased emitter junction, an abrupt termination of the space charge region is not possible due to the presence of mobile carriers in the space charge region.

For a large base device, the base charge $Q_{bc} \gg Q_e$ and forward

current characteristics computed from the conventional analysis⁽¹⁷⁾ are controlled only by the charge contained in the quasi-neutral base. The forward biased emitter junction space charge region was assumed to be depleted of carriers and the penetration of this region into the base was computed from the depletion theory. Thus the total charge

$$Q_b \approx Q_{bc} = A_e q \int_{x_{2e}}^{x_{1c}} N(x) dx \quad 4.3.6$$

It is not possible to extend the above approximation to shallow diffused devices, whose base widths are comparable to the width of space charge region at the emitter junction, thus requiring consideration of the stored charge Q_e .

A recent computer study of shallow diffused structures⁽⁴⁸⁾ has shown that boundaries of the quasi-neutral base region can be defined by considering the difference between the electrostatic potential obtained from neutral solution in the base and that evaluated from complete computer simulation of the device. Qualitatively, the boundaries were located where the difference between the electrostatic potentials was 2.5 mV, which corresponded to a hole concentration within 10% of the impurity concentration. This definition is arbitrary, however, and does not give sufficient information to compute the stored charge Q_e . Also since the method involves completely simulating the device, it is not easily applicable for modelling process.

A more systematic approach to the computation of forward current characteristics of shallow diffused devices is presented in the next section.

4.4 $I_c - V_{eb}$ Characteristics of Shallow Double Diffused Transistors

4.4.1 Development of a Procedure to Compute Forward Current Characteristics

Characterization of the base region of shallow diffused transistors in terms of majority carrier charge components associated with the space charge regions and the quasi-neutral base region was discussed in section 4.3. In the present section the method used to compute these components and to calculate the forward current characteristics of typical high frequency device profiles is discussed. A comparison of the results computed from conventional analysis is also presented. From these results, an estimate of the ratio of emitter-base space charge width to the metallurgical base width can be obtained quantitatively as a basis for the classification of a structure as a 'Shallow Diffused Device'. Devices falling under this category will have to be characterized by the modified regional approach or by a more detailed analysis. The accuracy of the modified method is ascertained by comparison of computed results with those obtained from computer simulation analysis.

The stored charge Q_e can be computed from Eqn. 4.3.3 if the hole density distribution in the space charge region is known. This can be accomplished by using the approximate solution algorithm developed in

Chapter 3 and by assuming the emitter junction to be characterized by a linear impurity grading. The validity of representing the distribution of impurities of a double diffused transistor near the vicinity of the emitter junction by a linear grading of impurities has been discussed in Chapter 3. This method is attractive because of the simplicity of the algorithm and the approximate solution; also it provides a continuity of carriers at the edge of the space charge layer and the quasi-neutral base. The hole distribution in this region can be computed from the auxiliary potential u_o and Boltzmann distribution, given by

$$\rho(x) = n_i \exp [u_n - u_o(x)] \quad x_{1e} \leq x \leq x_{2e} \quad 4.4.1$$

$$\text{where } u_n = \frac{q}{kT} \frac{V_{eb}}{2}$$

V_{eb} - is the applied emitter-base voltage

Limits of the space charge region are determined from the approximate solution.

Thus

$$Q_e = qA_e n_i \int_{x_{1e}}^{x_{2e}} \exp [u_n - u_o(x)] dx \quad 4.4.2$$

The stored charge Q_e in the collector region is assumed to be small and is neglected. The boundary of the quasi-neutral base on

the collector side is determined from the depletion theory.

The base charge Q_{bc} in the quasi-neutral region can be computed from a physical model representing the impurity profile in the base region. Profiles whose parameters are listed in Table 4.1 are assumed to follow a double Gaussian distribution of impurities. Assuming all the impurities are ionized at room temperature, the base charge Q_{bc} can be computed from

$$Q_{bc} = qA_e \int_{x_{2e}}^{x_{1c}} (N_{s1} e^{-\frac{x^2}{4D_1t_1}} - N_{s2} e^{-\frac{x^2}{4D_2t_2}} + N_B) dx \quad 4.4.3$$

In deriving a charge control relationship for forward current, ⁽⁴⁹⁾ $D_n(x)$ was substituted by an equivalent diffusion coefficient D_{nb} for electrons in the base. In a graded base transistor the diffusion coefficient becomes dependent on the impurity profile in the neutral region and field dependent in the space charge region. In modelling the base region, the assumption of a constant diffusion coefficient has been observed ⁽¹⁸⁾ to result in less accurate parameters evaluated for the model. Hence, the forward current characteristics presented in this section have not been subject to the constant diffusion coefficient approximation. Current densities can be computed as a function of applied voltage from Eqn. 4.2.9 and expressed as

$$J_c = - \frac{q n_i^2 \left\{ e^{\frac{qV_{eb}}{kT}} - e^{\frac{qV_{cb}}{kT}} \right\}}{\int_{x_{1e}}^{x_{1c}} \frac{D_n(x)}{D_n(y)} dx} \quad 4.4.4$$

$D_n(x)$ is computed from impurity density and the field dependent mobility data available from empirical approximations (51) given by,

$$\mu_1 = \frac{\mu_{\max} - \mu_{\min}}{1 + (N/N_{\text{ref}})^\alpha} + \mu_{\min} \quad 4.4.5$$

where μ_1 - mobility dependent on doping density

N - is the total doping density ($N_A + N_D$)

and

$$\mu_2 = \frac{\mu_0}{\left[1 + (\epsilon/\epsilon_c)^\beta \right]^{1/\beta}} \quad 4.4.6$$

where μ_2 - mobility dependent on the field

ϵ - electric field

Parameters for the above approximations are listed in Table 4.1.

Using Einstein's relationship, $D_n(x)$ is computed from Eqn. 4.4.5 for

$x_{2c} \leq x \leq x_{1c}$ and from Eqn. 4.4.6 $x_{1e} \leq x \leq x_{2c}$. Eqn. 4.4.4 can be evaluated only by numerical means.

Type of Carrier	$\mu_{\max}$ $\text{cm}^2/\text{V-sec}$	$\mu_{\min}$ $\text{cm}^2/\text{V-sec}$	μ_0 $\text{cm}^2/\text{V-sec}$	α	N_{Ref} atoms/cm ³	ϵ_c V/cm	β
Holes	495	47.7	487	0.76	6.3×10^{16}	1.95×10^4	1
Electrons	1330	65	1375	0.72	8.5×10^{16}	8×10^3	2

Table 4.1: Coefficients for the empirically derived position dependent mobility equation

4.4.2 Discussion of Results

From the procedure developed in the previous section, forward current characteristics have been computed for various profiles listed in Table 4.2. Profiles listed in the table range from devices of microwave capability to conventional high frequency transistors. Profile I is similar to the profile of a microwave transistor published by White and Thurston⁽⁵²⁾ with a cutoff frequency of 6 GHz. Profiles II and III are also typical examples of very high frequency devices with increasing base widths. 16BT2 corresponds to an actual high frequency device which was used in the modelling of the collector junction as presented in Chapter 2.

For comparison purposes, the results show forward current characteristics computed from the conventional analysis. In this approach, the base charge was computed from the quasi-neutral base region defined between depletion layer boundaries at the emitter and collector junctions. In order to compute the depletion layer semi widths for the forward biased emitter junction from Eqn. 2.4.4, the space charge capacitance was computed from the approximate solution method developed in Chapter 3. The computed results are also compared with the results obtained from computer simulation for two profiles.

As mentioned earlier computer simulation of devices consists of rigorously solving the current transport, continuity and space charge balance equations. The solution of these non-linear equations has been performed by a finite difference scheme algorithm developed

Table 4.2

Profile	N_{s1} atoms/cm ³	$\sqrt{4D_1 t_1}$ cm	N_{s2} atoms/cm ³	$\sqrt{4D_2 t_2}$ cm	N_B atoms/cm ³	X_{je} μ	X_{jc} μ	Remarks
Profile I	8×10^{20}	8.35×10^{-6}	2.0×10^{19}	1.348×10^{-5}	3.0×10^{15}	0.2	0.4	Parameters and
Profile II	8×10^{20}	1.648×10^{-5}	2×10^{19}	2.696×10^{-5}	3.0×10^{15}	0.4	0.8	junction depths
Profile III	8×10^{20}	3.298×10^{-5}	2×10^{19}	5.39×10^{-5}	3.0×10^{15}	0.8	1.6	computed from
16B # T2	3.597×10^{21}	4.137×10^{-5}	5.01×10^{17}	1.435×10^{-4}	2.53×10^{15}	1.5	3.3	theoretical profile

Table 4.2 Parameters of the Double Gaussian Mode for Extremely Narrow Base Transistors

originally by Arandjelovic⁽⁵⁴⁾, which was subsequently modified to simulate transistors by Treen⁽⁵⁵⁾. In this procedure, the analysis is carried out in one dimension without the need for classifying the device into different regions and the simulation is performed from emitter to collector contacts. The computed results from this method give a complete picture of the interactions occurring within the device.

Figs. 4.2-4.5 show computed results of forward current characteristics for profiles listed in Table 4.2. Results computed from the modified regional approach are observed to differ from those computed from conventional analysis for narrow base profiles and is most obvious for profile I which has a width ratio of 15%. This discrepancy decreases as going down from profile I to profile II and is almost indistinguishable for profiles III and 16BT2. The resulting difference is mainly due to the interpretation of the total base charge appearing in the generalized charge control Eqn. 4.2.11 from the two theories. In computing this parameter from conventional analysis, the stored majority carriers in the space charge region were completely neglected. However, the generalized Moll-Ross Eqn. 4.2.9 can be applied to any arbitrary limits placed on the boundaries of the quasi-neutral base region without the need for terminating at the edges of depletion layers as done in conventional analysis. In the modified analysis, limit x_1 in Eqn. 4.2.9 has been extended well into the space charge region on the emitter side, where the majority carrier contribution in the denominator of this equation

Fig. 4.2 GRAPH OF COLLECTOR CURRENT DENSITY vs APPLIED VOLTAGE

Fig. 4.3 Graph of $|J_c|$ vs. V_{eb} for Profile No. 2

Fig. 4.4 GRAPH OF $|J_c|$ vs Veb FOR PROFILE NO. 3

Fig. 4.5 Graph of $|J_c|$ vs. V_{eb} for 16B #T2 Profile

becomes negligible. From a charge control view point, this assumption accounts for the stored charge Q_e in the emitter space charge region, which becomes significant for shallow diffused devices. Further, excellent agreement obtained with the computer simulated results indicate the accuracy of the modified regional analysis in characterizing narrow base devices. This method also provides a more systematic analysis as opposed to arbitrary limits developed in a previous work⁽⁴⁹⁾.

From these computed results, it can be concluded that for devices whose profile is similar to that of profile III, the influence of stored charge Q_e in the space charge region can be neglected when computing the forward current characteristics. For this profile the width ratio (the ratio of equilibrium emitter space charge layer width to that of metallurgical base width) is 6%. This ratio can be used as a design figure in order to classify a "shallow diffused device". For ratios up to this value simple conventional theory can be used in terms of actual base region only; for larger ratios the effect of mobile charge in the emitter space charge layer is significant and more elaborate theory is necessary. The modified approach developed here should then be used and is probably the simplest method of analyzing the device.

In the next chapter, a method of evaluating parameters of a physical model from measured terminal performance of the device is presented. Devices used for this purpose are of extremely small

emitter space charge to base width ratios. Hence only conventional analysis was required for purposes of characterizing the base region.

CHAPTER 5

DETERMINATION OF PHYSICAL PARAMETERS OF

A GAUSSIAN MODEL FOR REAL DEVICES

5.1 Introduction

In the previous chapters, characterization of collector and base regions for double diffused transistors was carried out by representing the impurity distribution within the device by physical models obtained from an ideal solution of diffusion equations. For devices fabricated with the two-step diffusion process⁽¹³⁾, Gaussian impurity distributions were assumed to be valid. Terminal performance was computed from relationships derived from this impurity distribution model. Using the same basic relationships established in the earlier chapters, it is possible to evaluate parameters defining the physical model from terminal measurements made on real devices. The object of performing such an operation is to enable one to assess the validity of the model thus obtained by comparing the computed parameters with those measured on the device and also, to investigate the structure of an encapsulated device by nondestructive methods. A procedure for the determination of physical model parameters from terminal measurements for double diffused transistors has been presented previously^(17,21) by assuming that an exponential distribution of impurities could be used as an approximation to the actual donor

or acceptor impurity distribution in fabricated devices. Detailed analysis of this model⁽¹⁷⁾ has shown an exponential distribution of impurities to be a good representation of the 'tail' of classical Gaussian or Erfc distributions (i.e. for concentrations 10^{-3} below the surface value). Thus, being a good representation of the true impurity distribution components, the exponential model is adequate to model regions of double diffused transistors where the impurity density has fallen well below the surface concentration of impurities. However, for modern high frequency devices, characterized by heavily doped emitter-base regions and shallow junctions, physical parameters computed by assuming exponential distributions of impurities will result in considerable error; for accurate results a better model is essential and may be achieved with little additional complexity by using Gaussian rather than exponential distributions - a double Gaussian model is used through out this chapter. Also, the double Gaussian model is capable of accurate prediction of base sheet resistance. However, emitter sheet resistance cannot be predicted by this approach since the model is not generally valid in this region.

In Chapter 2, a single Gaussian model was shown to be adequate to model and characterize the collector junction region of a double diffused transistor. This distribution, obtained from ideal solution of diffusion equations, is well suited for modelling purposes, as it provides a basis for correlation of the device structure to process parameters⁽¹⁸⁾. The numerical procedure used to determine the

physical constants of a double Gaussian model from terminal measurements made on real devices^{*} is presented in this chapter. The validity of assuming an idealized impurity distribution is verified by comparing the computed physical parameters such as junction depths, collector background doping density and active base sheet resistance with the corresponding parameters measured on completed devices.

5.2 Procedure for Determination of Model Parameters from Terminal Measurements

The procedure adopted for computing model parameters from terminal measurements relies on proper interpretation of measured quantities in terms of an assumed impurity distribution defined by a mathematical model. In order to perform this operation, suitable care must be exercised while performing terminal measurements so as to conform to the assumptions made in deriving the related equations. Where permissible, an exponential approximation to the Gaussian impurity distribution has been made in order to facilitate model parameter determination process.

The physical model for a double diffused transistor represented by the double-Gaussian distribution of impurities can be analytically represented in normalized form from Eqn. 2.1.5 as

* 16B devices presented earlier in Chapter 2 are used for determining the model parameters in this chapter.

$$N(y) = N_{s2}(\xi e^{-y^2 m \gamma} - e^{-y^2 m} + \lambda) \quad 5.2.1$$

where

$$y = x/x_{jc}$$

$$\xi = N_{s1}/N_{s2}; \lambda = N_B/N_{s2}$$

$$m = x_{jc}^2/L_2^2$$

$$\gamma = L_2^2/L_1^2$$

$$L_1 = \sqrt{4D_1 t_1}; L_2 = \sqrt{4D_2 t_2}$$

In order to determine the impurity distribution of Eqn. 5.2.1, the 5 model parameters N_{s1} , N_{s2} , N_B , L_1 and L_2 have to be evaluated from the relationships in terms of terminal characteristics of the device. A straightforward application of the physical model of Eqn. 5.2.1 to interpret the electrical parameters measured at the terminals of the device results in complex relationships, thus, rendering the parameter determination process difficult, if not impossible. However, by making suitable approximations based on physical considerations, the process can be facilitated without introducing significant error in the final results. The development of a computer algorithm used for parameter determination is indicated below.

5.2.1 The Algorithm for Determination of Physical Model Parameters

The procedure followed is first to evaluate the parameters N_B , N_{s2} and L_2 associated with collector base junction; then to evaluate the remaining two parameters N_{s1} , L_1 of the emitter diffusion. Determination of the parameters associated with the base-collector region can be considerably simplified by neglecting the influence of the emitter diffusion on the total impurity distribution. From this assumption, justified in Chapter 2, the resulting single Gaussian distribution can be further simplified by approximating it by an exponential distribution of impurities at or near the base-collector junction; a method of determining the defining parameters of such an exponential model for the collector-base region has already been presented in Chapter 2. Using this method, and assuming a plane collector junction, the collector doping density N_B and characteristic length L of the exponential distribution are evaluated from capacitance measurements made at the terminals of the device. Values of N_B computed for different devices from the procedure outlined in Section 2.4.2, indicate very good agreement with those obtained from resistivity measurements. The collector junction depth x_{jc} is computed from junction breakdown voltage relationships derived by assuming a cylindrical exponential profile and show good agreement with junction depth data obtained by sectioning. The simple procedure presented in Chapter 2 is employed to evaluate N_B , L^* and

* L is used to designate the characteristic length of the exponential model.

x_{jc} for the devices under study.

Having determined the exponential model parameters, the corresponding parameters of a single-Gaussian base impurity distribution can be evaluated by equating the gradient of this model at the collector junction to that obtained from the exponential model. The surface concentration for this distribution is obtained by extrapolating the impurity distribution to the surface of the device. Thus, from the above sequence of computations the collector impurity density N_B and the parameters N_{s2} , L_2 of the base diffusion impurity distribution components in Eqn. 5.2.1 are known and the normalized parameters m and λ in Eqn. 5.2.1 are defined. The remaining parameters γ and ξ are evaluated from two sources of measured data, viz., the forward current characteristic I_C-V_{eb} for specified collector reverse bias V_{cb} , and the emitter-base capacitance. Choice of these electrical parameters was based on considerations of the ease and accuracy with which they can be measured and equations can be derived to characterize the required electrical properties concerned.

5.2.2 The Forward Current Transfer Relationship

Detailed analysis of the base region for a double diffused transistor, presented in Chapter 4, has shown forward current I_C-V_{eb} characteristics to be strongly influenced by the distribution of majority carriers in the base region. This dependency can be utilized to define the active base region from relationships derived in Chapter 4. The conventional "regional analysis" approach has been

used in this procedure to define the active base region of real devices, since the active base widths of devices investigated in this chapter were sufficiently large as compared to the emitter space charge region for the "regional analysis" to be accurate. With the assumption of a one dimensional n-p-n device structure operating under conditions of low level injection, with no carrier recombination in the base and with the collector reverse voltage low enough to neglect the avalanche multiplication effects, the I_c - V_{eb} relationship can be expressed in terms of a double-Gaussian model to give (after rearranging terms in Eqn. 4.4.4) the relationship:

$$\frac{q n_i^2 A_e \exp (q V_{eb} / k T)}{I_c} = x_{jc} N_s 2 \int_{y_1}^{y_2} \frac{(\xi e^{-y^2 m \gamma} - e^{-y^2 m} + \lambda)}{D_n(y)} dy \quad 5.2.2$$

where

$$y_1 = \frac{x_{je} + a_{2e}}{x_c} \quad 5.2.2.a$$

$$y_2 = \frac{x_{jc} - a_{1c}}{x_c} \quad 5.2.2.b$$

a_{2e} and a_{1c} are the distances of the junction edges of the space charge regions on the base side from the positions of the emitter and collector metallurgical junction respectively.

The space charge layer semiwidth a_{1c} can be determined from a single Gaussian model derived by neglecting the influence of the emitter diffusion in the vicinity of the collector junction, and by assuming the space charge region to be depleted of mobile carriers. The requirement of Poisson's equation that the electric field be continuous at the metallurgical junction, enables one to derive the desired relationship between the two components of depletion layer thickness a_{1c} and a_{2c} and can be expressed as

$$\lambda = \frac{\sqrt{\pi D_2 t_2}}{d_c} \left\{ \operatorname{erfc} \left(\frac{x_{jc} - a_{1c}}{\sqrt{4 D_2 t_2}} \right) - \operatorname{erfc} \left(\frac{x_{jc} + a_{2c}}{\sqrt{4 D_2 t_2}} \right) \right\} \quad 5.2.3$$

The total collector base depletion layer thickness is computed from the collector junction capacitance measured at the applied voltage V_{CB} maintained across the collector junction during $I_c - V_{eb}$ measurement and is given by

$$d_c = \frac{\epsilon A_c}{C_{tc}} \quad 5.2.4$$

The emitter junction in this analysis is assumed to be characterized by a linear grading of impurities in the vicinity of the emitter metallurgical junction. The gradient of this distribution is determined from reverse bias capacitance measurements described in Sec. 5.2.3. The forward bias capacitance of this junction corres-

ponding to the voltage at which the I_c-V_{eb} measurement is made is computed from the approximate solution algorithm developed in Chapter 3. The position of the emitter space charge layer edge a_{2e} is calculated by assuming complete depletion approximation to be valid for this purpose and is calculated from

$$a_{2e} = \frac{\epsilon A_e}{2C_{te}} \quad 5.2.5$$

where C_{te} is the forward bias capacitance computed from the approximate solution algorithm.

This method is simple to apply compared with previously published methods (17,18), where the forward biased junction capacitance was evaluated from transit time measurements. The active base region is assumed to be confined within the limits y_1 and y_2 .

The terms appearing on left hand side of Eqn. 5.2.2 are physical constants and measurable quantities. These quantities can be determined from I_c-V_{eb} measurements. Emitter diode characteristics can be obtained by direct dc measurement of collector current and emitter forward bias voltage. The emitter-diode characteristics are highly temperature dependent and hence the junction temperature must be maintained at the required temperature at which the intrinsic carrier density n_i is computed. At low current levels heat dissipation within the device can be neglected and the junctions are assumed to be at room temperature. However, at very low current levels, recombination

in the emitter space charge layer dominates and causes the diode law to depart from $\exp(qV_{eb}/kT)$ relationship. In order to ensure that measurements are performed within the $\frac{q}{kT}$ region, $I_c - V_{eb}$ measurements must be made over a wide range. From the slope of a plot of $\log |I_c|$ vs $|V_{eb}|$, the $\frac{q}{kT}$ region can be determined. For devices used in this chapter, this region extended over several decades. The $I_c - V_{eb}$ characteristic for a typical device is shown in Fig. 5.1. This relationship is used in conjunction with the interpretation of emitter-base junction capacitance data in order to define the two unknown parameters ξ and γ . The capacitance data allows the slope of the impurity density distribution at the emitter junction to be determined.

Device 16B #T4

Fig. 5.1 Graph of I_C vs. V_{eb}

5.2.3 Interpretation of Emitter-Base Junction Capacitance

Studies of idealized impurity distributions by Thomas⁽¹⁸⁾ have shown that for small reverse bias, the emitter junction of a double diffused transistor is better approximated by a symmetrical linear junction than by an abrupt junction. This fact was further ascertained in Chapter 3 from measurements performed on real devices. The linear junction property of diffused emitter junctions can be used to determine the gradient of impurities in the vicinity of the emitter junction from reverse bias capacitance measured at the terminals of the device. From Eqn. 3.2.2, the gradient of impurities in the vicinity of emitter junction can be computed by knowing the slope of measured $1/C_{te}$ versus V_{eb} plot. The slope of the impurity density distributions of the double Gaussian model, evaluated at the emitter junction, is then specified equal to this value determined from capacitance measurements. The slope of the Gaussian model at $x = x_{je}$ is given by the first derivative of Eqn. 5.2.1 as

$$S_1 = \left. \frac{dN}{dy} \right|_{y=y_e} = 2y_e m N_{s2} \left\{ \gamma \xi e^{-y_e^2 m \gamma} - e^{-y_e^2 m} \right\} \quad 5.2.6$$

$$\text{where } y_e = \frac{x_e}{x_c}$$

Parameters ξ and γ can now be evaluated by simultaneous solution of Eqns. 5.2.2 and 5.2.6. In this solution a position dependent empirical approximation⁽⁵¹⁾ is used for $D_n(y)$.

5.3 Evaluation of the Model Parameters by An Iterative Method

All analytical relationships necessary for the determination of the model parameters have been presented in the preceding section. Because of the complexity of the expressions involved, a numerical iterative procedure^{*} has been developed to evaluate the model parameters. The following data are used in the determination of model parameters:

- (1) A_e and A_c
- (2) Collector capacitance measured over a wide range of applied voltages.
- (3) Collector breakdown voltage BV_{CBO} .
- (4) $qn_i^2 \exp(qV_{eb}/kT) / I_c$ and C_{te} at forward voltage V_{eb} .
- (5) Gradient of impurities S_1 at the emitter junction evaluated from emitter capacitance measurements.

Emitter area A_e and collector area A_c were determined from the mask pattern used for fabrication of the devices. Since the procedure to evaluate N_B , L and x_{jc} has already been presented in Chapter 2, it will be assumed that these parameters are available as input data for the further modelling process. The following sequential procedure is used in evaluating model parameters:

* This algorithm was carried by a computer program written in FORTRAN IV language.

- (1) Using measured reverse bias capacitance values at the collector junction, parameters N_B and L of the single exponential model are evaluated from Poisson's equation through a minimization of error process as described in Chapter 2.
- (2) Assuming a cylindrical geometry for the edges of the collector junction, the collector junction depth x_{jc} is computed from simultaneous solution of equations derived for avalanche breakdown of the junction as described in Chapter 2.
- (3) Neglecting the influence of the emitter diffusion near the vicinity of the collector junction, a single Gaussian model is interpolated through x_{jc} , such that the slope of the Gaussian model at x_{jc} is same as that of the exponential model determined in step 1. Thus from an exponential model given by

$$N(x) = N_B (e^{-x'/L} - 1)$$

the slope at the collector junction is determined from

$$\left. \frac{dN(x)}{dx} \right|_{x'=0} = \frac{N_B}{L} \quad 5.3.1$$

For a single Gaussian model, written as

$$N(x) = N_{s2} e^{-x^2/L^2} + N_B$$

The slope at the collector junction is given by

$$\left. \frac{dN(x)}{dx} \right|_{x=x_{jc}} = \frac{2x_{jc} N_B}{L_2^2} \quad 5.3.2$$

Equating Eqns. 5.3.1 and 5.3.2

$$L_2^2 = 2x_c L \quad 5.3.3$$

The surface concentration corresponding to base diffusion is computed from

$$N_{s2} = - N_B e^{\left(x_{jc}^2 / L_2^2 \right)} \quad 5.3.4$$

From Eqns. 5.3.3 and 5.3.4 normalized parameters m and λ are computed.

- (4) The values of depletion layer semi widths a_{1c} and a_{2c} are next computed from Eqn. 5.2.3. An initial value for a_{1c} is assumed and a_{2c} is determined by subtracting a_{1c} from the total depletion layer width d_c . The two components of the depletion layer are next substituted in Eqn. 5.2.3 and the two sides of the equation are compared. If they do not agree a different value for a_{1c} is chosen and the procedure is repeated until mutually agreeing results are obtained. Knowing a_{1c} , the normalized boundary of the active base region at the collector end is com-

puted from Eqn. 5.2.2.b.

- (5) Using the junction gradient S_1 obtained from reverse bias capacitance measurements at the emitter junction, the "approximate solution" algorithm developed in Chapter 3 is applied to this junction. The forward capacitance computed from this method is employed to compute the depletion layer semi width a_{2e} from Eqn. 5.2.5.
- (6) The emitter diffusion parameters ξ and γ are next computed by an iterative procedure as follows.
 - (a) Parameter ξ is set to an initial value.
 - (b) An initial value for γ is next chosen. The position of the emitter junction is located as a point where there is total compensation of impurities.

Thus,

$$N_{s2e} \left(-\frac{x_e^2}{L_2^2} \right) \simeq N_{s1e} \left(-\frac{x_e^2}{L_1^2} \right)$$

or

$$x_e^2 = \frac{L_2^2 \ln \xi}{(\gamma-1)} \quad 5.3.5$$

The normalized boundary y_1 of the active base region on the emitter side of the base can now be computed from Eqn. 5.2.2.a.

- (c) The above values ξ , γ and y_1 are substituted into Eqn.

5.2.2 and the right hand side of the equation is evaluated by numerical integration.

- (d) Two sides of the $I_c - V_{eb}$ relationship of Eqn. 5.2.2 are compared. If they do not agree, a different value for γ is chosen and steps (b) through (d) are repeated until mutually agreeing results are obtained.
- (e) The parameters computed in the previous steps are substituted into Eqn. 5.2.6 to give a corresponding value for the slope S_1 of the impurity distribution model computed at the emitter junction; this is compared with the value of slope determined from emitter capacitance measurements.
- (f) If the values of impurity density slope in (e) do not agree, a different value for ξ is chosen and steps (b) through (f) are repeated until mutually consistent results are obtained.

A computer program has been developed to perform the above algorithm in FORTRAN language; a listing is presented in Appendix 5. This program requires the parameters of the single exponential model N_B , L and x_{jc} as input data. Fig. 5.2 illustrates the flow chart for the parameter evaluation procedure.

5.4 Discussion of Results

The method described in the preceding section for determination of physical parameters has been applied to the type 16B transistor device samples used in the modelling of the collector region in

Fig. 5.2 Flow Chart for computing parameters
of a double Gaussian model from terminal
measurements.

Chapter 2. The necessary measured data detailed at the beginning of section 5.3 were obtained from direct measurement of these parameters and are listed in Table 5.1. The Boonton model 75D direct capacitance bridge was used for the junction capacitance measurements. Strays associated with the leads and the can of the device were eliminated by balancing the instrument with a similar dummy can inserted across the test terminals of the instrument. Current levels for the $I_c - V_{eb}$ characteristics were maintained in the microampere range. Values of N_B , L and x_{jc} listed in Table 2.2 have been used as input parameters. Application of the iterative procedure resulted in the values of physical parameters listed Table 5.2.

The results obtained are for a range of devices whose substrate resistivity lie from 2.0 Ω -cm to 0.3 Ω -cm and all the devices were fabricated on the same production line. These devices were of non-epitaxial collector structure. Table 5.3 lists junction depths and base widths of these devices obtained from sectioning and staining the device.

Junction depths and base widths computed from the model agree extremely well with sectioned data. Since all the devices were fabricated on the same production line, one would expect the parameters associated with diffusion of impurities to be constant for all the devices. These parameters in a Gaussian model correspond to diffusion lengths L_1 and L_2 and surface concentrations N_{s1} and N_{s2} related to emitter and base diffusions. However, parameters computed

Device Sample	$V_{cb} + V_b$ Volts	N_B atoms/cm ³	L μ	x_{jc} μ	A_e $cm^2 \times 10^{-4}$	A_c $cm^2 \times 10^{-4}$	C_{tc} pF	a_{2e} μ	$\frac{I_c}{qn_i^2 \exp\left(\frac{qV_{eb}}{kT}\right)} \times 10^{-16}$ cm ² /sec	S_1 atoms/cm ⁴
16B/T2	5.50	2.35×10^{15}	0.312	3.27	1.532	4.941	2.583	0.493	2.460	2.70×10^{22}
16B/T4	5.58	1.11×10^{16}	0.349	3.05	1.532	4.738	5.1	0.466	2.480	3.24×10^{22}
16B/T5	5.56	1.07×10^{16}	0.330	3.08	1.532	4.738	5.44	0.543	1.204	2.75×10^{22}
16B/T10	5.60	2.25×10^{16}	0.342	2.80	1.532	4.738	6.43	0.587	1.433	2.11×10^{22}

Table 5.1: Input Data for Evaluation of Model Parameters

Device Sample	$V_{cb} + V_b$ Volts	L_2 μ	N_{s2} atoms/cm ³	ξ	γ	m	λ $\times 10^{-3}$	L_1 μ	x_{je} μ	x_{jc} μ	w_b μ
16B#T2	5.50	1.429	4.76×10^{17}	2327.7	11.03	5.23	5.31	.4302	1.37	3.27	1.90
16B#T4	5.58	1.459	8.77×10^{17}	2752.2	9.31	4.37	12.63	.4781	1.62	3.05	1.43
16B#T5	5.56	1.426	1.138×10^{18}	934.9	5.63	4.66	9.40	.4006	1.41	3.08	1.67
16B#T10	5.60	1.384	1.41×10^{18}	242.6	3.98	4.11	15.90	.6938	1.46	2.80	1.34

Table 5.2: Physical parameters derived from data of Table 5.1 by iterative method of section 5.3.

from the model show that they are not constant and vary with the device structure.

Device Sample	x_{je}^* μ	x_{jc}^* μ	W_B μ
16B#T2	1.5	3.3	1.8
16B#T4	1.5	3.0	1.5
16B#T5	1.5	3.0	1.5
16B#T10	1.5	2.7	1.2

Table 5.3: Junction depths obtained from bevelling and staining.

* Junction depths were located by assuming the metallurgical junction was located at the edge of stained emitter and base regions. Accuracy of measuring this location is $\pm 0.15\mu$ (half a sodium light fringe). However, depletion regions in the vicinity of emitter and collector junctions have been reported in the literature⁽⁵³⁾ to appear as dark unstained region. Hence, an additional uncertainty is introduced in the interpretation of stained regions obtained from standard staining techniques. However, in Chapter 2, computation of collector junction breakdown voltage from two extreme interpretations of the depth of the stained region (x_1 determined on the assumption that stained region extends to the metallurgical junction; x_{ji} assumes staining to the edge of depletion layer boundary in the base region) showed little difference except in the devices (16BT2) with most lightly doped background region. The results for the 16BT2 devices suggest that the junction depth x_{jc} lies between the values of x_1 , x_{ji} and perhaps is closer to the stained depth x_1 .

In fact one could expect such a variation due to non linearities existing in the diffusion process⁽¹⁸⁾. Anomalous effects occurring during diffusion of impurities cause the resulting impurity distribution to depart considerably from the ideal distribution in heavily doped regions of the device. Thus, surface concentration of impurities computed from extrapolating ideal distributions to the surface of the device will depart from expected values. This deviation is less pronounced for the base diffusion process. Computed values of base diffusion length L_2 listed Table 5.2 show the expected consistency. However, extrapolated surface concentrations corresponding to base diffusion do not have the same degree of consistency.

Device Sample	$\rho_{ab}^{\dagger}$ K Ohms/ $\square$	ρ_{ab}^* K Ohms/ $\square$
16B#T2	1.85	2.10
16B#T4	1.97	2.05
16B#T5	1.82	2.25
16B#T10	3.2	3.13

Table 5.4: Sheet resistance of active base region.

$\dagger$ obtained from four point probe measurement

* computed results

For the emitter diffusion the extrapolated impurity distribution near the surface of the device cannot possibly be of significance as the resulting surface concentrations from the model are higher than solid solubility limits. The deviation of idealized distributions from real impurity distribution is primarily due to non linear effects occurring in the heavily doped emitter region. However, this is of no consequence for the present purpose.

Deviation of computed results near the surface of the device does not inhibit the applicability of the model in correlating the terminal performance and physical parameters of the device. This being the primary aim, applicability of the model in a systematic design procedure can be demonstrated by computing active base sheet resistance. Detailed analysis of diffused layers has shown⁽¹⁸⁾ active base sheet resistance to be critically dependent on the structure of the base region and is commonly employed as a process control parameter. Values of the sheet resistance of the active base region computed from a procedure developed in an earlier work⁽¹⁸⁾ are listed in Table 5.4. Measured sheet resistance values were obtained from a test pattern provided on the slice. Computed results show very good agreement with the measured data, indicating that the model gives accurate representation of the base region of the double diffused structure. These independent checks give additional confidence in the validity of the model and of the procedure adopted for parameter determination.

CHAPTER 6

CONCLUSIONS

6.1 Summary and General Conclusions

The work presented in this thesis has been mainly concerned with development of a systematic procedure for both analysis and design of double diffused bipolar transistors. This objective has been realized in terms of a physical model, which is a representation of the main features of the internal structure and is of value as a basis of analysis of internal electronics of the device. Successful physical modelling of double diffused bipolar transistors has been demonstrated by assuming a double Gaussian model for impurity distribution in the device. Accuracy of the modelling process is verified from the degree of correlation obtained with experimentally measured parameters. Commonly employed theory for device operation is found to be inadequate to characterize small geometry devices. Characterization of such devices can be performed by an approximate solution approach, which provides a good agreement with results obtained from computer simulation of these devices. Further details of achievements in the work presented in this thesis are summarized below.

Chapter 2: A method for evaluating parameters of a collector junction model from junction capacitance and breakdown voltage measurements made at the terminals of the device is presented. By neglecting the

influence of emitter diffusion on the total impurity distribution near the vicinity of the collector junction, it was possible to approximate the resulting single Gaussian distribution of impurities by an exponential model. The accuracy of such a model for the physical characterization of the device structure was demonstrated by practical determination of defining parameters of the model (collector background doping density N_B , exponential characteristic length L and collector junction depth x_{jc}). From terminal performance data parameters N_B and L were evaluated from capacitance-voltage relationships derived in terms of the exponential model and the model parameter determination was carried out by process of computer fitting between this model and measured data. This operation was achieved by minimization of mean square voltage error between measured and computed voltage, for a voltage range over which the best functional fit to measured capacitance-voltage data was obtained. Approximately 20 to 40 data points were required for this purpose. Parameters evaluated from the method provided consistent results. Good agreement obtained between N_B values computed from the model and the values obtained from 4-point probe measurements appear to validate both the choice of the model for a planar transistor collector junction, and the method of determining the model parameters from capacitance data.

Utilizing the parameters of the exponential model, and the collector junction depth x_{jc} obtained from bevelling and staining procedures, computation of collector junction breakdown voltage BV_{CBO}

provided excellent agreement with measured breakdown voltage. Uncertainties existing in the interpretation of the stained region made little difference in the final computed results. Conversely, collector junction depth evaluated from breakdown voltage measurements, using the same basic relationships derived for breakdown voltage computations, agree extremely well with sectioned data. It is felt that method presented is valuable for deriving physical parameters by non-destructive means and to define a model for further characterization of the collector junction.

Chapter 3: A simple approximate method is presented for modelling a forward biased p-n junction by taking into account the presence of holes and electrons in the space charge region. This method enables one to characterize a forward biased junction by a more realistic theory than commonly employed complete depletion theory. The procedure developed was based on linearizing the Poisson's equation by a suitable function approximating the true electrostatic potential distribution in the space charge region. This operation was carried out by generating an auxiliary potential function of the form $U_0 = Ay + By^2$ to describe electron and hole densities in the space charge region satisfying Boltzmann statistics. Parameters of the auxiliary potential $U_0(y)$ were derived from relationships established with the use of boundary conditions which were obtained from physical considerations. Comparison of computed space charge layer properties from this method show good agreement with those obtained from rigorous

computer simulation methods⁽²⁹⁾. An important feature of the method is the capability of computing the capacitance-voltage characteristics for forward biased junctions. Due to the space charge neutralization, the space charge capacitance computed from the approximate solution method developed in this chapter tends to decrease with increasing forward bias voltages as compared to the results of previously formulated methods^(29,36,38). A specific application of this method was found in characterizing the forward biased emitter junction of a double diffused transistor.

Chapter 4: A quantitative analysis for characterization of forward current characteristics of shallow diffused bipolar transistors from a regional approach has been presented. The approach developed here is found to provide excellent agreement with those obtained from computer simulation methods^(54,55), yet retains simplicity of analysis. This analysis was performed by developing a generalized Moll-Ross equation in which total base majority carrier charge contributing was computed from components of stored charge in the device obtained from a regional consideration. Under normal active mode of the device, stored charges in the emitter space charge region and quasi-neutral base region are found to be the most significant contributors in characterizing the current transport properties of narrow base devices. However, the stored charge in the emitter space charge region becomes of less significance for devices with large base widths owing to large stored charge in the quasi neutral base region.

For purpose of assessing these effects, stored charge in the emitter space charge region was computed from approximate solution theory developed in Chapter 3. Results computed for theoretical profiles operating under low level injection conditions show that for devices whose width ratio (defined as the ratio of emitter space charge layer width to that of metallurgical base width) greater than 6%, charge stored in the emitter space charge region becomes significant in characterizing the forward current properties of the device, requiring the use of detailed Moll-Ross relationship.

Chapter 5: From relationships established between electrical parameters and the device structure in the base and the collector regions, an algorithm for computation of parameters of a double Gaussian model from measurements made at the terminals of the device has been presented in this chapter. The procedure developed relies on the capability of an exponential model to represent the impurity distribution in the collector region of the device. Using the procedure developed in Chapter 2, parameters of the exponential model evaluated from collector capacitance data were subsequently used in the evaluation of parameters for the base Gaussian distribution, by equating the gradient of impurities obtained at the collector junction from both the models. Parameters of the second Gaussian distribution corresponding to emitter diffusion in a double diffused transistor were evaluated from emitter capacitance and forward current characteristics measured at the terminals of the device. Use of 'approximate method'

of Chapter 3 is essential as a means of evaluating emitter capacitance under forward bias conditions. Physical parameters such as junction depths and substrate doping density computed from this procedure show good agreement with those measured on the device. However, extrapolation of the model to the surface of the device results in impurity concentrations in excess of solid solubility limits. The actual non-Gaussian impurity distribution in this region results mainly due to non linear processes occurring in the heavily doped region during impurity diffusion which are normally encountered in typical double diffused transistors. Non linear processes such as those associated with solid solubility limit and field aided diffusion (occurs at high impurity concentration) require a more detailed analysis if this region is to be characterized. Detailed analysis of this region has shown⁽¹⁸⁾ that a double Gaussian model can be approximated to more complex distributions when the impurity concentration has fallen below 10^{19} atoms/cm³. In a typical double diffused transistor, doping densities in the vicinity of the emitter junction and in most of the base region are well below the threshold value quoted above. The adequacy of the model in representing base region is further shown by agreement obtained between computed active base sheet resistance with those measured on the device. Parameters evaluated from this procedure were observed to be independent of biasing conditions within the limits of the basic assumptions made. The method employed for determination of physical parameters for such

a model has been found to give consistent results and to afford a measure of self checking of the assumptions made in both the approximate theory of internal electronic processes and the model parameter determination process itself.

To conclude, the accuracy of agreement obtained between deduced and directly measured parameters of the device, support the contention that a double Gaussian model and the procedure adopted to evaluate the corresponding model parameters provide a good basis for analysis and design of double diffused transistors.

6.2 Suggestion for Further Work

6.2.1 Modelling of the Emitter Junction

In this thesis, analysis and modelling of emitter junction was carried out by assuming a simple linearly graded impurity profile approximation. Such an approach is adequate to model devices with relatively large impurity gradient in the emitter region. However, for devices with smaller impurity gradient, the approximate solution method presented in Chapter 3 could result in introducing errors due to the departure of the impurity distribution in the vicinity of the emitter junction from a linearly graded profile. For such devices, representation of impurity profile by a pseudo exponential model⁽³¹⁾ has been shown to yield good results when used for characterizing emitter junction. Development of an approximate solution procedure using the exponential model is, therefore, suggested for further work.

6.2.2 A Model for Emitter Region of a Double Diffused Transistor

While essential aims of this study are considered to have been met, in the interest of simplicity more exact representation of impurity distributions in the heavily doped regions of the emitter of the device has been ignored. Consequently, the idealized impurity distribution modelling process was limited only to the base and the collector regions and is unable to provide information on the properties of the emitter region. As the primary objective of device modelling is to yield information of the structure as well as the electrical properties of a device, a study is therefore suggested with the objective of developing a more realistic model for the emitter region in order to take into account non-linear processes⁽¹⁵⁾ occurring during diffusion of impurities in a heavily doped region.

6.2.3 Modelling of Extremely Small Geometry Devices

Modelling procedures developed in this thesis have been applied only to relatively large geometry devices as compared to modern high frequency microwave devices. Although methods to characterize base region of such small geometry devices has been presented in this thesis, no attempt was made to apply these procedures to a real device. Based on the methods presented in this thesis, development of a procedure to evaluate model parameters of a real small geometry (viz. base widths less than 0.5μ) high frequency device is recommended for further work in order to ascertain the capability of

modelling procedures developed in this thesis for such device.

6.2.4 Junction Depth Measurements from Sectioning and Staining

Accuracy of the modelling procedures relies on their capability to correlate electrical parameters measured at the terminals of a device to the device structure. A method used to check accuracy of these procedures has been to evaluate important device geometry parameters (such as junction depths) from terminal measurements and compare them with sectioned data. Hence, the importance of determining the junction depths accurately can be recognized. Using the standard copper-sulphate or nitric acid staining process, regions adjoining the metallurgical junction have been reported⁽⁵³⁾ to appear as dark stained regions, thus introducing an uncertainty in the exact location of the metallurgical junction. This could be a serious problem, especially, for small geometry devices. A further study in interpretation of stained regions to locate metallurgical junction accurately would be useful.

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APPENDIX 1

VOLTFIT - PROGRAM TO COMPUTE PARAMETERS OF AN
EXPONENTIAL MODEL FROM MEASURED CAPACITANCE -
VOLTAGE DATA

1.1 PURPOSE

Parameters N_B and L of an exponential model are evaluated by computer model fitting of measured and computed capacitance - voltage data. This operation is implemented by minimizing the deviation of mean square voltage error.

1.2 USER PROCEDURE

Input data to the program are read in the following order:

```
READ      Q, EP, N, N1
READ      AC, AN, VB, BN, M
READ      CS(k), k = 1, M
READ      VS(k), k = 1, M
```

where

Q	Electronic Charge = 1.6×10^{-19} coulombs
EP	Dielectric Constant = 1.064×10^{-12}
N	Number of Model Parameters to be determined
N1	Dummy Variable
AC	Collector Area

AN	Slope of $1/C_{tc}^2$ vs. V_a plot
VB	Initial value for substrate doping density
M	Number of measured Capacitance - voltage data
CS	Measured Capacitance data
VS	Measured Voltage data

1.3 EXTERNAL PROGRAMS REQUIRED

A subroutine to minimize a function with at least two variables is required. The program developed is designed to operate with system subroutine ZANGWILL

1.4 METHOD OF EVALUATION

See Chapter 2

1.5 SPECIAL CONSIDERATIONS

Computational time required can be reduced by using an initial value for N_B evaluated from the method described by Thomas⁽¹⁷⁾.


```

C      PROGRAM VOLTFIT
C      THIS PROGRAM COMPUTES PARAMETERS OF AN EXPONENTIAL MODEL
C      FROM CAPACITANCE -VOLTAGE DATA BY MINIMIZING THE MEAN
C      SQUARE DEVIATION OF THE VOLTAGE. THE PROGRAM UTILIZES
C      THE SYSTEM SUBROUTINE ZANGWLL TO DETERMINE THE MINIMUM
C      OF THE MEAN SQUARE VOLTAGE ERROR.
C      PROGRAMMER RAMESH SISRSI*****
C      DIMENSION X(5),CT(50),GRX(5),W1(4),W2(4),W3(2),W4(2),W5(2)
C      1,SCALE(2),ICONST(2),RCONST(4),VS(50),CS(50)
C      COMMON VT(50),CK(50),VB,M1
C      EXTERNAL FUN
C      REAL L1
C      N CORRESPONDS TO NUMBER OF MODEL PARAMETERS TO BE DETERMINED
C      READ (5,100),Q,EP,N,N1
C      WRITE(6,103)
C      WRITE(6,111)
C      DO 3 I=1,N
C      READ(5,108)
C      WRITE(6,108)
C      INPUT VALUES ARE READ IN*****
C      READ(5,102),AC,AN,VB,PN,M
C      READ(5,104),(CS(K),K=1,M)
C      READ(5,105),(VS(K),K=1,M)
C      Y1=(EP*AC)/CS(1)
C      A3=AC**3
C      AS=12.0/(Q*(EP**2)*AN*A3)
C      WRITE(6,110),VB,EN,AS
C      L1=EN/AS
C      X(1)=Y1/L1
C      X(2)=(Q*BN*(Y1**2))/EP
C      INPUT VALUES FOR SYSTEM SUBROUTINE ZANGWLL ARE SET UP.
C      RCONST(1)=1.0E-08
C      RCONST(2)=0.5
C      RCONST(3)=3.0
C      RCONST(4)=0.1
C      SCALE(1)=1.0
C      SCALE(2)=1.0
C      ICONST(1)=1000
C      ICONST(2)=108
C      N2=1
C      M1=17
C      4 DO 5 K=1,M1
C      CT(K)=CS(K)
C      5 VT(K)=VS(K)
C      DO 2 K=1,M1
C      2 CK(K)=CT(1)/CT(K)
C      WRITE(108,99)
C      SYSTEM ROUTINE ZANGWLL WHICH MINIMIZES A FUNCTION
C      WITHOUT CALCULATING DERIVATIVES IS USED.

```

```

CALL ZNGWLL(FUN, 2, X, FMIN, PCNST, ICNST, SCALE, NOFEV, IEND, W1, W2, W3
1, W4, W5)
WRITE(6, 106), N2, VT(1), VT(M1)
L1=Y1/X(1)
BN=(EP*X(2))/(Q*(Y1**2))
WRITE(6, 107), BN, L1
CALL FUNCT(2, X, GRX, VAL)
WRITE(6, 91), GRX(1), GRX(2), VAL
M1=M1+4
IF(M1.GT.M) GO TO 3
N2=N2+1
GO TO 4
3 CONTINUE
91 FORMAT(2X, 'SLOPE1=', E14.8, 2X, 'SLOPE2=', E14.8, 2X, 'MDEV=',
1E14.8, '//)
99 FORMAT(1H1)
100 FORMAT(4G.0)
102 FORMAT(5G.0)
103 FORMAT(13X, '*****SINGLE EXPONENTIAL MODEL*****', ///)
104 FORMAT(7G.0)
105 FORMAT(10G.0)
106 FORMAT(12, '****', 5X, 'VOLTAGE RANGE=', F8.4, 2X, 'TO', F8.4, 2X, 'VOLTS',
1//)
107 FORMAT(5X, 'SUBSTRATE DOPING DENSITY=', E10.4, 5X, 'L1=', E10.4, //)
108 FORMAT('DEVICE 16BT1')
110 FORMAT(///, 4X, 'VB=', F8.4, 3X, 'NB INITIAL=', E10.4, 3X,
1'LINEAR SLOPE=', E10.4, //)
111 FORMAT(3X, 'INVESTIGATION OF VARIATION OF VOLTAGE RANGE ON MODEL PA
1RAMETERS', //)
CALL EXIT
END
SUBROUTINE FUNCT(IGO, X, GRX, VAL)
THIS SUBROUTINE COMPUTES THE MAGNITUDE OF THE SLOPE OF
THE MINIMIZING FUNCTION AT MINIMUM POINTS.
C
C
DIMENSION X(1), GRX(1)
COMMON VT(50), CK(50), VB, M
SUM=0.
GSUM=0.
RSUM=0.
XT=M
PRINT 99
99 FORMAT(1H1)
PRINT 98
98 FORMAT(12X, 'INVOLTAGE          CVOLTAGE          ERROR', //)
DO 1 I=1, M
CSCH=1./SINH(0.5*CK(I)*X(1))
CO TH =1./TANH(0.5*CK(I)*X(1))
TE=1./(CK(I)*X(1))
TD=(CK(I)**2)*(0.5*CO TH-TE)
TS=X(2)*TD-VB-(0.026*X(1)*CK(I))
TR=(VT(I)-TS)/VT(I)

```

```
      PRINT 100,VT(I),TS,TR
100  FORMAT(9X,3(F12.6,5X),//)
      SUM=SUM+(TR**2)
      DT=(0.25*(CSCH**2)-(TE**2))
      DT=(CK(I)**3)*DT
      DR=(TR/VT(I))*DT
      GSUM=GSUM+DR
      RSUM=RSUM+(TR/VT(I))*TD
1  CONTINUE
      VAL=SUM/XT
      GRX(1)=(2.*X(2)*GSUM)/XT
      GRX(2)=- (2.*RSUM)/XT
      RETURN
      END
      FUNCTION FUN(N,X)
C  THIS SUBROUTINE CONTAINS THE FUNCTION TO BE MINIMIZED.
      DIMENSION X(1)
      COMMON VT(50),CK(50),VB,M
      SUM=0.
      XT=M
      DO 1 I=1,M
      COTH=1./TANH(0.5*CK(I)*X(1))
      TE=1./(CK(I)*X(1))
      TD=(CK(I)**2)*(0.5*COTH-TE)
      TS=X(2)*TD-VB-(0.026*X(1)*CK(I))
      TR=(VT(I)-TS)/VT(I)
      SUM=SUM+(TR**2)
1  CONTINUE
      FUN=SUM/XT
      RETURN
      END
```

APPENDIX 2

BRAKEXPO - PROGRAM TO COMPUTE AVALANCHE BREAKDOWN
VOLTAGE OF A CYLINDRICAL EXPONENTIAL JUNCTION

2.1 PURPOSE

This program computes the breakdown voltage of an exponentially graded junction by assuming a cylindrical edge region. The program requires parameters N_B , L evaluated from capacitance-voltage data and the junction depth x_{jc} .

2.2 USER PROCEDURE

Input data to the program are read in the following order:

```
READ      GI(I),  I = 1, 15
READ      GQ(I),  I = 1, 15
READ      N, Q, EP, AE, AH, BE, BH
READ      AE1, AH1, BE1, BH1, AH2, BH2
READ      RJ, AS, BN, Z1
```

where

```
GI(I)      Weighted coefficients of 15 point Gaussian
            integration formula
GQ(I)      Abscissa's for the above integration formula
```

N Number of input data

Q Electronic Charge

EP Dielectric Constant

Following are the parameters of the ionization rates

AE = 3.8×10^6

AH = 2.25×10^7

BE = 1.75×10^6

BH = 3.26×10^6

AE1 = 7.03×10^5

AH1 = 1.582×10^6

BE1 = 1.231×10^6

BH1 = 2.036×10^6

AH2 = 6.71×10^5

BH2 = 1.693×10^6

RJ = Junction depth

AS = Gradient of the impurities at the metallurgical
 junction

BN = Background doping density

Z1 = Critical electric field of a linearly graded
 junction

2.3 METHOD OF EVALUATION

See Chapter 2

2.4 ERROR MESSAGES

Appropriate error messages are printed out when the program diverges and computation is aborted.

```

C      PROGRAM BRAKEXPO
C      PROGRAM TO COMPUTE JUNCTION BREAKDOWN AND FIELD DISTRIBUTION
C      OF A CYLINDRICAL JUNCTION WITH EXPONENTIAL GRADING OF
C      OF IMPURITIES.
C      PROGRAMMER RAMESH SIRSI*****
      REAL L1
      COMMON GI(15),GQ(15),AL,E1,AS,RJ,E2,BD,AE,AH,BE,BH,CN,CL,Q,EP,
1      IT1,T2,T3,TA(15),AE1,BE1,AH1,BH1,AH2,BH2,ET1,ET2
      READ(5,100),(GI(I),I=1,15)
      READ(5,100),(GQ(I),I=1,15)
      READ(5,130),N,Q,EP,AE,AH,BE,BH
      READ(5,130),AE1,AH1,BE1,BH1,AH2,BH2,DUM
      WRITE(6,120)
      ET1=1.75E+05;ET2=4.0E+05;Z2=1.0E+05
      DO 1 I=1,N
      READ(5,122)
      WRITE(6,122)
122  FORMAT('DEVICE 16B# 2')
      WRITE(6,125)
C      INPUT PARAMETERS FOR THE PROGRAM ARE READ IN.
      READ(5,131),RJ,AS,BN,Z1
      EI=Z1*Z2
C      SET INITIAL VALUE OF FIELD ASSUMING A SYM-LINEAR JUNCTION
      R1=(2.*EP*EI)/(Q*AS)
      R1=SQRT(R1)
      R1=R1*0.5
      R1=(RJ-R1)/RJ
      L1=BN/AS
      AL=RJ/L1
      BD=EXP(-RJ/L1)
      C1=1./BD
      CN=(Q*BN*C1*RJ)/(EP*(AL**2))
      CL=(AL**2)*BD/2.
      J=0
      L=0
      KA=0
      KB=0
C      AN INITIAL VALUE FOR ELECTRIC FIELD AT THE METALLURGICAL
C      JUNCTION IS SET BY ASSUMING A LINEARLY GRADED JUNCTION.
2      EI=Z1*Z2
      E1=EI
      K1=0
      K2=0
C      SEMIWIDTH R1 IS COMPUTED FROM METALLURGICAL JUNCTION FIELD
C      EXPRESSION.
      CALL RNWTON(R1,K1)
      IF(K1.EQ.1) GO TO 1
C      SEMIWIDTH R2 IS COMPUTED FROM FIELD CONTINUITY CONDITIONS
C      AT THE JUNCTION
      CALL RDEPL(R1,R2,K2)

```

```
      IF(K2.EQ.1) GO TO 1
C     INTEGRAL OF IONIZATION COEFFICIENT IS COMPUTED
      CALL FION(R1,R2,AP)
      RE=(1.0-AP)
      R=ABS(RE)
      IF(L.EQ.1) GO TO 12
      IF(R.LT.0.01) GO TO 10
C     CONDITION FOR ONSET OF AVALNCH BREAKDOWN IS CHECKED.
      IF(J.GT.100) GO TO 9
      CR=AP**(.1/.8.)
      Z1=Z1/CR
      J=J+1
      GO TO 2
10  L=1
      J=0
      GO TO 15
12  ER=(ER1-ER2)/ER1
      IF(ABS(ER).LT.1.0E-08) GO TO 14
15  IF(R.LT.0.001) GO TO 14
      IF(J.GT.100) GO TO 9
      IF(KA.EQ.1) GO TO 5
      IF(J.GT.0) GO TO 6
7   D1=RE
      ER1=Z1
      F1=Z1;Z1=Z1+0.1*Z1;J=J+1
      ER2=Z1
      GO TO 2
6   C=RE*D1
      IF
      IF(KB.EQ.1) GO TO 4
      B=ABS(D1)
      IF(B.GT.R) GO TO 7
      KB=1
4   D1=RE;ER1=Z1
      F1=Z1;Z1=Z1-0.1*Z1;J=J+1
      ER2=Z1
      GO TO 2
3   KA=1
      F2=Z1
8   ER1=Z1
      Z1=0.5*(F1+F2)
      ER2=Z1
      J=J+1
      GO TO 2
5   C=RE*D1
      IF(C.LT.0.) GO TO 11
      D1=RE;F1=Z1
      GO TO 8
11  F2=Z1
```



```

      GO TO 8
14 K=3
C   PROGRAM COMPUTES BREAKDOWN VOLTAGE
      BV=SINTEG(R2,R1,K)
      BV=BV*RJ
      X1=RJ*R1;X2=RJ*R2
      WRITE(6,115),RJ,AS,L1,X1,X2,BV
      GO TO 1
      9 PRINT 110
      1 CONTINUE
100 FORMAT(5G.0)
110 FORMAT(10X,30HNO CONVERGENCE IN MAIN PROGRAM,/)
115 FORMAT(6(E10.4,2X),/)
120 FORMAT(10X,'BREAKDOWN COMPUTATIONS ASSUMING EXPONENTIAL JUNCTION'
      1,/)

125 FORMAT(/,5X'RJ          AS          L1          X1          X2
      1          BV',/)
130 FORMAT(7G.0)
131 FORMAT(4G.0)
      CALL EXIT
      END
      SUBROUTINE RNWTON(R1,K1)
C   THIS SUBROUTINE COMPUTES DEPLETION LAYER SEMIWIDTH USING
C   NEWTON-RAPHSON METHOD.
      COMMON GI(15),GQ(15),AL,E1,AS,RJ,E2,BD,AE,AH,BE,BH,CN,CL,Q,EP,
      1T1,T2,T3,TA(15),AE1,BE1,AH1,BH1,AH2,BH2,ET1,ET2
      J=0
      1 IF(J.GT.100) GO TO 2
      C2=R1*AL
      C3=EXP(-C2)
      EJ=CN*(C3*(C2+1.0)-EXP(-AL)*(AL+1.0)+CL*(R1**2-1.))
      RE=(E1-EJ)/E1
      ET=EJ-E1
      R=ABS(RE)
      IF(R.LT.0
      DE=(CN*(AL**2)*R1)*(BD-C3)
      R1
      J=J+1
      GO TO 1
      2 WRITE(6,100)
      K1=1
      4 CONTINUE
      RETURN

100 FORMAT(16HERROR FROM RNEWT,/)
      END
      SUBROUTINE RDEPL(R1,R2,K2)
C   THIS SUBROUTINE COMPUTES DEPLETION LAYER SEMIWIDTH R2 FROM
C   FIELD CONTINUITY CONDITIONS.
      COMMON GI(15),
      1T1,T2,T3,TA(15),AE1,BE1,AH1,BH1,AH2,BH2,ET1,ET2
      R2=R1+3.0

```

```
J=0
L=0
K=0
C1=EXP(-R1*AL)
B2=BD/2.
C2=1./AL
HL=C1*C2*(R1+C2)+B2*(R1**2)
1 RH=EXP(-R2*AL)*C2*(R2+C2)+B2*(R2**2)
R
R=ABS(RE)
IF(R.LT.0.001) GO TO 10
IF(J.GT.100) GO TO 20
IF(K.EQ.1) GO TO 15
IF(J.GT.0) GO TO 5
6 F1=R2
D1=RE
R2=R2+0.1*R2
J=J+1
GO TO 1
5 C=RE*D1
IF(C.LT.0.) GO TO 7
IF(L.EQ.1) GO TO 2
B=ABS(D1)
IF(B.GT.R) GO TO 6
L=1
2 F1=R2
D1=RE
R2=R2-0.1*R2
J=J+1
GO TO 1
7 K=1
F2=R2
9 R2=0.5*(F1+F2)
J=J+1
GO TO 1
15 C=RE*D1
IF(C.LT.0.) GO TO 8
D1=RE
F1=R2
GO TO 9
8 F2=R2
GO TO 9
20 WRITE(6,100)
K2=1
10 RETURN
100 FORMAT(16HERROR FROM RDEPL,/)
END
SUBROUTINE FION(R1,R2,AP)
COMMON GI(15),GQ(15),AL,E1,AS,RJ,E2,BD,AE,AH,BE,BH,CN,CL,Q,FP,
1T1,T2,T3,TA(15),AE1,BE1,AH1,BH1,AH2,BH2,ET1,ET2
```

```

C=(R2+R1)*0.5
D=(R2-R1)*0.5
T1=EXP(-R1*AL)
T2=R1**2
T3=R1*T1
K=1
DO 10 I=1,15
X=C+D*GQ(I)
10 TA(I)=SINTEG(X,R1,K)
K=2
AP=SINTEG(R2,R1,K)
RETURN
END
FUNCTION SINTEG(X,Y,K)
COMMON GI(15),GQ(15),AL,E1,AS,RJ,E2,BD,AE,AH,BE,BH,CN,CL,Q,EP,
1 T1,T2,T3,TA(15),AE1,BE1,AH1,BH1,AH2,BH2,ET1,ET2
C1=0.5*(X+Y)
C2=0.5*(X-Y)
A=0.
DO 10 I=1,15
Z=C1+C2*GQ(I)
RT=FIELD(Z,K,I)
RT=RT*GI(I)
10 A=RT+A
SINTEG=A*C2
RETURN
END
FUNCTION FIELD(Z,K,I)
COMMON GI(15),GQ(15),AL,E1,AS,RJ,E2,BD,AE,AH,BE,BH,CN,CL,Q,EP,
1 T1,T2,T3,TA(15),AE1,BE1,AH1,BH1,AH2,BH2,ET1,ET2
C1=EXP(-Z*AL)
C2=Z**2
ER=(CN/Z)*(AL*(T3-Z*C1)+CL*(T2-C2)+(T1-C1))
ER=ABS(ER)
IF(ER.GT.ET2) GO TO 2
IF(ER.GT.ET1) GO TO 3
AS1=AE;AC=AH;BS=BE;CS=BH
GO TO 6
2 AS1=AE1;AC=AH2;BS=BE1;CS=BH2
GO TO 6
3 AS1=AE1;AC=AH1;BS=BE1;CS=BH1
6 IF(K.EQ.3) GO TO 4
IF(K.EQ.1) GO TO 10
C=EXP(-BS/ER)
D=EXP(-TA(I)*RJ)
PE=RJ*AS1*C*D
GO TO 5
10 PE=AS1*EXP(-BS/ER)-AC*EXP(-CS/ER)
GO TO 5
4 PE=ER
5 FIELD=PE
RETURN
END

```

APPENDIX 3

SIMPLIFIED APPROXIMATE SOLUTION METHOD FOR CHARACTERIZATION OF p-n JUNCTIONS

A3.1 INTRODUCTION

In the approximate solution method presented in Chapter 3, the space charge region is assumed to contain both the majority and the minority carriers. Unknown parameters of the approximate solution model were implicitly related by virtue of this assumption and these parameters were evaluated by iterative methods. However, the model parameters determination process can be simplified by assuming the space charge region to contain only fixed charges due to ionized impurities and mobile majority carriers. Using this assumption, it is possible to reduce the number of iterative steps by deriving explicit solutions for some of the unknown parameters. Such an assumption can be applied to characterize junctions when operated under low forward bias voltages. Also, equations derived from this assumption provide a better understanding of the relationships between the model parameters. A method to evaluate parameters of the approximate solution model by the simplified approach is presented in the next section.

A3.2 SIMPLIFIED APPROXIMATE SOLUTION METHOD

In order to derive necessary relationships, the boundary conditions established in Section 3.3.1 are assumed to be valid.

Neglecting the minority carriers, the normalized Poisson's equation can be expressed as

$$\frac{d^2 u}{dy^2} = - \frac{q x_o^2 n_i}{kT\epsilon} \left\{ \frac{a_o x_o}{n_i} y - \exp(\xi_h) \exp(u_o) \right\} \quad A3.1$$

At $y = \pm 1$, from the charge neutrality boundary condition at the edges of the space charge region,

$$(i.e. \quad \frac{d^2 u}{dy^2} = 0, \quad y = \pm 1)$$

the following relationship can be established.

$$\frac{a_o x_o}{n_i} = \exp(\xi_b) \exp(u_o) \quad \text{at } y = 1 \quad A3.2$$

Substituting auxillary potential $u_o = Ay + By_2$ in the above equation

$$\frac{a_o x_o}{n_i} = \exp(\xi_b) \exp(A + B) \quad A3.3$$

or

$$B = \ln \left(\frac{a_o x_o}{n_i} \right) - (A + \xi_b) \quad A3.4$$

Using the third boundary condition described in Chapter 3,
we have

$$u'_0 = (\pm 1) = \text{slope of the neutral solution at } y = \pm 1. \quad \text{A3.5}$$

Differentiating A3.2,

$$\left. \frac{du}{dy} \right|_{y=1} = 1 \quad \text{A3.6}$$

Thus, from A3.5

$$A + 2B = 1 \quad \text{A3.7}$$

Using A3.4

$$A = 2 \left\{ \ln \left(\frac{a x_0}{\eta_i} \right) - \xi_b \right\} - 1 \quad \text{A3.8}$$

or

$$\xi_b = \ln \left(\frac{a x_0}{\eta_i} \right) - \frac{1}{2}(1 + A) \quad \text{A3.9}$$

Thus by defining A, parameters B and ξ_b can be determined from equations A3.7 and A3.9. The remaining parameters x_0 , A and Q_T can be evaluated by using the iterative procedure described in Section 3.3.2.

A3.3 ANALYSIS OF THE SPACE CHARGE DENSITY DISTRIBUTION AT THE EDGES OF THE SPACE CHARGE REGION

Net space charge distribution within the transition region of a p-n junction can be expressed as:

$$\rho(x) = q \{N(x) - n + p\} \quad A3.10$$

where

$$N(x) = N_D - N_A$$

Using Boltzmann statistics for distribution of carriers and an auxillary potential function of the form $u_o = Ay + By^2$, the above equation for a linearly graded junction can be expressed in a normalized form as

$$\rho(y) = qn_i \left\{ \frac{a_o x_o}{i} y - 2 \exp(\xi_b) \sinh(Ay + By^2) \right\} \quad A3.11$$

In order to analyze the shape of the charge density distribution defined by the above equation, under conditions of low forward bias voltages, the minority carrier density is assumed to be engligible. Thus, the relationships derived in the previous section can be employed to investigate the shape of the charge distribution near the edges of the transition region.

Defferentiating eq. A3.11

$$\frac{d\rho(y)}{dy} = qn_i \left\{ \frac{a_o x_o}{i} - 2 \exp(\xi_b) \cosh(Ay + By^2) (A + 2By) \right\}$$

A3.12

$$\left. \frac{dp(y)}{dy} \right|_{y=1} = q\eta_i \left\{ \frac{a_{oo}}{i} - 2 \exp(\xi_b) \cosh(A+B) (A+2B) \right\}$$

A3.13

From eqn. A3.3, the above equation can be alternatively expressed as

$$\left. \frac{dp(y)}{dy} \right|_{y=1} = q\eta_i 2 \exp(\xi_b) \{ \sinh(A+B) - (A+2B) \cosh(A+B) \}$$

A3.14

Using eqn. A3.7, the above equation at $y = 1$ can be rewritten as

$$\left. \frac{dp(y)}{dy} \right|_{y=1} = 2\eta_i \exp(\xi_b) \{ \sinh(A+B) - \cosh(A+B) \}$$

A3.15

For values of $A+B \gg 1$

$$\sinh(x) \approx \cosh(x)$$

Hence,

$$\left. \frac{dp(y)}{dy} \right|_{y=1} = 0$$

Thus, for low forward voltages, where $A + B \gg 1$, the space charge distribution has a zero gradient at the edges of the transition region.

APPENDIX 4

APROXSOL - PROGRAM TO COMPUTE PARAMETERS OF THE
APPROXIMATE SOLUTION MODEL FOR A GIVEN APPLIED VOLTAGE

4.1 PURPOSE

This program computes parameters of the approximate solution model which takes into account presence of mobile carriers in the space charge region of a linearly graded junction.

4.2 USER PROCEDURE

Input data to the program are read in the following order:

READ GRAD, XI, QT, APLVOL, C1, N

where

GRAD	Gradient of the junction
XI	Initial value for the space charge region semiwidth
QT	Initial value for the total charge in one half of the space charge region
APLVOL	Applied voltage (+ ve for forward bias and -ve for reverse bias)
C1	Normalizing parameter for QT
N	Number of Input parameters.

4.3 METHOD OF EVALUATION

See Chapter 3

4.4 ERROR MESSAGES

Approximate error messages are printed out when the program diverges.

```
C   PROGRAM APROXSOL
C   THIS PROGRAM COMPUTES PARAMETERS OF THE APPROXIMATE SOLUTION
C   MODEL FOR A GIVEN APPLIED VOLTAGE.
C   PROGRAMMER RAMESH SIRSI*****
COMMON /BLOCK1/GQ(15),GI(15)//Q,EP,A,B,X0,GRAD,EIN,TA(15)
Q=1.6E-19;EP=1.064E-12;EIN=1.38E+10
C   INPUT DATA TO THE PROGRAM ARE READ IN.
READ(5,100),GRAD,XI,QT,APL VOL,C1,N
DO 11 I=1,N
WRITE(5,104),I
WRITE(5,101),GRAD,APL VOL
APLN=APL VOL/0.052
Z1=QT/C1
J=0;K=0;L=0;K1=0
C   INITIAL VALUE FOR TOTAL CHARGE IN ONE HALF OF THE SPACE CHARGE
C   QT IS SET.
1  QT=Z1*C1
C   PARAMETERS OF THE MODEL FOR THE GIVEN CHARGE QT ARE COMPUTED.
CALL APSOL(XI,QT,DEL UN,K1)
IF(K1.EQ.1) GO TO 11
XI=X0
RE=(APLN-DEL UN)/APLN
R=ABS(RE)
C   COMPUTED APPLIED VOLTAGE FROM POISSON'S EQUATION IS
C   CHECKED AGAINST INPUT VOLTAGE DATA.
IF(R.LT.0.0001) GO TO 2
IF(J.GT.100) GO TO 3
IF(K.EQ.1) GO TO 4
IF(J.GT.0) GO TO 5
6  A1=Z1;D1=RE;Z1=Z1+0.1*Z1
J=J+1
GO TO 1
5  C=RE*D1
IF(C.LT.0.0) GO TO 7
IF(L.EQ.1) GO TO 8
B=ABS(D1)
IF(B.GT.R) GO TO 6
L=1
8  A1=Z1;D1=RE;Z1=Z1-0.1*Z1
J=J+1
GO TO 1
7  K=1;A2=Z1
9  Z1=0.5*(A1+A2)
J=J+1
GO TO 1
4  C=RE*D1
IF(C.LT.0.) GO TO 10
A1=Z1;D1=RE
GO TO 9
10 A2=Z1
```

```
GO TO 9
3 WRITE(5,102)
2 WRITE(5,103),X0,A,B,QT
  APLVOL=APLVOL+0.1
1
100 FORMAT(6G.0)
101 FORMAT('GRAD=',E10.4,5X,'APPLIED VOLTAGE=',E10.4,/)
102 FORMAT('NO CONVERGENCE IN MAIN PROGRAM',/)
103 FORMAT('X0=',E10.4,3X,'A=',F6.3,3X,'B=',F7.3,3X,'QT=',E10.4,/)
104 FORMAT('TRIAL NO:',I2,/)
  END
SUBROUTINE APSOL(XI,QT,DELU,K1)
C   THIS PROGRAM COMPUTES PARAMETERS OF THE APPROXIMATE SOLUTION
C   MODEL FOR A GIVEN TOTAL CHARGE QT.
  COMMON /BLOCK1/GQ(15),GI(15)//Q,EP,A,B,X0,GRAD,FIN,TA(15)
  C1=1.0E-04
  X0=XI
  FEO=QT/EP
  Z1=X0/C1
  J=0;K=0;L=0;M=0;K1=0
1  X0=Z1*C1
  A=((FEO*X0)/0.026)-1.0
  BI=0.5*(1.-A)
  UI=ALOG((GRAD*X0)/EIN)-(0.5+(A/2.))
  CALL SELECT(UI,BI,DELU,K1)
  IF(K1.EQ.1)
    EINC=EIN*EXP(DELU)
    CON=X0*EINC*Q
    CT=(GRAD*X0)/(2.*EINC)
    QN=CON*(CT-2.*SINTEG(1.0,0.0,K))
    RE=1.0-(QN/QT);R=ABS(RE)
    IF(R.LT.1.0E-04) GO TO 2
    IF(J.GT.100) GO TO 3
    IF(L.EQ.1) GO TO 4
    IF(J.GT.0) GO TO 5
  6  A1=Z1;D1=RE;Z1=Z1+0.1*Z1
    J=J+1
    GO TO 1
  5  C=RE*D1
    IF(C.LT.0.) GO TO 7
    IF(M.EQ.1) GO TO 10
    B=ABS(D1)
    IF(B.GT.R) GO TO 6
    M=1
  10  A1=Z1;D1=RE;Z1=Z1-0.1*Z1
    J=J+1
    GO TO 1
  7  L=1;A2=Z1
  8  Z1=0.
    J=J+1
```

```
      GO TO 1
4  C=RE*D1
   IF(C.LT.0.) GO TO 9
   A1=Z1;D1=RE
   GO TO 8
9  A2=Z1
   GO TO 8
3  WRITE(5,103)
   K1=1
   GO TO 15
2  DELU=DELU*0.052
   CT=(X0*GRAD)/EINC
17 Y1=0.0;Y2=1.0
   CONST=-(Q*(X0**2.)*EINC)/(0.026*EP)
   C=0.5*(Y2+Y1);D=0.5*(Y2-Y1)
   K=1
   DO 16 I=1,15
   Y=C+D*GQ(I)
16 TA(I)=SINTEG(Y,Y1,K)
   K=2
   UNP=CONST*((CT/6.)-2.*SINTEG(Y2,Y1,K))+A
   DELU=ALOG(GRAD*X0)/EIN)-UNP
15 CONTINUE
103 FORMAT('NO CONVERGENCE IN X0')
   RETURN
   END
   FUNCTION FUN(X,K,I)
   COMMON Q,EP,A,B,X0,GRAD,EIN,TA(15)
   IF(K.EQ.2) GO TO 1
   FUN=SINH(A*X+B*(X**2.))
   GO TO 2
1  FUN=TA(I)
2  CONTINUE
   RETURN
   END
   FUNCTION SINTEG(X,Y,K)
   COMMON /BLOCK1/GQ(15),GI(15)
   C=(X+Y)*0.5
   D=(X-Y)*0.5
   A=0.0
   DO 1 I=1,15
   Z=C+D*GQ(I)
   RT=FUN(Z,K,I)
   RT=RT*GI(I)
1  A=RT+A
   SINTEG=A*D
   RETURN
   END
   SUBROUTINE SELECT(UI,BI,DELU,K1)
   COMMON Q,EP,A,B,X0,GRAD,EIN,TA(15)
```

```
J=0;Y2=0.0
T2=GRAD*X0
1 CALL RNEW T(UI,BI,K1)
  IF(K1.EQ.1) GO TO 2
  FUA=EXP(UI)-T2/(2.*EIN*SINH(A+B))
  IF(ABS(FUA).LT.1.0E-08) GO TO 2
  ER=UI-Y2
  IF(ABS(ER).LT.1.0E-06) GO TO 2
  IF(J.GT.100) GO TO 3
  DFUA=EXP(UI)
  Y2=UI
  UI=UI-(FUA/DFUA)
  J=J+1
  GO TO 1
3 WRITE(5,100)
  K1=1
2 DELU=UI
100 FORMAT('NO CONVERGENCE IN DELU')
  RETURN
  END
  SUBROUTINE RNEW T(UI,BI,K1)
  COMMON Q,EP,A,B,X0,GRAD,EIN,TA(15)
  J=0;Y2=0.0
  T2=GRAD*X0
1 FB=(A+2.*BI)*COSH(A+BI)-(T2/(2.*EIN*EXP(UI)))
  IF(ABS(FB).LT.1.0E-08) GO TO 2
  ER=BI-Y2
  IF(ABS(ER).LT.1.0E-06) GO TO 2
  IF(J.GT.100) GO TO 3
  DFB=SINH(A+BI)*(A+2.*BI)+2.*COSH(A+BI)
  Y2=BI
  BI=BI-(FB/DFB)
  J=J+1
  GO TO 1
3 WRITE(5,100)
  K1=1
2 B=BI
100 FORMAT('NO CONVERGENCE IN B')
  RETURN
  END
  BLOCK DATA
  COMMON/BLOCK1/GQ(15),GI(15)
  DATAGQ/-.98799251802,-.93727339240,-.84820658341,
1-.72441773136,-.57097217261,-.39415134708,-.201194094,0.0,
2.201194094,.39415134708,.57097217261,.72441773136,.84820658341,
3.93727339240,.98799251802/
  DATA GI/.030753242,.07036604749,.10715922047,
1.13957067793,.16626920582,.18616100002,.19843148533,.20257824193,
2.19843148533,.18616100002,.16626920582,.13957067793,.10715922047
3,.07036604749,.0307532420/
  END
```

APPENDIX 5

GAUSSIAN - PROGRAM TO COMPUTE PARAMETERS OF A
DOUBLE GAUSSIAN MODEL

5.1 PURPOSE

This program computes parameters of a double Gaussian model from electrical parameters measured at the terminals of a transistor

5.2 USER PROCEDURE

Input data to the program is read in the following order:

```
READ      GC(I), I = 1, 9
READ      GQ(I), I = 1, 15
READ      GI(I), I = 1, 15
READ      EP, Q, RN, TK, TH, N
READ      XC, AC, AE, CC, A2E, EL
READ      BN, AS, El, TB, VC
```

where

GC	Coefficients of Erfc polynomial
GQ	Coefficients of the Gaussian integration formula
GI	Abscissas for Gaussian integration formula
EP	Dielectric constant

Q	Electronic Charge
RN	= 6.3×10^{16}
Tk	= 0.026
TH	= 0.76
N	Number of device parameters to be determined
XC	Collector junction depth
AC	Collector area
AE	Emitter area
CC	Collector capacitance
A2E	Emitter depletion layer semiwidth
EL	Characteristics length of the exponential model
BN	Background doping density
AS	Gradient of impurities at the emitter junction
E1	= $\frac{I_c}{q n_i^2 \exp\left(-\frac{qV_{eb}}{kT}\right)}$
TB	Dummy parameter
VC	Applied voltage at the collector junction

5.3 METHOD OF EVALUATION

See Chapter 5

5.4 ERROR MESSAGES

Appropriate error messages are printed out by the program.

```

C      PROGRAM GAUSSIAN
C      THIS PROGRAM COMPUTES THE PARAMETERS OF A PHYSICAL MODEL
C      FOR A DOUBLE DIFFUSED TRANSISTOR ASSUMING A DOUBLE GAUSSIAN
C      IMPURITY DISTRIBUTION FROM TERMINAL MEASUREMENT DATA.
C      PROGRAMMER RAMESH SIRSI*****
C
      REAL L1,L2
      COMMON GC(9),GC(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,AIC,
1XE,XC,BL,SX,ST,ZF,F1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,EL,TA(15)
      PY=3.141592
C      COEFFICIENTS OF GAUSSIAN INTEGRATION FORMULA ARE READ IN.
      READ(5,1001),(GC(I),I=1,9)
      READ(5,1001),(GI(I),I=1,15)
      READ(5,1001),(GQ(I),I=1,15)
      READ(5,1002),EP,Q,RN,TK,TH,N
      WRITE(6,1016)
      DO 1 I=1,N
C      INPUT DATA TO THE PROGRAM ARE READ IN.
      READ(5,1005),XC,AC,AE,CC,A2E,EL
      READ(5,1005),BN,AS,E1,TB,VC
      DC=(EP*AC)/CC
      L=0
C      DEPLETION LAYER SEMI-WIDTHS ARE COMPUTED FROM POISSONS EQUATION.
      CALL COLECT (A1,BD)
      EL=BD
      AIC=A1
      XQ=XC**2
      SX=XQ/L2
      J2=0
      K2=0
      N2=0
      M2=0
      ZW=1.0E+05
      Z1=1.
32 ZE=Z1*ZW
      XF=X1-0.1*X1
      XF=XF**2
      UL=1.+(L2*ALOG(ZE))/XF
      UH=5.*UL
      L=1
C      PROGRAM COMPUTES THE EMITTER JUNCTION DEPTH FROM EMITTER-DIODE LAW
      CALL SELECT (UL,UH,L,GA)
C      SLOPE COMPUTED AT EMITTER JUNCTION FROM THE MODEL IS COMARED WITH

```

```
C      SLOPE DETERMINED FROM EMITTER CAPACITANCE DATA
      CALL SLOPE(RE)
      R=ABS(RE)
      IF(R.LT.1.0E-05) GO TO 25
      IF(J2.GT.200) GO TO 43
      IF(N2.EQ.1) GO TO 35
      IF(J2.GT.0) GO TO 30
31  P1=RE
      Q1=Z1
      Z1=Z1+0.1*Z1
      J2=J2+1
      GO TO 32
30  C=RE*P1
      IF(C.LT.0.) GO TO 33
      IF(K2.EQ.1) GO TO 34
      B=ABS(P1)
      IF(B.GT.R) GO TO 31
      K2=1
34  P1=RE
      Q1=Z1
      Z1=Z1-0.1*Z1
      J2=J2+1
      GO TO 32
33  N2=1
35  IF(M2.EQ.1) GO TO 36
      M2=1
      Q2=Z1
37  Z1=0.5*(Q1+Q2)
      J2=J2+1
      GO TO 32
36  C=RE*P1
      IF(C.LT.0.) GO TO 38
      P1=RE
      Q1=Z1
      GO TO 37
38  Q2=Z1
      GO TO 37
25  CALL TRANST(DF)
      L1=L2/GA
      AL=XE/XC
      L2=SQRT(L2)
      L1=SQRT(L1)
      WB=XC-XE
      CS2=ZE*CS1
      YE=XE+A2E
      YC=XC-AIC
      YN=YE/XC
      YM=YC/XC
      DN=(XC/(YC-YE))*AMOB(YM,YN)
      WRITE(6,1015),I
      WRITE(6,1050)
      WRITE(6,1020),VC,L2,BN,CS1,CS2,GA,AL
      WRITE(6,1051)
      WRITE(6,1021),XC,XE,WB,AIC,A2E,L1,DN,DF
      GO TO 1
```

```

43 WRITE(6,200)
200 FORMAT(10X,20HERROR DUE DIVERGENCE,/)
1 CONTINUE
1001 FORMAT(5G.0)
1002 FORMAT(6G.0)
1005 FORMAT(6G.0)
1015 FORMAT(10X,9HDEVICE NO,12,/)
1016 FORMAT(1H1,35X,54HPHYSICAL MODEL PARAMETERS WITH A GAUSSIAN DISTRI
1BUTION,/////)
1020 FORMAT(10X,F10.2,5X,6(E10.4,5X),/)
1021 FORMAT(10X,8(E10.4,5X),/)
1050 FORMAT(10X,96H      VC      L2      BN      CS
12      CS1      GA      AL,/)
1051 FORMAT(10X,96H      XC      XE      WB      A1
1C      A2E      L1      DN,/)
CALL EXIT
END
SUBROUTINE SELECT(UL,UH,L,X)
XT=5.*UH
X=UL
YA=X
CALL DIODE(X,E1,RE)
X=UH
19 YB=X
CALL DIODE(X,E2,RE)
C=E1*E2
IF(C.GE.0) GO TO 20
17 X=0.5*(YA+YB)
CALL DIODE(X,ER,RE)
IF(ABS(RE).LT.1.0E-05) GO TO 26
R=ABS((YB-YA)/YB)
IF(R.LE.1.0E-06) GO TO 26
C=E1*ER
IF(C.GE.0.) GO TO 15
YB=X
E2=ER
GO TO 17
15 YA=X
E1=ER
GO TO 17
20 X=1.5*X
IF(X.GT.XT) GO TO 25
GO TO 19
25 WRITE(6,200)
200 FORMAT(9H      ERROR)
26 RETURN
END
SUBROUTINE DIODE(X,ER,RE)
C THIS SUBROUTINE EVALUATES THE VALUE OF EMITTER JUNCTION DEPTH FROM
C EMITTER DIODE LAW
REAL L2

```

```
COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,AIC,
1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,FL,TA(15)
```

```
ST=X
XR=-(L2*ALOG(ZE))/(1.-X)
XE=SQRT(XR)
Y1=(XE+A2E)/XC
Y2=(XC-AIC)/XC
RH=SINTEG(Y2,Y1)
RH=ABS(RH)
HL=AE/(E1*CS1*XC)
HL=ABS(HL)
ER=RH-HL
RE=ER/HL
RETURN
END
```

```
FUNCTION FUN(X)
```

```
REAL L2
```

```
COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,AIC,
1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,EL,TA(15)
```

```
X=X**2
R=EXP(-X*SX)-ZE*EXP(-X*SX*ST)-BL
R1=CS1*(EXP(-X*SX)+ZE*EXP(-X*SX*ST)+BL)
D=(R1/RN)**TH
D=D+1.
DN=TK*((1265./D)+65.)
FUN=R/DN
RETURN
END
```

```
SUBROUTINE SLOPE(ER)
```

```
REAL L1,L2
```

```
COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,AIC,
1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,EL,TA(15)
```

```
L1=L2/ST
T1=EXP(-XR/L1)/L1
T2=EXP(-XR/L2)/L2
RH=2.*CS1*XE*(ZE*T1-T2)
RH=ABS(RH)
HL=AS
ER=(HL-RH)/HL
RETURN
END
```

```
SUBROUTINE COLECT (A1,BD)
```

C THIS SUBROUTINE EVALUATES THE COLLECTOR JUNCTION DEPLETION LAYER
C SEMI WIDTHS.

```
REAL L2
```

```
COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,AIC,
1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,EL,TA(15)
```

```
X=XC**2
L2=2.*XC*EL
BD=EXP(-X/L2)
CS1=BN/BD
DT=L2/4.
CT=SQRT(PY*DT)
CD=SQRT(L2)
C1=0.0001
J=0
K=0
L=0
L1=0
RS=2.0
10 A1=RS*C1
   A2=DC-A1
   X1=XC-A1
   X2=XC+A2
   RH=(CT/DC)*(ERFC(X1/CD)-ERFC(X2/CD))
   RH=ABS(RH)
   RE=(BD-RH)/BD
   R=ABS(RE)
   IF(R.LT.0.01) GO TO 15
   IF(J.GT.100) GO TO 40
   IF(K.EQ.1) GO TO 26
   IF(J.GT.0) GO TO 5
20 D1=RE
   B1=RS
   RS=RS+0.1*RS
   J=J+1
   GO TO 10
5  C=RE*D1
   IF(C.LT.0.) GO TO 25
   IF(L1.EQ.1) GO TO 6
   B=ABS(D1)
   IF(B.GT.R) GO TO 20
   L1=1
6  D1=RE
   B1=RS
   RS=RS-0.1*RS
   J=J+1
   GO TO 10
25 K=1
26 IF(L.EQ.1) GO TO 28
   L=1
   B2=RS
30 ER=(B2-B1)/B2
   ER=ABS(ER)
   IF(ER.LT.0.001) GO TO 15
   RS=0.5*(B1+B2)
   J=J+1
```

```
GO TO 10
28 C=RF*D1
  IF(C.LT.O.) GO TO 35
  D1=RE
  B1=RS
  GO TO 30
35 B2=RS
  GO TO 30
40 WRITE(6,100)
15 CONTINUE
100 FORMAT(14HERROR IN FIELD)
  RETURN
  END
  FUNCTION SINTEG(X,Y)
  COMMON GC(9),GQ(15),GI(15)
  B=ABS(X)
  C=ABS(Y)
  C1=0.5*(B+C)
  C2=0.5*(B-C)
  A=0.
  DO 10 I1=1,15
  Z=C1+C2*GQ(I1)
  RT=FUN(Z)
  RT=RT*GI(I1)
10 A=RT+A
  SINTEG=A*C2
  RETURN
  END
  FUNCTION AMOB(X,Y)
  COMMON GC(9),GQ(15),GI(15)
  C=(X+Y)*0.5
  D=(X-Y)*0.5
  A=0.
  DO 10 I=1,15
  Z=C+D*GQ(I)
  RT=FUMB(Z)
  RT=RT*GI(I)
10 A=A+RT
  AMOB=A*D
  RETURN
  END
  FUNCTION FUMB(X)
  REAL L2
  COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,AIC,
1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,EL,TA(15)
  X=X**2
  R=EXP(-X*SX)+ZE*EXP(-X*SX*ST)+BL
  D1=CS1*R
  D=(D1/RN)**TH
```



```
D=D+1.
DN=TK*((1265./D)+65.)
FUMB=DN
RETURN
END
SUBROUTINE TRANST(DF)
REAL L2
COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2F,A1C,
1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,FL,TA(15)
Y1=(XE+A2E)/XC
Y2=(XC-A1C)/XC
C=ABS(Y2)
D=ABS(Y1)
A=(C+D)*0.5
B=(C-D)*0.5
K=1
DO 10 I=1,15
X=A+B*GQ(I)
10 TA(I)=CINTEG(C,X,K)
K=2
RH=XQ*CINTEG(C,D,K)
DF=ABS(RH)
RETURN
END
FUNCTION CINTEG(X,Y,K)
COMMON GC(9),GQ(15),GI(15)
B=ABS(X)
C=ABS(Y)
C1=0.5*(B+C)
C2=0.5*(B-C)
A=0.
DO 10 I1=1,15
Z=C1+C2*GQ(I1)
RT=FUNC(Z,K,I1)
RT=RT*GI(I1)
10 A=RT+A
CINTEG=A*C2
RETURN
END
FUNCTION FUNC(X,K,I)
REAL L2
COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,A1C,
1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
2,X2,VC,BN,EL,TA(15)
X=X**2
R=EXP(-SX*X)-ZE*EXP(-SX*X*ST)-BL
IF(K.EQ.1) GO TO 15
R1=TA(I)/R
GO TO 10
```

```
15 RS=FXP(-SX*X)+ZE*EXP(-SX*X*ST)+EL
    D1=CS1*RS
    D=(D1/RN)**TH
    D=D+1.
    DN=TK*((1265./D)+65.)
    R1=R/DN
10 FUNC=R1
    RETURN
    END
    FUNCTION ERFC(X)
    REAL L2
    COMMON GC(9),GQ(15),GI(15),DC,EP,Q,RN,TK,TH,L2,A2E,AIC,
    1XE,XC,BL,SX,ST,ZE,E1,CS1,AS,XQ,XR,AE,TB,PY,CD,CT,X1
    2,X2,UC,BN,EL,TA(15)
    DIMENSION GD(9)
    DO 1 I=1,9
1 GD(I)=GC(I)
    DO 2 I=1,8
2 GD(I+1)=GD(I+1)+X*GD(I)
    T=X*GD(9)
    ERFC=EXP(T)
    RETURN
    END
```