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TITLE OF THESIS/TITRE DE LA THÈSE STRESSES DEVELOPED AT AN ASEISMIC CONTINENTAL MARGIN

UNIVERSITY/UNIVERSITÉ

CARLETON UNIVERSITY

DEGREE FOR WHICH THESIS WAS PRESENTED/
GRADE POUR LEQUEL CETTE THÈSE FUT PRÉSENTÉE M.Sc.

YEAR THIS DEGREE CONFERRED/ANNÉE D'OBTENTION DE CE DEGRÉ 1977

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**LA THÈSE A ÉTÉ
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STRESSES DEVELOPED AT
AN ASEISMIC CONTINENTAL MARGIN

by
R. STEPHENSON, B.Sc. (Hon.)

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfilment of the requirements for
the degree of Master of Science

Department of Geology
Carleton University
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21 June 1977

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1977

The undersigned hereby recommend to
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acceptance of this thesis,
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in partial fulfilment of the requirements for
the degree of Master of Science.

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7 July 1977

Abstract

The two-dimensional elastostatic stress distribution at an aseismic continental margin is analyzed using the finite element method. The margin is modelled from geological and geophysical data pertaining to Atlantic-type continental margins of rifted origin.

The effects of the differential gravitational load due to topography are calculated by considering the total contribution of the body forces to the equilibrium solution. An Airy-Heiskanen isostatic model with compensation to gravity provided by basal buoyancy forces is assumed. Stresses due to horizontal compressive forces applied at the oceanward end of the cross-section, to rapid sedimentary loading, and to the lateral contrast in temperature gradient across the margin are also computed. Consideration of stress-relaxation processes occurring in the crust allows a physical interpretation of the purely elastostatic solution to the problem.

The results of the analysis show two interesting features of the stress distribution at an aseismic continental margin. The topography induces modifications favourable to horizontal deviatoric tension beneath the continental shelf and to horizontal deviatoric compression beneath the ocean basin. The lateral variation in elastic properties across the margin produces stress concentrations in the crust, and to a lesser extent in the mantle,

below the lower continental slope and continental rise. These results (taken together permit some preliminary speculation on the evolution of aseismic margins into active subduction zones during the course of the geotectonic cycle.

Acknowledgement

The assistance of Dr. G. Ranalli is greatly appreciated. Discussions with various persons from Carleton, Dalhousie, and Durham Universities and the Earth Physics Branch of the federal Department of Energy, Mines, and Resources also proved rewarding. Dr. C. Beaumont of Dalhousie University was particularly helpful. The cooperation of numerous members of the Carleton University Computing Centre is gratefully acknowledged.

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1. Introduction

Continental margins form one of the major topographic features of the Earth's surface and are of two general types: those which are related to active lithosphere plate boundaries and possess much of the Earth's present seismicity, and those which are intraplate structures and are essentially aseismic. According to current geotectonic theories both kinds of margins have a role in mountain-building: the aseismic type as the main site of sediment accumulation and the seismic type as the locus of orogenic metamorphism, deformation, and upheaval. The connection is made by hypothesizing that the former "fail" and become active zones of plate subduction. It is therefore of interest to better understand the stress systems developed at aseismic continental margins during the geotectonic cycle.

Stresses induced by various kinds of loads acting on or within a continuous structure can be numerically determined using the finite element method, in which the body is subdivided into a "finite" number of "elements". The equilibrium state of the body under prescribed boundary conditions and loads is then approximately solved by considering only the points of interconnection of the elements. The technique was originally developed by aeronautical engineers (Turner et al. 1956) and its greatest advantage

is its suitability of application to bodies of complex shape and inhomogeneous internal structure. The finite element method was first applied to large-scale geologic problems by Douglas (1970) and since has been used extensively as a modeling tool in studying both small-scale and large-scale geotectonic processes (eg. Stephansson and Berner 1971, Beaumont and Lambert 1972, Service and Douglas 1973, Bischke 1974, 1976, Cobbold 1977, McCowan et al. 1977).

In this thesis the two-dimensional elastostatic stress distribution at an Atlantic-type aseismic continental margin is considered. Local modifications to regional lithospheric stresses are mainly due to loads acting near the surface of the plate: the topography of the margin, sedimentary loading, and the lateral contrast in temperature gradient. Consequently, a model of crustal dimension only is required and this has given some rationale to the choice of pure elasticity as its rheology.

2. The Finite Element Method

2.1 Introduction

The finite element method is a numerical method of stress analysis and its basis is the subdivision of a continuous structure into a finite number of "elements" connected at a finite number of "nodes" (cf. Zienkiewicz 1971). The equilibrium condition of the continuum is then satisfied at the nodes only and is represented by a system of equations, known collectively as the "stiffness" equation, expressed conveniently in matrix form as

$$\{F\} - [K] \{\delta\} = 0 \quad (2-1)^*$$

The rank of the matrices will depend on the number of nodes and the spatial dimensions. $\{F\}$ contains all components of force acting at each node; similarly $\{\delta\}$ contains displacements. The coefficient matrix $[K]$ represents the stiffness of the structure, a property which obviously incorporates a constitutive relationship with certain appropriate physical characteristics of the idealized continuum.

* The notations $\{ \}$ and $[\]$ refer to column (or row, by context) and square (or rectangular) matrices respectively.

The solution to the numerical problem, as expressed by equations (2-1), is obtained, as for the analytical approach, by the application of prescribed boundary conditions; the advantage of the numerical formulation is that much greater sophistication is possible so that complex shapes and configuration of physical properties, inhomogeneous and anisotropic structure are easily modelled. Furthermore, a broad range of rheological behaviour, linear and non-linear, can be accommodated by the method. However, for the purposes of this research only two-dimensional stress analysis in a linear elastic continuum has been considered and a general description of such follows; the primary source is Zienkiewicz (1971).

2.2 Formulation of Stiffness Equation

2.2.1 Elemental characteristics. It is first necessary to explicitly define displacement within a given element as a function of the nodal displacements; in general this function is described by

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N] \{\delta\}^e \quad (2-2)$$

where u and v are the components of displacement at any point within the element and $\{\delta\}^e$ contains the components of each nodal displacement (Figure 1). The elements of $[N]$, which depend on position, are called shape functions since they are intrinsically related to the shape and nature of the types of elements used. In

Figure 1. Displacement in a triangular element.

two-dimensional elastic analysis an appropriate element-type is the simple three-noded triangle for which a linear shape function is sufficient.

For such an element (Figure 1), displacement can be specifically described by the linear polynomials

$$u = \beta_1 + \beta_2 x + \beta_3 y \quad (2-3i)$$

$$v = \beta_4 + \beta_5 x + \beta_6 y \quad (2-3ii).$$

It is evident that six such expressions, defining each component of the nodal displacements, can be found by inserting the appropriate sets of nodal coordinates into equations (2-3). The polynomial coefficients are then solved in terms of the nodal displacements, $\{\delta\}^e$, and the functions $[N]$ are known. (This and subsequent steps are developed in greater detail in sections A2.3.1-8.)

The strain, $\{\epsilon\}$, within the given element is, by definition, related to displacement by simple differential operators contained here in matrix $[E]$:

$$\{\epsilon\} = [E] \begin{Bmatrix} u \\ v \end{Bmatrix} \quad (2-4i)$$

where

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} \quad (2-4ii).$$

Substitution of (2-2) into (2-4i) results in

$$\{\epsilon\} = [B] \{\delta\}^e \quad (2-5i)$$

where

$$[B] = [E] [N] \quad (2-5i).$$

The variables x and y in $[N]$ vanish due to the differential operators in $[E]$ so that strain, given by (2-5i), is constant throughout the element.

Stress is related to strain by matrix $[D]$ which contains appropriate elastic parameters. If initial strains, $\{\epsilon_0\}$, (for instance, those due to thermal effects) are present, and if it is assumed that there exists a known system of initial residual stresses, $\{\sigma_0\}$, the general elastic equation can be stated as

$$\{\sigma\} = [D] (\{\epsilon\} - \{\epsilon_0\}) + \{\sigma_0\} \quad (2-6)$$

where the components of $\{\sigma\}$ are analogous to those of $\{\epsilon\}$. In terms of the finite element construction, equation (2-6) is seen by (2-5i) to be equivalent to

$$\{\sigma\} = [D] [B] \{\delta\}^e - [D] \{\epsilon_0\} + \{\sigma_0\} \quad (2-7).$$

2.2.2 Elemental stiffness equation. For a given element the force term of the elemental stiffness equation, $\{F\}^e$, contains six nodal force components which must be statically equivalent to the elemental boundary stresses and the distributed load. In terms of the physical energy requirements of the system this is explicitly satisfied by equating the external and internal work done by arbitrary virtual nodal displacements, $d\{\delta\}^e$. (Figure 1). The former, done by the nodal forces, is simply

$$W_e = (d\{\delta\}^e)^T \{F\}^e \quad (2-8)$$

The internal work per unit volume is done by stresses and the distributed load, here (due to body forces, throughout the element and is

$$W_i^V = d\{\epsilon\}^T \{\sigma\} - d\{V\}^T \{p\} \quad (2-9)$$

where $\{p\}$ contains body force components. For the arbitrary nodal displacements the elemental displacement and strain in equation (2-9) are known, by equations (2-2) and (2-5i), to be

$$d\{V\} = [N]d\{\delta\}^e \quad (2-10i)$$

and

$$d\{\epsilon\} = [B]d\{\delta\}^e \quad (2-10ii)$$

By substitution of equations (2-10) into (2-9) and transposition, the expression for total internal work is obtained by integrating over the volume of the element:

$$W_i = (d\{\epsilon\}^e)^T \left(\int [B]^T \{\sigma\} d(vol) - \int [N]^T \{p\} d(vol) \right) \quad (2-11)$$

Equating W_i and W_e and using the expression for $\{\sigma\}$ given by (2-7) results in

$$\{F\}^e = \{F\}_{\epsilon_0}^e - \{F\}_{\sigma_0}^e + \{F\}_p^e = [k]^e \{\delta\}^e \quad (2-12i)$$

where forces due to initial strains are

$$\{F\}_{\epsilon_0}^e = \int [B]^T [D] \{\epsilon_0\} d(vol) \quad (2-12ii),$$

those due to the initial stress system are

$$\{F\}_{\sigma_0}^e = \int [B]^T \{\sigma_0\} d(\text{vol}) \quad (2-12iii),$$

those due to body forces are

$$\{F\}_p^e = \int [N]^T \{p\} d(\text{vol}) \quad (2-12iv),$$

and the elemental stiffness matrix is

$$[k]^e = \int [B]^T [D] [B] d(\text{vol}) \quad (2-12v).$$

It should be recognized that $[k]^e$ is a 6 by 6 matrix comprising nine 2 by 2 submatrices; each submatrix, k_{rs}^e , relates forces at node r to displacements at node s .

2.2.3 Stiffness of the whole structure. Equation (2-2) can be stated to apply to the whole structure by expanding $[N]$ with the insertion of appropriate zero elements:

$$\begin{Bmatrix} U \\ V \end{Bmatrix} = [N]_0 \{\delta\} \quad (2-13)$$

where $\{\delta\}$ contains all nodal displacements. An analogous matrix, $[B]_0$, can also be constructed. The equivalence of internal and external work is then considered for the entire structure after the application of external concentrated nodal forces, $\{F\}_R^*$.

* In the geologic context these are usually distributed loads which are expressed as static equivalents concentrated at boundary nodes by simple integration over the appropriate boundary surface (section A2.3.2).

Operations similar to those of the elemental analysis are performed with the analogous result that the structural stiffness matrix, $[K]$, is

$$[K] = \int [B]_0^T [D] [B]_0 d(\text{vol}) \quad (2-14)$$

where the integration is carried out over the volume of the structure. It follows from the comparison of equations (2-12v) and (2-14) that the submatrix of $[K]$ that relates nodes r and s is

$$K_{rs} = \sum_{q=1}^n k_{rs}^q \quad (2-15i)$$

where there are n elements in the structure and k_{rs}^q is the appropriate submatrix of the elemental stiffness matrix for element q . If nodes r and s do not both occupy an element q then

$$k_{rs}^q = 0 \quad (2-15ii)$$

The construction of equations (2-12ii) and (2-14) indicates that $[K]$ is intrinsically symmetric; moreover, it follows from equations (2-15) that it is a diagonally banded matrix with the bandwidth dependent on the arbitrary numbering of the nodes throughout the structure. The numerical value of the bandwidth is controlled by the elements(s) for which the integral difference between the assigned node numbers of two of its(their) nodes is maximum.

2.2.4 Equilibrium of the finite element structure.

The structural stiffness equation,

$$\{F\} = [K]\{\delta\} \quad (2-1),$$

where $\{F\}$ is the sum of all components of the various force contributions for each node, expresses the equilibrium condition of the structure only approximately. Whereas the true equilibrium of a system exists when its total potential energy is at a minimum, the finite element solution achieves such a minimum within the constraint of an assumed displacement pattern (Zienkiewicz 1971, p. 27). (An important implication of the energy consideration is that stiffness matrices are characteristically positive-definite.) The solution to equations (2-1) would be expected to converge toward the exact solution as the structural elements approach infinitesimal dimension; the hypothetical basis of this prediction is embodied in a set of convergence criteria, the theory of which is paramount in finite element studies (see Zienkiewicz 1971 for references).

A general application of the convergence criteria for the present case is the requisite compatibility of strains. In the formulation of section 2.2.3 no allowance was made for a possible contribution to the total work by the development of discontinuities between adjacent elements. It is therefore a requirement of the chosen shape functions that only finite strains exist at elemental interfaces: a condition satisfied if displacement at the interfaces is continuous. It is known from equations (2-3) that such displacement varies linearly subject to the displacement of the de-

fining nodes; as such it is independent of the element under consideration and the condition is met.

2.3 Solution of the Stiffness Equation

2.3.1 Boundary conditions. The stiffness matrix is inherently singular since its determinant is zero; consequently an infinite number of solutions exists for the linear system of equations (2-1). Physically, these describe rigid body movements which do not alter the state of stress in the structure. The application of appropriate boundary conditions prohibits such movements and renders $[K]$ non-singular. It is clear that at least one node needs to be constrained in one direction and a different node / constrained in the other to totally eliminate the possible occurrence of non-unique rigid translation as well as rotation; in algebraic terms this makes two equations trivial and allows their removal from the system (2-1).

2.3.2 Inversion of $[K]$. Equations (2-1) are solved for displacements $\{\delta\}$ by inversion of the stiffness matrix $[K]$. Many inversion techniques are known (eg. Westlake 1968) but consideration must be given to FORTRAN suitability and to the nature of the finite element problem. The most popular techniques are the Gauss elimination and Gauss-Seidel iterative methods (Zienkiewicz 1971); the latter is used mainly for non-linear problems. A procedure which

is particularly straightforward for banded, symmetric, positive-definite matrices such as $[K]$ is the Choleski method (Westlake 1968, p. 13) which is used here along lines developed by Dean (1972).

2.3.3 Stress solution. Once nodal displacements are known by solving (2-1), the stress for each element is calculated directly using equation (2-7). Since elemental strain is constant by equation (2-5i), stress discontinuities apparently exist at element boundaries; consequently the stress tensor is assumed to be effective at the elemental centroid. It follows that if an initial stress distribution is to be considered it must also be assumed to be constant throughout each element. Although elemental strain is not directly known, it is implicitly calculated in equation (2-7) into which (2-5i) was substituted and could be computed if desired.

3. Characteristics of Aseismic Continental Margins

3.1 Introduction

Continental margins not associated with active plate boundaries are known as aseismic, or passive, continental margins because of their relative stability. The term "Atlantic-type" has also been used (Heezen 1974) but should be reserved to describe margins which bound oceans having a median ridge and which originated by continental rifting and associated transform faulting. The origins of the aseismic inner margins of back-arc basins such as the Japan and Okhotsk Seas are more poorly understood (Karig 1971, Packham and Falvey 1971, Hilde et al. 1977).

The geologic evolution of an aseismic continental margin typically involves stable, continuous subsidence concentrated at the base of the continental slope. The characteristic smooth topographic relief of the margin is a surficial expression of a substantial accumulation, often more than 10 km, of essentially undeformed sediments beneath the slope and, particularly, continental rise (which extends from the base of the slope to the abyssal plain). The internal structure of this thick stratigraphic sequence is controlled by sedimentary processes but it is the geotectonic processes responsible for the subsidence which are of interest here.

For the purposes of finite element modelling a cross-section through a rifted Atlantic-type margin with a well-developed continental rise-sedimentary ridge complex has been constructed. Although the cross-section is meant to be representative it is based mainly on numerous geophysical and geological studies of the western margin of the North Atlantic, particularly in the region off Nova Scotia.

3.2 Geotectonic Development of Atlantic-type Continental Margins

3.2.1 Evolutionary cycle. The comparison of Kay's (1951) reconstruction of the Appalachian geosyncline with sections of the United States Atlantic margin (Drake et al. 1959) led to the now commonly accepted relationship between aseismic continental margins, sediment accumulation of geosynclinal form and dimension (Dietz and Holden 1966, Mitchell and Reading 1969, Wang 1972), and eventual mountain-building (Dietz 1966, Dewey 1969, Dewey and Bird 1970).

In the context of plate tectonics the evolution of an aseismic continental margin into a Pacific-type, or subducting, plate margin is fundamental to the geotectonic cycle (eg. Wilson 1966). Within this framework, numerous models have been proposed to explain the gross structural and lithologic characteristics of orogenic belts (eg. Bird and Dewey 1970, Dewey and Kidd 1974, Kennedy 1976).

However, the basic requisite geodynamic processes are as yet poorly understood; of most immediate interest to aseismic continental

margins are those processes relating to subsidence and basin formation and to the presumed subsequent "failure" of the margin. While there has been virtually no quantitative study of the latter, a brief introduction to ideas relating to the former follows.

3.2.2 Subsidence and basin formation. Walcott (1972) has shown that a substantial accumulation of sediment can be deposited at a continental edge because of the response of an elastic lithosphere to the sediment load. Loading would occur as progradation at the continental slope and subsidence of the shelf would be a consequence of the flexural rigidity of the lithosphere. However, at most aseismic continental margins, such as the Atlantic margin of North America, progradation is not a predominant sedimentological process (Heezen 1974); furthermore, the predicted gravity anomalies do not compare with those generally found (eg. Worzel 1968, Emery et al. 1970). Better correspondence is observed where sediment supply and rate of sedimentation are very large; two such locations are the Niger delta (Walcott 1972) and the Amazon cone (Cochran 1973). For the continental margin off southern France, although it does not have these characteristics, Watts and Ryan (1976) have attempted to remove effects predicted by a flexure model and have concluded there remains a major component of subsidence caused by other processes. As well, formation of major sedimentary basins, as evidenced by the geologic record, occurs not only at continental margins but also entirely within

"stable" continental platforms (e.g. Bott 1976a). The rate of subsidence for both types of basins often shows monotonic decay through time (Sleep 1971, Keen and Keen 1973, King et al. 1975, Sweeney 1977) suggesting that a single causative mechanism, or set of mechanisms, may be common to both environments.

In general, the exponential behaviour and resultant basin geometry may be a consequence of the isostatic response of a viscoelastic lithosphere to a sedimentary load (Beaumont 1977). Nevertheless, a pre-existing topographic depression is required and isostatic considerations, independent of lithosphere rheology, limit the stratigraphic form and thickness of potential basins.* Basin formation is therefore likely the product of an "initiating" mechanism, which creates a depression and/or contributes to later subsidence, and the response of the lithosphere to the subsequent loading.

Although processes such as crustal necking, loading by intrusions, and subcrustal erosion have also been suggested (Sleep and Snell 1976), thermal contraction is the most commonly cited initiating mechanism and was originally used to model the total sub-

* It has long been known (e.g. Bucher 1933) that, if approximate isostatic equilibrium is maintained throughout the process, potential basin thickness, being a function of the depth of the initial depression, is much less than observed sequences; the possible initial depth is limited by the observed prevalence of shallow-water sediments in ancient successions.

sidence at an aseismic continental margin (Sleep 1971): the topographic anomaly associated with a mid-ocean ridge is essentially an isostatically compensated thermal expansion and there is a progressive contraction of the lithosphere as the newly-rifted margin drifts away from the ridge. The result is subsidence of the continental shelf; erosion, therefore thinning, of continental crust occurs during the initial uplift and rifting stages. This last feature of the model, the ability to produce a crustal depression, has allowed its application to continental basins such as the Michigan (Sleep and Snell 1976, Haxby *et al.* 1976) and Paris basins (Foucher 1976), although for these the occurrence of a preceding subcrustal thermal event must be implicitly assumed. The subsidence rate decay constant observed for the western North Atlantic marginal basin as well as other basins is of the order of 50 my and compares favourably to that expected for subsidence due entirely to thermal contraction; the latter is predicted on the basis of subsidence of the sea-floor as it spreads away from the ridge axis (Sleep 1969) and the interpretation of heat flow data associated with such oceans (Sclater and Francheteau 1970). Consequently, the determination of the contribution to subsidence by isostatic response, compared to that due to thermal contraction, is somewhat complicated (eg. Sleep and Snell 1976).

A second possible initiating mechanism, particularly relevant to continental margins, is related to the topographically-induced

gravitational instability at a continental edge (Bott 1971a) and may contribute to the formation of fault-bounded, narrow, continental shelf basins which are observed at some aseismic margins (eg. Bott and Watts 1970). Intrinsic to the process is the occurrence of intracrustal flow (suggested to be due to "hot creep") at a depth of as little as 10 km beneath the shelf. The model has been supported by an elastic finite element study, with the load due solely to the topography, which predicts shear stresses of up to 37 MNm^{-2} in the crucial region (Bott and Dean 1972). Recently, Bott (1976b) has extended the model to apply as well to narrow continental basins which are characterized by faulting, graben formation, and spasmodic subsidence. Basins produced in this way, depending on their size, may initiate a regional lithospheric loading response resulting in more generalized, continuous, subsidence.

3.3 A Cross-section of an Atlantic-type Continental Margin

3.3.1 Bathymetry. The chosen representative cross-section of an aseismic continental margin, to which a finite element mesh was approximated, is illustrated by Figure 2. The ocean depth of 5 km compares to that assumed by Worzel (1974), based on hypsographic curves (eg. Defant 1961), for the "standard" ocean column (4.8 km). Heezen (1974) has summarized the major surficial characteristics of Atlantic-type margins: considerable variations in shelf, slope,

Figure 2. Cross-section of a rifted Atlantic-type continental margin. Isostatic mass deficiency across the margin is shown as a percentage of "Airy" mass of the standard continental or oceanic column. Refer to Table 1 for material properties of zones: (1) sedimentary layer, (2) continental crust, (3) transitional crust, (4) oceanic crust, and (5) mantle.

and rise gradients/widths exist and the values used here fall within the given ranges. Most of the variation can be attributed to the particular margin's age and/or mode of origin. Figure 2 represents a rifted as opposed to "transform" (Keen and Keen 1973, Scrutton 1976) Atlantic-type continental margin. Although the value used for the gradient of the continental slope (approximately 2.9°) is comparable to that reported by King (1975) southeast of Nova Scotia and Newfoundland, the gradient assigned to the continental rise (approximately 1.1°) is slightly larger than would be expected because it was desired, for practical reasons, to minimize the total length of the finite element model.

3.3.2 Layer velocities and densities: deep structure. Most information pertaining to the deep structure of continental margins is provided by seismic velocity profiles interpreted in conjunction with gravity and magnetic data (eg. Keen et al. 1975, Sobczak 1975, Talwani and Eldholm 1973, Featherstone et al. 1977). Gravity studies indicate that Airy-Heiskanen isostatic equilibrium is essentially maintained across rifted Atlantic-type margins but with small deviations occurring near the base of the continental slope and top of the continental rise (Emery et al. 1970, Scrutton 1976). The internal constraint on the geometry of the deep structure of such margins afforded by this observation has been an important consideration in constructing the representative cross-section (Figure 2).

Densities assigned to layers in the model are accurate to 100 kgm^{-3} only; seawater is therefore assumed to have a density of 1000 kgm^{-3} . The sedimentary layer, overlying continental and oceanic "basement" crust, consists of material of widely varying degrees of consolidation; however, it is considered to have a single density of 2300 kgm^{-3} . The basis of this approximation is the well-defined 8 km thick sedimentary complex at the continental margin off Nova Scotia documented by Keen et al. (1975) which appears to be homogeneous with respect to its observed compressive (P) wave velocity of 3.6 kms^{-1} . Nafe and Drake (1957, 1963) have studied the factors affecting the velocity of elastic waves in sedimentary material; and the well-known empirically-derived relationship between the density of water-saturated sediments and P-wave velocity, commonly referred to as the Nafe-Drake curve (1963), indicates the chosen density for the observed velocity. Moreover, 2300 kgm^{-3} has often been used by authors as the density of the sedimentary layer in gravity studies of continental margins (eg: Worzel and Shurbet 1955, Drake et al. 1959, Worzel 1965, Sobczak and Weber 1973, Sobczak 1975). The largest error in using a single "standard" density for all sedimentary material occurs for the 1 km thick layer overlying the abyssal plain (oceanic layer 1) which, in reality, is undoubtedly less dense (Worzel 1974). It should also be noted that, although this layer is beyond the resolution of the finite element mesh (Figure 3), its presence is

Figure 3. Finite element mesh approximating the representative cross-section of Figure 2. All quadrilaterals represent two triangular elements separated by a positively sloping (upper left to lower right) diagonal.

reflected in the densities assigned to the appropriate elements in order to maintain the Airy-Heiskanen isostatic model.

Although the crustal layer of Figure 2, the basement to the sedimentary complex, consists of continental and oceanic as well as transitional-type material, it is assigned a constant density of 2900 kgm^{-3} . The continental crust is assumed to be unlayered and, although seismic velocities generally tend to increase with depth, recent compilations of data indicate average P-wave velocities of 6.36 kms^{-1} (Worzel 1974) and 6.35 kms^{-1} (Goodacre, 1972). In oceanic crust the relatively thin layer 2 has been neglected and only the average P-wave velocity of layer 3 has been considered; Worzel (1974) calculates 6.70 kms^{-1} and Bott (1971b) gives the range $6.40\text{--}7.00 \text{ kms}^{-1}$. The values reported by Keen et al. (1975) are comparable. A "Nafe-Drake" velocity-density curve for metamorphic and igneous rocks in Ludwig et al. (1970) indicates continental and oceanic crustal densities for the appropriate velocities of 2840 kgm^{-3} and 2650 kgm^{-3} (interpreted by Worzel 1974) from which the assumed value, rounded upwards, is derived. The lateral variation in crustal seismic velocities across the continental margin, in view of the assumed constant density, will be reflected by the choice of Young's modulus for the different crustal regions (section 3.3.3) and is particularly relevant to the transition zone where P-wave velocities are anomalously high (Emery et al. 1970, Mayhew et al. 1970, Keen et al. 1975).

The mantle in the representative cross-section has a density of 3300 kgm^{-3} based on the crust/mantle density contrast of 400 kgm^{-3} determined from gravity studies in continental regions of isostatic equilibrium (Woollard 1966). (A smaller value, 200 kgm^{-3} , calculated from regression analysis of similar data in Canada (Goodacre 1972) is explained by the assumption of a layered continental crust.)

The chosen densities allow the construction of continental and oceanic sections such that both columns have equal mass and crustal thicknesses are close to average values. The exact nature and origin of the transition zone remains problematic (eg. Talwani and Eldholm 1973, Keen 1974, Keen et al. 1975) but its deep structure can be modelled from simple considerations. The mass distribution interpreted from the associated gravity anomalies is here approximated by maintaining the standard oceanic basement thickness beneath an 8 km thick sedimentary basin to its deepest point which, therefore, coincides with the area of maximum mass deficiency. This maximum, illustrated in Figure 2, as a proportion of the "Airy" mass of the oceanic or continental column, is 2.1% which compares favourably to that calculated by Emery et al. (1970) beneath the rise of eastern North America at a depth of 40 km. The often postulated basement ridge near the continental shelf break (Burk 1968, Rabinowitz 1974) and resultant gravity anomaly have been neglected.

3.3.3 Elastic and thermal coefficients. It is well-known that the velocities of elastic body waves are given by

$$v_p^2 = \frac{(\lambda + 2\mu)}{\rho} \quad (3-1i)$$

and

$$v_s^2 = \frac{\mu}{\rho} \quad (3-1ii)$$

where ρ is the density and λ and μ are Lamé's coefficients of the medium. Expressed in terms of Poisson's ratio, ν , and Young's modulus, E , equations (3-1) become

$$v_p^2 \rho = \frac{E(1-\nu)}{(1+\nu)(1+2\nu)} \quad (3-2i)$$

and

$$v_s^2 \rho = \frac{E}{2(1+\nu)} \quad (3-2ii).$$

Poisson's ratio for the medium can be determined from the seismic velocities by combining equations (3-2) such that

$$\nu = \frac{(v_p/v_s)^2 - 2}{2[(v_p/v_s)^2 - 1]} \quad (3-3).$$

There are few simultaneous observations of both v_p and v_s reported for the crust; however, seismological data for the upper mantle in the northwest Pacific Ocean (Sutton and Walker 1972) indicate $\nu = 0.25$ by equation (3-3). A similar value is computed from average v_p and v_s , 8.1 kms^{-1} and 4.7 kms^{-1} , compiled by Manghnani

et al. (1974) for the upper mantle in both continental and oceanic regions. Geophysical (Earth structure) models, experimental data, and petrologic and geochemical considerations all suggest a similar Poisson's ratio for upper mantle and crustal material (eg. Green and Liebermann 1976, Jordan and Anderson 1974) and $\nu = 0.25$ is therefore accepted for these layers in the present model. The sedimentary layer is presumably less rigid and may have a higher Poisson's ratio. The assumed P-wave velocity, 3.6 kms^{-1} , has a corresponding V_s , determined from an empirical relationship for shallow-water marine sediments (Nafe and Drake 1957) of 2.0 kms^{-1} resulting in $\nu = 0.28$ by equation (3-3). In the model, Poisson's ratio of the sedimentary layer is founded upwards to 0.30. Bott and Dean (1972) and Service and Douglas (1973) used $\nu = 0.25$ for elastic crust/mantle finite element models, Stephansson and Berner (1971) used $\nu = 0.30$ in an elastic model, and Bischke (1976) assumed $\nu = 0.33$ for both elastic lithosphere and elasticoviscous (Maxwell) asthenosphere. However, there is some question as to the applicability of such low values to geotectonic models because of the implicit large-scale bulk compression. A Poisson's ratio close to 0.5 approximates incompressibility and has been used mainly in non-linear and viscoelastic finite element studies (eg. Neugebauer and Breitmeyer 1975, Bischke 1974). For elastic models there appear to be no implicit analytical problems incurred by assuming a low

Poisson's ratio (N. H. Sleep 1976, oral communication), but the results must be interpreted within the rheological constraint.

The assumed densities and Poisson's ratios, in conjunction with appropriate P-wave velocities, allows the computation of Young's moduli by equation (3-2i) for the sedimentary layer, continental, transitional, and oceanic crust, and mantle. The results are summarized in Table 1. The P-wave velocity for transitional crust, 7.4 kms^{-1} , is taken from Keen *et al.* (1975); its horizontal extent and position are adapted from Keen *et al.* (1975), Worzel (1968) and King (1975).

Table 1 also lists the volume coefficients of thermal expansion, α , assumed to be constant through the mantle and the non-mantle regions, estimated from the compilation of Skinner (1966) following the assessment for upper mantle material of Bottinga and Allegre (1976). The coefficient is extremely temperature-dependent and the slightly lower value indicated for crustal and sedimentary materials reflects lower temperatures as much as structural/chemical variation.*

* Forsyth (1977) has modelled the change in α in the lithosphere as a linear decrease from $3.5 \cdot 10^{-5} \text{ }^{\circ}\text{K}^{-1}$ at the surface to $2.5 \cdot 10^{-5} \text{ }^{\circ}\text{K}^{-1}$ at a depth of 200 km. It has not been possible to incorporate this information in the present study; however, the change in results consequent on adopting Forsyth's model would be expected to be very small.

Table 1. Material properties of the representative cross-section.
Zones refer to Figure 2.

Zone	Layer	V_p (kms^{-1})	ρ (kgm^{-3})	E (10^5 MNm^{-2})	ν	α ($10^{-5} \text{ }^\circ\text{K}^{-1}$)
1	sediment	3.6	2300	0.22	0.30	2.5
2	crust { continental transitional oceanic	6.4	2900	1.0	0.25	2.5
3		7.4	2900	1.3	0.25	2.5
4		6.7	2900	1.1	0.25	2.5
5	mantle	8.1	3300	1.8	0.25	3.5

4. Boundary Conditions and Load Cases

4.1 Introduction

Opposite components of at least two nodes must be constrained in order to suppress non-unique rigid motions (section 2.3.1); once this requirement is met the remaining boundary conditions can take the form of further specified nodal displacements or of concentrated boundary nodal forces. Mixing of displacement and force boundary conditions results in unnecessary problems of interpretation and for the finite element formulation used here superposition of external forces onto constrained nodes is physically and mathematically meaningless. Furthermore, for elastic models, stress results proximal to such nodes are almost always unreliable; depending on the size and degree of convergence of the finite element mesh more distal results may similarly be unpredictably modified. For a simple point load on a "cratonic" block with constrained edges Service and Douglas (1973) have shown that results are better when the edges are arbitrarily maintained in the vertical plane by what they consider to be "residual tectonic and lateral gravity forces" (p. 4).

In the present study the stress distribution due to topographic variation at a continental margin is of great importance; for such a case the model is loaded by body forces rather than externally

applied forces and the results are therefore extremely sensitive to the chosen boundary conditions. Also, the finite element mesh used here has, for practical purposes, an element to total mesh size ratio considered to be relatively low. For these reasons it was decided prior to starting the analysis to avoid the use of displacement boundary conditions as much as possible. Changes in the results dependent on boundary condition type is discussed in section 5.1.5.

4.2 Boundary Stresses

4.2.1 Body forces and isostatic loads The exact nature of body forces, here taken to be the distributed load due to gravity, is poorly understood; nevertheless, in a finite element formulation they can be approximated directly (section A2.3.4) or by use of a body force potential function (Zienkiewicz 1971, p. 58). However, the effect of the internal work done by these forces during any virtual displacement (section 2.2.2) leads to structural instability unless somehow compensated. For small-scale structures with a small total gravitational effect, the body forces can be approximately maintained by a horizontally-constrained model base but this is not possible for models of crustal dimension. In the present study of a continental margin it is primarily the differential load due to gravity which is of interest. Bott and Dean (1972) neglected body forces when examining the effects of a regional hori-

zontal stress field across a continental margin. However, when considering the effects of the gravitational load they dealt with the body force problem by using a density contrast model which required that the margin be in a state of perfect Airy-Heiskanen isostatic equilibrium everywhere.

Consideration of the internal work done by a distributed external load (Zienkiewicz 1971, pp. 23, 25) allows the total effect of body forces to be included. The external load is due to the lithostatic pressure at the base of the model and, if the model is assumed to be an elastic plate "floating" on a viscous substratum, this is equivalent to the buoyancy pressure if Airy-Heiskanen isostasy is interpreted in terms of Archimedes's principle. The lithostatic pressure, expressed as a force per unit area, is

$$F_{\ell}^{\Delta} = g \sum \rho_j h_j \quad (4-1)$$

where ρ_j is the density of layer j having height h_j and g is the gravitational acceleration, assumed constant. If the elastic plate is subject to a point load there will be an additional term due to the buoyancy of the displaced "fluid" (Walcott 1970) and the total "Airy" pressure will be

$$F_a^{\Delta} = F_{\ell}^{\Delta} - \rho_m dg \quad (4-2)^*$$

* A similar, but unclearly written, expression is given by Service and Douglas (1973).

where ρ_m is the substratum density and d is the equilibrium vertical displacement at the base of the plate. F_g^Δ is concentrated at each of the basal nodes as an applied boundary force by simple integration, assuming unit thickness of the model, whereas the displacement-dependent term requires a modification to the structural stiffness matrix (sections A2.3.2, 5). A consequence of the application of equation (4-2) is that isostatic compensation is regionalized (Heiskanen and Vening Meinesz 1958, pp. 137-142).

There are two inherent problems in this formulation: first, equation (4-2) is only approximate because the displacement-dependent term is derived from considerations of a vertical point load applied to a uniform elastic plate. Here the possible disturbance due to the non-uniform distribution of the elastic properties is ignored. The non-uniform geometry is rationalized because the major load in question is implicitly the geometry itself. Secondly, although Service and Douglas (1973) indicate that equation (4-2) ensures immediate isostatic response in a model which will always be in isostatic equilibrium, the continental margin presently under study is not initially in such a state. Consequently, the lithostatic pressure is not constant along the base of the model but varies linearly under the continental slope and rise reaching a minimum at the slope-rise "break". The chosen boundary forces have been directly calculated (section A2.3.2). Alternatively, a

"dynamic" boundary condition might have been imposed by assuming pressure to remain constant, despite the deficiency of overburden mass, as a result of uplifting forces attempting to restore equilibrium. Such forces would be due to flow in the substratum induced by the inherent pressure gradient. The static boundary condition implies that the elastic plate, in a state of flexure, has simply been truncated at the base of the model. Flexural studies suggesting that the "effective" elastic thickness of the lithosphere is of the same order as the thickness of the present model (eg. Cochran 1973, Fulton and Walcott 1975) lend some support to the assumption of the static basal boundary condition.

4.2.2 Standard surface load. Since the total overburden pressure is used to calculate basal boundary forces it is necessary to apply the static force equivalents of the overlying mass, seawater, at the appropriate surface nodes. The continental shelf is assumed to be at sea-level and is therefore stress-free. On the slope and rise the hydrostatic pressure, acting normal to the boundary surface, is resolved into horizontal and vertical components, reducing the basal force required to retain a zero net sum of vertical forces by 0.02-0.12%. Such a small modification, although made, is negligible.

(Strictly (Walcott 1970), the change in water depth during an equilibrium displacement of the elastic plate results in an addition-

al term in equation (4-2):

$$F_a^\Delta = F_\ell^\Delta - \rho_m dg + \rho_w d'g \quad (4-3)$$

where ρ_w is water density and d' is the displacement at the surface of the plate. For a point load on a uniform plate, the conditions for which the equation is exact, $d=d'$. Here, because of the various approximations, the term has been neglected.)

4.2.3 Edge-forces and displacement boundary conditions. The ends of the continental margin section are constrained by the physical presence of adjacent lithosphere. To find the boundary forces to maintain this condition, for a finite element model, Service and Douglas (1973) used a reverse technique: for a uniform crustal block, having no differential loading, the required forces were calculated from an initial computer run in which horizontal displacement at the ends was prohibited. Although regarded by Service and Douglas as being residual tectonic and gravitational in origin, the resultant forces are mostly the latter. The magnitude of such forces is a function of Poisson's ratio in a perfectly elastic body. They may be considered to be due to the horizontal component of the overburden pressure, well-known to be

$$F_h^\Delta = \frac{\nu}{1-\nu} F_\ell^\Delta \quad (4-4)$$

where ν is Poisson's ratio, acting in the adjacent lithosphere.

Horizontal forces calculated by simple integration of equation (4-4) along the appropriate edge surface (section A2.3.2) are considered to be the "standard" vertical boundary condition for the present model. Resultant horizontal displacements, although minimal, are not exactly zero mainly because of the differential load but also because of the necessity of constraining at least two nodes in order to obtain physical/mathematical stability. In the model the two basal corner nodes are fixed and, in an effort to minimize rotation about these corners, following a suggestion by Sturgul et al. (1976), the basal centre node is constrained to move vertically only.

4.3 Loading at an Aseismic Continental Margin

4.3.1 Topography and compensation. For the general boundary conditions specified no further external applied forces are required to model the implicit load due to self-gravitation; although the body, or gravitational, forces are vertical the effect of the topography results in some horizontal loading because of the heterogeneous variation in lithostatic pressure gradient across the margin. The presence of such a "spreading stress" has been postulated for some time (eg. Gutenberg 1927, 1936). Figure 4 shows the horizontal component of the lithostatic pressure calculated from equation (4-4) as a function of depth for the continental and oceanic columns at the model ends. The discontinuity at $z = 2$ km is due to the

Figure 4. Horizontal pressure at the continental margin model. Area represents force (unit model thickness assumed): (A) spreading force due to margin topography, 5930 MN; (B) increase in effective spreading force due to overlying seawater, 410 MN; (C) compensation due to hydrostatic pressure along slope and rise, 40 MN; (D) arbitrary spreading force due to ridge topography, 6300 MN. Net sum of horizontal forces is 0 MN.

change in Poisson's ratio at the crust/sediment interface. The curves converge at $z = 34.5$ km because of the prescribed condition of constant "Airy" mass for both standard columns. The area between them is representative of the spreading force at the margin, the effects of which are of interest, and has a magnitude of 5940 MN. The inability of the model to include the overlying seawater, the mass of which contributes to the lithostatic pressure, increases the effective spreading force. This effect is practically negligible; moreover, it is partly compensated by the horizontal component of the hydrostatic pressure, acting normal to the model boundary, on the continental shelf and slope as is indicated in Figure 4. The discontinuity in the curve of the compensating pressure is due to the abrupt change in the topographic gradient at the slope-rise break. It should also be noted that since there is a mass deficiency under the slope and rise of the margin the total spreading force is implicitly greater than shown; however, the problem is tractable only in terms of the net boundary forces acting on the entire structure.

Bott and Dean (1972) have explained normal faulting and narrow basin formation on continental shelves in terms of the differential spreading stress and Meissner (1974) has speculated that the associated forces may "initiate or at least support those lateral movements which are observed at subduction zones" (p. 68). Frank (1972, p. 290) has noted that such pressures are, except in the case of the Tibetan Plateau, "just balanced" by the spreading stress

effected by a 50-100 km thick oceanic lithosphere modelled as a glacier analogue; the pressures would be due to mid-ocean ridge topography. Assuming isostatic equilibrium in the ocean basins and across ridges there are lateral variations in vertical density gradients in oceanic lithosphere and the resultant spreading stress would be concentrated above the level of isostatic compensation. Frank proposed an extreme upper level to be the (presumed horizontal) lithosphere base. This scheme is used here to infer horizontal balancing forces acting on the oceanic edge of the finite element model. The responsible spreading stress is maximized at the surface (Figure 4) and is totally compensated at $z = 40$ km, the model base. The magnitude of the maximum pressure, 0.36 MNm^{-2} at $z = 5$ km, the surface, was chosen arbitrarily in order to balance the topographic spreading force of the model.

4.3.2 Sedimentation. The present elastic model is only a static depiction of a dynamic system and the simulation of on-going sedimentary loading and the determination of its effects is problematic. The approach of arbitrarily displacing appropriate surficial nodes vertically is rejected because the results would be very difficult to interpret in terms of time-dependent subsidence and isostatic response mechanisms. Instead, instantaneous progradation of the shelf is assumed to occur. Progradation is not a predominant sedimentary process at most such margins and any available sediment will normally supply basins forming as a response to pre-

viously discussed mechanisms. Progradation would occur as a consequence of a rapid increase in sedimentation rate and could therefore be considered to occur in a geologically short time. Moreover, where thick stratigraphic wedges due to progradation are known to occur (ie., at major river deltas) they are typically associated with large positive gravity anomalies indicating their mass may not be in complete isostatic equilibrium. Alternatively, if the evolution of a continental margin has progressed to the point when subsidence has ceased, but substantial amounts of sediment are still available, then progradation might be expected to occur and the effects of its load can at least be viewed as external to those of the tectonically-induced subsidence (section 3.2.2).

In the present case, progradation was assumed to extend the continental shelf to a point above the initial slope-rise break. The sedimentary surface then was assumed to slope continuously to a point a few kilometres beyond the initial beginning of the abyssal plain. The static force equivalents of this distributed load were applied at the appropriate nodes and are illustrated schematically in Figure 10. No extra compensation was made at the model base, since the load is assumed to be applied rapidly relative to isostatic response times.

4.3.3 Temperature distribution. It was first demonstrated by Revelle and Maxwell in 1952, and since confirmed (eg. Lee and

Uyeda 1965), that, 'in stable regions, oceanic and continental heat flow are approximately the same. Bullard (1952) proposed that the equality was due to similar mean chemical compositions of crust and upper mantle beneath both types of regions to a depth of 150 km. Because of the obvious enrichment of radioactive elements in continental crust, this explanation implied that temperatures in the oceanic upper mantle were higher than their continental counterparts and the implicit thermal contrast at continental margins became a matter of geotectonic speculation (eg. MacDonald 1963, 1965, Pollack 1967). However, thermal modelling of oceanic lithosphere in the context of sea-floor spreading has allowed the removal of the chemical constraint of Bullard (Sclater and Francheteau 1970). The early models of this type were published by McKenzie (1967) and Sleep (1969); numerous variations have followed and are briefly reviewed by Forsyth (1977). Their implications are such that the petrologic state, and therefore physical structure and thermal evolution of the upper mantle beneath continents and oceans, is significantly dissimilar (Froidevaux et al. 1977). There remains some question whether seismological observations support (eg. Jordan 1975) or deny (eg. Hales 1976) this hypothesis.

Since the continental part of the cross-section is mainly crustal, a good estimate of the basal temperature can be derived from a simple steady-state conductive thermal model (eg. Jaeger 1965). Blackwell (1971) has considered thermal data for the eastern United States

and determined that temperatures at 40 km depth, 5 km below the crust/mantle boundary, are in the range 741-891°K. Goodacre (1972) has compiled temperature-depth estimates interpreted from magnetic, seismic, electromagnetic, and heat flow data in stable continental regions and concluded that temperatures near the crust/mantle boundary range from 800 to 1000°K. For the main thermal models of oceanic lithosphere Forsyth (1977) points out there is little difference in predicted isotherms at the depths under consideration; for a sea-floor age of 80-120 my the corresponding temperatures at a depth of 35 km range 825-675°K. It is obvious that the petrologic/structural dissimilarities in continental and oceanic upper mantle has minimal effect on the temperature distribution of a crustal model as thin as the present one. Rather, the predominant factor is simply the topography and resultant lateral depth contrast; the oceanic thermal gradient would be expected to be greater. In the present model the temperature distribution is approximated by assuming temperatures of 273°K at the surface, 773°K at the base, and deriving the intermediate values by simple linear interpolation.

In the finite element formulation the thermoelastic contribution to the resultant stresses can be included by consideration of the elemental load induced by thermal strains, assumed to be contained in the initial strain matrix of equation (2-6), the components of which are:

$$\{\epsilon_0\} = (1 + \nu) \begin{bmatrix} \alpha T' \\ \alpha T' \\ 0 \end{bmatrix} \quad (4-5)$$

where α is the coefficient of thermal expansion, ν is Poisson's ratio, and T' is an initial elemental temperature change. If it is assumed that at the time of initial continental break-up, the entire margin, the continental as well as proto-oceanic parts, had no lateral thermal contrast (ie., the whole system was uniformly heated), then the contrast implicit in the presumed temperature distribution has developed as the margin itself has developed. The observation that the time scale of the decay of heat flow in continental regions is an order of magnitude smaller than for oceanic regions (Sclater and Francheteau 1970) supports this simple model. Since the entire margin was previously hotter than at present the temperature profile of the continental column is taken to be "standard" and is subtracted from temperatures elsewhere at equal depth to produce a temperature contrast scheme suitable for the finite element formulation. The maximum "initial temperature change" calculated as such occurs at the top of the oceanic column which is, in fact, the part of the model which would be expected to have cooled the most since the origin of the margin. The assumed constant basal temperature, 773°K, is a minimum but its choice does not greatly affect the T' -distribution because of the method of computation. The possible thermal effects of subsidence and sedimentation and the associated thermoelastic stresses (eg. Haxby and Turcotte 1976) have not been considered.

5. Results and Discussion

5.1 Results of Applied Loads

5.1.1 Effects of self-gravitation. The case for which the load is implicit in the standard boundary conditions (section 4.3.1) has a resultant equilibrium displacement pattern due to the effect of gravitational forces acting within the non-uniformly shaped structure. The vertical displacement for the model top and base is shown in Figure 5. The basal displacement is near zero everywhere, ranging from +3.7 m under the shelf edge to -9.4 m under the continental rise, and is virtually negligible relative to surface subsidence which is -175 m at the continentward edge decreasing almost linearly to -92.7 at the other end. Basal subsidence essentially corresponds to the zone of deficient boundary forces which implies that the "dynamic" basal boundary condition (section 4.2.1) may have been more suitable. Other boundary effects are also evident: notable is that the surface displacement does not level off as it gets farther from the margin. The vertical strain associated with these displacements is of the order 10^{-3} , a value higher than usually observed for tectonic strain and indicative of the drawbacks of an elastic model. Horizontal strains are closer to realistic values; the maximum horizontal displacement is +7.8 m at the base of the crust beneath the continental slope. Although the displacement data cannot be interpreted literally, the pattern

Figure 5. Vertical displacement for gravitational load case:
(A) surficial displacement, (B) basal displacement.

of the horizontal displacements is shown in Figure 6 in order to physically illustrate the effect of the spreading stress: oceanward motion is concentrated near the base of the continental crust while a return "flow" is observed near the surface beneath the shelf.

The resultant principal stresses are, as would be intuitively expected, nearly vertical and horizontal everywhere (Figure 7). The vertical stress is essentially the overburden pressure, and ranges from 8.8 to 1131 MNm⁻² with increasing depth, and the horizontal stress is a function of the overburden pressure and Poisson's ratio. By convention, the vertical stress is referred to as σ_z , the horizontal as σ_x , and the stress perpendicular to the plane of the model (assuming plane strain) as σ_y , which is known to be

$$\sigma_y = \nu(\sigma_x + \sigma_z) \quad (5-1)$$

where ν is Poisson's ratio. In order to isolate the effects of the topography the computed stresses are compared to the theoretical elemental stresses, S_z and S_x , calculated by

$$S_z = F_l \Delta \quad (5-2i)$$

and

$$S_x = \frac{\nu}{1-\nu} S_z \quad (5-2ii)$$

Figure 6. Horizontal nodal displacements for gravitational load case. Displacements range from -6.5 to 7.8 m.

Figure 7. Principal stress distribution and R_0 for gravitational load case. Where principal stress axes differ more than 2.5° from x, z-axes they are circled. Axis length is proportional to magnitude of stress which ranges from 8.8 to 1131 MNm⁻².

where F_l^{Δ} is the overburden, or lithostatic pressure at the elemental centroid. Superimposed on the plot of the principal stresses are contours representing the ratio

$$R_G = \frac{\sigma_z/\sigma_x}{s_z/s_x} = \frac{\sigma_z/\sigma_x}{(1-\nu)/\nu} \quad (5-3).$$

R_G greater than unity implies that modifications to the theoretical stress distribution induced by the topography are favourable to horizontal tension; $R_G < 1.0$ implies a trend toward compression. Where the trajectory of σ_z deviates from the vertical by more than 2.5° it has been noted. The maximum such deviation is 4.9° which occurs in the crust below the lower continental slope. The more complex pattern in this region is related to the error incurred in the estimate of R_G as the angular deviation increases. Wherever $R_G < 1.0$ it is implicit that the minimum horizontal stress is σ_y because of equation (5-1); however, it follows that R_G calculated on the basis of σ_z/σ_y remains less than one.

The elemental shear stress,

$$\tau = \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \quad (5-4),$$

is obviously due in large part to the lithostatic pressure as well and ranges from 8.8 to 404 MNm⁻² increasing with depth. A theoretical shear stress, T , can be predicted knowing the maximum and minimum principal stresses by equations (5-2) and the ratio

$$R_T = \frac{T}{T} \quad (5-5)$$

Figure 8. Principal stress distribution and R_r for gravitational load case. Where principal stress axes differ more than 2.5° from x, z-axes they are circled. Axis length is proportional to magnitude of stress which ranges from 2.5 to 1431 MN/m².

is plotted in Figure 8. The computed shear stresses of adjacent elements tend to straddle the true stress curve (Zienkiewicz 1971); R_T has therefore been calculated for quadrilaterals comprising two elements rather than for each triangular element.

5.1.2 Regional horizontal stress field. In this case the spreading stress of the standard load case is compensated by horizontal forces applied to the oceanic end of the model (section 4.3.1). The stress distribution (Figure 9) is not altered substantially, the principal stresses are nearly vertical and horizontal everywhere and all elements are in a state of deviatoric horizontal tension. R_Q is plotted as before and reveals a negligible modification of the pattern in the continental crust relative to that for the gravitational load (Figure 7). The main feature is the deflection of the $R_Q < 0.9$ zone in the crust beneath the rise into the mantle beneath the slope. The small values shown for the oceanward, upper part of the model are obvious effects of the applied forces.

5.1.3 Vertical loading. The surficial and basal vertical displacements due to the load of prograded sedimentation (section 4.3.2) are illustrated in Figure 10. Also shown are the displacement curves if those of the gravitational load (Figure 5) are subtracted; the result is that the curves nearly coincide with the greatest deflection occurring below the centre of gravity of the

Figure 9. Principal stress distribution and R_0 for balanced spreading stress load case. Where principal stress axes differ more than 2.5° from x, z-axes they are circled. Axis length, is proportional to magnitude of stress which ranges from 8.7 to 1122 MNm $^{-2}$.

Figure 10. Concentrated load vectors and vertical displacement for prograded shelf load case: (A) surficial displacement, (B) basal displacement. If displacement curves for the gravitational load case (Figure 5) are subtracted, net surficial and basal vertical displacement for the prograded shelf case correspond to curve (B) except where shown by the dashed line.

distributed load. (The concentrated nodal forces, statically equivalent to the distributed load, are also shown in the diagram.) The maximum difference in the curves, 6 mm (just over 1% of the total), occurs where the sediment thickness is greatest, not where subsidence is greatest, and appears to be related to the higher Poisson's ratio assumed for the sedimentary layer, though may also be related to the isostatic mass deficiency maximized at the same point.

The principal stress distribution produced by the vertical load includes a significant region of compressive horizontal deviatoric stress, as would be intuitively expected, near the top of the model below the load. The deviatoric stresses are shown in Figure 11. The third deviatoric stress, σ'_y , is implicitly tensile everywhere because of equation (5-1) and, wherever the condition

$$\frac{2-\nu}{1+\nu} \sigma_z > \sigma_x > \frac{1+\nu}{2-\nu} \sigma_z \quad (5-6)$$

exists, both σ'_z and σ'_x are compressive. The region in which the greatest compressive deviatoric stress is sub-horizontal (ie., it is σ'_x) is indicated on the plot. In this region there is a relative concentration of shear stress with absolute values, compared to the standard gravitational load, increasing from 69 MNm^{-2} to 75 MNm^{-2} at the indicated point. Poisson's ratio, because of

Figure 11. Deviatoric principal stress distribution for prograded shelf load case. Axis length is proportional to magnitude of stress which ranges from 0.01 to 551 MNm². Tensile stress is indicated by '+' which refers to the axis one end of which is opposite the base of the 'T'. Zones where the greatest compressive stress is σ_x (sub-horizontal) are bracketed. Position of greatest elemental shear stress is marked by square.

its control on the relation (5-6), inhibits the formation of horizontal compressive deviatoric stress in much of the sedimentary layer.

5.1.4 Thermal contrast. Because of the schematic nature of the calculated T distribution, the uncertainty in the choice of thermal expansion coefficients, and the assumption that they remain constant over large regions, the results of the thermal load case are to be taken as a rough approximation only. The "initial" thermal strains are introduced into the model as negative nodal forces in the non-continental part of the margin; the largest resultant modifications to the computed stress pattern occur near the surface mainly beneath the middle and lower slope and upper continental rise where absolute horizontal tension is observed (Figure 12). The orientation of the principal stresses remains essentially horizontal and vertical throughout, an indication of the preponderance of the body forces which are large compared to the thermal load. In the oceanic crust the orientation begins to rotate slightly towards a horizontal greatest compression state. Superimposed on the stress distribution is the shear stress ratio, R_T , calculated as before by equation (5-5) but with T being the value observed for the gravitational load case. R_T depends on an increase in vertical compression relative to horizontal compression; a significant stress concentration in the vicinity of the lower slope and upper rise results.

Figure 12. Principal stress distribution and R for thermal contrast load case. Where principal stress axes differ more than 2.5° from x, z-axes they are circled. Axis length is proportional to magnitude of stress which ranges from 0.1 to 1128 MNm⁻². Tensile stress is indicated by 'T' which refers to the axis one end of which is opposite the base of the 'T'.

5.1.5 Alternate boundary conditions. The imposition of displacement boundary conditions, by fixing all the edge nodes of the model and releasing the basal centre node, does not greatly affect the results of the differential gravitational load case. Absolute displacements are modified only in the vicinity of the fixed boundary; the relative vertical strain across the margin remains essentially the same as for the standard boundary conditions and the resultant principal stresses, resolved in terms of R_0 , show the same general patterns (Figure 13). (Nevertheless, regions in which the stresses are adversely affected by the fixed boundary nodes are clearly present thus supporting the preference for force boundary conditions.)

When the margin is vertically loaded by the prograded continental shelf more differences are evident. Although the relative vertical strains remain quite similar, the horizontal displacement pattern is sensitive to the boundary conditions. The flexure induced by the load causes the ends of the model to rotate slightly about the basal corner nodes for the standard case. The exclusion of rotation by fixing the vertical boundary nodes results in the development of tensile horizontal stresses at the surface away from the load. The implication is that the true wavelength of the induced flexure is longer than the model and the changed boundary conditions are not directly responsible for the discrepancy in results; rather, the physical inadequacy of the model appears to increase its sensitivity to boundary conditions.

Figure 13- Principal stress distribution and R_0 for gravitational load case with fixed edge boundary conditions. Where principal stress axes differ more than 2.5° from x, z-axes they are circled. Axis length is proportional to magnitude of stress which ranges from 0.3 to 1088 MNm⁻². Tensile stress is indicated by '+' which refers to the axis one end of which is opposite the base of the 'T'.

5.2 Implications and Constraints Provided by Results

5.2.1 Local stress concentrations. Several localized but notable stress concentrations are evident in Figure 7. Anomalously high values of R_0 beneath the continental shelf at shallow depth and near the continentward edge of the model are a consequence of a deficiency in the proximal boundary forces. The horizontal displacement of the edge in this area reaches almost -6.5 m (Figure 6) and represents the severest violation of the vertical boundary condition of zero lateral movement (Section 4.2.3). These displacements are physically related to the spreading stress in the continental crust of the model but they imply the necessary existence of greater compensating boundary forces. Such forces, which would be over and above those due to the horizontal component of the overburden pressure, can be interpreted to be a result of a spreading stress associated with the topography of the continental landmass.

Other local anomalies are related to the crustal transition zone which has a higher Young's modulus than either continental or oceanic crust in the model (Table 1). Although this fact results in greater elemental stiffness in the transition zone, it is exhibited here as a relative increase in R_0 because of the positive correlation of stress to strain due to Young's modulus (section A2.3.3). This suggests that the increase may be a consequence of the large element sizes in the model: every Element

in the crustal transition zone possesses at least one node on a layer interface. Furthermore, the anomalies are observed at the normal/transitional crust boundaries which are modelled as distinct interfaces but which are more likely to be gradational in nature (eg. Keen et al. 1975). Minor effects such as those discussed here will not be further considered.

5.2.2 Deviatoric tension and spreading stress. A study of fault plane solutions for predominantly shallow intraplate earthquakes shows that, in general, lithosphere plate interiors are in a state of horizontal deviatoric compression, particularly in oceanic crust older than 10-20 my (Sykes and Sbar 1973). Although there are several examples of earthquakes due to normal faulting on the continental shelves of aseismic margins, the observed seismicity is generally incompatible with the results of the present model: the principal stress distribution effected by the differential topographic load under standard boundary conditions is one of horizontal deviatoric tension everywhere. Similarly incompatible are in situ stress measurements in the upper continental crust. Ranalli and Chandler (1975) report that, except for sedimentary cover rocks, horizontal stresses are usually in excess of the lithostatic pressure.

The nature of the computed stress distribution (Figure 7) is, of course, the result of elastic response to the predominant load

of the body forces. The end boundary forces, themselves a function of Poisson's ratio and the lithostatic pressure, are not directly responsible for the magnitude of the computed horizontal principal stresses since a similar stress pattern is observed for the boundary condition of fixed edge nodes (Figure 13) where there are no (nor could there be) horizontal applied forces. The effect of the differential gravitational load across the margin is, however, discernible by consideration of R_0 which measures the excess or deficiency in the vertical and horizontal principal stresses. The result (Figure 7) shows that the topography is favourable to a decrease of horizontal compression in continental crust, particularly near the surface, and to an increase of horizontal compression in the more oceanward continental rise, oceanic crust, and mantle. However, the changes are small compared to the effect of the body forces. Bott and Dean (1972), using an elastic finite element model, were able to produce a stress field with actual deviatoric tension on the continental side of the margin and deviatoric compression on the oceanic side. However, they neglected absolute body forces and considered only the lateral density contrast in their model. Such an approach implicitly assumes that initial stresses due to gravitational forces have been relaxed to a hydrostatic state. The implications of this assumption are discussed in section 5.2.4.

5.2.3 Non-vanishing yield strength in the upper lithosphere.

In situ stress measurements show the presence of significant remanent

stresses suggesting that rocks of the upper crust, at least, possess a non-vanishing yield strength in the geological time scale (Ranalli and Chandler 1975). The existence of large wavelength gravity anomalies in ancient cratonic interiors has been similarly interpreted (Jeffreys 1976). If it is accepted that the strength of the crust in the uppermost few kilometres of the present model is such that the stress differences due to gravity are sustained indefinitely, then the tectonic role of the differential load at a continental margin is negligible. (It is perhaps noteworthy in this context that the deflection produced by the load of the prograded shelf (Figure 10) is very similar to that expected for a uniform thin elastic plate in that corresponding surface and basal vertical displacements are nearly equal everywhere despite the presence of the lateral inhomogeneities. It may be that this is more a consequence of the assumption of a "fluid" substratum (section 4.2.1) than it is an indication of the minimal effect of the lateral inhomogeneities.)

~~There is~~ some, although sparse, seismic evidence supporting the present results which show horizontal deviatoric tension in the continental crust beneath the shelf and slope (Sykes and Sbar 1973). Recently, Basham et al. (1977) have reported a compatible fault plane solution for the June 1975 Beaufort Sea earthquake, the focus of which was in the lower crust beneath the shelf edge.

However, the observed seismicity strongly suggests a state of deviatoric horizontal compression in oceanic crust older than 10-20 my. This precludes an interpretation of the present results based solely on elasticity, unless the assumption is made that large compressive forces are being applied to the lithosphere plate at the mid-oceanic ridge. A maximal estimate of the forces associated with the spreading stress of the ridge topography (section 4.3.1) is clearly inadequate for this purpose (section 5.1.2). The physical processes required to produce sufficient forces are unclear; nevertheless, their implied necessity would have some bearing on plate driving mechanisms.

5.2.4 Pre-existing hydrostatic stress state. The question of whether or not the rocks of the upper crust possess a finite yield strength is a contentious one. ~~In situ stress measurements~~ (discussed in the previous section) refer only to the very topmost crust and are, moreover, susceptible to numerous modifications making their interpretation difficult. Similarly, large wavelength gravity anomalies in continental interiors (cf. Khan 1977) are possibly the result of dynamic processes occurring in the asthenosphere. The results of the present analysis may, therefore, be interpreted by assuming that the pre-existing elastostatic stress differences due to gravity have vanished through time as a result of crustal flow (cf. Collette 1976). This would effectively remove the apparent incompatibility of the observed seismicity in the oceanic crust.

Stresses in the continental block existing before rifting and ocean basin formation may have been near-hydrostatic if non-elastic processes in the lithosphere have allowed stress relaxation in the time-span involved. The age of the aseismic continental margin is one order of magnitude less than the pre-existing continent. The younger oceanic crust may respond by flow to the body forces during an early ductile stage. Therefore the modifications to the stress field induced by the topography and lateral inhomogeneities at the margin may be considered as superimposed on a hydrostatic state. This assumption is implicit in the approach of Bott and Dean (1972) who computed a maximum shear stress of 37 MNm^{-2} at 10 km depth in the continental crust. For the present results the absolute difference in predicted and computed shear stress,

$$\Delta T = T - T_p \quad (5-7)$$

is plotted in Figure 14. (It should be noted that $\Delta T < 0.0$ does not represent a net deficiency in shear stress relative to the predicted value.) Rather $|\Delta T|$ is the excess shear caused by modifications to the standard configuration; the sense of the modification is such that horizontal compression is favoured. The pattern is relatively more complex than that of Bott and Dean (because of the introduction of a more complicated geometry and distribution of elastic parameters) and the magnitudes of ΔT in

Figure 14. Principal stress distribution and ΔT for gravitational load case. Where principal stress axes differ more than 2.5° from x, z-axes they are circled. Axis length is proportional to magnitude of stress which ranges from 8.8 to 1131 MNm⁻². ΔT is given in MNm⁻². Position of vertical section of Figure 15 is marked by the arrow.

Figure 15. Variation in $\Delta\tau$ with depth for the vertical section marked by the arrow in Figure 14. Analogous results from Bott and Dean (1972) are shown by the dashed curve.

the analogous region are smaller by an order of magnitude. Figure 15 shows the relative variation of shear stress with depth for the model of Bott and Dean and for the present model through a vertical section of the continental crust (marked by the arrow in Figure 14). Bott and Dean used displacement boundary conditions and the discrepancy in the results seems to be most likely due to this difference in choice of boundary conditions compared to the present analysis. The absolute magnitudes of shear stresses due to topography, therefore, remain somewhat questionable but are almost certainly no greater in magnitude than 10 MNm^{-2} . What is most important is that the assumption of initial hydrostatic stresses allows the conclusion to be made that the differential load at an aseismic continental margin does result in horizontal deviatoric tension in the continental crust and deviatoric compression elsewhere thereby reconciling the results of the present analysis with seismic observations.

The results can be construed as an argument in favour of stress relaxation in the lithosphere as a whole in the time scale of 10^3 my; however, it can be argued that elastic stresses are effectively maintained in the upper lithosphere over periods of 10^2 my and less.

In retrospect, it appears that the consideration of the total gravitational effects in an elastostatic analysis is not necessary because of the inadequacy of the rheological model. Nevertheless, it has been shown that a meaningful interpretation of the results

of this approach can be made in the light of reasonable physical processes. The more direct approach of Bott and Dean (1972) requires assumptions which limit the choice of boundary conditions and, potentially, the amount of information provided by the solution.

5.2.5 Incipient subduction. The geometry of the continental margin redistributes the stresses associated with the sedimentary load in such a way that the zone of deviatoric compression due to near-surface downward flexure is mostly oceanward of the centre of gravity of the load (Figure 11). The position of the maximum shear stress associated with this zone, 75 MNm^{-2} , has been similarly displaced. The effects of the sedimentary load are superimposed on the results of the elastostatic gravity-induced stresses which may be assumed to be modifying a hydrostatic stress state (section 5.2.4). In general, the magnitudes of the shear stresses for each case are comparable. However, where the maximum compression is sub-horizontal as a result of the sedimentary load the principal axes have been transposed relative to the gravitational load; consequently a zone of shear stress concentration is present and the computed magnitudes are representative of the effect of the prograded shelf only. Similarly, the major result of the thermal load case is the concentration of shear stress, relative to the stresses induced by the body forces, in the same vicinity (Figure 13). It is interesting to note that

the tensile stresses produced along the continental slope by the thermal contrast coincide with deviatoric compressive stresses associated with the sedimentary load.

Compressive forces applied at the oceanward end of the model are seen to minimally affect the stress distribution in the continental crustal block; they do, however, modify the stress distribution on the oceanic side of the model. This is a consequence of the geometry of the margin and the broad lateral variation in elastic properties. In nature, it is not unreasonable to assume that there are such horizontal compressive forces acting on the oceanic lithosphere of a spreading sea. The assumption is supported by the observed seismicity and is implicit in the topography of mid-ocean ridges and postulated plate driving mechanisms. The forces applied in the present analysis, inferred from spreading stress considerations (section 4.3.1), are small compared to those which might reasonably be associated with plate driving mechanisms. The results, which are modified by the topography of the margin, show a deflection of the horizontal compression-favourable region beneath the shelf-rise break from the crust into the subcontinental mantle. The effect of the hypothesized thermal contrast and rapid shelf progradation is mainly the concentration of shear stress in the upper crust and sedimentary complex beneath the lower continental slope and the continental rise. It is generally accepted that, during the course of the geotectonic cycle, the lithosphere

in the vicinity of an aseismic continental margin fails thereby initiating plate subduction. If it can be assumed that the controlling stresses are intrinsic to the tectonic environment, then the results of the present analysis clearly suggest that the most likely locus of failure is at the continental rise and that subduction would involve lithosphere very close to the continental crust.

Many authors have speculated about the evolution of Atlantic-type margins into those of Pacific-type. Results presented here show that there is quantitative support to this general scheme but that, in detail, there is a major incompatibility. In many tectogeologic reconstructions (eg. Dewey and Bird 1970) subduction is assumed to occur much farther oceanward than suggested here. A major stress concentration, the source of which is not readily apparent (but which could exist in the lower lithosphere), would be required in the appropriate region. On the other hand, the loads examined in this analysis are considered to be essentially intrinsic to the geotectonic cycle. Since the resultant stress distribution is broadly conformable with the proposed scheme of development, it may reasonably be concluded that the loads examined here are important factors in the evolution of aseismic continental margins.

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Appendices

A1. Summary of Symbols and Matrices

α	volume coefficient of linear thermal expansion
$\{\beta\}$	constants of elemental displacement polynomials
γ	angle between a boundary and the x-axis
$\{\delta\}, \{\delta\}^e$	structural, elemental nodal displacements
$d\{\delta\}^e$	set of arbitrary virtual nodal displacements
δ_{ij}	Kronecker delta (index notation)
Δ	elemental area (except when used as a superscript)
$\{\epsilon\}$	elemental strain tensor
$\{\epsilon_0\}$	elemental initial (eg. thermal) strain tensor
ϵ_{ij}	strain tensor (index notation)
θ	cubical dilatation
λ, μ	Lamé's coefficients
ν	Poisson's ratio
ρ	density
$\{\sigma\}$	elemental stress tensor
$\{\sigma_0\}$	elemental initial stress tensor
σ	normal stress
σ_{ij}	stress tensor (index notation)
$\sigma_x, \sigma_y, \sigma_z$	elemental principal stresses closest to x, y, z-axes
τ	shear stress or elemental maximum shear stress

- $\Delta\tau$ difference between desired and a reference elemental shear stress
- ϕ angle between principal stress axis and x-axis
- a, b, c (with nodal subscripts) elemental geometry parameters
- $[A]$ nodal coordinate matrix for one component of elemental displacement
- $[A]_0$ above expanded to accommodate two components of elemental displacement
- $[B]$ matrix relating elemental strain to nodal displacements
- $[B]_0$ above expanded to accommodate all structural elements
- d, d_i equilibrium vertical displacement of an elastic plate, at node i
- $[D]$ matrix relating elemental stress to elemental strain
- $[E]$ matrix relating elemental strain to elemental displacement
- E Young's modulus
- $\{F\}, \{F\}^e$ structural, elemental nodal forces
- $\{F\}_{\epsilon_0}^e, \{F\}_{\sigma_0}^e, \{F\}_p^e$ components of $\{F\}^e$ due to initial strains, initial stresses, body forces
- $\{F\}_a, \{F\}_\ell, \{F\}_R$ components of $\{F\}$ due to "Airy" buoyancy force, lithostatic pressure, external applied forces
- $F_a^\Delta, F_b^\Delta, F_h^\Delta, F_\ell^\Delta$ "Airy" buoyancy force per unit area (pressure), boundary force per unit area, horizontal component of lithostatic pressure, lithostatic pressure
- $F_a^i, F_b^i, F_h^i, F_\ell^i$ static equivalents of above (force) acting at node i
- F^x, F^y (with nodal subscripts) x, y components of elemental nodal force
- g gravitational acceleration
- h vertical thickness
- $[K], [K]^e$ structural, elemental stiffness matrices
- K_{rs}, k_{rs}^e submatrices of above relating nodes r and s

$[L]$	isostatic stiffness matrix
$[N]$	matrix of elemental shape functions
$[N]_0$	above expanded to accommodate all structural elements
$[N]_1$	one-dimensional contraction of $[N]$
$\{p\}$	elemental distributed load
P	(with elemental subscript) midpoint of boundary edge of boundary element
R_σ, R_τ	ratio of elemental principal stress, elemental shear stress to reference values
S_x, S_z	elemental principal stress reference values
T	elemental shear stress reference value
T	elemental initial temperature change
$[]^T$	transpose matrix
u, v	x, y components of elemental displacement; (with nodal subscripts) x, y components of elemental nodal displacements
V_p	compressive (P) wave velocity
V_s	shear (S) wave velocity
w_e, w_i	elemental external, internal work done by $d\{\delta\}^e$
w_i^V	internal work per unit volume
x, y	plane of generalized finite element formulation
x, z	plane of continental margin model

A2. Computer Programs

A2.1 Main Programs

All computer programs were written in FORTRAN IV compatible with the Carleton University Computing Centre compiler (version F00). Program INFIN performs the essential matrix operations of the finite element method for two-dimensional elastic continua, as described in Chapter 2, as well as handling all requisite input and output. Other programs were written to perform specific tasks pertaining to the preparation of input and the manipulation of the output data of INFIN. The former category includes, for example, programs to generate the element connections for the finite element mesh, to check for error in the mesh digitization, and to compute appropriate nodal boundary forces. Programs to deal with the output of INFIN include those which compute deviatoric stresses, add and subtract stress tensors, calculate R_0 ratios, and, most importantly, plot the principal stress distribution. The latter was adapted from a program given by Zienkiewicz (1971).

A documentation of program INFIN follows.

A2.2 Program INFIN: Input/Output

A2.2.1 Formatted input. Data defining the finite element mesh are input through unit 4 as follows:

1. One record (20A4):

TITLE...name of finite element mesh

2. One record for mesh parameters (5I5):

NNODE...number of nodes

NELEM...number of elements

NTYPE(1)...number of elastic zones

NTYPE(2)...number of thermal zones

MBW...semi-bandwidth (If MBW = 0, it is calculated from the raw mesh data.)

3. One record for each node (NNODE records; M = 1, NNODE)
(5X, 2F10.0):

COORD(M,1)...x-coordinate of node M in km

COORD(M,2)...z-coordinate of node M in km

4. One record for each element (NELEM records; M = 1, NELEM)
(5X, 5I5, 2F10.0, 4X, 1I):

NOD(M,1)...smallest node number in element M

NOD(M,2)...next anti-clockwise node of element M

NOD(M,3)...next anti-clockwise node of element M

LTYPE(M,1)...elastic zone of element M

LTYPE(M,2)...thermal zone of element M

RHO...density of element M (written onto unit 11)

TEMP...initial temperature change of element M (written onto unit 11) in °K

ICOL(K)...if non-zero, element M is a basal element of column K; ICOL(K) is overprinted with M; K is incremented so that the total number of columns is NCOL.

Job and load case data are input through unit 5 as follows:

1. One record (I3):

JOBS...number of load cases to be solved; all the following records are repeated, as a group, JOBS times.

2. One record (20A4):

TITLE...name of load case

3. One record for load case options (1611):

IPRINT(1)...if non-zero, nodal mesh data output to unit 6.
 IPRINT(2)...if non-zero, element mesh data output to unit 6.
 IPRINT(3)...if non-zero, elastic load case mesh data output to unit 6.
 IPRINT(4)...if non-zero, thermal load case mesh data output to unit 6.
 IPLANE...if non-zero, plane stress is considered; if zero, plane strain is considered.
 IISOS...if non-zero, subroutine AIRY is called; adds a displacement dependent isostatic term to the system stiffness.
 IBODY...if non-zero, body forces due to gravity are included in the load case.
 ITHERM...if non-zero, initial strains due to thermal dilatation/contraction are included in the load case.
 ISTRES...if non-zero, initial stress, assumed constant within elements, are included in the load case. (The initial stress distribution would be generated by an immediately prior run.)
 IPN(1)...if non-zero displacements solution is output to unit 7. (By default it is output to unit 6.)
 IPN(2)...if non-zero, principal stress solution is output to unit 1F2. (By default it is output to unit 6.)
 IPN(3)...if non-zero, stress tensor solution is output to unit 1F3. (By default it is not output.)
 ITENS...if non-zero, stress tensor solution is output to unit 6.
 ID...if non-zero, input of elastic data is suppressed; elastic data of previous load case is saved. (If non-zero on load case 1, ID is ignored.)
 IS...if non-zero, input of isostatic data is suppressed; isostatic term of previous load case is saved. (If non-zero on load case 1 or if IISOS = 0, IS is ignored.)
 ISAVE...if non-zero, stress tensor solution is saved as initial stress distribution input to immediately following load case, where ISTRES must be non-zero.

4. One record for each node with a boundary condition (NDISP nodes, NDISP calculated from input) plus one terminating record (15, 12, 8X, 2F10.0, 2E15.0):

N...boundary node number (stored permanently in array IDISP(K); K < NDISP); N > NNODE indicates termination of boundary condition input.

IFIX(K)...if zero, displacement of N is fixed; if 1, x-displacement of N is fixed; if 2, z-displacement of N is fixed.

DISP(K,1)...x-displacement of N in metres (If IFIX(K) $\neq 0$, $\neq 1$; DISP(K,1) is ignored.)
 DISP(K,2)...z-displacement of N in metres (If IFIX(K) $\neq 0$, $\neq 2$; DISP(K,2) is ignored.)
 FORCE(1,N)...x-applied force at N in dynes
 FORCE(2,N)...z-applied force at N in dynes

5. One record for each elastic zone (NTYPE(1) records; M = 1, NTYPE(1)); skipped if load case > 1, ID $\neq 0$ (5X, E10.0, F10.0):

ELACON(M,1)...Young's modulus of zone M in dyn cm^{-2}
 ELACON(M,2)...Poisson's ratio of zone M

6. One record for each thermal zone (NTYPE(2) records; M = 1, NTYPE(2)); skipped if I THERM = 0 (5X, E15.0):

THERM (M)...linear coefficient of volumetric thermal expansion of zone M in $^{\circ}\text{K}^{-1}$

7. One record for isostatic data, skipped if IISOS = 0; skipped if load case > 1, IS $\neq 0$ (2F10.0):

YBASE...y-coordinate of model base in kilometres
 RHOSUB...density of submodel material in g cm^{-3}

8. One record for output file information; must be included (2IS):

IF2...output unit of principal stress solution if IPN (2) $\neq 0$.
 IF3...output unit of stress tensor solution if IPN(3) $\neq 0$.

A2.2.2 Binary Input/Output. Although the use of temporary

binary files increases execution time, they are used here to reduce the required central memory for execution. Unit 11, in particular, was introduced as a late modification, to specifically allow INFIN, as it is written and dimensioned, to be executed with a memory core not exceeding the normal limit established by the Carleton Computing Centre.

* Input, calculations, and output of INFIN are performed in cgs units.

The stress tensor solution is written to unit 8 in subroutine TENSr and is read, depending on options ISTRES, IPN(3), ITEMS, and ISAVE in subroutines BFORSE and TENSr. Thermal (initial) strains are calculated and written to unit 9 in BFORSE and are read in TENSr, depending on option ITHERM. Elemental areas and geometry matrices [B] are calculated and written to unit 10 in BFORSE and are read in subroutines ASSTIF and TENSr. Elemental densities and temperatures are input through unit 4 and written to unit 11 by the main program and are read in subroutines JOBPAr and BFORSE.

A2.3 Program INFIN: Theory

A2.3.1 Main program unit. The main program unit inputs the digitized model and calls subroutines in a loop controlled by JOBS, the number of load cases. Information transferral is by a blank common block and common blocks A, D, S. Parameters JOB, the load case, ID, IS, and ERROR, the error flag, are used as arguments to various subroutines.

A2.3.2 Subroutine JOBPAr. Subroutine JOBPAr inputs the load case options and boundary conditions, including applied forces; optionally outputs the same. Included in the force term of the structural stiffness equation (2-1) are external applied forces $\{F\}_R$. For geologic models these are usually a result of a distributed boundary load and if $F_b^A(l)$ is a distributed load

(force per unit area; recalling unit thickness for the model)

acting perpendicular to the boundary, ℓ , then the static equivalent concentrated force at boundary node i is

$$F_b^i = \int_{P_k}^{P_n} F_b^\Delta(\ell) d\ell \quad (A2-1)$$

where P_k and P_n are the midpoints of adjacent boundary elements k and n , both containing node i . (P_k or P_n may correspond with i if i is at a corner of the mesh.) If ℓ is at angle γ_k with x along element k and γ_n with x along n then the x, y components of the boundary force are given by

$$F_x^i = \sin \gamma_k \int_{P_k}^{P_i} F_b^\Delta(\ell) d\ell + \sin \gamma_n \int_{P_i}^{P_n} F_b^\Delta(\ell) d\ell \quad (A2-2i)$$

and

$$F_y^i = \cos \gamma_k \int_{P_k}^{P_i} F_b^\Delta(\ell) d\ell + \cos \gamma_n \int_{P_i}^{P_n} F_b^\Delta(\ell) d\ell \quad (A2-2ii)$$

where P_i is at node i .

A2.3.3 Subroutine GEND. Subroutine GEND computes the constitutive matrix $[D]$. In general, stress is related to strain, in matrix notation*, by

$$\{\sigma\} = [D]\{\epsilon\} - \{\epsilon_0\} + \{\sigma_0\} \quad (2-6)$$

* Notation will change depending on convenience and context.

Ignoring, for the present, the terms for initial stress and strain, $\{\sigma_0\}$ and $\{\epsilon_0\}$, for the case of pure elastic behaviour in an isotropic medium, equation (2-6), in index notation, becomes

$$\sigma_{ij} = \lambda \delta_{ij} + 2\mu \epsilon_{ij} \quad (A2-3i)$$

where δ_{ij} is the Kronecker tensor, λ and μ are Lamé's coefficients, and

$$\theta = \epsilon_{ii} = \epsilon_{11} + \epsilon_{22} + \epsilon_{33} \quad (A2-3ii).$$

The stress and strain matrices of equation (2-6) are of the form:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} \quad (A2-4i)$$

$$\{\epsilon\} = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ 2\epsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{Bmatrix} \quad (A2-4ii).$$

In two-dimensional analysis the special cases of plane strain and plane stress are encountered. Plane strain is defined where displacement in the third dimension, u_3 , is zero. For the strain tensor, $\epsilon_{ij} = 0$; $i = 3$ or $j = 3$ and the stress components of (A2-4i) are given directly by (A2-3i):

$$\sigma_{11} = (\lambda + 2\mu)\epsilon_{11} + \lambda\epsilon_{22} \quad (A2-5i)$$

$$\sigma_{22} = \lambda \epsilon_{11} + (\lambda + 2\mu) \epsilon_{22} \quad (\text{A2-5ii})$$

$$\sigma_{12} = 2\mu \epsilon_{12} \quad (\text{A2-5iii})$$

$$\sigma_{33} = \lambda (\epsilon_{11} + \epsilon_{22}) \quad (\text{A2-5iv})$$

$$\sigma_{13} = \sigma_{23} = 0 \quad (\text{A2-5v}).$$

Note that σ_{33} can be non-zero but will not be considered further until section A2.3.8. Since

$$2\mu = \frac{E}{1+\nu} \quad (\text{A2-6i})$$

and

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad (\text{A2-6ii}),$$

where E and ν and Young's modulus and Poisson's ratio, equations (A2-5i, ii, iii) become

$$\sigma_{11} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \epsilon_{11} + \frac{E\nu}{(1+\nu)(1-2\nu)} \epsilon_{22} \quad (\text{A2-7i})$$

$$\sigma_{22} = \frac{E\nu}{(1+\nu)(1-2\nu)} \epsilon_{11} + \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \epsilon_{22} \quad (\text{A2-7ii})$$

$$\sigma_{12} = \frac{E}{2(1+\nu)} 2\epsilon_{12} \quad (\text{A2-7iii}).$$

Therefore the coefficient matrix $[D]$ of equation (2-6) for plane strain is given by

$$[D] = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \quad (\text{A2-8}).$$

The condition of plane stress is satisfied where $\sigma_{ij} = 0$; $i = 3$ or $j = 3$. Since $\sigma_{33} = 0$, equation (A2-3i) can be rearranged, using identities (A2-6), to give

$$\epsilon_{33} = -\frac{\nu}{1-\nu} (\epsilon_{11} + \epsilon_{22}) \quad (\text{A2-9});$$

ϵ_{33} is a function of the other strains because of the two dimensional nature of the associated shape functions. Using (A2-9) and (A2-6) the stress components of (A2-4i) are given directly by (A2-3i):

$$\sigma_{11} = \frac{E}{1-\nu^2} \epsilon_{11} + \frac{E\nu}{1-\nu^2} \epsilon_{22} \quad (\text{A2-10i})$$

$$\sigma_{22} = \frac{E\nu}{1-\nu^2} \epsilon_{11} + \frac{E}{1-\nu^2} \epsilon_{22} \quad (\text{A2-10ii})$$

$$\sigma_{12} = \frac{E}{2(1+\nu)} 2\epsilon_{12} \quad (\text{A2-10iii}).$$

In the matrix form of (2-6):

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (\text{A2-11}).$$

Subroutine GEND computes the elements of $[D]$ by (A2-8) or (A2-11), depending on which case is under consideration, for each element of the finite element model having different assigned elastic properties (E and ν).

A2.3.4 Subroutine BFORSE. Subroutine BFORSE computes the geometry matrix [B] for each element and computes the final form of matrix {F} of the stiffness equation. The former is stored temporarily in array B, the latter permanently in FORCE. The totals of the force components are summed and output for information only..

Matrix [B] contains coefficients relating displacement to strain in a given element and its formation rests on the assumption of elemental displacement as a linear function of the nodal coordinates as discussed in general in Chapter 2. Recall equations (2-3):

$$u = \beta_1 + \beta_2 x + \beta_3 y \quad (2-3i)$$

$$v = \beta_4 + \beta_5 x + \beta_6 y \quad (2-3ii)$$

where u and v are the components of displacement within an element (Figure 1) and β_1 -6 are appropriate constants. Equations (2-3), in matrix notation, are

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{bmatrix} \{\beta\} \quad (A2-12i)$$

where

$$\{\beta\} = \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{Bmatrix} \quad (A2-12ii)$$

For any triangular element with nodes i, j, m having coordinates

$x_{i,j,m}$ and $y_{i,j,m}$, the nodal displacements $u_{i,j,m}$ and $v_{i,j,m}$

are known directly from (A2-12i). The resultant equations can be arranged such that:

$$\begin{Bmatrix} u_i \\ u_j \\ u_m \end{Bmatrix} = [A]_u \begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} \quad (\text{A2-13i})$$

and

$$\begin{Bmatrix} v_i \\ v_j \\ v_m \end{Bmatrix} = [A]_v \begin{Bmatrix} \beta_4 \\ \beta_5 \\ \beta_6 \end{Bmatrix} \quad (\text{A2-13ii}).$$

The coefficient matrices of (A2-13) are equivalent:

$$[A]_u = [A]_v = [A] = \begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{bmatrix} \quad (\text{A2-14}).$$

Although implicitly unnecessary later computation is made more efficient if $[A]$ is inverted at this point, solving equations (A2-13) for $\{\beta\}$ in terms of nodal displacements. Using a standard method of matrix inversion (the cofactor method):

$$[A]^{-1} = \frac{1}{\det [A]} \begin{bmatrix} a_i & a_j & a_m \\ b_i & b_j & b_m \\ c_i & c_j & c_m \end{bmatrix} \quad (\text{A2-15i})$$

where (implying cyclic permutation of the subscripts)

$$a_i = x_j y_m - x_m y_j \quad (\text{A2-15ii})$$

$$b_i = y_j - y_m \quad (\text{A2-15iii})$$

$$c_i = x_m - x_j \quad (\text{A2-15iv}).$$

Equations (A2-13) become:

$$\begin{Bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{Bmatrix} = [A]^{-1} \begin{Bmatrix} u_i \\ u_j \\ u_m \end{Bmatrix} \quad (\text{A2-16i})$$

and

$$\begin{Bmatrix} \beta_4 \\ \beta_5 \\ \beta_6 \end{Bmatrix} = [A]^{-1} \begin{Bmatrix} v_i \\ v_j \\ v_m \end{Bmatrix} \quad (\text{A2-16ii}).$$

The partial differential equation relating strain to displacement in a homogeneous, isotropic medium is, in matrix form,

$$\{\epsilon\} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \{u\} \quad (\text{A2-17})$$

where $\{\epsilon\}$ is of the form given by (A2-4ii). Equations (A2-16) are independent and can easily be combined by expansion of $[A]^{-1}$ by interspersal of zero elements:

$$\{\beta\} = [A]_0 \{\delta\}^e \quad (\text{A2-18i})$$

where

$$[A]_0 = \frac{1}{\det[A]} \begin{bmatrix} a_i & 0 & a_j & 0 & a_m & 0 \\ b_i & 0 & b_j & 0 & b_m & 0 \\ c_i & 0 & c_j & 0 & c_m & 0 \\ 0 & a_i & 0 & a_j & 0 & a_m \\ 0 & b_i & 0 & b_j & 0 & b_m \\ 0 & c_i & 0 & c_j & 0 & c_m \end{bmatrix} \quad (\text{A2-18ii})$$

and, as in Chapter 2,

$$\{\delta\}^e = \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_m \\ v_m \end{Bmatrix} \quad (\text{A2-18iii})$$

Equations (A2-12i) and (A2-18i) are inserted into (A2-17) to give the desired form:

$$\{\epsilon\} = [B] \{\delta\}^e \quad (\text{2-5i})$$

where

$$[B] = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{bmatrix} [A]_0$$

$$= \frac{1}{\det[A]} \begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_i & b_i & c_j & b_j & c_m & b_m \end{bmatrix} \quad (\text{A2-19})$$

Matrix [B] is seen to be purely a function of the nodal coordinates and independent of x and y. The determinant of [A] is easily calculated if, in physical terms, the local origin is translated to the coordinates of node i so that

$$[A] = \begin{bmatrix} 1 & 0 & 0 \\ 1 & x_j - x_i & y_j - y_i \\ 1 & x_m - x_i & y_m - y_i \end{bmatrix} \quad (\text{A2-20})$$

and

$$\det[A] = \det[A] = (x_j - x_i)(y_m - y_i) - (x_m - x_i)(y_j - y_i) \quad (\text{A2-21})$$

which is equal to twice the area of the element. Any such translation is arbitrary and does not affect the values of the elements of [B].

To find the final form of {F} it is necessary to compute the individual elemental contributions due to initial strains, initial stress and gravity, recalling that

$$\{F\}^e = \{F\}_{\epsilon_0}^e - \{F\}_{\sigma_0}^e + \{F\}_p^e = [k]^e \{\delta\}^e \quad (2-12i).$$

Forces due to initial strains are

$$\{F\}_{\epsilon_0}^e = \int [B]^T [D] \{\epsilon_0\} d(\text{vol}) \quad (2-12ii).$$

If it is assumed that causes other than temperature changes can be neglected, then the initial strain matrix, $\{\epsilon_0\}$, is easily found. Equation (A2-3i) is replaced by the linear thermoelastic stress-strain relation (eg. Boley and Weiner 1960):

$$\sigma_{ij} = \lambda \delta_{ij} \theta + 2\mu \epsilon_{ij} - \delta_{ij} (3\lambda + 2\mu) \alpha T \quad (A2-22)$$

where α is the coefficient of linear thermal expansion and T is a temperature change. Exactly analogous operations as were made in determining the contents of [D] in section A2.3.3, equations (A2-8) and (A2-11), while not actually modifying [D] in either case, result in

$$\{\epsilon_0\} = \begin{Bmatrix} \epsilon_{0x} \\ \epsilon_{0y} \\ 2\epsilon_{0xy} \end{Bmatrix} = (1+\nu) \begin{Bmatrix} \alpha T \\ \alpha T \\ 0 \end{Bmatrix} \quad (4-5)$$

for the case of plane strain, and

$$\{\epsilon_0\} = \begin{Bmatrix} \alpha T \\ \alpha T \\ 0 \end{Bmatrix} \quad (\text{A2-23})$$

for plane stress. In equation (2-12ii) both [B] and [D] are independent of x and y; further, considering the two-dimensional nature of the analysis, assuming a constant unit thickness in the third dimension, z,

$$\int d(\text{vol}) = \int dx dy dz = \int dx dy = \Delta, \quad (\text{A2-24})$$

where Δ is the area of the element and is known by (A2-21) to be one-half the determinant of [A]. Therefore (2-12ii) becomes

$$\{F\}_{\epsilon_0}^e = [B]^T [D] \{\epsilon_0\} \Delta \quad (\text{A2-25})$$

and can be evaluated directly.

Nodal forces due to initial stress within an element are given by:

$$\{F\}_{\sigma_0}^e = \int [B]^T \{\sigma_0\} d(\text{vol}) \quad (\text{2-12iii}).$$

$\{\sigma_0\}$ must be known and if it is a function of x and y the integration must be carried out explicitly. However, if such stresses are small compared to resultant stresses, they may be assumed to be constant within the element and their evaluation becomes direct, given by

$$\{F\}_{\sigma_0}^e = [B]^T \{\sigma_0\} \Delta \quad (A2-26).$$

Nodal forces due to a distributed body force are given by:

$$\{F\}_P^e = \int [N]^T \{p\} d(\text{vol}) \quad (2-12iv).$$

Recall that $[N]$ is the matrix of shape functions and that its components are linear functions of x and y :

$$\{v\} = [N] \{\delta\}^e \quad (2-2).$$

Therefore the integration in (2-12iv) must be carried out explicitly. The body force of interest is gravity which will be assumed to act in one direction only ($-y$). For simplicity, the analysis can be made unidimensionally. The contents of the suitably contracted form of $[N]$ can be derived from the substitution of (A2-16i) into the appropriate form of (A2-12i):

$$v = \{1 \ x \ y\} [A]^{-1} \begin{Bmatrix} v_i \\ v_j \\ v_m \end{Bmatrix} \quad (A2-27).$$

It is immediately obvious from comparison to equation (2-2) that

$$\begin{aligned} \{N\}_1^T &= \left([A]^{-1} \right)^T \begin{Bmatrix} 1 \\ x \\ y \end{Bmatrix} \\ &= \frac{1}{2\Delta} \begin{Bmatrix} a_i + b_i x + c_i y \\ a_j + b_j x + c_j y \\ a_m + b_m x + c_m y \end{Bmatrix} \end{aligned} \quad (A2-28)$$

with the three resultant values pertaining to nodes i , j , and m respectively. The non-zero (y) component of $\{p\}$ is the gravitational force and is equivalent to $-\rho g$ where ρ is the elemental density and g is the gravitational acceleration, both assumed to be constant within a given element. Considering node i only, and recalling (A2-24), equation (2-12iv) becomes

$$F_p^i = \frac{-\rho g}{2\Delta} \int (a_i + b_i x + c_i y) dx dy \quad (A2-29).$$

If a local origin is assumed to be at the elemental centroid (as before, any such translation of the coordinate origin is arbitrary and does not affect the physical system), then, for a triangular element,

$$\int x dx dy = \int y dx dy = 0 \quad (A2-30).$$

By definition,

$$x_i + x_j + x_m = y_i + y_j + y_m = 0 \quad (A2-31).$$

Substitution of (A2-31) into (A2-15ii) and comparison with (A2-21) reveals that

$$a_i = a_j = a_m = \frac{2\Delta}{3} \quad (A2-32).$$

With these conditions the nodal forces due to gravity can be evaluated directly. Equation (A2-29) becomes

$$\begin{aligned} F_p^i &= \frac{-\rho g}{2\Delta} \left[\int a_i dx dy + \int b_i x dx dy + \int c_i y dx dy \right] \\ &= \frac{1}{3} \rho g \Delta = F_p^j = F_p^m \end{aligned} \quad (A2-33).$$

The nodal forces calculated here by equations (A2-25), (A2-26), and (A2-33) are added into the structural force matrix, $\{F\}$, which also contains the external applied concentrated boundary loads given by (A2-1) in section A2.3.1.

A2.3.5. Subroutine AIRY. Subroutine AIRY calculates the additional stiffness terms due to isostatic considerations. The Airy-Heiskanen isostatic system can be interpreted as an elastic plate "floating" on a substratum. It has been shown in section 4.2.1 that the force per unit area due to buoyancy acting at the base of the plate is

$$F_a^\Delta = F_\ell^\Delta - \rho_m dg \quad (4-2)$$

where ρ_m is the substratum density, d is the equilibrium vertical displacement at the base of the plate, and g is the gravitational acceleration, assumed constant throughout; F_ℓ^Δ , being equivalent to the basal lithostatic pressure, is given by

$$F_\ell^\Delta = g \sum \rho_j h_j \quad (4-1)$$

where ρ_j is the density of layer j having thickness h_j . F_a^Δ represents a distributed boundary load; its static force equivalent acting at basal node i , assuming the base is parallel to the x -axis, is calculated by equation (A2-1) and is

$$F_a^i = F_\ell^i - \int_k^n \rho_m d_i g dx \quad (A2-34)$$

where, as before, P_k and P_n are the x-coordinates of the midpoints of adjacent elements. If the equilibrium displacement at i , d_i , is assumed constant from P_k to P_n then (A2-34) becomes

$$F_a^i = F_\ell^i - d_i \rho_m g (P_k - P_n) = F_\ell^i - d_i L_i \quad (A2-35).$$

Equation (A2-35), expressed in matrix notation, and expanded to have equivalent rank to the complete structural system, is

$$\{F\}_a = \{F\}_\ell - [L] \{\delta\} \quad (A2-36)$$

where $[L]$ is a diagonal matrix and $L_j = 0$ for any non-basal node j . $\{F\}_a$ is added to the stiffness equation (2-1) such that

$$\{F\} + \{F\}_\ell = ([K] + [L]) \{\delta\} \quad (A2-37).$$

$\{F\}_\ell$ and $[L]$ are known exactly. $\{F\}_\ell$ forms part of the general boundary conditions of the problem and is, in general, input by subroutine JOBPART as an intrinsic part of the external load $\{F\}_R$; $[L]$ is computed by subroutine AIRY and is added to the stiffness matrix, $[K]$, during execution of subroutine ASSTIF.

A2.3.6 Subroutine ASSTIF. Subroutine ASSTIF computes elemental stiffness matrices, $[k]^e$, and successively adds them into the structural stiffness matrix $[K]$; adds the isostatic stiffness matrix, $[L]$, to $[K]$; and modifies the resultant system of equations according to the prescribed displacement boundary conditions.

The elemental stiffness matrix is given by

$$[k]^e = \int [B]^T [D] [B] d(\text{vol}) \quad (2-12v)$$

Recalling equation (A2-24) and that matrices $[B]$ and $[D]$ are constant within an element, (2-12v) becomes

$$[k]^e = [B]^T [D] [B] \Delta \quad (A2-38)$$

where, as before, Δ is elemental area; equation (A2-38) gives $[k]^e$ exactly. The elemental stiffness equation,

$$\{F\}^e = [k]^e \{\delta\}^e \quad (2-12i),$$

rewritten for any element q , having nodes i , j , and m , is

$$\begin{Bmatrix} F_i \\ F_j \\ F_m \end{Bmatrix}^q = \begin{bmatrix} k_{ii} & k_{ij} & k_{im} \\ k_{ji} & k_{jj} & k_{jm} \\ k_{mi} & k_{mj} & k_{mm} \end{bmatrix}^q \begin{Bmatrix} \delta_i \\ \delta_j \\ \delta_m \end{Bmatrix}^q \quad (A2-39i)$$

where the displacement term is equivalent to that of (A2-18iii), the force term is analogous, and, in general, the stiffness coefficient k_{rs}^q , for two-dimensional analysis, is a submatrix of the form

$$k_{rs}^q = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix}_{rs}^q \quad (A2-39ii).$$

Submatrix k_{rs}^q relates, for element q , the forces developed at node r due to displacements at node s ; the analogous structural stiffness submatrix K_{rs} is simply the sum of the elemental submatrices which exist for all n elements in the structure:

$$K_{rs} = \sum_{q=1}^n k_{rs}^q \quad (2-15i)$$

If nodes r and s do not both occupy an element q then

$$k_{rs}^q = 0 \quad (2-15ii)$$

It is obvious that K_{rs} will be non-zero if and only if both nodes cooccupy at least one element somewhere in the total structure: a condition which includes the case where r equals s . Furthermore, equations (A2-39) show that $[K]$ is a symmetric matrix; therefore most of its elements can be ignored as being either non-unique or zero. The resultant semi-banded matrix is stored in the unidimensional array STIFF by means of indexing statement function INDX. The semi-bandwidth, MBW, is clearly controlled by the element or elements having the maximum integral difference between any two of its nodes.

As discussed in section 2.3.1 $[K]$ is inherently singular; i.e., its inverse is non-existent. The application of the displacement boundary conditions makes appropriate equations of the system

$$\{F\} = [K] \{d\} \quad (2-1)$$

trivial and allows their removal. Instead of actually removing

the equations, inconvenient for a digital computer because it would require shifting of a large number of memory locations, a technique credited to Payne and Irons (Zienkiewicz 1971) is used: the diagonal coefficient of $[K]$ for the relevant equation is multiplied by a large number, eg. 10^{15} , and the corresponding term in $\{F\}$ is replaced by the product of the prescribed displacement and the same large number; when the modified system of equations is solved the resultant displacement will be very close to its predefined value because of the scalar geometry.

A2.3.7 Subroutine ONESOL. The stiffness equation,

$$\{F\} = [K] \{\delta\} \quad (2-1)$$

is solved for $\{\delta\}$ by the inversion of $[K]$ using a standard method, the Choleski method (see section 2.3.2): consider the general system of equations

$$\{C\} = [A] \{x\} \quad (A2-40i);$$

if $[A]$ is symmetric and can be factored as

$$[A] = [U]^T [U] \quad (A2-40ii)$$

where $[U]$ is an upper triangular matrix, then the elements of $[U]$ can be computed using basic matrix arithmetic:

$$u_{11} = \sqrt{a_{11}} \quad (A2-41i),$$

$$u_{11} = a_{11}/u_{11} \quad (A2-41ii),$$

$$u_{ii} = \sqrt{a_{ii} - \sum_{m=1}^{(i-1)} u_{mi}^2}; \quad i > 1 \quad (A2-41iii),$$

$$u_{ij} = \frac{(a_{ij} - \sum_{m=1}^{(i-1)} u_{mi} u_{mj})}{u_{ii}}; \quad j > i \quad (A2-41iv),$$

and

$$u_{ij} = 0; \quad i > j \quad (A2-41v).$$

The substitution of equation (A2-40ii) into (A2-40i) gives

$$\{\dot{C}\} = [U]^T \{Y\} \quad (A2-42i)$$

where

$$\{Y\} = [U] \{X\} \quad (A2-42ii).$$

and, since the elements of $[U]$ are known by equations (A2-41),

$\{Y\}$ is computed directly from (A2-42i) given that

$$y_1 = c_1 / u_{11} \quad (A2-43i)$$

and

$$y_i = \frac{(c_i - \sum_{m=1}^{(i-1)} u_{mi} y_m)}{u_{ii}}; \quad i > 1 \quad (A2-43ii)$$

thus allowing the analogous solution of (A2-42ii) for $\{X\}$:

$$x_n = \frac{y_n}{u_{nn}} \quad (A2-44i)$$

and

$$x_i = \frac{(y_i - \sum_{m=i+1}^n u_{im} x_m)}{u_{ii}}; \quad i < n \quad (A2-44ii).$$

The elements of $\{Y\}$ and $\{X\}$ successively overwrite those of $\{C\}$ and $\{Y\}$ as they are calculated so that no additional computer storage is required for the displacement solution matrix which is therefore found in array FORCE.

A2.3.8 Subroutine TENSr. Subroutine TENSr computes elemental stress tensors from the displacement solution and successively transforms them to the principal system for output.

Recall the general linear stress-strain relation

$$\{\sigma\} = [D] (\{\epsilon\} - \{\epsilon_0\}) + \{\sigma_0\} \quad (2-6);$$

the expression for elemental stress, applied at the centroid of the triangular element, is given by the substitution of equation (2-5i) into (2-6) giving

$$\{\sigma\} = [D][B]\{\delta\}^e - [D]\{\epsilon_0\} + \{\sigma_0\} \quad (2-7).$$

The initial stress term, $\{\sigma_0\}$, was assumed to be constant within the element and is known exactly; the other terms, stress due to nodal displacements and stress due to initial strain, are computed directly.

The two-dimensional stress tensor, $\{\sigma\}$, of the form given by equation (A2-4i), is transformed for convenience into the

principal system. The normal and shear stresses, σ and τ , are
(Ranalli and Gale 1976, p. 78)

$$\sigma = \sigma_{11} \cos^2 \phi + \sigma_{22} \sin^2 \phi + 2\sigma_{12} \sin \phi \cos \phi \quad (\text{A2-45i})$$

and

$$\tau = -\sigma_{11} \sin \phi \cos \phi - \sigma_{12} \sin^2 \phi + \sigma_{21} \cos^2 \phi + \sigma_{22} \cos \phi \sin \phi \quad (\text{A2-45ii})$$

where ϕ is the angle between the σ_{11} direction and the normal of the plane of computation. Using elementary trigonometric identities, equations (A2-45) reduce to:

$$\sigma = \frac{1}{2}(\sigma_{11} + \sigma_{22}) + \frac{1}{2}(\sigma_{11} - \sigma_{22}) \cos 2\phi + \sigma_{12} \sin 2\phi \quad (\text{A2-46i})$$

and

$$\tau = \sigma_{12} \cos 2\phi + \frac{1}{2}(\sigma_{22} - \sigma_{11}) \sin 2\phi \quad (\text{A2-46ii}).$$

Shear stress vanishes in the principal system; therefore (A2-46iii), equated to zero, gives the orientation of one of the principal axes:

$$\tan 2\phi = \frac{2\sigma_{12}}{\sigma_{11} - \sigma_{22}} \quad (\text{A2-47}).$$

The other axis is orthogonal and it follows that the values of the respective principal stresses are given directly by equation (A2-46i):

$$\sigma_1 = \frac{1}{2}(\sigma_{11} + \sigma_{22}) + \frac{1}{2}(\sigma_{11} - \sigma_{22}) \cos 2\phi + \sigma_{12} \sin 2\phi \quad (\text{A2-48i})$$

and

$$\sigma_2 = \frac{1}{2}(\sigma_{11} + \sigma_{22}) - \frac{1}{2}(\sigma_{11} - \sigma_{22}) \cos 2\phi - \sigma_{12} \sin 2\phi \quad (\text{A2-48ii}).$$

It is obvious from equations (A2-48) that

$$\sigma_2 = \sigma_{11} + \sigma_{22} - \sigma_1 \quad (A2-49).$$

In subroutine TENSUR, equations (A2-47, 45i, 49) are evaluated in the given order.

For the case of plane strain a value for σ_{33} is also required: recall the appropriate stress equations, equations (A2-5). If ϵ_{11} and ϵ_{22} are expressed in terms of stress by the inversion and combination of equations (A2-5i) and (A2-5ii) and then substituted into (A2-5iv) the latter expression becomes, using identities (A2-6), and recalling that $\sigma_{13} = \sigma_{23} = 0$,

$$\sigma_{33} = \nu (\sigma_{11} + \sigma_{22}) = \nu (\sigma_1 + \sigma_2) = \sigma_3 \quad (A2-50).$$

where ν is Poisson's ratio and σ_1 and σ_2 are the previously defined principal stresses.

A2.4 Program INFIN: Listing

```

IMPLICIT DOUBLE PRECISION(A-H,O-Z)
COMMON FORCE(2,350),COORD(350,2),NOD(600,3),ELACON(8,2),D
1(6,3,3),LTYPE(600,2),THERM(8),IPRINT(4),IPN(3),NTYPE(2),IISOS,
2ITHERM,IBODY,ISTRES,IPLANE,ITENS,NNODE,NELEM,MBW,ISAVE
COMMON/A/ STISOS(50),IBASE(50),NBASE
COMMON/D/ DISP(60,2),IFIX(60),IDISP(60),NDISP
COMMON/S/ STIFF(13800)
INTEGER ERROR,ICOL(50),TITLE(20)
C
C INPUT DIGITAL MODEL
C
ERROR=0
MBW=0.
READ(4,403) TITLE
READ(4,404) NNODE,NELEM,NTYPE,MBW

```

```

      NX=2*NNODE
      WRITE(6,200)
      WRITE(6,403)
      WRITE(6,201) NNODE,NELEM,NTYPE
      DO 10 M=1,NNODE
10    READ(4,405) (COORD(M,J),J=1,2)
      NCOL=1
      DO 20 M=1,NELEM
      READ(4,406) (NOD(M,J),J=1,3), (LTYPE(M,J),J=1,2), RHO,TEMP,ICOL(NCOL)
      WRITE(11) RHO,TEMP
      IF(ICOL(NCOL).EQ.0) GO TO 19
      ICOL(NCOL)=M
      NCOL=NCOL+1
19    IF(LTYPE(M,2).EQ.0) LTYPE(M,2)=1
20    CONTINUE
      READ(5,100) JOBS
C
C CALCULATE BANDWIDTH, CHECK DIMENSIONS
C
      IF(MBW.NE.0) GO TO 3
      DO 30 M=1,NELEM.
30    MBW=MAX0(IABS(NOD(M,1)-NOD(M,2)), IABS(NOD(M,1)-NOD(M,3)), IABS(NOD(
      1M,2)-NOD(M,3)), MBW)
      MBW=2*(MBW+1)
3    WRITE(6,202) MBW,JOBS
      IF((NX*MBW).GT.13800) ERROR=1
      IF(ERROR.EQ.1) WRITE(6,303) ERROR
      IF(ERROR.EQ.1) GO TO 99
C
C CALL SUBROUTINES FOR EACH LOAD CASE
C
      DO 90 JOB=1,JOBS
      CALL JOBP(JOB,ID,IS,ICOL)
      IF((ID.NE.0).AND.(JOB.GT.1)) GO TO 1
      CALL GEND
1    CALL BFORSE(JOB)
      IF((IISOS.EQ.0).OR.((IS.NE.0).AND.(JOB.GT.1))) GO TO 2
      CALL AIRY(ERROR)
2    CALL ASSTIF
      CALL ONESOL(ERROR,NX)
      IF(ERROR.NE.0) GO TO 89
      CALL TENS(ICOL)
      GO TO 90
89    WRITE(6,303) ERROR
90    CONTINUE
99    STOP
C
C FORMAT STATEMENTS
C

```

```

100 FORMAT(I3)
200 FORMAT(1H1,"PROGRAM INFIN3 -- R.STEPHENSON 75-76")
201 FORMAT(////1H0,"MODEL PARAMETERS",//,"NO.OF NODES=",I5,/, "NO.OF EL
  ELEMENTS=",I5,/, "NO.OF ELASTIC ZONES=",I5,/, "NO.OF THERMAL ZONES=",I
  25)
202 FORMAT(1X,/, "MAX.SEMI-BANDWIDTH=",I5,/, "NO.OF LOAD CASES=",I5)
303 FORMAT(////1H0,"JOB ABORTED AT ERROR",I5)
403 FORMAT(20A4)
404 FORMAT(5I5)
405 FORMAT(5X,2F10.0)
406 FORMAT(5X,5I5,2F10.0,4X,I1)
  END

```

```

SUBROUTINE JOBP(JOB, ID, IS, ICOL)
  IMPLICIT DOUBLE PRECISION(A-H,O-Z)
  COMMON FORCE(2,350),COORD(350,2),NOD(600,3),ELACON(6,2),D(6,3,3),L
  1TYPE(600,2),THERM(8),TPRINT(4),IPN(3),NTYPE(2),IISOS,ITHERM,IBODY,
  2ISTRES,IPLANE,ITENS,NNODE,NELEM,MBW,ISAVE
  COMMON/D/ DISP(60,2),IFIX(60),IDISP(60),NDISR
  DIMENSION ICOL(50),DUM(2)
  INTEGER TITLE(20)
  REWIND 8,9,10,11

```

```

C
C INPUT/OUTPUT LOAD CASE TITLE, OPTIONS
C

```

```

  READ(5,403) TITLE
  WRITE(6,101) JOB
  WRITE(6,403) TITLE
  READ(3,102) IPRINT,IPLANE,IISOS,IBODY,ITHERM,ISTRES,IPN,ITENS,ID,I
  1S,ISAVE
  IF(IPLANE.EQ.0) WRITE(6,206)
  IF(IPLANE.NE.0) WRITE(6,207)
  IF(IISOS.NE.0) WRITE(6,208)
  IF(IBODY.NE.0) WRITE(6,209)
  IF(ITHERM.NE.0) WRITE(6,210)
  IF(ISTRES.NE.0) WRITE(6,211)
  WRITE(6,300)

```

```

C
C ZERO AND INPUT BOUNDARY CONDITIONS AND LOADING
C

```

```

  DO 9 M=1,60
    IFIX(M)=0
    DO 9 I=1,2
      9 DISP(M,I)=0.0
    DO 10 M=1,NNODE
      DO 10 I=1,2
        10 FORCE(I,M)=0.0

```

```

NDISP=1
1 READ(5,104) N,IFIX(NDISP), (DISP(NDISP,J),J=1,2),DUM
IF(N.GT.NNODE) GO TO 8
IF(IFIX(NDISP).EQ.0) GO TO 6
IDISP(NDISP)=N
NDISP=NDISP+1
6 DO 7 I=1,2
7 FORCE(I,N)=DUM(I)
GO TO 1

```

```

C
C OUTPUT LOAD CASE DATA; ADJUST UNITS
C

```

```

IF(IPRINT(1).EQ.0) GO TO 2
WRITE(6,200)
WRITE(6,201)
NDISP=NDISP-1
DO 20 M=1,NNODE
IFAX=0
DO 17 I=1,NDISP
IF(IDISP(I).NE.M) GO TO 17
IFAX=IFIX(I)
GO TO 18
17 CONTINUE
18 IF(IFAX) 12,11,13
11 WRITE(6,202) M,(COORD(M,J),J=1,2),(FORCE(J,M),J=1,2)
GO TO 20
12 WRITE(6,205) M,(COORD(M,J),J=1,2),(DISP(I,J),J=1,2),(FORCE(J,M),J=
11,2)
GO TO 20
13 IF(IFAX.EQ.2) GO TO 15
WRITE(6,204) M,(COORD(M,J),J=1,2),DISP(I,1),(FORCE(J,M),J=1,2)
GO TO 20
15 WRITE(6,203) M,(COORD(M,J),J=1,2),DISP(I,2),(FORCE(J,M),J=1,2)
20 CONTINUE
2 DO 30 M=1,NDISP
DO 30 I=1,2
30 DISP(M,I)=DISP(M,I)*1.0E2
IF(JOB.GT.1) GO TO 16
DO 35 M=1,NNODE
DO 35 I=1,2
35 COORD(M,I)=COORD(M,I)*1.0E5
16 IF(IPRINT(2).EQ.0) GO TO 3
WRITE(6,212)
NCOL=1
DO 40 M=1,NELEM
READ(11) RHO,TEMP
IF(ICOL(NCOL).NE.M) GO TO 40
WRITE(6,218) NCOL
NCOL=NCOL+1

```

```

40 WRITE(6,213) M, (NOD(M,J), J=1,3), (LTYPE(M,J), J=1,2), RHO, TEMP
REWIND 11

```

```

C INPUT/OUTPUT ELASTIC, THERMAL DATA

```

```

C

```

```

3 IF(ID.NE.0) GO TO 51
  K1=NTYPE(1)
  K2=NTYPE(2)
  DO 50 M=1,K1
50 READ(5,105) (ELACON(M,J), J=1,2)
51 IF(ITHERM.EQ.0) GO TO 4
  DO 60 M=1,K2
60 READ(5,106) THERM(M)
  4 IF(IPRINT(3).EQ.0) GO TO 5
    WRITE(6,214)
    DO 70 M=1,K1
70 WRITE(6,215) M, (ELACON(M,J), J=1,2)
  5 IF((ITHERM.EQ.0).OR.(IPRINT(4).EQ.0)) GO TO 99
    WRITE(6,216)
    DO 80 M=1,K2
80 WRITE(6,217) M, THERM(M)

```

```

C FORMAT STATEMENTS

```

```

C

```

```

99 RETURN
101 FORMAT(1H1,"LOAD CASE",I4," IS...")
102 FORMAT(16I1)
104 FORMAT(I5,I2,8X,2F10.0,2E15.0)
105 FORMAT(5X,E10.0,F10.0)
106 FORMAT(5X,E15.0)
200 FORMAT(1X," NODE      X-COORDINATE      Z-COORDINATE      X-DISPLACEMENT
1 Z-DISPLACEMENT:  X-APPLIED FORCE      Z-APPLIED FORCE")
201 FORMAT(1X,"              (KM)              (KM)              (METRES)
1 (METRES)              (DYNES)              (DYNES)",//)
202 FORMAT(1X,I5,2(5X,F10.4),2(4X,"NOT SPECIFIED"),3X,D15.8,2X,D15.8)
203 FORMAT(1X,I5,2(5X,F10.4),4X,"NOT SPECIFIED",7X,F10.4,3X,D15.8,2X,D
115.8)
204 FORMAT(1X,I5,2(5X,F10.4),7X,F10.4,4X,"NOT SPECIFIED",3X,D15.8,2X,D
115.8)
205 FORMAT(1X,I5,2(5X,F10.4),2(7X,F10.4),3X,D15.8,2X,D15.8)
206 FORMAT(1H0,"CASE OF PLANE STRAIN",//)
207 FORMAT(1H0,"CASE OF PLANE STRESS",//)
208 FORMAT(1X,"ISOSTATIC TERM INCLUDED IN STIFFNESS MATRIX")
209 FORMAT(1X,"BODY FORCES INCLUDED")
210 FORMAT(1X,"THERMAL STRAIN INCLUDED AS INITIAL STRAIN")
211 FORMAT(1X,"INITIAL STRESS DISTRIBUTION CONSIDERED")

```

```

212 FORMAT(///1H0,"ELEMENT      DEFINING NODES      ELASTIC TYPE  THER
1MAE TYPE      DENSITY(G/CM3)      TEMP.CHANGE(DEG:C)",//)
213 FORMAT(1X,I5,6X,I5,"",I5,"",I5,I5,2(5X,"TYPE",I5),7X,F10.3,10X,F
110.3)
214 FORMAT(///1H0,2X,"ELEMENT TYPE      ELASTIC PARAMETERS:  YOUNGS MOD
1ULUS(DYNES/CM2)      POISSONS RATIO",//)
215 FORMAT(6X,I5,T49,E11,4,T71,F11.4)
216 FORMAT(///1H0,2X,"ELEMENT TYPE      COEFFICIENT OF LINEAR THERMAL E
1XPANSION(DEG.C-1)",//)
217 FORMAT(6X,I5,T50,E15.4)
218 FORMAT(/,1H0,I2," **",/)
300 FORMAT(1H1)
403 FORMAT(20A4)
END

```

```

SUBROUTINE GEND
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
COMMON FORCE(2,350),COORD(350,2),NOD(600,3),ELACON(6,2),D(6,3,3),L
1TYPE(600,2),THERM(8),IPRINT(4),IPN(3),NTYPE(2),IISOS,ITHERM,IBODY,
2ISTRES,IPLANE,ITENS,NNODE,NELEM,MBW,ISAVE

```

```

C
C FORM NTYPE ELASTICITY MATRICES FOR PLANE STRESS OR PLANE STRAIN
C

```

```

      K=NTYPE(1)
      DO 10 M=1,K
      IF(IPLANE.NE.0) GO TO 1
      COMP1=ELACON(M,1)*(1.0-ELACON(M,2))/((1.0+ELACON(M,2))*(1.0-2.0*EL
1ACON(M,2)))
      COMP2=ELACON(M,2)/(1.0-ELACON(M,2))
      GO TO 2
      1 COMP1=ELACON(M,1)/(1.0-ELACON(M,2)**2)
      COMP2=ELACON(M,2)
      2 D(M,1,1)=COMP1
      D(M,2,2)=COMP1
      D(M,1,2)=COMP1*COMP2
      D(M,2,1)=COMP1*COMP2
      D(M,3,3)=COMP1*(1.0-COMP2)*0.5
      D(M,1,3)=0.0
      D(M,2,3)=0.0
      D(M,3,1)=0.0
      D(M,3,2)=0.0
10 CONTINUE
      RETURN
      END

```

```

SUBROUTINE BFORSE(JOB)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)

```



```

COMMON FORCE(2,350),COORD(350,2),NOD(600,3),RIACON(6,2),D(6,3,3),L
1TYPE(600,2),THERM(8),IPRINT(4),IPN(3),NTYPE(2),IISOS,ITHERM,IBODY,
2ISTRES,IPLANE,ITERM,ENODE,NELEM,MBW,ISAVE
DIMENSION XF(2,3),SUM(2),B(3,6),ENITE(3)

```

```
DO 9 M=1,2
```

```
9 SUM(M)=0.0
```

```
DO 80 M=1,NELEM
```

```
READ(11) RHO,TEMP
```

```
IF(JOB.GT.1) GO TO 2
```

```
C
```

```
C COMPUTE B MATRIX
```

```
C (THIRD NODE IS 'K' RATHER THAN 'M' AS IN SECTION A2.3.4)
```

```
C ESTABLISH LOCAL COORDINATE SYSTEM/ COMPUTE ELEMENTAL AREA FACTOR
```

```
C
```

```
I=NOD(M,1)
```

```
J=NOD(M,2)
```

```
K=NOD(M,3)
```

```
AJ=COORD(J,1)-COORD(I,1)
```

```
AK=COORD(K,1)-COORD(I,1)
```

```
BJ=COORD(J,2)-COORD(I,2)
```

```
BK=COORD(K,2)-COORD(I,2)
```

```
DEL=1.0/ABS((AJ*BK)-(AK*BJ))
```

```
IF(M.GT.1) GO TO 1
```

```
DO 10 I=1,3
```

```
DO 10 J=1,6
```

```
10 B(I,J)=0.0
```

```
1 B(1,1)=(BJ-BK)*DEL
```

```
B(1,3)=BK*DEL
```

```
B(1,5)=-BJ*DEL
```

```
B(2,2)=(AK-AJ)*DEL
```

```
B(2,4)=-AK*DEL
```

```
B(2,6)=AJ*DEL
```

```
B(3,1)=(AK-AJ)*DEL
```

```
B(3,2)=(BJ-BK)*DEL
```

```
B(3,3)=-AK*DEL
```

```
B(3,4)=BK*DEL
```

```
B(3,5)=AJ*DEL
```

```
B(3,6)=-BJ*DEL
```

```
DEL=0.5/DEL
```

```
WRITE(10) DEL,((B(I,J),J=1,6),I=1,3)
```

```
GO TO 3
```

```
2 READ(10) DEL,((B(I,J),J=1,6),I=1,3)
```

```
C
```

```
C COMPUTE BODY FORCES
```

```
C
```

```
3 IF(IBODY.EQ.0) GO TO 4
```

```
DO 20 I=1,3
```

```
XF(1,I)=0.0
```

```
XF(2,I)=-RHO*327.0*DEL
```

```
20 FORCE(2,NOD(M,I))=FORCE(2,NOD(M,I))+XF(2,I)
```

C
C COMPUTE THERMAL STRAIN FORCES
C

```

4 IF (ITHERM.EQ.0) GO TO 6
  L1=LTYPE(M,1)
  L2=LTYPE(M,2)
  COMP1=1.0
  IF (IPLANE.NE.0) GO TO 5
  COMP1=COMP1+ELACON(L1,2)
5 DO 30 I=1,2
30 EINIT(I)=COMP1*THERM(L2)*TEMP
  EINIT(3)=0.0
  WRITE(9) (EINIT(J),J=1,2)
  DO 50 I=1,2
  DO 50 J=1,3
    N=2*J-2+I
    XF(I,J)=0.0
    DO 40 L=1,3
    DO 40 K=1,3
40 XF(I,J)=XF(I,J)+(B(L,N)*D(L1,L,K)*EINIT(K))
  XF(I,J)=DEL*XF(I,J)
50 FORCE(I,NOD(M,J))=FORCE(I,NOD(M,J))+XF(I,J)

```

C
C COMPUTE INITIAL STRESS FORCES
C

```

6 IF (ISTRES.EQ.0) GO TO 80
  READ(8) (EINIT(J),J=1,3)
  DO 70 I=1,2
  DO 70 J=1,3
    N=2*J-2+I
    XF(I,J)=0.0
    DO 60 K=1,3
60 XF(I,J)=XF(I,J)+(B(K,N)*EINIT(K))
  XF(I,J)=DEL*XF(I,J)
70 FORCE(I,NOD(M,J))=FORCE(I,NOD(M,J))+XF(I,J)
80 CONTINUE

```

C
C OUTPUT NET X,Z FORCES
C

```

DO 85 M=1,NNODE
DO 85 I=1,2
85 SUM(I)=SUM(I)+FORCE(I,M)
  WRITE(6,100) SUM
  RETURN
100 FORMAT(1H0,/,/, " TOTAL X-FORCES=",E15.4, " DYNES",/,1X,"TOTAL Z-FOR
  CES=",E15.4, " DYNES")
END

```

```

SUBROUTINE AIRY(ERROR)
  IMPLICIT DOUBLE PRECISION(A-H,O-Z)
  COMMON FORCE(2,350),COORD(350,2),NOD(600,3),ELACON(6,2),D(6,3,3),I
  1TYPE(600,2),THERM(8),IPRINT(4),IPN(3),NTYPE(2),IISOS,ITHERM,IBODY,
  2ISTRES,IPLANE,ITENS,NNODE,NELEM,MBW,ISAVE
  COMMON/A/ STISOS(50),LBASE(50),NBASE
  INTEGER ERROR

```

```

C
C INPUT ISOSTATIC DATA/ COMPUTE ADDITIONAL BASAL STIFFNESS
C

```

```

  READ(5,100) YBASE,RHOSUB
  YBASE=YBASE*1.0E5
  NBASE=0
  DO 10 M=1,NNODE
    IF(COORD(M,2).NE.YBASE) GO TO 10
    NBASE=NBASE+1
    LBASE(NBASE)=M
  10 CONTINUE
  STISOS(1)=((COORD(LBASE(2),1)-COORD(LBASE(1),1))*0.5)*981.0*RHOSUB
  N=NBASE-1
  DO 20 I=2,N
    J=I+1
    K=I-1
    IF(((COORD(LBASE(K),1)).GT.(COORD(LBASE(I),1))).OR.((COORD(LBASE(I
    1),1)).GT.(COORD(LBASE(J),1)))) ERROR=3
    IF(ERROR.EQ.3) GO TO 99
  20 STISOS(I)=((COORD(LBASE(J),1)-COORD(LBASE(K),1))*0.5)*981.0*RHOSUB
  STISOS(NBASE)=((COORD(LBASE(NBASE),1)-COORD(LBASE(N),1))*0.5)*981.
  10*RHOSUB
  ERROR=0
  99 RETURN
  100 FORMAT(2F10.0)
  END

```

```

SUBROUTINE ASSTIF
  IMPLICIT DOUBLE PRECISION(A-H,O-Z)
  COMMON FORCE(2,350),COORD(350,2),NOD(600,3),ELACON(6,2),D(6,3,3),L
  1TYPE(600,2),THERM(8),IPRINT(4),IPN(3),NTYPE(2),IISOS,ITHERM,IBODY,
  2ISTRES,IPLANE,ITENS,NNODE,NELEM,MBW,ISAVE
  COMMON/A/ STISOS(50),IBASE(50),NBASE
  COMMON/D/ DISP(60,2),IFIX(60),IDISP(60),NDISP
  COMMON/S/ STIFF(13800)
  DIMENSION XK(6,6),B(3,6)
  INDX(I,J,K)=(I-1)*K+J-I+1
  REWIND 10
  DO 90 M=1,NELEM
    READ(10) DEL,((B(I,J),J=1,6),I=1,3)

```

```

C
C FORM ELEMENTAL STIFFNESS MATRIX, XK
C

```

```

      LT=LTYPE(M,1)
      DO 30 I=1,6
      DO 30 J=1,6
      XK(I,J)=0.0
      IF(I.GT.J) GO TO 2
      DO 20 L=1,3
      DO 20 K=1,3
20  XK(I,J)=XK(I,J)+(B(L,I)*D(LT,L,K)*B(K,J))
      XK(I,J)=DEL*XK(I,J)
      GO TO 30
      2  XK(I,J)=XK(J,I)
30  CONTINUE
C
C  ADD XK INTO STRUCTURAL STIFFNESS MATRIX (UNIDIMENSIONAL ARRAY STIFF)
C
      DO 40 I=1,3
      KA=(NOD(M,I)-1)*2
      KB=(I-1)*2
      DO 40 J=1,3
      LA=(NOD(M,J)-1)*2
      LB=(J-1)*2
      DO 40 K=1,2
      KC=KA+K
      KD=KB+K
      DO 40 L=1,2
      LC=LA+L
      LD=LB+L
      IF(KC.GT.LC) GO TO 40
      STIFF(INDX(KC,LC,MBW))=STIFF(INDX(KC,LC,MBW))+XK(KD,LD)
40  CONTINUE
90  CONTINUE
C
C  ADD DISPLACEMENT-DEPENDENT ISOSTATIC TERM INTO STIFF
C
      IF(IISOS.EQ.0) GO TO 3
      DO 35 N=1,NBASE
      M=2*IBASE(N)
35  STIFF(INDX(M,M,MBW))=STIFF(INDX(M,M,MBW))+STISOS(N)
C
C  ADJUST FOR DISPLACEMENT BOUNDARY CONDITIONS
C
      3  DO 36 N=1,NDISP
      M=IDISP(N)
      DO 36 I=1,2
      IF(IFIX(N)) 12,36,11
11  IF(IFIX(N).NE.I) GO TO 36
12  K=2*M-2+I
      K=INDX(K,K,MBW)

```

```

STIFF(K)=STIFF(K)*1.0E15
FORCE(I,M)=STIFF(K)*DISP(N,I)
36 CONTINUE
RETURN
END

```

```

SUBROUTINE ONESOL(ERROR,NX)
IMPLICIT DOUBLE PRECISION(A-H,O-Z)
COMMON FORCE(2,350),COORD(350,2),NOD(600,3),ELACON(6,2),D(6,3,3),L
1TYPE(600,2),THERM(8),IPRINT(4),IPN(3),NTYPE(2),IISOS,ITHERM,IBODY,
2ISTRES,IPLANE,ITENS,NNODE,NELEM,MBW,ISAVE
COMMON/S/,STIFF(13800)
DIMENSION A(13800),U(13800),X(700),Y(700),C(700),DUM(2)
EQUIVALENCE(A,U,STIFF),(FORCE,X,Y,C)
INTEGER ERROR
INDX(I,J,K)=(I-1)*K+J-I+1

```

```

C
C COMPUTE BANDED FACTOR MATRIX U
C

```

```

U(1)=SQRT(A(1))
DO 15 I=2,MBW
15 U(I)=A(I)/U(1)
DO 30 I=2,NX
SUM=0.0
L=I-1
DO 20 M=1,L
N=M+MBW-1
IF(I.GT.N) GO TO 20
SUM=SUM+U(INDX(M,I,MBW))**2
20 CONTINUE
SUM=A(INDX(I,I,MBW))-SUM
IF(SUM.LT.0.0) ERROR=4
IF(ERROR.EQ.4) GO TO 99
U(INDX(I,I,MBW))=SQRT(SUM)
N1=I+1
N2=I+MBW-1
DO 30 J=N1,N2
SUM=0.0
DO 25 M=1,L
N=M+MBW-1
IF(J.GT.N) GO TO 25
SUM=SUM+U(INDX(M,I,MBW))*U(INDX(M,J,MBW))
25 CONTINUE
30 U(INDX(I,J,MBW))=(A(INDX(I,J,MBW))-SUM)/U(INDX(I,I,MBW))

```

```

C
C COMPUTE Y
C

```

```

      Y(1)=C(1)/U(1)
      DO 40 I=2,NX
        L=I-1
        SUM=0.0
        DO 35 M=1,L
          N=M+MBW-1
          IF (1.GT.N) GO TO 35
          SUM=SUM+U(INDX(M,I,MBW))*Y(M)
        35 CONTINUE
      40 Y(I)=(C(I)-SUM)/U(INDX(I,I,MBW))

C
C COMPUTE X
C
      X(NX)=Y(NX)/U(INDX(NX,NX,MBW))
      DO 50 J=2,NX
        I=NX+1-J
        L=I+1
        SUM=0.0
        DO 45 M=L,NX
          N=I+MBW-1
          IF (M.GT.N) GO TO 45
          SUM=SUM+U(INDX(I,M,MBW))*X(M)
        45 CONTINUE
      50 X(I)=(Y(I)-SUM)/U(INDX(I,I,MBW))

C
C OUTPUT DISPLACEMENT SOLUTION DATA
C
      WRITE(6,100)
      DO 95 I=1,NNODE
        DO 90 J=1,2
          90 DUM(J)=FORCE(J,I)*1.0E-2
          WRITE(6,101) I,DUM
          IF (IPN(1).NE.0) WRITE(7,102) I,DUM
        95 CONTINUE
      ERROR=0

C
C FORMAT STATEMENTS
C
      99 RETURN
      100 FORMAT(1H1,"SOLUTION DATA",///,41X,"NODAL POINT",10X,"X-DISPLACEMENT",
        10X,"Z-DISPLACEMENT",/,65X,"(METRES)",13X,"(METRES)",//)
      101 FORMAT(42X,15,16X,E11.5,10X,E11.5)
      102 FORMAT(1X,14,2E11.5)
      END

      SUBROUTINE TENSr(ICOL)
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      COMMON FORCE(2,350),COORD(350,2),NOD(600,3),ELACON(6,2),D(6,3,3),L
      ITYPE(600,2),THERM(8),IPRINT(4),IPN(3),NTYPE(2),IISOS,ITHERM,IBODY,

```

```

2ISTRES, IPIANE, ITENS, NNODE, NELEM, NEW, ISAVE
DIMENSION EDISP(6), SIGMA(3), B(3,6), EINIT(3), ICOL(50)
DATA CONST/57.29578/
REWIND 8,9,10

```

```

C
C INPUT OUTPUT UNITS
C
  READ(5,104) IF2, IF3
  WRITE(6,100)
  WRITE(6,101)
  NCOL=1
  DO 50 M=1, NELEM
    READ(10) DEL, ((B(I,J), J=1,6), I=1,3)
C
C FORM ONE-DIMENSIONAL ELEMENTAL DISPLACEMENT MATRIX
C
  DO 10 I=1,3
    DO 10 J=1,2
      K=2*I-2+J
      10 EDISP(K)=FORCE(J, NOD(M,I))
C
C COMPUTE ELEMENTAL STRESS TENSOR
C
  DO 20 I=1,3
    20 SIGMA(I)=0.0
    K=LTYPE(M,1)
    DO 30 L=1,3
      DO 30 I=1,3
        DO 30 J=1,6
          30 SIGMA(L)=SIGMA(L)+D(K,L,I)*B(I,J)*EDISP(J)
C
C COMPUTE AND ADD THERMAL STRAIN STRESSES
C
  IF(ITHERM.EQ.0) GO TO 1
  READ(9) (EINIT(J), J=1,2)
  EINIT(3)=0.0
  DO 35 I=1,3
    EDISP(I)=0.0
    DO 35 J=1,3
      35 EDISP(I)=EDISP(I)+(D(K,I,J)*EINIT(J))
    DO 40 I=1,3
      40 SIGMA(I)=SIGMA(I)-EDISP(I)
C
C ADD INITIAL STRESSES
C
  1 IF(ISTRES.EQ.0) GO TO 2
  READ(8) (EINIT(J), J=1,3)
  DO 45 I=1,3
    45 SIGMA(I)=SIGMA(I)+EINIT(I)
  GO TO 6

```

```

2 IF((IPN(3).NE.0).OR.(ITENS.NE.0).OR.(ISAVE.NE.0)) WRITE(8) (SIGMA(
  1J),J=1,3)
C
C TRANSFORM TO PRINCIPAL SYSTEM
C (ALPHA IS PHI OF SECTION A2.3.8)
C
  6 IF((SIGMA(1)-SIGMA(2)).EQ.0.0) GO TO 3
  ALPHA=(2.0*SIGMA(3))/(SIGMA(1)-SIGMA(2))
  COMPI=0.0
  IF(ALPHA.LT.0.0) COMPI=180.0
  ALPHA=0.5*(DATAN(ALPHA)*CONST+COMPI)
  GO TO 4
3 ALPHA=45.0
4 ALCON=ALPHA/CONST
  SIGMA(3)=(SIGMA(1)*(DCOS(ALCON)**2))+(SIGMA(2)*(DSIN(ALCON)**2))+(
  1SIGMA(3)*DSIN(2.0*ALCON))
  SIGMA(2)=SIGMA(1)+SIGMA(2)-SIGMA(3)
  SIGMA(1)=SIGMA(3)
  IF(IPLANE.EQ.0) SIGMA(3)=ELACON(K,2)*(SIGMA(1)+SIGMA(2))
C
C CALCULATE ELEMENTAL SHEAR STRESS
C
  IF(IPLANE.NE.0) SIGMA(3)=(SIGMA(1)+SIGMA(2))*0.5
  COMPI=DMAX1(SIGMA(1),SIGMA(2),SIGMA(3))
  COMP2=DMIN1(SIGMA(1),SIGMA(2),SIGMA(3))
  COMPI=(COMPI-COMP2)*0.5
  IF(IPLANE.NE.0) SIGMA(3)=0.0
C
C OUTPUT STRESS SOLUTION DATA
C
  IF(ICOL(NCOL).NE.M) GO TO 5
  WRITE(6,103) NCOL
  NCOL=NCOL+1
  5 WRITE(6,102) M,ALPHA,SIGMA,COMPI
  IF(IPN(2).NE.0) WRITE(6,102) M,ALPHA,SIGMA,COMPI
50 CONTINUE
  IF((ITENS.EQ.0).AND.(IPN(3).EQ.0)) GO TO 99
  IF(ITENS.NE.0) WRITE(6,200)
  REWIND 8
  DO 60 M=1,NELEM
  READ(8) (SIGMA(J),J=1,3)
  IF(ITENS.NE.0) WRITE(6,201) M,SIGMA
  IF(IPN(3).NE.0) WRITE(6,201) M,SIGMA
  60 CONTINUE
C
C FORMAT STATEMENTS
C
  99 RETURN

```



```
100 FORMAT(1H0,///,"STRESSES ARE DYNES/CM2",/,"TENSILE STRESS IS POSIT  
1/VE",/,"ALPHA IS POSITION OF PS(1), DEGREES COUNTERCLOCKWISE FROM  
2HORIZONTAL AXIS")  
101 FORMAT(1H0,///,"    ELEMENT    ALPHA    .PS(1)    PS(2)  
1 PS(3)    MAX.SHEAR",//)  
102 FORMAT(4X,I5,F10.2,1X,3(2X,E11.4),4X,E11.4)  
103 FORMAT(/,1H0,12," **",/)  
104 FORMAT(2I5)  
200 FORMAT(1H0,///,"    ELEMENT    S(XX)    S(ZZ)  
1 S(XZ)",//)  
201 FORMAT(4X,I5,11X,3(2X,E11.4))  
END
```

END

5

08

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FIN