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Performance Analysis of Client-Server Systems with Multi-threaded Clients

by

Jingping Lin

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of
Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University,
Ottawa, Ontario, Canada

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**Performance Analysis of Client-Server Systems
with Multi-threaded Clients**

submitted by
Jingping Lin, B.Eng

In partial fulfillment of the requirements
for the degree of Master of Engineering

Thesis Co-Supervisor

Thesis Co-Supervisor

Chair, Department of Systems and
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Carleton University
January 12, 1994

Abstract

This thesis introduces an analytical performance model named *Rendezvous Network with Multi-Threaded Clients (RNMTC)* for performance analysis of client/server system. RNMTC is an extension of the *Stochastic Rendezvous Network (SRVN)*, a previously developed model for tasking systems with *rendezvous (RV)* communication style. An approximate analytical solution method and a simulator were developed for RNMTC and implemented in the thesis. Techniques for the computation of performance bounds for the RNMTC model are also investigated.

The analytic solution of the RNMTC model is based on hierarchical decomposition. At the higher level, a Markov Chain (MC) model is built such that the MC states correspond to all possible combinations of client execution states; at the lower level, each MC state is represented by an SRVN model with simple clients.

The analytic solution for RNMTC is applied to a number of test cases, the most important being a large case study of a distributed database running on a Tandem system. The analytic results are compared with two simulation results: one from a “detailed simulation model” which captures the hardware architecture of a Tandem computer system and the software behavior of the relational database system, and one from an “abstract simulation model” used to analyze RNMTC models. The “abstract error” is defined as the difference between the detailed and abstract simulation results while the “analytic approximation error” is defined as the difference between the abstract simulation and analytic results.

Performance bounds that are based on a set of very loose assumptions about the stochastic behaviour of the system are investigated. Techniques for the computation of an upper bound and a lower bound on performance for a number of specific RNMTC models are presented.

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List of Symbols and Terminology

RNMTC = Rendezvous Networks with Multi-Threaded Client

SRVN = Stochastic RendezVous Network

RV = RendezVous

QN = Queueing Network

PN = Petri Net

MC = Markov Chain

“in RV” phase = execution phase of an accepting task, when the requester is blocked is blocked waiting for a reply from the acceptor (also named first phase in SRVN and RNMTC)

“post RV” = execution phase of an accepting task, executed in parallel with the requester (also named second phase in SRVN)

C_a = total number of classes in the client system

$C_l(i)$ = total number of clients in class i

$N(i)$ = total number of client tasks in one client of class i

K = total number of servers in the system

$U(j)$ = total number of entries in server task j

$c(i, t)$ = average computation time for client task t in class i per job cycle

$s(i, e, p)$ = average service time in entry e during phase p ($p = 1$ or $p = 2$) of server task i per visit

$S(i, e)$ = average service time in entry e of server task i in two phases per visit

$$S(i, e) = s(i, e, 1) + s(i, e, 2)$$

$v(i, t, j, u(j))$ = average number of visits made by client task t in class i to entry u of server task j per job cycle

$v(i, t, j)$ = average number of visits made by client task t in class i to server task j per job cycle

$$v(i, t, j) = \sum_{u=1}^{U(j)} v(i, t, j, u(j))$$

$y(i, e, p, j, d)$ = mean number of RV requests made by (server) task i in entry e during phase p to entry d in task (server) j .

Chapter 1

Introduction

1.1 Research Motivation

Computer system performance in terms of speed and efficiency is one of the major factors which influence the design, development, configuration and tuning of a system. It is desirable to evaluate the performance of a computer system including both software and hardware components, because we want to make sure that the system is suitable for its intended applications, and that it satisfies the performance requirements. Performance evaluation is also useful to assess if a computer system operates near its optimal capacity under given resource constraints. With the enhancement in size and complexity of current computer systems, performance evaluation becomes more and more important and is strongly required throughout the life time of a system.

There are a number of system performance measures, such as, cycle time, response time, throughput, utilization, speedup, reliability, etc. The specific performance measures of interest is dependent on the types of the system. For example, response time is more crucial than other performance measures in a real time system, whereas throughput is a better description of performance for a batch processing system.

Different performance evaluation techniques may be appropriate in different stages of the lifetime of the system. For example, *measurement* is an appropriate approach once a system is built and instrumented, whereas *modelling* may be the only performance evaluation technique that may be used before the construction of the system. This thesis is concerned with the second approach to performance evaluation.

A model is an abstract representation of the system under study. By analyzing and evaluating the model, the behavior of the system can be inferred. An appropriate model should be simpler than the real world and relatively easy to analyze. Therefore, the construction of a model requires a process of considerable abstraction. Some characteristics of the system must be kept in the model because they are the major factors which affect the performance of the system. On the other hand, some other characteristics may be ignored to make the model simpler, if they have less impact on system performance. Unfortunately, there is no explicit definitions about whether a feature should be included in the model or not. The building of an appropriate model is mostly dependent on the ability and experience of the modeler.

In terms of modeling, the simulation modeling plays an important practical role in performance evaluation. More details of a system can be aptly captured in a simulation model than in analytical models and thus the simulation results are closer to the real world. However, the disadvantage of simulation is its high cost of developing the models and of running the experiments.

Using inexpensive analytical models may be a viable alternative to the analyst or designer for estimating system performance. Analytical modelling has the following advantages:

- It has much lower cost than simulation modeling.
- It provides insight on the performance characteristics of the system under study.
- Compared to the measurement methods which can be applied only to existing systems, analytical modeling methods not only can evaluate an existing system, but also can predict the performance of a future system.

Analytical modeling has also some disadvantages. It may have lower accuracy than simulation due to modeling restrictions imposed on analytical models to insure their feasibility and simplicity. Also, the size of the model has a strong impact on the complexity of analytical solutions, which makes some methods intractable for systems of a practical size.

A brief review of some known analytical models is given in the following.

Queueing Network (QN) is a well developed practical tools for computer system performance analysis [Lav89] [LZGS84]. A QN includes service centers which normally represent hardware resources, and clients which represent software jobs. The service centers are shared by

the clients. Each client obtains service for a given interval of time from a service center. After the service, the client leaves the service center and then may proceed to another service center. Each service center is associated with a queue in which the clients wait for service. There may be multiple service levels in each server queue and different service disciplines, such as FIFO (first come, first served), PS (processor sharing), may be used. Clients can be divided into different classes. Clients with statistical identical characteristics, such as routing probabilities, service demand, are put into the same class. The only dependence among clients is that they compete for server resources. The clients do not synchronize or communicate with one another. If the number of clients in the QN system is fixed, the QN is called *closed*. On the other hand, an *open* QN allows clients that complete their service to depart from the system and allows new client(s) to enter the system. Figure 1.1 is an example of a queueing system in which the CPU and the I/O devices are modelled by different kinds of service centers.

Figure 1.1. Example of queueing system

Numerous solution methods to solve queueing models have been developed. A survey of analytical computer performance modeling technique with an emphasis on developments of prac-

tical importance can be found in [Lav89].

QN is a powerful analytical model to evaluate system performance, but has also limitations [VZL87]. It fails to capture the impact of system concurrence and the communication and synchronization among software jobs, since a QN server center plays a sole role: supplying service. In a QN, each client is served by one service center at a time. The multi-level service requests produced by a server which calls another server while serving a client cannot be modeled directly by QN.

A *Petri-Net* (PN) is another kind of analytical model [AMBC84] [Mol82]. A PN contains a set of places, a set of transitions and a number of tokens. The tokens reside in places and may move from one place to another caused due to the firing of transitions. A PN is very efficient in representing flow models of concurrent software. Figure 1.2 shows a flow model for a simple system with concurrent tasks.

Figure 1.2. Flow model for a simple concurrent system

Figure 1.3 is a simple example of a PN system that is equivalent to the structure of Figure 1.2.

Figure 1.3. Example of Petri-Net system

The disadvantage of a PN is that it is weak in modeling contention for resources and may give rise to the well known problem of state space explosion.

Stochastic Rendezvous Networks (SRVN) in [MWNP88] [MWNP89] [PW91] [WNMP88] [WNPM89] and [Woo89] is a relatively new model which combines the advantages of both QNs and PNs. A SRVN is very appropriate in modelling client/server systems with rendezvous (RV) style of communication among tasks. Since this thesis extends SRVN, the model is presented in more details than QN and PN models.

Intertask rendezvous communication is supported by many systems. In such a system, a task (client) sending a message to another task (server) is blocked until it gets a reply from the server, and then it resumes execution in parallel with the server. Examples include Remote Procedure Calls (RPC) in distributed system that can be modeled by a SRVN model. Another example is represented by the large class of transaction processing systems.

An SRVN model includes a set of communicating tasks (see Figure 1.4). A task receiving rendezvous (RV) service requests has a FIFO input queue, where RV requests from clients are

waiting to be served. At the beginning of its cycle, the server checks its input queue. If the queue is empty, it waits on the “receive” primitive. If the queue is not empty, the server selects the first request from queue to serve and starts an execution cycle. After a first phase of service, the server sends a reply message back to the client which is blocked waiting for response from the server. Upon receiving the reply, the client is released. The server may then do a second phase of service in parallel with the client, for example some “house-keeping” work which can process in parallel with the client. After the second phase, the server finishes the current cycle and then the server goes back to check its input queue.

Servers may have multiple entries each of which can represent a different behavior with different workload. Hierarchical service is possible to be represented by SRVN model. A server which is serving a client may send nested requests to other servers.

An example of SRVN model from [Pet91] is shown in Figure 1.4. The model represents a

Figure 1.4. Example of SRVN model

hierarchical database system. There are two kinds of servers: local server and global server. Each local server has its own set of clients. Both local server and global server have multiple entries to offer different service for different request types. Most of the client requests can be satisfied in their local server, only a small proportion of the requests need service also from the global server which is called by local server. While the global server is processing the request, both the local server which sends the request to the global server and the client which initials the request are blocked.

An SRVN model is very effective in modelling concurrent system in which tasks use rendezvous communication. An SRVN captures the following features of a system: nested service offered by the hierarchy of servers, change of request classes by the clients, and parallel execution between clients and servers. However the clients have a single thread of execution and simple behavior.

Rendezvous Networks with Multi-Threaded Client (RNMTC) is an extension of the SRVN model introduced in this thesis, where each client is represented by a precedence graph and may have a variable number of execution threads. The RNMTC model is presented formally in Chapter 2. The model of a client is a precedence graph composed of a number of nodes and directed edges. The execution flow may consist of multiple concurrent execution threads whose number can vary due to fork and join operations. An execution thread represents an independent flow of control and may request services from servers. The communication between clients and servers is by rendezvous. The servers are similar to those in SRVN models. When a particular thread is blocked in Rendezvous with a server, other threads of the same client may continue their execution, thus enhancing concurrency and performance

The papers known from literature on modelling concurrent software system, which are the closest to our approach are [TB86] and [KME89]. Thomsian and Bay have used hierarchical decomposition method to handle parallel software characterized by a precedence graph [TB86]. Another decomposition technique for modelling concurrent software with looping constructs is introduced by A. Kapelnikov [KME89]. None of these works deals with the synchronous client-server interaction and the nested service that is aptly captured in the model presented in this thesis.

The important features of the RNMTC model are summarized here:

- *Nested Service*: An important feature of the model is its capability of modelling nested service that exists in systems where a software server uses the services of other servers to provide the requested service to the client.

- *Modelling Flexibility*: Both system software and hardware components can be modelled.

- *Multi-Phase Behavior of Servers*: The server execution can be characterized by two phases, an in-rendezvous phase and another optional post-rendezvous phase.

- *Multiple Multi-threaded Clients*: The model can handle multiple application programs each of which may be multi-threaded. Classes of clients are also supported.

The first three features are available in the SRVN model whereas the last one is a new contribution made by the research presented in the thesis.

The analytic solution for the RNMTC model is based on hierarchical decomposition: at the higher level, system behavior is represented by a Markov Chain (MC) model whose states correspond to all possible combinations of client execution states; at the lower level, the system is represented by SRVN models with simple clients, for which existing approximate MVA solution were introduced in several papers, as for example [PW91] [WNPM89] [PW93] [Woo89] and [Pet93]. The last one is incorporated in the algorithm for RNMTC models in this thesis.

Again a number of test cases have been examined. Comparisons between the outputs of the analytic solution and simulation results are made for checking the accuracy of the analytic solution. The results obtained from these comparisons are quite encouraging. The error rate is less than 5% in most of the cases. The computation time for the model has also been observed to be orders of magnitude smaller than the simulation runs.

This thesis also investigates bounding techniques for the certain types of RNMTC models. Computation of approximate analytic solution is based on a number of assumptions, such as, the exponential distribution of execution time and service time. Bounds have weaker assumptions about the stochastic behavior of the system. For some systems, the distributions for service times and thread execution times may be unknown. The bounds are the only performance measures that can be computed in such a situation. Techniques for the computation of an upper bound and a lower bound on completion time of clients are developed.

The main result of the thesis constitute a paper [PMLH94] that will be presented at the International Workshop on Modeling, Analysis and Simulation of Computer and Telecommunication Systems, MASCOTS'94, to be held in Durham, North-Carolina, January 31 - February 2, 1994.

1.2 Outline of Thesis

This thesis is divided into seven chapters. Chapter 2 describes formally the RNMTC model. Chapter 3 describes a simulator for RNMTC models built on top of the PARASOL simulation kernel [Nei_1]. The simulator is used to assess the accuracy of the approximate analytic solution method and to check the correctness and tightness of bounds for RNMTC models. A guide for using the simulator is presented in Appendix. Chapter 4 introduces the analytic solution method for RNMTC models. The hierarchical decomposition mechanism is discussed. Techniques for the computation of bounds on performance are discussed in chapter 5. The upper bounds and lower bounds for different kinds of system are derived. A discussion of the implementation of the techniques are included through BNR prolog [BNR88] along with some test results. Chapter 6 assesses the accuracy of the analytic solution through many test cases. A set of fork/join test cases and a large model for a On-Line Transaction Processing system are discussed. The purpose of various kinds of test cases is to show that the analytic solution method is applicable to a large range of systems. Chapter 7 presents the conclusions and future work.

Chapter 2

RNMTC Model

The *Rendezvous Network with Multi-Threaded Client* (RNMTC) model consists of two submodels: the *client submodel* and the *server submodel*. The client submodel contains a set of clients, each one represented by a precedence graph which consists of a set of nodes with precedence relationship among them. The server submodel is a set of SRVN-like servers communicating by rendezvous. A server task may receive a request either from a client task or from another server.

Since the RNMTC model is an extension of SRVN model, we review the definition of the latter first.

2.1 The SRVN Model

SRVN stands for *Stochastic Rendezvous Networks*. SRVN is a performance model initially proposed in [Woo89] which can model concurrent networks with *rendezvous* (RV) style of communication.

The SRVN is represented by a “communication graph” (see Figure 2.2) where the nodes are tasks and the edges are RV requests.

There are two types of tasks: *reference task* and *non-reference task*. In Figure 2.2, T_1 and T_2 are called reference tasks whereas other tasks are non-reference tasks. Reference tasks loop endlessly and function as pure clients. During its execution, a reference task may make service requests to non-reference tasks.

The communication between tasks in SRVN model is by RV. Figure 2.1 shows the behavior of a client and a server over time. After executing on its own CPU, the client makes a request to a server, then it is blocked. The client will resume operation only after receiving a reply from the server. All rendezvous requests are queued in a single FIFO queue at the server. The server accepts the first request from its queue and executes an *in-RV* phase. During this phase, the server may make nested requests to other servers. After ending the first phase and sending a reply to the client, the server may enter its *post-RV* phase which is parallel with the execution of the client. In this second phase, the server executes on its CPU and can make requests to other servers. At the end of the second phase the current cycle is completed. The server starts a new service if there are queue requests or becomes receive-blocked waiting for another request to arrive.

Figure 2.1. SRVN task behavior

The execution cycle of a non-reference task is characterized by two phases of execution: an "in-rendezvous" phase (phase 1) and a "post-rendezvous" phase (or phase 2). In either phase, a non-reference task may execute on their own and/or request services from other non-reference tasks. The reference tasks only have one phase: phase 2.

A non-reference task or server may have multiple *entries* with different workload characteristics. Different requests may arrive at different entries of a server according to different service requirements. For example in Figure 2.2, T_4 has two entries.

Figure 2.2. Example of an SRVN model

Concurrency is one of the major characteristics in SRVN models. In Figure 2.2, T_1 and T_2 are concurrent tasks both of which send service requests to server T_4 , which raises contention for T_4 .

The SRVN model is characterized by the following parameters:

$s(i, e, p)$ = mean computation time of task i in entry e during phase p ; ($p = 1$ or 2).

$y(i, e, p, j, d)$ = mean number of RV requests made by task i in entry e during phase p to entry d in task j .

In Figure 2.2, the values in square brackets represent the computation times in the first phase and second phase of the corresponding entry. The values in round brackets on each edges are the mean number of RV requests from one task (entry) to another.

Each task phase is a sequence of identical exponentially distributed stochastic execution periods, separated by RV requests to other tasks.

On average, an entry e in phase p of task i makes $Y(i, e, p) = \sum_j \sum_d y(i, e, p, j, d)$ requests to other tasks in one cycle. The mean duration of an execution period or time slice between two requests is $\mu(i, e, p)^{-1} = s(i, e, p) / (Y(i, e, p) + 1)$. After each time slice, the event following every execution period is selected randomly by the task: either makes a request to entry d in task j with probability $y(i, e, p, j, d) / (Y(i, e, p) + 1)$ or completes the cycle with probability $1 / (Y(i, e, p) + 1)$.

SRVN can model software servers as well as hardware servers(e.g. CPU). If several tasks are collocated on the same CPU, the impact of the processor contention must be modelled. We can model the processor contention by transferring to the processor the execution time of the software tasks collocated on it. Since each task phase may have a different execution time, the SRVN task representing the processor will have several entries.

For example in Figure 2.3, T_1 , T_2 , T_3 and T_4 are collocated on the same CPU. The CPU is modelled as an SRVN server (called CPU server) which has only “in RV” phase and is a pure server due to the fact it never makes requests to other servers. If a task T_i , entry e in phase p makes $Y(i, e, p)$ requests to other non-reference tasks, it will perform $Y(i, e, p) + 1$ computation periods during the current phase. Therefore the mean visit ratio from T_i to CPU server is $Y(i, e, p) + 1$. If the computation time $s(i, e, p)$ is transferred to entry d in the CPU server, then the service time of entry e takes the value $s(i, e, p) / (Y(i, e, p) + 1)$.

An example is shown in Figure 2.4 which is the SRVN model for Figure 2.3. T_3 has totally 1.5 mean requests to software servers (T_2 and T_4). It performs 2.5 computations one cycle in average. Its computation time is transferred to entry 3 in CPU server. Time slice in entry 3 is $3/2.5 = 1.2$.

By modelling a hardware resource as an SRVN server, hardware performance measures, such as utilization, queue length, can be obtained by solving the SRVN model.

Figure 2.3. Tasks collocated on one CPU

Figure 2.4. SRVN model for processor contention

2.2 Definition of the RNMTC model

As mentioned before, an RNMTC model consists of a set of complex clients (each described by a precedence graph) which are requesting services from a set of SRVN-like servers. If several clients are described by similar precedence graphs and request the same servers with the same mean visit numbers, they can be grouped in a “class of clients”, as will be shown in section 2.2.2.

2.2.1 Client Precedence Graph

A precedence graph is a collection of nodes and directed edges. Each node represents the execution of a program component. Node execution time is the sum of node computation time, service time at servers and queueing delay due to contention for servers. *Computation time* is the time that a node consumes on its own. The edges represent precedence relationship among nodes. A node is called a *source node* of edge k if the node is the origin of the edge k (e.g. node i in Figure 2.5). A node is called *destination node* of edge k on which edge k terminates (e.g. node j in Figure 2.5).

Figure 2.5. Simple precedence graph

The end of the edge with an arrow is called the head of the edge while the opposite end of edge is called the tail of the edge.

For the edges terminating on a single node, there are two basic relationships: AND and OR. If the edges do not have any symbols, the destination node can not be active until all the source nodes connected with the edges are finished. This is an AND relationship (Figure 2.6 a). If symbol “+” appears on the head of the edges, the destination node can be initiated after any one of the

source nodes completes its execution. This is called an OR relationship (Figure 2.6 b). In order to represent more complex precedence relationships, edges can be grouped hierarchically by drawing ellipses around them (Figure 2.6 c). In Figure 2.6 c, the edges included in the UNION (in ellipses) are connected by OR, and the result of the UNION and the remaining edges are connected by AND.

Figure 2.6. Edge terminating at the same node

For the edge emanating from a single node, there are also two basic relationships: AND and EXCLUSIVE-OR. If there are no symbols on the edges, all the destination nodes are to be activated after the source node is completed. This is an AND relationship (Figure 2.7 a). For the EXCLUSIVE-OR case, a probability p_i is associated with each edge. After the source node completes, only one of the destination nodes is activated. Destination nodes $1, \dots, n$ are initiated with the probabilities $p_1 \dots p_n$ (Figure 2.7 b). Edges emanating from the same node can also be combined hierarchically by applying the UNION relationship (Figure 2.7 c). For example, in Figure 2.7 c, the edges included in the UNION are connected by AND, and the UNION result and the remain-

ing edges are connected by OR.

Figure 2.7. edge emanating from the same node

There is also another kind of relationship called LOOP that may exist in a precedence graph (Figure 2.8). After node i is finished, either node k may be active with probability $1 - p$ or node j is initiated again with loop probability p . Inside the loop any graph structure is allowed.

Figure 2.8. Loop relationship

A precedence graph describing an RNMTTC client may include some or all of the basic rela-

tionships presented above. Each RNMTTC client is executed cyclically. The execution of a client is characterized by a number of different states. Each state corresponds to a set of particular tasks (nodes) that are active concurrently. In Figure 2.9 (a), are shown two simple graphs to give examples of single-threaded and multi-threaded execution states. The client in Figure 2.9 (a) has three execution states $\{a_1, a_2, a_3\}$, all of them single-threaded. On the other hand, Figure 2.9 (b) expresses an AND relationship. After node b_1 finishes execution, the execution thread is forked to two parallel branches and then join them before starting a new cycle. The client in Figure 2.9 (b) has four execution states $\{b_1, b_2b_3, b_2, b_3\}$, only the first state b_1 is single-threaded, while the last three states are multi-threaded, since the last three occur after a fork but before the corresponding join.

Figure 2.9. Simple precedence graph model

A more complex example of a client graph that executes cyclically is given in Figure 2.10 in which multiple relationships are included. And the loop may also exist. After the termination of a node, the operation routine may loop back with probability p_l , or invoke its successor with probability $1 - p_l$.

In a precedence graph, a node which has a successor node but has no predecessor node is defined as an *initialization node*, a node which has a predecessor node but has no successor node is called *termination node*, and a node which has both a predecessor node and a successor node is named *middle node*. Node 1 in Figure 2.10 is a initialization node, Node 8 is a termination node

and the rest are middle nodes. Another special node is called *isolation node* which has neither a predecessor nor a successor node.

Figure 2.10. Precedence graph model

2.2.2 RNMTC Model Parameters

As stated before, an RNMTC model contains a set of clients and a set of servers. Each node in a client precedence graph is called *client task*. Figure 2.11 shows a simple RNMTC model with one client. During the execution of the client according to its precedence graph, some of client tasks may make service requests to servers. The communication between a client task and a server is by RV. When a client task is being served by a server, the client task is blocked until the server sends a reply message. A client task computes for a time, and then either decides to send another request to a server or to terminate itself, activating its successor(s). After the execution of the ter-

mination node, a new cycle will be started.

Figure 2.11. Simple RNMTC model

Figure 2.12 illustrates an RNMTC model with multiple clients.

Each client is described by an independent precedence graph. All clients execute concurrently, sharing the same set of servers. The contention for servers exists not only within a client but also among different clients. If several clients are described by identical precedence graphs

and they request exactly the same servers, then the clients can be grouped in the same "class"..

Figure 2.12. Multiple client RNMTC model

The parameters of an RNMTC model are the computation times and mean visit numbers of client tasks to servers and the service times and mean visit numbers of servers to other servers. The following notations will be used for RNMTC model parameters.

C_a = total number of classes in the client system

$C(i)$ = total number of clients in class i

$N(i)$ = total number of client tasks in one client of class i

K = total number of servers in the system

$U(j)$ = total number of entries in server j

$c(i, t)$ = average computation time for client task t in class i per job cycle

$s(i, e, p)$ = average service time in entry e during phase p ($p = 1$ or $p = 2$) of server i per visit

$S(i, e)$ = average service time in entry e of server i in two phases per visit

$$S(i, e) = s(i, e, 1) + s(i, e, 2)$$

$v(i, t, j, u(j))$ = average number of visits made by client task t in class i to entry u of server j per job cycle

$v(i, t, j)$ = average number of visits made by client task t in class i to server j per job cycle

$$v(i, t, j) = \sum_{u=1}^{U(j)} v(i, t, j, u(j))$$

$y(i, e, p, j, d)$ = mean number of RV requests made by server task i in entry e during phase p to entry d in server task j .

The principal performance measures for an RNMTTC model are the cycle time for each of its clients. All other measures (throughputs, server utilizations, etc.) can be derived from client cycle times and model parameters.

Chapter 3

RNMTC Simulator

3.1 Motivation for Simulation

Simulation is an important tool for performance evaluation. Simulation involves constructing a model for the behavior of a system processing an appropriate abstraction of the workload. The major advantage of simulation is its generality and flexibility. A number of important issues must be considered in simulation.

1. It must be decided what to simulate and what not to simulate and at what level of detail. Simply duplicating the detailed behavior of the system is usually unnecessary and prohibitively expensive.

2. A careful experimental design is essential to keep the simulation cost down.

Simulation is frequently used to examine the accuracy of an analytic approximate algorithm such that the modeler has an idea if the accuracy of the algorithm is in acceptable limits. This is the main motivations for building the RNMTC simulator in this thesis. The analytic results of the approximate method for RNMTC models introduced in Chapter 4 will be compared with simulation results for a large number of test cases in Chapter 6. Another use of the RNMTC simulator is to assess the correctness of the upper bounds and lower bounds for certain classes of RNMTC models studied in this thesis.

The RNMTC simulator is built on top of a distributed system simulator kernel called PARASOL, which is briefly presented in the next section.

3.2 Brief Introduction to PARASOL

PARASOL is a simulation tool developed at the School of Computer Science, Carleton University. As a simulator, PARASOL allows the user to define the description and the behavior of both hardware and software for distributed and/or parallel computer systems. It can be used to predict the performance of the distributed/parallel systems or as a prototyping tool. The detail description of PARASOL simulator system can be found in [Nei_1] and [Nei_2] and this section gives only a brief introduction.

- Organization

Figure 3.1. PARASOL organization

Figure 3.1 shows the logical organization of the PARASOL simulator. The kernel is the implementation of PARASOL simulator and written entirely in C programming language. It supports two user interfaces: genesis tasks and PARASOL library. Genesis is a standard, PARASOL system defined task interface which is called automatically when the PARASOL system is started. This is the first task which allows the user to define the simulated hardware specification and the initial configuration of the system with respect to simulated application tasks.

The PARASOL library contains the functions called by the user defined tasks and the hardware specification. When the system is running, it will simulate the behaviors of user defined

tasks running on the user described hardware system.

Hardware Specification

The PARASOL can simulate a set of hardware resources, such as CPUs, buses and links between different hardware components.

The PARASOL supplies a set of methods of how to define the simulated system hardware. This can be done by “genesis” task by means of calling three functions: build_node, build_bus and build_link. Nodes must be constructed before the construction of buses and links.

Task Management and Communication

After the hardware is built, the tasks will be assigned on the system. The tasks can model either hardware or software components. It is the responsibility of the user to create, activate and manage all the tasks except one task - *genesis* which is managed by PARASOL.

The PARASOL supplies the methods of creating, resuming, killing and suspending tasks. The tasks can sleep, be awakened and migrated. The tasks can communicate with each other by sending/receiving message and broadcasting.

Statistic Report

In order to obtain the desired performance measures, PARASOL provides for the automatic capture of hardware device utilization statistics by means of the stat_flag arguments in “build” instructions. To meet user’s requirement, PARASOL also supplies a mechanism for two types of user defined statistical data. The first is “TIME” used to capture device utilization statistics and the second is “POP” used for population statistic, such as mean response time, mean waiting time, etc.

3.3 Architecture and Implementation of the RNMTCSimulator

The RNMTCSimulator is a discrete events simulator. The RNMTCSimulator gives the abstract simulation results when analytic solution is discussed.

Function of RNMTC Simulator

The RNMTC model may have multiple classes of client and each class may contain multiple clients. Each client is described by a precedence graph which may have the following task (node) types: initialization task, middle task, termination task and isolation task.

A client precedence graph may include the following types of precedence relationships, as described in section 2.2.1: AND, OR, Exclusive-OR, UNION and LOOP.

The RV communication between tasks is also simulated. The servers can have multiple entries and multiple phases.

Implementation of RNMTC Simulator

Each client has its own “manager” PARASOL task which does not consume any CPU time. Each “manager” has the following responsibilities:

(a) Activate the tasks belonging to the client which the “manager” is in charge of. The manager sends a message to all “initialization tasks” of this client to start an instance of the simulation representing a client in RNMTC system.

(b) Record the job performance measures after the “manager” receives the completion message from all its termination tasks.

In order to reduce the storage requirements for data structures, the redundancy of information is reasonably reduced as much as possible. Each individual task has its own identifier including task-class, task-client, task-name and task-id. The last one is assigned by the PARASOL kernel, whereas the first three are defined by user. However, the same parameters for each task in the same class, such as mean computation time, mean visit ratio to servers, are stored only once. Different clients in a class share the parameter data.

After the simulation is activated by genesis, each “manager” task starts activating and controlling its own subsystem. By looking at the task identifier with the same class and client, the “manager” can find its client tasks and get the task parameters from a data record.

The data specification for the simulated system is inputted by a program named “datainput”

and RNMTC simulator reads data and runs the simulation according to the data specification.

The RNMTC simulator was extensively used to analyze different test case that will be presented in Chapter 5 and Chapter 6.

Instructions for running the RNMTC simulator can be found in Appendix.

Chapter 4

Approximate Analytic Solution of RNMTC

4.1 Solution Introduction

The solution for RNMTC models introduced in this chapter is based on hierarchical decomposition: at the higher level, the system behavior is represented by a Markov Chain (MC) model whose states corresponding to all possible combinations of clients executions states; at the lower level, an SRVN with simple clients corresponds to each MC state.

Several MVA approximate solutions for SRVN were previously introduced. The solution presented in [PW91] and [Pet93] was used here.

Various decomposition methods are known for the analysis of concurrent software systems described by precedence graphs. For example, [TB86] introduces a algorithm which uses a decomposition method to obtain the cycle time of a tasking system modelled by a simple precedence graph without loop or probabilistic branching. Also in [KME89] is introduced a modelling methodology for the analysis of distributed software modelled by a complex precedence graph with loops and branching running on a hardware system modelled by QNs. The approach presented in this thesis is similar to that in [TB86] and [KME89] with the following differences:

- In our case the high level model represents a set of independent clients that may belong to several classes each one described by a precedence graph, instead of a single instance of a tasking system modelled by a single precedence graph.
- The lower level models are SRVNs instead of QNs, where the servers act also possibly as

clients to other servers (allowing for multi-layered service), and the service can be offered in two distinct phases (one “in RV” while the server is processing requests from a client task and the client task is blocked and the second “post RV” which is active in parallel with the client task).

The model of RNMTC is solved by hierarchical decomposition: at the higher level, the system behavior is represented by a *Markov Chain* (MC) model and at the lower level, the system is represented by SRVN models as described in the following subsections.

4.2 Higher Level by MC Model

MC can represent the behavior of a number of client systems. The states of MC correspond to all possible combinations of client execution states expressed in terms of enabled graph nodes. The transition among states are determined as a function of precedence relationships among the states. The transition rates μ among the states of the MC are determined from the throughput of the system in processing various combinations of client tasks. Figure 4.1 is a single client subsystem with node 1 and node 2 execute in parallel and then join in node 3.

Figure 4.1. A single client system

The MC corresponding to the execution of the system is given in Figure 4.2. In this simple case, the MC is acyclic, except for the final transition from the completion state to the initial state. This implies the repetitive execution of the client system. States S_1 , S_2 and S_3 are multi-threaded since they occur after a fork but before the corresponding join, whereas state S_4 is single-threaded.

If a system consists of multiple classes, MC becomes more complicated. Figure 4.3 is one example of such a system. Class 1 has 2 clients while class 2 has only one client. The MC graph

Figure 4.2. MC for simple system

for this system is shown in Figure 4.4.

Figure 4.3. Precedence graph with multiple classes

Figure 4.4. MC for multiple client system

In Figure 4.4, the following notations are used. Let $CT(i, j, k)$ be the client task (node) k in

client j of class i . Thus, for example, client task 3 of client 1 from class 2, in Figure 4.3, can be denoted as CT(2, 1, 3). Each MC state includes a number of client tasks which may be members of different classes and/or different clients. Figure 4.4 shows the MC corresponding to these concurrent clients shown in Figure 4.3. In the MC figure, the client tasks have been renumbered as follows:

$$1 = \text{CT}(1, 1, 1), 2 = \text{CT}(1, 1, 2), 3 = \text{CT}(1, 2, 1), 4 = \text{CT}(1, 2, 2),$$

$$5 = \text{CT}(2, 1, 1), 6 = \text{CT}(2, 1, 2), 7 = \text{CT}(2, 1, 3)$$

There are a lot of cyclic arcs among different states due to the fact that individual client runs independently. The client tasks in each state may compete for servers. The queueing delay due to contention for servers is obtained from the lower level model.

4.3 Lower level by SRVN Model

At the lower level, each MC state can be viewed as an SRVN model in which the client tasks active in that state are reference tasks in SRVN, while the RNMTTC servers are non-reference tasks in SRVN. The servers are called directly or indirectly by the client tasks in this state.

A few approximate MVA solution methods for SRVN models with simple clients were introduced in [MWNP88] [MWNP89] [PW91] [WNMP88] [WNPM89] and [Woo89] for various classes of SRVN models. Each method solves iteratively a set of nonlinear equations relating tasks and entry throughputs, mean duration of phases and mean waiting for queueing and for service of the RV requests. These methods take a common MVA-like approach, but differ in the way they compute the queueing delays. In the algorithm for RNMTTC model is embedded the SRVN method presented in [Pet93]. The main MVA equations are reviewed below.

In SRVN terminology, both the client tasks and the servers are named *tasks*. The client tasks are reference tasks, and their throughputs represent the main performance results of the model. For notation consistency it is assumed that a reference task has one entry with a single "post RV" phase.

The following introduction referenced from [PW91] and [Pet91] gives the basic procedure to compute performance measures for SRVN system.

The mean duration of phase p of task i , entry e , including the waiting for RV requests, is given exactly by:

$$x(i, e, p) = s(i, e, p) + \sum_j \sum_{d \in E(j)} y(i, e, p, j, d) (w(i, j, d) + x(j, d, 1)) \quad (4-1)$$

where $E(j)$ is the set of entries of task j , and $w(i, j, d)$ is the queuing delay suffered by a RV request from task i , entry e , phase p to task j entry d .

The throughput f_r of a reference task r (where $r = 1, R$) is given by the following exact equation, due to the fact that, by definition, a reference task has one entry with a second phase only:

$$f_r = 1 / x(r, 1, 2) \quad (4-2)$$

The throughput f_j of a non-reference task is given as the sum of its entry throughputs ϕ_{jd} :

$$f_j = \sum_{d \in E(j)} \phi_{jd} \quad (4-3)$$

The entry throughput ϕ_{jd} of a non-reference task j is related to the throughputs of its clients by the synchronization imposed by RV, as shown in the following traffic equations:

$$\phi_{jd} = \sum_i \sum_{e \in E(i)} \sum_p y(i, e, p, j, d) \phi_{ie}, \quad j = R + 1, N; d \in E(j) \quad (4-4)$$

By solving the set of equations (4-4), the non-reference throughputs can be obtained directly as a linear combination of the reference throughputs, in the form:

$$\phi_{jd} = \sum_{r=1}^R a_{jdr} f_r \quad j = R + 1, N \quad (4-5)$$

The queuing delay $w(i, j, d)$ suffered by i at server j for entry d due to other requests (from task k to entry g) found in service in phase 1 or 2, or in queue is computed as:

$$\begin{aligned} w(i, j, d) = & \sum_{k=1}^N \sum_{g \in E(j)} (A_{ijd}^{kg1} (v(j, g, 1) + x(j, g, 2)) + A_{ijd}^{kg2} v(j, g, 2) \\ & + B_{ijd}^{kg} (x(j, g, 1) + x(j, g, 2))) \end{aligned} \quad (4-6)$$

where v_{jgp} is the mean residual life of server j , entry g , phase p , and A and B are the arrival-instant probabilities defined for a system in steady-state as follows:

A_{ijdp}^{ksp} = the probability that a RV request from i arriving to task j for entry d finds j busy executing entry g phase p to serve a request from another task k

B_{ijdp}^{ksp} = the probability that a RV request from i arriving to j for entry d finds in queue a request from other task k to entry d of j

The SRVN algorithm repeats the following procedure, until reaching convergence. First consider that phase durations x and throughputs f are known, and compute new values for the arrival instant probabilities and for the waiting w (4-6). Then, by using the new waiting, new values are determined for phase durations from (4-1) and for throughputs from (4-2), (4-5) and (4-3).

4.4 Decomposition Method

The solution for RNMTTC model is based on the following assumptions:

- All execution time of client tasks and service time of servers are exponentially distributed.
- The server queue has a FIFO discipline to serve its clients.

The analytic method obtains an approximate, but reasonably accurate performance results for RNMTTC model. The approximate analytic solution for RNMTTC proposed in this research proceeds according to the following steps:

a) Construct the high level MC model based on the precedence graph. Each state in MC represents combinations of client execution states.

b) Determine transition rates μ from the throughputs corresponding to each client task in the state and derive the transition rate matrix $Q = [\mu]$.

-- If no requests are made to servers by any of client task in the state, the transition rate is the inverse of the execution time for each client task. For example, in Figure 4.1, client task 2 calls servers while other client tasks do not. The transition rate $\mu_1(1)$ for client task 1 in state s_3 (Figure 4.2) is the inverse of client task 1 execution time.

– If some or all client tasks call servers, get the transition rate as reference task throughputs obtained by solving the SRVN model. For example, in Figure 4.2, in order to get the transition rates for client tasks in state s_1 , the SRVN solver is called to derive the throughputs for each client task in the state.

c) Build the MC balance equation $YQ = 0$, where Y is a set of state probabilities $p_s(i)$. Solve the system of linear equations to get $p_s(i)$ for state i .

d) Compute the desired performance measures for the client system (as described in section 4.5).

The usage of RNMTC solver can be found in Appendix.

4.5 Performance Measures

Different performance measures can be obtained for single client RNMTC system and multiple client RNMTC system respectively. The following sections present the output from the analytic solution method.

4.5.1 Performance Measures for Single Client RNMTC System

In Single client RNMTC system, the overall performance and the performance on individual state and task can be computed as follows:

- Cycle time for the client T_c

Let $\sum \mu(1)$ be the total outgoing transition rate from state 1 in MC, $p_s(1)$ be state probability in state 1, $p_1(s)$ is the loop probability from state s to state 1 caused by the loop construct in precedence graph. Thus, according to the flow balance theory,

$$T_c = 1 / ((p_s(1) \sum \mu(1)) \cdot \prod_{i=2}^S (1 / (1 - p_1(i))))$$

where S is the total number of states in MC.

- System throughput X

$$X = 1 / T_c$$

- State i active time $S(i)$

$S(i)$ is the time spent on state i

$$S(i) = T_c p_s(i)$$

- Parallelism distribution $P_p(n)$

$P_p(n)$ is a probability that n tasks are active simultaneously during the client execution.

Clearly, $\sum_{i=1}^S p_s(i) = 1$. Therefore,

$$P_p(n) = \sum_{i \in R} P_s(i)$$

Where R is the set of states in which the number of concurrent tasks is n .

- Average parallelism A_p

Average parallelism A_p is the average number of concurrent tasks during the job execution.

$$A_p = \sum_{n=1}^N (n P_p(n))$$

where N is the total number of client tasks in the client system.

- Client task t active time $e(t)$. (t is a node in precedence graph).

$$e(t) = T_c \sum_{i \in E} P_i(i)$$

where E is the set of states in which client task t is executing.

- Client task t utilization $U(t)$

$U(t)$ is the utilization of client task t as the percentage of time client task t is “busy” (computation or waiting for RV replies) with respect to the completion time of the whole precedence graph.

$$U(t) = e(t) / T_c$$

4.5.2 Performance Measures for Multiple Clients RNMTC System

In multiple client system, we are more interested in the performance of individual client.

- Client Throughput $X_f(g)$ in class g

Due to the fact that all clients in a class are statistically identical, we only need to compute the throughput for a client, let this be client 1. Select a reference client task t in client 1 in the precedence graph. The node t is visited only once in one instance. $\mu(i, t)$ is the transition rate of termination of node t from MC state s .

$$X_f(g) = \sum_{i \in Y} (P_s(i) \mu(i, t))$$

Figure 4.5 shows 2 classes of clients with one client each. They have the same structure of precedence graph but have different computation time for client tasks so that they belong to different classes.

Figure 4.5. Multiple client system

Figure 4.6 shows the MC for the two classes system shown in Figure 4.5. In order to compute the throughput $X_f(2)$ for class 2, node 4 is selected as a reference node of the client. Node 4 is active in state S_3 and state S_4 . $\mu(S_3, 4)$ is the transition rate corresponding to the termination of node 4 in state S_3 while $\mu(S_4, 4)$ is the transition rate corresponding to the termination of node 4 in state S_4 . $P_S(S_3)$ and $P_S(S_4)$ are state S_3 and S_4 probabilities, respectively. Therefore,

$$X_f(2) = P_S(S_3) \mu(S_3, 4) + P_S(S_4) \mu(S_4, 4)$$

Figure 4.6. MC states

- Class throughput $X_c(g)$ in class g

If there are $C_f(g)$ clients in class g , class throughput is expressed as follows:

$$X_c(g) = C_f(g) X_f(g)$$

- Client cycle time $T_{cf}(g)$ in class g

$$T_{cf}(g) = 1 / X_f(g)$$

- Parallelism distributions $P_p(n)$

This is the same definition and computation as for the single client system.

$$P_p(n) = \sum_{i \in R} p_s(i)$$

- Average parallelism A_p

Average parallelism A_p is the average number of concurrent tasks during the job execution.

$$A_p = \sum_{n=1}^Z (n P_p(n))$$

where Z is the total number of client tasks in the multiple client RNMTC system.

$$Z = \sum_{i=1}^{C_s} c(i) N(i)$$

Chapter 5

Bounds

5.1 Motivation

Analytic techniques are used widely in the performance evaluation of concurrent systems. Test cases described in chapter 6 will prove that the algorithm presented in Chapter 4 produces a reasonable degree of accuracy in evaluating system performance. The algorithm, however, is based on assumptions about distribution of execution times and service times which may not always be satisfied on certain environments. Performance bounds which are based on weaker assumptions about system behavior can be used in such a situation.

The bounds presented in this chapter are applicable to systems with fixed computation and service time and general distributions of computation times and FIFO service at servers. These weaker conditions make the bounds robust and applicable to a wide range of systems characterized by rendezvous style communications. For systems in which the distribution of computation times are unknown, the exact analytic analysis may be intractable and bounds may be the only performance result that can be computed for such systems.

Upper bounds and lower bounds on task throughputs for a SRVN model are presented in [MWNP91]. The authors extract a set of bounds for several kinds of system: system with single reference task or multiple reference tasks, systems characterized by processor contention etc. But [MWNP91] only discusses the bounds based on the assumption that each server has only a single entry and the tasks have no precedence relationship. This research expands the bounds analysis to RNMTTC model characterized by multiple entry servers and single multi-threaded client. The

servers are restricted to be pure servers; that is, the server submodel has a single level without nested requests between servers. Computation of the upper and lower bounds on client completion times for certain classes of the RNMTTC are presented in this chapter. The following specific types of RNMTTC system are studied in this thesis:

- Deterministic system (section 5.3)
- Systems with fixed computation times and service times (section 5.3.1)
- Systems with general computation and service time distribution (section 5.3.2)
- Systems with probabilistic branching (section 5.4)
- Systems with loop construct (section 5.5)

5.2 Parameters

As stated in section 2.2, a node precedence graph is used to model a client system. A client cycle starts with the start of execution of initialization node(s) (or initialization client task) and ends with the completion of execution of termination node(s) (or termination client task). In order to compute the performance bounds for a client, some notations must be defined first. Some of these are already defined in section 4.4 in the content of the approximate analysis. However for the sake of clarity, these are listed here again.

N = total number of client tasks in the client

T_c = client cycle time

=> the difference between the time at which the initialization client tasks start execution and the time at which all termination client tasks complete

$c(t)$ = average computation time for client task t

$e(t)$ = average execution time for client task t

$e(t)$ includes $c(t)$ and also service time and queueing delay

$v(t, j, u)$ = average number of visits (or visit ratio) made by client task t to entry u of server j

$v(t, j)$ = average number of visits (or visit ratio) made by client task t to server j

$$v(t, j) = \sum_{u=1}^U v(t, j, u)$$

where U is total number of entries in server j . If only a single entry exist, $v(t, j, u)$ can be expressed as $v(t, j)$.

$e_s(t)$ = time at which client task t starts to execute

$e_f(t)$ = time at which client task t completes execution

$t < \bullet l \Rightarrow$ client task t is a direct predecessor of client task l

$t < \bullet \bullet l \Rightarrow$ client task t is a direct or indirect predecessor of client task l (which means that $t < \bullet l$ or $t < \bullet z$ and $z < \bullet \bullet l$)

$t \bullet > l \Rightarrow$ client task t is a direct successor of client task l

$t \bullet > \bullet l \Rightarrow$ client task t is a direct or indirect successor of client task l (which means that $t \bullet > l$ or $t \bullet > z$ and $z \bullet > \bullet l$)

$\mathcal{P}(t)$ = set of all direct predecessors of client task t

$\mathcal{P}^*(t)$ = set of all predecessors (direct or indirect) of client task t

$\mathcal{S}(t)$ = set of all direct successors of client task t

$\mathcal{S}^*(t)$ = set of all successors (direct or indirect) of client task t

$C(t)$ = concurrency set for client task t (set of all client tasks that can execute concurrently with client task t)

K = total number of servers in the system

$s(i, e, p)$ = average service time for entry e in phase p ($p = 1$ or $p = 2$) at server i per visit

$s(i, e)$ = average service time for entry e at server i (both phases included)

$$s(i, e) = s(i, e, 1) + s(i, e, 2)$$

$P(t_1, t_2, k)$ = probability that client task t_1 is delayed by client task t_2 at server k ,

where $t_2 \in C(t_1)$

The bounds analysis is based on the following lemma.

Lemma 1. $P(t_1, t_2, k) \leq \min(1, \frac{v(t_2, k)}{v(t_1, k)})$

Proof: A technique similar to the one presented in [MWNP92] is used for proving the lemma. Consider client task t_1 and client task t_2 make requests to server k at which FIFO discipline is exploited. The mean number of visits made during one job cycle by t_1 and t_2 are given by $v(t_1, k)$ and $v(t_2, k)$ respectively.

• Case 1: $\frac{v(t_2, k)}{v(t_1, k)} > 1$

$P(t_1, t_2, k)$ is a probability that cannot exceed 1.

• Case 2: $\frac{v(t_2, k)}{v(t_1, k)} \leq 1$

By contradiction, suppose that $P(t_1, t_2, k) > v(t_2, k) / v(t_1, k)$. Let $q(t_1, t_2)$ be the mean number of visits during one job cycle of t_1 in which t_1 found t_2 already enqueued or in service at server k .

Since the mean number of visits made by t_2 during one job cycle is $v(t_2, k)$, we have:

$$q(t_1, t_2) \leq v(t_2, k) \tag{5-1}$$

Now $P(t_1, t_2, k) = q(t_1, t_2) / v(t_1, k)$

Thus $P(t_1, t_2, k) > v(t_2, k) / v(t_1, k)$ implies

$$q(t_1, t_2) > v(t_2, k)$$

This is in contradiction with 5-1.

Based on the analysis of case 1 and case 2, we get

$$P(t_1, t_2, k) \leq \min(1, \frac{v(t_2, k)}{v(t_1, k)}) \quad \blacksquare$$

5.3 Bounds on the Performance of Deterministic System

A deterministic system is characterized by an AND parallelism: after the completion of a task, all of its successors start to execute. Each client task or server is assumed to be executing on its own processor so that there is no contention for hardware. But contention for software servers may exist.

First, we discuss the lower bound and upper bound on client task execution times in the system.

The lower bound is based on the optimistic situation in which each client task is served without any queueing delay at a server. Therefore, the lower bound on execution time for client task t , $e^-(t)$, includes the task's own computation time $c(t)$ and the total first phase service time at each server visited by t . Thus,

$$e(t) \geq e^-(t) = c(t) + \sum_{k=1}^K \sum_{u=1}^U v(t, k, u) s(k, u, 1) \quad (5-2)$$

The upper bound on the execution time for client task t , $e^+(t)$, not only includes all the terms included in 5-2, but also the queueing delay experienced by client task t at each server. In the worst case, whenever client task t arrives at server k , the server is executing its longest post RV phase corresponding to a previous request; this term is expressed by $\sum v(t, k) \max(s(k, u, 2))$. Each task y in the concurrency set $C(t)$ can introduce a queueing delay for t at any server. The upper bound on this queueing delay experienced by t per visit of k is given by $P^+(t, y, k) s(k, u)$, where $P^+(t, y, k)$ is the upper bound on the probability that y is enqueued at k when t arrives. If y visits multiple entries of k then u is that entry for which the average service time (including both phases)

is the highest. Thus,

$$e^+(t) = c(t) + \sum_{k=1}^K \sum_{u=1}^U v(t, k, u) s(k, u, 1) \\ + \sum_{k=1}^K v(t, k) \max_{u=1}^U (s(k, u, 2)) + \sum_{k=1}^K v(t, k) \sum_{y \in C(t)} \sum_{u=1}^U P^+(t, y, k) s(k, u)$$

Where $P(t, y, k)$ is the probability of client task t being delayed by y at server k . As shown in lemma 1:

$$P(t, y, k) \leq P^+(t, y, k) = \min(1, \frac{v(y, k)}{v(t, k)}).$$

Therefore,

$$e(t) \leq e^+(t) = c(t) + \sum_{k=1}^K \sum_{u=1}^U v(t, k, u) s(k, u, 1) \\ + \sum_{k=1}^K v(t, k) \max_{u=1}^U (s(k, u, 2)) \\ + \sum_{k=1}^K v(t, k) \sum_{u=1}^U \sum_{y \in C(t)} \min(1, \frac{v(y, k)}{v(t, k)}) s(k, u) \quad (5-3)$$

The bound on client completion time is based on the execution times of its client tasks and is presented next.

5.3.1 Bounds on Systems with Fixed Computation Times and Service Times

Inequalities are derived in previous subsection for computing bounds on the execution time of individual client task execution times. This section discusses the bounds on client completion time for systems in which client task computation times and service times of servers are fixed.

Since a client task t can start execution only when all its predecessors have completed. We have:

$$e_f^-(t) = \max_{i \in \mathcal{P}(t)} \{e_f^-(i)\}$$

The time at which t completes is given by

$$e_f(t) = e_s(t) + e(t)$$

Since a job completes when all its tasks have completed, we obtain the following expression for the job completion time:

$$T_c = \max_{i=1}^N \{e_f(i)\} \quad (5-4)$$

Let $e_s^+(t)$ be the latest starting time for client task t , $e_f^+(t)$ be the latest completion time for t . Client task t can start only when all its predecessors have completed execution, thus

$$e_s^+(t) = \max_{i \in \mathcal{P}(t)} \{e_f^+(i)\}$$

The upper bound on completion time of t is equal to the sum of its latest starting time and the upper bound on its execution time:

$$e_f^+(t) = e_s^+(t) + e^+(t) \quad (5-5)$$

From 5-3 and 5-5, we get

$$\begin{aligned} e_f^+(t) &= e_s^+(t) + c(t) + \sum_{k=1}^K \sum_{u=1}^U v(t, k, u) s(k, u, 1) \\ &\quad + \sum_{k=1}^K v(t, k) \max_{u=1}^U (s(k, u, 2)) \\ &\quad + \sum_{k=1}^K v(t, k) \sum_{u=1}^U \sum_{y \in \mathcal{C}(t)} \min \left(1, \frac{v(y, k)}{v(t, k)} \right) s(k, u) \end{aligned}$$

Since the client completes execution when its last client task completes, the upper bound on

T_c is given by the following expression:

$$T_c^+ = \max_{i=1}^N \{e_f^+(t)\} \quad (5-6)$$

The derivation of a lower bound on T_c is presented next:

Let $e_f^-(t)$ be the earliest completion time for client task t , $e_s^-(t)$ be the earliest starting time for t . Then,

$$e_s^-(t) = \max_{i \in \mathcal{A}(t)} \{e_f^-(t)\}$$

Then, the lower bound on execution time of t is given by

$$e_f^-(t) = e_s^-(t) + e^-(t) \quad (5-7)$$

Where $e^-(t)$ is the lower bound on the execution time of t .

From 5-2 and 5-7 we get

$$e_f^-(t) = e_s^-(t) + c(t) + \sum_{k=1}^K \sum_{u=1}^U v(t, k, u) s(k, u, 1)$$

The lower bound on client cycle time is then computed as:

$$T_c^- = \max_{i=1}^N \{e_f^-(t)\} \quad (5-8)$$

Computation of bounds for system with general computation and service time distributions is presented next.

5.3.2 Bounds on Systems with General Computation and Service Time Distribution

The bounds derived from fixed time systems are not valid for systems in which the computation time of client tasks and service time of servers are not fixed. A class of “fork-join” systems in which the client task execution times and server service times (in each phase) are characterized by IFR (Increasing Failure Rate) distribution is considered here.

Consider a client with $n+2$ client tasks from t_0 to t_{n+1} . The precedence graph for the client is shown in Figure 5.1. Client tasks t_0 and t_{n+1} have no requests to any servers while the client tasks from t_1 to t_n may have requests to servers. Each client task executes on its own processor.

Figure 5.1. Concurrent task in unbounded system

The expected client cycle time is given by

$$T_c = e(t_0) + E(\lambda) + e(t_{n+1})$$

Where $E(\lambda)$ is the expected time interval measured from the termination of client task t_0 to

the beginning of client task t_{n+1} .

The client cycle time is thus bounded as described below.

$$T_c \leq T_c^+ = e^+(t_0) + e^+(t_{n+1}) + E^+(\lambda) \quad (5-9)$$

$$T_c \geq T_c^- = e^-(t_0) + e^-(t_{n+1}) + E^-(\lambda) \quad (5-10)$$

In order to obtain the upper and lower bounds on T_c , we need to compute $E^+(\lambda)$ and $E^-(\lambda)$.

Clearly,

$$E(\lambda) = E[\max(e^-(t_1), e^-(t_2), \dots, e^-(t_n))] \quad (5-11)$$

Where $e^-(t_i)$ ($i = 1, \dots, n$) is a random variable for the execution time of client task t_i .

Noting that λ is the random variable for the time interval measured from the termination of t_0 to the beginning of t_{n+1} , we get

$$\lambda = \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n)) \quad (5-12)$$

Clearly,

$$\lambda \leq e^-(t_1) + e^-(t_2) + \dots + e^-(t_n)$$

By using 5-11 and noting that $e^+(t_i)$ is an upper bound on $e(t_i)$, we get:

$$E(\lambda) \leq E^+(\lambda) = e^+(t_1) + e^+(t_2) + \dots + e^+(t_n) = \sum_{m=1}^n e^+(t_m) \quad (5-13)$$

Therefore, from inequalities 5-9 and 5-13, we get

$$T_c^+ = e^+(t_0) + e^+(t_{n+1}) + \sum_{m=1}^n e^+(t_m) \quad (5-14)$$

Because client task t_0 and t_{n+1} have no requests to servers, their execution times only include their computation times. That is,

$$e^+(t_0) = c(t_0) \text{ and}$$

$$e^+(t_{n+1}) = c(t_{n+1})$$

Therefore, from 5-14, the upper bound on T_c can be described by

$$T_c^+ = c(t_0) + c(t_{n+1}) + \sum_{m=1}^n e^+(t_m) \quad (5-15)$$

Note that $e^+(t_i)$ ($i = 1, \dots, n$) can be found by using 5-3.

Following the derivation of 5-3, one observes that

$$\begin{aligned} e(t) \leq e^+(t) = c(t) + \sum_{k=1}^K \sum_{u=1}^U v(t,k,u) s(k,u,1) \\ + \sum_{k=1}^K v(t,k) R_k^+ + \sum_{k=1}^K Q^+(t,k) \end{aligned} \quad (5-16)$$

Where $\sum_{k=1}^K Q^+(t,k)$ is upper bound of the queueing delay (cover all visits) experienced by client task t at server k with other concurrent client tasks.

$$\sum_{k=1}^K Q^+(t,k) = v(t,k) \sum_{u=1}^U \sum_{y \in c(t)} P^+(t,y,k) S(k,u) \quad (5-17)$$

R_k^+ is the upper bound on the expected remaining time for the completion of the post RV phase of service due to a previous server request.

Since the second phase service times at any entry u of k is IFR distributed, the remaining service time when t arrives is upper bounded by $s(k,u,2)$ [Kle]. Thus,

$$R_k^+ = \max_{u=1}^U (s(k,u,2)) \quad (5-18)$$

Now by using Lemma 1, equation 5-16, 5-17 and 5-18, T_c^+ can be derived as in 5-15.

The lower bound T_c^- can be derived in the following manner.

From equation 5-12, we can get

$$\lambda = \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n))$$

Thus $\lambda \geq e^-(t_i)$ for $i = 1, \dots, n$ and $E(\lambda) \geq E[e^-(t_i)]$ for $i = 1, \dots, n$.

Noting that $e^-(t_i)$ is a lower bound on $E[e^-(t_i)]$, we get:

$$E(\lambda) \geq E^-(\lambda) = \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n))$$

Now by using 5-10, we get:

$$T_c^- = e^-(t_0) + e^-(t_{n+1}) + \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n)) \quad (5-19)$$

Since $e^-(t_0) = c(t_0)$ and $e^-(t_{n+1}) = c(t_{n+1})$, we get:

$$T_c^- = c(t_0) + c(t_{n+1}) + \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n)) \quad (5-20)$$

Note that $e^-(t_i)$ ($i = 1, \dots, n$) can be found by using 5-2.

The system shown in Figure 5.1 is a single layer system. The upper bound T_c^+ and lower bound T_c^- derived in 5-15 and 5-20 for such a system can be expanded from single layer to multi-

ple layers shown in Figure 5.2.

Figure 5.2. Multiple layer system

There are L layers in the system. There are n_l concurrent client tasks in layer l . $t_{l,i}$ in Figure 5.2 is client task i in layer l . In each layer, the concurrent client tasks join at the same client task (e.g. $t_{2,0}, \dots, t_{L,0}$) and the concurrent client tasks in next layer fork from these client tasks. Client tasks $t_{1,0}, \dots, t_{L,0}$ have no service requests to servers whereas other client tasks may make calls to servers.

Let T_{cl} be the completion time for layer l . T_{cl} is the time between the beginning of client task $t_{l,0}$ to the termination of client task $t_{l+1,0}$. T_{cl}^+ and T_{cl}^- are upper bound and lower bound on T_{cl} respectively and can be found by using 5-15 and 5-20.

$$T_{cl}^+ = c(t_{l,0}) + c(t_{l+1,0}) + \sum_{m=1}^{n_l} e^+(t_{l,m}) \quad (5-21)$$

$$T_{cl}^- = c(t_{l,0}) + c(t_{l+1,0}) + \max(e^-(t_{l,1}), e^-(t_{l,2}), \dots, e^-(t_{l,n_l})) \quad (5-22)$$

where $c(t_{l+1,0})$ in 5-21 and 5-22 is equivalent to $c(t_{n+1})$ in 5-15 and 5-26.

Let T_c be the completion time for the multiple layer system. Let T_c^+ and T_c^- are upper bound and lower bound on T_c . Noting that the execution times of client tasks from $t_{2,0}, \dots, t_{L,0}$ appear in the completion time for the layers, we get:

$$T_c^+ = \sum_{l=1}^L T_{cl}^+ - T_0$$

Where

$$T_0 = \sum_{l=2}^L c(t_{l,0}) \quad (5-23)$$

From 5-21 and 5-23 we get

$$T_c^+ = \sum_{l=1}^L (c(t_{l,0}) + c(t_{l+1,0}) + \sum_{m=1}^{n_l} e^+(t_{l,m})) - \sum_{l=2}^L c(t_{l,0}) \quad (5-24)$$

From 5-20, we can derive the lower bound on T_c .

$$T_c^- = \sum_{l=1}^L T_{cl}^- - T_0$$

From 5-22 and 5-23 we can get

$$\begin{aligned} T_c^- &= \sum_{l=1}^L (c(t_{l,0}) + c(t_{l+1,0}) + \max(e^-(t_{l,1}), e^-(t_{l,2}), \dots, e^-(t_{l,n_l}))) \\ &\quad - \sum_{l=2}^L c(t_{l,0}) \end{aligned} \quad (5-25)$$

5.4 Bounds on the Performance of Systems with Probabilistic Branching

In a precedence graph with probabilistic branching, the client tasks may have OR relationships with one another. All of the successors of a client task may not start execution after the completion of the client task. There is a branching probability b_j associated with the execution of each successor j of a source task i .

Consider a simple system shown in Figure 5.3 first.

Figure 5.3. Simple probabilistic branching

In this system, assume there is no server submodel and after the termination of client task t_0 , only one of its successors t_i ($i = 1, \dots, n$) starts execution. There are multiple execution paths in the system, such as $t_0 \rightarrow t_1 \rightarrow t_{n+1}$, $t_0 \rightarrow t_n \rightarrow t_{n+1}$. Path $t_0 \rightarrow t_i \rightarrow t_{n+1}$ is called path i . Let T_{ci} be the completion time for the system when path i is taken. The expected completion time T_c for this system is then given by

$$T_c = b_1 T_{c1} + b_2 T_{c2} + \dots + b_n T_{cn} = \sum_{i=1}^n b_i T_{ci} \quad (5-26)$$

Therefore

$$T_c^+ = \sum_{i=1}^n b_i T_{ci}^+ \quad (5-27)$$

and

$$T_c^- = \sum_{i=1}^n b_i T_{ci}^- \quad (5-28)$$

Now consider a more complicated probabilistic branching system in Figure 5.4.

Figure 5.4. Probabilistic branching

The initialization client task is t_0 and the termination client task is t_{n+1} . Neither of them makes any request to servers. Other client tasks may make requests to servers during their execution. There are two classes of successor client tasks: AND class and OR class. For an AND class (χA) all the client tasks will start execution upon the completion of their predecessors. The client tasks from t_1 to t_n in Figure 5.4 belong to the AND class client tasks. The client tasks in the OR class (χO) may or may not start execution after their predecessors finish. $t_{1,1}$ and t_{q,n_q} are two examples of OR class client tasks. The OR class client tasks are grouped with an ellipse. There are

Q groups in the system. Each group has a probability associated with it ($b_1, \dots, b_Q$) and group q contains n_q multiple client tasks. For example, the branching probability for group 1 is b_1 and group 1 has n_1 client tasks from $t_{1,1}$ to t_{1,n_1} . After the termination of t_0 , group q is chosen to execute with a probability b_q and all the client tasks in group q are activated. Note that

$$\sum_{q=1}^Q b_q = 1.$$

The concurrency set of client task $t_{q,j}$ includes client tasks in both classes: all of the client tasks in $\mathcal{XA}$ and the OR class client tasks $t_{q,k}$ which are in group q .

Let T_{cq} be the completion time for system shown in Figure 5.4 when group q is executed. Also let T_{cq}^+ and T_{cq}^- be upper and lower bounds on T_{cq} . Using 5-15, we get

$$T_{cq}^+ = c(t_0) + c(t_{n+1}) + \sum_{m=1}^n e^+(t_m) + \sum_{i=1}^{n_q} e^+(t_{q,i}) \quad (5-29)$$

Using 5-20 yields

$$\begin{aligned} T_{cq}^- &= c(t_0) + c(t_{n+1}) \\ &+ \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n), e^-(t_{q,1}), \dots, e^-(t_{q,n_q})) \end{aligned} \quad (5-30)$$

Let T_c be the completion time for the system shown in Figure 5.4. From 5-27 and 5-29, we get

$$T_c^+ = \sum_{q=1}^Q b_q (c(t_0) + c(t_{n+1}) + \sum_{m=1}^n e^+(t_m) + \sum_{i=1}^{n_q} e^+(t_{q,i})) \quad (5-31)$$

Noting that $\sum_{q=1}^Q b_q = 1$, 5-31 yields

$$T_c^+ = c(t_0) + c(t_{n+1}) + \sum_{m=1}^n e^+(t_m) + \sum_{q=1}^Q b_q \sum_{i=1}^{n_q} e^+(t_{q,i}) \quad (5-32)$$

From 5-28 and , we get

$$T_c^- = \sum_{q=1}^Q b_q (c(t_0) + c(t_{n+1}) + \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n), e^-(t_{q,1}), \dots, e^-(t_{q,n_q}))) \quad (5-33)$$

Again because $\sum_{q=1}^Q b_q = 1$, 5-33 yields

$$T_c^- = c(t_0) + c(t_{n+1}) + \sum_{q=1}^Q b_q \max(e^-(t_1), e^-(t_2), \dots, e^-(t_n), e^-(t_{q,1}), \dots, e^-(t_{q,n_q})) \quad (5-34)$$

The bounds derived in 5-32 and 5-34 for the system shown in Figure 5.4 can be expanded to systems with the multiple layers. Figure 5.5 is a multiple layer system and there may be some probabilistic branching in each layer. There are L layers in Figure 5.5. The "join" task in one layer serves as the "fork" task in the next layer. The fork/join tasks do not make service requests to servers that may be visited by other client tasks. The number of AND class tasks in layer l is n_l . A client task $t_{l,j}$ is j th client task in the AND class of layer l . In Figure 5.5 group(l,q) is the q th group in layer l . There are Q_l groups of OR tasks in layer l and the q th group in layer l contains n_{lq} client tasks. b_{lq} is the branching probability for group q in layer l . An OR class client task $t_{l,q,k}$ is the k th client task in group q of layer l .

Let T_{cl} be the completion time for layer l and T_c be the completion time for the multiple layer system. From 5-32 and 5-34, we can get the upper and lower bounds for layer l ($l = 1, \dots, L$). $c(t_{l+1,0})$ in 5-35 and 5-36 is equivalent to $c(t_{n+1})$ in 5-32 and 5-34. Thus T_{cl}^+ is given in the following manner:

Figure 5.5. Multiple layer probabilistic branching system

$$T_{cl}^+ = c(t_{l,0}) + c(t_{l+1,0}) + \sum_{m=1}^{n_l} e^+(t_{l,m}) + \sum_{q=1}^{Q_l} b_{lq} \sum_{i=1}^{n_{lq}} e^+(t_{l,q,i}) \quad (5-35)$$

and T_{cl}^- is given by:

$$T_{cl}^- = c(t_{l,0}) + c(t_{l+1,0}) + \sum_{q=1}^{Q_l} b_{lq} \max(e^-(t_{l,1}), e^-(t_{l,2}), \dots, e^-(t_{l,n_l}), e^-(t_{l,q,1}), \dots, e^-(t_{l,q,n_{lq}})) \quad (5-36)$$

From 5-35, the upper bound on T_c can be derived. Noting that the execution times of client tasks from $t_{2,0}, \dots, t_{L,0}$ already appear in the completion time for the layers, we get:

$$T_c^+ = \sum_{l=1}^L T_{cl}^+ - T_0 \text{ and}$$

$$T_c^- = \sum_{l=1}^L T_{cl}^- - T_0$$

Where

$$T_0 = \sum_{l=2}^L c(t_{l,0}) \quad (5-37)$$

From 5-35 and 5-37, we get

$$T_c^+ = \sum_{l=1}^L (c(t_{l,0}) + c(t_{l+1,0}) + \sum_{m=1}^{n_l} e^+(t_{l,m}) + \sum_{q=1}^{Q_l} b_{lq} \sum_{i=1}^{n_{lq}} e^+(t_{l,q,i})) - \sum_{l=2}^L c(t_{l,0}) \quad (5-38)$$

And from 5-36 and 5-37, the lower bound on T_c can be derived.

$$\begin{aligned}
T_c^- = & \sum_{l=1}^L (c(t_{l,0}) + c(t_{l+1,0})) \\
& + \sum_{q=1}^{Q_l} b_{lq} \max(e^-(t_{l,1}), e^-(t_{l,2}), \dots, e^-(t_{l,n_l}), e^-(t_{l,q,1}), \dots, e^-(t_{l,q,n_{lq}}))) \\
& - \sum_{l=2}^L c(t_{l,0})
\end{aligned} \tag{5-39}$$

5.5 Bounds on System with Simple Loops

Consider a system with a simple loop construct as shown in Figure 5.6. t_i is a client task in a precedence graph and may make requests to servers. In this system, each time t_i completes execution, either the system terminates itself with probability $1 - p_l$ or another iteration of the loop is started with the loop probability p_l . Let τ denote the expected execution time for the system. Thus,

Figure 5.6. General loop

$$\tau = e(t_i) + p_l \tau \text{ therefore,}$$

$$\tau = e(t_i) / (1 - p_l)$$

If bounds on $e(t_i)$ are known, $e^-(t_i) \leq e(t_i) \leq e^+(t_i)$, the bounds on τ are computed in the following manner.

$$\tau^+ = e^+(t_i) / (1 - p_l) \quad (5-40)$$

and

$$\tau^- = e^-(t_i) / (1 - p_l) \quad (5-41)$$

Consider, for example, the system shown in Figure 5.7. In this system, the server submodel may include multiple servers. Client task t_0 and t_{n+1} have no requests to servers and client tasks from t_1 to t_n may make service request to servers. .

Figure 5.7. Loop system

Equation 5-15 and 5-20 provides the upper bound and lower bound on completion time of

client system without loop shown in Figure 5.1. The system in Figure 5.7 has loop construct. After the completion of t_{n+1} , the execution routing may loop to t_0 with loop probability p_l or terminate with probability $1 - p_l$. By using 5-40 and 5-41. The upper bounds and lower bounds on T_c can be obtained as follows:

$$T_c \leq T_c^+ = (c(t_0) + c(t_{n+1}) + \sum_{m=1}^n e(t_m)) / (1 - p_l) \quad (5-42)$$

$$T_c \geq T_c^- = (c(t_0) + c(t_{n+1}) + \max(e(t_1), \dots, e(t_n))) / (1 - p_l) \quad (5-43)$$

5.6 Computation of the Bounds

The computations of upper bound and lower bounds are implemented by using a technique based on interval arithmetic as presented in the BNR prolog language [BNR88]. The numerical bounds is examined by the simulation results. A brief description of the interval arithmetic concept and how it is used to compute the bounds are presented.

The concept of interval arithmetic is centered around a data type called *interval*. An interval is an object that represents a real number which lies between an upper and a lower bound. The bounds of the real number define its range which may vary during the lifetime of a computation.

BNR Prolog [BNR88] developed at Bell Northern Research supplies the interval arithmetic capability as a special feature of the language. The interval arithmetic in BNR Prolog allows us to express arithmetic relationships among intervals and offers a mechanism for solving sets of equations and inequalities. The BNR Prolog interpreter evaluates the set of equations and inequalities and computes the interval which satisfy the set of relationships expressed by equations and inequalities. During the computation, intervals can change their values only by narrowing. Both raising the lower bound and lowering the upper bound are called narrowing operation.

The following examples will explain further the interval arithmetic feature of BNR Prolog. The predicate `range` in the programs sets the initial upper and lower bounds for both the intervals `_A` and `_B`. The predicate `print_interval` is used to print the intervals `_A` and `_B` at the end of the computations. The result intervals for `_A` and `_B` satisfy the inequalities specified. The interval arithmetic in BNR prolog is explained further with the following examples.

Example 1 (see Figure 5.8)

Program:

```
range (_A, [3, 5]),  
range (_B, [7, 9]),  
_A >= _B.  
nl, print_interval(_A).  
nl, print_interval (_B).
```

Result:

NO

Figure 5.8. Example 1

The result range is NULL ranges, for *_A* and *_B* that satisfy the given constraints can not be found.

Example 2 (see Figure 5.9)

Program:

```
? range (_A, [1,2]),  
range (_B, [3,5]),  
_A =< _B,  
nl, print_interval(_A),  
nl, print_interval (_B).
```

Result:

[1.0, 2.0001]

[3.0, 5.0001]

YES

Figure 5.9. Example 2

In this example, no narrowing is performed, because the original intervals satisfy the constraints.

Example 3 (see Figure 5.10)

Program:

```
? range (_A, [4, 8]),  
range (_B, [6, 9]),  
_A >= _B,  
nl, print_interval(_A),  
nl, print_interval(_B).
```

Result:

[6.0, 8.0001][6.0, 8.0001]

YES

Intervals
(before applying
inequality)

Intervals
(after applying
inequality)

Figure 5.10. Example 3

Narrowing is performed in this example. In order to satisfy the relationship $_A \geq _B$, the lower bound for $_A$ is raised and the upper bound for $_B$ is lowered.

The interval arithmetic feature of BNR Prolog can be efficiently used for the computation of performance bounds for the RNMTC system. As discussed in the previous section, a system is characterized by a set of inequalities corresponding to upper and lower bounds. Each inequality corresponding to upper and lower bounds on parameters such as $e(t_i)$ and T_c are converted into a prolog subgoal, when the program executes, each of these subgoals is evaluated and an interval $T_c^- \leq T_c \leq T_c^+$ that satisfies all the inequalities is produced.

The following is an example BNR prolog program used for the system shown in Figure 5.16. The loop probability p_l is set at 0.1. The number of concurrent tasks is 4.

Program:

loopn4pl :-

% loopn4pl: bound for loop with that the number of concurrent tasks is 4 and loop probability is 0.1.

range(_T, [0, _]).

_c0 is 1.0, _c1 is 1.0, _c2 is 1.0, _c3 is 1.0, _c4 is 1.0, _c5 is 1.0,

% _ci is the computation time for client task i (i = 0,...5)

_s11 is 1.0, _s12 is 1.0,

% _s11 is the service time for first phase of server and _s12 is the service time for second phase of server

_P1 is 0.1,

_v11 is 1.0, _v21 is 1.0, _v31 is 1.0, _v41 is 1.0,

% _vi1 is the average number of visits made by client task i (i = 1,..4) to server

$$_e_i \text{ is } (_c_i + _v_{1i} * _s_{1i}),$$

$$_e_2 \text{ is } (_c_2 + _v_{21} * _s_{11}),$$

$$_e_3 \text{ is } (_c_3 + _v_{31} * _s_{11}),$$

$$_e_4 \text{ is } (_c_4 + _v_{41} * _s_{11}),$$

% $_e_i$ is the lower bound on execution time of client task i ($i = 1, \dots, 4$)

$$_P_{121} \text{ is } (\min(1, _v_{21}/_v_{11})),$$

$$_P_{131} \text{ is } (\min(1, _v_{31}/_v_{11})),$$

$$_P_{141} \text{ is } (\min(1, _v_{41}/_v_{11})),$$

$$_P_{211} \text{ is } (\min(1, _v_{11}/_v_{21})),$$

$$_P_{231} \text{ is } (\min(1, _v_{31}/_v_{21})),$$

$$_P_{241} \text{ is } (\min(1, _v_{41}/_v_{21})),$$

$$_P_{311} \text{ is } (\min(1, _v_{11}/_v_{31})),$$

$$_P_{321} \text{ is } (\min(1, _v_{21}/_v_{31})),$$

$$_P_{341} \text{ is } (\min(1, _v_{41}/_v_{31})),$$

$$_P_{411} \text{ is } (\min(1, _v_{11}/_v_{41})),$$

$$_P_{421} \text{ is } (\min(1, _v_{21}/_v_{41})),$$

$$_P_{431} \text{ is } (\min(1, _v_{31}/_v_{41})),$$

% P_{tyl} is the probability of client task t being delayed by y at server

$$_e_1^+ \text{ is } (_c_1 + _v_{11} * _s_{11} + _v_{11} * _s_{12} + _P_{121} * (_s_{11} + _s_{12}) + \\ _P_{131} * (_s_{11} + _s_{12}) + _P_{141} * (_s_{11} + _s_{12}))$$

$$_e_2^+ \text{ is } (_c_2 + _v_{21} * _s_{11} + _v_{21} * _s_{12} + _P_{211} * (_s_{11} + _s_{12}) + \\ _P_{231} * (_s_{11} + _s_{12}) + _P_{241} * (_s_{11} + _s_{12}))$$

$$_e_3^+ \text{ is } (_c_3 + _v_{31} * _s_{11} + _v_{31} * _s_{12} + _P_{311} * (_s_{11} + _s_{12}) + \\ _P_{321} * (_s_{11} + _s_{12}) + _P_{341} * (_s_{11} + _s_{12}))$$

$$_e_4^+ \text{ is } (_c_4 + _v_{41} * _s_{11} + _v_{41} * _s_{12} + _P_{411} * (_s_{11} + _s_{12}) + \\ _P_{421} * (_s_{11} + _s_{12}) + _P_{431} * (_s_{11} + _s_{12}))$$

$$_e_{min} \text{ is } (\max(_e_1^-, _e_2^-)),$$

$$_e_{min} \text{ is } (\max(_e_{min}, _e_3^-)),$$

$$_e_{min} \text{ is } (\max(_e_{min}, _e_4^-)),$$

% $_e_{min}$ is the maximum among $_e_1^-$, $_e_2^-$, $_e_3^-$ and $_e_4^-$

$$_T >= (_c_0 + _c_5 + _e_{min}) / (1 - _P_1),$$

$$_T <= (_c_0 + _c_5 + _e_2^+ + _e_3^+ + _e_4^+ + _e_5^+) / (1 - _P_1),$$

nl, print_interval($_T$).

result:

[4.4444,42.223]

YES

5.7 Test Cases

A number of test cases have been studied. The computation of the bounds is realized by using BNR prolog and the bounds are compared with simulation results. The duration of simulation is set sufficiently long to ensure a confidence interval of the order of $\pm 1\%$ at a confidence level of 99%. The major motivation for the test cases is the investigation of the BNR prolog implementation of the bound computations. A detailed study of the tightness of the bounds is beyond the scope of the thesis.

5.7.1 Deterministic System

A deterministic system is shown in Figure 5.11.

Figure 5.11. Test case of multiple layer system

There are 2 layers of client tasks. The computation time for all client tasks is 1.0. Client tasks from $t_{1,1}$ to $t_{1,n}$ and from $t_{2,1}$ to $t_{2,n}$ make a single service request to each entry in all the servers. There are n concurrent client tasks in each layer. The parameters for servers are shown in Figure 5.11. In each case, three different values of n , ($n = 1, 4, 8$) are experimented.

- Fixed computation time and service time

In this case, the computation time for client tasks and service time for servers are fixed. Figure 5.12 shows the bounds, for the sake of comparison the simulation results are included. It is interesting to note that the completion time T_c obtained from the simulating is approximately in the middle of bounds T_c^+ and T_c^- .

Figure 5.12 Test result for system with fixed computation time and service time

- Exponential distribution of computation and service time

The system shown in Figure 5.11, with exponential distribution of computation and service time is investigated. Figure 5.13 shows the result for this case. the lower bound seems to be closer to the mean completion time in comparison to the upper bound.

Figure 5.13. Test case for exponential distribution

5.7.2 Probabilistic Branching

A system with probabilistic branching shown in Figure 5.14 is tested.

Figure 5.14. Test case of probabilistic branching

There is a single server called by t_1 , t_2 and t_3 with number of visits made to the server by each client task is 1. Client task t_3 is an AND class task and starts execution upon the completion of t_0 . Client tasks t_1 and t_2 are OR class tasks. After the termination of t_0 , either t_1 starts or t_2 starts. Client tasks t_0 and t_4 have no service requests to server. The computation times for client task t_2 is set 5.0 whereas the computation times for all of other client tasks are set 1.0. The parameters for the server is shown in Figure 5.14.

The results are shown in Figure 5.15. The lower bound seems to be tighter than the upper

bound.

Figure 5.15. Test result of probabilistic branching

5.7.3 System with Loop

A test case for a system with a single loop is presented next. The system is shown in Figure 5.16.

Figure 5.16. Test case of loop system

Client task t_0 and t_{n+1} have no requests to server while other client tasks may make requests to the server and the mean number of visit to the server is 1.0. The computation times and service time are exponential distributed. The computation times for all client tasks are 1.0. The parameters for server client is given in Figure 5.16. After the termination of the client task t_{n+1} , the system may either loop back to client task t_0 with a loop probability p_l or terminate.

Figure 5.17 shows the results for 4 concurrent client tasks. The test is repeated with $n = 8$

and the result are presented in Figure 5.18.

Figure 5.17. Test result for loop with 4 concurrent client tasks

Figure 5.18. Test result for loop with 8 concurrent client tasks

For both cases the lower bound seems to be quite tight. The tightness seems looser with the increasing of p_l .

Chapter 6

Test Cases for Analytic Solution Method

RNMTC model have been used with a number of different test cases and the result of the approximate analysis for each case is compared with that of simulation experiments. Both simpler test cases and a large transaction processing system are analyzed. The simulation experiments were run for sufficiently long durations of time to yield a confidence interval of the order of $\pm 1\%$ at a confidence level of 99%. The error rate ϵ is captured according to the following computation:

$$\epsilon = \frac{\text{output of analytic solver} - \text{simulation result}}{\text{simulation result}}$$

The performance measures derived from the analytic solver reside in reasonable range. The results show that the error rates for most of test cases are less than 10%. Section 6.1 shows the simpler cases and the results, and section 6.2 is the analysis of large transaction processing tandem database system.

This chapter is organized as the following way: section 6.1 gives the general test cases, such as balanced system, impact of the number of concurrent tasks, impact of layer of client tasks, unbalanced system, loops, probabilistic branching and processor contention. Section 6.2 presents a large RNMTC model of a distributed database system for two classes of client queries. The identify of bottleneck is also studied.

6.1 General Test Cases

A set of general test cases have been made to examine the accuracy and efficiency of the

approximate analysis for RNMTTC system. Each of the test cases is a representation of a specific kind precedence graph, such as, AND, OR, probabilistic branching and loop, etc.

6.1.1 Model 1: Balanced System

Balanced system have balanced precedence graph and SRVN graph. The computation time and service time are also evenly assigned. The balanced system under test is shown in Figure 6.1. All computation time, service time and visit ratio from client tasks to server s_1 and server s_2 are set 1, the only variable is the visit ratio from servers s_1 and s_2 to server s_3 . Table 1.1 on page 87 shows the error rate ϵ between the result from analytic solution method and the result from simulation corresponding to different values of visit ratio. Figure 6.2 presents ϵ as a function of the utilization of server s_3 . For the balanced system, the relative error seems to be low for very low and very high values of s_3 utilization whereas the highest ϵ ($\epsilon = -8.273$) is obtained at an intermediate value of s_3 utilization.

Figure 6.1. Balanced system

Figure 6.2. Error rate for balanced system

6.1.2 Model 2: Impact of the Number of Concurrent Tasks

In this section, a simple concurrent system is analyzed. The concurrence of the system is

increased by forking new concurrent tasks. Figure 6.3 is a RN-ATC model for such a system. The analytic solution is applied to the system with different number of concurrent tasks. All the computation times and service time are set 1 while visit ratios from client tasks to the server $v(i,1,1,1)$ ($i = 1,...,n$) is a variable.

Figure 6.3. Impact of number of concurrent tasks

The experiments were repeated with different number of concurrent tasks. Table 2.1 on page 87, Table 2.2 on page 88 and Table 2.3 on page 89 give the numerical results for client cycle time from analytic solution and simulation and the error rate. The error rate is insensitive with the number of concurrent tasks for large range of variation of visit ratio.

6.1.3 Model 3: Impact of layer of Client Tasks

In this test case, the impact of multiple layers of join-fork is investigated. The client tasks in

each layer have the same structures and parameters. All concurrent tasks t_{ij} ($i = 1, \dots, n; j = 1, \dots, 5$) call servers s_1 and s_2 with visit ratio is set 1. Execution time for client tasks and service time for server s_i ($i = 1, 2, 3$) in both first phase and second phase are set 1. Figure 6.4 is the structure of the system. Table 3.1 on page 89 shows the numerical error rate for different number of layers. From the result, one can see that the error rate is fairly insensitive to the number of join-fork layers.

Figure 6.4. Variation of client task layers

6.1.4 Model 4: Unbalanced System

The test cases in section 6.1.2 and 6.1.3 are generally balanced. This section tests an unbal-

2 of /de 2

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(ANSI and ISO TEST CHART No. 2)

anced system shown in Figure 6.5. The computation time, service time and visit ratio are unevenly distributed. The parameters are shown in Figure 6.5.

Figure 6.5. Unbalanced System

The mean cycle time of the client is 175.38 for analytic solution methods while it is 171.822 for simulation. The error rate is low with value 2.08%.

6.1.5 Model 5: Loops

Figure 6.6 is a system with loop. After the termination of client task t_8 , either t_9 is enabled with probability 0.1 or t_2 is initiated again with loop probability 0.9. All parameters are given in Figure 6.6.

Figure 6.6. Loops

The mean cycle time is 168.38 and 183.66 for analytic solution method and simulation respectively. Error rate is -9.07%.

6.1.6 Model 6: Probabilistic Branching

Figure 6.7 is a single thread system with probabilistic branching. After the termination of client task t_1 , there are three possible executing routines with different probabilities.

Figure 6.7. Probabilistic branching

The test result shows the error rate is 0.056% derived from analytic solution method result 5.6055 and simulating result 5.6024.

6.1.7 Model 7: Processor Contention

For the above examples, all test cases are based on the assumption that each client task and server has its own processor so that there is no contention for hardware.

This section shows how the analytic solution method can be used to evaluate the impact of hardware contention. Figure 6.8 is RNMTC model in which client task t_1 , t_8 and server s_1 have their own processor whereas client task t_i ($i = 2, \dots, 7$) may compete for processors. The number of processors for client task t_i ($i = 2, \dots, 7$) is varied from 1 to 6. The homogeneous processors are shared equally by t_i ($i = 2, \dots, 7$). For example, if the number of processor is 3, t_2 and t_3 , t_4 and t_5 , t_6 and t_7 , each pair shares one CPU. As stated in section 2.2, processor can be modeled as a srvn server. The computation time of a client task which is allocated on a processor can be converted to the service time of the processor.

Figure 6.8. Processor contention

Figure 6.9 is a RNMTC model corresponding to Figure 6.8 for 3 processors. The processors (CPU₁, CPU₂ and CPU₃) are viewed as SRVN servers. The computation time of t_i ($i = 2, \dots, 7$) is set 0 because computation time of a client task is converted to service time of the processor which the client task is allocated on. The visit ratio from client task to CPU server is 2 due to that total visit ratio from the client task to server is 1. Table 4.1 on page 90 shows analytic result and simu-

lation result, the error rate is less than 10%.

Figure 6.9. RNMT model for processor contention

6.1.8 Discussions

A number of cases have been tested to examine the efficiency and accuracy of analytic solution method. The test results show that the method can be used in various kinds of system structure, such as, node precedence graph with high degree concurrence, multiple layer fork and join, etc. The impact of processor contention can be also captured with analytic solution method by modelling processor as an SRVN server. Table 5.1 shows the minimum and maximum error rate ϵ obtained with some of models, as well as the average of the absolute value error rate for each test suite. Most of error rates between analytic solution method result and simulation result are less than 10%.

Table 1.1: Simulation and Approximate results for the Balanced System (Model 1)
 $c(l, t) = 1, (t = 1, \dots, 7); v(l, t, 1, 1) = v(l, t, 2, 1) = 1, (t = 1, \dots, 7); s(i, 1, 1) = s(i, 1, 2) = 1, (i = 1, 2);$
mean error rate = 3.53%

Case	$y(i, 1, 1, 3, 1)$ $i = 1, 2$	approx.	simulation	error rate (%)
1	0	4.2833	4.2841	-0.019
2	0.001	16.3116	16.2159	0.59
3	0.2	16.6388	18.1395	-8.273
4	0.5	20.4627	21.9848	-6.923
5	0.7	23.5954	24.9417	-5.398
6	1	28.8153	29.7841	-3.253
7	1.5	38.2658	38.4570	-0.497
8	2	48.1654	47.6266	1.131
9	3	68.5057	66.4952	3.023
10	4	89.1406	85.7318	3.976
11	6	130.7089	125.2500	4.358
12	20	423.4034	403.5800	4.912

Table 2.1: Simulation and Approximate results when the number of concurrent tasks = 3 (Model 2)
 $c(l, t) = 1; s(l, 1, 1) = s(l, 1, 2) = 1, (i = 1, 2);$
mean error rate = 2.321%

Case	$v(i, 1, 1, 1)$ $i = 1, \dots, n$	approx.	simulation	error rate (%)
1	0.2	4.4183	4.4465	-0.634
2	0.4	5.3154	5.2605	1.044
3	0.6	6.3438	6.1956	2.393
4	0.8	7.4317	7.2023	3.186
5	1	8.5518	8.2607	3.525

Table 2.1: Simulation and Approximate results when the number of concurrent tasks = 3 (Model 2)

$c(1, t) = 1; s(1, 1, 1) = s(1, 1, 2) = 1, (i = 1, 2);$
mean error rate = 2.321%

Case	$v(i, 1, 1, 1)$ $i = 1, \dots, n$	approx.	simulation	error rate (%)
6	1.2	9.6914	9.3530	3.618
7	1.5	11.4235	11.0359	3.513
8	3	20.2548	19.7260	2.681
9	5	32.1679	31.5596	1.928
10	20	122.0496	121.2140	0.689

Table 2.2: Simulation and Approximate results when the number of concurrent tasks = 5 (Model 2)

$c(1, t) = 1; s(1, 1, 1) = s(1, 1, 2) = 1, (i = 1, 2);$
mean error rate = 2.356%

Case	$v(i, 1, 1, 1)$ $i = 1, \dots, n$	approx.	simulation	error rate (%)
1	0.2	5.2839	5.3003	-0.309
2	0.4	6.9320	6.7660	2.454
3	0.6	8.7498	8.4499	3.550
4	0.8	10.6343	10.2470	3.780
5	1	12.5529	12.1125	3.636
6	1.2	14.4918	14.0258	3.323
7	1.5	17.4235	16.9116	3.027
8	3	32.2546	31.6480	1.917
9	5	52.1679	51.5298	1.238
10	20	202.0496	201.3940	0.326

Table 2.3: Simulation and Approximate results when the number of concurrent tasks = 10 (Model 2)

$c(l, t) = 1; s(l, l, 1) = s(l, l, 2) = 1, (i = 1, 2);$
mean error rate = 2.546%

Case	$v(i, l, l, l)$ $i = 1, \dots, n$	approx.	simulation	error rate (%)
1	0.2	7.3025	7.0994	2.861
2	0.4	10.9335	10.4546	4.581
3	0.6	14.7489	14.1976	3.883
4	0.8	18.6329	18.0590	3.178
5	1	22.5515	21.9621	2.684
6	1.2	26.4906	25.8867	2.333
7	1.5	32.4226	31.8301	1.862
8	3	62.2540	61.7002	0.898
9	5	102.1676	101.5210	0.637
10	20	417.6715	402.239	3.836

Table 3.1: Simulation and Approximate results for the impact of client task layers (Model 3)

$c(l, t) = 1; v(l, t, l, l) = v(l, t, 2, l) = 1; s(i, l, 1) = s(i, l, 2) = 1, (i = 1, 3);$
 $y(i, l, 1, 3, l) = 1; y(i, l, 2, 3, l) = 0;$
mean error rate = -3.58%

Case	number of layers	approx.	simulation	error rate (%)
1	1	28.8153	29.7841	-3.253
2	2	56.6307	58.6460	-3.433
3	3	84.4460	87.6300	-3.633
4	5	140.0768	145.4340	-3.684
5	9	251.3382	260.9020	-3.666
6	12	334.7842	348.2460	-3.866

Table 4.1: Simulation and Approximate results for the impact of processor contention (Model 7)

$c(I,t) = 1(t = 1, \dots, 8); v(I,t,1,1) = v(I,t,2,1) = 1; s(I,1,1) = s(I,1,2) = 1;$

mean error rate = 2.393%

Case	number of processors	approx.	simulation	error rate ϵ (%)
1	1	51.2278	51.16	0.132
2	2	35.3333	38.2458	-7.615
3	3	30.3030	33.1757	-8.659
4	6	27.2549	30.1163	-9.501

Table 5.1: Accuracy of Analytic Solution

Model	Minimum ϵ (%)	Maximum ϵ (%)	Average ϵ (%)
1	-0.019	-8.273	3.53
2	-0.309	4.581	
3	-3.253	-3.866	3.59%
7	0.132	-7.615	

6.2 RNMTTC Model of a Distributed Database System

In this section, a large case study is presented, where RNMTTC model is built for a distributed database Tandem On-Line Transaction Processing (OLTP) system. The starting point in building the analytic model was the simulation model described in [HMT92] which captures the hardware architecture of a Tandem computer system and software behavior of the relational database system NonStopSQL running on a Tandem multiprocessor machine [Zel90] and [Bor88]. This is called "detailed simulation model". A set of RNMTTC models are built to evaluate the performance of Tandem system by applying analytic solution. The RNMTTC simulator is used to build a "abstract" model to analyze RNMTTC models by simulation. The difference between the detailed and abstract simulation results gives the "*abstraction error*", while the difference between the abstract simulation and analytic results is called the "*analytic approximation error*".

6.2.1 Tandem Distributed Database System

A Tandem machine is a share nothing multiprocessor system on which NonStopSQL is implemented. NonStopSQL is a distributed relational database management system which achieves high performance through integration of SQL functions with operating system functions as disk I/O, virtual memory, disk cache management, etc. System integration and movement of SQL functions downward from the client reduces message traffic and device utilization by putting SQL optimizations at the lower level of the system [Bor88].

Figure 6.10 shows the basic architecture of the Tandem system. Up to 16 loosely-coupled processors can be interconnected by dual high-speed buses named Dynabus to form a single multiprocessor system. The transfer rates between processors is high (over 10 Mb/sec for 8 or more processors). Based on the principles of a share nothing architecture, each CPU has its own memory and I/O bus dedicated to interfacing to the outside world via I/O controllers (disk controllers, terminal controllers, etc.). The data transfers on the I/O bus occurs at high speed (4 Mb/sec) through direct memory access (DMA). System resources are managed by a message-based operating system which provides communication between processes executing on the same or on different processor, making the distribution of hardware component transparent. I/O devices are managed by system-level processes. Tandem files which contain SQL tables and indices can be horizontally partitioned across several disks into file fragments.

The software architecture is based on a client/server model, where client (or requesting) processes send their requests to server processes through messages, and then wait for a reply from the server. Therefore almost all communications with few exceptions is performed in a rendezvous manner. The main software components of the Tandem OLTP system modelled in [HMT92] are displayed in Figure 6.10.

Disk Process (DP) is the I/O process that manages a disk volume, acting as I/O server for files resident on that volume. DP integrates several important operating system and database management functions: performs disk I/O through “driver” subroutines, supports access to files records, manages disk cache through cooperation with the operating system virtual memory management mechanism and implements simple SQL operations taking care of locking and of logging procedures.

Line Handler (LH) is responsible for I/O to and from the users’s terminal by providing the transmission protocol. During its operation, LH handles incoming requests and outgoing responses for up to three users.

Terminal Control Process (TCP) provides screen formatting, data validation, screen sequencing and data mapping for up to 32 terminal. TCP receives user requests from LH and forwards them to application processes. It also gets responses from the application processes and sends them back to LH. It should be mentioned that TCP uses a nonblocking communication style, being a notable exception from the RV communication style.

Pathmon provides address information to TCP for inter-process communication.

Application Process (APP) receives user requests from TCP, decomposes them into simple SQL operations which are then sent one by one to the appropriate DP for execution. After receiving the reply (or replies) from DP, APP assembles the final response for the user and sends it back to TCP.

Executor Server Process (ESP) is an optional component used only for very complex query with potential for internal parallelism [Zel90]. In the case of partitioned files (SQL tables or indices) located on different disk volumes, several DPs can work in parallel on different processors on behalf of the same query. Thus an APP managing such a complex query will start several ESPs which will execute in parallel parts of the same query with the help of DPs. At the end, APP col-

Figure 6.10. Tandem System Architecture

lects all partial results and assembles the final response. For efficiency reasons, the number of ESPs used is the number of file fragments, and an ESP corresponding to a fragment runs on the same processor as the DP used to perform the low level SQL operations.

It should be mentioned that fault-tolerance, one of the important features of the Tandem architecture which is achieved by hardware and software redundancy, is not considered in the thesis.

6.2.2 RNMTTC model for Tandem System

The RNMTTC model of a Tandem database is built for two classes of client queries: *light readers* and *join query*. Some of the components of the Tandem architecture from Figure 6.10 are represented as servers and others as client tasks, depending on their roles in the system.

The hardware resources (CPU, I/O devices) are naturally represented as pure servers. The software component DPs are also modelled as servers which receive service requests from APPs. In order to perform their functions DPs make in turn requests to CPUs and disks (DKs). Each DP may have several entries with different workload corresponding to different functions performed by DP (read a database block or read the primary index). The number of visits that DP makes to DK depends on the *cache_hit_ratio* which is a model parameter with different values for every file fragment. Each CPU also has a number of entries with different computation times corresponding to various requests. All servers in the models have only one phase which is “in RV” phase.

The remaining software processes are included in the client submodel. PATHMON, TCP, LH and APPs are modelled as client tasks which make requests to servers. In order to reduce the model size, PATHMON, TCP and LH are integrated into one node in client submodel.

The precedence graphs describing the client tasks, and the model parameters (computation time and service time, visit ratios) are determined in a way similar to the Software Performance Engineering methodology proposed by C. Smith in [Smi90]: first build software execution models for a few representative workload scenarios, then characterize their software and hardware resource requirements and then compute the parameters for the performance model of the whole system.

In the distributed database model, the desirable performance measure is the user response time which is the time between that the user issues a query and that the user receives the answer from the system.

6.2.3 RNMTC Model for Light Reader Operation

The software execution model for light reader operation shown in Figure 6.11 represents a light reader that retrieves an item from a database table based on a key value entered by the user. The database table is partitioned in three fragments located on three different disks and the search key is the primary key of the file. The query chooses one fragment depending on the primary key value from which it will retrieve the required item. The data is responded to the user.

Figure 6.11. Light Reader Query Execution Model

Figure 6.12 is the RNMTC model for light reader execution model with that cache ratio is

The diagram illustrates a network topology with three classes of clients and three CPUs. Class 1 has 3 clients, Class 2 has 3 clients, and Class 3 has 4 clients. Each class has a root node t_1 and a child node t_2 . The nodes are connected to data processing units (DP1, DP2, DP3) and data keys (DK1, DK2, DK3). The CPUs (CPU1, CPU2, CPU3) are connected to the DP and DK units. The diagram shows the flow of data and control signals, with numerical values indicating the magnitude of the signals.

Class 1 (3 clients): Root node t_1 [0.2] connects to child node t_2 [0]. t_2 connects to DP1, DP2, and DP3. DP1 connects to DK1. DK1 connects to CPU1 and CPU2. CPU1 contains packets [0.0030, 0], [0.0047, 0], [0.0206, 0], [0.0076, 0]. CPU2 contains packets [0.0030, 0], [0.0016, 0], [0.0206, 0], [0.0076, 0].

Class 2 (3 clients): Root node t_1 [0.2] connects to child node t_2 [0]. t_2 connects to DP2, DP3, and CPU2. DP2 connects to DK2. DK2 connects to CPU2.

Class 3 (4 clients): Root node t_1 [0.2] connects to child node t_2 [0]. t_2 connects to DP3, CPU3, and CPU2. DP3 connects to DK3. DK3 connects to CPU3. CPU3 contains packets [0.0030, 0], [0.0206, 0], [0.0076, 0].

Control Signals: Arrows labeled with values (2), (0.3), (0.0333), (1.0), (4.5, 0), (5.5, 0) indicate the flow of control signals between the nodes and units.

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The test case includes total 10 clients. Each client submodel has two nodes executed in sequence and is single-threaded. The first node represents LH, TCP and PATHMON whereas the second node corresponds the other operation in the execution model.

The 10 clients are distributed on 3 cpus: three on CPU₁, three on CPU₂ and four on CPU₃. Three disk processes (DP₁, DP₂ and DP₃) are also allocated on the three cpus. Each DP has two entries: entry 1 for the case of finding the data in cache and entry 2 for the operation of reading from DK. Each CPU also has multiple entries with different computation time for various requests either from client tasks or from servers. Figure 6.13 shows the mean response time for light readers with varied cache hit ratio. The response time is larger for a low cache hit ratio than that for a high cache hit ratio, since the chance to find a required data block in buffer is lower when cache hit ratio is low and thus move visits to the disk are necessary. The abstraction error is between 0.15% and 10%, and the analytic approximation error is less than 3%.

Figure 6.13. Response time in function of the cache hit ratio

6.2.4 RNMTC Model for Join Query Operation

The *join query* operation is a heavy operation. One of the execution plans for parallel joins presented in [Zel90], assuming that a table small enough to be cached in memory is joined with a large table partitioned in three fragments residing on three different disks. Three ESP processes spawned by the application process can work in parallel. Figure 6.14 represents a join query-model.

Figure 6.15 is the RNMTC model for parallel join query execution model. The cache hit ratio is 0.2. There are 5 client tasks in the join client submodel. Client tasks t_3 , t_4 and t_5 in client submodel represent the three ESPs. DPs and CPUs also have more entries than that in pure light reader model to serve different requirements both form light readers and join query user. For instance, the last entry in CPU1 (workload is [0.0140, 0]) is called by one of ESPs, t_3 .

In this test case, we combine two classes clients in one system: one is join query, and another one is light readers. The join query user shares the system with a variable number of light readers (from 3 to 8). The model which includes two classes is shown in Figure 6.16. The cache hit ratio for join is 0.2 whereas it is 0.7 for light readers.

Figure 6.17 shows the response time for the two classes of clients. The abstraction error is less than 10% for light readers and less than 7% for join query, and the analytic approximation error is less than 3% for both classes.

Figure 6.14. Parallel join query execution model

Figure 6.15. RNMTTC model for join query model

Figure 6.16. RNMTC model for two classes

Figure 6.17. Response time for two classes of clients

6.2.5 Hardware and Software Bottleneck

The RNMTC model can be used to better understand the behavior of the system and to identify trouble spot that may cause performance problems. In the following example, the system behavior is studied under an increasing number of clients (all are light readers) until approaching

saturation. Figure 6.18 shows the increase in response time with the number of clients in the case of a cache hit ratio of 0.7. (The approximation error is less than 3%).

Figure 6.18. Light reader's response time in function of the number of clients

A relatively large cache hit ratio means that the number of visits to disk is relatively low, and thus the CPU is highly utilized and will become a bottleneck under heavy load. This reasoning is confirmed by Figure 6.19, where is plotted the utilization of servers CPU₁, DP₁ and DK₁ in function of the number of clients.

Following a similar reasoning we could assume that, if the cache hit ratio is low than the disk will become heavily loaded and will be a potential bottleneck. However, the RNMTC results invalidate this assumption. Figure 6.20 shows the increase in server utilization with the number of clients when the cache hit ratio is 0.3.

Figure 6.19. Utilization of servers in case of hardware bottleneck

None of the hardware servers is heavily loaded: both CPU₁ and DK₁ are utilized under 60%. In this case the system bottleneck is a software server, i.e., the disk process DP1. A software bottleneck can appear in any systems where a software process plays simultaneously the role of client and server: the “busy” time as a server includes periods when the process itself becomes a

client to other servers. Thus, even if the software server is blocked waiting to be served by other resources, it is nevertheless unable to accept another request from its input queue, being perceived as “busy” by its own clients. Understanding the behavior of the system helps to decide what measures are necessary to alleviate the bottleneck. The software bottleneck is a complex phenomenon involving more than one component, and therefore, more difficult to understand and identify than a hardware bottleneck. While the latter can be identified directly from the model parameters, the former can be diagnosed only from the results of an appropriate performance model.

Figure 6.20. Utilization of servers in case of software bottleneck

6.2.6 Result Analysis

The test result for Tandem system indicates that the analytic solution is applicable for real

system. RNMTTC is used to model the distributed database system. Two simulation models were used to validate the analytic results: a “detailed simulation model” that captures the hardware architecture of a Tandem computer system and the software behavior of the relational database system NonStopSQL, and an “abstract” RNMTTC simulation model. The difference between the abstract simulation and analytic results gives the “analytic approximation error”. The abstraction error was found to be less than 10%, and the analytic approximation error less than 3% for the test cases analyzed in the research. The RNMTTC model of the distributed database is also used to study a case of hardware and of software bottleneck.

Chapter 7

Conclusions

7.1 Research Summary and Contributions

With the increasing of system size and structure complexity, performance becomes a more and more important issue during the lifetime of system design. This research is motivated by the need for getting better analytical performance models for concurrent software systems and studying the analytic solution which has relatively high accuracy and low cost to predict and analyze system performance.

Based on a previously developed SRVN model and approximate solution for the SRVN model, this thesis developed a new model RNMTC and implemented an approximate analytic solution for this model. RNMTC is a combination of a server submodel which includes a number of servers and client submodel which is composed of a set of independent clients. The servers in the server submodel can represent both software components and hardware components and are used by the clients who compete for servers. Each system client is represented by a precedence graph and may contain multiple concurrent execution threads. An execution thread is an independent flow of control. The number of the threads can vary due to fork and join operations. The client task nodes in each client execute according to the corresponding precedence relationship. During the execution, the client task may make requests to servers. The communication between client task and server is synchronous by RV. After sending a request, the client task is blocked. Therefore the requesting execution flow is also blocked. After receiving a reply, the client task either makes another request or resume its execution. A server may make nested requests to other servers.

The analytic solution for the RNMTC model is based on hierarchical decomposition. A MC model describes the system at the higher level to represent all possible combinations of client execution states. For each MC state an SRVN model is solved at the lower level by approximate MVA methods previously developed in [Pet91] to get state probabilities and throughputs. Finally, the higher level MC is solved to get performance measures for the system.

A fair number of test cases, both abstract and practical, are analyzed by analytic solution and the results are compared with that from simulations. The error rates for all test cases are less than 10%.

Performance upper and lower bounds are also important for performance analysis. If the exact performance measures can not be derived in some cases, bounds may be the only measure for predicting system performance. Expressions for upper bounds and lower bounds are derived for some specific kinds of RNMTC model. The tightness of the bounds is also examined by comparing the bounds with simulation results.

The execution time experienced with the three analysis techniques, i.e. simulation, analytic solution and bound method, were very different. As expected, the simulation was the longest, taking several hours for each test case (the longest was 9 hours). The analytic algorithm took one to two order of magnitude less time than the simulation, depending strongly on the size of the MC model. The running time for the computation of bounds was of a few minutes for each test case.

A paper [PMLH94] has been produced from the thesis, which will be presented at the International Workshop on Modeling, Analysis and Simulation of Computer and Telecommunication Systems, MASCOTS'94, to be held in Durham, North-Carolina, January 31 - February 2, 1994.

7.2 Further Work

That the exponential state explosion of the MC model suffered by analytic method for RNMTC models and the computation time required by the analytical algorithm limits its applicability to systems with a low number of clients. For example, in the case of the RNMTC model from Figure 6.16 of the database system with two classes of clients, by increasing with one the number of heavy readers, the number of the MC states grows from 2304 to over 20000 states. Future work is necessary to implement a new encoding for the MC states, aggregating similar

states and reducing considerably the memory space required for the MC.

Another direction for future research is in the study of bounds. The expressions on bounds derived in this thesis are relatively loose, especially for some upper bounds, further work needs to be done to improve the tightness. This thesis considered a number of different RNMTC models. Extension of the bounding technique to other types of generic RNMTC model warrant further investigation.

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Appendix

Usage of RNMTC Simulator and analytic Solver

1. Directories

All the packages are in directory: `~jingping/rnmtc`.

Under `~jingping/rnmtc`, there are two subdirectories: “analysis” which includes the analytic solver and “simulation” which includes the RNMTC simulator.

2. Usage of RNMTC Simulator

Under `~jingping/rnmtc/simulation`, there are two subdirectories: “data” which includes the data of the simulation system and “source” which includes the source code of simulator.

File “Makefile” is for compiling the source code and generating executable code. There are two executable codes should be generated: “datainput” and “mtms.sun4”. To create them, do the following under `~jingping/rnmtc/simulation`:

1. `make datainput`
2. `make mtms.sun4`

2.1. Data Input

First, the data for the system under simulation must be built. In order to do this, type:

“datainput” under `~jingping/rnmtc/simulation`.

After typing this, you will be promoted for the data, such as task name, server name, execution time and precedence relationship. The task name and server name are digits and can not be same for different tasks in the same client.

“model name” is the test suite number which you will use when you run the simulator.

The data will be saved in `~jingping/rnmtc/simulation/data`.

Note: please check the limit for each data in source code “`~jingping/rnmtc/simulation/source/datainput.c`”, so the input data will not exceed the limit.

2.2. Simulation

After inputting the data, the simulation can be started. To start simulation, type:

“`mtms.sun4`” under `~jingping/rnmtc/simulation`.

When asked “model name”, input the model name which is under simulation.

The information how to decide the simulation random seeds, period and etc. can be referenced in [Nei_1] and [Neil_2].

3. Usage of RNMTc analytic Solver

Under `~jingping/rnmtc/analysis`, there are two subdirectories: “data” which includes the data of the system and “source” which includes the source code of the solver.

File “Makefile” is for compiling the source code and generating executable code. There are two executable codes should be generated: “datainput” and “Algo”. To create them, do the following under `~jingping/rnmtc/analysis`:

1. `make datainput`

2. `make Algo`

3.1. Data Input

The data input for analytic solver is very similar with that in simulator. The data is stored in

~jingping/rnmtc/analysis/data.

3.2. Start Algo

After inputting the data, you can use the solver to get the performance measures for a certain model, to do so, type:

“Algo” under ~jingping/rnmtc/analysis.

When asked “model name”, input the model name which is under tested.

END

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FIN