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Ductility and Moment Redistribution in Continuous FRP Reinforced Concrete Beams

by

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B.Sc. (Isfahan Univ. of Tech., Iran)

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A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
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Department of Civil and Environmental Engineering

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in Civil Engineering
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for Dr. J.L. Humar, Chair,
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July 1997

Abstract

Ductility is a desirable behaviour of reinforced concrete structures. It permits a structure to give warning before failure, and to absorb and dissipate energy under seismic loads. Furthermore, in indeterminate structures, ductility allows formation of plastic hinges and redistribution of internal stresses and moments, which in its turn is advantageous for the overall behaviour of the structure. Ductility and moment redistribution in reinforced concrete structures are commonly believed to be the consequences of yielding of steel reinforcement. Since FRP materials do not yield, there is uncertainty with regard to the achievement of ductility and moment redistribution in FRP reinforced concrete members.

The current study is primarily focused on the issues of ductility and moment redistribution in FRP reinforced concrete beams. During the course of the study, ten 6.5 *m* long continuous two-span concrete beams were tested to evaluate their ductility and moment redistribution capacity. Two beams were conventional reinforced concrete members while the remaining eight beams were reinforced for flexure with CFRP rebars. Two of the latter eight were under-reinforced while the remaining six were over-reinforced. To investigate the effect of concrete confinement on ductility, the compression zone of four of the over-reinforced beams was reinforced with CFRP grids (known as NEFMAC) for confinement. The shear reinforcement for two of the FRP over-reinforced beams comprised NEFMAC grids while the other eight beams had conventional steel U-stirrups.

Based on theoretical analyses, a simplified bi-linear model for moment-curvature relationship of FRP reinforced concrete sections was proposed. Results of the conducted experimental work were analysed for ductility and moment redistribution. Acceptable pseudo-ductile behaviour was observed for all the FRP over-reinforced concrete test beams. Consequently, based on the analytical and experimental results, new definitions for “curvature pseudo-ductility factor” and “deflection pseudo-ductility factor” were proposed for FRP reinforced concrete beams. Results of the experimental program and nonlinear finite element analysis confirmed full plastic redistribution and high ductility in the conventional reinforced concrete beams. The FRP over-reinforced beams achieved almost 50% of the maximum possible redistribution while the FRP under-reinforced beams did not show any ductility. On the other hand, confinement of the compression zone did not have any influence on either ductility or moment redistribution in the present tests. In addition to ductility and moment redistribution, the shear strength of FRP reinforced concrete beams was also investigated. Using a number of available experimental works, the applicability of two existing methods (BRI and JSCE) for calculation of shear strength of FRP reinforced concrete members was examined. It was noted that one of the methods is highly conservative while the other method often overestimates the shear strength of FRP reinforced concrete beams. A new expression for calculation of shear strength of FRP reinforced concrete members was proposed and its applicability was assessed. The proposed method, albeit empirical, is safe and not excessively conservative.

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Chapter 1

Introduction

1.1 General

Many steel reinforced concrete structures have experienced serious damage due to contact with sulphates, chlorides and other corrosive environments. The problem has created huge expenses for repair, rehabilitation and replacement of damaged structures in Canada, the United States and other parts of the world. To solve the problem, various solutions have been proposed, tested and executed, e.g., protection of steel through epoxy coating, cathodic protection and use of impermeable membranes. However, due to the inadequate performance of previous protective measures, it has been realized lately that the complete substitution of steel bars with a material non-susceptible to corrosion may be the ultimate solution.

Fibre Reinforced Plastic (FRP) has been considered as an alternative to steel in reinforced concrete structures since two decades ago. The main advantage of FRP materials is their excellent resistance against corrosion. Since the 1980's, FRP bars and tendons have been used in some reinforced and prestressed concrete bridges and structures in North America, Japan and Europe.

Although extensive experimental and analytical studies have been conducted on different aspects of the behaviour of FRP reinforced concrete structures, nevertheless some major issues remain unresolved, among them ductility and moment redistribution in indeterminate structures. Although lack of ductility in FRP reinforced concrete beams has been recognized by researchers, and some methods have been proposed to achieve a limited amount of ductility, very few studies have been conducted on moment redistribution in indeterminate FRP reinforced concrete members and on the efficiency of the methods proposed to date.

1.2 Problem Definition

Ductility is a desirable and advantageous behaviour of reinforced concrete structures. It could be defined as the ability of a structural member to withstand large deformations while concurrently undergoing reduction in stiffness, but without concomitant decrease in load carrying capacity. Ductility permits a structure to give warning before failure, and to absorb and dissipate energy under seismic loads. Furthermore, in indeterminate structures, ductility allows formation of plastic hinges and redistribution of internal stresses and moments.

Redistribution of moments in continuous beams and slabs is a logical consequence of ductility. Therefore, ductility and moment redistribution could be viewed in the same light. In other words, where relevant, achieving ductility leads to moment redistribution and vice versa. Moment redistribution in continuous beams permits structures to carry higher loads than those obtained from elastic analysis. Furthermore, for a certain load it allows the negative moment section and the positive moment section to be designed for a smaller and larger amount of moments, respectively. However, the increased load carrying capacity is neglected normally in practical designs in order to provide a secure margin for unexpected higher loads.

Achievement of ductility in FRP reinforced concrete structures, and consequently achievement of redistribution of stresses and moments in indeterminate concrete structures reinforced with FRP, requires further investigation because FRP materials are essentially linear elastic up to failure and lack yield and plastic flow characteristics. In the other words, FRP reinforced concrete beams hardly exhibit any nonlinear moment-curvature response. Therefore, they fail without providing sufficient indication of impending failure and it is generally believed that they lack the ability for any redistribution of stresses and moments. The foregoing belief is primarily based on theoretical considerations and on empirical knowledge gained from the behaviour of steel reinforced concrete members.

In the current experimental and analytical study, an attempt will be made to determine whether any noticeable ductility and moment redistribution exist in FRP reinforced concrete beams with different ratios of reinforcement. Furthermore, the effectiveness of some methods for improvement of ductility and moment redistribution in FRP continuous reinforced concrete beams will be investigated. To the writer's knowledge, this is the first systematic study of the subject of moment redistribution and ductility in continuous FRP reinforced concrete structures.

1.3 Objectives and Scope

The objectives of this study are:

1. Investigation of the moment-curvature relationship in continuous FRP reinforced concrete beams.
2. Investigation of the load-deflection response of continuous FRP reinforced concrete beams.

3. Investigation of ductility in FRP reinforced concrete beams.
4. Studying the possibility and the extent of moment-redistribution in continuous CFRP reinforced concrete beams in two cases: a) Under-reinforced beams, b) Over-reinforced beams.
5. Determining the effectiveness of CFRP shear reinforcement and its effect on ductility and moment-redistribution.
6. Studying the possibility and the extent of moment redistribution in continuous CFRP reinforced concrete beams with confined compression zone.
7. Examining the suitability and accuracy of the nonlinear finite element method for the analysis of FRP reinforced indeterminate beams.
8. Making design recommendations for achieving moment redistribution in FRP reinforced structures.

The scope of this study is restricted to two-span continuous concrete beams reinforced for flexure with CFRP rebars and for shear with steel or CFRP reinforcement. However, to provide a basis for comparison, a part of this study will be dedicated to two-span continuous beams longitudinally reinforced with conventional steel bars. The beams will be analysed by a finite-element-based computer program in each case, and the analytical and experimental results will be compared. It is not the purpose of this study to either formulate new finite element procedures for application to FRP reinforced concrete or to improve existing methods of analysis.

Chapter 2

Literature Review

2.1 General

Corrosion of reinforcement is a serious problem in steel reinforced concrete structures. The problem becomes critical if the reinforcement is highly stressed under service conditions. An ordinary reinforced concrete structure would lose part of its resistance capacity if it experienced prolong exposure to corrosive elements such as salts, acids, etc. This problem and its consequences have sometimes been considered not only an engineering problem but also a serious social problem (Hamada, 1992).

Repair and replacement of damaged concrete structures has caused millions of dollars of loss in North America as well as in other parts of the world. The cost of rehabilitation or repair of existing parking structures has been estimated as \$4 to \$6 billion in Canada (Bedard, 1992). In the United States, more than 40% of highway bridges need to be replaced or rehabilitated, according to the statistics of the Federal Highway Administration (Saadatmanesh and Ehsani, 1991). The repair cost of highway bridges in the United States is estimated at nearly \$50 billion, while for fixing all the concrete structures in the country, the astronomical figure of \$1 to

\$3 trillion has been forecast (Bedard, 1992).

Of course, some techniques have been developed and used to prevent corrosion in reinforced concrete. Among them epoxy coating of bars, polymer impregnation of concrete and cathodic protection might be mentioned, but each of these techniques have only partially succeeded (Razaqpur and Kashef, 1993). To fully eliminate the problem, attention has been directed to the complete replacement of steel rebars with new corrosion-resistant materials. Fibre Reinforced Plastics (FRP) are highly resistant to saline and alkaline environments, consequently they have become the subject of extensive research and development as possible substitute for both steel reinforcement and prestressing tendons.

2.2 FRP Materials

FRP materials essentially are made of two basic components, fibres and resin. The fibres, which are ideally elastic, brittle and very strong, are the main load resisting component of FRP. Depending on the type of the fibre, the diameter of the fibres is in the range of 5 to 25 micron (Rostasy, 1993).

The resin basically acts as a cohesive environment to keep the fibres together. In fact, the role of the resin (matrix) in FRP may be resembled to the role of cement paste in concrete, therefore, the matrices with low strength do not significantly influence the elastic modulus and final strength of the composite. In the other words, the strength and the modulus of elasticity of composite are mainly affected by the fibre properties and its volumetric ratio within the composite (Rostasy, 1993).

The fibre may be of glass, carbon, aramid or vinylon, where the composite product would be known as GFRP, CFRP, AFRP and VFRP, respectively. It should be pointed out that basalt fibres have been produced and used in Russia for a

number of years. It is claimed that they are comparable to glass fibres with regard to their strength and elastic modulus but they possess greater chemical stability and durability, particularly in alkaline environments (Aslanova and Resnyansky, 1995). The subsequent discussion, however, will be confined to the more common and widely produced fibres around the world.

Glass Fibres

Glass fibres may be classified into 4 categories (Razaqpur and Kashef, 1993);

1. E-glass: the most common in the market with low alkali content, used in the construction industry, (the modulus of elasticity, $E = 70\text{GPa}$, the ultimate strength, $\sigma_u = 1500 - 2500\text{MPa}$, and the ultimate strain, $\epsilon_u = 1.8\% - 3\%$).
2. Z-glass: high resistance to alkali attack, used in cement mortars and concrete as a reinforcing fibre.
3. A-glass: high alkali content, going out of production.
4. S-glass: used in aerospace technology and space research. where extra high strength and high modulus of elasticity are needed, ($E = 87\text{GPa}$ and $\sigma_u = 3900\text{MPa}$).

Note that new and improved types of glass are constantly being introduced into the market.

Carbon Fibres

Carbon fibres may be classified into two categories (Razaqpur and Kashef, 1993);

1. Synthetic fibres known by the chemical name polyacrylonitrile (PAN), have

very high strength (up to 3700MPa), but are expensive. There are three types of PAN carbon fibres. Type I, the stiffest carbon fibre and with the highest modulus of elasticity, ($E = 380\text{GPa}$, $\sigma_u = 2000\text{MPa}$ and $\epsilon_u = 0.5\%$). Type II, the strongest carbon fibre, ($E = 240\text{GPa}$, $\sigma_u = 2800\text{MPa}$ and $\epsilon_u = 1\%$), and finally Type III the least stiff fibre of this category with its strength being between Type I and Type II.

2. Pitch-based fibres, originally obtained from the distillation of coal, are cheaper than PAN fibres but have lower strength and modulus, ($E = 37.5 - 140\text{GPa}$, $\sigma_u = 765 - 2350\text{MPa}$ and $\epsilon_u = 2.1\%$). Recently, higher moduli pitch-based carbon fibres have been introduced and it is expected that soon the elastic modulus of carbon fibres would exceed that of steel.

Carbon fibres have excellent resistance to alkaline and acid environments and are considered to be highly chemically stable under a whole range of conditions.

Aramid Fibres

Aramid, an abbreviated term for aromatic polyamide, is made from para-type fibres with straight-chain benzene nuclei (Minosaku, 1992). Aramid basically is a man-made fibre, was first produced by DuPont in Germany under the name of Kevlar. Aramid fibres are almost fully crystalline, strong in the longitudinal direction but have weaker bonds in the transverse direction, (Kasperkiewicz et al, 1993). There are four types of kevlar, among them, Kevlar 49 has been designed for reinforcement applications, with the characteristics of $E = 120\text{GPa}$, $\sigma_u = 2700 - 3500\text{MPa}$ and $\epsilon_u = 2.0\% - 2.7\%$. Representatives of Kevlar products are TWARON which is produced by Aramid Maatschappij of Japan in cooperation with the AKZO Company of Germany, KEVLAR of DuPont of USA, and TECHNORA of Teijin Ltd. of Osaka Japan (Razaqpur and Kashef, 1993),

Matrix

The matrix (resin) could be a thermosetting or a thermoplastic compound. The thermosetting resins will harden upon application of heat and cannot be re-liquefied. The thermoplastic resins can be liquefied by heat and solidified by cooling. Polyesters, vinyl esters and epoxies are thermosetting resins, while polyvinyl chloride (PVC), polyethylene and polypropylene are thermoplastic resins (Bedard, 1992).

FRP Rods

At present, there are various manufacturers in the world supplying FRP rods. In Japan, more than 10 companies are manufacturing and supplying various types of FRP materials and rods (Tsuji, 1993). Some of the most famous rods (bars) which are readily available in the market are as follows (Razaqpur and Kashef, 1993, and Erki and Rizkalla, 1993).

1. LEADLINE: is produced by Mitsubishi Chemical Company of Japan from pitch-based carbon fibre, with the characteristics of $E = 147GPa$, $\sigma_u = 2550MPa$ and $\epsilon_u = 1.3\%$. The CFRP rods are produced in round shape from 1 to 17 mm diameter, and in deformed shape from 5 to 17 mm.

2. FiBRA-Rod: is produced by the Mitsui Construction Company of Japan from Kevlar 49, with the characteristics of $E = 65GPa$, $\sigma_u = 1290 - 1310MPa$ and $\epsilon_u = 2.1\%$.

3. TECHNORA: is produced by the Teijin Corporation of Japan from Aramid, with the characteristics of $E = 53GPa$, $\sigma_u = 1864MPa$ and $\epsilon_u = 3.6\%$.

4. CFCC: Carbon fibre composite cable is made by Tokyo Rope from PAN type carbon fibres. CFCC are produced in diameters from 3 to 40 mm with the strength

of 10 to 1100 kN.

5. ISOROD[®] is produced by the Pultrall Inc. of Thetford Mines in Quebec, from glass fibres and polyester resin, with the characteristics of $E = 44.8 GPa$, $\sigma_u = 700 MPa$.

6. C-Bar[®] is produced by Marshall Industries Composites of Jacksonville, Florida. The bar comprises E-glass fibres embedded in a urethane modified vinylester. The surface geometry of this bar is practically identical to that of a conventional reinforcing bar, but it has an $E = 41.5 GPa$ and a $\sigma_u = 660 MPa$.

Notice that this is an evolving field and many new products are constantly being introduced in the market. Consequently, there is no pretension of completeness in the foregoing review.

2.3 Basic Characteristics of FRP Reinforcement

2.3.1 Strength

a) Tensile Strength

FRPs normally have high tensile strength that exceeds the strength of conventional steel rebars. Their high strength makes them suitable as reinforcement for concrete structures, especially for pre-tensioned and post-tensioned concrete members. The tensile strength of FRPs is based basically on the tensile strength, volume ratio, size, and cross-sectional area of fibres. It is also affected by the bond performance of the matrix.

Yamasaki et al (1993) in their experiment on different kinds of fibre reinforced

materials reported tensile strengths of 1100 to 2200 *MPa* and elastic moduli of 100 to 150 *GPa*, for carbon fibre bars. For glass fibre bars, they reported tensile strengths of 900 to 1100 *MPa* and a modulus of elasticity of approximately 45 *GPa*. They also reported tensile strengths of 1350 to 1650 *MPa* and an elastic modulus of approximately 60 *GPa* for aramid fibre bars.

Malvar (1995) conducted some tensile tests on four types of GFRP bars with different deformations but identical diameters. Five specimens were chosen for each type of GFRP bars. He concluded that deep indentations and the resulting kinks in the longitudinal fibres will reduce the bar strength. For his specimens, he reported the ultimate strength between 450 to 710 *MPa*, and the modulus of elasticity between 28 to 48 *GPa*.

It should be noted that when the diameter of a FRP bar increases, its strength does not proportionally increase (Saadatmanesh, 1994, and Mochizaki et al, 1993). This characteristic is due to the fact that the fibres stress transfer efficiency decreases with their distance from the surface of the bar. However, for normal size bars, the tensile strength fluctuation is within 2 - 7 % (Mochizuki et al, 1993).

b) Compressive Strength

Although there are few studies available on the compressive behaviour of FRP bars, commonly their compressive strength is not taken into consideration. Bedard (1992) stipulated that in general, the absolute compressive strength of FRP rods are lower than their tensile strength, i.e. Isorod bars have a compressive strength of 600 *MPa* compared to their tensile strength of 700 *MPa*. Kobayashi and Fujisaki (1995) confirmed in their test on short columns reinforced with CFRP that under pure axial load, the contribution of FRP to the compressive strength of the column is negligible.

2.3.2 Non-Conductivity Property

FRPs have excellent non-conductivity property. In other words, they are magnetically and electrically neutral. Therefore, the use of FRP reinforced concrete would be very beneficial in a hospital's magnetic resonance imaging test chamber, in the guideways for magnetic levitation trains (Tarricone, 1993), and in airport runways and radar centres.

2.3.3 Fatigue Behaviour

Fatigue is a characteristic of a wide-range of structural materials and its disregard may cause some unexpected failure, especially in components exposed to high levels of cyclic loads or stresses. Compared to steel, the fatigue behaviour of FRP materials is excellent, i.e., for stresses less than 1/2 of their ultimate strength they do not fail due to fatigue (Ehsani and Saadatmanesh, 1993). Furthermore, it has been shown that the fatigue strength of CFRP and AFRP exceeds that of high strength prestressing wire (Rostasy, 1993).

Rahman et al (1993) in a series of tests on NEFMAC grids (a two or three dimensional grid of FRP, with carbon fibres in Type-C and with 74% carbon fibres and 26% glass fibres in Type-CG), investigated their tensile fatigue behaviour by cycling the load between 10% and 30% of the tensile strength with a 5 Hz frequency. They reported that after 4 million cycles, the Type-CG and Type-C retained 92% and 93% of their original tensile strength, respectively.

2.3.4 Creep Behaviour

The creep rupture phenomenon basically exists for all structural materials, however it is of no practical relevance if the creep strain is a small percentage of the initial strains. Overall, the creep behaviour of composites is very good, i.e., most of the commercially available fibres do not creep (Char et al, 1994).

Budelmann and Rostasy (1993) stipulated that for the determination of the admissible prestress of FRP elements, the creep rupture behaviour should be taken into account, and should be investigated by tests. Rahman et al (1993) reported that the creep deformations in NEFMAC are quite small. Applying a tension load equal to 40% of the tensile strength of NEFMAC for 175 days, they measured creep strains of Type-CG and Type-C NEFMAC to be equal to 2.7% and 1.8% of their initial strains, respectively.

Yamasaki et al (1993) reported that after one year from initial loading, the creep of carbon and glass fibres would be stopped, while for aramid fibre bars it will keep increasing even after one year. Furthermore, they showed that for deformed FRP bars, creep strain is larger than that for similar smooth FRP bars.

2.3.5 Bond Behaviour

Bond characteristic is very important for any reinforcement in concrete. In the case of FRP bars, although in very early investigations poor bond strength was reported for glass fibre reinforcement (Wines and Hoff, 1966), more recently acceptable bond strength has been reported for most FRP bars.

Nawy and Neuwerth (1977) in their experiment on twenty simply-supported beams observed the crack size and pattern under applied loads from which they

concluded that good mechanical bond between GFRP bars and concrete exists.

Yamasaki et al (1993) in their tests on CFRP, GFRP and AFRP rods with a diameter of 8 mm reported that the bond stress of straight FRP rods is almost equal to that of round steel bars. Furthermore, for the straight FRP bars covered by sand particles, the bond stress is almost equivalent to that of deformed steel bars.

Mashima and Iwamoto (1993) concluded from their tests that the bond strength of FRP rods is adequate for concrete structures. Furthermore, they did not observe any deterioration of bond property in CFRP, GFRP and VFRP rods under increased loads, but they observed some reduction in the bond of AFRP rods. They also obtained a bond strength of approximately 10 MPa for FRP rods with uneven surfaces.

Mashima and Iwamoto also conducted a series of tests on the influence of freezing and thawing on bond strength. They concluded that there was not any significant loss of bond strength in glass, vinylon and carbon FRPs, but aramid FRPs gradually lost some of their bond with the progress of freezing and thawing.

Kanakabo et al (1993) conducted a comprehensive set of experiments on bond performance of FRP bars. The first group of tests comprised simple bond tests carried out by pulling out a single bar from a concrete block. The second group involved a bond splitting test on cantilever type specimens. Finally, the third group consisted of an anti-symmetrical loading test for actual FRP reinforced beams. They concluded that the conventional formula which is used for deformed steel bar could be used for bond splitting strength of the bearing resistant type bar of FRPs. Furthermore, they found that for friction resistant type bar of FRP, the bond splitting strength is 80% to 95% of the value calculated by the formula. They also concluded that for the cantilever type specimens without lateral reinforcement, the bond behaviour showed the same tendency as the simple bond test results.

Malvar (1995) conducted some bond tests on four types of GFRP bars with different deformations but with identical diameter of 19 *mm*. The objective of the tests was to determine configuration-independent local bond stress-slip data for use in constitutive material models. During the pull-out tests, he exerted confining pressure on the specimens via a confining ring, varying from 3.4 to 31 MPa. He finally concluded that small surface deformations (about 5.4% of the nominal diameter of bar) are sufficient to develop maximum bond stresses up to five times of the concrete tensile strength. He also reported the bond stress of steel bars under the same confinement as the GFRP rebars to be 1.2 to 1.5 times greater than the bond stress of GFRP rebars.

Benmokrane et al (1996) conducted a series of experiments on the bond strength of GFRP reinforcing bars in concrete. They tested twenty concrete beams with the span length of 1.1 *m* as well as five pull-out specimens, reinforced with GFRP or steel bars. The results indicated that the diameter of bars affects the bond behaviour of GFRP and steel reinforcement. They also concluded that the bond strength of GFRP bars is less than that of steel reinforcement (60 – 90%), and attributed it to the differences between the characteristics of the surface deformations of GFRP and steel bars. In addition, they concluded that the bond strength obtained from beam tests is less than that from pull-out tests (55 – 95%), depending on the diameter of reinforcing bar. It is important to mention that the bond strength is not merely a function of the type of the fibre, but is strongly affected by the bar size, geometry, surface deformations and roughness and by other geometrical and mechanical ideosyncracies of the bar.

Ehsani et al (1996) conducted a comprehensive analytical and experimental study on 102 specimens to develop design guidelines for bond of GFRP rebars to concrete. They constructed and tested 48 beam specimens, 18 pull-out specimens and thirty-six 90° hooked rebar specimens. They found out no significant effect of embedment length on the stresses attained by GFRP bars until the loaded-end slip

reached 0.38 *mm* or the free-end slip reached 0.064 *mm*, hence, they recommended the allowable slips at the loaded-end and the free-end of GFRP rebars to be limited to 0.38 *mm* and 0.064 *mm*, respectively. Furthermore, they suggested some modifications to the existing guidelines for the development length of straight and hooked steel bars in order to make them applicable to GFRP reinforcing bars.

2.3.6 Bend of FRP Rods

For FRP rods, it seems that if they are to be used in concrete as reinforcement, they have to be bent for anchoring of longitudinal reinforcement, stirrups and hooks. But the bending process of FRP rods is much more difficult than that of steel rebars and it cannot be carried out in the field for existing FRP bars. Currently, bent FRP bars can be obtained by ordering them to be manufactured in the factory.

The tensile strength of the bent part is significantly less than that of straight part of FRP rods. As a matter of fact, when stress is applied on the bent part, the fibres fracture violently, causing tensile strength of the bent parts to drop to 30 - 70% of that of the straight parts of FRP rods (Mochizuki et al, 1993). It is also observed that for hooked bars with longer radius of bend, higher stiffness could be achieved. Furthermore, smaller radius of bend causes breaking of the bent portion of the bar at very small loads (Ehsani and Saadatmanesh, 1993). The reason for the reduced strength of the bends is the pressure exerted along the bend due to curvature, and the intrinsic weakness of fibres perpendicular to their axis.

Maruyama et al (1993) observed in their tests that CFRP and AFRP rods all ruptured at the bent portion. Furthermore, the location of the rupture was the beginning of the bent and on the loading side. They reported that the tensile strength of the bent portion of FRP rods will decrease hyperbolically if the curvature of the bend increased, i.e. the tensile strength of FRP bent rods was reported to be

about 65%, 60% and 50% of the tensile strength of the straight rods, for the bend radii of 25 mm, 15 mm and 5 mm, respectively. They also reported higher bend strengths for higher strength concrete.

Generally speaking, the bond strength of the better designed and produced FRP bars is adequate for applications in concrete structures. Sand coating can be used to further increase bond strength whenever the need arises.

2.3.7 Thermal Expansion

The thermal expansion characteristics of steel and concrete are very close, i.e., their thermal expansion coefficients are: $\alpha_{steel} = 12 \times 10^{-6} (1/^\circ C)$ and $\alpha_{concrete} = 10 \times 10^{-6} (1/^\circ C)$. The coefficient of thermal expansion of FRP rods is mostly different from concrete, and may be summarized as follows (Mashima and Iwamado, 1993): $\alpha_{CFRP} = (0.6 - 1.0) \times 10^{-6} (1/^\circ C)$, $\alpha_{GFRP} = (9 - 10) \times 10^{-6} (1/^\circ C)$ and $\alpha_{VFRP} = 4.4 \times 10^{-6} (1/^\circ C)$. The worst case is that of aramid with its thermal expansion coefficient being negative, i.e. $\alpha_{AFRP} = [(-6) - (-2)] \times 10^{-6} (1/^\circ C)$.

It should be pointed out that due to lack of transverse fibre in FRP rebars, their transverse thermal expansion may be much larger than their longitudinal thermal expansion (Gentry and Hudak, 1996). In an experiment conducted by Gentry and Hudak, the longitudinal thermal expansion of two types of GFRP rebars was measured as 4.8×10^{-6} and 8.2×10^{-6} while their transverse thermal expansion was 38×10^{-6} and 32×10^{-6} , respectively.

For Type-CG of NEFMAC, it is observed that the coefficient of thermal expansion is about $8.4 \times 10^{-6} (1/^\circ C)$, and is almost constant in the temperature range of $-30^\circ C$ to $+50^\circ C$, (Rahman et al, 1993). This could be quite favourable for bridge decks in many parts of Canada which experience a wide range of temperature

changes.

2.3.8 Sensitivity to Temperature Changes

Most modern plastics exhibit excellent stability in a wide range of temperatures between -68°C to $+107^{\circ}\text{C}$; nevertheless, it is observed that the flexural strength of FRP bars at temperatures in excess of 204°C decreases significantly, (Brown and Bartholomew, 1993).

Rahman et al (1993) in their experiments on NEFMAC Type-CG, reported a decrease of 9.5% and 2.5 % in the ultimate tensile strength and modulus of elasticity, respectively, for a rise in temperature from room temperature to $+50^{\circ}\text{C}$. They also reported 11 % decrease in tensile strength and 2.5 % decrease in modulus of elasticity for NEFMAC Type-CG, when room temperature dropped to -30°C .

Bryen et al (1996) conducted an experimental program on the short-term behaviour of CFRP prestressed concrete beams under low temperature (-27°C). They tested six 3.0 m long concrete beams prestressed with 8 mm diameter CFRP Lead-line bars. They concluded that the ultimate strength of CFRP bars bonded within concrete is unaffected by short-term exposure to temperature as low as -27°C .

2.4 Behaviour of FRP Reinforced Concrete Beams

Much research has been conducted on the feasibility of using FRP rebars instead of steel reinforcement in reinforced concrete beams. Various aspects of the strength and the serviceability of FRP reinforced concrete beams have been studied to date.

Nawy et al (1971) conducted an experimental study on twenty simply-supported

1.98 *m* long, 90 *mm* wide and 180 *mm* high beams. The beams were reinforced either with steel or fibre glass and were loaded to failure under four point bending. They found that fibre glass reinforced beams had three to five times more cracks than similarly designed steel reinforced beams, indicating a good mechanical bond between the fibre glass bars and the concrete. They also found no significant difference between the initial cracking moment of fibre glass and the steel reinforced concrete beams. They noted, however, that crack widths of fibre glass reinforced beams were larger than those of steel reinforced beams under the same load. Furthermore, they found that the final deflection of fibre glass reinforced concrete beams was at least three times greater than that of steel reinforced concrete beams. Therefore, they concluded that for a specific magnitude of crack width or deflection, fibre glass reinforced beams have much higher safety against failure than steel reinforced concrete beams. Of course, the higher safety means that serviceability will govern the design and that the full strength of glass fibre rods cannot be utilized.

Saadatmanesh and Ehsani (1991) tested five 3.05 *m* long rectangular beams and one T-beam reinforced with fibre glass or steel rebars. For the calculation of the ultimate strength of GFRP reinforced beams, they used conventional methods of analysis of steel reinforced concrete beams and found good agreement between the analytical and experimental results. Therefore, they concluded that the classical theories for the flexural analysis of steel reinforced concrete beams could be used for the analysis of FRP reinforced concrete beams with good accuracy. They also found a large number of well-distributed cracks in GFRP reinforced concrete beams, and attributed it to the good mechanical bond between GFRP bars and concrete.

Zia et al (1992) conducted an experimental study on the flexural and shear behaviour of concrete beams reinforced with 3-D continuous carbon fibre fabric which was made from PAN-type continuous fibres. They tested nine 2.44 *m* long simply-supported concrete beams, three of them were reinforced with 3-D fabric and 4 of them were reinforced with steel. They observed in the beams reinforced with

3-D fabric much more vertical cracks and much larger deflection compared to those reinforced with steel.

Okamoto et al (1992) tested six reinforced and prestressed beams using braided aramid FRP rods. They concluded that braiding fibres significantly improve the bond behaviour between FRP rebars and concrete. They also concluded that partial prestressing is effective for the control of deflection of FRP reinforced beams.

Faza and GangaRao (1992) conducted an experimental and analytical study on the deflection behaviour of FRP reinforced concrete beams. They suggested a modified moment of inertia to be used in lieu of the effective moment of inertia in the expressions suggested by the ACI code for the calculation of deflection in cracked reinforced concrete beams. Ali (1995) found the suggested expression to be inapplicable to CFRP reinforced concrete members.

Daniali (1992) tested a total of 30 GFRP reinforced concrete beams to investigate the development length of FRP bars. The beams were 3.05 m long with a cross section of $203 \times 457 \text{ mm}$, and the bars had deformed surfaces. He observed four modes of failure during his tests, including tension failure of the main reinforcement, pull out failure, splitting failure, and failure due to creep deformation. From his tests he suggested development length of 305 mm (12 in) and 440 mm (18 in) for #4 and #6 GFRP rebars, respectively.

Brown and Bartholomew (1993) tested six GFRP reinforced concrete beams, each with a length of 762 mm and a cross section of $152 \times 152 \text{ mm}$. The beams were simply-supported and were symmetrically loaded approximately at one-third of the span. They found that the ultimate strength of FRP reinforced beams could be predicted by the strength design method of steel reinforced concrete beams. They also found that FRP reinforced beams had more extensive cracking and larger deflection (three to four times larger) than the steel reinforced concrete beams.

Brown and Bartholomew also conducted a series of pull-out tests, concluding that GFRP reinforcing bars have a bond strength of two-thirds that of steel bars.

Issa et al (1993) conducted an experimental study of transfer length in GFRP pretensioned concrete. They tested twelve $152 \times 102 \times 2590$ mm concrete specimens, seven reinforced with fibreglass and five reinforced with steel. They concluded that the bond between fibreglass and concrete is stronger than that between steel and concrete, and that the transfer length of GFRP bars is smaller than that of steel bars. He also proposed a transfer length equal to 28 times of the nominal diameter of GFRP tendons in prestressed concrete.

Larralde et al (1993) conducted a study on the bond and slip of FRP rebars in concrete. In their experimental work, 12 FRP reinforced and 8 steel reinforced concrete cylinders were used with various embedment length of reinforcement. They concluded that the bond strength of FRP rebars was weaker than that of steel rebars with the same diameter and embedment length. They also reported greater slip at failure of FRP rebars compared to that of steel rebars.

Nanni (1993) conducted an analytical and experimental study on the flexural behaviour and design of FRP reinforced concrete members. He observed higher maximum moment capacity of FRP reinforced concrete beams than that of steel reinforced concrete beams, even with less reinforcement. However he mentioned lack of flexural rigidity and ductility of FRP reinforced concrete beams as their essential disadvantages. He finally recommended a procedure for the flexural design of FRP reinforced concrete beams by the ultimate strength or by the working stress method.

Saadatmanesh (1994) reported an experimental study on two beams reinforced with GFRP. One of the beams had longitudinal reinforcement much larger than the balanced reinforcement while the other one was under-reinforced. For the over-

reinforced concrete beam, he measured the high value of 0.0056 for the maximum strain of compression concrete at failure. Furthermore, he reported a nonlinear load-deflection relationship for the over-reinforced beam and a linear load-deflection response for the under-reinforced beam. Clearly, the nonlinearity is induced by the nonlinear behaviour of concrete under high level of stress.

Benmokrane et al (1996) conducted an experimental and analytical study on the flexural response of GFRP reinforced concrete beams. They tested eight 3.3 *m* long simply supported concrete beams, half reinforced with GFRP bars, the other half with steel reinforcement. Comparison was made between the load carrying capacities, load-deflection responses, and flexural rigidities of GFRP/steel reinforced beams and it was shown that the ACI formulas could be adapted and modified for studying the flexural behaviour of GFRP reinforced concrete beams.

2.5 Ductility of Reinforced Concrete Members

Ductility of conventional reinforced, prestressed and partially prestressed concrete, and FRP reinforced and prestressed concrete has been discussed in the literature. Furthermore, some analytical and experimental studies have been conducted regarding the ductility of reinforced or prestressed concrete members.

Naaman et al (1986) conducted an analytical and experimental study on the ductility of partially prestressed concrete members. They proposed some modifications to the concept of reinforcing index and the definition of yield curvature. Based on these modifications, they developed a nonlinear analytical model for investigation of ductility in partially prestressed concrete beams. In their experimental study they tested twelve partially prestressed concrete beams. The beams were simply-supported on a 2.75 *m* span, and each had a cross section of 114 × 228 *mm*. They concluded that for all kinds of steel reinforced, prestressed and partially prestressed

concrete beams, ductility decreases with increase in the reinforcing index. Furthermore, they noted that increasing the compressive strength of concrete decreases the ductility.

Cohn and Riva (1991) conducted a comprehensive analytical and parametric study on the ductility of structural concrete sections including reinforced, prestressed and partially prestressed concrete sections. They took into consideration different material parameters, i.e., different stress-strain law for concrete in compression and in tension, different stress-strain law for conventional and prestressed steel, and different bond-slip constitutive law for reinforcement. They also took into consideration different geometric parameters, i.e., different shape of the section, different tension and compression reinforcement indices, and different ratios for prestressing to total reinforcing steel. Finally, they proposed some modifications for ACI Code to ensure at least 20% moment redistribution for reinforced/prestressed sections of any shape and material combinations.

Naaman (1993) conducted a study on the issue of ductility in FRP prestressed and partially prestressed concrete beams. He stipulated that large deflection is not necessarily accompanied by good ductility. He stated that to account for elastic and inelastic energy at failure, a new definition of ductility is needed. Finally, he proposed a few methods to improve ductility in FRP prestressed concrete.

In another study conducted by Naaman and Jeong (1995), a new energy-based definition was proposed for ductility index, which could be used for both steel and FRP reinforced and prestressed concrete beams. The new definition expressed the ductility index as “the ratio of the total energy to the elastic energy at the failure state of the beam.” Using the new definition, they calculated the ductility indices of some test beams with conventional prestressing strands or FRP tendons. For prestressed concrete beams with steel, they used both the conventional and the new energy-based definitions of ductility index and obtained almost the same values.

Vijay et al (1996) in a separate study utilized the energy-based definition of ductility proposed by Naaman and Jeong to investigate the ductility indices of some of their GFRP under-reinforced and over-reinforced concrete test beams. They calculated different ductility indices at different load levels for each case, i.e. ductility indices equal to 1.4 and 2.5 respectively at 90% and 99.8% of the ultimate load for the under-reinforced beams, and 1.65 and 8.0. respectively, at 95% and 100% of the ultimate load for the over-reinforced beams.

Ali (1995) conducted an analytical and experimental study to develop a new concept for achieving ductility in FRP reinforced concrete members. In Phase I of his experimental study, he constructed four rectangular 4.2 *m* long beams with cross section of 200 × 400 *mm*. The beams were simply supported on a span of 4.0 *m* and were loaded in four-point bending. Two of the beams were reinforced with steel bars while the others were internally reinforced with layers of Tensar UX-1700 grids and externally reinforced with CFRP sheets bonded to the bottom of the beam. Tensar grids are made of polypropylene and are produced primarily for soil reinforcement (Koerner, 1990). They possess relatively high strength (200 MPa) but have relatively low modulus of elasticity (2 - 3 GPa). In Phase II, he constructed three 4.2 *m* long inverted T-beams with a height of 400 *mm*, web width of 200 *mm*, flange width of 400 *mm* and flange thickness of 100 *mm*. The beams were simply supported on a span of 4.0 *m* under four-point loading. Two of the beams were reinforced with the combination of CFRP rods and Tensar UX-1700 HT grids while the third one was reinforced only with CFRP rods. He used concrete with a compression strength of 35.0 MPa and 80.2 MPa for the beams of Phase I and Phase II, respectively. Ali concluded that a complete ductile flexural failure for FRP reinforced concrete beams is possible. His tests showed that if a combination of a stiff and strong reinforcement with a flexible but relatively strong reinforcement is used in concrete beams, large deformation before failure could be achieved without any major loss in flexural strength.

Mufti et al (1996) proposed the use of “deformability factor” instead of ductility factor, in FRP reinforced concrete beams. They defined the deformability factor as the ratio of curvature at ultimate to curvature at $\epsilon_c = 0.001$, where ϵ_c is strain in concrete. The deformability factor is a design tool but it has no influence on the response of a structure.

Further investigations about improvement of ductility in FRP reinforced concrete beams have been carried out by some researchers, which will be presented in Section 3.6.

2.6 Moment Redistribution in Reinforced Concrete

As reported by Park and Paulay (1975), Glanville and Thomas in 1935 were the first researchers who conducted an extensive investigation about moment redistribution in reinforced concrete beams. Since then other researchers have conducted experimental and analytical studies about the same subject. Based on those studies, the codes have established specific provisions and recommendations to allow for moment redistribution in continuous beams and frames.

Mattock (1959) conducted an experimental study on two series of continuous reinforced concrete beams. In Series 1, he tested four two-span continuous beams with cross section of $102 \times 229 \text{ mm}$ and length of 3.962 m . The beams were loaded unsymmetrically at the centre of one span only. In Series 2, he tested three two-span T-beams with height of 356 mm , flange width of 279 mm , flange thickness of 89 mm and web width of 114 mm . Each span was 4.572 m long and the beams were loaded symmetrically with concentrated loads at midspan. Mattock found out that in most of those beams, redistribution of moment took place even when

the steel stresses were below the yield stress. Furthermore, he reported the same amount of redistribution in the experimental beams at failure as calculated from theory. Finally he reported up to 14% increase in failure load compared to calculated (theoretical) failure load. Note that what Mattock stated about the occurrence of moment redistribution before yielding of steel reinforcement in his beams could be considered as evidence for the feasibility of moment redistribution in continuous FRP reinforced concrete beams.

Lazaro and Richards (1973) developed a linear approximation approach to analyze continuous steel reinforced concrete beams and frames and to account for moment redistribution. The model was based on the stiffness method but accounting for the stiffness changes in those members whose cross sections were cracked or had changed to a hinge. The model was verified using some full-scale and model tests of continuous beams and frames. The experimental and analytical results were almost identical from the view points of the hinge locations and the redistribution of moments.

Scholz (1993) conducted an analytical study on moment redistribution in continuous reinforced concrete beams. He proposed a new definition for moment-redistribution by normalizing the ultimate moment with respect to the maximum elastic moment of continuous beam under uniformly distributed load. Furthermore, he proposed a simple formula to account for moment-redistribution for design purposes. He also examined the effect of the beam slenderness in his analytical works. In another study Scholz (1993) proposed some expressions to account for ductility and redistribution simultaneously.

Tezuka et al (1995) conducted an unique experimental study on moment redistribution of continuous beams reinforced or prestressed with FRP. They tested two concrete beams reinforced with AFRP and CFRP, and three concrete beams prestressed with AFRP, CFRP and steel. The beams had a total length of 4.4 *m*

and a cross section of $120 \times 200 \text{ mm}$, while each of the 2.0 m spans was loaded with a concentrated load at a distance of 900 mm from the middle support. All beams were over-reinforced to make sure that compression failure took place prior to the tensile failure of FRP at the middle support. Tezuka et al reported up to 29.7% decrease in negative moment and up to 22.6% increase in positive moment due to redistribution. Furthermore, they reported up to about 44% increase in the failure load of the experimental beams compared to that calculated on the basis of elastic analysis.

Kodur and Campbell (1996) conducted an analytical study to develop a method for determination of the redistribution of moment in two-span continuous concrete beams. They proposed some expressions for design purposes, but since their study was restricted to prestressed concrete, their results could not be utilized in conventional reinforced concrete beams.

The preceding review shows that a limited number of studies have been conducted on the ductility and redistribution characteristics of FRP reinforced and/or prestressed concrete. It is also evident that achievement of these desirable characteristics in FRP reinforced concrete needs adoption of procedures different from those used in conventional reinforced concrete. The reason for the latter could be better explained by examining some fundamental concepts pertaining to the behaviour of reinforced concrete. In the following chapter, some of those concepts are examined in detail.

Chapter 3

Fundamental Concepts

3.1 General

Ductility and redistribution of stresses and moments are two important and desirable behaviours of reinforced concrete members. The most important advantages of ductility in a reinforced concrete member are as follows (Park and Paulay, 1975):

(1) It permits plastic distribution of internal stresses and large deformations prior to the collapse of the member. Therefore, the brittle failure of the member is prevented.

(2) It provides the necessary conditions for the redistribution of moments and stresses. In other words, the possible distribution of internal forces, especially bending moments, in the design of statically indeterminate structures is based on the ductility of the critical sections in the member.

(3) For structures subjected to dynamic loads, e.g. structures under seismic loads, ductility could provide the appropriate conditions for energy absorption and

dissipation by post-elastic deformations. Therefore, ductile structures could be designed for lower seismic forces with safety against collapse.

From the aforementioned advantages of ductility, it is evident that redistribution of stresses and moments is a consequence of ductility. In the other words, for any concrete structure with sufficient ductility, redistribution of stresses and moments will take place. Note that large deformation prior to the failure of a member is not necessarily accompanied by good ductility. Naaman (1993) states that the inelastic part of deformations are rather essential for ductile behavior while Razaqpur and Ali (1996) argue that the change of stiffness without loss of strength is the essential feature of ductile behaviour, provided the structure is not subjected to seismic loads. In other words, they disagree with the notion that inelastic deformations are *sine qua non* of ductility. Regardless of the definition of ductility, it is important to consider in design the purpose for which it is needed.

3.2 Ductility in Steel Reinforced Concrete

3.2.1 Moment-Curvature Relation

For a reinforced concrete section, curvature, ϕ , could be expressed as, (Park and Paulay, 1975):

$$\phi = \frac{\epsilon_c}{kd} = \frac{\epsilon_s}{d(1-k)} = \frac{(\epsilon_c + \epsilon_s)}{d} \quad (3.1)$$

where; kd = neutral axis depth, ϵ_c = concrete strain in the extreme compression fiber, ϵ_s = tension steel strain, and d = effective depth of section.

As the moment increases along a member, the curvature varies because of the variation of the strains and the neutral axis depth. For evaluating the moment-curvature relationship, often the so-called Hognestad's parabolic equation (Park and

Figure 3.1: *Moment-curvature relationship for steel reinforced concrete*

Paulay, 1975) for the compressive stress-strain relation of concrete has been utilized. For a better understanding of the moment-curvature relationship in different cases, a number of curves have been drawn for an identical concrete section, but with different amounts of reinforcement. The moment-curvature relationship for different amounts of tensile reinforcement is shown in Fig. 3.1, or in non-dimensional form in Fig. 3.2. In these figures, ρ represents the reinforcement ratio, ω the reinforcement index ($\omega = \rho f_y / f'_c$), and M the bending moment, with f_y being the yield stress of steel reinforcement and f'_c being the compressive strength of concrete. Note that the illustrated moment-curvature relations are based on idealized elastic-fully-plastic behaviour for the steel.

It can be observed in Fig. 3.1 that for lightly reinforced sections in which reinforcing steel yields prior to failure of the concrete in compression, the plastic curvature covers a vast region after first yield of steel, while the moment rises very slowly to a maximum due to an increase in the internal lever arm. On the other

Figure 3.2: *Non-dimensional moment-curvature relationship for steel reinforced concrete*

hand, for heavily reinforced sections and unconfined concrete, the crushing of concrete commences before the yielding of steel, and the plastic portion of curvature practically vanishes.

3.2.2 Ductility Factor

The ductility of a reinforced concrete member is commonly expressed as the ratio of the ultimate deformation to the yield deformation. The ratio is usually defined in terms of curvature, rotation, or deflection as follows:

$$\mu_{\phi} = \frac{\phi_u}{\phi_y} \quad \text{or} \quad \mu_{\theta} = \frac{\theta_u}{\theta_y} \quad \text{or} \quad \mu_{\Delta} = \frac{\Delta_u}{\Delta_y} \quad (3.2)$$

where, ϕ , θ and Δ stand for curvature, rotation and deflection, respectively, and the subscripts y and u stand for yield and ultimate, respectively. Among these definitions, μ_{ϕ} is the most popular and is called the “curvature ductility factor”,

Figure 3.3: *Effect of compression steel on moment-curvature relationship*

where ϕ_y and ϕ_u are the curvatures at yield and ultimate, respectively. Assuming a value of 0.004 for ultimate compression strain of concrete, ϵ_{cu} , it was determined that for tension steel ratios of 0.020, 0.015, 0.010 and 0.005, the curvature ductility factors are approximately 4, 6, 10 and 23, respectively, (Park and Paulay, 1975). Figures 3.3 and 3.4 show that if the other variables of section properties are held constant, an increase in the compression steel content, ρ' , or in the ϵ_{cu} , increases the ductility of the section. Also Figures 3.1 and 3.5 show that an increase in the tension steel content, or in the steel yield strength, decreases the ductility of the section. On the other hand, the curvature ductility factor is significantly reduced by the existence of axial load, (Fig. 3.6).

Furthermore, it is shown that ductility could be increased by the introduction of prestressing, (Cohn and Bartlett, 1982, and Naaman et al, 1986). Also Cohn and Riva (1991) in a comprehensive study investigated that for the T and I sections, increasing the size of the compressive flange, increases the ductility of the section.

Figure 3.4: Effect of ϵ_{cu} on moment-curvature relationship

Figure 3.5: Effect of steel yielding stress on moment-curvature relationship

Figure 3.6: *Effect of compression force on moment-curvature relationship*

Moreover, increasing f'_c values increases the ductility factor under a constant ρ value, and decreases it under a constant $\omega = \rho f_y / f'_c$ value.

In relating the preceding results to FRP reinforced concrete members, it can be argued that due to lack of yielding and the high strength of reinforcement, such members behave similar to conventional reinforced concrete reinforced with high strength steel.

3.2.3 Ductility Requirement in the Code for Flexural Sections

CSA Standard A23.3-94 (CSA A23.3, 1994) prescribes that for flexural members without axial loads, the area of tension reinforcement shall be such that the following

equation is satisfied for the section when the axial load is taken equal to zero.

$$\frac{c}{d} \leq \frac{700}{700 + f_y} \quad (3.3)$$

where c is distance from extreme compression fibre to neutral axis. For rectangular section using a rectangular stress block for concrete in compression, combination of the force equilibrium equation with this equation, leads to the following

$$\rho_{max} = \left(\frac{\phi_c}{\phi_s}\right) \alpha_1 \beta_1 \frac{f'_c}{f_y} \frac{700}{700 + f_y} \quad (3.4)$$

where ϕ_c and ϕ_s are resistance factors for concrete and reinforcement, respectively, β_1 is the ratio of the depth of the equivalent compression stress block of concrete to the depth of the neutral axis at ultimate, α_1 is the ratio of average stress in rectangular compression block to the specified concrete strength, and ρ_{max} is the maximum permissible ratio of tension reinforcement.

The balanced reinforcement ratio, ρ_b , could be calculated from balanced strain condition as

$$\rho_b = \alpha_1 \beta_1 \frac{f'_c}{f_y} \frac{700}{700 + f_y} \quad (3.5)$$

Comparing Eqs. 3.4 and 3.5, the value of ρ_{max} corresponds approximately to $0.71\rho_b$.

To provide an appropriate ductility for a member, the ACI 318-89 (ACI 318, 1989) limits the maximum reinforcement ratio, ρ_{max} , for a singly reinforced section to,

$$\rho_{max} = 0.75\rho_b \quad (3.6)$$

and for a doubly reinforced concrete section to,

$$\rho_{max} = 0.75\rho_b + \rho' \frac{f'_{sb}}{f_y} \quad (3.7)$$

where ρ' is the ratio of the compression reinforcement, i.e. A'_s/bd , A'_s is the area of compression steel, b (d) is section width (depth), and f'_{sb} is the stress in the compression steel at balanced condition.

The Commentary of ACI 318-89 (1989) mentions that since moment redistribution is dependent on adequate ductility in hinge regions, the amount of tension reinforcement in those regions should be limited to $0.5\rho_b$. Therefore, in flexural members of statically indeterminate structures, redistribution of moments is conditional upon;

$$\rho \leq \rho_{max} = 0.5\rho_b + \rho' \quad (3.8)$$

The latter expression will ensure $\phi_u/\phi_y > 3$ for sections without compression steel, and $\phi_u/\phi_y > 4$ for sections with compression steel, assuming $\rho'/\rho = 0.5$ and $\epsilon_{cu} = 0.003$.

It is important to mention that while restricting ρ to below ρ_{max} ensures tension failure in beams, in columns brittle compression failure cannot be prevented. It is axiomatic that columns are permitted by the codes to fail in brittle mode while beams are not.

3.3 Redistribution in Steel Reinforced Concrete Beams

3.3.1 Concept of Redistribution

Redistribution of stresses and moments in continuous beams and frames is a natural consequence of the nonlinear behaviour of moment-curvature relationship. In other words, the nonlinear relationship between the moment and the curvature of a section causes that section to change its stiffness with increasing level of moment. Since the moment along a beam generally varies, the relative stiffness will change accordingly and this will lead to redistribution of moments. Note that at lower levels of moment, the moment-curvature relationship is basically linear and therefore redistribution

becomes significant only at high levels of load, particularly after the reinforcement begins to yield. It also means that sections under high level of load will undergo plastic rotation. However, sections undergoing plastic rotations are not necessarily on the verge of failure. As a matter of fact, such sections will adjust themselves to the new conditions and will further rotate without any significant increase in their stresses while allowing the sections under lower moment to carry the additional load. Accordingly, the elastic diagram for distribution of moments along the beam would not be valid any more, and a new distribution of moments should be considered.

It is evident that moment redistribution causes the beam to carry higher load. The extent of the load, more than the elastic capacity, which can be carried depends on the extent of the plastic rotation which the beam is capable to undergo. As stated earlier, under certain conditions the codes allow moment redistribution to be considered in design, but in practice often the increase of load due to redistribution of moments is ignored and is regarded as an additional safety margin.

3.3.2 Moment Redistribution in the Code

ACI 318-89 (ACI 318, 1989) stipulates that negative moments at supports of continuous flexural members are permitted to be increased or decreased up to a maximum percentage of

$$20 \left(1 - \frac{\rho - \rho'}{\rho_b} \right) \quad (3.9)$$

The modified negative moments at supports should be used for modification of other negative or positive moments at any other section along the span. The Commentary of ACI 318-89 (1989), however, stipulates that the aforementioned percentage for moment redistribution is conditional upon adequate ductility in plastic hinge regions of the flexural member.

CSA Standard A23.3-94 (CSA A23.3, 1994) on the other hands, allows for any

increase or decrease of negative moments calculated by elastic analysis at the supports of continuous flexural members, with a percentage not more than 20% nor more than a maximum percentage of

$$30 - 50 \left(\frac{c}{d} \right) \quad (3.10)$$

where c is the depth of neutral axis at ultimate load and d is the effective depth of the section.

3.4 Ductility in FRP Reinforced Concrete

3.4.1 Curvature and Moment-Curvature Relation

The previously described basic concepts for curvature and moment-curvature relationship can be used for FRP reinforced concrete sections, provided in the expressions such as Eq. 3.1, ϵ_s is replaced by ϵ_f , where ϵ_f is the tensile strain in the FRP reinforcement. There is, however, a significant difference between FRP reinforced concrete and steel reinforced concrete, i.e. FRP under-reinforced concrete sections will fail before concrete can reach its maximum strain at the extreme compression fibre.

Analytical and empirical investigations performed by some researchers have shown that classical theories and expressions for analysis of steel reinforced concrete beams based on the strain compatibility method could be used with good accuracy for FRP reinforced concrete sections under bending. Thus, again in the present study the Hognestad's parabola for stress-strain behavior of concrete was used to evaluate the moment-curvature relationship of FRP reinforced sections.

The moment-curvature relationships for a typical rectangular concrete section reinforced with different types and ratios of FRP are illustrated in Fig. 3.7, or

Figure 3.7: *Moment-curvature relationship for FRP reinforced concrete*

in a non-dimensional form in Fig. 3.8. It is observed that the moment-curvature relationship is basically linear, with a limited non-linear portion. For comparison, the moment-curvature curves for both the steel and the FRP reinforced concrete are shown in Fig. 3.9, or in non-dimensional form in Fig. 3.10. Notice the nonlinearity in the moment-curvature relationship of carbon reinforced section at higher moment levels. This nonlinearity is due to the nonlinear behaviour of concrete and it might be enough to allow for some moment redistribution as reported by Tezuka et al (1995).

Figure 3.8: *Non-dimensional moment-curvature relationship for FRP reinforced concrete*

3.4.2 Balanced Reinforcement Ratio and Mode of Failure

Although FRP materials do not show a specific yield behaviour, the balanced reinforcement ratio can be defined for FRP reinforced concrete. The balanced condition represents the circumstance where concrete in compression reaches its ultimate strain simultaneously with the rupture of FRP in tension. If the amount of FRP used is less than the balanced reinforcement, flexural failure of the section would be initiated by the rupture of FRP in tension. In this case the FRP would exhibit linear behavior and the concrete would not reach its ultimate strain; therefore, the complete nonlinear behavior of concrete would not be utilized. On the other hand, if the amount of FRP exceeds the balanced reinforcement, the failure of the flexural section commences with the crushing of concrete in the compression zone. In

Figure 3.9: Comparison between moment-curvature relationship of steel and FRP reinforced concrete

this case, the concrete can realize its ultimate strain at the extreme compression fiber, therefore it will exhibit the maximum sustainable nonlinear behavior. Both modes of failure would be sudden and brittle (Dolan, 1991), nevertheless, relatively, compression failure would be less dramatic than tension failure.

Kakizawa et al (1993) stated that both compression concrete failure and tension failure of FRPs have some proponents. The former claim that compression failure mode is preferable for FRP reinforced concrete members because the failure of the member proceeds more gradually at the ultimate state compared to tension failure mode. The latter assert that brittle failure of FRP should be accepted in FRP reinforced concrete members because of economy.

Figure 3.10: Comparison between non-dimensional moment-curvature relationship of steel and FRP reinforced concrete

In the opinion of the writer, unlike a steel reinforced section, sections with more FRP reinforcement than the balanced reinforcement, (over-reinforced sections), may be desirable because of the following reasons. First, an FRP over-reinforced section will undergo more inelastic deformations than an under-reinforced section because of more inelastic deformations in the concrete under compression. Secondly, over-reinforced section ensures that considerable cracks and deformations occur prior to failure, giving more warning about impending failure. In other words, an over-reinforced section exhibits some kind of pseudo-ductile behaviour, at least from the safety and warning points of view. Thirdly, an over-reinforced design basically leads to a reduction in the dimensions of the section, which may be advantageous from the architectural and the economical points of view. The latter may be only feasible if deflection control requirements could be satisfactorily met.

3.4.3 Ductility of FRP Reinforced Concrete Members

The mechanical properties of FRP reinforced concrete beams is quite different from those reinforced with steel, i.e., there is no recognizable ductility in FRP reinforced concrete members, especially if ductility is defined based on plastic energy dissipation. The main reason is that FRP materials do not show any plastic deformation, therefore their failure is elastic and brittle. Concrete also generally does not undergo any significant plastic deformation, except when confined. Accordingly, the mechanical property of FRP reinforced concrete members is basically elastic, with little plastic deformation (Okamura et al, 1993). Indeed, one of the major drawbacks of FRP reinforced concrete is its poor sectional and member ductility (Naaman, 1993). However, despite the lack of intrinsic plasticity in FRP, FRP over-reinforced concrete sections may permit moment redistribution due to cracking and inelasticity in concrete.

As Naaman (1993) mentioned, many researchers have assumed that ductility requirement can be satisfied with FRP reinforced concrete sections since large deflection can be achieved. However, this notion of ductility could be quite dangerous, since large deflection may be due to the low modulus of elasticity of FRP. Clearly, large deformation does not necessarily represent good ductility, even though ductility is often accompanied by large deformation.

3.5 Confinement of Compression Concrete

If the compression zone of concrete is confined with some closely spaced stirrups or spirals, the stress-strain characteristic will distinctly change, and strength and ductility of concrete will considerably improve. Experimental study conducted by Ziara et al (1995) on 12 conventionally under-reinforced concrete beams confirmed

Figure 3.11: Stress-strain relationship of confined concrete, Kent and Park (1971)

that the presence of confinement increases the ductility of the concrete beams, although this study was not concerned with the effect of confinement on the ductility of essentially non-ductile concrete beams, i.e. over-reinforced concrete beams. Different models have been proposed for prediction of the stress-strain relationship of confined concrete, some of which could be used for a wide range of conditions while others may have more limited application.

Kent and Park (1971) suggested a model for concrete columns confined with steel hooks which did not include any strength enhancement in concrete, as shown in Fig. 3.11. The characteristics of this model are as follows:

For $\epsilon_c \leq 0.002$

$$f_c = f'_c \left[\frac{2\epsilon_c}{0.002} - \left(\frac{\epsilon_c}{0.002} \right)^2 \right] \quad (3.11)$$

for $0.002 < \epsilon_c \leq \epsilon_{20}$

$$f_c = f'_c [1 - Z(\epsilon_c - 0.002)] \quad (3.12)$$

for $\epsilon_c > \epsilon_{20}$

$$f_c = 0.2\epsilon_c \quad (3.13)$$

Figure 3.12: Stress-strain relationship of confined concrete, Sheikh and Uzumeri (1978)

where,

$$Z = \frac{0.5}{\frac{3+0.002f'_c}{f'_c-1000} + \frac{3}{4}\rho_s\sqrt{\frac{b''}{S_h}} - 0.002} \quad (3.14)$$

f'_c = cylinder strength in psi. ρ_s = ratio of transverse steel to volume of confined core, b'' = width of core and S_h = spacing of hoops.

Sheikh and Uzumeri (1978) suggested another model with some strength enhancement, as shown in Fig. 3.12. The characteristics of this model are as follows:

$$f'_{cc} = k_s f_{co} \quad (3.15)$$

$$k_s = 1.0 + \frac{B^2}{140000 P_{oc}} \left[\left(1 - \frac{nc^2}{5.5B^2} \right) \left(1 - \frac{S}{2B} \right) \right] \sqrt{\rho_s f'_s} \quad (3.16)$$

$$P_{oc} = 0.85 f'_c (A_{oc} - A_s) \quad (3.17)$$

$$\epsilon_{s1} = 80 k_s f'_c \times 10^{-6} \quad (3.18)$$

$$\frac{\epsilon_{s2}}{\epsilon_o} = 1 + \frac{248}{c} \left[1 - 5.0 \left(\frac{S}{B} \right)^2 \right] \frac{\rho_s f'_s}{\sqrt{f'_c}} \quad (3.19)$$

Figure 3.13: Stress-strain relationship of confined concrete, Saatcioglu and Razvi (1992)

$$\epsilon_{s85} = 0.225\rho_s\sqrt{\frac{B}{S}} + \epsilon_{s2} \quad (3.20)$$

where f'_{cc} = compression strength of confined concrete, f'_{co} = compression strength of unconfined concrete, that is roughly equal to $0.85f'_c$, k_s = strength enhancement coefficient, B = core dimension, P_{oc} = concentric capacity of unconfined concrete in the core, n = number of longitudinal bars in one side of the section minus one, c = center to center distance between the longitudinal bars, S = spacing of transverse reinforcement, ρ_s = volumetric ratio of transverse steel, f'_s = stress in transverse steel at peak stress of confined concrete (for practical cases, f'_s = yield stress of transverse reinforcement, f'_y), A_{oc} = section area of core concrete, A_s = section area of longitudinal reinforcement, and ϵ_o = strain corresponding to the maximum stress in plain concrete. It should be noted that in the preceding equations, units are MPa, N and mm.

Saatcioglu and Razvi (1992) proposed a model for stress-strain relationship of

confined concrete as illustrated in Fig. 3.13. The model consists of a parabola for the ascending branch and a linear portion for the descending branch. More details about this model could be found in the study conducted by Saatcioglu et al (1995). The characteristics of this model are as follows:

$$f_c = f'_{cc} \left[2 \left(\frac{\epsilon_c}{\epsilon_1} \right) - \left(\frac{\epsilon_c}{\epsilon_1} \right)^2 \right]^{1/(1+2K)} \leq f'_{cc} \quad (3.21)$$

$$f'_{cc} = f'_{co} + k_1 f_{le} \quad (3.22)$$

$$f_{le} = k_2 f_l \quad (3.23)$$

$$k_1 = 6.7(f_l)^{-0.17} \quad (3.24)$$

$$k_2 = 0.26 \sqrt{\left(\frac{b_c}{S} \right) \left(\frac{b_c}{S_l} \right) \left(\frac{1}{f_l} \right)} \leq 1.0 \quad (3.25)$$

$$f_l = \frac{\Sigma A_s f_{yt} \sin \alpha}{S b_c} \quad (3.26)$$

$$\epsilon_1 = \epsilon_{o1}(1 + 5K) \quad (3.27)$$

$$\epsilon_{85} = 260\rho\epsilon_1 + \epsilon_{o85} \quad (3.28)$$

$$\rho = \frac{\Sigma A_s}{S(b_{cx} + b_{cy})} \quad (3.29)$$

$$K = \frac{k_1 f_{le}}{f'_{co}} \quad (3.30)$$

where f_c = the stress of concrete for ascending parabola, f'_{cc} = compression strength of confined concrete, f'_{co} = compression strength of unconfined concrete, f_l = average lateral pressure, f_{le} = equivalent uniform lateral pressure, b_c = core dimension, S = spacing of transverse reinforcement, S_l = spacing of laterally supported longitudinal reinforcement, A_s = section area of each transverse reinforcement, α = the angle between the transverse reinforcement and the side of section, f_{yt} = the yield strength of transverse reinforcement, ϵ_{o1} = the strain corresponding to peak stress of unconfined concrete (in the absence of experimental data ϵ_{o1} could be taken equal to 0.002), ϵ_{o85} = the strain at 85% strength level beyond the peak stress of unconfined

concrete (in the absence of experimental data ϵ_{o85} could be taken equal to 0.0038), and finally b_{cx} and b_{cy} are the core dimensions.

It should be mentioned that some other models for stress-strain relationship of confined concrete can be found in the literature; one of the latest one is the model proposed by Attard and Setunge (1996) based on some experimental data, but most of them seem to be essentially the same as the preceding models with some limited changes.

Although all the aforementioned models were primarily proposed for confinement of concrete columns, the model proposed by Saatcioglu and Razvi seems to be equally applicable to concrete beams. The reason is that this model is the only one which accounts for rectangular stirrups with non-equal sides, which is very common in reinforced concrete beams. Furthermore, the effects of different arrangements of reinforcement are also taken into consideration in the model. Saatcioglu and Razvi (1992) showed that the model is capable of predicting the experimental data for square and rectangular sections confined with rectilinear hoops, stirrups, cross ties and combination of different types of lateral reinforcement. Therefore, their proposed model seems to be more suitable for evaluation of stress-strain relationship of confined compression zone in reinforced concrete beams.

It must be pointed out that the preceding models were derived empirically from the results of tests involving steel ties or hoops. Currently, very minimal data is available for concrete confined by FRP ties or hoops. The main reason is that bending of FRP bars causes a significant reduction in their tensile load carrying capacity, and as such FRP ties are not commercially produced. But an attempt will be made in the current investigation to use FRP grids for confinement and stirrups.

3.6 Improvement of Ductility

3.6.1 Hybrid Fibre Composites

As stated earlier, lack of ductility in FRP reinforced concrete members is primarily due to the linear elastic behaviour of FRP up to failure. Therefore, as a basic solution, attempts have been made to produce a ductile FRP reinforcement.

Bakis et al (1996) developed a procedure for fabricating and testing of hybrid fibre FRP rods, with pseudo-ductile behaviour. In his investigation, some 11 mm diameter rods were manufactured using various combinations of E-glass, polyvinyl alcohol (vinylon), aramid (kevlar 49), and carbon fibres in an unsaturated polyester resin matrix. The study was conducted in 5 groups, group A with 36% glass and 13% carbon, group B with 24% glass, 23% PVA and 9% carbon, group C with 60% glass and 5% carbon, group D with 36% glass and 13% carbon, and finally group E with 27% PVA, 17% aramid and 6% carbon. The carbon fibres were concentrated in the core for groups A and B, spread over the surface for groups C and E, and dispersed through the cross section for group D. Bakis et al recorded the results of stress-strain relationship of groups A and D as shown in Fig. 3.14. It could be observed from Fig. 3.14 that for group A the carbon fibres failed at approximately 735MPa or 1.4% strain, while prior to this event, the stress-strain behaviour was essentially linear. Furthermore, the glass fibres continued to carry load until they failed at about 630MPa, or 4.6% strain. Also for group D, a partial failure of the carbon fibres occurred at about 525MPa or 1% strain, while additional carbon fibres gradually failed after the primary failure. The final failure took place at approximately the same stress and at a strain of 2.4%.

As a conclusion for their study, it could be said that the hybrid composites could exhibit the desired pseudo-ductility, although for achieving better pseudo-

Figure 3.14: *Stress-strain relationship of hybrid fibre composites, Bakis et al (1996)*

ductile behaviour and warning of impending failure, more investigation regarding the optimum combination and arrangement of fibres is needed.

Tamuzs et al (1996) conducted a different study on the ductility of hybrid fibre composite reinforcement. They used braided FRP wound around a thin cylindrical shell core to develop an FRP material which is rather stiff in the first elastic part of its stress-strain relationship, and becomes ductile close to failure, with some strain hardening before failure and an elongation of at least 3% at maximum load. A cylindrical shell core made of glass fibre reinforced polyester was tested in their investigation. The idea was that the braided fibres could compress the central core material after break up of the resin. Hence, after the collapse of the core material at a certain pressure, the braided fibres will straighten and increase the elongation

Figure 3.15: Stress-strain relationship for aramid composite, Tamuzs et al (1996)

of FRP rod. Tamuzs et al conducted their study for specimens with and without core and with different angle of braided fibres, ϕ . They concluded that increasing the angle of braided fibres decreases the modulus of elasticity and tensile strength of the specimen significantly. At the same time, the modulus of elasticity decreases for specimens without core while the ultimate strain increases. Fig. 3.15 shows the stress-strain relationship of some of the specimens tested by Tamuzs et al. The figure reveals (notice the yield-like plateau) that a ductile behaviour could be achieved with braided FRP, although it is accompanied by some decrease in the modulus of elasticity and ultimate strength.

3.6.2 Hybrid Reinforcement System

Use of hybrid reinforcement system was suggested as an alternative method for achieving ductility in FRP reinforced concrete beams. Razaqpur and Ali (1996) suggested a hybrid FRP reinforcement system comprising a low modulus polymeric reinforcement, such as high density polypropylene grid (HDPP), and CFRP. They tested three 4.2m long high strength inverted T-beams under four point bending. Two of the beams were hybrid reinforced while the third one was reinforced just with CFRP and was considered as a control beam. The ratio of CFRP for all beams was less than the balanced reinforcement ratio. It was observed that for hybrid reinforced beams after the rupture of CFRP occurred, the beams kept carrying load, albeit the load was lower than before the rupture of CFRP, (Fig. 3.16). Although the HDPP reinforcement underwent large strain after the rupture of CFRP, it was capable of carrying a significant amount of load. Finally the hybrid reinforced beams failed in a ductile manner and they had at least three times more deflection at failure compared to the control beam. This concept is in theory sound but practically a higher stiffness propylene would be needed to reduce the rather large amount of HDPP currently required in order to achieve a reasonable load carrying capacity after the failure of the main CFRP reinforcement.

3.6.3 Combining Bonded and Unbonded FRP

Some researchers have proposed varying the bonding characteristics of multiple FRPs to achieve pseudo-plastic deformation characteristics and ductility (JSCE, 1993). The idea is that if all the reinforcing bars are bonded, they will break at the maximum strength. On the other hand, if both bonded and unbonded reinforcing bars are used, the bonded bars will fail at maximum load while the unbonded bars will fail at higher deflection and curvature, but at lower load than the maximum

Figure 3.16: Load-deflection curve of a typical hybrid reinforced beam. (Razaqpur and Ali, 1996)

load. This procedure provides an apparently ductile behaviour in concrete beams reinforced with such system.

3.6.4 Confinement of Compression Concrete

As mentioned in Section 3.5, confined concrete possesses ductile behaviour. It is evident that brittle behaviour of FRP reinforced concrete beams is due to lack of ductility in both FRP reinforcement and in concrete. Therefore, if concrete behaves in a ductile fashion, and if the section is allowed to fail due to concrete compression failure, a ductile behaviour for FRP reinforced flexural member would be expected. Hence the confinement of the compression zone is another possible method for achieving ductility in FRP reinforced concrete members.

Figure 3.17: Load-displacement curve of FRP reinforced beam with confined concrete, Taniguchi et al (1993)

To confirm this idea, Taniguchi et al (1993) conducted an experimental study on six concrete beams prestressed with CFRP, while the compression concrete was confined with CFRP ties with circular or rectangular shapes. They found good ductility for the prestressed test beams with confined concrete. One of the load-displacement curves of their test beams is shown in Fig. 3.17.

An alternative method for rendering concrete ductile was explored by Naaman et al (1992). They used discontinuous fibres in a cement slurry, SIFCON (Slurry-Infiltrated Fibre CONcrete). It is well established that the use of such fibres in concrete improves the stress-strain relationship and the ductility of concrete. Therefore, FRP reinforced concrete beams with SIFCON in compression zone would fail in a ductile manner, provided that failure is initiated by concrete in the compres-

sion zone. The tests conducted by Naaman et al revealed a drastic improvement in ductility through this method.

In the present investigation the concept of confinement of compression zone will be further investigated to determine whether it can be used to achieve and/or to increase moment redistribution in continuous FRP reinforced concrete beams.

The review of literature and the results of the first principles in the present study reveal that many of the issues related to ductility and moment redistribution in FRP reinforced concrete members remain unresolved and there is no consensus on whether ductility could be achieved indeed in such structures. This stems in part from our concept of ductility which is formed by the observed behaviour of conventional steel reinforced concrete structures. Clearly, due to the entirely different mechanical properties of FRP from those of steel, one cannot expect FRP reinforced concrete structures to replicate the behaviour of the corresponding steel reinforced concrete structures. However, by testing and analyzing the behaviour of FRP reinforced concrete members of different types, one could aim for the development of structural configurations and reinforcement regimes that would meet the same (or nearly the same) design and performance requirements as currently applied to steel reinforced concrete structures. It is with the latter philosophy in mind that the current investigation is pursued.

Chapter 4

Experimental Program

4.1 General

The main objective of the current experimental study is investigation of ductility and moment redistribution in CFRP reinforced concrete beams. Furthermore, it is intended to experimentally evaluate the effect of the amount of CFRP reinforcement and the effect of confinement of compression concrete on the ductility and redistribution of moments in continuous beams. It is also intended to examine the flexural and shear behaviour of continuous beams which are longitudinally and transversely reinforced with CFRP.

Two-span continuous beams will be tested to investigate ductility and moment redistribution simultaneously although ductility alone could be investigated through tests on simply-supported beams. In each case two nominally identical specimens will be tested to obtain more confidence in the results and to reduce the effect of defects and/or errors on the interpretation of the results. As a reference and for comparison, two beams will be longitudinally reinforced with steel while the remaining beams will be longitudinally reinforced with CFRP. Transversely, all beams will be

reinforced with steel except two which will contain CFRP grids as shear reinforcement. For all the beams, the moment-curvature relationship and their ductility will be investigated, the possible moment redistribution will be evaluated, and deflection and crack width at various load stages will be measured.

4.2 Test Specimens

Ten two-span beams each with a total length of 6.5 *m*, a clear span of 3.0 *m*, and a cross section of 250 × 350 *mm* will be tested. The beams are categorized as follows:

- (a) Two steel reinforced concrete beams (under-reinforced), used as reference (control) beams.
- (b) Two CFRP reinforced concrete beams (under-reinforced).
- (c) Two CFRP reinforced concrete beams (over-reinforced).
- (d) Two CFRP reinforced concrete beams with confined compression zone, using CFRP grids (NEFMAC) as confinement reinforcement.
- (e) Two CFRP reinforced concrete beams with NEFMAC grids as both shear and confinement reinforcement.

4.3 Theoretical Flexural Behaviour of Test Beams

The beams will be loaded symmetrically by applying a concentrated load at the middle of each span. The loading and the corresponding elastic bending moment and shear diagrams are shown in Fig. 4.1.

Figure 4.1: *Elastic bending moment and shear diagrams for a two-span beam under central concentrated loads*

4.3.1 Elastic Behaviour

The bending moments from elastic analysis are $M_e^- = 0.188PL$ and $M_e^+ = 0.156PL$, where M_e^- and M_e^+ stand for maximum elastic negative and positive moments, respectively. Hence in an elastic analysis, the ratio of the maximum negative moment to maximum positive moment is

$$\frac{M_e^-}{M_e^+} = 1.2 \quad (4.1)$$

From equilibrium

$$M_e^+ + \frac{1}{2}M_e^- = \frac{P_e L}{4} \quad (4.2)$$

where P_e is the maximum load based on elastic behaviour.

Substituting Eq. 4.1 into Eq. 4.2 and re-arranging the terms

$$P_e = \frac{1}{L}(4M_e^+ + 2.4M_e^+) = 6.4\frac{M_e^+}{L}$$

or

$$P_e = 5.33\frac{M_e^-}{L} \quad (4.3)$$

4.3.2 Inelastic (Plastic) Behaviour

Suppose that the negative and positive moment regions have equal flexural capacity, and suppose that the required conditions for plastic behaviour, i.e. for moment redistribution, exist. It follows that at ultimate state

$$M_p^- = M_p^+ \quad (4.4)$$

where M_p^- and M_p^+ stand for maximum plastic negative and positive bending moments, respectively.

Let, $M_p^- = M_e^- = 1.2M_e^+$, then

$$M_p^+ + \frac{1}{2}M_p^- = \frac{P_p L}{4} \quad (4.5)$$

where P_p is the maximum load at the plastic (failure) stage. Substituting $1.2M_e^+$ for M_p^+ in Eq. 4.5,

$$P_p = \frac{1}{L}(4.8M_e^+ + 2.4M_e^+) = 7.2 \frac{M_e^+}{L}$$

or

$$P_p = 6.0 \frac{M_e^-}{L} \quad (4.6)$$

Dividing Eq. 4.6 by Eq. 4.3, the ratio of ultimate plastic load to elastic load would be

$$\frac{P_p}{P_e} = 1.125 \quad (4.7)$$

It means that if full redistribution were to take place for the test beams, they would withstand 12.5% more load than that based on elastic analysis. Accordingly, the proposed test beams will be designed for the preceding level of plastic capacity.

4.4 Material Properties from Ancillary Tests

As stated earlier, the test beams flexural (longitudinal) reinforcement will be either conventional mild steel or CFRP rebars. Similarly, the shear reinforcement will be either steel U-stirrups or CFRP grids known by the trade name NEFMAC. The NEFMAC grids will also be used as confinement reinforcement in the compression zone of some of the beams. In the following, the material properties of the various types of reinforcement and the concrete that was used in the construction of the test beams will be given. Most of the properties were obtained from ancillary tests

conducted during the course of this investigation. In some cases the manufacturers recommended values were used. However, the use of the latter values will not affect the outcome and/or conclusions of the tests.

4.4.1 Longitudinal Steel Reinforcement

Longitudinal steel reinforcement was used only in the first two test beams. It consisted of *No.* 20 steel bars with a nominal yield strength of 400 MPa. However, after completion of the tests on the steel reinforced concrete beams, five test coupons were obtained from the end regions of the tested beams. Two strain gauges were attached to each coupon and they were tested in tension using the 400 kip Tinius Olsen universal testing machine.

The stress-strain relationship of each coupon is shown in Fig. 4.2. The recorded strain values of the two strain gauges in each coupon are shown in the figure which indicate close agreement between the two strain gauge readings. It should be mentioned that after yielding of the bars, the strain gauges were not able to monitor the strain, hence the maximum elongation of the bars was measured manually after each test.

The results of the tests are summarized in Table 4.1. In this table, f_y = yield stress, ϵ_y = yield strain, E_s = modulus of elasticity of steel, σ_u = ultimate stress, and ϵ_u = maximum elongation. From Table 4.1, the average yield stress and modulus of elasticity of longitudinal reinforcing steel are 462 MPa and 203.6 GPa, respectively.

4.4.2 Transverse Steel Reinforcement

No. 10 steel bars were used as transverse reinforcement in eight test beams. Five straight legs of different stirrups were cut and used as tension coupons. The results

Figure 4.2: Stress-strain relationship of steel bar tension coupons

Table 4.1: *Results of tension tests on longitudinal reinforcing steel coupons*

Sample	$f_y, (MPa)$	ϵ_y	$E_s, (MPa)$	$\sigma_u, (MPa)$	ϵ_u
1	457.6	0.00226	202.5	726.9	0.25
2	466.3	0.00232	201.0	680.6	0.20
3	463.4	0.00224	206.7	735.6	0.17
4	457.6	0.00227	201.6	729.8	0.16
5	466.3	0.00226	206.3	677.7	0.19
Average	462.2	0.00227	203.6	710.1	0.19

of the tests are tabulated in Table 4.2.

Table 4.2: *Results of tests on transverse reinforcing steel coupons*

Sample	$f_y, (MPa)$	$\sigma_u, (MPa)$	ϵ_u
1	458.5	756.8	0.21
2	507.5	747.9	0.20
3	489.7	770.0	0.18
4	445.2	734.6	0.20
5	445.2	739.0	0.20
Average	469.2	749.7	0.20

From this table, the average yield stress of transverse reinforcing steel is 469 MPa. No strain gauges were used in these tests, hence the yield strain and the modulus of elasticity could not be recorded. Therefore, a modulus of $E_s = 200.0$ GPa will be assumed for design purposes.

4.4.3 FRP Rods

CFRP Leadline rebars were used as longitudinal reinforcement in eight test beams. The rods had indented surfaces with diameter of 8 or 10 *mm*. Attempt was made to measure the tensile strength of the bars in the laboratory using two 250 *mm* long steel grips. Hence, the 8 *mm* CFRP rods were centered in the grips and the grips were filled up with grout with a minimum compressive strength of 60 MPa. Strain gauges were installed on the middle section of the bars. Unfortunately, the bars slipped at a stress of 2100 MPa. However, the modulus of the bars was found to be $E_{cf} = 146.5$ GPa, which is in very close agreement with the value of 147.0 GPa recommended by the manufacturer.

Since the tension coupon tests did not result in the ultimate strength of the CFRP rebars and due to lack of proper grips for further tests, the properties of CFRP rebars were assumed according to the producer's recommendations (Mitsubishi Chemical Corporation, 1996). The manufacturer's guaranteed minimum tensile strength and elastic modulus for CFRP rods are 2550 MPa and 147 GPa, respectively. Note that the actual strength may be much higher.

4.4.4 NEFMAC Grids

CFRP NEFMAC grids were used as confining and shear reinforcement in some of the test beams. The grids consisted of C6 bars with 50 *mm* spacing in both directions. Due to lack of proper grips or chucks, the grids were not tested for their properties, instead, their properties were extracted from the Standard Specifications of NEFMAC recommended by the manufacturer. According to those specifications, the tensile strength and elastic modulus of C6 NEFMAC grid are 1200 MPa and 100 GPa, respectively.

4.4.5 Concrete

Concrete was cast in two phases. Ready mix concrete was ordered from the plant and was delivered to the laboratory. Separate tests were carried out to obtain the concrete properties in each phase.

Phase I

Phase I involved the construction of the first six test beams, i.e two steel reinforced beams, two FRP under-reinforced beams and two FRP over-reinforced beams (without confinement). Concrete with $f'_c = 50$ MPa, slump of 100 mm and maximum aggregate size of 15 mm was ordered from the ready made plant. The delivered concrete was made with superplasticizers and silica fume. During the casting of the beams, a total of 15 standard concrete cylinders ($150 \times 150 \times 300$ mm) were cast and cured under the same condition as the test beams. To obtain smooth and leveled surfaces, the cylinders were capped with the sulphur compounds prior to testing.

Two cylinders were tested under compression after 7 days and another two were tested after 14 days. The 7-day compressive strengths of the cylinders were 36.6 and 40.5 MPa, and the 14-day compressive strengths were 47.7 and 50.5 MPa.

Five cylinders were tested for 28-day compressive strength. A compressometer (containing an LVDT) was attached to the surface of each cylinder prior to test. The load and the corresponding changes in the length of the compressometer were recorded using a simple data acquisition system. The stress-strain curves of the tested cylinders are shown in Fig. 4.3, except for cylinder IV for which the correct strains could not be recorded. It should be mentioned that because of relatively high strength of concrete, the descending branch of stress-strain curves could not be captured with the existing testing machine in the laboratory. The compressive strengths of the cylinders (f'_c) and their corresponding strains (ϵ_0) are given in Table

a) Cylinder I

b) Cylinder II

c) Cylinder III

d) Cylinder V

Figure 4.3: *Stress-strain relationship of concrete (Phase I) at 28 days*

4.2. The table also contains the values of E_c , the modulus of elasticity of concrete. E_c is based on the secant modulus which is the slope of a line connecting two points on the curve. According to ASTM C469 (ASTM, 1991), the lower point corresponds to 50 micro-strain while the upper point corresponds to a stress equal to 40% of maximum stress. Disregarding the results of Cylinder IV, the mean values of the

Table 4.3: *Results of tests on the concrete cylinders at 28 days (Phase I)*

Cylinder	$f'_c, (MPa)$	ϵ_0	$E_c, (MPa)$
1	56.2	0.00218	37910
2	51.2	0.00234	40960
3	53.1	0.00206	35820
4	34.7	—	—
5	57.6	0.00203	39520

28-day compressive strength of concrete and its corresponding strain were found to be $f'_c = 54.5$ MPa and $\epsilon_0 = 0.00215$, respectively. Furthermore, the mean value of modulus of elasticity of concrete was found to be $E_c = 38550$ MPa. Notice that if we use the following expression which is recommended by ACI:

$$E_c = 5000\sqrt{f'_c} \quad (4.8)$$

we will get $E_c = 36910$ MPa, which is close to the experimental value. On the other hand, if we use the recommendations of CSA A23.3-94,

$$E_c = \left(3300\sqrt{f'_c} + 6900\right) \left(\frac{\gamma_c}{2300}\right)^{1.5} \quad (4.9)$$

where γ_c is the density of concrete in kg/m^3 , then assuming $\gamma_c = 2500$ kg/m^3 , we will get $E_c = 35430$ MPa, which is 8% less than the measured value.

Three cylinders were tested concurrently with the beam tests at the age of 5 months. Longitudinal strains of the cylinders were measured using a compressometer

and two strain gauges. The stress-strain curves of the cylinders (up to the maximum stress) are shown in Fig. 4.4. To be able to calculate the Poisson's ratio (ν) of the concrete during the test of the beams, two strain gauges were laterally attached on each of the first two cylinders. The variation of lateral versus longitudinal strains are shown in Fig. 4.5. The results of the latter tests are shown in Table 4.3.

Table 4.4: Results of tests on 5 months old concrete cylinders

Cylinder	f'_c , (MPa)	ϵ_0	E_c , (MPa)	ν
1	73.4	0.00265	45030	0.18
2	81.8	0.00270	46020	0.22
3	76.5	0.00241	43160	—

From Table 4.3, the mean values of $f'_c = 77.2$ MPa, $\epsilon_0 = 0.0026$, $E_c = 44740$ MPa, and $\nu = 0.2$ are obtained. Note that again if we use the ACI code expression of Eq. 4.8, we will get the value of $E_c = 43930$ MPa which is in close agreement with the experimental value. On the other hand, the corresponding value from CSA recommendation (Eq. 4.9) is $E_c = 40680$ MPa, which is substantially less than the experimental value.

Using the obtained experimental values of E_c and ν and substituting in the well-known expression $G = E/2(1 + \nu)$, we may calculate the value of $G_c = 18640$ MPa for shear modulus of concrete.

In addition to the preceding tests, two cylinders at the age of 28 days and one cylinder at the age of 150 days were tested for tension using the split cylinder test according to ASTM C496 (ASTM, 1991). From these tests, the tensile strengths of concrete cylinders were found to be 3.36 and 3.45 MPa at 28 days and 4.23 MPa at 150 days. However, the results may not be reliable enough because they are less than 10% of the compressive strength and the number of cylinders tested was too small.

Figure 4.4: *Stress-strain relationship of concrete (Phase I) during the test of the beams*

a) Cylinder I

b) Cylinder II

Figure 4.5: *Lateral versus longitudinal strain of concrete cylinders during the test of the beams*

Phase II :

Phase II involved the construction of the remaining four test beams, i.e. FRP over-reinforced beams with confined compression concrete. Once again, concrete with the following specifications were ordered from the ready mix plant: $f'_c = 50$ MPa, maximum aggregate size = 12.5 mm and slump = 100 mm. A total of 12 cylinders were cast and cured under the same conditions as the test beams, two of those cylinders were tested under compression after 7 days. The 7-day compression strengths of the cylinders were 39.9 and 35.7 MPa.

Four cylinders were tested in compression at the age of 28 days, and their stress-strain curves are shown in Fig. 4.6. The important mechanical properties of these cylinders are given in Table 4.4. From this table, the mean values of $f'_c = 65.6$ MPa, $\epsilon_0 = 0.00236$, and $E_c = 34100$ MPa are obtained. During the tests of the last

Table 4.5: *Results of tests on the concrete cylinders at 28 days (Phase II)*

Cylinder	$f'_c, (MPa)$	ϵ_0	$E_c, (MPa)$
1	67.4	0.00240	35330
2	68.6	0.00249	35180
3	64.8	0.00225	36510
4	61.5	0.00228	29300

four beams, another four concrete cylinders (at the age of 2 months) were tested under compression. The stress-strain curves of the latter cylinders are shown in Fig. 4.7 while Table 4.5 presents a summary of the test data. It can be seen that the compressive strength of Cylinders 2 and 4 are much less than the 28-day compressive strength of the concrete, which is highly unusual, therefore, the results of these cylinders were discarded. However, the E_c values of all the cylinders were used to calculate the mean value of the elastic modulus. Accordingly, a mean value of $f'_c = 70.2$ MPa, $\epsilon_0 = 0.00224$ and $E_c = 42000$ MPa will be used for the concrete

a) Cylinder I

b) Cylinder II

c) Cylinder III

d) Cylinder IV

Figure 4.6: Stress-strain relationship of concrete (Phase II) at 28 days

a) Cylinder I

b) Cylinder II

c) Cylinder III

d) Cylinder IV

Figure 4.7: *Stress-strain relationship of concrete (Phase II) during the test of the beams*

Table 4.6: *Results of the tests on the concrete cylinders during the test of the beams (Phase II)*

Cylinder	$f'_c, (MPa)$	ϵ_0	$E_c, (MPa)$
1	67.9	0.00230	38250
2	50.1	0.00147	43560
3	72.6	0.00218	44000
4	50.1	0.00159	42200

at the time of testing of the beams.

To determine the tensile strength of the concrete, two cylinders at the age of 28 days were tested using the split cylinder test. The tensile strengths of concrete cylinders were found to be 4.1 and 4.3 MPa. However, the results may not be considered reliable because they are less than 10% of the corresponding compressive strength and the number of cylinders were too few to sufficiently account for the usual variability encountered in the tensile strength testing of concrete.

4.5 Design and Details of Test Beams

It was mentioned earlier that a total of 10 beams in 5 groups were designed and tested. In each case one duplicate specimen was tested to ensure consistency in the results and acquisition of data in the event of unexpected failure of one of the beams. We shall use mnemonic notation as follows to designate the 10 beams: Beams SR1, SR2, CRU1, CRU2, CRO1, CRO2, CRCS1, CRCS2, CRCN1 and CRCN2. In this notation SR denotes Steel Reinforced beams while CR stands for CFRP Reinforced beams, O(U) means Over(Under)-reinforced and CRCS(CRCN) is used for CFRP over-Reinforced beams with Confined concrete and with Steel(NEFMAC) as shear

reinforcement. The 1 and 2 beams in each group are nominally identical. Full details of the beams are given in subsequent sections.

4.5.1 Beams Dimensions and Geometry

It was initially intended to design and test full scale beams in order to be able to obtain reliable results for immediate application in practice. However, considering practical limitations of the laboratory, i.e. available space, forms, jack capacity and casting of concrete, the decision was made to select dimensions in the lowest range of the practical projects. Based on the preceding limitations, 3 *m* long span was chosen with an extra 0.25 *m* end length for the purpose of fitting the support plates and for embedment of longitudinal reinforcement. Accordingly, 6.5 *m* long two-span beams were selected. For the cross-section of the beams, attempt was made to select dimensions not too small to be far from practical size, and not too large to create difficulties in formwork and handling of concrete in the laboratory. Accordingly, a cross-section of 250 × 350 *mm* was chosen.

4.5.2 Design and Detailing

The test beams were designed using conventional methods and the general provisions described in concrete text books and codes. Although preliminary design was based on some assumptions for material properties, the actual material properties as described in section 4.4 were as follows:

a) Concrete

$f'_c = 77 \text{ MPa}$: for the first 6 beams and $f'_c = 70 \text{ MPa}$: for the last 4 beams,
 $\epsilon_{cu} = 0.0035$.

b) Longitudinal steel reinforcement

$$f_y = 462 \text{ MPa} \quad \text{and} \quad E_s = 203.6 \text{ GPa.}$$

c) Transverse steel reinforcement

$$f_y = 469 \text{ MPa} \quad \text{and} \quad E_s = 203.6 \text{ GPa.}$$

d) CFRP rebars

$$f_{fu} = 2550 \text{ MPa} \quad \text{and} \quad E_{cf} = 147 \text{ GPa.}$$

e) NEFMAC grids

$$f_{fu} = 1200 \text{ MPa} \quad \text{and} \quad E_{cf} = 100 \text{ GPa.}$$

In design of the beams, identical bending moment capacities were provided for both positive and negative moment regions. Furthermore, the beams were designed for shear using existing code provisions and adequate shear reinforcement was provided to avoid premature shear failure. After the presentation of the test results later, the adequacy of those provisions will be examined and discussed.

Figs. 4.8 to 4.12 show the reinforcement details of the test beams reinforced with steel, CFRP (under-reinforced), CFRP (over-reinforced), CFRP + confining NEFMAC grids, and CFRP + confining and shear NEFMAC grids, respectively. The full design of the test beams is provided in Appendix A. The basic characteristics of the design are given in Table 4.7 for each group of the test beams, assuming $d = 300\text{mm}$ for all cases. In this table, W_s and W_f stand for the total weight of the longitudinal reinforcement for steel and CFRP rebars, respectively.

4.5.3 Description of the Beams

Beams SR1 and SR2 are conventional reinforced concrete beams (Fig. 4.8) with 1500 mm^2 longitudinal steel reinforcement in each of the positive and negative moment

Table 4.7: Design characteristics of test beams

Beam Type	$A_s(A_f)$ mm^2	ρ	ρ_b	ρ/ρ_b	M_{max} $kN.m$	A_f/A_s (%)	W_f/W_s (%)
steel reinf.	1500	0.02	0.0569	0.35	193.2	100	100
CFRP under-reinf.	92.2	0.0123	0.0028	0.44	68.8	6.1	1.7
CFRP over-reinf.	502.6	0.0067	0.0028	2.39	217.4	33.5	6.8

regions, and transverse steel stirrups ($A_v = 200mm^2$) at 125 mm spacing. These are control beams and are used as reference for the other beams.

As shown in Fig. 4.9, beams CRU1 and CRU2 are under-reinforced with CFRP bars. They are longitudinally reinforced with $2\phi 8$ Leadline rods ($A_f = 92.2 mm^2$) at the top and bottom of the section, and transversely with steel stirrups ($A_v = 200mm^2$) spaced at 150 mm throughout the beam.

Beams CRO1 and CRO2 are CFRP over-reinforced concrete beams which are longitudinally reinforced with $7\phi 10$ CFRP rods ($A_f = 502.6 mm^2$) at the critical positive and negative moment zones, and transversely with steel stirrups ($A_v = 200mm^2$) spaced at 100 mm as shown in Fig. 4.10. The preceding six beams had the same concrete strength and were cast at the same time.

Beams CRCS1 and CRCS2 are CFRP over-reinforced concrete beams with confined compression zone and steel stirrups (Fig. 4.11). The longitudinal reinforcement at the critical positive and negative moment regions was $3\phi 8$ & $5\phi 10$ ($A_f = 497.6 mm^2$) and the transverse reinforcement was identical to that of beams CRO1 and CRO2. Concrete in the positive and negative moment regions of these beams was confined using NEFMAC grids, with a height of 200 mm and spaced at 50 mm.

Beams CRCN1 and CRCN2 are CFRP over-reinforced concrete beams with con-

Figure 4.8: Details of reinforcement for steel reinforced concrete test beams (Beams SR1 and SR2)

finer concrete throughout the beam length, and with NEFMAC grids as transverse reinforcement (Fig. 4.12). Grids of $200 \times 300 \text{ mm}$ were cut from NEFMAC and were put at the intervals of 50 mm to provide shear reinforcement and to make sure that effective and sufficient confinement has been provided.

4.6 Construction of the Test Beams

The test beams were constructed in two phases, using similar procedures.

Phase 1 : Construction of Beams SR1, SR2, CRU1, CRU2, CRO1, and CRO2

The construction commenced with the making of the plywood forms. Strain gauges were attached to the longitudinal steel or CFRP reinforcement at three loca-

Figure 4.9: Details of reinforcement for CFRP under-reinforced concrete beams (Beams CRU1 and CRU2)

Figure 4.10: Details of reinforcement for CFRP over-reinforced concrete beams (Beams CRO1 and CRO2)

Figure 4.11: Details of reinforcement for CFRP over-reinforced concrete beams with confining NEFMAC grids (Beams CRCS1 and CRCS2)

Figure 4.12: Details of reinforcement for CFRP over-reinforced concrete beams with confining and shear NEFMAC grids (Beams CRCN1 and CRCN2)

Figure 4.13: *The reinforcement cage of Beam SR1*

tions along the beam, i.e right under the load points and over the middle support. The strain gauges were bonded with high accuracy and extreme care. Furthermore, a thin layer of silicone coating was applied throughout the gauge length and on the soldered connections to protect them against impact, moisture and damage during casting.

The reinforcement cages were constructed with extreme care and then placed in the forms. The inside of the forms were brushed with oil to facilitate their removal after the casting and curing of the concrete. Plastic chairs were used to maintain the cover of 38 mm over the stirrups at all sides. Fig. 4.13 and 4.14 show the constructed reinforcement cages of Beams SR1 and CR01, respectively.

The concrete was supplied by a local concrete plant in accordance with the design specifications. The maximum aggregate size was $\frac{5}{8}$ in (15 mm) and the ordered slump was 100 mm. Superplasticizer and silica fume were used in the concrete to obtain

Figure 4.14: *The reinforcement cage of Beam CRO1*

good workability without increasing the water/cement ratio. For all the beams, the concrete was poured in three lifts, and was properly vibrated using an electric vibrator and tamping with a steel rod. After finishing the concrete to a smooth level surface, it was covered with plastic sheets. During casting, the slump and the air content of the mix were measured and recorded as 100 mm and 0.2%, respectively. The concrete was cured at ambient temperature ($15 - 30^{\circ}\text{C}$) and the surface was kept wet for at least 10 days. The forms were removed after two weeks and the curing was continued under ambient conditions. Although the ordered concrete strength was 50 MPa, the actual strength from standard cylinders was found to be 54.5 MPa at 28 days and 77.2 MPa after 5 months (during the tests). Fig. 4.15 shows the casting of the concrete for this set of the beams.

Phase 2 : Construction of Beams CRCS1, CRCS2, CRCN1, and CRCN2

The formwork from Phase I was reused to construct the remaining four beams.

Figure 4.15: *Casting of concrete for the first set of beams*

Since high quality plywood was utilized in making the forms, the wood did not warp or noticeably deform due to previous usage. Prior to the assembly of the cages, strain gauges were attached to the longitudinal CFRP reinforcement, and the steel and /or NEFMAC transverse reinforcement, and were coated with a thin layer of silicone. Extreme care was exercised during the construction process of the reinforcement cages to minimize the concrete cover on the reinforcing NEFMAC grids in the compression zone and to maximize the effectiveness of the confinement of the concrete. Once again, 38 mm chairs were used to position the reinforcement cages accurately within the formwork. Figs. 4.16 and 4.17 show the constructed cages for Beam CRCS1 and CRCN2, respectively. The remaining steps in the assemblage of cages and casting of concrete were the same as in Phase I.

The maximum size of aggregates in the latter mix was $\frac{1}{2}$ in (12.7 mm). A 55 MPa concrete was ordered to a local concrete plant, however, the actual strength from standard cylinders was found to be 65.6 MPa at 28 days and 70.2 MPa after 2

Figure 4.16: *The reinforcement cage of Beam CRCS1*

Figure 4.17: *The reinforcement cage of Beam CRCN2*

Figure 4.18: *The constructed beams prior to testing*

months, i.e. at the time of testing of beams in Phase II.

For all the test beams, strain gauges and Demec points were glued on the surface of the concrete. Furthermore, the surface of the concrete was white-washed and a grid was drawn on it in order to trace the crack pattern during the test. Fig. 4.18 shows some of the beams after construction but prior to testing.

4.7 Test Equipment and Instrumentation

The following equipments and instrumentation were used in the present testing program:

1. Two MTS closed-loop servo-controlled electro-hydraulic jacks as actuators, with the capacities of 110 kip (489 kN) and 150 kip (667 kN), and stroke lengths of

Figure 4.19: *Equipments used for test beams*

10 in (250 mm) and 6 in (150 mm), respectively.

2. Two MTS high-capacity force transducers as load cells with capacities of 250 kN and 500 kN.
3. Knife-edge bearing plates with roller supports.
4. MTS automatic data acquisition system.
5. Twenty channels of linear variable differential transducer (LVDT).
6. Portable hand microscope with 10 micron accuracy.
7. Demec gauges.

Fig. 4.19 schematically shows the typical disposition of the various equipment and instrumentation used for each beam.

Measurement of deflection throughout the beams was made using 20 LVDTs.

The LVDTs were installed under the beam at 0.5 *m* interval on both sides of the cross-section. The base of each LVDT was fixed to the ground while the top portion was in contact with the bottom of the beam. Furthermore, to check the results of LVDTs, four digital deflectometers were attached to the bottom of each beam right under the load points and at a distance of one meter from the middle support.

Since the test beams were statically indeterminate, reactions at the end supports were measured using high-capacity load cells. As stated earlier, the capacities of the load cells were 250 and 500 kN, with their respective stiffnesses being 3.9 and 5.8 *GN/mm*. The maximum displacement of the load cells at full capacity was 0.08 *mm*, and at the maximum expected load was less than 0.04 *mm*. In other words, the load cells were sufficiently stiff to prevent appreciable reaction (or extra moment) produced by unequal settlement of the supports. To make sure that the settlement of the supports during the loading of the beams was indeed negligible, two dial gauges (with the accuracy of 10 micron) were attached to each support and the settlement was monitored during the loading process. At the end of each test, it was affirmed that there was no significant unequal settlement during the loading process.

Measurement of strains in the longitudinal reinforcing steel, CFRP rebars, and NEFMAC grids was made using electrical resistance strain gauges with gauge length of 5 *mm*. The strain gauges were sensitive to 0.01 micro strain with a standard deviation of 2%. For all ten test beams, strain gauges were attached to the longitudinal reinforcement at the maximum positive and negative moment regions, i.e. two strain gauges on the tensile reinforcement and one strain gauge on the compressive reinforcement at mid-spans and at the middle support. Fig. 4.20 shows the typical positions of the nine strain gauges on the longitudinal reinforcement of the test beams.

Strain gauges on transverse reinforcement were used for Beams CRCS1, CRCS2, CRCN1 and CRCN2 only. For beams CRCS1 and CRCS2, six strain gauges were

Figure 4.20: *Typical locations of the nine strain gauges on longitudinal reinforcement of the test beams*

attached to the steel stirrups located at a distance of 150 *mm* to the right of the middle support, and on stirrups located midway between the load points and the middle support (see detail A in Fig. 4.21). Furthermore, eight strain gauges were attached to the horizontal and vertical ribs of the NEFMAC grids located 100 *mm* and 200 *mm* from the middle support (see detail B in Fig. 4.21).

In beams CRCN1 and CRCN2 (see detail A in Fig. 4.22), seven strain gauges were attached to the NEFMAC grids situated 100 *mm* from the middle support. Furthermore, four strain gauges were placed on the horizontal and vertical legs of NEFMAC grids in the confined compression zone at the section located 200 *mm* from the middle support (detail B in Fig. 4.22). In addition, a total of six strain gauges were affixed to the vertical legs of NEFMAC grids located midway between the load points and the middle support (see detail C in Fig. 4.22). The purpose of the latter was to measure the strain in the shear reinforcement and to determine the contribution of the shear reinforcement to the shear resistance of the member.

For each test beam, three strain gauges were attached to the surface of concrete at the maximum positive and negative moment sections. The locations of the gauges were on the side of the beam and at 50 *mm* from the extreme compression fibre for

Figure 4.21: *Locations of strain gauges on the transverse reinforcement of Beams CRCS1 & CRCS2*

Beams SR1, SR2, CRO1 and CRO2, and at 20 mm from the extreme compression fibre in the remaining beams.

Since the key to moment redistribution in FRP over-reinforced concrete members would be the in-elasticity of concrete, it was decided to install Demec gauges on the concrete in the compression zone of the beams. These gauges were intended to provide data in the event of the failure of electrical resistance gauges during the loading process. The data could then be used to obtain the depth of the neutral axis and the maximum compressive strain reached in the concrete. Towards that end, Demec disks were glued to the surface of concrete at thirty different locations as shown in Fig. 4.23. The Demec gauge length was 200 mm. It should be pointed out that during the testing because of some difficulties that arose, the strain measurements at all these locations could not be taken.

Figure 4.22: Location of strain gauges on transverse reinforcement for Beams CRCN1 & CRCN2

Figure 4.23: Typical locations of Demec points

Figure 4.24: *Typical test setup*

4.8 Test Setup

The tests were conducted in the structural engineering facility of the Civil Engineering Laboratories of Carleton University. The steel frame supporting the actuators were fixed to the 1.0 m thick laboratory strong floor. The load cells were placed under the knife-edge plates and on the roller supports while the roller supports were setting directly on the strong floor. The middle support was designed as a pin support. Four rods bolted to the upper plate of the middle support were used to prevent the test beams from lateral movement. The LVDTs and the dial gauges were fixed to the floor. Fig. 4.23 shows a close view of the setup. In this figure, part of the frames, the actuators, the middle support, the LVDTs, and one of the end supports can be seen.

4.9 Loading and Data Acquisition Procedures

In order to be able to capture the descending branch of the load-displacement curve, the load was applied through displacement control. The rate of loading (i.e. the rate of applied displacement) and the operation of the jacks could be easily controlled by the PC computer attached to the data acquisition system. The selected rate of loading was 0.01 mm/sec for all the tests. In addition, the start of recording of data, the intervals between the recording times, and any pause or end of recording could be easily controlled by the computer. The interval between the recording times was chosen 10 seconds for all the tests. All data including, the values of the loads, the strokes of actuators, the reactions, the displacements and the strains could be read on the screen throughout the duration of the test.

Since some quantities were manually measured during the test, including some displacements along the beam and over the supports, concrete strain by the Demec gauges, and the crack widths by a hand microscope, the loading was stopped at various stages. The load increments were mostly between 10 kN to 25 kN. Further explanation about the loading of each beam will be given in the next chapter.

Chapter 5

Experimental Results

5.1 General

It may be recalled that the beams were tested under concentrated mid-span loads using quasi-static monotonic loading. During the tests, the complete behaviour of the beams was carefully monitored. The reactions of the end supports, the strains of the reinforcement and the concrete, and the displacement at different locations were recorded by an automatic data acquisition system. The strains in the compression zone of the concrete were also measured using Demec gauges. The crack patterns were traced and the crack widths were measured using a hand microscope with least count of 10 micron.

In this chapter, the complete behaviour of the beams during the test will be described. Furthermore, the load-displacement curves, the crack patterns, the crack widths, the strain variation on the concrete surface and in the longitudinal and transverse reinforcement will be shown. As mentioned earlier, for each beam a duplicate specimen was tested. Accordingly, the results of the two beams in each case will be discussed concurrently.

5.2 Beams Behaviour and Observations

5.2.1 Beams SR1 and SR2

Specimens in this set were designed as control beams. It may be recalled that these beams were conventionally under-reinforced with steel, with ratio of $\rho/\rho_b = 0.4$, where ρ and ρ_b are the actual and the balanced reinforcement ratios. Beam SR1 was the first of the ten test beams to be tested, hence some preliminary checks were carried out to ensure accurate operation of the instrumentation and the setup. The checks were conducted by applying a maximum load of 30 kN and 100 kN and subsequently some minor adjustments were made to the setup. Note that the predicted failure load for this beam was about 400 kN.

The full test of Beam SR1 was started by applying load increments of 20 kN on each span, up to a total load of 200 kN, thereafter, the load increments were changed to 10 kN. At each load stage, the dial gauges were read and recorded, the cracks were traced and the critical crack widths were measured. Also at some load stages, the measurement of the strains on concrete were taken using the Demec gauges. It was observed that the first flexural crack developed in the negative moment region at a load of 20 kN. Later, the first flexural crack in the positive moment region took place at about 30 kN. After a load of approximately 60 kN, the vertical cracks gradually started to become inclined toward the middle support and the loading points. The inclined cracks, typically known as shear cracks, formed at an angle of approximately 45° to the horizontal. At $P = 300$ kN, there were 12 cracks in the positive moment region of each span, and 10 in the negative moment region. The distance between the cracks varied from 150 to 200 mm. At 340 kN, a major shear crack started to develop between the middle support and the load point in right span. The crack rapidly approached to the loading point at an angle of approximately 30° , and caused shear failure at 357 kN. Although the design shear resistance of the beam

was 450 kN, it failed prematurely and unfortunately did not reach its design flexural capacity. This issue will be discussed later in Chapter 7. Consequently, the desired flexural failure did not occur for this beam. After the test, it was found out that the technician had inadvertently set the computer clock wrongly which caused the loss of data for loads greater than 250 kN. Therefore, for loads greater than 250 kN, the only available data was the manual recording of the dial gauges. Fig. 5.1 shows Beam SR1 at failure.

To prevent premature shear failure in Beam SR2, special bandages were designed and fabricated as external shear reinforcement. Each bandage basically consisted of two 25 *mm* diameter vertical steel rods and two horizontal plates. The bars were bolted to the ends of the plates, with the plates being tightly in contact with the top and bottom faces of the beam. Prior to the testing of Beam SR2, the bandages were installed on the beam between the load points at 300 *mm* spacing. Fig. 5.2 shows the actual arrangement of the bandages on the beam.

The test of Beam SR2 was started with load increments of 20 kN up to a load of 300 kN on each span, thereafter the increments were reduced to 10 kN. The first flexural crack appeared over the middle support at a load of 15 kN, while the first flexural crack under the load points was observed at 25 kN. The number of cracks increased up to 120 kN, then the cracks started to get wider and deeper without much change in their number. At 250 kN there were 14, 15 and 7 cracks in the positive moment regions of the right and the left spans, and in the negative moment region, respectively. The crack spacing varied between 100 to 200 *mm*.

At 350 kN the maximum deflection under the load points was about 13 *mm*, then the displacements started to increase significantly and faster, signalling the yield of the tensile reinforcement. The rate of displacement increase became even higher at 380 kN, which was interpreted as the beginning of yield in the compression reinforcement. When the load reached 400 kN, very large displacements

Figure 5.1: *Beam SR1 at failure*

were observed with minor changes in the loads. The test was continued without any major change in the load up to a maximum displacement of 120 mm and then it was stopped. Fig. 5.2 shows Beam SR2 under a load of 400 kN, where the large curvature illustrates the full plastic behaviour of the beam.

5.2.2 Beams CRU1 and CRU2

The two specimens in this set were designed as CFRP under-reinforced concrete beams. The ratio of ρ/ρ_b for this set was 0.44 while the actual amount of reinforcement was only 6.1% of that of the previous steel reinforced concrete beams, ($A_f/A_s = 6.1\%$). The weight of the FRP used as reinforcement in Beams CRU1 and CRU2 was 1.7% of the weight of the longitudinal steel reinforcement in Beams SR1 and SR2.

Figure 5.2: Full plastic behaviour of Beam SR2 under load $P=400\text{ kN}$

Beam CRU1 was loaded in 10 kN increments. The first flexural crack occurred over the middle support at 8 kN. At 60 kN, the number of cracks under the loads in the right and the left spans and over the middle support were 4, 5 and 4, respectively. The maximum depth of the cracks at 20 kN, 40 kN and 80 kN was found to be approximately 200, 250 and 300 mm, respectively. Whenever a new crack appeared in the beam, the load dropped slightly, but then it increased again. At 80 kN, a new deep crack occurred 550 mm from the left support while a similar crack formed 550 mm from the right support at 90 kN. At 100 kN, shear cracks appeared and jointed the flexural cracks at 45° . Since the cracks were not intercepting any stirrups and the compression zone was very limited (less than 40 mm high), it was concluded that maybe it is a sign of premature flexural + shear failure. Therefore, a couple of bandages were attached to the beam as external shear stiffeners. At load $P=146\text{ kN}$, a loud bang was heard from the beam with a large drop in the load of the left span. However, the right span kept carrying the load up to about 160 kN, then a big

bang was heard, followed by a large drop in the load. Fig. 5.3 shows Beam CRU1 under a load of 60 kN. This figure also contains a close picture of data acquisition system.

Beam CRU2 was loaded the same as the preceding beam but with an unloading and reloading excursion. The first flexural crack formed over the middle support at 10 kN. However, despite the formation of the flexural cracks at 10 kN load, major increase in the displacements took place after 25 kN. At 70 kN, the beam was unloaded to zero load while a maximum residual displacement of about 5 mm was observed. When the beam was reloaded to 70 kN and beyond, no change of stiffness was observed due to the unloading and reloading excursion. Again, to avoid possible premature shear failure, some bandages were attached to the beam after 100 kN load. Up to 100 kN, the number of cracks was limited to 5 in each of the positive and negative moment regions. However, at 105 kN, two major cracks occurred 400 mm from the end supports. At 130 kN, another two major cracks developed between the load points and the middle support, causing the loads to drop about 15 kN each. The beam continued to carry the load up to $P = 170$ kN, then a big bang was heard, signalling the complete failure of the beam, i.e. the rupture of the longitudinal FRP reinforcement over the middle support. Fig. 5.4 shows Beam CRU2 at failure.

5.2.3 Beams CRO1 and CRO2

Specimens in this set were designed as CFRP over-reinforced concrete beams. The ratio of ρ/ρ_b for this set was 2.61 while their reinforcement amount was 33.5% of that of the steel reinforced concrete beams, ($A_f/A_s = 33.5\%$). On the other hand, the weight of the FRP reinforcement in the latter beams was 6.8% of the weight of the longitudinal steel reinforcement in Beams SR1 and SR2.

Beam CRO1 was loaded with increments of 25 kN up to 300 kN; thereafter the

Figure 5.3: *Beam CRU1 at load $P = 60$ kN*

Figure 5.4: *Beam CRU2 at failure*

load increment was reduced to 10 kN. The first crack occurred over the middle support at a load of 30 kN. As the load increased, vertical cracks developed at other locations along the beam. At 80 kN, there existed 8 and 5 flexural cracks in the positive and negative moment regions, respectively, with the crack spacing being approximately 150 mm. Later, the vertical cracks gradually became inclined toward the loading points and the middle support, forming typical flexure-shear cracks, with the inclination of the shear cracks being approximately 45° to the longitudinal axis of the beam. To assess the extent of damage at the service load level, the beam was loaded up to 150 kN then unloaded to zero. At zero load, a maximum displacement of approximately 4 mm was recorded. Upon reloading, the displacements increased linearly and when the load again reached 150 kN, the displacements were approximately the same as those prior to unloading. This indicates that loading-unloading follows a linear response. At 180 kN, a shear crack in the left span started to develop quickly and approached the loading point, indicating possible shear failure of the beam at 190 kN. Therefore, loading was stopped and some bandages were attached to the beam to avoid shear failure. The loading was subsequently resumed until 350 kN. At 350 kN, it was noticed that the beam was undergoing rapid deflection accompanied by minor decrease in the load and by crackling noise. When the load dropped to 320 kN (8% less than the maximum load), the maximum displacement became 50% higher than that corresponding to 350 kN. At that moment, a loud noise was heard with a major drop in the load. The sound was due to the compression failure of concrete over the middle support. Then the beam adjusted itself to the new condition and continued to carry more load. The phenomenon of load drop and regaining of stiffness repeated itself a number of times with further increase in displacements. Finally, the test was stopped at 250 kN while the corresponding maximum displacement was approximately 250% more than that of the maximum load. Fig. 5.5 shows the plastic behaviour of Beam CRO1 after the maximum load.

Beam CRO2 was loaded similarly to Beam CRO1 but without unloading. The

Figure 5.5: *Plastic behaviour of Beam CRO1 after the maximum load*

first flexural crack occurred vertically at 20 kN over the middle support. The formation and development of cracks in this beam were similar to those of the preceding beam. To prevent premature shear failure, some bandages were attached to the beam at a load of 175 kN. At that load, the maximum crack width reached 1.0 mm. The maximum load on each span reached 357 kN with the corresponding maximum displacement being 40 mm. Immediately after the maximum load, the beam began to undergo higher displacements and the load dropped. For instance, at 300 kN the maximum displacement was approximately 70 mm. After 300 kN, the load dropped sharply but the beam quickly regained its stiffness and was able to sustain higher load. This phenomenon repeated itself a number of times with displacement increasing continuously. Finally the test was stopped at a maximum displacement of about 100 mm. Fig. 5.6 shows the behaviour of Beam CRO2 under the maximum load of 350 kN.

Figure 5.6: *Beam CRO2 under load $P=350$ kN*

5.2.4 Beams CRCS1 and CRCS2

Specimens in this set were designed the same as Beams CRO1 and CRO2, but with some confining CFRP grids, known as NEFMAC, placed in the compression zone in the maximum moment regions. The behaviour of this set of beams was expected to be fundamentally the same as the preceding set with some differences due to confinement.

The first flexural crack in Beam CRCS1 occurred over the middle support at 30 kN. At 75 kN there were 4 flexural cracks in each of the positive moment regions and 3 at negative moment zone, while the average distance between the cracks was about 300 mm. At 125 kN the number of the cracks had increased to 8, 5 and 3 in the parts of the right and left spans between the end supports and the load points and in the region between the load points, respectively. At 150 kN the maximum crack width reached 1.0 mm, and at 270 kN, a large crack suddenly occurred at

a distance of 200 *mm* from the right support and moved up to 40 *mm* from the top surface of the beam, causing a drop of 10 kN in the load on the right span. Subsequently, a similar crack formed at a distance of 200 *mm* from the left support and propagated up to 70 *mm* from the top surface of the beam. Once again to prevent premature shear failure, some bandages were attached to the beam. At 325 kN a large 45° inclined crack appeared half-way between the middle support and the left load point. At a slightly higher load a similar crack formed in the right span. Eventually, the concrete in the compression zone over the middle support crashed and spalled off within a limited height of 20 *mm*. Accordingly, the loads dropped to 300 kN and even lower, but then began to increase slowly accompanied by high rate of displacement. The phenomenon of load drop and regaining of strength recurred several times until the test was stopped at a maximum displacement of 70 *mm*. Fig 5.7 shows the deformation of Beam CRCS1 over the middle support after the maximum load. Notice in Fig. 5.7 that in contrast to Beams CRO1 and CRO2 (Figs. 5.5 and 5.6) only a small number of bandages are attached to the beam. It should be emphasized that the application of the bandages was a precautionary measure and was intended to ensure flexural failure.

Beam CRCS2 was loaded similarly to Beam CRCS1, but with an extra unloading and reloading excursion. The first flexural crack occurred in the negative moment region while at 50 kN, there had formed two 250 *mm* high cracks over the middle support, and two and one 220 *mm* high cracks under the right and the left load points, respectively. The number of cracks in the preceding regions increased to 4, 5 and 5 at 110 kN, and 4, 9, and 8 at 200 kN, respectively. The maximum crack width reached 1.0 *mm* at 150 kN. At 225 kN, the beam was unloaded to zero and then reloaded to the latter load level. It was observed that after unloading, a maximum displacement of 4 *mm* remained in the beam. However, during the reloading of the beam, the displacements increased linearly and finally at 225 kN reached the same value as in the first load cycle. At 230 kN a new crack formed at a distance of

Figure 5.7: *Plastic behaviour of Beam CRCS1 after the maximum load*

230 mm from the left support. The flexural-shear crack, half vertical and half 45° inclined, reached up to 60 mm from the top surface of the beam. The same type of crack occurred in the right span at 280 kN, following which a few bandages were attached to the beam. The beam continued to carry load up to 352.5 kN with the corresponding maximum displacement being 35 mm, then with the advent of the crushing of the compression concrete over the middle support, the loads dropped. Again, relatively large deflections were observed at lower loads and finally the test was stopped at a maximum displacement of 90 mm. Fig. 5.8 shows the deformed Beam CRCS2 under a load of 340 kN on each span.

5.2.5 Beams CRCN1 and CRCN2

Specimens in this set were reinforced entirely with CFRP materials. The longitudinal reinforcement of the beams was the same as in Beams CRCS1 and CRCS2, but

Figure 5.8: *Beam CRCS2 under load $P = 340 \text{ kN}$*

NEFMAC grids were used as shear and confining reinforcement.

Beam CRCN1 was loaded with increments of 25 kN. The first flexural crack was observed at 25 kN while at 75 kN, there were 7, 11 and 4 deep cracks in the right and left spans between the end supports and the load points, and between the load points, respectively. These numbers increased to 12, 14 and 6 at 100 kN, and to 14, 16 and 7 at 175 kN, respectively. These cracks were spaced at approximately 100 mm. At 230 kN, two new flexural cracks were formed at a distance of 200 mm from the end supports. The cracks were vertical up to 200 mm and then became inclined at 45° to the beam axis. The cracks on this beam were much narrower, i.e. at the high load of 300 kN, the maximum crack width just reached 1.0 mm. The beam reached a maximum load of 375 kN without any sign of shear failure, therefore, unlike the previous test beams, no bandages were used. At 375 kN the load on the right span started to drop slowly. However, the left span carried higher load up to 400 kN, then the crushing of compression concrete occurred and the

load dropped. However, the drop in the load was very gradual, i.e. when the loads decreased from 375 kN to 310 kN, the maximum displacement almost doubled. At 310 kN, a complete failure of the compression concrete took place and the loads dropped dramatically. Finally, after some fluctuations in the loads, the test was stopped. Fig. 5.9 shows Beam CRCN1 under a load of 250 kN.

The first flexural crack in Beam CRCN2 occurred at 20 kN. With increasing load, the cracks increased in number and became inclined toward the loading points and the middle support. At 100 kN, there were 9, 9 and 4 cracks, respectively, in the right and left spans between the load points and the end supports and in the region between the load points. These numbers increased to 12, 12 and 5 at 125 kN, to 14, 15 and 6 at 160 kN, and to 17, 16 and 8 at 250 kN. At 250 kN, the beam was unloaded to zero and then reloaded. The unloading-reloading process did not have any noticeable influence on the overall behaviour of the beam. At 270 kN, a large flexural-shear crack formed at a distance of 200 mm from the left support. The crack was accompanied with a loud noise and a 10 kN drop in the load on the left span. At 280 kN, a similar crack developed at a distance of approximately 200 mm from the right support. The maximum crack width reached 1.0 mm at 280 kN. Spalling of concrete cover in the compression zone in the vicinity of the middle support was observed when the load reached 350 kN. However, the beam continued to carry higher load up to 365 kN, followed by a smooth drop with noticeable change in displacements. Finally at 305 kN, the loads dropped dramatically and continued to fluctuate; at that juncture, the test was stopped. Fig. 5.10 shows the plastic deformation of Beam CRCN2 after the maximum load. It is significant to observe the large amount of deformation that this beam experienced before failure despite the lack of any reinforcement.

Figure 5.9: *Beam CRCN1 at load $P = 250$ kN*

Figure 5.10: *Plastic deformation of Beam CRCN2 at failure*

5.3 Load-Displacement Curves

The displacements of each beam at 10 different locations (0.5 *m* apart) were measured using LVDTs. The variations of load versus the maximum displacement under the load for all the test beams (except for Beam SR1 for which a major part of its data was lost) are shown in Fig. 5.11 to 5.19. Each figure exhibits the load-midspan deflection curves of the right and left spans of the beam.

The deflected shape of the beams at various load levels are shown in Fig. 5.20 to 5.29. The displacements are shown for load levels corresponding to 20%, 40%, 60%, 80% and 100% of P_{max} , where P_{max} is the maximum load sustained by each beam.

From the serviceability point of view, the ratio of the maximum deflection to the span length (δ/L) should be compared to the maximum allowable ratios permitted by the code. Depending on the type of construction and the function of the structural member, the allowable ratios in the code range from $L/480$ to $L/180$. Noting that the serviceability requirements in the code are based on the service loads, i.e. unfactored dead load plus live load, while the maximum load (design load) is calculated based on the factored loads, it may be reasonable to assume that the service load is only 40% to 50% of the design factored loads. It is recognized that the latter percentage and the attendant maximum deflection under service conditions is also dependent on the ratio of the dead load to the live load.

With the preceding considerations in mind and by reference to Figs. 5.11 to 5.19 or 5.20 to 5.29, we observe that at 50% of the maximum load, the maximum displacements are 6 , 14, 17, 15, 13, 11, 11, 12 and 12 *mm*, respectively, for Beams SR2, CRU1, CRU2, CRO1, CRO2, CRCS1, CRCS2, CRCN1 and CRCN2. Hence the ratio of δ/L for Beam SR2 is $6/3000 = 1/500$, which satisfies the most stringent deflection requirements of the code. On the other hand, if the average values of de-

flections are taken for the two nominally identical test beams in each set, the ratio of δ/L would be 1/194 for Beams CRU1 and CRU2, 1/214 for Beams CRO1 and CRO2, 1/273 for Beams CRCS1 and CRCS2, and 1/250 for Beams CRCN1 and CRCN2. It means that under the worse scenario case, the tested FRP reinforced concrete beams do not satisfy the more stringent code requirements for deflection. It should be noted that among the FRP reinforced test beams, the largest deflection was experienced by the FRP under-reinforced beams and the least by the FRP over-reinforced beams with confinement. Therefore, the deflection of FRP reinforced concrete beams may be limited by using over-reinforced sections with a higher amount of reinforcement as well as confinement of compression concrete.

5.4 Crack Formation and Propagation

The formation of cracks in reinforced concrete members is in general due to volume changes of concrete caused by shrinkage and thermal effects, dead and live loads, imposed displacements (support settlement) and misfit of the members. However, generally the cracks due to the aforementioned causes, with the exception of load effects, could be minimized by enhancing the quality of concrete and by employing proper construction procedures. Therefore, in reinforced concrete structures cracks caused by tensile stresses due to loads are given greater attention from the point of view of analysis and design.

In the current experiment, extreme care was exercised during the casting and curing of concrete as well as during the tests. Therefore, it was anticipated that the effect of the non-structural factors on the formation and propagation of the cracks would be small. Nevertheless, before the testing of Beams CRU1, CRU2, CRCS1, CRCS2, CRCN1 and CRCN2, some transverse cracks were observed. However, the maximum width of these cracks before testing of those beams was measured to be

0.1 mm. Furthermore, the cracks were basically in the compression zone of concrete, so they closed upon loading.

During the test, it was observed that the first cracks were always flexural cracks. The cracks formed at early stages of loading, i.e. at a load between 5% to 10% of the maximum load. Immediately after the formation of the first crack in each beam, new cracks continued to form up to about 50% of the maximum load, but the rate of increase in the number of cracks decreased with increased levels of load, and the cracks gradually became deeper and wider. Concurrently, the initial flexural cracks became inclined following essentially the principle compressive stress trajectories at higher loads. During the last 30% of loading of each beam, seldom any new crack formed, but the existing cracks became wider with minor changes in the position of the neutral axis.

5.4.1 Crack Pattern

The crack pattern of all the test beams at maximum load is shown in Figs. 5.30 to 5.34. It can be seen in these figures that: (1) The number of cracks in the positive moment regions is larger than that in the negative moment region. This may be attributed to the wider expanse of the positive moment region along the beam. (2) The number of cracks in the FRP under-reinforced beams is much less than that in FRP over-reinforced beams. (3) The distribution and propagation of the cracks in Beams CRCN1 and CRCN2 are better than those in the other over-reinforced test beams. The latter is due to the close spacing of the transverse reinforcement, i.e. 50 mm intervals of NEFMAC grids in Beams CRCN1 and CRCN2. (4) The cracks in the FRP under-reinforced beams are primarily flexural while the over-reinforced beams exhibit a combination of flexural and flexural-shear cracks. (5) There is an overall similarity in the crack patterns of Beam SR1 and SR2 (Fig. 5.30) and Beams CRCN1 and CRCN2 (Fig. 5.34), albeit the number of cracks in the latter two beams

is larger than in the former beams. Also the flexural cracks are higher in the latter beams and are more closely spaced. (6) Beams CRCS1 and CRCS2 have fewer cracks than Beams CRCN1 and CRCN2 at failure, but the maximum deflection of the two sets of beams at the same load level appear to be nearly the same.

5.4.2 Crack Width

For the measurement of the crack width, a microscope with accuracy of 10 micron was used. The microscope was able to measure crack widths up to 1.00 mm. Figs. 5.35 to 5.44 display the crack width versus load curves for all the test beams. The graphs show the maximum measured crack width in both the positive and the negative moment regions and at different load levels, i.e. at different ratios of P/P_{max} .

The maximum permitted crack width for reinforced concrete members is not stipulated explicitly in the code. However, the Gergely-Lutz expression in Eq. 5.1 has been the basis of the crack control requirement in both the CSA A23.3-94 and the ACI 318-89.

$$w = c\beta f_s \sqrt[3]{d_c A} \quad (5.1)$$

where w = the maximum crack width (mm), β = ratio of distance to the neutral axis from the extreme tension fibre to that from the centroid of the main reinforcement, f_s = stress in steel at specified load calculated by elastic cracked section theory (MPa), d_c = thickness of concrete cover measured from the extreme tension fibre to the centre of the bar located closest thereto (mm), A = effective tension area of concrete surrounding the main tension reinforcement and having the same centroid as that reinforcement, divided by the number of bars (mm²), and c = a numerical constant determined from statistical analysis of experimental data whose value could be taken as 11×10^{-6} mm²/N. Based on the Gergely-Lutz expression, the code gives

the following equation for crack control:

$$z = f_s \sqrt[3]{d_c A} \times 10^{-3} = \frac{w}{c\beta} \times 10^{-3} \quad (5.2)$$

where z is a quantity limiting the distribution of flexural reinforcement. The code limits the value of z to 30 kN/mm and 25 kN/mm , respectively, for interior and exterior exposure under normal conditions. If β is assumed to be 1.2 for general design, it can be concluded that the z values in the code implicitly limit the maximum crack width to 0.40 mm and 0.33 mm for interior and exterior exposure, respectively.

It should be stated again that the crack control calculations in the code are based on service loads and that the service loads normally do not exceed 50% of the design factored load. From Figs. 5.35 and 5.36, the maximum crack widths in Beams SR1 and SR2 at 50% of the maximum load are 0.37 mm and 0.58 mm , respectively. However, the maximum crack width for Beam SR1 may be increased to 0.45 mm which corresponds to 50% of the maximum expected flexural failure load, i.e. at 50% of 400 kN . Hence, it can be seen that the maximum measured crack width in the present steel reinforced beams are approximately 30% higher than the code limitations. On the other hand, the assumption that the service loads are 50% of the design factored load is somewhat arbitrary, for in reality this ratio can vary between 30 – 50%. If we use an average ratio of 40%, then the maximum crack width would drop to a more acceptable level. Conversely, we can use the results in Fig. 5.36 to find the load corresponding to a maximum crack width of 0.4 mm which would be approximately 40% of P_{max} . Now, we can use the latter ratio for the crack width limits in the FRP reinforced beams.

It can be seen in Figs. 5.37 to 5.44 that the maximum measured crack width for all the FRP reinforced concrete test beams, except for Beams CRCN1 and CRCN2, at 50% of the maximum load is generally close to or more than 1.0 mm . However, for Beams CRCN1 and CRCN2, it is about 0.70 mm which is comparable to the maximum crack width of steel reinforced concrete beams. At 40% of the maximum

load, the maximum crack width in Beam CRCN1 is only 0.45 *mm* which is nearly the same as in the steel reinforced beams at the same load ratio. This shows that from the maximum crack width point of view, the best behaviour was exhibited by Beams CRCN1 and CRCN2 which may be attributed to their over-reinforcement and to the close spacing of their transverse reinforcement. It can be argued that since FRP reinforcement does not corrode, the maximum permissible crack width could be higher for FRP reinforced concrete beams (Kaneko et al, 1993). Also the crack width of beams such as Beams CRCN1 and CRCN2 could be further decreased by reducing the size of the longitudinal reinforcement and by considering better distribution and arrangement for the longitudinal bars.

5.5 Beams Reactions

As mentioned earlier, load cells were used at the end supports of the beams to measure beam reactions. Due to the indeterminacy of the test beams, effort was made to detect potential unequal settlement of the supports. Fortunately the load cells under the end supports were stiff and did not undergo any noticeable displacement, i.e. the maximum theoretical displacement of the load cells under the maximum loads on the beams was less than 0.04 *mm*. The large steel section used as the middle support was also quite stiff, accordingly support settlement was not a problem. To ascertain the validity of the preceding suppositions, dial gauges with accuracy of 10 micron were installed at different locations on all the supports and their displacement at different load stages was recorded. Based on those measurements, it was concluded that there was not any significant uneven settlement of the supports for any of the beams, i.e. the maximum recorded uneven settlement of the supports was less than 0.1 *mm*.

Based on the load cells readings, the variation of end support reactions versus

the load is shown in Figs. 5.45 to 5.53, respectively, for Beams SR2 to CRCN2. Each graph contains two curves corresponding to the right and left support reaction. It should be pointed out that although the graphs appear to be linear, in reality they are not. The reason is that for a two-span continuous beam such as the test beams, the extent of the deviation of the support reactions from linearity is expected to be very small, i.e. it can vary from $R = 0.312P$ for fully elastic condition to $R = 0.333P$ for fully plastic condition, where R is the end reaction and P is the applied load on each span. In the following chapter, the reactions will be further examined in order to determine the extent of moment redistribution in the test beams.

5.6 Strain Variation

Although the test beams are indeterminate, knowing the end support reactions from the load cell readings permitted the calculation of the internal forces, i.e. the bending moment and shear at any particular section. The latter allows one to determine the internal stresses and strains. However, to be able to measure the strains directly, some strain gauges were attached to the reinforcement and on the concrete surface, and were connected to the data acquisition system. Therefore, the strains at selected locations were measured directly and their variations were plotted versus the load.

5.6.1 Longitudinal Reinforcement and Concrete

Five millimeter long electrical resistance strain gauges were attached to the longitudinal reinforcement at the maximum positive and negative moment sections of the test beams as described in Section 4.8 and shown in Fig. 4.20. Furthermore, 30-mm long strain gauges were glued to the surface of the concrete on the side of the beams as described in Section 4.8. The variation of the strain versus applied load in the

longitudinal reinforcement and on the concrete at the maximum positive and negative moment sections of the test beams is shown in Figs. 5.54 to 5.80. Notice that not all the attached strain gauges were functioning during the test, therefore, the graphs do not contain the strain in some of the tensile or compressive longitudinal reinforcement bars.

In Figs. 5.54 to 5.56, the yield of the longitudinal tensile and compressive steel reinforcement can be easily distinguished for Beam SR2. However, since FRP materials do not yield, no yield plateau can be observed for FRP reinforcement. It should be noted that since the failure strain of the CFRP reinforcement is quite high and due to the limitation in the maximum extension of the strain gauges, the high level strains in CFRP reinforcement could not be always recorded. Therefore, the strains of CFRP under-reinforced concrete beams, i.e. Beams CRU1 and CRU2, at high levels of strains, particularly close to failure of the beams, are not shown realistically in Figs. 5.57 to 5.62.

5.6.2 Transverse Reinforcement

As was mentioned earlier, no strain gauges were attached to the transverse reinforcement of the first six test beams, i.e. Beams SR1, SR2, CRU1, CRU2, CRO1 and CRO2. However, 5-mm long strain gauges were attached to the steel shear reinforcement and the confining NEFMAC grids at different locations of Beams CRCS1 and CRCS2 as shown in Fig. 4.20. In addition, 5-mm long strain gauges were attached to the transverse shear and confining NEFMAC grids at different locations in Beams CRCN1 and CRCN2 as shown in Fig. 4.21. The variation of the strain in the transverse reinforcement versus the applied loads is shown in Figs. 5.81 to 5.85 for Beam CRCS1, Figs. 5.86 to 5.90 for Beam CRCS2, Figs. 5.91 to 5.95 for Beam CRCN1, and Figs. 5.96 to 5.100 for Beam CRCN2.

From Figs. 5.81 to 5.100, it can be seen that the maximum strain in the confining NEFMAC grids in all four beams reached up to 4000 micro-strain, which corresponds to 400 MPa stress. It means that the NEFMAC grids and their intervals have effectively confined the concrete in the compression zone, both in Beams CRCS1 and CRCS2, and in Beams CRCN1 and CRCN2. It is significant to observe that the load-strain graphs of the NEFMAC grids are neither linear nor bilinear in all cases. The nonlinear variations in Figs. 5.97 and 5.98 are particularly interesting, for they are similar to the behaviour of the steel stirrups in Fig. 5.89. This nonlinearity is due to progressive development of cracks and the loss of shear resistance of concrete. But despite the relatively high level of strain in the grids, they were rarely stressed more than 50% of their failure stress. The approximately 400 MPa stress reached in some of the grids is essentially equal to the yield stress of the steel stirrups but since the modulus of NEFMAC is only half that of steel, the corresponding strain is double the steel yield strain. Despite these high strain levels, the overall behaviour of the beams with NEFMAC grids as shear and/or confining reinforcement was not adversely affected.

In summary, the experimentally measured data for the test beams were reported in this chapter. The chapter essentially presented the raw data as obtained from the data acquisition system, with some amount of interpretation whenever warranted. In the next chapter, we will use the data in the present chapter to analyse the beams, with particular emphasis on ductility and moment redistribution.

Figure 5.11: Load-displacement curve for Beam SR2

Figure 5.12: Load-displacement curve for Beam CRU1

Figure 5.13: Load-displacement curve for Beam CRU2

Figure 5.14: Load-displacement curve for Beam CRO1

Figure 5.15: Load-displacement curve for Beam CRO2

Figure 5.16: Load-displacement curve for Beam CRCS1

Figure 5.17: Load-displacement curve for Beam CRCS2

Figure 5.18: Load-displacement curve for Beam CRCN1

Figure 5.19: Load-displacement curve for Beam CRCN2

Figure 5.20: Deflected shape of Beam SR2 at different loads (up to 98% of P_{max})

Figure 5.21: Deflected shape of Beam SR2 at different loads (up to 100% of P_{max})

Figure 5.22: Deflected shape of Beam CRU1 at different loads

Figure 5.23: Deflected shape of Beam CRU2 at different loads

Figure 5.24: Deflected shape of Beam CRO1 at different loads

Figure 5.25: Deflected shape of Beam CRO2 at different loads

Figure 5.26: Deflected shape of Beam CRCS1 at different loads

Figure 5.27: Deflected shape of Beam CRCS2 at different loads

Figure 5.28: Deflected shape of Beam CRCN1 at different loads

Figure 5.29: Deflected shape of Beam CRCN2 at different loads

Figure 5.30: Crack pattern of Beams SR1 and SR2 at failure

Figure 5.31: Crack pattern of Beams CRU1 and CRU2 at failure

Figure 5.32: Crack pattern of Beams CRO1 and CRO2 at failure

Figure 5.33: Crack pattern of Beams CRCS1 and CRCS2 at failure

Figure 5.34: Crack pattern of Beams CRCN1 and CRCN2 at failure

Figure 5.35: Crack width at various ratios of the maximum load for Beam SR1

Figure 5.36: Crack width at various ratios of the maximum load for Beam SR2

Figure 5.37: Crack width at various ratios of the maximum load for Beam CRU1

Figure 5.38: Crack width at various ratios of the maximum load for Beam CRU2

Figure 5.39: Crack width at various ratios of the maximum load for Beam CRO1

Figure 5.40: Crack width at various ratios of the maximum load for Beam CRO2

Figure 5.41: Crack width at various ratios of the maximum load for Beam CRCS1

Figure 5.42: Crack width at various ratios of the maximum load for Beam CRCS2

Figure 5.43: Crack width at various ratios of the maximum load for Beam CRCN1

Figure 5.44: Crack width at various ratios of the maximum load for Beam CRCN2

Figure 5.45: End support reactions versus load for Beam SR2

Figure 5.46: End support reactions versus load for Beam CRU1

Figure 5.47: End support reactions versus load for Beam CRU2

Figure 5.48: End support reactions versus load for Beam CRO1

Figure 5.49: End support reactions versus load for Beam CRO2

Figure 5.50: *End support reactions versus load for Beam CRCS1*

Figure 5.51: *End support reactions versus load for Beam CRCS2*

Figure 5.52: End support reactions versus load for Beam CRCN1

Figure 5.53: End support reactions versus load for Beam CRCN2

Figure 5.54: *Strain versus load for negative moment region over middle support of Beam SR2*

Figure 5.55: Strain versus load for positive moment region under load point in Beam SR2

Figure 5.56: Strain versus load for positive moment region under load point in Beam SR2

Figure 5.57: *Strain versus load for negative moment region over middle support of Beam CRU1*

Figure 5.58: *Strain versus load for positive moment region under load point in Beam CRU1*

Figure 5.59: Strain versus load for positive moment region under load point in Beam CRU1

Figure 5.60: Strain versus load for negative moment region over middle support of Beam CRU2

Figure 5.61: Strain versus load for positive moment region under load point in Beam CRU2

Figure 5.62: Strain versus load for positive moment region under load point in Beam CRU2

Figure 5.63: Strain versus load for negative moment region over middle support of Beam CRO1

Figure 5.64: Strain versus load for positive moment region under load point in Beam CRO1

Figure 5.65: *Strain versus load for positive moment region under load point in Beam CRO1*

Figure 5.66: *Strain versus load for negative moment region over middle support of Beam CRO2*

Figure 5.67: Strain versus load for positive moment region under load point in Beam CRO2

Figure 5.68: Strain versus load for positive moment region under load point in Beam CRO2

Figure 5.69: Strain versus load for negative moment region over middle support of Beam CRCS1

Figure 5.70: *Strain versus load for positive moment region under load point in Beam CRCS1*

Figure 5.71: Strain versus load for positive moment region under load point in Beam CRCS1

Figure 5.72: Strain versus load for negative moment region over middle support of Beam CRCS2

Figure 5.73: Strain versus load for positive moment region under load point in Beam CRCS2

Figure 5.74: Strain versus load for positive moment region under load point in Beam CRCS2

Figure 5.75: *Strain versus load for negative moment region over middle support of Beam CRCN1*

Figure 5.76: Strain versus load for positive moment region under load point in Beam CRCN1

Figure 5.77: Strain versus load for positive moment region under load point in Beam CRCN1

Figure 5.78: Strain versus load for negative moment region over middle support of Beam CRCN2

Figure 5.79: Strain versus load for positive moment region under load point in Beam CRCN2

Figure 5.80: *Strain versus load for positive moment region under load point in Beam CRCN2*

Figure 5.81: Strain versus load for confining NEFMAC grid at 100 mm from middle support, Beam CRCS1

Figure 5.82: *Strain versus load for confining NEFMAC grid at 200 mm from middle support, Beam CRCS1*

Figure 5.83: Strain versus load for steel stirrup at 150 mm from middle support, Beam CRCS1

Figure 5.84: Strain versus load for steel stirrup at 750 mm ($L/4$) from middle support, right span of Beam CRCS1

Figure 5.85: Strain versus load for steel stirrup at 750 mm ($L/4$) from middle support, left span of Beam CRCS1

Figure 5.86: Strain versus load for confining NEFMAC grid at 100 mm from middle support, Beam CRCS2

Figure 5.87: Strain versus load for confining NEFMAC grid at 200 mm from middle support, Beam CRCS2

Figure 5.88: *Strain versus load for steel stirrup at 150 mm from middle support, Beam CRCS2*

Figure 5.89: Strain versus load for steel stirrup at 750 mm ($L/4$) from middle support, right span of Beam CRCS2

Figure 5.90: Strain versus load for steel stirrup at 750 mm ($L/4$) from middle support, left span of Beam CRCS2

Figure 5.91: Strain versus load for confining ribs of NEFMAC grid at 100 mm from middle support, Beam CRCN1

Figure 5.92: Strain versus load for confining ribs of NEFMAC grid at 200 mm from middle support, Beam CRCN1

Figure 5.93: Strain versus load for shear ribs of NEFMAC grid at 200 mm from middle support, Beam CRCN1

Figure 5.94: Strain versus load for shear ribs of NEFMAC grid at 750 mm ($L/4$) from middle support, right span of Beam CRCN1

Figure 5.95: Strain versus load for shear ribs of NEFMAC grid at 750 mm ($L/4$) from middle support, left span of Beam CRCN1

Figure 5.97: Strain versus load for confining ribs of NEFMAC grid at 200 mm from middle support, Beam CRCN2

Figure 5.98: *Strain versus load for shear ribs of NEFMAC grid at 200 mm from middle support, Beam CRCN2*

Figure 5.99: Strain versus load for shear ribs of NEFMAC grid at 750 mm ($L/4$) from middle support, right span of Beam CRCN2

Figure 5.100: Strain versus load for shear ribs of NEFMAC grid at 750 mm ($L/4$) from middle support, left span of Beam CRCN2

Chapter 6

Analysis of Ductility and Moment Redistribution in the Test Beams

6.1 General

The importance of ductility in reinforced concrete members was discussed earlier in Chapter 3. The main advantages of ductility in reinforced concrete structures were stated as prevention of brittle failure and warning of impending failure, providing the necessary conditions for the redistribution of moments and stresses, and furnishing the appropriate conditions for energy absorption and dissipation by post-elastic deformations, especially for structures under seismic loads.

Conventional under-reinforced concrete structures under flexure benefit from ductility and its consequences. It is widely believed, however, that FRP reinforced concrete members suffer from brittle failure and lack of ductility, although few studies have been actually conducted to methodically substantiate these claims. This chapter is mainly concerned with the fundamental aspects of ductility in FRP rein-

forced concrete members, and in particular with their non-linear moment-curvature relationship. An attempt will be made to show the existence of a pseudo-ductile behaviour in FRP reinforced concrete sections. Furthermore, a bi-linear model for moment curvature relationship of FRP reinforced concrete sections will be proposed, also a new definition will be proposed for the ductility factor of FRP reinforced sections. Finally, the current test beams will be analyzed for their ductility and moment redistribution in the light of the basic principles that are presented in this chapter.

6.2 Moment-Curvature Relation

The moment-curvature relationship of steel reinforced concrete beams with various geometric properties and reinforcement ratios was illustrated in Figs. 3.1 to 3.6. It was observed in those figures that conventional under-reinforced concrete sections exhibit highly ductile behaviour, especially those with lower reinforcement ratios. For that reason the codes of practice normally prohibit over-reinforcing of conventional reinforced concrete members under flexure.

Similarly, the moment-curvature relationship of FRP reinforced concrete beams with different ratios of reinforcement and different FRP materials was illustrated in Figs. 3.7 and 3.8. From those figures it could be seen that the moment-curvature relationship of FRP reinforced concrete sections is mainly linear, with a limited non-linear portion at higher moments. This limited non-linearity is markedly noticeable in FRP over-reinforced sections, especially in sections with higher ratios of reinforcement. This observation leads to the conclusion that FRP over-reinforced concrete beams under flexure, especially those with higher reinforcement ratios, are preferable from the ductility point of view. Coincidentally, as discussed earlier in Sec. 3.4.2, FRP over-reinforced concrete beams also offer other advantages, including smaller deflection under service loads.

Table 6.1: *Some characteristics of moment-curvature curves for CFRP reinforced concrete section ($b = 300$ mm, $h = 500$ mm, $d = 450$ mm, $f'_c = 25$ MPa, $\epsilon_{cu} = 0.003$, $f_{cu} = 1470$ MPa, $E_f = 130$ GPa)*

ρ	M_u $\times 10^8$ (N.mm)	ϕ_u $\times 10^{-4}$ (1/mm)	$\phi_{0.8}$ $\times 10^{-4}$ (1/mm)	slope of moment curvature curve $\times 10^{13}$ (N.mm ²)			
				before cracking	just after cracking	at $0.8M_u$	at ultimate
0.0005	0.426	0.266	0.211	7.841	0.160	0.159	0.159
0.0010	0.860	0.283	0.229	7.869	0.308	0.301	0.297
0.0025	2.053	0.312	0.239	7.953	0.711	0.619	0.520
0.0050	2.809	0.245	0.183	8.091	1.299	1.035	0.819
0.0100	3.569	0.185	0.137	8.359	2.294	1.662	1.192
0.0150	4.041	0.158	0.114	8.617	3.134	2.190	1.415
0.0200	4.378	0.143	0.101	8.867	3.865	2.655	1.554
0.0300	4.839	0.125	0.871	9.340	5.104	3.262	1.698

6.3 Proposed Moment-Curvature Model

To be able to assess the general behaviour of FRP reinforced concrete beams in flexure, the moment-curvature curves for rectangular concrete sections reinforced with either carbon or glass bars and containing different reinforcement ratios, i.e. $\rho = 0.0005$ to $\rho = 0.0300$, were calculated. The maximum permissible strain of compression concrete was taken as $\epsilon_{cu} = 0.003$. Selected results are tabulated in Tables 6.1 and 6.2 for carbon and glass reinforced concrete sections, respectively. Note that the selected values for tensile strength and modulus of elasticity of FRP reinforcement which were used in the calculations of the quantities in Tables 6.1 and 6.2 were taken from the experimental works carried out by Saadatmanesh (1994) and Ali (1996) for GFRP and CFRP rebars, respectively. Although the mechanical

Table 6.2: *Some characteristics of moment-curvature curves for GFRP reinforced concrete section ($b = 300 \text{ mm}$, $h = 500 \text{ mm}$, $d = 450 \text{ mm}$, $f'_c = 25 \text{ MPa}$, $\epsilon_{cu} = 0.003$, $f_{cu} = 1180 \text{ MPa}$, $E_f = 53.6 \text{ GPa}$)*

ρ	M_u $\times 10^8$ (N.mm)	ϕ_u $\times 10^{-4}$ (1/mm)	$\phi_{0.8}$ $\times 10^{-4}$ (1/mm)	slope of moment curvature curve $\times 10^{13}$ (N.mm ²)			
				before	just after	at	at
				cracking	cracking	$0.8M_u$	ultimate
0.0005	0.335	0.490	0.409	7.820	0.069	0.069	0.069
0.0010	0.700	0.531	0.432	7.828	0.134	0.130	0.127
0.0025	1.479	0.494	0.388	7.851	0.317	0.286	0.356
0.0050	1.986	0.360	0.282	7.889	0.598	0.508	0.443
0.0100	2.614	0.266	0.199	7.965	1.102	0.893	0.723
0.0150	3.033	0.224	0.167	8.041	1.553	1.209	0.922
0.0200	3.350	0.199	0.148	8.115	1.967	1.469	1.083
0.0300	3.816	0.170	0.121	8.262	2.709	2.003	1.311

characteristics of both GFRP and CFRP range over a relatively wide band, the objective here is to propose a model independent of the actual values of those characteristics, consequently, the full range of the variations in the mechanical properties of FRP materials was not utilized in the current investigation.

The results show that before cracking, the moment-curvature is essentially linear and exhibits high rigidity compared to after cracking. At cracking the moment initially drops, then it increases with increasing curvature, but with a lower rigidity than before and exhibiting minor non-linearity. With further increase in moment, the non-linearity becomes more noticeable. The non-linearity becomes particularly visible with increased reinforcement ratio, ρ . It can be seen from the preceding tables that the latter is true for all reinforcement ratios and for both carbon and

glass FRP reinforced beams. More specifically, the following comments can be made:

1 . The slope of the moment-curvature curve, or the rigidity of the section at $M = 0.80M_u$, is within 47% to 52% of the sum of the slopes just after cracking and at ultimate. Note that M_u in the present thesis refers to the nominal ultimate capacity. In other words, the rigidity of the section at $0.8M_u$ is the average of the rigidities of the section after cracking and at the ultimate point.

2 . The tangent to the moment-curvature curve at points close to the cracking moment essentially pass through the origin.

According to the preceding observations, the approximate moment-curvature relationship illustrated in Fig. 6.1 is proposed for FRP reinforced concrete sections. To obtain this model for any particular section, by reference to Fig. 6.1 the following steps need to be followed:

- 1 . Calculate the moment and the curvature at cracking, M_{cr} and ϕ_{cr} , [point A].
- 2 . Calculate the moment and the curvature at ultimate, M_u and ϕ_u , [point D].
- 3 . Calculate the curvature corresponding to $0.8M_u$, $\phi_{0.8}$, [point C].
- 4 . Locate point B on the line OC by setting $M = M_{cr}$.

The resulting curve $OABCD$ is the proposed moment-curvature model.

The effect of the deviation of the broken line OAB from the straight line OB (Fig. 6.1), on the deflection calculation based on the moment-curvature curve may not be too significant; therefore, this deviation may be neglected and a bi-linear model may be used, (lines OC and CD in Fig. 6.1). This bi-linear model can be used easily for manual deflection calculations.

Figure 6.1: *Proposed model for moment-curvature relationship of FRP reinforced concrete*

The proposed model was used for reinforcement ratios from 0.0005 to 0.0300, and the percentage reduction in the rigidity of the section after point C, $\tan \beta_2 = EI_2$ (Fig. 6.1), to the rigidity after cracking but before point C, $\tan \beta_1 = EI_1$, are tabulated in Tables 6.3 and 6.4, for sections reinforced with carbon and glass, respectively.

Regarding the balanced reinforcement ratios for carbon and glass reinforced concrete sections, ($\rho_b = 0.00258$ for carbon with $F_u = 1470$ MPa and $E_f = 130$

Table 6.3: *The effect of reinforcement ratio on the percentage loss of stiffness in carbon reinforced concrete section*

ρ	$EI_1 \times 10^{13}$ ($N.mm^2$)	$EI_2 \times 10^{13}$ ($N.mm^2$)	loss of stiffness (%)
0.0005	0.160	0.158	1.6
0.0010	0.305	0.296	3
0.0025	0.683	0.577	15
0.0050	1.222	0.929	24
0.0100	2.109	1.430	32
0.0150	2.841	1.798	37
0.0200	3.478	2.083	40
0.0300	4.500	2.439	46

GPa, and $\rho_b = 0.00184$ for glass with $F_u = 1180$ MPa and $E_f = 53.6$ GPa; assuming $f'_c = 25$ MPa), it can be observed that the reduction in the stiffness for $\rho \leq 0.5\rho_b$ is less than four percent, for both carbon and glass reinforced concrete. Therefore, it is proposed to consider just one straight line, (line OD in Fig. 6.1), as an approximate moment-curvature relationship for the FRP reinforced concrete sections with $\rho \leq 0.5\rho_b$. On the other hand, for $\rho > \rho_b$, the percentage loss of stiffness can be from 15% to 46% for carbon reinforced section and from 5% to 34% for glass reinforced section.

6.4 Ultimate Compressive Strain of Concrete

It is well known that the curvature of a reinforced concrete section is mainly dependent on the maximum compressive strain of concrete at ultimate limit state, ϵ_{cu} . The 1989 ACI Code recommended $\epsilon_{cu} = 0.003$ and the 1994 CSA Code recom-

Table 6.4: *The effect of reinforcement ratio on the percentage loss of stiffness in glass reinforced concrete section*

ρ	$EI_1 \times 10^{13}$ ($N.mm^2$)	$EI_2 \times 10^{13}$ ($N.mm^2$)	loss of stiffness (%)
0.0005	0.0683	0.0682	0.15
0.0010	0.132	0.128	3
0.0025	0.307	0.270	12
0.0050	0.572	0.477	17
0.0100	1.042	0.811	22
0.0150	1.450	1.067	26
0.0200	1.821	1.275	30
0.0300	2.486	1.645	34

mended $\epsilon_{cu} = 0.0035$ are conservative. Park and Paulay (1975) stated that it would be reasonable to take a value higher than $\epsilon_{cu} = 0.003$, for the ultimate curvature calculations. They suggested that a value of $\epsilon_{cu} = 0.004$ could be used for ultimate curvature calculations. Blume et al (1964) recommended a value of $\epsilon_{cu} = 0.004$ for ultimate curvature calculations assuming unconfined concrete. In an experimental study conducted by Saadatmanesh (1994) on concrete beams reinforced with glass fiber reinforced plastics bars, it was reported that for a beam with $\rho \geq \rho_b$ ($\rho = 0.0155$), the concrete crushed at a measured strain of approximately 0.0056. Therefore, he assumed a maximum concrete strain of 0.005 for his analytical moment curvature curves. Assuming the latter for ultimate concrete compressive strain, the slope of the different regions of the moment-curvature curve, for various reinforcement ratios of CFRP and GFRP reinforced concrete section were calculated. Again it was determined that the slope of the moment-curvature curve at $M = 0.8M_u$ is approximately equal to the average of the slopes of the curve after cracking and at ultimate moment. Accordingly, the proposed bi-linear model could be fitted to a

moment-curvature relationship, even with $\epsilon_{cu} = 0.005$. It means that the general assumptions of the model are not affected not only by the reinforcement ratio and the type of FRP, but also by the maximum compressive strain considered for concrete at failure.

The percentage reduction in the rigidity of the second linear part of the model, to the rigidity of the first part after cracking, are tabulated in Table 6.5 and 6.6, for CFRP and GFRP reinforced concrete, respectively. Comparison between Table 6.3 and Table 6.5, or between Table 6.4 and Table 6.6, reveals that higher amounts of ultimate compressive strain of concrete enhance the non-linearity of the response of the FRP over-reinforced concrete sections.

Table 6.5: *Effect of reinforcement ratio on the percentage loss of stiffness in carbon reinforced concrete with $\epsilon_{cu} = 0.005$*

ρ	$EI_1 \times 10^{13}$ ($N.mm^2$)	$EI_2 \times 10^{13}$ ($N.mm^2$)	loss of stiffness (%)
0.0050	1.163	0.647	44
0.0100	2.004	0.899	55
0.0150	2.721	1.066	61
0.0200	3.332	1.168	65
0.0300	4.336	1.315	70

6.5 Pseudo-Ductility Factor

As stated earlier in Chapter 3, for steel reinforced concrete members the ductility factor is commonly defined as the ratio of ultimate deformation to the yield deformation (Eq. 3.2). However, it is evident that the conventional definition for the ductility factor cannot be applied to FRP reinforced concrete sections since FRP

Table 6.6: *Effect of reinforcement ratio on the percentage of the loss of stiffness in glass reinforced concrete with $\epsilon_{cu} = 0.005$*

ρ	$EI_1 \times 10^{13}$ ($N.mm^2$)	$EI_2 \times 10^{13}$ ($N.mm^2$)	loss of stiffness (%)
0.0025	0.304	0.253	17
0.0050	0.553	0.377	32
0.0100	0.996	0.584	41
0.0150	1.384	0.729	47
0.0200	1.738	0.840	52
0.0300	2.356	0.985	58

materials do not exhibit a yield point. For this reason, it has been stated that the concept of ductility may have to be revised for FRP reinforced structures (Naaman, 1993), and the necessary requirements of the codes for ductility control may need to be carefully reviewed for flexural sections.

Here a curvature pseudo-ductility factor, μ_ϕ , for FRP reinforced concrete beams is defined as the ratio of the slope of the first linear part to the slope of the second linear part of the previously proposed moment-curvature model (Fig. 6.1), i.e.

$$\mu_\phi = \frac{EI_1}{EI_2} \quad (6.1)$$

This factor is actually the ratio of the rigidity of the FRP reinforced section corresponding to moments within the first and the second segments of the bi-linear moment-curvature curve. Higher pseudo-ductility factor implies higher non-linearity in the flexural response of a structural member. This factor is calculated in Table 6.7 for different reinforcement ratios and different ϵ_{cu} values, and it ranges between 1.0 to 3.3.

Table 6.7: *Pseudo-ductility factor of glass or carbon reinforced concrete section with a given ρ and ϵ_{cu}*

ρ	CFRP		GFRP	
	$\epsilon_{cu} = 0.003$	$\epsilon_{cu} = 0.005$	$\epsilon_{cu} = 0.003$	$\epsilon_{cu} = 0.005$
0.0005	1.02	1.02	1.00	1.00
0.0010	1.03	1.03	1.03	1.03
0.0025	1.18	1.18	1.14	1.20
0.0050	1.32	1.80	1.20	1.47
0.0100	1.47	2.23	1.28	1.70
0.0150	1.58	2.56	1.36	1.90
0.0200	1.67	2.85	1.43	2.07
0.0300	1.85	3.30	1.51	2.39

6.6 Moment Redistribution

The proposed bi-linear model will account for some redistribution of stresses in indeterminate structures. In other words, when moment in a member exceeds the maximum moment in the first linear part of the model, its rigidity would become smaller than its earlier value and moment distribution within the member would no longer follow the elastic behaviour.

In order to illustrate the typical procedure for the analysis of an indeterminate beam based on the bi-linear moment-curvature model, a continuous two-span beam with concentrated loads at mid-spans (similar to the test beams in the current study) is selected. For both the negative and the positive moment regions of the beam, the bi-linear moment-curvature relationship with rigidities of EI_1 and EI_2 , respectively, for the first and second branches of the curve, is assumed. Let $EI_1/EI_2 = \beta$, and suppose that the moments corresponding to the end points of the first and second

parts of the moment-curvature curve are M_0 and M_u , respectively. Note that in the proposed model $M_0 = 0.8M_u$.

For the analysis of the beam due to symmetry only half of the beam needs to be considered. As long as the maximum moment at the middle support is less than M_0 , the entire beam will have rigidity EI_1 , hence an elastic moment of $3Pl/16$ will develop at that support. Once the support moment exceeds M_0 , for some parts of the beam the rigidity will change to EI_2 , therefore; the beam will begin to behave nonlinearly. Henceforth ($P > P_0 = 16M_0/3l$), the beam may be analysed by taking advantage of the principle of virtual force. The beam and its necessary bending moment diagrams for using that principle are illustrated in Fig. 6.2. In this figure, M_{xi} (Fig. 6.2b) represents the bending moment diagram when a unit moment is applied at the released left support, while M_{xp} (Fig. 6.2a) represents the bending moment diagram when a unit load acts at mid-span.

Let us assume that nonlinearity in the beam develops in two stages. In stage 1, it is assumed that part of the negative moment region with length a (the shaded area in Fig. 6.2c) has rigidity EI_2 while the maximum positive moment is still less than M_0 . Based on compatibility requirements

$$\delta_{11}M' + \delta_{10}P = 0 \quad (6.2)$$

where, δ_{11} and δ_{10} , respectively, are the rotations of the released middle support due to the applied unit moment and the unit load. The latter quantities can be calculated using

$$\delta_{11} = \int_l \frac{M_{xi} \cdot M_{xi}}{EI} dx \quad (6.3)$$

$$\delta_{10} = \int_l \frac{M_{xp} \cdot M_{xi}}{EI} dx \quad (6.4)$$

Using the moment diagrams in Fig. 6.2

$$\delta_{11} = \frac{1}{EI_2} \int_0^a \left(1 - \frac{x}{l}\right) \cdot \left(1 - \frac{x}{l}\right) dx + \frac{1}{EI_1} \int_a^l \left(1 - \frac{x}{l}\right) \cdot \left(1 - \frac{x}{l}\right) dx \quad (6.5)$$

Figure 6.2: Bending moment diagrams used for virtual force method

$$\delta_{10} = -\frac{1}{EI_2} \int_0^a \left(1 - \frac{x}{l}\right) \cdot \frac{x}{2} dx - \frac{1}{EI_1} \int_a^{l/2} \left(1 - \frac{x}{l}\right) \cdot \frac{x}{2} dx - \frac{1}{EI_2} \int_{l/2}^l \left(1 - \frac{x}{l}\right) \cdot \frac{l-x}{2} dx \quad (6.6)$$

Assuming $EI_2 = 1$ and $EI_1 = \beta$ ($\beta > 1.0$) and performing the integrations we obtain

$$\delta_{11} = \frac{l}{3} \left\{ \beta + \left(1 - \frac{a}{l}\right)^3 \cdot (1 - \beta) \right\} \quad (6.7)$$

$$\delta_{10} = \frac{a^2}{2} (1 - \beta) \cdot \left(\frac{1}{2} - \frac{a}{3l}\right) - \frac{l^2}{16} \quad (6.8)$$

Note that due to the assumption of $EI_1 = 1$, the values of δ_{11} and δ_{10} in the latter equations are only indicative of the relative displacements. However, the assumption does not affect the value of M' obtained from Eq. 6.2.

Based on linear distribution of the bending moment (Fig. 6.2c), it could be

written that

$$M' = \frac{(2M_0 + Pa)l}{2(l-a)} \quad (6.9)$$

Substituting Eq. 6.9 into Eq. 6.2 gives

$$P = -\frac{M_0 l}{l-a} / \left(\frac{\delta_{10}}{\delta_{11}} + \frac{al}{2(l-a)} \right) \quad (6.10)$$

During stage 1, if the negative moment at the fixed support does not reach the maximum flexural capacity of the section, M_u , and the positive moment under the load point exceeds M_0 , the rigidity of part of the positive moment region would drop to EI_2 . Let us call the latter situation stage 2. Suppose that the expansion of the central region of the beam with rigidity EI_2 from the load point towards the left and right are given by b and c , respectively (see shaded areas in Fig. 6.2d). Again taking advantage of the principles of virtual force and enforcing the compatibility requirements, after some manipulations the following expressions could be obtained

$$\delta_{11} = \frac{l\beta}{3} + \frac{l}{3}(1-\beta) \left\{ \left(1 - \frac{a}{l}\right)^3 - \left(1 - \frac{b'}{l}\right)^3 + \left(1 - \frac{c'}{l}\right)^3 \right\} \quad (6.11)$$

$$\delta_{10} = \frac{5\beta l^2}{48} - \frac{l^2}{6} - \frac{1-\beta}{2} \left\{ \frac{1}{3l} (a^3 - b'^3 - c'^3) - \frac{1}{2} (a^2 - b'^2) - c' (l - c') \right\} \quad (6.12)$$

where

$$b' = l/2 - b \quad (6.13)$$

$$c' = l/2 + c \quad (6.14)$$

Again, from linear representation of the bending moment diagram, a relationship analogous to Eq. 6.9 could be derived between P and a . Furthermore, the following expressions could be deduced for b and c

$$b = \left\{ \frac{Pl/2 - M' - 2M_0}{2(M' - M_0)} \right\} a \quad (6.15)$$

$$c = \left\{ \frac{P + 2M'/l}{P - 2M'/l} \right\} b \quad (6.16)$$

Based on the foregoing discussions and equations, the following strategy may be chosen for evaluating the reactions and bending moments (and in fact, the issue of

moment redistribution) of the two-span continuous beam with a bi-linear moment curvature relationship.

1. Choose a small value for a .
2. Calculate δ_{11} and δ_{10} from Eqs. 6.7 and 6.8.
3. Calculate P from Eq. 6.10.
4. Calculate M' from Eq. 6.9.
5. If
 - $M' > M_u$, select a smaller value for a and go to step 2.
 - $M' \approx M_u$ stop. The calculated P and M' from step 3 and 4 would be, respectively, the maximum load and the negative moment at the middle support. All other quantities can be calculated as usual.
 - $M' < M_u$ and $M_c = (Pl/4 - M'/2) < M_0$, select a higher value for a and go to step 2.
 - $M' < M_u$ and $M_c \geq M_0$, select a higher value for a and go to step 6.
6. Calculate b and c from Eqs. 6.15 and 6.16.
7. Calculate δ_{11} and δ_{10} from Eqs. 6.11 and 6.12.
8. Calculate P from Eq. 6.10.
9. Calculate M' from Eq. 6.9.
10. If
 - $M' < M_u$ and $M_c < M_u$, choose a higher value for a and go to step 6.
 - $(M' \approx M_u \text{ and } M_c < M_u)$ or $(M' < M_u \text{ and } M_c \approx M_u)$, stop. The calculated P and M' from step 8 and 9 would be, respectively, the maximum load and the negative moment at the middle support. All other quantities can be calculated as usual.

A computer program was developed based on the preceding algorithm and the percentages of redistribution of positive and negative moments were calculated for a

Table 6.8: *Pseudo-ductility factor and moment redistribution of GFRP reinforced concrete section with a given ρ and ϵ_{cu} for a two-span beam under concentrated load in mid-spans*

ρ	$\epsilon_{cu} = 0.003$			$\epsilon_{cu} = 0.005$		
	μ_ϕ	Redistribution (%) for:		μ_ϕ	Redistribution (%) for:	
		M^-	M^+		M^-	M^+
0.0005	1.00	0.0	0.0	1.00	0.0	0.0
0.0010	1.03	-0.7	0.4	1.03	-0.7	0.4
0.0025	1.14	-2.1	1.2	1.20	-2.8	1.7
0.0050	1.20	-2.8	1.7	1.47	-5.4	3.3
0.0100	1.28	-3.6	2.2	1.70	-7.3	4.4
0.0150	1.36	-4.4	2.6	1.90	-8.6	5.2
0.0200	1.43	-5.1	3.0	2.07	-9.6	5.8
0.0300	1.51	-5.8	3.5	2.39	-11.3	6.8

two-span beam under concentrated load in mid-spans, assuming the reinforcement ratios and the pseudo-ductility factors listed in Table 6.7. The calculated values of moment redistribution are shown in Table 6.8 and 6.9 for concrete sections reinforced with GFRP and CFRP, respectively. Note that in these tables, the percentage of moment redistribution has been taken as the percentage discrepancy between the bending moments obtained from the linear-elastic analysis and the nonlinear analysis.

6.7 Ductility of Tested Beams

6.7.1 Steel Reinforced Concrete Beams

The first two tested beams, i.e. Beams SR1 and SR2 were steel under-reinforced concrete beams, therefore; they showed highly ductile behaviour as expected. The

Table 6.9: *Pseudo-ductility factor and moment redistribution of CFRP reinforced concrete section with a given ρ and ϵ_{cu} for a two-span beam under concentrated load in mid-spans*

ρ	$\epsilon_{cu} = 0.003$			$\epsilon_{cu} = 0.005$		
	μ_ϕ	Redistribution (%) for:		μ_ϕ	Redistribution (%) for:	
		M^-	M^+		M^-	M^+
0.0005	1.02	-0.6	0.3	1.02	-0.6	0.3
0.0010	1.03	-0.7	0.4	1.03	-0.7	0.4
0.0025	1.18	-2.5	1.5	1.18	-2.5	1.5
0.0050	1.32	-4.0	2.4	1.80	-8.0	4.8
0.0100	1.47	-5.4	3.2	2.23	-10.5	6.3
0.0150	1.58	-6.4	3.8	2.56	-11.9	7.2
0.0200	1.67	-7.0	4.2	2.85	-12.8	7.7
0.0300	1.85	-8.3	5.0	3.30	-14.0	8.4

ductility of these beams could be observed in Fig. 5.11. From that figure, the “displacement ductility factor” may be calculated as $\mu_\Delta = \Delta_u / \Delta_y = 7$. Furthermore, in Fig. 5.20 large increase in displacement could be observed while the load marginally increased from $0.98P_{max}$ to P_{max} .

The moment-curvature relationship of a concrete section reinforced similarly to Beams SR1 and SR2 is shown in Fig. 6.3. In this figure, we observe the typical ductile behaviour of conventional under-reinforced concrete sections. Using Fig. 6.3, the “curvature ductility factor” of the section could be calculated as $\mu_\phi = \phi_u / \phi_y \approx 7$.

6.7.2 FRP Under-Reinforced Concrete Beams

For assessing the ductility of FRP under-reinforced concrete beams, i.e. Beams CRU1 and CRU2, the load-displacement curves in Figs. 5.12 and 5.13 will be considered. From these figures, it could be seen that after the maximum load, there

Figure 6.3: *Moment-curvature relationship for Beams SR1 and SR2*

is a sharp drop in the load without any change in the corresponding displacement. In other words, the failure is sudden and brittle, so the main sign of ductility which is warning before impending failure did not happen in the FRP under-reinforced concrete beams.

The moment-curvature relationship of an FRP under-reinforced concrete section, similar to Beams CRU1 and CRU2, is shown in Fig. 6.4. This figure is representative of the moment-curvature relationship of FRP under-reinforced concrete sections which do not show any sign of non-linear behaviour. In other words, the “curvature pseudo-ductility factor” as defined in Section 6.5 may be calculated as $\mu_\phi = EI_1/EI_2 = 1.02$, which shows almost no change in the stiffness of the section during the loading process.

Figure 6.4: *Moment-curvature relationship for Beams CRU1 and CRU2*

6.7.3 FRP Over-Reinforced Concrete Beams

To evaluate the ductility of the FRP over-reinforced concrete beams, the load-displacement curves of Beams CRO1 and CRO2 (Figs. 5.14 and 5.15) will be considered. From these figures it could be seen that up to the maximum load, there is no sign of impending failure, i.e. no significant variation in the slope of the load-displacement curve is noticeable. However, after the maximum load, a gradual drop in the load could be noticed, following which the load remains constant, or slightly increases, while the displacement increases more rapidly. In other words, the full failure does not occur until a noticeable displacement takes place with minor changes in the load, with the load actually dropping compared to its maximum value. This phenomenon may be better understood by considering Figs. 5.24 and

5.25. In Fig. 5.24 it could be seen that in Beam CRO1, when the load dropped from P_{max} to $0.92P_{max}$, the maximum displacement increased from 37 mm to 54 mm. In other words, with only 8% drop in the maximum load, the beam maximum displacement increased by 46%. Similarly it could be seen in Fig. 5.25 that when the applied load on Beam CRO2 dropped from P_{max} to $0.84P_{max}$, the maximum displacement of the beam increased from 38 mm to 70 mm. In other words, the beam experienced 84% increase in the maximum displacement with only 16% drop in the maximum load.

The foregoing behaviour could be regarded as ductility in indeterminate FRP reinforced concrete structures because it gives a reasonable warning of impending failure before the full collapse of any critical member in an indeterminate structure. In other words, if members in an indeterminate structure which possesses the foregoing characteristic reach their maximum capacities, a sudden and brittle failure would not happen. Instead, those members will undergo relatively large deformations with much lower stiffnesses while carrying lower load. Meanwhile, other members within the same structure will compensate for the lost carrying capacity of the critical members by sustaining higher loads. (Note that this is basically the process of redistribution of stresses between the various parts of the total structure). The noticeable variation in the displacement of the critical members may provide adequate warning of impending failure. It is recognized that ductility in the present context does not involve large plastic deformations and consequently does not imply the same notion as in conventional reinforced concrete members.

The moment-curvature relationship of an FRP over-reinforced concrete section, such as Beams CRO1 and CRO2, is shown in Fig. 6.5. The figure shows the trend towards non-linearity, especially at higher loads. The proposed curvature pseudo-ductility factor (Eq. 6.1) from Fig. 6.5 can be calculated as: $\mu_\phi = EI_1/EI_2 = 1.35$. This amount of ductile behaviour in the member permits a noticeable amount of redistribution of stresses and moments in an indeterminate structure. The latter

Figure 6.5: *Moment-curvature relationship for Beams CRO1 and CRO2*

could be verified using the algorithm described in Section 6.6 for the CFRP over-reinforced tested beams, which leads to -4.3% and $+2.6\%$ redistribution for the maximum negative and positive moments, respectively. Note that these values are within the range of the experimental values which will be presented in Table 6.13.

Based on the load-displacement behaviour of FRP over-reinforced concrete beams, a new definition may be given for “deflection pseudo-ductility factor” of FRP reinforced concrete flexural members, namely

$$\mu_{\Delta} = \frac{\Delta_2}{\Delta_1} \quad (6.17)$$

where, Δ_2 is the maximum displacement of the flexural member when load drops to $0.8P_{max}$ in the descending branch of the load-deflection curve, and Δ_1 is the

maximum displacement corresponding to the maximum load, P_{max} .

It is evident that higher values of the deflection pseudo-ductility factor implies higher increase in the displacement of the flexural member within the 20% drop of the maximum load, which provides a wider margin of safety and warning prior to complete failure of the structure. This factor is approximately $\mu_{\Delta} = 1.75$ for Beams CRO1 and CRO2, and 1.0 for Beams CRU1 and CRU2. The latter value explains lack of ductility and is in accordance with the observed sudden failure of Beams CRU1 and CRU2.

6.7.4 FRP Over-Reinforced Concrete Beams with Confinement

To evaluate the effect of confinement of compression concrete on the ductility of FRP over-reinforced concrete beams, consider the load-displacement curves of Beams CRCS1, CRCS2, CRCN1 and CRCN2 in Figs. 5.16 to 5.19, respectively. From these figures no positive effect of confinement of compression concrete could be seen on the ductile behaviour of the beams. From the aforementioned figures or from Figs. 5.26 to 5.29, it could be seen that for all beams with confined compression zone, except Beam CRCS1, the deflection pseudo-ductility factor, μ_{Δ} , is approximately the same as that of Beams CRO1 and CRO2. Therefore, in the case of the present tested beams the confinement of compression concrete did not increase the pseudo-ductility of FRP over-reinforced concrete beams.

One reason for the ineffectiveness of the confining reinforcement may be the rather small depth of the compression zone in the beam. Given the relative size of the grid aperture to the depth of the compression zone, the ineffectiveness of the confining reinforcement is not unexpected. It must be mentioned that the original design of the test beams was based on 50 MPa concrete, while the actual strength

of the concrete in the beams was found to be 70 MPa at the time of testing. This difference between the design and the actual strength caused further reduction in the depth of the compression zone at ultimate. Since others (Taniguchi et al, 1993) have reported that confinement of compression zone increases ductility, the concept needs to be further investigated.

6.8 Flexural Analysis of Test Beams

6.8.1 General

In this section, the theoretical flexural strength of all tested beams will be calculated, and the results will be compared to the corresponding experimental data. Furthermore, based on the measured reactions with load cells, the actual moments in the critical positive and negative moment regions under the maximum load and consequently the percentage of moment redistribution will be calculated. The relevant characteristics of the tested beams for present calculations are tabulated in Table 6.10. Note that the effective depth, d , in the table is based on the measurements taken after the failure of the beams.

6.8.2 Beams without Confinement

In the calculation of the flexural capacities, the equivalent rectangular stress block for concrete will be used in accordance with the code recommendations. Therefore, the reinforcement ratio at balanced condition, ρ_b , could be calculated as

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \times \frac{\epsilon_{cu}}{\epsilon_{cu} + f_y/E_s} \quad (6.18)$$

where, the extreme compression strain of concrete at ultimate, ϵ_{cu} , will be taken as 0.0035. When Eq. 6.18 is used for FRP reinforced concrete sections, f_y and E_s

Table 6.10: The characteristics of the tested beams

Beam Name	f_y (MPa)	f_u (MPa)	$E_s(E_f)$ (GPa)	b (mm)	d (mm)	$A_s(A_f)$ (mm ²)	f'_c (MPa)
SR1 and SR2	462	710	203.6	250	295	1500	77
CRU1 and CRU2	—	2550*	147.0*	250	300	92.2	77
CRO1 and CRO2	—	2550	147.0	250	275	502.6	77
CRCS1 and CRCS2	—	2550	147.0	250	280	497.3	70
CRCN1 and CRCN2	—	2550	147.0	250	280	497.3	70

*These values are based on the manufacturer's recommendations.

should be replaced by f_u and E_f , respectively.

If a section is under-reinforced, the depth of the equivalent rectangular stress block, a , can be calculated as:

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad (6.19)$$

and the flexural strength of the section at ultimate is

$$M_r = A_s f_y \left(d - \frac{a}{2} \right) \quad (6.20)$$

where A_s and f_y should be replaced by A_f and f_u , respectively, for FRP under-reinforced concrete sections. Note that in general the assumption of an equivalent rectangular stress block, with block parameters given by the code, is not applicable to FRP under-reinforced sections. Unlike conventional reinforced concrete where yielding of steel reinforcement generally ensures failure of concrete at ultimate, the lack of yielding in FRP may prevent concrete from reaching its maximum strain capacity. Accordingly, the concrete stress distribution in FRP under-reinforced could vary from linear to parabolic, while the code block parameters are derived on the basis of concrete reaching its maximum strain and the stress distribution being basically parabolic.

In the case of the present FRP under-reinforced beams, the depth of the neu-

tral axis at ultimate was very small and it is unlikely that the calculated strength would be substantially affected by the actual shape of the stress distribution in the compression zone.

For FRP over-reinforced concrete sections, using first principles and strain compatibility requirements, the following expressions could be used

$$a^2 + \left(\frac{\epsilon_{cu} A_f E_f}{0.85 f'_c b} \right) a - \frac{\epsilon_{cu} A_f E_f \beta_1 d}{0.85 f'_c b} = 0 \quad (6.21)$$

$$f_f = \epsilon_{cu} E_f \left(\frac{\beta_1 d - a}{a} \right) \quad (6.22)$$

$$M_r = A_f f_f \left(d - \frac{a}{2} \right) \quad (6.23)$$

where, f_f is the stress in tensile FRPs at ultimate.

Finally the maximum concentrated loads at mid-spans based on full elastic behaviour of the beam could be calculated as

$$P_e = 5.333 \frac{M_r^-}{l} \quad (6.24)$$

where l is the length of each span.

Taking advantage of Eqs. 6.18 to 6.24, using the information of Table 6.10 and assuming $\beta_1 = 0.65$, the calculations for the first six test beams (beams without confinement) has been carried out and the results are tabulated in Table 6.11.

6.8.3 Beams with Confinement

Although it was stated earlier that the confinement reinforcement did not noticeably affect the ductility of the beams, nevertheless in this section we will consider its effect on the ultimate strength of the beams. The effect of confinement in compression concrete could be taken into consideration by means of any acceptable model for

Table 6.11: *The results of calculations for unconfined test beams*

Beam Name	ρ_b	ρ	ρ/ρ_b	a (mm)	M_r (kN.m)	P_e (kN)
SR1 and SR2	0.0559	0.0203	0.36	42.35	189.76	337.35
CRU1 and CRU2	0.0028	0.0012	0.44	14.37	68.84	122.39
CRO1 and CRO2	0.0028	0.0073	2.61	45.83	189.06	336.11

confinement, e.g. the model proposed by Saatcioglu and Razvi (1992) described in Section 3.5. A very important problem in beams with confined compression concrete is their balanced condition. The balanced reinforcement ratio for beams with confined concrete is not the same as that of unconfined beams. Here, an attempt will be made to account for the effect of confinement on the balanced reinforcement ratio and on the ultimate strength.

It is evident that confinement of compression concrete is only useful for over-reinforced concrete beams, while over-reinforcing could be assessed based on conventional balanced reinforcement ratio for beams with unconfined concrete, i.e. Eq. 6.18. However, such a section could be under-reinforced or over-reinforced after spalling of cover concrete, therefore a new balanced condition would emerge. To approach the latter condition it would be reasonable to assume that as long as the strain of the extreme compression fibre of concrete is less than the ultimate strain of unconfined concrete (i.e. $\epsilon_c \leq \epsilon_{cu}$), there would be no major difference between the behaviour of a confined and an unconfined beam. However, once the spalling of the cover concrete takes place, the confined concrete with its own characteristics introduces a new set of conditions which require reconsideration of the balanced condition.

Suppose, f'_c = the compressive strength of unconfined concrete, ϵ_{cu} = the maximum permitted compressive strain for unconfined concrete, f'_{cc} = the compressive

Figure 6.6: *Strain diagram at balanced condition for a rectangular beam with confined compression concrete*

strength of confined concrete, ϵ_{ccu} = the maximum permitted compressive strain for confined concrete, $K_s = f'_{cc}/f'_c$ = the factor of strength enhancement (strength gain), b_c = width of confined concrete, and d' = depth of cover on confined compression concrete. Fig. 6.6 shows the section of the beam and the strain distribution at balanced condition after spalling of the cover concrete.

From equilibrium of forces on the section

$$A_s f_y = 0.85 K_s f'_c b_c \beta_1 c_{cb} + 0.85 f'_c (b - b_c) \beta_1 c_b \quad (6.25)$$

and from linear strain distribution

$$c_{cb} = (d - d') \frac{\epsilon_{ccu}}{\epsilon_y + \epsilon_{ccu}} \quad (6.26)$$

$$c_b = c_{cb} \frac{\epsilon_{cu}}{\epsilon_{ccu}} \quad (6.27)$$

Substituting Eqs. 6.26 and 6.27 into Eq. 6.25 and simplifying the expression, the following expression is obtained for $\rho_{b,con.}$, the balanced reinforcement ratio of a

rectangular beam with confined concrete.

$$\rho_{b,con.} = 0.85\beta_1 \frac{f'_c}{f_y} \frac{d-d'}{bd} \left[K_s b_c + (b-b_c) \frac{\epsilon_{cu}}{\epsilon_{ccu}} \right] \frac{\epsilon_{ccu}}{\epsilon_y + \epsilon_{ccu}} \quad (6.28)$$

If the confined section is under-reinforced compared to a section with no concrete cover beyond the confining reinforcement, ($\rho_{con.} = A_s/b(d-d') \leq \rho_{b,con.}$), the section could be analysed using Eqs. 6.19 and 6.20 but using f'_{cc} and $(d-d')$ instead of f'_c and d . However, if the confined section is still over-reinforced even after spalling of cover concrete, ($\rho_{con.} > \rho_{b,con.}$), the following expression could be written using the compatibility and equilibrium conditions:

$$A_s f_s = 0.85 f'_c a_c \left[K_s b_c + (b-b_c) \frac{\epsilon_{cu}}{\epsilon_{ccu}} \right] \quad (6.29)$$

Using the strain diagram and Hooke's law

$$f_s = E_s \epsilon_{ccu} \frac{\beta_1 (d-d') - a_c}{a_c} \quad (6.30)$$

Substituting Eq. 6.30 into 6.29

$$a_c^2 + \frac{A_s E_s \epsilon_{ccu}}{0.85 f'_c [K_s b_c + (b-b_c) \cdot \epsilon_{cu}/\epsilon_{ccu}]} [a_c - \beta_1 (d-d')] = 0 \quad (6.31)$$

Once the parameter a_c is calculated by solving Eq. 6.31, the ultimate flexural strength of the section can be calculated from the following expression

$$M_r = A_s f_s \left[(d-d') - \frac{a_c}{2} \right] \quad (6.32)$$

Notice that if the longitudinal reinforcement is FRP, the parameters A_s , f_y , f_s , ϵ_y and E_s , respectively, should be replaced by A_f , f_u , f_f , ϵ_{fu} and E_f in Eqs. 6.25 to 6.32.

To account for the effect of confinement in Beams CRCS1, CRCS2, CRCN1, and CRCN2, the model of Saatcioglu and Razvi for stress-strain relationship of confined concrete (as discussed in Chapter 3) is selected. Using the information in Table 6.10

and Eqs. 3.21 to 3.30 and considering Figure 3.13, for the aforementioned confined test beams the details are as follows:

$$f'_{co} = 70 \text{ MPa}, \quad S = 50 \text{ mm}, \quad \text{and} \quad b_c = 200 \text{ mm}$$

$$f'_{cc} = f'_{co} + k_1 f_{le}$$

$$f_{le} = k_2 f_l$$

$$f_l = \Sigma A_s f_{yt} \sin \alpha / (S b_c)$$

where α = the angle between the transverse reinforcement and the side of section and S = spacing of transverse reinforcement. Since 5 ribs of NEFMAC grids are located transversely in confined compression zone and the section area of each rib is 17.5 mm^2

$$\Sigma A_s = \Sigma A_N = 5 \times 17.5 = 87.5 \text{ mm}^2$$

Furthermore, in Figs. 5.81 and 5.86, it can be observed that the maximum recorded strain in the confining ribs of NEFMAC grids corresponds to stresses of 300 to 400 MPa. Therefore a value of 300 MPa (which is 25% of the ultimate stress of NEFMAC) seems to be reasonable for confining NEFMAC reinforcement. Hence;

$$f_l = (87.5 \times 300) / (50 \times 200) = 2.625 \text{ MPa}$$

$$k_1 = 6.7(f_l)^{-0.17} = 5.686$$

$$k_2 = 0.26 \sqrt{(b_c/S) \cdot (b_c/S_1) \cdot (1/f_l)} \leq 1.0$$

$$k_2 = 0.26 \sqrt{(200/50) \cdot (200/50) \cdot (1/2.625)} = 0.642$$

$$f_{le} = K_2 f_l = 0.642 \times 2.625 = 1.685 \text{ MPa}$$

$$f'_{cc} = 70 + 5.686 \times 1.685 = 79.6 \text{ MPa}$$

$$\epsilon_1 = \epsilon_{01}(1 + 5K)$$

$$K = k_1 f_{le} / f'_{co} = 9.58/70 = 0.137$$

$$\epsilon_1 = 0.002(1 + 5 \times 0.137) = 0.00337$$

$$\epsilon_{85} = 260 \rho \epsilon_1 + \epsilon_{085}$$

$$\rho = \Sigma A_s / S(b_{cx} + b_{cy}) = (7 \times 17.5) / 50(200 + 50) = 0.0098$$

$$\epsilon_{085} \approx 0.0038$$

$$\epsilon_{85} = 260 \times 0.0098 \times 0.00337 + 0.0038 = 0.0124$$

- Check mode of failure after spalling of cover concrete

cover concrete: $d' = 20mm$

$$K_s = f'_{cc}/f'_c = 79.6/70 = 1.137$$

Suppose: $\epsilon_{cu} = 0.0035$ and $\epsilon_{ccu} = \epsilon_{085} = 0.0124$ (worst condition),

from Eq. 6.28: $\rho_{b,con.} = 0.00567$

$$\rho_{con.} = 497.3/(250 \times 210) = 0.0095$$

Since $\rho_{con.} > \rho_{b,con.}$, the section remains over-reinforced even after spalling of the cover concrete.

For the analysis of the section two different cases, i.e. before and after spalling of cover concrete, may be considered. The case with the higher bending moment capacity would give the maximum capacity of the beam.

a) Before spalling of cover concrete

In this case, the beam behaves as an unconfined concrete beam, therefore Eqs. 6.21 to 6.23 govern, which yield $a = 48mm$, $f_f = 1436MPa$, and $M_r = 182.8kN.m$.

b) After spalling of cover concrete

Suppose that the maximum strain of confined compression concrete is 0.005, even though such a high compressive strain at a level of 20 mm from the bottom or top face of the beam was not measured in the actual tests, then from Eq. 6.31: $a_c = 52.28mm$, from Eq. 6.30: $f_f = 1641mm$, and from Eq. 6.32: $M_r = 190.9kN.m$.

Therefore, the maximum bending moment capacity of the section is $M_r = 190.9MPa$ and the maximum concentrated loads on mid-spans assuming full elastic behaviour would be $P_e = 5.333M_e^-/l = 339.3kN$.

6.9 Measured versus Calculated Bending Moments and Failure Loads

In the preceding section, the bending moment capacities and the maximum loads based on full elastic behaviour of the beams were calculated. However, for steel reinforced concrete beams, a full plastic behaviour is expected, therefore; the failure load may be predicted using Eq. 4.7 which gives $P_p = 1.125P_e$. Furthermore, for FRP over-reinforced concrete beams semi-plastic behaviour is expected, therefore their failure load may be predicted assuming the increase in its load capacity beyond the elastic limit to be 50% of the corresponding increase that could be achieved under full plastic condition, (i.e. $P_p = 1.06P_e$). The FRP under-reinforced concrete beams did not exhibit any plastic behaviour, hence their failure load can be predicted by calculating their maximum load under full elastic condition.

The maximum measured bending moments (over the middle support or under the load points) of test beams, based on the reactions of the end supports, were calculated and are given in Table 6.12. Furthermore, the measured and predicted failure loads are given in the same table, and a comparison between the measured and the calculated bending moment capacities and failure loads is made. In Table 6.12, R_A stands for recorded end support reaction, and subscripts “*exp*” and “*anal.*”, respectively, stand for experimental and analytical results.

Results of Table 6.12 show that for Beam SR2, the experimental failure load and its corresponding maximum bending moment is about 6% more than the corresponding predicted failure load (based on full plastic behaviour) and bending moment capacity. The difference may be attributed to ignoring strain-hardening of reinforcing steel in the calculations.

The table shows a large discrepancy between the experimental and the analytical

Table 6.12: A comparison between measured and calculated moments and failure loads

Beam Name	R_A (kN)	$P_{u,exp}$ (kN)	$M_{max,exp}$ (kN.m)	$P_{u,anal.}$ (kN)	$M_{u,anal.}$ (kN.m)	$\frac{P_{u,exp}-P_{u,anal.}}{P_{u,anal.}}$ (%)	$\frac{M_{max,exp}-M_{u,anal.}}{M_{u,anal.}}$ (%)
SR2	134.8	402.5	202.2	379.5	189.8	6.0	6.5
CRU1	46.0	146.0	81.0	122.4	68.8	19.3	17.7
CRU2	53.5	170.3	95.0	122.4	68.8	39.1	38.1
CRO1	112.3	349.3	187.1	356.3	189.1	-2.0	-1.1
CRO2	114.8	356.3	190.1	356.3	189.1	0.0	0.5
CRCS1	104.2	325.0	174.9	359.7	190.9	-9.6	-8.4
CRCS2	113.9	352.5	187.1	359.7	190.9	-2.0	-2.0
CRCN1	121.1	375.3	199.7	359.7	190.9	4.3	4.6
CRCN2	118.2	365.4	193.5	359.7	190.9	1.6	1.4

results of FRP under-reinforced concrete beams, i.e. Beams CRU1 and CRU2. The difference could be attributed to the difference between the actual tensile strength of CFRP longitudinal rods and the recommended tensile strength of 2550 MPa. As mentioned in Chapter 4, we could not determine the actual strength of the CFRP rods and we used the recommended value of 2550 MPa by the manufacturer. However, this value was recommended as a minimum expected tensile strength of CFRP rods. It is important to mention that in a recent study conducted by Fam et al (1997), the tensile strength for Leadline CFRP bars (the same as the longitudinal reinforcement in the current study) has been taken as 2950 MPa. Fortunately the preceding difference in the tensile strength of longitudinal reinforcement does not affect the strength calculations for FRP over-reinforced concrete beams.

The results show a very good agreement between the experimental and the analytical failure loads and moments for FRP over-reinforced concrete beams (without confinement), i.e. Beams CRO1 and CRO2. It means that the application of conventional calculations of ultimate flexural capacity of reinforced concrete beams is

applicable to FRP over-reinforced concrete beams.

Results of Table 6.12 show minor differences between the experimental and the predicted failure loads and moments for all the FRP over-reinforced beams with confined concrete except Beam CRCS1, which inexplicably failed prematurely at a load equal to the elastic load of the unconfined beam. It means that the selected model for stress-strain relationship of confined concrete is suitable for the behaviour of compression zone of concrete beams with confinement.

6.10 Moment Redistribution

For evaluating the amount of moment redistribution in the test beams, the measured positive and negative bending moments are tabulated in Table 6.13. These moments are actually calculated using the measured reaction values. The table also contains the calculated positive and negative moments based on elastic analysis (which means without any redistribution) and based on the maximum measured loads. In other words, the calculated elastic moments in the table, M_e^- and M_e^+ are, respectively, equal to $0.188P_{u,exp}l$ and $0.156P_{u,exp}l$, where l is the span length.

The results in Table 6.13 show that Beam SR2 reached its maximum expected redistribution capacity. For the FRP over-reinforced concrete beams (with or without confinement), it can be observed that about 40% to 50% of the maximum possible redistribution took place compared to a conventional under-reinforced concrete beam. However, the effect of confinement on redistribution is not noticeable from the results. Finally, the results show negligible redistribution of moments for FRP under-reinforced concrete beams.

Table 6.13: *Moment redistribution percentages in the test beams*

Beam Name	$M_{u,exp}^+$ (kN.m)	$M_{u,exp}^-$ (kN.m)	M_e^+ (kN.m)	M_e^- (kN.m)	Moment Redistribution (%)	
					$\frac{M_{u,exp}^+ - M_e^+}{M_e^+}$	$\frac{M_{u,exp}^- - M_e^-}{M_e^-}$
SR2	202.2	-199.4	188.4	-227.0	7.2	-12.2
CRU1	69.0	-81.0	68.3	-82.3	1.0	-1.6
CRU2	80.2	-95.0	79.7	-96.0	0.6	-1.0
CRO1	168.5	-187.1	163.5	-197.0	3.1	-5.0
CRO2	172.2	-190.1	166.7	201.0	3.3	-5.4
CRCS1	156.3	-174.9	152.1	-183.3	2.8	-4.6
CRCS2	170.9	-187.1	165.0	-198.8	3.6	-5.9
CRCN1	181.7	-199.7	175.6	-211.7	3.5	-5.7
CRCN2	177.3	-193.5	171.0	-206.1	3.7	-6.1

6.11 Summary and Conclusion

Although FRP materials have a linear stress-strain relationship, FRP reinforced concrete sections may exhibit ductile behavior. Such ductile behavior will become more evident if the amount of reinforcement is increased. However, for an FRP under-reinforced concrete section, the non-linearity in the moment-curvature curve is too weak to permit ductility. On the other hand, for FRP over-reinforced sections, the non-linearity in the moment-curvature curves can be clearly seen. Accordingly, for such sections, some degree of ductile behaviour is expected. However, the traditional definition of ductility factor for steel reinforced concrete sections cannot be applied to FRP reinforced concrete sections, even though the main criteria of ductility are satisfied.

A simplified model for moment-curvature relationship of FRP reinforced concrete sections was presented based on analytical studies. According to this model,

the moment-curvature curve may be assumed to comprise two linear segments. To draw the proposed moment-curvature relationship for a given section, first the resisting moment and curvature of the section at ultimate need to be calculated, followed by the curvature calculation corresponding to 80% of the ultimate moment.

The ratio of the slope of the second linear part to the slope of first linear part of the model is defined as the curvature pseudo-ductility factor. Higher amounts of pseudo-ductility factor imply more ductile behavior for an FRP reinforced concrete section.

Such bi-linear moment curvature relationship will cause non-linear behaviour in the whole structure. In other words, redistribution of stresses and moments would be an inevitable consequence of such behaviour. To analyze structures composed of members with bi-linear stiffness, the necessary relations were derived for the analysis of a two-span continuous beam, based on the virtual force method. An algorithm was proposed to use the foregoing equations for the assessment of redistribution in such beams.

It is recommended that a reinforcement ratio between $2\rho_b$ to $3\rho_b$ be selected for FRP reinforced flexural members. This amount of reinforcement seems to provide adequate ductility to the member. Notice that the proposed reinforcement ratio is less than half of the normal reinforcement ratio in ordinary flexural steel reinforced concrete sections.

Results of the conducted experimental work were analysed in this chapter with emphasis on the issues of ductility and moment redistribution. The procedure for evaluating the bending moment capacity of concrete sections reinforced with FRP rebars was shown and the strategy for analysing sections with confinement was presented. Good agreement between the predicted and the experimental bending moment capacities and the failure loads was observed in most of the cases.

Acceptable pseudo-ductile behaviour was observed for all FRP over-reinforced concrete test beams. A new definition for “deflection pseudo-ductility factor” was proposed. Almost 50% of the maximum possible redistribution of a fully ductile response was observed for all the FRP over-reinforced concrete beams. The confinement of the compression concrete did not noticeably affect redistribution but it increased the ultimate capacity or failure load compared to its expected capacity without consideration of confinement.

Full redistribution was experienced for conventional under-reinforced continuous concrete beams. However, it was observed that redistribution of moments could occur from the early stages of loading and not necessarily right after the first yield of reinforcing steel and at the first critical section.

Chapter 7

Shear Strength of FRP Reinforced Concrete Beams

7.1 General

The shear behaviour of FRP reinforced concrete beams and columns cannot be evaluated using existing code expressions for steel reinforced concrete members because the latter are predicated on the assumption of stress redistribution due to yielding of longitudinal and transverse reinforcement. However, FRP materials do not yield and due to their lower modulus, shear cracks in FRP reinforced concrete may be wider, leading to reduction in shear resistance due to the aggregate interlock mechanism. The lack of strong dowel action in FRP leads to a further reduction in the shear resistance of FRP reinforced concrete.

Despite these difficulties, recently the Building Research Institute (BRI, 1995) of Japan Ministry of Construction and the Japan Society of Civil Engineers (JSCE, 1995) each recommended some equations for evaluating the shear strength of FRP

reinforced concrete beams and columns. In arriving at those recommendations, the detailed basis of their development was not provided in the English language literature. The purpose of this study is to examine the validity of the recommended expressions by comparing their predictions with available experimental data. Finally, based on the ACI procedure for evaluating the shear strength of conventional reinforced concrete beams, a new expression is proposed here for shear strength of FRP reinforced concrete beams. The predictions of the proposed method are compared with the results of the BRI and the JSCE methods.

The shear resistance of the FRP reinforced concrete test beams in the current study will be evaluated using the aforementioned guidelines of BRI and JSCE methods. Furthermore, the proposed method will be used for evaluating the shear resistance of the test beams, and the results of different methods will be compared.

Although this chapter is basically focusing on the shear behaviour of FRP reinforced concrete beams, the shear behaviour of the test beams reinforced with steel, i.e. Beams SR1 and SR2, will be also evaluated using the current code procedure. Furthermore, the results will be compared with the experimental values and the differences will be discussed.

7.2 Current Guidelines

7.2.1 The BRI Method

According to BRI (1995), the ultimate shear strength (V_u) of FRP reinforced concrete beams and columns is given by

$$V_u = \text{Min}\{0.8V_{u1}, 0.9V_{u2}\} \quad (7.1)$$

where

$$V_{u1} = bj \left\{ \frac{0.115k_u k_p (\sigma_B + 180)}{M/Qd + 0.12} + 2.7\sqrt{\rho_w \sigma_w} \right\} \quad (7.2)$$

$$V_{u2} = bj \left\{ \frac{0.115k_u k_p (\sigma_B + 180)}{M/Qd + 0.12} + 2.7\sqrt{\rho'_w \sigma_w} \right\} \quad (7.3)$$

$$k_p = 0.82(100\rho_t E_f / E_s)^{0.23} \quad (7.4)$$

$$\rho'_w = (E_f / E_s) \rho_w \quad (7.5)$$

b = beam (column) width, j = internal lever arm of the section (and may be taken equal to $7/8$ of effective depth, d), k_u = correction factor dependent on d and may be taken as 0.72 , k_p = correction factor dependent on ρ_t , ρ_t = tensile reinforcement ratio, M/Qd = shear span ratio where M is bending moment and Q is shear force, ρ_w = shear reinforcement ratio, ρ'_w = equivalent shear reinforcement ratio, σ_B = compressive strength of concrete and σ_w = tensile strength of shear reinforcement at bend section. Notice that in the preceding expressions force is measured in kg and length in cm .

7.2.2 The JSCE Method

According to JSCE (1995), the design shear capacity, V_{ud} , of FRP reinforced concrete beams, is the smaller of V_{ud1} and V_{ud2} , where V_{ud1} is calculated as follows:

$$V_{ud1} = V_{cd} + V_{ad} + V_{sd} \quad (7.6)$$

$$V_{cd} = \beta f'_{cd} X_e b_w \quad (7.7)$$

$$\beta = 0.2(\sigma'_n / f'_{cd})^{0.7} \quad (7.8)$$

$$X_e = \left\{ 1 - 0.8(\rho_{fw} E_{fw})^{-0.2} \right\} \left\{ 1 + (\sigma'_n / f'_{cd})^{0.7} \right\} X \quad (7.9)$$

$$\rho_{fw} = A_{fw} / (b_w s_f) \quad (7.10)$$

$$V_{ad} = \beta_p \beta_{pE} f'_{cd}{}^{1/3} (h - X_e) b_w \quad (7.11)$$

$$\beta_p = 1 - 5(\sigma'_n/f'_{cd}) \geq 0.0 \quad (7.12)$$

$$\beta_{pE} = 0.24 \left\{ \frac{\rho_f E_f + 10\rho_{fw} E_{fw}}{5000k} + 0.66 \right\} \geq 0.40 \quad (7.13)$$

$$\rho_f = A_f/(b_w d) \quad (7.14)$$

$$k = 1 - (\sigma'_n/f'_{cd})^{0.1} \quad (7.15)$$

$$V_{sd} = A_{fw} E_{fw} \epsilon_{fw} (h - X_e) / (\tan \theta_{cr} S_f) \quad (7.16)$$

$$\theta_{cr} = 45 \left\{ 1 - (\sigma'_n/f'_{cd})^{0.7} \right\} \quad (7.17)$$

$$\epsilon_{fw} = 0.0001 \sqrt{f'_{cd}} \sqrt{\frac{\rho_t E_f}{\rho_{fw} E_{fw}}} \left[1 + \left(\frac{\sigma'_n}{f'_{cd}} \right)^{0.2} \right] \quad (7.18)$$

where, V_{cd} = design shear capacity provided by the un-cracked concrete portion of the section, V_{ad} = design shear capacity of diagonally cracked portion of the section, V_{sd} = design shear capacity provided by shear reinforcement, f'_{cd} = design compressive strength of concrete, b_w = web width, σ'_n = axial compressive stress, X_e = depth of compressive field of concrete at ultimate limit state, A_{fw} = total amount of area of shear reinforcement over the interval S_f , E_{fw} = modulus of elasticity of shear reinforcement, X = elastic neutral axis depth of cracked transformed section, A_f = area of tensile reinforcement, E_f = modulus of elasticity of tensile reinforcement, θ_{cr} = angle of diagonal cracks, and ϵ_{fw} = ultimate strain of shear reinforcement. Note that in Eqs. 7.6 to 7.18, force is measured in N and length in mm .

The quantity V_{ud2} is the design shear capacity assuming shear reinforcement failure, i.e. diagonal tension failure, and is calculated as follows:

$$V_{ud2} = V_{co} - \beta_m(V_{co} - V_{cd}) + \beta_m V_{ad} + \beta_m V_{sd} \quad (7.19)$$

$$V_{co} = \beta f'_{cd} X_o b_w + \beta_p \beta_{pE} f'_{cd}^{1/3} (h - X_o) b_w \quad (7.20)$$

$$\beta_{pE} = 0.24 \left\{ \frac{\rho_f E_f}{5000k} + 0.66 \right\} \geq 0.40 \quad (7.21)$$

$$\beta_m = f_{bud} / (E_{fw} \epsilon_{fw}) \quad (7.22)$$

$$X_o = \left\{ 1 + (\sigma'_n/f'_{cd})^{0.7} \right\} X \quad (7.23)$$

where, X_o = depth of compressive field of concrete in the state of occurrence of diagonal cracks, and f_{bud} = tensile strength of shear reinforcement at bend section.

7.3 Selected Experimental Beams

Twenty two beams from experimental works carried out by various researchers were selected. The detailed dimensions and material properties of the selected beams are given in Table 7.1. In this table, the reinforcement type is identified by G = glass, S = steel, and C = carbon. Also, F_u = ultimate strength of FRP or yield stress of steel, L = span length, E = elastic modulus, A_f (A_s) = area of longitudinal (transverse) reinforcement, h = section height, and b = section width. Where the required characteristics of the beams and/or the materials were not specified, some reasonable values were assumed.

Complete details of the selected beams can be found in the work of the original investigators as follows. Beams 1 to 3: Faza (1991), Beams 4 and 5: Saadatmanesh (1994) and Saadatmanesh and Ehsani (1991), Beams 6 to 9: Vijay et al (1996), Beams 10 to 13: Al-Salloum et al (1996), Beams 14 and 15: Alsayed et al (1996), and Beams 16 to 22: Zhao et al (1995).

7.4 Predicted versus Experimental Shear Capacity

The maximum shear force at failure for each beam was extracted from the experimental data and is denoted by $V_{u,exp}$ in Table 7.2. It should be pointed out that for each beam which failed in flexure, the latter force is less than its ultimate shear resistance, but the actual magnitude of the ultimate shear resistance cannot be

Table 7.1: Characteristics of selected beams

No.	b mm	h mm	d mm	L/d	f' _c MPa	Longitudinal Reinforcement				Shear Reinforcement				
						Type	A _f mm ²	F _u MPa	E GPa	Type	S mm	A _v mm ²	F _u MPa	E GPa
1	152.4	305.0	279.4	3.274	51.8	2Gϕ 25.2	1006.5	552	48.3	Gϕ9.5	152	141.9	552	48.3
2	152.4	305.0	279.4	3.274	44.9	2Gϕ 25.2	1006.5	552	48.3	Gϕ9.5	152	141.9	552	48.3
3	152.4	305.0	279.4	3.274	34.5	2Gϕ 21.9	774.2	552	48.3	Sϕ6.3	152	64.52	400	200.0
4	205.0	450.0	405.0	3.20	31.25	4Gϕ 18.8	1290	1179	54.2	Sϕ10	100	157	414	200.0
5	205.0	450.0	405.0	3.20	30.0	2Sϕ 22	380	480	200.0	Gϕ10	300	157	1179	54.2
6	150.0	300.0	264.5	1.89	44.8	2Gϕ 19	567	655	54.2	Gϕ9.5	101.6	142	655	54.2
7	150.0	300.0	264.5	1.89	44.8	2Gϕ 19	567	655	54.2	Gϕ9.5	152.4	142	655	54.2
8	150.0	300.0	264.5	1.89	31.0	2Gϕ 12.7	254	655	54.2	Gϕ9.5	101.6	142	655	54.2
9	150.0	300.0	264.5	1.89	31.0	2Gϕ 12.7	254	655	54.2	Gϕ9.5	152.4	142	655	54.2
10	200.0	210.0	157.5	7.937	31.3	4Gϕ 19	1134	700	35.63	Sϕ8	120	100	286	200.0
11	200.0	260.0	210.7	5.933	31.3	4Gϕ 12.7	507	886	43.37	Sϕ8	120	100	286	200.0
12	200.0	300.0	247.5	5.051	40.7	2Gϕ 19	567	700	35.63	Sϕ8	120	100	286	200.0
13	200.0	250.0	197.5	6.329	40.7	4Gϕ 19	1134	700	35.63	Sϕ8	120	100	286	200.0
14	200.0	360.0	311.0	3.23	35.53	3Gϕ 19	851	565	35.63	Gϕ6.35	150	63.3	565	35.63
15	200.0	360.0	311.0	3.23	39.55	3Sϕ 16	603	400	200.0	Gϕ6.35	150	63.3	565	35.63
16	150.0	300.0	250.0	3.0	34.3	2Cϕ 19	568	1124	105.0	Gϕ6	90	56.5	1100	39.0
17	150.0	300.0	250.0	3.0	34.3	4Cϕ 19	1136	1124	105.0	Gϕ6	90	56.5	1100	39.0
18	150.0	300.0	250.0	3.0	34.3	4Cϕ 19	1136	1124	105.0	Cϕ6	90	56.5	1300	100.0
19	150.0	300.0	250.0	3.0	34.3	3Cϕ 19	852	1124	105.0	Gϕ6	90	56.5	1100	39.0
20	150.0	300.0	250.0	3.0	34.3	2Cϕ 19	568	1124	105.0	Sϕ6	90	63.4	368	200.0
21	150.0	300.0	250.0	2.0	34.3	2Cϕ 19	568	1124	105.0	Sϕ6	90	63.4	368	200.0
22	150.0	300.0	250.0	4.0	34.3	2Cϕ 19	568	1124	105.0	Sϕ6	90	63.4	368	200.0

deduced from the experimental data. On the other hand, for beams which failed in shear, $V_{u,exp.}$ represents their ultimate shear resistance. To distinguish between the two modes of failure, in Table 7.2 the beams failing in flexure are separated from those failing in shear. For the beams which failed in flexure, if a method predicted the shear capacity to be larger than $V_{u,exp.}$, we cannot ascertain whether the method is safe or not, therefore we simply indicate the particular method's prediction to be "inconclusive". On the other hand, if a method predicted a shear capacity less than the observed $V_{u,exp.}$, we indicate the ratio of the experimental to the observed value. Clearly, in the latter case the predicted capacity is "conservative".

In Table 7.2 the predicted shear capacities of the selected beams by the BRI and JSCE methods are given. Observe that the calculated values are based on the bend strength of the stirrup being 50% of the tensile strength of the straight bar. In the literature (Mochizuki et al 1993) it is stated that the bend strength can vary from 30 - 70% of the straight bar strength. In the present calculations, however, regardless of the value of the bend strength, the calculated shear capacities were governed by diagonal compression failure, which is not affected by stirrup strength.

The results in Table 7.2 show that there is a significant difference between the observed experimental capacities and the corresponding values calculated by the BRI method. Note that 11 of the 22 beams failed in flexure which means that their shear strength was higher than the $V_{u,exp.}$ values recorded in Table 7.2. The BRI method consistently underestimated the capacity of these beams as well as of all the other beams which failed in shear. Due to the relatively high conservatism inherent in this method, designs based on this method will be safe but not necessarily economical.

For the beams that failed in flexure, the JSCE method correctly predicted higher shear capacity than the flexural capacity. For the 11 beams that failed in shear, however, for 7 of them this method overestimated the shear capacity. Accordingly, in its present form this method may lead to unsafe design.

7.5 Proposed Method

The ACI method for evaluating the ultimate shear strength V_u of conventional reinforced concrete is the basis of the proposed method. That is

$$V_u = V_c + V_s \quad (7.24)$$

$$V_c = 0.166\sqrt{f'_c}b_wd \quad (7.25)$$

$$V_s = \frac{A_v f_v d}{S} \quad (7.26)$$

where, V_c = shear capacity provided by concrete at ultimate state, and V_s = shear capacity provided by shear reinforcement at ultimate state, b_w = section web width, d = effective depth, A_v = area of shear reinforcement over the interval S , and f_v is the stress in shear reinforcement at ultimate state. For FRP reinforced concrete we propose that f_v be calculated from

$$f_v = \frac{f_u}{\alpha\phi} \quad (7.27)$$

f_u = tensile strength of shear reinforcement (MPa), ϕ is the diameter of shear reinforcement (mm), and α ($1/mm$) is a coefficient. For $f_u \leq 2000$ MPa , α is given by

$$\alpha = \frac{0.27}{1 - (f_u - 400)/2000} \quad (7.28)$$

Note that α is an empirical coefficient and was obtained by fitting some of the experimental data with the prediction of Eq. 7.27.

The proposed method was used to evaluate the shear strength of the experimental beams which failed in shear. The results are given in Table 7.2 and are compared to the results of the BRI and the JSCE methods. Note that except for beam 14,

Table 7.2: Predicted versus experimental shear strength of FRP reinforced beams

Beam No.	$V_{u,exp.}$ (kN)	$V_{u,BRI}$ (kN)	$V_{u,JSCE}$ (kN)	$V_{u,prop.}$ (kN)	$\frac{V_{u,exp.}}{V_{u,BRI}}$	$\frac{V_{u,exp.}}{V_{u,JSCE}}$	$\frac{V_{u,exp.}}{V_{u,prop.}}$	Observed Mode of Failure
1	89.1	66.5	112.2	102.7	1.34	Incon.	Incon.	Flexure
2	81.3	62.4	106.0	99.2	1.30	Incon.	Incon.	Flexure
3	61.8	52.1	244.2	69.4	1.19	Incon.	Incon.	Flexure
4	190.0	130.6	561.6	173.9	1.45	Incon.	1.09	Flexure
5	135.0	104.0	122.0	132.0	1.30	1.11	1.02	Flexure
10	54.0	35.7	167.4	47.6	1.51	Incon.	1.13	Flexure
11	80.0	39.1	193.5	63.7	2.05	Incon.	1.26	Flexure
12	95.0	48.2	245.3	81.3	1.97	Incon.	1.17	Flexure
13	92.0	42.9	214.9	64.9	2.14	Incon.	1.42	Flexure
16	86.1	55.5	77.7	105.7	1.55	1.11	Incon.	Flexure
20	120.0	57.4	202.0	74.9	2.09	Incon.	1.60	Flexure
6	126.8	87.0	135.4	93.1	1.46	0.94	1.36	Shear
7	115.0	80.7	104.8	87.8	1.43	1.10	1.31	Shear
8	123.2	68.3	112.1	113.7	1.80	0.85	1.08	Shear
9	123.3	62.1	84.9	77.7	1.99	1.08	1.59	Shear
14	68.54	57.0	93.2	101.2	1.20	0.74	0.68	Shear
15	100.41	75.4	112.5	101.2	1.33	0.89	0.99	Shear
17	113.0	60.7	98.8	105.7	1.86	1.14	1.07	Shear
18	125.9	78.3	139.9	105.7	1.61	0.90	1.19	Shear
19	116.2	58.3	88.5	105.7	1.99	1.31	1.13	Shear
21	123.3	74.6	202.0	74.9	1.65	0.61	1.65	Shear
22	73.3	50.0	202.0	74.9	1.47	0.36	0.98	Shear

the results are practically on the safe side for all the other cases, and the degree of conservatism is less than the BRI method.

7.6 Shear Strength of FRP Reinforced Test Beams

The FRP reinforced beams tested in the current investigation can be divided into four categories from the shear reinforcement point of view. The characteristics of each category are given in Table 7.3.

Table 7.3: *Characteristics of shear reinforcement of FRP reinforced test beams*

Category No.	Beam Name	Longit. Reinf.	Trans. Reinf.	A_v (mm^2)	S (mm)
1	CRU1 and CRU2	CFRP	Steel	200	150
2	CRO1 and CRO2	CFRP	Steel	200	100
3	CRCS1 and CRCS2	CFRP	Steel	200	100
4	CRCN1 and CRCN2	CFRP	CFRP	87.5	50

Although the final mode of failure of the aforementioned test beams was flexural, it may be recalled from Chapter 5 that signs of premature shear failure were detected and accordingly in some of the beams bandages were used as external shear reinforcement. Hence, the expected shear failure load of Beams CRU1 and CRU2 was estimated to be 110 kN, while the shear failure load of Beams CRO1, CRO2, CRCS1 and CRCS2 was approximately 190 kN. However, for Beams CRCN1 and CRCN2 no shear failure was detected, hence shear bandages were not applied. The latter beams failed in flexure at 373 kN, which means that their shear capacity was greater than 373 kN.

The shear strength of the foregoing beams was calculated using the different

available methods and is given in Table 7.4.

Table 7.4: Comparison between the predictions of different methods for shear strength of FRP reinforced concrete test beams

Category No.	$V_{u,exp.}$	$V_{u,BRI}$	$V_{u,JSCE}$	$V_{u,prop.}$	$V_{u,exp.}$	$V_{u,exp.}$	$V_{u,exp.}$
	(kN)	(kN)	(kN)	(kN)	$V_{u,BRI}$	$V_{u,JSCE}$	$V_{u,prop.}$
1	110.0	108.5	542.8	168.8	1.01	0.20	0.65
2	190.0	115.6	805.3	182.1	1.64	0.24	1.04
3	190.0	113.8	773.1	180.7	1.67	0.25	1.05
4	> 373.0	150.9	409.2	380.5	> 2.47	> 0.91	> 0.98

Table 7.4 shows that the BRI method overestimates the shear failure loads of the test beams in most of the cases, while the JSCE method severely underestimates it. The proposed method gives a reasonable prediction of the shear strength of the FRP over-reinforced beams.

It is useful to assess the relative contribution of concrete and shear reinforcement to the shear resistance of the FRP reinforced test beams by using the experimental results. Since Beams CRCN1 and CRCN2 were the only test beams with strain gauges attached to their transverse reinforcement and no bandages applied to them, such assessment is only feasible for these beams. From Figs. 5.91 to 5.100 and using the data of those strain gauges which worked perfectly, a stress of approximately 400 MPa may be appropriate for the vertical ribs of the NEFMAC grids at the maximum load. Therefore, the contribution of shear reinforcement to the shear resistance of the latter two beams would be 196 kN, which leads to maximum contribution of concrete to the shear resistance being 177 kN. The latter is clearly greater than the 103.5 kN which is obtained from Eq. 7.25. However, it should be noted that the proposed expression by ACI (Eq. 7.25) is a safe minimum value for shear resistance of concrete, and can be exceeded in practice.

7.7 Shear Strength of Test Beams Reinforced with Steel

Although this chapter is basically concerned with the evaluation of the shear strength of FRP reinforced concrete beams, the shear strength of the two test beams reinforced with steel, i.e. Beams SR1 and SR2 will be discussed in this section. The reason is mainly the occurrence of premature shear failure in Beam SR1 at 357.0 kN, while shear failure in Beam SR2 was avoided by the provision of external shear stiffeners.

Beams SR1 and SR2 were longitudinally and transversely reinforced with steel reinforcement. The basic characteristics of the stirrups in these beams were: $A_v = 200\text{mm}^2$, $S = 125\text{mm}$, and $f_y = 471\text{MPa}$. Taking advantage of the conventional method of ACI for shear strength calculation, i.e. Eqs. 7.24 to 7.26 and ignoring the strength reduction factors, we obtain $V_u = 329.7\text{kN}$ or $P_u = 479.3\text{kN}$. Notice that basically the same equations have been adopted in Section 11.3 of CSA A23.3-94 (CSA, 1994) and the relevant method is referred to as the simplified method for shear design in flexural regions.

However, the CSA code recommends a more general method of shear design based on the compression field theory (Collines and Mitchell, 1991). In this method, the factored shear resistance of the member, V_{rg} , can be determined using the following equations

$$V_{rg} = V_{cg} + V_{sg} + \phi_p V_p \leq 0.25\phi_c f'_c b_w d_v + \phi_p V_p \quad (7.29)$$

$$V_{cg} = 1.3\lambda\phi_c\beta\sqrt{f'_c}b_w d_v \quad (7.30)$$

$$V_{sg} = \frac{\phi_s A_v f_y d_v \cot \theta}{S} \quad (7.31)$$

where, V_{cg} = factored shear resistance attributed to concrete, V_{sg} = factored shear resistance provided by shear reinforcement, ϕ_c , ϕ_s and ϕ_p = resistance factors respec-

tively for concrete, steel reinforcement, and prestressing tendons, V_p = component in the direction of applied shear of the effective prestressing force, b_w = minimum effective web width within effective depth d , d_v = distance measured perpendicular to the neutral axis between the resultants of the tensile and the compressive force due to flexure, but not less than $0.9d$, λ = factor to account for low density concrete, θ = angle of inclination of diagonal compressive stresses to the longitudinal axis of the member, and β = factor accounting for shear resistance of cracked concrete. The values of θ and β for each design section may be taken from Table 11.1 or Figure 11.1 of the code based on the ratio of $v_f/\lambda\phi_c f'_c$ and the longitudinal strain, ϵ_x , where

$$v_f = \frac{V_f - \phi_p V_p}{b_w d_v} \quad (7.32)$$

$$\epsilon_x = \frac{0.5(N_f + V_f \cot \theta) + M_f/d_v - A_p f_{po}}{E_s A_s + E_p A_p} \leq 0.002 \quad (7.33)$$

where, M_f = factored moment at section, N_f = factored axial load normal to the cross section occurring simultaneously with V_f , A_p = area of prestressing tendons in tension zone, f_{po} = stress in prestressing tendons when stress in surrounding concrete is zero, and E_p = modulus of elasticity of tendons.

Based on the foregoing equations, Beams SR1 and SR2 may be analysed for shear as follows (note that in the following calculations, the effect of resistance factors is not taken into consideration since the material characteristics and the loads are known in the test beams).

Assume: $P_{max} = 357.0kN$, then $V_{max} = 0.688P_{max} = 245.6kN$ and $M_{max} = 0.188P_{max}l - V_f d_v = 140.6kN$.

If $\theta = 45^\circ$, then from Eq. 7.33, $\epsilon_x = 0.0023 > 0.002$, hence, $\epsilon_x = 0.002$.

From Eq. 7.32, $v_f = 3.97MPa$, and from Table 11.1 of the code, $\beta = 0.143$ and, $\theta = 43^\circ$.

From Eq. 7.30 and 7.31, $V_{cg} = 100.93kN$, and $V_{sg} = 200.01kN$, therefore $V_{rg} = 300.94kN$, which gives: $P_{max} = V_{max}/0.688 = 437.4kN$.

It means that the CSA prediction of shear resistance of Beams SR1 and SR2 is

more than 357 kN. Let us consider the following conditions:

Let: $P_{max} = 440.0kN$, then, $V_{max} = 302.72kN$, and $M_{max} = 173.24kN$.

If $\theta = 42.5^\circ$, then from Eq. 7.33, $\epsilon_x = 0.00288 > 0.002$, hence, $\epsilon_x = 0.002$.

From Eq. 7.32, $v_f = 4.89MPa$, and from Table 11.1 of the code, $\beta = 0.1398$, and $\theta = 42.46^\circ$ ✓

From Eq. 7.30 and 7.31, $V_{cg} = 98.68kN$, and $V_{sg} = 203.83kN$, therefore, $V_{rg} = 300.94kN$, and $P_{max} = 439.7kN$ ✓

The foregoing calculations show that both the general method of CSA and the simplified method of ACI and CSA overestimate the shear resistance of Beams SR1 and SR2. The estimated shear capacities by the two methods are 23% and 34% higher than the experimental value. The overestimating of the code methods is surprising for conventional reinforced concrete beams and it may be attributed to the indeterminacy of the experimental beams while the codes expressions in shear are primarily based on experimental data from simply supported beams. This finding needs further investigation, but its implication can be serious in practice.

In summary, in this chapter the applicability of the BRI and the JSCE methods to the calculation of shear strength of FRP reinforced concrete members was studied. Comparison of experimental data with the results of these methods indicated that no matter which combination of FRP material and steel is used for flexural and shear reinforcement, the BRI method is highly conservative. Hence, designs based on this method are safe but may be uneconomical. The JSCE method overestimates the shear strength of FRP reinforced concrete beams in many cases. Since the latter is on the unsafe side, the method needs further examination before application in practice. The proposed method in this paper, which is partially based on the ACI method, is less conservative than the BRI method, but further work is needed to give it the theoretical underpinning needed before its adoption in practice. Finally, studying the shear behaviour of concrete test beams reinforced with steel indicated that the conventional expressions of code noticeably overestimate the shear strength

of Beams SR1 and SR2. Since the code predictions are on the non-conservative side, further study is warranted in this regard.

Chapter 8

Finite Element Analysis of Test Beams

8.1 General

Nonlinear finite element procedure will be used to analyze the test beams. Such an analysis, once verified, could be utilized in the future as a tool for further investigation of the effect of some relevant parameters on the ductility and moment redistribution characteristics of continuous concrete beams and other structures reinforced with FRP.

In the present investigation the program NONLACS, developed at Carleton University (Razaqpur and Nofal, 1988), will be used to study the effect of some parameters on moment redistribution in FRP reinforced concrete continuous beams. In this chapter, the program NONLACS will be briefly described together with the constitutive relations and method of analysis that it employs. Subsequently, the program will be used to analyze a number of continuous FRP reinforced concrete

beams tested by others and the beams tested in the current study. The analytical and experimental results will be compared in order to ascertain the degree of accuracy that can be achieved using this program.

8.2 Computer Program NONLACS

8.2.1 Basic Characteristics

The computer program NONLACS (NON-Linear Analysis of Concrete and Steel), developed by Razaqpur and Nofal (1988), was adopted from the program FELARC (Finite Element Layered Analysis of Reinforced Concrete). The program FELARC originally was developed by Ghoneim and Ghali (1978), and later on modified by Razaqpur in 1982. The program NONLACS can be used for the nonlinear analysis of any reinforced or prestressed concrete structures as well as composite steel-concrete structures. However, such structures must be composed of members which could be modelled as thin shell, plate bending, membrane, or combinations of such elements.

NONLACS can take into account the effects of creep, shrinkage and temperature changes. Moreover it accounts for material nonlinearity in both steel and concrete; it accounts for changes in the stiffnesses of the elements due to cracking of concrete and other material nonlinearities. It also takes into account the strain-hardening in steel and the tension-stiffening of concrete. NONLACS can accept several time intervals for time-dependent analysis. Therefore, for more realistic analysis, dead load (and prestressing loads) could be applied in one time interval and live load in another. However, if only instantaneous analysis under monotonically increasing load is required, only one time interval needs to be chosen but the load could be applied incrementally. Note that the capability of NONLACS for accepting incremental analysis may be used for load reversal analysis under cycling loading. In

such a case (or even in the case of cyclic variations of temperature), an appropriate number of increments should be selected to permit application of negative loads (reversed loads). NONLACS does not take into account fatigue effects, geometric nonlinearity and the change in bond stresses between adjacent surfaces (i.e., the bond-slip of the reinforcement in concrete).

NONLACS can analyze thin plate steel structures. For steel structures (or other materials analyzed using the theory of plasticity), the common yield criteria such as the Von-Mises, the Tresca, the Mohr-Coulomb and the Drucker-Prager criteria could be utilized. This program contains different types of finite elements, i.e. quadrilateral facet shell element, which could be used for both plane and plate bending problems, boundary element, which could be used if the reactions of the supports are required or if elastic foundation exists, one-dimensional element (bar element), and shear-connector element.

Quadrilateral Facet Shell Element

The Quadrilateral Facet Shell Element (QFSE) is an isotropic shell which is formulated to account for the coupled in-plane and bending actions. The in-plane element in QFSE, however, could be of type QLC3 (Quadrilateral Linear Cubic element) (Ghoneim and Ghali, 1982), or RQUAD4 (Refined QUADrilateral membrane element with 4 corner nodes) (Razaqpur and Nofal, 1990), whereas the plate bending element could be RBE (Rectangular Bending Element) (Zienkiewicz, 1971), or IDKQ (the Improved Discrete Kirchhoff Quadrilateral plate bending element) (Chinniah, 1985). For the problems that will be analyzed with NONLACS in the current study, only RQUAD4 and IDKQ are chosen for in-plane and plate bending elements, respectively.

It should be noted that each quadrilateral facet shell element could be divided

into layers through its thickness, with each layer being considered in a state of plane stress. Furthermore, the layers could be of different materials (concrete, steel, FRP *etc.*) and each concrete layer can be reinforced separately. The number of layers may be selected according to the type of problem, i.e., one layer for plane or membrane problems, and five to eight layers for plate bending problems.

8.2.2 Material Modelling in NONLACS

a) Concrete model in compression

The stress-strain relationship of concrete in NONLACS consists of two branches (Fig. 8.1). The ascending branch is based on Saenz's (1964) equation

$$\sigma = \frac{E_0 \epsilon}{1 + \left(\frac{E_0}{E_{sc}} - 2 \right) \frac{\epsilon}{\epsilon_{cu}} + \left(\frac{\epsilon}{\epsilon_{cu}} \right)^2} \quad (8.1)$$

The descending branch is modelled with Smith-Young's (1955) equation

$$\sigma = \sigma_c \left(\frac{\epsilon}{\epsilon_{cu}} \right) . e^{[1-(\epsilon/\epsilon_{cu})]} \quad (8.2)$$

where E_0 = initial tangent modulus, E_{sc} = secant modulus of elasticity at peak stress, ϵ_{cu} = strain at peak stress, σ_c = concrete compressive strength, and σ and ϵ are stress and strain, respectively. Using the concept of equivalent strain (Darwin and Pecknold, 1977), the program resolves any state of combined in-plane stress into the directions of the material axes of orthotropy. For each of those directions, the latter stress-strain relations are used. Depending on the ratio of stresses or strains in the directions of orthotropy, E_{sc} and ϵ_{cu} will change. The failure criterion is based on the Kupfer and Gerstle biaxial failure envelope (Ghoneim and Ghali, 1982).

Figure 8.1: *The stress-strain relationship of concrete in compression (used in NON-LACS)*

b) Concrete model in tension

The bi-linear tensile stress-strain relationship employed in NONLACS is shown in Fig. 8.2. The ascending branch represents the behaviour of concrete in tension before cracking where the stress and strain are smaller than the cracking stress (σ_t) and the cracking strain (ϵ_{cr}), respectively. The descending branch represents the behaviour of concrete after cracking. In reality, the tension-stiffening effect of the model (descending branch), assumes zero stiffness for concrete after cracking, but the concrete between the cracks is assumed to be capable of transferring tensile stress. The ultimate tensile strain permitted in concrete, ϵ_{tu} , is generally much larger than the cracking strain, ϵ_{cr} , but good results can be obtained using $\epsilon_{tu} = 10\epsilon_{cr}$.

Figure 8.2: *The stress-strain relationship of concrete in tension (used in NONLACS)*

c) Modelling of steel reinforcement

The stress-strain relationship of steel bars and prestressing tendons in NONLACS is modelled with a bi-linear curve, (Fig. 5.3). The first linear part represents the behaviour of steel before yield while the second part represents the strain-hardening of the material. If loading and unloading occurs, the reduction in yield strength, i.e. the so-called Bauschinger's effect, may be considered as illustrated in Fig. 8.3.

8.3 Verification of the Program NONLACS for FRP Reinforced Concrete

Svecova (1994) conducted a comprehensive study to verify the applicability of the program NONLACS to concrete beams and slabs reinforced with FRP. She chose a total of eight concrete members which had been tested previously in the laboratory by other researchers. The members included one FRP reinforced beam tested by Faza and GangaRao (1991), two concrete beams reinforced with a proprietary FRP

Figure 8.3: *The stress-strain relationship of steel (used in NONLACS)*

sheet (known as FORCA TOW SHEET) tested by TONEN Corporation of Japan (1992), one conventional reinforced concrete beam strengthened with epoxy bonded CFRP unidirectional laminates tested by Meier and Kaiser (1992), one concrete beam pre-tensioned with AFRP tendons tested by Sumitomo Construction Co., one partially prestressed concrete T-beam which was reinforced with CFRP tendons, tested by Naaman et al (1992), and finally two high strength concrete slabs reinforced with layers of CFRP and AFRP meshes, tested by Kawakami (1992). All eight of the aforementioned members were simply-supported.

For the beam tested by Faza and GangaRao, Svecova compared the load-deflection relationship from NONLACS with the experimental results. She found that the finite element results closely agree with the experimental data up to failure. However, the failure load predicted by NONLACS was 19% more than the experimental value.

For the first beam tested by TONEN Corporation of Japan, Svecova found

very close agreement between the results of NONLACS and the experimental load-deflection data, while the failure load by finite element was only 3% more than the experimental failure load. However, for this beam the finite element overestimated the strains after cracking. For the second beam tested by TONEN, very close agreement between the results of NONLACS and the measured deflection was reported, but the failure load predicted by the finite element was about 45% more than that found in the experiment. This was attributed to the delamination of the sheet from the soffit of the beam in the experiment. In the analysis, the interfacial shear failure was ignored.

For the beam of Meier and Kaiser, close agreement between the experimental and analytical load-deflection curves was reported. The failure load and the maximum deflection at midspan from NONLACS were respectively 6.6% and 2.2% more than those observed in the test.

For the beam tested by Sumitomo Construction Co., Svecova found good agreement between the experimental and the analytical load-deflection curves, except for some minor disparity after cracking. The failure load obtained from NONLACS was 22.3% higher than the failure load in the experiment, which was attributed to the probable initial cracking of concrete.

For the partially prestressed beam tested by Naaman et al, Svecova reported favorable results before cracking of the beam. However, after cracking NONLACS predicted a stiffer response compared to the experimental load-deflection curve. The failure load predicted by NONLACS was 11.5% lower than the experimental value.

For the FRP reinforced slabs tested by Kawakami, favorable agreement was observed between the finite element and the experimental results. The NONLACS results for failure load was 7.2% higher and 1.6% less than the corresponding experimental failure loads for the two slabs, respectively. Since these slabs were cast

with high strength concrete, Svecova concluded that NONLACS is also capable of predicting the behaviour of FRP reinforced members containing high strength concrete.

Since the preceding analysis covered a relatively wide range of FRP reinforced concrete structures, it is reasonable to assume that NONLACS would be a suitable candidate for analyzing continuous FRP reinforced beams. Nevertheless, the program reliability in this respect will be further examined in the next section.

8.4 Analysis of Existing Continuous FRP Reinforced Concrete Beams

8.4.1 General

The objective of this section is to present the essential steps which would be followed to analyze the results of the current experimental work. It is also intended to verify the applicability of the program NONLACS to the analysis of continuous FRP reinforced concrete beams, and to investigate the capability of NONLACS to predict redistribution of moments.

To the writer's knowledge, prior to the current study, the study conducted by Tezuka et al (1995) is the only one in the open literature on the moment redistribution of continuous FRP reinforced concrete beams. Tezuka et al in their experimental work tested five two-span continuous beams reinforced or pre-tensioned with FRP. Here, two of the tests which are FRP reinforced continuous beams (test beams # 1 and # 3) are selected for analysis by NONLACS.

Figure 8.4: Detail of test specimen (Tezuka et al beams)

8.4.2 Description of the Beams

The beams were each two-span continuous beams with a cross section of 120×200 mm and a clear span length of 2000 mm, (Fig. 8.4). Four AFRP- $\phi 9$ bars and four CFRP- $\phi 10.5$ bars were used in beam # 1 and beam # 3, respectively. The mechanical properties of AFRP and CFRP used in those beams are shown in Table 8.1. The compressive strength of concrete for both beams was 40 MPa. Both beams

Table 8.1: Material properties of reinforcement in Tezuka et al beams

Type of Reinforcement	Nominal Diameter (mm)	Sectional Area (mm ²)	Nominal Tensile Strength (MPa)*	Young's Modulus (GPa)*	Ultimate Elongation (%)
AFRP- $\phi 9$	9	63	1520	70	2.17
CFRP- $\phi 10.5$	10.5	55.7	1900	140	1.36

*The original units were not in SI system.

Figure 8.5: Reactions and bending moments for test beams of Tezuka et al based on elastic analysis

were designed and reinforced in such a way that compressive failure of concrete at the middle support (negative moment region) would initiate beam failure.

For the beam shown in Fig. 8.4, using any conventional method for linear-elastic analysis of beams, e.g. moment-distribution method, the reactions and bending moments could be obtained as shown in Fig. 8.5. We would compare the moments in the latter figure with the corresponding moments obtained from nonlinear analysis.

8.4.3 Nonlinear Analysis with NONLACS

Description of the Mesh and the Procedure of Analysis

Taking advantage of symmetry, half of each beam was modelled and analyzed. A relatively fine mesh with 40 quadrilateral facet shell elements and 63 nodes was

Figure 8.6: *The finite element mesh used for the analysis of Tezuka et al beams by NONLACS*

chosen for each beam, as shown in Fig. 8.6. The height of the beam was divided into two finite elements. Furthermore, some boundary elements were added at support positions to calculate the reactions. Thirty load steps were used, with the last ten load steps being increased in very small increments.

As mentioned earlier in the literature review, FRP materials cannot be fully utilized in compression. To account for this behaviour, some minor changes were made in NONLACS to ignore the effect of FRP reinforcement in compression. Note that all the FRP reinforcement was modelled as linear elastic up to failure.

Comparison of Results

a) Moment-Curvature Relationship

Fig. 8.7 and Fig. 8.8 show the moment-curvature relationship of the middle support of Beams # 1 and # 3, respectively. The figures contain the experimental curve, the curve from nonlinear analysis conducted by Tezuka et al, and the curve obtained from nonlinear analysis by NONLACS. Both figures show close agreement between the experimentally measured curvatures and the corresponding nonlinear finite element prediction of NONLACS. Furthermore, it could be seen that NONLACS has

Figure 8.7: *Experimental and analytical moment-curvature relationship of the middle support for Tezuka et al Beam # 1*

predicted the moment-curvature relationship of the beams relatively better than the nonlinear procedure used by Tezuka et al. The difference between the analytical and experimental results immediately after cracking, at a moment of approximately 4 kN.m (Fig. 8.7), is due to the analytical procedure employed. The experiments were conducted using displacement control while the analysis employs load control where the load increases monotonically and is not allowed to drop. As far as the subsequent behaviour and ultimate capacity of the beams is concerned, this aspect of the response of the beams is of little practical significance.

Figure 8.8: *Experimental and analytical moment-curvature relationship of the mid-support for Tezuka et al Beam # 3*

b) Failure Load

The failure loads for the two beams obtained from the nonlinear analysis by NONLACS and the corresponding experimental failure loads are shown and compared in Table 8.2. The results of Table 8.2 show that the failure loads predicted by NONLACS are 2.4% and 15.0% more than the experimental failure loads for beams # 1 and # 3, respectively. Although the 15% difference is somewhat large, nevertheless NONLACS predicted the overall beam behaviour satisfactorily. Note that NONLACS assumes perfect bond between the concrete and the FRP reinforcement, but for very high strength FRPs, such as CFRP, this assumption needs further examination because bond failure could lead to premature failure of a beam. In the case of the two beams analyzed here, the exact mode of failure was not reported.

Table 8.2: *The failure loads of Tezuka et al beams predicted by NONLACS*

Beam No.	Specimen	Failure Load (kN)		Discrepancy %
		Experiment	NONLACS	
1	RC-AF	46.9	48.02	+2.4
3	RC-CF	56.8	65.34	+15.0

c) Moment Redistribution

The bending moments at the middle support and at the mid-spans obtained from nonlinear analysis by NONLACS are shown in Table 8.3. This table also contains the corresponding bending moments based on a linear-elastic analysis up to failure, i.e. according to the bending moment distribution in Fig. 8.5. Note that the percentage discrepancy between the bending moments obtained from the linear-elastic analysis and the nonlinear analysis could be considered as the percentage of moment redistribution, as indicated in Table 8.3.

Table 8.3: *Ultimate moments for Tezuka et al beams obtained from NONLACS*

Beam No.	Failure Load (kN)	At the Middle Support $M^-(kN.m)$			At the Loading Point $M^+(kN.m)$		
		nonlinear	linear	$\frac{M_{na}^- - M_{la}^-}{M_{la}^-}$	nonlinear	linear	$\frac{M_{na}^+ - M_{la}^+}{M_{la}^+}$
		M_{na}^-	M_{la}^-	(%)	M_{na}^+	M_{la}^+	(%)
1	48.02	-15.34	-18.42	-16.7	15.33	13.64	12.4
3	65.34	-23.55	-25.06	-6.0	19.39	18.56	4.5

Note : The subscripts "la" and "na" stand for "linear analysis" and "nonlinear analysis", respectively.

Table 8.4 is similar to Table 8.3 but the moment redistribution is calculated on the basis of the experimental results of Tezuka et al.

Table 8.4: *Experimental results for Tezuka et al beams*

No.	Ultim. Load (kN)	At the Middle Support			At the Loading Point		
		$M^-(kN.m)^*$			$M^+(kN.m)^*$		
		measured M_e^-	computed [†] M_a^-	$\frac{M_e^- - M_a^-}{M_a^-}$ (%)	measured M_e^+	computed [†] M_a^+	$\frac{M_e^+ - M_a^+}{M_a^+}$ (%)
1	46.9	-15.50	-18.05	-14.1	14.72	13.34	10.3
3	56.8	-15.30	-21.78	-29.7	19.72	16.09	22.6

*The original units were not in SI system.

†Computed elastically at the measured ultimate load.

Note : The subscripts "e" and "a" stand for "experimental" and "analytical", respectively.

Table 8.3 shows that for Beam # 1 (reinforced with AFRP), the program NONLACS predicted -16.7% and $+12.4\%$ redistribution for negative and positive bending moments, respectively. These values are comparable to the corresponding -14.1% and $+10.3\%$, respectively, reported by Tezuka et al (Table 8.4). However Tables 8.3 and 8.4 show that for the Beam # 3 (reinforced with CFRP), NONLACS predicted -6.0% and $+4.5\%$ redistribution respectively for negative and positive bending moments, which are far from those reported in the experiment of Tezuka et al (-29.7% and $+22.6\%$ for the redistribution of negative and positive bending moments, respectively). This discrepancy may be attributed to the possible bond failure in Beam # 3 and the inability of NONLACS to account for this problem. Clearly this problem will need further investigation. On the other hand, such a large amount of moment redistribution for FRP reinforced concrete beams is rather unexpected. Given the experience with conventional over-reinforced concrete beams, the latter level of redistribution needs closer scrutiny.

It may be useful to mention that Tezuka et al in their analytical work utilized a nonlinear procedure to predict the bending moments of the middle support and mid-spans. However, based on their analysis they reported very low percentages of

redistribution which differed greatly from the experimental values. They attributed this problem to the predominant diagonal shear cracks near the middle support created because of insufficient shear reinforcement. The results of their nonlinear analytical work are shown in Table 8.5.

Table 8.5: Results of nonlinear analysis conducted by Tezuka et al

Name of Specimen	Ultimate Load (kN)	At the Middle Support $M^-(kN.m)^*$			At the Loading Point $M^+(kN.m)^*$		
		nonlinear	linear	$\frac{M_{na}^- - M_{la}^-}{M_{la}^-}$	nonlinear	linear	$\frac{M_{na}^+ - M_{la}^+}{M_{la}^+}$
		M_{na}^-	M_{la}^-	(%)	M_{na}^+	M_{la}^+	(%)
RC-AF	41.5	-15.79	-15.89	-0.6	11.87	11.77	0.8
RC-CF	54.0	-20.50	-20.70	-0.9	15.40	15.30	0.6

*The original units were not in SI system.

Note : The subscripts "la" and "na" stand for "linear analysis" and "nonlinear analysis", respectively.

Notice that the nonlinear analytical results of Tezuka et al (Table 8.5) indicate that prediction of moment redistribution in continuous FRP reinforced concrete beams is not a simple matter. NONLACS predictions are better but further investigation is needed to establish its capabilities in predicting the full response of FRP reinforced concrete structures.

Figure 8.9: The finite element mesh used for the analysis of test beams by NONLACS

8.5 Nonlinear Analysis of Test Beams with NON-LACS

8.5.1 Modelling of Beams

Each test beam was discretized by finite elements as shown in Fig. 8.9. Taking advantage of symmetry, half of the beam was modelled and divided into 40 quadrilateral facet shell elements with 63 nodes. The size of elements were chosen smaller in the regions with expected higher stresses, i.e. the positive and negative moment regions. In addition to conventional elements, some appropriate boundary elements were added at support positions.

The material properties of concrete, steel and FRP were taken from the results of Chapter 4 and used in the analysis. The 3×3 Gauss integration rule was used for each quadrilateral facet shell element. Only instantaneous analysis with monotonically increasing load was carried out in the program, using thirty load steps with the last ten load steps involving very small increments.

Figure 8.10: *Experimental and analytical load-displacement curves for Beam SR2*

8.5.2 Comparison of the Experimental and the Analytical Results

a) Load-Displacement Curve

Fig. 8.10 to 8.14 show the experimental and the analytical load-displacement curves for Beams SR2, CRU1 and CRU2, CRO1 and CRO2, CRCS1 and CRCS2, and CRCN1 and CRCN2, respectively. The figures contain the experimental curves for the beams of each group, and the curve obtained from analysis by NONLACS. Notice that the experimental curves in the foregoing figures were taken as the average of the load-displacement response of the right and the left spans of each beam.

Figure 8.11: *Experimental and analytical load-displacement curves for Beams CRU1 and CRU2*

Fig. 8.10 shows good agreement between the experimentally measured displacements and the corresponding nonlinear finite element prediction for Beam SR2. However, NONLACS predicted the displacement response of the beam only up to the maximum load. Since the procedure of loading in the experimental work was displacement control, the test beams response after the maximum load could be captured but the finite element procedures in NONLACS are not designed to obtain the structure response beyond the peak load.

Fig. 8.11 shows poor agreement between the experimental and the analytical load-displacement responses of Beams CRU1 and CRU2. However, as mentioned in the preceding chapters, since Beams CRU1 and CRU2 had very low amount of

Figure 8.12: *Experimental and analytical load-displacement curves for Beams CRO1 and CRO2*

reinforcement, they cracked during handling and moving prior to their testing. The effect of the initial cracks is not taken into account in the analysis. Hence, the disagreement between the analytical and the experimental curves may be attributed to the difference among the initial conditions of the actual beams and their finite element model. While one could introduce initial cracks in the analysis to simulate pre-cracked concrete, such refinements are beyond the scope of the current work. Notice, however, that the curves are basically parallel which means that change in the initial conditions of the analysis could move the two curves closer.

Fig. 8.12 shows close agreement between the experimental and the analytical load-displacement curves for Beams CRO1 and CRO2. Furthermore, Figs. 8.13

Figure 8.13: *Experimental and analytical load-displacement curves for Beams CRCS1 and CRCS2*

and 8.14 show good agreement between the experimental and the analytical load-displacement responses of Beams CRCS1 and CRCS2, and CRCN1 and CRCN2, respectively. However for the latter cases, the analytical curves deviate from the experimental curves at higher loads. This may be attributed to the local bond failure of the longitudinal FRP rods at higher loads, and the inability of NONLACS to account for bond failure. Overall, the prediction of NONLACS for the load-displacement responses of the over-reinforced concrete beams seems satisfactory.

Figure 8.14: *Experimental and analytical load-displacement curves for Beams CRCN1 and CRCN2*

b) Failure Load

The failure loads for the test beams obtained from nonlinear analysis by NONLACS and the corresponding experimental failure loads are shown and compared in Table 8.6. From this table it is seen that the differences between the experimental and the analytical failure load are low in most of the cases. The maximum difference is observed for Beam CRU2 and CRCS1. However, as mentioned earlier, the ultimate tensile strength of FRP bars (which is very important in the evaluation of the bending strength of FRP under-reinforced concrete beams) could be higher than the minimum value recommended by the manufacturer, a value which was utilized in

Table 8.6: *The failure loads of test beams predicted by NONLACS*

Group No.	Beams Name	Failure Load (kN)		Discrepancy %
		Experiment	NONLACS	
1	SR2	402.5	384.1	-4.6
2	CRU1	146.0	143.0	-2.1
	CRU2	170.3	143.0	-16.0
3	CRO1	349.3	358.2	+2.5
	CRO2	356.3	358.2	+0.5
4	CRCS1	325.0	361.8	+11.3
	CRCS2	352.5	361.8	+2.6
5	CRCN1	375.3	361.8	-3.6
	CRCN2	365.4	361.8	-1.0

the analytical study. Furthermore, the difference between the failure loads of Beams CRU1 and CRU2, which are nominally identical, also makes it difficult to predict their capacity by the finite element or any other method of analysis. Table 8.6 shows 11.3% difference between the analytical and the experimental failure loads of Beam CRCS1. As mentioned in Chapter 5, Beam CRCS1 failed prematurely. Overall, the predicted failure loads of NONLACS may be deemed to be satisfactory and reliable.

c) Moment Redistribution

The bending moments at the middle support and at the mid-spans of the test beams obtained from finite element analysis by NONLACS are shown in Table 8.7. This table also contains the corresponding bending moments based on linear-elastic analysis, i.e. according to bending moments distribution in Fig. 4.1.

Comparison of the results of Table 8.7 and Table 6.11 indicates good agreement

Table 8.7: *Ultimate moments and the corresponding redistribution for test beams obtained from NONLACS*

Beam Name	At the Middle Support $M^-(kN.m)$			At the Loading Point $M^+(kN.m)$		
	nonlinear	linear	$\frac{M_{na}^- - M_{la}^-}{M_{la}^-}$	nonlinear	linear	$\frac{M_{na}^+ - M_{la}^+}{M_{la}^+}$
	M_{na}^-	M_{la}^-	(%)	M_{na}^+	M_{la}^+	(%)
SR2	-186.2	-216.6	-14.0	195.0	179.8	8.5
CRU1 , CRU2	-78.1	-79.5	-1.8	68.2	67.5	1.0
CRO1 , CRO2	-188.7	-202.0	-6.6	174.3	167.6	4.0
CRCS1,CRCS2	-190.5	-204.1	-6.7	176.1	169.3	4.0
CRCN1,CRCN2	-190.5	-204.1	-6.7	176.1	169.3	4.0

Note : The subscripts “la” and “na” stand for “linear analysis” and “nonlinear analysis”, respectively.

between the redistribution percentages predicted by NONLACS and the measured values. In other words, NONLACS confirms the occurrence of approximately 50% of the maximum possible moment redistribution for FRP over-reinforced concrete test beams and negligible amount of redistribution in FRP under-reinforced concrete beams.

In conclusion, the finite element analytical results as well as the experimental results confirm that some moment redistribution can be expected in FRP over-reinforced concrete beams. Therefore, whenever FRP is used as the main reinforcement for concrete structures, it is preferable to over-reinforce the section, especially if the structure is indeterminate. Of course, other advantages of FRP over-reinforced concrete beams with regard to serviceability (as discussed in Chapter 5) may also preclude under-reinforcement.

Chapter 9

Summary, Conclusions, and Recommendations for Future Work

9.1 Summary

The current study is primarily focused on the issues of ductility and moment redistribution in FRP reinforced concrete beams. During the course of the study, ten 6.5 *m* long continuous two-span concrete beams were tested to evaluate their ductility and moment redistribution capacity. Two beams were conventional reinforced concrete members while the remaining eight beams were reinforced for flexure with CFRP rebars. Two of the latter were under-reinforced while the remaining six were over-reinforced. The compression zone of four of the over-reinforced beams was reinforced with CFRP grids (known as NEFMAC) for confinement. The objective of the latter was to investigate the effect of concrete confinement on ductility. The shear reinforcement for two of the FRP over-reinforced beams comprised NEFMAC

grids while the other eight beams had conventional steel U-stirrups. All the beams were loaded by a point load applied at the middle of each span. During the test, beam deflection and the strain in the reinforcement and the concrete were measured for various levels of load.

The analysis of the test results was commenced by focusing on ductility in FRP reinforced concrete beams. Based on theoretical analyses, a simplified bi-linear model for moment-curvature relationship of FRP reinforced concrete sections was proposed. It was concluded that a bi-linear moment-curvature relationship would induce nonlinearity in the structure and that such nonlinearity would cause redistribution of internal stresses. Results of the conducted experimental work were analysed for ductility and moment redistribution. Acceptable pseudo-ductile behaviour was observed for all the FRP over-reinforced concrete test beams. Consequently, based on the analytical and experimental results, new definitions for “curvature pseudo-ductility factor” and “deflection pseudo-ductility factor” were proposed for FRP reinforced concrete beams.

Results of the experimental program confirmed full plastic redistribution and high ductility in the conventional reinforced concrete beams. The FRP over-reinforced beams achieved almost 50% of the maximum possible redistribution while the FRP under-reinforced beams did not show any ductility. On the other hand, confinement of the compression zone did not have any influence on either ductility or moment redistribution in the present tests.

In addition to ductility, the shear strength of FRP reinforced concrete beams was also investigated. The applicability of two existing methods (BRI and JSCE) for calculation of shear strength of FRP reinforced concrete members was studied and discussed, using a number of available experimental works. It was noted that no matter which combination of FRP material and steel is used for flexural and shear reinforcement, one of the methods is highly conservative while the other method

often overestimates the shear strength of FRP reinforced concrete beams. A new expression for calculation of shear strength of FRP reinforced concrete members was proposed and its applicability was assessed. The proposed method, albeit empirical, is safe and not excessively conservative. In assessing the shear behaviour of the tested beams, the CFRP shear reinforcement (NEFMAC grids) performed well and there was no indication of shear distress or failure prior to the flexural failure of the particular test beams.

To evaluate the capability of the nonlinear finite element programs in predicting the behaviour of statically indeterminate FRP reinforced concrete structures, an existing nonlinear finite element program, named NONLACS, was used. Minor modifications were made to the program to account for the characteristics of FRP reinforcement. The applicability of NONLACS to the analysis of beams tested by other researchers was demonstrated. The same program was used to analyse the two-span continuous FRP reinforced beams tested in the present investigation. The results of the nonlinear analysis were compared with the corresponding experimental data, with good agreement between the two sets of results.

9.2 Conclusions

The present investigation supports the following conclusions:

1. Moment redistribution in continuous beams is not only dependent on the inelastic behaviour of the reinforcement but also on the in-elasticity of concrete which plays a significant role in this regard. Therefore, moment redistribution is possible in continuous FRP reinforced beams and frames, albeit the FRP reinforcement behaves linearly throughout and the extent of redistribution is less than in conventional steel reinforced concrete.

2. FRP under-reinforced concrete beams could not in general satisfy serviceability requirements, i.e. the current crack width and the deflection limitations would be exceeded. However, FRP over-reinforced concrete beams may be comparable to conventional reinforced concrete beams from the serviceability point of view.

3. Theoretical analysis of the moment-curvature relationship of various FRP reinforced concrete beams show that a bi-linear model can be used for the moment-curvature response. The model implies pseudo-ductile behaviour, especially for FRP over-reinforced concrete beams with high reinforcement ratio.

4. Observation of the load-displacement curves of the continuous FRP over-reinforced concrete test beams indicates that after the maximum load, the beams undergo large deflection without correspondingly large drop in the load. This behaviour is indicative of stress redistribution in continuous FRP over-reinforced concrete structures and could be deemed as a form of ductility.

5. The results of the current experimental work show the occurrence of approximately 50% of the maximum possible redistribution (compared to the conventional concrete beams reinforced with steel) for the tested FRP over-reinforced concrete beams.

6. The confinement of the compression concrete in the present tests had no impact on the amount of moment redistribution. However, it would appear that the confinement increased the compressive strength of the concrete and enhanced the flexural strength of the beams.

7. The increased flexural strength due to confinement could be predicted reasonably well by using the Saatcioglu-Razvi model for confined concrete.

8. Application of the current guidelines (BRI and JSCE) for evaluation of the shear strength of FRP reinforced concrete beams indicates that one method overesti-

mates while the other underestimates the shear strength of FRP reinforced concrete members.

9. The non-linear finite element program NONLACS is capable of analyzing continuous FRP reinforced concrete beams up to maximum load but is unable to predict the unloading response. NONLACS is capable of predicting the maximum load as well as the redistribution of moments with a reasonable degree of accuracy. The nonlinear finite element method could be used to investigate the effect of various parameters on the moment redistribution and ductility characteristics of FRP reinforced concrete structures, up to the point of maximum load. To be able to capture the post-peak response, such programs need to be modified for analysis by the displacement control method.

10. The results of the analysis of the test beams with finite element program NONLACS confirm the occurrence of moment redistribution in continuous FRP over-reinforced concrete beams.

11. Whenever FRP is selected as flexural reinforcement for a concrete structure, a reinforcement ratio of greater than $2.5\rho_b$ is recommended. This amount of reinforcement appears to satisfy serviceability requirements and to provide adequate ductility for the concrete members.

9.3 Recommendations for Future Work

1. Further experimental and analytical studies should be conducted to investigate the ductility of CFRP reinforced concrete beams with a wider range of reinforcement ratios and for both normal and high strength concrete.

2. Since the current experimental work was carried out using only CFRP rein-

forcement, similar experiments should be conducted on concrete beams reinforced with GFRP or AFRP to be able to formulate general design rules for ductility and moment redistribution in FRP reinforced concrete beams.

3. A series of continuous FRP reinforced concrete beams should be tested under different loading configurations in order to compare the maximum theoretical redistribution percentage with the actual achievable amount. For instance, in a two-span continuous beam under symmetrical point loads, the load may be applied at various distances from the middle support which would theoretically lead to different amounts of redistribution.

4. Although the present experimental investigation did not reveal any effect of confinement of compression concrete on ductility and redistribution in CFRP reinforced concrete beams, further experiments should be performed on concrete sections whose compressive zone would be comparable to the aperture of the confinement reinforcement. Furthermore, the effect of other methods of improving ductility, such as utilizing hybrid reinforcement, on the redistribution of stresses and moments in FRP reinforced concrete beams should be experimentally investigated.

5. It seems that there is an urgent need for a comprehensive analytical and experimental study on the behaviour of FRP reinforced concrete beams and frames under dynamic and seismic loads. In the other words, the capability of FRP reinforced members for energy absorption and energy dissipation under seismic loads need to be investigated.

6. Analytical studies should be carried out to derive and recommend guidelines for ductility demand, ductility capacity and the percentage of allowable moment-redistribution in FRP reinforced concrete beams.

7. Experimental and analytical investigation is needed to evaluate the shear strength of concrete beams longitudinally and/or transversely reinforced with FRP.

Single span four point loaded beams seem to be more appropriate for such investigation. The effect of moment, acting concurrently with the shear on a section, on the shear strength of FRP reinforced concrete beams requires more detailed investigation.

8. Finite element procedures should be developed for analysis by the displacement control method. In this manner, the descending branch of the load-deflection response of beams can be captured and the actual distribution of moments in the beam in the post-peak regime can be determined.

Appendix A

Design of Test Beams

A.1 Steel Reinforced Concrete Beams

$$f'_c = 77 \text{ MPa}$$

$$f_y = 462 \text{ MPa}$$

$$\beta_1 = 0.85 - 4.7 \times 0.08 < 0.65, \text{ hence: } \beta_1 = 0.65$$

$$\rho_b = 0.85\beta_1 \frac{f'_c}{f_y} \frac{\epsilon_{cu}}{\epsilon_{cu} + \frac{f_y}{E_s}}$$

$$\rho_b = 0.85 \times 0.65 \times \frac{77}{462} \times \frac{0.0035}{0.0035 + \frac{462}{203600}} = 0.0569$$

$$\text{Suppose : } \rho^- = \rho^+ \approx 0.35\rho_b = 0.01955$$

$$A_s^+ = A_s^- = \rho b d = 0.01955 \times 250 \times 300 = 1466 \text{ mm}^2$$

$$\text{Use 5 No. 20, } A_s = 5 \times 300 = 1500 \text{ mm}^2$$

$$\rho = 0.02, \frac{\rho}{\rho_b} = 0.351$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{1500 \times 462}{0.85 \times 77 \times 250} = 42.35 \text{ mm}$$

$$M_r = A_s f_y \left(d - \frac{a}{2} \right)$$

$$M_r = 1500 \times 462 \times \left(300 - \frac{42.35}{2}\right) = 193.22 \times 10^6 N.mm$$

$$= 193.22 kN.m$$

$$P_c = 5.333 \frac{M_r^-}{l} = 5.333 \times \frac{193.22 \times 10^6}{3000} = 343.5 \times 10^3 N = 343.5 kN$$

$$P_p = 6 \frac{M_r^-}{l} = 1.125 P_c = 386.4 kN$$

Design for Shear

$$R_A \times l - P \times \frac{l}{2} + M_B^- = 0$$

$$R_A \times 3000 - 386.4 \times 10^3 \times 1500 + 193.22 \times 10^6 = 0$$

$$R_A = 128793 N = 128.8 kN$$

$$V_{max} = 386.4 - 128.8 = 257.6 kN$$

$$V_c = 0.2 \lambda \phi_c \sqrt{f'_c} b_w d \text{ (According to CSA A23.3-94)}$$

$$V_c = 0.2 \times 1 \times 0.6 \times \sqrt{77} \times 250 \times 300 = 78975 N = 79.0 kN$$

$$V_s = 257.6 - 79.0 = 178.6 kN < 4V_c \text{ o.k.}$$

$$S = \frac{\phi_s A_s f_y d}{V_s}$$

$$\text{For No. 10 : } A_s = 2 \times 100 = 200 mm^2$$

$$S = \frac{0.85 \times 200 \times 471 \times 300}{178.6 \times 10^3} = 134.5 mm$$

$$S_{max} = \text{Min}\left\{\frac{d}{2}, 600 mm\right\} = 150 mm$$

Use No. 10 @125mm as stirrups.

A.2 FRP Reinforced Concrete Beams (Under-Reinforced)

For CFRP bars:

$$f_{fu} = 2550 \text{ MPa}$$

$$E_f = 147 \text{ GPa}$$

$$\rho_b = 0.85 \times 0.65 \times \frac{77}{2550} \times \frac{0.0035}{0.0035 + 2550/147000} = 0.0028$$

$$\text{Use } 2\Phi 8 = 2 \times 46.1 = 92.2 \text{ mm}^2$$

$$\rho = \frac{92.2}{250 \times 300} = 0.00123$$

$$\rho/\rho_b = 0.44 \text{ o.k.}$$

$$a = \frac{92.2 \times 2550}{0.85 \times 77 \times 250} = 14.37 \text{ mm}$$

$$M_r = 92.2 \times 2550 \times \left(300 - \frac{14.37}{2}\right) = 68.84 \times 10^6 \text{ N.mm} = 68.84 \text{ kN.mm}$$

$$P_e = 5.333 \frac{M_r}{l} = \frac{5.333 \times 68.84 \times 10^6}{3000} = 122.38 \times 10^3 \text{ N} = 122.38 \text{ kN}$$

For shear : Use No. 10 @150mm as stirrups.

A.3 FRP Reinforced Concrete Beams (Over-Reinforced)

$$\rho_b = 0.0028$$

$$\text{Suppose : } \rho \approx 2.5\rho_b = 0.007$$

$$\text{Use } 7\Phi 10 = 7 \times 71.8 = 502.6 \text{ mm}^2, (\rho = 0.0067)$$

$$f_f = E_f \epsilon_f = 0.0035 E_f \left(\frac{\beta_1 d - a}{a} \right)$$

$$A_f f_f = 0.85 f'_c ab$$

$$A_f \times 0.0035 E_f \left(\frac{\beta_1 d - a}{a} \right) = 0.85 f'_c ab$$

$$a^2 + \left(\frac{0.0035 A_f E_f}{0.85 f'_c b} \right) a - \frac{0.0035 A_f E_f}{0.85 f'_c b} \beta_1 d = 0$$

Substituting the appropriate parameters;

$$a^2 + 15.8a - 3081.7 = 0$$

$$a = 48.17 \text{ mm}$$

$$f_f = 0.0035 \times 147000 \times \frac{0.65 \times 300 - 48.17}{48.17} = 1568 \text{ MPa}$$

$$M_r = A_f f_f \left(d - \frac{a}{2} \right)$$

$$M_r = 502.6 \times 1568 \times \left(300 - \frac{48.17}{2} \right) = 217.44 \times 10^6 \text{ N.mm} = 217.44 \text{ kN.m}$$

$$P_e = 5.333 \frac{M_r}{l} = \frac{5.333 \times 217.44 \times 10^6}{3000} = 385.54 \times 10^3 \text{ N} = 385.54 \text{ kN}$$

For shear : Use No. 10 as stirrups, $A_v = 200 \text{ mm}^2$

$$R_A \times 3000 - 386.54 \times 10^3 \times 1500 + 217.44 \times 10^6 = 0$$

$$\therefore R_A = 120790 \text{ N} = 120.8 \text{ kN}$$

$$V_{max} = 386.54 - 120.8 = 265.7 \text{ kN}$$

$$V_c = 79.0 \text{ kN}$$

$$V_s = 265.7 - 79.0 = 186.7 < 4V_c \text{ o.k.}$$

$$S = \frac{\phi_s A_v f_y d}{V_s}$$

$$S = \frac{0.85 \times 200 \times 471 \times 300}{186.7 \times 10^3} = 129 \text{ mm}$$

$$S_{max} = \text{Min} \left\{ \frac{d}{2}, 600 \text{ mm} \right\} = 150 \text{ mm}$$

Use No. 10 @100mm as stirrups.

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IMAGE EVALUATION TEST TARGET (QA-3)

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