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System Performance Analysis With $\pi/4$ -QPSK In An Indoor Wireless Environment

by

Xiaoyu Wang M.Eng.

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of

Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University
Ottawa, Ontario, Canada, K1S 5B6

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**“System Performance Analysis With $\pi/4$ -QPSK
In An Indoor Wireless Environment”**

submitted by **Xiaoyu Wang**

in partial fulfillment of the requirements
for the degree of Master of Engineering

Dr. El-Tanany, Co-Supervisor

Dr. Mahmoud, Co-Supervisor

Chairman,
Department of Systems and Computer Engineering
Carleton University
November 20, 1994

Abstract

A transmitter and receiver design with $\pi/4$ -QPSK modulation is proposed for indoor wireless radio channels.

Over flat Rayleigh fading channels, three detection methods - limiter discriminator detection, delay detection and coherent detection - have been compared in terms of symbol error rate (SER) performance and circuit complexity. Coherent detection has the best performance among the three, but it is the most complex for implementation. Limiter discriminator detection and delay detection are found to be equivalent.

Over frequency selective Rayleigh fading channels, the delay detection has a SER over 0.1 due to severe intersymbol interference caused by the multipath characteristics of the channel. A maximum likelihood sequence estimation (MLSE) equalizer, using the Viterbi algorithm, is employed to improve the SER to 10^{-3} for a signal to noise ratio of 40 dB. The simulation results indicate that system performance improves with longer delay spreads, smaller fading rates, or lower order of Butterworth & Chebyshev 1 filters.

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List of Abbreviations and Acronyms

ACI	- Adjacent Channel Interference
ACT	- Adaptive Carrier Tracking Demodulator
AWGN	- Additive White Gaussian Noise
BER	- Bit Error Rate
BISDN	- Broadband Integrated Services Digital Network
BPF	- Bandpass Filter
BPSK	- Binary Phase Shift Keying
CCI	- Cochannel Interference
CIR	- Channel Impulse Response
DFE	- Decision Feedback Equalizer
DQPSK	- Differentially encoded Quadrature Phase Shift Keying
FFT	- Fast Fourier Transform
GMSK	- Gaussian filtered Minimum Shift Keying
GSM	- Global System for Mobile Telecommunications
ISI	- Intersymbol Interference
<i>kbps</i>	- 10^3 bits per second
<i>ksps</i>	- 10^3 symbols per second
LMS	- Least Mean Square
MLSE	- Maximum Likelihood Sequence Estimation
MMSE	- Minimum Mean Square Error
MSE	- Mean Square Error
MSK	- Minimum Shift Keying
NRZ	- Non-Returning Zero
<i>pdf</i>	- probability density function

PSK	- Phase Shift Keying
QAM	- Quadrature Amplitude Modulation
QPSK	- Quadrature Phase-Shift Keying
$\pi/4$-QPSK	- $\pi/4$ -shifted Quadrature Phase-Shift Keying
RLS	- Recursive Least Squares
SER	- Symbol Error Rate
SIR	- Signal to Interference Ratio
SNR	- Signal to Noise Ratio
SRRC	- Square Root Raised Cosine
TDMA	- Time Division Multiple Access
TIA	- Telecommunication Industry Association
VCO	- Voltage Controlled Oscillator
WSS	- Wide Sense Stationary

Mathematical Notation

The following symbols and notational conventions are used throughout the thesis. In some cases, the same symbol is used to denote different variables, but this should be obvious from the context of the discussion.

$\mathbf{A}$	- general matrix or vector
$\mathbf{A}^T$	- transpose of $\mathbf{A}$
$\mathbf{A}^*$	- conjugate of $\mathbf{A}$
$\mathbf{A}^{-1}$	- inverse of matrix $\mathbf{A}$
c	- speed of light (3×10^8 m/s)
$c(\tau; t)$	- time-varying channel impulse response
$E(\cdot)$	- expected value of the quantity within parenthesis
$e[k]$	- adaptive output error signal
$\exp(\cdot)$	- natural exponential function
f	- frequency variable
f_c	- carrier frequency
f_d	- maximum Doppler frequency shift
f_k	- tap coefficient of the channel estimator
f_{off}	- cutoff frequency of a filter
f_s	- symbol rate
g	- power ratio of the 2nd ray signal to 1st ray signal
I_k	- input data sequence of MLSE
$\hat{I}_k$	- estimated data sequence of MLSE
$Im(\cdot)$	- imaginary part of the quantity in parenthesis

$I_0(\cdot)$	- zeroth order modified Bessel function of the first kind
j	- imaginary number, square root of negative one
$J[k]$	- MSE function
$J_0(\cdot)$	- zeroth order Bessel function
k	- discrete time index
L_{data}	- length of the data segment of a frame
L_{frame}	- length of a frame
L_{train}	- length of a training segment
L	- channel memory length
$\ln(\cdot)$	- natural logarithm
$\log_{10}(\cdot)$	- logarithm to the base 10
$m(t)$	- received signal mean
n_k	- discrete noise sequence
$P(f)$	- frequency response of a filter
$p(t)$	- impulse response of a filter
$p(r_k)$	- probability density function of the random variables r_k
$\text{Re}(\cdot)$	- real part of the quantity in parenthesis
$r(t)$	- received signal envelope
$S(f)$	- received signal power spectrum
t	- continuous time
t_d	- delay between the 2nd ray and the main ray
T	- symbol period
v	- speed of portable
w	- weight factor of Kalman algorithm
$x_c(t)$	- received signal in-phase component
$x_s(t)$	- received signal quadrature component

$\alpha_n(t)$	- attenuation factor for n th path
β	- roll off factor of SRRC filter
Δ	- step size of LMS algorithm
$\delta(\tau)$	- Dirac impulse (delta) function
λ	- carrier wavelength
$\mu_k()$	- metric of Viterbi algorithm
ϕ_k	- received signal phase at kT instant
$\Delta\phi_k$	- signal phase shift between kT and $(k-1)T$
σ_n^2	- noise variance
σ_s^2	- average desired signal power
$\tau_n(t)$	- n^{th} path propagation time delay
$\sum_a^b$	- summation operator from a to b

Chapter 1

Introduction

Most current industry-driven research and development in wireless indoor communication is focused on frequency bands below 3 GHz. As the ever increasing demand for wireless access continues, new opportunities must be sought in higher bands, such as the 30 – 300 GHz millimeter bands [1]. These higher bands offer much wider information bandwidths and less spectrum congestion. The use of these bands also allows for wireless access to broadband integrated services at rates of tens or hundreds of megabits per second.

The characteristics of these channels and the currently available circuit technologies pose new challenges to the design of high speed transmission and reception strategies.

$\pi/4$ -QPSK is considered a suitable modulation scheme for an indoor wireless environment because of its power and spectral efficiency [2].

1.1 Thesis Objectives

This thesis proposes a transmitter and receiver design using $\pi/4$ -QPSK modulation. The main considerations are the choice of detection method and shaping filter. When the data rate is sufficiently high, the intersymbol interference severely degrades system performance unless a suitable equalization technique is applied. This thesis deals with the performance evaluation of a $\pi/4$ -QPSK modem that is suitable for implementation and use over millimeter bands.

1.2 Problem Statement

The three available detection methods, limiter-discriminator detection, delay detection and coherent detection, need to be investigated to show each method's advantages and disadvantages.

Two types of indoor wireless channels, the flat Rayleigh fading channel and the frequency selective Rayleigh fading channel, are investigated. For a selective Rayleigh

fading channel, because of the multipath characteristics and the Doppler frequency shift, the error performance of an unequalized system could be degraded significantly. Means of improving the system performance also need to be determined.

The use of a pair of square root raised cosine (SRRC) filters as the shaping filter in the transmitter and the post detection filter in the receiver can minimize the intersymbol interference. Because of its circuit complexity and implementation difficulty, the SRRC filters are not suitable for high frequency bands. The problem of whether conventional filter types, such as Butterworth filters, can be used as the shaping filter and the receiver filter is investigated, as well as what kind of performance that should be expected.

1.3 Summary of Results and Contributions

This thesis provides a transmitter and a receiver design with $\pi/4$ -QPSK modulation. In the receiver design, three detection methods are compared: limiter-discriminator detection, delay detection and coherent detection. Limiter-discriminator detection and delay detection are found to be equivalent. Comparison shows that coherent detector has a better SER performance (3 dB) than the other two methods. Since the coherent detection needs more complex circuitry, the delay detection is considered as a better choice.

System performance with a delay detector is studied in terms of SER, over a flat Rayleigh fading channel and a selective Rayleigh fading channel. For a flat Rayleigh fading channel, the delay detector performs very well, and the smaller fading rates yield better SER performance. For a frequency selective Rayleigh fading channel, the SER performance with a delay detector is not acceptable. Experiments show that a MLSE equalizer significantly improves performance.

For the above studies, SRRC filters are used as the shaping and receiver filters.

A system using 4th or 8th order Butterworth and Chebyshev type 1 filters as the shaping filter and the receiver filter respectively has also been studied. The results show that over a flat Rayleigh fading channel with 4th order filters, the delay detector gives the same level of performance as the MLSE equalizer does. Over a frequency selective

Rayleigh fading channel, the delay detector lost its capability for detection. The MLSE equalizer can improve the SER performance to 10^{-3} for a signal to noise ratio of 40 dB. The SER performance is found to be better with a smaller fading rate or a larger delay spread. For 8th order filters, the delay detector cannot perform acceptably in the two fading situations. A MLSE equalizer can be used to improve the SER performance.

1.4 Organization

Chapter 2 presents relevant background material and a survey of current publications in $\pi/4$ -QPSK modulation and equalization techniques.

Chapter 3 introduces the system model used in this thesis. Three detection methods, delay detection, limiter-discriminator detection and coherent detection, are presented. The operation of $\pi/4$ -QPSK modulation and MLSE equalization is described.

Chapter 4 gives the simulation background and describes the simulation of the system with BOSS (a Block Oriented System Simulator). The correctness of the simulation is verified by comparing the results with the published results under similar conditions and simulation parameters.

Simulation results for the performance of the system over flat and frequency selective Rayleigh fading channels are presented in Chapter 5. The impact of replacing the SRRC filter with a 4th or 8th order Butterworth or Chebyshev 1 filter is analyzed. The effects of fading rates and delay spreads on the system performance are also discussed.

The conclusions of this research and some questions for further research are given in Chapter 6.

Chapter 2

Background and Literature Survey

2.1 Characterization of Indoor Radio Channels

Indoor radio usually operates at frequencies between 30 and 300 GHz [3]. These channels provide a high information transport capability. They enable an indoor radio network to reach data rates up to 150 Mb/s, which can be used to access the Broadband Integrated Services.

Recently, extensive measurements have been carried out for indoor radio propagation characteristics over millimeter bands. They show that an indoor millimeter band channel can be statistically summarized as a time-varying multipath Rayleigh fading environment[3][4].

If a pulse is transmitted over a fading channel, the received signal appear as a train of pulses. This is called the *multipath* phenomenon, and is one of the characteristics of a fading channel.

The second characteristic is *time variation*. Because of the time variation in the structure of the medium, the amplitude, phase and delay of the received signal are also time-varying. Moreover, this time variation is unpredictable to the channel users. Therefore, it is reasonable to characterize the time-varying multipath channel statistically.

A signal can be represented, in general, as

$$s(t) = \text{Re} \left[u(t)e^{j2\pi f_c t} \right] \quad (2.1)$$

where $u(t)$ is the equivalent lowpass signal and f_c is the carrier frequency. $s(t)$ is to be transmitted over an indoor wireless channel.

The received signals are multiple propagation rays, and each ray has a different propagation delay and attenuation factor. Both the propagation delays and the attenuation

factors are time-varying. Thus the received bandpass signal can be expressed as

$$x(t) = \sum_n \alpha_n(t) s(t - \tau_n(t)) \quad (2.2)$$

where $\alpha_n(t)$ is the attenuation factor of the n th path and $\tau_n(t)$ is the propagation delay of the n th path. Substituting $s(t)$ in (2.2) with (2.1) yields

$$x(t) = \text{Re} \left(\left\{ \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} u[t - \tau_n(t)] \right\} e^{j2\pi f_c t} \right) \quad (2.3)$$

The equivalent lowpass received signal in (2.3) is

$$r(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} u[t - \tau_n(t)] \quad (2.4)$$

Since $r(t)$ is the response of a lowpass signal $u(t)$ transmitted through an equivalent lowpass channel, it follows that the equivalent low pass channel can be described by the time-varying impulse response

$$c(\tau; t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \delta(\tau - \tau_n(t)) \quad (2.5)$$

Thus, (2.5) is an appropriate definition of the equivalent lowpass impulse response for a fading channel.

Consider the transmission of an unmodulated carrier at frequency f_c . Then $u(t) = 1$ for all t , and the received signal given by (2.4) becomes

$$\begin{aligned} r(t) &= \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)} \\ &= \sum_n \alpha_n(t) e^{-j\theta_n(t)} \end{aligned} \quad (2.6)$$

where $\theta_n(t) = 2\pi f_c \tau_n(t)$ is the phase of the n th path signal. Thus, the received signal is the sum of n time-varying vectors with amplitudes $\alpha_n(t)$ and phases $\theta_n(t)$. It can be seen that $\theta_n(t)$ will change by 2π whenever τ_n changes by $1/f_c$. Since $1/f_c$ is a small number and $\theta_n(t)$ can change by 2π with relatively small variations of the medium. It is expected that the delay τ_n of every path will change at a different rate and in an

unpredictable manner. This implies that the received signal $r(t)$ in (2.6) can be modeled as a random process. When there are a large number of paths, $r(t)$ can be modeled as a complex-valued Gaussian random process by applying the central limit theorem [5]. The multipath propagation channel embodied in the received signal $r(t)$, given in (2.6), results in *signal fading*. This *fading* phenomenon is primarily a result of the time variations in the phase $\theta_n(t)$ and amplitude $\alpha_n(t)$. That is, the randomly time-varying phase $\theta_n(t)$ and amplitude $\alpha_n(t)$ can result in the paths add destructively which makes the received signal $r(t)$ weaker. At other times the paths adding constructively which makes the received signal stronger.

When the impulse response $c(\tau; t)$ of the channel is modeled as a zero mean complex-valued Gaussian process, the envelope $|c(\tau; t)|$ at any instant t is Rayleigh-distributed. In this case, the channel is called a *Rayleigh fading channel*. When the mean of $c(\tau; t)$ can no longer be modeled as zero, the envelope $|c(\tau; t)|$ has a Ricean distribution and the channel is termed a *Ricean fading channel*.

Fading channels can be characterized according to the channel impulse response $c(\tau; t)$. Its autocorrelation can be shown as

$$\phi_c(\tau_1; \tau_2; \Delta t) = \frac{1}{2} E(c^*(\tau_1, t) c(\tau_2; t + \Delta t)) \quad (2.7)$$

Because the phase, amplitude, and delay in each scattered ray are uncorrelated, the autocorrelation function can be expressed as a function of the difference in the delays. That is

$$\begin{aligned} \phi_c(\tau_1; \tau_2; \Delta t) &= \phi_c(\tau_1 - \tau_2; \Delta t) \\ &= \phi_c(\tau; \Delta t) \delta(\tau_1 - \tau_2) \\ &= \phi_c(\tau; \Delta t) \delta(\tau - \tau_0) \end{aligned} \quad (2.8)$$

where $\tau_0 = \tau_1 - \tau_2$.

If $\Delta t = 0$ then $\phi_c(\tau; \Delta t) = \phi_c(\tau)$. This is the average power output of the channel as a function of the time delay τ .

As with any autocorrelation function, $\phi_c(t)$ has its maximum at $\tau = 0$. The value of τ at which $\phi_c(t)$ is zero is called the *multipath spread* or *delay spread* T_m of the channel.

The Fourier transform of $\phi_c(\tau; \Delta t)$ in the variable τ is defined as $\phi_C(\Delta f; \Delta t)$, also called the space-frequency, space-time correlation function.

If $\Delta t = 0$ then $\phi_C(\Delta f; \Delta t) = \phi_C(\Delta f)$. $\phi_C(\Delta f)$ is known as the space-frequency correlation function of the channel. The range of Δf over which $\phi_C(\Delta f)$ is non-zero is referred to as the *channel coherence bandwidth* $(\Delta f)_c$. It is approximately equal to the reciprocal of the channel delay spread T_m , $(\Delta f)_c \approx \frac{1}{T_m}$.

When the bandwidth of the transmitted signal is smaller than $(\Delta f)_c$, the channel is said to be a *frequency non-selective* or *flat fading* channel. Otherwise, the channel is said to be a *frequency selective* fading channel.

2.1.1 Tapped Delay Line Channel Model

A time-varying frequency selective fading channel can be modelled as a tapped delay line with tap spacing $1/W$ and weight coefficients $\{c_n(t)\}$. The channel estimator used in the maximum likelihood sequence estimation (MLSE) equalizer is based on this model.

For a frequency selective fading channel, the band-limited equivalent lowpass signal $u(t)$ can be represented as

$$u(t) = \sum_{n=-\infty}^{\infty} u\left(\frac{n}{W}\right) \frac{\sin \pi W(t - n/W)}{\pi W(t - n/W)} \quad (2.9)$$

by the sampling theorem (Section 7.5 in [5]), where W is the bandwidth of $u(t)$. The Fourier transform of $u(t)$ is

$$\begin{aligned} U(f) &= \frac{1}{W} \sum_{n=-\infty}^{\infty} u\left(\frac{n}{W}\right) e^{-j2\pi f n/W} & |f| \leq W/2 \\ U(f) &= 0 & |f| > W/2 \end{aligned} \quad (2.10)$$

The received signal of (2.3) can be expressed as

$$r(t) = \int_{-\infty}^{\infty} C(f; t) U(f) e^{j2\pi f t} df \quad (2.11)$$

where $C(f; t)$ is the Fourier transform of the equivalent lowpass impulse response $c(\tau; t)$ of the fading channel. Substitution of $U(f)$ from (2.10) into (2.11) yields

$$r(t) = \frac{1}{W} \sum_{-\infty}^{\infty} u\left(\frac{n}{W}\right) c\left(t - \frac{n}{W}; t\right) \quad (2.12)$$

(2.12) has the form of a convolution sum. Hence it can be expressed in the alternative form

$$r(t) = \frac{1}{W} \sum_{-\infty}^{\infty} u\left(t - \frac{n}{W}\right) c\left(\frac{n}{W}; t\right) \quad (2.13)$$

By defining a set of time-variable channel coefficients as

$$c_n(t) = \frac{1}{W} c\left(\frac{n}{W}; t\right) \quad (2.14)$$

and using these channel coefficients, (2.13) gives

$$r(t) = \sum_{-\infty}^{\infty} u\left(t - \frac{n}{W}\right) c_n(t) \quad (2.15)$$

(2.15) for the received signal shows that the time-varying frequency selective channel can be modeled as a tapped delay line with tap spacing $1/W$ and tap weight coefficients $\{c_n(t)\}$. For all practical purposes the tapped delay line model for a selective fading channel can be truncated at $[T_m W] + 1$ taps (Figure 2.1), where T_m is the total multipath spread of the channel. $[T_m W]$ is called the memory length of the channel, denoted as L .

2.2 Choice of Modulation Methods

Because of the fading and multipath characteristics of indoor radio channels, a suitable modulation scheme should satisfy the following requirements[6]:

- (1) A power efficient amplifier may be used without affecting the efficiency gained by the modulation scheme.
- (2) Extremely low out-of-band power should be achieved after nonlinear power efficient amplification.

Figure 2.1 The tapped delay line model of a frequency selective channel.

(3) Synchronization should be achieved quickly, and should be immune to fast fading.

(4) It should permit adaptive equalization or other techniques for mitigating the effects of delay spread and fading to be implemented.

Power and bandwidth efficient digital communication systems traditionally use coherent detection. Although coherent systems perform well in stationary additive white Gaussian noise (AWGN) environments, and have a theoretical 1 to 3dB power efficiency advantage in Rayleigh and Ricean faded mobile systems, their performance may degrade significantly when disturbances such as multipath fading, Doppler shifts and other excessive forms of phase noise are present. These effects have been taken into consideration in the design of new, relatively narrow band digital cellular and digital mobile radio communication systems, i.e., systems which operate in the 1.2 to 50 kb/s range and have a Doppler shift in the range of 15 to 100Hz.

The best known methods of reducing envelope fluctuations with linear modulation are offset QPSK and QAM. However, this type of modulation may require coherent demodulation.

Constant envelope modulation techniques such as MSK and GMSK can be nonlinearly amplified without significant spectral regeneration, and can be differentially or discriminator detected. The disadvantage of these modulation techniques is that they may have a low spectral efficiency which does not always meet the requirements of standards for digital radio systems [6]. Attention is presently focused on the use of linear modulation techniques for nonlinearly amplified systems in order to meet the simultaneous power and spectral efficiency requirements.

In the search for alternative linear modulation systems which have low envelope fluctuation, Akaiwa and Nagata [2] and many others have studied $\pi/4$ -QPSK, a technique first introduced by Baker [7] in 1962. This modification of QPSK has carrier phase transitions which are restricted to $\pm\pi/4$ and $\pm3\pi/4$. As the phase does not undergo instantaneous $\pm\pi$ transitions as in QPSK, the envelope fluctuation is significantly reduced. Also, coherent as well as noncoherent detection can be applied to $\pi/4$ -QPSK. It has been

shown by Akaiwa and Nagata [2] that the spectral efficiency obtained by $\pi/4$ -QPSK is double that obtained by comparable two-level digital FM, GMSK, or other constant envelope modulation techniques.

Due to these attractive features of $\pi/4$ -QPSK, it has been chosen as the standard modulation technique for the emerging American Digital Cellular (ADC) second generation radio system [6].

2.3 MLSE Equalizer

Many adaptive equalizer structures have been proposed during the past three decades, such as the linear transversal equalizer (LTE), the decision feedback equalizer (DFE), the lattice equalizer and the maximum likelihood sequence estimator equalizer (MLSE) [5]. Various algorithms have been implemented using these equalizers structures, such as the least mean square (LMS) algorithm and many types of recursive least square (RLS) algorithms.

Among all equalizer structures and algorithms, the DFE based on the RLS algorithm, and the MLSE equalizer based on the Viterbi algorithm, are recommended for mobile communication systems [8]. W. Huang and T. S. Rappaport studied these two equalizers. Their results show that the performance of the MLSE and DFE equalizers are similar. This thesis considers only the MLSE equalizer with the Viterbi algorithm.

2.3.1 MLSE Equalizer

A MLSE equalizer is comprised of the Viterbi algorithm and a channel estimator, as shown in Figure 2.2.

Figure 2.2 MLSE equalizer with Viterbi algorithm

A sequence of information data is transmitted through a non-ideal band-limited channel with additive Gaussian noise. Let I_k represent the information symbol at the k th sampling instant. The received baseband signal sequence would be

$$r_k = \sum_{j=0}^L f_j I_{k-j} + n_k \quad (2.16)$$

where $\{f_k\}$ is the set of tap coefficients of the equivalent discrete time channel, $\{n_k\}$ is the additive Gaussian noise sequence, $\{r_k\}$ is a Gaussian variable with mean $\sum_{j=0}^L f_j I_{k-j}$, and L is the channel memory length, as shown in Figure 2.3. Therefore the joint probability density function of the random variables $r_1, r_2, \dots, r_N \equiv R_N$, conditioned on the transmitted data sequence $I_1, I_2, \dots, I_p \equiv \Upsilon_P$, is

$$p(R_N | \Upsilon_P) = K \exp \left[-\frac{1}{N_0} \sum_{k=0}^N \left[r_k - \sum_{j=0}^L f_j I_{k-j} \right]^2 \right] \quad (2.17)$$

The logarithm of $p(R_N | \Upsilon_P)$ is proportional to the quantity $J(\Upsilon_P)$, defined as

$$J(\Upsilon_P) = \sum_{k=0}^N \left[r_k - \sum_{j=0}^L f_j I_{k-j} \right]^2 \quad (2.18)$$

Fig. 2.3 The equivalent discrete time channel model with AWGN.

The maximum likelihood estimation of the symbol sequence $I_1, I_2, \dots, I_p$ is sequence of symbols that maximize $p(R_N | Y_P)$, or equivalently, minimize $J(Y_P)$. The method for determining the particular sequence $I_1, I_2, \dots, I_p$ which minimizes the quantity $J(Y_P)$ is an exhaustive search. That is, evaluate $J(Y_P)$ for all possible sequences and select the sequence that yields the minimum $J(Y_P)$. This signal processing method has been termed maximum likelihood sequence estimation (MLSE).

2.3.2 Viterbi Algorithm

An efficient method for implementing MLSE processing is the Viterbi algorithm.

The Viterbi algorithm was originally proposed for decoding convolutional codes by A. Viterbi [9]. Soon thereafter, the algorithm was used to yield maximum likelihood sequence decisions. This can be regarded as a specific form of dynamic programming. Its applicability to receivers over channels with intersymbol interference was noticed by Omura [10]. The adaptive version of the maximum likelihood receiver with the Viterbi

algorithm was proposed by Forney [11] and Ungerboeck [12].

The decision variables or metrics that are computed for the MLSE equalizer of the sequence $\{I_k\}$ are given by (2.18). The metrics in (2.18) can be computed recursively.

If a sequence $\{I_k\}$ of N symbols is transmitted, where each symbol is chosen from an M -symbol alphabet, there are M^N possible sequences and, hence, M^N metrics. The sequence which gives minimum metric is selected as the detected sequence. However, not all M^N metrics need be computed, since the finite-duration discrete time channel model shown in Figure 2.3 is equivalent to a convolutional encoder operating on the information sequence. Therefore, the Viterbi algorithm for decoding convolutional codes is an efficient method for optimum sequence estimation.

The Viterbi algorithm used in the simulation is described as follows [12]. Suppose a sequence $R_N = r_1, r_2, \dots, r_N$ is received, where r_k is given by (2.16). The algorithm is designed to estimate the input sequence $\xi_N = I_1, I_2, \dots, I_N$. The decision, based on the maximum likelihood criterion, is made in favor of the sequence ξ_N that maximizes the joint probability density function

$$p(R_N | \xi_N) = p(r_N, r_{N-1}, \dots, r_1 | I_N, I_{N-1}, \dots, I_1) \quad (2.19)$$

Since the additive noise sequence $\{n_k\}$ is white, the joint probability density $p(R_N | \xi_N)$ can be expressed as a product of marginal densities, i.e.,

$$p(R_N | \xi_N) = \prod_{k=1}^N p(r_k | I_k, I_{k-1}, \dots, I_{k-L}) \quad (2.20)$$

where, by definition, $I_k = 0$ for $k \leq 0$.

When there is no intersymbol interference, the signal r_k only depends on the symbol I_k and noise n_k . Therefore, the maximum of $p(R_N | \xi_N)$ over the symbol ξ_N is equivalent to maximizing each of the marginal densities $p(r_k | I_k)$. In other words, a sequence detection based on the maximum likelihood criterion is reduced to simple symbol-by-symbol detection. More importantly, however, is the fact that only M metrics are computed per signal interval and, hence, only MN metrics need be computed for the detection of N symbols instead of M^N .

In the presence of intersymbol interference that spans L symbols (L interfering components), the maximization of $p(R_N | \xi_N)$ can be performed recursively and efficiently by computing NM^{L+1} metrics. For sequence $r_1, r_2, \dots, r_{L+1}$, M^{L+1} joint probabilities

$$p(r_{L+1}, \dots, r_1 | I_{L+1}, \dots, I_1) = \prod_{k=1}^{L+1} p(r_k | I_k, I_{k-1}, \dots, I_{k-L}) \quad (2.21)$$

should be computed corresponding to the M^{L+1} possible sequences for the symbols $I_{L+1}, \dots, I_1$. In practice, multiplication of the probabilities in (2.21) can be avoided by taking the natural logarithm of both sides of the equation. It is observed that the next symbol r_{L+2} does not depend on the first symbol I_1 . This fact indicates that only those sequences which have the largest probability with respect to I_1 should be kept, and the others discarded. The M^{L+1} possible sequences of $I_{L+1}, \dots, I_1$ are subdivided into M^L groups correspondent to the possible choices of the symbols $I_{L+1}, \dots, I_2$. In each of these groups there are M sequences that differ only in the symbol I_1 . From the M sequences in each group, the sequence with the largest probability (with respect to I_1) is selected, and is represented by the following metric

$$\mu_1(\xi_{L+1}) \equiv \mu_1(I_{L+1}, \dots, I_1) = \max_{I_1} \sum_{k=1}^{L+1} \ln p(r_k | I_k, \dots, I_{k-L}) \quad (2.22)$$

The $M - 1$ other sequences in each M^L group are discarded. Thus M^L surviving sequences with metrics $\mu_1(I_{L+1})$ are left. At this point, if all M^L surviving sequences have the same value of I_1 , that value is the decision on I_1 . Otherwise the decision on I_1 can only be decided later.

Upon reception of r_{L+2} , the probabilities for $I_{L+2}, \dots, I_1$, or, equivalently, the logarithms of the probabilities, are computed as follows:

$$\begin{aligned} \ln p(r_{L+2}, \dots, r_1 | I_{L+2}, \dots, I_1) &= \ln p(r_{L+2}, \dots, r_2 | I_{L+2}, \dots, I_1) \\ &+ \sum_{k=1}^{L+1} \ln p(r_k | I_k, I_{k-1}, \dots, I_{k-L}) \end{aligned} \quad (2.23)$$

(2.23) gives the probabilities of M^{L+2} sequences of symbols $I_{L+2}, \dots, I_1$. So the maximal probability with respect to I_2, I_1 is computed by the following:

$$\begin{aligned} \max_{I_2, I_1} \ln p(r_{L+2}, \dots, r_1 | I_{L+2}, \dots, I_1) &= \max_{I_2} \ln p(r_{L+2}, \dots, r_2 | I_{L+2}, \dots, I_1) \\ &+ \max_{I_2, I_1} \sum_{k=1}^{L+1} \ln p(r_k | I_k, I_{k-1}, \dots, I_{k-L}) \end{aligned} \quad (2.24)$$

Since $r_{L+2}, \dots, r_2$ has nothing to do with I_1 , $\max_{I_2, I_1} \ln p(r_{L+2}, \dots, r_2 | I_{L+2}, \dots, I_1) = \max_{I_2} \ln p(r_{L+2}, \dots, r_2 | I_{L+2}, \dots, I_1)$. It should be noted that the summation of probabilities of the right hand side of (2.23) was computed previously in (2.21) with respect to I_1 . According to the principle of dynamic programming,

$$\begin{aligned} &\max_{I_2, I_1} \sum_{k=1}^{L+1} \ln p(r_k | I_k, I_{k-1}, \dots, I_{k-L}) \\ &= \max_{I_2} \left\{ \max_{I_1} \sum_{k=1}^{L+1} \ln p(r_k | I_k, I_{k-1}, \dots, I_{k-L}) \right\} \\ &= \max_{I_2} \mu_1(\xi_{L+1}) \end{aligned} \quad (2.25)$$

Again, the sequence which has the largest probability for I_2, I_1 has a metric $\mu_2(\xi_{L+2})$

$$\mu_2(\xi_{L+2}) \equiv \mu_2(I_{L+2}, \dots, I_1) = \max_{I_2, I_1} \ln p(r_{L+2}, \dots, r_1 | I_{L+2}, \dots, I_1) \quad (2.26)$$

(2.24) can be represented as

$$\mu_2(\xi_{L+2}) = \max_{I_2} [\ln p(r_{L+2}, \dots, r_2 | I_{L+2}, \dots, I_2) + \mu_1(\xi_{L+1})] \quad (2.27)$$

The same procedure may be used for computing $\ln p(r_{L+2}, \dots, r_2 | I_{L+2}, \dots, I_2)$.

$2M^{l+1}$ computations are needed to find $\mu_2(\xi_{L+2})$. If all M^L surviving sequences terminate with the same value of I_1 , a decision is made on symbol I_1 . Otherwise, the decision is deferred to a later stage.

This procedure continues with the reception of subsequent signal samples. Upon reception of r_{L+N} , the metric

$$\mu_N(\xi_{L+N}) = \max_{I_N} [\ln p(r_{L+N}, \dots, r_N | I_{L+N}, \dots, I_N) + \mu_{N-1}(\xi_{L+N-1})] \quad (2.28)$$

is used to compute the most probable sequence of $I_{L+N}, I_{L+N-1} \dots I_2, I_1$. $\mu_{N-1}(\xi_{L+N-1})$ is the metric with which the most probable sequence $I_{L+N-1}, I_{L+N-2} \dots I_2, I_1$ is found from $(N-1)M^{L+1}$ computations.

The computation time is NM^{L+1} to find the most probable sequence of $I_{L+N}, I_{L+N-1} \dots I_2, I_1$. Hence, a total of NM^{L+1} computations are needed for the detection of N symbols, for each symbol with M possible status.

As indicated above, the delay in detecting each information symbol is variable. In practice, a fixed delay is used by truncating the surviving sequences to the q most recent symbols where $q \gg L$. When the M^{L+1} surviving sequences of $\mu_{k+q}(\xi_{L+k+q})$ disagree on symbol I_k , the symbol with the highest probability is chosen.

2.3.3 Channel Estimator

The MLSE equalizer implemented via the Viterbi algorithm, as embodied in the metric computation given by (2.28), requires the coefficients $\{f_k\}$ of the equivalent discrete time channel. To accommodate a channel that is unknown or slowly time-varying, a channel estimator, connected in parallel with the detector, should be included [13], as shown in Figure 2.2.

The channel estimator is shown in Figure 2.4. It is a replica of the equivalent discrete time channel which models the intersymbol interference.

There are two approaches to adjusting the coefficients of the channel estimator. One is a statistical approach which uses the steepest descent algorithm to minimize the mean square error between the received data and the estimated data. Another is the least square approach which uses the least square algorithm to minimize the squared received data.

Steepest Descent Algorithm

The steepest descent algorithm minimizes the mean square error (MSE) between the actual received sequence $\{r_{k-q}\}$ and the estimated sequence $\{\hat{r}_{k-q}\}$ formed from the output of the Viterbi algorithm. The speed of convergence and the accuracy of the estimation are

controlled by the value of the adjustment parameter Δ in the same manner as a standard linear equalizer. A delay q , equal to the decision delay, is required to align the received sequence $\{r_k\}$ and the estimated sequence $\{\hat{r}_{k-q}\}$.

Every T seconds, a new bit of data r_{k-q} and a new information symbol $\hat{I}_{k-q}$ are fed into the channel estimator. An estimate $\hat{r}_{k-q}$ is formed by multiplying the sequence of symbols $\hat{I}_j$ stored in the tapped delay line by the corresponding tap gains $\hat{f}_j$ and by summing up the $L + 1$ products. An error signal $e^{(k)}$ is formed by taking the difference between the actual data sample r_{k-q} and the estimated data $\hat{r}_{k-q}$. This error, scaled by a factor Δ , is then multiplied by the symbol $\hat{I}_j$ at the appropriate tap. The resulting products are added to the corresponding values of the previous tap gains to form a new set of tap gains according to the following equation:

$$f_j^{(k+1)} = f_j^{(k)} + \Delta e^{(k)} \hat{I}_{k-q}, \quad j = 0, 1, \dots, L \quad (2.29)$$

In the receiver of Figure 2.2, these new tap gains are fed into the Viterbi Algorithm for the next computation.

(2.29) is the *steepest descent algorithm* for adjusting the tap gains of the channel estimator. The channel estimator with this algorithm is capable of tracking low variations in the channel response. It can be shown [13] that when the MSE between the received data r_k and the estimated data $\hat{r}_k$ is minimized, the resulting values for the tap gains of the channel estimator are the values of the sampled channel impulse response.

In the preceding discussion, the estimated information sequence at the output of the Viterbi algorithm is used in making adjustments to the channel estimator. For the start-up operation, a short training sequence should be sent to initially setting the tap gains.

The main advantage of the steepest descent algorithm is its computational simplicity. However, the price paid for this simplicity is slow convergence because the algorithm has only a single adjustable parameter for controlling the convergence rate.

Recursive Least Squares (Kalman) Algorithm

Indoor wireless channels suffer from the fading of multipath components due to receiver

Figure 2.4 The channel estimator structure

motion. This requires an adaptive equalizer with sufficient tracking abilities to follow these channel changes.

Because the least square approach can obtain faster convergence than the statistical approach, it is suitable for tracking indoor wireless channels.

It is convenient to express the recursive least square (RLS) algorithm in matrix form. The estimate of the information symbol from a linear equalizer can be defined as

$$\hat{I}(t) = \sum_{j=-K}^K c_j(t-1)v_{t-j} \quad (2.30)$$

where t is the current instant, $t-1$ is the previous instant and the range for j is 0 to $N-1$. Define

$$y(t) = v_{t+K} \quad (2.31)$$

then the estimate $\hat{I}(t)$ becomes

$$\begin{aligned} \hat{I}(t) &= \sum_{j=0}^{N-1} c_j(t-1)y(t-j) \\ &= C_N'(t-1)Y_N(t) \end{aligned} \quad (2.32)$$

where $C_N'(t-1)$ is the column vector of the equalizer coefficients $c_j(t-1)$, $j = 0, 1, \dots, N-1$, and $Y_N(t)$ is the column vector of the received signals $v(t-j)$, $t, j = 0, 1, \dots, N-1$.

Suppose the previous estimated coefficient vector is $C_N(t-1)$ and the inverse of the correlation matrix is $P_N(t-1)$. When a new signal v_{t+K} is received, $Y_N(t)$ is updated. The recursive computation to update $C_N(t)$ and $P_N(t)$ proceeds as follows:

symbol estimate: $\hat{I}(t) = Y_N'(t)C_N(t-1)$

error: $\epsilon_N(t) = I(t) - \hat{I}(t)$

Kalman gain vector:

$$K_N(t) = \frac{P_N(t-1)Y_N^*(t)}{w + Y_N'(t)P_N(t-1)Y_N^*(t)} \quad (2.33)$$

Update inverse of the correlation matrix:

$$P_N(t) = \frac{1}{w} \left[P_N(t-1) - K_N(t) Y_N'(t) P_N(t-1) \right] \quad (2.34)$$

Update coefficients:

$$\begin{aligned} C_N(t) &= C_N(t-1) + K_N(t) e_N(t) \\ &= C_N(t-1) + P_N(t) Y^*(t) e_N(t) \end{aligned} \quad (2.35)$$

where w is a weighting factor, $0 < w < 1$. The algorithm described above is called the RLS direct form or Kalman algorithm.

It is noted that the equalizer coefficients $C_N(t)$ change with time by an amount equal to the product of the error $e_N(t)$ and the Kalman gain vector $K_N(t)$. Since $K_N(t)$ is N -dimensional, each tap coefficient of the channel estimator is controlled by one element of $K_N(t)$. As a result, rapid convergence is obtained. This is especially important in tracking a time-varying channel. In contrast, the steepest descent algorithm, expressed in our present notation, is

$$C_N(t) = C_N(t-1) + \Delta Y_N^*(t) e_N(t) \quad (2.36)$$

The only variable parameter is the step size Δ .

In starting the algorithm, a unit vector for $C_N(t)$ is used.

2.4 Review of Current Research

This section reviews the detection and equalization techniques which have been proposed in the literature with $\pi/4$ -QPSK modulation.

2.4.1 Detection Techniques

The application of $\pi/4$ -QPSK for land mobile channels was proposed by Y. Akaiwa and Y. Nagata [2]. The receiver consists of a noncoherent detector using a limiter-discriminator and an integrate-and-dump filter. The BER performance of the four-level

detection, under static and fading conditions, is comparable to the BER of conventional digital frequency modulation systems. Their results prove that the linear modulation scheme $\pi/4$ -QPSK has twice the spectral efficiency of conventional digital FM in mobile radio channels and has a high immunity against fast fading.

C. L. Liu and K. Feher [14], and Goode, Kazecki, Dennis [15] compare limiter-discriminator detection, delay detection and coherent detection in static and Rayleigh fading channels. The main results of their comparisons are: The limiter-discriminator detector and the delay detector have similar BER performances; The delay detector is the most straight-forward detector to implement; The coherent detector gives the best performance in static and flat Rayleigh fading channels, but its implementation is the most complex because of carrier recovery. Their simulation shows that in a flat fading channel, a Doppler shift causes severe error floors, but in a frequency selective fading channel, the delay spread is the main cause of performance degradation.

S. Chen, Xueshu and G. J. Saulnier [16] used a delay detector over a flat Rayleigh fading channel. They found that the BER performance with a delay detector is sensitive to the sampling timing error and to the large carrier frequency offset, but insensitive to the rolloff factor of the SRRC filter. They proposed an algorithm for the correction of the sampling timing error and carrier frequency offset to improve the performance.

Y. Yamao, S. Saito, H. Suzuki and T. Nojima [17] compare QPSK and $\pi/4$ -QPSK modulation schemes. They used a postdetection selection diversity to improve the performance in a fading environment. Their results show that $\pi/4$ -QPSK has a slight advantage over QPSK for the power amplifier implementation. An adaptive carrier tracking demodulator is more suitable for mobile or handheld units because of the low power consumption, and a differentially coherent demodulator is better for the base station equipment.

V. Fung and T.S. Rappaport [18] also use a delay detector to study the BER performance over flat Rayleigh fading and frequency selective Rayleigh fading channels. Their results show that the BER performance over a flat fading channel is a function of

the signal to noise ratio and the receiver speed. In frequency selective Rayleigh fading channels, for the low power ratio of the main ray to second ray C/D ($C/D < 25$ dB), the BER performance is determined by the normalized delay τ/T , where τ is the second ray delay, and T is the symbol period. For high C/D , the BER is determined by the receiver speed.

C.L. Liu and K. Feher [19] present a theoretical analysis of BER performance for a $\pi/4$ -QPSK modem with delay detection over a frequency selective Rayleigh fading channel. Based on the probability density function of the phase difference between two consecutive symbols of a $\pi/4$ -QPSK signal, they develop a mathematical expression of BER. Co-channel interference is also considered.

2.4.2 Equalizers

An indoor wireless channel degrades the performance of a communication system by introducing intersymbol interference. However, equalization techniques can be used to mitigate this performance degradation. An adaptive equalizer updates its coefficients based on a recursive algorithm so that it can work in an unknown and time-varying indoor wireless radio channel.

S. Goode, H. Kazecki and D. Dennis [15] used a DFE to improve the performance of a two-ray fading channel. In order to train and follow the rapid channel variations, the RLS algorithm is used to update the equalizer's coefficients. Their results show that the equalizer performs better for larger main ray to second ray signal ratios, and performance with an equalizer is quite close to that with a coherent detector.

W. Huang and T.S. Rappaport [20] studied two adaptive equalizers, the DFE and the MLSE. Their system uses the $\pi/4$ -QPSK modulation scheme over a two-ray channel model. They found that, without equalizers the system error floor is about 0.2, but the two equalizers can reduce the error floor to the 10^{-3} range. The DFE and the MLSE are found to have comparable performance levels. Their simulation also indicate the equalizers work best when the mean power of different rays are equal or close.

T. Kohama, H. Kondoh and Y. Akaiwa [21] proposed a nonlinear equalizer ATDES (adaptive threshold detector with estimated sequence) for a system with $\pi/4$ -QPSK modulation scheme. The system uses a hard-limiter and a delay detector with a pair of SRRC filters over a two-ray channel. With equalization, the BER error floor is reduced from 10^{-1} to 10^{-2} .

2.4.3 Conclusion of Survey

The above survey indicates extensive research has been done for $\pi/4$ -QPSK systems which use a pair of SRRC filters as the shaping and receiver filters. Most of the research uses the delay detector over flat Rayleigh fading channels.

However, there is serious performance degradation with a delay detector over a frequency selective Rayleigh fading channel. From the survey, equalization is considered an efficient method to compensate for the degradation. DFE and MLSE equalizers[15][8] have been studied for $\pi/4$ -QPSK modulation over frequency selective Rayleigh fading channels. The research results are very impressive. But the results only show one aspect of the equalization. Goode, Kazecki and Dennis's results show the comparative performance between the DFE and the coherent detector. Huang and Rappaport's results show the performance of DFE and MLSE equalizers as a function of fading rates. It will be worthwhile to have a comprehensive investigation of the effects of fading rate and delay spread on the performance.

All the filters used in past research are SRRC filters. Indoor wireless systems use millimeter bands, and their data rates reach hundreds of megabits per second. Considering the circuit implementation difficulty for SRRC filters in these frequency bands and data rate, the possibility of using a conventional filter as the shaping and receiver filters is also a practical and unexplored topic.

Chapter 3

System Model

This chapter describes the system model in detail. The transmitter uses $\pi/4$ -QPSK modulation. In the receiver, three detection methods, limiter-discriminator detection, delay detection and coherent detection, are described. The Rayleigh fading channel simulator proposed by Jakes [22] is used. Because of the complexity of the Viterbi algorithm, an efficient implementation of the MLSE equalizer is developed. In the last section, the system over an indoor wireless channel is presented.

3.1 Transmitter with $\pi/4$ -QPSK Modulation

The block diagram of the transmitter model with $\pi/4$ -QPSK modulation is shown in Figure 3.1.

The two independent *Data Source* blocks generate two random binary data sequences: inphase S_i and quadrature S_q . The data rate of S_i and S_q is called the *symbol rate* of the system.

The *Signal Mapping* block transforms S_i and S_q into a phase signal $\Delta\phi_k$, according to Table 3.1.

S_i	S_q	$\Delta\phi_k$
1	1	$\pi/4$
1	-1	$-\pi/4$
-1	1	$3\pi/4$
-1	-1	$-3\pi/4$

Table 3.1 Phase table for $\pi/4$ -QPSK

Figure 3.1 The transmitter with $\pi/4$ -QPSK modulation.

Figure 3.2 Possible phase states of a $\pi/4$ -QPSK modulated baseband signal at sampling instants.

The outputs of *Signal Mapping* are an inphase impulse a_k and a quadrature impulse b_k , at $t = kT$, where T is the symbol period. a_k and b_k are determined recursively by the previous impulse a_{k-1} , b_{k-1} and the current phase signal $\Delta\phi_k$ as following:

$$\begin{aligned} a_k &= a_{k-1} \cos \Delta\phi_k - b_{k-1} \sin \Delta\phi_k \\ b_k &= a_{k-1} \sin \Delta\phi_k + b_{k-1} \cos \Delta\phi_k \end{aligned} \quad (3.1)$$

In the $a - b$ coordinate, a_k and b_k are always one of the eight positions marked by "x", shown in Figure 3.2. In contrast to the conventional QPSK with four phase states, $\pi/4$ -QPSK has eight different phase states. Their carrier phase changes between the current symbol and the previous symbol can only be $\pm\pi/4$ or $\pm3\pi/4$.

Suppose that initially $a_0 = 0$ and $b_0 = 1$. The phase is ϕ_0 in Figure 3.2. Now, symbol $(S_i, S_q) = (1, 1)$ is received from the *Data Source* at $t = T$. $\Delta\phi_1$ is $\pi/4$ according to Table 3.1. Then $a_1 = -1/\sqrt{2}$ and $b_1 = 1/\sqrt{2}$, according to (3.1), the phase state is moved to ϕ_1 , rotated $\pi/4$.

Each *Shaping Filter* in Figure 3.1 is a lowpass filter, which is used to limit the bandwidth of the $\pi/4$ -QPSK modulated signal. The shaping filter used in this thesis is SRRC, Butterworth or Chebyshev type 1 filter.

From [5], the raised cosine spectral function is given by

$$X(f) = \begin{cases} T & 0 \leq |f| \leq (1 - \beta)/2T \\ \frac{T}{2} [1 - \sin \{ \pi T(f - \frac{1}{2\pi})/\beta \}] & (1 - \beta)/2T \leq |f| \leq (1 + \beta)/2T \end{cases} \quad (3.2)$$

where β is the rolloff factor. Its raised cosine time function $x(t)$ is

$$x(t) = \frac{\sin \pi t/T}{\pi t/T} \frac{\cos \beta \pi t/T}{1 - 4\beta^2 t^2/T^2} \quad (3.3)$$

which is shown in Figure 3.3. It can be seen that $x(t)$ is equal to 0 at $t = kT$, where k is any integer except 0. This means that a raised cosine filtered baseband signal, when sampled at $t = kT$, is free from ISI.

Transformation of the square root of the raised cosine spectral function $X(f)$ into a time function yields the SRRC time function $p(t)$:

$$p(t) = \begin{cases} 1 - \beta + \frac{4\beta}{\pi} & t = 0 \\ \frac{\beta}{\sqrt{2}} \left[\left(1 + \frac{2}{\pi}\right) \sin\left(\frac{\pi}{4\beta}\right) + \left(1 - \frac{2}{\pi}\right) \cos\left(\frac{\pi}{4\beta}\right) \right] & t = \pm \frac{T}{4\beta} \\ \frac{\sin(\pi(1-\beta)t/T) + \frac{4\beta}{\pi} \cos(\pi(1+\beta)t/T)}{\pi t/T (1 - (2\beta t/T)^2)} & \text{other } t \end{cases} \quad (3.4)$$

Figure 3.3 A raised cosine filtering pulse

The detailed derivation of (3.4) is given in Appendix A.

In the standard model recommended by the Telecommunication Industry Association, a pair of SRRC filters with 0.35 roll-off factor are used for the shaping and receiver filters. The purpose of using SRRC filters is to receive an approximate raised cosine shaped signal which limits the ISI introduced by filtering to the minimum.

In Figure 3.1, the complex envelope of the baseband transmitted signal after the *Shaping Filters* is given by:

$$\begin{aligned}
 A(t) + jB(t) &= \sum_{k=0}^{\infty} (a_k + jb_k)p(t - kT) \\
 &= \sum_{k=0}^{\infty} p(t - kT)e^{j\phi_k}
 \end{aligned}
 \tag{3.5}$$

The final signal from the transmitter is

$$\begin{aligned}
 s(t) &= \text{Re} \left[(A(t) + jB(t))e^{j2\pi f_c t} \right] \\
 &= \sum_{k=0}^{\infty} p(t - kT)\cos(\omega_c t + \phi_k)
 \end{aligned}
 \tag{3.6}$$

where $p(t)$ is the time function of the SRRC filter.

3.2 Receiver

3.2.1 Introduction

With $\pi/4$ -QPSK modulation, the information is completely contained in the phase changes $\Delta\phi_k$ between two consecutive samples. Therefore only the phase change between two consecutive samples is needed for detection in the receiver.

Three detection schemes for a $\pi/4$ -QPSK system are considered in this thesis. They are:

- (1) limiter-discriminator detection;
- (2) linear delay detection;
- (3) coherent detection.

3.2.2 Limiter-Discriminator Detection

The block diagram of the limiter discriminator detection is shown in Figure 3.4.

A SRRC bandpass filter (BPF) with a center frequency ω_c is used to receive the desired signal with the carrier frequency ω_c . The output signal of BPF in the inphase and quadrature channels are:

Figure 3.4 The limiter-discriminator detector

$$\begin{aligned}
 x(t) &= \sum_{k=0}^{\infty} q(t - kT) \cos(\omega_c t - \phi_k + \theta(t)) \\
 y(t) &= \sum_{k=0}^{\infty} q(t - kT) \sin(\omega_c t + \phi_k + \theta(t))
 \end{aligned}
 \tag{3.7}$$

where ϕ_k is the $\pi/4$ -QPSK modulated phase signal, $\theta(t)$ is the phase shift caused by the channel and $q(t)$ is the received amplitude function.

Residual error can be caused by random amplitude variations from an exponentially modulated signal. A *hard-limiter* is used to remove the random amplitude variation without destroying the modulation[23]. The inphase and quadrature outputs of the hard limiter are:

$$\begin{aligned}
 I &= \sum_{k=0}^{\infty} \cos(\omega_c t + \phi_k + \theta(t)) \\
 Q &= \sum_{k=0}^{\infty} \sin(\omega_c t + \phi_k + \theta(t))
 \end{aligned}
 \tag{3.8}$$

The *FM discriminator* in Figure 3.4 extracts the instantaneous frequency of the received signal, then subtracts the carrier frequency. The output of the FM discriminator is the frequency deviation.

The *integrate-and-dump* filter integrates the input every symbol period. For example, it integrates the frequency deviation from $(k - 1)T$ to kT for the k th symbol period and outputs the phase increment from $t = (k - 1)T$ to $t = kT$. By sampling at $t = kT$, the phase change between $t = (k - 1)T$ and $t = kT$ is retrieved.

Before detection, the phase signal is reduced by *Modulo 2π* . If the output of the *integrate-and-dump* is beyond $[-\pi, +\pi]$, this will force the phase into $[-\pi, +\pi]$ for detection purposes.

The phase signal can now be detected by a 4-level threshold detector. The four thresholds are 0, $\pi/2$, $-\pi/2$, and π . The decision is made according to the following criteria:

$$\begin{aligned}
 0 \leq \Delta\phi_k \leq \pi/2 & \quad S_i = 1 \quad S_q = 1 \\
 \pi/2 \leq \Delta\phi_k \leq \pi & \quad S_i = 1 \quad S_q = -1 \\
 -\pi \leq \Delta\phi_k \leq -\pi/2 & \quad S_i = -1 \quad S_q = 1 \\
 -\pi/2 \leq \Delta\phi_k \leq 0 & \quad S_i = -1 \quad S_q = -1
 \end{aligned} \tag{3.9}$$

3.2.3 Delay Detection

Figure 3.5 illustrates the differential delay detector employing a delay line and a phase detector.

The receiver bandpass filter is a SRRC filter with centre frequency ω_c . The signal received by the BPF can be represented as

$$\sum_{k=0}^{\infty} q(t - kT) \cos(\omega_c t + \phi_k + \theta_k(t)) \tag{3.10}$$

where ϕ_k is the $\pi/4$ -QPSK modulated phase by the k th input data, $\theta_k(t)$ is the phase shift occurred during the k th symbol period and $q(t)$ is the received amplitude function. The signal will multiply with itself at a one symbol period delay

$$\sum_{k=0}^{\infty} q(t - (k - 1)T) \cos(\omega_c t + \phi_{k-1} + \theta_{k-1}(t)) \tag{3.11}$$

The bandwidth of the multiplier output is twice that of the transmitted signal, i.e., $(1 + \alpha)/T$.

The outputs of the multiplier then pass through an ideal brick-wall lowpass filter with a bandwidth of $(1 + \alpha)/T$. The baseband signals are now recovered. Its inphase and quadrature signals are:

Figure 3.5 The delay detector

$$\begin{aligned}
x(t) &= \sum_{k=0}^{\infty} q_2(t) \cos(\omega_c T + \phi_k - \phi_{k-1} + \theta_k(t) - \theta_{k-1}(t - T)) \\
y(t) &= \sum_{k=0}^{\infty} q_2(t) \sin(\omega_c T + \phi_k - \phi_{k-1} + \theta_k(t) - \theta_{k-1}(t - T))
\end{aligned} \tag{3.12}$$

where $q_2(t)$ is the amplitude function of the baseband signal.

Sampling $x(t)$ and $y(t)$ at $t = kT$ yields

$$\begin{aligned}
x_k &= \cos(\omega_c T + \Delta\phi_k + \Delta\theta_k) \\
y_k &= \sin(\omega_c T + \Delta\phi_k + \Delta\theta_k)
\end{aligned} \tag{3.13}$$

where $\Delta\phi_k = \phi_k - \phi_{k-1}$ and $\Delta\theta_k = \theta_k - \theta_{k-1}$. The phase change can be obtained as

$$\Delta\phi_k = \tan^{-1}\left(\frac{y_k}{x_k}\right) - \omega_c T - \Delta\theta_k \tag{3.14}$$

Since the phase shift $\theta_k(t)$ is slowly varying, the change between two consecutive samples, $\theta_k(t) - \theta_{k-1}(t - T)$, is normally much smaller than $\theta_k(t)$. Thus the delay detector can reduce the phase shifts.

A phase detector is then used to recover the data with the phase changes $\Delta\phi_k$. The decision criteria are the same as in section 3.2.3, given by (3.9).

3.2.4 Coherent Detection

The block diagram of a coherent detector is shown in Figure 3.6.

A bandpass filter with centre frequency ω_c is used to receive the required signal. The received signal is mixed with the carrier, then passes through the brick-wall LPF whose bandwidth is $(1 + \alpha)/2T$. The outputs of the LPFs are $\cos(\theta_k(t) + \phi_k)$ and $\sin(\theta_k(t) + \phi_k)$, where ϕ_k is the phase modulated by the k th symbol in the input and $\theta_k(t)$ is the phase shift introduced by the channel during the k th symbol period.

Using a 4-time frequency multiplier, $\cos(4\theta + 4\phi_k)$ and $\sin(4\theta + 4\phi_k)$ will be produced.

Figure 3.6 The coherent detector

Assume ϕ_k is 0. Then ϕ_{k-1} must be one of $\{n\pi/4; n = 1, 3, -3, -1\}$ according to $\pi/4$ -QPSK modulation. Thus

$$\begin{aligned}\cos(4\theta_k(t) + 4\phi_k) &= \cos(4\theta_k(t)) \\ \sin(4\theta_k(t) + 4\phi_k) &= \sin(4\theta_k(t)) \\ \cos(4\theta_{k-1}(t) + 4\phi_{k-1}) &= \cos(4\theta_{k-1}(t) \pm \pi) = -\cos(4\theta_{k-1}(t)) \\ \sin(4\theta_{k-1}(t) + 4\phi_{k-1}) &= \sin(4\theta_{k-1}(t) \pm \pi) = -\sin(4\theta_{k-1}(t))\end{aligned}\tag{3.15}$$

Since the phase shift $\theta_k(t)$ in the k th symbol does not vary much from the phase shift $\theta_{k-1}(t)$ in the $(k-1)$ th symbol, (3.15) shows that the output of the LPF during the k th symbol is opposite the output of LPF during the $(k-1)$ th symbol. It is easy to show that this is also true for all other possible ϕ_k and ϕ_{k-1} values.

A clock with a frequency that is half of the symbol rate is used to invert the negative sign of $\cos(4\theta_{k-1}(t) + \phi_{k-1})$ and $\sin(4\theta_{k-1}(t) + \phi_{k-1})$. This makes the following relations always true:

$$\begin{aligned}\cos(4\theta_k(t) + 4\phi_k) &= \cos(4\theta_k(t)) \\ \sin(4\theta_k(t) + 4\phi_k) &= \sin(4\theta_k(t)) \\ \cos(4\theta_{k-1}(t) + 4\phi_{k-1}) &= \cos(4\theta_{k-1}(t)) \\ \sin(4\theta_{k-1}(t) + 4\phi_{k-1}) &= \sin(4\theta_{k-1}(t))\end{aligned}\tag{3.16}$$

A *frequency divider* is used to divide the frequency of the input signals $\cos(4\theta_k)$ and $\sin(4\theta_k)$ by 4, to obtain $\cos \theta_k$ and $\sin \theta_k$. Then the coherent carrier with the recovered phase shift $\theta_k(t)$ is used to reduce the phase shift brought in by the channel, thereby reducing the detection errors caused by the phase shift to very low levels.

Ideally, the outputs of the LPFs are $\cos \phi_k$ and $\sin \phi_k$. Therefore the same differential phase detector as the one used in the delay detector can be used to recover the data.

3.3 Channel Model

The Rayleigh fading channel simulator proposed by Jakes [22] is used in our simulation.

Figure 3.7 The Rayleigh fading channel simulator

The simulator has N_0 low-frequency oscillators with frequencies equal to the Doppler shifts $\omega_m \cos(2\pi n/N_0)$, $n = 1, 2, \dots, N_0$, plus one oscillator with frequency ω_m . The amplitudes of all the components are made equal to unity except for the one with frequency ω_m , which is set equal to $1/\sqrt{2}$. The phases β_n are chosen appropriately so that the probability distribution of the resultant phase will be as close as possible to a uniform distribution. The block diagram of this simulator is shown in Figure 3.7, along with an illustration of the frequency spacings of the oscillators for $N_0 = 8$, where

$$\begin{aligned}\omega_m &= 2\pi v/\lambda \\ \omega_n &= \omega_m \cos \frac{2\pi n}{N} \\ N_o &= \frac{1}{2(\frac{N}{2} - 1)} \\ \alpha &= 0 \\ \beta_n &= \frac{\pi n}{N_o + 1}\end{aligned}\tag{3.17}$$

By taking advantage of trigonometric relationships, the oscillators are provided with a gain equal to $2 \cos \beta_n$ or $2 \sin \beta_n$. The outputs of the individual oscillators, with the appropriate gain factors, are first summed to produce in-phase $x_c(t)$ and quadrature $x_s(t)$ components. After mixing with the carrier, the final composite output signal $y(t)$ becomes a narrow-band signal centered on the carrier frequency ω_c , with Rayleigh fading characteristics:

$$\begin{aligned}y(t) &= x_c(t) \cos \omega_c t + x_s(t) \sin \omega_c t \\ x_c(t) &= 2 \sum_{n=1}^{N_0} \cos \beta_n \cos \omega_n t + \sqrt{2} \cos \alpha \cos \omega_m t \\ x_s(t) &= 2 \sum_{n=1}^{N_0} \sin \beta_n \cos \omega_n t + \sqrt{2} \sin \alpha \cos \omega_m t\end{aligned}\tag{3.18}$$

The phase of $y(t)$ should be random and uniformly distributed from 0 to 2π . This is accomplished by choosing $\alpha = 0$ and $\beta_n = \frac{\pi n}{N_0+1}$ [22].

3.4 MLSE Equalizer

Severe ISI is introduced into the received signal as a result of imperfect filter designs,

Figure 3.8 MLSE equalizer with Viterbi algorithm

multipath fading and incomplete knowledge of the channel characteristics. A MLSE equalizer can be used to reduce the ISI and provide better performance.

The block diagram for the MLSE equalizer is shown in Figure 3.8.

In the following discussion, it is assumed that the transmitted data is divided into frames and each frame consists of a training segment and a data segment. The memory length of the intersymbol interference is assumed to be one symbol.

The following describes the operation of the MLSE equalizer for one frame of data.

The first step is to make a start-up operation to track the channel variations by using Kalman Algorithm [5]. A short training sequence is transmitted for this procedure, in which the transmitted data is known to the receiver. For each received piece of data, the difference between it and the transmitted data is used to adjust the tap gain of the channel estimator until getting tracked to the channel.

The estimated channel then is used by the MLSE to estimate the information symbols. Suppose r_j and r_{j+1} are received. With the previously estimated symbol data $\hat{I}_{j-1}$, the channel coefficients $\{f_k\}$ are updated with the Kalman algorithm in the channel estimator. The estimated previous symbols and the channel coefficients are used by the Viterbi algorithm to estimate which data symbol I_j would be the most probable. The

received signals r_j and r_{j+1} can be represented as:

$$\begin{aligned} r_j &= f_0 I_j + f_1 \hat{I}_{j-1} + n_j \\ r_{j+1} &= f_0 I_{j+1} + f_1 I_j + n_{j+1} \end{aligned} \quad (3.19)$$

where $\{n_j\}$ is a sequence of statistically independent zero mean Gaussian noise. The sequence of $\pi/4$ -QPSK modulated data has the following relation

$$I_j = e^{j\phi_j} \quad (3.20)$$

where ϕ_j can be one of $\{0, \frac{\pi}{4}, \frac{2\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{6\pi}{4}, \frac{7\pi}{4}\}$, but the phase shift between I_j and $\hat{I}_{j-1}$ can only be one of $\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$. Because of these characteristics of $\pi/4$ -QPSK modulation, I_j has 4 possible realizations corresponding to $\hat{I}_{j-1}$. Also, I_{j+1} has 4 possible realizations with respect to each I_j . Consequently, we now need to compute $4*4=16$ metrics:

$$\begin{aligned} \mu_j(I_{j+1}, I_j) &= \min_{I_j} \sum_{k=j}^{L+j} \left(r_k - \sum_{i=0}^L f_i I_{k-i} \right)^2 \\ |\phi_j - \phi_{j-1}|, |\phi_{j+1} - \phi_j| &= \pm \frac{\pi}{4}, \pm \frac{3\pi}{4} \end{aligned} \quad (3.21)$$

where $I_j = 0$ for $j < 0$.

Assume that the phase of the estimated symbol $\hat{I}_{j-1}$ is $\pi/4$. The sequence $\hat{I}_{j-1}, I_j$ and I_{j+1} is termed a path. There are four possible paths terminating with $\phi_{j+1} = \frac{\pi}{4}$. The phase of I_j can be: $\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 0$. For the 4 possible paths, the I_j which results with minimum metric value from (3.21) is kept for further calculation, and the other three are discarded. Thus the metric for the surviving I_j is:

$$\begin{aligned} \mu_j(I_{j+1} = e^{j\frac{\pi}{4}}, I_j) &= \min_{I_j} \left[\sum_{k=j}^{L+j} \left(r_k - \sum_{i=0}^L f_i I_{k-i} \right)^2 \right] \\ \text{where } I_j &= e^{\pm j\frac{\pi}{2}}, e^{+j\pi}, e^{+j0} \end{aligned} \quad (3.22)$$

This process is repeated for $I_{j+1} = e^{j\frac{3\pi}{4}}, I_{j+1} = e^{-j\frac{3\pi}{4}}$ and $I_{j+1} = e^{-j\frac{\pi}{4}}$, and their surviving I_j s are also saved. The 4 surviving I_j values are stored in the

$j+1$ th column of matrix M , (See Figure 3.10) where the column number represents the number of the estimated symbol, and the row number represents the surviving I_j value. The numbers $\{1, 2, 3, 4, 5, 6, 7, 8\}$ are defined to represent the I_j values of $\left\{e^{j\frac{\pi}{4}}, e^{j\frac{2\pi}{4}}, e^{j\frac{3\pi}{4}}, e^{j\frac{4\pi}{4}}, e^{j\frac{5\pi}{4}}, e^{j\frac{6\pi}{4}}, e^{j\frac{7\pi}{4}}, 1\right\}$ respectively. For example, $\mu_j(I_{j+1} = e^{j\frac{3\pi}{4}}, I_j)$ is minimized when $I_j = e^{j\frac{2\pi}{4}}$. So, the element in the $j+1$ th column and 3rd row of M is 2.

When r_{j+2} is received, the four possible symbols yield 16 paths and 16 corresponding metrics, given as

$$\mu_{j+1}(I_{j+2}, I_{j+1}, I_j) = \mu_j(I_{j+1}, I_j) + \left(r_{j+2} - \sum_{i=0}^L f_i I_{j+2-i} \right)^2 \quad (3.23)$$

where $|\phi_{j+2} - \phi_{j+1}|, |\phi_{j+1} - \phi_j| = \pm \frac{\pi}{4}, \pm \frac{3\pi}{4}$

For the four paths terminating with $I_{j+2} = e^{j\frac{\pi}{2}}$, the I_{j+1} value which gives the minimum metric is saved. This procedure is repeated for $\{I_{j+2} = e^{j\pi}, e^{-j\pi}, e^{j0}\}$. Consequently, only 4 paths survive at this stage, and the surviving I_{j+1} values are saved in the $j+2$ th column of M .

The procedure is then repeated until r_{j+q} is received, where q is the required length for the trellis pass convergence. q should be larger than the memory length of the channel. Now we have an $8 \times (q+1)$ matrix which stores the path histories of the possible 8 states and demonstrates their tendency to merge at some time $j+i$ into a common path. This common path is the estimated data sequence.

If q is taken too small, the paths are not merged properly, and the estimated data will then bring in errors. Also, a much larger q is not a good choice, because if q is taken larger than the value at which the paths merge then the SER performance will not be improved, and it will require much more processing time.

After the estimate $\hat{I}_j$ is found, the first column of the matrix is deleted and all the columns are shifted to the left. The last column is used to store the new surviving paths.

Figure 3.9 The flow chart of MLSE equalizer implementation

Each estimated symbol is used to update the tap gains of the channel estimator with the Kalman algorithm. With the estimated tap gains, the new surviving values are found and stored in the last column of the matrix.

The procedure will go on until one frame of data is received. The same operation will be repeated for the rest of the data frames.

the surviving value for $I_{j+2} = \frac{6\pi}{4}$ is $\frac{3\pi}{4}$

the surviving value for $I_{j+1} = \frac{3\pi}{4}$ is $\frac{2\pi}{4}$

the surviving value for $I_j = \frac{2\pi}{4}$ is $\frac{3\pi}{4}$.

Because $I_{j-1} = \frac{3\pi}{4}$, the estimate for I_j is $\frac{2\pi}{4}$.

3.5 System Configuration

As described in Chapter 2, an indoor radio channel can be a flat Rayleigh fading channel or a multipath frequency selective Rayleigh fading channel, dependent on the fading condition. In this research, two kinds of fading channel are considered. For the multipath fading, a two-ray Rayleigh fading channel is used.

Figure 3.11(a) illustrates the system over a flat Rayleigh fading channel with a detector. The transmitter generates a $\pi/4$ -QPSK modulated complex signal. The signal undergoes a Rayleigh fading and additive white Gaussian noise. The receiver receives the distorted signal, and recovers the original data with a delay detector.

Figure 3.11(b) shows the system over a two-ray frequency selective Rayleigh fading channel. The $\pi/4$ -QPSK modulated complex signal from the transmitter passes through a two-path Rayleigh fading channel, where each path has independent Rayleigh fading and a different delay. Additive white Gaussian noise is also added before the receiver. The received signal has two optional processing methods. One is to use a delay detector as in Figure 3.11(a). The other is to use the MLSE equalizer to compensate for the intersymbol interference. The SER performance of these two methods is compared.

Figure 3.11 The block diagram of the simulation system.

Chapter 4

Simulation Model

This chapter presents the assumptions which this thesis based on, the simulation environment and the simulation parameters. Then, the simulation of the system in BOSS is presented. The verification of the simulation can be found in the last section.

4.1 Simulation Background

4.1.1 Assumptions

The system used in this thesis is based on the following assumptions:

- (1) The receiver is synchronized
- (2) Additive white Gaussian noise and a Rayleigh fading channel are employed
- (3) No co-channel or adjacent channel interference is considered

4.1.2 Simulation Environment

The $\pi/4$ -QPSK system is simulated with the Block Oriented System Simulator (BOSS) [24][25]. BOSS allows the user to represent communication systems in block diagrams, where each block simulates a real circuit for the signal processing operation. BOSS provides a large set of basic block modules which are used to simulate communication systems, such as a Butterworth filter, a multiplier, or a sampler.

The MLSE equalizer is not available in BOSS. Because there are a lot of array data storage and mathematical model implementation, it is not suitable to simulate in BOSS. Therefore it is simulated in the C programming language,

All simulations are run on SPARC workstations under the UNIX operating system.

4.1.3 Simulation Parameters

Fading Rate

The fading rate is defined as the ratio of the maximum Doppler frequency to the symbol rate of transmitted data, usually represented as $f_d T_s$. The Doppler frequency f_d is given by:

$$f_d = \frac{v_m}{\lambda} = \frac{v_m f_c}{c} \quad (4.1)$$

where

v_m is the moving speed of the receiver relative to the transmitter,

λ is the carrier wave length,

f_c is the carrier frequency,

c is the velocity of light.

In an indoor wireless communication environment, the system performance is very sensitive to the fading rate. Under the same condition, the larger the fading rate, the worse the performance would be.

Total Number of Symbols

In the BOSS simulator, a simulation will run until the required simulation time is reached. The total number of symbols is the product of the simulation time and the symbol rate.

The total simulation time is chosen to be 10^5 , which makes a simulation run for 2 days. In that period time the system can generate more than 100 errors (for $S/N \leq 30$ dB) to make the result accurate statistically. For $S/N = 40$ dB, more simulation runs are needed to have an accurate SER value.

Signal to Noise Ratio

The calculation of the received signal's average power is:

$$P_{av} = \frac{\sum_{i=1}^N S_i^2}{N} \quad (4.2)$$

and the average noise power is given by:

$$NoisePower = \frac{\sum_{i=1}^N N_i^2}{N} \quad (4.3)$$

where

N is the total number of samples during a simulation run

S_i is the i th sample of the received signal

N_i is the i th sample of the noise process.

The signal to noise ratio (S/N) is defined as:

$$S/N = 10 \log_{10} \left(\frac{P_{av}}{NoisePower} \right) \quad (4.4)$$

Frame Length and Percentage of Training Segment

The transmitted data is separated into frames. Each transmitted frame is partitioned into a training segment and a data segment, as shown in Figure 4.1. The percentage of the training segment is

$$P_{t/f} = \frac{L_{train}}{L_{frame}} * 100\% \quad (4.5)$$

where L_{frame} is the number of symbols in a frame, L_{train} is the number of symbols in the training segment of a frame, L_{data} is the number of symbols in the data segment of a frame, and $L_{data} = L_{frame} - L_{train}$.

Figure 4.1 The frame structure

The frame length is chosen according to the Doppler rate. Usually, the frame length should be much less than the inverse of the Doppler rates to ensure that the channel estimator can track channel variations. In the simulation, $0.05/f_d$ is used as the frame length.

Sampling Instant

The timing of the sampling of the received signal is important in minimizing the effects of ISI. For a flat fading channel, it is possible to sample at the instant of minimum ISI. But for a selective fading channel, no such perfect sampling instant exists and therefore the optimal sampling instant has to be determined empirically. This can be achieved by carrying out a large number of simulation runs with the sampling instant as the only parameter. The instant which yields the best overall BER performance will be the optimal sampling instant.

Roll-off Factor

In the case of SRRC filtering, a rolloff factor of 0.35 was used for both the shaping filter and the receiver filter. This choice is made to facilitate the comparison of our results with the results of [15].

Filter Order

For conventional filters, the filter order can be chosen between 1 and 20. With a higher

filter order, the transition region between the passband and stopband is reduced and the amount of ISI brought to the modulated signal is increased.

4th and 8th order filters are chosen for our simulation.

Weighting Factor for Kalman Algorithm:

The Kalman algorithm for channel tracking was introduced in Section 2. The weighting factor w is a constant to control the convergence of the algorithm. The value for w is valid between 0 to 1. With a smaller w , the algorithm converges faster. w can not be too small. But if w is too small, the algorithm will never converge, and the Kalman algorithm will lose tracking of the channel variations.

Number of Channel Taps

This parameter determines the structure of the channel simulator. As the number of taps is increased, the simulated channel will be closer to the real channel model but the computation time will increase significantly. The channel is usually represented by two rays or three rays. A two-ray channel is used in this thesis, so there are 2 taps[8].

Symbol Error Rate (SER)

The number of symbol errors is obtained by counting the received symbol estimates that differ from the corresponding transmitted symbols in each frame. The SER is defined as

$$SER = \frac{\sum_{i=1}^{N_{frame}} N_{error}(i)}{N_{frame} L_{data}} \quad (4.6)$$

where $N_{error}(i)$ is the number of symbol errors in the i th frame, N_{frame} is the number of frames in a simulation run, and L_{data} is the number of symbols in a frame.

The simulated SER is a statistical estimate of the average probability of symbol error. This estimate can generally be improved by increasing the number of transmitted symbols. In all simulations, each SER point is an average based on the total number of

symbols in the simulation run. For each simulation run, the parameter table indicates the total number of symbols in that simulation run, usually 10^5 .

Error Margin

There is a error margin table for each SER curve as a function of S/N. Each SER point is the average of 6 SER values under the same set of parameters. The error margin is the largest difference between the SER vaule and the average.

4.2 System Simulation

4.2.1 Transmitter

The BOSS simulation of the transmitter is shown in Figure 4.2.

$\pi/4$ -QPSK Source Encoder

$\pi/4$ -QPSK Source Encoder used in Figure 4.2 is presented in Figure 4.3.

Block Random Data generates two different pseudo-random pulse streams of true and false, with the set random number seed and the symbol rate.

Block Logical to Numeric transfers “true” into 1 and “false” into -1, and outputs a pulse stream of -1 and 1.

Block Make Complx combines two different data stream into a complex data stream.

Block Complex Phase outputs the phase signal ϕ of the complex input. Because the real part and imaginary part are -1 or 1, the phase only can be one of four values: $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{-3\pi}{4}, \frac{-\pi}{4}$. The phase signal ϕ is denoted as ϕ_1 in the first symbol period, ϕ_2 in the second symbol period, and so on, and ϕ_n in n th symbol period.

Block Multi Stage Delay produces a delay of one symbol period.

Block Adder outputs a phase signal Φ . It is denoted as Φ_1 in the first symbol period, Φ_2 in the second symbol period, and so on, and Φ_n in the n th symbol period. Then

Figure 4.2 The simulation of the transmitter in BOSS.

- (1) Output1 is a $\pi/4$ -QPSK modulated signal with SRRC shaping.
- (2) Output2 is a temporary signal.
- (3) Output3 is the original data sequence, kept for the comparison with the recovered data.

Figure 4.3 The $\pi/4$ -QPSK source encoder.

- (1) Output1 is a $\pi/4$ -QPSK modulated signal before the shaping filter.
- (2) Output2 is a temporary signal
- (3) Output3 is the original data sequence, kept for the comparison with the recovered data.

$\Phi_1 = \phi_1$, $\Phi_2 = \Phi_1 + \phi_2 = \phi_1 + \phi_2$, ..., and $\Phi_n = \Phi_{n-1} + \phi_n = \phi_1 + \phi_2 + \dots + \phi_n$. This is the $\pi/4$ -QPSK differential encoding process.

Block Modulo Real normalizes the phase into between $[-\pi, \pi]$ with modulo 2π , to prevent the phase accumulation from being too large.

Block e^{jX} takes e^{jX} function for input X, yields a_k, b_k of Figure 3.1.

In Figure 4.3, the first output (Output1) is a $\pi/4$ -QPSK modulated signal before the shaping filter, the second output (Output2) is a temporary signal, and the third output (Output3) is the original data sequence, kept for the comparison with the recovered data later.

Transmitter with SRRC Shaping

Block $\pi/4$ -QPSK Source Encoder generates a $\pi/4$ -QPSK modulated baseband signal.

Block Position In_Symbol and Logical to Numeric generate a sequence of impulses to sample the modulated signal.

Block Raised Cosine Filter functions as a raised-cosine filter or a SRRC filter with the different parameter options. In our simulation, the filter functions as a SRRC shaping filter. The output of the filter will be a $\pi/4$ -QPSK modulated baseband signal with SRRC filtering.

In Figure 4.2, the first output (Output1) is a $\pi/4$ -QPSK modulated signal with the SRRC shaping, the second output (Output2) is the temporary signal, and the third output (Output3) is the original data sequence.

4.2.2 Receiver with Delay Detector

Figure 4.4 shows the BOSS simulation block diagram for a delay detector.

Block Raised Cosine Filter functions as a lowpass SRRC receiver filter.

Block Combiner with One Symbol Delay delays one symbol period for the input signal and has its conjugate function, then multiplies with the original input signal. Its

Figure 4.4 The simulation of the delay detector.
(a)

Block combiner with one-symbol delay is given in the 2nd diagram.

Input of delay detector is the received signal.

Output of delay detector is the estimated data by the delay detector.

Figure 4.5 The simulation of the detection criteria.

Input is the output of the combiner with one-symbol delay.

Output is the estimated data by the delay detector.

output is a complex baseband signal, x_k , y_k of Equation 3.16.

Block Detection Module simulates the detection procedure of Equation 3.17 to recover information data, shown in Figure 4.5.

4.2.3 Rayleigh Fading Channel

Figure 4.6 shows the BOSS simulation diagram of a Rayleigh fading channel simulator, and the Branch module is given in detail in the Figure 4.7.

In Figure 4.7, 2_PI*F*T generates ω_n . Block e^{jX} and Real of Complx implement the function $\cos(\omega_n t)$. The following amplifier gives the coefficient 2 for $\cos(\omega_n t)$. For the last two amplifiers, the forward one gives the coefficient $\cos \beta_n$ and the backward one gives the coefficient $\sin \beta_n$. The forward output is $2\sin \beta_n \cos \omega_n t$ and the backward output is $2\cos \beta_n \cos \omega_n t$.

In the Rayleigh Channel module, the sum of the nine Branch outputs gives the $x_c(t)$ and $x_s(t)$ of Equation 3.18. The output of the Rayleigh Channel is a Rayleigh-distributed signal.

4.2.4 System Simulation

System over a Flat Rayleigh Fading Channel

Figure 4.8 shows the system simulation diagram in BOSS. The system uses a delay detector over a flat Rayleigh fading channel. A pair of SRRC filters are used as the shaping and the receiver filters.

The Transmitter with SRRC Shaping described in Section 4.2.1 generates a $\pi/4$ -QPSK modulated complex signal with SRRC shaping.

The transmitted signal passes through the Rayleigh Channel and is combined with complex white Gaussian noise from Cmplx White Noise module.

The Raised Cosine Filter is set to function as a SRRC lowpass receiver filter, and the delay detector is simulated by the Combiner with One-Symbol Delay and the Detection

Figure 4.6 The simulation of the Rayleigh fading channel.

Input is the signal without Rayleigh fading.

Output is the Rayleigh faded signal.

Figure 4.7 The detailed structure of branch block used in Rayleigh channel.

The left output is $2\cos\beta_n\cos\omega_nt$.

The right output is $2\sin\beta_n\cos\omega_nt$.

Figure 4.8 The simulation of the system over a flat Rayleigh fading channel.

Figure 4.9 The simulation of the system over a two-ray fading channel with a MLSE equalizer and a delay detector.

Figure 4.10 The simulation of the BER calculator.

Input1 is the estimated data with a delay detector.

Input2 is the original data kept for the comparison.

Input3 is the control signal for sampling.

Module. The Multi Stage Delay delays the original data by an amount equal to the time caused by the data processing from transmitter to finishing detection. The BER Calculator compares the original data stream with the estimated data stream and counts the number of symbol errors, shown in Figure 4.10.

The Position In_Symbol generates a control signal which is “true” at the specified time within each symbol, but otherwise is “false”. This specified time is the optimal sampling instant which is adjusted by the optimal sampling method. This control signal is used to control the sampling in the BER Calculator.

In the BER Calculator, the Sample&_Hold modules sample the estimated data streams and the original data streams when the control signal is “true”, and hold that value when the control signal changes to “false”. The Complex Error Counter compares the original data samples and the recovered data samples, and counts the number of differences, i.e., the number of symbol errors. It then prints out the SER.

System over a Frequency Selective Rayleigh Fading Channel

Figure 4.9 shows the system simulation diagram in BOSS. The system uses a delay detector and a MLSE equalizer over a two-ray Rayleigh fading channel.

The Transmitter with SRRC Shaping generates a $\pi/4$ -QPSK modulated complex envelope with SRRC shaping, which passes through two propagation paths with independent Rayleigh fading on each path. There is a time delay and amplitude variation between the main ray and the second ray. The delay is denoted as t_d , and g is used to denote the power ratio of the second ray to the main ray. These two ray signals and the additive Gaussian noise are combined. This is the input to the receiver with a SRRC filter.

The received signal is sampled and stored in a data file “sample.dat” by the Write Sample to File module for MLSE equalizer data processing. The original data is also sampled and stored in a data file “original.dat” for symbol error counting, after being delayed to synchronize with the received signal. The sampling rate is equal to the symbol rate. The sampling instant is controlled by Position In_Symbol which is set

to the optimal sampling instant in advance. The MLSE equalizer is simulated in C language as described in Section 3.4.

The rest of the simulation diagram is the same as that over the flat Rayleigh fading channel. The BER Calculator gives the symbol error rate performance for the delay detector. The SER with an MLSE equalizer is compared with the SER with a delay detector.

4.2.5 Verification of Simulation

In order to show the correctness of this simulation system, the simulation in [15] was performed. The simulation diagram of Figure 4.8 is used for the flat Rayleigh fading channel, with data rate = 48 kbps, Doppler frequency = 35Hz and roll-off factor = 0.35.

The eye diagram of the received $\pi/4$ -QPSK modulated signal over an ideal channel is shown in Figure 4.11. The phase constellation of $\pi/4$ -QPSK with a SRRC filter is shown in Figure 4.12. It is obvious that there are eight focused phase states.

The static SER performance with the delay detector is shown in Figure 4.13. The table gives the error margins for each average SER point. The error margin for each SER point is obtained by six simulation runs with different input sequence data under the same simulation parameters. Without fading, the system performance can be close to 10^{-3} for S/N of 10dB.

The system performance is shown in Figure 4.14 for the delay detector over a flat Rayleigh fading channel with Doppler rate of 35Hz. Over a flat Rayleigh fading channel, the delay detector has a SER of 10^{-4} for a S/N of 40 dB. The SER is reduced steadily as the S/N gets larger. These results agree very well with those in [15].

Figure 4.11 The eye diagram of the delay detector. (a) In-phase channel eye. (b) Quadrature channel eye.

Figure 4.12 The SRRC filtered $\pi/4$ -QPSK signal constellation.

S/N	1 dB	3 dB	5 dB	7 dB
Error Margin for DD	0.03534	0.00576	0.00588	0.00094

Figure 4.13 The static performance of the standard system.

S/N	10 dB	20 dB	30 dB	40 dB
Error Margin for DD	0.00802	0.00089	0.000040	0.00008

Figure 4.14 The flat fading performance of the standard system with 0.35 rolloff factor.

Chapter 5

Simulation Results

5.1 Introduction

A performance comparison of the three detection methods over flat Rayleigh fading channels is presented in Section 5.2.

In Section 5.3, results are given for the system using a pair of SRRC filters as the shaping and the receiver filters. The system performance is studied over flat Rayleigh fading channels and frequency selective Rayleigh fading channels. This includes the effects of the fading rate and the delay spread. A comparison of the delay detector and the MLSE equalizer is also presented.

Because of its design complexity and implementation difficulty, especially for high speed data transmission, a SRRC filter is not practical. Sections 5.4 and 5.5 give the $\pi/4$ -QPSK system performance using conventional shaping and receiver filters under two fading conditions. The most popular conventional filters used in communication systems, Butterworth and Chebyshev type 1 filters, are considered.

The measure of performance in this thesis is the average SER for a given S/N at the receiver. The accuracy of SER can generally be improved by increasing the number of transmitted symbols. In the following simulation results, each SER point is an average value of 6 simulation runs with the same simulation parameters. The only difference is the seed of the random binary data sequence. For each simulation, the parameter table indicates the number of symbols in each simulation run, usually 10^5 . A table is attached with the performance curves, illustrating the error margin (EM) of each SER point.

5.2 Comparison of Three Detection Methods

In order to compare the three detection methods, the simulation is performed over a static channel and a flat Rayleigh fading channel with $f_d T = 1.46 \times 10^{-3}$ and roll-off factor $= 0.35$. The system is simulated using MATLAB.

Figure 5.1, 5.2 and 5.3 are the eye diagrams of the limiter discriminator, the delay detector and the coherent detector respectively.

The static SER performance with the three detection methods (Figure 5.4) shows that the delay detector (DD) and the limiter-discriminator detector (LD) have very close performance curves. The coherent detector (CD) has about 3 dB advantage for the same signal to noise ratio. The simulation results obtained here agree very much with those in [15].

The system performance with the three detection methods over a flat Rayleigh fading channel with the fading rate of 1.46×10^{-3} is shown in Figure 5.5. Again, the delay detector and the limiter-discriminator detector have very close performance curves over a flat Rayleigh fading channel. The coherent detector has a similar performance for $S/N < 30$ dB, but becomes much worse than the delay detector and limiter-discriminator detector for $S/N > 30$ dB. The error margins contained in the table indicate a reasonable reliability of the results. These results are also consistent with those in [15].

The simulation also shows that there is no difference between the delay detector and the limiter-discriminator detector. In a limiter-discriminator detector, a discriminator is actually a phase differentiator, and the integration & dump filter produces the phase difference over one symbol period. Therefore, the two schemes are equivalent.

The delay detector is the easiest to implement because of its simplicity, while the coherent detector is the most complex since it requires a linear receiver and some additional circuitry to recover the coherent carrier phase.

Besides its hardware simplicity, the delay detection has another advantage. Because of phase ambiguity in coherent detection and phase shift eliminating capability of delay detection, delay detection has a lower error floor than the coherent detection caused by the Doppler shift [6]. The coherent detection, however, has lower power consumption.

Figure 5.1 The baseband 4-level eye diagram of the limiter-discriminator detector.

Figure 5.2 The eye diagram of the delay detector. (a) In-phase channel eye. (b) Quadrature channel eye.

Figure 5.3 The eye diagram of the coherent detector in inphase and quadrature channels.

Figure 5.4 The static performance with the three detection methods.

Figure 5.5 The flat fading performance comparison for the three detection methods.

5.3 Performance with SRRC Filters

5.3.1 Over Flat Rayleigh Fading Channels

Figure 4.8 shows a $\pi/4$ -QPSK system employing a delay-detector, with a pair of SRRC filters as the shaping and receiver filters, over a flat Rayleigh fading channel. The system's SER performance as a function of signal-to-noise ratio (S/N) and the fading rates ($f_d T$) is shown in Figure 5.6, where the rolloff factor of the SRRC filter is set to 0.35. The table gives the error margin (EM) for each SER point.

The performances for three different fading rates, 10^{-3} , 1.46×10^{-3} , 2×10^{-3} , are quite close, because the phase shift caused by the Doppler frequency over a flat Rayleigh fading channel can be eliminated by the delay detector. However the delay detector cannot eliminate the Doppler shift completely; therefore the SER performance worsens with a larger fading rate. Figure 5.6 indicates that the delay detector has quite a good performance for a range of Doppler spreads over a flat Rayleigh fading channel.

The error margin for each SER point can reach $\pm 7.5\%$. In order to measure the confidence level of the results, 20 simulations are run under the same simulation parameters with different random input sequences, where the signal to noise ratio is 20 dB, and the fading rate is 1.46×10^{-3} . The table of results is shown in Table 5.1. Taking the confidence interval as 15% of the mean value, the level of confidence in the results is 85%.

The average SER when using an MLSE equalizer is also studied over a flat Rayleigh fading channel. Its performance is compared in Figure 5.7 with the performance of the delay detector, where $f_d T = 0.146\%$, the number of symbols per simulation run = 10^5 , the rolloff factor = 0.35, the frame length = 60 symbols, the training length per frame = 7 symbols, and the number of channel taps = 2.

It is evident that the performance of the standard system with a delay detector over a flat Rayleigh fading channel is very good, because there is very little intersymbol interference introduced by SRRC filtering. Using an MLSE equalizer has no better

performance than using a delay detector because of the burst error. With MLSE equalization, the updated coefficients depend on the estimated data. If the estimated data is errorous, it will result in more errors in the following estimation. So the system performance with the delay detector is better than with the MLSE equalizer.

5.3.2 Over Frequency Selective Rayleigh Fading Channel

For a frequency selective Rayleigh fading channel, the simulation parameters are listed in Table 5.2.

Figure 5.8 gives the performance of the MLSE equalizer and the delay detector over a frequency selective Rayleigh fading channel.

It is evident that over a frequency selective Rayleigh fading channel, the system performance with a delay detector is quite unacceptable. Because the received sample is a combination of the current symbol and pervious symbols, if the current symbol remains dominant in the received sample, the delay detector can detect it. If the current symbol is weaker than the noise and the interference brought in by the previous symbol, the delay detector can not detect it.

On the other hand, the MLSE equalizer shows a strong capability for data recovery over the frequency selective Rayleigh fading channel. The MLSE equalizer uses a channel estimator to track the variations of the channel, and thereby reduces the interference from the previous symbol. This characteristic of the equalizer results in superior performance.

It is evident from Figures 5.7 and 5.8 that a MLSE equalizer improves the system's SER performance to the order of 10^{-3} for a S/N of 40dB. But a delay detector only has such good performance over flat Rayleigh fading channels, not over frequency selective Rayleigh fading channels.

Figure 5.9 shows the effects of the delay spread. In this experiment, $f_d T = 1.46 \times 10^{-3}$, the main ray to second ray signal ratio is 3dB, the second ray delay (t_d) is chosen as $0.5T, 0.75T, T$.

As can be seen in Figure 5.9, the delay detector performs unacceptably in all three

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cases, but the MLSE equalizer can reduce the SER from an order of 10^{-1} to an order of 10^{-3} for a S/R of 40dB.

In addition, system performance is more influenced by the longer delay between two rays. With the same main ray to second ray signal ratio, a longer delay causes more intersymbol interference from the previous symbols, and therefore more errors in the delay detection. But because the channel estimator is tapped with one symbol period, it can more easily track the channel with the longer delay. This probably explains why the system with MLSE equalizer performs better with longer delays.

Figure 5.10 shows the effects of the fading rate on system performance. In this experiment, the delay spread is T , main ray to second ray signal ratio is 3dB, and the fading rates are chosen at 2×10^{-3} , 1.46×10^{-3} , 10^{-3} . As illustrated in Figure 5.10, the delay detector has poor performance at all three fading rates. The MLSE equalizer improves the SER from an unacceptable level to an order of 10^{-3} for a S/N of 40db. It is noted that the system with a MLSE equalizer improves with a smaller fading rate. This result is expected, since tracking a randomly varying channel is easier at lower fading rates.

Figure 5.11 shows the average SER performance curve as a function of the percentage of training symbols in a frame. The vertical line on each SER point shows the range of 6 simulation results. In this experiment, $t_d = T$, the main ray to second ray signal ratio is 3dB, the fading rate is 1.46×10^{-3} , the frame size is 60, and the S/N is 20 dB. In this case, the SER improvement is quite slow for percentages larger than 15%. For percentages less than 10%, the percentage value strongly affects the SER performance. Therefore it is evident that values between 10% and 15% are suitable. Accordingly, the number of training symbols is set to 7 where the frame length is 60 symbols in this experiment, resulting in a percentage of training of 11.67%.

The data frame length should be chosen according to the Doppler rate f_d . Usually the frame length should be $\ll \frac{1}{f_d}$ to ensure that the channel estimator can track the channel variations. In our simulation, the *frame length* is chosen as $\frac{0.05}{f_d}$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for $f_d T = 10^{-3}$	0.00755	0.00126	0.00035	0.00008
EM for $f_d T = 1.46 \times 10^{-3}$	0.00802	0.00089	0.00040	0.00008
EM for $f_d T = 2 \times 10^{-3}$	0.00740	0.00110	0.00045	0.00007

Figure 5.6 The fading effect on performance over a flat Rayleigh fading channel.

No.	SER Values
1	0.00843
2	0.00787
3	0.00830
4	0.00814
5	0.00754
6	0.00827
7	0.00775
8	0.00853
9	0.00815
10	0.00798
11	0.00876
12	0.00795
13	0.00858
14	0.00737
15	0.00754
16	0.00830
17	0.00883
18	0.00851
19	0.00832
20	0.00788
mean	0.00815

Table 5.1 The list of simulation results with the same simulation parameters, but different input data sequences.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE	0.0082	0.00212	0.00056	0.00054
EM for delay detector	0.00802	0.00089	0.00040	0.00008

Figure 5.7 Performance comparison of the MLSE equalizer and the delay detector over a flat Rayleigh fading channel.

Shaping & receiver filter	SRRC
Doppler rates ($f_d T$)	1.46×10^{-3}
Total symbol number	10^5
Samples per symbol	16
Roll-off factor of SRRC filter	0.35
Signal to noise ratio(S/N)	10 ~ 40 dB
2nd ray delay t_d	T
two rays power ratio g	-3 dB
frame length	60 symbols
training length per frame	7 symbols
RLS weighting factor	0.9
number of channel taps	2

Table 5.2 Simulation parameters for frequency selective Rayleigh fading channels.

S/N	10 dB	20 dB	30 dB	40 dB
FM for MLSE	0.01024	0.00472	0.00112	0.00065
EM for delay detector	0.03074	0.02544	0.02883	0.02556

Figure 5.8 Performance with MLSE equalizer and delay detector over frequency selective Rayleigh fading channels.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $t_d = 0.50T$	0.01071	0.00450	0.00110	0.00065
EM for MLSE $t_d = 0.75T$	0.00987	0.00517	0.00093	0.00073
EM for MLSE $t_d = 1.00T$	0.01024	0.00472	0.00112	0.00065
EM for DD $t_d = 0.50T$	0.03524	0.02646	0.02648	0.02574
EM for DD $t_d = 0.75T$	0.03655	0.02820	0.02704	0.02812
EM for DD $t_d = 1.00T$	0.03074	0.02544	0.02883	0.02556

Figure 5.9 SER performance with SRRC filters over a frequency selective Rayleigh fading channel. $g=-3\text{dB}$, $f_d T = 1.46 \times 10^{-3}$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_dT = 2 \times 10^{-3}$	0.00985	0.00480	0.00096	0.00074
EM for MLSE $f_dT = 1.46 \times 10^{-3}$	0.01024	0.00472	0.00112	0.00065
EM for MLSE $f_dT = 10^{-3}$	0.00942	0.00506	0.00087	0.00056
EM for DD $f_dT = 2 \times 10^{-3}$	0.03156	0.02727	0.02803	0.02878
EM for DD $f_dT = 1.46 \times 10^{-3}$	0.03074	0.02544	0.02883	0.02556
EM for DD $f_dT = 10^{-3}$	0.03221	0.02604	0.02778	0.02723

Figure 5.10 SER performance with SRRC filters over a frequency selective Rayleigh fading channel. $g=-3\text{dB}$, $t_d = T$.

**Figure 5.11 SER performance vs. the percentage of training. $g=-3\text{dB}$,
 $t_d = T$, $f_d T = 1.46 \times 10^{-3}$, $S/N=20\text{ dB}$.**

5.4 Performance with Butterworth Filters

In this section, all filters used in the system model Figures 4.8 and 4.9 are replaced with Butterworth filters. Figure 4.8 simulates the system employing a delay detector and a MLSE equalizer over a flat Rayleigh fading channel. Figure 4.9 simulates the system over a selective Rayleigh fading channel with a delay detector and a MLSE equalizer.

The time functions of 4th order and 8th order Butterworth filter are shown in Figure 5.12, with the cutoff frequency f_{off} as half of the symbol rate. From Figure 5.12, it is noted that the value and position of the pulse peaks are dependent on the order of the Butterworth filters. Therefore for the different order of filter, the sampling instant of the predetection samples needs to be adjusted accordingly. The optimal instant can be reached by measuring the minimum SER with several sampling instants in simulation runs.

In addition, the pulses in Figure 5.12 are not equal to 0 at $t = kT + t_0$ where t_0 is the adjusted optimal sampling instant or the instant of peak value, T is the symbol period and k is any positive integer. This means that ISI is introduced by the shaping filter. Therefore, an equalizer is needed to reduce the ISI for a system with conventional filters for flat and frequency selective Rayleigh fading channels.

The effects of the fading rate and the delay spread on the system performance are also investigated.

5.4.1 Over Flat Rayleigh Fading Channels

The simulation parameters for this experiment are given in Table 5.3, where the fading rates are chosen at 2×10^{-3} , 10^{-3} , 5×10^{-4} , and the frame length $L_{frame} = 50, 100, 200$ symbols respectively. The necessary training length per frame is 15 symbols. In this set of tests, these parameters are different from the previous set, because the previous set of parameters was taken from a paper for the purpose of comparison. The SER performance as a function of the signal-to-noise ratio and the fading rates with 4th order and 8th order Butterworth filters is shown in Figures 5.13 and 5.14 respectively.

It can be seen from Figures 5.13 and 5.14 that the fading rate has little effect on the performance with a delay detector over a flat Rayleigh fading channel. For higher order filters, the system performance with a delay detector is worse, because there is more intersymbol interference brought by the signal pulse, as shown in Figure 5.12. The fading rate has a larger effect on the performance with a MLSE equalizer than with a delay detector.

For a 4th order Butterworth filter, as discussed in the previous section, a delay detector gives a good performance because the ISI introduced by the shaping filter is negligible. With MLSE equalization, the updated coefficients depend on the estimated data; if the estimated data is erroneous, it will result in more errors in the following estimation. So the system performance with the delay detector is better than with the MLSE equalizer.

However with an 8th order Butterworth filter as the shaping filter, which introduces more ISI than the 4th order filter, the MLSE equalizer gives better performance than the delay detector does (Figure 5.14). It is evident that the MLSE equalizer performance is better with higher order filters and smaller fading rates. Another interesting point in Figure 5.14 is that for a flat Rayleigh fading channel, the channel estimator usually only needs one tap, but with a 2-tap channel estimator, the MLSE equalizer can reduce the ISI from the previous symbol.

Fig. 5.12 The 4th and 8th order Butterworth shaping pulses.

Shaping & receiver filter	4th order Butterworth	8th order Butterworth
Timing for received data and original data	2.3T	4.1T
Doppler rates	$2 \times 10^{-3}, 10^{-3}, 5 \times 10^{-4}$	
Total symbol number	10^5	
Samples per symbol	16	
Cut-off frequency/symbol rate	0.5	
Signal to noise ratio(S/N)	10 ~ 40 dB	
frame length	50, 100, 200 symbols	
training length per frame	15 symbols	
RLS weighting factor	0.9	
number of channel taps	2	

Table 5.3 Simulation parameters for flat Rayleigh fading channels

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_d T = 2 \times 10^{-3}$	0.01278	0.00527	0.00127	0.00088
EM for MLSE $f_d T = 10^{-3}$	0.00978	0.00350	0.00176	0.00068
EM for MLSE $f_d T = 5 \times 10^{-4}$	0.00856	0.00188	0.0007	0.00017
EM for DD $f_d T = 2 \times 10^{-3}$	0.02516	0.00246	0.00118	0.00012
EM for DD $f_d T = 10^{-3}$	0.00563	0.00408	0.00015	0.00004
EM for DD $f_d T = 5 \times 10^{-3}$	0.00743	0.00274	0.00008	0.00002

Figure 5.13 SER performance with 4th order Butterworth filters over flat Rayleigh fading channels.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_d T = 2 \times 10^{-3}$	0.03245	0.00612	0.00234	0.00157
EM for MLSE $f_d T = 10^{-3}$	0.02966	0.00453	0.00178	0.00085
EM for MLSE $f_d T = 5 \times 10^{-4}$	0.03561	0.00365	0.00086	0.00021
EM for DD $f_d T = 2 \times 10^{-3}$	0.06534	0.00561	0.00458	0.00523
EM for DD $f_d T = 10^{-3}$	0.06126	0.00485	0.00412	0.00588
EM for DD $f_d T = 5 \times 10^{-4}$	0.05734	0.00511	0.00386	0.00547

Figure 5.14 SER performance with 8th order Butterworth filters over flat Rayleigh fading channels.

5.4.2 Over Frequency Selective Rayleigh Fading Channels

The simulation parameters are given in Table 5.4.

The SER performance, as a function of signal-to-noise ratio and fading rate, is shown in Figures 5.15 and 5.16. It can be seen that over a frequency selective Rayleigh fading channel, the system with the delay detector gives SER above 10^{-1} for all three cases. However, a MLSE equalizer can reduce this SER performance from 10^{-1} to 10^{-4} for a signal to noise ratio of 40dB. The SER performance improves with smaller $f_d T$.

The SER performance, as a function of delay spread, is given in Figures 5.17 and 5.18. In this experiment, $f_d T = 10^{-3}$, the main ray to second ray signal ratio is 3dB, and the second ray delay is varied from $0.50T$ to T . The delay detector again performs poorly in all three cases, whereas the MLSE equalizer improves system performance to 10^{-4} for a signal to noise ratio of 40dB. In addition, the SER performance improves with the longer delay with the MLSE equalizer, because the channel estimator is a T -tapped estimator. However, for the delay detector, the SER performance is reduced with a longer delay, because the more intersymbol interference is introduced.

Shaping & receiver filter	4th order Butterworth	8th order Butterworth
Timing for received data and original data	2.3T	4.1T
Doppler rates	$2 \times 10^{-3}, 10^{-3}, 5 \times 10^{-4}$	
Total symbol number	10^5	
Samples per symbol	16	
Cut-off frequency/symbol rate	0.5	
Signal to noise ratio(S/N)	10 ~ 40 dB	
2nd ray delay t_d	0.75T	
two rays power ratio	-3dB	
frame length	50, 100, 200 symbols	
training length per frame	15 symbols	
RLS weighting factor	0.9	
number of channel taps	2	

Table 5.4 Simulation parameters for frequency selective Rayleigh fading channels

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_dT = 2 \times 10^{-3}$	0.0156	0.00480	0.00155	0.00013
EM for MLSE $f_dT = 10^{-3}$	0.0126	0.00243	0.00085	0.00025
EM for MLSE $f_dT = 5 \times 10^{-4}$	0.0117	0.00188	0.00018	0.00003
EM for DD $f_dT = 2 \times 10^{-3}$	0.05237	0.05188	0.04834	0.04822
EM for DD $f_dT = 10^{-3}$	0.05422	0.04965	0.04560	0.04378
EM for DD $f_dT = 5 \times 10^{-4}$	0.04867	0.04534	0.03233	0.03212

Figure 5.15 SER performance with 4th order Butterworth filters over a frequency selective Rayleigh fading channel. $g=-3$ dB, $t_d=0.75T$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_d T = 2 \times 10^{-3}$	0.04212	0.02272	0.01835	0.00177
EM for MLSE $f_d T = 10^{-3}$	0.03856	0.02034	0.00167	0.00014
EM for MLSE $f_d T = 5 \times 10^{-4}$	0.03554	0.01822	0.00040	0.00008
EM for DD $f_d T = 2 \times 10^{-3}$	0.06645	0.05668	0.05644	0.05617
EM for DD $f_d T = 10^{-3}$	0.06227	0.05633	0.05612	0.05578
EM for DD $f_d T = 5 \times 10^{-4}$	0.05843	0.04383	0.04343	0.04328

Figure 5.16 SER performance with 8th order Butterworth filters over a frequency selective Rayleigh fading channel. $g=-3$ dB, $t_d=0.75T$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $t_d = 0.50T$	0.02587	0.00358	0.00114	0.00177
EM for MLSE $t_d = 0.75T$	0.0126	0.00243	0.00085	0.00025
EM for MLSE $t_d = 1.00T$	0.01043	0.000146	0.00040	0.00021
EM for DD $t_d = 0.50T$	0.04745	0.04571	0.04524	0.03734
EM for DD $t_d = 0.75T$	0.05422	0.04964	0.04560	0.04378
EM for DD $t_d = 1.00T$	0.04746	0.04544	0.03975	0.03847

Figure 5.17 SER performance with 4th order Butterworth filters over a frequency selective Rayleigh fading channel. $g = -3\text{dB}$, $f_d T = 10^{-3}$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $t_d=0.50T$	0.04087	0.02305	0.00174	0.00183
EM for MLSE $t_d=0.75T$	0.03856	0.02034	0.00167	0.00014
EM for MLSE $t_d=1.00T$	0.02347	0.00321	0.00097	0.00028
EM for DD $t_d=0.50T$	0.05749	0.05571	0.05392	0.05217
EM for DD $t_d=0.75T$	0.06227	0.05633	0.05612	0.05578
EM for DD $t_d=1.00T$	0.06723	0.05844	0.05723	0.05847

Figure 5.18 SER performance with 8th order Butterworth filters over a frequency selective Rayleigh fading channel. $g=-3\text{dB}$, $f_dT = 10^{-3}$.

5.5 Performance with Chebyshev type 1 filters

In this section, all the filters used in the system models in Figure 4.8 and Figure 4.9 are replaced with Chebyshev type 1 filters. Both flat and frequency selective Rayleigh fading channels are considered.

The time functions of 4th and 8th order Chebyshev type 1 filters are shown in Figure 5.19, with f_{off} = half the symbol rate and ripple = 1dB, where ripple is the maximum attenuation (in power dB) in the passband. Because the peak of the pulse is varied with the order of the Chebyshev type 1 filter, the sampling instant of predetection samples needs to be adjusted. The optimal instant can be reached by measuring the minimum SER with several sampling instants in simulation runs.

As with Butterworth filters, the impulse response is not equal to 0 at $t = kT + t_0$ (Figure 5.19), where t_0 is the adjusted optimal sampling instant, T is the symbol period and k is any positive integer. Compared to the 4th order filter, the ratio of the first peak to the second peak of 8th order is smaller. This means that more ISI is introduced by the shaping filter. Therefore, equalization techniques are needed for both flat and frequency selective Rayleigh fading channel models.

5.5.1 Over Flat Rayleigh Fading Channel

The simulation parameters for this experiment are given in Table 5.5.

Figures 5.20 and 5.21 display the SER curves for fading rates of 2×10^{-3} , 10^{-3} , 5×10^{-4} , with a frame length of 50, 100, 200 symbols respectively. The training length per frame is 15 symbols. Figures 5.20 and 5.21 show the performance with 4th and 8th order Chebyshev type 1 filters respectively.

Figures 5.20 and 5.21 give essentially the same information for Chebyshev type 1 filters as Figures 5.13 and 5.14 do for Butterworth filters. Compared with Chebyshev type filters, Butterworth filters have better performance.

Fig. 5.19 The 4th order and 8th order Chebyshev type 1 shaping pulses.

Shaping & receiver filter	4th order Chebyshev 1	8th order Chebyshev 1
Timing for received data and original data	2.7T	5.5T
Doppler rates	$2 \times 10^{-3}, 10^{-3}, 5 \times 10^{-4}$	
Total symbol number	10^5	
Samples per symbol	16	
Cut-off frequency/symbol rate	0.5	
Passband Ripple	1 dB	
Signal to noise ratio(S/N)	10 ~ 40 dB	
frame length	50, 100, 200 symbols	
training length per frame	15 symbols	
RLS weighting factor	0.9	
number of channel taps	2	

Table 5.5 Simulation parameters for a flat Rayleigh fading channel

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_d T = 2 \times 10^{-3}$	0.02934	0.00543	0.01835	0.00054
EM for MLSE $f_d T = 10^{-3}$	0.02774	0.00037	0.00167	0.00012
EM for MLSE $f_d T = 5 \times 10^{-4}$	0.00568	0.00032	0.00040	0.00008
EM for DD $f_d T = 2 \times 10^{-3}$	0.03468	0.00313	0.00155	0.00047
EM for DD $f_d T = 10^{-3}$	0.02784	0.00386	0.00075	0.00016
EM for DD $f_d T = 5 \times 10^{-4}$	0.02543	0.00088	0.00038	0.00013

Figure 5.20 SER performance with 4th order Chebyshev type 1 filters over a flat Rayleigh fading channel.

Figure 5.21 SER performance with 8th order Chebyshev type 1 filters over a flat Rayleigh fading channel.

Figure 5.20 indicates again that with 4th order Chebyshev type 1 filters, both the delay detector and the MLSE equalizer have good performance. The performance with MLSE equalizer is marginally worse for higher signal to noise ratios. Both curves are sensitive to the fading rate over flat Rayleigh fading channels. The performance improves with a smaller fading rate.

For 8th order Chebyshev type 1 filters, Figure 5.21 shows that the system performance degrades with use of delay detector, because the shaping filter brings in more intersymbol interference. With the MLSE equalizer, the system performance can be improved to 10^{-3} for a signal to noise ratio of 40dB.

5.5.2 Over Frequency Selective Rayleigh Fading Channel

The simulation parameters are listed in Table 5.6.

Figures 5.22 and 5.23 show the SER performance for different fading rates. These two figures provide the same kind of information for Chebyshev type 1 filters as Figures 5.15 and 5.16 do for Butterworth filters over a frequency selective Rayleigh fading channel. Over a frequency selective Rayleigh fading channel, SER with delay detector is near 0.5. With the MLSE equalizer, SER is reduced from 10^{-1} to 10^{-3} for a signal to noise ratio of 40dB. With lower fading rates, is greater the improvement in the SER performance.

Figures 5.24 and 5.25 show the performance curves for different delay spreads. In this experiment, the fading rate 10^{-3} , the main ray to second ray signal ratio is 3dB, but the second ray delay is varied from $0.50T$ to $1.00T$. Figures 5.24 and 5.25 provide the same information for Chebyshev type 1 filters as Figures 5.17 and 5.18 do for Butterworth filters. The delay detector's performance stays above 0.5 over a frequency selective Rayleigh fading channel. The MLSE equalizer can improve the performance to 10^{-3} for a signal to noise ratio of 40dB. Because the information is totally lost in the delay detector, no effect of the fading rates can be observed on the SER. But the system performance with the MLSE equalizer improves with a longer delay or smaller fading rate.

Shaping & receiver filter	4th order Chebyshev 1	8th order Chebyshev 1
Timing for received data and original data	2.7T	5.5T
Doppler rates	$2 \times 10^{-3}, 10^{-3}, 5 \times 10^{-4}$	
Total symbol number	10^5	
Samples per symbol	16	
Cut-off frequency/symbol rate	0.5	
Passband Ripple	1 dB	
Signal to noise ratio(S/N)	10 ~ 40 dB	
2nd ray delay t_d	0.75T	
two ray power ratio	-3 dB	
frame length	50, 100, 200 symbols	
training length per frame	15 symbols	
RLS weighting factor	0.9	
number of channel taps	2	

Table 5.6 Simulation parameters for frequency selective Rayleigh fading channels

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_d T = 2 \times 10^{-3}$	0.03415	0.003705	0.00084	0.00074
EM for MLSE $f_d T = 10^{-3}$	0.004468	0.003747	0.00078	0.00017
EM for MLSE $f_d T = 5 \times 10^{-4}$	0.004221	0.001750	0.00066	0.00014
EM for DD $f_d T = 2 \times 10^{-3}$	0.06634	0.06018	0.05868	0.05705
EM for DD $f_d T = 10^{-3}$	0.05443	0.05228	0.05217	0.05122
EM for DD $f_d T = 5 \times 10^{-4}$	0.04415	0.04314	0.04243	0.04206

Figure 5.22 SER performance with 4th order Chebyshev type 1 filters over a frequency selective Rayleigh fading channel. $g = -3$ dB, $t_d = 0.75T$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $f_dT = 2 \times 10^{-3}$	0.03568	0.005374	0.00444	0.00081
EM for MLSE $f_dT = 10^{-3}$	0.03544	0.002237	0.00114	0.00074
EM for MLSE $f_dT = 5 \times 10^{-4}$	0.02992	0.001661	0.00088	0.00036
EM for DD $f_dT = 2 \times 10^{-3}$	0.07411	0.06640	0.06533	0.06437
EM for DD $f_dT = 10^{-3}$	0.06631	0.06292	0.06197	0.06123
EM for DD $f_dT = 5 \times 10^{-4}$	0.06417	0.06244	0.06188	0.06107

Figure 5.23 SER performance with 8th order Chebyshev type 1 filters over a frequency selective Rayleigh fading channel. $g=-3\text{dB}$, $t_d=0.75T$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $t_d= 0.50T$	0.03732	0.004434	0.00154	0.00056
EM for MLSE $t_d= 0.75T$	0.004468	0.003747	0.00078	0.00017
EM for MLSE $t_d= 1.00T$	0.004407	0.001359	0.00064	0.00023
EM for DD $t_d= 0.50T$	0.04618	0.04529	0.04407	0.04210
EM for DD $t_d= 0.75T$	0.05443	0.05228	0.05217	0.05122
EM for DD $t_d= 1.00T$	0.05792	0.05483	0.05466	0.05285

Figure 5.24 SER performance with 4th order Chebyshev type 1 filters over a frequency selective Rayleigh fading channel. $g=-3\text{dB}$, $f_dT = 10^{-3}$.

S/N	10 dB	20 dB	30 dB	40 dB
EM for MLSE $t_d=0.50T$	0.03585	0.008473	0.00518	0.00116
EM for MLSE $t_d=0.75T$	0.03544	0.002237	0.00114	0.00074
EM for MLSE $t_d=1.00T$	0.03282	0.002168	0.00082	0.00025
EM for DD $t_d=0.50T$	0.06529	0.06318	0.06203	0.06145
EM for DD $t_d=0.75T$	0.06631	0.06292	0.06197	0.06123
EM for DD $t_d=1.00T$	0.07034	0.06545	0.06355	0.06277

Figure 5.25 SER performance with 8th order Chebyshev type 1 filters over a frequency selective Rayleigh fading channel. $g=-3\text{dB}$, $f_dT = 10^{-3}$.

Chapter 6

Summary and Discussion

6.1 Summary

Transmitter and receiver designs with $\pi/4$ -QPSK modulation over indoor wireless radio channels have been proposed. In the receiver design, three different detection methods were investigated over a flat Rayleigh fading channel. They are limiter-discriminator detection, linear delay detection and coherent detection. It was found that there is no performance difference between delay detection and limiter-discriminator detection. Coherent detection has a 3 dB better SER performance than the others, but the coherent detector is also the most complex. Because delay detection can provide good performance with a very simple design, it is considered to be a good detection method for a flat Rayleigh fading channel.

With a pair of SRRC filters as the shaping and receiver filter, the system was investigated over flat and frequency selective Rayleigh fading channels. Over flat fading channels, the delay detector can give a SER performance of 10^{-4} for a S/N of 40dB and the fading rate has little effect on this performance. Over frequency selective Rayleigh fading channels, the system performance degraded significantly with a delay detector because of the multipath channel characteristics. A MLSE equalizer with the Viterbi algorithm was used to improve the system performance, along with a channel estimator for tracking the channel variations. Simulations show that a delay detector gives a SER of 10^{-1} for all ranges of signal to noise ratio, and a MLSE equalizer can improve the system SER performance to 10^{-3} for a signal to noise ratio of 40dB. The system performance is better with smaller fading rates and larger delay spreads.

The system performance using conventional filters as the shaping and receiver filters was also investigated over flat and frequency selective Rayleigh fading channels. The conventional filters used in the experiments are 4th and 8th order Butterworth and Chebyshev type 1 filters. It is found that for the 4th order Butterworth and

Chebyshev 1 filters over flat Rayleigh fading channels, the delay detector gives a slightly better performance than the MLSE equalizer. Over frequency selective Rayleigh fading channels, the system performance with the delay detector is unacceptable, but a MLSE equalizer can improve the SER performance to 10^{-3} for an average signal to noise ratio of 40dB. For 8th order filters, the performance over flat Rayleigh fading channels is worse than for 4th order filters. The MLSE equalizer gives better performance than the delay detector. Over frequency selective Rayleigh fading channels, the system with MLSE equalizer can give a SER of 10^{-3} for an average signal to noise ratio of 40dB, instead of a SER of 0.5 with a delay detector. Comparing the two types of conventional filters, the system with Butterworth filters gives better performance than that with Chebyshev type 1 filters.

In summary, all the results indicate that the delay detector can be used over flat Rayleigh fading channels, where the intersymbol interference introduced by the shaping filter is negligible. However, over frequency selective Rayleigh fading channels, an equalizer is needed for acceptable performance. Conventional filters, such as Butterworth and Chebyshev type 1 filters, can be used to replace the SRRC filters as the shaping and receiver filter, to achieve a good performance.

6.2 Future Research

During the course of this research, a number of research topics were discovered but were considered beyond the scope of this thesis.

The simulation results show that the closer the second ray delay is to one symbol period, the better the SER performance will be. This is because the equalizer used is T-tapped. For delays less than half of a symbol period, the use of a fractionally spaced equalizer may be a better choice.

In a real communication system, cochannel interference and adjacent channel interference always exist. While this thesis focused on the intersymbol interference problem, it would be interesting to expand the investigation to other factors such as cochannel

interference and adjacent channel interference.

This research could also be extended to investigate channel models other than the two-ray channel, and eventually to actual measured channel response data.

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Appendix: A Square Root Raised Cosine Function

A derivation of the square root raised cosine function is given in this appendix.

The frequency response of a raised cosine filter is

$$X(f) = \begin{cases} T & 0 \leq |f| \leq (1 - \beta)/2T \\ \frac{T}{2} [1 - \sin \{ \pi T (f - \frac{1}{2T}) / \beta \}] & (1 - \beta)/2T \leq |f| \leq (1 + \beta)/2T \end{cases} \quad (\text{A.1})$$

where β is the rolloff parameter and T is the symbol period. The raised cosine time function $x(t)$ is

$$x(t) = \frac{\sin \pi t/T}{\pi t/T} \frac{\cos \beta \pi t/T}{1 - 4\beta^2 t^2/T^2} \quad (\text{A.2})$$

With a square root raised cosine frequency response $X(f)$, the frequency response of a SRRC filter is

$$\begin{aligned} P(f) &= \begin{cases} 1 & 0 \leq |f| \leq (1 - \beta)/2T \\ \sqrt{\frac{1}{2} [1 - \sin \{ \pi T (f - \frac{1}{2T}) / \beta \}]} & (1 - \beta)/2T \leq |f| \leq (1 + \beta)/2T \end{cases} \\ &= \begin{cases} 1 & 0 \leq |f| \leq (1 - \beta)/2T \\ \cos \left\{ \pi \frac{\beta - 1 + 2T|f|}{4\beta} \right\} & (1 - \beta)/2T \leq |f| \leq (1 + \beta)/2T \end{cases} \end{aligned} \quad (\text{A.3})$$

Taking the inverse Fourier transform of $P(f)$ gives:

$$\begin{aligned}
p(t) &= \int_0^{\frac{1-\beta}{2T}} e^{j2\pi ft} df + \int_{-\frac{1-\beta}{2T}}^0 e^{j2\pi ft} df \\
&+ \int_{\frac{1-\beta}{2T}}^{\frac{1+\beta}{2T}} \cos\left(\pi \frac{\beta-1+2Tf}{4\beta}\right) e^{j2\pi ft} df \\
&+ \int_{-\frac{1-\beta}{2T}}^{-\frac{1+\beta}{2T}} \cos\left(\pi \frac{\beta-1-2Tf}{4\beta}\right) e^{j2\pi ft} df
\end{aligned} \tag{A.4}$$

The 1st and 2nd terms of the right side of Equation A.4 are equal to

$$\frac{\sin \frac{\pi(1-\beta)t}{T}}{\pi t} \tag{A.5}$$

and the 3rd and 4th terms of the right side of Equation A.4 are equal to

$$\frac{\frac{\sin \pi \frac{1-\beta}{T} t}{\pi t} + \frac{\pi t}{\beta(2\pi t)^2} \cos \pi \frac{1+\beta}{T} t}{1 - \left(\frac{T}{4\beta t}\right)^2} \tag{A.6}$$

So that

$$\begin{aligned}
p(t) &= -\frac{\left(\frac{T}{4\beta t}\right)^2 \sin \pi \frac{1-\beta}{T} t + \frac{1}{4\beta t} \cos \pi \frac{1+\beta}{T} t}{\pi t \left(1 - \left(\frac{T}{4\beta t}\right)^2\right)} \\
&= \frac{\sin \pi \frac{1-\beta}{T} t + \frac{4\beta t}{T} \cos \pi \frac{1+\beta}{T} t}{\frac{\pi t}{T} \left(1 - \left(\frac{4\beta t}{T}\right)^2\right)}
\end{aligned} \tag{A.7}$$

For $t = 0$ and $t = \pm \frac{T}{4\beta}$, the values can be deduced by limit theory.

So the time response of a SRRC filter is

$$p(t) = \left\{ \begin{array}{ll} 1 - \beta + \frac{4\beta}{\pi} & t = 0 \\ \frac{\beta}{\sqrt{2}} \left[\left(1 + \frac{2}{\pi}\right) \sin\left(\frac{\pi}{4\beta}\right) + \left(1 - \frac{2}{\pi}\right) \cos\left(\frac{\pi}{4\alpha}\right) \right] & t = \pm \frac{T}{4\beta} \\ \frac{\sin(\pi(1-\beta)t/T) + \frac{4\beta t}{T} \cos(\pi(1+\beta)t/T)}{\pi t/T(1-(2\beta t/T)^2)} & \text{other } t \end{array} \right\} \quad (\text{A.8})$$

7

END

3 1 - 0 5 - 9 5

FIN