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LA THÈSE A ÉTÉ
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IMPLEMENTATION OF A PROBE-COMPENSATED
CYLINDRICAL NEAR-FIELD MEASUREMENTS
SYSTEM AT THE COMMUNICATION RESEARCH
CENTER, SHIRLEY BAY, CANADA

by

Simon R. Gauthier, B. Eng. /

A thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the degree of

Master of Engineering

Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada
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The undersigned recommend to the Faculty of Graduate Studies and
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"Implementation of a Probe-Compensated Cylindrical
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LIST OF SYMBOLS

$\vec{A}$	Vector potential
$\hat{a}$	Constant vector of unit length
$a_n(h), b_n(h)$	Amplitude weighting functions for the cylindrical wave expansion of the fields radiated by an arbitrary antenna
$\vec{B}$	Magnetic field density
C	Path of integration using the method of steepest descent
C_{nh}	Complex amplitude of a cylindrical wave with mode indices n and h
$C(\theta, \phi)$	Cross-polarization pattern of an antenna
$c_m(h), d_m(h)$	Amplitude weighting functions for the cylindrical wave expansion of the fields radiated by the probe antenna when it is oriented as in Fig. 4a
$c'_m(h), d'_m(h)$	Amplitude weighting functions for the cylindrical wave expansion of the fields radiated by the probe antenna when it is rotated by 90° about its longitudinal axis
CPR	Complex polarization ratio
$\vec{D}$	Electric flux density
D	Largest dimension of an antenna aperture
DFT	Discrete Fourier transform
$\vec{E}$	Far-field electric field intensity
FFT	Fast Fourier transform
$\vec{H}$	Far-field magnetic field intensity
h, n	Wavenumbers of test antenna and probe, respectively
$H_n^{(1)}(\Lambda r), H_n^{(2)}(\Lambda r)$	Hankel functions of first and second kind, respectively, of order n and argument (Λr) . Equivalent to $Z_n^{(3)}(\Lambda r)$ and $Z_n^{(4)}(\Lambda r)$.

LIST OF SYMBOLS (cont'd)

$I_n(h), I'_n(h)$	Two-dimensional Fourier integrals
$J_n(\Lambda r)$	Bessel function of the first kind of order n and argument (Λr)
k	Free-space wave propagation constant
$\vec{L}, \vec{M}, \vec{N}$	Independent vector solutions to Eqn. (2.4)
$\vec{m}_{nh}, \vec{n}_{nh}$	Cylindrical wave vectors
M	Total number of samples in elevation on the measurement cylinder
N	Can represent the highest significant angular harmonic in the cylindrical wave expansion of the fields of an antenna; or, the total number of samples taken in azimuth on the measurement cylinder
$N_n(\Lambda r)$	Bessel function of the second kind of order n and argument (Λr)
NBS	National Bureau of Standards, USA
P	Power dissipated in a network
Q	Quality factor
(r, ϕ, z)	Cylindrical coordinates
(R, ϕ, θ)	Spherical coordinates
RTEII	Real Time Executive (Series II) operating system (Hewlett-Packard)
SA2020	Scientific-Atlanta Series 2020 Antenna Analyzer
$v(r_o, \phi_o, z_o)$	Complex voltage data measured with a probe at coordinates (r_o, ϕ_o, z_o) on a cylinder enclosing an antenna. The prime notation refers to the probe rotated by 90° about its longitudinal axis.
$v'(r_o, \phi_o, z_o)$	
W_e, W_m	Respectively the time-average electric and time-average magnetic energy stored in a network at resonance.

LIST OF SYMBOLS (cont'd)

Z_w	Wave impedance
$Z_n^{(i)}(\Lambda r)$	Represents any one of the four cylindrical Bessel functions with indice n and argument (Λr)
∇	Del operator
Δz	Sampling increment in elevation (z)
$\Delta \phi$	Sampling increment in azimuth (ϕ)
Δh	Increment in wavenumber corresponding to z
$\Delta_n(h)$	Determinant of the system of equations to solve for the amplitude weighting functions of the fields radiated by an arbitrary antenna
δ_{nh}	Kronecker delta function
$\delta(\eta+h)$	Dirac delta function
ϵ	Permittivity
λ	Wavelength
ω	Angular frequency
μ	Permeability
σ	Conductivity
Λ	$(k^2 - h^2)^{1/2}$
ψ	Arbitrary scalar function satisfying Eqn. (2.8)
(ϕ_n, θ_m)	Respectively the azimuth and elevation coordinates of the test antenna fields radiation pattern

CHAPTER 1

INTRODUCTION OF GENERAL CONCEPTS

1.1 General Overview of Near-Field Measurements Systems

With the development of large space antennas, it has become impractical in many cases, and sometimes even impossible, to measure antenna radiation patterns on conventional far-field antenna ranges: for example, the distance to the radiating far field may be too long; or it may simply be impractical to move the antenna from its operating environment to the antenna range. Also, with the use of expensive satellite communication links, there is an increasing emphasis placed on the accuracy of antenna measurements. For these and other reasons, some new and powerful techniques have been developed by which an antenna far-field pattern is determined from measurements made in its radiating near-field region.

Conventional far-field measurement techniques implemented on antenna ranges have always had strong advantages:

- a) measurement procedures are relatively simple;
- b) the characteristics of the antennas are measured directly and no waiting period is required for complete data collection and far-field transformation;
- c) these techniques have wide acceptance by the industry as a result of many years of study and proven reliability. The techniques are well understood.

There are also, however, many disadvantages to far-field testing. One of the greatest disadvantages is that a considerable real estate investment is usually required for the facility. In addition, tests must be made on ranges which are sufficiently long to provide acceptable phase accuracy; on large facilities, it becomes most difficult to compensate for ground reflections and other outside interferences. The size of a facility may also dictate that it cannot be located in close proximity to the laboratory or manufacturing complex. In such cases, costs of transportation and liaison between two facilities are also incurred. Because tests are performed outside of a controlled environment, many unscheduled interruptions result from adverse weather conditions such as rain, wind, ice and snow. For many precision tests, solar deflectors must also be accounted for.

For these reasons, near-field testing has become increasingly attractive for certain applications in recent years and will become more so in years to follow. Its major advantages are:

- a) it permits a facility to be contained in a much smaller space, thereby reducing real estate investment;
- b) it provides a degree of physical security when performing military-related testing as well as preventing outside interferences and controlling unwanted reflections;
- c) no weather problems are encountered;
- d) it becomes much easier to locate such a facility in close proximity to the production line or laboratory, thereby reducing any expenses associated with a remote facility;

- e) there are no atmospheric loss problems;
- f) it can provide complete definition of antenna characteristics.

However, near-field measurement techniques also have some disadvantages. Their major drawback is actually the fact that they are not yet widely accepted or understood by the industry and many of the techniques have yet to be verified to the satisfaction of the antenna community. Further disadvantages are that complicated automated systems may be required and that computer processing of the data must be performed thereby resulting in some delay between data collection and availability of results. These computational problems, however, are not as formidable as one might think at first glance and it is possible to process large amounts of data both quickly and economically. A comparison of the advantages and disadvantages of current antenna measurements methods is presented as Table 1.

1.2 General Definition of Terms

The basic idea behind near-field measurement methods consists in extrapolating the far-field radiation pattern of an arbitrary antenna based on the measurements of amplitudes and phases taken in its near-field region. By definition, an antenna is a device for transforming a guided wave into a wave radiated in space, or vice-versa; the distribution of field strength about an antenna is, in general, a function of both the distance from the antenna and the angular coordinates. Since it is our objective to determine antenna

TABLE 1

Comparison of Antenna Measurements MethodsAdvantagesDisadvantagesFar-Field Method (Outdoor Range)

- Relatively simple procedures - No complicated analysis required - Faster, if only limited information is required (e.g. principal plane cuts) - Good for comparison measurements (gain, polarization, etc.) - Well accepted by industry; well recognized techniques	- Large distances required for high accuracy; result in expensive real estate investment - High towers required to eliminate ground reflections - Limited information usually obtained - Atmospheric losses at higher microwave frequencies and mm waves - Weather and interference problems - Accurate absolute measurements are difficult or impossible

Far-Field Method (Indoor Range)

- No weather problems - No interference (+ degree of physical security) - Convenience - Simplicity - Good for some comparison measurements - No complicated analysis	- Large expensive facility required - Accurate absolute measurements are difficult in many cases - Pattern and polarization data change as the antenna pair is rotated about the chamber axis - Limited in the size of antennas to be tested because of $2D^2/\lambda$ limitation and size of required "quiet zone" - Limited information: no vectorial or near-field data

Near-Field Method

- No interference (+ degree of physical security) - No weather problems - Relatively small space: no expensive real estate investment - No atmospheric loss problems - Well suited for absolute gain and polarization measurements	- Automated system required: cost is actually less than that of an anechoic chamber facility and possibly less than that of outdoor far-field range; but may be more complicated to operate - Large amounts of data taken and processed

TABLE 1 (cont'd)Advantages

- Complete pattern, gain, polarization and vectorial field data can be obtained, all at arbitrary distances
- Near-field antenna interactions are treatable
- Relatively inexpensive, particularly after initial installation
- In-situ measurements include effects of mounting, etc.
- A great deal of information available in a hurry during design stages
- High accuracy
- Special advantages for arrays

Disadvantages

- Computer analysis required
- Large apertures and wide beams pose some scanning problems which may be overcome with non-planar systems
- Not yet totally accepted by industry

radiation patterns, it is instructive first to consider the nature of the field regions surrounding an antenna.

In the region close to an antenna, the field will include a reactive component as well as a radiating component. The strength of this reactive component, however, decays rapidly with distance from the antenna so that it soon becomes insignificant compared to the strength of the radiating component. That region in space in which the reactive component of the field predominates is called the "reactive near-field region", and beyond this region, the radiating field predominates. That region in which the radiating field prevails can be further subdivided into the "radiating near-field region" and the "radiating far-field region". In the radiating near-field region, the angular distribution of radiated energy is dependent on the distance from the antenna, along with the relative phases and amplitudes of contributions from the various elements of the antenna; in the radiating far field, the relative phases and amplitudes of contributions from the various elements change very slowly with distance, and the angular distribution of radiated energy measured is essentially independent of the distance from the antenna.

Thus the space surrounding an antenna is composed of three regions: the reactive near-field region, the radiating near-field region and the radiating far-field region. The boundaries between these regions are not well defined, but for any antenna, the reactive near-field region extends only a short distance; the commonly accepted distance to the boundary between the reactive and radiating near-field

regions of an antenna is $\lambda/2\pi$, where λ is the wavelength of operation. Similarly, for electrically large antennas of the aperture type, the commonly used criterion to define the boundary between the radiating near-field and the radiating far-field regions is given by the so-called "Rayleigh distance" which is $2D^2/\lambda$, where D is the largest dimension of the aperture and λ the wavelength. Although the aforementioned criterion defining the distance to the far-field region is generally accepted and is used quite widely, one must always remember that it is an arbitrary choice and that it may be inadequate for some special situations. For example, if one must accurately measure patterns of antennas having very low sidelobes or if one must make accurate gain measurements of pyramidal horns which have large phase deviations across their apertures, the measurement distance may have to be much longer than $2D^2/\lambda$. What we refer to in this document as near-field measurements will be performed in the radiating near field of the antenna under test, i.e. at a distance greater than $\lambda/2\pi$ but much less than $2D^2/\lambda$.

1.3 Various Approaches to Near-Field Measurements

Two fundamentally different principles have been applied in attempts to extrapolate an antenna far-field pattern from near-field measurements. The first method was inspired by the principle that the complete electromagnetic field configurations surrounding an antenna can be computed if either the current or charge distribution over the antenna structure is known exactly. Stratton [1] has, in

the early forties, come up with a detailed presentation of this technique. However, except for very simple geometries, the fundamental formulation of the method is extremely difficult or impossible to apply without approximations relating to the nature of the fields or currents on the surface over which the integration analysis is performed. These approximations, which are necessary to permit evaluation of surface integrals, usually fall into one of the following areas [2]:

- 1) assuming negligible contributions of the fields or currents over some portion of the surface;
- 2) assuming all radiation flows outward normal to the surface;
- 3) assuming the electric and magnetic fields are linearly related as in a plane wave;
- 4) assuming small-angle approximations and thus limiting the angular region for locating the field point.

As shown in Reference [2], the source distribution method can be used to calculate far-field patterns of arbitrary geometry antennas with acceptable accuracy for the main beam and close-in sidelobes, in spite of the above assumptions. One of the major drawbacks, however, is that it theoretically requires an ideal probe, which is impossible to obtain in practice and consequently involves additional approximations. Better agreement between measured and calculated results can therefore be achieved through a second method called "the modal expansion technique".

This second method for determining far-field patterns from near-field measurements is based on expressing the antenna total electromagnetic field in terms of a modal expansion. In this technique, the measured data from an antenna near field are converted to a summation or spectrum of modes corresponding to wavenumbers in the coordinate system in which the near-field data is taken. The pattern of the sampling probe is also transformed to obtain a modal expansion in terms of wavenumbers. With these data, the weighting of the probe pattern on each component of the wavenumber spectrum of the near-field data can be eliminated on a term-by-term basis. This corrected wavenumber spectrum can then be used to obtain the far-field pattern for the test antenna. Thus one of the major difficulties in applying the source distribution method to determine ~~far-field~~ patterns, that of finding an ideal probe for measurement of the source distribution, is eliminated by the modal expansion method [2].

The modal expansion method is based on the fact that an arbitrary electromagnetic field pattern can be expressed as a linear combination of a set of orthogonal solutions in an appropriate coordinate system, and has accordingly been an area of considerable interest [2-9]. Three types of measurement surfaces - related to an equivalent coordinate system - are used in practice; these are the planar, cylindrical and spherical surfaces. The advantages and limitations of using each different measurement surface are discussed in Reference [3]; the choice of which system to use is finally based primarily on the type of antenna to be measured. The planar scanning

system has so far attracted the largest popularity due to its mathematical and computational simplicity; it is well suited to high gain antennas - especially phased array antennas for phase shifter adjustment and element interrogation - and requires no movement whatsoever of the test antenna. Its chief disadvantage, however, is that the pattern can be calculated only in a cone with an apex angle less than 180° without repeating the measurement. In addition, the dimensions of the near-field range can become prohibitive for large, wide-beam antennas. These disadvantages can be partially overcome with the circular cylindrical measurement surface. A cylindrical system is best suited to fan-beam or single plane omnidirectional antennas. The complete 360° azimuth pattern for all elevation angles, not including the spherical polar angles, can be computed from a single measurement. Although it requires about fifty percent more computation than the planar scanning technique, cylindrical scanning requires the least expensive antenna and probe positioning equipment for most antenna types. It should be noted here that both the planar and cylindrical surfaces exhibit computational efficiency in that all numerical integration can be performed using the fast Fourier transform (FFT) algorithm.

While cylindrical scanning is a significant improvement over the planar technique for many applications, it is spherical scanning that appears to have the most promise in the coming years. Conventional antenna positioners which orient the antenna in its spherical coordinate system can be used for spherical scanning measurements.

With positioners readily available, special positioners need not be designed as in the previous two cases. This technique has the further advantage that the entire sphere can be evaluated and there are no open sections, thereby suiting itself well to low gain and omnidirectional antennas. The computational requirements, however, are considered severe: the software required for a spherical system is considerably more complex than that for the two previous methods. However, a great deal of progress has been made in this area and standard product systems have become available during the past year to companies wishing to implement this measurement technique.

The near-field measurement system to be implemented at CRC was finally chosen, after serious consideration, to be of the circular cylindrical type. The major reasons which influenced this choice are:

- a) the implementation of a spherical scanning system would have been ideal; however, this method was discarded at the very beginning due to the complexity of the probe-compensated solutions to be obtained with respect to the time frame allocated;
- b) even if most of the antennas tested at CRC are primarily high-gain, a cylindrical scanning system is still preferred over a planar system based on an omni-present requirement to know the full 360° azimuth pattern of an antenna;
- c) the availability of a probe-arm mechanism and of an azimuth-over-elevation positioner - later described in Chapter 4 - makes cylindrical scanning most attractive:

the test antenna can be simply rotated in azimuth and

the probe moved in steps along a vertical line;

- d) finally, the implementation of a cylindrical near-field measurement system is considered academically more challenging than that of a planar scanning system.

Although the physical implementation of cylindrical scanning is fairly straightforward, three major approaches have been used, in the literature, to transform the measured near-field data to computed far-field characteristics. The first approach is basically the formulation of the Lorentz reciprocity theorem, reported by Brown and Jull [4] and by W. Leach [9]. In this approach, the test antenna and the probe are enclosed in proper surfaces so that a source-free volume is obtained; then, using the Lorentz reciprocity theorem for the source-free volume, the coupling equation is derived. The second approach, which we call the scattering matrix formulation, consists of different types of scattering matrices being used to derive the coupling equation; this method was first formulated by Yaghjian [5]: the matrices relate the amplitudes of waveguide modes and expansion coefficients by linear matrix transformations. These can be taken as definitions or derived from Maxwell's equations. Two types of scattering matrices can be used in the derivation of the coupling equation. The first, the source scattering matrix introduced by Yaghjian, uses expansion coefficients related to the modes based on the Hankel functions of the second kind and Bessel functions: the Hankel function expansion is applied for the radiated field and the Bessel

function expansion is used for the incident field. The second type of scattering matrix, the normal scattering matrix, was primarily designed for spherical scanning techniques [6], but may also apply to cylindrical scanning. The normal technique relates the amplitudes of the waveguide modes with the amplitudes of propagating incoming and outgoing modes in the space surrounding the antenna; thus the received signal depends only upon propagating incoming modes. From various discussions available in the literature, it appears that either scattering matrix formulation is equally applicable.

Finally, the third approach has only recently been presented by Borgiotti [7] as an integral formulation in which both the test antenna and the probe fields are represented as a superposition of plane waves (PWS). The author shows how the PWS approach, so effective in planar aperture problems, can be extended through a suitable modification of the formalism to cylindrical problems. This method now appears very attractive since it is analytically simple and numerically efficient, particularly in the treatment of the probe correction. However, since we became aware of this approach at only a late stage in the implementation process of the near-field measurement system, this technique was not considered in detail.

All three approaches, of course, theoretically give the same final result, but sometimes this cannot be seen directly from the above references due to the different notations. The agreement between the results obtained by the Lorentz reciprocity theorem formulation and the scattering matrix approach was demonstrated in

Reference [8]. As pointed out by Appel-Hansen, two major differences in these approaches should be noted. The first difference is related to reciprocity and the manner in which the transmitting and receiving characteristics of the antennas enter the coupling equation: in the Lorentz reciprocity theorem formulation, the transmitting characteristics of both antennas enter and reciprocity is assumed; in the scattering matrix formulation, it is convenient to use the transmitting characteristic of the transmitting antenna and the receiving characteristic of the receiving antenna, so that reciprocity is not assumed. Thus the scattering matrix formulation can be used for non-reciprocal antennas. The second difference is related to scattering: in the Lorentz reciprocity theorem formulation, multiple scattering between the test and probe antennas is neglected, but scattering or reradiation is taken into account in the beginning of the derivation. However, during the steps of derivation, reradiation is neglected in some integrals and gives no contribution in others, as shown in detail in Reference [9]. In the scattering matrix formulation, reradiation and multiple scattering are both completely neglected.

Since the radiation pattern of either the probe or the test antenna can only be determined in the "receiving mode" using the facilities available at CRC, we have opted to design our cylindrical near-field measurement system on the Lorentz reciprocity theorem approach. The fact that most antennas tested at CRC are reciprocal substantiates even more this final choice. This probe-compensated cylindrical near-field measurements system has been based on the original development work of W. Leach [9]; a summarized presentation

of the method is available in Reference [32]. However, the simplifications brought to the characterization of the probe antenna along with the overall implementation of the system using restricted facilities are original contributions which we believe are very valuable.

1.4 Definition of the Problem

It is required to implement a cylindrical near-field measurements system. The theory behind cylindrical near-field measurements is first presented in Chapter 2 of this thesis: the cylindrical wave expansion of an electromagnetic field, its far-field approximation solution, and the determination of an antenna far-field pattern from probe-compensated cylindrical near-field measurements are developed in succession. The major problem in the measurement of electromagnetic fields has always been that of sampling. Actually, this problem can be more accurately divided into two parts: sampling density and probe compensation, and they both constitute the main research theme of this thesis.

The sampling density is first addressed at the end of Chapter 2. It has generally been assumed, for example, that sampling distances on the order of a wavelength are sufficient to characterize a field; in fact, the sampling distance should be based on the spatial rate of change of the fields over any surfaces. This spatial rate of change is actually determined by the nature of the sources illuminating the surface of interest and by the distance separating the sources from the surface.

Regardless of the sampling density used, the accuracy of any calculated far field depends on the accuracy of the measured samples. Since an ideal probe is not physically realizable, any antenna used to measure the near field of another antenna will have an output proportional to a weighted average of the field over its effective aperture. Thus, the ability to compensate for the effects of a non-ideal probe is an important consideration in any near-field measurement scheme: in fact, failure to compensate for the effects of a non-ideal probe will always result in the filling of the nulls on the computed radiation pattern of the antenna under test. A simplified probe-compensation technique is presented in Chapter 3.

Results of the experimental work performed are covered in Chapter 4. It includes such matters as the design of a test antenna and the measurement of its radiation pattern, along with a detailed description of the near-field measurements set-up and procedures.

Finally, conclusions and suggestions for future work are presented in Chapter 5.

CHAPTER 2

THEORY OF NEAR-FIELD MEASUREMENTS

2.1 Introduction

Based on a method developed by W. Leach [9], the mathematical background for the determination of the far field of an antenna from probe-compensated near-field measurements on the surface of a cylinder completely enclosing the antenna is presented in this chapter. First, the three-dimensional cylindrical wave expansion of an electromagnetic field is derived, and it is shown that the cylindrical wave spectrum of the field radiated by any antenna can be determined uniquely from a knowledge of the tangential field on a cylinder containing the antenna. Then, using the method of steepest descent to evaluate the necessary integrals, a far-field approximation to the general expression is achieved. One finds, as expected, that the far-field electric field intensity and far-field magnetic field intensity are transverse to the radial direction of propagation and are mutually perpendicular.

It is further shown how the cylindrical wave spectrum of an antenna under test can be obtained from the output of an arbitrary probe used to measure the tangential electric field on the surface of a cylinder enclosing the antenna. Due to the non-ideal characteristics of the probe, some field compensation is required for accurate extrapolation of the far field, and this is achieved simply based on the knowledge of the cylindrical wave expansion of the probe radiated

field, when used as a transmitter. A spatial sampling criterion, which is intended to minimize the potentially large amount of near-field data required, is finally developed in detail. The major assumption made throughout these developments is that the antenna under test is not a high-Q or a supergain structure.

2.2 The Cylindrical Wave Expansion of an Electromagnetic Field

Within any closed domain of a homogeneous, isotropic, source-free medium, it can be shown that all vectors characterizing the electromagnetic field satisfy one and the same differential equation [10]:

$$\nabla^2 \vec{C} - \mu\epsilon \frac{\partial^2 \vec{C}}{\partial t^2} - \mu\sigma \frac{\partial \vec{C}}{\partial t} = 0 \quad (2.1)$$

where $\vec{C}$ denotes any such vector - specifically $\vec{E}$, $\vec{B}$, $\vec{D}$, $\vec{H}$, the vector potential and the Hertzian vectors. Because of the linearity of this wave equation, fields of arbitrary time variations can be constructed from harmonic solutions. Consequently, there is no loss of generality in the assumption that the vector $\vec{C}$ contains time only as a factor $e^{-j\omega t}$. By the operator ∇^2 acting on a vector, one must understand $\nabla^2 \vec{C} = \nabla(\nabla \cdot \vec{C}) - \nabla \times \nabla \times \vec{C}$. Therefore, Eqn. (2.1) can be rewritten as:

$$\nabla(\nabla \cdot \vec{C}) - \nabla \times \nabla \times \vec{C} + k^2 \vec{C} = 0 \quad (2.2)$$

where k is the propagation constant defined as:

$$k^2 = \omega^2 \mu \epsilon + j \mu \omega \sigma \quad (2.3)$$

In order to follow a similar derivation process and notation as that performed by Leach [9] - who has done most of the grassroot work in probe-compensated near-field measurements on a cylinder - we will apply the previous reasoning more specifically to the vector potential. Based on Eqn. (2.2), the sinusoidal steady-state form of the vector magnetic potential function satisfies the differential equation:

$$\nabla^2 \vec{A} + k^2 \vec{A} = 0 \quad (2.4)$$

A general solution to this equation can be obtained by using a technique described by Stratton [10], in which three independent vector solutions are constructed:

$$\vec{L} = \vec{\nabla} \psi \quad (2.5)$$

$$\vec{M} = \vec{\nabla} \times \hat{a} \psi \quad (2.6)$$

$$\vec{N} = (1/k) \vec{\nabla} \times \vec{M} \quad (2.7)$$

where $\hat{a}$ is any constant vector of unit length and ψ is a scalar function which satisfies the equation

$$\nabla^2 \psi + k^2 \psi = 0 \quad (2.8)$$

In fact, if $\vec{A}$ is replaced by $\vec{L}$, $\vec{M}$ or $\vec{N}$, one can easily verify that Eqn. (2.4) is indeed satisfied, subject to Eqn. (2.8). In addition,

since $\hat{a}$ is a constant vector, it follows that $\vec{M}$ is related to $\vec{L}$ and $\vec{N}$ by

$$\vec{M} = \vec{L} \times \hat{a} = \frac{1}{k} \vec{\nabla} \times \vec{N} \quad (2.9)$$

For one and the same generating function ψ , the vector $\vec{M}$ is then perpendicular to the vector $\vec{L}$, or $\vec{L} \cdot \vec{M} = 0$. The vector functions $\vec{L}$, $\vec{M}$ and $\vec{N}$ also have other notable properties that follow directly from their definitions:

- a) since both $\vec{M}$ and $\vec{N}$ are defined as the curl of a vector, they are solenoidal vector fields, i.e.

$$\vec{\nabla} \cdot \vec{M} = 0 \quad ; \quad \vec{\nabla} \cdot \vec{N} = 0 \quad (2.10)$$

- b) and since $\vec{L}$ is defined as the gradient of a scalar, it is a conservative vector field, or

$$\vec{\nabla} \times \vec{L} = 0 \quad ; \quad \vec{\nabla} \cdot \vec{L} = \nabla^2 \psi = -k^2 \psi \quad (2.11)$$

Therefore, solutions for $\vec{M}$, $\vec{N}$ and $\vec{L}$ may be obtained by first solving Eqn. (2.6) in any coordinate system of interest. In the present case, a solution is desired as a function of the cylindrical coordinates (r, ϕ, z) illustrated in Fig. 1. Stratton [11] studied the behaviour of the scalar function ψ over a cylinder. His work shows that by using the method of separation of variables particular solutions to Eqn. (2.8) in cylindrical coordinates are:

$$\psi_{nh}^{(i)}(r, \phi, z) = Z_n^{(i)}(\Lambda r) e^{jn\phi} e^{-jhz} \quad (2.12)$$

$$i = 1, 2, 3, 4$$

Figure 1. Coordinate system definition.

where n is any integer, h is any real number and Λ is defined as

$$\Lambda = (k^2 - h^2)^{1/2} \quad (2.13)$$

The function $Z_n^{(i)}(\Lambda r)$ is any one of the four cylindrical Bessel functions, also known as circular cylinder functions, of order n and argument (Λr) . More specifically,

$$Z_n^{(1)}(\Lambda r) = J_n(\Lambda r) \quad (2.14)$$

$$Z_n^{(2)}(\Lambda r) = N_n(\Lambda r) \quad (2.15)$$

$$Z_n^{(3)}(\Lambda r) = H_n^{(1)}(\Lambda r) \quad (2.16)$$

$$Z_n^{(4)}(\Lambda r) = H_n^{(2)}(\Lambda r) \quad (2.17)$$

- Note:
- 1) The function $N_n(\Lambda r)$ is the Bessel function of the second kind as defined in [11] and should not be confused with the previously mentioned vector solution $\vec{N}$.
 - 2) The functions $H_n^{(i)}(\Lambda r)$, $i = 1, 2$ are the Bessel functions of the third kind, commonly known as Hankel functions, and are defined by the relations

$$H_n^{(1)}(\Lambda r) = J_n(\Lambda r) + jN_n(\Lambda r) \quad (2.18)$$

$$H_n^{(2)}(\Lambda r) = J_n(\Lambda r) - jN_n(\Lambda r)$$

For the present case though, a solution for $\vec{A}$ is desired, which is valid in the region external to the smallest cylinder containing all sources defined by $r \geq r_a$. In order for $\vec{A}$ to satisfy the radiation condition at infinity, the large argument asymptotic expansion of the cylindrical Bessel function must represent surfaces of constant phase which propagate in the positive radial direction. For time variations of the form "exp(jwt)", the only function which satisfies this condition is the Hankel function of the second kind $H_n^{(2)}(\Lambda r)$, since it represents outward propagating waves. As given by Stratton [11], the large argument asymptotic expansion of this function is:

$$H_n^{(2)}(\Lambda r) \approx j^{n+1/2} \left(\frac{2}{\pi \Lambda r}\right)^{1/2} e^{-j\Lambda r} \quad (2.19)$$

In conclusion, with the choice of circular cylinder function completed, Eqn. (2.12) becomes:

$$\psi_{nh}^{(4)}(r, \phi, z) = H_n^{(2)}(\Lambda r) e^{jn\phi} e^{-jh z} \quad (2.20)$$

Examination of Eqn. (2.20) reveals the form of the linear combination of the vectors $\vec{M}$, $\vec{N}$ and $\vec{L}$ for the general solution to Eqn. (2.4); there are first two independent parameters "n" and "h" over which the linear combination must be formed. Since n is an integer, the combination will involve a summation over all "n"; on the other hand, since h is any real number, the combination will also be in the form of an integral over all real "h". Thus a solution for $\vec{A}$ can be expressed as [11]:

$$\vec{A}(r, \phi, z) = \frac{-1}{j\omega} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} [a_n(h) \vec{M}_{nh}^{(4)}(r, \phi, z) + b_n(h) \vec{N}_{nh}^{(4)}(r, \phi, z) + c_n(h) \vec{L}_{nh}^{(4)}(r, \phi, z)] dh \quad (2.21)$$

where $a_n(h)$, $b_n(h)$ and $c_n(h)$ are scalar amplitude weighting functions and where the factor $(\frac{-1}{j\omega})$ was chosen only to simplify the resulting expression for the electric field intensity, $\vec{E}$. In fact, based on the solution for $\vec{A}$, solutions for the magnetic and electric field intensities can be obtained from the relations:

$$\vec{H} = \frac{1}{\mu} \vec{\nabla} \times \vec{A} \quad (2.22)$$

$$\vec{E} = \frac{1}{j\omega\epsilon} \vec{\nabla} \times \vec{H} \quad (2.23)$$

where it has been assumed that the conductivity σ is zero in the region of interest. Using Eqns. (2.5), (2.6), (2.7) and (2.9), it follows that $\vec{E}$ and $\vec{H}$ are given by:

$$\vec{E} = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} [a_n(h) \vec{M}_{nh}^{(4)} + b_n(h) \vec{N}_{nh}^{(4)}] dh \quad (2.24)$$

$$\vec{H} = \frac{-k}{j\omega\mu} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} [a_n(h) \vec{N}_{nh}^{(4)} + b_n(h) \vec{M}_{nh}^{(4)}] dh \quad (2.25)$$

The absence of the vector $\vec{L}$ in these solutions follows from the fact that $\vec{L}$ has non-zero divergence, as shown in Eqn. (2.11), and thus cannot be used to represent $\vec{E}$ or $\vec{H}$ in a source-free region since $\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{H} = 0$. We now recall Eqn. (2.6):

$$\vec{M} = \vec{\nabla} \times \hat{a}\psi$$

Although the constant unit vector $\hat{a}$ may be chosen arbitrarily, it would be convenient to choose it so that $\vec{M}$ has no z-component. Now, if Eqn. (2.6) was rewritten in the form

$$\vec{M} = \vec{\nabla}\psi \times \hat{a} \quad (2.26)$$

one can see that the choice $\hat{a} = \hat{z}$ makes this true. Therefore, from Eqns. (2.6) and (2.7), it follows that the solutions for $\vec{M}$ and $\vec{N}$ can be written as:

$$\vec{M}_{nh}^{(i)} = \vec{m}_{nh}^{(i)} e^{jn\phi} e^{-jhz} \quad (2.27)$$

$$\vec{N}_{nh}^{(i)} = \vec{n}_{nh}^{(i)} e^{jn\phi} e^{-jhz} \quad (2.28)$$

where

$$\vec{m}_{nh}^{(i)} = \hat{r} \left(\frac{jn}{r} \right) z_n^{(i)}(\Lambda r) - \hat{\phi} \frac{\partial z_n^{(i)}}{\partial r} \quad (2.29)$$

$$\vec{n}_{nh}^{(i)} = -\hat{r} \left(\frac{jh}{k} \right) \frac{\partial z_n^{(i)}}{\partial r} + \hat{\phi} \left(\frac{nh}{kr} \right) z_n^{(i)}(\Lambda r) + \hat{z} \left(\frac{\Lambda^2}{k} \right) z_n^{(i)}(\Lambda r) \quad (2.30)$$

With the introduction of Eqns. (2.27) and (2.28) into Eqn. (2.24), the electric field intensity can be expressed as:

$$\vec{E} = \sum_{n=-\infty}^{\infty} e^{jn\phi} \int_{-\infty}^{\infty} [a_n(h) \vec{m}_{nh}^{(4)} + b_n(h) \vec{n}_{nh}^{(4)}] e^{-jhz} dh \quad (2.31)$$

where $\vec{m}_{nh}^{(4)}$ and $\vec{n}_{nh}^{(4)}$ represent respectively electric and

magnetic solutions transverse to the z-direction, or

$(TE)_z$ and $(TM)_z$. On any cylinder of constant radius r , each scalar component of this equation is in the form of a Fourier series in ϕ and a Fourier integral in z . Thus it follows that the inverse relationship

$$a_n(h) \vec{m}_{nh}^{(4)} + b_n(h) \vec{n}_{nh}^{(4)} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \vec{E}(r_1, \phi, z) e^{-jn\phi} e^{jhz} d\phi dz \quad (2.32)$$

must exist, where r_1 is the radius of the cylinder.

It follows that Eqn. (2.32) can be solved for $a_n(h)$ and $b_n(h)$ if it is separated into its scalar components and the resulting equations solved simultaneously. However, since there are only two quantities to be determined, only two components of the field $\vec{E}(r_1, \phi, z)$ need to be known. If these two components are chosen to be E_ϕ and E_z on the cylinder, the resulting equations are

$$a_n(h) \left[-\frac{\partial H_n^{(2)}}{\partial r} \right] + b_n(h) \left[\frac{nh}{kr} H_n^{(2)}(\Lambda r) \right] = I_\phi \quad (2.33)$$

$$b_n(h) \left[\frac{\Lambda^2}{k} H_n^{(2)}(\Lambda r) \right] = I_z \quad (2.34)$$

where

$$I_\phi = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} E_\phi(r_1, \phi, z) e^{-jn\phi} e^{jhz} d\phi dz \quad (2.35)$$

and

$$I_z = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} E_z(r_1, \phi, z) e^{-jn\phi} e^{jhz} d\phi dz \quad (2.36)$$

Solving these equations simultaneously yields:

$$a_n(h) = \frac{-I_\phi}{\frac{\partial}{\partial r_1} [H_n^{(2)}(\Lambda r_1)]} + \frac{nh I_z}{(\Lambda^2 r_1) \frac{\partial}{\partial r_1} [H_n^{(2)}(\Lambda r_1)]} \quad (2.37)$$

$$b_n(h) = \frac{I_z}{\left(\frac{\Lambda^2}{k}\right) H_n^{(2)}(\Lambda r_1)} \quad (2.38)$$

Then, a knowledge of the tangential electric field intensity on the surface of a cylinder containing all sources is sufficient to determine the fields at all points external to the cylinder. The solution involving the tangential magnetic field on the cylinder can be obtained similarly.

It should be mentioned here that in order to use Eqns. (2.37) and (2.38), the tangential electric field on the cylinder must be either known a priori or measured with an ideal probe. Obviously in practice, the accuracy of any calculated far field will depend on the accuracy of the measured near-field samples. Since an ideal probe is not physically realizable, any chosen antenna used to probe the near-field will have an output proportional not only to one vector but to a weighted average of the field over its effective aperture. Thus the ability to compensate for the effects of a non-ideal probe can be an important consideration in any near-field measurement scheme. Leach, in Reference [9], developed a new method for determining the functions $a_n(h)$ and $b_n(h)$ from actual measurements made on a cylinder

with an arbitrary probe. This method will be presented further in Section 2.4; it assumes only that the cylindrical wave expansion of the field radiated by the probe, when it is used as a transmitter, is known. In the development of a cylindrical near-field measurement system at CRC, we have further simplified this method by assuming reciprocity of the probe and by defining its polarization. As mentioned earlier, these developments will be presented later in Section 2.4. For the meantime, let us consider what simplification results if the field of an antenna is to be evaluated in the far field of a source.

2.3 Far-Field Approximation to the Cylindrical Wave Expansion

We have just developed, in the previous Section, a general expression for the electric field intensity external to a closed surface containing all sources. In general though, the evaluation of this expression is a formidable task. However, a considerable simplification results if the field is to be evaluated in the far-field region of the source.

For large radius r , the cylindrical wave vectors $\vec{m}_{nh}^{(4)}$ and $\vec{n}_{nh}^{(4)}$, which are defined in Eqns. (2.29) and (2.30), can be simplified considerably if the Hankel function and its partial derivative with respect to r are replaced by their large argument asymptotic expansion [11,12]. The large argument asymptotic expansion for $H_n^{(2)}(\Lambda r)$ has already been given in Eqn. (2.19); the corresponding expansion for $\frac{\partial H_n^{(2)}}{\partial r}$ can be obtained by taking the partial derivative of that equation with respect to r , which yields

$$\frac{\partial H_n^{(2)}(\Lambda r)}{\partial r} \approx j^{n-1/2} \left(\frac{2\Lambda}{\pi r}\right)^{1/2} e^{-j\Lambda r} \quad (2.39)$$

where the term involving $r^{-3/2}$ has been neglected for large r . The resulting expressions for the cylindrical wave vectors $\vec{m}_{nh}^{(4)}$ and $\vec{n}_{nh}^{(4)}$ are:

$$\vec{m}_{nh}^{(4)} = -\hat{\phi} j^{n-1/2} \left(\frac{2\Lambda}{\pi r}\right)^{1/2} e^{-j\Lambda r} \quad (2.40)$$

$$\vec{n}_{nh}^{(4)} = -(\hat{r}h - \hat{z}\Lambda) \frac{j^{n+1/2}}{k} \left(\frac{2\Lambda}{\pi r}\right)^{1/2} e^{-j\Lambda r} \quad (2.41)$$

Substituting these expressions into the general solution for $\vec{E}$ [Eqn. (2.31)] results in:

$$\begin{aligned} \vec{E} = & j^{1/2} \left(\frac{2}{\pi r}\right)^{1/2} \sum_{n=-\infty}^{\infty} j^n e^{jn\phi} \int_{-\infty}^{\infty} \Lambda^{1/2} [\hat{\phi} j a_n(h) \\ & - \frac{1}{k} (\hat{r}h - \hat{z}\Lambda) b_n(h)] e^{-j\Lambda r} e^{-jh z} dh \end{aligned} \quad (2.42)$$

This equation can be simplified by making the change in variable,

$$h = k \sin \alpha \quad (2.43)$$

by replacing Λ by its original value $\Lambda = (k^2 - h^2)^{1/2}$ and by converting the far-field point into spherical coordinates using the transformations

$$r = R \sin \theta \quad (2.44)$$

$$z = R \cos \theta \quad (2.45)$$

The result is then:

$$\vec{E} = j^{1/2} k^{3/2} \left(\frac{2}{\pi R \sin \theta} \right) \sum_{n=-\infty}^{\infty} j^n e^{jn\phi} \int_C [\hat{\phi} j a_n(k \sin \alpha) - (\hat{r} \sin \alpha - \hat{z} \cos \alpha) b_n(k \sin \alpha)] \cos^{3/2} \alpha e^{-jk R \sin(\alpha+\theta)} d\alpha \quad (2.46)$$

where C is the contour illustrated in Fig. 2.

The integral of Eqn. (2.46) was evaluated by Leach [9] using the method of steepest descent. Although fairly straightforward, the evaluation is much involved; it is reproduced here as Appendix A. The resulting expression for the electric field intensity is shown to be:

$$\vec{E} = \frac{-2k \sin \theta}{R} e^{-jkR} \sum_{n=-\infty}^{\infty} j^n e^{jn\phi} [\hat{\phi} a_n(k \cos \theta) - j(\hat{r} \cos \theta - \hat{z} \sin \theta) b_n(k \cos \theta)] \quad (2.47)$$

Since $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$, the above equation can therefore be separated into spherical components, which are:

$$E_R = 0 \quad (2.48)$$

$$E_{\theta} = \frac{-j k 2 \sin \theta}{R} e^{-jkR} \sum_{n=-\infty}^{\infty} j^n b_n(k \cos \theta) e^{jn\phi} \quad (2.49)$$

$$E_{\phi} = \frac{-2k \sin \theta}{R} e^{-jkR} \sum_{n=-\infty}^{\infty} j^n a_n(k \cos \theta) e^{jn\phi} \quad (2.50)$$

As would be expected, the far-field electric field intensity is transverse to the radial direction. The expressions for the

Figure 2. Contour of integration C .

far-field magnetic field intensity are obtained from the solution for the electric field intensity simply by interchanging the amplitude weighting functions $a_n(h)$ and $b_n(h)$ and by multiplying by the factor $-k/j\omega\mu$ as shown earlier in Eqns. (2.24) and (2.25). They are:

$$H_R = 0 \quad (2.51)$$

$$H_\theta = -E_\phi/\eta \quad (2.52)$$

$$H_\phi = E_\theta/\eta \quad (2.53)$$

where $\eta = (\mu/\epsilon)^{1/2}$. Thus the far-field magnetic field intensity is also transverse to the radial direction and is related to the solution for $\vec{E}$ by the vector equation

$$\vec{H} = \frac{\hat{R} \times \vec{E}}{\eta} \quad (2.54)$$

Since both $\vec{E}$ and $\vec{H}$ have no radial components and are mutually perpendicular, the far field comprises a spherical transverse electromagnetic field.

It can be seen from Eqns. (2.48) to (2.50) and (2.51) to (2.53), that the far field is determined only by those values of $a_n(h)$ and $b_n(h)$ for which $-k \leq h \leq k$. Recall in effect that $h = k \cos \theta$. It can therefore be concluded that the part of the near field for which $|h| > k$ represents evanescent waves (or reactive energy storage) in the vicinity of the antenna which in no way influence the far-field structure except to the extent that they are necessary to support a particular current distribution on the antenna.

2.4 Determination of the Far Field of an Antenna from Probe-Compensated Near-Field Measurements on a Cylinder Containing the Antenna

The ability to compensate for the effects of a non-ideal probe, as previously mentioned in Section 2.2, is an important consideration in any near-field measurement scheme. It will be shown, in this section, that the far field of an antenna can be calculated independently of the receiving properties of the probe. Based on a method developed by Leach [9], the only information which will be assumed about the probe is a knowledge of the cylindrical wave expansion of its radiated field, when used as a transmitter.

Consider Fig. 3. Let Σ_a be a cylinder of radius r_a that contains an arbitrary test antenna connected to signal generator A. Denote the field radiated by the test antenna by $\vec{E}_a(r)$ and $\vec{H}_a(r)$, defined as per Eqns. (2.24) and (2.25). Let this field be incident on a probe antenna whose reference origin O' is located at the point (r_0, ϕ_0, z_0) . Let the probe be connected via a waveguide feeder to generator B. Denote the field radiated by the probe when generator B is activated by $\vec{E}_b(r')$ and $\vec{H}_b(r')$, where r' is measured with respect to O' . Let the field scattered by the test antenna when generator B is activated be denoted by $\vec{E}_{as}(r)$ and $\vec{H}_{as}(r)$, and the field scattered by the probe when generator A is activated by $\vec{E}_{bs}(r')$ and $\vec{H}_{bs}(r')$.

It is desired to determine the signal induced across the terminals of generator B where only generator A is activated. If generator B is then replaced by a linear detector having an input

impedance equal to the output impedance of generator B, it can be shown that the far field of the test antenna can be calculated from the detector output voltage if its amplitude and phase are known as functions of ϕ_0 and z_0 over the cylinder of radius r_0 . It will be assumed that the amplitude weighting functions in the cylindrical wave expansion of the field radiated by the probe when generator B is activated are known.—A numerical method is developed, further in this thesis, by which one obtains this information from the measured far-field pattern (the two principal plane cuts only) of the probe antenna.—Also, without loss of generality, it will be assumed that generator B is matched to the waveguide feeder for the probe antenna.

In Fig. 3, let V be the volume bounded by the surface Σ_1 , Σ_b and Σ_∞ , where Σ_1 is the cylinder of radius r_1 , Σ_b is the surface lying just outside the shield enclosing generator B and cutting the waveguide feeder for the probe at S_b , and Σ_∞ is the sphere of infinite radius. Since V contains no sources by assumption, it follows from the Lorentz reciprocity theorem [13] that:

$$\oint_{\Sigma_1 + \Sigma_b + \Sigma_\infty} [(\vec{E}_a + \vec{E}_{bs}) \times (\vec{H}_b + \vec{H}_{as}) - (\vec{E}_b + \vec{E}_{as}) \times (\vec{H}_a + \vec{H}_{bs})] \cdot \hat{n} \, da = 0 \quad (2.55)$$

where all multiply scattered terms have been neglected. It should be noted here that, based on Silver [14], the Lorentz theorem is the basis for all calculations of the output of a receiving antenna in the presence of a transmitting antenna. As demonstrated by Kerr [13], the integral of Eqn. (2.55) will vanish identically over Σ_∞ and over

Σ_b , except for the area S_b . Also, by virtue of their definition,
 $E_{bs} = H_{bs} = 0$ over S_b . Thus, it follows that Eqn. (2.55) reduces to:

$$\begin{aligned}
 & \oint_{\Sigma_1} (\vec{E}_a \times \vec{H}_b - \vec{E}_b \times \vec{H}_a) \cdot (-\hat{r}) da \\
 & + \oint_{\Sigma_1} (\vec{E}_a \times \vec{H}_{as} - \vec{E}_{as} \times \vec{H}_a) \cdot (-\hat{r}) da \\
 & + \oint_{\Sigma_1} (\vec{E}_{bs} \times \vec{H}_b - \vec{E}_b \times \vec{H}_{bs}) \cdot (-\hat{r}) da \\
 & + \oint_{S_b} (\vec{E}_a \times \vec{H}_b - \vec{E}_b \times \vec{H}_a) \cdot (\hat{x}') da \equiv 0
 \end{aligned} \tag{2.56}$$

where the terms of the integral involving the products of the scattered fields, i.e.

$$\oint_{\Sigma_1} (\vec{E}_{bs} \times \vec{H}_{as} - \vec{E}_{as} \times \vec{H}_{bs}) \cdot (-\hat{r}) da \tag{2.57}$$

have been neglected. It has also been assumed that, in the integral over S_b , the contribution of the scattered terms $\vec{E}_{as}$ and $\vec{H}_{as}$ is negligible, so that $\vec{E}_b + \vec{E}_{as} \hat{=} \vec{E}_b$ and $\vec{H}_b + \vec{H}_{as} \hat{=} \vec{H}_b$. These assumptions are actually valid if the scattered fields over Σ_1 and S_b are small compared to the incident fields. Without them, the desired solution to Eqn. (2.55) would be impossible.

Let the four integrals in Eqn. (2.56) be denoted by I_1 , I_2 , I_3 and I_4 respectively. To evaluate I_1 , the cylindrical wave expansions of the fields over Σ_1 are assumed initially to be in the forms:

$$\vec{E}_a(r) = a_n(h) \vec{M}_{nh}^{(4)}(r) + b_n(h) N_{nh}^{(4)}(r) \quad (2.58)$$

$$\vec{H}_a(r) = \frac{-k}{j\omega\mu} [b_n(h) \vec{M}_{nh}^{(4)}(r) + a_n(h) N_{nh}^{(4)}(r)] \quad (2.59)$$

$$\vec{E}_b(r') = c_m(\eta) \vec{M}_{m\eta}^{(4)}(r') + d_m(\eta) N_{m\eta}^{(4)}(r') \quad (2.60)$$

$$\vec{H}_b(r') = \frac{-k}{j\omega\mu} [d_m(\eta) \vec{M}_{m\eta}^{(4)}(r') + c_m(\eta) N_{m\eta}^{(4)}(r')] \quad (2.61)$$

where h and η represent the wavenumbers for the test antenna and probe respectively.

After I_1 is evaluated for these fields, the result must be summed in m and n and integrated in η and h to obtain the final value of the integral. Leach [9, Appendix B] developed some vector translation theorems which made it possible to change the origin for $\vec{E}_b$ and $\vec{H}_b$ from O' to O . These vector translation theorems, reproduced for convenience in Appendix B, result in the following:

$$\begin{aligned} \vec{E}_b(r) = & \sum_{\ell=-\infty}^{\infty} (-1)^{\ell} H_{m+\ell}^{(2)}(\lambda r_o) e^{j\ell\phi_o} e^{j\eta z_o} \\ & \cdot (c_m(\eta) M_{-\ell\eta}^{(1)}(r) + d_m(\eta) N_{-\ell\eta}^{(1)}(r)) \end{aligned} \quad (2.62)$$

and

$$\begin{aligned} \vec{H}_b(r) = & \frac{-k}{j\omega\mu} \sum_{\ell=-\infty}^{\infty} (-1)^{\ell} H_{m+\ell}^{(2)}(\lambda r_o) e^{j\ell\phi_o} e^{j\eta z_o} \\ & \cdot (d_m(\eta) M_{-\ell\eta}^{(1)}(r) + c_m(\eta) N_{-\ell\eta}^{(1)}(r)) \end{aligned} \quad (2.63)$$

where $\lambda = (k^2 - \eta^2)^{1/2}$. Substitution of these expressions and those for $\vec{E}_a$ and $\vec{H}_a$ into the integrand for I_1 gives:

$$\begin{aligned}
 I_1 = & \frac{k}{j\omega\mu} \sum_{\ell=-\infty}^{\infty} (-1)^{\ell} H_{m+\ell}^{(2)}(\lambda r_0) e^{j\ell\phi_0} \\
 & \cdot e^{j\eta z_0} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \{ [a_n(h) d_m(\eta) + b_n(h) c_m(\eta)] \\
 & \cdot [\vec{M}_{nh}^{(4)} \times \vec{N}_{- \ell \eta}^{(1)} + \vec{N}_{nh}^{(4)} \times \vec{M}_{- \ell \eta}^{(1)}] \\
 & + [a_n(h) c_m(\eta) + b_n(h) d_m(\eta)] \cdot [\vec{M}_{nh}^{(4)} \times \vec{N}_{- \ell \eta}^{(1)} + \vec{N}_{nh}^{(4)} \times \vec{M}_{- \ell \eta}^{(1)}] \} \\
 & \cdot \hat{r} r_1 d\phi dz
 \end{aligned} \tag{2.64}$$

where all $\vec{M}$'s and $\vec{N}$'s are functions of (r, ϕ, z) .

In Reference [9], Leach also demonstrated that, from the orthogonality properties of the cylindrical wave vectors (presented in Appendix C), the terms involving the cross-products

$$(\vec{M}_{nh}^{(4)} \times \vec{N}_{- \ell \eta}^{(1)}) \cdot \hat{r} \quad \text{and} \quad (\vec{N}_{nh}^{(4)} \times \vec{M}_{- \ell \eta}^{(1)}) \cdot \hat{r}$$

had zero contribution to the integral. The remaining terms were then evaluated with the aid of Eqns. (C.3) and (C.4) resulting in:

$$\begin{aligned}
 I_1 = & \frac{k r_1}{j\omega\mu} \sum_{\ell=-\infty}^{\infty} (-1)^{\ell} H_{m+\ell}^{(2)}(\lambda r_0) e^{j\ell\phi_0} e^{j\eta z_0} \\
 & \cdot [a_n(h) c_m(\eta) + b_n(h) d_m(\eta)] \left\{ \frac{4\pi^2 \Lambda^3}{k} \delta_{n\ell} \delta(\eta+h) \right. \\
 & \cdot [J'_{-n}(\Lambda r_1) H_n^{(2)}(\Lambda r_1) - J_{-n}(\Lambda r_1) H_n^{(2)'}(\Lambda r_1)] \}
 \end{aligned} \tag{2.65}$$

where δ_{nl} is the Kronecker delta and $\delta(\eta+h)$ is the Dirac delta function, as defined in [15]. This expression can be simplified with the aid of the identity:

$$J'_{-n}(\Lambda r_1) H_n^{(2)}(\Lambda r_1) - J_{-n}(\Lambda r_1) H_n^{(2)'}(\Lambda r_1) = \frac{j2(-1)^n}{\pi \Lambda r_1} \quad (2.66)$$

Thus, I_1 further reduces to:

$$I_1 = \frac{8\pi\Lambda^2}{\omega\mu} H_{n+m}^{(2)}(\Lambda r_0) e^{jn\phi_0} e^{j\eta z_0} \delta(\eta+h) \cdot [a_n(h)c_m(\eta) + b_n(h)d_m(\eta)] \quad (2.67)$$

which is independent of r_1 . When this expression is summed over all m and n and integrated over all η and h , the result is:

$$I_1 = \frac{8\pi\Lambda^2}{\omega\mu} \sum_{n=-\infty}^{\infty} e^{jn\phi_0} \int_{-\infty}^{\infty} [a_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_0) + b_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_0)] e^{-jh z_0} dh \quad (2.68)$$

in which the sampling property of the impulse function $\delta(\eta+h)$ has been used to eliminate the integral in η .

Evaluation of integrals I_2 and I_3 can be done similarly using the appropriate cylindrical wave expansions, vector orthogonality properties of Appendix C, and vector translation theorems from Appendix B. Since multiple scattering is neglected, $\vec{E}_a$, $\vec{H}_a$, $\vec{E}_{as}$ and

$\vec{H}_{as}$ satisfy the homogeneous wave equation outside Σ_a . Thus, by the Lorentz reciprocity theorem, it follows, as shown step-by-step by Leach in [9], that the integral I_2 is identically zero. Similarly, I_3 also vanishes.

The integral for I_4 can be evaluated by assuming that the waveguide feeder for the probe will support only the dominant $(TE)_{10}$ mode. It should be noted here that this is not a restrictive assumption, for the same result would be obtained assuming any other mode or combination of modes; only the final constant of proportionality would differ. In a waveguide then, the assumed transverse fields can be expressed in the form:

$$\vec{E}_{az'}(x', y', z') = \kappa \cos \frac{\pi y'}{a} e^{j\beta x'} \quad (2.69)$$

$$\vec{H}_{ay'}(x', y', z') = \frac{\kappa}{Z_w} \cos \frac{\pi y'}{a} e^{j\beta x'} \quad (2.70)$$

$$\vec{E}_{bz'}(x', y', z') = \cos \frac{\pi y'}{a} e^{-j\beta x'} \quad (2.71)$$

$$\vec{H}_{by'}(x', y', z') = \frac{1}{Z_w} \cos \frac{\pi y'}{a} e^{-j\beta x'} \quad (2.72)$$

where β is the propagation constant in the waveguide, a is the width of the waveguide, and Z_w is the wave impedance. The fields have been normalized so that the electric field at the centre of the waveguide has unit magnitude where generator B alone is activated. Substituting these expressions into the integrand of I_4 , one obtains:

$$I_4 = \frac{-\kappa}{Z_w} \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} 2 \cos^2 \left(\frac{\pi y'}{a} \right) dx' dy'$$

$$= -ab \frac{\kappa}{Z_w} \quad (2.73)$$

where b denotes the height dimension of the waveguide. As shown in Eqn. (2.69), the factor κ represents the amplitude of the electric field induced in the probe waveguide when only generator A is activated. It is actually a function of the position of the probe antenna with respect to the test antenna. If generator B is now replaced by a matched linear detector and only generator A is activated, the detector output voltage will be proportional to κ . Thus the expression for I_4 can be rewritten as:

$$I_4 = -C \frac{ab}{Z_w} v(r_o, \phi_o, z_o) \quad (2.74)$$

where C is the constant of proportionality and $v(r_o, \phi_o, z_o)$ is the position-dependent detector output voltage.

It then follows, from all previous results, that Eqn. (2.56) reduces to:

$$C \frac{ab}{Z_w} v(r_o, \phi_o, z_o) = \frac{8\pi\Lambda^2}{\omega\mu} \sum_{n=-\infty}^{\infty} e^{jn\phi_o} \int_{-\infty}^{\infty} [a_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) + b_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_o)] e^{-jh z_o} dh \quad (2.75)$$

The detector output voltage can be normalized by choosing the constant of proportionality to be:

$$C = \frac{32\pi^3 Z_w k^2}{\omega \mu a b} \quad (2.76)$$

With this choice, the normalized detector output voltage then becomes:

$$v(r_o, \phi_o, z_o) = \frac{\Lambda^2}{4\pi^2 k^2} \sum_{n=-\infty}^{\infty} e^{jn\phi_o} \int_{-\infty}^{\infty} [a_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) + b_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_o)] e^{-jhz_o} dh \quad (2.77)$$

Examination of the above equation reveals that $v(r_o, \phi_o, z_o)$ is in the form of a Fourier series in ϕ_o and a Fourier integral in z_o .

Thus the equation has an inverse which is given by:

$$a_n(h) \sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_o) + b_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_o) = \frac{k^2}{\Lambda^2} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} v(r_o, \phi_o, z_o) e^{-jn\phi_o} e^{jhz_o} d\phi_o dz_o \quad (2.78)$$

This is the desired result! Equation (2.78) relates the cylindrical wave amplitude weighting functions $a_n(h)$ and $b_n(h)$ of an arbitrary test antenna to the two-dimensional Fourier transform of the output voltage $v(r_o, \phi_o, z_o)$ of a probe when the measurement surface is a cylinder of radius r_o .

If the cylindrical wave amplitude weighting functions for the probe antenna $c_m(-h)$ and $d_m(-h)$ are known, it follows that Eqn. (2.78) can be solved for $a_n(h)$ and $b_n(h)$, provided two independent measurements of $v(r_o, \phi_o, z_o)$ are made. Let $v'(r_o, \phi_o, z_o)$ represent the output

voltage of the probe when it is rotated by 90° about its longitudinal axis. If the rotated probe cylindrical wave amplitude weighting functions are denoted by $c'_m(-h)$ and $d'_m(-h)$, Eqn. (2.78) becomes:

$$\begin{aligned} a_n(h) &= \sum_{m=-\infty}^{\infty} c'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) + b_n(h) \sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \\ &= \frac{k^2}{\Lambda^2} \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} v'(r_0, \phi_0, z_0) e^{-jn\phi_0} e^{jhz_0} d\phi_0 dz_0 \end{aligned} \quad (2.79)$$

Thereafter, when Eqns. (2.78) and (2.79) are solved simultaneously, one finds that the functions $a_n(h)$ and $b_n(h)$ are given by:

$$\begin{aligned} a_n(h) &= \frac{k^2}{\Lambda^2 \Delta_n(h)} \left[I_n(h) \sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right. \\ &\quad \left. - I'_n(h) \sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \end{aligned} \quad (2.80)$$

$$\begin{aligned} b_n(h) &= \frac{k^2}{\Lambda^2 \Delta_n(h)} \left[I'_n(h) \sum_{m=-\infty}^{\infty} c'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right. \\ &\quad \left. - I_n(h) \sum_{m=-\infty}^{\infty} c'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \end{aligned} \quad (2.81)$$

where the factors $I_n(h)$ and $I'_n(h)$ are defined as:

$$I_n(h) = \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} v(r_0, \phi_0, z_0) e^{-jn\phi_0} e^{jhz_0} d\phi_0 dz_0 \quad (2.82)$$

$$I'_n(h) = \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} v'(r_0, \phi_0, z_0) e^{-jn\phi_0} e^{jhz_0} d\phi_0 dz_0 \quad (2.83)$$

and where

$$\begin{aligned} \Delta_n(h) = & \left[\sum_{m=-\infty}^{\infty} c_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \cdot \left[\sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \\ & - \left[\sum_{m=-\infty}^{\infty} c'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \cdot \left[\sum_{m=-\infty}^{\infty} d_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \quad (2.84) \end{aligned}$$

It is important to note here that the probe is assumed to respond predominantly to one linear polarization component, i.e. vertical or horizontal, so that the determinant of the system of equations $\Delta_n(h)$ is not zero; otherwise, a solution for $a_n(h)$ and $b_n(h)$ would not exist.

Equations (2.80) to (2.84) form the basis of the method for the determination of the far field of an arbitrary antenna from measurements made with a probe over the surface of a cylinder completely enclosing the antenna. By using these equations and determining the cylindrical wave amplitude weighting functions $a_n(h)$ and $b_n(h)$, the far field of the antenna can therefore be calculated from Eqns. (2.49) and (2.50). Since the far field is determined from only those values of $a_n(h)$ and $b_n(h)$ for which $-k \leq h \leq k$, it follows that the cylindrical wave amplitude weighting functions of the probe need to be known only for arguments inside this interval.

As we have just seen, probe-compensated near-field measurements take into account the directional effects of the probe antenna on the primary field radiated by the antenna under test. It may

sometimes seem, however, that probe compensation is not a significant consideration when the near-field measurement surface is a cylinder. As far as the azimuth pattern of the test antenna for fixed elevation angles is concerned, probe-compensation is indeed not significant unless one is interested in precision measurements; in fact, the probe is always pointing towards the axis of the measurement cylinder. However, probe correction becomes an important consideration when calculating the elevation pattern of the test antenna, since this pattern is predominantly determined by the axial variation of the near-field data on the cylinder. In any case, there is essentially no difference in the numerical work necessary to calculate the far-field patterns with or without probe compensation. Once the probe coefficients have been determined, the only difference between the two cases is the Hankel functions which enter the solution. Without probe compensation, both these functions and their first derivatives must be evaluated, while derivatives of the Hankel functions do not enter the probe-compensated solution.

Details on how to choose a suitable probe for near-field measurements and on how to determine its cylindrical wave amplitude weighting functions are presented in Section 3.3. But first, let's consider the spatial sampling requirements over the measurement cylinder surface.

2.5 Determination of a Spatial Sampling Criterion on the Cylinder Surface Completely Enclosing the Antenna

A problem of major importance in the measurement of electromagnetic fields is that of sampling. From A.C. Ludwig's original research work in this area [16], it has generally been assumed that sampling distances of the order of a wavelength are sufficient to characterize a field. In fact, the spatial rate of change of the fields over any surface is determined by the nature of the sources illuminating that surface and by the distance separating the sources from the surface. It is well-known that the near field of an antenna can vary rapidly with distance, especially in those cases where the fields are highly reactive. In contrast, the far field does not change appreciably over distance subtending an angle less than the half-power beamwidth.

The required sampling density of any electromagnetic field will also be a function of the numerical integration method used to calculate the far field from the sampled near-field data. Since the actual measurement of the near field of an antenna over some surface enclosing it potentially requires the accumulation of a large amount of data, the problem of determining an optimum spatial sampling rate becomes one of great significance; sampling rates which are too high would obviously result in the acquisition of more data than is necessary to characterize a given near field, while sampling rates that are too low would result in supplying data which may be meaningless.

We have already developed, in Section 2.2, the cylindrical wave expansion for an electromagnetic field. Therefore, in order to establish a criterion for the sampling of the near field on a cylinder enclosing the antenna, the variations of the field with the coordinates on the cylinder can be studied by simply examining the cylindrical wave expansion. It has been shown earlier that each component of the field on a cylinder can be written in the form of a Fourier series in the azimuth angle ϕ and of a Fourier integral in the axial distance z . Thus it follows that the results of the sampling theory of Fourier transform analysis can be used to establish a sample spacing criterion on the cylinder if upper bounds on the angular harmonic n and the wavenumber h can be determined. One means of establishing these bounds is by examining the antenna quality factor, or Q .

The Q of an antenna is an important overall parameter specifying the antenna performance and the inherent physical limitations of antenna size on the gain. In particular, a high Q value means that large amounts of reactive energy are stored in the near field of the antenna. This in turn implies large currents, high ohmic losses, narrow bandwidth, and a large frequency sensitivity. In Reference [17], Collin and Rothschild have developed a general method for evaluating the Q of an antenna. In essence, the method consists of subtracting the energy density associated with the radiation field from the energy density of the total field, thus enabling the reactive magnetic and electric energy to be computed. Specific application of the method

was also made to evaluate the Q of cylindrical modes; it is this special application that is of most interest to us.

One recalls, from basic electronics, that the quality factor or Q of an electrical network at resonance may be defined as:

$$Q = \frac{\omega W_T}{P} \quad (2.85)$$

where W_T is the time-average energy stored in the network, and P is the power dissipated in the network. When the network is not resonant, the input impedance is proportional to $P + 2j\omega(W_m - W_e)$, where W_m and W_e are respectively the time-average magnetic and electric energy stored in the network. To make the input impedance resistive, some additional energy storage must be added so that the net reactive energy vanishes. If it is now agreed that the network is always to be operated with an additional ideal lossless reactive element so that the input impedance is pure real, then the Q of the resultant network may be defined as:

$$Q = \frac{2\omega W}{P} \quad (2.86)$$

where W is the larger of W_m or W_e , and P is the power dissipation in the original untuned network. This is actually the usual situation when it comes to antennas; that is, the antenna is normally tuned to resonance by the addition of a reactive element. If the added reactive element does not dissipate any energy, the Q of the resultant system is then given by Eqn. (2.86), where P is now the total radiated power

and W is the larger of W_m , the time-average magnetic energy, or W_e , the time-average electric energy, stored in the near field around the antenna. Consequently, Eqn. (2.86) can be considered to be the upper bound on the Q of an antenna system that is tuned to resonance by the addition of a single reactive element. Any loss in the tuning element would reduce the Q below that given by Eqn. (2.86).

It should be mentioned here that high- Q antennas - that is, structures that have a large amount of reactive energy stored in their near field such as supergain antennas - are excluded from the discussion, since as shown by Harrington in Reference [18], no a priori upper bound on the required sampling rate for such antennas can be specified. The term "supergain antenna" is well-defined by Harrington [18]. And although it may be difficult in general to relate the Q of an antenna to the degree to which it is a supergain structure, Harrington showed that a supergain antenna is necessarily a high- Q antenna.

From Eqn. (2.86), the Q of a single cylindrical wave with mode indices " n " and " h " can therefore be expressed as:

$$Q_{nh} = \frac{2\omega W_{nh}}{P_{nh}} \quad (2.87)$$

It then follows logically that a spectrum of cylindrical waves can be represented by a linear combination which simply is a weighted average of the Q_{nh} over all n and all h . That is:

$$Q = \frac{\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} |C_{nh}|^2 P_{nh} Q_{nh} dh}{\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} |C_{nh}|^2 P_{nh} dh} \quad (2.88)$$

where $\{|C_{nh}|^2 P_{nh}\}$ are the weighting factors, the term C_{nh} being the complex amplitude of the cylindrical wave with mode indices n and h . Collin and Rothschild, in Reference [17], have evaluated P_{nh} and Q_{nh} for a single cylindrical wave radiated by an ideal, loss-free antenna of radius a . The expression "ideal, loss-free antenna of radius a " was originally defined by J. U. Chu [19] as "one having no energy storage for $R < a$, where R is the spherical radial distance from the centre of the smallest sphere of radius a completely enclosing the antenna". In the present cylindrical case, this would be interpreted as an antenna for which there is no energy storage for $r < a$, a being the radius of the smallest cylinder completely enclosing the antenna.

The Q of this ideal antenna then has to be less than or equal to that of any other loss-free antenna fitting into the cylinder $r = a$, since any field present for $r < a$ can only add to the energy storage. In Reference [17], Collin and Rothschild modelled an antenna as being a series of sources placed on the surface of an infinitely long cylinder of radius a . The radiated field for $r > a$ was then expressed as a superposition of $(TE)_z$ and $(TM)_z$ modes, having angular variation $e^{-jn\phi}$, n an integer. The expressions for P_{nh} and Q_{nh} for the $(TE)_z$ and $(TM)_z$ cylindrical waves were shown to be identical in either case and are given by:

$$P_{nh} = \frac{2(k^2 - h^2)}{\omega\mu} \quad (2.89)$$

$$\begin{aligned}
Q_{nh} = & \frac{\pi k^2}{4(k^2 - h^2)} \left\{ \frac{4\rho}{\pi} + [(n^2 + 1 - \rho^2)(J_n^2 + N_n^2)] \right. \\
& - [(n+1)J_n - \rho J_{n+1}]^2 \\
& \left. - [(n+1)N_n - \rho N_{n+1}]^2 \right\}
\end{aligned} \tag{2.90}$$

where $h < k$, J_i and N_i are Bessel functions of argument ρ , and $\rho = \Lambda a = (k^2 - h^2)^{1/2} a$. For $h \geq k$, the total radiated power P_{nh} is zero and therefore Q_{nh} is undefined. From Eqns. (2.89) and (2.90), one obtains the relation:

$$\left(\frac{k^2 - h^2}{k^2} \right) Q_{nh} = \frac{\pi}{\omega \epsilon} P_{nh} Q_{nh} \tag{2.91}$$

of which the right-hand term, aside from the factor $\pi/\omega\epsilon$, appears in the numerator of the general expression for Q in Eqn. (2.88). Collin and Rothschild have shown that the quantity $\left(\frac{k^2 - h^2}{k^2} \right) Q_{nh}$, when plotted versus the argument Λa , increases very rapidly when n becomes larger than Λa . It follows, from Eqn. (2.88), that Q can then become large if C_{nh} is not small for values of $n > \Lambda a$. Since $\Lambda \leq k$ for real h , it follows that the highest significant angular harmonic in the cylindrical wave expansion of the fields of an antenna which is not a high- Q structure is:

$$N = ka \tag{2.92}$$

As pointed out by Leach [9], if a is taken to be the radius of the

smallest sphere completely enclosing the antenna, then the result of Eqn. (2.92) agrees with the conclusions reached by Chu [19] and Harrington [18] in studying the physical limitations of antennas using spherical wave expansions. In general, however, the radius of the smallest cylinder completely enclosing an antenna will be less than or equal to the radius of the smallest sphere, the two being equal if the antenna is oriented so that its longest dimension is perpendicular to the axis of the cylinder.

A sample criterion for the azimuth angle ϕ on a cylinder enclosing an antenna can now be formulated by applying the Nyquist sampling criterion of Fourier transform theory to Eqn. (2.92). Assuming that N is the maximum angular harmonic in the cylindrical wave expansion of a given field, then it follows that the maximum angular separation between adjacent samples is:

$$\Delta\phi = \frac{\pi}{N} \quad (2.93)$$

In the present case, N must be chosen as the smallest integer greater than " ka ", a being, as mentioned earlier, the radius of the smallest cylinder completely enclosing the antenna.

A sample spacing criterion in the elevation plane, or z -direction, on the surface of a cylinder enclosing an antenna can be determined in a similar manner. It will be recalled that the term $P_{nh} Q_{nh}$ in Eqn. (2.88) increases rapidly for $|n| > (k^2 - h^2)^{1/2} a$. As we have mentioned earlier, it follows that $|C_{nh}|$ must be small for $h^2 > k^2 - \frac{n^2}{2a^2}$. This therefore implies that the highest significant wavenumber

in the cylindrical wave expansion of the field of an antenna - which is not a high-Q structure - is $|h| \leq k$. Thus the maximum z-sample spacing on the cylinder is given by the Nyquist spacing:

$$\begin{aligned}\Delta z &= \pi/k \\ &= \lambda/2\end{aligned}\tag{2.94}$$

since $k = 2\pi/\lambda$ in free space.

In the original development of the above two sample spacing criteria, Leach [9] pointed out an interesting comparison which is worth presenting here. If one considers the measurement cylinder as the smallest cylinder completely enclosing the antenna, then the arc length on the cylinder separating adjacent sample points when sampling in the azimuth direction is:

$$\begin{aligned}\Delta s &= a\Delta\phi \\ &= a \times \frac{\pi}{ka} \\ &= \lambda/2\end{aligned}\tag{2.95}$$

This is the same as the z-sample spacing, except that $\Delta s > \Delta z$ when sampling on a cylinder of radius greater than a .

It should be noted here that although the sample spacing criteria defined by Eqns. (2.93) and (2.94) are useful, they are in no way absolute, for it is impossible to predict a priori an exact

cut-off wavenumber for h in the cylindrical wave expansion of the fields radiated by a given antenna. In practice, these sample spacings should be used as guidelines in determining the sample spacing between measurement points, more conservative rates being instead chosen when believed required.

CHAPTER 3

NUMERICAL CONSIDERATIONS AND SYSTEMS SOFTWARE
IMPLEMENTATION3.1 Introduction

The numerical approach taken to solve for the far field of a given antenna, while compensating for the measuring probe, along with the description of the overall system software implementation, is presented in this chapter. In short, the near-field data processing developed on a Honeywell Sigma-9 computer employs two programs. The first program, called COEF, calculates the translated amplitude weighting functions for the probe from its far-field pattern data; these weighting functions can then be used for all measurements conducted at the same frequency and scan radius. The second program, named NEAR, extrapolates the far-field pattern of the test antenna from the measured cylindrical near-field data and the previously calculated probe coefficients. The actual method used in characterizing the probe and the one employed for evaluating the test antenna far field from measured data are presented in Sections 3.2 and 3.3 of this chapter.

It is worth mentioning here that the overall near-field measurement system was originally conceived to be implemented around the use of a Scientific-Atlanta Series 2020 Antenna Analyzer, commonly called SA2020. The SA2020 is a software-programmable instrumentation package; its digital processor consists of a Hewlett-Packard Series

21MX computer. The software interface required for data extraction and transfer onto magnetic tape and, further on, into the Sigma-9 system, consists of short routines which are described in detail in the last section of this chapter. But first, let's look at the evaluation of the probe compensation coefficients.

3.2 Evaluation of the Probe Compensation Coefficients

To evaluate Eqns. (2.80) and (2.81), it is necessary to know the amplitude weighting functions in the cylindrical wave expansion of the field radiated by the probe. However, since it is necessary to know these functions only for wavenumbers $|h| < k$, i.e. for an actual propagating wave, it is therefore possible to obtain them simply from a knowledge of the far field radiated by the probe.

The selection of a probe is, in itself, an important consideration in the development of the system, since an ill-conditioned probe will introduce serious errors in the compensation calculations. Richmond and Tice, in Reference [24], have investigated the desired characteristics for a probe in order to be suitable for microwave near-field measurements. In doing so, they have come up with some criteria of suitability:

- a) any distortion of the fields by the probe and associated equipment must not seriously affect the accuracy of the measurement;
- b) the aperture of the probe must be small enough to measure essentially the field at a point;

- c) the probe must have the desired polarization to a high degree of accuracy;
- d) the probe must deliver a signal voltage large enough to permit accurate measurements.

Ideally, it is then desired to measure the unperturbed tangential electric field on an arbitrarily shaped surface, where the probe output is directly proportional to the field intensity. It is axiomatic, however, that if the probe extracts energy from the field for measurement purposes, the field is no longer unperturbed. The problem is to bring about a compromise between the maximum allowable perturbation of the fields and the extraction of the minimum energy that will permit the desired identification of the characteristics of the fields.

A short dipole closely approximates the behaviour characteristics of an ideal probe. However, its broad pattern makes it very susceptible to picking up extraneous reflections from the anechoic chamber. Also, to obtain satisfactory results, a dipole probe must generally be fed by a coaxial cable and this requires a balun. In the microwave region, as reported by Joy and Bodnar [25], the open-ended waveguide and small pyramidal horn are most practical and, due to their availability and ease of fabrication, become most attractive. Therefore, based on these findings, the two probes selected for the near-field measurement system were simple open-ended waveguides: one WR229 probe, with aperture dimensions of 2.29 in. x 1.145 in. calibrated at 3.85 GHz for use on the range between 3.50-4.20 GHz; and

one WR159 probe, with aperture dimensions of 1.59 in. x 0.795 in. calibrated to 6.175 GHz for the 5.825-6.525 GHz operation. Both probes are made of rectangular waveguide and consequently are linearly polarized.

In fact, circularly polarized probes cannot be used for this measurement system. Looking back at the expression for the determinant of the system of equations [Eqn. (2.84)], it can easily be shown that $\Delta(90^\circ, 0^\circ) = 0$, when the probe is circularly polarized in the boresight direction. In such a case, no solution can be obtained for the far field of the test antenna in this direction since the probe correction becomes infinite. It should be noted here that nulls in the radiation pattern of any probe will also be sources of error since they will introduce large correction factors in the computed patterns. In general, it can be safely stated that the larger the correction a probe requires the greater the uncertainty in the computed patterns of the antennas under test. This follows from the fact that the detection system to which the probe is connected must have a finite signal-to-noise ratio which limits its dynamic range. If the probe correction has a variation that approaches this dynamic range, the accuracy of null depths and low-level portions of the calculated patterns may be affected.

Let's assume now that the probe antenna is oriented as shown in Fig. 4a. As previously shown in Section 2.3, the far-field electric field intensity radiated by the probe can be expressed as:

$$E_\theta(\theta, \phi) = j \sin \theta \sum_{n=-\infty}^{\infty} j^n d_n(k \cos \theta) e^{jn\phi} \quad (3.1)$$

Figure 4a. Coordinates reference system for probe antenna.

Figure 4b. Coordinates reference system for probe antenna rotated by 90° about its orientation axis.

$$E_{\phi}(\theta, \phi) = \sin \theta \sum_{n=-\infty}^{\infty} j^n c_n(k \cos \theta) e^{jn\phi} \quad (3.2)$$

where the constant factor $-2ke^{-jkR}/R$ has been suppressed in each equation, and $c_n(h)$ and $d_n(h)$, with $h = k \cos \theta$, are the amplitude weighting functions in the cylindrical wave expansion of the field radiated by the probe. If the probe is now rotated by 90° in the right-hand sense about its longitudinal axis as shown in Fig. 4b, the far-field electric field intensity radiated by the rotated probe can then be expressed as:

$$E'_{\theta}(\theta, \phi) = j \sin \theta \sum_{n=-\infty}^{\infty} j^n d'_n(k \cos \theta) e^{jn\phi} \quad (3.3)$$

$$E'_{\phi}(\theta, \phi) = \sin \theta \sum_{n=-\infty}^{\infty} j^n c'_n(k \cos \theta) e^{jn\phi} \quad (3.4)$$

where $c'_n(h)$ and $d'_n(h)$ are similarly defined. However, since it is necessary to resolve the measured probe data into Fourier series in the azimuth angle ϕ , some upper limit on the maximum angular harmonic for each probe must be established. It was pointed out in Chapter 2 that, in his study of the physical limitations of antennas, Harrington [18] has shown using spherical wave expansions that the highest significant angular harmonic in the field of an antenna is given by:

$$N_s = kr \quad (3.5)$$

where r is the radius of the smallest sphere completely enclosing the antenna. Thus, Eqns. (3.1) to (3.4) can be rewritten, in practice, as:

$$E_{\theta}(\theta, \phi) = j \sin \theta \sum_{n=-N_s}^{N_s} j^n d_n(h) e^{jn\phi} \quad (3.6)$$

$$E_{\phi}(\theta, \phi) = \sin \theta \sum_{n=-N_s}^{N_s} j^n c_n(h) e^{jn\phi} \quad (3.7)$$

$$E'_{\theta}(\theta, \phi) = j \sin \theta \sum_{n=-N_s}^{N_s} j^n d'_n(h) e^{jn\phi} \quad (3.8)$$

$$E'_{\phi}(\theta, \phi) = \sin \theta \sum_{n=-N_s}^{N_s} j^n c'_n(h) e^{jn\phi} \quad (3.9)$$

At 3.85 GHz, it follows that the smallest integer greater than " kr " for the WR229 open-waveguide probe is $N_s \doteq 3$. Similarly, at 6.175 GHz, using the WR159 probe, one obtains $N_s \doteq 3$.

In order to compensate for probing in the computation of the far field of a test antenna, the weighting functions $c_n(h)$, $d_n(h)$, $c'_n(h)$ and $d'_n(h)$ have to be evaluated with the argument:

$$\begin{aligned} h &= k \cos(180^\circ - \theta_m) \\ &= -k \cos \theta_m \end{aligned} \quad (3.10)$$

where θ_m is the test antenna elevation angle at which the far field is to be computed. Hence the notation $c_n(-h)$, $d_n(-h)$, $c'_n(-h)$ and $d'_n(-h)$ in Eqns. (2.80), (2.81) and (2.84).

As mentioned earlier, Eqns. (2.80) through (2.84) will provide a solution for the weighting functions $a_n(h)$ and $b_n(h)$ as long as the determinant of the system of equations $\Delta_n(h)$ is not zero. This will be achieved by assuming that the probe responds predominantly to one linear polarization component. Now, if one could somehow "fix" or specify one of the polarization components of the probe in such a way that the other would be negligible, then the evaluation of Eqns. (2.80), (2.81) and (2.84) would be much simplified since only two of the four probe weighting functions would actually be required.

This effect can be achieved by placing a "polarizing grid", also called a grating, at the mouth of the probe. Silver [26] and Marcuvitz [27] have detailed the theory of the field's behaviour in the presence of gratings. They showed that the space between the grating elements may be thought of as waveguides beyond cut-off for the electric vector parallel to the grating element. On the other hand, the grid has only a low insertion loss for the wave component orthogonal to the grating elements. The polarizing grid used in connection with the probe was formed by photoetching a sheet of copper clad fibreglass, with the geometry shown in Fig. 5. The fibreglass sheet is 0.003 in. thick, and a surface of 36 in. x 48 in. is coated with 2 ounces of copper. The dielectric constant of the fibreglass is roughly $\epsilon = 5$. For normal incidence, the insertion loss of the field component perpendicular to the grating elements has been measured to be less than 1 dB. The measured attenuation level, however, of the field component parallel to the grating is in the order of 20 dB.

W = width = .040 in.
 T = thickness $\leq$.001 in.
 S = spacing = .060 in.
 t = dielectric thickness = .003 in.

Figure 5. Design model of polarizing grid.

Higher levels of attenuation can be achieved by simply "piling up" two or more layers of grid, spaced $\lambda/4$ apart.

Figures 6 and 7 show both the measured H-plane copolar and cross-polar radiation patterns of the probe antenna, respectively, without and with polarizing grid attachment. It is easily seen that the addition of the grid has practically no effect on the copolar pattern (vertical component) of the field; however, the cross-polarization pattern (horizontal component) of the probe is substantially attenuated, down to a level where it can be neglected for measurement purposes. A similar situation holds for the E-plane cut radiation patterns.

It should be noted here that there is no universally accepted definition of "cross-polarization" at present, and that at least three different definitions have been used either explicitly or implicitly in the literature [28]. The IEEE standard definition of cross-polarization is "the polarization orthogonal to a reference polarization". For circular polarization, this is adequate, but for linear or elliptical polarization, the direction of the reference polarization must still be defined. In Reference [28], Ludwig presented the three alternative definitions, graphically represented in Fig. 8a, in the case of nominally linear polarization; these are:

Definition #1: in a rectangular coordinate system, one unit vector is taken as the direction of the reference polarization, and another as the direction of cross-polarization;

Figure 6a. H-plane copolar radiation pattern of probe antenna without grid attachment (WR229).

SCAN NR: 1 STEP ANG: .15 FREQ NR: 1 FREQ (MHz): 3850

Figure 6b. H-plane crosspolar radiation pattern of probe antenna without grid attachment (WR229).

Figure 7a. H-plane copolar radiation pattern of probe antenna with grid attachment (WR229).

Figure 7b. H-plane crosspolar radiation pattern of probe antenna with grid attachment (WR229).

TOP: Direction of the reference polarization
 BOTTOM: Direction of the cross-polarization

Figure 8a. Alternate polarization definitions.

Figure 8b. Antenna pattern measurement system.

Definition #2: in a spherical coordinate system, the same procedure is performed using the unit vectors tangent to the spherical surface;

Definition #3: reference and cross-polarization are defined to be what one measures when antenna patterns are taken in the usual manner.

According to Ludwig, the pattern measurement method (definition #3) is usually the best choice for describing antenna behaviour and, as such, is the most commonly used by antenna engineers. It is the one which was chosen for the present case. In the terminology used by Scientific Atlanta, this method may be implemented by mounting the antenna being tested, shown at the origin of the system as illustrated in Fig. 8b, on a model tower or an elevation-over-azimuth positioner with an auxiliary polarization axis [29]. The elevation angle is always zero (z-axis horizontal) so this axis is actually not required. Each pattern cut begins at $\theta = 0$ where the polarization axis is used to set the pattern cut angle ϕ by rotating the antenna being tested about the z-axis. The probe is rotated about its axis by a second polarization positioner to align the probe polarization parallel to the polarization of the antenna being tested for a principal polarization pattern, or orthogonal to the polarization of the antenna being tested for a cross-polarization pattern. Note that the orientation of the polarization of the antenna being tested, at the point $\theta = 0$, is the basic polarization reference direction by definition. Then a pattern is taken by varying θ by rotating in azimuth. This is equivalent to the probe traversing a great circle as illustrated in

Fig. 8b. The probe remains fixed about its axis so it retains the same relative orientation with respect to the unit vectors $\hat{i}_\theta$ and $\hat{i}_\phi$. Therefore, the measured pattern is given by:

$$M(\theta) = E(\theta, \phi) \cdot \{\sin \beta \hat{i}_\theta + \cos \beta \hat{i}_\phi\} \quad (3.11)$$

where $E(\theta, \phi)$ is the field of the transmitting antenna, and the pattern cut angle ϕ and the probe polarization angle β are fixed for a given pattern. For a transmitted field polarized in the $\hat{i}_y$ direction at $\theta = 0$, this alignment procedure leads to $\beta = \phi$ for a reference polarization pattern $R(\theta, \phi)$ and $\beta = \phi \pm 90^\circ$ for a cross-polarization pattern $C(\theta, \phi)$. Therefore, ignoring unimportant sign differences:

$$R(\theta, \phi) = E(\theta, \phi) \cdot \{\sin \phi \hat{i}_\theta + \cos \phi \hat{i}_\phi\} \quad (3.12)$$

and

$$C(\theta, \phi) = E(\theta, \phi) \cdot \{\cos \phi \hat{i}_\theta - \sin \phi \hat{i}_\phi\} \quad (3.13)$$

This is equivalent to:

$$\begin{aligned} \hat{i}_{\text{ref}} &\equiv \sin \phi \hat{i}_\theta + \cos \phi \hat{i}_\phi \\ &= (1 - \cos \theta) \sin \phi \cos \phi \hat{i}_x + \{1 - \sin^2 \phi (1 - \cos \theta)\} \hat{i}_y \\ &\quad - \sin \theta \sin \phi \hat{i}_z \end{aligned} \quad (3.14)$$

and

$$\begin{aligned} \hat{i}_{\text{cross}} &\equiv \cos \phi \hat{i}_\theta - \sin \phi \hat{i}_\phi \\ &= \{1 - \cos^2 \phi (1 - \cos \theta)\} \hat{i}_x - (1 - \cos \theta) \sin \phi \cos \phi \hat{i}_y \\ &\quad - \sin \theta \cos \phi \hat{i}_z \end{aligned} \quad (3.15)$$

This overall procedure makes it easy to avoid probe polarization misalignment which is a severe pitfall in cross-polarization measurements.

Now, one recalls that Eqns. (3.6) through (3.9) are in the form of a Fourier series in ϕ . Therefore, the amplitude weighting functions for a particular value of θ can be obtained by numerically evaluating the Fourier inversion integral from the measured fields. For example, the solution for $d_n(k \cos \theta)$ in Eqn. (3.6) is:

$$d_n(k \cos \theta) = \frac{1}{2\pi j^{n+1} \sin \theta} \int_{-\pi}^{\pi} E_{\theta}(\theta, \phi) e^{-jn\phi} d\phi \quad (3.16)$$

In the present case though, only the front hemisphere of the far field of the probe is actually known, i.e. $-\pi/2 \leq \phi \leq \pi/2$. In Reference [9], Leach demonstrated that Eqn. (3.6) approximates the measured data in the least square sense if, for field values that have been taken for an interval $-\phi_1 \leq \phi \leq \phi_1$, the $d_n(k \cos \theta)$ satisfy the system of equations:

$$\sum_{n=-N_s}^{N_s} d_n(k \cos \theta) \frac{\sin(m-n)\phi_1}{(m-n)\phi_1} = \frac{1}{2j^{m+1} \sin \theta} \int_{-\phi_1}^{\phi_1} E_{\theta}(\theta, \phi) e^{-jm\phi} d\phi \quad (3.17)$$

where $-N_s \leq m \leq N_s$. A similar system of equations applies to the evaluation of $c_n(h)$, $c'_n(h)$ and $d'_n(h)$. If $\phi_1 = 180^\circ$, each $d_n(k \cos \theta)$ is determined independently. However, for $\phi_1 < 180^\circ$, as would be the case over the front hemisphere around the probe, a set of $2N_s + 1$

simultaneous equations must be solved for each $d_n(k \cos \theta)$. Both the integration required in Eqn. (3.17) and the solution to these simultaneous equations can be accomplished easily with standard numerical techniques. The problem is how to economically perform the measurement of the far field of the probe on a sphere of constant radius about the probe.

In Reference [9], in the process of characterizing the probe, Leach measured the far field of the probe for numerous values of θ and ϕ on the front hemisphere around the probe using a standard azimuth turntable on which the probe is placed and an inverted swing carriage on which a receiving antenna (standard gain horn) is mounted, as shown in Fig. 9. This technique was originally developed at Scientific-Atlanta by Joy and Paris [30] and is designed to characterize a probe for any type of near-field measurements, either planar, cylindrical or spherical. Although fairly simple, this operation is very time-consuming since one has to compensate for all unwanted reflections in the chamber; it becomes especially more so if similar equipment is not available such as the case at CRC.

It should be mentioned here that probe compensation for a spherical near-field measurement system is usually negligible since the measuring probe is pointed at all times towards the centre of rotation of the antenna under test. On the other hand, planar scanning will require compensation in both θ and ϕ since the orientation of the probe is displaced in both elevation and azimuth from the centre of the measurement plane. Cylindrical scanning is, one could say,

Figure 9. Standard azimuth turntable/inverted swing carriage configuration.

half-way in between. As we mentioned earlier in Chapter 2, probe-compensation in azimuth is negligible, unless one is interested in precision measurements; however, probe correction is an important consideration in calculating the elevation pattern of the antenna under test, since the elevation pattern is predominantly determined by the axial variation of the near-field data on the cylinder.

Consider the sphere diagram in Fig. 10a which can represent the probe far-field electric field components orientation on a sphere of constant radius around the probe; consider also Fig. 10b which shows the probe orientation with a polarizing grid attachment and Fig. 10c which shows the same probe rotated by 90° about its longitudinal axis. Figure 10a represents graphically the fact that the electric field of the probe without a grid attachment has two orthogonal components, $E_\theta(\theta, \phi)$ and $E_\phi(\theta, \phi)$. Since the probe is not ideal, it will respond to a resultant of these two components at each point, proportionally to its complex polarization ratio (CPR). Now if a grid is inserted as in Fig. 10b, the probe becomes purely "vertically polarized" and should now respond strictly to the $E_\theta(\theta, \phi)$ component, or at least to a much higher degree than before. In other words, we have attenuated the cross-polarization component of the field, i.e. $E_\phi(\theta, \phi)$ to a level where it has become negligible, as shown previously in Fig. 7b. From Eqns. (3.6) to (3.9), it follows that:

$$E_\phi(\theta, \phi) \approx 0 \quad (3.18)$$

Where
 $E_{\theta} = f(\theta, \phi)$
 $E_{\phi} = f(\theta, \phi)$

Figure 10a. Orientation of field components over a sphere of constant radius.

Figure 10b. Probe orientation with polarizing grid attachment.

Figure 10c. Probe orientation with polarizing grid attachment once rotated by 90° about its longitudinal axis.

or $c_n(h) = 0$. Similarly, once the probe is rotated by 90° as in Fig. 10c, it can be seen easily that:

$$E'_\theta(\theta, \phi) \approx 0 \quad (3.19)$$

or $d'_n(h) = 0$. The resulting set of equations that characterizes the probe then consists only of:

$$E_\theta(\theta, \phi) = j \sin \theta \sum_{n=-N_s}^{N_s} j^n d_n(h) e^{jn\phi} \quad (3.20)$$

$$E'_\phi(\theta, \phi) = \sin \theta \sum_{n=-N_s}^{N_s} j^n c'_n(h) e^{jn\phi} \quad (3.21)$$

which means that according to Eqn. (3.17) one has to obtain measured data to solve for:

$$\begin{aligned} & \sum_{n=-N_s}^{N_s} d_n(k \cos \theta) \frac{\sin(m-n)\phi_1}{(m-n)\phi_1} \\ &= \frac{1}{2j^{m+1} \sin \theta} \int_{-\phi_1}^{\phi_1} E_\theta(\theta, \phi) e^{-jm\phi} d\phi \end{aligned} \quad (3.22)$$

$$\begin{aligned} & \sum_{n=-N_s}^{N_s} c'_n(k \cos \theta) \frac{\sin(m-n)\phi_1}{(m-n)\phi_1} \\ &= \frac{1}{2j^m \sin \theta} \int_{-\phi_1}^{\phi_1} E'_\phi(\theta, \phi) e^{-jm\phi} d\phi \end{aligned} \quad (3.23)$$

where once again $-N_s \leq m \leq N_s$.

With the right amount of measured data, the solutions for $d_n(h)$ and $c'_n(h)$ for values of $\theta_1 \leq \theta \leq (180^\circ - \theta_1)$ and $-\phi_1 \leq \phi \leq \phi_1$ can be obtained with the Gauss-Jordan method for solving simultaneous equations. In this case, however, due to financial constraints and to a low priority status for time allocation of the anechoic chamber facilities, the only data collected was the amplitude and phase values of the probe far field for the two principal plane cuts, i.e.

Case (A): $-\pi/2 \leq \phi \leq \pi/2$

; H-plane cut

$$\theta = \pi/2$$

Case (B): $\phi = 0$

; E-plane cut

$$0 \leq \theta \leq \pi$$

at increments of $\Delta\alpha = 0.1^\circ$. The corresponding radiation patterns are shown in Figs. 11 and 12. The cross-polarization patterns in Fig. 12 confirm that the cross-polar component of the field can be neglected in both cases.

The measurement set-up used to obtain these patterns is pictured in Fig. 13. Both the source antenna - which consisted of a standard gain horn with a grid attachment that reduced its cross-polarization level to -60 dB - and the probe (WR229) were set up inside the anechoic chamber. Because of reflections coming from all surroundings, the probe had to be installed on a spin table at the

Figure 11a. H-plane copolar radiation pattern of probe with
polarizing grid attachment (WR229).
Samples taken every 0.1° .

Figure 11b. E-plane copolar radiation pattern of probe with polarizing grid attachment (WR229).
Samples taken every 0.1°.

Figure 12a. H-plane crosspolar radiation pattern of probe with polarizing grid attachment (WR229).
Samples taken every 0.1° .

Figure 12b. E-plane crosspolar radiation pattern of probe with polarizing grid (WR229).
Samples taken every 0.1° .

NOTE: The phase centre of the probe antenna, which is theoretically at the mouth of an open-ended waveguide, has been aligned with the axis of rotation of the turntable.

ORIENTATION OF ANTENNAS

	PROBE	SOURCE
Case (A): H-plane cut (copolar)		
H-plane cut (cross-polar)		
Case (B): E-plane cut (copolar)		
E-plane cut (cross-polar)		

Figure 13. Probe patterns measurement set-up.

2

top of an $11\frac{1}{2}$ ' tower mounted on a standard azimuth turntable, in order to be at the centre of the "silence zone" created by the presence of heavier absorbing material on the floor, walls and ceiling. This is the only way to obtain the true pattern of any antenna, inherently limited by the design of the chamber. However, because of the height of the tower, the alignment of the probe phase centre with the centre of rotation of the table became most difficult to achieve. The final setting error is reflected in the measurement of the phase patterns for both the H-plane and E-plane cuts, shown in Figs. 14 and 15. In both patterns, the slight "slant" to the right represents an actual physical setting offset of about $1/16$ in.; better accuracy was impossible to achieve. This phase error though is believed to be minimal and well within limits of practical measurements (less than 10°).

As mentioned earlier, only the two principal plane cuts of the probe far field were measured. With this data, it is impossible to solve for the $d_n(h)$ and $c'_n(h)$ other than at an elevation of $\theta = 90^\circ$. However, it is believed that the rest of the information required can be obtained with sufficient accuracy through pattern multiplication, as long as it is assumed that the fields along the cut axis other than $\theta = \pi/2$ and $\phi = 0$ have a distribution function identical to that of the two major polar axes. Based on the relatively good symmetry and broad beamwidth of open-ended waveguides and small horns radiation patterns (refer, for example, to Figs. 11 and 12), the above assumption was used and the required values for $E_\theta(\theta, \phi)$ and $E'_\phi(\theta, \phi)$ were obtained for

Figure 14. Phase pattern in H-plane of the probe with grid attachment (WR229).
Samples taken every 0.1°.

Figure 15. Phase pattern in E-plane of probe with grid attachment (WR229).
Samples taken every 0.1° .

$0^\circ \leq \theta \leq 180$ and for $-90^\circ \leq \phi \leq 90^\circ$, at an increment of 0.1° .

These complex values were obtained, as we just said, through the principle of antenna pattern multiplication, numerically implemented in the program "FIELD" which is included as Appendix G. For the specific case of our WR229 probe, we produced an 1801×1801 array of complex field values which, due to its large size, required a disk storage allocation of 13,000 granules*. Obviously, knowledge of the field values at every tenth of a degree over the full front hemisphere of the probe was not required, but this was accomplished so as to create the forementioned array storage, named "FLDDATA", to account for future requirements from a user.

The probe amplitude weighting functions can now be found easily by solving Eqns. (3.22) and (3.23). This is done accurately through the use of a fourth computer program entitled "COEF", and included as Appendix H. This overall method of probe characterization has many advantages. First, it is simple to implement and can be performed with the use of on-site available antenna range equipment and computer processing facilities; second, only the two main polar cuts are required, which in this case means considerable savings in allocated measurement time. Another significant advantage is the fact that the insertion of a polarizing grid has cancelled the need for one to determine the complex polarization ratio (CPR) of either the probe or the measuring antenna. The CPR enables one to decompose a measured field into its azimuth and elevation components; and as

* A granule is 512 words, a word is 32-bit long.

shown in Reference [31], the determination of an antenna CPR can become quite frustrating and time-consuming since accurate phase measurements have to be performed.

Except for the fact that the values of $E_\theta(\theta, \phi)$ and $E_\phi(\theta, \phi)$ have to be calculated for the probe, the overall data processing in this near-field measurement system is actually substantially reduced, thereby allowing for greater speed and efficiency. In effect, since the probe weighting functions $c_n(h)$ and $d'_n(h)$ no longer need to be evaluated having been reduced to almost zero by the polarizing grid, Eqns. (2.80), (2.81) and (2.84) are further simplified to:

$$a_n(h) = \frac{-k^2}{\Lambda^2 \Delta_n(h)} I'_n(h) \sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \quad (3.24)$$

$$b_n(h) = \frac{-k^2}{\Lambda^2 \Delta_n(h)} I_n(h) \sum_{m=-\infty}^{\infty} c'_m(h) H_{n+m}^{(2)}(\Lambda r_0) \quad (3.25)$$

and

$$\Delta_n(h) = - \left[\sum_{m=-\infty}^{\infty} c'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \cdot \left[\sum_{m=-\infty}^{\infty} d'_m(-h) H_{n+m}^{(2)}(\Lambda r_0) \right] \quad (3.26)$$

The amplitude weighting functions of the probe enter the solutions for $a_n(h)$ and $b_n(h)$, given by Eqns. (3.24) and (3.25), in the form of a summation in m over the angular harmonics of the probe. In practice, the limits of this summation are given by Eqn. (3.5), i.e.

$-N_S \leq m \leq N_S$. The coefficients in these summations are the Hankel functions $H_{n+m}^{(2)}(\Lambda r_0)$, where $\Lambda r_0 = (k^2 - h^2)^{1/2} r_0 = k r_0 \sin \theta$ and θ is the

elevation angle of the test antenna at which the pattern is being computed. Obviously, an error factor, generated first by neglecting the cross-polarization pattern of the probe and second by interpolating its field values for cut axis other than $\theta = \pi/2$ and $\phi = 0$, will be present and is recognized, but it is also believed to be negligible throughout the processing.

3.3 Evaluating the Far Field of an Arbitrary Antenna from Measured Near-Field Data

It was demonstrated earlier in Section 2.3 that the far-field electric field intensity radiated by an arbitrary antenna over the surface of a sphere of constant radius R is given by [Recall Eqns. (2.48) to (2.50)]:

$$E_R = 0 \quad (3.27)$$

$$E_\theta = -j \frac{2k \sin \theta}{R} e^{-jkR} \sum_{n=-\infty}^{\infty} j^n b_n(k \cos \theta) e^{jn\phi} \quad (3.28)$$

$$E_\phi = \frac{-2k \sin \theta}{R} e^{-jkR} \sum_{n=-\infty}^{\infty} j^n a_n(k \cos \theta) e^{jn\phi} \quad (3.29)$$

If the constant factor $-2k e^{-jkR}/R$ is suppressed in each equation, as it was previously done in Section 3.2, the radial field component neglected, the far-field electric field intensity components of the antenna can then be expressed as:

$$E_\theta(\theta, \phi) = j \sin \theta \sum_{n=-\infty}^{\infty} j^n b_n(k \cos \theta) e^{jn\phi} \quad (3.30)$$

$$E_{\phi}(\theta, \phi) = \sin \theta \sum_{n=-\infty}^{\infty} j^n a_n(k \cos \theta) e^{jn\phi} \quad (3.31)$$

where, as one recalls, $a_n(k \cos \theta)$ and $b_n(k \cos \theta)$ are the amplitude weighting functions in the cylindrical wave expansion of the field radiated by the antenna. Equations (3.24) through (3.26) in the previous section summarize by themselves how the solutions for $a_n(h)$ and $b_n(h)$ - where $h = k \cos \theta$ - can be obtained from the output of a probe with polarizing grid. Aside from the factors involving the amplitude weighting functions characterizing the probe, these solutions primarily require the evaluation of the two integrals:

$$I_n(h) = \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} v(r_0, \phi_0, z_0) e^{-jn\phi_0} e^{jhz_0} d\phi_0 dz_0 \quad (3.32)$$

$$I'_n(h) = \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} v'(r_0, \phi_0, z_0) e^{-jn\phi_0} e^{jhz_0} d\phi_0 dz_0 \quad (3.33)$$

where $v(r_0, \phi_0, z_0)$ and $v'(r_0, \phi_0, z_0)$ represent the output voltage of the probe on the measurement cylinder of radius r_0 . The primed function is used throughout this paper to denote the probe output after it has been rotated by 90° about the longitudinal axis.

A diagram of the measurement cylinder with appropriate axis definitions and related parameters is shown in Fig. 16. Let the measurement cylinder be divided into a lattice of points with coordinates $(r_0, n\Delta\phi, m\Delta z)$ where $0 \leq n \leq N-1$ and $0 \leq m \leq M-1$, and N and M are positive integers. To exactly evaluate Eqns. (3.32) and (3.33)

Figure 16. Schematic of experimental set-up for cylindrical near-field scanning, showing coordinate system for measurement surface.

from the output voltages of the probe at these points, two conditions must be satisfied:

- a) first, v and v' must be zero when $z < 0$ and $z > (M-1)\Delta z$;
- b) second, v and v' must have no angular harmonic n greater than $\pi/\Delta\phi$ and must be wavenumber limited in h to a maximum wavenumber less than or equal to $\pi/\Delta z$.

The first condition is usually easy to meet, although it obviously cannot be implemented with all structure. However, if the test antenna is aligned in the cylinder so that it does not radiate appreciably in the $\pm z$ -direction, the first condition can be met approximately assuming M is chosen large enough. The second condition can be met if the antenna under test is not a high-Q structure and if the sample intervals, in azimuth $\Delta\phi$ and in elevation Δz , are chosen not too large, i.e. in accordance with the sampling criteria developed in Section 2.5.

Assuming that the preceding conditions are met, the integrals $I_n(h)$ and $I'_n(h)$ can then be evaluated as two two-dimensional discrete Fourier transforms (DFTs). A short summary on the theory of the one- and two-dimensional DFTs is included as Appendix D and should be of "refreshing" interest to the reader. It shows that Eqns. (3.32) and (3.33) can basically be replaced by a summation of discrete "transformed" probe complex voltage outputs, evaluated for all ordered pairs (n,h) . More specifically, the two integrals are numerically implemented as:

$$I(n, h) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} v(r_o, \phi_o, z_o) e^{-j(2\pi/N)n_1 \times n} e^{j(2\pi/M)m_1 \times m} \quad (3.34)$$

$$I'(n, h) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} v'(r_o, \phi_o, z_o) e^{-j(2\pi/N)n_1 \times n} e^{j(2\pi/M)m_1 \times m} \quad (3.35)$$

where n_1 and m_1 vary as

$$0 \leq n_1 \leq N-1 \quad (3.36)$$

and

$$0 \leq m_1 \leq M-1 \quad (3.37)$$

The DFT format of Eqns. (3.34) and (3.35) is obtained by replacing ϕ_o , h and z_o in Eqns. (3.32) and (3.33) by their equivalent definitions, i.e.

$$\phi_o = n_1 \Delta\phi = n_1 \left(\frac{2\pi}{N} \right) \quad (3.38)$$

$$z_o = m_1 \Delta z \quad (3.39)$$

$$h = m \Delta h \quad (3.40)$$

where Δh is defined as:

$$\Delta h = 2\pi/M\Delta z \quad (3.41)$$

The two DFT algorithms were originally programmed directly on the computer. This straightforward procedure, however, showed itself

to be most inefficient since $(N \times M)^2$ complex operations are required for the calculation of the DFT coefficients. For large values of N and M , as encountered in practice, this approach to the numerical implementation of the problem becomes computationally most uneconomical.

The fast Fourier transform (FFT) is a highly efficient procedure for computing the DFT of large series of discrete data samples. It takes advantage of the fact that the calculation of the coefficients of the DFT can be carried out iteratively, which results in a considerable savings of computation time. In more detail, the technique can be interpreted in terms of combining the DFTs of the individual data samples such that the occurrence times of these samples are taken into account sequentially and applied to the DFTs of progressively larger mutually exclusive subgroups of data samples, which are combined to ultimately produce the DFT of the complete series of data samples [20,21,22]. This explanation of the FFT algorithm adopted by Cochran et al. in Reference [20] is believed to be particularly descriptive for programming purposes.

For example, assuming that our two-dimensional data arrays consist of $N = 2^n$ and $M = 2^m$ samples, then about $(2nN) \times (2mM) = 4(N \times M) \log_2(N \times M)$ arithmetic operations will be required to evaluate all $(N \times M)$ associated DFT coefficients. For large N and M , this compares most favourably with the initial number of operations required when using straightforward programming procedures. In addition, it should be pointed out that the FFT not only reduces the computation

time but also substantially reduces round-off errors associated with these computations.

Another attractive feature of the FFT algorithm is that the computations are usually performed "in-place", i.e. the input data arrays are replaced by the output arrays after the calculations are completed. In the present case, the two-dimensional input arrays $v(r_0, n_1 \Delta \phi, m_1 \Delta z)$ and $v(r_0, n_1 \Delta \phi, m_1 \Delta z)$ are replaced on output by the arrays containing all values for $I_n(m \Delta h)$ and $I'_n(m \Delta h)$ respectively.

In the implementation of an FFT algorithm for our particular case, one must consider that the number of samples in both azimuth and elevation are unlikely to be always a factor of 2. When summing in the z-direction, the problem can be easily solved by the well-known technique of zero-filling [23]. However, when transforming in azimuth (ϕ -direction), the problem is different since one proceeds along a closed path. Consequently, a one-dimensional FFT algorithm was developed, which can transform an arbitrary number κ of complex data points, where κ only has to be non-prime for speed. This condition is definitely easier to achieve than the previous "factor of 2" dependence. The two-dimensional processing is then simply obtained by evaluating the integrals $I_n(h)$ and $I'_n(h)$ as series of two one-dimensional DFTs, as shown in Appendix D. Since the far-field expressions for $E_\theta(\theta, \phi)$ and $E_\phi(\theta, \phi)$ are evaluated at $h = k \cos \theta$, the values of θ corresponding to $h = m \Delta h$, based on Eqns. (3.40) and (3.41) are found by:

$$\frac{2\pi}{\lambda} \cos \theta = m \left(\frac{2\pi}{M\Delta z} \right) \quad (3.42)$$

and therefore the values θ_m are:

$$\theta_m = \cos^{-1} \left(\frac{m\lambda}{M\Delta z} \right) ; -\frac{M}{2} \leq m \leq \frac{M}{2} - 1 \quad (3.43)$$

A flow diagram depicting the evaluation procedures of an antenna far field based on near-field measurements is presented in Fig. 17.

As mentioned at the very beginning of this thesis, the cylindrical near-field measurement technique is most advantageous when applied to antennas for which it is desired to calculate the complete 360° far-field azimuth pattern; also, since it is impossible to measure the near field over a cylinder enclosing completely one antenna, this method is mostly suitable for structures such as fan-beam antennas and steerable arrays, which radiate narrow vertical beam patterns, predominantly in the annular region about the x-axis as in Fig. 16 ($\theta = \pi/2$). This region can be symmetrically defined as being limited in size by a critical angle θ_c such that $\theta_c < \theta \leq (\pi - \theta_c)$. In order for all θ_m defined in Eqn. (3.43) to lie within this interval, it follows, as explained by Leach in Reference [9], that Δz must satisfy the criterion:

$$\Delta z = \frac{\lambda}{2 \cos \theta_c} \quad (3.44)$$

Obviously, for $\theta_c \neq 0$, this condition violates the z-sample spacing criterion developed earlier in Chapter 2. One obvious way to solve

Figure 17. Flow-diagram depicting the evaluation of far field.

this problem is simply to choose $\Delta z \leq \lambda/2$, i.e. smaller than that specified by Eqn. (3.44) above and to ignore all calculations for $\theta \leq \theta_c$ and $\theta > (\pi - \theta_c)$. This, however, is not a very efficient solution since - due to the properties of the DFTs - the resolution of the resulting computed pattern for $\theta_c < \theta < (\pi - \theta_c)$ will be substantially reduced. Although this decrease in resolution can be compensated for by zero-filling the near-field data arrays [23], the process will increase the value of the total number of samples in elevation (M) which, in turn, will further increase computer storage requirements and computer processing time.

As far as we are concerned, the above technique of zero-filling is still acceptable, although inefficient in the sense that it does not make full use of the FFT algorithm as found by Leach in Reference [9]. In effect, Leach has developed an ingenious alternate solution to the problem. In summary, by using the FFT algorithm, the near-field data is "smoothed" in such a way that it can be resampled by numerical interpolation with the sample spacing specified by Eqn. (3.44). This operation becomes equivalent to actually "low-pass filtering" the data. The process is as follows:

- a) first, the near-field data arrays are transformed in z so that on output the wavenumber spacing is $\Delta h = 2\pi/M\Delta z$ as specified by Eqn. (3.41);
- b) second, only the elements in the transformed arrays corresponding to values of m such that

$$|m| \leq \frac{M\Delta z}{\lambda} \cos \theta_c \quad (3.45)$$

are inverse transformed in z generating new "filtered" arrays with M' elements where $M' < M$. It follows that these arrays are wavenumber limited in z such that the Nyquist sample spacing is now given by Eqn. (3.44);

- c) finally, the filtered arrays are augmented with $(M-M')$ zeros so that the final "smoothed" array still contains M elements in z . This zero-filling process will not affect in any way the accuracy of the subsequent FFT operations. Instead, the resolution will be improved on output since all M values of θ_m will now lie within the interval $\theta_c < \theta \leq (\pi - \theta_c)$.

This operation, as mentioned earlier, is equivalent to filtering the near-field data with an ideal low-pass filter having a cut-off wavenumber h_c of:

$$h_c = \frac{2\pi}{\lambda} \cos \theta_c \quad (3.46)$$

Once the low-pass filtering is performed, the evaluation of the two integrals $I_n(m\Delta h)$ and $I'_n(m\Delta h)$ can be easily achieved as pictured in Fig. 17.

Once the probe amplitude weighting functions are known and the calculations for $I_n(m\Delta h)$ and $I'_n(m\Delta h)$ are completed, the test antenna cylindrical wave amplitude weighting functions $a_n(m\Delta h)$ and $b_n(m\Delta h)$ are obtained through Eqns. (3.24) and (3.25); then, based on Eqns. (3.30) and (3.31), each $a_n(m\Delta h)$ and $b_n(m\Delta h)$ is multiplied

by the factors $j^n \sin \theta_m$ and $j^{n+1} \sin \theta_m$ respectively. Finally, using the FFT algorithm once again, the far-field electric field intensity components of the test antenna are calculated by simply performing the summations in n . The field components so computed will be $E_\theta(\theta_m, \phi_n)$ and $E_\phi(\theta_m, \phi_n)$ where:

$$\phi_n = \frac{2\pi n}{N} \quad ; \quad 0 \leq n \leq N-1 \quad (3.47)$$

and

$$\theta_m = \cos^{-1} \left[\frac{m \cos \theta_c}{(M/2)} \right]; \quad -\frac{M}{2} \leq m \leq \frac{M}{2} - 1 \quad (3.48)$$

The Fortran program NEAR, presented as Appendix I, represents the digital computer implementation of the above technique. It basically follows the flow-diagram presented earlier in Fig. 17. In addition to the complex near-field data $v(r_0, \phi_0, z_0)$ and $v'(r_0, \phi_0, z_0)$, the inputs to the program also consist of the frequency at which the test is being performed, the radius of the measurement cylinder, both the ϕ - and z -sample spacings, and in order to determine the angle $\theta_{c_{\min}}$ the vertical distance "Z" from the x-axis - or the axis fixed by $\theta = \pi/2$ - to the position in elevation where the first near-field sample is taken. The use of the FFT algorithm in the program results, as mentioned earlier, in the majority of the calculations being performed in place, i.e. the input near-field data arrays will be replaced after processing by the output far-field values.

3.4 System Software Implementation

The actual implementation of a cylindrical near-field measurement system at CRC has been designed around their newly acquired Scientific-Atlanta Series 2020 Antenna Analyzer, commonly referred to as SA2020. Because of the extensive use of this equipment in our measurement process, a short presentation of its characteristics and capabilities seems most appropriate.

The SA2020 is an automatic software-programmable instrumentation package developed to perform antenna radiation pattern recordings and other commonly-made antenna measurements in the 0.1 to 18.0 GHz range. The equipment basically consists of a microwave receiving subsystem, a microwave signal source, an antenna positioning control subsystem, a digital processor subsystem, and the required input/output capabilities. Complete manual control is also available when hands-on testing is required. The relationship of the four basic subsystems is presented in the simplified block diagram of Fig. 18.

The receiving subsystem provides relative amplitude and phase measurements of low level RF input signals from 0.1 to 18 GHz, with outputs to the digital processing subsystem or to manual display units. It will also provide accurate frequency measurement of the received signals, and can be located up to 150 feet from the system's main console.

The microwave signal source has a typical frequency accuracy of ± 13 MHz across the transmitting band, with a resolution of 1 MHz.

Figure 18. Simplified system block diagram showing relationship of major subsystems for the SA2020.

It has been specifically designed for antenna measurements applications where it is required to tune an RF source from a remote control location. Mounted near the transmitting antenna to minimize losses, it can be controlled from the operator console either manually or by the digital processor at distances up to 2500 feet.

The positioner control subsystem primarily consists of a programmable positioner control unit which, through a synchro select unit, can sequentially drive automatically up to six positioner axes. The programmable positioner control unit provides control, command and position angle tracking of the positioning system, which frees the digital processor to perform other tasks. Manual back-up is also available for this entire subsystem. The subsystem may be operated in either the "POSITION" or the "RECORD" mode and may be commanded to halt at any time by the digital processing subsystem. In the position mode, position input commands ($.01^\circ$ resolution) are provided by the digital processing subsystem to the positioner control subsystem. The latter selects the commanded axis and causes the positioner controller to drive the axis to the commanded input position as determined by synchro data feedback. In the record mode, the subsystem stores the record start angle (1° resolution), a scan direction, the record increment, the record speed, and the record stop angle for a selected axis. In this mode, the subsystem can be programmed for record increments of 0.1, 0.2, 0.5, 1.0 and 2.0° and one of fifteen preset record speeds.

The input/output subsystem consists of peripheral devices which provide the capability for operator interface with the system, assist in system programming, and provide hard copy output of measured and calculated data. It includes a Tektronix display terminal with teletype-style keyboard; a high-speed paper tape reader, a paper tape punch, a Versatec electrostatic printer/plotter, and a rectangular pattern recorder.

The digital processing subsystem provides the digital control of the instrumentation, stores the measured digital data, and performs the analysis and display of the data. It consists primarily of a Hewlett-Packard Series 21MX digital processor with a 32K 16-bit word memory, and a Hewlett-Packard Model 7900A 2.5 Megaword disc storage system. The software supplied includes the HP Real Time Executive (RTE II) operating system along with the Scientific Atlanta Series 2020 Antenna Analyzer software. The Series 21MX computer provides 128 standard instructions, including 80 instructions which emulate the HP2100 Series processor; 42 instructions for indexing, for byte and bit manipulation, and for byte word moves; and 6 single precision floating point instructions. There are four general purpose registers, two of which may be used as index registers. The HP Model 7900A disc drive is a random access moving-head dual disc memory device. It uses a permanent disc and a removable disc cartridge with a total on-line memory capacity of 2.5 million 16-bit words. The permanent (lower) disc is used in the SA2020 exclusively for software program storage

and the removable (upper) disc is reserved for recorded data, providing an on-line data capacity of 1.25 megaword and unlimited shelf storage.

As mentioned earlier, the mathematical process for the extrapolation of an antenna far-field radiation pattern based on its near-field measurements consists of two primary programs - one which computes the probe weighting functions required for compensation and one which evaluates the far-field pattern of the antenna under test. Due to the memory size required in the computations by the two primary programs, the overall measurement system processing has been implemented on CRC's Honeywell (CP-V) Sigma-9 computer. Consequently, a method had to be found to transfer the required measured data from the SA2020 memory bank to the Sigma-9's. For the evaluation of both the probe weighting functions and the test antenna far-field pattern, the "required" data consist of field amplitude, field phase and azimuth information.

The simultaneous data acquisition of amplitude, phase and azimuth on the SA2020 is normally performed through a Scientific-Atlanta-written program called "RASTER SCAN". This program actually acquires data - which includes the step angle in degrees, the frequency in MHz, the scan angle in degrees, the desired amplitude in dB, the amplitude of the reference signal in dBm, and the phase angle in degrees - by moving the positioner axes in a "scan" (azimuth) and "step" (elevation) fashion. Data is acquired by monitoring the position of the scan axis and recording the above-mentioned information at the prescribed angular increments. At the end of a scan, the

step axis is positioned to a new angle and another scan is taken.

A minimum of two step angles has to be processed; and up to ten frequencies may be used in data recording.

In designing the near-field measurement system around the SA2020, the following criteria had to be considered:

- a) All data is to be recorded generally at one single frequency (in the present case, either 3.850 GHz or 6.175 GHz) and is required at only one elevation angle from the horizontal, i.e. $\theta = \pi/2$. This applies for both the probe characterization and the actual near-field measurements. But, as mentioned earlier, the "RASTER SCAN" must perform scanning at a minimum of two step angles. In other words, less than half of the data recorded by the SA2020 is of any use, and ideally, only the useful data should need to be transferred to the Sigma-9.
- b) The only output device available is the paper tape punch. For large amounts of data (small scan increments), it is most impractical, especially when the Sigma-9 computer is equipped only with a back-up paper tape reader.* Somehow, the data must be put on a more practical "transferring" device, i.e. magnetic tape.
- c) As acquisition program, "RASTER SCAN", has specific format descriptions which, upon initialization, are passed to the Data File Manager (DFM) of the RTE II operating system.

These format descriptions are for maximum data recording and are modified by the acquisition program to reflect test file instructions with regard to the number of scans, the number of data points, the data to be recorded and the number of frequencies. Upon reception, DFM generates an internal subroutine called "DATA FILE PACK" (DFPK) which performs all the housekeeping necessary to actually store data on the removable disc, packed bit by bit.

The advantage of a "packed" write is that it is fast and uses the least length of tape per given data file. The disadvantage of a "packed" write though is that the packed data is of little use to a host computer, other than another similar system, unless the customer comes up with a compatible "unpacking" routine. Unpacking data requires a large amount of bit shuffling.

- d) Scientific-Atlanta Co. sells a "magnetic tape option" for the SA2020 system which includes a magnetic tape drive and the appropriate software routine that unpacks the antenna data from the data file prior to transferring it to magnetic tape. However, since CRC did not purchase this option, there are no means of actually unpacking the data onto tape. There is also very little information available from Scientific-Atlanta on how the data is originally packed on disc.

e) Finally, Hewlett-Packard magnetic tape drives are easily available in other CRC directorates. Therefore, if the required azimuth, amplitude and phase data could be extracted from the data file created by the RASTER SCAN and put unpacked on disc in an adjacent data file through the Hewlett-Packard RTE II system which includes its own File Manager mode, it should then be possible to transfer the information directly onto tape in an unpacked form.

It should be well understood here that the SA2020 system has two levels of software. As mentioned earlier, the RTE II is inherent to the Series 21MX processor and, superimposed to this and protected by a security code, is the Scientific-Atlanta-written antenna analyzer software. In the sell/purchase agreement with CRC, Scientific-Atlanta had included the SA2020 Software Manual which contains a description of all subroutines available in the system, some of which can retrieve data from packed files for special processing requirements. One of these subroutines, called DGET, can extract named amplitude or phase data from a designated scan in a data file.

From the understanding of the Hewlett-Packard RTE II system and of the File Manager Option (FMGR) of the Batch Spool Monitor, it has then been possible to generate an original Fortran program which uses the subroutine DGET - which in turn calls the SA subroutine Data File Unpack (DFUP) - to create an unpacked data file under any arbitrary 6-letter name on the relocatable disc, and containing only the required information. This program, called SRG1, is presented as

Appendix E. It should be noted that all data obtained through DGET is converted into integers, since the floating point arrangement on the HP 21MX and on the Sigma-9 computer is entirely different. This definitely eases the data transfer to the host computer but also limits the user to an octal 16-bit maximum numerical representation of 32767, the most significant bit being used for sign. Consequently, the amplitude and phase values are restricted to a two-decimal accuracy; for the same reason, azimuth values ranging from 180° to 360° are passed in file as negative azimuth values ranging from -180° to 0° .

Once on disc in an unpacked integer form, the data can easily be copied onto magnetic tape using any HP tape drive. In the HP system, data can be placed on tape in either fixed or variable length records depending on the "type" of the file itself. A Type 1 file, for example, has random access, fixed length 128-word records; a Type 2 file has random access, fixed length user-defined records, but not exceeding 128 words; a Type 3 file consists of variable length, user-defined records with sequential access and with extents (if necessary) provided. The File Management Program (FMP), which provides an interface between the user and the HP file management system, has seven standard file types. Types 1-3 have just been defined; Type 3 and greater files are all variable length records with sequential access and extents provided. However, the HP system will assign Types 4-7 to special purpose files as follows:

- a) Type 4: source programs
- b) Type 5: relocatable programs
- c) Type 6: memory-image programs

d) Type 7: absolute binary programs.

File types greater than seven may also be defined by the user; however, they will be treated by FMP as Type 3 files. When any file of Type 3 or greater is created, FMP writes an end-of-file (EOF) mark at the beginning of the file. As records are written to the file, the EOF is moved automatically to follow the last record.

As records are transferred from disc into magnetic tape, an end-of-record (EOR) mark, which consists of a 3/4 inch gap, will be printed on tape automatically at the end of each record. The EOF mark, which is a 3 inch gap, will be written on tape at the end of the entire data file. When more than one data file has to be transferred, a double EOF mark, i.e. a 6 inch gap, will require printing at the end of the entire listing; the HP drive will not read past a double EOF mark.

The data files created by SRG1 are of Type 3. This choice was made based on a requirement to have a "data extraction" program as general as possible such that a user is not limited to specific cases. Type 3 files have the advantage of having, upon creation, the remaining unused portion of an entire cartridge allocated when the exact size of the file to be open is not known. Any area that is left unused at the end of the processing may be returned with a truncating parameter when the file is closed.

Another problem encountered in the overall design of the measurement system is the fact that the Sigma-9 processor has a 32-bit word while all data transferred on tape by the HP computer consists

of 16-bit words. That means that one Sigma-9 word will actually consist of two HP ones. In order for the data to be meaningful, a "conversion" program is then required. The Fortran program CONVERSION enclosed as Appendix F has been developed to do just that. First, using cards supplied to that effect, the magnetic tape is read by the Sigma-9 and all information is dumped on a Sigma-9 disc file called "STORAGE", set at F:8. Using the Sigma-9 subroutine "BUFFER IN", the data is then read into memory. In order for the original information to be retained, each word is split in two using logical shift techniques; bit 15 of each new word is checked in order to retrieve the sign of the original information passed; each word and its sign is then converted back into floating point format and the data is outputted finally into a file called MTAPE, set at F:200. The program CONVERSION will also allow for more than one file to be processed, the output being written on MTAPEx set at F:(199+x), where x is the number (or sequence) of the file to be processed. A specific data file ends up containing three variable length records, one for each the azimuth, the amplitude and the phase. It should be noted that the maximum number of points in any record cannot exceed 1801; this is equivalent to a maximum azimuth sampling rate of 0.1° for a 180° scan, or of 0.2° increments for a complete 360° scan. It was found that these limits are more than sufficient to perform accurate calculation of both the probe characteristics and the far-field extrapolation.

After CONVERSION has been executed, the data is ready to be processed by either primary Sigma-9 programs, i.e. COEF and/or NEAR.

CHAPTER 4

EXPERIMENTAL WORK

4.1 Introduction

The impetus for this program has been directed towards the implementation, and testing of a low-cost cylindrical near-field measurement system using the available antenna facilities at CRC. Accordingly, the total required experimental work is divided into three major phases:

- 1) the design of a microwave probe and the measurement of its far-field radiation pattern, on an indoor range, in order to characterize it with appropriate weighting functions;
- 2) the design of a test antenna and the measurement of its far-field radiation pattern on an outdoor antenna range in order to be able to compare the actual measured far-field pattern and the computed one;
- 3) and finally, the accurate measurement of the test antenna near field and recording of the data so as to extrapolate mathematically its far field.

This Chapter is dedicated primarily to the analysis of the second step. In fact, the indoor measurement of the probe far-field pattern for characterization purposes - step one - has been reported earlier in Section 3.2. The procedure was relatively simple, although complicated by the fact that the anechoic chamber used was brand new and these measurements were the first being taken in it!

It was also the first time that the SA2020, and related controls, was being operated and many "bugs" had to be worked out. The overall process ended up being very time-consuming.

The third step unfortunately was not possible to complete due to the anechoic chamber being fully utilized - 100% of the time - since early February by SPAR Aerospace Ltd for the construction of the Anik C spacecraft. Consequently, this experimental work is included in Chapter 5 under areas for future work.

The design of the test antenna - step two - has been motivated by the need for some type of high-gain probing device with very low cross-polarization radiation pattern in order to calibrate the entire anechoic chamber (50'x50'x40') for cross-polarization levels of -45dB. A three-foot parabolic antenna was chosen. As in the case of the near-field probe, a polarization grid was to be inserted at the mouth of the aperture; but the actual development of a 3'x4' grid using photoetching techniques is a problem in itself. Results on the final design of the test antenna and on the measurement of its far-field pattern are presented in Section 4.2.

A practical setting to implement a cylindrical near-field measurements system has also been developed and is presented in detail in Section 4.3. As mentioned earlier, the system is based on the capabilities of the Scientific-Atlanta Series 2020 Antenna Analyzer and on one of its software routines, "RASTER SCAN". In the set-up, the near-field probe actually acts as the source and its position can be varied vertically by remote control along an I-beam

support; for each vertical setting, the test antenna facing the probe is rotated in azimuth and the amplitude and phase values of the radiated field are recorded by the analyzer at every interval $\Delta\phi$ and stored for further processing. An overall pattern accuracy of at least $\pm 0.5\text{dB}$ for the main lobe of the test antenna and for its first two sidelobes is expected to be achieved, both in elevation and azimuth.

4.2 Test Antenna Design and Measurement

The choice of an antenna as test subject for the near-field measurement system has been dictated by some of CRC's operational requirements. One of these requirements has been the need to calibrate an eight-foot parabolic reflector which is intended to be used as source antenna to propagate 6-4 GHz from the 220-foot level of an outside tower located at the end of CRC's 1300-foot long antenna range. Before this reflector can be mounted as a source antenna, it is necessary to measure its radiation pattern and cross-polarization level; this is done by "looking" at the eight-foot disk with antennas whose radiation patterns and cross-polarization levels are known. Since the desired cross-polarization level specification is -45dB , the required measuring antenna then has to have a cross-polarization characteristic of around -60dB in order to obtain any meaningful results. An additional constraint is that the design of this "measuring" antenna has to be performed at low-cost using already available in-house equipment and materials.

This antenna calibration is to be performed over the frequency bands 3.5-4.2 GHz and 5.825-6.525 GHz. Due to the length of the antenna range, one of the first requirements was to have an antenna with a maximum beamwidth of about 10° at the lowest frequency tested. This obviously eliminated many antenna configurations. A front-fed parabolic reflector appeared as an attractive choice. The only antenna of such kind available in-house was a low-cost three-foot reflector, manufactured by Spincraft; this then became the main component of our final test structure.

The final design of the test antenna is shown schematically in Fig. 19. It consists of the three-foot reflector mounted on a 4'x4' styrofoam block which supports both the feed and the polarizing grid; this configuration has the advantage of eliminating any need for strut supports which would only create additional cross-polarization in the radiation pattern of the antenna. To further minimize cross-polarization generating effects, a dummy cable has been added to compensate for any asymmetry in the design. The whole structure weighs approximately twenty-five pounds. In fact, the insides of the styrofoam block were carved out to reduce the weight factor to a minimum. The styrofoam itself has a relative dielectric constant of $\epsilon_r = 1.08$, thereby producing only negligible phase shift errors. The pyramidal horns used as primary feeds are "potted" in styrofoam plugs which permits focus tuning and easy replacement. A 4'x4' polarizing grid covers the entire front aperture of the reflector.

Figure 19. Test structure with styrofoam support for feed and polarizing grid (side view).

There were numerous steps followed in the development of this antenna structure. The first task was to determine the f/D ratio - focal length of parabola to diameter - which was unfortunately not documented. To do so, the diameter of the parabola was first accurately measured ($D = 34.625$ inches), and various f/D ratios, ranging from 0.375 to 0.450, were plotted on large-scale graph paper in increments of 0.005. The actual curvature of the disk was then measured, to an accuracy of $\pm 1/32$ in., and the results plotted against the previous curves and compared. These measurements were repeated four times on four different axes passing through the centre point of the parabola but oriented at 45° to each other. The disk f/D ratio was finally chosen as being equal to the theoretical plotted value which best fitted the measured data; it was: $f/D = 0.410$.

Once the f/D ratio of a reflector antenna has been determined, it then becomes possible to design a proper feed. References [33,34] contain significant information about feed design for parabolic reflector antennas. With the use of accurate graphs, it is possible to find some of the feed parameters, such as feed angle, corresponding space attenuation and feed 10-dB beamwidth. The basic five-step procedure is as follows:

- 1) Start with aperture edge illumination and f/D ratio;
- 2) Find feed angle using f/D ;
- 3) Find space attenuation using feed angle;
- 4) Subtract space attenuation from desired edge illumination to get feed taper;

5) Find feed 10-dB beamwidth using feed taper.

NOTE: Values for steps 2), 3) and 5) are obtained directly from plotted curves.

The aperture edge illumination is normally determined by the user and is based on whether the reflector will be designed for maximum gain or for a reduction in sidelobe levels. Ideally, a parabolic reflector can be represented with a linear constant aperture illumination. In practice, it is generally assumed that a typical feed pattern can be described as a cosine function of the angle measured away from the peak of the beam. For maximum gain, it has been found that the energy reflected should be distributed so that the field at the reflector edges is approximately 10 dB below that of the centre; for good (i.e. low-level) sidelobe performance, the field at the edge should be about 20 dB down. The minimization of sidelobe levels was obviously of prime consideration in the design of an antenna which was eventually going to be used as a calibrating device on an antenna range; however, good gain performance was also desirable. Consequently, a compromise was made and the aperture edge illumination was chosen as -15 dB.

From the feed design procedure described earlier and using the graphs provided in Reference [34], the following data characterizing the feed of the parabolic reflector was obtained:

1) Aperture edge illumination: -15 dB

f/D ratio = 0.410

2) Feed angle: 128°

- 3) Space attenuation: 2.9 dB
- 4) Feed taper required: -12.1 dB
- 5) 10-dB beamwidth: 58.2° off-axis or 116.4° total

As mentioned earlier, a pyramidal-shape - also called rectangular - horn had been chosen as a feed to the reflector. This is usually the case for most parabolic dishes since first, the waveguide used to feed the horn, or the coax-to-waveguide transition, is generally rectangular; and second, the radiation pattern of pyramidal horns can be easily shaped in either plane by simply varying the flare angles. Two horns were designed for the reflector: one at $f_{D1} = 3.850$ GHz (Horn #1) to cover the lower frequency band and another at $f_{D2} = 6.175$ GHz (Horn #2) for the upper frequency region of interest. The throat of horn #1 was to match a WR229 waveguide; a WR159 waveguide transition fed horn #2. The final dimensions of Horn #1 and horn #2 are presented in Figs. 20a and 20b.

The most difficult part in the design of the feed is actually the optimization of the aperture dimensions for a specific illumination; this is a matter of trial and error because of the necessity of fitting the horn to the waveguide transition, and it is done by varying the slant lengths of flare, or the flare angles, in the E- and H-planes. A major constraint in our design was that the feed could not exceed a total length of $2\frac{1}{2}$ inches; since the styrofoam plug has a maximum length of $6\frac{5}{8}$ " and the available transitions with flanges are about $2\frac{1}{2}$ to 3 inches long, this leaves us with roughly a one inch margin. Because of its wide application, the pyramidal horn

Figure 20a. Schematic drawing of Horn #1; $f_{D1} = 3.850$ GHz.

Figure 20b. Schematic drawing of Horn #2; $f_{D2} = 6.175$ GHz.

has long been the subject of extensive investigation. Nevertheless, for engineering applications, there is still a dependence on early empirical formulae [33] for the prediction of its radiation characteristics. These early formulae are based on the theory of Fraunhofer diffraction from a planar aperture and hence they are not sufficiently accurate. In Reference [36], however, Antar and Lo have presented a new approach for simply but accurately modelling pyramidal horns; their method is based on Kirchoff's diffraction theory and numerical integration using a two-dimensional Gauss-Legendre quadrature algorithm. This novel technique, numerically implemented at CRC in a Fortran program called YAYA, has been used as a primary tool in designing the feeds for the reflector.

For a specific design frequency, aperture dimensions and slant lengths for the flare - which are obtained at first through basic relations derived from the geometry of a pyramid - the program YAYA can output the flare angles of the horn in both E- and H-planes and the corresponding gain value. But most important, it also provides a calculated value of illumination (in dB) for any angle between zero degrees and 90°; 0° being on boresight and symmetry of pattern being here assumed. Therefore, through trial and error, various values of slant lengths of flare and aperture dimensions were inputted until an illumination of -12.1 dB at 64° off-axis (128° total feed angle) was obtained. The final values of the flare angles in both planes have been included in Fig. 20. This design procedure was first performed at $f_{D1} = 3.850$ GHz and later repeated at $f_{D2} = 6.175$ GHz.

The horns were consequently built in CRC's model shop using an electroplating process which resulted in high quality products.

As mentioned earlier, the test antenna was intended to calibrate an eight-foot parabolic reflector and, as such, required a low cross-polarization level. As for the case of the near-field probe, a polarizing grid was to be inserted in front of the reflector aperture to minimize its cross-polarization. The idea sounded fairly simple, but the design of a four-foot square polarizing grid - exact dimensions being 45"x45" - showed itself to be quite involved. The design procedures were similar to those for the design of the probe's grid described earlier in Chapter 3. A thin 45"x45" sheet of copper plated mylar was used as the material; a rubylith mask was first cut and the required lithographic processes then implemented. It was the first time that lithography was performed on a mask of such size at CRC and required convincing many people of the validity of the trials! Due to the sheer size of the grid, a new suction pump along with a special stainless steel cylinder and holding container had to be designed in order to uniformly coat the mask's negative with photoresist solution. The size of the grid even forced us to expose it to sunlight during the developing stage since large enough light banks were not available. The final results were surprisingly very good; and this technique is now commonly used at CRC in the making of arrays or lenses composed of a large number of thin elements.

The far-field radiation pattern of the test structure was measured on a conventional outdoor antenna range for performance at

3.85 GHz. As mentioned earlier, the range distance is 1300 feet; the source used for the measurements was a Scientific-Atlanta four-foot parabolic reflector fed by a pyramidal horn through an HP8690B sweeper. The resulting plots of copolar and cross-polar radiation patterns for both E- and H-planes are presented in Figs. 21 and 22. Due to the presence of the polarizing grid covering the entire front aperture of the reflector, the cross-polarization level of the antenna structure is, as expected, very low.

4.3 Near-Field Measurement Set-up

The theory of probe-compensated near-field measurements on a cylinder - and the practicality of the method - was verified by experimental work performed by W. Leach; his results are presented in Reference [9]; in summary, the computed radiation patterns match the measured patterns accurately to within ± 0.5 dB in azimuth and, in elevation, similar accuracy is achieved for at least the first two sidelobes. Comparable results are expected to be obtained using our new simplified method.

The practical set-up used in this present work for the actual near-field measurements is shown in Fig. 23. The test antenna is the low-cost one-meter parabolic reflector, mounted on styrofoam blocks, earlier described in Section 4.2. It is fed by a pyramidal horn with an aperture edge illumination of -15 dB. The test antenna is attached vertically on the face plate of an SA Model 5101 Azimuth

	<u>Azimuth (DEG)</u>		<u>Amplitude (dB)</u>
First sidelobes:	Left:	14	-19.0
	Right:	13	-18.5
Second sidelobes:	Left:	23	-30.0
	Right:	23	-28.5

GAIN OF TEST STRUCTURE: 26.1 dB

Figure 21a. H-plane copolar radiation pattern of test structure measured on a conventional outdoor range ($f = 3.847$ GHz).

NOTE: The cross-polarization level of the test structure could not be measured accurately below the -40 dB level, due to the "X-po" level of the range itself being at approximately that value.

Figure 21b. H-plane crosspolar radiation pattern of test structure measured on a conventional outdoor range ($f = 3.847$ GHz).

	<u>Azimuth (DEG)</u>		<u>Amplitude (dB)</u>
First sidelobes:	Left:	14	-17.5
	Right:	12	-17.0
Second sidelobes:	Left:	24	-28.0
	Right:	18	-25.5

Figure 22a. E-plane copolar radiation pattern of test structure measured on a conventional outdoor range (f= 3.847 GHz)

NOTE: The cross-polarization level of the test structure could not be measured accurately below the -40 dB level, due to the "X-po" level of the range itself being at approximately that value.

Figure 22b. E-plane crosspolar radiation pattern of test structure measured on a conventional outdoor range (f= 3.847 GHz)

Figure 23. Near-field antenna measurement set-up in anechoic chamber.

Positioner mounted sideways at the end of an 11½-foot tower bolted on top of a standard heavy-duty SA Model 53450 azimuth-over-elevation positioner. Great care should be taken to have the phase centre of the test structure aligned accurately with the centre of rotation of the heavy-duty turntable; the phase centre of the test antenna will be found, by trial and error, to be somewhere between the feed mouth and the front aperture of the dish. Across from the test antenna is the receiving probe, mounted on a Scientific-Atlanta Model DWG-59510/59511 Interface Control Aperture Probe mechanism similar to that shown schematically in Fig. 24; this mechanism is basically a 20-foot I-beam support along which a carriage-mounted probe, driven by remote control, can travel up and down a distance of eighteen feet; a synchro signal indicates the position of the probe to within ± 1 cm. The I-beam is scaled every tenth of a centimeter, enabling accurate positioning of the probe at any point on a vertical line parallel to the axis of rotation of the azimuth-over-elevation positioner. The probe mechanism is also aligned so that the longitudinal axis of the probe intersects the axis of rotation of the heavy-duty platform at about 1 meter.

At almost all times during the measurements, both the probe and the test antenna are contained within a 16-foot "zone of silence" created in the centre of the anechoic chamber by heavier absorbing materials on the walls and ceiling. To further minimize unwanted reflections, the surfaces of the probe mechanism, of the antenna tower and of the heavy-duty turntable itself are covered with a supply of 8-inch absorbing material.

Figure 24. Schematic close-up of the probe mechanism.

The near-field data $v(r_0, \phi_0, z_0)$ are first measured in amplitude and phase by setting the probe at a fixed vertical position and then by rotating the antenna platform in azimuth, thereby performing a RASTER SCAN with the SA2020. Once all values of $v(r_0, \phi_0, z_0)$ are recorded for a fixed vertical setting, i.e. a fixed value of z_0 , the probe is immediately rotated by 90° about its longitudinal axis and a RASTER SCAN is performed once again in order to record all values of $v'(r_0, \phi_0, z_0)$. This is done to ensure that all $v(r_0, \phi_0, z_0)$ and $v'(r_0, \phi_0, z_0)$ are taken exactly at the same elevation z_0 ; accuracy in r_0 is easily achieved by the physical positioning of the probe support tower at a fixed distance to the axis of rotation of the azimuth-over-elevation positioner; accuracy in ϕ_0 is achieved by the precision control exercised by the SA2020 over the heavy-duty platform ($\pm 0.1^\circ$) during the processing of a RASTER SCAN, as previously described. Once the values of $v'(r_0, \phi_0, z_0)$ are registered, the probe is "de-spun" to its original reference orientation, lowered automatically by the required Δz sample spacing, and the above procedure is then repeated. The initial and final vertical positions of the probe, while sampling in z , are set as being the points where the relative radiated field amplitude is down by -40 dB. Anything below that is considered to be zero, thereby fulfilling the condition whereby the fields must be zero for $z < 0$ and $z > (M-1)\Delta z$. It should be noted here that the determination of these two vertical points is most important since they finally determine the overall number of samples in elevation (M).

Also note that the near-field data being digitally recorded by the SA2020 primarily consist of amplitude, phase and azimuth information.

4.4 Near-Field Measurement Errors

In a near-field measurement system, the resulting computed radiation pattern of the subject antenna will only be acceptable as long as the near-field data processed has been accurately measured. Near-field measurement errors are most often produced in a complicated fashion. For example, a measurement receiver may be non-linear; this same receiver may also add a random amplitude and phase to the signal and may have a limited dynamic range. Misalignment in the probe position, shifts in rotary joints, poor A/D conversion may also generate multiple additional errors which will affect the final extrapolation results. No concise study of the effects of combined errors on the extrapolation results of a cylindrical near-field measurement system has yet ever been published. The latest available information on the subject, dated June 1980, is from the U.S. National Bureau of Standards where a research team, headed by C.F. Stubenrauch, were currently in the process of compiling data specifically relating to this topic. Published results should therefore be available soon. However, much data pertaining to error analysis of planar measurement systems is presently available in the literature. E.B. Joy, for example, - who heads the near-field facilities of the Georgia Institute of Technology - has published intensively on the subject. Some of his findings, particularly those relating to the types of errors

generated by signal processing/detection equipment, can easily apply to a cylindrical measurement system. A list of types of error could therefore be formulated as follows:

1) Random amplitude error:

- these are usually associated with measurement noise.

Random amplitude errors can also be generated at the source and in data conversion and recording equipment.

2) Amplitude error which is linear with respect to the true amplitude:

- this type of error is associated with the measurement receiver and data conversion equipment. It is one form of non-linearity possessed by almost all measuring devices.

3) Amplitude error which is quadratic with respect to the true amplitude:

- this error is another form of non-linearity found in most receivers and data conversion equipment;

4) Amplitude truncation with respect to the true amplitude:

- this type of error simulates the errors associated with the dynamic range limitation of the receiver, data conversion, equipment and data recording equipment;

5) Amplitude errors which are linear with time:

- this type of error accounts for the drift of equipment output with time. The most common source of this type

of error is the microwave source; however, receivers and data conversion equipment may also produce this type of error;

6) Random phase error:

- random phase errors are usually associated with the receiver as measurement noise. Random phase errors may occur in other ways also, such as random flexing of coaxial cables, random motion of the test antenna or of the probe antenna, noise in data conversion and data recording equipment. Random phase errors may also be caused by jitter of the phase-lock mechanism of the source.

7) Phase error which is linearly proportional to the true phase:

- primarily due to the receiver characteristics;

8) Phase error which is quadratic with respect to the true phase:

- this is another type of non-linearity associated with measurement receivers, data conversion equipment and data recording equipment;

9) Phase error which is quadratic with amplitude:

- the phase accuracy of a phase/amplitude receiver is a non-linear function of the amplitude of the received signal. For moderate and large amplitude signals, the phase error of the receiver is small and almost

independent of the signal strength. For low amplitude signals, the phase error increases rapidly with decreasing amplitude.

10) Phase error which is linear with time:

- drift of the receiver output and slow warping of mechanical positioners can cause a gradually increasing or decreasing phase error to be produced;

11) Near-field area truncation error:

- the theory of cylindrical near-field measurement states that a cylinder completely enclosing the antenna must be sampled. Such complete sampling is impossible to achieve, and something less than the entire near-field cylinder is measured.

12) Amplitude and phase errors due to the motion of the near-field probe:

- the test antenna is rotated in a continuous manner for any vertical probe position, and data is received and converted in a continuous manner. Data is recorded, however, only when the test antenna passes a specified sample point. Two factors limit the speed of measuring the near field: first, there is the speed of scan of the test antenna and, second, there is the time constant of the receiver; any variations or fluctuations in either will generate errors.

13) Multiplexing error:

- in some cases, such as for a phased array, the antenna being measured may possess multiple modes of operation: the near-field distribution corresponding to one polarization component of each mode could be measured during one coverage of the azimuth plane by the test antenna. As the test antenna is rotated, it can be switched from one mode to another so that the near-field samples for each mode are interleaved in time (and/or space). Clearly the receiver time constant is a limiting factor in this process.

14) Amplitude and phase errors due to reflections of fixed or different amplitudes and phases:

- reflections are always present to some degree on a far-field or near-field measurement range; reflected signals will either add or subtract with true signals, causing amplitude and phase errors;

15) Probe misalignment and probe position error:

- when the near-field probe is mounted on the near-field range, it is conceivable that the coordinate system defined for it does not exactly coincide with that defined for the test antenna; both probe and test antenna misalignment may occur;
- the measurement system is positioned to specified positions in a cylindrical plane in order to measure the response of the probe to the near field at these

specified points. Due to random mechanical vibrations, random thermal vibrations and random inaccuracies inherent in the positioning drives and indicators, the cylindrical coordinates are not accurately known.

These are only some of all possible errors which may occur strictly in the near-field data gathering phase of the whole measurement process. Errors associated with numerically rounded-off figures and other software processing inaccuracies are not considered here since they are generally at least one order of magnitude smaller than equipment generated errors.

As we mentioned in the introduction to this Chapter, the final testing of this cylindrical probe-compensated near-field measurement system has not yet been implemented. The reasons behind this shortfall, along with comments on predicted overall system performance and suggestions for future work, are presented in Chapter 5.

CHAPTER 5

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

The low-cost design of a cylindrical near-field measurement system - implemented around existing antenna measurement facilities and available equipment - has proven itself to be an engineering challenge. As was shown throughout this document, numerous problems, most of them unanticipated during the initial definition of the problem, surfaced throughout the implementation but were subsequently solved satisfactorily. To summarize, our cylindrical near-field measurement system has three major characteristics:

- 1) It uses a vertically polarized probe which eliminates the need for complex polarization ratio measurements; further characterization of the probe is based on the complex measurements (amplitude and phase) of its two main polar axes only.
- 2) It extrapolates the far-field of the antenna under test using simplified probe-compensated equations, due to the previous elimination of one of the probe far-field components.
- 3) Finally, all of the above is performed using present CRC antenna measurement and computer processing facilities.

Considering the relatively short time frame allocated to this project and taking into account a work implementation priority system

presently in effect at CRC by which the development, construction and testing of ANIK-class satellites and related parts/systems prevail, it is believed that the performances so far are very good and most promising. All indicate that a cylindrical probe-compensated near-field measurement system is feasible at low cost and can be implemented using currently available resources. At this point in time, the entire system is ready to be finally tested. The measuring equipment used in our system have all been employed previously in various other antenna tests and their technical characteristics are well known and understood; software programs SRG1, CONVERSION, FIELD, and COEF have already been tested in the probe characterization phase and have performed satisfactorily. The fifth and last piece of software, the program "NEAR" which actually performs the extrapolations, has been tested subroutine by subroutine and has produced excellent simulated results. All that remains is an actual probing of the test antenna near field and the mathematical extrapolation, using NEAR, of its far-field pattern. As mentioned earlier, it has been unfortunately impossible to complete this last portion of our experimental work. This was due mainly to CRC's anechoic chamber being fully booked, since early February and up until late August, 1981, by SPAR Aerospace Ltd. and other contracting firms involved in the construction of ANIK C - and ANIK D - class spacecrafts.

In Reference [32], Leach verified a similar measurement system on a ten-element resonant array of longitudinal shunt slots cut into the broad wall of a section of WR-90 waveguide. The slots were

designed so that the antenna radiated a Chebyshev pattern with side-lobe levels of -20 dB. The load end of the waveguide array was terminated in an adjustable short which was set for minimum input VSWR at the frequency of 9.68 GHz. Some of Leach's final results are shown in Fig. 25. The three-dimensional far-field amplitude pattern of E_ϕ , which was calculated from a set of measurements with the antenna mounted vertically in the cylinder - as would be the case in our system - is first displayed in Fig. 25a. Then, in Figs. 25b and 25c, the principal plane amplitude patterns representing the cuts $\phi = 0^\circ$ and $\theta = 90^\circ$ in Fig. 25a are displayed with the measured relative far-field patterns of the test antenna, for $-180^\circ \leq \phi \leq 180^\circ$ and $30^\circ < \theta < 150^\circ$. The cross-polarization pattern was apparently calculated also but was too low in amplitude to be interpreted meaningfully. As one can see, good accuracy is achieved from the extrapolation. In fact, Leach attributes the discrepancies between the measured and calculated patterns in these figures mainly to reflections caused by unshielded objects in the vicinity of the near-field range. We are most confident that similar performances can be achieved with our simplified probe-compensated system. As we mentioned earlier in Chapter 2, as far as the azimuth pattern of a test antenna at a fixed elevation angle is concerned, probe compensation is not a significant consideration and its effects could easily be neglected since the probe is always pointing toward the centre of the measurement cylinder. However, even if probe correction becomes important in calculating the elevation pattern of the antenna under test, our probe-compensation

Figure 25a. Three-dimensional representation of calculated far-field pattern for E_ϕ [32].

Figure 25b. Comparison of calculated [-] to measured [.....] amplitude patterns for a cut in elevation ($\phi = 0^\circ$). (Extracted from Reference [32])

Figure 25c. Comparison of calculated [-] to measured [.....] amplitude patterns for a cut in azimuth ($\theta = 90^\circ$). (Extracted from Reference [32])

algorithm - although simplified to allow for the intended "polarizing" of the probe - still provides adequate correction and, consequently, accurate results should be obtained.

The first recommendation to be made for future work would obviously be to test this cylindrical near-field measurement system. Also, it would be most interesting to characterize additional near-field probes - primarily small horns of different gain/directivity - in addition to the open-ended waveguide which was used and compare the accuracy of the final extrapolated results.

Along another line, findings made by E. Bello Ojeda and C.H. Walter [39], and published in late 1979, should be taken into consideration and analyzed more closely with respect to improving the accuracy of extrapolated results. One recalls that in making near-field computations, a priori knowledge of the cut-off wavenumber was estimated as being $N = ka$, where k is the free-space wavenumber and a is the radius of the smallest sphere or cylinder, as the case may be, enclosing the source. Up until now, the only available guidelines on which to base this estimation were those derived from work performed by Chu [19] and Collin and Rothschild [17], and which were presented earlier in Chapters 2 and 3. However, it has been found by Ojeda and Walter that using only wave harmonics lower than ka may be inadequate for some antenna problems. Consequently, they have proposed new cut-off criteria for cylindrical and spherical wave harmonics. These criteria could be used with the system and the results compared with earlier extrapolations that used the old guidelines.

Two other suggestions closely related to the subject of cylindrical near-field measurement systems and of great potential interest are first the study, development and comparison of Borgiotti's [7] new numerical approach for far-field evaluation based on measurements taken on a cylinder and, second, the development and possible implementation, using CRC's facilities, of a hybrid (planar/cylindrical) near-field measurement system [40] in which, contrary to the work performed at NBS, the planar portion of the system would use the novel plane-polar approach presented by Y. Rahmat-Samii et al. in Reference [41]. With the advent of large space reflectors, a hybrid measurement system, using planar techniques for the evaluation of the main beam and a cylindrical system to obtain the far-field sidelobes, offers enormous capabilities. The two new approaches presented in References [7] and [41] may respectively allow for simpler, more accurate cylindrical and planar near-field measurement systems.

In any case, near-field measurement techniques are definitely gaining more and more popularity and there is considerable evidence that the near-field approach will eventually become the method of antenna qualification in the future.

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APPENDICES

APPENDIX A

EVALUATION OF EQUATION (2.46) USING THE METHOD OF STEEPEST DESCENT

Recall Eqn. (2.46):

$$\begin{aligned} \vec{E} = j^{\frac{1}{2}} k^{3/2} \left(\frac{2}{\pi R \sin \theta} \right)^{\frac{1}{2}} \sum_{n=-\infty}^{\infty} j^n e^{jn\phi} \int_C [\hat{\phi} j a_n(k \sin \alpha) \\ - (\hat{r} \sin \alpha - \hat{z} \cos \alpha) b_n(k \sin \alpha)] \cos^{3/2} \alpha e^{-jkR \sin(\alpha+\theta)} d\alpha \end{aligned} \quad (A.1)$$

where C is the contour illustrated in Fig. 2. Each scalar component of the integral in the above expression is of the form

$$I = \int_C f(\alpha) e^{Rg(\alpha)} d\alpha \quad (A.2)$$

This integral can be evaluated for large R using the method of steepest descent. The method first consists of finding the point α_0 on contour C at which $g'(\alpha) = 0$. The contour C is then deformed into the path S which passes through α_0 and on which

$$\operatorname{Re}[g(\alpha)] \leq \operatorname{Re}[g(\alpha_0)] \quad (A.3)$$

$$\operatorname{Im}[g(\alpha)] = \operatorname{Im}[g(\alpha_0)]$$

The point α_0 is called the saddle point of the integral and the path S is the steepest descent path. If this path exists, the change in

variables

$$g(\alpha) = g(\alpha_0) - w^2 \quad (\text{A.4})$$

can be made, where w is real and α lies on S . Thus the integral can be transformed into:

$$I = e^{Rg(\alpha_0)} \int_{-\infty}^{\infty} f(\alpha) e^{-Rw^2} \frac{d\alpha}{dw} dw \quad (\text{A.5})$$

where α is a function of w defined implicitly by Eqn. (A.4).

The method of steepest descent refers to the first term in the asymptotic expansion about $R = \infty$ of the integral in Eqn. (A.5). This is shown by Clemmow [35] to be:

$$I = \pm \left(\frac{-2\pi}{Rg''(\alpha_0)} \right)^{1/2} e^{Rg(\alpha_0)} f(\alpha_0) \quad (\text{A.6})$$

where the ambiguity in sign must be resolved by examining:

$$\left[\frac{d\alpha}{dw} \right]_{\alpha=\alpha_0} = \pm \left[\frac{-2}{g''(\alpha_0)} \right]^{1/2} \quad (\text{A.7})$$

at the saddle point. The sign in Eqn. (A.7) must be chosen to make $d\alpha$ tangent to S at $\alpha = \alpha_0$. The corresponding sign is then used in Eqn. (A.6).

The function $g(\alpha)$ in the integral to be evaluated in Eqn. (A.1) is:

$$g(\alpha) = -j k \sin(\alpha + \theta) \quad (\text{A.8})$$

The saddle point is found by setting $g'(\alpha_0) = 0$, as mentioned earlier.

Thus:

$$g'(\alpha_0) = -jk \cos(\alpha_0 + \theta) = 0 \quad (\text{A.9})$$

The only solution to this equation which lies on contour C is:

$$\alpha_0 = \frac{\pi}{2} - \theta \quad (\text{A.10})$$

Consequently, it follows that at α_0 :

$$g(\alpha_0) = -jk \quad (\text{A.11})$$

The region of the complex α -plane for which $\text{Re}[g(\alpha)] \leq \text{Re}[g(\alpha_0)]$ is defined by the equation:

$$\cos(\alpha_r + \theta) \sinh \alpha_i \leq 0 \quad (\text{A.12})$$

where $\alpha = \alpha_r + j\alpha_i$. This region is the shaded region shown in Fig.

26. The steepest descent path must lie in this region to ensure convergence of the integral. On this path, the condition $\text{Im}[g(\alpha)] = \text{Im}[g(\alpha_0)]$ is also satisfied. Thus the equation for the path is:

$$\sin(\alpha_r + \theta) \cosh \alpha_i = 1 \quad (\text{A.13})$$

This path is sketched in Fig. 26.

To resolve the sign ambiguity in Eqn. (A.7), $d\alpha/dw$ must be calculated at the saddle point. At this point, $g''(\alpha_0) = jk$. Thus, using Eqn. (A.7), $d\alpha/dw$ is given by:

$$\begin{aligned} \left[\frac{d\alpha}{dw} \right]_{\alpha=\alpha_0} &= \pm \left(\frac{-2}{jk} \right)^{1/2} \\ &= \pm (1+j)(1/k)^{1/2} \end{aligned} \quad (\text{A.14})$$

Figure 26. Path of steepest descent.

Examination of Fig. 26 at $d = \alpha_0$ shows that:

$$\begin{aligned}\frac{d\alpha}{dw} &= (1 + j \frac{\partial \alpha_i}{\partial \alpha_r}) \frac{d\alpha_r}{dw} \\ &= (1+j) \frac{d\alpha_r}{dw}\end{aligned}\quad (A.15)$$

Since a positive increment in w corresponds to moving along S in the positive direction, it follows from Fig. 26 that $d\alpha_r/dw > 0$ at every point on S . Therefore, the positive sign in Eqn. (A.14) must be chosen in order for this equation to agree with Eqn. (A.15). Thus, for the problem at hand, Eqn. (A.6) becomes:

$$I = + \left(\frac{-2\pi}{g''(\alpha_0)} \right)^{1/2} e^{Rg(\alpha_0)} f(\alpha_0) \quad (A.16)$$

With the substitution of the above result in Eqn. (A.1), the far-field electric field intensity becomes:

$$\begin{aligned}\vec{E} &= \frac{-2k \sin \theta}{R} e^{-jkR} \sum_{n=-\infty}^{\infty} j^n e^{jn\phi} [\hat{\phi} a_n(k \cos \theta) \\ &\quad - j(\hat{r} \cos \theta - \hat{z} \sin \theta) b_n(k \cos \theta)]\end{aligned}\quad (A.17)$$

This is the desired expression!

APPENDIX B

VECTOR TRANSLATION THEOREMS FOR CYLINDRICAL VECTOR WAVE FUNCTIONS

In the derivation of the response of the probe used to measure the near-field of an antenna on a cylinder, it is necessary to be able to translate the reference coordinate of the probe to that of the antenna being tested. The necessary vector translation theorems are derived as follows.

Let the coordinates (r_0, ϕ_0, z_0) be the location of the origin of the coordinate system (r', ϕ', z') in the coordinate system (r, ϕ, z) as shown in Fig. 27. It is desired to express the cylindrical vector wave functions $\vec{M}_{nh}^{(4)}(r')$ and $\vec{N}_{nh}^{(4)}(r')$, earlier defined in Section 2.2, as functions of the coordinates (r, ϕ, z) .

In the system (r', ϕ', z') , the generating function $\psi_{nh}^{(4)}$ for these vectors is given by:

$$\psi_{nh}^{(4)}(r') = H_n^{(2)}(\Lambda r') e^{jn\phi'} e^{-jhz'} \quad (B.1)$$

This can be expressed as a function of (r, ϕ, z) by using Graf's addition theorem for the Hankel function, which states that:

$$H_n^{(2)}(\Lambda r') = e^{-jn\phi'} \sum_{m=-\infty}^{\infty} H_{n+m}^{(2)}(\Lambda r_0) J_m(\Lambda r) e^{jm(\phi - \phi_0)} \quad (B.2)$$

When this is substituted into Eqn. (B.1), the generating function is transformed into:

Figure 27. Coordinate system for the cylindrical wave translation theorems.

$$\psi_{nh}^{(4)}(r') = \sum_{m=-\infty}^{\infty} [H_{n+m}^{(2)}(\Lambda r_o) e^{jm\phi_o} e^{jhz_o}] J_m(\Lambda r) e^{-jm\phi} e^{-jhz} \quad (B.3)$$

The vectors $\vec{M}_{nh}^{(4)}$ and $\vec{N}_{nh}^{(4)}$ are obtained by the following operations on $\psi_{nh}^{(4)}$:

$$\vec{M}_{nh}^{(4)} = \vec{\nabla}' \times \hat{z} \psi_{nh}^{(4)} \quad (B.4)$$

$$\vec{N}_{nh}^{(4)} = \frac{1}{k} \vec{\nabla}' \times \vec{M}_{nh}^{(4)} \quad (B.5)$$

where the primes denote operations on the primed coordinates (r', ϕ', z') .

Since the del operator is invariant to a coordinate transformation, these become:

$$\vec{M}_{nh}^{(4)}(r') = \sum_{m=-\infty}^{\infty} [H_{n+m}^{(2)}(\Lambda r_o) e^{jm\phi_o} e^{jhz_o}] \cdot \vec{\nabla} \times [\hat{z} J_m(\Lambda r) e^{-jm\phi} e^{-jhz}] \quad (B.6)$$

$$\vec{N}_{nh}^{(4)}(r') = \frac{1}{k} \sum_{m=-\infty}^{\infty} [H_{n+m}^{(2)}(\Lambda r_o) e^{jm\phi_o} e^{jhz_o}] \cdot \vec{\nabla} \times \vec{\nabla} \times [\hat{z} J_m(\Lambda r) e^{-jm\phi} e^{-jhz}] \quad (B.7)$$

From the definitions of $\vec{M}_{nh}^{(1)}$ and $\vec{N}_{nh}^{(1)}$ earlier developed in Chapter 2, i.e.

$$\vec{M}_{nh}^{(1)}(r) = \vec{\nabla} \times [\hat{z} J_m(\Lambda r) e^{-jm\phi} e^{-jhz}] \quad (B.8)$$

$$= \vec{\nabla} \times \hat{z} \psi_{nh}^{(1)}(r)$$

$$\vec{N}_{nh}^{(1)}(r) = \frac{1}{k} \vec{\nabla} \times \vec{M}_{nh}^{(1)}(r) \quad (B.9)$$

it follows that Eqns. (B.6) and (B.7) can be reduced to:

$$\vec{M}_{nh}^{(4)}(r') = \sum_{m=-\infty}^{\infty} (-1)^m H_{n+m}^{(2)}(\Lambda r_o) e^{jm\phi_o} e^{jhz_o} \vec{M}_{-mh}^{(1)}(r) \quad (B.10)$$

$$\vec{N}_{nh}^{(4)}(r') = \sum_{m=-\infty}^{\infty} (-1)^m H_{n+m}^{(2)}(\Lambda r_o) e^{jm\phi_o} e^{jhz_o} \vec{N}_{-mh}^{(1)}(r) \quad (B.11)$$

Eqns. (B.10) and (B.11) are the desired translation theorems. They are valid for all $r \leq r_o$.

APPENDIX C

ORTHOGONALITY PROPERTIES OF THE CYLINDRICAL WAVE VECTORS

There are four orthogonality properties of the cylindrical wave vectors $\vec{M}$ and $\vec{N}$ over a cylinder of constant radius. These properties are:

Property A

$$\int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \vec{M}_{nh}^{(i)} \times \vec{M}_{m\eta}^{(j)} \cdot \hat{r} \, d\phi \, dz = 0 \quad \text{for all } m, n, \eta \text{ and } h \quad (\text{C.1})$$

This property follows from the fact that the vector $\vec{M}$ contains no z-component. Thus the product $\vec{M}_{nh}^{(i)} \times \vec{M}_{m\eta}^{(j)}$ has only a z-component which is zero when scalar-multiplied by the unit vector $\hat{r}$.

Property B

$$\int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \vec{N}_{nh}^{(i)} \times \vec{N}_{m\eta}^{(j)} \cdot \hat{r} \, d\phi \, dz = 0 \quad \text{for all } m, n, \eta \text{ and } h \quad (\text{C.2})$$

The integrand in Eqn. (C.2) can be simplified as follows:

$$\begin{aligned} \vec{N}_{nh}^{(i)} \times \vec{N}_{m\eta}^{(j)} \cdot \hat{r} &= \vec{N}_{m\eta}^{(j)} \cdot (\hat{r} \times \vec{N}_{nh}^{(i)}) \\ &= \left[\frac{nh\lambda^2}{k^2 r} Z_n^i(\Lambda r) Z_m^j(\lambda r) - \frac{m\eta\Lambda^2}{k^2 r} Z_n^i(\Lambda r) Z_m^j(\lambda r) \right] e^{j(m+n)\phi} e^{-j(\eta+h)z} \end{aligned}$$

This is identically zero when integrated with respect to ϕ and z unless $m = -n$ and $\eta = -h$. However, under these conditions, the term

in brackets is identically zero since $\lambda = \Lambda$ when $\eta = -h$. Thus, Property B follows immediately.

Property C

$$\begin{aligned} & \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \vec{N}_{nh}^{(i)} \times \vec{M}_{m\eta}^{(j)} \cdot \hat{r} \, d\phi \, dz \\ &= \frac{4\pi^2 \Lambda^3}{k} Z_n^i(\Lambda r) Z_{-n}^j(\Lambda r) \delta(\eta+h), \text{ for } m = -n \\ &= 0 \quad \text{otherwise.} \end{aligned} \quad (C.3)$$

The integrand of Eqn. (C.3) can be simplified as follows:

$$\begin{aligned} \vec{N}_{nh}^{(i)} \times \vec{M}_{m\eta}^{(j)} \cdot \hat{r} &= - \vec{N}_{nh}^{(i)} \cdot (\hat{r} \times \vec{M}_{m\eta}^{(j)}) \\ &= \frac{\lambda \Lambda^2}{k} Z_n^i(\Lambda r) Z_m^j(\lambda r) e^{j(m+n)\phi} e^{j(\eta+h)z} \end{aligned}$$

This is zero when integrated with respect to ϕ unless $m = -n$. In this case, the integral of the exponential term involving ϕ is 2π . The integral of the exponential term involving z results in the factor $2\pi\delta(\eta+h)$. Since this is zero for $\eta + h \neq 0$, it follows that the substitution $\eta = -h$ can be made in the rest of the expression. Thus Property C follows immediately.

Property D

$$\begin{aligned} & \int_{-\infty}^{\infty} \int_{-\pi}^{\pi} \vec{M}_{nh}^{(i)} \times \vec{N}_{m\eta}^{(j)} \cdot \hat{r} \, d\phi \, dz \\ &= \frac{-4\pi^2 \Lambda^3}{k} Z_n^i(\Lambda r) Z_{-n}^j(\Lambda r) \delta(\eta+h), \text{ for } m = -n \end{aligned} \quad (C.4)$$

= 0

otherwise.

This property follows immediately from Property C.

APPENDIX D

THE DISCRETE FOURIER TRANSFORM (DFT) [37]

A) The One-Dimensional Case

There are several techniques available for representing sequences or equivalently discrete-time systems including the convolution summation, the Fourier transform, and the z-transform. In the case where the sequence to be represented is periodic - and, as well, when the sequence is of finite duration - this sequence can be represented in a discrete Fourier series. Thus, consider the periodic sequence $x_p(n)$ with a period of N samples. Since $x_p(n)$ is periodic, it can be represented as:

$$x_p(n) = \sum_{k=-\infty}^{\infty} X_p(k) e^{j\left(\frac{2\pi}{N}\right)kn} \quad (D.1)$$

where the only possible frequencies of which $x_p(n)$ can be composed are $\omega_k = 2\pi k/N$, $-\infty < k < \infty$, since these are the only frequencies whose periods are integrally related to N . The quantity $X_p(k)$ in Eqn. (D.1) represents the amplitude of the sinusoid at frequency ω_k . Equation (D.1), however, contains much redundant information since the complex exponential at frequency $\omega_k = 2\pi k/N$ is indistinguishable from the complex exponential at frequency $\omega_k \pm mN = (2\pi/N)(k \pm mN)$, $0 < m < \infty$, because of the periodicity of the function $e^{j\theta}$ every 2π ; e.g.

$$e^{j\left(\frac{2\pi}{N}\right)kn} = e^{j\left(\frac{2\pi}{N}\right)(k \pm mN)n} ; 0 < m < \infty \quad (D.2)$$

Thus Eqn. (D.1) can be expressed in the form:

$$x_p(n) = \sum_{k=0}^{N-1} x_p(k) e^{j\left(\frac{2\pi}{N}\right)kn} \quad (D.3)$$

emphasizing the fact that there are only N distinct exponentials that are periodic of period N samples. For convenience, and to conform with popular notation, we rewrite Eqn. (D.3) as:

$$x_p(n) = \frac{1}{N} \sum_{k=0}^{N-1} x_p(k) e^{j\left(\frac{2\pi}{N}\right)kn} \quad (D.4)$$

where the division by $1/N$ has no bearing on the form of the representation. To obtain a relation $x_p(k)$ in terms of $x_p(n)$, we multiply both sides of Eqn. (D.4) by $e^{-j(2\pi/N)nm}$ and then sum over all n giving:

$$\sum_{n=0}^{N-1} x_p(n) e^{-j\left(\frac{2\pi}{N}\right)nm} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} x_p(k) e^{j\left(\frac{2\pi}{N}\right)n(k-m)} \quad (D.5)$$

Interchanging orders of summation on the right side of Eqn. (D.5) and using the relation:

$$\sum_{n=0}^{N-1} e^{-j\left(\frac{2\pi}{N}\right)n(k-m)} = \begin{cases} N & ; \text{ if } k = m \\ 0 & ; \text{ otherwise} \end{cases} \quad (D.6)$$

we obtain:

$$\sum_{n=0}^{N-1} x_p(n) e^{-j\left(\frac{2\pi}{N}\right)nm} = \sum_{k=0}^{N-1} x_p(k) U_o(k-m) \quad (D.7)$$

Reversing left and right sides of the equation and substituting the index k for m , we finally arrive at:

$$X_p(k) = \sum_{n=0}^{N-1} x_p(n) e^{-j(\frac{2\pi}{N})nk} \quad (D.8)$$

Equation (D.8) has been called the discrete Fourier transform (DFT) and Eqn. (D.4) is called the inverse discrete Fourier transform (IDFT).

B) The Two-Dimensional Case

A periodic signal in two dimensions is defined by the relations:

$$x_p(n_1, n_2) = x_p(n_1 + m_1 N_1, n_2 + m_2 N_2) \quad (D.9)$$

where N_1 is the period along the first dimension, N_2 is the period along the second dimension, and m_1 and m_2 are any integers. The subscript "p" in Eqn. (D.10) indicates the signal is periodic. Just as in one-dimension, two-dimensional signals can be exactly represented as a linear combination of a finite set of exponentials whose periods are subperiods N_1 and N_2 , i.e.

$$x_p(n_1, n_2) = \frac{1}{N_1} \frac{1}{N_2} \sum_{k_1=0}^{N_1-1} \sum_{k_2=0}^{N_2-1} X_p(k_1, k_2) e^{j(\frac{2\pi}{N_1})n_1 k_1} e^{j(\frac{2\pi}{N_2})n_2 k_2} \quad (D.10)$$

where the periodic Fourier coefficients $X_p(k_1, k_2)$ represent the amplitude of $x_p(n_1, n_2)$ at the two-dimensional frequency $\omega_1 = (2\pi/N_1)k_1$, $\omega_2 = (2\pi/N_2)k_2$. $X_p(k_1, k_2)$ may be readily determined by evaluating:

$$X_p(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_p(n_1, n_2) e^{-j(\frac{2\pi}{N_1})n_1 k_1} e^{-j(\frac{2\pi}{N_2})n_2 k_2} \quad (D.11)$$

Equations (D.10) and (D.11) are referred to, respectively, as the two-dimensional inverse discrete Fourier transform and as the two-dimensional discrete Fourier transform.

Many properties of Eqns. (D.10) and (D.11) may be derived [38]. Two important properties, however, concern the evaluation of these two equations as a series of one-dimensional DFTs and the extension of a DFT representation to finite duration signals. The first property may be seen by rewriting Eqn. (D.11) as:

$$X_p(k_1, k_2) = \sum_{n_1=0}^{N_1-1} e^{-j(\frac{2\pi}{N_1})n_1 k_1} \left[\sum_{n_2=0}^{N_2-1} x_p(n_1, n_2) e^{-j(\frac{2\pi}{N_2})n_2 k_2} \right] \quad (D.12)$$

The bracketed term constitutes a series of N_1 one-dimensional DFTs obtained by varying n_1 from 0 to (N_1-1) . If we call the result of each DFT " $g_p(n_1, k_2)$ ", then Eqn. (D.12) becomes:

$$X_p(k_1, k_2) = \sum_{n_1=0}^{N_1-1} e^{-j(\frac{2\pi}{N_1})n_1 k_1} g_p(n_1, k_2) \quad (D.13)$$

which is again a series of N_2 one-dimensional DFTs as k_2 is valid from 0 to (N_2-1) . The extension of the above argument to the evaluation of the inverse DFT is straightforward.

The property that a DFT representation provides an exact representation for a finite duration sequence $x(n_1, n_2)$ may be readily seen by examining Eqns. (D.10) and (D.11) for the case when:

$$x_p(n_1, n_2) = 0 \quad \begin{array}{l} n_1 < 0 ; n_1 > (N_1-1) \\ n_2 < 0 ; n_2 > (N_2-1) \end{array} \quad (D.14)$$

We call this finite sequence $x(n_1, n_2)$. Although $x(n_1, n_2)$ is no longer periodic, its value within a period is identical to that of its periodic counterpart and thus the DFT/IDFT representation provides an exact and convenient means for obtaining Fourier transform coefficients. The primary difference between $x(n_1, n_2)$ and the periodic sequence $x_p(n_1, n_2)$ is that the Fourier transform of $x(n_1, n_2)$ is a continuous function of ω_1 and ω_2 , whereas the Fourier transform of $x_p(n_1, n_2)$ is strictly a line spectrum at the appropriate frequencies.

```

0001 FTN4.L
0002 C
0003 C*****
0004 C
0005     PROGRAM SRG1
0006 C
0007 C
0008 C     THIS PROGRAM IS USED TO EXTRACT "PACKED" AZIMUTH,
0009 C     AMPLITUDE AND PHASE DATA FROM A DESIGNATED SCAN OF
0010 C     A FILE CREATED BY "RASTER SCAN". THE DATA ARE
0011 C     THEN UNPACKED AND TRANSFERED TO A FILE WHICH CAN
0012 C     SUBSEQUENTLY BE COPIED ONTO MAGNETIC TAPE AND PASSED
0013 C     TO THE SIGMA-9 COMPUTER.
0014 C
0015 C     NOTE THAT ALL INFORMATION IS PASSED IN INTEGER FORM,
0016 C     WHICH LIMITS THE USER TO A TWO-DECIMAL ACCURACY.
0017 C
0018 C     NOTE ALSO THAT THE TRANSFER FILENAME MUST CONTAIN
0019 C     SIX CHARACTERS.
0020 C
0021 C*****
0022 C
0023 C
0024     COMMON IVTEK(170),ISCOM(100),ITGN(128),IBMAX(128),
0025     COMMON IDFM(128)
0026     DIMENSION IRDCB(144),IRBUF(128),
0027     $MYCOM(128),
0028     $MYREC(2),MYWRD(2),MYBIT(2),
0029     $MYTYP(3),MYNAM(3),MYFMT(3),
0030     $NSFMT(3),NNAMS(2),NFMTS(2),IPTR(2),
0031     $IMOVD(2),
0032     $NAME(3),MLEFT(30)
0033     DIMENSION LESS1(17),IPWR2(14),INF1(1),INF3(1)
0034 C
0035     DIMENSION NX1(1802),IDCB1(144),NAM1(3),ISIZE(2)
0036 C
0037 C     NAME OF FILE TO BE PROCESSED
0038 C
0039 C
0040     WRITE(1,5)
0041     5 FORMAT(2X,"ENTER NAME OF FILE TO BE PROCESSED",/)
0042     READ(1,6) ISCOM(11),ISCOM(12),ISCOM(13)
0043     6 FORMAT(3A2)
0044 C
0045 C     WE NOW CALL THE SUBROUTINE DGET WHICH WILL
0046 C     EXTRACT THE AMPLITUDE AND PHASE INFORMATION
0047 C     FROM A DESIGNATED SCAN IN THE DATA FILE OF
0048 C     INTEREST
0049 C
0050 C     NOTE THAT ALL DATA COLLECTED WILL BE CONVERTED
0051 C     INTO INTEGER NUMBERS SO THAT THE CREATED FILE
0052 C     (ARRAY) WILL BE COMPATIBLE WITH SIGMA-9
0053 C     PROCESSING.
0054 C
0055 C     WE FIRST EXTRACT THE AZIMUTH DATA
0056 C

```

```

0057 C A.  INITIALIZATION
0058 C
0059     INST=1
0060     P1=1.0
0061     P2=1.0
0062     INAM=2HAA
0063     IDIR=1
0064     ISTAT=0
0065     CALL DGET(INST,LERR,P1,P2,INAM,IDIR,ISTAT)
0066 C
0067 C B.  RETURN AZIMUTH DATA
0068 C
0069     ALIM=90.0
0070     DO 30 I=1,1802
0071     INST=2
0072     CALL DGET(INST,LERR,P1,P2,IEOS,IDIR,ISTAT)
0073     IF(P1.GT.ALIM)GO TO 10
0074     NP1=IFIX(P1*100.0+0.5)
0075     NX1(I)=NP1
0076     GO TO 20
0077 10 P11=P1-360.0
0078     NP11=IFIX(P11*100.0+0.5)
0079     NX1(I)=NP11
0080 20 IF(IEOS.EQ.1)GO TO 40
0081 C
0082 30 CONTINUE
0083 C
0084 40 IK=I
0085 C
0086 C
0087 C     THE TRANSFER FILE "SGTF1" IS NOW CREATED
0088 C AND THE DATA CONTAINED IN "NX1" IS PASSED INTO IT.
0089 C
0090 C
0091 C     THE TRANSFER FILE "XXXXX" (GIVE IT A NAME) IS NOW
0092 C CREATED AND THE DATA CONTAINED IN "NX1" IS PASSED
0093 C INTO IT.
0094 C
0095     WRITE(1,41)
0096 41 FORMAT(2X,"NAME YOUR TRANSFER FILE (6 CHARACTERS)",/)
0097     READ(1,42) NAM1(1),NAM1(2),NAM1(3)
0098 42 FORMAT(3A2)
0099 C
0100 C
0101     ISIZE(1)=-1
0102     ITYPE=3
0103     ICR=3
0104     ISEC=0
0105 C
0106     CALL CREAT(IDCBI,IERR,NAM1,ISIZE,ITYPE,ISEC,ICR)
0107     IF(IERR.LT.0)STOP1
0108 C
0109 C     THE DATA IS TRANSFERRED
0110 C
0111     CALL WRITF(IDCBI,IERR,NX1,IK)
0112     IF(IERR.LT.0)STOP2

```



```

0113 C
0114 C
0115 DO 70 I=1,IK
0116 NX1(I)=0
0117 70 CONTINUE
0118 C
0119 C WE THEN OBTAIN THE AMPLITUDE INFORMATION
0120 C NOTE THAT THE SYSTEM MUST BE INITIALIZED AGAIN
0121 C
0122 C C. RETURN AMPLITUDE DATA
0123 C
0124 INST=1
0125 P1=1.0
0126 P2=1.0
0127 INAM=2HAA
0128 IDIR=1
0129 ISTAT=0
0130 CALL DGET(INST,LERR,P1,P2,INAM,IDIR,ISTAT)
0131 C
0132 DO 80 I=1,IK
0133 INST=2
0134 CALL DGET(INST,LERR,P1,P2,IEOS,IDIR,ISTAT)
0135 NP2=-(IFIX(P2*100.0))
0136 NX1(I)=NP2
0137 80 CONTINUE
0138 C
0139 CALL WRITE(IDCBI,IERR,NX1,IK)
0140 IF(IERR.LT.0)STOP3
0141 C
0142 C
0143 DO 100 I=1,IK
0144 NX1(I)=0
0145 100 CONTINUE
0146 C
0147 C WE NOW EXTRACT THE PHASE INFORMATION
0148 C PLEASE NOTE THAT A SYSTEM RE-INITIALIZATION
0149 C IS REQUIRED.
0150 C
0151 C D. RE-INITIALIZATION
0152 C
0153 INST=1
0154 P1=1.0
0155 P2=1.0
0156 INAM=2HAA
0157 IDIR=1
0158 ISTAT=0
0159 CALL DGET(INST,LERR,P1,P2,INAM,IDIR,ISTAT)
0160 C
0161 C E. RETURN PHASE DATA
0162 C
0163 DO 110 I=1,IK
0164 INST=2
0165 CALL DGET(INST,LERR,P1,P2,IEOS,IDIR,ISTAT)
0166 IP2=IFIX(P2*100.0)
0167 NX1(I)=IP2
0168 110 CONTINUE

```

```
0169 C
0170 CALL WRITE(IDCB1,IERR,NX1,IK)
0171 IF(IERR.LT.0)STOP4
0172 C
0173 C
0174 C WE THEN TERMINATE THE EXTRACTION PROCESS
0175 C
0176 INST=3
0177 CALL DGET(INST)
0178 C
0179 CALL LOCF(IDCB1,IERR,IWC,IWB,IOFF,JSC)
0180 IF(IERR.LT.0)STOP5
0181 ITRUN=JSC/2-IWB-1
0182 CALL CLOSE(IDCB1,IERR,ITRUN)
0183 IF(IERR.LT.0)STOP6
0184 C
0185 STOP
0186 END
```

```
** NO ERRORS** PROGRAM = 03019 COMMON = 00654
```

```

C
C*****
C
C          PROGRAM CONVERSION      (FORT4: SCONVERT)
C
C*****
C
C  THIS PROGRAM IS USED TO DECODE THE INFORMATION PUT
C  ON MAGNETIC TAPE BY THE HEWLETT-PACKARD 21MX COMPUTER.
C
C  THIS INFORMATION CONTAINS AZIMUTH, AMPLITUDE AND PHASE
C  DATA (MAX. OF 1801 POINTS EACH) OBTAINED THROUGH A
C  A RASTER SCAN ON THE SA2020 ANTENNA ANALYZER SYSTEM.
C  IT MAY ALLOW A USER TO DEFINE THE RADIATION PATTERN
C  CHARACTERISTICS OF AN ANTENNA UNDER TEST.
C
C  NOTE THAT THE SIGMA-9 COMPUTER WILL READ THE HP MAG
C  TAPE AND DUMP IT ON A DISC FILE CALLED "STORAGE".
C  THEREFORE BEFORE THIS PROGRAM IS USED, A NAME ATTRIBU-
C  TION (SET COMMAND) HAS TO BE PERFORMED IN THE FOLLOWING
C  MANNER:
C
C          SET F:8/STORAGE:IN
C          SET F:200/MTAPE:OUT
C
C  IF MORE THAN ONE FILE HAS TO BE PROCESSED, THE "SET"
C  COMMANDS WILL INCLUDE ADDITIONAL STEPS SUCH AS:
C
C          SET F:(199+x)/MTAPE:OUT
C
C  WHERE x IS THE NUMBER (OR SEQUENCE) OF THE FILE TO BE
C  PROCESSED.
C
C  THIS PROGRAM WILL RUN ON THE COMMAND:
C
C          RUN SCONVERT
C
C*****
C
C  DIMENSION IDATA(901), IHALF(1802), DATA(1802)
C  WRITE(108,1)
C  1  FORMAT(/,/,2X,'ENTER THE TOTAL NUMBER OF FILES TO BE'
C  2  'PROCESSED',/)
C  READ(105,2)NF
C  2  FORMAT(G)
C  DO 66 I=1,NF
C  WRITE(199+IC,10)IC
C  10  FORMAT(/,/,2X,'FILE NAME: MTAPE',I1,/)
C  N=901
C
C  THE SIGMA-9 SUBROUTINE "BUFFER IN" IS CALLED
C
C  DO 65 I=1,3
C  CALL BUFFER IN(6,1,IDATA,N,ISTAT,NUM)
C  50  GO TO(50,60,70,80),ISTAT
C
C  ONE WORD OF THE SIGMA-9 (32 BITS) ACTUALLY EQUALS

```

C TWO WORDS OF THE HP 21MX COMPUTER (16 BITS).
 C THEREFORE THE SIGMA-9 WORD HAS TO BE SPLIT IN HALF
 C IN ORDER FOR THE ORIGINAL INFORMATION TO BE
 C RETAINED.

00 L=1
 DO 01 IA=1,NUM
 IHALF(L)=ISL(IDATA(IA),-16)
 IHALF(L+1)=IAND(IDATA(IA),4ZFFFF)
 L=L+2
 01 CONTINUE

C
 C NWDS=L-2

C
 C WE NOW CHECK FOR THE SIGN OF THE INFORMATION PASSED
 C (BIT #15)

C
 DO 03 K=1,NWDS
 ISGN=IAND(IHALF(K),478000)
 IF (ISGN.NE.0) GO TO 02
 DATA(K)=IHALF(K)/100.0
 GO TO 03
 02 DATA(K)=-((IEOR(IHALF(K),820000FFFF)+1)/100.0
 03 CONTINUE

C
 C THE DATA IS OUTPUTED ON FILE

45 WRITE(199+JL,04)(DATA(K),K=1,NWDS)
 04 FORMAT(8F9.2)
 05 CONTINUE
 06 CONTINUE
 GO TO 100
 70 WRITE(108,71)
 71 FORMAT(/,/,/,20X,"END OF PROCESSING",/,/,/) --
 GO TO 100
 80 WRITE(108,81)
 81 FORMAT(/,2X,"ERROR",/)
 100 STOP
 END

```

C
C*****
C
C          PROGRAM FIELD          (FORT4:SFIELD)
C
C*****
C
C  THIS PROGRAM USES THE PRINCIPLE OF PATTERN MULTIPLICATION
C  IN ORDER TO CHARACTERIZE COMPLETELY THE FRONT HEMISPHERE
C  OF THE NEAR-FIELD PROBE BASED ON COMPLEX DATA (AMPLI-
C  TUDES AND PHASES) FROM ITS TWO MAIN POLAR CUTS.
C  WE ARE OBVIOUSLY ASSUMING THAT THESE TWO CUTS MOSTLY
C  DETERMINE THE OVERALL PATTERN OF THE PROBE: THIS IS TRUE
C  IN THE CASE OF MOST OPEN-WAVEGUIDE AND SMALL HORN
C  ANTENNAS.
C
C  THIS PROGRAM CAN PROCESS UP TO 1801 SAMPLE POINTS IN EACH
C  POLAR CUT, WHICH IS EQUIVALENT TO DEFINING THE FRONT
C  HEMISPHERE OF THE PROBE (-90 TO +90 DEGREES) EVERY 0.1
C  DEGREE.
C
C  DUE TO THE ENORMOUS SIZE OF STORAGE REQUIRED IN PROCESSING
C  AN 1801X1801 ARRAY, A SPECIAL DATA FILE HAD TO BE CREATED
C  IN THIS ACCOUNT EXCLUSIVELY FOR THAT PURPOSE. THIS DATA FILE IS
C  CALLED "FLDDATA" AND WILL CONTAIN ALL FINAL RESULTS. THIS FILE
C  IS NOT ACCESSIBLE BY EDIT SINCE IT IS WRITTEN ENTIRELY IN BINARY;
C  IT WILL BE HOWEVER ACCESSED BY THE PROGRAM "SCOEF" WHICH CALCUL-
C  ATES THE PROBE AMPLITUDE WEIGHTING FUNCTIONS.
C
C  THE FILE "FLDDATA" SHOULD NOT BE DELETED!!!!!!!
C
C  BEFORE USING "FIELD", THE COMMAND
C
C          BEGIN.
C
C  SHOULD BE GIVEN. THIS WILL IMPLEMENT THE FOLLOWING SETTINGS:
C
C          SET F:99/FLDDATA;RANDOM;OUTIN;SAVE
C          SET F:201/MTAPE1;IN
C          SET F:202/MTAPE2;IN
C
C  THIS PROGRAM WILL RUN ON THE COMMAND:
C
C          RUN SFIELD
C
C  NOTE: IT IS ASSUMED HERE THAT THE PROGRAMS "SRG1" (ON THE
C  SA2020 ANTENNA ANALYZER) AND "SCONVERSION" (ON THE SIGMA-9)
C  HAVE PREVIOUSLY BEEN RUN SUCCESSFULLY.
C
C*****
C
C  COMPLEX*8 ETHETA(1801),E1PHI(1801),FIELD
C
C  DIMENSION PHI(1801),VAP(1801),PHF(1801),PH11(1801),

```

```

2VAP1(1801),PHP1(1801),XXR(1801),XXI(1801),XXR1(1801),
3XXI1(1801)
R=57.29577951

```

```

C
C   AE INPUT THE MEASURED DATA: ANGLE, AMPLITUDE AND PHASE
C   VALUES. PHI, VAP, PHP ARE FOR THE AZIMUTH CUT, WHILE
C   THE PRIME NOTATION APPLIES TO THE CUT IN ELEVATION.
C
WRITE(108,5)
5 FORMAT(/,/,5X,"INPUT THE NUMBER OF SAMPLES TAKEN PER
3"CUT",/)
READ(105,6)NP
6 FORMAT(G)

```

```

C
READ(201,10)(PHI(I),I=1,NP)
10 FORMAT(8F9.2)
READ(201,11)(VAP(I),I=1,NP)
11 FORMAT(8F9.2)
READ(201,12)(PHP(I),I=1,NP)
12 FORMAT(8F9.2)
READ(202,13)(PHI1(I),I=1,NP)
13 FORMAT(8F9.2)
READ(202,14)(VAP1(I),I=1,NP)
14 FORMAT(8F9.2)
READ(202,15)(PHP1(I),I=1,NP)
15 FORMAT(8F9.2)

```

```

C
TEMP=VAP(1)
DO 30 I=2,NP
TEMPX=VAP(I)
IF(TEMPX.GE.TEMP)GO TO 30
TEMP=TEMPX
30 CONTINUE
DIFF=TEMP
TEMP=VAP1(1)
DO 40 I=2,NP
TEMPX=VAP1(I)
IF(TEMPX.GE.TEMP)GO TO 40
TEMP=TEMPX
40 CONTINUE
DIFF1=TEMP

```

```

C
DO 50 I=1,NP
VAP(I)=-VAP(I)+DIFF
VAP1(I)=-VAP1(I)+DIFF1
50 CONTINUE
DO 60 I=1,NP
XXR(I)=(10.**((VAP(I)/10.))*COS(PHP(I)/R)
XXI(I)=(10.**((VAP(I)/10.))*SIN(PHP(I)/R)
XXR1(I)=(10.**((VAP1(I)/10.))*COS(PHP1(I)/R)
XXI1(I)=(10.**((VAP1(I)/10.))*SIN(PHP1(I)/R)
ETHETA(I)=CMPLX(XXR(I),XXI(I))
E1PHI(I)=CMPLX(XXR1(I),XXI1(I))
60 CONTINUE

```

```

C
IREC1=0
DO 70 I=1,NP
DO 70 J=1,NP
FIELD=E1PHI(I)*ETHETA(J)
WRITE DISK 99,IREC1,FIELD

```

```
IREC1=IREC1+2
70 CONTINUE
REWIND 99
DO 80 J=1,NP
IREC1=0+(J*2-2)
IREC2=0+(J*2-2)
DO 80 I=1,NP
READ DISK 99,IREC1,FIELD
FIELD=FIELD*ETHETA(J)
```

```
C
C THE DATA IS OUTPUTED ON DISC
C
```

```
WRITE DISK 99,IREC2,FIELD
IREC1=IREC1+(NP*2)
IREC2=IREC2+(NP*2)
80 CONTINUE
```

```
C
REWIND 99
STOP
END
```

```

C
C*****
C
C      PROGRAM COEF      (FORT4: SCOE)
C
C*****
C
C      THIS PROGRAM USES MEASURED AND CALCULATED AZIMUTH,
C      AMPLITUDE AND PHASE VALUES OF THE NEAR-FIELD PROBE
C      RADIATION PATTERN IN ORDER TO CALCULATE THE AMPLITUDE
C      WEIGHTING FUNCTIONS WHICH WILL MATHEMATICALLY DEFINE
C      IT AND THEREBY ALLOW US TO COMPENSATE FOR ITS EFFECTS
C      DURING NEAR-FIELD MEASUREMENTS.
C
C
C      IT IS ASSUMED HERE THAT THE PROGRAMS "SRG1" (ON THE
C      SA2020 ANTENNA ANALYZER), "SCONVERSION" AND "SFIELD"
C      (ON THE SIGMA-9) HAVE BEEN PREVIOUSLY RUN SUCCESSFULLY.
C
C      BEFORE USING "COEF", THE COMMAND:
C
C          COEFBEGIN.
C
C      SHOULD BE GIVEN. THIS WILL IMPLEMENT:
C
C          !SET F:99/FLDDATA;RANDOM;IN
C          !SET F:201/MTAPE1;IN
C          !SET F:202/MTAPE2;IN
C          !SET F:300/PARAM1;OUT
C          !SET F:301/DCOE;OUT
C          !SET F:302/C1COEF;OUT
C
C      THIS PROGRAM WILL RUN ON THE COMMAND:
C
C          SCOEFX.
C
C      NOTE THAT THE DATA FROM THE H-PLANE CUT (COPOLAR) IS
C      STORED INTO MTAPE1 AND THAT OF THE E-PLANE CUT (COPOLAR)
C      INTO MTAPE2.
C
C*****
C
C      COMPLEX*8 EFLD(1801),FUNC(1801),A(7,7),RSIDE(7),
C      2WA(63),WK(7),DENOM,DEN1,FIELD,TEGRAL
C
C      COMMON PHI(1801),PHI1(1801)
C
C      WRITE(108,1)
C      1  FORMAT(/,5X,'THIS ROUTINE IS USED TO EVALUATE THE NEAR-
C      2'FIELD',/,5X,'PROBE COEFFICIENTS BASED ON ITS MEASURED
C      3'RADIATION PATTERN.',/)

```



```

WRITE(108,2)
2  FORMAT(/,/,5X,'INPUT THE NUMBER OF SUMMATIONS REQUIRED '
3  'IN DEFINING ',/,3X,'THE NEAR-FIELD PROBE (HINT: N=ka) ',
4  /,/)
  READ(108,3)NA
3  FORMAT(G)
  WRITE(108,4)
4  FORMAT(/,5X,'ENTER ALSO THE FREQUENCY AT WHICH THE POLAR '
5  'CUTS ',/,3X,'WERE PERFORMED, BOTH THE START AND FINISH '
6  'ANGLES (IN DEGREES) ',/,3X,'OF THE CUTS, THE INCREMENT '
7  '(IN DEG.) BETWEEN SAMPLES, AND THE TOTAL NUMBER ',/,3X,
8  'OF POINTS SAMPLED.',/,/)
  READ(105,5)F,STA,FIN,INC,NP
5  FORMAT(5G)

```

```

C
  R=57.29577951
  N1=-NA
  AINC=INC/R
  IR=0
20 IR=IR+1
  READ(202-IR,20)(PHI1(I),I=1,NP)
20 FORMAT(G)
  READ(201+IR,30)(PHI(I),I=1,NP)
30 FORMAT(G)
  WRITE(300,31)N1,NA,INC,NP
31 FORMAT(4G)

```

```

C
C   THE NUMBER OF POINTS TREATED (NP) IS CHECKED HERE SO
C   THAT IX=(NP-1)/3 RESULTS IN AN INTEGER. IF THE INITIAL
C   VALUE OF IX IS NOT AN INTEGER, NP IS INCREASED UP UNTIL
C   IT SATISFIES THE ABOVE CONDITION.

```

```

C
C   THIS IS MADE NECESSARY BECAUSE OF THE USE OF SIMPSON'S
C   SECOND RULE.

```

```

C
C
  AB=0.0
  IX=(NP-1)/3
  IDIFF=(IX*3)+1
  IF(IDIFF.EQ.NP)GO TO 33
  NP=NP+1
  IX=(NP-1)/3
  IDIFF=(IX*3)+1
  IF(IDIFF.EQ.NP)GO TO 32
  NP=NP+1
  IX=(NP-1)/3
  IDIFF=(IX*3)+1
  PHI(NP-1)=0
  EFLD(NP-1)=CMPLX(AB,AB)
32 PHI(NP)=0
  EFLD(NP)=CMPLX(AB,AB)

```

```

C
C
C   WE FIRST FORM A COMPLEX MATRIX OF LINEAR COEFFICIENTS
C   A(IM,IN) BY EVALUATING THE EXPRESSION SIN((IM-IN)*APHI1)/
C   ((IM-IN)*APHI1), WHERE APHI1 IS THE LIMIT OF INTEGRATION
C   IN AZIMUTH.

```

```

33 ABI=0.0
  ABR=1.0

```

```
K=NA+1
IREC1=0
```

```
C
DO 100 IKK=1,NP
C
DO 35 I=1,NP
READ DISC 99,IREC1,FIELD
EFLD(I)=FIELD
IREC1=IREC1+2
35 CONTINUE
C
DO 50 IM=N1,NA
DO 50 IN=N1,NA
T=(IM-IN)*APHI1
IF(T.NE.0)GO TO 40
A(IM+K,IN+K)=CMPLX(ABR,ABI)
GO TO 50
40 TO=SIN(T)/T
A(IM+K,IN+K)=CMPLX(TO,ABI)
50 CONTINUE
```

```
C
C WE THEN EVALUATE THE "RIGHT SIDE" OF OUR LINEAR EQUATIONS
C SYSTEM. IT CONSISTS OF THE INTEGRAL OF A FUNCTION
C  $F(X)=FIELD*EXP(-i*IM*X)$ , WHERE FIELD IS COMPLEX,
C POSITION-DEPENDENT, AND IS CONSECUTIVELY EQUAL TO THE
C ETHETA AND EIPHI VALUES OBTAINED PREVIOUSLY BY EXTRAPOLA-
C TION USING THE PROGRAM "FIELD".
```

```
C
C THIS NUMERICAL INTEGRAL USES SIMPSON'S SECOND RULE
```

```
C
DEN1=CMPLX(ABI,ABR)
THETA=(90.0-PHI1(IKK))/R
DO 60 IM=N1,NA
DENOM=1/((2*(DEN1*(IM+1-IR))*SIN(THETA))
CALL SIMPSONS(EFLD,AINC,IM,TEGRAL)
RSIDE(IM)=TEGRAL*DENOM
60 CONTINUE
```

```
C
C IT IS NOW POSSIBLE TO SOLVE FOR THE PROBE AMPLITUDE
C WEIGHTING COEFFICIENTS AT THE ELEVATION ANGLE THETA.
```

```
C
N=(2*NA)+1
IA=(2*NA)+1
M=1
IB=(2*NA)+1
IJOB=0
CALL LEQ2C(A,N,IA,RSIDE,M,IB,IJOB,WA,WK,IER)
```

```
C
C THE VALUES OF THE COMPLEX AMPLITUDE WEIGHTING FUNCTIONS
C CHARACTERIZING THE PROBE ARE RETURNED BY THE SUBROUTINE
C "LEG2C" INTO THE ARRAY RSIDE(IM).
```

```
C
IF(IER.EQ.129)GO TO 70
IF(IER.EQ.130)GO TO 80
GO TO 90
70 WRITE(108,75)
75 FORMAT(/,/, 'MATRIX A IS ALGORITHMICALLY SINGULAR',/,/)
GO TO 90
80 WRITE(108,85)
85 FORMAT(/,/,5X, 'MATRIX A IS ILL-CONDITIONED!! DOES NOT
```

```

      8' CONVERGE!!! ',/,/)
      90 WRITE(301+IR,95)(RSIDE(IM),IM=N1,NA) ~
      95 FORMAT(14G)

```

C
C

```

      IREC1=IREC1+(NP*2)
      100 CONTINUE

```

C
C

```

      IF(IR.NE.0)GO TO 110
      WRITE(108,105)
      105 FORMAT(/,5X,'THE COEFFICIENTS FOR THE FIELD ETHETA ARE '
      7'NOW',/,4X,'STORED IN FILE DCOEF (SET AT F:301)',/)
      WRITE(108,106)
      106 FORMAT(/,/,/)

```

C

```

      GO TO 10
      110 WRITE(108,111)
      111 FORMAT(/,5X,'THE COEFFICIENTS FOR THE FIELD E1PHI ARE '
      8',/,4X,'STORED IN FILE C1COEF (SET AT F:302) ',/),
      STOP
      END

```

C

C*****

C

SUBROUTINES

C

C*****

C

```

      SUBROUTINE SIMPSONS(EFLD,AINC,IM,TEGRAL)

```

C

```

      COMPLEX*8 EFLD(1801),FUNC(1801),TEG,TTEG,TEGRAL
      COMMON PHI(1801),PHI1(1801)

```

C

```

      AB=0
      R=57.29577951

```

C

```

      DO 10 KK=1,NP
      EXXR=REAL(EFLD(KK))
      EXXI=AIMAG(EFLD(KK))
      FR=EXXR*COS(IM*PHI(KK)/R)+EXXI*SIN(IM*PHI(KK)/R)
      FI=EXXI*COS(IM*PHI(KK)/R)-EXXR*SIN(IM*PHI(KK)/R)
      FUNC(KK)=CMPLX(FR,FI)

```

```

      10 CONTINUE

```

C

```

      TEGRAL=CMPLX(AB,AB)
      I=1
      J=2
      K=3
      DO 20 L=4,NP,3
      TEG=FUNC(I)+(3*FUNC(J))+(3*FUNC(K))+FUNC(L)
      TTEG=(3*AINC/8)*TEG
      TEGRAL=TEGRAL+TTEG
      I=I+3
      J=J+3
      K=K+3
      20 CONTINUE
      RETURN
      END

```

APPENDIX I: NEAR

PROGRAM NEAR

(FORT4;SNEAR)

THIS PROGRAM IS USED TO EXTRAPOLATE THE FAR-FIELD PATTERN OF AN ARBITRARY TEST ANTENNA BASED ON PROBE-COMPENSATED NEAR-FIELD MEASUREMENTS.

IT IS ASSUME HERE THAT THE PROGRAMS "SRG1", "CONVERSION", "FIELD", AND "COEF" HAVE ALL BEEN RUN SUCCESSFULLY.

BEFORE THIS PROGRAM IS RUN, THE COMMAND:

!NEARBEGIN.

MUST BE GIVEN. THIS WILL IMPLEMENT:

```
!SET F:99/DATAMAIN;IN
!SET F:300/PARAM1;IN
!SET F:301/DCOEF;IN
!SET F:302/C1COEF;IN
!SET F:401/BN;OUT
!SET F:402/AN;OUT
```

THE PROGRAM "NEAR" WILL RUN ON THE COMMAND:

!SNEARX.

IT SHOULD BE NOTED HERE THAT THE FILE "DATAMAIN" CONTAINS THE MEASURED AZIMUTH, AMPLITUDE AND PHASE DATA OBTAINED FROM THE SA2020.

THE PROGRAM WILL FIRST READ THE DATA PERTAINING TO WHEN THE PROBE IS IN IT'S ORIGINAL ORIENTATION. THEN IT WILL READ THE DATA PERTAINING TO THE PROBE ROTATED BY 90 DEGREES, BEFORE PROCEEDING TO THE NEXT ELEVATION INTERVAL.

CARE SHOULD THEN BE PAID TO ENSURE THAT THE DATA IS STORED IN "DATAMAIN" IN THE RIGHT FASHION.

THE FILE "DATAMAIN" ACTUALLY CONSISTS OF ALL "MTAPE" FILES CREATED BY THE PROGRAM "CONVERSION" WHEN THE NEAR-FIELD DATA IS PASSED INTO THE SIGMA-9.

THE FIELDS EXTRAPOLATED VALUES, I.E. ETHETA AND EPHI, WILL BE CONTAINED IN FILES "BN" AND "AN" RESPECTIVELY.

```
COMPLEX*8 VOLT(150,50),DATA(150),TRANSFORM(150),WORK(150),
2HANKY(-25:24,-5:5),PCOEF(-5:5),DET(-25:24),HANK,
```

3 3
OF / DE

3SUM,COMPL,COMPL1,CONST

DIMENSION XXR(150),XXI(150),EXXR(150),EXXI(150),YXXR(150),
4YXXI(150),ARG(150),VA(150,50),VPH(150,50),VA1(150,50),
5VPH1(150,50)

WRITE(108,980)
980 FORMAT(/,/,5X,'ENTER THE FREQUENCY OF THE TEST IN GHZ',/,/)
READ(105,981)F
981 FORMAT(G)
WRITE(108,982)
982 FORMAT(/,/,5X,'ENTER THE RADIUS (in meters) OF THE'
4' MEASUREMENT CYLINDER',/,/)
READ(105,983)R0
983 FORMAT(G)
WRITE(108,984)
984 FORMAT(/,/,5X,'ENTER THE VERTICAL'
7'DISTANCE (also in meters) BETWEEN THE POINT IN',/,5X,
8'ELEVATION AT WHICH THE FIRST ROW OF SAMPLES'
9' WERE TAKEN AND THE',/,5X,'LONGITUDINAL AXIS'
5' "X" OF THE TEST ANTENNA (theta=90 ; Phi=0)',/,/)
READ(105,985)Z
985 FORMAT(G)
WRITE(108,986)
986 FORMAT(/,/,5X,'ENTER THE SAMPLING INTERVAL IN'
2'ELEVATION DZ (in cm)',/,5X,'AND IN AZIMUTH'
3' IDPH (in deg)',/,/)
READ(105,987)DZ1,IDPH
987 FORMAT(2G)
WRITE(108,988)
988 FORMAT(/,/,5X,'ENTER FINALLY THE TOTAL NUMBER OF'
4'SAMPLE STEPS',/,5X,'DONE IN ELEVATION',/)
READ(105,989)NP
989 FORMAT(G)
PI=3.14159265358979
R=57.29577951
DZ=DZ1*1E-2
DPH=IDPH*2*PI/360.0
NP=2*PI/DPH
DH=2*PI/(M*DZ)
C=3.0E8
F=F*1E9
WL=C/F
AK=2*PI/WL
TANG=ATAN(Z/R0)
TTANG=90.0-(TANG*R)
WRITE(108,4)TTANG
4 FORMAT(/,5X,'ENTER THE VERTICAL CUT LIMIT ANGLE'
2'"THETA CRITICAL".',/,5X,'NOTE THAT THIS ANGLE HAS TO'
3'BE GREATER THAN:',F6.2,/)
READ(105,5)TTAC
5 FORMAT(G)
THETAC=TTAC/P
IF(TTAC.GT.TTANG)GO TO 7
WRITE(108,6)
6 FORMAT(/,/,/,10X,'ERROR',/,/,10X,'CRITICAL ANGLE'
4'TOO SMALL!!!',/,/)
GO TO 285
7 NRO=-NP/2

```

NTEST=NP/2
NDIF=NP-NTEST
IF((NDIF*2).EQ.NP)GO TO 997
NN1=NP/2
GO TO 998
997 NN1=(NP/2)-1
998 MM0=-MP/2
MTEST=MP/2
MDIF=MP-MTEST
IF((MDIF*2).EQ.MP)GO TO 999
MM1=MP/2
GO TO 8
999 MM1=(MP/2)-1

```

C
C

8. $IR=0$

CC

WE FIRST INPUT THE NEAR-FIELD DATA (MEASURED AMPLITUDE AND PHASE VALUES) OBTAINED FROM THE SA2020 ON THE MEASUREMENT CYLINDER.

C
C

THIS DATA SHOULD PRESENTLY BE STORED IN THE FILE ENTITLED
"DATA MAIN", SET AT F:99

C

DO 65 I=1,MP

C

DO 10 J=1,NP

```
10 READ(99,11)PHI
```

```
11 FORMAT(G)
```

C

DO 20 J=1,NP

```
20 READ(99,21)VA(I,J)
```

```
21  FORMAT(G)
```

C

DO 30 J=1,NP

```
30 READ(99,31)VPH(I,J)
```

31 FORMAT(G)

C

DO 40 J=1,NP

40 READ(99,41)PHI

41 FORMAT(G)

C

DO 50 J=1,NP

```
50 READ(99,51)VA1(I,J)
```

51 FORMAT(G)

३

DO 60 J=1,NP

```
60 READ(99,61)VPH1(I,J)
```

```
61 FORMAT(G)
```

C

65 CONTINUE

C
C

```

70 IF (IR.EQ.1) GO TO 85

```

```
WRITE(108,75)
```

```

75 FORMAT(//,2X,' WE ARE FIRST CALCULATING THE "ETHETA" '
5'COMPONENT OF THE ',//,2X,'TEST ANTENNA FAR-FIELD '
6'PATTERN. ',//,/)

```

C

DC 80 I=1,MP

```

DO 80 J=1, NP
XR=(10.0**((VA(I,J)/10.0))*COS(VPH(I,J)/R)
XI=(10.0**((VA(I,J)/10.0))*SIN(VPH(I,J)/R)
VOLT(I,J)=CMPLX(XR,XI)
80 CONTINUE

C
GO TO 95

C
85 WRITE(108,86)
86 FORMAT(/,/,2X,'WE ARE NOW CALCULATING THE "EPHI"
7"COMPONENT OF THE ',/,2X,'TEST ANTENNA FAR-FIELD
8"PATTERN. ',/,/)

C
DO 90 I=1, MP
DO 90 J=1, NP
XR=(10.0**((VA1(I,J)/10.0))*COS(VPH1(I,J)/R)
XI=(10.0**((VA1(I,J)/10.0))*SIN(VPH1(I,J)/R)
VOLT(I,J)=CMPLX(XR,XI)
90 CONTINUE

C
WE ARE FIRST LOW-PASS FILTERING THE DATA TO INCREASE THE
C RESOLUTION OF THE CALCULATED FIELDS IN THE ELEVATION REGION
C DELIMITED BY -THETAC < THEIA < THETAC
C
C
95 IMC=M*DZ*COS(THETAC)/WL+0.5
DZT=WL/(2*COS(THETAC))

C
CALL LPF(MP,NP,DZ,IMC,DZT,VOLT)

C
ONCE THE DATA IS PROPERLY "FILTERED", WE CAN EASILY
C PERFORM THE TWO-DIMENSIONAL FFT.
C
C
THIS 2D-FFT IS ACHEIVED USING THE SUBROUTINE "FOURG"
C WHICH PROCESSES ONE DIMENSION OF THE DATA ARRAY AT
C A TIME.
C
AB=0.0
DO 100 J=1, NP
DATA(J)=CMPLX(AB,AB)
TRANSFORM(J)=CMPLX(AB,AB)
WORK(J)=CMPLX(AB,AB)
100 CONTINUE

C
ISIGN=-1
N=NP
DO 130 I=1, MP
DO 110 J=1, NP
110 DATA(J)=VOLT(I,J)
CALL FOURG(DATA,N,ISIGN,WORK)
DO 120 J=1, NP
120 VOLT(I,J)=DATA(J)
130 CONTINUE

C
C
DO 135 I=1, MP
DATA(I)=CMPLX(AB,AB)
TRANSFORM(I)=CMPLX(AB,AB)
WORK(I)=CMPLX(AB,AB)
135 CONTINUE

```


C
C

```

      ISIGN=1
      N=MP
      DO 160 J=1,NP
      DO 140 I=1,MP
140   DATA(I)=VOLT(I,J)
      CALL FOURG(DATA,N,ISIGN,WORK)
      DO 150 I=1,MP
150   VOLT(I,J)=DATA(I)/N
160   CONTINUE

```

C
C
C
C
C
C
C
C
C
C

WE NOW HAVE ALL THE ELEMENTS TO SOLVE FOR THE
AMPLITUDE WEIGHTING FUNCTIONS $A_N(h)$ AND $B_N(h)$
OF THE TEST ANTENNA.

WE FIRST RECALL THE PROBE COEFFICIENTS -- AS
CALCULATED BY THE PROGRAM "COEF" -- IN ORDER TO
COMPENSATE FOR ITS EFFECTS.

```

      READ(300,170)N1,NA,AINCP,NPROB
170   FORMAT(4G)

```

C

```

      DO 210 IM=MM0,MM1
      THETA=ACOS(IM*COS(THETAC)/(MP/2.0))

```

C

```

      THETAT=180.0-(THETA*R)
      T1=THETAT*10.0
      T2=T1+0.5
      IT3=T2
      THETAS=IT3/10.0
      POINT=(THETAS/AINCP)+1.5
      IPOINT=POINT
      REWIND 301,302

```

C

```

      DO 180 KT=1,NPROB
      READ(302-IR,175)(PCOEF(KJ),KJ=N1,NA)
175   FORMAT(2G)
      IF(KT,EQ,IPOINT)GO TO 181
180   CONTINUE

```

C
C
C
C

WE THEN EVALUATE THE HANKEL FUNCTIONS WITH
ARGUMENT ZA AND INDICE NU

```

181   DO 182 IN=NNO,NN1
      DO 182 MI=N1,NA
      HANKY(IN,MI)=CMPLX(AB,AB)
182   CONTINUE
      DEL=AK*SIN(THETA)
      ZA=DEL*R0
      DO 185 IN=NNO,NN1
      DO 185 MI=N1,NA
      CALL HANKEL(ZA,MI,IN,HANK)
      HANKY(IN,MI)=HANK
185   CONTINUE

```

C
C
C
C

WE THEN CALCULATE THE DETERMINANT OF OUR SYSTEM OF
EQUATIONS.

```

DO 195 IN=NN0,NN1
SUM=CMPLX(AB,AB)
DO 190 MI=N1,NA
SUM=PCOEF(MI)*HANKY(IN,MI)+SUM
190 CONTINUE
DET(IN)=-SUM
195 CONTINUE

```

```

C
C IT IS NOW POSSIBLE TO CALCULATE THE AMPLITUDE WEIGHTING
C FUNCTIONS FOR THE FAR FIELD OF THE TEST ANTENNA.
C

```

```

IC=IM+1-MM0
DO 200 IN=NN0,NN1
IK=IN+1-NN0
CONST=(AK**2)/(DET(IN)*(DEL**2))
VOLT(IC,IK)=CONST*(-VOLT(IC,IK))
200 CONTINUE

```

```

C
C
C 210 CONTINUE
C

```

```

C THE FAR-FIELD PATTERN OF THE TEST ANTENNA -- ONE
C COMPONENT AT A TIME -- IS FINALLY OBTAINED
C USING THE FFT ALGORITHM TO SUM ALL AMPLITUDE
C WEIGHTING FUNCTIONS.
C

```

```

BA=1.0
N=MP
ISIGN=1
DO 240 IC=1,MP
IIM=IC+MM0-1
THETAM=ACOS(IIM*COS(THETAC)/(MP/2.0))
DO 220 IK=1,NP
COMPL=CMPLX(AB,BA)
COMPL1=COMPL**(IK+1-IR)
DATA(1K)=VOLT(IC,IK)*COMPL1*SIN(THETAM)
220 CONTINUE
CALL FOURG(DATA,N,ISIGN,WORK)
DO 230 IK=1,NP
230 VOLT(IC,IK)=DATA(1K)/N

```

```

C
C 240 CONTINUE
C

```

```

C
C DO 260 I=1,MP
C IIM=I+MM0-1
C THETAM=(ACOS(IIM*COS(THETAC)/(MP/2.0)))*R
C DO 260 J=1,NP
C IIN=J+NN0-1
C PHIN=(2*PI*IIN/NP)*R
C IF(1R.EQ.1)GO TO 255
C WRITE(401,250)THETAM,PHIN,VOLT(I,J)
250 FORMAT(2X,'ETHETA(',F5.2,',',F5.2,')=',G,
85X,G)
GO TO 260

```

```

C
C 255 WRITE(402,256)THETAM,PHIN,VOLT(I,J)
C 256 FORMAT(2X,'EPhi(',F5.2,',',F5.2,')=',G,

```

95X,G)-
260 CONTINUE

C IF(IR,EQ,1)GO TO 270
WRITE(108,265)
265 FORMAT(/,/,2X,'THE COMPLEX VALUES OF THE "ETHETA"
6'COMPONENT OF THE TEST ',/,2X,'ANTENNA FAR-FIELD'
7'PATTERN ARE NOW STORED ',/,2X,'IN A FILE "BN" SET '
8'AT F:401 ',/,/)
GO TO 280

C 270 WRITE(108,271)
271 FORMAT(/,/,2X,'THE COMPLEX VALUES OF THE "EPHI"
2'COMPONENT OF THE TEST ',/,2X,'ANTENNA FAR-FIELD PATTERN '
3'ARE NOW STORED ',/,2X,'IN THE FILE "AN" , SET '
4'AT F:402 ',/,/)
GO TO 285

C 280 IR=IR+1
GO TO 70

C 285 STOP
END

C *****

C SUBROUTINES

C *****

C SUBROUTINE LPF(MP,NP,DZ,IMC,DZT,VOLT)

C COMPLEX*8 VOLT(150,50),DATA(150),TRANSFORM(150),
2WORK(150)
DIMENSION XXR(150),XXI(150),EXXR(150),EXXI(150),
3YXXR(150),YXXI(150),ARG(150)
AB=0.0
MIL=IMC-1
MIH=MP-IMC+2

C DO 85 J=1,NP

C DO 10 I=1,MP
10 DATA(I)=VOLT(I,J)

C ISIGN=-1
N=MP
CALL FOURG(DATA,N,ISIGN,WORK)

C DO 20 I=1,MIL
20 DATA(I)=CMPLX(AB,AB)
DO 30 I=MIH,MP
30 DATA(I)=CMPLX(AB,AB)

C ISIGN=1
N=MP
CALL FOURG(DATA,N,ISIGN,WORK)

C DO 35 I=1,MP
XXR(I)=REAL(DATA(I)/N)

```

      XXI(I)=AIMAG(DATA(I)/N)
35  CONTINUE
C
      NDIM=MP
      EPS=.0001
      XMAX=DZ*(MP-1)
      DO 50 IK=1,MP
      DO 40 I=1,MP
      EXXR(I)=XXR(I)
      EXXI(I)=XXI(I)
      ARG(I)=DZ*I
40  CONTINUE
      X=DZ*IK
      IF(X.GE.XMAX)GO TO 75
      CALL AITKLAGR(X,ARG,EXXR,YR,NDIM,ERS,IER1)
      IF(IER1.NE.0)GO TO 55
      YXXR(IK)=YR
      CALL AITKLAGR(X,ARG,EXXI,YI,NDIM,EPS,IER2)
      IF(IER2.NE.0)GO TO 65
      YXXI(IK)=YI
50  CONTINUE
      GO TO 75
C
55  WRITE(108,60)
60  FORMAT(/,/,5X,'ERROR IN THE FIRST INTERPOLATION',/,/)
      GO TO 90
65  WRITE(108,70)
70  FORMAT(/,/,5X,'ERROR IN THE SECOND INTERPOLATION',/,/)
      GO TO 90
C
75  ICOUNT=IK-1
      DO 76 I=1,ICOUNT
76  VOLJ(I,J)=CMPLX(YXXR(I),YXXI(I))
      ICOUNT=ICOUNT+1
      DO 77 I=ICOUNT,MP
77  VOLT(I,J)=CMPLX(AB,AB)
C
85  CONTINUE
C
      RETURN
90  STOP1
      END
C
C*****
C
      SUBROUTINE FOURG (DATA,N,ISIGN,WORK)
C
C      ONE-DIMENSIONAL TRANSFORM OF COMPLEX DATA, ARBITRARY NUMBER OF
C      POINTS. N POINTS CAN BE TRANSFORMED IN TIME PROPORTIONAL TO
C      N*LOG(N) (FOR A NON-PRIME), WHEREAS OTHER METHODS TAKE N**2 TIME.
C      FURTHERMORE, BECAUSE FEWER ARITHMETIC OPERATIONS ARE PERFORMED,
C      LESS ERROR IS BUILT UP. THE TRANSFORM DONE IS--
C
C      DIMENSION DATA(N),TRANSFORM(N),WORK(N)
C      COMPLEX DATA,TRANSFORM,WORK
C      TRANSFORM(K) = SUM(DATA(J)*EXP(ISIGN*2*PI*I*(J-1)*(K-1)/N)),
C      SUMMED FROM J = 1 TO N FOR ALL K FROM 1 TO N. THE TRANSFORM
C      VALUES ARE RETURNED TO DATA, REPLACING THE INPUT. N MAY BE ANY
C      POSITIVE NUMBER, BUT IT SHOULD BE NON-PRIME FOR SPEED. ISIGN =
C      +1 OR -1. A -1 TRANSFORM FOLLOWED BY A +1 ONE (OR VICE VERSA)

```

```

C RETURNS N TIMES THE ORIGINAL DATA. WORK IS A ONE-DIMENSIONAL
C COMPLEX ARRAY OF LENGTH N USED FOR WORKING STORAGE.
C RUNNING TIME IS PROPORTIONAL TO N * (SUM OF THE PRIME FACTORS OF
C N). FOR EXAMPLE, N = 1960, TIME IS TO * 1960 * (2+2+2+5+7+7).
C NAIVE METHODS DIRECTLY IMPLEMENTING THE SUMMATION RUN IN TIME
C PROPORTIONAL TO N**2. AN UPPER BOUND FOR THE RMS RELATIVE ERROR
C IS 3 * 2**(-B) * SUM(F**1.5), WHERE B IS THE NUMBER OF BITS IN
C THE FLOATING POINT FRACTION AND THE SUM IS OVER THE PRIME
C FACTORS OF N.
C
C DIMENSION DATA(1), WORK(1), IFACT(32)
C TWOPI=6.283185307*FLOAT(ISIGN)
C FACTOR N INTO ITS PRIME FACTORS, NFACT IN NUMBER. FOR EXAMPLE,
C FOR N = 1960, NFACT = 6 AND IFACT(IF) = 2, 2, 2, 5, 7 AND 7,
C IF=0
C NPART=N
C DO 50 ID=1,N,2
C IDIV=ID
C IF (ID-1) 10,10,20.
10 IDIV=2
20 IQUOT=NPART/IDIV
C IF (NPART-IDIV*IQUOT) 40,30,40
30 IF=IF+1
C IFACT(IF)=IDIV
C NPART=IQUOT
C GO TO 20
40 IF (IQUOT-IDIV) 60,60,50
50 CONTINUE
60 IF (NPART-1) 80,80,70
70 IF=IF+1
C IFACT(IF)=NPART
80 NFACT=IF
C
C SHUFFLE THE DATA ARRAY BY REVERSING THE DIGITS OF THE INDEX.
C REPLACE DATA(I) BY DATA(IREV) FOR ALL I FROM 1 TO N. IREV-1 IS
C THE INTEGER WHOSE DIGIT REPRESENTATION IN THE MULTI-RADIX
C NOTATION OF FACTORS IFACT(IF) IS THE REVERSE OF THE REPRESENTATION
C OF I-1. FOR EXAMPLE, IF ALL IFACT(IF) = 2, THEN FOR I-1 = 11001,
C IREV-1 = 10011. A WORK ARRAY OF LENGTH N IS NEEDED.
C
C IP0=2
C IP3=IP0*N
C IWORK=1
C I3REV=1
C DO 110 I3=1,IP3,IP0
C WORK(IWORK)=DATA(I3REV)
C WORK(IWORK+1)=DATA(I3REV+1)
C IP2=IP3
C DO 100 IF=1,NFACT
C IPI=IP2/IFACT(IF)
C I3REV=I3REV+IPI
C IF (I3REV-IP2) 110,110,90
90 I3REV=I3REV-IP2
100 IP2=IPI
110 IWORK=IWORK+IP0
C IWORK=1
C DO 120 I3=1,IP3,IP0
C DATA(I3)=WORK(IWORK)
C DATA(I3+1)=WORK(IWORK+1)
120 IWORK=IWORK+IP0

```

```

C      PHASE-SHIFTED FOURIER TRANSFORM OF LENGTH IFACT(IF).
C      IPROD=IP1/IP0
C      IREM=N/(IFACT(IF)*IPROD)
C      DIMENSION DATA(IPROD,IFACT(IF),IREM),WORK(IFACT(IF))
C      COMPLEX DATA,WORK
C      DATA(I1,J2,I3) = SUM(DATA(I1,I2,I3) * W**(I2-1)), SUMMED OVER
C      I2 = 1 TO IFACT(IF) FOR ALL I1 FROM 1 TO IPROD, J2 FROM 1 TO
C      IFACT(IF) AND I3 FROM 1 TO IREM.
C      W = EXP(ISIGN*2*PI*I*(I1-1+IPROD*(J2-1))/(IPROD*IFACT(IF))).
C
      IF=0
      IP1=IP0
130     IF (IP1-IP3) 140,240,240
140     IF=IF+1
      IFCUR=IFACT(IF)
      IP2=IP1*IFCUR
      THETA=TWOPI/FLOAT(IFCUR)
      SINTH=SIN(THETA/2.)
      ROOTR=-2.*SINTH*SINTH
C
C      COS(THETA)-1, FOR ACCURACY
C
      ROOTI=SIN(THETA)
      THETA=TWOPI/FLOAT(IP2/IP0)
      SINTH=SIN(THETA/2.)
      WSTPR=-2.*SINTH*SINTH
      WSTPI=SIN(THETA)
      WMINR=1.
      WMINI=0.
      DO 230 I1=1,IP1,IP0
      IF (IFCUR-2) 150,150,170
150     DO 160 I3=I1,IP3,IP2
      J0=I3
      J1=I3+IP1
      TEMPR=WMINR*DATA(J1)-WMINI*DATA(J1+1)
      TEMPI=WMINR*DATA(J1+1)+WMINI*DATA(J1)
      DATA(J1)=DATA(J0)-TEMPR
      DATA(J1+1)=DATA(J0+1)-TEMPI
      DATA(J0)=DATA(J0)+TEMPR
160     DATA(J0+1)=DATA(J0+1)+TEMPI
      GO TO 220
170     IWMAX=IP0*IFCUR
      DO 210 I3=I1,IP3,IP2
      I2MAX=I3+IP2-IP1
      WR=WMINR
      WI=WMINI
      DO 200 IWORK=1,IWMAX,IP0
      I2=I2MAX
      SUMR=DATA(I2)
      SUMI=DATA(I2+1)
180     I2=I2-IP1
      TEMPR=SUMR
      SUMR=WR*SUMR-WI*SUMI+DATA(I2)
      SUMI=WR*SUMI+WI*TEMPR+DATA(I2+1)
      IF (I2=I3) 190,190,180
190     WORK(IWORK)=SUMR
      WORK(IWORK+1)=SUMI
      TEMPR=WR
      WR=WR*ROOTR-WI*ROOTI+WR

```

```

200 WI=TEMPR*ROOTI+WI*ROOTR+WI
   IWORK=1
   DO 210 I2=I3,I2MAX,IP1
   DATA(I2)=WORK(IWORK)
   DATA(I2+1)=WORK(IWORK+1)
210 IWORK=IWORK+IP0
220 TEMPR=AMINR
   WMINR=WMINR*WSTPR-WMINI*WSTPI+WMINR
230 WMINI=TEMPR*WSTPI+WMINI*WSTPR+WMINI
   IP1=IP2
   GO TO 130
240 RETURN
   END

```

C

C*****

C

```

SUBROUTINE HANKEL(ZA,MI,IN)

```

C

C

C

C

C

C

```

THIS IS THE FORMAL DEFINITION OF THE HANKEL FUNCTION
(ASYMPTOTIC EXPANSION) BASED ON THE BOOK BY G. ARFKEN
ENTITLED "MATHEMATICAL METHODS FOR PHYSICISTS",
ACADEMIC PRESS (p.517)

```

C

```

COMPLEX PART2,PART1,TOP,HANK

```

```

COMMON HANK

```

```

Z=ZA

```

```

NU=MI+IN

```

```

PI=3.141592654

```

```

U=4*(NU**2)

```

```

PHI=Z-((NU+.5)*(PI/2))

```

```

FP1=(U-1)*(U-9)/(2*((8*Z)**2))

```

```

FP2A=(U-1)*(U-9)*(U-25)*(U-49)

```

```

FP2B=24*((8*Z)**4)

```

```

FP2=FP2A/FP2B

```

```

FP3A=(U-1)*(U-9)*(U-25)*(U-49)*(U-81)*(U-121)

```

```

FP3B=720*((8*Z)**6)

```

```

FP3=FP3A/FP3B

```

```

FP=1-FP1+FP2-FP3

```

```

QP1=(U-1)/(8*Z)

```

```

QP2=(U-1)*(U-9)*(U-25)/(6*((8*Z)**3))

```

```

QP3A=(U-1)*(U-9)*(U-25)*(U-49)*(U-81)

```

```

QP3B=120*((8*Z)**5)

```

```

QP3=QP3A/QP3B

```

```

QP=QP1-QP2+QP3

```

```

PART2=CMPLX(FP,-QP)

```

```

AB=0

```

```

TOP=CMPLX(AB,-PHI)

```

```

PART1=CEXP(TOP)

```

```

HANK=((2/(PI*Z))**.5)*PART1*PART2

```

```

RETURN

```

```

END

```

END

0	1	H	0	9	H	8	2
---	---	---	---	---	---	---	---

FIN