1

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

National Library
of Canada

Bibliothèque nationale
du Canada

Canadian Theses Service

Service des thèses canadiennes

Ottawa, Canada
K1A 0N4

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

Performance Of $\pi/4$ -QDPSK Over Fading And Shadowed Mobile Radio Channels

by

Minn Thien Le

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University

September, 1990

© Minh Thien Le, 1990

National Library
of Canada

Bibliothèque nationale
du Canada

Canadian Theses Service Service des thèses canadiennes

Ottawa, Canada
K1A 0N4

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

ISBN 0-315-60462-X

The undersigned recommend to the Faculty of
Graduate Studies and Research acceptance of the thesis

**"Performance Of $\pi/4$ -QDPSK Over Fading
And Shadowed Mobile Radio Channels"**

submitted by
Minh Thien Le,
B. Eng.

in partial fulfillment of the requirements
for the degree of Master of Engineering

Thesis Supervisor

Chairman,
Department of Systems
and Computer Engineering

Carleton University

September, 1990

Mathematical Notation

T	symbol period
$m(x)$	local mean component
$r_0(x)$	fast fading component
$r(x)$	lognormal superimposed by fast fading signal envelope
E_b/N_0	bit energy to noise spectral density ratio
E_s/N_0	symbol energy to noise spectral density ratio
P_S	probability of symbol error
P_L	probability of symbol error lower bound
P_U	probability of symbol error upper bound
$Re\{z(t)\}$	real part of $z(t)$
$Im\{z(t)\}$	imaginary part of $z(t)$
N	number of wave components
e	natural number (equals to 2.7183)
f_m	maximum Doppler shift
f_l	lognormal fading rate
$s(t)$	wanted signal
$i(t)$	interfering signal
$n(t)$	noise
$J_0(x)$	ordinary Bessel function of zero order
I	in phase channel
Q	quadrature channel
q_I	output of the in phase channel
q_Q	output of the quadrature channel
A'	transpose of matrix A
K_A	covariance matrix of A
σ	standard deviation of local mean
σ_s^2	average signal power

Acknowledgments

I would like to express my gratitude to Professor A. U. Sheikh, my thesis supervisor, for his guidance and great encouragement throughout the course of this research.

I would also like to thank Dr. L. Mason of Communication Research Centre for several worthwhile discussions with regard to the theoretical analysis.

I am grateful to TRIO (Telecommunication Research Institute of Ontario) for the financial support.

Finally, I would like to thank my colleagues who provided me guidance during difficult times.

Table of Contents

Abstract	iii
Acknowledgments	iv
List of Figures	viii
List of Acronyms	ix
Mathematical Notation	x
Chapter 1 INTRODUCTION	1
1.1 Evolution in Mobile Radio Systems	1
1.2 Definition and The Problem	3
1.3 Thesis Objectives	4
1.4 Thesis Outline	5
Chapter 2 CELLULAR MOBILE RADIO SYSTEM	7
2.1 Introduction	7
2.2 Multipath Fading Channel	10
2.2.1 Short-term Fading	10
2.2.1.1 Scattering Model	11
2.2.1.2 Envelope Autocorrelation and Signal Spectrum	13

2.2.2 Long-term Fading	14
2.2.3 Short-term Fading and Long-term Fading Superimposition	15
2.2.4 Model of Fast Fading Signal Superimpose on Slow Fading Signal	15
2.3 Cochannel Interference	17
2.4 Modulation scheme selection	19
Chapter 3 THEORETICAL ANALYSIS	25
3.1 System Overview	25
3.2 Error Probability When the Envelope is Purely Rayleigh Distributed	29
3.2.1 The Bounding Approach	29
3.2.2 Formulation in the Rayleigh channel	31
3.2.3 Numerical Result	37
3.3 Error Probability When The Envelope is Rayleigh and Lognormally Distributed	38
Chapter 4 SIMULATION IMPLEMENTATION	46
4.1 Introduction to BOSS	47
4.2 Rayleigh Fading Simulation	48
4.2.1 Realization Method	48
4.2.2 BOSS Implementation	51
4.3 Simulation of Lognormal Fading Superimposed by Rayleigh Fading	54
4.4 Additive White Gaussian Noise	59
4.5 Transmitter Implementation	59
4.6 Receiver Implementation	60

4.7	Cochannel Interference Implementation	64
4.8	System Implementation and Parameters	66
4.8.1	Fixed parameters	66
4.8.2	Derived parameters	66
4.8.3	System parameters	68
Chapter 5	SIMULATION RESULTS	70
5.1	SER under only AWGN	71
5.2	SER under Rayleigh Fading	74
5.3	SER under Rayleigh-Lognormal Fading	75
5.4	SER under Rayleigh-Lognormal-Cochannel Interference Conditions	82
Chapter 6	CONCLUSIONS	89
6.1	Summary	89
6.2	Suggestions for further work	90
References		92
Appendix A	TABLES OF VALUES	96
A.1	SER in only AWGN Channel	98
A.2	SER in Rayleigh Fading Channel	99
A.3	SER in Rayleigh-Lognormal Fading Channel	101
A.4	SER in Rayleigh-Lognormal-Cochannel Interference Channel	105

List of Figures

Figure 2.1	Service area is divided into small cells in cellular systems	9
Figure 2.2	Standard deviation of local mean by various authors, after [7]	16
Figure 2.3	The long-term fading signal superimposed by the short-term fading signal . .	18
Figure 2.4	Space diagrams of conventional QDPSK and $\pi/4$ shifted QDPSK (with zero initial phase)	23
Figure 3.1	Block diagram of a QDPSK system in AWGN, fading channel subjected to interference conditions	27
Figure 3.2	Phasor plot diagram (of the phase difference) of $\pi/4$ -QDPSK	28
Figure 3.3	Receiver block diagram of QDPSK	30
Figure 3.4	Theoretical bounds of $\pi/4$ -QDPSK symbol error probability in a Rayleigh fading channel with two values of $f_m T$: a) $f_m T = 0.01$ b) $f_m T = 0.001$	39
Figure 3.5	Theoretical bounds of $\pi/4$ -QDPSK symbol error probability in a Rayleigh-lognormal fading channel with $f_m T$ and σ as parameters : a) $f_m T = 0.001$, $\sigma = 6$ dB b) $f_m T = 0.001$, $\sigma = 9$ dB c) $f_m T = 0.01$, $\sigma = 6$ dB d) $f_m T = 0.01$, $\sigma = 9$ dB .	43
Figure 3.6	Theoretical bounds of $\pi/4$ -QDPSK symbol error probability in a Rayleigh-lognormal channel subjected to cochannel interference with $f_m T$, σ , and CIR as parameters : a) $f_m T = 0.01$ b) $f_m T = 0.001$	45
Figure 4.1	Simulator that duplicates mobile radio spectrum (baseband equivalent)	50
Figure 4.2	Generation of Rayleigh fading signal showing the frequencies and magnitudes of the sinusoids.	52
Figure 4.3	BOSS block diagram to simulate Rayleigh fading with mobile radio spectrum. .	53

Figure 4.4	Internal structure of lognormal module showing the generation of lognormal fading	56
Figure 4.5	Generation of lognormal plus Rayleigh fading signal.	57
Figure 4.6	Decision regions of the $\pi/4$ shifted QDPSK detector	61
Figure 4.7	Internal structure of the transmitter module	62
Figure 4.8	Internal structure of the receiver module	63
Figure 4.9	The interfering signal subjects to the same fading conditions as does the desired signal	65
Figure 4.10	The simulation block diagram of a QDPSK system under Rayleigh and lognormal fading conditions	67
Figure 5.1	SER performance of $\pi/4$ -QDPSK under AWGN	72
Figure 5.2	Comparison of BER performance of QDPSK and GMSK in an AWGN channel	73
Figure 5.3	Comparison of simulation and analytical results for $\pi/4$ -QDPSK in a Rayleigh fading channel: a) $f_m T = 0.01$ b) $f_m T = 0.001$	76
Figure 5.4	Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh fading channel for different $f_m T$	77
Figure 5.5	Comparison of simulation and analytical results for $\pi/4$ -QDPSK in a Rayleigh-lognormal channel: a) $\sigma = 6$ dB b) $\sigma = 9$ dB	79
Figure 5.6	Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal channel with $\sigma = 6$ dB	80
Figure 5.7	Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal channel with $\sigma = 9$ dB	81

Figure 5.8	Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal fading channel for different values of σ ($f_m T = 0.01$)	83
Figure 5.9	Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal fading channel for different values of σ ($f_m T = 0.001$)	84
Figure 5.10	Comparison of simulation and analytical results of $\pi/4$ -QDPSK in a Rayleigh-lognormal-interference channel: a) CIR = 15 dB, $\sigma = 6$ dB, $f_m T = 0.001$ b) CIR = 20 dB, $\sigma = 9$ dB, $f_m T = 0.001$	85
Figure 5.11	Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal-interference channel ($\sigma = 6$ dB)	86
Figure 5.12	Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal-interference channel ($f_m T = 0.001$)	87

List of Acronyms

AWGN	additive white Gaussian noise
BER	bit error rate
BOSS	Block Oriented System Simulator
BPF	bandpass filter
CNR	carrier to noise ratio
CIR	carrier to interference ratio
DCPSK	differentially coherent phase shift keying
FDMA	frequency division multiple access
FM	frequency modulation
GMSK	Gaussian filtered minimum shift keying
LPF	lowpass filter
<i>pdf</i>	probability density function
PSK	phase shift keying
QDPSK	quadrature differential phase shift keying
SNR	signal to noise ratio
SER	symbol error rate
TFM	tamed frequency modulation
TDMA	time division multiple access

Mathematical Notation

T	symbol period
$m(x)$	local mean component
$r_0(x)$	fast fading component
$r(x)$	lognormal superimposed by fast fading signal envelope
E_b/N_0	bit energy to noise spectral density ratio
E_s/N_0	symbol energy to noise spectral density ratio
P_S	probability of symbol error
P_L	probability of symbol error lower bound
P_U	probability of symbol error upper bound
$Re\{z(t)\}$	real part of $z(t)$
$Im\{z(t)\}$	imaginary part of $z(t)$
N	number of wave components
e	natural number (equals to 2.7183)
f_m	maximum Doppler shift
f_l	lognormal fading rate
$s(t)$	wanted signal
$i(t)$	interfering signal
$n(t)$	noise
$J_0(x)$	ordinary Bessel function of zero order
I	in phase channel
Q	quadrature channel
q_I	output of the in phase channel
q_Q	output of the quadrature channel
A'	transpose of matrix A
K_A	covariance matrix of A
σ	standard deviation of local mean
σ_s^2	average signal power

σ_i^2	average interfering signal power
σ_n^2	average noise power
ϕ_n	carrier phase as a function of symbols
$\phi_s(t)$	phase of the transmitted signal as a function of time
$\phi_i(t)$	phase of the interfering signal as a function of time
$\Delta\phi_n$	phase difference as a function of symbols
$\rho(\tau)$	autocorrelation coefficient
ω_c	radian carrier frequency
f_c	carrier frequency (in Hz)
Γ	average carrier to noise ratio
Λ	average carrier to interference ratio
Δ	average interference to noise ratio
$\gamma(t)$	carrier to noise ratio random variable
$\delta(t)$	interference to noise ratio random variable

Chapter 1

INTRODUCTION

1.1 Evolution in Mobile Radio Systems

Mobile radio communication has gone through significant developments since the date Marchese Guglielmo Marconi successfully demonstrated the possibility of mobile radio communication by establishing radio transmission between a land-based station and a tugboat over an 18 mile path. The two elements in mobile radio over which a user has the least control are the time variant channel and the choice of frequency band. In the early days of mobile radio when the number of users was substantially low and spectral utilization factor was of no consequence, the researchers concentrated only on the mobile propagation mechanisms, phenomena which distinguish the mobile radio channel from other channels such as point-to-point fixed link. Lately, with rapid growth in mobile communications, channel utilization has become the foremost concern.

Until recently, analog FM transmission has been the predominant mode of information transmission because of its high immunity to the channel fading. The chief

disadvantage of FM compared to AM and other narrowband schemes, however, is that a larger bandwidth is required. To ensure high quality FM voice transmission, the channel spacing is specified as high as 30 KHz in the 150 MHz band or 25 KHz in the 450 MHz and 850 MHz bands. Transmission bandwidth to speech bandwidth ratio around 5 makes this mode of transmission spectrally inefficient. Several spectrally efficient schemes have been devised to improve spectral utilization. Cellular mobile radio, where high spectral utilization is achieved by extensive frequency reuse, is another practical solution to the problem of congestion. Cell splitting, a process of partitioning the service areas into smaller and smaller cells so that different frequency sets can be reused more often, has also been considered an important strategy towards increase in the system capacity without requiring an additional spectrum.

Frequency reuse and cell splitting have resulted in considerable improvement in the system capacity, but without additional spectral allocations analog cellular systems operating in several densely populated areas will reach their capacity soon. Although the Federal Communication Commission (FCC) allocated an additional 10 MHz (in 800 — 900 MHz bands) for mobile radio applications in July 1986, yet even this band is in danger of being saturated. Similar allocations have been made by Department of Communication (DOC) in Canada. In order to provide for the projected growth of users, and the variety of proposed services, new approaches to capacity increase must be found. The proposal on second generation digital cellular system is a step in the right direction.

The second generation digital cellular transmission systems have been endorsed by most telecom operators. On several aspects, digital technology is more efficient and flexible than analogue. In the present day applications require transmission of speech and

data, and integration of speech and data transmission become a basic system requirement. In view of the need for this integration in mobile systems, conversion of analogue voice to digital voice is inevitable. Many modulation scheme spectrally more efficient than the conventional FM scheme with similar system performance exist. Among these digital modulation techniques, tamed frequency modulation (TFM) [1] and Gaussian filtered minimum shift keying (GMSK) [2], being used in Europe and Japan, have proved to provide satisfactory services.

The use of information in digital form allows the implementation of time division multiple access (TDMA), which perhaps is the new and most important technique being exploited in second generation of mobile radio in providing high capacity systems. In North America, a combination scheme NBTDMA/NBFDMA (narrowband TDMA/narrowband FDMA) divides the allocated spectrum into eight hundred and thirty three 30 KHz channels. Within each 30 KHz band, 3 TDM channels are available at full rate (or 6 TDM channels at half rate). The bit rate to be achieved with acceptable bit error rate (BER) in each 30 KHz channel is 48 kbits/sec. The modulation scheme adopted as the US digital cellular mobile communications standard is $\pi/4$ shifted quadrature differential phase shift keying ($\pi/4$ -QDPSK) [3].

1.2 Definition and The Problem

In a typical mobile radio propagation situation, the received signal will show fading consisting of very rapid fluctuations around the mean signal level which has relatively slow variations. The short-term variations are caused by multipath propagation, and the amplitude distribution of the signal is usually approximated very closely by the Rayleigh distribution [4, 5, 6]. The mean value variations are accounted by shadow effects, which

are caused by the nature of local environment such as buildings, trees, hills, etc., and are usually found to be lognormally distributed [7, 4, 8]. In theoretical work the mean value is normally assumed constant, i.e., the signal envelope distribution is assumed to be pure Rayleigh. This simplifies the mathematical analysis considerably, but it will seriously limit the accuracy of any attempt to predict the performance of a system operating in a realistic environment.

Several authors have studied $\pi/4$ -QDPSK, in which, however, the channel is always assumed to be purely Rayleigh fading channel. D. L. Schilling & C. K. Pauw derived a formula for M-DPSK (multiple differential phase shift keying, general case of $\pi/4$ -QDPSK) in a *slow* Rayleigh fading channel [9]. To include the influence of an arbitrary fading rate, J. Horikoshi *et al.* gave a bound for $\pi/4$ -QDPSK in a Rayleigh fading channel subjected to cochannel interference [10]. K. Feher & Y. Guo recently carried out an experimental study of $\pi/4$ -QDPSK in a Rayleigh fading and delay spread channel corrupted by cochannel interference [11]. Of all the studies, the effect of shadowing was a noticeable omission. R. C. French included the shadow effects to some of his studies [12, 13], but his method yields only the *prediction* of the loss in error performance for digital FM systems and not for a particular system like $\pi/4$ -QDPSK.

The exclusion of the shadow effects from the available results cause inaccuracies in the evaluation of the system performance. For realistic environments, the channel shadowing must be included in evaluation of $\pi/4$ -QDPSK system performance.

1.3 Thesis Objectives

The main objective of this thesis is to evaluate the $\pi/4$ -QDPSK performance in a flat Rayleigh-lognormal fading channel subjected to cochannel interference. The

performance will be measured in terms of symbol error rates (SER). This objective is accomplished by:

1. Establishing the $\pi/4$ -QDPSK system model which includes fast Rayleigh fading, shadowing, and cochannel interference.
2. Simulating the system on the computer to obtain the results which are expressed in terms of symbol error rates.
3. Theoretically analyzing the $\pi/4$ -QDPSK system in a previously described environment. This will consist of reviewing the existing analytical methods for the systems of our interest, and extending them to the case of our study.
4. Comparing the simulation and the analytical results to confirm the reliability of the simulation model.
5. Presenting the results in graphical formats, interpreting the outcomes, from which extracting the individual influence of Rayleigh fading, shadowing, and cochannel interference on the system performance.

1.4 Thesis Outline

The thesis is divided into five chapters including the current chapter 1 on introduction.

Chapter 2 provides fundamental principles of a mobile radio channel characteristics. The probability density function (*pdf*) of the signal envelope when subjected to either short-term or long-term fading, or when subjected to both types of fadings are derived. The chapter ends with a detailed description of $\pi/4$ -QDPSK modulation scheme.

Chapter 3 presents an analysis method to obtain a bound for the $\pi/4$ -QDPSK symbol error probability in a flat Rayleigh-lognormal fading channel subjected to cochannel interference. The analytical results are shown for two different Doppler shifts, two shadow spread values, and various carrier-to-interference ratios (CIR).

Chapter 4 covers details of the simulation implementation for the system under investigation. The realization method of Rayleigh/Rayleigh superimposed by lognormal channels are provided before actual implemented modules are constructed. The rationale behind the choices of important simulation parameters such as Rayleigh fading rate, lognormal fading rate is also given in this chapter

Chapter 5 gathers the simulation results and compares them to the analytical ones which were obtained in chapter 3. The final results are arranged into appropriate set of curves which form the basis of discussion.

Chapter 6 summaries the thesis work and suggests some areas for further research.

Tables of all the values derived from theoretical analysis and simulation are recorded in Appendix A.

Chapter 2

CELLULAR MOBILE RADIO SYSTEM

2.1 Introduction

In mobile radio channel, the signal travels from the transmitter to the receiver via more than one path due to the reflection and the scattering from buildings, trees, and other obstacles along the path. The radio waves, which arrive from different directions, combine constructively or destructively to result in fluctuating resultant. As the vehicle moves, the phase relationship between the constituent signal components changes causing fluctuations in the signal amplitude and the resultant phase. This rapid fluctuation, known as fast fading, can be viewed from the envelope variation in the time domain or from the Doppler spread in the frequency domain.

Over short distances, the mean of the faded signal remains constant, but varies over greater distances as the vehicle moves. This slower change, often called shadowing, is caused by the large scale variations in the terrain profile between the transmitter and the receiver signal path. The net effect of fast fading and shadowing is to produce a slowly

varying signal superimposed by a rapid and severe fading signal. Random variation of the received signal makes it necessary to use statistical communication theory to characterize the signal behaviour.

Due to shortage of spectrum in mobile radio systems, frequency bands can be reused in several ways. In cellular systems, the service area is broken into a number of smaller areas, or cells, usually of regular polygon such as the equilateral, the square, or the regular hexagon in which case a plane can be covered with no gaps or overlaps. Figure 2.1 illustrates the cellular concept with the cells having the regular hexagon shapes. Each cell has its own base station for communication with mobile users in its own cell. Since each station covers a local area or a cell, it is possible to reuse the same frequency simultaneously in other cells located at a distance to control interference of the service area. A cellular system could be designed with square or equilateral triangular cells, but regular hexagon cells are preferred because it is closer to a circle and requires fewer number of base stations to serve a given coverage area.

In the early days, the base stations were located sufficiently far apart to minimize cochannel interference. However, the demand for mobile radio has now reached the point where channels must be reused at shorter distances to increase the system capacity with the result that significant cochannel interference may result. This work does not address the problem of containment of interference. We address, however, the effect of cochannel interference on the performance of a particular modulation scheme.

In this chapter, we will look at a short-term fading model, which establishes several important relationships describing the received signal, e.g the first and second order statistics of the signal envelope and the nature of the frequency spectrum. We will

Figure 2.1 Service area is divided into small cells in cellular systems

also study the first order statistic of the shadowed signal level. Finally, a description of a suitable modulation scheme, $\pi/4$ shifted QDPSK, is given. The study of this modulation scheme is important because this modulation format has been chosen for the second generation digital cellular systems.

2.2 Multipath Fading Channel

Multipath fading phenomenon in mobile radio is a complex process whose characterization can be studied comprehensively only through the establishment of accurate channel models. The channel variations can be broken up into two parts: the short-term fading (or rapid fading), whose effects are usually described by the error rate performance as a function of mean signal-to-noise ratio (SNR), and the long-term fading (or slow fading), which changes the mean path attenuation significantly over a long period. It will be seen in this section that, the statistical model of the short-term fading can be regarded to be conditional upon the instantaneous values of the mean path described by the long-term statistics.

2.2.1 Short-term Fading

Radio signal experiences fast fading when the mobile receiver moves through the radio wave field. Fast fading is consequence of the movement of the mobile in a multipath environment. In suburban areas, there may be a direct line of sight present, in which case there is a dominant component, however in urban areas, with high buildings, the direct line of sight may not exist. This subsection describes a scattering model which is widely accepted by most researchers. The statistical properties of the signal waveforms such as its *pdf*, autocorrelation, and spectrum will also be given.

2.2.1.1 Scattering Model Clarke [5] proposed a simple scattering model for the signal reception in urban environment, which establishes several important relationships describing the received signal, e.g the first and second order statistics of the signal envelope and the nature of the frequency spectrum.

The scattering model is based on a number of assumptions:

- the received signal consists of a large number of independent randomly distributed phase components (with no direct component) — this is realistic in heavily built up areas, where there is no direct line from the base station to the vehicle.
- the phase angles are assumed to have a uniform *pdf* in $[0, 2\pi]$ — this assumption is reasonable at VHF or above where the wavelength is short enough to ensure that small changes in path length result in significant changes in the RF phase.
- the plane waves are vertically polarized with horizontal angles of arrival and phase angles are statistically independent.
- the angle of arrival is also random and its *pdf* depends on the scattering characteristic of the environment.

The transmitted signal is assumed to be an unmodulated carrier of a radian frequency ω_c (rad/sec). The received signal is assumed to be made up of a limited number of wave components, N .

The vertical field component can be written as :

$$E_z = E_0 \sum_{n=1}^N c_n \cos(\omega_c t + \theta_n) \quad (2.1)$$

where

- $E_0 c_n$ is the real amplitude of n^{th} wave in the z field,

- θ_n is the phase shift of the n^{th} wave,
- c_n are normalized such that $\sum_{n=1}^N \langle c_n^2 \rangle = 1$.

If N is large ($N \geq 10$), from the central limit theorem it follows that E_z is approximately Gaussian random variable. This Gaussian process may be written in terms of two quadrature components say x_c , x_s , both being Gaussian distributed random process

$$E_z = x_c(t) \cos \omega_c t - x_s(t) \sin \omega_c t, \quad (2.2)$$

where

$$x_c(t) = E_0 \sum_{n=1}^N c_n \cos(\omega_n t + \phi_n), \quad (2.3)$$

$$x_s(t) = E_0 \sum_{n=1}^N c_n \sin(\omega_n t + \phi_n), \quad (2.4)$$

$$\theta_n = \omega_n t + \phi_n. \quad (2.5)$$

When $x_c(t)$ and $x_s(t)$ are both Gaussian processes, it has been shown [14] that the envelope is Rayleigh distributed. Let $r(t)$ and $\theta(t)$ be the envelope and the phase of the received signal respectively then

$$r(t) = \sqrt{x_c^2(t) + x_s^2(t)} \quad (2.6)$$

and

$$\theta(t) = \arctan\left(\frac{x_s}{x_c}\right) \quad (2.7)$$

The envelope r can be shown to be Rayleigh distributed, and the phase θ is uniformly distributed in the interval $[0, 2\pi]$

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), & r \geq 0; \\ 0, & \text{otherwise} \end{cases} \quad (2.8)$$

where σ^2 is the mean power $\left(= \frac{E_0^2}{2}\right)$.

$$p(\theta) = \frac{1}{2\pi} \quad \theta \in [0, 2\pi] \quad (2.9)$$

2.2.1.2 Envelope Autocorrelation and Signal Spectrum The autocorrelation coefficient for the vertical component envelope $|E_z|$, based on the scattered wave multipath model derived by Clarke, was found to be [5]

$$\rho_{|E_z|}(\tau) = J_0^2(2\pi f_m \tau) \quad (2.10)$$

where f_m is the maximum Doppler shift $(f_m = \frac{V}{c} f_c = \frac{V}{\lambda})$ (c is the velocity of light), and τ is the separation time.

Cross correlation between two signals r_1 and r_2 received by two antennas located a distance d apart can also be considered. This cross correlation is given by:

$$\rho_{r_1 r_2}(d) = J_0^2(\beta d) \quad (2.11)$$

where β is the phase shift.

When expressed in complex form $E_z(t) = x(t) + jy(t)$, the autocorrelations of the real and imaginary components of $E_z(t)$ are also found to have the forms of Bessel functions:

$$\langle x(t)x(t+\tau) \rangle = \langle y(t)y(t+\tau) \rangle = J_0^2(2\pi f_m \tau) \quad (2.12)$$

When either the transmitter or the receiver is in motion, the components of the received signal experience a Doppler shift, the frequency shift being related to the spatial

angle between the direction of arrival of that component and the direction of vehicle motion, or in other words on vector velocity between the transmitter and receiver.

The power spectrum $S_{E_z}(f)$ of the signal envelope for the E_z field can be derived by taking the Fourier transform of the autocorrelation function. For an omnidirectional vertical monopole antenna and the *pdf* of the incident wave angle is uniform over $[0, 2\pi]$, the RF Doppler spectrum is found to have the form

$$S_{E_z}(f) = \frac{K}{f_m} \left[1 - \left(\frac{f - f_c}{f_m} \right)^2 \right]^{-\frac{1}{2}}, \quad (2.13)$$

where f_c is the carrier frequency, f_m is the maximum Doppler shift, and K is a constant.

2.2.2 Long-term Fading

Measurements reported by Reudink [7], Okumura *et al.* [15], and French [16] suggest that the local mean of the signals, received at a given range and frequency, and in similar environmental areas, when measured over larger distances and expressed in dB, follows a normal distribution. Let x be the local mean, expressed in dB, then the *pdf* of x is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left[-\frac{(x - \mu)^2}{2\sigma^2} \right], \quad (2.14)$$

where μ is the average local mean or median (also expressed in dB), which may be predicted if transmitted power and path loss are known.

When expressed on a linear scale, the local mean follows a lognormal distribution. Let $z = 10^{\frac{x}{20}}$, then the *pdf* of z is found to be

$$f(z) = \frac{\log e}{z\sqrt{2\pi}\sigma} \exp \left[-\frac{(20 \log z - \mu)^2}{2\sigma^2} \right] \quad (2.15)$$

The standard deviation σ is a function of frequency and terrain as shown in Figure 2.2, reproduced from Figure 21 of Reudink [7] with the result of R. French [16] added to the Figure. At 800–900 MHz, typical values of σ fall within the range 5 — 9 dB, with more variation in suburban and hilly terrain.

2.2.3 Short-term Fading and Long-term Fading Superimposition

In previous section, we have seen that over a relatively small distance (a few tens of wavelengths), the signal is well described by Rayleigh statistics, with the local mean over a somewhat larger area being lognormally distributed. The overall distribution, when examined in larger areas, is expected to be a mixture of Rayleigh and lognormal. Suzuki [17], Hansen and Meno [4] suggest a statistic for the mobile radio signal in the form of a Rayleigh distribution with a lognormal distributed mean.

The expression suggested by Suzuki is

$$p(r) = \int_0^{\infty} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \frac{1}{\sigma\lambda\sqrt{2\pi}} \exp\left[-\frac{(20\log\sigma - \mu)^2}{2\lambda^2}\right] d\sigma \quad (2.16)$$

that is, the integral of the Rayleigh distribution over all possible values of σ , weighted by the *pdf* of σ , and this was an attempt to provide a transition from local to global statistics. λ is the standard deviation of the signal mean.

This expression will be used in our analysis when deriving the probability of symbol error in presence to Rayleigh and lognormal fading conditions.

2.2.4 Model of Fast Fading Signal Superimpose on Slow Fading Signal

According to Lee and Yeh [8], the received signal envelope can be represented as a product of a stationary Rayleigh process with a slowly varying mean signal level. Let

Figure 2.2 Standard deviation of local mean by various authors, after [7]

$r(x)$ be the impaired signal, then $r(x)$ can be written as

$$r(x) = m(x) r_0(x) \quad (2.17)$$

where

1. $r_0(x)$ is a stationary Rayleigh stochastic process with unit mean
2. $m(x)$ is the mean value of $r(x)$

$m(x)$ varies much more slowly than $r_0(x)$. While the variation of $r_0(x)$ is seen over a distance of 40λ , the variation of $m(x)$ is obvious only over a distance of at least 400λ .

If $m(x)$ is lognormally distributed then $r(x)$ is also lognormally distributed when the short-term fluctuation is averaged out over proper distances.

Figure 2.3 graphically illustrates the meaning of equation (2.17).

Based on equation (2.17), the superimposed signal can be generated by first producing the fast fading $r_0(x)$ with unity mean, having Rayleigh distributed amplitude, and multiply it by much more slow fading $m(x)$, whose *pdf* follows lognormal distribution with the mean equal to the mean of the $r(x)$.

2.3 Cochannel Interference

Cochannel interference is produced when unintended signal(s) generated from several transmitter geographically distributed using the same carrier frequency appear at a receiver. In cellular systems, as a result of extensive reuse of available frequencies, cochannel interference becomes a dominant factor to influence the system performance. Cell splitting process, which increases the number of channels by reducing the cell radii, makes cochannel interference an unomitable factor. It is known that in such systems the

Figure 2.3 The long-term fading signal
superimposed by the short-term fading signal

performance is limited by carrier-to-interference ratio. Knowing the minimum required carrier-to-interference ratio to produce satisfactory results, a designer is able to locate the stations at appropriate distances in order to reduce interference as well as to maximize the system capacity.

Assuming that $s(t)$ is the desired signal and $i(t)$ is the interfering signal, the relationship between them can be expressed by the following equations:

$$s(t) = \exp(j(\omega_c t + \phi_s(t))) \quad (2.18)$$

$$i(t) = r \exp(j(\omega_c t + \phi_i(t))) \quad (2.19)$$

where $\phi_s(t)$ and $\phi_i(t)$ are the phase-modulation components of the desired and interfering signal respectively, and r is the ratio of the amplitude of the interfering signal to that of the desired signal.

The wanted and unwanted signals often travel through similar terrains. It is therefore reasonable to assume that both signal envelopes follow the same distribution, i.e. lognormal superimposed by Rayleigh distribution.

2.4 Modulation scheme selection

In multipath radio channels, the envelope bearing information is nearly destroyed because of the large fluctuations in the signal magnitude caused by fast fading. Any amplitude modulation scheme is regarded as unsuitable for mobile radio. Rather, many frequency and phase modulated transmissions schemes have been devised because of their greater immunity to fading conditions.

Digital FM schemes have been used in mobile data communications [1, 2, 18, 19]. FM systems, however, occupy wide bandwidth, and result in low spectrum efficiency.

Linear modulation methods such as QPSK have been studied in the past few years to take advantage of its efficient spectrum characteristics [20, 21].

Under fast fading conditions, coherent demodulation in PSK, which requires carrier phase recovery, needs complex and expensive circuits. This imposes the potentially high cost penalty in the receiver implementation.

A clever technique to get around the phase synchronization problem is to use phase comparison detection. In this method, the current symbol is evaluated taking the previous symbol (phase) as reference. Thus, the difference of the two consecutive phases is detected rather than the absolute value of the phase itself. This kind of detection named differentially coherent phase shift keying (DCPSK) requires differential encoding at the transmitter.

Voelcker [22] has shown that DCPSK system requires about 3 dB more power than a coherent PSK system for identical performance. Thus any physically realizable coherent scheme can approach a 3 dB improvement only under the most favourable conditions. However, a phase-average scheme which has too long a memory (compared to the phase stability of the medium) can degrade the performance of a PSK system severely. Another advantage of differential detection over coherent detection is that the former results in lower irreducible bit error rate (caused by random FM effect in fast fading) [23].

$\pi/4$ shifted QDPSK versus conventional QDPSK

In M-ary DCPSK (M phase level DCPSK), the error rate increases with the increase in the number of phase levels, because of smaller decision regions. The

disadvantage of using higher value of M (number of phase levels) is that higher SNR is required to obtain a given BER. Its advantage, however, lies in that it requires narrower channel bandwidth for the same data rate. The trade-off is thus between spectral efficiency and transmission power. Four phase DCPSK ($M=4$), or quadriphase DCPSK or QDPSK, show the most promise among the multi-level phase modulation schemes on the basis of SNR and throughput [20, 24].

In coherent QPSK, each symbol conveys two binary bits. The symbols are encoded into four possible phases in the modulated carrier, usually of equal distance apart in the complex plane. A typical set of phases is

$$\phi_n = \frac{n\pi}{2} \quad (n = 0, 1, 2, 3)$$

Another common set of phases is

$$\phi_n = \frac{(2n-1)\pi}{4} \quad (n = -1, 0, 1, 2)$$

In *differential* QPSK, each symbol also represents two binary bits. The symbols however reside in the phase difference $\Delta\phi_n$ of the two consecutive modulated carrier phases. In conventional QDPSK, the set of the phase differences is

$$\Delta\phi_n = \frac{n\pi}{2} \quad (n = 0, 1, 2, 3)$$

A possible mapping between the set of symbols and the set of the phase difference values is

$$\Delta\phi = \begin{cases} 0 & \longleftrightarrow \text{symbol 0} \\ \pi/2 & \longleftrightarrow \text{symbol 1} \\ \pi & \longleftrightarrow \text{symbol 2} \\ 3\pi/2 & \longleftrightarrow \text{symbol 3} \end{cases}$$

If the initial carrier's phase is set to zero, the subsequent carrier's phases will take the values in the set $\{0, \pm\pi/2, \pi\}$. The transition from one phase to another in the set can occur for any current phase. This is shown in the space diagram of conventional QDPSK in Figure 2.4a.

In $\pi/4$ shifted QDPSK, the set of the phase differences is

$$\Delta\phi_n = \frac{(2n-1)\pi}{4} \quad (n = -1, 0, 1, 2)$$

A possible mapping between the set of symbols and the set of the phase difference values is

$$\Delta\phi = \begin{cases} \pi/4 & \longleftrightarrow \text{symbol 0} \\ 3\pi/4 & \longleftrightarrow \text{symbol 1} \\ -3\pi/4 & \longleftrightarrow \text{symbol 2} \\ -\pi/4 & \longleftrightarrow \text{symbol 3} \end{cases}$$

Again, assuming zero initial carrier phase, the subsequent phases will take their values from the set $\{0, \pm\pi/4, \pm\pi/2, \pm3\pi/4, \pi\}$. Figure 2.4b shows such phase transition in the space diagram of $\pi/4$ shifted QDPSK.

From Figure 2.4, it is seen that conventional QDPSK has wider phase change, which means wider signal envelope dynamic range than in $\pi/4$ shifted QDPSK. Due to nonlinear amplification, the signal spectrum characteristics is always degraded at the output of the power amplifier. In general, the degradation increases for a scheme with wider signal envelope dynamic range. From Figure 2.4, it is seen that $\pi/4$ shifted QDPSK is superior to conventional QDPSK in this respect, and thus is chosen the standard modulation scheme in cellular mobile radio system. In our study, the effect of nonlinear amplification is ignored, therefore the error rate performances of conventional

**Figure 2.4 Space diagrams of conventional QDPSK
 and $\pi/4$ shifted QDPSK (with zero initial phase)**

QDPSK and $\pi/4$ shifted QDPSK would be exactly the same. The two names thus will be used interchangeably where there is no risk of confusing, and specified where such differentiation is needed.

Chapter 3

THEORETICAL ANALYSIS

This chapter presents an analytical approach to obtain a bound for the $\pi/4$ -QDPSK error probability in a flat Rayleigh fading and lognormal shadowed channel in the presence of cochannel interference. Section 3.1 gives the description of the overall system. Section 3.2 reviews the symbol error bound formula derived by J. Horikoshi *et al.* [10], when the signal envelope is Rayleigh distributed. In the last section, lognormal variation of the signal mean is introduced, and the symbol error probability bounds for Rayleigh-lognormal channel are developed.

3.1 System Overview

The block diagram of the system under investigation is shown in Figure 3.1. The input data α_i to the modulator are the information symbols whose values are from the set $\{0, 1, 2, 3\}$. The modulator produces the modulated signal $v(t)$, described in the

complex form as:

$$v(t) = \text{Re}\{\exp(j(\omega_c t + \phi_s(t)))\} \quad (3.1)$$

where ω_c is the radian carrier frequency, $\phi_s(t)$ is the phase determined by the data sequences.

Let $\Delta\phi$ be the difference of the two consecutive phases, $\phi_s(t)$ and $\phi_s(t-T)$, then in $\pi/4$ -QDPSK, it is determined as a function of binary data as follows:

$$\Delta\phi = \begin{cases} \pi/4 & \longleftrightarrow & 11 \\ -\pi/4 & \longleftrightarrow & 10 \\ 3\pi/4 & \longleftrightarrow & 01 \\ -3\pi/4 & \longleftrightarrow & 00 \end{cases} \quad (3.2)$$

The corresponding signal space diagram (of the phase difference) is shown in Figure 3.2.

The transmitted phase $\phi_s(t)$ thus has the form

$$\phi_s(t) = \sum_{t=-\infty}^{\infty} \frac{n\pi}{4} p(t - iT) \quad (3.3)$$

with $n \in \{0, 1, \dots, 7\}$ and $p(t)$ is a rectangular unit pulse of duration T (the symbol period).

Denote $s(t)$ the signal being distorted by the fading channel, $i(t)$ the interference signal from another base station using the same carrier frequency and identical modulation scheme, and $n(t)$ the additive white Gaussian noise. The input $e(t)$ to the BPF at the receiver thus equals the sum of the three described components

$$e(t) = s(t) + i(t) + n(t) \quad (3.4)$$

**Figure 3.1 Block diagram of a QDPSK system in AWGN,
fading channel subjected to interference conditions**

Figure 3.2 Phasor plot diagram (of the phase difference) of $\pi/4$ -QDPSK

3.2 Error Probability When the Envelope is Purely Rayleigh Distributed

3.2.1 The Bounding Approach

The following analysis is based on the detection of signal using the two in-phase and quadrature branches. The detection block diagram of QDPSK is shown in Figure 3.3.

Let E_I and E_Q be the events that the samplers make erroneous decisions on channel I and Q respectively. The probability of symbol error, denoted P_S , can be expressed as $P_S = \text{Prob}(E_I \cup E_Q)$, where relation $E_I \cup E_Q$ is read as event E_I or event E_Q . For any two events E_I and E_Q , the following inequalities hold

$$\begin{aligned} \max\{\text{Prob}(E_I), \text{Prob}(E_Q)\} &\leq P_S = \text{Prob}(E_I \cup E_Q) \\ &\leq \text{Prob}(E_I) + \text{Prob}(E_Q) \end{aligned} \quad (3.5)$$

Assume that the probabilities of the bit error on each channel I and Q are the same, i.e.,

$$\text{Prob}(E_I) = \text{Prob}(E_Q) \quad (3.6)$$

Then (3.5) becomes

$$\begin{aligned} P_L = \text{Prob}(E_I) = \text{Prob}(E_Q) &\leq P_S = \text{Prob}(E_I \cup E_Q) \\ &\leq 2P_L = P_U \end{aligned} \quad (3.7)$$

The lower bound is thus actually the *bit error rate*, and the upper bound is *twice the bit error rate*. Since the two bounds are related, only one bound, for instance the lower bound P_L needs to be evaluated. In reference to Figure 3.2, the bit error probability

Figure 3.3 Receiver block diagram of QDPSK

(which can be inspected based on either channel output; here output of channel I is chosen) can be expressed as

$$\begin{aligned}
 P_L &= \frac{1}{2} \cdot \text{Prob}\left[P_L \mid \pm \frac{\pi}{4}\right] + \frac{1}{2} \cdot \text{Prob}\left[P_L \mid \pm \frac{3\pi}{4}\right] \\
 &= \frac{1}{2} \cdot \text{Prob}[q_I < 0, \cos \Delta\phi_s > 0] \\
 &\quad + \frac{1}{2} \cdot \text{Prob}[q_I > 0, \cos \Delta\phi_s < 0],
 \end{aligned} \tag{3.8}$$

where q_I is the output of channel I. This relationship will be used to determine the symbol error probability lower bound P_L , which means that $\text{Prob}[q_I < 0]$ and $\text{Prob}[q_I > 0]$ have to be found for various transmitted symbols. These probabilities will be derived in the next subsection.

3.2.2 Formulation in the Rayleigh channel

Using the lowpass complex envelope representation, the components in equation 3.4 can be written as

$$s(t) = \text{Re}\{z_s(t) \exp(j(\omega_c t + \phi_s(t)))\} \tag{3.9}$$

$$i(t) = \text{Re}\{z_i(t) \exp(j(\omega_c t + \phi_i(t)))\} \tag{3.10}$$

$$n(t) = \text{Re}\{z_n(t) \exp(j\omega_c t)\}. \tag{3.11}$$

Here $\phi_i(t)$, similar to $\phi_s(t)$, is the phase determined by the data sequences of the interference signal.

When the channel is Rayleigh fading, i.e., the signal $s(t)$ and the interference $i(t)$ envelopes are both Rayleigh distributed, it is well known that $z_s(t)$ and $z_i(t)$ can be modeled as zero mean complex Gaussian processes [25]. Also the lowpass equivalent $z_n(t)$ of the zero mean Gaussian noise $n(t)$ has been proved to be a zero mean complex Gaussian process [25].

The signal passing through the BPF is expressed as

$$e(t) = \text{Re}\{z_1(t) \exp(j\omega_c t)\}, \quad (3.12)$$

where

$$z_1(t) = z_s(t)e^{j\phi_s(t)} + z_i(t)e^{j\phi_i(t)} + z_n(t). \quad (3.13)$$

If ω_c and T are selected so as to satisfy $\omega_c T = 2n\pi$ (n is an integer), the one time slot delayed signal $e(t-T)$ is

$$\begin{aligned} c(t-T) &= \text{Re}\{z_1(t-T) \exp(j\omega_c(t-T))\} \\ &= \text{Re}\{z_1(t-T) \exp(j\omega_c t)\} \\ &= \text{Re}\{z_2(t) \exp(j\omega_c t)\} \end{aligned} \quad (3.14)$$

where

$$z_2(t) = z_1(t-T). \quad (3.15)$$

The LPF output of I-channel detector $q_I(t)$ is expressed as a function of $z_1(t)$ and $z_2(t)$ [26]

$$\begin{aligned} q_I(t) &= \frac{1}{2} \text{Re}[z_1(t)z_2^*(t)] \\ &= \frac{1}{4}[z_1^*(t)z_2(t) + z_1(t)z_2^*(t)]. \end{aligned} \quad (3.16)$$

By setting sample timing at $t = nT$, (3.16) becomes

$$q_I = \frac{1}{4}[z_1^*(nT)z_2(nT) + z_1(nT)z_2^*(nT)] \quad (3.17)$$

We define two Gaussian vectors $\mathbf{A}$ and $\mathbf{Z}$ as

$$\mathbf{A}' = [z_s \quad z_i \quad z_n] \quad (3.18)$$

and

$$\mathbf{Z}' = [z_1 \quad z_2] \quad (3.19)$$

($\mathbf{A}'$ here denotes tranpose of $\mathbf{A}$). Their covariance matrices are computed as follows (the angle brackets denote the average with respect to the time)

$$\begin{aligned} \mathbf{K}_A &= \frac{1}{2} \langle (\mathbf{A} - \langle \mathbf{A} \rangle)^* (\mathbf{A} - \langle \mathbf{A} \rangle)' \rangle \\ &= \begin{bmatrix} \sigma_s^2 & 0 & 0 \\ 0 & \sigma_i^2 & 0 \\ 0 & 0 & \sigma_n^2 \end{bmatrix} \end{aligned} \quad (3.20)$$

where

$$\begin{aligned} \sigma_s^2 &= \frac{1}{2} \langle z_s, z_s^* \rangle \\ \sigma_i^2 &= \frac{1}{2} \langle z_i, z_i^* \rangle \\ \sigma_n^2 &= \frac{1}{2} \langle z_n, z_n^* \rangle \end{aligned} \quad (3.21)$$

and

$$\begin{aligned} \mathbf{K}_Z &= \frac{1}{2} \langle (\mathbf{Z} - \langle \mathbf{Z} \rangle)^* (\mathbf{Z} - \langle \mathbf{Z} \rangle)' \rangle \\ &= \begin{bmatrix} \sigma^2 & \sigma^2 \rho^* \\ \sigma^2 \rho & \sigma^2 \end{bmatrix} \end{aligned} \quad (3.22)$$

where

$$\sigma^2 = \sigma_s^2 + \sigma_i^2 + \sigma_n^2 \quad (3.23)$$

$$\sigma^2 \rho = \sigma_s^2 \rho_s(T) e^{j\Delta\phi_s} + \sigma_i^2 \rho_i(T) \langle e^{j\Delta\phi_i} \rangle + \sigma_n^2 \rho_n(T) \quad (3.24)$$

Here $\rho_s(\tau)$, $\rho_i(\tau)$, $\rho_n(\tau)$ are the correlation coefficient functions of the processes $z_s(t)$, $z_i(t)$, $z_n(t)$ respectively. If $S_s(f)$, $S_i(f)$, $S_n(f)$ are the equivalent lowpass power density spectra of the corresponding processes after passing through the BPF, then their

correlation coefficient functions are the inverse Fourier transforms of the power spectra

$$\begin{aligned}\rho_s(\tau) &= \frac{1}{\sigma_s^2} \int_{-\infty}^{\infty} S_s(f) \exp(j2\pi\tau f) df \\ \rho_i(\tau) &= \frac{1}{\sigma_i^2} \int_{-\infty}^{\infty} S_i(f) \exp(j2\pi\tau f) df \\ \rho_n(\tau) &= \frac{1}{\sigma_n^2} \int_{-\infty}^{\infty} S_n(f) \exp(j2\pi\tau f) df\end{aligned}\quad (3.25)$$

$\Delta\phi_s, \Delta\phi_i$ represent the phase differences between two consecutive samples, i.e.,

$$\Delta\phi_d = \phi_d(nT) - \phi_d((n-1)T) \quad (d = s, i) \quad (3.26)$$

Assume the interference signal is another $\pi/4$ -QDPSK modulated signal with equally probable symbols, then $\langle e^{j\Delta\phi_i(T)} \rangle = \frac{1}{4} \sum_{n=-1}^2 e^{j\frac{2n-1}{4}\pi} = 0$ holds. Then (3.23-3.24) is simplified to

$$\sigma^2 \rho = \sigma_s^2 \rho_s(T) e^{j\Delta\phi_s} + \sigma_n^2 \rho_n(T) \quad (3.27)$$

Now let us transform vector $\mathbf{Z}$ into vector $\mathbf{U}' = [u_1, u_2]$ by the transformation

$$\mathbf{U} = \mathbf{LZ}, \quad (3.28)$$

where

$$\mathbf{L} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (3.29)$$

The covariance matrix of U can be computed as follows

$$\begin{aligned} \mathbf{K}_U &= \mathbf{L} \mathbf{K}_Z \mathbf{L}^T \\ &= \begin{bmatrix} 2\sigma^2 + \sigma^2\rho + \sigma^2\rho^* & \sigma^2\rho - \sigma^2\rho^* \\ \sigma^2\rho^* - \sigma^2\rho & 2\sigma^2 - \sigma^2\rho - \sigma^2\rho^* \end{bmatrix} \end{aligned} \quad (3.30)$$

From (3.17) and (3.28), q_I becomes

$$q_I = \frac{1}{8}(U_1 U_1^* - U_2 U_2^*) \quad (3.31)$$

The join pdf $p(U_1, U_2)$ can be obtained as [25]

$$\begin{aligned} p(U_1, U_2) &= \frac{1}{(2\pi)^2 4\sigma^4 (1 - |\rho|^2)} \exp \left\{ -\frac{1}{8\sigma^4 (1 - |\rho|^2)} \right. \\ &\quad \cdot \frac{1}{4\sigma^4 (1 - |\rho|^2)} [K_{11} U_1 U_1^* + K_{22} U_2 U_2^* + \\ &\quad \left. K_{21} (U_1^* U_2 - U_1 U_2^*)] \right\}, \end{aligned} \quad (3.32)$$

where

$$\begin{aligned} K_{11} &= 2\sigma^2(1 + \rho_{re}) \\ K_{22} &= 2\sigma^2(1 - \rho_{re}) \\ K_{21} &= -2j\sigma^2\rho_{im} \end{aligned} \quad (3.33)$$

which are directly obtained from (3.30). ρ_{re} , ρ_{im} are real and imaginary parts of ρ .

Let us make another transformation on vector U by setting

$$U_1 = R_1 \exp(j\theta_1) \quad (3.34a)$$

$$U_2 = R_2 \exp(j\theta_2). \quad (3.34b)$$

Then from (3.31), (3.34a) and (3.34b), q_I becomes

$$q_I = \frac{1}{8}(R_1^2 - R_2^2) \quad (3.35)$$

From (3.32) and (3.34), the join *pdf* of $R_1, \theta_1, R_2, \theta_2$ can be worked out :

$$p(R_1, \theta_1, R_2, \theta_2) = \frac{R_1 R_2}{(2\pi)^2 4\sigma^4 (1 - |\rho|^2)} \exp \left\{ -\frac{1}{8\sigma^4 (1 - |\rho|^2)} \right. \\ \left. \cdot [K_{11} R_1^2 + K_{22} R_2^2 - 2j K_{21} \sin(\theta_1 - \theta_2)] \right\} \quad (3.36)$$

The join *pdf* of R_1, R_2 , is thus

$$p(R_1, R_2) = \int_0^\infty \int_0^\infty p(R_1, \theta_1, R_2, \theta_2) d\theta_1 d\theta_2 \\ = \frac{R_1 R_2}{4\sigma^4 (1 - |\rho|^2)} I_0 \left(\frac{2\rho_{im} R_1 R_2}{4\sigma^4 (1 - |\rho|^2)} \right) \\ \cdot \exp \left\{ -\frac{1}{4\sigma^4 (1 - |\rho|^2)} [(1 - \rho_{re}) R_1^2 + (1 + \rho_{re}) R_2^2] \right\} \\ (0 \leq R_1, R_2 < \infty) \quad (3.37)$$

From (3.35) we have

$$\text{Prob}[q_I < 0] = \text{Prob}[R_1 < R_2] = \frac{1}{2} \left(1 - \frac{\rho_{re}}{\sqrt{1 - \rho_{im}^2}} \right) \quad (3.38)$$

Similarly,

$$\text{Prob}[q_I > 0] = \frac{1}{2} \left(1 + \frac{\rho_{re}}{\sqrt{1 - \rho_{im}^2}} \right) \quad (3.39)$$

Combine (3.38) and (3.39), (3.8) becomes

$$P_L = \frac{1}{4} \left\{ 2 - \frac{1}{2} \left[\frac{\rho_{re}(\pi/4)}{\sqrt{1 - \rho_{im}^2(\pi/4)}} + \frac{\rho_{re}(\pi/4)}{\sqrt{1 - \rho_{im}^2(\pi/4)}} - \frac{\rho_{re}(3\pi/4)}{\sqrt{1 - \rho_{im}^2(3\pi/4)}} - \frac{\rho_{re}(-3\pi/4)}{\sqrt{1 - \rho_{im}^2(-3\pi/4)}} \right] \right\} \quad (3.40)$$

3.2.3 Numerical Result

In a multipath fading environment, $\rho_s(\tau)$ is found to have the form of a Bessel function (see chapter 2, section 2.1):

$$\rho_s(\tau) = J_0(2\pi f_m \tau) \quad (3.41)$$

For simplicity, we consider only the case of uncorrelated noise:

$$\rho_n(T) = 0 \quad (3.42)$$

In view of (3.41) and (3.42), (3.40) is simplified to

$$P_L = \frac{1}{2} \left[1 - \frac{\sigma_s^2 J_0^2(2\pi f_m T)}{\sqrt{2\sigma^4 - \sigma_s^4 J_0^2(2\pi f_m T)}} \right] \quad (3.43)$$

where

$$\sigma^2 = \sigma_s^2 + \sigma_i^2 + \sigma_n^2 \quad (3.44)$$

Define Γ the average CNR, and Λ the average CIR

$$\Gamma = \frac{\sigma_s^2}{\sigma_n^2}, \quad \Lambda = \frac{\sigma_s^2}{\sigma_i^2} \quad (3.45)$$

then (3.43) can be rewritten as

$$P_L = \frac{1}{2} \left(1 - \Gamma \Phi^{-\frac{1}{2}} J_0(2\pi f_m T) \right) \quad (3.46)$$

where

$$\Phi = 2 \left(\Gamma + \frac{\Gamma}{\Lambda} + 1 \right)^2 - \Gamma^2 J_0^2(2\pi f_m T)$$

In the case of no cochannel interference, i.e., $\Lambda \rightarrow \infty$, (3.46) reduces to

$$P_L = \frac{1}{2} \left(1 - \Gamma \tilde{\Phi}^{-\frac{1}{2}} J_0(2\pi f_m T) \right) \quad (3.48)$$

where

$$\tilde{\Phi} = 2(\Gamma + 1)^2 - \Gamma^2 J_0^2(2\pi f_m T)$$

Equation (3.48) is plotted with two values of $f_m T$: 0.01 and 0.001. The plot is shown in Figure 3.4a and 3.4b. The exact numerical values are found in the table in appendix A.

3.3 Error Probability When The Envelope is Rayleigh and Lognormally Distributed

When the mean of the signal envelope is Rayleigh faded with a mean lognormally distributed, then the mean power of $z_s(t)$ (measured over relatively short distances) is no longer constant over long distances. The quantity σ_s^2 thus becomes a random variable $\sigma_s^2(t)$ having a lognormal distribution. It is therefore necessary to include the *pdf* of σ_s^2 to completely characterize the received signal. However, because the variable involved in the equation 3.46 is the signal to noise ratio $\frac{\sigma_s^2}{\sigma_n^2}$, hence rather than obtaining the *pdf* of σ_s^2 itself, it is more useful to establish the *pdf* of the random variable $\frac{\sigma_s^2}{\sigma_n^2}$.

Figure 3.4 Theoretical bounds of $\pi/4$ -QDPSK symbol error probability in a Rayleigh fading channel with two values of $f_m T$: a) $f_m T = 0.01$ b) $f_m T = 0.001$

Let $\gamma(t) = \frac{\sigma_s^2(t)}{\sigma_n^2(t)}$ and Γ be the mean of $\gamma(t)$

$$\Gamma = \langle \gamma(t) \rangle = \langle \frac{\sigma_s^2}{\sigma_n^2} \rangle \quad (3.50)$$

then the *pdf* of γ can be derived as follows:

$$\begin{aligned} p(\gamma) &= p(\sigma_s^2) \frac{d\sigma_s^2}{d\gamma} \\ &= p(\sigma_s^2) \sigma_n^2 \\ &= \frac{\log e}{\sigma_s^2 \sqrt{2\pi}\sigma} \exp \left[-\frac{(10 \log \sigma_s^2 - 10 \log \langle \sigma_s^2(t) \rangle)^2}{2\sigma^2} \right] \sigma_n^2 \\ &= \frac{\log e}{\gamma \sqrt{2\pi}\sigma} \exp \left[-\frac{((10 \log \sigma_s^2 - 10 \log \sigma_n^2) - (10 \log \langle \sigma_s^2(t) \rangle - 10 \log \sigma_n^2))^2}{2\sigma^2} \right] \\ &= \frac{\log e}{\gamma \sqrt{2\pi}\sigma} \exp \left[-\frac{(10 \log \gamma - 10 \log \Gamma)^2}{2\sigma^2} \right] \end{aligned} \quad (3.51)$$

Similarly, the *pdf* of the random variable $\delta = \frac{\sigma_i^2}{\sigma_n^2}$ (namely the interference to noise ratio) can be proved to be

$$p(\delta) = \frac{\log e}{\delta \sqrt{2\pi}\sigma} \exp \left[-\frac{(10 \log \delta - 10 \log \Delta)^2}{2\sigma^2} \right] \quad (3.52)$$

where $\Delta = \langle \delta \rangle = \langle \frac{\sigma_i^2}{\sigma_n^2} \rangle$.

Assuming the rate of change of the signal envelope due to the newly introduced lognormal fading is slow compared with fast Rayleigh fading such that the signal envelope remains constant over a period of T seconds (symbol period), the wanted results can be obtained by integrating the expression in equation (3.46) over the *pdf* of $\frac{\sigma_s^2}{\sigma_n^2} = \gamma$ and $\frac{\sigma_i^2}{\sigma_n^2} = \delta$. Thus we have

$$P_L = \int_0^\infty \int_0^\infty P_L \left(\frac{\sigma_s^2}{\sigma_n^2} = \gamma, \frac{\sigma_i^2}{\sigma_n^2} = \delta \right) p(\gamma) p(\delta) d\gamma d\delta \quad (3.53)$$

where the first term of the the integrand is the same as that in equation 3.46 with Γ replaced by γ , and Γ/Λ replaced by δ . That is

$$P_L\left(\frac{\sigma_s^2}{\sigma_n^2} = \gamma, \frac{\sigma_i^2}{\sigma_n^2} = \delta\right) = \frac{1}{2}\left(1 - \gamma\Phi^{-\frac{1}{2}}J_0(2\pi f_m T)\right) \quad (3.54)$$

where $\Phi = 2(\gamma + \delta + 1)^2 - \gamma^2 J_0^2(2\pi f_m T)$, $p(\gamma)$ and $p(\delta)$ are given as in 3.51 and 3.52 respectively.

The integrated interval is positive, thus the bounding characteristics do not change after the integration, i.e., P_L in (3.53) remains being the lower bound. The same conclusion can be applied to the upper bound which is easily seen to equal twice P_L , where P_L is given in (3.53).

When there is no cochannel interference, (3.53) is reduced to a single integral

$$P_L = \int_0^{\infty} P_L\left(\frac{\sigma_s^2}{\sigma_n^2} = \gamma\right) p(\gamma) d\gamma \quad (3.55)$$

where

$$P_L\left(\frac{\sigma_s^2}{\sigma_n^2} = \gamma\right) = \frac{1}{2}\left(1 - \gamma\tilde{\Phi}^{-\frac{1}{2}}J_0(2\pi f_m T)\right) \quad (3.56)$$

$$\tilde{\Phi} = 2(\gamma + 1)^2 - \gamma^2 J_0^2(2\pi f_m T)$$

The integrals in (3.53) and (3.55) involve quite a number of complex functions and are difficult to simplify. The integrals are therefore numerically evaluated using the computer software Mathematica. Mathematica uses adaptive integration method to approximate the expression. It takes the necessary number of points to achieve the required accuracy. It looks for the singular point of the integrand, and will take more samples if the integrand is changing rapidly in that region. With those features, P_L in

(3.53) and (3.55) can be easily evaluated by Mathematica with good accuracy and little effort. The intervals to be integrated are specified (in Mathematica) as those appearing in the integral, i.e., $[0, \infty]$.

Equation (3.55) (no cochannel interference case) is evaluated for two values of normalized fading rates: $f_m T = 0.01, 0.001$; and two values of standard deviation of the mean: $\sigma = 6, 9$ dB. The results are shown in Figure 3.5a, 3.5b, 3.5c, 3.5d.

Equation (3.53) is evaluated for CIR of 5, 10, 15, 20 dB; $f_m T = 0.01, 0.001$; $\sigma = 6, 9$ dB. The results are shown in Figure 3.6a and 3.6b. From Figure 3.6, it is seen that interference has a strong effect on SER performance. The irreducible symbol error rate is several order of magnitude higher than in the non-interference case.

The results given in this chapter are for reference purpose only. They will be used to check the simulation results, which are presented in chapter 5. Further discussion therefore is deferred until final results are presented in chapter 5.

Figure 3.5 Theoretical bounds of $\pi/4$ -QDPSK symbol error probability in a Rayleigh-lognormal fading channel with $f_m T$ and σ as parameters : a) $f_m T = 0.001$, $\sigma = 6$ dB b) $f_m T = 0.001$, $\sigma = 9$ dB c) $f_m T = 0.01$, $\sigma = 6$ dB d) $f_m T = 0.01$, $\sigma = 9$ dB

Figure 3.5 (cont'd)

Figure 3.6 Theoretical bounds of $\pi/4$ -QDPSK symbol error probability in a Rayleigh-lognormal channel subjected to cochannel interference with $f_m T$, σ , and CIR as parameters : a) $f_m T = 0.01$ b) $f_m T = 0.001$

Chapter 4

SIMULATION IMPLEMENTATION

This chapter describes the simulation implementation in a bottom-up fashion, i.e., low level modules are outlined first, followed by discussion on the higher level modules, and the complete system.

The software package used to implement the simulation, BOSS (*Block Oriented Systems Simulator*), is introduced in the section 4.1. The “sum of nine sinusoids” Rayleigh fading channel model is briefly outlined, and its implementation is given in section 4.2. The implementation of the long-term fading signal when accompanied by the short-term fading signal is based on Lee’s model and given in section 4.3. Implementations of the channel as well as the structure of the transmitter, the receiver, and the interference are given in detail.

All the signals are implemented using their lowpass complex envelope equivalents.

4.1 Introduction to BOSS

BOSS (*Block Oriented Systems Simulator*) [27] is a general purpose simulation package designed to run under the VMS operating system on the DEC VAXstation or under the UNIX operating system on the Sun-3 Workstations. BOSS assumes that modules, subsystems, and systems can be represented in a hierarchical block diagram form, i.e., the existing models are used to construct new models. BOSS provides a model library which contains a large number of functional blocks for the simulation of communication systems. In order to simulate a system, a block diagram of the system must be constructed by connecting blocks in the desired topology.

The majors components of the BOSS software package are written in LISP while the simulation program to be executed is written by BOSS in FORTRAN based on the specified block diagram.

BOSS is discrete time simulation package (i.e. signal waveforms only exist at uniformly spaced sample times called samples). A module interacts with other modules only at sample times and only through a visible set of input and output signals. Also, a module may contain user specified internal parameters controlling module operation. The values of the internal module parameters do not change during the execution of a simulation.

A simulation in BOSS starts generating signal samples as specified by the system block diagram at time equal zero, and continues generating samples separated by the simulation step time (DT) until the simulation stop time (STOP-TIME).

During the simulation, selected values can be collected and saved on a signal file which is used to display the simulation results for analysis purposes later.

BOSS has the capabilities to display the simulation results in a variety of formats including time domain plots (time plots, scatter diagram etc.), frequency domain plots (amplitude, phase and power spectral plots etc.), and other displays (histograms, correlation plots etc.).

BOSS Limitation

During the simulation course, a software bug was discovered. The bug occurs when the 3 dB bandwidth parameter in any lowpass filter module provided by BOSS is too narrow compared to the sampling bandwidth. Specifically, if the filter bandwidth to sampling bandwidth ratio $< 10^{-4}$ then the filter will not function properly.

Due to this limitation, the difficulty arises when the fading processes are too slow compared to the data rate. As a consequence, wider fading bandwidths for such processes are used to solve the problem. Further details will be discussed when the simulation parameters are specified later in the subsequent sections.

4.2 Rayleigh Fading Simulation

4.2.1 Realization Method

Jakes [28] describes a method to simulate mobile radio fading that produces random phase modulation, a Rayleigh fading envelope, and a time-averaged, discrete approximation to the power spectrum of the form in equation 2.13.

We will present here the realization of the simulation method without going through the mathematical development, which can be found in [28]. Some modifications are made so that the output is an equivalent complex baseband of the passband multipath interference.

Figure 4.1 shows a block diagram of such a simulator.

According to Jakes, N_0 oscillators ($N_0 \geq 8$) with frequencies equal to the Doppler shifts $\omega_m \cos\left(\frac{2\pi n}{N}\right)$ ($N = 4N_0 + 2$; $n = 1, 2, \dots, N_0$) plus one frequency ω_m are used to generate signals frequency shifted from a carrier frequency ω_c (0 Hz carrier frequency in our baseband representation). The phases β_n and the value α are chosen appropriately so that the probability distribution of the resultant phase will be as close as possible to a uniform distribution, $1/2\pi$. The outputs of the individual oscillators, with appropriate gain factors, are summed to produce in-phase (x_c) and quadrature (x_s) bands, and then combined to produce the complex baseband output signal $y(t)$.

From the block diagram we get

$$\begin{aligned} x_c(t) &= 2 \sum_{n=1}^{N_0} \cos \beta_n \cos \omega_n t + \sqrt{2} \cos \alpha \cos \omega_m t \\ x_s(t) &= 2 \sum_{n=1}^{N_0} \sin \beta_n \cos \omega_n t + \sqrt{2} \sin \alpha \cos \omega_m t \end{aligned} \quad (4.1)$$

Hence

$$\langle x_c^2 \rangle = 2 \sum_{n=1}^{N_0} \cos^2 \beta_n + \cos^2 \alpha \quad (4.2)$$

$$= N_0 + \cos^2 \alpha + \sum_{n=1}^{N_0} \cos 2\beta_n \quad (4.3)$$

$$\langle x_s^2 \rangle = 2 \sum_{n=1}^{N_0} \sin^2 \beta_n + \sin^2 \alpha \quad (4.4)$$

$$= N_0 + \sin^2 \alpha - \sum_{n=1}^{N_0} \cos 2\beta_n \quad (4.5)$$

$$\langle x_c x_s \rangle = 2 \sum_{n=1}^{N_0} \cos \beta_n \sin \beta_n + \sin \alpha \cos \alpha \quad (4.6)$$

$$N = 4N_0 + 2$$

$$\beta = \frac{\pi n}{N_0} \quad n = 1, 2, \dots, N_0$$

$$\omega_0 = \omega_m \cos\left(\frac{2\pi n}{N}\right)$$

Figure 4.1 Simulator that duplicates mobile radio spectrum (baseband equivalent)

The phase of $y(t)$ must be random and uniformly distributed from zero to 2π ; this may be accomplished in several ways provided $\langle x_c^2 \rangle \approx \langle x_s^2 \rangle$ and $\langle x_c x_s \rangle \approx 0$. Choose $N_0 = 8$, $\alpha = \frac{\pi}{4}$, $\beta_n = \frac{\pi n}{N_0} = \frac{\pi n}{8}$ then (4.2–4.6) become

$$\langle x_c^2 \rangle = \langle x_s^2 \rangle = 8.5 \quad (4.7)$$

since $\sum_{n=1}^8 \cos\left(\frac{2\pi n}{8}\right) = 0$, and $\langle x_c x_s \rangle \approx 0$.

Figure 4.2 shows the frequencies and magnitudes of the sinusoids when the parameter are chosen as such.

Another independent Rayleigh fading signal can be generated by providing additional phase shifts to the oscillator. The following phase shifts are used :

$$\gamma_n = \frac{\pi n}{9} \quad (n = 1, 2, \dots, N_0) \quad (4.8)$$

4.2.2 BOSS Implementation

The block diagram in Figure 4.2 implemented in BOSS is shown in Figure 4.3. In order to simplify the layout of the system, the two magnitudes (shown on the two arrows going out from the oscillators) are combined into one complex magnitude. The result is that only one adder is needed. A gain module is provided to normalize the mean square of the output to unity.

Block descriptions and Parameter values

The COS module generates a sinusoidal waveform of specified amplitude, frequency, and phase. The output of the module is $A \cos(2\pi F t + Q)$, where A is the amplitude, F is the frequency in Hertz, Q is the phase in radians. By numbering the

Figure 4.2 Generation of Rayleigh fading signal showing the frequencies and magnitudes of the sinusoids.

Figure 4.3 BOSS block diagram to simulate Rayleigh fading with mobile radio spectrum.

COS modules from 1 to 9, the parameter values of the nine modules, taken directly from those shown in Figure 4.2, are recorded in the table 4.1. Also appearing in the table are the phase shifts when generating the additional independent Rayleigh fading signal.

4.3 Simulation of Lognormal Fading Superimposed by Rayleigh Fading

Generation of lognormal fading signal

The lognormal fading signal is generated from a normally distributed signal which is then transformed by the function $z = 10^{\frac{x}{10}}$. The varying rate of the signal is controlled by filtering the signal using a lowpass filter. Figure 4.4 shows the internal structure of the lognormal fading module. The GAUSS GENERATOR module produces random samples having Gaussian probability distribution with zero mean and standard deviation σ . This normally distributed signal is filtered by the LOWPASS FILTER module. The Butterworth filter was chosen, but any other types of filter can also be used. This control mechanism allows the generation of the signal at any desired rate which is relatively slow compared to the Rayleigh fading.

Lognormal-plus-Rayleigh fading signal

The lognormal superimposed on the Rayleigh fading signal is produced by taking the product of the Rayleigh fading signal, described in section 4.2, and the lognormal fading signal, previously outlined. The corresponding BOSS block diagram is shown in Figure 4.5.

The LOGNORMAL+RAYLEIGH FADING module simulates a Rayleigh fading signal with its mean varies slowly according to the lognormal distribution. The mean square of the composite signal is normalized to unity which is obtained over a large

COS Module	Amplitude (V)	Frequency (Hz)	Phase shift (radians) of Rayleigh 1	Phase shift (radians) of Rayleigh 2
1	$1.848 + j 0.765$	$0.983 f_m$	0	0.3490
2	$1.414 + j 0.414$	$0.932 f_m$	0	0.6981
3	$0.765 + j 1.848$	$0.350 f_m$	0	1.0472
4	$0.000 + j 2.000$	$0.739 f_m$	0	1.3963
5	$-0.765 + j 1.848$	$0.603 f_m$	0	1.7453
6	$-1.414 + j 1.414$	$0.446 f_m$	0	2.0943
7	$-1.848 + j 0.765$	$0.274 f_m$	0	2.4434
8	$-2.000 + j 0.000$	$0.092 f_m$	0	2.7925
9	$1.000 + j 1.000$	$1.000 f_m$	0	3.1416

Table 4.1 The parameter values of the COS modules.

f_m is the input maximum Doppler frequency.

**Figure 4.4 Internal structure of lognormal module
showing the generation of lognormal fading**

Figure 4.5 Generation of lognormal plus Rayleigh fading signal.

number of samples. Within a smaller number of samples, the Rayleigh statistics of the signal is observed to also have a mean of unity.

Parameter Descriptions

The long-term fading rate f_l should be chosen such that the long-term fading process, as its name implies, effects the signal slowly over a long period of time. The term 'slowly' fading means 'slow relative to the short-term fading (or fast fading)'. To exhibit the long-term fading characteristic, as opposed to the short-term fading, f_l should be much smaller than f_m .

In actual channel, the long-term variation could be as low as seventy times more than the short-term variation [8]. However, it is not practical to simulate such a slow process because the simulation time will be excessively long. As long as the relative rate between the slow (short-term) fading and the fast (short-term) fading is small, the results will not vary notably. The difference caused by different relative fading rates is the number of simulated samples needed to obtain equivalent results. The slower the long-term signal varies compared to the short-term signal, the more samples are required to produce comparable effects on probability of error. The value $f_m/f_l = 10$ is seen to be a reasonable ratio for such purposes. It permits the distinction between the Rayleigh and lognormal fading signals without taking too much simulation time.

When the Rayleigh fading rate is one thousand times slower than the symbol rate ($f_m T = 0.001$), the desired lognormal fading rate is much too slow to be produced by any of BOSS lowpass filter modules. In this case, the lognormal fading rate f_l is increased to a minimum value at which a BOSS filter module works properly. The maximum value that ratio f_m/f_l can attain without causing the filter failure is 2.5. This

value is therefore chosen to be one of the simulation parameter value.

4.4 Additive White Gaussian Noise

The BOSS module COMPLX GAUSS WHITE NOISE is used to simulate zero mean complex white Gaussian noise. The noise spectrum spreads constantly over the entire sampling bandwidth, i.e., from -4 Hz to 4 Hz in our simulation (detail on the sampling bandwidth is referred to section 4.8.3). Since the noise has zero mean, the power equals its variance. The real and imaginary part of the complex noise each has variance $\sigma_n^2/2$, thus the total power noise power is σ_n^2 . Because the carrier power is fixed (at 1 Watt), the noise variance actually determines the *CNR*.

4.5 Transmitter Implementation

The structure of the transmitter is shown in Figure 4.7. The SYMBOL GENERATOR module produces symbols (namely 0, 1, 2, and 3) at equal probability of occurrence. The SYMBOL-TO-PHASE SHIFT module converts the symbols to the equivalent phase changes according to the following table:

Data Symbol	Phase change $\Delta\Phi$
0	$\pi/4$
1	$3\pi/4$
2	$-3\pi/4$
3	$-\pi/4$

Table 4.2 Phase change as a function of data symbol in $\pi/4$ shifted QDPSK prototype

The DELAY module simply delays the signal by T seconds (T is the symbol period). The output of the adder thus equals the previous phase value plus the current phase shift value produced by the SYMBOL-TO-PHASE SHIFT module. This is the absolute phase to be sent through the channel. Finally, the module PHASE-TO-COMPLX produces a complex number whose phase is the module input, and whose magnitude equals to *unity*.

4.6 Receiver Implementation

The QDPSK demodulator operation is simply the reverse of that of the modulator. The structure of the receiver is shown in Figure 4.8. Basically, the detector module extracts the phases of the two consecutive received (complex) signals and computes their difference. The PHASE SHIFT-TO-SYMBOL module converts the phase difference to the corresponding symbols based on the decision region displayed in Figure 4.6. If the phase difference is between 0 and $\pi/2$, the received signal is detected as the symbol 0; if the phase difference is between $\pi/2$ and π , the received signal is detected as the symbol 1, etc. Finally, the detected symbols are compared with the originally transmitted symbols, and the symbol error probability is calculated.

It should be pointed out that the receiver measures the incoming phases at midpoint of the symbol duration, i.e., if a symbol is sent out at time nT , then the detector will compute the difference of the phases based on samples taken at times $(n-1/2)T$ and $(n+1/2)T$.

Some authors analyzed the system based on the detection of each bit on independent quadrature channels (I and Q channels). In actual systems, lowpass filters are used to remove high frequency signals, in our case the twice carrier frequency signal, result-

Figure 4.6 Decision regions of the $\pi/4$ shifted QDPSK detector

Figure 4.7 Internal structure of the transmitter module

Figure 4.8 Internal structure of the receiver module

ing in the signal detection at baseband. When simulating at baseband, the inclusion of those lowpass filters is not necessary.

Should in the simulation model a lowpass filter be included at the receiver to suppress the noise and other undesired signals, its effect would be equivalent to that of the *bandpass filter*. In case the lowpass filter bandwidth B is much greater than the signal bandwidth ($B \gg 1/T$), the noise samples at T seconds apart are still uncorrelated, thus the results will not change significantly from the non-filter case. However, if the lowpass filter bandwidth is less than the signal bandwidth ($B < 1/T$), then the noise samples at T seconds interval become correlated, will give poor results for differential modulation systems. In addition, much too narrow a lowpass filter bandwidth will degrade the system performance because of excessive intersymbol interference.

4.7 Cochannel Interference Implementation

The interfering signal is considered to be another $\pi/4$ shifted QDPSK modulated signal originating independently from another base station. This signal also subjects to the same fading conditions as does the desired signal. Therefore, the cochannel interference can be implemented as depicted in Figure 4.9. The modules QDPSK TXT 1 and QDPSK TXT 2 generate signals independent of each other by using a different seed in the random number generator. Besides, while the power of the desired signal is fixed (equal to *unity*), the power of the interfering signal is varied in order to vary CIR. The signal produced by RAYLEIGH + LOGNORMAL 2 module should be independent from that in RAYLEIGH + LOGNORMAL 1 module. The module RAYLEIGH + LOGNORMAL 2 is implemented follows the description in section 4.2.

Figure 4.9 The interfering signal subjects to the same fading conditions as does the desired signal

Furthermore, we have assumed that the standard deviation parameters in both modules RAYLEIGH + LOGNORMAL 1 and RAYLEIGH + LOGNORMAL 2 are identical.

4.8 System Implementation and Parameters

The BOSS block diagram for the complete system under investigation is shown in Figure 4.10. Following is the list of the parameters and their values used in the simulation.

4.8.1 Fixed parameters

The two fixed parameters are the symbol period and the signal power. For simplicity, the symbol period is chosen to be 1 second, and the signal power is chosen to be 1 Watt.

4.8.2 Derived parameters

The parameters which related to the symbol period and the signal power are to be derived from them. They are:

Normalized fading bandwidth f_m We want to run the simulation at two normalized fading rates: $f_m T = 0.01$ and $f_m T = 0.001$. They correspond to a vehicle speed of 300 km/h and 30 km/h respectively for a 48 kbits/sec system having a carrier frequency of 850 MHz. The former fading rate value may be too high for a practical system, but is of interest from the theoretical point of view. The symbol period T is chosen to be 1 second (see section 4.8.1), therefore the two normalized values of f_m used in the simulation are $f_m = 0.01$ and $f_m = 0.001$.

Figure 4.10 The simulation block diagram of a QDPSK system under Rayleigh and lognormal fading conditions

Normalized lognormal fading rate f_l This parameter is chosen as follows: when $f_m T = 0.01$ then $f_l T = 0.001$; when $f_m T = 0.001$ then $f_l T = 4 \times 10^{-4}$. This is due to the limitation in filter implementation in BOSS as explained in section 4.3.

Carrier-to-interference ratio CIR This value is set at 5, 10, 15, and 20 dB.

Carrier-to-noise ratio CNR This value is set at 5 to 60 dB in steps of 5 dB.

Standard deviation σ This value is set at 6 and 9 dB, which correspond to typical shadow spread in suburban and urban areas respectively.

4.8.3 System parameters

We add here the two system simulation parameters: DT and STOP-TIME. DT is the time interval between signal samples and STOP-TIME is the time duration of the simulation.

The sampling rate, $1/DT$, is chosen so that it is greater than twice the highest frequency present in the signals. In our system, the highest rate signal, which is the QDPSK modulated signal, is normalized to 1 ($1/T = 1$). DT is therefore chosen to be 1/8 seconds. This ensures that the aliased power falling into the simulation bandwidth is small compared to the noise power in the same bandwidth.

The STOP-TIME value is chosen carefully to obtain accurate estimates of symbol error rate without taking excessive simulation time. Usually, the simulation time duration is set so that a minimum of 100 symbol errors occur. In our case, however, the symbol error rate is not the only factor which effects the simulation time duration. In fact, the resulting symbol error probability is often quite high, thus does not create problem in setting the number of symbols per run. The burden rather lies while realization of very slowly varying processes such as the long-term fading process which needs many

more number of samples to simulate. When $f_i T = 0.001$, STOP-TIME is set at 100,000 seconds or higher. When $f_i T = 4 \times 10^{-4}$ then STOP-TIME is set at 500,000 seconds or higher. Because only one run per point is carried out, the error rates are collected at cumulative number of symbols, and examined to ensure that they eventually converge to a steady value. Another word, at some point the series made up by these collected error probabilities will have insignificant fluctuations around a value which is recorded as the ultimate result for the run.

Chapter 5

SIMULATION RESULTS

This chapter presents the simulation results. The first section, 4.1, gives the symbol error rate (SER) in presence of AWGN. Section 4.2 shows the SER when the system is subjected to Rayleigh fading. Section 4.3 displays the SER when the channel has both Rayleigh fading and lognormal shadowing. The final section, section 5.4, gives the SER results when the impairment such as Rayleigh fading, lognormal shadowing, and cochannel interference are all present.

The analytical bounds found in chap 3 are also shown along with the simulation SER to aid the reader in verifying the correctness of both analytical and simulation results. Due to a large number of simulated curves, only a few comparisons of the simulation and analytical results are made in this chapter. The check for the other results are referred to the tables in the appendix 1.

5.1 SER under only AWGN

The system is first simulated in the presence of only Gaussian noise. The SER performance of QDPSK in AWGN has been analytically studied by several authors, one of them Fleck and Trabca work is significant [29]. The theoretical results obtained from [29] and simulation results are compared in Figure 5.1. Figure 5.1 reveals an excellent agreement between the two results, and therefore asserts the accuracy of the simulation model and the software.

To get a feel of how well QDPSK system performs in presence of AWGN, the error rate performance of QDPSK system is compared with that of GMSK (Gaussian Minimum Shift Keying) system with coherent detection. This is shown in Figure 5.2. The theoretical curve of bit error rate (BER) versus E_b/N_0 for QDPSK system is taken from [29]. The GMSK system has a normalized bandwidth of the predetection rectangular bandpass filter $B_r T = 0.63$, and a normalized 3dB-down bandwidth of the premodulation Gaussian lowpass filter $B_b T = 0.25$. The measured static BER in nonfading environment in this case is approximated by [2] as

$$P_e(\gamma) = \frac{1}{2} \operatorname{erfc}(\sqrt{0.68\gamma}) \quad (5.1)$$

where $\gamma = E_b/N_0$, and erfc is the complementary error function. Figure 5.2 shows that QDPSK required 0.7 dB more than GMSK at high BER ($> 10^{-2}$) but this difference narrows at lower BER ($< 10^{-2}$). This is due to the use of 4 phases to transmit the signal in QDPSK. The trade-off is the narrower bandwidth required in QDPSK compared to that in GMSK. Also, while the need for carrier synchronization is bypassed in QDPSK, it remains a burden in GMSK where the coherent detection scheme is used.

Figure 5.1 SER performance of $\pi/4$ -QDPSK under AWGN

Figure 5.2 Comparison of BER performance of QDPSK and GSMK in an AWGN channel

5.2 SER under Rayleigh Fading

The system under Rayleigh fading was simulated for two values of $f_m T$: 0.01 and 0.001. The results of the simulation are first checked by plotting them with the theoretical bounds (refer to chapter 3) in Figure 5.3a and Figure 5.3b. The simulated curves are seen to lie nicely within the bounds, again supporting the reliability of the results. The simulation results for $f_m T = 0.01$, 0.001 and also for the case of no fading (theoretical) are shown in the same graph (Figure 5.4) so that the effect of fast fading can be examined. From the plot in Figure 5.4, the following observations are made:

1. Rayleigh fading caused by random multipath propagation results in a great degradation of SER performance. For example, for a SER of $1\text{E-}2$, the required CNR, in the presence of AWGN only, is about 10 dB, but in fast fading environment, at the normalized fading rate of $f_m T = 0.001$, is more than 22 dB, a difference of more than 12 dB. At lower symbol error rate, this difference increases. At SER of $1\text{E-}3$ and normalized fast fading rate $f_m T = 0.001$, the additional power over AWGN only case is about 20 dB. This is because as the CNR increases, the effect of fast fading becomes more dominant than the noise, which causes the increase in degradation of the system.
2. Fast fading results in irreducible symbol error rate, which is the lowest symbol error rate that can be achieved when the CNR goes to infinity. The irreducible SER is determined by the rate of the fading: the more severe the fading, the higher the irreducible SER. At $f_m T = 0.01$, the irreducible SER is about $2\text{E-}3$, but at $f_m T = 0.001$, the irreducible SER is only $1.5\text{E-}5$.
3. Between the curves impaired by Rayleigh fading, however, the SER performance is

not much effected by the fading rate at low CNR. From the plot, it is observed that the SERs for both Rayleigh fading curves are almost the same for CNR up to 18 dB.

Points 2) and 3) can be explained as follows: The error rate at any time is the result of Gaussian noise and the fast fading. At high CNR, i.e., the noise power is low, the error rate is mainly due to fading, which is the envelope level crossing rate as seen in the time domain or random shifts in the spectrum as seen in the frequency domain. As the CNR approaches infinity, fast fading remains the only factor determining the system error rate. This fading mechanism is not influenced by the amount of the signal power, thus setting a limit for the error rate that can be achieved. This is the consequence of phase carrying the information. At low CNR, however, the error rate is mostly contributed by the thermal noise, therefore the degradation due to fading does not make much difference. Another word, the overall error rates for different fading rates will be almost the same at low CNR.

5.3 SER under Rayleigh-Lognormal Fading

The simulation system was run with two values of standard deviations: $\sigma = 6$ and 9 dB. The normalized fading rates as in the previous simulation were used, i.e., $f_m T = 0.01$ and $f_m T = 0.001$, hence four sets of curves were produced. To verify the results, two curves in the set of four are chosen and plotted with the analytical curves (from chapter 3) in Figure 5.5a and Figure 5.5b. The simulated curves are seen to lie between the analytical curves for lower and upper bounds.

Next, the results are grouped together in different categories so that the effect of each fading component can be examined. Figure 5.6 and Figure 5.7 show the influence of the fast fading upon the SER performance for fixed standard deviation of the mean (σ

Figure 5.3 Comparison of simulation and analytical results for $\pi/4$ -QDPSK in a Rayleigh fading channel: a) $f_m T = 0.01$ b) $f_m T = 0.001$

Figure 5.4 Simulated SER performance of $\pi/4$ -QDPSK
in a Rayleigh fading channel for different $f_m T$

= 6, 9 dB, respectively). It can be seen that different Doppler shifts still result in quite distinguished irreducible SER, as seen earlier in non-lognormal fading channel case.

Figures 5.8 and 5.9 show the influence of shadowing for fixed fast fading rate. From the Figures, the following conclusions are drawn :

1. The degradation due to shadowing is low at low CNR (or high error rate), and increases with increasing CNR (or decreasing error rate). Comparing for example in Figure 5.8, the two curves $\sigma = 0$ dB and $\sigma = 6$ dB at $f_m T = 0.01$ in Figure for instance. For $1E-1$, and $\sigma = 6$ dB, the system degrades by 2 dB, but to obtain a SER of $1E-2$, an additional CNR of almost 5 dB is required. The explanation given in section 5.2 can be used again here to account for this phenomenon: at low CNR, the noise becomes one of the dominant factors affecting the error probability. When the noise is so high that it causes most of the errors, the effect of the fading becomes negligible. As the CNR increases, the effect of the noise becomes less significant, the effect of the fading becomes more considerable and it makes a noticeable difference in performance in presence and absence of lognormal fading.
2. Shadowing has considerable degradation effect on the SER performance. Higher fluctuation in the signal mean produces higher error rate. The error probability curve shifts to the right about 4 dB for $\sigma = 6$ dB and about 8 dB for $\sigma = 9$ dB at moderate values of CNR (between 15 to 35 dB). The values of standard deviations $\sigma = 6$ dB and $\sigma = 9$ dB represent typical variation of the signal mean in the suburban and urban areas respectively. The loss in the SER performance due to shadowing is thus increases in going from rural to suburban, to urban areas.
3. At very high CNR (> 50 dB), the system shows an irreducible error rate, which is

Figure 5.5 Comparison of simulation and analytical results for $\pi/4$ -QDPSK in a Rayleigh-lognormal channel: a) $\sigma = 6$ dB b) $\sigma = 9$ dB

Figure 5.6 Simulated SER performance of $\pi/4$ -QDPSK
in a Rayleigh-lognormal channel with $\sigma = 6$ dB

Figure 5.7 Simulated SER performance of $\pi/4$ -QDPSK
in a Rayleigh-lognormal channel with $\sigma = 9$ dB

accounted for by the effect of fast fading. This shows that the system performance in a Rayleigh-lognormal fading channel is limited by fast fading only. This is because the random FM effect produced by shadowing is negligible compared to the one produced by fast fading.

4. For $f_m T = 0.001$ at high CNR, there is some inaccuracy due to the limited number of symbols used in the simulation (10^6 symbols) compared to the SER of 10^{-5} . The actual number of errors that occurred in this case is approximately 20 (to give a SER of about 2×10^{-5}) which is too small to produce accurate results.

5.4 SER under Rayleigh-Lognormal-Cochannel Interference Conditions

The simulation was run with four different values of CIR : 5, 10, 15 and 20 dB. For each value of CIR, same values of $f_m T$ (fading rate) and σ (standard deviation) used in previous simulation are used to make a comparison. Total of 16 sets of curves were produced in this case. To verify the results, two curves in the set of 16 are chosen and plotted with the analytical curves (obtained from chapter 3) in Figure 5.10a and Figure 5.10b. It is seen from the Figures that the simulated curves lie well within the theoretical bounds.

The results are grouped and shown in two different ways in Figure 5.11 and Figure 5.12 to observe the effect of each impairment to the SER performance. In Figure 5.11, σ is fixed at 6 dB so that the effect of fast fading can be inspected. In Figure 5.12, a Doppler shift is chosen ($f_m T = 0.001$), so that the influence of shadowing and interference can be studied. From Figure 5.11, the following remarks are brought out :

2

of/de

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Figure 5.8 Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal fading channel for different values of σ ($f_m T = 0.01$)

Figure 5.9 Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal fading channel for different values of σ ($f_m T = 0.001$)

Figure 5.10 Comparison of simulation and analytical results of $\pi/4$ -QDPSK in a Rayleigh-lognormal-interference channel: a) $\text{CIR} = 15 \text{ dB}$, $\sigma = 6 \text{ dB}$, $f_m T = 0.001$ b) $\text{CIR} = 20 \text{ dB}$, $\sigma = 9 \text{ dB}$, $f_m T = 0.001$

Figure 5.11 Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal-interference channel ($\sigma = 6$ dB)

Figure 5.12 Simulated SER performance of $\pi/4$ -QDPSK in a Rayleigh-lognormal-interference channel ($f_mT = 0.001$)

1. The SER performance is strongly susceptible by interference. For instance, the irreducible SER in the case $\text{CIR} = 20 \text{ dB}$, $\lambda = 6 \text{ dB}$, is about $5\text{E-}2$, which is several order of magnitude higher than $1.5\text{E-}5$, the irreducible SER in the non-interference case.
2. the presence of interference removes the large difference of the irreducible SERs for the two fading rates. For example, without the interference, the irreducible SERs for $f_m T = 0.01, 0.001$ are $2\text{E-}3, 1.5\text{E-}5$, respectively. When $\text{CIR} = 15 \text{ dB}$, the irreducible SERs for $f_m T = 0.01, 0.001$ are $1.2\text{E-}1, 1.1\text{E-}1$, which is quite indistinguishable. This observation indicates that the system is quite insensitive to the fading rate in the presence of interference.

From Figure 5.12, it can be seen that lognormal fading has some recognizable effect on the error rate performance when the interference is weak, say $\text{CIR} = 20 \text{ dB}$, but virtually none when the interference is strong, say $\text{CIR} = 5, 10 \text{ dB}$.

While different fading rates have very little effect on the system performance in the presence of the interference, different standard deviations of shadowing produce quite divergent outcomes. Thus it can be stated in a general sense that the system is somewhat more sensitive to shadowing than it is to fast fading *in a cochannel interference environment*. This shows that the variation of the signal, when subjected to cochannel interference, is somewhat more dominant to the system performance when it takes place over the long term than in the short term.

Chapter 6

CONCLUSIONS

6.1 Summary

The error rate performance of $\pi/4$ -QDPSK system in a flat Rayleigh-lognormal fading channel subject to cochannel interference has been studied. The simulation model was established, the system was simulated, and the simulation results were obtained. A theoretical analysis was developed to produce SER lower and upper bounds. The simulation and analytical results were shown to agree well with each other, hence strongly supporting the correctness of both the simulation and the analysis.

To implement the Rayleigh fading channel, the “sum-of-nine-sinusoids” model is chosen. To simulate the long-term fading superimposed by the short-term fading, a model suggested by Lee [8] has been adapted.

The system was simulated for two values of Doppler shifts: $f_m = 0.01/T$, $0.001/T$, two values of standard deviations: $\lambda = 6, 9$ dB, and four values of CIR: 5, 10, 15, 20 dB.

In a Rayleigh-lognormal fading channel, the shadowing impairment with standard deviation of 6 dB degrades the system by 2 dB (at low CNR) and 4 dB (at high CNR). The shadowing impairment with standard deviation of 9 dB deteriorates the system even more, 5 dB (at low CNR) and 8 dB (at high CNR). The irreducible error rate in this condition (i.e., no interference) nevertheless does not depend on the shadowing, but rather on the Doppler shift. The higher the Doppler shift, the higher error probability the system attains.

When cochannel interference is introduced, in addition to the existing Rayleigh-lognormal fading channel, the SER performance gets much worse. Among the source of impairments being considered, the interference now becomes the limiting factor of the system. Both the Doppler shift f_m and the shadow spread σ are seen to have very little effect when the interference is present.

6.2 Suggestions for further work

The other impairments which may have significant effects on the system performance are frequency selectivity, intersymbol interference, and adjacent channel interference.

Frequency selectivity is considered important when the delay between the signal paths is relatively long comparable to the time slotted for one symbol.

Adjacent channel interference, though often has less effect than cochannel interference, may substantially alter the system behaviour when many more channels are assigned within a small region and should be studied.

As much as the channel characteristics is concerned, methods of processing the received signal to improve performance should be examined. The two well known signal processing techniques are adaptive equalization and diversity.

Capacity of cellular system is also becoming a major concern. The capacity is limited by the presence of interference in multipath frequency selective environment. Work on capacity of interference limited system is therefore recommended.

References

- [1] F. de Jager and C. B. Dekker, "Tamed Frequency Modulation - A Novel Approach to Achieve Spectral Economy in Digital Transmission," *IEEE Trans. on Commun.*, vol. COM-26, no. 5, pp. 534-542, May 1978.
- [2] K. Murota and K. Hirade, "GMSK Modulation for Digital Mobile Radio Telephony," *IEEE Trans. on Commun.*, vol. COM-29, no. 7, pp. 1044-1050, July 1981.
- [3] Engineering Dept., EIA, *Cellular System: Dual-Mode Subscriber Equipment - Network Equipment Compatibility Specification*, Dec. 1989.
- [4] F. I. Meno, "Mobile Radio Fading in Scandinavian Terrain," *IEEE Trans. on Veh. Technol.*, vol. VT-26, no. 4, Nov. 1977.
- [5] R. M. Clarke, "A Statistical Theory of Mobile Radio Reception," *Bell Syst. Tech. J.*, vol. 47, pp. 957-1000, July-Aug 1968.
- [6] M. J. Gans, "A Power-Spectral Theory of Propagation in the Mobile Radio Environment," *IEEE Trans. on Veh. Technol.*, vol. 47, no. 1, pp. 27-38, Feb. 1972.
- [7] D. O. Reudink, "Properties of Mobile Radio Propagation above 400 MHz," *IEEE Trans. on Veh. Technol.*, vol. VT-23, no. 4, pp. 143-160, Nov. 1974.
- [8] W. C. Y. Lee and Y. S. Yeh, "On the Estimation of the Second-order Statistics of Log-normal fading in mobile environment," *IEEE Trans. on Commun.*, vol. COM-22, no. 6, June 1974.

- [9] C. K. Pauw and D. L. Schilling, "Probability of Error for M-ary PSK and DPSK on a Rayleigh Fading Channel," *IEEE Trans. on Commun.*, vol. COM-36, no. 6, pp. 755-756, June 1988.
- [10] J. Horikoshi, H. Horie, and T. Morinaga, "QDPSK Error Rate Considerations with Cochannel Interference in Land Mobile Communications," *Trans. of I.E.C.E. of Japan*, vol. J64-B, no. 7, pp. 698-705, July 1981.
- [11] Y. Guo and K. Feher, "Performance Evaluation of Differential Pi/4-QDPSK Systems in a Rayleigh Fading/Delay Spread/CCI/AWGN Environment," in *Proc. IEEE Veh. Tech. Conf. 40th*, pp. 420-424, May 1990.
- [12] R. C. French, "Error Rate Predictions and Measurements in the Mobile Radio Data Channel," *IEEE Trans. on Veh. Technol.*, vol. VT-27, no. 3, pp. 110-116, Aug. 1978.
- [13] R. C. French, "The Effect of Fading and Shadowing on Channel Reuse in Mobile Radio," *IEEE Trans. on Veh. Technol.*, vol. VT-28, no. 3, pp. 171-181, Aug. 1979.
- [14] S. O. Rice, "Mathematical Analysis of Random Noise," *Bell Syst. Tech. J.*, vol. 24, p. 45, 1945.
- [15] Y. Okumura, "Field Strength and its Variability in VHF and UHF land Mobile Radio Services," *Rev. Elect. Comm. Lab.*, vol. 16, Sept-Oct 1968.
- [16] R. C. French, "Radio Propagation in London at 462 MHz," *Radio and Electron. Eng.*, vol. 46, no. 7, pp. 333-336, July 1976.
- [17] H. Suzuki, "A Statistical Model for Urban Radio Propagation," *IEEE Trans. on Commun.*, vol. COM-25, no. 7, pp. 673-680, July 1977.
- [18] Y. Akaiwa, "Performance of Baseband Band Limited Multilevel FM with Discrimination Detection for Digital Mobile Telephony," *Trans. of I.E.C.E. of Japan*, vol. E-64, pp. 463-

469, July 1981.

- [19] K. Honma, "On a Method of Constant Envelope Modulation for Digital Mobile Communication," in *Proc. IEEE ICC '80*, pp. 24.1.1–24.1.5, June 1980.
- [20] Y. Akaiwa and Y. Nagata, "Highly Efficient Digital Mobile Communication with a Linear Modulation Method," *IEEE J. Select. Areas Commun.*, vol. SAC-5, no. 5, pp. 890–895, June 1987.
- [21] F. G. Jenks, P. D. Morgan, and C. S. Warren, "Use of Four-level Phase Modulation for Digital Mobile Radio," *IEEE Trans. Electromagn. Compat.*, vol. EMC-14, pp. 113–128, Nov. 1972.
- [22] H. B. Voelcker, "Phase-Shift Keying in Fading Channels," *Proc. Inst. Electr. Eng.*, vol. 107, part B, pp. 31–38, Jan. 1960.
- [23] K. Kinoshita, M. Hata, and H. Nagabuchi, "Evaluation of 16 kbit/s Digital Voice Transmission for Mobile Radio," *IEEE Trans. on Veh. Technol.*, vol. VT-33, pp. 321–327, Nov. 1984.
- [24] J. A. Tarallo and G. I. Zysman, "Modulation Techniques for Digital Cellular Systems," in *Proc. IEEE Veh. Tech. Conf. 38th*, pp. 245–248, Sept. 1988.
- [25] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGraw-Hill Inc., 1966.
- [26] J. Horikoshi and T. Morinaga, "Multipath Interference Characteristics of DPSK in Land Mobile Digital Communications," *Trans. of I.E.C.E. of Japan*, vol. J65-B, no. 12, pp. 1559–1560, Dec. 1982.
- [27] E. Komp, "Block-Oriented Systems Simulator (BOSS)," in *IEEE Military Commun. Conf.*, vol. 3, pp. 36.1.1–36.1.10, 1986.

- [28] W. C. Jakes, ed., *Microwave Mobile Communications*. New York: John Wiley & Sons Inc., 1973.
- [29] J. T. Fleck and E. A. Trabka, "Error Probabilities of Multiple-State Differentially Coherent Phase-Shift Keyed Systems in the presence of White, Gaussian Noise," in *Investigation of Digital Data Communication Systems* (J. G. Lawton, ed.), Rep. UA-1420-S-1, 1961.

Appendix A

TABLES OF VALUES

This appendix records the theoretical bounds, and the simulated probability of symbol error as a function of CNR with $f_m T$ (normalized fading rate), σ (standard deviation of the local mean), and CIR (carrier-to-interference ratio) as parameters. Throughout the appendix, P_L and P_U denotes the analytical lower bound and upper bound of symbol error probability following the method given in chapter 3. P_S denotes the symbol error probability obtained from the simulation.

In the case of only AWGN channel, the system was simulated until 100 errors occurred.

In the case of AWGN/Rayleigh fading channel, the number of symbols simulated was 10^5 or higher.

In the case of AWGN/Rayleigh-lognormal fading and/or cochannel interference channel, the number of symbols simulated was higher for $f_m T = 0.01$ and was

10^6 or higher for $f_m T = 0.001$.

A.1 SER in only AWGN Channel

CNR (dB)	P_{th}	P_s
0	0.4762	0.470
1	0.4255	0.417
2	0.3703	0.363
3	0.3120	0.305
4	0.2529	0.247
5	0.1956	0.188
6	0.1429	0.138
7	9.754E-2	9.15
8	6.123E-2	5.76E-2
9	3.468E-2	N/A
10	1.730E-2	1.72E-2
11	7.357E-3	7.35E-3
12	2.565E-3	2.30E-3
13	6.967E-4	N/A
14	1.383E-4	N/A

Table A.3 SER in an AWGN channel

Note: P_{th} is the theoretical symbol error probability obtained from Fleck and Trabka [29].

A.2 SER in Rayleigh Fading Channel

CNR (dB)	P_L	P_U	P_S
5	0.1819	0.3639	0.308
10	8.11E-2	0.1622	0.139
15	3.02E-2	6.04E-2	5.2E-2
20	1.07E-2	2.14E-2	1.9E-2
25	4.11E-3	8.22E-3	7.3E-3
30	1.97E-3	3.95E-3	3.7E-3
35	1.30E-3	2.60E-3	2.5E-3
40	1.08E-3	2.16E-3	2.2E-3
45	1.01E-3	2.03E-3	2.0E-3
50	9.95E-4	1.99E-3	2.0E-3
55	9.88E-4	1.97E-3	2.0E-3
60	9.86E-4	1.97E-3	2.0E-3
70	9.85E-4	1.97E-3	N/A

Table A.4 SER in a Rayleigh fading channel for $f_m T = 0.01$

SER in Rayleigh Fading Channel (cont'd)

CNR (dB)	P _L	P _U	P _S
5	0.1815	0.3630	0.305
10	8.04E-2	0.1608	0.132
15	2.93E-2	5.86E-2	4.7E-2
20	9.76E-3	1.95E-2	1.6E-2
25	3.14E-3	6.29E-3	5.0E-3
30	1.00E-3	2.01E-3	1.6E-3
35	3.25E-4	6.51E-4	5.1E-4
40	1.09E-4	2.19E-4	1.5E-4
45	4.14E-5	8.29E-5	5.1E-5
50	1.98E-5	3.97E-5	2.8E-5
55	1.30E-5	2.60E-5	2.2E-5
60	1.08E-5	2.17E-5	1.8E-5
70	9.96E-6	1.99E-5	1.5E-5

Table A.5 SER in a Rayleigh fading channel for $f_m T = 0.001$

A.3 SER in Rayleigh-Lognormal Fading Channel

CNR (dB)	P_L	P_U	P_S
5	0.1970	0.3940	0.322
10	0.1102	0.2204	0.180
15	5.21E-2	0.1043	8.6E-2
20	2.16E-2	4.32E-2	4.2E-2
25	8.40E-3	1.68E-3	1.7E-3
30	3.48E-3	6.95E-3	6.8E-3
35	1.79E-3	3.58E-3	N/A
40	1.24E-3	2.48E-3	2.4E-3
45	1.07E-3	2.13E-3	N/A
50	1.01E-3	2.02E-3	2.0E-3
55	N/A	N/A	N/A
60	N/A	N/A	2.0E-3

Table A.6 $\sigma = 6$ dB, $f_m T = 0.01$

SER in Rayleigh-Lognormal Fading Channel (cont'd)

CNR (dB)	P _L	P _U	P _S
5	0.2061	0.4123	0.353
10	0.1314	0.2629	0.227
15	7.41E-2	0.1483	0.131
20	3.72E-2	7.44E-2	6.4E-2
25	1.69E-2	3.38E-2	3.0E-2
30	7.30E-3	1.46E-2	1.4E-2
35	3.30E-3	6.59E-3	7.3E-3
40	1.78E-3	3.55E-3	3.7E-3
45	1.25E-3	2.49E-3	N/A
50	1.07E-3	2.13E-3	2.1E-3
55	1.01E-3	2.02E-3	N/A
60	9.90E-4	1.98E-3	2.0E-3

Table A.7 $\sigma = 9$ dB, $f_m T = 0.01$

SER in Rayleigh-Lognormal Fading Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	1.97E-1	3.93E-1	3.3E-1
10	1.10E-1	2.19E-1	1.8E-1
15	5.10E-2	1.02E-1	8.4E-2
20	2.07E-2	4.14E-2	3.4E-2
25	7.45E-3	1.49E-2	1.2E-2
30	2.51E-3	5.02E-3	4.2E-3
35	8.20E-4	1.64E-3	1.3E-3
40	2.68E-4	5.36E-4	3.9E-4
45	9.15E-5	1.83E-4	1.3E-4
50	3.58E-5	7.16E-5	3.6E-5
55	1.81E-5	3.61E-5	1.9E-5
60	1.25E-5	2.49E-5	1.6E-5
65	1.07E-5	2.13E-5	1.5E-5
70	1.01E-5	2.02E-5	N/A

Table A.8 $\sigma = 6$ dB, $f_m T = 0.001$

SER in Rayleigh-Lognormal Fading Channel (cont'd)

CNR (dB)	P _L	P _U	P _S
5	2.06E-1	4.12E-1	3.3E-1
10	1.08E-1	2.61E-1	2.1E-1
15	7.30E-2	1.46E-1	1.2E-1
20	3.64E-2	7.27E-2	5.8E-2
25	1.60E-2	3.20E-2	2.6E-2
30	6.35E-3	1.27E-2	1.1E-2
35	2.33E-3	4.65E-3	4.4E-3
40	8.05E-4	1.61E-3	1.6E-3
45	2.72E-4	5.44E-4	4.8E-4
50	9.40E-5	1.98E-4	1.7E-4
55	3.68E-5	7.36E-5	5.6E-5
60	1.84E-5	3.68E-5	1.9E-5
65	1.25E-5	2.50E-5	1.5E-6

Table A.9 $\sigma = 9$ dB, $f_m T = 0.001$

A.4 SER in Rayleigh-Lognormal-Cochannel Interference Channel

CNR (dB)	P_L	P_U	P_S
5	2.72E-1	5.44E-1	4.6E-1
10	2.34E-1	4.68E-1	4.0E-1
15	2.16E-1	4.31E-1	3.6E-1
20	2.09E-1	4.17E-1	3.4E-1
25	2.06E-1	4.12E-1	3.4E-1
30	2.06E-1	4.11E-1	3.4E-1
35	2.05E-1	4.10E-1	3.4E-1
40	2.05E-1	4.10E-1	3.4E-1

Table A.10 CIR = 5 dB, $\sigma = 6$ dB, $f_m T = 0.01$

CNR (dB)	P_L	P_U	P_S
5	2.35E-1	4.69E-1	4.1E-1
10	1.78E-1	3.56E-1	3.2E-1
15	1.48E-1	2.95E-1	2.6E-1
20	1.35E-1	2.70E-1	2.3E-1
25	1.30E-1	2.60E-1	2.2E-1
30	1.29E-1	2.57E-1	2.2E-1
35	1.28E-1	2.56E-1	2.2E-1
40	1.28E-1	2.56E-1	2.2E-1

Table A.11 CIR = 10 dB, $\sigma = 6$ dB, $f_m T = 0.01$

SER in Rayleigh-Lognormal-Cochannel Interference Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	2.13E-1	4.26E-1	3.7E-1
10	1.42E-1	2.84E-1	2.6E-1
15	1.00E-1	2.00E-1	1.8E-1
20	8.10E-2	1.62E-1	1.4E-1
25	7.40E-2	1.48E-1	1.3E-1
30	7.15E-2	1.43E-1	1.2E-1
35	7.05E-2	1.41E-1	1.2E-1
40	7.00E-2	1.40E-1	1.2E-1

Table A.12 CIR = 15 dB, $\sigma = 6$ dB, $f_m T = 0.01$

CNR (dB)	P_L	P_U	P_S
5	2.03E-1	4.06E-1	3.5E-1
10	1.23E-1	2.46E-1	2.3E-1
15	7.30E-2	1.46E-1	1.4E-1
20	4.88E-2	9.76E-2	9.1E-2
25	3.92E-2	7.83E-2	7.06E-2
30	3.58E-2	7.15E-2	6.37E-2
35	3.47E-2	6.93E-2	6.12E-2
40	3.43E-2	6.85E-2	6.04E-2

Table A.13 CIR = 20 dB, $\sigma = 6$ dB, $f_m T = 0.01$

SER in Rayleigh-Lognormal-Cochannel Interference Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	2.80E-1	5.59E-1	4.7E-1
10	2.47E-1	4.93E-1	4.2E-1
15	2.28E-1	4.56E-1	3.9E-1
20	2.20E-1	4.40E-1	3.7E-1
25	2.17E-1	4.33E-1	3.6E-1
30	2.17E-1	4.33E-1	3.6E-1
35	2.16E-1	4.31E-1	3.6E-1
40	2.15E-1	4.30E-1	3.6E-1

Table A.14 CIR = 5 dB, $\sigma = 9$ dB, $f_m T = 0.01$

CNR (dB)	P_L	P_U	P_S
5	2.49E-1	4.97E-1	4.3E-1
10	2.03E-1	4.05E-1	3.6E-1
15	1.75E-1	3.50E-1	3.1E-1
20	1.62E-1	3.23E-1	2.8E-1
25	1.56E-1	3.12E-1	2.7E-1
30	1.54E-1	3.08E-1	2.6E-1
35	1.54E-1	3.07E-1	2.6E-1
40	1.50E-1	3.00E-1	2.6E-1

Table A.15 CIR = 10 dB, $\sigma = 9$ dB, $f_m T = 0.01$

SER in Rayleigh-Lognormal-Cochannel Interference Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	2.28E-1	4.56E-1	4.0E-1
10	1.71E-1	3.42E-1	3.1E-1
15	1.34E-1	2.68E-1	2.5E-1
20	1.15E-1	2.29E-1	2.1E-1
25	1.06E-1	2.11E-1	1.9E-1
30	1.03E-1	2.05E-1	1.8E-1
35	1.01E-1	2.02E-1	1.8E-1
40	1.00E-1	2.00E-1	1.8E-1

Table A.16 CIR = 15 dB, $\sigma = 9$ dB, $f_m T = 0.01$

CNR (dB)	P_L	P_U	P_S
5	2.15E-1	4.30E-1	3.8E-1
10	1.51E-1	3.02E-1	2.8E-1
15	1.06E-1	2.12E-1	2.0E-1
20	8.08E-2	1.61E-1	1.6E-1
25	6.85E-2	1.37E-1	1.3E-1
30	6.35E-2	1.27E-1	1.2E-1
35	6.20E-2	1.24E-1	1.1E-1
40	6.00E-2	1.20E-1	1.1E-1

Table A.17 CIR = 20 dB, $\sigma = 9$ dB, $f_m T = 0.01$

SER in Rayleigh-Lognormal-Cochannel Interference Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	2.72E-1	5.44E-1	4.4E-1
10	2.34E-1	4.68E-1	3.8E-1
15	2.16E-1	4.31E-1	3.4E-1
20	2.08E-1	4.16E-1	3.3E-1
25	2.06E-1	4.11E-1	3.2E-1
30	2.05E-1	4.10E-1	3.2E-1
35	2.05E-1	4.09E-1	3.2E-1
40	2.04E-1	4.08E-1	3.2E-1

Table A.18 CIR = 5 dB, $\sigma = 6$ dB, $f_m T = 0.001$

CNR (dB)	P_L	P_U	P_S
5	2.34E-1	4.68E-1	3.9E-1
10	1.78E-1	3.55E-1	3.0E-1
15	1.47E-1	2.94E-1	2.4E-1
20	1.34E-1	2.68E-1	2.2E-1
25	1.30E-1	2.59E-1	2.1E-1
30	1.28E-1	2.56E-1	2.1E-1
35	1.28E-1	2.55E-1	2.0E-1
40	1.27E-1	2.54E-1	2.0E-1

Table A.19 CIR = 10 dB, $\sigma = 6$ dB, $f_m T = 0.001$

SER in Rayleigh-Lognormal-Cochannel Interference Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	2.13E-1	4.25E-1	3.6E-1
10	1.41E-1	2.82E-1	2.4E-1
15	9.95E-2	1.99E-1	1.7E-1
20	8.00E-2	1.60E-1	1.3E-1
25	7.30E-2	1.46E-1	1.2E-1
30	7.05E-2	1.41E-1	1.1E-1
35	6.95E-2	1.39E-1	1.1E-1
40	6.95E-2	1.39E-1	1.1E-1

Table A.20 CIR = 15 dB, $\sigma = 6$ dB, $f_m T = 0.001$

CNR (dB)	P_L	P_U	P_S
5	2.03E-1	4.05E-1	3.4E-1
10	1.23E-1	2.45E-1	2.1E-1
15	7.20E-2	1.44E-1	1.2E-1
20	4.80E-2	9.60E-2	8.0E-2
25	4.53E-2	7.66E-2	6.2E-2
30	3.49E-2	6.98E-2	5.6E-2
35	3.38E-2	6.76E-2	5.4E-2
40	3.30E-2	6.60E-2	5.3E-2

Table A.21 CIR = 20 dB, $\sigma = 6$ dB, $f_m T = 0.001$

SER in Rayleigh-Lognormal-Cochannel Interference Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	2.79E-1	5.58E-1	4.5E-1
10	2.46E-1	4.92E-1	3.9E-1
15	2.28E-1	4.56E-1	3.6E-1
20	2.20E-1	4.39E-1	3.5E-1
25	2.16E-1	4.32E-1	3.4E-1
30	2.15E-1	4.30E-1	3.4E-1
35	2.15E-1	4.30E-1	3.4E-1
40	2.15E-1	4.30E-1	3.4E-1

Table A.22 CIR = 5 dB, $\sigma = 9$ dB, $f_m T = 0.001$

CNR (dB)	P_L	P_U	P_S
5	2.48E-1	4.96E-1	4.0E-1
10	2.03E-1	4.05E-1	3.3E-1
15	1.75E-1	3.50E-1	2.9E-1
20	1.61E-1	3.22E-1	2.6E-1
25	1.56E-1	3.11E-1	2.5E-1
30	1.54E-1	3.07E-1	2.5E-1
35	1.53E-1	3.06E-1	2.4E-1
40	1.53E-1	3.05E-1	2.4E-1

Table A.23 CIR = 10 dB, $\sigma = 9$ dB, $f_m T = 0.001$

SER in Rayleigh-Lognormal-Cochannel Interference Channel (cont'd)

CNR (dB)	P_L	P_U	P_S
5	2.28E-1	4.55E-1	3.7E-1
10	1.71E-1	3.41E-1	2.8E-1
15	1.34E-1	2.67E-1	2.2E-1
20	1.14E-1	2.27E-1	1.9E-1
25	1.05E-1	2.10E-1	1.7E-1
30	1.02E-1	2.03E-1	1.5E-1
35	1.01E-1	2.01E-1	1.6E-1
40	1.00E-1	2.00E-1	1.6E-1

Table A.24 CIR = 15 dB, $\sigma = 9$ dB, $f_m T = 0.001$

CNR (dB)	P_L	P_U	P_S
5	2.15E-1	4.30E-1	3.6E-1
10	1.51E-1	3.01E-1	2.5E-1
15	1.05E-1	2.10E-1	1.8E-1
20	7.95E-2	1.59E-1	1.3E-1
25	6.75E-2	1.35E-1	1.1E-1
30	6.30E-2	1.26E-1	1.0E-1
35	6.10E-2	1.22E-1	9.8E-2
40	6.05E-2	1.21E-1	9.7E-2

Table A.25 CIR = 20 dB, $\sigma = 9$ dB, $f_m T = 0.001$

END

1|2|0|4|9|1

FIN