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**LA THÈSE A ÉTÉ
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Measurements of Mean Velocities and Reynolds Stresses in a Circulation Controlled Flow

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BASc.(University of Ottawa)

*A Thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the degree
of Master of Engineering*

DEPARTMENT OF MECHANICAL AND AERONAUTICAL ENGINEERING
CARLETON UNIVERSITY
OTTAWA, ONTARIO
CANADA

December 1985

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submitted by Jean Agiomavritis, in partial fulfillment of the requirements for the degree of
Master of Engineering.

Thesis Supervisor

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Abstract

This thesis is concerned with measurements of mean flow and turbulence parameters in the shear flow downstream of a slot, from which air is injected tangentially to the surface to provide boundary layer control.

A hot-wire signal analysis has been derived and tested experimentally on a plane wall jet apparatus to determine its applicability and limitations. Experimental results for mean velocity and Reynolds stresses obtained from the plane wall-jet apparatus agree with those of other investigators. Modifications to an existing Boundary Layer Control apparatus have been made and necessary software for a Data Acquisition System has been developed to collect data and control the experiments.

The two-dimensional characteristics of the flow in the modified Boundary Layer Control apparatus have been improved using boundary layer suction and the quality of the flow was checked by static pressure measurements, spanwise mean velocity hot-wire measurements and by two flow visualization schemes. Mean velocity and Reynolds stress hot-wire data have been obtained for three different cylinder momentum coefficients, with the plane wall momentum coefficient and the tunnel free stream velocity held constant. All data have been analysed using a simplified version of the hot-wire signal analysis.

The measurements have extended the data available for flowfields similar to that observed on the rounded trailing edge of airfoils which utilize tangential blowing for boundary layer and circulation control.

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Nomenclature

A	- y intercept of hot-wire calibration curve.
B	- slope of hot-wire calibration curve.
C	- y intercept of hot-wire calibration curve.
$C_{\mu c}$	- cylinder wall-jet momentum coefficient.
$C_{\mu w}$	- plane wall-jet momentum coefficient (B.L.C. apparatus).
C_P	- static pressure coefficient, $(P - P_{\infty} / \frac{1}{2} \rho_{\infty} U_{\infty}^2)$; $P_{\infty} = \text{atm.}$
d	- orifice diameter of flowmeter (Equa. 3.11), - also cylinder diameter.
D	- pipe diameter of orifice plate flowmeter (Equa. 3.11).
$\sqrt{e^2}$	- linearized RMS value corresponding to $\sqrt{e_{br}^2}$, (Equa. 3.5).
$\sqrt{e_{br}^2}$	- RMS of the AC component of bridge output voltage of hot-wire signal.
E	- instantaneous voltage.
$\bar{E}_{lin}$	- linearized voltage corresponding to $\sqrt{e_{br}^2}$, time mean value (Equa. 3.3).
$\bar{E}_{br}$	- bridge output voltage of hot-wire anemometer, time mean value.
$\bar{E}_0$	- bridge output voltage at zero flow.
E	- velocity of approach factor orifice plate flowmeter (Equa. 3.11).
$\bar{E}_{tr}$	- pressure transducer time mean output voltage.
h	- pressure head across the orifice, (Equa. 3.11). or - pitch factor of hot-wire probe.
k	- yaw factor of hot-wire probe, (Equa. 3.10).
$\dot{m}$	- mass flow rate per unit span, (Equa. 3.13).
$\dot{M}$	- mass flow rate (lbm/hr), (Equa. 3.11).
P	- static pressure.
P_{C1}	- static pressure at the upstream end of tunnel contraction
P_{C2}	- static pressure at the downstream end of tunnel contraction
P_j	- static pressure at the cylinder wall-jet exit.
P_0	- plenum pressure.
P_t	- total pressure at point of maximum velocity.

ΔP	-	pressure drop between the chamber and the calibration point.
P_{∞}	-	static pressure of freestream.
Q	-	resultant flow velocity.
R_o	-	cold wire resistance at reference temperature.
S	-	slope of linearized calibration curve of hot-wire probe, slanted probe.
S^*	-	slope of linearized calibration curve of hot-wire probe, normal probe.
t	-	jet slot thickness, (plane still air wall-jet apparatus).
$\bar{t}$	-	mean jet slot thickness, (cylinder wall-jet B.L.C. apparatus)
ΔT	-	temperature change in flow field.
T_j	-	temperature of the jet air.
U_{∞}	-	tunnel freestream velocity.
U	-	mean velocity.
U_{eff}	-	effective cooling velocity.
U_B	-	sensor based velocity bi-normal to wire.
U_N	-	sensor based velocity normal to wire.
U_T	-	sensor based velocity tangential to wire.
u_e	-	external flow velocity.
u, v, w	-	time mean velocity components in x, y, z directions.
u', v', w'	-	fluctuating velocity components in x, y, z directions.
$\overline{u'v'}, \overline{u'w'}, \overline{v'w'}$	-	Reynolds shear stress components.
V_n	-	velocity normal to the porous end plates.
w	-	air density at high pressure tapping (Equa. 3.11).
X	-	downstream distance from plane wall-jet slot (B.L.C. apparatus), also downstream distance from plane wall-jet slot, (still air wall-jet apparatus).
Y	-	distance from cylinder surface.

- Z - correction multiplier for orifice plate flowmeter (Equa. 3.11),
or
- spanwise distance from the centreline of cylinder and plane wall-jets (B.L.C. apparatus).

Greek Symbols

- α - hot-wire probe angle (the complementary angle to the angle between the probe stem and the wire sensor (Figure 19);
- also pitch angle (Figure 16).
 ζ, ξ, η - fixed coordinate system of hot-wire probe, (Figure 19).
 θ - distance from the cylinder slot exit, specifies measuring station.
 ρ - density of air during calibration.
 ρ_{∞} - density of air at freestream conditions.
 ρ_j - density of jet air.
 ψ - orientation angle of hot-wire sensor (i.e. angle through which the hot-wire probe is rotated about its stem, Figure 19).

Στους γονείς μου Γεωργία και Φώτη

Chapter 1

Introduction

1.1 Background

When flows encounter a region with unfavourable pressure gradient, that is if the pressure increases downstream, the retarded fluid particles cannot penetrate too far into the region of increased pressure owing to their low kinetic energy. Strong adverse pressure gradient effects overcome momentum and shear stress effects which tend to keep the fluid from moving forward. The flow can no longer remain attached to the solid boundaries and it separates from them. Further downstream of the point of flow separation, the adverse gradient effects tend to push the fluid near the boundaries backwards thus causing a reverse flow.

Flow separation has detrimental effects on the aerodynamic characteristics of the device involved because it is associated with large pressure drag and as a result, loss of lift. Boundary layer control (B.L.C.) is a means of preventing or delaying flow separation. The B.L.C. concept is particularly attractive to high lifting surfaces, such as aircraft wings and it has been extensively studied as a means of minimizing drag at cruising speeds, of maximizing lift at landing and takeoff and minimizing unsteady flow effects in general^[41].

High performance fighter aircraft, VSTOL aircraft, carrier based aircraft and helicopters are the most prominent contenders for B.L.C. applications. The two most promising methods of B.L.C. include suction and blowing.

Boundary layer control by suction is the process in which low kinetic energy fluid is removed from the boundary layer by applying suction through series of slots or perforations on the solid boundaries. The resulting fuller velocity profile shape is such that the shear stresses near the wall are greater and the flow can advance further in the region of increased pressure.

The boundary layer control by blowing concept relies on the re-energization of the low kinetic energy fluid near the wall for delaying flow separation and decreasing drag. When high energy fluid is injected tangentially into the boundary layer, it increases its kinetic energy and the resulting robust velocity profile shape is such that it can advance further into the increased pressure region. Tangential injection of high energy fluid at the rounded

trailing edge of airfoils is a promising way of achieving circulation control. The injected fluid remains attached to the rounded surface by virtue of the Coanda effect as illustrated in Figure 2. Air for blowing can be supplied from the gas turbine engine compressor bleed. The delay of flow separation increases the circulation around the airfoil and as a result its lift. By varying the amount of blowing, it is possible to achieve circulation control (C.C.) around the airfoil. The flow around a circulation controlled airfoil is illustrated in Figure 1.

Due to the lack of a sharp trailing edge and the associated Kutta condition, circulation control by tangential injection may be achieved at low blowing rates. High lift coefficients and low effective drag make this scheme very attractive for fixed-wing STOL aircraft and rotorcraft.

This study is concentrated on the understanding of the flow field similar to that observed downstream of the injection slot of a rounded trailing edge of a circulation controlled airfoil.

1.2 Nature of Flow

The flowfield associated with application of tangential injection is relatively complex because it involves the mixing of a turbulent wall-jet with a turbulent boundary layer in a strong adverse pressure gradient on a highly curved surface.

Figure 3 shows a typical wall-jet over a curved surface in a moving stream. The velocity profiles downstream of the slot are usually not symmetrical and the flow is not self preserving. The possible behavior of wall-jets in a freestream on an adverse pressure gradient has been described by Dvorak^[17], see Figure 4.

The development of the flow may proceed in a number of different ways; namely;

- a) The velocity minimum is completely entrained by the wall-jet.
- b) Both the velocity minimum and the velocity maximum persist.
- c) Both the velocity minimum and the velocity maximum disappear.

Flow separation can take place by either of two distinct mechanisms;

The flow can separate from the solid boundary.

The velocity minimum, if it persists, can be deepened by a sufficiently strong adverse pressure gradient until reverse flow occurs and the external flow separates from the

wall-jet flow.

The two major factors affecting the flow development are severity of the adverse pressure gradient and the momentum defect of the upstream boundary-layer as compared with the jet momentum.

1.3 Previous Research

Within the broad definition of a turbulent wall-jet, a wide range of flows and phenomena have been researched. In this section, only a small number of wall-jet flows that have similarities with the flowfield described in Section 1.2 will be presented.

Kruka and Eskinazi^[40] have carried out noteworthy research on wall-jets in a moving stream. *Irwin*^[32] in later work has reported measurements including mean velocity, turbulence stresses, triple and quadruple velocity correlations, intermittency and spectra of the longitudinal turbulence intensity. *Launder and Rodi*^[42] found Irwin's data suitable for testing numerical methods. *Erian*^[20] has investigated the influence of pressure gradient on turbulent flows with asymmetric mean velocity, using a plane wall-jet in a free stream.

Wilson and Goldstein^[53] have investigated surface curvature effects on the development of a two-dimensional jet. The wall-jet in their experiments, developed around the convex surface of a circular cylinder in still surroundings. Surface curvature effects on flow development have also been studied by *Guittou and Newman*^[29], *Giles et al.*^[26] with wall-jet flows over logarithmic spiral surfaces in still air.

Considerable work on wall-jets on plane and curved surfaces in still surroundings or in a free stream has been carried out at McGill University. *Launder and Rodi*^[42] provide an extensive review of work on turbulent wall-jets and evaluation of experimental data for use in computational schemes.

Attempts for developing calculation methods for prediction of the development of wall-jets in arbitrary pressure gradients have been made by *Kind*^[37] and *Dvorak*^[17] among others. *Kind*^[37] has compared his calculation method with the experimental data of *McGahan*^[44], *Kind*^[37] and *Fekete*^[23]. These authors could only do limited tests on their methods due to limited amount and range of experimental data suitable for use as test cases. Later work of *Kind et al.*^[38] provided measurements of flows with tangential injection and comparison with two prediction methods. This work involved profiles in eighteen different two-dimensional flows over plane and curved surfaces with a variety of adverse pressure gradients. The data

4

were compared with an integral method of *Kind*^[37] which used four parameters to describe the velocity profiles, and a finite difference method of *Dvorak*^[17], using an eddy viscosity model to establish shear stress in the flow. Both calculation schemes showed weaknesses in predicting the flow near the separation points. The authors^[38] concluded that additional flow field data was desirable to further assess capabilities of prediction methods and to guide efforts to improve them. In complex flows of this type, agreement within about 10% of velocity is considered successful^[38].

The present work attempts to provide measurements of mean velocity and turbulence in a flow field very similar to that described by *Kind et al.*^[38]. These measurements are intended to assist in the testing of calculation methods.

1.4 Purpose of the Current Study

The main purpose of the present study is to provide suitable data for testing of prediction methods in flows with tangential injection. The progression from planning to completion of this task involved the following major steps;

- i) Familiarization with the principles of hot-wire anemometry and implementation of a data acquisition system for gathering data.
- ii) Derivation of hot-wire equations and experimentation to the equations.
- iii) Measurements of mean velocity and turbulence in a plane wall-jet and comparison of results with that of other experimentors.
- iv) Modification to the boundary layer control apparatus.
- v) Experimentation on the boundary layer control apparatus.

Chapter 2

Description of Apparatus

The first section of this chapter includes a brief description of a plane wall-jet apparatus designed for studies of plane wall-jet flows in still surroundings. The second section involves a detailed description of a boundary layer control apparatus designed for studies of flows developed downstream of the tangential injection slot, on a circulation controlled wing section with cylindrical trailing edge. This section also includes descriptions of all the supporting equipment and instruments used in carrying out the present experiments.

2.1 Plane Wall-Jet Apparatus

A plane wall-jet apparatus, which was already in existence, was used in the first stages of this study. A photograph of this apparatus is shown in Figure 6.

Air from a plenum chamber discharged tangentially to a 38.1 cm long and 30.5 cm wide polished steel plate, through a slot of thickness t equal to 1.587 mm. The slot thickness varied less than ± 0.12 mm across the entire 30.5 cm span. The chamber was continuously supplied with air from both sides by a centrifugal blower. Two 58.4 cm $\times$ 20.3 cm side plexiglass plates assist in maintaining two-dimensional flow conditions downstream of the jet exit.

An aluminium bracket clamped between the side plates provides support to the traversing mechanism which was driven by a stepper motor. In this arrangement, the transverse gear moved a guide tube along an axis perpendicularly to the flow surface of the apparatus. A drilled aluminium block allowed probes to be supported on the guide tubes near the flow surface, to be aligned with the flow and to be rolled at any desired angle. Figure 11 shows the traversing gear fitted to the wall-jet apparatus.

Suthanthiran^[50] provides a more detailed description of the wall-jet apparatus together with a number of checks of flow two-dimensionality. His flow visualization studies and mean velocity measurements showed that the flow is two-dimensional, except near the side walls.

2.2 Boundary Layer Control Apparatus

The boundary layer control (B.L.C.) apparatus used in the present study is a modified version of that used in previous studies of boundary layer control with tangential injection^[38]. A schematic diagram of this "modified" B.L.C. apparatus is shown in Figure 8, and its components will be presented in following sub-sections.

2.2.1 Wind Tunnel

The wind tunnel used for this study is a closed circuit low-speed facility located at Carleton University. A single stage fan powered by a 50 hp electric motor provided the drive of the wind tunnel. The maximum wind speed at the tunnel test section was 150 mph at approximately 1500 rpm fan speed. The pitch of the fan blades was set to 30°. A 12:1 contraction ratio with five screens in the settling chamber gave a longitudinal turbulent level at the test section of approximately 0.15%^[27].

The test section freestream velocity was computed according to the following wind tunnel calibration relationship;

$$\frac{P_{C1} - P_{C2}}{0.91} = \frac{1}{2} \rho_{\infty} U_{\infty}^2 \quad (2.1)$$

where P_{C1} and P_{C2} are the static pressures at the upstream and downstream ends of the tunnel contraction respectively. The pressure differential was monitored by an inclined (1:2 slope) single tube micro-manometer filled with oil of a specific gravity of 0.826 and sensitivity of ± 0.5 mm. Adjustments of the tunnel speed were possible by varying the speed of the fan. Previous calibration measurements showed that the wind speed across the test section was approximately constant ($\pm 3\%$).

2.2.2 Test Section

All the components of the B.L.C. apparatus were enclosed in a 20 in by 30 in by 6 ft long removable test section that could be fitted in the wind tunnel circuit. The test section is made from wood with plexiglass front and back windows.

2.2.3 Plane Wall and Cylinder Wall-Jets

The back wall of the test section was fitted with a blowing slot as shown in Figure 8, mainly for purposes of an earlier experiment. The vertical slot has a mean exit thickness of

0.034 in (0.86 mm), and the thickness variation was estimated to ± 0.001 in (0.0254 mm) across 14 in of its 19.5 in (49.5 cm) span. The wall-jet plenum chamber and its sharp slot lip were made of polished mild steel. Air from a centrifugal blower, ducted through flexible 2 in diameter hoses, enters the plenum chamber through 2 in diameter openings at either end. The plane wall was fitted with a row of 39 static pressure taps ($\frac{1}{32}$ in I.D.) placed at various spacings downstream of the jet exit. Additional spanwise rows of static taps ($\frac{1}{32}$ in I.D.) at distances of 7 in and 11 in downstream of the slot exit, enable spanwise static pressure measurements. An iron-constantan, type J, thermocouple was installed in the wall plenum chamber to monitor the air temperature.

A 5 in diameter cylinder was mounted with its center at the same plane as the wall slot as shown in Figure 8. The cylinder slot had a mean thickness of 0.033 in (0.84 mm) and its variation was estimated to ± 0.001 in (0.0254 mm) across the central 14 in and 0.002 in (0.050 mm) across the full span. The slot lip was squared off to a thickness of 0.01 in and it was highly polished. Twenty seven static pressure taps (1 mm I.D.) were fitted at various distances around the circumference of the cylinder at mid span. Air from a centrifugal blower unit entered the cylinder plenum chamber through 2 in diameter openings at either end as shown in Figure 9. Again an iron-constantan thermocouple was fitted in the cylinder plenum chamber to monitor the air temperature. The angular position of the jet exit plane with respect to the wall slot could be adjusted by rotating the cylinder around its vertical axis. In the present study, the plane of the cylinder slot exit coincided with that of the wall-jet slot as shown in Figure 8. Eight bolts, four at each end of the cylinder running through the end wall porous plates held the cylinder firmly in place.

2.2.4 Porous Endwalls

Two porous bronze plates, 11.5 in by 15 in were mounted on the wall of the working section at either end of the cylinder. Suction boxes were fitted to each end porous plate with 2 in diameter openings through which air could be sucked. Four Dustbane, type PC-1, heavy duty vacuum cleaner units, two connected in parallel per opening, provided the suction as shown in Figure 9. The purpose of the two porous plates was to suppress, to the maximum extent possible, end-effects and thus to obtain two-dimensional flow conditions.

2.2.5 Forebody

Previous investigations of the shear layer downstream of a tangential injection slot on a circulation-controlled circular cylinder in a freestream have produced reasonable mean

velocity profiles (*Kind et al.*^[38]). Although these measurements provided data for flow cases where fluid from a wall-jet mixes with an upstream boundary layer in a strong adverse pressure gradient, the boundary layer approaching the blowing slot was very thin, this is not typical of many applications where B.L.C. is used.

It was later understood that a forebody extending upstream of the circular cylinder would produce a thicker boundary layer at the slot, thus making the flow conditions more realistic. *W. Kelly*^[34] worked towards the development of a forebody shape for the existing boundary layer control apparatus, using a computer solution for incompressible flow over airfoils of arbitrary section. His work produced a forebody shape that satisfied the design considerations of favourable or moderately adverse pressure gradients near the slot.

This shape was also adopted in constructing a forebody for the present study. The forebody consisted of a "Lexan" skin, ($\frac{1}{16}$ in thick) screwed to 4 ($\frac{1}{4}$ in thick) plexiglass ribs. One row of static pressure taps was fitted at mid-span around the forebody. The forebody is shown in Figures 8, 11 and 12. Two bolts, one on each end near the nose of the forebody, running through the upper and lower ends of the test section kept the forebody in place.

2.2.6 Traversing Gear

The traversing gear used in the plane wall-jet apparatus has already been mentioned in Section 2.1. A stepper motor, DISA type 52C01, transfers movement to a toothed guide tube via a 10-teeth pinion gear. With the stepper motor set at a 10:1 gear ratio, the probe movement was 0.025 mm/step. The selection of 10:1 gear ratio minimizes gear slip and enables traversing of very thin layers. The stepper motor was controlled by a sweep drive unit (DISA 52B01). The sweep drive unit enables remote control and offers a choice of both forward and backward sweep, variable sweep time, together with upper and lower stop capabilities. It was quickly realized that traversing could be controlled by the ADACS system if the "stop" connector of the sweep drive unit was properly connected to a system actuation relay. A considerable time was spent in perfecting the control of the traversing, making it as flexible as possible. The software developed for this function was incorporated in the ADACS computer programs, U&RMS and M&RMS described in Appendix D.

Although the traversing gear described earlier was suitable for measurements on the still air wall-jet apparatus, it had to be modified considerably for measurements on the boundary layer control apparatus.

A new traversing mechanism was designed for traversing a hot-wire probe across the

boundary layer developed downstream of the slot exit of the boundary layer control apparatus. This mechanism is shown in Figure 10 and the photograph of Figure 11. In this arrangement, the stepper motor was placed inside of the cylinder jet away from the end wall porous plates. Two steel supports fastened inside the tunnel supported both the stepper motor and the probe holder. The probe can be set at any desired roll angle. A protractor allows measurement of the roll angle. Two rubber cushions, 3 mm thick, were placed on both ends of the steel supports to damp vibrations transmitted through the tunnel structure.

A simple kinematic analysis of the stepper motor, probe holder arrangement of the traversing gear is carried out in Appendix A. Thus, given the guide tube displacement it was possible to determine the corresponding probe sensor movement. This analysis was easily incorporated in the data acquisition and control software developed for these experiments, (see Appendix A).

2.2.7 Orifice-Plate Flowmeters

Orifice-plate flowmeters were used for mass flow measurements in the cylinder wall-jet and the suction system. The flowmeters were designed in accordance with British Standard 1042^[10]. The orifice diameter, d , was $1\frac{3}{16}$ in. Two pressure taps, one at a distance, D , upstream of the orifice and one at a distance, $\frac{D}{2}$, downstream of the orifice enabled measurements of pressure changes across the orifice. A dial thermometer fitted downstream of the orifice monitored the temperatures of the flowing air. Figure 9 shows the orifice-plate flowmeters in place. Mass flow measurement procedures will be presented in Chapter 3.

2.2.8 Manometers and Pressure Transducer

A multitube manometer board consisting of fifty tubes, in the vertical position, was used to measure static pressure distributions around the cylinder and forebody, and the static pressure inside both jet plenum chambers. During any particular run, all pressure lines leading to the board could be clamped off, thus "freezing" the pressure distribution shape for inspection or for taking data. The manometer board could be read to an accuracy of approximately ± 0.05 in of water. Another three multitube manometers consisting of twenty tubes in the vertical position, were used to measure static pressure distributions along the back wall of the test section. These manometers could be read to an accuracy of approximately ± 0.05 in of water.

2.2.9 Blowers

The cylinder plenum chamber was supplied by a unit consisting of two 1 hp vacuum cleaner units arranged in parallel and fitted in a wooden casing. Two screens, one at the suction side and the other at the blowing side were installed in an attempt to filter the air before it entered the plenum. The wall blowing slot was supplied with air by a heavy duty Dustbane blower unit, with a flow rate of 92 cubic feet/minute at full speed. This unit was also equipped with a 656 sq. in dry air filter. Each porous end-plate was connected to the suction side of two similar Dustbane blower units, connected in parallel. The power and hence the flow rate of the blowers was controlled by single variacs each connected to different circuits of the laboratory. A schematic diagram of the blowing air and suction system is shown in Figure 9.

2.2.10 Hot-Wire Anemometer

A constant temperature anemometer, Thermo Systems Inc. Model 1053A, was used to measure mean and fluctuating flow velocities in the present study. This type of anemometer utilizes a feedback controlled wheatstone bridge circuit for maintaining the sensor at constant temperature. The typical operating range of the sensor is 150° C to 260° C. The three arms of the wheatstone bridge have a constant resistance and the fourth arm is the sensor. Any variation in the sensor resistance due to changes in fluid velocity over the sensor is corrected by automatic re-adjustment of the current through the sensor. The output of the anemometer is the voltage required to drive the necessary current through the sensor to maintain its temperature at constant value.

The anemometer unit is also equipped with a TSI, model 1052, linearizer, for linearizing the bridge voltage versus flow velocity relationship.

The selection of the sensor operating temperature is limited by the mechanical properties of the sensor material. High operating temperatures improve sensor response to flow velocity fluctuations and minimizes temperature effects (Bridshaw^[6]). The two types of hot-wire probes used for the present measurements were DISA single miniature probes, the 55P11 normal probe and the 55P12 45° slant probe. The probe sensor is a tungsten wire of 5 μ m diameter and 1.25 mm in length, welded to the tips of the probe prongs. These probes could operate at a maximum temperature of 300° C, allowing a maximum overheat ratio of about 1.9. This value was used throughout the present experiments. Additional instruments used in conjunction with the anemometer unit include a Tektronix, type 2213.

oscilloscope, a Brüel and Kjaer electronic voltmeter, type 2425 and a DISA, type 55D31 digital voltmeter. The oscilloscope was used for optimizing the response of the bridge output signal and for ensuring that the anemometer operation was always stable. The DISA 55D31 voltmeter has a 0.1 to 100 s time constant selection capability and an accuracy of 0.1% of full scale ± 1 digit. Its resolution is 1 mV and has a $\pm 10\%$ of indicated time, time constant accuracy. This voltmeter is designed primarily for measurements of mean flow velocities during studies of flow phenomena using a hot-wire anemometer.

The Brüel and Kjaer voltmeter could perform RMS, average, $\pm$ peak and maximum peak measurements on signals with voltage levels from 100 μ V to 300 V in the 0.5 Hz to 500 kHz frequency range. When used for RMS measurements, the time constants in the fast and slow modes, give averaging times of 0.27 and 3 s respectively. Coaxial cables were used for connecting the various instruments to minimize noise. A schematic diagram of the instruments and their appropriate connections is shown in Figure 13.

2.2.11 Calibration Apparatus

A calibration apparatus was available for this study; it is shown in Figure 15 and in the photograph of Figure 7. The flow was produced by a variable-speed centrifugal blower that supplied air to a settling chamber from which it discharged through an axi-symmetric nozzle of 1.5 in diameter. Three screens were fitted inside the settling chamber to damp out flow fluctuations. An iron-constantan, type J, thermocouple was placed inside the chamber to monitor air temperature during the calibration process.

A probe mounting mechanism provided means for probe alignment with the flow and allowed for probe roll, yaw and pitch relative to the flow direction.

The hot-wire probe was placed in the potential core of the jet, 4.3 cm downstream of the nozzle-exit plane, and on the nozzle centre line. The flow velocity was monitored by a pressure transducer connected to a static tap in the side wall of the settling chamber. The pressure transducer is a Rosemount model 831A capacitance type and its output voltage, E_{tr} , was monitored by a Hewlett-Packard digital voltmeter with a 1000 mV to 1000V range and a variable sampling rate. The transducer was calibrated regularly against an inclined micro-manometer similar to the one described before. The sensitivity of this transducer was about 0.12 mm of water per 1 mV output voltage. Calibration drifts of less than 3% were detected over a 2 month period. The static tap was previously calibrated against a pitot-static tube placed in the jet at the aforementioned probe position.

2.2.12 Automatic Data Acquisition and Control Systems (ADACS)

The Hewlett-Packard, model 3954A, is a computer based system with precision measurement and control capabilities. This system was utilized in the present study for gathering data from a hot-wire probe and for control of probe traversing. The system includes a specialized software support package that is designed to use the system instruments to achieve specific operations and functions. The system's components are described separately.

System Computer

The system computer is the HP-85A desktop computer. The computer is interfaced with the system control unit via the 82937A HP-IB I/O (input/output) cord. The Hewlett-Packard (HP-85A) computer has a total memory of 32 K and it can be programmed using a version of the BASIC programming language. Its features include a cathode ray tube display, an internal printer, a tape drive auto-start and error trapping.

Digital Voltmeters

The ADACS system voltmeters are the Hewlett-Packard HP-3456A and the HP-3437A. The HP-3456A digital voltmeter is a $3\frac{1}{2}$ to $6\frac{1}{2}$ digits, integrating voltmeter, with 100 nV sensitivity. Its capabilities include DC and RMS AC voltage measurements in the range of 0 to 100 V. It also has selectable integrating times from 0.01 to 100 power line cycles, and it eliminates noise by an analog input filter and digital averaging. Its read/write memory can hold up to 350 readings with a specified delay between readings. The HP-3456A voltmeter can be programmed and triggered through the front panel (externally), internally or through software.

Data Acquisition/Control Unit

The 3497 Data Acquisition/Control Unit combines precision measurement and control capabilities. Its standard features include a built in real time clock, front panel display/controls and HP-IB interfacing. The control unit is equipped with twenty channels that can be used for data acquisition and sixteen relay actuator channels. In the present experiments, two channels were used for data acquisition and one channel for control. A schematic diagram of the ADACS system connected to the hot-wire anemometer unit is shown in Figure 13.

Chapter 3

Experimental Procedures

3.1 Experimentation on the Plane Wall-Jet Apparatus (Still Surroundings)

The Plane wall-jet apparatus described in Section 2.1 and shown in Figure 6, was used in the earlier stages of this study for the following tasks;

- (i) Familiarization with hot-wire measuring techniques.
- (ii) Development and testing of ADACS computer programs.
- (iii) Evaluation of hot-wire signal analysis.
- (iv) Comparison of results to those of previous investigators.

A short description of the experimental procedures for carrying out the aforementioned tasks will be described in the following subsections.

3.1.1 Familiarization with Hot-Wire Measuring Techniques

Familiarization experiments involved calibration of hot-wire probes and linearization of the anemometer output signals using the TSI 1052 electronic signal linearizer. The linearizer coefficients were calculated and set according to the procedures outlined in the TSI linearizer manual. A series of experiments was carried out with and without the electronic linearizer, and results were compared. The calibration and setting of the linearizer coefficients had to be repeated about every two hours due to the linearizer output signal drifting with time. The use of the electronic signal linearizer was a long and tedious process.

It was realized that an easier and more accurate method had to be employed for linearizing the anemometer output signal. The TSI FORTRAN version, linearization program was made available by Yowakim^[55]. The linearizer coefficients calculated by the program were set on the linearizer. A simple test of the program was performed by comparing mean velocity measurements taken with and without the linearizer. A listing and a sample output program, that can be used as a future reference is included in Appendix B.

3.1.2 Development and Testing of ADACS Computer Programs

The HP-305A data acquisition system was used for sampling of both the time-mean ($\bar{E}_{br}$) and the RMS ($\sqrt{e^2_{br}}$) of the output hot-wire output signal. The ADACS program developed in this study used system subprograms as a basis for sampling the hot wire signals. The function of these subprograms was to program and trigger the HP-3456 volt meter. Thus, the system subroutine "Avdcv" and the "Acv" were used for measurements of $\bar{E}_{br}$ and $\sqrt{e^2_{br}}$ respectively. A set of data taken in the wall-jet at $x/t = 93.3$ showed a considerable scatter when the subroutines were first tried. This led to a series of experiments in the wall-jet apparatus with the objective of obtaining repeatable hot wire measurements in a short period of time, "optimum sampling rate". In these experiments, the hot-wire probe was kept approximately at the point of the maximum shear stress of the boundary layer at $x/t = 93.3$, (i.e. at that point the scatter of the data was maximum)

Experimentation showed that by increasing the number of power line cycle (PLC) integration times of the HP3456 voltmeter, better results could be obtained. Thus, the voltmeter was programmed for maximum PLC integration time 100. Large PLC integration time increases the accuracy, repeatability and reduces the noise. At 100 PLC integration time, sampling the noise reaction was 75 dB and the error in the measurements was approximately 0.0015%. Thus, for a complete sampling of $\bar{E}_{br}$ and $\sqrt{e^2_{br}}$ at 100 PLC integration time and 60 Hz power line frequency, it took the voltmeter about 1.7 s. Further experimentation showed that by increasing the number of readings/trigger, the results were more repeatable. Thus the voltmeter was programmed to average 999 readings/trigger. For better accuracy, the voltmeter was triggered thirty times for sampling $\bar{E}_{br}$ and thirty times for sampling $\sqrt{e^2_{br}}$. The total time spent at the "optimum sampling rate" was 102 s for each point. The errors involved in sampling both $\bar{E}_{br}$ and $\sqrt{e^2_{br}}$ were less than 0.5% at this sampling rate.

The number of times that the voltmeter is triggered can be adjusted through the software. The programs developed to sample the hot-wire signals and control the traversing in wall-jet apparatus experiments are earlier versions of the program "U&RMS" and "M&RMS" shown in Appendix D.

The evaluation of the hot-wire signal analysis will follow in Section 4.3. The nomenclature and probe orientation during experiments with the plane wall-jet apparatus is shown in Figure 14. A set of results obtained from the plane wall-jet will be presented and compared

to results of other investigators.

3.1.3 Hot-Wire Calibration and Linearization Methods

The hot-wire probes were calibrated using the calibration apparatus shown in Figures 7 and 15. Before collecting data, the settling chamber and the probe support device were leveled and the base of the apparatus was clamped to the working bench. In this arrangement the flow direction through the nozzle was kept horizontal; i.e. parallel to the x -axis of the coordinate system shown in Figure 15. The probe mounting mechanism kept the probe rigidly in the potential core of the jet, 4.3 cm downstream of the nozzle-exit plane and on the nozzle center line. The probe mounting mechanism was equipped with a protractor system so that the probe could be set at zero pitch and zero yaw angles during calibration. The inside of the nozzle was smoothed to ensure that structural irregularities did not caused flow separations. The turbulence intensity at the maximum flow speed of 70 ms^{-1} was approximately 2%. Yowakim^[55] and Kelly^[34] have also found that the flow through the same settling chamber was essentially one-dimensional. Intentional changes of roll, yaw and pitch angles of the probe provided estimation on the effect of probe misalignment on the $\overline{E}_{br}$ measurements. Less than 1% change in $\overline{E}_{br}$ were detected when probe angles were varied to $\pm 2^\circ$ at high flow speeds.

Having ensured good flow one-dimensionality and probe alignment, actual data could be collected. In this study, twenty simultaneous measurements of the mean bridge output voltage, $\overline{E}_{br}$, and the pressure transducer output voltage, $\overline{E}_{tr}$, were taken during calibration. It was believed that a better approximation of the $\overline{E}_{br}$ versus flow velocity, U , relationship would be obtained using a large number of data points especially at the low velocity region. The flow velocity was estimated by the following relation;

$$u = \sqrt{\frac{2\Delta P}{\rho}} \quad (3.1)$$

The differential pressure, ΔP , was measured with a pressure transducer and its output signal, $\overline{E}_{tr}$, was related to ΔP by a calibration process. The ADACS system was programmed to average a total of 2000 points for both the $\overline{E}_{br}$ and $\overline{E}_{tr}$ signals at each data set. The simultaneous input of the flow temperature provided a correction to the air density and thus the flow velocity calculation. The calibration procedure took approximately twenty minutes from start to completion. The temperature of the air increased an average of 4°C during this period and temperature effects on hot-wire measurements were not considered. Sufficient time was allowed between readings for establishing steady-state conditions. The

relationship between flow velocity and the bridge output voltage is described by King's law as;

$$\overline{E}_{br}^2 = A + Bu^n \quad (3.2)$$

Values of n are such as to provide a better fit to the nonlinear relationship; typically $n = 0.45, 0.5$.

A direct relationship between the bridge voltage and the flow velocity may be obtained by either passing $\overline{E}_{br}$ through an electronic signal linearizer or by computational methods. Although the anemometer unit was equipped with an electronic signal linearizer (TSI model 1052), it was preferred to use a computer program for signal linearization, which would increase the speed of the measurements. The TSI FORTRAN linearization program was converted to the ADACS Basic language version. A listing of the computer program developed for calibration of hot-wire probes "CALIB*" is shown in Appendix B. This program approximates the response of the anemometer bridge output with a fourth degree polynomial and converts $\overline{E}_{br}$ to linearized voltages, $\overline{E}_{lin}$, by the following relation;

$$\overline{E}_{lin} = S \left(A_0 + A(\overline{E}_{br} - \overline{E}_0) + B(\overline{E}_{br} - \overline{E}_0)^2 + C(\overline{E}_{br} - \overline{E}_0)^3 + D(\overline{E}_{br} - \overline{E}_0)^4 \right) \quad (3.3)$$

where $\overline{E}_0$ and $\overline{E}_{br}$ are the bridge output at zero and non-zero values, respectively, of the effective cooling velocity, S is the slope of $\overline{E}_{lin}$ vs u curve and A_0, A, B, C, D are the coefficients of the polynomial calculated by the computer program. Thus, $\overline{E}_{lin}$ may be related to the flow velocity as;

$$\overline{E}_{lin} = Su \quad (3.4)$$

The same linearization procedure was the procedure followed by Yowakim^[55] and Kelly^[34], who had also converted the TSI Fortran linearization program to ADACS Basic language, with excellent results.

The method of computing the linearized RMS signal $\sqrt{e_{lin}^2}$ from the measured RMS of the bridge output signal e_{br}^2 is outlined in the following; assume that for low turbulence cases,

$$\sqrt{e_{lin}^2} = \Delta \overline{E}_{lin} \quad \text{and} \quad \sqrt{e_{br}^2} = \Delta \overline{E}_{br}$$

or

$$\sqrt{e_{lin}^2} = \frac{\Delta \overline{E}_{lin}}{\Delta \overline{E}_{br}} \sqrt{e_{br}^2}$$

taking the limits would give

$$\sqrt{e_{lin}^2} = \frac{\partial \bar{E}_{lin}}{\partial \bar{E}_{br}} \sqrt{e_{br}^2}$$

Differentiation of equation 3.3 determines $\partial \bar{E}_{lin} / \partial \bar{E}_{br}$ which if substituted in the above relationship will give the final expression for $\sqrt{e_{lin}^2}$ (Yowakim^[55]). Thus;

$$\begin{aligned} \sqrt{e_{lin}^2} = S \bigg(& A + 2B(\bar{E}_{br} - \bar{E}_o) + 3C(\bar{E}_{br} - \bar{E}_o)^2 \\ & + 4D(\bar{E}_{br} - \bar{E}_o)^3 \bigg) \sqrt{e_{br}^2} \end{aligned} \quad (3.5)$$

As a check on the method, various voltage signals from the anemometer bridge unit were input to the TSI electronic signal linearizer and its outputs compared well with the values calculated from equations 3.4 and 3.5.

3.1.4. Determination of the Sensitivity Coefficients

Studies by Champagne^[13] and Mojola^[45] have demonstrated the importance of the yaw sensitivity coefficient, k , in accounting for the tangential cooling velocity encountered with inclined probes. Jørgensen^[33] showed that in turbulent flows the overall directional characteristics of the probe must be composed of a pitch correction factor, h , in addition to the yaw correction factor, k .

Figure 16 shows an arbitrary orientation of a hot-wire probe with respect to the flow direction. Then the effective cooling velocity acting on the sensor is given by Jørgensen^[33] as;

$$u_{eff}^2 = u_x^2 + k^2 u_y^2 + h^2 u_z^2 \quad (3.6)$$

where k and h are the yaw and pitch correction factors respectively.

If the pitch angle, $\theta = 0$, equation 3.6 becomes;

$$u_{eff}^2 = u_x^2 + k^2 u_y^2$$

or

$$u_{eff}^2(\alpha) = u_{\theta=0}^2 (\cos^2 \alpha + k^2 \sin^2 \alpha) \quad \text{for } \theta = 0 \quad (3.7)$$

If the yaw angle, $\alpha = 0$, equation 3.6 becomes;

$$u_{eff}^2 = u_x^2 + h^2 u_z^2$$

or

$$u_{eff}^2(\theta) = u_{\alpha=0}^2(\cos^2 \theta + h^2 \sin^2 \theta) \quad \text{for } \alpha = 0 \quad (3.8)$$

If u is substituted with u_{eff} in equation 3.4;

$$\bar{E} = S u_{eff}$$

Substituting equations 3.7 and 3.8 into equation 3.4 it becomes;

$$\frac{\bar{E}(\alpha)^2}{S^2} = \frac{\bar{E}(0)^2}{S^2} (\cos^2 \alpha + k^2 \sin^2 \alpha)$$

or

$$k = \frac{1}{\sin \alpha} \left(\frac{\bar{E}(\alpha)^2}{\bar{E}(0)^2} - \cos^2 \alpha \right)^{\frac{1}{2}} \quad (3.9)$$

Similarly

$$h = \frac{1}{\sin \theta} \left(\frac{\bar{E}(\theta)^2}{\bar{E}(0)^2} - \cos^2 \theta \right)^{\frac{1}{2}} \quad (3.10)$$

The values for α , θ selected in this study were 0, 90°, as suggested by Jørgensen^[33]. The probes were visually aligned with the flow and slight misalignments had little effect on the results since large angles were selected (i. e. 0, 90°). The values of the sensitivity coefficients obtained in this study were $k = 0.236$ and $h = 1.174$ for an average of five different probes (normal and slanted).

3.2 Experimentation on the B.L.C. Apparatus

3.2.1 Mass Flow Rate Measurements

The suction and blowing flow rates were calculated by using the measured pressure differences across the orifice-plates in the following equation^[10];

$$\dot{M} = 359.1 \times c \times z \times d^2 \times E \times \sqrt{h} \times \sqrt{w} \quad (3.11)$$

where

$\dot{M}$ = mass flow rate, (lbm/hr)

c = discharge coefficient=0.608 here

d = orifice diameter, (inches)

E = velocity of approach factor = $\frac{1}{\sqrt{1-m^2}}$; $m = \left(\frac{d}{D}\right)^2$

D = pipe diameter, (inches)

h = pressure difference across orifice plate, (inches of water)

w = density of fluid at upstream pressure tapping, (lbm/ft³)

z = correction multiplier (Table C.1)

In order to obtain the momentum flux coefficients for the wall and cylinder jets, it was first necessary to determine the slot thicknesses. The mean slot thicknesses, $\bar{t}$, were determined by dividing the volume flow rate supplied to each slot in still air conditions by the velocity corresponding to isentropic expansion from plenum pressure to ambient pressure. Thus, the slot thickness was determined according to the following relation;

$$\bar{t} = \frac{\dot{m}}{\rho_j V_j}; \quad \rho_j = \frac{P_j}{RT_j} \quad (3.12)$$

where

$\dot{m}$ = mass flow rate per unit span

V_j = velocity of jet

P_j = static pressure at jet exit

T_j = temperature of jet air

ρ_j = density of jet air

From isentropic relations the jet velocity is given by the following relation;

$$V_j = \sqrt{\frac{2\gamma RT_j \left[\left(\frac{P_0}{P_j} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]}{\gamma - 1}} \quad (3.13)$$

where P_0 = plenum total pressure.

The mean thickness was found to be 0.87 mm and 0.90 mm for the cylinder and the plane wall slots respectively. Thus, knowing the mean slot thicknesses the momentum flux coefficients were calculated by the following relation;

$$C_\mu = \frac{(\dot{m}^2 / \rho_j \bar{t})}{\frac{1}{2} \rho_\infty U_\infty^2 d} \quad (3.14)$$

where

ρ_∞ = density of freestream air

U_∞ = velocity of freestream air = 21 ms⁻¹ here

d = cylinder diameter

The C_μ values attained in this study were 0.1, 0.2 and 0.22 for the cylinder jet and 0.11 for the plane wall-jet. A calibration between the orifice-plate pressure difference and the plane wall plenum chamber pressures was established at actual operating temperature and freestream velocity for each cylinder momentum coefficient (C_{μ_c}) and wall-jet momentum coefficient (C_{μ_w}) combination. During actual test running, the orifice-plates were removed from the blowing air supply lines and the C_μ 's were determined using the measured plenum chamber pressures. This procedure was adopted to reduce blower power requirements thus minimizing heating of the blowing air and attendant errors in the hot-wire data. Figure 17 shows the calibration curves for both cylinder and wall slots. A computer program, MFLOW, was developed to calculate, as accurately as possible, both the mass flow rate and the slot thickness of the cylinder and wall slots. The jet exit velocities, V_j , calculated from equation 3.13 were approximately checked with pitot tube (1 mm O.D.) measurements placed at the slot exits. The slot thicknesses were also approximately checked by feeler gauges. All the necessary measured input parameters, a short description and a listing of the "MFLOW" program are shown in Appendix C.

3.2.2 End-Wall Boundary Layer Control

When the blowing on the cylinder is activated, the separation of the flow is not symmetrical about the cylinder-forebody chord. Thus the flow conditions are similar to those observed in circulation controlled wings (see Figure 1), where the separation of the flow at the side of the slot occurs later than the separation from the other side. When a circulation controlled two-dimensional wing section is tested in a wind tunnel, the circulation around it causes a tendency for the vorticity to be shed where the wing penetrates the boundary layer on the floor and ceiling of the wind tunnel. Thus the bound vorticity shed downstream of the wing leading edge introduces three-dimensional effects into the flow.

In this study, in order to ensure two-dimensional flow conditions downstream at the forebody and at the vicinity of the cylinder slot exit, suction was applied to the porous end-walls. The suction intended to reduce the thickness of the boundary layer and reduce tendency for bound vorticity, near the cylinder (i.e. where vorticity is more pronounced). The amount of suction required to ensure two-dimensionality of the flow, with only the circulation controlled cylinder in similar flow conditions as in the present study, was determined by Goode^[27]. He describes that the suction requirement corresponds to approximately two times the approaching turbulent boundary layer deficit mass flow and in his

case, V_n/U_∞ was equal to about 0.015 for each plate, where V_n is the velocity component normal to the porous end plate. In this study, although the suction requirements were lighter due to the presence of the forebody, limitations of suction equipment did not allow for higher suction rates to be achieved. The V_n/U_∞ ratio here was about 0.013 for each porous plate.

3.2.3 Flow Quality Checks

This section includes descriptions of a number of flow quality checks performed in this study in order to provide strong direct or indirect evidence that the flow downstream of the cylinder slot, achieved reasonably two-dimensional conditions. Good flow two-dimensionality is essential if the data are to be used for validation of two-dimensional flow calculation methods.

Flow Alignment and Visualization

With the test section in the wind tunnel circuit carefully leveled in all directions and clamped in place, separate leveling of all B.L.C. apparatus components were necessary. First the two end-walls porous plates were faired into the section's end-walls and the cylinder was adjusted normal to the porous end plates. The plane wall-jet in this arrangement was at a fixed distance of 5.25 in from the center of the cylinder. The test section walls were all smooth. The forebody was fitted upstream of the cylinder and fixed in place with its leading edge plane normal to the section's end-walls as shown in Figures 8, 11 and 12.

Having ensured that the apparatus was aligned with the tunnel freestream flow direction, a grid of tufts of 0.75 in long black thread were taped at various positions along the surface of the forebody. This was done primarily to ensure that the boundary layer developing on the forebody did not separate before it reached the cylinder jet exit, and also to examine the spanwise flow development on the forebody. No separation of the flow was detected and the tufts failed to indicate any spanwise flow irregularities.

Another tuft visualization scheme was used downstream of the cylinder and wall-jets. Tufts of 0.75 in long black thread were glued in a square grid formation 0.75 in apart as shown in Figures 31 to 33. One grid covered the vertical side wall of the working section for a distance of 11 in downstream of the wall-jet. Grids of tufts on both the ceiling and floor of the working section covered an area of 11.5 in $\times$ 6 in just downstream of the porous end plates, as shown in Figures 31 to 33.

With the tufts in place, a series of trial and error experiments were conducted to determine the best roughness element configuration for tripping, and thus making fuller the boundary layer that developed on the forebody. Tripping wires of 1.2 mm and 2.1 mm O.D's and strips of dymo-tape with a capital V, were first tried at several upstream positions across the entire 50 cm span of the forebody. By observing the movement of the tufts, when the wind tunnel was switched on and at the highest cylinder jet momentum coefficient of $C_{\mu_e} = 0.22$, it was possible to evaluate the effectiveness of the tripping devices. It was found that tripping of the boundary layer upstream improved the two-dimensionality of the flow mainly because the resulting turbulent boundary layer is more stable. A considerable time was devoted in obtaining the best roughness configuration. The roughness arrangement adopted for further experimentation consisted of a 50 cm $\times$ 23 cm piece of sand paper mounted on the leading edge of the forebody. It should be noted that because the primary concern in this study was to obtain good flow two-dimensionality conditions downstream of the cylinder jet.

Surface flow visualization on the cylinder was also attempted in order to provide evidence on the spanwise uniformity of the separation point. A thin film of glycerin with titanium oxide mixture, was applied just downstream of the cylinder slot exit. The static pressure taps on the cylinder were previously covered with a strip of adhesive tape starting at a distance $\theta = 12^\circ$ from the jet exit. The flow was kept until the pattern developed by the glycerin mixture remained unchanged. Then, the flow was turned off so that the pattern could be photographed. A discussion on the flow visualization results will follow in Chapter 5.

Pressure Measurements

Static pressure measurements was another means that was used for providing evidence about the flow two-dimensionality. Spanwise static pressure at distances of $x = 7$ in and $x = 11$ in on the side wall downstream of the wall-jet exit, were recorded at all C_{μ_e} , C_{μ_s} combinations with an accuracy of about 2%, using water filled manometers. Results and discussion of these measurements will follow in Chapter 5.

Still Air Measurements

Previous work of Kind *et al.*^[38] on B.L.C. apparatus, had revealed that random short wavelength fluctuations of the maximum total pressure across the span of the cylinder jet occurred under still air conditions.

In this study, this effect was also examined with the cylinder jet issuing in still air surrounding a pitot tube of 1 mm I.D. which was traversed across the central 40 cm of the cylinder span and at three downstream stations; namely at $\theta = 32^\circ$, 62° , and 92° from the cylinder slot exit. Simultaneous measurements of the maximum total pressure and the cylinder plenum pressure were taken at $0.5 \text{ cm} \pm 3\%$, spanwise intervals using two single tube tilting manometers.

The pressures could be estimated with an accuracy of about 1.5%. Results and discussion on these measurements will follow in Chapter 5.

In still air surroundings, the static pressure distribution on the cylinder was also recorded using the multitube manometer board, with an accuracy of about 2%. These results will be later compared to those of *Fekete*^[23].

Spanwise Mean Velocity Profile Measurements

Spanwise mean velocity measurements were another means used for providing evidence on the flow two-dimensionality downstream of the cylinder jet exit and in the close vicinity of the cylinder. A normal hot-wire probe was traversed through the shear layer that developed on the cylinder, at various downstream stations and at spanwise distances of $z = -1$, -14 and $+14 \text{ cm}$. The cylinder momentum coefficients were $C_{\mu_c} = 0.1$, 0.2 and 0.22 while the wall-jet momentum coefficient was constant at $C_{\mu_w} = 0.11$. Results and discussion will follow in Chapter 5.

3.2.4 Repeatability and Operating Flow Conditions

Control of Flow Parameters

The wind tunnel freestream velocity (U_∞) was monitored via a tilting micro-manometer as described in Section 2.2.8. In this study, the freestream velocity was held constant at $U_\infty = 21 \text{ ms}^{-1}$, a value set by *Kind et al.*^[38] in all experiment runs. The primary reason for choosing this particular freestream velocity was that cylinder momentum coefficients of $C_{\mu_c} = 0.22$ maximum could be attained with the available equipment. The Reynolds number with respect to the cylinder diameter, corresponding to this velocity was about 180,000. The variation of U_∞ was less than 1% during a typical experimental run of one hour. The side wall jet momentum coefficient was also held constant at $C_{\mu_w} = 0.11$ in all experiment runs. This was because earlier experiments by *Gooden*^[27] did not show significant effect of the wall-jet momentum coefficient on the cylinder velocity distribution.

The effect of higher C_{μ_w} on the mean velocity and longitudinal turbulence at $\theta = 12^\circ$ on the cylinder has been also investigated in this study. The cylinder jet momentum coefficient, C_{μ_c} , values attained in the present experiments were 0.1, 0.2 and 0.22. A calibration relationship between the mass-flow rate supplied to the wall and cylinder plenum chambers and the corresponding pressure in the plenum (described in Section 3.2.1), allowed close monitoring of the C_{μ_c} and C_{μ_w} via manometers.

The suction rate through each porous end-wall plate was held constant at approximately 60 c.f.m. With all the flow parameters set accordingly sufficient time, approximately five minutes, was allowed for steady-state flow conditions to be reached. By simultaneously monitoring the flow parameters and the static pressure distribution downstream of the cylinder slot, it was found the same flow conditions could be repeated accurately. Measurements of mean velocity profiles at the same station and at different time periods verified the repeatability of the flow conditions.

Temperature Measurements

One of the major difficulties in using hot-wires for measurements is the effect of fluid temperature changes on the heat transfer from the sensors. Since the hot-wire responds both to velocity and temperature fluctuations, elimination of the latter helps to ensure accurate velocity measurements.

Although the flow temperature in the present study could not be fully controlled, due to limitations of the equipment, an attempt was made to estimate the temperature changes. Early experiments had shown that the wind tunnel freestream temperature varied a maximum of 2°C over a period of two hours. The air temperature inside the side wall-jet plenum chamber was monitored with an accuracy of $\pm 1^\circ \text{C}$ via a thermocouple. In a typical experimental run, at $C_{\mu_w} = 0.11$, the side wall-jet plenum temperature was about 7°C higher than the freestream air temperature. The temperature in the cylinder plenum at $C_{\mu_c} = 0.1$ was 5°C higher than the freestream air temperature, at $C_{\mu_c} = 0.2$ it was about 10°C higher and at $C_{\mu_c} = 0.22$ it was about 15°C higher than freestream air temperature. Since the aforementioned increases in temperatures was the result of heating by the blower motors, which could not be controlled, in all experimental runs it was tried to obtain the same temperature differences. This would ensure good repeatability in the measurements even if errors were to be introduced by the temperature changes.

- Bradshaw ^[6] describes that one means of minimizing temperature effects is to run the

wire at as high a temperature as possible. Hence, the hot-wire sensor temperature was set to 300° C, which corresponds to an overheat ratio of 1.9 in all experiment runs. Higher overheat ratios could not be sustained by the tungsten wires used in this study. Although the plenum chamber air temperatures and the freestream air temperature could easily be monitored during the experiment, the actual temperature of the flow resulting from the mixing of the cylinder jet air with the freestream air was not known. According to the TSI anemometer manual, the overheat ratio of a hot-wire sensor is given by the following relation:

$$\text{Overheat ratio} = \frac{R_h}{R_c} = 1 + q_c(T_s - T_e) \quad (3.15)$$

where

q_c = temperature coefficient for resistance of sensor

($\approx 0.0058/^{\circ}\text{C}$ for the tungsten wire used in this experiment)

R_h = operating resistance of sensor

R_c = resistance of sensor when at fluid temperature

T_s = operating temperature of sensor

T_e = temperature of the operating environment

An estimation of the temperature difference between the cylinder wall-jet air and the freestream air may be obtained by rewriting equation 3.15, as:

$$\Delta T = \frac{\Delta R}{R_o q_c} \quad 3.16$$

where

ΔT = temperature difference between the freestream and cylinder wall-jet air

ΔR = difference in cold wire resistance at ΔT

R_o = cold wire resistance at reference temperature

The reference temperature and the corresponding cold wire resistance were assumed the same as those of the freestream air. Thus, an estimation of the actual flow temperature was obtained by traversing a cold-wire through the boundary layer downstream of the cylinder jet. The cold resistances were measured by adjusting the variable "Probe Resistance Decades" on the anemometer until the meter needle did not deflect when the

"Res Meas" control was quickly pressed and released. Results and discussion on temperature effects will follow in Chapter 5.

Zeroing of the Hot-Wire Probes

The traversing linkage mechanism used for these experiments has been described in Section 2.2.6 and it is shown in Figures 10 and 11. The hot-wire probes were brought to the first measuring position, "zeroed", with good repeatability by following the empirical experimental procedure: with the anemometer in the "Stand-By" position and no flow the prongs of a normal hot-wire probe were moved tangentially to the surface. When the prongs touched the steel cylinder surface, the movement was stopped and the orientation of the sensor was checked visually using a mirror. The protractor system in the back of the probe holder tube was adjusted to 90° roll angle ($\psi = 90^\circ$) at this position.

The movement was then reversed and the prongs were moved an average of 0.05 mm from the cylinder surface. This position was defined as the first measuring point. When 45° slanted probes were used with their prong plane normal to the cylinder surface as shown in Figure 18, the first measuring point is defined at 0.55 mm from the cylinder surface to the probe axis. Before proceeding with the actual measurements, the bridge voltage at zero flow $\bar{E}_0$ was checked with the $\bar{E}_0$ obtained during calibration. All distances needed for solving the kinematics of the traversing linkage were measured with a micrometer.

Mean Velocity and Turbulence Measurements

Mean velocity and turbulence data were collected at eight stations downstream of the cylinder slot exit; namely at $\theta = 2^\circ, 12^\circ, 22^\circ, 42^\circ, 52^\circ, 62^\circ$ and 72° . The spanwise position of the probe was $Z = -1 \text{ cm}$ from the centerline, in all experiments. The choice of $Z = -1 \text{ cm}$ as the measuring position at all stations was because three-dimensionality effects would be less severe at the midspan of the cylinder. Moreover, it was necessary to keep some distance from the static pressure taps located on the surface of the cylinder to eliminate possible flow disturbances resulting from them.

A large number of data points were collected at each measuring station in an attempt to better define the minima and maxima of both the mean velocity and turbulence profiles. The need for a large number of data points at measuring stations close to the slot exit was more pronounced because there the gradients of the profiles were larger and the shear layer was thinner. The computer programs developed to collect data and to traverse the probes allowed changes of the traversing step size during the experiments. Thus, once

the probes were "zeroed" and the flow conditions established, the experiment was fully computer controlled and it could be easily interrupted.

Measurements using Normal Probes

Mean velocities were measured using a single normal hot-wire probe. The prong plane of the probe was parallel to the cylinder surface, $\psi = 90^\circ$, as shown in Figure 18. With this arrangement, a better spatial resolution could be achieved than with a 45° slanted probe and more data could be collected in the vicinity of the cylinder surface.

The ADACS computer was programmed to sample at the optimum rate described in Section 3.1.2. The ADACS computer program version, "U&RMS", calculated the actual Y distance of the probe from the cylinder surface, the mean velocity, u , and the longitudinal turbulence, $\sqrt{u'^2}$. A short description and a listing of the program "U&RMS" is shown in Appendix D.

Measurements using 45° Slanted Probes

The shear stresses $\overline{u'v'}$ were measured with slanted probes. The probe orientation was such that the probe prongs plane was normal to the cylinder surface as shown in Figure 18. Once the probe was traversed at this orientation ($\psi = 0^\circ$), it was brought back to the zero position, rolled 180° and the same procedure repeated.

The ADACS computer program version "M&RMS", was also designed to sample at the optimum rate and calculated the distance, Y , from the cylinder surface and displayed the $\overline{E_{br}}$ and $\sqrt{e'^2_{br}}$ at each measuring point. Part of the program "M&RMS" is described in Appendix D.

During the experimental runs, the ADACS system and all the supporting instruments were closely checked to ensure that the experiment proceeded satisfactorily. By observing the hot-wire signal on the oscilloscope, it was possible to detect experimental malfunctions. In such a case, the traversing was repeated. All the data were transferred to the University's mainframe computer system for processing.

Chapter 4

Hot-Wire Signal Analysis

4.1 Introduction

Hot-wire anemometer signal analysis has been the subject of many research projects in the past. One of the most notable is the analysis by Acrivellis^[1]. He describes that with his method, a conventional hot-wire anemometer and a commercially available slanting hot wire probe as well as a simple measuring setup may be used to determine all velocity and Reynolds stress components, in multi-dimensional flow fields of any turbulent intensity with no sacrifice of accuracy. Acrivellis's method has been extensively studied by Yowakim^[55] who found errors in its formulation. Therefore, he^[55] directed his efforts towards the derivation of a similar hot-wire signal analysis with the orientation of the hot-wire probe axis normal to the freestream flow direction.

One of the tasks of this thesis was to derive hot-wire equations for the coordinate system shown in Figure 19, following similar guidelines as those described by Yowakim^[55]. In the present work the probe axis is approximately aligned with the nominal flow direction. This would enable measurements of all mean velocity and Reynolds stress components in the flows presented in this study, by using a single 45° slant probe or a combination of 45° slant and normal hot-wire probes.

4.2 Derivation of Hot-Wire Equations for 3-D Flows

The method described here is based on measurements with a single 45° hot-wire probe rotated at as many angles as are required for determining all the unknowns, or a combination of slant and normal probes. The resulting system of equations relating the square output hot-wire signal to mean or turbulence quantities can be solved simultaneously to determine these quantities. Figure 19 shows a 45° slant hot-wire probe in an arbitrary angular position in the flow field. The velocity components of this field are $u + u'$, $v + v'$ and $w + w'$ parallel to the axes x , y and z of the inertial coordinate system. Primes indicate fluctuating components. The ξ , ζ , η coordinate system of Figure 19 is a probe-based system. The ξ -axis or the probe-based velocity, U_N , is perpendicular to the hot-wire, forming angle α with the probe axis. The angle, α , is 45° for a 45° slant probe or 0° for a normal probe.

The axes ζ and η , and the corresponding velocity components, u_T and u_B , are defined parallel and bi-normal to the hot-wire. Each angular position of the hot-wire is obtained through rotation of the α° slant probe about its axis. The probe-based coordinate system moves with it. The transformation from inertial to probe-based coordinate system requires a rotation of the x, y, z coordinate system on the x -axis, probe-axis, through the angle ψ , and on the z -axis through the angle α . Since this is an orthogonal transformation, the velocity components are transformed in the same way as the coordinates. Hence, the instantaneous velocity components are;

$$u_N = (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \quad (4.1)$$

$$u_T = -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \quad (4.2)$$

$$u_B = (v + v') \sin \psi + (w + w') \cos \psi \quad (4.3)$$

where u_N , u_T , u_B are the normal, tangential and binormal velocity components defined previously.

The effective cooling velocity acting on the sensor depends on the yaw and pitch angles that the sensor makes with the freestream flow direction. Jørgensen^[24] has defined the effective cooling velocity acting on the sensor as;

$$u_{eff}^2 = u_N^2 + k^2 u_T^2 + h^2 u_B^2 \quad (4.4)$$

where k is the yaw factor and h is the pitch factor. The method of determination of these factors has been mentioned in Section 3.1.4.

The linear relationship between effective cooling velocity and hot-wire output voltage is;

$$E = S u_{eff} \quad (4.5)$$

where S is the slope of the linearized plot of E vs. u_{eff} .

From equations 4.4 and 4.5,

$$u_{eff} = (u_N^2 + k^2 u_T^2 + h^2 u_B^2)^{\frac{1}{2}} \quad (4.6)$$

or

$$\begin{aligned} u_{eff} &= \left[\{ (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \}^2 \right. \\ &\quad + k^2 \{ -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \}^2 \\ &\quad \left. + h^2 \{ (v + v') \sin \psi + (w + w') \cos \psi \}^2 \right]^{\frac{1}{2}} \\ &= \frac{E(\psi, \alpha)}{S} \end{aligned} \quad (4.7)$$

The square root term of equation 4.7 is not expanded into a binomial series and then time-averaged as in the case of the conventional method of determination, but it is first squared and thereafter time-averaged. Then, for the squared output signal;

$$\frac{\overline{E^2}(\psi, \alpha)}{S^2} = au^2 + bv^2 + cw^2 + duv + fvw + guw$$

or

$$\frac{\overline{E}(\psi, \alpha)}{S} = \sqrt{au^2 + bv^2 + cw^2 + duv + fvw + guw} \quad (4.8)$$

where

$$a = \cos^2 \alpha + k^2 \sin^2 \alpha$$

$$b = \sin^2 \alpha \cos^2 \psi + k^2 \cos^2 \alpha \cos^2 \psi + h^2 \sin^2 \psi$$

$$c = \sin^2 \alpha \sin^2 \psi + k^2 \sin^2 \psi \cos^2 \alpha + h^2 \cos^2 \psi$$

$$d = \sin 2\alpha \cos \psi - k^2 \sin 2\alpha \cos \psi$$

$$f = -\sin 2\psi \sin^2 \alpha - k^2 \sin 2\psi \cos^2 \alpha + h^2 \sin 2\psi$$

$$g = k^2 \sin \psi \sin 2\alpha - \sin \psi \sin 2\alpha$$

The only assumption involved in obtaining equation 4.8 was that for small turbulence intensity ($\approx 20\%$), the turbulent terms were considered very small compared with the time-mean velocity terms and they were omitted. Detailed intermediate steps involved in obtaining equation 4.8 from the expansion of equation 4.7 are shown in Appendix F.

From equation 4.8 one can get the time-mean velocity components of flow from mean ($\overline{E}$) output signal of the hot-wire anemometer. To determine the three mean velocity

components from equation 4.8, one has to establish a system of three independent linear equations, by taking measurements for three suitable ψ values with a slanted hot-wire probe at each point in the flow.

If 'e' is considered to be the fluctuating component of the linearized output hot-wire signal, it can be related to the linearized instantaneous and time averaged signals E and $\bar{E}$ as;

$$\frac{e(\psi, \alpha)}{S} = \frac{E(\psi, \alpha) - \bar{E}(\psi, \alpha)}{S} \quad (4.9)$$

Substituting expressions of $E(\psi, \alpha)/S$ and $\bar{E}(\psi, \alpha)/S$ into equation 4.9 and after manipulation and time averaging we can arrive at the final form to be used in the data reduction.

Therefore,

$$\frac{\bar{e}^2(\psi, \alpha)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1b_1 \overline{u'v'} + 2a_1c_1 \overline{u'w'} + 2b_1c_1 \overline{v'w'} \right] \quad (4.10)$$

where

$$Q = \frac{\bar{E}(\psi, \alpha)}{S} = \left[au^2 + bv^2 + cw^2 + duv + fvw + guw \right]^{\frac{1}{2}}$$

$$a_1 = 2au + dv + gw$$

$$b_1 = 2bv + fw + du$$

$$c_1 = 2cw + gu + fv$$

The assumption involved in this analysis was for low turbulence intensity ($\approx 20\%$). Higher order terms such as $\overline{u'^4}$, $\overline{u'^2v'^2}$, $\overline{u'v'^2}$ etc. were neglected since for low turbulence intensity, they will be small compared with the terms obtained in equation 4.10

Equation 4.10 gives results for the Reynolds stresses (in 3-D flow). This expression is valid for both slanted and normal hot-wire probes, provided that the turbulence intensity is less than 20%. Coefficients a_1 , b_1 , c_1 should be computed at each local position in the flow field and for every probe orientation angle, ψ . Finding all six Reynolds stress components requires that a system of six linear independent equations be established by selecting suitable α and ψ values for the hot-wire probe. Detailed intermediate steps involved in arriving at equation 4.10 from equation 4.9 are shown in Appendix F.

4.2.1 Determination of Mean and Fluctuating Quantities at Different (u, v)

Position 1 ($\psi = 0^\circ$, $\alpha = 45^\circ$)

From equation 4.8;

$$a = \frac{1}{2} + \frac{1}{2}k^2 = \frac{1}{2}(1 + k^2)$$

$$b = \frac{1}{2} + \frac{1}{2}k^2 = \frac{1}{2}(1 + k^2)$$

$$c = h^2$$

$$d = 1 - k^2$$

$$f = 0$$

$$g = 0$$

Therefore,

$$\frac{\overline{E}(0, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + \frac{1}{2}(1 + k^2)v^2 + h^2w^2 + (1 - k^2)uv \right]^{\frac{1}{2}}$$

or for 2-D flow;

$$\frac{\overline{E}(0, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + \frac{1}{2}(1 + k^2)v^2 + (1 - k^2)uv \right]^{\frac{1}{2}}$$

From equation 4.10;

$$a_1 = 2au + dv + gw$$

$$b_1 = 2bv + fw + du$$

$$c_1 = 2cw + gu - fv$$

or

$$a_1 = (1 + k^2)u + (1 - k^2)v$$

$$b_1 = (1 + k^2)v + (1 - k^2)u$$

$$c_1 = 2h^2w$$

$$\frac{\overline{e^2}(0, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1b_1 \overline{u'v'} + 2a_1c_1 \overline{u'w'} + 2b_1c_1 \overline{v'w'} \right]$$

or, for 2-D flow;

$$\frac{\overline{e^2}(0, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + 2a_1b_1 \overline{u'v'} \right]$$

Position 2 ($\psi = 90^\circ$, $\alpha = 45^\circ$)

From equation 4.8;

$$a = \frac{1}{2}(1 + k^2)$$

$$b = h^2$$

$$c = \frac{1}{2}(1 + k^2)$$

$$d = 0$$

$$f = 0$$

$$g = k^2 - 1$$

$$\frac{\overline{E}(90, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + h^2v^2 + \frac{1}{2}(1 + k^2)w^2 + (k^2 - 1)uw \right]^{\frac{1}{2}}$$

or for 2-D flow;

$$\frac{\overline{E}(90, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + h^2v^2 \right]^{\frac{1}{2}}$$

From equation 4.10;

$$a_1 = (1 + k^2)u + (k^2 - 1)w$$

$$b_1 = 2h^2v$$

$$c_1 = (1 + k^2)w + (k^2 - 1)u$$

$$\frac{\overline{e^2}(90, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1b_1 \overline{u'v'} + 2a_1c_1 \overline{u'w'} + 2b_1c_1 \overline{v'w'} \right]$$

or for 2-D flow;

$$\frac{\overline{e^2}(90, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + 2a_1b_1 \overline{u'v'} \right]$$

Position 3 ($\psi = 180^\circ$, $\alpha = 45^\circ$)

From equation 4.8;

$$a = \frac{1}{2}(1 + k^2)$$

$$b = \frac{1}{2}(1 + k^2)$$

$$c = h^2$$

$$d = k^2 - 1$$

$$f = 0$$

$$g = 0$$

$$\frac{\overline{E}(180, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + \frac{1}{2}(1 + k^2)v^2 + h^2w^2 + (k^2 - 1)uv \right]^{\frac{1}{2}}$$

or for 2-D flow;

$$\frac{\overline{E}(180, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + \frac{1}{2}(1 + k^2)v^2 + (k^2 - 1)uv \right]^{\frac{1}{2}}$$

From equation 4.10;

$$a_1 = (1 + k^2)u + (k^2 - 1)v$$

$$b_1 = (1 + k^2)v + (k^2 - 1)u$$

$$c_1 = 2h^2w$$

$$\frac{\overline{e^2}(180, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1b_1 \overline{u'v'} + 2a_1c_1 \overline{u'w'} + 2b_1c_1 \overline{v'w'} \right]$$

or for 2-D flow;

$$\frac{\overline{e^2}(180, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + 2a_1b_1 \overline{u'v'} \right]$$

Position 4 ($\psi = 270^\circ$, $\alpha = 45^\circ$)

From equation 4.8;

$$a = \frac{1}{2}(1 + k^2)$$

$$b = h^2$$

$$c = \frac{1}{2}(1 + k^2)$$

$$d = 0$$

$$f = 0$$

$$g = 1 - k^2$$

$$\frac{\overline{E}(270, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + h^2v^2 + \frac{1}{2}(1 + k^2)w^2 + (1 - k^2)uw \right]^{\frac{1}{2}}$$

or for 2-D flow;

$$\frac{\overline{E}(270, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + h^2v^2 \right]^{\frac{1}{2}}$$

From equation 4.10;

$$a_1 = (1 + k^2)u + (1 - k^2)w$$

$$b_1 = 2h^2v$$

$$c_1 = (1 + k^2)w + (1 - k^2)u$$

$$\frac{\overline{e^2}(270, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1b_1 \overline{u'v'} + 2a_1c_1 \overline{u'w'} + 2b_1c_1 \overline{v'w'} \right]$$

or for 2-D flow;

$$\frac{\overline{e^2}(270, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + 2a_1b_1 \overline{u'v'} \right]$$

Position 5 ($\psi = 45^\circ$, $\alpha = 45^\circ$)

From equation 4.8;

$$a = \frac{1}{2}(1 + k^2)$$

$$b = \frac{1}{4}(1 + k^2) + \frac{1}{2}h^2$$

$$c = \frac{1}{4}(1 + k^2) + \frac{1}{2}h^2$$

$$d = 0.707(1 - k^2)$$

$$f = -\frac{1}{2}(1 + k^2) + h^2$$

$$g = 0.707(k^2 - 1)$$

$$\frac{\overline{E}(45, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + \left(\frac{1}{4}(1 + k^2) + \frac{1}{2}h^2 \right)v^2 + \left(\frac{1}{4}(1 + k^2) + \frac{1}{2}h^2 \right)w^2 + 0.707(1 - k^2)uv + \left(-\frac{1}{2}(1 + k^2) + h^2 \right)vw + (0.707(k^2 - 1))uw \right]^{\frac{1}{2}}$$

or for 2-D flow;

$$\frac{\overline{E}(45, 45)}{S} = \left[\frac{1}{2}(1 + k^2)u^2 + \left(\frac{1}{4}(1 + k^2) + \frac{1}{2}h^2 \right)v^2 + (0.707(1 - k^2))uv \right]^{\frac{1}{2}}$$

From equation 4.10;

$$a_1 = (1 + k^2)u + 0.707(1 - k^2)v + 0.707(k^2 - 1)w$$

$$b_1 = \left(\frac{1}{2}(1 + k^2) + h^2 \right)v + \left(h^2 - \frac{1}{2}(1 + k^2) \right)w + 0.707(1 - k^2)u$$

$$c_1 = \left(\frac{1}{2}(1 + k^2) + h^2 \right)w + 0.707(k^2 - 1)u + \left(h^2 - \frac{1}{2}(1 + k^2) \right)v$$

$$\frac{\overline{e^2}(45, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1b_1 \overline{u'v'} + 2a_1c_1 \overline{u'w'} + 2b_1c_1 \overline{v'w'} \right]$$

or for 2-D flow;

$$\frac{\overline{e^2}(45, 45)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + 2a_1b_1 \overline{u'v'} \right]$$

Position 6 ($\psi = 0^\circ$, $\alpha = 0^\circ$) normal probe

From equation 4.8;

$$a = 1$$

$$b = k^2$$

$$c = h^2$$

$$d = 0$$

$$f = 0$$

$$g = 0$$

$$\begin{aligned}\frac{\bar{E}(0, 0)}{S} &= [au^2 + bv^2 + cw^2]^{\frac{1}{2}} \\ &= [u^2 + k^2v^2 + h^2w^2]^{\frac{1}{2}}\end{aligned}$$

or for 2-D flow;

$$\frac{\bar{E}(0, 0)}{S} = [u^2 + k^2v^2]^{\frac{1}{2}}$$

From equation 4.10;

$$a_1 = 2u$$

$$b_1 = 2k^2v$$

$$c_1 = 2h^2w$$

$$\frac{\bar{e}^2(0, 0)}{S^2} = \frac{1}{4Q^2} [a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1b_1 \overline{u'v'} + 2a_1c_1 \overline{u'w'} + 2b_1c_1 \overline{v'w'}]$$

or for 2-D flow;

$$\frac{\bar{e}^2(0, 0)}{S^2} = \frac{1}{4Q^2} [a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + 2a_1b_1 \overline{u'v'}]$$

Position 7 ($\psi = 90^\circ$, $\alpha = 0^\circ$) normal probe

From equation 4.8;

$$a = 1$$

$$b = h^2$$

$$c = k^2$$

$$d = 0$$

$$f = 0$$

$$g = 0$$

$$\frac{\overline{E}(90, 0)}{S} = \left[u^2 + h^2 v^2 + k^2 w^2 \right]^{\frac{1}{2}}$$

or for 2-D flow;

$$\frac{\overline{E}(90, 0)}{S} = \left[u^2 + h^2 v^2 \right]^{\frac{1}{2}}$$

From equation 4.10;

$$a_1 = 1$$

$$b_1 = 2h^2 v$$

$$c_1 = 2k^2 w$$

$$\frac{\overline{e^2}(90, 0)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + c_1^2 \overline{w'^2} + 2a_1 b_1 \overline{u'v'} + 2a_1 c_1 \overline{u'w'} + 2b_1 c_1 \overline{v'w'} \right]$$

or for 2-D flow;

$$\frac{\overline{e^2}(90, 0)}{S^2} = \frac{1}{4Q^2} \left[a_1^2 \overline{u'^2} + b_1^2 \overline{v'^2} + 2a_1 b_1 \overline{u'v'} \right]$$

4.3 Evaluation of the Hot-Wire Signal Analysis

In this study, an attempt has been made to obtain all mean velocity components and Reynolds stresses, using the analysis presented in section 4.2 and data from the wall-jet apparatus. This also provides a check of the analysis.

Mean Velocities:

The equations derived in section 4.2 for determining the mean velocities u , v and w , using a slanted probe at positions 1 to 5, include products of the mean velocities such as uv , vw and uw . Thus, six independent equations are needed to solve for the mean velocities at each point of the flow. These six equations can be obtained by a single slanted probe rotated to six different positions or by a combination of slanted and normal probes. Yowakim^[55] has concluded that it is preferable to use a slanted probe to obtain 5 equations and a normal probe to obtain the sixth equation. Acrivillelis^[1] also uses a normal probe for obtaining a sixth equation.

The solution of the system of equations in their full form was first tried using an IMSL library routine to solve the matrix, on the University's main computer. The result obtained for v and w were unrealistic. By examining the terms of the equations it was realized that addition of equations for positions 1 with 3 and 2 with 4 eliminated the mean velocity products. Thus two equations were obtained with the unknown terms u^2 , v^2 and w^2 as follows;

$$\frac{\overline{E^2}(0, 45) + \overline{E^2}(180, 45)}{S^2} = (1 + k^2)u^2 + (1 + k^2)v^2 + 2h^2w^2 \quad (4.11)$$

$$\frac{\overline{E^2}(90, 45) + \overline{E^2}(270, 45)}{S^2} = (1 + k^2)u^2 + 2h^2v^2 + (1 + k^2)w^2 \quad (4.12)$$

A third equation can be provided by position 6 using a normal probe as;

$$\frac{\overline{E^2}(0, 0)}{S^2} = u^2 + k^2v^2 + h^2w^2 \quad (4.13)$$

Solution of the above system of equations also resulted in unrealistic values of v and w . Similar problems in determining v and w were encountered by Kelly^[34] who used the

method described by Acrivillelis^[1] and data from the same wall-jet apparatus. The sensitivity of the method in obtaining v and w may be explained as follows; if values of v and w are small then $\overline{E^2}(0, 45) \approx \overline{E^2}(0, 180)$ or $\overline{E^2}(0, 90) \approx \overline{E^2}(0, 270)$. When the signals are very similar, this introduces singularities in the solution of the system of equations. Kelly^[34] concluded that for this wall-jet apparatus $\overline{E^2}(0, 45) \approx \overline{E^2}(0, 180)$ and $\overline{E^2}(0, 90) \approx \overline{E^2}(0, 270)$, was indeed the case. Since the flow in the wall-jet achieved good two-dimensionality (Sethanthiran^[50]), w was dropped from equations 4.11 and 4.12, and the efforts were directed towards the determination of v alone.

Adding equation 4.11 to 4.12 we obtain;

$$\frac{\overline{E^2}(0, 45) + \overline{E^2}(180, 45) + \overline{E^2}(90, 45) + \overline{E^2}(270, 45)}{S^2} = 2(1 + k^2)u^2 + (2h^2 + (1 - k^2))v^2 \quad (4.14)$$

Equation 4.14 and equation 4.11 (with $w = 0$) were solved by an iterative process as outlined below. First, v^2 was assumed to be zero and equation 4.14 was solved for u^2 . Using this value of u^2 , v^2 was calculated using equation 4.11. The iterations were repeated until the values of u^2 and v^2 converged (after about twenty iterations). The values of v obtained by this approach showed a large scatter and they were about ten percent of the values of u . All calculations were performed using a computer program, "TEST" shown in Appendix E. If v and w are eliminated from the equation for position 6 then;

$$\frac{\overline{E^2}(0, 0)}{S^2} = u^2 \quad (4.15)$$

Equation 4.15 provides a check on the analysis. Although v and w were not determined accurately by the analysis, u was within $\pm 3\%$ of the value obtained by equation 4.15. Figure 20 shows a good agreement in the determination of u with a normal and a slanted probe. It is believed that the analysis can provide more accurate estimations of v and w , if their magnitudes are larger and $\overline{E^2}(0, 45) \neq \overline{E^2}(180, 45)$, $\overline{E^2}(90, 45) \neq \overline{E^2}(270, 45)$. Due to time limitations, it was not possible to test the analysis further using data from other flow fields.

Reynolds Stresses:

At first, attempts were made to determine all six components of the Reynolds stresses by solving the system of six equations derived in Section 4.2 for positions 1 to 6. The results obtained were not realistic mainly because the values of v and w were not determined accurately and they influenced the calculations. Yowakim^[55] has found that elimination of v from the Reynolds stress calculations provided better results.

If the equation for position 3 is subtracted from the equation for position 1, u , $\overline{v'^2}$ and $\overline{u'^2}$ are eliminated giving a direct relationship for the shear stress $\overline{u'v'}$ as follows;

$$\begin{aligned} \frac{\overline{e^2}(0, 45) - \overline{e^2}(180, 45)}{S^2} &= \frac{(1+k^2)^2 u^2 \overline{u'^2} + (1-k^2)^2 u^2 \overline{v'^2} + 2(1+k^2)(1-k^2) u^2 \overline{u'v'}}{2(1+k^2)u^2} \\ &\quad - \frac{(1+k^2)^2 u^2 \overline{u'^2} (k^2-1)^2 u^2 \overline{v'^2} - 2(1+k^2)(k^2-1) u^2 \overline{u'v'}}{2(1+k^2)u^2} \\ &= 2(1-k^2) \overline{u'v'} \end{aligned}$$

or

$$\overline{u'v'} = \frac{\overline{e^2}(0, 45) - \overline{e^2}(180, 45)}{2(1-k^2)S^2} \quad (4.16)$$

Equation 4.16 is in agreement with that derived by Acrivillelis^[1] and Rodi^[47]. Similarly it can be shown that;

$$\overline{u'w'} = \frac{\overline{e^2}(90, 45) - \overline{e^2}(270, 45)}{2(1-k^2)S^2} \quad (4.17)$$

For $v = w = 0$ the equations for positions 6 and 7 give a direct relation for the longitudinal turbulence, $\overline{u'^2}$, as follows;

$$\frac{\overline{e^2}(90, 0)}{S^2} = \overline{u'^2}; \quad \frac{\overline{e^2}(0, 0)}{S^2} = \overline{u'^2} \quad (4.18)$$

Equation 4.18 is in agreement with that of Rodi^[47] and it provides a check on the method.

Addition of the equations of positions 1 and 3 eliminates $\overline{u'v'}$ and the resulting equation is in terms of $\overline{u'^2}$ and $\overline{v'^2}$. Thus;

$$\begin{aligned} &\frac{\overline{e^2}(180, 45) + \overline{e^2}(0, 45)}{S^2} \\ &= \frac{(1+k^2)^2 u^2 \overline{u'^2} + (1-k^2)^2 u^2 \overline{v'^2} + 2(1+k^2)(1-k^2) u^2 \overline{u'v'} + (1+k^2)^2 u^2 \overline{u'^2}}{2(1+k^2)u^2} \\ &\quad + \frac{(k^2-1)^2 u^2 \overline{v'^2} + 2(1+k^2)(k^2-1) u^2 \overline{u'v'}}{2(1+k^2)u^2} \\ &= (1+k^2) \overline{u'^2} + \frac{(1-k^2)^2}{(1+k^2)} \overline{v'^2} \end{aligned} \quad (4.19)$$

By substituting equation 4.18 into 4.19, $\overline{v'^2}$ can be obtained as follows;

$$\overline{v'^2} = \frac{[\overline{e^2}(180, 45) + \overline{e^2}(0, 45)](1 + k^2)^2}{S^2(1 - k^2)^2} - \frac{(1 + k^2)^2 \overline{e^2}(90, 0)}{S^2(1 - k^2)^2} \quad (4.20)$$

Similarly, addition of the equations for position 1 and 3 eliminates $\overline{u'w'}$ and the resulting equation is in terms $\overline{u'^2}$ and $\overline{w'^2}$. Thus;

$$\begin{aligned} & \frac{\overline{e^2}(90, 45) + \overline{e^2}(270, 45)}{S^2} \\ &= (1 + k^2)\overline{u'^2} + \frac{(k^2 - 1)^2}{(1 + k^2)}\overline{w'^2} \end{aligned} \quad (4.21)$$

By substituting equation 4.18 into 4.21, $\overline{w'^2}$ can be obtained as follows;

$$\overline{w'^2} = (1 + k^2) \frac{[\overline{e^2}(90, 45) + \overline{e^2}(270, 45)]}{S^2(k^2 - 1)^2} - \frac{(1 + k^2)^2 \overline{e^2}(90, 0)}{S^2(k^2 - 1)^2} \quad (4.22)$$

where "*" indicates values for normal probes.

The Reynolds stresses were calculated according to equations 4.16 to 4.20 using the computer program "TEST" listed in Appendix E. The results obtained for mean velocity and Reynolds stress components, using this simplified version of the analysis, are shown in Figures 20 to 26. The hot-wire signal analysis presented in this thesis has shown sensitivity in calculating v , w and all six Reynolds stress terms when used in its full form. Further evaluation of the analysis is necessary before any concrete conclusions are made about its accuracy and applicability.

4.4 Error Analysis

4.4.1 Effects of "v" on Reynolds Stress Calculations

As discussed in Section 4.3, v was not included in the Reynolds stress calculations because it could not be estimated accurately. In this sub-section, an attempt will be made to determine the effect of v on the calculation of the shear stress, $\overline{u'v'}$.

The shear stress, $\overline{u'v'}$, was first calculated by equation 4.16, and it was compared to the shear stress, $\overline{u'v'}$, obtained from the solution of the system of six equations for positions 1 to 6. In the second method the value of u was that obtained by a single normal probe and v was first assumed to vary linearly with Y as follows;

$$v = kY$$

4.23

where k is a constant. For $k = 1$ in equation 4.23 the difference, in $\overline{u'v'}$ obtained by the two methods, increased from 1% at $Y \approx 0$ to about 10% at $Y = Y_{m/2}$ and 40% at $Y > Y_{m/2}$. When $k = 0.1$, the differences in $\overline{u'v'}$ were 10% lower as might be expected. It was realized that the assumption of $v = kY$ was not appropriate and that it introduced substantial differences into calculations of $\overline{u'v'}$. It was then decided to use the values for v reported by Irwin^[32] for calculation of $\overline{u'v'}$ by the six equation analysis. The differences in $\overline{u'v'}$ values given by the two methods were of the order of 3% up to the position $Y = Y_{m/2}$, and increased to about 10% at $Y > Y_{m/2}$.

4.4.2 Effects of Probe Spatial Resolution

The effect of spatial resolution of the slanted probes on the determination of mean and turbulence quantities was estimated as follows; the $\overline{E}_{br}$ and $\sqrt{e^2_{br}}$ signals obtained at the point of the maximum shear stress ($Y = 0.6 \text{ mm}$), were replaced with those obtained at a distance $Y = 0.7 \text{ mm}$. The results when compared showed that the change in u was about 1.5%, $\sqrt{u'^2}$ changed about 5%, $\sqrt{v'^2}$ also changed about 5% and $\overline{u'v'}$ changed about 8%. These results show that spatial resolution has only a small influence on the calculation of the mean velocity, u , but substantially affects the values determined for the Reynolds stresses. Bearing in mind that the 45° slanted probe spans a distance, $\Delta Y \approx 1 \text{ mm}$ when $\psi = 0^\circ$ or 180° , the errors on Reynolds stresses due to this effect could easily be of order of $\pm 20\%$, at the station where these measurements were taken.

4.4.3 Effects of Signal Sampling Errors

Although both $\overline{E}_{br}$ and $\sqrt{e^2}$ were sampled with an accuracy of about 0.5 % as described in Section 3.1.2, a test was carried out to determine the errors introduced by signal changes. When $\overline{E}_{br}$ values were intentionally changed by 0.7 %, the mean velocity, u , changed by 1.0%, with $\sqrt{u'^2}$ and $\sqrt{v'^2}$ by 2.0% and $\overline{u'v'}$ by 7.0%. When $\sqrt{e^2_{br}}$ values were changed by 1.2%, $\sqrt{u'^2}$ and $\sqrt{v'^2}$ changed by 1% and $\overline{u'v'}$ changed by 4.0%.

This test indicates that the Reynolds stress results depend more on changes in the $\overline{E}_{br}$ signals than on the $\sqrt{e^2_{br}}$ signals.

To determine the effect of sampling frequency on the turbulence measurements, the following relation was used to determine the smallest eddy size that could be estimated

from this sampling technique;

$$\text{eddy size} = \text{frequency of turbulence} \times \text{mean flow velocity}$$

For a given sampling frequency, fluctuation frequencies up to one fifth of the sampling frequency can be measured. For a typical mean flow velocity of about 25 m/s the eddy size was estimated to be of the order of one thousand smaller than the distance $Y_{m,2}$. This indicates that at the "optimum sampling rate" it is possible to measure almost all the turbulence in the flow. The effect of sampling frequency on the shear stress is not expected to be large since the contribution of small eddies to shear stress is small^[57]

Chapter 5

Results and Discussion

5.1 Results from the Plane Still Air Wall-Jet Apparatus

The plane still air wall-jet apparatus results were non-dimensionalized in a similar fashion as the results of other investigators of wall-jet flows. Figure 20 shows mean velocity profiles obtained with a normal and a slanted probe. These profiles are asymmetric with respect to the point of maximum velocity, a result typical of wall-jet flows^[30]. It is evident from the graph that slanted probes are not suitable for measurements near the walls because the probes must be positioned far enough from the wall to allow them to be rolled through 360 degrees. As a result, the analysis presented in Section 4.2 is not suitable for near the wall measurements. Figure 21 shows a mean velocity profile obtained with a normal probe at $x/t = 93.23$. This profile agrees well with those obtained by *Kind* and *Suthanthiran*^[39] and *Kelly*^[34] for the same wall-jet apparatus, and that of *Irwin*^[32] obtained for a wall-jet in a moving stream. The agreement of the non-dimensionalized mean velocities obtained at different distances downstream of the wall-jet slot exit indicates that the flow reaches a self preserving state.

Results for longitudinal turbulence obtained at $x/t = 93.23$, are plotted in Figure 22, and agree well with those obtained by *Kind* and *Suthanthiran*^[39] and *Kelly*^[34]. The longitudinal turbulence is plotted in Figure 23, non-dimensionalized with the maximum mean velocity, u_m . The curve peaks at the correct distance from the wall, but has a maximum value of $\sqrt{u'^2}/u_m$, about 15% higher than the results obtained by *Wilson* and *Goldstein*^[53]. *Kelly*^[34] ascribes that the difference in the turbulence intensities may be related to the fact that the wall-jet growth rate for this apparatus is higher than the conventionally accepted $dY_{m/2}/dx$ values of about 0.07^[9]. Figure 26 shows that the results for the wall-jet growth rate obtained in this study fall in the same line with the results obtained by *Kelly*^[34]. The value of $dY_{m/2}/dx$ is about 0.88 which is close to the value of 0.85 obtained by *Wilson* and *Goldstein*^[53]. The fact that the results obtained for the wall-jet growth rate agree with those of *Kelly*, indicates that $Y_{m/2}$ values were determined correctly. The $\sqrt{v'^2}/u_m$ profile shown in Figure 23 is similar to that of *Eskinazi* and *Eriän*^[22], but

it does not have any distinct maximum as that obtained by Irwin^[32]. The $\sqrt{u'^2}$, u_m and $\sqrt{u'^2}/u_m$ curves meet at a distance of about $Y/Y_{m/2} = 1.6$ as those of Irwin^[32].

Figure 25 shows a shear stress profile obtained at $x/t = 93.23$ plotted with profiles of other investigators. All plots are in fair agreement in the region below the point of maximum shear stress, but have different maxima and drop differently in the region beyond the point of maximum shear stress. Launder and Rodi^[43] have identified that among the available sets of data there is a considerable scatter in the shear stress profiles. In a comparison of the shear stress profiles of Wilson and Goldstein^[53], Guitton^[28], Giles et al.^[26] and Bradshaw and Gee^[9], they show that the differences in the shear stresses are large at the region beyond the point of maximum shear stress, but they do not discuss this effect. The fact that the maximum shear stress point obtained in this study does not coincide with that of Kelly^[34], may be explained as follows; according to Wilson and Goldstein^[53], slot Reynolds number effects alter the viscous dissipation rates of the turbulence affecting the value of the maximum shear stress to about 15%. Thus, differences in the slot Reynolds number achieved in this set of data, with the corresponding slot Reynolds number achieved by Kelly^[34], may have caused the difference in the value of the maximum shear stress. The point of $\overline{u'v'} = 0$, extrapolated from the plot of Figure 25, does not coincide with the point where $du/dy = 0$. A result typical of non-symmetric mean velocity profile flows^[30]. In this set of data, the point of $\overline{u'v'} = 0$ lies closer to the wall than the point where $du/dy = 0$. The same result has been described by Kelly^[34] for the same wall-jet apparatus. Irwin^[32], Wilson and Goldstein^[53], Eskinazi and Erian^[22], Tailland and Mathieu^[51] and Beguier^[4] among others, reported the same result for wall-jet flows. Results obtained for $\overline{u'w'}$ are plotted and shown in Figure 24 along with the results of $\overline{u'v'}$. The fact that $\overline{u'w'}$ is not zero, as is the case for two-dimensional flows, shows that some flow three-dimensionalities were present.

The results obtained for the wall-jet apparatus provided confidence for carrying out further measurements for the B.L.C. apparatus.

5.2 Results from the B.L.C. Apparatus

5.2.1 Flow Quality

Still Air Measurements:

The maximum total pressure distribution of the cylinder jet flow under still air conditions, exhibited random short wave length fluctuations along the span. *Kind et al.*^[38] have reported similar measurements for the same cylinder and they have concluded that these fluctuations in total pressure are an unavoidable consequence of the inherent instability of the flow in the outer region of a wall-jet with convex curvature. *Guillon and Newman*^[29] also report similar fluctuations of the total pressure and they concluded that they are due to slot lip imperfections. The results obtained at $\theta = 32^\circ$, 62° and 92° are shown in Figure 27. The cylinder surface static pressure distribution in still surroundings is shown in Figure 28. The results are in good agreement with the results of *Fekete*^[23] for the region from $\theta = 40^\circ$ to 110° . The range variation in the pressure near the slot in the present data may be due to slot irregularities. *Fekete*^[23] reports a number of pressure distributions depicting similar variations of the pressure near the slot, when he varied the slot thickness and the Reynolds number.

It is suspected that the presence of the forebody may be the reason why there is no pressure drop in the region of $\theta' = 110^\circ$ to $\theta = 150^\circ$. Furthermore the flow will not separate after $\theta = 150^\circ$ and pressure drop will not occur.

Static Pressure Measurements:

The static pressure distribution over the cylinder surface and the forebody for $C_{\mu c} = 0.1, 0.2, 0.22$; $C_{fw} = 0.11$ and $U_\infty = 21 \text{ ms}^{-1}$ are shown in Figure 29. The pressure distributions over the forebody for all $C_{\mu c}$ values show that the forebody design requirement for about zero pressure gradient near the slot is met. The pressure distribution also shows that at higher $C_{\mu c}$ values, the flow is accelerated around the cylinder and forebody because of an increase in the circulation around the forebody. At the injection slot the flow is re-energized and it separates at $\theta = 62^\circ$ for $C_{\mu c} = 0.2, 0.22$. It is also evident from the graph that the pressure distributions for $C_{\mu c} = 0.2, 0.22$ are very similar. This indicates that relatively small changes of $C_{\mu c}$ do not influence the flowfield significantly. For $C_{\mu c} = 0$, the flow separated at about $\theta = 32^\circ$.

Static Pressure Distribution on the Plane Wall:

The static pressure distribution obtained at the $x = 7$ in and $x = 11$ in stations downstream of the slot in the plane wall-jet and at spanwise distances of $z = -7$ in to $z = +7$ in are shown in Figure 30 for $C_{\mu c} = 0.1, 0.22$. Results show that the flow on the rear wall is accelerated near the ceiling and the floor, and it is two-dimensional over about the central six inches. At $x = 11$ in, the conditions are more severe as the flow approaches separation. This suggests that the flow over the side wall does not achieve as good two-dimensionality as that reported by Gooden^[27]. His results have shown that for $C_{\mu c} = 0.3$, $C_{\mu w} = 0.0$ and with suction on, the spanwise pressure distribution varied to about 2%. In the present study, it is evident that the presence of the forebody increases the bound vorticity which is shed downstream, introducing flow deterioration. Moreover, the suction is not adequate to completely eliminate three-dimensional effects in the region away from the cylinder.

Flow Visualization:

The effect of end-wall suction is shown in six photographs taken of the tuft study; for $C_{\mu c} = 0.1, 0.2, 0.22$, shown in Figures 31, 32 and 33. The tufts show clearly that end-wall suction improves the flow two-dimensionality on the cylinder and on the wall. For $C_{\mu c} = 0.1$, the tufts show best alignment since the shedding vorticity is less for lower $C_{\mu c}$ values. The tufts show that the flow is two-dimensional for $x \leq 7$ in on the plane wall.

The surface flow visualization study results are shown in Figures 34 to 37. It is evident from the photographs that the point of separation is not uniform spanwise. For $C_{\mu c} = 0.1$, the separation point spanwise uniformity is best. This visualization scheme suggests that the flow away from the mid-span region has higher momentum and it separates later than the flow in the mid-span. Gravity effects have certainly influenced the quality of the surface visualization study.

Mean Velocity Profile Measurements:

Mean velocity profiles downstream of the cylinder slot exit are shown in Figures 38 to 40, for three different spanwise stations. These profiles show fair agreement for $C_{\mu c} = 0.1, 0.22$ and at stations close to the separation point, more departure from two-dimensionality is evident. At the separation point, the profiles show a large departure from two-dimensionality. Figure 39 indicates reasonable two-dimensionality for a span of about 6.7 in (17 cm). Further away from mid-span, the profiles are distinctly fuller. This agrees with the surface

visualization study which showed that separation occurs somewhat later at the region away from the mid-span.

The flow quality checks have demonstrated that the flow is two-dimensional for about 6.7 in in the mid-span of the cylinder. The nature of the flow is such that it is difficult to obtain two-dimensional flow conditions at the cylinder separation region. Improvements of the two-dimensional characteristics of the flow are desirable.

Temperature Effects

The temperature difference between the cylinder jet air and the freestream air, measured using the cold-wire method described in Section 3.2.4 and the approximation described by equation 3.16, was no more than $6^\circ \text{C} \pm 8\%$ for the most severe conditions of $C_{\mu c} = 0.22$ at $\theta = 2^\circ$ and at Y of about 1 mm. The temperature differences away from the cylinder wall and at all downstream measuring stations ($\theta > 2^\circ$) were less than 6° .

The fact that the temperatures measured were lower than the supply air temperature, measured at the inlet of the cylinder plenum, may be due to the rapid mixing of the cylinder wall-jet air with the freestream air. An estimate of the maximum percentage error introduced by temperature variation is about 4% based on the following relationship [56] (see nomenclature, Section 3.2.4);

$$\% \text{ error} = \frac{2\Delta T}{T_s - T_e} \times 100$$

5.2.2 Mean Velocity and Reynolds Stress Measurements

The mean velocity component, u , and the Reynolds stresses in this study have been calculated according to the analysis described in Chapter 4, using equations 4.15 to 4.21. All the data were transferred to the University's main frame computer system for processing. The computer program, DATARED, shown in Appendix D was developed to perform the calculations and another program, SEP, was developed to prepare separate files for plotting of the results.

Mean Velocities:

The mean velocity profiles measured at seven stations downstream of the cylinder slot exit for $C_{\mu c} = 0.1$ and at eight stations for $C_{\mu c} = 0.2, 0.22$ are presented in Figures 41, 42 and 43 respectively. The flow downstream of the cylinder slot does not achieve a

self preserving state because the ratio of the maximum velocity, u_m , to the velocity of the external flow, u_e , varies along the flow. The mean velocity profiles are presented in a similar fashion to those of *Kind et al.*^[38]. Mean velocity results at several measuring stations and at different cylinder momentum coefficients are tabulated in Appendix G.

For all cylinder momentum coefficients and at $\theta = 2^\circ, 12^\circ, 22^\circ$ and 32° , the shear layer was not thick enough to enable measurements inside the inner layer, while from $\theta = 42^\circ$ onward more points in the inner layer could be obtained. All mean velocity profiles show distinct maxima and minima until $\theta = 52^\circ$ for $C_{\mu c} = 0.2, 0.22$. The velocity maxima and minima tend to disappear as the shear layer approaches separation. The resulting shear layers are similar to conventional turbulent boundary layers. The mean velocity profiles have a flatter maximum point region than the profiles obtained in the plane-wall jet apparatus. *Dvorak*^[17] explains that mean velocities profiles can exhibit both a maximum and minimum because the injected fluid cannot immediately entrain the entire upstream boundary layer fluid. The mean velocity results show that the injected fluid loses its momentum as it flows downstream of the cylinder wall, due to wall viscous effects and due to mixing with the slower moving external flow. In wall-jet flows over convex surfaces, such as the present, the outer flow is unstable causing rapid mixing of the jet fluid with the external flow^[35]. This results in a rapid decay of the velocity maximum and increase the growth rate of the wall-jet. *Giles et al.*^[26], *Guilton and Newman*^[29], *Wilson and Goldstein*^[53] and *Kind et al.*^[38] have demonstrated that streamwise curvature has a large effect on growth rate of wall-jets. This effect, although desirable, because the velocity minimum does not persist, deepens and promotes separation as illustrated in Figure 4, results in a relatively rapid decaying of the maximum velocity. With rapidly decaying maximum velocity of the wall-jet, the flow separates relatively early from the cylinder surface. The point of separation occurs at about $\theta = 62^\circ$ for $C_{\mu c} = 0.1$ and at about $\theta = 72^\circ$ for $C_{\mu c} = 0.2, 0.22$.

The mean velocity profiles measured with $C_{\mu c} = 0.22$ are quite similar to those obtained at $C_{\mu c} = 0.2$. The comparison shows that small C_{μ} changes only affect the flow in the region close to separation. At upstream stations, the flow is almost unaffected by small changes in $C_{\mu c}$. The small differences in the mean velocity measurements at $C_{\mu c} = 0.2, 0.22$ also indicate that consistent results may be obtained from the present B.L.C. apparatus with good repeatability. Figure 44 shows that for $C_{\mu c}$ at $\theta = 12^\circ$, the variation of $C_{\mu w}$ has very little effect on the cylinder mean velocity profile. This result agrees with *Gooden*^[27] who concluded that the variation of $C_{\mu w}$ did not have significant effects on the mean velocity profiles measured on the cylinder, and external flow is essentially the same for different $C_{\mu w}$.

values.

The results obtained in this study cannot be compared in detail to results reported by *Kind et al.*^[38], because the flow conditions and geometry are different in the present study, i.e. the angular position of the cylinder jet plane with respect to the mean flow direction is 90° in this study and the corresponding angular position was 32° in their^[38] study. Moreover, the presence of the forebody has introduced more differences in the flowfield.

5.3 Discussion

5.3.1 Reynolds Normal Stresses

The normal Reynolds stress profiles measured at seven stations downstream of the cylinder slot exit for $C_{\mu c} = 0.1$ and at eight stations for $C_{\mu c} = 0.2$ and 0.22 are presented in Figures 45 to 54. The maximum mean velocity, u_m , has been used as a non-dimensionalizing velocity parameter while the distance from the wall, Y , has been kept dimensional, as in the mean velocity results.

At the measuring station of $\theta = 2^\circ$, results for $\overline{v'^2}$ were not obtained because the thickness of the jet is very small and it was not possible to obtain results for the jet flow region using the slanted probe. The normal Reynolds stress, $\sqrt{\overline{v'^2}}/u_m$, profiles presented here show significant differences from those obtained in the plane wall-jet apparatus. The $\sqrt{\overline{v'^2}}/u_m$ profiles presented here are significantly larger than that obtained in the plane wall-jet apparatus. *Eskog* and *Yeh*^[21] state that the wall curvature influences the turbulence quantities to a significant extent. Moreover, *Guillon* and *Newman*^[29] have concluded that wall curvature dramatically increases the turbulence component, $\overline{v'^2}$, as compared with $\overline{u'^2}$. This may be explained in terms of the instability to radial fluctuations of the flow in the outer region of wall-jets having convex curvature.

The turbulence intensity, $\sqrt{\overline{v'^2}}/u_m$, is larger than the longitudinal turbulence intensity, $\sqrt{\overline{u'^2}}/u_m$, in the shear layer at all measuring stations upstream of the separation point and at all cylinder momentum coefficients. Both $\sqrt{\overline{v'^2}}/u_m$ and $\sqrt{\overline{u'^2}}/u_m$ intensities become progressively larger as the flow develops downstream of the slot, because the mean velocity is decreasing in the adverse pressure gradient downstream of the slot. *Wilson* and *Goldstein*^[53] also report $\sqrt{\overline{v'^2}}/u_m$ intensities larger than $\sqrt{\overline{u'^2}}/u_m$ in a wall-jet over a circular cylinder flow and *Guillon* and *Newman*^[29] report some deviation in $\sqrt{\overline{v'^2}}/u_m$ and $\sqrt{\overline{u'^2}}/u_m$ for a jet over logarithmic spiral surfaces.

At stations $\theta = 2^\circ, 12^\circ, 22^\circ$ and 32° , both turbulence intensities $\sqrt{u'^2}/u_m$ and $\sqrt{v'^2}/u_m$ are very similar for all $C_{\mu c}$ values. At measuring stations close to the separation point, the turbulence intensities depend significantly on the $C_{\mu c}$ value. Thus, at $C_{\mu c} = 0.1$ and at $\theta = 42^\circ, 52^\circ, 62^\circ$, the turbulence intensities are considerably larger than the corresponding intensities for $C_{\mu c} = 0.2$ and 0.22 . The turbulence intensities, $\sqrt{u'^2}/u_m$ and $\sqrt{v'^2}/u_m$ obtained for $C_{\mu c} = 0.2, 0.22$ are in good agreement at the measuring stations $\theta = 12^\circ, 22^\circ, 32^\circ$ and 42° . At $\theta = 52^\circ$, $\sqrt{u'^2}/u_m$ profiles are similar while the $\sqrt{v'^2}/u_m$ profile for $C_{\mu c} = 0.22$ is slightly larger. At $\theta = 62^\circ$, it is shown clearer that $\sqrt{v'^2}/u_m$ intensity is larger for $C_{\mu c} = 0.22$. This indicates that at stations close to the flow separation, the turbulence intensities are more reliant on small variations of $C_{\mu c}$. The $\sqrt{u'^2}/u_m$ intensities at $\theta = 62^\circ, 72^\circ$ for $C_{\mu c} = 0.2$ and 0.22 , show some differences with $\sqrt{u'^2}/u_m$ being lower at $C_{\mu c} = 0.22$, presumably because the flow is slightly farther from the separation. The turbulence intensity profiles do not show any large differences in the region outside the shear layer at measuring stations before the separation point. The turbulence intensities, $\sqrt{u'^2}/u_m$ and $\sqrt{v'^2}/u_m$ of the external flow for $C_{\mu c} = 0.1$ reaches a constant value of 1.5% at $\theta = 2^\circ$, 1.9% at $\theta = 22^\circ$, 2.8% at $\theta = 42^\circ$ and 15% at the separation point of $\theta = 62^\circ$. When $C_{\mu c}$ is increased to 0.2, the corresponding intensities are lower, i.e. 1.3% at $\theta = 2^\circ$, 1.6% at $\theta = 22^\circ$ and about 10% at the separation point. The large increase in the turbulence intensities in the region close to separation may reflect fluctuation of the external inviscid flow. The maxima of the turbulence intensity profiles occur at approximately the same distance from the wall as those of Wilson and Goldstein^[53], Guitton and Newman^[29] and Irwin^[32].

Quantitatively, the results for the turbulence intensities are lower than those available in the literature for wall-jets over convex surfaces in still air surroundings. This may be due to the fact that mixing is less vigorous in wall-jet flows with the external flow, than in wall-jet flows in still surroundings.

Figures 52, 53 and 54 show results of $\sqrt{u'^2}/u_m$, $\sqrt{v'^2}/u_m$ and $\sqrt{w'^2}/u_m$ for three different flow conditions. These results show large differences between $\sqrt{u'^2}$ and $\sqrt{v'^2}$, $\sqrt{w'^2}$ while the differences between $\sqrt{v'^2}$ and $\sqrt{w'^2}$ are smaller. The deviation of these results is attributed partly to three-dimensional effects and partly to inaccuracies in the measurements. The larger deviation of $\sqrt{u'^2}$ from $\sqrt{v'^2}$ and $\sqrt{w'^2}$ may be due to the fact that $\sqrt{v'^2}$ and $\sqrt{w'^2}$ were calculated using signals from both a normal and a slanted probe, according to equations 4.20, 4.21 of Chapter 4 whereas $\sqrt{u'^2}$ was measured with a normal probe. Moreover, spatial resolution effects are more pronounced when a slanted probe is used, because the probe

was perpendicular to the wall as shown in Figure 18.

The results presented in Figures 52, 53 and 54 indicate that $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{w'^2}$ are approximately equal, indicating that the turbulence is definitely not isotropic. The longitudinal turbulence profile, $\sqrt{u'^2}/u$, obtained at $\theta = 12^\circ$, $C_{\mu c} = 0.1$ and at $C_{\mu w} = 0.11$ and 0.2 are presented in Figure 50. These results show that variation of $C_{\mu w}$ does not affect the turbulence intensities downstream of the cylinder slot exit, as was the case for the mean velocity profiles discussed previously.

Tabulated results for both $\sqrt{u'^2}/u_m$ and $\sqrt{v'^2}/u_m$ obtained in this study are included in Appendix G.

5.3.2 Reynolds Shear Stresses

The Reynolds shear stress profiles measured at six stations downstream of the cylinder slot exit for $C_{\mu c} = 0.1$ and at seven stations for $C_{\mu c} = 0.2, 0.22$ are presented in Figures 55 to 60. The square of the maximum velocity, u_m^2 , has been used as a non-dimensionalizing parameter for the shear stresses and Y has been kept dimensional as in the turbulence and mean velocity graphs. Shear stress profiles for all $C_{\mu c}$ values show no significant scatter between successive measuring points at all measuring stations upstream of the separation point. This is because the turbulence intensities before the point of separation were small, thus, it is expected that the hot-wire signals are sampled more accurately. At $\theta = 52^\circ, 62^\circ$ for $C_{\mu c} = 0.1$ and at $\theta = 62^\circ, 72^\circ$ for $C_{\mu c} = 0.2, 0.22$, the scatter in the data increased significantly. It is evident from the graphs that it was not possible to obtain many data points in the negative shear stress region near the wall, especially at measuring stations close to the slot exit. This is because the shear layer being very thin, did not allow for measurements close to the wall using a slanted probe. More points could be obtained in the negative shear layer at measuring stations downstream of the slot exit, because the shear layer progressively thickened farther away from the slot exit.

The shear stress profiles obtained for $C_{\mu c} = 0.2, 0.22$ are in fair agreement with each other. At $\theta = 12^\circ$ to 52° , the profiles for $C_{\mu c} = 0.2$ and 0.22 are virtually the same, with slight differences in their maxima. As the separation point is approached at measuring stations of $\theta = 62^\circ, 72^\circ$, the two profiles show some differences. Thus, at $\theta = 62^\circ$ for $C_{\mu c} = 0.22$ the shear stress profile has a larger maximum than the profile of $C_{\mu c} = 0.2$. The profile for $C_{\mu c} = 0.2$ shows that the shear stress is close to zero near the wall, indicating that the flow separates earlier for $C_{\mu c} = 0.2$ than for $C_{\mu c} = 0.22$. At $\theta = 72^\circ$, the profiles

for $C_{\mu c} = 0.2$ and $C_{\mu c} = 0.22$ both show a significant increase in shear stress some distance from the wall. As expected, the values near the wall are fairly close to zero.

The shear stress profiles obtained for $C_{\mu c} = 0.1$ are similar to those obtained for $C_{\mu c} = 0.2$ with the profiles for $C_{\mu c} = 0.2$ having slightly higher maxima, at measuring stations of $\theta = 12^\circ$ to 42° . At $\theta = 52^\circ, 62^\circ$, the shear stress profiles for $C_{\mu c} = 0.1, 0.2$ are considerably different because the flow separates earlier at lower $C_{\mu c}$ values. All the shear stress profiles for $C_{\mu c} = 0.1, 0.2$ and 0.22 between $\theta = 12^\circ$ and 42° follow closely the sign of the mean velocity gradient as expected. Thus, the shear stresses cross the zero axis at distances from the wall close to those where $du/dy = 0$. All shear stress profiles tend to follow the same general trend which can be described as follows.

The profiles cross the zero axis at approximately the same point where $u = u_{max}$, they are positive for the wall-jet flow region. They cross the zero axis again at the point where $u \approx u_{min}$ and have a short negative shear stress region. The zero axis is again crossed at approximately the point of $du/dy = 0$ for the external flow. This can be seen better in profiles for $C_{\mu c} = 0.1$ at $\theta = 12^\circ, 22^\circ$ and for $C_{\mu c} = 0.2$ at $\theta = 12^\circ$. The points where $\overline{u'v'} = 0$ lie further from the surface than the points where $du/dy = 0$. *Bequier*^[4] also demonstrates that the shear stress profiles follow closely the sign of du/dy for both the shear layer and the external flow region, but in his case the positions of $\overline{u'v'} = 0$ lie closer to the wall with respect to the positions of $du/dy = 0$. *Erian*^[20] describes that $\overline{u'v'} = 0$ does not necessarily occur closer to the wall with respect to the position of $du/dy = 0$ and has demonstrated experimentally that for wall-jet flows which experience adverse pressure gradients, the location of $\overline{u'v'} = 0$ is further from the wall with respect to the $du/dy = 0$ position. The present flow experiences a strong streamwise curvature and this may explain the fact that $\overline{u'v'} = 0$ lies further from the wall with respect to the $du/dy = 0$ position.

As the wall-jet gradually loses its momentum, the $\overline{u'v'} = 0$ points lie considerably further from the wall with respect to the positions of $du/dy = 0$. *Eskinazi* and *Erian*^[22] refer to the region between $\overline{u'v'} = 0$ and $du/dy = 0$ as areas of negative turbulence production or zones of opposing shear. *Hinze*^[30] explains that in these regions $\overline{u'v'}$ and du/dy have opposite signs and their product yields a negative contribution to the rate of change of the kinetic energy of the turbulence. The negative contribution does not necessarily imply a gain of energy by the mean motion at the cost of the energy of the turbulence.

Figures 61, 62 and 63 show measurements of $\overline{u'w'}$ for $C_{\mu c} = 0.1$ at $\theta = 32^\circ, 42^\circ$ and for $C_{\mu c} = 0.2$ at $\theta = 42^\circ$ together with the corresponding measurements of $\overline{u'v'}$. All profiles

show that $\overline{u'w'}$ is close to zero outside the shear flow. Inside the shear flow region, the measurements give non-zero values for $\overline{u'w'}$; this indicates imperfect two-dimensionality of the flow. Note however that the $\overline{u'w'}$ values are substantially less than the $\overline{u'v'}$ values at corresponding positions in the shear flow.

Figure 64 shows shear stress profiles obtained in the present study, non-dimensionalized with u_m^2 and Y with $Y_{1/2m}$, (see Nomenclature in Figure 5). These non-dimensionalized shear stress profiles do not show a good agreement, for $C_{\mu c} = 0.2$ at $\theta = 22^\circ$ and for $C_{\mu c} = 0.1, 0.2$ for $\theta = 42^\circ$. This may be explained by the fact that the flow is not reaching a self-preserving state. The results show the same general trends as those of Irwin^[32] for a wall-jet in a freestream and those of Wilson and Goldstein^[53] in a plane wall-jet flow in still air surroundings. The results for the shear stresses obtained in this study, although having demonstrated the same trends as those of previous works, are subjected to errors due to probe spatial resolution. Errors due to probe spatial resolution were unavoidable in the present flow because the shear layer is thin. Certainly more information is needed to assess the magnitude of the errors introduced by the probe spatial resolution effects in the measurements of the shear stresses. Due to the time limitations, it was not possible to investigate further the effect of the probe spatial resolution on the shear stress measurements.

5.4 Conclusions

The hot-wire signal analysis derived in this study has shown sensitivity in calculating the mean velocity components v , w in a plane wall-jet flow in still surroundings. Its derivation involves the assumption of turbulence levels of equal to or less than twenty percent. The sensitivity of the analysis in calculating v , w has forced simplifications on the analysis in the calculation of the Reynolds stresses. The analysis in its simplified form agrees with the analyses of other investigators. Further testing of the analysis is necessary before concrete conclusions are drawn about its applicability. The analysis when tested with data for a plane wall-jet flow has produced similar results with those of other investigators.

The modifications designed for the B.L.C. apparatus have produced a thicker boundary layer downstream of the cylinder slot, which facilitates measurements of both mean velocities and Reynolds stresses. The measurements of $\overline{v'^2}$ and $\overline{u'v'}$ are subjected to probe spatial resolution errors because the probe is oriented normal to the surface of the cylinder. In the mean velocity and longitudinal turbulence data, probe spatial resolution effects are

not significant because the probe was oriented parallel to the surface of the cylinder. The flow quality experiments have shown that the flow two-dimensionality has been affected by the presence of the forebody and improvements are desirable. The flow quality experiments have shown that the flow is two-dimensional for the central 6.7 in of the cylinder span and 18 cm span of the plane wall downstream of the plane wall-jet. The flow also shows a spanwise non-uniform separation.

Temperature effects have not been considered in the analysis of the results. Despite the errors that may have been introduced in the results, it has been demonstrated that repeatable measurements can be obtained for the B.L.C. apparatus. Mean velocity and Reynolds stress results are insensitive to small cylinder momentum coefficient changes at stations close to the slot exit, but are sensitive at stations close to the separation. The shear stress results follow the same trends as those of other investigators and compare well with wall-jet results available in the literature.

So far as the author is aware, the present experiment provides the first mean velocity and Reynolds stress data for different momentum coefficients, for cases where flows consisting of a wall-jet mixing with an upstream boundary layer in a strong adverse pressure gradient, develop into a conventional boundary layer. The results presented in this work, provide data for an important and complex flow, which can be used in testing of prediction methods.

5.5 Recommendations for Future Work

One can easily check the derivation of the analysis by following the step by step detailed derivation included in Appendix F. The analysis may be tested in its full form with results obtained from the simplified equations, 4.15 to 4.21. The computer program, TEST, listed in Appendix E can be followed for these comparisons.

Unless the mean velocities, v , w , can be computed and checked, they should not be included in the Reynolds stress calculations because they introduce large errors. The flow quality in the B.L.C. apparatus may be improved to some extent if more suction is applied through the porous end plates. With the forebody already mounted in the B.L.C. apparatus and by using the ADACS computer programs listed in Appendix D, it is a simple matter to proceed with further experimentation. The results obtained in this study may be compared to similar computational schemes as those of *Kind et al.*^[38].

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Diagrams and Figures

FLOW AROUND A CIRCULATION-CONTROLLED AIRFOIL

Figure 1 (Ref. 27)

Figure 2 : Tangential Blowing over a Rounded Trailing Edge Airfoil (Ref. 18)

Figure 3 : Flow near an Injection Slot (Ref. 35)

POSSIBLE TYPES OF FLOW DEVELOPMENT DOWNSTREAM OF AN INJECTION SLOT

Figure 4 (Ref. 17)

Nomenclature for Cylinder Wall-Jet Velocity Profile

Figure 5 (Ref. 27)

Figure 6 : Photograph of the Plane Wall-Jet Apparatus

Figure 7 : Photograph of the Calibration Apparatus

Figure 8 Boundary Layer Control Apparatus (B.L.C.), with Forebody in place.

Figure 9 : Schematic Diagram of Blowing and Suction System

Figure 10 Traverse Gear Schematic Diagram (B.L.C. Apparatus)

Figure 11: Photograph of the Traversing Gear in Place

Figure 12: Photograph of the Rear-side of B.L.C. Apparatus

Figure 13 : Schematic Diagram of Instrumentation

Figure 14: Nomenclature for the Plane Still-Air Wall-Jet Velocity Profile and Probe Orientation During Measurements

Figure 15: Schematic Diagram of the Calibration Apparatus

Figure 16: Probe Orientation for k , h Measurements (Ref. 33)

Figure 17 : Cylinder and Plane Wall-Jet Slot Calibration Curve (B.L.C. Apparatus)

slanted probe: Prong Plane // to X, Y plane

at $\psi = 0^\circ$, $\psi = 180^\circ$

normal probe: Prong Plane perp. to X, Y plane

at $\psi = 0^\circ$, $\psi = 180^\circ$

Figure 18: Probe Orientation for Measurements in the B.L.C. Apparatus

Figure 19 Hot Wire Probe Coordinate System and Nomenclature

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No 2)

Results for the Plane Wall-Jet Apparatus .
(Still Surroundings)

Figure 20 Wall Jet Mean Velocity Profiles, Plane Still-air Wall Jet

Figure 21 Wall Jet Mean Velocity Profiles, Plane Still-air Wall Jet

Figure 22 Wall Jet Longitudinal Turbulence Profiles, Outer Layer Plane Still-air Wall Jet

Figure 23 Normal Stress Profiles, Plane Still-air Wall Jet $\frac{z}{l} = 93.23$

Figure 24 Wall Jet Shear Stress Profiles Plane Still-air Wall Jet $\frac{x}{l} = 93.23$

Figure 25 Shear Stress Profiles

Figure 26 Wall Jet Growth, Plane Still-air Wall Jet

Results for the B.L.C. Apparatus

Figure 27: Spanwise Distribution of Maximum Total Pressure in Wall-Jet on Cylinder in Still Air

Figure 28: Surface Static Pressure Distribution on the Cylinder in Still Surroundings

Figure 29 Forebody and Cylinder Static Pressure Distribution

Figure 30

SUCTION OFF

SUCTION ON

Figure 31 : Flow Visualization, Effect of Boundary Layer Suction

$$C_{\mu_r} = 0.10; \quad C_{\mu_w} = 0.11; \quad U_{\infty} = 21 \text{ ms}^{-1}$$

SUCTION OFF

SUCTION ON

Figure 32 : Flow Visualization, Effect of Boundary Layer Suction

$$C_{\mu} = 0.20, \quad C_{\mu_s} = 0.11; \quad U_{\infty} = 21 \text{ ms}^{-1}$$

SUCTION OFF

SUCTION ON

Figure 33 Flow Visualization, Effect of Boundary Layer Suction

$$C_{\mu} = 0.22, C_{\mu_{\text{max}}} = 0.11; U_{\infty} = 21 \text{ m s}^{-1}$$

Figure 35: Surface Flow Visualization
 $C_{\mu_c} = 0.20$, $C_{\mu_w} = 0.11$; $U_{\infty} = 21 \text{ ms}^{-1}$

Figure 34: Surface Flow Visualization
 $C_{\mu_c} = 0.10$, $C_{\mu_w} = 0.11$; $U_{\infty} = 21 \text{ ms}^{-1}$

Figure 36 : Surface Flow Visualization

$C_{\mu_c} = 0.22$; $C_{\mu_w} = 0.11$; $U_{\infty}^* = 21 \text{ ms}^{-1}$

Figure 37 : Surface Flow Visualization

$C_{\mu_c} = 0.22$, $C_{\mu_w} = 0.00$; $U_{\infty} = 0.00 \text{ ms}^{-1}$

Figure 38 Spanwise Mean Velocity Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

$C_{\mu_s} = 0.1$; $C_{\mu_w} = 0.11$; $U_\infty = 21.0 \text{ ms}^{-1}$

$\theta = 62^\circ$

- $\odot$ $Z = -14 \text{ cm}$
- $\triangle$ $Z = +14 \text{ cm}$
- $+$ $Z = +10 \text{ cm}$
- $\square$ $Z = -7.5 \text{ cm}$
- $\times$ $Z = -10 \text{ cm}$

Figure 39 Spanwise Mean Velocity Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

Figure 40 Spanwise Mean Velocity Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

Figure 41 Mean Velocity Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

$C_{\mu_e} = 0.2; C_{\mu_w} = 0.11$
 $\odot \quad \theta = 2^\circ$
 $\Delta \quad \theta = 12^\circ$
 $+ \quad \theta = 22^\circ$
 $\square \quad \theta = 32^\circ$
 $\times \quad \theta = 42^\circ$
 $\diamond \quad \theta = 52^\circ$
 $\times \quad \theta = 62^\circ$
 $\times \quad \theta = 72^\circ$

Figure 42 Mean Velocity Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

$\odot$ $\theta = 2^\circ$
 $\triangle$ $\theta = 12^\circ$
 $+$ $\theta = 22^\circ$
 $\square$ $\theta = 32^\circ$
 $\times$ $\theta = 42^\circ$
 $\diamond$ $\theta = 52^\circ$
 $\times$ $\theta = 62^\circ$
 $\times$ $\theta = 72^\circ$

$C_{\mu} = 0.22$, $C_{\mu} = 0.11$

Figure 43 Mean Velocity Profiles. Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

Figure 44 : Mean Velocity Profiles for Different Values of C_{μ_w}
(B.L.C. Apparatus)

Figure 45 Longitudinal Turbulence Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

$\theta = 62^\circ$

x

Figure 45 (cont.)

Figure 46 Longitudinal Turbulence Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

$\theta = 72^\circ$

Figure 46 (cont.)

Figure 47 Longitudinal Turbulence Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (normal probe)

$\theta = 72^\circ$

Figure 47 (cont.)

Figure 48 : Longitudinal Turbulence Profiles for Different Values of C_{μ_w} (B.L.C. Apparatus)

$C_{\mu_c} = 0.1; C_{\mu_w} = 0.11$

$\theta = 12^\circ$

$\theta = 22^\circ$

$\theta = 32^\circ$

$\theta = 42^\circ$

$\theta = 52^\circ$

Figure 49 Normal Stress Profiles ($\frac{V'v'}{U_m}$). Downstream the Cylinder Slot, B.L.C. Apparatus (slanted probe)

$\theta = 62^\circ$

Figure 49 (cont.)

$C_{\mu} = 0.2; C_{\mu_0} = 0.11$
 $\Delta \quad \theta = 12^\circ$
 $\triangle \quad \theta = 22^\circ$
 $\square \quad \theta = 32^\circ$
 $\times \quad \theta = 42^\circ$
 $\diamond \quad \theta = 52^\circ$
 $\times \quad \theta = 62^\circ$

Figure 50 Normal Stress Profiles ($\frac{\sqrt{\tau_{xx}}}{u_m}$), Downstream the Cylinder Slot, B.L.C. Apparatus (slanted probe)

$\bar{x}$ $\theta = 72^\circ$

Figure 50 (cont.)

Figure 51 Normal Stress Profiles ($\sqrt{\frac{v'^2}{u_m}}$), Downstream the Cylinder Slot, B.L.C. Apparatus (slanted probe)

$\bar{x} \quad \theta = 72^\circ$

Figure 51 (cont.)

Figure 52 Normal Stress Profiles, Downstream the Cylinder Slot. B.L.C. Apparatus (slanted probe)

Figure 53 Normal Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

- Δ $\theta = 12^\circ$
- $+$ $\theta = 22^\circ$
- $\square$ $\theta = 32^\circ$

$$C_{\mu_c} = 0.1; \quad C_{\mu_w} = 0.11$$

Figure 55 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

$\times \quad \theta = 42^\circ$
 $\diamond \quad \theta = 52^\circ$
 $\times \quad \theta = 62^\circ$

$C_{\mu_c} = 0.1; C_{\mu_w} = 0.11$

Figure 56 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

- Δ $\theta = 12^\circ$
 $+$ $\theta = 22^\circ$
 $\square$ $\theta = 32^\circ$

$$C_{\mu_c} = 0.2; C_{\mu_w} = 0.11$$

Figure 57 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

- $\times \quad \theta = 42^\circ$
 $\diamond \quad \theta = 52^\circ$
 $\times \quad \theta = 62^\circ$
 $\times \quad \theta = 72^\circ$

$$C_{\mu_c} = 0.2; C_{\mu_w} = 0.11$$

Figure 58 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

Δ $\theta = 12^\circ$
 $+$ $\theta = 22^\circ$
 $\square$ $\theta = 32^\circ$

$C_{\mu_e} = 0.22$; $C_{\mu_w} = 0.11$

Figure 59 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

- x $\theta = 42^\circ$
- ◇ $\theta = 52^\circ$
- x $\theta = 62^\circ$
- x $\theta = 72^\circ$

$C_{\mu_c} = 0.22; C_{\mu_w} = 0.11$

Figure 60 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

Figure 61 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

Figure 63 Shear Stress Profiles, Downstream the Cylinder Slot, B. L. C. Apparatus (slanted probe)

Figure 64 Shear Stress Profiles

APPENDICES

Appendix A Traversing Gear Design.

A.1 Traversing Gear Kinematic Analysis

Figure A.1 shows the traversing linkage with the probe moved from its original position G, to D. Point W is the point that the pinion gear and guide tube connect when the probe is at zero (i.e. G) position. Point O is the point around which the support tube (OG) pivots. Point A is the point at which the guide tube is pinned to the support tube (OG). Distances OW, WA and OG have to be measured before traversing. Thus, if 'adv' is the advancement of the guide tube during a traversing step;

$$WN = WA + adv$$

From triangle AOW, by the cosine rule;

$$WA^2 = OA^2 + OW^2 - 2OA OW \cos \phi$$

$$\phi = \cos^{-1} \left(\frac{OA^2 + OW^2 - WA^2}{2OA OW} \right)$$

Applying the sine rule;

$$\epsilon = \sin^{-1} \left(\frac{OA}{WA} \sin \phi \right)$$

From triangle ONW,

by the cosine rule;

$$ON^2 = OW^2 + WN^2 - 2OW WN \cos \epsilon$$

by the sine rule

$$\gamma = \sin^{-1} \left(\frac{WN}{ON} \sin \epsilon \right)$$

and

$$\delta = 180 - \gamma - \epsilon$$

and

$$AN = WN - WA$$

$$\zeta = \sin^{-1} \left(\frac{AN}{OA} \sin \delta \right)$$

and from A'ON triangle

$$\psi = 90 - \delta$$

From triangle OA'N;

$$k = \sin^{-1} \left(\frac{ON}{OA'} \sin \psi \right)$$

and

$$OA = OA'$$

and

$$\lambda = 180 - \kappa - \psi$$

$$\mu = \lambda + \zeta$$

then from triangle DOG, applying the cosine rule

$$GD^2 = OD^2 + OG^2 - 2OG OD \cos \mu$$

and

$$OG = OD$$

then

$$GD = \sqrt{2OG^2 - 2OG^2 \cos \mu}$$

Therefore, by knowing the advancement of the guide at any instant we can find the actual distance of probe, (GD).

If the probe is traversed backwards the analysis is the same but $W_N = W_A - adv$; $\psi = 90 + \delta$.

A.2 Traversing Gear Error Analysis

To determine the accuracy of the determination of the distance from the cylinder surface, Y, calculated following the analysis above, the results were compared to results from a graphical solution and all distances and angles were checked.

Having ensured that the analysis gave correct results, the error analysis was performed. Thus, the measured distances OW, WA and OG were intentionally changed to determine

the difference in Y determined from the analysis. Thus, when AW was changed 2% the distance Y changed 0.5% (0.02 mm), when OW was changed by 1.5% Y changed 0.8% (0.03 mm), when OA was changed by 1.5% the change in Y was 1.8% (0.07 mm) and an OG change of 1.5% resulted in Y changing by 1% (0.04 mm).

Because the distances OA , OG and AW could be measured by a micrometer, the errors were minimized. The overall error in Y is estimated to be less than 0.03 mm. Effects of traversing pinion slip were minimized by using a 10:1 gear ratio as described in Section 2.2.6. Errors in Y determination resulting from arc length, were minimized since Y_{max} was 1.5 mm and the support tube length, OG , was kept as long as possible, typically 40 cm.

Figure A.1 Traverse Gear Geometry

Appendix B

Computer Programs for Calibration of Hot-Wire Probes

B.1 Computer Program "CALIB"

A brief description for the main parameters of the program and its statements are given below along with the flow chart and the program listing.

C0 ()	Coefficients used for linearization of bridge mean and RMS signals.
C1 ()	Calculated linearized mean bridge voltage output, $\bar{E}_{lin}$.
E1 ()	Percent error in linearization plot.
F0 ()	Average transducer output voltage.
F1 ()	Flow velocity calculated in ms^{-1} .
I0 ()	Slope of transducer calibration curve; (i.e. $I0 = 0.1626$).
K0 ()	Transducer voltage output, read by the HP 3456A voltmeter.
L1 ()	Coefficient to be used in setting TSI electronic linearizer unit.
N0	Number of data pairs to be read; input $\equiv 20$.
N8	Normalized flow velocities.
P0 ()	Average mean bridge output voltage, $\bar{E}_{br}$.
S6	Slope of $\bar{E}_{br}$ vs U curve.
T0 ()	Bridge mean output voltage, read by the HP 3456A voltmeter.

Y0 () Flow temperature; input degrees Celsius.

Program Description

line 4	Description
10 - 160	Program title, function and date of experiment.
170 - 240	Deceleration variables and dimensioning of arrays.
320 - 370	Input number of data sets to be read and the slope of transducer calibration curve.
490 - 510	Initialization of ADACS system and set HP 3456A voltmeter to take readings.
520 - 590	Acquisition and averaging of mean bridge output signals, using Channel 1.
600 - 790	Acquisition and averaging of pressure transducer output voltage and calculation of flow velocity in ms^{-1} .
800 - 820	Print $\bar{E}_{br}$, flow velocity (ms^{-1}) and corresponding temperature in $^{\circ}C$. Press "continue" key to proceed with next set of readings.
910 - 1090	$\bar{E}_{br}$ vs U (ms^{-1}) plotting routine.
1160 - 2360	Body of linearization routine, converted to BASIC from FORTRAN version of TSI manual.
2370	Print of coefficients to be used in latter linearization of mean and RMS bridge signals.
2670	Print of coefficients for setting linearizer coefficients in TSI linearizer unit.

- 3140 Print the slope of linearizer curve, $\bar{E}_{lin}$ vs U .
- 3230 - 3590 Plotting of $\bar{E}_{lin}$ vs U .
- 3620 - 4540 Subroutine "init", to initialize ADACS system.
- 4550 - 4800 Subroutine "AVDCV", to average DC voltage from bridge output.

Figure B.1 Flowchart of Computer Program "CALIB*"

```

10 REM *****
20 REM PROGRAM (CALIB*)
30 REM *****
40 REM This program determines
50 REM the LINEARIZER coeff's
60 REM for HOT-WIRE anemometer
70 REM *****
80 !
81 DISP "INPUT DATE"
82 INPUT D9
83 PRINT "*****"
84 PRINT "CALIBRATION APR-" ; D9
85 PRINT "*****"
86 PRINT
87 PRINT
90 !
100 OPTION BASE 1
110 REAL A(8,8), C1(21), C0(8), E1(
21), F1(21), L1(4)
120 REAL N8(21), P0(21)
130 REAL R(21), S(9), W(21), X5(8,8
), V0(21), T(21), F0(21)
140 DIM X1(21), X7(21), T0(30)
150 REAL U(21), Q(21), A0(30)
160 REAL X(21), W0(21)
170 !
180 REM *****
190 REM F1=VELOCITY (m/s)
200 REM P0=BRIDGE VOLTAGE (volt)
210 REM *****
220 !
230 !
240 O1=4
250 DISP "Enter number of DATA P
OINTS (0=N)"
260 INPUT N0
270 DISP "Enter slope of TRANSDU
CER calib. (i.e = .1626)"
280 INPUT I0
290 !
300 !
310 PRINT USING 320
320 IMAGE "VEL(m/s)", 5X, "B. VOLT"
, 5X, "TEMP(C)"
330 PRINT USING 340
340 IMAGE /
350 !
360 REM *****
370 REM Main program (calib.)
380 REM using DATA-ACQUISITION
390 REM *****
400 !
410 !
420 P1=3456 ! DIGITAL VOLTMETER
430 !
440 GOSUB 3390 ! Subroutine "ini
t"
450 FOR I=1 TO N0

```

```

452 !
460 P1=1
470 S0=0
480 FOR I1=1 TO 20
490 GOSUB 4310 ! Subrt "Avdcw"
500 T0(I1)=P @ S0=S0+T0(I1) @ NE
    XT I1 @ P0(I)=S0/20
510 P1=2 ! CHANNEL

511 Q0=0
512 FOR M=1 TO N0
513 !
514 !
520 GOSUB 4310 ! Subrt "Avdcw"
521 K0(M)=P
522 Q0=Q0+K0(M)
523 NEXT M
524 F0(I)=Q0/20
530 F0(I)=P
540 F0(I)=F0(I)*(-1)
550 Y=F0(I)
560 W0(I)=F0(I)-Y
561 DISP "Input TEMPERATURE(C)"
562 INPUT Y0
570 W0(I)=W0(I)*1000
571 D1=353.266953/(273.1+Y0)
572 F1(I)=4.024861738*SQR(W0(I))*
    I0/D1)
580 !
590 PRINT USING 600 ; F1(I),P0(I
    ),Y0
600 IMAGE 000.000,3X,000.0000000
    ,4X,DD.DD
610 !
620 DISP "Press 'CONT' key to re
    ad next point " @ DISP "NEXT
    POINT IS=",I+1 @ BEEP 80,22
    0
630 PAUSE
640 NEXT I
650 !
660 ! *****
670 ! START PLOTTING ROUTINE
680 ! *****
690 !
700 DISP "Input (max) xaxis i.e
    x9=?"
710 DISP "x9=?"
720 INPUT X9
730 DISP "Input (max) yaxis i.e
    y9=?"
740 DISP "Y9=?"
750 INPUT Y9
760 DISP "STEP IS??"
770 DISP "T7=??"
780 INPUT T7
790 !
800 PEN 1 @ GCLEAR
810 SCALE 0,X9,0,Y9

```



```

1420 FOR I=1 TO L
1430 FOR J=1 TO L
1440 I2=J-1+I
1450 A(I,J)=R(I2)
1460 NEXT J
1470 !
1480 A(I,K1)=S(I)
1490 NEXT I
1500 !
1510 FOR I=1 TO L
1520 A(K1,I)=-1
1530 K2=I+1
1540 !
1550 FOR J=K2 TO K1
1560 A(K1,J)=0
1570 NEXT J
1580 !
1590 !
1600 C=1/A(1,I)
1610 !
1620 FOR I1=2 TO K1
1630 FOR J=K2 TO K1
1640 A(I1,J)=A(I1,J)-A(I1,I)*A(1,J)*C
1650 NEXT J
1660 NEXT I1
1670 !
1680 !
1690 FOR I1=1 TO L
1700 FOR J=K2 TO K1
1710 A(I1,J)=A(I1+1,J)
1720 NEXT J
1730 NEXT I1
1740 NEXT I
1750 !
1760 !
1770 FOR I=1 TO L
1780 X5(I,K1-2)=A(I,K1)
1790 NEXT I
1800 !
1810 !
1820 IF N9>=01 THEN 2080
1830 !
1840 !
1850 !
1860 !
1870 N9=N9+1
1880 J=2*N9
1890 R(J)=0
1900 R(J+1)=0
1910 S(N9+1)=0
1920 !
1930 !
1940 !
1950 FOR I=1 TO N8
1960 Q2=P0(I)^(J-1)
1970 R(J)=R(J)+Q2*W(I)
1980 Q3=P0(I)^J
1990 R(J+1)=R(J+1)+Q3*W(I)
2000 Q4=P0(I)^N9

```

```

2010 S(N9+1)=S(N9+1)+F1(I)*Q4*W(
I)
2020 NEXT I
2030 !
2040 !
2050 GOTO 1390
2060 PRINT USING 2070
2061 PRINT
2065 PRINT USING 2070
2070 IMAGE "The 5 coefficients a
re"
2071 PRINT
2080 M1=O1+1
2090 FOR I=1 TO M1
2100 C0(I)=A(I,K1)
2110 Q=P0(N0)^(I-1)
2120 X5(I,4)=X5(I,4)*Q
2130 PRINT C0(I)
2140 NEXT I
2150 !
2160 FOR J2=2 TO N0
2170 C9=C0(1)
2180 FOR I=2 TO 5
2190 Q5=P0(J2)^(I-1)
2200 C9=C9+C0(I)*Q5
2210 NEXT I
2220 !
2230 !
2240 C1(J2)=C9*10/F1(N0)
2250 E1(J2)=(C9-F1(J2))*100/F1(J
2)
2260 NEXT J2
2270 !
2280 !
2290 S6=10/F1(N0)
2300 !
2310 FOR I=1 TO 4
2320 L1(I)=X5(I+1,4)*S6
2330 NEXT I
2340 !
2350 !
2360 PRINT USING 2370
2370 IMAGE "//
2380 !
2390 PRINT USING 2400
2400 IMAGE "The linearized coeff
icients are:"
2410 !
2420 FOR I=1 TO 4
2430 PRINT USING 2440 ; L1(I)
2440 IMAGE 3X,000.0000
2450 NEXT I
2460 !
2470 !
2480 !
2490 FOR I=2 TO N0
2500 N8(I)=F1(I)*S6
2510 PRINT USING 2520
2520 IMAGE ///
2530 !

```

```

2540 PRINT USING 2550
2550 IMAGE "FLOW",4X,"NORM FLOW"
2560 !
2570 FOR I=2 TO N0
2580 N8(I)=F1(I)*S6
2590 PRINT USING 2600 ; F1(I),N8
      (I)
2600 IMAGE DD.DDDD,3X,DD.DDDD.
2610 NEXT I
2620 !
2630 !
2640 PRINT USING 2650
2650 IMAGE //
2660 PRINT USING 2670
2670 IMAGE "CAL.OUT",3X,"%ERROR"
2680 !
2690 FOR I=2 TO N0
2700 PRINT USING 2710 ; C1(I),E1
      (I)
2710 IMAGE DDD.DDDD,3X,DDD.DDDD
2720 NEXT I
2730 !
2740 DISP "DO YOU WISH TO LINEAR
      IZE SIGNAL MATHEMATICALY?"
2750 DISP "IF (YES) H1=1"
2760 DISP "IF (NO) H1 #1"
2770 !
2780 DISP "H1=?"
2790 INPUT H1
2800 IF H1#1 THEN GOTO 3360
2810 !
2820 !
2830 !
2840 !
2850 !
2860 !
2870 !
2880 PRINT
2890 PRINT
2900 PRINT "THE SLOPE IS=";S6
2910 PRINT
2920 !
2930 !
2940 !
2950 ! *****
2960 ! PLOT LIN.B.VOLT vs VELOC.
2970 ! *****
2980 !
2990 DISP "Press 'CONT' to plot"
3000 DISP "  results"
3010 !
3020 PAUSE
3030 BEEP 80,300
3040 !
3050 DISP "Ready to input values
      "
3060 DISP "  needed for plotting"
3070 !
3080 DISP "VEL (max)=?"
3090 DISP "  V9=?"

```

```

3100 INPUT V9
3110 DISP "LIN.B.VOLT (max)=?"
3120 DISP " B9=?"
3130 INPUT B9
3140 DISP " VELOCITY STEP=?"
3150 DISP "V8=?"
3160 INPUT V8
3170 DISP "LIN.B.VOLT STEP=?"
3180 DISP "B8=?"
3190 INPUT B8
3200 !
3210 PEN 1 @ GCLEAR
3220 SCALE 0,V9,0,B9
3230 XAXIS 0,V8,0,V9
3240 YAXIS 0,B8,0,B9
3250 FOR I=1 TO N0
3260 F1(I)=0
3270 C1(I)=0
3280 PLOT F1(I),C1(I)
3290 NEXT I
3300 !
3310 !
3320 !
3330 !
3340 !
3350 !
3360 END
3370 !
3380 ! *****
3390 ! SUBPROGRAM Init hp85
3400 ! *****
3410 !
3420 !
3430 ! P1=Dvmeter
3440 DIM Q7$(23) ! DEVICE NAMES
3450 !
3460 ! INITIALIZE DEVICE ADDRESS

3470 !
3480 R0=709 ! 349A SCANNER ADDRE
S
3490 R1=722 ! 3456 DIGITAL VOLTM
E
3500 R2=724 ! 3437A SYSTEM VOLTM
ETER ADDRESS
3510 R3=0 ! ERROR FLAG
3520 R4=0 ! USED IN SUBROUTINE "
Strip"
3530 Q3=R0 DIV 100 ! INTEGER DIV
IDE,i.e. Bus=IP(Scn/100)
3540 Q4=R1
3550 R1$="0000000000000000"
3560 ON ERROR GOTO 4010 ! IF P1
HAS NOT BEEN DEFINED
3570 IF P1=3497 THEN R1=R0 ! PAS
S PARAMETER USED HERE
3580 IF P1<>3456 AND P1<>3497 TH
EN 4010 ! Abort
3590 OFF ERROR
3600 !

```

```

3610 !
3620 !
3630 ! VERIFY THAT INTERFACE IS
      HP-IB
3640 !
3650 ON ERROR GOTO 3720
3660 STATUS 03,0 ; 06
3670 OFF ERROR
3680 !
3690 ! PRINT ERROR MESSAGE TO CR
      T IF NO INTERFACE OR NO HP-
      IB
3700 !
3710 IF 06=1 THEN 3830 ! OK
3720 OFF ERROR
3730 !
3740 CLEAR
3750 DISP USING "/",K" ; "CAUTION
      HP-IB INTERFACE #",,,," NOT
      HERE"
3760 DISP USING "/",K" ; "PROGRAM
      EXECUTION TERMINATED"
3770 BEEP
3780 STOP
3790 !
3800 ! INITIALIZE 82937 HP-IB IN
      TERFACE, RESET INTERFACE
3810 !
3820 !
3830 ABORTIO 03 ! INITIALIZES IN
      TERFACE AND BUS
3840 RESET 03 ! SENDS DEVICE CLE
      AR (DCL)
3850 !
3860 !
3870 ! CHECK FOR EQUIPMENT ON BU
      S AT ALL ADDRESSES AND PRIN
      T DEVICE NAMES
3880 !
3890 !
3900 CLEAR @ DISP USING "/",2X,K,
      DD,/,/,/ ; "EQUIPMENT PRESEN
      T ON BUS ";03
3910 FOR 02=0 TO 30
3920 OUTPUT 03*100+02 USING "#"
      ; ! ADDRESS DEVICE TO LISTE
      N
3930 RESUME 03 ! DROP ATN
3940 STATUS 03,2 ; 05 ! SEE IF I
      T LISTENED
3950 IF BINAND(05,3)=0 THEN 3980
      ! NOT THERE
3960 GOSUB 4060 ! CALL DEV-NAME
3970 DISP USING 3990 ; 07$,02
3980 NEXT 02
3990 IMAGE 37A,"at address",22
4000 GOTO 4040 ! EXIT
4010 OFF ERROR
4020 R3=2
4030 R$="Init"

```

```

4040 RETURN
4050 ! SUBEND
4060 ! SUB DEV-NAME
4070 ! Q7$=DEVICE NAME
4080 Q7$=""
4090 IF Q2<>04 MOD 100 THEN 4110
4100 Q7$="3456A Digital voltmete
r"
4110 IF Q2<>R2 MOD 100 THEN 4130
4120 Q7$="3437A System voltmeter
"
4130 IF Q2<>21 THEN 4150
4140 Q7$="hp85A System computer"
4150 IF Q2<>R0 MOD 100 THEN 4170
4160 Q7$="3497A Scanner Timer"
4170 IF R1=R0 THEN Q7$=Q7$&" ,DVM
"
4180 IF Q7$<>" " THEN 4200
4190 Q7$="Device Unknown"
4200 !
4210 RETURN
4220 ! SUBEND
4230 !
4240 !
4250 ! *****
4260 ! End of "init" SUB'NE
4270 ! *****
4280 !
4290 !
4300 ! -----

4310 ! Start "Avdcdv" SUBROUTINE
4320 ! -----
4330 !
4340 ! SUBPROGRAM Avdcdv) hp85
4350 ! QA 12/7/84 GAC
4360 ! P1=Channel
4370 !
4380 IF P1<>-1 THEN OUTPUT R0 ; "
AC";INT(P1)
4390 IF R1=R0 THEN 4430
4400 OUTPUT R1 ; "HR41000STI6STGT
3"
4410 ENTER R1 ; P
4420 GOTO 4520 ! RETURN
4430 OUTPUT R1 ; "VR5VA1VF1VD5VC0
VS0VW0S01VN999VT3"
4440 P=0
4450 FOR Q4=1 TO 100
4460 ENTER R1/USING "#,K" ; Q5
4470 P=P+Q5
4480 NEXT Q4
4490 OUTPUT R1 ; "S00" ! EXIT SYS
TEM OUTPUT MODE
4500 P=P/999
4510 GOTO 4520 ! RETURN
4520 RETURN
4530 ! SUBEND
4540 !
4550 !

```

SAMPLE OUTPUT FROM COMPUTER PROGRAM (CALIB*)

B-1

CALIBRATION AFB- 11

VEL. m/s	E VOLT	TEMP. C
0.000	0.000000	00.00
0.011	0.000000	00.00
0.022	0.000000	00.00
0.033	0.000000	00.00
0.044	0.000000	00.00
0.055	0.000000	00.00
0.066	0.000000	00.00
0.077	0.000000	00.00
0.088	0.000000	00.00
0.099	0.000000	00.00
0.110	0.000000	00.00
0.121	0.000000	00.00
0.132	0.000000	00.00
0.143	0.000000	00.00
0.154	0.000000	00.00
0.165	0.000000	00.00
0.176	0.000000	00.00
0.187	0.000000	00.00
0.198	0.000000	00.00
0.209	0.000000	00.00
0.220	0.000000	00.00
0.231	0.000000	00.00
0.242	0.000000	00.00
0.253	0.000000	00.00
0.264	0.000000	00.00
0.275	0.000000	00.00
0.286	0.000000	00.00
0.297	0.000000	00.00
0.308	0.000000	00.00
0.319	0.000000	00.00
0.330	0.000000	00.00
0.341	0.000000	00.00
0.352	0.000000	00.00
0.363	0.000000	00.00
0.374	0.000000	00.00
0.385	0.000000	00.00
0.396	0.000000	00.00
0.407	0.000000	00.00
0.418	0.000000	00.00
0.429	0.000000	00.00
0.440	0.000000	00.00
0.451	0.000000	00.00
0.462	0.000000	00.00
0.473	0.000000	00.00
0.484	0.000000	00.00
0.495	0.000000	00.00
0.506	0.000000	00.00
0.517	0.000000	00.00
0.528	0.000000	00.00
0.539	0.000000	00.00
0.550	0.000000	00.00
0.561	0.000000	00.00
0.572	0.000000	00.00
0.583	0.000000	00.00
0.594	0.000000	00.00
0.605	0.000000	00.00
0.616	0.000000	00.00
0.627	0.000000	00.00
0.638	0.000000	00.00
0.649	0.000000	00.00
0.660	0.000000	00.00
0.671	0.000000	00.00
0.682	0.000000	00.00
0.693	0.000000	00.00
0.704	0.000000	00.00
0.715	0.000000	00.00
0.726	0.000000	00.00
0.737	0.000000	00.00
0.748	0.000000	00.00
0.759	0.000000	00.00
0.770	0.000000	00.00
0.781	0.000000	00.00
0.792	0.000000	00.00
0.803	0.000000	00.00
0.814	0.000000	00.00
0.825	0.000000	00.00
0.836	0.000000	00.00
0.847	0.000000	00.00
0.858	0.000000	00.00
0.869	0.000000	00.00
0.880	0.000000	00.00
0.891	0.000000	00.00
0.902	0.000000	00.00
0.913	0.000000	00.00
0.924	0.000000	00.00
0.935	0.000000	00.00
0.946	0.000000	00.00
0.957	0.000000	00.00
0.968	0.000000	00.00
0.979	0.000000	00.00
0.990	0.000000	00.00

The linearized coefficients are

3860
4 1841
- 3234
5 6480

FLOW	NOFF	FLOW
3 6105		1436
3 8817		7256
3 9520		9321
4 5734	1	0759
5 0293	1	1832
2 7177	1	7187
3 3234	2	1931
13 7438	3	9980
15 7632	3	7084
13 3482	4	3160
20 9200	4	8999
22 9616	5	3313
24 3635	5	7135
27 4839	5	5675
30 7399	6	2867
34 5537	6	1289
37 1786	6	7464
38 4658	6	2845
42 5072	10	0000

CAL	OUT	ERROR
	1376	-4 2023
	7484	3 2285
	3134	-1 9999
1	0763	0350
1	1656	-1 4850
1	6246	-5 0692
2	0987	-4 3050
2	9382	-2 2620
3	6724	-1 9707
4	2984	-1 4090
4	9081	2076
5	3556	4565
5	7364	0825
5	5799	8028
7	3424	7635
8	2190	1 1091
8	8158	7930
9	3380	5764
9	9878	0218

THE SLOPE IS = 1.235254075513

C Calculate velocity of flow from transducer voltage readings

C -----

C

DO 7 I=1,NUMBER

KELV=273.0

T(=KELV+TEMP(I)

C

SSS=.013888*VOLT(I)*T(I)

FLOW(I)=.74648*SQRT(SSS)

XX(I)=FLOW*(1.OE3)

KX(=XX(I)

FLOW(I)=FLOAT(KXX(I))*(1.OE-3)

7

CONTINUE

C

C

C

DO 9 I=1,NUMBER

WRITE(6,102)VOLT(I),FLOW(I),POWER(I),TEMP(I)

9

CONTINUE

102

FORMAT(4F14.4)

C

C

ZEROE=POWER(I)

ZEROF=FLOW(1)

C

C

DO 10 I=1,NUMBER

FLOW(I)=FLOW(I)-ZEROF

10

POWER(I)=POWER(I)-ZEROE

C

WEIGHT(1)=100

WEIGHT(2)=80

WEIGHT(3)=60

WEIGHT(4)=40

WEIGHT(3)=20

```
      WEIGHT(4)=10
      NUM1=NUMBER-1
C
C
      DO 15 I7,NUM1
15    WEIGHT(I)=1
      WEIGHT(NUMBER)=100
C
      DO 20 I=1,3
20    R(I)=0.0
C
      DO 25 I=1,2
25    S(I)=0.0
C
C
      DO 30 I=1,NUMBER
      R(1)=R(1)+WEIGHT(I)
      R(2)=R(2)+WEIGHT(I)*POWER(I)
      S(1)=S(1)+WEIGHT(I)*FLOW(I)
      S(2)=S(2)+WEIGHT(I)*POWER(I)*FLOW(I)
C
30    CONTINUE
C
      N1=1.
35    L=N1+1
      K1=L+1
C
      DO 45 I=1,L
      DO 40 J=1,L
      I2=J-1+I
40    A(I,J)=R(I2)
45    A(I,K1)=S(I)
C
      DO 75 I=1,L
      A(K1,J)=-1.
```

```

      K2=I+1
C
      DO 50 J=K2,K1
50    A(K1,J)=0.0
      C=1./A(1,I)
C
      DO 60 I1=2,K1
      DO 55 J=K2,K1
55    A(I1,J)=A(I1,J)-A(I1,J)*A(1,J)*C
60    CONTINUE
C
      DO 70 I1=1,L
      DO 65 J=K2,K1
65    A(I1,J)=A(I1+1,J)
70    CONTINUE
75    CONTINUE
C
      DO 80 I=1,L
80    X52(I,K1-2)=A(I,K1)
C
      IF(N1.GE.ORDER) GO TO 90
82    N1=N1+1
      J=2*N1
      R(J)=0.0
      R(1)=0.0
      S(N1+1)=0.0
C
      DO 85 I=1,NUMBER
      R(J)=R(J)+(POWER(I)**(J-1))*WEIGHT(I)
      R(J+1)=R(J+1)+(POWER(I)**J)*WEIGHT(I)
85    S(N1+1)=S(N1+1)+FLOW*(POWER(I)**N1)*WEIGHT(I)
C
      GO TO 35
C
90    MLOOP=ORDER+1

```

```

C
DO 95 I=1,MLOOP
  COEFF(I)=A(I,K1)
  X52(I,4)=X52(I,4)*(POWER(NUMBER)**(I-1))
95  CONTINUE
C
DO 105 JK=2,NUMBER
  CALFLO=COEFF(1)
C
DO 100 I=2,5
100  CALFLO=CALFLO+COEFF(*POWER(JK)**(I-1))
    CALFL(JK)=CALFLO+10/FLOW(NUMBER)
C
105  ERROR(JK)=(CALFLO-FLW()*100/FLW(JK))
    SPAN=10/FLW(NUMBER)
C
DO 110 I=1,4
  LINCOF(I)=X52((I+1),4)*SPAN
110  CONTINUE
    WRITE(6,22)
22   FORMAT('LINEARIZED COEFFICIENTS')
    WRITE(6,103)(LINCOF(I),I=1,4)
103  FORMAT(//,'A=',F10.6,' B=',F10.6,' C=',F10.6,' D=',F10.6)
C
C
    WRITE(6,1000)
1000 FORMAT(/,7X,'FLOW',8X,'NORM FLOW',4X,'CALC OUTPUT',4X,'PCT ERROR
1')
C
DO 115 I=2,NUMBER
  NORMFLOW(I)=FLOW(I)*SPAN
  WRITE(6,1001)FLOW(I),NORMFL(I),CALFL(I),ERROR(I)
115  CONTINUE
1001 FORMAT(F14.5,2F14.3,F14.2)
C
  STOP
  END

```

Appendix C

Computer Program MFLOW

The computer program "MFLOW" calculates the mass flow rate across the orifice by applying equation 3.11. In equation 3.11, h is measured directly while the two correction factor multiplier is obtained by interpolation between the values of Table C.1. The fluid density, w is found from measurements of static head and temperature at the high pressure orifice tapping.

The velocity at the jet exit was found by assuming isentropic expansion from plenum to atmospheric conditions. Thus;

$$\frac{P_o}{P_j} = \left(1 + \frac{\gamma - 1}{\gamma} M_j^2\right)^{\frac{\gamma}{\gamma - 1}} \quad (C.1)$$

or

$$\frac{P_o}{P_j} = \left(1 + \frac{\gamma - 1}{\gamma} \frac{V_j^2}{c_j^2}\right)^{\frac{\gamma}{\gamma - 1}}$$

or

$$\frac{P_o^{\frac{\gamma - 1}{\gamma}}}{P_j^{\frac{\gamma - 1}{\gamma}}} = 1 + \frac{\gamma - 1}{\gamma} \frac{V_j^2}{c_j^2}$$

or

$$\frac{V_j^2}{c_j^2} = \frac{\frac{P_o^{\frac{\gamma - 1}{\gamma}}}{P_j^{\frac{\gamma - 1}{\gamma}}} - 1}{\frac{\gamma - 1}{\gamma}} \quad \text{and } c = \sqrt{\gamma RT}$$

Then,

$$V_j^2 = \frac{2\gamma RT \left[\left(\frac{P_o}{P_j} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]}{\gamma - 1} \quad (C.2)$$

where

T = plenum temperature $\approx$ temperature of air through orifice

P_o = total plenum pressure

P_j = pressure at jet exit

T was recorded with a thermometer downstream of the orifice and P_j is the static pressure reading on the cylinder, 2 mm downstream of the cylinder jet exit plane.

Table C.1

M Difference across orifice in mm of H_2O	Z M_R	Multipliers		
		M_D	M_E	
2	1.014	1.007	1.000	
4	1.013	1.007	0.996	
8	1.011	1.007	0.994	
12	1.010	1.007	0.992	
14	1.010	1.007	0.990	
16	1.009	1.007	0.988	
18	1.009	1.007	0.986	
20	1.008	1.007	0.984	
24	1.008	1.007	0.981	
28	1.008	1.007	0.978	
30	1.007	1.007	0.977	
35	1.007	1.007	0.973	
40	1.006	1.007	0.969	
45	1.006	1.007	0.965	
50	1.006	1.007	0.962	

M_R = Reynolds number correlation factor

M_D = pipe size correlation factor

M_E = expansibility correlation factor

```

C
C   This program calculates the MASS-FLOW rate through the
C   orifice ,the exit JET VELOCITY and cylinder SLOT THICKNESS
C
C
C   *****
C
C   DH .....Head,difference across orifice(in H2O)
C   Z .....Multipliers (from charts)
C   HHPR .....Head at high press. side of orifice(in)
C   T .....Temp. at orifice exit (F)
C   HJ .....Head at jet exit (in)
C   PPLEN .....Plenum pressure (in)
C   PATM .....Atmospheric pressure (Kg/m2)
C   MFR .....Mass flow rate through orifice(lbm/min)
C   DENSJ .....Air density at jet exit
C   PO .....Plenum pressure
C   VJ .....velocity at jet exit.
C   TMEAN .....Mean slot thickness
C
C   *****
C
C
C   REAL DH,Z,HHPR,T,HJ,PATM,PPLEN,MFR,DENSJ,PO,VJ
C   REAL DENS,TMEAN,VEL,PTUB,FLAG,TJK,MFRM
C   FLAG=1
C
C
C   READ(5,*)DH,Z,HHPR,T,HJ,PATM,PPLEN,PTUB
C
C
C   TEMP=T+459.69
C   DENS=(2.*HHPR*5.2+14.7*144)/(53.3*TEMP)
C   MFR=5.477758158*Z*SQRT(DH)*SQRT(DENS)
C   MFRM=MFR*.00756
C   PRESJ=HJ*25.400+PATM

```

```
TJK=(T-32)*.55555+273.
PRINT*,TJK
DENSJ=9.81*PRESJ/(287*TJK)
PO=PPLEN*25.4*PATM
VJ=2.*1.4*287*TJK*((PO/PRESJ)**.285714285-1.)
VJ=VJ/.4
VJ=SQRT(VJ)
C   PO=SQRT(PPLEN)
MFRM=MFR*.00756
TMEAN=MFRM/ (.495*DENSJ*VJ)
C   To find velocity from pitot tube measurement
C
C
VEL=SQRT((2.*PTUB*9806*.0254)/(DENSJ))
C
C
WRITE(6,10)
10  FORMAT('The HEAD difference across orifice is:')
WRITE(6,20)AH
20  FORMAT(F10.4)
WRITE(6,40)
40  FORMAT(//)
WRITE(6,1)DENS,MFR
1   FORMAT('DENSITY=',F10.5,5X,'MASS FLOW=',F10.5)
WRITE(6,2)DENSJ,VJ
2   FORMAT('DENS. JET=',F10.5,5X,'VELOCITY JET=',F10.5)
WRITE(6,30)VEL
30  FORMAT('VELOCITY P. TUBE=',F10.6)
WRITE(6,4)TMEAN
4   FORMAT('TMEAN=',F10.6)
FLAG=FLAG+1
IF(FLAG.LE.9) GOTO 28
A  STOP
END
```

Appendix D

ADACS Computer Programs U&RMS and M&RMS

A brief description of the program, U&RMS, and its statements are given below along with the flowchart and the program listing. The computer program, M&RMS, is a slightly modified version of the program, U&RMS. The M&RMS program is used for sampling the hot-wire signals with a slanted probe and outputs the distance, Y (mm), $\bar{E}_{br}$ and $\sqrt{e^2}$. The program, U&RMS, samples the hot-wire signals when a normal probe is used and outputs Y (mm), calculates the mean velocity, u , according to equation 4.15 of Chapter 4 and $\sqrt{u'^2}$ according to equation 4.18 of Chapter 4.

Because the two programs are identical in all other aspects, listing of the program, M&RMS, is not included.

Description of U&RMS Program

Parameter	Description
$w_9; w_1; w_3; w_2$	Distances $OG; OA; AW; OW$ in cm to be used in traversing gear kinematic analysis
k_1	Initial number of traversing gear steps
A_9	Actual guide tube advance for initial gear steps specified
N_0	Initial number of data points to be taken
N	Total number of data sets
$A_2; A_1$	Input number of times that the HP 3456A voltmeter will be triggered for RMS measurements; and the corresponding number for mean voltage measurements
$A_0; B; C; D; C$	Coefficients determined from program "CALIB*" to be used for linearization of signal
S	Slope of linearized $\bar{E}_{lin}$ vs U curve, determined from program, "CALIB"
$P_6; T_6$	ψ and α angles of probe
T_5	Measured angular distance from cylinder slot
B_5	Cylinder momentum coefficient, $C_{\mu c}$
$P_0(); F_0()$	$\bar{E}_{lin}; \sqrt{e'^2}$

<i>Line</i>	<i>Description</i>
250-280	Date of experiment
320-390	Input traversing gear dimensions
430-610	Specify the initial traversing gear size
750-840	Input the number of data points per set and number of sets
850-950	Specify how many times the HP 3456A voltmeter has to be triggered for measuring RMS and mean voltages
980-1140	Input coefficients and slope found from program, "CALIB*", to be used to linearize signals
1170-1360	Specify the probe orientation, the measuring station and the cylinder momentum coefficient
1420-1780	Loop to sample signals
1800-2050	Traversing gear kinematic analysis to determine actual probe distance, Y (mm), at each point of the flow
2060-2360	Relay actuator software to control movement of probe
3390-3650	Subroutine "Avdev" to sample mean voltages at the rate specified in Chapter 3
3650-3850	Subroutine "Acv" with modifications to sample RMS signal at the rate specified in Chapter 3

Figure D1: Flowchart of Computer Program U&RMS

```

10 |
20 | PROGRAM (U&RMS)
30 |
40 |
50 | *****
60 | THIS PROGRAM CALCULATES
70 | MEAN AND TURBULENT
80 | QUANTITIES USING
90 | A NORMAL H.W. PROBE
100 | ( June 25 85 )
110 | *****
120 |
130 |
140 |
150 |
160 |
170 | CHANEL 2 MEASURES (RMS)
180 | CHANEL 1 MEASURES (MEAN)
190 |
200 | REAL Q1,Q2,W1,W2,W3,P9,M1
210 | REAL W4,Q2,P8,M0,A9,M2
220 | PRINT "*****"
230 |
240 | PRINT "Mean @ Long Turbulenc
    | e profile"
250 | DISP "DATE IS"
260 | INPUT D9
270 | PRINT "JULY-";D9;"-85"
280 | PRINT
290 | PRINT "*****"
300 |
310 | PRINT
320 | PRINT "Traversing Gear Dime
    | nsions (cm)"
330 | PRINT
340 | PRINT "
350 | PRINT "OG=" @ INPUT W9@ PRIN
    | T W9
360 | PRINT "OA=" @ INPUT W1@ PRIN
    | T W1
370 | PRINT "AW=" @ INPUT W3@ PRIN
    | T W3
380 | PRINT "OW=" @ INPUT W2@ PRIN
    | T W2
390 | PRINT @ PRINT
400 | PRINT "
410 |
420 |
430 | DISP "*****"
440 | DISP "Ready to input the ini
    | tial number of gear steps"
450 | DISP "(for each (Y, step))"
460 | DISP "(i.e. how many gear ste
    | ps you want before you move"
470 | DISP "to the next measuring
    | position)"
480 | DISP "*****"
490 |

```

```

500 !
510 !
520 DISP "K1=???"
530 INPUT K1
540 DISP "*****"
550 DISP "Ready to input the initial gear advancement (i.e. A9)"
560 DISP "This represents the total length of the gear arm."
570 DISP "to be advanced at each step i.e 5 steps of gear = 0.1mm"
580 DISP "*****"
590 !
600 PRINT "Initial Gear advance = " @ INPUT A9@ PRINT A9
610 !
620 PRINT
630 PRINT "....."
640 !
650 ! <<<<<<<<<<>>>>>>>>>
660 !
670 !
680 DIM T0(3,100),F0(3,100)
690 DIM P0(3,100),F0(3,100)
700 DIM A(4,5),X(5),A1(3),B1(3)
710 DIM T1(100)
720 !
730 !
740 !
750 PRINT "The initial #.of data points is="
760 DISP "N0=??"
770 !
780 INPUT N0@ PRINT N0
790 PRINT
800 !
810 PRINT "# OF Data sets="
820 DISP "N=??"
830 INPUT N@ PRINT N
840 !
850 PRINT
860 PRINT "....."
870 PRINT @ PRINT
880 PRINT "Input # of (RMS) averages"
890 DISP "A2=??"
900 INPUT A2@ PRINT A2
910 PRINT "Input # of mean averages"
920 DISP "INPUT A1"
930 DISP "A1="
940 INPUT A1@ PRINT A1
950 !
960 PRINT
970 PRINT "....."
980 PRINT
990 PRINT "Linearized coeff's are:"
1000 DISP "INPUT A0"

```



```

1010 PRINT "A0="
1020 INPUT A0 PRINT A0
1030 PRINT "A="
1040 INPUT A0 PRINT A
1050 PRINT "B=" @ INPUT B0 PRINT
B
1060 PRINT "C=" @ INPUT C0 PRINT
C
1070 PRINT "D=" @ INPUT D0 PRINT
D
1080 !
1090 PRINT "
1100 PRINT
1110 PRINT "Slope=" @ INPUT S0 P
PRINT S
1120 PRINT
1130 PRINT "E(0)=" @ INPUT E00 P
PRINT E0
1140 !
1150 !
1160 PRINT "
1170 DISP "Psi=? @ Theta=?"
1180 DISP "Psi=?"
1190 INPUT P6
1200 DISP "Theta=?"
1210 INPUT T6
1220 PRINT "Psi=", P6, "Theta=", T6
1230 !
1240 PRINT
1250 PRINT "*****"
*****"
1260 PRINT
1270 PRINT "The measuring station
n (Theta on cylinder) is="
1280 !
1290 INPUT T50 PRINT T5
1300 PRINT "The blowing rate (Cm
) cyl="
1310 INPUT B50 PRINT B5
1320 PRINT
1330 PRINT "*****"
*****"
1340 PRINT @ PRINT
1350 !
1360 !
1370 PRINT USING 1380
1380 IMAGE 3X, "#", 4X, "Y(mm)", 5X
, "U/U0", 5X, "SQR02"
1390 PRINT
1400 !
1410 ! #####
##
1420 P1=3456 ! DIGITAL VOLTMETER
1430 GOSUB 2500
1440 !
1450 M2=0
1460 !
1470 FOR J=1 TO N
1480 FOR I=1 TO N0
1490 !

```

```

1500 S1=0
1510 P1=1
1520 FOR J8=1 TO A1
1530 I
1540 GOSUB 3460 : SUB "Acdecv"
1550 T1(J8)=P
1560 S1=S1+T1(J8)
1570 NEXT J8
1580 T0(J,I)=S1/A1
1590 I
1600 I
1610 P2=T0(J,I)-E0
1620 I ///////////////////////////////////////////////////
1630 I
1640 P0(J,I)=A*P2+B*P2^2+C*P2^3+
D*P2^4+A4
1650 I ///////////////////////////////////////////////////
1660 I
1670 P0(J,I)=P0(J,I)*S
1680 S0=0
1690 P1=2
1700 FOR I8=1 TO A2
1710 GOSUB 3740 : Subr"Acv"
1720 F8(I8)=P
1730 S0=S0+F8(I8)
1740 NEXT I8
1750 F0(J,I)=S0/A2
1760 F2=A+2*B*(T0(J,I)-E0)+3*C*(
T0(J,I)-E0)^2+4*D*(T0(J,I)-
E0)^3
1770 F0(J,I)=S*F0(J,I)*F2
1780 F0(J,I)=F0(J,I)/S
1790 I *****
1800 I *****
1810 I
1820 W4=W3+M2
1830 I
1840 I
1850 I
1860 Q1=ACS((W1^2+W2^2-W3^2)/(2*
W2*W1))
1870 Q2=ASN(W1*SIN(Q1)/W3)
1880 P9=SQR(W2^2+W4^2-2*W2*W4*CO
S(Q2))
1890 I
1900 P8=ASN(W4*SIN(Q2)/P9)
1910 P7=PI-P8-Q2
1920 W5=W4-W3
1930 M0=ASN(W5*SIN(P7)/W1)
1940 M5=.5*PI-P7
1950 M4=ASN(P9*SIN(M5)/W1)
1960 M4=PI-M4
1970 M3=PI-M4-M5
1980 M=M3+M0
1990 M1=SQR(2*W9^2-2*W9^2*COS(M)
)
2000 I
2010 M1=M1*10
2020 J6=P0(J,I)/S

```

```

2030 J7=J6.21
2040 PRINT USING 2050 . I.M1.J7.
      F0(J,I)
2050 IMAGE DD DD.X,DD DDDDD,2X,0
      DD DDD,2X,DD.DDDD
2060 !
2070 IF I<N0 THEN M2=M2+A9
2080 !
2090 ! *****
2100 !
2110 IF I=N0 THEN GOTO 2220
2120 FOR K0=1 TO K1
2130 CLEAR 709
2140 OUTPUT 709 : "D04,1"
2150 CLEAR 709
2160 OUTPUT 709 : "D04,1"
2170 CLEAR 709
2180 NEXT K0
2190 WAIT 4000
2200 NEXT I
2210 IF I=N0 THEN PRINT
2220 IF I=N0 THEN DISP "INPUT #
      OF GEAR STEPS " @ BEEP 50,4
      00
2230 IF I=N0 THEN PRINT
2240 IF I=N0 THEN PRINT
2250 IF I=N0 THEN PRINT
2260 IF I=N0 THEN DISP "K1=?"
2270 IF I=N0 THEN INPUT K1
2280 IF I=N0 THEN PRINT "Gear ad
      vance"
2290 IF I=N0 THEN PRINT "A9=?"
2300 IF I=N0 THEN INPUT A9
2310 IF I=N0 THEN PRINT A9
2320 IF I=N0 THEN DISP "INPUT #
      OF NEXT DATA SET"
2330 IF I=N0 THEN DISP "N0=?"
2340 IF I=N0 THEN INPUT N0
2350 IF I=N0 AND J=N THEN PRINT
2360 IF I=N0 THEN PRINT "END OF
      DATA ACQUISITION"
2370 GOTO 1480
2380 !
2390 !
2400 IF I=N0 AND J=N THEN PRINT
      "END*****"
2410 !
2420 !
2430 !
2440 !
2450 !
2460 NEXT J
2470 !
2480 !
2490 ! *****
2500 ! SUBPROGRAM Init hp85
2510 ! *****
2520 !
2530 !

```

```

2540 ! P1=Dvmeter
2550 DIM Q7$(23) ! DEVICE NAMES
2560 !
2570 ! INITIALIZE DEVICE ADDRESS

2580 !
2590 R0=709 ! 349A SCANNER ADDRESS
2600 R1=722 ! 3456 DIGITAL VOLTMETER ADDRESS
2610 R2=724 ! 3437A SYSTEM VOLTMETER ADDRESS
2620 R3=0 ! ERROR FLAG
2630 R4=0 ! USED IN SUBROUTINE "Strip"
2640 O3=R0 DIV 100 ! INTEGER DIVIDE, i.e. Bus=IP(Scn/100)
2650 O4=R1
2660 R1$="0000000000000000"
2670 ON ERROR GOTO 3130 ! IF P1 HAS NOT BEEN DEFINED
2680 IF P1=3497 THEN R1=R0 ! P1'S PARAMETER USED HERE
2690 IF P1<>3456 AND P1<>3497 THEN 3130 ! Abort
2700 OFF ERROR
2710 !
2720 !
2730 !
2740 ! VERIFY THAT INTERFACE IS HP-IB
2750 !
2760 ON ERROR GOTO 2830
2770 STATUS O3,0 ; O6
2780 OFF ERROR
2790 !
2800 ! PRINT ERROR MESSAGE TO CRT IF NO INTERFACE OR NO HP-IB
2810 !
2820 IF O6=1 THEN 2950 ! OK
2830 OFF ERROR
2840 !
2850 CLEAR
2860 DISP USING "/,K" ; "CAUTION HP-IB INTERFACE #",O6," NOT HERE"
2870 DISP USING "/,K" ; "PROGRAM EXECUTION TERMINATED"
2880 Z6=P0(1,I)
2890 BEEP
2900 STOP
2910 !
2920 ! INITIALIZE 82937 HP-IB INTERFACE, RESET INTERFACE
2930 !
2940 !
2950 ABORTIO O3 ! INITIALIZES INTERFACE AND BUS
2960 RESET O3 ! SENDS DEVICE CLEAR (DCL)

```

```

2970 !
2980 !
2990 ! CHECK FOR EQUIPMENT ON BUS
      S AT ALL ADDRESSES AND PRINT
      T DEVICE NAMES
3000 !
3010 !
3020 CLEAR @ DISP USING "/,2X,K,
      DD,/,/" ; "EQUIPMENT PRESEN
      T ON BUS ";03
3030 FOR 02=0 TO 30
3040 OUTPUT 03*100+02 USING "*"
      ; ! ADDRESS DEVICE TO LISTE
      N
3050 RESUME 03 ! DROP ATN
3060 STATUS 03,2 ; 05 ! SEE IF I
      T LISTENED
3070 IF BINAND(05,3)=0 THEN 3100
      ! NOT THERE
3080 GOSUB 3180 ! CALL DEV-NAME
3090 DISP USING 3110 ; Q7$,02
3100 NEXT 02
3110 IMAGE 37A,"at address",22
3120 GOTO 3160 ! EXIT
3130 OFF ERROR
3140 R3=2
3150 R$="Init"
3160 RETURN
3170 ! SUBEND
3180 ! SUB DEV-NAME
3190 ! Q7$=DEVICE NAME
3200 Q7$=""
3210 IF 02<>04 MOD 100 THEN 3230
3220 Q7$="3456A Digital voltmete
      r"
3230 IF 02<>R2 MOD 100 THEN 3250
3240 Q7$="3437A System voltmeter
      "
3250 IF 02<>21 THEN 3270
3260 Q7$="hp85A System computer"
3270 IF 02<>R0 MOD 100 THEN 3290
3280 Q7$="3497A Scanner Timer"
3290 IF R1=R0 THEN Q7$=Q7$&"DVM
      "
3300 IF Q7$<>" THEN 3320
3310 Q7$="Device Unknown"
3320 !
3330 RETURN
3340 ! SUBEND
3350 !
3360 !
3370 ! *****
3380 ! End of "init" SUB'NE
3390 ! *****
3400 !
3410 !
3420 ! -----
3430 ! Start "Avdcv" SUBROUTINE
3440 ! -----

```

```

3450 !
3460 ! SUBPROGRAM Avdcv hp85
3470 ! QA 12/7/84 GAC
3480 ! P1=Channel
3490 !
3500 IF P1<>-1 THEN OUTPUT R0 ; "
AC";INT(P1)
3510 IF R1=R0 THEN 3550
3520 OUTPUT R1 ; "HR41000STI6STGT
3"
3530 ENTER R1 ; P
3540 GOTO 3640 ! RETURN
3550 OUTPUT R1 ; "VR3VA1VF1VD5VC0
VS0VW0S01VN999VT3"
3560 P=0
3570 FOR 04=1 TO 999
3580 ENTER R1 USING "#,K" ; 05
3590 P=P+05
3600 NEXT 04
3610 OUTPUT R1 ; "S00" ! EXIT SYS
TEM OUTPUT MODE
3620 P=P/999
3630 GOTO 3640 ! RETURN
3640 RETURN
3650 ! SUBEND
3660 !
3670 !
3680 ! *****
3690 ! End of subroutines
3700 ! *****
3710 ! Subroutine "Acvdc"
3720 ! *****
3730 !
3740 ! SUBPROGRAM Acv hp85A
3750 !
3760 ! P1=Channel
3770 !
3780 IF P1<>-1 THEN OUTPUT R0 ; "
AC";INT(P1)
3790 OUTPUT R1 ; "HF21000STI6STGT
3" ! FILTER OFF
3800 ! OUTPUT R1 ; "HF21000STI6S
TGT3" ! FILTER ON
3810 ENTER R1 ; P
3820 GOTO 3830 ! RETURN-Reading
IN P
3830 RETURN
3840 ! SUBEND
3850 !
3860 ! *****
3870 ! END OF SUBROUTINE
3880 ! *****
3890 !

```

AUGU- 2 -85

Traversing Gear Dimensions (cm)

OG=
40.85OA=
16.35AW=
20.7OW=
26.4Initial Gear advance=
.006The initial # of data points is=
16# OF Data sets=
3Input # of (RMS) avars
30Input # of mean avars
50

Linearized coeff's are

A0=
3 2983950086E-3A=
.96621823251B=
9 2939511398C=
-3.47118457849D=
2.87892285316Slope=
.188909385757E(0)=
2 7262735Psi= 180
Theta= 45

The measuring station (Theta on
cylinder) is=

53
The block rate = 1=

			RMS
1.00	0.00000	5.579	6622
2.00	1.4991	5.530	7041
3.00	2.9982	5.538	7425
4.00	4.4972	5.604	8185
5.00	5.9963	5.719	9036
6.00	7.4954	5.809	9725
7.00	8.9945	5.833	1.0390
8.00	1.04936	5.896	1.0977
9.00	1.19927	5.933	1.1537
10.00	1.34917	5.917	1.1989

Gear advance

10=20

1.00	1.34917	5.893	1.2051
2.00	1.59982	5.877	1.2862
3.00	1.84887	5.858	1.3551
4.00	2.09872	5.769	1.3903
5.00	2.34856	5.708	1.4253
6.00	2.59841	5.654	1.4450
7.00	2.84826	5.528	1.4612
8.00	3.09811	5.391	1.4319
9.00	3.34796	5.250	1.3892
10.00	3.59781	5.092	1.3326
11.00	3.84766	5.016	1.2821
12.00	4.09751	5.896	1.2102
13.00	4.34737	5.798	1.1224
14.00	4.59722	5.742	1.0388
15.00	4.84708	5.671	.9539
16.00	5.09693	5.656	.8827
17.00	5.34679	5.648	.7702
18.00	5.59665	5.708	.6611
19.00	5.84651	5.788	.5778
20.00	6.09637	5.887	.5252

Gear advance

20=

1.00	6.09637	5.875	5163
2.00	6.34623	5.974	4740
3.00	6.59610	6.044	4318
4.00	6.84596	6.117	4090
5.00	7.09583	6.182	3785
6.00	7.34570	6.265	3515
7.00	7.59557	6.344	3362
8.00	7.84544	6.382	3183
9.00	8.09531	6.441	2935
10.00	8.34519	6.504	2711

Gear advance

H9=??

.02

1.00	6	34519	6.487	2725
2.00	8	84495	6.602	2368
3.00	9	34472	6.651	2173
4.00	9	84449	6.696	2069
5.00	10	34428	6.719	2020
6.00	10	84408	6.746	1969
7.00	11	34388	6.777	1951
8.00	11	84370	6.809	1934
9.00	12	34353	6.830	1927
10.00	12	84337	6.852	1952
11.00	13	34323	6.869	1937
12.00	13	84310	6.886	1937
13.00	14	34298	6.898	1954
14.00	14	84286	6.929	1928
15.00	15	34279	6.935	1928


```

READ(5,*)K,H,S,KN,HN,SN,NO
READ(5,*)(EL(1,I) , I=1,L)
READ(5,*)(EL(3,I) ,I=1,L)
READ(5,*)(EL(2,I) ,I=1,L)
READ(5,*)(EL(4,I) ,I=1,L)
READ(5,*)(EL(5,I) ,I=1,L)
READ(5,*)(EL(6,I) ,I=1,L)
READ(5,*)(EFL(1,I) ,I=1,L)
READ(5,*)(EFL(3,I), I=1,L)
READ(5,*)(EFL(2,I), I=1,L)
READ(5,*)(EFL(4,I) ,I=1,L)
READ(5,*)(EFL(5,I) ,I=1,L)
READ(5,*)(EFL(6,I) ,I=1,L)
READ(5,*)(V5(I) ,I=1,L)
READ(5,*)(U5(I) ,I=1,L)

C
C *****
C
38 IF(OPTION.EQ.1) GO TO 411
C
C <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<>>>>>>>>>>>>>>>>>>>>>>>>>
C This portion linearizes the H.W signals
C <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<>>>>>>>>>>>>>>>>>>>>>>>>>
C
C READ(5,*)AO,A1,B1,C1,D1,E0,AON,A1N,B1N,C1N,D1N,EON
C
C DO 1 J=1,5
DO 119 I=1,L
READ(5,*)(E(J,I))
READ(5,*)(EF(J,I))
PRINT*,E(J,I)
119 CONTINUE
1 CONTINUE
C
DO 41 J=1,5
DO 42 I=1,L
READ(5,*)EF(J,I)
PRINT*,EF(J,I)
42 CONTINUE
41 CONTINUE
C
C DO 2 J=1,4
DO 3 I=1,L
```


3 OF DE 3

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

```

A(1,2)=1.+K**2
A(1,3)=2.*(H**2)
A(2,1)=1.+K**2
A(2,2)=2.*(H**2)
A(2,3)=1.+K**2
A(3,1)=1.
A(3,2)=KN**2
A(3,3)=HN**2
B(1)=((EL(1,I)**2)+(EL(3,I)**2))/(2.*S**2)
B(2)=((EL(2,I)**2)+(EL(4,I)**2))/(2.*S**2)
B(3)=(EL(5,I)**2)/(SN**2)
      CALL SOLVE
U(I)=SQRT(B(1))
V(I)=B(2)
W(I)=B(3)
WRITE(6,12) U(I),V(I),W(I)
12   FORMAT(2X,"U=",F10.5,4X,"VSQ=",F10.5,4X,"WSQ=",F10.5)
345  CONTINUE
C
C *****
C   Calculation of ( U , V ) using slanted probes only.
C   ( solution of equations 1+3, 2+4)
C *****
C
WRITE(6,102)
102  FORMAT(/,"MEAN VELOCITIES (2-D) USING SLANTED PROBE")
C
DO 346 I=1,L
  A(1,1)=1.*K**2
  A(1,2)=1.+K**2
  A(2,1)=1.+K**2
  A(2,2)=2.*H**2
  B(1)=((EL(1,I)**2)+(EL(3,I)**2))/(S**2)
  B(2)=((EL(2,I)**2)+(EL(4,I)**2))/(S**2)
C
  CALL SOLVE
C
  U2S(I)=SQRT(B(1))
  V2S(I)=B(2)
C
  WRITE(6,103) U2S(I),V2S(I)
103  FORMAT(2X,"U2S=",F10.5,4X,"V2S=",F10.5)
C
346  CONTINUE

```

```

C
C *****
C      Calculation of mean velocities ( U, V ) using a normal
C      probe . Solution of equations of position 6 & 7
C *****
C
C
C      WRITE(6,104)
104  FORMAT(/,"MEAN VELOCITIES(2-D) USING NORMAL PROBE")
C
C      DO 347 I=1,L
C          A(1,1)=1
C          A(1,2)=KN**2
C          A(2,1)=1.
C          A(2,2)=HN**2
C          B(1)=EL(5,I)**2/SN**2
C          B(2)=EL(6,I)**2/SN**2
C
C      CALL SOLVE
C
C      U2N(I)=B(1)
C      V2N(I)=B(2)
C
C      WRITE(6,105)U2N(I),V2N(I)
105  FORMAT(2X,"U2N=",F10.5,4X,"V2N=",F10.5)
C
347  CONTINUE
C
C *****
C      This section of program calculates mean and turbulent
C      quantities using the ( X-wire ) method
C *****
C
C      WRITE(6,99)
99   FORMAT(/," The mean and turbulent quantities (X-probe)")
C      WRITE(6,991)
991  FORMAT(/)
C      WRITE(6,101)
101  FORMAT(5X,"U",15X,"u'2",12X,"v'2",10X,"u'v'")
C
C
C      DO 11 I=1,L

```

```

C      XX=EFL(1,I)**2
      XX1=EFL(3,I)**2
      DEN=4.0*(S**2)
      UVX(I)=(XX-XX1)/DEN
      WRITE(6,121)U1(I),US1(I),US2(I),UVX(I)
121    FORMAT(2X,F10.5,4X,F10.5,4X,F10.5,4X,F10.5)
11    CONTINUE
C
C
C      *****
C      This portion calculates      turbulent quantities assuming
C      V & W  are ZERO
C      *****
C
C
      WRITE(6,14)
14    FORMAT(/,"Calculation of turbulence assuming (V=W=0)")
      WRITE(6,144)
144    FORMAT(/)
      DO 228 I=1,L
          UVZ=(EFL(1,I)**2-EFL(3,I)**2)/(2.*(1.-K**2)*S**2)
          UWZ=(EFL(4,I)**2-EFL(2,I)**2)/(2.*(1.-K**2)*S**2)
          U2Z=(EFL(5,I)**2)/(SN**2*(1.+KN**2))
          U2Z=SQRT(U2Z)
          V2Z=(1.+K**2)*(EFL(1,I)**2+EFL(3,I)**2)
          FOL=2.*(1.-K**2)**2*S**2
          V2Z=(V2Z/FOL)-(EFL(5,I)**2*(1+KN**2))/(SN**2)
          V2Z=SQRT(V2Z)
C      U5(I)=EL(5,I)/S
      WRITE(6,8989)U2Z,V2Z,UVZ,UWZ
8989  FORMAT(X,"u2=",F8.5,3X,"v2=",F8.5,3X,"w2=",F8.5,3X,"u'v'="
1,F8.5,3X,"u'w'=",F8.5)
228  CONTINUE
C
C
C      *****
C      Calculation of ( U @ V ) using a slanted probe.
C      Equations 1+3 @ 2+4 are solved by an iteration procedure
C      *****
C
      WRITE(6,15)
15    FORMAT(/,"Calculation (U & V) using SLANTED probe only")
      WRITE(6,151)

```



```

151  FORMAT(/)
      F=0.0
C      I=0.0
          DO 10 I=1,L
      C1=(EL(1,I)**2+EL(2,I)**2+EL(3,I)**2+EL(4,I)**2)/(2.*S**2)
      C2=.5*(1.+K**2)+H**2
      C3=(EL(2,I)**2+EL(4,I)**2)/(S**2)
C      C4=(EL(5,I)**2+EL(6,I)**2)/(2.*SN**2)
          VSQ(1)=0.0
20      USQ(I)=(C1-C2*VSQ(I))/(1.+K**2)
C
          VSQ(I)=(C3-(1.+K**2)*USQ(I))/(2.*H**2)
C      WSQ(I)=(C4-USQ(I)-.5*(KN**2+HN**2)*VSQ(I))/(.5*(HN**2+KN**2))
          F=F+1
      IF(F.LE.NO) GO TO 20
          WRITE(6,9990)USQ(I),VSQ(I)
9990  FORMAT(2X,E12.5,2X,E12.5)
C
C      IF(M.LE.NO) GO TO 20
C      M=M+1
10      CONTINUE
C
C      *****
C      Solution of the turbulence quantities using full
C      equations. ( W is not included )
C      *****
C
      DO 1250 I=1,L
      WRITE(6,2008)U5(I),V5(I)
2008  FORMAT('U5=',F10.3,5X,'V5=',F10.3)
      PAR=1.+K**2
      PARN=1.-K**2
      PARNN=K**2-1
C
      QQ1=2.*PAR*(U5(I)**2+V5(I)**2)+4.*PARN*U5(I)*V5(I)
      AA1=PAR*U5(I)+PARN*V5(I)
      BB1=PAR*V5(I)+PARN*U5(I)
C
      QQ2=2.*PAR*(U5(I)**2+V5(I)**2)+4.*PARNN*U5(I)*V5(I)
      AA2=PAR*U5(I)+PARNN*V5(I)
      BB2=PAR*V5(I)+PARNN*U5(I)
C
      QQ3=U5(I)**2+KN**2*V5(I)**2
      AA3=U5(I)**2

```

```

BB3=KN**4*V5(I)**2
BB4=U5(I)*V5(I)*KN**2

```

```

A(1,1)=AA1**2/QQ1
A(1,2)=BB1**2/QQ1
A(1,3)=2.*AA1*BB1/QQ1

```

```

A(2,1)=AA2**2/QQ2
A(2,2)=BB2**2/QQ2
A(2,3)=2.*AA2*BB2/QQ2

```

```

A(3,1)=AA3/QQ3
A(3,2)=BB3/QQ3
A(3,3)=BB4/QQ3

```

```

B(1)=(EFL(1,I)**2)/(S**2)
B(2)=(EFL(3,I)**2)/(S**2)
B(3)=(EFL(5,I)**2)/(SN**2)

```

```

CALL SOLVE

```

```

WRITE(6,565)B(1),B(2),B(3)
565  FORMAT(2X,"u'2=",F10.5,2X,"y'2=",F10.5,2X,"u'v'=",F10.5)
1250  CONTINUE
STOP
END

```

```

/ SUBROUTINE **SOLVE** TO SOLVE SYSTEM OF EQN'S

```

```

Subroutine to compute the coefficients of the Influence Function

```

```

SUBROUTINE SOLVE
COMMON A(3,3),U(3)

```

```

C *** Search leading column of the matrix for the pivotal element

```

C

```

DO 4 K=1,2
IA=K
IB=K+1
DO 1 I=IB,3
AA=ABS(A(IA,K))
AB=ABS(A(I,K))
IF(AB.GT.AA) IA=I

```

1 CONTINUE

C

C *** Change rows when necessary

C

```

IF(IA.EQ.K) GO TO 3
DO 2 J=K,3
AA=A(K,J)
A(K,J)=A(IA,J)
A(IA,J)=AA

```

2 CONTINUE

```

AB=U(K)
U(K)=U(IA)
U(IA)=AB

```

C

C *** Check for zero pivotal element, then proceed with forward elimination

C

```

3 IF(ABS(A(K,K)).LE.1.OE-8) GO TO 7
DO 4 I=IB,3
AA=A(I,K)/A(K,K)
U(I)=U(I)-AA*U(K)
DO 4 J=IB,3
A(I,J)=A(I,J)-AA*A(K,J)

```

4 CONTINUE

C

C *** Solve for the unknowns by back-substitution

C

```

IF(ABS(A(3,3)).LE.1.OE-8) GO TO 7
U(3)=U(3)/A(3,3)
DO 6 K=2,3
I=4-K
AA=U(I)
IA=I+1
DO 5 J=IA,3
AA=AA-A(I,J)*U(J)
5 CONTINUE
U(I)=AA/A(I,I)

```

```
6  CONTINUE  
   RETURN  
7  WRITE(6,8)  
8  FORMAT(//2X,50H THE SOLUTION MATRIX HAS A ZERO DETERMINANT PROGR  
   1AM STOPPED)  
   RETURN  
   END
```

Program (DATARED)

```

C
C
C *****
C This program calculates turbulence quantities using the
C equations of Chapter 4 of the Thesis, in their simplified form.
C This program was used for calculations of all mean and turbulence
C quantities..
C *****
C
C
C
C
C CHARACTER FNAME*64, ANS*1
C CHARACTER*64 FOUT
C
C DIMENSION US1(100),US2(100),UVX(100),YN(100),UVO(100),YS(100)
C DIMENSION EFL(3,100),USQ(100),U1(100),EL(3,100),U11(100)
C DIMENSION US11(100),WS(100)
C REAL K
C
C PRINT*, 'DO YOU WANT TO CALCULATE (w2) component??'
C PRINT*, 'IF *YES* OPTION =0'
C READ*,OPTION
C
C PRINT*, 'ENTER NUMBER OF DATA POINTS (max=100)'
C READ*,N
C PRINT*, 'ENTER SLOPE OF LINEARIZED CURVE (slanted probe)'
C READ*,S
C PRINT*, 'ENTER VALUE OF (K) COEFFICIENT'
C READ*,K
C PRINT*, 'Enter Max velocity(m/s)'
C READ*,UMAX
C PRINT*, 'Enter Min velocity(m/s)'
C READ*,UMIN
C PRINT*, 'Enter Ymax'
C READ*,YMAX
C PRINT*, 'Enter Ymin'
C READ*,YMIN
C
C Read in Linearized (rms) and Mean voltages
C position (1) refers to (psi=0,theta=45 deg)
C position (2) refers to (psi=180 ,theta=45 deg)
C
C PRINT *, 'Please provide the INPUT file name'
C READ (*,9080)FNAME
9080 FORMAT(A)

```

C
C
C

Opening the input file

OPEN (5, FILE=FNAME, STATUS='OLD', ERR=999)
GO TO 990

999 PRINT *, 'Error in opening file :', FNAME
PRINT *, 'File does not exist'
PRINT *, 'Do you want to enter new file name (Y/N)?'
READ (*, '(A)') ANS
IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') THEN
PRINT *, 'Please provide the INPUT file name'
READ (*, 9080) FNAME
OPEN (5, FILE=FNAME, STATUS='OLD', ERR=999)
ELSE
PRINT *, 'I am unable to help you.'
STOP
ENDIF

C
C

990 READ(5, *) (YN(I), I=1, N)
READ(5, *) (UO(I), I=1, N)
READ(5, *) (USQ(I), I=1, N)
C READ(5, *) (YS(I), I=1, N)
READ(5, *) (EFL(1, I), I=1, N)
READ(5, *) (EFL(2, I), I=1, N)
READ(5, *) (EL(2, I), I=1, N)

C
C
C
C
C

Opening the output file

PRINT *, 'Do you want the output to be printed on a file(Y/N)?'
READ (*, '(A)') ANS
IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') THEN
PRINT *, 'Please provide the OUTPUT file name'
READ (*, 9000) FOUT
9000 FORMAT(A)
OPEN (6, FILE=FOUT, STATUS='NEW', ERR=998)
END IF
GO TO 991
998 PRINT *, 'Error in opening file:', FOUT
PRINT *, 'File already exists'
PRINT *, 'Do you want to overwrite file (Y/N)?'

```

READ (*, '(A)') ANS
IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') THEN
  OPEN (6, FILE=FOUT, STATUS='OLD', ERR=999)
ELSE
  PRINT*, 'Enter new file name:'
  READ (*, 9000) FOUT
  OPEN (6, FILE=FOUT, STATUS='NEW', ERR=999)
ENDIF

```

C

991 CONTINUE

C

C

DO 4 I=1,N

C

C

C

C

```

*****
Calculation of (v'2) Turbulence component
*****

```

```

UU=EFL(1,I)**2/(S**2)
UUU=EFL(2,I)**2/(S**2)
UPA=(UU+UUU)*(1.+K**2)**2
UPAF=UPA/((1.-K**2)**2)
UNC=(1.+K**2)**2/(1.-K**2)
UNF=UNC*USQ(I)**2
US2(I)=(UPAF-UNF)
US2(I)=SQRT(US2(I))

```

C

IF (OPT.EQ.0.0) THEN

C

C

C

C

C

```

*****
Calculation of (w'2) Turbulence component
*****

```

```

VC1=(1.+K**2)/(S**2)
VCN=(1.+K**2)/((K**2-1)**2)
WS(I)=(EFL(1,I)**2+EFL(2,I)**2)/(S**2)
WS(I)=WS(I)*VC1
WS(I)=WS(I)-VCN*(USQ(I)**2)
WSND=WS(I)/UMAX
END IF

```

C

C

```

C
C *****
C Calculation of Shear stresses (u'v' & u'w')
C *****

      XX=EFL(1,I)**2
      XX1=EFL(2,I)**2
      DEN=2.0*(S**2)*(1.-K**2)
      UVX(I)=(XX-XX1)/DEN
      UVX(I)=UVX(I)*(-1.)

C
      V2U=US2(I)/(UUO(I)*21.)
      V2UMAX=US2(I)/UMAX
C
      V2UMIN=US2(I)/UMIN
C
      U2U=USQ(I)/(UUO(I)*21.)
      U2UMAX=USQ(I)/UMAX
C
      U2UMIN=USQ(I)/UMIN
C
C
C
      UVMAX=UVX(I)/(UMAX**2)
      UVMIN=UVX(I)/(UMIN**2)

C
C
C
      UUMIN=UUO(I)*21./UMIN
      UUMAX=UUO(I)*21./UMAX
C
      YNMAX=YN(I)/YMAX
      YNMIN=YN(I)/YMIN
      YS(I)=YN(I)+.55
      YSMAX=YS(I)/YMAX
      YSMIN=YS(I)/YMIN

C
      IF(OPTION.EQ.0) THEN
        V2UMAX=WSND
      END IF

C
C *****

      WRITE(6,16) YN(I),UUO(I),U2UMAX,YS(I),V2UMAX,UVMAX
4      CONTINUE
C
16  FORMAT(8(2X,F10.6))

      STOP
      END

```



```

C      Program (SEP)
C      *****
C      This Program is used in conjunction with (DATARED) Program
C      for plotting the results
C      *****
C
C      CHARACTER FNAME*64, ANS*1
C      CHARACTER*64 FOUT
C      DIMENSION A(90,17),TITLE(80)
C      PRINT*,'TITLE??'
C      READ12,(TITLE(I),I=1,80)
12  FORMAT(80A1)
C
C      Opening the output file
C
C      PRINT *, 'Do you want the output to be printed on a file(Y/N)?'
C      READ (*, '(A)') ANS
C      IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') THEN
C          PRINT *, 'Please provide the OUTPUT file name'
C          READ (*,9000)FOUT
9000      FORMAT(A)
C          OPEN (6,FILE=FOUT,STATUS='NEW',ERR=996)
C          END IF
C          GO TO 991
996  PRINT*,'Error in opening file:',FOUT
C      /PRINT*,'File already exists'
C      PRINT*,'Do you want to overwrite file (Y/N)?'
C      READ (*, '(A)') ANS
C      IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') THEN
C          OPEN (6,FILE=FOUT,STATUS='OLD',ERR=999)
C      ELSE
C          PRINT*,'Enter new file name:'
C          READ (*,9000)FOUT
C          OPEN (6,FILE=FOUT,STATUS='NEW',ERR=999)
C      ENDIF
C
C
C      991  WRITE(6,12)(TITLE(I),I=1,80)
C          PRINT*,'TITLE FOR X-AXIS??'
C          READ12,(TITLE(I),I=1,80)
C          WRITE(6,12)(TITLE(I),I=1,80)
C          PRINT*,'TITLE FOR Y-AXISS??'

```

```

READ12,(TITLE(I),I=1,80)
WRITE(6,12)(TITLE(I),I=1,80)
PRINT*,'GIVE # OF X-SET AND # OF Y_SETS (1 1)'
READ*,NX,NY
PRINT*,'ENTER # OF DATA POITS'
READ*,NDATA
WRITE(6,14)NDATA,NX,NX
14 FORMAT(3I5,2X, '/')
PRINT*,'GIVE XMIN,XMAX,XDIV AND YMIN,YMAX,YDIV'
READ*,XMIN,XMAX,NXDIV,YMIN,YMAX,NYDIV
WRITE(6,21)XMIN,XMAX,NXDIV,YMIN,YMAX,NYDIV
21 FORMAT(2F10.3,I5,2F10.3,I5,2X, '/')
WRITE(6,22)
22 FORMAT(3I,'3',3I,'1',2X, '/')
C
PRINT*,'GIVE COMBINATION'
READ*,N1,N2
C
C
PRINT *, 'Please provide the INPUT file name'
READ (*,9080)FNAME
9080 FORMAT(A)
C
C Opening the input file
C
OPEN (5,FILE=FNAME,STATUS='OLD',ERR=999)
GO TO 990

999 PRINT*,'Error in opening file :',FNAME
PRINT*,'File does not exist'
PRINT*,'Do you want to enter new file name (Y/N)?'
READ (*, '(A)') ANS
IF (ANS .EQ. 'Y' .OR. ANS .EQ. 'y') THEN
PRINT *, 'Please provide the INPUT file name'
READ (*,9080)FNAME
OPEN (5,FILE=FNAME,STATUS='OLD',ERR=999)
ELSE
PRINT*,'I am unable to help you:'
STOP
ENDIF
C
990 CONTINUE
C
C

```

Table G.10: Results from B.L.C. Apparatus Experiments

$$\theta = 42^\circ; C_{\mu c} = 0.2; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{w'^2}{u_{max}^2}}$	$\frac{u'w'}{U_{max}^2}$
0.000000	1.225000	0.078405	0.550000	0.139116	-0.002826
0.151010	1.330000	0.089943	0.701010	0.146978	-0.001116
0.302020	1.998000	0.105602	0.852020	0.153082	0.000145
0.453030	2.290000	0.104854	1.003030	0.155927	0.000759
0.604040	2.406000	0.105119	1.154040	0.156088	0.000593
0.755050	2.472000	0.105878	1.305050	0.156587	0.000229
0.906060	2.536000	0.106245	1.456060	0.163563	0.000298
1.057070	2.565000	0.108813	1.607070	0.167406	0.000150
1.208080	2.565000	0.109291	1.758080	0.170345	-0.000115
1.359090	2.565000	0.110922	1.909090	0.175191	-0.000259
1.510100	2.572000	0.115593	2.060100	0.176846	-0.000179
1.661110	2.561000	0.116850	2.211110	0.181138	-0.000249
1.812120	2.538000	0.119273	2.362120	0.183357	-0.000213
1.963130	2.506000	0.120395	2.513130	0.186085	-0.000174
2.114140	2.480000	0.121114	2.664140	0.186786	-0.000292
2.265150	2.445000	0.122212	2.815150	0.187318	-0.000123
2.416160	2.406000	0.123811	2.966160	0.186384	0.000073
2.567180	2.348000	0.124735	3.117180	0.185124	0.000010
2.718190	2.294000	0.124682	3.268190	0.182868	0.000066
2.869880	2.204000	0.122010	3.519880	0.174215	0.000071
3.221560	2.130000	0.118968	3.771560	0.161403	0.000538
3.473250	2.057000	0.115248	4.023250	0.148056	0.000583
3.724950	1.989000	0.108996	4.274950	0.134718	0.000473
3.976640	1.916000	0.101735	4.526640	0.118211	0.000554
4.228330	1.853000	0.089158	4.778330	0.107742	0.000562
4.480030	1.813000	0.079630	5.030030	0.093436	0.000304
4.731720	1.794000	0.068974	5.281720	0.081229	0.000183
4.983420	1.781000	0.064389	5.533420	0.070503	0.000336
5.235120	1.782000	0.057917	5.785120	0.064959	0.000180
5.486820	1.790000	0.052866	6.036820	0.061922	0.000169
5.738520	1.813000	0.048913	6.288520	0.057139	0.000132
5.990220	1.834000	0.046369	6.540220	0.055215	0.000112
6.241930	1.854000	0.044470	6.791930	0.049577	0.000061
6.493630	1.891000	0.039452	7.043630	0.045094	0.000021
6.745350	1.931000	0.034146	7.295350	0.041786	0.000026
7.248770	1.950000	0.030417	7.798770	0.036303	0.000035
7.752200	1.960000	0.027555	8.302200	0.033715	0.000002
8.255650	1.961000	0.025704	8.805650	0.031765	0.000018
8.759100	1.964000	0.024254	9.309100	0.030953	-0.000002
9.262560	1.943000	0.023304	9.812560	0.031046	0.000036
9.766030	1.927000	0.022706	10.316030	0.031070	0.000002
10.269510	1.918000	0.022545	10.819510	0.029651	0.000036
10.773010	1.907000	0.022095	11.323010	0.030018	-0.000008
11.276520	1.896000	0.022021	11.826520	0.029092	0.000007
11.780030	1.888000	0.022015	12.330030	0.029322	0.000007
12.283570	1.878000	0.021754	12.833570	0.028760	0.000029
12.787110	1.870000	0.021591	13.337110	0.028528	0.000022
13.290670	1.866000	0.021925	13.840670	0.029109	0.000043
13.794250	1.858000	0.021195	14.344250	0.027836	0.000008
14.297840	1.850000	0.021066	14.847840	0.028531	0.000040
14.801450	1.841000	0.021034	15.351450	0.028468	0.000033
15.305070			15.855070		

Appendix F

Hot-Wire Signal Analysis

Based on the coordinate system shown in Figure 4.1 before and according to the theory by M. Acrivelllis^[1], the instantaneous velocity components can be expressed as:

$$\begin{aligned} u_N &= (u + u') \cos \alpha + \left[(v + v') \cos \psi - (w + w') \sin \psi \right] \sin \alpha \\ u_T &= -(u + u') \sin \alpha + \left[(v + v') \cos \psi - (w + w') \sin \psi \right] \cos \alpha \\ u_B &= (v + v') \sin \psi + (w + w') \cos \psi \end{aligned} \quad (4.3)$$

where

u_N — normal velocity component

u_T — tangential velocity component

u_B — binormal velocity component

and

primes ($'$) indicate fluctuating velocity components.

From cooling law^[33], the effective cooling velocity u_{eff} is;

$$u_{eff}^2 = u_N^2 + k^2 u_T^2 + h^2 u_B^2 \quad (4.4)$$

The linear relationship between effective cooling velocity and hot-wire output voltage is:

$$E = S u_{eff} \quad (4.5)$$

where

E — output voltage

S — slope of linearized plot of E vs. u_{eff} .

From equations 4.4 and 4.5,

$$u_{eff} = \left(u_N^2 + k^2 u_T^2 + h^2 u_B^2 \right)^{\frac{1}{2}} \quad (4.6)$$

or

$$\begin{aligned}
u_{eff} &= \left[\{ (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \}^2 \right. \\
&\quad + k^2 \{ -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \}^2 \\
&\quad \left. + h^2 \{ (v + v') \sin \psi + (w + w') \cos \psi \}^2 \right]^{\frac{1}{2}} \\
&= \frac{E(\psi, \alpha)}{S}
\end{aligned} \tag{4.7}$$

After squaring both sides;

$$\begin{aligned}
u_{eff}^2 &= \left[\{ (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \}^2 \right. \\
&\quad + k^2 \{ -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \}^2 \\
&\quad \left. + h^2 \{ (v + v') \sin \psi + (w + w') \cos \psi \}^2 \right]
\end{aligned}$$

or

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} &= u_{eff}^2 \\
&= \left[\{ (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \}^2 \right. \\
&\quad + k^2 \{ -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \}^2 \\
&\quad \left. + h^2 \{ (v + v') \sin \psi + (w + w') \cos \psi \}^2 \right]
\end{aligned}$$

$$\begin{aligned}
u_{eff}^2 &= \frac{E^2(\psi, \alpha)}{S^2} \\
&= (u + u')^2 \cos^2 \alpha + \sin^2 \alpha [(v + v') \cos \psi - (w + w') \sin \psi]^2 \\
&\quad + 2(u + u') \cos \alpha [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \\
&\quad + k^2 [(u + u')^2 \sin^2 \alpha + [(v + v') \cos \psi - (w + w') \sin \psi]^2 \cos^2 \alpha \\
&\quad - 2(u + u') \sin \alpha [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha] \\
&\quad + h^2 [(v + v')^2 \sin^2 \psi + (w + w')^2 \cos^2 \psi + 2(v + v')(w + w') \sin \psi \cos \psi]
\end{aligned}$$

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} = & (u + u')^2 \cos^2 \alpha \\
& + \sin^2 \alpha \left[(v + v')^2 \cos^2 \psi + (w + w')^2 \sin^2 \psi - 2(v + v')(w + w') \cos \psi \sin \psi \right] \\
& + 2(u + u') \cos \alpha \sin \alpha (v + v') \cos \psi - 2(u + u')(w + w') \cos \alpha \sin \alpha \sin \psi \\
& + k^2 \left[(u + u')^2 \sin^2 \alpha \right. \\
& + \left. \left[(v + v')^2 \cos^2 \psi + (w + w')^2 \sin^2 \psi - 2(v + v')(w + w') \sin \psi \cos \psi \right] \cos^2 \alpha \right. \\
& - \left. 2(u + u') \sin \alpha \cos \alpha \cos \psi (v + v') + 2(u + u')(w + w') \sin \psi \sin \alpha \cos \alpha \right] \\
& + h^2 \left[(v + v')^2 \sin^2 \psi + (w + w')^2 \cos^2 \psi + 2(v + v')(w + w') \sin \psi \cos \psi \right]
\end{aligned}$$

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} = & (u^2 + 2uu' + u'^2) \cos^2 \alpha \\
& + \sin^2 \alpha \left[(v^2 + 2vv' + v'^2) \cos^2 \psi + (w^2 + 2ww' + w'^2) \sin^2 \psi \right. \\
& - \left. 2(vw + vw' + v'w + v'w') \cos \psi \sin \psi \right] \\
& + 2 \cos \alpha \sin \alpha \cos \psi (uv + uv' + u'v + u'v') \\
& - 2 \cos \alpha \sin \alpha \sin \psi (uw + uw' + u'w + u'w') \\
& + k^2 \left[(u^2 + 2uu' + u'^2) \sin^2 \alpha + (v^2 + 2vv' + v'^2) \cos^2 \psi \cos^2 \alpha \right. \\
& + (w^2 + 2ww' + w'^2) \sin^2 \psi \cos^2 \alpha \\
& - \left. 2(vw + vw' + v'w + v'w') \cos \psi \sin \psi \cos^2 \alpha \right. \\
& - \left. 2(uv + uv' + u'v + u'v') \sin \alpha \cos \alpha \cos \psi \right. \\
& + \left. 2(uw + uw' + u'w + u'w') \sin \alpha \cos \alpha \sin \psi \right] \\
& + h^2 \left[(v^2 + 2vv' + v'^2) \sin^2 \psi + (w^2 + 2ww' + w'^2) \cos^2 \psi \right. \\
& + \left. 2(vw + vw' + v'w + v'w') \sin \psi \cos \psi \right]
\end{aligned}$$

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} = & u^2 \cos^2 \alpha + 2uu' \cos^2 \alpha + u'^2 \cos^2 \alpha \\
& + v^2 \cos^2 \psi \sin^2 \alpha + 2vv' \cos^2 \psi \sin^2 \alpha + v'^2 \cos^2 \psi \sin^2 \alpha \\
& + w^2 \sin^2 \psi \sin^2 \alpha + 2ww' \sin^2 \psi \sin^2 \alpha + w'^2 \sin^2 \psi \sin^2 \alpha \\
& - 2vw \cos \psi \sin \psi \sin^2 \alpha - 2vw' \cos \psi \sin \psi \sin^2 \alpha \\
& - 2v'w \cos \psi \sin \psi \sin^2 \alpha - 2v'w' \cos \psi \sin \psi \sin^2 \alpha \\
& + 2uv \cos \alpha \sin \alpha \cos \psi + 2uv' \cos \alpha \sin \alpha \cos \psi \\
& + 2u'v \cos \alpha \sin \alpha \cos \psi + 2u'v' \cos \alpha \sin \alpha \cos \psi \\
& - 2uw \cos \alpha \sin \alpha \sin \psi - 2uw' \cos \alpha \sin \alpha \sin \psi \\
& - 2u'w \cos \alpha \sin \alpha \sin \psi - 2u'w' \cos \alpha \sin \alpha \sin \psi \\
& + k^2 u^2 \sin^2 \alpha + 2k^2 uu' \sin^2 \alpha + k^2 u'^2 \sin^2 \alpha \\
& + k^2 v^2 \cos^2 \psi \cos^2 \alpha + 2k^2 vv' \cos^2 \psi \cos^2 \alpha + k^2 v'^2 \cos^2 \psi \cos^2 \alpha \\
& + k^2 w^2 \sin^2 \psi \cos^2 \alpha + 2k^2 ww' \sin^2 \psi \cos^2 \alpha + k^2 w'^2 \sin^2 \psi \cos^2 \alpha \\
& - 2k^2 vw \cos \psi \sin \psi \cos^2 \alpha - 2k^2 vw' \cos \psi \sin \psi \cos^2 \alpha \\
& - 2k^2 v'w \cos \psi \sin \psi \cos^2 \alpha - 2k^2 uv \sin \alpha \cos \alpha \cos \psi \\
& - 2k^2 uv' \sin \alpha \cos \alpha \cos \psi - 2k^2 u'v \sin \alpha \cos \alpha \cos \psi \\
& - 2k^2 u'v' \sin \alpha \cos \alpha \cos \psi + 2k^2 uw \sin \alpha \cos \alpha \sin \psi \\
& + 2k^2 uw' \sin \alpha \cos \alpha \sin \psi + 2k^2 u'w \sin \alpha \cos \alpha \sin \psi \\
& + 2k^2 u'w' \sin \alpha \cos \alpha \sin \psi \\
& + h^2 v^2 \sin^2 \psi + h^2 v'^2 \sin^2 \psi + 2h^2 vv' \sin^2 \psi \\
& + h^2 w^2 \cos^2 \psi + h^2 w'^2 \cos^2 \psi + 2h^2 ww' \cos^2 \psi \\
& + 2h^2 vw \sin \psi \cos \psi + 2h^2 vw' \sin \psi \cos \psi \\
& + 2h^2 wv' \sin \psi \cos \psi + 2h^2 v'w' \sin \psi \cos \psi
\end{aligned}$$

Noting that $2 \cos \theta \sin \theta = \sin 2\theta$,

we have;

$$\begin{aligned}
 \frac{E^2(\psi, \alpha)}{S^2} = & u^2 \cos^2 \alpha + 2uu' \cos^2 \alpha + u'^2 \cos^2 \alpha + v^2 \cos^2 \psi \sin^2 \alpha \\
 & + 2vv' \cos^2 \psi \sin^2 \alpha + v'^2 \cos^2 \psi \sin^2 \alpha + w^2 \sin^2 \psi \sin^2 \alpha \\
 & + 2ww' \sin^2 \psi \sin^2 \alpha + w'^2 \sin^2 \psi \sin^2 \alpha - vw \sin 2\psi \sin^2 \alpha \\
 & - vw' \sin 2\psi \sin^2 \alpha - v'w \sin 2\psi \sin^2 \alpha - v'w' \sin 2\psi \sin^2 \alpha \\
 & + uv \sin 2\alpha \cos \psi + uv' \sin 2\alpha \cos \psi + u'v \sin 2\alpha \cos \psi \\
 & + u'v' \sin 2\alpha \cos \psi - uw \sin 2\alpha \sin \psi - uw' \sin 2\alpha \sin \psi \\
 & - u'w \sin 2\alpha \sin \psi - u'w' \sin 2\alpha \sin \psi \quad 1^{st} \text{ term} \\
 & + k^2 u^2 \sin^2 \alpha + 2k^2 uu' \sin^2 \alpha + k^2 u'^2 \sin^2 \alpha \\
 & + k^2 v^2 \cos^2 \psi \cos^2 \alpha + 2k^2 vv' \cos^2 \psi \cos^2 \alpha + k^2 v'^2 \cos^2 \psi \cos^2 \alpha \\
 & + k^2 w^2 \sin^2 \psi \cos^2 \alpha + 2k^2 ww' \sin^2 \psi \cos^2 \alpha + k^2 w'^2 \sin^2 \psi \cos^2 \alpha \\
 & - 2k^2 vw \cos \psi \sin \psi \cos^2 \alpha - 2k^2 vw' \cos \psi \sin \psi \cos^2 \alpha \\
 & - 2k^2 v'w \cos \psi \sin \psi \cos^2 \alpha - 2k^2 v'w' \cos \psi \sin \psi \cos^2 \alpha \\
 & - k^2 uv \sin 2\alpha \cos \psi - k^2 uv' \sin 2\alpha \cos \psi \\
 & - k^2 u'v \sin 2\alpha \cos \psi - k^2 u'v' \sin 2\alpha \cos \psi \\
 & + k^2 uw \sin 2\alpha \sin \psi + k^2 uw' \sin 2\alpha \sin \psi \\
 & + k^2 u'w \sin 2\alpha \sin \psi + k^2 u'w' \sin 2\alpha \sin \psi \quad 2^{nd} \text{ term} \\
 & + h^2 v^2 \sin^2 \psi + h^2 v'^2 \sin^2 \psi + 2h^2 vv' \sin^2 \psi \\
 & + h^2 w^2 \cos^2 \psi + 2h^2 ww' \cos^2 \psi + h^2 w'^2 \cos^2 \psi \\
 & + h^2 vw \sin 2\psi + h^2 vw' \sin 2\psi + h^2 wv' \sin 2\psi \\
 & + h^2 v'w' \sin 2\psi \quad 3^{rd} \text{ term}
 \end{aligned}$$

Let

$$\begin{aligned}
 a &= \cos^2 \alpha + k^2 \sin^2 \alpha \\
 b &= \sin^2 \alpha \cos^2 \psi + k^2 \cos^2 \alpha \cos^2 \psi + h^2 \sin^2 \psi \\
 c &= \sin^2 \alpha \sin^2 \psi + k^2 \cos^2 \alpha \sin^2 \psi + h^2 \cos^2 \psi \\
 d &= \sin 2\alpha \cos \psi - k^2 \sin 2\alpha \cos \psi \\
 f &= -\sin 2\psi \sin^2 \alpha - k^2 \sin 2\psi \cos^2 \alpha + h^2 \sin 2\psi \\
 g &= k^2 \sin 2\alpha \sin \psi - \sin 2\alpha \sin \psi
 \end{aligned}$$

Rearranging;

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} = & u^2 [\cos^2 \alpha + k^2 \sin^2 \alpha] \\
& + v^2 [\cos^2 \psi \sin^2 \alpha + k^2 \cos^2 \psi \cos^2 \alpha + h^2 \sin^2 \psi] \\
& + w^2 [\sin^2 \psi \sin^2 \alpha + k^2 \sin^2 \psi \cos^2 \alpha + h^2 \cos^2 \psi] \\
& + uv [\cos \psi \sin 2\alpha - k^2 \cos \psi \sin 2\alpha] \\
& + vw [-\sin 2\psi \sin^2 \alpha - k^2 \sin 2\psi \cos^2 \alpha + h^2 \sin 2\psi] \\
& + uw [-\sin \psi \sin 2\alpha + k^2 \sin \psi \sin 2\alpha]
\end{aligned}$$

or

$$\begin{aligned}
& au^2 + bv^2 + cw^2 + duv + fvw + guw \\
& + 2uu' \cos^2 \alpha + u'^2 \cos^2 \alpha + 2vv' \cos^2 \alpha \sin^2 \alpha \\
& + v'^2 \cos^2 \psi \sin^2 \alpha + 2ww' \sin^2 \psi \sin^2 \alpha + w'^2 \sin^2 \psi \sin^2 \alpha \\
& - uv' \sin 2\psi \sin^2 \alpha - v'w \sin 2\psi \sin^2 \alpha - v'w' \sin 2\psi \sin^2 \alpha \\
& + uv' \cos \psi \sin 2\alpha + u'v \cos \psi \sin 2\alpha + u'v' \cos \psi \sin 2\alpha \\
& - uw' \sin \psi \sin 2\alpha - u'w \sin \psi \sin 2\alpha - u'w' \sin \psi \sin 2\alpha \\
& + 2k^2 uu' \sin^2 \alpha + k^2 u'^2 \sin^2 \alpha + 2k^2 vv' \cos^2 \psi \cos^2 \alpha \\
& + k^2 v' \cos^2 \psi \cos^2 \alpha + 2k^2 ww' \sin^2 \psi \cos^2 \alpha + k^2 w'^2 \sin^2 \psi \cos^2 \alpha \\
& - k^2 vw' \sin 2\psi \cos^2 \alpha - k^2 v'w \sin 2\psi \cos^2 \alpha - k^2 v'w' \sin 2\psi \cos^2 \alpha \\
& - k^2 uv' \sin 2\alpha \cos \psi - k^2 u'v \sin 2\alpha \cos \psi \\
& - k^2 u'v' \sin 2\alpha \cos \psi + k^2 uw' \sin 2\alpha \sin \psi \\
& + k^2 u'w \sin 2\alpha \sin \psi + k^2 u'w' \sin 2\alpha \sin \psi \\
& + h^2 v'^2 \sin^2 \psi + 2h^2 vv' \sin^2 \psi + 2h^2 ww' \cos^2 \psi \\
& + h^2 w'^2 \cos^2 \psi + h^2 vw' \sin 2\psi + h^2 v'w \sin 2\psi \\
& + h^2 v'w' \sin 2\psi \\
& = \frac{E^2(\psi, \alpha)}{S^2}
\end{aligned}$$

or

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} = & au^2 + bv^2 + cw^2 + duv + fvw + guw \\
& + u'^2 \left[\cos^2 \alpha + k^2 \sin^2 \alpha \right] \\
& + v'^2 \left[\cos^2 \psi \sin^2 \alpha + k^2 \cos^2 \psi \cos^2 \alpha + h^2 \sin^2 \psi \right] \\
& + w'^2 \left[\sin^2 \psi \sin^2 \alpha + k^2 \sin^2 \psi \cos^2 \alpha + h^2 \cos^2 \psi \right] \\
& + u'v' \left[\cos \psi \sin 2\alpha - k^2 \cos \psi \sin 2\alpha \right] \\
& + v'w' \left[-\sin 2\psi \sin^2 \alpha - k^2 \sin 2\psi \cos^2 \alpha + h^2 \sin 2\psi \right] \\
& + u'w' \left[-\sin \psi \sin 2\alpha + k^2 \sin \psi \sin 2\alpha \right] \\
& + uu' \left[2 \cos^2 \alpha + 2k^2 \sin^2 \alpha \right] \\
& + vv' \left[2 \cos^2 \psi \sin^2 \alpha + 2k^2 \cos^2 \psi \cos^2 \alpha + 2h^2 \sin^2 \psi \right] \\
& + ww' \left[2 \sin^2 \psi \sin^2 \alpha + 2k^2 \sin^2 \psi \cos^2 \alpha + 2h^2 \cos^2 \psi \right] \\
& + vw' \left[-\sin 2\psi \sin^2 \alpha - k^2 \sin 2\psi \cos^2 \alpha + h^2 \sin 2\psi \right] \\
& + v'w \left[-\sin 2\psi \sin^2 \alpha - k^2 \sin 2\psi \cos^2 \alpha + h^2 \sin 2\psi \right] \\
& + uv' \left[\cos \psi \sin 2\alpha - k^2 \cos \psi \sin 2\alpha \right] \\
& + u'v \left[\cos \psi \sin 2\alpha - k^2 \cos \psi \sin 2\alpha \right] \\
& + uw' \left[-\sin \psi \sin 2\alpha + k^2 \sin \psi \sin 2\alpha \right] \\
& + u'w \left[-\sin \psi \sin 2\alpha + k^2 \sin \psi \sin 2\alpha \right]
\end{aligned}$$

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} = & au^2 + bv^2 + cw^2 + duv + fvw + guw \\
& + au'^2 + bv'^2 + cw'^2 + du'v' + f'v'w' + gu'w' \\
& + 2auu' + 2bvv' + 2cww' \\
& + duv' + du'v + fvw' + f'v'w + guw' + gu'w
\end{aligned}$$

$$\begin{aligned}\frac{E^2(\psi, \alpha)}{S^2} &= au^2 + bv^2 + cw^2 + duv + fvw + guw \\ &\quad + au'^2 + bv'^2 + cw'^2 + du'v' + f v'w' + gu'w' \\ &\quad + 2auu' + 2bvv' + 2cww' \\ &\quad + d(uv' + u'v) + f(vw' + v'w) + g(uw' + u'w)\end{aligned}$$

Let

$$D = 2auu' + 2bvv' + 2cww' + d(uv' + u'v) + f(vw' + v'w) + g(uw' + u'w)$$

or

$$\begin{aligned}\frac{E(\psi, \alpha)}{S} &= \left[a(u^2 + u'^2) + b(v^2 + v'^2) + c(w^2 + w'^2) \right. \\ &\quad \left. + d(uv + u'v') + f(vw + v'w') + g(uw + u'w') + D \right]^{\frac{1}{2}}\end{aligned}$$

Let

$$Q = (au^2 + bv^2 + cw^2 + duv + fvw + guw)^{\frac{1}{2}}$$

Then:

$$\frac{E^2(\psi, \alpha)}{S^2} = \left(D + au'^2 + bv'^2 + cw'^2 + du'v' + f v'w' + gu'w' \right) + Q$$

$$\begin{aligned}\frac{E(\psi, \alpha)}{S} &= \sqrt{Q^2 + D + au'^2 + bv'^2 + cw'^2 + du'v' + f v'w' + gu'w'} \\ &= \left[Q^2 \left(1 + \frac{D}{Q^2} + \frac{u'^2 a + v'^2 b + w'^2 c}{Q^2} + \frac{u'v'd + v'w'f + u'w'g}{Q^2} \right) \right]^{\frac{1}{2}} \\ &= Q \left[\left(1 + \frac{D}{Q^2} + \frac{u'^2 a + v'^2 b + w'^2 c}{Q^2} + \frac{u'v'd + v'w'f + u'w'g}{Q^2} \right) \right]^{\frac{1}{2}}\end{aligned}$$

Expanding the final expression into binomial series; first degree. i.e.

$$(1+x)^{\frac{1}{2}} = 1 + \frac{x}{2}$$

Then,

$$\frac{E(\psi, \alpha)}{S} = Q \left[\left(1 + \frac{au'^2 + bv'^2 + cw'^2}{2Q^2} + \frac{du'v' + fv'w' + gu'w'}{2Q^2} \right) + \frac{D}{2Q^2} \right]$$

time averaging

$$\frac{\overline{E}(\psi, \alpha)}{S} = Q \left[\left(1 + \frac{a\overline{u'^2} + b\overline{v'^2} + c\overline{w'^2}}{2Q^2} + \frac{d\overline{u'v'} + f\overline{v'w'} + g\overline{u'w'}}{2Q^2} \right) \right]$$

Note:

D — drops out when expression is time-averaged because it includes terms of the type uu' , vv' (products of mean and fluctuating velocity terms).

For very small turbulence intensity ($\approx 20\%$), turbulent terms are very small so that they can be dropped out (smaller than time-mean velocity component).

Therefore

$$\frac{\overline{E^2}(\psi, \alpha)}{S^2} = Q^2 = au^2 + bv^2 + cw^2 + duv + fvw + guw \quad (4.8)$$

If (e) is considered to be the fluctuating component of the linearized output signal from hot-wire it can be related to the linearized instantaneous and time averaged signals, E and $\overline{E}$ by;

$$\frac{e(\psi, \alpha)}{S} = \frac{E(\psi, \alpha) - \overline{E}(\psi, \alpha)}{S}$$

Substituting expressions of $\frac{E(\psi, \alpha)}{S}$ and $\frac{\overline{E}(\psi, \alpha)}{S}$ into the latter equation we have;

$$\begin{aligned}
\frac{e(\psi, \alpha)}{S} &= Q \left[\left(1 + \frac{au'^2 + bv'^2 + cw'^2}{2Q^2} + \frac{du'v' + fv'w' + gu'w'}{2Q^2} \right) + \frac{D}{2Q^2} \right] \\
&\quad - Q \left[\left(1 + \frac{\overline{au'^2} + \overline{bv'^2} + \overline{cw'^2}}{2Q^2} + \frac{\overline{du'v'} + \overline{fv'w'} + \overline{gu'w'}}{2Q^2} \right) \right] \\
&= Q + \frac{au'^2}{2Q} + \frac{bv'^2}{2Q} + \frac{cw'^2}{2Q} + \frac{du'v'}{2Q} + \frac{fv'w'}{2Q} + \frac{gu'w'}{2Q} + \frac{D}{2Q} \\
&\quad - Q - \frac{\overline{au'^2}}{2Q} - \frac{\overline{bv'^2}}{2Q} - \frac{\overline{cw'^2}}{2Q} - \frac{\overline{du'v'}}{2Q} - \frac{\overline{fv'w'}}{2Q} - \frac{\overline{gu'w'}}{2Q}
\end{aligned}$$

or

$$\begin{aligned}
\frac{e(\psi, \alpha)}{S} &= \frac{1}{2Q} \left[a(u'^2 - \overline{u'^2}) + b(v'^2 - \overline{v'^2}) + c(w'^2 - \overline{w'^2}) + d(u'v' - \overline{u'v'}) \right. \\
&\quad \left. + f(v'w' - \overline{v'w'}) + g(u'w' - \overline{u'w'}) + D \right]
\end{aligned}$$

Let

$$\begin{aligned}
A &= a(u'^2 - \overline{u'^2}) + b(v'^2 - \overline{v'^2}) + c(w'^2 - \overline{w'^2}) \\
B &= d(u'v' - \overline{u'v'}) + f(v'w' - \overline{v'w'}) + g(u'w' - \overline{u'w'}) \\
D &= 2auu' + 2bvv' + 2cww' + g(uw' + u'w) + d(uv' + u'v) + f(vw' + v'w)
\end{aligned}$$

or

$$\frac{e(\psi, \alpha)}{S} = \frac{1}{2Q} [A + B + D]$$

Squaring final equation;

$$\{A + B + D\}^2 = (A^2 + B^2 + D^2 + 2AB + 2AD + 2BD)$$

and

$$\begin{aligned}
A^2 &= \left[a(u'^2 - \overline{u'^2}) + b(v'^2 - \overline{v'^2}) + c(w'^2 - \overline{w'^2}) \right]^2 \\
&= a^2(u'^2 - \overline{u'^2})^2 + b^2(v'^2 - \overline{v'^2})^2 + c^2(w'^2 - \overline{w'^2})^2 \\
&\quad + 2ab(u'^2 - \overline{u'^2})(v'^2 - \overline{v'^2}) + 2ac(u'^2 - \overline{u'^2})(w'^2 - \overline{w'^2}) \\
&\quad + 2bc(v'^2 - \overline{v'^2})(w'^2 - \overline{w'^2}) \\
&= a^4(u'^4 - 2u'^2\overline{u'^2} + (\overline{u'^2})^2) + b^2(v'^4 - 2v'^2\overline{v'^2} + (\overline{v'^2})^2) \\
&\quad + c^2(w'^4 - 2w'^2\overline{w'^2} + (\overline{w'^2})^2) \\
&\quad + 2ab(u'^2v'^2 - u'^2\overline{v'^2} - \overline{u'^2}v'^2 + \overline{u'^2}\overline{v'^2}) \\
&\quad + 2ac(u'^2w'^2 - u'^2\overline{w'^2} - \overline{u'^2}w'^2 + \overline{u'^2}\overline{w'^2}) \\
&\quad + 2bc(v'^2w'^2 - v'^2\overline{w'^2} - \overline{v'^2}w'^2 + \overline{v'^2}\overline{w'^2})
\end{aligned}$$

$$\begin{aligned}
B^2 &= \left[d(u'v' - \overline{u'v'}) + f(v'w' - \overline{v'w'}) + g(u'w' - \overline{u'w'}) \right]^2 \\
&= d^2(u'v' - \overline{u'v'})^2 + f^2(v'w' - \overline{v'w'})^2 + g^2(u'w' - \overline{u'w'})^2 \\
&\quad + 2df(u'v' - \overline{u'v'})(v'w' - \overline{v'w'}) + 2dg(u'v' - \overline{u'v'})(u'w' - \overline{u'w'}) \\
&\quad + 2fg(v'w' - \overline{v'w'})(u'w' - \overline{u'w'}) \\
&= d^2 \left((u'v')^2 - 2u'v'\overline{u'v'} + (\overline{u'v'})^2 \right) + f^2 \left((v'w')^2 - 2v'w'\overline{v'w'} + (\overline{v'w'})^2 \right) \\
&\quad + g^2 \left((u'w')^2 - 2u'w'\overline{u'w'} + (\overline{u'w'})^2 \right) \\
&\quad + 2df \left((u'v')v'w' - u'v'\overline{v'w'} - \overline{u'v'}v'w' + \overline{u'v'}\overline{v'w'} \right) \\
&\quad + 2dg \left((u'v')u'w' - u'v'\overline{u'w'} - \overline{u'v'}u'w' + \overline{u'v'}\overline{u'w'} \right) \\
&\quad + 2fg \left((v'w')u'w' - v'w'\overline{u'w'} - \overline{v'w'}u'w' + \overline{v'w'}\overline{u'w'} \right)
\end{aligned}$$

$$\begin{aligned}
D^2 &= \left[2auu' + 2bv'v' + 2cww' + guw' + gu'w + duv' + du'v + fvw' + fv'w \right]^2 \\
&= 4a^2u^2u'^2 + 4b^2v^2v'^2 + 4c^2w^2w'^2 + g^2u^2w'^2 + g^2u'^2w^2 \\
&\quad + f^2v^2w' + f^2v'^2w^2 + d^2u^2v'^2 + d^2u'^2v^2 \\
&\quad + 8abuvu'v' + 8acuwu'w' + 4agu^2u'w' + 4agu'^2uw \\
&\quad + 4afuu'vw' + 4afuu'v'w + 4adu^2u'v' + 4aduvu'^2 \\
&\quad + 8bcvww'w' + 4bgvuv'w' + 4bgvv'u'w + 4bfv^2v'w' \\
&\quad + 4bfvv'^2w + 4bdvuv'^2 + 4bdv^2v'u' + 4cgw^2u'w' \\
&\quad + 4cgwu'u'^2 + 4cfvw'w'^2 + 4cfw^2w'v' + 4cdww'u'v' \\
&\quad + 4cdww'u'v' + 2g^2uu'ww' + 2gfuw'^2v + 2gfuw'v'w \\
&\quad + 2gdu^2w'v' + 2gduw'u'v + 2gfu'vw'w + 2gfw^2u'v' \\
&\quad + 2gdu'uww + 2gdu'^2vw + 2f^2ww'vv' + 2dfuv'vw' \\
&\quad + 2dfu'w'v^2 + 2dfv'^2uw + 2dfu'vv'w + 2d^2uv'u'v
\end{aligned}$$

$$\begin{aligned}
2AB &= 2 \left[a(u'^2 - \overline{u'^2}) + b(v'^2 - \overline{v'^2}) + c(w'^2 - \overline{w'^2}) \right] \\
&\quad \times \left[d(u'v' - \overline{u'v'}) + f(v'w' - \overline{v'w'}) + g(u'w' - \overline{u'w'}) \right] \\
&= 2 \left[ad(u'^2 - \overline{u'^2})(u'v' - \overline{u'v'}) + af(u'^2 - \overline{u'^2})(v'w' - \overline{v'w'}) \right.
\end{aligned}$$

$$\begin{aligned}
& + ag(u'^2 - \overline{u'^2})(u'w' - \overline{u'w'}) + bd(v'^2 - \overline{v'^2})(u'v' - \overline{u'v'}) \\
& + bf(v'^2 - \overline{v'^2})(v'w' - \overline{v'w'}) + bg(v'^2 - \overline{v'^2})(u'w' - \overline{u'w'}) \\
& + cd(w'^2 - \overline{w'^2})(u'v' - \overline{u'v'}) + cf(w'^2 - \overline{w'^2})(v'w' - \overline{v'w'}) \\
& + cg(w'^2 - \overline{w'^2})(u'w' - \overline{u'w'}) \Big]
\end{aligned}$$

$$\begin{aligned}
2AD &= 2 \left[a(u'^2 - \overline{u'^2}) + b(v'^2 - \overline{v'^2}) + c(w'^2 - \overline{w'^2}) \right] \\
&\times \left[2auu' + 2bvv' + 2cww' + g(uw' + u'w) + d(uv' + u'v) + f(vw' + v'w) \right] \\
&= 2 \left[2a^2uu'(u'^2 - \overline{u'^2}) + 2abvv'(u'^2 - \overline{u'^2}) \right. \\
&\quad + 2acww'(u'^2 - \overline{u'^2}) + ag(u'^2 - \overline{u'^2})(uw' + u'w) \\
&\quad + af(u'^2 - \overline{u'^2})(vw' + v'w) + ad(u'^2 - \overline{u'^2})(uv' + u'v) \\
&\quad + 2abuu'(v'^2 - \overline{v'^2}) + 2b^2vv'(v'^2 - \overline{v'^2}) \\
&\quad + 2bcww'(v'^2 - \overline{v'^2}) + bg(v'^2 - \overline{v'^2})(uw' + u'w) \\
&\quad + bf(v'^2 - \overline{v'^2})(vw' + v'w) + bd(v'^2 - \overline{v'^2})(uv' + u'v) \\
&\quad + 2acuu'(w'^2 - \overline{w'^2}) + 2bcvv'(w'^2 - \overline{w'^2}) \\
&\quad + 2c^2ww'(w'^2 - \overline{w'^2}) + cg(w'^2 - \overline{w'^2})(uw' + u'w) \\
&\quad \left. + cf(w'^2 - \overline{w'^2})(vw' + v'w) + cd(w'^2 - \overline{w'^2})(uv' + u'v) \right]
\end{aligned}$$

$$\begin{aligned}
2BD &= 2 \left[d(u'v' - \overline{u'v'}) + f(v'w' - \overline{v'w'}) + g(u'w' - \overline{u'w'}) \right] \\
&\times \left[2auu' + 2bvv' + 2cww' + g(uw' + u'w) + d(uv' + u'v) + f(vw' + v'w) \right] \\
&= 2 \left[2adu u'(u'v' - \overline{u'v'}) + 2bdv v'(u'v' - \overline{u'v'}) \right. \\
&\quad + 2dcw w'(u'v' - \overline{u'v'}) + dg(u'v' - \overline{u'v'})(uw' + u'w) \\
&\quad + df(u'v' - \overline{u'v'})(vw' + v'w) + d^2(u'v' - \overline{u'v'})(uv' + u'v) \\
&\quad + 2afu u'(v'w' - \overline{v'w'}) + 2bfv v'(v'w' - \overline{v'w'}) \\
&\quad + 2cfw w'(v'w' - \overline{v'w'}) + fg(v'w' - \overline{v'w'})(uw' + u'w) \\
&\quad + f^2(v'w' - \overline{v'w'})(vw' + v'w) + fd(v'w' - \overline{v'w'})(uv' + u'v) \\
&\quad + 2agu u'(u'w' - \overline{u'w'}) + 2bgv v'(u'w' - \overline{u'w'}) \\
&\quad + 2gcw w'(u'w' - \overline{u'w'}) + g^2(u'w' - \overline{u'w'})(uw' + u'w) \\
&\quad \left. + gf(u'w' - \overline{u'w'})(vw' + v'w) + gd(u'w' - \overline{u'w'})(uv' + u'v) \right]
\end{aligned}$$

Therefore

$$\frac{\overline{e^2(\psi, \alpha)}}{S^2} = \frac{1}{4Q^2}(A^2 + B^2 + D^2 + 2AB + 2AD + 2BD)$$

Time averaging the terms AD and BD will be zero because they contain terms of the form vv' , uu' , uw' , ww' etc. throughout which when time averaged give zero.

Therefore

$$\frac{\overline{e^2(\psi, \alpha)}}{S^2} = \frac{1}{4Q^2}[\overline{A^2} + \overline{B^2} + 2\overline{AB} + \overline{D^2}]$$

Terms $\overline{A^2}$, $\overline{B^2}$, and $2\overline{AB}$ contain quantities of higher order, i.e. $\overline{u'^4}$, $\overline{u'^2v'^2}$, $\overline{u'^2v'^2}$ etc. and for low turbulence level, i.e. $\approx 20\%$ they may be neglected.

Term $\overline{D^2}$ contains quantities of the order $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$, $\overline{v'u'}$, $\overline{v'w'}$ etc. and they are kept.

$$\frac{\overline{e^2(\psi, \alpha)}}{S^2} = \frac{1}{4Q^2}(\overline{D^2})$$

Factoring out the $\overline{D^2}$ term we have;

$$\begin{aligned} & u'^2 [4a^2u^2 + d^2v^2 + g^2w^2 + 4aguw + 4aduv + 2gdvw] \\ & + v'^2 [4b^2v^2 + d^2u^2 + f^2w^2 + 4bdv + 4bfv + 2dfuw] \\ & + w'^2 [4c^2w^2 + g^2u^2 + f^2v^2 + 4cguw + 4cfv + 2gfuw] \\ & + u'v' [8abuv + 4afuw + 4adu^2 + 4bgvw + 4bdv^2 + 2gfw^2 + 2gduw + 2dfv + 2d^2uv] \\ & + u'w' [8acuw + 4agu^2 + 4afuv + 4cgw^2 + 4cdv + 2g^2uw + 2gduv + 2gfv + 2dfv^2] \\ & + v'w' [8bcvw + 4bgvu + 4bfv^2 + 4cfw^2 + 4cdv + 2gfuw + 2gdu^2 + 2f^2vw + 2dfuv] \end{aligned}$$

$\overline{D^2}$ can be expressed as;

$$\overline{D^2} = (a_1u' + b_1v' + c_1w')^2$$

where

$$a_1 = 2au + dv + gw$$

$$b_1 = 2bv + fw + du$$

$$c_1 = 2cw + gu + fv$$

or

$$\begin{aligned}
u_{eff} &= \left[\{ (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \}^2 \right. \\
&\quad + k^2 \{ -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \}^2 \\
&\quad \left. + h^2 \{ (v + v') \sin \psi + (w + w') \cos \psi \}^2 \right]^{\frac{1}{2}} \\
&= \frac{E(\psi, \alpha)}{S} \tag{4.7}
\end{aligned}$$

After squaring both sides;

$$\begin{aligned}
u_{eff}^2 &= \left[\{ (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \}^2 \right. \\
&\quad + k^2 \{ -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \}^2 \\
&\quad \left. + h^2 \{ (v + v') \sin \psi + (w + w') \cos \psi \}^2 \right]
\end{aligned}$$

or

$$\begin{aligned}
\frac{E^2(\psi, \alpha)}{S^2} &= u_{eff}^2 \\
&= \left[\{ (u + u') \cos \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \}^2 \right. \\
&\quad + k^2 \{ -(u + u') \sin \alpha + [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha \}^2 \\
&\quad \left. + h^2 \{ (v + v') \sin \psi + (w + w') \cos \psi \}^2 \right]
\end{aligned}$$

$$\begin{aligned}
u_{eff}^2 &= \frac{E^2(\psi, \alpha)}{S^2} \\
&= (u + u')^2 \cos^2 \alpha + \sin^2 \alpha [(v + v') \cos \psi - (w + w') \sin \psi]^2 \\
&\quad + 2(u + u') \cos \alpha [(v + v') \cos \psi - (w + w') \sin \psi] \sin \alpha \\
&\quad + k^2 [(u + u')^2 \sin^2 \alpha + [(v + v') \cos \psi - (w + w') \sin \psi]^2 \cos^2 \alpha \\
&\quad - 2(u + u') \sin \alpha [(v + v') \cos \psi - (w + w') \sin \psi] \cos \alpha] \\
&\quad + h^2 [(v + v')^2 \sin^2 \psi + (w + w')^2 \cos^2 \psi + 2(v + v')(w + w') \sin \psi \cos \psi]
\end{aligned}$$

APPENDIX G

TABULATED RESULTS

B.L.C. Apparatus

Table G.1: Results from B.L.C. Apparatus Experiments

$$\theta = 12^\circ; C_{\mu c} = 0.1; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{u'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	2.658000	0.120989	0.550000	0.150423	0.011041
0.098210	3.131000	0.112852	0.648210	0.133648	0.013186
0.196430	3.299000	0.109681	0.746430	0.122063	0.014674
0.294640	3.302000	0.111247	0.844640	0.100913	0.016186
0.392860	3.223000	0.118634	0.942860	0.188186	0.016776
0.491080	3.097000	0.121394	1.041080	0.185560	0.017044
0.589300	2.992000	0.122138	1.139300	0.179235	0.016214
0.687520	2.868000	0.124682	1.237520	0.167800	0.014995
0.785750	2.729000	0.120102	1.335750	0.157860	0.013585
0.883970	2.555000	0.117839	1.433970	0.141348	0.011419
0.982200	2.357000	0.112151	1.532200	0.128564	0.009508
1.080430	2.187000	0.103799	1.630430	0.109722	0.007212
1.178660	2.015000	0.092939	1.728660	0.092964	0.005085
1.276890	1.879000	0.081754	1.826890	0.076174	0.003068
1.375120	1.763000	0.071062	1.925120	0.064343	0.001611
1.473360	1.681000	0.062497	2.023360	0.056244	0.000587
1.571590	1.628000	0.054837	2.121590	0.056764	0.000023
1.669830	1.604000	0.051369	2.219830	0.053819	-0.000431
1.768070	1.592000	0.049774	2.318070	0.052692	-0.000634
1.866310	1.588000	0.049295	2.416310	0.051031	-0.000756
1.964560	1.589000	0.049001	2.514560	0.049535	-0.000813
2.062800	1.603000	0.049233	2.612800	0.048216	-0.000809
2.161050	1.616000	0.048891	2.711050	0.048833	-0.000800
2.259300	1.634000	0.048930	2.809300	0.047040	-0.000848
2.357550	1.649000	0.048180	2.907550	0.047079	-0.000854
2.455800	1.666000	0.047798	3.005800	0.046711	-0.000849
2.554050	1.681000	0.047429	3.104050	0.046492	-0.000845
2.652310	1.694000	0.047394	3.202310	0.045692	-0.000820
2.750560	1.712000	0.047209	3.300560	0.045572	-0.000804
2.848820	1.725000	0.046602	3.398820	0.045536	-0.000781
2.947080	1.738000	0.046915	3.497080	0.044139	-0.000763
3.045340	1.753000	0.046568	3.595340	0.043965	-0.000763
3.143600	1.767000	0.046252	3.693600	0.044034	-0.000770
3.241870	1.782000	0.045992	3.791870	0.043009	-0.000720
3.340130	1.799000	0.045603	3.890130	0.042550	-0.000710
3.438400	1.811000	0.045355	3.988400	0.041034	-0.000670
3.536670	1.821000	0.044674	4.086670	0.040688	-0.000648
3.634940	1.851000	0.044288	4.184940	0.038932	-0.000600
3.733220	1.875000	0.043480	4.283220	0.038439	-0.000586
3.831510	1.932000	0.041210	4.381510	0.035573	-0.000497
4.224610	1.973000	0.038529	4.774610	0.032124	-0.000402
4.470320	2.010000	0.036899	5.020320	0.028231	-0.000306
4.716040	2.039000	0.033173	5.266040	0.025776	-0.000230
4.961770	2.056000	0.031255	5.511770	0.022675	-0.000180
5.207520	2.071000	0.028662	5.757520	0.021007	-0.000122
5.453270	2.082000	0.026338	6.003270	0.019075	-0.000097
5.699040	2.094000	0.023892	6.249040	0.017034	-0.000065
5.944810	2.103000	0.021681	6.494810	0.015265	-0.000052
6.190580	2.108000	0.017013	6.740580	0.013764	-0.000025
6.436350	2.110000	0.012776	6.986350	0.011373	-0.000007
6.682120	2.098000	0.010018	7.232120	0.010601	-0.000007
6.927890	2.090000	0.009368	7.477890	0.008941	-0.000003
7.173660	2.082000	0.008702	7.723660	0.008358	-0.000001
7.419430	2.069000	0.008447	7.969430	0.008437	-0.000003
7.665200	2.055000	0.008175	8.215200	0.008055	-0.000001
7.910970	2.045000	0.007844	8.460970	0.008022	-0.000002
8.156740	2.033000	0.008201	8.706740	0.007950	-0.000006
8.402510	2.025000	0.007963	8.952510	0.007399	-0.000004
8.648280	2.013000	0.007725	9.198280	0.007672	-0.000001
8.894050	2.007000	0.007868	9.444050	0.007587	-0.000001
9.139820	1.994000	0.007558	9.689820	0.007948	-0.000002
9.385590	1.986000	0.007375	9.935590	0.007940	0.000000
9.631360	1.977000	0.007309	10.181360	0.007894	0.000000
9.877130	1.968000	0.007394	10.427130	0.007834	0.000000
10.122900	1.959000	0.007297	10.672900	0.007769	-0.000002

Table G.2: Results from B.L.C. Apparatus Experiments

$$\theta = 12^\circ; C_{\mu c} = 0.2; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{v'^2}{u_{max}^2}}$	$\frac{\overline{u'v'}}{U_{max}^2}$
0.000000	3.316000	0.102149	0.550000	0.161765	0.008850
0.098210	3.281000	0.103637	0.648210	0.169725	0.010522
0.196430	3.964000	0.102919	0.746430	0.182632	0.012775
0.294640	4.296000	0.096589	0.844640	0.200015	0.015369
0.392860	4.278000	0.102229	0.942860	0.205003	0.017055
0.491080	4.171000	0.108608	1.041080	0.204247	0.017693
0.589300	4.025000	0.110836	1.139300	0.200652	0.017527
0.687520	3.859000	0.113963	1.237520	0.193953	0.017036
0.785750	3.666000	0.113535	1.335750	0.182075	0.015354
0.883970	3.500000	0.111873	1.433970	0.168570	0.013490
0.982200	3.316000	0.111577	1.532200	0.155845	0.012090
1.080430	3.163000	0.108796	1.630430	0.134053	0.009346
1.178660	3.032000	0.106542	1.728660	0.107355	0.006820
1.276890	2.864000	0.103006	1.826890	0.078134	0.004573
1.375120	2.725000	0.098525	1.925120	0.042401	0.002735
1.473360	2.530000	0.090509	2.023360	0.013329	0.001338
1.571590	2.340000	0.079893	2.121590	0.021621	0.000426
1.669830	2.187000	0.069462	2.219830	0.014741	0.000002
1.768070	2.064000	0.058173	2.318070	0.031866	-0.000331
1.866310	1.995000	0.051112	2.416310	0.037901	-0.000520
1.964560	1.958000	0.046373	2.514560	0.041161	-0.000590
2.062800	1.951000	0.044196	2.612800	0.040630	-0.000624
2.161050	1.953000	0.042454	2.711050	0.041286	-0.000640
2.259300	1.981000	0.041509	2.809300	0.041338	-0.000661
2.357550	1.996000	0.040787	2.907550	0.040655	-0.000671
2.455800	2.015000	0.039861	3.005800	0.040651	-0.000664
2.554050	2.030000	0.039228	3.104050	0.040296	-0.000659
2.652310	2.048000	0.038787	3.202310	0.040049	-0.000647
2.750560	2.067000	0.038058	3.300560	0.039942	-0.000637
2.848820	2.086000	0.037830	3.398820	0.039410	-0.000619
2.947080	2.102000	0.037228	3.497080	0.038913	-0.000604
3.045340	2.118000	0.036937	3.595340	0.038228	-0.000567
3.143600	2.134000	0.036370	3.693600	0.037662	-0.000559
3.241870	2.152000	0.036069	3.791870	0.037293	-0.000542
3.340130	2.167000	0.035446	3.890130	0.036640	-0.000521
3.438400	2.176000	0.034913	3.988400	0.035555	-0.000491
3.536670	2.193000	0.035046	4.086670	0.033966	-0.000459
3.634940	2.208000	0.034616	4.184940	0.033285	-0.000444
3.733220	2.222000	0.033743	4.283220	0.032621	-0.000415
3.831490	2.256000	0.032339	4.381490	0.029182	-0.000338
4.224610	2.291000	0.030569	4.774610	0.025423	-0.000264
4.470320	2.330000	0.028456	5.020320	0.022192	-0.000197
4.716040	2.362000	0.025834	5.266040	0.020361	-0.000147
4.961770	2.386000	0.023703	5.511770	0.017617	-0.000106
5.207520	2.406000	0.021916	5.757520	0.016419	-0.000083
5.453270	2.418000	0.020525	6.003270	0.013332	-0.000059
5.699040	2.431000	0.018972	6.249040	0.011138	-0.000043
5.944810	2.434000	0.017466	6.494810	0.009585	-0.000028
6.190580	2.444000	0.014966	6.985400	0.006307	-0.000017
6.436350	2.443000	0.013316	7.476030	0.001905	-0.000008
7.191710	2.433000	0.012040	7.966710	0.003321	-0.000007
7.911430	2.420000	0.012317	8.461430	0.005603	-0.000008
8.403190	2.412000	0.012174	8.953190	0.006273	-0.000006
8.895010	2.401000	0.011898	9.445010	0.006160	-0.000007
9.386870	2.389000	0.011910	9.936871	0.006502	-0.000005
9.878780	2.377000	0.011954	10.428780	0.006914	-0.000005
10.370730	2.365000	0.011830	10.920730	0.006794	-0.000007
10.862740	2.357000	0.011675	11.412740	0.006434	-0.000006
11.354790	2.343000	0.011873	11.904790	0.006997	-0.000007
11.846890	2.337000	0.011822	12.396890	0.007092	-0.000006
12.339040	2.327000	0.012011	12.889040	0.007375	-0.000006
12.831240	2.315000	0.011653	13.381240	0.006984	-0.000004
13.323490	2.303000	0.012069	13.873490	0.007547	-0.000004
13.815790	2.297000	0.012001	14.365790	0.007608	-0.000004
14.308140	2.288000	0.011797	14.858140	0.007188	-0.000003

Table G.3: Results from B.L.C. Apparatus Experiments

$$\theta = 12^\circ; C_{\mu c} = 0.22; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{u'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	3.209000	0.116047	0.550000	0.155420	0.009949
0.098210	3.347000	0.109632	0.648210	0.176553	0.012065
0.196430	4.080000	0.108100	0.746430	0.188452	0.014057
0.294640	4.525000	0.102509	0.844640	0.198142	0.015411
0.392860	4.534000	0.110589	0.942860	0.197571	0.016070
0.491080	4.430000	0.113583	1.041080	0.194012	0.016074
0.589300	4.278000	0.116556	1.139300	0.185650	0.015046
0.687520	4.088000	0.119660	1.237520	0.173240	0.013586
0.785750	3.906000	0.120388	1.335750	0.159350	0.011873
0.883970	3.721000	0.118415	1.433970	0.145869	0.010276
0.982200	3.528000	0.118304	1.532200	0.130083	0.008621
1.080400	3.357000	0.114415	1.630400	0.110690	0.006718
1.178660	3.215000	0.111660	1.728660	0.080276	0.004518
1.276890	3.059000	0.107932	1.826890	0.047144	0.003030
1.375120	2.864000	0.103330	1.925120	0.033709	0.001549
1.473360	2.655000	0.094217	2.023360	0.045125	0.000653
1.571590	2.444000	0.083126	2.121590	0.038358	0.000104
1.669830	2.272000	0.071857	2.219830	0.021580	-0.000214
1.768070	2.136000	0.061112	2.318070	0.020629	-0.000427
1.866310	2.047000	0.052065	2.416310	0.033710	-0.000541
1.964560	1.995000	0.046411	2.514560	0.038332	-0.000579
2.062800	1.990000	0.043864	2.612800	0.039304	-0.000593
2.161050	1.994000	0.041864	2.711050	0.040139	-0.000592
2.259300	2.009000	0.040708	2.809300	0.040114	-0.000607
2.357550	2.026000	0.039349	2.907550	0.040650	-0.000617
2.455800	2.038000	0.038801	3.005800	0.039779	-0.000608
2.554050	2.060000	0.038515	3.104050	0.039003	-0.000592
2.652310	2.074000	0.037400	3.202310	0.039180	-0.000569
2.750560	2.097000	0.036927	3.300560	0.039691	-0.000591
2.848820	2.118000	0.036612	3.398820	0.040130	-0.000619
2.947080	2.135000	0.036103	3.497080	0.039596	-0.000686
3.045340	2.153000	0.035685	3.595340	0.038879	-0.000667
3.143600	2.168000	0.035170	3.693600	0.038242	-0.000638
3.241870	2.184000	0.034804	3.791870	0.037655	-0.000629
3.340130	2.200000	0.034594	3.890130	0.036744	-0.000611
3.438400	2.213000	0.034182	3.988400	0.035434	-0.000462
3.536670	2.223000	0.033731	4.086670	0.034506	-0.000442
3.634940	2.243000	0.033126	4.184940	0.033737	-0.000415
3.733220	2.262000	0.032959	4.283220	0.033283	-0.000412
3.831490	2.297000	0.031353	4.381490	0.030093	-0.000324
4.224610	2.333000	0.029216	4.774610	0.025756	-0.000202
4.470320	2.373000	0.026821	5.020320	0.022012	-0.000109
4.716040	2.410000	0.024588	5.266040	0.019962	-0.000065
4.961770	2.436000	0.022465	5.511770	0.018633	-0.000032
5.207520	2.455000	0.020178	5.757520	0.018175	-0.000006
5.453270	2.470000	0.018887	6.003270	0.016184	0.000017
5.699040	2.482000	0.017160	6.249040	0.015224	0.000032
5.944810	2.491000	0.015563	6.494810	0.014389	0.000045
6.190580	2.501000	0.015412	6.740580	0.010664	0.000059
6.436350	2.500000	0.012142	7.478030	0.011925	0.000068
7.419710	2.491000	0.009437	7.969710	0.014756	0.000046
7.911430	2.478000	0.007723	8.461430	0.019445	-0.000018
8.403190	2.464000	0.007167	8.953190	0.019529	-0.000020
8.895010	2.455000	0.006749	9.445010	0.019437	-0.000019
9.386870	2.443000	0.006762	9.936871	0.019200	-0.000023
9.878780	2.430000	0.006652	10.428780	0.019352	-0.000018
10.370730	2.419000	0.006495	10.920730	0.019223	-0.000015
10.862740	2.404000	0.006378	11.412740	0.019096	-0.000015
11.354790	2.396000	0.006357	11.904790	0.018951	-0.000018
11.846890	2.386000	0.006319	12.396890	0.018835	-0.000017
12.339040	2.377000	0.006473	12.889040	0.018800	-0.000017
12.831240	2.364000	0.006178	13.381240	0.018711	-0.000016
13.323490	2.357000	0.006034	13.873490	0.018483	-0.000012
13.815790	2.342000	0.006221	14.365790	0.018575	-0.000013
14.308140	2.336000	0.006281	14.858140	0.018810	-0.000014

Table G.4: Results from B.L.C. Apparatus Experiments

$$\theta = 22^\circ; C_{\mu c} = 0.1; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{u'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	2.197000	0.097378	0.550000	0.147610	0.000014
0.097610	2.370000	0.102071	0.647610	0.158682	0.001449
0.195220	2.491000	0.103340	0.745220	0.171943	0.004243
0.292840	2.568000	0.104607	0.842840	0.179046	0.007030
0.390460	2.591000	0.106924	0.940460	0.185730	0.009114
0.488080	2.639000	0.110085	1.038080	0.189613	0.010671
0.585700	2.615000	0.111965	1.135700	0.192624	0.012334
0.683330	2.609000	0.117736	1.233330	0.188806	0.012697
0.780950	2.590000	0.118737	1.330950	0.186571	0.012818
0.878590	2.553000	0.120987	1.428590	0.186550	0.013239
0.976220	2.512000	0.122592	1.526220	0.183106	0.013365
1.073850	2.447000	0.122240	1.623850	0.175233	0.012690
1.171490	2.375000	0.121693	1.721490	0.168032	0.011801
1.269130	2.301000	0.120572	1.819130	0.160580	0.010972
1.366780	2.238000	0.119600	1.916780	0.149568	0.009635
1.464420	2.168000	0.115874	2.014420	0.144851	0.008991
1.562070	2.103000	0.112793	2.112070	0.136634	0.007832
1.659720	2.021000	0.107368	2.209720	0.128280	0.006795
1.757370	1.935000	0.100863	2.307370	0.121950	0.005765
1.855030	1.862000	0.093917	2.405030	0.110159	0.004505
1.952690	1.807000	0.085352	2.502690	0.098493	0.003154
2.050350	1.767000	0.078664	2.600350	0.090541	0.001929
2.148010	1.739000	0.075741	2.698010	0.077647	0.001145
2.245680	1.715000	0.072757	2.795680	0.070607	0.000562
2.343350	1.689000	0.066873	2.893350	0.071509	0.000314
2.441020	1.675000	0.062018	2.991020	0.068311	-0.000078
2.538690	1.668000	0.060091	3.088690	0.066047	-0.000394
2.636370	1.667000	0.057551	3.186370	0.067520	-0.000546
2.734050	1.668000	0.056257	3.284050	0.066816	-0.000740
2.831730	1.659000	0.055104	3.381730	0.066971	-0.000731
2.929410	1.675000	0.055364	3.479410	0.066334	-0.000798
3.027100	1.688000	0.055504	3.577100	0.064949	-0.000844
3.124790	1.701000	0.055228	3.674790	0.064146	-0.000874
3.222480	1.708000	0.054721	3.772480	0.063109	-0.000864
3.320170	1.722000	0.054166	3.870170	0.062094	-0.000866
3.417870	1.738000	0.053711	3.967870	0.062147	-0.000846
3.515570	1.755000	0.053687	4.065570	0.061664	-0.000797
3.613270	1.772000	0.052285	4.163270	0.069827	-0.000730
3.710970	1.785000	0.052496	4.260970	0.068834	-0.000718
3.808670	1.812000	0.051212	4.358670	0.065306	-0.000585
3.906370	1.865000	0.049017	4.456370	0.062952	-0.000532
4.004070	1.907000	0.046493	4.554070	0.060802	-0.000460
4.101770	1.942000	0.043729	4.651770	0.046379	-0.000389
4.200000	1.969000	0.041760	4.749470	0.042375	-0.000317
4.297700	1.989000	0.038954	4.847170	0.039677	-0.000234
4.395400	2.004000	0.036832	4.944870	0.035776	-0.000111
4.493100	2.018000	0.035672	5.042570	0.032878	-0.000093
4.590800	2.027000	0.033173	5.140270	0.030801	-0.000051
4.688500	2.043000	0.027595	5.237970	0.028446	-0.000023
4.786200	2.048000	0.024560	5.335670	0.028072	0.000066
4.883900	2.044000	0.022945	5.433370	0.024637	0.000043
4.981600	2.036000	0.021417	5.531070	0.025481	-0.000009
5.079300	2.026000	0.021009	5.628770	0.022774	0.000038
5.177000	2.016000	0.021397	5.726470	0.021797	0.000029
5.274700	2.004000	0.020648	5.824170	0.022499	0.000031
5.372400	1.990000	0.021079	5.921870	0.021686	0.000013
5.470100	1.980000	0.020430	6.019570	0.022107	0.000039
5.567800	1.985000	0.020201	6.117270	0.021983	0.000035
5.665500	1.969000	0.019968	6.214970	0.021774	0.000035
5.763200	1.948000	0.019973	6.312670	0.021981	0.000032
5.860900	1.940000	0.020071	6.410370	0.021090	0.000040
5.958600	1.927000	0.019892	6.508070	0.021787	0.000048
6.056300	1.920000	0.019818	6.605770	0.021365	0.000060
6.154000	1.910000	0.019683	6.703470	0.021256	0.000045
6.251700	1.900000	0.019172	6.801170	0.021887	0.000046

Table G.5: Results from B.L.C. Apparatus Experiments

$$\theta = 22^\circ; C_{\mu c} = 0.2; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{v'^2}{u_{max}^2}}$	$\frac{u'v'}{U_\infty^2}$
0.000000	2.481000	0.084757	0.550000	0.163327	-0.000612
0.097610	2.917000	0.095972	0.647610	0.171913	0.002894
0.195220	3.294000	0.097870	0.745220	0.183229	0.005962
0.292840	3.486000	0.100816	0.842840	0.195269	0.008929
0.390460	3.603000	0.108218	0.940460	0.202310	0.011163
0.488080	3.636000	0.108069	1.038080	0.211181	0.013469
0.585700	3.621000	0.114091	1.135700	0.211414	0.014176
0.683330	3.620000	0.119010	1.233330	0.211072	0.014953
0.780950	3.580000	0.122127	1.330950	0.210285	0.015238
0.878590	3.525000	0.125547	1.428590	0.206929	0.015232
0.976220	3.469000	0.126460	1.526220	0.203943	0.015217
1.073850	3.394000	0.127854	1.623850	0.197863	0.014546
1.171490	3.299000	0.129561	1.721490	0.190496	0.014194
1.269130	3.204000	0.129971	1.819130	0.178237	0.012865
1.366780	3.087000	0.127299	1.916780	0.169358	0.011487
1.464420	3.003000	0.126399	2.014420	0.156498	0.010279
1.562070	2.916000	0.124835	2.112070	0.127972	0.006764
1.659720	2.809000	0.121275	2.209720	0.124511	0.006604
1.757370	2.700000	0.117307	2.307370	0.090056	0.003515
1.855030	2.589000	0.110871	2.405030	0.096572	0.004043
1.952690	2.462000	0.106210	2.502690	0.091189	0.003698
2.050350	2.387000	0.096092	2.600350	0.081544	0.002773
2.148010	2.310000	0.090822	2.698010	0.064410	0.001590
2.245680	2.250000	0.084307	2.795680	0.062036	0.001067
2.343350	2.218000	0.080360	2.893350	0.058497	0.000756
2.441020	2.176000	0.074229	2.991020	0.055312	0.000306
2.538690	2.146000	0.068761	3.088690	0.053853	-0.000098
2.636370	2.121000	0.062964	3.186370	0.054867	-0.000339
2.734050	2.103000	0.058097	3.284050	0.056888	-0.000415
2.831730	2.092000	0.054080	3.381730	0.056280	-0.000497
2.929410	2.095000	0.051722	3.479410	0.054905	-0.000572
3.027100	2.097000	0.049745	3.577100	0.054185	-0.000584
3.124790	2.106000	0.049238	3.674790	0.052341	-0.000600
3.222480	2.114000	0.047809	3.772480	0.050423	-0.000571
3.320170	2.132000	0.047417	3.870170	0.048857	-0.000548
3.417870	2.148000	0.046401	3.967870	0.048107	-0.000535
3.515570	2.165000	0.045607	4.065570	0.046944	-0.000500
3.613270	2.182000	0.044804	4.163270	0.045968	-0.000480
3.710970	2.204000	0.043757	4.260970	0.045238	-0.000448
3.808670	2.243000	0.041922	4.358670	0.041229	-0.000345
3.906370	2.287000	0.039771	4.456370	0.037000	-0.000290
4.004070	2.324000	0.036659	4.554070	0.035063	-0.000246
4.101770	2.354000	0.034069	4.651770	0.030360	-0.000145
4.200000	2.377000	0.031757	4.749470	0.027914	-0.000115
4.297700	2.392000	0.029750	4.847170	0.025391	-0.000079
4.395400	2.406000	0.028252	4.944870	0.023864	-0.000072
4.493100	2.418000	0.025738	5.042570	0.022924	-0.000059
4.590800	2.420000	0.024517	5.140270	0.021020	-0.000047
4.688500	2.423000	0.020920	5.237970	0.020318	-0.000037
4.786200	2.419000	0.018732	5.335670	0.018831	-0.000033
4.883900	2.406000	0.017786	5.433370	0.018096	-0.000021
4.981600	2.388000	0.017223	5.531070	0.018525	-0.000028
5.079300	2.372000	0.017138	5.628770	0.018480	-0.000024
5.177000	2.358000	0.016971	5.726470	0.018030	-0.000022
5.274700	2.340000	0.016937	5.824170	0.017556	-0.000024
5.372400	2.321000	0.016742	5.921870	0.017555	-0.000019
5.470100	2.305000	0.016553	6.019570	0.017683	-0.000015
5.567800	2.292000	0.016641	6.117270	0.016774	-0.000009
5.665500	2.272000	0.016441	6.214970	0.017083	-0.000013
5.763200	2.261000	0.016652	6.312670	0.016255	-0.000006
5.860900	2.244000	0.016297	6.410370	0.016328	-0.000004
5.958600	2.233000	0.017675	6.508070	0.014613	-0.000004
6.056300	2.220000	0.016118	6.605770	0.016702	-0.000004
6.154000	2.208000	0.016155	6.703470	0.016713	-0.000006

Table G.6: Results from B.L.C. Apparatus Experiments

$$\theta = 22^\circ; C_{\mu c} = 0.22; C_{\mu w} = 0.11$$

$Y_c (mm)$ (norm. probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	$Y (mm)$ (sl probe)	$\sqrt{\frac{u'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	2.748000	0.145885	0.550000	0.107543	0.002743
0.097610	3.426000	0.100696	0.647610	0.168009	0.005160
0.195220	3.734000	0.107709	0.745220	0.177970	0.007501
0.292840	3.831000	0.114180	0.842840	0.183261	0.010020
0.390460	3.873000	0.120678	0.940460	0.186390	0.011296
0.488008	3.843000	0.126474	1.038008	0.188088	0.012426
0.585700	3.788000	0.128265	1.135700	0.191298	0.013421
0.683330	3.719000	0.129433	1.233330	0.192090	0.013966
0.780950	3.626000	0.132845	1.330950	0.187774	0.013903
0.878590	3.509000	0.131672	1.428590	0.186608	0.013782
0.976220	3.421000	0.130949	1.526220	0.184027	0.013422
1.073850	3.328000	0.132062	1.623850	0.177476	0.012715
1.171490	3.238000	0.129776	1.721490	0.173016	0.012129
1.269130	3.165000	0.128831	1.819130	0.160020	0.010964
1.366780	3.065000	0.125823	1.916780	0.149862	0.009755
1.464420	2.975000	0.122964	2.014420	0.141267	0.008804
1.562070	2.893000	0.119394	2.112070	0.132510	0.007807
1.659720	2.776000	0.115343	2.209720	0.118465	0.006310
1.757370	2.696000	0.110156	2.307370	0.109905	0.005408
1.855030	2.594000	0.104195	2.405030	0.097497	0.004277
1.952690	2.520000	0.098617	2.502690	0.085276	0.003200
2.050350	2.443000	0.091280	2.600350	0.073423	0.002153
2.148010	2.377000	0.084748	2.698010	0.064553	0.001310
2.245680	2.308000	0.077388	2.795680	0.059428	0.000700
2.343350	2.257000	0.071067	2.893350	0.058631	0.000284
2.441020	2.225000	0.064406	2.991020	0.057410	-0.000045
2.538690	2.193000	0.057796	3.088690	0.058902	-0.000250
2.636370	2.178000	0.054314	3.186370	0.057109	-0.000422
2.734050	2.174000	0.049951	3.284050	0.056871	-0.000471
2.831730	2.180000	0.048230	3.381730	0.055601	-0.000625
2.929410	2.193000	0.046821	3.479410	0.052556	-0.000555
3.027100	2.202000	0.045628	3.577100	0.050836	-0.000523
3.124790	2.214000	0.044678	3.674790	0.049581	-0.000523
3.222480	2.230000	0.043517	3.772480	0.048172	-0.000496
3.320170	2.248000	0.042899	3.870170	0.046108	-0.000479
3.417870	2.266000	0.042022	3.967870	0.045719	-0.000462
3.515570	2.283000	0.040883	4.065570	0.044983	-0.000436
3.613270	2.294000	0.040003	4.163270	0.043652	-0.000413
3.710970	2.312000	0.039258	4.260970	0.042390	-0.000373
3.808670	2.345000	0.037349	4.358670	0.039291	-0.000304
3.906370	2.375000	0.035506	4.456370	0.035245	-0.000226
4.004070	2.405000	0.033332	4.554070	0.032853	-0.000180
4.101770	2.438000	0.031246	4.651770	0.027482	-0.000106
4.200000	2.453000	0.029100	4.750000	0.025032	-0.000075
4.298230	2.475000	0.026551	4.848230	0.023982	-0.000049
4.396460	2.490000	0.024657	4.946460	0.022840	-0.000046
4.494690	2.498000	0.022935	5.044690	0.021422	-0.000034
4.592920	2.502000	0.021507	5.142920	0.020073	-0.000027
4.691150	2.511000	0.019361	5.241150	0.019414	-0.000026
4.789380	2.502000	0.017975	5.339380	0.017591	-0.000022
4.887610	2.491000	0.017410	5.437610	0.017695	-0.000024
4.985840	2.473000	0.017253	5.535840	0.017245	-0.000021
5.084070	2.460000	0.016886	5.634070	0.017599	-0.000019
5.182300	2.444000	0.016741	5.732300	0.017161	-0.000018
5.280530	2.425000	0.016344	5.830530	0.016929	-0.000015
5.378760	2.410000	0.016221	5.928760	0.016786	-0.000010
5.476990	2.394000	0.016086	6.026990	0.016988	-0.000006
5.575220	2.383000	0.015989	6.125220	0.016894	-0.000003
5.673450	2.369000	0.016148	6.223450	0.016442	0.000000
5.771680	2.354000	0.016137	6.321680	0.016197	-0.000005
5.869910	2.337000	0.016365	6.419910	0.015846	-0.000007
5.968140	2.330000	0.015882	6.518140	0.016226	-0.000004
6.066370	2.311000	0.015557	6.616370	0.016603	-0.000002
6.164600	2.301000	0.015751	6.714600	0.016208	0.000004

Table G.7: Results from B.L.C. Apparatus Experiments

$$\theta = 42^\circ; C_{\mu c} = 0.1; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{v'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	0.476000	0.083294	0.550000	0.186675	-0.009045
0.251680	0.506000	0.088858	0.801680	0.194763	-0.008520
0.503360	1.030000	0.132451	1.053360	0.152602	-0.007206
0.755050	1.313000	0.138095	1.305050	0.141888	-0.003025
1.006730	1.445000	0.127737	1.556730	0.150638	0.000689
1.258410	1.523000	0.124369	1.808410	0.166096	0.003619
1.510100	1.587000	0.125553	2.060100	0.174046	0.007032
1.761780	1.577000	0.118737	2.311780	0.187089	0.008590
2.013470	1.592000	0.122353	2.563470	0.188742	0.010863
2.265150	1.576000	0.123304	2.815150	0.192922	0.012464
2.516840	1.563000	0.124626	3.066840	0.194133	0.013079
2.768530	1.517000	0.128862	3.318530	0.186300	0.013190
3.020210	1.466000	0.130106	3.570210	0.177445	0.011992
3.271900	1.420000	0.127644	3.821900	0.171317	0.011564
3.523590	1.383000	0.126292	4.073590	0.159368	0.009889
3.775280	1.345000	0.125209	4.325280	0.141828	0.007869
4.026980	1.309000	0.122230	4.576980	0.122427	0.006305
4.278670	1.252000	0.115443	4.828670	0.107121	0.004614
4.530360	1.211000	0.106607	5.080360	0.098232	0.003425
4.782060	1.181000	0.098989	5.332060	0.093896	0.002415
5.033760	1.155000	0.090048	5.583760	0.087475	0.001581
5.285460	1.149000	0.084865	5.835460	0.085215	0.000962
5.537160	1.147000	0.079301	6.087160	0.083409	0.000659
5.788860	1.150000	0.076065	6.338860	0.086163	0.000223
6.040560	1.180000	0.072042	6.590560	0.085087	0.000213
6.292270	1.224000	0.071695	6.842270	0.075531	0.000345
6.543970	1.270000	0.068644	7.093970	0.073987	0.000634
6.795690	1.338000	0.068132	7.345690	0.064278	0.000316
7.047400	1.382000	0.061698	7.597400	0.061142	0.000434
7.299110	1.436000	0.067227	7.849110	0.065258	0.000363
7.550820	1.457000	0.052294	8.100820	0.048194	0.000446
7.802530	1.480000	0.048160	8.352530	0.045580	0.000181
8.054240	1.510000	0.041394	8.604240	0.043793	0.000125
8.305950	1.519000	0.037045	8.855950	0.040553	0.000155
8.557660	1.523000	0.035544	9.107660	0.040858	0.000100
8.809370	1.525000	0.033253	9.359370	0.040229	0.000085
9.061080	1.525000	0.031443	9.611080	0.038004	0.000179
9.312790	1.519000	0.030928	9.862790	0.038568	0.000196
9.564500	1.517000	0.029938	10.114500	0.040019	0.000170
9.816210	1.514000	0.028877	10.366210	0.039707	0.000191
10.067920	1.515000	0.029307	10.617920	0.039655	0.000197
10.319630	1.511000	0.030318	10.869630	0.038362	0.000252
10.571340	1.508000	0.028503	11.121340	0.039086	0.000274

Table G.8: Results from B.L.C. Apparatus Experiments

$$\theta = 42^\circ; C_{\mu c} = 0.1; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{w'^2}{u_{max}^2}}$	$\frac{u'w'}{U_{max}^2}$
0.000000	0.476000	0.083294	0.550000	0.136464	0.000697
0.251680	0.506000	0.088858	0.801680	0.200157	0.002299
0.503360	1.030000	0.132451	1.053360	0.189768	0.004002
0.755050	1.313000	0.138095	1.305050	0.170488	0.002775
1.006730	1.445000	0.127737	1.556730	0.166781	0.001413
1.258410	1.523000	0.124369	1.808410	0.163736	0.001080
1.510100	1.587000	0.125553	2.060100	0.165429	0.000085
1.761780	1.577000	0.118737	2.311780	0.175099	-0.001176
2.013470	1.592000	0.122353	2.563470	0.177968	-0.000949
2.265150	1.576000	0.123304	2.815150	0.187291	-0.001446
2.516840	1.563000	0.124626	3.066840	0.188550	-0.001668
2.768530	1.517000	0.128862	3.318530	0.187536	-0.001288
3.020210	1.466000	0.130106	3.570210	0.183560	-0.001553
3.271900	1.420000	0.127644	3.821900	0.180736	-0.001240
3.523590	1.383000	0.126292	4.073590	0.176486	-0.001083
3.775280	1.345000	0.125209	4.325280	0.163099	-0.000462
4.026980	1.309000	0.122230	4.576980	0.149585	-0.000231
4.278670	1.252000	0.115443	4.828670	0.138853	-0.000171
4.530360	1.211000	0.106607	5.080360	0.129078	0.000188
4.782060	1.181000	0.098989	5.332060	0.124687	-0.000030
5.033760	1.155000	0.090048	5.583760	0.117429	0.000125
5.285460	1.149000	0.084865	5.835460	0.105408	0.000077
5.537160	1.147000	0.079301	6.087160	0.102242	0.000196
5.788860	1.150000	0.076065	6.338860	0.098077	0.000107
6.040560	1.180000	0.072042	6.590560	0.099487	-0.000099
6.292270	1.224000	0.071695	6.842270	0.094573	0.000353
6.543970	1.270000	0.068644	7.093970	0.087719	0.000195
6.795680	1.338000	0.068132	7.345680	0.083476	-0.000025
7.047380	1.382000	0.061698	7.597380	0.078253	-0.000172
7.299090	1.436000	0.057227	7.849090	0.068424	0.000046
7.550800	1.457000	0.052294	8.100800	0.068369	0.000019
7.802510	1.480000	0.048160	8.352510	0.060651	0.000158
8.054220	1.510000	0.041394	8.604220	0.054757	0.000210
8.305930	1.519000	0.037045	8.855930	0.053160	0.000128
8.557640	1.523000	0.035544	9.107640	0.047769	0.000087
8.809350	1.525000	0.033253	9.359350	0.045306	0.000080
9.061060	1.525000	0.031443	9.611060	0.044980	0.000094
9.312770	1.519000	0.030928	9.862770	0.042726	0.000105
9.564480	1.517000	0.029938	10.114480	0.043961	0.000130
9.816190	1.514000	0.028877	10.366190	0.042614	0.000149
10.067900	1.515000	0.029307	10.617900	0.041454	0.000089
10.319610	1.511000	0.030318	10.869610	0.041451	0.000115
10.571320	1.508000	0.028503	11.121320	0.040952	0.000108

Table G.9: Results from B.L.C. Apparatus Experiments

$$\theta = 42^\circ; C_{\mu c} = 0.2; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_{\infty}}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{v'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{\infty}^2}$
0.000000	1.225000	0.078405	0.550000	0.189024	-0.007378
0.151010	1.330000	0.089943	0.701010	0.184065	-0.009345
0.302020	1.498000	0.105602	0.852020	0.161943	-0.005244
0.453030	2.290000	0.104854	1.003030	0.149108	-0.002333
0.604040	2.406000	0.106119	1.154040	0.155068	-0.000307
0.755050	2.472000	0.106878	1.305050	0.160957	0.002273
0.906060	2.536000	0.106245	1.456060	0.168862	0.004069
1.057070	2.565000	0.108813	1.607070	0.175911	0.006042
1.208080	2.565000	0.109291	1.758080	0.182689	0.007599
1.359090	2.565000	0.110922	1.909090	0.186858	0.009028
1.510100	2.572000	0.115593	2.060100	0.188999	0.010269
1.661110	2.561000	0.116850	2.211110	0.190970	0.010878
1.812120	2.538000	0.119273	2.362120	0.192200	0.011605
1.963130	2.505000	0.120395	2.513130	0.193762	0.012349
2.114140	2.480000	0.121114	2.664140	0.194091	0.012711
2.265150	2.445000	0.122212	2.815150	0.193022	0.012829
2.416160	2.406000	0.123811	2.966160	0.187585	0.012243
2.567180	2.348000	0.124735	3.117180	0.182598	0.011932
2.718190	2.294000	0.124682	3.268190	0.178571	0.011429
2.869200	2.204000	0.122010	3.419200	0.166434	0.009830
3.020210	2.130000	0.118968	3.570210	0.152903	0.008175
3.171220	2.057000	0.115248	3.721220	0.137241	0.006550
3.322230	1.989000	0.108996	3.872230	0.119561	0.004604
3.473240	1.916000	0.101735	4.023240	0.099312	0.002903
3.624250	1.853000	0.089158	4.174250	0.087143	0.001834
3.775260	1.813000	0.079630	4.325260	0.080889	0.001427
3.926270	1.794000	0.068974	4.476270	0.076415	0.000964
4.077280	1.781000	0.064389	4.627280	0.066479	0.000393
4.228290	1.782000	0.057917	4.778290	0.063171	0.000237
4.379300	1.790000	0.052866	4.929300	0.067558	0.000106
4.530310	1.813000	0.048913	5.080310	0.064111	0.000137
4.681320	1.834000	0.046369	5.231320	0.060206	0.000099
4.832330	1.854000	0.044470	5.382330	0.047859	0.000109
4.983340	1.891000	0.039452	5.533340	0.041831	0.000098
5.134350	1.931000	0.034146	5.684350	0.036500	0.000122
5.285360	1.950000	0.030417	5.835360	0.033576	0.000059
5.436370	1.960000	0.027555	5.986370	0.032375	0.000014
5.587380	1.961000	0.025704	6.137380	0.029813	0.000041
5.738390	1.954000	0.024254	6.288390	0.028994	0.000039
5.889400	1.943000	0.023304	6.439400	0.028267	0.000036
6.040410	1.927000	0.022706	6.590410	0.029036	0.000047
6.191420	1.918000	0.022545	6.741420	0.028747	0.000045
6.342430	1.907000	0.022095	6.892430	0.027887	0.000061
6.493440	1.896000	0.022021	7.043440	0.032894	0.000235
6.644450	1.888000	0.022015	7.194450	0.027453	0.000066
6.795460	1.878000	0.021754	7.345460	0.027216	0.000087
6.946470	1.870000	0.021591	7.496470	0.027193	0.000098
7.097480	1.866000	0.021925	7.647480	0.027864	0.000109
7.248490	1.858000	0.021195	7.798490	0.027167	0.000116
7.399500	1.850000	0.021066	7.949500	0.025337	0.000076
7.550510	1.841000	0.021034	8.100510	0.025949	0.000115

Table G.10: Results from B.L.C. Apparatus Experiments

$$\theta = 42^\circ; C_{\mu c} = 0.2; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{w'^2}{u_{max}^2}}$	$\frac{u'w'}{U_\infty^2}$
0.000000	1.225000	0.078405	0.550000	0.139116	-0.002826
0.151010	1.330000	0.089943	0.701010	0.146978	-0.001116
0.302020	1.998000	0.106602	0.852020	0.153082	0.000145
0.453030	2.290000	0.104854	1.003030	0.155927	0.000759
0.604040	2.406000	0.106119	1.154040	0.156088	0.000593
0.755050	2.472000	0.106878	1.305050	0.158587	0.000229
0.906060	2.536000	0.106245	1.456060	0.163563	0.000298
1.057070	2.565000	0.108813	1.607070	0.167406	0.000150
1.208080	2.565000	0.109291	1.758080	0.170345	-0.000115
1.359090	2.565000	0.110922	1.909090	0.175191	-0.000259
1.510100	2.572000	0.115593	2.060100	0.176846	-0.000179
1.661110	2.561000	0.116850	2.211110	0.181138	-0.000249
1.812120	2.538000	0.119273	2.362120	0.183357	-0.000213
1.963130	2.506000	0.120395	2.513130	0.186085	-0.000174
2.114140	2.480000	0.121114	2.664140	0.186786	-0.000292
2.265150	2.445000	0.122212	2.815150	0.187318	-0.000123
2.416160	2.406000	0.123811	2.966160	0.186384	0.000073
2.567180	2.348000	0.124735	3.117180	0.185124	0.000010
2.718190	2.294000	0.124682	3.268190	0.182868	0.000066
2.869880	2.204000	0.122010	3.519880	0.174215	0.000071
3.221560	2.130000	0.118968	3.771560	0.161403	0.000538
3.473260	2.057000	0.115248	4.023260	0.148056	0.000583
3.724960	1.989000	0.108996	4.274960	0.134718	0.000473
3.976640	1.916000	0.101735	4.526640	0.118211	0.000564
4.228330	1.853000	0.089158	4.778330	0.107742	0.000562
4.480030	1.813000	0.079630	5.030030	0.093436	0.000304
4.731720	1.794000	0.068974	5.281720	0.081229	0.000183
4.983420	1.781000	0.064389	5.533420	0.070503	0.000336
5.235120	1.782000	0.057917	5.785120	0.064959	0.000180
5.486820	1.790000	0.052866	6.036820	0.061922	0.000169
5.738520	1.813000	0.048913	6.288520	0.057139	0.000132
5.990220	1.834000	0.046369	6.540220	0.055215	0.000112
6.241930	1.854000	0.044470	6.791930	0.049577	0.000061
6.493630	1.891000	0.039452	7.296360	0.045094	0.000021
7.248770	1.931000	0.034146	7.798770	0.041786	0.000026
7.752200	1.950000	0.030417	8.302200	0.036303	0.000036
8.255650	1.960000	0.027555	8.805650	0.033715	0.000002
8.759100	1.961000	0.025704	9.309100	0.031765	0.000018
9.262560	1.954000	0.024254	9.812560	0.030953	-0.000002
9.766030	1.943000	0.023304	10.316030	0.031046	0.000036
10.269510	1.927000	0.022706	10.819510	0.031070	0.000002
10.773010	1.918000	0.022545	11.323010	0.029651	0.000036
11.276520	1.907000	0.022095	11.826520	0.030018	-0.000008
11.780030	1.896000	0.022021	12.330030	0.029092	0.000007
12.283570	1.888000	0.022015	12.833570	0.029322	0.000007
12.787110	1.878000	0.021754	13.337110	0.028760	0.000029
13.290670	1.870000	0.021591	13.840670	0.028528	0.000022
13.794250	1.866000	0.021925	14.344250	0.029109	0.000043
14.297840	1.858000	0.021195	14.847840	0.027836	0.000008
14.801450	1.850000	0.021066	15.351450	0.028531	0.000040
15.305070	1.841000	0.021034	15.855070	0.028468	0.000033

Table G.11: Results from B.L.C. Apparatus Experiments

$$\theta = 62^\circ; C_{\mu c} = 0.1; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{v'^2}{u_{max}^2}}$	$\frac{\overline{u'v'}}{U_\infty^2}$
0.000000	0.087000	0.087575	0.550000	0.097562	-0.001302
0.101310	0.090000	0.090418	0.651310	0.095083	-0.001266
0.202620	0.099000	0.092263	0.752620	0.095922	-0.001432
0.303930	0.114000	0.097603	0.853930	0.093064	-0.001316
0.405240	0.121000	0.100237	0.955240	0.090640	-0.001489
0.506560	0.133000	0.102302	1.056560	0.091539	-0.001336
0.607870	0.138000	0.106901	1.157870	0.087714	-0.001561
0.709190	0.138000	0.107500	1.259190	0.090653	-0.001343
0.810510	0.146000	0.112026	1.360510	0.089093	-0.002157
0.911840	0.153000	0.114753	1.461840	0.086882	-0.001643
1.013150	0.174000	0.126926	1.563150	0.090131	-0.002316
1.114490	0.196000	0.145354	1.664490	0.082533	-0.002536
1.215830	0.218000	0.154525	1.765830	0.104936	-0.003686
1.317190	0.250000	0.173915	1.867190	0.097444	-0.002061
1.418560	0.284000	0.193304	1.968560	0.093976	-0.002469
1.519940	0.300000	0.210312	2.069940	0.091377	-0.000664
1.621330	0.352000	0.232703	2.171330	0.098372	-0.001423
1.722740	0.381000	0.249795	2.272740	0.123593	0.001664
1.824170	0.412000	0.264133	2.374170	0.147091	0.002223
1.925600	0.457000	0.281678	2.475600	0.164632	0.007107
2.027050	0.449000	0.287154	2.577050	0.196151	0.011324
2.128520	0.537000	0.317124	2.678520	0.162631	0.009234
2.229990	0.574000	0.327257	2.779990	0.208274	0.020100
2.331480	0.574000	0.330227	2.881480	0.220013	0.022930
2.432990	0.658000	0.346447	2.982990	0.269097	0.033197
2.534500	0.679000	0.344455	3.084500	0.290386	0.042042
2.636030	0.752000	0.384463	3.186030	0.161134	0.026920
2.737580	0.767000	0.341307	3.287580	0.264380	0.033766
2.839140	0.781000	0.354173	3.389140	0.291735	0.045613
2.940710	0.808000	0.331268	3.490710	0.325253	0.057072
3.042300	0.821000	0.352003	3.592300	0.274477	0.051552
3.143900	0.841000	0.336907	3.693900	0.307664	0.049135
3.245510	0.850000	0.312551	3.795510	0.265614	0.044343
3.347140	0.906000	0.313345	3.897140	0.308903	0.058297
3.448780	0.878000	0.295401	3.998780	0.268235	0.042306
3.550440	0.891000	0.307548	4.095440	0.232781	0.040045
3.652110	0.892000	0.266456	4.197110	0.313472	0.051885
3.753800	0.893000	0.245811	4.298800	0.244688	0.033242
3.855500	0.903000	0.239351	4.395500	0.277467	0.040703
3.957240	0.884000	0.202449	4.497240	0.280693	0.039479
4.058940	0.866000	0.183628	4.598940	0.316250	0.045553
4.160600	0.863000	0.200594	4.695600	0.253556	0.034267
4.262320	0.842000	0.171318	4.797320	0.249694	0.029220
4.364030	0.834000	0.164974	4.899030	0.234319	0.026083
4.465740	0.811000	0.137496	4.995740	0.230331	0.023322
4.567450	0.816000	0.138784	5.097450	0.212003	0.020061
4.669160	0.817000	0.132629	5.199160	0.190023	0.014832
4.770870	0.825000	0.130937	5.295870	0.173984	0.012462
4.872590	0.846000	0.128550	5.397590	0.155820	0.009537
4.974300	0.866000	0.128054	5.499300	0.161775	0.010134
5.076010	0.900000	0.120870	5.596010	0.149303	0.008489
5.177720	0.934000	0.138805	5.697720	0.143360	0.006530
5.279430	0.967000	0.144839	5.799430	0.136276	0.009039

Table G.12: Results from B.L.C. Apparatus Experiments

$$\theta = 62^\circ; C_{\mu c} = 0.2; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{v'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	0.697000	0.194553	0.550000	0.114997	-0.011132
0.253270	0.803000	0.206959	0.803270	0.134141	-0.011070
0.506560	0.965000	0.229486	1.056560	0.152441	-0.015641
0.759850	1.092000	0.227699	1.309850	0.176793	-0.017759
1.013160	1.272000	0.236143	1.563160	0.179450	-0.018756
1.266490	1.398000	0.217741	1.816490	0.212408	-0.021618
1.519820	1.434000	0.226215	2.069820	0.190317	-0.018796
1.773170	1.526000	0.198608	2.323170	0.204536	-0.015806
2.026530	1.510000	0.167983	2.576530	0.223826	-0.012236
2.279900	1.556000	0.161859	2.829900	0.211862	-0.011325
2.533300	1.535000	0.170706	3.083300	0.198112	-0.007518
2.786700	1.540000	0.177809	3.336700	0.181475	-0.004174
3.040110	1.517000	0.166939	3.590110	0.182447	-0.00764
3.293540	1.514000	0.158777	3.843540	0.186211	-0.000663
3.546980	1.492000	0.161342	4.096980	0.164972	0.003980
3.800440	1.479000	0.165599	4.350440	0.161556	0.004535
4.053900	1.448000	0.161660	4.603900	0.157483	0.004524
4.307390	1.413000	0.158434	4.857390	0.163852	0.004839
4.560880	1.381000	0.156366	5.110880	0.156393	0.005619
4.814390	1.357000	0.154392	5.364390	0.152284	0.005354
5.067910	1.325000	0.151429	5.617910	0.152490	0.005504
5.321450	1.297000	0.147255	5.871450	0.152811	0.005140
5.575000	1.277000	0.138453	6.125000	0.155164	0.004532
5.828560	1.243000	0.135776	6.378560	0.149380	0.004130
6.082140	1.212000	0.129581	6.632140	0.149196	0.003562
6.335730	1.198000	0.125239	6.885730	0.152341	0.004668
6.589340	1.178000	0.119335	7.139340	0.144991	0.003467
6.842960	1.167000	0.113322	7.392960	0.144077	0.002876
7.096600	1.161000	0.106506	7.646600	0.135044	0.001944
7.350210	1.150000	0.091186	7.900210	0.124308	0.000444
7.603810	1.161000	0.084882	8.153810	0.110189	0.000173
7.857400	1.181000	0.074847	8.407400	0.098598	-0.000472
8.111000	1.201000	0.073179	8.661000	0.085436	-0.000718
8.364600	1.236000	0.067760	8.914600	0.076145	-0.000837
8.618200	1.285000	0.062606	9.168200	0.070587	-0.001037
8.871800	1.315000	0.055022	9.421800	0.067466	-0.001068
9.125400	1.341000	0.062133	9.675400	0.063041	-0.001194
9.379000	1.356000	0.044617	9.929000	0.059892	-0.000945
9.632600	1.368000	0.038970	10.182600	0.058238	-0.000815
9.886200	1.377000	0.035503	10.436200	0.059265	-0.000880
10.139800	1.376000	0.033746	10.689800	0.048243	-0.000696
10.393400	1.385000	0.031959	10.943400	0.045016	-0.000585
10.647000	1.387000	0.031200	11.197000	0.039800	-0.000406
10.900600	1.384000	0.030080	11.450600	0.037428	-0.000363
11.154200	1.382000	0.030934	11.704200	0.035813	-0.000274

Table G.13: Results from B.L.C. Apparatus Experiments

$$\theta = 62^\circ; C_{\mu c} = 0.22; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{v'^2}{u_{max}^2}}$	$\frac{\overline{u'v'}}{U_{max}^2}$
0.000000	0.937000	0.177364	0.550000	0.243985	-0.026248
0.253270	1.151000	0.202060	0.803270	0.256126	-0.029839
0.506560	1.294000	0.216625	1.056560	0.223288	-0.023859
0.759850	1.435000	0.213502	1.309850	0.211048	-0.021443
1.013160	1.539000	0.185756	1.563160	0.232780	-0.021711
1.266490	1.635000	0.160601	1.816490	0.212512	-0.014032
1.519820	1.706000	0.163045	2.069820	0.204140	-0.011453
1.773170	1.701000	0.161312	2.323170	0.205054	-0.009227
2.026530	1.709000	0.155681	2.576530	0.204734	-0.005109
2.279910	1.720000	0.150194	2.829910	0.200813	-0.000811
2.533300	1.706000	0.151902	3.083300	0.205577	0.000665
2.786700	1.688000	0.162351	3.336700	0.200356	0.001441
3.040110	1.662000	0.155188	3.590110	0.203974	0.004352
3.293540	1.639000	0.157046	3.843540	0.199889	0.007448
3.546980	1.617000	0.156116	4.096980	0.201124	0.008404
3.800440	1.592000	0.155819	4.350440	0.208202	0.009086
4.053900	1.552000	0.155089	4.603900	0.205300	0.009080
4.307390	1.507000	0.152852	4.857390	0.206471	0.010192
4.560880	1.475000	0.149452	5.110880	0.200921	0.009760
4.814390	1.439000	0.145460	5.364390	0.200113	0.009760
5.067910	1.407000	0.142434	5.617910	0.194077	0.009152
5.321450	1.381000	0.137940	5.871450	0.188568	0.009034
5.575000	1.354000	0.132254	6.125000	0.186000	0.008475
5.828560	1.329000	0.125343	6.378560	0.177368	0.007543
6.082140	1.302000	0.118355	6.632140	0.173242	0.006725
6.335730	1.292000	0.111279	6.885730	0.169909	0.006395
6.589340	1.274000	0.104815	7.139340	0.161028	0.005310
6.842960	1.266000	0.098331	7.392960	0.155662	0.004251
7.096600	1.252000	0.091858	7.646600	0.149271	0.003612
7.350310	1.247000	0.077240	7.900310	0.130655	0.002268
7.603910	1.247000	0.071927	8.153910	0.116751	-0.000843
7.857510	1.283000	0.068125	8.407510	0.099472	-0.000003
8.111280	1.305000	0.068125	8.661280	0.093951	-0.000496
8.365050	1.330000	0.062004	8.915050	0.082348	-0.001060
8.618820	1.349000	0.055631	9.168820	0.082877	-0.001349
8.872590	1.388000	0.049521	9.422590	0.071647	-0.001056
9.126360	1.411000	0.043225	9.676360	0.066088	-0.000891
9.380130	1.422000	0.038054	9.930130	0.057183	-0.000696
9.633900	1.418000	0.034441	10.183900	0.055721	-0.000690
9.887670	1.425000	0.032519	10.437670	0.050783	-0.000628
10.141440	1.432000	0.032245	10.691440	0.047438	-0.000500
10.395210	1.426000	0.031035	10.945210	0.047559	-0.000465
10.648980	1.429000	0.028776	11.198980	0.043842	-0.000363
10.902750	1.430000	0.029172	11.452750	0.044830	-0.000458
11.156520	1.431000	0.028729	11.706520	0.042711	-0.000355
11.410290	1.427000	0.028427	11.960290		

Table G.14: Results from B.L.C. Apparatus Experiments

$$\theta = 72^\circ; C_{\mu c} = 0.2; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{u'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	0.100000	0.042100	0.550000	0.182197	-0.003594
0.234390	0.155000	0.059001	0.784390	0.189091	-0.003393
0.468790	0.179000	0.064461	1.018790	0.202050	-0.002792
0.703200	0.207000	0.092493	1.253200	0.202670	-0.003243
0.937620	0.228000	0.106373	1.487620	0.212858	-0.004144
1.172050	0.267000	0.127146	1.722050	0.227722	-0.003447
1.406490	0.285000	0.135533	1.956490	0.237625	-0.002614
1.640930	0.340000	0.160446	2.190930	0.243858	-0.001840
1.875380	0.371000	0.176674	2.425380	0.260431	0.001619
2.109850	0.407000	0.189401	2.659850	0.280109	0.003797
2.344320	0.464000	0.219332	2.894320	0.276741	0.009043
2.578800	0.491000	0.229768	3.128800	0.294206	0.013089
2.813290	0.575000	0.259389	3.363290	0.298329	0.018548
3.047780	0.541000	0.246182	3.597780	0.318013	0.020023
3.282290	0.634000	0.280553	3.832290	0.311364	0.027892
3.516810	0.681000	0.290373	4.066810	0.290993	0.024224
3.751330	0.746000	0.322339	4.301330	0.273964	0.030956
3.985870	0.803000	0.347386	4.535870	0.245494	0.037257
4.220410	0.847000	0.341971	4.770410	0.236949	0.037506
4.454970	0.873000	0.347083	5.004970	0.225265	0.042124
4.689530	0.902000	0.336861	5.239530	0.222207	0.036893
4.924100	0.945000	0.353600	5.474100	0.185590	0.035634
5.158690	1.025000	0.347497	5.708690	0.157469	0.035514
5.393280	1.084000	0.387561	5.943280	0.095690	0.035531
5.627880	1.129000	0.368588	6.177880	0.086427	0.030974
5.862490	1.148000	0.339850	6.412490	0.103246	0.032375
6.097110	1.177000	0.350290	6.647110	0.100063	0.035714
6.331740	1.185000	0.320795	6.881740	0.087917	0.027942
6.566380	1.216000	0.290868	7.116380	0.178528	0.033612
7.035690	1.257000	0.308120	7.585690	0.096045	0.023172
7.505040	1.269000	0.268713	8.055040	0.158051	0.029868
7.974440	1.289000	0.245813	8.524440	0.138609	0.022405
8.443870	1.289000	0.225657	8.993870	0.163847	0.023038
8.913340	1.285000	0.207525	9.463340	0.178114	0.023230
9.382860	1.270000	0.216683	9.932860	0.138573	0.019631
9.852420	1.265000	0.197440	10.402420	0.164208	0.020433
10.322020	1.237000	0.183121	10.872020	0.162679	0.018884
10.791650	1.198000	0.174909	11.341650	0.145979	0.015733
11.261350	1.184000	0.173941	11.811350	0.132590	0.014849
11.731000	1.160000	0.165828	12.281000	0.126384	0.013288
12.200850	1.139000	0.146792	12.750850	0.126569	0.010988
12.670670	1.128000	0.143840	13.220670	0.101128	0.008501
13.140540	1.111000	0.127629	13.690540	0.107773	0.007545
13.610450	1.107000	0.120939	14.160450	0.090724	0.006367
14.080400	1.110000	0.114075	14.630400	0.089613	0.006044
14.550400	1.123000	0.105763	15.100400	0.076521	0.003552
15.020450	1.129000	0.107178	15.570450	0.058210	0.003148
15.490000	1.139000	0.106790	16.040000	0.050799	0.002713

Table G.15: Results from B.L.C. Apparatus Experiments

$$\theta = 72^\circ; C_{\mu c} = 0.22; C_{\mu w} = 0.11$$

Y (mm) (norm probe)	$\frac{u}{U_\infty}$	$\sqrt{\frac{u'^2}{u_{max}^2}}$	Y (mm) (sl probe)	$\sqrt{\frac{u'^2}{u_{max}^2}}$	$\frac{u'v'}{U_{max}^2}$
0.000000	0.145000	0.074250	0.550000	0.183261	-0.000582
0.234390	0.199000	0.087878	0.784390	0.190948	-0.000591
0.468790	0.247000	0.104482	1.018790	0.192543	-0.001004
0.703200	0.278000	0.116874	1.253200	0.207339	-0.000623
0.937620	0.322000	0.135463	1.487620	0.205052	-0.000423
1.172050	0.367000	0.157200	1.722050	0.220170	0.001290
1.406490	0.464000	0.187483	1.956490	0.220626	0.003364
1.640930	0.501000	0.209034	2.190930	0.209978	0.000281
1.875380	0.559000	0.231346	2.425380	0.239897	0.007267
2.109850	0.646000	0.255962	2.659850	0.218383	0.010618
2.344320	0.692000	0.265431	2.894320	0.226805	0.010890
2.578800	0.702000	0.274581	3.128800	0.268579	0.020671
2.813290	0.825000	0.333023	3.363290	0.174462	0.020535
3.047780	0.825000	0.308191	3.597780	0.233352	0.022832
3.282290	0.968000	0.360835	3.832290	0.027986	0.022384
3.516800	0.966000	0.325102	4.066800	0.206023	0.035502
3.751330	1.023000	0.340414	4.301330	0.180747	0.035440
3.985870	1.059000	0.322025	4.535870	0.133174	0.027293
4.220410	1.120000	0.344588	4.770410	0.155198	0.046108
4.454970	1.179000	0.328808	5.004970	0.066326	0.037153
4.689530	1.214000	0.337970	5.239530	0.149418	0.045653
4.924100	1.287000	0.300591	5.474100	0.148364	0.033861
5.158690	1.346000	0.291718	5.708690	0.049375	0.041014
5.393280	1.322000	0.296068	5.943280	0.144293	0.020899
5.627880	1.349000	0.292204	6.177880	0.128098	0.020489
5.862490	1.364000	0.285728	6.412490	0.120638	0.021256
6.097110	1.385000	0.260099	6.647110	0.178317	0.022864
6.331740	1.411000	0.238649	6.881740	0.173464	0.017061
6.566380	1.398000	0.231727	7.116380	0.156877	0.016541
7.035690	1.403000	0.215734	7.585690	0.176036	0.019180
7.505040	1.410000	0.198910	8.055040	0.192048	0.019609
7.974440	1.384000	0.199811	8.524440	0.193654	0.021183
8.443870	1.368000	0.183504	8.993870	0.194427	0.019286
8.913340	1.334000	0.175789	9.463340	0.197761	0.019398
9.382860	1.299000	0.170102	9.932860	0.191335	0.017841
9.852420	1.292000	0.165465	10.402420	0.175153	0.015827
10.322020	1.244000	0.157288	10.872020	0.172234	0.015232
10.791660	1.219000	0.147113	11.341660	0.166868	0.014079
11.261350	1.204000	0.138976	11.811350	0.143219	0.010226
11.731080	1.182000	0.128261	12.281080	0.139071	0.008964
12.200850	1.174000	0.116442	12.750850	0.123343	0.006796
12.670670	1.172000	0.105882	13.220670	0.113014	0.004836
13.140540	1.173000	0.097253	13.690540	0.099760	0.003908
13.610450	1.182000	0.092936	14.160450	0.090501	0.002840
14.080400	1.194000	0.088350	14.630400	0.092569	0.002575
14.550400	1.212000	0.082319	15.100400	0.083815	0.002219
15.020450	1.220000	0.079575	15.570450	0.063994	0.001356
15.490550	1.243000	0.071148	16.040550	0.061587	0.000897

END

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