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19, April, 1985

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James E Moore

**A Comparison of the Calculation
Schemes for Computing Weld Cooling**

by

James E. Moore
B. Eng. (Mechanical)

A thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the Degree of

Master of Engineering

Department of
Mechanical and Aeronautical Engineering
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April, 1985

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H. H. Sarin as an

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Department of Mechanical
and Aeronautical Engineering

Abstract

The calculation schemes available for computing weld cooling behaviour are examined in this thesis. These schemes range from the closed form analytical relationships of Rosenthal/Adams to a comprehensive finite element approach (FEA). In view of the importance of the 1500 to 100 °C cooling time $t_{15/1}$ (heat-affected zone cold cracking models) and the 800 to 500 °C cooling time $t_{8/5}$ (heat-affected zone hardness and strength models) in welding research and practice, these parameters are singled out for special consideration. Closed form analytical and FEA cooling times are compared with experimentally measured values for a bead-on-thick plate weld. In addition, a parametric FEA approach is used to examine the significance of the inherent assumptions associated with the closed form analytical relationships on weld cooling.

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List of Symbols

A_n	— Parameter of convenience $A_n = U_n^2 / (U_n + (gh/k)^2 + (2gh/k))$
A/D	— Analogue to Digital
a_1	— Length of the heat source in the front (a_1) and rear (a_2) of the welding arc (m)
b	— Half the width of the heat source (m)
C	— Heat capacity ($J/kg \cdot ^\circ C$)
$C\rho$	— Volumetric heat capacity ($J/m^3 \cdot ^\circ C$)
C_{ij}	— Thermal "capacitance" matrix
c	— Heat source depth (m)
D	— Governing differential equations
e	— Element
f	— View factor
$f(\Omega, t)$	— Forcing function
f_1	— Fraction of energy distributed in the front (f_1) and rear (f_2) portions of the welding arc (m)
FEM	— Finite element method
FEA	— Finite element approach
FZ	— Fusion zone
g	— Workpiece thickness (m)
g_c	— Saturation thickness (m)
h	— Surface heat transfer coefficient ($W/m^2 \cdot ^\circ C$)
H	— Enthalpy (J/m^3)
HAZ	— Heat-affected zone
H_f	— Heat of fusion (J/m^3)
H_t	— Heat of transformation (J/m^3)
I	— Welding current (Amperes)
[K]	— Generalised FEM stiffness matrix
K_{ij}	— Thermal "stiffness" matrix
K_0	— Bessel function of the second kind and zero order
k	— Thermal conductivity ($W/m \cdot ^\circ C$)
L	— Differential operator
MP	— Melting point temperature of the material ($^\circ C$)
N	— Basis functions
n	— Normal vector
p	— nodal point
Q	— Heat generation rate in the material (W/m^3)
$\dot{Q}$	— Heat input rate (W)

q	— Heat flow or thermal load (W/m^2)
$[R]$	— "Effective" load vector
R_i	— Thermal loading vector
r	— Radial distance from heat source (m)
RAM	— Random access memory
SMA	— Shielded metal arc weld
T	— Temperature ($^{\circ}C$)
$[T]$	— Vector of unknown temperatures ($^{\circ}C$)
T_a	— Ambient Temperature ($^{\circ}C$)
$\hat{T}$	— Exact temperature field ($^{\circ}C$)
t	— Time (s)
U_n	— Eigenvalues which satisfy $\tan U_n = 2khgU_n / (k^2U_n^2 - g^2h^2)$
V	— Welding voltage (Volts)
v	— Welding speed (m/s)
w/o	— Weight percent composition (%)
(x, y, z)	— Coordinate distances for fixed axes system (m)
(ξ, η, ζ)	— Coordinate distances for moving axes system (m)
α	— Thermal diffusivity (m^2/s)
Γ	— Body surface in (x, y, z) space
ϵ	— Emissivity
η	— Welding efficiency
Θ	— FEM solution scheme
ρ	— Density (kg/m^3)
σ	— Stefan - Boltzmann's constant ($5.669 \times 10^{-8} W/m^2 K^4$)
$\lambda (= 1/2\alpha)$	— Parameter of convenience (s/m^2)
ξ	— Distance in front of heat source (m)
ξ	— Non-dimensional parameter ($t/\Delta t$)
$t_{8/5}$	— Cooling times from 800 to 500 $^{\circ}C$ (s)
$t_{15/1}$	— Cooling times from 1500 to 100 $^{\circ}C$ (s)
Ω	— Solution domain in (x, y, z) space
ω	— Weighting function
2D	— Two-dimensional heat flow
2.5D	— Two and one-half dimensional heat flow
3D	— Three-dimensional heat flow

Chapter 1

Introduction

Fusion welding is extensively used for joining materials in structural applications because of many advantages over alternative joining procedures such as mechanical fasteners. A welded joint may be thought of as a continuous extension of the base metal. Therefore, load transfer along and across the welded joint is uniform whereas with mechanical fasteners the transfer is localized. In addition, the welding option provides greater design flexibility and freedom in practice.

If a weldment is viewed by designers as an extension of the base material, then it must be defect-free and have mechanical properties equal to or exceeding the base material. In practice, welds are often located at discontinuities where stresses are biaxial or triaxial and where stress levels are often more intense than in the base metal. Therefore, in a general sense, better mechanical properties are required in the weldment than in the base material. Fortunately, the strength of defect free steel welds usually exceeds that of the base material. However, defects such as cracks are often encountered and weld toughness is usually less than that of the base material. The appearance of weld cracks and low toughness are of considerable concern to designers and have been a subject of intense research for many years.

The mechanical properties of a welded joint are in part influenced by the thermal cycle. The highly localized heat input of the welding arc causes severe thermal gradients which, in turn, lead to microstructural changes in the heat affected zone (HAZ) of the base material. The mechanical properties to a large extent are determined by the microstructural changes. For example, on rapid cooling, hardened microstructures (i.e. martensite, bainite) appear as a result of austenite decomposition. Such structures have high strength but low toughness. At the same time, residual stresses form as a result of shrinkage (thermal contraction). The shrinkage often exceeds the yield strain and causes plastic deformation to occur. However, martensite and bainite have relatively limited plastic capacity and cracks form when the ductility of the region is exhausted. In addition, the whole process is aggravated by the fact that hydrogen is absorbed during the welding process. The absorption of hydrogen increases the cracking susceptibility of the

HAZ although the mechanism is not totally understood. This is known as hydrogen induced "cold" cracking (sometimes under other circumstances termed "delayed" cracking because the cracks often form some 48 to 72 hours after the weld has been deposited)

In view of the importance of HAZ cold cracking and mechanical properties, there has been extensive research in this area - particularly in Japan. Based on many years of experimental research, engineers at the Nippon Steel Laboratories have recently proposed semi-empirical methods of predicting heat-affected zone hardness levels and preheat thresholds for preventing cracking [1-4]. The introduction of these methods to the welding industry has generally been recognized as a very significant step towards ensuring high quality welds in practice.

In both the hardness and preheat model the weld cooling behaviour must be calculated or otherwise determined. In principle, the HAZ undergoes a different thermal history from that of the fusion zone (FZ). However, Adams [5] and others, have shown that the thermal cycle at the interface of the HAZ and the FZ, where cracking and hardening are most severe, is not significantly different from that at the centerline. Hence, in welding research and practice the centerline thermal history is generally viewed as characteristic of the weld as a whole. As a matter of convenience it is customary (in Japan and elsewhere [6, 7]) to utilize cooling parameters that have been extracted from the centerline thermal cycle. The centerline cooling time from 800 to 500 °C — $t_{8/5}$ is used in the hardness and strength models [6, 8], while the cooling time from 1500 to 100 °C — $t_{15/1}$ has evolved as the most relevant cooling parameter for the cold cracking models [1, 2] (see Figure 1). The 800 to 500 °C temperature range is said to be appropriate for hardness models because this is the region where austenite decomposes to hardened bainite or martensite. On the other hand, the 1500 to 100 °C cooling time is used in the preheat model because this is the cooling period when hydrogen escapes from the weldment. So important is the role of hydrogen in the formation of cold cracks that it alone is basis for determining the preheat threshold.

Both models are relatively quantitative except that it has not yet been established how $t_{8/5}$ and $t_{15/1}$ should be calculated. The Japanese have proposed nomogram methods for determining the cooling parameters. However, these are awkward in practice and not adaptable to routine computer or other calculation procedures. It is, therefore, apparent that a reliable and convenient scheme for calculating these thermal parameters is of considerable importance and is an appropriate research topic.

There are basically two approaches for calculating weld cooling parameters — the conventional closed form analytical approach and the more recent numerical schemes (i.e.

Figure 1 (a) Schematic representation of a centerline ($y = 0$) thermal history to define $\tau_{8/5}$ and $\tau_{15/1}$. (b) Schematic representation of a bead-on-plate weld used for computing the transient temperature fields $T(\xi, y, z, t)$, where (x, y, z) is a fixed axes system, (ξ, y, z) is an axes system which moves with the heat source, MP denotes the melting point isotherm, FZ the fusion zone, and HAZ the heat-affected zone.

the finite difference and finite element methods). The closed form analytical approach is based on the theory of moving heat sources originated by Rosenthal [9] and Rykalin [10]. Developments since that time include the Adams [5], and Graville [11] modifications and an alternative formulation by Tanaka [12]. This list could be extended but is considered relatively complete for computing the $\tau_{8/5}$ and $\tau_{15/1}$ thermal parameters.

Among the numerical schemes available, the finite element approach (FEA) is the most flexible and comprehensive. This method, therefore, was selected for study in this investigation. A multi-purpose finite element program, especially tailored for weld analysis, has been assembled by Goldak *et al* [13]. Details of the finite element equation formulation and the heat source model are presented in Chapters 5 and 6. Cooling times from the closed form analytical relationships (five in all cited) are compared to the FEA computed values and, in turn, to experimental measurements in this investigation.

Over the past decade, there has been considerable interest in numerical techniques for analysing both the thermal and mechanical changes that occur during welding [14 - 20]. The numerical approach eliminates most of the underlying assumptions associated with the analytical solutions (that tend to limit their usefulness). In particular, the FEA offers considerable promise for predicting the cooling parameters. At present, the implementation of an FEA technique requires considerable time and expense. However, with rapidly decreasing computer costs [21] and the availability of automatic mesh generation algorithms [22, 23] this situation is changing. In the near future, finite element weld analysis may well be used on a routine basis in many situations.

Nevertheless, the closed form solutions provide a useful framework for rationalizing weld cooling behaviour. At the same time, where only a calculator or perhaps a personal computer is available, cooling times generated from the closed form relationships can be quite useful. However, before accepting these calculated cooling times with confidence, it is necessary to have considerable experimental experience and a reasonable understanding of the significance of assumptions imposed on the solutions. Specifically, the assumptions are as follows:

- a) the heat source is regarded as a point of infinite temperature moving along the surface of the workpiece (see Figure 1);
- b) fixed thermal properties must be used even though it is well known that they are quite temperature dependent;
- c) convective and radiative heat transfer are usually neglected;
- d) weld reinforcement is assumed insignificant;
- e) the workpiece is large enough in the $x - y$ directions that boundary planes do not

effect the cooling behaviour

None of these assumptions is necessary in the finite element analysis used in this investigation. Yet all assumptions can be imposed on the FEA and then extracted one by one to test their significance. It is not possible to test the point source assumptions in any other way. Experimentally, the assumptions can be only evaluated collectively. On the other hand, in a FEA study it is possible to separately evaluate the significance of each assumption independently. Such a study is found in this thesis.

The measured cooling parameters $\tau_{3/5}$ and $\tau_{15/1}$ of a bead on thick plate weld are compared with the FEA computed and closed form analytical values. The results of this comparison are used to evaluate the calculation schemes available for weld cooling, at least for the bead-on-thick plate situation of this investigation. Then, the assumptions described above are imposed on the FEA solution, are removed one by one. In this way, the significance of each factor that affects the weld cooling behaviour is evaluated.

Chapter 2

Experimental Work --

In research and practice, the bead-on-thick plate configuration (Figure 1) is a basic weld test. It is almost universally used for examining cooling and HAZ hardening response, although more severe configurations (e.g. Tee and Y groove tests) are also used to measure the HAZ cracking threshold of a steel.

The cooling parameters (i.e. $\tau_{8/5}$ and $\tau_{15/1}$) of a bead-on-thick plate test were selected for study in this investigation. The workpiece consisted of low carbon steel (0.18 w/o C, 0.89 w/o Mn, 0.29 w/o Si) plate $500 \times 500 \times 38$ mm in size. The shielded metal arc (SMA) process was used to deposit a central weld along the surface of the workpiece (voltage $V = 30$ volts, current $I = 265$ amps and welding speed $v = 3.8$ mm/s — Figure 1). The test conditions were selected to conform with the experimental work of Bennett [24] as a means of examining the reproducibility of cooling time measurements in different laboratories. The workpiece was suspended, well away from the work table, by placing insulating supports under the corners to ensure free convection at the surfaces. In this way, almost the entire workpiece was exposed to ambient laboratory atmospheric conditions ($T_a = 20.5$ °C). This precaution was probably unimportant for $\tau_{8/5}$ measurements but omission might well have affected the $\tau_{15/1}$ parameter where convective heat transfer becomes significant.

A Pt-Pt /13% Rh thermocouple was "harpooned" into the weld pool to measure the transient centerline thermal cycle. This procedure is an established technique and is widely used in practice [8, 24]. An Omega-CJ cold-junction compensating device was used to ensure a true temperature signal and the response was recorded on an analogue x-y recorder and with a digital data acquisition system.

The microprocessor data acquisition system consisted of an analogue-digital (A/D) unit and an Apple II computer accompanied by appropriate software [25]. The software was assembled to control the sampling rate of the digital output from the A/D converter and for processing this data into meaningful temperature readings. A relatively rapid sampling rate resulted in an apparently continuous digitized thermal cycle (Appendix B). Two identical welding experiments were carried out to measure the complete centerline thermal

cycle. The results from these duplicate runs agreed to within a few percent of each other (Figure 2, Table 1). Furthermore, the average measured cooling times extracted from the thermal cycle ($\tau_{8/5} = 8.1 \text{ s}$, $\tau_{15/1} = 100 \text{ s}$) agreed reasonably well with the measurements reported by Bennett [24] ($\tau_{8/5} = 7.0 \text{ s}$, $\tau_{15/1} = 100 \text{ s}$) — an encouraging confirmation of Bennett's findings. The reproducibility of the results (cooling times) and the agreement with Bennett's measurements suggested that the experimental information was reliable. The closed form and FE analysis (cooling time calculations) of this experimental situation are contained in Chapters 4–6.

Table — 1

**Experimental Cooling Time Measurements
(Bead-on-thick Plate Welds)**

Measurement	$\tau_{8/5}$ (sec.)	$\tau_{15/1}$ (sec.)
1	7.9	96
2	8.4	105
Bennett, A. P. [24]	7	100

Voltage = 30 volts, Current = 265 amps, Welding speed = 3.81 mm/s, $T_o = 20.5^\circ\text{C}$, Plate Size 500 × 500 × 38 mm

Figure 2

Experimental centerline cooling curve Welding parameters voltage $V = 30 \text{ volts}$, current $I = 265 \text{ amps}$, welding speed $v = 3.81 \text{ mm/s}$, initial plate temperature $T_0 = 20.5^\circ\text{C}$, plate size $500 \times 500 \times 38 \text{ mm}$

Chapter 3

Heat Flow Theory

The principal mode of heat transfer from the welding arc is thermal conduction. The fundamental relationship which governs heat transfer by conduction originates by satisfying the conservation of energy (i.e. the net energy conducted into the body + the heat generated within the body = the change in internal energy). This can be written in differential form giving Fourier's relationship for three dimensional heat flow:

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) + Q - C\rho \frac{\partial T}{\partial t} = 0 \quad (3.1)$$

where $T(x, y, z, t)$ is the temperature at the point (x, y, z) at time t ($^{\circ}C$),
 $Q(x, y, z, t)$ is the heat generation rate in the material (W/m^3),
 k is the thermal conductivity ($W/m \cdot ^{\circ}C$), which is a measure of the energy transfer rate through the medium,
 $C\rho$ is the specific heat ($J/m^3 \cdot ^{\circ}C$), which is a measure of the energy that is absorbed moving through the medium,
and ρ is the density of the material (kg/m^3).

In addition to heat transfer by conduction, some of the heat is transferred by convection and radiation to the surroundings. In some cases, by selecting appropriate boundary conditions when solving Equation (3.1), these additional modes of heat transfer can be taken into account. However, their inclusion considerably increases the complexity (and difficulty) of obtaining a useful solution.

For the low carbon steel used in this investigation, isotropic thermal properties (i.e. independent of direction) are assumed. Equation (3.1) can, therefore, be simplified to

$$\nabla^2 T(x, y, z, t) + Q(x, y, z, t) = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (3.2)$$

where the thermal diffusivity of the material is defined as:

$$\alpha = \frac{k}{C\rho} \quad (m^2/s) \quad (3.3)$$

It is apparent from Equation (3.2) and (3.3) that, to determine the transient temperature field during welding, the thermal properties (conductivity (k), specific heat ($C\rho$), and

diffusivity (α) of the workpiece must be known. However, they vary considerably over the temperature range of a typical weld [26] (see Figures 3 and 4). Unfortunately, published data for the thermal properties of low carbon carbon steels in the high temperature region are not available beyond 1200 °C. Therefore, the existing data are usually extrapolated into the high temperature region (i.e. $T > 1200$ °C). This method implies that the specific heat and conductivity values are the same above and below the melting point temperature ($T_{mp} = 1480$ °C) which is clearly unrealistic. Some accommodation of a more realistic model is possible with the FEA (see Chapters 5 and 6). In addition to the thermal properties, the density of the material is required. Since the density of steel is not a strong function of temperature (i.e. varies less than 7% between room and melting point temperature) a mean value of 7800 kg/m³ is used for all subsequent analyses.

It is also apparent from Equation (3.1) that the rate of energy transfer to the workpiece from the heat source is needed. The rate of energy transfer or heat input rate is calculated from the welding parameters according to the relationship:

$$\dot{Q} = \eta VI \quad (Watts) \quad (3.4)$$

where V is the voltage, I is the current, and η the process efficiency. The process efficiency is a measure of the heat loss in the vicinity of the welding arc, which is primarily due to intense radiation. It is process dependent and measurements suggest that a value of $\eta = 0.8$ should be applied to the SMA process used in this investigation [27]. The efficiency is clearly one of the most important parameters in the analysis and at the same time there is less than complete confidence in the measured values available in the literature. In fact, there is a substantial need for more research in this area. For further discussion of the significance of the heat input see reference [27]. Applying a value of $\eta = 0.80$, a heat input rate of $\dot{Q} = 6360$ Watts is calculated for the experimental conditions (Chapter 2) of this investigation.

Closed form solutions to the Fourier equation are provided in Chapters 4 together with the $\tau_{8/5}$ and $\tau_{15/1}$ cooling time calculations. The FEA solution to this problem is described in Chapter 5. Experimental cooling time values (Chapter 2) are compared with closed form analytical (Chapter 4) and FEA (Chapter 6) values.

Figure 3 Variation of specific heat capacity, C_p , with temperature for a low carbon steel [26].

Figure 4 Variation of thermal conductivity, k , and thermal diffusivity, α , with temperature for a low carbon steel [26].

Chapter 4

Closed Form Analytical Centerline Cooling Relationships

Much of the analytical heat transfer theory for welding is based on work originally presented by Rosenthal [9] and by Rykalin [10]. A basic bead-on-plate welding situation such as that shown in Figure 1 can be used to describe the Rosenthal/Rykalin solution to the Fourier Equation (3.2). The heat flow behaviour for a given welding situation is first characterized as either three-dimensional (3D) or two-dimensional (2D) (see Figure 5). If, for the majority of the weld thermal cycle, there is significant heat flow in the through-thickness direction, the heat flow is said to be 3D. This is the case for relatively shallow penetration welds (e.g. conventional arc processes) on "thick" plate. Alternatively, if the heat flow is primarily parallel to the surface of the workpiece (i.e. in the $x - y$ plane) the process is considered 2D. 2D heat flow applies to both deep penetration welds (i.e. electron beam, laser, and plasma arc) where the depth-to-width ratio exceeds ten [7], and in conventional arc processes on thin plate.

The heat source is assumed to be either a line heat source (2D) or a point heat source (3D) travelling at constant speed along a semi-infinite plate. The solutions to the Fourier equation for these situations is based on the concept of a quasi-stationary temperature distribution. The term "quasi-stationary" means that the temperature field does not change with respect to the moving heat source. Two coordinate systems are necessary, one fixed (x, y, z) and the other (ξ, y, z) moving along with the heat source as shown in Figure 1. This moving system is related to the fixed coordinate system by the transformation equation: $\xi = x - vt$.

Where the heat flow is 3D the temperature field is given by:

$$T(r, \xi) = T_0 + \frac{\dot{Q}}{2\pi kr} \exp\left(\frac{-v(\xi + r)}{2\alpha}\right) \quad (4.1)$$

where

$$r = \sqrt{\xi^2 + y^2 + z^2} \quad \text{and} \quad \xi = x - vt$$

T_0 — is the initial temperature of the workpiece ($^{\circ}\text{C}$)

r — is the distance measured from the moving heat source.

Figure 5

Two-dimensional and three-dimensional welding situations: (a) "thin" plate workpiece (2D) and (b) "thick" plate deep penetration weld (2D) and a "thick" plate shallow penetration weld (3D).

For relatively "thin" plates or deep penetration welds where the temperature gradients in the thickness direction are negligible, the 2D quasi-stationary temperature distribution is given by:

$$T(r, \xi) = T_o + \frac{\dot{Q}}{2\pi k g} \exp\left(\frac{-v\xi}{2a}\right) K_o\left(\frac{vr}{2a}\right) \quad (4.2)$$

where

K_o --- is a modified Bessel function of the second kind and zero order and,

g --- is the plate thickness (m).

It is apparent from Equations (4.1) and (4.2) that, even though the thermal properties are a strong function of temperature, fixed values must be selected when applying the closed form analytical relationships. The selection of "fixed" values has been the subject of some debate. In this investigation the thermal conductivity value of $k = 25 \text{ W/m}^\circ\text{C}$ and the volumetric specific heat of $C\rho = 6 \times 10^6 \text{ J/m}^3 \cdot ^\circ\text{C}$ suggested by Tanaka [12] has been selected. Many investigators find the application of the Tanaka values is necessary if calculated cooling times are to match measured values [1,2, 28]. In such cases, k and $C\rho$ are not used as material properties but as parameters to fit the calculated cooling times to the experimental measurements.

The 800 to 500 $^\circ\text{C}$ cooling time can be extracted from either Equation (4.1) or (4.2) to obtain the formulations proposed by Adams [5]. A 3D time temperature relationship is generated on setting $y = 0$ and $z = 0$ in Equation (4.1). It can then be rearranged for cooling times as follows:

$$r_{3D} = \frac{1}{2\pi k} \left(\frac{\dot{Q}}{v} \right) \left[\frac{1}{(T_f - T_o)} - \frac{1}{(T_i - T_o)} \right] \quad (4.3)$$

where $T_i = 800^\circ\text{C}$, $T_f = 500^\circ\text{C}$ for calculating $r_{3/3}$. By applying the input welding parameters (Chapter 2) used in this investigation, a cooling time of $r_{3/3} = 8.5$ seconds is calculated. This calculation assumes that heat flows from the source in the ξ , y , and z directions and that the weld temperature drops below 500 $^\circ\text{C}$ (for $r_{3/3}$) before heat reaching the lower surface of the workpiece (z direction) begins to influence the cooling time. The heat flow is considered initially to be three-dimensional. 3D heat flow is a good assumption for calculating the 800 to 500 $^\circ\text{C}$ cooling time of the welding situation in this investigation (justification below) but not a good assumption for calculating $r_{15/1}$. As the heat transfer process proceeds, heat eventually reflects from the lower surface and decreases the gradients in the through-thickness direction. Heat then flows primarily in the plane of the workpiece ($x - y$ direction). The heat flow at this point is two-dimensional. Under these conditions the following relationship can be obtained from the Adams' cooling rate

formulations [5]. This is accomplished by simplifying Equation (4.2) to the centerline (i.e. $y = 0$) cooling relationship and then arranging it for cooling times as follows:

$$\tau_{2D} = \frac{1}{4\pi k C \rho} \left(\frac{Q}{v g} \right)^2 \left[\frac{1}{(T_f - T_o)^2} - \frac{1}{(T_i - T_o)^2} \right] \quad (4.4)$$

where $T_i = 1500^\circ\text{C}$, $T_f = 100^\circ\text{C}$ for calculating $\tau_{15/1}$. This equation yields a calculated cooling time for $\tau_{15/1}$ of 163 seconds. In actual fact the 2D relationship is a considerable approximation because the heat flow always begins 3D and then degenerates to a 2D morphology while Equation (4.4) assumes that the process is entirely 2D. The change in the heat flow morphology is clearly shown in the FEA generated cross-sectional isotherms shown in Figure 6.

Adams suggested the concept of a saturation thickness for distinguishing 2D and 3D situations. For experiments where the workpiece thickness is less than the saturation thickness, the 2D heat flow relationship (Equation 4.4) is the best approximation while the 3D relationship (Equation 4.3) should be used for workpiece thicknesses in excess of the saturation value. The original Adams [5] saturation thickness rearranged for cooling times is as follows:

$$g_c = \sqrt{\left(\frac{Q}{2v C \rho} \right) \left[\frac{1}{(T_f - T_o)} + \frac{1}{(T_i - T_o)} \right]} \quad (4.5)$$

Substituting the welding parameters (Chapter 2) used in this investigation into this relationship, a saturation thickness for $\tau_{8/5}$ of $g_c = 22 \text{ mm}$ and for $\tau_{15/1}$ a value of $g_c = 43 \text{ mm}$ is obtained. This clearly shows that Equation (4.3) should be used for $\tau_{8/5}$ while Equation (4.4) used for $\tau_{15/1}$. These relationships are quite useful in practice because they can be easily handled with a calculator or pocket computer. However, they are approximations especially where there is mixed 2D/3D (i.e. so called 2.5D) heat flow, although a graphical correction method has been suggested by Jhaveri *et al* [29]. At the same time all the assumptions listed previously (Chapter 1) that apply to the closed form solutions are imposed on these relationships.

A more general series solution to the Fourier equation was proposed in the original Rosenthal theory that applies to all heat flow regions (2D, 2.5D, 3D) [9]:

$$T = T_o + \frac{Q}{2\pi k} \left\{ \frac{1}{vt} + 2 \sum_{n=1}^{\infty} \frac{1}{r_n} \exp(\lambda v(vt - r_n)) \right\} \quad (4.6)$$

where

$$r_n = \sqrt{\xi^2 + (2ng)^2}$$

It should be noted that this is the centerline cooling modification of the original relationship and is, therefore, stated in a slightly different form than in Rosenthal's presentation

Figure 6

FEA generated isotherms to show the transition from 3D to 2D heat flow. Workpiece cross section 38 seconds after the arc has passed. (Welding parameters: $V = 30 \text{ volts}$, $I = 265 \text{ amps}$, $v = 3.81 \text{ mm/s}$, $g = 33 \text{ mm}$, and $T_0 = 20.5^\circ \text{C}$).

The solution degenerates to a 3D solution for large thicknesses and to a 2D solution for thin workpieces. Both $\tau_{8/5}$ and $\tau_{15/1}$ can be calculated from this one relationship and there is no need to use the saturation thickness concept. Using the welding parameters of this investigation (Chapter 2), calculated cooling times for $\tau_{8/5}$ of 8.5 seconds and for $\tau_{15/1}$ of 163 seconds are obtained. It is apparent that an iterative-technique must be used to obtain the cooling parameters since time cannot be isolated in terms of temperature. A small pocket or personal computer is, therefore, necessary ($\approx 2K$ bytes RAM with efficient programming techniques) and for this reason it has not received as much attention as the Adams relationship until recently. On the other hand, all the closed form analytical assumptions (Chapter 1) are imposed on the solution.

To some extent, the assumptions that convection and radiation effects are small are eased by including a mean surface heat transfer coefficient in the solution. A 2D relationship has been proposed by Graville [11] that can be used for calculating $\tau_{15/1}$:

$$T = T_0 + \frac{\dot{Q}}{2\pi k g v} \sqrt{\frac{\pi}{2\lambda t}} \exp\left(\frac{-ht}{\lambda k g}\right) \quad (4.7)$$

A cooling time of $\tau_{15/1}$ of 156 seconds is calculated with this relationship for the welding situations described in Chapter 2. This calculation can be performed since most $\tau_{15/1}$ calculations in practice can be approximated by 2D heat flow and this is a widely used relationship for this purpose. On the other hand a mean convection - radiation coefficient must be used, although it is well known to be temperature sensitive. For longer time - low temperature cooling (i.e. $\tau_{8/5}$), a convective coefficient of $h = 16.7 W/m^2 \cdot ^\circ C$ suggested by Yurioka [2] and Graville [11] has been used. This value corresponds to the value of the coefficient determined from isothermal plate cooling experiments at about $300^\circ C$. Apart from this, all other assumptions listed in Chapter 1 are imposed on the solution.

In an approach quite different from that of Rosenthal, Tanaka [12], published the following relationship in a Japanese paper:

$$T = T_0 + \frac{\dot{Q}}{\pi k g} \exp\left(\frac{v^2 t}{2\alpha}\right) \sum_{n=0}^{\infty} A_n K_0\left[\frac{vt}{g} \sqrt{U_n^2 + \left(\frac{vg}{2\alpha}\right)^2}\right] \quad (4.8)$$

where U_n is an eigenvalue which satisfies

$$\tan U_n = 2khgU_n / (k^2 U_n^2 - g^2 h^2)$$

and

$$A_n = U_n^2 / (U_n^2 + (gh/k)^2 + (2gh/k))$$

At present, no publication in English has outlined the limitations or presented a derivation of Tanaka's relationship. Therefore, the nature of the assumptions and approximations

were not known until now. To confirm the final form given in Equation (4.8) a comprehensive derivation (by the author) is included in Appendix A. From this derivation, all the assumptions listed previously are found to be imposed on this relationship except that it contains a mean surface heat transfer coefficient. It is, however, a seemingly more versatile relationship than the Rosenthal formulation and, in particular, can be modified to calculate cooling times ($\tau_{8/5}$, $\tau_{15/1}$) in the presence of cooling uniformly preheated plates. It is, however, mentioned to emphasize the need for including the Tanaka approach in this investigation. The preheat modification is beyond the scope of the present study although it is included in this author's derivation for completeness in Appendix A.

For the welding parameters selected for this study, calculated cooling times of $\tau_{15/1} = 7.9$ seconds and $\tau_{8/5} = 153$ seconds are obtained from the Tanaka relationship. For longer time — low temperature cooling (i.e. $\tau_{15/1}$), a convective coefficient of $h = 16.7 \text{ W/m}^2 \cdot \text{C}$ has been applied. At high temperatures (i.e. $\tau_{8/5}$), a convective coefficient of $h = 84 \text{ W/m}^2 \cdot \text{C}$ suggested by Yurioka [2] has been used (which is based on experimental results). It is apparent that an iterative solution together with an eigenvalue iterative evaluation and series solution for evaluating the Bessel function requires the use of a computer. With reasonably efficient programming techniques, about 5K bytes RAM are required which is possible on many small portable pocket, briefcase or personal computers.

The centerline relationships presented in this chapter have been implemented in a Fortran language computer program (Appendix C). The program is now available for calculating the closed form analytical cooling parameters $\tau_{8/5}$ and $\tau_{15/1}$ for any general welding situation. It was used in this investigation for comparison with the experimental and FEA values. The results of the closed form calculations and measured values are summarized in Table 2.

Table — 2

**A Comparison of Experimental and Closed
Form Analytical Cooling Times**

Method	Thermal Conductivity $W/m \cdot C$	Surface Heat Transfer Coeff. $W/m^2 \cdot C$	$\tau_{8/5}$ (s)	$\tau_{15/1}$ (s)
Experimental ⁽¹⁾ (1)	—	—	7.9	96
(2)	—	—	8.4	105
(Av)	—	—	8.1	100
Experimental ⁽¹⁾ (Bennett, A.P. [24])	—	—	7	100
Adams (3D) ⁽²⁾	25	—	8.5	—
(2D) ⁽³⁾	25	—	—	163
Graville (2D) ⁽⁴⁾	25	16.7	—	156
Rosenthal (2.5D) ⁽⁵⁾	25	—	8.5	163
Tanaka (2.5D) ⁽⁶⁾	25	83.5	7.9	—
(2.5D) ⁽⁶⁾	25	16.7	—	153

Notes

- (1) Experimental Conditions: $V = 30$ volts (4) Equation (4.7)
 $I = 265$ amps, $g = 38$ mm, $v = 3.81$ mm/s (5) Equation (4.6)
- (2) Equation (4.3) (6) Equation (4.8)
- (3) Equation (4.4)

Chapter 5

Finite Element Analysis

5.1 Finite Element Equation Formulation

In recent years, the numerical techniques have received considerable attention for analysing the heat flow behaviour during welding. It is an important area of development because of the limited usefulness of the closed form analytical solutions to accurately model the thermal history. In particular, the finite element method (FEM) can eliminate most of the assumptions described previously (Chapter 1) that have to be imposed on the closed form solutions. Hence, the nonlinear transient heat flow behaviour of the welding process can be modelled more realistically. The FEM for nonlinear transient heat flow is based on the conservation of energy — i.e. the solution to Fourier's heat conduction equation. It is formulated according to the following presentation.

5.1.1 Weighted Residual Method

The finite element formulation governing the conductive heat flow in a medium can be obtained using the weighted residual method. The welding heat flow problem is governed by Fourier's differential heat conduction Equation (3.1) and is of the following form:

$$L[\hat{T}(\Omega, t)] - f(\Omega, t) = 0 \quad (5.1)$$

Where L is a differential operator, $f(\Omega, t)$ represents the forcing function, and $\hat{T}(\Omega, t)$ is the solution to the temperature field in the domain Ω at time t . The weighted residual method uses an approximate solution $T(\Omega, t)$ to the exact solution $\hat{T}(\Omega, t)$. For example, the exact temperature field can be approximated by a series of polynomials. Substituting an approximate solution for the unknown exact solution, Equation (5.1) becomes:

$$L[T(\Omega, t)] - f(\Omega, t) = \epsilon \quad (5.2)$$

Since $T(\Omega, t)$ is not an exact solution, Equation (5.2) will not be equal to zero but to some residual value ϵ . The goal is to determine $T(\Omega, t)$ so that the weighted residual over the entire domain is zero [30 - 32]. Therefore, integrating the weighted residual given in

Equation (5.2) over the domain gives:

$$\int_{\Omega} \{L[T(\Omega, t)] - f(\Omega, t)\} \omega_i d\Omega = 0 \quad (5.3)$$

where ω_i is the weighting function. The domain Ω can be discretized into "elements" or subdomains since Equation (5.3) holds for the entire domain.

For the FEM a set of piecewise polynomial functions called interpolation "basis" or "shape" functions is used to approximate the exact solution $T(\Omega, t)$ and are of the form:

$$T(\Omega, t) = N_1(\Omega)T_1(t) + N_2(\Omega)T_2(t) + \dots + N_p(\Omega)T_p(t) \quad (5.4)$$

or

$$T(\Omega, t) = \sum_{i=1}^p N_i T_i$$

where T_i are the unknowns representing the temperatures located at the "nodal" points, p , for each element in the domain, and N_i are the element level basis functions. In this way, the equation to be minimized can be written as:

$$\sum_{e=1}^n \int_{\Omega_e} \{L[\sum_{i=1}^p N_i T_i] - f(\Omega, t)\} \omega_i d\Omega_e = 0 \quad (5.5)$$

where the subscript, e , refers to each element in the domain. To take the weighting function ω_i as the element level basis functions N_i , constitutes the well known "Galerkin's" weighted residual method. Adopting this method, Equation (5.5) can be rewritten in final form as:

$$\sum_{e=1}^n \int_{\Omega_e} \{L[\sum_{i=1}^p N_i T_i] - f(\Omega, t)\} N_i d\Omega_e = 0 \quad (5.6)$$

5.1.2 Equation Formulation

From Galerkin's weighted residual method (i.e. using the basis functions for the weighting functions) the following equation must be satisfied:

$$\int_{\Omega} N^T D(T) d\Omega = 0 \quad (5.7)$$

where $D(T)$ represents the governing differential heat flow equation. Applying Galerkin's weighted residual method to Fourier's heat flow Equation (3.1) gives:

$$\sum_{e=1}^n \sum_{i=1}^p \left(\int_{\Omega_e} N_i^T \left\{ \frac{\partial}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) \right\} d\Omega_e + \int_{\Omega_e} N_i^T \left(Q - C \rho \frac{\partial T}{\partial t} \right) d\Omega_e \right) = 0 \quad (5.8)$$

The integral or "weak" form of Equation (5.8) can be obtained by applying Green's identity:

$$\sum_{\epsilon=1}^n \sum_{i=1}^p \left(- \int_{\Omega_{\epsilon}} \frac{\partial N_i^T}{\partial x} \left(k_x \frac{\partial T}{\partial x} \right) + \frac{\partial N_i^T}{\partial y} \left(k_y \frac{\partial T}{\partial y} \right) + \frac{\partial N_i^T}{\partial z} \left(k_z \frac{\partial T}{\partial z} \right) d\Omega_{\epsilon} \right. \\ \left. + \int_{\Gamma_{\epsilon}} N_i^T \left(k_n \frac{\partial T}{\partial n} \right) d\Gamma_{\epsilon} + \int_{\Omega_{\epsilon}} N_i^T \left(Q - C \rho \frac{\partial T}{\partial t} \right) d\Omega_{\epsilon} \right) = 0 \quad (5.9)$$

5.1.3 Application of the Boundary Conditions

The surface heat losses are due to a combination of convective and radiative heat transfer. The heat transfer by convection at the boundary surface to the surroundings at temperature T_0 is given by Newton's law of cooling [33]:

$$q_c = -k_n \frac{\partial T}{\partial n} = h(T - T_0) \quad (5.10)$$

For heat exchange by radiation from the boundary to the surroundings, the Stefan - Boltzmann equation [34] is used:

$$q_r = \sigma \epsilon f (T^4 - T_0^4) \quad (5.11)$$

where σ is Stefan - Boltzmann constant, ϵ is the total hemispherical emissivity $0 < \epsilon \leq 1$, f is the view or shape factor (for this analysis a non-participating medium was assumed, i.e. $f = 1$). For radiative heat transfer a linearized form of the Stefan - Boltzmann equation can be used to obtain an "effective" convective heat transfer coefficient as follows:

$$q_r = [\sigma \epsilon (T^2 + T_0^2)(T + T_0)](T - T_0) \\ = h_r(T - T_0) \quad (5.12)$$

Therefore, combining the convective, radiative, and surface thermal loading:

$$q = q_s + (h_c + h_r)(T - T_0) \quad (5.13)$$

and applying the piecewise polynomial basis functions:

$$T(\Omega, t) = \sum_{j=1}^p N_j(\Omega) T_j(t) \quad (5.14)$$

and

$$T'(\Omega, t) = \sum_{j=1}^p N_j'(\Omega) T_j(t) \quad (5.15)$$

to Equation (5.9) yields the following:

$$\sum_{\epsilon=1}^n \sum_{i=1}^p \sum_{j=1}^p \left\{ \int_{\Omega_{\epsilon}} \left\{ \frac{\partial N_i^T}{\partial x} \left(k_x \frac{\partial N_j}{\partial x} \right) + \frac{\partial N_i^T}{\partial y} \left(k_y \frac{\partial N_j}{\partial y} \right) + \frac{\partial N_i^T}{\partial z} \left(k_z \frac{\partial N_j}{\partial z} \right) \right\} T_j(t) d\Omega_{\epsilon} \right. \\ \left. + \int_{\Gamma_{\epsilon}} N_i^T (q_s + (h_r + h_c)(N_j T_j - T_0)) d\Gamma_{\epsilon} - \int_{\Omega_{\epsilon}} N_i^T (Q - C \rho N_j \frac{\partial T_j}{\partial t}) d\Omega_{\epsilon} \right\} = 0 \quad (5.16)$$

To obtain the FEM formulation at the global level, Equation(5.16) is integrated over each element. In terms of the thermal "capacitance" matrix, $C_{i,j}$, thermal "stiffness" matrix, $K_{i,j}$, and thermal loading vector R_i , where:

$$C_{i,j} = \int_{\Omega} N_i^T C_p N_j d\Omega \quad (5.17)$$

$$K_{i,j} = \int_{\Omega} N_i^T k_{ij} N_j d\Omega + \int_{\Gamma} N_i^T (h_r + h_c) N_j d\Gamma \quad (5.18)$$

and

$$R_i = - \int_{\Omega} N_i^T Q d\Omega + \int_{\Gamma} N_i^T (q_s - (h_r + h_c) T_o) d\Gamma \quad (5.19)$$

Equation (5.16) can be written for the i th equation as:

$$C_{i,j} \frac{\partial T_j(t)}{\partial t} + K_{i,j} T_j(t) + R_i = 0 \quad (5.20)$$

When the domain is discretized into elements, this will now be converted to a set of simultaneous ordinary differential equations that must be solved for each time step.

5.1.4 Transient Heat Flow Analysis

To this point, the finite element equation formulation of the heat conduction problem has been presented in the space-time domain, but has not been explicitly written for the time domain. Since the final equation results in an identical expression, the time-marching formulation is treated separately.

For the time-marching formulation, consider a typical "element" of duration Δt with nodal values t_n and t_{n+1} . Linear basis functions can be written in terms of the local variable, ξ , since only first order derivatives are required:

$$N_1 = 1 - \frac{t}{\Delta t} = 1 - \xi \quad (5.21)$$

and

$$N_2 = \frac{t}{\Delta t} = \xi \quad (5.22)$$

(Note: By definition $\xi = t/\Delta t$, therefore, $0 \leq \xi \leq 1$) The linearly interpolated temperatures and their time derivatives are given by:

$$\begin{aligned} T(t) &= N_1(t_n)T(t_n) + N_2(t_{n+1})T(t_{n+1}) \\ &= (1 - \xi)T(t_n) + \xi T(t_{n+1}) \end{aligned} \quad (5.23)$$

and

$$\begin{aligned} \frac{\partial T(t)}{\partial t} &= \frac{\partial N_1}{\partial t} T(t_n) + \frac{\partial N_2}{\partial t} T(t_{n+1}) \\ &= \frac{1}{\Delta t} (T(t_{n+1}) - T(t_n)) \end{aligned} \quad (5.24)$$

Applying the weighted residual method to Equation (5.20):

$$\int_0^1 \omega_j \left[C_{i,j} \frac{\partial T_j}{\partial t} + K_{i,j} T_j + R_i \right] d\xi = 0 \quad (5.25)$$

and substituting Equations (5.21 - 5.24) gives:

$$\begin{aligned} \int_0^1 \omega_j d\xi \left(\frac{C_{i,j}}{\Delta t} (T_j(t_{n+1}) - T_j(t_n)) \right) + K_{i,j} \left(\int_0^1 \omega_j d\xi - \int_0^1 \omega_j \xi d\xi \right) T_j(t_n) \\ + K_{i,j} \int_0^1 \omega_j \xi d\xi T_j(t_{n+1}) + \int_0^1 \omega_j R_i d\xi = 0 \end{aligned} \quad (5.26)$$

Now, dividing through by $\int_0^1 \omega_j d\xi$ and letting $\Theta = \frac{\int_0^1 \omega_j \xi d\xi}{\int_0^1 \omega_j d\xi}$, Equation (5.26) becomes:

$$\left\{ \frac{C_{i,j}}{\Delta t} + K_{i,j} \Theta \right\} T_j(t_{n+1}) + \left\{ K_{i,j} (1 - \Theta) - \frac{C_{i,j}}{\Delta t} \right\} T_j(t_n) + R_i = 0 \quad (5.27)$$

where

$$R_i = \frac{\int_0^1 \omega_j R_i d\xi}{\int_0^1 \omega_j d\xi} \quad (5.28)$$

Solving for R_i , where:

$$R_i = (1 - \xi) R_i(t_n) + \xi R_i(t_{n+1}) \quad (5.29)$$

and by applying the weight residual method as before it can be shown that:

$$\tilde{R}_i = (1 - \Theta) R_i(t_n) + \Theta R_i(t_{n+1}) \quad (5.30)$$

Therefore, the finite element equation formulation for the heat conduction in a solid is given by:

$$\left\{ \frac{C_{i,j}}{\Delta t} + K_{i,j} \Theta \right\} T_j(t_{n+1}) + \left\{ K_{i,j} (1 - \Theta) - \frac{C_{i,j}}{\Delta t} \right\} T_j(t_n) + (1 - \Theta) R_i(t_n) + \Theta R_i(t_{n+1}) = 0 \quad (5.31)$$

The value for Θ can vary between $0 \leq \Theta \leq 1$, and its value determines the solution scheme for the set of equations produced when the time domain is discretized into elements (i.e. for $\Theta = 1$, the implicit; $\Theta = \frac{2}{3}$, Galerkin's; $\Theta = \frac{1}{2}$, central-difference or Crank Nicolson; and $\Theta = 0$, explicit; solution schemes are used). In this investigation the implicit time-marching scheme was selected for the finite element analysis.

Since there are i nodes per element, Equation (5.31) creates a set of i equations. These assembled for each element to form a set of algebraic equations of the following form:

$$[\mathbf{K}][\mathbf{T}] = [\mathbf{R}] \quad (5.32)$$

The $[\mathbf{K}]$ is given by the matrix of coefficients of $T_j(t_{n+1})$ and the "effective" load vector $[\mathbf{R}]$ is composed of the remaining terms in Equation (5.31). The system of algebraic

equations (Equation(5.32)) is solved for the temperatures at every node for each time step t_{n+1} . The solution to Equation (5.32) in this investigation is obtained by using a Frontal Solver Package originally developed by Irons [31]. For nonlinear material properties (i.e. conductivity and specific heat) and boundary conditions (i.e. convection and radiation) an iterative solution technique must be adopted for each time step. The finite element formulation presented in this chapter for transient heat flow has been implemented in a computer program called *ASGARD* especially tailored for weld analysis by Goldak *et al* [13].

The finite element formulation presented to this point is quite general and can be applied to any transient thermal heat flow problem. In this investigation, the finite element formulation has been adopted to study the heat flow behaviour during welding. After the subdivision of the workpiece into "elements" to form a mesh, Equation (5.31) is applied to each element to determine the heat flow behaviour. The thermal loads (R in Equation (5.31)) are obtained from the contribution to the nodal point loads generated from a special heat source model. Relatively extensive databases are necessary to handle the nonlinear thermal conductivity and heat capacity behaviour (K_{ij} and C_{ij} in Equation (5.31)). In addition, a knowledge of the nonlinear convective - radiative boundary conditions is required to calculate K_{ij} in Equation (5.31). A detailed description of the foregoing follows in Chapter 6. Also, the FEA computed $\tau_{8/3}$ and $\tau_{13/1}$ cooling times for the experimental conditions described in Chapter 2 are presented.

Chapter 6

Finite Element Weld Model

6.1 Two-Dimensional PEA Approximation

In general, the nonlinear transient heat flow behaviour during welding is fully three dimensional. Fourier's 3D heat flow relationship (Equation (3.1)) can be rearranged in terms of the moving coordinate system (ξ, y, z) (Figure 1) as follows:

$$\frac{1}{v^2} \frac{\partial}{\partial t} \left(\alpha_x \frac{\partial T}{\partial t} \right) + \frac{\partial}{\partial y} \left(\alpha_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\alpha_z \frac{\partial T}{\partial z} \right) + \frac{Q}{C\rho} - \frac{\partial T}{\partial t} = 0 \quad (6.1)$$

where $\xi = x - vt$. If the welding speed, v , is high compared to the diffusion rate, α , of the material, the third term of Equation (6.1) can be neglected. This implies that the heat flow in the longitudinal direction (i.e. along the x direction) is negligible, and the problem can be reduced to a two-dimensional analysis (i.e. $y - z$ plane). This assumption reduces the analysis to a 2D temperature field and, in turn, decreases the computing costs by several decades. Andersson [14] has shown that for most steel welding situations (except for high heat low speed cases) neglecting the longitudinal heat flow causes little error. A 2D ($y - z$) analysis has, therefore, been adopted in this investigation.

A reference plane and FEM mesh are fixed at some point in the workpiece as shown in Figure 7. The heat source model moves along the top of the plate and eventually interacts with the reference plane. Symmetry of the geometry at the centerline of the weld reduces the computations to one-half of the weld cross-section. The FEM mesh consists of 2D eight node quadrilateral and six-node triangular (grading) elements arranged as shown in the Figure 7. The temperature field for both element types is approximated with piecewise quadratic polynomials. Therefore, the concentration of the elements is higher in the region where the temperature gradients are severe and lower farther from the heat source.

6.2 FEA Investigation of the Point Source Assumptions

Initially, all of the closed form analytical point source assumptions (Chapter 1) were imposed on the FEA model. The following assumptions describe the FEA point source

Figure 7 Finite element weld analysis: (a) schematic representation of the cross-sectional reference plane concept and, (b) the mesh used for the transient heat flow analysis.

configuration.

- a) Fixed thermal properties, where the Yurioka [3] values of $k = 25 \text{ W/m}^\circ\text{C}$ and $C\rho = 6 \times 10^6 \text{ J/m}^3 \cdot ^\circ\text{C}$ are used.
- b) A point heat source, modelled by a single nodal load of strength $\dot{Q}$ on the surface of the workpiece.
- c) The latent heats of fusion and transformation are neglected.
- d) The effects of convection and radiation heat transfer to the surrounding are neglected.
- e) Weld reinforcement is assumed insignificant.
- f) The workpiece width in the y direction is sufficiently large that the bounding planes do not effect the weld cooling behaviour.

These assumptions were removed one by one to evaluate the significance of each point source assumption independently. After all the assumptions were removed, the cooling times $\tau_{8/5}$ and $\tau_{15/1}$ of the fully assembled FEA weld model were compared with the closed form analytical values and, in turn, with the experimental measurements.

6.3 Thermal Properties of the Material

With all the assumptions listed above imposed on the the FEA point source model, cooling times of $\tau_{8/5} = 9.4 \text{ s}$ and $\tau_{15/1} = 172 \text{ s}$ are computed. It should be noted that they are similar to the values found with the closed form analytical models (e.g. Rosenthal values, 8.5 s, and 163 s respectively). At the same time, they are somewhat high compared to the experimental values of $\tau_{8/5} = 8.1 \text{ s}$ and $\tau_{15/1} = 100 \text{ s}$ as shown in Table 3.

Following this, variable thermal properties were added into the FEA computations. The database for the thermal properties was generated from Figures 3 and 4. In addition, to account for weld pool stirring, a high thermal conductivity ($k = 120 \text{ W/m}^\circ\text{C}$) was applied above the melting point temperature. This ensured rapid heat transfer in this region. When variable thermal properties were introduced into the FEA computations the cooling times calculated were dramatically reduced ($\tau_{8/5} = 4.9 \text{ s}$, $\tau_{15/1} = 135 \text{ s}$). A comparison of centerline time-temperature curve for both "fixed" and variable thermal properties is shown in Figure 8. It appears that variable thermal properties have a very significant effect on the computed thermal history.

6.4 Distributed Heat Source Model

Following the thermal property study, a distributed heat source was added to the

Figure 8

FEA point source generated centerline cooling curves computed using: (a) constant thermal properties ($k = 25 \text{ W/m}^\circ\text{C}$, and $C_p = 6 \times 10^6 \text{ J/m}^3 \cdot ^\circ\text{C}$) and, (b) variable thermal properties — see Figures 3 and 4. Welding parameters: $V = 30 \text{ volts}$, $I = 265 \text{ amps}$, $v = 3.81 \text{ mm/s}$, $g = 38 \text{ mm}$, and $T_o = 20.5^\circ\text{C}$.

computations. The most advanced heat source model available is the double ellipsoidal configuration suggested by Goldak [35] (Figure 9). The shape of the double ellipsoid simulates relatively closely the "tear-drop" shape of the weld pool. The model assumes that the energy distribution is Gaussian in all three dimensions (i.e. maximum intensity at the center of the heat source diminishing towards the weld pool edges). The heat flux distribution can be written as:

$$q_i(\xi, y, z) = \frac{6\sqrt{3}f_i\dot{Q}}{a_i b c \pi \sqrt{\pi}} \exp\left[-3\left(\frac{\xi^2}{a_i^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)\right] \quad (6.2)$$

The subscript, i , is used to refer to either the front ($i = 1$) or the rear ($i = 2$) portion of the ellipsoidal heat source. The additional terms (not yet defined in Equation (6.1)) are:

- f_i — the fractions of heat input deposited in the front and rear portions of the ellipsoid (where $f_1 + f_2 = 2$);
- a_i — are the lengths of the heat source, in the front (a_1) and the rear (a_2) of the welding arc.
- b — half the width of the heat source.
- c — depth of the heat source.

The ellipsoid dimensions were obtained from the non-dimensional methods suggested by Christensen *et al* [27] ($a_1 = 9\text{ mm}$, $a_2 = 16\text{ mm}$, $b = 6\text{ mm}$, $c = 6\text{ mm}$) and checked experimentally (see Section 6.9). The fraction of energy allocated to the front ellipsoid was assumed to be 60 percent of the total heat input.

The 3D heat source model can easily be applied to the 2D finite element heat flow analysis. This is accomplished by allowing the heat source to pass through the reference plane. In this way, a surface thermal load, which is elliptically distributed on the plane, is obtained for each time increment. If small enough time-steps are used, the total heat flux deposited should be approximately equal to the gross heat input ($\dot{Q} = 6360\text{ Watts}$). From this investigation, it was found that ten time-steps for the front and eight time steps for the rear portions of the heat source were sufficient to capture the "ellipsoidal" distribution of the flux. When the ellipsoidal heat source was introduced into the FEA computations, no significant changes in the calculated cooling times were found ($\tau_{2/5} = 4.8\text{ s}$, $\tau_{15/1} = 130\text{ s}$) — Table 3. This was presumably because of the relatively small weld, thick workpiece conditions. These findings were unexpected and are not necessarily the case in other welding situations.

Figure 9

FEA heat source model: (a) double ellipsoid heat source configuration together with the power distribution function along the ξ axis. ($a_1 = 9\text{ mm}$, $a_2 = 16\text{ mm}$, $b = 6\text{ mm}$, $c = 6\text{ mm}$) and (b) experimental "tear-drop" shape of the weld pool which is closely modelled using the double ellipsoid heat source configuration purposed by Goldak *et al* [35].

6.5 Heat Loss to the Surroundings

From the exposed surfaces of the workpiece, heat exchange takes place with the surrounding environment in the form of free-convection and radiation. Vinokurov [36] suggested a combined convective - radiative heat transfer coefficient of:

$$h = 0.00241 \epsilon T^{1.61} \quad W/m^2 \cdot ^\circ C \quad (6.3)$$

should be applied to all surfaces outside the ellipsoid. However, the inclusion of surface heat transfer using Vinokurov's relationship did not appreciably change the cooling time from 800 to 500 $^\circ C$ ($\tau_{8/5} = 4.7 s$), nor was the cooling time from 1500 to 100 $^\circ C$ significantly affected ($\tau_{15/1} = 122 s$) — Table 3. Presumably this reflects the fact that it is a thick plate situation where the heat flow is conduction dominated. On the other hand, convection plays a dominant role on the 1500 to 100 $^\circ C$ cooling time in other welding situations.

6.6 Latent Heats of Fusion and Transformation

To account for the latent heats of fusion and transformation, the volumetric heat capacity values of the material were modified. The final composite thermal properties used in the FEA are given in Figures 10 and 11. The heats of fusion ($2.1 \times 10^9 J/m^3$) and transformation ($3.3 \times 10^8 J/m^3$) were distributed over the melting and transformation temperature ranges respectively, i.e. included in the volumetric heat capacity [26, 37]. The volumetric heat capacity values were determined from the enthalpy, H ($C_p = \frac{\partial H}{\partial t}$) [38, 39].

The inclusion of the latent heats reduced the peak temperatures and decreased the severity of the thermal gradients (Figure 12). On the other hand, the "overall" cooling time (i.e. $\tau_{15/1}$) did not change appreciably since the energy absorbed on heating was released on cooling. This is evident from the FEA computations. When the latent heat of fusion was included, $\tau_{8/5}$ changed appreciably ($\tau_{8/5} = 6.0 s$), but the cooling time $\tau_{15/1}$ remained unchanged ($\tau_{15/1} = 122 s$) — Table 3. When the latent heat of transformation was included, the cooling time $\tau_{8/5}$ increased to 6.6 seconds but once again the overall cooling time of $\tau_{15/1} = 122 s$ was unaffected — Table 3. The effect of the latent heats on the overall cooling curve is shown in Figure 12.

6.7 Weld Reinforcement

The final addition to the FEA model was weld reinforcement (Figure 13). Due to the similarity of the filler and base material, the same thermal properties were applied to

Figure 10

The volumetric heat capacity and latent heats of fusion and transformation arranged for FEA computer implementation. In addition, the enthalpy - temperature relationship used in the FEA program [26, 37].

Figure 11

The thermal conductivity - temperature relationship used in the FEA program. Note that a high conductivity in the molten region is used to simulate heat transfer by stirring [26, 38].

Figure 12 FEA generated centerline cooling curve with (a) no phase change effects, (b) latent heat of fusion and, (c) with both the heats of fusion and transformation included.

Figure 13

Cross-section of weld bead. Welding parameters: $V = 30$ volts, $I = 265$ amps, $v = 3.81$ mm/s, $g = 33$ mm, and $T_o = 20.5$ °C).

the reinforcement. Including the weld reinforcement in the FEA computations noticeably increased the cooling time 800 to 500 °C ($\tau_{8/5} = 7.2$ s), but there was no change in the 1500 to 100 °C ($\tau_{15/1} = 122$ s) cooling time as shown in Table 3. The centerline thermal cycle with all the point source assumptions removed is given in Figure 14. In addition, the effects of the point source assumptions on the weld cooling parameters are summarized in Table 3.

6.8 FEA Cooling Times

When all the point source assumptions (Chapter 1) are removed, the FEA computations for the welding parameters given in Chapter 2 yield cooling times of $\tau_{8/5} = 7.2$ seconds and $\tau_{15/1} = 122$ seconds. This result (result 1, Table 4) was calculated using a temperature dependent surface heat transfer coefficient suggested by Vinokurov [36]. The 1500 to 100 °C cooling time result is somewhat high compared to the experimental value of $\tau_{15/1} = 100$ s as shown in Table 4. This indicated a need for an alternative approach to obtain a “combined” convective – radiative heat transfer coefficient. Therefore, a convective coefficient of $12 \text{ W/m}^2 \text{ °C}$ suggested by Andersson [14], combined with the Stefan-Boltzmann equation ($\epsilon = 0.9$ for steel [40]) was substituted in place of Vinokurov’s relationship. A comparison of the effective heat transfer coefficient of these two approaches is given in Figure 15. The conventional approach did not significantly affect the cooling time from 800 to 500 °C ($\tau_{8/5} = 7.0$ s), but did lead to a much improved value for the cooling time from 1500 to 100 °C ($\tau_{15/1} = 102$ s). This result (result 2, Table 4) compares favourably with both the experimental values of this investigation ($\tau_{8/5} = 8.1$ s and $\tau_{15/1} = 100$ s) and with those of Bennett ($\tau_{8/5} = 7$ s and $\tau_{15/1} = 100$ s). A comparison of the experimental, FEA and closed form analytical cooling times is summarized in Table 4.

6.9 Weld Pool Dimensions

The FEA provides the “overall” transient thermal response (i.e. both heating and cooling behaviour for every node) during welding, from which the critical isotherms defining the HAZ and MP isotherm can be extracted (Figure 7). The FEA computations yielded a weld pool width of 14 mm, a depth of 6.5 mm, and a length of 25 mm. The experimental weld pool shape was measured from the cross-sectional and surface profile photographs shown in Figures 9 and 13. The fusion zone cross-section is semi-elliptical in shape, of estimated width 16 mm and depth 5 mm. A measured weld pool length of 27 mm was obtained from surface profile photograph. These values are very close to the FEA results when one considers the complexity of the weld pool physics.

Figure 14

Centerline cooling curves: (a) experimental, (b) FEA generated. Welding Parameters: $V = 30$ volts, $I = 265$ amps $v = 3.31$ mm/s, $g = 38$ mm, $T_0 = 20.5$ °C; thermal properties: volumetric heat capacity, heats of fusion (2.1×10^9 J/m³) and transformation (3.3×10^8 J/m³) — see Figure 11; thermal conductivity — see Figure 12; combined surface heat transfer coefficient $h = 0.00241 \epsilon T^{1.61}$ W/m² °C (Vinokurov [36], $\epsilon = 0.9$ [40]); arc efficiency $\eta = 0.8$ — see Figure 10.

Figure 15

A comparison of the "combined" convective - radiative heat transfer coefficient for (a) Vinokurov's relationship $h = 0.00241 \epsilon T^{1.61} \text{ W/m}^2 \cdot \text{°C}$ and (b) Andersson's convection value of $h = 12 \text{ W/m}^2 \cdot \text{°C}$ [14] with Stefan - Boltzmann relationship (using an emissivity of $\epsilon = 0.9$ [40]) — Equation (5.12)

Table — 3

**FEA Calculations to Show the Effect of Each Assumption
(column headings) on the Cooling Times $\tau_{8/5}$ and $\tau_{15/1}$**

Method	Thermal Cond.(k)	Heat Source	Surface Transfer	Heat of Fusion	Heat of Trans.	Reinforce- ment	$\tau_{8/5}$ (s)	$\tau_{15/1}$ (s)
Adams (3D) ⁽¹⁾	mean ⁽⁴⁾	Point	No	No	No	No	8.5	—
Adams (2D) ⁽²⁾	▪	▪	▪	▪	▪	▪	—	163
FEM ⁽³⁾	▪	▪	▪	▪	▪	▪	9.4	172
FEM	Var. ⁽⁵⁾	▪	▪	▪	▪	▪	4.9	135
FEM	Var.	Dist. ⁽⁶⁾	▪	▪	▪	▪	4.8	130
FEM	Var.	Dist.	Var. ⁽⁷⁾	▪	▪	▪	4.7	122
FEM	Var.	Dist.	Var.	Yes ⁽⁸⁾	▪	▪	6.0	122
FEM	Var.	Dist.	Var.	Yes	Yes ⁽⁹⁾	▪	6.6	122
FEM	Var.	Dist.	Var.	Yes	Yes	Yes ⁽¹⁰⁾	7.2	122
Expt. ⁽¹¹⁾							8.1	100

Notes

(1) Equation (4.3)	(7) $h = 0.00241 \epsilon T^{1.61} W/m^2 \cdot ^\circ C$ [36], and $\epsilon = 0.9$ [40]
(2) Equation (4.4)	(8) $H_f = 2.1 \times 10^9 J/m^3$ [26]
(3) Figure 5	(9) $H_t = 3.3 \times 10^8 J/m^3$ [26]
(4) $k = 25 W/m \cdot ^\circ C$	(10) Figure 7 and 13
(5) Figure 3 and 4	(11) Experimental conditions - Chapter 2
(6) Double Ellipsoid [35]
(Figure 9)	

Table — 4

**A Comparison of Experimental, FEA and Closed
Form Analytical Cooling Times**

Method	Thermal Conductivity $W/m \cdot ^\circ C$	Surface Heat Transfer Coeff. $W/m^2 \cdot ^\circ C$	$\tau_{8/5}$ (s)	$\tau_{15/1}$ (s)
Experimental ⁽¹⁾ (1)	—	—	7.9	96
(2)	—	—	8.4	105
(Av)	—	—	8.1	100
Experimental ⁽¹⁾ (Bennett, A.P. [24])	—	—	7	100
Adams (3D) ⁽²⁾	25	—	8.5	—
(2D) ⁽³⁾	25	—	—	163
Graville (2D) ⁽⁴⁾	25	16.7	—	156
Rosenthal (2.5D) ⁽⁵⁾	25	—	8.5	163
Tanaka (2.5D) ⁽⁶⁾	25	83.5	7.9	—
(2.5D) ⁽⁶⁾	25	16.7	—	153
FEA	Var ⁽⁷⁾	⁽⁸⁾ 0.00241 $\epsilon T^{1.61}$	7.2	122
FEA	Var ⁽⁷⁾	⁽⁹⁾ 12 (convection) $\epsilon = 0.9$ (radiation)	7.0	102

Notes

(1) Experimental Conditions: $V = 30$ volts	(6) Equation (4.8)
$I = 265$ amps, $g = 38$ mm, $v = 3.81$ mm/s	(7) Variable thermal properties
(see Figures 3 and 4)
(2) Equation (4.3)	(8) Reference [36, 40]
(3) Equation (4.4)	(9) Reference [14, 40]
(4) Equation (4.7)	
(5) Equation (4.6)	

Chapter 7

Discussion

7.1 800 to 500 °C Cooling Times

The calculated cooling times $\tau_{8/5}$ for the closed form analytical centerline relationships are relatively close to the average measured value ($\approx 5\%$ high on average). These results should not be interpreted too literally since the selection of the thermal properties ($k = 25 \text{ W/m}^\circ\text{C}$ and $C\rho = 6 \times 10^6 \text{ J/m}^3^\circ\text{C}$) is somewhat arbitrary. These values are considered appropriate for this case although different values could lead to substantial differences between measured and calculated cooling times. Values as high as $41 \text{ W/m}^\circ\text{C}$ have been used by some investigators [3, 8]. It should be noted that $k = 25 \text{ W/m}^\circ\text{C}$ is not the mean thermal conductivity of the material, but is a best fit parameter for matching experimental measurements to the cooling relationships. This 3D heat flow behaviour for $\tau_{8/5}$ is considered an ideal case for the point source relationship, i.e. a relatively fast cooling weld on a thick plate. It is the author's experience that, for the point source model, reasonably reliable results are obtained if $k = 25 \text{ W/m}^\circ\text{C}$ and $\tau_{8/5} < 10 \text{ s}$. Beyond 10 seconds convective - radiative effects, bead size, etc. begin to play a major role in the cooling behaviour. For relatively slow cooling welds on thin plates, calculated values can exceed the measured values by a considerable amount.

The FEA calculated cooling times $\tau_{8/5}$ were found to be somewhat less ($\approx 10\%$) than the average measured value. However, the agreement is quite reasonable in view of the fact the arc efficiency value of $\eta = 0.80$ is the subject of some doubt.

7.2 1500 to 100 °C Cooling Times

The analytical point source models were on average, some 50 to 60% in excess of the measured $\tau_{15/1}$ parameter. This was the case even when surface heat transfer was included in the calculation (Equations (4.7) and (4.8)). It would appear that the point source relationships have only limited usefulness when calculating $\tau_{15/1}$, at least for the welding conditions of this investigation. In order to match calculated cooling times with the experimental values, the surface heat transfer coefficient (used in Graville's and Tanaka's

relationship of $h = 16.7 W/m^2 \cdot ^\circ C$) could be increased. This, however, can only be contemplated after much experimental and calculation experience and even then such a change has doubtful value. The $\tau_{15/1}$ values calculated by all methods (except the FEA result 2, Table 3) exceed the measured values by a substantial amount. In particular, the FEA value (result 1, Table 3) exceed the measured value by some 22%. This is due to the fact that Vinokurov's relationship for the convection - radiation contribution gives too small a heat transfer coefficient at low temperatures (Figure 15). As shown in Figure 14, most of the cooling time in the $\tau_{15/1}$ parameter occurs at lower temperatures. As previously discussed, convective - radiative data covering weld cooling at low temperatures is not well documented and, in any event, is quite likely to vary considerably from one experiment to another. On the other hand, it is perhaps surprising that the calculated values are this close to the measured ones. The findings of the FEA value, using Vinokurov's relationship (result 1, Table 3), indicate the need for an alternative approach to obtain the effective heat transfer coefficient. Therefore, a convective coefficient of $12 W/m^2 \cdot ^\circ C$ suggested by Andersson [14], combined with the Stefan - Boltzmann equation ($\epsilon = 0.9$ for steel [40]) was substituted in place of Vinokurov's relationship. Using this combination, a value of $\tau_{15/1}$ of 102 seconds is calculated which compares favourably with the average measured value of 100 seconds. At the same time this does not change $\tau_{8/5}$ appreciably. Obviously, there is a need for further investigation of weld cooling behaviour in the low temperature region. It is an important subject since this is the region where HAZ cold cracks appear and it is well established that cracks are sensitive to cooling time [2, 3].

7.3 FEA Point Source Assumptions

The FEA proved to be very effective in assessing the significance of the assumptions in the point source model. In addition, the FEA investigation provided several surprising and very enlightening findings. These are documented in the following.

7.3.1 800 to 500 $^\circ C$ Cooling Time

When all the assumptions of the point source model (i.e. thermal conductivity $k = 25 W/m \cdot ^\circ C$, no surface heat transfer, no latent heats, and no weld reinforcement) are imposed on the FEA, the computed 800 to 500 $^\circ C$ cooling time is 9.4 seconds. It is interesting to note the close agreement with the calculated value ($\tau_{8/5} = 8.5 s$) using the Adams relationship. When variable thermal properties are applied to the FEA, the cooling time is drastically reduced to 4.9 seconds (Figure 8). The inclusion of a distributed heat

source produces no appreciable effect on the cooling time ($\tau_{8/5} = 4.8 \text{ s}$). Furthermore, the inclusion of surface heat transfer to the surroundings does not significantly change the cooling time ($\tau_{8/5} = 4.7 \text{ s}$). This is presumably because of the relatively small weld, thick workpiece conditions. On the other hand, distributing the thermal load and surface heat losses are known to significantly effect the cooling behaviour for higher heat input, thin workpiece welds. However, including the latent heat of fusion, increases $\tau_{8/5}$ to 6 seconds and the heat of transformation further to 6.6 seconds. The latent heats have a strong effect on weld cooling. Because of this, it is interesting to note that the closeness of the point source value of Adams $\tau_{8/5} = 8.5 \text{ s}$ for which the latent heats are neglected agrees reasonably well with the measured value of $\tau_{8/5} = 8.1 \text{ s}$. Evidently, the choice of the thermal conductivity value ($k = 25 \text{ W/m}^\circ\text{C}$) compensates for the latent heats. It is not clear, however, that this is the case in all welding situations. The final addition to the FEM model of weld reinforcement increases the cooling time $\tau_{8/5}$ to 7.2 seconds. This value compares favourably with the experimental measured value of 8.1 seconds. It is evident that weld reinforcement has a significant effect on cooling at higher temperatures, i.e. $\tau_{8/5}$. Apparently, the longer the heat path from the weld reinforcement to the base material, the greater the delay in the cooling.

7.3.2 1500 to 100 °C Cooling Time

The outcome for 1500 to 100 °C cooling times is similar to that for the cooling times from 800 to 500 °C. When the point source model assumptions are imposed, a computed $\tau_{15/1}$ of 172 seconds is obtained. This compares closely with the value ($\tau_{15/1} = 163 \text{ s}$) calculated by the Adams relationship, but does not agree with the experimental value of 100 seconds. When variable thermal properties are introduced into the FEA, the cooling time is drastically reduced ($\tau_{15/1} = 135 \text{ s}$) (Figure 8). However, neither the inclusion of a distributed heat source ($\tau_{15/1} = 130 \text{ s}$) nor surface heat transfer ($\tau_{15/1} = 122 \text{ s}$) significantly changes the cooling times. As with $\tau_{8/5}$, the inclusion of a distributed heat source and surface heat transfer is expected to be minor due to the small weld, thick workpiece conditions. In addition, the overall cooling time $\tau_{15/1}$, is not noticeably affected by including the latent heats and weld reinforcement ($\tau_{15/1} = 122 \text{ s}$). However, the latent heats are not expected to change the cooling times significant since the energy absorbed on heating is released on cooling at the higher temperatures (i.e. latent heat of fusion at 1500 °C, and heat of transformation at 700 °C). It is, therefore, expected that these factors could have a greater effect on higher temperature cooling (i.e. $\tau_{8/5}$) but a lesser effect where most of the time is in the lower temperature region $\tau_{15/1}$.

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Chapter 8

Conclusions and Future Research

8.1 Conclusions

- (1) The experience of this investigation would suggest that $\tau_{8/5}$ values generated with the point source relationships are relatively accurate for low heat fast cooling ($\tau_{8/5} < 10$ s) welds. On the other hand $\tau_{15/1}$ values generated with these same relationships considerably exceed measured values.
- (2) While the FEA computed $\tau_{8/5}$ is in good agreement with measured values, the $\tau_{15/1}$ calculated cooling time exceeds the experimental values by some 22 % (Vinokurov surface transfer coefficient). However, the FEA computed $\tau_{15/1}$ is in good agreement when the Andersson convective coefficient $h_c = 12 \text{ W/m}^2 \text{ } ^\circ\text{C}$ and a emissivity of $\epsilon = 0.9$ are used. It is suggested that low temperature weld cooling behaviour needs more study to handle this problem with confidence.
- (3) The FEA study showed that variable thermal properties, the latent heats of fusion and transformation, and reinforcement all have a significant effect on $\tau_{8/5}$. The effects of the distributed heat source and surface heat transfer are small for welds where the cooling time is $\tau_{8/5} < 10$ s.
- (4) On the other hand, the most significant effects in computing $\tau_{15/1}$ appear to be variable thermal properties and convective heat transfer. The latent heats, source distribution and reinforcement are not significant for low heat fast cooling welds ($\tau_{15/1} < 10$ s).

8.2 Future Research

This investigation of a bead-on-thick plate weld (i.e. low heat - fast cooling welds where $\tau_{8/5} < 10$ s) is considered to be relatively complete. It is, however, suggested that it be followed by a study of a high heat - slow cooling weld (i.e. $\tau_{8/5} \approx 100$ s). In this latter study, surface heat transfer, weld-pool shape, and reinforcement would be expected to play greater roles in the weld cooling behaviour. Also, the determination of the heat transfer coefficient for low temperature cooling needs further investigation if this problem is to be handled with confidence.

The finite element results, of this investigation, reveal that cooling behaviour is very sensitive to the thermal properties used. However, as thermal properties for low carbon steel are not available at high temperatures, these should be determined if the accuracy of the calculated temperature field is to be improved. In addition, it is valid to assume that the rapid decrease in computer costs and advancements of theoretical models, should make it possible to perform full 3D FEA on an economical basis in the near future. The availability of thermal properties mentioned above coupled with more accessible full 3D FEA will lead to improved accuracy of computed temperature fields.

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Appendix A

Tanaka's 2.5D Temperature Distribution due to a Moving Heat Source

S. Tanaka [12], presented a 2.5D temperature relationship for a finite thick plate due to a moving point heat source. The approach taken is quite different from that of Rosenthal [9] and Rykalin [10], although, there are similarities in the model assumed. However, both surface heat transfer and cooling preheated workpieces are taken into account in this relationship and it is, therefore, more comprehensive than any of the closed form relationships published in English at the present time. There is, however, no English reference to the Tanaka relationship and the assumptions necessary to arrive at the final form are not well understood. The following derivation is, therefore, presented to shed light on Tanaka's relationship. The final form obtained for the "quasi-stationary" temperature distribution is of the following form:

$$T = T_o + \frac{\dot{Q}}{\pi k g} e^{-\lambda v t} \sum_{m=0}^{\infty} A_n \left(\cos \frac{U_n z}{g} + \frac{gh}{k U_n} \sin \frac{U_n z}{g} \right) \left[K_o \left[\frac{r}{g} \sqrt{U_n^2 + \left(\frac{vg}{2\alpha} \right)^2} \right] \right. \\ \left. + 2(T_p - T_o) \sum_{m=0}^{\infty} A_n \left(\cos \frac{U_n z}{g} + \frac{gh}{k U_n} \sin \frac{U_n z}{g} \right) \left(\frac{\sin U_n}{U_n} - \frac{gh}{k U_n^2} \cos(U_n - 1) \right) e^{-U_n^2 \alpha t / g^2} \right] \quad (1)$$

where U_n is an eigenvalue satisfying:

$$\tan U_n = 2ghkU_n / (k^2 U_n^2 - g^2 h^2) \quad (2)$$

and

$$A_n = U_n^2 / (U_n^2 + (gh/k)^2 + (2gh/k)) \quad (3)$$

The derivation is presented in two sections; part (A) involves the temperature distribution due to the moving heat source with convective boundary conditions applied, and part (B) takes into account the effect of uniform preheating. The final solution can be expressed as the sum of the temperature distribution due to the welding heat and preheating.

(A) Temperature Distribution of a Moving Heat Source

Restating the thermal heat conduction equation given previously in Chapter 3:

$$\frac{\partial^2 T(x, y, z, t)}{\partial x^2} + \frac{\partial^2 T(x, y, z, t)}{\partial y^2} + \frac{\partial^2 T(x, y, z, t)}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T(x, y, z, t)}{\partial t} \quad (A.1)$$

Consider the moving coordinate system (ξ, y, z) , as shown in Figure 1, which is related to a fixed or stationary reference coordinate system (x, y, z, t) by $\xi = x - vt$. It can be shown that

$$\left(\frac{\partial T}{\partial t}\right)_s = \frac{\partial T}{\partial \xi} \cdot \frac{\partial \xi}{\partial t} + \left(\frac{\partial T}{\partial t}\right)_m = -v \frac{\partial T}{\partial \xi} + \left(\frac{\partial T}{\partial t}\right)_m \quad (\text{A.2})$$

where the subscripts s and m refer to the stationary and the moving coordinate systems respectively. For quasi-stationary conditions (i.e. a constant temperature distribution with respect to a moving coordinate system) the term $\left(\frac{\partial T}{\partial t}\right)_m$ must be equal to zero. Therefore, Equation (A.1) can be rewritten as:

$$\frac{\partial^2 T(\xi, y, z)}{\partial \xi^2} + \frac{v}{\alpha} \frac{\partial T(\xi, y, z)}{\partial \xi} + \frac{\partial^2 T(\xi, y, z)}{\partial y^2} + \frac{\partial^2 T(\xi, y, z)}{\partial z^2} = 0 \quad (\text{A.3})$$

Assuming that the general solution of Equation (A.3) is of the form:

$$T(\xi, y, z) = e^{-\lambda v \xi} \Psi(\xi, y, z) \quad (\text{A.4})$$

and using the separation of variables technique, the solution of Equation (A.4) becomes:

$$T(\xi, y, z) = e^{-\lambda v \xi} F(\xi, y) Z(z) \quad (\text{A.5})$$

Differentiating Equation (A.5) with respect to (ξ, y, z) and substituting the appropriate terms back into Equation (A.3) yields:

$$e^{-\lambda v \xi} \left[\left(\frac{\partial^2 F}{\partial \xi^2} - (\lambda v)^2 F \right) Z + \frac{\partial^2 F}{\partial y^2} Z + F \frac{d^2 Z}{dz^2} \right] = 0 \quad (\text{A.6})$$

Now dividing through by $e^{-\lambda v \xi} Z(z) F(\xi, y)$ gives:

$$\frac{1}{F} \left[\frac{\partial^2 F}{\partial \xi^2} + \frac{\partial^2 F}{\partial y^2} - (\lambda v)^2 F \right] + \frac{1}{Z} \left[\frac{d^2 Z}{dz^2} \right] = 0 \quad (\text{A.7})$$

Since $F(\xi, y)$ is independent of $Z(z)$, let:

$$\frac{1}{Z} \left[\frac{d^2 Z}{dz^2} \right] = -\eta^2 \quad (\text{A.8})$$

and

$$\frac{1}{F} \left[\frac{\partial^2 F}{\partial \xi^2} + \frac{\partial^2 F}{\partial y^2} - (\lambda v)^2 F \right] = \beta^2 \quad (\text{A.9})$$

therefore, $\beta^2 - \eta^2 = 0$

(i) Solving for Equation (A.8) where:

$$\frac{d^2 Z}{dz^2} + \eta^2 Z = 0 \quad (\text{A.8})$$

which is subjected to the homogeneous convective boundary conditions:

$$-k \frac{dZ}{dz} + HZ = 0 \quad \text{at} \quad z = 0 \quad (\text{A.10})$$

and

$$k \frac{dZ}{dz} + HZ = 0 \quad \text{at} \quad z = g \quad (\text{A.11})$$

It can be shown that the solution is given by:

$$Z(\eta_n, \xi) = \eta_n \cos(\eta_n z) + H \sin(\eta_n z) \quad (\text{A.12})$$

where the eigenvalues η_n are the roots of the following transcendental equation:

$$\tan(\eta_n g) = 2\eta_n h / (\eta_n^2 - H^2) \quad (\text{A.13})$$

and the norm $N(\eta_n)$ is given by:

$$\frac{1}{N(\eta_n)} = \frac{2}{\eta_n^2 g + H^2 g + 2H} \quad (\text{A.14})$$

(Note $H = h/k$). For the above solution, the convective heat transfer coefficient on the upper and lower surfaces of the plate are equal. The solution of Equation (A.8) can be carried out equally as well for $h_1 \neq h_2$, but for simplicity (and lack of reliable convective heat transfer coefficients for welding) the assumption that $h_1 = h_2$ is applied. See section (B) for a detailed solution of Equation (A.8)).

(ii) Solving for $F(\xi, y)$ given in Equation (A.9):

$$\frac{\partial^2 F}{\partial \xi^2} + \frac{\partial^2 F}{\partial y^2} - (\beta_m^2 + \lambda^2 v^2) F = 0 \quad (\text{A.15})$$

Equation (A.9) can be expressed in the form of Bessel's differential equation when transformed into cylindrical coordinates [41]:

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \frac{\partial F}{\partial r} + \frac{1}{r^2} \frac{\partial^2 F}{\partial \phi^2} - (\beta_m^2 + \lambda^2 v^2) F = 0 \quad (\text{A.16})$$

where $r^2 = \xi^2 + y^2$, $\phi = \tan^{-1}(y/\xi)$, and $F = F(\beta_m, r)$. Assuming there is no ϕ dependence (i.e. $F(r, \phi)$ is symmetric in form) the general solution is given by:

$$F(\beta_m, r) = C_1 J_0[ir(\beta_m^2 + \lambda^2 v^2)^{\frac{1}{2}}] + C_2 Y_0[ir(\beta_m^2 + \lambda^2 v^2)^{\frac{1}{2}}] \quad (\text{A.17})$$

where J_0 and Y_0 are zero order Bessel functions of the first and second kind respectively. In terms of a modified Bessel function where the argument is a real quantity, the following relationship can be used:

$$I_n(x) = i^{-n} J_n(ix) \quad (\text{A.18})$$

and

$$K_n(x) = i^{n+1} \frac{\pi}{2} [J_n(ix) + Y_n(ix)] \quad (\text{A.19})$$

where $n = 0$ and $x = r(\beta_m^2 + \lambda^2 v^2)^{\frac{1}{2}}$. Therefore,

$$F(\beta_m, r) = C_3 I_0 [r(\beta_m^2 + \lambda^2 v^2)^{\frac{1}{2}}] + C_4 K_0 [r(\beta_m^2 + \lambda^2 v^2)^{\frac{1}{2}}] \quad (\text{A.20})$$

Applying the boundary conditions, (a) for a semi-infinite plate:

$$\lim_{r \rightarrow \infty} \frac{\partial T}{\partial r} = 0 \quad (\text{A.21})$$

and (b) for a point heat source:

$$\lim_{r \rightarrow 0} -k(2\pi r) \frac{\partial T}{\partial r} = \dot{Q} \quad (\text{A.22})$$

where $\dot{Q}$ is the effective heat input (*Watts*). The boundary condition given in (a) reveals that the coefficient C_3 must be equal to zero, and from (b) it can be shown that $C_4 = \dot{Q}/2\pi k$, therefore, Equation (A.20) can be written as:

$$F(\beta_m, r) = \frac{\dot{Q}}{2\pi k} K_0 [r(\beta_m^2 + \lambda^2 v^2)^{\frac{1}{2}}] \quad (\text{A.23})$$

The general solution of Equation (A.3) [41] can be written in terms of (ξ, r, z) as:

$$T(r, z, t) = e^{-\lambda v \xi} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} F(\beta_m, r) \frac{Z(\eta_n, z)}{N(\eta_n)} \int_{z'=0}^z Z(\eta_n, z') f(z') dz' \quad (\text{A.24})$$

Substituting $F(\beta_m, r)$ and $Z(\eta_n, z)$ into Equation (A.24) and noting that $\eta_n^2 = \beta_m^2$ gives:

$$T(r, z) = T_0 + \frac{\dot{Q}}{2\pi k g} e^{-\lambda v \xi} \sum_{m=0}^{\infty} \frac{[\beta_m \cos(\beta_m z) + H \sin(\beta_m z)]}{N(\beta_m)} K_0 [r(\beta_m^2 + \lambda^2 v^2)^{\frac{1}{2}}] \quad (\text{A.25})$$

where

$$\tan \beta_m g = \frac{2\beta_m H}{\beta_m^2 - H^2} \quad (\text{A.26})$$

and

$$\frac{1}{N(\beta_m)} = \frac{2}{\beta_m^2 g + H^2 g + 2H} \quad (\text{A.27})$$

Substituting $N(\beta_m)$ into Equation (A.25) and defining $U_n = \beta_m/g$, the final form of the temperature distribution due to a moving point heat source can be presented:

$$T = T_0 + \frac{\dot{Q}}{\pi k g} e^{-\lambda v \xi} \sum_{n=0}^{\infty} A_n \left(\cos \frac{U_n z}{g} + \frac{g h}{k U_n} \sin \frac{U_n z}{g} \right) K_0 \left[\frac{r}{g} \sqrt{U_n^2 + \left(\frac{v g}{2a} \right)^2} \right] \quad (\text{A.28})$$

where U_n is an eigenvalue satisfying:

$$\tan U_n = 2gkhU_n / (k^2 U_n^2 - g^2 h^2) \quad (\text{A.29})$$

and

$$A_n = U_n^2 / (U_n^2 + (gh/k)^2 + (2gh/k)) \quad (A.30)$$

(B) Cooling Behaviour of a Uniformly Preheated Plate

The cooling behaviour of a uniformly preheated plate is also governed by Fourier's differential heat conduction Equation (A.1), but is subject to a different set of boundary and initial conditions. The cooling of a uniformly preheated plate is essentially a one-dimensional heat flow problem, therefore, the heat conduction equation reduces to:

$$\frac{\partial^2 T(z, t)}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T(z, t)}{\partial t} \quad (B.1)$$

The boundary conditions of the workpiece on the top surface is:

$$-k \frac{dT}{dz} + h_1 T = h_1 T_0 \quad \text{at } z = 0 \quad (B.2)$$

and bottom surface is:

$$k \frac{dT}{dz} + h_2 T = h_2 T_0 \quad \text{at } z = g \quad (B.3)$$

and the initial temperature distribution in the plate is:

$$T(z, t = 0) = T_p \quad \text{for } 0 \leq z \leq g \quad \text{at } t = 0$$

Using the method of separation of variables where $T(z, t) = Z(z) \cdot \Gamma(t)$, Equation (B.1) can be expressed as:

$$\frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} = \frac{1}{\alpha \Gamma(t)} \frac{d\Gamma(t)}{dt} = -\beta^2 \quad (B.4)$$

Since the L.H.S. is only a function of (z) and the R.H.S. is only a function of (t) , Equation (B.4) must be equal to a constant, say $-\beta^2$. Therefore,

$$\frac{d\Gamma}{dt} + \alpha \beta^2 \Gamma = 0 \quad (B.5)$$

for which the general solution is given by:

$$\Gamma(t) = C_1 e^{-\alpha \beta^2 t} \quad (B.6)$$

and

$$\frac{d^2 Z(z)}{dz^2} + \beta^2 Z(z) = 0 \quad (B.7)$$

for which the general solution is given by:

$$\begin{aligned} Z(z) &= C_2 e^{i\beta z} + C_3 e^{-i\beta z} \\ &= D_1 \cos(\beta z) + D_2 \sin(\beta z) \end{aligned} \quad (B.8)$$

Applying the convective boundary condition at $z = 0$ where

$$-\frac{dZ}{dz} + HZ = 0 \quad \text{at} \quad z = 0 \quad (B.9)$$

and assuming that $H_1 = H_2$, it can be shown that; $D_1 = \beta$ and $D_2 = H$, therefore, Equation (B.8) becomes

$$Z(z) = \beta \cos(\beta z) + H \sin(\beta z) \quad (B.10)$$

Since the convective boundary condition must also be satisfied at $z = g$, i.e.

$$\frac{dZ}{dz} + HZ = 0 \quad \text{at} \quad z = g \quad (B.11)$$

the following relationship is obtained:

$$\tan \beta_m g = \frac{2\beta_m H}{\beta_m^2 - H^2} \quad (B.12)$$

The homogeneous solution of $T(z, t) = Z(z) \cdot \Gamma(t)$ is a special case of the Sturm-Liouville problem, for which the solution is given by:

$$T(z, t) = \sum_{m=0}^{\infty} C_m Z(\beta_m, z) \cdot \Gamma(t) \quad (B.13)$$

but since $\Gamma(t) = e^{-\alpha \beta_m^2 t}$, Equation (B.13) becomes:

$$T(z, t) = \sum_{m=0}^{\infty} C_m Z(\beta_m, z) e^{-\alpha \beta_m^2 t} \quad (B.14)$$

where $Z(\beta_m, z)$ are the eigenfunctions and the initial condition is given by:

$$f(z) = \sum_{m=0}^{\infty} Z(\beta_m, z) = T_p - T_o \quad (B.15)$$

Utilizing the orthogonality property of the eigenfunctions:

$$\int_0^g Z(\beta_m, z) Z(\beta_n, z) dz = \begin{cases} 0, & \text{for } m \neq n; \\ N(\beta_m), & \text{for } m = n \end{cases} \quad (B.16)$$

It can be shown that:

$$C_m = \frac{1}{N(\beta_m)} \int_0^g Z(\beta_m, z') f(z') dz' \quad (B.17)$$

where $N(\beta_m)$ is defined as the norm:

$$N(\beta_m) = \int_0^g [Z(\beta_m, z)]^2 dz \quad (B.18)$$

and now substituting into Equation (B.13) results in:

$$T(z, t) = \sum_{m=0}^{\infty} \left[\frac{1}{N(\beta_m)} \int_0^g Z(\beta_m, z') f(z') dz' \right] Z(\beta_m, z) e^{-\alpha \beta_m^2 t} \quad (B.19)$$

The above solution is valid for $t > 0$, and as $t \rightarrow 0$ the temperature distribution in the plate approaches the temperature of the surroundings. Solving for $N(\beta_m)$ where $H_1 = H_2$ (see Ref. [41])

$$\frac{1}{N(\beta_m)} = \left[\beta_m^2 g + \frac{2}{H^2} g + 2H \right] \quad (B 20)$$

and substituting $Z(\beta_m, z)$, and $f(z')$ into Equation (B 21) gives

$$\begin{aligned} T(z, t) &= \sum_{m=0}^{\infty} \frac{1}{N(\beta_m)} \int_0^g (\beta_m \cos(\beta_m z') + H \sin(\beta_m z')) (T_p - T_o) dz' (\beta_m \cos(\beta_m z) + H \sin(\beta_m z)) e^{-\alpha \beta_m^2 t} \\ &= 2(T_p - T_o) \sum_{m=0}^{\infty} \frac{1}{N(\beta_m)} \left[\sin(\beta_m g) - \frac{Hg}{\beta_m} \cos(\beta_m g) + \frac{H}{\beta_m} \right] (\beta_m \cos(\beta_m z) + H \sin(\beta_m z)) e^{-\alpha \beta_m^2 t} \end{aligned} \quad (B 21)$$

$$\tan \beta_m g = \frac{2\beta_m H}{\beta_m^2 - H^2} \quad (B 12)$$

Rearranging Equation (B.21) to obtain the form presented by S. Tanaka by using the relationship, $U_n = \beta_m g$ gives:

$$T(z, t) = 2(T_p - T_o) \sum_{n=0}^{\infty} A_n \left(\cos \frac{U_n z}{g} + \frac{gh}{kU_n} \sin \frac{U_n z}{g} \right) \left[\frac{\sin U_n}{U_n} - \frac{Hg}{U_n^2} \cos(U_n - 1) \right] e^{-\alpha U_n^2 t / g^2} \quad (B 22)$$

where U_n is an eigenvalue satisfying:

$$\tan U_n = 2gkhU_n / (k^2 U_n^2 - g^2 h^2) \quad (B 23)$$

$$A_n = U_n^2 / (U_n^2 + (gh/k)^2 + (2gh/k)) \quad (B 24)$$

The final solution is expressed as the sum of temperature distributions due to both a moving point heat source and to a uniformly preheated plate (which is given in Equations (1-3). The centerline temperature distribution is obtained by substituting $y = 0$, $z = 0$ and $T_p = T_o$ into Equations(1), which reduces to:

$$T = T_o + \frac{\dot{Q}}{\pi k g} \exp\left(\frac{v^2 t}{2\alpha}\right) \sum_{n=0}^{\infty} A_n K_0 \left[\frac{vt}{g} \sqrt{U_n^2 + \left(\frac{vg}{2\alpha}\right)^2} \right] \quad (4)$$

where:

$$\tan U_n = 2khgU_n / (k^2 U_n^2 - g^2 h^2) \quad (2)$$

$$A_n = U_n^2 / (U_n^2 + (gh/k)^2 + (2gh/k)) \quad (3)$$

Appendix B

Experimental Data Acquisition System

An analogue $x-y$ recorder and a digital acquisition system were utilized for the collection of the experimental measurements reported in this investigation (Chapter 2). The microprocessor data acquisition system consisted of a 12 bit analogue-digital (A/D) unit built *in house* and a Apple II computer with appropriate software generated by the author [25].

This system (Figure 16) records the millivolt output from the thermocouple against time. A specific software program was written to control the collection of millivolt readings and also the processing of these readings into time-temperature measurements. Data collection commences at the point the thermocouple is injected into the weld pool (i.e. sampling begins when a threshold temperature of 50 °C is exceeded). The sampling rate is programmed to decrease with time, from a maximum rate of 100 to 1 readings per second after 200 seconds has elapsed. After sampling, the data collected is converted into corresponding temperature readings using a reference data table. These readings are then stored directly on diskette for future retrieval and analysis. In addition, a graphical plot of the experimental time-temperature cooling curve is displayed on the screen to show the overall results of the test. A listing of the experimental data acquisition program follows.

Figure 16

Schematic representation of the experimental data acquisition system

```

100 D$ = CHR$(4)
120 CALL - 936 REM CLEAR SCREEN
140 PRINT "EXPERIMENTAL DATA ACQUISITION PROGRAM"
160 REM AUTHOR J E MOORE
180 REM DATE 16/NOV/84
200
220 REM EXPERIMENTAL DATA ACQUISITION PROGRAM
240
260
280 REM THERMOCOUPLE REFERENCE TABLE PT/PT - 13%RH
300 DATA 0,13,26,38,50,61,72,82,92,102,112
320 DATA 121,131,140,149,158,167,175,184,192,201
340 DATA 209,217,226,234,242,250,258,265,273,281
360 DATA 289,296,304,311,319,326,334,341,348,356
380 DATA 363,370,378,385,392,399,406,413,420,427,435
400 DATA 434,441,448,455,462,469,476,483,490,496,504
420 DATA 503,510,517,523,530,537,544,550,557,563,570
440 DATA 570,577,583,590,596,603,609,616,622,629,635
460 DATA 635,642,648,654,661,667,673,680,686,692,698
480 DATA 698,705,711,717,723,729,736,742,748,754,760
500 DATA 760,766,772,778,784,790,796,802,808,814,820
520 DATA 820,826,832,838,844,850,856,862,868,873,879
540 DATA 879,885,891,897,902,908,914,920,925,931,937
560 DATA 937,943,948,954,960,965,971,977,982,988,993
580 DATA 993,999,1005,1010,1016,1021,1027,1032,1038,1043,1049
600 DATA 1049,1055,1060,1065,1071,1076,1082,1087,1093,1098,1104
620 DATA 1104,1109,1114,1120,1125,1131,1136,1141,1147,1152,1157
640 DATA 1157,1163,1168,1173,1179,1184,1189,1195,1200,1205,1211
660 DATA 1211,1216,1221,1227,1232,1237,1242,1248,1253,1258,1263
680 DATA 1263,1269,1274,1279,1285,1290,1295,1300,1306,1311,1316
700 DATA 1316,1321,1327,1332,1337,1342,1348,1353,1358,1363,1368
720 DATA 1368,1374,1379,1384,1389,1395,1400,1405,1410,1415,1421
740 DATA 1421,1426,1431,1436,1442,1447,1452,1457,1463,1468,1473
760 DATA 1473,1478,1484,1489,1494,1499,1505,1510,1515,1520,1526
780 DATA 1526,1531,1536,1542,1547,1552,1557,1563,1568,1573,1578
800 DATA 1578,1584,1589,1594,1600,1600
820
840 REM CONSTANTS
860
880 WRITE = 52480 REM PRINT FORMAT
900 POKE 49284,0: REM SELECT CHANNEL 1 SLOT 0
920 ZERO = 0: CVN = 49280: REM CONVERSION
940 AD = 49281: REM DATA STORAGE LOCATION
960 NS = 250: REM NUM OF SAMPLES
980 START = 0: O: J = 0: QI = 0: QY = 0: K = 0: I = 0: KK = 0
1000 DLAY(1) = 6: DLAY(2) = 40: DLAY(3) = 325: DLAY(4) = 900
1020 ST(1) = 8: ST(2) = 27.06: ST(3) = 182: ST(4) = 0
1040 REM SAMPLING TIMES(ST) SEC
1060 DT(1) = ST(1) / NS: DT(2) = ST(2) / NS
1080 DT(3) = ST(3) / NS: DT(4) = ST(4) / NS
1100 DIM THERMO(750), REFTABLE(255)
1120 FOR I = 0 TO 255: READ REFTABLE(I): NEXT I
1140 X = FRE(0): REM MEMORY MANAGEMENT
1160
1180 REM WELDING PARAMETERS
1200
1220 PRINT: PRINT "WELDING PARAMETERS" PRINT
1240 PRINT "PREHEAT TEMPERATURE( DEG. C)": INPUT PT
1260 PRINT "WELDING VOLTAGE (VOLTS)": INPUT VES
1280 PRINT "WELDING CURRENT (AMPS)": INPUT IEB
1300 PRINT "WELDING SPEED (CM/SEC)": INPUT V
1320 PRINT "PLATE THICKNESS (CM)": INPUT H
1340 PRINT " "
1360
1380 REM SAMPLING
1400
1420 VTAB 16: PRINT "AD READING"
1440 TI = 5: REM START SAMPLING
1460 POKE CVN,ZERO: START = PEEK(AD)

```

```

1480 VTAB 16: HTAB 16: PRINT START
1500 IF START < TI THEN 1460
1520 :
1540 FOR J = 1 TO 3
1580 FOR I = 1 TO NS
1600 K = NS * (J - 1) + I
1620 POKE CVH,ZERO:THERMO(K) = PEEK(AD)
1660 FOR PAUSE = 0 TO DLAY(J): NEXT PAUSE
1680 NEXT I
1720 NEXT J
1740 :
1760 PRINT "END OF SAMPLING": PRINT
1780 PRINT "DO YOU WISH TO PROCESS THIS RUN (YES/NO)": INPUT A$
1800 IF A$ = "NO" THEN END
1820 :
1840 REM SAVE DATA FILE ON DISK
1860 :
1880 D$ = CHR$(4): REM CTRL-D
1900 PRINT "NAME OF DATA FILE FOR DISK STORAGE?": INPUT Z$
1920 PRINT D$;"OPEN ";Z$
1940 PRINT D$;"WRITE ";Z$
1960 FOR J = 1 TO 3 * NS
1980 PRINT THERMO(J): NEXT J
2000 REM SAVE ONLY AD READING TO CONSERVE DISK SPACE
2020 PRINT PT: PRINT VEB: PRINT IEB
2040 PRINT V: PRINT H
2060 PRINT NS: PRINT DT(1): PRINT DT(2): PRINT DT(3): PRINT DT(4)
2080 PRINT D$;"CLOSE ";Z$
2100 :
2120 REM CONVERT AD READING INTO TEMPERATURE MEASUREMENTS (COMPRESSED)
2140 PRINT : PRINT "PLEASE WAIT PROCESSING DATA"
2160 FOR I = 1 TO (3 * NS)
2180 THERMO(I) = REFTABLE(THERMO(I))
2200 NEXT I
2220 :
2240 PRINT : PRINT "DO YOU WANT A GRAPHICAL DISPLAY"
2260 PRINT "OF THE DATA (YES/NO)?": INPUT B$
2280 IF B$ = "NO" THEN 2840
2300 :
2320 REM LOAD REFERENCE CURVE
2340 HGR2
2360 PRINT CHR$(4):"BLOAD TEMP GRID, A$4000"
2380 :
2400 REM GRAPHIC DISPLAY
2420 :
2440 FOR K = 1 TO 3
2460 FOR J = 1 TO NS
2480 QY = 170 - 160 * THERMO(NS * (K - 1) + J) / 1500
2500 QX = 38 + 200 * (DT(K) * J + KK) / 100
2520 IF QX < 38 THEN 2600
2540 IF QY < 10 OR QY > 191 THEN 2600
2560 IF QX > 278 THEN 2640
2580 HPLLOT QX,QY
2600 NEXT J
2620 KK = KK + ST(K): NEXT K
2640 INPUT A$: IF A$ < > "" THEN 2660
2660 TEXT
2680 GOTO 2840
2700 PRINT D$;"PR$":4
2720 PRINT D$;"IN$":4
2740 INPUT " ":T$
2760 PRINT D$;"IN$":0
2780 PRINT D$;"PR$":0
2800 PRINT T$
2820 RETURN
2840 PRINT : PRINT "DO YOU WANT A HARDCOPY OF THE DATA COLLECTED"
2860 INPUT A$
2880 IF A$ = "NO" THEN END
2900 WRITE = 62480: REM PRINTER FORMATING

```

```

2920 PRINT CHR$(4);"PR#1"; CHR$(13);
2940 PRINT CHR$(27);"M"; CHR$(10); REM PAGESIZE
2960 :
2980 IF B$ = "NO" THEN 3040
3000 POKE 1913,66: REM GRAPHIC HARDCOPY
3020 PRINT CHR$(17): REM CTRL-Q FOR PRINTOUT
3040 :
3060 REM OUTPUT TEMPERATURE DATA
3080 PRINT CHR$(27); CHR$(66); CHR$(8): REM LINE SPACING
3100 PRINT CHR$(0): REM CARRIAGE RETURN
3120 PRINT CHR$(14): REM NORMAL PRINT
3140 PRINT "EXPERIMENTAL DATA COLLECTED (SUMMARY) FILE NAME= ";Z$
3160 PRINT : PRINT "TEMPERATURE(DEG.C)    TIME(SEC.)"
3180 PRINT : PRINT :KK = 0
3200 PRINT CHR$(15): REM CONDENSED PRINT
3220 FOR K = 1 TO 3
3240 FOR J = 0 TO (NS + 13) STEP 25
3280 FOR I = 1 TO 25 STEP 2:TIME = DT(K) + (J + 1) + KK
3300 CALL WRITE:TIME;F6.3," ":
3320 NEXT I
3340 PRINT CHR$(0)
3360 FOR I = 0 TO 24 STEP 2
3370 QI = NS + (K - 1) + J + I
3375 IF QI > NS THEN 3400
3380 CALL WRITE:TEMPNO(QI);F6.1," ":
3400 NEXT I
3420 PRINT CHR$(0)
3440 PRINT CHR$(0)
3460 NEXT J
3480 KK = KK + ST(K)
3500 NEXT K
3520 END

```

Appendix C

Analytical Centerline Relationships

The closed form analytical centerline relationships presented in Chapter 2 (i.e. Equations (4.1) to (4.8)) are implemented in a Fortran computer program. This program called *Thermal* enables the calculation of the closed form analytical cooling parameters, $r_{8/3}$ and $r_{13/1}$ for any general welding situation. It is used in this investigation for comparison with the experimental and FEA values. In addition, the centerline time-temperature data generated can be displayed graphically using an accompanying program called *WTC_Plot*. The following contains the program listing:

```

*
* *****
*
*           PROGRAM      T H E R M A L
*
* *****
*
* DESCRIPTION:  This program calculates the thermal history of a weld
*               using various heat flow models.  The analytical schemes
*               are based on the equations derived by Rosenthal, Adams,
*               Graville, Yarioka, and Tanaka.  The following algorithm
*               can be used to calculate and plot the time-temperature
*               curve for various welding situations with or without
*               uniform preheat.
*
* AUTHOR:      J. E. MOORE
* DATE:        MAY, 25, 1984
*
* PROGRAM THERMAL
*
* PURPOSE:     The main program reads the input welding parameters and
*               then calls the various heat flow models to calculate the
*               weld thermal cycle for the prescribed input welding
*               parameters.  The output is printed in a format which is
*               acceptable for plotting using 'WTC_PLOT'.
*
* *****
*
* IMPLICIT LOGICAL(A-Z)
*
* INTEGER NPTS, I, J, NMODELS, LIMIT, COUNT
*
* REAL A, AMP, B, DTP, ETA, G, H, K, LAMBDA, PI, Q, QP, TG, TP, TEMP2, CP,
1  PHARG, V, VOLT, TMP(7, 50), TIM(7, 54), DTIM(4), TMPSRCH(4), DTIME
*
* COMMON /MOD/ G, H, K, LAMBDA, PI, Q, TG, V, TMP, TIM, DTIM,
*          TMPSRCH, CP
* COMMON /PRNT/ A, B, DTP, PHARG
* COMMON /SRCH/ TEMP2, DTIME, LIMIT, COUNT
*
* DATA PI, LIMIT, NPTS, NMODELS, TMPSRCH, DTIM
1  /3.1415927, 50, 50, 7,
2  1500., 800., 500., 100., .5, 1., 5., 50./
*
* PRINT*, 'Input welding parameters:'
* PRINT*, 'Voltage (Volts), Current (Amps), Efficiency (0<Eff<1)'
* PRINT*, 'Welding Speed (m/s), Plate Thickness (m)'
* PRINT*, 'Preheat and Surrounding Temperatures (Deg. C)'
* PRINT*, 'Local Preheating Width (Enter zero for uniform preheat)'
* READ(5, '(20G)') VOLT, AMP, ETA, V, G, TP, TG, B
* PRINT*, 'Enter Thermal Properties:'
* PRINT*, 'Thermal Conductivity (J/m.C.s), Diffusivity (m2/s)'
* PRINT*, 'Surface Conductance of plate (J/m2.C.s)'
* READ(5, '(20G)') K, A, H
*
* DO 10 J=1, NMODELS
*   DO 20 I=1, NPTS
*     TMP(J, I)=0.0
*     TIM(J, I)=0.0
20  CONTINUE
10  CONTINUE

```

```

C
C   COMPUTE AND PRINTOUT WELDING PARAMETERS
C
      CP=K/A
      Q=ETA*VOLT*AMP
      QP=Q/V
      LAMBDA=0.5/A
      DTP=TP-T0
      PHARG=-H/(LAMBDA*K*G)
*
      WRITE(6,190)
190  FORMAT(/.25X,'PROGRAM  THERMAL')
      WRITE(6,200)VOLT,AMP,ETA
200  FORMAT(/.5X,'Voltage=',F8.0,' Volts      Current=',F8.3,
1' Amps      Efficiency=',F4.3)
      WRITE(6,210)V,G
210  FORMAT(5X,'Weld speed=',F6.4,' m/s      Plate thickness=',F6.4,' m')
      WRITE(6,220)TP,B
220  FORMAT(5X,'Preheat temp.=',F5.1,' Deg. C      Local preheat width=',
*F5.4,' m')
      WRITE(6,230)Q,QP
230  FORMAT(5X,'Heat input=',F7.1,' Watts      Heat input per unit',
1' thickness=',E10.4,' J/m')
      WRITE(6,240)K,A
240  FORMAT(5X,'Thermal conductivity=',F6.2,' J/(m.s.C)      Thermal',
1' diffusivity=',E10.4,' m2/s')
      WRITE(6,250)H
250  FORMAT(5X,'Surface heat transfer coefficient=',F6.2,
*      ' J/m2.s.C',///)
*
      CALL MODEL1(NPTS)
      CALL MODEL2(NPTS)
      CALL MODEL3(NPTS)
      CALL MODEL4(NPTS)
      CALL MODEL5(NPTS)
      CALL MODEL6(NPTS)
      CALL MODEL7(NPTS)
      CALL DATOUT(VOLT,AMP,ETA,TP,NMODELS,NPTS)
      STOP
      END
*
* *****#*****
* *
* *      SUBROUTINE      S E A R C H      *
* *
* *****#*****
*
* DESCRIPTION:  This subroutine is called by MODEL to determine the
*               upper time limit of the time-temperature thermal cycle
*               for plotting.  An iteration technique is used to find
*               the time at which any particular temperature occurs.
*               (ie. T1500, T800, T500, or T100)
*
* *****#*****
*
*      SUBROUTINE SEARCH(TEMP,TIME,TEMP1,DTIM,FLAG)
*
*      IMPLICIT LOGICAL(A-Z)
*
*      INTEGER FLAG,LIMIT,COUNT
*

```

```

      REAL TEMP,TEMP1,TEMP2,DTEMP,ERROR,TIME,DTIME,DTIM
      COMMON /SRCH/ TEMP2,DTIME,LIMIT,COUNT
      ERROR=TEMP-TEMP1
      IF (FLAG.EQ.0) THEN
        TEMP2=0.0
        DTIME=DTIM
        FLAG=1
        COUNT=1
        LIMIT=50
        IF (ERROR.LT. 0.0) FLAG = 4
      ENDIF
      IF (FLAG.EQ. 4 ) THEN
        IF (ERROR.LT. 0.0) THEN
          TIME = TIME - DTIM
          GO TO 10
        ELSE
          FLAG=1
        ENDIF
      ENDIF
      DTEMP=(TEMP2-TEMP)
      TEMP2=TEMP
      IF (ERROR.GT.0.0 .AND. FLAG.EQ.1) THEN
        IF (ABS(DTEMP).LT.20) DTIME=2*DTIME
        TIME=TIME+DTIME
        GOTO 10
      ENDIF
      IF (ABS(ERROR).GT.0.1) THEN
        DTIME=DTIME/2
        IF (ERROR.GT.0.0) THEN
          TIME=TIME+DTIME
        ELSE
          TIME=TIME-DTIME
        ENDIF
        FLAG=2
        GOTO 10
      ENDIF
      FLAG=3
10    CONTINUE
      IF (COUNT.GT.LIMIT) THEN
        PRINT*,'          *** SUBROUTINE SEARCH ***'
        PRINT*,'*** ITERATION LIMIT EXCEEDED *** LIMIT= ',LIMIT
        STOP
      ENDIF
      IF (TIME.LE.0) TIME=1.E-7
      COUNT=COUNT+1
      RETURN
      END

```

```

      SUBROUTINE  P R E H E A T

```

```

* DESCRIPTION: This subroutine is used to take into account surface
*               and conductive heat losses due to preheating. The code
*               is based on the equations derived by Graville and
*               Yarielka, for uniform or locally preheated plates.

```



```

* *****
*
*   SUBROUTINE PREHEAT(TEMP,TIME)
*
*   IMPLICIT LOGICAL(A-Z)
*
*   REAL TEMP,TIME,DTP,PHARG,A,B,COEFF1
*
*   COMMON /PRHT/ A,B,DTP,PHARG
*
*   COEFF1=DTP*EXP(PHARG*TIME)
*   IF (B.EQ.0.0) THEN
*     TEMP=TEMP+COEFF1
*   ELSE
*     TEMP=TEMP+COEFF1*0.5*B/SQRT(2*A*TIME+B*B)
*   ENDIF
*   RETURN
*   END
*
* *****
*
*   SUBROUTINE      D A T O U T
*
* *****
*
* DESCRIPTION:  This subroutine outputs the time-temperature data
*              in a format which is acceptable for plotting using
*              'CAL_PLOT' library program.
*
* *****
*
*   SUBROUTINE DATOUT(VOLT,AMP,ETA,TP,NMODELS,NPTS)
*
*   IMPLICIT LOGICAL(A-Z)
*
*   REAL TMP(7,50),TIM(7,54),TIM15T01(7),TIM8T05(7),
* *   VOLT,AMP,ETA,Q,V,G,TP,K,A,H,LAMBDA,PI,TO
*
*   INTEGER I,J,NMODELS,NPTS
*   CHARACTER FID10*10
*
*   COMMON /MOD/ G,H,K,LAMBDA,PI,Q,TO,V,TMP,TIM,DTIM,
* *   TMPSECH,CP
*   COMMON /PRHT/ A,B,DTP,PHARG
*
*   FID10='*WTC_DATA'
*   DO 5 J=1,NMODELS
*     WRITE(6,90) J
*     TIM15T01(J)=TIM(J,NPTS+4)-TIM(J,NPTS+1)
*     TIM8T05(J)=TIM(J,NPTS+3)-TIM(J,NPTS+2)
*     WRITE(6,100) TIM15T01(J),TIM8T05(J)
*   WRITE(6,92)
*     DO 7 I=1,NPTS
*       TIM(J,I)=TIM(J,I)-TIM(J,NPTS+1)
*       WRITE(6,95) TIM(J,I),TMP(J,I)
*     CONTINUE
*   CONTINUE
*   OPEN (UNIT=10,FILE=FID10,STATUS='UNKNOWN',ERR=1000)
*   WRITE(10,105,ERR=1002) NMODELS,NPTS
*   WRITE(10,110,ERR=1002) VOLT,AMP,ETA,Q,V,G,TP,K,A,H
*   WRITE(10,115,ERR=1002) TIM15T01(J),J=1,NMODELS

```

```

WRITE(10,115,ERR=1002) TIMST05(J),J=1,NMODELS
DO 10 I=1,NPTS
    WRITE(10,115,ERR=1002) TIM(J,I),J=1,NMODELS
    WRITE(10,115,ERR=1002) TMP(J,I),J=1,NMODELS
10  CONTINUE
    CLOSE (UNIT=10,STATUS='KEEP',ERR=1004)
    GO TO 1010
1000 PRINT*, ' *** ERROR IN OPENING UNIT 10 *WTC_DATA ***'
    STOP
1002 PRINT*, ' *** ERROR IN WRITING TO UNIT 10 *WTC_DATA ***'
    STOP
1004 PRINT*, ' *** ERROR IN CLOSING UNIT 10 *WTC_DATA'
    STOP
1010 CONTINUE
90  FORMAT(/,18I,'WELD THERMAL HISTORY FOR MODEL - ',I2,/)
92  FORMAT(18I,'TIME (Sec.)',5I,'TEMPERATURE (Deg. C)',/)
95  FORMAT(15I,F12.2,10I,F10.1)
100 FORMAT(15I,'Cooling Time from 1500 TO 100 Deg.C',F9.2,' Sec.',
*      /,15I,'Cooling Time from 800 TO 500 Deg.C',F9.2,' Sec.',/)
105 FORMAT(1I,2I5)
110 FORMAT(1I,4E15.8/,1I,6E15.8)
115 FORMAT(10F9.2)
    RETURN
    END

```

```

*
* *****
*
*      FUNCTION      E I P K O
*
* *****
*
* DESCRIPTION:  This function subprogram is called by MODEL to
*               calculate a besseel function of the first order by
*               passing the arguments associated with Ko(arg2) and
*               exp(arg1).
*
* *****
*
*      DOUBLE PRECISION FUNCTION EIPKO(ARG1,ARG2)
*
*      IMPLICIT DOUBLE PRECISION (A-Z)
*
*      REAL ARG1,ARG2
*
*
*      MODIFIED BESSEL FUNCTION 0<ARG1<=2
*
*      IF (ARG2.GT.0 .AND. ARG2.LE.2.) THEN
*          I=ARG2/2.
*          Y=ARG2/3.75
*          I0=1 +3.5156229*Y**2 +3.0899424*Y**4 +1.2067492*Y**6
*          * +.2659732*Y**8 +0.0360768*Y**10+0.0045813*Y**12
*          K0=-DL0G(X)*I0 - .5*721566 +.42278420*I**2 +.23069756*I**4
*          * +.03488590*I**6 +.00262698*I**8 +.00010750*I**10
*          * +.00000740*I**12
*          EIPKO=EIP(ARG1)*K0
*      ELSE
*
*      MODIFIED BESSEL FUNCTION 2<ARG2<=128
*

```

```

      IF (ARG2.GT.2. .AND. ARG2.LE.128.) THEN
        I=2./ARG2
        COEFF=1.25331414 -.07832358*I +.02189568*I**2
        *      -.01062446*I**3 +.00587872*I**4 -.00251540*I**5
        *      +.00053208*I**6
        EIPK0=EXP(ARG1-ARG2)*COEFF/SQRT(ARG2)
      ELSE
        C  MODIFIED BESSEL FUNCTION  ARG2>128
        *
        PI=3.141592654
        EIPK0=SQRT(0.5*PI/ARG2)*EXP(ARG1-ARG2)
      ENDIF
    ENDIF
  RETURN
END

*
* *****
* *
* *      SUBROUTINE      M O D E L 1
* *
* *****
*
* DESCRIPTION:  This subroutine calculates the weld centerline thermal
*               cycle using Rosenthal's three-dimensional heat flow
*               equation in conjunction with Graville's uniform and local
*               preheat equations.
*
* *****
*
* SUBROUTINE MODEL1(NPTS)
*
* IMPLICIT LOGICAL(A-Z)
*
* INTEGER FLAG,I,NPTS,J
*
* REAL COEFF1,DTP,K,PI,TEMP,DTIM(4),DTIM1,G,H,LAMBDA,
* *    TMP(7,50),TIM(7,54),TO,V,Q,TMPSRCH(4),A,B
*
* COMMON /MOD/ G,H,K,LAMBDA,PI,Q,TO,V,TMP,TIM,DTIM,
* *    TMPSRCH,CP
* COMMON /PRHT/ A,B,DTP,PHARG
*
* PRINT*,'MODEL1'
* J=1
* COEFF1=Q/(2*PI*K*V)
* DO 10 I=1,4
*   TIM(J,NPTS+I)=COEFF1/(TMPSRCH(I)-TO)
*   IF (DTP.LT.0.1) GO TO 21
*   FLAG=0
*   REPEAT 20, WHILE (FLAG.NE.3)
*     TEMP=TO+COEFF1/TIM(J,NPTS+I)
*     CALL PREHEAT(TEMP,TIM(J,NPTS+I))
*     CALL SEARCH(TEMP,TIM(J,NPTS+I),TMPSRCH(I),DTIM(I),FLAG)
20   CONTINUE
21   CONTINUE
10  CONTINUE
  DTIM1=TIM(J,NPTS+4)-TIM(J,NPTS+1)
  DO 11 I=1,NPTS
    TIM(J,I)=DTIM1*(FLOAT(I-1)/(NPTS-1))**2+TIM(J,NPTS+1)
    TMP(J,I)=TO+COEFF1/TIM(J,I)
  
```

```

      IF (DTP.GT.0.1) CALL PREHEAT(TMP(J,I),TIM(J,I))
11  CONTINUE
    RETURN
    END

*
* *****
* *
* *          SUBROUTINE  M O D E L 2
* *
* *****
*
* DESCRIPTION:  This subroutine is called by MAIN to calculate the
*               weld centerline thermal cycle using Rosenthal's two-
*               dimensional heat flow model, where only conduction
*               to the base metal is considered. (ie. no surface losses)
*               This equation is used in conjunction with Graville's
*               uniform and local preheat equation.
*
* *****
*
* SUBROUTINE MODEL2(NPTS)
*
*   INTEGER FLAG,I,NPTS,J
*
*   REAL COEFF1,DTP,K,PI,TEMP,TMP(7,50),TIM(7,54),DTIM(4),Q,
*   *     TO,V,TMPSECH(4),I1,EXPKO,COARG,LAMBDA,CP,A,DTIM1,G,H,B
*
*   COMMON /MOD/ C,H,K,LAMBDA,PI,Q,TO,V,TMP,TIM,DTIM,
*   *           TMPSECH,CP
*   COMMON /PRHT/ A,B,DTP,PHARG
*
*   PRINT*,'MODEL2'
*   J=2
*   COEFF1=Q/(2*PI*K*G)
*   COARG=LAMBDA*V*V
*   DO 10 I=1,4
*     FLAG=0
*     TIM(J,NPTS+I)=(Q/(V*G))**2/((4*PI*K*CP)*(TMPSECH(I)-TO)**2)
*     REPEAT 20, WHILE (FLAG.NE.3)
*       I1=COARG+TIM(J,NPTS+I)
*       TEMP=TO+COEFF1*EXPKO(I1,I1)
*       IF (DTP.GT.0.1) CALL PREHEAT(TEMP,TIM(J,NPTS+I))
*       CALL SEARCH(TEMP,TIM(J,NPTS+I),TMPSECH(I),DTIM(I),FLAG)
20  CONTINUE
10  CONTINUE
*
*   DTIM1=TIM(J,NPTS+4)-TIM(J,NPTS+1)
*   DO 11 I=1,NPTS
*     TIM(J,I)=DTIM1*(FLOAT(I-1)/(NPTS-1))**2+TIM(J,NPTS+1)
*     I1=COARG+TIM(J,I)
*     TMP(J,I)=TO+COEFF1*EXPKO(I1,I1)
*     IF (DTP.GT.0.1) CALL PREHEAT(TMP(J,I),TIM(J,I))
11  CONTINUE
    RETURN
    END

*
* *****
* *
* *          SUBROUTINE  M O D E L 3
* *
* *****

```

```

*****
* DESCRIPTION: This subroutine is called by MAIN to calculate the
* weld centerline thermal cycle using Rosenthal's two-
* dimensional heat flow model, where conduction to the
* base metal and surface heat losses are considered.
* This equation is used in conjunction with Graville's
* uniform and local preheat equation.
*
*****

SUBROUTINE MODEL3(NPTS)
  IMPLICIT LOGICAL(A-Z)
  INTEGER FLAG,I,NPTS,J
  REAL COEFF1,DTP,K,DTIM(4),PI,TEMP,TMP(7.50),TIM(7.54),G,DTIM1,Q,
  * TO,V,TMPSECH(4),I1,I2,KIPKO,COARG1,COARG2,LAMBDA,CP,H,A,B
  COMMON /MOD/ G,H,K,LAMBDA,PI,Q,TO,V,TMP,TIM,DTIM,
  * TMPSECH,CP
  COMMON /PRHT/ A,B,DTP,PHARG
  PRINT*, 'MODEL3'
  J=3
  COEFF1=Q/((2*PI*K*G)
  COARG1=LAMBDA*V*V
  COARG2=V*SQR((LAMBDA*V)**2+2*H/(K*G))
  DO 10 I=1,4
    FLAG=0
    TIM(J,NPTS+I)=Q/((2*PI*K*V)*(TMPSECH(I)-TO))
    REPEAT 20, WHILE (FLAG.NE.3)
      I1=COARG1*TIM(J,NPTS+I)
      I2=COARG2*TIM(J,NPTS+I)
      TEMP=TO+COEFF1*KIPKO(I1,I2)
      IF (DTP.GT.0.1) CALL PREHEAT(TEMP,TIM(J,NPTS+I))
      CALL SEARCH(TEMP,TIM(J,NPTS+I),TMPSECH(I),DTIM(I),FLAG)
20    CONTINUE
10    CONTINUE
  DTIM1=TIM(J,NPTS+4)-TIM(J,NPTS+1)
  DO 11 I=1,NPTS
    TIM(J,I)=DTIM1*(FLOAT(I-1)/(NPTS-1))+2*TIM(J,NPTS+1)
    I1=COARG1*TIM(J,I)
    I2=COARG2*TIM(J,I)
    TMP(J,I)=TO+COEFF1*KIPKO(I1,I2)
    IF (DTP.GT.0.1) CALL PREHEAT(TMP(J,I),TIM(J,I))
11  CONTINUE
  RETURN
  END

*****
* SUBROUTINE      M O D E L 4
*
*****
* DESCRIPTION: This subroutine calculates the weld centerline thermal

```

```

*      cycle using Rosenthal's and Adams approximation to
*      Rosenthal's two-dimensional heat flow equation in
*      conjunction with Graville's uniform and local preheat
*      equations.
*
* *****
*
*      SUBROUTINE MDEL4(NPTS)
*
*      IMPLICIT LOGICAL(A-Z)
*
*      INTEGER FLAG,I,NPTS,J
*
*      REAL COEFF1,DTP,K,DTIM(4),PI,TEMP,A,DTIM1,Q,H,B,
*      *      TMP(7,50),TIM(7,54),TO,V,TMPSRCH(4),G,LAMBDA,CP
*
*      COMMON /MOD/ G,H,K,LAMBDA,PI,Q,TO,V,TMP,TIM,DTIM,
*      *      TMPSRCH,CP
*      COMMON /PRHT/ A,B,DTP,PHARG
*
*      PRINT*,'MODEL4'
*      J=4
*      COEFF1=Q/((2*K*V*G+SQRT(2*PI*LAMBDA)))
*      DO 10 I=1,4
*          TIM(J,NPTS+I)=(Q/(V*G))**2/((4*PI*K*CP)*(TMPSRCH(I)-TO)**2)
*          IF (DTP.LT.0.1) GO TO 21
*          FLAG=0
*          REPEAT 20, WHILE (FLAG.NE.3)
*              TEMP=TO+COEFF1/SQRT(TIM(J,NPTS+I))
*              CALL PREHEAT(TEMP,TIM(J,NPTS+I))
*              CALL SEARCH(TEMP,TIM(J,NPTS+I),TMPSRCH(I),DTIM(I),FLAG)
20      CONTINUE
21      CONTINUE
10      CONTINUE
*
*      DTIM1=TIM(J,NPTS+4)-TIM(J,NPTS+1)
*      DO 11 I=1,NPTS
*          TIM(J,I)=DTIM1*(FLOAT(I-1)/(NPTS-1))**2+TIM(J,NPTS+1)
*          TMP(J,I)=TO+COEFF1/SQRT(TIM(J,I))
*          IF (DTP.GT.0.1) CALL PREHEAT(TMP(J,I),TIM(J,I))
11      CONTINUE
*      RETURN
*      END
*
* *****
*
*      SUBROUTINE MDEL5
*
* *****
*
*      DESCRIPTION: This subroutine calculates the weld centerline thermal
*      cycle using Graville's and Yuricka's two-dimensional heat
*      equation with a Bessel function approximation, in
*      conjunction with Graville's uniform and local preheat
*      equations.
*
* *****
*
*      SUBROUTINE MDEL5(NPTS)

```

```

*
*   IMPLICIT LOGICAL(A-Z)
*
*   INTEGER FLAG,I,NPTS,J
*
*   REAL COEFF1,DTP,K,DTIM(4),PI,TEMP,CP,A,PHARG,DTIM1,Q,
*   *   TMP(7,50),TIM(7,54),TO,V,TMPSRCH(4),G,LAMBDA,H,B
*
*   COMMON /MOD/ G,H,K,LAMBDA,PI,Q,TO,V,TMP,TIM,DTIM,
*   *   TMPSRCH,CP
*   COMMON /PRHT/ A,B,DTP,PHARG
*
*   PRINT*,'MODEL5'
*   J=5
*   COEFF1=Q/(2*K+V*G*SQRT(2*PI*LAMBDA))
*   DO 10 I=1,4
*       FLAG=0
*       TIM(J,NPTS+1)=Q/((2*PI*K+V)*(TMPSRCH(I)-TO))
*       REPEAT 20, WHILE (FLAG.NE.3)
*       TEMP=TO+COEFF1*EXP(PHARG*TIM(J,NPTS+1))/SQRT(TIM(J,NPTS+1))
*       IF (DTP.GT.0.1) CALL PREHEAT(TEMP,TIM(J,NPTS+1))
*       CALL SEARCH(TEMP,TIM(J,NPTS+1),TMPSRCH(I),DTIM(I),FLAG)
20      CONTINUE
10      CONTINUE
*
*   DTIM1=TIM(J,NPTS+4)-TIM(J,NPTS+1)
*   DO 11 I=1,NPTS
*       TIM(J,I)=DTIM1*(FLOAT(I-1)/(NPTS-1))+2*TIM(J,NPTS+1)
*       TMP(J,I)=TO+COEFF1*EXP(PHARG*TIM(J,I))/SQRT(TIM(J,I))
*       IF (DTP.GT.0.1) CALL PREHEAT(TMP(J,I),TIM(J,I))
11      CONTINUE
*   RETURN
*   END
*
* *****
* *
* *   SUBROUTINE   M O D E L 5
* *
* *****
*
* DESCRIPTION: This subrountine calculates the weld centerline thermal
* cycle using Rosenthal's 2.5-dimensional heat flow
* equation where only conduction to the base plate is
* is considered, and is used in conjunction with Graville's
* uniform and local preheat equations.
*
* *****
*
*   SUBROUTINE MODEL5(NPTS)
*
*   IMPLICIT LOGICAL(A-Z)
*
*   INTEGER FLAG,N,NPTS,I,J
*
*   REAL COEFF1,DTP,K,DTIM(4),PI,TEMP,RN,A,CP,DTIM1,B,H,PHARG,
*   *   TMP(7,50),TIM(7,54),TO,V,Q,TMPSRCH(4),G,LAMBDA,SUM,
*   *   CO1,CO2,XI,DELTA
*
*   COMMON /MOD/ G,H,K,LAMBDA,PI,Q,TO,V,TMP,TIM,DTIM,
*   *   TMPSRCH,CP
*   COMMON /PRHT/ A,B,DTP,PHARG

```

```

PRINT*, 'MODEL6'
J=6
COEFF1=Q/(2*PI*K)
CO1=LAMBDA*V
CO2=4*G*G
DO 10 I=1,4
  FLAG=0
  TIM(J,NPTS+I)=(Q/(V*G))**2/((4*PI*K*CP)*(TMPSRCH(I)-T0)**2)
  REPEAT 20, WHILE (FLAG.NE.3)
    SUM=0.0
    XI=TIM(J,NPTS+I)*V
    DO 30 N=1,50
      RN=SQRT(XI*XI+CO2*N*N)
      DELTA=EXP(CO1*(XI-RN))/RN
      SUM=SUM+DELTA
      IF (DELTA.LT.1E-7) GO TO 31
30    CONTINUE
    WRITE(6,100) N
31    CONTINUE
    TEMP=T0+COEFF1*(1/XI+2*SUM)
    IF (DTP.GT.0.1) CALL PREHEAT(TEMP,TIM(J,NPTS+I))
    CALL SEARCH(TEMP,TIM(J,NPTS+I),TMPSRCH(I),DTIM(I),FLAG)
20    CONTINUE
10    CONTINUE
*
DTIM1=TIM(J,NPTS+4)-TIM(J,NPTS+1)
DO 11 I=1,NPTS
  SUM=0.0
  TIM(J,I)=DTIM1*(FLOAT(I-1)/(NPTS-1))**2+TIM(J,NPTS+1)
  XI=TIM(J,I)*V
  DO 40 N=1,40
    RN=SQRT(XI*XI+CO2*N*N)
    DELTA=EXP(CO1*(XI-RN))/RN
    SUM=SUM+DELTA
    IF (DELTA.LT.1E-8) GO TO 41
40    CONTINUE
41    CONTINUE
    TMP(J,I)=T0+COEFF1*(1/XI+2*SUM)
    IF (DTP.GT.0.1) CALL PREHEAT(TMP(J,I),TIM(J,I))
11    CONTINUE
100  FORMAT(6X, 'MODEL6 - NONCONVERGENT SERIES: NUM. OF TERMS=', I4)
RETURN
END

* *****
* *
* *      SUBROUTINE M O D E L 7
* *
* *****
*
* DESCRIPTION: This subroutine calculates the weld centerline thermal
*              cycle using Yuricka's solution to the three-dimensional
*              heat distribution equation due to a point source moving
*              on a preheated plate of finite thickness.
*              Note: Compile with the ADP (Automatic Double Precision)
*                   option.
*
* *****
*
SUBROUTINE MODEL7(NPTS)

```



```

      IMPLICIT LOGICAL(A-Z)

      INTEGER INTER, NTERMS, NLIMIT, NDIV, I, N, FLAG, NPTS, J

      REAL*8 AN(50), C, CO1, CO2, CO3, CO4, COEFF1, DELTA, DIFF, DTP, EIPK0,
      *   LAMBDA, PI, Q, SUM1, SUM2, TOL1, TOL2, TEMP, TMPSECH(4), DTIM(4),
      *   TMP(7,50), TIM(7,54), TO, U(50), UN, UNINC, Y1, Y2, V, X1, X2, DTIM1,
      *   TT, CP, ANDIFF, G, H, K, A, B

      COMMON /MOD/ G, H, K, LAMBDA, PI, Q, TO, V, TMP, TIM, DTIM,
      *   TMPSECH, CP
      COMMON /PRHT/ A, B, DTP, PHARG

      DATA INTER, NTERMS, NDIV, TOL1, TOL2, SUM1, SUM2, ANDIFF
      *   /0,50,100,1E-8,1E-6,0,0,1E11/

      PRINT*, 'MODEL7'
      J=7
      COEFF1=Q/(PI*K*G)
      CO1=LAMBDA*V*V
      CO2=V/G
      CO3=(V*G*LAMBDA)**2
      CO4=2*LAMBDA*G*G
      NLIMIT=NTERMS
      C=G*H/K
      UN=C

      DO 10 N=1, NTERMS
        DO 20 I=1, NDIV
          INTER=0
          UNINC=PI/(2*NDIV)
          UN=UN+UNINC
          Y1=2*C+UN/(UN+UN-C*C)
          Y2=TAN(UN)
          DIFF=Y1-Y2
          IF (DIFF.LT.0) THEN
            IF (ABS(DIFF).GT.TOL1) THEN
              UNINC=UNINC/2.
              IF (DIFF.GT.0) THEN
                UN=UN+UNINC
              ELSE
                UN=UN-UNINC
              ENDIF
              Y1=2*C+UN/(UN+UN-C*C)
              Y2=TAN(UN)
              DIFF=Y1-Y2
              INTER=INTER+1
              IF (INTER.GT.30) THEN
                WRITE(*,200) INTER
                GO TO 10
              ENDIF
            GO TO 30
          ENDIF
          U(N)=UN
          AN(N)=UN+UN/(UN+UN+C*C*2*C)
          UN=PI*N
          IF (N.GT.1) ANDIFF=AN(N)-AN(N-1)
          IF (ABS(ANDIFF).LT.TOL2) THEN
            NLIMIT=N
            GO TO 40
          ENDIF
        GO TO 10
      GO TO 10
    
```

```

      ENDIF
20      CONTINUE
      WRITE(*,210) UN
10      CONTINUE
40      CONTINUE
200     FORMAT(5X,'MODEL-7 Non-convergent value for eigenvalue after',
      *15,' iterations',/,',for the interval from ',E11.4)
210     FORMAT(5X,'MODEL-7 Unsatisfied eigenvalues for the interval from',
      *E11.4)
      *
      *
      DO 5 I=1,4
        FLAG=0
        TIM(J,NPTS+I)=Q/((2*PI*K*V)*(TMPSRCH(I)-TO))
        REPEAT 50, WHILE (FLAG.NE.3)
          SUM1=0.0
          X1=C01+TIM(J,NPTS+I)
          DO 60 N=1,NLIMIT
            X2=C02+TIM(J,NPTS+I)+SQRT(U(N)*U(N)+COS)
            DELTA=AN(N)*RIPK0(X1,X2)
            SUM1=SUM1+DELTA
            IF (DELTA.LT.1E-7) GO TO 61
60          CONTINUE
61          CONTINUE
          IF (DTP.GT.0.1) THEN
            SUM2=0.0
            DO 70 N=1,NLIMIT
              TT=AN(N)/U(N)*(SIN(U(N))-C*(COS(U(N))-1)/U(N))
              DELTA=TT*EXP(-U(N)*U(N)+TIM(J,NPTS+I)/C04)
              SUM2=SUM2+DELTA
              IF (DELTA.LT.1E-7) GO TO 71
70            CONTINUE
          ENDIF
71          CONTINUE
          TEMP=TO+COEFF1*SUM1+2*DTP*SUM2
          CALL SEARCH(TEMP,TIM(J,NPTS+I),TMPSRCH(I),DTIM(I),FLAG)
50        CONTINUE
5      CONTINUE
      *
      DTIM1=TIM(J,NPTS+4)-TIM(J,NPTS+1)
      DO 80 I=1,NPTS
        SUM1=0.0
        TIM(J,I)=DTIM1*(FLOAT(I-1)/(NPTS-1))+2*TIM(J,NPTS+1)
        X1=C01+TIM(J,I)
        DO 90 N=1,NLIMIT
          X2=C02+TIM(J,I)+SQRT(U(N)*U(N)+COS)
          DELTA=AN(N)*RIPK0(X1,X2)
          SUM1=SUM1+DELTA
          IF (DELTA.LT.1E-8) GO TO 91
90        CONTINUE
91        CONTINUE
        IF (DTP.GT.0.1) THEN
          SUM2=0.0
          DO 100 N=1,NLIMIT
            TT=AN(N)/U(N)*(SIN(U(N))-C*(COS(U(N))-1)/U(N))
            DELTA=TT*EXP(-U(N)*U(N)+TIM(J,I)/C04)
            SUM2=SUM2+DELTA
            IF (DELTA.LT.1E-7) GO TO 101
100        CONTINUE
101        CONTINUE
      ENDIF
      *

```

TMP(J,I)=T0+COEFF1*SUM1+2*DIP*SUM2

80

CONTINUE
RETURN
END

Appendix D

Graphical Display of Results

A general purpose two-dimensional plotting routine was written to graphically display the results of this investigation. This program called *Plot-2D* uses Fortran - callable plotting subroutines found in the *CALPLOT* library. Its capability has been demonstrated by the 2D graphs presented in this thesis. The input data file structure is documented within the program listing presented below, (for further details readers are referred to the *CALPLOT* Reference Manual):

```

*
* *****
*
*               PROGRAM  P L O T - 2 D
*
* *****
*
* DESCRIPTION   This program is a limited purpose two dimensional
*               plotting routine using CAL_PLOT library subroutines
*
* AUTHOR       J E MOORE
* DATE         15 SEPT 84
*
* *****
*
*               PROGRAM PLOT2D
*
*
*               INTEGER INTEQ(5),LINTYP(5),NCURVES,NPTS,I,J,K,
*               *       FLAG,XDIV,YDIV,NDECI,NDECY,NVARI(5),NVARY(5),
*               *       SYMTYP(5),NIVAR,NYVAR
*
*               REAL    XMIN,XMAX,YMIN,YMAX,XVAR(5,99),YVAR(5,99),
*               *       SIZE,XSIZE,YSIZE,X,Y,DX,DY,XSCALE,YSCALE,
*               *       XINC,YINC,ILEN,YLEN,XVALUE,YVALUE,XPOINT(101),
*               *       YPOINT(101),FACTO
*
*               CHARACTER ANS*1,FID10*30,TITLE*70,STRING*8,LIN*1,
*               *       XLABEL*40,YLABEL*40,PNT*1,SYN*1
*
*               DATA FLAG,LINTYP,INTEQ,SYMTYP
*               * /0, 0,1,2,4,5, 17,17,17,17,17, 0,15,2,3,4/
*
* *****
*
*               DATA FILE STRUCTURE
*
* *****
*
*               File Format.
*
*               line1  Plot Title (<70 characters)
*               line2  XLABEL    (<40 characters)
*               line3  YLABEL    (<40 characters)
*               line4  NPTS,NIVAR,NYVAR
*               line5  XMIN,XMAX,XDIV, YMIN,YMAX,YDIV
*               line6  NDECI, NDECY
*               line7-(NPTS+7)
*                   X1 X2... X5      Y1 Y2... Y5
*
*               How To Run PLOT2D_SI
*
*               !FORTRAN PLOT2D_SI
*               !LINK *G OVER *L(UNSAT=:CALPLOT.LIB)
*               !*L
*
* *****

```

```

*      Dictionary for INTEGER variables
*
*.....
*
*      FLAG      Flag for overlay plots
*      I,J,K      DO Loop counters
*      LINTYP     Type of line to be plotted
*                (default or user selected)
*      NCURVES    Number of curves to be plotted (<5)
*      NDECI,NDECY Number of decimal places for the x,y axes
*      NPTS       Number of data points to be plotted (<100)
*      NIVAR,NYVAR Array used to store the (x,y) variables
*      NVARI,NVARY Number of I and Y variables in inputfile (<5)
*      SYMTYP     Symbol type to be plotted
*
*.....
*
*      Dictionary for REAL variables
*
*.....
*
*      DI,DY      Length of the tick marks
*      SIZE       Character size plotted
*      IINC,YINC   Distance tick marks are apart (in data units)
*      ILEN,YLEN   Length of the x and y axes ( in inches)
*      IMAX,YMAX   Maximum value to be plotted on the x,y axes
*      IMIN,YMIN   Minimum value to be plotted on the x,y axes
*      IPOINT,    : Array containing the (x,y) curve data
*      YPOINT     currently being plotted
*      IVAR,YVAR   Array containing all the data from the
*                inputfile (ie. I(NIVAR,NPTS), Y(NYVAR,NPTS))
*
*.....
*
*      Dictionary for CHARACTER variables
*
*.....
*
*      FID10:     File name opened with unit = 10
*      TITLE:     Plot title
*      LIN:       For the data points joined by a line
*      PNT:       Data points plotted only
*      SYM:       Symbols to be plotted
*      XLABEL:    String containing the X and Y axes labels
*      YLABEL
*
*.....
*
C      Open UNIT 10 and read in data file to be plotted
*
*.....
*
1050 CONTINUE
5   FID10='
PRINT*, 'What is the name of your plotting file'
READ (5,FMT='(A30)') FID10
IF (FID10.EQ.' ') GO TO 5
*
OPEN (UNIT=10,FILE=FID10,STATUS='OLD',ERR=1000)

```

```

READ(10,'(A70)') TITLE
READ(10,'(A40)') ILABEL
READ(10,'(A40)') YLABEL
READ(10,'(30G)',ERR=1002) NPTS,NIVAR,NTVAR
IF (NPTS GT 99 .OR. NIVAR GT 5 .OR. NIVAR GT 5) THEN
  PRINT*,'Too many data points or variables (modify source)'
  STOP
ENDIF
IF (FLAG EQ 1) THEN
  IPOINT(NPTS+1) = IMIN
  YPOINT(NPTS+1) = YMIN
ENDIF
READ(10,'(30G)',ERR=1002) XMIN,XMAX,IXDIV,YMIN,YMAX,YDIV
READ(10,'(30G)',ERR=1002) NDECI,NDECY
IF (NDECI GT 5 .OR. NDECI GT 5)
* PRINT*,' Number of decimals places limited to 5'
IF (NDECI LT 0 .OR. NDECY LT 0)
* PRINT*,' Number of decimals cannot be less than 0'
DO 10 J=1,NPTS
  READ(10,210,ERR=1002)(XVAR(I,J),I=1,NIVAR),(YVAR(K,J),K=1,NTVAR)
10 CONTINUE
210 FORMAT(10(30G))
CLOSE (UNIT=10,STATUS='KEEP',ERR=1004)
GO TO 1010
1000 PRINT*,' *** ERROR IN OPENING UNIT 10 ***'
STOP
1002 PRINT*,' *** ERROR IN READING FROM UNIT 10 ***'
STOP
1004 PRINT*,' *** ERROR IN CLOSING UNIT 10 ***'
STOP
1010 CONTINUE
*
*****
*
C Question user on graph options
*
*****
*
PRINT*,'Enter the number of curves to be plotted on the graph'
READ (5,'(20G)') NCURVES
PRINT*,'Enter the (X,Y) data sets to be plotted'
PRINT*,'(ie. variable location (X1,Y1,Y2,Y3))'
PRINT*,' would be 1,1 1,2 1,3)'
READ (5,210) ((NVAR1(J),NVAR2(J)),J=1,NCURVES)
PRINT*,'Do you want to plot symbols at each data point (Y/N)?'
READ(5,'(A1)') SYM
IF (SYM EQ 'Y') THEN
  PRINT*,'Do you want the default Symbol selection (Y/N)?'
  READ (5,'(A1)') ANS
  IF (ANS EQ 'N') THEN
    PRINT*,'Enter the symbol number for each curve (0-15)'
    READ (5,210) (SYNTYP(J),J=1,NCURVES)
  ENDIF
ENDIF
PRINT*,'Do you want to connect the points with a line (Y/N)?'
READ(5,'(A1)') LIN
IF (LIN EQ 'Y') THEN
  PRINT*,'Do you want the default line selection (Y/N)?'
  READ (5,'(A1)') ANS
  IF (ANS EQ 'N') THEN
    PRINT*,'Enter the data pair (LINTYP,INTEQ) for each curve'
    READ (5,210) (LINTYP(J),INTEQ(J)),J=1,NCURVES
  ENDIF
ENDIF

```

```

        ENDIF
    ENDIF
    PRINT*, 'Would you like to only plot point data (Y/N)?'
    READ(5, '(A1)') PNT
    IF (PNT .EQ. 'Y') THEN
        LIN='Y'
        DO 12 I=1,5
            LINTYP(I)=1
            INTEQ(I)=16
12        CONTINUE
    ENDIF
    IF (FLAG .EQ. 1) GO TO 1075
    PRINT*, 'Do you want to change default plot size?'
    READ(5, '(A1)') ANS
    IF (ANS .EQ. 'Y') THEN
        PRINT*, 'Enter scaling factor'
        READ(5, '(20G)') FACTO
    ELSE
        FACTO = 1.0
    ENDIF

*
*****
*
C    Prompt for Device and set Window size for plotting
C    to 11 x 8.5 inches
*
*****
*
CALL START ('?', 'INCHES')
CALL PAGE (11.0, 8.5, 0.0, 0.0)
CALL FACTOR (FACTO)
*

SIZE=0.125
ISIZE = 7.5
YSIZE = 8.5
DX = XMAX - XMIN
DY = YMAX - YMIN
ISCALE = DX / ISIZE
YSCALE = DY / YSIZE
XINC = DX / XDIV
YINC = DY / YDIV
XLEN = ISIZE * 2.0
YLEN = YSIZE * 2.00
*
*****
*
C    Plot Border around graph and draw axes
*
*****
*
CALL PLOT (1.0, 1.45, 3)
CALL PLOT (10.5, 1.45, 2)
CALL PLOT (10.5, 8.0, 2)
CALL PLOT (1.0, 8.0, 2)
CALL PLOT (1.0, 1.45, 2)
*

CALL PLOT(2.0, 2.00, 3)
CALL PLOT(XLEN, 2.00, 2)
CALL PLOT(2.0, 2.00, 2)
CALL PLOT(2.0, YLEN, 2)
CALL PLOT(2.0, 2.00, 2)

```



```

*
*****
*
C   Put tick marks on axes
*
*****
*
DO 22 I=1,XDIV
    I = 2.0 + (XSIZE / XDIV) * I
    CALL PLOT(I,2.00,3)
    CALL PLOT(I,2.10,2)
22  CONTINUE
*
DO 25 I = 1, YDIV
    Y = 2.00 + (YSIZE / YDIV) * I
    CALL PLOT(2.0,Y,3)
    CALL PLOT(2.1,Y,2)
25  CONTINUE
*
*****
*
C   Put scale on X and Y axes
*
*****
*
Y = 1.93 - SIZE
DO 26 I = 0, XDIV
    I = 2.0 + (XSIZE / XDIV) * I
    IVALUE = IINC * I + IMIN
    CALL NUMBER (I,Y,SIZE,IVALUE,0.0,NDECY)
26  CONTINUE
*
X = 2.10 - 6 * SIZE
DO 27 I= 0, YDIV
    Y = 1.90 + (YSIZE / YDIV) * I
    YVALUE = YINC * I + YMIN
    IF (NDECY .LT. 0) NDECY=0
    IF (NDECY .GT. 5) NDECY=5
    WRITE(STRING,'(F5.1)') NDECY, YVALUE
    CALL SYMBOL (I,Y,SIZE,STRING,0.0,5)
27  CONTINUE
*
*****
*
C   Put Title, Xlabel, Ylabel on graph
*
*****
*
CALL SYMBOL (2.25,7.70,0.150,TITLE,0.0,70)
CALL SYMBOL (3.75,1.53,0.15,XLABEL,0.0,40)
CALL SYMBOL (1.25,2.00,0.15,YLABEL,90.0,40)
*
*****
*
C   Draw Lines, Symbols, or data points
*
*****
*
CALL PLOT (2.0,2.00,-3)
XPOINT(NPTS+1) = IMIN
YPOINT(NPTS+1) = YMIN
1075 CONTINUE

```

```

XPOINT(NPTS+2) = ISCALE
YPOINT(NPTS+2) = YSCALE
DO 30 J=1,NCURVES
  DO 40 I=1,NPTS
    XPOINT(I) = XVAR(NVARI(J),I)
    YPOINT(I) = YVAR(NVARI(J),I)
    IF (SYM .EQ. 'Y') THEN
      X = (XPOINT(I) - XMIN) / ISCALE
      Y = (YPOINT(I) - YMIN) / YSCALE
      CALL SYMBOL (X,Y,0.075,SYMTYP(J),0.0,-1)
    ENDIF
  40 CONTINUE
  IF (LIN .EQ. 'Y') THEN
    CALL LINE (XPOINT,YPOINT,NPTS,1,LINTYP(J),INTEQ(J))
  ENDIF
30 CONTINUE
*
*
CALL PLOT (0.0,0.0,1)
PRINT*,
PRINT*,
1100 PRINT*, 'Do you want to overlay another data set (Y/N)'
READ (5, '(A1)') ANS
IF (ANS .EQ. 'Y') THEN
  FLAG = 1
  GO TO 1050
ENDIF
IF (ANS .NE. 'N') GO TO 1100
*
*
CALL PLOT (0.0,0.0,999)
*
*
STOP
END

```

2 2
OF / DE

END

21-02-86

FIN