Working Memory and Solution Procedures for Single-digit Subtraction and Multiplication

by

Catherine Lovelace

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Abstract

Adults (n = 48) solved single-digit multiplication (6x9) or subtraction problems (8-3) in combination with two types of memory loads (i.e., visual patterns or non-words) to investigate how the phonological loop and visual-spatial sketch pad are involved in the production of simple arithmetic problems. As memory load difficulty increased, performance on the arithmetic problems decreased. Remembering a visual pattern interfered significantly more than remembering non-words with solving subtraction problems, suggesting the VSSP was used to solve subtraction problems. Participants reported using retrieval (phonological trials) and decomposition (visual trials) to solve subtraction problems. Arithmetic performance on multiplication problems did not vary as a function of the working memory loads. Participants only reported using retrieval, suggesting working memory resources were not used when solving the multiplication problems. Results indicated that processing simple arithmetic problems required the use of working memory resources and the type of sub-system used was dependent upon operation and participants' solution procedures.

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Table of Contents

List of Figures

List of Appendices

Working Memory and Solution Procedures for Single-digit Subtraction and Multiplication

Mental arithmetic is an important, basic, cognitive skill in adults and one of the key subject areas taught in elementary educational settings. Consequently, it is of special interest to investigate the underlying cognitive processes that people use to solve simple mental arithmetic problems. Research has shown that mental arithmetic performance is influenced by both the organization and representation of simple arithmetic facts in long-term memory and the cognitive processing of that information in working memory (De Rammelaere, Stuyven & Vandierendonck, 1999, 2001; Lemaire, Abdi & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000).

Working memory can be defined as 'active memory'. It contains information that the cognitive system can presently access, including information from long-term memory. Working memory allows people to organize and retain information about their immediate experiences, support the acquisition of new information, and solve problems (Miyake & Shah, 1999). The rationale of the present research was to explore the contribution of working memory resources in the production of simple mental arithmetic problems such as 6x9 and 8-3. Specifically, this study addressed how modality-specific subsystems of working memory (i.e., phonological and visual) are involved in the production of simple mental arithmetic problems, and which of the subsystems are used for processing different operations in simple mental arithmetic, such as multiplication and subtraction. DeStefano and LeFevre (2004) reviewed the literature on the role of working memory in the solution of arithmetic problems. Their conclusions suggest that all components of working memory (i.e., phonological, visual and executive functioning) play an important role in the mental processing and solution of arithmetic problems.

There were three goals in the present study. The first goal was to replicate the findings that simple mental arithmetic does require working memory resources. The second goal was to replicate findings from Lee and Kang (2002) that the role of working memory systems in the solution of simple mental arithmetic is an operation specific relationship. The third goal was to investigate whether the selection and execution of solution procedures in simple mental arithmetic influences the use of working memory resources

Resources in Mental Arithmetic

For many decades, adults were assumed to solve single-digit arithmetic problems through the automatic retrieval of stored arithmetic facts from long-term memory (Ashcraft, 1992; Campbell, 1995; Baroody, 1984). Automatic retrieval of stored facts does not require the use of cognitive resources because automatic processing is fast, effortless, and does not interfere with other cognitive processes (Lemaire et al., 1996). However, more recent research has shown that adults solve even simple arithmetic problems through the use of a variety of different procedures and strategies other than automatic retrieval (Geary, Frensch, & Wiley, 1993; Geary & Wiley, 1991; Hecht, 1999; LeFevre, Bisanz et al., 1996; LeFevre, Sadesky, & Bisanz, 1996; Svenson, 1985). Consequently, the use of procedures and strategies to solve arithmetic problems is likely to require the use of cognitive resources, on the assumption that strategies are controlled, effortful, and interfere with other cognitive processes (Lemaire et al., 1996).

Two categories of theories have emerged regarding the number of cognitive capacities or resources involved in the execution of mental tasks; single-capacity and multiple-resource theories. The idea that cognitive performance, such as mental arithmetic, is dependent upon the availability of mental resources contained in one large ‘pool’ of limited mental capacity comes from single-capacity resource theories (Kahneman & Treisman, 1984; Lemaire et al., 1996). These theories suggest that the more information there is to be processed in a task the greater the amount of cognitive resources will be required to complete the task. When the situation demands that a person must perform two tasks concurrently, the amount of mental resources will be reduced, and as a result, performance will begin to suffer. This single-capacity theory has been challenged throughout the literature by many researchers proposing a multiple-resource approach (Wickens, 1980, 1984). The multiple-resource theory states that cognitive performance, such as mental arithmetic, is limited by several different resources. This theory suggests that the use of different 'pools' of limited resources, such as those used for encoding, response production, and processing can limit arithmetic performance, rather than the amount of information to be processed (Lemaire et al., 1996). The multiple-resource theory has difficulty defining the type of the resources a particular task is expected to use. In order to specify the types of resources that will be used, a theoretical framework must be utilized. One theoretical framework that is able to hold up to the specifications of multiple-resources is Baddeley's Working Memory Model (Baddeley & Hitch, 1974; Miyake & Shah, 1999).

Baddeley's Working Memory Model

Baddeley's multi-component model of working memory is an example of a multiple-resource theory. Baddeley refers to working memory as a system for the temporary storage and processing of information in a variety of cognitive tasks. The multi-component model consists of three limited-capacity inter-dependent systems: the central executive and two subsidiary slave systems, the phonological loop and the visual-spatial sketch pad (Baddeley, 1986, 1996, 2001; Baddeley & Hitch, 1974; Logie, 1995). The functions of the central executive are to: (1) monitor the allocation of attentional resources to the different processing stages (encoding, calculation, response) and to the sub-systems during cognitive activities, (2) plan and sequence activities, and (3) regulate the activities of the phonological loop and the visual-spatial sketchpad (VSSP). The phonological-articulatory loop is a specialized component that stores, rehearses and maintains verbal or speech-based information. The visual-spatial sketchpad (VSSP) is specialized for the storage and maintenance of visual and spatial information. These components make it a useful conceptual framework within which to study cognitive processes such as simple mental arithmetic (DeStefano & LeFevre, 2004; Furst & Hitch, 2000; Lemaire et al., 1996; Logie & Baddeley, 1987; Seitz & Schumann-Hengsteler, 2000). This framework is useful in distinguishing the specific types of mental resources, such as phonological and visual, that may be involved in adults' mental arithmetic performance.

To investigate the multi-component model of working-memory processes, researchers often use the dual-task methodology. In dual-task studies, people perform a modality-specific, secondary task (e.g., articulatory suppression) concurrently with a primary task (e.g., solution of an arithmetic problem). If the secondary task accesses the same resources of working memory as the original task, then performance on the original task should drop off as the processing of both tasks becomes too demanding, because of the use of cognitive resources by the secondary task. Dual-task studies are able to provide convincing evidence on the specific cognitive processes that are involved in the solution of arithmetic problems because they can be used to isolate the roles of the different working memory components (Baddeley, 2001; De Rammelaere et al., 1999, 2001; DeStefano & LeFevre, 2004; Lemaire et al., 1996; Seitz & Schumann-Hengsteler, 2000). DeStefano and LeFevre (2004) have shown that the majority of mathematical cognition-working memory research has used the multi-component model developed by Baddeley and colleagues (Baddeley, 1986, 1996; Baddeley & Hitch, 1974; Logie, 1995). The present study used the dual-task methodology in order to investigate Baddeley's multi-component working memory model.

Evidence for the Role of Working Memory in Single-Digit Mental Arithmetic

The existing research on the role of working memory in simple mental arithmetic can be organized according to the type of arithmetic task the participants were required to perform; production or verification. In a production task, participants must generate the answer to an arithmetic problem presented (e.g., 3x4 = 12). In a verification task, participants must verify whether the answer that is presented with the arithmetic problem is correct or not (e.g., 3x4 = 15, True or False).

Verification task experiments. Ashcraft, Donley, Halas, and Vakali (1992) used a simple verification task combined with three modality-specific secondary tasks: articulatory suppression (i.e., repeat a letter) to disrupt and load up the rehearsal system of the phonological loop, and two central executive interference tasks (e.g., generate words beginning with a particular letter, or alphabetize four presented letters). Ashcraft and colleagues (1992) did not include a control (neutral) condition, such as, only verifying the addition problems without performing a secondary task. Twenty adult participants verified the answers to single-digit addition problems at three levels of complexity (low, medium, high) and multi-digit addition problems at two levels of complexity, carry (e.g., 24+19 = 43) and no carry (e.g., 11+10 = 22). Articulatory suppression did not appear to have an effect on the verification of single-digit addition problems, suggesting that phonological working memory resources were not being used; however, the verification of addition problems did appear to require the use of central executive resources. The effect of problem complexity was similar at all three levels of secondary tasks, although there was a trend for a greater effect of central executive interference on the most difficult problems. Conclusions are difficult to draw from this research as there is no control condition. Without a control or neutral condition, it is problematic to determine whether the phonological load actually interfered with the verification of the problem or whether the interference was just greater with the central executive loads than with the phonological load.

Participants in Ashcraft et al. (1992) could have been using pure automatic retrieval of basic facts from long-term memory and therefore not using any subsystem resource. This result suggests that participants' access to permanently stored arithmetic facts and numerical knowledge in long-term memory is controlled via the central executive component of the working memory system. Alternatively, it is not that phonological working memory resources are not being used in the verification of single-digit addition problems, but rather they are being used much less than central executive resources. There is no way to chose between these possibilities, as Ashcraft and colleagues (1992) did not include a comparative no-load condition.

Lemaire et al. (1996) used a simple verification task combined with three modality-specific secondary tasks: two articulatory suppression tasks (e.g., repeating the word "the") and ("repeat the letter set "abcdef") to disrupt and load up the rehearsal system of the phonological loop, and a central executive interference task (e.g., random letter generation from the set "abcdef"). Lemaire et al. (1996) also included a control (neutral) condition, where participants only performed the arithmetic tasks without performing a secondary task (e.g., articulatory suppression or central executive interference). Sixty adult participants verified the answers to both "true" and "false" single-digit addition and multiplication problems at two levels of difficulty, easy (e.g., 4+3 = 10) and hard (e.g., 8x7 = 54). When participants verified "true" problems, the size of the problem difficulty effect increased in both of the secondary conditions when compared to the control conditions. This suggests that the verification of "true" single-digit multiplication and addition problems requires both phonological and central executive resources. These modality-specific effects are greater on the most difficult problems, perhaps suggesting that those types of problems were more difficult to retrieve directly from long term memory and therefore required more working memory resources from a subsystem than easy problems. Therefore in situations where part of the working memory resources had to be devoted to a modality-specific secondary task (articulatory suppression or random letter generation), the more difficult problems became even harder to verify.

The important finding in Lemaire et al. (1996) is that the effect of working memory load is not a constant across problem difficulty, even within single-digit arithmetic. This finding means that the secondary tasks did not add a constant amount of additional processing, but added different amounts of processing depending upon problem types (easy vs. hard). When participants verified "false" problems, the size of the problem difficulty effect increased in only one of the secondary conditions when compared to the control conditions. Suggesting, that the verification of "false" single-digit multiplication and addition problems requires central executive resources but not phonological resources. This suggests that as in Seitz and Schumann-Hengsteler (2000) and in Ashcraft et al. (1992), participants in Lemaire et al. could have been using pure automatic retrieval of basic facts from long-term memory in order to verify "false" sums and therefore not requiring the use of any subsystem resource. The patterns of results in Lemaire et al. are consistent with previous findings on single-digit arithmetic problems; the central executive appears to play a crucial role in simple arithmetic, phonological resources tend to be used when facts are not automatically retrieved from long-term memory, and secondary tasks do not add a constant amount of additional processing, but add different amounts of processing depending upon factors such as problem type (easy vs. hard) or operation (Lee & Kang, 2002).

De Rammelaere et al. (1999) replicated the procedures of Lemaire et al. (1996) with some important modifications. They explored whether the conclusions of Lemaire et al. (1996) would still hold true if modifications were made (e.g., new stimuli, standard secondary tasks and a new task). De Rammelaere et al. still used a simple verification task, combined with modified modality-specific secondary tasks. The first task was an articulatory suppression task (e.g., repeating the word "the"). However, in contrast to Lemaire et al. (1996), participants in De Rammelaere et al. repeated the word "the" continuously instead of every 2 seconds, to better disrupt and load up the rehearsal system of the phonological loop. The central executive interference task used in Lemaire et al. (1996) was random letter generation from a series every two seconds. In contrast, De Rammelaere et al. required participants to say one random letter from the entire alphabet at a rate of one per second. The final secondary task was a "pure" central executive interference task (e.g., random time interval generation task). Participants were required to tap an unpredictable rhythm, in order to load the central executive while not interfering with one of the slave systems. De Rammelaere et al. (1999) also included a control (neutral) condition, where participants only performed the arithmetic tasks without performing a secondary task (e.g., articulatory suppression or central executive interference).

Forty adult participants verified the answers to both "true" and "false" single-digit addition and multiplication problems at two levels of difficulty, small split: +1 (e.g., 8+4 = 13) and large split: +5 (e.g., 8+4 = 17). De Rammelaere et al. (1999) found that phonological resources were not being used in the verification of single-digit addition or multiplication problems for either "true" or "false" problems. De Rammelaere et al.'s results provide additional support for the conclusion that phonological resources are not used in the verification of "false" problems (Lemaire et al., 1996). However, the findings concerning "true" problems were different. In contrast to Lemaire et al. (1996), De Rammelaere et al. found that a load on the phonological loop had no effect on the latencies of "true" problems. A plausible explanation for these different findings is that the articulatory suppression task used by Lemaire et al. (1996) was not a 'pure' phonological task but interfered with central executive resources. Therefore it appeared as though phonological resources were being used when in fact they were "disguised" central executive resources. Moreover, De Rammelaere et al. provided additional empirical evidence for the crucial role of the central executive in the solution of single-digit arithmetic. These findings replicate as well as extend the results from Lemaire et al. (1996), because they were found by using a 'pure' central executive task that does not interfere with any of the slave systems (e.g., random interval generation).

De Rammelaere et al. (2001) investigated the contradictory results concerning the role of phonological resources in the verification of "true" single-digit addition and multiplication sums (De Rammelaere et al., 1999; Lemaire et al., 1996). They used a simple verification task, combined with two modality-specific secondary tasks: an articulatory suppression task (e.g., repeating the word "the") to disrupt and load-up the rehearsal system of the phonological loop, and a "pure" central executive interference task (e.g., random time interval generation task). Participants were required to tap an unpredictable rhythm, in order to load the central executive while not interfering with one of the slave systems. De Rammelaere and colleagues (2001) also included a control (neutral) condition, where participants only performed the arithmetic tasks without performing a secondary task (e.g., articulatory suppression or central executive interference). Thirty adult participants verified the answers to "true" single-digit addition and multiplication problems at two levels of difficulty, small split: +1 or -1 (e.g., 8+4 = 13 or 11) and large split: +9 or -9 (e.g., 8+4 = 21 or 3).

De Rammelaere and colleagues (2001) found that articulatory suppression had no effect on the verification of "true" single-digit addition and multiplication problems, supporting the conclusion that phonological resources are not used in the verification of "true" problems (De Rammelaere et al., 1999). On the contrary, the random interval generation task had a negative effect on the verification of simple addition and multiplication problems. This finding extends the conclusions of Lemaire et al. (1996) and De Rammelaere et al. (1999) who argued that central executive working memory resources play a critical role in the verification of simple arithmetic problems.

Production task experiments. Seitz and Schumann-Hengsteler (2000) used a production task combined with four modality-specific secondary tasks: irrelevant speech (e.g., listening to a female Armenian voice) and articulatory suppression (e.g., repeating the word "lemonade") to disrupt and load up the rehearsal system of the phonological loop, visual-spatial tapping (e.g., tapping of the figure 8) to disrupt and occupy resources of the visual-spatial sketchpad and a central executive interference task (e.g., random letter generation). Seitz and Schumann-Hengsteler (2000) also included a control (neutral) condition, where participants performed neutral tapping (e.g., random tapping) to carry out a non-specific secondary task. Twelve adult participants solved, and vocalized the answer to multiplication problems at two levels of difficulty, single-digit (e.g., 3x4) and double digit (e.g., 8x17).

For single-digit problems, no modality-specific effect was found, suggesting that visual-spatial and phonological resources were not used in the solution of single-digit multiplication problems. Participants in this study may have been using automatic retrieval of basic facts from long-term memory and therefore not requiring the use of any subsystem resources. This finding is consistent with Baddeley's (1996) model that access to permanent information does not require the intervention of the "slave" subsystems of working memory. However, mentally solving single-digit multiplication problems did appear to require the use of central executive resources. This suggests that access to permanently stored arithmetic facts and numerical knowledge in long-term memory is controlled via the central executive component of the working memory system. Although the focus of this paper is single-digit arithmetic, Seitz and Schumann-Hengsteler (2000) also found that solving multi-digit multiplication problems required resources from the phonological loop and the central executive components of working memory, but not the visual-spatial sketchpad.

Lee and Kang (2002) used a production task combined with two modality-specific secondary tasks: articulatory suppression (e.g., repeating a non-word string) to disrupt and load up the rehearsal system of the phonological loop, and visual-spatial suppression (e.g., holding the shape and location of an abstract figure in memory) to disrupt and occupy resources of the visual-spatial sketchpad. Lee and Kang (2002) also included a control (neutral) condition, where participants only performed the arithmetic task without performing a secondary task (e.g., articulatory or visual-spatial suppression). Ten adult participants solved, and typed the answer to randomly intermixed single-digit multiplication (e.g., 7x4 =

28) and single-digit subtraction (e.g., 7-4 = 3) problems. Their results showed that dual tasks involving different subsystems of working memory have a separate effect on single-digit multiplication and subtraction problems. Articulatory suppression had a negative effect on multiplication performance but not on subtraction performance, whereas visual-spatial suppression had a negative effect on subtraction performance but not on multiplication performance. Therefore these findings suggest that the role of working memory subsystems in the solution of single-digit arithmetic is operation specific. These findings also suggest that, unlike the participants in Seitz and Schumann-Hengsteler (2000), participants were not using pure automatic retrieval of basic facts from long-term memory and therefore required the use of phonological and visual-spatial working memory resources.

In summary, the production and verification of single-digit mental arithmetic problems appears to involve multiple resources of cognitive processing or all three components of the working memory system proposed by Baddeley (1986, 1996; Baddeley & Hitch, 1974).

Central Executive

Seitz and Schumann-Hengsteler (2000), Ashcraft et al. (1992), Lemaire et al. (1996), and De Rammelaere et al. (1999, 2001) found that mentally solving single-digit multiplication and addition problems did appear to require the use of central executive resources. These results suggest that the central executive has a general effect on the processing of simple arithmetic facts but it does not tell us which aspects of the central executive are important to arithmetic or in what way they contribute. One explanation is that participants in these studies could have been using automatic retrieval of basic facts from long-term memory in order to verify and produce the sums. This explanation is consistent with Baddeley's (1996) model in that access to permanent information does not require the intervention of the "slave" subsystems of working memory, and therefore access to permanently stored arithmetic facts and numerical knowledge in long-term memory is controlled solely via the central executive component of the working memory system.

Phonological Loop

Unlike the consensus on the role of central executive resources, researchers do not agree on the role that phonological resources have in the verification and production of mental arithmetic. Seitz and Schumann-Hengsteler (2000), Ashcraft et al. (1992) and De Rammelaere et al. (1999, 2001) found that articulatory suppression had no effect on the verification or production of single-digit addition and multiplication sums. These results suggest that participants in these studies could have been using retrieval of basic facts from long-term memory and therefore did not require the use of any subsystem resource (e.g., phonological loop), relying on central executive resources. In contrast, Lee and Kang (2002) found that articulatory suppression had a negative effect on the production of single-digit multiplication performance and Lemaire et al. (1996) found that it had a negative effect on both single-digit addition and multiplication problems. The findings in Lee and Kang and Lemaire at al. suggest a role for phonological resources in the verification and production of simple addition and multiplication problems. An explanation for these reverse findings is that participants in Lee and Kang and Lemaire et al. were not using automatic retrieval of simple arithmetic facts from long-term memory, but using one or more solution strategies requiring the use of memory resources other than central executive (e.g., phonological loop).

Visual-Spatial Sketch Pad (VSSP)

Only two papers looking at single-digit arithmetic were found that included the measurement of the VSSP within their study (Lee & Kang, 2002; Seitz & Schumann-

Hengsteler, 2000). Similar to the role of the phonological loop, the results on the role of the VSSP in the production of simple arithmetic are mixed. Lee and Kang (2002) found that visual-spatial suppression (e.g., holding the shape and location of an abstract figure in memory) interfered with subtraction performance, suggesting that visual-spatial resources were being used in the solution of single-digit subtraction problems. An explanation for these results is participants were not using automatic retrieval of simple arithmetic facts from long-term memory, but were using one or more strategies, therefore requiring the use of memory resources other than the central executive (e.g., visual-spatial resources). Seitz and Schumann-Hengsteler (2000) found no evidence for the role of visual-spatial resources (e.g., tapping the figure 8) in the production of single-digit multiplication sums. This finding suggested that participants in Seitz and Schumann-Hengstler could have been using automatic retrieval of basic facts from long-term memory, therefore not requiring the use of memory resources other than central executive resources. These results should be interpreted with caution as participants from Lee and Kang (2002) were from Hong Kong and participants from Seitz and Schuman-Hengstler (2000) were from Germany. Cultural differences in language and education may have influenced the results.

A secondary explanation for the contradictory findings on the role of visual-spatial resources in the production of simple mental arithmetic problems is the lack of similarity among the visual-spatial tasks. It is possible that the suppression task used in Lee and Kang (2002) (i.e., holding the shape and location of an abstract figure in memory) did interfere with visual-spatial resources, whereas the suppression task used in Seitz and Schumann-Hengsteler (2000; i.e., tapping the figure 8) did not. This suggests that tapping the figure 8, and holding an image in memory may actually be accessing different components of the visual-spatial sketchpad. For example, tapping the figure 8 may be accessing a more spatial component of the VSSP were as the recall of a visual object may be accessing a more visual component of the VSSP. Another possibility for the contradictory VSSP findings is that participants in Lee and Kang (2002) typed their responses to the arithmetic problems, whereas participants in Seitz and Schumann-Hengsteler (2000) spoke their responses.

Models of Numerical Cognition

Existing models of numerical cognition differ on whether the cognitive processing of arithmetic problems involves one or multiple resources. According to the abstract code model proposed by McCloskey (1992), people encode the operands of a problem, such as 7+3, into a single internal abstract code, perform the calculation, and then assemble an answer by activating appropriate output codes, such as saying the answer. One implication of the abstract code model is that the various stages of numerical processing (encoding, calculation, and response generation) are independent. The type of codes used in one stage does not influence processing in other stages (McCloskey, 1992).

In contrast, Dehaene and his colleagues (Dehaene, 1992, 1997; Dehaene & Cohen, 1995) proposed a triple-code model. The main difference between the triple-code model and the abstract code model is that within the triple-code model, Arabic (visual), verbal-auditory (phonological), and magnitude codes are differentially involved in various kinds of arithmetic processing. According to multiple-code models, people may encode and represent the operands in an internal phonological or visual-magnitude code. This internal code may interact with codes used during response assembly (calculation) and response production (Lee & Kang, 2002). Therefore, the triple-code model links specific tasks to specific internal codes (Dehaene & Cohen, 1995). For example, Lee and Kang (2002) suggest that multiplication is regarded as a rote verbal memory task that utilizes the auditory verbal code, whereas subtraction is performed using the analog magnitude code. The research that was reviewed in the present paper and in DeStefano and LeFevre (2004), suggests that solution of mental arithmetic problems often involves a variety of mental structures, codes, and processes. Therefore, the abstract code model is probably not adequate to cover the full range of arithmetic processing that people use because multiple codes do appear to be involved.

The present research will address two hypotheses that stem from the multi-component, multiple-resource theoretical framework: (1) The modality-specific subsystems of working memory (phonological loop and VSSP) are involved in the production of solutions to simple mental arithmetic problems (e.g., 6x9 and 8-3), and (2) the specific subsystem that is used for processing simple mental arithmetic problems is 'selected' based on the type of operation being performed. Specifically, a phonological memory load task will interfere and significantly delay the performance of multiplication problems but not subtraction problems and a visual memory load task will interfere and significantly delay the performance of subtraction problems but not multiplication.

Solution Procedures in Mental Arithmetic

A striking feature of human cognition is that people use multiple strategies to accomplish most cognitive tasks that require problem solving (Lemaire & Reder, 1999). The concept of strategy is important for describing and understanding cognitive processing in arithmetic because it is "a procedure that is invoked in a flexible, goal-oriented manner and that influences the selection and implementation of subsequent procedures" (Bisanz & LeFevre, 1990, p. 236).

For example, an adult who decides to use the standard subtraction algorithm to calculate the answer to a difficult subtraction problem would have used his or her knowledge of arithmetic to select an approach that would maximize his or her probability of success. Furthermore, after much practice with arithmetic problems, the adult's solution procedure may have changed and he or she is now able to obtain the answer to the same problem automatically (Bisanz & LeFevre, 1990). Solution procedures can differ in their accuracy, in the amounts of time needed for execution, in their memory demands, and in the range of problems to which they apply. The same participant on a number of different problems can select different solution procedures. Furthermore, adaptive solution procedure choices allow people to meet situational demands and overcome limited knowledge and processing resources (Siegler, 1988b; Siegler & Shipley 1995). The mechanisms inherent within Siegler and Shipley's (1995) adaptive strategy choice model (ASCM) may be used to describe and explain multiple procedure use in adult arithmetic performance (Campbell & Timm, 2000; Hecht, 1999, 2002; LeFevre & Liu, 1997). According to the ASCM model, information about arithmetic problems is stored as distributions-of-associations between problems and procedures. A particular procedure will be selected if the strength of the association between a particular problem (e.g., 2x4) and a particular procedure (e.g., retrieval) is high. The strength of the association depends upon prior experience with that problem and if the selected procedure can be used accurately and efficiently (Siegler & Shipley, 1995). Therefore, even if working memory resources are not abundantly available, a non-retrieval procedure may be used if it can be used accurately and efficiently (Hecht 1999, 2002). However, non-retrieval solution procedures are considerably slower and more error prone than retrieval based solutions (Geary et al., 1993; LeFevre et al., 1996). Thus non-retrieval solution procedures will place a heavy load on working memory resources and participants who use non-retrieval solution procedures will show working memory load interference (Hecht, 2002; Seyler, Kirk & Ashcraft).

A non-retrieval or backup solution procedure is any procedure other than direct retrieval that is used to solve the problem. Examples of backup solution procedures used to solve simple subtraction problems typically involve counting or decomposition (Geary et al., 1993). Counting can be used in two ways, counting-down (e.g., 8-3 = counting down in increments "7", "6", "5") or counting-up (e.g., 9-7 = counting up in increments "8", "9"). Decomposition refers to breaking up or 'decomposing' the original problem into a different, more easily manageable problem such that a person can than use retrieval to access the answer (e.g., 13-7 = 10-7 = 3+3 = 6). Examples of backup solution procedures used to solve simple multiplication problems typically involve derived-facts, repeated addition and the nines rule (LeFevre, Bisanz, et al., 1996). Derived-fact is similar to decomposition and refers to the transformation of the original problem into a different, more easily manageable problem such that a person can than retrieve the answer (e.g., 7x8 = 7x7 = 49+7). Repeated addition refers to adding an operand the appropriate number of times (e.g., adding 6+6 for 6x2). The nines rule refers to algorithms specific to the nine-time's problems. The rule is: subtract one from any number times nine and that is the first number in the answer to the problem (e.g., 9x4 = (4-1=3) = 36).

Evidence in support of the view that adults use multiple solution procedures to solve arithmetic operations has come from experimental research in which individuals provided self-reports of their solution procedures, on a trial-by-trial basis (Geary et al., 1993; Geary & Wiley, 1991; LeFevre, Bisanz, et al., 1996; LeFevre et al., 1996). In these studies, adults reported using procedures other than direct retrieval on addition, subtraction and multiplication problems. For example, adults reported using counting to solve problems such as 7+4 (e.g., "8, 9, 10, 11") decomposition to solve problems such as 15-9 (e.g., "15-10+1") and derived-facts to solve problems such as 7x8 (e.g., "7x7+7"). The frequency with which adults report using procedures other than retrieval varies across studies and individuals. However, adults report using a greater frequency of back-up solution procedures for addition and subtraction problems than for multiplication problems (LeFevre, Smith-Chant, Hiscock, Daley & Morris, 2003; Smith-Chant & LeFevre, 2003). The multiple-procedure evidence from research based on self-reports has challenged existing theories in which adults only retrieve answers directly from memory by activating associative links between operations and solutions (Ashcraft, 1987, 1992; Campbell, 1985, 1987, 1995; Geary, Brown, & Samaranayake, 1991; LeFevre, Bisanz, & Mrkonjic, 1988).

The present research addressed the hypotheses that adults use solution procedures other than direct retrieval to solve simple multiplication and subtraction problems, and those non-retrieval solution procedures will tax working memory resources. The limited availability of working memory resources will interfere with the execution of non-retrieval solution procedures used in the solution of simple multiplication and subtraction problems. The use of solution procedures such as counting (Hecht, 2002) and decomposition (Seyler, Kirk & Ashcraft) will place a heavy load on participants' visual and phonological working memory resources. Therefore, participants who use non-retrieval solution procedures will show working memory load interference.

Hypotheses

Based on previous research in this area, the present study addressed three hypotheses: (1) The processing and solution of simple mental arithmetic uses working memory resources; (2) Working memory subsystems have a differential effect on arithmetic operation. Specifically, a phonological memory load task will interfere and significantly delay the performance of multiplication problems but not subtraction problems and a visual memory load task will interfere and significantly delay the performance of subtraction problems but not multiplication. And, (3) the limited availability of working memory resources will interfere with the execution of non-retrieval solution procedures used in the solution of simple multiplication and subtraction problems. Specifically, the use of solution procedures such as counting (Hecht, 2002) and decomposition (Seyler, Kirk & Ashcraft, 2003) will place a heavy load on participants' phonological and visual working memory resources. Therefore, participants who use non-retrieval solution procedures will show greater working memory load interference than those participants who used retrieval.

Method

Participants

A total of 48 adults participated in this experiment. The participants were randomly assigned to one of two operation conditions: multiplication (N=24), or subtraction (N=24). The participants (22 males; 26 females) ranged in age from 18 to 54, with a median age of 21 years. Forty of the participants reported having received their elementary and secondary education in Canada. The remaining eight participants reported receiving their elementary and secondary education in another country (i.e., China, Egypt, Mexico, South Africa and England). All participants had a minimum of high school education. All participants were recruited from announcements posted at Carleton University, and received 2 course credits as partial fulfillment for introductory psychology course requirements or a $20 cash honorarium as compensation for participation. Treatment of the people within this study was in accordance with the ethical guidelines of the Canadian Psychological Association.

Materials

Arithmetic stimuli. All participants solved a total of 224 computer-presented simple arithmetic problems (multiplication or subtraction), with an equal number of problems in both the phonological (112 problems) and visual (112 problems) memory conditions. The total problem set was comprised of 56 multiplication problems for the dual task condition and 28 problems for the arithmetic control condition, with single-digit multiplicands (e.g., 6x9) and their reverse (e.g., 9x6).; or 56 subtraction problems for the dual task condition and 28 problems for the memory control condition, which contained a mixture of double-digit and single-digit minuends with single-digit subtrahends (e.g., 15-6, 6-5). The multiplication problem set contained an equal number of small (e.g., products < 26) and large (e.g., products > 26) problems for both the easy and hard memory load conditions. None of the problems contained zero answers (e.g., 6x0), ties (e.g., 6x6), or ones (e.g., 7x1). The subtraction problem set contained an equal number of small (e.g., no-borrow, 7-6) and large (e.g., borrow, 16-7) problems. None of the problems contained subtrahends with zeros or ones (e.g., 14-0, 12-1), or ties (e.g., 6-6). The multiplication and subtraction problems were divided into the large and small size groupings based on previous research in the area (Seitz & Schuman-Hengsteler, 2000; Seyler et al., 2003). A production task where participants typed their responses to the arithmetic problems was used in the present experiment in order to try and replicate the findings from Lee & Kang (2002).

Phonological memory task. In both operations, the phonological memory task items were consonant-vowel-consonant (c-v-c) non-words (e.g., GUB). The cvcs were created so as not to spell a word, proper name, or common abbreviation, to ensure participants' use of phonological rehearsal. Random combinations of one cvc (easy load) and three cvc (hard load) word strings were presented to each participant in the phonological conditions (e.g., GUB MEP NAL). Each randomly generated non-word string was presented randomly with a multiplication problem to each participant in the phonological interference or dual task condition. In the phonological control condition (28 trials), the randomly generated hard non-word string (3 CVC's) was presented with a random number. Participants were asked to recite and remember the non-word string for later recall.

Visual memory task. In both operations, the visual memory task items were various patterns of five (easy load) and eight (hard load), three inch shaded blocks, arranged within a 2 x 4 (+1row of 2) or 4 x 4 respectively block grid pattern. The visual patterns were created so as not to form recognizable objects or letters, in order to discourage participants from using a verbal label as a memory aid. The visual patterns were similar to the patterns from the Visual Patterns Test (Della Sala, Gray, Baddeley & Wilson, 1997). The patterns were presented randomly with a multiplication problem to each participant in the visual interference or dual task condition. In the visual control condition (28 trials), randomly generated hard patterns (8 blocks) were presented with a random number. Participants were asked to look at the pattern and remember it for later replication.

Ability test. All participants solved an 8-minute paper and pencil arithmetic fluency test (French, Ekstrom, & Price, 1963). In the fluency test, participants solved multi-digit addition, multiplication and subtraction operations as quickly and accurately as possible.

Arithmetic fluency is defined as the total number of problems solved correctly on the test. This test was used as a measure of participant's basic arithmetic skill level, in order to compare the results across participants and groups on an equivalent measure.

Questionnaire. All participants filled out a paper and pencil Math Background and Interests Questionnaire. The questionnaire requests information about their educational background, procedures they use on arithmetic problems, their thoughts and feelings about arithmetic and the language(s) they speak (see Appendix A).

Apparatus. The arithmetic problems were presented horizontally at the center of a 15-inch monitor that is controlled by a Dell Celeron desktop computer, running at 1.8 GHz. The experiment was programmed and run on E-Prime software. The computer program recorded response times with accuracy within one millisecond, errors and the visual pattern responses. Timing was initiated with the presentation of the arithmetic problem on the screen and is terminated with a key press.

Participants typed in the answers to the arithmetic problems and recorded their solution procedures using a Targus USB keypad and they used the mouse to replicate the visual patterns. The experimenter recorded participant responses to the phonological loads by typing their responses on the keyboard.

Procedure

Each participant was tested individually at Carleton University in a quiet room in the CACR laboratory. Half of the participants were randomly assigned to the multiplication group and the other half of the participants were randomly assigned to the subtraction group. All participants completed both a phonological interference condition and a visual interference condition. Each working memory task section was sub-divided into three conditions, the dual task condition and two control conditions; arithmetic only and memory load only. During the experiment, these conditions were randomly mixed together and presentation of phonological and visual memory loads was counter balanced among participants.

The instructions for all conditions stated that arithmetic problems can be solved in many different ways and encouraged participants to solve the arithmetic problems and the memory loads as quickly and accurately as possible. Participants received four practice trials in each of the three load conditions. Both groups received additional instructions that explained after they answered each arithmetic problem, they would be asked to describe how they solved that problem (see Appendix B for complete instructions). The experimenter read through the instructions with each participant. No feedback was given on experimental trials.

Before each trial began, the word "READY" was shown in the center of the screen for 1.5 s. The types of tasks that followed were similar for all participants across working memory loads (phonological and visual) but were different for participants across arithmetic operation depending upon the group the participant was in (multiplication or subtraction).

After participants completed the computer presented tasks, they solved the arithmetic fluency test. In the fluency test, participants solved multi-digit addition, multiplication, and subtraction operations with instructions to answer as quickly and accurately as possible. The test was timed per page in 2-minute intervals by the experimenter using a stopwatch. The experimental session concluded with participants filling out the questionnaire, receiving their compensation and being debriefed about their experimental session. Duration of the experimental session was approximately 1 hour and 45 minutes, with a break at midpoint.

Phonological Interference Condition

Dual task condition (56 trials). As shown in Appendix C(i), in this condition, participants completed two different tasks at the same time, and were instructed as such. After the "READY" prompt was displayed, a non-word string with either one or three cvcs appeared in the center of the screen for 2 s, followed by the arithmetic problem (multiplication or subtraction). The problem remained on the screen for 4 s, while the participant responded by typing in the answer using the USB number keypad. Then, "Recall the Words" appeared on the screen until the experimenter recorded the participant's recall of the non-word string on the keyboard. Immediately afterwards, "How did you solve the math problem" appeared and remained on the screen until the participant chose the solution procedure that best described how they solved the arithmetic problem, from a list of four choices that appeared on the screen. The participant typed their response (i.e., pressed #1 for retrieval) on the USB keypad.

Arithmetic only (28 trials). As shown in Appendix C(ii), in this control condition, participants only answered the arithmetic problems. To control for overall verbalizations, the non-word strings were not shown, as in the dual task. To eliminate the memory load of the non-word task, participants saw three asterisks and were not required to give a response of any kind. After the "READY" prompt was displayed, three asterisks appeared in the center of the screen for 2 s, followed by the arithmetic problem (multiplication or subtraction). The problem remained on the screen for 4 s while the participant responded by typing in the answer using the USB number keypad. Then the three asterisks appeared on the screen again for 2 s. Immediately afterward, "How did you solve the math problem", appeared and remained on the screen until the participant chose the solution procedure that best described how they solved the arithmetic problem from a list of four choices. The participant typed their response on the USB keypad.

Memory load only (28 trials). As shown in Appendix C(iii), in this control condition, participants only completed the phonological load, and were instructed as such. To control for arithmetic presentation, a random single-digit or double digit number was shown in place of the arithmetic problem. To eliminate additional mental processing, participants were not required to give a response of any kind. After the "READY" prompt was displayed, a pseudo-word string with three cvcs appeared in the center of the screen for 2 s, followed by the random number. The number remained on the screen for 4 s, to make it comparable to the dual task interval. Then "Recall the Words" appeared on the screen until the experimenter recorded the participant's recall of the non-word string on the keyboard. Immediately afterwards, three asterisks appeared and remained on the screen for 2 s. This was to replace participant's self-reports, as in the dual task condition.

Visual Interference Condition

Dual task condition (56 trials). As shown in Appendix C(iv), after the "READY" prompt was displayed, a grid pattern of either five or eight shaded blocks appeared on the computer screen for 2 s, followed by the arithmetic problem (multiplication or subtraction). The problem remained on the screen for 4 s or until the participant responded by typing in the answer using the USB number keypad. Then a corresponding empty grid appeared on the screen. The participant then replicated the visual pattern shown previously, by 'clicking' on the empty blocks in the grid with the left mouse button. The grid remained on the screen until the participant clicked the appropriate number of boxes. Immediately afterward, "How did you solve the math problem", appeared and remained on the screen until the participant chose the solution procedure that best described how they solved the arithmetic problem from a list of four choices. The participant typed their response on the USB keypad.

Arithmetic only condition (28 trials). As shown in Appendix C(v), in this control condition, participants only answered the arithmetic problems, and were instructed as such. To control for overall visualizations, the visual patterns were not shown, as in the dual task condition. To eliminate the memory load of the visual patterns, participants saw an empty grid and were not required to give any type of response. After the "READY" prompt was displayed, an empty 4 x 4 grid appeared on the screen for 2 s, followed by the arithmetic problem (multiplication or subtraction). The problem remained on the screen for 4 s while the participant responded by typing in the answer using the USB number keypad. Then the empty grid appeared again on the screen for 2 s. Immediately afterward, "How did you solve the math problem", appeared and remained on the screen until the participant chose the solution procedure that best described how they solved the arithmetic problem from a list of four choices. The participant typed their response on the USB keypad.

Memory load only (28 trials). As shown in Appendix C(vi), after the "READY" prompt was displayed, a grid pattern of eight shaded blocks appeared on the screen for 2 s. To control for arithmetic presentation, a random single-digit or double digit number was shown in place of the arithmetic problem. To eliminate additional mental processing, participants were not required to give any type of response. The number remained on the screen for 4 s, to make it comparable to the dual task interval. Then the corresponding empty grid appeared on the screen. The participant then replicated the visual pattern shown previously, by 'clicking' on the empty blocks in the grid with the left mouse button. The grid remained on the screen until the participant 'clicked' the appropriate number of boxes. Immediately afterward, three asterisks appeared and remained on the screen for 2 s. This was to replace participant's self-reports, as in the dual task condition.

Classification of Self-Reports

On each trial that an arithmetic problem was solved, the participant, based upon the choices given by the experimenter, classified their solution procedure by typing their response on the USB keypad. Descriptions and examples of each procedure choice were given to the participants during the instructions and practice trials. Four main categories or choices given to the participants, similar to Geary et al. (1993) and LeFevre et al. (1996), were used for the classification of both the multiplication and subtraction solution procedures: (1) retrieval, (2) decomposition or transformation, (3) counting or repeated addition and (4) other (see Appendix C).

Retrieval refers to the direct retrieval of basic facts from long-term memory (e.g., retrieving "56" automatically to solve 7x8). Participants were encouraged to choose this procedure if they just remembered or just knew the answer without any additional steps or processing to arrive at the answer. Decomposition or transformation refers to breaking up the original problem into a different, more easily manageable problem (e.g., 13-7 = 10-7 = 3+3 = 6). Participants were encouraged to choose this procedure if they were taking apart the original problem or using a more familiar problem to arrive at the answer (e.g., 7x8 = 7x7 = 49+7). Counting refers to counting up or down (e.g., 12-9 = "9", "10", "11", "12"). Participants were encouraged to choose this procedure if they were counting in increments on a mental number line. Repeated addition refers to adding an operand the appropriate number of times (e.g., adding 6+6 for 6x2). Participants were encouraged to choose this procedure if they were adding one of the multiplicands the same number of times as the other would tell them to. Other procedures refer to anything that did not fit into the above categories.

Participants were encouraged to choose this procedure if they were using any other strategy not listed, if they were guessing or did not respond to the arithmetic problem.

Results

Results are presented in three main sections. The first section includes the analyses for the arithmetic solution times and their error rates, the second section contains the analyses for the memory load error rates, and the final section contains the solution procedure analyses.

Due to a computer logging error, participant memory load responses were not recorded for the easy load phonological condition in both operation groups. Therefore, for the error analyses, only the control and hard load conditions were used. However the experimenter noted that participants made very few errors in the easy phonological condition.

Arithmetic Problems: Solution Times and Error Rates

Overall, the mean score on the arithmetic fluency test was 78 ($SD = 25$). Participants' arithmetic skill was in the average range, and did not differ significantly from the expected mean for this population, $t\ (47) = .55$, (i.e., 80, LeFevre et al., 2003). The arithmetic fluency scores of the two groups of participants (multiplication and subtraction) were not significantly different from one another (78 vs. 76), $t\ (46) = -.26$, $p > .05$. Therefore, any significant differences attributable to the type of operation a participant would be solving would not be the result of differences in arithmetic skill level.

The present experiment was designed with subtraction and multiplication as a between-groups variable because the hypothesis was to examine the differential effects of working memory loads on arithmetic. Therefore, the overall comparison between subtraction and multiplication is not useful for the present hypothesis. Furthermore, people are doing dissimilar things when solving multiplication or subtraction problems. There is evidence in the literature and in the present study that people use different strategies and are therefore processing and solving these operations as separate and unique processes. For example, the majority of participants reported using retrieval in the solution of multiplication problems in the present study (see also LeFevre, Bisanz et al., 1996; Smith-Chant & LeFevre, 2003) whereas they reported using various back-up solution procedures to solve subtraction problems in the present study (see also Seyler et al., 2003). In order to compare the effects of working memory loads within each arithmetic operation, mean solution times (correct trials only) and mean percentage of errors for the arithmetic problems were analyzed for each operation in separate 2 (memory: visual vs. phonological) x 3 (load: control vs. easy vs. hard) x 2 (problem size: large vs. small) analyses of variance, with repeated measures on all three factors.

Subtraction Problems. As expected, participants solved the no-borrow subtraction problems significantly faster and more accurately than the borrow subtraction problems (1441 ms vs. 2078 ms), $F(1, 23) = 188.90$, $MSE = 154455$, $p < .001$, and (2% vs. 12%), $F(1, 23) = 37.77$, $MSE = .016$, $p < .001$. The problem size effect is a pervasive finding in the solution of mental arithmetic problems (Geary et al., 1993; LeFevre et al., 1996).

As predicted, participants solved subtraction problems significantly more slowly with a visual memory load than with a phonological memory load (1815 ms vs. 1704 ms), $F(1, 23) = 7.29$, $MSE = 122375$, $p < .05$. These results suggest that the visual memory load interfered significantly more with the mental processing of subtraction problems than did the phonological memory load. Therefore, participants used their visual working memory sub-system when solving subtraction problems significantly more than they used their phonological working memory system. Furthermore, subtraction solution times varied with load, $F(2, 46) = 6.21$, $MSE = 38261$, $p < .01$. Pairwise comparisons (LSD, $p < .001$) indicated that participants solved subtraction problems in the control and easy dual task conditions significantly faster than in the hard dual-task condition, (1731 ms and 1731 ms vs. 1817 ms. Error rates also varied with load, $F(2, 46) = 3.82$, $MSE = .002$, $p < .05$, Participants solved the subtraction problems in the easy dual task condition more accurately than in the hard dual-task condition, (6% vs. 8%). The error rate in the control condition (7%) did not differ significantly from either the easy or hard conditions. These results suggest that the hard memory load condition interfered significantly more with the processing of subtraction problems and required more working memory resources than did the easy load and control conditions.

Although the two-way interaction (memory x load) was not significant for solution times, the pattern of results provides some useful information about how the type of load is related to subtraction performance, $F(2, 46) = 2.73$, $MSE = 24928$, $p = .08$, $CI = 64.5$ ms. As shown in Figure 1, the visual load had a greater impact on solution times than the phonological load only in the easy condition. In the hard condition, the solution times for the phonological load were not significantly different from those with the visual load. As discussed later in the paper, the hard phonological load was very difficult (participants made a substantial number of errors). Thus, there may have been both a phonological and a more general attentional demand in that condition. In summary, although the overall pattern of these results does not exactly replicate the findings from Lee and Kang (2002), the pattern of results for the visual memory load was as predicted. Specifically, participants solved subtraction problems significantly more slowly with a visual memory load than with a phonological memory load.

The two-way interaction (memory x load) was significant for arithmetic errors, $F(2, 46) = 3.81$, $MSE = .003$, $p < .05$, CI = 1.8%. As shown in Figure 2, the pattern of results was similar to that shown in the response times. In general, the visual memory load did interfere with subtraction more than the phonological load, particularly under the hard load condition. As, the memory load difficulty increased, participant's performance on the subtraction problems became slower.

A significant two-way interaction (memory x problem size) for solution times was obtained, $F(1, 23) = 6.98$, $MSE = 25215$, $p < .05$, CI = 64.8 ms. As shown in Figure 3, there was no difference between the phonological and visual memory conditions for the no-borrow subtraction problems. However, as the subtraction problems became more difficult and required more working memory resources, participants' performance in the visual memory condition was slower than in the phonological condition. This supports the previous findings that participants were using visual working memory resources to solve subtraction problems.

In summary, the overall patterns of results for the subtraction problems were not as predicted. However, the visual memory load did significantly interfere more with the solution of subtraction problems than did the phonological load, particularly when the visual load and subtraction problems became more difficult and required more working memory resources. Therefore, participants used their visual working memory sub-system when solving subtraction problems significantly more than they used their phonological working memory system. Contrary to what was expected, the phonological load did interfere with the solution of subtraction problems. However these findings were not as pervasive or consistent.

Figure 1: Subtraction Solution Times (Memory x Load)

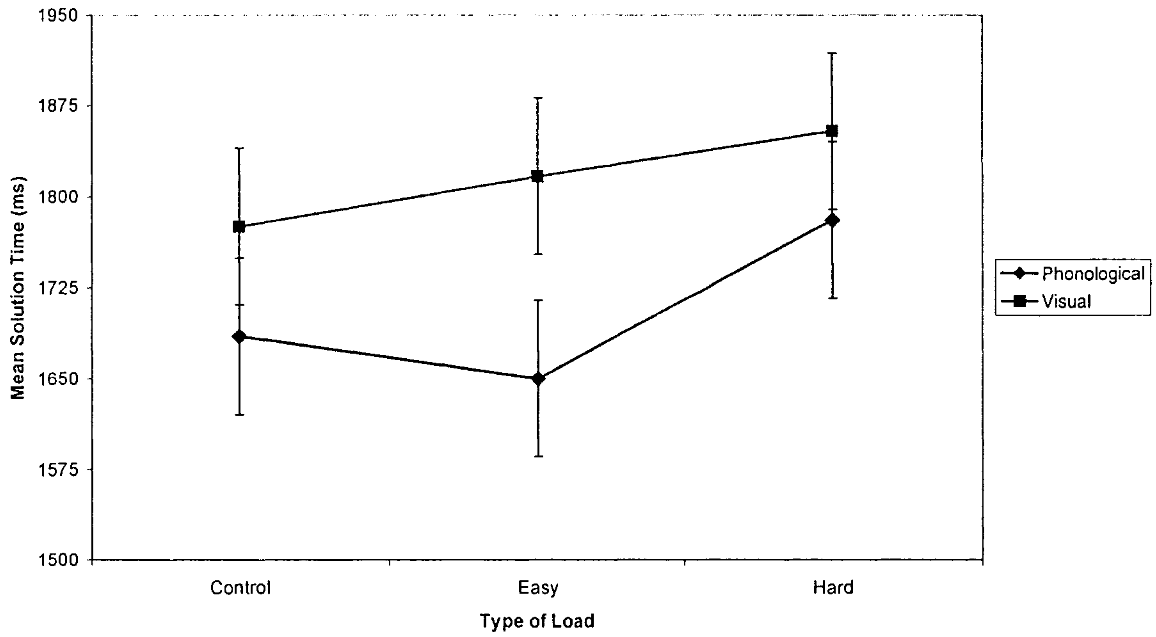


Figure 2: Subtraction Error Rates (Memory x Load)

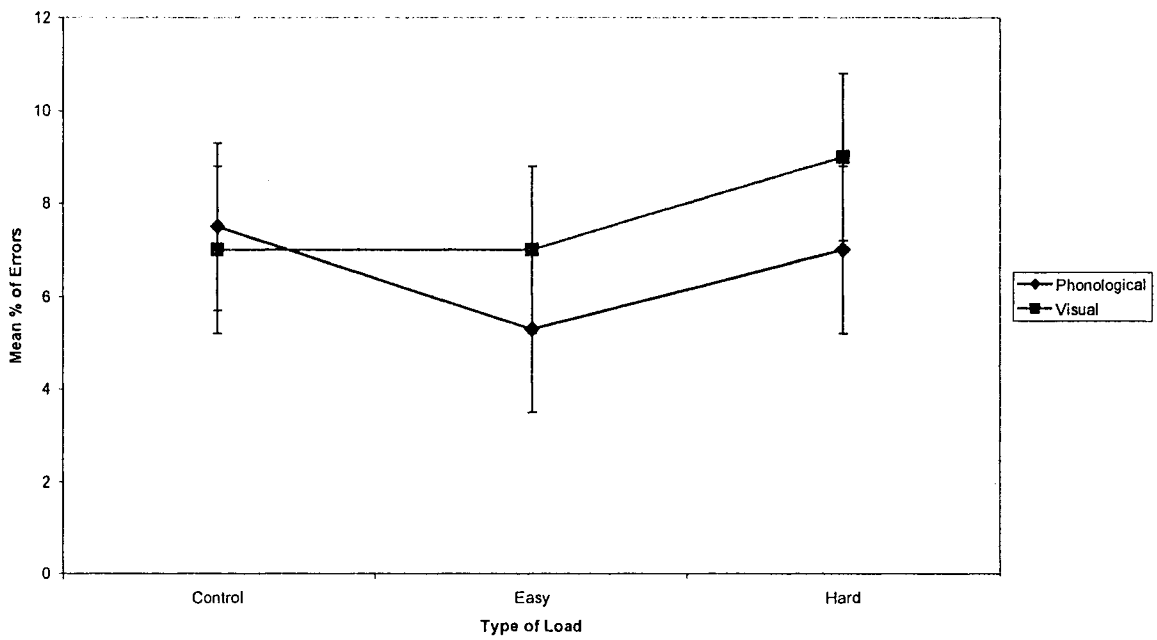
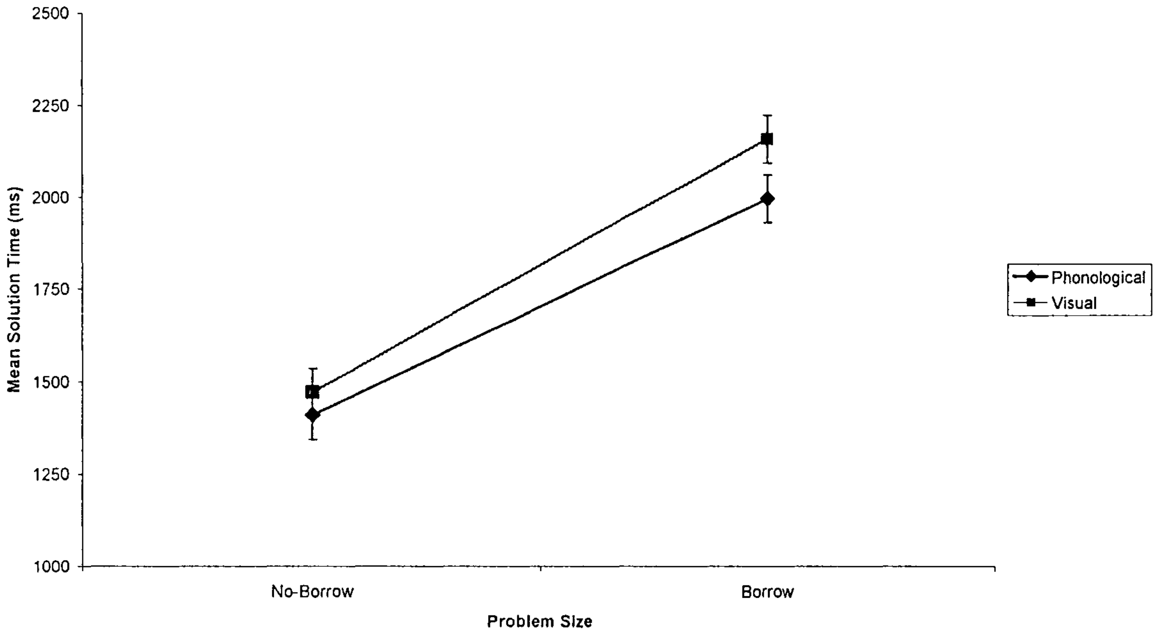


Figure 3: Subtraction Solution Times (Memory X Problem Size)



Multiplication Problems. As expected, participants solved the small multiplication problems significantly faster and more accurately than when they solved the large multiplication problems (1663 ms vs. 2104 ms), $F(1, 23) = 224.82$, $MSE = 62108$, $p < .001$, and (2% vs. 13%), $F(1, 23) = 36.61$, $MSE = .023$, $p < .001$. The problem size effect is a pervasive finding in arithmetic research (Geary et al., 1993; LeFevre et al., 1996).

Pairwise comparisons (LSD, $p < .01$) revealed that participants solved multiplication problems in the easy dual task condition (1845 ms) significantly faster than they solved multiplication problems in the control (1890ms) and hard dual task conditions (1916ms), $F(2, 46) = 6.21$, $MSE = 38261$, $p < .01$. Hence, participants' performance in the easy condition was better than in the control condition when they were only solving a multiplication problem. This result may have occurred due to a priming or arousal effect, therefore preparing the participant for more efficient problem solving. The pattern of results for the errors did not reach statistical significance (7% and 7% vs. 9%), suggesting that participants made the same number of errors in each of the load conditions.

No other main effects or interactions reached statistical significance. This lack of differences across conditions suggests that the specific type of working memory condition (i.e., visual or phonological) did not influence participants' solution times or error rates when solving the multiplication problems Therefore, participants did not appear to rely differentially on phonological versus visual working memory when they solved multiplication problems.

Summary. In summary, it was hypothesized that the working memory subsystems would be differentially involved in the solution of multiplication versus subtraction problems. Overall, the patterns of results for the arithmetic solution times and error rates were not as hypothesized. However, the patterns of results suggest that the visual working memory load did interfere with participants' solution of subtraction problems. Participants were using their visual working memory resources significantly more than their phonological resources to solve subtraction problems particularly when the load difficulty and problem size increased. When participants solved the multiplication problems, there was no difference between the phonological and visual interference. The type of memory load task did not affect the mental solution of multiplication problems, suggesting that participants did not use a notable amount of working memory resources when solving the multiplication problems. However participants were using some working memory resources, possibly central executive resources as the type of load did significantly interfere. There were two unpredicted findings. The first is that the phonological load did interfere with the solution of subtraction problems. However these findings were not as pervasive or consistent. The second unpredicted finding is that performance in the phonological condition improved when they solved an easy dual-task load compared to only solving an arithmetic problem. This may have occurred due to a priming or arousal effect, therefore preparing the participant for more efficient problem solving.

An additional solution time analysis was conducted on only those trials for which both the arithmetic problem and the memory load tasks were done correctly. The pattern of results is identical to the above solution time analysis, however, due to the large percentage of errors on the phonological memory load task and the missing data in the easy phonological load condition many trials were not able to be included in the analysis and therefore the results would have skewed towards the visual condition. Therefore the solution time analysis, where trials were selected for only the correct arithmetic problem was used.

Secondary Task Error Rates

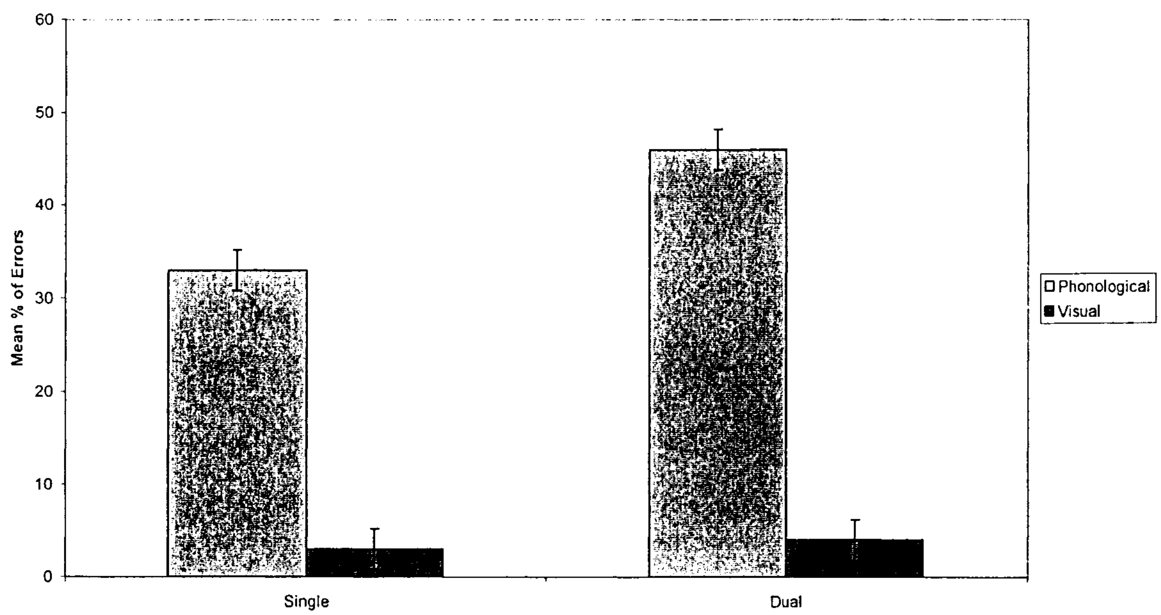
Mean percentage of errors for the secondary task loads were analyzed using a 2 (memory: visual vs. phonological) x 2 (load: control vs. hard) x 2 (operation: subtraction vs. multiplication) analysis of variance, with repeated measures on the first two factors, and operation as the between subjects factor.

As predicted, participants remembered the memory load in the single task (control) condition more accurately than they remembered the hard dual task (19% vs. 28%), $F(1, 46) = 160.64$, $MSE = .046$, $p < .001$. This result suggests that solving a subtraction or multiplication problem while simultaneously remembering a hard memory load was significantly more taxing on participants' working memory than only remembering the hard memory load.

The main effect of memory indicates that participants completed the visual memory task with significantly fewer errors than the phonological memory task (4% vs. 44%), $F(1, 46) = 62.68$, $MSE = .006$, $p < .001$. This main effect is qualified by a significant two-way interaction of (memory task x load), $F(1, 46) = 58.84$, $MSE = .006$, $p < .001$, CI = 2.2%. As shown in Figure 4, participants had significantly more difficulty solving the phonological load than the visual load. Furthermore, when participants were required to use more working memory resources (i.e., dual task) their performance on the memory load task became worse.

In summary, the results for the secondary task error rates were not expected. The results suggest that for participants in both the subtraction and multiplication conditions, the phonological memory condition was significantly more difficult than the visual memory condition. However, the significantly low error rate on the visual task shows that all of the interference caused by the load is in participants' arithmetic performance. This supports the hypothesis that the participants were using their visual working memory to solve subtraction problems. Conversely, the significantly high error rate on the phonological task makes it difficult to interpret whether the interference is occurring in the arithmetic performance or in the memory load performance. This suggests that the phonological load used in the present study may actually be a measure of the central executive component of memory because participants were using some working memory resources that were not differential.

Figure 4: Secondary Task Errors (Memory x Load)



Solution Procedures

Mean percentage of solution procedure use for the arithmetic problems was analyzed using 2 (memory: visual vs. phonological) x 3 (load: control vs. easy vs. hard) x 2 (problem size: large vs. small) x 3 (strategy: retrieval vs. decomposition vs. counting) x 2 (operation: subtraction vs. multiplication) analysis of variance, with repeated measures on the first four factors and operation as the between subjects factor.

Overall, participants reported using retrieval to solve the arithmetic problems significantly more often than they reported using decomposition or counting, with counting being the least significantly reported strategy (84% vs. 11% and 4%) $F(2, 92) = 358.44$, $MSE = 3181$, $p < .001$. A significant three-way interaction (memory x strategy x operation) was obtained, $F(2, 92) = 5.14$, $MSE = 469$, $p < .01$, CI = 5.6%. As shown in Figure 5a, participants only reported using retrieval to solve the multiplication problems and they used it equally on both the visual and phonological trials. As shown in Figure 5b, participants reported using retrieval and decomposition to solve the subtraction problems on the majority of the trials for both the visual and phonological conditions, with retrieval being reported more often for phonological trials and decomposition reported more often for visual trials.

These results support the arithmetic solution time and error rate results. In particular, the results for the multiplication problems suggested that participants did not use a note worthy amount of working memory resources in the processing and solution of multiplication facts. The finding that participants reported using retrieval on 97% of the arithmetic trials lends support to the idea that retrieval is an automatic process of retrieving the response from long-term memory and therefore limiting the use of working memory resources to little or none. These findings also suggest that the phonological task was measuring participants' central executive functioning. Furthermore, the solution time and error rate results for the subtraction problems suggested that participants used visual working memory to process and solve the problems. The finding that participants reported using back-up solution procedures such as, decomposition and counting, particularly in the visual condition, suggests that decomposition is a visually based solution procedure and that the use of solution procedures uses up a significant proportion of working memory resources.

There was also a significant three-way interaction of (strategy x problem size x operation), $F(2, 92) = 12.68$, $MSE = 2459$, $p < .001$, CI = 12.8%. As shown in Figures 6a and 6b, the significant problem size differences appear only in the solution of subtraction problems. Participants reported using retrieval significantly more frequently on the no-borrow subtraction problems than on the borrow problems. Furthermore, participants reported using back-up solution procedures, especially decomposition significantly more frequently on the borrow subtraction problems than on the no-borrow problems. These findings suggest that participants have the smaller, less difficult subtraction problems stored in memory, but must solve the larger more difficult ones with procedures other than direct retrieval.

In summary, the results for the solution procedures illustrate that participants reported using retrieval on the majority of their trials, then decomposition and counting respectively. Participants' solving the multiplication problems with retrieval lends support to the idea that retrieval is an automatic process of retrieving the response from long-term memory and therefore limiting the use of working memory resources and using central executive functions. Furthermore, participants using back-up solution procedures such as, decomposition and counting, to solve subtraction problems, especially in the visual condition suggests that decomposition is a visually based solution procedure and that the use of solution procedures uses up a significant proportion of working memory resources.

Figure 5a: Multiplication Solution Procedures (Memory x Strategy x Operation)

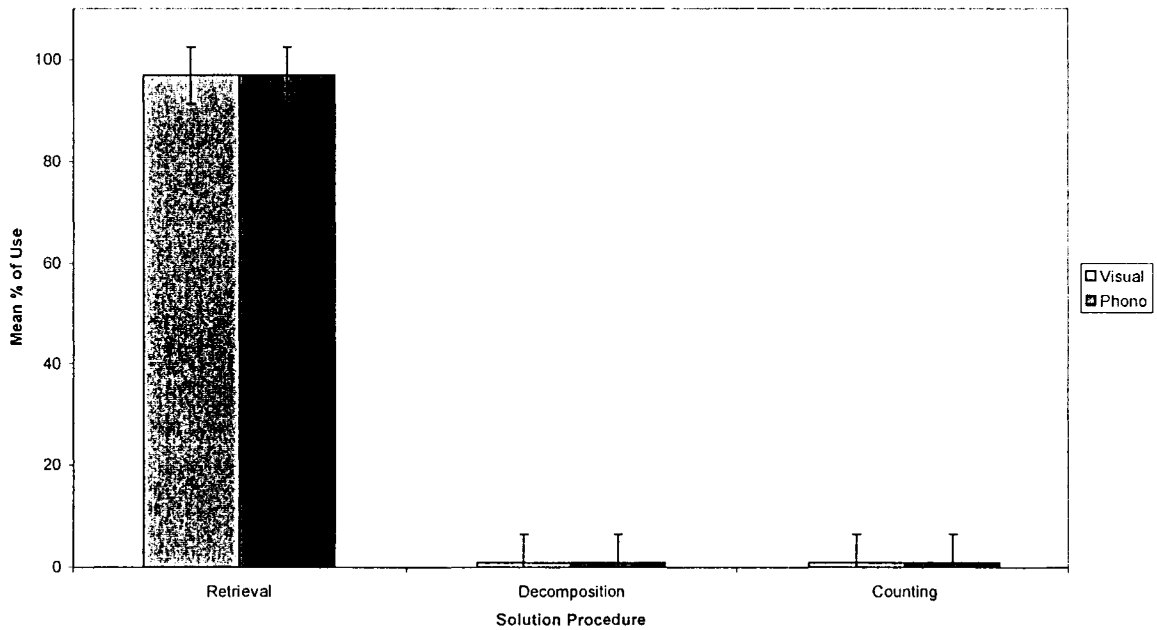


Figure 5b: Subtraction Solution Procedures (Memory x Strategy x Operation)

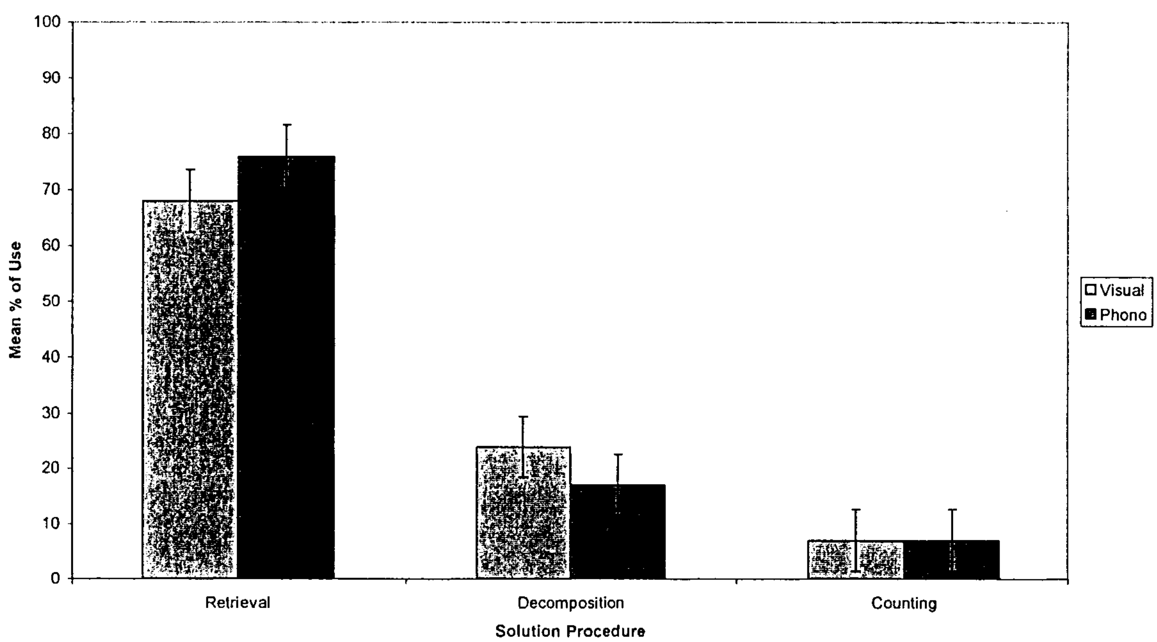


Figure 6a: Multiplication Solution Procedures (Strategy x Problem Size x Operation)

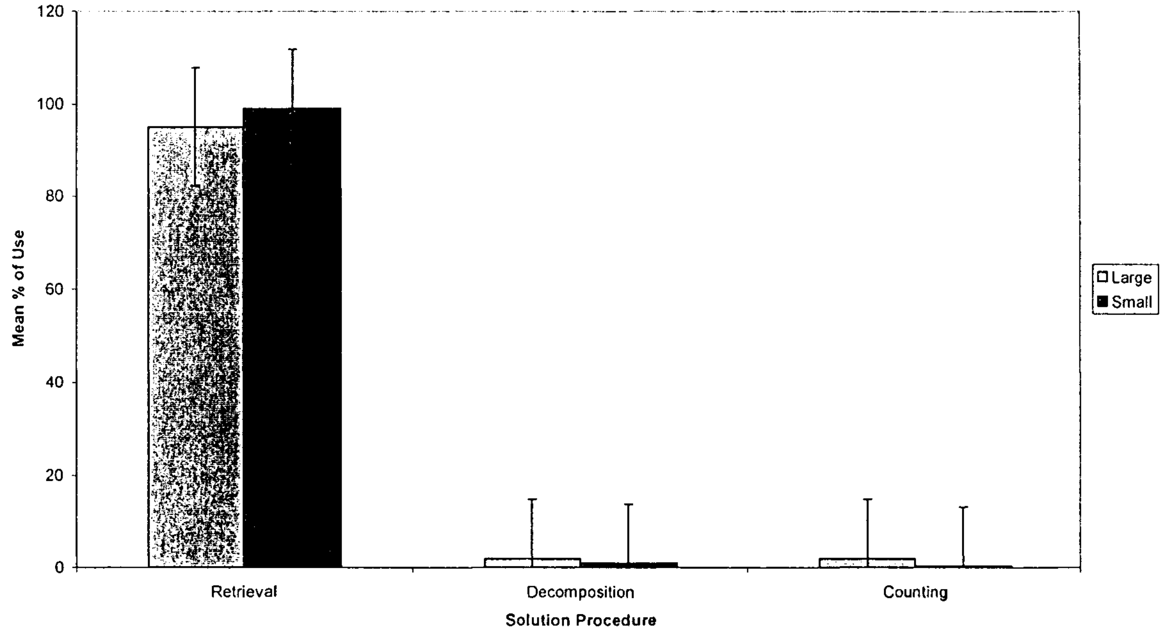
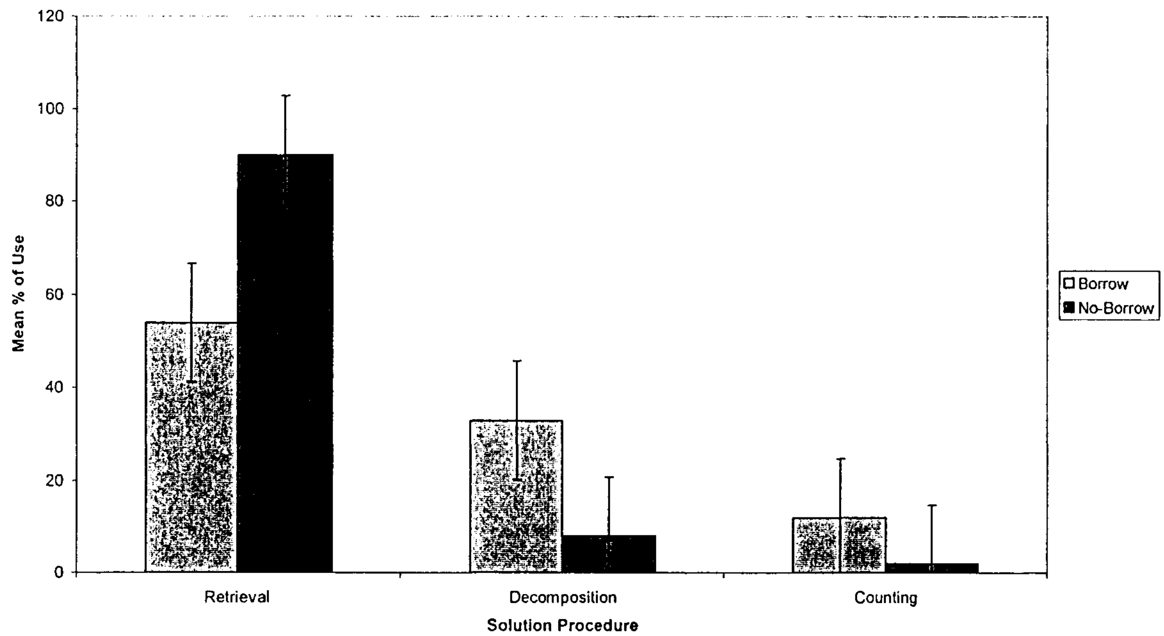


Figure 6b: Subtraction Solution Procedures (Strategy x Problem Size x Operation)



Discussion

The rationale of the present research was to explore the contribution of working memory resources in the production of simple mental arithmetic problems. Specifically, how the modality specific subsystems of working memory (i.e., phonological loop and VSSP) are involved in the processing and solution of different arithmetic operations such as, multiplication and subtraction problems. The hypotheses for the present research were: (1) That the processing and solution of simple mental arithmetic problems does require working memory resources; (2) To replicate the findings of Lee and Kang (2002) that the role of working memory systems in the solution of simple mental arithmetic is an operation specific relationship; and (3) The selection and execution of back-up solution procedures (i.e., decomposition and counting) influences the type of working memory subsystem that is used.

Hypothesis 1: Simple mental arithmetic uses working memory resources. This hypothesis was supported by the arithmetic solution time and error rates, as well as the error rates for the secondary tasks. Results from the present study show that participants did use working memory resources to solve the arithmetic problems. Specifically, the patterns of solution times and error rates for the arithmetic problems show that as the amount of information participants were required to remember increased (i.e., control, easy, hard) the more interference there was with the processing and solution of the arithmetic problems. Furthermore, results from the secondary task error rates indicate that participants remembered the memory load in the single task (control) condition more accurately than they remembered the hard dual task. This result suggests that solving a subtraction or multiplication problem while simultaneously remembering a hard memory load was significantly more taxing on participants' working memory than only remembering the hard memory load.

Hypothesis 2: Working memory subsystems have a differential effect on arithmetic operation. This hypothesis was only partially supported. When participants solved subtraction problems, their solution times and error rates revealed that the visual memory task (i.e., remembering a visual pattern) interfered significantly more than the phonological memory task (i.e., remembering 3 cvc pseudo-words) in the dual-task load conditions and in particular when solving the borrow problems. These findings suggest that participants used their visual working memory system significantly more often than their phonological sub-system when solving mental subtraction problems. When participants solved the multiplication problems, there was no difference between the phonological and visual interference, however there was non-specific working memory interference suggesting the possibility of central executive resources being used. Contrary to the hypothesis, both the visual and the phonological memory load interfered with the solution of subtraction problems. However, the phonological results are not pervasive. The type of memory load task did not affect the mental solution of multiplication problems, suggesting that participants did not use a notable amount of working memory resources when solving the multiplication problems. This finding, although contrary to Lee and Kang (2002) may be explained due to the cultural differences between their participants educated in China and the participants in this study. The participants in Lee and Kang (2002) may have displayed phonological interference when solving multiplication problems because they have multiplication facts stored as phonological codes in long term memory. However, participants in this study would not have been educated in the same rote verbal memorization as the participants in Lee and Kang (2002) and therefore were retrieving their facts using a more general (non-specific) working memory.

Hypothesis 3: Solution procedures influence working memory. This hypothesis was supported. Participants only reported using retrieval to solve the multiplication problems and they used it equally on both the visual and phonological trials. While participants reported using retrieval and decomposition to solve the subtraction problems on the majority of the trials for both the visual and phonological conditions, with retrieval being reported more often for phonological trials and decomposition reported more often for visual trials.

These results support the arithmetic solution time and error rate results. In particular, the results for the multiplication problems suggested that participants did not use a note worthy amount of working memory resources in the processing and solution of multiplication facts. The finding that participants reported using retrieval on 97% of the arithmetic trials is consistent with Baddley's (1996) model that access to permanent information does not require the intervention of the working memory sub-systems. Therefore, the phonological condition was measuring participants' central executive functioning. Furthermore, the solution time and error rate results for the subtraction problems suggested that participants used visual working memory to process and solve the problems. The finding that participants reported using back-up solution procedures such as, decomposition and counting, especially in the visual condition suggest that decomposition is a visually based solution procedure and that the use of solution procedures uses up a significant proportion of working memory resources. Participants reported using retrieval significantly more frequently on the no-borrow subtraction problems than on the borrow problems. Furthermore, participants reported using back-up solution procedures, especially decomposition significantly more frequently on the borrow subtraction problems than on the no-borrow problems. These findings suggest that participants have the smaller, less difficult subtraction problems stored in long-term memory, but must solve the larger more difficult ones with procedures other than direct retrieval.

To conclude, the processing of simple mental arithmetic problems does require the use of working memory resources. However the type of sub-system that is used is partially dependent upon the operation being solved and the type of solution procedures participants are using. These findings are consistent with Dehaene et al.'s (1995) triple-code model. According to multiple-code models, people may encode and represent the operands in an internal phonological or visual-magnitude code. This internal code may interact with codes used during response assembly (calculation) and response production (Lee & Kang, 2002). Therefore, the triple-code model links specific tasks to specific internal codes (Dehaene & Cohen, 1995). For example, this research suggests that subtraction is performed using the visual-magnitude code. The results also suggest that subtraction problems may involve the auditory verbal code, and that the central executive is involved in the solution of simple arithmetic problems. However due to the difficulty of the phonological task, interpretation of the phonological load results must be cautioned.

Limitations

One of the main limitations of the present study is that there was missing data in one of the conditions (i.e., easy phonological load) which limited the error analyses that could be performed. Another limitation is that the phonological task appeared to be significantly more difficult than the visual memory task. Conclusions about the phonological results must be interpreted with caution as it appeared that many participants could not complete the task correctly. Finally, the control memory loads only used the hard load. Again, this limited the type of analyses that could be done with the control loads. Had there been an easy and hard load control there might have been more significant differences among the load conditions.

Future studies in this area should address the above limitations as well as investigate a sample of participants that do not use retrieval to solve any simple arithmetic problems. Or, use more difficult arithmetic problems, forcing participants to use back-up solution procedures and therefore accessing working memory. Furthermore, future research in this area should investigate whether other operations such as division and addition use differential sub-systems of working memory.

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Appendix A: Math Background and Interest Questionnaire

MATH BACKGROUND AND INTERESTS QUESTIONNAIRE

1. In which province did you receive your education (or country if not Canada)?

Elementary	Secondary	Post-Secondary (University)

2. Age: ____________________________ (in years)

3. Sex: male female

4. What languages do you speak?

English		French		Other (please specify):	

5. Which do you consider your first language (i.e., the one you are the most proficient at speaking)?

English		French		Other (please specify):	

6. If you were in French immersion in school, please answer the following questions:

(a) I was in French immersion from grades		to	
(b) I studied math in French from grades		to	

7. Do you wear / have you ever worn corrective lenses (i.e., glasses or contacts)? YES NO

   If **yes,** for what purpose do / did you wear them (e.g., driving, reading):

8. Have you ever had a seizure? YES NO

   If **yes,** how frequently do you have seizures (i.e., number per week or year):

9. Have you ever had a head injury (e.g., concussion)? YES NO

   If **yes**, how severe was the head injury?

1	2	3	4	5	6	7
mild			moderate			severe

10. Are you colour blind or colour weak (i.e., red & green deficiency only)? YES NO

If **yes,** rate the extent of your colour blindness:

1	2	3	4	5	6	7
mild			moderate			severe

11. Please rate your visual-motor ability (e.g., catching a ball; walking a balance beam):

1	2	3	4	5	6	7
very poor			average			very good

MATHEMATICS

1. When did you start using a calculator? (e.g., Grade 6, first year university):____________________

2. How frequently do you use a calculator to do calculations such as 6 x 9?

1	2	3	4	5	6	7
never			sometimes			always

3. How frequently do you use a calculator to do calculations such as 34 + 97 + 18?

1	2	3	4	5	6	7
never			sometimes			always

4. Please rate your level of basic mathematical skill (e.g., skill at calculation):

1	2	3	4	5	6	7
very low			moderate			very high

5. Please rate your level of mathematical skill in more complex areas of mathematics (e.g., calculus, algebra):

1	2	3	4	5	6	7
very low			moderate			very high

6. Do frequently do you AVOID situations involving mathematics?

1	2	3	4	5	6	7
almost always			sometimes			almost never

7. How often do you find that situations involving mathematics make you nervous?

1	2	3	4	5	6	7
almost always			sometimes			almost never

8. How difficult was mathematics for you in high school?

1	2	3	4	5	6	7
extremely difficult			moderately difficult			not at all difficult

SOLVING SIMPLE ARITHMETIC PROBLEMS

1. How frequently do you solve math problems such as 3 x 9 by switching the numbers around so that the **larger** number is placed first (9 x 3)?

1	2	3	4	5
never	rarely	sometimes	often	always

2. How frequently do you solve math problems such as 7 x 6 by switching the numbers around so that the **smaller** number is placed first (6 x 7)?

1	2	3	4	5
never	rarely	sometimes	often	always

3. Overall, please estimate what proportion of the time you solve simple multiplication problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

4. Overall, please estimate what proportion of the time you solve simple addition problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

5. Overall, please estimate what proportion of the time you solve subtraction problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

6. Overall, please estimate what proportion of the time you solve division problems in ways *other than* by automatically remembering the answers?

1	2	3	4	5
never	rarely	sometimes	often	always

Appendix B: Instructions for Participants

Instructions for participants: Arithmetic Control Condition

In this experiment you will be doing 4 types of tasks.
Each task has a set of 4 practice problems.
Your first task is Practice Problems for Math Only

1. Each trial will begin with a READY screen.
2. Then you will see ***
3. Then the math problem will appear.
Solve it as quickly and accurately as you can,
and TYPE the answer on the keypad.
don't correct or change your answer once you've typed it in
Then just wait until you see the next screen.
4. You will then see *** again.
5. The final screen will ask
"How did you solve the math problem."
Choose the BEST description from the following:
(1)Retrieved= You have the answer memorized.
"I just remembered" or "I just new the answer"
(2)Added/Counted= You added or counted. e.g.7x3(7+7+7)
(3)Transformed= You 'transformed' or changed the problem
into another one that you knew.
e.g.8x9(You transformed it into 8x8+8)
(4)Other= You did something else not listed

Press ENTER to begin...

Instructions for participants: Phonological Easy Load Condition

This task is Practice Problems for Math+Memory

1. Each trial will begin with a READY screen.
2. Then you will see a non-word (e.g. VEP)
I want you to read this word aloud and REMEBER it
because you will have to repeat it later.
3. Then the math problem will appear.
Solve it as quickly and accurately as you can,
and TYPE the answer on the keypad.
don't correct or change your answer once you've typed it in
Then just wait until you see the next screen.
4. You will then see "Recall The Words".
I want you to say the word you were remembering.
5. The final screen will ask
"How did you solve the math problem."
Choose the BEST description from the following:
(1)Retrieved= You have the answer memorized.
"I just remembered" or "I just new the answer"
(2)Added/Counted= You added or counted. e.g.7x3(7+7+7)
(3)Transformed= You 'transformed' or changed the problem
into another one that you knew.
e.g.8x9(You transformed it into 8x8+8)
(4)Other= You did something else not listed

Press ENTER to begin....

Instructions for participants: Phonological Memory Control Condition

This task is Practice Problems for Memory Only

1. Each trial will begin with a READY screen.
2. Then you will see 3 non-words (e.g. VEP GUP FOD)
I want you to read these words aloud and REMEBER them
because you will have to repeat them later.
3. Then a random number will appear. You don't do anything.
Just wait until you see the next screen.
4. You will then see "Recall The Words".
I want you to say the 3 words you were remembering.
5. The final screen will have ***

Press ENTER to begin....

Instructions for participants: Phonological Hard Load Condition

This task is Practice Problems for Math+Memory

1. Each trial will begin with a READY screen.
2. Then you will see 3 non-words (e.g. VEP GUP FOD)
I want you to read these words aloud and REMEBER them
because you will have to repeat them later.
3. Then the math problem will appear.
Solve it as quickly and accurately as you can,
and TYPE the answer on the keypad.
don't correct or change your answer once you've typed it in
Then just wait until you see the next screen.
4. You will then see "Recall The Words".
I want you to say the words you were remembering.
5. The final screen will ask
"How did you solve the math problem."
Choose the BEST description from the following:
(1)Retrieved= You have the answer memorized.
"I just remembered" or "I just new the answer"
(2)Added/Counted= You added or counted. e.g.7x3(7+7+7)
(3)Transformed= You 'transformed' or changed the problem
into another one that you knew.
e.g.8x9(You transformed it into 8x8+8)
(4)Other= You did something else not listed

Press ENTER to begin....

Instructions for participants: Visual Easy Load Condition

This task is Practice Problems for Math+Memory

1. Each trial will begin with a READY screen.
2. Then a grid with a total of 10 boxes will appear.
FIVE of the boxes are filled in to make a random pattern.
You have to look at the pattern and REMEMBER it
as you will have to replicate it later on a Blank grid.
3. Then the math problem will appear. Solve it as quickly
and accurately as you can, and TYPE the answer on the keypad.
don't correct or change your answer once you've typed it in
Then just wait until you see the next screen.
4. A Blank grid will appear. Using the left button on the mouse
replicate the pattern you were remembering by "clicking" on the
empty boxes on the screen to fill them.
once you click on a box you can NOT unclick it
5. The final screen will ask
"How did you solve the math problem."
Choose the BEST description from the following:
(1)Retrieved= You have the answer memorized.
"I just remembered" or "I just new the answer"
(2)Added/Counted= You added or counted. e.g.7x3(7+7+7)
(3)Transformed= You 'transformed' or changed the problem
into another one that you knew.
e.g.8x9(You transformed it into 8x8+8)
(4)Other= You did something else not listed

Press ENTER to begin....

Instructions for participants: Visual Memory Control Condition

This task is Practice Problems for Memory Only

1. Each trial will begin with a READY screen.
2. Then a grid with a total of 16 boxes will appear.
EIGHT of the boxes are filled in to make a random pattern.
You have to look at the pattern and REMEMBER it
as you will have to replicate it later on a Blank grid.
3. Then a random number will appear. You don't do anything.
Just wait until you see the next screen.
4. A Blank grid will appear. Using the left button on the mouse
replicate the pattern you were remembering by "clicking" on the
empty boxes on the screen to fill them.
once you click on a box you can NOT unclick it
5. The final screen will have ***

Press ENTER to begin...

Instructions for participants: Visual Hard Load Condition

This task is Practice Problems for Math+Memory

1. Each trial will begin with a READY screen.
2. Then a grid with a total of 16 boxes will appear.
EIGHT of the boxes are filled in to make a random pattern.
You have to look at the pattern and REMEMBER it
as you will have to replicate it later on a Blank grid.
3. Then the math problem will appear. Solve it as quickly
and accurately as you can, and TYPE the answer on the keypad.
don't correct or change your answer once you've typed it in
Then just wait until you see the next screen.
4. A Blank grid will appear. Using the left button on the mouse
replicate the pattern you were remembering by "clicking" on the
empty boxes on the screen to fill them.
once you click on a box you can NOT unclick it
5. The final screen will ask
"How did you solve the math problem."
Choose the BEST description from the following:
(1)Retrieved= You have the answer memorized.
"I just remembered" or "I just new the answer"
(2)Added/Counted= You added or counted. e.g.7x3(7+7+7)
(3)Transformed= You 'transformed' or changed the problem
into another one that you knew.
e.g.8x9(You transformed it into 8x8+8)
(4)Other= You did something else not listed

Press ENTER to begin....

Appendix C(i-vi): Experimental Conditions

Appendix C(i): Phonological Dual Task Trials

Easy Load

GUB	6 x 9 =	Recall the Word	How did you solve the math problem?

Hard Load

GUB MEP NAL	9 - 4 =	Recall the Words	How did you solve the math problem?

Appendix C(ii): Arithmetic Control Trial (Phonological)

* * *

9 – 4 =

* * *

How did you solve the math problem?

Appendix C(iii): Phonological Memory Load Control

GUB MEB NAL

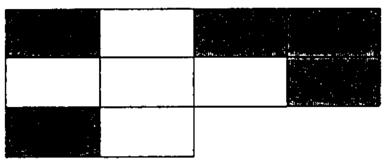
21

Recall the Words

* * *

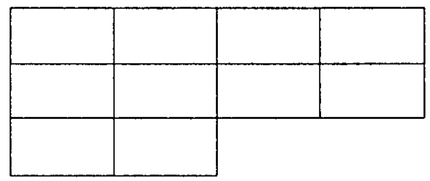
Appendix C(iv): Visual Dual Task Trials

Easy Load



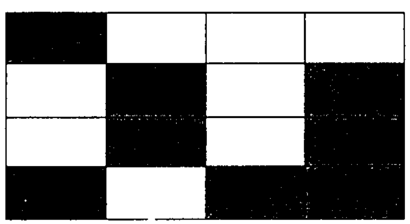
6 x 9 =

Replicate the Pattern



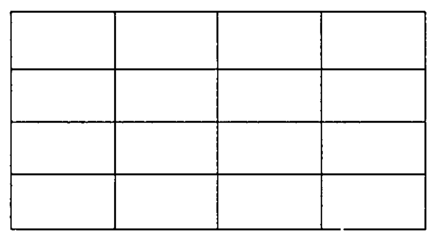
How did you solve the math problem?

Hard Load



9 - 4 =

Replicate the Pattern



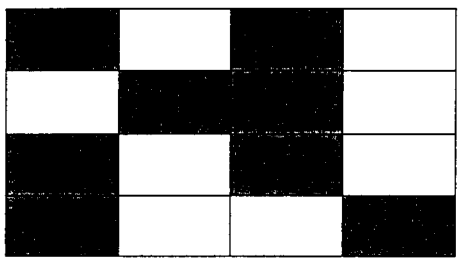
How did you solve the math problem?

Appendix C(v): Arithmetic Control Trial (Visual)

$9 - 4 =$

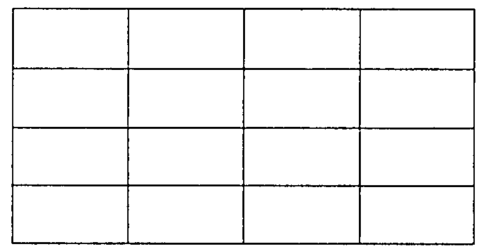
How did you solve the math problem?

Appendix C(vi): Visual Memory Load Control



6

Replicate the Pattern



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