INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI[®]

Modal Testing and Model Updating of a Steel Bridge Pier Frame

Submitted by

Shougui Huang, B.A. Sc

Hefei University of Technology

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment
of the requirement for the degree of Master of Applied Science

Department of Civil and Environmental Engineering

Carleton University

Ottawa, Ontario

Shougui Huang, 2005

The Master of Applied Science in Civil Engineering is a joint program with University of
Ottawa, administered by Ottawa-Carleton Institute of Civil Engineering

Library and
Archives Canada

Bibliothèque et
Archives Canada

0-494-08374-3

Published Heritage
Branch

Direction du
Patrimoine de l'édition

395 Wellington Street
Ottawa ON K1A 0N4
Canada

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file *Votre référence*

ISBN:

Our file *Notre référence*

ISBN:

NOTICE:

The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

AVIS:

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l'Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L'auteur conserve la propriété du droit d'auteur et des droits moraux qui protègent cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.

Canada

Department of Civil and Environmental Engineering

The undersigned recommend to the Faculty of Graduate Studies and Research acceptance
of the thesis

**Modal Testing and Model Updating of a Steel Bridge Pier
Frame**

Submitted by

Shougui Huang

in partial fulfillment of the requirement for the degree of Master of Applied Science

Chair, Department of Civil and
Environmental Engineering

Thesis Supervisor

Carleton University

Ottawa, Ontario, Canada

August, 2005

Abstract

Modal testing of a steel bridge pier frame is carried out in the laboratory to gain insight on the sensitivity and dynamic characteristics of actual old structure systems, which have been subjected to deterioration from operation in the field. The procedure and theoretical basis of updating the theoretical finite element model by modal testing results are presented in the thesis. The updated finite model can be used to assess of the capacity and performance and remaining service life of the structure. The study of the environmental influences and uncertainties on field vibration modal characteristics is part of a long-term monitoring project of the Confederation Bridge at Carleton University. The present study compliments the Confederation Bridge monitoring project to make advance in the practical application of vibration based structural health monitoring techniques as structural condition assessment tools.

Acknowledgements

I would like to thank Professor David T. Lau for his instruction, his time and encouragement throughout this research. It was a great pleasure to work and learn from this knowledgeable professor.

Thanks are due to the staff of Civil Engineering laboratory in Carleton University for their help during the modal testing of the bridge pier frame.

I would like to express my thanks to my colleague, Nicolas Londono, Ryan Deng, and Viet Phung for their help and encouragement.

The test frame was provided for research with assistance from Institute for Research on Construction (IRC) of the National Research Council of Canada (NRC) and Public Works and Government Services Canada. Their assistance and help in this research are gratefully acknowledged.

I would like to take this opportunity to express my deepest appreciations to my family for their love, encouragement and patience. Without their support, I would not have the opportunity to finish this research work.

Contents

ABSTRACT	III
ACKNOWLEDGEMENTS	IV
CHAPTER 1 INTRODUCTION	11
1.1 INTRODUCTION	11
1.2 OBJECTIVES AND SCOPE.....	15
CHAPTER 2 THEORETICAL BASIS OF MODAL TESTING	17
2.1 INTRODUCTION	17
2.2 THEORETICAL MODAL ANALYSIS.....	18
<i>2.3 Method of Experimental Modal Analysis.....</i>	<i>21</i>
<i>2.3.1 The Analysis of Frequency Response Function (FRF)</i>	<i>21</i>
<i>2.3.2 The Analysis of Least-Square Complex Exponential method (LSCE)</i>	<i>24</i>
CHAPTER 3 FINITE ELEMENT ANALYSIS OF THE STEEL FRAME	29
3.1 DESCRIPTION OF THE STEEL FRAME	29
3.2 PRELIMINARY FINITE ELEMENT MODEL	30
3.3 PRELIMINARY FINITE ELEMENT ANALYSIS RESULTS.....	32
CHAPTER 4 MODAL TESTING OF STEEL FRAME.....	35
4.1 INTRODUCTION	35
4.2 MODAL TESTING PROCEDURES	35
<i>4.2.1 Modal Testing Configurations</i>	<i>37</i>
<i>4.2.2 Modal Testing Instrumentation.....</i>	<i>39</i>

4.2.3 <i>Experiment Setup and Procedure:</i>	41
4.3 EXPERIMENT MODAL ANALYSIS	43
4.3.1 <i>Method of Modal Analysis</i>	43
4.3.2 <i>Data Acquisition and Analysis</i>	43
4.3.3 <i>The Results of Modal Analysis</i>	44
CHAPTER 5 MODEL CORRELATION AND UPDATING	50
5.1 INTRODUCTIONS	50
5.2 PRELIMINARY ADJUSTMENTS TO FE STRUCTURAL MODEL	51
5.2.1 <i>Modeling the support boundary conditions</i>	51
5.2.2 <i>Modeling the rigidity at connection joints</i>	52
5.2.3 <i>Determine the range of approximate springs' constant values</i>	53
5.3 REDUCTIONS OF FINITE ELEMENT DATA	56
5.4 COMPARISON OF THE MEASURED AND FE MODEL DATA	58
5.5 THE THEORETICAL BASIS OF CORRECTION OF THE STIFFNESS MATRIX	59
5.5.1 <i>Correction of Eigenvector Matrix</i>	59
5.5.2 <i>Correction of Stiffness Matrix</i>	62
5.6 THE RESULTS AND PROCEDURE OF MODEL CORRELATION	66
CHAPTER 6 SUMMARY	72
6.1 SUMMARY AND CONCLUSIONS	72
REFERENCE	75
APPENDIX FIGURES OF FREQUENCY RESPONSE FUNCTION	78

List of Figures

Figure 3.1 The Old Steel Pier Frame	29
Figure 3.2 Finite Element Model of Bridge Steel Pier Frame	31
Figure 3.3 Mode Shape 1 (45.998 Hz).....	32
Figure 3.4 Mode Shape 2 (52.492 Hz).....	33
Figure 3.5 Mode Shape 3 (59.056 Hz).....	33
Figure 3.6 Mode Shape 4 (68.88 Hz).....	34
Figure 3.7 Mode Shape 5 (89.275 Hz).....	34
Figure 4.1 Impact Hammer Used as Exciter in Modal Testing	40
Figure 4.2 Accelerometer.....	40
Figure 4.3 Analyzer	41
Figure 4.4 Locations and Directions of Acceleration Response Measurement	42
Figure 4.5 Modal mode shape 1 from modal testing	45
Figure 4.6 Modal mode shape 2 from modal testing	46
Figure 4.7 Modal mode shape 3 from modal testing	47
Figure 4.8 Modal mode shape 4 from modal testing	48
Figure 4.9 Modal mode shape 1 from modal testing	49
Figure 5.1 Boundary conditions model of the steel frame.....	52
Figure 5.2 Models of Joint Rigidity and Boundary Conditions of the Test Frame	53
Figure 5.3 Mass Matrix $[M]$ of the Test Frame in 3D Paragraph (unit: kilogram)	68
Figure 5.4 Stiffness Matrix $[K]$ of the Test Frame in 3D Paragraph (unit: kn/m)	69
Figure 0.1 Frequency Response Function at Node 10 of Case 1	78
Figure 0.2 Frequency Response Function at Node 6 of Case 1	79

Figure 0.3 Frequency Response Function at Node 5 of Case 1	80
Figure 0.4 Frequency Response Function at Node 4 of Case 1	81
Figure 0.5 Frequency Response Function at Node 3 of Case 1	82
Figure 0.6 Frequency Response Function at Node 10 of Case 2	83
Figure 0.7 Frequency Response Function at Node 22 of Case 2	84
Figure 0.8 Frequency Response Function at Node 23 of Case 2	85
Figure 0.9 Frequency Response Function at Node 4 of Case 2	86
Figure 0.10 Frequency Response Function at Node 24 of Case 2	87
Figure 0.11 Frequency Response Function at Node 10 of Case 3	88
Figure 0.12 Frequency Response Function at Node 11 of Case 3	89
Figure 0.13 Frequency Response Function at Node 12 of Case 3	90
Figure 0.14 Frequency Response Function at Node 4 of Case 3	91
Figure 0.15 Frequency Response Function at Node 13 of Case 3	92
Figure 0.16 Frequency Response Function at Node 10 of Case 4	93
Figure 0.17 Frequency Response Function at Node 18 of Case 4	94
Figure 0.18 Frequency Response Function at Node 17 of Case 4	95
Figure 0.19 Frequency Response Function at Node 4 of Case 4	96
Figure 0.20 Frequency Response Function at Node 16 of Case 4	97
Figure 0.21 Frequency Response Function at Node 10 of Case 5	98
Figure 0.22 Frequency Response Function at Node 21 of Case 5	99
Figure 0.23 Frequency Response Function at Node 20 of Case 5	100
Figure 0.24 Frequency Response Function at Node 4 of Case 5	101
Figure 0.25 Frequency Response Function at Node 19 of Case 5	102

Figure 0.26 Frequency Response Function at Node 10 of Case 6	103
Figure 0.27 Frequency Response Function at Node 9 of Case 6	104
Figure 0.28 Frequency Response Function at Node 8 of Case 6	105
Figure 0.29 Frequency Response Function at Node 4 of Case 6	106
Figure 0.30 Frequency Response Function at Node 7 of Case 5	107

List of Table

Table 3-1 Modal Frequencies with Fixed Condition at Supports	32
Table 4-1 Modal Testing Procedure	42
Table 4-2 Modal Frequencies from modal testing.....	44
Table 5-1 Modal Frequencies with fixed conditions at supports.....	55
Table 5-2 Modal Frequencies with approximate fixed conditions at supports.....	55
Table 5-3 The Analysis Result after Modification	56
Table 5-4 Stiffness Matrix $[k_a]$ (unit: kn/m)	70
Table 5-5 Stiffness Matrix $[k]$ (unit: kn/m).....	70
Table 5-6 Mass Matrix $[m_a]$ (unit: kilogram).....	70

Chapter 1 INTRODUCTION

1.1 Introduction

Modal testing of an old steel bridge pier frame is carried out in the laboratory to gain insight on the sensitivity and characteristics of modal test results as measurement indicators to deterioration of typical old bridge structures by eliminating the uncertainties and influences on the test results due to fluctuations in the field environment. The important consideration of the environmental influences and uncertainties on field vibration modal characteristics is part of a long-term monitoring project of Confederation Bridge at Carleton University. The present study compliments the Confederation Bridge monitoring project to make advance in the practical application of vibration based structural health monitoring techniques as structural condition assessment tools. The difficulties and challenges in modeling accurately the actual conditions and behavior of the old bridge pier compared to the theoretical computer model determined from the construction drawings and specifications for the purpose of structural conditions evaluation are also addressed in the present study.

The aging of bridge infrastructure has led to operational and structural safety concerns of many of these existing old structures. The upgrading of aging and deteriorated bridges is among the top priorities of many levels of government in Canada in dealing with the maintenance and renewal problems of the transportation network infrastructure within their jurisdictions. The need of reliable methodologies for accurate assessment of the

performance and remaining service life of existing old bridges is well recognized with increasing urgency in recent years.

Inspection procedures have been developed and followed to assess the structural conditions of existing old bridges. The typical approach of combining visual inspection with detailed structural analysis, such as finite element method, has been followed for years in bridge condition assessment and performance evaluation practices of existing old bridge structures. Regular periodic visual bridge inspection, sometimes aided by non-destructive testing methods, can provide insight and valuable information on the conditions of the structure. However, the effectiveness of visual inspection is often limited to surface damage of critical parts of the structures, especially large bridges, which makes visual inspection assessment method unreliable. Visual inspection alone cannot provide the detailed information on the capacity of the structure. On the other hand, the reliability of the finite element analysis results of existing bridges depends on the accuracy and realistic representation of the structure model to the actual field conditions of the structure and assumptions used in the analysis, which are in turn dependent on available material information and data, such as modulus of material, conditions of structure member joint connections, and actual boundary conditions of the structural supports of the structures in the field. In the application of vibration based structural health monitoring technique, the actual structural behavior of even a new structure in the field can be different from the theoretical assumed behavior because of the differences between the actual structure and the nominal design values and assumptions adopted in the design process. For deteriorated bridge structures, this problem of discrepancy between the structural model and the actual bridge behavior is

even greater and thus has significant implications on the use of field measurement results, such as field modal testing data and information for structural condition assessment. Without accurate information on the actual conditions and behavior of existing old structures in the field under their operating environment, it is very difficult to accurately predict the performance, reserved capacity and remaining service life of aging structures. Accurate determination of the strength capacity and remaining service life of aging bridges and other structures provides crucial information for the development of rational and cost effective repair and rehabilitation strategies for the aging infrastructures in the society.

The development of modal testing method in conjunction with model updating technique with particular emphasis on civil engineering application brings forth a promising viable approach to solve some of the aforementioned structural assessment problems. In modal dynamic testing of a structure, both the excitation forces and the responses are measured. After processing and analysis of the measured data, dynamic properties, such as modal frequencies, modal damping and modal shapes of the structure, can be extracted from the measured data by modal analysis techniques. The extracted dynamic properties based on the measured data are more accurate indication of the conditions of the structure. For example, the measured vibration frequencies are directly related to the actual stiffness properties of the tested structure. Theoretically, by using information based on dynamic properties extracted from modal testing, computer models of the structure can be refined to accurately represent the behavior of the aged bridge structure. The calibrated updated computer model of existing bridge structures can then be used to predict remaining capacities and service life of the facilities, based on which rehabilitation strategies can be

devised. However, in order to do so, careful consideration should be applied that the data and results from modal testing correctly reflect the actual properties of the structure. The influence of other factors on the accuracy of modal test results representative of the conditions of the structure must be eliminated.

In engineering practice, it is common that the experimental modal testing data collected in field under operating environment of the test structure are different from the expected results determined from design specifications or tests conducted under the controlled laboratory condition. There are two aspects associated with the differences. One aspect is the inherent differences between the as-built structure and the idealized structural model based on design specifications, and the further expansion of these differences over time because of gradual deterioration of the structure. The other aspect is the influence of random environmental factors or unknown processes not easily quantified on the quality of the modal testing results. Modal testing of old bridge structures, excited either by controlled force or other ambient excitations, is usually carried out in the field without any control of the background environmental conditions that appear as background noise in the measured data. The environmental noises can contaminate and affect the quality of the response signals. For example, temperature change around the structure can noticeably affect the dynamic properties of the structure. Consequently, in order to use modal testing results as the basis for structural condition assessment, the influence of the environment on the measured data must be carefully taken into account in the assessment process.

In light of the problems of using modal testing techniques as the basis for structural assessment in civil engineering applications, an actual steel bridge pier bent retained from

a demolished bridge is tested in the laboratory. Laboratory modal testing eliminates the influence of random environment noise in the field, and thus allows the study to focus on the modal testing and data analysis techniques to quantify the dynamic properties of the old bridge structure associate with deteriorations resulting from operation of the structure.

1.2 Objectives and Scope

The main objective of the present experimental study is to gain insight and knowledge on the practicality and challenges of applying modal testing structural assessment techniques to the evaluation of actual field deteriorated structures. A steel pier frame from a demolished old bridge is studied. Information and insights gained on obtaining the dynamic characteristics of the old steel frame help engineers to better and more accurately assess the performance of aging bridge structures. The modal dynamic properties of the steel bridge frame determined from the theoretical finite element analysis models of the structure based on nominal construction specifications are compared with the experimental modal testing results. Model updating of the computer analysis model of the steel frame by using the modal testing results are investigated to establish calibrated model of the structure that better captures its actual behavior and deteriorated condition. From the research results, suggestions on modeling old structures are presented.

The thesis is divided into six chapters. Chapter2 presents the theoretical basis and derivations of the modal testing method and numerical data analysis procedures. Chapter-3 gives a description of the steel bridge pier frame used in the present study and finite element model of the test structure. Chapter 4 describes the experimental set up of the

steel bridge pier frame for modal testing in the laboratory. The modal test results are presented. Chapter 5 presents the structural analysis model updating of the bridge pier frame by calibration and refinement of the finite element structure model of the test frame using the modal testing data. Chapter 6 gives a summary and conclusions of the study, and makes some suggestions for future research.

Chapter 2 THEORETICAL BASIS OF MODAL TESTING

2.1 Introduction

Modal testing is an experimental technique that can be used to derive the modal parameters of a structure system, such as mode shapes, vibration frequencies and modal damping [1-10]. Modal testing can be divided into three steps: test structure setup, frequency response function measurement and modal parameter identification. A discussion of the setup of the steel pier frame is presented in Chapter 4. Frequency response function represents the relationship between the vibration responses of the structure with the excitation input as a function of the excitation frequency. In modal testing, both excitation and responses of the test structure are measured to generate frequency response function (FRF). A set of different frequency response functions can be established by measuring the vibration response at different locations of the test structure and also by varying the excitation input location. Modal parameter identification is then carried out to extract the desired dynamic characteristics of the test structure by numerical analysis of these frequency response functions (FRF) obtained from experimental modal testing. This process is referred to as experimental modal analysis.

Before the discussion of the application of the modal testing results, the two different modal analysis procedures from which the modal parameters can be derived are presented. Experimental modal analysis obtains modal parameters from measured FRF data. Theoretical modal analysis derives modal parameters from numerical structural model, which consists of mass, stiffness and damping matrices.

Modal testing data are valuable and have applications in many aspects of engineering practices. Some applications use the modal data directly, while other applications use the data for further analysis. A detailed discussion on the use and application of modal test data is presented in chapter 5. A few example of general applications of modal testing data are listed as follows

- Updating finite element model
- Modification of Structural dynamic characteristics
- Forced response prediction
- Structural damage detection

2.2 Theoretical Modal Analysis

A clear understanding of the theoretical basis of modal testing is important for its successful implementation later in this research. Here, the theory of modal testing is discussed to give insight of experiment modal testing procedures followed in the laboratory test discussed later in this thesis. Theoretical vibration analysis of a structural model can be divided into three stages as follows

1. Establishing a numerical structural model to represent a structural system in terms of its mass, stiffness and damping factors, which are referred to as the physical structural properties of the analyzed system.
2. Theoretical analysis of the numerical structural model leads to a description of the structure's behavior by a set of vibration mode shapes and corresponding frequencies and modal damping factors, which are referred to as modal properties of the structure.

3. Deriving from structural vibration behavior under given external excitation forces, the structure's response behavior given in terms of a set of vibration response functions in the frequency domain can be obtained, which are referred to as frequency response functions.

The dynamic response of an undamped multiple degrees-of-freedom (MDOF) system is represented by Equation 2.1 as follows

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \quad \text{Equation 2.1}$$

where $[M]$ is the $n \times n$ mass matrix, $[K]$ is the $n \times n$ stiffness matrix, $\{x(t)\}$ and $\{\ddot{x}(t)\}$ denote the $n \times 1$ time-varying displacement and acceleration vector respectively, $\{f(t)\}$ is the $n \times 1$ time-varying force vector. For the free vibration case, $\{f(t)\} = \{0\}$ in Equation 2.1 gives

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \quad \text{Equation 2.2}$$

A solution of Equation 2.2 can be written in the form expressed by Equation 2.3.

$$\{x(t)\} = \{X\}e^{i\omega t} \quad \text{Equation 2.3}$$

where $\{X\}$ is a $n \times 1$ time-independent amplitude vector.

Substituting Equation 2.3 into Equation 2.2 leads to

$$([K] - \omega^2[M])\{X\}e^{i\omega t} = \{0\} \quad \text{Equation 2.4}$$

The only non-trivial solution for Equation 2.4 is

$$\det([K] - \omega^2[M]) = 0 \quad \text{Equation 2.5}$$

Modal natural frequencies ω_r ($r=1, 2, \dots, n$) where n is the number of system degree-of-freedom and a set of unique mode shape displacement shape vectors $\{\psi_r\}$ ($r=1, 2, \dots, n$) corresponding to the modal frequencies can be obtained by solving Equation 2.5.

Modal analysis of the forced vibration case represented by Equation 2.1 leads to the derivation of the frequency response function of the system. Expressing the input excitation forcing function and the displacement response in the solution of Equation 2.1 can be derived using Equation 2.3 and Equation 2.6.

$$\{x(t)\} = \{X\}e^{i\omega t} \quad \text{Equation 2.3}$$

$$\{f(t)\} = \{F\}e^{i\omega t} \quad \text{Equation 2.6}$$

where $\{F\}$ and $\{X\}$ are $n \times 1$ time-independent complex amplitude vector of the excitation force and displacement responses, respectively.

Substituting Equation 2.3 and Equation 2.6 into Equation 2.1 gives

$$([K] - \omega^2[M])\{X\}e^{i\omega t} = \{F\}e^{i\omega t} \quad \text{Equation 2.7}$$

or

$$\{X\} = ([K] - \omega^2[M])^{-1}\{F\} \quad \text{Equation 2.7a}$$

In theoretical modal analysis with given or known stiffness and mass properties and input excitation force of the systems, the displacement response in frequency domain can be found by Equation 2.7a.

On the other hand, in experimental modal analysis the interpretation of Equation 2.7a is different in that both the displacement response $\{X\}$ and the input excitation $\{F\}$ are measured in this case, so that Equation 2.7a is rewritten as follows

$$\{X\} = [\alpha(\omega)]\{F\} \quad \text{Equation 2.7b}$$

where $[\alpha(\omega)] = ([K] - \omega^2[M])^{-1}$ is $n \times n$ receptance frequency response function matrix for the system.

The element in this frequency response function matrix, $\alpha_{jk}(\omega)$, is give by:

$$\alpha_{jk}(\omega) = \left(\frac{X_j}{F_k} \right) \quad \text{Equation 2.8}$$

The receptance $\alpha_{jk}(\omega)$ represents the ratio of the displacement response at DOF 'j' to the single input force applied at DOF 'k', which is the same as the displacement response at DOF 'j' due to a unit input force applied at DOF 'k'.

Although the response variable in the above derivation is displacement, similar frequency response function can be derived for velocity and acceleration. Therefore, frequency response function may be defined in terms of displacement, velocity, or acceleration. The definitions of the different forms of frequency response function are presented as follows.

$$\text{Receptance: } \alpha_{jk}(\omega) = \frac{X_j(\omega)}{F_k(\omega)} = \frac{\text{Displacement}}{\text{Force}} \quad \text{Equation 2.9}$$

$$\text{Mobility: } Y_{jk}(\omega) = \frac{\dot{X}_j(\omega)}{F_k(\omega)} = \frac{\text{Velocity}}{\text{Force}} \quad \text{Equation 2.10}$$

$$\text{Accelerance: } A_{jk}(\omega) = \frac{\ddot{X}_j(\omega)}{F_k(\omega)} = \frac{\text{Acceleration}}{\text{Force}} \quad \text{Equation 2.11}$$

where $X_j(\omega) = \int_{-\infty}^{\infty} x_j(t) e^{-i\omega t} dt$, $F_k(\omega) = \int_{-\infty}^{\infty} f_k(t) e^{-i\omega t} dt$, $x_j(t)$ is the displacement response at DOF 'j' in time domain, $f_k(t)$ is the input force at DOF 'k' in time domain.

2.3 Method of Experimental Modal Analysis

2.3.1 The Analysis of Frequency Response Function (FRF)

In the discussion above, the procedure of theoretical modal analysis and the derivation of the frequency response function have been presented. Further analysis of frequency

response function to obtain the modal information contained in frequency response function is presented next.

A receptance FRF of a damped MDOF structural system can be derived from the equation of motion of the MDOF system to give the relationship between modal parameters and frequency response functions. The free vibration equation of motion of MDOF system is given by Equation 2.12 as follows

$$[M]\{\ddot{x}\} + [K]\{x\} + i[H]\{x\} = \{0\} \quad \text{Equation 2.12}$$

Where $[H] = \mu[M] + \nu[K]$ is the proportional structural damping matrix, μ, ν are real constant coefficients of the mass and stiffness proportional damping terms respectively. The $\{x\}$ and $\{\ddot{x}\}$ are the response vector of displacement and acceleration respectively.

The receptance FRF of Equation 2.12 can be given in Equation 2.13 by following the procedure presented from Equation 2.1 to Equation 2.7b.

$$[\alpha(\omega)] = ([K] + i[H] - \omega^2[M])^{-1} \quad \text{Equation 2.13}$$

or

$$[\alpha(\omega)]^{-1} = ([K] + i[H] - \omega^2[M]) \quad \text{Equation 2.14}$$

Pre-multiplying both sides by $[\Phi]^T$ and post-multiplying both sides by $[\Phi]$ gives

$$[\Phi]^T [\alpha(\omega)]^{-1} [\Phi] = [\Phi]^T ([K] + i(\mu[M] + \nu[K]) - \omega^2[M]) [\Phi] \quad \text{Equation 2.15}$$

where $[\Phi]$ is mass normalized mode shapes vector.

$$[\Phi]^T [M] [\Phi] = [I]$$

$$[\Phi]^T [M] [\Phi] = \begin{bmatrix} \ddots & \omega_r^2 & \ddots \end{bmatrix}$$

Rearranging Equation 2.15 gives

$$[\Phi]^T [\alpha(\omega)]^{-1} [\Phi] = ([\ddot{\omega}_r^2] + i(\mu[I] + \nu[\ddot{\omega}_r^2]) - \omega^2[I]) \quad \text{Equation 2.16}$$

$$[\alpha(\omega)] = [\Phi] ([\ddot{\omega}_r^2] + i(\mu[I] + \nu[\ddot{\omega}_r^2]) - \omega^2[I])^{-1} [\Phi]^T \quad \text{Equation 2.17}$$

$$[\alpha(\omega)] = [\Phi] \left[\ddot{\omega}_r^2 \left(1 + i\left(\nu + \frac{\mu}{\omega_r^2}\right) \right) - \omega^2 \right]^{-1} [\Phi]^T \quad \text{Equation 2.18}$$

From Equation 2.18, the modal frequency can be rewritten as follows

$$\lambda_r^2 = \omega_r^2 (1 + i\eta_r), \quad \eta_r = \nu + \frac{\mu}{\omega_r^2}, \quad (r=1, 2, \dots, n) \text{ where } r \text{ is the degree-of-freedom, } \lambda_r \text{ is the}$$

frequency of the structure system, the ω_r is the natural frequency of the r th mode.

Substituting λ_r^2 into Equation 2.18, the receptance matrix of the system can be determined from the modal data of the system as follows

$$[\alpha(\omega)] = [\Phi] \begin{bmatrix} \lambda_1^2 - \omega^2 & & \\ & \lambda_2^2 - \omega^2 & \\ & & \ddots \\ & & & \lambda_n^2 - \omega^2 \end{bmatrix}^{-1} [\Phi]^T \quad \text{Equation 2.19}$$

$$[\Phi] = \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \dots & & & \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nn} \end{bmatrix} \quad \text{where } [\Phi] \text{ is the matrix of the mass normalized mode shapes.}$$

The receptance function $\alpha_{jk}(\omega)$ can be written as follows

$$\alpha_{jk}(\omega) = \frac{\phi_{j1}\phi_{k1}}{\lambda_1^2 - \omega^2} + \frac{\phi_{j2}\phi_{k2}}{\lambda_2^2 - \omega^2} + \dots + \frac{\phi_{jn}\phi_{kn}}{\lambda_n^2 - \omega^2} \quad \text{Equation 2.20}$$

Equation 2.20 can be rewritten by using the modal parameter as follows

$$\alpha_{jk}(\omega) = \sum_{r=1}^n \frac{r A_{jk}}{\lambda_r^2 - \omega^2} = \left\{ {}_1 A_{jk} \quad {}_2 A_{jk} \dots {}_n A_{jk} \right\} \begin{Bmatrix} \frac{1}{\lambda_1^2 - \omega^2} \\ \frac{1}{\lambda_2^2 - \omega^2} \\ \dots \\ \frac{1}{\lambda_n^2 - \omega^2} \end{Bmatrix} = \left\{ A_{jk} \right\}^T \left\{ \frac{1}{\lambda_r^2 - \omega^2} \right\} \quad \text{Equation 2.21}$$

Therefore the p th column of the matrix $[\alpha(\omega)]$ can be written as follows

$$[\alpha(\omega)]_p = \begin{Bmatrix} \alpha_{p1}(\omega) \\ \alpha_{p2}(\omega) \\ \dots \\ \alpha_{pn}(\omega) \end{Bmatrix} = [A_p] \begin{Bmatrix} \frac{1}{\lambda_1^2 - \omega^2} \\ \frac{1}{\lambda_2^2 - \omega^2} \\ \dots \\ \frac{1}{\lambda_n^2 - \omega^2} \end{Bmatrix} \quad \text{Equation 2.22}$$

$$\text{where } [A_p] = \begin{bmatrix} \phi_{p1}\phi_{11} & \phi_{p2}\phi_{12} & \dots & \phi_{pn}\phi_{1n} \\ \phi_{p1}\phi_{21} & \phi_{p2}\phi_{22} & \dots & \phi_{pn}\phi_{2n} \\ \dots & \dots & \dots & \dots \\ \phi_{p1}\phi_{n1} & \phi_{p2}\phi_{n2} & \dots & \phi_{pn}\phi_{nn} \end{bmatrix} = \begin{bmatrix} {}_1 A_{p1} & {}_2 A_{p1} & \dots & {}_n A_{p1} \\ {}_1 A_{p2} & {}_2 A_{p2} & \dots & {}_n A_{p2} \\ \dots & \dots & \dots & \dots \\ {}_1 A_{pn} & {}_2 A_{pn} & \dots & {}_n A_{pn} \end{bmatrix} \quad \text{Equation 2.23}$$

It is obvious that $[A_p]$ contains the all the elements of the modal shapes in $[\Phi]$. Therefore, theoretically it is possible from $[A_p]$ to derive modal frequencies and modal shapes. A detailed discussion about the methods to extract modal parameters from FRF is presented next.

2.3.2 The Analysis of Least-Square Complex Exponential method (LSCE)

Modal test data can be acquired in the forms of either frequency response function or impulse response function. The typical methods of experiment modal analysis can be divided into two categories of either frequency domain analysis or time domain analysis. However, there is a simple relationship that links these two analysis approaches. The

frequency response function derived in the frequency domain can be shown as the Fourier transform of the impulse response function in the time domain, and vice versa. An indirect method in time domain Least Square Complex Exponential method [11-13] (LSCE) is applied in the experiment modal analysis of the test data in this study. In frequency domain, the FRF in terms of receptance $\alpha_{jk}(\omega)$ for a linear, viscously damped system can be obtained by following the procedure presented from Equation 2.1 to Equation 2.20 as follows

$$\begin{aligned}\alpha_{jk}(\omega) &= \sum_{r=1}^n \left(\frac{{}_r A_{jk}}{\omega_r \xi_r + i(\omega - \omega_r \sqrt{1 - \xi_r^2})} + \frac{{}_r A_{jk}^*}{\omega_r \xi_r + i(\omega + \omega_r \sqrt{1 - \xi_r^2})} \right) \\ &= \sum_{r=1}^{2n} \frac{{}_r A_{jk}}{\omega_r \xi_r + i(\omega - \omega_r)}\end{aligned}\quad \text{Equation 2.24}$$

where

ω_r is the natural frequency of the r th mode

ξ_r is the damping ratio of the r th mode

${}_r A_{jk}$ is the residue corresponding to the r th mode

* denotes complex conjugate

$$\omega_r' = \omega_r \sqrt{1 - \xi_r^2}$$

$$\omega_{r+n}' = -\omega_r'$$

$${}_{r+n} A_{jk}^* = {}_r A_{jk}$$

The corresponding impulse response function (IRF) obtained by an inverse Fourier transform of the corresponding frequency response function (FRF) is given in Equation 2.25

$$h_{jk}(t) = \sum_{r=1}^{2n} A_{jk} e^{s_r t} \quad \text{Equation 2.25}$$

For convenience, $h_{jk}(t), A_{jk} e^{s_r t}$ are replaced by $h(t), A_r$ respectively. Equation 2.25 can be rewritten as follows

$$h(t) = \sum_{r=1}^{2n} A_r e^{s_r t} \quad \text{Equation 2.26}$$

$$s_r = -\omega_r \xi_r + i\omega_r'$$

The time response real value $h(t)$, over the total duration of measurement $L\Delta t$ equally spaced at time interval Δt , are shown as follows

$$\begin{aligned} h(0) &= \sum_{r=1}^{2n} A_r e^0 = \sum_{r=1}^{2n} A_r \\ h(1\Delta t) &= \sum_{r=1}^{2n} A_r e^{s_r(\Delta t)} = \sum_{r=1}^{2n} A_r V_r \end{aligned} \quad \text{Equation 2.27}$$

.....

$$h(L\Delta t) = \sum_{r=1}^{2n} A_r e^{s_r(L\Delta t)} = \sum_{r=1}^{2n} A_r V_r^L$$

$$V_r = e^{s_r(\Delta t)} \quad \text{Equation 2.28}$$

The values of A_r and V_r are unknowns. The solutions of A_r and V_r can be obtained by Prony's method [14] developed in 1795 by Prony. For an underdamped system, there exists a polynomial in V_r of order L with coefficients β such that the following expression can be verified

$$\beta_0 + \beta_1 V_r^1 + \beta_2 V_r^2 + \dots + \beta_L V_r^L = 0 \quad \text{Equation 2.29}$$

where $\beta_0, \beta_1, \beta_2, \dots, \beta_L$ are real coefficients.

In order to calculate the coefficients $\beta_0, \beta_1, \beta_2 \dots \beta_L$, Equation 2.27 are multiplied by $\beta_0, \beta_1, \beta_2 \dots \beta_L$ and summed. The result is given as follows

$$\sum_{j=0}^L \beta_j h_j = \sum_{j=0}^L (\beta_j \sum_{r=1}^{2n} A_r V_r^j) = \sum_{r=1}^{2n} (A_r \sum_{j=0}^L \beta_j V_r^j) \quad \text{Equation 2.30}$$

Substituting Equation 2.29 into Equation 2.30 yields:

$$\sum_{j=0}^L \beta_j h_j = 0, \text{ for each } V_r = e^{s_r(\Delta t)} \quad \text{Equation 2.31}$$

Equation 2.31 can be rewritten as follows

$$\sum_{j=0}^{2n-1} \beta_j h_j = -h_{2n}, \text{ by setting } \beta_{2n} = 1, L = 2n \quad \text{Equation 2.32}$$

Equation 2.32 can be expanded as follows

$$\{h_0 \quad h_1 \quad h_2 \quad \dots \quad h_{2n-1}\} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \beta_{2n-1} \end{Bmatrix} = -h_{2n} \quad \text{Equation 2.33}$$

Repeating the procedure presented from Equation 2.24 to Equation 2.33 and using a different set of IRF data $\{h_1 \quad h_2 \quad h_3 \quad \dots \quad h_{2n}\}$ gives Equation 2.34. The IRF $\{h_1 \quad h_2 \quad h_3 \quad \dots \quad h_{2n+1}\}$ has the same number of data as that in Equation 2.33 but starts with a time interval shift Δt .

$$\{h_1 \quad h_2 \quad h_3 \quad \dots \quad h_{2n}\} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \beta_{2n-1} \end{Bmatrix} = -h_{2n+1} \quad \text{Equation 2.34}$$

Repeating the application of the previous procedure yields a full set of $2n$ equations as follows

$$\begin{bmatrix} h_0 & h_1 & h_2 & \dots & h_{2n-1} \\ h_1 & h_2 & h_3 & \dots & h_{2n-1} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ h_{2n-1} & h_{2n} & h_{2n+1} & \dots & h_{4n-2} \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \beta_1 \\ \cdot \\ \cdot \\ \beta_{2n-1} \end{Bmatrix} = - \begin{Bmatrix} h_{2n} \\ h_{2n+1} \\ \cdot \\ \cdot \\ h_{4n-1} \end{Bmatrix} \quad \text{Equation 2.35}$$

The β_j can be obtained by solving Equation 2.35 and the V_r can be calculated from Equation 2.29. Knowing V_r , the residual A_r can be calculated from Equation 2.36. The modal constants and phase angles can be determined from Equation 2.27. Equation 2.27 can be rewritten as follows by setting $L = 2n$.

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ V_1 & V_2 & V_3 & \dots & V_{2n} \\ V_1^2 & V_1^2 & V_1^2 & \dots & V_{2n}^2 \\ \cdot & \cdot & \cdot & \dots & \cdot \\ V_1^{2n-1} & V_1^{2n-1} & V_3^{2n-1} & \dots & V_{2n}^{2n-1} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ \cdot \\ A_{2n} \end{Bmatrix} = - \begin{Bmatrix} h_0 \\ h_1 \\ h_2 \\ \cdot \\ h_{2n-1} \end{Bmatrix} \quad \text{Equation 2.36}$$

From Equation 2.13 to Equation 2.36, the complex exponential method is presented to process one IRF to extract the dynamic parameters. It is quite straightforward to extend this procedure to process several IRFs simultaneously. This leads to the Least-Squares Complex-Exponential-method (LSCE).

Chapter 3 FINITE ELEMENT ANALYSIS OF THE STEEL FRAME

3.1 Description of the Steel Frame

The steel frame, shown in Figure3-1, is part of the old Perley Bridge. This frame was located at 33rd axis of the old Bridge. The old Perley Bridge which has been demolished is a steel structure bridge with concrete slab deck. It was originally built in 1931 to cross the Ottawa River connecting Hawkesbury, Ontario and Grenville, Quebec. In 1996, the old Perley Bridge was demolished and replaced by a new construction Longsault Bridge. Some structural components were retained from the old bridge during the demolishing process for research purposes. The steel frame used in the present study is one of them. Over the years of its service life, the old Perley Bridge had undergone several retrofit modifications and repairs. The numerous modifications and repairs to the frame at different stages make it difficult and challenging to establish a precise finite element model for its performance evaluation. This is a commonly encountered problem in old bridge structures.

Figure 3.1 The Old Steel Pier Frame

3.2 Preliminary Finite Element Model

Research and advances in the development of finite element analysis procedures in the last three decades have led to confidence in the modeling of complex structure by finite element models. In dynamic analysis, the important aspects of an accurate structural model by the finite element method are the accurate representation and modeling of the mass, stiffness and damping properties of the structure. Comparatively, the mass properties of a structural system can be estimated more easily and more accurately than the stiffness and damping behavior. For a linear elastic structure under static loading conditions, the stiffness properties of typical structural members can usually be modeled with a high degree of accuracy based on well-established engineering principles. However, under dynamic loading conditions, the modeling of the behavior is affected by additional energy dissipation properties of the structure and may be more significantly influenced by the boundary or support conditions of the structure. This may lead to higher level of discrepancy of the finite element analysis. Consequently even without the complication of structural deterioration, it is typical there are some slight discrepancies between the finite element structural model and the actual structural behavior. In structural condition assessment applications, it is important to account this discrepancy before the finite element model of the structure can be used to determine whether any damage or deterioration to the structural condition has occurred. This can be achieved through a calibration process or model updating methodology.

Prior to the modal testing of the steel pier frame, a finite element analysis of the frame is carried out to obtain its theoretical modal frequencies and mode shapes. The information

is used to determine the optimum locations for bridge vibration response measurements and locations of the exciter for the modal tests.

The finite element model of the steel frame is shown in Figure3-2. The frame is modeled by beam elements. It is assumed fixed at the floor supports. The connection joints between the members are considered as rigid. In this preliminary baseline structural model, based on inspection of the frame, a reduction of 5% of the cross sectional area is assumed for the structural members to account for the corrosion loss of the member sections.

Figure 3.2 Finite Element Model of Bridge Steel Pier Frame

3.3 Preliminary Finite Element Analysis Results

A preliminary modal analysis of the structure is performed to extract the first 5 modes of the structure. The results of the first five modes are listed in Table1, and are illustrated in Figures 3.3 to 3.7.

Table 3-1 Modal Frequencies with Fixed Condition at Supports

TABLE: Modal Periods And Frequencies				
(fixed feet U1=U2=U3=U4=U5=U6=0)				
OutputCase	Step	Period	Frequency	CircFreq
		Sec	Hz	rad/sec
MODAL	Mode 1	0.02174	45.998	289.01
MODAL	Mode 2	0.019051	52.492	329.82
MODAL	Mode 3	0.016933	59.056	371.06
MODAL	Mode 4	0.014518	68.88	432.79
MODAL	Mode 5	0.011201	89.275	560.93

Figure 3.3 Mode Shape 1 (45.998 Hz)

Figure 3.4 Mode Shape 2 (52.492 Hz)

Figure 3.5 Mode Shape 3 (59.056 Hz)

Figure 3.6 Mode Shape 4 (68.88 Hz)

Figure 3.7 Mode Shape 5 (89.275 Hz)

Chapter 4 MODAL TESTING OF STEEL FRAME

4.1 Introduction

As discussed earlier, computer models of the steel frame based on design specification by the finite element method is not generally expected to be able to represent the exact behavior and characteristics of the structure without undergoing proper model updating procedure. Modal testing is a reliable method to obtain information of the actual dynamic properties of the structure, such as mass, stiffness, natural frequencies, mode shapes, and modal damping values. These vibration properties collectively reflect the behavior and actual condition of the structure. Therefore, the results from modal testing conducted under different load conditions can be used to calibrate and update the theoretical model of the structure. The model, after updated by using the experimental data, can be used to predict the behavior of the structure with high degree of accuracy in the future.

4.2 Modal Testing Procedures

Modal testing is an experimental technique that measures both excitation and responses of the test structure to determine the resonant frequencies, mode shapes and modal damping of the structure. In modal testing a frequency response function (FRF) as the relationship between the vibration responses of the structure with the excitation input as a function of the excitation frequency is established. A set of different frequency response

functions can be established by measuring the vibration response at different locations and also by varying the excitation input location.

A systematic approach to obtain the FRF is to repeat the procedure by applying the excitation at each of the measurement locations one at a time. For each case of excitation location, the responses at all the pre-selected measurement locations are recorded. By repeating this procedure of applying the excitation at each measurement location of the structure and taking response measurements throughout the structure, a set of frequency response functions are obtained, which can be put in a symmetric matrix form. In practice, however, it is not necessary to carry out all the test case to obtain the entire matrix of FRF. The modal model of the tested structure can be derived theoretically from single one row or an array of the FRF matrix, as shown in Equation 2.17.

Recent research and development have led to many advances on modal testing and analysis technique. In recently developed modal testing procedures, the excitation can be single point input or multiple point input applied at different directions. The excitation force can be applied within a selected frequency band, which can be transient, random or white noise. However, the excitation generated by a simple impact hammer is still the most efficient approach to gain information of the dynamic properties of the structure. By exciting the structure at carefully selected locations, desired vibration modes of different frequencies can be excited to contribute to the response of the test structure. Since in most modal testing setup, the number of measurement instruments is generally less than the number of measurement locations, the vibration responses of the test structure are measured by roving the accelerometers around the structure to determine a full row or an array of the FRF matrix. With the modal testing data expressed in the FRF matrix,

numerical modal analysis, such as curve fitting approach, can then be carried out to extract the modal parameters of the structure. This process is commonly known as experimental modal analysis. The modal parameters constitute a modal model of the test structure.

The steps of a typical modal testing of a structure are outlined as follows

- Establish a structural model of the test structure. Determine the frequency band of interest of the test structure
- Select the type of excitation force for modal testing. Select the location for the application of the excitation force. Determine measurement locations of the response.
- A set of frequency response functions are measured and stored during the modal testing. Subsequent modal analysis is carried out to identify the modal parameters of the test structure

4.2.1 Modal Testing Configurations

Although conducting an impact modal testing is fast and convenient, there are several important requirements that must be taken into account in order to achieve accurate results. They include considerations in the test procedure to account for pre-trigger delay, force and exponential windowing, and validity or acceptability of the test results.

Because the impulse signal from an impact load test lasts for only a short duration, it is important to capture the impact loading and the resulting response for later modal

analysis of the data. To ensure all signals are captured, a pre-trigger delay in the data collection process is applied.

Two common time domain windows used in impact modal testing are the force and exponential windows. These windows are applied to the signals after they are sampled and before analysis of the data by Fast Fourier transform procedure. The Force Window is used to remove noise from the impulse (force) signal. Ideally, an impulse signal is non-zero only during a small duration within the sampling window, and should be zero over the remaining of the window time period. Any non-zero measured data of the excitation force function following the impulse signal in the sampling window is assumed to be measurement noise. The force window preserves the data samples taken in the vicinity of the impulse, and filter out the noise from the measured force signals sufficient away from the impulse. The Exponential Window is applied to the impulse response signals to reduce leakage in the spectral analysis of the response data signals. In the spectral analysis by Fast Fourier Transform, it is assumed that the signal is periodic in the transform window. To be periodic, the waveform must have no discontinuity at its beginning or end in the transform window. Signals that are considered as periodic in the transform window must meet one of the following requirements:

1. Signals are completely contained within the transform window.
2. Cyclic signals complete an integer number of cycles within the transform window.

If a time signal is not periodic in the transform window, when it is transformed to the frequency domain, a smearing of its spectrum will occur. This is called leakage. Leakage distorts the spectrum and affects accuracy of the results. Therefore, if the response signal

in an impact test decays to zero (or near zero) before the end of the sampling window, there is no leakage, and no special windowing is required. On the other hand, if the response does not decay to zero before the end of the sampling window, an exponential window is used to reduce the leakage effects in the response spectrum. The application of exponential window adds artificial damping to all of the modes of the structure in a known manner. This artificial damping can be subtracted from the modal damping estimates after the curve fitting procedure during modal data analysis.

4.2.2 Modal Testing Instrumentation

The selected instruments used for the modal testing of the steel frame are listed as follows

- **Exciter**

An instrumented hammer as shown in Figure4-1 is used to excite the steel frame. The hammer on impact produces a force pulse to the test structure. At the hammer tip, a force transducer is connected to the computer with a built-in FFT analyzer. The hammer has interchangeable tips made of steel, plastics or rubber. The hardness of the tip and the surface of the structure determine the frequency range of the input force. The impact hammer used in the tests in this study is a PCB model with a frequency range of 0 to 750 Hz with the rubber hammer tip attached. The sensitivity of the measurement signal is 2.22 mV/N.

Figure 4.1 Impact Hammer Used as Exciter in Modal Testing

- **Motion Transducers**

Five accelerometers as shown in Figure 4-2 are used to measure the vibration response of the structure caused by the impact force excitation. The accelerometers used are piezoelectric type with a frequency response range of 0.5 Hz to 2000 Hz and a sensitivity of 300mV/g.

Figure 4.2 Accelerometer

- **Measurement and Analysis System**

An 8 channel computer-based FFT-analyzer as shown in Figure 4-3 is used to record and analyze the data.

Figure 4.3 Analyzer

4.2.3 Experiment Setup and Procedure:

The modal testing of the steel frame was carried out in the civil engineering laboratory of Carleton University. The base of the test frame was fixed to the one-meter-thick reinforced concrete strong floor. The locations for the application of the impact force and acceleration measurement are shown in Figure 4.4. While impact excitation is applied at Node 4 in the horizontal in-plane direction of the frame, the accelerometers are roved around the structure to capture the responses of the frame. The roving procedure is summarized in Table 4-1. The analyzer collects the experimental data. Preliminary analysis is carried out on the analyzer to assess the quality of the data. The figures of FRF are shown in Appendix.

Location and Direction of Accelerometers

Figure 4.4 Locations and Directions of Acceleration Response Measurement

Table 4-1 Modal Testing Procedure

Modal testing Procedure					
Case1 (impact force at node 4, horizontal direction)					
Location of accelerometers	node 3	node 4	node 5	node 6	node 10
Accelerometer sensor	5	4	3	2	1
Case2 (impact force at node 4, horizontal direction)					
Location of accelerometers	node 24	node 4	node 23	node 22	node 10
Accelerometer sensor	5	4	3	2	1
Case3 impact force at node 4, horizontal direction)					
Location of accelerometers	node 13	node 4	node 12	node 11	node 10
Accelerometer sensor	5	4	3	2	1
Case4 (impact force at node 4, horizontal direction)					
Location of accelerometers	node 16	node 4	node 17	node 18	node 10
Accelerometer sensor	5	4	3	2	1
Case5 (impact force at node 4, horizontal direction)					
Location of accelerometers	node 19	node 4	node 20	node21	node 10
Accelerometer sensor	5	4	3	2	1
Case6 (impact force at node 4, horizontal direction)					
Location of accelerometers	node 7	node 4	node8	node9	node 10
Accelerometer sensor	5	4	3	2	1

Subsequent modal analysis is carried out by using the computer software “LMS CADA-x”. The dynamic properties of the test steel frame are extracted from the modal testing data. Comparisons of the experimental results with the theoretical values from structural model of the frame are presented here.

4.3 Experiment Modal Analysis

In modal analysis of the experimental impact test data, one of the objectives is to determine the actual dynamic characteristics of the test structure. Extensive experimental modal testing and analysis procedures have been developed for mechanical and aerospace engineering applications in the last few decades. More recently, the development has been extended to applications of civil engineering structures.

4.3.1 Method of Modal Analysis

Modal test data can be acquired in the forms of either frequency response function or impulse response function. The typical methods of modal analysis can be divided into two categories: frequency domain analysis and time domain analysis. The Least-Square Complex Exponential method (LSCE) is applied in the modal analysis of the test data in this study. The LSCE is an indirect method in time domain to process several IRFs simultaneously.

4.3.2 Data Acquisition and Analysis

For the test, primary digital data acquisition is performed by a Horizon spectrum analyzer with 8 input channels. The input channels are used to acquire the excitation and response

time histories. The analyzer calculates the FRF so that the quality of the measurements can be assessed during the testing.

The duration of data acquisition for each excitation event was 25.6 s, giving a frequency resolution of 0.039Hz, which meets requirement to have sufficient frequency resolution for accurate modal parameter estimation later. The frequency response functions are calculated by the spectrum analyzer using the H_1 estimator [15-16]. Five excitation events are averaged in the frequency domain to produce FRF at each test point to reduce the effects of extraneous noise.

After the acquisition of the FRF data, modal parameter estimation is performed to estimate modes of vibration of the structure. The time domain MDOF method LCSE is utilized. This is an MDOF curve fitting method that simultaneously analyses all FRFs corresponding to a single reference point. The methodology of LCSE has been discussed above from Equation 2.14 to Equation 2.15.

4.3.3 The Results of Modal Analysis

The estimated modal properties of the first five modes of vibration of the steel frame are presented in Table 4-2. It is noted of the five identified modes with frequencies lower than 51 Hz, the estimated modal damping ratios are between 0.17% and 0.43%.

Table 4-2 Modal Frequencies from modal testing

pole no	stab type	frequency	frequency units	damping (%)	description
Mode 1	s	24.869	Hz	0.429	
Mode 2	s	31.17	Hz	0.163	
Mode 3	s	37.114	Hz	0.237	
Mode 4	s	42.366	Hz	0.151	
Mode 5	s	50.881	Hz	0.174	

The figures of the first five mode shapes are presented in Figures 4-5 to 4-9.

Figure 4.5 Modal mode shape 1 from modal testing

Figure 4.6 Modal mode shape 2 from modal testing

Figure 4.7 Modal mode shape 3 from modal testing

Figure 4.8 Modal mode shape 4 from modal testing

Figure 4.9 Modal mode shape 1 from modal testing

Chapter 5 MODEL CORRELATION AND UPDATING

5.1 Introductions

The modal properties and parameters obtained from modal testing are accurate representation of the actual behavior of the test structure. Based on this premise, the modal data can be used to calibrate computer models of a structure that can then be used to more accurately predict the behavior and performance of the structure under its operating environment in the field. In the calibration process of the study here, the finite element model of the test structure is calibrated or updated by using the modal testing data presented in chapter 4.

The specific objective of model updating in this thesis is primarily to develop the updating procedure for establishing a robust and accurate model by using modal testing data. Because the steel test frame used in this study comes from a part of an old bridge, the calibrated computer finite element model of the structure thus includes the effects of deterioration due to exposure to the environment and its operating service on its structural behavior and performance.

The accuracy of a computer analysis or finite element model of a structure is influenced by a number of factors

- Accuracy in the representation of the support boundary conditions of the structure
- Discrepancies in the actual physical and material properties of the structure from the theoretical model as specified or assumed in its design
- Accurate representation of the damping behavior
- Modeling of the joints or connections of the structure

5.2 Preliminary Adjustments to FE Structural Model

After initial comparison of the test results with the results obtained from the preliminary finite element model of the steel frame, it is observed that the differences in the first five natural frequencies between the measured and the predicted values for each of the modes is greater than 10%. It is also observed that the calculated analytical frequencies from finite element model of the test structure are higher than the experimental measured results, which indicate the stiffness properties of the finite element model of the structure is overestimated compared to the actual behavior of the frame.

A preliminary model updating process is considered here which focuses on some uncertainties in representing the actual behavior of the test frame due to the following issues:

1. The degree of rigidity of joints in the test frame and their accurate representation
2. The fixity of the base supports of the frame and their representation

5.2.1 Modeling the support boundary conditions

The actual boundary conditions of a structure in the field or even in this case of the test setup in the laboratory are generally different from the ideal support boundary conditions assumed in the structural model for design. Generally the boundary condition can ideally be divided into four cases: fixed, hinged, roller or free support conditions. The actual boundary conditions usually are not exactly represented by one of the four ideal cases, but fall between the ideal cases. In order to determine the accurate boundary conditions, a general boundary condition model for the test structure is introduced to more accurately represent the actual state of the structure.

The general boundary condition model consists of 3 springs attached to the base of the column. The spring constants K_1 , K_2 and K_3 of the boundary condition model represent the degree of restriction to horizontal translation, vertical translation, and rotation, respectively. (See figure 5.1)

Figure 5.1 Boundary conditions model of the steel frame

5.2.2 Modeling the rigidity at connection joints

It is sometimes a challenge to determine correctly the actual rigidity condition of the joints or intersections in a structure. In this study of the test frame, six springs are incorporated at the joints of the frame to simulate the connection rigidity conditions of the steel frame (See figure 5.2). Two types of spring constants, K_4 and K_5 , represent the rigidity of the joints.

Figure 5.2 Models of Joint Rigidity and Boundary Conditions of the Test Frame

5.2.3 Determine the range of approximate springs' constant values

Theoretically a fixed boundary condition at the support is represented by infinite or large values of the spring constants of $K1$, $K2$ and $K3$. In the calibration process here, the spring constants $K1$, $K2$ and $K3$ of the structural model with fixed support boundary conditions are adjusted to give approximate identical natural frequencies. In the calculation, the two sets of natural frequencies are considered to be identical if the difference between the two results is within 1%.

In subsequent step of the calibration process, the individual spring constant $K1$, $K2$ and $K3$ are separately adjusted to give a more accurate representation of the fixity condition or behavior of the base support of the frame. In discussion here, the procedure of calibrating the spring constant $K1$ for the degree of fixity in the horizontal translation direction is summarized as follows:

1. The displacement degrees-of-freedom $U1$, $U2$, $U3$, $U4$, $U5$ and $U6$ are set to zero to represent the fixed boundary condition (See figure 5.1)
2. Modal analysis is carried out to extract the natural frequencies of the frame under the fixed boundary condition. The first five natural frequencies are used as the reference for the next step (See Table 5.1).
3. The displacement degrees-of-freedom $U2$, $U3$, $U5$ and $U6$ are set to zero. A nonzero initial guess for stiffness value $K1$ is introduced at the two supports of the frame which corresponds to degree-of-freedom $U1$ and $U4$. Modal analysis of the modified structural model is performed to obtain the natural frequencies of the frame. The first five natural frequencies are selected to compare with the natural frequencies obtained from the previous step.
4. The spring constant parameter $K1$ is adjusted to match the first five natural frequencies with the corresponding reference frequencies. The value of $K1$ can be used to represent the boundary condition of $U1=U4=0$

The adjustments of the spring constant parameters $K2$ and $K3$ to represent the boundary conditions of $U2=U5=0$ and $U3=U6=0$ respectively can be obtained in the same way as the procedure for determining the value of $K1$. In general, these values of $K1$, $K2$ and $K3$, which represent the fixed boundary condition, are greater than those corresponding to the real conditions of the steel frame. The notations $K1_{MAX}$, $K2_{MAX}$ and $K3_{MAX}$ are used to denote these values. (See Table 5.1).

Similarly, a corresponding set of K_4 and K_5 can be determined to represent the rigid joint behavior of the test frame. To determine the correct spring constants K_4 and K_5 to represent the actual joint rigidity behavior of the test frame, the following procedures are followed:

1. Modal analysis is carried out to extract the natural frequencies of the steel frame assuming rigid joints and fixed boundary conditions. The first five natural frequencies are used as the reference for the next step.
2. The spring constant parameters K_4 or K_5 , are adjusted so that the first five natural frequencies are the same as the reference frequencies. These values of K_4 or K_5 are then used to represent the rigid joint condition of the structure (See Table 5.1 5.2).

Table 5-1 Modal Frequencies with fixed conditions at supports

TABLE: Modal Periods And Frequencies				
(fixed supports U1=U2=U3=U4=U5=U6=0)				
OutputCase	Step	Period	Frequency	CircFreq
		Sec	Cyc/sec	rad/sec
MODAL	Mode 1	0.02174	45.998	289.01
MODAL	Mode 2	0.019051	52.492	329.82
MODAL	Mode 3	0.016933	59.056	371.06
MODAL	Mode 4	0.014518	68.88	432.79
MODAL	Mode 5	0.011201	89.275	560.93

Table 5-2 Modal Frequencies with approximate fixed conditions at supports

TABLE: Modal Periods And Frequencies				
(k1=4e10, k2=4e10, k3=1e10,k4=5e8,k5=5e8,rigid at beam & column joints)				
OutputCase	Step	Period	Frequency	CircFreq
		Sec	Cyc/sec	rad/sec
MODAL	Mode 1	0.021787	45.9	288.4
MODAL	Mode 2	0.019063	52.457	329.6
MODAL	Mode 3	0.016987	58.868	369.88
MODAL	Mode 4	0.01457	68.633	431.23
MODAL	Mode 5	0.011237	88.992	559.15

The actual support and joint behavior is between totally fixed and totally free conditions. By using the measured first five natural frequencies as the baseline references, the structural model is calibrated by adjusting the base support spring constant parameters and joint spring constant to have the same frequencies as the measured results. The optimal spring constant values are determined by trial and error. The final spring constant values are $K1 = 6 \times 10^8$, $K2 = 5.5 \times 10^7$, $K3 = 1.3 \times 10^7$, $K4 = 5 \times 10^6$ and $K5 = 5 \times 10^6$. After modification, the frequency shift to lower ends. The theoretical analysis results are presented in Table 5.3. It is observed that these adjustments shift the frequencies of the test frame lower.

Table 5-3 The Analysis Result after Modification

TABLE: Modal Periods And Frequencies				
(k1=6e8, k2=5.5e7, k3=1.3e7,k4=5e6,k5=5e6)				
OutputCase	Step	Period	Frequency	CircFreq
		Sec	Cyc/sec	rad/sec
MODAL	Mode 1	0.041593	24.042	151.06
MODAL	Mode 2	0.031351	31.897	200.42
MODAL	Mode 3	0.025758	38.823	243.93
MODAL	Mode 4	0.023731	42.14	264.77
MODAL	Mode 5	0.019933	50.169	315.22

5.3 Reductions of Finite Element Data

Because of the limitation of the number of sensor instruments, e.g. accelerometers and the limited capacity of FFT analyzer, available for taking measurements in the modal testing, the dimension or size of the modal testing model is typically significantly smaller than that of finite element model of the test structure. The number of measured modes and degrees-of-freedom are usually less than that of the FE model. In order to compare

results from the modal testing with the finite element model results, reduction of the finite element model is necessary.

A process of system equivalent reduction expansion process (SEREP) [17] is introduced to solve this problem. The full finite element model system is reduced to the same size as the measured data. This reduction makes it possible to compare the measured and FE data.

The SEREP method is presented as follows:

The modal transformation equation can be written as:

$$\begin{Bmatrix} \{x_a\} \\ \{x_b\} \end{Bmatrix} = \{x_n\} = [U_n] \{p\} = \begin{bmatrix} [U_a] \\ [U_b] \end{bmatrix} \{p\} \quad \text{Equation 5.1}$$

where

$\{x_n\}$ is the vector of the physical coordinates

$\{x_a\}$ is the vector of retained coordinates

$\{x_b\}$ is the vector of reduced coordinates

$[U_n]$ is the modal transform matrix

$\{p\}$ is the modal coordinates vector

the transformation relationship between the physical model coordinates and the modal coordinates is expressed as follows:

$$\{x_a\} = [U_a] \{p\} \quad \text{Equation 5.2}$$

$$[U_a]^T \{x_a\} = [U_a]^T [U_a] \{p\} \quad \text{Equation 5.3}$$

$$\{p\} = \left[[U_a]^T [U_a] \right]^{-1} [U_a]^T \{x_a\} = [U_a]^k \{x_a\} \quad \text{Equation 5.4}$$

$$\text{where } [U_a]^k = \left[[U_a]^T [U_a] \right]^{-1} [U_a]^T$$

Substituting Equation 5.4 into Equation 5.1 gives

$$\{x_n\} = [U_n] [U_a]^g \{x_a\} = \begin{bmatrix} [U_a] \\ [U_b] \end{bmatrix} [U_a]^g \{x_a\} \quad \text{Equation 5.5}$$

From the Equation 5.5 the SEREP transformation matrix can be given as follows:

$$[T] = [U_n] [U_a]^g \quad \text{Equation 5.6}$$

The reduced mass and stiffness are the calculated as follows:

$$\begin{aligned} [M_a^s] &= [T]^T [M_n] [T] \\ [K_a^s] &= [T]^T [K_n] [T] \end{aligned} \quad \text{Equation 5.7}$$

where

$[M_n]$ is the original mass matrix

$[K_n]$ is the original stiffness matrix

The advantages of the SEREP procedure are as follows:

- The reduced system has the same frequencies and mode shapes as that of the original full system.
- The quality of the results is independent to the selection of the degrees-of-freedom in the reduced system.

5.4 Comparison of the Measured and FE Model Data

A comparison of the measured and FE model results is carried out in the modal domain.

The most commonly used method is by means of modal assurance criterion (MAC) [18]

$$\text{MAC} (\{\Phi_x\}_j, \{\Phi_a\}_j) = \frac{|\{\Phi_x\}_i^T \{\Phi_a^*\}_j|^2}{(\{\Phi_x\}_i^T \{\Phi_x^*\}_i) (\{\Phi_a\}_i^T \{\Phi_a^*\}_i)} \quad \text{Equation 5.9}$$

The Modal Assurance Criterion (MAC) is used to estimate the degree of correlation between mode shape vectors. The MAC is often used to pair modes shapes derived from analytical analysis with those obtained experimentally.

5.5 The Theoretical Basis of Correction of the Stiffness Matrix

The measured data obtained from a modal testing are different from the theoretical analysis model. The measured modes are not always orthogonal to the analytical mass matrix. Therefore the measured modes need modification before model updating. The following sections discuss the modification of measured eigenvector matrix and stiffness matrix of the structural system [19].

5.5.1 Correction of Eigenvector Matrix

The measured modes are not usually orthogonal to the analytical mass matrix. Assuming the analytical mass matrix is correct, modification to the measured modes is needed to meet the requirement of orthogonality. This condition can be imposed by determining the matrix that minimizes a weighted Euclidean norm defined by

$$J = \|[N]([\Phi] - [\Phi_m])\| = \sum_{i=1}^n \sum_{k=1}^m \left[\sum_{j=1}^n [N]_{ij} ([\Phi]_{jk} - [\Phi_m]_{jk}) \right]^2 \quad \text{Equation 5.10}$$

subject to the orthogonality condition

$$[\Phi]^T [M_a] [\Phi] = [I] \quad \text{Equation 5.11}$$

where

$$[N] = [M_a]^{1/2}$$

$[M_a]$ is the analytical mass matrix

$[\Phi_m]$ is the measured eigenvector matrix

$[N]_{ij}, [\Phi]_{ij}, [\Phi_m]_{ij}$ are the (i, j) elements of the matrices $[N]$, $[\Phi]$, $[\Phi_m]$.

respectively

m is the number of measured eigenvectors

n is the number of degrees of freedom in the analytical model

Equation 5.11 represents the constraints of the corrected eigenvector matrix $[\Phi]$. The Lagrange Multiplier method uses these constraint equations to produce the augmented function to be minimized as

$$J = \sum_{i,j,h=1}^n \sum_{k=1}^m \{ [N]_{ij} ([\Phi]_{jk} - [\Phi_m]_{jk}) [N]_{ih} ([\Phi]_{hk} - [\Phi_m]_{hk}) \} \\ + \sum_{i,h=1}^m \gamma_{ih} \left\{ \sum_{k=1}^m ([\Phi]_{ji} [M_a]_{jk} [\Phi]_{kh} - \delta_{ih}) \right\} \quad \text{Equation 5.12}$$

where δ_{ij} represents the Kronecker delta. The first term of Equation 5.12 comes from Equation 5.10, the bracketed expression in the second term represents the equations generated from Equation 5.11. The γ_{ih} are the Lagrange Multipliers, which can be formed into a matrix $[\Gamma]$. The symmetry in equation 5.11 implies there are only $\frac{1}{2}m(m+1)$ independent equations out of a possible m^2 . Thus there is some redundancy in Equation 5.12, which may alternatively be viewed as lack of uniqueness of the matrix $[\Gamma]$. The Lagrange Multiplier matrix, $[\Gamma]$, may be forced to be unique by introducing the constraint of symmetry so that

$$[\Gamma] = [\Gamma]^T \quad . \quad \text{Equation 5.13}$$

The augmented function given by equation 5.12 is now minimized with respect to the nm elements of the corrected eigenvector matrix, $[\Phi]_s$. Differentiating Equation 5.12 with respect to each of these elements and setting the result to zero gives

$$\begin{aligned} \frac{\partial J}{\partial [\Phi]_{rs}} = & \sum_{i,h=1}^n [N]_{ir} [N]_{ih} ([\Phi]_{hs} - [\Phi_m]_{hs}) + \sum_{i,j=1}^n [N]_{ij} ([\Phi]_{js} - [\Phi_m]_{js}) [N]_{ir} \\ & + \sum_{h=1}^m \gamma_{sh} \left\{ \sum_{k=1}^n [M_a]_{rk} [\Phi]_{kh} \right\} + \sum_{i=1}^m \gamma_{is} \left\{ \sum_{j=1}^n [\Phi]_{ji} [M_a]_{jr} \right\} = 0 \end{aligned} \quad \text{Equation 5.14}$$

Because $[N]$ is symmetric, the first two terms can be simplified by using

$$\sum_{i=1}^n [N]_{ir} [N]_{ih} = [M_a]_{rh}, \text{ where } [M_a]_{rh} \text{ is the } (r, h) \text{ element of the mass matrix } [M_a].$$

The last two terms in Equation 5.14 can be combined because of the symmetry of $[\Gamma]$ and $[M_a]$. Equation 5.14 can be written as follows

$$\frac{\partial J}{\partial [\Phi]_{rs}} = 2 \sum_{h=1}^n [M_a]_{rh} ([\Phi]_{hs} - [\Phi_m]_{hs}) + 2 \sum_{h=1}^m \sum_{k=1}^n [M_a]_{rk} [\Phi]_{kh} \gamma_{sh} = 0 \quad \text{Equation 5.15}$$

The first term in Equation 5.15 represents the (r, s) element of the matrix $2[M_a]([\Phi] - [\Phi_m])$ and the second term represent the (r, s) element of the matrix $2[M_a][\Phi][\Gamma]$. Thus the nm equations represented by Equation 5.15 may be written in matrix form as

$$2[M_a]([\Phi] - [\Phi_m]) + 2[M_a][\Phi][\Gamma] = 0$$

or

$$[\Phi] - [\Phi_m] + [\Phi][\Gamma] = 0 \quad \text{Equation 5.16}$$

Collecting together terms in $[\Phi]$ gives

$$[\Phi][I] + [\Gamma] = [\Phi_m] \quad \text{Equation 5.17}$$

Equation 5.17 gives some insight into the physical significance of the Lagrange Multiplier matrix. It represents an error between the measured and the corrected eigenvectors. It is assumed that $[I + \Gamma]$ is non-singular which is the case if the measured eigenvector matrix almost satisfies the orthogonality constraint, in which case $[\Gamma]$ should be small. Under this assumption Equation 5.17 may be written as

$$[\Phi] = [\Phi_m]([I] + [\Gamma])^{-1} \quad \text{Equation 5.18}$$

Substituting the expression for $[\Phi]$ given by Equation 5.18 into the mass orthogonality constraint, Equation 5.11 gives

$$([I] + [\Gamma])^{-1} [\Phi_m]^T [M_a] [\Phi_m] ([I] + [\Gamma])^{-1} = [I]$$

Pre and post multiplying by $[I + \Gamma]$ and taking the square root produces

$$([I] + [\Gamma]) = ([\Phi_m]^T [M_a] [\Phi_m])^{1/2} \quad \text{Equation 5.19}$$

Thus the Lagrange Multiplier matrix may also be viewed as an error in the orthogonality constraint. Finally the corrected eigenvector matrix is obtained by substituting the expression for the Lagrange Multiplier matrix, equation 5.18, into equation 5.17 to obtain the corrected eigenvector matrix as

$$[\Phi] = [\Phi_m] ([\Phi_m]^T [M_a] [\Phi_m])^{1/2} \quad \text{Equation 5.2}$$

5.5.2 Correction of Stiffness Matrix

The modified modal parameters can now be used to correct the analytical stiffness matrix. The mass matrix, $[M_a]$, is assumed to be exact. There are two constraints, represented by Equation 5.21 and Equation 5.22, on the corrected stiffness matrix.

If K represents the corrected stiffness matrix then the equation of motion of the model of the structure implies that

$$[K][\Phi] = [M_a][\Phi][\Lambda] \quad \text{Equation 5.21}$$

where $[\Lambda]$ is an $(m \times m)$ diagonal matrix with the measured natural frequencies squared on the diagonal. Thus equation 5.21 represents nm constraint equations on the n^2 elements of the corrected stiffness matrix $[K]$.

The symmetry of $[K]$ produces $\frac{1}{2}n(n+1)$ further independent constraint equations given by

$$[K]^T = [K] \quad \text{Equation 5.22}$$

The function to be minimized and related in some way to the difference between the corrected stiffness matrix and the analytically derived stiffness matrix is given in Equation 5.23

$$J = \frac{1}{2} \left\| [N]^{-1} ([K] - [K_a]) [N]^{-1} \right\| = \frac{1}{2} \sum_{i,j=1}^n \left[\sum_{h,k=1}^n [N]_{ih}^{-1} ([K]_{hk} - [K_a]_{hk}) [N]_{kj}^{-1} \right]^2 \quad \text{Equation 5.23}$$

where

$$[N] = [M_a]^{1/2} \text{ as before}$$

$$[N]_{ij}^{-1}, [K]_{ij}, [K_a]_{ij} \text{ are the } (i, j) \text{ elements of the matrices } [N]^{-1}, [K], [K_a]$$

The weighting by the inverse of the matrix $[N]$ allows, to some extent, for differences in the magnitudes of elements of the mass and stiffness matrices. The augmented function to be minimized, as required by the Lagrange Multiplier method, is given by

$$\begin{aligned}
J = & \frac{1}{2} \sum_{i,j=1}^n \left[\sum_{h,k=1}^n [N]_{ih}^{-1} ([K]_{hk} - [K_a]_{hk}) [N]_{kj}^{-1} \right]^2 + \sum_{i,j=1}^n \gamma_{kij} ([K]_{ij} - [K_a]_{ji}) \\
& + 2 \sum_{i=1}^n \sum_{j=1}^n \gamma_{\Lambda ij} \sum_{h=1}^n ([K]_{ih} [\Phi]_{hj} - [M]_{ih} [\Phi]_{hj} \omega_{mj}^2) = 0
\end{aligned}
\tag{Equation 5.24}$$

where ω_{mj} denotes the j th measured natural frequency and the Lagrange multipliers $\gamma_{\Lambda ij}$ and γ_{kij} can be expressed as elements of the matrices $[\Gamma_{\Lambda}]$ and $[\Gamma_k]$. The first term represents the function from Equation 5.23, the second term represents the equation of motion constraint, Equation 5.21, and the third term represents the symmetry constraint, Equation 5.24. In a similar way to the orthogonalisation of the measured modes the stiffness matrix symmetry constraint will produce a unique solution for the Lagrange Multipliers only if the following constraint is added

$$[\Gamma_k] = -[\Gamma_k]^T \tag{Equation 5.25}$$

Differentiating equation 5.24 with respect to the (r, s) element of the corrected stiffness matrix, $[K]_s$, gives the (r, s) element of the following matrix equation

$$[M_a]^{-1} ([K] - [K_a]) [M_a]^{-1} + 2[\Gamma_{\Lambda}] [\Phi]^T + 2[\Gamma_k] = 0 \tag{Equation 5.26}$$

The skew-symmetry of $[\Gamma_k]$, Equation 5.25, can be used to eliminate $[\Gamma_k]$ from Equation 5.26 to give, after some rearrangement to make $[K]$ the subject of the equation

$$[K] = [K_a] - [M_a] [\Gamma_{\Lambda}] [\Phi]^T [M_a] - [M_a] [\Phi] [\Gamma_{\Lambda}]^T [M_a] \tag{Equation 5.27}$$

Post multiplying Equation 5.26 by $[\Phi]$ and using orthogonality of the corrected eigenvector matrix and the equation of motion, Equation 5.20, gives the following equation for $[\Gamma_{\Lambda}]$

$$[M_a][\Phi][\Lambda] = [K_a][\Phi] - [M_a][\Gamma_\Lambda] - [M_a][\Phi][\Gamma_\Lambda]^T[M_a][\Phi] \quad \text{Equation 5.28}$$

Although Equation 5.28 represents n^2 equations for the $n \times m$ unknown elements of $[\Gamma_\Lambda]$, it is difficult to proceed without a further assumption that $[\Gamma_\Lambda]^T[M_a][\Phi]$ is symmetric, and thus

$$[\Gamma_\Lambda]^T[M_a][\Phi] = [\Phi]^T[M_a][\Gamma_\Lambda] \quad \text{Equation 5.29}$$

Using this assumption in Equation 5.28 produces

$$[M_a][\Phi][\Lambda] = [K_a][\Phi] - [M_a]([I] + [\Phi][\Phi]^T[M_a])[\Gamma_\Lambda] \quad \text{Equation 5.30}$$

Simplification of Equation 5.30 requires the inversion of $([I] + [\Phi][\Phi]^T[M_a])$. The following identity may be proved by direct calculation: for any idempotent matrix $[A]$, that is $[A]^2 = [A]$,

$$([I] + [A])^{-1} = ([I] - \frac{1}{2}[A])$$

In this case here

$$([I] + [\Phi][\Phi]^T[M_a])^{-1} = ([I] - \frac{1}{2}[\Phi][\Phi]^T[M_a]) \quad \text{Equation 5.31}$$

Using this identity Equation 5.30 becomes, after collecting terms together and rearranging to make $[\Gamma_\Lambda]$ the subject of the equation,

$$[\Gamma_\Lambda] = [M_a]^{-1}[K_a][\Phi] - \frac{1}{2}[\Phi][\Phi]^T[K_a][\Phi] - \frac{1}{2}[\Phi][\Lambda] \quad \text{Equation 5.32}$$

From this definition of $[\Gamma_\Lambda]$ it is easy to check that $[\Gamma_\Lambda][M_a][\Phi]$ is symmetric and thus the assumption was justified. This expression for $[\Gamma_\Lambda]$ is now substituted into Equation 5.27 to obtain, after collecting terms together, the following updated stiffness matrix $[K]$

$$\begin{aligned}
[K] = & [K_a] - [K_a][\Phi][\Phi]^T[M] - [M][\Phi][\Phi]^T[K_a] \\
& + [M][\Phi][\Phi]^T[K][\Phi][\Phi]^T[M] + [M][\Phi][\Lambda][\Phi]^T[M]
\end{aligned}$$

Equation 5.33

where

$[K]$ is the updated stiffness matrix

$[K_a]$ is the original analytical stiffness matrix

$[M_a]$ is the mass matrix

$[\Phi]$ is mode shapes, normalized with mass matrix $[M]$

$[\Lambda]$ is measured eigenvalue matrix

5.6 The Results and Procedure of Model Correlation

The procedure of model correlation includes the following steps:

1. Before modal testing, preliminary theoretical analysis of the structural system is carried out to obtain the baseline modal parameters as the reference information. In this step, the theoretical mass, the stiffness and the eigenvalue matrices are determined.
2. Modal parameters, such as mode shapes and modal frequencies, are obtained by experimental modal analysis of modal testing data of the structure.
3. Preliminary adjustments are made to the theoretical structure model by adjusting the stiffness matrix. Details of this step have been presented in Section 5.2.
4. Before comparing the experiment mode shapes and modal frequencies with those from the theoretical analysis, the reduction of the finite element model is necessary to make the finite element model be the same size as the modal testing

model. A process of system equivalent reduction expansion process (SEREP) is carried out as discussed in Section 5.4. A new reduced mass and stiffness matrix are obtained in this step.

5. The criterion MAC is adopted to identify the experiment mode shapes and modal frequencies with the corresponding ones from the theoretical analysis results.
6. The stiffness is updated by using mode shapes and modal frequencies obtained from modal testing. The updating method is presented in Section 5.5

The updating of a five-degrees-of-freedom model is carried out by following the above procedures. The mass matrix $[M]$ and stiffness matrix $[K]$ shown in Figure 5-3 and Figure 5-4 respectively are the full size (74x74) from theoretical model. The mass matrix $[m_a]$ and stiffness matrix $[k_a]$ showed in Table 5-4, 5-5 respectively is of size (5x5) from the reduction of theoretical mass matrix $[M]$ and stiffness matrix $[K]$ model.

Figure 5.3 Mass Matrix $[M]$ of the Test Frame in 3D Paragraph (unit: kilogram)

Figure 5.4 Stiffness Matrix $[K]$ of the Test Frame in 3D Paragraph (unit: kn/m)

The stiffness matrix $[k_a]$ (5×5) obtained from the reduction of the full theoretical Stiffness matrix $[K]$ is shown in Table 5-4.

The $[m_a]$ (5×5) obtained from the reduction of the full theoretical mass matrix $[M]$ is shown in Table 5-6.

The updated stiffness matrix $[k]$ is shown in Table 5-5.

Table 5-4 Stiffness Matrix $[k_a]$ (unit: kn/m)

k_a					
	1	2	3	4	5
1	4.23E+10	-2.54E+10	-1.61E+10	2.93E+10	-8.07E+09
2	-2.54E+10	5.12E+10	-6.13E+10	3.02E+10	2.59E+09
3	-1.61E+10	-6.13E+10	1.56E+11	-1.16E+11	7.80E+09
4	2.93E+10	3.02E+10	-1.16E+11	9.42E+10	-8.88E+09
5	-8.07E+09	2.59E+09	7.80E+09	-8.88E+09	1.74E+09

Table 5-5 Stiffness Matrix $[k]$ (unit: kn/m)

k					
	1	2	3	4	5
1	4.23E+10	-2.54E+10	-1.61E+10	2.93E+10	-8.07E+09
2	-2.54E+10	5.12E+10	-6.13E+10	3.02E+10	2.59E+09
3	-1.61E+10	-6.13E+10	1.56E+11	-1.16E+11	7.80E+09
4	2.93E+10	3.02E+10	-1.16E+11	9.42E+10	-8.88E+09
5	-8.07E+09	2.59E+09	7.80E+09	-8.88E+09	1.74E+09

Table 5-6 Mass Matrix $[m_a]$ (unit: kilogram)

m_a					
	1	2	3	4	5
1	6.04E+05	-2.96E+05	-3.07E+05	4.51E+05	-1.16E+05
2	-2.96E+05	6.87E+05	-9.13E+05	4.96E+05	2.51E+04
3	-3.07E+05	-9.13E+05	2.39E+06	-1.78E+06	1.27E+05
4	4.51E+05	4.96E+05	-1.78E+06	1.42E+06	-1.34E+05
5	-1.16E+05	2.51E+04	1.27E+05	-1.34E+05	2.63E+04

The updated stiffness matrix k is almost same as the stiffness matrix $[k_a]$, because the $[k_a]$ is the reduction of the full theoretical Stiffness matrix $[K]$ in five degrees-of-freedom and the $[K]$ has been preliminarily corrected by modal testing data, which is presented in Section 5.2.

Chapter 6 SUMMARY

6.1 Summary and Conclusions

Modal testing of an old steel bridge pier frame retained from the old demolished Perley Bridge has been carried out in the laboratory to obtain the sensitivity and dynamic characteristics of the structure system. Because modal testing is conducted in a controlled laboratory environment, the uncertainties and influences on the modal parameter results due to fluctuations in the field environment can be eliminated. Therefore the modal test results reported herein reflect the deterioration conditions of the old bridge structure. The experience and insight gained from this study are unique and most valuable.

The important consideration of the environmental influences and uncertainties on field vibration modal characteristics is part of a long-term bridge monitoring and assessment project of the Confederation Bridge at Carleton University. The present study compliments the Confederation Bridge monitoring project to make advance in the practical application of vibration based structural health monitoring techniques as effective structural condition assessment tools. Using the measured data obtained from modal testing of the steel bridge pier frame, the restraint conditions of the supports of the test pier frame and the rigidity condition of the joints of the pier frame have been calibrated in a preliminary structural model of the test frame. It is then demonstrated that a system updating procedure can be applied to establish an accurate structure model of the frame, which reflects its deteriorated conditions, for future practical engineering

application of damage detection and condition assessment. The difficulties and challenges in modeling accurately the actual conditions and behavior of the old bridge pier compared to the theoretical computer model determined from the construction drawings and specifications for the purpose of structural conditions evaluation are addressed in this study.

Based on comparison of measured modal testing results and theoretical analysis, it is shown that a structural model, established on design and construction specifications, and common engineering assumptions, typically does not represent the behavior of an aged structure properly. The theoretical finite element structural model typically overestimates the rigidity of the structure. The major problems of the finite element model mainly arise due to the following factors

- Inaccurate representation of support boundary conditions
- Inaccurate modeling of the joints

Inaccuracy or discrepancies in the modeling of the foundation support conditions of the structure has significant influences on the behavior of the structure with more pronounced effects on the lower vibration modes. In comparison, the actual modeling of the rigidity conditions of the joints in the frame has influences more evenly on all the modes. It is also found that modeling of damping of the frame does not affect critically the results in this study, because the steel pier frame has very light damping.

Old bridge structures often have undergone repairs and rehabilitations sometime during service life. Accurate and effective model updating procedures which can very accurately represent the structural conditions of the structures for the purpose of application for structural assessment are needed. The finite model updated by the modal testing data

which accurately reflects the conditions of the structure can then be used to assess the capacity and performance, and remaining service life of the structure, which is an important part in the design of any repair or retrofit scheme for it.

REFERENCE

- [1] Ewins, D. J., "Model Testing: Theory and Practice", Research Studies Press Ltd., England, 1984
- [2] Maia, N.M.M., Silva, J.M.M., He, J., Lieven, N.A.J., Lin, R.M., Skingle, G.W., To, W.M., Urgueira, A.P.V., "Theoretical and Experimental modal analysis", Research Studies Press, 1997
- [3] Dimarogonas, A. "Vibration for Engineers", Prentice-hall Inc. 1996
- [4] Harris, C.M. "Shock and Vibration Handbook" McGraw-Hill, 1995
- [5] Corelli, D. and Brown, D.L. "Impact Testing Considerations", Proceedings of the 17th International Modal Analysis Conference, Kissimmee, FL, USA, 518. 1999
- [6] Broch, J. T. "Mechanical Vibration and Shock Measurements", Bruel & Kjaer, 1980
- [7] Maia, M.M.M. "Modal Identification Methods in the Frequency Domain", NATO Science Series E: Application Sciences, 363, 251-264, 1999
- [8] Maia, M.M.M. and Ewins, D.J. "A New Approach for the Modal Identification of Lightly Damped Structures", Mechanical Systems and Signal Processing, #92), 173-193, 1989
- [9] Batill, S.M. and Holkamp, J.J. "Parameter Identification of Discrete Time Series Models for Structural Response Prediction", AIAA Journal, 27(11), 1636-1649, 1989.
- [10] Budwantord, B. and Jezequel. L. "Comparison of Time Domain Modal Identification Methods", Proceeding of the 8th International Modal Analysis Conference, Kissimmee, FL, 540-546, 1990.

- [11] Brown, D. L., Allemang, R. J., Zimmerman, R., Mergeay, M., "Parameter Estimation Techniques for Modal Analysis", SAE Technical Paper Series, No. 790221, 1979
- [12] Spitznogle, F. R., Quazi, A. H., "Representation and Analysis of Time-Limited Signals Using a Complex Exponential Algorithm ", The Journal of the Acoustical Society of America, Vol.47, No. 5 (Part 1), 1970, pp. 1150-1155.
- [13] Spitznogle, F. R., et al, "Representation and Analysis of Solar Signals, Vol.1 : Improvements in the Complex Exponential Signal Analysis Computational Algorithm", Texas Instruments, Inc. No. U1-829401-5, Office of Naval Research Contract No. N00014-69-C0315, 1971, 37 pp.
- [14] Prony, R., "Essai Experimental Et Analitique Sur Les Lois De La Dilatabilite Des Fluides Elastiques Et Sur Celles De La Force Expansive De La Vapeur De L'Eau Et De La Vapeur De L'Alcool, A Differentes Temperatures", Journal de L'Ecole Polytechnique (Paris), Vol. 1, Cahier 2, Floreal et Prairial, An. III, 1795, pp. 24-76
- [15] Mitchell, L. D., Cobb, R. E., Deel, J. C. and Luk, Y. W., "An Unbiased Frequency Response Function Estimator ", International Journal of Analytical and Experimental Modal Analysis, Jan. 1988, pp. 12-19.
- [16] Goyder, H. G. D., "Frequency Response Testing a Non-linear Structure in a Noisy Enviroment with a Distorting Shaker", Proceedings of the Spring Conference, Institute of Acoustics, 1981, pp. 37-40.
- [17] O'Callahan, J., Avitable, P., Riemer, R., "System Equivalent Reduction Expansion Process", Proceedings of the 7th International Modal Analysis Conference (IMAC VII), Las Vegas, Nevada, U. S. A., 1989, PP. 29-37.

- [18] Allemang, R. J., Brown, D. L., "A Correlation Coefficient For Modal Vector Analysis", Proceedings of the 1st International Modal Analysis Conference (IMAC I), Orlando, Florida, U. S. A., 1982, pp. 110-116.
- [19] Friswell, M.I., Mottershead, J.E., "Finite Element Model Updating in Structure Dynamic", Kluwer Academic Publishers, 1995, pp. 126-134

APPENDIX Figures of Frequency Response Function

Figure 0.1 Frequency Response Function at Node 10 of Case 1

Figure 0.2 Frequency Response Function at Node 6 of Case 1

Figure 0.3 Frequency Response Function at Node 5 of Case 1

Figure 0.4 Frequency Response Function at Node 4 of Case 1

Figure 0.5 Frequency Response Function at Node 3 of Case 1

Figure 0.6 Frequency Response Function at Node 10 of Case 2

Figure 0.7 Frequency Response Function at Node 22 of Case 2

Figure 0.8 Frequency Response Function at Node 23 of Case 2

Figure 0.9 Frequency Response Function at Node 4 of Case 2

Figure 0.10 Frequency Response Function at Node 24 of Case 2

Figure 0.11 Frequency Response Function at Node 10 of Case 3

Figure 0.12 Frequency Response Function at Node 11 of Case 3

Figure 0.13 Frequency Response Function at Node 12 of Case 3

Figure 0.14 Frequency Response Function at Node 4 of Case 3

Figure 0.15 Frequency Response Function at Node 13 of Case 3

Figure 0.16 Frequency Response Function at Node 10 of Case 4

Figure 0.17 Frequency Response Function at Node 18 of Case 4

Figure 0.18 Frequency Response Function at Node 17 of Case 4

Figure 0.19 Frequency Response Function at Node 4 of Case 4

Figure 0.20 Frequency Response Function at Node 16 of Case 4

Figure 0.21 Frequency Response Function at Node 10 of Case 5

Figure 0.22 Frequency Response Function at Node 21 of Case 5

Figure 0.23 Frequency Response Function at Node 20 of Case 5

Figure 0.24 Frequency Response Function at Node 4 of Case 5

Figure 0.25 Frequency Response Function at Node 19 of Case 5

Figure 0.26 Frequency Response Function at Node 10 of Case 6

Figure 0.27 Frequency Response Function at Node 9 of Case 6

Figure 0.28 Frequency Response Function at Node 8 of Case 6

Figure 0.29 Frequency Response Function at Node 4 of Case 6

Figure 0.30 Frequency Response Function at Node 7 of Case 5