INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI[®]

**SPACE-TIME PROCESSING TECHNIQUES WITH APPLICATION TO
BROADBAND WIRELESS SYSTEMS**

by

Sébastien Roy

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy

Ottawa-Carleton Institute for Electrical and Computer Engineering
Faculty of Engineering
Department of Systems and Computer Engineering

Carleton University
Ottawa, Ontario, Canada K1S 5B6
Copyright © 2000 by Sébastien Roy

**National Library
of Canada**

**Acquisitions and
Bibliographic Services**

**395 Wellington Street
Ottawa ON K1A 0N4
Canada**

**Bibliothèque nationale
du Canada**

**Acquisitions et
services bibliographiques**

**395, rue Wellington
Ottawa ON K1A 0N4
Canada**

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-58259-0

Canada

The undersigned recommend to the Faculty of Graduate Studies and Research
acceptance of this thesis

Space-Time Processing in Fixed Broadband Wireless Systems

submitted by

Sébastien Roy, B.Sc.A., M.Sc.

in partial fulfillment of the requirements for the degree of Doctor of Philosophy

Thesis Supervisor

Prof. David D. Falconer

External Examiner

Dr. Jack Winters

Chair, Department of Systems and Computer Engineering

Carleton University

August, 2000

Abstract

This thesis consists of a study of space-time receivers used at base stations to

- a. mitigate co-channel interference and
- b. provide robustness against fading.

Many results herein are of a fundamental nature and could apply generally to any cellular narrowband or broadband wireless system including mobile systems. However, their application to a specific real-world system — the Local Multipoint Communication System or LMCS which is characterized by fixed subscribers, broadband operation and millimeter wavelengths — is also considered. The contributions can be classified into three broad themes:

1. The effect of channel envelope correlation between the array elements of a space-time receiver has been carefully assessed through the use of two analytical, three-dimensional propagation models applicable to both narrowband and wideband contexts. Furthermore, a comprehensive field measurement experiment has been carried out to help validate and refine these models.
2. As an upper bound on optimal array performance, a novel analysis of the matched-filter bound is presented which includes the effect of correlation. It is known that the performance of the best possible diversity receiver, the array maximum-likelihood receiver, is bounded by the MFB. Specifically, the main two novel aspects of our analysis are
 - the inclusion of spatial correlation through our proposed *concatenated array-equivalent channel*;

- a treatment of the Rician fading case based on the Nakagami distribution (and the fact that it can closely approximate the Rice distribution).

3. A novel S-T processing architecture has been studied which is designed to exploit the multi-user information available at the base station (i.e. joint detection). *Multi-user decision-feedback space-time processing* is a system whereby decisions from other users are coupled through adaptive feedback filters into the desired user's receiver in order to remove not only the post-cursor ISI but also the post-cursor CCI from in-cell interferers. This architecture therefore exploits the multiuser information to obtain more effective interference nulling at the cost of increased complexity. Numerical results are provided comparing this architecture with various linear and non-linear array structures in both a generic single-cell scenario and broadband wireless multi-cell contexts.

Acknowledgements

A large number of people have provided invaluable help in the realization of this thesis. First of all, thanks are due to my thesis supervisor, David D. Falconer, who generously provided advice and moral support without counting the hours. He also provided a pleasant research context which was flexible enough to accommodate the natural development of this somewhat rebellious student's personal research style.

Thanks are also due to Robert Bultitude for many helpful discussions regarding propagation issues and channel modeling and to Raymond Bérubé and the rest of the "CRC team" — Pierre Melançon and Tyler Smith — whose combined efforts were instrumental in completing a very ambitious measurement experiment.

At Carleton, gratitude must be extended to the two students who contributed in various ways to the measurement experiment and propagation study. Lorelei Villanueva worked under contract during the summer of '98 and wrote several of the routines that would later serve to extract impulse responses and otherwise post-process the measurement data. She also patiently and extensively surveyed the measurement area on bike and is largely responsible for transmission site selection.

Ehab Armanious also provided some code and agreed to play the essential role of equipment operator for the latter part of the summer of '99.

I am also indebted to many fellow students for their camaraderie as well as numerous stimulating discussions (both technical and non-technical): Ranjiv Saini, Nausheen Naz, M. Tariq Ali, Abdelgader Legnain, Philippe Garvie, Marco Lavoie and others.

Finally, my deepest gratitude goes to my wife Chantal and my son Nicolas for their patience and constant support. My parents, Aurella and Armand, also deserve mention for their unswerving faith in my capacities.

Dedication

A Chantal, l'amour de ma vie

Contents

1	Introduction	1
1.1	Digital Wireless Communication Networks	1
1.2	Microwave fixed broadband wireless digital network access	5
1.3	Methods of upgrading capacity: antenna arrays	6
1.4	Thesis objectives	8
1.5	Thesis overview	8
1.6	Thesis contributions	9
2	Background & Literature Survey	11
2.1	Adaptive Antenna Arrays and Beamforming	11
2.1.1	Reception-side Beamforming	11
2.2	Wideband arrays	18
2.3	Separability of signals by an array	21
2.4	Other Forms of Linear Diversity Combining	23
2.4.1	Maximal Ratio Combining	23
2.4.2	Equal Gain Combining	24
2.4.3	Selection Diversity	24
2.5	Non-linear Diversity Combining	25

2.5.1	Maximum-Likelihood Diversity Reception	25
2.5.2	Array Decision-Feedback Processing	28
2.5.3	Multi-user Decision-Feedback Processing	30
2.6	Evaluating Array Performance	30
2.6.1	Efficient Monte-Carlo Simulation	31
2.6.2	Analytical Results in Interference-Free Systems	34
2.6.3	Analytical Results in Interference-Limited Systems	35
2.6.4	BER evaluation and the distribution of residual interference .	42
2.6.5	Impact of training methods	47
2.7	Fixed wireless systems and LMCS	61
2.7.1	Asynchronous Transfer Mode (ATM)	63
2.7.2	Channelization	65
2.8	Vector Channel Modeling	65
2.9	Conclusion	70
3	Characterization of the typical LMCS vector channel	72
3.1	Analytical model - Rayleigh fading	73
3.2	Scattering geometries	80
3.2.1	Cylinder of scatterers (hollow model)	81
3.2.2	3-D circular gaussian model (full model)	83
3.3	Delay spread shape	86
3.4	Rician fading	87
3.5	Numerical results	92
3.5.1	Effect of subscriber position	92
3.5.2	Effect of subscriber antenna beamwidth	93
3.5.3	Effect of subscriber antenna front-to-sidelobe ratio	98

3.5.4	Effect of angle to subscriber	100
3.5.5	Effect of base station height	101
3.5.6	Effect of scattering region size	102
3.5.7	Effect of frequency separation	103
3.6	Monte-Carlo simulation issues	107
3.7	Measurements	108
3.7.1	Angular sweep measurements	109
3.7.2	Correlation measurements	120
3.8	Conclusion	150
4	The Matched-Filter Performance Bound on Optimal Space-Time Processing	152
4.1	The matched-filter bound in Rayleigh fading	153
4.2	The matched-filter bound in Rician fading	159
4.3	Numerical Results	162
4.3.1	Rayleigh fading	162
4.3.2	Rician fading	168
4.3.3	Effect of power delay profile shape	168
4.3.4	Numerical results based on propagation model	170
4.4	Conclusion	177
5	Optimal linear and decision-feedback array performance	179
5.1	Cross-decision-feedback arrays	180
5.2	Single-cell Generic Numerical Results	192
5.2.1	Effect and selection of BER evaluation method	192
5.2.2	Single-cell 2-antenna, 2-user system	193

5.2.3	Single-cell 4-antenna, 4-user system	196
5.2.4	Impact of filter lengths	198
5.3	Impact of training	202
5.4	Diversity order / XDF connectivity tradeoff	202
5.4.1	Impact of delay profile shape	206
5.5	Numerical Results Pertaining to Broadband Wireless Systems	210
5.5.1	Results with uncorrelated fading	210
5.5.2	Diversity order / XDF connectivity tradeoff	215
5.5.3	Comparison with non-SDMA scenarios	215
5.5.4	Results with fading correlation	216
5.6	Conclusion	218
6	Conclusion	232
6.1	Philosophical underpinnings and justification	232
6.2	Discussion of results and contributions	236
6.3	Applicability of results	240
6.3.1	Applicability to fixed wireless systems operating in other bands	240
6.4	Applicability to mobile wireless	241
6.5	Suggestions for future work on space-time processing	242
6.5.1	Analytical performance assessment of space-time processors .	242
6.5.2	Impact of training	243
6.5.3	Multi-user decision-feedback systems	244
A	Map of transmitter sites in Parkwood Hills	246
B	Site-by-site fitting of full model to measurement results	252

C	Solution of a matrix Wiener-Hopf system of equations	262
D	Spectral factorization of a matrix autocorrelation sequence	265
E	Matrix of DC coefficients	269
F	Closed-form expression of the beta CDF	271

List of Tables

3.1	Delay profiles and corresponding D factor.	88
5.1	Performance of space-time processors in SMDA (2 co-channel users/sector) and non-SDMA (1 co-channel user/sector) scenarios; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	217

List of Figures

2.1	Typical receiver architecture.	12
2.2	The narrowband linear combiner structure.	13
2.3	Probability density function for elevation angle β as proposed by Aulin.	68
2.4	Probability density function for elevation angle β as proposed by Parsons and Turkmani.	68
3.1	Geometry of the three-dimensional cylinder-of-scatterers model with the presence of a secondary scattering area. S marks the position of the subscriber, A_1 and A_2 are the two base station antennas.	81
3.2	Angular received power density for the hollow model using a range of 500 m and a scattering cylinder radius of 70 m. For simplicity, the model has been “flattened”, i.e. the maximum elevation angle $\beta_m = 0^\circ$ and the base station height is zero.	84
3.3	Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 10λ . Base station is at (0, 0); array axis indicated by arrows.	93
3.4	Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 50λ . Base station is at (0, 0); array axis indicated by arrows.	94

3.5	Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 100λ . Base station is at $(0, 0)$; array axis indicated by arrows.	94
3.6	Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 500λ . Base station is at $(0, 0)$; array axis indicated by arrows.	95
3.7	Fading envelope correlation as a function of subscriber position; full model, interelement separation of 10λ . Base station is at $(0, 0)$; array axis indicated by arrows.	95
3.8	Fading envelope correlation as a function of subscriber position; full model, interelement separation of 50λ . Base station is at $(0, 0)$; array axis indicated by arrows.	96
3.9	Fading envelope correlation as a function of subscriber position; full model, interelement separation of 100λ . Base station is at $(0, 0)$; array axis indicated by arrows.	96
3.10	Fading envelope correlation as a function of subscriber position; full model, interelement separation of 500λ . Base station is at $(0, 0)$; array axis indicated by arrows.	97
3.11	Envelope correlation vs. subscriber antenna beamwidths for 5 interelement separations using the hollow model; $m = 1000m$, $\delta_0 = \frac{\pi}{2}$	98
3.12	Envelope correlation vs. subscriber antenna beamwidths for 5 interelement separations using the full model; $m = 1000m$, $\delta_0 = \frac{\pi}{3}$	99
3.13	Envelope correlation vs. subscriber antenna beamwidths for 3 FTS ratios.	99

3.14 Envelope correlation vs. angle to subscriber for various subscriber antenna beamwidths using the hollow model at a distance $m = 1000\text{m}$ and using a separation of 100λ	100
3.15 Envelope correlation vs. angle to subscriber for various subscriber antenna beamwidths using the full model at a distance $m = 1000\text{m}$ and using a separation of 100λ	101
3.16 Envelope correlation vs. interelement spacing for three subscriber antenna beamwidths and two base station heights using the full model.	102
3.17 Envelope correlation vs. interelement spacing using the hollow model with $\delta_0 = \frac{\pi}{2}$	103
3.18 Envelope correlation vs. interelement spacing using the hollow model with $\delta_0 = \frac{\pi}{3}$	104
3.19 Envelope correlation vs. interelement spacing using the full model with $\delta_0 = \frac{\pi}{2}$	104
3.20 Envelope correlation vs. interelement spacing using the full model with $\delta_0 = \frac{\pi}{3}$	105
3.21 Envelope correlation vs. frequency f_2 (f_1 being fixed at 29.5 GHz) for five values of τ_0 using an exponential delay profile.	105
3.22 Envelope correlation vs. frequency f_2 (f_1 being fixed at 29.5 GHz) for five values of τ_0 using a gaussian delay profile.	106
3.23 Envelope correlation vs. frequency f_2 (f_1 being fixed at 29.5 GHz) for five values of τ_0 using a double-spike delay profile.	106
3.24 Angular power distribution derived from the Parkwood Hills measurements (dBs).	116

3.25 Angular power distribution derived from the Kanata measurements (dBs).	119
3.26 Measurement setup.	121
3.27 Block diagram of receiver for wideband correlation measurements. . .	123
3.28 Block diagram of transmitter for wideband correlation measurements.	124
3.29 Trailer unit serving as mobile transmitter.	127
3.30 Details of turntable assembly and transmitter up-converter.	129
3.31 Azimuth pattern of transmit antenna.	130
3.32 Azimuth pattern of receive antennas.	131
3.33 Layout of the penthouses on the roof of the Chesterton Towers building.	132
3.34 The antenna platform in operation atop the Chesterton Towers pent- house.	134
3.35 Side view of first penthouse showing chimney.	135
3.36 Aerial photograph of Chesterton Towers.	136
3.37 Correlation matrix $\hat{\mathbf{R}}_{12}$ for a spacing of 10λ with the Tx located at 37 Woodmount.	140
3.38 Histogram of ε_1	144
3.39 Histogram of ε_2	145
3.40 Histogram of ε_3	146
3.41 Histogram of ε_4	147
3.42 Scattering region size parameter σ_r vs. subscriber-to-base distance m .	149
3.43 Three transmitters at different distances from the base.	150
4.1 System model for maximum-likelihood diversity reception in the ab- sence of CCI.	154

4.2	MFB outage probability in a 2-element array as a function of E_b/N_0 , delay spread d and correlation ρ	164
4.3	MFB average BER in a 2-element array as a function of E_b/N_0 , delay spread d and correlation ρ	165
4.4	MFB outage probability in a 2-element array as a function of RMS delay spread d and correlation ρ	166
4.5	MFB average BER in a 2-element array as a function of RMS delay spread d and correlation ρ	167
4.6	MFB outage probability as a function of K factor, correlation ρ and delay spread d	169
4.7	MFB average BER as a function of K factor, correlation ρ and delay spread d	169
4.8	MFB performance as a function of delay spread shape and correlation ρ in a 2-element array with $d = \frac{1}{3}$	171
4.9	Cell / sector plan for Monte-Carlo simulations with detail of base station array topologies.	172
4.10	MFB outage probability in a two element array as a function of delay spread d and interelement separation d_x	173
4.11	MFB average BER in a two element array as a function of delay spread d and interelement separation d_x	174
4.12	MFB outage probability in a four element array as a function of delay spread d and interelement separation d_x	175
4.13	MFB average BER in a four element array as a function of delay spread d and interelement separation d_x	176
5.1	Matrix channel model for space-time reception in a multi-user system.	182

5.2	Portion of a one-antenna, $N + 1$ -user cross-decision-feedback receiver structure corresponding to signal 0.	184
5.3	Performance of a linear space-time processor produced with 3 different BER evaluation methods with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of $d = 0.1$ symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$). . .	194
5.4	Performance of various space-time processors with 2 elements and 1 interferer as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol duration; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	195
5.5	Performance of various space-time processors with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol duration; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	197
5.6	Performance of various space-time processors with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of $\frac{1}{2}$ symbol duration; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	199

5.7	Performance of various space-time processors with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 2 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	200
5.8	Performance of linear space-time processors with 4 elements and 3 interferers as a function of SIR and of filter length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	201
5.9	Performance of linear space-time processors with 3 taps per element, 4 elements and 3 interferers as a function of SIR and of training sequence length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	203
5.10	Performance of linear space-time processors with 6 taps per element, 4 elements and 3 interferers as a function of SIR and of training sequence length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	204
5.11	Performance of linear space-time processors with 11 taps per element, 4 elements and 3 interferers as a function of SIR and of training sequence length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	205

5.12	Average BER performance of linear, x2df and xdf systems in the presence of 3 equal-power interferers for $M=2$ to 5; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	207
5.13	Average BER performance of linear space-time processing with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile as a function of delay profile shape.	209
5.14	Average BER performance of decision-feedback space-time processing with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile as a function of delay profile shape.	209
5.15	Average BER performance of fully-connected XDF space-time processing with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile as a function of delay profile shape.	220

5.16 Geographical system plan used in the multicell simulations. Four square cells are shown, each being divided into 4 sectors. The base station of interest, located in the upper left-hand sector of the lower right-hand cell, is identified by a “B”. Assuming perfect sector isolation and neglecting interferers beyond the first tier, co-channel users in up to 8 sectors (in addition to the sector of interest) can affect reception. It can be seen that in three of these sectors, interferers can occasionally, depending on their position with respect to their own base, illuminate the base through their mainlobe. Interferers from the remaining 5 sectors (grayed out in the figure), will always come in through their backlobe. The most severe interferer is, of course, the one sharing the sector of interest with the desired user.	221
5.17 Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	222
5.18 Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 4$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	223
5.19 Performance of various 2-antenna space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 4$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	224

5.20	Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 0.1$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	225
5.21	Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$ and the in-sector users are undergoing Rician fading; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).	226
5.22	Average BER performance of linear, x2df and xdf systems for $M = 2$ to 5.	227
5.23	Average BER performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 0.1$ in the presence of fading correlation as a function of interelement spacing d_x	228
5.24	Outage probability performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 0.1$ in the presence of fading correlation as a function of interelement spacing d_x	229
5.25	Average BER performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$ in the presence of fading correlation as a function of interelement spacing d_x	230

5.26	Outage probability performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$ in the presence of fading correlation as a function of interelement spacing d_x	231
A.1	Map of transmitter sites; North is towards the right of the page and the basestation building (Chesterton Towers) is circled. Sites which were found to result in insufficient received power during data collection are marked as red dots; the green dots mark the remaining sites.	247
A.2	Lower left quadrant of map.	248
A.3	Lower right quadrant of map.	249
A.4	Upper left quadrant of map.	250
A.5	Upper right quadrant of map.	251
B.1	Site 1 - 37 Woodmount.	253
B.2	Site 2 - 11 Charkay.	253
B.3	Site 3 - Benson 53 ft. away from Seymour.	254
B.4	Site 4 - 1 Kilmory.	254
B.5	Site 5 - 3 Kilmory.	254
B.6	Site 6 - 71 Tiverton.	254
B.7	Site 7 - 58 Newbury.	254
B.8	Site 8 - 42 Newbury.	254
B.9	Site 14 - 15 Bowmoor.	255
B.10	Site 15 - Wigan and Inverness.	255
B.11	Site 37 - 40 Walgate.	255
B.12	Site 39 - 11 Charkay.	255

B.13 Site 43 - 28 Mulvaugh.	255
B.14 Site 49 - 48 Tiverton.	255
B.15 Site 53 - 17 Lynhaven.	256
B.16 Site 54 - 5 Rockfield.	256
B.17 Site 65 - 7 Inverness.	256
B.18 Site 66 - 27 Inverness.	256
B.19 Site 90 - 35 Seymour.	256
B.20 Site 102 - 114 Fourseasons.	256
B.21 Site 109 - Fourseasons & Shadetree.	257
B.22 Site 110 - Coolspring & Viewmount.	257
B.23 Site 117 - 25 Southview.	257
B.24 Site 129 - 18 Welkin.	257
B.25 Site 130 - Chesterton & Assiniboine.	258
B.26 Site 131 - Meadowlands & Peter.	258
B.27 Site 133 - 46 Laird.	259
B.28 Site 143 - Canter & Bayne.	259
B.29 Site 144 - Bayne & David.	259
B.30 Site 145 - 5 David.	259
B.31 Site 146 - 23 David.	259
B.32 Site 147 - 35 David.	259
B.33 Site 150 - 26 Harris.	260
B.34 Site 151 - 14 Harris.	260
B.35 Site 152 - 3A Joan.	260
B.36 Site 153 - Rossland & Pender.	260
B.37 Site 155- 26 Withrow.	260

B.38 Site 159 - Tower & St-Helene.	260
B.39 Site 162 - East end of Cotes des Neiges.	261
B.40 Site 163 - 27 Lotta.	261
B.41 Site 203 - 118 Rossland.	261
B.42 Site 205 - 96 Rossland.	261
B.43 Site 212 - 79 Tower.	261
B.44 Site 218 - 41 Palsen.	261

Table of Notation

α	azimuth ray departure angle at subscriber
A	mainlobe-to-sidelobe ratio
(α_0, β_0)	direction of look of subscriber antenna
a_i	incident power wave in element i
$a_i^{(j)}[k]$	overall discrete impulse response for user i at the output of feedforward filter j
$\mathbf{a}^{(j)}[k]$	vector of all users' overall impulse response at the output of feedforward filter j
$\mathbf{A}[k]$	$N + 1 \times N + 1$ matrix where the columns are the $\mathbf{a}^{(j)}[k], j = 0, \dots, N$
$\mathbf{a}_n$	$N \times 1$ vector of actual transmitted symbols by all users at time n
$A_j(\mathbf{k}_i)$	response of array element j to free space signal i
ATM	asynchronous transfer mode
$\hat{\mathbf{a}}_n$	$N \times 1$ vector of possible (as estimated by receiver) transmitted symbols by all users
β	elevation ray departure angle at subscriber
$B(m, n)$	Beta function
BER	bit error rate
c	speed of light constant (3.0×10^8 m/s)

$\mathbf{C}$	matrix of channels from all $N + 1$ users to all M elements of array
$\mathbf{c}_0(t), \mathbf{c}_0$	spatial signature (vector channel) of desired user at time t
CDMA	code-division multiple-access
$\mathbf{c}_i(t), \mathbf{c}_i$	spatial signature (vector channel) of user i at time t
$\hat{\mathbf{c}}_i$	estimated spatial signature of i th user
δ	azimuth ray arrival angle at base station
D	delay spread shape factor
$\mathbf{D}$	$N \times N$ matrix of cross-signature coefficients, excluding desired user
(δ_0, ϵ_0)	arrival direction at base station corresponding to line-of-sight
Δ_a	size of the array aperture, in meters
DAVIC	digital audio-visual council
$\mathbf{D}_d$	$N + 1 \times N + 1$ matrix of cross-signature coefficients
$\delta[k]$	Kronecker delta function
$d_n[k]$	equalized, combined and scaled channel impulse response of n th user
$\tilde{d}_n[k]$	interfering channel coefficients
DOA	direction of arrival
$\Delta\omega$	frequency separation $ \omega_2 - \omega_1 $
Δx	spatial separation $\ \mathbf{x}_2 - \mathbf{x}_1\ $
ϵ	elevation arrival angle at base station
ε	minimum achievable mean-square error
ε_0	minimum achievable mean-square error for user 0
E_0	root-mean-square value of the magnitude of the baseband complex envelope of the electric field E_z
$e_1(t), e_1$	real part of $E_t^{(1)}(t)$
$e_2(t), e_2$	imaginary part of $E_t^{(1)}(t)$

$e_3(t), e_3$	real part of $E_l^{(2)}(t)$
$e_4(t), e_4$	imaginary part of $E_l^{(2)}(t)$
ε_{DF}	minimum achievable mean-square error by a decision-feedback adaptive processor
ε_L	minimum achievable mean-square error by linear adaptive processor
$E_l^{(1)}(t)$	baseband complex envelope of electric field at frequency ω_1 and position $\mathbf{x}_1$
$E_l^{(2)}(t)$	baseband complex envelope of electric field at frequency ω_2 and position $\mathbf{x}_2$
EVP	eigenvector projection
E_z	vertical component of the electric field
f	frequency in Hertz
${}_2F_1(a, b; c; x)$	Gauss hypergeometric function
FDMA	frequency-division multiple-access
FTS	Front-to-sidelobe
$f_{ij}[n]$	discrete impulse response of cross-feedback filter from signal i to processor j
$f_\tau(\tau)$	power delay profile
$[\gamma_0]$	matrix of DC coefficients of $\mathbf{M}[k]$
Γ_i	reflection coefficient in element i
GSMI	generalized sample matrix inversion
$\Gamma(x)$	Gamma function
$\Gamma_{\tilde{M}}(x)$	multivariate Gamma function
$G_b(\delta, \epsilon)$	base station antenna element power pattern in azimuth and elevation
$G_s(\alpha, \beta)$	subscriber antenna element power pattern in azimuth and elevation

$G_T(\omega)$	transmit filter frequency response
$G_{tot}(\omega)$	combined transmit and receive filter frequency response
$G_R(\omega)$	receive filter frequency response
HTP	Hung-Turner projection
$\mathbf{I}$	identity matrix
$I_0(x)$	modified Bessel function of zeroth order
INR	interference-to-noise ratio
IP	Internet protocol
$\mathbf{k}$	wavenumber vector
K	number of training symbols or snapshots used to form a weight vector solution in block adaptation methods
K	Rician K factor – ratio of specular component power to fading component variance
λ	wavelength
Λ	canonical (diagonal) form of a square matrix
λ_i	i th eigenvalue of a matrix
LMCS	local multipoint communication system (canadian acronym)
LMDS	local multipoint distribution service (U. S. acronym)
LSMI	loaded sample matrix inversion
M	number of sub-waves from the same direction of arrival
M	number of antenna elements in array
μ_0	optimal SINR at the output of an ideal, infinite-length space-time processor when user 0 is the desired signal
m_1	variance of e_1
m_2	covariance of e_1 and e_3

m_3	covariance of e_1 and e_4
$\mathbf{M}[k]$	loaded correlation matrix equal to $\mathbf{R}[k] + \mathbf{I}\delta[k]$
$\mathbf{M}^+[k], \mathbf{M}^-[k]$	causal and anticausal factor matrices of $\mathbf{M}[k]$, i.e. $\mathbf{M}[k]$ results from the discrete matrix convolution of $\mathbf{M}^+[k]$ and $\mathbf{M}^-[k]$
MBAA	multiple-beam antenna array
n	discrete time index corresponding to the n th symbol period
N	number of co-channel interferers in cell or sector <i>or</i> number of active signals sharing a common carrier (including desired) minus one
N	number of waves with different directions of arrival
$\mathbf{N}$	matrix of noise eigenvectors
NIU	network interface unit
$\mathbf{P}$	$N \times N$ diagonal matrix whose diagonal elements are the received interferer powers normalized to the desired signal power (see $\mathcal{P}_i$)
$\hat{\mathbf{P}}$	$N+1 \times N+1$ diagonal matrix of interferer powers; same as $\tilde{\mathbf{P}}$ but with element (0,0) (desired signal power) set to 0
$\tilde{\mathbf{P}}$	$N+1 \times N+1$ diagonal matrix whose i th diagonal element is the received powers of the i th active user
ϕ_0	size of antenna pattern aperture corresponding to the mainbeam measured in a two-dimensional plane intersecting the antenna position and the direction of look
ϕ_{3dB}	half-power beamwidth
pdf	probability density function
P_e	probability of error (same as BER)
P_i	Total received power at array from i th user

$\mathcal{P}_i$	received power of the i th interferer normalized to the desired signal power, i.e. $\mathcal{P}_i = \frac{P_i}{P_0}$
$\mathbf{P}_S^\perp$	projection matrix projecting onto the subspace orthogonal to subspace S
$Q(x)$	Q function
QPSK	quaternary phase-shift keying
$\hat{\mathbf{R}}$	input array vector covariance matrix estimated by averaging over a number of input signal snapshots
r	radius of scattering area or distance from the subscriber to a scattering point
ρ	normalized SINR
ρ_0	normalized SINR for user 0 (normalized with respect to the ideal SINR)
r_1	envelope (magnitude) of $E_t^{(1)}(t)$
R_{11}	complex variance of $E_t^{(1)}(t)$
R_{12}	covariance of $E_t^{(1)}(t)$ and $E_t^{(2)}(t)$
r_2	envelope (magnitude) of $E_t^{(2)}(t)$
R_{22}	complex variance of $E_t^{(2)}(t)$
ρ_e	envelope cross-correlation coefficient
ρ_p	power cross-correlation coefficient
R_e	covariance of envelopes r_1 and r_2
$r_{ij}[k]$	discrete cross-correlation function between the impulse responses of user i and user j 's channels
$\mathbf{R}[k]$	$N + 1 \times N + 1$ channel correlation matrix where element (i, j) is $r_{ij}[k]$
RMS	root-mean-square
$\mathbf{R}_{xx}(t), \mathbf{R}_{xx}$	Received signal short-term covariance matrix at time t

SDMA	space-division multiple-access
$\mathbf{S}$	matrix of signal eigenvectors
σ^2, σ_n^2	white gaussian noise power at each array element output (after front-end but prior to processing)
σ_I^2	residual interference power
σ_s^2	data symbol variance
$\hat{s}_i[n]$	estimate of n th received symbol from user j after slicing
τ	delay variable
τ_0	mean delay spread
S_{ij}	S -parameter between array elements i and j
$\tilde{s}_i[n]$	estimate of n th received symbol from user j prior to slicing
SINR	signal-to-interference-plus-noise ratio
SMI	sample matrix inversion
STDSMI	space-time decomposition sample matrix inversion
T	symbol period
TDMA	time-division multiple-access
$\mathbf{U}$	matrix of eigenvectors
$\text{vec}(\mathbf{X})$	vector operator: forms a vector by concatenating the columns of its matrix argument
$\mathbf{w}(t), \mathbf{w}$	weight vector of narrowband array at time t
W	effective signal bandwidth
W_c	coherence bandwidth
$\omega_i^{\text{opt}}(t)$	impulse response of feedforward filter in the equalizer where signal i is the desired signal
$\omega_i(t)$	optimal feedforward filter impulse response for user i

$\mathbf{x}$	a position (x, y, z) in 3D space
x2df	multi-user decision feedback with 2 feedback filters
XDF	Cross (multi-user) decision-feedback
$\mathbf{X}[k]$	result of the discrete matrix convolution of $\mathbf{M}^{-}[k]$ and $\mathbf{A}[k]$
$\mathbf{x}(t), \mathbf{x}$	received signal vector at output of array elements at time t
ξ_{ii}	user i 's self-signature coefficient or fading coefficient
ξ_{ij}	cross-signature coefficient between users i and j
XTSMI	extended training sample matrix inversion
$y_i(t)$	output of i th feedforward filter
z	scattering point height
$\mathbf{Z}$	$M \times M$ array mutual impedance matrix
Z_C	characteristic impedance of the transmission lines connected to the array elements' outputs
z_m	maximum scattering point height

Chapter 1

Introduction

A system cannot be analyzed into parts. If you do, what you end up with is not contained within the original whole. It is created by the act of analysis.

David Bohn, physicist

1.1 Digital Wireless Communication Networks

It is said that the 1990s are the decade of wireless communications. Indeed, over the last decade, the number of subscribers to wireless communication services has grown exponentially. It is estimated that in the first decades of the twenty-first century there will be as many wireless telephone subscribers as conventional wireline subscribers [1]. The number of cellular users in the US has grown from 5.1 million to 23.3 million subscribers from 1990 to 1994 and it is projected that the number will reach 46.9 million by the year 2000. First generation cellular systems (analog) are reaching the point of saturation and providers are scrambling to keep up with the demand. Second

generation digital cellular systems are now in their initial stages of deployment while third generation systems are in the planning stages.

In parallel with the growing demand for wireless services, an evolutionary change is also taking place. Cellular phones are evolving beyond the telephone paradigm to become universal portable gateways into the digital universe [2], providing a wide variety of services to subscribers on the move. This evolution is designated by a number of acronyms: *personal communication services* (PCS), *personal communication networks* (PCN), or *universal portable communications* [3],[4],[5]. Furthermore, a plethora of new wireless services are emerging, most of which involve multi-media and relatively large bandwidth requirements. Spurred by the development of the Internet, the growth of telecommuting and SOHO (Small Office Home Office) and the increasing popularity of bandwidth-hungry services (e.g. video-on-demand, teleconferencing), various systems are being deployed or designed to provide broadband wireless network access.

Consumer appetite for wireless communications has exceeded all forecasts and first generation systems around the globe are approaching or reaching capacity. However, spectrum space is a limited resource. The current response to this phenomenon is two-fold:

1. Second and later-generation systems and new services are being located at increasingly higher frequency bands which are still relatively free of traffic. Indeed, the wide-spread usage of millimeter bands has until recently been delayed due to the hardware difficulties and costs involved. However, the development of RF technology has kept in step with the times and systems operating in the 28-30 GHz band are being tested now while others are in various stages of development.

2. Both existing and proposed systems are being designed or retrofitted to support techniques that bring more efficient spectrum usage, i.e. increased capacity.

In this present-day context, the future development of wireless networks faces many difficult challenges. Two issues stand out as being of foremost importance, and it will be seen that they are strongly related problems:

1. **The hostility of the wireless channel.** The opening words in Jakes' classic reference book [6] describe eloquently the magnitude of the problem, as it was perceived by the pioneers of the field:

Nature is seldom kind. One of the most appealing uses for radio-telephone systems—communication with people on the move—must overcome radio transmission problems so difficult they challenge the imagination. A microwave radio signal transmitted between a fixed base station and a moving vehicle in a typical urban environment exhibits extreme variations in both amplitude and apparent frequency. Fades of 40 dB or more below the mean level are common, with successive minima occurring about every half wavelength (every few inches) of the carrier transmission frequency. A vehicle driving through this fading pattern at speeds up to 60 mi/hr can experience random signal fluctuations occurring at rates of 100-1000 Hz, thus distorting speech when transmitted by conventional methods. These effects are due to the random distribution of the field in space, and arise directly from the motion of the vehicle. If the vehicle is stationary, the fluctuation rates are orders of magnitude less severe.

The sentence ending with "...thus distorting speech when transmitted by con-

ventional methods” immediately suggests the use of digital transmission which can benefit from sophisticated error-correction coding and lends itself better to various signal processing techniques designed to combat fading and shadowing. This certainly constitutes one of the motivating factors behind the rapid transition to digital in wireless networks. Jakes also implicitly suggests that fixed wireless systems should experience less severe fading effects than mobile systems. The focus of the present work being on millimeter-wavelength fixed wireless systems, it should be noted that measurements have shown that fading effects in these systems were surprisingly important [7]. This is due to the smallness of the wavelength which magnifies the effect of scatterer motion in multipath fading. Furthermore, very small objects (such as raindrops) become scatterers and very small surface irregularities result in rough surface scattering in this band [8]. Parsons and Bajwa sum up nicely the magnitude of the challenge in the context of wideband, high frequency operation [9]:

...in heavily built-up areas, particularly at higher operating frequencies, multipath propagation is probably the single most destructive influence.

2. **The increasingly crowded spectrum space.** It is possible to accommodate large numbers of users through spatial reuse of frequencies. In effect, the limited reach of radio waves (whose power decreases with the square of the distance in free space) is used to advantage, since it allows many users to utilize the same channel provided they and their respective base stations are separated from each other by a sufficient distance. In fact, this is the basis of the cellular concept [10]. Nonetheless, as the number of users increases, interference from

users of the same channel (*co-channel interference* or CCI) becomes a significant impediment. Adjacent channels are also a source of interference due to spectral overlap. This is termed *adjacent channel interference* or ACI.

It is a generally accepted fact that wireless cellular systems are interference-limited. Specifically, co-channel interference from the closest cells where channels are re-used and adjacent channel interference are the most important limiting factors. It follows that the implementation of techniques to reduce interference or to reduce the impact of interference in existing systems is equivalent in many cases to boosting capacity. Another aspect of the capacity issue is designated as the multiple-access problem and consists of allocating in the most efficient manner the limited spectrum space amongst users in a dynamic fashion. Basic multiple access strategies include TDMA (Time Division Multiple Access), FDMA (Frequency Division Multiple Access) and CDMA (Code Division Multiple Access) as well as combinations thereof [1], [3].

1.2 Microwave fixed broadband wireless digital network access

This work is concerned mainly with broadband digital access which is likely a major axis for development in the next few years and where the capacity problem is compounded by the magnitude of the bit rates required by each individual user. Specifically, we will be interested herein in fixed wireless broadband access networks including but not limited to those falling under the designations LMCS (Local Multipoint Communication System) and LMDS (Local Multipoint Distribution Service)

¹and how they can be enhanced by the use of digital beamforming / adaptive array techniques. Designed to operate in the vicinity of 28-30 GHz and consuming up to a full GHz of bandwidth, LMCS-class system are specified to support asymmetric traffic flow, offering up to 10 times more bandwidth on the downlink than on the uplink.

LMCS systems are now under their initial stages of deployment which implies that issues such as coverage are considered more relevant than capacity. However, the projected market growth shows clearly that efficient spectrum usage will become a priority in the near future. Furthermore, should techniques allowing capacity increases by an order of magnitude or more become widely available, LMCS-class systems could be relocated to lower, narrower bands where hardware problems are less serious, propagation is better known and less hostile and the manufacturing cost of RF components is significantly less. Unlicensed or licensed bands in the vicinity of 2 GHz and 5 GHz are strong possibilities.

1.3 Methods of upgrading capacity: antenna arrays

Sectoring and *cell splitting* are commonly-used methods to add capacity to existing systems. They allow channel reuse at a closer range but require a more costly infrastructure and may create additional control (handoff) overhead. Cell splitting is possibly the most expensive option since it requires the installation of complete new base stations as well as the expenses (rental of area on top of high-rise, etc.) and

¹LMCS and LMDS designate the same systems with the distinction that the former is the preferred acronym in Canada while the latter is predominantly employed in the U.S.

potential legal complications associated with new radio transmission sites. *Adaptive spatial diversity* at the base station is a third option that is arguably the most promising way to significantly augment capacity. An adaptive array can simultaneously combat fading, noise and interference (both CCI and ACI), hence allowing more users to coexist. Typical arrays work by minimizing a single performance measure — e.g. the mean-square error between the output and the desired signal — and will therefore implicitly allocate more degrees of freedom to the most severe impediment, be it fading or interference.

Winters, Salz and Gitlin [14] have shown that with 2 or 3 elements, an array can double the capacity of IS-54 mobile radio systems while with 5 elements, it can be increased 7-fold (frequency reuse factor of 1). Adaptive arrays can also be exploited in a more sophisticated manner by forming highly narrow beams targeted at each individual user, thus allowing channel *reuse within cell*. It effectively adds a new dimension (space) in the multiple-access scheme and the capacity gains in space are multiplicative with respect to the capacity in time and frequency [12]. Antenna arrays can be conceived in a modular fashion and can be introduced in existing systems in an incremental fashion, progressively increasing the diversity order as the traffic grows. However, to fully realize the potential promise of spatial diversity, it is necessary to carefully select parameters such as the array topology, the adaptation techniques, etc. Also, the introduction of an array modifies considerably the multiple-access problem and requires changes in the existing (non-array) multiple-access protocols.

1.4 Thesis objectives

The present work is a study of the performance of space-time receivers used at the base station on the uplink to jointly mitigate co-channel interference and provide robustness against fading. While many results herein can readily be extended to mobile communications, the emphasis is on fixed broadband wireless access systems such as LMCS.

Basically, the approach is to investigate a number of interrelated system parameters — e.g. array topology, space-time processing strategy (linear, decision-feedback, multi-user decision-feedback, maximum-likelihood), number of co-channel carriers per sector, etc. — and examine their impact on performance while promoting whenever possible fundamental understanding and results of broad applicability.

1.5 Thesis overview

Chapter 2 provides a detailed literature review and background on space-time processing, analytical and numerical methods to assess their performance and propagation modeling in the context of array-enhanced communication systems.

Chapter 3 explores propagation issues and how they related to fading correlation observed between elements of an array as a function of array topology (e.g. interelement spacing). Two analytical propagation models are formulated and they are used to illustrate the impact of various parameters on interelement correlation. An empirical section follows based on an outdoor measurement experiment which ultimately serves to validate and refine one of the theoretical model.

Chapter 4 presents a novel analysis of the matched-filter bound in the context of wideband arrays which includes the effects of fading correlation and supports the

inclusion of a static component.

Finally, chapter 5 addresses linear and decision-feedback space-time processing. Numerical results are presented illustrating the impact of training (assuming that sample matrix inversion is employed to compute the mean-square-error-minimizing weights) and filter length on linear space-time processing. An analysis is provided yielding a closed-form expression for the minimum-achievable mean-square error in ideal infinite-length multi-user decision-feedback space-time processors with an arbitrary subset of users in the feedback loop. This formula is then exploited to provide comparative numerical results spanning a wide range of space-time processing strategies from linear to fully-connected decision-feedback processing.

Chapter 6 provides concluding remarks, a discussion of results and suggestions for further research.

1.6 Thesis contributions

1. Two detailed macrocell-type (local scattering) three-dimensional propagation models of the stochastic type (i.e. which assume a very large number of scatterers) were formulated for both Rayleigh and Rician fading for computing correlation coefficients in a wideband array.
2. An extensive outdoor measurement experiment was carried out and served as a basis for an empirical study of fading correlation in a wideband array. This served to validate and refine (by quantifying several parameters) one of the theoretical propagation models.
3. A procedure was devised for the removal of mutual impedance effects in array measurements.

4. New analytical matched-filter bound results were produced (average BER and outage probability) which are applicable to both the Rayleigh-fading and the Rician-fading vector channels with or without fading correlation. The novel aspects here are:

- the inclusion of spatial correlation through the proposed *concatenated array-equivalent channel*.
- the treatment of the Rician case based on the Nakagami distribution.

5. Some semi-analytical results were computed illustrating the impact of training (i.e. finite training sequence length) on a linear space-time processor.

6. A new formula was derived for the MMSE achievable by an optimal infinite-length multi-user decision-feedback space-time processor where an arbitrary subset of users is part of the feedback loop.

7. Extensive numerical results were plotted based on the above formula comparing the performance of space-time processors with varying degrees of cross-feedback connectivity in:

- a generic Rayleigh-fading single-cell scenario;
- a multi-cell broadband wireless scenario patterned after LMCS (highly directional subscriber antennas, full frequency reuse in every sector and every cells, square cells with 90 degree sectors) with the inclusion of carrier reuse within each sector.

Chapter 2

Background & Literature Survey

Ce que l'on conçoit bien s'énonce clairement et les mots pour le dire arrivent aisément.

Nicolas Boileau

I couldn't do it. I couldn't reduce it to freshman level. That means we don't really understand it.

Richard Feynman

2.1 Adaptive Antenna Arrays and Beamforming

2.1.1 Reception-side Beamforming

A digital adaptive antenna array or digital beamformer consists of an array of individual elements, independent receiver front-ends for each element, and a combiner—i.e.

Figure 2.1: Typical receiver architecture.

a device capable of combining the information from all receivers in such a manner as to maximize a performance criterion and produce some useful output. The current trend is toward all-digital implementation. Digitization of the signals occurs as close as possible to — sometimes immediately after — the antenna elements. Accordingly, the combiner is typically implemented using some form of Digital Signal Processing (DSP) technology. In general, each receiver provides two bit streams for the in-line and quadrature components of the received signal at the element (see figure 2.1). These are then individually processed and then combined to yield the desired output.

There is an important distinction to be made between narrowband arrays designed for flat fading and broadband arrays (the so-called “space-time” arrays) [15] designed for frequency-selective fading environments. For the sake of simplicity and unless indicated otherwise, general array concepts will be presented hereafter in the context of narrowband systems.

Most narrowband arrays are based on the linear structure of fig. 2.2. The combiner is simply a weight and sum structure which is capable of enhancing a desired signal and eliminating unwanted signals by provoking constructive and destructive interference effects. Given the received signal vector $\mathbf{x}(t)$ and the weight vector $\mathbf{w}(t)$, the output

Figure 2.2: The narrowband linear combiner structure.

of the combiner is simply

$$y(t) = \mathbf{w}^H(t)\mathbf{x}(t) \quad (2.1)$$

In multipath fading environments, an array can provide robustness against fading due to spatial diversity effects. Indeed, the signals at each element will not fade simultaneously provided that the elements are sufficiently spaced to decorrelate the fading envelopes. For a carrier wavelength of λ , it is generally assumed in the literature that separations of $\frac{\lambda}{2} - \lambda$ for subscriber-side arrays and $8\lambda - 40\lambda$ for base station-side arrays are the minimum required for this purpose [13] [14].

In systems affected by noise and fading only (i.e. no interference), the optimal combining scheme is maximal-ratio combining [16] where each branch is weighted in proportion to its SNR. This is merely the application in the spatial dimension of the matched-filter principle. However, multiuser wireless systems are typically interference-limited. In this context, it is not so obvious to determine which combining scheme is optimal. For the time being, it is sufficient to state that the optimal *linear*

combiner in the *mean-square sense* is the spatial equivalent of the Wiener-Hopf filter [17], [18] and its weight vector is given by

$$\mathbf{w}(t) = \alpha \mathbf{R}_{\mathbf{xx}}^{-1}(t) \mathbf{c}_0(t) \quad (2.2)$$

where $\mathbf{R}_{\mathbf{xx}}(t) = \langle \mathbf{x}(t) \mathbf{x}^H(t) \rangle$ is the received signal covariance matrix, $\mathbf{c}_0(t)$ is the vector channel of the desired signal and α is a scalar constant. The covariance matrix can also be expressed as a function of the channel vectors as

$$\mathbf{R}_{\mathbf{xx}} = \mathbf{C} \bar{\mathbf{P}} \mathbf{C}^H + \mathbf{I} \sigma^2, \quad (2.3)$$

where

$$\mathbf{C} = [\mathbf{c}_0, \mathbf{c}_1, \dots, \mathbf{c}_N], \quad (2.4)$$

and the matrix of signal powers is defined

$$\bar{\mathbf{P}} = \text{diag}(P_0, P_1, \dots, P_N). \quad (2.5)$$

Optimum combining was first studied by Bogachev and Kiselev [19] who derived the probability density function of the SINR and the average BER for the case of a single dominant interferer when the desired channel is subjected to Rayleigh fading. Subsequently, Winters [20] studied optimum combining in the context of digital mobile radio, employing Monte Carlo techniques to obtain average BER curves with multiple interferers and the desired channel fading characterized as Rayleigh.

When a large number of signals are impinging on the array, each one is arriving from a different direction and they are all affected by distinct scattering fields. The vector of the array's received signals in response to any single transmitted signal is

referred to as the *spatial signature* of the signal. It corresponds to a time-varying, stochastic vector channel associated with the signal. It is the sum of the array's responses to the individual paths carrying energy from the signal source.

Based on the far-field assumption, signals at each element are superpositions of plane waves, each of which obeys the wave equation:

$$s(\mathbf{x}, t) = \text{Re} [Ae^{j(\omega t - \mathbf{k} \cdot \mathbf{x})}] , \quad (2.6)$$

where the vector $\mathbf{k}$ is called the wavenumber vector; its magnitude is equal to the number of radians per meter of length that the monochromatic plane wave exhibits in the direction of propagation [21]:

$$|\mathbf{k}| = \frac{2\pi}{\lambda} = \frac{\omega}{c} \quad (2.7)$$

It can be seen that the planes where $\mathbf{k} \cdot \mathbf{x}$ is constant — planes of constant phase — are perpendicular to vector $\mathbf{k}$. It follows that in a free space environment, the spatial signature of a narrowband signal i is simply a series of phase shifts corresponding to the direction of arrival (DOA) of a single plane wave¹:

$$\mathbf{c}_i = \frac{1}{\sum_{j=1}^N A_j^2(\mathbf{k}_i)} [A_1(\mathbf{k}_i)e^{-j(\mathbf{k}_i \cdot \mathbf{x}_1)}, A_2(\mathbf{k}_i)e^{-j(\mathbf{k}_i \cdot \mathbf{x}_2)}, \dots, A_N(\mathbf{k}_i)e^{j(\mathbf{k}_i \cdot \mathbf{x}_N)}] \quad (2.8)$$

where $A_j(\mathbf{k}_i)$ is the response of array element j to signal i , $\mathbf{x}_n$ is the position in three-dimensional space of the n th array element, N is the number of array elements and $\mathbf{k}_i$ is the wavenumber vector.

In such a case and provided the array topology is known, it is possible to identify and separate up to M distinct signals with M elements. However, two signals whose

¹Note that the spatial signature is defined here as *normalized*, i.e. $|\mathbf{c}_i| = 1$.

DOAs are very close in angle may not be separable. The resolution capability of an array is a function of its spatial extent (aperture). The larger the aperture, the more precise is the array resolving power.

With multipath fading, the situation is slightly different. The spatial signature of a signal corresponds not to a single DOA but to a complex pattern with possibly many lobes of some importance. In principle, the fact remains that an M element array with M combiners (sets of weights) can separate M distinct signals; it can be seen that this amounts to resolving a linear system of M equations with M unknowns (zero-forcing solution). Equivalently, a single combiner is capable of extracting one (desired) signal and nulling $M - 1$ interferers. However, if multipath fading is present, some portion of the array's degrees of freedom will be consumed to combat fading, thus reducing somewhat the interference-suppression capability. Winters, Salz and Gitlin [22] [23] showed that with an array of $K + M$ elements employing optimum combining, $M - 1$ interferers could be nulled out while achieving a $K + 1$ diversity improvement against multipath fading. Furthermore, it was determined in [14] that an optimum combiner first allocates degrees-of-freedom to interference rejection. Leftover DOF's, if any, are employed to combat fading.

Unfortunately, perfect interference nulling is not possible in reality since it would require perfect knowledge of the vector channels of all signals. The optimal achievable SINR μ_0 by a narrowband MMSE linear combining array is given by

$$\mu_0(t) = P_0 \mathbf{c}_0^H(t) \mathbf{R}_{I+N}(t)^{-1} \mathbf{c}_0(t). \quad (2.9)$$

where P_0 is the desired signal's power, $\mathbf{c}_0$ is the desired signal's signature and $\mathbf{R}_{I+N}$ is the interference plus noise covariance matrix which has the following structure

$$\mathbf{R}_{I+N} = \sum_{i=1}^N P_i \mathbf{c}_i \mathbf{c}_i^H + \sigma^2 \mathbf{I} = \mathbf{C} \hat{\mathbf{P}} \mathbf{C}^H + \mathbf{I} \sigma^2, \quad (2.10)$$

where

$$\hat{\mathbf{P}} = \text{diag}(0, P_1, P_2, \dots, P_N). \quad (2.11)$$

It should be noted that the minimum achievable array MSE is equal to the interference-plus-noise power. It follows that the MMSE solution (2.2) is the same, within a scaling factor, as the minimum SINR solution. In fact, there is a third performance criterion, the minimum noise variance (MNV) criterion [31], which also produces the same solution within a scaling factor.

The zero-forcing (ZF) criterion, however, results in a slightly different solution. In zero-forcing, the objective is to bring down to zero—i.e. completely null— all interferers. Therefore, additive noise plays no part in this solution and the optimal weight vector can be expressed

$$\mathbf{w} = \beta \left(\sum_{i=1}^N P_i \mathbf{c}_i \mathbf{c}_i^H \right)^{-1} \mathbf{c}_0. \quad (2.12)$$

Obviously, the ZF and MMSE solutions are equivalent in the noiseless case. While it may seem that the MMSE solution is always better (since it maximizes the SINR), it would appear that ZF solutions often exhibit faster convergence and are less sensitive to errors in the estimation of the covariance matrix (see section 2.6.5).

Since channel and covariance matrix estimates will always be less than perfect, a real system's SINR will always be less than (2.9).

It may not be necessary to completely null interferers in digital wireless communications. A reduction in power of a few dBs might be enough [27] to get a reasonable BER on the desired channel, especially if the array is employed in conjunction with

error-correction coding [24]. In typical cellular systems, one can expect to encounter several interferers whose power is in the vicinity of that of the desired signal (in-cell interferers or first-tier interferers) and many more interferers whose power is an order of magnitude less than the desired signal (out-of-cell interferers or second-tier interferers) [25]. Consequently, the array will have to tolerate more than N interferers [27]; this is possible but the interferers will be only partially nulled and the SINR at the output will suffer a significant degradation [28], [29]. However, even a marginal improvement in SINR can bring forth large increases in the system capacity. Generally, the number of high-power interferers (i.e. interferers whose power is in the vicinity of, or higher than the desired signal) should never exceed the number of nulls even though a larger total number of interferers can be tolerated [28].

2.2 Wideband arrays

In the wideband context, channel vectors and the covariance matrix become functions of frequency as well as time. It is appropriate then that the weight vector also become a function of frequency. This translates in practice to the implementation of a filter / equalizer in each branch instead of a single complex weight or phase shifter. The resulting system is referred to as a *space-time equalizer* or a *space-time processor*.

Two conditions may require the introduction of a wideband array. The first and most obvious case is where the band of interest is wide enough to entail *frequency-selective fading* effects, i.e. time-dispersion with significant energy beyond the symbol period. The second case is where there is sufficient separation between the highest and the lowest frequencies in the desired signal to obtain an incoherent phase relationship from one end of the array to the other [30]. The first condition can be seen as

time spreading significant enough to cause intersymbol interference, i.e. the impulse response has significant energy beyond the symbol duration with high probability. In the second case, the extent in space of the array (its *aperture*) is such that there can be a delay of one symbol duration or more between the signals received at the two ends of the array. It can be seen that this requires wideband operation even if the signals are individually flat fading at each element.

Mathematically the first condition requiring wideband operation can be stated

$$B > B_c, \quad (2.13)$$

where B is the bandwidth occupied by the desired signal, including transmit and receive filters, and B_c is the coherence bandwidth of the wireless channel at the specified carrier frequency. Many different definitions of the coherence bandwidth are employed in the literature. It will be assumed herein that the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9 [1].

The second condition is defined

$$\Delta_a > \frac{c}{B}, \quad (2.14)$$

where Δ_a is the aperture of the array and c is the speed of light. The condition is stated rather conservatively and it should be noted that for the narrowband assumption to hold in a strict fashion, we need $\Delta_a \ll \frac{c}{B}$.

The second condition is of limited practical interest² in scattering environments. Indeed, for any realistic aperture size — i.e. arrays capable of fitting on a single high-

²This condition is included for the sake of completeness and to dispel any confusion that might arise as to what constitutes a proper definition of a wideband array.

rise rooftop — the bandwidth required to trigger the second condition is so large that severe frequency-selective fading effects would also be present. In other words, the first condition is always more restrictive than the second, except in very extreme situations such as *distributed arrays* having aperture sizes of hundreds of meters.

For consistency with the narrowband case, the overall power in the spatial signature will be normalized to unity. Hence

$$\int_{-\frac{B}{2}}^{\frac{B}{2}} \mathbf{c}_i^H(f) \mathbf{c}_i(f) df = 1. \quad (2.15)$$

The optimal array output SINR for a wideband MMSE array with infinite-length filters is expressed [31]

$$\mu_0 = \frac{P_0 \int_{-B/2}^{B/2} [\mathbf{c}_0^H(f) \mathbf{R}_{I+N}^{-1}(f) \mathbf{c}_0(f)]^2 df}{\int_{-B/2}^{B/2} \mathbf{c}_0^H(f) \mathbf{R}_{I+N}^{-1}(f) \mathbf{c}_0(f) df}. \quad (2.16)$$

Analysis and simulation can often be simplified by dividing the band of interest into N_b frequency bins of equal width Δ_b where $\Delta_b \ll B_c$. Since each bin can be considered a flat-fading sub-channel, the optimal SINR can be expressed

$$\mu_0 = \frac{P_0 \sum_{n=1}^{N_b} [\mathbf{c}_0^H[n] \mathbf{R}_{I+N}^{-1}[n] \mathbf{c}_0[n]]^2}{\sum_{n=1}^{N_b} \mathbf{c}_0^H[n] \mathbf{R}_{I+N}^{-1}[n] \mathbf{c}_0[n]}. \quad (2.17)$$

It can be observed that μ_0 as expressed above is a random variable parameterized on the sub-channel correlations between pairs of frequency bins and array elements. Therefore, propagation modeling yielding correlation as a function of space and frequency separations is sufficient to fully characterize the SINR.

Furthermore, the minimum achievable MSE can be expressed as a function of the channel spectra as follows:

$$\varepsilon_L = \sigma_a^2 T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \left[\mathbf{I} + \frac{\sqrt{\tilde{\mathbf{P}}} \mathbf{C}^H(f) \mathbf{C}(f) \sqrt{\tilde{\mathbf{P}}} \sigma_a^2}{N_0} \right]_{11}^{-1} df. \quad (2.18)$$

Since the minimum MSE is proportional to the power of the residual noise and interference at the output of the array receiver, we can relate it to the output SINR. The following is the exact relationship [160]

$$\text{SINR}_{\text{out}} = \mu_0 = \frac{1}{\text{MSE}} - 1. \quad (2.19)$$

2.3 Separability of signals by an array

The resolution of the array becomes less of an issue in multipath fading environments since two very close sources might result in very different spatial signatures due to scattering effects [27]. Regardless of spatial positions, there is a small probability that any two users will have inseparable spatial signatures. The separability of two signals i and j can be quantified by a spatial correlation coefficient — hereafter referred to as the *cross-signature coefficient* — defined as the inner product of their signatures [29]

$$\zeta_{ij} = \mathbf{c}_i^H \mathbf{c}_j \quad (2.20)$$

Likewise, the inner product of a signature with itself ζ_{ii} will be referred to as the *self-signature coefficient* or the *fading coefficient*.

It was also shown in [29] that the output SINR of an optimum combiner is directly related to the set of cross-signature coefficients. In the single interferer case, the SINR μ_0 can be expressed as follows

$$\mu_0 = \frac{P_0}{\sigma^2} \left[\zeta_{11} - \frac{P_1}{\sigma^2 + \zeta_{11} P_1} |\zeta_{01}|^2 \right] \quad (2.21)$$

where P_0 is the power of the desired signal, P_1 is the power of the interferer, σ^2 is the gaussian noise power, ζ_{00} is the desired signal's fading coefficient and ζ_{01} is the cross-signature coefficient between the desired user's channel and the interferer's channel. In the general case of N interferers, the formula becomes

$$\mu_0 = \frac{P_0 |\mathbf{D}_d|}{\sigma^2 |\mathbf{D}|} \quad (2.22)$$

where

$$\mathbf{D} = \begin{bmatrix} \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1N} \\ \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \zeta_{N1} & \zeta_{N2} & \cdots & \zeta_{NN} \end{bmatrix} \quad (2.23)$$

and

$$\mathbf{D}_d = \begin{bmatrix} \zeta_{00} & \zeta_{01} & \zeta_{02} & \cdots & \zeta_{0N} \\ \zeta_{10} & \zeta_{11} & \zeta_{12} & \cdots & \zeta_{1N} \\ \zeta_{20} & \zeta_{21} & \zeta_{22} & \cdots & \zeta_{2N} \\ \vdots & \vdots & \ddots & \vdots & \\ \zeta_{N0} & \zeta_{N1} & \zeta_{N2} & \cdots & \zeta_{NN} \end{bmatrix} \quad (2.24)$$

It should be noted that the above formula was originally derived for non-fading (line-of-sight) environments. It remains valid in a slow fading context as an expression of the SINR averaged over a period shorter than the coherence time of the channel.

The cross-signature coefficient and its relationship to the array SINR constitute useful tools for dynamic channel allocation in reuse-within-cell systems. Indeed, they can be employed to periodically evaluate the *separability* by the array of sets of signals; should two signatures become so correlated that the array can no longer effectively separate them, one of the corresponding signals must be assigned to a different carrier. See [32] and [33] for SDMA protocols based on this concept.

2.4 Other Forms of Linear Diversity Combining

Simpler linear combiners are often considered when some performance penalty is acceptable and / or where cost is a critical constraint. While all these combining schemes are suboptimal in the presence of interferers, their performance may be very close to optimal in certain system contexts.

2.4.1 Maximal Ratio Combining

Maximal Ratio Combining (MRC) is equivalent to optimal MMSE combining when only white noise is present. It follows that in interference-free environments, it is the optimal combiner. Each branch is simply weighted in proportion to its SNR and all branches are co-phased and summed. The main difficulty in implementing the maximal ratio concept in its truest form is the need for coherent reception, i.e. perfect knowledge of the vector channel's amplitudes and phases. This is, however, a somewhat less severe requirement than with optimal combining where both the array covariance matrix and the desired channel must be estimated. Aside from being less computationally intensive than optimum combining (which requires matrix inversion and vector-matrix multiplication), MRC lends itself rather well to analysis in many application contexts (see section 2.6).

It has been observed in [34] and elsewhere that an MMSE equalizer operating in interference-limited environments is equivalent to a *weighted sum of the outputs of all signals' matched filters*. An MRC combiner is really a spatial matched-filter since its output is proportional to the total desired signal energy. Therefore, it can be stated that given a desired signal and N interferers, the optimal MMSE combiner is equivalent to a linear combination of N MRC combiners. Hence

$$\mathbf{w} = \alpha \mathbf{R}^{-1} \mathbf{c}_0 = \sum_{i=0}^N a_i \mathbf{c}_i \quad (2.25)$$

The same principle can be readily extended to the broadband case by incorporating frequency dependence in eq. (2.25).

2.4.2 Equal Gain Combining

Since this scheme simply co-phases and sums up the output of all branches without individual weighting, it does away with the requirement for channel estimation.

It has been shown that for small arrays, the performance of equal-gain combining was reasonably close to that of MRC [6, p. 317] in fading, interference-free environments. In fact, in the limiting case of an infinite number of branches, the difference in average SNR between MRC and EGC is only 1.05 dB.

2.4.3 Selection Diversity

This minimalist diversity strategy consists of selecting only one branch out of the array, the one with the highest SNR. It has the advantage of reducing hardware complexity substantially since only one receiver is required instead of M . However, partial reception circuitry must be implemented on all branches to permit SNR monitoring.

Classical performance results with selection diversity can be found in [6], [35]. In [36], it was pointed out that practical implementations of selection diversity are *not* based on selecting the highest SNR branch but rather the highest *signal-plus-noise* power at the output of a sampled filter, where noise power is a random variable. Results presented therein show improved performance with respect to conventional selection diversity.

In [37], a generalization of selection diversity was proposed whereby the two (three) branches with the two (three) highest SNR are selected and summed (forming a small EGC). It was demonstrated that in some cases, second-order and third-order selection diversity exhibited performance approaching that of MRC or EGC. These methods could constitute an interesting solution for those cases where a level of performance somewhere between selection diversity and equal-gain combining is required.

2.5 Non-linear Diversity Combining

Given the additional information provided by diversity reception, it is generally known that non-linear processing can yield a substantial performance gain compared to linear combining methods. This gain, however, comes at a cost in terms of implementation complexity. Therefore, elaborate diversity receivers such as maximum-likelihood receivers may not be well suited for LMCS-class systems where speed is essential to keep up with the high bit rates. However, simpler non-linear processing (such as decision-feedback space-time processing) might constitute a realistic alternative in the near future. Furthermore, the study of non-linear methods can serve as a benchmark against which to gauge the complexity/performance tradeoffs of various linear combiners.

2.5.1 Maximum-Likelihood Diversity Reception

By definition, the maximum-likelihood receiver is the best possible receiver, assuming equiprobable data. In the interference-free, non-diversity case ($M = 1$), the maximum-likelihood receiver's structure is well-known [38]; a matched-filter followed by a T-spaced sampler, a whitening filter and a maximum likelihood sequence esti-

mator (which can be implemented as a form of Viterbi decoder [39]). Equivalently, the whitening filter can be removed if a modified metric is used in the MLSE [40]. Both derivations show that the output of a matched filter sampled at symbol rate yields a set of *sufficient statistics* to perform optimum reception.

The diversity version of the maximum-likelihood receiver was first derived and studied by Van Etten [41], [42]. He showed that in the presence of CCI and ISI, the outputs of a bank of N *multiple matched-filters* (space-time matched filters), where each filter is matched to one of the signal sources, constitutes a set of sufficient statistics for M-L reception. Each matched-filter output is followed by a symbol-rate sampler before being piped to a vector form of the MLSE (implementable with the vector Viterbi algorithm).

The performance of an M-L receiver is notoriously difficult to assess through analysis. However, an approximate value for the BER in the narrowband case as a function of the channel coefficients can easily be obtained with this formula [11]:

$$P_e \approx e^{-\frac{d_{min}^2 \rho_0}{4}}, \quad (2.26)$$

where ρ_0 is the desired signal SNR at each branch averaged over the fading. In turn, the minimum Euclidian distance d_{min}^2 can be calculated as follows

$$d_{min}^2 = \min_{\substack{\mathbf{a}_n, \hat{\mathbf{a}}_n \\ \mathbf{a}_n \neq \hat{\mathbf{a}}_n}} |\mathbf{C}\mathbf{P}(\mathbf{a}_n - \hat{\mathbf{a}}_n)|^2, \quad (2.27)$$

where $\mathbf{C}$ is the $M \times (N+1)$ normalized channel coefficient matrix, $\mathbf{P}$ is the previously

defined $(N + 1) \times (N + 1)$ matrix expressing the sources' transmitted power and $\mathbf{a}_n$ is an $(N + 1) \times 1$ vector of possible transmitted symbols from all sources. Therefore, $d_{\min}^2$ is obtainable by exhaustively searching through all $\mathbf{a}_n$ and $\hat{\mathbf{a}}_n$.

In both narrowband and wideband systems, a useful and easily obtained bound on the performance of the maximum-likelihood receiver is the *matched-filter bound* [51]. Quite simply, the matched-filter bound is the performance of the system when there is no ISI and no CCI, just additive noise. Obviously, even the best possible receiver can never exceed that level of performance. However, a properly implemented vector M-L receiver should be able to approach (but not necessarily reach) the MFB. Furthermore, the MFB in many system contexts is amenable to analytical treatment.

Many works have resorted to the MFB as a benchmark to establish the relative performance of proposed equalizer structures [52], [53], [54]. Typically, these relied on Monte Carlo methods to obtain the average BER performance over an ensemble of channels. Mazo [55], however, was able to derive analytically the MFB in the single-branch case with two-path Rayleigh fading and AWGN. This result was extended to an arbitrary number of paths by Ling [56] whose work also includes the cases of correlated paths and uncorrelated diversity.

An analytical derivation of the MFB for broadband diversity in uncorrelated Rayleigh fading was provided by Clark et al. [25], [57] based on a Karhunen-Loève expansion of the channel spectra. This derivation has the distinction of not being based on a time-domain model of multiple time-discrete paths, but rather on a more general frequency-domain representation which does not require that each path be considered individually, i.e. computational load is independent of the number of paths if the frequency correlation function is known. Consequently, this form of modeling is adequate for *continuously dispersive* (CD) channels as opposed to *discrete-path* (DP)

channels.

Finally, Yip and Ng [58] have obtained a semi-analytical solution for the MFB of a BPSK communication link in a Rician fading channel. They have considered both the CD and the DP channel types but not diversity nor correlation. Although they provide a closed-form expression for the average BER, it is in the form of an infinite series which can be subject to instability and / or slow convergence when the Rice factor is high. Therefore, they provide an alternative solution based on numerical integration.

2.5.2 Array Decision-Feedback Processing

It is possible to improve the performance of a broadband linear space-time processor without undue complexity by adding a *decision-feedback filter*. Performance is improved because decision-feedback removes post-cursor ISI as long as all decisions are correct. However, DF processing suffers from error propagation as soon as a single detection error is made. Proper use of training sequences is therefore critical to minimize error events.

While linear equalization and decision-feedback equalization [60] were historically used mostly for removing ISI in strictly band-limited channels or channels suffering some form of spectral distortion, they were also found useful in combating cross-talk / cyclostationary interference [62], [63], [64]. In wireless applications, equalizers have been applied to remove ISI induced by frequency-selective multipath fading [61], [62]. In this context, it has been found that the additional power of decision-feedback equalizers was often required because of linear equalizers' sensitivity to spectral nulls.

Most attempts at analytical performance assessments of DF processing assume that all decisions are correct. This removes considerable analytical difficulty since

one can simply remove the post-cursor ISI outright. The seminal work of Salz [60] provides an expression for the minimum achievable MSE by a decision-feedback equalizer operating in an interference-free channel as a function of the channel spectrum:

$$\varepsilon_{DF} = \sigma_a^2 \exp \left\{ -T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \ln \left[\frac{\sigma_a^2}{N_0} P_0 |C_0(f)|^2 + 1 \right] df \right\} \quad (2.28)$$

This work was inspired in turn by earlier publications by Price on zero-forcing DFE [65].

Optimum infinite-length space-time processing based on linear equalization (LE) and decision-feedback equalization (DFE) and the corresponding minimum MSE have been studied by Balaban and Salz [66], [67] and subsequently by Li and Ding [68] for channels with additive gaussian noise only. Notably, Li and Ding rederived the results of Balaban and Salz by showing that a space-time processor was equivalent to a single equalizer (i.e. no diversity) by means of a *single channel equivalent model*.

Numerous works have been published concerning the application of DF space-time processing to channels with both AGN and interference. Petersen and Falconer [63], [64] studied the minimum MSE of one-antenna DFE and LE in strictly band-limited channels. Li, Winters and Sollenberger [69] produced a unified derivation of the minimum MSE for space-time DFE and LE with and without bandwidth restrictions. They proceeded by first obtaining the MMSE for the one-antenna case and used the single channel equivalent model to extend their result to multiple antennas. Also, Ariyavisitakul, Winters and Lee [70] wrote an extensive unified treatment of space-time DFE and LE processing, which also includes the effects of filter lengths. Hence for space-time DF processing in the presence of interferers, we have from [70]

$$\varepsilon_{DF} = \exp \sigma_a^2 \left\{ -T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} \ln [1 + \mathbf{c}_0^H(f) \mathbf{R}_{I+N}^{-1}(f) \mathbf{c}_0(f)] df \right\} \quad (2.29)$$

It is noteworthy that Smee and Beaulieu [71] recently produced a groundbreaking analytical evaluation of the BER of a single-antenna DFE combating noise and ISI only which includes the effects of error propagation. It is most likely that this solution can be readily extended to the multiple antennas case by use of the single channel equivalent model. However, the exact evaluation of the BER in the presence of CCI remains an open problem.

2.5.3 Multi-user Decision-Feedback Processing

Fulghum and Duel-Hallen [73], [74] and also Tidestav et al. [75] proposed the use of cross-feedback filters in multi-user systems to couple decisions from other users into the desired user's DFE thus potentially eliminating some portion of the post-cursor CCI. In [72], [73], different degrees of cross-feedback connectivity were studied via simulation for a quasi-narrowband channel (dispersion limited to 2 symbol periods). Their results show that in a 2-antenna system with a single interferer, a gain of nearly 2.5 dB is achievable by the addition of cross-feedback.

2.6 Evaluating Array Performance

Important performance measures for adaptive arrays such as average BER, average SINR and BER outage probability are most often obtained through Monte Carlo simulation, especially in the broadband case. However, a number of special cases exist for which analytical solutions have been found. Furthermore, the recent progress in the application of multivariate statistical theory to the study of arrays has made fully analytical solutions more widely applicable than ever before.

However, there are always cases when simulation is required, especially when non-

ideal systems are being studied, e.g. imperfect channel estimates, limited number of taps, etc. Typically, idealistic analytical results serve as benchmarks against which to compare simulations in addition to providing unique insights and understanding into the problem at hand.

2.6.1 Efficient Monte-Carlo Simulation

Typical BER-oriented simulations of wireless systems need in the vicinity of 100000 iterations while the study of very low BER systems requires literally millions of iterations or more. Furthermore, taking into account the fact that not only the desired signal undergoes fading but also the interferers', the number of degrees of freedom in the simulation is increased by a factor of N and statistical significance demands an increase in the number of iterations by the same factor. It is therefore crucial to streamline the simulation algorithm in order to get reasonably quick results. If some portions of the system can be simplified through analysis, so much the better.

The simulation problem can be attacked using one of two angles.

1. Start by attempting to implement the actual system in software with the same structure that would be used in reality, i.e. calculating actual tap values, using channel estimation procedures, etc. This approach has the advantage that any reality-based constraint can be easily incorporated (such as limited-precision representation of tap values, nonlinearities in the LNAs or phase noise in the local oscillators). However, both the implementation time and the simulation time are likely to be prohibitive for simple systems and downright unmanageable for complex ones.
2. Implement the *equations* describing the BER obtainable with an *ideal system*

for a particular realization of the channels. An ideal system here entails

- (a) perfect knowledge of channels and array covariance matrix;
- (b) infinite arithmetic precision;
- (c) infinite number of taps in all branches in the broadband case;
- (d) perfectly white, uncorrelated noise;
- (e) zero convergence time in adaptive systems;
- (f) total absence of nonlinearities in the array front-end.

The bulk of the simulation then becomes simply the production of random realizations of channels. It should be noted that by avoiding the consideration of individual transmitted symbols, the result obtained is likely to be approximate unless sophisticated methods are employed to obtain the pdf of the residual interference (see 2.6.4) in which case the elegance and simplicity of this type of simulation are lost.

The two simulation approaches described above will be referred to in the text as α -simulations and β -simulations respectively. The approach favored here is the second one with a few nuances. Initially, simulations are to be implemented for ideal systems using the appropriate analytical formulas. Then the *relative* impact of the channel and covariance matrix estimation methods are studied for incorporation in the initial results. It will be seen that the said impact can be separated nicely from the optimum performance index, leading to some interesting insights. Furthermore, the analytical methods developed herein can easily be extended to deal with finite filter lengths.

Narrowband Simulations

The narrowband case is somewhat trivial since each element of a vector channel is simply a complex coefficient drawn from the appropriate distribution (Rayleigh, Rice or Nakagami [47]). However, in the case of correlated fading, the work load increases substantially since generating correlated variates requires a linear transformation (i.e. a vector-matrix multiplication) at every iteration [43]. For example, if the covariance matrix of a Rayleigh fading vector channel $\mathbf{y}$ is Σ_M and $\Sigma_M^{\frac{1}{2}}$ is its “square root” obtained by Cholesky factorization, then one instance of $\mathbf{y}$ can be obtained from a vector of uncorrelated, zero-mean complex gaussian variates $\mathbf{x}$ as follows

$$\mathbf{y}[n] = \Sigma_M^{\frac{1}{2}} \mathbf{x}[n] \quad (2.30)$$

To simulate Rician fading, the fading components are treated as described above and then added to the fixed component.

Unfortunately, the correlation coefficients are functions of the subscribers’ positions and the scattering environment. A realistic assessment over all possible subscriber mappings in a cell requires that we take into account this additional degree of freedom, again resulting in a corresponding increase in the number of iterations.

Broadband Simulations

The problem is compounded when broadband channels are considered. Many more degrees of freedom are required in the simulation to model frequency-selective fading. The presence of coefficient correlations in channel modeling further complicates the problem. Let us examine this issue in turn for both time-domain and frequency-domain simulations.

If one elects to use time-domain simulation, the most likely channel model is a

tapped-delay line of sufficient length. This approach is appropriate for α -simulations but totally inadequate for idealized infinite-length β -simulations. Even if the widely used *wide-sense stationary, uncorrelated scattering* (WSSUS) [76] assumption is applied, Hoecher [77] has shown that the taps would exhibit correlation due to the effect of transmit and receive filtering. Generating correlated variates at each iteration from (2.30) can become a monumental computational effort if the number of taps is large. Furthermore, it would seem that computing the correlation coefficients between taps in a wideband array requires multidimensional Monte Carlo integration [80] and preliminary trials have resulted in serious numerical instability problems.

In the frequency-domain approach (most appropriate for infinite-length filtering), the spectrum is divided into narrow bins whose width is much smaller than the coherence bandwidth. The bins are then modeled individually as flat-fading, but again their respective coefficients will be correlated. Clark [25] has used a Karhunen-Loève expansion³ to obtain an orthogonal basis for the channel spectra. This allows the simulation to proceed with the generation of uncorrelated variates only. Clark's approach is based on specifying the power delay profile of the channel and, consequently the frequency correlation function (the two form a Fourier-transform pair) and assumes that channels are uncorrelated across branches.

Other works have also proposed using K-L expansions for the purpose of efficient time-domain simulations in non-diversity ($M = 1$) environments [78], [79].

2.6.2 Analytical Results in Interference-Free Systems

In the noise-only case (stationary interference), the distribution of the output SNR of a maximal ratio combiner in flat Rayleigh fading can be found in Jakes [6]. A

³Also known as the *covariance basis* [26].

closed-form expression for the average BER obtained with coherent MRC and incoherent envelope detection combining in Rician fading was published by Lindsey [44] in 1964. Matched filter detection and correlated, binary signalling waveforms were employed in the coherent combiner while the non-coherent combiner was studied in conjunction with M-ary signalling. Subsequently, Lindsey [45] obtained an expression for the average BER of a binary communication link employing equal power, correlated signalling waveforms and incoherent square-law detection in a diversity receiver with Rician fading channels. Abu-Dayya and Beaulieu [46] have derived the pdf of the output SNR and the average BER of an equal-gain combiner operating in slow, non-selective Rician fading for both coherent PSK modulation and non-coherent FSK. In the same work, the possibility of modeling a Rician fading environment by an appropriately chosen Nakagami model is examined and formulas are provided for the parameters of the equivalent Nakagami distribution. A number of more recent analytical results have focused on Nakagami-fading channels [47]. In [48] and [49], expressions were presented for the the average BER for coherent and differentially coherent diversity reception of PSK signals for many specific cases in Nakagami fading. In a similar vein, a method for computing the average BER in non-coherent diversity combining (post-detection) over Nakagami-fading channels is presented in [50].

2.6.3 Analytical Results in Interference-Limited Systems

It is more difficult to find closed-form expressions in the case where co-channel interferers (cyclostationary interference) are present. Furthermore, when such expressions are presented, they are often not exact. One specific assumption that is often used is that the interferers are non-fading. Another commonly-used assumption is the gaussianity of residual interference. The use of the latter assumption arises from the

well-known difficulties involved in dealing with the exact pdf of the interference.

Bogachev and Kiselev [19] in their analysis of optimum combining with a single interferer obtained a closed-form expression for the average BER with coherent detection of PSK by assuming that the interferer is non-fading. Winters [20] carries this assumption forward by reusing Bogachev and Kiselev's result and also treats interferers like noise in his analysis of the SINR of a Maximal Ratio Combiner on the basis that the Rayleigh fading channel vectors are complex gaussian variables.

Some recent works in the area of optimum combining have profited from results and advances in multivariate statistical theory. Shah and Haimovich [82] have shown that the SINR of an optimum combiner subjected to M *equal-power* interferers (where M is greater than or equal to the number of antennas) and in the *absence of noise* follows a scaled version of the *null distribution* of Hotelling's T^2 statistic [81]. Closed-form expressions for the average SINR and the average BER are also provided. The latter is derived from the SINR distribution by assuming gaussianity of the residual interference.

Shah and Haimovich's contribution is based in large part on the Wishart distribution which constitutes a multivariate generalization of the Chi-square distribution. Given a complex, zero-mean multigaussian $M \times N$ matrix⁴ $\mathbf{X}$ whose N columns are independent and identically distributed according to a correlation matrix Σ , its distribution function [84, p. 54] is

⁴The complex multigaussian distribution discussed here assumes that for $\mathbf{X} = \mathbf{A} + j\mathbf{B}$,

$$\langle \mathbf{A}\mathbf{A}^H \rangle = \langle \mathbf{B}\mathbf{B}^H \rangle \quad \langle \mathbf{A}\mathbf{B}^H \rangle = \langle \mathbf{B}\mathbf{A}^H \rangle$$

and was originally studied by Goodman [83].

$$f_{\mathbf{X}}(\mathbf{X}) = \pi^{-NM} |\Sigma|^{-N} \text{etr} [-\Sigma^{-1} \mathbf{X} \mathbf{X}^H], \quad (2.31)$$

where

$$\text{etr} \langle \cdot \rangle = e^{\text{tr} \langle \cdot \rangle}. \quad (2.32)$$

Equivalently, we can express the fact that the $M \times N$ matrix $\mathbf{X}$ follows a complex normal law with covariance matrix σ and mean vector $\mathbf{m}_x$ by the following succinct notation:

$$\mathbf{X} \approx CN_M(N, \sigma, \mathbf{m}_x). \quad (2.33)$$

Assuming that $\mathbf{m}_x = 0$, the matrix $\mathbf{R} = \mathbf{X} \mathbf{X}^H$ is said to follow a complex Wishart distribution with parameter matrix Σ and N degrees of freedom, denoted as $CW_M(N, \Sigma)$. The distribution of $\mathbf{R}$ is expressed

$$f_{\mathbf{R}}(\mathbf{R}) = \frac{|\mathbf{R}|^{N-M} |\Sigma|^{-N} \text{etr} [-\Sigma^{-1} \mathbf{R}]}{\tilde{\Gamma}_M(N)}, \quad (2.34)$$

where the complex multivariate gamma function $\tilde{\Gamma}_M(N)$ is defined as

$$\tilde{\Gamma}_M(N) = \pi^{M(M-1)/2} \prod_{i=1}^M \Gamma(N - i + 1). \quad (2.35)$$

It is also sometimes useful to know the distribution of $\mathbf{R}^{-1}$. By a simple transformation of r.v.'s and knowing that the Jacobian of $f(\mathbf{A} \rightarrow \mathbf{A}^{-1})$ is $|\mathbf{A}|^{M+1}$, we can express the *inverted Wishart distribution* as

$$f_{\mathbf{R}^{-1}}(\mathbf{R}^{-1}) = \frac{|\Sigma|^{-N} \text{etr} [-\Sigma^{-1} \mathbf{R}^{-1}]}{\tilde{\Gamma}_M(N) |\mathbf{R}^{-1}|^{N+1}}. \quad (2.36)$$

In [82], it was recognized that the interference-only covariance matrix of an optimal combiner in a Rayleigh fading environment, having the structure

$$\mathbf{R}_I = \sum_{i=1}^N \mathbf{c}_i \mathbf{c}_i^H \quad (2.37)$$

follows a complex Wishart law if and only if

1. the interferers have equal power;
2. the interferers' vector channels are all identically correlated, i.e. are characterized by the same covariance matrix Σ ;
3. the number of interferers N is greater than or equal to the number of antenna elements M .

It was further assumed in [82] that $\Sigma = \mathbf{I}_M$, i.e. the fading processes are uncorrelated across the array.

A property of Wishart matrices is that if $\mathbf{R}$ is $CW_M(N, \Sigma)$ and $\mathbf{c}$ is any vector then the ratio

$$\frac{\mathbf{c}^H \Sigma^{-1} \mathbf{c}}{\mathbf{c}^H \mathbf{R}^{-1} \mathbf{c}} \sim \chi^2(N), \quad (2.38)$$

is a χ^2 variable with N degrees of freedom and is independent of $\mathbf{c}$ [17], [81]. Now, if we make $\mathbf{c} = \mathbf{c}_0$ a complex gaussian vector (since the desired user's channel is assumed to also undergo Rayleigh fading), then the quadratic form $\mathbf{c}_0^H \Sigma^{-1} \mathbf{c}_0$ is a scaled χ^2 variable, this time with M degrees of freedom. It follows that the SINR $\mu_0 = \mathbf{c}_0^H \mathbf{R}^{-1} \mathbf{c}_0$ is a scaled ratio of χ^2 variables and its distribution corresponds to a scaled version of the Snedecor F-distribution [81] (and in this particular case also corresponds to a scaled null distribution of the Hotelling T^2 statistic) and is given [82]:

$$f_{\mu_0}(\mu_0) = \frac{\Gamma(N+1)}{\Gamma(M)\Gamma(N+1-M)} \left(\frac{P_0}{P}\right)^{N+1-M} \frac{\mu_0^{M-1}}{\left(\frac{P_0}{P} + \mu_0\right)^{N+1}}. \quad (2.39)$$

While this appears to be a promising result, it is noteworthy that it does not submit itself readily to generalization to include white noise, unequal power interferers and / or fading correlation, at least not without altering the nature of the Wishart distribution itself. Actually, it is possible to extend it to include desired channel fading correlation and even interferer fading correlation (as long as all interferers are identically correlated) but such a result by itself is of limited practical interest if white noise cannot be accounted for.

In a related work by Gao and Smith [85], [86], the Wishart density was bypassed completely in order to take advantage of an advanced statistical result by Kathri [88] which directly (with some simplifications and scaling) provides a closed-form for the distribution of the SINR in the interference-only case but with unequal power interferers:

$$f_{\mu_0}(\mu_0) = \tilde{\Gamma}_M(1+N) \{\tilde{\Gamma}_M(N) \tilde{\Gamma}_1(M)\}^{-1} |\mathbf{P}|^{-M} q^{M(1+N)} \mu_0^{M-1} (1 + q\mu_0)^{-(1+N)} \\ \times {}_1F_0(1+N; \mathbf{T}, \mathbf{S}) \quad (2.40)$$

where the matrix $\mathbf{P}$ is the matrix of normalized interferer powers

$$\mathbf{P} = \text{diag} \left(\frac{P_1}{P_0}, \frac{P_2}{P_0}, \dots, \frac{P_n}{P_0} \right), \quad (2.41)$$

and $\tilde{\Gamma}_M(a)$ is the complex multivariate gamma function as defined in (2.35) and $q > 0$ is an arbitrary constant. The difficulty in evaluating this expression stems from the presence of the ${}_1F_0(\cdot)$ term which is an hypergeometric function with two matrix arguments $\mathbf{T}$ and $\mathbf{S}$ defined as

$$\mathbf{T} = \mathbf{I}_N - q\mathbf{P}^{-1}, \quad (2.42)$$

$$\mathbf{S} = \text{diag}((1 + q\mu_0)^{-1}, \mathbf{I}_{M-1}). \quad (2.43)$$

Hypergeometric functions of matrix arguments constitute a long-standing analytical device used by statisticians [81], [88] whose definition involves infinite sums of *zonal polynomials* [81], [89]. Until recently, they have been rather difficult to evaluate numerically. Gao and Smith in [85], [86] have taken advantage of an alternate formulation by Gross and Richards [90] for these functions which relates them to classical hypergeometric functions. Through a series of elaborate mathematical manipulations involving evaluation of a limit by repeatedly applying L'Hôpital's rule and Laplace's expansion theorem, they have been able to express the desired distribution in the form

$$f_{\mu_0}(\mu_0) = \sum_{i=1}^N \alpha_i (1 + \mathcal{P}_i \mu_0)^{-2}, \quad (2.44)$$

where

$$\mathcal{P}_i = \frac{P_i}{P_0}, \quad (2.45)$$

and

$$\alpha_i = (-1)^{N-M} \mathcal{P}_i^{N-M+1} \sum_{\substack{1 \leq i_1 < \dots < i_{M-1} \leq n \\ i_j \neq i}} \mathcal{P}_{i_1} \mathcal{P}_{i_2} \dots \mathcal{P}_{i_{M-1}} \left[\prod_{j \neq i} (\mathcal{P}_j - \mathcal{P}_i) \right]^{-1}. \quad (2.46)$$

To simplify the computation of the coefficients, a recursion procedure was employed in [85]. However, if any two interferers have nearly equal power, this formulation is subject to numerical instability.

In a subsequent work, Smith, Gao and Clark [87] explored the possibility of incorporating the effect of noise by observing that white gaussian noise is equivalent to an infinite number of weak, uncorrelated interferers. In [85] and [86], special cases of (2.44) were considered and it was proven that the reliability function (i.e. the probability that the SINR is greater than a threshold; the cdf of $-\mu_0$) when there is a finite number K of weak interferers can be expressed

$$F_{-\mu_0}(x) = P(\mu_0 \geq x) = \frac{\sum_{i=0}^{M-1} D_i^{(K)} x^i}{\prod_{j=1}^N (1 + \mathcal{P}_j x)}, \quad (2.47)$$

where $D_i^{(K)}$ is the coefficient of x^i in $\prod_{j=1}^N (1 + \mathcal{P}_j x)$. This formulation has the advantage of tolerating equal power interferers and was used as the basis for the equivalent interference model of additive gaussian noise. This was done by assuming that there are K interferers with power $P_i = \frac{\sigma^2}{K}$ and normalized power $\mathcal{P}_i = \frac{\sigma^2}{K P_0}$ and letting K tend towards infinity. Thus, we have

$$\lim_{K \rightarrow \infty} \prod_{j=1}^{K+N} (1 + \mathcal{P}_j x) = \lim_{K \rightarrow \infty} \left(1 + \frac{\sigma^2 x}{K P_0}\right)^K \prod_{j=1}^N (1 + \mathcal{P}_j x), \quad (2.48)$$

where N is the number of real interferers. It can be seen that this limit is of the well-known form [91]

$$\lim_{k \rightarrow \infty} \left(1 + \frac{a}{k}\right)^k = e^{-a}. \quad (2.49)$$

By invoking the limit and substituting in (2.47), we obtain

$$F_{-\mu_0}(x) = e^{-\frac{\sigma^2 x}{P_0}} \sum_{i=1}^M \frac{A_i(x)}{(i-1)!} \left(\frac{\sigma^2 x}{P_0} \right)^{i-1}, \quad (2.50)$$

where

$$A_i(x) = \begin{cases} 1, & M \geq N + i \\ \frac{1 + \sum_{j=1}^{M-i} C_j x^j}{\prod_{n=1}^N (1 + P_n x)}, & M < N + i \end{cases} \quad (2.51)$$

and

$$C_i = \sum_{1 \leq l_1 < \dots < l_i \leq n} P_{l_1} P_{l_2} \dots P_{l_i}. \quad (2.52)$$

With no interferers, one can obtain from (2.51) the well-known pdf of the SNR of maximal ratio combining [6, p. 319], [16, p. 781]:

$$f_{\mu_0}(\mu_0) = -\frac{dF_{-\mu_0}(\mu_0)}{d\mu_0} = \frac{\mu_0^{M-1} e^{-\frac{\mu_0 \sigma^2}{P_0}}}{\left(\frac{\sigma^2}{P_0} \right)^M (M-1)!}. \quad (2.53)$$

Winters, Salz and Giltin [14] employed another approach to obtain the pdf of the MSE of a narrowband array under the assumption of infinite signal-to-noise ratio (noiseless case). Their derivation is based on formula (2.18) and makes use of determinant theory to prove that the MSE is chi-square distributed with M degrees of freedom when all signatures are uncorrelated and have equal power.

2.6.4 BER evaluation and the distribution of residual interference

While it is in most cases relatively easy to obtain the average and the distribution of the MMSE (and, equivalently, the SINR), obtaining the average BER is more problematic. That is because calculating the BER in an exact fashion requires knowledge

of the distribution of the residual interference which is a function of the specific symbols transmitted by all sources.

Let us first consider the equalized and combined channel coefficient expressed as

$$d_n[k] = \int_{-\infty}^{\infty} \sqrt{Q_n} \mathbf{w}^H(f) \mathbf{c}_n(f) e^{j2\pi f k T} df \quad (2.54)$$

where k is the discrete time index (assuming symbol rate sampling) and n identifies the signal source ($n = 0, \dots, N$) where $n = 0$ denotes the desired user. Without loss of generality, we can assume that the tap coefficients are scaled to satisfy $d_0[0] = 1$. Let us also define the interfering coefficients

$$\tilde{d}_n[k] = \begin{cases} 0, & \text{if } n = 0 \text{ and } k = 0 \\ d_n[k], & \text{elsewhere} \end{cases} \quad (2.55)$$

In a QPSK or BPSK system the BER is expressed

$$P_e = \int_0^{\infty} p_I(x) \left\{ Q\left(\frac{1-x}{\sqrt{\sigma^2}}\right) + Q\left(\frac{1+x}{\sqrt{\sigma^2}}\right) \right\} dx \quad (2.56)$$

where the distribution of the residual interference $p_I(x)$ is symmetrical with zero mean and σ^2 is the variance of the noise. In the broadband case and assuming static channels, the residual interference I is a sum of discrete r.v's:

$$I = \text{Re} \left[\sum_{n,k} x_n[k] \tilde{d}_n[k] \right], \quad (2.57)$$

with a mean of zero and a variance given by

$$\sigma_I^2 = \sigma_a^2 \sum_{n,k} \left(\text{Re} [\tilde{d}_n[k]] \right)^2 + \sigma_b^2 \sum_{n,k} \left(\text{Im} [\tilde{d}_n[k]] \right)^2 \quad (2.58)$$

where σ_a^2 is the average symbol energy before transmitter amplification in the in-line rail and σ_b^2 is its counterpart in the quadrature rail. For BPSK and QPSK, σ_a^2 and σ_b^2 are equal and can be assumed to be equal to unity without loss of generality.

The naive (exhaustive) approach to computing $p_I(x)$ is computationally demanding since it needs to be calculated over the ensemble of possible transmitted symbols⁵ and the ensemble of channel responses. For this reason, various approximations and bounds are often useful. Here is a representative sample of these approximate BER calculation methods, listed in order of increasing precision:

1. Saltzberg's bound

This simple bound [92] is employed heavily in the literature [25], [52], [54], [66], but typically is in error by a factor of 2 to 10 [93]. The Saltzberg bound is expressed

$$P_e \leq \frac{1}{2} \exp \left[-\frac{1}{2(\sigma_I^2 + \sigma_n^2)} \right]. \quad (2.59)$$

2. Assumption of gaussianity

By assuming that residual interference follows a gaussian distribution, the simple BER formulas used for AWGN channels become applicable. For example, with QPSK modulation we have:

$$P_e \approx Q \left(\sqrt{\frac{1}{\sigma_I^2 + \sigma_n^2}} \right). \quad (2.60)$$

In all likelihood, this approximation is closer to the exact value when the number of interferers is large and / or when the BER is high (in the vicinity of 10^{-3}).

⁵If the nominal channel length is L symbols, the alphabet size is A and the number of signals is N , there are A^{NL} combinations of interest.

For static channels, the interference pdf is typically close to binomial [71] and therefore can be closely approximated by a gaussian distribution [94]. However, there are cases where the interference pdf becomes quite intricate and does not resemble a gaussian distribution at all [96]⁶. Furthermore, the gaussian pdf has infinite tails while the interference pdf is peak-limited. Since the tails of the distribution are critical in BER calculations (especially with relatively low BERs, i.e. $\leq 10^{-4}$), the accuracy of the gaussian approximation can be notably improved by truncating the tails (see below).

3. Assumption of gaussianity with peak limiting

Given a single realization of the channels, the constructive addition of all interfering channel coefficients yields the maximum possible value of the interference term at the receiver input. This constitutes an appropriate truncation point for the interference pdf. Hence

$$p_I(x) = \begin{cases} C_I \exp \left[-\frac{1}{2} \left(\frac{x}{\sigma_I} \right)^2 \right], & |x| \leq I_p, \\ 0, & |x| > I_p. \end{cases} \quad (2.61)$$

where

$$C_I = \left\{ \sqrt{2\pi}\sigma_I \left[1 - 2Q \left(\frac{I_p}{\sqrt{2\sigma_I^2}} \right) \right] \right\}^{-1} \quad (2.62)$$

and the truncation point is

⁶It would appear that the binomial assumption is valid for extremely slowly decaying channels with hundreds of symbols contributing to the ISI [96] or, equivalently, for very large numbers of interferers of roughly equal power. Neither case is an accurate universal representation of practical systems.

$$I_p = A_I \sum_{n,k} |\text{Re}(\tilde{d}_n[k])| + A_Q \sum_{n,k} |\text{Im}(\tilde{d}_n[k])| \quad (2.63)$$

where A_I is the maximum amplitude of the constellation on the real axis while A_Q is the maximum amplitude on the imaginary axis.

Results presented by Clark [25] suggest that this method is very precise in many practical contexts at least for typical BERs ($10^{-3} - 10^{-5}$).

4. High-precision methods

The more accurate methods are generally both algorithmically complex and computationally demanding. A number of such methods are currently in use and assessing their respective merits is a daunting task. A comparative evaluation in terms of accuracy and complexity can be found in a paper by Helstrom [95]. Many such methods aim to calculate directly an approximate BER rate and bypass the pdf of the interference itself (although it is implicitly part of the derivation) [93], [95]. Exceptions include Metzger's algorithm [96] which approximates the interference pdf by a series of weighted Dirac pulses. The precision of this algorithm is a function of the chosen number of pulses used in approximating the pdf. Its computational effort is only proportional to the number of symbols $2L$ contributing to the ISI⁷. It was successfully employed by Clark [25] (to obtain results for broadband diversity in the presence of CCI and ISI) who found by testing that its accuracy was within 1% of the exact average BER.

Another approach which seeks to estimate directly the pdf of the interference is

⁷the exhaustive approach requires an effort proportional to M^{2L} where M is the number of points in the signalling constellation.

the method of moments as presented by Kavehrad [97]. It has the advantage of providing a given degree of precision using much fewer moments than the older Gauss Quadrature Rules (GQR) method [98] (as applied to BER calculations by Benedetto et al. [99]). The method of moments was successfully employed by Lo in the study of broadband equalization in the presence of CCI and ISI [100]. However, it is stated in [97] that this method loses much of its efficiency if the pdf of the interference is spiky. It is of interest, then, that Metzger [96] and previously Hill and Blanco [101] found cases where the pdf is very intricate and spiky.

In one of the latest works on this topic, Beaulieu [93] expresses the bit error rate as an infinite series. His approach is based on expressing the noise complementary pdf as an exact or approximate Fourier series. He provides results for ISI, CCI and both as well as analytical bounds on the accuracy of the estimate.

2.6.5 Impact of training methods

As will be seen later, the system context studied herein is a *packet-based* digital wireless communication system. Furthermore, it will be demonstrated that the length of a packet is likely to be much smaller than the coherence time of the channel. In other words, a vector channel can be assumed fixed over the duration of a packet. Hence, packets from a given user will rarely arrive in consecutive slots but rather will be dispersed in time with a significant interarrival delay. This entails that a mechanism must be provided to estimate the array covariance matrix and the desired user's channel in every packet. Furthermore, no tracking capability is required since the estimates can be assumed valid for the duration of the packet. Therefore, we will be concerned exclusively with *block adaptation* methods (as opposed to symbol-by-

symbol tracking) where a block is assumed to correspond to a packet.

The preferred estimation mechanism will be the use of a known sequence of training symbols to be transmitted with every packet. Presumably, it is possible to get rid of training sequence overhead by using a *blind adaptation* algorithm. However, these methods are not always robust, generally complex, and most have prohibitive convergence times. Many algorithms used in blind adaptation and blind estimation require some structure in the signal such as constant modulus (e.g. CMA), narrow angle spread (e.g. ESPRIT, MUSIC) or slightly different carrier frequencies [102]. Consequently, those algorithms often break down in a multipath fading environment. While recent developments in *parametric array processing* have addressed the problem of coherent signals (which arise as a result of multipath in most wireless systems), the field is not yet considered mature as many issues of practical importance remain to be resolved. For example, parametric methods require a multidimensional search and therefore typically have a high computational cost. Other signal structure-based approaches, exploiting higher-order statistics (e.g. fourth-order cumulants [103], [104]) or high-order cyclic spectra of cyclostationary signals [105] require a considerable amount of data in order to yield a reliable result⁸

LMCS systems probably constitute a good testing ground for some form of blind adaptation because the subscribers are static. However, implementation of blind methods in the near future seems unreasonable for a number of reasons.

⁸It should be noted that the references cited in this discussion are with respect to blind DOA estimation methods as opposed to blind beamforming. However, the two problems are *equivalent* as noted in [119]; indeed, knowledge of the DOAs of all impinging signals plus knowledge of noise power is sufficient to form an optimal beam in the mean-square sense or the zero-forcing sense. It is, however, possible to form an optimal beam without explicit knowledge of DOAs; only the spatial signature of the desired signal and the array covariance matrix are required. Most blind methods rely on some geometric argument and must therefore estimate all DOAs to compute spatial signatures and the array covariance matrix.

1. Since we are in the early stages of LMCS development, the excess bandwidth required by training data is readily available and hence the usual argument in favor of blind methods — namely, spectrum efficiency — does not yet apply. Furthermore, training symbols are generally needed for synchronization purposes anyway and can thus also serve for array adaptation.
2. Unlike some first generation cellular standards, the LMCS protocols and developing standards can easily be adapted to provide training sequences in every packet.
3. Since LMCS operates at relatively high bit rates, adaptation algorithms should have minimal complexity. Unfortunately, blind adaptation algorithms are characterized by a much higher complexity than their non-blind counterparts.
4. In most cases, it is not clear how a blind algorithm performs if the number of signals is close or equal to the number of antenna elements.
5. As discussed above, the field of blind adaptation is still relatively young; indeed, it is said in [106] about blind array processing methods applied to personal communications:

It is important to note that the various optimal and/or suboptimal proposed methods are of current research interest, and that in practice, many challenging and interesting problems remain to be addressed.

Therefore, although blind adaptation is a wide and active area of research, we must conclude that it does not seem a promising avenue for our goals and nothing more will be said about it here.

In selecting an appropriate training / adaptation scheme, it is important to keep in mind some system aspects specific to the LMCS / fixed-subscriber context:

1. Although the channels undergo significant fading effects [7], the fading rate is likely to be much less than in a mobile system.
2. Since the subscribers are fixed, the covariance matrix of a spatial signature (averaged over the small-scale fading) will be almost static in most cases. This makes LMCS systems good candidates for subspace methods⁹ since the underlying eigenstructure of the channels changes very slowly. In other words, the increased complexity of subspace methods can be justified because the required weight update frequency is so low.
3. In systems like these, the main impediment that an array should combat is interference and more specifically changes in the *interfering traffic patterns* as interfering users get on and off the network at random times. More information is readily available about in-cell interferers since the slots and channels that they use are assigned by the base station itself. Moreover, some degree of synchronization among intra-cell users is likely. Out-of-cell interferers, however can start transmitting at any odd time, i.e. they are not synchronized with the base station of interest. Furthermore, the power of out-of-cell interferers at the receiver can be in the vicinity of that of the desired signal due to shadowing effects and the full reuse of carriers in neighboring cells¹⁰. Whenever out-of-cell interferers arrive in an asynchronous fashion and are therefore not present for

⁹This statement pertains to subspace decompositions performed on the medium-term covariance matrix (averaged over the short-term fading) and assumes that multipath fading is tracked by conventional, i.e. non-subspace, means.

¹⁰All major LMCS proposals suggest a frequency reuse factor of 1.

the whole duration of the packet, they will be referred to as *part-time interferers*.

In a multibeam array where several packets must be received simultaneously and effectively separated, three major factors will affect performance relative to the optimal MMSE solution:

1. The length and structure of the training sequence. Desirable properties for a family of training sequences include low autocorrelation at delays other than zero and low crosscorrelation among themselves. The latter is desirable to allow resolution of many packets simultaneously (which requires all resolved packets to use different training sequences).
2. The position of the training sequence. It can be situated at the beginning, in the middle or at the end of a packet. In a system which uses storage to process the whole packet at once, the training sequence need not be in the preamble. One could argue, however, that if the channels are fixed for the duration of the packet, it does not matter where the training sequence is located. However, if there is a slight variation in the channels over the length of a packet, placing the training sequence in a midamble will be preferable to either a preamble or a postamble. Furthermore, it might be important to take into account the presence of part-time interferers which affect only a portion of the packet. If the array is to null such interferers effectively, the training sequence would have to lie at least in part in the region of the packet affected by the part-time interferer. Again, a midamble would be better than either a preamble or a postamble although the best choice for combating part-time interferers would be a distributed training sequence, i.e. training symbols spread evenly at regular intervals throughout the packet.

3. The method of training, i.e. how the training information is processed to yield a set of array weights.

To evaluate the impact of training and estimation with respect to the optimum ideal solution, one parameter of importance is the *normalized SINR* ρ_0 , i.e.

$$\rho_0 = \frac{SINR_{real}}{SINR_{ideal}} \quad (2.64)$$

Unless otherwise stated, the normalized SINR does not include the effect of estimating the desired signal signature.

In the discussion to follow about training and estimation schemes, it will be assumed that K symbols in the packet are known and constitute a training sequence. The training symbols may or may not have some desirable statistical properties.

Here is a list of relevant training methods in increasing order of sophistication (approximately):

1. **Sample Matrix Inversion (SMI)**¹¹ The expression “sample matrix” in the name of this method simply refers to the estimation of the covariance matrix by averaging over a series of K received sample vectors (or *snapshots*), i.e.

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^K \mathbf{x}_k \mathbf{x}_k^H. \quad (2.65)$$

The desired users’ signature must also be estimated with the help of the known training sequence $\mathbf{p}$ of length K . The simplest estimation method is to simply correlate the received snapshots with the corresponding training symbols, i.e.

¹¹Also known as Direct Matrix Inversion (DMI)

$$\hat{\mathbf{c}}_0 = \frac{1}{K} \sum_{k=1}^K p_k \mathbf{x}_k. \quad (2.66)$$

The SMI weight vector solution is then simply

$$\mathbf{w}_{SMI} = \alpha \hat{\mathbf{R}}^{-1} \hat{\mathbf{c}}_0 \quad (2.67)$$

While the sample matrix can be derived using more than K snapshots (since it does not require knowledge of the transmitted symbols), both the sample matrix and the desired signature *must* be estimated using the same set of snapshots. Indeed, even though the sample matrix could be made more precise, the fact that its estimation is inconsistent with the estimation of the signature leads to a significantly worse solution in (2.67). Hence, it is not possible to use more than K snapshots in estimating both the covariance matrix and the signature (at least not in a trivial way).

It is well known that if the number of training snapshots K is less than the number of interferers and / or the number of interferers is much less than the number of elements, the sample matrix is likely to be *singular* in which case the solution (2.67) doesn't exist. However, this potential problem can be circumvented by using the Moore-Penrose pseudo-inverse $\hat{\mathbf{R}}^+$ instead of the conventional inverse $\hat{\mathbf{R}}^{-1}$. Hence

$$\mathbf{w}_{GSMI} = \alpha \hat{\mathbf{R}}^+ \hat{\mathbf{c}}_0 = \alpha (\mathbf{R}^H \mathbf{R})^{-1} \mathbf{R}^H \hat{\mathbf{c}}_0 \quad (2.68)$$

Since $\mathbf{R}^+ = \mathbf{R}^{-1}$ whenever $\mathbf{R}$ is nonsingular, this solution can be applied indiscriminately and will be termed henceforth *generalized sample matrix inversion*

(GSMI). However, GSMI requires two additional matrix-matrix multiplication operations with respect to SMI.

The normalized SINR of SMI is given

$$\rho_{SMI} = \frac{\left[\mathbf{c}_0^H \hat{\mathbf{R}}_{I+N}^{-1} \mathbf{c}_0 \right]^2}{\mathbf{c}_0^H \hat{\mathbf{R}}^{-1} \mathbf{R}_{I+N} \hat{\mathbf{R}}_{I+N}^{-1} \mathbf{c}_0} \frac{1}{\mathbf{c}_0^H \mathbf{R}_{I+N}^{-1} \mathbf{c}_0} \quad (2.69)$$

As a classical result, Reed et al. [107] found that the normalized SINR of SMI is beta-distributed with parameters $(2(K - N + 2), 2(N - 1))$. Also, the expected normalized SINR is $\langle \rho_{SMI} \rangle = \frac{K-N+2}{K+1}$.

2. Loaded Sample Matrix Inversion (LSMI)

From the above formula for the average normalized SINR $\langle \rho \rangle$, it can be shown that $K = 2N - 3$ snapshots are required to reach the 3dB loss point with SMI. For large arrays, the number of snapshots $K \approx 2N$ can become prohibitive. Its slow convergence rate is due to the important fluctuations of the noise related eigenvalues which distort the adapted pattern. To circumvent this limitation, Gabriel [108], Cox et al. [109] and Carlson [110] as well as others have suggested adding a small constant to the diagonal elements of the covariance matrix prior to inversion. Hence

$$\hat{\mathbf{w}}_{LSMI} = \left[\hat{\mathbf{R}}^{-1} + \alpha \mathbf{I} \right] \mathbf{c}_0 \quad (2.70)$$

where the loading constant α has the effect of compressing the smallest eigenvalues (the noise eigenvalues) in the inverted matrix and of limiting the range of their fluctuations. One immediate benefit of this method is the fact that the diagonal loading will make a singular covariance matrix invertible.

Several authors have studied the performance of LSMI (eq. (2.70)) through simulations [110], [111], [112]. A lesser-known analytical study of LSMI was produced by Cheremisin [113]. He has shown that for a number of snapshots K greater than the number of interferers M , the normalized SINR is beta-distributed with parameters $(2(K-M+1), 2M)$. Therefore, the pdf is expressed

$$f_{\rho}(\rho) = \frac{\Gamma(K+1)}{\Gamma(K-M+1)\Gamma(M)} \rho^{K-M} (1-\rho)^{M-1} \quad (2.71)$$

where $0 < \rho \leq 1$ and the diagonal load meets the condition $\sigma^2 < \alpha \leq \lambda_M$ where λ_M is the smallest interferer eigenvalue. From this, the expected normalized SINR is found to be $\langle \rho_{LSMI} \rangle = \frac{K-M+1}{K+1}$. Therefore, the 3dB loss point can be reached with only $K_{3dB} = 2M - 1$ snapshots which constitutes in certain cases a huge improvement on SMI. A remarkable aspect of this result is that, provided the loading meets the aforementioned condition, the SINR does *not* depend on the number of elements N . So, while the number of snapshots required to achieve a given level of performance with SMI is linearly proportional to N , with LSMI it is independent of N but linearly proportional to M .

While LSMI seems to provide greatly enhanced performance at almost no cost in complexity (simply adding the loading term to the diagonal), some quipros apply. Indeed, optimal performance demands that the condition on the loading term be met. In some systems, this will not be a problem since the number of interferers will be known and their power level will be much higher than the thermal noise level. It is then simply a matter of determining the typical noise power at the receiver and setting the load α just slightly higher. In more unpredictable contexts, however, it may be necessary to perform an

eigendecomposition on the estimated covariance matrix to estimate the values of σ^2 and λ_M . Needless to say, this augments significantly the complexity of the training procedure.

Furthermore, the pdf of ρ suggests that the performance of LSMI is only better than SMI for $M < N$. In fact, if $M \geq N$ (overloaded operation) the loading procedure is likely quite useless because no individual noise eigenvalues will show up in the covariance matrix.

3. Extended Training Sample Matrix Inversion (XTSMI)

As discussed above under the SMI heading, the estimation of the covariance matrix does not depend on the length of the training sequence and could theoretically be made over the entire duration of a packet. However, since the desired signature vector can only be estimated over the known training sequence, this would result in inconsistent estimates and would actually make performance worse in spite of the improved precision of the covariance matrix.

Wang and Poor [114] have recently proposed an algorithm based on eigenanalysis which extends the estimation of the desired signature vector over the entire length of the packet. It can be employed with the covariance matrix estimated over the entire packet without provoking statistical consistency problems.

The derivation of this algorithm proceeds from an elementary observation. For a narrowband array receiving a known training sequence $\mathbf{p} = (p_1, p_2, \dots, p_K)$, the i th symbol sent can be estimated at the receiver by

$$p_i \approx \mathbf{x}^H[i] \mathbf{w}, \quad (2.72)$$

where $\mathbf{x}[i]$ is the array received vector at time index i and

$$\mathbf{w} = \hat{\mathbf{R}}^{-1} \hat{\mathbf{c}}_0 \quad (2.73)$$

where $\hat{\mathbf{R}}$ and $\hat{\mathbf{c}}_0$ are estimated from the K training symbols in accordance with (2.65) and (2.66). Generalizing to the entire training sequence we have

$$\mathbf{p} \approx \Gamma^H \mathbf{w} = \Gamma^H \hat{\mathbf{R}}^{-1} \mathbf{c}_0, \quad (2.74)$$

where

$$\Gamma = [\mathbf{x}[1], \mathbf{x}[2], \dots, \mathbf{x}[K]]. \quad (2.75)$$

Since noise contributes nothing to the estimation process and will rather diminish accuracy, we then replace $\hat{\mathbf{R}}^{-1}$ by its eigendecomposition but without noise. Hence

$$\hat{\mathbf{R}}^{-1} \approx \mathbf{U}_s \Lambda_0^{-1} \mathbf{U}_s^H, \quad (2.76)$$

where $\mathbf{U}_s$ is the matrix of signal eigenvectors and

$$\Lambda_0 = \text{diag}[\lambda_1 - \sigma^2, \lambda_2 - \sigma^2, \dots, \lambda_M - \sigma^2] \quad (2.77)$$

where $\lambda_1, \dots, \lambda_M$ are the M largest eigenvalues of R . Multiplying both sides of (2.74) by $(\mathbf{U}_s^H \Gamma \Gamma^H \mathbf{U}_s)^{-1} \mathbf{U}_s^H \Gamma$ and using (2.76), we get

$$(\mathbf{U}_s^H \Gamma \Gamma^H \mathbf{U}_s)^{-1} \mathbf{U}_s^H \Gamma \mathbf{p} = \Lambda_0^{-1} \mathbf{U}_s \hat{\mathbf{c}}_0 \quad (2.78)$$

which after minor algebraic manipulations is equivalent to

$$\hat{\mathbf{c}}_0 = \mathbf{U}_s \mathbf{\Lambda}_0 (\mathbf{\Gamma}^H \mathbf{U}_s)^+ \mathbf{p} \quad (2.79)$$

This estimate of the signature is based on an eigendecomposition of the covariance matrix estimated over the entire packet. It can be used advantageously to form the Wiener weight vector in the usual way (SMI) in conjunction with an estimate $\hat{\mathbf{R}}$ also taken over the entire packet. It was shown in [114] that a substantial gain could be incurred by XTSMI over standard SMI. In their simulation, they used the IS-54/136 slot length of 162 symbols, 14 training symbols and BPSK modulation. With 10 antenna elements and 5 interferers, each of which being 6 dB below the desired signal, the BER curve of XTSMI is slightly closer to the exact MMSE curve than it is to the SMI curve. The paper also extends the algorithm to the wideband (ISI) case but simulations seem to show that the improvement with respect to SMI is not as great as in the narrowband case.

4. Eigenvector Projection (EVP)

The underlying concept of the eigenvector projection method is the separation of the observation space into an interference and a noise subspace. The weights are then found by projecting the received signal vectors into the noise (interference-free) subspace. Two fundamental characteristics of this method are

- (a) the noise power does not play a role in computing the solution;
- (b) the desired signal is filtered out before computing the weight solution.

In other words, this algorithm is focused on the interferers and attempts to null them completely, thus tending towards the zero-forcing optimal solution.

The desired signal signature must be estimated still because as explained in [115], it is required by the preprocessor which filters out the desired signal from the data vectors. Once this step is performed the filtered data vectors are used to estimate the interference-plus-noise covariance matrix. An eigendecomposition follows revealing the following structure

$$\hat{\mathbf{R}}_{I+N} = \mathbf{U}\mathbf{A}\mathbf{U}^H = \mathbf{S}\mathbf{S}^H + \sigma^2\mathbf{N}\mathbf{N}^H \quad (2.80)$$

where the columns of $\mathbf{S}$ are M eigenvectors spanning the interference subspace and the columns of $\mathbf{N}$ are the noise eigenvectors. The eigenvectors $\mathbf{S}$ are used to form a projection matrix corresponding to the space *orthogonal* to the interference subspace, i.e.

$$\mathbf{P}_S^\perp = \mathbf{I} - \mathbf{S}\mathbf{S}^H. \quad (2.81)$$

The projection matrix is then employed to obtain the weight vector

$$\mathbf{w}_{EVP} = \mathbf{P}_S^\perp \mathbf{c}_0 \quad (2.82)$$

Empirical evaluations of the performance of the EVP as an interference canceling method have been presented in [115], [116], [117]. In addition, Gierull [118] has shown that in the infinite interference-to-noise ratio (i.e. the noiseless case), the pdf of the normalized SINR of EVP is exactly the same as LSML. Furthermore, even if the INR is finite, the pdf is well approximated by (2.71) except if the INR is very low (≤ 5 dB) and the number of snapshots is very small (≤ 6).

The fact that the performance of EVP is for all practical intents and purposes the same as LSMI stems from the fact that both methods reduce the impact of noise on the weight vector solution. In other words, they both tend towards the zero-forcing solution. There is a nuance, however, in that the LSMI method does not get rid of noise completely but rather reduces its impact. EVP, on the other hand, concentrates on the interference-only subspace and noise's influence is felt only because the estimation of the interference subspace is imperfect. This explains why, when the number of snapshots $K = 6$ and $INR = 5$ dB, the performance of EVP is slightly superior to LSMI [118].

While the performance of EVP does not depend on the choice of a loading term α , it does depend on the estimation of the number of interferers M (i.e. the number of large eigenvalues). If the estimated number is too small, some interferers will not be nulled. If the number is estimated too large, then there will be fewer dimensions left onto which to project the data vectors which might lead to partial desired signal nulling.

Furthermore, the preprocessing and the eigendecomposition required by EVP make it a computationally demanding algorithm. However, it may be the best method to deal with part-time interferers. Indeed, both SMI and (to a lesser degree) LSMI will only partially null part-time interferers if operating in the presence of noise. This is because those interferers will be perceived as less powerful than they actually are. Under the MMSE / maximum SINR criterion, this will lead to partial nulling. On the other hand, EVP is a true zero-forcing class algorithm since it attempts to completely null *all* interferers regardless of the noise power (provided that the interference power does not fall low enough to be mistaken for noise).

2.7 Fixed wireless systems and LMCS

The Local Multipoint Communication System (LMCS)¹² is a broadband wireless access service operating in the 28 GHz range and capable of delivering to homes and businesses high-bandwidth communication services such as wireless cable TV, Internet access, video telephony, video conferencing, video-on-demand and other multimedia services [120].

In order to accommodate the large bit rates required, Industry Canada has reserved a block of spectrum of 1 GHz for LMCS while in the U. S. the allocated spectrum is slightly larger. Licenses have been granted to several service providers to cover both urban and rural markets [121], [122].

Several different system concepts have been proposed for LMCS. Commercial base station equipment as well as NIUs and antennas are available from companies such as Harris, Nortel and Lucent. Proposed channelization schemes are always based on hybrid FDMA/TDMA structures. Indeed, CDMA was initially ruled out due to the enormous chip rates which would be required to achieve a decent processing gain and the ensuing hardware-related difficulties. While there are now proposals for CDMA systems operating under such conditions, FDMA/TDMA is arguably still the major option (although it might be losing ground since it is faced with a huge installed base of CDMA technology in other bands which may imply economies of scale as the technology migrates to LMCS). It is generally assumed, based on the nature of LMCS services, that the bandwidth usage will be asymmetrical with strong emphasis on the downlink side. An 850/150 MHz downlink/uplink split is typical [121].

LMCS is characterized by fixed subscribers, large bit rates and difficult propa-

¹²The acronym LMCS is specific to Canada and the equivalent system class in the U. S. is referred to as LMDS (Local Multipoint Distribution System).

gation conditions [123], [124], [125], [127]. Indeed, vegetation and rain attenuation [126] are a major concern at 28 GHz. To boost gain and reduce interference, highly directional antennas (beamwidth of $5^\circ - 10^\circ$) can be used at the subscribers. Furthermore, proposals are generally based on 4 sectors and full frequency reuse in every cell (sometimes in every sector). It has also been proposed to use polarization isolation between adjacent cells and / or sectors. However, measurements and trials in this band have shown significant depolarization effects and hence, poor cross-polarization isolation [126], [123], [124]. Alternatively, the proposed Cellular Vision system uses a combination of cross-polarization isolation and carrier frequency offsets to reduce sector-to-sector interference.

It is likely that a coverage of at least 90% will be required. However, a preliminary study conducted by Hewlett-Packard [128] has shown that without resorting to special techniques, cells with a radius of 2 km could be 60% covered with a base station height of 40 ft and 80% covered with a base station height of 80 ft.

Cells will likely overlap to mitigate somewhat the expected coverage problems. This will allow remote locations to select the best possible air link from a handful of base stations. This macrodiversity advantage can be realized in a fixed fashion but is more problematic if dynamic diversity is demanded. Indeed, this would require either installing more than one antenna per subscriber to accommodate all possible base stations, or somehow constantly realigning the subscriber antenna everytime there is a base station switch.

To achieve better coverage and / or to mitigate CCI, an array of omnidirectional elements can be employed at the base station for digital beamforming. Highly directional antennas, or weighted linear combinations of them, can also be used at bases to fulfill the same goals. The effective number of sectors would then be much larger

than 4 (e.g. 12 or more).

2.7.1 Asynchronous Transfer Mode (ATM)

In order to enforce connection-based quality-of-service (QoS) (e.g. bit-rate and latency constraints), it is generally assumed that the LMCS physical layer will serve to support ATM (Asynchronous Transfer Mode) traffic. This would also have the advantage of unifying wired and wireless networks into a single protocol [120].

The ATM protocol is based on very small packets called *cells* (53 bytes including a 5-byte header and a 48-byte payload) so that it can support traffic with strong real-time constraints (i.e. thus emulating the performance of switched, dedicated connection networks such as the PSTN) as well as the more traditional packet-based traffic which is characterized by looser latency constraints and bursty transfers. In fact, one of the dominating features of ATM is its ability to guarantee different levels of quality of service (QoS) for different connections¹³. In general, four service classes are supported [129]:

1. **Constant Bit-Rate (CBR)** for real-time applications (e.g. voice) which require enforcing a real-time delay constraint and delay jitter. The bit rate is specified and it is assumed that the application offers traffic at this constant bit rate in a continuous fashion. Bounds on the acceptable cell loss rate and the transfer delay are also specified when the connection is established. CBR traffic is deterministic and characterized by a constant cell inter-arrival time.
2. **Variable Bit-Rate (VBR)** targeted mainly at video applications, frame relay and similar applications which have known and generally predictable bursty

¹³As a product of the telephone companies' research labs, ATM is *connection-oriented* like its ancestor, the PSTN.

traffic characteristics. VBR traffic is well characterized by an ON/OFF source. During the ON period the inter-arrival time is constant while during the OFF period no cells are generated.

3. **Available Bit-Rate (ABR)** connections use a feedback loop to adjust their bit rate to the level of congestion in the network. This is useful to minimize cell loss rate but the bit-rate varies dynamically. A minimum cell rate (MCR) can be specified so that ABR connections will provide at least the MCR plus additional bandwidth if any is left over after VBR and CBR connections have been accounted for.
4. **Unspecified Bit-Rate (UBR)** is the lowest service class and does not guarantee any minimum throughput. UBR connections operate on a best-effort basis after all other service classes have been allocated and can be stalled for prolonged periods of time; they are similar in essence to Web browsing connections over the Internet.

In the context of LMCS, it is expected that only the first three classes need to be supported in initial implementations.

However, ATM was originally designed for very low BER fiber channels and therefore standard ATM cells must be “encapsulated” into larger packets to support the additional overhead required by wireless channels [130], [131]. Furthermore, it would seem that IP is gaining ground as a possible alternative to ATM. This is due to its widespread usage, the maturity of the technology, the low-cost of IP equipment and the readily available pool of expertise. Also, the upcoming IPv6 standard provides some ATM-like features, such as service classes and guaranteed QoS.

2.7.2 Channelization

The DAVIC standard [131] for LMCS proposes the use of narrowband uplink channels occupying a bandwidth of 1.4583 MHz and carrying an effective bit rate of 1.8875 Mbit/s using QPSK modulation. Assuming a total uplink bandwidth of 300 MHz, a given sector can accommodate up to 205.72 uplink channels. Each of these channels is divided in frames and slots (TDMA) and can accommodate several distinct connections. Some slots can be reserved for contention / link establishment. Each downlink channel occupies a bandwidth of 40 MHz and carries an effective bit rate of 53.7582 Mbit/s. Assuming a total downlink bandwidth of 850 MHz, a given sector could accommodate up to 21.25 downlink channels.

It is expected that some significant portion of the LMCS traffic is of “broadcast” type (e.g. wireless cable) which implies that a given downlink channel can service many subscribers simultaneously (point-to-multipoint connection).

2.8 Vector Channel Modeling

The evaluation of the capacity and the performance of cellular systems enhanced by antenna arrays requires a propagation model capable of providing the direction of arrival of every wave component or, equivalently, fading envelope crosscorrelation coefficients between every pair of elements. In much of the literature pertaining to antenna diversity, it is assumed that the fading envelopes are decoupled (uncorrelated) provided elements are sufficiently separated ($15\lambda - 40\lambda$) at the base. This assumption needs to be nuanced if it is to hold up to a close reality check. Indeed, correlation coefficients depend on many other factors besides antenna separation including the position of the subscriber with respect to the array (range and angle), the shape of

the scattering region and the transmit and receive antenna patterns. In certain cases, it is possible to observe very high correlation with separations as large as 1000λ [132]. Furthermore, antenna arrays' aperture will typically be constrained at least by the size of the rooftop surface on which the base station is situated (e.g. at 28 GHz, 10 meters $\approx 1000\lambda$). In the interest of cost-effectiveness, the entire array might be built on a single substrate, in which case the aperture size might be in the vicinity of $10 - 20\lambda$.

From the above discussion it can be surmised that in practical systems fading processes will normally exhibit significant correlation across the array. For accurate assessment of performance, it is therefore important to take correlation into account. An adequate propagation model then becomes necessary to quantify the correlation matrices of all impinging signals.

It has been established that fading correlation has an effect not only on the gain against fading but also on the interference-suppression capability of an array [13]. In addition, the system Shannon capacity is also affected by fading correlation [133].

Ossana [134] produced the first statistical model for fading waveforms based on the superposition of a single direct ray and a single reflected ray. This model was compared with measurements with generally good agreement. However, some discrepancies with the model could be observed at very low frequencies and at frequencies near the maximum Doppler shift. This prompted Clarke [135] to improve on Ossana's work using a different model originally proposed by Gilbert [136]. The new model postulated the superposition of a large number N of vertically polarized, horizontally traveling plane waves with equal amplitudes. The phase of each ray is assumed to be random and uniformly distributed between 0 and 2π . The direction of arrival of the rays at the subscriber is also uniformly distributed between 0 and 2π . If the sub-

scriber is in motion, this model yields a Doppler spectrum that is reasonably close to measured spectra although significant discrepancies remain. This model suggests a scattering geometry where scatterers are uniformly distributed on a ring at a fixed distance from the subscriber.

The assumption of a continuum of scatterers at an equal distance from the subscriber (denoted variously as *local scattering model*, *macrocell model* or *ring-of-scatterers model*) is widely used in the literature [6]. Aulin was the first to show that the introduction of a third dimension, extruding the ring of scatterers into a cylinder, brought better agreement with empirical results [137]. Let us define α and β as r.v.'s corresponding to the azimuthal and the elevation ray angle, respectively. In addition to having a uniform ray density in the azimuthal plane ($f_\alpha(\alpha) = \frac{1}{2\pi}$), Aulin proposed the following ray density in elevation at the subscriber:

$$f_\beta(\beta) = \begin{cases} \frac{\cos\beta}{2\sin\beta_m} & |\beta| \leq \beta_m \leq \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases} \quad (2.83)$$

This density is rather flat, dropping to zero abruptly at the specified maximum elevation angle β_m (see figure 2.3). It was chosen mostly for mathematical convenience since it allows derivation of closed-form solutions. Parsons and Turkmani perfected the three-dimensional model by proposing a more realistic (smoother) density in elevation (see figure 2.4) [139]:

$$f_\beta(\beta) = \begin{cases} \frac{\pi}{4\beta_m} \cos\left(\frac{\pi}{2} \frac{\beta}{\beta_m}\right) & |\beta| \leq \beta_m \leq \frac{\pi}{2} \\ 0 & \text{elsewhere} \end{cases} \quad (2.84)$$

Crosscorrelations at the base station can be computed from such propagation models by finding the angular ray arrival pdf at the base station end. For example

Figure 2.3: Probability density function for elevation angle β as proposed by Aulin.

Figure 2.4: Probability density function for elevation angle β as proposed by Parsons and Turkmani.

the covariance matrix (averaged over short-term fading¹⁴) of user i at the base is expressed

$$\begin{aligned}\Sigma_i &= \langle \mathbf{x}_i(t) \mathbf{x}_i^H(t) \rangle \\ &= \int_{\delta_0 - \Delta}^{\delta_0 + \Delta} \mathbf{a}_i(\phi) \mathbf{a}_i^H(\phi) d\phi\end{aligned}\quad (2.85)$$

where $\mathbf{a}(\phi)$ is the response of the array to the component of signal i coming from angle ϕ , 2Δ is the total angle spread and δ_0 is the main (or central) direction of arrival. Furthermore, a specific element of Σ_i can be expressed as follows [134], [138] provided the array topology is constrained to a 2D plane parallel to the (x, y) plane:

$$\sigma_i[k, l] = P_i \int_{\delta_0 - \Delta}^{\delta_0 + \Delta} \exp \left[j2\pi \frac{d[k, l]}{\lambda} \cos(\phi - \theta[k, l]) \right] f_\phi^{(i)}(\phi) d\phi \quad (2.86)$$

where $\theta[k, l]$ is the angle¹⁵ of the axis joining elements k and l , $d[k, l]$ is the distance between elements k and l , λ is the wavelength and $f_\phi^{(i)}(\phi)$ is the angular pdf of arriving power from user i .

Various distributions $f_\phi(\phi)$ have been proposed to compute analytically the cross-correlations and / or crosscovariances between array elements. Cox proposed using the n th power of a simple cosine function, i.e.

$$f_\phi(\phi) = Q \{ \cos(2\pi[\phi - \delta_0]) + 1 \}^n \quad (2.87)$$

¹⁴Formally, one must distinguish between

1. the short-term covariance matrix $\hat{\mathbf{R}}$ which is obtained by averaging over a range of symbols short enough that the channels can be considered fixed;
2. the medium-term covariance matrix Σ which is averaged over the short-term (multipath) fading;
3. the long-term covariance matrix which is averaged over both multipath fading and shadowing.

¹⁵measured from the same origin angle as ϕ

where Q is a normalizing constant. It can be observed that as n grows large, this distribution tends towards gaussianity. Accordingly, some authors have also proposed the use of a true gaussian pdf. Winters and Salz [13] employed a simpler flat distribution (constant over a determined angular spread) in order to obtain closed-form expressions for the cross-correlation coefficients.

As an alternative to imposing a ray arrival pdf at the array, a scattering region shape can be specified to relate the ray departure pdf's to the ray arrival distribution. The only model to our knowledge to obtain crosscorrelations based on this scheme is the one by Parsons and Turkmani [139]. It is helpful in the LMCS context to proceed in this fashion since the antenna pattern of the subscriber needs to be accounted for.

2.9 Conclusion

This chapter presented an overview of space-time processing concepts and strategies centering around the use of such beamformers at the base station on the uplink. Highlights include:

- basic array operation concepts in free-space, flat fading and frequency-selective fading environments;
- brief descriptions of the main linear space-time processing methods;
- descriptions of maximum-likelihood space-time processing and the matched-filter bound concept;
- an overview of conventional and multi-user array decision-feedback processing;
- an extensive overview of numerical and analytical methods to evaluate the performance of array-enhanced communication links.

Some attention was also given to related issues:

- A brief description of fixed wireless systems and the LMCS system context is presented, including an overview of protocol and channelization issues which have an impact on the introduction of arrays.
- As a foundation for chapter 3, a brief overview of previous works in the area of vector channel modeling is provided.

Chapter 3

Characterization of the typical LMCS vector channel

The reasonable man tries to adapt to the world around him;

The unreasonable man attempts to adapt the world to himself;

Therefore, any progress is due to the unreasonable man.

George Bernard Shaw

Since a frequency-domain approach is adopted for all analytical and simulation work herein and the wideband case must be considered, a propagation model should be designed to yield correlation as a function of separation in space and separation in frequency. The model proposed here is largely based on [139], [140] with the difference that it is derived for the wideband context and incorporates the effect of arbitrary antenna patterns. Furthermore, a novel scattering geometry is proposed in section 3.2.2. The wideband treatment is inspired by the methodology of Jakes [6] which he employed to obtain the fading envelope correlation as a function of frequency

separation.

3.1 Analytical model - Rayleigh fading

Let us assume that transmission occurs in a band significantly larger than the coherence bandwidth and comprised of the frequencies between ω_{min} and ω_{max} . The electric field at a point in space $\mathbf{x}$ and at frequency ω arising from a subscriber's signal which has undergone frequency-dispersive scattering is generally expressed as

$$E_Z(\omega, t, \mathbf{x}) = E_0 \sum_{n=1}^N \sum_{m=1}^M C_{nm} \cos(\omega t + \omega_n t - \omega \tau_{nm} - \mathbf{k}_n \cdot \mathbf{x}). \quad (3.1)$$

The n th wave leaving the scattering region around the subscriber at elevation angle β_n and azimuth angle α_n is composed of M subwaves with propagation delays τ_{nm} . There are therefore N waves with distinct directions of arrivals, each of which being composed of M subwaves with distinct propagation delays. The Doppler frequency ω_n in the context of fixed wireless systems like LMDS is due to motion of scatterers only. Since this model is not concerned with temporal variations (it will be applied to Monte Carlo simulation) and the effect of Doppler frequency on the correlation functions is negligible, it will be assumed that $\omega_n = 0$.

Let us define the r.v.'s α and β as the azimuthal and elevation angle defining the direction of departure of a ray from the subscriber site. The isotropic scattering assumption is extended to three dimensions by imposing the following density for the azimuth and elevation components of the departure angle

$$f_{\alpha,\beta}(\alpha, \beta) = \begin{cases} \frac{1}{8\beta_m} \cos\left(\frac{\pi}{2} \frac{\beta}{\beta_m}\right) & |\beta| \leq \beta_m \leq \frac{\pi}{2}, \quad 0 \leq \alpha \leq 2\pi, \\ 0 & \text{elsewhere,} \end{cases} \quad (3.2)$$

as proposed by Parsons and Turkmani [139], [140].

The dot product $\mathbf{k}_n \cdot \mathbf{x}$ corresponds to the delay and phase shift associated with the position $\mathbf{x}$. The coefficients C_{nm} are defined as

$$C_{nm}^2 = G_b(\delta_n, \epsilon_n) G_s(\alpha_n, \beta_n) f(\delta_n, \epsilon_n, \tau_{nm}) d\delta d\epsilon d\tau \quad (3.3)$$

where δ_n and ϵ_n are the elevation and azimuth angles of arrival at the base station. G_b and G_s are the 3D antenna patterns of the base station element, $\mathbf{x}$ and the subscriber and $f(\delta_n, \epsilon_n, \tau_{nm})$ is the discrete ray arrival density as a function of arrival angles and delay. It will be seen later that specification of the shape of the scattering region around the subscriber implicitly defines a geometric relationship between departure angles (α_n, β_n) and arrival angles (δ_n, ϵ_n) . Equation (3.3) can thus be expressed either as a function of arrival angles or as a function of departure angles¹.

To derive the lag-frequency cross-correlation function², the electric field at two points in position-frequency space is expressed in quadrature notation:

$$\omega_1, \mathbf{x}_1 \rightarrow E_Z(\omega_1, \mathbf{x}_1, t) = e_1 \cos(\omega_1 t) - e_2 \sin(\omega_1 t), \quad (3.4)$$

$$\omega_2, \mathbf{x}_2 \rightarrow E_Z(\omega_2, \mathbf{x}_2, t) = e_3 \cos(\omega_2 t) - e_4 \sin(\omega_2 t). \quad (3.5)$$

In accordance with the Central Limit Theorem and assuming for now that there is no line-of-sight between the transmitter and the observation point (Rayleigh-type fading), the $e_i(t)$ are Gaussian random variables³ and are expressed as follows:

¹We will keep using all 4 angles in many expressions both for conciseness and because it is desirable to pursue the initial development before specifying the scattering geometry.

²*Lag* here is employed to denote separation in space and should not be confused with *delay*.

³This assumption is generally assumed to be valid if the number of effective rays contributing to the field is equal to or greater than 6.

$$e_1 = E_0 \sum_{n,m} C_{nm} \cos(-\omega_1 \tau_{nm} - \mathbf{k}_n \cdot \mathbf{x}_1), \quad (3.6)$$

$$e_2 = E_0 \sum_{n,m} C_{nm} \sin(-\omega_1 \tau_{nm} - \mathbf{k}_n \cdot \mathbf{x}_1), \quad (3.7)$$

$$e_3 = E_0 \sum_{n,m} C_{nm} \cos(-\omega_2 \tau_{nm} - \mathbf{k}_n \cdot \mathbf{x}_2), \quad (3.8)$$

$$e_4 = E_0 \sum_{n,m} C_{nm} \sin(-\omega_2 \tau_{nm} - \mathbf{k}_n \cdot \mathbf{x}_2), \quad (3.9)$$

$$(3.10)$$

Alternatively, the complex baseband equivalents [16, p.152] of $E_Z(\omega_1, \mathbf{x}_1, t)$ and $E_Z(\omega_2, \mathbf{x}_2, t)$ can be respectively expressed as:

$$E_l^{(1)} = e_1 + j e_2, \quad (3.11)$$

$$E_l^{(2)} = e_3 + j e_4. \quad (3.12)$$

Consequently, the complex envelope correlation between these two field components, including the effects of transmit and receive filtering, is defined

$$\begin{aligned} R_{12} &= \langle E_l^{(1)} E_l^{(2)*} \rangle G_{tot}(\omega_1) G_{tot}(\omega_2)^* \\ &= \langle (e_1 + j e_2)(e_3 - j e_4) \rangle G_{tot}(\omega_1) G_{tot}(\omega_2)^* \\ &= 2 [\langle e_1 e_3 \rangle + j 2 \langle e_2 e_3 \rangle] G_{tot}(\omega_1) G_{tot}(\omega_2)^*, \end{aligned} \quad (3.13)$$

where $\langle \cdot \rangle$ denotes expectation and

$$G_{tot}(\omega) = G_T(\omega) G_R(\omega), \quad (3.14)$$

is the combined transmit and receive filter spectrum.

The definition in eq. (3.13) implies the following:

$$\langle e_1 e_3 \rangle = \langle e_2 e_4 \rangle, \quad (3.15)$$

$$\langle e_2 e_3 \rangle = -\langle e_1 e_4 \rangle, \quad (3.16)$$

$$\langle e_1 e_2 \rangle = \langle e_3 e_4 \rangle = 0, \quad (3.17)$$

and

$$\text{Re}(R_{12}) = 2\langle e_1 e_3 \rangle G_{tot}(\omega_1) G_{tot}(\omega_2)^*, \quad (3.18)$$

$$\text{Im}(R_{12}) = 2\langle e_2 e_3 \rangle G_{tot}(\omega_1) G_{tot}(\omega_2)^*. \quad (3.19)$$

Making the number of rays tend towards infinity⁴, the self-correlation of the envelope can be evaluated:

$$R_{11} = \langle E_t^{(1)} E_t^{(1)*} \rangle |G_{tot}(\omega_1)|^2 \quad (3.20)$$

$$\begin{aligned} &= E_0^2 |G_{tot}(\omega_1)|^2 \int_{-\pi}^{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} G_{b_1}(\delta, \epsilon) G_s(\alpha, \beta) f_{\delta, \epsilon}(\delta, \epsilon) d\delta d\epsilon \\ &= |G_{tot}(\omega_1)|^2 [\langle e_1^2 \rangle + \langle e_2^2 \rangle] \\ &= |G_{tot}(\omega_1)|^2 [2\langle e_1^2 \rangle], \end{aligned} \quad (3.21)$$

and if the two base station antenna patterns G_{b_1} and G_{b_2} are identical, $R_{11} = R_{22}$.

For the remainder of the development, the transmit and receive filters will be assumed ideal and the corresponding factor(s) will be omitted for the sake of clarity and conciseness.

In the same fashion, it can be shown that

⁴This generally corresponds to the *continuously-dispersive* (CD) channel type.

$$\begin{aligned}
R_{12} &= \left\langle E_l^{(1)}(t) E_l^{(2)}(t)^* \right\rangle = E_0^2 \int_{-\pi}^{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{G_{b_1}(\delta, \epsilon) G_{b_2}(\delta, \epsilon) G_s(\alpha, \beta)} d\delta d\epsilon \\
&\quad \cdot \int_0^{\infty} f_{\delta, \epsilon, \tau}(\delta, \epsilon, \tau) \exp [j \{ (\omega_1 - \omega_2) \tau + \mathbf{k} \cdot \mathbf{x}_1 - \mathbf{k} \cdot \mathbf{x}_2 \}] d\tau. \quad (3.22)
\end{aligned}$$

Since the choice of a coordinate system is arbitrary, $\mathbf{x}_1$ and $\mathbf{x}_2$ can both be assumed to lie on a line parallel to the abscissa without loss of generality (i.e. both element positions lie in a horizontal plane and are on a line parallel to the x axis)⁵. Hence:

$$\begin{aligned}
\mathbf{k} \cdot \mathbf{x}_1 - \mathbf{k} \cdot \mathbf{x}_2 &= \frac{\omega_1}{c} \cos \delta \left(\frac{\Delta x}{2} \right) - \left(-\frac{\Delta x}{2} \right) \frac{\omega_2}{c} \cos \delta \cos \epsilon \\
&= \frac{(\omega_1 + \omega_2) \Delta x}{2c} \cos \delta \cos \epsilon. \quad (3.23)
\end{aligned}$$

To further develop equation (3.22), it will be assumed that the distribution of power in terms of arrival angles is independent from the distribution of power in delay. Hence:

$$f_{\delta, \epsilon, \tau}(\delta, \epsilon, \tau) = f_{\delta, \epsilon}(\delta, \epsilon) f_{\tau}(\tau). \quad (3.24)$$

In addition, let the expected delay spread be of exponential shape:

$$f_{\tau}(\tau) = \frac{1}{\tau_0} e^{-\frac{\tau}{\tau_0}}. \quad (3.25)$$

Then equation (3.22) becomes, assuming omnidirectional base station antennas:

⁵For simplicity, it is assumed in developments throughout this chapter that all elements in the array are in the same horizontal plane. The concepts presented, however, can be readily extended to the general case (non-zero vertical spacing) which is illustrated in figure 3.1. Nevertheless, it was found in [140] that vertical spacing is much less effective than horizontal spacing in decoupling the fading envelopes; for that reason, we will limit our study exclusively to horizontal spacing.

$$\begin{aligned}
R_{12} &= \langle E_l^{(1)} E_l^{(2)*} \rangle \\
&= E_0^2 \frac{1}{1 - j\tau_0 \Delta\omega} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} G_s(\alpha, \beta) f_{\delta, \epsilon}(\delta, \epsilon) e^{j(\mathbf{k}_1 \cdot \mathbf{x}_1 - \mathbf{k}_2 \cdot \mathbf{x}_2)} d\delta d\epsilon,
\end{aligned} \tag{3.26}$$

where $\Delta\omega = |\omega_2 - \omega_1|$. This equation defines the medium-term covariance matrix Σ when delay spread is exponential and can only be further developed by specifying a scattering geometry.

The four real gaussian variables $e_1 \dots e_4$ are characterized by the joint pdf

$$f_{e_1 \dots e_4}(e_1, \dots, e_4) = \frac{e^{-\frac{1}{2} \mathbf{e} \mathbf{A} \mathbf{e}^T}}{\sqrt{(2\pi)^4 |\mathbf{A}|}}, \tag{3.27}$$

where $\mathbf{e} = [e_1, \dots, e_4]^T$ and

$$\mathbf{A} = \begin{bmatrix} \langle e_1^2 \rangle & 0 & \langle e_1 e_3 \rangle & -\langle e_2 e_3 \rangle \\ 0 & \langle e_1^2 \rangle & \langle e_2 e_3 \rangle & \langle e_1 e_3 \rangle \\ \langle e_1 e_3 \rangle & \langle e_2 e_3 \rangle & \langle e_1^2 \rangle & 0 \\ -\langle e_2 e_3 \rangle & \langle e_1 e_3 \rangle & 0 & \langle e_1^2 \rangle \end{bmatrix}. \tag{3.28}$$

By performing the following transformation of variables

$$e_1 = r_1 \cos \theta_1, \quad e_2 = r_1 \sin \theta_1, \tag{3.29}$$

$$e_3 = r_2 \cos \theta_2, \quad e_4 = r_2 \sin \theta_2,$$

we obtain the pdf

$$f_{r_1, r_2, \theta_1, \theta_2}(r_1, r_2, \theta_1, \theta_2) = \frac{r_1 r_2}{(2\pi m_1)^2 (1 - l^2)}$$

$$\exp \left[-\frac{r_1^2 + r_2^2 - 2r_1 r_2 l \cos(\theta_2 - \theta_1 + \phi)}{2m_1(1-l^2)} \right], \quad (3.30)$$

where

$$m_1 = \langle e_1^2 \rangle, \quad m_2 = \langle e_1 e_3 \rangle, \quad m_3 = \langle e_1 e_4 \rangle, \quad (3.31)$$

$$\tan \phi = \frac{m_3}{m_2}, \quad l^2 = \frac{m_2^2 + m_3^2}{m_1^2}. \quad (3.32)$$

Using the above four-fold density, we can express the covariance of the envelopes (i.e. the magnitudes r_1 and r_2) at the two frequencies and positions. We have

$$R_e(\omega_1, \omega_2, \Delta x) \triangleq \langle r_1 r_2 \rangle = \int_0^\infty \int_0^\infty r_1 r_2 f_{r_1, r_2}(r_1, r_2) dr_1 dr_2, \quad (3.33)$$

where $f_{r_1, r_2}(r_1, r_2)$ can be obtained by integrating (3.30):

$$\begin{aligned} f_{r_1, r_2}(r_1, r_2) &= \int_0^{2\pi} \int_0^{2\pi} f_{r_1, r_2, \theta_1, \theta_2}(r_1, r_2, \theta_1, \theta_2) d\theta_1 d\theta_2 \\ &= \frac{r_1 r_2}{m_1^2(1-l^2)} \exp \left[-\frac{r_1^2 + r_2^2}{2m_1(1-l^2)} \right] I_0 \left(\frac{r_1 r_2}{m_1} \frac{l}{1-l^2} \right), \end{aligned} \quad (3.34)$$

where $I_0(x)$ is the modified Bessel function of zero order.

Using (3.34), the integral in (3.33) can be carried out to yield

$$R_e(\omega_1, \omega_2, \Delta x) = \frac{E_0^2 \pi}{4} {}_2F_1 \left(-\frac{1}{2}, -\frac{1}{2}; 1; \frac{l^2}{4} \right). \quad (3.35)$$

A simple, second-order approximation results from expanding the hypergeometric function

$$R_e(\omega_1, \omega_2, \Delta x) \approx \frac{E_0^2 \pi}{4} \left(1 + \left(\frac{l^2}{4} \right) \right). \quad (3.36)$$

Finally, we get the envelope correlation coefficient by normalization:

$$\rho_e(\omega_1, \omega_2, \Delta x) \approx \frac{\pi}{16 - 4\pi} \left(\frac{m_2^2 + m_3^2}{m_1^2} \right). \quad (3.37)$$

It is sometimes also useful to know the cross-correlation of the squared envelopes, i.e. the signal powers. It can be obtained by evaluating the integral

$$\langle r_1^2 r_2^2 \rangle = \int_0^\infty \int_0^\infty r_1^2 r_2^2 f_{r_1, r_2}(r_1, r_2) dr_1 dr_2, \quad (3.38)$$

which has the following closed-form solution:

$$\langle r_1^2 r_2^2 \rangle = 4E_0^4 (1 + l^2) m_1^2. \quad (3.39)$$

Normalization will yield the power cross-correlation coefficient

$$\rho_p(\omega_1, \omega_2, \Delta \mathbf{x}) = l^2. \quad (3.40)$$

It is interesting to note that the approximate ρ_e differs from the exact ρ_p by only a factor $\frac{\pi}{16-4\pi}$. In fact, numerical tests have shown that ρ_p is, for all intents and purposes, equivalent to the *exact* ρ_e (based on (3.35)), i.e. when plotted the two curves are visually indistinguishable. It is therefore appropriate to use ρ_p in lieu of the second-order approximation to ρ_e .

3.2 Scattering geometries

From the general formulas derived in the previous section, a scattering geometry must be specified for the analysis to proceed. It will be assumed that the base station

Figure 3.1: Geometry of the three-dimensional cylinder-of-scatterers model with the presence of a secondary scattering area. S marks the position of the subscriber, A_1 and A_2 are the two base station antennas.

is sufficiently high that only scattering in the vicinity of the mobile needs to be considered. Furthermore, only single-bounce scattering will be considered.

3.2.1 Cylinder of scatterers (hollow model)

The cylindrical scattering model utilizes the geometry and distribution originally proposed by Parsons and Turkmani in a narrowband context [139], [140] (see figure 3.1 and eq. (3.2)). The scatterers are distributed along the surface of a vertical cylinder around the subscriber whose extent in elevation above and below the subscriber position is finite and defined by the maximum elevation angle β_m and the cylinder radius r . The distance from base to subscriber is denoted m and the base station height is denoted h_b . Furthermore, let the main (central) direction of arrival (the line-of-sight) be defined in azimuth and elevation by the pair of angles (δ_0, ϵ_0) .

A second cylinder can be employed to model a secondary scattering area which

translates at the base as a secondary main direction-of-arrival. Additional cylinders can be introduced as necessary to model such propagation effects if they are expected to be significant in the system under study. For a simulation-oriented propagation model also employing secondary scattering areas to obtain an angular power distribution, see [141].

However, we will restrict the following development to a single scattering area for the sake of clarity.

Now that the shape of the scattering region is defined, trigonometry allows us to relate arrival angles (azimuth, elevation) = (δ, ϵ) at the base station to departure angles (azimuth, elevation) = (α, β) at the subscriber:

$$\delta = \delta_0 - \arcsin \left(\frac{r \sin \alpha}{\sqrt{m^2 + r^2 - 2mr \cos \alpha}} \right), \quad (3.41)$$

$$\epsilon = -\arcsin \left(\frac{h_b - r \tan \beta}{\sqrt{m^2 + r^2 - 2mr \cos \alpha + (h_b - r \tan \beta)^2}} \right). \quad (3.42)$$

Furthermore, it can be shown that

$$E(\alpha, \beta) = \cos \delta \cos \epsilon = \frac{m \cos \delta - r \cos(\alpha - \delta_0)}{\sqrt{m^2 + r^2 - 2mr \cos \alpha + (h_b - r \tan \beta)^2}}. \quad (3.43)$$

We can then rewrite (3.26) as a function of departure angles only, thus avoiding the need for an explicit definition of $f_{\delta, \epsilon}(\delta, \epsilon)$:

$$R_{12} = E_0^2 \frac{\pi}{8|\beta_m|(1 - j\tau_0\Delta\omega)} \int_{-\beta_m}^{\beta_m} \int_{-\pi}^{\pi} G_s(\alpha, \beta) \cos \left(\frac{\pi}{2} \frac{\beta}{\beta_m} \right) e^{j(\mathbf{k}_1 \cdot \mathbf{x}_1 - \mathbf{k}_2 \cdot \mathbf{x}_2)} d\alpha d\beta, \quad (3.44)$$

where there only remains to express the argument of the exponential as a function of α and β . By substituting (3.23), we get

$$\begin{aligned}
R_{12} &= E_0^2 \frac{\pi}{8|\beta_m|(1-j\tau_0\Delta\omega)} \int_{-\beta_m}^{\beta_m} \int_{-\pi}^{\pi} G_s(\alpha, \beta) \cos\left(\frac{\pi}{2} \frac{\beta}{\beta_m}\right) \cdot \\
&\quad \exp\left(j \frac{(\omega_1 + \omega_2)\Delta x}{2c} \cos\delta\right) d\alpha d\beta \\
&= \frac{E_0^2}{2} \frac{1}{1-j\tau_0\Delta\omega} \int_{-\pi}^{\pi} G_s(\alpha, \beta) \exp\left(j \frac{(\omega_1 + \omega_2)\Delta x}{2c} E(\alpha, \beta)\right) d\alpha d\beta. \quad (3.45)
\end{aligned}$$

where the δ and ϵ variables can be considered placeholders for (3.41) and (3.42), thus completing the conversion to departure angles.

Likewise, the self-moment definition in (3.20) can be expressed

$$R_{xx} = \frac{\pi E_0^2}{16\beta_m} \int_{-\beta_m}^{\beta_m} \int_{-\pi}^{\pi} G_{b_x}(\delta, \epsilon) G_s(\alpha, \beta) \cos\left(\frac{\pi}{2} \frac{\beta}{\beta_m}\right) d\alpha d\beta = \frac{E_0^2}{2}, \quad (3.46)$$

where x is either 1 or 2 and (3.41) and (3.42) should be substituted to perform numerical integration.

It should be noted that if the model is reduced to two dimensions (i.e. $\beta_m = 0$) and the lag is null, the hollow model reduces to the Jakes wideband model [6]. Likewise, if the frequency separation is 0, the hollow model reduces to a form highly related — but not identical — to the Parsons & Turkmani model [140].

3.2.2 3-D circular gaussian model (full model)

This model introduces another degree of freedom by making the radius r a random variable. Therefore, the positions of scatterers are not constrained to a 2D surface but rather fill a 3D volume.

This is done to remedy one very unrealistic feature of the hollow model. Indeed, the angular pdf of received power at the base has the aspect of figure 3.2. Very little power is received from the main DOA while it peaks dramatically at the edges of

Figure 3.2: Angular received power density for the hollow model using a range of 500 m and a scattering cylinder radius of 70 m. For simplicity, the model has been “flattened”, i.e. the maximum elevation angle $\beta_m = 0^\circ$ and the base station height is zero.

the angle spread region. In a real system, not only would the pdf be smoother but the power would peak at the main DOA. In choosing between the hollow and the full model for a specific simulation run, one should take into account that while the full model is likely to produce more realistic results, the improvement might not be significant enough to justify the added computational complexity. Ultimately, the decision should be made based on the particular aspects of the system the simulation is meant to highlight.

Taking the position of the subscriber as the origin, we start by specifying the position of a scatterer by its cartesian coordinates (x, y, z) . Making x and y independent gaussian variables with the same variance σ^2 results in a circular 2D gaussian pdf:

$$f_{x,y}(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (3.47)$$

Knowing that the radius r and the azimuth angle α are the polar equivalents of x and y , i.e.

$$r = \sqrt{x^2 + y^2}, \quad \alpha = \arg(x + jy), \quad (3.48)$$

A simple transformation of variables yields their densities [142, p.146]

$$f_r(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}, \quad f_\alpha(\alpha) = \frac{1}{2\pi}. \quad (3.49)$$

The height variable can be independently specified according to the same density used previously for the elevation angle β :

$$f_z(z) = \begin{cases} \frac{\pi}{4z_m} \cos\left(\frac{\pi}{2} \frac{z}{z_m}\right), & z \leq z_m \leq \frac{\pi}{2}, \\ 0 & \text{elsewhere.} \end{cases} \quad (3.50)$$

The elevation angle β is then geometrically defined as

$$\beta = \arctan\left(\frac{z}{r}\right). \quad (3.51)$$

By performing a transformation of r.v.'s, the joint pdf of β and r is obtained:

$$f_{r,\beta}(r, \beta) = \frac{\pi \cos \beta}{4r z_m} \cos\left(\frac{\pi}{2} \frac{r \tan \beta}{z_m}\right) \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}, \quad (3.52)$$

from which the self-moment can be expressed in such a way as to be numerically integrable:

$$\begin{aligned} R_{11} = & \frac{E_0^2}{8z_m} \int_0^\infty \int_{-\pi}^\pi \int_{-\arctan \frac{z_m}{r}}^{\arctan \frac{z_m}{r}} G_s(\alpha\beta) \frac{\cos \beta}{r} \cos\left(\frac{\pi}{2} \frac{r \tan \beta}{z_m}\right) \frac{r}{\sigma^2} \\ & \cdot e^{-\frac{r^2}{2\sigma^2}} d\beta d\alpha dr \end{aligned} \quad (3.53)$$

where again δ and ϵ are placeholders for (3.41) and (3.42).

Likewise, the cross-moment can be expressed

$$R_{12} = E_0^2 D \int_0^\infty \int_{-\pi}^\pi \int_{-\arctan \frac{z_m}{r}}^{\arctan \frac{z_m}{r}} \frac{1}{8z_m r \sec \beta} \cos\left(\frac{\pi r \tan \beta}{2 z_m}\right) \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \cdot e^{j \frac{(\omega_1 + \omega_2) \Delta x}{2c} E(\alpha, \beta)} d\alpha d\beta dr, \quad (3.54)$$

where D is a factor related to the power delay profile. In the case of exponential delay profile with RMS delay spread of T_0 , we have

$$D = \frac{1}{1 - j\tau_0 \Delta\omega}, \quad (3.55)$$

as was implicitly specified in eq. (3.45).

3.3 Delay spread shape

It was seen that the moments' expressions include a factor D which embodies the shape of the power delay profile. Although Jakes has found that the delay spread shape has little impact on the fading envelope frequency correlation function, array performance studies herein will be performed over a small, representative set of simple power delay profiles, as previously done by Clark [25], Glance and Greenstein [147] and others.

Consider the delay spectrum shapes listed in table 3.1 and specified according to an RMS delay spread of τ_0 .

1. The exponential spectrum is representative of continuous delay profiles obtained in urban area measurements [143], [146].

2. The gaussian delay spread represents the class of two-sided delay spectra. By virtue of the Central Limit Theorem, it can occur as a result of including the effects of many cascaded filters at both the transmit and receive end. Indeed, convolving the radio channel impulse response with several similar filter characteristics will tend to yield a gaussian-like result. Furthermore, the gaussian characteristic has been stipulated as a delay spread shape model in a number of analytical works [147], [57]. Also, many empirically measured delay profiles tending to be exponential in shape are characterized by a smooth, although steep, slope up. In other words, the envelope of a measured impulse response is often shaped like a Gamma distribution, i.e. somewhere between the gaussian and the exponential shape [148]. In that sense, the exponential and the gaussian shapes can be seen as two theoretical extremes which can be helpful in characterizing a whole continuum of realistic delay spreads.
3. The double-spike spectrum is typical of mountainous terrain and overwater paths; it also leads to worst-case performance for a given τ_0 in the absence of equalization [147].

The delay spread shape factor D was obtained for all three delay spectra by substituting the delay spread distribution into (3.22) and integrating with respect to τ .

3.4 Rician fading

If there exists a line-of-sight between subscriber and base (a significant possibility in fixed wireless), the electric field at one of the base station antennas is made up of a specular and a fading component, i.e.

shape	$p(\tau)$	D
exponential	$\frac{1}{\tau_0} e^{-\frac{\tau}{\tau_0}}$	$\frac{1}{1-j\tau_0\Delta\omega}$
gaussian	$\frac{1}{\sqrt{2\pi\tau_0}} e^{-\frac{1}{2}\left(\frac{\tau}{\tau_0}\right)^2}$	$e^{-\frac{(\Delta\omega)^2\tau_0^2}{2}}$
double-spike	$\frac{1}{2} [\delta(\tau) + \delta(\tau - 2\tau_0)]$	$\frac{1}{2} [1 + e^{j(\Delta\omega)2\tau_0}]$

Table 3.1: Delay profiles and corresponding D factor.

$$E_l^{(1)} = E_{l_s}^{(1)} + E_{l_f}^{(1)}, \quad (3.56)$$

where the static component $E_{l_s}^{(1)}$ is a constant and the fading component $E_{l_f}^{(1)}$ is a zero-mean, complex gaussian r.v.

The self-moment expression then becomes

$$\begin{aligned}
 R_{11}^{los} &= \langle E_l^{(1)} E_l^{(1)*} \rangle |G_{tot}(\omega_1)|^2 \\
 &= \langle (E_{l_s}^{(1)} + E_{l_f}^{(1)}) (E_{l_s}^{(1)} + E_{l_f}^{(1)})^* \rangle |G_{tot}(\omega_1)|^2 \\
 &= 2A^2 + R_{11},
 \end{aligned} \quad (3.57)$$

where $A = \sqrt{2}|E_{l_s}|$. Furthermore, we have

$$E_{l_s}^{(1)} = \sqrt{\frac{E_0^2 K}{2}} G_{b_1}(\delta_0, \epsilon_0) G_s(\alpha_0, \beta_0) G_{tot}(\omega_1) \quad (3.58)$$

where the Rice factor K is the ratio of the received power in the line-of-sight component versus the average power of the fading component (i.e. $K = \frac{A^2}{m_1}$), (δ_0, ϵ_0) are the arrival angles corresponding to the line-of-sight and (α_0, β_0) are the corresponding departure angles. It can be verified from figure 3.1 that $\alpha_0 = \pi - \delta_0$ and $\beta_0 = -\epsilon_0$. This ratio is constant over the band of interest and is calculated by assuming unity gain at both transmit and receive antennas. The K factor is therefore an “air” value; the

effective LOS received power will vary according to patterns and the angle-of-arrival of the direct ray.

Likewise, the cross-moment expression is

$$\begin{aligned} R_{12}^{los} &= \langle E_l^{(1)}(t) E_l^{(2)}(t)^* \rangle G_{tot}(\omega_1) G_{tot}(\omega_2)^* \\ &= \sqrt{G_{b_1}(\delta_0, \epsilon_0) G_{b_2}(\delta_0, \epsilon_0)} G_{tot}(\omega_1) G_{tot}(\omega_2) \frac{E_0^2 K}{2} + R_{12}. \end{aligned} \quad (3.59)$$

Unfortunately, derivation of the envelope correlation coefficient is non-trivial in Rician fading. Indeed, the approach employed in the Rayleigh fading case becomes intractable when the complex gaussian variables involved are non-central.

However, the power cross-correlation coefficient ρ_p can be derived and will be found useful in fitting the analytical models to measurement data.

Without loss of generality, we can assume that the static component has zero phase at element 1. Hence:

$$r_1^2 = (e_1 + A)^2 + e_2^2, \quad (3.60)$$

$$r_2^2 = (e_3 + A \cos \phi)^2 + (e_4 + A \sin \phi)^2, \quad (3.61)$$

where ϕ is the phase of the static component at element 2. Observing that r_1^2 is a non-central chi-square variate with 2 degrees of freedom and non-centrality parameter A^2 , we wish to find its expectation conditioned on e_3 and e_4 . The variable $r_1^2|e_3, e_4$ is still a non-central chi-square but its non-centrality parameter is $\left(\frac{m_2 e_3}{m_1} + \frac{m_3 e_4}{m_1} + A\right)^2 + \left(\frac{m_2 e_3}{m_1} - \frac{m_3 e_4}{m_1}\right)^2$ where $m_1 = \langle e_1^2 \rangle$, $m_2 = \langle e_1 e_3 \rangle$ and $m_3 = \langle e_1 e_4 \rangle$.

It can be shown that

$$\langle r_1^2 | e_3, e_4 \rangle = \left(\frac{m_2 e_3}{m_1} + \frac{m_3 e_4}{m_1} + A \right)^2 + \left(\frac{m_2 e_3}{m_1} - \frac{m_3 e_4}{m_1} \right)^2 + 2m_1 - 2\frac{m_2^2 e_3^2}{m_1^2} - 2\frac{m_3^2 e_4^2}{m_1^2}. \quad (3.62)$$

Knowing that the expectation of a product can be expressed in terms of a conditional expectation as follows [142]:

$$\langle r_1^2 r_2^2 \rangle = \langle \langle r_1^2 | e_3, e_4 \rangle r_2^2 \rangle, \quad (3.63)$$

we can substitute (3.62) in the above, expand and take the expectation term-by-term to obtain:

$$\langle r_1^2 r_2^2 \rangle = A^4 + 4A^2 (m_2 \cos \phi + m_3 \sin \phi) + 4A^2 m_1 + 4m_1^2 (1 + l^2), \quad (3.64)$$

where l^2 is as defined in (3.32).

The correlation coefficient is by definition

$$\rho_p = \frac{\langle r_1^2 r_2^2 \rangle - \langle r_1^2 \rangle \langle r_2^2 \rangle}{\sqrt{(\langle r_1^4 \rangle - \langle r_1^2 \rangle^2) (\langle r_2^4 \rangle - \langle r_2^2 \rangle^2)}}, \quad (3.65)$$

where $\langle r_1^2 \rangle = \langle r_2^2 \rangle = A^2 + 2m_1$ and $\langle r_1^4 \rangle = \langle r_2^4 \rangle = A^4 + 8A^2 m_1 + 8m_1^2$.

Thus, we have

$$\rho_p = \frac{l^2 m_1 + \frac{A^2}{m_1} (m_2 \cos \phi + m_3 \sin \phi)}{m_1 + A^2}, \quad (3.66)$$

and it can be verified that it reduces to (3.40) when $A = 0$. Rewriting the above in terms of the rician K factor, we obtain

$$\rho_p = \frac{l^2 m_1 + K (m_2 \cos \phi + m_3 \sin \phi)}{1 + m_1 K}. \quad (3.67)$$

Although $\langle r_1^2 r_2^2 \rangle$ is always positive, ρ_p can be negative and its sign is partly determined by the angle ϕ . In fact, the angle ϕ can have a relatively large impact on ρ_p . However, while it is easy to determine ϕ from the subscriber position when the static component is, in fact, a line-of-sight component, it becomes problematic when the static component is the sum of many paths reflected off static scatterers⁶ which may or may not include an actual LOS component.

It follows that formula (3.67) is inappropriate when the static component is a vector sum since this makes the angle ϕ unpredictable (unless the amplitudes, phases and DOA's of all components are known). It would therefore be desirable to somehow "average out" the effect of ϕ in (3.67).

However, subtlety is called for here since averaging over ϕ will null the effect of the K factor (since both $\sin \phi$ and $\cos \phi$ have zero average if ϕ is uniformly distributed between 0 and 2π). Observing that the K factor term in (3.67) can be either positive or negative and will introduce an oscillation in ρ_p as separation increases with frequency depending on ϕ , it makes sense to seek to characterize the envelope of these oscillations. Indeed, the latter is independent of ϕ .

To achieve this, either maximization or minimization of ρ_p over ϕ is possible. Since the measurement data will be processed in such a way as to find the maximum ρ_p , we will do the same with our analysis. Using calculus-of-variations techniques, we find

$$\rho_{pmax} = \frac{l^2 + K \left(\sqrt{m_2^2 + m_3^2} \right)}{1 + m_1 K}. \quad (3.68)$$

⁶Arguably, this is the predominant situation in systems with fixed subscribers such as LMCS/LMDS.

3.5 Numerical results

Unless otherwise specified, the curves presented in this section have the following parameters:

- scattering cylinder radius $r = 70\text{m}$ or standard deviation $\sigma_r = 70\text{m}$ in the case of the full model;
- base station antennas are omnidirectional;
- base station height $h_b = 35\text{m}$;
- maximum elevation angle at the subscriber $\beta_m = 10^\circ$ or, for the full model, maximum scattering volume elevation $z_m = 15\text{m}$;
- if the subscriber pattern is not omnidirectional, the front-to-sidelobe ratio is 20 dB;
- carrier frequency is 29.5 GHz;
- distance from base is 1000 m.

Since the effect of spatial separation is nearly completely decoupled from the effect of frequency separation in the band of interest, all curves assume zero frequency separation unless otherwise specified.

3.5.1 Effect of subscriber position

Figures 3.3-3.6 illustrate the variation of the fading envelope correlation coefficient using the hollow model as a function of subscriber position in a $3\text{km} \times 3\text{km}$ cell with a two-element base station array. In figure 3.3, the interelement spacing is

Figure 3.3: Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 10λ . Base station is at (0, 0); array axis indicated by arrows.

relatively small (10λ) and therefore correlation is generally high. However, there are two regions broadside of the array and relatively close to it where correlation is low. As the interelement spacing is augmented in the subsequent figures, the regions of low correlation progressively grow until they fill most of the cell except the immediate vicinity of the array axis (fig. 3.6).

Figures 3.7-3.10 provide the same information as the previous set, but using the full model.

3.5.2 Effect of subscriber antenna beamwidth

The influence of the subscriber antenna beamwidth, which is varied between 0 and 250° , on correlation is shown in figures 3.11 and 3.12 using the hollow model and the full model, respectively. The distance between the base and the subscriber is 1 km and the subscriber is exactly broadside for figure 3.11, i.e. $\delta_0 = \frac{\pi}{2}$, while it is at $\frac{\pi}{3}$ for

Figure 3.4: Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 50λ . Base station is at $(0, 0)$; array axis indicated by arrows.

Figure 3.5: Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 100λ . Base station is at $(0, 0)$; array axis indicated by arrows.

Figure 3.6: Fading envelope correlation as a function of subscriber position; hollow model, interelement separation of 500λ . Base station is at $(0, 0)$; array axis indicated by arrows.

Figure 3.7: Fading envelope correlation as a function of subscriber position; full model, interelement separation of 10λ . Base station is at $(0, 0)$; array axis indicated by arrows.

Figure 3.8: Fading envelope correlation as a function of subscriber position; full model, interelement separation of 50λ . Base station is at $(0, 0)$; array axis indicated by arrows.

Figure 3.9: Fading envelope correlation as a function of subscriber position; full model, interelement separation of 100λ . Base station is at $(0, 0)$; array axis indicated by arrows.

Figure 3.10: Fading envelope correlation as a function of subscriber position; full model, interelement separation of 500λ . Base station is at $(0, 0)$; array axis indicated by arrows.

figure 3.12.

While we generally expect correlation to go up as the antenna beamwidth is reduced, it is noteworthy that in both figures the correlation drops — i.e. the general trend is reversed — when the beamwidth gets close to zero. This is because the FTS is relatively low at 20 dB. Indeed, the sidelobe is essentially an omnidirectional pattern 20 dB down from the narrow main beam. If the latter is very narrow (e.g. on the order of 1-3 degrees), its total power is actually less than the total power in the sidelobe (since the latter spans the entire 360°). It follows that the sidelobe then becomes dominant thus yielding low correlations at the base.

The hollow model curves (figure 3.11) have an additional intriguing characteristic in that for some interelement spacings, ρ_e drops with increasing beamwidth, reaches a low, and starts to rise again in the vicinity of 150° . This is due to the fact that, as the pattern expands towards the lateral edges of the scattering cylinder (i.e. the portions of the cylinder at right angles to the direction of look), a higher concentration

Figure 3.11: Envelope correlation vs. subscriber antenna beamwidths for 5 interelement separations using the hollow model; $m = 1000m$, $\delta_0 = \frac{\pi}{2}$.

of energy is created in those regions as illustrated by the received pattern in figure 3.2. Therefore, there is a smooth transition between a single medium-sized beam to twin narrow beams which results in higher correlations at the base. As the pattern expands further, the relative importance of those lateral spikes in the arrival pattern is reduced since more energy is coming from the middle back of the cylinder. Hence, the correlation drops back down as the subscriber antenna pattern gets close to zero.

3.5.3 Effect of subscriber antenna front-to-sidelobe ratio

Figure 3.13 shows the variation in ρ_e versus the subscriber antenna beamwidth for FTS ratios of 20, 30 and 40 dBs. It clearly shows that the drop in correlation for very low beamwidths is related to the sidelobe level, as postulated in the last section. Indeed, the importance of the said drop diminishes as the FTS is increased until it vanishes when the FTS is at 40 dB.

Figure 3.12: Envelope correlation vs. subscriber antenna beamwidths for 5 interelement separations using the full model; $m = 1000m$, $\delta_0 = \frac{\pi}{3}$.

Figure 3.13: Envelope correlation vs. subscriber antenna beamwidths for 3 FTS ratios.

Figure 3.14: Envelope correlation vs. angle to subscriber for various subscriber antenna beamwidths using the hollow model at a distance $m = 1000\text{m}$ and using a separation of 100λ .

3.5.4 Effect of angle to subscriber

Figures 3.14 and 3.15 illustrate the variation in correlation when only the angle to the subscriber relative to the array axis is varied between the end-fire direction (0°) and broadside (90°) using the hollow and the full model, respectively.

It may a priori be surprising that in both figures, the 1° and the omnidirectional curves are almost identical (while the 15° curve is the farthest). As explained in the previous 2 sections, this is due to the fact that with a FTS of 20 dB, the sidelobe becomes the dominant pattern when the beam is very narrow. Therefore, the main-beam's impact becomes insignificant at 1° and the omnidirectional sidelobe behavior underlies the plotted curves.

Figure 3.15: Envelope correlation vs. angle to subscriber for various subscriber antenna beamwidths using the full model at a distance $m = 1000\text{m}$ and using a separation of 100λ .

3.5.5 Effect of base station height

Although both models include the base station height relative to the subscriber as a parameter, it has effectively very little effect on the computed correlation. As noted by Parsons and Turkmani [140], it becomes a more influential parameter when base station elements are spaced vertically instead of / in addition to horizontally. However, vertical spacing is not considered within the scope of this thesis since it is much less effective in decorrelating fading envelopes.

To illustrate how little effect the base station height has, figure 3.16 plots ρ_e as a function of horizontal interelement separation for three different subscriber antenna patterns and two extreme values of base station heights. The solid curves correspond to a height of 15m, while the dashed curves correspond to $h_b = 300\text{m}$ ⁷.

⁷While the planned Tokyo tower is expected to reach 1km in height, there are very few buildings in the world, if any, capable of supporting a base station at a height of 300m (the actual Tokyo tower (333m) is one of them as is the world's tallest building, the Petronas tower (452 m) in Kuala Lumpur,

Figure 3.16: Envelope correlation vs. interelement spacing for three subscriber antenna beamwidths and two base station heights using the full model.

While the models are relatively insensitive to base station height, their applicability to real-world scenarios is highly conditioned on this parameter. First, the local scattering assumption (i.e. scattering activity occurs around the subscriber, not the base) makes sense only if the base is significantly higher than most of the buildings in the cell. Second, the scattering region will *shrink* in size and may eventually vanish completely as the base station height is increased.

3.5.6 Effect of scattering region size

Figures 3.17-3.20 illustrate the rather pronounced effect of altering the size of the scattering region. The basic rule here is that the larger the scattering area, the lower the correlation. A base-subscriber distance of $m = 1000m$ is employed in all 4 figures. Figures 3.17 and 3.18 were generated using the hollow model with $\delta_0 = \frac{\pi}{2}$ and $\delta_0 = \frac{\pi}{3}$, respectively. Likewise, figures 3.19 and 3.20 were generated using the full model with Malaysia). Such an extreme figure was required to observe a significant change in correlation.

Figure 3.17: Envelope correlation vs. interelement spacing using the hollow model with $\delta_0 = \frac{\pi}{2}$.

$\delta_0 = \frac{\pi}{2}$ and $\delta_0 = \frac{\pi}{3}$, respectively.

3.5.7 Effect of frequency separation

In the LMCS frequency band and as noted earlier, the effect of spatial separation on fading envelope correlation is for all intents and purposes independent from the effect of frequency separation. Both aspects can therefore be studied separately. Consequently, this section illustrates the variations in ρ_e when the spatial separation is zero.

Keeping frequency 1 fixed at 29.5GHz, the correlation coefficient is plotted versus frequency 2, which spans 29.5 GHz-30 GHz, in figures 3.21-3.23. Figure 3.21 is generated using an exponential delay profile with various values of τ_0 , while figures 3.22 and 3.23 correspond to the gaussian and double-spike delay profiles, respectively.

It should be noted also that since the spatial separation is zero, the choice of model (hollow or full) is irrelevant and will not affect the resulting curves.

Figure 3.18: Envelope correlation vs. interelement spacing using the hollow model with $\delta_0 = \frac{\pi}{3}$.

Figure 3.19: Envelope correlation vs. interelement spacing using the full model with $\delta_0 = \frac{\pi}{2}$.

Figure 3.20: Envelope correlation vs. interelement spacing using the full model with $\delta_0 = \frac{\pi}{3}$.

Figure 3.21: Envelope correlation vs. frequency f_2 (f_1 being fixed at 29.5 GHz) for five values of τ_0 using an exponential delay profile.

Figure 3.22: Envelope correlation vs. frequency f_2 (f_1 being fixed at 29.5 GHz) for five values of τ_0 using a gaussian delay profile.

Figure 3.23: Envelope correlation vs. frequency f_2 (f_1 being fixed at 29.5 GHz) for five values of τ_0 using a double-spike delay profile.

3.6 Monte-Carlo simulation issues

The above described propagation models are meant to provide adequate parameterization of a vector channel to generate independent realizations applicable to Monte Carlo simulation. Given a wideband vector channel $\mathbf{c}_0$ of N elements, it can be expanded into a matrix of flat-fading coefficients by dividing the signal bandwidth into N_b “bins” of equal width B_b and of sufficiently small width to ensure flat-fading behavior (i.e. $B_b \ll B_c$ where B_c is the coherence bandwidth). Hence, the resulting frequency-space channel matrix is expressed

$$\mathbf{C}_0 = \begin{bmatrix} c_0(f_{min} + \frac{B_b}{2}) & c_0(f_{min} + \frac{3B_b}{2}) & \cdots & c_0(f_{max} - \frac{B_b}{2}) \end{bmatrix} \quad (3.69)$$

where each element in this $N \times N_b$ matrix is a complex gaussian variate with zero mean in the case of Rayleigh fading and non-zero mean in the case of Rician fading. All entries form a complex joint multigaussian process fully characterized by the matrix of pairwise complex covariances and the vector of means:

$$\mathbf{R}_0 = \langle \text{vec}(\mathbf{C}_0) \text{vec}(\mathbf{C}_0)^H \rangle \quad (3.70)$$

$$\mathbf{m}_0 = \langle \text{vec}(\mathbf{C}_0) \rangle = \begin{cases} 0 & \text{for Rayleigh fading,} \\ \mathbf{f}(K) & \text{for Rician fading,} \end{cases} \quad (3.71)$$

where the i th element of $\mathbf{f}(K)$ is a function of the Rice factor K and is equal to $E_{I_i}^{(i)}$ as defined in the eq. (3.58).

Given $\mathbf{R}_0$ and $\mathbf{m}_0$, generation of channel variates can proceed either by the brute force approach defined by eq. (2.30) or when simulation speed is essential, an orthogonal expansion can be carried out beforehand as discussed under “Broadband

Simulations” in section 2.6.1. More details on the latter are provided in the next chapter.

3.7 Measurements

The proposed channel models are targeted specifically in this thesis at the study of diversity-enhanced millimeter-wave broadband wireless systems with fixed subscribers (i.e. LMCS). However, it appears that in some typical operating environments, there is very little scattering activity and most of the links are characterized by an unobstructed line-of-sight. While such a case might still lead to significant multipath fading effects for some links (since a single reflected path suffices), fading correlation at the array is likely to be very high.

It follows that the proposed propagation models are most useful in the more difficult case of a scattering-rich environment where a significant portion of the links have no clear line-of-sight to the base. This constitutes in many ways a worst-case scenario and therefore its in-depth study is justified.

Subscriber-side LMCS impulse response measurement data characterizing two typical suburban areas was analyzed to determine:

- the degree of scattering activity;
- the distribution of power versus angle of arrival at the subscriber.

The first item aims to determine if and when correlation modeling is required in the LMCS context. The second item is useful in validating and / or refining the isotropic assumption which is central to our proposed models.

The impulse response measurements used for these characterizations were measured in Ottawa in 1997 by the Communications Research Center (CRC) and Carleton

University at 29.5 GHz in the residential areas of Parkwood Hills and Kanata between a typical hub transmitter and many receive locations through the use of directional antennas at both ends. Many of these impulse responses, especially in Parkwood Hills, were from non line-of-sight (NLOS) locations where the direct path from transmitter to receiver was obstructed by trees or buildings.

Another series of impulse response measurements was carried out in Parkwood Hills by CRC and Carleton University in 1999 specifically for the requirements of this thesis. In this case a subscriber-side transmitter with a suitable directional antenna was employed to illuminate a two-element array at the base. In order to characterize fading correlation as a function of spatial separation, impulse response measurements were taken simultaneously from both receive antennas for many transmitter locations and ten different interelement spacings at the base.

3.7.1 Angular sweep measurements

The experimental setup for the 1997 campaign consists of a transmitter equipped with a 23 degree beamwidth antenna located on top of a high-rise building (base) and a narrow-beamwidth transmitter located at one of a number of different locations throughout the “cell” within a radius of 1 km.

Although the objectives of the measurement portion of this thesis are related to the uplink, the 1997 results are useful by virtue of channel reciprocity. In other words, power received at the subscriber site from direction θ is equivalent to the power that would be radiated in that direction by the subscriber and reflected to the base station if the direction of the link was reversed (provided the carrier frequency is the same for both uplink and downlink).

Furthermore, we are interested in characterizing non-LOS sites to see how well the

isotropic scattering assumption made in the NLOS part of the propagation models holds up in a real environment.

Measurements proceed from recording the received signal resulting from transmitting repeatedly a 63-chip pseudo-random bit sequence (PRBS) at a rate of 40 Mchips/s. Taking a channel “shapshot” consists of capturing a portion of the received signal corresponding to 6 consecutive sounding sequences, i.e. 378 chips. Post-processing of this data then yields an estimate of the channel impulse response over a bandwidth of 40 MHz.

At the transmitter, the 40 Mbps PRBS is phase (QPSK) modulated onto a 950 MHz carrier and upconverted successively to 5.2 GHz and 29.5 GHz. This signal is then passed through 1 GHz-wide bandpass filter, amplified to +24 dBm and fed to the transmit antenna. All oscillators in the transmit system are locked onto a single rubidium standard for frequency stability.

At the receiver, the signal is picked up, suitably amplified and downconverted successively to 5.2 GHz and 950 MHz. The 950 MHz IF signal is amplified and applied to an in-phase, 3 dB power splitter whose outputs are mixed with the in-phase and quadrature components of a locally generated 950 MHz carrier for downconversion to baseband. The resulting baseband I and Q components are then amplified before being piped to the input of a 200 megasample per second (100 Ms/s per channel for both I and Q) 8-bit A/D converter. A standard PC is used to control the sampling and the motion of the mechanical turntable controlling the azimuthal bearing of the antenna.

Impulse response estimation

The 63-chip sounding signal is created by QPSK-modulating a 127-bit pseudo-random bit sequence (i.e. 2 bits/chip). The chosen PRBS for channel sounding is part of the class of m-sequences (with $m = 127$) which are characterized by autocorrelation properties that greatly facilitate the process of impulse response estimation. Indeed, the periodic autocorrelation function is two-valued and is given by

$$R[k] = \begin{cases} 1 & k = lN_c, \\ -\frac{1}{N_c} & k \neq lN_c, \end{cases} \quad (3.72)$$

where $N_c = 2^r - 1$ is the length (period) of the sequence. If N_c is large, we have

$$R[k] \approx \sum_{i=-\infty}^{\infty} \delta[k - iN_c]. \quad (3.73)$$

In the noiseless case, the sampled received signal $r[n]$ is simply the discrete convolution of the sounding sequence $p[n]$ and the overall channel impulse response $c[n]$ (assuming for now that the channel is static):

$$r[n] = p[n] * c[n]. \quad (3.74)$$

Convolving both sides by the conjugate of the sounding sequence we get

$$\begin{aligned} r[n] * p^*[n] &= c[n] * p[n] * p^*[n] \\ &\approx c[n] * \sum_{i=-\infty}^{\infty} \delta[k - iN_c] \\ &= \sum_{i=-\infty}^{\infty} c[n - iN_c], \end{aligned} \quad (3.75)$$

which constitutes a sequence of impulse response estimates of the channel provided that the impulse response length is less than the sounding sequence period N_c .

It has been assumed thus far that the sampling rate was equal to the chip rate. For proper accuracy in practice, the sampling rate is generally higher than the chip rate and in the '97 campaign it was in fact a non-integer multiple of the chip rate, i.e. 2.5 samples per chip. To get an integer number of samples per chip the received sequence is first interpolated by a factor of 2, thus yielding 5 samples per chip.

Furthermore, carrying out true convolution with finite sequences leads to undesirable “tail effects”. Therefore, the preferred convolution-based procedure is to

1. first interpolate the received sequence and the sounding sequence to bring them both to the same rate which should be an integer multiple of the chip rate;
2. perform the convolution as a spectral multiplication in the frequency domain, e.g.

$$\tilde{c}[n] \approx \mathcal{F}^{-1}\{\mathcal{F}\{p[n]\} \times \mathcal{F}\{r[n]\}^*\}, \quad (3.76)$$

where $\mathcal{F}\{\cdot\}$ is the discrete Fourier transform and $\mathcal{F}^{-1}\{\cdot\}$ is the inverse discrete Fourier transform.

For our analysis of the 1997 measurement data, it was deemed preferable to utilize least-square estimation of the impulse response in lieu of the above described method. While it is harder to implement, it has the advantage of standing on a formal mathematical foundation and of being optimal (in the least-square sense).

The method we describe was first suggested by Crozier [151] but our development differs somewhat from the original. In the following, we will again assume that the sampling rate is equal to the chip rate.

The received signal can be expressed as follows:

$$r[k] = \sum_{i=0}^{N_c-1} c(iT, kT)p[k-i] + \nu_k, \quad (3.77)$$

where $c(\tau, t)$ is the time-varying impulse response of the channel.

Rewriting the above in vector notation, we have

$$\mathbf{r}_k = \mathbf{P}_k \mathbf{c}_k + \nu_k, \quad (3.78)$$

where

$$\mathbf{P}_k = \begin{bmatrix} p[0] & p[N_c-1] & p[N_c-2] & \cdots & p[1] \\ p[1] & p[0] & p[N_c-1] & \cdots & p[2] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p[N_c-1] & p[N_c-2] & \cdots & \cdots & p[0] \end{bmatrix}, \quad (3.79)$$

and

$$\mathbf{c}_k^T = [c(0, kT), c(T, kT), \dots, c((N_c-1)T, kT)]. \quad (3.80)$$

Ignoring noise, the best least-square fit to this system of equation can be obtained via the pseudo-inverse of $\mathbf{P}_k$:

$$\mathbf{c}_k = \mathbf{P}_k^+ \mathbf{r}_k = (\mathbf{P}_k^H \mathbf{P}_k)^{-1} \mathbf{P}_k^H \mathbf{r}_k. \quad (3.81)$$

Since every received signal snapshot includes six consecutive sequence lengths, it is possible and preferable to process more than one sequence at a time. Indeed, the convolution between channel and sounding sequence is continuous and infinite. Therefore, it is preferable to process a longer sequence in one shot in order to diminish artifacts due to truncation.

Because of prohibitive memory requirements, it was deemed impractical to process all 6 sequences at once. Consequently, processing was performed on blocks of 3 sequences. Furthermore, experiments have shown that using longer blocks in the estimation procedure does indeed provide significantly better accuracy.

This method yields effectively six impulse response estimates. These are then processed using a modified version of a noise thresholding method first described by Sousa et al. [143] which is itself based on CFAR thresholding (Constant False Alarm Rate) previously developed for removing false echoes from radar data [144].

The objective of the noise thresholding method is to remove as much of possible of the thermal noise, keeping only the true channel echoes. If all 1890 samples⁸ in a six-sequence long estimate were due to noise alone, the noise variance could be estimated as one-half of the mean square value of the sample amplitudes. This estimate is almost perfect for our purposes except that it is significantly distorted by the valid echoes whose power is typically much larger.

To reduce the effect of the few very large samples due to valid echoes (they typically appear in less than 5% of samples), the noise power σ_N^2 is estimated based on the median of the samples.

Knowing that the amplitude of a complex white gaussian noise sample $n[k]$ is Rayleigh distributed, the probability that the amplitude exceeds some level ν is given by

$$P(n[k] > \nu) = \exp\left(-\frac{\nu^2}{2\sigma_N^2}\right). \quad (3.82)$$

The estimated median level σ_m can be found by letting $\nu = \sigma_m$ and equating

⁸After interpolation, the total number of samples in one snapshot is 6 sequences $\times$ 63 chips/sequence $\times$ 5 samples/chip = 1890 samples

(3.82) with 0.5, thus yielding

$$\sigma_N = \frac{\sigma_m}{\sqrt{4}} \approx 0.85\sigma_m. \quad (3.83)$$

The standard deviation is thus 85% of the median in the absence of channel echoes. This estimation procedure is robust against the very large samples corresponding to the valid echoes since the median is insensitive to their actual magnitudes. See [143] for further arguments validating this approach.

Once σ_N has been determined, the threshold can be set to $\nu = \eta \cdot \sigma_N$ where η is determined from a specified constant false alarm rate P_{fa} by using (3.82) to give

$$P_{fa} = \exp\left(-\frac{\eta^2}{2}\right). \quad (3.84)$$

The thresholding process can be improved by exploiting the fact that 6 impulse response estimates are available for each channel “snapshot”. A sample point is declared as a valid echo if:

- a. the sample is above the threshold level in all 6 delay profiles;
- b. at least one neighboring point meets condition a.

The value of such a valid sample is computed by taking the mean of the amplitudes and phases of the corresponding samples in all 6 IRs. The remaining points are assumed to be noise only and are set to zero.

Parkwood Hills

The Parkwood Hills area is predominantly populated by one- and two-floor houses, a few high-rise apartment buildings, and roughly 20% tree coverage, both deciduous and

Figure 3.24: Angular power distribution derived from the Parkwood Hills measurements (dBs).

coniferous. The hub transmitter sits atop a tall high-rise apartment building which is itself on a small hill making it one of the highest points in all of Ottawa (approximately 141 m above sea level and 38 m above ground). Measurements were taken at 8 main sites between 0.5 and 1 km from the transmitter. Multiple measurements were taken at each main site at different antenna heights and different locations within the immediate vicinity (up to several meters away from the “central” location). At 52 of these locations, angular sweeps were performed by taking impulse response snapshots at every one of a set of uniformly spaced bearings spanning the entire 360 degrees. For all but three of these locations, measurements were taken at 60 azimuthal bearings in steps of 6 degrees using a receive antenna with a 12 degree beamwidth. At the remaining three locations, a 2 degree beamwidth cassegrain reflector (± 1 deg. in both azimuth and elevation) antenna was employed and therefore measurements were taken at 360 bearings to ensure a complete, unbroken azimuthal scan.

For each bearing, a snapshot of 6 sequence lengths was recorded. An impulse response estimate was produced through the least-squares method. The total power in the impulse response was computed in order to produce the angular power density in figure 3.24. It is noteworthy that there is a discontinuity between 0 and 359 degrees; this is due to the fact that those two measurements are separated by a significant amount of time (the time required to complete the entire sweep) during which the scattering environment may have changed significantly. It shows a significant amount of scattering activity with non-negligible amount of power arriving from virtually all directions. While there are too few measurement sites to postulate an analytical distribution (e.g. using the Kolmogorov-Smirnoff goodness-of-fit procedure) with any reasonable degree of statistical significance, we can still utilize this figure to roughly assess the degree of validity of the isotropic scattering assumption (which is fundamental to both propagation models presented in this chapter).

It can be seen clearly that there is energy impinging from virtually all directions and that no single direction stands out as clearly dominant. These two observations support the isotropic assumption. However, there is some emphasis in the direction of the base and in the direction directly opposite the base, making the overall density shape somewhat elliptical instead of circular (i.e. isotropic).

It should be pointed out that models based on isotropic scattering tacitly assume that the line-of-sight is blocked; this certainly wasn't the case for all measurement sites (about half had an unobstructed LOS). This fact accounts for the extra energy in the vicinity of 0° .

The emphasis near 180° can be explained in one of two ways:

- There is a direct line-of-sight to some large reflectors and / or scatterers behind the subscriber. At these wavelengths, most surfaces are "rough" most of

the time [8] and therefore act like scatterers rather than specular reflectors. However, the direction of arrival being roughly the same as the direction of departure in this case, there is a higher probability of encountering specular behavior ("smooth" surface in the electromagnetic sense) which would reflect more energy than any other bearing (where arrival and departure angles are not so closely aligned).

- There is a non-negligible back-lobe in the antenna pattern so that when it is pointed in the vicinity of 180° , there is some energy leaked from the direct line-of-sight through the back of the antenna.

In the first case, the backside emphasis is real, while in the second case it is illusory, being merely an artifact of the measurement antenna. Either way, the measurement results in light of the above rationale do not disprove the isotropic assumption, but rather provide some support for its validity.

It seems clear that both the isotropic assumption and the propagation models derived herein (which assume a very large number of scatterers) apply best to sites which are characterized by a large degree of random scattering activity (as opposed to sites with only one or two significant paths to the base which would be better modeled in a deterministic fashion). Therefore, the modeling techniques described here may not be appropriate for all typical sites, but rather are targeted at the most problematic sites. Indeed, it is the heavily scattered links which are most likely to have long impulse responses (which may require equalization), and to fade frequently and rapidly. Hence, it makes sense to focus modeling efforts on these problem sites bearing in mind that system parameters derived from such models will most likely be more than adequate for the remaining sites.

For a different perspective, consider figure 3.25 which is an angular power dis-

Figure 3.25: Angular power distribution derived from the Kanata measurements (dBs).

tribution derived from the Kanata measurements where most sites were LOS. The measurement range here was from 1 to 3.5 km. The transmit antenna was mounted on the rooftop of a 30 m high building (which, incidentally, is part of the Newbridge Networks headquarters) and pointed towards a nearby residential area. It is the same vertically polarized horn antenna used in Parkwood Hills. The received antenna was the 2 deg. cassegrain reflector antenna used for part of the Parkwood Hills measurements. A total of 111 measurements were performed, each of which including a 360-step angular sweep. However, only 79 sites had enough receive power to be included in the analysis leading to figure 3.25.

The target area is a mixture of small commercial industrial and commercial buildings, woods, houses and low-rise apartment buildings. Measurements sites were located at 50 ft. intervals along one of three arcs centered on the transmitter. The inner arc is along a circular road which is bordered by a small forest on its inner

side which extends almost all the way to the transmitter. Its outer side marks the beginning of a mostly residential area into which the two larger arcs are drawn.

Because the transmitter antenna is at a relatively low elevation and the signal has to cross the forest to reach the target area, the main mode of propagation is diffraction over tree tops. It is known that this type of propagation favors straight paths so that nearly all energy is emerging from the forest traveling along radial lines extending from the transmitter. Furthermore, there is little vegetation beyond the inner arc. These factors conspire to reduce scattering activity around the subscriber to a minimum, except for the immediate vicinity of the line-of-sight.

This is therefore representative of the kind of area for which the models proposed here are ill-suited.

3.7.2 Correlation measurements

A second series of measurements was carried out in the summer and fall of 1999 in order to validate and/or refine the proposed propagation models. The experimental setup consists of a pair of approximately omnidirectional receiving antennas positioned on top of a high-rise building (base) and a narrow-beamwidth transmitter located in a number of different locations throughout the “cell” within a radius of 1 km (see fig. 3.26).

The measurements were performed by transmitting repeatedly a 511-chip pseudo-random bit sequence (PRBS) at a rate of 40 Mchips/s. Hence, we will be characterizing a 40 MHz channel. Furthermore, the carrier frequency was 29.5 GHz.

Since the PRBS is much longer than in the angular sweep measurements (and this yields a higher SNR for the estimated impulse response), only 4 sequence lengths are captured per snapshot. Also, the impulse response estimates are extracted in this

Figure 3.26: Measurement setup.

case by the correlation method since the more complicated least-squares approach would be both difficult to implement for 511 chips and unnecessary in light of the higher gain provided by a longer PRBS.

We know that the observed correlations are a function of many parameters:

1. interelement spacing;
2. frequency separation;
3. base station height;
4. shape of scattering region;
5. base and subscriber antenna patterns;
6. position of subscriber within cell (distance from base and angle from broadside).

Originally, a large number of transmitter locations (219) were selected in order to provide adequate coverage of the last item. The density of transmitter sites throughout the site was made as uniform as possible except for the end-fire regions where fewer sites were selected. Indeed, those areas are less interesting since it is known that array correlations are likely to be very high and relatively insensitive to interelement

spacing. In fact, for those reasons it is unlikely that an array would be operated with subscribers in its end-fire region where separation of co-channel signals is more difficult and robustness against fading is considerably reduced. A more likely scenario for a linear array is to have it cover a 90-degree sector whose edges are 45 degrees away from broadside. The vast majority of transmitter locations was therefore selected within 45 degrees of broadside.

For each location, measurements have been taken for a set of 10 representative spacings:

$$\{10\lambda, 15\lambda, 20\lambda, 30\lambda, 40\lambda, 50\lambda, 100\lambda, 250\lambda, 500\lambda, 750\lambda\}, \quad (3.85)$$

where λ is the carrier wavelength.

For a given separation and orientation of the pair of base station antennas, the correlations can be computed from the measured data as follows:

1. Record sequence lengths of received data *simultaneously*⁹ at both base station antennas. Repeat the acquisition process 200 times with sufficient time (approx. 5 s [145]) between consecutive snapshots to ensure statistical independence¹⁰
2. Estimate channel impulse response at both elements.
3. Perform noise thresholding.
4. Compute the discrete Fourier transform of both impulse responses.

⁹Measurements can be taken in alternance from one antenna then from the other as long as the entire process is completed within a period significantly shorter than the channel correlation time.

¹⁰Snapshots were actually taken every second in order to bring the data collection time to a manageable level. This will result in very little correlation in most cases where there is a high amount of scattering activity, e.g. if vegetation plays a major role. Otherwise, the presence of correlation is offset somewhat by the large number of snapshots (200).

Figure 3.27: Block diagram of receiver for wideband correlation measurements.

5. Dividing the channel bandwidth into narrow bins (much smaller than the coherence bandwidth), compute bin-to-bin correlations.
6. Compare results with analytical model.

To avoid duplicating the receiver circuitry, a RF switch is used to select which antenna is being observed at any given time. It is held in one position while the 4 sequence lengths are acquired at a sampling rate of 160 MHz (4 samples per chip) and then flipped to the second position for acquisition from the other antenna. There is some “dead time” between the end of the first acquisition period and the beginning of the second since the switching occurs at a rate of 8 kHz. The entire acquisition process takes approximately 176 μ s. The channel can be assumed static over this duration¹¹. A block diagram of the receiver test-set is shown in fig. 3.7.2.

The transmitter set-up (shown in fig. 3.7.2) includes a PRBS generator which

¹¹Previous work has established the channel correlation time at 0.0425s[149]

where a_i is the incident power wave in the i th element and Γ_i is the reflection coefficient defined as

$$\Gamma_i = \frac{\sum_{j=1}^N S_{ij} a_j}{a_i}. \quad (3.87)$$

The reflected power wave is $\sum_{j=1}^N S_{ij} a_j$ and the S_{ij} are the readily measurable S-parameters.

The S-parameters are related to the mutual and self-impedance by the following:

$$\mathbf{S} = (\mathbf{Z} - \mathbf{Z}_c)(\mathbf{Z} + \mathbf{Z}_c)^{-1}, \quad (3.88)$$

where $\mathbf{Z}$ is the mutual impedance matrix and $\mathbf{Z}_c$ is defined

$$\mathbf{Z}_c = \text{diag}(Z_c, Z_c, \dots, Z_c), \quad (3.89)$$

and Z_c is the characteristic impedance of the connecting transmission lines.

The “true” correlation between elements k and l takes only the incident waves into account, i.e.

$$R_{kl} = \langle a_k a_l^* \rangle. \quad (3.90)$$

If the S-parameters are known, the measured correlation can be expressed

$$\begin{aligned} \hat{R}_{kl} &= \langle I_k I_l^* \rangle = \langle a_k a_l^* (1 - \Gamma_k)(1 - \Gamma_l)^* \rangle \\ &= R_{kl} - \sum_{j=1}^N S_{kj} R_{jl} - \sum_{j=1}^N S_{lj}^* R_{jk} + \sum_{i=1}^N \sum_{j=1}^N S_{ki} S_{lj}^* R_{ij}. \end{aligned} \quad (3.91)$$

Therefore, the removal of mutual and self-impedance effects amounts to resolving a system of N^2 linear equations.

In the planned measurements experiments, $N = 2$ and the system to resolve would comprise only 4 equations. Preliminary tests with the network analyzer have shown that for the set of spacings under consideration, there is no measurable mutual coupling, i.e. $S_{12} = S_{21} = 0$ for all intents and purposes. However, the reflection coefficients are quite high (S_{11} in the vicinity of -10 dB in the band of interest) and will induce an antenna-specific bias (i.e. a frequency-dependent gain is measured which is significantly different even between antennas of the same make and model). Therefore, eq. (3.91) applies.

While $\hat{R}_M$ is affected by the self-impedance of the measurement antennas even in the absence of mutual impedance, the normalized correlation coefficients ρ_e and ρ_p are not affected in this case. Indeed, it can be shown that the correction factors due to self-impedance only cancel out in the calculation of correlations.

Details of the transmitter setup

For mobility, the entire transmitter apparatus (as detailed in the block diagram in fig. 3.28) is mounted in a self-contained trailer unit (see photo in fig. 3.29). Power is provided by an on-board operator. For quick setup upon arrival at a designated site, the foldable antenna mast is mounted on an azimuthal, servo-powered turntable. This allows the transmit antenna to be precisely steered in azimuth via computer control. Furthermore, the trailer is provided with hand-operated jacks in order to stabilize the transmitter platform during measurements.

However, such stabilization does not necessarily imply that the platform is level especially if the ground is significantly inclined. Therefore, the base of the antenna mast is also equipped with a two degree-of-freedom articulation which allows the on-board computer to control the lateral and longitudinal inclination of the mast.

Figure 3.29: Trailer unit serving as mobile transmitter.

The turntable assembly and the transmitter up-converter are pictured in figure 3.30. When unfolded, the mast brings the antenna to a total height of 4.9 meters.

The Tx antenna is vertically polarized with a half-power beamwidth of 8 degrees in azimuth. Its pattern is depicted in figure 3.31.

Details of receiver setup

As described in 3.7.2, the unique receiver circuit is shared by the two receive antennas through the use of a fast RF switch. Each antenna is connected to one of the switch's inputs via a 15-foot long piece of high-grade "K" cable.

The antennas themselves are biconicals housed in a cylindrical low-loss dielectric radome approximately 5 inches high and 3 inches in diameter. These are omnidirectional antennas specified for the 12-40 GHz range. Their pattern is depicted in figure 3.32.

Sections of wood planks are assembled together on site using screws and winged nuts to form a flat, linear wooden base which is laid out on the ground a few inches away from the edge of the roof where the receiver is set up. There are 3 mounting holes near one edge of this wooden platform for the fixed, or "reference" antenna. Ten similar sets of mounting holes are provided across the length of the platform to secure the second antenna at any of the ten predetermined positions with respect to the reference antenna (see 3.7.2). Since the largest spacing considered is 750 wavelengths and the wavelength at 29.5 GHz is $3/295$ m, the total length of the platform is nearly 8 meters.

The building selected to act as the base station site in the Parkwood Hills area is the same three-winged apartment tower (Chesterton Towers) used for the angular sweep measurements. As pointed out earlier, it is the highest building in the vicinity.

Figure 3.30: Details of turntable assembly and transmitter up-converter.

Figure 3.31: Azimuth pattern of transmit antenna.

Figure 3.32: Azimuth pattern of receive antennas.

Figure 3.33: Layout of the penthouses on the roof of the Chesterton Towers building.

Indeed, Parkwood Hills is an older residential area where, with the exception of a handful of apartment buildings, nothing exceeds two stories. As seen in figure 3.33, there is an approximately hexagonal-shaped penthouse sitting atop the building's core. The roof of this penthouse stands at a height of 140.7 meters above sea level and it has one edge facing east which is long enough to install the wooden measurement platform.

This is indeed where the platform was installed (as indicated by the thin rectangle in fig. 3.33) to collect data from all the Tx sites located eastwards from Chesterton Towers. The RF switch, receiver circuitry and acquisition computer were all mounted on a small equipment rack. It was positioned as far away behind the antenna platform as possible — given the relatively short length of the K cables — to minimize the

effect of the rack and the human operator on the antenna patterns. The approximate location of the rack is marked by a small black square in fig. 3.33.

A metallic ledge, about 6 inches wide and extending a few inches above roof level, runs along the entire perimeter of the roof. Since this ledge can act as a reflector and disturb the experiment, the antennas were mounted on small masts, about 2 feet high and made of PVC pipes and pipe fittings. These masts, when mounted on the wood platform and provided that the latter is positioned close enough to the ledge (10-20 cm), should provide ample clearance. Indeed, the elevation pattern of the antennas does not extend very far below the horizon (half-power beamwidth is ± 6 degrees) and they will therefore not "see" the ledge.

The picture in figure 3.34 shows the antenna masts, the wood platform and the ledge during operation.

Unfortunately, there were other undesirable objects at this receiver site which could not be dealt with. There is a second square penthouse which sits atop the roof of the first one directly behind the receiver installation (see fig. 3.33). On the northern edge of the roof, a brick chimney stands about 2 meters tall. It is completed by an additional 1.5 meters of metallic tubing (see figure 3.35). Also, a second length of metallic pipe runs horizontally at a height of approximately 1.3 meters from the brick chimney to the middle of the wall of the second penthouse which is situated behind the equipment rack. This can be seen clearly from the aerial photograph in figure 3.36.

For the transmitter sites west of Chesterton Towers, the receiver was installed on top of the second penthouse. From this higher vantage point (144.8 meters above sea level), there are no interfering objects except the rack and the operator. Although the incoming signal has to travel over the entire length of the west wing before reaching

Figure 3.34: The antenna platform in operation atop the Chesterton Towers penthouse.

Figure 3.35: Side view of first penthouse showing chimney.

Figure 3.36: Aerial photograph of Chesterton Towers.

the receiver, the latter is sufficiently high that interfering effects (due to the far edge of the said wing) are not a major concern. It should be noted that there was not enough room to accommodate the full length of the platform. Therefore, the largest spacing (750 wavelengths) was dropped when measuring signals from the western Tx sites.

Data collection

For most Tx sites, data collection was performed according to the six-step sequence detailed in section 3.7.2 for all ten spacings (nine for western sites). However, a large proportion of the selected sites resulted in insufficient power at the base because of the low gain of the omnidirectional receiver antennas and the rapid attenuation of the signal at the wavelength of interest (especially when communicating through vegetation).

After having performed the measurement procedure for the first spacing for a given Tx site, the Rx operator would use the rack-mounted acquisition computer to estimate the impulse response of the channel. If the SNR was deemed too poor (e.g. main peak is less than 10 dB above the noise floor), the remaining measurements at this site were canceled and the Tx operator instructed to proceed to the next site.

Of the 219 sites originally surveyed, 141 were thus rejected in the field due to insufficient power. These are indicated as red dots in the maps in Appendix A while the 68 “valid” sites are marked as green dots.

Of the remaining sites, 22 more were rejected during data analysis because the margin over noise turned out to be insufficient for proper processing.

Data analysis

Numerical trials using the measurement models have shown that the effects of frequency separation and spatial separation are nearly independent in the band of interest. Therefore, the two aspects can be studied separately with minimal impact. Likewise, visual inspection of the correlation matrix estimates calculated from measurements shows that the behavior of the correlation coefficient with respect to frequency spacing is unexpected and apparently random. This is due in part to the presence of a static component which is a vector sum of many static paths and to some limitations of the experimental setup. This will be elaborated upon below.

The frequency correlation behavior could be studied from the measurement data in isolation from the spatial separation aspect by correlating data from a single antenna with itself. However, such results would be no different from those of previous non-array sounding experiments carried out by other researchers. Therefore, the analysis of the measurement data was limited to the spatial aspect. The wideband channel sounding will still be found useful since it will provide enough flexibility to eliminate undesirable perturbations due to the static component, thus increasing overall precision.

Instead of the straightforward channel spectrum, it was decided that analysis would proceed based on the power spectral density of the estimated channel impulse response because:

1. At these frequencies and with the measurement equipment employed, the phase information is unreliable. This is especially true when considering the phase difference between the two channels since they are not actually measured simultaneously but in alternance.

2. The analytical formula used to tie the measurements to a theoretical model is based on power correlation, not envelope correlation. Indeed, it was found that in the presence of a static component, the analysis of envelope correlation was mathematically intractable (see section 3.4).

Given the sample rate of 160 MHz, a 160 MHz wide PSD is obtained after IR estimation, noise thresholding, DFT and squaring. It was found empirically that this PSD usually has a triangular envelope. In fact, its decaying tails have little margin over noise. Because of this and since the essence of the 40 Mchips sounding signal is supported on an interval of 80 MHz (two sidebands of 40 MHz), the upper and lower 40 MHz of the PSD were truncated (i.e. not considered in the processing).

The remaining central 80 MHz for both channels were divided in 25 bins of 3.2 MHz. Since this is narrow enough to be considered flat fading, the content of each bin was averaged down to a single coefficient, thereby allowing the representation of a channel PSD snapshot by a vector of 25 real numbers. Let $\mathbf{v}_{1k}$ denote the length 25 vector representing the k th snapshot (out of 200) on channel 1. Collecting all snapshots for this channel together forms a 25×200 matrix:

$$\mathbf{V}_1 = [\mathbf{v}_{11}, \mathbf{v}_{12}, \dots, \mathbf{v}_{1k}]. \quad (3.92)$$

To study the correlation between channel 1 (ant. 1) and channel 2 (ant. 2), an estimate of the bin-to-bin correlation matrix is computed according to

$$\hat{\mathbf{R}}_{12} = \mathbf{V}_1 \mathbf{V}_2^T. \quad (3.93)$$

One such matrix, for the smallest antenna spacing (10 wavelengths) at site #1 (37 Woodmount st.) is depicted in figure 3.37. What is a priori surprising about it is

Figure 3.37: Correlation matrix $\hat{\mathbf{R}}_{12}$ for a spacing of 10λ with the Tx located at 37 Woodmount.

that it does not peak along the diagonal, i.e. when the frequency separation between channel 1 and 2 is zero.

Looking at all the collected measurement data, it is found that maxima and minima occur anywhere in the matrix with no apparent structure or pattern. This is explainable if we consider the static component. As pointed out earlier, there is practically always a static (i.e. time invariance) component in a fixed wireless system of this kind whether a clear line-of-sight exists or not. This static component is the vector sum of all paths which involve only static scatterers and / or reflectors. It follows that the phase of this component varies across the entire length of the receiving array in an entirely unpredictable manner (as discussed in 3.4). The said phase is also frequency-dependent although again, the relationship vs. frequency is unknowable (unless the position and relative importance of every reflector and scatterer is

known).

We've seen in section 3.4 that the phase difference between the static component at antenna 1 and the same at antenna 2 can have a large impact on the power correlation coefficient. This explains the seemingly random behavior of the correlation matrix estimates.

While the randomness in these matrices is now explained, there still remains to find a way to extract useful information from them. The retained method is rooted in two observations:

1. Oscillatory behavior in frequency (as observed in fig. 3.23) or in space (such as in figs. 3.17 and 3.18) is indicative of a relative scarcity of significant rays in the channel impulse response. Indeed, a small number of components will lead to visible constructive / destructive interference effects as the phase relationships vary. If the number of individual components rises, those interference effects tend to cancel out, thus dampening the oscillations. Despite its assumption of a large number of scatterers, the hollow model leads to this kind of oscillatory behavior because all scattering activity takes place at a fixed distance from the subscriber. In that case, the hollow model is appropriate when we wish to approximate channels with few components while the opposite is true of the full model.
2. There exists a significant static component which is a vector sum of several rays causes unpredictable variations (which are a function of the phase difference ϕ) of the correlation with respect to both separation in space and in frequency.

The oscillations and variations described in both observations are impossible to match exactly in theory using a stochastic approach. In other words the locations

of the peaks and valleys are unpredictable. Therefore, for the purpose of fitting a theoretical curve to the measurements, we will strive to extract the *upper envelope* of the empirical correlation vs. spacing curve.

The naive approach to this would be to find the maximum in each correlation matrix estimate and to plot these vs. spacing. However, it was found that each individual estimate in the correlation matrix is somewhat noisy, despite being computed over 200 snapshots. Therefore, it was found that averaging over a small neighborhood (e.g. 5×5 samples) around the maximum in the matrix was a preferable approach.

For each given site, this approach provides a ten-point correlation curve covering the ten spacings used in the experiment. The challenge is then to find an equivalent theoretical curve using equation (3.67) and the hollow or full model. Since it is not desirable to attempt modeling of the oscillations, the full model was selected. To select the best set of parameters, the following procedure was devised:

1. All parameters which had a known value during the experiment are constrained to the same values for the theoretical curve: the distance from subscriber to base m , the angle to the subscriber δ_0 , the interelement separation Δx and the beamwidth of the Tx antenna.
2. If the measurement log indicates the presence of a line-of-sight, the K factor is initially set based on the distance m according to

$$K(dB) = 24 - 0.032m. \quad (3.94)$$

This formula embodies measurement results obtained in the LMCS band by Papazian et al. [124] in a similar suburban area with foliated trees.

If the measurement log indicates no line-of-sight, the K factor is initially set to 0.

This value of K is approximate since it is known that K can be greater than 0 in the absence of a LOS because of the existence of static indirect paths. Likewise, the real K factor in a LOS situation can be above or below the value given by (3.94) since the formula is merely an indication of the average behavior.

3. The parameter σ_r , which controls the size of the scattering region is manually adjusted in steps of 10 meters until a reasonable fit to the empirical data is achieved.
4. If necessary, the K factor is adjusted to improve the fit.

The empirical and theoretical curves (obtained with the above procedure) for all 44 retained sites are shown in Appendix B.

To measure the goodness-of-fit, commonly employed tools such as the Kolmogorov-Smirnoff statistic are not appropriate since our work focuses on the evaluation of a correlation coefficient as a function of many parameters and not on its distribution¹². Instead, the distributions of various error measures between individual empirical and theoretical points on the correlation curves are relied upon as indicators of the quality of the fit.

The most straightforward error measure is simply the difference between the empirical and the theoretical coefficient, i.e.

$$\varepsilon_1 = \rho_p^{(empirical)} - \rho_p^{(theoretical)}, \quad (3.95)$$

¹²A distribution-oriented approach would have required the number of snapshots at any given site to be increased by an order of magnitude and some amount of spatial averaging involving small displacements of the Tx antenna (i.e. a few wavelengths to a few feet) would have been desirable.

Figure 3.38: Histogram of ε_1 .

and its histogram is plotted in figure 3.38. Except for a few outliers, it is entirely supported on the interval $[-0.5, 0.5]$ and heavily concentrated around $\varepsilon_1 = 0$. Its average is 0.0012 and its standard deviation is 0.1612.

The second error measure considered is the squared error, i.e.

$$\varepsilon_2 = |\varepsilon|^2, \quad (3.96)$$

and the corresponding histogram is depicted in figure 3.39. The average of ε_2 is 0.0259 while its standard deviation is 0.0635.

It is also of interest to consider error measures normalized to the value of the empirical coefficient, i.e.

Figure 3.39: Histogram of ε_2 .

Figure 3.40: Histogram of ε_3 .

$$\varepsilon_3 = \frac{\varepsilon_1}{\rho_p^{(empirical)}}, \quad \varepsilon_4 = \frac{\varepsilon_2}{\rho_p^{(empirical)^2}}, \quad (3.97)$$

Figure 3.40 show the histogram of ε_3 ; its mean is -0.0049 and the standard deviation is 0.1076. Likewise, figure 3.41 depicts the histogram of ε_4 which has a mean of 0.0116 and a standard deviation of 0.0364.

For all these statistics, the point corresponding to zero spacing was not considered since it would have introduced an optimistic bias by virtue of the fact that its error is always 0 (since $\rho_p = 1$ both empirically and theoretically).

For 27 out of the 44 sites, the K factor was set according to (3.94). Of the remainder, 16 sites were processed with $K = 0$ of which only one was marked in

Figure 3.41: Histogram of ε_4 .

the log as an LOS site¹³. Only in two additional instances was it found necessary to modify K from its initial value and then only by a small amount (2 dB and 4.6 dB).

The Tx antenna beamwidth in the model was set at 8 degrees which corresponds to the half-power beamwidth of the antenna used in the field. The front-to-sidelobe ratio was set to 25 dB which is roughly equal to the ratio between the half-power point and the maximum sidelobe level of the field antenna.

Only the parameter σ_r was effectively allowed to vary arbitrarily in order to achieve the best possible fit. It became obvious a posteriori that there was a relationship between the chosen σ_r and the distance from the base. In fact, the σ_r vs. distance plot in figure 3.42 shows clearly that the relationship is linear. The best linear least-squares fit to these points is given by

$$\sigma_r = 17.036 + 0.2105m. \quad (3.98)$$

One possible explanation for this phenomenon arises when the alignment of the Tx antenna in elevation is considered. Indeed, if the Tx site is close to the base, the Tx antenna is tilted slightly upwards. Therefore, a significant fraction of the energy in the main beam will avoid potential scatterers, especially distant ones, simply because the beam picks up altitude as it travels towards the base. Distant sites, on the other hand, have their antenna pointed nearly horizontally and the main beam is projected directly into the suburban maze of vegetation and houses.

Another aspect of the measurement setting at the root of the relation in (3.98) is revealed by observing the relative distances between the Tx and Rx and the scatterers. Consider for example figure 3.43. It shows three Tx sites at different distances from the base station. All three have scattering rings of the same radius. It can be seen

¹³According to the formula, its K factor should have been 9.6 dB.

Figure 3.42: Scattering region size parameter σ_r vs. subscriber-to-base distance m .

Figure 3.43: Three transmitters at different distances from the base.

clearly that for the closest site (site A), most reflected rays will travel a much larger total distance than LOS or near-LOS rays. If the Tx site is a little further away (site B), the differences in length between reflected paths and the LOS path are considerably reduces and they become hardly significant in the case of site C.

It can be concluded that, in general, the closer a transmitter is to the base, the faster reflected rays attenuate as the scattering radius increases.

3.8 Conclusion

This chapter has presented two three-dimensional analytical propagation models for the study of correlations in the fading vector channels that arise at a base station-side receiving antenna array. These models have the following novel characteristics:

1. While the models are fairly general, they were developed and are applied herein to the study of high-frequency broadband wireless systems and more specifically, LMCS-class systems operating in the 28-30 GHz band.
2. In order to study correlations in wideband channels, formulas were derived providing correlation as a function of both separation in space and in frequency. Numerical trials have shown that the effects of frequency separation and spatial separation are nearly independent in the LMCS frequency band. Therefore,

both aspects according to the proposed models can be treated separately with very little impact.

3. To the best of our knowledge, these are the first models of their kind which accommodate an arbitrary antenna pattern at the transmitter. The study of the relation between the Tx antenna beamwidth and the correlation yielded some interesting insights, notably that the correlation does not always rise monotonically with decreasing beamwidth, depending on the FTS ratio.
4. While the geometry of the hollow model has been borrowed from the work of Parsons & Turkmani, the full model is an entirely new scattering geometry which is more realistic since the scattering radius is no longer fixed but variable.
5. A novel formula was derived for the power cross-correlation in the presence of a static component, making both models usable in the representation of both Rayleigh and Rician environments.
6. Measurement data has shown good agreement with the full model; as good as could be expected, given the stochastic nature of the channel and the limitations of the experiment (relatively few sites, no spatial averaging at a given site, etc.). Furthermore, the fitting process has revealed a linear relationship between the scattering region size parameter σ_r and the distance from the base m .

Chapter 4

The Matched-Filter Performance Bound on Optimal Space-Time Processing

Competition is a sin.

John D. Rockefeller

From the sublime to the ridiculous, there is but one step.

Napoléon

The maximum-likelihood diversity receiver is by definition the best possible receiver under two conditions:

1. data symbols are equiprobable;
2. perfect knowledge of the channels is assumed in the past, present and future (see section 2.5.1).

While the maximum-likelihood receiver's implementation complexity is often prohibitive, the matched-filter bound on its performance is a useful benchmark against which to compare linear and non-linear receiver architectures. In fact, it is in that role analogous to the Shannon capacity with the difference that the latter is meant to include the potential power of coding¹.

4.1 The matched-filter bound in Rayleigh fading

Since the matched-filter bound neglects the effects of ISI and CCI, we need only concern ourselves with one vector channel, namely the desired user's. With the goal of performing an orthogonal expansion, we first form a *concatenated array-equivalent channel* $J(f)$ in the frequency domain, including the effects of transmit and receive filters $G_T(f)$ and $G_R(f)$:

$$J(f) = \sum_{n=1}^N H_n(f - 2nW), \quad (4.1)$$

where W is the nominal bandwidth of the channel, N is the number of array elements and

$$H_n(f) = \begin{cases} \sum_{k=-\infty}^{\infty} G_T(f - kW)G_R(f - kW)c_n(f - kW), & |f| \leq W, \\ 0, & \text{elsewhere.} \end{cases} \quad (4.2)$$

This is simply an artificially constructed spectrum made up of the spectra of the individual branches' channels placed in a side-by-side arrangement. It is supported on the interval $[f_{min} = -W, \dots, f_{max} = (2N + 1)W]$. The branch spectra must be

¹While the Shannon capacity is always theoretically attainable, the same is not true of the matched-filter bound

Figure 4.1: System model for maximum-likelihood diversity reception in the absence of CCI.

sufficiently spaced to avoid overlap. The formulation above uses a spacing of $2W$ which is adequate provided the excess bandwidth is no more than 100%.

The standard formulation for the maximum-likelihood receiver requires that the noise be white before the received signal is processed. Implicitly, this means that the receive filter must be chosen to perform the function of *noise whitening*. Now, if $G_R(f)$ is a raised-cosine filter and the noise in the channel is assumed white (i.e. its spectrum is flat over the entire bandwidth B occupied by the signal), then the noise after filtering and symbol-spaced sampling will also be white thanks to the overlapping tails. If on the other hand the sampling rate is higher than the symbol rate, the receive filter would have to be flat over a larger bandwidth in order to avoid coloring the noise.

Alternatively, the receiving filter could be simply left out of the formulation if we assume that it is part of the maximum-likelihood receiver (which is simply a matched-filter in the absence of CCI and ISI). It can be concluded that noise whitening is only an issue if the channel noise spectrum itself is not flat in which case a whitening filter $W(f)$ is necessary (see fig. 4.1).

An orthogonal basis for $J(f)$ can be obtained by applying a Karhunen-Loève

expansion, i.e. solving the integral equation

$$\int_{f_{\min}}^{f_{\max}} R_J(f_1, f_2) \phi_i(f_2) df_2 = \lambda_i^{(J)} \phi_i(f_1), \quad (4.3)$$

where $R_J(f_1, f_2)$ is the autocorrelation function of $J(f)$, the ϕ_i form the sought-after orthonormal basis and the λ_i are the corresponding eigenvalues. Indeed, it is a well-known fact from the theory of integral equations that the eigenfunctions $\phi_i(f)$ are orthonormal and satisfy [142, p. 414]:

$$R_J(f, f) = \sum_{n=1}^{\infty} \lambda_i^{(J)} |\phi_i(f)|^2. \quad (4.4)$$

By dividing the frequency interval supporting $J(f)$ into an appropriate number of bins N_b of equal width Δ_b (where $\Delta_b \ll B_c$), a discrete version of equation (4.3) can be obtained for numerical processing. It also makes sense to limit the number of eigenfunctions to N_b since the discretized channel can have no more than N_b degrees of freedom. Hence, we have:

$$\mathbf{R}_J \mathbf{D} \Phi = \Lambda \Phi. \quad (4.5)$$

where

$$\mathbf{D} = \text{diag} \left[\frac{3}{8} \Delta_b, \frac{7}{6} \Delta_b, \frac{23}{24} \Delta_b, \Delta_b, \Delta_b, \dots, \Delta_b, \Delta_b, \frac{23}{24} \Delta_b, \frac{7}{6} \Delta_b, \frac{3}{8} \Delta_b \right], \quad (4.6)$$

$$\Phi = \begin{bmatrix} \phi_1(f_{\min} + \frac{B_b}{2}) & \phi_2(f_{\min} + \frac{B_b}{2}) & \dots & \phi_{N_b}(f_{\min} + \frac{B_b}{2}) \\ \phi_1(f_{\min} + \frac{3B_b}{2}) & \phi_2(f_{\min} + \frac{3B_b}{2}) & \dots & \phi_{N_b}(f_{\min} + \frac{3B_b}{2}) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(f_{\max} - \frac{B_b}{2}) & \phi_2(f_{\max} - \frac{B_b}{2}) & \dots & \phi_{N_b}(f_{\max} - \frac{B_b}{2}) \end{bmatrix}, \quad (4.7)$$

$$\mathbf{R}_J = \begin{bmatrix} R_J(f_{\min} + \frac{\Delta f}{2}, f_{\min} + \frac{\Delta f}{2}) & R_J(f_{\min} + \frac{\Delta f}{2}, f_{\min} + \frac{3\Delta f}{2}) & \cdots & R_J(f_{\min} + \frac{\Delta f}{2}, f_{\max} - \frac{\Delta f}{2}) \\ R_J(f_{\min} + \frac{3\Delta f}{2}, f_{\min} + \frac{\Delta f}{2}) & R_J(f_{\min} + \frac{3\Delta f}{2}, f_{\min} + \frac{3\Delta f}{2}) & \cdots & R_J(f_{\min} + \frac{3\Delta f}{2}, f_{\max} - \frac{\Delta f}{2}) \\ \vdots & \vdots & \ddots & \vdots \\ R_J(f_{\max} - \frac{\Delta f}{2}, f_{\min} + \frac{\Delta f}{2}) & R_J(f_{\max} - \frac{\Delta f}{2}, f_{\min} + \frac{3\Delta f}{2}) & \cdots & R_J(f_{\max} - \frac{\Delta f}{2}, f_{\max} - \frac{\Delta f}{2}) \end{bmatrix}. \quad (4.8)$$

The non-uniform weighting of the diagonal elements of $\mathbf{D}$ corresponds to a numerical quadrature rule with the same order as the extended version of Simpson's rule [153].

Since the matrices are square, the discretized K-L expansion can be seen as a standard eigendecomposition on the correlation matrix $\mathbf{R}_J$, i.e.:

$$\mathbf{R}_J \mathbf{D} = \Phi^H \Lambda \Phi. \quad (4.9)$$

In the absence of CCI and ISI, the BER is merely a function of the instantaneous bit energy. For a given channel realization, the bit energy is $E_b = Wz$ where

$$\begin{aligned} z &= \sum_{n=1}^N \int_{-\frac{W}{2}}^{\frac{W}{2}} |H_n(f)|^2 df \\ &= \int_{-\frac{NW}{2}}^{\frac{NW}{2}} |J(f)|^2 df. \end{aligned} \quad (4.10)$$

Expressed in terms of the eigenvalues of the channel(s) as defined in eq. (4.3), z becomes

$$z = \sum_{n=1}^N \sum_{i=1}^{\kappa_H} \lambda_i^{(H)} |a_{n,i}|^2$$

$$\begin{aligned}
 &= \sum_{i=1}^{\kappa_J} \lambda_i^{(J)} |b_i|^2 \\
 &= \sum_{i=1}^{N_J} \sum_{j=1}^{\mu_i} \lambda_i^{(J)} |c_{i,j}|^2,
 \end{aligned} \tag{4.11}$$

where the $\lambda_i^{(H)}$ s are the eigenvalues of a single branch channel's frequency correlation function, the $\lambda_i^{(J)}$ s are the corresponding eigenvalues for the concatenated array-equivalent channel $J(f)$, κ_H is the number of non-zero eigenvalues of the $H_n(f)$, κ_J is the number of non-zero eigenvalues of $J(f)$, the $a_{n,i}$ s, b_i s and $c_{i,j}$ s are complex independent unit-variance gaussian variates and μ_i is the algebraic multiplicity of the i th eigenvalue².

It should be noted that in practice, the number of significant eigenvalues κ_J will be relatively small. It corresponds to the number of "degrees of freedom" in the concatenated channel and should be roughly equal to the number of discrete frequency bins required to accurately model the frequency-selective behavior times the number of branches.

It follows that z is a sum of chi-square variables with different variances. It will be referred to henceforth as a *non-uniform chi-square random variable*. If the fading is Rayleigh, the individual chi-square variates are all central and the characteristic function of z is given:

$$\Psi_z(jv) = \prod_{i=1}^{N_J} \left(\frac{1}{1 - j2\pi v \lambda_i} \right)^{\mu_i}. \tag{4.12}$$

Using Heaviside's method [91], the characteristic function can be expanded in partial fractions resulting in

²In practice, one can typically make the simplifying assumption that all eigenvalues have an algebraic multiplicity of 1 since it is usually the case in finite-precision numerical processing.

$$\Psi_z(jv) = \sum_{i=1}^{N_J} \sum_{j=1}^{\mu_i} \frac{A_{i,j}}{(1 - j2\pi v \lambda_i)^j}, \quad (4.13)$$

where

$$A_{i,\mu_i-k} = \frac{1}{(-\lambda_i)^k k!} \left[\frac{\partial^k}{\partial x^k} \left\{ \prod_{\substack{j=1 \\ j \neq i}}^{N_J} \left(\frac{1}{1 - \lambda_j x} \right)^{\mu_j} \right\} \right]_{x=1/\lambda_i}. \quad (4.14)$$

The pdf of z can be obtained by taking the Fourier transform of (4.13):

$$f_z(z) = \mathcal{F}[\Psi_z(jv)] = \sum_{i=1}^{N_J} \sum_{j=1}^{\mu_i} \frac{A_{i,j}}{\lambda_i^j (j-1)!} z^{j-1} e^{-\frac{z}{\lambda_i}}. \quad (4.15)$$

For BPSK and QPSK, the instantaneous BER as a function of z is

$$P_e(z) = \frac{1}{2} \operatorname{erfc} \left[\sqrt{\frac{Wz}{N_0}} \right]. \quad (4.16)$$

The outage probability is then:

$$\begin{aligned} P_{out} = P(P_e \geq P_{e0}) &= \int_0^{z_0} f_z(z) dz \\ &= \sum_{i=1}^{N_J} \sum_{j=1}^{\mu_i} A_{i,j} \left(1 - e^{-z_0/\lambda_i} \sum_{k=0}^{j-1} \frac{(z_0/\lambda_i)^k}{k!} \right), \end{aligned} \quad (4.17)$$

where $P_{e0} = \frac{1}{2} \operatorname{erfc} \left[\sqrt{\frac{Wz_0}{N_0}} \right]$. The above integral was solved using formula (3.351-1) in [154].

The average BER can also be obtained:

$$\begin{aligned}
 \langle P_e \rangle &= \frac{1}{2} \int_0^\infty f_z(z) \operatorname{erfc} \left[\sqrt{\frac{Wz}{N_0}} \right] dz \\
 &= \frac{1}{2} \sum_{i=1}^{N_J} \sum_{j=1}^{\mu_i} A_{i,j} \left[1 - \frac{1}{\sqrt{\pi}} \sum_{k=0}^{j-1} \frac{\Gamma(\frac{1}{2} + k)}{k!} \sqrt{\frac{\frac{W}{N_0} \lambda_i}{\left(\frac{W}{N_0} \lambda_i + 1\right)^{2k+1}}} \right],
 \end{aligned} \tag{4.18}$$

where the integral was solved using integration by parts, the derivative of the error function, the integral of $p_z(z)$ as in (4.17) and formula (3.351-3) in [154].

The average BER formula is a generalization (to include correlated fading) of the one obtained by Clark [25], [57] for uncorrelated fading. It is also consistent with other previous works as reported in [25].

4.2 The matched-filter bound in Rician fading

Obtaining a closed-form expression for the average BER in Rician fading is problematic at best. For example, Yip and Ng [58] have published Rician-fading MFB results with no diversity nor correlation. Their average BER formula is an infinite series which is subject to numerical instability (i.e. does not converge or converges very slowly) for high K factors³. For those cases, their recommended solution is to resort to numerical integration in the characteristic function domain.

The problem is that there is no known closed-form expression for the density of a non-uniform and non-central chi-square r.v.. Indeed, in Rician fading the characteristic function of z is

$$\Psi_z(jv) = \prod_{i=1}^{\kappa_J} \frac{1}{1 - j2\pi v \lambda_i} e^{\frac{jv}{1 - j2\pi v \lambda_i}}, \tag{4.19}$$

³It is noteworthy that the high K factor case is the most relevant for the study of LMCS systems.

and it does not lend itself readily to partial fraction expansion as was done for the Rayleigh-fading case.

However, the more general Nakagami [47] fading distribution is much easier to work with in this context, includes Rayleigh fading as a special case and can closely approach the Rician distribution.

Indeed, the Nakagami distribution is part of the Gamma family of distributions and can therefore lend itself to the partial fractions expansion approach used in 4.1. It was found in [47] that a very good approximation of the Rician distribution can be obtained via the following relationship between the Rice K factor and the Nakagami parameter m :

$$K = \frac{\sqrt{m^2 - m}}{m - \sqrt{m^2 - m}}, \quad m > 1, \quad (4.20)$$

$$m = \frac{(K + 1)^2}{2K + 1} = 1 + \frac{K^2}{2K + 1}. \quad (4.21)$$

The fact that the power of a Nakagami-fading envelope channel can be modeled as a sum of i.i.d. gaussian r.v.'s was observed and exploited in [59] in the analysis of maximal ratio combining in correlated Nakagami channels. However, the analysis in [59] was limited to the narrowband case.

Given a single Nakagami flat-fading signal, the output at branch i is given by

$$x_i(t) = a_i s(t) + n_i(t), \quad (4.22)$$

where the envelope a_i has a marginal pdf given by

$$f_{a_i}(r) = \frac{2}{\Gamma(m_i)} \left(\frac{m_i}{\Omega_i} \right)^{m_i} r^{2m_i-1} e^{-m_i r^2 / \Omega_i}, \quad (4.23)$$

the fading parameter m_i denotes the Nakagami family and $\Omega_i = \langle a_i^2 \rangle$.

By a transformation of r.v.'s, we find that the corresponding instantaneous SNR $\gamma_i = a_i^2 \frac{E_s}{N_{0i}}$ is given by

$$f_{\gamma_i}(x) = \frac{1}{\Gamma(m_i)} \left(\frac{m_i}{\Gamma_i} \right)^{m_i} x^{m_i-1} e^{-m_i x / \Gamma_i}, \quad (4.24)$$

where $\Gamma_i = \langle \gamma_i \rangle = \Omega_i \frac{E_s}{N_{0i}}$ is the SNR averaged over the fading.

It can be seen that the distribution in (4.24) is equivalent to a central chi-square distribution with $2m_i$ degrees-of-freedom and where the variance of the underlying gaussians is $\sigma^2 = \frac{\Gamma_i}{2m_i}$.

It follows that the SNR of a Nakagami fading signal can be modeled as the sum of the squares of m_i i.i.d. *complex* gaussian r.v.'s. It is this fact that makes the Nakagami approximation to Rician fading amenable to the partial fractions expansion approach, provided that m_i is constrained to the integer domain.

We start by forming a concatenated array-equivalent channel $J_1(f)$ as in the Rayleigh case:

$$J_1(f) = \sum_{n=1}^N H_n(f - 2nW), \quad (4.25)$$

where the individual frequency components of $H_n(f)$ are Rayleigh fading.

Furthermore and although this model is Rayleigh fading, we define its autocorrelation function directly from the parameters of the Rician problem at hand. From chapter 3, we know that correlations in the Rician case are separable into a static and a fading component; hence, we have:

$$R_{J_1}(f_1, f_2) = R_f^{(fading)}(f_1, f_2) + R_f^{(static)}(f_1, f_2), \quad (4.26)$$

where the fading and static components are computed as described in chapter 3.

Assuming that the K factor is the same for all branches and for all frequencies⁴ and denoting the equivalent Nakagami m parameter (found from (4.24)) simply as m , we finally form a “meta” concatenated array-equivalent channel:

$$J_m(f) = \sum_{k=1}^m \frac{1}{m} J_1(f - 2(N+1)kW) \quad (4.27)$$

where individual copies of $J_1(f)$ are uncorrelated among themselves. It follows that:

$$\mathbf{R}_J = \begin{bmatrix} \frac{1}{m}\mathbf{R}_{J_1} & 0 & \cdots & 0 \\ 0 & \frac{1}{m}\mathbf{R}_{J_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{1}{m}\mathbf{R}_{J_1} \end{bmatrix}. \quad (4.28)$$

It can be verified that the sum of the squares of the same frequency component in all copies of $J_1(f)$ is in fact the square of a Nakagami r.v with parameter m .

This “meta” concatenated model has the advantage that it is now in the form of an equivalent Rayleigh fading system. It follows that the average BER and the outage probability can be obtained from it by following the procedure outlined in 4.1.

4.3 Numerical Results

4.3.1 Rayleigh fading

Figure 4.2 shows matched-filter bound outage probability curves for a 2-element array in a Rayleigh fading environment obtained from (4.17). Figure 4.3 shows average BER curves in the same context. It should be noted that all outage probability results in

⁴This is a reasonable assumption provided that all antenna elements at the base are omnidirectional and that the receiver’s operation is based on symbol-spaced sampling. Indeed, the sampled spectrum will be “flattened” by virtue of the overlapping tails.

this chapter are based on an outage BER of 10^{-3} . An exponential delay profile is assumed with the RMS delay spread d (normalized to the symbol period) taking on 6 different values. Likewise, the correlation coefficient ρ between the two elements covers a set of 6 values for a total of 36 curves. The $d = 0$ curve corresponds to a flat-fading channel while the improvement observed when d is increased is due to the corresponding increase in intrinsic frequency diversity. In other words, a long RMS delay spread implies more degrees-of-freedom in the frequency-selective fading process resulting in a diversity effect in frequency (in addition to the two-branch spatial diversity) which provides robustness against fading. As noted by Clark [25], [57], the asymptotic limit on this intrinsic diversity performance gain (obtained by letting d tend towards infinity) is the same as the performance for non-fading channels ($C_n(f) = 1$, for all n). This is due to the ergodicity of the $C_n(f)$ (i.e. $R_{C_n}(\Delta f) \rightarrow 0$ as $\Delta f \tau_0 \rightarrow \infty$) which implies that they become deterministic in the limiting case.

The performance decreases with increasing spatial correlation since this reduces the spatial diversity benefit until, when $\rho = 1$, the system becomes equivalent to a single-antenna (non-diversity) receiver. In this case, the second antenna serves only to double the input signal power. Furthermore, for systems which already provide a large amount of frequency diversity, decreasing the spatial correlation from 1 to 0 provides only marginal performance improvement.

In figures 4.4 and 4.5, outage probability and average BER curves are shown as a function of RMS delay spread d and correlation ρ for $E_b/N_0 = 4$ dB and $E_b/N_0 = 6$ dB. These figures illustrate how reducing correlation (whenever possible) can improve performance much the same way that augmenting d would, although up to a certain limit. For example, at $E_b/N_0 = 6$ dB, the same outage probability of 10^{-2} can be obtained with 0 correlation and $d = 0.65$ or with full correlation and $d = 1.55$.

Figure 4.2: MFB outage probability in a 2-element array as a function of E_b/N_0 , delay spread d and correlation ρ .

Figure 4.3: MFB average BER in a 2-element array as a function of E_b/N_0 , delay spread d and correlation ρ .

Figure 4.4: MFB outage probability in a 2-element array as a function of RMS delay spread d and correlation ρ .

Figure 4.5: MFB average BER in a 2-element array as a function of RMS delay spread d and correlation ρ .

4.3.2 Rician fading

Figures 4.6 and 4.7 show outage probability and average BER curves for the two-branch matched-filter bound in a Rician fading environment. Curves are provided for three correlation levels (0, 0.75 and 1) and four values of the Nakagami m -parameter (1, 3, 6 and 14) corresponding to Rician K -factors of 0 (Rayleigh case), 6.48 dB, 10.20 dB and 14.23 dB. It can be seen that a strong static component improves performance in fading channels dramatically. In fact, the $K = 14.23$ dB curve in figure 4.5 is less than 1 dB away from the “no fading” curve.

It is worth observing that with the Nakagami approximation, incrementing the m parameter is equivalent to increasing the number of diversity branches. In fact, the performance is equivalent to that of m mutually independent copies of the array combined together with received power at each branch being divided by m . Hence, it can be concluded that introducing a static component is approximately equivalent performance-wise to increasing the degree of spatial diversity.

4.3.3 Effect of power delay profile shape

It was observed in [25] that the delay profile shape (exponential, gaussian and double-spike) can in some cases have a significant effect on the matched-filter bound performance. Indeed, it was shown therein that the gaussian delay spread provided always slightly better performance (typically by 1 dB or less) than the exponential profile and the gap between the two grew with increasing RMS delay spread and shrank with increasing number of branches. The former effect is easily explained by considering that at very small RMS delay spreads, any delay profile shape tends towards an ideal impulse and therefore very little difference in behavior can be observed. The latter is due to the larger relative importance when N is large of the spatial diversity effect

Figure 4.6: MFB outage probability as a function of K factor, correlation ρ and delay spread d .

Figure 4.7: MFB average BER as a function of K factor, correlation ρ and delay spread d .

with respect to the frequency diversity effect. Indeed, if we consider the characteristic function defining the eigenmodes of the fading process for a single-branch receiver in either the Rayleigh or the Rician case (Nakagami approximation), it will have the form of (4.13). If the number of branches is augmented and there is zero correlation, then the available symbol energy is a sum of N i.i.d. r.v.'s with the said characteristic function. It follows that as N grows large, the distribution of the sum tends towards a gaussian distribution irrespective of the delay profile shape (as captured by the relative weights of the eigenmodes) by virtue of the central limit theorem.

The double-spike profile typically yields performance slightly inferior to the exponential shape. Furthermore, its performance versus the RMS delay spread will reach a limit at the point where the two spikes become far enough apart to be resolvable independently within the receiver's bandwidth⁵. For $B = 1.5$, this limit occurs at $d = \frac{1}{3}$ and this is precisely the case illustrated in figure 4.8. This figure illustrates

- the best achievable performance for the double-spike profile and
- the effect of correlation on the differences in performance among the three delay profiles.

4.3.4 Numerical results based on propagation model

Consider a $2 \text{ km} \times 2 \text{ km}$ square cell divided into 4 sectors (see figure 4.9) where each sector is serviced by a linear array at the base station. Monte-Carlo simulations were performed over the subscriber's position in a sector (assuming that the position is uniformly distributed over the entire surface of the sector) with correlations being computed based on the full model. Both a 2-element array (such as in the lower

⁵The same is also true of any delay profile made up of a few independently fading paths.

Figure 4.8: MFB performance as a function of delay spread shape and correlation ρ in a 2-element array with $d = \frac{1}{3}$.

Figure 4.9: Cell / sector plan for Monte-Carlo simulations with detail of base station array topologies.

sectors in figure 4.9) and a 4-element uniformly spaced linear array (such as in the upper sectors in figure 4.9) were considered for various values of the interelement separation d_x . The base station height was set at 40 m, the scattering region height z_m at 15 m and the subscriber antenna beamwidth at 10 degrees. The carrier frequency was fixed at 29.5 GHz. The scattering region size parameter σ_r varied as a function of distance to base according to the empirically derived relation (3.98). Furthermore, Rayleigh fading was assumed because it constitutes the worst case.

Figure 4.10 and 4.11 show outage probability and average BER performance for a 2 element array for 6 different interelement separations. It is interesting to observe that varying the correlation (by increasing d_x) in this case yields a wider performance range than was observed in figures 4.2 and 4.3 by varying ρ from 0 to 1. This is especially visible for $d = 0$. It is because the simulations employed complex correlation coefficients (computed from the propagation models) and therefore negative correlation was allowed.

Figures 4.12 and 4.13 show similar curves for 4-element, uniformly-spaced linear

(a) $d = 0, 0.25, 1$.

(b) $d = 0.1, 0.5, 2$.

Figure 4.10: MFB outage probability in a two element array as a function of delay spread d and interelement separation d_x .

Figure 4.11: MFB average BER in a two element array as a function of delay spread d and interelement separation d_x .

Figure 4.12: MFB outage probability in a four element array as a function of delay spread d and interelement separation d_x .

arrays as a function of delay spread and interelement spacing. Despite the increased number of branches, the interelement spacing still has a large impact on performance when d is small. However, if we consider these curves in conjunction with the other numerical results in this chapter, we can safely conclude that the impact of the array topology (i.e. d_x) on the MFB performance is much less significant if

- there is a significant static component and / or;
- there is a large amount of intrinsic frequency diversity in the channel (i.e. d is large).

Figure 4.13: MFB average BER in a four element array as a function of delay spread d and interelement separation d_x .

4.4 Conclusion

The analysis of the chapter has yielded matched-filter performance bounds on optimal space-time processing in both Rayleigh and Rician spatially correlated frequency-selective fading. Specifically, the 2 major novel concepts of the analysis are:

- the *concatenated array-equivalent channel* concept which allows us to introduce the effect of correlation on the bounds;
- a treatment of the Rician case which is based on the Nakagami distribution.

Numerical results were presented showing the influence of several parameters (correlation, delay profile shape, K factor) on the MFB for a two-element array. Furthermore, additional results were produced based on the full propagation model of chapter 3 (with empirically validated parameters) comparing the MFB performance of various 2-element and 4-element array topologies in a $2 \text{ km} \times 2 \text{ km}$ square cell with 4 sectors.

A number of insights emerge from the study of the analysis and the graphs herein:

- Increasing the K factor is approximately equivalent, performance-wise, to increasing the degree of spatial diversity⁶.
- Increasing the correlation is equivalent to decreasing the degree of spatial diversity. In the limiting case, N fully-correlated branches are equivalent to a single branch with the signal power being multiplied by N .

⁶This refers to an approximate mathematical equivalence highlighted by the use of Nakagami r.v.'s to approximate Rician fading. This equivalence, which may constitute a convenient insight, does not have any underlying physical meaning.

- Although the interelement spacings in an array can have a large effect on performance (by virtue of the fading correlations thus induced), this effect becomes less significant when the intrinsic frequency diversity is large (say for $d > 0.5$).
- While augmenting the amount of frequency diversity (i.e. making d larger) improves performance according to the MFB, frequency-based diversity is not as readily exploitable by real-world space-time processors as spatial diversity. Indeed, any linear or decision-feedback processor will enhance the thermal noise when attempting to equalize a highly frequency-selective fading channel. Augmenting spatial diversity (either by adding branches or by augmenting the interelement separation, thereby lowering correlation) is therefore preferable to the introduction of additional intrinsic frequency diversity.
- Reducing correlation (when possible) by increasing interelement spacing constitutes an attractive avenue to increase performance since it is cheaper than adding branches and the diversity gain is of the spatial variety, i.e. it is readily and fully exploitable by linear and decision-feedback processors (as opposed to intrinsic frequency diversity).

Chapter 5

Optimal linear and decision-feedback array performance

Nothing in this world is more powerful than an idea whose time has come.

Victor Hugo

This chapter investigates space-time receiver architectures in multi-user wireless systems where optimal idealized (infinite-length) space-time filtering is applied to minimize the mean-square error. First, a novel analysis of multi-user decision-feedback array processing is presented. Numerical results follow comparing the performance of various optimal linear and nonlinear array structures in

- a single-cell generic Rayleigh fading scenario;
- various multi-cell fixed broadband wireless scenarios characterized by Rayleigh or Rician fading, log-normal shadowing and narrow-beamwidth subscriber antennas.

Additionally, a number of parameters have been targeted to determine their impact on the simulation results. Specifically, we will look at the impact of:

- BER evaluation methods;
- the training method and the training sequence length;
- the RMS delay spread;
- the shape of the delay profile (exponential, gaussian or double-spike);
- the amount of fading correlation.

5.1 Cross-decision-feedback arrays

The use of cross-feedback filters in multi-user systems to couple decisions from other users into the desired user's receiver was originally proposed by Duel-Hallen [166] in a CDMA context, and subsequently by Fulghum and Duel-Hallen [167], [73] for narrowband TDMA systems and also by Tidestav et al. [75]. The advantage of this type of system is the potential elimination of all (full cross-feedback coupling) or part of (partial coupling) the post-cursor CCI in addition to the post-cursor ISI (which is suppressed by the standard decision-feedback filter).

The analytical contribution of this section is the derivation leading to an expression for the minimum achievable MSE by space-time processors equipped with cross-feedback filters in addition to standard DF, i.e. self-feedback filters. This analysis was previously the object of two conference papers: In [159], the *fully-connected* case (where cross-feedback filters are implemented for all significant interferers) was addressed while in [168], the analytical treatment was extended to include partial connectivity systems. The end result is a formula expressing the minimum-achievable

MSE as a function of channel spectra in the same spirit as (2.29) and other previously derived MMSE formulas [60], [69], [70].

Where decision-feedback is concerned, these derivations have traditionally relied on *spectral factorization theory* [160] to solve a Wiener-Hopf system of linear equations of the form

$$r[k] = a[k] + \sum_{l=-\infty}^0 r[k-l]a[l], \quad \text{for } k \leq 0. \quad (5.1)$$

where $r[k]$ is a correlation function. This approach was originated by Salz [60] who in turn was inspired by earlier work on zero-forcing decision-feedback equalization by Price [65].

For our purposes, it has been necessary to work with a matrix version of (5.1). To our knowledge, the only previous work where a matrix Wiener-Hopf analysis was presented in a similar context is a paper by Falconer and Foschini [161] on DFE applied to QAM systems. In that case, the 2×2 matrix structure is due to the independent treatment of the two quadrature carriers. In this derivation, the $N \times N$ matrix structure is necessary because all filtered impulse responses are anticausal thanks to the cross-feedback filters.

We proceed by initially treating the single-antenna case. The resulting formula is extended to space-time receivers by applying the single channel equivalent model first proposed in [68] and employed in [69].

Under consideration is a wireless communications system with an M -element array at the receiver and N co-channel interferers (in addition to desired signal 0) as illustrated in figure 5.1. The received signal is expressed in vector notation as:

Figure 5.1: Matrix channel model for space-time reception in a multi-user system.

$$\mathbf{x}(t) = \sum_{n=0}^N \sum_{k=-\infty}^{\infty} \mathbf{c}_n(t - kT) s_n[k] + \mathbf{n}(t), \quad (5.2)$$

where

$$\begin{aligned} \mathbf{x}(t) &= [x_1(t), \dots, x_M(t)]^T, \\ \mathbf{c}_n(t) &= [c_{n1}(t), \dots, c_{nM}(t)]^T, \\ \mathbf{n}(t) &= [n_1(t), \dots, n_M(t)]^T, \end{aligned}$$

and $s_n[k]$ is user n 's k th transmitted symbol.

The overall channel impulse response $c_{nm}(t)$ between user n and the output of array element m includes the radio channel impulse response $h_{nm}(t)$ and the transmit pulse shaping filter $g_t(t)$. The receive pulse shaping filter, if any, is absorbed into the feedforward filter and is thus not part of the channel response. Hence:

$$c_{nm}(t) = g_t(t) * h_{nm}(t), \quad (5.3)$$

where $*$ denotes convolution.

In a one-antenna, $N + 1$ user system, the equalizer / temporal processor for user 0 is shown in figure 5.2. Although such an architecture with a single antenna and $N \gg 0$ would not offer interesting performance, it is studied as a stepping stone in the analysis before proceeding to the more adequate multi-antenna case.

The output of the n th feedforward filter is

$$y_n(t) = \int w_n(t) x(t - \tau) d\tau. \quad (5.4)$$

In what follows, it will be assumed that feedback filters are implemented for the first N' signals (where $N' < N$) in addition to the desired signal. Therefore, the

Figure 5.2: Portion of a one-antenna, $N + 1$ -user cross-decision-feedback receiver structure corresponding to signal 0.

remaining $N - N'$ interfering signals can only be reduced by the feedforward filter.

It follows that the output of equalizer 0, prior to slicing, is

$$\begin{aligned}
 \tilde{s}_0[k] &= y(kT) - \sum_{n=0}^{N'} \sum_{i=1}^{\infty} f_{n0}[i] \tilde{s}_0[k-i] \\
 &= \sum_{n=0}^N \sum_{i=-\infty}^{\infty} \int w_0(\tau) c_n(iT - \tau) d\tau s_n[k-i] + \int w_0(\tau) n(kT - \tau) d\tau \\
 &\quad - \sum_{n=0}^{N'} \sum_{i=1}^{\infty} f_{n0}[i] \tilde{s}_0[k-i].
 \end{aligned} \tag{5.5}$$

Assuming all decisions are correct, the optimal choice for the classical feedback filter is the one that completely eliminates postcursor ISI:

$$f_{00}[k] = \int w_0(\tau) c_0(kT - \tau) d\tau. \tag{5.6}$$

Likewise, the optimal choice for the cross-feedback filter $f_{n0}[k]$ will completely eliminate postcursor CCI caused by signal n :

$$f_{n0}[k] = \int w_0(\tau) c_n(kT - \tau) d\tau, \quad n = 0, \dots, L. \tag{5.7}$$

The MSE at the equalizer output is then

$$\begin{aligned}
 \varepsilon \{w_0(t)\} &= \langle |\tilde{s}_0[k] - s_0[k]|^2 \rangle \\
 &= \sigma_s^2 \left| \int w_0(\tau) c_0(-\tau) d\tau - 1 \right|^2 \\
 &\quad + \sigma_s^2 \sum_{i=-\infty}^{-1} \left| \int w_0(\tau) c_0(iT - \tau) d\tau \right|^2 + \sigma_s^2 \sum_{n=1}^{N'} \sum_{i=-\infty}^0 \left| \int w_0(\tau) c_n(iT - \tau) d\tau \right|^2 \\
 &\quad + \sigma_s^2 \sum_{n=N'+1}^N \sum_{i=-\infty}^{\infty} \left| \int w_0(\tau) c_n(iT - \tau) d\tau \right|^2 + N_0 \int |w_0(\tau)|^2,
 \end{aligned} \tag{5.8}$$

where $\sigma_s^2 = \langle |s_n[k]|^2 \rangle, \forall n, k$. Based on this expression, calculus-of-variations techniques can be employed to obtain the optimal $w_0(t)$, i.e. the $w_0(t)$ which minimizes the MSE, which is expressed

$$w_0^{\text{opt}}(t) = \frac{\sigma_s^2}{N_0} c_0^*(-t) - \sum_{n=0}^{N'} \sum_{k=-\infty}^0 a_n^{(0)}[k] c_n^*(kT-t) - \sum_{n=N'+1}^N \sum_{k=-\infty}^{\infty} a_n^{(0)}[k] c_n^*(kT-t), \quad (5.9)$$

where

$$a_n^{(0)}[k] = \int w_0^{\text{opt}}(\tau) c_n(kT - \tau) d\tau. \quad (5.10)$$

Multiplying both sides of (5.9) by $c_n(kT - t)$, integrating and using definition (5.10), we obtain

$$a_i^{(0)}[k] = r_{i0}[k] - \sum_{l=-\infty}^0 \sum_{j=0}^{N'} a_j^{(0)}[l] r_{ij}[k-l] - \sum_{l=-\infty}^{\infty} \sum_{j=N'+1}^N a_j^{(0)}[l] r_{ij}[k-l], \quad (5.11)$$

where

$$r_{ij}[k] = \frac{\sigma_s^2}{N_0} \int c_i(kT - \tau) c_j^*(t) dt. \quad (5.12)$$

By virtue of the Poisson sum formula, we can express the above correlation function in the frequency domain as follows:

$$r_{ij}(\omega) = \sum_{l=-\infty}^{\infty} r_{ij}(l) e^{-j\omega l T} = \sum_{k=-\infty}^{\infty} \frac{\sigma_s^2}{N_0 T} C_i \left(\frac{\omega}{2\pi} - \frac{k}{T} \right) C_j^* \left(\frac{\omega}{2\pi} - \frac{k}{T} \right). \quad (5.13)$$

We will denote the Fourier transform of the one-sided sequences $a_n^{(0)}[k], (n = 0, \dots, N')$ and the two-sided sequences $a_n^{(0)}[k], (n = N' + 1, \dots, N)$ respectively, as

$$\begin{aligned}
a_n^{-(0)}(\omega) &= \sum_{k=-\infty}^0 a_n^{(0)}[k]e^{-jk\omega}, \quad \text{for } n = 0, \dots, N', \\
a_n^{(0)}(\omega) &= \sum_{k=-\infty}^{\infty} a_n^{(0)}[k]e^{-jk\omega}, \quad \text{for } n = N' + 1, \dots, N.
\end{aligned} \tag{5.14}$$

We can then rewrite (5.11) in the frequency domain:

$$a_i^{(0)}(\omega) = - \left\{ \left(a_0^{-(0)}(\omega) - 1 \right) r_{i0}(\omega) + \sum_{j=1}^{N'} a_j^{-(0)}(\omega) r_{ij}(\omega) + \sum_{j=N'+1}^N a_j^{(0)}(\omega) r_{ij}(\omega) \right\}, \tag{5.15}$$

which can be conveniently rewritten as two vector equations:

$$\mathbf{b}^{(0)}(\omega) = - \left\{ \left(a_0^{-(0)}(\omega) - 1 \right) \mathbf{r}(\omega) + \mathbf{R}_A(\omega) \mathbf{c}^{(0)}(\omega) + \mathbf{R}_B(\omega) \mathbf{b}^{(0)}(\omega) \right\}, \tag{5.16}$$

where

$$\begin{aligned}
\mathbf{r}(\omega) &= [r_{N'+1,0}(\omega), \dots, r_{N,0}(\omega)]^T, \\
\mathbf{b}^{(0)}(\omega) &= [a_{N'+1}^{(0)}(\omega), \dots, a_N^{(0)}(\omega)]^T, \\
\mathbf{c}^{(0)}(\omega) &= [a_1^{-(0)}(\omega), \dots, a_{N'}^{-(0)}(\omega)]^T, \\
\mathbf{R}_A(\omega) &= [r_{ij}(\omega)]_{j=1, \dots, N'}^{i=N'+1, \dots, N}, \\
\mathbf{R}_B(\omega) &= [r_{ij}(\omega)]_{j=N'+1, \dots, N}^{i=N'+1, \dots, N},
\end{aligned}$$

and

$$\mathbf{d}^{(0)}(\omega) = -\mathbf{R}_C(\omega) \mathbf{d}^{(0)}(\omega) + \mathbf{R}_C(\omega) \mathbf{u} - \mathbf{R}_D(\omega) \mathbf{b}^{(0)}(\omega), \tag{5.17}$$

where

$$\begin{aligned}
\mathbf{u} &= [1, 0, \dots, 0]^T, \\
\mathbf{d}^{(0)}(\omega) &= \left[a_0^{-(0)}(\omega), \dots, a_{N'}^{-(0)}(\omega) \right]^T \\
&= \left[a_0^{(0)}(\omega), \mathbf{c}^{(0)}(\omega)^T \right]^T, \\
\mathbf{R}_C(\omega) &= [r_{ij}(\omega)]_{j=0, \dots, N'}^{i=0, \dots, N'}, \\
\mathbf{R}_D(\omega) &= [r_{ij}(\omega)]_{j=N'+1, \dots, N}^{i=0, \dots, N'}.
\end{aligned}$$

From (5.16), we have

$$\mathbf{b}^{(0)}(\omega) = \left[1 - a_0^{-(0)}(\omega) \right] [\mathbf{R}_B(\omega) + \mathbf{I}]^{-1} \mathbf{r}(\omega) - [\mathbf{R}_B(\omega) + \mathbf{I}]^{-1} \mathbf{R}_A(\omega) \mathbf{c}^{(0)}(\omega), \quad (5.18)$$

which, by letting

$$\mathbf{Q}(\omega) = [\mathbf{R}_B(\omega) + \mathbf{I}]^{-1} \mathbf{R}_D^H(\omega), \quad (5.19)$$

and noting that

$$\mathbf{R}_D^H(\omega) = [\mathbf{r}(\omega), \mathbf{R}_A(\omega)], \quad (5.20)$$

leads to the expression

$$\mathbf{b}^{(0)}[k] = \mathbf{Q}[k] \mathbf{u} - \sum_{l=-\infty}^0 \mathbf{Q}[k-l] \mathbf{d}^{(0)}[l]. \quad (5.21)$$

After substituting the above in (5.17), we get

$$\mathbf{d}^{(0)}(\omega) + [\mathbf{R}_C(\omega) - \mathbf{V}(\omega)] \mathbf{d}^{(0)}(\omega) = [\mathbf{R}_C(\omega) - \mathbf{V}(\omega)] \mathbf{u}, \quad (5.22)$$

where

$$\begin{aligned} \mathbf{V}(\omega) &= \mathbf{R}_D(\omega)\mathbf{Q}(\omega) \\ &= \mathbf{R}_D(\omega) [\mathbf{R}_B(\omega) + \mathbf{I}]^{-1} \mathbf{R}_D^H(\omega). \end{aligned} \quad (5.23)$$

Letting $\mathbf{X}(\omega) = \mathbf{R}_C(\omega) - \mathbf{V}(\omega)$ and $\mathbf{X}[k] = \mathcal{F}^{-1}[\mathbf{X}(\omega)]$ (where $\mathcal{F}^{-1}[\cdot]$ denotes the inverse Fourier transform), we have

$$\mathbf{d}^{(0)}[k] + \sum_{l=-\infty}^0 \mathbf{X}[k-l]\mathbf{d}^{(0)}[l] = \mathbf{X}[k]. \quad (5.24)$$

Combining this equation with the same from the other equalizers, we get

$$\mathbf{D}[k] + \sum_{l=-\infty}^0 \mathbf{X}[k-l]\mathbf{D}[k] = \mathbf{X}[l], \quad (5.25)$$

where

$$\mathbf{D}[k] = [\mathbf{d}^{(0)}[k], \mathbf{d}^{(1)}[k], \dots, \mathbf{d}^{(N)}[k]], \quad (5.26)$$

which is a matrix Wiener-Hopf system of equations. Such a system is solvable via spectral factorization theory (see Appendix C) and yields the MMSE formula:

$$\epsilon_0^{\text{opt}} = \sigma_s^2 \left[\exp \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln [\mathbf{X}(\omega) + \mathbf{I}] d\omega \right\} \right]_{11}^{-1}, \quad (5.27)$$

where $\exp[\cdot]$ and $\ln[\cdot]$ are scalar operators mapped to a matrix, e.g.

$$\exp[\mathbf{A}] = \begin{bmatrix} \exp(a_{11}) & \cdots & \exp(a_{1P}) \\ \vdots & \ddots & \vdots \\ \exp(a_{P1}) & \cdots & \exp(a_{PP}) \end{bmatrix}, \quad (5.28)$$

and $[\cdot]_{11}^{-1}$ is element (1,1) of the inverse of its matrix argument. By changing the ordering of the users in defining $\mathbf{X}(\omega)$ and by virtue of symmetries inherent to a

matrix inversion, it can be verified that the other elements along the diagonal of the inverted matrix correspond to the MMSE's of the other N' users involved in the feedback loop. Hence:

$$\epsilon_i^{\text{opt}} = \sigma_s^2 \left[\exp \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln [\mathbf{X}(\omega) + \mathbf{I}] d\omega \right\} \right]_{ii}^{-1}, \quad \text{for } i = 0, \dots, N', \quad (5.29)$$

provided that each of the other N' users' processor is also fed by crossfeedback filters from all the same $N' + 1$ users (and none other).

It follows that the average MMSE over all $N' + 1$ users' processors is simply

$$\bar{\epsilon}^{\text{opt}} = \frac{\sigma_s^2}{N' + 1} \text{tr} \left[\exp \left\{ \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln [\mathbf{X}(\omega) + \mathbf{I}] d\omega \right\} \right]^{-1}, \quad \text{for } i = 0, \dots, N'. \quad (5.30)$$

While the above results (i.e. (5.27) and (5.30)) were derived for a single-antenna system, it can be readily shown as before by virtue of the *single channel equivalent model* (see [159] and [69]) that it remains valid in the multi-antenna case. The only difference is that the correlation matrices are then defined based on

$$r_{ij}(\omega) = \frac{\sigma_s^2}{N_0 T} \sum_{m=1}^M \sum_{k=-\infty}^{\infty} C_{i,m} \left(\frac{\omega}{2\pi} - \frac{k}{T} \right) C_{j,n}^* \left(\frac{\omega}{2\pi} - \frac{k}{T} \right), \quad (5.31)$$

where M is the number of antennas at the receiver and $C_{i,m}(\omega)$ is the Fourier transform of the channel impulse response from user i to array element m .

In [70], a procedure is devised (through a combination of analysis and empirical fitting) to compute the number of feedforward and feedback filter taps required for a DFE space-time processor to have near optimal (i.e. infinite-length) performance. Specifically, the required number of feedforward filter taps is expressed:

$$N_0^{(ff)} = C_0 + A_0 + 1, \quad (5.32)$$

where C_0 is the number of taps in the *causal* portion of the filter, A_0 is the number of taps in the *anticausal* portion and

$$C_0 \approx (L - 1)N \frac{\bar{\gamma}}{10}, \quad (5.33)$$

$$A_0 \approx (L - 1) + (L - 1)(N + 1) \frac{\bar{\gamma}}{10}, \quad (5.34)$$

where N is the number of co-channel interferers, L is the total length of the impulse responses in symbol periods and $\bar{\gamma}$ is the average SNR (in decibels) at the input of the processor.

It follows that the required length of the feedback filter to completely cancel the postcursors of the desired signal is $L - 1 + C_0$.

It was demonstrated in [70] that these filter lengths are sufficient to yield BER performance within 5% of the infinite-length level.

Observing that a multi-user MMSE DF processor is equivalent to a standard DF processor once the postcursors of the interferers have been removed, the above result can be readily extended to the multi-user decision-feedback case. Hence, the number of feedforward filter taps is given by (5.32) and in general the number of taps in a feedback filter coupling decisions from user n to user 0 is

$$N_{n0}^{(fb)} = L - 1 + C_0, \quad (5.35)$$

where L_n is the length in symbol periods of the impulse response of the n th user's channel. This is sufficient to completely cancel the postcursor portion of interferer n .

5.2 Single-cell Generic Numerical Results

Monte Carlo simulations were performed for a 4-antenna, 4-user system comparing the performance of linear, decision-feedback and cross-decision-feedback space-time processing with varying degrees of cross-feedback connectivity. Channel spectra were efficiently produced at each iteration using a set of precomputed orthogonal basis functions [25]. Furthermore, Rayleigh fading was assumed with uncorrelated envelopes between the elements. Unless otherwise noted, all channels are characterized by an exponential delay profile with a RMS delay spread of 1 symbol duration.

5.2.1 Effect and selection of BER evaluation method

In [25], numerical experiments have shown that in most instances of linear wideband S-T processing:

1. There is very little difference between various BER evaluation methods and the exact BER (as calculated using Metzger's method — see section 2.6.4). The notable exception occurs when the RMS delay spread d is very small, e.g. $d < 0.5$. In other words, the “spread” between any BER evaluation methods augments with the reduction in RMS delay spread since such a reduction implies a reduction in the number of significant residual interference components (see 2.6.4).
2. The peak-limited gaussian method in particular gives nearly exact performance in all cases.

Figure 5.3 compares three BER evaluation methods (peak-limited gaussian, gaussian and Saltzberg's bound) for a linear S-T processor when $d = 0.1$. High-precision

methods (such as Metzger's and the method of moments) are noticeably absent from the comparison because they bring very little precision improvements above the peak-limited method (according to empirical results such as those in [25]).

Therefore, given the objectives of this thesis and the lack of tangible precision gain (as indicated by various empirical experiments), the additional implementation complexity of high-precision methods was deemed unwarranted. Additionally, figure 5.3 shows that even with a very small d , the gaussian method is almost as precise as the peak-limited gaussian method. Based on these observations and in light of the added simulation / computation time required by the peak-limited method, the gaussian method was selected for all subsequent results in this chapter.

5.2.2 Single-cell 2-antenna, 2-user system

Figure 5.4 compares the performance of a handful of S-T processing strategies in a 2-antenna system in the presence of a single interferer when the signal-to-interference ratio (SIR) is varied between -20 and 10 dB. White noise power is kept at 1% of the interference power, i.e. $\text{INR} = P_1/\sigma^2 = 20$ dB. The "linear", "df" and "xdf" curves stand for linear, decision-feedback and 2-user cross-decision-feedback s-t processing, respectively. Two additional curves are included as benchmarks: the "df-no CCI" curve represents DF performance in the absence of CCI while the "mfb" (matched filter bound) curve is an optimal bound on maximum-likelihood space-time processing based on the assumption that all CCI and ISI has been successfully eliminated by the receiver while the total desired signal energy remains exploitable. It should be noted that the two benchmark curves are not plotted versus SIR (since there is no interference) but rather versus SNR which, according to our INR figure of 20 dB, ranges from 0 to 30 dB on the horizontal axis.

(a)

(b)

Figure 5.3: Performance of a linear space-time processor produced with 3 different BER evaluation methods with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of $d = 0.1$ symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.4: Performance of various space-time processors with 2 elements and 1 interferer as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol duration; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

In the single interferer case, it can be observed that cross-decision feedback brings an improvement over decision-feedback processing which is about half as important (in terms of SIR gain in dB) as the improvement gained in moving from linear to decision-feedback processing.

5.2.3 Single-cell 4-antenna, 4-user system

Figure 5.5 compares the performance of various space-time processing strategies in the presence of three equal-power interferers when the signal-to-interference ratio (SIR) is varied between -20 and 10 dB, i.e. $P_1 = P_2 = P_3$ and $\text{SIR} = P_0/P_1$. This corresponds to a single-cell SDMA scenario with 4 co-channel users where the transmit powers are adjusted to compensate the differences in propagation loss (i.e. static power control). The S-T strategies considered form a superset of the ones treated in the 2-user case. The “linear”, “df”, “df-no interf.” and “mfb” curves are as defined previously. The “x2df” and “x3df” curves represent partially-connected XDF processors with 2 and 3 feedback filters, respectively (classical decision-feedback + 1 or 2 cross-feedback filters) while “xdf” stands for a fully-connected cross-decision-feedback processor.

For the “x2df” and “x3df” processors, two strategies were used to assign feedback filters to interferers. Either the feedback filters are assigned dynamically to the strongest interferer(s) first (“x2df-dyn.” and “x3df-dyn.” curves), or the assignment is simply performed at random or is static (“x2df-stat.” and “x3df-stat.” curves)¹. It can be seen that dynamic optimal assignment affords a slight performance gain with respect to random assignment.

If we consider dynamic assignment only, we see that moving from “df” to “x2df”

¹Since all interferers have the same average power in this scenario, random and static assignment of cross-feedback filters constitute equivalent strategies performance-wise

(a)

(b)

Figure 5.5: Performance of various space-time processors with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol duration; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

yields an SIR gain roughly equal to the SIR gain obtained by moving from linear to “df”. Likewise, the SIR gains between “x3df” and “x2df” and between “xdf” and “x3df” are of the same order. Furthermore, the “xdf” curve is less than 1 dB away from the CCI-free decision-feedback curve.

Figures 5.6 and 5.7 illustrate the performance of the same set of space-time processors for $d = 0.5$ and $d = 2$, respectively. In all three cases, ($d = 0.5, 1, 2$) approximately the same SIR spread is observed between linear and CCI-free decision-feedback curves. In the $d = 0.5$ case, the “xdf” curve is a bit further away (about 2 dB) from the CCI-free DF curve. This is logical since, with shorter impulse responses, the feedback filters can’t remove as much CCI and ISI energy. Furthermore, we notice that the entire curve set suffers a 1 dB loss when $d = 0.5$ versus $d = 1$ while with $d = 2$ we observe a 1 dB gain. This is simply due to the fact that longer impulse responses offer more intrinsic frequency diversity which the S-T processors can exploit to combat fading.

5.2.4 Impact of filter lengths

While all other results in this section are based on infinite-length filters, this section will look specifically at the degradation in performance suffered by linear processors when the filters are truncated at a given point. Symbol-spaced taps are assumed and the system under study is a 4-element array with 4 users. Figure 5.8 shows that a filter length of 11 is enough to get nearly optimal performance.

(2)

(b)

Figure 5.6: Performance of various space-time processors with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of $\frac{1}{2}$ symbol duration; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.7: Performance of various space-time processors with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 2 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.8: Performance of linear space-time processors with 4 elements and 3 interferers as a function of SIR and of filter length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

5.3 Impact of training

We now examine the impact of training on a 4-element, 4-user processor with finite filter lengths. However, some reflexion is required before PDF formulas from 2.65 (which are derived for narrowband processors) can be applied.

As far as the adaptation algorithm is concerned, a finite-length equalizer with N_T taps is essentially the same as a narrowband array with as many branches. It follows that a space-time processor with M elements and N_T taps / branch adapted via SMI will have the same normalized SINR PDF as an equalizer with MN_T taps.

The figures in this section show performance curves for SMI adaptation at various percentile points. To produce these, we relied on the CDF of the normalized SINR ρ . Since ρ is beta-distributed, a closed-form finite-series expression for the CDF of a beta distribution was derived and it is outlined in Appendix F.

Figures 5.9, 5.10 and 5.11 illustrate the impact of training loss on both average BER and outage probability for filter lengths N_T of 3, 6 and 11 taps, respectively.

The figures show that an average loss of 8-10 dB with respect to the asymptotic case (i.e. $K \rightarrow \infty$) is to be expected when the length K of the training sequence is approximately equal to the total number of taps MN_T . This loss is reduced to 2-3 dB when the length of the training sequence is doubled.

5.4 Diversity order / XDF connectivity tradeoff

For the same context (4 equal-power users), figure 5.12 compares the “linear”, “x2df” and “xdf” receivers for $M = 2$ to 5. It shows that the “xdf” receiver with M elements achieves significantly better performance than the linear receiver with $M + 1$ elements. In fact, “xdf” with $M = 3$ exhibits performance which is almost as good

(a)

(b)

Figure 5.9: Performance of linear space-time processors with 3 taps per element, 4 elements and 3 interferers as a function of SIR and of training sequence length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.10: Performance of linear space-time processors with 6 taps per element, 4 elements and 3 interferers as a function of SIR and of training sequence length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.11: Performance of linear space-time processors with 11 taps per element, 4 elements and 3 interferers as a function of SIR and of training sequence length when impulse responses are characterized by an exponential delay profile and an RMS delay spread of 1 symbol durations; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

as a linear processor with 5 elements (and, incidentally, slightly superior to “x2df” with 4 elements). These observations are not only valid in an average sense but also hold in the outage probability sense, as shown in 5.12(b).

Therefore, in this specific scenario (4 equal-power interferers in a Rayleigh-fading environment), the implementation of fully-connected decision-feedback is better, performance-wise, than adding one element to an equivalent linear processor, in addition to avoiding the receiver-hardware replication that the latter implies.

It should be noted that for the “x2df” curves, we have considered only random assignment² in order to avoid idealizing the performance of the x2df processor. Indeed, although dynamic assignment involves little additional cost / complexity, it may not always be feasible. For example, decisions from some interferers may not be available because they are communicating with another base station (i.e. the system employs macrodiversity / overlapping cells). Or, if the system occupies an unlicensed band, there might be co-channel interferers inside the cell which are in fact part of an independent, competing system. Finally, decisions from the strongest interferer might be known to be unreliable (thanks to error-detection coding; or the said interferer might itself have been assigned to a less powerful, i.e. linear, receiver because its link is operating at a lower quality-of-service requirement) and it is therefore preferable to assign the cross-feedback filter elsewhere to avoid error propagation.

5.4.1 Impact of delay profile shape

Figures 5.13, 5.14 and 5.15 compare the average BER obtained in a 2 to 5 elements, 4 user single-cell system for various representative delay profiles:

²Again, it is equivalent to a static assignment strategy in this case since all interferers have the same average power.

(a)

(b)

Figure 5.12: Average BER performance of linear, x2df and xdf systems in the presence of 3 equal-power interferers for $M=2$ to 5; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

- double-spike with $d = 0.5$;
- exponential with $d = 1$;
- exponential with $d = 2$;
- gaussian with $d = 1$;
- gaussian with $d = 2$.

It should be noted that since the receiver under study performs symbol-spaced sampling, the two impulses in the double-spike delay profile will be completely resolved if d is at least 0.5. It follows that no further improvement in performance can be gained by lengthening the double-spike impulse response beyond $d = 0.5$.

Figure 5.13 shows the effect of delay profile on linear processing. Since the double-spike profile offers little intrinsic diversity³, its performance is clearly lagging behind the other two profiles. Moreover, the gaussian profile affords a slight gain (less than 1 dB) over the exponential profile.

In figure 5.14, it can be seen that the picture is virtually identical for decision-feedback processing.

Fully-connected XDF (fig. 5.15), however, is slightly different. The gap between $d = 1$ and $d = 2$ curves and the gap between exponential and gaussian profiles are both slightly larger than in fig. 5.13 and 5.14.

³If the two spikes are resolved by the receiver, the diversity gain is that of a narrowband array with twice the actual number of branches.

Figure 5.13: Average BER performance of linear space-time processing with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile as a function of delay profile shape.

Figure 5.14: Average BER performance of decision-feedback space-time processing with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile as a function of delay profile shape.

5.5 Numerical Results Pertaining to Broadband Wireless Systems

As an example of the benefits of “deep embedding” of antenna arrays in broadband wireless systems, this section studies the performance of various linear and non-linear space-time processing schemes in broadband wireless systems where 2 co-channel users are active in each sector. Hence, the user capacity of the system is double that of a conventional system and the array is necessary to spatially discriminate co-channel users within the same sector (i.e. SDMA).

5.5.1 Results with uncorrelated fading

Square cells are assumed with 4 sectors per cell. All carriers are reused in every sector and sector-to-sector isolation is assumed to be absolute. Furthermore, the subscribers are fixed and employ directional antennas with a 3 degree beamwidth and a front-to-backlobe (FTB) ratio of 20 dB. This entails that a fixed power level exists in every direction outside the main lobe and that it is 20 dB below the peak mainlobe power (i.e. the power at the direction of look). Measurement results in the LMCS band (i.e. in the vicinity of 29 GHz) [127] were employed to establish the variance of the log-normal shadowing at 14 dB. All signals also undergo Rayleigh fading and are subject to a form of static power control based on path loss only. Indeed, it is reasonable to expect that the power would be at least adjusted upon installation of the subscriber antenna according to the distance from the base station.

Using the above parameters, Monte-Carlo simulations were performed by applying to the channel realizations the optimal infinite-length formulas for the minimum MSE for various S-T processing schemes. Channel spectra were efficiently produced at each

iteration using a set of precomputed orthogonal basis functions [25]. Furthermore, the fading process is characterized by uncorrelated envelopes between the elements. An exponential power delay profile is assumed with a delay spread of 1 symbol duration.

Only first-tier interferers were considered since preliminary trials have indicated that second tier interferers have negligible effect. Under these conditions, the base station of interest is affected by 8 other sectors which implies that 18 co-channel signals are involved in the simulation. Of these, ten are always illuminating the base from their backlobe while the remainder will occasionally come in through their mainlobe (see figure 5.5). Positions for all 18 users are randomly generated at every iteration based on a uniform PDF across the surface of their respective sectors. Received powers prior to shadowing and fading are computed according to a propagation exponent of 4.5 [127] which is adequate for non-line-of-sight links.

Average BER curves are shown in figure 5.17a for 2, 4, 8 and 16 array elements. The curves are approximate since they are computed assuming gaussianity of the residual interference. As before, the “linear”, “df” and “xdf” curves stand for linear, decision-feedback and fully-connected cross-decision-feedback s-t processing, respectively. In the latter case, all 18 users’ signals are processed by feedback filters under the assumption that decisions for the out-of-cell signals are somehow available at the base station. This is possible if, for instance, adjacent base stations will supply over wireline the received data streams from these interferers upon request. However, this can represent a significant overhead and bandwidth occupation over the cabled network linking bases to their local hub. The “x2df” curve corresponds to a simpler cross-decision-feedback system where only the in-sector interferer and the desired signal are assigned a feedback filter. The “df-no interf.” curve is simply a benchmark decision-feedback system with no co-channel interference.

Clearly, the “x2df” is the best complexity-performance tradeoff as well as being the most practical solution since it does not require information exchange between base stations. However, the difference in performance between the various schemes diminishes as the number of antenna elements augments and it becomes almost negligible when $M = 16$. It is also interesting to note that the “x2df” system offers a performance gain with respect to standard decision-feedback (“df”) which is in most cases comparable to that offered by “df” with respect to linear space-time processing.

It is interesting to note also that the “xdf” curve hugs the benchmark “df-no interf.” curve all the way through the error floor affecting the other systems when $M = 2$ and $M = 4$.

Outage probability curves are shown in figure 5.17b where the outage event is defined as $P(P_e > 10^{-3})$.

It can be deduced from figure 5.17 that with only 2 antennas, acceptable performance (e.g. $P_e \approx 10^{-4}$) can probably be obtained by an “x2df” system if E_b/N_0 is high and/or some form of error-correction coding is employed. However, this is true only for systems whose performance is very close to ideal, i.e. negligible loss due to training, numerical precision and finite filter lengths. Nonetheless, the picture improves if, for example, the in-sector users have a clear line-of-sight to the base or if the carriers are underutilized, i.e. only a small subset of the interferers are generally active simultaneously.

Of course, for a frequency reuse factor of $1/M$ (i.e. M co-channel users per sector and full reuse of carriers in every sector and cell), at least M elements are required. However, with this minimal configuration the spatial degrees of freedom are completely exploited for discrimination of the in-sector co-channel signals and temporal processing alone is left to deal with ISI and the out-of-sector interference sources.

Our simulation results indicate that the addition of one or a few more elements (than the minimum M) might be desirable especially once real-world imperfections (i.e. training loss, finite numerical precision, etc.) are factored in.

Figure 5.18 shows a set of average BER and outage probability curves similar to the one depicted in figure 5.17 except that the RMS delay spread is 4 symbols. In general, the curves exhibit an improvement in required E_b/N_0 of up to 1.5 dB versus figure 5.17 thanks to the increase in intrinsic frequency diversity due to the longer impulse responses. However, the error floor affecting all processors except “xdf” occurs at a somewhat higher BER in 5.18(a).

The 2-antenna case when $d=4$ exhibits the largest spread of all results between the various S-T processors (and specifically between “x2df” and “xdf”). Figure 5.19 populates this spread by showing performance curves of intermediate partial-connectivity S-T processors, i.e. “x3df”, “x12df”, etc. It can be seen that nearly all processors between “x2df” and “x12df” have nearly identical (i.e. undistinguishable) performance curves. As the number of feedback filters augments beyond 12, performance quickly approaches that of “xdf”.

Figure 5.20 illustrates the performance of the same set of S-T processors when $d = 0.1$. Since the intrinsic frequency diversity is much lower than in the previous 2 cases, the performance is worse. Furthermore, the performance spread between linear and “xdf” is much smaller since feedback filters have little excess delay spread to work with (and therefore can bring little improvement above optimal linear processing).

In all partial connectivity decision-feedback systems, the assignment of cross-feedback filters was done in a static fashion, i.e. the filters were assigned first to the users whose *average power* was higher. It would also be possible to have a dynamic assignment which tracks the power changes due to shadowing and fading. However,

simulation trials have shown that dynamic assignment in this scenario brings about negligible performance improvement, i.e. the curves are close enough that they are impossible to distinguish by eye. This is reasonable given that the in-cell interferer is typically an order of magnitude above the other interference sources and it will normally be assigned the first feedback filter in both dynamic (except for very rare exceptional events) and static assignment strategies.

In all the above discussed scenarios, we have assumed that no user had a line-of-sight to the base of interest (i.e. all users were undergoing Rayleigh fading). In figure 5.21, however, the line-of-sight case is explored. Out-of-sector interferers are considered too distant to have a line-of-sight and they are therefore still characterized by Rayleigh fading. Furthermore, the relative importance of the line-of-sight component (as embodied by the K factor) characterizing the Rician fading of the two in-sector users diminishes appropriately with distance. Setting the cell size to $2 \text{ km} \times 2 \text{ km}$, the K factors were calculated according to

$$K(\text{dB}) = 24 - 0.032d_b, \quad (5.36)$$

where d_b is the distance to the base in meters. This formula embodies empirical results obtained in the LMCS band by Papazian et al. [124] and was also employed in chapter 3 (see (3.94)). The curves are virtually identical to figure 5.17a when the E_b/N_0 is low (below 20 dB). Above that level for $M = 2$ and $M = 4$, the linear, “df” and “x2df” processors exhibit slightly inferior performance versus the all-Rayleigh case, while “xdf” and the benchmark curve are slightly superior.

5.5.2 Diversity order / XDF connectivity tradeoff

In figure 5.22, we compare the impact of going from linear to non-linear processing versus adding one element to the array for array sizes of $M = 2$ to 5. Due to the large number of interferers (with respect to the values of M considered) in this scenario, the results are not quite as positive as in figure 5.12. Nonetheless, fully-connected cross-decision-feedback processing (xdf) with M elements yields better performance than linear processing with $M + 1$ elements if the E_b/N_0 is high enough. It is interesting to note that the E_b/N_0 threshold — at which xdf becomes better than a linear processor with an additional element — is shrinking as the number of elements goes up.

These observations on the relative performance of linear and XDF processors are valid only when considering the average behavior of the system. Indeed, they break down when outage probability is considered.

5.5.3 Comparison with non-SDMA scenarios

Table 5.1 constitutes an attempt at assessing the amount of additional space-time processing complexity required to effectively handle the doubling of capacity with respect to a standard non-SDMA full-frequency reuse system. All figures are for Rayleigh fading and exponential delay profile with RMS delay spread of 1 symbol period.

Table 5.1(a) presents average BER performance for the non-SDMA case with one and two antenna elements and the 2-cochannel-users-per-sector case with two and three antenna elements, for E_b/N_0 levels of 24 dB and 30 dB. Table IB presents outage probabilities (with the outage BER being set at 10^{-3}) for the same set of system scenarios. It is seen that even with the reduced interference levels in the non-SDMA case, a one-antenna processor yields relatively poor performance in a

full-frequency reuse scenario. However, the addition of error-control coding should make this approach acceptable. Furthermore, the addition of a second element helps improve the picture appreciably, with linear processing at $E_b/N_0 = 30$ dB yielding a BER in the vicinity of 10^{-4} and an outage probability of 0.16%.

It is interesting to note that 2 elements in the 2 co-channel-users SDMA scenario are enough to slightly surpass the average BER performance of the 1 element non-SDMA case. However, 3 element processors yield much more interesting results in this case.

This set of results seems to indicate that generally, space-time processors with 1 more element than there are co-channel users per sector are the minimum configuration for decent performance (e.g. $\text{BER} \sim 10^{-5} - 10^{-4}$) in this type of system (square cells with 4 sectors, subscriber beamwidths of 5 degrees and FTB=20 dB, RMS delay spread=1) if coding is not employed. Two notable exceptions are the “x2df” and the “xdf” processors which, with two elements and at an E_b/N_0 of 30 dB, do provide adequate performance for low quality links if combined with low-complexity links (average BERs of 8.58×10^{-4} and 2.31×10^{-4} and outage probabilities of 5.8% and 4.3%, respectively).

5.5.4 Results with fading correlation

For the same 2-user per sector multicell setting, the effect of fading correlation was studied assuming a 4-element linearly spaced array (where fading correlation is modeled as in 4.3.4). Since the impact of correlation will be felt more strongly when the impulse responses are short (and, consequently, there is a lack of intrinsic frequency diversity), we first look at the $d = 0.1$ case.

The average BER curves in figure 5.23 and the outage probability curves in figure

	1 co-channel user/sector (non-SDMA)				2 co-channel users/sector (SDMA)			
	$M = 1$		$M = 2$		$M = 2$		$M = 3$	
E_b/N_0	24 dB	30 dB	24 dB	30 dB	24 dB	30 dB	24 dB	30 dB
linear	0.0204	0.0027	7.37×10^{-4}	1.02×10^{-5}	0.0288	0.0051	0.0019	9.41×10^{-5}
df	0.0102	7.67×10^{-4}	2.71×10^{-4}	3.91×10^{-6}	0.0176	0.0022	8.91×10^{-4}	4.95×10^{-5}
x2df	0.0102	7.67×10^{-4}	2.71×10^{-4}	3.91×10^{-6}	0.0047	8.58×10^{-4}	4.24×10^{-4}	1.40×10^{-5}
xdf	0.0100	5.98×10^{-4}	2.60×10^{-4}	8.54×10^{-7}	0.0088	2.31×10^{-4}	3.45×10^{-4}	3.60×10^{-7}

(a)

	1 co-channel user/sector (non-SDMA)				2 co-channel users/sector (SDMA)			
	$M = 1$		$M = 2$		$M = 2$		$M = 3$	
E_b/N_0	24 dB	30 dB	24 dB	30 dB	24 dB	30 dB	24 dB	30 dB
linear	0.680	0.214	0.114	0.0013	0.86	0.344	0.2517	0.0083
df	0.477	0.0632	0.0487	2.00×10^{-4}	0.742	0.134	0.142	0.0027
x2df	0.477	0.0632	0.0487	2.00×10^{-4}	0.584	0.058	0.0795	8.33×10^{-4}
xdf	0.474	0.0603	0.0477	2.0×10^{-4}	0.579	0.043	0.072	1.67×10^{-4}

(b)

Table 5.1: Performance of space-time processors in SMDA (2 co-channel users/sector) and non-SDMA (1 co-channel user/sector) scenarios; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

5.24 show that correlation has a very significant effect in this case. For the high correlation case when the interelement separation d_x is λ , the performance is reduced to the level of a 2-antenna base station. At the other extreme, it can be seen that a 100λ separation is almost as positive as complete decorrelation, there being an E_b/N_0 loss of only 1 dB versus the uncorrelated case.

The $d = 1$ case is examined in figures 5.25 (average BER) and 5.26 (outage probability). While the performance loss due to high correlation $d_x = 1$ is now less important (than that found when $d = 0.1$), the impact of correlation is still very profound despite the presence of intrinsic frequency diversity. In fact, the E_b/N_0 loss versus the uncorrelated case can be as much as 8 dB for linear processing. However, it is interesting to note that the decision-feedback schemes (“df”, “x2df” and “xdf”) offer more robustness against fading correlation with the worst-case (i.e with $d_x = \lambda$) E_b/N_0 loss being reduced to 5 dB.

5.6 Conclusion

The analysis herein yielded an exact formula for the minimum-achievable MSE by an optimal infinite-length multi-user decision-feedback space-time processor with any degree of cross-feedback connectivity as a function of channel spectra.

It follows that previously derived formulas for classical decision-feedback and fully-connected cross-decision-feedback are in fact special cases of the present result.

All sets of simulation curves in section 5.2 seem to show that fully-connected XDF processing is reasonably close to CCI-free DF performance. Therefore, XDF processing is very effective at further reducing CCI in addition to the spatial discrimination performed by the linear (feedforward) processing. However, it is worth noting that

XDF is less effective at reducing CCI when the impulse responses are very short (e.g. $d = 0.1$, see fig. 5.20) since there is then hardly any post-cursor energy to remove. On the other hand, the effect on performance is minor for RMS delay spreads as low as $d = 0.5$ (see fig. 5.6).

It is of interest from a system design perspective that varying the degree of cross-feedback connectivity seems to uniformly fill the performance gap between standard DF and fully-connected XDF. It follows that this type of structure can be applied to achieve any desired performance / complexity tradeoff within this range. This entails that progressively augmenting the degree of cross-feedback connectivity constitutes an attractive incremental upgrade path in existing systems.

The performance of linear, DF and XDF processors was also studied in the context of a multicell broadband wireless system with fixed subscribers employing highly directional antennas. In this system scenario, the space-time processor had to contend with 2 co-channel users per sector as well as a large number of significant out-of-cell interferers. It was seen that an XDF system with as few as 2 feedback filters can bring about an improvement in BER versus standard DF roughly equivalent to the improvement obtained by standard DF versus linear processing. It was also observed that a fully-connected XDF processor (where even the out-of-cell signals are part of the feedback loop) is not subject to the error floor which affects the other processors.

Furthermore, simulation results indicate that in many significant cases, fully-connected XDF processing achieves BER performance equal to or better than linear processing with one additional element. It can therefore be applied to reduce RF hardware complexity (by avoiding the receiver front-end replication required by an additional antenna element) at the cost of increased numerical complexity.

Figure 5.15: Average BER performance of fully-connected XDF space-time processing with 4 elements and 3 interferers as a function of SIR when impulse responses are characterized by an exponential delay profile as a function of delay profile shape.

Figure 5.16: Geographical system plan used in the multicell simulations. Four square cells are shown, each being divided into 4 sectors. The base station of interest, located in the upper left-hand sector of the lower right-hand cell, is identified by a “B”. Assuming perfect sector isolation and neglecting interferers beyond the first tier, co-channel users in up to 8 sectors (in addition to the sector of interest) can affect reception. It can be seen that in three of these sectors, interferers can occasionally, depending on their position with respect to their own base, illuminate the base through their mainlobe. Interferers from the remaining 5 sectors (grayed out in the figure), will always come in through their backlobe. The most severe interferer is, of course, the one sharing the sector of interest with the desired user.

(a)

(b)

Figure 5.17: Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.18: Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 4$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.19: Performance of various 2-antenna space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 4$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.20: Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 0.1$; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

(a)

(b)

Figure 5.21: Performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$ and the in-sector users are undergoing Rician fading; (a) average BER, (b) outage probability ($P_{out} = P(BER > 10^{-3})$).

Figure 5.22: Average BER performance of linear, x2df and xdf systems for $M = 2$ to 5.

(a)

(b)

Figure 5.23: Average BER performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 0.1$ in the presence of fading correlation as a function of interelement spacing d_x .

(a)

(b)

Figure 5.24: Outage probability performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 0.1$ in the presence of fading correlation as a function of interelement spacing d_x .

(a)

(b)

Figure 5.25: Average BER performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$ in the presence of fading correlation as a function of interelement spacing d_x .

(a)

(b)

Figure 5.26: Outage probability performance of various space-time processors in a 2-user per sector SDMA system where all channels are characterized by a normalized RMS delay spread $d = 1$ in the presence of fading correlation as a function of interelement spacing d_x .

Chapter 6

Conclusion

6.1 Philosophical underpinnings and justification

This thesis was written as a study of a set of interrelated system issues pertaining to the use, in broadband wireless systems, of adaptive space-time processing techniques applied to the uplink at the base station in order to

- mitigate co-channel interference via spatial discrimination;
- provide robustness against multipath fading and shadowing via diversity;
- increase capacity via carrier reuse (i.e. SDMA).

In the early stages of the research process, it was determined that the preferable approach would combine:

- an emphasis on the development of widely-applicable theoretical results;
- an application of said results whenever appropriate to a real-world system context.

The latter's role was filled by LMCS which, with its large bandwidth, high frequency of operation, fixed subscribers and its bright commercial future, is a worthy representative of the "broadband wireless" class of systems.

One of the key system aspects we chose to focus on was the impact of fading correlation on space-time processing performance. While the literature has addressed this before in the context of mobile cellular (e.g. see [13]), the uniqueness of the present contribution lies in part in a number of system aspects: a broadband wireless system with fixed subscribers, the presence of wideband channels, the presence of static components (either line-of-sight, static multipath vector or both) from a significant number of subscriber sites, operation in the little-known band around 29 GHz and the use of highly directional subscriber antennas.

The fading correlation efforts presented herein are also novel in both the magnitude and the unique nature of the study. Most contributions on the subject fall near one of the two ends of a spectrum ranging from system-level with rudimentary modeling of propagation aspects (e.g. the "wedge" arrival PDF in [13] and the cosine-shaped PDF in [170]) to fully empirical characterization of joint fading processes in a highly specific context (a unique indoor layout or outdoor setting). Few papers (like [170] and [171]) combine thorough analytical modeling with empirical results in an attempt to reconcile the two. For example, [13] and [140] are exclusively analytical, albeit useful, efforts.

Two essential facts can be extracted from these classical papers to illuminate the forthcoming discussion:

- The correlation coefficient can get as high as 0.5 before any serious effect on performance is noted, and

- the degradation in performance with correlation in an adaptive array that combats both fading and cochannel interference is only slightly worse than the degradation in an equivalent system combating fading alone (see [13]).

The latter statement can be rationalized by considering that the capacity of an array to combat interference alone (i.e. “zero-forcing” operation) is totally independent of correlation. Indeed, computing the optimal array output boils down to solving a system of k equations in l unknowns regardless of the amount of correlation¹.

We also note a result of interest in Lee [170] where it was found that with two horns having half-power beamwidths of 24° at the base station and operating at 836 MHz, a separation of 70λ was sufficient to lower correlation below the 0.5 mark.

Given such results, what purpose does further study of the impact of fading correlation serve? Indeed, one could postulate that system designers need only space array elements farther than $70\text{--}100\lambda$ apart to eliminate the correlation concern. Three objections can be raised to this statement in the context of this thesis.

- First, practical considerations may preclude the possibility of spacing antennas that far apart. Indeed, for a CCI combating array to offer really interesting performance, it should have more than 2 elements. If the number of elements is large, it may not be practical or it may be costly to have the antenna elements standing independently from one another. For example, an economical and promising avenue for implementation of S-T processing in LMCS and other millimeter-wavelength systems consists in building an integrated array on a single monolithic substrate. In LMCS, interelement spacing would then be of

¹We are assuming here that increased correlations can be due to reduced spacings and / or reduced scattering activity but the interelement spacings cannot be brought down to 0 if there are to be measurable phase shifts on impinging wave fronts allowing the array to separate signal sources

the order of a single wavelength. For such cases, system designers will need tools and models to appropriately quantify correlation-impacted performance.

- Secondly, the degree of correlation is highly dependent upon the position of the subscriber in the cell (and the specifics of the surrounding scattering environment). Numerical trials in the preliminary stages of this thesis have shown that even with very large interelement separations (i.e. up to 1000λ — which is more than 10 meters at LMCS frequencies) there are a few subscriber positions (e.g. at the array’s broadside near the edge of the cell) which will yield large correlation coefficients. This is a concern because the subscribers are fixed, i.e. subscribers assigned to these “problem spots” suffer significant link quality degradation on a permanent basis because of correlation effects.
- Thirdly, LMCS is characterized by the presence of static components and the use of narrow beamwidth antennas. Both factors contribute to raising the level of correlation above that of an equivalent Rayleigh fading / omnidirectional context (which is almost universally assumed in the literature).

The correlation-related portion of this work is also unique in the way it integrates with other parts of the thesis. For example, while the impact of correlation on linear processing is somewhat commonly dealt with in the literature, this is the only work to our knowledge to look at the impact of correlation on the matched-filter bound on maximum-likelihood performance with continuously dispersive wideband channels. Chapter 5 also provides results on the correlation-impacted performance of a whole family of decision-feedback systems.

Another system aspect which was deemed important was the multi-user context. Under this heading, it was intuited that multi-user information could be exploited by

space-time processors in the pursuit of one of two objective:

- (a) To improve performance (from the perspective of BER and / or outage probability);
- (b) To reduce complexity at the cost of reduced performance.

The multi-user decision-feedback structure had been previously suggested by other authors as an answer to objective (a). In the spirit of our original commitment towards fundamental, widely-applicable results, we contributed a formula for the minimum mean-square error achievable by such a structure with an arbitrary degree of cross-feedback connectivity as a function of channel spectra.

Two architectures were also devised to target objective 2, but these were left out of the thesis due to lack of time and the secrecy requirements of pending patent applications.

6.2 Discussion of results and contributions

Chapter 3 provides two subscriber-local scattering models yielding correlations as a function of spatial and frequency separation. The hollow model is based on the three-dimensional geometry of Parsons & Turkmani [140]. It can be succinctly described as an extrusion in the third dimension of Clarke's classic "ring of scatterers" model. The full model brings in a new geometry where the scattering area radius is also a random variable (in addition to departure angles in elevation and azimuth). It was seen that the hollow model leads to oscillatory correlation curves (e.g. as a function of separation) and is therefore better suited to the modeling of channels with relatively few rays. The full model, on the other hand, provides smooth curves which are appropriate to the modeling of continuously-dispersive channels.

A novel formula was derived providing the power cross-correlation coefficient ρ_P in the presence of a static component, thus making both full and hollow model usable in the representation of Rayleigh and Rician environments. It was observed that the power cross-correlation is highly dependent on the angle ϕ which is the phase difference of the static component between elements 1 and 2.

However, in the LMCS context the static component is likely to be a vector sum and this makes ϕ unpredictable. To remove the influence of ϕ for the purpose of fitting the models to empirical data, another formula was derived giving the absolute value of ρ_P maximized over ϕ .

Many curves were provided for the two analytical models in order to examine the effect of:

- subscriber position;
- subscriber antenna beamwidth;
- subscriber antenna front-to-sidelobe ratio;
- angle to subscriber;
- base station height;
- scattering region size;
- interelement spacing and
- frequency separation for all three representative delay profiles (gaussian, exponential and double-spike).

Notable observations include:

- The base station height has no effect on the models. In reality, the pointing of subscriber antenna at a steep elevation angle (given that it has a narrow elevation pattern) will most certainly affect the shape of the scattering region, although it is not clear at this point how this could be modeled.
- The correlation does not always rise monotonically with decreasing antenna beamwidth (see fig. 3.11). However, it will tend towards monotonic behavior if the front-to-sidelobe ratio is augmented. A rationale for this is presented in 3.5.2.

The analysis of angular sweep measurements carried out in 1997 has shown that the isotropic scattering assumption is not too far from reality in the Parkwood Hills area at 29.5 GHz at least if we consider ensemble averaging over many subscriber locations (see 3.7.1 for detailed discussion). This served as an empirical justification for the isotropic shape of both the full and hollow model.

Despite the many practical limitations of the correlation measurements experiment, the measurement data (collected from 44 sites) shows very good agreement with the full model. Indeed, the mean-squared error between empirical and theoretical correlation coefficients is only 0.0259 while the normalized mean-squared error is 1.16%.

The fitting process also revealed a linear relationship between the scattering region size parameter σ_r and the distance from the base m .

These results made it possible to set the parameters of the full model so that it could serve as a measurement-validated tool in subsequent chapters.

In chapter 4, derivations are produced of the matched-filter bound on optimal space-time processing in both Rayleigh and Rician spatially correlated frequency-selective fading. Numerical results were presented showing the influence of several

parameters (correlation, delay profile shape, RMS delay spread, K factor) on the MFB for a 2-element array. Further results compare the MFB performance for 2- and 4-element uniformly-spaced linear arrays as a function of the interelement spacing. These results are based on utilization of the full model with empirically-validated parameters.

While results indicate that both additional spatial diversity (obtained by adding elements) and additional intrinsic frequency diversity (obtained by lengthening the impulse response) have a positive impact on MFB performance, frequency-based diversity is not as readily exploitable by linear and decision-feedback processors. Indeed, increasing frequency diversity through longer impulse responses induces additional ISI which cannot be equalized by linear, DF or XDF processors without enhancing thermal noise. It is of interest then that a reduction of correlation (when possible) is equivalent to a “free” addition of spatial diversity.

It was shown in chapter 5 that XDF with 2 feedback filters could bring an improvement over decision-feedback of the same order as the improvement gained by going from linear to decision-feedback provided that $d \geq 0.5$. Since the second feedback filter has the same complexity as the self-feedback filter, the cost/performance impact of adding the second feedback filter is the same as that of adding the first one.

It was also shown that in many cases, fully-connected XDF could achieve performance equal to or better than linear processing with one additional element provided that d is not too small (e.g. $d \geq 0.5$). In the multicell scenario (where the in-cell interferer is stronger by an order of magnitude than the other co-channel interferers), the E_b/N_0 needs to be above a certain threshold for the above statement to hold. It is interesting that the threshold lowers when the diversity order M is increased.

However, in the single-cell scenario with $d = 1$, an M -element fully-connected

XDF processor is not only always better than an $M + 1$ -element linear processor, it is also significantly better than the “x2df” processor regardless of SIR.

It follows that implementing XDF can be a cost-effective alternative to adding an element (with the implied hardware replication) to a linear processor.

6.3 Applicability of results

Although this thesis was developed in the context of LMCS systems operating in the 28-30 GHz band, many results are broadly applicable. This section means to clarify which parts of the thesis can be readily applied in other contexts such as mobile wireless or fixed wireless systems in other bands.

6.3.1 Applicability to fixed wireless systems operating in other bands

Since this thesis was initially fleshed out, the objectives of commercial LMCS developers have changed. Indeed, the system was initially defined to serve residential customers, offering wireless internet and other broadband services. This is one of the basic assumptions carried out in much of the work herein. However and in light of heavy coverage problems in the LMCS band uncovered by various investigators, LMCS development is now targeted specifically at establishing links to business offices for wide-area networking.

The type of residential application envisioned herein is now considered for fixed wireless systems operating in lower bands where coverage problems may be less severe and RF components are likely cheaper. Specifically, bands in the vicinity of 10-12 GHz and 2-5 GHz are considered.

Most of our results can be readily extended to such lower bands except of course for the measurement-related results. The propagation models have exactly the same value regardless of the band under consideration but it is to be expected that correlation results and scattering activity would be significantly different. Nonetheless, the measurement results can provide a rough starting point for initial experimentation in other bands until similar measurement data becomes available for the bands of interest.

However, we would postulate that the parameters for the full model which are derived from analysis of the measurement data are generally *not* applicable in other bands. Other results which rely on these parameters are found in sections 4.3.4 and 5.5.4 and their validity in other bands is questionable.

The remainder of this thesis is completely independent of the band. For example, results for the broadband wireless multi-cell scenario in chapter 5 (except section 5.5.4) are directly applicable in any band. In fact, all such results are even independent of cell dimensions because of the relative transmit power assumptions made therein. One exception, however, is the Rician case treated in figure 5.21. In that case, the cell size is fixed and the K factor is varied according to a formula which is specific to the LMCS band.

6.4 Applicability to mobile wireless

All results are applicable to TDMA mobile wireless systems except

- results which rely on the measurements (sections 3.7, 4.3.4, 5.5.4);
- results which assume fixed subscribers with directional antennas (section 5.5).

For example, results in section 5.2 (which pertain to the single-cell generic Rayleigh fading scenario) do not assume either fixed subscribers or directional antennas.

6.5 Suggestions for future work on space-time processing

6.5.1 Analytical performance assessment of space-time processors

It would be desirable to examine the possibility of extending the analytical techniques discussed in the background section (see section 2.6.3). While it is possible to express analytically the PDF of the SINR at the output of a linear narrowband processor in the presence of white noise and multiple uncorrelated Rayleigh-fading interferers with arbitrary powers, it would be of interest to extend this type of method to

- include wideband processing;
- include the impact of correlation;
- express not only the PDF of the SINR, but also the average BER (e.g. deriving the latter from the former by assuming gaussianity of the residual interference).

These extensions would make the multivariate analytical approach more widely applicable and enable, in many cases, rapid performance assessment bypassing completely the need for lengthy Monte-Carlo simulations. It would also be of interest to apply this approach (if possible) to decision-feedback and XDF systems.

6.5.2 Impact of training

It was shown in 2.6.5 that an elegant procedure exists to determine analytically the PDF of the SINR training loss in narrowband linear processors using block adaptation. Since there is an equivalence, from the perspective of the adaptation algorithm, between an equalizer and spatial processors, it is possible with some reflexion to apply the known PDFs for training loss to space-time processors. To the best of our knowledge, section 5.2.5 is the first utilization of these PDFs for wideband processors.

Two aspects of training loss theory stand out as important:

- The PDF of the training loss is independent from the individual users' powers and spatial signatures. It is thus also independent from the PDF of the ideal SINR (see item 1 above) which should make it possible to combine the two to yield the PDF of the real SINR, including impact of training.
- The PDF of the training loss as described herein is based on the estimation of the spatial covariance matrix alone, i.e. it assumes that the desired signal signature is perfectly known.

In light of this, it would be interesting to

- attempt to combine the PDF of the idealized SINR with the training loss PDF and possibly go on to derive approximate or exact average BER or outage probability expressions. The latter would probably not be closed-forms but they might be relatively simple integral representations suited to numerical evaluation;
- extend the theory to include the effect of desired signal signature estimation;

- based on the latter, derive the PDF for other training methods such as XTSMI (see 2.6.5);
- investigate the possibility of applying training loss theory to decision-feedback and cross-decision-feedback systems.

6.5.3 Multi-user decision-feedback systems

- Formulate a structure combining trellis coding with multi-user decision-feedback systems possibly and provide an analytical or semi-analytical assessment of its performance. This is non-trivial since pre-decoding decisions in such systems are unreliable and hence inappropriate for the feedback loop. Utilizing post-decoder decisions also poses a problem since there is a multi-symbol decoding delay. For conventional decision-feedback, this problem is typically solved using one of three approaches:
 - transmitter precoding [172];
 - interleaver / deinterleaver pair to introduce the delay required for reliable feedback [173], [174];
 - use of soft decisions for feedback [175], [176].

Transmitter precoding requires a priori channel information which may not be available in the target wireless system. The interleaving approach, aside from its added complexity, requires a predictive structure which is suboptimal for finite feedforward filter lengths. Finally, the use of soft decisions is highly unreliable even if not as prone to causing error propagation as hard tentative decisions.

Whether one of these approaches or a new structure altogether is the preferable

choice for multi-user decision-feedback (XDF) remains an open question.

- Combine multi-user decision-feedback with successive interference cancellation and provide an analysis of its performance.
- Incorporate error propagation in the study of the performance of XDF.

Appendix A

Map of transmitter sites in Parkwood Hills

Figure A.1: Map of transmitter sites; North is towards the right of the page and the basestation building (Chesterton Towers) is circled. Sites which were found to result in insufficient received power during data collection are marked as red dots; the green dots mark the remaining sites.

Figure A.2: Lower left quadrant of map.

Figure A.3: Lower right quadrant of map.

Figure A.4: Upper left quadrant of map.

Appendix B

Site-by-site fitting of full model to measurement results

Figure B.1: Site 1 - 37 Woodmount.

Figure B.2: Site 2 - 11 Charkay.

Figure B.3: Site 3 - Benson 53 ft. away from Seymour.

Figure B.4: Site 4 - 1 Kilmory.

Figure B.5: Site 5 - 3 Kilmory.

Figure B.6: Site 6 - 71 Tiverton.

Figure B.7: Site 7 - 58 Newbury.

Figure B.8: Site 8 - 42 Newbury.

Figure B.9: Site 14 - 15 Bowmoor.

Figure B.10: Site 15 - Wigan and Inverness.

Figure B.11: Site 37 - 40 Walgate.

Figure B.12: Site 39 - 11 Charkay.

Figure B.13: Site 43 - 28 Mulvaugh.

Figure B.14: Site 49 - 48 Tiverton.

Figure B.15: Site 53 - 17 Lynhaven.

Figure B.16: Site 54 - 5 Rockfield.

Figure B.17: Site 65 - 7 Inverness.

Figure B.18: Site 66 - 27 Inverness.

Figure B.19: Site 90 - 35 Seymour.

Figure B.20: Site 102 - 114 Fourseasons.

Figure B.21: Site 109 - Fourseasons & Shadetree.

Figure B.22: Site 110 - Coolspring & Viewmount.

Figure B.23: Site 117 - 25 Southview.

Figure B.24: Site 129 - 18 Welkin.

Figure B.25: Site 130 - Chesterton & Assiniboine.

Figure B.26: Site 131 - Meadowlands & Peter.

Figure B.27: Site 133 - 46 Laird.

Figure B.28: Site 143 - Canter & Bayne.

Figure B.29: Site 144 - Bayne & David.

Figure B.30: Site 145 - 5 David.

Figure B.31: Site 146 - 23 David.

Figure B.32: Site 147 - 35 David.

Figure B.33: Site 150 - 26 Harris.

Figure B.34: Site 151 - 14 Harris.

Figure B.35: Site 152 - 3A Joan.

Figure B.36: Site 153 - Rossland & Pender.

Figure B.37: Site 155- 26 Withrow.

Figure B.38: Site 159 - Tower & St-Helene.

Figure B.39: Site 162 - East end of Cotes des Neiges.

Figure B.40: Site 163 - 27 Lotta.

Figure B.41: Site 203 - 118 Rossland.

Figure B.42: Site 205 - 96 Rossland.

Figure B.43: Site 212 - 79 Tower.

Figure B.44: Site 218 - 41 Palsen.

Appendix C

Solution of a matrix Wiener-Hopf system of equations

Given a system of equations of the form

$$\mathbf{R}[k] = \mathbf{A}[k] + \sum_{l=-\infty}^0 \mathbf{R}[k-l]\mathbf{A}[l], \quad \text{for } k \leq 0, \quad (\text{C.1})$$

where $\mathbf{R}[k]$ is a matrix correlation sequence, we wish to express $\mathbf{A}[k]$ as a function of $\mathbf{R}[k]$.

Let

$$\mathbf{M}[k] = \mathbf{R}[k] + \mathbf{I}\delta[k], \quad (\text{C.2})$$

where $\mathbf{I}$ is the identity matrix and $\delta[k]$ is the Kronecker delta function. Since $\{\mathbf{M}[k]\}_{-\infty}^{\infty}$ is a matrix correlation sequence, it can be represented as the discrete matrix convolution of an anticausal matrix sequence $\{\mathbf{M}^{-}[k]\}_{-\infty}^0$ and a causal matrix sequence $\{\mathbf{M}^{+}[k]\}_{k=0}^{\infty}$ [60], [160], [26] (see Appendix D for proof), i.e.

$$\mathbf{M}[k] = \sum_{j=0}^{\infty} \mathbf{M}^+[j] \mathbf{M}^-[k-j] \quad \text{for all } k. \quad (\text{C.3})$$

Let the matrix sequence $\{\mathbf{X}[k]\}_{-\infty}^{\infty}$ be defined from

$$\mathbf{R}[k] = \sum_{j=0}^{\infty} \mathbf{M}^+[j] \mathbf{X}[k-j] \quad \text{for all } k. \quad (\text{C.4})$$

By substitution of (C.3) and (C.4) into (C.1), we obtain

$$\sum_{j=0}^{\infty} \mathbf{M}^+[j] \left\{ \mathbf{X}[k-j] - \sum_{n=-\infty}^0 \mathbf{M}^-[n-k-j] \mathbf{A}[n] \right\} = 0. \quad (\text{C.5})$$

It can be shown through basic algebraic manipulations that a solution of (C.5) can be obtained by solving

$$\mathbf{X}[k] = \sum_{n=-\infty}^0 \mathbf{M}^-[n-k] \mathbf{A}[n] \quad k \leq 0. \quad (\text{C.6})$$

Taking the two-sided Fourier transform of (C.4) yields

$$\mathbf{R}(\omega) = \mathbf{M}^+(\omega) \mathbf{X}(\omega). \quad (\text{C.7})$$

Likewise, the one-sided transform of (C.6) is expressed

$$\mathbf{X}^-(\omega) = \mathbf{M}^-(\omega) \mathbf{A}^-(\omega), \quad (\text{C.8})$$

and (C.7) also yields $\mathbf{X}^-(\omega)$ as

$$\mathbf{X}^-(\omega) = [\mathbf{M}^+(\omega)^{-1} \mathbf{R}(\omega)]_-, \quad (\text{C.9})$$

where the operator $[\cdot]_-$ stands for the anticausal part of its argument. Thus, by observing that $\mathbf{M}(\omega) = \mathbf{M}^+(\omega) \mathbf{M}^-(\omega)$ and $\mathbf{M}(\omega) = \mathbf{R}(\omega) + \mathbf{I}$, we can deduce that

$$[M^+(\omega)^{-1}\mathbf{R}(\omega)]_- = \mathbf{M}^-(\omega) - [\gamma_0]^{-1}, \quad (\text{C.10})$$

where $[\gamma_0]$ stands for the matrix of DC coefficients of $\mathbf{M}^+(\omega)$. It follows that

$$\mathbf{A}(\omega) = \mathbf{I} - \mathbf{M}^-(\omega)^{-1} [\gamma_0]^{-1}, \quad (\text{C.11})$$

and $[\gamma_0]$ can be found (see [60] and Appendix E) by

$$\begin{aligned} [\gamma_0]^2 &= \exp \left\{ \frac{1}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \ln [\mathbf{R}(\omega) + \mathbf{I}] d\omega \right\} \\ &= (\mathbf{I} - \mathbf{A}[0])^{-1}. \end{aligned} \quad (\text{C.12})$$

where $\exp[\cdot]$ and $\ln[\cdot]$ are scalar operators mapped to a matrix, e.g.

$$\exp[\mathbf{A}] = \begin{bmatrix} \exp(a_{11}) & \cdots & \exp(a_{1P}) \\ \vdots & \ddots & \vdots \\ \exp(a_{P1}) & \cdots & \exp(a_{PP}) \end{bmatrix}. \quad (\text{C.13})$$

The preceding development can be seen as a matrix extension of the work in Appendix A of [60].

Appendix D

Spectral factorization of a matrix autocorrelation sequence

Theorem D.1 *Given a matrix autocorrelation sequence*

$$\mathbf{R}[k] = \{r_{ij}[k]\}_{i,j=0}^M, \quad (\text{D.1})$$

where

$$r_{ij}[k] = \sum_{l=-\infty}^{\infty} c_i[l]c_j^*[k-l]. \quad (\text{D.2})$$

It can be factored in the frequency domain into a causal and anticausal part such that

$$\mathbf{R}(\omega) = \mathbf{R}^+(\omega)\mathbf{R}^-(\omega), \quad (\text{D.3})$$

which, in the time domain is expressed

$$\mathbf{R}[k] = \sum_{l=0}^{\infty} \mathbf{R}^+[l]\mathbf{R}^-[k-l] \quad (\text{D.4})$$

where $R^+[k] = 0$ for $k < 0$ and $R^-[k] = 0$ for $k > 0$

Proof While it is well-known that an autocorrelation sequence such as

$$r_{ii}[k] = \sum_{l=-\infty}^{\infty} c_i[l]c_i^*[k-l], \quad (D.5)$$

can be factored into a causal and anticausal part

$$r_{ii}[k] = \sum_{l=0}^{\infty} r_i^+[l]r_i^-[k-l], \quad (D.6)$$

where $r_i^+[k] = 0$ for $k < 0$ and $r_i^-[k] = 0$ for $k > 0$.

The non-trivial aspect of the extension of the factorizability to matrix correlation sequences resides in the fact that, while the diagonal elements are autocorrelation sequences (and therefore factorizable by virtue of (D.5)), the off-diagonal elements are cross-correlation sequences. It is therefore necessary as a first step to prove that the latter are also factorizable into causal and anticausal parts.

The said proof relies on the general form of Parseval's relation:

$$\sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \tilde{X}_1(e^{j\omega})\tilde{X}_2^*(e^{j\omega})d\omega, \quad (D.7)$$

where $\tilde{X}_1(e^{j\omega})$ and $\tilde{X}_2(e^{j\omega})$ are the spectra of the discrete-time signals $x_1[n]$ and $x_2[n]$. If the latter are the sampled versions of continuous-time signals $x_1(t)$ and $x_2(t)$, then the relation can be reformulated as a function of the unsampled spectra $X_1(\omega)$ and $X_2(\omega)$:

$$\sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n] = \frac{1}{2\pi T} \sum_{k=-\infty}^{\infty} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} X_1\left(\omega - \frac{2\pi k}{T}\right) X_2^*\left(\omega - \frac{2\pi k}{T}\right) d\omega. \quad (D.8)$$

Given

$$r_{ij}[k] = \sum_{n=0}^{\infty} r_{ij}^+[n] r_{ij}^-[k-n], \quad (\text{D.9})$$

where $r_{ij}^+[k] = 0$ for $k < 0$ and $r_{ij}^-[k] = 0$ for $k > 0$, our intent is to prove that $r_{ij}^+[k]$ and $r_{ij}^-[k]$ exist.

We can assume without loss of generality that both $r_{ij}^+[k]$ and $r_{ij}^-[k]$ are $L + 1$ samples long which implies that $r_{ij}[k]$ is $2L + 1$ samples long.

By virtue of Parseval's relation and introducing a time shift of L samples, we have:

$$r_{ij}[-L] = r_{ij}^+[0] r_{ij}^-[-L] = \frac{1}{2\pi T} \sum_{l=-\infty}^{\infty} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} C_i \left(\omega - \frac{2\pi l}{T} \right) C_j^* \left(\omega - \frac{2\pi l}{T} \right) e^{-j\omega L} d\omega. \quad (\text{D.10})$$

Likewise, if the time shift is incremented we get:

$$r_{ij}[1-L] = \sum_{n=0}^1 r_{ij}^+[n] r_{ij}^-[1-L-n] = \frac{1}{2\pi T} \sum_{l=-\infty}^{\infty} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} C_i \left(\omega - \frac{2\pi l}{T} \right) C_j^* \left(\omega - \frac{2\pi l}{T} \right) e^{-j\omega(L-1)} d\omega. \quad (\text{D.11})$$

Continuously incrementing the time shift until it reaches L yields a set of $2L + 1$ equations in $2L + 2$ unknowns. Such a system has an infinite number of solutions which proves that the spectral factorization of a cross-correlation sequence in causal and anticausal parts is possible.

The same procedure applies to the factorization of a matrix autocorrelation sequence. Assuming without loss of generality that all individual auto- and cross-correlation sequences are $2L + 1$ samples long, it can be verified that an $N \times N$ matrix sequence will lead to a system of $N^2(2L + 1)$ equations in $N^2(2L + 2)$ unknowns. Since such a system is solvable, this concludes the proof.

APPENDIX D. SPECTRAL FACTORIZATION OF A MATRIX AUTOCORRELATION. . . 268

QED.

Appendix E

Matrix of DC coefficients

Theorem E.1 *Given the spectral factorization of a matrix autocorrelation sequence*

$$\mathbf{M}(\omega) = \mathbf{M}^+(\omega)\mathbf{M}^-(\omega), \quad (\text{E.1})$$

the matrix of DC coefficients of $\mathbf{M}^+(\omega)$ (or, equivalently of $\mathbf{M}^-(\omega)$ since they are the same) is given by

$$[\gamma_0]^2 = \exp \left\{ \frac{1}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \ln [\mathbf{M}(\omega)] d\omega \right\}. \quad (\text{E.2})$$

Proof Given that $\ln [\cdot]$ represents an element-by-element mapping of the natural logarithm operator to its matrix argument, $\ln \mathbf{M}(\omega)$ can be expanded in a two-sided Fourier series¹:

$$\ln \mathbf{M}(\omega) = \sum_{n=-\infty}^0 [\gamma_n^-] e^{jn\omega} + \sum_{n=0}^{\infty} [\gamma_n^+] e^{jn\omega}, \quad (\text{E.3})$$

¹It should be pointed out that each element of $\ln [\mathbf{M}(\omega)]$ is expanded individually into a Fourier series; the ensemble of resulting series is then grouped together conveniently into a single matrix expression.

where $[\gamma_n^-]$ and $[\gamma_n^+]$ are the anticausal and causal coefficient matrices. Exploiting the definition of Fourier series, we can write:

$$[\gamma_0^-(\omega)] [\gamma_0^+(\omega)] = \exp \left\{ \frac{1}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \ln [\mathbf{M}(\omega)] d\omega \right\}, \quad (\text{E.4})$$

where the integration and the exponentiation are applied individually to each matrix element.

Given that $[\gamma_0^-] = [\gamma_0^+] = [\gamma_0]$, we have:

$$[\gamma_0(\omega)]^2 = \exp \left\{ \frac{1}{2\pi} \int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} \ln [\mathbf{M}(\omega)] d\omega \right\}. \quad (\text{E.5})$$

QED.

Appendix F

Closed-form expression of the beta CDF

Given a beta-distributed r.v. with parameters (α, β) , its PDF is defined:

$$f_x(x) = \frac{\Gamma\left(\frac{\alpha+\beta}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta}{2}\right)} x^{\frac{\alpha}{2}-1} (1-x)^{\frac{\beta}{2}-1}, \quad (\text{F.1})$$

and its CDF is given by

$$F_x(x) = \frac{\Gamma\left(\frac{\alpha+\beta}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta}{2}\right)} \int_0^x z^{\frac{\alpha}{2}-1} (1-z)^{\frac{\beta}{2}-1} dz. \quad (\text{F.2})$$

Applying integration by parts, we obtain

$$F_x(x) = \frac{\Gamma\left(\frac{\alpha+\beta}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta}{2}\right)} \left[\frac{x^{\frac{\alpha}{2}}(1-x)^{\frac{\beta}{2}-1}}{\frac{\alpha}{2}} + \frac{\frac{\beta}{2}-1}{\frac{\alpha}{2}} \int_0^x z^{\frac{\alpha}{2}}(1-z)^{\frac{\beta}{2}-2} dz \right], \quad (\text{F.3})$$

where it can be observed that the remaining integral is of the same form as the original except that the exponent of z has been incremented while the exponent of $(1-z)$ has been decremented. It follows that integration by parts can be applied

iteratively until the last integral term contains only a power of z and is thus expressible in closed form:

$$F_x(x) = \frac{\Gamma\left(\frac{\alpha+\beta}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)\Gamma\left(\frac{\beta}{2}\right)} \left[\frac{x^{\frac{\alpha}{2}}(1-x)^{\frac{\beta}{2}-1}}{\frac{\alpha}{2}} + \frac{\frac{\beta}{2}-1}{\frac{\alpha}{2}} \left[\frac{x^{\frac{\alpha}{2}+1}(1-x)^{\frac{\beta}{2}-2}}{\frac{\alpha}{2}+1} + \dots \right. \right. \\ \left. \left. \dots + \frac{\frac{\beta}{2}-2}{\frac{\alpha}{2}+1} \left[\frac{x^{\frac{\alpha}{2}+2}(1-x)^{\frac{\beta}{2}-3}}{\frac{\alpha}{2}+2} + \frac{x^{\frac{\alpha}{2}+\frac{\beta}{2}}}{\left(\frac{\alpha}{2}+\frac{\beta}{2}-1\right)\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)} \right] \right] \right]. \quad (\text{F.4})$$

By inspection, the latter can be reduced to a finite series expression:

$$F_x(x) = x^{\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2} + \frac{\beta}{2} + 1\right) \sum_{n=0}^{\frac{\beta}{2}-1} \frac{(1-x)^{\frac{\beta}{2}-1-n} x^n}{\Gamma\left(\frac{\alpha}{2} + n + 1\right) \Gamma\left(\frac{\beta}{2} - n\right)}. \quad (\text{F.5})$$

Bibliography

- [1] T. S. Rapaport, *Wireless Communications*. Upper Saddle River: Prentice Hall, 1996.
- [2] N. Negroponte, *Being Digital*. New York: Random House, 1995.
- [3] K. Pahlavan and A. H. Levesque, *Wireless Information Networks*. New York: Wiley, 1995.
- [4] D. C. Cox, "Universal digital portable communications," *Proc. IEEE*, vol. 73, no. 4, April 1987.
- [5] D. C. Cox, "Wireless network access for personal communications," *IEEE Commun. Mag.*, vol. 30, no. 12, Dec. 1992.
- [6] W. C. Jakes et al., *Microwave Mobile Communications*. Piscataway: IEEE Press, 1974.
- [7] W. Honcharenko, J. P. Kruys, D. Y Lee and N. J. Shah, "Broadband wireless access," *IEEE Commun. Mag.*, vol. 35, no. 1, Jan. 1997.
- [8] P. Beckmann and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*. New York: The MacMillan Co., 1963.

- [9] J. D. Parsons and A. S. Bajwa, "Wideband characterisation of fading radio channels," *IEE Proc. Pt. F*, vol. 129, no. 2, Apr. 1982.
- [10] V. H. Macdonald, "The cellular concept," *BSTJ*, vol. 58, no. 1, Jan. 1979.
- [11] J. H. Winter and J. Salz, "On the capacity of radio communication systems with diversity in a Rayleigh fading environment," *IEEE J. Select. Areas in Commun.*, vol. SAC-5, pp. 871-878, June 1987.
- [12] J. Litva and T. K.-Y. Lo, *Digital Beamforming in Wireless Communications*. Boston: Artech House, 1996.
- [13] J. Salz and J. H. Winters, "Effect of fading correlation on adaptive arrays in digital wireless communications," in *Proc. ICC'93*,
- [14] J. H. Winters, J. Salz and R. D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Commun.*, vol. 42, no. 2/3/4, Feb/Mar/Apr 1994.
- [15] A. J. Paulraj and C. B. Papadias, "Space-time processing for wireless communications," *IEEE Signal Proc.*, pp. 49-83, Nov. 1997.
- [16] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.
- [17] S. Haykin, *Adaptive Filter Theory*. Upper Saddle River: Prentice-Hall, 1996.
- [18] S. T. Alexander, *Adaptive Signal Processing*. New York: Springer-Verlag, 1986.
- [19] V. M. Bogachev and I. G. Kiselev, "Optimum combining of signals in space-diversity reception," *Telecommun. Radio Eng.*, vol. 34/35, p. 83, Oct. 1980.

- [20] J. H. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE J. Sel. Areas Comm.*, vol. SAC-2, no. 4, July 1984.
- [21] Don H. Johnson and Dan E. Dudgeon, *Array Signal Processing: Concepts and Techniques*. Englewood Cliffs: Prentice Hall, 1993.
- [22] J. H. Winters, J. Salz and R. D. Gitlin, "The capacity of wireless communication systems can be substantially increased by the use of antenna diversity," in *Proc. ICUPC'92*, pp. 2.01.1-2.01.5.
- [23] J. H. Winters, J. Salz and R. D. Gitlin, "Adaptive antennas for digital mobile radio," *Adaptive Antenna Systems Symposium, 1992*, in *Proceedings of the IEEE Long Island Section*, pp. 81-86.
- [24] C. L. Despins, D. D. Falconer and S. A. Mahmoud, "Compound strategies of coding, equalization, and space diversity for wide-band TDMA indoor wireless channels," *IEEE Trans. Vehic. Tech.*, vol. 41, no. 4, Nov. 1992.
- [25] M. V. Clark, *Diversity and Equalization in Digital Cellular Radio*, PhD dissertation, University of Canterbury, Christchurch, New Zealand, 1992.
- [26] B. Picinbono, *Random Signals and Systems*. Upper Saddle River: Prentice-Hall, 1993.
- [27] J. H. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE JSAC*, vol. SAC-2, no. 4, July 1984.
- [28] J. Cui, D. D. Falconer and A. U. H. Sheikh, "SINR of antenna array with a large number of interfering users," in *proc. VTC'96*, vol. 3, pp. 1486-90.

- [29] H.-C. Lin, "Spatial correlations in adaptive arrays," *IEEE Trans. Ant. Prop.*, vol. AP-30, no. 2, March 1982.
- [30] J. E. Hudson, *Adaptive Array Principles*. Stevenage: Peregrinus, 1981.
- [31] R. A. Monzingo and T. W. Miller, *Introduction to Adaptive Arrays*. New York: Wiley, 1980.
- [32] C. Farsakh, K. Pensel and J. A. Nossek, "An SDMA system based on spatial covariances," in *Proc. VTC'98*, pp. 1171-1175, Ottawa, Canada (1998).
- [33] T. Ohgane, Y. Ogawa and K. Itoh, "A Study on a channel allocation scheme with an adaptive array in SDMA," in *Proc. VTC'97*, pp. 725-729, Phoenix AZ, USA (1997).
- [34] B. R. Petersen and D. D. Falconer, "Equalization in cyclostationary interference," Technical Report SCE-90-01, Dept. of Systems and Computer Engineering, Carleton University, Jan. 1990.
- [35] J. N. Pierce, "Theoretical diversity improvement in frequency-shift keying," *Proc. IRE*, vol. 46, pp. 903-910, May 1958.
- [36] E. A. Neasmith and N. C. Beaulieu, "New Results on Selection Diversity," *IEEE Trans. Comm.*, vol. 46, no. 5, pp. 695-704, May 1998.
- [37] T. Eng, N. Kong and L. B. Milstein, "Comparison of diversity combining techniques for Rayleigh-fading channels," *IEEE Trans. Comm.*, vol. 44, no. 9, pp. 1117-1129, Sept. 1996.

- [38] G. D. Forney, "Maximum-likelihood sequence estimation of digital waveforms in the presence of intersymbol interference," *IEEE Trans. Info. Theory*, vol. IT-18, pp. 363-378, May 1972.
- [39] G. D. Forney, "The Viterbi algorithm," *Proc. IEEE*, vol. 61, pp. 268-278, Mar. 1973.
- [40] G. Ungerboeck, "Adaptive maximum-likelihood receiver for carrier modulated data transmission systems," *IEEE Trans. Comm.*, vol. COM-22, pp. 624-635, May 1974.
- [41] W. Van Etten, "An optimum linear receiver for multiple channel digital transmission systems," *IEEE Trans. Comm.*, Aug. 1975.
- [42] W. Van Etten, "Maximum-likelihood receiver for multiple channel transmission systems," *IEEE Trans. Comm.*, Feb. 1976.
- [43] L. Devroye, *Non-Uniform Random Variate Generation*. New York: Springer-Verlag, 1992.
- [44] W. Lindsey, "Error probabilities for Rician fading multichannel reception of binary and N-ary signals," *IEEE Trans. Info. Theory*, vol. IT-10, pp. 339-350, Oct. 1964.
- [45] W. Lindsey, "Error probability for incoherent diversity reception," *IEEE Trans. Info. Theory*, vol. IT-11, pp. 491-499, Oct. 1965.
- [46] A. A. Abu-Dayaa and N. C. Beaulieu, "Microdiversity on Rician Fading Channels," *IEEE Trans. Comm.*, vol. 42, no. 6, pp. 2258-2267, June 1994.

- [47] M. Nakagami, "The m -distribution, a general formula of intensity distribution of rapid fading," in *Statistical Methods in Radio Wave Propagation*, W.G. Hoffman, Ed. Pergamon: Oxford, England, 1960.
- [48] E. K. Al-Hussaini and A. M. Al-Bassiouni, "Performance of MRC diversity systems for the detection of signals with Nakagami fading," *IEEE Trans. Comm.*, vol. COM-33, pp. 1315-1319, Dec. 1985.
- [49] V. A. Aalo, "Performance of maximal-ratio diversity systems in correlated Nakagami-fading environment," *IEEE Trans. Comm.*, vol. 43, pp. 1134-1143, Feb./Mar./Apr. 1995.
- [50] F. Patenaude, J. H. Lodge and J.-Y. Chouinard, "Noncoherent diversity reception over Nakagami-fading channels," *IEEE Trans. Comm.*, vol. 46, no. 8, pp. 985-991, Aug. 1998.
- [51] R. W. Lucky, J. Salz and E. J. Weldon, '*Principles of Data Communications*. New York: McGraw-Hill, 1968.
- [52] W. C. Wong and L. J. Greenstein, "Multipath fading models and adaptive equalizers in microwave digital radio," *IEEE Trans. Comm.*, vol. COM-32, pp. 928-934, Aug. 1984.
- [53] R. A. Valenzuela, "Performance of adaptive equalization for indoor radio communications," *IEEE Trans. Comm.*, vol. COM-37, pp. 291-293, Mar. 1989.
- [54] P. Balaban and J. Salz, "Dual diversity combining and equalization in digital cellular mobile radio," *IEEE Trans. Vehic. Tech.*, vol. VT-40, pp. 342-354, May 1991.

- [55] J. E. Mazo, "Exact matched filter bound for two-beam Rayleigh fading," *IEEE Trans. Comm.*, vol. COM-39, pp. 1027-1030, July 1991.
- [56] F. Ling, "Matched filter-bound for time-discrete multipath Rayleigh fading channels," *IEEE Trans. Comm.*, vol. 43, no. 2/3/4, pp. 710-713, Feb./Mar./Apr. 1995.
- [57] M. V. Clark, L. J. Greenstein, W. K. Kennedy and M. Shafi, "Matched filter performance bounds for diversity combining receivers in digital mobile radio," *IEEE Trans. Vehic. Tech.*, vol. 41, pp. 356-362, Nov. 1992.
- [58] K.-W. Yip and T.-S. Ng, "Matched filter bound for multipath Rician-fading channels," *IEEE Trans. Comm.*, vol. 46, no. 4, pp. 441-445, Apr. 1998.
- [59] M. Z. Win and J. H. Winters, "On maximal ratio combining in correlated Nakagami channels with unequal fading parameters and SNR's among branches: an analytical framework," in Proc. WCNC'99, New Orleans, Louisiana, Sept. 1999.
- [60] J. Salz, "Optimum mean-square decision feedback equalization," *BSTJ*, vol. 52, no. 8, pp. 1341-1371, Oct. 1973.
- [61] P. Monsen, "Feedback equalization for fading dispersive channels," *IEEE Trans. Info. Theory*, pp. 56-64, Jan. 1971.
- [62] P. Monsen, "MMSE equalization of interference on fading diversity channels," *IEEE Trans. Comm.*, vol. COM-32, pp. 5-12, Jan., 1984.

- [63] B. R. Petersen and D. D. Falconer, "Minimum mean square equalization in cyclostationary and stationary interference—analysis and subscriber line calculations," *IEEE JSAC*, vol. 9, pp. 931-940, Aug. 1991.
- [64] B. R. Petersen and D. D. Falconer, "Suppression of adjacent-channel, co-channel and intersymbol interference by equalizers and linear combiners," *IEEE Trans. Comm.*, vol. COM-42, pp. 3109-3118, Dec. 1994.
- [65] R. Price, "Nonlinearly feedback-equalized PAM vs. capacity for noisy filter channels," Record of IEEE Int. Conf. on Comm. 1972, Philadelphia, Pa., June 1972, pp. 22-12 to 22-17.
- [66] P. Balaban and J. Salz, "Optimum diversity combining and equalization in digital data transmission with applications to cellular mobile radio—Part I: theoretical considerations," *IEEE Trans. Comm.*, vol. COM-40, pp. 885-894, May 1992.
- [67] P. Balaban and J. Salz, "Optimum diversity combining and equalization in digital data transmission with applications to cellular mobile radio—Part II: numerical results," *IEEE Trans. Comm.*, vol. COM-40, pp. 895-907, May 1992.
- [68] Y. Li and Z. Ding, "A simplified approach to optimum diversity combining and equalization in digital data transmission," *IEEE Trans. Comm.*, vol. COM-43, pp. 2285-2288, Aug. 1995.
- [69] Y. Li, J. H. Winters and N. R. Sollenberger, "Optimum spatial-temporal equalization for diversity receiving systems with co-channel interference," *Proc. ICC. 98*, 1998.

- [70] S. L. Ariyavisitakul, J. H. Winters and I. Lee, "Optimum space-time processors with dispersive interference—unified analysis and required filter span," submitted for publication.
- [71] J. E. Smee and N. C. Beaulieu, "Error-rate evaluation of linear equalization and decision feedback equalization with error propagation," *IEEE Trans. Comm.*, vol. 46, no. 5, pp. 656-665, May 1998.
- [72] T. L. Fulghum, *Adaptive Multielement Decision Feedback Receiver Structures for Narrowband Multiuser Detection in a Frequency Selective Channel*, PhD dissertation, North Carolina State University, May, 1998.
- [73] T. L. Fulghum and A. Duel-Hallen, "Adaptive multielement DFE receivers for narrowband multiuser detection," *Proc. VTC'98*, pp. 41-45, May 1998.
- [74] A. Duel-Hallen, "A family of multiuser decision-feedback detectors for asynchronous CDMA channels," *IEEE Trans. on Communications*, vol. 43, pp. 421-435, 1995.
- [75] C. Tidestav, A. Ahlen and M. Sternad, "Narrowband and broadband multiuser detection using a multivariable DFE," in *Proc. PIMRC'95*, pp. 732-736, 1995.
- [76] P. A. Bello, "Characterization of randomly time-variant linear channels," *IEEE Trans. Comm. Sys.*, vol. 11, pp. 360-393, Dec. 1963.
- [77] P. Hoeher, "A statistical discrete-time model for the WSSUS multipath channel," *IEEE Trans. Vehic. Tech.*, vol. 41, pp. 461-468, Nov. 1992.

- [78] M. Visintin, "Karhunen-Loève expansion of a fast Rayleigh fading process," *Elec. Letters*, vol. 32, pp. 1712-1713, Aug. 1996.
- [79] K.-W. Yip and T.-S. Ng, "Karhunen-Loève expansion of the WSSUS channel output and its application to efficient simulation," *IEEE J. Select. Areas Comm.*, vol. 15, no. 4, pp. 640-646, May 1997.
- [80] S. Roy and D. D. Falconer, "A diversity channel model using multiple tapped-delay lines characterized by lag-delay correlations," in *Proc. Antennas and Electromag. conf., ANTEM'98*, Ottawa, Canada, Aug. 1998.
- [81] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*. New York: Wiley, 1982.
- [82] A. Shah and A. M. Haimovich, "Performance analysis of optimum combining in wireless communications with Rayleigh fading and cochannel interference," *IEEE Trans. Comm.*, vol. 46, no. 4, pp. 473-479, April 1998.
- [83] N. R. Goodman, "Statistical analysis based on certain multivariate complex distributions," *Ann. Math. Statist.*, pp. 152-177, 1963.
- [84] M. S. and C. G. Khatri, *An introduction to multivariate statistics*. New York: Elsevier, 1979.
- [85] H. Gao and P. J. Smith, "Exact SINR calculations for optimum linear combining in wireless systems," *Prob. Eng. Inform. Sci.*, to be published.
- [86] H. Gao, *The Design and Analysis of Some Computationally Intensive Communication Systems*, Ph. D. dissertation, Inst. Statistics and Operations Res., Victoria Univ. of Wellington, Wellington, New Zealand, June 1996.

- [87] H. Gao, P. J. Smith and M. V. Clark, "Theoretical reliability of MMSE linear diversity combining in Rayleigh-fading additive interference channels," *IEEE Trans. Comm.*, vol. 46, no. 5, pp. 666-672.
- [88] C. G. Khatri, "On certain distribution problems based on positive definite quadratic functions of in normal vectors," *Ann. Math. Statist.*, vol. 37, pp. 468-479, 1966.
- [89] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Ann. Math. Statist.*, vol. 35, pp. 475-501, 1964.
- [90] K. I. Gross and D. St. P. Richards, "Total positivity, spherical series, and hypergeometric functions of matrix argument," *J. Approx. Theory*, vol. 59, pp. 224-226, 1989.
- [91] G. B. Thomas and R. L. Finney, *Calculus and Analytic Geometry*. Reading, Mass.: Addison-Wesley, 1984.
- [92] B. R. Saltzberg, "Intersymbol interference error bounds with applications to ideal bandlimited signalling," *IEEE Trans. Info. Theory*, vol. IT-14, pp. 563-568, July 1968.
- [93] N. C. Beaulieu, "The evaluation of error probabilities for Intersymbol and cochannel interference," *IEEE Trans. Comm.*, vol. 39, no. 12, Dec. 1991.
- [94] R. E. Walpole, *Introduction to Statistics*. New York: Macmillan, 1982.
- [95] C. W. Helstrom, "Calculating error probabilities for intersymbol and cochannel interference," *IEEE Trans. Comm.*, vol. COM-34, no. 5, pp. 430-435, May 1986.

- [96] K. Metzger, "On the probability density of intersymbol interference," *IEEE Trans. Comm.*, vol. COM-35, no. 4, pp. 396-402, Apr. 1987.
- [97] M. Kavehrad and M. Joseph, "Maximum entropy and the method of moments in performance evaluation of digital communication systems," *IEEE Trans. Comm.*, vol. COM-34, no. 12, pp. 1183-1189, Dec. 1986.
- [98] G. H. Golub and J. H. Welsch, "Calculations of gauss quadrature rules," *Math. Comput.*, vol. 26, pp. 221-230, Apr. 1969.
- [99] S. Benedetto et al., "Error probability in the presence of intersymbol interference and additive noise for multilevel digital signals," *IEEE Trans. Comm.*, vol. COM-21, pp. 181-190, Mar. 1973.
- [100] N. W. K. Lo, *Adaptive Equalization for a Multipath Fading Channel in the Presence of Interference*, PhD dissertation, Dept. of Systems & Computer Eng., Carleton University, Ottawa, Canada, May 1994.
- [101] F. S. Hill and M. A. Blanco, "Random geometric series with intersymbol interference," *IEEE Trans. Info. Theory*, vol. IT-19, pp. 326-335, May 1973.
- [102] J. Cui, D. D. Falconer and A. U. H. Sheikh, "Blind adaptation of antenna arrays based on small frequency offsets," *IEEE Trans. Comm.*, vol. 46, no. 1, Jan. 1998.
- [103] Chen and Y.-S. Lin, "DOA estimation by fourth-order cumulants in unknown noise environments," In *Proc. IEEE ICASSP*, vol. 4, pp. 296-299, Minneapolis, MN, Apr. 1993.

- [104] B. Porat and B. Friedlander, "Direction finding algorithms based on high-order statistics," *IEEE Trans. SP*, vol. SP-39, pp. 2016-2024, Sept. 1991.
- [105] S. Shamsunder and G. Giannakis, "Signal selective localization of nongaussian cyclostationary sources," *IEEE Trans. SP*, vol. 42, pp. 2860-2864, Oct. 1994.
- [106] H. Krim and M. Viberg, "Two decades of array signal processing research," *IEEE Signal Processing Magazine*, pp. 67-94, July 1996.
- [107] I. Reed, J. D. Mallett and L. E. Brennan, "Rapid convergence rate in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 10, no. 6, pp. 853-562, 1974.
- [108] W. F. Gabriel, "Using spectral estimation techniques in adaptive array processing," *IEEE Trans. Antennas Propagation*, vol. AP-34, no. 3, pp. 291-300, Mar. 1986.
- [109] H. Cox, R. M. Zeskind and M. N. Owen, "Robust adaptive beamforming," *IEEE Trans. Acoust. Speech Signal Proc.*, vol. ASSP-35, no. 5, pp. 1365-1376, Oct. 1987.
- [110] B. D. Carlson, "Covariance matrix estimation errors and diagonal loading in adaptive arrays," *IEEE Trans. AES*, vol. 24, no. 4, pp. 397-401, July 1988.
- [111] R. L. Dilsavor and R. L. Moses, "Analysis of modified SMI method for adaptive array weight control," *IEEE Trans. Signal Processing*, vol. SP-41, no. 2, pp. 721-726, Feb. 1993.
- [112] M. W. Ganz, R. L. Moses and S. L. Wilson, "Convergence of SMI and the diagonally loaded SMI algorithms with weak interference," *IEEE Trans. Antennas Propagation*, vol. AP-38, no. 3, pp. 394-399.

- [113] O. P. Cheremisin, "Efficiency of adaptive algorithms with regularized sample covariance matrix," *Radiotechnol. Electron. (Russia)*, vol. 2, no. 10, pp. 1933-1941, 1982 (in Russian).
- [114] X. Wang and H. V. Poor, "A new adaptive algorithm for interference suppression in wireless communications," in *Proc. VTC'98*, Ottawa, Canada, pp. 1655-1659. May 1998.
- [115] A. M. Haimovich and Y. Bar-Ness, "An eigenanalysis interference canceller," *IEEE Trans. Signal Proc.*, vol. 39, no. 1, Jan. 1991.
- [116] T. K. Citron and T. Kailath, "Eigenvector methods and beamforming," in *Proc. IEEE ICASSP*, 1984.
- [117] U. Nickel, "Application of array signal processing to phased array radar," pp. 467-474 in J.L. Lacoume et al. (eds), *Signal Processing IV: Theory and Applications*. North-Holland:Elsevier Science, 1988.
- [118] C. H. Gierull, "Statistical analysis of the eigenvector projection method for adaptive spatial filtering of interference," *IEE Proc.-Radar, Sonar and Navig.*, vol. 144, no. 2, pp. 57-63, Apr. 1997.
- [119] C. H. Gierull, "Performance analysis of fast projections of the Hung-Turner type for adaptive beamforming," *Signal Processing, Part I*, vol. 50, no. 1-2, pp. 17-29, 1996.
- [120] G. M. Stamatelos and D. D. Falconer, "Millimeter radio access to multimedia services via LMDS," in *proc. Globecom'96*, London, 1996.

- [121] Federal Communications Commision, "Third notice of proposed rulemaking and supplementary tentative decision", July 13, 1995.
<http://www.fcc.gov/wtb/wirehome.html>
- [122] Industry Canada, "Local Multipoint Communications Systems (LMCS) in the 28 GHz range: policy, authorization procedures and evaluation criteria," 1996.
<http://info.ic.gc.ca/ic-data/telecom/telecom-e.html>
- [123] P. B. Papazian and G. A. Hufford, "Time variability and depolarization of the Local Multipoint Distribution Service radio channel," in *proc. Wireless Communications Conf.*, 1997.
- [124] P. B. Papazian G. A. Hufford, R. J. Achatz and R. Hoffman, "Study of the Local Multipoint Distribution Service radio channel," *IEEE Trans. Broadcasting*, vol. 43, no. 2, pp. 175-184, June 1997.
- [125] S. Y. Seidel and H. W. Arnold, "28 GHz Local Multipoint Distribution Service (LMDS): strengths and challenges," *Virginia Tech. Symposium*, June 1995.
- [126] D. A. Gray, "A broadband wireless access system at 28 GHz," in *proc. Wireless Communications Conf.*, 1997.
- [127] R. Bultitude et al., "Radio propagation data pertinent to the design of LMCS systems operating at 28 GHz," Draft technical report, Canadian Institute for Telecommunications Research (CITR), 1998.
- [128] A. F. Elfaraie and M. Shakouri, "Propagation measurements at 28 GHz for coverage evaluation of Local Multipoint Distribution Service," in *proc. Wireless Communications Conf.*, 1997.

- [129] N. Giroux, "A tutorial on ATM traffic management," *Can. J. Elect. & Comp. Eng.*, vol. 21, no. 3, 1996.
- [130] D. Raychaudhuri and N. D. Wilson, "ATM-based transport architecture for multiservices wireless personal communication networks," *IEEE J. Select. Areas Comm.*, vol. 12, no. 8, Oct. 1992.
- [131] DAVIC (Digital Audio-Visual Council) "DAVIC 1.2 specification part 08: lower layer protocols and physical interfaces LMDS baseline document nr. 17 revision 2.0," June 21, 1996. <http://www.davic.org>
- [132] S. Roy and D. D. Falconer, "A three-dimensional wideband propagation model for the study of base station antenna arrays with application to LMCS," in *proc. VTC'98*, Ottawa, Canada, May 1998.
- [133] D. S. Shiu, G. J. Foschini, M. J. Gans and J. M. Kahn, "Fading correlation and its effect on the capacity of multi-element antenna systems," submitted to *IEEE Trans. Comm.*, Jan. 1998.
- [134] J. F. Ossana, Jr., "A model for mobile radio fading due to building reflections: theoretical and experimental waveform power spectra," *Bell Syst. Tech. J.*, vol. 43, pp. 2935-2971, Nov. 1964.
- [135] R. H. Clarke, "A statistical theory of mobile-radio reception," *Bell Syst. Tech. J.*, vol. 47, pp. 957-1000, 1968.
- [136] E. N. Gilbert, "Energy reception for mobile radio," *Bell Syst. Tech. J.*, vol. 44, pp. 1779-1803, Oct. 1965.

- [137] T. Aulin, "A modified model for the fading signal at a mobile radio signal," *IEEE Trans. Vehic. Tech.*, vol. VT-28, no. 3, pp. 182-203, 1979.
- [138] M. J. Gans, "A power-spectral theory of propagation in the mobile- radio environment," *IEEE Trans. Vehic. Tech.*, vol. VT-21, no. 1, pp. 27-38, 1972.
- [139] J. D. Parsons and A. M. D. Turkmani, "Characterisation of mobile radio signals: model description," *IEE Proceedings-I*, vol. 138, no. 6, pp. 549-556, Dec. 1991.
- [140] J. D. Parsons and A. M. D. Turkmani, "Characterisation of mobile radio signals: base station crosscorrelations," *IEE Proceedings-I*, vol. 138, no. 6, pp. 557-565, Dec. 1991.
- [141] J. J. Blanz and P. Jung, "A flexibly configurable spatial model for mobile radio channels," *IEEE Trans. Commun.*, vol. 46, no. 3, pp. 367-371, March 1998.
- [142] A. Papoulis, *Probability, Random Variables, and Stochastic Processes*. New-York: Mc-Graw-Hill, 1991.
- [143] E. S. Sousa, V. M. Jovanović and Christian Daigneault, "Delay spread measurements for the digital cellular channel in Toronto," *IEEE Trans. Vehic. Tech.*, vol. 43, no. 4, pp. 837-847, Nov. 1994.
- [144] H. Rohling, "Radar CFAR thresholding in clutter and multiple target situations," *IEEE Trans. Aerospace Electron. Syst.*, vol. AES-19, no. 4, pp. 608-621, July 1983.
- [145] N. Naz and D. D. Falconer, "Temporal Variations Characterization for Fixed Wireless at 29.5 GHz," to be presented at VTC2000-Spring, Tokyo, May 2000.

- [146] D. C. Cox, "Distributions of multipath delay spread and average excess delay for 910-MHz urban mobile radio paths," *IEEE Trans. Antennas Propagat.*, vol. 23, pp. 206-213, July 1975.
- [147] B. Glance and L. J. Greenstein, "Frequency-selective fading effects in digital mobile radio with diversity combining," *IEEE Trans. Comm.*, vol. COM-31, pp. 1086-1094, Sept. 1983.
- [148] J. F. Mastengelo et al., "A new wideband high frequency channel simulator," *IEEE Trans. Comm.*, vol. 45, no. 1, pp. 26-34, Jan. 1997.
- [149] N. Naz, *Temporal variation characterization for LMCS channel and system design implications*, M. Eng. dissertation, Carleton University, Ottawa, Canada, 1999.
- [150] R. L. Peterson, R. E. Ziemer and D. E. Borth, *Introduction to Spread Spectrum Communications*. Englewood Cliffs: Prentice-Hall, 1995.
- [151] S. N. Crozier, *Short-block data detection techniques employing channel estimation for fading time-dispersive channels*, PhD dissertation, Carleton University, Ottawa, Canada, 1990.
- [152] M. Rammal et al. "Study of mutual coupling effects on radiated patterns of antenna arrays," *IEE Proc.-Microw. Antennas Propag.*, vol. 144, no. 5, pp. 389-391, Oct. 1997.
- [153] W. H. Press et al., *Numerical Recipes in C, 2nd ed.* Cambridge: Cambridge University Press, 1992.

- [154] I.S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*. San Diego: Academic Press, 1994.
- [155] D. Zwillinger et al., *Standard Mathematical Tables and Formulae*. Boca Raton: CRC Press, Inc., 1996.
- [156] H. Bateman et al., *Higher Transcendental Functions*. New York: McGraw-Hill, 1953.
- [157] H. Uhlig, "On singular Wishart and singular multivariate Beta distributions," *Ann. Stat.*, vol. 22, no. 1, pp. 395-405.
- [158] J. A. Díaz-Garcia, R. Gutierrez Jáimez and K. V. Mardia, "Wishart and Pseudo-Wishart Distributions and Some Applications to Shape Theory," *J. Multivariate Anal.*, vol. 63, pp. 73-87. 1997.
- [159] S. Roy and D. D. Falconer, "Multi-user decision-feedback space-time equalization and diversity reception," in *proc. VTC'99 spring*, Houston, U.S., May 1999.
- [160] J. M. Cioffi, G. P. Dudevoir, M. V. Eyuboglu and G. D. Forney, Jr., "MMSE decision-feedback equalizers and coding — part I: equalization results," *IEEE Trans. Comm.*, vol. 43, pp. 2583-2594, Oct. 1995.
- [161] D. D. Falconer and G. J. Foschini, "Theory of minimum mean-square-error QAM systems employing decision feedback equalization," *BSTJ*, v. 52, no. 10, pp. 1821-1849, Dec. 1973.

- [162] C. Farsakh and J. A. Nossek, "Spatial covariance based downlink beamforming in an SDMA mobile radio system," *IEEE Trans. Comm.*, vol. 46, no. 11, pp. 1497-1506, Nov. 1998.
- [163] D. Gerlach, *Adaptive Transmitting Antenna Arrays at the Base Station in Mobile Radio Networks*, PhD dissertation, Stanford University, Stanford, U. S., August 1995.
- [164] S. Roy and D. D. Falconer, "Modeling the narrowband base station correlated diversity channel," in *proc. CTMC'99*, Vancouver, Canada, June 1999.
- [165] C. G. Khatri, P. R. Krishnaiah and P. K. Sen, "A note on the joint distribution of correlated quadratic forms," *J. Stat. Plan. Infer.*, vol. 1, pp. 299-307, 1977.
- [166] A. Duel-Hallen, "A family of multiuser decision-feedback detectors for asynchronous CDMA channels," *IEEE Trans. Comm.*, vol. 43, pp. 421-435, 1995.
- [167] T. L. Fulghum, *Adaptive Multielement Decision Feedback Receiver Structures for Narrowband Multiuser Detection in a Frequency Selective Channel*, PhD dissertation, North Carolina State University, May 1995.
- [168] S. Roy and D. D. Falconer, "Multi-user decision-feedback space-time processing with partial cross-feedback connectivity," in *Proc. VTC'2000-Spring*, Tokyo, Japan, May 2000.
- [169] S. Roy and D. D. Falconer, "The effect of fading correlation on base station antenna arrays in LMCS systems," presented at *1998 Annual CITR Research Conf.*, Ottawa, August 1998.

- [170] W. C.-Y. Lee, "Effect on correlation between two mobile radio base-station antennas," *IEEE Trans. Vehic. Tech.*, vol. VT-22, no. 4, pp. 130-140, Nov. 1973.
- [171] F. Adachi et al., "Crosscorrelation between the envelopes of 900 MHz signals received at a mobile radio base station site," *IEE Proc. part F*, vol. 133, no. 6, pp. 506-512, Oct. 1986.
- [172] M. Tomlinson, "New automatic equalizers employing modulo arithmetic," *Electronics Letters*, vol. 42, pp. 138-139, March 1971.
- [173] M. V. Eyoboglu, "Detection of coded modulation signals on linear, severely distorted channels using decision-feedback noise prediction with interleaving," *IEEE Trans. Comm.*, vol. 36, pp. 401-409, April 1988.
- [174] K. Zhou, J. G. Proakis and F. Ling, "Decision-feedback equalization of time-dispersive channels with coded modulation," *IEEE Trans. Comm.*, vol. 38, pp. 18-24, Jan. 1990.
- [175] O. Agazzi and N. Seshadri, "On the use of tentative decisions to cancel intersymbol interference and nonlinear distortion (with application to magnetic recording channels)," *IEEE Trans. Inform. Theory*, vol. 43, pp. 394-408, March 1997.
- [176] S. H. Muller, W. H. Gerstacker and J. B. Huber, "Reduced-state soft-output trellis-equalization incorporating soft feedback," *IEEE GLOBECOM'96*, London, UK, Nov. 1996, pp. 95-100.