INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI[®]

**Design and Manufacture of an Experimental System for the
Analysis of Splashing and Freezing Phenomena**

by

Mark Ermenc, B.Eng.

A thesis submitted to the Faculty of Graduate Studies and Research in partial
fulfilment of the requirements for the degree of

Master of Applied Sciences

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

Department of Mechanical and Aerospace Engineering

Carleton University

Ottawa, Ontario

Canada

September 2001

National Library
of Canada

Acquisitions and
Bibliographic Services

395 Wellington Street
Ottawa ON K1A 0N4
Canada

Bibliothèque nationale
du Canada

Acquisitions et
services bibliographiques

395, rue Wellington
Ottawa ON K1A 0N4
Canada

Your file Votre référence

Our file Notre référence

The author has granted a non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of this thesis in microform, paper or electronic formats.

The author retains ownership of the copyright in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de cette thèse sous la forme de microfiche/film, de reproduction sur papier ou sur format électronique.

L'auteur conserve la propriété du droit d'auteur qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

0-612-68312-5

**The undersigned recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis**

**Design and Manufacture of an Experimental System for the Analysis of
Splashing and Freezing Phenomena**

submitted by

Mark Ermenc, B.Eng.

**in partial fulfilment of the requirements for
the degree of Master of Applied Sciences**

Thesis Supervisor

Chair, Dept. of Mechanical and Aerospace Engineering

**Carleton University
September 2001**

Abstract

This work describes the design, manufacture and testing of a system to investigate splashing and freezing phenomena. The system was designed to study both pure splashing and mixed splashing and freezing behaviour. This system was used to drop liquid droplets onto a solid surface in a variety of low temperature conditions. Temperature and humidity were monitored, and system isolation was maintained. During warm trials, a phenomenon was noted in the distribution of splashing droplets referred to as a “dead zone”, a region where the droplets did not strike. The interaction of impinging droplet and sessile droplet that creates the “dead zone” phenomena is discussed as well. During cold trials, several distinct behaviour regimes of splashing and freezing interaction were observed.

Acknowledgements

There are several people whose assistance was invaluable to this work. I would like to extend a very special thanks to the following:

John Gaydos, Ph.D, P.Eng, for his assistance throughout the work as my advisor. His assistance in both the theoretical and practical sides of this work was invaluable.

YunZhen Zhou, M.Eng., for the design and assembly of the digital vision system.

Rajko Ermenc, Class A Leader Mechanic (retired), for his advice in the design of the coolant reservoir.

John Ermenc, B.A., B.Sc., for his assistance in the selection of the plexi-glass thickness, and assistance in selection of several laboratory components.

Andrea Dingledine, B.Eng., for her continued support and assistance in editing.

Nancy Powell, without whom I fear this work, as well as much of the department, would have been pure chaos.

To all the technicians at Carleton University, I would like to extend a special thank you. Few people anywhere demonstrate such a willingness to help, such competence in their field, and such humility when outside of it. I especially thank:

Terry Goodwin, Health and Safety Officer, for his continued advice in the design, manufacture and testing of the system.

Jim Sliwka, for his assistance in the design and manufacture of the system.

Alex Proctor, for his assistance in the design and manufacture of the system.

All these fine people deserve both thanks and praise for their hard work and attention to detail.

Table of Contents

1.0 Introduction	1
1.1 Overview	1
1.2 Overview of Phenomena	2
1.3 Relevance to Industry	2
1.4 Objectives	3
2.0 Analysis of Design	4
2.1 Experimental Objectives	4
2.2 Initial Design Considerations	4
2.3 Initial Variable Analysis	6
2.4 Experimental Isolation	7
2.5 Delivery of Test Liquid	8
2.6 Vibration Isolation	9
2.7 View Angle	10
2.8 Design of Drop Path	11
2.8.1 Droplet Diameter	11
2.8.2 Drop Velocity	12
2.8.3 Limiting Factors to Droplet Velocity	15
2.8.4 Design of Impact Plate	17
2.9 The Cooling System	18
2.9.1 Chiller Selection	19
2.9.2 Assembly of the Coolant System	26
2.9.3 Coolant System Testing and Optimization	28
2.9.4 Falling Droplet Heat Transfer	32
2.10 Design of Temperature Monitoring system	39
2.11 Design of Humidity Monitoring system	42
2.11.1 Humidity Monitoring	42
2.11.2 Humidity Control	43
2.12 Overview of Digital Vision System	44
2.12.1 Hardware Overview	44
2.12.2 Software Overview	45
2.13 Evaporation Considerations	46
2.14 Boundary Layer Considerations	48
2.15 System Cost	49

3.0 Analysis of Theory	50
3.1 Summary of Background Theory	51
3.1.1 Scope	51
3.1.2 Microscopic versus Macroscopic	51
3.1.3 Relevant Parameters	52
3.1.4 Surface Tension	52
3.1.5 Hydrophilic and Hydrophobic	53
3.1.6 Young's Equation	54
3.1.7 Line Tension	55
3.1.8 Capillary Pressure	56
3.1.9 Laplace's Equation	57
3.1.10 Mathematical Manipulations	58
3.2 Summary of Pendant Drop Theory	59
3.2.1 Definition	59
3.2.2 Pendant Drop Force Balance	59
3.2.2 Capillary Pressure	60
3.2.3 Drop Shape Based on Surface Tension	61
3.2.4 Fluid Density (Gravity Effects)	61
3.2.5 Size of Droplet at Separation	61
3.3 Summary of Free Droplet Theory	62
3.3.1 Definition	62
3.3.2 Free Droplet Force Balance	62
3.3.3 Air Resistance	63
3.3.4 Separation Disturbances	65
3.4 Summary of Sessile Drop Theory	66
3.4.1 Definition	66
3.4.2 Sessile Drop Force Balance	66
3.4.3 Variation in Properties (Laplace's Equation)	67
3.4.4 Drop Shape Based on Surface Tension	68
3.4.5 Hydrophilic and Hydrophobic Surfaces	69
3.5 Summary of Spreading Drop Theory	69
3.5.1 Definition	69
3.5.2 Spreading Drop Force Balance	69
3.5.3 Precursor Flow	70
3.5.4 Roughness Effects	71
3.5.5 Quasi-Static Flow Assumption	71
3.5.6 Theoretical Contact Angle	71

3.6 Summary of Droplet Impact Theory	74
3.6.1 Definition	74
3.6.2 Impact Dynamics	75
3.6.3 Impact Velocity	77
3.6.4 Energy Loss During Impact	77
3.6.5 Air Entrainment	79
3.6.6 Surface Roughness Effects	79
3.7 Summary of Splashing and Freezing Interaction Theory	80
3.7.1 Definition	80
3.7.2 Effects of Cooling Liquid	81
3.7.3 Roughness Generation through Ice Formation	81
3.7.4 Shear Friction Heating and Re-freezing	82
3.7.5 Hydrophilic / Hydrophobic Behaviour of Surface	83
3.7.6 Speed of Frost Line	84
3.7.7 Speed of Water Line	85
3.7.8 Change in Surface Area	86
4.0 Experimental Procedure.....	87
4.1 Data Acquisition	87
4.2 Image Processing	88
5.0 Test Results.....	89
5.1 Pure Splashing Results	89
5.2 Splashing and Freezing Result	95
5.3 Alternate Test Liquids	98
5.3.1 Test Liquid Properties	98
5.3.2 Splashing Results	100
5.3.3 Splashing and Freezing Results	101
5.4 Sample Images	102
6.0 Future Work.....	107
7.0 References.....	109

List of Figures

Fig.1 Final Light Path Selection	10
Fig.2 Convective Heat Transfer	20
Table 1 Tabulated results of Heat Transfer Calculations	25
Fig.3 Coolant System Flow Patterns	27
Fig.4 Alternate Rack Configuration	28
Fig.5 Initial Cooling Trial	29
Fig.6 Insulated Cooling Trial	29
Fig.7 Short-Term Warming	31
Fig.8 Long-Term Warming	31
Fig.9 Effect of interfacial shear stress	38
Fig.10 Thermocouple Calibration Data	41
Fig.11 Concave Surface Evaporation Testing	46
Fig.12 Convex Surface Evaporation Testing	47
Fig.13 Young's Equation	54
Fig.14 Pendant Drop Force Balance	60
Fig.15 Free Droplet Force Balance	63
Fig.16 Sessile Drop Force Balance	66
Fig.17 Spreading Drop Force Balance	70
Fig.18 Contact Angles on an Incline	72
Fig.19 Spreading of a Freezing Droplet	86
Fig.20 An 804mm droplet fall height onto Al foil on glass, no cooling	93
Fig.21 A 548mm droplet fall height onto painted glass, no cooling	94
Fig.22 Photographs of Torroidal Ice Cup	102
Fig.23 Captures of Torroidal Ice Cup Behaviour	103
Fig.24 Splash Imaging, Droplets 1 to 4	104
Fig.25 Splash Imaging, Droplets 5 to 8	105
Fig.26 Precursor Flow and Low Splashing Images.	106
Fig.A1 Design of Containment Box	114
Fig.D1 Design of Light Path	118
Fig.D2 Alternate Configuration of Light Path	119
Fig.F1 Design of Cooling Rack	127
Table H1 Tabulated results of Heat Transfer Calculations	138
Fig.I1 Ambient Conditions	139
Fig.P1 Initial Image Capture	169
Fig.P2 Image + Sobell	170
Fig.P3 Image + Sobell + Negate	171
Fig.P4 Image + Sobell + Negate + Subtract 10	172
Fig.P5 Blob Analysis	173

List of Appendices

Appendix A: Design of the Containment Box	112
Appendix B: Design of the Support Rack	115
Appendix C: Test of 0.0075" ID Tubing	116
Appendix D: Design of Light Path	117
Appendix E: Investigation of Air Resistance	120
Appendix F: Design of the Cooling System	126
Appendix G: Conductive Heat Transfer Calculations	131
Appendix H: Convective Heat Transfer Corrections	133
Appendix I: Ambient Conditions	139
Appendix J: Labour Estimate	140
Appendix K: Pricing Estimate	141
Appendix L: Discussion of Parameters	142
Appendix M: Overview of Mathematical Formulations	146
Appendix N: Analysis of a Related Work	149
N.1 Scope of Article	149
N.2 Application of Theory	149
N.3 Analysis of Theory	150
N.4 Analysis of Mathematics	150
Appendix O: Laboratory Procedure	153
O.1 Operational Hazards	153
O.2 Operational Methods	157
O.3 Operational Procedure	160
O.3.1 Setup Procedure	160
O.3.2 Single Droplet Analysis	161
O.3.3 Multiple Droplet Analysis	163
O.3.4 Single Imaging	163
O.3.5 Sequence Imaging	164
O.3.6 Image Processing	165
O.3.7 Shutdown Procedure	166
Appendix P: Effect of Image Processing Operations	168

List of Symbols

A_n	Area of object N
a	Acceleration
d	Diameter
E_K	Kinetic energy
E_P	Potential energy
E_T	Total internal energy
F	Force
Gr	Grasshof Number
g	Acceleration due to gravity, taken as 9.81 m/s^2 or 32.174 ft/s^2
m	Mass
N	Number of moles
Nu	Nusselt Number
P	Pressure
Pr	Prandtl Number
Q	Heat
R	Radius
Ra	Rayleigh's Number
Re	Reynold's Number
S	Entropy
T	Temperature
t	Time
U	Bulk fluid velocity
V_a	Velocity of particle or body A
V_n	Volume of object N
Z	Height
μ_j	Chemical potential energy of the j^{th} component
γ	Surface tension
ρ	Density
μ	Viscosity

It is possible to have situations where confusion may arise from these symbols, particularly in the duplicity of velocity and volume. Where possible, text has been added to assist in clarification. Most of these variables are described in more detail in Appendix L: Discussion of Parameters.

1.0 Introduction

This work describes the design, manufacture and testing of a system to investigate splashing and freezing phenomena. It provides the beginning of a theoretical bridge to practical industrial solutions of the problems induced by these phenomena.

1.1 Overview

This work explains the design and intended operational procedures for an experimental system intended to investigate the interactions of the splashing and freezing phenomena. This system is capable of dropping liquid onto a plate at very controlled rates. The plate and surrounding air can also be cooled. This permits the analysis of splashing and freezing phenomena both in isolation and in interaction. Included also is a summary of the relevant theory to assist investigators in their analysis. This work provides the knowledge base required to carry out the work that may someday lead to an understanding of the interaction between splashing and freezing. From this base, future work will be required in the investigation, and this work provides some guidance for what direction that work should take.

This approach may lead to a theoretical understanding of the problems indicated above, especially aircraft icing. Current work in the field seems almost exclusively based on solving the problem experimentally. The aim of this work is to begin a combined

theoretical and experimental investigation that may someday bridge the gap to the best solution of the problem.

1.2 Overview of Phenomena

Splashing is simply the break-up of a droplet of liquid as it impacts a surface. Splashing can be done at relatively low speeds, depending on the surface impacted. This phenomenon has been extensively studied.

Freezing is simply the transformation of a liquid to a solid as energy is removed via heat. This phenomenon is a classical one that can be considered well understood.

Splashing and freezing is the interaction of the two phenomena. The interaction has not been studied extensively, hence the need for an apparatus to analyze it.

1.3 Relevance to Industry

Splashing and freezing phenomenon have several applications throughout industry. Aircraft icing, for example, is a major concern of the aerospace industry, and solving that problem requires an understanding of the interactions between splashing and freezing. Automobile windshields suffer similar effects in northern climates. Metal plating is another example of splashing and freezing interaction in an industrial setting.

Pure splashing is of great concern to even more industries. Steel rolling mills require oil sprays, which induce splashing. Computer printers and several bio-medical and pharmaceutical works require exact knowledge of when splashing phenomena will occur.

1.4 Objectives

This work has four primary objectives:

- 1) Describe the design and manufacture of the system.

This will permit future investigators to gain insight into the nature of the system, and thus any bias inherent to the work. This material is summarized in section 2.

- 2) Summarize the background theory.

This will give future investigators an introduction to the theoretical material required for future work in this field. This theory is summarized in section 3.

- 3) Present a guide to the operation of the system.

This will be required for reproduction of the experiment. This guide is summarized in section 4.

- 4) Present the interesting results obtained from preliminary investigation.

There are three key results of interest. Droplet interaction during splashing, the Dead Zone phenomenon, and the regimes of splashing and freezing. These results are presented in section 5.

2.0 Analysis of Design

2.1 Experimental Objectives

The system had to be designed to investigate splashing and freezing phenomena. To do so, it dropped liquid droplets onto a plate. The distance, plate surface, and droplet size were carefully controlled. The resultant distributions of droplets were then analyzed to obtain information about the splashing phenomenon. Splashing and freezing interaction was analyzed by cooling the plate and monitoring the plate temperature. The droplet distribution was studied as in the pure splashing case.

2.2 Initial Design Considerations

Before any engineering design could be started, the principal considerations had to be laid out. In this case, the design considerations were mostly flexible, so the initial considerations were as follows:

- Investigation of phenomena
- Separation of phenomena
- Versatility of design
- Use of digital vision system in parallel design
- Low cost

The system had to be capable of investigating three phenomena: splashing, freezing and the interaction of splashing and freezing. The primary design consideration was thus that the system could perform this analysis.

The system had to be capable of investigating the phenomena separately. Further, isolation from various sources of contamination was required.

The system had to be versatile. Wherever possible, it had to permit adaptation towards the analysis of any variables that proved of interest. Further, the system had to be able to be modified to match cases of industrial interest. For example, the system had to be able to accept a variety of liquids and impact surfaces. This gave the system the capability to analyze emulsified contaminants, and the effects of roughness and surface curvature.

The system was designed in parallel with the digital vision system that was used. Details of the digital vision system can be found in the Master's of Engineering thesis of

Zhou (2000). This hardware system permitted images to be captured through the use of a CCD camera.

Cost of components and labour is an obvious part of industrial design that is often missed in research and development work. While the total cost of the system would not be low, whenever a lower cost approach would not jeopardize the final accuracy of the system it was used. This led to in-house manufacture of several components.

2.3 Initial Variable Analysis

The first decision made was which variables had to be controlled, and to what extent. The first variable requiring control was obviously chemical purity. The liquid used obviously had to be uncontaminated during the procedure. It was deemed preferable for the system to use water as its operating liquid. To maintain purity, the system required a method of delivery that would not contaminate the sample. It also required a containment enclosure to prevent the experiment from being contaminated by dust, oils or other airborne contaminants. Droplet control was also required. The droplet volume had to be reproducible, and the flow rate variable. The impact surface also needed some consideration, so as to prevent chemical interaction or significant roughness. If there were a substantial roughness to the plate, it would cause large errors in calculation when assuming a flat, essentially smooth plate. Roughness had to be adjustable, so that it could be analyzed separately. Temperature control also had to be included in the design of the

system. Temperature knowledge was considered more important than control, but the plate had to be brought below the freezing point of the liquid used. Humidity control was also considered, but after an analysis of the complexity of maintaining an exact humidity, it was decided that humidity knowledge would suffice. It was expensive to maintain an exact humidity level, and thus was generally not considered in investigations of splashing behaviour. Rough humidity control had to be possible, however, in order to investigate the effect experimentally.

2.4 Experimental Isolation

The system had to prevent contamination by dust, oils or other aerosol contaminants. Further, since the droplets may be very small, the system had to enclose stagnant air, to ensure that air currents did not affect the results. To ensure all of these effects, a containment box was decided upon to enclose the system. This box had to be airtight, and still had to permit imaging. The final design was a cubic plexiglass containment box, 1.2m to a side, with access doors in the front. The plexiglass permitted imaging through the box, was low in chemical reactivity, and did not out-gas contaminants. The box was manufactured in-house. Further information on the design can be found in Appendix A: Design of the Containment Box. To support elements within the box, a support rack was added. Details of the rack can be found in Appendix B: Design of the Support Rack.

2.5 Delivery of Test Liquid

The system had to deliver the test liquid to the experiment without contamination. It had to have a variable speed delivery, and the resultant droplets had to be of reproducible size. The final design selection was the Masterflex pumping system. It used rollers to squeeze the tubing, which ran from the sample container to the containment box. This forced the fluid to move without contacting the fluid in any way. Chemical non-reactivity was easily ensured by selection of the tubing material. The system could thus be modified to use new test liquids by simply changing the tubing. The conventional system of a mechanically operated syringe was limited by volume. The Masterflex system offered the possibility of uninterrupted long-term pumping to permit ice accretion in large layers.

To ensure the resultant droplets were of reproducible volume, testing was done with the tubing end squarely cut. Droplets were permitted to form on the end of the tubing at low flow rates, and separate due to gravity forces. It was found that the droplets were sufficiently reproducible if the end was clamped in place to prevent vibration. An example of the flow rate testing which led to the choice of tubing diameter can be seen in Appendix C: Test of 0.0075" ID Tubing. The pump selection was as follows:

Masterflex Low Speed Precision Standard Pump (7521-50)

RPM:	1 to 100
Input:	115 Vac (50-60 Hz), 3 Amp
Shipping Weight:	14.8 lb (7.3 kg)
Design Load:	Up to 4 standard pump heads
Control:	Manual Analog or by RS-232

Tygon Tubing

ID:	0.0075" (0.19mm) or 0.0095" (0.24mm)
OD:	0.0795" (2.02mm) or 0.0819" (2.08mm)

2.6 Vibration Isolation

Vibration could cause error in the system by altering the droplet size and trajectory. To minimize vibration, all pumps were supported separately from the containment box. The containment box was placed on two pieces of ½" (12.7mm) plywood, cut 122cm square. These were placed on a vibration isolation support, providing both thermal and vibrational isolation. The support separated the isolated surface via an inflated rubber membrane. The digital vision system contained its own vibration isolation table.

2.7 View Angle

The droplet distribution had to be seen from a top-down look angle. That would permit an analysis of the radial distribution in splashing. Being able to vary the look angle to a side view permitted analysis of pendant and sessile droplets, as well as studying the texture of ice accretion. Thus, a variable look angle system was adopted. The look angle could be changed with the use of a mirror placed within the system. The final geometry used is visible in figure 1, below. Additional details of the design, as well as lighting concerns are available in Appendix D: Design of the Light Path.

Fig.1 Final Light Path Selection

2.8 Design of Drop Path

2.8.1 Droplet Diameter

The intent of the experiment was to investigate the interaction of multiple droplets. This was investigated by viewing an accumulation of droplets. Minimizing the droplet diameter permitted the minimum area of surface to be used. This permitted the maximum magnification of the digital vision system, which in turn minimized the error due to image capture. Thus, the diameter of the droplet was kept small. A droplet diameter of 5mm or less led to the use of the smallest diameter tubings commonly available for the Masterflex pump system.

Frohn and Roth, (p.83, Frohn, 2000) describe a calculation for the droplet size created by permitting a droplet to break free from a vertical tube. This is the method of droplet creation used in this system. The final droplet radius (d) is thus found as:

$$d = [6 (d_{\text{tubing}}) (\sigma) / (\rho_{\text{drop}}) (g)]^{1/3} \quad [2.8.1.1]$$

where d_{tubing} is the outer diameter of the tubing, σ is the surface tension of the droplet, and ρ_{drop} is the density of the drop. For this system, the equation becomes:

$$d = [6 (0.00208\text{m or } 0.00202\text{m}) (0.07275 \text{ N/m}) / (998 \text{ kg/m}^3) (9.81\text{m/s}^2)]^{1/3}$$

$$d = 4.53 \text{ mm for the larger tubing.}$$

$d = 4.48 \text{ mm}$ for the smaller tubing.

These values are roughly confirmed by experimental measurements, made by weighing several droplets and averaging, which give a droplet mass of 0.033g. For a sphere, the volume can be calculated as:

$$V = (4/3) \pi R^3 = m / \rho \quad [2.8.1.2]$$

where V is the volume of the droplet, R is the droplet radius, m is the mass of the droplet and ρ is the density of the droplet. This yields:

$$R = [(3 m) / (4 \pi \rho)]^{1/3} \quad [2.8.1.3]$$

$$\begin{aligned} d &= 2 \times [(3) (0.033 \times 10^{-3} \text{ kg}) / (4) \pi (998 \text{ kg/m}^3)]^{1/3} \\ &= 4 \text{ mm} \end{aligned}$$

2.8.2 Drop Velocity

The drop path was designed to be of variable height. The upper support on the rack could be adjusted to place the tube that formed the droplet nearly onto the plate for spreading analysis, and up to the top of the containment box for splashing analysis. This gave the drop just over one metre to accelerate. A ballistic analysis would define the drop impact velocity as:

$$U^2 = U_o^2 + 2gz \quad [2.8.2.1]$$

The initial velocity at separation (U_o) was negligible. This occurred because the droplet was permitted to form off the end of the tubing, grow and separate under gravity forces. This was done to ensure a reproducible drop size. Thus, the droplet cannot be ejected from the tubing if this droplet accuracy is to be preserved. Consequently, the ballistic impact velocity must be:

$$\begin{aligned} U &= (2gz)^{1/2} & [2.8.2.2] \\ &= (2 (9.81 \text{ m/s}^2) (1\text{cm to } 106.7\text{cm}))^{1/2} \\ &= (0.5 \text{ m/s) to (4.57m/s)} \end{aligned}$$

It is important to note that this velocity estimate does not account for air friction. To estimate the effects of air friction, several analyses are possible, depending on the droplet diameter. Range and Feuillebois (p.16, JCI 203, 1998) give a description of several of these methods. They describe experimental results for a droplet of 4mm diameter with the altered expression:

$$U = (g/A (1 - e^{-2AZ}))^{1/2} \quad [2.8.2.3]$$

where $A = 3 (c_f) (\rho_{\text{air}}) / 8 (\rho_{\text{drop}}) (R_o)$ and $c_f = 0.796$

For a droplet diameter of 4.5mm:

$$\begin{aligned} A &= 3 (0.796) (1.2 \text{ kg/m}^3) / 8 (998 \text{ kg/m}^3) [(0.0045\text{m})/2] [2.8.2.4] \\ &= 0.1595 \text{ /m} \end{aligned}$$

$$\begin{aligned} U &= [(9.81\text{m/s}^2) / (A)(1-e^{(-2)(A)(1.067\text{m})})]^{1/2} [2.8.2.5] \\ &= 14.6 \text{ m/s} \end{aligned}$$

This result makes little sense. Air resistance cannot increase the velocity of the droplet by a factor of three, so another approach was taken. A somewhat lengthy elementary analysis was performed, the details of which are summarized in Appendix E: Investigation of Air Resistance. This investigation returns a velocity value of 4.426 m/s with air resistance, and 4.429 m/s without.

This leads one to the conclusion that air friction will cause a very small effect for this size of droplet or larger, for this fall height or lower.

Another point about these velocity calculations that needs consideration is that a droplet moving at approximately 4 m/s is a slow moving droplet of about 14 kilometers per hour. This, of course, it is not a match for rain droplets at terminal velocity, or aircraft wings, or automotive windshields, or many other industrial phenomena. Nonetheless, this gives the opportunity to analyze the theory of droplet impact, spreading and splashing. Lower velocities means slower spreading, which means more images can be taken for

study. Further, it permits the splashing to be contained to smaller samples, permitting more magnification on the camera, while still imaging the entire splash zone.

To match such cases, non-dimensional parameters such as the Reynold, Prandtl and Weber numbers should be considered for matching. As these terms are all dependent on velocity in different ways, either the most important term must be matched, or the velocity must be matched. If the velocity must be matched, droplet initial velocity can be raised by increasing the flow rate to generate a non-zero initial U_0 . This will induce a spray, however, which will create situations of multiple droplet interaction, and reduce the droplet diameter.

2.8.3 Limiting Factors to Droplet Velocity

To ensure that the calculated velocity would actually be attained, the terminal velocity of the droplet, and the critical instability velocity of the droplet were calculated.

The terminal velocity is the maximum velocity of the droplet in free air. It was calculated by performing a force balance, the details of which are summarized in Appendix E: Investigation of Air Resistance. The resultant equation reduced to:

$$U_{\text{terminal}} = [4 D_{\text{drop}} ((\rho_{\text{droplet}} / \rho_{\text{air}}) g - 1) / 3 C_D]^{1/2} \quad [2.8.3.1]$$

The C_D value, however, was dependant on the droplet velocity, and an iterative solution was required. The iterations were performed, using the graph of C_D values for a sphere presented in the literature, (p.295, Daugherty & Franzini, 1977). This iteration returned a result of 11m/s and a Reynold's Number based on diameter of approximately 3400. As the droplet velocities in the system are not expected to reach beyond 4 m/s, the droplets would not be restricted by terminal velocity.

The critical instability velocity of the droplet is the velocity at which drag forces overcome surface tension forces, and the droplet disintegrates. The balance of surface tension and drag forces is given in the literature, (p.15, Frohn and Roth, 2000), as:

$$2 \pi R_{\text{drop}} \sigma_{\text{drop}} = C_D \rho_{\text{air}} U_{\text{drop}}^2 \pi R_{\text{drop}}^2 / 2 \quad [2.8.3.2]$$

This can be reduced to a velocity or diameter dependent form. To investigate the maximum speed sustainable before the droplet breaks up, the velocity reduction was used. The expression becomes:

$$U_{\text{critical}} = [8 \sigma_{\text{drop}} / D_{\text{drop}} \rho_{\text{air}} C_D]^{1/2} \quad [2.8.3.3]$$

Again, this is an iterative solution as it includes both velocity and the drag coefficient. Iterated as before, it yields a critical velocity of 16.5 m/s at a Reynolds Number based on diameter of about 5000. As this is above the terminal velocity, this break-up will not be a consideration for the system. To investigate the maximum droplet

size permissible at the velocities created by the system, the diameter reduction was used.

The expression becomes:

$$D_{\text{critical}} = 8 \sigma_{\text{drop}} / U_{\text{drop}}^2 \rho_{\text{air}} C_D \quad [2.8.3.4]$$

This formulation indicates that for a velocity of 4 m/s, a droplet diameter of 72mm or larger will break up under drag forces, with the 72mm droplet having a Reynold's Number, based on diameter, of about 19,000. Obviously, as the droplet to be used was only 4mm in diameter, it was well below the critical instability diameter.

2.8.4 Design of Impact Plate

The impact plate is the surface upon which the droplets impact after their fall. . To ensure pure study of spreading and splashing phenomena, surface roughness and chemical reactivity of the plate had to be minimized. These factors influenced the droplet spreading and splashing behaviour

In order to minimize surface roughness and chemical reactivity, gold was selected as the material for the impact plate. Gold does not form an oxide layer, and so its surface roughness could be kept low. It also provided a surface of low chemical reactivity. Obviously, there was no need for a specimen of monolithic gold, so a gold plated glass plate was used. The plates were manufactured in-house at Carleton University, in the

Fabrication Labs of fourth floor Minto Case. In general, in order to generate sufficient adhesion between the glass and the gold, a thin layer of titanium or chrome was first sputtered onto the surface, followed by the layer of gold. In this case, titanium was used, at the discretion of the manufacturer, Mr. L. Berndt. Two plates were prepared. In the first, the layers were approximately 300 Angstroms thick. In the second, the layers were approximately 2,000 to 3,000 Angstroms thick. The thinner layered plate was translucent, thus allowing under-lighting of the experiment, should the desire arise. The thicker layered plate functioned as a mirror.

In both cases, the plates were prepared in a similar manner. Washings with hydrogen peroxide, ammonium hydroxide and de-ionized water were used to clean the glass of organics. The glass was then dried by washing with isopropyl alcohol, and blow-drying with filtered nitrogen gas. The titanium was then sputtered onto the surface, and shortly thereafter, the gold.

The plates measured 10.2cm square.

2.9 The Cooling System

In order to study freezing phenomena, the system had to be cooled to below the freezing point of the test liquid. To cool the system, several possible design solutions were considered. Spraying with liquid nitrogen was considered too extreme, as were

many other rapid cooling techniques. Ideally, the plate would be maintained at its low temperature. To do this, it was possible to use small “Peltier Units”. These small electric devices can provide precise temperature control, acting as a heat pump to add or remove heat. The cost of creating a system with enough strength, however, proved prohibitive. The conventional alternative involved cooling a liquid and pumping it through a piping network that would act to cool the system. The system would then require a reservoir of fluid, a chiller, pumps and a piping system. The details of the design of the coolant system can be found in Appendix F: Design of the Cooling System.

In cases where more radical cooling was required, dry ice (solid CO₂) was used as a cooling agent, packed around the area of the impact plate. If merely in the vicinity, it caused a very cold boundary layer to be trapped, allowing for better cooling of the plate. As it was packed more and more in contact with the supports, it provided more and more cooling. Actually placing the dry ice in contact with the impact plate made for very extreme cooling. By variations, this method could be used to study extreme temperatures.

2.9.1 Chiller Selection

The cooling system was driven by the chiller, which cooled the reservoir fluid. In order to size the chiller, the expected heat loss had to be estimated. Assuming convection and conduction will play significant roles in heat transfer, the first check was to see if conduction would limit heat transfer. This check can be found in detail in Appendix G:

Conductive Heat Transfer Calculations. The result was as expected, convection, not conduction limits heat transfer. Thus, the heat transfer by convection was calculated.

Applying conventional convection heat transfer analysis:

Fig.2 Convective Heat Transfer

$$T_{o1} = T_{air} = 20^{\circ}\text{C}$$

$$T_{o2} = T_{fluid} = -20^{\circ}\text{C}$$

$$L = t_{pipe} = 0.0023 \text{ m}$$

$$K = 54 \text{ W/m}^{\circ}\text{C}$$

$$Q = T_{o1} - T_{o2} / [(1/Ah_1) + (L/AK) + (1/Ah_2)] \quad [2.9.1.1]$$

To use this approach, the heat transfer coefficients h_1 and h_2 must be calculated as follows:

$$Q = Ah_1 (T_{o1} - T_1) \quad [2.9.1.2]$$

$$= Ah_2 (T_2 - T_{o2}) \quad [2.9.1.3]$$

$$= AK (T_1 - T_2) / L \quad [2.9.1.4]$$

Even with the three auxiliary equations, this leaves four equations in the five unknowns:

Q, T_1, T_2, h_1, h_2 .

To solve this, we assume common literature values for h_1, h_2 :

For a 0.02 m Diameter Cylinder:

Air: $h_1 = 8 \text{ W/m}^2\text{°C}$

Water: $h_2 = 741 \text{ W/m}^2\text{°C}$

$$\begin{aligned}
 Q &= T_{o1} - T_{o2} / [(1/Ah_1) + (L/AK) + (1/Ah_2)] & [2.9.1.5] \\
 &= 20^\circ\text{C} - (-20^\circ\text{C}) / [(1/(0.155\text{m}^2)(8\text{W/m}^2\text{°C})) + (0.0023\text{m}/(0.155\text{m}^2)(54 \text{ W/m }^\circ\text{C})) \\
 &\quad + (1/(0.155\text{m}^2)(741 \text{ W/m}^2\text{°C}))] \\
 &= 49 \text{ W}
 \end{aligned}$$

Find T_1, T_2 by rearranging auxiliary equations:

Let $q = Q/A$

$$\begin{aligned}
 T_1 &= -(q / h_1) + T_{o1} & [2.9.1.6] \\
 &= -(49\text{W} / (0.155 \text{ m}^2)(8 \text{ W/m}^2\text{°C})) + 20^\circ\text{C} \\
 &= -19.559^\circ\text{C}
 \end{aligned}$$

$$\begin{aligned}
 T_2 &= (q / h_2) + T_{o2} & [2.9.1.7] \\
 &= (49\text{W} / (0.155 \text{ m}^2)(741 \text{ W/m}^2\text{°C})) - 20\text{°C} \\
 &= -19.573\text{°C}
 \end{aligned}$$

To check:

$$\begin{aligned}
 Q &= AK (T_1 - T_2) / L & [2.9.1.8] \\
 &= (0.155\text{m}^2) (54 \text{ W/m}^2\text{°C}) (-19.559\text{°C} - -19.573\text{°C}) / (0.0023\text{m}) \\
 &= 49 \text{ W}
 \end{aligned}$$

Thus, we expect 49W of heat transfer from the rack when first started in 20°C ambient air.

Using a similar procedure, we can determine the heat transfer from the coil.

Coil characteristics:

$$\begin{aligned}
 A_{\text{coil}} &= \pi(d_{\text{coil}}) L_{\text{coil}} & [2.9.1.9] \\
 &= \pi(3/8") 50' \\
 &= 0.456 \text{ m}^2
 \end{aligned}$$

$$K_{\text{cu}} = 83 \text{ W/m}^2\text{°C}$$

Coil heat transfer:

$$\begin{aligned}
 Q &= T_{o1} - T_{o2} / [(1/Ah_1) + (L/AK) + (1/Ah_2)] & [2.9.1.10] \\
 &= 20^\circ\text{C} - (-20^\circ\text{C}) / [(1/(0.456\text{m}^2)(8\text{W/m}^2^\circ\text{C})) + (0.0023\text{m}/(0.456\text{m}^2)(83\text{W/m}^\circ\text{C})) \\
 &\quad + (1/(0.456\text{m}^2)(741\text{ W/m}^2^\circ\text{C}))] \\
 &= 144\text{ W}
 \end{aligned}$$

Thus, we expect 140W of heat transfer from the coil when first started in 20°C air.

This creates a load of nearly 200 W on start-up.

A chiller was found that produced 170 W cooling power at -20°C . This chiller was determined suitable due to the fact that the start-up transient would be short, as the air in the containment box would rapidly cool. Furthermore, the system would not normally be run at -20°C , and the chiller would not necessarily need to meet the demand exactly due to the size of the reservoir.

For the alternate configuration with the coolant plate reservoir, the heat transfer was calculated. In the analysis of the coolant plate, it was assumed that the insulation would block the heat path from all sides but the open top, and as such, the area of the top alone was used. There will be some leakage through the insulation, but the insulation will trap a large boundary layer of generally still cold air.

Coolant plate characteristics:

$$\begin{aligned} A_{\text{coil}} &= 13.5'' \times 8.375'' \\ &= 0.0729 \text{ m}^2 \end{aligned}$$

$$K_{\text{cu}} = 180 \text{ W/m}^\circ\text{C}$$

$$\begin{aligned} L &= 1/32'' \\ &= 0.000794 \text{ m} \end{aligned}$$

Coolant plate heat transfer:

$$\begin{aligned} Q &= T_{o1} - T_{o2} \quad / \quad [(1/Ah_1) + (L/AK) + (1/Ah_2)] & [2.9.1.11] \\ &= 20^\circ\text{C} - (-20^\circ\text{C}) \quad / \quad [(1/(0.0729\text{m}^2)(8\text{W/m}^2\ ^\circ\text{C})) + (0.000794\text{m}/(0.0729\text{m}^2)(180\text{W/m} \\ &\quad ^\circ\text{C})) + (1/(0.0729 \text{ m}^2)(741 \text{ W/m}^2\ ^\circ\text{C}))] \\ &= 31 \text{ W} \end{aligned}$$

In order to ensure that the heat transfer calculations were accurate, several correction factors were investigated. The correction factors are empirical relations for curves fit to experimental data for various conditions by various experimenters. The relations from the work of McAdams and the work of Churchill and Chu were presented in Ozisik, p.431. The specific case of a horizontal, flat plate is given with McAdams

relations in Ozisik, p.437. The details of these calculations are given in Appendix H:

Convective Heat Transfer Corrections. The results are summarized in Table 1, below.

Table 1 Tabulated results of Heat Transfer Calculations

Method	Required Cooling (W)		
	Coolant Pipe	Coolant Plate	Coil
Convection	49	32	140
McAdams	310	160	800
Churchill and Chu	230	140	470
McAdams (Flat Plate)	140	74	370

The values were thus very disparate between the corrections. The corrections were by far the worst-case scenario. After carefully inspecting the criterion, the flat plate correction seemed the correction most accurately matching the conditions for the rack. The chiller could produce the required 140.W of rack cooling for that case. As such it was decided to manufacture the system with the given chiller, and test to see if the system could handle the load of the rack. The coil would be used only for transients. The results of these tests are discussed in section 5.3, which details the final system transients for the rack. Transients for the cooling plate are comparable, if slightly quicker.

The chiller selected was an FTS/Kinetics Flexi-Cool FC55A01 with RS-232 control.

2.9.2 Assembly of the Coolant System

The coolant system was then assembled using the above components. The flow within the reservoir was designed to use the coolest possible liquid. The pumps were bottom-intake submersible pumps, which pumped fluid from the bottom of the reservoir. The chiller coil was placed near the intakes. As can be seen in fig.3, the fluid returns were placed high above the intakes, above the fluid level. The returning fluid would then splash into the reservoir, providing mixing to improve heat transfer, and re-cooling. The returned fluid would tend to float, however, being warm, and would only return to the pumps when cooling again increased its density.

The coil flow was selected as top-to-bottom. This made the coolest part of the coil the uppermost region, where the droplet was forming. This would cool the droplet, and the coil would ensure the entire drop path remained cool.

Fig.3 Coolant System Flow Patterns

The rack was designed with the inlet and outlet on the same end. This system causes more flow along the nearer branches, as the flow path is shorter. This leads to a temperature gradient along the rack, as shown in Fig.4. As the rack was designed to be

short, and the test point very localized, this was considered acceptable. The rack was also designed, however, to permit the feed or the return to be switched with one of the plugs. This would permit the rack temperature gradient to be removed for large samples.

Fig.4 Alternate Rack Configuration

2.9.3 Coolant System Testing and Optimization

The system was then manufactured and tested to see if insulation would be required. Without insulation, the system took several hours to cool, and never cooled to a sufficiently cold temperature. The test was performed with the pumps off, so any heat loss was through the reservoir walls. The results of that first test can be seen in Fig.5, below, along with the insulated results, for comparison.

Fig.5 Initial Cooling Trial

Fig.6 Insulated Cooling Trial

Fig.6, above, summarizes the results of the same test after the insulation was installed. The transient again took several hours, but this time yielded much more favorable results. This clearly demonstrated the need for insulation on the containment box.

To investigate if further heat losses could be prevented and permit colder operations, the warming transient was inspected.

The graphs below show the system behaviour during warming. Fig.7 gives a detail of the short-term behaviour, while Fig.8 extends the perspective. The test data was gathered by stopping the chiller, then measuring the temperature of the coolant reservoir. The pumps were run to generate the worst-case situation. During this time, the ambient temperature and humidity was recorded for comparison, and is also displayed. The ambient temperature is given as the average of the two digital thermometers listed as A and B, and is thus listed as AB(degC) in the graphs below.

Fig.7 Short-Term Warming

Fig.8 Long-Term Warming

2.9.4 Falling Droplet Heat Transfer

As is evidenced by the heat transfer calculations of section 2.9.1 the coil transferred a large amount of heat into the coolant reservoir. This analysis investigated the effects of heat transfer to a chilled droplet (0°C) from ambient air (20°C) during the time period of a typical experiment. If the amount was low, the cooling coil would be used only rarely. This would permit the reservoir to warm much more slowly, and stabilize to colder temperatures.

According to Ozisik, (p.420, Ozisik, 1985) if:

$$Gr / Re^2 \gg 1 \quad [2.9.4.1]$$

then free convection dominates, where:

$$Gr_L = g \beta (T_\infty - T_w) L^3 / \psi^2 \quad [2.9.4.2]$$

In this case, interpolating values for air at 20°C: (p.736, Ozisik, 1985)

$$\begin{aligned} \psi &= [9.49 \times 10^{-6} + (15.68 - 9.49) \times 10^{-6} \times (293.15 - 250) / (300 - 250)] \text{ m}^2/\text{s} \\ &\sim 14.83 \times 10^{-6} \text{ m}^2/\text{s} \end{aligned}$$

$$Pr = 0.722 - (0.722 - 0.708) \times (293.15 - 250) / (300 - 250)$$

$$\sim 0.7099$$

$$k = [0.02227 + (0.02624 - 0.02227) \times (293.15 - 250) / (300 - 250)] \text{ W/m}^\circ\text{C}$$

$$= 0.02570 \text{ W/m}^\circ\text{C}$$

$$\beta \sim 1/T \sim 1/293.15\text{K}$$

$$\sim 3.411 \times 10^{-3} / \text{K}$$

$$g = 9.81 \text{ m/s}^2$$

$$T_\infty = 20^\circ\text{C}$$

$$T_w = 0^\circ\text{C}$$

$$L = D = 0.0045 \text{ m for the droplet}$$

$$L = 0.4 \text{ m for the impact plate}$$

Applying these values to equation 2.9.4.2, above, yields:

$$Gr_D \sim 277 \quad \text{for the droplet}$$

$$Gr_L \sim 1.95 \times 10^8 \quad \text{for the plate}$$

Thus, in this case, free convection will dominate at the plate. Further, the boundary layer can be investigated using the criterion (p.426, Ozisik, 1985) for the Rayleigh number (p.421, Ozisik, 1985). This criterion states that the boundary layer will remain laminar if the Rayleigh number is less than 10^9 .

$$\begin{aligned} Ra &= Gr_L Pr & [2.9.4.3] \\ &\sim 1.38 \times 10^8 < 10^9 \end{aligned}$$

This confirms that the plate boundary layer will be laminar. With this result, further confidence is obtained in the calculations for heat transfer of the plate.

For the droplet, when the drop is forming, its velocity is zero. As such, the Re_D is also zero, and the free convection situation occurs.

An analysis of free convection on a single, isothermal sphere gives the following expression for the Nusselt number (p.447, Ozisik, 1985):

$$Nu_m = h_m D / k \quad [2.9.4.4]$$

$$= 2 + 0.43 (Ra_D)^{1/4} \quad \text{For } 1 < Ra_D < 10^5 \text{ and } Pr \sim 1 \quad [2.9.4.5]$$

With the experiments performed, the system tended to form drops in 20s. However, since the droplet was forming during this period, and was thus very small for the initial period, the heat transfer can be assumed to be much less. In order to estimate this, the area was averaged over the period, and therefore, halved. This result will still be high, as the droplet was in contact with the cold fluid, and this heat transfer is neglected. The result is as follows:

$$Q = \dot{Q} t = (1/2) h_m A \Delta T t \quad [2.9.4.6]$$

$$\begin{aligned}
&= (1/2) [(k / D) (2 + 0.43 Ra_D^{1/4})] [4\pi (D/2)^2] \Delta T t \\
&= (1/2)(0.02570 \text{ W/m}^\circ\text{C})(2 + 0.43 (277 \times 0.7099)^{1/4})(\pi \times 0.0045\text{m})(20^\circ\text{C})(20\text{s}) \\
&= 0.2624 \text{ J}
\end{aligned}$$

$$Q = m C_p \Delta T \quad [2.9.4.7]$$

$$\Delta T = Q / [4/3 \pi (D/2)^3 \rho_{H_2O}] C_{p_{H_2O}} \quad [2.9.4.8]$$

$$\begin{aligned}
&= (0.2624 \text{ J}) / [\pi/6 (0.0045\text{m})^3 (998\text{kg/m}^3) (4.2178 \times 10^3 \text{ J/kg}^\circ\text{C})] \\
&= 1.3^\circ\text{C}
\end{aligned}$$

Thus it can be seen that the droplet obtains little energy from its surroundings during formation, and thus the temperature near the drop formation is of minor concern.

Forced convection during the fall will further warm the droplet, however evaporative cooling will ameliorate this warming.

For forced convection, the expression for the Nusselt number changes to: (p.447, Ozisik, 1985)

$$Nu_m = 2 + (0.4 Re^{1/2} + 0.06 Re^{2/3}) Pr^{0.4} (\mu_\infty / \mu_w)^{1/4} \quad [2.9.4.9]$$

where μ_∞ is the viscosity of the air far from the droplet, and μ_w is the viscosity of the air at the droplet. As these viscosities are effectively the same, the fraction becomes one and is discarded, as recommended for the analysis of a vapour.

For the falling droplet, a time scale of 0.45s was used.

$$\begin{aligned}
 Q &= \dot{Q} t = h_m A \Delta T t & [2.9.4.10] \\
 &= [(k / D) (2 + (0.4 \text{ Re}^{1/2} + 0.06 \text{ Re}^{2/3}) \text{Pr}^{0.4})] [4\pi (D/2)^2] \Delta T t \\
 &= [(0.02570 \text{ W/m}^\circ\text{C}) (2 + (0.4(1333)^{1/2} + 0.06(1333)^{2/3}) (0.7)^{0.4})] [(\pi \times 0.0045 \text{ m})] \\
 &\quad \times (20^\circ\text{C}) (0.45 \text{ s}) \\
 &= 0.06855 \text{ J}
 \end{aligned}$$

$$Q = m C_p \Delta T \quad [2.9.4.11]$$

$$\begin{aligned}
 \Delta T &= Q / [4/3 \pi (D/2)^3 \rho_{\text{H}_2\text{O}}] C_{\text{P}_{\text{H}_2\text{O}}} & [2.9.4.12] \\
 &= (0.06855 \text{ J}) / [\pi/6 (0.0045 \text{ m})^3 (998 \text{ kg/m}^3) (4.2178 \times 10^3 \text{ J/kg}^\circ\text{C})] \\
 &= 0.34^\circ\text{C}
 \end{aligned}$$

Thus, the air transfers very little of its energy to the droplet during the fall.

The droplet's evaporative cooling was calculated as well. To calculate this, an analysis was used based on the Reynold's Analogy. The calculation is a long process, as follows:

$$T = (T_i - T_o) e^{(-t/\tau)} + T_o \quad [2.9.4.13]$$

$$\begin{aligned}
 \tau &= (m c_p / h A) / (1 + \rho_a h_{fg} (h_d/h) (w_{1w} - w_{1o}) / (1 - w_{1w})(T_1 - T_o)) \\
 & & [2.9.4.14]
 \end{aligned}$$

$$m = (4/3) \pi R^3 \rho_w \quad [2.9.4.15]$$

$$A = 4 \pi R^2 \quad [2.9.4.16]$$

$$w_{1w} = P_{1w} (M_1/M_2) / (P - P_{1w}(1 - M_1/M_2)) \quad [2.9.4.17]$$

$$w_{1o} = P_{1o} (M_1/M_2) / (P - P_{1o}(1 - M_1/M_2)) \quad [2.9.4.18]$$

$$P_{1w} = P_v(T_w) \quad [2.9.4.19]$$

$$P_{1o} = \{RH\} P_v(T_o) \quad [2.9.4.20]$$

$$h_d/h = (D_{12})^{4/3} / V^{1/3} K \quad [2.9.4.21]$$

$$h = Nu_d K / d \quad [2.9.4.22]$$

$$Nu_d = D \sigma_w (v/\alpha)^{1/3} / \rho_a V \quad [2.9.4.23]$$

In equations 2.9.4.13 through 2.9.4.23:

T is the final droplet temperature (in °C).

T_o is the ambient temperature (20 °C).

T_i is the initial droplet temperature (4 °C).

R is the droplet radius (0.002265 m).

ρ_w is the density of water (1002.28 kg/m³).

ρ_a is the density of air (1.2951 kg/m³).

C_p for air is (4217.8 J/kg K).

k for air is (0.024255 W/m K).

h_{fg} for air is (2454100 J/kg).

V is the velocity of the droplet (4.43m/s). This is the impact velocity. As it is the fastest the droplet travels, it is thus the most extreme case.

D₁₂ is a diffusion parameter (0.000026 m²/s).

M₁ is the molar mass of water (18 kg/mol).

M₂ is the molar mass of air (28.97 kg/mol).

P_v is the vapour pressure (813 Pa at 3 °C), (2339 Pa at 20 °C).

{RH} is the relative humidity (0.5 is an average value from experimental results).

P is the ambient pressure (101300 Pa). This assumes a standard atmosphere.

t is the time of flight (0.45 s).

ν is the kinematic viscosity of air ($1.2585 \times 10^{-5} \text{ m}^2/\text{s}$)

α for air is ($0.00176605 \text{ m}^2/\text{s}$).

σ_w is the interfacial shear stress at the liquid-vapour interface (in N/m^2).

The final temperature is thus dependent on the interfacial shear stress at the liquid-vapour interface. As we know that the shear stress between a liquid and a vapour must be low, we can investigate the effect of varying shear stress with Fig.9:

Fig.9 Effect of interfacial shear stress

The final result of the analysis is that the temperature increases very slightly. This change is quite small compared to the temperature gradient, which will drive the heat transfer to the cold plate, and is thus of little concern. The fact that the temperature increases indicates that the cold droplet in a warm environment gathers condensation, rather than evaporates. This leads to the negative evaporative cooling.

2.10 Design of Temperature Monitoring system

The temperature monitoring system had to be simple, yet rugged and flexible. Two methods were considered. Thermocouples could be placed throughout the system and feed back through a selector switch to a digital reader. Alternatively, thermometers could be placed at the points of interest, and monitored from the exterior. The thermocouple system was more convenient, however it would lead to many wires passing through the containment box surface. Furthermore, it would be more difficult to adjust, move, add or replace thermocouples than thermometers. Finally, thermometers were generally less expensive than thermocouples for comparable accuracy. For all these reasons, thermometers were often used instead of thermocouples.

Five points of interest were monitored. The temperature of the liquid reservoir before the pump was measured by one thermometer immersed in the liquid. The temperature of the air in the region of droplet formation was measured by another

thermometer suspended there. A high accuracy thermometer resting in boundary layer above the plate surface measured the temperature there. This is reasonable, because the plate boundary layer was more than two centimeters thick. (See section 2.14 for details). The thermometer sensor area was thus immersed in the boundary layer. The impact plate temperature was monitored by a thermocouple contacting the upper surface, near an unused corner to provide rapid response. The temperature of the box overall was measured by a thermometer placed against the inside surface of the box, and several lower quality digital thermometers placed around the inside of the box. The temperature of the coolant reservoir was monitored by the display of the thermocouple probe, which the chiller used for feedback control.

This layout was eventually modified for further accuracy. An insulating open-topped box was used to trap cool air around the rack/plate assembly to both help define a thicker boundary layer and maintain an even temperature. Into this area a test tube containing the coolant liquid was placed, with a thermometer resting in that fluid. This gave a more accurate reading, though it proved to respond slower.

Ambient temperature was measured by any of the replacement thermometers kept outside the box for future addition or replacement.

The plate thermocouple was doubled up to ensure accuracy. The thermocouples were calibrated by moving two thermocouple probes together through the boundary layer above liquid nitrogen. Variation in probe geometries was dealt with by pausing at each

reading point to give the thermocouples time to come to rest. This provided a wide range of temperature, to ensure that the thermocouples behaved similarly in all conditions. In addition, checks against room temperature and ice formed the comparison points to ensure the thermocouples were properly calibrated. The thermocouple readings were plotted against each other. Any error is evidenced by an angle from 45 degrees in Fig.10, below:

Fig.10 Thermocouple Calibration Data

As can be seen from the graph, there is little deviation between the readings, and thus both thermocouples show good agreement.

2.11 Design of Humidity Monitoring system

2.11.1 Humidity Monitoring

In order to determine the feasibility of controlling humidity, research included contacting the National Art Gallery, several museums, and archaeological groups including the Canadian Conservation Institute. The resulting research indicated that humidity control was an expensive and imprecise operation. For example, the National Art Gallery used a system that could only measure and control the humidity to within +/- 3% relative humidity, and that varied by season.

As a result, the notion of humidity control was discarded, and a humidity monitoring system was adopted. The system consisted of three digital humidity meters placed inside the box. The meters were Mercury PreciseTemp Indoor Digital Thermometers. They displayed temperatures from 32°F to 122°F with a resolution of 0.1°F. They displayed relative humidity from 20% to 99% with a resolution of 1%. The ambient humidity of the laboratory was found to vary substantially, as can be seen in Appendix I: Ambient Conditions.

2.11.2 Humidity Control

Humidity can be increased by misting distilled water into the containment box. In practice, it was discovered that a fine mist spread over the walls of the containment box did not adversely affect the analysis, and increased the humidity to at least 50% within five minutes. To reduce high humidity, a Danby DPAC8099 8000BTU air conditioner and 75 pint dehumidifier was installed. Dehumidifier operation heated the air, however, and as such it was found to be useful to vent the exhaust into a container of dry ice. In high humidity conditions, condensation on the test plate was undesirable.

The effect of humidity was also investigated. By inspecting evaporation analysis, it can be rapidly determined that the concentration gradient drives evaporation. Thus, a high external concentration of water in the air means little evaporation from the water droplet. Thus, a high relative humidity means a low amount of evaporation.

Evaporative cooling of the falling drop is desirable, however evaporation of the droplet while on the test surface is not. As such, the system should have high humidity near the test plate, but low humidity within the remainder of the test cell. The addition of the insulated, open-topped box to contain the cold air around the plate traps cold air in place around the coil. Because cold air has a lower capacity to support water, the relative humidity increases. Thus, the relative humidity at the plate can easily be found to be 20% higher than elsewhere in the test cell.

2.12 Overview of Digital Vision System

The digital vision system designed by Zhou included four essential components. The digital camera was a Dalsa CCD with a 1" capture area yielding 1024x1024 pixel resolution. The camera lens was a Nikon ED AF Micro Nikkor 70-180mm 2:4.5-5.6 D. The support system used to position the camera was as described in his thesis. The software system used included Matrox Intelcam, Matrox Inspector, and the ADSA software packages. The selection of this system provided a known hardware system that was readily available. By pursuing the work of Zhou, a known hardware system of obtaining images was granted. All image processing work described in this work is derived from the tools made available by this selection, but the work of Zhou does not include this image processing work.

2.12.1 Hardware Overview

The digital vision system was placed on the recommended vibration isolation system, which, in turn, was placed on a table which was reinforced with 1½" angle-iron brackets to support the added weight.

A Matrox Genesis card was installed in the Pentium II computer, and a second monitor was added. The right monitor was thus operated by the Genesis card, and was the only display area capable of operating the imaging software.

2.12.2 Software Overview

The Matrox Intellicam software uses the Genesis card to display continuous feed from the camera, or obtain single images.

The Matrox Inspector software can use the Genesis card to obtain sequences of images, or perform digital image processing.

The ADSA software uses a digital image of a drop to solve Laplace's equation of capillary for the surface tension given a set of experimentally generated droplet edge points. It is discussed in detail in the work of O.I. del Rio and A.W. Neumann. (p.136, JCIS 196, 1997)

The use of this software in experimental operation of the system is given in more detail in Appendix O: Laboratory Procedure.

2.13 Evaporation Considerations

In order to ensure that the test liquid would not evaporate excessively during the test, evaporation testing was done to determine a rate of evaporation in the laboratory. The evaporation was performed by filling a test tube, and weighing it as time passed. The test tube was kept within the laboratory, in a corner away from both traffic and the apparatus. The results of the testing are summarized in Fig.11, below:

Fig.11 Concave Surface Evaporation Testing

The trend line describes a line of best fit for the data, with equation. The initial test region shows a different rate of evaporation than the later rate. This change may have to do with the ambient humidity of the region. This time period contained both high ambient humidity, and a period in which water was boiled within the laboratory.

Evaporation testing was later conducted with a sessile drop of approximately 1cm radius on a petri dish. The results of this evaporation testing showed a marked increase in evaporation in this configuration. The petri dish evaporated in one hour what the test tube evaporated in one week. Humidity and temperature were similar between the two tests, comparable to ambient conditions. Tests gave similar results within and outside of the containment box. The differences lay in the layout of the tests themselves. The test tube provided both vapour containment and a concave meniscus. The petri dish provided very little vapour containment and a convex meniscus. Experimentally, the boundary layer will provide some vapour containment, but the meniscus of the droplets will definitely be convex. As such, these values are more appropriate. These results can be seen in Fig.12.

Fig.12 Convex Surface Evaporation Testing

We further note that the experiments had a time scale of approximately three minutes. Therefore, the evaporation is very small, and can be neglected.

2.14 Boundary Layer Considerations

The thermal boundary layer thickness was calculated using an equation from Ozisik. (p.426, Ozisik, 1985):

$$\delta_t(x)/x = 3.93 \text{ Pr}^{-1/2} (0.952 + \text{Pr})^{1/4} \text{Gr}_x^{-1/4} \quad [2.14.1]$$

Where $\text{Pr} = 0.715$

$\text{Gr}_x = 6.63 \times 10^7$ for steel, and 2.6×10^9 for copper

Equation 2.14.1 yields values of 2.3 cm for the plate boundary layer, and 1.6 cm for the coil boundary layer thickness. This indicates that the impacted droplet will remain within the boundary layer, and thus remain cooled by the plate.

The calculations in 2.9.4 showed that the boundary layer would indeed be laminar. The addition of the insulated, open-topped box further ensured that the air surrounding the droplets would be chilled.

2.15 System Cost

As this work was designed for research, not industrial manufacture, cost was a secondary consideration. The details of system cost are available in Appendix J: Labour Estimate, and Appendix K: Pricing Estimate. The overall system cost is \$6,600. This figure is in year 2000 dollars, and does not include the digital vision system. The system is currently unique, and thus no comparison may be made with industry.

3.0 Analysis of Theory

This section summarizes the essential theory required to pursue investigations into the issue of splashing and freezing phenomena. This section is intended as a primer to convey the basics of the theory to future investigators. It represents the bulk of the knowledge required to begin work in this field.

There is no single, definitive work on the subject. Much of the material in this section has been drawn from a variety of sources. The most recommended works for understanding spreading phenomena are the works of S. Middleman. His works An Introduction to Mass and Heat Transfer, Principles of Analysis and Design and Modeling Axisymmetric Flows; Dynamics of Films, Jets and Drops are excellent works from which much of the description of sessile and spreading droplets are drawn. The splashing phenomena is described in many works, but the impact of droplet to surface is well described in the Journal of Colloid Interface Science article “Influence of surface roughness on liquid impact” by K. Range and F. Feuillebois.

There is surprisingly little research done in the interaction of splashing and freezing phenomena. That is why it is important to perform investigations into the theory, and why the system is required.

Further research will, of course, be required to bring investigators to an understanding of current research efforts in the field, and to remain current.

3.1 Summary of Background Theory

3.1.1 Scope

The study of splashing and freezing phenomena includes a study of two fields: thermodynamics as well as a study of fluid mechanics. As any investigator rapidly realizes, the two have a junction point in the analysis of system energy. Each field uses the same analysis techniques of control volumes and steady flow energy analysis. It is at this junction that the study of splashing and freezing phenomena takes place.

3.1.2 Microscopic versus Macroscopic

One of the important concepts to recognize in the study of splashing and freezing phenomena is the scaling of effects. Macroscopic effects are effects that are large, and microscopic effects are effects that are small. In general parlance, macroscopic effects are effects that occur with a length scale of even a small fraction of a droplet diameter, or include some fraction of the droplet's fluid volume. Microscopic effects are effects that occur at an atomic level, such as the attractions between molecules, or the roughness associated with the atomic packing structure of a solid surface.

Most of the factors governing the development of a droplet's behaviour are macroscopic, but some microscopic effects are important, specifically the attractions between molecules that create energetically preferable macroscopic actions.

3.1.3 Relevant Parameters

There are many parameters used in the analysis of droplets. The primary independent variables include temperature, chemical potential and surface tension. Most systems in the spreading regime find the combination of T , μ_j , and γ to be defining. Falling droplets require gas viscosity, and kinetic and potential energy. For more information about these parameters, refer to Appendix L: Discussion of Parameters.

3.1.4 Surface Tension

A critically important concept in the understanding of splashing and freezing phenomena is surface tension. A liquid surface is an interface between that liquid and any solids, liquids or gasses it may be in contact with. Energetically, it is usually preferable to minimize this interfacial area. As such, the liquid seeks to reduce its surface area. The force that seeks to restore the fluid to a minimum surface area is known as surface tension.

Surface tension can be seen in many physical systems, and is relatively easy to visualize. Surface tension is the reason that droplets form their characteristic curved shapes. Surface tension can be measured by several means. The system was designed to use the measurement system outlined in the work of Zhou. This system uses the same digital vision system used by the experimental system and the ADSA software.

The reference for surface tension is a water droplet in air. Thus, if a surface tension is listed for a solid, this will be the surface tension value γ^{SL} for a water droplet on the solid in air at standard temperature and pressure.

3.1.5 Hydrophilic and Hydrophobic

With most liquid-surface combinations, the most variable and critical factor determining the behaviour is the liquid-solid interfacial energy. Naturally, stability demands that systems tend to reduce their energy. The relative energy of the surfaces thus defines the actions the system will take.

The term hydrophilic is used for a liquid drop to solid surface combination that energetically prefers spreading. As such, the solid-liquid and liquid-gas interfacial energies are low, while the solid-gas interfacial energy is high. This type of system tends to promote spreading.

The term hydrophobic is used for a liquid drop to solid surface combination that energetically dislikes spreading. As such, the solid-liquid and liquid-gas interfacial energies are high, while the solid-gas interfacial energy is low. This type of system tends to discourage spreading.

3.1.6 Young's Equation

Young's equation of capillary describes the balance of forces at a three-phase contact line for a liquid droplet resting on a solid, and surrounded by its vapour. The forces are shown in Fig.13. This figure shows a two-dimensional cross-section, and one must visualize the contact point shown as actually a line of contact between the liquid, solid and vapour.

Fig.13 Young's Equation

The force balance along the solid thus becomes:

$$\gamma^{SV} = \gamma^{LV} \cos \theta + \gamma^{SL} \quad [3.1.6.1]$$

where γ^{SV} is the surface tension between the solid and vapour phases, γ^{LV} is the surface tension between the liquid and vapour phases, γ^{SL} is the surface tension between the solid and liquid phases, and θ is the contact angle.

This equation is known as Young's equation.

3.1.7 Line Tension

Line tension describes what is currently a controversial phenomena in the literature of fluid mechanics. If one imagines the perimeter of a droplet resting on a solid in top view, one can imagine this perimeter as a curved line. It has been noticed that the system tends to minimize the length of this line. The restoring force on this line is thus termed line tension. There is as of yet, no uncontested work, which defines the value of this restoring force.

3.1.8 Capillary Pressure

Capillary pressure is an important concept in understanding the spreading of low-volume flows such as those associated with this system. Capillary pressure is the force that draws a liquid through a small pipe. The action of this force can be observed by lowering a small tube into a reservoir of liquid. The liquid will draw up the tube, forming a rest level higher inside the tube than in the rest of the reservoir. This action is capillary pressure in action.

Capillary pressure results from surface tension. The reason the fluid climbs is exactly the same as why the meniscus climbs. To form a balance within Young's equation, the fluid must rest at a given contact angle. As the fluid surface attempts to curve sufficiently to form this angle, the surface gains more energy, due to the increasing surface energy. To minimize this energy, the fluid level rises to straighten the surface. These two actions generate the phenomenon referred to as capillary pressure. The fluid level continues to rise until the force of gravity on the rising column balances the capillary pressure. Thus, the total energy becomes balanced, with surface energies balanced by potential energies.

In cases, such as mercury, where the meniscus drops away, the same interaction occurs, until the buoyancy forces balance the system. Essentially, the meniscus is being pushed upwards by the weight of the reservoir fluid outside the capillary tube.

Occasionally, the investigator will encounter a value called the capillary number. It represents the balance between viscous resistance and surface-tension induced flow. This value requires careful definition if multiple liquids are involved. For example, according to Cox, (p.169, JFM 168, 1986), the capillary number for fluid A displacing fluid B is defined as follows:

$$Ca = \mu_A U / \gamma \ll 1 \quad [3.1.8.1]$$

And

$$\lambda Ca = \mu_B U / \gamma \ll 1 \quad [3.1.8.2]$$

where λ is the viscosity ratio: μ_B / μ_A , U is the fluid velocity, and γ is the tension between the interface liquids, assuming it is large enough that interfacial tension dominates viscous effects.

3.1.9 Laplace's Equation

Laplace's equation is a governing equation for the calculation of the variation of parameters within a fluid. Laplace's equation is commonly written as:

$$P^L - P^V = (1/R_1 + 1/R_2) \gamma^{LV} \quad [3.1.9.1]$$

where the pressure difference across the surface of the droplet ($P^L - P^V$) is thus directly related to the mean curvature of the surface ($1/R_1 + 1/R_2$) and the surface tension (γ^{LV}). R_1 and R_2 represent radii of curvature in perpendicular directions along the surface of the interface, measured from inside the droplet.

Thus, a very large radius of curvature, such as the apex of a sessile drop, supports very little pressure in a fluid. A very small radius of curvature, such as the side of a sessile drop, supports a very high pressure in a fluid.

If the centre of curvature lies within the gas, the sign of the pressure difference is reversed, and a high curvature supports a very low pressure in the liquid.

3.1.10 Mathematical Manipulations

Several formulations of equations were found that could be used to simplify the mathematics that were required by the investigation of splashing and freezing phenomena. These are discussed in Appendix M: Overview of Mathematical Formulations. They generally involve a transformation from an energy minimum formulation to an entropy maximum formulation.

Another consideration is the use of mass transfer analysis. For this analysis, the reader would be well advised to consult R. Byron Bird's Transport Phenomena. (p.636, Bird, 1924). While an older work, its chapter 21: Interphase Transport in Multicomponent Systems is still very valid thermodynamics. Another, more current work of note is Stanley Middleman's An Introduction to Mass Transfer and Heat Transfer. (p.615, Middleman, 1998.) His chapter 16: Simultaneous Heat and Mass Transfer takes the analysis to the level required to study these phenomena.

3.2 Summary of Pendant Drop Theory

3.2.1 Definition

A pendant drop is a hanging drop. It is a drop supported by surface tension. A pendant drop is traditionally supported from a tube, which supplied the liquid necessary to form the drop.

3.2.2 Pendant Drop Force Balance

A pendant drop is in static equilibrium. As such, the forces acting on the droplet must remain in balance. As the only appreciable forces occur in the vertical direction, they appear on Fig.14.

Fig.14 Pendant Drop Force Balance

As can be seen from Fig.14, the surface tension forces must balance the gravity forces. If the volume and density of the droplet can be accurately measured, then the surface tension forces can be accurately known. This means that pendant droplets can be used to determine surface tension. By studying the droplet just before separation, the maximum surface tension can be identified.

3.2.2 Capillary Pressure

Capillary pressure causes the drop liquid to rise up the sides of the tube, making the three-phase contact line occur on the outside of the tube above the tube's end. This is the reason the tube appears imbedded within the drop. This will occur unless the tube material is highly hydrophobic.

3.2.3 Drop Shape Based on Surface Tension

Energetically speaking, the drop's preferential shape is a perfect sphere. The drop attempts to move itself to that shape using surface tension. Acting against this shape is the force of gravity, which is acting downwards. The force of gravity distorts the droplet into an oblong shape. This distortion continues until the droplet gains sufficient surface area that surface tension balances gravity.

3.2.4 Fluid Density (Gravity Effects)

At the bottom of the droplet, the liquid has different characteristics than at the top. Due to the compression of the fluid above, the pressure is higher at the bottom of the droplet. This higher pressure demands a higher surface curvature according to Laplace's equation. This variation, while slight, explains the characteristic shape of the drop tip.

3.2.5 Size of Droplet at Separation

The size of the droplet at separation is a critical piece of information for the investigation. With a very small volume flow into the droplet, the droplet can be assumed to separate when the surface tension forces are no longer capable of expanding to accommodate the expanding gravity forces. At this critical volume, which will depend on

the geometry of the tube and the characteristics of the liquid, the drop will separate from the tube. The key point is that this droplet volume will be highly reproducible. With a slow pumping, the droplets will be the same size with every experiment.

The method for calculating the droplet diameter after release is described in section 2.8.1, above.

3.3 Summary of Free Droplet Theory

3.3.1 Definition

A free droplet is an unsupported drop, usually surrounded by gas. It is a droplet in free-fall. We will assume that the droplet is falling through a gas, else the problem becomes trivial.

3.3.2 Free Droplet Force Balance

A free droplet is not in static equilibrium. As such, the forces acting on the droplet will not remain in balance. The unbalanced force will provide acceleration for the droplet. As the only appreciable forces occur in the vertical direction, they appear on figure 15.

Fig.15 Free Droplet Force Balance

As can be seen from Fig.15, the resistance caused by air friction distorts the droplet shape. This distortion is similar to the distortion evidenced in pendant drops, above. This friction-generated resistance also slows the droplet.

3.3.3 Air Resistance

As one would expect, the air resistance of a droplet is highly dependant on the droplet's shape. More specifically, it is highly dependant on the wetted area of the droplet. This is the total surface area of the droplet. It is also highly dependant on the profile area of the droplet. This is the area of the largest cross-section as seen from a point in the direction of motion of the droplet. These dependencies cause the system to react in the following way:

As the wetted area of the droplet grows, so does the drag it generates, and as such, so does the air resistance.

Increased air resistance causes the droplet to stretch out in space, growing in length, and wetted area while reducing in profile area.

As the profile area of the droplet shrinks, so does the drag it generates, and as such, so does the air resistance.

Decreased air resistance causes the droplet to contract towards a sphere under the action of surface tension forces.

This interaction defines that the actions must balance. As profile area is more significant than wetted area, the desired shape is very long and narrow. Surface tension, however, will prevent the system from deforming to this point. Air resistance and surface tension balance the shape of the droplet. The tendencies of the system are more important in the damping of disturbances.

The critical speed at which the droplet's drag forces overcome its surface tension forces, and it loses cohesion can be calculated using the equation described in section 2.8.1, above.

3.3.4 Separation Disturbances

The separation of the droplet is a violent one, from the droplet's perspective. The elongation produced by hanging the droplet caused a large surface tension that is no longer balanced. This unbalanced surface tension thus draws the neck of the drop back into the droplet, and this action causes the droplet to overshoot a spherical shape. The action is similar to the way a balloon is commonly propelled by pulling and releasing the tab. The droplet would then oscillate until viscous damping settled out the action. However, the droplet is a free droplet by this time, and as the droplet gains velocity, drag forces increase. As the oscillations alter the droplet's shape, they alter its profile and wetted areas.

As the droplet is flattened in side view, it presents high profile and wetted areas. Thus, the drag is high, and the droplet is subjected to strong drag forces, which seek to streamline it. The more flattened the droplet is, the stronger these forces act. This dampens the oscillations of the droplet.

As the droplet is elongated in side view, it presents low profile, but high wetted area. Thus, the drag forces are weak, and surface tension may overcome them to propagate further oscillations, until all forces eventually come to balance.

3.4 Summary of Sessile Drop Theory

3.4.1 Definition

A sessile droplet is a droplet resting on a surface. It is supported by the normal force provided by a solid surface.

3.4.2 Sessile Drop Force Balance

A sessile drop is in static equilibrium. As such, the forces acting on the droplet must remain in balance. As the only appreciable forces occur in the vertical direction, they appear on figure 16.

Fig. 16 Sessile Drop Force Balance

As can be seen from Fig.16, the normal forces must essentially balance the gravity forces. The contribution of the surface tension here is small. This balance, of course, means that the sample can be weighed to obtain its mass. This can permit accurate volume estimation if density is known. It can also provide accurate density measurement if the volume can be accurately found. This is important, because the ADSA software relies on this fact by obtaining volume information through an edge detection methodology.

More importantly, Young's equation and Laplace's equation can be used on a sessile drop. These equations can provide important information about the droplet. If the edge of the droplet can be accurately measured, then the surface tension forces can be accurately determined by integrating Laplace's equation. This means that sessile droplets can be used to determine surface tension.

3.4.3 Variation in Properties (Laplace's Equation)

As pressure is one of the controlling variables of the properties of a liquid, if the pressure inside a droplet varies, then so do the properties of the liquid. Laplace's equation indicates that the pressure within a liquid will vary with the curvature of the surface. As the curvature of droplets varies over their height, so too must their properties. Density varies with pressure, which varies with curvature. Thus, high curvature near the bottom

of a sessile drop indicates high pressure. This agrees with an intuitive sense of hydrostatic pressure.

Naturally, all pressure-dependant parameters will vary along the height of the liquid, not just density.

3.4.4 Drop Shape Based on Surface Tension

The drop shape of a sessile drop strongly depends on the surface tension forces. For a droplet of very high surface tension, the drop will tend to be round, becoming, in some extreme cases, a perfect sphere with only an infinitesimally small point of contact with the supporting substrate. For a droplet of very low surface tension, the drop will tend to be flat, becoming, in some extreme cases, a thin layer of fluid coating the entire surface.

The contact angle of the droplet at the three phase contact point is also very important. The contact angle for a sessile drop will be the static contact angle. This angle can be used in Young's equation to determine information about the relative values of the interfacial energies, through the surface tensions.

3.4.5 Hydrophilic and Hydrophobic Surfaces

Hydrophilic and hydrophobic surfaces cause the surface tension forces at the liquid-solid interface to decrease or increase respectively. This change in surface tension forces causes a change in the drop shape, as described above.

3.5 Summary of Spreading Drop Theory

3.5.1 Definition

A spreading drop is a sessile drop that is expanding in contact area. This is usually due to volume addition or reaction with the solid substrate.

3.5.2 Spreading Drop Force Balance

A spreading drop is not in static equilibrium, nor is it commonly in dynamic equilibrium. It is, however, usually subjected to a fairly small unbalanced force. As such, its spreading is usually slow. In some cases, however, such as strongly hydrophilic substrates, the spreading can prove to be quite rapid.

Fig.17 Spreading Drop Force Balance

As can be seen from Fig.17, the normal forces still balance the gravitational forces. Consequently, the spreading is radial from the drop centre. The capillary pressure and the internal pressure of the liquid overbalance the surface tension forces, causing spreading. As this spreading continues, the droplet's bulk volume reduces, and the precursor film grows, until the forces again come into balance. The growth is slowed by friction and viscous losses. These losses can be appreciable in the thin precursor film.

3.5.3 Precursor Flow

When the droplet expands, a very thin layer of liquid expands ahead of the visually observable contact point. Due to capillary pressure, this layer is self-propagating. As the droplet expands, it expands onto fluid, thus reducing the resistance by friction. In essence, the droplet's expansion is self-lubricating.

3.5.4 Roughness Effects

The precursor flow will fill in small dents in the surface as it expands. According to Cox, (p1, JFM 131,1983) in cases where the surface is noticeably grooved, the precursor flow will show a preference to grow along the grooves. If the flow is forced to expand perpendicular to the grooves, the macroscopic contact angle will evidence some hysteresis as the droplet jumps between equilibrium positions (i.e. between a drop with an advancing contact angle Y_a to a drop with a contact angle $Y < Y_a$ after the jump). This stick-slip behaviour causes a slower time-averaged expansion than for flow along the grooves.

3.5.5 Quasi-Static Flow Assumption

It is important to realize that spreading analysis commonly assumes a quasi-static flow. This means that the flow is assumed to occur very slowly, and viscous losses are neglected. In a faster flow, viscous losses as the drop expands onto the precursor layer, and the precursor layer expands onto the solid will be more relevant. In very high-speed spreading, many of the conventional spreading analysis assumptions need to be revisited.

3.5.6 Theoretical Contact Angle

A theoretical contact angle is an important description of the interaction between surface and liquid. This is the angle at the contact point between the fluid and the substrate for a system in equilibrium. A theoretical contact angle is used in Young's equation of capillary. The actual contact angle will tend not to equal the theoretical

contact angle if the liquid is expanding, contracting or in some dynamic mode of motion. An advancing contact angle and a receding contact angle are sometimes given for liquids. The actual expanding contact angle, Y_a , is the contact angle in a case where the droplet is spreading, i.e. the droplet bulk fluid is moving towards the contact point. The actual receding contact angle, Y_r , is the contact angle in a case where the droplet bulk fluid is moving away from the contact point. This hysteresis is important in cases where the droplet is vibrating. If the vibrations keep the contact angle between the advancing and receding angles, the contact line will not move. If not, the contact line will expand when the contact angle achieves the advancing angle, and contract when the contact angle achieves the receding angle.

In cases where the droplet is at rest on an incline, the contact angles will be exceptionally different from a case where the droplet is on a level surface. Examine the case in figure 18.

Fig.18 Contact Angles on an Incline

As can be seen from Fig.18, The droplet does not maintain one uniform contact angle throughout its perimeter. In order for the system to be in static stability, there must be no net force along the surface. That requires that the shear component of the normal reaction be balanced. To vary the force balance of Young's equation, the contact angle must be varied. As the reaction is a product of the gravity force, and thus the mass of the droplet, larger droplets will require a more extreme reaction than smaller ones. As the incline grows, the shear component rises from zero, to a maximum at ninety degrees incline. If the contact angles need to be varied beyond the maximum theoretical contact angles for the system, the contact angle resists, the droplet can no longer produce the required shear force, and the droplet begins to slide down the slope.

The effects of gravity on contact angle are described in the work of M.R. Sages and C.A. Ward, in their two articles on the matter. (p.3651 and 3661, JCP 109, 1998) The reader is encouraged to investigate Appendix N: Analysis of a Related Work, before perusing this work, however.

The theoretical contact angle of a system is described in the work of Jordan G. Petrov and Peter G. Petrov. (p.143, *Colloids and Surfaces* 64,1992) A further discussion is available in the work of R.V. Sedev, C.J. Budziak, J.G. Petrov, and A.W. Newmann. (p.1, JCIS 159, 1993) Their work, entitled Dynamic Contact Angles at Low Velocities, is an excellent reference for the concept of contact angles, and regimes that do not involve high velocity splashing.

3.6 Summary of Droplet Impact Theory

3.6.1 Definition

Droplet impact involves the impact of a free droplet with either a liquid-vapour or solid-vapour surface. Droplet impact differs from droplet spreading in the rate of contact area expansion.

Microscopic splashing is the break-up of the droplet into several smaller droplets as kinetic forces overcome surface tension forces. This occurs with nearly every droplet impact of sufficient velocity.

Macroscopic splashing also involves the break-up of the droplet into several smaller droplets. In macroscopic splashing, however, these droplets land far enough away from the droplet so as not to be re-absorbed in the spreading and retracting of the droplet.

The essential difference between these two descriptions is whether one droplet or several exists when the system comes to rest.

When splashing is referred to within this work, it shall refer to macroscopic splashing.

3.6.2 *Impact Dynamics*

When the droplet impacts with the solid plate, the kinetic energy of the particles within the droplet begins to change, as the normal force of the plate applies a very large unbalanced force, however it takes some time for this action to stop the entire droplet. Initially, only the liquid at the point of contact is stopped. The remainder of the droplet is unaffected, and continues downwards.

Range and Feuillebois (p16, JCIS 203, 1998) describe what follows. The motion of the bulk droplet compresses the liquid near the point of contact, increasing the pressure of the liquid there. Eventually the local water pressure reaches what some refer to as the water hammer pressure. This is the point at which the pressure no longer increases locally, and instead, the line defining the compressed volume expands as the adjacent liquid is now forced to compress. As the droplet continues downward, the compressed volume grows. Pressure information in a fluid is limited by the speed of sound in the liquid, and thus, this is the maximum speed of this pressure wave. This pressure wave travels upwards into the undeformed droplet, while the undeformed droplet continues downwards at its initial speed. As the droplet was highly curved, indeed nearly spherical, the rate of growth of the contact area is very great. As long as it remains higher than the speed of sound within the liquid, the compressed liquid cannot escape, and continues growing. As the droplet continues its fall and impact, the curvature of the droplet lessens, and the expansion speed decreases. Eventually, the line of compressed liquid will reach the point of contact, and begin travelling up the sides of the droplet. As soon as it does so,

it permits the compressed liquid to jet out the periphery. The pressure wave continues upwards, carrying information about the impact, until it reaches the apex of the drop, slowing the approach of the liquid it passes. As the droplet continues, it spreads, following the initial jet.

The droplet geometry during the rest of the impact is described well in the work of Xiaoguang Zhang and Osman A. Basaran. (p.166, JCIS 187, 1997) Their work also summarizes much of the work in the field of impact analysis, and is quite recommended.

Another description of the geometry, which includes a discussion of emulsified droplets, is given in the work of B. Prunet-Foch, F.Legay, M.Vignes-Adler, and C.Delmotte. (p.151, JCIS 199, 1998) While the work should not be used as an overview of work in the field, the discussion of the results of the first few milliseconds after impact are good.

An excellent overview of the physics and mathematics of the impact is given in Stanley Middleman's work Modeling Axisymmetric Flows. (p.184, Middleman, 1995) The reader is directed to chapter 6-5: Impact of a Drop with a Solid Surface. This work is highly recommended for its clarity of presentation.

3.6.3 Impact Velocity

The impact velocity is an important consideration. Lower velocities will tend to change the characteristics of impact. Surface tension will be more dominant, and less splashing will occur. Higher velocities will leave less time for slow-acting phenomena to affect the splashing. At very high velocities, surface deformation can become a consideration.

3.6.4 Energy Loss During Impact

Once the pressure wave has traveled to the apex of the drop, the droplet begins to vibrate as the impact activates the droplet's many vibrational modes. This vibration causes the system to lose some energy, but tends to occur so slowly that little energy is lost in this way before the bulk droplet has begun spreading.

The flow regime in spreading is laminar. Very low velocities and flow thicknesses yield very low Reynold's numbers. These flows tend to be analyzed more within the realm of thermodynamics than conventional fluid mechanics. In splashing, this is not always so. The very high velocities associated with splashing flows are held in check only by the thickness of the layers. Usually the flows are very thin, yielding large viscous losses as wall shear slows the flow. Flow turbulence, however, poses a method of energy

dissipation in thicker flows, such as within the bulk droplet during spreading, although it is not the dominant energy dissipation.

Energy loss to viscosity is the dominant method of kinetic energy loss in splashing flows. Velocities in splashing are very high compared to either spreading flows, where the precursor flow, which lubricates the spreading, has a velocity that is negligible. The spreading of liquid layers along the impact surface creates large viscous losses, until rebound velocities or air entrainment break contact between liquid and wall. The effects of viscosity on spreading behaviour is described in the work of L. Bacri, G. Debregeas and F. Brochard-Wyart. (p.6708, *Langmuir* 12, 1996) The work discusses droplets placed, and permitted to fall small distances, but does not cover large falls with splashing behaviour.

Thermodynamic considerations play a small role in such rapid rate actions unless there is a large temperature gradient. However, in splashing and freezing interaction, they become much more critical, as many parts of the behaviour revolve around fluid that is very near to freezing. One interesting discussion comes about in *Surface Cooling by an Impinging Water Drop*, by M. Pasandideh-Fard, S.D. Aziz, S. Chandra and J. Mostaghimi. (p.1, HD99-89, 1999) This work describes the effect of droplets on surface temperature. It also gives images of the droplet profile during deformation, and is useful for visualization.

3.6.5 Air Entrainment

As the droplet spreads, the precursor flow attempts to grow in front of the leading edge of the drop's bulk. As the velocity of spreading increases, there is increasing resistance to the growth of the precursor layer through viscosity and friction. As this resistance grows, the length of the precursor film reduces. Eventually, the spreading occurs so quickly that no precursor film can grow. At this point, the bulk liquid expands directly onto the surface, instead of the precursor film. If there is roughness on the surface, and there usually is, then air entrainment during the expansion generally occurs. Air entrainment occurs as the drop expands over a gap or lowering in the surface. The rate of expansion prevents the liquid from settling and replacing the air, thus trapping a pocket of air beneath the drop.

During splashing, a high rate of expansion is the norm, inducing air entrainment.

3.6.6 Surface Roughness Effects

Surface roughness has several effects that tend to promote splashing. First, surface roughness promotes air entrainment. This entrainment breaks contact between the liquid and the solid, while reducing the loss of liquid drop kinetic energy to viscous dissipation at the solid surface. For the liquid to splash, the drop must expand to the point where surface tension forces thin the layer so much that the liquid breaks up into smaller

droplets. In order to overcome the surface tension, the liquid has to maintain kinetic energy as long as possible.

Larger surface roughness also generates the possibility of fluid “jumping” from an irregularity in the surface. If the liquid flow is turned from the surface by the irregularity, it will break contact from the solid surface, and act as above.

Very large surface roughness can provide an asymmetry in spreading behaviour. The initial contact of the drop with the plate may not be at a right angle to the droplet’s path, or the plate may contain ridges and grooves that slow spreading in certain directions and enhance it in others.

3.7 Summary of Splashing and Freezing Interaction Theory

3.7.1 Definition

Freezing is the state transformation from liquid to solid. The action of freezing requires heat transfer. The interaction between splashing and freezing usually requires a cooled liquid and super-cooled solid surface. As splashing occurs, heat transfers from the liquid to the solid surface, which causes the liquid to freeze. This heat transfer effect will be most pronounced with metallic, heat-conducting surfaces. For surfaces that possess low thermal conductivity, one can imagine a completely different set of behaviours.

3.7.2 Effects of Cooling Liquid

As most liquids cool, their densities increase. This will tend to reduce the volume of the system and increase its viscosity. In the case of water, the solid crystal form uses more space than the associated liquid, and so there is a temperature (4°C for water at 1atm pressure) at which the water reaches maximum density.

As the liquid cools, it also increases in viscosity and surface tension.

As the liquid spreads, it cools, transferring heat to the impact plate, or lower layers of ice, formed from previous droplet impacts and freezings. As the droplet spreads, its kinetic energy keeps the molecules from aligning into a crystal. In effect, the motion of the droplet prevents the liquid from freezing. This motion permits the liquid to super-cool. The liquid cools below the freezing temperature, yet remains liquid. As soon as the liquid stops moving, however, it freezes in place.

3.7.3 Roughness Generation through Ice Formation

Roughness on the surface and shear friction slows the liquid in contact with the surface to a stop. The liquid thus slowed freezes, and essentially becomes part of the solid surface. The liquid just over top, slowed by viscosity, is now effectively in contact with the new solid surface. It slows, cools and freezes. Areas where air is entrained do not cool

nearly as quickly, and spread farther, becoming thinner. Splashed droplets freeze to the surface, leaving sections of virgin plate between them and the rest of the drop. Macroscopic roughness is thus created by the ice formation. This roughness traps or re-directs later flows. This, in turn affects the ice formation patterns, which affect later flows, and so on.

This action increases the effective roughness of the plate surface, which increases the chances of and effects of splashing.

The mathematics of the growth and decay of ice without the addition of additional droplets is discussed by Lock. (p.54, Lock, 1990) This gives the thermodynamics of ice formation, but does not cover the dynamics of the impacting drop, or its effects on freezing. For conditions that are only slightly below freezing, where the droplet may come to rest before freezing over, this may suffice. Another work to consider in tandem is the work of David N. Anderson; *Methods for Scaling Icing Test Conditions*. (p.1, AIAA 95-0540, 1995). This work discusses scaling methods in some detail.

3.7.4 Shear Friction Heating and Re-freezing

As new droplets spread across the ice, the convective and conductive heat transfer out of the ice, and the shear friction across the ice can melt the outermost layers of ice. This can cause the contours of the ice surface near the impact point to shift slightly before

the liquid cools and re-freezes. When the additional volume freezes, it defines the new ice contour. As the ice insulates heat transfer from the liquid to the plate, the droplet tends to freeze slower than the previous droplet. This tends to remove the most radical surface roughness features first. Spines and ridges tend to be thin, and easily broken or melted as a consequence. This behaviour is most common when the droplet bulk liquid is warm, as it offers more heat to the system. The warmer the water, the longer it remains liquid, and the more ice it melts.

3.7.5 Hydrophilic / Hydrophobic Behaviour of Surface

As the temperature of the surface decreases, the liquid tends to adhere to the surface as a solid more rapidly, thus making the surface act more hydrophilic. However, the overall spreading is curtailed, making the surface act more hydrophobic. As such, it is wrong to model an extremely cold surface as a hydrophilic or hydrophobic one. Furthermore, the system can and will be frozen into a non-equilibrium state. Splashing will spread a droplet much further than quasi-static expansion would. The thin, widely spread fluid rapidly freezes into place, preventing surface tension forces from returning it to an equilibrium state. As the system has a large kinetic energy, and little time, many effects are neglected. Indeed, a very hydrophobic liquid/surface combination would tend to increase air entrainment, and thus increase splashing. This would induce an opposing effect to the increased surface tension of such a system, which would tend to reduce

expansion. Similarly, hydrophilic surfaces would have reversed, but similarly opposing effects.

3.7.6 Speed of Frost Line

The frost line is the dividing surface between liquid and frozen liquid. Initially, as ice begins to form, the frost line forms along the contact area with the impact plate. This initial ice layer insulates the surface, slowing heat transfer to the surface. As spreading continues, the bulk of the liquid passes over the frost line, and its thermal inertia prevents the frost line from immediately expanding upwards into the liquid. That is, the liquid in contact with the ice is warm, and must be cooled before freezing can occur. Furthermore, it receives heat from convection and conduction with the bulk liquid as the bulk liquid passes by. The heat transfer to the ice, however, eventually overcomes the liquid in contact with the ice, and freezes it. The frost line then chases the contact line, as the solid surface draws large amounts of energy from the liquid. The frost line thus lags the contact line by the time required to transfer enough energy out to freeze the liquid. As such, the speed of the frost line depends on the speed of the contact line. The frost line has the same velocity the contact line did at any given radial distance. In some cases, especially when the flow is slow, the contact line may not lag the contact point at all. It may advance up the leading edge of the flow slightly, forming a roughness effect which would lead to air entrainment as the bulk fluid flowed over top, dropped to the surface, and freezing took place again. This would lead to a macroscopic roughening effect.

Once the bulk of the liquid is past, the frost line can begin to move upwards. As such, the frost line will reach the surface of the droplet near the point of impact first. As it spreads outwards, it chases the contact line at a fair distance. As the frost line passes any disturbances in the liquid, they are frozen into place. However, the flow is almost guaranteed to be supercritical. As such, surface ripples will not flow back towards the centre. This prevents some potential roughness effects. Humidity tends to condense onto the ice and plate, however, forming a frosting of fine ice crystals, which adds roughness.

3.7.7 Speed of Water Line

The water line denotes the contour for the farthest expansion of the droplet. It normally relates to the contact point, but in some cases where the frost line does not lag the contact line, it would be the contact point of liquid-gas farthest from the impact point.

The speed of the water line is critical in determining the final configuration of the system. It defines the flow velocity, which determines the effect roughness will have. It also defines whether the frost line will lag the water line, and by how much. The more lag the frost line has, the further the droplet will eventually spread, and the thinner and more uniform the resulting ice layer will be.

3.7.8 Change in Surface Area

As the surface area of the droplet grows, the liquid-solid contact area grows. As heat transfer is dependent on area, it is obvious that the rate of heat transfer increases as the spreading continues. The layer of ice that forms immediately on the surface insulates the upper layer somewhat, slowing heat transfer as heat transfer takes place. The droplet growth also causes the volume to spread out over the contact area, thinning the droplet, and leaving less heat to be drawn out at any given point in order to freeze the droplet.

Thus, the heat transfer accelerates as the droplet expands, and decelerates as heat transfer successfully grows the ice layer. These conflicting tendencies affect the rate of heat transfer, though heat transfer continues drawing out heat, until the droplet freezes through. Because of both the thickness and velocity at the edge, the droplet should freeze through in the centre before freezing through in the edge, as depicted in Fig. 19.

Fig. 19 Spreading of a Freezing Droplet

4.0 Experimental Procedure

4.1 Data Acquisition

The experiments performed in the laboratory to obtain the images that were analyzed, as well as the analysis method used, are all described in detail in Appendix O: Laboratory Procedure. This appendix serves as an operating manual for the system, and any future operators are strongly encouraged to investigate it.

The procedure used was essentially as follows:

- 1) Plate chilled, and temperatures are recorded.
- 2) Pump engaged, droplet permitted to form, break free, free fall and impact surface from the tubing end which is set at a known height.
- 3) As each droplet impacts, an image is captured and recorded electronically using the digital vision system.
- 5) After approximately five to seven drops, the system interaction is established.
- 6) After ten drops, the experiment is ended, and the pumps are disengaged.
- 7) Temperatures are recorded, the containment box is opened and the plate is cleaned.
- 8) Humidity concerns are addressed, and the containment box is sealed. Additional tests may be obtained by returning to 1).
- 9) Image processing of the data occurs.

4.2 Image Processing

The goal of the image processing was to convert the graphical images to numeric data. To do this, the image had to be first digitally cleaned to obtain the edges of the droplets. Then the droplet edges had to be analyzed to obtain area and centroid information. This resultant information could then be analyzed to obtain a mass distribution pattern.

The effects of the various image processing techniques used can be seen in detail in Appendix P: Effect of Image Processing Operations.

The resultant data was then analyzed to provide information about the trends of behaviour through the various phenomena considered.

5.0 Test Results

Testing with the system has provided several preliminary results. The analysis of splashing and freezing phenomena is far from complete, but the results that were obtained are given here. The raw data of the experimental work consists of images of droplets that may not be reproduced here without near total loss of accuracy. As any interpretation of the data relies heavily on image processing, any interpretation would be heavily biased. Thus only a few representative images have been included that the reader may understand the interpretations presented here. Also presented are qualitative descriptions of the trends discovered, to ease understanding of the results of the investigation.

5.1 Pure Splashing Results

Pure splashing results were obtained from the system during testing. Results of interest included the actions of the first and second droplet.

In every test, the first droplet to impact the plate spread, and then retracted to form a sessile drop. It was incredibly rare to see any visible splashing results, regardless of whether the plate was pre-wetted or not, rough or smooth, and regardless of the plate material. At the velocities considered (less than 4.5m/s), the surface tension of the water droplets overcame the kinetic forces associated with the impact.

The second droplet, however, generally induced splashing, regardless of whether the plate was pre-wetted or not, rough or smooth, warm or cool, and almost regardless of fall height. The impact of liquid on liquid induced splashing.

The theory explained this partially by the surface angle. Instantaneously, at impact, the falling droplet sees either a sloped impact surface, or a point impact surface, depending on whether or not the falling droplet impacted on the apex of the sessile droplet. If the apex was impacted, the falling droplet and solid substrate will squeeze the sessile droplet, causing liquid to jet out the sides of the first drop. Meanwhile, the edges of the falling droplet will rebound from the angles of the sessile droplet, causing additional radial movement of fluid. If the apex was not struck, the sessile drop and the falling droplet recoil in opposite directions, away from the point of contact. The part of the sessile droplet beneath the point of impact, however, is heavily pressurized, which causes radial jetting. The interesting aspect, however, is that the theory does not explicitly state whether the splashed mass comes from the impinging or sessile droplet. An assumption that the splashed mass comes primarily from the sessile droplet is a hypothesis based on the observations of impact with a sessile droplet that was frozen.

When the falling droplet impacted a frozen sessile droplet, even one with an equilibrium shape, the free drop merely spread over the surface. This would imply that spreading due to apex impact is negligible. Splashing thus appears to be primarily a function of the sessile drop being squeezed into jetting.

There also appears to be a critical size of sessile droplet that causes splashing. When the sessile drop is approximately the same volume as the falling drop, there tends to be splashing. When the sessile drop is much smaller or larger, there is little splashing.

In the case of a large sessile drop, the falling drop becomes lost in the large sessile drop. The radial acceleration given by the impact is insufficient to generate the critical escape velocity where the liquid's inertia can overcome its surface tension and splash.

In the case of a small sessile drop, the falling drop envelops the sessile drop. While the impact begins the motion of the sessile drop, the fluid must accelerate, and as it does so, the contact point between the drops moves as well, initially at speeds greater than the free droplet's velocity due to the curvature. Once this contact point passes the edge of the sessile drop, the falling droplet may entrain a certain amount of air due to the pressurization of the air, but then will contact the plate away from the sessile drop. Thus, the sessile drop must now accelerate the falling droplet as well, and the slowing of the radial flow tends to reduce the velocity below the critical escape velocity where the liquid's inertia can overcome its surface tension and splash.

Another interesting phenomena to occur is the dead zone, a gap between the primary sessile droplet and the splashed droplets. For any number of drops where the system still splashes, there is a radial zone after the initial drop in which there is essentially no volume of fluid. Fig.20 shows that the experimental data, while noisy, demonstrates this trend. The composite shows the initial data, and the relative noise, justifying the multiple trials. The average shows the dead zone relatively clearly in the

area between one and five initial droplet radii. The scaling parameter of initial droplet radii allows results at different droplet sizes to be compared. Fig.21 shows the results for a case where the fall distance was halved, the surface was made hydrophobic, and the surface roughness was removed. All splashing results clearly demonstrate a dead zone.

The graphs below clearly demonstrate the dead zone phenomenon. Fig.20 is experimental data for a droplet falling 804 mm onto aluminum foil pressed onto glass with no cooling. Fig.21 is experimental data for a droplet falling 548 mm onto painted glass with no cooling. Both show the fraction of total mass found at any given radial distance, measured in initial droplet radii. Despite the different fall heights and surface characteristics, the dead zone is clearly visible at a radial distance of two to four initial droplet radii. In both cases, an initially clean surface was subjected to numerous drops falling along the same path. The droplets impacted first the plate, then each other. The tests show the results after ten droplets, though the dead zone is visible even after the first droplet impacts a sessile drop. After three to four droplets, behaviour of all further droplets is essentially identical until a very large residual sessile drop is formed. Later droplets continued to produce splashing results until the volume of the central sessile droplet was sufficiently large to contain the kinetic forces with viscous friction and surface tension. After this critical size was reached, splashing was rare, and the sessile drop merely expanded with the added volume of each free droplet. In its expansion, the sessile drop grew across the dead zone and then absorbed previously splashed droplets, further increasing its rate of growth. All of these results show high reproducibility.

Fig.20 An 804mm droplet fall height onto Al foil on glass, no cooling

Fig.21 A 548mm droplet fall height onto painted glass, no cooling

5.2 Splashing and Freezing Result

At all ranges of velocity that we studied, and for all surfaces we studied, the system has a predictable behaviour. Five general situations were observed.

At warm (i.e. room temperature, water at 20°C) temperatures, the first droplet impacts and remains cohesive. The second droplet impacts with the first, inducing splashing. The splashing fluid seems to leave a small sessile droplet with a dead zone, followed by a ring of splashed droplets. The exact size of the sessile droplet varies, but is usually too small to induce splashing of the third droplet. The fourth droplet often splashes, but after this, the chaos of the system makes it impossible to make predictions. If the second droplet did not impact directly with the first, and the off-centre splashing biased the system, the bias will be carried over for many droplets, and will be visible in the droplet distribution for many impacts afterwards.

At cool (i.e. near freezing point, water at 0°C) temperatures, the liquid of the sessile droplet tends to have a high viscosity, and as such, splashing is greatly reduced, quickly leading to a large sessile drop. Drop impact rate is important here, as the warm free drop will warm the cool sessile drop when they impact. If the resulting sessile drop has enough time to cool, the behaviour will be the same. If the sessile drop is impacted again before it can cool, then the system behaves more like the warm temperature system described above.

At cold (i.e. slightly below freezing point, water at -5°C) temperatures, the droplets land, recede into a sessile drop, and freeze over. If the impacting droplets impact rapidly, and strike the sessile drop before it freezes over, the interaction strongly resembles cool temperature impacts, as described above. If the sessile droplets can freeze completely, the ice formed becomes the new surface, and the next droplet to land acts as if it were the first droplet. That is, even despite the shape of the ice, it does not splash. The impacting droplet spreads out, forms an equilibrium layer, and then freezes over. This continues, with the droplets taking longer and longer to freeze over due to the reduction in heat transfer to the cold plate because of the increasing thickness of ice. Eventually the behaviour shifts, assuming the drop impact rate remains constant.

At very cold (i.e. strongly below freezing point, water at -20°C) temperatures, the first droplet lands, recedes slightly, but freezes over before it can do so fully, leaving a droplet with a slightly torroidal shape. That is, the edges are thicker than the centre. The resulting “cup” is filled in by the next drop, which may also spread over the edge. However, the shape of the first drop will curtail the spreading of the second drop. If the drops are rapid, the third drop may impact with some liquid at the centre of the “cup”. This tends to build the “cup” as ice accumulates on the inside walls of the cup, but is spread out of the centre by drop impacts before it can freeze. If the drops are slow, the second drop will have frozen over completely, and the third and subsequent drops add on to the top in a relatively even layer, slowly building both the height and diameter of the ice. Eventually the ice gets large enough to make a droplet act like the first droplet, however, since heat transfer through ice is limited, the time to freezing for each

additional drop increases. This means that behaviour similar to a rapid drop rate begins to take effect, and then behaviour similar to a cool drop situation begins to take effect. The number of drops required to generate this change in behaviour is strongly dependent on the temperature of the impacting free droplets.

At extremely cold (i.e. far below freezing point, water at -50°C) temperatures, the first droplet lands, and freezes while spreading. The resulting ice layer will then crack as the ice continues to cool and in so doing, contracts. The second drop spreads over the first, and freezes as it does so. The cracking seems not to have any macroscopic effect on the spreading. If the droplets are rapid, they spread over the smooth surface of ice before cracking, temporarily warming the lower layer, and delaying the cracking as the ice continues to accumulate. Eventually the ice layer slows heat transfer, causing a shift through the regimes as described above.

This spreading and freezing phenomena is obvious both from the ice formation and the initial droplet diameter. In the freezing cases, the droplet is much larger because the droplet freezes before it can retract into a sessile drop. Typical initial droplet areas average to 295 pixels^2 for low drop heights and 647 pixels^2 for high drop heights. With freezing, these average to 5679 pixels^2 instead. This order of magnitude increase is due to the droplet freezing at its most spread condition.

These are not all the possible situations, however. There will also be other situations one can readily imagine. At cryogenic temperatures, for instance, the freezing line would travel faster than the motion of the droplet, capturing a partially spread

droplet. Similarly, at very hot temperatures, reduced viscosity and boiling would affect the spreading phenomena.

5.3 Alternate Test Liquids

In order to test the effects of contamination, test liquids with a different surface tension and viscosity were used. The prime consideration was to modify the fluid properties and analyze what effect they had on the impact behaviour. This analysis would add generality to the analysis and permit an understanding of how contaminated liquids may behave.

The test liquids tested were Pentadecane and Decyl Alcohol.

5.3.1 Test Liquid Properties

According to Jasper, (p.914, 920 and 948, JPCRD, 1972) the surface tension of the fluids was:

Water (20°C):	72.88 $\times 10^{-3}$ N/m
Pentadecane (20°C):	27.07 $\times 10^{-3}$ N/m
Decanol	26 (Approx.) $\times 10^{-3}$ N/m
Acetone (20°C):	21.78 $\times 10^{-3}$ N/m

Acetone is listed to describe the possibility of surface interaction with a contaminated surface. Acetone, then distilled water was used as a cleaning wash of the surface between tests.

It can be seen that the test liquids thus have surface tensions well below that of water. Thus, splashing results can be expected to have a different behaviour. The behaviour of the test liquids is discussed in sections 5.3.4 and 5.3.5, below.

One important note for the operation of the system is that fluids with a lower surface tension tend towards a faster drop rate. The pump must be turned down to maintain a common drop rate between test liquids.

According to the 1980 CRC Handbook of Chemistry and Physics, the melting point of the liquids is:

Water	0°C
Pentadecane	9°C
Decanol	7°C

The test liquids were chosen to have melting points near to each other, and above zero so as to permit the system to easily drop the test liquid through various splashing and freezing situations.

Though exact values could not be obtained in the literature, it was apparent by observation that the test liquids also had moderately high viscosities when compared with water. The exact viscosity values were not of critical concern in the experimental analysis. Merely the trend was investigated.

The test liquids, especially the pentadecane, demonstrated hydrophilic behaviour.

5.3.2 Splashing Results

The primary result of interest in the splashing of the alternate test liquids was the reduced dead zone around the sessile drop.

The primary droplet tends towards larger spreading and splashing due to the lower surface tension of the alternate test liquids. In fact, given time, the alternate test liquids will spread over the entire area of the plate, and so tests must be done with some speed.

Because of this spreading behaviour, the alternate test liquids tend to form a large, thin sessile droplet rapidly. This tends to change the splashing behaviour as the droplet critical size is reached rapidly. In viscous test liquids, however, when the droplet impacts a very thin film, there is some splashing. The spreading liquid only partly meets resistance, and partly spreads over top of the layer. The shear action drags some stagnant

liquid along, forming a ramp, which lifts the flow up. Some of the liquid then can separate if the surface tension is low. This leads to a critical layer thickness, above which the impacting droplet is entirely contained, and below which splashing occurs.

From the tests, it is apparent that surface tension and viscosity are important characteristics in determining the droplet behaviour.

5.3.3 Splashing and Freezing Results

The splashing and freezing results for the alternate test liquids demonstrate the same situations as water.

The alternate test liquids clearly demonstrate very cold freezing. A torroidal ice cap is produced, filled, and covered by droplets. The droplets then begin to impact a warmer and warmer surface as the ice reduces the heat transfer to the plate. Eventually, the droplets no longer freeze, and then splashing results.

5.4 Sample Images

The sample images displayed here are given to attempt to convey how the above results came about. The images are generally unprocessed captures, and each includes some comments describing the conditions of the test. Of the hundreds of images captured from dozens of tests, only a few are presented here for brevity. The order of images is always left to right, then top to bottom.

Fig.22 Photographs of Torroidal Ice Cup

These images are photographs taken of testing on the gold plated test plate under very cold conditions. The first drop lands, spreading to the condition shown in the left image. Then, the fluid recedes, however the liquid in contact with the surface has frozen, maintaining the contact area. As the liquid recedes, it is frozen into the torroidal ice cap shown in the center image. When the next drop lands, it fills in the ice cup, and freezes, creating a smooth surfaced ice layer, which is shown in the right image.

Fig.23 Captures of Torroidal Ice Cup Behaviour

These images are captures taken of testing on the gold plated test plate under very cold conditions, similar to those shown in Fig.23. Here, the plate is at -25°C , and the local humidity is 44%. The ambient humidity is 39%. Note the ice cup formed by the first drop, and how it is filled in by the second. Then the third drop lands, forming a less pronounced ice cup. The fourth drop fills the ice cup, but also spreads over the entire ice surface because the ice cup of the third droplet is not pronounced enough to contain it. The fifth drop cannot form an ice cup, because of the thickness of the ice layer. Thus the fifth and sixth drops spread over the surface and then freeze. This is why there is no splashing. This demonstrates the change of regimes of splashing and freezing behaviour.

Fig.24 Splash Imaging, Droplets 1 to 4

This series describes the impact of warm droplets under cool conditions. The plate boundary layer temperature was 0°C , and the plate was being fed by water at -9.2°C . The relative humidity of the boundary layer was 25%, and the relative humidity of the test cell was 45%. The test fluid was water at 20°C . Trace freezing took 18 minutes to occur.

Fig.25 Splash Imaging, Droplets 5 to 8

A continuation of the series in Fig.25, note how the early biasing from drop 2 moved the sessile droplet and created a continuing bias maintained throughout the drops. The final distribution still shows the bias from the chaos of drop 2. This is common.

Fig.26 Precursor Flow and Low Splashing Images.

These images demonstrate the precursor flow. Between image one and two, no droplet has impacted, but time has passed. Here, the test liquid is Decyl Alcohol, which is hydrophilic. This causes the sessile drop to produce an enhanced precursor flow. The next images show the reduced splashing because of the large sessile droplet. The temperature is 9.4°C , which is near the freezing point of the Decyl Alcohol.

6.0 Future Work

It should be obvious to the reader that more work is required in this field. This work describes the design, manufacture and testing of an experimental system capable of investigating several phenomena. These phenomena need to be investigated in more detail. Splashing and freezing interactions must be understood if a bridge is to be made between theoretical fluid mechanics and real-world solutions to industrial problems.

Presented in this work are some interesting observations that are either not described, or not adequately described in the literature. Further research must be done on these observations to investigate their exact nature, and publish such findings. Specifically, investigations must be performed on:

- The splashed mass primarily originating from the sessile droplet as opposed to the impinging one. Is it a behavior only visible in certain low velocity impact systems?
- The “Dead Zone” phenomenon and how splashing interaction creates a dead zone, how fluid parameters influence its size, and how a wider range of impinging velocities affects the zone.
- The freezing behaviour regimes as a function of both temperature, drop rate, and the effects of large droplet impinging velocities.

One area of study that will require additional work is the statistical analysis of the droplets. As data is produced for large numbers of tests, each with variable numbers of droplets, a statistical system of analyzing this data will need to be perfected. This system may build upon the analysis performed herein, however it will need to be tailored to the parameters governing the exact investigations performed. These methods may be useful in forming an understanding of the macroscopic processes involved in the phenomena.

Another possible area of study that will require additional consideration is the application of chaos theory mathematics to the phenomena. Powerful methods exist for analyzing the motion of the liquid particles. These methods may be useful in forming an understanding of the microscopic processes involved in the phenomena.

As a last comment, the performance indices of the test cell cannot be matched to every case that may be of industrial significance. For instance, the velocities that can be obtained in the test cell are too far below that of an aircraft to match Reynolds or Mach numbers. As such, analysis will need to be done for any application of this work as to what the most relevant parameters are and how best to match them. For some cases, it may be necessary to modify or replace the system with one more specialized to meeting the criterion of interest.

7.0 References

Theses:

- 1 YunZhen Zhou, Design of a Digital Vision System for Measuring Surface Tension, 2000, Carleton University, Ottawa.

Lab Manuals:

- 2 75.101, Introductory Physics 1, 1994, Carleton University, Ottawa.

Books:

- 3 M.N. Ozisik, Heat Transfer, A Basic Approach, 1985, M^cGraw Hill, Toronto.
- 4 Howard Reiss, Methods of Thermodynamics, 1965, Blaisdell, Toronto.
- 5 Herbert B. Callen, Thermodynamics, 1962, John Wiley & Sons, New York.
- 6 I.M.Gelfand, Calculus of Variations, 1963, Prentice-Hall, Toronto.
- 7 John G. Kirkwood, Chemical Thermodynamics, M^cGraw-Hill, Toronto.
- 8 A. Frohn & N.Roth Dynamics of Droplets, 2000, Springer, New York.
- 9 Robert L. Daugherty & Joseph B Franzini, Fluid Mechanics with Engineering Applications, 1977, M^cGraw-Hill, Toronto.
- 10 Robert C. Weast, CRC Handbook of Chemistry and Physics, 1980, CRC, Florida.
- 11 G.S.H. Lock, The Growth and Decay of Ice, 1990, Cambridge University Press, Cambridge.

- 12 R. Byron Bird, Warren E. Stewart & Edwin N. Lightfoot, Transport Phenomena, 1924, John Wiley & Sons, New York.
- 13 S. Middelman, An Introduction to Mass and Heat Transfer, Principles of Analysis and Design, 1998, John Wiley & Sons, New York
- 14 S. Middleman, Modeling Axisymmetric Flows; Dynamics of Films, Jets and Drops, 1995, Academic Press, San Diego.

Articles:

- 15 R.G. Cox, *The spreading of liquid on a rough solid surface*, Journal of Fluid Mechanics, vol.131, p1, 1983.
- 16 J.G. Petrov, P.G. Petrov, *Forced advancement and retraction of polar liquids on a low energy surface*, Colloids and Surfaces, vol.64, p141, 1992.
- 17 R.G. Cox, *The dynamics of the spreading of liquids on a solid surface. Part 1: Viscous flow*, Journal of Fluid Mechanics, vol.168, p.169, 1986.
- 18 K. Range & F. Feuillebois, *Influence of surface roughness on liquid drop impact*, Journal of Colloid and Interfacial Science, vol. 203, p.16, 1998.
- 19 C.A. Ward & M.R. Sages, *Effect of gravity on contact angle: A theoretical investigation*, Journal of Chemical Physics, vol. 109, p3651, 1998.
- 20 C.A. Ward & M.R. Sages, *Effect of gravity on contact angle: An experimental investigation*, Journal of Chemical Physics, vol. 109, p3661, 1998.
- 21 Joseph J. Jasper, *The Surface Tension of Pure Liquid Compounds*, Journal of Physics and Chemistry Reference Data, vol.1, No.4, p.841, 1972.

- 22 O.I. del Rio & A.W. Neumann, *Axisymmetric Drop Shape Analysis: Computational Methods for the Measurement of Interfacial Properties from the Shape and Dimensions of Pendant and Sessile Drops*, Journal of Colloid and Interface Science, Vol.196, p.136, 1997.
- 23 Xioguang Zhang & Osaman A. Basaran, *Dynamic Surface Tension Effects in Impact of a Drop with a Solid Surface*, Journal of Colloid and Interface Science, Vol.187, p.166, 1997.
- 24 B. Prunet-Foch, F. Legay, M. Vignes-Adler & C. Delmotte, *Impacting Emulsion Drop on a Steel Plate: Influence of the Solid Substrate*, Journal of Colloid and Interface Science, Vol.199, p.151, 1998.
- 25 L. Bacri, G. Debregeas & F. Brochard-Wyart, *Experimental Study of the Spreading of a Viscous Droplet on a Nonviscous Liquid*, Langmuir, Vol.12, p.6708, 1996.
- 26 R.V. Sedev, C.J. Buziak, J.G. Petrov & A.W. Neumann, *Dynamic Contact Angles at Low Velocities*, Journal of Colloid and Interface Science, Vol.159, p.1, 1993.
- 27 M. Pasandideh-Fard, S.D. Aziz, S. Chandra & J. Mostaghimi, *Surface Cooling by an Impinging Water Drop*, Proceedings of the 33rd National Heat Transfer Conference, 1999, Albuquerque, New Mexico, HTD99-89, 1999.
- 28 David N. Anderson, *Methods for Scaling Icing Test Conditions*, Proceedings of the 33rd Aerospace Science Meeting, Reno, January 1995, AIAA 95-0540, 1995.

Appendix A: Design of the Containment Box

The system would require a containment box around the experiment to prevent contamination by dust, oils or other aerosol contaminants. The containment box would thus have to be essentially airtight, and would also have to be transparent to permit the camera to view the experiment. To permit a high degree of versatility, the design would have to be large. Space for large samples, and instrumentation would have to be included. In addition, the system would have to permit a vertical drop distance sufficient to induce splashing. The system was thus designed to be at least a metre in each direction.

Only custom enclosures could be found in the volume range desired, so the decision was made to manufacture the box in-house.

The material selection for the box was relatively straightforward. Glass and plastics were immediately considered, as was a metal box with an optical window. Optical glass and crystal windows, however, proved to be quite expensive. Investigation showed that plexiglass offered acceptable transparency. Little difference was found with glass, and so there was no need for optical glass or crystal. After investigation, the best pricing information was as follows:

3/8" thickness Plexiglass sheets:	Canus Plastics:	\$265 per 4'x 8' sheet.
1/2" thickness Plexiglass sheets:	Canus Plastics:	\$365 per 4'x 8' sheet.
1/2" thickness Lexan Plastic sheets:	Canus Plastics:	\$627 per 4'x 8' sheet.

1/4" thickness Stainless Steel sheets: Metal Supermarkets: \$455.62 per 4' x 4' sheet.

3/8" thickness Stainless Steel sheets: Loucon Metal: \$739 per 4' x 4' sheet.

Both the high cost and the fact that metal would be a poor thermal barrier caused the concept of a metal box to be discarded. The notion of a glass box posed problems with the fragility of handling. In the end, this concept was dropped in favor of a plastic box. Plexiglass was an immediate consideration, as was Lexan plastic. The added strength of Lexan plastic, however, was not required for the application, so the lower cost of Plexiglass was preferred. The 1/2" (1.27cm) sheets were selected and ordered.

Fastening was performed by means of 1" (2.54cm) screws. Holes were drilled and tapped every 10cm. The sheets were then assembled, with readily available lengths of 1/4" (6.35mm) polyethylene tubing placed between the Plexiglass sheets. By tightening the screws, the compressed tubing provided an air-tight seal. The construction required 96 screws in total.

The front face was left open, to permit access doors. One vertical strip of Plexiglass, 15.2cm wide was mounted flush to the front face for the doors to seal against. These doors were simply 61cm x 122cm sheets of Plexiglass, mounted to the box with 3 1/2" (8.89cm) Zinc plated broad butt hinges by bolts that were cut to 2.5cm, six per hinge, two hinges per door.

To fasten the doors closed, $\frac{3}{4}$ " (1.9cm) diameter eye bolts were fastened to the 15.2cm wide strip. Holes were then cut through the doors to permit them to pass, and pins were fashioned out of 4" (10.16cm) common nails.

Polyethylene $\frac{1}{2}$ " (1.27cm) ID Tubing sliced in half then made the seal running inside the eye hooks down the vertical strip, and across the other edges. The tubing was held in place by clear packing tape when the doors were open.

On any bolt through the Plexiglass panels, seals were ensured via an interference fit of Teflon tape wrapped around the bolt.

Fig.A1 Design of Containment Box

Appendix B: Design of the Support Rack

In order to support the tube end, mirror, cooling coil and thermometers, a support system would have to be placed into the containment box. This support system would have to be both modular and variable. Components could later have been added or moved, and the support system would have to adapt. As such, a very flexible support system would have to be designed.

The selection for the support system included the use of modular pipes and clamps as sold by Cole-Parmer. The final system includes two vertical aluminum rods, which run from the bottom to top of the box interior. These were cut from stock in-house, as solid rods of the desired length were not available. Solid rods were chosen for their strength, as these rods would not only support apparatus, but further reinforce the box. Circular mounting “feet” clamp to the rod and screw into the box, preventing movement of the rods. Affixed by clamps to these rods are tubes, which in turn support the apparatus within the box. Affixed to these rods are short arms, which hold the mirror and thermometers. Additional rods or arms could be easily added to the system to support further apparatus. The copper cooling coil was hung from the rods and tubes with clear packing tape to permit easy adjustment of the coil geometry.

Appendix C: Test of 0.0075" ID Tubing

Tygon Tubing Diameter: 0.0075" ID, 0.0795" OD

Operating Fluid: Tap Water

Masterflex Pump Setting: 10 (100rpm)

The testing was done with the tube pumping into a beaker covered with aluminum foil to prevent evaporation. The line after the last point on the graph leads to a point taken seven hours after the initial testing began. The variations near the start of the test are relatively minor. The average flow rate over the entire period is 0.0686 g/s.

Fig.C1 Flow rate of smallest tubing

Appendix D: Design of Light Path

In order to view the sample from a top-down perspective, a mirror was required to redirect the line of sight of the camera, see Fig. D1. The light path was then traced from the camera to the mirror and to the sample. From there, it could also be continued if additional lighting was required. The light path continued past the impact surface to a location outside the box. This location is where additional light input, or a white imaging backdrop was placed to resolve dark images. The advantage of a clear containment box was that diffuse light was easy to input without adding heat to the system. As can be readily seen from Fig. D1, a small movement and angling of the mirror permitted the light path to be moved considerably. This permitted the light path to be moved, to accommodate changes to the system. This provided the system a high degree of flexibility. The difficult part of constructing the light path was designing a support that permitted the mirror to be moved as necessary, yet support it precisely. The mirror was not in contact with any cooling apparatus, so condensation was not a problem. If it ever were, a heating unit could be attached to the back face of the mirror.

During testing of the system, it became apparent that the images obtained by the camera were easier to process without imaging the coil. As such, the light path was modified to an alternate configuration shown in Fig. D2. This configuration had the added benefit that the droplet passed through the field of view of the camera in free-fall, permitting the possibility of capturing images of the drop in free fall.

Effect in light path of small variance in mirror placement

Fig.D1 Design of Light Path

Fig.D2 Alternate Configuration of Light Path

A white-surface imaging backdrop was later selected, as the system proved to be capable of imaging reflections off even unpolished aluminum surfaces. The opaque white surface blocked direct light. However, side lighting from fluorescent lamps placed within or beside the containment box proved to be quite sufficient to permit imaging of the droplets. For darker samples, fluorescent lighting was added in front of the white imaging surface. This provided additional light along the light path. Fluorescent lighting was found to provide a diffuse light with a smooth intensity. This allowed lighting without glare. In addition, aluminum foil was placed on the box to make the inner surface reflective, so as to maximize the effect of latent light.

In all cases, alignment of the light path was confirmed with a small laser.

Appendix E: Investigation of Air Resistance

Starting from an elementary force analysis, we write:

$$F = (\text{mass}) \times (\text{acceleration}) \quad [\text{E.1}]$$

$$= m \, dU/dt \quad [\text{E.2}]$$

$$= m \, g - C_D(\text{Sphere}) \, 1/2 \rho_a U^2 \, 1/4 \pi d^2 \quad [\text{E.3}]$$

where $C_D(\text{Sphere})$ is the drag coefficient, defined as $F_d / 1/2 \rho_a U^2 \, 1/4 \pi d^2$

F_d is the drag force

U is the droplet velocity

ρ_a is the density of the air the droplet is falling through

For spherical, small aerosol particles: (p.295, Daugherty & Franzini, 1977)

$$C_D(\text{Sphere}) = 24/Re_D (1 + 0.158 Re_D^{2/3}) \text{ for } Re_D < 10^3 \quad [\text{E.4}]$$

For this case, where a velocity of $U < 5 \text{ m/s}$ is expected, we obtain:

$$\begin{aligned} Re_D &< \rho_a U d / \mu_a \quad [\text{E.5}] \\ &< (1.205 \text{ kg/m}^3) (5 \text{ m/s}) (4.5 \times 10^{-3} \text{ m}) / (1.8 \times 10^{-5} \text{ Ns/m}^2) \\ &< 1506 \end{aligned}$$

We may obtain an analytical solution for small Re_D by approximating the Reynolds number as:

$$C_D(\text{Sphere}) = 24/Re_D \quad [E.6]$$

Thus, the force balance equation becomes:

$$m \, dU/dt = \quad mg - C_D(\text{Sphere}) \, 1/2 \rho_a U^2 \, \pi R_o^2 \quad [E.7]$$

$$dU/dt = \quad g - C_D(\text{Sphere}) \, 1/2 \rho_a U^2 \, \pi R_o^2 / (\rho_w \, 4/3 \pi R_o^3) \quad [E.8]$$

$$dU/dt + \quad C_D(\text{Sphere}) \, 3/8 \, (\rho_a / \rho_w) \, (1/R_o) \, U^2 = g \quad [E.9]$$

$$dU/dt + \quad (24\mu_a/\rho_a \, U \, 2R_o) \, 3/8 \, (\rho_a / \rho_w) \, (1/R_o) \, U^2 = g \quad [E.10]$$

$$dU/dt + \quad 9/2 \, (\mu_a/\rho_w R_o^2) \, U = g \quad [E.11]$$

$$dU/dt + \quad c \, U = g \quad [E.12]$$

This is an ordinary differential equation, which has a solution of the form:

$$U(t) = \quad A \, e^{-ct} + g/c \quad [E.13]$$

where A is a constant to be determined.

Analysis of the initial conditions shows that the droplet begins at rest, and so:

$$U(t = 0) = 0 \quad [E.14]$$

$$Ae^0 + g/c = 0 \quad [E.15]$$

$$A = -g/c \quad [E.16]$$

Thus the solution becomes:

$$U(t) = \frac{g}{c} (1 - e^{-ct}) \quad [E.17]$$

For a water droplet of 2.5mm in air, c becomes:

$$\begin{aligned} c &= \frac{9}{2} \mu_a / \rho_w R_o \quad [E.18] \\ &= \frac{9}{2} (1.8 \times 10^{-5} \text{ Ns/m}^2) / (998 \text{ kg/m}^3)(4.5 \times 10^{-3} \text{ m}) \\ &= 4.0 \times 10^{-3} / \text{s} \end{aligned}$$

As c increases with decreasing R_o , air friction will be dominated by inertia for larger drops. Thus, air friction will be less important for large drops, and the calculated results are applicable for droplets of a given size or larger.

To get a more exact analysis, we integrate to obtain an expression for distance as a function of time:

$$S(t) = g/c (t - (1/c)[e^{-ct} - 1]) \quad [E.19]$$

where e^{-ct} can be expressed as $1 + (-ct) + (-ct)^2/2! + (-ct)^3/3! \dots$

Selecting the leading orders of the time expansion, the distance is thus approximated as:

$$S(t) = g t^2 / 2 + c g t^3 / 6 \quad [E.20]$$

Note that when air friction is ignored, c goes to zero as the viscosity term goes to zero. This causes the above equation to become the standard ballistic equation.

Solving for t when c is 0.004/s and $S(t)$ is 1m, we obtain:

$$t = 0.4516 \text{ s}$$

$$U(0.4516\text{s}) = 4.426 \text{ m/s}$$

$$Re_D = 1333$$

Conversely for a purely ballistic equation where air friction is ignored:

$$t = 0.4515 \text{ s}$$

$$U(0.4515\text{s}) = 4.429 \text{ m/s}$$

$$Re_D = 1334$$

This leads one to the conclusion that air friction will cause a very small effect for this size of droplet or larger, for this fall height or lower.

Terminal Velocity:

To ensure the velocities were possible, the terminal velocity had to be calculated. The terminal velocity is the maximum velocity of the droplet in free air. It was calculated by performing a force balance, as follows:

$$F_{\text{Buoyancy}} + F_{\text{Drag}} = F_{\text{Gravity}} \quad [\text{E.21}]$$

$$\rho_{\text{air}} V_{\text{drop}} + C_D \rho_{\text{air}} U_{\text{drop}}^3 A_{\text{drop}} / 2 = m_{\text{drop}} g \quad [\text{E.22}]$$

where ρ_{air} is the density of the air, V_{drop} is the volume of the droplet, C_D is the drag coefficient of the droplet, U_{drop} is the speed of the droplet, A_{drop} is the cross-sectional area of the droplet with respect to the velocity vector, m_{drop} is the mass of the droplet and g is the acceleration due to gravity.

Assuming a spherical droplet for the calculation of drag, this becomes:

$$\rho_{\text{air}} (4/3 \pi R_{\text{drop}}^3) + C_D \rho_{\text{air}} U_{\text{drop}}^3 (\pi R_{\text{drop}}^2) / 2 = \rho_{\text{droplet}} (4/3 \pi R_{\text{drop}}^3) g \quad [\text{E.23}]$$

where R_{drop} is the radius of the droplet. This can be reduced to:

$$U_{\text{terminal}} = [4 D_{\text{drop}} ((\rho_{\text{droplet}} / \rho_{\text{air}}) g - 1) / 3 C_D]^{1/2} \quad [\text{E.24}]$$

The C_D value, however, is dependent on the droplet velocity, and an iterative solution was required. The iterations were performed, using the graph of C_D values for a sphere presented in the literature, (p.295, Daugherty & Franzini, 1977). This iteration returned a result of 11m/s and a Reynold's Number based on diameter of approximately 3400.

As the droplet velocities in the system are not expected to reach beyond 4 m/s, the droplets will not be restricted by terminal velocity.

Appendix F: Design of the Cooling System

Initially, the design of the system required that the plate itself be maintained at the below freezing temperature. It was decided that -20°C was a minimum temperature for testing. The consideration of air temperature, however, prompted a design that could chill the air temperature if desired. This was performed by the inclusion of a large coil that wound around the drop path. If the air temperature was to be controlled, fluid from the reservoir could be pumped through the coil to chill the air temperature. The plate cooling itself was performed by a rack of steel pipe upon which the plate would sit directly. If a small plate was to be used, a metal support plate could be placed on the rack, and the plate upon that. The rack itself was designed to be modular, so that it could be lengthened or shortened by adding or removing standardized lengths. In order to feed liquid to the rack and coil, polyethylene tubing, wrapped in insulation, was used.

Steel pipe was used for the rack as it fulfilled the requirements of low cost, high damage tolerance and low chemical reactivity. The coil was made of a 50' length of flexible copper tubing. This copper tubing could be bent to any shape required by the system, or by redesign of the system. Barbed hose nipple adapters were soldered onto each end to permit mating with the polyethylene tubing. Polyethylene tubing was selected due to its high flexibility, even at very low temperatures, its low chemical reactivity and its ease of sealing. Polyethylene tubing permits an excellent seal with indoor/outdoor caulking, and its flexibility transmits few loads to such seals.

reservoir, and, in turn, chill an aluminum plate that would support the impact plate. The aluminum plate was held in place by clamps such that it remained in contact with the reservoir fluid.

This entire reservoir was placed within rigid Styrofoam insulation to prevent heat conduction, and the walls of the insulation were kept high enough to trap a large boundary layer of chilled air.

The coolant reservoir contained the bath of chilled fluid that was pumped through the system. It contained the pumps that moved the fluid and the chiller that cooled the fluid. The first selection required was that of the operating fluid. A liquid was selected for ease of containment and use. By definition the fluid had to be able to flow below the freezing point of water. A mixture of 50% Ethylene Glycol and 50% Water (by volume) was available from Cole-Parmer. This mixture was very safe, pure, posed no reactivity problems, and had a listed freezing point of -35°C . However, in the study of the extremely cold regimes, (see section 5.2 for details) dry ice was packed around the system to drop the temperature to below -50°C . Under these conditions, the mixture did not freeze, and so the actual freezing point of the mixture is unknown.

In order to size the reservoir, the volume of the rack and coil was calculated.

The volume of the rack:

$$V_{\text{rack}} = V_{\text{lines}} + V_{\text{branches}} \quad [\text{F.1}]$$

$$= N_{\text{lines}}(\pi/4(d_{\text{line}})^2 L_{\text{line}}) + N_{\text{branches}}(\pi/4(d_{\text{branch}})^2 L_{\text{branch}})$$

$$= 2[\pi/4(3/8'')^2 15'] + 6[\pi/4(3/8'')^2 8']$$

$$= 8.615 \text{ in}^3$$

$$= 0.14 \text{ L}$$

The volume of the coil:

$$V_{\text{coil}} = \pi/4(d_{\text{coil}})^2 L_{\text{coil}} \quad [\text{F.2}]$$

$$= \pi/4(3/8'')^2 50'$$

$$= 66.268 \text{ in}^3$$

$$= 1.09 \text{ L}$$

The volume of the connecting tubing:

$$V_{\text{tubes}} = N_{\text{tubes}}(\pi/4(d_{\text{tube}})^2 L_{\text{tube}}) \quad [\text{F.3}]$$

$$= 4[\pi/4(3/8'')^2 5']$$

$$= 26.507 \text{ in}^3$$

$$= 0.43 \text{ L}$$

Total Fed Volume:

$$\begin{aligned}
 V_{\text{tot}} &= V_{\text{rack}} + V_{\text{coil}} + V_{\text{tubes}} & [\text{F.4}] \\
 &= 0.14 \text{ L} + 1.09 \text{ L} + 0.43 \text{ L} \\
 &= 1.66 \text{ L}
 \end{aligned}$$

Thus, the pumps would supply a volume of 1.66 L with chilled fluid from the reservoir. Consultation with a mechanic gave a rule of thumb for fitting the reservoir volume. To provide adequate fluid to the pumps and provide a good thermal reservoir, a volume ratio of approximately 50:1 to 100:1 was thus selected. This meant that the reservoir would be between 83L and 166L in capacity.

After some pricing was done, it was decided to manufacture the reservoir box in-house, using the same ½" Plexiglass sheets as the containment box. The reservoir box was designed to fit conveniently on the support table beside the chiller. The final reservoir box sizing was 15" high x 22" x 22". This provided a volume of 7260 in³, or 119 L. After leaving a gap of approximately 2 3/8 in for air, the reservoir would contain 100 L of fluid, minus the volume of the pumps. The sizing was on the smaller end, but this was considered acceptable, because there were to be two pumps, one to feed the coil, the other to feed the rack. This would allow for independent operation of the systems. The reservoir box was then insulated. When assembled and tested, the reservoir, isolated from the rest of the system, generated temperatures of -30°C.

Appendix G: Conductive Heat Transfer Calculations

A very high number was expected, indicating convection would be the limiting form of heat transfer.

To estimate heat transfer area:

Estimate rack area:

Neglect tubing curvature (will yield lower answer)

Assume plate covers entire rack (will yield higher answer)

$$\begin{aligned}
 A_{\text{rack}} &= (8'' \times 15'')/\text{side} \times (2 \text{ sides}) & [G.1] \\
 &= 240 \text{ in}^2 \\
 &= 0.155 \text{ m}^2
 \end{aligned}$$

Estimate temperature gradient:

Assuming worst case, maximum temperature gradient.

Minimum plate temperature: -20°C

Assume air has not been cooled: $+20^{\circ}\text{C}$

$$\begin{aligned}
 \Delta T &= (20^{\circ}\text{C}) - (-20^{\circ}\text{C}) & [G.2] \\
 &= 40^{\circ}\text{C}
 \end{aligned}$$

Estimate thermal conductivity:

Assuming it will be similar to 0.5%C steel

$$K = 54 \text{ W/m}^\circ\text{C}$$

Estimate wall thickness:

For standard type 40 3/8" steel pipe, the wall thickness is 0.091 in

$$x = 0.091 \text{ in} = 0.0023 \text{ m}$$

Estimate conductive heat transfer:

$$\begin{aligned} Q_{\text{conduction}} &= K A_{\text{plate}} \Delta T / x & [\text{G.3}] \\ &= (54 \text{ W/m}^\circ\text{C}) (0.155 \text{ m}^2) (40^\circ\text{C}) / (0.0023 \text{ m}) \\ &= 145,565 \text{ W} \end{aligned}$$

This number is very large, as was expected. Therefore, conduction will not limit heat flow. Instead, convective analysis must be performed.

Appendix H: Convective Heat Transfer Corrections

In order to ensure that the heat transfer calculations were accurate, several correction factors were investigated. The correction factors are empirical relations for curves fit to experimental data for various conditions by various experimenters. The relations from the work of McAdams and the work of Churchill and Chu were presented in Ozisik, (p.431, Ozisik, 1985). The specific case of a horizontal, flat plate is given with McAdams relations in Ozisik, (p.437, Ozisik, 1985).

McAdams correction:

$$Q = h A (T_o - T_w) \quad [H.1]$$

Where:

$$h = Nu_m K / L \quad [H.2]$$

$$Nu_m = c (Gr_L \times Pr)^n \quad [H.3]$$

$$Gr_L = g \beta (T_o - T_w) L^3 / \nu^2 \quad [H.4]$$

In this case:

$$g = 9.81 \text{ m/s}^2$$

$$\beta = 1/T_o$$

$$T_w = -20^\circ\text{C} = 253.15 \text{ K}$$

$$T_o = 20^\circ\text{C} = 293.15 \text{ K}$$

To adequately describe the system, the square root of area was selected as a length scale.

$$\begin{aligned} L &= (0.155\text{m}^2)^{1/2} && \text{for the steel rack} \\ &= (0.456\text{m}^2)^{1/2} && \text{for the copper coil} \\ &= (0.0729\text{m}^2)^{1/2} && \text{for the cooling plate} \end{aligned}$$

From Ozisik, p736:

$$\psi = 12.59 \times 10^{-6} \text{ m}^2/\text{s} \quad \text{at } 275 \text{ K} \quad [\text{H.5}]$$

Furthermore, as the air flow over the plate will be laminar, in effect nearly zero, we can use the presented coefficients:

$$c = 0.59$$

$$n = 0.25$$

as these values of c and n hold for $10^4 < Gr_L Pr < 10^9$.

Thus:

$$\begin{aligned}
 Gr_L &= g \beta (T_o - T_w) L^3 / \psi^2 & [H.6] \\
 &= (9.81 \text{ m/s}^2) (1/293.15\text{K}) (293.15\text{K} - 253.15\text{K}) (L)^3 / (12.59 \times 10^{-6} \text{ m}^2/\text{s})^2 \\
 &= 5.153 \times 10^8 & \text{for the steel rack} \\
 &= 2.600 \times 10^9 & \text{for the copper coil} \\
 &= 1.662 \times 10^8 & \text{for the cooling plate}
 \end{aligned}$$

Again from Ozisik:

$$Pr = 0.715 \quad \text{at } 275 \text{ K} \quad (\text{p.736})$$

Thus:

$$\begin{aligned}
 Gr_L Pr &= 3.68 \times 10^8 & \text{for the steel rack} \\
 &= 1.86 \times 10^9 & \text{for the copper coil} \\
 &= 1.19 \times 10^8 & \text{for the cooling plate}
 \end{aligned}$$

These values are within the upper edge of the correction, and may cause some inaccuracy.

$$Nu_m = c (Gr_L \times Pr)^n \quad [H.7]$$

$$= 0.59 (Gr_L Pr)^{0.25} \quad [H.8]$$

$$\begin{aligned}
 &= 81.7 && \text{for the steel rack} \\
 &= 122.5 && \text{for the copper coil} \\
 &= 61.6 && \text{for the cooling plate}
 \end{aligned}$$

$$h = \text{Nu}_m K / L \quad [\text{H.9}]$$

From Ozisik:

$$K = 0.2426 \text{ W/m}^\circ\text{C} \quad \text{at } 275 \text{ K} \quad (\text{p736})$$

$$\begin{aligned}
 h &= 50.37 \text{ W/m}^2\text{C} && \text{for the steel rack} \\
 &= 44.01 \text{ W/m}^2\text{C} && \text{for the copper coil} \\
 &= 55.35 \text{ W/m}^2\text{C} && \text{for the cooling plate}
 \end{aligned}$$

$$\begin{aligned}
 Q &= h A (T_o - T_w) && [\text{H.10}] \\
 &= 310 \text{ W} && \text{for the steel rack} \\
 &= 800 \text{ W} && \text{for the copper coil} \\
 &= 160 \text{ W} && \text{for the cooling plate}
 \end{aligned}$$

This posed the distinct problem of over 1 kW of cooling power required for the rack and coil. As the value was so large, and so different from the initial calculated requirements, additional corrections were investigated.

Churchill and Chu correction:

$$\text{Nu}_m = 0.68 + [0.67 (\text{Gr}_L \text{Pr})^{1/4}] / [1 + (0.492/\text{Pr})^{9/16}]^{4/9} \quad [\text{H.11}]$$

$$\begin{aligned} &= 61.62 && \text{for the steel rack} \\ &= 92.39 && \text{for the copper coil} \\ &= 54.43 && \text{for the cooling plate} \end{aligned}$$

$$\begin{aligned} h &= 37.97 \text{ W/m}^2\text{°C} && \text{for the steel rack} \\ &= 25.61 \text{ W/m}^2\text{°C} && \text{for the copper coil} \\ &= 48.91 \text{ W/m}^2\text{°C} && \text{for the cooling plate} \end{aligned}$$

$$\begin{aligned} Q &= 240 \text{ W} && \text{for the steel rack} \\ &= 470 \text{ W} && \text{for the copper coil} \\ &= 140 \text{ W} && \text{for the cooling plate} \end{aligned}$$

This produced a result of over 700 W of cooling required for the rack and coil.

McAdams correction for a flat, horizontal plate:

$$\text{Nu}_m = 0.27 (\text{Gr}_L \text{Pr})^{1/4} \quad [\text{H.12}]$$

$$\begin{aligned} &= 37.40 && \text{for the steel rack} \\ &= 56.07 && \text{for the copper coil} \\ &= 28.2 && \text{for the cooling plate} \end{aligned}$$

Note that the copper coil is not a flat, horizontal plate, and so the results may be inaccurate for the coil.

h	$= 23.05 \text{ W/m}^2\text{°C}$	for the steel rack
	$= 20.14 \text{ W/m}^2\text{°C}$	for the copper coil
	$= 25.34 \text{ W/m}^2\text{°C}$	for the cooling plate
Q	$= 140 \text{ W}$	for the steel rack
	$= 370 \text{ W}$	for the copper coil
	$= 74 \text{ W}$	for the cooling plate

This produced a result of over 500W of cooling required for the rack and coil. The results of all the calculations are summarized in the table below:

Table H1 Tabulated results of Heat Transfer Calculations

Method	Required Cooling (W)		
	Coolant Pipe	Coolant Plate	Coil
Convection	49	32	140
McAdams	310	160	800
Churchill and Chu	230	140	470
McAdams (Flat Plate)	140	74	370

Appendix I: Ambient Conditions

This graph describes typical ambient conditions for the laboratory. As can be readily seen, the ambient humidity varies greatly over even very short intervals. Seasonal variations also cause general trends in the data, but the humidity can vary over one day sufficiently to require attention. The ambient conditions must thus be assessed on a day-by-day basis.

Fig.II Ambient Conditions

Appendix J: Labour Estimate

The labour estimate provides an estimate of the work required to manufacture the system presented in herein. A pricing estimate is also provided, but the pricing information is accurate only to year 2000/2001 prices, and should be investigated before ordering.

Design Labour

Design labour is not required for the manufacture of this system, though permitting two hours of design labour to interpret the work presented herein is recommended.

Manufacturing Labour

Manufacture of Containment Box		12 h (Plexi-glass cutting, tap & dye)
Assembly of Containment Box	2 persons x	2 h (Unskilled Labour)
Manufacture of Coolant Reservoir		1 h (Plexiglas cutting)
Manufacture of Cooling Rack		2 h (Pipefitting)
Manufacture of Cooling Coil		1 h (Soldering, Pipefitting)
Assembly of Cooling system		4 h (Caulking, insulating)
Reinforcement of support table		<u>3 h (Machining, carpentry)</u>
		25 h Total

Appendix K: Pricing Estimate

Object	Unit Cost	Number	Total Cost
<i>Liquid delivery system</i>			
Containment Beaker (Pyrex 500ml, pack of 6)	\$26	1	\$26
Masterflex Pump	\$1190	1	\$1190
Masterflex Pump Head	\$738	1	\$738
Masterflex Pump Cartridges	\$80	4	\$320
Tubing (roll)	\$64	2	\$128
<i>Containment Box</i>			
Plexiglass (4'x8' sheet, 3/8" thick)	\$265	4	\$1060
Screws (1", Box of 100)	\$2.99	1	\$2.99
Hinges, pack of 2	\$8.69	2	\$17.38
Tygon Tubing (3/16" ID, roll)	\$32	1	\$32
Eye bolts	\$0.39	4	\$1.54
4" Common Nails (price per pound $\approx$ 100)	\$1.99	0.04	\$0.08
Plywood (4'x8' sheet, 3/4" thick)	\$40.06	1	\$40.06
<i>Cooling system</i>			
Plexiglass (4'x8' sheet, 3/8" thick)	\$265	1	\$265
Methylene Chloride (1L)	\$27.10	1L	\$27.10
Chiller	\$1630	1	\$1630
Pumps (Little Giant NK-2)	\$189	2	\$378
Polyethylene Tubing (3/8" ID, 5/8" OD, per foot)	\$0.88	25	\$2.20
Indoor/Outdoor Caulk (tube)	\$5.49	1	\$5.49
Copper Tubing (50' length, 3/8" OD Flexible)	\$23.30	1	\$23.30
Barbed Hose Nipple Adapters (1/4" NPT-3/8" ID)	\$2.29	8	\$18.32
Nipples (6" 1/4" NPT)	\$0.87	6	\$5.22
Nipples (2" 1/4" NPT)	\$0.49	16	\$7.84
Tees (1/4" NPT)	\$3.10	12	\$37.40
Unions (1/4" NPT)	\$12	6	\$72
Plugs (1/4" NPT)	\$0.52	2	\$1.04
Teflon Tape (roll)	\$3.50	1	\$3.50
Rack Supports (1" Angle Iron, price per foot)	\$0.51	3	\$1.53
Thermometer (-38 to 42°C, $\pm$ 0.2°C)	\$37.50	2	\$75
Thermometer (-40 to 50°C, $\pm$ 1°C)	\$8.95	3	\$23.85
Humidity Monitor	\$30	3	\$90
<i>Lighting system</i>			
Mirror (Protected Aluminum)	\$32.80	1	\$32.80
Fluorescent Lamps	\$34	2	\$68
Compact Fluorescent Lamp	\$27	1	\$27
<i>Support Rack</i>			
Aluminum Rods (1/2" d, price per foot)	\$1.00	8	\$8
Steel Rod (750mm)	\$28	2	\$56
Steel Rod (250mm, pack of 2)	\$30	2	\$60
Foot Clamps (Pack of 4)	\$34	1	\$34
90° Clamps (Pack of 5)	\$35	2	\$70
Board Clamps (Pack of 2)	\$32.50	1	\$32.50
Thermometer Clamps	\$16	2	\$32

Note: Pricing information is not given for the Digital Vision System. For this information, see (1).
 Total system price: \$6609.14 plus applicable shipping, handling and taxes.

Appendix L: Discussion of Parameters

Capital letters are commonly used for total bulk phase values. Small letters are commonly used for specific values defined either per unit mass, (such as specific entropy, $s = S/M$ where M is total mass), or per mole, (such as specific molar entropy, $s = S/N$ where N is the total number of moles.)

In dynamic cases, the values are commonly replaced with gradients. Several parameters, such as entropy, enthalpy, height and so on, are meaningless individually. It is the difference between these values that are of interest.

Total internal energy is usually indicated by E_T . Total internal energy is a commonly used variable in energy analysis. It accounts for all gravitational, thermal, chemical and kinetic energies within a system. Total internal energy is dependant on nearly every other parameter.

Temperature is usually indicated by T . Temperature is a macroscopic variable, which describes an average of the microscopic vibration of molecules within the bulk material.

Chemical potential energy is usually indicated by μ_j . The subscript indicates that it is the chemical potential energy of the j^{th} component. Chemical potential energy is a

macroscopic variable, which describes the average reactivity of a chemical species. Care must be taken not to confuse this term with viscosity, which is defined below.

Surface tension is usually indicated by γ . Some authors prefer the use of σ . Superscripts are commonly used to indicate what the surface is between. For example, γ^{SL} is the surface tension of a surface between a solid and a liquid, etc.

Surface tension is a macroscopic variable, which describes either the surface tension (force) which resists surface deformation, or the surface energy (units of J/m^2) required to create new surface.

Viscosity is usually indicated by μ . This usually leads to little confusion, as it is rarely used in energy equations. Some authors, especially in chemical engineering, prefer the use of η for viscosity. Viscosity is a macroscopic variable that describes the resistance of a fluid to shear deformation. It is significantly higher in liquids than in gasses. Care must be taken not to confuse this term with the chemical potential energy, which is defined above.

Kinetic energy is usually indicated by E_K , though it is occasionally written as T , K or KE . Kinetic energy describes the momentum energy of the material. It may be used macroscopically or microscopically, though it is usually applied to the bulk material. Kinetic energy is dependant on velocity.

Velocity is usually indicated by U for bulk fluids, and by V for particles and bodies. Often the average velocity is used, which is indicated by a bar above the symbol. Velocity is the rate of change of position with time. The description of velocity often requires several co-ordinates to accurately describe it in three-space. Care must be taken not to confuse this term with volume, which is defined below.

Potential energy is usually indicated by E_P , though it is occasionally written as U , P or PE . Potential energy describes the gravitational potential energy of the material. It may be used macroscopically or microscopically, though it is usually applied to the bulk material. Potential energy is dependant on height.

Height is usually indicated by Z . Often the average height is used, which is measured from the centre of mass. Height is position above a reference. The difference in height between droplet and impact plate is usually the height of concern for falling droplets.

Entropy is usually indicated by S . Entropy is a macroscopic variable, which describes disorder within the bulk material.

Pressure is usually indicated by P . Pressure is a macroscopic variable, which describes compression within the bulk material.

Volume is usually indicated by V . Volume is a macroscopic variable, which describes the dimensions of the bulk material. Care must be taken not to confuse this term with velocity, which is defined above.

Number of moles is usually indicated by N . Number of moles is a macroscopic variable which describes the quantity of molecules within the bulk material.

Appendix M: Overview of Mathematical Formulations

An interesting mathematical manipulation occurs in the change from an energy minimum formulation to the entropy maximum formulation of the fundamental equations. This manipulation was best described by Callen, in his work: *Thermodynamics*. (1962) Therein, he demonstrates this manipulation for several relations, including:

Euler Equation:

$$S = (1/T)U + (P/T)V - \sum_{k=1}^r (\mu_k/T)N_k \quad [\text{M.1}]$$

Gibbs-Duhem relation:

$$U d(1/T) + V d(P/T) - \sum_{k=1}^r N_k d(\mu_k/T) = 0 \quad [\text{M.2}]$$

The Euler equation is a relatively simple property relation that is derived from the definition of the homogeneous first order property:

$$U(\lambda S, \lambda X_1, \dots, \lambda X_N) = \lambda U(S, X_1, \dots, X_N) \quad [\text{M.3}]$$

The form most commonly used is that simplified for a simple compressible system which appears as:

$$U = TS - PV + \mu_1 N_1 + \dots + \mu_r N_r \quad [\text{M.4}]$$

This yields a relatively simple to use energy balance that does not require derivations, and can generate information on a well-known system. It is common to require more information with this equation than is experimentally available, however, and so boundary conditions must often be placed.

The Gibbs-Duhem relation is another energy balance that is often useful when a second relation between the intensive parameters is required. Integrating the Gibbs-Duhem relation can provide sufficient information to replace the use of one of the three equations of state that are needed to generate the Euler equation. By taking the derivative of a formulation of the Euler equation, and simplifying using a general relation:

$$dU = T dS - P dV + \mu dN_1 + \dots + \mu dN_r \quad [\text{M.5}]$$

It can be rapidly determined that, for a simple, one component system:

$$S dT - V dP + N d\mu = 0 \quad [\text{M.6}]$$

Another very powerful use of the Gibbs-Duhem relation, however, is the isolation of the chemical potential energy, in the following form:

$$d\mu = -s dT + v dP \quad [M.7]$$

This form permits analysis of the chemical potential energy in differential form, thus yielding a powerful tool for setting up an analysis of a system.

The work of Callen also discusses the use of Legendre transforms. These transforms can separate the intrinsic variables, forming several powerful descriptions:

- Helmholtz Potential (Replaces S by T as independent)
- Enthalpy (Replaces V by P as independent)
- Gibbs Free Energy (Replaces S by T and V by P simultaneously)
- Grand Canonical Potential (For a single-component system)

Of these formulations, for the investigation herein, the most interesting is the Grand Canonical Potential, which has the following form:

$$dU [T, \mu] = - S dT - P dV - N d\mu \quad [M.8]$$

This formulation proves to be convenient in several instances.

Appendix N: Analysis of a Related Work

N.1 Scope of Article

In the research required for the design of the system, an article was found that was of some interest. The article, by Ward and Sages (p.3651, JCP 109,1998), describes the effect of gravity on contact angle. Furthermore, it summarizes the analysis of a droplet under normal gravity conditions. This analysis made the article of interest. Some issues were discovered during the analysis of the mathematics that made the results of mention to future investigators intending to use this information.

N.2 Application of Theory

Using the analysis found in this article, spreading, splashing and freezing phenomenon can be extrapolated to cases with varied gravity, such as high-altitude analysis, or spacecraft design. Cases with effects that simulate gravity, such as centripetal acceleration, can be similarly analyzed.

The analysis of the equilibrium of a droplet appears very useful for analyzing spreading flows. Most of the work deals with equilibrium conditions, however, and is thus not largely useful for direct analysis of splashing flows.

N.3 Analysis of Theory

The article outlines three conditions of equilibrium; Young's equation, Laplace's equation and a relation between the chemical potentials of the fluid components and the gravitational intensity.

The article uses the conditions of equilibrium to determine the effects of having a zero-gravity environment, and a relation to determine the difference in contact angles between a liquid above a gas and a liquid below a gas. The theory is supported by experimental data where possible.

The use of the fundamental equations to verify experimental conditions, and predict new conditions is an excellent idea. Investigators into the interaction between splashing and freezing should perform similar methodology in theoretical analysis.

N.4 Analysis of Mathematics

Several questions remain as to the mathematics contained within the article. As correspondence is still ongoing, comment will be withheld as to the veracity of the analysis within the article. The following, however, has been noted:

- The discussion of line tension on p.3652 speculates on a modification to accepted equations, but does not exemplify such modifications. It is difficult

to see exactly what terms the author is considering adding, and hence what their effects would be on the other equations.

- The diagram on p.3653 used to describe the variables includes assumptions that are only valid for virtual displacements, but the associated work uses integrations of the displacements that could violate these assumptions.
- Numerous subscript and superscript errors are discovered in analysis of the work, such as:
 - Equation 4,5: T should read T^j from the identities in equation 25.
 - Equation 17: z should read z_i for the identity of potential energy.
 - Equation 24: n_i should read n_i^j from the substitution from equation 13.
 - Equation 32: The second ϕ_b should be a z_b .
- The derivation of equation 30 involves breaking up the $d(R_1 x_j)$ term in equation 26, yet the given substitutions cannot perform such an action.
- Equation 44 relies on the presumption that equation 1 yields a constant value if taken on either side of the free surface. That is to say, the chemical potential is the same in the liquid as in the gas. This assumption may not be widely applicable.

- The derivation of equation 53 yields a format that is not theoretically intuitive. While the results may hold empirical significance for a given test cell, they are difficult to compare to other works. Equating $2\gamma^{LV}$ may have been a more intuitive and useful methodology.
- Following the substitutions involved in generating equation 70 leads to a $+n^{SV}s^{SV}$ term, not a $-n^{SV}s^{SV}$ term.

Appendix O: Laboratory Procedure

O.1 Operational Hazards

The laboratory environment, as it was designed to be operated, poses very few hazards to the investigators. Few of the materials used have any significant toxicity, and none pose critical hazard. The system itself was designed not to be fragile, however improper handling of the system can damage components.

The coolant fluid was contained, and there should be little need for direct contact, however should the need arise, the fluid is not hazardous to touch. Replacement fluid should remain available to replace any evaporation losses or to replace the fluid if it somehow becomes contaminated significantly. The pumps are fairly robust, and the system displays a low reactivity, so contamination of the coolant is not a major concern.

The plate surface is non-reactive, as well as the cooling rack. If the test liquid is varied, it is recommended that a “splash guard” be placed beneath the rack if the liquid is reactive to the styro-foam insulated open-topped box, or the plexi-glass beneath. Test samples of the styro-foam and plexi-glass are available in the lab, and should always be tested with new operating liquids. The simplest recommendation for a splash guard would be to purchase an 11”x 17” Teflon cookie pan. The Teflon surface displays exceptionally low reactivity, the splash should be contained within its area, and the rack was designed to fit inside one. The lip will contain any fluid, and if the splashing area is

wider than the pan, two pans can be used, with one rack support inside each, using a teflon tape to seal the joint between the pans.

The polyethylene tubing used to pump the coolant fluid and the test liquid shows low reactivity. To ensure that any operating fluid does not violate their integrity, the reactivity table contained in the laboratory should be consulted. It was located within the documentation associated with the Masterflex pump system. Additionally, a small test piece of tubing should be placed within a small test volume of the liquid and given a 24 hour period to react. If no noticeable reaction has occurred, the new liquid may be used. If a reaction has occurred, use the reactivity table to select another tubing material.

The Masterflex pump system is highly modular, and nearly any soft tubing material of appropriate diameters may be used. It is highly recommended that the tubing used to pump the coolant not be changed, as doing so would require draining the coolant system, cleaning it thoroughly with isopropyl alcohol and scrapings, then re-sealing it with the new tubing while the system is dry. Use at least two layers of silicone caulking if possible, and permit ample drying time between layers.

To maintain integrity of the seals, do not apply loads to the silicone seals. Avoid moving the coolant reservoir while full, as the seals at the corners may not hold under such heavy loads.

The coolant freezing point is listed as -35°C . If the coolant fluid freezes, the expansion of the water may damage the coolant reservoir. In order to prevent this, the lower limit function of the chiller may be used. This value will prevent the chiller from operating if the thermocouple reads a value below the set limit. The thermocouple was placed at the bottom of the tank, at approximately the same level as the pump intakes. In operation, there has not been a problem with such freezing.

If for any reason the coolant reservoir requires draining, use the following procedure:

- a) Unplug both pumps, and turn off chiller.
- b) Open containment box and remove to storage any plates.
- c) Place a wooden stool within the box, and rest one of the empty coolant storage boxes on it. Position it so that it rests under the junction of the cooling coil and the polyethylene tubing.
- d) Open the cap of the coolant storage box, and pull the mouth out from the box.
- e) Carefully separate the polyethylene tubing from the cooling coil by pulling them apart and twisting.
- f) Quickly place the polyethylene tubing within the mouth of the box.
- g) Plug in the coolant coil pump. The cord can be identified by the labels located at the point where the cords enter the coolant reservoir.
- h) When the box is nearly filled, unplug the pump, and close the cap of the coolant storage box.

- i) Remove the coolant box and return to d) if required. Normally, four boxes of coolant fluid are used.
- j) To replace the fluid, simply lift the lid and use the “tap” fittings on the storage boxes. They may be rested on the corners of the reservoir, and then the taps opened to permit them to pour into the reservoir.

When handling the chiller coil, do so carefully. Do not permit it to strike objects, especially if cold. If it is to be reshaped, permit it to return to room temperature before doing so whenever possible.

The thermocouple in the coolant reservoir is sealed in place, and should not be removed. If it must be removed, drain the coolant reservoir and use care in its removal.

The coolant coil can be reshaped as necessary, however permit the copper to warm to room temperature first. Furthermore, drain the coil if possible, as it will make the coil easier to handle.

The rack assembly is rugged, and may be adjusted as necessary. Simply disconnect and drain the rack before modifying it, and re-cover any threads that are loosened with teflon tape before re-tightening them. Remember to begin by opening the unions if segments are to be removed.

The containment box doors are easily removed by pushing up the pins with one of the nails normally used to fasten the doors.

If the containment box requires disassembly, be certain to have at least two people available, as well as a drill with a Red Robertson (square) screwdriver attachment. Furthermore, disassemble and remove all interior components first before commencing the disassembly.

O.2 Operational Methods

While the operational hazards describe methods to use that avoid damage to the system or investigator, the operational methods describe methods for improving the quality of the results.

When placing the test plate, place the test plate onto the support plate, and the support plate onto the coolant rack.

For freezing analysis, chill samples overnight in a common refrigerator freezer. This ensures their temperature will be well below freezing, and gives a greater temperature gradient than the coolant system alone. The coolant system will assist in keeping the sample from warming up as quickly.

The cooling transient of the chiller and rack is very slow. Once the system has warmed, it takes several hours to cool to minimum temperature again. For this reason it is recommended that the system be allowed to cool over night. The warming transient is substantially quicker. With the coil active, the reservoir warms in minutes. Without the coil active, however, the system stabilizes to a plate temperature of between -6.7°C and -7.3°C when the ambient air temperature is 20°C . Thus, the coil should only be used for transients where the air temperature in the containment box is of importance. For most tests, leaving the chiller and rack operating continuously will be sufficient. The rack, however, tends to ice over during such operation. To de-ice it, merely turn off the rack pump. When it de-ices, turn on the rack pump again, and leave 15 to 20 minutes for the system to re-equalize.

Use the Matrox Inspector software only for capturing sequences of images. The Matrox Inspector software cannot be used to provide real-time viewing of the system. When capturing a sequence, do not click on the window containing the sequence before saving the sequence. Use the save function instead. Matrox Inspector will revert that sequence to only the last frame if the window is clicked on. Save the sequence of images as an AVI so that the sequence can be viewed with any appropriate viewer.

Use the Intelcam software for capturing only single images. The Intelcam software can also be used to provide real-time viewing of the system. This function is very useful for positioning the plate and for zooming and focussing the lens.

Realistically, a video sequence can only be taken of a single droplet impact. This is due to the memory requirements of the computer. In order to obtain a decent frame rate, the sequence must be stored in system RAM. The frame rate is limited by the camera hardware, and the Digitizer Control Format (.dcf file). The maximum frame rate currently possible is approximately 17 frames per second. This translates to one capture every 59ms. To image multiple droplet impacts, image one drop, then save the sequence. Begin a new capture, and capture the second drop. With the .0075" ID tubing, the droplets impact in approximately 25s intervals. The results of testing this tubing can be found in Appendix C: Test of 0.0075" ID Tubing. Similar results occur with the 0.099" ID tubing, though it tends to a droplet formation interval of 20s. If this time is insufficient to set up the captures, turn down the pump speed, or stop the pump between droplets.

To ease the digital processing, capture an image of the plate before the first droplet, then after each droplet impact. These images can then be subtracted to show the effects of each drop.

To calibrate the imaging magnification, capture an image of an opened set of calipers. The magnification of the system can be easily obtained by comparing the distance on the image to the known distance on the calipers.

Attempt to fill the image as much as possible with the test plate. The more pixels in size an object is, the less imaging error will play a role in the final results.

O.3 Operational Procedure

O.3.1 Setup Procedure

The general setup procedure for operating the system is simple. Very little of the setup has a critical order, so steps may mostly be performed in any convenient order.

- Turn on chiller power bar, chiller, and plug in rack pump.
- Set chiller set point to desired temperature. (Usually -60°C)

The chiller and rack pump should be permitted to operate continuously. If they are turned off, the cooling system will require several hours to cool the reservoir.

- Turn on computer power bar, then computer itself.
- Turn on camera power bar, then camera power supply.
- Turn on light power bar.
- Log on to computer, select Intelcam.
- Select Genesis board operation.
- Load .dcf file
- Set to continuous grab mode.
- Remove lens cover from camera lens.

- Zoom and focus camera lens.
 - If the lens will not focus, use the slider to move the camera support.
 - If the picture will not appear, check the f-stop.
 - If the rack will not appear on the screen, adjust the mirror angle
- Obtain test plate. (From dust-free storage, or refrigerated storage)
- Open containment box by drawing out pins and opening door.
- Place test plate on rack.
- Adjust thermometer so that it touches test plate.
- Adjust mirror if necessary.
- Adjust lighting if necessary.
- Close containment box by closing door and inserting pins.
 - If difficulties arise in fitting the door, try opening other door and closing both doors simultaneously.
- Allow system to stabilize.
- Save one image as TestN0.mim in a folder named with the current date.

O.3.2 Single Droplet Analysis

A single droplet may be analyzed by either single or sequence imaging. Single imaging takes one image of the droplet at the time of selection. Sequence imaging takes a series of images a set time delay apart, commencing at the time of selection. Single

imaging can conceivably be used several times on a single impact, but due to the time constraints, only three types of images can realistically be taken.

An “impact” image can be obtained just after impact. This image should demonstrate the initial impact location and should capture the droplet breaking open. This image is taken by saving the image as soon as the impact is observed. The image will then be taken at a time equal to the reaction speed of the individual. While this may seem absurdly imprecise, it can actually be calibrated statistically with a simple experiment outlined in the first year engineering physics course. (75.101 Lab Manual, 1994)

A “spread” image can also be taken once the droplet has first stabilized. This image should demonstrate the maximum diameter of spreading. This image is selected once the droplet has impacted, and the image is no longer changing rapidly.

A “rest” image can also be taken once the droplet has finally stabilized. This image should demonstrate the rest position of the droplet. This image is selected once the droplet has finally come to rest. This may take as much as 20 seconds in pure splashing experiments with water.

By analyzing the differences between the impact and spread images, the spreading dynamics can be seen. By analyzing the differences between the spread and rest images, the restoring forces can be seen.

A sequence of images is probably more useful for analyzing a single drop. As the images can be taken 59 ms apart, the first image will show the droplet at no more than 59 milliseconds from impact. Further images at regular intervals permit the analysis of the droplet evolution. Edge velocities can be calculated by comparing measurements.

O.3.3 Multiple Droplet Analysis

Analyzing multiple droplets is essentially similar to analyzing a single drop. Due to the large amount of data, however, single imaging is more likely to be of use than sequence imaging. By maintaining consistent imaging for each droplet, the effect of cumulative droplets can be examined. In pure splashing measurements, this allows for liquid-liquid impacts, at various surface angles and curvatures as the liquid drop on the plate expands. In the splashing and freezing measurements, this allows for an effective increase in the roughness of the plate, as well as an increased area for heat transfer.

O.3.4 Single Imaging

To obtain a single image, perform the following steps once the computer is on-line and in two-monitor mode and the camera is on:

- 1) Start Intelcam.
- 2) Select card: Genesis Dig/Mod.

- 3) Move the Intelcam window to the Genesis monitor. (Right monitor)
- 4) Select Continuous Grab. (This can be found in the Digitizer menu)
- 5) Load DCF: Continuous.dcf. (Browse: There are several backup copies. Use the one in C:\, or the one in M:\)
- 6) The screen will display the camera view. Focus the camera appropriately.
- 7) Right click on the camera view and select Save As, or stop the image, and select Save As. (File menu)
- 8) Save the file appropriately. (.mim or .jpg)

O.3.5 Sequence Imaging

To obtain a sequence of images, perform the following steps once the computer is on-line and in two-monitor mode and the camera is on:

- 1) Start Matrox Inspector 3.2.
- 2) Select Video Card: Genesis.
- 3) Move the Inspector window to the Genesis monitor. (Right monitor)
- 4) Select Sequence. (File Menu, Grab submenu)
- 5) Indicate save to computer memory.
- 6) Enter number of frames. (17 frames yields 1s of imaging at fastest imaging)
- 7) Enter time between frames. (59 ms or more. Enter 0 to select fastest imaging)
- 8) Click next.
- 9) When ready to image, click Start.

- 10) DO NOT CLICK ON IMAGE.
- 11) Select Save As. (File menu)
- 12) Save the file appropriately. (.avi)

O.3.6 Image Processing

To process an image from captured data to mass distribution, the following procedure may be used:

- 1) Capture image and save as .mim so as not to lose information.
- 2) Start Matrox Inspector, selecting VGA operation.
- 3) Move the window to fill the left (VGA driven) monitor display.
- 4) Open the .mim file.
- 5) Perform the Sobell Filter Operation. (Tools Menu, Filter. Sobell is in the given list.)
- 6) Perform a Negation Operation. (Tools Menu, Arithmetic. Negate is in the given list.)
- 7) Perform a Subtract 10 Operation. (Tools Menu, Arithmetic. Subtract is in the given list, the value to be subtracted must be changed from the default of 0.)
- 8) Perform the Blob Analysis function on the image. A set-up blob analysis is available as M:/Blob.bst
- 9) Save the Result as a text file Result N.txt where N is the .mim image name.

- 10) Close Matrox Inspector and Open Microsoft Excell. An image processing worksheet was created as M:/Blob.xls
- 11) Open Result N.txt in Wordpad.
- 12) Cut everything from Wordpad except the top line.
- 13) Paste into Microsoft Excell in the first four columns of the worksheet.
- 14) The worksheet also requires values for an initial drop. It is recommended that average values of the first droplet of several tests in a given configuration be used to minimize error.

Images demonstrating the effect of the various image processing functions can be seen in Appendix O: Effect of Image Processing Operations. These images make the reasons for using them obvious.

O.3.7 Shutdown Procedure

Very little about the shutdown procedure is time-critical or order-critical. The system was designed to be rugged, and it was designed for continuous operation.

- Open containment box by drawing out pins and opening door.
- Adjust thermometer so that it no longer touches test plate.
- Remove test plate, and place on counter.
- Clean any excess test liquid and condensed water from containment box.

- Close containment box by closing door and inserting pins.
 - If difficulties arise in fitting the door, try opening other door and closing both doors simultaneously.
- Allow test liquid to melt, then dry and clean with an isopropyl alcohol wash.
 - Note that the test plates provided are glass coated with gold. Gold was selected specifically so that it does not produce an oxide layer, thus the plate may be left to war and dry for long periods, IE: overnight.
- Place plate within a dust enclosure. (A ziplock bag or such will suffice)
- Place lens cover on camera lens.
- Turn off light power bar.
- Turn off camera power supply, then camera power bar.
- Turn off computer, then computer power bar.

If the chiller is to be turned off as well, continue. Note that the chiller should be left operating if the system is to be used the following day.

- Turn off chiller.
- Turn off chiller power bar.
- Unplug rack pump

Appendix P: Effect of Image Processing Operations

To demonstrate the effects of the various image processing operations used, images of each stage are given here. These images are .jpg images of the operations, and so quality is slightly below what can be expected during actual analysis. Darker images have been brightened and sharpened by 20% so as to allow details to be printed. In the actual procedure, the Initial Image Capture and Initial + Sobell images are nearly black.

Fig.P1 Initial Image Capture

This is the original image as it was captured by the Digital Image System. No image processing has yet taken place.

Fig.P2 Image + Sobell

This is the image after the Sobell operation has occurred. The Sobell operation is a derivative operation. It replaces rate of change of value in the original as value in the current. Thus areas with high rate of change are given high (light) values. This has the effect of making the edges bright, and the background black.

Fig.P3 Image + Sobell + Negate

This is the image after the Negate function is then used. The Negate function simply reverses the grey scale. Light areas become dark, and dark images light. This allows the noise points to be clearly seen as black dots.

Fig.P4 Image + Sobell + Negate + Subtract 10

This is the final step of the image processing, where the Subtract function is used. This function subtracts a constant value from all pixels. On a 256 scale, a constant of 10 is used to clear the noise. Dark points that drop below zero roll back to 256, and are lost in the bright background. The subtraction thus buries the digital noise. Using a larger constant would excessively degrade the edge data.

Fig.P5 Blob Analysis

The image analysis software fits lines to contours in the data. By selecting a fitting criterion that includes large differences, the noise points remain buried, while the edge data is fit. The analysis software can then be used to give various information about the contoured areas.