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OPTIMUM DESIGN OF INTERNAL FLOW PASSAGES

WITH SPECIFIC REFERENCE TO WIND

TUNNEL CONTRACTIONS

by

Mikhail Nageb Mikhail

B.Eng., B.Sc., M.Eng.

A Thesis Submitted to the Faculty of Graduate
Studies in Partial Fulfillment of the
Requirements for Ph.D. in Engineering

Department of Mechanical and Aeronautical Engineering
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October 1976

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The undersigned hereby recommend to the Faculty of Graduate Studies and Research, Carleton University, acceptance of the thesis entitled, "Optimum Design of Internal Flow Passages with Specific Reference to Wind Tunnel Contractions", submitted by M.N. Mikhail in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Mechanical Engineering.

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ABSTRACT

A study aimed at developing a design method for contracting ducts for low speed wind tunnels is presented. The optimum shape for a contraction has been defined as that which avoids separation at entry and, using the shortest possible length, satisfies the flow quality requirements in the test section.

A numerical scheme has been developed based on the method of lines, for the solution of the governing inviscid flow equations in an axisymmetric duct of arbitrary shape. A computer code based on that scheme has been built and tested, and is used in the present study to calculate the flow inside a geometrical model of a settling chamber, contraction, and test section combination.

The choice of a contraction contour shape is based on the hypothesis of the possibility of controlling the flow characteristics inside a duct by varying the duct wall curvature distribution. This hypothesis proved to be successful.

It has been shown that by optimizing the duct curvature distribution, it is, in fact, possible to reduce the contraction length to about one half of that presently used in practice, while maintaining the high quality of test section flow required by modern aerodynamic testing. The results show that a contraction with an area ratio of eight with conditions typical for modern tunnels can be as short as one inlet radius.

Both Stratford's criterion for turbulent boundary-layer separation and a computer code based on a lag-entrainment method of boundary-layer calculation have been used to check on the viscous flow behaviour in the contraction. It is found that while the contraction inlet is a region of possible boundary-layer separation if the contraction is too short, the boundary-layer flow is always able to overcome the adverse pressure gradient near the contraction exit, and that region is quite safe from possible boundary-layer separation (provided re-laminarization does not occur).

The effects of the contraction ratio, the Reynolds number scale, and the boundary-layer origin on the contraction performance and its optimum length were investigated and are presented. It is found that, at low Reynolds number scale conditions, the possibility of boundary-layer re-laminarization represents severe limitations on the contraction length. In the range investigated ($4 < CR < 12$), the optimum length for the contraction is found to decrease as the contraction ratio increases.

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M.N. Mikhail

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NOMENCLATURE

A Geometrical parameter

B Geometrical parameter

C_f Local skin friction coefficient

$C_f = \tau_w / (\frac{1}{2} \rho U^2)$, where τ_w is the wall shear stress.

C_p Pressure coefficient $C_{p1} = \frac{p - p_1}{\frac{1}{2} \rho U_1^2} = 1 - (Q_w)_1^2 / U_1^2$

and $(C_p)_e = 1 - (Q_w)^2 / (Q_w)_{\max}^2$.

CR Contraction ratio (area ratio of the contracting duct).

H Boundary layer shape factor

K Vorticity parameter as defined in Section 3.1. Also acceleration parameter defined on page 74.

k Aerodynamic loss factor for screen (see page 5).

L Non-dimensional contraction length*, $L = \ell / R_1$, $L = L_1 + L_e$

L_1 Non-dimensional contraction inlet length*, $L_1 = \frac{x_{1p} - x_1}{R_1}$

L_e Non-dimensional contraction exit length*, $L_e = \frac{x_e - x_{1p}}{R_1}$

ℓ Contraction length

ℓ_1 Contraction inlet length*, $\ell_1 = x_{1p} - x_1$

ℓ_e Contraction exit length*, $\ell_e = x_e - x_{1p}$

MS Margin of safety for boundary layer separation, defined on page 79.

m, N, n-Geometric parameters

P Non-dimensional pressure $P = \frac{p}{\frac{1}{2} \rho U_1^2}$

p Flow static pressure

Q Non-dimensional flow speed, $Q = q / U_1$

NOMENCLATURE (Cont'd)

q Flow speed

R Contraction radius

R_e Reynolds number

(RH-LH) Right hand side minus left hand side of Stratford's criterion for turbulent separation (Eqn. 5.1).

r Radial co-ordinate

S Distance along the contraction contour

S_0 Distance from the boundary-layer origin to the contraction inlet

U Non-dimensional axial velocity component, $U = \frac{u}{U_i}$

u Velocity component in the axial direction.

V Non-dimensional radial velocity component, $V = \frac{v}{U_i}$

v Radial velocity component

W Non-dimensional flow component in the circumferential direction, $W = \frac{w}{U_i}$

w Circumferential velocity component.

X Non-dimensional axial distance*, $X = x/R_1$, measured from inlet to settling chamber unless otherwise specified.

x Axial co-ordinate*

Z Non-dimensional axial distance*, $Z = \frac{x - x_1}{l}$, $0 < Z < 1$

Z_1 Non-dimensional axial distance*, $Z_1 = \frac{x - x_1}{l_1}$, $0 < Z_1 < 1$

Z_2 Non-dimensional axial distance*, $Z_2 = \frac{x - x_{1p}}{l_e}$, $0 < Z_2 < 1$

δ Boundary layer thickness

θ Boundary layer momentum thickness

ρ Flow mass density

ν Kinematic viscosity

- Ψ Non-dimensional Stokes stream function, $\Psi = \psi / U_1 R_1^2$
 ψ Stokes stream function
 ξ Transformed axial co-ordinate, $\xi = X$
 η Transformed radial co-ordinates, $\eta = \frac{r}{R_w}$
 ΔU Flow uniformity, defined as velocity deviation from its nominal value presented as percent of the nominal value
-

* See illustrations of Fig. 14(a) and Fig. 14(b)

SUBSCRIPTS

- c Along centreline
 e At contraction exit
 i At contraction inlet
 ip Geometrical inflection point
 o Test section conditions
 r First derivative in radial direction
 rr Second derivative in radial direction
 s Settling chamber conditions
 w At contraction wall
 x First derivative in axial direction
 xx Second derivative in axial direction
 η First derivative with respect to η
 $\eta\eta$ Second derivative with respect to η
 ξ First derivative with respect to ξ
 $\xi\xi$ Second derivative with respect to ξ
 $\xi\eta$ Second mixed derivative with respect to both ξ and η

SUPERSCRIPTS

() Number in parenthesis indicates number of
differentiation

First derivative with respect to Z

Second derivative with respect to Z

2

1. INTRODUCTION

1.1 WIND TUNNEL CONTRACTION

Wind tunnels usually have a contracting duct fitted upstream of their working section. The existence of these contractions is intended to serve the very important purpose of ensuring uniform, steady, low turbulence flow to the test section. Figures 1(a) and 1(b) respectively show a picture and an airline diagram, to illustrate the design of a modern large low-speed wind tunnel (Ref. 1).

The importance of the contracting duct as an element of a wind tunnel circuit is discussed in the following sections. The choice of the area ratio of the contraction is usually based on economical considerations. The factors affecting the choice of the contraction ratio are discussed below. Once the contraction ratio is selected the contraction design problem reduces to a choice for both the contraction length and the contraction contour shape. Since the contraction is a complicated and costly item to manufacture, it is obvious that its length should be kept as short as possible, subject to a shape being chosen that will ensure uniform, steady flow in the test section.

The use of a shorter contraction represents direct cost savings corresponding with savings in both the tunnel circuit length and tunnel shell area. It should be noted that a shorter contraction, in many cases, would enable a larger first diffuser area ratio and consequently, a larger area for the first corner of the return circuit wind tunnel, which is very beneficial to circuit efficiency.

The present thesis addresses the problem of optimum contraction design. An optimum contraction is defined as the shortest possible contraction to satisfy specified requirements of test section flow quality.

1.1.1 EFFECT OF CONTRACTION ON TUNNEL FLOW

The existence of a contracting duct upstream of the test section reduces spatial irregularities which may exist in the main flow, and reduces the relative turbulence intensity in the airstream. Equally important, the existence of a contraction maintains the airspeed in most of the wind tunnel circuit substantially lower than the speed in the test section. This results in lower aerodynamic losses in corners and passages and permits the installation of a heat exchanger and anti-turbulence devices such as screens and honeycomb with low pressure drop penalties.

The sources for spatial non-uniformity of the tunnel mean flow could be (Ref. 2):

- i) Poor corner vane design.
- ii) Boundary-layer growth in the return circuit, and separations caused by an excessive diffuser angle; or excessively short contraction.
- iii) Poor fan or straightener-vane design.

With the trend towards diffusing the flow rapidly, just upstream of the settling chamber, the wide-angle diffuser could act as an important source of flow non-uniformity unless carefully designed.

The existence of a contracting duct, with consequent increase in the mean speed smooths out the flow irregularities. In fact, Prandtl (Ref. 3) was the first to point out that decrease in the longitudinal flow irregularities can be obtained by considerations of the Bernoulli equation. Although the flow is irregular, and therefore rotational, we can apply the Bernoulli equation separately for each streamline, i.e. $u_2^2 = (2/\rho)(p_1 - p_2) + u_1^2$. The pressure drop or the energy gain is the same for all streamlines;

therefore, the change in the longitudinal velocity from streamline to streamline is $u_2 \delta u_2 = u_1 \delta u_1$ or $\delta u_2 / \delta u_1 = u_1 / u_2 = 1/CR$ (Ref. 4). This means that the longitudinal velocity irregularities expressed as a fraction of the mean velocity is reduced in proportion to the square of the duct contraction ratio.

The contraction effect on the free stream turbulence has been the subject of a number of studies, as we see, for example, in References 2 - 10. Again, it was Prandtl (Ref. 3) who - with great physical insight - was the first to consider the effect of a contracting stream on the components of the velocity fluctuations. In a semi-quantitative theory, Prandtl showed that the contraction exerts a selective effect on the components of turbulence. While the longitudinal component expressed as a fraction of the mean velocity reduces in proportion to the square of the contraction ratio, the transverse components - again expressed as a fraction of the mean velocity - is only reduced in proportion to the square root of the contraction ratio. One would expect that turbulence isotropy would be recovered in the test section.

1.1.2 CHOICE OF CONTRACTION RATIO

For a certain wind tunnel, the test section size, range of air-speeds and required flow quality, are usually defined by the kind of tests speculated for that tunnel. It is clear that the above requirements can be achieved by an infinite number of geometric varieties for the tunnel circuit, and optimization of the tunnel geometric parameters to achieve the best economy is necessary. One of the most important parameters for such optimization studies is the area ratio of the contracting duct, the contraction ratio (CR):

From an economic point of view, increasing the contraction ratio has two opposite effects. On the one hand, higher construction cost is associated with higher contraction ratio due to the increase in the circuit size and, consequently, higher shell and foundation costs. On the other hand, increasing the contraction ratio reduces the power required to drive the tunnel significantly. A reduction in the required driving power represents savings in both capital cost of the drive and in tunnel running cost.

Generally speaking, increasing the contraction ratio, increases the tunnel circuit physical dimensions which result in lower flow dynamic pressure and, consequently, lower pressure losses all around the circuit. Moreover, higher contraction ratio, not only reduces the losses across each turbulence screen, but also, requires fewer screens to achieve the same flow quality. Table 1.1 (Ref. 11) gives some estimates of screen power losses for various contraction ratios with screens chosen to reduce an entering airflow of 6% turbulent intensity (a typical figure for a return circuit wind tunnel) to a working section flow with 0.15% turbulence.

The sketch of Figure 2 represents the expected trends of the effect of the contraction on both tunnel shell cost and drive cost. An optimum value for the contraction ratio at which the total capital cost is a minimum as shown in Figure 2, can usually be found.

TABLE 1.1

Some estimates of screen power losses for various contraction ratios with screens chosen to reduce an entering air flow of 6% turbulent intensity to a working section flow of 0.15% turbulence (Ref. 11).

Contraction Ratio	Required No. of (k=1) Screens*	Turbulence Intensity % in Working Section	<u>Power Required With Screens</u> <u>Power Required Without Screens</u>
5.0	5	0.13	2.05
7.5	4	0.12	1.37
10.0	3	0.13	1.18
12.5	2	0.14	1.09
15.0	2	0.12	1.05

* Losses in screens can be expressed as:

Aerodynamic loss = $k\bar{q}$, where $\bar{q}$ is the dynamic head of the approaching flow

1.2 AVAILABLE CONTRACTION DESIGN METHODS

It may appear that there is no problem in designing even a short duct to accelerate the air stream; indeed this would be the case if the air is accelerating continuously, but an elementary argument can show that due to the wall curvature, regions of adverse pressure gradient have to appear near the wall at both inlet and outlet of a finite length contraction. If the adverse pressure gradient is allowed to become severe enough to cause boundary-layer separation, this could degrade the flow quality in the test section.

To avoid boundary-layer separation, in the absence of quantitative methods of judgement, longer and longer contractions have been used; a situation similar to the use of a high factor of safety (or factor of ignorance as it is sometimes called) to account for the lack of knowledge in many engineering design areas.

Due to the absence of a satisfactory method of contraction design, existing contractions have been designed either by eye, or by applying necessary modifications to one of the available methods, modifications which are unlikely to rise above the level of "intelligent" guesswork. Either way the final design depends on the ingenuity and experience of the designer and the design process is more art than science.

By reviewing the available methods of contraction design, one faces the surprising fact that most of the effort has been directed towards the mathematical solution of the inviscid flow equations with little regard to the physical conditions of a real wind tunnel contraction.

The studies available in the literature are concerned with both two dimensions and axisymmetric contractions. Since two-dimensional contractions are of little interest, only the axisymmetric contraction design

methods are reviewed in the following section.

In an axisymmetric duct with irrotational, incompressible, inviscid flow, Stokes stream function ψ - as will be shown later in this report - has to satisfy the Stokes-Beltrami equation: (Ref. 12):

$$\psi_{xx} + \psi_{rr} - \frac{1}{r} \psi_r = 0$$

where the velocity components are related to the stream function as:

$$u = \frac{1}{r} \psi_r$$

$$v = -\frac{1}{r} \psi_x$$

Applying the "separation of variables" technique for the solution of partial differential equation - see Lamp (Ref. 12) and Bossel (Ref. 13) - it can be assumed that:

$$\psi(r, x) = f(r) g(x)$$

Depending on the choice of solution constant, we obtain different types of solutions. In general, three possible series solutions may be constructed:

$$\psi_1(r, x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} F_0^{(2n-2)}(z) r^{2n}}{2^{2n-2} n [(n-1)!]^2}$$

$$\psi_2(r, x) = \frac{r^2}{2} + r \sum_{n=1}^{\infty} J_1(M_n r) \left[a_n \exp(M_n x) + b_n \exp(-M_n x) \right]$$

$$\psi_3(r, x) = \frac{r^2}{2} + r \sum_{n=1}^{\infty} I_1(M_n r) \left[c_n \sin(M_n x) + d_n \cos(M_n x) \right]$$

The J_n are the ordinary, and the I_n the modified Bessel functions, M_n 's are eigen-values while $F_0(x)$ is the velocity distribution on the axis.

Solution ψ_1 is restricted to families of velocity distributions on the axis that permit analytical expression of their derivatives. The velocities at entrance and exit - a finite distance apart - are neither uniform nor parallel. While solution ψ_2 is aperiodic, solution ψ_3 is periodic in the axial direction. The assumption $c_n = 0$ results in parallel flow at entrance and exit, but the velocity distribution at these stations is necessarily non-uniform. A uniform velocity distribution could be achieved by choosing $d_n = 0$ at the cost of non-parallel flow at entrances and exit. The fact that the entrance and exit velocity distributions in a contraction cannot be both uniform and parallel if entrance and exit are a finite distance apart, is a basic result of potential flow theory and not of particular solutions used (Ref. 13).

Thwaites (Ref. 14, 1946) assumed that the flow velocity potential is constant on each of the inlet and exit planes of the contraction, and hence for a contraction whose length is π - relative to inlet radius of 1 - he obtained the periodic solution:

$$\psi = \frac{1}{2} d_0 R^2 + R \sum_{n=1}^N \frac{d_n}{n} \cos(nx) I_1(nR),$$

which is a special case of solution ψ_3 obtained above. The coefficients d_n for $n=0 \rightarrow N$ are chosen such that the solution would satisfy some imposed conditions. Two types of contractions are calculated, Thwaites' contraction No. 1 in which the velocity along the walls increases monotonically, and

Thwaites' contraction No. 2 in which the inlet velocity is uniform. The shape and velocity distributions of each contraction are shown in Figure 3.

It is clear that the above shown periodic solution is not valid when the contraction is connected to the straight sections of the settling and test sections. The invalidity of the solution in a real situation is, simply, due to the absence of periodic conditions and due to the elliptic nature of the governing equations.

Bossel (Ref. 13, 1969) used a trial-and-error procedure for the calculations of the coefficients d_n in Thwaites' solution. In this method the series solution can be truncated after N terms and if the wall contour is R_j , prescribed at N points x_j , we get a system of N -equations for the unknown Fourier coefficients as:

$$\sum_{n=1}^N \frac{d_n}{n} R_j I_1(NR_j) \cos(nx_j) = \psi_w - d_0 \frac{R_j^2}{2}$$

Once the d_n 's are known, the stream function and velocities follow in the entire flow field.

The design procedure then starts by choosing an "arbitrary" shape for the contraction contour R_j , and then using a computer routine, to find the Thwaites' coefficients which make the wall stream surface fit the chosen shape. The flow velocity can then be calculated and examined to check the regions of adverse pressure gradient. The wall shape can be modified accordingly, counting on the hypothesis that local changes in the wall shape produce, essentially, local changes in the flow field.

The solution in this case is still periodic, although the author of Ref.13 claims to have found that the effects of periodicity are not significant. This method is best suited to check a proposed contraction design rather than to create a new one.

Two important features for the above-mentioned design methods (Thwaites and Bossel) are: first, the solution is periodic which - as mentioned before - does not represent a practical wind tunnel situation; second, the contraction length is arbitrarily fixed $L = \pi$. This fixed length arises from the choice of a form for the solution which, although mathematically convenient, does not represent any practically significant criterion.

Reviewing the literature reveals that the choice of a velocity distribution along the duct center-line has been the basis of a number of methods for contraction design (Ref. 15 to Ref. 20). If the velocity along the center line ($r = 0$) is chosen as $u = F_0(x)$, the whole flow field can be deduced from that function, by radial expansion, as:

$$u(x,r) = \sum_{n=0}^{\infty} \frac{(-1)^n r^{2n}}{2^{2n} (n!)^2} F_0^{(2n)}(x)$$

$$v(x,r) = \sum_{n=1}^{\infty} \frac{(-1)^n 2n r^{2n-1}}{2^{2n} (n!)^2} F_0^{(2n-1)}(x)$$

and

$$\psi = \int_0^r r u(x,r) dr$$

The velocity, $F_0(x)$ along the axis of the contraction can be chosen arbitrarily as long as it permits analytical expression of its derivatives, and satisfies the following conditions (Ref. 16):

i) It conforms to normal axial velocity patterns, converting smoothly from the inlet velocity at the beginning of the contraction to the outlet velocity at the end of the contraction. Since the contraction is theoretically of infinite length, "beginning" and "end" will, in the theoretical context, be taken to mean conditions at $x = -\infty$ and $x = +\infty$ respectively.

ii) All its successive differential quotients tend to zero as $x \rightarrow +\infty$.

Once the center-line velocity is chosen the whole flow field can be deduced from the above equations. The different streamlines are then drawn and one of them is chosen for the contraction contour. The contractions designed by this method are of infinite length and have to be truncated and faired with the straight inlet and exit segments.

Contractions designed by choosing an axial velocity distribution were first proposed by Tsien (Ref. 15, 1943). He used a center line velocity in the form:

$$F_0(x) = 0.55 + 0.90 \int_0^x \sqrt{2\pi} e^{-(x^2/2)} dx$$

The specific numerical values used are chosen to yield an exit velocity of unity and to provide the velocity ratio required. Tsien's center line velocity distribution together with the chosen contraction shape are shown in Figure 4. It is clear that the chosen $F_0(x)$ provide equal flow acceleration from the inlet to the middle and from the middle

to the exit; it is not clear though, whether this has been done on purpose. This distribution was used to design the contraction cone for a large pressurized tunnel at GALCIT. The contraction shape chosen is the "last monotone" streamline, i.e. the last streamline away from the center line along which the velocity is always increasing which means no adverse pressure gradient regions. It is clear from Figure 4 that the slope of the wall at inlet is not zero and a great deal of shape modification is required at that region. This means that the solution is not valid anymore, especially in the inlet part which is shown later to be the critical region.

A similar method has been used by Szczeniowski (Ref. 17, 1943) to design a wind tunnel contraction. He used a simpler model for the velocity distribution along the contraction axis in the form:

$$f_0(x) = \left(\frac{CR + 1}{2}\right) u + \left(\frac{CR - 1}{2}\right) u \tanh \frac{x}{a}$$

where CR is the contraction ratio, u and a are arbitrary velocity and linear scaling parameters, respectively.

Cohen and Ritchie (Ref. 16, 1962) have used a similar expression to that of Szczeniowski to design a 7.4:1 area ratio contraction for a wind tunnel at Northampton College of Advanced Technology, England. They added an extra term in the axial velocity distribution to facilitate more control over the resulting streamline shapes. Their axial velocity distribution takes the general form:

$$f_0(x) = A + b_1 \tanh(k_1 x) + b_2 e^{-k_2 x^2}$$

The parameters in this expression are chosen to advance the point of inflexion of the wall to an earlier stage of development of the flow and increase the "conversion factor" of the upstream half of the contraction compared with the downstream half of the contraction. By the conversion factor is meant the amount of increase in the velocity in a part of the contraction relative to the total increase in the contraction as a whole. In this study (Ref. 16), using conversion factors of 0.625 and 0.375 for the upstream and downstream halves respectively - instead of 0.5 and 0.5 - results in much better convergence characteristics of the solution, and complete elimination of an adverse pressure gradient near the working section for all values of stream functions investigated(?) Among the streamlines obtained, the one with "acceptable" adverse pressure gradient at inlet is picked as a basis for the contraction contour. The shape of the contraction contour and the velocity distribution are shown in Figure 5.

Shima (Ref. 18, 1964) presented a generalization to the above method in which the analysis was made for an arbitrary velocity distribution along the axis.

More recently, Tsien's velocity distribution which has the general form:

$$F_0(x) = a + b \int_0^x e^{-(c^2 x^2)} dx$$

has been used with slight modification by Barger and Bowen (Ref. 19, 1972). They derived a recursion expression for the derivatives of that expression, and used a value of $c = 1$ instead of $c = 1/\sqrt{2}$ used by Tsien.

The resulting shape has an early inflection point, approximately at 0.20 of the contraction length. The total length is about five times the inlet radius for a contraction ratio of about ten.

In Ref. 20, 1973 Barger presents a method in which the streamlines can be constructed from an assumed velocity distribution along the axis by the use of the streamline curvature equations. Using an iterative procedure, the successive streamlines are computed until a specified mass flow is obtained (unless the solution becomes unstable before the mass flow condition is met). In such a case the last stable streamline can be taken as the wall contour, unless the solution displays an "unacceptable" pressure gradient along the wall.

A different approach has been presented by Whitehead, Wu and Waters (Ref. 21, 1951) for the design of a contraction of finite length with parallel sections upstream and downstream of it.

To design an axisymmetric contraction, the method starts by choosing a shape for a two-dimensional channel. The two-dimensional design is based on a boundary chosen in the hodograph plane to give an "acceptable" pressure gradient at inlet and outlet. The axisymmetric flow in a duct of the same shape as the two-dimensional design is solved approximately, using the 2-D velocity potential and stream function as independent variables. The approximate solution depends on the close similarity which exists between the shapes of the streamlines for the two flows. It was found that adverse pressure gradients are usually less than those for the two-dimensional flow and that favourable pressure gradients are increased. Boundary shapes which are satisfactory for two-dimensional channels therefore, should have an extra margin of safety when used for a contraction of

circular cross section. The procedure for a contraction design as given by these authors (Ref. 21) is lengthy and involves mathematical difficulties.

The major concern of all the above-discussed contraction design methods is either to obtain a contour for a monotonic wall velocity distribution - which is not feasible practically - or to limit the adverse pressure gradients within undefined "acceptable" levels. The main disadvantage of these methods, then, is that they emphasize techniques for mathematical solutions of the inviscid flow equations with little regard to practical problems involving viscous flow. The result is that the necessary practical modifications in most cases invalidated the theoretical solution.

Batchelor and Shaw (Ref. 22, 1944) were the first to define a limit for an acceptable adverse pressure gradient. Their criterion of a pressure gradient equivalent to that in a seven degree included angle conical diffuser could be criticized, though it was the best they could have done at that time.

The Batchelor and Shaw method is based on a choice of the flow acceleration:

$$u \frac{du}{dx} = f(x)$$

where $f(x)$ could be any smooth function with zero values at both ends of the contraction. They chose

$$f(x) = k \sin^2 (\pi x/L)$$

where k is a constant which depends on the required contraction ratio and L is the contraction length. By assuming one dimensional flow in the contraction i.e. $uA = \text{constant}$, the solution for the cross sectional area (A) distribution is:

$$\left(\frac{1}{A^2} - \frac{1}{A_1^2} \right) = \left(\frac{1}{A_e^2} - \frac{1}{A_1^2} \right) \left(\frac{x}{L} - \frac{1}{2\pi} \sin^2 (\pi x/L) \right)$$

Even with small contraction ratios, the contraction boundaries defined by expressions of the type given above, though giving a gradual and smooth transition to the working section, may not converge too rapidly near the inlet. Batchelor and Shaw calculated the flow field inside such a contraction boundary by 'hand' using a relaxation technique, and thus determined the velocity distribution along the streamlines. The streamline along which the fall in velocity at inlet was not more severe than that in a seven degree conical diffuser was then, chosen as a boundary for the contraction. Figure 6 shows streamlines and the velocity distribution calculated by Batchelor and Shaw. This method was used to design a number of successful (but long) contractions for low speed wind tunnels.

During the course of the present work, the results of two interesting studies have been added to the literature Ref. 23, 1974 and Ref. 24, 1975. In these studies a more practical approach to the wind tunnel contraction design problem has been taken. In both of these studies Stratford's criterion (Ref. 25, 1959) has been used to check the approach to separation of the turbulent boundary layer at the contraction inlet. In both of these studies the aim was the choice of a contraction contour

- out of a particular family of contours - which resulted in the shortest contraction length without risking flow separation.

Chemielewski (Ref. 23, 1974) followed Batchelor and Shaw in specifying an average streamwise acceleration distribution $f(x)$ in the convergent segment in the form:

$$f(x) = \left[\frac{1}{2} \left\{ 1 - \cos 2\pi \left(\frac{x-x_1}{L} \right)^n \right\} \right]^m$$

where n and m are controlling parameters used for optimizing the contraction length. The resulting contour can be expressed as:

$$\left(\frac{R_i}{R} \right)^4 = (CR^2 - 1) \frac{F(x)}{F(x_1 + L)} + 1$$

where: $F(x) = \int_{x_1}^x f(x) dx$

The inviscid flow calculations inside the defined contour were done using a finite difference over relaxation scheme developed by Hoffman (Ref. 26). Once the potential flow solution was available, computation of the viscous flow proceeded. Preliminary efforts were made in that study to use the axisymmetric version of the Cebeci-Smith method (Ref. 27). Because of the complexity and time-consuming set-up requirements of the Cebeci-Smith method, the simpler alternative of the Stratford turbulent separation criterion was used.

It was found that the Stratford criterion gave separation results in close agreement with the Cebeci-Smith method but tended to

be conservative from a design standpoint by predicting separation to occur slightly earlier, i.e., further upstream.

In that study, (Ref. 23) the contraction length was optimized and presented as a function of the contraction ratio - as shown in Figure 7 - for the conditions of inlet radius of 10 inches and exit Reynolds number per foot of 10^6 /ft. Under these conditions, the results shown in Figure 7, suggest that the required length for a contraction of area ratio of eight - as an example - is 2.6 times its inlet radius.

This study for the first time has presented a rational approach for contraction design, and demonstrated the feasibility of using a simple separation criterion (Ref. 25) for discriminating between acceptable and unacceptable wall-pressure distributions.

The chosen family of contours in Ref. 25 are characterized by too rapid area change at inlet, and what seems to be unnecessarily long exits. The choice of the length then, is controlled by the inlet separation requirement and consequently the resulting contraction length is more than what one would hope for and can actually achieve (see the results of the present study).

Morel (Ref. 24, 1975) chose a one-parameter family of wall shapes for his study. The contraction contours were formed of two cubic arcs joined smoothly together at an inflection point. The inviscid, incompressible flow calculations were made using a computer program of the stream line curvature type (developed by General Motors Detroit Diesel - Allison Division as an extension of the original computer program of Katzanis (Ref. 28)). Stratford's criterion for turbulent separation (Ref. 25) was used as a guide for possible separation. Using the Stratford

criterion with some geometrical consideration led to a separation criterion at inlet of $C_{p_i} = 0.39$ (for $R_{D_i} = 10^6$). It has been judged that no separation is expected at exit and maximum pressure coefficient C_{p_e} near the exit was used as a measure to flow non-uniformity at the contraction exit.

Design charts, as shown in Figure 8, are presented, which yield a shape parameter and nozzle length for an arbitrary choice of maximum wall-pressure coefficients at both inlet (as an indication of danger of separation) and at exit, which is related to exit velocity non-uniformity.

The design criteria used in that study (Ref. 24) result in the rather surprising finding, that nozzles with large contraction ratio, require smaller length for the same flow quality in the test section, than nozzles with lower contraction ratio.

To satisfy the design criteria of that study, namely no flow separation at inlet and maximum difference between the wall velocity and center-line velocity of two percent of the test section speed, the required contraction length for a contraction ratio of nine, is 1.7 times the inlet radius with the inflection point at 0.45 of the contraction length. The results were for a Reynolds number based on inlet diameter of 10^6 .

Comparisons between the results of the present study and that of the previous studies are presented later in the text.

The above has been a review of the published studies on the axisymmetric contraction problem during a period of time of over 32 years since the first study published in February 1943 (Ref. 15) to the most recent publication of June 1975 (Ref. 24). The present approach to the problem is introduced in a following section.

(NOTE: Additional recently published study (Ref. 47) is discussed

in Appendix V)

2. PRESENT APPROACH TO THE CONTRACTION DESIGN PROBLEM - DESIGN CRITERIA

The present work was initiated some time ago by the realization that the available methods of wind tunnel contraction design are far from being satisfactory.

Most of the available design methods are indirect methods, in which a closed form solution for the inviscid flow have been tried, and one of the resulting streamlines is then chosen as a boundary for the contraction. Seeking a closed form solution necessitates imposing unrealistic conditions. The applicability of the resulting analysis to an actual contraction is then limited.

The introduction of numerical analysis techniques encourages a direct method of solution. In this a boundary shape is specified inside which the flow parameters are determined, usually by some numerical technique. Batchelor and Shaw's study (Ref. 22, 1944) was the first in that direction, followed by the studies of Ref. 23, 1974 and Ref. 24, 1975. Because of the flexibility and restriction-free characteristics of numerical techniques, the present work was channelled towards the solution of the direct problem.

In the present study, the contraction shape is based on optimizing the curvature distribution of the contraction wall. The curvatures of the streamlines - as created by the curvature of the contraction wall - are responsible for both the adverse pressure gradient and, hence, possibly boundary-layer separation and flow non-uniformity at exit. Hence, it is possible to control the flow most effectively by controlling the wall curvature distribution.

It is known that whenever a contracting duct is fitted between a settling chamber and a test section, regions of adverse pressure gradient have to appear at the contraction wall near both its inlet and exit. If the

adverse pressure gradient is severe enough to cause the boundary-layer flow to separate in these regions, this would introduce unsteadiness to the test section flow, which is clearly an undesirable characteristic for an aerodynamic test facility.

There have been cases where such boundary-layer separations have occurred. For example, Batchelor and Shaw (Ref. 22) reported the existence of inlet separation in both a 4:1 contraction leading to a 9' x 7' test section and the contraction of a 1/8 -scale model of the same tunnel. The separation in the full-scale tunnel caused intermittent fluctuations in the test section total pressure of magnitude as large as 10 percent. No mention is made of the length of that contraction, but from the aerodynamic outline given, one would estimate the length to be 1.7 times the inlet radius.

Another known case of flow separation in a contraction is that of the R.A.E. 4ft x 3ft experimental low-turbulence wind tunnel (Refs. 30 and 31). The tunnel is of octagonal cross section, and the settling chamber contains nine removable screens followed by a contraction of 31.2 contraction ratio. The contraction length is only 1.45 times an equivalent inlet radius. The central part of the contraction is a frustrum of an octagonal pyramid with an angle of 100 degrees between opposite walls (see Figure 9). The inlet and exit parts of the contraction were designed on the basis of model tests, to give a monotonic pressure gradient along the walls (?) A change of section shape from a regular octagonal to the irregular octagonal of the working section is made in the final part of the contraction.

Measurements of the flow in the contraction of the full-scale tunnel showed that the contraction cone with monotonic increase of velocity along its walls to the value in the working section was an ideal which could not be fully attained in practice. It was found that there was some rise in pressure in the initial part of the contraction which caused boundary-layer separation.

There were also regions of high velocity along the fillets just ahead of the working section which did not produce any marked ill effects. The separation and flow reversal at the contraction inlet produced fluctuations in total pressure of the test section flow and flashes of high-intensity turbulence spreading a considerable distance from the working section walls. It was found that the flow separation at the contraction inlet depended on the number of damping screens installed. A region of flow reversal occurred on the concave walls at the start of the contraction when all nine damping screens were fitted. It was found later that this separation was absent when only two (the first and last) damping screens were fitted, but was present again when the last one was removed. The wide angle diffuser, settling chamber and contraction, had been designed on the basis of the results of a series of tests on a model of about 1/13th scale. The results of these tests were reported in Ref. 32, and no separation was found on the model. In order to facilitate the measurements, the model tests were made at higher wind speeds than in the full-scale tunnel, which might be a plausible explanation for the conflicting results.

To avoid the danger of boundary-layer separation, contraction designers have relied on using larger lengths to provide a more gradual area change. It would be useful at this point to look at both the literature and at available wind tunnel designs to compare the values of a contraction length suggested and used (see Table 2.1 and Table 2.2 below).

It is clear from Table 2.1 and Table 2.2 that there is no answer to the question of how long certain contractions need to be, even more ambiguous is the answer to the question of the best shape for a contraction contour.

The above discussion suggests that an "optimum" contraction is the shortest possible contraction inside which the flow does not suffer from any separation and which provides uniform flow to the test area.

In the following section we are going to investigate some ways to define a sensible shape for the contraction contour to achieve the shortest possible contraction.

TABLE 2.1

TYPICAL WIND TUNNEL CONTRACTION LENGTH AS IT APPEARS IN THE LITERATURE

AUTHOR AND REF. NO.	CR	L	REMARKS
Thwaites (Ref. 14, 1946)		3.14	Length is independent of CR
Cohen and Ritchie (Ref. 16, 1969)	7.4	2.3	It has been used in a contraction with $R_1 = 55.4$ in. and $U_0 = 200$ ft/sec.
Shima (Ref. 18, 1964) recomputed Tsiens' (Ref. 15, 1943) nozzle	9	4.67	Nozzle was tested by Ripkin (Ref. 29) for $R_1 = 9$ in. $U_0 = 18$ ft/sec. $(R_D)_e = 0.91 \times 10^6$
Bossel (Ref. 13, 1969)	16	3.14	The contraction was constructed and tested at $(R_D)_e = 6.28 \times 10^5$ and $R_1 = 30$ cm.
Barger and Bowen (Ref. 19, 1972)	10	5	
Chemielewski (Ref. 23, 1974)	2-25 8 (for example)	1.5-4 2.6	Analytical study with $R_1 = 10$ in. and $(R_e/D)_e = 10^6$ /ft.
Morel (Ref. 24, 1975)	4-16 4-9 (for example)	1.5-2.5 1.7	Analytical study. For $(R_D)_1 = 10^6$ and allowable maximum pressure coefficient at exit of 0.06

TABLE 2.2

TYPICAL WIND TUNNEL CONTRACTION LENGTH AS IT IS
IN OPERATING OR PROPOSED TUNNELS.

TUNNEL	CR	L	INLET DIAMETER AND TEST SECTION SPEED	REMARKS
NAE 30 ft x 30 ft* VSTOL - Tunnel (Canada)	5.85	2.28	$D_i = 82$ ft. $U_o = 180$ ft/sec. (operational)	operational
Boeing - Vertol VSTOL - Tunnel USA	5.11	1.49	$D_i = 51$ ft. $U_o = 440$ ft/sec.	operational
Lockheed/Georgia Low Speed Tunnel USA	3.4 2.06	1.21 1.46	$D_i = 58$ ft. $D_i = 31.5$ ft. $U_o = 370$ ft/sec.	operational
NLR LST 6 x 8 m The Netherlands	9.0	2.0	$D_i = 77$ ft. $U_o = 325$ ft/sec.	1/10 - scale models have been success- fully tested. Presently being rede- signed for a variety of working sec- tion sizes
GUK (Germany)	7 2.51	2.0 1.8	$D_i = 93$ ft. $U_o = 180$ ft/sec. $D_i = 35.2$ ft. $U_o = 427$ ft/sec.	proposed

* The method of Ref. 17 was used in the design of the contraction of the NAE 30 ft. x 30 ft. Wind Tunnel.

2.1 SHAPE OF A CONTRACTION CONTOUR

In literature there are three studies which define a contraction contour. Two of them (Ref. 22, 1944 and Ref. 23, 1974) derive the contraction shape from some assumed one-dimensional flow acceleration, and the third (Ref. 24, 1975), uses a shape which consists of two cubic arcs joined smoothly together at an inflection point.

Herein the objective is to choose a shape for the contraction in such a way as to control the development of the adverse pressure gradient along the contraction walls at its inlet and exit regions. The chosen shape should have the flexibility to enable shortening the contraction as much as possible and still meet the design criteria.

Let us consider the following argument for the existence of the adverse pressure gradients. Consider two streamlines - see Figure 10 - (1) near the wall, and (2) along the center of a duct. Far upstream, in the constant area duct we will assume that the streamlines are parallel and that the total pressure and static pressure are the same at both points (1) and (2). When the flow enters a contracting segment of the duct, the streamlines near the wall will start to curve towards the center-line. The curvature of the streamlines requires higher pressure near the wall than that towards the center-line, i.e. a positive radial pressure gradient near the exit. Near the inlet, where the velocity at point (2) has not changed appreciably from that at (2), the static pressure also has not changed. This leads us to the conclusion that $P_2' = P_2 = P_1$, but $P_1' > P_2'$, then $P_1' > P_1$ and an adverse pressure gradient must have developed along the wall near the entrance of the converging duct. A similar argument holds at the exit region where the streamlines begin to straighten.

The pressure gradient across a streamline at any point is proportional to the curvature of the streamline and the square of the velocity at that point. Since, near the wall, the streamlines take almost the shape of the wall, it would be effective to control the adverse pressure gradient by controlling the curvature of the wall. Near the inlet and exit, the wall slope is small, and the curvature can be approximated by the second derivative of the wall co-ordinates.

From the above discussion, it is clear that one may control the flow in the duct to a large extent by controlling the distribution of the wall "curvature".

For the purpose of the present study a distribution of the form:

$$R'' = -A(1-BZ) \sin\left(\frac{1}{1+n} 2\pi(Z)\right) \quad (1)$$

has been selected to yield zero R'' at both inlet and exit. In the above expression, A is an amplitude parameter which is defined by the required change in Radius $(R_1 - R_0)$, while B is a parameter, the adjustment of which ensures the required wall slope at exit (zero slope in the present case). The key parameter, n, controls the location of the inflection point (i.e. the point at which $R''=0$). For $n=0$, the inflection point lies right at the mid-length (it follows that B must be zero) and the distribution is a simple sine-function shape. With small positive values of n, the inflection point moves upstream - toward the wide end - and

with negative n , the inflection moves downstream (relative to the mid-length point). Moving the point of inflection upstream would result in increasing the magnitude of the wall curvature at inlet relative to that at outlet, which means a more rapid contraction at inlet. Moving the point of inflection downstream will result in an opposite effect.

The contraction geometry can be obtained by numerically integrating R'' twice,

$$R'' = -A (1-BZ) \sin \left(2\pi(Z) \frac{1}{1+n} \right)$$

$$R' = -A \int_0^Z (1-BZ) \sin \left(2\pi(Z) \frac{1}{1+n} \right) dZ \quad .l$$

$$R = R_1 - A \int_0^Z \int_0^Z (1-BZ) \sin \left(2\pi(Z) \frac{1}{1+n} \right) dZ \quad dZ \quad .l^2$$

Once the position of the inflection point relative to the total contraction length (Z_{ip}) is defined, n is determined by:

$$n = - \left(1 - \log(Z_{ip}) / \log(0.5) \right)$$

With n known, B can be calculated to give zero slope at exit. If the slope is zero at inlet, then B can be calculated from:

$$B = \frac{\int_0^1 \sin \left(2\pi(Z) \frac{1}{1+n} \right) dZ}{\int_0^1 Z \sin \left(2\pi(Z) \frac{1}{1+n} \right) dZ}$$

For a given inlet radius and required contraction ratio, A can be calculated from:

$$A = \frac{R_1 \left(1 - \frac{1}{\sqrt{CR}} \right) / \ell^2}{\int_0^1 \int_0^1 (1-BZ) \sin \left(2\pi(Z) \frac{1}{1+n} \right) dZ \quad dZ}$$

Figure 11(a) shows the effect of n on the position of the inflection point and on the parameter B. Figure 11 (b) shows the magnitude parameter A as a function of the contraction ratio and the position of the inflection point. The effect of the position of the inflection point on both the "curvature" distribution and the contraction contour shape is shown in Figures 12(a) and (b).

The logic behind the above method of selecting the contraction contour is the flexibility in affecting the flow adverse pressure gradient at inlet and outlet through the control of the magnitude of the wall curvature in these regions. For a certain inlet radius and contraction ratio, for the same relative position of the point of inflection, shortening the contraction raises the magnitude of R" at both inlet and outlet as shown in Figure 13. One would continue shortening the contraction until flow separation at either one end or the other is approached. At that point in the calculation, one would fix the length, and by moving the inflection point away from the region of expected separation, would ease the pressure gradient in that region and increase it toward the other end. By adjusting the inflection point position, one would expect that the shortest possible contraction would be the one inside which the flow would separate at both ends simultaneously, provided that the test section flow uniformity were still maintained.

A mathematical model and computer code are then required to carry out the above described proposal.

3. ANALYTICAL MODEL

3.1 EQUATIONS GOVERNING THE INVISCID FLOW

A cylindrical co-ordinate system (x, r, B) is best suited for the description of an axially-symmetric flow field. If u, v and w are the velocity components in the axial, radial and circumferential directions respectively, the steady Navier-Stokes equations for axisymmetric flow ($\frac{\partial}{\partial \theta} = 0$), in the absence of body and viscous forces, are as follows:

In the axial direction or x direction,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = - \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (3.1)$$

In the radial or r direction,

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial r} - \frac{w^2}{r} = - \frac{1}{\rho} \frac{\partial p}{\partial r} \quad (3.2)$$

In the circumferential or θ direction,

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial r} + \frac{vw}{r} = 0 \quad (3.3)$$

The condition of mass continuity for steady ($\frac{\partial}{\partial t} = 0$) incompressible ($\rho = \text{constant}$), and axisymmetric flow ($\frac{\partial}{\partial \theta} = 0$) is:

$$\frac{\partial u}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} (rv) = 0 \quad (3.4)$$

This equation is satisfied by the introduction of Stokes' stream function, ψ , such that the axial and radial velocity components are defined as:

$$u = \frac{1}{r} \frac{\partial \psi}{\partial r}$$

and

$$v = -\frac{1}{r} \frac{\partial \psi}{\partial x}$$

(3.5)

Using the definitions of equation (3.5), equations (3.1) and (3.2) can be combined in the form (Hall, Ref. 33),

$$\frac{\partial \psi}{\partial x^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial r^2} = -K \frac{dK}{d\psi} + \frac{r^2}{3} \frac{dp_t}{d\psi} \quad (3.6)$$

where K is proportional to the circulation and defined as:

$$K = \omega r$$

and p_t is the total (stagnation) pressure, defined as:

$$p_t = p + \frac{1}{2} \rho (u^2 + v^2 + w^2)$$

Both p_t and K are constants along a stream surface.

In the general case of rotational, swirling flow, equations (3.3) and (3.6) describe the flow motion. In case of no swirl, $w = 0$, equation (3.3) is identically zero, and equation (3.6) with $K = 0$, reduces to:

$$\psi_{xx} - \frac{1}{r} \psi_r + \psi_{rr} = \frac{r^2}{\rho} \frac{dp_t}{d\psi} \quad (3.7)$$

is sufficient with suitable boundary conditions to describe the flow.

Moreover, for irrotational flow, $p_t = \text{constant}$ along all stream surfaces, and the governing equation is:

$$\psi_{xx} - \frac{1}{r} \psi_r + \psi_{rr} = 0 \quad (3.8)$$

which is known as the Stokes-Beltrami equation.

3.2 FORMULATION OF THE PROBLEM

3.2.1 GEOMETRICAL MODEL

The geometrical model used in this study is shown in Figure 14(a), Figure 14(b) illustrates the geometrical parameters and nomenclature. The model consists of a constant area segment of radius R_1 and length $L_S R_1$ to represent the wind tunnel settling chamber. It is followed by the contraction segment with contour defined as $R = R(x)$ and length $\ell = L R_1$. The contraction ends at the beginning of the test section which is represented by a constant area segment with radius $R_0 = R_1 / CR$ and length $L_0 R_0$.

Typically, the settling chamber length L_S is about one. A heat exchanger and a number of turbulence screens usually occupy part of the settling chamber. The test section length, L_0 , is typically 4-5. Typical values of L are given in both Table 2.1 and Table 2.2; actually the objective of the present study is to choose $R(x)$ in such a way as to minimize the length L maintaining the required flow quality.

3.2.2 NORMALIZING OF THE GOVERNING EQUATIONS

a) Length Scale

Let the contraction inlet radius (the settling chamber radius) be a reference dimension. One may write:

$$R = \frac{r}{R_1}, \quad X = \frac{x}{R_1}, \quad L = \frac{\ell}{R_1}$$

b) Velocity Scale

One selects the axial inlet velocity U_1 defined as:

$$U_1 = \frac{\bar{Q}}{A_1}$$

as a reference velocity,

where Q is volume flow rate, and A_1 is the contraction inlet area, $A_1 = \pi R_1^2$,

then:

$$U = \frac{u}{U_1}, \quad V = \frac{v}{U_1} \quad \text{and} \quad W = \frac{w}{U_1}$$

where, U, V, W are the non-dimensional velocity components in the axial, radial and circumferential directions, respectively.

c) Pressure Scale

Let the dynamic pressure at inlet ($\frac{1}{2} \rho U_1^2$) be a reference pressure such that:

$$P_t = \frac{p_t}{\frac{1}{2} \rho U_1^2} = \frac{p}{\frac{1}{2} \rho U_1^2} + \frac{\frac{1}{2} \rho (u^2 + v^2 + w^2)}{\frac{1}{2} \rho U_1^2}$$

$$P_t = P + U^2 + V^2 + W^2$$

where P_t and P are the non-dimensional total pressure and the static pressure respectively.

d) Normalized Equations

With the non-dimensional Stokes' stream function

$$\Psi = \frac{\psi}{U_1 R_1^2}$$

The governing equation for non-swirling, rotational flow, can be written as:

$$\frac{\partial^2 \Psi}{\partial X^2} - \frac{1}{R} \frac{\partial \Psi}{\partial R} + \frac{\partial^2 \Psi}{\partial R^2} = \frac{-R}{2} \frac{dP_t}{d\Psi} \quad (3.9)$$

The axial and radial velocity components are defined as:

$$U = \frac{1}{R} \frac{\partial \Psi}{\partial R}$$

$$V = -\frac{1}{R} \frac{\partial \Psi}{\partial X} \quad (3.10)$$

One should notice that $P_t = P_t(\Psi)$ i.e. total pressure is constant along a stream surface.

3.2.3 Boundary Conditions

Along the contraction center-line, one chooses $\Psi = 0.00$

Since the contraction surface is a stream surface, Ψ is constant and equal to its value at the entrance, where $\Psi = 0.50$.

At the contraction inlet we can specify a radial distribution for Ψ , or equivalently, a radial distribution for the axial velocity U , since

$$\Psi = \int_0^1 UR dR$$

To start the integration, one would need a radial distribution for $\frac{\partial \Psi}{\partial X}$ at inlet to start a downstream integration, but the nature of the

elliptic equation describing the flow, forbids definition of such a distribution, since this would result from the downstream conditions of the flow. Alternatively, one might assume that, far downstream in a constant area duct (namely, the working section exit, say) the streamlines become parallel and hence $\frac{\partial \Psi}{\partial X} = 0$. One would need to iterate, then, on $\frac{\partial \Psi}{\partial X}$ at entry, to obtain parallel flow downstream well in the constant area section.

Mathematically, the boundary conditions can be expressed as:

$$\begin{aligned} \text{At } R = 0 & \quad \Psi = 0 \\ \text{At } R = R_w & \quad \Psi = 0.5 \\ \text{At } X = 0 & \quad \Psi(R) = \int_0^R U R dR \end{aligned} \quad (3.11)$$

where $U = U(R)$ is given.

$$\text{At } X = (L_s + L + L_o) \quad (\text{see Figure 14})$$

$$\frac{\partial \Psi}{\partial X} = 0.$$

The differential equation (3.9), with the definition (3.10), and the above boundary conditions, are just sufficient to describe the flow in the axisymmetric duct. In the following section, a numerical solution scheme is presented, and the computer program used is described.

4. NUMERICAL SOLUTION SCHEME AND COMPUTER PROGRAM

4.1 NUMERICAL SOLUTION SCHEME

4.1.1 TRANSFORMATION OF THE EQUATIONS

The task is to solve equation (3.9) subject to the boundary conditions of equation (3.11) inside a boundary with geometry as shown in Figure 14, for a variety of contraction contour shapes. Solution of equation (3.9) inside a boundary of general shape involves difficulties - and, consequently, potential for numerical errors - in setting the boundary conditions numerically irrespective of the method of numerical solution. For this reason it is decided to transform the physical plane into a plane in which the duct has fixed boundaries irrespective of the contraction contour shape.

Let us consider the co-ordinates η and ξ defined as:

$$\eta = \frac{R}{F(X)} = \frac{R}{R_w} \quad \text{and} \quad \xi = X \quad (4.1)$$

where: $F(X) = R_w$ is the boundary shape. Any boundary shape is transformed into a rectangle bounded by $\eta = 0$ and $\eta = 1$ as shown in Figure 15.

Using transformation (4.1) and letting,

$$F' = \frac{dF(X)}{dX}, \quad \text{and} \quad F'' = \frac{d^2 F(X)}{dX^2}, \quad \text{equation (3.9) for irrotational}$$

flow ($P_t = \text{Constant i.e. } \frac{dP_t}{d\psi} = 0$) transforms into:

$$\Psi_{\xi\xi} = C\Psi_{\eta} + D\Psi_{\eta\eta} + E\Psi_{\eta\xi} \quad (4.2)$$

where:

$$C = \eta \left(\frac{FF'' - 2F'^2}{F^2} \right) + \frac{1}{\eta F^2}$$

$$D = - \left[\left(\eta \frac{F'}{F} \right)^2 + \frac{1}{F^2} \right]$$

$$E = 2\eta \frac{F'}{F}$$

The transformed boundary conditions are:

$$\text{At } \eta = 0$$

$$\Psi(\xi, 0) = 0$$

$$\text{At } \eta = 1$$

$$\Psi(\xi, 1) = 0.5$$

$$\text{At } \xi = 0$$

$$\Psi(0, \eta) = \int_0^{\eta} U_0 d\eta \quad (4.3)$$

$$= \frac{1}{2} \eta^2 \quad \text{for uniform flow at entry}$$

$$\text{At } \xi = L_T$$

$$\Psi_{\xi}(L_T, \eta) = 0$$

where L_T is the total length:

$$L_T = L_s + L + L_o$$

4.1.2 METHOD OF SOLUTION

Among the possible solution schemes for partial differential equations of the boundary value type, the method of lines possesses the attractive combination of accuracy with the requirement of only small computer core size. The basic feature of the method of lines (MOL) as presented by Jones, South, and Klunker (Ref. 34), is that our rectangular region may be divided into strips by dividing lines, parallel to the ξ - axis. At each line, the derivatives with respect to η are replaced by finite-difference approximations, and derivatives with respect to ξ remain continuous. Thus, our partial differential equation is reduced to a system of $2N$ first order ordinary equations, where N is the number of lines used. These equations can be integrated using one of the standard techniques, such as Runge-Kutta or predictor-corrector methods.

The boundary value problem is solved by the "shooting technique," which involves estimating the unknown streamline "slopes" (Ψ_{ξ}) at the initial point and integrating the ordinary differential equations across to the end point. The required boundary condition at the end point, $\Psi_{\xi} = 0$, can then be satisfied by iterating on the missing initial conditions. It is also shown in Ref. 34 that although the ordinary differential equations are inherently unstable, accurate solutions can be readily obtained by MOL. It is shown in Ref. 34 that, if the region of interest is divided into only a few strips by the dividing lines, accurate solutions can be obtained by using high-order finite-difference approximations.

The advantage of MOL over grid techniques is that an order of magnitude of fewer unknowns is required to complete the solution. Grid

techniques solve for the unknown Ψ at each grid point; MOL solves for the unknown Ψ_{ξ} at one end of each dividing line. This facilitates the solution in terms of smaller storage requirements and, hence a smaller machine may be used.

The disadvantage of MOL compared with grid techniques is that a solution may not be as accurate. The accuracy of the grid techniques can be improved by reducing the grid size (this, of course, required a bigger computer core size and takes more computation time). Theoretically, the accuracy of the grid techniques is limited only by round-off errors, so that by using double precision, a result of high accuracy may be obtained.

As pointed out earlier, the MOL system of ordinary differential equations is inherently unstable and the instability becomes worse if the finite-difference strip size is too small; thus one has to accept results using a certain strip size at which the instability is insignificant.

The accuracy and stability of MOL are studied in detail by Jones, South, and Klunker (Ref. 34). The important points of this study are summarized as follows:

1. The error due to the instability of the MOL ordinary differential equations may be expected to grow large, in proportion with $\exp(NX)$, where X is the distance integrated and N is the number of lines.
2. Accurate solutions may be obtained with a five-point difference scheme (equation 4.4) provided N and X are not too large.

A detailed consideration of the solution of the two-dimensional Laplace's equation:

$$\Psi_{xx} + \Psi_{yy} = 0$$

by MOL (Ref. 34) showed that an accuracy of better than 1% on Ψ_x is obtained even with a coarse finite-difference and integration step size ($N = 3$ with $\delta x = 0.25$).

In the present study computer experiments were made to use more than three lines (in addition to the two boundary lines, the center-line and the wall) which were just enough for the use of the five-point difference scheme (equation 4.4) recommended for accuracy (Ref. 34). Due to the growth in the solution instability, the results showed that it was better to use only three lines.

The first and second derivatives were replaced by a five-point difference scheme as follows:

$$\Psi_{\eta}(\xi, \eta) = \frac{4}{3} \left(\frac{\Psi(\xi, \eta + \delta\eta) - \Psi(\xi, \eta - \delta\eta)}{2\delta\eta} \right) \quad (4.4a)$$

$$- \frac{1}{3} \left(\frac{\Psi(\xi, \eta + 2\delta\eta) - \Psi(\xi, \eta - 2\delta\eta)}{4\delta\eta} \right)$$

and

$$\Psi_{\eta\eta}(\xi, \eta) = \frac{4}{3} \left(\frac{\Psi(\xi, \eta + \delta\eta) + \Psi(\xi, \eta - \delta\eta) - 2\Psi(\xi, \eta)}{\delta\eta^2} \right) - \frac{1}{3} \left(\frac{\Psi(\xi, \eta + 2\delta\eta) + \Psi(\xi, \eta - 2\delta\eta) - 2\Psi(\xi, \eta)}{4\delta\eta^2} \right)$$

(4.4b).

The mixed differential $\Psi_{\xi\eta}$, is expressed in finite difference notation as in equation (4.4a) with Ψ_{ξ} substituted for Ψ .

The truncation error in equation (4.4a) is of the order of $O(\delta\eta^4 \Psi^{(5)})$, and that of equation (4.4b) is of the order of $O(\delta\eta^4 \Psi^{(6)})$.

To apply equations (4.4) on the line next to the center line (line (1) in Figure 16), the image line is used. But, since it is not possible to do so when applying on line (3), the following alternative finite-difference scheme is used:

$$\delta\eta \frac{\partial \Psi_3}{\partial \eta} = -\frac{1}{4} (\Psi_3 - \Psi_w) + \frac{3}{2} (\Psi_3 - \Psi_2) - \frac{1}{2} (\Psi_3 - \Psi_1) + \frac{1}{12} (\Psi_3 - \Psi_c) \quad (4.5a)$$

for the first derivative, where $\Psi_w = 0.5$ and $\Psi_c = 0$. For the second derivative:

$$\begin{aligned} \frac{\delta\eta^2}{2} \frac{\partial^2 \Psi_3}{\partial \eta^2} = & -\frac{1}{6} (\Psi_2 - \Psi_3) + \frac{1.75}{3} (\Psi_1 - \Psi_3) - \frac{1}{4} (\Psi_0 - \Psi_3) \\ & + \frac{0.125}{3} (\Psi_1 - \Psi_3) + \frac{1.25}{3} (\Psi_w - \Psi_3) \end{aligned} \quad (4.5b)$$

is used. The truncation error in expression (4.5a), is of the order of $O[(\delta\eta)^5 \Psi^{(5)}]$ and that in (4.5b) is of the order of $O[(\delta\eta)^6 \Psi^{(6)}]$.

Transforming equation (4.2) into a set of six coupled ordinary first order differential equations for the six unknown functions $(\Psi_1, \frac{\partial \Psi_1}{\partial \xi} \text{ for } i = 1, 2, 3)$ has been done as follows:

$$\begin{aligned} \text{Let: } Y(1) &= \Psi_1 & Y(4) &= \frac{\partial \Psi_1}{\partial \xi} \\ Y(2) &= \Psi_2 & Y(5) &= \frac{\partial \Psi_2}{\partial \xi} \\ Y(3) &= \Psi_3 & Y(6) &= \frac{\partial \Psi_3}{\partial \xi} \end{aligned} \quad (4.6)$$

Equation (4.2) can be written as follows:

$$\begin{aligned} Y_{\xi}(1) &= Y(4) \\ Y_{\xi}(2) &= Y(5) \\ Y_{\xi}(3) &= Y(6) \\ Y_{\xi}(4) &= C(1) \Psi_{\xi}(1) + D(1) \Psi_{\eta\eta}(1) + E(1) \Psi_{\xi\eta}(1) \\ Y_{\xi}(5) &= C(2) \Psi_{\eta}(2) + D(2) \Psi_{\eta\eta}(2) + E(2) \Psi_{\xi\eta}(2) \\ Y_{\xi}(6) &= C(3) \Psi_{\eta}(3) + D(3) \Psi_{\eta\eta}(3) + E(3) \Psi_{\xi\eta}(3) \end{aligned} \quad (4.7)$$

This is a set of ordinary differential equations where $C(1)$, $D(1)$ and $E(1)$ are C , D , and E as defined in equation (4.2) and calculated

on line i , $i = 1, 2$, and 3 . The values $\Psi_{\eta}(i)$, $\Psi_{\eta\eta}(i)$ and $\Psi_{\xi\eta}(i)$ are calculated from the appropriate expressions of (4.4) or (4.5), as a function of $Y(J)$, $J = 1, 2, \dots, 6$.

This set of the first order simultaneous differential equations (equation 4.7) is integrated using a Predictor-Corrector method with Runge-Kutta starting procedure for which an efficient subroutine, DBASHER, is available in the Scientific Programs Library at Carleton University Computer Center. The Predictor-Corrector method used is comparable to the classic Runge-Kutta method in accuracy, but is significantly faster, as only two derivative evaluations, as opposed to four, are required per step.

To perform the integration, the values of $Y(4)$, $Y(5)$ and $Y(6)$ at the initial point are missing. One assumes them to carry out the integration to the end point where $Y(4)$, $Y(5)$, and $Y(6)$ must be zero. One would iterate on the missing values, until error on the final conditions is sufficiently small. Let us call our initial missing conditions ϕ_i , $i = 1, 2, \dots, N$, and the errors are ϵ_i , $i = 1, 2, \dots, N$. (N is the number of lines, 3 in our case). We need to minimize $\sum_{i=1}^N \epsilon_i^2$ with respect to ϕ_i , $i = 1, \dots, N$. (Note that in this case, the number of unknowns is equal to the number of conditions, which is not always necessary).

The Powell method (Ref. 35) for minimizing the sum of squares - which is one of the best methods available (Ref. 34) - is used in the present study. The Powell minimization computer routine was obtained from the National Research Council Computer Library and was used in this study.

Unfortunately, by starting the integration at one end, errors due to instability grow too large before the other end is reached.

to allow any meaningful results. To avoid integrating the equations over large distances, it was decided to divide the total length, L_T , into a number of integration segments and provide matching conditions to the solutions inside different segments at their interface. A number of segments has been tried, but it was found that six integration segments - see Figure 16 - are necessary to keep the instability errors insignificant without restraining the duct total length or its area ratio.

One should emphasize that the integration segments discussed above and shown in Figure 16, are not necessarily related to the physical geometry of the duct under consideration. The length of each of these integration segments is arbitrary, and is independent of the geometry, as long as their total length adds up to the actual total length. The computer code requires the length of each integration segment as input data. Although different integration segment lengths were tried, the results presented in this study were all obtained with each of the constant area parts at contraction inlet and exit represented by an integration segment, and the middle four integration segments were of equal length and independent of the particular contraction geometry considered.

The step size of the forward integration in each integration segment was defined by the number of integration steps in each segment. To decide on an integration step size, a number of experimental computer runs were made with a different number of steps per segment. It was found that there was no appreciable gain of accuracy upon increasing the number of steps in each segment to more than five; and five integration steps per segment were used to obtain the results presented here.

On each line - see Figure 16 - the unknowns are; Ψ_ξ at station I, Ψ and Ψ_ξ at station III and station V, and Ψ at station VII. In total we have $6N$ unknowns. At the same time, we also have $6N$ conditions resulting from applying the condition of continuity for both Ψ and Ψ_ξ on each line at stations II, IV, and VI.

The Powell routine solves for values of the unknowns, which minimize the sum of squares of error functions. The errors in our case are the mismatching between values of both Ψ and Ψ_ξ on each side of stations II, IV and VI on each line. In this case the error in Ψ was normalized by $U_1 R_1^2$, and the error in Ψ_ξ by $U_1 R_1$. The minimum sum of the squares of the normalized errors is typically of the order of 10^{-6} .

For linear problems of the kind considered here, the Powell routine is capable of reaching the optimum values for the unknowns essentially in one iteration; allowing for more iterations would improve the solution, but not significantly.

Our problem now with the complications introduced by dividing the duct into different integration segments, reduced to one of finding $6N$ unknowns - (18 unknowns in this case where $N = 3$). Once these unknowns are found the value of the stream function Ψ can be obtained by direct integration of equation (4.7) using a standard integration routine. This still represents savings in both the required core size and computation time over a standard grid technique where the unknown Ψ are values in the whole region.

Once Ψ is known, the flow velocity along both the centre line and the contraction wall (and similarly anywhere) may be calculated as follows:

At the centre line: $V = 0$ from symmetry

$$U_c = \frac{1}{R} \psi_R \bigg|_{R=0} = (\psi_{RR})_{R=0} = \frac{1}{F^2} (\psi_{\eta\eta})_{\eta=1}$$

Using equation (4.4b) to express $\psi_{\eta\eta}$,

$$U_c = \frac{8Y(1) - 0.5Y(2)}{3(\delta\eta)^2 F^2}$$

where $\delta\eta = 0.25$ and F is the relative contraction radius at each point.

On the wall:

$$U_w = \frac{1}{R} \psi_R \bigg|_{R=F} \quad \text{but } \psi_R = \frac{1}{F} \psi_{\eta}$$

therefore,

$$U_w = \frac{1}{F^2} (\psi_{\eta})_{\eta=1}$$

To calculate ψ_{η} at the boundary, the following one-sided difference scheme was used. The axial component of velocity at the wall is:

$$U_w = \frac{1}{12\delta\eta F^2} (25\psi_w - 48Y(3) + 36Y(2) - 16Y(1))$$

The radial component at the wall is (since the wall is a streamline):

$$V_w = F U_w$$

and the flow velocity along the wall is then:

$$Q_w = (u_w^2 + v_w^2)^{1/2}$$

The above analysis is valid for any axisymmetric duct with continuous and smooth contour.

4.2 COMPUTER PROGRAM

A FORTRAN computer code to solve the inviscid, incompressible flow in an axisymmetric duct of arbitrary area distribution has been developed on the basis of the numerical scheme presented in the previous section. The development of that code to a state in which it can be used with confidence required an effort over a considerable period of time. "Debugging" and testing the subroutines individually was done on the Carleton University IBM 1620 machine. The first major difficulty encountered was during the initial trials of assembling the program on the Carleton University SIGMA IX computer. The source of the trouble was the numerical instability in the differential equations as discussed in Section 4.1.2. Originally, the effect of this instability had been underestimated, but no meaningful results were obtained until the corrective procedure of dividing the domain of interest into segments was taken. The development of the code to a working condition was done essentially on the NRC IBM360/67 from the NAE terminal at Uplands, Ottawa,* and continued on the Carleton SIGMA IX where a test case - as described later in this section - was run successfully. The other challenging difficulty was to get the code to run on the DSMA** computer. The size of the program was far exceeding the capacity of the Xerox 530 machine. A complete reconstruction of the program was necessary in which the program was divided into four different segments in addition to the main-line. Each of the four segments to be called in the active core area by the main-line, only when needed during execution otherwise

* The helpful suggestions of Mr. D. Jones, head of the NAE computer group at the High Speed Aerodynamics Lab are greatly appreciated.

** Dilworth, Secord, Meagher and Associates Limited, Consulting Engineers.

it would be stored on disc. Major changes in many subroutines were necessary in order to reduce the total program size to make it fit within the available computer capacity. The most important changes have been made to the Powell minimization routine. Changes included the removal of all unused options which were available in the routine,⁴ and deleting many logic statements. A considerable reduction of the number of variables used was accomplished by using the same name for more than one variable whenever possible. As the program stands now, the modified Powell minimization routine serves as a main-line which controls the rest of the sub-programs. The current structure of the program is shown in Figure 17 and described in the following section.

4.2.1 PROGRAM STRUCTURE AND DESCRIPTION OF SUBROUTINES

As shown in Figure 17 the program consists of main-line and four segments. Segment 1 consists of subroutines FUNC, BASHER, and DERIVE, segment 2 contains only subroutine SOLVD, segment 3 contains both subroutine SHAPE and subroutine QSF, while segment 4 consists of subroutine TEST only.

The main-line is essentially a modified version of the Powell minimization routine obtained from the NRC Computer Library. The main-line reads the input data as described in Appendix III. It calls in segment 3, then calls subroutine SHAPE to calculate the shape parameters F , F' , F'' and S as functions of X . Subroutine SHAPE calls subroutine QSF several times to perform integration processes. The calculated shape parameters are printed and then stored in arrays in a common area of the memory.

Segment 3 is then brought in and subroutine FUNC is called. Subroutine FUNC divides the domain of interest into 6 regions as described before. It sets up the initial conditions of integration at the start of each integration region. As a first guess, these initial conditions are set as if the flow were one-dimensional. Subroutine FUNC calls subroutine BASHER to perform the simultaneous integrations of the differential equations. Subroutine BASHER, in turn, calls subroutine DERIVE to provide the values of the variables' first derivatives at each point of the forward integration. Subroutine FUNC calculates the error functions to be minimized, at the end of each integration region and returns them to the minimization routine contained in the main-line. Subsequently, FUNC is called to calculate the error functions corresponding with different initial conditions defined by the minimization routine. The number of FUNC calls is at least the number of unknown initial conditions plus one (19 in our case) for a linear problem. After that, segment 2 is called and subroutine SOLVD is used for inverting a matrix (18 x 19) as a part of the minimization process. Once the correct initial conditions are obtained to minimize the error functions, subroutine FUNC is called to compute and print out U_w , U_c , C_{P_w} , C_{P_c} , as function of the axial distance. These quantities are also stored in a common area of the memory for subsequent use as input for viscous flow calculations. In the last part of the calculation, segment 4 is called, and subroutine TEST is used to check on boundary-layer behaviour, using the available geometric and inviscid flow data. As will be described later, subroutine TEST uses Stratford's criterion for turbulent separation (Ref. 25) to check on possible inlet boundary

layer separation, then it checks on possible boundary layer re-laminarization going through the region of favourable pressure gradient. The subroutine then uses Stratford's criterion to check on possible exit boundary layer separation after the calculation of an equivalent length, as will be described later.

If required, the geometrical data and the inviscid flow velocity distribution along the wall can be punched on cards to be used as input data for more detailed boundary layer calculations.

4.2.2 PROGRAM TESTING

It was necessary, before using the computer code in any systematic study of changing duct variables, to establish the validity and accuracy of its results. It was decided that Thwaites' contraction No. 2 - as described below - would make a good case for comparing the results of the present numerical scheme with an analytical solution.

As mentioned before in section 1.2, Thwaites' periodic series solution (Ref. 14) is:

$$\Psi = \frac{1}{2} d_o R^2 + R \sum_{n=1}^N \frac{d_n}{n} \cos(nX) I_1(nR)$$

from which the velocity components are:

$$\bar{U} = d_0 + \sum_{n=1}^N d_n \cos(nX) I_0(nR)$$

$$V = \sum_{n=1}^N d_n \sin(nX) I_1(nR)$$

The coefficients, d_n , are chosen to ensure as uniform a velocity distribution across the wide end as possible, taking only the first four terms of the series (i.e. $N = 3$). This can be done by considering the irrotationality condition: $\frac{\partial U}{\partial R} - \frac{\partial V}{\partial X} = 0$; if $\frac{\partial V}{\partial X} = 0$ then $\frac{\partial U}{\partial R} = 0$. Thus by choosing the coefficients, d_n , in the expression:

$$\frac{\partial V}{\partial X} = \sum_{n=1}^N n d_n \cos(nX) I_1(nR)$$

to produce the form:

$$\frac{\partial V}{\partial X} = -A (1 + \cos(x))^N, A > 0$$

the first $2N-1$ derivatives of V with (respect to X at $X = \pi$ and any radius) are zero, and the $2N$ -th derivative is negative. It is clear that this will ensure a very gradual contraction at first, and help in making the velocity across the wide end ($X = \pi$) as uniform as possible. The above condition at $X = \pi$ and $R = 1$, yields the coefficients:

$$d_1 = 1, \quad d_2 = 0.28424 \quad \text{and} \quad d_3 = 0.02859$$

and the wall streamlines go through both points $X = \pi$, $R = 1$. And $X = 0$, $R = 0.35$ yields $d_0 = 1.04722$.

With ψ_w and d_n ($n = 0, 1, 2, 3$) known, the ordinates of the contraction contour are found as the solutions of the equation,

$$\psi_w = \frac{1}{2} d_0 R^2 + R \sum_{n=1}^3 \frac{d_n}{n} \cos(nX) I_1(nR)$$

For each value of R ($1 \geq R \geq 0.35$) the above equation reduces to a cubic equation in $\cos X$ and hence X can be obtained. Once the co-ordinates of boundary points are known the velocity components can be calculated from the expressions given above. Table 4.1 shows the contraction co-ordinates and velocity distribution along both the boundary and the center line. Velocities are calculated using the above expression for ψ and the chosen coefficients. The velocity is not normalized, to facilitate comparison with Thwaites' results shown in Table 4.4. It should be noted that difficulty was encountered near the wide end. Corresponding to $R = 0.99$ it is found that shown in Table 4.1 that $X = 2.409$, and corresponding to $R = 1$, it was found that $X = \pi$. It was difficult to find X corresponding to R between 1.0 and 0.99, and an approximation was used in that region.

For the numerical solution, R , R' , and R'' were required as a function of X . R' and R'' are computed from the calculated R distribution using finite difference expressions similar to those given in section 4.1.2. Table 4.2 contains R , R' and R'' as functions of the distance X from the wide end. These data were used to compute the flow numerically, using

the developed computer program. The computation starts from the wide end. The obtained velocity distribution along both the contraction and the centerline are shown in Table 4.3 together with the deviation of these velocities from that obtained using Thwaites' series solution.

The comparison of Table 4.3 shows excellent agreement between the present numerical and the analytical results. The largest differences appear at the wide end of the contraction where the numerical calculation started with the highest difference of 0.9%. This, actually, is completely opposite to the expected trend of the numerical inaccuracy. One can relate these differences to an inaccuracy in defining the contour shape at inlet. The reason for that is the slow variation of radius with distance at the wide end. The way we defined the contour was to find X corresponding with some R and extreme difficulty has been associated with relating definite X to some input R (for $R \approx 1.0$). One would expect that this has produced some inaccuracy in calculating the velocity components, using an inaccurate X and R in the series solution. A proof of that claim is that if the wall velocity calculated numerically is compared with the series solution as presented by Thwaites (Ref. 14), and given in Table 4.4, instead of the series solution calculated here, we get different results. For example, at the point of maximum error (0.9%) as calculated before, where $R = 0.99$, the comparison of the numerical wall velocity with that of the series solution of Table 4.4 shows a difference of 0.00%.

The results of the above test case confirm the adequacy and the good accuracy of the numerical scheme used and suggest that the computer program is ready to be used in a detailed study of the contraction design problem to be described in the following sections.

TABLE 4.1

THWAITES' (REF. 14) CONTRACTION NO. 2

CONTOUR AND VELOCITY DISTRIBUTION

X	R	q_w	q_c
.0000000	.3500000	2.4352551	2.3600537
.3302227	.3600000	2.3035217	2.2333335
.4639372	.3700000	2.1822248	2.1170213
.5645707	.3800000	2.0702735	2.0100281
.6478494	.3900000	1.9667152	1.9114026
.7199213	.4000000	1.8707148	1.8203110
.7839677	.4100000	1.7815380	1.7360201
.8418950	.4200000	1.6985367	1.6578831
.8949557	.4300000	1.6211374	1.5853275
.9440269	.4400000	1.5488312	1.5178451
.9897521	.4500000	1.4811651	1.4549839
1.0326208	.4600000	1.4177348	1.3963402
1.0730160	.4700000	1.3581786	1.3415530
1.1112442	.4800000	1.3021722	1.2902986
1.1475557	.4900000	1.2494242	1.2422858
1.1821581	.5000000	1.1996719	1.1972526
1.2152260	.5100000	1.1526783	1.1549621
1.2469080	.5200000	1.1082289	1.1152001
1.2773320	.5300000	1.0661295	1.0777725
1.3066091	.5400000	1.0262038	1.0425028
1.3348366	.5500000	.9882911	1.0092305
1.3621005	.5600000	.9522454	.9778093

TABLE 4.1 (Continued)

X	R	q_w	q_c
1.3884771	.5700000	.9129332	.9481056
1.4140349	.5800000	.8852327	.9199969
1.4388357	.5900000	.8540322	.8933712
1.4629354	.6000000	.8242295	.8681256
1.4863850	.6100000	.7957308	.8441655
1.5092316	.6200000	.7684498	.8214037
1.5315183	.6300000	.7423073	.7997598
1.5532856	.6400000	.7172302	.7791595
1.5745709	.6500000	.6931513	.7595338
1.5954098	.6600000	.6700085	.7408192
1.6158359	.6700000	.6477447	.7229562
1.6358811	.6800000	.6263069	.7058896
1.6555764	.6900000	.6056464	.6895681
1.6749516	.7000000	.5857183	.6739435
1.6940359	.7100000	.5664811	.6589708
1.7128582	.7200000	.5478966	.6446077
1.7314472	.7300000	.5299296	.6308143
1.7498318	.7400000	.5125481	.6175530
1.7680413	.7500000	.4957225	.6047883
1.7861060	.7600000	.4794263	.5924861
1.8040523	.7700000	.4636354	.5806141

TABLE 4.1 (Concluded)

X	R	q_w	q_c
1.8219283	.7800000	.4483284	.5691412
1.8397543	.7900000	.4334867	.5580374
1.8575732	.8000000	.4190942	.5472733
1.8754267	.8100000	.4051378	.5368204
1.8933609	.8200000	.3916076	.5266504
1.9114276	.8300000	.3784970	.5167348
1.9296857	.8400000	.3658031	.5070449
1.9482032	.8500000	.3535277	.4975514
1.9670603	.8600000	.3416772	.4882234
1.9863519	.8700000	.3302646	.4790282
2.0061938	.8800000	.3193101	.4699302
2.0267288	.8900000	.3088432	.4608899
2.0481371	.9000000	.2989053	.4518620
2.0706524	.9100000	.2895527	.4427931
2.0945852	.9200000	.2808625	.4336182
2.1203639	.9300000	.2729392	.4242550
2.1486039	.9400000	.2659275	.4145937
2.1802399	.9500000	.2600318	.4044807
2.2167998	.9600000	.2555548	.3936832
2.2610614	.9700000	.2529780	.3818130
2.3190080	.9800000	.2531774	.3681119
2.4085247	.9900000	.2582407	.3505968
3.1383460	1.0000000	.2895613	.3028718

TABLE 4.2

GEOMETRICAL PARAMETERS OF THWAITES' (REF. 14)

CONTRACTION NO. 2

X	R	R'	R''
.0000000	1.0000000	.0000000	.0000000
.0765398	1.0000000	.0000000	.0000000
.1570796	1.0000000	.0000000	.0000000
.2356194	.9999889	-.0002287	-.0060136
.3141593	.9999381	-.0011749	-.0182246
.3926991	.9997645	-.0035897	-.0456014
.4712389	.9992983	-.0089229	-.0942078
.5492767	.9982311	-.0193197	-.1764873
.6283185	.9960521	-.0378293	-.3034911
.7088583	.9919713	-.0684983	-.4877597
.7853982	.9848503	-.1160321	-.7316151
.8639380	.9723190	-.1842832	-1.0063638
.9424728	.9553672	-.2726926	-1.2448924
1.0210176	.9300785	-.3716751	-1.2537590
1.0995574	.8971967	-.4625542	-1.0186097
1.1780972	.8581254	-.5269369	-.6055219
1.2566371	.8153206	-.5575546	-.1852985
1.3351769	.7713125	-.5580101	.1326121
1.4137167	.7280698	-.5399001	.3305844
1.4922565	.6868133	-.5093257	.4352976
1.5707963	.6482108	-.4731068	.4791609

TABLE 4.2 (Concluded)

X	R	R'	R''
1.6493361	.6125457	-.4349983	.4668586
1.7278760	.5798736	-.3971549	.4744224
1.8064158	.5501227	-.3607454	.4515726
1.8849556	.5231550	-.3263432	.4240157
1.9634954	.4988021	-.2941771	.3950412
2.0420352	.4768863	-.2642779	.3665127
2.1205750	.4572321	-.2365664	.3394574
2.1991149	.4396729	-.2109032	.3144114
2.2776547	.4240543	-.1871195	.2916236
2.3561945	.4102358	-.1650338	.2711753
2.4347343	.3980912	-.1444625	.2530525
2.5132741	.3875088	-.1252252	.2371895
2.5918139	.3783905	-.1071480	.2234953
2.6703538	.3706520	-.0900645	.2118710
2.7488936	.3642214	-.0738158	.2022195
2.8274334	.3590391	-.0582506	.1944521
2.9059732	.3550573	-.0432240	.1884929
2.9845130	.3522391	-.0285965	.1842808
3.0630528	.3505585	-.0142327	.1817216
3.1415927	.3500000	.0000000	.1809381

TABLE 4.3

COMPARISON BETWEEN THE VELOCITY DISTRIBUTION OBTAINED BY NUMERICAL SOLUTION
AND THAT OBTAINED BY THE ANALYTICAL SERIES
SOLUTION INSIDE THWAITES' (REF. 14) CONTRACTION NO. 2

THWAITES' SERIES SOLUTION

PRESENT NUMERICAL SOLUTION

X	R	q_c	q_w	q_c	q_w	$\frac{q_c - q_w}{q_c} \%$	$\frac{q_w - q_c}{q_w} \%$
.000000	.3500000	2.3600537	2.4352551	2.35956	2.43529	0.0208	0.001
.0765398	.3505585	2.3526815	2.4276022	2.35252	2.42754	.0068	0.003
.1570796	.3522391	2.3307139	2.4047903	2.33067	2.40464	.0017	0.004
.2356194	.3550573	2.2945927	2.3672552	2.29459	2.36715	.0000	0.004
.3141593	.3590391	2.2450389	2.3157076	2.24507	2.31560	.0014	0.009
.3926991	.3642214	2.1830305	2.2511112	2.18304	2.25102	.002	0.004
.4712389	.3706520	2.1097725	2.1746519	2.10973	2.17459		0.003
.5497787	.3783905	2.0266600	2.0877016	2.02671	2.08763		0.003
.6283185	.3875088	1.9352372	1.9917764	1.93536	1.99173	.006	0.002
.7068583	.3980912	1.8371320	1.8884910	1.83695	1.88851		0.001
.7853982	.4102358	1.7341092	1.7795124	1.73392	1.77951		0.0
.8639380	.4240543	1.6278241	1.6665114	1.62776	1.66661	0.004	0.006
.9424778	.4396729	1.5199770	1.5511205	1.51993	1.55125		0.008

TABLE 4.3 (Continued)

X	R	q_c	q_w	q_c	q_w	$\frac{q_c - q_w}{q_c}$	$\frac{q_w - q_c}{q_w}$
1.0210176	.4572321	1.4121714	1.4348890	1.41212	1.43506	0.012	0.012
1.0995574	.4768863	1.3058969	1.3192484	1.30582	1.31951	0.006	0.020
1.1780972	.4988021	1.2024979	1.2054817	1.20241	1.20583	0.029	0.029
1.2566371	.5231550	1.1031483	1.0947015	1.10305	1.09512	0.038	0.038
1.3351769	.5501227	1.0088340	.9878373	1.00871	0.98837	0.013	0.054
1.4137167	.5798736	.9203427	.8856364	0.92020	0.88629	0.074	0.074
1.4922565	.6123457	.8382607	.7886731	0.83810	0.78946	0.100	0.100
1.7278760	.7280698	.6334341	.5333511	0.69454	0.61318	0.022	0.168
1.8064158	.7713125	.5790804	.4615914	0.63331	0.53446	0.028	0.028
1.8849556	.8153206	.5313760	.3978864	0.57134	0.39887	0.007	0.247
1.9634954	.8581254	.4899608	.3438658	0.48996	0.34452	0.140	0.140
2.0420352	.8971967	.4543944	.3016349	0.45443	0.30175	0.038	0.038
2.1205750	.9300785	.4241805	.2728804	0.42428	0.27229	0.023	0.216
2.1991149	.9553672	.3987902	.2574275	0.39898	0.25612	0.508	0.508
2.2776547	.9731990	.3776841	.2526788	0.37789	0.25084	0.728	0.728
2.3561945	.9848503	.3603288	.2547875	0.36048	0.25264	0.042	0.843
2.4347343	.9919713	.3462178	.2602790	0.34639	0.25793	0.049	0.092
2.5132741	.9960521	.3348736	.2667836	0.33507	0.26442	0.059	0.886
2.5918139	.9982311	.3258662	.2729660	0.32607	0.27075	0.062	0.812

TABLE 4.3 (Concluded)

X	R	q _c	q _w	q _c	q _w	$\frac{q_c - q_w}{q_c}$	$\frac{q_w - q_c}{q_w}$
2.6703538	.9992983	.3188141	.2782042	0.31902	0.27625	0.064	0.702
2.7488936	.9997645	.3133887	.2823033	0.31359	0.28065	0.064	0.585
2.8274334	.9999381	.3093148	.2853030	0.30948	0.28399	0.053	0.460
2.9059732	.9999889	.3063709	.2873551	0.30650	0.28634	0.042	0.353
2.9845130	.9999984	.3043873	.2886456	0.30446	0.28787	0.024	0.269
3.0630528	.9999979	.3032443	.2893428	0.30325	0.28872	0.002	0.214
3.1415927	1.0000000	.3028712	.2895616	0.30277	0.28899	0.033	0.197

TABLE 4.4

DATA FOR THWAITES' CONTRACTION NO. 2

TAKEN FROM REF. 14

X	R	q _w	q _c
0	0.35	2.435	2.360
0.464	0.37	2.182	2.117
0.720	0.4	1.871	1.820
0.990	0.45	1.480	1.455
1.335	0.55	0.988	1.009
1.595	0.65	0.673	0.741
1.768	0.75	0.496	0.605
1.948	0.85	0.354	0.498
2.129	0.93	0.272	0.421
2.261	0.97	0.253	0.382
2.408	0.99	0.258	0.351
2.488	0.995	0.265	0.338
3.142	1.0	0.289	0.306

5. INITIAL RESULTS

5.1

SOME PRELIMINARY INVISCID FLOW RESULTS

The starting point for the inviscid flow calculation is considered to be the inlet to the settling chamber. A typical settling chamber length equal to the settling chamber radius - as shown in Figure 14 - is used in this study.

Computer experiments were performed to define a distance inside the test section, which the flow has to travel before it becomes essentially parallel. The results suggest that a distance of one local diameter is sufficient for the flow to become uniform and parallel. This is in agreement with the conclusions of Ref. 23 and Ref. 24.

After the development of the computer program and after tests on its accuracy were complete, it was necessary to subject the philosophy behind the choice of the contraction contours to some practical tests. It was important to ensure that changes in the controlling parameters actually resulted in the expected effects on the flow field.

For a fixed contraction ratio, the effect of changing the contraction length (but keeping the relative position of the inflection point fixed) on the R'' distribution, and consequently on the contraction shape, is shown in Figures 18 and 19. The velocity along the contraction wall for the above contractions, given in Figure 19 for contraction lengths of $L = 1, 2$ and 3 , shows that as the contraction is shortened, the adverse pressure gradient on the wall becomes steeper at both contraction ends. This indicates that there is a limit beyond which shortening the contraction results in a pressure

gradient severe enough to cause boundary-layer separation.

The effects of the position of the inflection point on the R'' distribution, and consequently, on the shape of the contraction contour, are shown in Figures 20(a) and (b) for a fixed contraction ratio of 8 and a fixed length, $L = 2$. Figure 21 shows the corresponding effect on the wall velocity, from which the anticipated trend is clear. For a fixed length, moving the inflection point downstream, results in lowering the adverse pressure gradient at inlet, but raising it at exit, and vice-versa. This means that if the flow in some contraction configuration is expected to separate at inlet - say - it would be possible to prevent the expected separation by moving the inflection point downstream without increasing the contraction length, provided that this would not separate the flow at exit.

The above reported results have confirmed the expected trends and verified the logic behind the specific choice of contour shape.

In an attempt to evaluate the adequacy of the present contraction contours in comparison with some analytically derived contours, Thwaites' contraction No. 2 (Ref. 14) was chosen for study. The availability of a closed form of the stream function, from which the contour shape and the wall velocity can be calculated, was the reason for the choice of that contraction for comparison purposes. The shape of Thwaites' contraction No. 2 and its R'' distribution are shown in Figure 22. Although the comparison should be made for the same contraction length, it was found that Thwaites' contraction has practically zero curvature for some length at

the inlet, and that the area change is very slow at exit. For that reason, and for the comparison to be fair to Thwaites' contraction, it was decided to compare it with a contraction of length 2.8 inlet radii (Thwaites' contraction length is $L = 3.14$). It was decided to keep the position of the inflection point at the same station, which is at about 0.42 of the contraction length. The comparison between the two wall contours and the corresponding R'' distributions is also shown in Figure 22. The resulting wall velocities in both contractions are compared in Figure 23, which shows that for the same contraction ratio, the same contraction length and same position of the inflection point, our suggested contour produces less severe adverse pressure gradients at both inlet and exit of the contraction.

5.2 VISCOUS FLOW CONSIDERATIONS

Only through the consideration of the viscous flow in the boundary layers near the contraction wall, will it be possible to judge the acceptability of results of the inviscid flow solution.

Inlet Reynolds No. Scale for Large Wind Tunnels

For consideration of the viscous flow, one needs to define physical contraction conditions; more precisely, we need to define an appropriate Reynolds number scale, i.e. $R_D = \frac{\rho_1 D_1 U_1}{\mu_1}$ where "1" refers to conditions at inlet to the contraction. Table 5.1 (a) shows data for some large existing and anticipated wind tunnels. The data in Table 5.1 (b) has been chosen as representative of that of a modern large low-speed wind tunnel and is used in the present study.

Boundary Layer Calculation

The prediction of turbulent boundary-layer growth and separation can be accomplished using one of many available methods. The large number of available methods is generally divided into three groups: differential methods, integral methods, and simple methods. Differential methods involve detailed solution of the boundary-layer partial differential equations. (see Ref. 36 for details of a number of these methods). These methods, in general, are lengthy and require large computer capacity but have the advantage of fully accounting for boundary-layer development history (in most cases) and yielding detailed boundary-layer structure both along and across the flow. This has been judged as an unnecessary complication for the

present purposes. In the integral methods of boundary layer prediction, only momentum integral or energy integral equations (i.e. ordinary differential equations) are solved, which means that the boundary layer equations are satisfied in some mean sense instead of at each point in the boundary layer (Ref. 36). It follows that these methods are easier to apply and are much faster to compute compared with the differential methods. Moreover, such methods yield sufficient information (rate of growth and skin friction) for our purposes.

The Lag-Entrainment method (Ref. 37) has combined the convenience of the integral methods with the accuracy expected from the best differential methods. Ref. 37 compares the accuracy of this method with that of the better differential methods; with favourable results. For that reason it was decided to use a computer program for predicting turbulent boundary layers in compressible 2-D or axisymmetric flow using Green's Lag-Entrainment method (Ref. 37) for the calculation of the boundary layers along the contraction walls.*

What are called simple methods, are actually just criteria derived from boundary-layer structural considerations to predict separation using inviscid flow data. Among these methods, Stratford's criterion (Ref. 25) is provably the most dependable, and has proved to predict separation reasonably accurately but conservatively (Ref. 23, Ref. 38, and Ref. 39). Stratford's criterion, besides its accuracy, is convenient for calculation purposes; hence, it will be used in this study. Then a more detailed Lag-Entrainment boundary-layer calculation will follow for verification purposes.

* A listing and computer deck for this program was made available by one of its authors, Mr. D.J. Weeks of RAE, Bedford, via the NAE at Uplands.

TABLE 5.1

a) DATA, AS FAR AS IT IS KNOWN, FOR SOME LARGE EXISTING AND ANTICIPATED WIND TUNNELS

TUNNEL NAME	T/S DIMENSION (m)	T/S SPEED (m/sec.)	CR	P ₁ (bar)	T ₁ (°F)	D ₁ (ft)	U ₁ (ft/sec.)	RD
NAE	9.1 x 9.1 (30' x 30')	55.	5.85	1.	100.	82.	30.	1.3 x 10 ⁷
GUK	9.5 x 9.5	55.	7.	1.	100.	93.	26.	1.3 x 10 ⁷
NLR	8 x 6	100.	9.	1.	100.	77.	37.	1.6 x 10 ⁷
ONERA F ₁	4.5 x 3.5	120.	12.	4.	100.	51.	33.	3.7 x 10 ⁷
RAE	5 x 4.2	109.	16	3.	100.	68.	22.	2.5 x 10 ⁷

b) DATA USED IN THE PRESENT STUDY

Contraction Ratio CR: 8

Settling Chamber D₁: 80 ft.U₁: 30 ft/sec.P₁: 1 barT₁: 100 °FRD: 1.3 x 10⁷

5.2.1 STRATFORD'S CRITERION FOR TURBULENT SEPARATION

Using Stratford's criterion to predict turbulent boundary-layer separation in two-dimensional incompressible flow, requires the use of only the pressure distribution along the surface and the Reynolds' number scale. Stratford's method is based upon dividing of the boundary layer into outer and inner regions. The outer is an historical region in which the pressure rise just causes a lowering of the dynamic pressure profile. In the presence of a rapid pressure rise, the shear forces in the outer part of the boundary layer are small compared with either the inertia forces or the pressure gradient hence, the shear forces are neglected in that region, even though vorticity is present. In the inner layer, on the other hand, the inertia forces are small, so that the mean velocity profile is distorted by the pressure gradient until the latter is largely balanced by the transverse gradient of shear stresses.

By formulating the above argument, with the use of $\frac{1}{n}$ - power law for the velocity distribution in the boundary layer outer region, the following condition at separation is reached:

$$\frac{(n+1)^{(n+1)/4} (n+2)^{1/2}}{(n-2)^{(n-2)/4}} C_p^{(n-2)/4} \left(S \frac{dC_p}{ds} \right)^{1/2} = 11.342 B (10^{-6} R_s)^{0.10} \quad (5.1)$$

where B is an empirical factor, which is found (experimentally) to be as follows:

$$B = 0.66 \quad \text{for } d^2_p/dS^2 < 0$$

$$B = 0.73 \quad \text{for } d^2_p/dS^2 > 0$$

S , in the above expressions, is the distance along the surface measured from an equivalent leading-edge. R_S is the Reynolds number based on the local distance S and the peak velocity. The value of n is suggested (Ref. 38) to be calculated from: $n = \log_{10} R_S$, i.e., $n = n(S)$.

For Reynolds' numbers of the order of 10^6 , the above expression reduces to:

$$C_p (S dC_p/dS)^{1/2} = k (10^{-6} R_S)^{0.10} \quad \text{for } C_p \leq \frac{4}{7} \quad (5.2)$$

$$\text{where } k = 0.39 \quad d^2_p/dS^2 \geq 0$$

$$= 0.35 \quad d^2_p/dS^2 \leq 0$$

The full expression (equation 5.1) has been used in this study.

It has been proven that the above criterion is consistently dependable, although it predicts separation to occur earlier than in practice in cases where it has been tested (Ref. 23, Ref. 38 and Ref. 39) i.e. it is a conservative criterion.

Stratford's criterion applies for an adverse pressure gradient, turbulent boundary-layer flow starting from the leading-edge. From that point of view, there is no problem in using it to predict the separation of the flow at entry to the contraction by assuming that the boundary

layer is turbulent after the last screen in the settling chamber, and by using a Reynolds' number based on the inlet velocity and the arc length measured along the contour starting at the last screen.

A problem, however, appears in trying to apply the criterion to the region of adverse pressure gradient near the contraction exit. The criterion as it is presently formulated is strictly not applicable in that highly non-equilibrium region, since the flow has experienced considerable acceleration before it enters the adverse pressure gradient region. To cope with a similar situation, Stratford (Ref. 25) suggested the use of a distance S from a virtual origin. The flow with favourable gradient is replaced with one at constant pressure and with a mainstream velocity equal to the peak mainstream velocity at the beginning of the adverse pressure-gradient region. The criterion of equivalence is the value of the boundary-layer momentum thickness, θ , at the point of peak velocity. Stratford (Ref. 25) presents an expression that is empirically derived for the calculation of the equivalent distance from the leading-edge. But since the momentum losses are generally different in the present convergent axisymmetric flow than in the two-dimensional flow considered by Stratford, an expression for the equivalent distance in an axisymmetric flow is derived in Appendix I, Part 1, and reads as follows:

$$S_{eq.} = \int_0^{S_m} \left(\frac{R}{R_m} \right)^{5/4} \left(\frac{U}{U_m} \right)^{27/7} dS \quad (5.3)$$

where the subscript m indicates the position of maximum velocity.

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5.2.2 POSSIBLE RE-LAMINARIZATION OF THE TURBULENT BOUNDARY LAYER

The flow along the contraction wall (see Figure 19, for example) will decelerate initially to a minimum value near the contraction inlet, and then will accelerate rapidly - and overshoot - to a maximum value near the exit before it decelerates again to the test section conditions.

In a typical large facility contraction (i.e. with high Reynolds' number conditions) no special attention was really necessary in the accelerated flow region. In the present work, where the task is to shorten the contraction as much as possible, flow acceleration could become sufficiently large to affect the turbulent structure of the boundary-layer. If the flow acceleration became high enough, boundary-layer re-laminarization might occur (for example see Refs. 40, 41, 42, 43, and 44). If the boundary layer does become laminar, its ability to stand the subsequent adverse pressure gradient at exit would be substantially reduced. The reverse transition of the boundary-layer from turbulent to laminar, as indicated by the studies referred to above, is a slow and gradual process, which makes the possibility of complete re-laminarization of the boundary-layer flow in a contraction unlikely. The major concern here about re-laminarization is, then, that it would probably invalidate the separation criterion adopted at outlet.

Although one would expect that re-laminarization probably depends upon most of the same parameters as the natural transition from

laminar to turbulent boundary layer flow; Reynolds number pressure gradient, free-stream turbulence, Mach number, heat transfer, wall curvature, roughness, etc., re-laminarization has been correlated in the literature with only one parameter, K defined as $K = \frac{v}{U^2} \cdot \frac{dU}{dx}$ (or with a combination of K and local skin friction coefficient, C_f).

As a result of the fact that re-laminarization is gradual and a "slow" process, it is not possible to associate its occurrence with a specific local value of K. Experiments (Refs. 41 and 44) have shown that when K exceeds a value of about 2×10^{-6} , laminarization effects will become significant. In the present study a check on the value of K on the contraction wall has been built into the computer code. The value of $K = 2.0 \times 10^{-6}$ has been taken as an indication of a possible change in the boundary-layer turbulent structure and, hence, presents a limit on the application of Stratford's turbulent criterion for predicting separation at the contraction exit. It was not judged to be valid to use Stratford's criterion for laminar boundary-layer downstream of such a condition.

The general procedure for checking the boundary-layer behaviour in the contraction after the inviscid flow has been calculated, is to use Stratford's criterion to check for possible separation at inlet through the adverse pressure gradient region; then, in the favourable pressure region, to check the value of K at each point to ensure that no re-laminarization might occur. From the contour shape and the velocity

an equivalent distance to a virtual origin is calculated to be used in Stratford's criterion to check possible boundary-layer separation within the adverse pressure gradient region at outlet.

The use of Stratford's criterion served as a first order indication of the effect of the various geometric parameters, then more detailed boundary-layer calculations were made using the Lag-Entrainment method of Green for predicting the boundary-layer parameters along the contraction contour. A brief account of this viscous flow calculation scheme is given in the next section.

5.2.3 BOUNDARY LAYER CALCULATION SCHEME

A computer program for predicting turbulent boundary-layers in compressible 2-D or axisymmetric flow developed by Green, Weeks and Brooman has been used to check the boundary layer behaviour in the contraction. This Lag-Entrainment approach is an integral method involving three coupled ordinary differential equations, momentum-integral, entrainment, and a rate equation for the entrainment coefficient. In Ref. 37, the predictions of that method are compared with a wide range of experimental data including a representative sample from the Stanford Conference. Its predictions of the Stanford (Ref. 36) test cases appear to be as good as, or better than, those of the methods placed in the first class by the Conference Evaluation Committee. This method allows for the effects of longitudinal surface curvature and lateral stretching of the boundary-layer flow.

The code requires the input of the contraction co-ordinates normalized by a reference length (the inlet radius in this case), the

contraction surface curvature, and the inviscid flow velocity distribution along the wall normalized by a reference velocity (inlet axial velocity U_1 in the present case). It is required also, to input the reference Reynolds' number scale and the stagnation temperature.

To start the computations, the program requires an initial value for both the momentum thickness, θ , and the shape factor, H . It has been assumed that the leading-edge of the turbulent boundary-layer is at the beginning of the flow, i.e. at the last screen, and by using the method of Appendix I, θ is calculated at a distance of 0.2 (i.e. $\Delta x/R_1 = 0.2$) and that value of θ/R_1 is taken as an initial value for the Lag-Entrainment computation. An initial value for H is assumed to be the equilibrium shape factor, $H = 1.4$ for flat plate turbulent flow.

Computer experiments on the effects of variations in these initial values on the development of the boundary-layer showed that these effects disappear in two or three computation steps $\Delta x/R = 0.4$ (i.e. in a distance of about $10^3 \theta_1$ for the Reynolds' number conditions of the present study.

Computation yields the distribution along the contour of the following parameters: local skin friction coefficient C_f ; the boundary-layer shape factor, H ; the non-dimensional momentum thickness, θ_1 , the displacement thickness, δ_1 , and the Reynolds' number, $R\theta_1$ based on momentum thickness.

Figures 24 (b), (c), (d), and (e) show a plotted sample of the program output for the case shown in Figure 24 (a); i.e. a 'short' contraction approaching optimum conditions. Figures 24 (b) through (d) show local C_f , θ and $R\theta$. Figure 24 (e) shows the effect of longitudinal curvature and lateral stretching on the predicted local skin friction coefficient, C_f .

6. CONTRACTION SHAPE OPTIMIZATION

6.1 RESULTS BASED ON STRATFORD'S CRITERION

A series of computer runs has been made in order to answer the main question of how short the contraction can be made. A variation of both length and position of inflection point in the contraction wall with specifications as shown in Table 5.1 (b), was made.

The contraction geometry is such that a settling chamber of length equal to its radius preceding the contraction is allowed for and the calculation is carried out up to one exit diameter into the constant area test section.

In searching for the minimum length at which no inlet separation occurs, the results followed the expected trend initially, i.e. the length decreased rapidly as the inflection point was moved downstream until $Z_{ip} = 0.55$ was reached - see Figure 25. After that, however, the inflection point has practically no effect on the minimum length in the range $Z_{ip} = 0.55 - 0.65$. For $Z_{ip} > 0.65$, moving the inflection point downstream results in the requirement of greater length to avoid inlet separation. This may sound unexpected, but a careful look at the characteristics of the chosen family of contours shows that these results are not unreasonable. This can best be explained by referring to Figure 26. For the same contraction length, moving the inflection point towards the exit reduces the maximum value of R'' at inlet, until the inflection point reaches 0.55 of the length. Moving it further downstream results in an increase in $|R''|_{\max}$. (see also Figure 26).

This confirms the sensitivity of the flow at inlet to the wall curvature

$|R''|$. Figure 27 shows that the maximum value of the wall pressure

coefficient at inlet closely follows variations similar to that of $R''_{\max}$ when the position of the inflection point is changed. Only the "turning point" of the curves on Figure 27 seems to be at about $Z_{ip} = 0.60$ compared with $Z_{ip} = 0.55$ in Figure 26. As a general result, the position of maximum C_{pw} is consistently downstream of the position of the maximum wall R'' . Figure 28 shows that, irrespective of the position of the maximum wall C_p , separation occurs (approximately) when $(C_{pw})_{\max} = 0.5$.

The position of the start of the predicted separation consistently happens upstream of the position of the maximum C_{pw} . The value of C_{pw} at the beginning of separation lies in the narrow range of 0.42 - 0.45. Figure 29 shows that the flow can stand more wall curvature without inlet separation as the position of maximum R'' is moved downstream.

Let us consider now the adverse pressure-gradient region at the contraction exit. The original scheme was to shorten the contraction and to push the inflection point downstream so as to increase the magnitude of the outlet adverse pressure gradient until the boundary layer there was on the point of separating. The results of applying Stratford's criterion for turbulent separation at exit, as described in Section 5.2.1, results in the fact that it is not possible actually to achieve boundary-layer separation near the contraction exit for reasons discussed in the following section.

The pressure coefficient to be used in Stratford's criterion is referred to the peak wall velocity upstream of the adverse pressure gradient region. Hence, the resulting magnitude of the maximum C_p at outlet is considerably smaller than that at inlet.

Figure 30 shows one extreme case as an example - a case with inlet separation. An even more important factor is the shape of the wall C_p curve - see Figure 30. At inlet, as C_p increases $\frac{dC_p}{ds}$ also, in general, increases, and separation would occur if $C_p \left(\frac{dC_p}{ds}\right)^{1/2}$ exceeds a certain value. At exit the shape of the pressure distribution is such that as C_p increases, $\frac{dC_p}{ds}$ decreases, and for separation $C_p \left(\frac{dC_p}{ds}\right)^{1/2}$ has to exceed a relatively higher value which corresponds to a generally higher Reynolds' number, (compared with the inlet region). For that reason, conditions that lead to exit separation are extremely difficult to achieve.

It is true that shortening the contraction and/or moving the inflection point downstream would result in a higher pressure gradient at exit (see Figures 31, and 32), but also, due to the more rapid acceleration, the boundary layer in the adverse pressure gradient region is very much thinner, as indicated by the shorter equivalent length shown in Figures 31 and 32.

Using Stratford's criterion, separation would be expected in regions where the value of the left-hand side (LH) of Equation 5.1 equals or exceeds that of the right-hand side (RH). The value of the difference (RH-LH) can be used as an indication of the margin of safety before separation would occur. Separation would occur if $(RH-LH) \leq 0$.

Figure 33 shows a plot of the minimum value of (RH-LH) - which we refer to as the margin of safety (MS) - at the contraction exit, as a function of the position of the inflection point, while the contraction length is held constant at $L = 1.15$ - which is quite short, see Table 2.1 and Table 2.2. As shown in Figure 33, the margin of safety is reduced by moving

the inflection point downstream while the length is kept constant.

The above discussion indicates that the more rapid acceleration which results from moving the inflection point downstream produces not only more severe adverse pressure gradient at exit, but also results in a thinner boundary-layer at the entry to that exit region, which would thus be able to stand higher adverse pressure gradients. The net result of the two opposing effects - as Figure 33 indicates - is a reduction in the margin of safety (MS). The reduction of that margin of safety due to inflection point movement towards the contraction exit as shown in Figure 33 is, however, not significant. This would suggest that, in general, the contraction exit is safe as far as turbulent boundary-layer separation is concerned.

In all cases considered so far, the test on the possibility of re-laminarization of the turbulent boundary-layer going through the rapid acceleration region before exit did not indicate any re-laminarization possibility.

From the above-reported results, it seems that inlet separation is the dominant factor when it comes to the selection of a contraction from our particular family of contours. Using Stratford's criterion to judge the possibilities of turbulent boundary-layer separation, the results presented in Figure 25 suggest that the best member of the presently-used family of contours to be used with the specified conditions, is the one with a length of 1.3 of its inlet radius with its inflection point anywhere between 0.55 - 0.60 of its length.

6.2

BOUNDARY-LAYER CALCULATION RESULTS

The computer code being based on integral methods is relatively fast. A typical run to compute the boundary-layer characteristic along the contraction wall starting at the beginning of the settling chamber through to one contraction exit diameter into the test section, will typically take less than 5 minutes on a Xerox 530 machine. Including allowances for such secondary influences as streamline longitudinal curvature, and flow lateral stretching, can easily double the computation time.

A series of computer runs for geometrics with parameters in the range suggested by the application of Stratford's criterion has been carried out. A summary of the results obtained is shown in Figures 34 and 35. The trends of the effect of the contraction length and the position of the inflection point are in complete agreement with the trends indicated by the results of applying the simple criterion of Stratford. Only the results of applying Stratford's criterion seem to be conservative. Stratford's criterion results in the conclusion that with a contraction length of $L = 1.2$, inlet separation is expected, while the boundary-layer calculations show that a contraction of that length is safe from separation. This is clear by comparing the results shown in Figure 25 with those on both Figures 34 and 35.

To choose a contraction - the shortest contraction free of inlet separation - on the basis of the boundary-layer calculations, we must decide upon the criterion for separation. Actually, for two-dimensional and axisymmetric flows, the separation point is defined as the point where the wall shear stress is equal to zero, i.e. where $C_f = 0$, and this parameter is used to predict the separation point, when a differential method

is used for boundary-layer calculations. In integral methods, on the other hand, the shape factor, H is usually used to locate the separation point. In integral methods, as flow approaches separation, the value of H increases rapidly. Separation of the flow is assumed to occur when H reaches a value between 1.8 and 2.4 (Ref. 38, Ref. 39 and Ref. 46).

In the present calculation, a value for the skin friction less than 5×10^{-4} has been obtained. Although it did not go to zero, the shape factor exceeded the value of 2, which indicates possible separation. For the present purpose then, a shape factor, $H > 1.8$ will be considered as a useful indicator of possible separation.

Based on the results obtained and shown in Figure 35, and using the separation criterion mentioned above, the shortest contraction would be the one with $L = 1.2$. As the results shown in Figure 35 indicate, contractions of length 1.2 times their inlet radius can be used without fearing boundary-layer separation if the inflection point is located in the range of 0.55 - 0.65 of its length. Figure 35 also shows that it would be better, i.e. there would be more margin of safety before separation, if $Z_{ip} = 0.60$ were used.

7. MODIFIED FAMILY OF CONTOURS

7.1 A MODIFIED CONTRACTION CONTOUR SHAPE

The experience gained through working with the originally chosen "curvature" distribution and the trends of the results obtained have been a great help in choosing a modified distribution which overcomes the difficulties previously encountered.

The main difficulty with the previous R'' - distribution was the change of the shape of that distribution at inlet when the inflection point is pushed downstream beyond a certain limit. That change in shape is such that the curvature increases very slowly at inlet which represents inefficient utilization of the inlet segment length.

The second difficulty encountered during computer experimentation with the chosen family of contours was the inability to change the length of the exit segment independently. To shorten the length of the exit part it was necessary either to shorten the total length, keeping the relative position of the inflection point, or to move the inflection point downstream at the same total length. Both alternatives - beyond certain limits - yielded undesirable effects on the flow in the inlet region. (See Figures 36 (a) - 37 and Appendix IV).

The required properties in a modified contraction shape, as a direct consequence of the above discussion, would be the ability to control both inlet length - i.e. length from inlet to inflection point and exit length - i.e. from inflection point to exit - individually, to

achieve the best possible results. The shape at inlet should provide a rapid increase of curvature from its zero value at entry in order to utilize the contraction length to its full extent. The results of section 6 show the importance of keeping the maximum magnitude of R'' as low as possible; this can be achieved most effectively by distributing the curvature over the total contraction length instead of concentrating it over a narrow range. Also, there are some indications from the previous results, that the flow is able to tolerate a higher magnitude of R'' when the maximum value is moved downstream.

Actually, from the successful correlation between the magnitude and trends of both the R'' - distribution and the resulting C_p - distribution, (as it is clear from the results of section 6) one should be able to specify requirements for the R'' - distribution from known requirements of C_p - distribution. As indicated by Stratford's criterion for turbulent separation, what is required is to keep a quantity $SP = C_p \left(S \frac{dC_p}{dS} \right)^{1/2}$ within limits before separation can occur. This means that where C_p is small, $\frac{dC_p}{dS}$ can be large, as C_p increases $\frac{dC_p}{dS}$ should decrease. Generally, one would like to keep SP at its maximum possible value throughout the adverse pressure gradient region. The above consideration and discussion lead one to postulate that the R'' distribution in the inlet region up to the inflection point should be as shown in Figure 38.

In a search for a mathematical function that would have the required shape and at the same time contain some adjustable parameters to control both the rapidity of R'' magnitude increase at the start and even more important, to control the location of the maximum R'' magnitude, the following functional form is found to possess these requirements.

$$R'' = -A(1 + Bz) \left[\sin (\pi z) \right]^N, \quad 0 \leq z \leq 1$$

In this function, for $N < 1$, as N decreases the "squareness" of the function increases, i.e. decreasing N provides a sharp increase in R'' magnitude at the entrance to the segment. B is a parameter to control the location of $R''_{\max}$. For $B = 0$, the maximum is at $z = 0.5$, whereas increasing B , moves the maximum location downstream; decreasing it (i.e. $B < 0$) has a reverse effect. The effects of the parameters B and N on the R'' distribution shape are shown in Figures 39(a) and 39(b) respectively.

A similar function for the R'' distribution in the exit part of the contraction, i.e. from the inflection point to the exit plane, is judged to be suitable due to the flexibility it provides for changing the exit length independently of the inlet part, also it facilitates changing the shape of the distribution, independent of the length.

The R'' distribution over the contraction length is:

$$R'' = -A_1 (1 + B_1 Z_1) \left[\sin (\pi Z_1) \right]^N$$

$$0 \leq Z_1 \leq 1$$

and:

$$R'' = A_2 (1 + B_e Z_2) \left[\sin (\pi Z_2) \right]^N$$

$$0 \leq Z_2 \leq 1$$

(7.1)

The parameters B_1 , B_e , N_1 , and N_e are to be chosen.

The imposed condition of zero slope at exit starting with zero slope at inlet defines the relative amplitude A_{21} as:

$$A_{21} = \frac{A_2}{A_1} = \frac{\int_0^1 (1 + B_1 Z_1) [\sin(\pi Z_1)]^{N_1} dZ_1}{\int_0^1 (1 + B_e Z_2) [\sin(\pi Z_2)]^{N_e} dZ_2} \quad (7.2)$$

The slope is to be calculated from:

$$\frac{R'}{A_1} = - \left\{ \int_0^{Z_1} (1 + B_1 Z_1) [\sin(\pi Z_1)]^{N_1} dZ_1 \right\} \cdot l_1$$

for $0 \leq Z_1 \leq 1$

and,

$$\begin{aligned} \frac{R'}{A_1} = & - \left[\int_0^1 (1 + B_1 Z_1) [\sin(\pi Z_1)]^{N_1} dZ_1 \right] \cdot l_1 \\ & + A_{12} \left[\int_0^{Z_2} (1 + B_e Z_2) [\sin(\pi Z_2)]^{N_e} dZ_2 \right] \cdot l_e \end{aligned} \quad (7.3)$$

for $0 \leq Z_2 \leq 1$

The co-ordinates of the contour can be calculated by integration of the slope;

$$R = R_1 + A_1 \int_0^x \frac{R'}{A_1} dx \quad (7.4)$$

where the amplitude parameter A_1 can be calculated from:

$$A_1 = R_1 \left(1 - \frac{1}{\sqrt{CR}} \right) / \int_0^L \frac{R'}{A_1} dx \quad (7.5)$$

The above choice for R'' contains a large number of controlling parameters: B_i , B_e , N_i , N_e , as well as the inlet length ($X_{ip} - X_i$), and exit length ($X_e - X_{ip}$). These latter two parameters are equivalent to the original total length and position of the inflection point, except that in the presently modified scheme the choice of that length would not affect the shape of the R'' distributions. In the new family we have, of course, the extra controlling parameters: B_i , N_i , B_e and N_e to provide flexibility in choosing R'' shape in both regions independently.

To incorporate the new shape in the computer code, it was necessary to modify only the subroutine "SHAPE". The input to the program, besides the inlet flow data, inlet radius and contraction ratio, should include inlet length, exit length, inlet B_i and N_i and exit B_e and N_e .

The presently modified shape has been used in a wide range of parametric experiments on the computer, starting, however from the previously established results.

7.2

INITIAL RESULTS OBTAINED WITH THE MODIFIED
FAMILY OF CONTOURS

The modified "curvature" distribution described by equation 7.1 has been used in a parametric study on the computer to assess the effects of the different geometric parameters on the flow behaviour inside the contraction.

The way in which the R'' -distribution for the modified family of contours is defined (Equation (7.1)) makes it possible to regard the contraction as consisting of two geometrical sections. An inlet section extends from the contraction inlet to its inflection point, and an exit section extends from the inflection point to the contraction exit. The length and shape of the R'' -distribution of each of these sections can be varied independently.

A word has to be said here regarding the continuity of the contraction wall at the joining planes at the inlet and outlet and at the inflection position. As indicated by the transformed flow governing equation, (Equation 4.2) which contains the duct geometrical parameters, the flow is affected by the R , R' and R'' distributions of the duct wall. Hence, as far as the flow is concerned, the duct is continuous as long as its wall R , R' and R'' are continuous. No attempt has been made to ensure continuity of the slope of the R'' -distribution at the joining planes.

One has to emphasize here that the above-mentioned geometrical sections are not necessarily related to the integration segments described in Section 4.1.2 and shown in Figure 16. Considering the contraction to consist of two geometric sections is just a convenient way to describe the contraction shape. Once this is done, a solution for the flow inside the

defined shape is sought. To facilitate the numerical solution, the whole domain of interest is divided into integration segments which could or could not be related to physical geometrical divisions. The argument of Section 4.1.2 and Figure 16 is still valid independently of how we define the contraction shape.

The following two subsections describe the effects of the inlet geometric parameters, i.e. N_1 , B_1 and L_1 , and the exit geometric parameters, i.e. N_e , B_e and L_e , on the flow field. In the following parametric study nominal values for the parameters are assumed to be $N_1 = N_e = 1.0$, $B_1 = B_e = 0.0$, $L_1 = 0.8$ and $L_e = 0.6$. The only exception is made when studying the effects of both B_1 and B_e ; in that case nominal values for $N_1 = N_e = 0.1$ are considered, because effects of B_1 and B_e on the R'' -distribution are more significant at smaller N_1 and N_e respectively.

7.2.1

EFFECTS OF INLET SECTION GEOMETRIC PARAMETERS ON THE FLOW FIELD

The effects of the geometric parameters of the contraction inlet section, namely N_1 , B_1 , and L_1 , on the flow field are first examined. The effect of N_1 on the shape of the contraction R'' -distribution is shown in Figure 40. Only the R'' -distribution in the inlet section is affected by changing N_1 , with no change in the exit section distribution.

The effect of N_1 on the pressure coefficient at the contraction inlet $(Cp)_1$ is shown in Figure 41 which shows that the maximum value of $(Cp)_1$ can be reduced by decreasing the value of N_1 , i.e. providing a lower magnitude of R'' for the same contraction length. These results are in agreement with the original expectations. Figure 42 shows the effect of N_1 on the distribution of $(RH-LH)_1$, where RH and LH are the right hand

side and left hand side of Stratford's criterion Equation (5.1) respectively. Boundary layer separation is expected when $(RH-LH)$ equal or are less than zero. As shown in Figure 42 separation would be expected for $N_1 = 3.0$. Boundary layer separation thus becomes less likely as N_1 decreases. Effects of N_1 on the flow in the contraction exit region are found to be insignificant. Table 7.1 and Table 7.2 show the effect of N_1 on $(Cp)_e$ and on $(LH-RH)_e$ respectively in the adverse pressure gradient region at exit.

The effect of varying B_1 on the shape of the contraction R'' -distribution is shown in Figure 43. Varying B_1 changes both the shape and maximum magnitude of R'' in the inlet section. It only affects the R'' magnitude in the exit section. As B_1 increases from negative to positive, the position of maximum R'' magnitude moves downstream and the effect of that on the pressure coefficient at contraction inlet is shown in Figure 44. Figure 44 shows that a lower value for maximum $(Cp)_1$ can be achieved by increasing B_1 . Figure 45 shows that increasing B_1 has a favourable effect on the viscous flow, as one would expect from its effect on $(Cp)_1$ as shown in Figure 44. The effects of B_1 on both $(Cp)_e$ and $(RH-LH)_e$ in the adverse pressure gradient region at the exit are shown in Table 7.3 and Table 7.4 respectively. These effects are not significant and are mainly because of the change in the R'' magnitude in the contraction exit region.

The contraction inlet section length has a significant effect on the magnitude of R'' in both inlet and exit section as shown in Figure 46. The corresponding effects on the inlet flow are shown in Figure 47 and Figure 48. Figure 47 shows the effect of L_1 on the pressure coefficient in the contraction inlet region $(Cp)_1$; as L_1 decreases, the maximum $(Cp)_1$ increases rapidly. Figure 48 shows that the minimum value of $(RH-LH)_1$,

i.e. $(MS)_1$ decreases drastically with L_1 for such major steps in shortening. Figures 49 and 50 show the effect of the contraction inlet length on the flow in the contraction exit section and initial part of the working section. The effects of L_1 on $(Cp)_e$ are shown in Figure 49 which shows a significant effect on the magnitude of the velocity overshoot, however, the effect of L_1 on $(MS)_e$ is found to be unimportant as shown in Figure 50.

7.2.2 EFFECT OF EXIT SECTION GEOMETRIC PARAMETERS ON THE FLOW FIELD

The effects of the exit geometrical parameters N_e , B_e and L_e on the contraction flow field are next examined and presented here.

The effect of N_e on the shape of the contraction R'' -distribution is shown in Figure 51. N_e changes the R'' -distribution in the exit section only. The consequent effect of N_e on the exit pressure coefficient is shown in Figure 52. The effect of N_e on the boundary layer margin of safety at exit $(MS)_e$ is shown in Table 7.5, and clearly has only a minor effect. The effects of N_e on the inlet flow parameters $(Cp)_1$ and $(LH-RH)_1$ are generally insignificant as can be seen from examining Tables 7.6 and 7.7.

The effect of B_e on the contraction R'' -distribution is shown in Figure 53. Changing B_e essentially changes both the shape and magnitude of R'' -distribution in the contraction exit section. However, it only changes the R'' magnitude in the inlet section to preserve the zero exit slope condition. As B_e increases from the value of -1.0 to a value of 1.0, the position of maximum R'' magnitude in the exit section moves downstream, and the maximum value of R'' is reduced in both contraction inlet and exit regions. Figure 54 shows that as far as $(Cp)_e$ is concerned, it is beneficial to use lower values of B_e . Also Figure 55 shows a slightly higher boundary layer margin

of safety for $B_e = -1$, than that for $B_e = 1.0$. The effects of B_e on the flow in the inlet section are shown in Figures 56 and 57.

The use of $B_e = 1.0$ results in a lower magnitude of R'' in the inlet section than that corresponding to $B_e = -1.0$, consequently $(Cp)_1$ is lower and $(RH-LH)_1$ is higher in the case of $B_e = 1.0$ as shown in Figures 56 and 57 respectively. Thus we see opposing effects at exit and entry for the influence of B_e .

The effects of changing the contraction exit section length L_e are shown in Figures 58 through 63. Figure 58 shows the effect of shortening the exit section length on the "curvature" distribution. As expected, shorter L_e results in higher R'' at not only the exit region but also in the inlet region. The resulting effect on $(Cp)_e$ is shown in Figure 59. As expected, higher and higher values for $(Cp)_e$ are obtained due to shortening L_e . The effect of shorter exit segments on the quantity $(RH-LH)_e$ is shown in Figure 60. The effect of decreasing L_e on the boundary layer separation margin of safety $(MS)_e$ at exit is shown in Figure 61, and indicates that although $(MS)_e$ is reduced by decreasing the length L_e , the exit boundary layer is quite safe from separation even at the extremely short exit length of $L_e = 0.1$. The effects of shortening L_e on the flow in the contraction inlet region are shown in Figures 62 and 63. Figure 62 shows higher $(Cp)_1$ for shorter L_e , and Figure 63 shows significant reduction in $(MS)_1$ as L_e is drastically decreased.

One should indicate that shortening the exit section length to extremely small values as indicated above, forces most of the flow acceleration to occur in a short distance and hence necessitates using a smaller integration step size near the contraction exit.

7.2.3

RE-EXAMINATION OF DESIGN CRITERIA

The design criteria as discussed earlier in the text define the optimum contraction as the shortest possible one to satisfy the function of providing the working area of the test section with steady, uniform flow. Avoiding boundary layer separation at contraction inlet and exit is considered enough assurance of steadiness of the flow providing, also, that flow entering the contraction section from the settling chamber is steady. Results obtained so far show the possibility of reducing the length of the contraction exit section significantly without fear of boundary layer separation or re-laminarization. Such an extreme reduction of the contraction exit section length would have a degrading effect on the quality of the flow in the initial part of the test section due to the velocity overshoot involved. A quantitative measure to an acceptable level of flow non-uniformity has thus to be defined.

The high quality of modern aerodynamic testing calls for a high degree of flow uniformity. Uniformity, defined as the excess, or deficiency, in local velocity from the nominal velocity as a fraction of the nominal value, is required to be maintained within 0.25% in the part of the test section used for model testing in recent wind tunnels.

The requirement of 0.25% flow uniformity in the test section has then been considered as a criterion in the following parts of the present study.

TABLE 7.1

EFFECT OF N_1 ON $(Cp)_e$

$CR = 8.0,$ $L_1 = 0.8,$ $L_e = 0.6$

$B_1 = 0.0,$ $N_e = 1.0,$ $B_e = 0.0$

X	$(Cp)_e$	
	$N_1 = 0.1$	$N_1 = 3.0$
2.33	0.0126	0.0121
2.40	0.0671	0.0694
2.56	0.1118	0.1123
2.72	0.1192	0.1195
2.88	0.1205	0.1207
3.04	0.1207	0.1209

TABLE 7.2

EFFECT OF N_1 ON $(RH-LH)_e$

$CR = 8.0,$ $L_1 = 0.8,$ $L_e = 0.6$

$B_1 = 0.0,$ $N_1 = 1.0,$ $B_e = 0.0$

x	$(RH-LH)_e$	
	$N_1 = 0.1$	$N_1 = 3.0$
2.33	10.56	10.56
2.40	9.50	9.48
2.56	9.84	9.85
2.72	10.35	10.35
2.88	10.70	10.70
3.04	10.95	10.95

TABLE 7.3

EFFECT OF B_1 ON $(Cp)_e$

$CR = 8.0,$ $L_i = 0.8,$ $L_e = 0.6$

$N_i = 0.1,$ $N_e = 0.1,$ $B_e = 0.0$

X	$(Cp)_e$	
	$B_1 = -1.0$	$B_1 = 1.0$
2.40	0.0677	0.0669
2.56	0.1133	0.1273
2.72	0.1210	0.1372
2.88	0.1224	0.1389
3.04	0.1226	0.1391

TABLE 7.4

EFFECT OF B_1 ON $(RH-LH)_e$

$CR = 0.8,$ $L_1 = 0.8,$ $L_e = 0.6$

$N_1 = 1.0,$ $B_e = 0.0,$ $N_e = 1.0$

X	$(RH-LH)_e$	
	$B_1 = -1.0$	$B_1 = 1.0$
2.40	9.40	9.33
2.56	9.79	9.62
2.72	10.31	10.22
2.88	10.67	10.62
3.04	10.93	10.90

TABLE 7.5

EFFECT OF N_e ON $(MS)_e$

$CR = 8.0,$ $L_1 = 0.8,$ $L_e = 0.6$

$N_1 = 1.0,$ $B_1 = 0.0,$ $B_e = 0.0$

N_e	$(MS)_e$
0.1	9.34
3.0	9.13

TABLE 7.6

EFFECT OF N_e ON $(C_p)_e$

$CR = 8.0,$ $L_1 = 0.8,$ $L_e = 0.6$

$N_1 = 1.0,$ $B_1 = 0.0,$ $B_e = 0.0$

X	WALL $(C_p)_1$	
	$N_e = 0.1$	$N_e = 3.0$
0.000	0.0000	0.0000
0.20	0.0069	0.0069
0.40	0.0185	0.0186
0.60	0.0433	0.0434
0.80	0.0993	0.0995
1.00	0.2294	0.2294
1.07	0.3039	0.3035
1.14	0.3721	0.3713
1.21	0.4187	0.4172
1.28	0.4327	0.4301
1.35	0.4052	0.4008
1.41	0.3251	0.3177
1.49	0.1730	0.1610
1.56	-0.8914	-0.1072

TABLE 7.7

EFFECT OF N_e ON $(LH-RH)_1$

$CR = 8.0,$ $L_1 = 0.8,$ $L_e = 0.6$

$N_1 = 1.0,$ $B_1 = 0.0,$ $B_e = 0.0$

X	$(RH-LH)_1$	
	$N_1 = 0.1$	$N_1 = 3.0$
0.20	8.49	8.49
0.40	9.06	9.06
0.60	9.27	9.26
0.80	8.79	8.79
1.00	6.73	6.24
1.07	3.31	3.33
1.14	2.31	2.37
1.21	3.32	3.43

7.3

SELECTION OF CONTRACTION SHAPE PARAMETERS

In selecting the best values for the different parameters involved, parameters for both the inlet section and exit section have been selected independently. It was assumed that the flow in the inlet region is affected primarily by the shape of the inlet segment, and the exit segment shape would have only a second order effect on the inlet flow. A similar assumption was used with regard to exit flow.

Nothing in the above arguments should be taken to deny the upstream effect of the exit segment shape on the inlet flow, or the effect of the shape of inlet segment on the flow downstream of it. All that is implied is that the local effects dominate and the far end would have effects of second order. Although the initial results presented in Section 7.2 are in general agreement with the above assumption, it also shows that, it is not strictly true, and the effects of geometric parameters of one end of the contraction on the flow on the other end should be carefully watched.

The shape parameters for the contraction inlet section are to be chosen based on inlet flow separation considerations. The shape of the exit section is to be chosen on the basis of exit flow separation (which is unlikely), consideration of test section flow uniformity, and/or the possibility of boundary layer re-laminarization.

The values for the shape parameters which are eventually arrived at are: $B_i = 1.5$, $N_i = 0.01$, $B_e = 0.0$, and $N_e = 3.0$. In presenting the results which led to each of the above values, the other parameters are held constant at their finally chosen values, while the specific parameter under consideration is changed over a wide range. It is clear that quite a few iterations are actually performed to reach the final set of values for the parameters, but only the last iteration is discussed here to make the presentation manageable.

7.3.1

SHAPE PARAMETERS FOR THE INLET SECTION

The parameter B_1 controls the position of maximum R'' magnitude at inlet. As B_1 increases from negative, to zero, to positive values, the position of maximum R'' moves from upstream of the inlet segment middle point to its middle point, to downstream of it, as shown previously in Figure 39(a) and Figure 43. The best value for B_1 is that which keeps the contraction inlet safe from boundary layer separation. A good measure of safety of separation is the Margin of Safety (MS). Boundary-layer separation occurs when MS becomes less than, or equal to, zero. An increase in MS positive value indicates an increase of the margin of safety to separation and consequently a greater possibility of decreasing the segment length before separation can occur. Table 7.8 shows the effect of B_1 on (MS). These results, together with that previously presented in Figure 43, Figure 44 and Figure 45, show that improved contraction performance at inlet is expected as B_1 increases. At $B_1 = 1.5$, the maximum magnitude of R'' in the inlet region is already pushed all the way downstream and very little change in geometry can be realized by increasing B_1 beyond 1.5. The value $B_1 = 1.5$, was then, chosen for the present contraction.

The shape of the R'' -distribution corresponding to $B_1 = 1.5$ is shown in Figure 76 and agrees with the expectations based on the results of Section 6.

It is useful at that point to check the validity of the assumption made that changes in the inlet section shape as a result of changing B_1 would have only second order effects on the flow at the exit region. Table 7.8 shows the effect of changing B_1 on the maximum wall velocity at exits in ratio to the test section velocity, and on $(MS)_e$. This, together with the results of Tables 7.3 and 7.4, indicates the validity of that assumption.

Table 8.9, together with the previously presented data in Figures 41 and 42, shows the effect of N_1 on the flow parameters in the contraction. Smaller N_1 results in a less severe pressure gradient at inlet. This is in agreement with the original expectations. Decreasing N_1 below 0.01 results in very little change in R'' -distribution and contraction geometry and this value, $N_1 = 0.01$, was chosen for the present contraction.

The effects of N_1 on exit flow parameters are insignificant as shown in Table 7.1, Table 7.2, and Table 7.9.

TABLE 7.8

EFFECT OF THE CONTRACTION INLET SHAPE PARAMETER " B_1 " ON
FLOW PARAMETERS AT BOTH CONTRACTION INLET AND EXIT

CR = 8.0

$R_1 = 40 \text{ ft}$

$U_1 = 30 \text{ ft/sec}$

$L_1 = 0.65$

$L_e = 0.45$

$N_1 = 0.01$

$N_e = 3.0$

$B_e = 0.0$

B_1	$(MS)_1$	Q_{max}/U_o	$(MS)_e$
-1.00	-1.4	1.125	8.6
-0.50	-0.5	1.131	8.4
0.0	-0.0	1.134	8.4
0.50	0.3	1.136	8.4
1.00	0.5	1.137	8.4
1.50	0.7	1.138	8.3

TABLE 7.9

EFFECT OF INLET SHAPE PARAMETER " N_1 " ON FLOW
PARAMETERS AT BOTH CONTRACTION INLET AND EXIT

CR = 8.0

$R_1 = 40$ ft

$U_1 = 30$ ft/sec

$L_1 = 0.65$

$L_e = 0.45$

$B_1 = 1.5$

$B_e = 0.0$

$N_e = 3.0$

N_1	$(MS)_1$	$(MS)_e$	$(Q)_{max}/U_o$
3.0	-5.7	8.4	1.14
1.0	-1.6	8.3	1.14
0.2	0.5	8.3	1.14
0.01	0.7	8.3	1.14

7.3.2

SHAPE PARAMETERS FOR THE EXIT SECTION

The choice of the exit segment shape and length would be based on the consideration of avoiding separation of the flow through the adverse pressure gradient region, and/or on consideration of achieving uniform flow in the test area.

Figures 64 through 68 show the effects of the exit shape parameters B_e and N_e on the contraction flow parameters.. These figures, together with Figures 68 and 69, which show the effect of shortening the contraction exit segment to an extremely short length, confirm the conclusion reached before that the contraction exit region is quite safe from boundary layer separation.

In the absence of possible boundary layer separation at exit, the boundary layer margin of safety at exit $(MS)_e$ loses its significance as a criterion for the choice of best shape parameters. The flow uniformity in the test section becomes our criterion for the selection of the shape of the contraction exit providing no boundary layer re-laminarization would occur.

It would be useful at that point to consider the characteristics of the decay of the flow non-uniformity with distance inside the constant area test section. Figures 72 and 73 show the decay of the flow non-uniformity as a function of the distance inside the test section measured from the contraction exit. The above results suggest that the decay in the flow non-uniformity can be represented by the exponential form $e^{-\alpha x}$ where the coefficient of decay at the wall α_w and that at the centreline α_c are generally different and each of them is a weak function of the position. An average value for α_w and α_c over the distance of one exit radius in the test section measured from the contraction as function of the geometrical parameters is shown in Table 7.10.

The above results show that the geometrical parameters of the contraction exit section not only affect the flow uniformity at the contraction exit, but also affect the decay rate of the flow non-uniformity inside the test section. However, the latter effect is rather negligible, and the trends of the shape parameters' effect on the flow uniformity at exit is preserved inside the test section.

Based on the above results values for B_e and N_e were chosen on the basis of flow uniformity at test section inlet for a fixed contraction length. The length of the contraction exit section is, then, defined by a required absolute value for the flow uniformity in the test section.

Table 7.11 shows the effect of B_e on the exit flow uniformity. Reducing B_e from a positive value to zero, to negative values, moves the position of the maximum exit R'' in the upstream direction away from the test section which results - as expected - in better flow uniformity at the contraction exit. This effect, however, is weak. It should be noticed that moving B_e to its extreme negative value of -1.0 would have an appreciable degrading effect on the inlet separation margin of safety $(MS)_1$ as shown in Figures 57 and 66.

Since positive B_e degrades the flow uniformity to the test section, while negative B_e unfavourably affects the inlet flow separation margin of safety, it would seem reasonable to keep the peak of the exit R'' in the middle of the exit segment by choosing $B_e = 0.0$.

To study the effect of N_e on the exit flow uniformity, B_e was fixed at its zero value and N_e is changed from $N_e = 0.01$ to $N_e = 3.0$. The effect of changing N_e on the exit flow uniformity is shown in Figure 70.

Although increasing N_e improves the exit flow uniformity that effect is weak. Hence, a rather arbitrary choice for $N_e = 3.0$ was made.

TABLE 7.10

EFFECT OF BOTH N_e AND B_e ON FLOW NON-UNIFORMITY

DECAY FACTOR α_c and α_w *

CR = 8.0, L = 1.0, $Z_{1P} = 0.65$, $B_1 = 1.5$, $N_1 = 0.01$

$B_e = 0.0$			$N_e = 3.0$		
N_e	α_w	α_c	B_e	α_w	α_c
0.5	4.11	3.43	-1.0	3.53	3.56
1.0	4.08	3.46	-0.5	3.53	3.56
2.0	3.93	3.45	0.0	3.58	3.56
3.0	3.58	3.57	0.5	3.61	3.56

* The non-uniformity along the centreline decay as $e^{-\alpha_c x}$ and along the wall as $e^{-\alpha_w x}$.

TABLE 7.11

EFFECT OF B_e ON FLOW UNIFORMITY
AT CONTRACTION EXIT

CR = 8.0, L_1 = 0.65, L_e = 0.35

B_1 = 1.5, N_1 = 0.01, N_e = 3.0

B_e	PERCENT FLOW UNIFORMITY	
	Along Wall	Along Centreline
-1.0	2.37	-6.45
-0.5	2.70	-7.17
0.0	3.00	-7.70
0.5	3.16	-7.74

7.4

THE CONTRACTION LENGTH (L)

In the preceding section a choice has been made of the shape of the R'' distribution for both the inlet and exit sections separately. Using the selected parameters for the contraction shape, the length of the contraction is yet to be defined. The following two sections are devoted to the selection of the shortest possible contraction to satisfy the design criteria.

7.4.1

THE EXIT SECTION LENGTH (L_e)

The choice of a length for the exit section of the contraction depends strongly on the required flow quality in the test section. Table 7.12 shows the flow uniformity at different stations in the test section as a function of the exit segment length. Study of the results presented in Table 7.12 show that, at the same station, the flow uniformity is worse along the centreline than that along the wall. This would mean that the centreline flow uniformity would be the dominating factor in choosing the contraction exit length.

As shown in Table 7.13 the contraction exit length has an appreciable effect on the separation margin at both inlet and exit. Since the exit end is safe from separation anyway, only the effect on the inlet separation should be watched, together with the effect on the flow uniformity in the test section while choosing the exit length.

Table 7.14 shows the required length for the contraction exit section L_e as a function of the flow settling length in the test section, i.e. the length allowed for the flow to settle to the required flow uniformity of 0.25%. Table 7.14 shows a strong dependence of the

contraction exit length on that "unused" length of the test section.

For a settling length of 1.5 times the test section radius, L_e can be as short as 0.15. But for a settling length of only 0.5 times the test section radius, the contraction exit segment length L_e should be as large as one.

As an example of a current practice, if one local radius in the test section is left for the flow to settle upon leaving the contraction, a contraction exit section length of 0.45 would be required. This length has been chosen in the subsequent studies.

7.4.2 THE INLET SECTION LENGTH (L_i)

The choice of the inlet section length would depend primarily on the possible separation of the turbulent boundary-layer in the inlet region. At the same time the effect of the inlet length on the working area flow uniformity should be carefully watched. Table 7.15 shows the effect of inlet length on the flow uniformity at the beginning of the previously chosen work area, which is one local radius from the contraction exit. The above results show a rather weak effect.

The effect of the inlet length on the boundary-layer separation margin of safety is shown in Figure 74 both for the case of exit length of 0.35 and of 0.45. For the chosen exit length of 0.45, the minimum inlet length, according to Stratford's criterion to avoid inlet boundary-layer separation, is 0.65.

Some boundary-layer calculations were made for different inlet section lengths with the exit length fixed at 0.45. The purpose

of these runs was to confirm that in a contraction with an inlet section as short as 0.65 the flow would not separate as indicated by Stratford's criterion; further to investigate if further reduction in that length would be possible. A summary of the results is shown in Figure 75. These results again show that Stratford's criterion is a little on the conservative side. It also shows that if we adopt $H = 1.8$ as an indication of possible boundary-layer separation, the inlet contraction section could be as short as 0.60 times the inlet radius without expecting the flow to separate.

It should be reported here that some difficulties were encountered in using the Lag-Entrainment program to calculate the boundary-layer flow in the region of high flow acceleration near the contraction exit. The difficulties were due to the failure of the forward integration subroutine - subroutine VINT - to carry the calculation through the high flow acceleration region. However, the failure of the program at exit should not affect the validity of the predictions through the inlet region.

TABLE 7.12

THE EFFECT OF THE CONSTRUCTION EXIT SECTION LENGTH (L_e)

ON TEST SECTION FLOW UNIFORMITY

CR = 8.0, $L_1 = 0.7$, $L_T/S = 4.5$, $R_e = 1.5$, $N_1 = 0.01$, $B_e = 0.0$, $W_e = 3.0$

ΔU_w = percent flow uniformity along the wall.

ΔU_c = percent flow uniformity along centerline.

Distance Inside Test Section as Fraction of L_T/S	$L_e = 0.20$		$L_e = 0.30$		$L_e = 0.40$		$L_e = 0.45$	
	ΔU_w	ΔU_c	ΔU_w	ΔU_c	ΔU_w	ΔU_c	ΔU_w	ΔU_c
0.0	12.40	-12.39	6.85	-9.46	4.56	-6.22	2.25	-5.51
0.1	1.86	-2.79	1.10	-1.99	0.73	-1.31	0.43	-1.07
0.2	0.31	-0.60	0.19	-0.42	0.13	-0.27	0.08	-0.21
0.3	0.05	-0.12	0.03	-0.08	0.02	-0.05	0.02	-0.04
0.4	0.01	-0.02	0.01	-0.02	0.00	-0.01	0.00	-0.01

TABLE 7.12 (Concluded)

Distance Inside Test Section as Fraction of $L_{T/S}$	$L_e = 0.50$		$L_e = 0.60$		$L_e = 0.70$		$L_e = 1.00$	
	ΔU_w	ΔU_c	ΔU_w	ΔU_c	ΔU_w	ΔU_c	ΔU_w	ΔU_c
0.0	3.13	-4.64	2.14	-3.21	1.50	-3.17	0.61	-1.3
0.1	0.51	-0.94	0.35	-0.65	0.25	-0.54	0.11	-0.22
0.2	0.09	-0.20	0.06	-0.13	0.044	-0.10	0.02	-0.04
0.3	0.02	-0.04	0.01	-0.03	0.01	-0.02	0.00	-0.01
0.4	0.00	-0.01	0.00	-0.00	0.00	-0.00	0.00	-0.00

TABLE 7.13

EFFECT OF THE CONTRACTION EXIT SECTION LENGTH (L_e)
ON FLOW SEPARATION MARGIN OF SAFETY AT BOTH INLET AND EXIT

$CR = 8.0$, $B_1 = 1.5$, $N_1 = 0.01$

$L_1 = 0.70$, $B_e = 0.0$, $N_e = 3.0$

L_e	$(MS)_1$	$(MS)_e$
0.20	-0.94	7.12
0.30	0.44	7.48
0.40	0.68	8.43
0.50	1.17	8.99
0.60	1.58	9.49
0.70	2.57	9.60
1.00	3.67	10.25

TABLE 7.14

REQUIRED LENGTH FOR THE CONTRACTION EXIT SECTION TO ACHIEVE
A 0.25% FLOW UNIFORMITY IN THE TEST SECTION AS A FUNCTION OF
ALLOWED FLOW SETTling LENGTH IN THE TEST SECTION.

$$\begin{aligned} L_i &= 0.7, & B_i &= 1.5, & N_i &= 0.01 \\ & & B_e &= 0.0 & N_e &= 3.0 \end{aligned}$$

FLOW SETTling LENGTH
TEST SECTION RADIUS

REQUIRED L_e

1.5

0.15

1.0

0.45

0.5

1.0

TABLE 7.15

EFFECT OF THE CONTRACTION INLET SECTION LENGTH (L_1)
ON THE FLOW UNIFORMITY IN THE TEST SECTION

CR = 8.0, $B_1 = 1.5$, $N_1 = 0.01$

$B_e = 0.0$, $N_e = 3.0$

(a) - $L_e = 0.45$

L_1 PERCENT FLOW UNIFORMITY AT DISTANCE OF 0.9
 R_o FROM CONTRACTION EXIT

	ΔU_w	ΔU_c
0.45	0.12	-0.26
0.50	0.12	-0.25
0.55	0.11	-0.24
0.60	0.10	-0.24
0.65	0.10	-0.23

(b) - $L_e = 0.35$

L_1 PERCENT FLOW UNIFORMITY AT DISTANCE OF 0.9
 R_o FROM CONTRACTION EXIT

	ΔU_w	ΔU_c
0.60	0.17	-0.35
0.65	0.16	-0.34
0.70	0.16	-0.33
0.75	0.15	-0.33

7.5 THE CHOSEN CONTRACTION - COMPARISONS

The R'' distribution and the contour shape of the present contraction are shown in Figures 76 and 77. The velocity distribution along both the centreline and the wall are shown in Figure 78, while the expected flow uniformity distribution in the test section is presented in Figure 79.

Comparisons between the chosen contractions of the original and modified families of contours are shown in Figures 80 - 83. The inflection point in both geometries lies at a distance, from the contraction inlet, of about 0.6 of the contraction length.

The shape of R'' distribution of the contraction contour used in the study of Ref. 23 is shown in Figure 37. In comparison with the present R'' shape shown in Figure 76, the two shapes are in complete disagreement. This explains the difference in the required contraction length. The present study shows that the required length for a given condition is less than half the length shown to be necessary in the study of Ref. 23.

In the study presented in Ref. 24, the contraction contours consist of two cubic arcs matched at the inflection point. The R'' distribution, then, is linear at both inlet and exit with discontinuity at the inflection point. This R'' distribution near the contraction inlet is not far from what is used here, though, using the design criteria given in the reference resulted in a contraction longer than the contraction resulted from the present study.

Data presented in both Tables 2.1 and 2.2 show contraction length of 100 percent or more longer than it needs to be according to the present results.

As an example, let us consider the contraction of the NAE 30 ft. Wind Tunnel (Ref. 1) as representative of a large modern tunnel which has

the same inlet Reynolds number scale as the case studies here. The NAE Wind Tunnel contraction ratio is 5.85 and its length $L = 2.28$. The present study - as shown in section 8 - indicates the need of a relative length $L = 1$. for the same contraction ratio. This would mean a potential saving of more than 50 ft. of the contraction length. A reduction of the contraction length of that order represents a significant saving in the tunnel total length, in the tunnel shell construction cost, in the required drive cost and running power cost, and in many cases might allow for possible tunnel circuit modifications accompanied by better tunnel efficiency.

8. PARAMETRIC ANALYSIS

The individual effects of the different parameters on the contraction performance and its required length were investigated. In carrying out this parametric study, the contraction shape parameters were kept the same as that defined in the previous section, namely, the inlet parameters $B_i = 1.5$, $N_i = 0.01$, and the exit parameters $B_e = 0.0$, and $N_e = 3.0$. The contraction length was varied to keep the required performance. For each parameter reasonable steps away from the standard conditions considered previously were taken. This would bracket all likely design cases.

8.1 EFFECT OF CONTRACTION RATIO

(a) Considerations of Inlet Separation

The analysis of Appendix III (a) shows that, for the same contraction performance regarding boundary-layer separation at inlet, the required contraction length as a function of the contraction ratio takes the form:

$$L \sim \sqrt{1 - \frac{1}{\sqrt{CR}}} \quad (8.1)$$

Figure 84 shows the required contraction length as a function of the contraction ratio. The validity of the form of equation (8.1) was examined by a number of computer runs at discrete contraction ratios (2, 4, 8, 12, and 16). The results of this numerical check out - as shown in Figure 84 - are in agreement with equation (8.1) and bracket the minimum lengths.

Comparison of the present results with that of Ref. 23, shows general agreement of the shape of the $L - CR$ Curve as shown in Figure 85. The results presented in Figure 85 of both the study of Ref. 23 and the present results were based solely on considerations of inlet separation.

(b) Considerations of Test Section Flow Uniformity

The variation in the contraction length with the contraction ratio as shown in Figure 84 was found to result in variations in the test section flow quality as shown in Figure 86. The variations shown in the flow uniformity in the test section are more than one can consider as equivalent performance of the contractions.

In fact, relation (8.1) was derived on the basis of considerations of boundary-layer separation at inlet, hence, it should be used to scale only the inlet length L_1 . In the same time, the exit length L_e should be chosen on the basis of achieving the required flow uniformity in the test section. Computer experiments were performed to define L_e required to achieve the same flow uniformity as in the standard case. The variations of the contraction inlet length, exit length, and the total length as a function of the contraction ratio are shown in Fig. 87. These results indicate that for the same performance, the required contraction length decreases as the contraction ratio increases ($4 \leq CR \leq 12$). That conclusion is in agreement with the findings of Ref. 24.

Fig. 88 shows the inflection point position as a function of the contraction ratio.

8.2 EFFECT OF REYNOLDS NUMBER SCALE

(a) Considerations of Inlet Separation

The analysis of Appendix III (b) shows that, for the same contraction performance regarding boundary-layer separation at inlet, the contraction length varies with Reynolds number scale according to:

$$L \sim R_e^{-0.06} \quad (8.2)$$

Figure 89 shows the variation of the required contraction length as a function of the Reynolds number based on inlet conditions. The changes in the Reynolds number are an order of magnitude up and down. The results of the computer runs - as shown in Figure 89 - confirmed the form of the analytical predictions.

None of the previous published works studied the Reynolds number effect on the required contraction length.

(b) Considerations of Boundary-Layer Re-Laminarization

At the lowest Reynolds number scale considered here, namely Reynolds number based on inlet conditions and inlet diameter, $(R_D)_1 = 1.3 \times 10^6$, the contraction length required to avoid boundary-layer separation at the contraction inlet is $L = 1.26$ as shown in Figure 89.

A check on the flow acceleration parameter, K , along the contraction wall showed that it exceeded its previously chosen critical value of $K = 2 \times 10^{-6}$ over a considerable length of the contraction wall. To avoid that situation the contraction length has to be increased to at least $L = 1.8$ as the results shown in Figures 90(a) and 90(b) suggest.

(c) Considerations of Test Section Flow Uniformity

Since the contraction length at the lowest Reynold's number investigated is governed by the requirement of avoidance of boundary-layer re-laminarization, the final choice of the contraction length at the different Reynolds numbers, and the inflection point position are shown in Fig. 91(a). The corresponding flow uniformity in the test section is shown in Fig. 91(b). In going from $(R_D)_1 = 13 \times 10^6$ to $(R_D)_1 = 130 \times 10^6$, the contraction inlet length L_1 scale according to relation (8.2), while the required exit length L_e , for the same degree of flow uniformity in the test section, stays the same at $L_e = 0.45$.

8.3 EFFECT OF THE BOUNDARY-LAYER ORIGIN

In the analysis presented in the earlier chapters of this thesis, the turbulent boundary-layer was assumed to start from an origin at a distance of $S_0 = 1$ from the contraction inlet. The effect of a variation in that distance was studied. In effect this approximately represents a change of position of the most downstream screen.

(a) Considerations of Inlet Separation

The analysis of Appendix III(c) suggests that the required contraction length, L , as a function of the distance S_0 can be expressed as:

$$L \sim S_0^{0.22} \quad (8.3)$$

Numerical checks on the above relation proved it to be applicable as shown in Figure 92.

(b) Considerations of Test Section Flow Uniformity

The variations of L as a function of S_0 as shown in Figure 92 result in variations in the test section flow uniformity as shown in Figure 93. To keep the same contraction performance, i.e. no boundary-layer separation at inlet and equivalent flow uniformity in the test section, the contraction exit section length L_e is kept the same at $L_e = 0.45$, while the inlet length L_i is scaled according to relation (8.3). The variations of both the contraction length L , and the position of the inflection point Z_{ip} as a function of the distance S_0 are shown in Fig. 94(a) and 94(b) respectively.

9. CONCLUSIONS

- 1 - The equations describing the inviscid incompressible flow in an axisymmetric duct of arbitrary area distribution were expressed in terms of transformed co-ordinates in which the duct has fixed straight boundaries. The duct shape parameters appear in the transformed equation rather than in the boundary conditions which give an insight into the effect of the different shape parameters, and greatly simplify numerical solution. The developed computer code, based on the method of lines, was successfully tested and proved to be efficient and accurate.
- 2 - A criterion for the design of wind tunnel contractions was developed which relates the contraction contour shape and its length to the required test section flow steadiness and uniformity. An optimum contraction was defined as the shortest possible contraction to satisfy the design criterion.
- 3 - The proposed approach of controlling the flow parameters through the control of the duct wall curvature distribution was proved to be successful, and facilitated a limited optimization of the contraction contour shape. Optimization of the contraction wall curvature distribution results in a contraction contour shape that satisfies the design criteria in a much smaller length than the present state-of-the-art indicates. As an example, a contraction for a typical large modern tunnel with a contraction ratio of eight can be made as short as one half its inlet diameter. Existing wind tunnel contractions are, typically, twice as long.

4 - The required degree of flow uniformity in the test section, and the allowable length of a settling region before this degree of uniformity is required, are an important parameter in defining the necessary contraction length.

5 - The effects of different geometric and flow parameters on the contraction performance and on the minimum contraction length required to meet the design criteria were studied. The parameters investigated are: the duct contraction ratio, the Reynolds number scale, and the boundary-layer origin. It was found that possible boundary-layer re-laminarization at low Reynolds number conditions represents a severe limitation on the contraction length.

The optimum contraction length was found to decrease as the contraction ratio increases. This mainly because the length of the exit section required to produce the same flow uniformity in the test section is smaller at higher contraction ratio.

The length variations in the range investigated ($4 \leq CR \leq 12$) is small.

10. GENERAL COMMENTS

a) Contraction Cross-Section Shape Change

A contraction connected to a circular settling chamber (a necessary requirement if the wind tunnel circuit is pressurized) and rectangular (or square) test section has to undergo a shape change from axisymmetric cross section at inlet to three-dimensional at exit. The analytical treatment of a three-dimensional flow, in particular the viscous part of the flow calculations and the prediction of the "corner" boundary-layers, is virtually impossible. For that reason the effects of the change in the contraction shape can only be checked through an experimental program.

The common practice for the design of wind tunnel contraction with transition of the cross section shape from circular to rectangular is to design an axisymmetric duct with the same entry conditions and the required area ratio. The shape change can then be introduced gradually in such a way as to keep the cross section area distribution equivalent to that of the axisymmetric design (Ref. 24). It has been always argued that maintaining the area distribution during a cross sectional shape change would approximately keep the results of an axisymmetric solution valid.

The present study has indicated that the flow in the contraction inlet region is not very sensitive to the shape of the contraction at exit, for that reason one would not expect strong effect of exit shape change on the inlet boundary layers. An experimental investigation would be mainly required to check the effect of shape change on local flow, in particular near the corners and on the flow quality in the test section. Some experimental data including flow visualization in a contraction undergoing shape change was reported in Ref. 48.

b) Effect of Boundary-Layer Displacement Thickness

In the present case where the boundary-layer displacement thickness is small compared to the contraction wall radius as shown in Figure 24(c), it is possible to correct for the effect of the existence of the boundary-layer on the core inviscid flow by displacing the contraction contour outward by an amount equivalent to the local boundary-layer displacement thickness. In the study of Ref. 47, the effect of this correction to the contraction contour was investigated. It was concluded that such correction can be completely ignored without any appreciable change in the contraction performance.

c) The Re-Laminarization Problem

As shown in section 8.2 (b), at low Reynolds number conditions the choice of the contraction length is governed by the requirements of avoiding possible boundary-layer re-laminarization, and this leads to an excessive contraction length compared to a higher Reynolds number case. If the requirements of avoiding boundary-layer re-laminarization in choosing the contraction length is ignored, a quick calculation shows that flow separation is likely to occur in case the boundary-layer does indeed become laminar.

As an alternative to avoiding re-laminarization by increasing the contraction length, it might be possible to demote re-laminarization by adding roughness to the contraction wall. An investigation into the suitability of such a technique could result in saving the need of long contractions in low Reynolds number applications.

d) Effect of Wall Manufacturing Tolerances

The particular numerical solution technique employed in the present study does not allow the study of the effect of local deviations in the contraction wall due to tolerances in its fabrication. Generally, very localized irregularities are not expected to affect the overall performance of the contraction. The results presented in Figure 80, suggest that the effect of moderate variations in the contraction wall curvature distribution on the contraction performance is not significant.

e) Analysis of Existing Wind Tunnels

The computer routine developed for the present study can be used in analyzing contractions of operating wind tunnels. The analysis can be compared with measured data whenever such data exists. Such comparison can illustrate the effect of shape change and three-dimensional effects near the contraction exit.

f) Comparisons of Curvature Distribution

The shape of the contraction wall curvature distribution recommended as a result of the present study is compared with the curvature distribution of the contraction contours of the studies of Ref. 23 and Ref. 24. The sketch of Figure 95 shows the general shapes of the curvature distribution. The present study recommends a curvature distribution which is distributed over the entire inlet region with its maximum magnitude to be as close to the inflection point as possible. In the exit region a "peaky" distribution is recommended.

From the sketch of Figure 95, it can be seen that the curvature distribution used in the study of Ref. 23 is almost the opposite to that recommended here. The curvature shape is "peaky" at inlet and almost flat at exit.

In Ref. 24, a linear curvature distribution with discontinuity at the inflection point, as shown in Figure 95, is used. In the inlet region the maximum curvature distribution is at the inflection point, but it starts slowly at the contraction inlet. In the exit section, the curvature is "peaky" with its maximum magnitude at the inflection point. This would help attaining good flow uniformity at exit, although it adversely affects the flow at inlet.

The study of Ref. 47 recommends a uniformity distributed curvature in the inlet region with a short exit region.

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Courtesy of DSMA)

NAE 30 FT. V/STOL WIND TUNNEL,
CUTTER AVIA

FIG.1(a)

FIG. 1(b) - AIRLINE DIAGRAM FOR NRC 30-FT WIND-TUNNEL (REF. 1)

Fig. 2 - TYPICAL WIND TUNNEL CAPITAL COST VARIATION
AS FUNCTION OF THE AREA RATIO OF THE
CONTRACTING DUCT

(a) Thwaites' Contraction No. 1

(b) Thwaites' Contraction No. 2

FIG. 3 - SHAPE AND VELOCITY DISTRIBUTION ALONG CONTRACTION WALL AND ITS CENTRELINE FOR BOTH THWAITES' CONTRACTION NO. 1 IN WHICH THE WALL VELOCITY INCREASES MONOTONICALLY, AND THWAITES' CONTRACTION NO. 2 IN WHICH THE INLET VELOCITY IS UNIFORM (REF. 14)

(a)

(b)

FIG.4 - TSIEN'S CONTRACTION (REF. 15)

(a) Assumed velocity along the contraction axis

(b) The resulting streamlines in the meridian x , r plane together with the velocity profiles at different sections

FIG.5 - CONTRACTION CONTOUR SHAPE AND VELOCITY DISTRIBUTION ALONG WALL AND AXIS OF COHEN AND RITCHIE 7.4:1 CONTRACTION (REF. 16)

FIG. 6 - STREAMLINES AT CONTRACTION INLET AND VELOCITIES ALONG THEM AS
CALCULATED BY BATCHELOR AND SHAW (REF. 22)

FIG.7 - CHMIELEWSKI'S SEPARATION BOUNDARY FOR THE CONTRACTION FAMILY CORRESPONDS TO, $n = 4.0$, $m = 0.8$, WITH $(R_e/D)_e = 10^6/\text{FT}$, $R_1 = 10$ IN. (REF. 23)

CR = 4

CR = 9

CR = 16

FIG. 8 - MOREL'S DESIGN CHARTS (REF. 24). NOTE THAT x IS THE POSITION OF MATCHING POINT OF THE TWO CUBIC ARCS AS A FRACTION OF THE CONTRACTION LENGTH.

FIG.10 - TYPICAL STREAMLINES PATTERN IN A CONTRACTING DUCT

FIG.11(a) - THE GEOMETRICAL PARAMETERS z_{1p} AND B AS A FUNCTION OF THE EXONENT FACTOR n .

FIG.11(b) - THE MAGNITUDE PARAMETER $\frac{A}{R_1}$ AS A FUNCTION OF THE CONTRACTION RATIO, CR

FIG.12(a) - THE EFFECT OF THE INFLECTION POINT POSITION ON THE SHAPE OF R'' -DISTRIBUTION. NOTE THAT THE MAGNITUDE PARAMETER A INCREASES AS THE INFLECTION POINT MOVES UPSTREAM (SEE FIG.11(b))

FIG.12(b) - THE EFFECT OF INFLECTION POINT POSITION ON THE SHAPE OF THE CONTRACTION CONTOUR

FIG.13 - THE EFFECT OF SHORTENING THE CONTRACTION ON
 R'' DISTRIBUTION FOR THE SAME CONTRACTION RATIO
 AND SAME RELATIVE POSITION OF THE INFLECTION POINT

FIG.14(a) - THE MODEL USED FOR SETTLING CHAMBER, CONTRACTION, AND TEST SECTION.

TYPICAL VALUES FOR THE LENGTH OF THE DIFFERENT PARTS ARE: $L_s \approx 1.$, $L = 2-2.5$, and $L_o = 4.-5.$ THE SHAPE OF THE CONTRACTION $R(x)$ IS TO BE DETERMINED

FIG. 14(b) - ILLUSTRATIONS OF THE NOMENCLATURE

FIG. 15 - (a) THE PHYSICAL x - r PLANE

(b) THE TRANSFORMED ξ - η PLANE. $\xi = x$, $\eta = \frac{R}{F(x)}$

FIG.16 - SCHEME FOR NUMERICAL SOLUTION

(a) The working domain divided into segments. Arrows show the direction of integration, matching conditions applied at stations where two arrows meet.

(b) The working domain is divided into five equally spaced lines parallel to E-axis along which the governing equations are integrated

FIG.1.7 - THE STRUCTURE OF THE COMPUTER CODE

FIG.18 - EFFECT OF THE CONTRACTION LENGTH ON
 R'' DISTRIBUTION

FIG.19 - EFFECT OF THE CONTRACTION LENGTH ON THE VELOCITY DISTRIBUTION ALONG THE CONTRACTION WALL

FIG.20(a) - R'' DISTRIBUTION FOR DIFFERENT
INFLECTION POINT POSITIONS

FIG.20(b) - CONTRACTION CONTOURS FOR DIFFERENT INFLECTION POINT POSITIONS

FIG.21 - WALL VELOCITY DISTRIBUTIONS FOR THE CONTRACTIONS OF FIG.20

FIG. 22 - COMPARISON BETWEEN THWAITES' CONTRACTION NO. 2 AND PRESENT CONTRACTION

FIG. 23 - COMPARISON BETWEEN THE WALL VELOCITY IN BOTH THWAITES' CONTRACTION AND THE PRESENT CONTRACTION (Axial Distance X the Same as that in

FIG. 22)

FIG.24(a) - WALL VELOCITY DISTRIBUTION AND INLET CONDITIONS USED AS AN INPUT TO THE LAG-ENTRAINMENT BOUNDARY LAYER COMPUTER CODE.

FIG. 24(b) - LOCAL SKIN FRICTION COEFFICIENT FOR THE CASE SHOWN IN FIG. 24(a)

FIG. 24(c) - THE BOUNDARY LAYER MOMENTUM THICKNESS θ FOR THE CASE SHOWN

IN FIG. 24(a)

FIG.24(d) - LOCAL REYNOLDS NO. BASED ON MOMENTUM THICKNESS (R_θ)
FOR THE CASE SHOWN IN FIG. 24(a)

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FIG. 24(e) - THE EFFECT OF INCLUDING LONGITUDINAL CURVATURE AND LATERAL STRETCHING ON LOCAL SKIN FRICTION COEFFICIENT *

FIG.25 - INLET SEPARATION BOUNDARY AS FUNCTION OF THE CONTRACTION LENGTH AND INFLECTION POINT POSITION

FIG.26 - INLET SEPARATION BOUNDARY (dashed line) AS
 FUNCTION OF THE INFLECTION POINT POSITION
 AND MAX $|R''|$ AT INLET

FIG.27 - INLET SEPARATION BOUNDARY (dashed line) AS
 FUNCTION OF INFLECTION POINT POSITION AND
 MAX. $(CP)_w$ AT INLET

FIG.28 - SEPARATION BOUNDARY AS FUNCTION OF MAX. (C_p)_w AND THE POSITION OF ITS OCCURRENCE

FIG.29 - INLET SEPARATION BOUNDARY AS FUNCTION OF MAX $|R''|$ AT INLET AND THE POSITION OF ITS OCCURRENCE

FIG. 30 - TYPICAL WALL PRESSURE COEFFICIENTS AT BOTH CONTRACTION INLET AND EXIT

FIG.31 - EFFECT OF CONTRACTION LENGTH ON THE EQUIVALENT DISTANCE AND MAX. WALL PRESSURE COEFFICIENT NEAR CONTRACTION EXIT.

FIG.32 - EFFECT OF POSITION OF THE INFLECTION POINT
ON THE EQUIVALENT DISTANCE AND MAX. WALL
PRESSURE COEFFICIENT NEAR EXIT

FIG. 33 - EFFECT OF THE POSITION OF THE INFLECTION POINT
 ON BOUNDARY-LAYER MARGIN OF SAFETY NEAR
 CONTRACTION EXIT

FIG.34 - MIN. VALUE OF THE LOCAL SKIN FRICTION
 COEFFICIENT IN THE INLET REGION AS CALCULATED
 BY THE BOUNDARY LAYER METHOD OF GREEN ET AL.
 (REF. 37)

FIG.35 - EFFECT OF THE POSITION OF THE INFLECTION POINT
 ON THE MAXIMUM VALUE FOR THE SHAPE FACTOR H AT
 INLET

FIG. 36(a) - CONTRACTION CONTOUR

FIG. 36(b) - R" DISTRIBUTION FOR THE CONTRACTIONS OF FIG. 36(a)

FIG. 36(c) - WALL PRESSURE COEFFICIENT IN THE INLET
REGION OF CONTRACTIONS SHOWN IN FIG. 36(a)

FIG.37 - R'' DISTRIBUTION FOR ONE OF THE FAMILY OF CONTOURS
CONSIDERED IN REF. 23 WITH $m = 0.8$, $n = 4$. (See page 17 of the
present report for definition of m and n)

FIG.38 - SUGGESTED R'' DISTRIBUTION IN THE INLET REGION
OF THE MODIFIED FAMILY OF CONTOURS

FIG.39(a) - EFFECT OF "B" ON THE SHAPE OF R'' DISTRIBUTION;
 THE MAGNITUDE C IS ADJUSTED SUCH THAT THE AREA
 UNDER THE CURVE IS THE SAME IN ALL CASES

FIG.39(b) - EFFECT OF THE EXPONENT "N" ON "R" DISTRIBUTION

FIG. 40 - EFFECT OF VARIATION OF N_1 ON R'' -DISTRIBUTION.
X MEASURED FROM CONTRACTION INLET

FIG. 41 - EFFECT OF VARIATIONS OF N_1 ON THE PRESSURE COEFFICIENT AT CONTRACTION INLET

FIG. 42 - EFFECT OF VARIATION OF N_1 ON STRATFORD'S CRITERION (LH-RH) AT CONTRACTION INLET

FIG. 43 - EFFECT OF VARIATION OF B_1 ON R'' -DISTRIBUTION
X MEASURED FROM CONTRACTION INLET

FIG. 44 - EFFECT OF VARIATION OF B_1 ON THE PRESSURE COEFFICIENT AT CONTRACTION INLET

FIG. 45 - EFFECT OF VARIATION OF " B_1 " ON STRATFORD'S CRITERION (RH-LH) AT INLET

FIG. 46 - EFFECT OF VARIATION OF CONTRACTION INLET SECTION LENGTH L_1 ON R'' -DISTRIBUTION X MEASURED FROM INLET TO THE LONGEST CONTRACTION

FIG. 47 - EFFECT OF THE CONTRACTION INLET SECTION LENGTH L_1 ON THE PRESSURE COEFFICIENT AT INLET

FIG. 48 - EFFECT OF VARIATION OF THE CONTRACTION INLET SECTION LENGTH " L_1 " ON STRATFORD'S CRITERION (RH-LH) AT INLET

FIG. 49 - EFFECT OF VARIATION OF CONTRACTION INLET SECTION LENGTH L_1 ON PRESSURE COEFFICIENT - NEAR THE CONTRACTION EXIT. X IS MEASURED FROM SETTLING CHAMBER INLET IN EACH CASE.

FIG. 50 - EFFECT OF VARIATION OF CONTRACTION INLET SECTION'S LENGTH " L_1 " ON EXIT BOUNDARY LAYER MARGIN OF SAFETY $(MS)_e$

FIG. 51 - EFFECT OF VARIATION OF CONTRACTION EXIT SECTION SHAPE PARAMETER " N_e " ON R'' -DISTRIBUTION. X IS MEASURED FROM CONTRACTION INLET

FIG. 52 - EFFECT OF VARIATION OF CONTRACTION EXIT SECTION SHAPE PARAMETER " N_e " ON THE EXIT PRESSURE COEFFICIENT

FIG. 53 - EFFECTS OF VARIATION OF EXIT SHAPE PARAMETER " B_e " ON CONTRACTION R'' -DISTRIBUTION. X IS MEASURED FROM CONTRACTION INLET

FIG. 54 - EFFECT OF VARIATION OF EXIT SHAPE PARAMETER
" B_e " ON EXIT PRESSURE COEFFICIENT $(C_p)_e$

FIG. 55 - EFFECT OF VARIATION OF EXIT SHAPE PARAMETER " B_e " ON STRATFORD'S CRITERION $(RH-LH)$ AT EXIT

FIG. 56 -- EFFECT OF VARIATION OF EXIT SHAPE PARAMETER " B_e " ON INLET PRESSURE COEFFICIENT $(C_p)_i$

FIG. 57 - EFFECT OF VARIATION OF EXIT SHAPE PARAMETER "B_e" ON STRATFORD'S CRITERION (RH-LH) AT CONTRACTION INLET

FIG. 58 - EFFECT OF VARIATION OF EXIT LENGTH " L_e " ON R'' -DISTRIBUTION. X IS MEASURED FROM CONTRACTION INLET.

FIG. 59 - EFFECT OF VARIATION OF CONTRACTION EXIT SECTION'S LENGTH " L_e " ON EXIT PRESSURE COEFFICIENT $(C_p)_e$

FIG. 60 - EFFECT OF VARIATION OF EXIT SECTION LENGTH
" L_e " ON STRATFORD'S CRITERION $(RH-LH)$ AT EXIT

FIG. 61 - EFFECT OF VARIATION OF EXIT SECTION LENGTH
 $"L_e"$ ON EXIT BOUNDARY LAYER MARGIN OF SAFETY
 (MS)_e

FIG. 62 - EFFECT OF EXIT SECTION LENGTH " L_e " ON INLET PRESSURE COEFFICIENT $(C_p)_1$

FIG. 63 - EFFECT OF VARIATION OF EXIT SECTION LENGTH " L_e "
ON INLET BOUNDARY LAYER MARGIN OF SAFETY (MS)₁

CR = 8.0 $L_1 = 0.65$ $L_e = 0.35$
 $N_1 = 0.01$ $B_1 = 1.5$ $N_e = 3.0$

FIG. 64 - EFFECT OF " B_e " ON EXIT BOUNDARY LAYER MARGIN OF SAFETY

FIG. 65 - EFFECT OF " B_e " ON MAXIMUM WALL VELOCITY

FIG. 66 - EFFECT OF THE MODIFIED FAMILY OF CONTOUR'S GEOMETRIC PARAMETER " B_e " ON BOUNDARY LAYER MARGIN OF SAFETY AT INLET

FIG. 67 - EFFECT OF N_e ON THE BOUNDARY LAYER MARGIN OF SAFETY AT BOTH INLET AND EXIT OF THE CONTRACTION

FIG. 68 - EFFECT OF THE MODIFIED FAMILY OF CONTOUR'S GEOMETRIC PARAMETER " N_e " ON MAXIMUM WALL VELOCITY OVERSHOOT

FIG.69 - EFFECT OF THE EXIT SEGMENT LENGTH OF THE MODIFIED FAMILY OF CONTOURS ON MAX. WALL VELOCITY AND BOUNDARY LAYER MARGIN OF SAFETY AT EXIT.

FIG.70 - EFFECT OF THE GEOMETRICAL PARAMETER " N_e "
 OF THE MODIFIED FAMILY OF CONTOURS ON FLOW
 NON-UNIFORMITY AT INLET TO TEST SECTION

FIG.71 - EFFECT OF EXIT SEGMENT LENGTH ON FLOW
 NON-UNIFORMITY AT INLET TO TEST SECTION

FIG.72 - FLOW NON-UNIFORMITY AS FUNCTION OF DISTANCE
 IN THE TEST SECTION MEASURED FROM CONTRACTION
 EXIT

FIG.73 - FLOW NON-UNIFORMITY ON LOGARITHMIC SCALE
AS FUNCTION OF DISTANCE IN THE TEST SECTION
MEASURED FROM CONTRACTION EXIT

FIG.74 - EFFECT OF THE INLET SEGMENT LENGTH OF THE MODIFIED FAMILY OF CONTOURS ON BOUNDARY LAYER MARGIN OF SAFETY AT INLET

FIG.75 - EFFECT OF INLET SEGMENT LENGTH ON MINIMUM LOCAL SKIN FRICTION AND ON MAXIMUM SHAPE FACTOR IN INLET REGION

FIG. 76 - R'' DISTRIBUTION FOR THE CHOSEN CONTRACTION

FIG. 77 - THE CHOSEN CONTRACTION CONTOUR, X MEASURED FROM CONTRACTION INLET

FIG. 78 - VELOCITY DISTRIBUTION FOR CONTRACTION SHOWN IN FIG. 77.

FIG.79 - FLOW NON-UNIFORMITY IN THE TEST SECTION
FOR THE CONTRACTION OF FIG.77, X MEASURED
FROM INLET TO TEST SECTION

FIG. 80 - R'' DISTRIBUTION FOR THE SELECTED MEMBERS OF BOTH THE ORIGINAL AND MODIFIED FAMILIES

FIG. 81 - COMPARISON BETWEEN THE CONTRACTION CONTOUR SHAPES OF THE ORIGINAL AND MODIFIED FAMILIES.

FIG. 82 - VELOCITY ALONG THE WALL OF BOTH ORIGINAL AND MODIFIED CONTRACTIONS

FIG.83 - PRESSURE COEFFICIENT ALONG THE WALL OF THE INLET REGION FOR THE CONTRACTIONS SHOWN IN FIG. 81

FIG. 84 - CONTRACTION LENGTH AS A FUNCTION OF THE DUCT CONTRACTION RATIO, BASED ON CONSIDERATION OF INLET SEPARATION ONLY.

FIG. 85 - THE CONTRACTION LENGTH AS FUNCTION OF THE CONTRACTION RATIO;
COMPARISON BETWEEN THE PRESENT RESULTS AND THOSE OF REF. 23.
THESE RESULTS ARE BASED ONLY ON CONSIDERATIONS OF INLET SEPARATION.

FIG. 86 - VARIATIONS OF TEST SECTION FLOW UNIFORMITY
WITH CONTRACTION RATIO

FIG.87 - VARIATIONS OF THE CONTRACTION OPTIMUM LENGTH AS A FUNCTION OF THE CONTRACTION RATIO.

FIG. 88 - THE OPTIMUM POSITION FOR THE INFLECTION POINT AS A FUNCTION OF THE CONTRACTION RATIO (Based on Results Shown in Fig. 87).

FIG. 89 - EFFECT OF REYNOLDS NUMBER ON THE REQUIRED CONTRACTION LENGTH.
BASED ON APPLICATION OF STRATFORD'S CRITERION AT INLET.

FIG. 90(a) EFFECT OF THE CONTRACTION LENGTH ON
 THE ACCELERATION PARAMETER K .
 $K > 2 \times 10^{-6}$ INDICATES POSSIBLE
 BOUNDARY-LAYER RE-LAMINARIZATION.

FIG. 90(b) EFFECT OF THE INFLECTION POINT POSITION ON THE ACCELERATION PARAMETER K .

FIG. 91(a) VARIATIONS OF THE CONTRACTION LENGTH AND POSITION OF THE INFLECTION POINT AS A FUNCTION OF THE REYNOLDS NUMBER SCALE

FIG. 91(b) FLOW UNIFORMITY AT DIFFERENT INLET REYNOLD'S NUMBER FOR THE CONTRACTION LENGTH SHOWN IN FIG. 91(a).

FIG. 92 - THE EFFECT OF THE BOUNDARY-LAYER LEADING EDGE POSITION ON THE REQUIRED CONTRACTION LENGTH BASED ONLY ON CONSIDERATIONS OF INLET SEPARATION

FIG. 93 - VARIATIONS OF THE TEST SECTION FLOW UNIFORMITY WITH S_o , WHEN CONTRACTION LENGTHS ARE AS SHOWN IN FIG. 92.

FIG. 94(a) THE OPTIMUM CONTRACTION LENGTH AS A FUNCTION OF S_o , ON THE BASIS OF THE SAME FLOW UNIFORMITY IN THE TEST SECTION.

FIG. 94(b) - INFLECTION POINT POSITION AS A FUNCTION OF s_o .
CONTRACTION LENGTH AS SHOWN IN FIG. 94(a).

FIG. 95 - COMPARISON BETWEEN THE SHAPE OF THE CONTRACTION CONTOUR'S CURVATURE DISTRIBUTION USED IN DIFFERENT STUDIES

FIG. 96 - THE OPTIMUM CONTRACTION LENGTH AS FUNCTION OF THE EXIT RADIUS AS GIVEN IN REF. 47

APPENDICES

APPENDIX I

1. THE EQUIVALENT LENGTH PROBLEM

For an axisymmetric flow, the following is a derivation for an expression of an equivalent distance the flow would have to travel at constant mean velocity in a constant area duct in order that the boundary layer momentum thickness θ_m at point x_m would be equivalent to that of a flow with velocity distribution U_1 in a duct with radius R .

The boundary layer momentum integral equation is (Ref. 45)

$$\frac{d\theta}{dx} + \left(\frac{H+2}{U_1} \frac{dU_1}{dx} + \frac{1}{R} \frac{dR}{dx} + \frac{1}{S} \frac{dS}{dx} \right) \theta = \frac{\tau_w}{\rho U_1^2}$$

By assuming local coefficient of friction in the form:

$$C_f = 0.0450 \left(\frac{\nu}{U_1 \delta} \right)^{0.25}$$

and power-law velocity distribution in the form:

$$\frac{U}{U_1} = \left(\frac{y}{\delta} \right)^{1/7}$$

which leads to $H = 9/7$, and $\theta/\delta = 7/72$, for the incompressible flow case, Sibulkin (Ref. 45), obtained the following solution:

$$\delta_m^{5/4} = \frac{0.289}{R_m^{5/4} U_m^{115/28}} \int_0^X R_m^{5/4} U_m^{27/7} dX \quad (2)$$

assuming $\delta = 0$ at $X = 0$.

For $U_1 = U_m = \text{constant}$, and $R = R_m = \text{constant}$ and $X = X_{eq}$, we

get:

$$\delta_m^{5/4} = \frac{0.289}{R_m^{5/4} U_m^{115/28}} R_m^{5/4} U_m^{27/7} X_{eq} \quad (3)$$

Comparing equations (2) and (3), and considering that θ is a fixed fraction of δ , we get the following expression for the equivalent distance X_{eq} :

$$X_{eq} = \int_0^{X_m} \left(\frac{R}{R_m} \right)^{5/4} \left(\frac{U_1}{U_m} \right)^{27/7} dX \quad (4)$$

2. THE BOUNDARY LAYER MOMENTUM THICKNESS

As a consequence of assuming a 1/17-power law for the velocity distribution in the boundary layer, θ and δ are related (Ref. 45) as:

$$\theta = \frac{7}{72} \delta$$

Using the expression given by equation (2) for δ , the momentum thickness θ at any point at distance X , assuming $\theta = 0$ at $X = 0$, can be given by:

$$\theta = \frac{7}{72} \left[\frac{0.298 v^{1/4}}{R^{5/4} U_1^{115/28}} \int_0^X R^{5/4} U_1^{27/7} dx \right]^{4/5} \quad (5)$$

APPENDIX II

R" - DISTRIBUTION FOR THE FAMILY OF CONTRACTIONS OF REF. 23

In Ref. 23 the shape of the contraction is given by:

$$\left[\frac{R_1}{R} \right]^4 = (CR^2 - 1) \frac{F(x)}{F(L)} + 1 \quad (1)$$

where:

$$F(x) = \int_0^x f(x) dx$$

and,

$$f(x) = \left\{ \frac{1}{2} \left[1 - \cos 2\pi \left(\frac{x}{L} \right)^n \right] \right\}^p$$

For $R_1 = 1$, eqn. (1) can be rewritten as:

$$R = \frac{1}{[\alpha F_x + 1]^{1/4}} \quad (2)$$

$$\text{where } \alpha = \frac{CR^2 - 1}{F(L)}$$

Differentiating equation (2) twice results in,

$$\frac{d^2 R}{dx^2} = \frac{5}{16} \alpha^2 F^{12} (\alpha F + 1)^{-9/4} - \frac{1}{4} \alpha F'' [\alpha F + 1]^{-5/4} \quad (3)$$

where:

$$\alpha = \frac{CR^2 - 1}{L \int_0^L f(x) dx} = \frac{CR^2 - 1}{L \int_0^L f\left(\frac{x}{L}\right) d\left(\frac{x}{L}\right)}$$

$$F = L \int_0^L f\left(\frac{x}{L}\right) d\left(\frac{x}{L}\right)$$

$$F' = f(x) = \left\{ \frac{1}{2} \left[1 - \cos 2\pi \left(\frac{x}{L} \right)^n \right] \right\}^p$$

$$F'' = \frac{d}{dx} f(x)$$

$$= \frac{pn\pi}{L} \left(\frac{x}{L} \right)^{n-1} \left\{ \frac{1}{2} \left[1 - \cos 2\pi \left(\frac{x}{L} \right)^n \right] \right\}^{p-1} \sin 2\pi \left(\frac{x}{L} \right)^n$$

APPENDIX III

THE EFFECT OF DIFFERENT PARAMETERS ON BOUNDARY-LAYER SEPARATION AT THE CONTRACTION INLET

The following is an approximate analytical approach to assess the effect of different parameters on the contraction performance. The parameters considered here are: the contraction ratio, the Reynolds number scale, and the distance between the turbulent boundary-layer leading edge and the contraction inlet. The analysis is based solely on Stratford's criterion for inlet separation.

ANALYSIS

- At the point of separation, according to Stratford's criterion, equation (5.2):

$$C_p \left(S \frac{dc_p}{ds} \right)^{1/2} = k (10^{-6} R_{es})^{0.1} \quad \text{where } k \text{ is constant,}$$

which can be written as:

$$C_p \left(S \frac{dc_p}{ds} \right)^{1/2} \sim R_s^{0.1} \quad (1)$$

- Assuming that:

$$C_p \sim (R'')^n \quad (2)$$

An analysis of the available numerical results shows that n is not strictly constant; a mean value of $n = 0.6$ is found to be representative ($0.8 \leq L \leq 1.3$).

- Keeping the shape of the R'' distribution the same, the magnitude of R'' at each point is related to the contraction length by:

$$R'' \sim \frac{1}{L^2} \quad (3)$$

(a) Effect of Contraction Ratio

- Assuming that for values of CR's of practical interest, the distance S of equation (1) above will not change significantly, it follows that the right-hand side of relation (1) is constant, and to keep the left-hand constant, c_p should be kept the same. This would require keeping R'' the same (see relation (2)) by adjusting the contraction length, L .

$$\begin{aligned} \Delta R &= R_1 - R_0 = R_1 \left(1 - \frac{1}{\sqrt{CR}}\right) \\ &= \int_0^L \int_0^x R'' dx dx \quad \text{with } dx = l dz \end{aligned}$$

then,

$$\begin{aligned} R_1 \left(1 - \frac{1}{\sqrt{CR}}\right) &= \int_0^L \int_0^x R'' dz dz \\ &= \frac{L}{R_1} \int_0^L \int_0^x R_1 R'' dz dz \end{aligned}$$

For the same R'' distribution, the above integral is constant,

hence:

$$L^2 \sim 1 - \frac{1}{\sqrt{CR}} \quad \left(\text{since } L = \frac{l}{R_1}\right)$$

or,

$$L \sim \sqrt{1 - \frac{1}{\sqrt{CR}}} \quad (4)$$

This relation is plotted in Figures 84 and 85.

(b) Effect of Reynolds Number Scale

For small changes in contraction length, S is effectively constant

and then, $R_s \sim R_{D1}$. Also $c_p \left(S \frac{dC_p}{dS} \right)^{1/2} \sim (C_p)^{1.5}$.

This means that relation (1) can be expressed as:

$$(C_p)^{1.5} \sim R_{D1}^{0.1} \quad (5)$$

$$\text{— but } c_p \sim (R'')^{0.6}; \text{ and } R'' \sim \frac{1}{L^2}$$

$$\text{Then, } c_p \sim \frac{1}{(L)^{1.2}} \quad (6)$$

Thus relation (5) can be written:

$$\frac{1}{(L)^{1.8}} \sim R_{D1}^{0.1}$$

or,

$$L \sim R_{D1}^{-0.06} \quad (7)$$

This relation is shown in Figure 86.

(c) Effect of Boundary-Layer Leading Edge Position

We assume that $S = S_0 + \bar{S}$, where S_0 is the distance from the boundary-layer leading edge to the contraction inlet, and $\bar{S}$ is the distance from contraction inlet to point of probable separation. From the numerical results, $\bar{S} \approx 0.05$.

Considering the R.H.S. of relation (1) then

$$R_{eS} \sim S^{0.1}$$

Considering the L.H.S. of relation (1) we have noted previously that $\left(\frac{dC_p}{dS}\right)_{sep} \sim (C_p)_{sep}$, hence,

$$C_p \left(S \frac{dC_p}{dS}\right)^{0.5} \sim (C_p)^{1.5} (S)^{0.5}$$

Relation (1) then reduces to:

$$(C_p)^{1.5} (S)^{0.5} \sim (S)^{0.1}$$

or,

$$(C_p)^{1.5} \sim (S)^{-0.4}$$

And from relation (6) i.e. $C_p \sim \frac{1}{(L)^{1.2}}$, then, $(L)^{-1.8} \sim (S)^{-0.4}$

or,

$$L \sim S^{0.22}$$

(8)

This relation is shown in Figure 87.

APPENDIX IV

COMMENTS ON THE ORIGINAL FAMILY OF CONTOURS

The following is a brief comment on the results obtained using the original family of contours described in section 2.1. The results obtained using this family of contours is described in section 6. These results show that the inlet conditions, as far as boundary layer separation is concerned, are improving when the inflection point is moved downstream up to about 0.6 of the contraction length. Moving the inflection point further downstream results in a reverse effect. That reverse effect is, actually, the main shortcoming of the present choice of a contraction contour. The data shown in Figure 26 explains partly the reason for that trend reverse, the maximum wall curvature at inlet - or more precisely maximum (R'') - is larger than $Z_{ip} = 0.65$ than that when $Z_{ip} = 0.55$ at the same total length. This was not expected, and the function chosen for R'' has gone through a change in the shape at these extreme positions of the inflection point.

Since Figure 25 shows that for a contraction length of $L = 1.3$, inlet separation occurs with $Z_{ip} = 0.65$ but not at $Z_{ip} = 0.55$, a comparison between the shape of both contraction contours is shown in Figure 36 (a). Their R'' distribution and the resulting inlet c_p distribution are shown in Figures 36 (b) and 36 (c) respectively (Note the correlation between both R'' and C_p distributions). In Figure 36 the solid line belongs to the $Z_{ip} = 0.55$ contraction, while the dashed line belongs to that with $Z_{ip} = 0.65$.

Stratford's criterion predicts separation for the pressure distribution given by the dashed line in Figure 36 (c) and no separation for

the solid line distribution in the same figure. The reason is due to the fact that in the case of the dashed line distribution the boundary layer travels a greater distance and hence gets thicker before it enters the region of higher adverse pressure gradient. Actually, by such a slow start we waste a valuable region where the boundary layer is still relatively thin and can stand an even higher adverse pressure gradient.

For the purpose of comparison, Thwaites contraction No. 2, as shown in Figure 22 starts slowly which requires the concentration of the curvature in a narrow region, which represents a poor utilization of the length.

As noted before, the study of Ref. 23 results in relatively long contractions. To relate these results to the curvature distribution of the contraction wall the R'' - distribution of the contraction for which so-called optimized results are reported have been calculated numerically according to the derivation in Appendix II. The R'' distribution is shown in Figure 37. In this distribution the very slow start has actually wasted about half the entry region without appreciable area change. Most of the convergence has been forced, then, to occur in a narrow region giving rise to the requirement of high local curvature for a given length, and consequently the requirement of longer length to reduce the actual maximum value to limits where boundary layer separation can be avoided. It is clear, also, from Figure 37 that the curvature in the exit region is relatively low and that region is, then, unnecessarily long.

APPENDIX V

REVIEW OF BORGER'S STUDY (REF. 47)

After the present work had been concluded an additional study on the same subject was found. In Ref. 47, Borger used a polynomial of the fifth degree to describe the contraction contour. The coefficients of the polynomial were chosen to give an optimum contraction contour. For the calculation of the inviscid flow, the contraction walls are represented by vortex sheets. The limits of the vortex sheets are formed by source and sink discs (Ref. 49-50). The boundary-layer calculations were carried out using a method by Bradshaw et al (Ref. 51). The criterion used for no separation was $C_f \geq 0.002$ in the inlet region, and $C_f \geq 0.0025$ in the exit region. The boundary-layer calculations in the exit region start at location where the air speed is maximum. A small boundary-layer thickness is assumed at that location. The flow uniformity requirement is set at 0.1% at the contraction exit. For a contraction ratio of eight, the contraction length is found to be $L = 1.65$ with the inflection point at $X_{ip} = 0.65$. The results presented (Ref. 47) are in agreement with the present results in that the required contraction length decreases as the contraction ratio increases in the range $CR \geq 4$. The contraction length as a function of the contraction exit radius (for inlet radius of one) is shown in Figure 95.

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