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✓
TECHNIQUES FOR THE SYNTHESIS
OF WIDEBAND, SERIES-FED
ARRAYS OF MICROSTRIP PATCH RADIATORS ✓

by
Peter C. Strickland, M.Sc

Thesis submitted to the faculty of Graduate Studies and
Research in partial fulfillment of the requirements for
the degree of:

Doctor of Philosophy

Department of Electronics
Faculty of Engineering
Carleton University
Ottawa, Canada
August 1989

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TECHNIQUES FOR THE SYNTHESIS
OF WIDEBAND, SERIES-FED
ARRAYS OF MICROSTRIP PATCH RADIATORS

Submitted by P.C. Strickland
in partial fulfillment of the requirements for the
degree of Doctor of Philosophy

Prof J.S. Wight

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ABSTRACT

The work reported in this document represents a comprehensive investigation of techniques devised by the author for the synthesis of wideband, series-fed arrays of microstrip patch radiators. The investigation deals with two principle aspects of the array design problem:

- 1) the development of new broadband pattern synthesis techniques
- 2) the development of individual radiators or sub-arrays having increased bandwidth.

Two novel pattern synthesis techniques have been investigated. The first is unique in that it allows the power pattern to be optimised in a least mean square error sense over the desired operating band of the array. This is much more effective than alternative approaches which constrain such factors as the source norm or superdirectivity ratio and do not ensure a useful pattern bandwidth. The second pattern synthesis approach investigated was a numerical optimization whereby the sum of the pattern errors at several discrete frequencies is minimized. This approach allows different limits on pattern error to be imposed on different regions thus optimally tailoring the pattern to a given application.

A very interesting technique for the synthesis of broadband radiating elements has been investigated, and demonstrated. This approach, based on filter theory, requires that a radiating element or sub-array be broken up into multiple electrically coupled resonators allowing the quality factor and resonant frequency of each to be specified for the desired bandpass characteristic. A second approach focusses on making the elements non-resonant. Conceptually these elements are series-fed and have the same characteristic impedance as the feed line. The only discontinuities, on the feed line, are then the small radiation conductances and fringing capacitances at the radiating edges. The non-resonant condition can be achieved by periodically loading the high impedance sections such that their characteristic impedance matches that of the low impedance sections. Alternatively the substrate thickness may be varied along the array such that the characteristic impedance remains constant.

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GLOSSARY OF NOTATIONS

c	3×10^8 m/s
C	Capacitance Per Unit Length
C_o	Lumped Capacitance
δE	First Variation of E
$[f]$	Discrete Source Matrix
$F(k\eta)$	Array Space Factor
$[g]$	Discrete Field Matrix
g^*	Complex Conjugate of $[g]$
$\bar{g}$	Normalized Radiation Conductance
(g)	Radiation Conductance
$ g $	Norm of $[g]$
h	Microstrip Substrate Thickness
$I(Z')$	Current Distribution
k	Wavenumber
L	Aperture Length
L_o	Lumped Inductance
Q	Quality Factor of Resonator
R_L	Excess Loss Ratio
T_N	Chebyshev Polynomial
T^{-1}	Inverse of $[T]$
T^H	Hermitian Conjugate of $[T]$
v_i	Junction VSWR
V_n^+	Forward Wave Amplitude
V_n^-	Reverse Wave Amplitude
w	Microstrip Line Width

$\bar{Y}$	Normalized Admittance
Z_n	Characteristic Impedance
τ	Superdirectivity Ratio
Γ	Reflection Coefficient
ϵ_e	Effective Dielectric Constant
ϵ_0	Permittivity of Free Space
ϵ_r	Relative Permittivity
μ_0	Permeability of Free Space
η	Directional Cosine $\eta_x^2 + \eta_y^2 + \eta_z^2 = 1$
η_x	$\sin \theta \cos \phi$
η_y	$\sin \theta \sin \phi$
η_z	$\cos \theta$
ϕ	Phase Length
$[\]_{N \times M}$	$N \times M$ Matrix
$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$	Wave Amplitude Transmission Matrix

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CHAPTER I
INTRODUCTION

1.1 GENERAL

The concept of the series-fed microstrip patch array has been in the open literature for over a decade. The practical application of such arrays has however been severely limited primarily due to the narrow pattern bandwidth achieved with conventional implementations. This is unfortunate because the low profile, light weight and low manufacturing cost of these arrays would otherwise make them very attractive for use in a wide range of applications. This thesis research examines the limitations on pattern bandwidth and investigates means by which this bandwidth can be increased.

A conventional series-fed microstrip patch array is illustrated in figure 1.1.1. The radiating elements are low impedance, half wavelength, rectangular microstrip resonators. These resonators or patches as they are often called, are connected by sections of higher impedance microstrip line whose length determines the phase shift between radiating elements. The transmission line model typically used to represent each section of the array is illustrated in figure 1.1.2.. As shown, the discontinuities between the high impedance and low impedance microstrip lines are modeled as shunt conductances accounting for radiation, and changes in the length of both the high impedance and low impedance lines

to account for fringing fields. The radiation conductance increases as the width of the resonator increases thus providing a means for controlling the intensity of illumination across the aperture.

In a series-fed array, the amplitude and phase at each radiator depends strongly on the wave transmission parameters of each transmission line segment preceding it in the array. There are many factors which cause these parameters to vary, introducing errors in the aperture illumination. These factors include:

- (1) etching tolerances
- (2) uncertainty in the substrate thickness
- (3) uncertainty in the dielectric constant
- (4) inadequate modeling of the discontinuities
- (5) changes in frequency
- (6) expansion or contraction
- (7) change in dielectric constant with temperature

As an example of the potential impact of these factors, consider an array designed for a microwave landing system azimuth antenna [1]. Figure 1.1.3(a) shows the elevation pattern of the array in the far field. Figure 1.1.3(b) shows the same pattern but with a 4% change in dielectric constant. This change has introduced large sidelobes below the horizon which are not acceptable in the

microwave landing system application. The array in this case was designed using a conventional Fourier synthesis technique.

The impact of amplitude and phase errors in the aperture illumination on the far field pattern of the array depends strongly on the pattern synthesis technique used and the bandwidth of the radiating elements. In this thesis, unique synthesis techniques are described which minimize the sensitivity of the radiation pattern to errors in aperture illumination. Microstrip radiating elements which have significantly wider bandwidths than the conventional rectangular patch are also described in this report.

Figure 1.1.1 Conventional Series-Fed Microstrip Patch Array

Figure 1.1.2 Transmission Line Model for Rectangular Patch Radiator

Figure 1.1.3 Power Pattern of Series-Fed Microstrip Patch Array. Fourier Synthesis

(a) Freq = 5.06 GHz; $\epsilon_r = 2.33$

Figure 1.1.3 Power Pattern of Series-Fed Microstrip Patch Array Using Fourier Synthesis

(b) Freq = 5.06 GHz; $\epsilon_r = 2.42$

1.2 BACKGROUND

1.2.1 Techniques of Pattern Synthesis

It can be shown [2] that an aperture illumination exists which will allow an array of any finite size to produce any desired pattern with an arbitrary precision. There is however a practical limit to the precision to which a desired pattern can be produced. This limit results because attempts to increase the precision result in a rapid variation in amplitude and phase across the array which is generally impossible to implement. Further, such an array illumination would be extremely sensitive to frequency and environmentally induced changes. The synthesis problem here is thus that of finding physically realizable aperture illuminations which provide adequate approximations to the desired pattern.

The usual measure of pattern sensitivity found in the literature is the superdirectivity ratio γ . The reciprocal of the superdirectivity ratio is a measure of the relative concentration of power within the visible region.

The superdirectivity ratio is given by

$$\tau = \frac{\int_{-\infty}^{\infty} |F(k\eta)|^2 d\eta}{\int_{-1}^1 |F(k\eta)|^2 d\eta} \quad 1.2.1.1$$

where $F(k\eta)$ is the array space factor and η is the directional cosine. Rhodes [3] indicates that this is not always a good measure of sensitivity and suggests the following form which is an approximation to antenna Q:

$$\tau_{\alpha} = \frac{\int_{-\infty}^{\infty} |F(k\eta)|^2 |1 - \eta^2|^B d\eta}{\int_{-1}^1 |F(k\eta)|^2 (1 - \eta^2)^B d\eta} \quad 1.2.1.2$$

The parameter B which controls the weighting function is defined by Rhodes [3] for various sources. It is clear, regardless of the expression used, that large lobes in the invisible region correspond to excitations which are usually difficult to synthesize and sensitive to errors.

The principle synthesis techniques applicable to series-fed microstrip patch arrays are:

- (1) Fourier Synthesis
- (2) Woodward Synthesis
- (3) Optical Synthesis
- (4) Discrete Least Mean Square Synthesis

The Fourier synthesis technique is used to synthesize a continuous source either linear or two dimensional and of infinite extent which will yield an approximation to a far field pattern which is pre-defined both in amplitude and phase. Provided that the discrete sources of the series-fed array have separations of half a wavelength or less, these sources will yield a reasonable approximation to a continuous distribution, and the Fourier synthesis technique can thus be used. The continuous distribution is simply sampled at the locations of the discrete sources. This sampled distribution is then used to define the relative amplitudes and phases of the elements. Consider the array illustrated in figure 1.2.1.1. If we assume that the illumination can be approximated by a continuous source along the Z axis then the array space factor $F(k \cos \theta)$ is given approximately by

$$F(k \cos \theta) = \int_{-\infty}^{\infty} I(Z') \exp [jkZ' \cos \theta] dz' \quad 1.2.1.3$$

Figure 1.2.1.1 Linear Array Geometry

The array space factor is readily observed to be simply the Fourier transform of the aperture illumination $I(Z')$. The aperture illumination required for a particular array space factor is thus given by the inverse transform:

$$I(Z') = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k \cos \theta) \exp[-jZ' k \cos \theta] d(k \cos \theta)$$

1.2.1.4

Since the far field pattern is defined only over the visible range $(-k \leq k \cos \theta \leq k)$ and typically only in magnitude, equation 1.2.1.4 can yield an infinite number of solutions for the aperture illumination. Since the illumination is defined over infinite extent by equation 1.2.1.4 it must be truncated in order to yield a physically realizable solution. The degradation in the far field pattern introduced by this truncation may be reduced by two means [4]. The first is to use the far field phase of the truncated illumination along with the required intensity pattern in a second Fourier synthesis to define a new illumination. This may be done iteratively resulting in a final truncated illumination with an improved far field pattern. The second technique is to extend the pattern into the invisible region. In the invisible region the pattern given by the truncated illumination is used and in the visible region the desired pattern is used. This

technique can also be used iteratively and simultaneously with the first technique.

In the Woodward synthesis technique [5], the aperture illumination is defined by a set of basis functions whose amplitudes are selected such that the desired far field pattern is achieved exactly at a finite number of points. These points are given by:

$$k \cos \theta = \frac{2n\pi}{L} \quad n = 0, \pm 1, \pm 2 \dots \pm N$$

1.2.1.5

where $N = L/(\text{wavelength})$ and L is the aperture length. The aperture illumination is given by the Fourier series:

$$I(Z') = \sum_{-N}^N A_n \exp(-j2n\pi Z'/L) \quad 1.2.1.6$$

The far field pattern resulting from this aperture illumination is, from 1.2.1.1:

$$F(k \cos \theta) = \sum_{-N}^N a_n \int_{-L/2}^{L/2} \exp[-jz'(2n\pi/L - k \cos \theta)]$$

1.2.1.7

Each term in this series corresponds to a $\sin x/x$ term giving:

$$F(k\cos\theta) = \sum_{-N}^N a_n \sin (n\pi - Lk\cos\theta/2)/(n\pi - Lk\cos\theta/2)$$

1.2.1.8

Since the peak of each $\sin x/x$ term corresponds to a null on each of the other terms in the series, the far field can be synthesized exactly at these points by setting the weights (a_n) according to:

$$a_n = F(2n\pi/L) \quad 1.2.1.9$$

There have been many extensions of Woodward's method published and it is worthwhile discussing the impact of some of these. The first such extension was proposed by Woodward and Lawson [6] and showed that the far field could in fact be synthesized exactly at an arbitrary number of points. As the number of points increases however the stored energy in the aperture increases making the illumination difficult to realize. This is discussed further in section 1.2.4. An extension of the Woodward synthesis proposed by Taylor [7] allows the nulls in the far field pattern to be synthesized exactly. R.F. Hyneman and R.M. Johnson [8] described a technique whereby the pattern error in the visible region could be reduced without introducing large lobes in the invisible region and thus without increasing the stored energy.

The optical method of pattern synthesis [9] is based on geometrical optics and thus yields relatively crude far field patterns particularly in the case of short apertures. The technique is quite useful however for two reasons. The first is that the magnitude of the aperture illumination can be arbitrarily pre-defined. The synthesis technique yields only the phase distribution required given a desired far field pattern and a particular aperture intensity distribution. Secondly the technique yields either a monotonically increasing or a monotonically decreasing phase gradient. In the case of series-fed arrays this tends to minimize the sensitivity of the beamshape to changes in frequency or propagation velocity.

In the optical synthesis procedure it is assumed that the energy radiated by an infinitesimal section of the aperture is directed in a ray in the direction defined by the local phase gradient. The phase gradient is increased or decreased monotonically such that the radiated power is distributed according to the desired far field intensity pattern. This is defined by the following equation [4]:

$$\int_0^{z'} |I(z')|^2 dz' = b \int_{k \cos \theta_1}^{k \cos \theta} |F(k \cos \theta)|^2 d(k \cos \theta)$$

The normalization parameter b is determined by setting $\theta = \theta_2$ and $z = L$ in the above equation. The far field pattern exists only between θ_1 and θ_2 . Note that this synthesis technique does not provide any control over the far field phase. In order to improve the far field pattern Walter [4] has proposed that the far field phase resulting from the optically derived illumination be used in a Fourier synthesis thus yielding a slightly altered illumination with a more accurate far field pattern.

Consider now the discrete least mean square synthesis approach as described by Mautz and Harrington [10]. The relationship between the source (f) and the field it produces (g) can be expressed by:

$$[T]f = g \quad 1.1.1.11$$

where f is the source vector:

$$f = [f_n] \text{ Nx1} \quad 1.1.1.12$$

and g is the vector defining the field at M points:

$$g = [g_m] \text{ Nx1} \quad 1.1.1.13$$

and $[T]$ is the matrix with elements given by:

$$T_{mn} = \exp [jk x_n \cos \theta_m] \quad 1.1.1.14$$

Typically the field is defined at many more points than the source and the least mean square error solution is given by:

$$f = [T^H T]^{-1} T^H g \qquad 1.1.1.15$$

where T^H is the transpose conjugate (Hermitian conjugate) of T .

1.2.2 Constrained Optimization

There has over the years been a considerable amount of research performed in the area of constrained optimization of the power pattern of a radiating aperture. The principal thrust of this work has been towards deriving physically realizable excitations as opposed to super gain type solutions as discussed in section 1.2.1. It is well recognized however that as the various measures of the superdirectivity such as the source norm or Q increase the antenna pattern bandwidth tends to be reduced. It is reasonable then to ensure a relatively low source norm or Q if a wide pattern bandwidth is required. This however does not ensure that the pattern bandwidth is maximized in any sense. The relationship between these parameters and pattern bandwidth is different for different antennas. Furthermore it is impossible except through trial and error to determine what the constraint should be in order to achieve a desired pattern bandwidth.

Perhaps the most relevant examples of constrained power pattern optimization are those of Mautz and Harrington [10] and Rhodes [11]. Mautz and Harrington considered the case of discrete arrays with a constraint on the source norm. The source norm is defined as the square root of the sum of the squares of the absolute values of the voltage at each element:

$$\text{Norm} = \left[\sum_N |f_N|^2 \right]^{\frac{1}{2}} \quad 1.2.2.1$$

The source norm was constrained to be less than a specific threshold while the mean square pattern error at M discrete points was minimized.

Rhodes [11] considered the case of a continuous aperture distribution which results in minimum mean square pattern error subject to a constraint on Taylor's superdirectivity ratio τ . The nature of the superdirectivity ratio was discussed in section 1.2.1. It was found that when the aperture is short the pattern error is relatively insensitive to the superdirectivity ratio. Large increases in τ were required for small reductions in the pattern error. The impact of constraints on τ was more significant for relatively long apertures.

An alternative approach to the synthesis of realizable aperture illuminations has been taken by Richards and Lo [12] and Ashmead [13]. In these analyses the elements of the array were assumed to have random errors in excitation. The resulting radiation pattern which is also random was optimized in a probabilistic sense yielding a desirable nominal excitation. As in the other case discussed above, this approach does tend to increase the pattern bandwidth relative to that achieved with an

unconstrained optimization. The approach does not however guarantee a desirable trade-off between mid-band pattern error and pattern bandwidth.

1.2.3 Radiator Bandwidth

The bandwidth of a conventional rectangular microstrip patch radiator is quite narrow particularly when a thin substrate with high permittivity is used. Figure 1.2.3.1 from [14] indicates that at 1 GHz with a relative dielectric constant of 2.32 and a substrate thickness of 0.159 cm the bandwidth is limited to approximately 1% for a voltage standing wave ratio of less than two. Modified patch shapes have been developed by many corporations in order to achieve increased bandwidth. However in general these have not been reported in the open literature. A few examples of such modified patches are illustrated in figure 1.2.3.2. These include opening a large hole in the center of the patch, removing squares from the corners and using pentagonal structures.

Recently the concept of using electromagnetically coupled parasitic radiating elements to increase the patch bandwidth has appeared in the literature [15, 16, 17]. The two basic approaches which have been used are:

- 1) stack radiators with slightly different resonant frequencies on top of each other
- 2) place the parasitic resonators beside the fed element

The stacking of the resonators obviously has the advantage of allowing a higher element density but is clearly more difficult to fabricate. Typically these techniques allow a

bandwidth 2-3 times that of the single radiator without parasitic elements to be achieved.

Relatively little research has been performed in the area of wideband arrays of electrically coupled resonators. The work which has been done in this area has concentrated on the design of log periodic arrays [18, 19]. While these arrays have achieved very wide impedance bandwidths, the pattern bandwidth has been very narrow. The arrays have been electrically large and have produced narrow beams which are highly frequency dependent in both direction, and shape.

An important aspect of wideband radiator design which must not be overlooked is that of optimal impedance matching. Fano [20] has shown that it is impossible to perfectly match a reactive load over a finite bandwidth. Instead a wide bandwidth match can be achieved only at the expense of a significant reflection coefficient over the band considered. Often however this reflection coefficient can be made sufficiently low over the desired bandwidth to meet the requirements of a given application. Matthaei, Young and Jones [21] have applied conventional microwave filter theory to the problem of achieving a broadband match to a load which can be modeled as a series or parallel R,L,C circuit. The basis of their approach is that the reactive component of the load impedance is resonated out

at mid-band and this serves as the first resonator of a filter designed to give the desired bandwidth. This approach has been applied to the problem of corporate-fed microstrip patch radiators by Van de Capelle [22].

Figure 1.2.3.1 Variation of Bandwidth with Frequency for Different Dielectric Substrate Antennas from [14]

(a) Raytheon Corporation

(b) Ball Aerospace

(c) Harris Corporation

Figure 1.2.3.2 Three Examples of Patch Radiators with Increased Bandwidth

1.3 OBJECTIVES AND ORGANIZATION

The pattern bandwidth of a series-fed microstrip patch array is typically limited both by the impedance bandwidth of the individual radiating elements and by the characteristics of the synthesized aperture illumination. It is the principle aim of this thesis to develop techniques which will reduce both of these limitations thus allowing practical series-fed arrays with wide pattern bandwidths to be designed.

Two novel pattern synthesis techniques have been investigated. Both methods have the potential to yield aperture illuminations with far field power patterns which are relatively insensitive to the operating frequency. The first and most elegant technique is a least mean square error approach. This technique minimizes the sum of the pattern errors at several discrete frequencies in the operating band. The second technique is a numerical optimization approach. In this case an aperture illumination synthesized by means of a conventional algorithm is perturbed yielding a relatively insensitive illumination.

The thesis also investigated two approaches to obtaining broadband radiating elements. The first approach has been called the Filter Theory Approach mainly because

in one of its forms it makes use of conventional microwave filter theory. It is interesting to note that a series-fed microstrip patch array of directly coupled rectangular resonators and a $H_i z - L_o z$ microstrip filter have the same structure. The difference of course is that in one case radiation loss is desirable and in the other it is undesirable. A short section of a long series-fed array will typically have sufficiently low radiation loss that the transfer function can be determined while neglecting radiation loss. Filter theory can then be applied to synthesize the desired bandpass characteristic of that short array section. This being applied to each array section ensures a desirable return loss characteristic from the complete array.

In a second implementation of the filter theory approach low impedance resonators are coupled by high impedance resonators but these are configured such that the resulting array is electrically small. Resonator dimensions are then optimized yielding a broadband radiating element.

An alternative approach to producing a broadband radiating element involves making the radiator non-resonant. If each of the radiators in a series-fed array has the same characteristic impedance as the feed line, then the radiators are non-resonant and need not be a half-

wavelength long. The only discontinuities are then the radiation conductances which are small. One approach used to make the characteristic impedances of all sections equal was to use periodic loading provided by narrow capacitive stubs. Provided that these stubs are sufficiently narrow, their radiation losses will be negligible. A second approach involved varying the substrate thickness such that the characteristic impedances of all transmission lines were equal.

The investigation of the broadband radiating element has had three phases. The first was development of the basic theory behind the element performance. The second was a numerical simulation and the third was fabrication and measurement. This methodology has also been used in developing the filter theory approach to array and element design. In the case of the pattern synthesis techniques the theory was developed and this was followed by verification by means of numerical simulation.

CHAPTER II
ARRAY PATTERN SYNTHESIS

2.1 LEAST MEAN SQUARE ERROR WITH PATTERN CONSTRAINTS AT MULTIPLE FREQUENCIES

2.1.1 Introduction

The pattern synthesis technique described here is unique in that it allows the power pattern to be optimized in a very useful sense over the desired operating band of the array. As discussed in previous sections, constraints on such artificial parameters as source norm and superdirectivity ratio do not in fact ensure a useful pattern bandwidth. The procedure described here however ensures optimum performance over the desired bandwidth. The procedure is quite general and can be applied to both shaped beam and pencil beam patterns and all types of arrays or apertures.

The concept which is implemented here is quite simple and yet very powerful. It can be described in the following way. First the desired far field pattern is established at an arbitrary number of discrete frequencies in the desired operating band. The desired pattern may be the same at all frequencies or may be different at any number of the frequencies. Initially we will assume that the desired pattern is defined both in amplitude and phase. Later we will consider the case where amplitude only is defined. The second step is to establish the relationship

between the excitation at mid-band and the far field pattern at each frequency. In some cases this turns out to be a trivial operation and in other cases it requires careful modeling of the array. In this analysis the source is assumed to be an array of discrete elements. These discrete elements could of course represent a sampled continuous distribution. The samples of the far field pattern can be loaded into a single vector $[g]$ such that the first m samples represent the pattern at one frequency and the subsequent groups of m samples correspond to other frequencies in the desired bandwidth.

2.1.2 Theory

The relationship between the excitation at midband $[f]$ and the field vector $[g]$ is described by [10] the transformation $[T]$.

$$[T]_{M \times N} f = g \quad 2.1.1$$

where $[f]$ is the vector describing the source at mid-band

$$f = [f_n]_{N \times 1} \quad 2.1.2$$

and g is the vector describing the far field pattern at M points on the frequency-direction plane.

$$g = [g_m]_{M \times 1} \quad 2.1.3$$

In the case of a pattern defined with a fixed source distribution, $[T]$ is the matrix with elements:

$$T_{mn} = \exp [jkx_n \cos \theta_m] \quad 2.1.4$$

where k = wavenumber

x_n = element position along array

θ_m = sample direction relative to endfire

In the case of a source distribution which changes with frequency:

$$T_{mn} = A_{mn} \exp[j(k_m x_n \cos \theta_m + \phi_{mn})]$$

2.1.5

The wavenumber now depends on which group of pattern samples is considered and the variables A_{mn} and ϕ_{mn} define the change in amplitude and phase respectively at each element versus frequency. In the simplest case we may consider a series-fed array with elements loosely coupled to a feed line and the end of the feed line terminated in a matched load. We then have:

$$A_{mn} \approx \text{Constant} \quad 2.1.6$$

$$k_m = \frac{2\pi f}{c} \quad 2.1.7$$

$$\phi_{mn} = x_n B_m \quad 2.1.8$$

Where B_m is the phase constant on the feed line associated with the m th pattern sample. Typically the number of points M defined on the frequency-direction plane will be much larger than the number of source points N defined. In this case there is in general no exact solution to (2.1.1) however we may find the mid-band excitation f which minimizes the mean square error:

$$E = || [T]f - g ||^2 \quad 2.1.9$$

This can be minimized in the usual manner [23] starting with the identity:

$$||v||^2 = v^H v$$

$$\begin{aligned} E &= (f_t [T_t] - g_t)^* ([T] f - g) \\ &= f^H [T^H T] f - g^H [T] f - f^H [T^H] g + g^H g \end{aligned} \quad 2.1.10$$

where A^H is the Hermitian conjugate of A and
 A_t is the transpose of A

Taking the first variation with respect to f and setting this equal to zero we obtain:

$$\delta E = (f^H [T^H T] - g^H [T]) \delta f = 0 \quad 2.1.11$$

Since δf is arbitrary we have:

$$f^H [T^H T] = g^H [T] \quad 2.1.12$$

Taking the transpose-conjugate of each side we obtain:

$$[f^H [T^H T]]^H = [g^H [T]]^H \quad 2.1.13$$

$$\text{Identity: } [ABC]^H = C^H B^H A^H$$

Therefore:

$$[T^H T] f = [T^H] g \quad 2.1.14$$

$$f = [T^H T]^{-1} [T^H] g \quad 2.1.15$$

Taking the first variation of (2.1.10) with respect to f^H , we obtain:

$$\delta E = \delta f^H ([T^H T] f - [T^H] g) = 0 \quad 2.1.16$$

Since δf^H is arbitrary we obtain:

$$[T^H T] f = [T^H] g \quad 2.1.17$$

$$f = [T^H T]^{-1} [T^H] g \quad 2.1.18$$

Since (2.1.18) is identical to (2.1.15), this makes the first variation of the pattern error with respect to the source zero and according to [10] this represents the minimum mean square error solution.

2.1.3 Computational Results

The performance of the least mean square error pattern synthesis with multiple pattern constraints at different frequencies has been evaluated by means of a numerical example. The case considered here is that of 28 series-fed elements loosely coupled to a common transmission line. The elements are assumed to have omnidirectional patterns and thus the synthesized pattern may be considered to be the array factor of a real array. The pattern which was synthesized in a least mean square sense was a uniform excitation with a null ten degrees wide centered at broadside. The center frequency for the array is 5 GHz and bandwidths of $\pm 10\%$ and $\pm 15\%$ are considered. The 28 elements are assumed to form a linear array with a mean spacing of 4 cm between elements.

A program has been written in FORTRAN to perform the least mean square error synthesis. The program which is included as Appendix A has been written to be completely general in application. Field distributions at two different frequencies in the desired band are read from data files FOR007.DAT and FOR010.DAT respectively. Source phase errors resulting from frequency deviations from mid-band are read from file FOR011.DAT. The source positions are read from file FOR008.DAT and the synthesized mid-band source excitations are written into file FOR009.DAT. When

the program is run the operator is prompted to enter the two design frequencies, the number of source elements and the number of field points specified at each frequency. The program was written to run on a VAX VMS system having PLXY plotting software. The routines could, however, easily be altered to run on other systems. In addition to yielding the source distribution at mid-band, the program also produces a plot of the synthesized radiation pattern at any chosen frequency. Note that the program must be used iteratively, that is it is run once to find an initial estimate of the source distribution and the resulting source locations are used in the next iteration.

The program was run initially to find the source distribution which optimized the pattern in a least mean square error sense at mid-band. This case serves as our reference against which the impact of pattern constraints at multiple frequencies can be observed. In selecting field sample points, some thought must be put into the relative contributions to mean square error from various portions of the pattern. For example a point 30 dB down in the null will contribute much less to the mean square error than a point outside the null. Without considering this effect, a person could equally space the field samples in space. This, however, results in a very poor null since points below -10 dB result in negligible contribution to the error. In the example considered here, excellent

results were achieved by defining 70 points uniformly distributed outside the null and 155 points uniformly distributed inside the null. For good results the field should be defined at points more closely spaced than $(\lambda)/D$ radians where (λ) is the wavelength and D is the total array length. As an alternative to the non-uniform field sampling, a weighting function may be applied to the error contributions. For simplicity this was not done here.

Figure 2.1.3.1 shows the pattern obtained at mid-band. There are no extraneous lobes within the null which is roughly 20 dB down over its 10 degree width. Outside the null there is a fluctuation of roughly ± 1 dB with most of the ripple close to the sharply rising edges of the null. Figures 2.1.3.2 and 2.1.3.3 show the effect of a 10% increase and decrease in the frequency respectively. In both cases the null becomes filled in considerably (to the -13 dB level at one end) and the pattern outside the null becomes severely distorted with a variation of roughly ± 3 dB. Figures 2.1.3.4 and 2.1.3.5 show the effect of a 15% increase and decrease in frequency respectively. In this case the pattern becomes even more severely distorted and the null becomes filled at one end to the -11 dB level. Note that the pattern is not shown squinted.

Figure 2.1.3.1 Least Mean Square Error Synthesis
Optimized at Mid-Band; Pattern at
Mid-Band

Figure 2.1.3.2 Least Mean Square Error Synthesis
Optimized at Mid-Band; Pattern at
 $1.1 f_0$

Figure 2.1.3.3 Least Mean Square Error Synthesis
Optimized at Mid-Band; Pattern at
 $0.9 f_0$

Figure 2.1.3.4 Least Mean Square Error Synthesis
Optimized at Mid-Band Pattern at
 $1.15 f_0$

Figure 2.1.3.5 Least Mean Square Error Synthesis
Optimized at Mid-Band; Pattern at
 $0.85 f_0$

The program for Least Mean Square Error Pattern Synthesis was run again this time using pattern constraints at multiple frequencies. In the first case the pattern was optimized at $1.05 f_0$ and $0.95 f_0$ simultaneously. Figure 2.1.3.6 shows the pattern of the resulting source distribution plotted at mid-band. As in the case of the mid-band optimization, the null depth is again approximately -20 dB. The pattern is not quite as good as that of Figure 2.1.3.1 however having a bit more ripple and a slightly narrower null. Figures 2.1.3.7 and 2.1.3.8 show the resulting patterns at $1.1 f_0$ and $0.9 f_0$. The pattern remains stable at these frequency offsets, maintaining the -20 dB null depth and having only slightly increased ripple inside and outside the null. This is a dramatic improvement over the case of mid-band optimization where the pattern is severely distorted when the frequency is offset by 10%. A second optimization was performed, this time at $1.075 f_0$ and $0.925 f_0$. The pattern at mid-band is shown in Figure 2.1.3.9. Again the pattern is just about as good as that obtained with mid-band optimization. Figures 2.1.3.10 and 2.1.3.11 show the resulting patterns at $1.15 f_0$ and $0.85 f_0$ respectively. The pattern continues to remain intact with a null of roughly -20 dB. The pattern ripple has increased further however to roughly +/- 2.5 dB at $0.85 f_0$. This performance is far superior to that obtained with mid-band optimization.

Figure 2.1.3.6 Least Mean Square Error Synthesis
Optimized at $1.05 f_0$ and $0.95 f_0$
Pattern at Mid-Band

Figure 2.1.3.7 Least Mean Square Error Synthesis
Optimized at $1.05 f_0$ and $0.95 f_0$
Pattern at $1.1 f_0$

Figure 2.1.3.8 Least Mean Square Error Synthesis
 optimized at $1.05 f_0$ and $0.95 f_0$
 Pattern at $0.9 f_0$

Figure 2.1.3.9 Least Mean Square Error Synthesis
Optimized at $1.075 f_0$ and $0.925 f_0$
Pattern at Mid-Band

Figure 2.1.3.10 Least Mean Square Error Synthesis
Optimized at $1.075 f_0$ and $0.925 f_0$
Pattern at $1.15 f_0$

Figure 2.1.3.11 Least Mean Square Error Synthesis
Optimized at $1.075 f_0$ and $0.925 f_0$
Pattern at $0.85 f_0$

Figures 2.1.3.12 and 2.1.3.13 show the aperture phase distribution for the case of mid-band optimization and optimization at $1.075 f_0$ and $0.925 f_0$ respectively. The phase distributions can be seen to be quite similar with the most difference appearing at elements 12, 13 and 21. Figures 2.1.3.14 and 2.1.3.15 show the amplitude distributions for the cases of mid-band optimization and dual-frequency optimization respectively. Again the distributions are quite similar with significant differences at elements 5, 12, 13 and 19-28. It is clear that the dual-frequency optimization has resulted in only small changes in the aperture distribution but these changes result in tremendous improvements in the array performance.

As a final point in the discussion of the least mean square error synthesis with constraints at multiple frequencies, the nature of the amplitude distributions should be discussed. In figures 2.1.3.14 and 2.1.3.15, the amplitude distribution spans roughly 50 dB across the array. This is not desirable for most practical array designs since most of the energy is radiated in a small number of elements. The reason why this particular distribution was obtained is that the far field phase was defined as zero at all field points. What is required is a means of defining the far field phase such that a more desirable amplitude distribution is obtained. The Optical

Synthesis Technique which is discussed elsewhere in this thesis provides a means for doing this. With this technique the amplitude distribution is pre-defined as desired and the optical synthesis is performed resulting in a source distribution whose pattern will approximate the desired pattern in magnitude at the design frequency. The phase of this pattern can then be used as the field phase for the least mean square error synthesis. This technique is discussed further in section 2.2.2. Note that far field phase could alternatively be used as a variable to further reduce the mean square error.

Figure 2.1.3.12 Least Mean Square Error Synthesis
Optimized at Mid-Band; Source Phase
Distribution

Figure 2.1.3.13 Least Mean Square Error Synthesis
Optimized at $1.075 f_0$ and $0.925 f_0$
Source Phase Distribution

Figure 2.1.3.14 Least Mean Square Error Synthesis
Optimized at Mid-Band; Source
Amplitude Distribution

Figure 2.1.3.15 Least Mean Square Error Synthesis
Optimized at $1.075 f_0$ and $0.925 f_0$
Source Amplitude Distribution

2.2 NUMERICAL OPTIMIZATION

2.2.1 Introduction

Numerical optimization is a very useful approach to antenna pattern synthesis since it allows all available parameters to be varied in order to obtain the desired pattern characteristics. For example in addition to element excitations, element positions may also be optimized. While numerical optimization is widely used in antenna pattern synthesis, it is typically applied to optimize the pattern at a particular frequency. This can result in a very narrow pattern bandwidth or even super-gain type solutions which may be impractical to implement even over a narrow bandwidth. In this chapter a numerical optimization procedure is discussed which minimizes the sum of the pattern errors at several discrete frequencies across the desired operating band. This procedure is discussed here within the context of the series-fed microstrip patch array used for a Microwave Landing System (MLS) antenna.

The maximum practical width of a rectangular patch radiator is limited to approximately 0.5 of one wavelength in the dielectric due to excitation of the first transverse mode under the patch. Since the radiation conductance (g) increases with patch width, this sets a limit on the

fraction of the incident power which can be radiated by a single patch for a fixed feed line impedance.

Consequently, in order to avoid excessive power dissipation by the termination at the end of the travelling wave feed it is necessary to choose an aperture illumination which is maximum in intensity towards the input of the array and is not sharply peaked.

2.2.2 Synthesis of Efficient Excitation

Several different design techniques have been proposed [4, 24, 25] which can be used to synthesize a required far field pattern while ensuring that the aperture intensity distribution yields an efficient array. Each of these techniques allows the aperture intensity distribution to be pre-defined with the aperture phase being varied to obtain the required power pattern. The optical synthesis technique described by Walter [4] and first proposed by Chu [9] produces a phase gradient which is monotonically increasing or decreasing along the array. As a consequence of this the first order response to a change in frequency or dielectric permittivity is a squinting of the beam. Other aperture illuminations which may yield the same power pattern have been found to generally exhibit more pattern deterioration when frequency or permittivity are changed. In the optical synthesis procedure, it is assumed that the energy radiated by each element is directed in a ray in the

direction defined by the local phase gradient. This can be expressed by:

$$\int_0^{z'} |A(z')|^2 dz' = b \int_{k \cos \theta_1}^{k \cos \theta} |F(k \cos \theta)|^2 d(k \cos \theta)$$

(2.2.2.1)

where $A(z')$ is the aperture illumination and $F(k \cos \theta)$ is the far field pattern which is constrained to exist only between θ_1 and θ_2 . The normalization parameter b is determined by setting $\theta = \theta_2$ and $z = L$. This synthesis technique being based on the assumptions of geometric optics does not yield power patterns which are sufficiently accurate for applications such as the microwave landing system antenna. A method proposed by Walter [4] and used here is to use the far field phase resulting from the optically derived illumination in a Fourier synthesis thus yielding a slightly altered illumination with a more accurate far field pattern.

The combined optical-Fourier technique has been used to synthesize the excitation for an azimuth antenna for a microwave landing system (MLS). The methods used for fitting the patch widths and separations to the required excitation are given elsewhere [26, 27] and this aspect of

the design process will not be elaborated upon here. The simulated far field power pattern of the array and the ideal far field power pattern are given in figure 2.2.1. The corresponding intensity distribution of the aperture illumination is given in figure 2.2.2.

Figure 2.2.1 Ideal Power Pattern and Pattern
Obtained by Optical-Fourier Synthesis

Figure 2.2.2 Aperture Intensity Distribution

2.2.3 Pattern Sensitivity

At frequencies below S band, honeycomb type substrates are typically used in microstrip arrays providing a low stable dielectric constant and satisfactory dimensional tolerances. Above S band the low dielectric thickness makes honeycomb substrates impractical and PTFE/fiberglass materials are generally used. Unfortunately the permittivity of these materials is a strong function of temperature and also varies from batch to batch. The change in permittivity and/or changes in frequency cause the phase gradients on the array to be altered from those derived in the synthesis described above. The effects of this on the power pattern for the MLS antenna are illustrated in Figure 2.2.3. Under worst case combinations of frequency and dielectric constant, considerable energy is radiated below the horizon. The resulting ground reflection can cause navigational errors in the MLS application.

The optical-Fourier technique described above, like most pattern synthesis methodologies, derives an aperture illumination which if implemented exactly will produce a desired far field pattern. If, however, the illumination contains errors or changes in some manner there is no guarantee that the pattern will be satisfactory in its particular application.

Figure 2.2.3 Power Pattern of Optical-Fourier
Synthesized Illumination.
(a) $f = 5.03$ GHz, $\epsilon_r = 2.29$
(b) $f = 5.09$ GHz, $\epsilon_r = 2.37$

A numerical optimization procedure has been used here to perturb the optical-Fourier illumination minimizing the sensitivity of the power pattern to changes in frequency and dielectric constant.

The optimization program follows the following procedure:

- 1) Initial array dimensions derived from the Optical-Fourier synthesis are read from data files.
- 2) Wave transmission matrices are computed for each of the transmission lines and discontinuities in the array.
- 3) The transmission matrices are used to compute the power and phase radiated at each discontinuity.
- 4) The radiation pattern is computed.
- 5) Steps 2-4 are repeated at mid-band, the upper band edge and the lower band edge.
- 6) At each frequency the difference between the desired pattern and the actual pattern is determined and at each angle the error is weighted by a coefficient selected for the given application).

- 7) The error function is computed using steps 2 through 6 for the width of one radiator increased by a small amount and again for the width decreased by a small amount. If the sum of the weighted pattern errors is reduced in either case, then this dimension is permanently altered.
- 8) Step 7 is repeated for each of the radiator widths and then each of the inter-radiator separations is similarly perturbed.
- 9) Once all of the widths and separations have been perturbed once, the entire cycle (steps 2-8) is repeated until a minimum value of weighted error is reached.
- 10) If the minimum value of pattern error attained (local minimum) is not sufficiently low then each of the dimensions is perturbed by a selected value and the optimization is repeated. This search routine allows a global minimum or satisfactory local minimum to be found.

The measure of pattern error has been tailored to the MLS application with heavy weight placed on sidelobes below the horizon and little weight placed at angles more than 25° above the horizon. The optimization procedure used minimizes the sum of the pattern errors under two sets of conditions. These are high dielectric constant with frequency at the high end of the band ($\epsilon_r = 2.37$; $F = 5.09$ GHz) and low dielectric constant with frequency at the low end of the band ($\epsilon_r = 2.29$; $F = 5.03$ GHz). Each patch is separately perturbed and the effect of the change on the pattern error is calculated. All changes which reduce the error are incorporated. Patch widths and separations are modified alternately and the cycle is continued until a local minimum of the error is reached. As the local minimum is approached the size of the perturbations is reduced. Figure 2.2.4 shows the power pattern of the perturbed excitation under worst case conditions of frequency and dielectric constant.

Figure 2.2.4 Power Pattern of Optimized Illumination
(a) $f = 5.03$ GHz, $\epsilon_r = 2.29$
(b) $f = 5.09$ GHz, $\epsilon_r = 2.37$

2.3 DISCUSSION

In Sections 2.1 and 2.2, it has been demonstrated that the pattern synthesis technique used to determine the array excitation has a large impact on the bandwidth over which the desired beam shape can be maintained. Conventional approaches to pattern synthesis such as Fourier and Woodward techniques do not even consider the impact of bandwidth or random errors on the resulting patterns. As indicated in Section 1.2.2, techniques which constrain the source norm or superdirectivity ratio have been developed. These parameters cannot however, in general, be related to any measure of pattern error. The techniques described in Sections 2.1 and 2.2 of this thesis overcome this limitation allowing pattern error to be minimized over the desired operating bandwidth.

Section 2.1 has described a Least Mean Square Error Solution having multiple sets of pattern constraints corresponding to different frequencies in the operating band. A numerical example was used to demonstrate how dramatic improvements in the radiation pattern at the frequency band edges can be obtained. The impact on the pattern error at mid-band is very small. Section 2.2 describes a pattern synthesis approach in which a numerical optimization minimizes the sum of the pattern errors at several discrete frequencies. This approach allows

optimization of patterns described in amplitude only, with far field phase being optimized for best amplitude pattern.

While the pattern synthesis techniques described in this chapter can increase the pattern bandwidth of a phased array considerably, there remains a limit on array pattern bandwidth imposed by the impedance bandwidths of the individual radiating elements. A high Q radiating element will reflect much of the power incident on it at the edges of the Frequency band thus altering the array excitation. The reflected energy may result in a high reflection coefficient at the array input or may be radiated by other elements which then have undesired amplitude and phase.

In the following chapter new techniques are described which allow the design of broadband microstrip patch radiating elements for both series-fed and corporate-fed arrays. These techniques include the following:

- 1) The use of multiple electrically coupled resonators optimized for maximum bandwidth.
- 2) The synthesis of matching networks for series-fed elements such that wideband Chebychev or Butterworth characteristics are obtained.

- 3) The weighting of elements in sub-arrays in order to synthesize Chebyshev, Butterworth or other responses.
- 4) The creation of non-resonant radiating elements either by periodic loading of high impedance lines or by variation of the substrate thickness.

The combined use of these new radiating elements and the pattern synthesis techniques described in Sections 2.1 and 2.2 will allow very broadband microstrip patch arrays to be produced.

CHAPTER III

SYNTHESIS OF ARRAY AND ELEMENT IMPEDANCE CHARACTERISTICS

3.1 INTRODUCTION

The series-fed microstrip patch array is identical in geometry to a stepped impedance bandpass filter composed of half wavelength microstrip resonators. It is reasonable then to assume that the interesting transfer functions obtained with bandpass filters can also be obtained with series-fed arrays. For example it could be very useful to synthesize series-fed arrays having Chebyshev or Butterworth type responses. The principle difference between the series-fed array and the stepped impedance filter is that in the case of the array radiation losses are desirable and maximized while in the case of the filter these losses are undesirable and are minimized.

Section 3.2 of this chapter investigates short series-fed arrays configured to be electrically small such that they may be used as radiating elements in larger arrays. This configuration is illustrated in Figure 3.1.2. It is demonstrated both by analysis and measurement that very broad impedance and pattern bandwidths are achievable with this type of element. Clearly conventional filter theory can not be applied here for two reasons. The first is that the radiation loss is high and the second is that the last element is not in general terminated. The approach used here, is a numerical optimization whereby the dimensions of each resonator are fine tuned until the

impedance bandwidth is maximized. The fact that the element is electrically small ensures that the pattern will be well behaved over a band of frequencies.

It is clear that an entire series-fed array of N elements can not in general be synthesized as an N element bandpass filter using conventional filter theory due to the high radiation losses. One approach however which is investigated in section 3.3 is to divide the array into sub-arrays each of which can be synthesized as a bandpass filter. The number of elements in the sub-array is chosen to be small enough that the radiation losses can be ignored with little impact on the transfer functions. The sub-arrays can be amplitude weighted and phased relative to each other in order to synthesize an arbitrary beam shape. Such an array is illustrated in Figure 3.1.1. This beam is constrained to fall within the sub-array pattern. The case where all of the sub-arrays are connected by half wavelength lines is a particularly interesting and useful one. This corresponds to a narrow broadside beam. Typically broadside beams are avoided in series-fed arrays since this resonance condition generally results in very narrow impedance bandwidth. With the filter theory approach described here a broad impedance bandwidth can be ensured while implementing a broadside beam.

An implementation which combines the concepts of sections 3.2 and 3.3 is illustrated in Figure 3.1.3. In this case the electrically small sub-arrays of section 3.2 are combined into a series-fed array. The impact of this is that the array may be frequency scanned having a very attractive scan rate with frequency. Figure 3.1.4 illustrates a configuration in which the same elements are series-fed with little frequency sensitivity.

Figure 3.1.1 Series-Fed Array Designed as a Stepped Impedance Filter

**Figure 3.1.2 A Direct Coupled Multiple Resonator
Radiating Element**

Figure 3.1.3 A Frequency Scanned Array of Broadband Sub-Arrays

**Figure 3.1.4 Insensitive Series-Fed Array of
Broadband Elements**

3.2 COUPLED RESONATOR RADIATING ELEMENTS

3.2.1 Introduction

In recent years a number of articles have appeared in notable journals describing radiating elements combining multiple electromagnetically coupled resonators [28, 29, 30]. The purpose of combining multiple resonators is to increase the bandwidth of the driven element. Parasitic resonators are added on top of or adjacent to the electrically driven resonator. Direct electrical coupling is also possible and has been used to form electrically large arrays including shaped beam series-fed arrays and log-periodic arrays [31]. In this section direct electrically coupled resonators are considered producing wideband, electrically small radiating elements. Since these radiators produce wide beams which are insensitive to the transmit frequency, they are suitable for use in wideband phased arrays. It is shown that an array of narrow electrically coupled resonators can achieve much wider bandwidth than a single wide resonator of the same surface area. Alternatively wide resonators can be combined for even wider bandwidth.

2

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

An equivalent circuit for the coupled resonator radiating element is given in Figure 3.2.1. The element is modeled as alternating high impedance and low impedance transmission lines with shunt conductances at the junctions in order to account for radiation. Fringing fields are accounted for in the usual manner by modifications of the line lengths [33].

Figure 3.2.1 Equivalent Circuit of Electrically Coupled Resonator Radiating Element

Consider now only the n th section of the stepped impedance line as illustrated in Figure 3.2.2. As shown the section can be considered to be a cascade of a transmission line section a shunt conductance and a junction discontinuity. The wave amplitude transmission matrix for the transmission line section is given by:

$$\begin{bmatrix} a_n^+ \\ a_n^- \end{bmatrix} = \begin{bmatrix} e^{j\beta l} & 0 \\ 0 & e^{-j\beta l} \end{bmatrix} \begin{bmatrix} d_n^+ \\ d_n^- \end{bmatrix}$$

3.2.1

where β is the propagation constant along the line of impedance z_n , and l is the line length.

For the junction discontinuity we have:

$$\begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix} = \frac{1}{2} \begin{bmatrix} A_n & B_n \\ B_n & A_n \end{bmatrix} \begin{bmatrix} a_{n+1}^+ \\ a_{n+1}^- \end{bmatrix}$$

3.2.2

where $A_n = r_n + \frac{1}{r_n}$ $B_n = r_n - \frac{1}{r_n}$

$$r_n = \left[\frac{z_{n+1}}{z_n} \right]^{1/2}$$

For the shunt conductance simple calculations yield the following wave transmission matrix

$$\begin{bmatrix} d_n^+ \\ d_n^- \end{bmatrix} = \begin{bmatrix} (1 + \bar{g}/2) & (\bar{g}/2) \\ (-\bar{g}/2) & (1 - \bar{g}/2) \end{bmatrix} \begin{bmatrix} c_n^+ \\ c_n^- \end{bmatrix}$$

3.2.3

Note that the wave transmission parameters are normalized wave amplitudes given by:

$$a_n^+ = \frac{v_n^+}{\sqrt{Z_n}} \quad 3.2.4$$

Figure 3.2.2 Equivalent Circuit of n^{th} Section of Stepped Impedance Line

Cascading the shunt conductance and step discontinuity we obtain

$$\begin{bmatrix} d_n^+ \\ d_n^- \end{bmatrix} = \frac{1}{2} \begin{bmatrix} (1 + \bar{g}/2) & (\bar{g}/2) \\ (-\bar{g}/2) & (1 - \bar{g}/2) \end{bmatrix} \begin{bmatrix} A_n & B_n \\ B_n & A_n \end{bmatrix} \begin{bmatrix} a_{n+1}^+ \\ a_{n+1}^- \end{bmatrix}$$

3.2.5

$$= \frac{1}{2} \begin{bmatrix} (A_n + \bar{g}/2 (A_n + B_n)) & (B_n + \bar{g}/2 (A_n + B_n)) \\ (B_n - \bar{g}/2 (A_n + B_n)) & (A_n - \bar{g}/2 (A_n + B_n)) \end{bmatrix} \begin{bmatrix} a_{n+1}^+ \\ a_{n+1}^- \end{bmatrix}$$

3.2.6

Cascading this with the transmission line section we obtain:

$$\begin{bmatrix} a_n^+ \\ a_n^- \end{bmatrix} = \frac{1}{2} \begin{bmatrix} e^{jB_l} & 0 \\ 0 & e^{-jB_l} \end{bmatrix}$$

$$\begin{bmatrix} (A_n + \bar{g}/2 (A_n + B_n)) & (B_n + \bar{g}/2 (A_n + B_n)) \\ (B_n - \bar{g}/2 (A_n + B_n)) & (A_n - \bar{g}/2 (A_n + B_n)) \end{bmatrix} \begin{bmatrix} a_{n+1}^+ \\ a_{n+1}^- \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} e^{jB_l} (A_n + \bar{g}/2 (A_n + B_n)) & e^{jB_l} (B_n + \bar{g}/2 (A_n + B_n)) \\ e^{-jB_l} (B_n - \bar{g}/2 (A_n + B_n)) & e^{-jB_l} (A_n - \bar{g}/2 (A_n + B_n)) \end{bmatrix} \begin{bmatrix} a_{n+1}^+ \\ a_{n+1}^- \end{bmatrix}$$

$$= \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \begin{bmatrix} a_{n+1}^+ \\ a_{n+1}^- \end{bmatrix}$$

3.2.7

The expression given above defines the transfer function of one segment of the array. The transfer function for the entire array is given by cascading the wave transmission matrices for each of the segments.

$$\begin{bmatrix} a_0^+ \\ a_0^- \end{bmatrix} = \prod_{n=0}^N \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \begin{bmatrix} a_{N+1}^+ \\ a_{N+1}^- \end{bmatrix}$$

3.2.8

3.2.2 Theoretical Results

A program has been written, in Fortran, which optimizes the lengths and widths of each resonator in order to minimize the conjugate mismatch loss over a prescribed bandwidth. This loss is given by:

$$L_C = 10 \log_{10} \left[\frac{|1 - \Gamma_S \Gamma_L|^2}{(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)} \right]$$

3.2.9

where Γ_S = source reflection coefficient
 Γ_L = load reflection coefficient

The source impedance is defined as the complex conjugate of the radiator input impedance at the center of the band. The conjugate mismatch loss is thus zero at mid-band. The formulation of equations (3.2.1) through (3.2.8) has been used to evaluate the input impedance at each frequency. The optimization program, which is included as Appendix B, performs the following algorithm:

- (1) Initial values of all dimensions are assumed. Suitable initial values are lengths equal to the mid-band resonant lengths and widths midway between the upper and lower constraints.
- (2) The conjugate mismatch loss is evaluated at points around the center frequency. The error function is the sum of the mismatch loss at these points.
- (3) The width or length of one resonator is changed and the error function is re-evaluated. If the error is not reduced then the dimension is returned to its original value.
- (4) Task (3) is repeated many times such that all dimensions are altered as many as 40 times.
- (5) The optimization is terminated after a specified number of iterations. The mismatch loss is plotted versus frequency and the optimized dimensions are printed.

In order that junction reactances may be neglected at this point, all lengths are evaluated as percentages of the resonant lengths at mid-band. A model of the radiation conductance versus patch width is required by the optimization routine however. This model is specific to the substrate characteristics and frequency considered. Assuming the parallel resonator

configuration of Figure 3.1.2, with a spacing of 5 mm between resonators mutual coupling can be ignored [32]. Mutual coupling can also be ignored in the configuration of Figure 3.1.1 due to the large E-plane element spacing.

It should be noted that in this chapter each of the transmission line sections, wide or narrow, is called a resonator. The wide resonators are also called radiators and the narrow resonators are also called connecting resonators.

A frequency of 2.2 GHz is assumed and a Duroid 5880 PTFE-fiberglass substrate 0.060" thick is assumed. This substrate has a dielectric constant of 2.2. Experimental results discussed in section 3.2.3 yielded the following empirical model for the radiation conductance at one edge:

$$g = 2.835 \times 10^{-4} w^{1.44} \text{ mho} \quad 3.2.10$$

where w is the patch width in centimeters. This model is of course only valid for patch widths sufficiently narrow that only the dominant mode is excited. The first higher order mode is the transverse resonance mode which can be excited if the patch width exceeds approximately half of a wavelength in the dielectric.

$$w_{TR} = \frac{c}{2 f/\epsilon_r} = 4.6 \text{ cm} \quad 3.2.11$$

In the optimization the maximum patch width has been constrained to be 4.0 cm. The minimum resonator width has been constrained to be greater than 2 mm. Narrower lines are difficult to fabricate using the techniques considered here.

The optimization routine has been run for radiators composed of one, three, five and seven resonators. The resulting optimum resonator lengths are listed in Table 3.2.1 below. In each case the first and every subsequent odd resonator converged onto the maximum width. The connecting resonators converged onto the minimum width. This is not unreasonable since the radiation conductance increases rapidly with patch width and thus the bandwidth of individual resonators increases rapidly with patch width.

Resonator Number	Number of Resonators			
	1	3	5	7
1	1.0	0.997	0.991	0.988
2	---	1.48	1.419	1.497
3	---	0.912	0.942	0.923
4	---	---	1.446	1.366
5	---	---	0.907	1.002
6	---	---	---	1.502
7	---	---	---	1.141

Table 3.2.1 Resonator Lengths Relative to Their
Mid-Band Resonant Length

The conjugate mismatch loss for these four cases is plotted in Figures 3.2.3 through 3.2.6. The -3 dB bandwidth is:

ONE RESONATOR	,	Bandwidth = 3.1%
THREE RESONATORS	,	Bandwidth = 3.5%
FIVE RESONATORS	,	Bandwidth = 6.0%
SEVEN RESONATORS	,	Bandwidth = 7.9%

While little improvement in bandwidth is obtained with three resonators, seven resonators result in a factor of 2.5 increase in bandwidth. As previously noted, these results assume that the patch is conjugately matched at mid-band. This is appropriate for a single resonator however in the case of the multiple resonator configurations, the impedance spirals on the Smith chart over the bandwidths considered here. It will be shown in the following section that large improvements in bandwidth can be achieved by matching to the center of the impedance circle. This is a standard broad-banding technique and the disadvantage of course is a modest reflection coefficient at mid-band.

Typically microwave filters are designed from coupled resonators each of which is resonant at mid-band. The characteristics of the radiators considered here however are quite different from bandpass filters and

coupled resonators with equal resonant frequencies do not in fact produce elements with the broadest possible bandwidth. In order to demonstrate this, the optimization routine was run for the case of seven resonators but the resonator lengths were not allowed to vary. As in the previous cases the first and subsequent odd resonators converged onto the maximum width while the even numbered resonators converged onto the minimum width. The conjugate mismatch loss for this case is plotted in Figure 3.2.7. As illustrated the -3 dB bandwidth is only 3.3% as opposed to 7.9% which is achieved if the resonator lengths are allowed to vary.

Figure 3.2.3 Single Resonator Radiator
Conjugate Mismatch Loss

Figure 3.2.4 Three Resonator Radiator
Conjugate Mismatch Loss

Figure 3.2.5 Five Resonator Radiator
Conjugate Mismatch Loss

Figure 3.2.6 Seven Resonator Radiator
Conjugate Mismatch Loss

Figure 3.2.7 Seven Resonator Radiator Equal
Resonant Frequencies Conjugate
Mismatch Loss

3.2.3 Radiator Fabrication and Experimental Results

3.2.3.1 Patch Characterization

In order to develop a model for the radiation conductance of the resonators, a sequence of test patches was fabricated and evaluated. The patches were fabricated on the same Teflon/Fiberglass substrate on which the seven resonator element was fabricated. Resonators with widths of 1 cm, 2 cm, 3 cm, 4 cm, 5 cm and 6 cm were constructed. The resonators were tested singly, without mutual coupling since mutual coupling was assumed to be negligible at the H plane spacings considered here. Published reports verify that this a reasonable assumption in this case [32]. Table 3.2.3.1 below lists the effective dielectric constant, L_{e0} and resonant length for each of the resonators at 2.2 GHz. L_{e0} is the effective increase in length due to an open end. L_{res} is the resonant length of the resonator.

W (cm)	ϵ_e	L_{e0} (mm)	$L_{res.}$ (cm)
1	1.97	0.79	4.70
2	2.05	0.85	4.59
3	2.07	0.85	4.57
4	2.10	0.85	4.53
5	2.12	0.84	4.51
6	2.13	0.84	4.50

Table 3.2.3.1 Test Elements - Resonant Length

The patches were fed from the ground plane side by an sma connector. The center pin of the connector goes through the substrate and contacts the edge of the resonator at the center of one side. The input impedance for each resonator was measured on a network analyzer at the resonant frequency. At resonance the input impedance is equal to $2g$. The input impedance is given in Table 3.2.3.2 below along with the calculated radiation conductance for one edge of the patch.

w (cm)	$\bar{z}$ ($z_0 = 50$)	g (mho) (one edge)
1	22.0	4.545×10^{-4}
2	13.0	7.692×10^{-4}
3	7.0	1.429×10^{-3}
4	4.8	2.083×10^{-3}
5	3.9	2.564×10^{-3}
6	3.3	3.030×10^{-3}

Table 3.2.3.2 Radiation Conductance

Typically a model of the following form is used for the radiation conductance of microstrip patches:

$$g = k w^{\alpha} \quad (3.2.3.1)$$

This model is used here and is fitted exactly to the measured values for 2 cm and 4 cm radiators:

$$\begin{aligned}\alpha &= \ln [g_1/g_2] / \ln [w_1/w_2] & (3.2.3.2) \\ &= 1.44\end{aligned}$$

$$\begin{aligned}k &= g_1/w^\alpha & (3.2.3.3) \\ &= 2.835 \times 10^{-4}\end{aligned}$$

$$\text{Therefore } g = 2.835 \times 10^{-4} w^{1.44} \quad (3.2.3.4)$$

In equation (3.2.3.4) the patch width is in cm. This model and the measured data are both plotted in Figure 3.2.3.1. The model is clearly adequate over the 2 cm to 4 cm range of patch widths which is of interest here. The fit is however poor for patch widths of 1 cm and 6 cm. Note that patches as wide as 6 cm can in this case support a transverse resonant mode. Excitation of this mode is generally avoided by centering the feed on the broad edge.

Figure 3.2.3.1 Measured Radiation Conductance and Model for Patch Radiators

3.2.3.2 Element Design

The theoretical results presented in section 3.2.2 indicate that very large bandwidths can be achieved with radiating elements composed of seven resonators. A seven resonator element with the configuration illustrated in Figure 3.1.1 was designed, fabricated and tested and the results are given in this section. A center frequency of 2.2 GHz was selected for this design. This resulted in manageable dimensions with modest sensitivity to dimensional tolerances. In order to ensure that the element would respond as a simple small radiator, producing a single broad band beam in both the E and H planes, the maximum resonator width was constrained to be 2 cm. The minimum resonator width was constrained to be 1 mm. Running the optimization program for this case resulted in the following dimensions:

L_1	=	1.000	W_1	=	2 cm
L_2	=	1.407	W_2	=	1 mm
L_3	=	0.867	W_3	=	2 cm
L_4	=	1.330	W_4	=	1 mm
L_5	=	1.021	W_5	=	2 cm
L_6	=	1.444	W_6	=	1 mm
L_7	=	1.078	W_7	=	2 cm

The lengths of the resonators (L_n) are listed as the ratio of the electrical length to the mid-band resonant electrical length.

The radiator was fabricated on a random fiber Teflon/Fiberglass substrate. Copper tape was used for the metalization. The material was a Duroid 5880 with a dielectric thickness of 0.062" and a relative dielectric constant of 2.2. The following expressions were used to characterize the microstrip lines [33].

For the narrow resonators:

$$\epsilon_e = \frac{\epsilon_r + 1}{2} \left[1 - \frac{1}{2H'} \left[\frac{\epsilon_r - 1}{\epsilon_r + 1} \right] \left[\frac{\ln \pi}{2} + \frac{1}{\epsilon_r} \frac{\ln 4}{\pi} \right] \right]^{-2}$$

3.2.3.5

$$H' = \frac{Z_0 [2(\epsilon_r + 1)]^{\frac{1}{2}}}{119.9} + \frac{1}{2} \left[\frac{\epsilon_r - 1}{\epsilon_r + 1} \right] \left[\frac{\ln \pi}{2} + \frac{1}{\epsilon_r} \frac{\ln 4}{\pi} \right]$$

3.2.3.6

For the wide resonators:

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left[1 + 10 \frac{h}{w} \right]^{-0.555}$$

3.2.3.7

Evaluating these yields:

$$\begin{aligned} \epsilon_e &= 1.74 & w &= 1 \text{ mm} \\ \epsilon_e &= 2.05 & w &= 2 \text{ cm} \end{aligned}$$

The resonant lengths are given by:

$$L_R = \frac{c}{2f\epsilon_e^{1/2}} - 2 L_{eo} \quad 3.2.3.8$$

where L_{eo} is the effective increase in the length of the wide resonators due to the open end. This is given by [33]

$$L_{eo} = 0.412 h \frac{\epsilon_e + 0.3}{\epsilon_e - 0.258} \frac{w/h + 0.262}{w/h + 0.813} \quad 3.2.3.9$$

This expression is valid to within about 5% for a wide range of width-height ratios [33].

Evaluating these expressions we obtain the following resonator lengths:

$$\begin{aligned} L_1 &= 46.04 \text{ mm} \\ L_2 &= 72.73 \text{ mm} \\ L_3 &= 39.70 \text{ mm} \\ L_4 &= 68.75 \text{ mm} \\ L_5 &= 47.04 \text{ mm} \\ L_6 &= 74.64 \text{ mm} \\ L_7 &= 49.75 \text{ mm} \end{aligned}$$

A dimensional drawing of the seven resonator element is given in Figure 3.2.3.2. The 90 degree bends in the narrow resonators were chamfered such that the chamfered edge has a length of 0.183 times the line width. The effective length of the corner was modeled as 0.19 times the substrate thickness [33]. An illustration of the mask for the resonator is given in figure 3.2.3.3. The radiator is fed from the ground plane side by an sma connector whose center pin passes through the substrate and contacts the end of resonator number 7.

It should be noted that connections to the wide resonators are made alternately at the sides and ends (see Figure 3.2.3.2). This is done in order to allow the mid-points of the wide resonators to be kept in a line. The element thus remains quite compact and rectangular. The end connections are well modeled by Touchstone and other computer aided design routines. No good model exists at this time for the side connection. As we will see in the next section, the impact of this on the experimental results was, in this case, small.

All dimensions are in millimeters

Figure 3.2.3.2 Seven Resonator Radiating Element

Figure 3.2.3.3 Mask for 7 Resonator Element

3.2.3.3 Impedance Characteristics

The seven resonator radiating element detailed in Figure 3.2.3.1 was fabricated as described in Section 3.2.3.2 and its impedance characteristics were observed over a band of frequencies using an HP8754A Network analyzer with optional frequency doubler. A reference short was produced by soldering copper foil over the end of an SMA connector of the same type as that used to feed the radiator. This was produced such that the short would appear at the location of the ground plane on the radiator. This short was used to establish the reference plane for the impedance measurement. The measured input impedance is plotted in Smith chart form in Figure 3.2.3.4. The wideband performance can be attributed to the fact that the impedance trajectory with frequency forms a tight loop thus maintaining a good match for frequencies on the loop. The center frequency is in error by only 4.5% being 2.1 GHz as opposed to the design frequency of 2.2 GHz. This small error can be attributed to a combination of many factors including:

- 1) Fabrication Tolerances
- 2) Mutual Coupling
- 3) Junction Effects
- 4) Tolerance on Permittivity

The most significant of these factors is the modeling of the junction of the low impedance and high impedance lines. In particular referring to Figure 3.2.3.1 the junctions where the high impedance line joins at the side of the low impedance line. It is likely that the effective length of the low impedance line is extended considerably in this configuration since the straight line path is diagonal across the resonator. The important observation is the very broadband nature of this radiator which is evidenced by Figure 3.2.3.4. The theoretical conjugate mismatch loss as given by the program in Appendix A is plotted in Figure 3.2.3.5. over a 10% bandwidth. Matched at mid-band, the resonator has a mismatch loss of approximately 3 dB at the band edges. This performance is verified by the measured results given in Figure 3.2.3.6, also conjugately matched at mid-band.

An effective means of increasing the impedance bandwidth of an arbitrary load is to intentionally mismatch the source at mid-band resulting in a lower peak VSWR over a wide bandwidth. The impedance trajectory of

the seven resonator element is ideal for this type of approach since it circles a point which could be the conjugate match point. In this case the center of the trajectory is approximately

$$Z = (50\Omega) (1.4 - j0.4)$$

Using the complex conjugate of this as the source impedance the resulting mismatch loss is plotted in Figure 3.2.3.7. The result is tremendous in comparison to the case of the mid-band match. In this case a peak mismatch loss of 0.25 dB is achieved over a 5% bandwidth. With a conjugate match at mid-band the peak mismatch loss over the same band is 1.25 dB. Outside the 5% bandwidth, the mismatch loss is reduced only slightly when this broadbanding technique is used. Note that the impedance trajectory of a single resonator will not turn a tight circle and thus can not be broadband matched in this way.

As further evidence of the effectiveness of the electrically coupled resonator approach, the mismatch loss of a single resonator resonant at mid-band and of the same width as those considered here is given in Figure 3.2.3.8. The 0.25 dB mismatch loss bandwidth is only 0.8% as compared to 5% for the seven resonator element mismatched at mid-band.

Figure 3.2.3.4 Input Impedance of Seven Resonator Element Versus Frequency

Figure 3.2.3.5 Conjugate Mismatch Loss Seven Resonator
Element Matched at Mid-Band

Figure 3.2.3.6 Seven Resonator Radiating Element
Measured and Theoretical Performance

Figure 3.2.3.7 Seven Resonator Radiating Element
Broadband Match

Figure 3.2.3.8 Single Resonator Conjugate Mismatch Loss

3.2.3.4 Measured Radiation Patterns

It has been demonstrated in Sections 3.2.3.1 through 3.2.3.3 that very large bandwidths can be obtained with radiating elements composed of multiple electrically coupled resonators. In order for such an element to be useful in phased arrays, however, it must also be demonstrated to have a broad beam directed nominally broadside to the ground plane and having low sidelobes.

The seven resonator element described in Section 3.2.3.2 has been tested on a far field antenna range and its E and H plane patterns have been recorded. Figure 3.2.3.9 is the measured E plane pattern (electric field vector and beam peak are in the plane of the pattern cut). The -3dB beamwidth in this plane is roughly 80 degrees. As we would expect this is a beamwidth typical of a single microstrip patch radiator on a thin PTFE substrate and is more than adequate for most phased array applications.

Figure 3.2.3.10 gives the H plane pattern (the magnetic field vector and the beam peak are in the plane of the pattern cut). The pattern is plotted at mid-band (2.1 GHz) and at 2.0 GHz. The source level was reduced at the lower frequency so that the plots would be

displaced and thus easily discriminated. The -3dB beamwidth in both cases is roughly 64 degrees. It is clear however that as the frequency deviates from mid-band the beam squints slightly from the broadside position. For the operational bandwidth of this radiator the squinting ($\pm 10^\circ$ at $\pm 15\%$ frequency deviation) will typically have negligible impact on phased array performance. Broader bandwidth elements can however be produced by using any of:

- a) More resonators
- b) Thicker substrate
- c) Lower substrate permittivity

The broader bandwidth could lead to excessive beam squint when the radiator configuration tested here is used. The beam squint can be eliminated however by having electrically coupled resonators on both sides of the resonator to which the input connection is made.

Figure 3.2.3.9 Seven Resonator Element E Plane Pattern

Figure 3.2.3.10 Seven Resonator Element H Plane Pattern

3.3 Applying Filter Theory to Element Response

3.3.1 Introduction

Consider a single series-fed rectangular patch radiator of characteristic impedance Z_C . This patch is fed by a transmission line having characteristic impedance $Z = Z_L$ and is terminated by Z_L . Assume that the radiation conductance at the termination end is included in Z_L and that the radiation conductance at the input end is G_R . The input admittance in this case is given by:

$$Y_{in} = G_R + Y_L \left[\frac{1 + j (Z_L/Z_C) \tan \beta l}{1 + j (Z_C/Z_L) \tan \beta l} \right]$$

3.3.1

Depending on whether $Z_L \gg Z_C$ or $Z_C \gg Z_L$ the loaded radiator may be modeled as a parallel RLC or a series RLC circuit. Typically $Z_L \gg Z_C$ and the parallel RLC model applies. We have then:

$$Y_{in} \approx G_R + Y_L + j [\tan \beta l]/Z_C \quad 3.3.2$$

Near resonance we have: $\delta\phi = \beta l - \pi$

$$Y_{in} \approx G_R + Y_L + j \delta\phi/Z_C \quad 3.3.3$$

$$\approx G_R + Y_L + j Y_C \pi \delta\omega/\omega_0 \quad 3.3.4$$

For a parallel R,L,C circuit we have:

$$Y_{in} \approx 1/R + j (1/\omega L)^2 (\delta\omega/\omega_0) \quad 3.3.5$$

Consider now what happens if we invert the impedance given by (3.3.4) using a quarter wavelength transformer. We then have the impedance given by:

$$Z_{IN}' \approx [G_R + Y_L + j Y_C \pi \delta\omega/\omega_0] \quad 3.3.6$$

This impedance now has the characteristics of a series R,L,C:

$$Z_{IN} \approx R + j 2L\delta\omega \quad 3.3.7$$

Matthaei, Young and Jones [21] have applied conventional microwave filter theory to the problem of achieving a broadband match to a load which can be modeled as a series or parallel R,L,C circuit. The basis of their approach is that the reactive component of the load impedance is resonated out at the mid-band frequency, and this resonant circuit serves as the first resonator of a filter designed to give the required bandwidth. This approach was applied to the problem of corporate-fed microstrip patch radiators by Pues and Van de Capelle [22].

The work in [22] found that for a 10 dB minimum in band return loss the broadband matching technique provided an increase in bandwidth by a factor of 3.9 for the radiating element and matching network considered there. Fano [20] has shown that it is impossible to perfectly match a reactive load over a finite bandwidth. Instead a wide bandwidth match can be achieved only at the expense of a large reflection coefficient over the band considered. As [22] has shown however substantial increases in bandwidth can sometimes be achieved while maintaining acceptable return loss. In [22] two additional resonators, in addition to the resonant load, are used in the matching network. Reference [21] indicates that one or two added resonators can provide substantial increases in bandwidth while diminishing returns make larger numbers impractical.

3.3.2 Matching Network Design

In the case of the series-fed array, we must not only find the values of the matching network components but we must find a way to incorporate these between the array elements. Further we must ensure that the matching network itself does not result in excessive radiation. One attractive configuration is a matching network composed of shorted stubs and connecting lines, each a quarter wavelength long at mid-band. Such matching

networks are described in [21] and illustrated in Figure 3.3.1.

For convenience the design procedure of [21] will be described here. The first step is to determine the factors R_A and δ which describe the load. R_A is the input impedance (real) at the center frequency and δ is the ratio of this to the magnitude of the imaginary part of the input impedance at the band edges. Note that δ is the reciprocal of the Q . The next step is to find an appropriate low pass filter prototype which can be transformed into the desired bandpass matching network. Note that in this report we will consider only three resonator filters where one resonator is the radiating element itself. The reason for this is that there is insufficient space between radiating elements for additional matching sections. As we will see a smaller number of sections will not allow equal output and input impedance. Suppose for argument that we have $R_A = 33.33$ and $\delta = 0.3$ and a three resonator filter. Consulting Figure 4.09-7 in [21] we obtain the low pass prototype element values for a Chebyshev impedance matching network:

$$\begin{aligned} g_0 &= 1 \\ w_1 &= 1 \\ g_1 &= 3.33 \\ g_2 &= 0.520 \\ g_3 &= 2.12 \\ g_4 &= 0.397 \end{aligned}$$

The series stub Z_1 , the shunt stubs Y_k , and the connecting lines $Y_{k,k+1}$ are all a quarter-wavelength long at midband

Figure 3.3.1 Impedance-Matching Network Formed From Stubs and Connecting Lines
From [21]

Note that w_1' is the upper edge of the passband as illustrated in Figure 3.3.2 and the element values are as defined in Figure 3.3.3. Note that in the case of the low pass filter prototype g_3 is the normalized source impedance for optimum performance. We will see that when transforming to the bandpass network the source impedance can be given any desired value. From Figure 4.09-3 in [21] we find that the maximum in-band attenuation will be 1.2 dB. Figure 4.09-4 in [21] indicates that the minimum in-band attenuation will be 0.9 dB. The design equations which yield the stub and connecting line admittances are given in Table 11.09-1 of [21]. Note that in this case we have a stub at the input to the radiator, followed by a quarter wave connecting line and a second stub. The stubs are also a quarter wavelength long at mid-band. Evaluating the expressions for an output impedance of 100 ohms yields the following values for a 30% bandwidth:

$$Y_2 = 0.055 \text{ mhos}$$

$$Y_{23} = 0.011 \text{ mhos}$$

$$Y_3 = 0.025 \text{ mhos}$$

Note that the impedance of the first stub in this example is too low and will result in excessive radiation.

Simplifying the design equations in [21] for the case $n=3$ (two matching stubs) yields the following admittances for the stubs and connecting lines:

Figure 3.3.2 Response of Low Pass Prototype
From [21]

Figure 3.3.3 A Load with a Low-Pass Impedance
Matching Network (case of $n=4$)
From [21]

$$Y_2 = G_A (N_{23} - J_{23}/G_A) \quad 3.3.7$$

$$Y_3 = G_A (C_3'' \tan \theta_1 + N_{23} - J_{23}/G_A)$$

$$Y_{23} = G_A (J_{23}/G_A)$$

$$C_3'' = C_3 - g_2$$

$$C_3 = g_3 g_4 \frac{R_A}{R_B}$$

$$\frac{J_{23}}{G_A} = (C_3/g_3)^{1/2}$$

$$N_{23} = \left[(J_{23}/G_A)^2 + (g_2 \tan \theta)^2 \right]^{1/2}$$

$$\theta = \frac{\pi}{2} \left(1 - \frac{w}{2} \right)$$

where w = fractional bandwidth

G_A = Load Admittance ($1/R_A$)

R_B = Desired input impedance

Consider now a square radiator, 10 cm long on each side, fabricated on a PTFE/Glass substrate. The substrate is Duroid 5880 with $\epsilon_r = 2.2$ and a dielectric thickness of 0.060". Measurements indicate that the radiator is resonant at 1006 MHz. The radiator impedance is inverted with a 50 Ω line one quarter wavelength long at 1006 MHz. The line has a width of 4.66mm and an

effective dielectric constant of 1.8781. This gives a length, ignoring discontinuities of:

$$L = \frac{c}{4f\epsilon_e^{1/2}} = \frac{3 \times 10^8}{(4.024 \times 10^9) (1.3704)} \quad 3.3.8$$

$$= 5.44 \times 10^{-2} \text{m}$$

The input impedance has been measured with the radiator terminated in 50Ω and looking into the impedance inverter. The results are given in Table 3.3.1.

Frequency (MHz)	Impedance (Normalized to 50Ω)
955	1.7 -j 2.10
965	1.4 -j 1.55
975	1.3 -j 1.20
985	1.25 -j 0.75
995	1.2 -j 0.38
1005	1.2 +j 0.0
1015	1.2 +j 0.38
1025	1.3 +j 0.72
1035	1.4 +j 1.20
1045	1.7 +j 1.60

Table 3.3.1 Input Impedance of 10cm square radiator on 0.060" Duroid 5880 after 50Ω impedance inverter

A bandwidth of 965 MHz to 1045 MHz with a center frequency of 1005 MHz has been assumed. This results in the following values for R_A and δ :

$$\begin{aligned} R_A &= 60\Omega \\ \delta &= 0.762 \end{aligned}$$

Consulting figure 4.09-7 in [21] we obtain the low pass prototype elements values for a Chebyshev matching network with three resonators and the appropriate value of δ :

$$\begin{aligned} g_0 &= 1 \\ w_1' &= 1 \\ g_1 &= 1.31 \\ g_2 &= 0.923 \\ g_3 &= 0.90 \\ g_4 &= 0.77 \end{aligned}$$

Evaluating equations (3.3.7) we have

$$\begin{aligned} \theta &= 1.5083 \text{ rad} \\ C_3 &= 0.8316 \\ C_3'' &= -0.0914 \\ J_{23}/G_A &= 0.96125 \\ N_{23} &= 14.7755 \\ Y_2 &= 0.2302 \text{ mhos} \\ Y_3 &= 0.2059 \text{ mhos} \\ Y_{23} &= 0.01602 \text{ mhos} \end{aligned}$$

radiation conductances. This can be done either by simply scaling the impedances or by using entirely different filter designs.

It is quite clear that narrow broadside beams can be produced by the means described above if the various sub-arrays are combined by sections of line which are one half wavelength long at mid-band. Due to the coherent addition of reflections from the various sub-arrays the bandwidth will however be reduced relative to that of a single sub-array. It is also possible to produce shaped beams simply by controlling the phase shift between sub-arrays. The only limitation here is that the shaped beam is constrained to fall within the sub-array pattern. Each sub-array can be considered to be a single element and this array of elements can be excited to obtain the desired array factor. The resulting far field pattern is then the product of the sub-array pattern and the array factor.

Since amplitude and phase can be controlled at each sub-array, most any synthesis technique can be used to determine the required sub-array excitations. It is recommended however that the optical synthesis technique described by Chu [9] be used as a starting point. This is because this technique produces a phase gradient which is monotonically increasing or decreasing along the

3.3.3 Theoretical and Experimental Results

In order to correct the line lengths of the matching network for discontinuities at the impedance steps, junctions and line ends, the entire network has been optimized on Touchstone (trademark of EESof Ltd.). The final dimensions are given in the circuit file of Table 3.3.3. The theoretical return loss of the radiator with the resulting Chebyshev matching network is given in Figure 3.3.5. The return loss characteristic of the same radiator except with a single quarter wavelength impedance matching section is given in figure 3.3.6. It is clear that the Chebyshev match sacrifices mid-band return loss for a better return loss over a wide bandwidth. At the 10 dB return loss level the bandwidth is increased by a factor of 2.5.

The network was fabricated according to the optimized dimensions using copper tape on the Duroid 5880 substrate. The measured return loss characteristic is given in Figure 3.3.7. The response is quite close to that predicted (Figure 3.3.5) except that increased ripple exists in the passband. This can be attributed to manufacturing tolerances and approximations made in modeling such as neglecting radiation loss in the stubs. The results are very good however and clearly indicate the broad impedance bandwidth achievable with this matching technique.

```

DIM      FREQ MHZ
        RES OH
        ANG DEG
        LNG CM

CKT      MSUB ER=2.2 H=0.1524 T=1.8E-03 RHO=0.71 RGH=0.0
        MSTEP 13 1 W1=0.466 W2=10.00
        NLIN 1 2 W=10.05 L=10.05
        RES 1 0 R=550
        RES 2 0 R=550
        MSTEP 2 3 W1=10.00 W2=0.466
        NLIN 3 5 W=0.466 L # 5.1 5.10169 5.5
        MCROS 4 5 6 7 W1=1.847 W2=0.466 W3=1.847 W4=0.329
        MLEF 4 W=1.847 L # 10. 10.07250 11.
        MLEF 6 W=1.847 L # 10. 10.07391 11.
        NLIN 7 10 W=0.329 L # 4.0 5.60810 6.0
        MCROS 9 10 11 12 W1=1.622 W2=0.329 W3=1.622 W4=0.466
        MLEF 9 W=1.622 L # 10. 10.15229 11.
        MLEF 11 W=1.622 L # 10. 10.15907 11.
        DEF2P 13 12 TRANS

FREQ     SWEEP 905 1105 5

OUT       TRANS DB[S11] GR1

GRID      RANGE 905 1105 50
          GR1 -20 0 5

TERM      ZO=50.0

OPT       RANGE 999 1050
          TRANS DB[S11]<-12.00

```

Table 3.3.3 Touchstone File for Radiator
with Chebyshev Match

Figure 3.3.4 Radiator With Chebyshev Matching Network

Figure 3.3.5 Return Loss Characteristics of Radiator With Chebyshev Matching Network

Figure 3.3.6 Return Loss Characteristics of Radiator with Quarter Wavelength Transformer Matching

Figure 3.3.7 Measured Return Loss Characteristic of Radiator With Chebyshev Matching Network

3.4 SUB-ARRAY SYNTHESIS

3.4.1 Introduction

It is an interesting observation that a series-fed array of directly coupled microstrip patch radiators is identical in geometry to a stepped impedance microwave filter implemented in microstrip. Both devices are composed of alternating high impedance and low impedance lines. The principal difference between them is that in the case of the antenna, radiation losses are desirable and in the case of the filter they are a nuisance and minimized.

Stepped impedance filters are generally implemented with all sections resonant at mid-band. The band-pass characteristic is controlled by means of the impedances of the various sections. Series-fed arrays on the other hand typically have the low impedance sections resonant at mid-band while the lengths of the high impedance sections are varied in order to obtain the desired phase distribution across the aperture. The patch widths are varied to control the intensity distribution on the array.

In order to apply microwave filter theory to the design of series-fed arrays the lengths of the high

impedance sections must be such that they are resonant at mid-band. This is of course a very important case in antenna design corresponding to a broadside beam at mid-band. Typically this resonant case is avoided in antenna design because it generally results in a very narrow impedance bandwidth. Instead series-fed arrays are generally implemented with at least several degrees of beam squint in order to avoid the resonance condition. The effect of this is that the reflections from the many discontinuities tend to combine with random phase at the input port resulting in low VSWR. Often, however, it is undesirable to implement a beam squint at mid-band. For example on a side-looking airborne radar this would require that the antenna stick out from the aircraft in order to obtain a broadside beam.

Filter theory suggests that it is not necessary to have a beam squint in order to obtain a broad impedance bandwidth. It will be shown that more than adequate impedance bandwidth can be obtained in typical applications. The basic approach taken is to design short sections of the series-fed array to be bandpass filters with the desired bandpass characteristic. Over these short sections or subarrays the radiation losses can be ignored with little impact on the bandpass characteristic. Aperture intensity taper can be applied by designing the various sub-arrays to have different

radiation conductances. This can be done either by simply scaling the impedances or by using entirely different filter designs.

It is quite clear that narrow broadside beams can be produced by the means described above if the various sub-arrays are combined by sections of line which are one half wavelength long at mid-band. Due to the coherent addition of reflections from the various sub-arrays the bandwidth will however be reduced relative to that of a single sub-array. It is also possible to produce shaped beams simply by controlling the phase shift between sub-arrays. The only limitation here is that the shaped beam is constrained to fall within the sub-array pattern. Each sub-array can be considered to be a single element and this array of elements can be excited to obtain the desired array factor. The resulting far field pattern is then the product of the sub-array pattern and the array factor.

Since amplitude and phase can be controlled at each sub-array, most any synthesis technique can be used to determine the required sub-array excitations. It is recommended however that the optical synthesis technique described by Chu [9] be used as a starting point. This is because this technique produces a phase gradient which is monotonically increasing or decreasing along the

array. The result is that the first order response to a change in frequency or dielectric permittivity is a squinting of the beam. With other synthesis techniques a similar change could result in a break-up of the main beam or other severe beam shape distortion. The algorithm for the optical synthesis technique is described in Chapter I. As discussed by Walter [4] it is necessary to follow the optical synthesis by a more exact technique which will optimize the beam shape while not eliminating the desirable properties obtained with the optical approach. For example a Fourier synthesis can be performed using the desired far field intensity distribution and the far field phase distribution obtained by means of the optical synthesis.

The procedure described here for the synthesis of the bandpass characteristic of the sub-arrays follows that given by Matthaei, Young and Jones [21] for stepped impedance filters. The basic approach is to equate the junction Voltage Standing Wave Ratios (VSWRs) to those derived for stepped impedance transformers with the desired transfer function. Note that the bandwidth of the filter will be half that of the transformer having the same number of sections and the same junction VSWRs because the section length of the filter is twice that of the transformer.

Consider the equivalent circuit for a series-fed array of rectangular patches given in Figure 3.3.1.1. The junction VSWRs are given by:

$$V_i = \begin{cases} z_{i-1}/z_i & ; \quad z_{i-1}/z_i > 1 \\ z_i/z_{i-1} & ; \quad z_i/z_{i-1} > 1 \end{cases}$$

3.4.1.1

Equating the junction VSWRs of the filter with those of the stepped impedance transformer results in filter section impedances which compared to those of the transformer have every second impedance step inverted:

$$\text{etc.} \quad z_1' = z_1 \quad z_2' = z_1^2/z_2 \quad z_3' = z_1^2 z_3/z_2^2$$

3.4.1.2

or

$$\text{etc.} \quad z_1' = 1/z_1, \quad z_2' = z_2/z_1^2 \quad z_3' = z_2^2/(z_1^2 z_3)$$

The excess loss ratio of a microwave filter is given by:

$$R_L = \frac{\text{Incident Power} - \text{Load Power}}{\text{Load Power}} \quad 3.4.1.3$$

$$= \frac{|\Gamma|^2}{1 - |\Gamma|^2} \quad 3.4.1.4$$

(a)

(b)

Figure 3.4.1.1 Sub-Array Geometry
a) Illustration
b) Equivalent Circuit

where Γ is the input reflection coefficient. Following the convention of filter theory, a radiating element will be said to have a maximally flat response if its excess loss ratio follows a Butterworth polynomial. This is defined by equation 3.4.1.5 below,

$$\frac{|\Gamma|^2}{1 - |\Gamma|^2} = \left| \frac{R - 1}{2 \sqrt{R}} \sin^{N-1} 2\beta l \right|^2$$

3.4.1.5

where the sub-array has $N-1$ resonators and thus N impedance steps.

The first step in designing a wideband sub-array is to determine the radiation conductance required from that section. This will depend on the overall array length, the sub-array length and the desired aperture intensity distribution. Knowing the required radiation conductance and the desired input impedance the desired ratio R of the input to output impedance of the equivalent transformer, as will demonstrated here, can be estimated. The average impedance step will be $R/(N-1)$. The normalized section impedances have been computed by Matthaei, Young and Jones [21] and are included as tables 3.4.1.1 and 3.4.1.2.

As an alternative to the Butterworth implementation described above, the excess loss ratio may be equated to the Chebyshev polynomial yielding an equal ripple type response. In this case we have:

$$\frac{|\Gamma|^2}{1 - |\Gamma|^2} = k^2 T_N^2 (\sin \beta l / u) \quad 3.4.1.6$$

where Γ = reflection coefficient
 k = constant to be determined
 T_N = Chebyshev polynomial
 β = propagation constant
 l = length of each resonator
 u = constant to be determined

The normalized section impedances for this case have also been tabulated by Matthaei, Young and Jones [21] and are not included here. Relative to the maximally flat implementation the equal ripple implementation will have a wider bandwidth for a given value of R , number of resonators and maximum in-band VSWR.

A	n = 5		n = 6			n = 7			n = 8			
	Z_1	Z_2	Z_1	Z_2	Z_3	Z_1	Z_2	Z_3	Z_1	Z_2	Z_3	Z_4
1.5	1.01277	1.07904	1.00636	1.04540	1.14960	1.00318	1.02570	1.09628	1.00158	1.01438	1.06041	1.15872
2.0	1.02201	1.13908	1.01096	1.07904	1.26929	1.00547	1.04448	1.17039	1.00273	1.02481	1.10571	1.28658
2.5	1.02931	1.18816	1.01458	1.10608	1.37082	1.00727	1.05944	1.23157	1.00363	1.03307	1.14243	1.39558
3.0	1.03539	1.23002	1.01759	1.12884	1.45995	1.00878	1.07195	1.28415	1.00438	1.03997	1.17355	1.49162
3.5	1.04061	1.26672	1.02018	1.14861	1.53996	1.01007	1.08275	1.33055	1.00503	1.04590	1.20071	1.57813
4.0	1.04521	1.29954	1.02246	1.16613	1.61292	1.01121	1.09229	1.37227	1.00560	1.05114	1.22490	1.65722
4.5	1.04932	1.32931	1.02450	1.18191	1.68026	1.01223	1.10085	1.41030	1.00611	1.05583	1.24678	1.73039
5.0	1.05305	1.35663	1.02635	1.19631	1.74297	1.01315	1.10863	1.44534	1.00658	1.06009	1.26681	1.79870
6.0	1.05962	1.40549	1.02961	1.22186	1.85731	1.01479	1.12240	1.50837	1.00740	1.06762	1.30252	1.92356
7.0	1.06530	1.44845	1.03243	1.24413	1.96010	1.01620	1.13436	1.56414	1.00812	1.07414	1.33381	2.03617
8.0	1.07032	1.48696	1.03493	1.26395	2.05396	1.01746	1.14496	1.61440	1.00875	1.07992	1.36177	2.13926
9.0	1.07482	1.52196	1.03717	1.28186	2.14066	1.01859	1.15451	1.66032	1.00932	1.08513	1.38714	2.23474
10.0	1.07892	1.55413	1.03921	1.29822	2.22148	1.01962	1.16322	1.70270	1.00984	1.08987	1.41041	2.32393
15.0	1.09531	1.68600	1.04740	1.36450	2.56378	1.02375	1.19830	1.87818	1.01194	1.10895	1.50543	2.70350
20.0	1.10760	1.78804	1.05356	1.41497	2.84017	1.02688	1.22484	2.01581	1.01354	1.12335	1.57860	3.01198
25.0	1.11753	1.87251	1.05855	1.45628	3.07621	1.02942	1.24645	2.13089	1.01484	1.13507	1.63889	3.27666
30.0	1.12592	1.94524	1.06277	1.49152	3.28448	1.03158	1.26482	2.23080	1.01594	1.14502	1.69087	3.51111
35.0	1.13322	2.00950	1.06646	1.52243	3.47223	1.03347	1.28087	2.31965	1.01692	1.15371	1.73661	3.72308
40.0	1.13969	2.06729	1.06973	1.55006	3.64407	1.03515	1.29518	2.40004	1.01778	1.16146	1.77770	3.91762
45.0	1.14552	2.12000	1.07268	1.57510	3.80311	1.03667	1.30812	2.47372	1.01857	1.16845	1.81513	4.09813
50.0	1.15084	2.16856	1.07538	1.59807	3.95162	1.03805	1.31996	2.54192	1.01928	1.17485	1.84958	4.26701
60.0	1.16027	2.25588	1.08017	1.63911	4.22331	1.04052	1.34106	2.66530	1.02056	1.18624	1.91145	4.57684
70.0	1.16847	2.33312	1.08434	1.67513	4.46845	1.04268	1.35951	2.77519	1.02168	1.19620	1.96609	4.85724
80.0	1.17575	2.40267	1.08805	1.70736	4.69297	1.04460	1.37597	2.87473	1.02269	1.20507	2.01523	5.11474
90.0	1.18230	2.46613	1.09139	1.73661	4.90095	1.04634	1.39087	2.96605	1.02359	1.21310	2.06003	5.35379
100.0	1.18828	2.52464	1.09444	1.76343	5.09522	1.04793	1.40450	3.05064	1.02442	1.22043	2.10129	5.57761

Table 3.4.1.1 Maximally Flat Quarter Wave Transformers
From [34]

MAXIMALLY FLAT QUARTER-WAVE TRANSFORMERS

R	n=2	n=3	n=4		R	n=2	n=3	n=4	
	Z ₁	Z ₁	Z ₁	Z ₂		Z ₁	Z ₁	Z ₁	Z ₂
1.00	1.00000	1.00000	1.00000	1.00000	12.50	1.88030	1.38110	1.17961	2.21603
1.25	1.05737	1.02829	1.01405	1.07223	15.00	1.96799	1.41512	1.19506	2.35186
1.50	1.10668	1.05202	1.02570	1.13512	17.50	2.04531	1.44475	1.20847	2.47169
1.75	1.15016	1.07255	1.03568	1.19120	20.00	2.11474	1.47108	1.22035	2.58072
2.00	1.18921	1.09068	1.04444	1.24206	25.00	2.23607	1.51650	1.24078	2.77447
2.50	1.25743	1.12177	1.05933	1.33204	30.00	2.34035	1.55498	1.25803	2.94423
3.00	1.31607	1.14793	1.07176	1.41051	40.00	2.51487	1.61832	1.28632	3.23492
4.00	1.41421	1.19071	1.09190	1.54417	50.00	2.65915	1.66978	1.30920	3.43136
5.00	1.49535	1.22524	1.10801	1.65686	60.00	2.78316	1.71540	1.32853	3.69752
6.00	1.56509	1.25439	1.12153	1.75529	80.00	2.99070	1.78522	1.36025	4.06810
8.00	1.68179	1.30219	1.14356	1.92323	100.00	3.16228	1.84359	1.38591	4.38263
10.00	1.77828	1.34089	1.16129	2.06509					

Table 3.4.1.2 Maximally Flat Quarter Wave Transformers From [21]

3.4.2 Design

In this section two resonant series-fed arrays are designed. Each of the arrays is eight elements long and has an insertion loss due to radiation of two decibels. These are intended to be representative sections of longer arrays having higher insertion loss and thus higher efficiency. One of the arrays is designed as two identical sub-arrays in series. Each sub-array thus has seven resonators and is designed to have a maximally flat transfer function. The second array has eight patches of identical widths and separations and thus serves as a reference.

The arrays are designed at a center frequency of 5.06 GHz because this results in a manageable array length of approximately 30 cm and radiating elements at this frequency have been well characterized by the author. The substrate is assumed to be Duroid 5880 with a relative dielectric constant of 2.2 and a thickness of 0.062 inches.

Extensive characterization [1] was performed in order to obtain the following empirical model for the radiation conductance of one side of one patch.

$$g = (0.7) (1.449 (w-w_c)/L_{ao})^2 / 90.0 \quad 3.4.2.1$$

where w = patch width
 w_c = connecting line width
 L_{ao} = wavelength in free space

For the resonant patch spacings on the substrate which were considered, the resonant patch length is given by

$$L_R = 0.5c / (f \epsilon_e^{1/2}) - 2 D_R \quad 3.4.2.2$$

$$D_R = (((w - w_c)/D)^{0.6167} (D) (0.0334)) \quad 3.4.2.3$$

where D = substrate thickness

D_R = effective length increase due to fringing field

ϵ_e = effective dielectric constant

The resonant connecting line length is given by:

$$L_C = 0.5 c / (f \epsilon_e^{1/2}) - 2 D_S \quad 3.4.2.4$$

$$D_S = (((w - w_c)/D)^{1.5702} (D) (0.02518)) \quad 3.4.2.5$$

Equations 3.2.3.5 through 3.2.3.9 are used to find the effective dielectric constant of a line of a particular width.

The radiation loss incurred at a single radiator is given approximately by

$$\text{Loss} = -10\log_{10}(1-2gZ_0) \quad 3.4.2.6$$

where Z_0 is the connecting line impedance and g is the radiation conductance of each radiating edge.

In order to obtain 0.25 dB loss at each radiator we thus require:

$$g = (1-10^{-0.025})/(2Z_0) \quad 3.4.2.7$$

Assuming a 50 ohm feed impedance we obtain:

$$g = 5.594 \times 10^{-4}$$

From 3.3.2.1 we obtain:

$$w = w_C + (L_{a0}/1.449)(128.57g)^{1/2} \quad 3.4.2.8$$

If we assume a connecting line impedance of 50Ω then we have:

$$\begin{aligned} w &= 0.00496 + (0.04092)(0.2682) \quad 3.4.2.9 \\ &= 1.593 \times 10^{-2} \text{ m} \end{aligned}$$

The width of the resonators on the reference array will thus be 1.593 cm. Finding the resonant length and connecting line length we have:

$$\begin{aligned} D_R &= 0.174 \text{ mm} \\ \epsilon_e &= 2.06 (\epsilon_e \text{ for radiator}) \\ Z_O &= 20.81 \Omega \\ L_R &= 2.03 \text{ cm} \\ D_S &= 0.835 \text{ mm} \\ L_C &= 1.98 \text{ cm} \\ \epsilon_{ec} &= 1.904 (\epsilon_e \text{ for connecting line}) \end{aligned}$$

We must now determine the dimensions of the array elements based on the Butterworth filter sub-arrays. Referring to Table 3.4.1.1 we have a seven resonator filter and thus the $n=7$ column is relevant. The impedance ratio R will dictate the insertion loss due to radiation from the sub-array. Consider the case $R=100$.

$$\begin{aligned} Z_O &= 50\Omega; & Z_1 &= 52.4\Omega; & Z_2 &= 39.1\Omega; & Z_3 &= 84.9\Omega \\ Z_4 &= 25.9\Omega; & Z_5 &= 84.9\Omega; & Z_6 &= 39.1\Omega; & Z_7 &= 52.4\Omega \end{aligned}$$

In table 3.4.2.1 below the line width, effective dielectric constant and resonant length is entered for each of these resonators

Impedance (Ω)	Line Width (mm)	ϵ_r	Res. Length (mm)
50	4.96	1.904	N/A
52.4	4.63	1.895	21.53
39.1	7.09	1.950	21.12
84.9	2.02	1.802	21.09
25.9	12.19	2.022	20.52

Table 3.4.2.1

Resonator Data for Butterworth Sub-Array

Four arrays have been fabricated and tested, and the results are discussed in Section 3.3.3. The masks for the four arrays are illustrated in Figures 3.4.2.1 through 3.4.2.4. These correspond to a single maximally flat sub-array, a single uniform sub-array, two cascaded maximally flat sub-arrays and two cascaded uniform sub-arrays respectively.

Figure 3.4.2.1 Single Maximally Flat Sub-Array

Figure 3.4.2.2 Single Uniform Sub-Array

Figure 3.4.2.3 Two Cascaded Maximally Flat Sub-Arrays

ALL
DIMENSIONS
IN
MILLIMETERS

NOT
DRAWN
TO
SCALE

Figure 3.4.2.4 Two Cascaded Uniform Sub-Arrays

3.4.3 Theoretical and Measured Results

The arrays discussed in section 3.4.2 have been simulated using the Touchstone software package. Both arrays of equal width patches and arrays of maximally flat sub-arrays have been analyzed. The impedances of the resonators in the maximally flat sub-arrays are given in table 3.4.2.1. In the arrays of equal width patches, alternating 15.93 mm and 4.96 mm line widths are assumed. As discussed in section 3.4.2 a 0.06 inch substrate thickness is assumed.

Figure 3.4.3.1 illustrates the hypothetical case of a single maximally flat sub-array without radiation loss. The return loss in this case exceeds 36 dB at points 0.7 GHz from mid-band. Figure 3.4.3.2 shows the theoretical plots of S11 and S12 for the same sub-array except this time with the radiation loss considered. Note that a simple ideal transmission line model was used. The insertion loss is seen to be very uniform and of magnitude equal to 1 dB over a band of roughly 2 GHz. The return loss exceeds 19 dB over a band of roughly 1.9 GHz. The radiation loss has thus resulted in a slight mismatch over the passband. In comparison, the same parameters plotted for the uniform sub-array in Figure 3.4.3.3 indicate frequencies of considerable mismatch over the same band. In particular at 0.9 GHz

from mid-band the return loss is only 11 dB. When two such sub-arrays are cascaded, in this case phased for a beam squint of 23° , the mismatch becomes quite severe. Figure 3.4.3.4 indicates only 6.5 dB return loss at 0.9 GHz from mid-band and 7 dB return loss at 0.3 GHz from mid-band. In comparison two cascaded maximally flat sub-arrays result in a minimum of 14.5 dB return loss over a 1.8 GHz band and 13 dB return loss over a 2.5 GHz band. As illustrated in figure 3.4.3.5, the insertion loss is roughly 2 dB over a 2.5 GHz bandwidth.

When multiple sub-arrays are connected in series, the return loss is degraded both in the case of the equal patch widths and the maximally flat sub-arrays. The weighting on the sub-arrays however continues to ensure a significant improvement in the return loss over the pass band. In addition to improving the return loss of the array, the use of maximally flat sub-arrays results in lower local reflections resulting in less sensitivity of the pattern to frequency and the number and nature of connected sub-arrays.

3.4.4 Experimental Results

In order to verify the performance of the maximally flat sub-array concept, a number of arrays have been fabricated and tested. In particular four arrays were

produced; a single four element uniform array, a single four element maximally flat array, two cascaded uniform arrays and two cascaded maximally flat arrays. The dimensions of these arrays are as given in Section 3.4.2 and the spacing between cascaded arrays is one quarter wavelength resulting in roughly 23 degrees of beam squint. The return loss is plotted for each of these arrays over the 3.1 GHz to 7.1 GHz band with the output port terminated in 50 ohms in each case. The plots are given in Figures 3.4.4.1 through 3.4.4.4 for the cases of a single uniform array, a single maximally flat array, cascaded uniform arrays and cascaded maximally flat arrays respectively. The results need very little discussion as they are quite close to the theoretical results discussed above. As would be expected, the deviation in dB from theoretical increases at higher return loss since only minor perturbations are required to alter the response at these points. The close match to the theoretical results is a consequence of careful characterization of the radiating elements which was done previously [1].

As discussed in section 3.4.1, a principal disadvantage of the maximally flat sub-array in array design is that the relatively broad sub-array pattern can result in grating lobes being excited. This is because the weighting of the aperture resulting from the

Figure 3.4.3.1 Theoretical S11
Single Maximally Flat Sub-
Array Without Radiation Loss

Figure 3.4.3.2 Theoretical S11, S21
Single Maximally Flat Sub-Array

Figure 3.4.3.3 Theoretical S11, S21
Single Uniform Sub-Array

Figure 3.4.3.4 Theoretical S11, S21
Two Cascaded Uniform Sub-Arrays

Figure 3.4.3.5 Theoretical S_{11} , S_{21}
Two Cascaded Maximally Flat
Sub-Arrays

Figure 3.4.4.1 Measured Return Loss
Single Uniform Sub-Array

Figure 3.4.4.2 Measured Return Loss
Single Maximally Flat
Sub-Array

Figure 3.4.4.3 Measured Return Loss
Two Cascaded Uniform Sub-
Arrays

Figure 3.4.4.4 Measured Return Loss
Two Cascaded Maximally
Flat Sub-Arrays

maximally flat radiator configuration results in a broader beam than that of the uniform array of the same length. The spacing of the sub-arrays then results in grating lobes in the sub-array pattern. This is a problem which is commonly encountered when sub-arrays are used in phased array design. Interlacing of the sub-arrays can alleviate this problem. Alternatively, the array configuration of Figure 3.1.3 can be used. In this case the sub-array weighting is in the azimuth plane while a narrow frequency scanned beam is formed in the elevation plane.

As an example of the potential grating lobe problem, two linear arrays each containing three cascaded sub-arrays, in phase, with 6 dB higher weighting on the center array were analyzed. In one case the sub-arrays were uniformly weighted and in the other maximally flat sub-arrays were assumed. These sub-arrays were as defined in section 3.4.2. The theoretical radiation patterns are given in figures 3.4.4.5 and 3.4.4.6. The maximally flat sub-array weighting can be seen to increase the peak sidelobe level in this case from -17 dB to -8 dB.

Figure 3.4.4.5 Three Cascaded Maximally Flat Sub-Arrays, Radiation Pattern

Figure 3.4.4.6 Three Cascaded Uniform Sub-Arrays, Radiation Pattern

CHAPTER IV
NON-RESONANT RADIATING ELEMENTS

4.1 Introduction

In order to obtain arrays with very wide bandwidth, it is proposed that the characteristic impedance of the patch be made equal to that of the feed line. In a conventional series-fed microstrip patch array this would imply that there be no steps in the line width and thus virtually no radiation. Here however we propose to step the line width as required for radiation but to load the various sections with capacitive stubs thus raising the characteristic impedance to some constant value.

Two potential implementations of this wideband array geometry are given in Figures 4.1.1(a) and (b). In (a) the feed line is narrow and the radiators are wide patches. The feed line as well as the patches must be periodically loaded in this case. The loading is tapered off at the ends in order to match the input and output impedances to the desired values, probably 50 ohm. In (b) which is perhaps the more likely implementation the feed line is wide and the radiators are the transitions to the narrow sections. Only the narrow sections are loaded and this loading is such that the characteristic impedance of the narrow sections is equal to that of the feed line.

The feed line is tapered at the ends to match to the desired input and output impedances. Note that since none of the sections is resonant, they need not be of 180 degrees electrical length. Unlike conventional patch radiators then we can independently control the phase of each and every edge thus allowing broader patterns in the E plane and more pattern control.

In the simplest possible implementation of this type of array, the loading is provided by narrow stubs less than one quarter wavelength long. Provided that these stubs are sufficiently narrow their radiation losses will be negligible. The input impedance of a single open circuited stub is given by:

$$Z_{in} = -jZ_0 \cot \beta l \quad 4.1.8$$

If the spacing between the stubs is small compared to the wavelength then the loaded line will act as a uniform line of characteristic impedance [35]:

$$Z_C = (\mu_0 \epsilon_0 \epsilon_e)^{\frac{1}{2}} (C + C_0/d)^{-1} \quad 4.1.9$$

where ϵ_e is the effective dielectric constant, C is the capacitance per unit length without loading and C_0/d is the added capacitance per unit length due to the loading.

Typically in a series-fed array the beam pointing direction is a function of frequency due to the frequency dependence of the relative element phases. In many applications this does not pose a problem. In a very broadband array implemented with non-resonant elements, this beam squint could pose a problem in some situations. There are a number of conventional methods to eliminate this beam squint including end-to-end arrays as illustrated in figure 4.1.2(a), multiple corporate-fed arrays 4.1.2(b) and periodic phase shifters (c). In (a) the beams from the two back to back arrays tend to scan in opposite directions so that the beam peak remains in the desired direction regardless of frequency. In (b) the corporate feed dictates the beam position while the "sub-array" beams are allowed to squint with frequency. The resulting beam pointing direction is independent of frequency. In (c) the phase shift can compensate for a desired frequency. Note that with non-resonant elements the architecture of (c) yields a very broadband phase scanned array.

of/de

3

The phase velocity will also be altered by the loading according to:

$$v_p = \left[(C + \frac{C_0}{d}) L \right]^{-\frac{1}{2}} \quad 4.1.10$$

where L is the inductance per unit length. Assuming quasi-TEM behaviour we have:

$$L = (\mu_0 \epsilon_0 \epsilon_e)^{\frac{1}{2}} Z_0 \quad 4.1.11$$

Figure 4.1.1 Arrays of Non-Resonant Elements

Typically in a series-fed array the beam pointing direction is a function of frequency due to the frequency dependence of the relative element phases. In many applications this does not pose a problem. In a very broadband array implemented with non-resonant elements, this beam squint could pose a problem in some situations. There are a number of conventional methods to eliminate this beam squint including end-to-end arrays as illustrated in figure 4.1.2(a), multiple corporate-fed arrays 4.1.2(b) and periodic phase shifters (c). In (a) the beams from the two back to back arrays tend to scan in opposite directions so that the beam peak remains in the desired direction regardless of frequency. In (b) the corporate feed dictates the beam position while the "sub-array" beams are allowed to squint with frequency. The resulting beam pointing direction is independent of frequency. In (c) the phase shift can compensate for a desired frequency. Note that with non-resonant elements the architecture of (c) yields a very broadband phase scanned array.

(a) Back to Back Arrays

(b) Multiple Corporate-Fed Arrays

(c) Broadband Phase Scanned Array

Figure 4.1.2 Series-Fed Arrays with Frequency Independent Beam Pointing

4.2 Periodic Structures

It is useful to recall at this point some basic theory of periodic structures. This theory which is well documented in the literature [35] is directly applicable to the radiating element considered here.

The non-resonant radiating element can be modeled as a periodically loaded high impedance transmission line terminated at each end by a low impedance transmission line in parallel with the shunt radiation conductance. The periodically loaded line can be divided into identical unit cells as illustrated in Figure 4.2.1. For simplicity the unit cell can be assumed here to be symmetrical. Consider now the wave amplitude transmission matrix for the unit cell.

$$\begin{bmatrix} C_n^+ \\ C_n^- \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} C_{n+1}^+ \\ C_{n+1}^- \end{bmatrix}$$

For the periodically loaded line to act as a simple transmission line we require:

$$V_{n+1} = e^{-\tau d} V_n \quad 4.2.1$$

$$C_{n+1}^+ = e^{-\tau d} C_n^+ \quad 4.2.2$$

$$C_{n+1}^- = e^{-\tau d} C_n^- \quad 4.2.3$$

Figure 4.2.1 Periodically Loaded Line and Its Unit Cell

When the sum of the forward and reverse waves satisfies 4.2.1 this sum is called a Block Wave. Solving 4.2.1 we obtain [35]:

$$\cosh \tau d = \cos \theta - \frac{\bar{B}}{2} \sin \theta \quad 4.2.4$$

where τ is the propagation constant for Block waves

$$\theta = k_0 d$$

k_0 is the propagation constant on the line

d is the unit cell length

$\bar{B}$ is the normalized stub admittance

In general we have:

$$\tau = \alpha + jB \quad 4.2.4$$

For the special case:

$$\tau = \alpha ; B = 0$$

we have:

$$\cos \theta - \frac{\bar{B}}{2} \sin \theta > 1 \quad 4.2.5$$

Similary for:

$$\tau = j\beta ; \alpha = 0$$

we have:

$$-1 < \cos \theta - \frac{\bar{B}}{2} \sin \theta < 1 \quad 4.2.6$$

Also for:

$$\tau = \alpha + j \pi/d$$

we have:

$$\cos \theta - \frac{\bar{B}}{2} \sin \theta < -1 \quad 4.2.7$$

Equation 4.2.6 gives the condition for unattenuated Block Waves. This is satisfied for sufficiently small stub spacings and also has solutions at harmonic frequencies. The input impedance for the line loaded with capacitive stubs is given by:

$$\frac{\bar{Z}}{\beta} = \left[\frac{2 \sin \theta + \bar{Y} \tan \theta_S \cos \theta - \bar{Y} \tan \theta_S}{2 \sin \theta + Y \tan \theta_S \cos \theta + Y \tan \theta_S} \right]^{1/2} \quad 4.2.8$$

In the low frequency limit we have:

$$\frac{\bar{Z}}{\beta} = \left[\frac{\theta}{\theta + Y\theta_S} \right]^{1/2} \quad 4.2.9$$

4.3 Theoretical Results

In order for the narrow connecting line of the non-resonant element to appear to have the same characteristic impedance as the wide lines over a wide band of frequencies, the spacing between the loading stubs must be much less than a wavelength. In practical implementations however the stub spacing must be a significant fraction of one wavelength. This is due to several factors including:

- (1) the radiating edges must not be obstructed
- (2) radiation from the stubs must be minimized
- (3) the stubs have finite widths

In order that the radiating edges of the wide lines not be obstructed, the stubs should be as far as possible from the radiating edges. Radiation from the stubs will become a problem if the sum of the stub widths is not much less than the wide line width. We will consider two stub spacings here, these being $1/16$ and $1/8$ of a wavelength at a frequency of 2.2 GHz respectively. The stub spacing which is practical in a given case depends primarily on the frequency and substrate thickness.

The first case to be considered has a stub spacing of $1/16$ wavelength at 2.2 GHz. The circuit model which

was assumed is given in Figure 4.3.1. For $Z_0 = 16$ ohms and using expression 4.2.8 we have:

$$\begin{aligned}\bar{Z}_B^2 &= 0.1024 \\ &= \frac{0.765 + \bar{B}(0.924) - \bar{B}}{0.765 + \bar{B}(0.924) + \bar{B}}\end{aligned}\quad 4.3.1$$

$$\begin{aligned}\bar{B} &= 2.515 \\ &= 0.5 \tan \theta_S \\ \theta_S &= 78.76^\circ\end{aligned}\quad 4.3.2$$

This radiator configuration has been analyzed using the Touchstone software package on an IBM personal computer. The Touchstone file is given in Figure 4.3.2 and a plot of the return loss is given in Figure 4.3.3. This data indicates that the element has the sharply defined passbands and stop bands characteristic of periodically loaded structures. The upper edge of the lower passband is at approximately 2375 MHz. As we would expect the return loss is maximum in the vicinity of 2.2 GHz. The reason for this excellent return loss which is over 35 dB is that the stub length was calculated to give the required loading at this frequency. Since the stubs are relatively long and thus quite non-linear with frequency the return loss degrades at lower and higher frequencies. The return loss is in excess of 9 dB over a 30% bandwidth.

Figure 4.3.1 Circuit Model for Non-Resonant Element, Stub Spacing = $1/16$ wavelength at 2.2 GHz

```

! FILE NRPATCHS
!
DIM          FREQ MHZ
            ANG DEG
CKT
    TLIN 1 2 Z=50.0 E=11.25 F=2200.
    TLIN 2 3 Z=50.0 E=22.5 F=2200.
    TLIN 3 4 Z=50.0 E=22.5 F=2200.
    TLIN 4 5 Z=50.0 E=22.5 F=2200.
    TLIN 5 6 Z=50.0 E=22.5 F=2200.
    TLIN 6 7 Z=50.0 E=22.5 F=2200.
    TLIN 7 8 Z=50.0 E=22.5 F=2200.
    TLIN 8 9 Z=50.0 E=22.5 F=2200.
    TLIN 9 10 Z=50.0 E=11.25 F=2200.
    TLOC 2 0 Z=100.0 E=78.76 F=2200.
    TLOC 3 0 Z=100.0 E=78.76 F=2200.
    TLOC 4 0 Z=100.0 E=78.76 F=2200.
    TLOC 5 0 Z=100.0 E=78.76 F=2200.
    TLOC 6 0 Z=100.0 E=78.76 F=2200.
    TLOC 7 0 Z=100.0 E=78.76 F=2200.
    TLOC 8 0 Z=100.0 E=78.76 F=2200.
    TLOC 9 0 Z=100.0 E=78.76 F=2200.0
    DEF2P 1 10 TRANS
FREQ
    SWEEP 1600. 2500. 10.
OUT
    TRANS DB[S11] GR1
TERM
    ZO=16.49
GRID
    RANGE 1600 2500 100
    GR1 -30 0 5

```

Figure 4.3.2 Touchstone File for Analysis of Non-Resonant Radiator with Eight Loading Stubs

Figure 4.3.3 Return Loss Characteristic of Non-Resonant Radiator with 8 Loading Stubs

Since the return loss degrades slowly at lower frequencies, the element is usable over a much broader bandwidth.

The second non-resonant radiator configuration to be considered is given in Figure 4.3.4. In this case only four loading stubs have been used and their lengths have been optimized for minimum return loss over the band 2.0 GHz to 2.4 GHz. This optimization was also performed using the Touchstone software package. The Touchstone file is given in Figure 4.3.5. In this case an actual microstrip circuit implementation was analyzed whereas in the previous case an ideal transmission line model was analyzed. The analysis in this case includes the effects of the step discontinuities and T junctions. The impedance of the feed line was chosen to be 16.49 ohms which corresponds to a line width of 4.2 cm on the substrate considered. This substrate is a Duroid 5880 material manufactured by Rogers Corporation. A dielectric thickness of 0.120" is assumed and the relative dielectric constant is 2.2.

The return loss is plotted in Figure 4.3.6 over the band 1650 MHz to 2700 MHz. This data exhibits the well defined passbands and stopbands characteristic of periodically loaded structures. In particular the lower passband has an upper limit of approximately 2475 MHz.

As we would expect the return loss is best in the 2000 MHz to 2400 MHz band where the optimization was performed. The return loss is better than 6 dB however over the band 1750 MHz to 2475 MHz. This is a 34% bandwidth. As discussed in the case of the element with 8 stubs, the return loss degrades slowly at lower frequencies and rapidly at higher frequencies.

ALL DIMENSIONS IN MILLIMETERS

SUBSTRATE IS 0.120 INCH THICK DUROID 5880

Figure 4.3.4 Circuit Model for Non-Resonant
Radiator with Four Loading Stubs

```

! NAME NRPATCH2
! PURPOSE ANALYSIS OF MICROSTRIP IMPLEMENTATION OF NON-RESONANT PATCH
DIM
    FREQ MHZ
    RES OH
    ANG DEG
    LNG CM
CKT
    MSUB ER=2.2 H=0.3175 T=1.8E-03 RHO=0.71 RGH=0.0
    MTEE 2 3 4 W1=1.0 W2=1.0 W3=0.2
    MTEE 5 6 7 W1=1.0 W2=1.0 W3=0.2
    MTEE 8 9 10 W1=1.0 W2=1.0 W3=0.2
    MTEE 11 12 13 W1=1.0 W2=1.0 W3=0.2
    MSTEP 15 1 W1=4.2 W2=1.0
    MSTEP 14 16 W1=1.0 W2=4.2
    MLEF 4 W=0.2 L # 0.0 1.81012 2.58
    MLEF 7 W=0.2 L # 0.0 0.71884 2.58
    MLEF 10 W=0.2 L # 0.0 0.70842 2.58
    MLEF 13 W=0.2 L # 0.0 1.81167 2.58
    MLIN 1 2 W=1.0 L=0.35
    MLIN 3 5 W=1.0 L=0.7
    MLIN 6 8 W=1.0 L=0.7
    MLIN 9 11 W=1.0 L=0.7
    MLIN 12 14 W=1.0 L=0.35
    DEF2P 15 16 TRANS
FREQ
    SWEEP 1600. 2900. 25.
OUT
    TRANS DB[S11] GR1
TERM
    Z0=16.49
OPT
    RANGE 2000. 2400.
    TRANS DB[S11]<-20.

```

Figure 4.3.5 Touchstone File For Analysis of Non-Resonant Radiator with Four Loading Stubs

Figure 4.3.6 Return Loss Characteristic of
Non-Resonant Radiator with Four
Loading Stubs

4.4 Experimental Results

A "non-resonant radiating element" with four loading stubs has been analyzed, optimized, fabricated and tested. This element is illustrated in Figure 4.4.1. A regular microstrip patch radiator with the same dimensions but without the loading stubs has also been analyzed. This radiator is illustrated in Figure 4.4.2. The radiators were analyzed using the Touchstone software package and the program listing is given in Figure 4.4.3. The return loss characteristic of the non-loaded element is illustrated in Figure 4.4.4. The 10 dB return loss bandwidth is in this case approximately 125 MHz. The theoretical return loss of the non-resonant radiating element is plotted in Figure 4.4.5. The 10 dB return loss bandwidth is in this case approximately 650 MHz. The addition of four short narrow loading stubs has thus increased the radiator bandwidth by a factor of 5.2 in the series-fed configuration. This is an increase in percentage bandwidth from 5.7 percent to 27. percent. This is remarkable bandwidth for a microstrip patch radiator or any other type of radiating element. There is clearly reason to believe that virtually unlimited bandwidths can be achieved simply by addition of increasing numbers of loading stubs. In practice the bandwidth is limited by the length of the loading sections, the density of stubs achievable and how close

the stubs can come to the radiating edges. In this case there is virtually no coupling between stubs or between any stub and any radiating edge. The simple model of Figure 4.4.3 is thus sufficient to predict the element behaviour.

A non-resonant element with four loading stubs has been fabricated and tested. This element is identical to that analyzed above. The element was fabricated on a Teflon/Fiberglass substrate, 0.062 inches thick. The material used was Duroid 5880 which is manufactured by Rogers Corporation of Chandler Arizona USA. The material was clad on one side with 1 oz/ft² copper. The microstrip circuit was cut from copper tape which was applied to the Teflon substrate prior to cutting. This technique resulted in satisfactory tolerances due to the relatively low frequency and thick substrate. SMA connectors with microstrip tabs were soldered onto the ends of the circuit board. The 0.062 inch substrate thickness results in a good match to the sma connector.

The return loss of the fabricated patch was measured over the 1.7 GHz to 2.7 GHz band. The analog outputs from a network analyzer were used to drive an analog x,y plotter. The measured results are given in Figure 4.4.6. The measured return loss is very close to the theoretical characteristic down to the -20 dB level.

Below this level many factors contribute to altering the measured response including fabrication tolerances, connector mismatch and raw material tolerances. The two lobes in the null region do appear but their levels are higher than those theoretically predicted.

The measured non-resonant element demonstrates the wide bandwidth achievable with the loaded line approach. The element also demonstrates the sharp cut-off of the passband at the high frequency end and the relatively slow cut-off at low frequencies which is a general characteristic of periodically loaded transmission lines.

ALL DIMENSIONS IN MILLIMETERS

Figure 4.4.1 Non-Resonant Radiating Element

ALL DIMENSIONS IN MILLIMETERS

Figure 4.4.2 Radiator Without Loading Stubs

```

! NAME NRPATCHB
DIM      FREQ MHZ
          RES OH
          ANG DEG
          LNG CM
CKT
MSUB ER=2.2 H=0.1549 T=1.8E-03 RHO=0.71 RGH=0.0
MTEE 1 3 2 W1=0.49 W2=0.49 W3=0.2
MLEF 2 W=0.2 L # 0.5 0.91296 1.75
MLIN 3 6 W=0.49 L # 0.9 0.99384 1.1
MLIN 6 10 W=0.49 L=2.3532
MTEE 10 12 11 W1=0.49 W2=0.49 W3=0.2
MLEF 11 W=0.2 L # 0.5 1.20232 1.75
MLIN 12 13 W=0.49 L # 0.9 0.98950 1.2
MSTEP 13 14 W1=0.49 W2=4.2
MLIN 14 15 W=4.2 L=4.60544
MSTEP 15 16 W1=4.2 W2=0.49
MLIN 16 17 W=0.49 L # 0.9 0.95472 1.2
MTEE 17 19 18 W1=0.49 W2=0.49 W3=0.2
MLEF 18 W=0.2 L # 0.5 1.30738 1.75
MLIN 19 22 W=0.49 L # 0.9 1.18877 1.3
MLIN 22 26 W=0.49 L=2.0
MTEE 26 28 27 W1=0.49 W2=0.49 W3=0.2
MLEF 27 W=0.2 L # 0.5 0.84109 1.75
DEF2P 1 28 TRANS
FREQ
SWEEP 1700. 2900. 25.
OUT
TRANS DB[S11] GR1
OPT
RANGE 2100. 2500.
TRANS DB[S11]<-27.5
GRID
GR1 -35 0 5

```

Figure 4.4.3 Touchstone Program for Analysis and Optimization of "Non-Resonant Radiator"

Figure 4.4.4 Theoretical Return Loss.
Radiator Without Loading Stubs

Figure 4.4.5 Theoretical Return Loss
Non-Resonant Radiating Element

Figure 4.4.6 Measured Return Loss
Non-Resonant Radiating
Element

4.5 VARIABLE SUBSTRATE THICKNESS

If the substrate thickness beneath each patch radiator is increased relative to that beneath the feed line, then the characteristic impedance of the feed line and radiators can be made equal. A series-fed array of this type can be modelled as a constant impedance transmission line loaded by the radiation conductances and fringe capacitances of the patch radiators. Since the radiators are virtually non-resonant, very large bandwidth can be obtained using this configuration.

As experimental verification of this array concept, two radiators were fabricated. The first, a conventional patch radiator, illustrated in Figure 4.5.1, is the reference against which the non-resonant radiator is evaluated. The non-resonant radiator is illustrated in Figure 4.5.2. In each case, the radiator is printed on 0.010" thick FR-4 fiberglass material. The fiberglass is suspended above an aluminum ground plane resulting in a dielectric which is primarily air. The substrate dielectric constant may be evaluated using the quasi-static approximation:

$$\epsilon_r' = \frac{d_1 + d_2}{d_1/\epsilon_1 + d_2/\epsilon_2} \quad 4.5.1$$

where d_1 is the thickness of the fiberglass, d_2 is the thickness of the air gap, ϵ_1 is the fiberglass permittivity and ϵ_2 is the air gap permittivity.

The dimensions of the printed radiator are identical in the two cases. These dimensions are:

Feed line width:	10.62mm
Patch width:	30.73mm
Patch length:	62.80mm

The substrate thickness for the feed line and resonant radiator is 2.286 mm. This yields a 50 ohm feed line impedance. The substrate thickness under the non-resonant radiator is 6.35 mm. This also results in a 50 ohm radiator characteristic impedance.

The return loss characteristic of the reference radiator was measured and is given in Figure 4.5.3. The resonant frequency is roughly 2.28 GHz. The return loss characteristic is non-symmetrical due to the added discontinuities which appear at the microstrip to coax transitions. A bandwidth measured to the 10dB return loss points of 2100 MHz to 2450 MHz was obtained. The return loss characteristic of the non-resonant radiator is given in Figure 4.5.4. In this case no evidence of a resonance condition exists. The return loss is greater than 10dB from D.C. to approximately 5.2 GHz.

The ultra-broad bandwidth obtained with the non-resonant radiator suggests that this type of element could be used to make series-fed arrays covering many octaves. A potential log-periodic array concept using this type of radiator is illustrated in figure 4.5.5. In this implementation of a log periodic array, beam width and direction are roughly independent of frequency. Beam width depends on the overall length and relative size of adjacent radiators and beam direction depends on inter-element spacing.

Figure 4.5.1 Reference For Evaluation of Non-Resonant Radiator Concept

ALL DIMENSIONS IN MILLIMETERS

Figure 4.5.2 Non-Resonant Radiator with Varying Substrate Thickness

Figure 4.5.3 Return Loss Characteristic of Reference Radiator

Figure 4.5.4 Return Loss Characteristic of Non-Resonant Radiator

Figure 4.5.5 Log-Periodic Array of Non-Resonant Radiating Elements

CHAPTER V
CONCLUSIONS AND FUTURE WORK

5.1 CONCLUSIONS

This thesis contains a comprehensive treatment of the principal problems associated with the synthesis of broadband series-fed arrays of microstrip patch radiators. The work has been categorized into two basic areas. The first is the synthesis of theoretical aperture excitations having radiation patterns which are insensitive to random and deterministic amplitude and phase errors in the array feed. The second area is the development of broadband radiating elements. This work has resulted in the development of novel techniques for pattern synthesis and the design of broadband microstrip patch radiators. While these techniques have been developed in the context of series-fed microstrip patch radiators, they are applicable to a much broader range of antenna design problems. The techniques described have been evaluated extensively using numerical methods and where appropriate the numerical evaluations have been verified by experiment.

Conventional pattern synthesis techniques such as Fourier, Woodward and discrete least mean square error techniques have been previously devised to provide a good fit to a desired pattern provided that the required excitation is implemented exactly. It is possible in fact to define an excitation which given any finite sized aperture will produce the desired pattern exactly. In

general, however, the pattern becomes increasingly sensitive to errors in the excitation as the pattern fit improves. In series-fed arrays the excitation at individual radiators is very sensitive to frequency, effective dielectric constant of the feed line and other factors. Pattern synthesis techniques are thus required which optimize the pattern over the range of these variables. Two such techniques have been investigated here. The first is a solution which results in the least mean square error in the complex radiation pattern over any desired number of discrete angles and frequencies. This is an extension of the conventional least mean square error method [10]. The second technique is a numerical optimization whereby the sum of the pattern errors at several discrete frequencies is minimized. This second technique while being less elegant is more versatile since it can be applied either in the case where far field phase is defined or without penalty in the case where the far field phase is arbitrary. With either of the synthesis techniques the pattern error can be weighted in proportion to its relative importance in the various angular and frequency regions.

The performance of the pattern synthesis techniques has been evaluated by means of numerical examples. In each case it was found that dramatic reductions in the pattern error were obtained at the band edges while in comparison to conventional synthesis techniques only small degradations in pattern error were obtained at mid-band. The least mean square error synthesis is the preferred technique in cases where a required far field phase is defined. This is because the optimization technique can not guarantee an absolute minimum error without the inclusion of an exhaustive search. When the far field phase is arbitrary, however, the numerical optimization is likely to result in lower pattern error due to the elimination of this constraint.

The bandwidth of a conventional rectangular microstrip patch radiator is quite narrow particularly for a thin substrate with high permittivity. While modified patch shapes have been developed by many corporations in order to achieve increased bandwidth, experiments performed by the author indicate that in general only small improvements are obtained. Recent work using electromagnetically coupled parasitic radiating elements [15, 16, 17] has obtained increases in bandwidth by factors of 2 - 3. In this thesis, however,

factors as high as 6 are reported with potential for further improvement.

A very interesting technique for the synthesis of broad-band radiating elements has been investigated, and demonstrated. This approach, based on filter theory, requires that a radiating element or sub-array be broken up into multiple electrically coupled resonators allowing the quality factor and resonant frequency of each to be specified for the desired bandpass characteristic. An element composed of four such electrically coupled patch radiators has been demonstrated. A bandwidth over six times that of a single radiator was obtained. In the case of sub-arrays, a procedure has been described allowing Butterworth or Chebyshev bandpass characteristics to be approximated.

Conventional filter theory was also applied to the problem of obtaining a broadband impedance match to series-fed microstrip patches. It was found that the addition of two matching stubs in front of each resonator could increase the bandwidth of a typical radiator by a factor of 2.5.

A second approach to the synthesis of broadband radiating elements focuses on making the elements non-resonant. Conceptually, these elements are series-fed and have the same characteristic impedance as the feed line. The only discontinuities are then the small radiation conductances. The non-resonant condition can be achieved by periodically loading the high impedance sections such that their characteristic impedance matches that of the low impedance sections. It is demonstrated that the addition of two small loading stubs on each side of a typical series-fed radiator will increase its impedance bandwidth by a factor of roughly 4.7.

An alternative method was also investigated for obtaining a non-resonant microstrip patch radiating element. This technique involves varying the substrate thickness such that the characteristic impedance of the feed line and that of the radiators are equal. The array can thus be modelled as a constant impedance transmission line loaded periodically by the discontinuities at the end of the radiators. These discontinuities consist of a radiation conductance and fringe capacitance. Using this type of radiator it should be possible to design series-fed arrays having a bandwidth of several octaves. For example, a single conventional radiator was measured to have 10dB return loss over the band 2104 MHz to 2450 MHz. When the ground plane spacing under the patch was increased to make its characteristic

impedance 50 ohms, greater than 10dB return loss was obtained from DC to 5.2 GHz.

5.2 FUTURE WORK

Some research areas related to this thesis which deserve additional work include the following:

- 1) In the case of the non-resonant microstrip patch radiator with varying substrate thickness the discontinuities must be characterized. This would allow computer simulation of the performance of arrays of these elements.
- 2) A multi-octave bandwidth series-fed array could be designed using non-resonant radiators and a log-periodic approach. The array would be fed at the high frequency end and terminated at the low frequency end.
- 3) A four patch directly coupled multi-resonator element was demonstrated to have an increase in bandwidth of a factor of 6 over a single element of the dimensions used. It would be useful to evaluate the bandwidths achievable with larger numbers of resonators.

- 4) In this thesis a broadband radiator concept was introduced which involves periodically loading the feed line making the radiator non-resonant. A practical implementation was demonstrated, however, other implementations are possible and the ultimate limitations have not been established.

- 5) Insensitive aperture excitations can be obtained by constraining the superdirectivity ratio or the source norm. These approaches do not guarantee however that the pattern error is minimized over a given set of conditions. A numerical example could be used to illustrate this point as well as the relative performance of these approaches.

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APPENDIX A
FORTRAN PROGRAM FOR
LEAST MEAN SQUARE ERROR PATTERN SYNTHESIS
WITH PATTERN CONSTRAINTS AT MULTIPLE
FREQUENCIES

A PROGRAM FOR LEAST MEAN SQUARE ERROR PATTERN SYNTHESIS

PROGRAM LMSE10

TO RUN USE THE FOLLOWING COMMANDS:

FOR LMSE

LINK LMSE+CMINV+CMPROD+C2MINV+CMHER+SYS\$SYSTEM:PLTUSL/LIB

RUN LMSE

RUN SYS\$SYSTEM:PLXY

PLT>PLTDAT.VEC

PLT>EXIT

PRINT PLTDAT.PLT/NOFEED

THIS PROGRAM IS A COMPLETELY GENERAL PATTERN SYSTHESIS PROGRAM FOR A LINEAR ARRAY. THE FAR FIELD PATTERN MAY BE DEFINED IN BOTH AMPLITUDE AND PHASE AT AN ARBITRARY NUMBER OF POINTS (LARGER THAN THE NUMBER OF DISCRETE SOURCE POINTS) AND AT EACH OF TWO FREQUENCIES. THE FAR FIELD PATTERN IS READ FROM FILE FOR007.DAT FOR THE FIRST FREQUENCY AND FILE FOR010.DAT FOR THE SECOND FREQUENCY. THE SOURCE POSITIONS ARE READ FROM FILE FOR008.DAT. THE SYNTHESIZED SOURCE VALUES ARE WRITTEN INTO FILE FOR009.DAT. THE FIELD FORMAT IS ANGLE IN DEGREES FROM ENDFIRE FOLLOWED BY A COMPLEX VALUE. THE FAR FIELD PATTERN AT THE SECOND FREQUENCY IS STORED IN FOR010.DAT. THE SOURCE PHASE ERROR CORRESPONDING TO THE FREQUENCY CHANGE IS READ FROM FILE FOR011.DAT.

NOTE THAT MATRIX DIMENSIONS MUST BE THE SAME IN THE MAINLINE AND SUBROUTINE PROGRAMS!!!!

```
REAL XSOURCE(450), ANGLE(450), THE, RE, IM, K, NRE, NIM, XP(400)
REAL ANG, YM, YP(400), K1, K2, FREQ1, FREQ2, ERR(40)
INTEGER N, NSOURCE, MFIELD, IR(40000), IC(40000), M, MFIELD1, MFIELD2, MARK
COMPLEX FIELD(450, 1), T(450, 40), THER(40, 450), PROD(450, 450)
COMPLEX PROD2(450, 450), PHAC
```

XSOURCE - VECTOR OF SOURCE POSITIONS IN METERS FROM A REFERENCE

ANGLE - VECTOR OF ANGLE OF FAR FIELD SAMPLE FROM END FIRE

THE - PHASE ANGLE FOR A GIVEN FIELD ANGLE AND SOURCE POINT

RE, IM REAL AND IMAGINARY COMPONENTS OF PHASER

K - WAVENUMBER

T - MATRIX RELATING FIELD TO SOURCE [T][SOURCE]=[FIELD]

THER - HERMITTIAN CONJUGATE OF T (TRANPOSE CONJUGATE)

PROD - PRODUCT OF TWO MATRICES

PROD2 - ANOUTHER PRODUCT OF TWO MATRICES

```
TYPE *, 'ENTER THE NUMBER OF SOURCE POINTS'
```

```
READ *, NSOURCE
```

```
TYPE *, 'ENTER THE NUMBER OF FIELD POINTS AT FREQUENCY 1'
```

```
READ *, MFIELD1
```

```
DO N=1, NSOURCE
```

```
    READ(008, *) XSOURCE(N)
```

```
END DO
```

```

TYPE *, 'ENTER THE DESIGN FREQUENCY 1 IN HZ'
READ *, FREQ1
TYPE *, 'ENTER THE NUMBER OF FIELD POINTS AT FREQ 2'
READ *, MFIELD2
TYPE *, 'ENTER FREQ 2 IN HERTZ'
READ *, FREQ2
DO M=1, MFIELD1
    READ(007, *) ANGLE(M), FIELD(M, 1)
PRINT *, ANGLE(M), FIELD(M, 1)
    ANGLE(M)=ANGLE(M)*1.745329252E-02
END DO
DO M=1, MFIELD2
    N=M+MFIELD1
PRINT *, N
    READ(010, *) ANGLE(N), FIELD(N, 1)
PRINT *, N, ANGLE(N)
    ANGLE(N)=ANGLE(N)*1.745329252E-02
END DO
MFIELD=MFIELD1+MFIELD2
K IS WAVENUMBER
K1=2.094395102E-08*FREQ1
K2=2.094395102E-08*FREQ2
THIS SECTION COMPUTES THE TRANSFORMATION MATRIX
THE NUMBERING SYSTEM IS AS SHOWN T(M, N)
M IS VERTICAL COUNTER N IS HORIZONTAL COUNTER

```

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

THIS SECTION READS THE PHASE ERROR

```

DO N=1, NSOURCE
    READ(011, *) M, ERR(N)
    RE = COS(ERR(N))
    IM = SIN(ERR(N))
    ERR(N) = CMPLX(RE, IM)
END DO

DO N=1, NSOURCE
    DO M=1, MFIELD
        IF(M .LE. MFIELD1) THEN
            THE = K1*XSOURCE(N)*COS(ANGLE(M))
        ELSE
            THE=K2*XSOURCE(N)*COS(ANGLE(M))
        END IF
        RE = COS(THE)
        IM = SIN(THE)
        T(M, N) = CMPLX(RE, IM)
        IF(M .GT. MFIELD1) THEN
            T(M, N) = T(M, N)*ERR(N)
        ELSE
            T(M, N) = T(M, N)/ERR(N)
        END IF
        PRINT *, M, N, T(M, N)
    END DO
END DO

```

THIS SECTION COMPUTES THE SYSTHEZIZED SOURCE VECTOR SOURCE(N)

CALL CMHER(MFIELD, NSOURCE, T, THER)

THIS SECTION CALCULATES THE HERMITIAN CONJUGATE

```
DO N=1, NSOURCE
  DO M=1, MFIELD
    THER(N, M)=CMPLX(REAL(T(M, N)), -AIMAG(T(M, N)))
  END DO
END DO
```

```
DO M=1, NSOURCE
DO N=1, MFIELD
PRINT *, 'M, N, THER', M, N, THER(M, N)
END DO
END DO
CALL CMPROD(NSOURCE, MFIELD, MFIELD, NSOURCE, THER, T, PROD)
```

THIS CODE CALCULATES THE MATRIX PRODUCT

```
DO N=1, NSOURCE
  DO M=1, NSOURCE
    PROD(M, N)=CMPLX(0., 0.)
  END DO
END DO
DO N=1, NSOURCE
  DO M=1, NSOURCE
    DO NC=1, MFIELD
      PROD(M, N)=PROD(M, N)+THER(M, NC)*T(NC, N)
    END DO
  END DO
END DO
```

```
CALL C2MINV(PROD, NSOURCE, DETRM, IR, IC)
DO M=1, NSOURCE
DO N=1, NSOURCE
PRINT *, 'M N INV(M, N)', M, N, PROD(M, N)
END DO
END DO
CALL CMPROD(NSOURCE, NSOURCE, NSOURCE, MFIELD, PROD, THER, PROD2)
```

THIS CODE CALCULATES THE MATRIX PRODUCT

```
DO N=1, MFIELD
  DO M=1, NSOURCE
    PROD2(M, N)=CMPLX(0., 0.)
  END DO
END DO
```

```
DO N=1,MFIELD
  DO M=1,NSOURCE
    DO NC=1,NSOURCE
      PROD2(M,N)=PROD2(M,N)+PROD(M,NC)*THER(NC,N)
    END DO
  END DO
END DO
```

```
DO M=1,NSOURCE
  DO N=1,MFIELD
    PRINT *, 'M N D(M,N)', M, N, PROD2(M,N)
  END DO
END DO
DO N=1,2
PRINT *, 'M FIELD(M,1)', N, FIELD(M,1)
END DO
CALL CMPROD(NSOURCE,MFIELD,MFIELD,1,PROD2,FIELD,PROD)
```

THIS CODE EVALUATES THE MATRIX PRODUCT

```
DO N=1,1
  DO M=1,NSOURCE
    PROD(M,N)=CMPLX(0.,0.)
  END DO
END DO
DO N=1,1
  DO M=1,NSOURCE
    DO NC=1,MFIELD
      PROD(M,N)=PROD(M,N)+PROD2(M,NC)*FIELD(NC,N)
    END DO
  END DO
END DO
```

THIS SECTION PRINTS OUT THE OUTPUT
AND WRITES IT TO FOR009.DAT

```
DO M=1,NSOURCE
  PRINT *, M, PROD(M,1)
  WRITE(009,*)PROD(M,1)
END DO
```

THIS SECTION PLOTS THE SYNTHESIZED PATTERN

```

PRINT *, 'INPUT PLOT FREQ RATIO 1.00X(FREQ1)'
READ *, KRAT
K=K1*KRAT
PRINT *, '2*ERR TYPE 1, NEG 2*ERR TYPE 2, ELSE TYPE 0'
READ *, MARK
IF(MARK .EQ. 1) THEN
  DO N=1, NSOURCE
    PROD(N, 1) = PROD(N, 1)*ERR(N)*ERR(N)
  END DO
END IF
IF(MARK .EQ. 2) THEN
  DO N=1, NSOURCE
    PROD(N, 1) = PROD(N, 1)/(ERR(N)*ERR(N))
  END DO
END IF

CALL PLOTST(, 'CM', 0)
CALL FACTOR(2, 0)
CALL PLOT(3.5, 4.0, -3)
CALL PLOT(0.0, 0.0, 3)
DO N=1, 360
  ANG=(N*180./360.)*1.74533E-02
  RE=0.0
  IM=0.0
  DO M=1, NSOURCE
    THE=K*XSOURCE(M)*COS(ANG)
    PRINT *, 'ANG SOURCE XSOURCE THE', ANG, M, XSOURCE(M), THE
    NRE=COS(THE)
    NIM=SIN(THE)
    PHAC=CMPLX(NRE, NIM)
    PHAC=PHAC*PROD(M, 1)
    RE=RE+REAL(PHAC)
    IM=IM+AIMAG(PHAC)
  END DO
  XP(N)=N*9.0/360.0
  YP(N)=RE*RE+IM*IM
END DO
YM=0.0
DO N=1, 360
  IF(YP(N) .GT. YM) THEN
    YM=YP(N)
  END IF
END DO
DO N=1, 360
  YP(N)=6.0+2.0*ALOG10(YP(N)/YM)
  IF(YP(N) .LT. 0.0) THEN
    YP(N) = 0.0
  END IF
  CALL PLOT(XP(N), YP(N), 2)
  PRINT *, N, XP(N), YP(N)
END DO

```

```
CALL AXIS(0.0,0.0, 'THETA (DEGREES)', -15.9, 0.0, 0.0, 20.0)
CALL AXIS(0.0,0.0, 'ARRAY FACTOR (DB)', 17.6, 0.0, 90.0, -30.0, 5.0)
CALL SYMBOL(2.2, -1.0, 0.2, 'LMSE PATTERN', 0.0, 12)
CALL SYMBOL(4.4, -1.0, 0.2, 'SYNTHESIS', 0.0, 9)
```

```
STOP
END
```

THE FOLLOWING SUBROUTINE COMPUTES THE PRODUCT OF TWO
COMPLEX MATRIXES MX1(M1,N1) MX2(M2,N2)

```
SUBROUTINE CMPROD(M1,N1,M2,N2,MX1,MX2,MPROD)
INTEGER M1,N1,M2,N2
COMPLEX MX1(250,250),MX2(250,250),MPROD(250,250)
```

THIS SUBROUTINE FINDS THE PRODUCT OF TWO ARBITRARY
TWO DIMENSIONAL COMPLEX MATRIXES

M1,N1 ARE THE VERTICAL AND HORIZONTAL SIZES OF THE
LEFT HAND MATRIX
M2,N2 ARE THE VERT AND HOR SIZES OF THE RIGHT HAND MATRIX

```
DO N=1,N2
  DO M =1,M1
    MPROD(M,N)=CMPLX(0.0,0.0)
  END DO
END DO
DO N=1,N2
  DO M=1,M1
    DO NC=1,N1
      MPROD(M,N) = MPROD(M,N) + MX1(M,NC)*MX2(NC,N)
    END DO
  END DO
END DO
RETURN
END
```

```
SUBROUTINE CMHER(MC,NC,TM,TMHER)
```

THIS SUBROUTINE CALCULATES THE HERMITIAN CONJUGATE OF
A MATRIX. (TRANSPOSE-CONJUGATE)

```
INTEGER N,NC,M,MC
COMPLEX TMHER(250,250),TM(250,250)
DO N=1,NC
  DO M=1,MC
    TMHER(N,M) = CMPLX(REAL(TM(M,N)), -AIMAG(TM(M,N)))
  END DO
END DO
RETURN
END
```

APPENDIX B

ELECTRICALLY COUPLED RESONATOR OPTIMIZATION PROGRAM

C
C
C
C
C
C
C
C
C

THIS PROGRAM OPTIMIZES THE LENGTHS AND WIDTHS OF EACH OF THE
RESONATORS IN A SERIES-FED ARRAY IN ORDER TO MAXIMIZE THE
BANDWIDTH. EACH SECTION IS ASSUMED INITIALLY TO BE RESONANT
AT THE SPECIFIED CENTER FREQUENCY. A MODEL DESCRIBING THE
RADIATION CONDUCTANCE VERSUS PATCH WIDTH MUST BE ENTERED
INTO THE CODE.

REAL F, ER, D, T, WC, W(100), LR(100), LC(100), S(100), WCN(100)
REAL EEFF, Z, LAO, DS(100), LCE(100), LAC, DPH(100), EEC(100), ZCN(100)
REAL R, I, DW(100), LRE(100), ZC, LAP, ZZ, EMIN, DEW, FO
REAL RECO(100), REE, KKK, NORM(100), RLO(100), CAL
REAL E1(100), E2(100), DR, YD(100), EM, ROD, ERR, DELR, MR
REAL PH1(100), PH2(100), XS, X1(100), X2(100), RR, RL, IL, XXP(1000)
REAL A, AR, K, G(100), PH, RE, IM, XP, YP, EXT, EXL, YYMAX, YYP(1000)

C

COMPLEX AT(100), AL(100), BL(100), BT(100), CL(100), CT(100)
COMPLEX DL(100), DT(100), AP(100), BP(100), CP(100), DP(100)
COMPLEX AC(100), BC(100), CC(100), DC(100), APT(100), BPT(100)
COMPLEX CPT(100), DPT(100), AAP(100), BBP(100), CCP(100), DDP(100)
COMPLEX VP, VM, V1(100), V2(100), VP2, VM2, CALC
COMPLEX ZBAR(100), ZIN, ZBR, ZINF, QAMA(100), QAG

C
C

FO CENTER FREQUENCY
N NUMBER OF ELEMENTS IN THE ARRAY
F FREQUENCY IN HZ
ER RELATIVE DIELECTRIC CONSTANT
D DIELECTRIC THICKNESS IN METERS
T COPPER THICKNESS IN METERS
WC INPUT FEED LINE WIDTH IN METERS
A ATTENUATION PER METER AS VOLTAGE RATIO <1
AR 1/A
W(M) PATCH WIDTH
LR(M) PATCH LENGTH
LC(M) CONNECTING LINE LENGTH FROM ELEMENT M TO M+1
LAO FREE SPACE WAVELENGTH
S(M) APX CENTER TO CENTER PATCH SPACING
K MUTUAL COUPLING FACTOR
EEFF EFFECTIVE DIELECTRIC CONSTANT
ZC IMPEDANCE OF INPUT LINE
G(M) RADIATION CONDUCTANCE OF ONE SIDE OF PATCH
NORMALIZED TO THE CONNECTING LINE
DS(M) EFFECTIVE INCREASE IN PATCH SEPARATION
DUE TO ONE DISCONTINUITY
LCE(M) EFFECTIVE LINE LENGTH B/N PATCH M AND M+1
LAC CONNECTING LINE WAVELENGTH
DPH(M) PHASE LENGTH OF CONNECTING LINE
AC(M)...DC(M) CONNECTING LINE TRANSMISSION MATRIX
R, I, RR REAL AND IMAGINARY PARTS OF COMPLEX NUMBERS
AP(M)...DP(M) SHUNT CONDUCTANCE TRANSMISSION MATRIX
DR EFFECTIVE INCREASE IN PATCH LENGTH DUE TO ONE DISCONTINUITY
LRE(M) EFFECTIVE PATCH LENGTH
AT(M)...DT(M) TRANSMISSION MX OF JUNCTION AT LOAD END OF PATCH
AL(M)...DL(M) TRANS. MX OF JUNCTION AT SOURCE END OF PATCH
LAP WAVELENGTH ON PATCH
APT(M)...DPT(M) TRANSMISSION MX OF PATCH TRANSMISSION LINE
VP, VM VOLTAGE OF WAVE TRAVELING TOWARD LOAD AND SOURCE
VP2, VM2 "
RL RETURN LOSS(DB)
IL INSERTION LOSS(DB)

```

C      E1(M),E2(M) MAG OF VOLTAGE AT LOAD AND SOURCE ENDS OF PATCH
C      V1(M),V2(M)  VOLTAGE AT LOAD AND SOURCE ENDS OF PATCH
C      PH1(M),PH2(M) PHASE OF VOLTAGE AT LOAD AND SOURCE ENDS OF PATCH
C      XS POSITION COUNTER
C      X1(M),X2(M)  POSITION OF LOAD AND SOURCE ENDS OF PATCH RELATIVE
C                   TO THE LOAD END OF PATCH 1
C      MR REAL VERSION OF INTEGER
C      M,N,P INTEGER COUNTERS
C      PH PHASE USED IN PLOTTING
C      RE,IM REAL AND IMAGINARY PARTS OF EXCITATION
C      XP,YP X AND Y PLOTTING POSITIONS

```

INITIAL CONDITIONS

THE LENGTHS ARE FRACTIONS OF THE MID-BAND
RESONANT LENGTH

```

LCE(1) = 1.0
LCE(2) = 1.0
LCE(3) = 1.0
LRE(1) = 1.0
LRE(2) = 1.0
LRE(3) = 1.0
LRE(4) = 1.0
W(1) = 1.0E-02
W(2) = 1.0E-02
W(3) = 1.0E-02
W(4) = 1.0E-02
WCN(1) = 1.0E-02
WCN(2) = 1.0E-02
WCN(3) = 1.0E-02
WCN(4) = WC
FO = 2.2E09
ER = 2.2
N = 4
D = 0.00157
T = 3.556E-05
WC = 2.41E-03

```

THE OPTIMIZATION LOOP BEGINS HERE

```

ND = 1
EMIN = 10000.
DO LCO = 1,40
  ND = ND+1
  IF (ND .EQ. 3) THEN
    ND = 1
  END IF
  DEW = .2E-03
  IF (LCO .LE. 10) THEN
    DEW = 1.0E-03
  ELSE
    IF (LCO .LE. 20) THEN
      DEW = 0.5E-03
    ELSE
      IF (LCO .LE. 30) THEN
        DEW = 0.4E-03
      ELSE
        IF (LCO .LE. 40) THEN
          DEW = 0.2E-03
        END IF
      END IF
    END IF
  END IF

```

```

ELSE
  IF (LCO .LE. 50) THEN
    DEW = 0.5E-03
  ELSE
    DEW = 0.05E-03
  END IF
END IF
END IF
END IF
DO NC = 1, 2*N-1
  DO WCO = 1, 3
    SET = 2
    SET2 = 2
    IF (ND .EQ. 1) THEN
      IF (NC .LE. N) THEN
        W(NC) = W(NC) + (WCO-2.)*DEW
        SET = 1
        IF (W(NC) .GT. 2.E-02) THEN
          IF (WCO .EQ. 3) THEN
            SET = 2
            W(NC) = W(NC) - (WCO-2.)*DEW
          END IF
        END IF
      END IF
    ELSE
      SET2 = 1
      WCN(NC-N) = WCN(NC-N) + (WCO-2.)*DEW/2.
      IF (WCN(NC-N) .LT. 1.E-03) THEN
        IF (WCO .EQ. 1) THEN
          SET2 = 2
          WCN(NC-N) = WCN(NC-N) - (WCO-2.)*DEW/2.
        END IF
      END IF
    END IF
  END IF
ELSE
  IF (NC .LE. N) THEN
    LRE(NC) = LRE(NC)*(1.+0.003*(WCO-2.))
  ELSE
    LCE(NC-N) = LCE(NC-N)*(1.+0.003*(WCO-2.))
  END IF
END IF
DO FC = 1, 5
  F = (1. + (FC-3.)*0.01)*FO

```

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C

THE COMPUTATION SECTION BEGINS HERE

MODEL

ENTER PATCH MODEL INTO THIS SECTION

THIS SECTION CALCULATES THE CONDUCTANCES AND THE EFFECTIVE LENGTHS OF THE CONNECTING LINES. THE MODEL USED HERE SHOULD BE MODIFIED ACCORDING TO AVAILABLE EMPIRICAL DATA.

```

LAO = 3.0E08/F
DO M = 1, N
  G(M) = 1.43E-04*((W(M)*100.)**1.64)
END DO

```

C
C
C

THIS SECTION CALCULATES THE WAVE TRANSMISSION PARAMETERS

```

C      FOR THE CONNECTING LINES
C
      DO M = 1,N
          CALL SMIC(WCN(M),F,D,T,ER,EEC(M),ZCN(M))
      END DO
      ZCN(N) = 50.
      DO M = 1,N-1
          DPH(M) = 3.14159*F/FO*LCE(M)
          BC(M) = CMPLX(0.0,0.0)
          CC(M) = CMPLX(0.0,0.0)
          R = COS(DPH(M))
          I = SIN(DPH(M))
          AC(M) = CMPLX(R,I)
          I = -I
          DC(M) = CMPLX(R,I)
      END DO

C
C      THIS SECTION CALCULATES THE WAVE TRANSMISSION PARAMETERS
C      FOR THE SHUNT CONDUCTANCES
C
      DO M=1,N
          IF(M.EQ. 1)THEN
              ZZ = 50.0
          ELSE
              ZZ = ZCN(M-1)
          END IF
          R = 1.0 + G(M)*ZZ/2.0
          AP(M) = CMPLX(R,0.0)
          R = G(M)*ZZ/2.0
          BP(M) = CMPLX(R,0.0)
          R = -R
          CP(M) = CMPLX(R,0.0)
          R = 1.0 - G(M)*ZZ/2.0
          DP(M) = CMPLX(R,0.0)
      END DO
      DO M=1,N
          R= 1.0+G(M)*ZCN(M)/2.0
          AAP(M) = CMPLX(R,0.0)
          R = G(M)*ZCN(M)/2.0
          BBP(M) = CMPLX(R,0.0)
          R = -R
          CCP(M) = CMPLX(R,0.0)
          R = 1.0 - G(M)*ZCN(M)/2.0
          DDP(M) = CMPLX(R,0.0)
      END DO

C
C      THIS SECTION CALCULATES THE WAVE TRANSMISSION PARAMETERS
C      OF THE PATCH TRANSMISSION LINES AND DISCONTINUITIES
C
      DO M =1,N
          IF (M.EQ. 1)THEN
              ZZ = 50.0
          ELSE
              ZZ = ZCN(M-1)
          END IF
          CALL SMIC(W(M),F,D,T,ER,EEFF,Z)
          R = (ZZ + Z)/(2.0*ZZ)
          AT(M) = CMPLX(R,0.0)
          DT(M) = AT(M)
          R = (Z + ZCN(M))/(2.0*Z)
          AL(M) = CMPLX(R,0.0)
          DL(M) = AL(M)
          R = (ZZ - Z)/(2.0*ZZ)
      END DO

```

```

BT(M) = CMPLX(R,0,0)
CT(M) = BT(M)
R = (Z - ZCN(M))/(2.0*Z)
BL(M) = CMPLX(R,0,0)
CL(M) = BL(M)
LAP = 3.0E08/(F*SQRT(EEFF))
DPH(M) = 3.14159*F/F0*LRE(M)
BPT(M) = CMPLX(0,0,0,0)
CPT(M) = CMPLX(0,0,0,0)
R = COS(DPH(M))
I = SIN(DPH(M))
APT(M) = CMPLX(R,I)
I = -I
DPT(M) = CMPLX(R,I)
END DO

```

C
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C

THIS SECTION DETERMINES THE VOLTAGES AT EACH END OF EACH PATCH. THE WAVE TRAVELING TOWARDS THE LOAD HAS VOLTAGE VP AND THE WAVE TRAVELING TOWARDS THE SOURCE HAS VOLTAGE VM. WE START AT THE LOAD WITH VP = (1,0,0,0) VM = (0,0,0,0). THE VOLTAGE AT THE LOAD END OF PATCH 1 IS V1(1). THE VOLTAGE AT THE SOURCE END OF PATCH 1 IS V2(1). PATCH 1 IS THE PATCH CLOSEST TO THE LOAD. THE TOTAL VOLTAGE V1(M) OR V2(M) IS GIVEN BY THE SUM VP+VM WHERE VP AND VM ARE COMPLEX.

```

VP = CMPLX(1,0,0,0)
VM = CMPLX(1,0,0,0)
DO M = 1,N
  VP2 = AP(M)*VP + BP(M)*VM
  VM2 = CP(M)*VP + DP(M)*VM
  VP = VP2
  VM = VM2
  V1(M) = VP + VM
  VP2 = AT(M)*VP + BT(M)*VM
  VM2 = CT(M)*VP + DT(M)*VM
  VP = VP2
  VM = VM2
  VP2 = APT(M)*VP + BPT(M)*VM
  VM2 = CPT(M)*VP + DPT(M)*VM
  VP = VP2
  VM = VM2
  VP2 = AL(M)*VP + BL(M)*VM
  VM2 = CL(M)*VP + DL(M)*VM
  VP = VP2
  VM = VM2
  VP2 = AAP(M)*VP + BBP(M)*VM
  VM2 = CCP(M)*VP + DDP(M)*VM
  VP = VP2
  VM = VM2
  V2(M) = VP + VM
  IF (M .LT. N) THEN
    VP2 = AC(M)*VP + BC(M)*VM
    VM2 = CC(M)*VP + DC(M)*VM
    VP = VP2
    VM = VM2
  END IF
END DO

```

C
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C

THE COMPUTATION SECTION ENDS HERE AND THE ERROR FUNCTION IS TERMINATED BELOW

C
C

```

      ZBAR(FC) = (VP+VM)/(VP-VM)
      END DO
      ZIN = ZBAR(3)
      ERR = 0.0
      GAG = CONJG(ZBAR(3))
      GAG = (GAG - 1.0)/(GAG + 1.0)
      DO FC = 1,5
        GAMA(FC) = (ZBAR(FC)-1.0)/(ZBAR(FC)+1.0)
        RECO(FC) = CABS((ZBAR(FC) - 1.0)/(ZBAR(FC) + 1.0))
        CALC = (1.0,0.0)
        CALC = CALC - GAG*GAMA(FC)
        CAL = (CABS(CALC))**2.
        CAL = CAL/(1.0-RECO(FC)**2.)
        CAL = CAL/(1.0-CABS(GAG)**2.)
        CAL = 10.0*ALOG10(CAL)
        ERR = ERR + CAL
        IF (WCO .EQ. 2) THEN
          PRINT *, 'CONJ MISMATCH LOSS',FC,CAL
        END IF
      END DO

```

C

```

      IF (ERR .GT. EMIN) THEN
        IF (ND .NE. 1) THEN
          IF (NC .LE. N) THEN
            LRE(NC)=LRE(NC)/(1.+0.003*(WCO-2.))
          ELSE
            LCE(NC-N)=LCE(NC-N)/(1.+0.003*(WCO-2.))
          END IF
        END IF
        IF (NC .LE. N) THEN
          IF (SET .EQ. 1) THEN
            W(NC) = W(NC) - (WCO-2.)*DEW
          END IF
        ELSE
          IF (SET2 .EQ. 1) THEN
            WCN(NC-N)=WCN(NC-N)-(WCO-2.)*DEW/2.
          END IF
        END IF
        ELSE
          EMIN = ERR
          PRINT *, 'EMIN =', EMIN
          ZINF = ZIN*50.
        END IF
      END DO

```

```

      PRINT *, 'FINISHED CHANGING RESONATOR ', NC
    END DO
    PRINT *, 'FINISHED OUTSIDE LOOP ', LCO

```

```

  END DO
  PRINT *, 'ERROR = ', EMIN
  PRINT *, 'OPTIMUM RESONATOR LENGHTS'
  PRINT *, 'LRE(1)', LRE(1)
  PRINT *, 'LRE(2)', LRE(2)
  PRINT *, 'LRE(3)', LRE(3)
  PRINT *, 'LRE(4)', LRE(4)
  PRINT *, 'LCE(1)', LCE(1)
  PRINT *, 'LCE(2)', LCE(2)
  PRINT *, 'LCE(3)', LCE(3)
  PRINT *, 'OPTIMUM RESONATOR WIDTHS'
  PRINT *, 'W(1) = ', W(1)
  PRINT *, 'WCN(1) = ', WCN(1)
  PRINT *, 'W(2) = ', W(2)

```


END

18/02-91

FIN