DEMAND FOR CANADIAN BANK NOTES BY DENOMINATION:
A COMBINED SHARE/REGRESSION APPROACH

by

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the Faculty of Graduate Studies and Research
in partial fulfilment of
the requirements for the degree of

Master of Management Studies

School of Business

Carleton University

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Acceptance Sheet

The undersigned hereby recommends to the Faculty of Graduate
Studies and Research acceptance of the thesis.

DEMAND FOR CANADIAN BANK NOTES BY DENOMINATION:
A COMBINED SHARE/REGRESSION APPROACH

submitted by

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in partial fulfilment of the requirements for
the degree of master of Management Studies

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Abstract

The purpose of this thesis is to develop a model to estimate the demand for bank notes by denomination by using behavioural variables that capture consumers' preference for certain denominations, combined with variables prescribed by classical macroeconomics.

The combined share/regression model (combined model) created in this thesis does explain the demand for bank notes by denomination, and provides a viable approach to model building. However, the behavioural variables available for use in this thesis did not make a significant contribution to the explanatory power of the combined model. The combined model's ability to explain consumer's preference for certain denominations would be improved if more behavioural data on bank note usage were available. It is recommended that the Bank of Canada (the Bank) conduct periodic surveys to collect data on consumer's attitudes towards bank note usage, and that the Bank also gather data on emerging alternative means of payment.

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1. Introduction

The Bank Act of Canada prescribes four major functions for the Bank of Canada (the Bank), namely: monetary policy, banking services, management of Government of Canada debt, and the issuing of bank notes. The latter function involves satisfying the public's demand for additional bank notes and replacing worn notes withdrawn from circulation. The cost of issuing notes represents forty-three percent of the total operating costs of the Bank of Canada, of which eighteen percent of the cost is related to bank note production¹. Therefore, the Bank of Canada has a continuing interest in managing the note issue function in order to control its expenditures.

A key planning activity associated with the management of the note issue function is forecasting the total demand² for bank notes and the mix of note denominations. From a management perspective, bank note denominations represent the different products that a central bank must manufacture to meet the public's demand. The activities surrounding the manufacturing of bank note denominations are similar to those of a classical production/sales problem in terms of production, storage, and transportation. However, since the Bank of Canada is the sole provider of bank notes, and since bank notes cannot be considered normal commodities, the standard approach to production and operations management must be modified to estimate the demand for bank notes by denomination.

¹ 1995 Annual Report of the Governor of the Bank of Canada to the Minister of Finance.

² The demand for bank notes is defined as the number of bank notes in circulation. Demand for bank notes and bank note circulation are used interchangeably.

The purpose of this thesis is to develop a framework for estimating the demand for Canadian bank notes by denomination. Once the demand for bank notes by denomination can be estimated, several management issues related to bank note manufacturing can be pursued. In particular, in the short term, knowing the demand for the denominational mix of bank notes facilitates the planning of production and transportation of bank notes as well as the scheduling of personnel for processing. In the medium term, management may want to monitor new note production and inventories. Finally, in the long term, forecasting of bank note demand by denomination will aid research into the introduction of new denominations, the acquisition of new technologies, and changes in payment habits on bank note demand.

The thesis is organized as follows. A statement of the research problem is presented in Section 2, followed by an analysis of the literature in Section 3. The formalized research question is then presented in Section 4, and the proposed methodology, data analysis, findings and conclusions are described in Sections 5 through 7.

2. Problem Statement

The Bank of Canada has to actively manage the bank note issue function. Thus a continuing improvement in its knowledge and understanding of the demand for bank notes, in total and by denomination, is important to the Bank. The Bank of Canada model for estimating bank note flows using annual data was originally formulated by Bilkes (1981) and revised by Cloutier (1985) in order to incorporate data by denomination and by Bank of Canada Agency location. It is the opinion of some of the Bank's management personnel that this model is not fully responsive to changes in the demand environment, and that it does not provide enough information to guide the decision of when to change the denominational mix.

The main objective of this thesis is to determine whether or not a model can be developed to estimate the demand for bank notes by denomination by using variables that would capture consumers' preference for certain denominations in addition to the variables prescribed by classical macroeconomics. The Bank of Canada will benefit from this thesis since it will lead to a better understanding of the role played by each denomination in circulation. This thesis requires a critical examination of the literature on bank note demand and denomination mix to determine whether new developments might offer a methodology for developing such a model. The results of this thesis may be used in conjunction with already existing models to better understand the role played by different denominations in circulation, and ultimately to provide demand forecasts.

3. Literature Review

The need for accurate predictions of the volume of bank notes required to meet the public's demand has given rise to numerous empirical studies over the past two decades. The literature on forecasting the demand for bank notes is examined primarily to determine whether its application to individual denominations may provide an approach to estimating the share of each bank note in circulation. Bank notes in circulation refers to the number of bank notes held by the general public or by financial institutions. Furthermore, once this share is derived, a method for estimating the total demand for bank notes is required in order to obtain denominational totals.

Most of the studies demonstrate that the total demand for bank notes has been viewed as part of the demand for money where the explanatory variables are derived from monetary theory. The approaches to deriving the demand for bank notes via monetary theory are investigated and categorized as monetary theoretic approaches. It will be shown that the monetary theoretic approaches are most frequently implemented using some form of regression model. Also presented are models based on principles other than monetary theory, which are classified as non-monetary theoretic approaches. These approaches are subdivided into time series and share models. The work on share models includes methods based on bank note characteristics, note-coin boundaries, and efficient payment.

3.1 Monetary Theoretic Approaches

Currency demand models are often developed to mirror the transaction and hoarding motives for holding money. As a result, the demand for bank notes is often assumed to be dependent on the level of nominal income and interest rates. Modelled as linear or logarithmic linear equations, the variables included in these currency demand models are selected using the concept of minimizing the transaction and opportunity costs of holding money (Boeschoten, 1991; Boeschoten and Fase, 1992; Fase, 1981; Fase and Van Nieuwkerk, 1977; Browne, 1981; Viren, 1992; Laflèche, 1994). The basic regression approach is first investigated in which the only explanatory variables are either gross national product (GNP) or consumption, and nominal interest rates. Extensions of the latter were created in an attempt to capture the effects of financial innovations and high inflation.

3.1.1 Basic Approach

In the currency demand equations, which are derived by regression, the two most common explanatory variables are GNP or as an alternative to GNP, consumption expenditure, together with short run interest rate. GNP or consumption expenditure is used to proxy the volume of cash transactions (Boeschoten, 1991; Fase, 1981) whereas the interest rate variable attempts to capture the cost of holding money (Boeschoten and Fase, 1992, Laflèche, 1994). A system of note-demand equations are used to determine the demand for each denomination and are mainly related to one another through a possible covariance of disturbances (Manski

and Goldin, 1982).

Among central banks, De Nederlandsche Bank has carried out extensive research on the demand for bank notes. Boeschoten (1991) completed a multi country analysis on the developments in bank note circulation forecasting and new payment systems. According to Boeschoten (1991), sixty percent of the central banks surveyed use some more or less advanced econometric forecasting model. Boeschoten (1991) combined the data of fifteen countries in order to construct a semi-loglinear relationship among several variables³ and concluded that one of the most important economic factors that affect the total demand for the value of notes and coins in circulation is consumption. Furthermore, Boeschoten (1991) briefly examines the denominational structure. Although he concludes that most of the countries he surveyed follow the assumption that the denominations should approach as much as possible a geometric distribution, with equal multiplicative distances, he does not suggest a method for determining the most efficient denominational mix.

In an earlier study, Fase (1981) presented examples of forecasting models studied at the De Nederlandsche Bank. Specifically, Fase (1981) analyzed two categories of forecasting methods: causal or regression models and ARIMA models. Using the regression approach, Fase postulated a model where the circulation for a particular denomination mainly depends

³ Boeschoten (1991) assumed that the value of currency in circulation outside banks is a function of: nominal consumption, the average of short and long term interest rates, the rate of inflation, total of direct and indirect taxes divided by GNP to proxy the taxation rate, the number of automatic teller machines and cash dispensers per capita, the number of point-of-sale terminals per capita, and the share of the value of credit card transactions in nominal consumption.

on nominal national income as a proxy for the volume of transactions. This was also hypothesized by Fase and Van Nieuwkerk (1977). The time series approach proposed by Fase (1981) will be investigated later.

In a more recent Canadian study, Laflèche (1994) assumed that the growth in the value of bank notes held by the public is explained by the growth of real income, prices, interest rates, and the number of automatic teller machines (ATMs). Laflèche (1994) found a positive relationship between the growth of currency and the growth in disposable income and prices and a negative relationship between interest rates and currency demand. Furthermore, Laflèche (1994) concluded that ATMs have a negative effect on the demand for bank notes because although ATMs increase the accessibility of cash for individuals, there is less of a need to hold large amounts of cash. This ATM variable had the least effect on Laflèche's currency-demand equation because when she omitted the ATM variable, the results of the analysis did not change.

Although the basic approach allows one to predict the total demand for bank notes, it somewhat underestimates the demand due to a lack of other explanatory variables. These variables, such as ATMs and credit cards, could improve the specification of the currency demand model as opposed to solely relying on GNP or consumption. Furthermore, there is a disagreement as to whether or not consumption is more representative than GNP as a proxy for the volume of cash transactions (Fase, 1981; Bilkes, 1981, Laflèche 1994). Using consumption as a proxy is justified by considering that currency is demanded by consumers

to carry out transactions on goods and services whereas in the case of using GNP, it is not reasonable to assume that more currency will be required for increases in inventories of goods or investments in capital goods (Bilkes, 1981, Laflèche, 1994). Laflèche (1994) found that personal disposable income provided better results than gross domestic product (GDP) because the former represents the income actually available to individuals for spending. While the basic approach is useful only in those countries with stable inflation, the other extreme, that is, countries with high inflation, may be overlooked.

3.1.2 Extension to Capture The Effects of Financial Innovations

Financial innovations such as credit, debit, and smart cards have the effect of reducing the amount of currency (bank notes and coins) which is required in purchase transactions. Therefore, financial innovations should be factored into currency demand models. Unfortunately, this has been somewhat difficult to model due to the problems associated with measuring, for example, the number of transactions made with such cards. Data on these transactions are not always available. According to Viren (1992), proxies in the form of linear time trends are often used to capture the effects of credit card transactions. As Viren (1992) notes, this is a very crude procedure and does not allow for any quantitative assessments of the impact of the financial innovations.

Gomme (1985) analyzed the implications of credit cards on the demand for Canadian currency by adding a credit card variable to an already existing total currency demand model.

Essentially, the following variables were used to proxy credit card transactions: 1) the dollar value of the retail volume of bank credit card transactions, 2) the dollar value of the gross value of transactions and, 3) the outstanding balance on all bank credit card accounts. Gomme (1985) concluded that the coefficient on the bank credit card variable was insignificant in all regressions suggesting that the variable adds very little explanation to movements in the currency demand.

In a more recent study, Viren (1992) was able to obtain data on credit card transactions in Finland during the last two decades and to create a total currency demand model. In addition to evaluating the effects of financial innovations, Viren (1992) also investigated the impact of ATMs on the total demand for currency. Viren (1992) concluded that financial innovations do indeed have a negative effect on the demand for currency whereas ATMs have a positive effect. Although the impact of ATMs may be felt differently by the various denominations, no attempt has been made so far to quantify the effects of financial innovations on the demand for bank notes by denomination.

3.1.3 Extension to Capture the Effects of High Inflation

Manski and Goldin (1982) state that the basic monetary theoretic approach is not satisfactory in a high inflation context. A country with very high inflation would have a list of bank note denominations which is frequently changing because of eroding bank note purchasing powers. Manski and Goldin (1982) state that the validity of a denomination-specific demand equation

estimated for time series relies on the stability over time of the properties of the denomination and that such a system of denomination-specific equations provides little guidance in forecasting note demand when a new denomination is introduced.

Manski and Goldin (1982) hypothesized that the total demand for currency is determined by the number of cash transactions (GNP was used as a proxy), that payments are made efficiently, and the velocity⁴ of circulation of a denomination depends on the opportunity cost of holding it. By means of a regression model, Manski and Goldin (1982) were able to represent the total demand for currency and concluded that the real note value, the position in the currency list, and the opportunity cost of holding a note explain most of the substantial observed variation in note demand between denominations and over time. In Canada, prices have not increased as drastically as in Israel (the country of study in the Manski and Goldin paper) as shown by the relatively stable denomination list. Therefore, this model would not be appropriate in the Canadian context.

The monetary theoretic approach and its extensions appears to be more concerned with estimating the total demand for bank notes than the denominational mix. In those studies which made provisions for estimating the demand by denomination, the demand for the "last" denomination was obtained by subtraction in order for the individual denominational demands to sum to the total demand. These drawbacks suggest that the current monetary theoretic

⁴ Velocity of money: represents the average amount of "work" done by a unit of money; the number of times an average dollar changes hand per unit of time or the ratio of nominal national income to the quantity of money (Chiang, 1974).

approach may be a limited estimating device for deriving the demand for bank notes by denomination, and thus strengthen the motivation for finding a better denomination mix estimation method.

3.2 Non-Monetary Theoretic Approaches

The main distinction between the monetary-theoretic and the non-monetary theoretic approach is that the former is a causal approach where the total demand for bank notes is explained by certain economic related variables. Alternatively, the non-monetary theoretic approaches presented below concentrate primarily on estimating the denominational mix by methods which do not rely on economic variables but rather on denomination specific variables such as the "distance" between denominations and the average life of a bank note.

This section is divided into two sub-sections, beginning with an examination of a time series approach to forecasting the demand by denomination, and ending with analysis of methods for estimating the share by denomination.

3.2.1 Autoregressive Integrated Moving Average

Instead of forecasting with a standard regression or causal model, an alternative is to use an Autoregressive Integrated Moving Average (ARIMA) approach. Essentially, the ARIMA approach relies on the past behaviour of the variable being modelled/forecasted instead of (as

in regression) on the use of explanatory variables suggested by economic theory such as GNP or consumption. Thus it can be regarded as a sophisticated extrapolation mechanism (Kennedy, 1992).

Fase (1981) employed an ARIMA model to forecast the demand for each denomination on a weekly and monthly basis since data on related economic variables (i.e. national income) was not available in the country under study. According to Fase, forecasts of seasonal fluctuations are reasonably accurate and knowing the time profile of the future bank note circulation tells management what may happen if no action is taken.

Many econometricians acknowledge that ARIMA models are efficient summaries of the time dependencies in the data, and are useful benchmarks for forecasting, but they do not consider them to be powerful explanatory tools because they must assume that no change occurs in the fundamental causes underlying the regular patterns. Therefore, econometricians view ARIMA models as complements to traditional structural modelling (Kennedy, 1992), rather than full alternatives.

3.2.2 Share Models

This section presents three alternative perspectives to determining the most efficient denominational mix that a central bank should produce to meet the public's demand. Specifically, these are methods based on bank note characteristics, note-coin boundaries, and

the theory of efficient payment.

The characteristics approach was adapted by Kohli (1988) for the Swiss National Bank, based on an earlier study of market shares in the context of competing ski resorts (Morey, 1981). According to Kohli (1988), bank notes of different denominations serve different purposes and are perceived differently by the public, that is, they have different characteristics. Small valued denominations tend to be more liquid whereas large valued denominations are often used for large payments and as a store of value. Therefore, it would not be accurate to state that different denominations are perfect substitutes for one another.

Kohli (1988) states that individuals have definite ideas about the denominations they wish to hold and that the observed currency mix is determined by their views towards the characteristics of the different bank notes. The characteristics may be, for example, the denomination itself, the physical size of the bank note, the associated risk in terms of counterfeiting, and/or the distribution of the various denominations. Thus Kohli (1988) asserts that the share of each denomination is explained by the characteristics of that particular bank note.

Kohli (1988) claims that the share model he developed allows him to predict the demand for any particular denomination, to assess the role of each characteristic, and to predict the demand for a bank note that does not yet exist. Although all of his share equations are identical for each denomination in terms of structure and coefficient values, the shares of each

denomination will differ as a result of differences in the values of the characteristics. Kohli's (1988) analysis indicated that the public tends to favour bank notes of large physical size, that increases in interest rates and in velocity increase the shares of small denominations, and that increases in the price level and in real income favour large denominations.

Note-coin boundary methods provide another tool of currency planning. The primary purpose of these methods is to suggest when a bank note should be replaced by coin. With sustained inflation, as lower denomination notes experience decreasing purchasing powers they begin to be used more frequently in market transactions and less frequently as a store of value. Therefore, the note remains in circulation longer than it should from the point of view of its physical quality. In such cases, these low denomination notes are essentially treated as coins by the public. The Canadian \$1 note is a good example of that phenomenon. The \$1 notes were remaining relatively longer in circulation than the other denominations and when deposited by financial institutions, the \$1 notes were in such poor condition that the majority of them were destroyed. In 1989, the Bank of Canada replaced the \$1 note with a coin.

A note-coin boundary method developed by Koeze (1988) suggests that as the ratio of the average life of a note to the mean time between issue and return approaches unity, a bank note should be replaced by a coin. When the average life is shorter than the mean time between issue and return, bank notes become worn before the central bank has had the opportunity to withdraw and replace them with new bank notes. Furthermore, Koeze (1988) suggests that converting notes to coins should be made with lags between each conversion

and public opinion should also be taken into account.

According to Kingston (1989), Koeze's (1988) technique cannot be applied blindly to Canadian data. This technique suggests that even the \$1,000 note could be replaced by a coin because this denomination remains in circulation for longer periods of time than the other denominations. This kind of method does not explain the demand for bank notes in total or by denomination.

Payne (1982) developed a "D-Metric" heuristic which allows one to determine, in particular, the volumes required for each denomination and the volumes of new denominations. Payne's D-Metric approach assumes that if one has knowledge of the average day's pay (D), and nothing else, it is possible to specify the appropriate note and coin denominational structure for any currency. As the average day's pay increases through time with inflation, the purchasing power of each denomination steadily declines. Bennett and Bilkes (1991) applied Payne's concept to Canadian data and found that with an average take-home pay of about \$100 in 1990, the lowest bank note denomination should have been \$5 and the highest coin denomination should be \$2. In 1996 this became the case in Canada.

Although the D-Metric method appears to eliminate much of the guesswork of currency management in determining when to change the denomination structure, most currency systems have not readily been adjusted to the structures suggested by the D-Metric heuristic. This method does not seem to consider the preferences of the public on the denomination

structure.

Finally, the theory of efficient payment methods assumes that payments by consumers are made with the minimum number of coins and bank notes for any amount paid. The model proposed by Cramer (1982) attempts to explain the extent to which the issuing authorities of the Dutch currency system succeeded in meeting the need for bank notes efficiently by the denomination mix. Cramer assumed that the demands of the public reflect the needs for efficient payment. Multiple solutions of efficient combinations can be obtained as the manner of payment changes. Specifically, Boeschoten and Fase (1989) analyzed the effects on the share of bank notes and coins in circulation by altering the manner of payment in three ways: 1) efficient payment without change, 2) payment with the next highest denomination, change being returned in an efficient way, and 3) a combination of the latter two in which for each amount the minimum number of notes and coins is taken. Boeschoten and Fase (1989) concluded that the shares of the denominations of notes and coins differed significantly depending on the manner of payment. Furthermore, they found that the demand for currency (notes and coins) does not necessarily decline with a more efficient denominational mix. Although theoretically, the notion of efficient payment is desirable, it does not necessarily reflect how the public actually makes a payment with currency.

As demonstrated in the literature review, there has been extensive research on explaining the total demand for bank notes. The literature also suggests that macroeconomic indicators such as GNP or consumption are mainly used to estimate the total demand for currency. Although

these monetary-theoretic models could be used to estimate the demand for bank notes by denomination, there is evidence that other factors, namely behavioural factors, might impact consumer's propensity to use certain bank note denominations. The monetary theoretic models discussed in the literature review do not capture the behavioural aspect of bank note demand. Furthermore, the literature review suggests that there is currently no methodology developed for combining both economic related factors with behavioural factors for estimating the demand for bank notes by denomination. It will be shown that Kohli's (1988) share model offers an opportunity for expanding his methodology for combining both economic and behavioural factors.

4. Research Question

The research question studied in this thesis is how to estimate the demand for bank notes by denomination taking into account both behavioural and macroeconomic factors.

5. Combined Share/Regression Model

The combined share/regression model (combined model) developed in this thesis offers an alternative to the purely monetary theoretic approach to estimating the demand for bank notes and focuses on estimating the demand for bank notes by denomination. The combined model is comprised of two components. The share component of the combined model was influenced by the methodology adopted by Kohli (1988), which is derived from marketing research and explicitly models the public's preference for specific bank note characteristics. The regression component of the combined model is derived from econometric models and accommodates the effect of certain important monetary theoretic variables similar to those described in the literature review.

This section begins with background information on Kohli's (1988) model, followed by a theoretical description of the combined model. Finally, the variables used in the combined model are described, and estimation issues related to the combined model are discussed.

5.1 Background Information

Kohli (1988) states that given the total demand for bank notes, the share of a denomination in circulation can be determined by the characteristics of that particular denomination. To Kohli (1988) a bank note characteristic could be the size of the bank note, the colour of the bank note, or some other feature that might distinguish one denomination of a bank note from

another. Kohli's (1988) model must, of course, satisfy the natural constraint that the sum of the demand for bank notes by denomination must equal the total demand for bank notes. The utility approach Kohli (1988) used can be represented as:

$$\max_{M_1, \dots, M_{J+1}} [U(M_1, \dots, M_{J+1}, a_{11}, \dots, a_{1J+1}, \dots, a_{K1}, \dots, a_{KJ+1})]$$

subject to $\sum_i M_i = M$.

where M is the total number of bank notes in circulation, M_i is the number of bank notes of denomination i in circulation, and a_{ik} is the value of characteristic k associated with bank note of denomination i , $i = 1, \dots, J+1$. Note that the number of denominations is denoted $J+1$ for future notational convenience.

To approximate the utility function, the constant elasticity of substitution (CES) functional form was used (Kennedy, 1992), namely $U(*) = \sum_i M_i^{(\sigma-1)/\sigma} h(a_{1i}, \dots, a_{Ki})$, where $h(*)$ is some function of the K characteristics, and σ is a positive constant. A standard Lagrangian optimization of this utility function yields the following expression for the bank note shares:

$$s_i = M_i/M = h(i)^\sigma / \sum_j h(j)^\sigma,$$

where s_i is the share of bank note i in circulation, and $h(i) = h(a_{1i}, \dots, a_{Ki})$. Kohli (1988) approximated $h(i)$ by means of a loglinear form:

$$\ln h(i) = \alpha_0 + \sum_k \alpha_k \ln a_{ki},$$

where α_0 is a common intercept and α_k is the coefficient of characteristic k .

If important characteristics were missing and were considered uncorrelated with the included ones, Kohli (1988) noted that their effect could be partially accounted for by means of a share specific constant, i.e., by writing the loglinear approximation of $h(i)$ with denomination specific intercept in the form $\ln h(i) = \alpha_0 + \sum_k \alpha_k \ln a_{ik}$.

Substitution of the common intercept form of $h(i)$ into the share form yields:

$$\begin{aligned} s_i &= \exp(\sigma\alpha_0 + \sigma\sum_k \alpha_k \ln a_{ik}) / [\sum_j \exp(\sigma\alpha_0 + \sigma\sum_k \alpha_k \ln a_{ij})] \\ &= \exp(\beta_0 + \sum_k \beta_k \ln a_{ik}) / [\sum_j \exp(\beta_0 + \sum_k \beta_k \ln a_{ij})] \end{aligned} \quad [1]$$

where s_i is the share of denomination i in circulation to the total volume of notes circulation,

$\beta_k = \sigma\alpha_k$ ($k = 0, \dots, K$), and a_{ik} is the value of characteristic k associated with bank note of denomination i , $i = 1, \dots, I+1$.

As noted by Kohli (1988), all the share equations [1] are identical in form, and involve identical parameters. The differences in the shares of the $I+1$ denominations are due only to differences in the values of the characteristics in the numerator. The individual demands for the bank notes of denomination i are allowed to differ purely through the different values of their characteristics. Thus, Kohli (1988) described his model as completely characteristic driven.

Kohli's (1988) choice of characteristics was somewhat arbitrary. Basically, he began by considering a large number of characteristics and then determined those that significantly contributed to explaining demand. He estimated the values of the parameters of his non-linear

model using maximum likelihood, and tested specific model hypotheses using the large sample form of the likelihood ratio test. As a result, he found six characteristics to be significant:

- 1) the distribution of the various denominations, i.e. the spread between denominations defined as $a_{it} = (L_i^2 + U_i^2)$ where $L_i = d_{i,t}/d_i$ and $U_i = d_{i,t+1}/d_i$, d_i being the face value of the i th denomination;
- 2) the physical size of each bank note;
- 3) the purchasing power of each bank note, i.e., price level weighted by the face value of the denomination;
- 4) the relative position of each denomination, i.e., income per capita weighted by the face value of the denomination;
- 5) the variations in the storage demand for bank notes, i.e., velocity of money weighted by the face value of the denomination;
- 6) the opportunity cost of holding money, i.e., interest rate weighted by the face value of the denomination.

The nature of Kohli's (1988) model allows for differences in denominational share only through differences in the "characteristics" of each denomination. The first two characteristics are distinct from the last four in that they are "pure" characteristics of a bank note denomination. Specifically, the value of the characteristic is different for each denomination. The last four are attempts by Kohli (1988) to create "share" characteristics from economic variables by multiplying by the natural logarithm of the face value of the denomination in question.

Kohli's (1988) system of share equations automatically satisfies the natural constraint that the sum of the demand for bank notes by denomination must equal the total demand for bank notes. Thus the system can be solved by omitting one equation, and evaluating its share later as a residual. However, one could instead take the relative shares (s/s_{i+1}) of the bank notes to a reference note which can be taken as the $(I+1)$ th note, without any loss of generality. On taking logarithms of the relative shares s/s_{i+1} , $i = 1, \dots, I+1$, the system [1] of I -relative share equations can be written as:

$$\ln (s/s_{i+1}) = \sum_k \beta_k \ln (u_k / u_{k+1}), i = 1, \dots, I+1. \quad [2]$$

Unlike Kohli's (1988) non-linear specification, the set of equations represented by [2] is linear in the parameters $\beta_k = \sigma \alpha_k$. The dependent variables represent the relative share of each denomination in circulation, and the independent variables are the relative values of the individual characteristic, measured relative to a reference bank note. Use of the denomination

specific intercept version of $h(i)$ merely adds an intercept to each of the I -relative shares given in equation [2].

Under the relative share model [2], it can be seen that Kohli's (1988) synthetic share characteristics, consisting of macroeconomic indicators weighted by denomination, namely GDP price deflator, GDP per capita, velocity, and interest rate, are ineffective since their economic content cancels out. For example, if interest rate (weighted by denomination) is used as a characteristic in equation [2], the following equation would result:

$$\begin{aligned} \ln(s/s_{t-1}) &= \sigma[\alpha_1 \ln(\text{interest rate} * \text{denomination } i / \text{interest rate} * \text{denomination } I+I) + \alpha_2 \dots] \\ &= \sigma[\alpha_1 \ln(\text{denomination } i / \text{denomination } I+I) + \alpha_2 \dots] \end{aligned} \quad [3]$$

In equation [3], the actual interest rates cancel leaving only the relative share of denomination i as an explanatory variable. Thus in Kohli's (1988) case, the characteristics a_{3i} , a_{4i} , a_{5i} , and a_{6i} all reduce to the same ratio of denomination values. Though this proof of the weakness of Kohli's (1988) chosen variable set has been based on the relative share formulation of the model, it applies in full to Kohli's (1988) representation since the relative share formulation (together with $\sum s_i = 1$) is one-to-one equivalent to Kohli's (1988) form.

One can conclude that since the coefficients of individual characteristics are the same between denominations under a relative share formulation, characteristics which are truly specific to the denomination are required in order for a share model of Kohli's (1988) type to be valid.

If variables, such as GDP or consumption, which do not differ between denomination are to be used, then it becomes necessary to move away from the “share model” formulation by relaxing the constraint that coefficients of the same characteristics are constant across denominations. Thus, Kohli’s (1988) share model should be corrected and expanded to include denomination specific characteristics and variables prescribed from macroeconomic theory.

5.2 Combined Share/Regression Model Formulation

By expanding Kohli’s (1988) original concept, a new model was developed, the combined share/regression model. The combined share/regression approach, which incorporates features of both a pure share model and a regression type econometric model, will be used to see whether or not an improved methodology can be developed to estimate the relative share of a bank note denomination in circulation. It can be argued that the relative shares of different bank notes are likely to be differentially affected by variables such as GDP or consumption, so it is unlikely that a pure share model will be able to explain all the observed variation in denominational mix over time.

The combined model is designed to capture the impact of consumers’ behaviour as well as the influence of basic macroeconomic indicators on the demand for bank notes. It is comprised of two components, a share component which incorporates those characteristics which are denomination specific, and a regression component that builds in the

macroeconomic variables. The share component includes characteristics that differ by denomination and whose impact on each denomination is entirely due to differences in the value of the characteristic. An example of a share characteristic would be the influence of ATMs on the use of bank notes. Since ATMs are only stocked with \$5, \$10, and \$20 in Canada, then one would expect that these machines would have influenced the demand for these bank notes over time. Although the information expressed in the former example would be useful for determining the influence of ATMs on the demand for bank notes, currently there is no information on the number of bank notes issued for each denomination from banking machines. As noted earlier, coefficients in the share part of the model are the same across denominations.

The regression component of the combined model contains variables whose values are the same regardless of the denomination but whose impact is different across denomination. An example of such a variable is the influence of the performance of the Canadian economy on the demand for bank notes. One would expect that as the economy improves, Canadians are willing to spend more thus increasing the demand for bank notes. It is natural to suppose that this effect will not be uniform across all denominations, so that in the regression part of the combined model, regression coefficients will differ by denomination. For the purposes of clarity in describing the combined model, the share component has "characteristics" whereas the regression component has "variables".

The combined model corrects and moves beyond Kohli's (1988) proposal and properly captures the effect of economic variables. Furthermore, the combined model accounts for the restriction that all shares sum to one. It has the following formulation:

$$\ln y_i = (s_i/s_{i+1}) = \beta_{0i}^* + \sum [\beta_k^* (a_{ki} / a_{ki+1})] + \sum \delta_j x_j + \gamma_j d_j + \epsilon \quad [4]$$

where y_i = the relative share of the volume of denomination i in circulation to the volume of denomination $i+1$ in circulation

β_{0i}^* = intercept for denomination i , $i = 1, \dots, J+1$, given by $\beta_{0i} - \beta_{0i+1}$

a_{ki} = value of characteristic k for denomination i , $i = 1, \dots, J+1$

β_k^* = characteristic coefficient for denomination i , $i = 1, \dots, J+1$, given by $\beta_k - \beta_{k+1}$

x_j = value of variable j , $j = 1, \dots, J$, (J being the number of regression variables)

δ_j = coefficient of the variable for x_j for denomination i , $i = 1, \dots, J+1$.

The regression component of the combined model may also contain dummy variables, d_j (with coefficient δ_j) to indicate presence or absence of certain factors. The combined model incorporates denomination specific intercepts, since it is unlikely that all explanatory characteristics and variables will be identified and included in the combined model. Note that if only a single (common) intercept term is included in the combined model for the shares of each denomination, then this intercept will cancel out of the relative share form given in equation [4]. Denomination specific intercepts, however, will remain as in [4].

5.3 Definition of Characteristics and Variables Used in the Combined Share/Regression Model

This section documents the data used in the combined model and provides a detailed description of the characteristics and variables used in developing a preliminary demand model for bank notes by denomination.

5.3.1 Data Documentation

The time period for the data analysis is from 1960 to 1995 in which annual data was used for the dependent and independent variables. The dependent variables in system [4] are defined as the logged relative shares, $\ln(s_i/s_{I+1})$, where s_i is the share of the volume of denomination i in circulation and s_{I+1} is the share of the volume of denomination $I+1$ in circulation. Denomination $I+1$ is the reference denomination and is represented by the \$5 bank note in the combined model. The independent variables are described in detail below. The sources and raw data of the dependent variables, the share characteristics and the regression variables used in the combined model are presented in Appendix C.

The Canadian denomination mix of bank notes in 1995 was the \$2, \$5, \$10, \$20, \$50, \$100, \$1,000. In the model, the shares of \$50 and \$100 notes in circulation were combined because these two denominations behave similarly in terms of growth. Also preliminary runs of the combined model revealed a high correlation between the \$50 and \$100 notes, which led to

unstable estimation of the regression parameters. The \$1,000 note was excluded from the analysis because it is not used for everyday transactions. The \$1,000 note is generally used for settling large-value transactions, auctions, gifts, and inheritances, and is not subject to the same influences as the smaller denominations that are the subject of this thesis.

5.3.2 Characteristics Used in the Combined Model

It is difficult to find “pure” share characteristics for the denominations currently available in Canada. As mentioned earlier, one of Kohli’s (1988) “pure” characteristics was the spread between denominations. This characteristic was not used in this thesis because the spread between denominations remains constant over time and its effect on the demand for bank notes is taken up by the combined model with denomination specific intercepts.

In an effort to obtain other pure characteristics, all major Canadian banks were approached and asked to provide specific information on bank notes and ATMs. For example, a useful piece of data would have been to know the algorithm used for the mix of denominations dispensed for a withdrawal from an ATM. The results of the inquiry were limited as the banks do not report this type of information.

The following are the characteristics used in the share component of the combined model. In order to meet the requirement of being denomination specific, the value of these characteristics all vary by denomination.

a) Change and transaction note characteristics

Bank note denominations are often viewed as serving different purposes, such as making change, a medium for transactions, and a store of value. An exploratory attempt to model these different characteristics of bank notes has been made by combining the idea of Payne's (1982) D-Metric equivalent of bank notes, the information on the average value of a credit card transaction, and the consumer price index (CPI). The D-Metric equivalents for bank notes are $D/10$ which is equivalent to the \$2, $D/5 \approx \$5$, $D/2 \approx \$10$, $D \approx \$20$, $2D \approx \$50$, $5D \approx \$100$. The D-Metric equivalent are based on the theory that the face value of bank notes follow a geometric distribution and there is a system of efficient payment.

While the original D-Metric idea was based on the "average day's pay", it is felt that the average day's pay does not reflect cash use as people are no longer paid in cash. A more meaningful competitor for cash use is the credit card. Thus, instead of using average day's pay as the quantity in the D-Metric concept, it is replaced by the average value of a credit card.

Cutoffs between the change, transaction, and value ranges indicate which denomination(s) fall within which category. The change/transaction cut-off point is assumed to be midway between $D/10$ and $D/5$, and the transaction/value cut-off point is assumed to be midway between $D/2$ and D . The choice of the cut-off points is

based on a perception of the use for these bank notes over time. For example, in 1960 it is reasonable to assume that the \$20 was used as a store of value and that the \$5 note was used for transactions. In 1995, the \$20 note is now used mainly for transactions whereas the \$5 note is used for change.

Since data on the average value of a credit card transaction is only available beginning in 1977, this is the starting point, $D = \$30.50$ (D being the benchmark value). From there, the average value of a credit card transaction is deflated by the CPI to 1960 and inflated to 1995. As the value of the D-Metric equivalent bank notes change over time, the different denominations move in and out of the different ranges (change, transaction, value). Thus, the denomination is given a value of 1 if it is either a change, transaction, or value note and a zero if it is not. Because a bank note can belong to one (and only one) of the change, transaction, or value categories at any one time, only two of these share characteristics were included in the combined model, that is, the change and transaction characteristics. Finally, it is assumed that as the real value of bank note denominations decline over time, then the demand for larger bank notes will increase.

The characteristics considered in this thesis do not comprise an exhaustive list and should be viewed as exploratory. There are other more recent financial innovations which could influence the demand for bank notes. In particular, the introduction of a debit card system in Canada in the early 1990's and the pilot project of an electronic purse by Mondex launched

by Royal Bank, CIBC, and Hong Kong Bank of Canada in September 1996. Although there is insufficient data available on debit cards, it is expected that debit cards and electronic purses will have a negative impact on the demand for bank notes. Cheques were also not included in this research because, according to the Canadian Payments Association (1994), the data on cheques includes purchases of goods and services, and payments on commercial loans. Therefore, this data would not be representative of the use of cheques as an alternative means of payment.

5.3.3 Regression Variables Used in the Combined Model

As mentioned earlier, it can be argued that the relative shares of different bank notes are likely to be affected by economic indicators and thus it is unlikely that a pure share model will be able to explain all the observed variation in denominational mix over time. Thus the following regression variables were considered for the combined model.

a) Real Consumption

The literature review indicated that most of the studies used either, GNP, GDP, or consumption to capture the impact of economic activity on the demand for bank notes; in this thesis, real consumption will be used to capture economic activity. Given the length of time period of the data it is worthwhile to segregate the real growth in the economy from changes in prices. Statistics Canada (1995) defines real

consumption as personal expenditures on consumer goods and services in 1986 dollars, that is, new goods or services acquired/consumed by households for their consumption. Real consumption is used instead of real GDP because it more closely represents cash expenditures made by consumers than does GDP. GDP includes, in addition to consumer spending, expenditures that are unlikely to be made in cash, such as business investment and spending by public administrations. Growth in the economy usually implies increases in spending on consumer goods. Furthermore, preliminary runs of the combined model revealed that real consumption did a better job at explaining the demand for bank notes than did real GDP. Thus, it is assumed that increases in real consumption lead to increases in the demand for bank notes.

b) Implicit price index (consumption)

Since the effects of real economic activity excludes the effect of prices, it is only reasonable to have a separate variable for the impact of changes in price levels on the demand for bank notes. The implicit price index⁵ for consumption with 1986=100 is used in the combined to capture the impact of changes in price levels on the demand for bank notes independent of their impact as represented by the change and transaction characteristics. It is expected that as the price level increases, the demand for bank notes will increase.

c) Short run interest rates

Short run interest rates are used to capture the opportunity cost of holding money. The short run interest rate used in the combined model is the real 90-day commercial paper rate (nominal rate minus the inflation rate). It is expected that as interest rates rise, the demand for \$50 (combined with \$100) bank notes declines since there are incentives for the consumer to hold interest-bearing assets.

d) Underground economy dummy

The underground economy includes all production both legal and illegal that is hidden from income tax. Most of these underground transactions are settled by cash because of its anonymity. Laflèche (1994) states that there is little doubt that cash transactions associated with the underground economy most likely increase the demand for currency. Thus, a variable is needed to capture the possible impact of legal activities in the underground economy on the demand for cash. This thesis does not take into account those activities in the underground economy which are considered illegal.

Spiro (1993) attempted to estimate a presumed increase in the underground economy after noting a significant increase in the ratio of the currency stock to household spending since the introduction of the Goods and Services Tax (GST) in 1991. A dummy variable has been added to the combined model to capture the impact on the

demand for \$50 bank notes of the growth in legal activities in the underground economy as a result of the GST. This dummy variable takes the value 1 after the introduction of the GST (value of 0 before). It is assumed that the GST resulted in a increase in underground economic activity and thus leads to an increase in the demand for \$50 (combined with the \$100) notes.

e) Number of ATMs installed

Technology is usually included in regression models as a linear term in time. However, financial innovations are far from being linear. The number of ATMs installed registered growth rates up to 45 per cent in 1989. Thus adding the number of ATMs installed as a variable in the combined model could capture the impact of this one type of financial innovation. Most of the currency models found in the literature review added this variable to represent the impact of financial innovations in banking. It is expected that as the number of ATMs installed increases, the demand for \$10 and \$20 notes increases.

f) Dummy variable to capture the effect of withdrawing the \$1 note from circulation

A dummy variable was added to the model to capture the effect on the \$2 note of the Bank of Canada's withdrawal of the \$1 note from circulation in 1989, the dummy is defined as $d=1$ for the \$2 denomination, and $d=0$ for all other denominations. It is

assumed that there was no effect of the withdrawal of the \$1 note on the other denominations.

g) Regression variable considered but not included in the combined model

The lagged dependent variable was not included in the combined model for two reasons. Firstly, most of the currency models in the literature search did not include a lagged dependent variable. Secondly, the share of bank notes in circulation changes very slowly over time thus including the last period's share as an explanatory variable may explain most of the demand for bank notes. It is felt that an explanation beyond using the last period's share would be more useful in explaining the demand for bank notes.

5.4 Matrix Formulation For The Combined Share/Regression Model

5.4.1 The "pure" share model

The share component of the combined model is characterized by coefficients which are equal among all denominations; it is the value of the characteristics which differ by denomination. To estimate the share coefficients of the combined model, it is easier to begin with an explanation of a share model with unequal coefficients (SUCM). The SUCM has a similar formulation to the multivariate multiple regression model with the difference that the values

of the independent variables vary by denomination. Thus, the SUCM is typical of Zellner's (1962) Seemingly Unrelated Regression (SUR) model with coefficients that are estimated by means of generalized least squares (GLS). To simplify the exposition, the explanation begins with the combined model that corresponds to the logged form of Kohli's (1988) model (equation [4]), and will initially ignore the restriction that the shares must sum to one. In other words, the share model, $z_i = \ln(s_i)$, will be described initially as if all $I+1$ shares are functionally independent. The constraint will be added later. Thus,

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{I+1} \end{bmatrix} = \begin{bmatrix} X_1 & 0 & 0 & \dots & 0 \\ 0 & X_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & X_{I+1} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{I+1} \end{bmatrix} + \epsilon, \quad [5]$$

or

$$z = X_E \beta + \epsilon, \quad [6]$$

where the individual matrices, X_i , $i=1, \dots, I+1$, in equation [5] contain T realizations of the independent variables (characteristics) corresponding to each share. The matrix X_E in equation [6] is the block diagonal matrix of dimension $((I+1)T \times (I+1)K)$ containing the values of the independent characteristics (including a single intercept), and β is a column vector of length $(I+1)K$ containing the i individual K -vectors of parameters, β_i , $i = 1, \dots, I+1$. The vector z is a column vector of length $(I+1)T$ containing all $I+1$ vectors of the realization of each logged share z , $z_i = \ln s_i$, $i=1, \dots, I+1$.

For a share model with equal coefficients (SECM), which comprises part of the combined model, the values of the independent variables are also denomination specific, but the coefficients do not vary by denomination. To achieve the SECM, the restriction that the coefficients have common (equal) values is imposed during the estimation process. Instead of externally applying the constraint that the share coefficients must be equal, it is easier to collapse the independent variable matrix (X_E) so that the block diagonal matrix in the SUCM now becomes a stacked matrix. The share model with equal coefficients (SECM) has the following formulation:

$$z = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_{I+1} \end{bmatrix} \beta_S + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_{I+1} \end{bmatrix} \quad [7]$$

$$= X_S \beta_S + \epsilon \quad [8]$$

where X_S is a stacked matrix of dimensions $(I+1)T \times K$, and β_S denotes the common (equal) value of $\beta_1 = \beta_S = \dots = \beta_{I+1} = \beta_E$. Note that in this formulation, each intercept term has the same coefficient for each denomination, so that the denomination specific intercepts of equation [4] must be considered separately. The denomination specific intercepts will in fact be included in the "regression" component of the combined model.

The GLS/SUR estimate of the coefficient β_S is given by $\beta_S = (X_S' V_S^{-1} X_S)^{-1} (X_S' V_S^{-1} z)$, where V_S is the (estimated) covariance matrix of the disturbances. Note that the constraint that the share sum to one is being ignored for now so that in this case, V_S is non-singular for the $I+1$ disturbances.

5.4.2 The multivariate regression model

The regression part of the model is typical of multivariate multiple regression models where the parameters are estimated via $I+1$ univariate multiple regressions. The regression coefficient vectors which are different for each denomination, are correlated, and tests involving different vectors must take account of these correlations.

A multivariate multiple regression model is exhibited in [9], namely

$$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{I+1} \end{bmatrix} = \begin{bmatrix} X & 0 & 0 & \dots & 0 \\ 0 & X & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & X \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{I+1} \end{bmatrix} + \epsilon_R, \quad [9]$$

or

$$z = X_R \beta_R + \epsilon_R, \quad [10]$$

where X_R is a block diagonal matrix of dimension $(I+1)T \times (I+1)K$, with each identical block containing the denomination specific intercepts (if these are to be included) and values of the independent variables. In equation [10], β_R is a column vector of length $(I+1)K$ containing the $I+1$ individual K -vectors of parameters, β_i , $i = 1, \dots, I+1$. The vector z is a column vector of length $(I+1)T$ containing all $I+1$ vectors of the realization of each logged share z , $z_i = \ln s_i$, $i = 1, \dots, I+1$.

The distributional assumptions of classical multivariate regression applied to the current problem are that:

1) The rows of the error vector ϵ_R corresponding to a given denomination are independent, i.e., no serial correlation over time $t=1, \dots, T$.

2) Elements of ϵ_R corresponding to the same time are correlated. For a given time t , the covariance matrix of the errors corresponding to the $I+1$ denominations will have a covariance matrix V_R that is constant, i.e., that does not vary over time.

Under these assumptions, it can be shown that separate ordinary least squares (OLS) estimates of the coefficient β_i , $i=1, \dots, I+1$ are efficient. However, to compare different β 's, their inter-correlations must be taken into account.

5.4.3 The Combined Share/Regression Model

The combined model is comprised of a share part with equal coefficients and a multivariate multiple regression part with varying coefficients. Again, the restriction that shares must sum to one is ignored for now. The combined model has the following form:

$$z = (X_S, X_R) \begin{bmatrix} \beta_S \\ \beta_R \end{bmatrix} + \epsilon \quad [11]$$

where the parameter matrix β_s refers to the equal coefficients of the share part and β_R corresponds to the coefficients of the regression part. As noted in Section 5.4.1, a combined model featuring denomination specific intercepts can be accommodated by including these in the “regression” part of the model.

From now on, the combined model will be written $z = X\beta + \epsilon$ where $X = (X_s, X_R)$ and $\beta = (\beta'_s, \beta'_R)$. The combined model assumes that the shares of the volume of bank notes in circulation by denomination are independent across time, which is tantamount to assuming that all important explanatory characteristics and variables have been included in the combined model. The complications involved in weakening this assumption are considerable, and beyond the scope of this thesis.

5.5 Parameter Estimation

5.5.1 Formulating the Share Constraint

In the above outline of the combined model, the restriction that the shares must sum to one was ignored. In obtaining estimators of β , this constraint must be satisfied. This will be accomplished in this thesis by measuring all denomination shares relative to a reference denomination, thus reducing the number of equations in the system from $I+1$ to I . For a given time period, this can be accomplished by pre-multiplying the $(I+1) \times 1$ vector of logged shares by a matrix $C'_{(i^*)}$, where the subscript i^* indicates the reference row:

$$C_{(I^*)} = \begin{array}{c} \left| \begin{array}{ccccc} 1 & 0 & 0 & \cdot & 0 \\ 0 & 1 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 0 \\ -1 & -1 & -1 & \cdot & -1 \\ 0 & 0 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 1 \end{array} \right| \begin{array}{l} \\ \\ \\ \\ I+1 \text{ rows} \\ \leftarrow i^* \text{ row} \\ \\ \\ \end{array} \end{array} \quad [12]$$

I columns

As noted earlier, the reference category for constructing relative shares can be taken to be category $i^* = I+1$, without loss of generality. This transformation is equivalent to subtracting term by term the $(I+1)$ th equation from each of the preceding I equations. Henceforth, the corresponding transformation matrix will be denoted C . Equation [11], is pre-multiplied by $(C' \otimes I_T)$, where C is as defined above, and where $\otimes$ denotes the Kronecker matrix product, with I_T a $T \times T$ identity matrix. This product applies the C' transformation to all shares at each of the T time periods.

For two matrices A and B of dimension $m \times n$ and $p \times q$, the Kronecker product is defined by the $mp \times nq$ matrix

$$A \otimes B = \begin{array}{c} \left| \begin{array}{ccc} a_{11}B & \dots & a_{1n}B \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_nB & \dots & a_{nn}B \end{array} \right| \end{array}$$

Then, equation [11] can then be written as:

$$\mathbf{y} = (\mathbf{C}' \otimes \mathbf{I}_T) \mathbf{z} = (\mathbf{C}' \otimes \mathbf{I}_T) (\mathbf{1}, \mathbf{X}_C) \begin{bmatrix} \beta_0 \\ \beta_C \end{bmatrix} + (\mathbf{C}' \otimes \mathbf{I}_T) \epsilon, \quad [13]$$

where the elements of $\mathbf{y}$ now represent logged relative shares, $\ln(s_{it}/s_{t+1,t})$.

The matrix $(\mathbf{1}, \mathbf{X}_C)$ represents the combined share/regression design matrix $\mathbf{X} = (\mathbf{X}_S, \mathbf{X}_R)$, and makes explicit the vector of ones corresponding to the common intercept of the “pure” share model. Similarly, the combined parameters (β_S', β_R') have been re-written (β_0', β_C') . The unity vector representing the intercept is of dimension $(I+1) \times T$ and can be written as $\mathbf{1} \equiv \mathbf{1}_{(I+1)} \otimes \mathbf{1}_T$. Using the rule of multiplying Kronecker products, $(\mathbf{C}' \otimes \mathbf{I}_T)(\mathbf{1}_{(I+1)} \otimes \mathbf{1}_T) = (\mathbf{C}' \mathbf{1}_{(I+1)}) \otimes (\mathbf{I}_{T(I+1)} \mathbf{1}_T) = 0$, since $(\mathbf{C}' \mathbf{1}_{(I+1)}) = 0$. The latter condition is evident as the elements in each row of $\mathbf{C}'$ sum to zero. Thus it can be seen that the common intercept part of the “pure” share model disappears under the relative share formulation, i.e., β_0 is not estimable. As noted earlier, if denomination specific intercepts are required, they can be entered in the regression part of the model. Equation [13] can thus be written as

$$\begin{aligned} \mathbf{y} &= (0, [\mathbf{C}' \otimes \mathbf{I}_T] \mathbf{X}_C) \begin{bmatrix} \beta_0 \\ \beta_C \end{bmatrix} + (\mathbf{C}' \otimes \mathbf{I}_T) \epsilon \\ &= (\mathbf{C}' \otimes \mathbf{I}_T) \mathbf{X}_C \beta_C + (\mathbf{C}' \otimes \mathbf{I}_T) \epsilon, \end{aligned} \quad [14]$$

where $\mathbf{X}_C$ contains share variables (characteristics), regression variables, and (possibly) denomination specific intercepts.

Note that pre-multiplying X_c by $(C' \otimes I_T)$ is the same as subtracting the row of the X_c matrix corresponding to share $I+1$ (the reference) from each of the other shares, for each time period $t = 1, \dots, T$.

5.5.2 Estimation and Inference

As noted in Section 5.4, the “pure” share model can be estimated using Zellner’s (1962) SUR methodology, which is an application of GLS, with an estimated covariance matrix. The same approach can be used for the combined model formulated for logged relative shares.

Following the GLS method, the estimate of β_c in [14] is given by

$$\beta_c = [\{ (C' \otimes I_T) X_c \}' [Var \epsilon^*]^{-1} \{ (C' \otimes I_T) X_c \}]^{-1} \cdot [\{ (C' \otimes I_T) X_c \}' [Var \epsilon^*]^{-1} \{ (C' \otimes I_T) y \}] \quad [15]$$

where $\epsilon^* = (C' \otimes I_T) \epsilon$.

Also needed is

$$\begin{aligned} Var \epsilon^* &= (C' \otimes I_T) (Var \epsilon)^{-1} (C' \otimes I_T) \\ &= (C' \otimes I_T) (\sum \otimes I_T) (C \otimes I_T) \\ &= (C' \sum C) \otimes I_T, \\ &= V \end{aligned} \quad [16]$$

where $\sum$ is the covariance matrix of the disturbances in the combined model.

In Appendix A it is shown that the corresponding values of the estimated β_c parameters are invariant to the choice of denomination used as the reference bank note.

Equations [15] and [16] provide estimates of the β 's under the implicit assumptions that Σ , the covariance matrix of the logged shares, is known. In practice, this is not the case, and Σ has to be estimated. Initially, one can assume that V , the (estimated) covariance matrix of the transformed disturbances, is proportional to the identity matrix, i.e., the same assumption used for ordinary least squares (OLS). It is well known that the first iterate on $V = \sigma^2 I$ provides a consistent estimator of β_C . However, the OLS situation yields biased estimates of the true covariance matrix, so it is customary to construct an estimate of V from the OLS residuals, and to proceed to an estimated GLS step. This one-step estimator yields consistent estimators of β_C and V , i.e., for large samples, the one-step estimator is adequate. For smaller samples, one can continue the iteration by using the residuals from the one-step estimator regression to compute a second iterate of an estimate of V . This process can be continued until the regression coefficients converge. In this thesis, the results reported are all based on the one-step estimator, obtained by a simple iteration following an initial OLS step.

To test the characteristics and variables in the combined model, a large sample t-statistic is required. This can be achieved by dividing each individual estimate of β_C (namely β_{C1}), by the square root of its corresponding estimated variances. The estimated variance of β_C is given by $\text{var}(\beta_C) = \{(C' \otimes I_T)X_C\}' [\text{Var } \epsilon^*]^{-1} \{(C' \otimes I_T)X_C\}^{-1}$. The estimated variance of the disturbances is given by $\text{Var } \epsilon^* = (C' \sum C) \otimes I_T$, where the estimate of $C' \sum C$ is obtained from the OLS residuals. The diagonals of $\text{var}(\beta_C)$ give the estimated variances of the β_{C1} 's.

6. Findings

The combined model was tested in order to assess its ability to explain the demand for bank notes by denomination based on the relative share of a specific denomination in circulation. This involved determining whether the regression component explains significant variations over and above the share component (i.e. is a pure share model tenable).

Overall, the combined model did explain the demand for bank notes by denomination. However, most of the demand by denomination is explained by the economic variables, the share component marginally added to the explanatory powers of the model.

The Figures 1 through 5 below show the estimated share of a bank note in circulation (i.e., held by the public) and the actual share of that denomination in circulation. A dotted line in the figure represents the estimated share of denomination i in circulation (s_i) as predicted by the combined model. The estimated shares are the back-transformations from the predicted logged relative shares. A solid line represents the actual share of a denomination in circulation, i.e., those notes held by the public. It is the raw data upon which the estimation of the parameters of the combined model are based.

For the \$2, \$5, \$10, \$20, and \$50 (combined with \$100) notes as shown in Figures 1 to 5 respectively, the combined model did a good job at estimating the share of these notes in circulation over the entire estimation period with minor over- and under-estimations.

The share model presented earlier did not produced a good fit with an R^2 of 31% whereas the regression model did produce a good fit with an R^2 of 95%. When the share and regression models are combined the fit was slightly improved, resulting in an R^2 of 97%. The R^2 used in this thesis is the version proposed by Willet and Singer (1988), modified for separate intercepts. It should be noted that the choice of an appropriate R^2 is not a simple topic; refer to Appendix B for the reasoning behind the choice of the modified Willet and Singer (1988) R^2 .

Table 1 below shows the parameter estimates and corresponding t-values of all the characteristics and variables included in the combined model.

Table 1
Parameter Estimates and T-Values

	Parameter values	t-values
Characteristics:		
transaction notes	0.479	1.62
change notes	-0.187	-3.00
Variables:		
Real consumption		
\$2	0.330	2.90
\$10	0.222	2.03
\$20	1.161	7.99
\$50(combined with \$100)	0.260	1.91
Implicit price index (consumption)		
\$2	-0.005	-3.06
\$10	-0.013	-10.23
\$20	-0.008	-4.82
\$50(combined with \$100)	0.012	6.40
Real 90-day commercial paper rate		
\$50(combined with \$100)	-0.005	-0.65
Underground economy dummy		
\$50(combined with \$100)	-0.117	-2.17
Number of ATMs		
\$10	0.00007	1.50
\$20	0.00004	0.85
Withdrawal of \$1 bank note dummy	\$2	0.251
Intercept		
\$2	-3.505	-2.64
\$10	-1.696	-1.34
\$20	-13.081	-7.72
\$50 (combined with \$100)	-4.764	-3.01
Willet and Singer R ² modified for separate intercepts	0.968	

As shown by the parameter values and t-statistics in Table 1, the change note characteristic indicates that as the real value of bank notes decline over time, the demand for smaller denominations decreases. The consumption variable indicates that increases in real economic growth lead consumers to increase their usage of all denominations, whereas an increase in price levels leads to an increase in the demand for higher denomination bank notes. The negative sign of the underground economy dummy parameter is contrary to the assumption made earlier that the introduction of the GST may have stimulated growth in the underground economy thus leading to an increase in the demand for \$50 (combined with \$100) notes. Since there is no exact measure of the underground economy, it is possible that the GST may have caused only a blip in the growth of underground economic activity and thus there no impact on the demand for higher denomination bank notes. The short run interest rates and the number of ATMs installed did not have a significant on the demand for bank notes. This is not surprising because very little interest is paid on demand deposits and the number of ATMs installed does not necessarily reflect their usage.

7. Conclusion

In this thesis a framework for a combined model was developed and it was demonstrated that the combined model did explain the demand for bank notes by denomination. Most of the demand was explained by economic variables and not by means of behavioural factors. Thus, a pure share model is not tenable, a conclusion that runs counter to Kohli's (1988) findings.

The combined model did a good job at estimating the share of all denominations with minor over- and under-estimations. Declines in the real value of a bank notes, measure by a fall in the share characteristics, lead to a decrease in the demand for bank notes used for making change. It is possible that the price level variable absorbed some of the explanatory power of the share characteristics thus resulting in a less significant impact on the demand for bank notes used for everyday transactions. As expected, increases in economic growth, as captured by the real consumption variable, did have a positive measurable impact on all denominations. Contrary to expected, increases in price level had a negative impact on most denominations except for the \$50 (combined with \$100) notes. It is possible that increases in the price levels does not necessarily imply growth in the demand for smaller denominations. Increases in underground activity, as represented by a dummy variable, had a negative impact on the demand for \$50 notes, contrary to what was expected. It is possible that the dummy variable does not entirely represent the impact of the GST on the growth of underground economic activity. Finally, short term interest rates did not capture the hoarding motives for holding money and the number of ATMs did not capture the effect of financial innovations

on the demand for bank notes.

In order to use the combined model to its fullest potential, that is, combining both behavioural and economic factors, several issues need to be addressed. There is a need for behavioural type data for denomination specific characteristics. Although the results of the change and transaction characteristics are promising, these characteristics are only exploratory in nature. Refinements to the proposed characteristics are required in order to better represent the different functions of bank notes (change, transaction, store of value). Furthermore, with emerging financial innovations in the banking industry (debit cards, electronic purses), it will be imperative to capture data on these types of transactions to assess public's preferences towards bank notes. To capture this data and to track changes in attitudes towards bank notes, the Bank of Canada may want to communicate information needs to the financial institutions who are participating in these new technologies and also to perform periodic surveys of bank note usage by the public. Finally, if cash is the medium of exchange for legal activities in the underground economy there is a need to better estimate the size of the underground economy and to study how its change impacts the demand for bank notes.

Appendix A

Proof of Invariance of β_C to the Reference Category

The proof begins with equation [15] for the parameter estimate β_C obtained using transformation matrix C which uses the $(I+1)$ th denomination as the reference. The key point in the proof is given in Lemma 1, namely:

Lemma 1

Any matrix C (of the form given in equation [12]) having reference row i^* , can be generated from another matrix D of the same form, having reference row j , by

$$C = DA$$

where A is a non-singular $I \times I$ matrix.

Proof:

Let $L(D)$ represent the column space of D , and let $c_1, \dots, c_I$, $d_1, \dots, d_I$ and $a_1, \dots, a_I$ represent the columns of C , D , and A respectively. Then each $c_i \in L(D)$ since $c_i = Da_i$. Also if A is non-singular, C must be of full column rank since DA is the product of a matrix having full column rank (D) and a non-singular matrix (A). Since the columns of D are orthogonal to the unit vector, so must be the columns of C . Thus C is a matrix of the required form.

Now assume that C can be constructed such that $c_1, \dots, c_l$ is a basis for $L(D)$, using singular A , i.e., without loss of generality, assume a_1 is a linear combination of $a_2, \dots, a_l$. This implies that c_1 is a linear combination of $c_2, \dots, c_l$. Then C is of rank $l-1$, and hence is not a basis. This is a contradiction and hence A must be non-singular. Q.E.D.

Theorem

The estimated parameter vector β_c in equation [15] is invariant to the choice of the reference category.

Proof:

Equations [15] and [16] can be written more compactly as:

$$\begin{aligned} \beta_c &= \{[(C' \otimes I_T) X_c]^{-1} [Var \epsilon^*]^{-1} [(C' \otimes I_T) X_c]^{-1} \cdot [(C' \otimes I_T) X_c]^{-1} [Var \epsilon^*]^{-1} [(C' \otimes I_T) y]\} \\ &= [X_c' \{C(C' \sum C)^{-1} C' \otimes I_T\} X_c]^{-1} \cdot [X_c' \{C(C' \sum C)^{-1} C' \otimes I_T\} y] \end{aligned} \quad [17]$$

Next substitute DA for C in equation [17] noting that $C(C' \sum C)^{-1}$

$$= D A (A' D' \sum D A)^{-1} A' D'$$

$$= D a a^{-1} (D' \sum DA)^{-1} (A')^{-1} A' D'$$

$$= D (D' \sum D)^{-1} D', \text{ since } A \text{ is non-singular. Therefore,}$$

$$\beta_c = [X' \{D(D' \sum D)^{-1} D' \otimes I_T\} X]^{-1} \cdot [X' \{D(D' \sum D)^{-1} D' \otimes I_T\} y]$$

Thus, it does not matter whether or not D is used instead of C , that is, it does not matter which reference category is chosen.

The estimate β_c is invariant to choice of reference category Q.E.D.

Note 1

The invariance proved above does not carry over to the method used by some economists, that is, to the strategy of reducing the set of $I+1$ shares to an estimable set by dropping an equation. The latter strategy is equivalent to pre-multiplying by a C^* matrix of the form

$$C^* = \begin{bmatrix} I_1 \\ 0' \end{bmatrix}.$$

where, for sake of illustration, the last row is taken as the reference row and is set to zero. The column space of C^* is not the same as that of C and D , since the elements in the columns of C^* do not sum to zero, i.e., they are not orthogonal to the unit vector. Thus, one cannot find A to take C^* into C .

Different C^* 's, based on locating the row of zeros in different places, produce different β_c estimates, which are different from those given by equation [17]. In the technique used in this thesis, all C and D matrices have the same column space which is orthogonal to the unit vector, which effectively says that there is no information on a common intercept. However, full information on the regression parameters of the predictor variables are retained.

Note 2

The invariance proved above involves a known covariance matrix which is not directly estimable. What can be estimated is $\mathbf{C}'\Sigma\mathbf{C}$ (or $\mathbf{D}'\Sigma\mathbf{D}$), which is done iteratively from the residuals at the previous iteration. Since the estimates $\mathbf{C}'\Sigma\mathbf{C}$ and $\mathbf{D}'\Sigma\mathbf{D}$ also satisfy the relation $\mathbf{C}'\Sigma\mathbf{C} = \mathbf{A}'\mathbf{D}'\Sigma\mathbf{D}\mathbf{A}$, the above argument also applies to the case of an iteratively estimated covariance matrix.

Appendix B

1.0 R^2 for Separate Intercept Models

The combined model formulation in this thesis is based on Zellner's (1962) SUR, which is nothing but an application of GLS as explained in the body of the thesis. McElroy (1977) devised an R^2 statistic for SUR models with separate intercept terms for each SUR equation. Her work was also discussed by Neudecker and Windmeijer (1991). However, the difficulty with using McElroy's R^2 is that it includes the variance-covariance matrix which changes as variables are added or removed from a model, thus rendering comparisons of R^2 for the different models invalid, in the sense that adding a variable is no longer guaranteed to increase (or at least to not decrease) a model's R^2 . In addition, McElroy's R^2 measures fit in the space of the GLS transformed variables, which is not the variable scale understood by practitioners. This latter obstacle can be overcome by using Willet and Singer's (1988) R^2 which is based on the proportion of variance explained in terms of the original untransformed variables, i.e., it incorporates residuals based on the untransformed data. It should be noted that Willet and Singer's (1988) residuals do incorporate the efficient parameter estimates obtained via the SUR/GLS, so that R^2 for different models must still be compared with care.

2.0 R^2 for Single Intercept Model

As discussed in Section 5.4.1, a combined model having a single intercept (non-denomination specific) intercept term results in a SUR/GLS analysis of a transformed model having no intercept. As stated by Kvalseth (1985), R^2 measures for no-intercept models must be chosen and applied with caution. While it is possible to define an R^2 for a no-intercept model that satisfies $0 \leq R^2 \leq 1$, Kvalseth (1985) suggests that the "usual" R^2 be used when comparing intercept models with no-intercept models. This suggestion is based on the notion that even if the "usual" R^2 for a no-intercept model is negative, this can still be regarded as evidence of a poor fit.

Appendix C

Sources and raw data of the dependent and independent variables

1.0 Average Volume of Notes in Circulation

Source: Financial Services Department, Bank of Canada, millions of pieces

Data	\$2	\$5	\$10	\$20	\$50	\$100
1960	26.785	28.160	50.698	32.137	2.862	3.896
1961	27.884	29.064	50.598	33.650	2.927	3.948
1962	29.138	29.946	51.725	35.639	3.010	4.042
1963	30.220	30.776	52.204	37.656	3.061	4.074
1964	31.631	31.885	53.345	39.671	3.147	4.140
1965	33.444	33.743	56.399	42.720	3.314	4.336
1966	35.652	35.677	61.350	45.018	3.554	4.544
1967	38.612	38.240	64.510	50.606	3.868	4.854
1968	40.759	39.984	65.977	55.400	4.090	5.135
1969	43.430	41.766	68.387	61.333	4.516	5.787
1970	45.710	43.577	69.613	67.376	4.780	6.235
1971	48.529	45.768	71.826	75.678	5.242	6.974
1972	52.669	49.665	77.553	88.470	6.107	8.353
1973	57.517	54.362	83.225	102.681	7.082	10.055
1974	63.688	58.794	90.940	121.752	7.559	11.507
1975	70.365	62.483	97.934	142.544	8.481	13.178
1976	76.653	64.415	101.571	159.801	10.289	14.886
1977	81.107	66.905	104.408	174.731	12.144	16.491
1978	85.849	69.534	108.611	190.464	14.269	19.180
1979	90.757	71.729	111.435	203.260	16.699	21.959
1980	95.756	73.868	113.445	213.261	19.093	24.336
1981	101.240	75.802	113.453	223.897	22.179	27.062
1982	103.884	77.254	109.982	234.378	25.369	30.024
1983	108.455	79.567	107.676	249.050	29.620	34.874
1984	112.301	84.775	105.737	263.896	34.125	39.700
1985	116.355	90.726	106.903	279.993	38.873	44.399
1986	122.981	100.296	107.134	290.975	43.989	49.641
1987	133.870	108.199	108.758	301.759	49.019	55.044
1988	143.323	116.923	111.861	312.643	54.728	60.920
1989	160.108	126.937	111.687	319.633	59.519	66.640
1990	181.978	134.423	109.755	331.830	62.373	71.280
1991	191.475	147.782	102.585	348.210	66.287	77.289
1992	198.223	145.971	102.950	352.192	69.776	86.340
1993	204.810	140.963	99.678	355.330	72.419	96.138
1994	211.167	138.111	96.664	366.676	74.027	105.110
1995	222.052	137.630	94.274	366.639	73.045	109.950

2.0 Share Characteristics

Sources:

Average value of a credit card transaction: Canadian Bankers' Association DB38PUBLIC

CPI: CANSIM series P484549

	change	change	transaction		transaction	value		
	\$2	cut-off	\$5	\$10	value cut-off	\$20	\$50	\$100
D-Metric equiv.	d/10		d/5	d/2		d	2d	5d
1960	\$1.50	\$2.25	\$3.01	\$7.51	\$11.27	\$15.03	\$30.05	\$75.13
1961	\$1.52	\$2.28	\$3.04	\$7.61	\$11.41	\$15.22	\$30.44	\$76.09
1962	\$1.53	\$2.30	\$3.07	\$7.67	\$11.51	\$15.35	\$30.69	\$76.73
1963	\$1.55	\$2.33	\$3.11	\$7.77	\$11.65	\$15.54	\$31.08	\$77.69
1964	\$1.58	\$2.37	\$3.16	\$7.90	\$11.85	\$15.80	\$31.59	\$78.98
1965	\$1.61	\$2.42	\$3.22	\$8.06	\$12.09	\$16.12	\$32.23	\$80.58
1966	\$1.65	\$2.48	\$3.30	\$8.25	\$12.38	\$16.50	\$33.00	\$82.51
1967	\$1.71	\$2.56	\$3.42	\$8.54	\$12.81	\$17.08	\$34.16	\$85.40
1968	\$1.77	\$2.66	\$3.54	\$8.86	\$13.29	\$17.72	\$35.44	\$88.61
1969	\$1.84	\$2.76	\$3.69	\$9.21	\$13.82	\$18.43	\$36.86	\$92.14
1970	\$1.93	\$2.89	\$3.85	\$9.63	\$14.45	\$19.26	\$38.53	\$96.32
1971	\$1.99	\$2.99	\$3.98	\$9.95	\$14.93	\$19.91	\$39.81	\$99.53
1972	\$2.05	\$3.07	\$4.10	\$10.24	\$15.36	\$20.48	\$40.97	\$102.42
1973	\$2.14	\$3.22	\$4.29	\$10.72	\$16.08	\$21.45	\$42.89	\$107.23
1974	\$2.31	\$3.46	\$4.61	\$11.53	\$17.29	\$23.05	\$46.10	\$115.26
1975	\$2.56	\$3.84	\$5.12	\$12.81	\$19.22	\$25.62	\$51.24	\$128.10
1976	\$2.84	\$4.26	\$5.68	\$14.19	\$21.29	\$28.38	\$56.76	\$141.91
1977	\$3.05	\$4.58	\$6.10	\$15.25	\$22.88	\$30.50	\$61.00	\$152.50
1978	\$3.32	\$4.99	\$6.65	\$16.62	\$24.93	\$33.23	\$66.47	\$166.17
1979	\$3.63	\$5.44	\$7.25	\$18.13	\$27.20	\$36.27	\$72.53	\$181.34
1980	\$3.99	\$5.98	\$7.98	\$19.95	\$29.92	\$39.89	\$79.79	\$199.47
1981	\$4.49	\$6.73	\$8.98	\$22.44	\$33.67	\$44.89	\$89.78	\$224.44
1982	\$4.98	\$7.46	\$9.95	\$24.88	\$37.32	\$49.76	\$99.53	\$248.82
1983	\$5.26	\$7.89	\$10.52	\$26.31	\$39.46	\$52.62	\$105.23	\$263.08
1984	\$5.49	\$8.24	\$10.99	\$27.46	\$41.20	\$54.93	\$109.86	\$274.65
1985	\$5.71	\$8.56	\$11.42	\$28.55	\$42.82	\$57.10	\$114.20	\$285.50
1986	\$5.95	\$8.92	\$11.89	\$29.73	\$44.59	\$59.45	\$118.91	\$297.27
1987	\$6.21	\$9.31	\$12.42	\$31.04	\$46.56	\$62.08	\$124.16	\$310.41
1988	\$6.46	\$9.69	\$12.91	\$32.29	\$48.43	\$64.57	\$129.15	\$322.87
1989	\$6.78	\$10.17	\$13.56	\$33.90	\$50.86	\$67.81	\$135.62	\$339.04
1990	\$7.10	\$10.66	\$14.21	\$35.52	\$53.28	\$71.04	\$142.07	\$355.18
1991	\$7.50	\$11.25	\$15.00	\$37.51	\$56.26	\$75.02	\$150.03	\$375.08
1992	\$7.61	\$11.42	\$15.23	\$38.07	\$57.11	\$76.15	\$152.29	\$380.73
1993	\$7.75	\$11.63	\$15.51	\$38.77	\$58.16	\$77.55	\$155.10	\$387.74
1994	\$7.77	\$11.66	\$15.54	\$38.85	\$58.28	\$77.70	\$155.41	\$388.52
1995	\$7.94	\$11.91	\$15.87	\$39.69	\$59.53	\$79.37	\$158.74	\$396.86

1977 D=\$30.50 (average value of a credit card transaction)

3.0 Regression Variables

	real consumption	Implicit price index (consumption)	90-day commercial paper rate	Number of ATMs installed in Canada	underground economy dummy	Withdrawal of \$1 note dummy
	CANSIM series D20488	CANSIM series D20557	CANSIM series B14039	Canadian Bankers' Association DB247Public		
1960	101,455	25.4	3.64	0	0	0
1961	102,669	25.5	2.93	0	0	0
1962	108,009	25.9	4.06	0	0	0
1963	112,802	26.5	3.70	0	0	0
1964	119,203	26.9	3.87	0	0	0
1965	126,425	27.5	4.65	0	0	0
1966	133,092	28.5	6.03	0	0	0
1967	138,425	29.7	5.54	0	0	0
1968	144,642	31.0	6.61	0	0	0
1969	152,075	32.3	7.65	0	0	0
1970	155,116	33.4	7.14	0	0	0
1971	164,327	34.2	4.27	0	0	0
1972	176,672	35.7	4.90	0	0	0
1973	189,897	38.0	6.98	0	0	0
1974	200,889	41.9	10.36	0	0	0
1975	210,369	46.4	7.73	40	0	0
1976	224,105	49.8	9.10	80	0	0
1977	231,190	53.5	7.42	120	0	0
1978	239,063	57.5	8.52	160	0	0
1979	245,965	62.4	11.79	200	0	0
1980	251,344	68.6	13.02	532	0	0
1981	257,129	76.3	18.63	861	0	0
1982	250,316	84.1	14.23	1,230	0	0
1983	258,904	89.4	9.37	1,960	0	0
1984	270,854	92.9	10.92	2,790	0	0
1985	284,923	96.3	9.45	3,628	0	0
1986	297,478	100.0	9.20	4,512	0	0
1987	310,453	104.0	8.23	5,269	0	0
1988	324,301	107.9	9.46	6,176	0	0
1989	335,284	113.0	12.12	8,970	0	1
1990	338,717	117.9	13.10	11,708	0	1
1991	333,396	123.6	8.98	13,155	1	1
1992	337,619	125.2	6.83	14,576	1	1
1993	342,858	127.4	4.77	16,058	1	1
1994	352,913	128.2	5.29	16,927	1	1
1995	357,827	130.2	7.13	17,670	1	1

Appendix D

Combined Share/Regression Program

```

This is a Gauss program which estimates coefficients of the combined model.
/* MMRM (Multivariate Multiple Regression Model Section)*/
/* this loads in one X variable of regression type */
    load xi[35,1] = h:\sylvie\thesis\mtx\cons.asc;
    load xr[35,1] = h:\sylvie\thesis\mtx\renov.asc;
/* this is the dummy variable for capturing the impact of elimination of the $1 note */
    load dm[35,1] = h:\sylvie\thesis\mtx\dumy.asc;
/* create an individual intercept variable */
    indint = ones(35,1);
/* Now create a block diagonal matrix for the individual intercepts and for the regression
variables. Notation follows that of Neudecker and Windmeijer */
    rg = eye(5);
    xw = rg.*indint;
    xx2 = rg.*dm;
    xx3 = rg.*xi;
    xx4 = rg.*xr;
/* The following matrices are loaded into the program. There is one matrix for each
denomination (l=5). Each matrix is comprised of 35 observations (1960-1994), one intercept,
and at least one characteristic (35 x #) matrix). The value of the characteristic varies by
denomination. */
/* SECM (Share Equal Coefficient Model Section) */
load a2[35,3] = h:\sylvie\thesis\mtx\atmcc2.asc;
load a5[35,3] = h:\sylvie\thesis\mtx\atmcc5.asc;
load a10[35,3] = h:\sylvie\thesis\mtx\atmcc10.asc;
load a20[35,3] = h:\sylvie\thesis\mtx\atmcc20.asc;
load a50[35,3] = h:\sylvie\thesis\mtx\atmcc50.asc;
load shr[175,1] = h:\sylvie\thesis\mtx\shralcb.asc;
    Y = ln(shr);
    xx1=(a2la10la20la50la5);
    xx=xx1-xx2-xx3-xx4;
    c1 = eye(4);
    let c2[4] = -1;
    c3 = c1-c2;
    I35=eye(35);
    C = c3.*I35;
    W = C*Y;
    ZX = C*xx;
    ZW = C*xw;
/* Now drop first element in xx as the intercept is removed by C. Next, drop dummies for

```

all but the S2 bill and drop the first three columns of the renovation variables thus keeping S20, S50 & S100.

```

ZX = ZX[.,2 3 4 9 10 11 12 16 17];
/* Now for the matrix of intercepts, we drop the last column, which corresponds to a
redefinition of the intercept parameters (beta1 - betaref -> beta1*) */
ZW = ZW[.,1 2 3 4];
/* Here ZX and ZW correspond to N and W's Z and W. */
Z = ZX-ZW;
/* Initial Parameter Values */
V = eye(175);
VT = C*V*C';
Vi = inv(VT);
Betaold=inv(Z'*Vi*Z)*Z'*Vi*W;
/* "OLS parameter estimates (last note as reference)": Betaold:*/
Covz=(res*res')/19;
/* Correlation matrix corresponding to the C*Omega*C' covariance matrix*/
se=sqrt(diag(Covz));
corrz=covz./se;
corrz=corrz./se;
"Correlation matrix corresponding to C*Omega*C'";
corrz;
Vthat = covz*.135;
Vihat = inv(Vthat);
Betanew = inv(Z'*Vihat*Z)*Z'*Vihat*W;
"GLS/SUR parameter estimates (last note as reference)":
Betanew;
/* Calculation for the asymptotic t-values */
vbetahat = inv(Z'*Vihat*Z);
sebeta = sqrt(diag(vbetahat));
asyt = betanew ./ sebeta;
"The asymptotic t values are";
asyt';
pvalt = cdfc(abs(asyt),19);
pval = 2*pvalt;
"The p-values are";
pval';
/* Correlation Matrix */
COR = vbetahat ./ sebeta;
COR = (COR') ./ sebeta;
/* R SQR */
/* The following is W-S R^2 for SUR with separate intercepts*/
Nn = 135 -(1/35)*ones(35,35);
what = Z'*betanew;

```



```

komegal=eye(4).*.Nn;
R3WS = (what*Komegal*what)/(W'*Komegal*W);
"R^2: Willet & Singer modified for separate intercept";
R3WS;

/* The following section is a back-calculation of the fitted shares */
w = z*betanew;
expw = exp(w);
rshare = (reshape(expw,4,35))';
refshare = 1/(ones(35,1) + rshare*ones(4,1));
share1 = rshare[..1].*refshare;
share2 = rshare[..2].*refshare;
share3 = rshare[..3].*refshare;
share4 = rshare[..4].*refshare;

/* we now have share1, share2, and share3, etc. (the reference i.e., refshare) in original
untransformed units, as estimated by the model. Now put them together and calculate an
overall R^2 */
shares = share1|share2|share3|share4|refshare;
/*"The fitted shares are:"
shares;

/* The following section calculates R^2 in original untransformed units */
shrm = (reshape(shr,5,35))';
shrm1 = shrm[..1];
shrm2 = shrm[..2];
shrm3 = shrm[..3];
shrm4 = shrm[..4];
shrm5 = shrm[..5];
shrm1 = shrm1 - ones(35,1).*(ones(1,35)*shrm1./35);
shrm2 = shrm2 - ones(35,1).*(ones(1,35)*shrm2./35);
shrm3 = shrm3 - ones(35,1).*(ones(1,35)*shrm3./35);
shrm4 = shrm4 - ones(35,1).*(ones(1,35)*shrm4./35);
shrm5 = shrm5 - ones(35,1).*(ones(1,35)*shrm5./35);
shrm = shrm1|shrm2|shrm3|shrm4|shrm5;
R2orig = 1 - (((shr-shares)*(shr-shares))./(shrm'*shrm));
"R^2 in original untransformed share units";
R2orig;

```

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