Implementation of a Drop Test Impact Rig for Dynamic Testing of High Speed Craft Shock Mitigation Seats and Extraction of Modal Parameters

by

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Abstract

The motivation for this work is human-factors-influenced vibration attenuation using high-speed craft (HSC) shock-mitigating seats. The high-g loads occurring at the seat-deck interface locations during slam impacts in moderate to high seas results in serious potential for injury to the occupants. Since various designs and types of shock-mitigating seats are available, quantifying their shock attenuation characteristics can be challenging. The need for a standard testing platform and experimental analysis to investigate a seat's effectiveness forms the major objective of research being carried out by Carleton University's Applied Dynamics Laboratory (ADL) in partnership with Defence Research and Development Canada-Atlantic (DRDC-Atlantic). A drop tower was designed and manufactured by the ADL for testing seats in order to characterize their shock mitigating effectiveness by simulating the severe conditions of a slam impact at sea through the use of singular impact testing. Further, in order to identify the seats' dynamic parameters from drop test data, the Eigensystem Realization Algorithm (ERA), a modal-analysis-based system identification method, was applied to two commercial shock-mitigating seats provided by DRDC-Atlantic. The technique was shown to successfully extract the damping ratio as well as the damped and undamped natural frequencies of the seats from impact test data. The dynamic properties of the seats derived from the ERA can be tabulated vis-a-vis their vibration performance metrics illustrated in the thesis, which can subsequently inform decisions related to the design and/or procurement of commercial seats.

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List of Acronyms

Acronyms	Definition
ACV	Air Cushioned Vehicle
ABS	American Bureau of Shipping
ADL	Applied Dynamics Laboratory
AIMU	American Institute of Marine Underwriters
ATD	Anthropomorphic Test Dummy
BS	Bureau of Standards
DAQ	Data Acquisition
DC	Direct Current
DFT	Discrete Fourier Transform
DoF	Degree of Freedom
DNV	Det Norske Veritas
DRI	Dynamic Response Index
EAF	Energy Absorbing Foam

EMA	Experimental Modal Analysis
ERA	Eigensystem Realization Algorithm
ERA/DC	Eigensystem Realization Algorithm with Data Correlation
FFT	Fast Fourier Transform
FRF	Frequency Response Function
GRP	Glass Reinforced Plastic
HSC	High Speed Craft
IC	Impact Count
IDFT	Inverse Discrete Fourier Transform
ICI	Impact Count Index
IFFT	Inverse Fast Fourier Transform
IMO	International Maritime Organization
IRF	Impulse Response Function
ISO	International Standardization Organization
LSCE	Least Squares Complex Exponential
LVDT	Linear Variable Differential Transformer
MEMS	Micro Electro-Mechanical System
MIMO	Multiple Input Multiple Output
MSV	Mode Singular Value

NASA	National Aeronautics and Space Administration
NeXt-ERA	Natural Excitation Technique Eigensystem Realization Algorithm
PSD	Power Spectral Density
RQI	Ride Quality Index
SDoF	Single Degree of Freedom
SIMO	Single Input Multiple Output
SVD	Singular Value Decomposition
SWATH	Small Wateplane Area Twin Hull
TMDS	Terminal Data Management System
VDV	Vibration Dose Value
WBV	Whole Body Vibration

Chapter 1

Introduction

1.1 Motivation

High-speed crafts (HSC) find good application in the defence, leisure and workboats industry. Their ruggedness, high speed, maneuverability and, responsiveness justify their prevalence in naval environments. Thus, it becomes critically important to ensure the health and safety of the HSC operators and passengers in all conditions including high seas and during long duration runs.

An operator could be exposed to shock loading having peaks up to 20g and thus the issue of spinal/thorax injury susceptibility arises. The drop-impact test rig, designed and manufactured by the Applied Dynamics Lab. (ADL), Carleton University, aims to investigate the dynamics of such shocks by simulating a high-g environment on the suspension seat as experienced in a typical real-world operation. The whole approach of singular impact testing on the drop tower incorporates a long-term vision to be able to propose better seat testing methodologies, dynamic modelling, and simulation of human-seat interaction under various operational environment conditions, and to help the concerned agencies with design support to mitigate injury risk. Another critical objective for the project undertaken by

Carleton University's ADL is to assist the sponsoring agency, Defence Research and Development Canada-Atlantic (DRDC-Atlantic), in the selection of naval shock-mitigating seats that are well-suited to attenuating occupant injury risks under various operating conditions encountered by the Royal Canadian Navy. Towards this end, a subsequent system identification technique was applied to determine the modal parameters of the seats' dynamics from the data obtained from drop-tests.

1.2 Project Objectives

The primary objective of this project is to develop simple and effective methodologies for testing the shock-mitigating effectiveness of HSC suspension seats for reducing occupant injury risks due to high-g accelerations during slam impacts of the HSC.

A comprehensive literature survey on testing methodologies revealed the effectiveness of drop tests and a lack of testing equipment/rigs which could simulate the high-g loads experienced by operators as a result of slam impacts of the HSC in high seas. A singular impact event capable of generating such high-g loads was identified as a potential testing method. To realize this, a drop-test rig was designed to replicate singular impact events and study the dynamic behaviour of the seat, while also being a monolithic and balanced structure under impacts. The drop tower could test various seats and thus help characterize their dynamic performance in attenuating slam impact loads.

Since characterizing the seats was the other major intent, this involved finding the modal parameters of a given seat which define its shock mitigating characteristics. As the only available information from drop-testing is typically acceleration

signal data, the parameter extraction procedure called for an advanced system identification technique which would be enough to determine accurate modal parameters that describe the behaviour of the seats. Moreover, through this work it was sought to undertake a cost-effective and alternate technique to test the seat components physically in order to determine the system parameters. Individual component testing is more reliable and accurate but comes at the cost of fixture development, assembly/disassembly of components, and test time. A goal of this project was to determine the system parameters with reasonable accuracy by using established modal analysis identification methods. After investigating the available identification techniques, it was inferred that the Eigensystem Realization Algorithm (ERA) technique was well-suited to this objective. An important precursor to applying the system identification technique was to have a proper signal post-processing roadmap to generate the impulse response functions (IRFs) since they contain the system's characteristics and are the starting points for the ERA identification technique. The system identification procedure was applied to two commercial shock mitigation seats. It must be mentioned here that the goal of this work is not to compare the shock attenuation effectiveness between the two seats. The work is focused on the establishment of a procedure to test the seats on the commissioned drop-test rig.

1.3 Thesis Outline

The progression of this thesis has been made in such a way as to familiarize the reader with the research conducted towards the objectives cited in the previous section.

Chapter 2 details the various kinds of HSC used in the naval industry. It also describes the types used in the navy. The loads experienced by the HSC during slam

impacts and their effect on the boat performance are detailed. The chapter also outlines the negative effects of vibration on humans, underpinning the importance of shock mitigation concepts. The severity of such impacts are a major concern for operator effectiveness and a literature survey done to assess the risks is detailed in Section 2.2. Section 2.4 lists and explains some standard indices used to measure vibration and its effects on the human body, which could be used as governing criteria towards the procurement of seats. Shock mitigating seats that reduce the loads experienced by the user at the deck-seat interface are critical to the design process and Section 2.5 of the chapter describes the concepts related to such seats. Section 2.5 of the chapter also lists and explains the different shock-mitigating seats used on HSCs. Of particular interest are the Shockwave and Ullman Dynamics seats which were used as representative HSC suspension seats in this project. Brief descriptions of them are given in the last section of the chapter.

Chapter 3 presents generic descriptions of standard vibration testing methodologies. Pertinent to the project, a literature review related to drop testing is presented to account for the efficacy of the process to represent slam impacts. Section 3 of the chapter describes the construction of the Carleton University Drop Tower along with the sensors used and data acquisition (DAQ) system for acquiring test data. Section 3.4 deals with the mathematical model of the drop impact test and derives an expression of the impact pulse width.

Raw signals acquired from drop tests need to be pre-processed prior to their use with the developed system identification methodology. Chapter 4 outlines the process used for partial clean-up of the signals. It also describes the process used to obtain displacement and velocity profiles from acceleration data and, frequency and impulse response functions from the derived data. A multi-input-multi-output

(MIMO) mathematical model of a typical shock-mitigating seat was derived first in Cartesian coordinates and then in the state-space domain. The state-space domain's system matrix that contains the system information is the matrix on which most system identification methods are based.

Chapter 6 describes some standard system identification methods used for modal parameter identification explored in this work. Because of the disadvantages encountered in most methods, the ERA was found to be relatively more applicable to the case at hand and hence was the technique of choice to identify the seat parameters.

The penultimate chapter presents the results of modal parameters extraction of both representative seats considered and their analyses.

The final chapter comprises concluding remarks for the established test procedure for HSC shock mitigation seats, contributions of the project to the field of HSC seat study and recommendations to improve upon this work in the future.

Chapter 2

High Speed Craft, Shock Mitigation Seats, and Human Factors

2.1 Introduction

High Speed Craft (HSC) have become increasingly important in modern times owing to their inherent high speed and responsiveness capabilities. They are primarily used as patrol and rescue craft by navies and coastguards around the world, and in fast ferrying, leisure, and sports industries. It has been estimated that North American shipyards garner 30% market share of the worldwide construction orders for HSC [1], thus proving to be an important maritime industry. Although there is no standard definition of a HSC, a marine vessel is referred to as a HSC if it is capable of going in excess of 25 knots. Some HSC can achieve service speeds in the range of 40-60 knots. However, speed is not the sole criterion to classify a HSC. According to the International Maritime Organization (IMO), a HSC is a vessel which can sail at a speed greater than or equal to

$$3.7\Delta^{0.1667} \text{ m/s}$$

where Δ [m³] is the displacement corresponding to the design waterline.

HSC can be broadly classified into two categories, air-supported and displacement

<i>Craft Type</i>	<i>Applicable Length</i>
Mono-hull	$< 427 \text{ ft}$
Multi-hull	$< 328 \text{ ft}$
Surface Effects Ship (SES)	$< 295 \text{ ft}$
Hydrofoil	$< 197 \text{ ft}$

Table 2: HSC Classification [2].

types [1]. Air supported craft are air cushion vehicles (ACV), surface-effect Ships (SES) and foil supported craft such as hydrofoils and jetfoils. Displacement type vessels are conventional monohull, catamaran, trimaran, small waterplane-area twin-hull (SWATH), and air lubricated hulls. The monohull type HSC is mostly used in the military and in particular, the planing hull craft, finds increasing military applications owing to its versatility. When the planing hull craft moves at high speeds, the bow lifts out of the water surface and is dynamically supported by the air-flow underneath it, while the stern is in a displacement mode and is supported by the sea surface. The various kinds of HSC are shown in Figure 1.

In addition to this, the American Bureau of Shipping classifies HSC based on lengths [2], according to Table 2.

2.2 Slam Impact Experienced on HSC

When a HSC travels on rough or head seas, it lances off the waves by being momentarily airborne and then violently drops onto the sea surface, thereby resulting in a phenomenon called ‘slamming’. The loadings due to slam impact affect the structural integrity of the boat’s hull, both locally and globally. The adverse effects of slamming have become important in marine vessel design and research since HSC are increasingly being made lighter by the use of composite materials in order to enable high speeds at higher load carrying capacity [3].

Slamming usually happens when the following conditions are simultaneously satisfied [4]:

- The relative motion of the boat’s keel at the bow approaching the sea surface is a positive quantity; and
- The same relative value exceeds an empirical critical value of $0.0928\sqrt{gL}$, where L is the length of the hull.

Typically, a large slam occurs within a pulse width of 200 milliseconds. The dynamic response as a result of slam causes intense vibrations or ‘shudder’ at the deck, and can lead to severe fatigue loading cases of the structure during frequent seakeeping operations [5]. The most-frequently-occurring deck accelerations due to slam are within the 8g–10g range, while the most violent ones at 20g occur less frequently and with lesser pulse-width [[6], [7]]. The American Bureau of Shipping notes that, typically, three slamming events affect the hull structure of a vessel: bottom slamming, bowflare slamming, and stern slamming [8]. The hull experiences scantling due to the impact loads generated by the high-pressure peaks when the hull slams onto the waves. The severities of slam impacts are dependent on the following [8]

- intensity and pulse-width of impact pressure;
- spatial duration of impact pressure; and
- bow's strength and geometry.

The American Bureau of Shipping (ABS) guide [8] provides guidelines for hull design against typical slamming events. However, it ignores the vibratory response of the hull upon slamming. In high speed catamarans, wet-deck slamming is predominant which occurs when the cross deck impacts on the water surface in large waves [5]. For structural design, slam load analysis and design have been carried out using the Det Norske Veritas (DNV) static sag-moment rules [9] and finite element analysis of hydrodynamic loads [[5] [3]]. The static analysis ignores wave dynamics. Influences of vessel speed, sea states, and ride-control system on a 98 m *Incat* high speed catamaran during slam events were studied by identification of slam events using strain gauges on the hull and assessing the impacting surface's vertical velocity relative to the waves [10]. It was found that vessel speed has a direct bearing on the frequency of slams recorded, while the relative vertical velocity was seen to be weakly correlated with the number of slams recorded during the sea trials. The boat's ride control system functioned effectively for speeds above 35 knots. Prediction of slamming loads using vertical velocity and investigation of wet deck slamming of catamarans can be found in [11]. Hydroelastic analysis encompassing fluid-structure interactions has been successfully applied to model the behaviour of flexible high-speed patrol boats in irregular sea states [12].

2.3 Biodynamic Risks Associated with HSC Slamming

At low frequencies and lower vibration magnitudes, the human body can be modelled as a linear lumped parameter mechanical system as depicted in Figure 2. The low-frequency (3–6 Hz range) thorax-abdomen section is a primary area of concern when injury risks to varying vibration levels are assessed [13]. Whole body vibration (WBV) experienced during rough rides pose considerable health risks to the human body. It depends on the motion duration, direction, frequency, magnitude, and variabilities related to biodynamics, posture and, physiology. The most common injury is to the spinal vertebrae discs which results in back pain. Moreover, the sitting posture adds to the stress on musculature and the lumbar spine’s intervertebral discs. Personnel on HSC travelling on long-duration hauls can also be subject to physical fatigue which affects their physical and cognitive performance. Fatigue occurs when occupants use their legs as a way to absorb repeated shock loadings from the deck [14]. The muscles run in an overdriven state, leading to higher heart rates and oxygen consumption. In addition, perceived fatigue in HSC personnel was also found to be greater during night-time operations as opposed to day shifts [15]. The sitting posture of an occupant also has considerable bearing on back injuries, with the relationship being linear [16]. In addition, lateral supports on some shock-mitigating seats can also lead to injuries due to sideways spinal deflection under the action of large lateral forces, as seen in Figure 3. Vertical motions with frequencies around 1 Hz and some higher frequencies also contribute to the severity of WBVs. Also, additional loading of head mounted gear and other equipment strapped onto the body of riders contribute to ride discomfort.

The general procedure of recording occupant injury is through self-reporting.

A study conducted by the US Naval Health Research Center on Special Operations craftsmen in 2000 revealed that 65% reported some one or more kinds of injury which needed treatment, 95% of which were attributed to HSC operations. Most of the injuries were related to the lower back spinal area, knee, and shoulder [18]. Severe slam impacts with high deck acceleration peaks can also lead to lumbar fracture. Thus, in addition to optimal structural design of the boat to reduce slam impact induced vibration effects, additional solutions are implemented through the use of shock mitigating seats to further isolate the occupants and reduce injury rates.

2.4 Vibration Measurement

Various metrics of measuring and quantifying vibration in regards to WBV are used. This section lists and describes the application of the standard indices. Almost all methods use the values captured by mounted accelerometers to measure the dynamic accelerations.

2.4.1 Peak Accelerations

Peak magnitudes of acceleration, although basic in concept, serve as important indicators of vibrations. Peak amplitude in time histories recorded during a test are first tabulated and then averaged using a peak-to-trough approach [19]. Commonly used averages of the peaks, referred to as one-third, one-tenth, and one-hundredth peak accelerations are obtained using the expression below

$$A_{1/n} = \left(\frac{1}{N/n}\right) \sum_{t=1}^{N/n} A_t$$

where A_t is the amplitude of the acceleration peak; N is the number of peak magnitudes; and $n = 3, 10$ or 100 as appropriate. Though simple in application, the approach is subject to the user's choice of selection of amplitudes.

2.4.2 Ride Quality Index

The ride quality index (RQI) for two separate testing events is defined by the equation

$$RQI_{II-I} = 1 - \frac{A_{II}}{A_I}$$

where A_{II} & A_I are the recorded accelerations in a time history from the two events [19]. As is evident from the above expression, RQI is a relative index. It is plotted as shown in Figure 4. Positive values indicate better ride quality than negative ones and vice-versa, with superior ride comfort for higher positive values. Negative values are indicative of higher potential for injury.

2.4.3 Impact Count Index

The impact count index (ICI) was developed as a simple yet sensitive measure to quantify repeated shock (RS) events for slamming experienced at sea. The technique makes use of acceleration magnitudes and frequency content analyses to construct an impact count (IC) histogram. From the IC, the total number of shocks are expressed as percentages below a threshold which are then displayed in the form of an ICI plot. Reading the acceleration time history, an peak value corresponding to a shock is first assigned to a bin, with the suitable range of each bin being 0.2g. The procedure is followed for different values of shocks to generate a cumulative series of bins containing those impact magnitudes, thereby giving the IC [20]. This result is then graphed as a sequence of impact counts (ordinate) for each bin (abscissa). Low magnitude bins representative of regular boat motion are discarded and the plot is shown beginning from a suitable bin value. The percentages of total shocks below a given threshold are then plotted against the bins to gauge the relative performances between different seats.

2.4.4 Vibration Dose Value

Vibration dose value (VDV) is a measure of accumulated exposure to periods of varying magnitudes of accelerations in a particular timespan, and is expressed as

$$VDV = \left[\int_{t=0}^T a^4(t) dt \right]^{1/4} \quad (\text{ms}^{-1.75})$$

where a is the acceleration profile. The time dependency of VDV corresponds to a decrease of 1.5 dB in magnitude per doubling of the exposure [21]. According to the EU Physical Agents directives, VDV over an 8 hour exposure at sea should not exceed 9.1 m/s^{1.75}.

2.4.5 Seat Effective Amplitude Transmissibility

The seat effective amplitude transmissibility (SEAT) value is one of the most popular ways of assessing the shock isolation attribute of a suspension seat. SEAT values can be obtained from experimentally derived seat transmissibility functions for a variety of excitation spectra. The SEAT value is defined as

$$SEAT\% = \frac{\text{Vibration on seat}}{\text{Vibration on deck}} \times 100$$

where the numerator/denominator can be the rms/VDV of the recorded input/output signals. SEAT values can be computed to estimate the seat cushion vibration using transmissibility measurements derived for the seat-person combination [22].

2.4.6 Dynamic Response Index

The dynamic response index (DRI) analytically assesses the effect of peak vertical accelerations on a human body in terms of its spinal compression and then relates this effect to the probability of injury [23]. It is derived by considering a simple 1-DoF second order mass-spring-dashpot system with the connection parameters being those of a human body's spine and is expressed as

$$DRI_z = \frac{\omega_n^2 \delta_{max}}{g}$$

where $\omega_n = 52.9$ rad/sec is the average natural frequency of the spinal column; δ_{max} is the maximum compression of the spine; and g is the acceleration due to gravity. The limit, according to specifications, is set at a DRI value of 18 which corresponds to a 5% probability of injury.

2.5 Shock Mitigating Seats

In regards to HSC, there are three major types of shock-mitigating seats as listed below

- saddle type seats (sprung or unsprung);
- conventional sprung seats; and
- supportive seats with a sprung squab (cushion).

Designing shock mitigating seats is a challenging task because both stiffness and damping behaviour have to be looked into to provide optimum comfort at varying levels of seakeeping impacts. The human body is capable of absorbing loadings in the range of 3g–4g with just a simple foam seat. Beyond this range, a sprung seat is required to attenuate the shocks. Some seats have sprung squabs which could be electrically controlled to enable raising/lowering the seat to enable a combination of standing/sitting positions based on rider comfort [24]. The convention followed in BS 6841 and ISO 2631-1 for predicting seating discomfort is shown Figure 6. The shock-mitigating seats dealt with in this research are of the suspension type, wherein the spring supports the sprung mass (human body+seat-pan mass), and the damper acts as the energy dissipative component to control the vibratory response to excitations. The thumb rule for designing marine shock-mitigating seats is to avoid

having the seat vibration’s resonant frequency be in the 3–6 Hz resonant frequency range of the human body. The spring element is generally made “soft” so as to reduce the effect of shock loadings on the rider [26]. Care is taken to prevent bottoming-outs so that there is no amplification of load on the occupant. Careful consideration is also given to damping behaviour in some suspension seats. Damping needs to be higher for sudden shock loads to prevent high displacements, hence avoiding bottoming-out. Also, damping should not be high enough so as to transmit greater shock load to the occupant. Thus an optimal level of damping is generally applied for such cases.

The two seats studied and tested in this project are passive commercial shock mitigating seats. They are discussed in the subsequent sections.

2.5.1 Shockwave Marine Suspension Seat

The Shockwave seat comes equipped with the marine version of the Float FOX air-spring suspension with a maximum travel of 203 mm. The suspension includes a speed-sensitive damper for providing progressive stiffness based on velocity and changing pressure, and an adjustable air-pump for varying load and sea states. A proprietary Samurai Sealing sealing system is present in the system to prevent loss of air between different adjustment levels. The seat-pan includes an EAF (energy absorbing foam) padding for added comfort during rides.

2.5.2 Ullman Dynamics Compact Seat

The Ullman Dynamics Compact seat (Figure 7) is designed to optimise body posture. It is described as a semi-active type of seat by the manufacturer, wherein the spring becomes progressively stiffer with higher loads. However, it should be noted that from the standard classifications in literature [27], it is a passive seat. The dashpot

consists of a nitrogen damper as part of the system. It has been designed to fit into compact areas and is made of non-corrosive materials. Its structure is made up of fibre reinforced plastic, aluminum (5077), and stainless steel (316) components. The seat is usually mounted onto a glass reinforced plastic (GRP) base. The spring element of the seat offers a maximum vertical displacement of 150 mm and consists of two parallel flexible composite boards, the ends of which are fixed to the seat structure's steel brackets at an angle.

Figure 1: HSC types [1]

Figure 2: Equivalent lumped parameter representation of the human body [13].

Figure 3: Spinal deflection due to lateral seat support when exposed to high lateral forces [17].

Figure 4: RQI plot example [19].

Figure 5: Plots showing (a) IC and; (b)ICI [20].

Figure 6: Basi-centric axes of person on a seat [25].

Figure 7: Ullman Dynamics Compact marine seat (*image courtesy: Ullman Dynamics*).

Chapter 3

Vibration Testing and the Carleton University Drop Tower

3.1 Introduction

Vibration testing is the process of assessing the characteristics of a structure when subjected to shakers and shock tests. A structure is generally subjected to vibration tests in order to meet regulatory standards, qualify desired performance, validate design specifications, and test against structure degradation (fatigue, shock, etc.). Given the risks posed due to vibration, as was highlighted in Chapter 3, well developed measurement techniques, advanced measurement and testing devices, and analyses methods have been developed in industry and academia to help study noise and vibration related phenomena [28]. This chapter focuses on the various testing devices available for vibration evaluation. Further, a literature survey is presented to assess test devices suitable for testing shock-mitigating suspension seats. Section 3 of the chapter deals with the design and manufacturing of the Carleton University Drop Tower.

Vibration testing is widely used in the aerospace, automotive, construction,

and electronics industries. It comprises the excitation of a structure by applying a forcing function and then measuring its response(s) for further analyses. This is usually done through the use of vibration testing methods like shaker tests, impact hammer tests, shock tests, and drop test. Standard vibration tests are sinusoidal testing, random testing, force testing, and shock testing.

3.1.1 Sinusoidal Testing

Sinusoidal vibration is a widely used method of exciting the low frequency modes below 40 Hz. There are two ways of sine testing. In the first, called sine sweeping, testing is done at a log rate of 1–6 octaves/min and a fixed amplitude to reduce fatigue cycles [29]. The second type, called sine dwelling, involves testing wherein the frequency is kept constant and the test runs for a fixed number of cycles [30].

For requirements when the vibration level is to be kept constant with respect to a sinusoidal excitation, the vibration signal at the output could be fed into a servo loop to control the excitation from the driver [13]. The required frequency sweep is automated in the control oscillator and regulation speed in the control loop is fixed such that it is greater than the rate at which resonances occur. To perform vibration tests at high frequencies and lower exciting forces, a piezoelectric accelerometer is generally used as an exciter owing to its passive nature (i.e., obeying the reciprocity theorem). This type of testing cycles up and down repetitively between the frequency limits for a specified time or number of sweep cycles to ensure that adequate reliability levels are attained [31].

Sinusoidal testing is commonly used for product quality assurance and also to test products under severe conditions. However, it is barely representative of the real world operating conditions and is mostly used as a preliminary testing technique [31].

3.1.2 Random Testing

In random vibration testing, the excitation signals are specified based on the power spectral density (PSD) of the excitation signal and the testing time [30]. The severity of such a test could be measured by the highest PSD value (i.e, at resonance). Random vibration testing is conducted so as to mimic a real world failure process wherein most frequency components interact in a non-linear fashion [13]. Wideband random testing is done to excite the natural modes of a structure simultaneously. The general idea of doing such a test is to first define the limits of vibration based on collected data from a real world operation.

To avoid the effect of the structure's resonant modes when it is excited by a random vibration signal, a spectrum shaper with attenuation-adjusted parallel band-pass filters is generally used. In conjunction, a real-time frequency analyzer is used to compare the observed spectrum to that of the desired excitation so that the spectrum shaper can shape the actual excitation to the desired spectrum [13]. This system can also be used to linearize the excitation if so desired. However, the system has a downside in that it is passive and cannot be used for real-time control of the excitation spectrum. Also, owing to the presence of a large number of analog electronic components, the system tends to be cumbersome and bulky. Modern digital techniques can synthesize the actual signal close to the spectrum characteristics of the desired excitation spectrum.

In order to avoid the disadvantages associated with the foregoing random vibration method, sweep random vibration testing is used. It is similar to swept sine testing and the same regulating and control characteristics can be used, while inheriting the statistical properties of a random signal.

3.1.3 Force Testing

In force testing, a transducer is fixed between the worktable and the structure. Mechanical impedance (Z) can then be expressed as a relation when the structure is set into motion upon excitation by an input force. This concept of mechanical impedance in force testing enables the determination of modal properties and evaluation of compliance [13].

Mechanical impedance (Z) is expressed as

$$Z = \frac{F}{v} \quad (1)$$

where F is the excitation force and v is the resulting velocity. The inverse of mechanical impedance is called *mechanical mobility* and is expressed as M .

The mechanical impedance (Z_k) and mobility (M_k) of a spring element with stiffness k are given as

$$Z_k = \frac{k}{j\omega} \quad (2)$$

$$M_k = \frac{j\omega}{k} \quad (3)$$

For a damper with damping coefficient c , its impedance and mobility are expressed as

$$Z_c = c \quad (4)$$

$$M_c = \frac{1}{c} \quad (5)$$

The advantage of using velocity as the dynamic parameter in impedance/mobility is that it requires less dynamic range of the sensor than using displacement/acceleration [13].

For the purpose of force testing, the structure is excited through an impedance head [13]. The impedance device has two sensors. The first one gives a signal proportional to the input force and the second one measures the motion. To measure mechanical impedance, the velocity level of the measuring point is made constant and the force recorded gives the magnitude of impedance as

$$|Z| = \frac{|F|}{|v|} \quad (6)$$

Mobility is useful when the objective is to find out the system's modal parameters. For measuring the mobility, the excitation force level is kept constant and the modulus of velocity level is measured as

$$|M| = \frac{|v|}{|F|} \quad (7)$$

A phase meter is used for measuring the phase difference between force and velocity.

3.2 Drop Testing: Literature Survey

Since the deck acceleration arising from a slam impact event experienced by planing boats is usually in the range of 8– 20 gs, drop testing was considered as a viable method to test the shock-mitigating seats and is a useful experimental approach to design for reliable operation [32]. A comprehensive literature review was conducted to look at some of the drop testing devices used in shock-mitigation-related research.

Conventional methods of testing are: (1) at-sea testing, and (2) laboratory testing. At-sea testing does give accurate results but at the cost of time, manpower, repeatability, and the need for elaborate equipment. Laboratory evaluation thus is a good alternative to simulating such an environment using singular representative

testing events, while also mitigating the demerits faced during at-sea testing. Work related to seat-testing has been done in the fields of high speed marine craft, mine-proof vehicles, as well as the aerospace industry. Most of the test-related work in this area, albeit in different industries, could be carried over to HSC seat testing based on similar principles.

A method somewhat similar to the testing approach dealt with in this thesis was followed at the Massachusetts Institute of Technology to investigate the mitigation of mechanical shocks on high speed planing boats [33]. One of the major objectives of the study was to develop and investigate methods for shock mitigation and testing methodologies for HSC seats by designing and validating an inexpensive drop tower. The subject was a STIDD800v5 seat used on US Navy boats.

The fabricated drop tower consisted of a reinforced steel frame drop table, linearly guided by the use of steel posts and sleeves. The table section was raised using a 1-ton chain-fall hoist. A quick-release hook similar to the ones used for dropping life-boats was used for lifting the table before an impact. A wide range of shock pulse shapes could be obtained by varying the tile material type (density, durometer and geometry) the table was dropped upon. Drops were performed using lumped mass weights varying from 180 lbs to 205 lbs to replace a human being. The drop heights used to generate the acceleration response range (5g–8g) were varied between 6 inch to 8 inch.

SigLab Version 2.3 software was used to record and process accelerometer data from the impacts. The instrumentation consisted of the SigLab system, a Dell laptop, PCB signal conditioners, ICP piezo-accelerometers, and an FFT box. The sensors were interfaced with a MATLAB software application for post-processing. However,

displacement was not measured to evaluate the shock absorbers action during impact owing to the nature of the study.

Work of the same type was carried out at the Virginia Polytechnic University which involved the construction of a seat testing rig to evaluate the dynamics of a shock mitigating seat in a controlled environment [34]. The rig allowed motion along the vertical direction. Its outer frame supporting structure of the rig was made of aluminum. The structure was made rigid by using diagonal and horizontal braces secured to the four vertical uprights on the corners. An inner carriage with rollers, made of 80/20 preformed aluminum was implemented to provide the vertical motion. The outer frame was bolted onto a 3 square feet, 1 inch thick solid steel plate which was bolted to the floor. The shock mitigating seat (STIDD 800v5) was mounted to a solid steel plate of 1 inch thickness. This steel plate was bolted to the inner mobile carriage to simulate vertical motion on the seat.

A hydraulic actuation system, made by MTS, was employed to simulate high vertical acceleration. The system was able to simulate event waveforms including both single and multiple frequency. The actuator (MTS Model 242.09) had a stroke of 2 inch and a force capacity of 2200 lbf. It had a built-in load cell and LVDT for accurate measurement of force and displacement. The actuator was equipped with a digital MTS 407 servo controller, providing displacement/force control through a single channel to the actuator. An MTS 505.20 SilentFlo hydraulic power unit, capable of a flow rate of 20 gal/min, was employed to provide pressurized fluid to the actuation system.

Instrumentation on the system comprised three accelerometers, an LVDT, a string potentiometer, a dSPACE digital signal processor, and a laptop. Interfacing

for the sensor signals was implemented using MATLAB/Simulink and dSPACE Control Desk. The accelerometers were positioned on the seat floor, seat-pan, and external weights. A PCB MEMS 3741D4HB10G uniaxial accelerometer was glued to the steel plate (seat floor) using vibration resistant superglue to ensure a strong hold. A PCB MEMS 3713D1FD20G triaxial accelerometer was glued to the underside of the seat-pan to measure accelerations along horizontal and vertical directions. A PCB 352C65 shear piezoelectric uniaxial accelerometer was used to measure the acceleration of the mass on top of the seat cushion using Industrial-grade wax to glue it to the mass. A string potentiometer was used to measure the position and velocity of the shock mitigation system. It was mounted to the bottom of the seat pan with the string attached to the steel base floor of the rig. The LVDT provided real-time position values of the actuator on a feedback loop. Signals from these sensors were routed to the dSPACE Autobox via a DS2201ADC analog to digital converter. The AutoBox provided digital signal processing.

Another project conducted at the University of Nevada, Las Vegas, involved the design and construction of two drop towers for characterizing the behaviour of shock mitigation seats on combat vehicles during mine-blast threats [35]. One of the drop towers stood at 12 feet on a steel platform which weighed in excess of 300 lbs. This had two robust bearing shafts located on either side, which functioned as linear guides for the vertical drop of the seat-pan. The second drop tower was lighter in construction in that it used a four-point guided aluminum seat-pan, with four bearing shafts located on each of the corners of the platform. This was specifically done to achieve less noisy input signals and more controlled descent of the platform as opposed to the former drop tower.

For signal capture and instrumentation, an SAE seat-pad accelerometer was

mounted at the seat-butt interface of the cushion. A 2 mV/g Dytran shock accelerometer, capable of upto 2500 g peak, was mounted onto the impact plate because of its noise-reduction and high sensitivity characteristics. A combination of a 1 inch thick rubber base in a 6 inch X 6 inch impact base along with a thin gel elastomer impact gel on the striking plate's bottom surface was used to smooth out the acceleration profiles. A 1-D passive anthropomorphic dummy, developed by the University of Southampton, was used to mimic the damping features of a human body with the natural frequency of the dummy being close to that of a human body at about 5 Hz. The dummy was strapped onto the seat's lumbar support to prevent any transverse motion.

A study which employed drop testing of helicopter seats was jointly conducted between the Beijing University of Aeronautics and Astronautics and the China Helicopter Research and Development Institute [36]. This study aimed to evaluate energy-absorption capacity of a new-type of a helicopter crash-worthy seat and the human response under impact conditions. The drop tower consisted of two vertical rails to guide the seat carriage vertically. The seat carriage was connected to an electric winch which was equipped with an electromagnetic hook for smooth release. A plunger was bolted to the underside of the carriage which was guided to a cylindrical pool of water upon impact. The cylinder had holes whose diameter could be altered to achieve a controlled signal profile.

An anthropomorphic test device (ATD) was secured to the seat for drop-testing. The instrumentation was rather elaborate owing to the inclusion of ATD for testing. Accelerometers were attached at the head and chest positions to measure the respective acceleration of the occupant. One accelerometer was mounted to the seat pan to measure its acceleration. To measure the input pulse, an accelerometer was fixed

on the bottom of the carriage. A pelvic load cell was used to measure the lumbar compressive force. A displacement transducer was mounted between the seat pan and the carriage to measure the relative motion. Two dynamic strain gauges were fixed on the frame to record the dynamic strain response. The LMS DIFAIII system was used for data acquisition of output signals at 102 Hz. A high-speed camera was employed to monitor the impact at 500 fps. System TEMA was used for post-processing of data.

From the literature survey, it was concluded that the proposed design of the Carleton University drop tower was very much in line with the principles of drop testing of seats. Also, drop testing was the preferred choice for testing the seats since it excites a wide range of vibration modes and, broadband excitation methods tend to produce the best linear system for the measured input-output signals by linearizing the frequency response functions [28].

3.3 Carleton University Drop Tower

3.3.1 Design and Manufacturing

The Carleton University drop tower is shown in Figure 8, and its key parts are labelled in Figure 9(a). Sizing was based upon preliminary hand calculations for critical parts of the drop tower, size and footprint of the seat specimens, and the standard metal components available from suppliers. Both metric and imperial units have been used in the drawings since the slider-rail assembly used the metric system. The manufacturing was done in-house at the Mechanical and Aerospace Engineering department's machine shop at Carleton University.

Starting at the foundation of the rig, the base consists of a 2 inch thick steel plate (Figure 9(a)). Eight threaded holes for 3/8 inch bolts were made on the top-side of the base plate to fix the four angles. The angles secure a soft polyurethane sheet (3/4 inch thick) along with a 5 inch thick rubber block, both of which together act as an impacting base which absorbs impact energy partially to smooth out the impact-acceleration pulsewidth. Right angle faced diagonal pieces made of steel were bolted to the four corners of the base plate. These served as diagonally-extended bases to mount the vertical posts on. An assembly of an S200 X 27 steel I-beam welded along its cross-sectional profile to a 3/4 inch steel plate was tightly bolted to the base plate using 1 inch bolts. Two gussets were welded between the 3/4 inch thick plate and the rear of the I-beam to provide resistance against dynamic moments during impacts, as shown in Figure 9(a). To provide additional rigidity to the welded assembly of the I-beam-plate sub-assembly and the base plate, two 1/2 inch thick steel angles were rigidly secured such that they were bolted together with the lower part of the I-beam's web with a 1 inch hardened grade-8 steel bolts, and their bases were tightly secured to the assembly with four 1 inch grade-8 steel bolts going into the base plate. The angles also serve in resisting lateral moments of the I-beam during impacts.

An NSK slider-rail assembly was chosen for the intended dynamic loads. The linear rail was mounted eccentrically on one flange of the I-beam as shown in Figure 9(a). It was secured to the flange section using Allen bolts, wedge washers, and nuts. A 3/8 inch thick steel plate with drilled holes was also welded to the front-facing top surface of the I-beam to mount an electric winch on.

A carriage to mount the shock mitigation seats on, was manufactured by assembling machined steel plates as shown in Figure 9(a). The middle 1 inch thick

seat plate had a pattern of threaded holes drilled into its top surface to rigidly secure the shock mitigation seats with bolts. Its lateral faces had three 1/2 inch threaded holes. The facing plates (3/4 inch thickness) of the carriage had a circular pattern of clearance holes on opposite quadrants to enable easy swivelling of the seat-plate. This pattern of holes enables swivelling of the middle seat plate, which can be bolted angularly to the indexed plates using three hardened grade-8 steel bolts on each of the laterally opposite sides. The angular variation of the seat plate will allow the evaluation of lateral and longitudinal forces experienced by the occupant. Further, the front- and rear- facing plates were tightly bolted to the bottom (3/4 inch thickness) steel plate using grade-8 steel 7/16 inch bolts. The facing plates were additionally secured to the bottom plate of the carriage using four angles (3/8 inch thickness) to provide additional rigidity. The rear-facing plate had a pattern of clearance holes drilled into its surface so that two NSK sliders could be secured to it after sliding them through the steel rail. The combined slider-rail assembly enables an almost free fall of the carriage. A D-hook with 1/2 inch thread, fabricated for reliable release of the carriage during drop tests, was tightly screwed to the top surface of the rear-facing plate. A hard polyurethane bumper assembly was bolted to the underside of the carriage-assembly's bottom plate which would impact upon the soft rubber block resting on the base plate to help produce a smoother pulse upon impact.

An electric clutch-type winch of 1200 lbs capacity was mounted to the welded plate on top of the I-beam to provide for safe lifting of the carriage-seat-occupant weight (Figure 9(a)). The snap link at the free end of the winch's steel cable was connected to a mechanical Release-Matic quick-release hook (Figure 9(b)) which functions as the trigger for free fall of the carriage when pulled by a string attached to it. An additional safety feature to secure the carriage in the event of a release-hook malfunction was integrated into the device. It comprised a secondary winch of the

same capacity as the first one, secured to a designed assembly fixed to the I-beam's web, a bracket designed to bear the lifting load, and a locking frustum which slots in the hole of the bracket during the need of an emergency support to the carriage (Figure 10).

A plastic-coated wire mesh was wrapped around and secured to the drop-rig's vertical posts to prevent the expulsion of any small fasteners or components while conducting tests.

To mount the dummy mass replacing an occupant, a saddle was designed and fabricated to allow for the vertical motion of the dummy mass while restricting its sideways motion. This saddle-dummy mass assembly was secured using 10 inch long 1/2 inch bolts for the Shockwave seat (Figure 11(a)) and with ratcheting straps for the Ullman Dynamis seat (Figure 9(a)).

3.3.2 Instrumentation and Data Acquisition

The instrumentation part of the test-rig consists of seven uniaxial MEAS Model 4000A 3rd generation silicon MEMS accelerometers, one Penny-Giles linear potentiometer, a voltage supply unit, a DAQ system and, a personal computer. This is illustrated in Figure 11. Three accelerometers were mounted on orthogonal surfaces of a cuboid to measure accelerations of the dummy mass along the vertical, lateral, and longitudinal axes as shown in Figure 11. This assembly was tightly mounted with screws to the underside of the seat pan.

A second set of similarly mounted accelerometers was fixed on the seat-pan to measure accelerations of the seat-pan in the three orthogonal directions (Figure 11). An accelerometer was also mounted onto the bottom side of the carriage to measure

Figure 8: Complete assembly of the Carleton University drop tower.

its vertical acceleration. The Penny-Giles linear potentiometer was mounted to the air-spring shock absorber using collar brackets on the tubes. However, due to the large displacements observed from the accelerometer integrated displacement data, the linear sensor was removed subsequently from the Shockwave seat's shock

absorber as it only had a maximum range of 76.1 mm.

The DAQ system used was a National Instruments NI USB-6289 M Series board. Interfacing was done through LabView-2010 and the signals were acquired at a sampling rate of 2 kHz. Six channels for data acquisition were programmed on LabView's block diagram format and the signals were presented in real-time using its graphical display interface (Figure 12). The PC used for interfacing was an Intel Dual-Core system running Windows-7 OS. Signals from the sensors were recorded using the block diagram and were logged into MS Excel using the TDMS data logger function for post-processing and subsequent signal analysis.

3.4 Mathematical Modelling of Drop Impact

Impact can be modelled on the principles of classical mechanics or based on the signal form of the input impulse. Modelling based on both methods are presented in this section.

Assuming that the carriage drops from a height h , its velocity v_1 just before impact is given by

$$v_1 = \sqrt{2gh} \quad (8)$$

where g is the acceleration due to gravity ($= 32.174 \text{ ft/s}^2$). Assuming inelastic impact,

$$v_2 \approx 0 \quad (9)$$

From Newton's second law, average acceleration during impact is the time rate of change of velocity. Thus, acceleration due to impact is

$$a = \frac{dv}{dt} = \frac{v_1}{\Delta T} = \frac{\sqrt{2gh}}{\Delta T} \quad (10)$$

where ΔT is the pulsewidth of the impact.

A more accurate representation could be made by considering the coefficient of restitution e , i.e.,

$$e = \frac{v_2}{v_1} = \frac{\sqrt{2gh_{rebound}}}{\sqrt{2gh_{compression}}} = \frac{\sqrt{h_{rebound}}}{\sqrt{h_{compression}}}$$

Since an accelerometer captures the impact response in the drop test, a better way to model the impact process is by applying a half-sine/cosine curve for acceleration and velocity output. A half-sine curve is a close approximation of the pulse shape obtained from the accelerometer's response.

An equation for recorded half-cosine velocity is written as

$$v = \frac{v_2 - v_1}{2} \cos\left(\frac{\pi t f_s}{N}\right) \quad (11)$$

where $\frac{v_2 - v_1}{2}$ is the velocity amplitude with v_1 (−ve) and v_2 (+ve) being the velocities before and after the impact respectively, f_s is the sampling frequency, and N is the number of samples recorded within the half-cosine pulse. Pertinent to drop tests

$$|v_2| = e|v_1|$$

and this results in

$$v_2 - v_1 = |v_1|(1 + e)$$

$$v = \frac{|v_1|(1+e)}{2} \cos\left(\frac{\pi t f_s}{N}\right) \quad (12)$$

To obtain an equation for acceleration recorded by the accelerometer upon impact, the expression of velocity is differentiated with respect to time, resulting in

$$a = \frac{dv}{dt} = \frac{-|v_1|(1+e)}{2} \left(\frac{\pi f_s}{N}\right) \sin\left(\frac{\pi t f_s}{N}\right) \quad (13)$$

The peak value of the acceleration profile is given by the amplitude part of the acceleration expression derived above. Thus,

$$a_m = \left| \frac{-|v_1|(1+e)}{2} \left(\frac{\pi f_s}{N}\right) \right| \quad (14)$$

Knowing that $\Delta T = N f_s^{-1}$, with ΔT being the pulse-width, and $v_1 = \sqrt{2gh}$, the final expression of the peak acceleration upon impact is

$$a_m = \frac{(1+e)\sqrt{2gh}}{2} \frac{\pi}{\Delta T} \quad (15)$$

Dividing a_m by g to obtain acceleration in terms of g , results in

$$a_g = \sqrt{\frac{h}{2g}} \frac{\pi (1+e)}{\Delta T} \quad (\text{g units}) \quad (16)$$

The value of e from displacement profiles of the carriage comes to around 0.82 for the steady state region. A typical drop test impact event had pulse width in the range of 50 msec which can be adjusted by varying the material of the impacting surfaces. Table 3 gives the accelerations for the observed pulsewidth and the derived coefficient of restitution.

Table 3: Theoretical peak accelerations for different drop heights.

Drop height (inch)	Acceleration ($e = 0.82$) (g units)
6	10.08
12	14.25
18	17.45
24	20.15
30	22.53

3.5 Conclusion

A summary of standard vibration tests was presented in this chapter. Further, a comprehensive literature survey was undertaken so as to determine standard tests suited to simulate high-g slam events at sea. It was concluded that drop testing is an effective testing method. A drop impact event was mathematically modelled to determine expected accelerations. A drop tower was designed, manufactured and, properly instrumented with the required sensors. A data acquisition interface was programmed with LabView to acquire the signals which were stored in MS Excel format using the TDMS data-logger function.

(a) Drop Tower

(b) Saddle on Ullman seat

Figure 9: Photos showing [a] Drop tower components; and [b] Saddle on Ullman seat.

(a) Secondary lifting mechanism

(b) Seat bracket and frustum locking arrangement

Figure 10: Photos showing [a] Secondary lifting mechanism; and [b] seat bracket and frustum locking arrangement.

(a)

(b)

(c)

(d)

Figure 11: A set of photos showing: [a] accelerometer on dummy-mass; [b] accelerometer on seat-pan; [c] accelerometer on carriage; and [d] LabView DAQ.

(a) Block diagram

(b) Acceleration signals

Figure 12: Plots showing [a] LabView block diagram for DAQ; and [b] Acceleration signals acquisition.

Chapter 4

Pre-processing Experimental Data

4.1 Introduction

The drop impact results obtained from the tests described in Chapter 3 showed signs of noise corruption owing to some structural vibration and needed to be filtered before the data could be fed into the parameter extraction algorithm. A suitable noise reduction strategy was applied so as to maintain accuracy of the processed data. Since all the measured signals were accelerations and displacement data was also needed, an accurate frequency-domain integration was used to obtain the displacements of the seat-pan and dummy mass.

Modal analysis involves measurement of the frequency response functions (FRF) and the impulse response functions (IRF) of a structural system. This chapter explains the concepts of both and the procedures for deriving them from measured input and output data. From the underlying design of the seats, a generalized two DoF model of a typical shock mitigating seat was developed. The model assumes the absence of dry friction in the seat mechanism while taking into account the viscous damping. This helped in identifying the FRFs and IRFs that needed to be measured so that the sensor placement could be implemented accordingly. Further to processing the

data, impulse and frequency responses were obtained, which are required as inputs to the ERA parameter identification procedure.

4.2 Signal Processing

Typical acceleration profiles obtained from drop tests are shown in Figure 13 for a 6 inch drop. It is to be noted that the signals are unprocessed. As is observed, the

Figure 13: Raw acceleration signals from a 6 inch drop of the Shockwave seat.

signals seem to be corrupted by noise from structural vibration. The first big spike observed right after 4 sec is attributed to the moment when the hook is released. The region of interest covers the waveforms after 4.2 sec.

The mean value of each signal was subtracted to remove the gravity offset. Subsequently, any trends in the signals were removed by detrending since the signal window required for analysis is of a finite length and the discrete Fourier transform (DFT) generally works for a signal of infinite length. Detrending a signal removes discontinuities arising from interpolated points in the finite signal and makes it more conducive for fast Fourier transform (FFT) analyses. The low dynamic variations in time in a signal are traced by detrending. Detrending involves identifying a linear interpolation function that fits the original waveform best [37].

The data was then filtered at 30 Hz to remove the unwanted noise resulting from structural vibration. A fourth order dual low pass Butterworth filter was used for this purpose. The filtered signals are shown in Figure 14

4.3 Derivation of Displacement Waveform from Acceleration

A displacement waveform was needed to gauge the maximum displacement of the seat-pan. Another purpose of obtaining displacement profiles was to derive the impulse and frequency responses for the intended extraction algorithm. To this end, a linear potentiometer (76.1 mm stroke) was initially chosen. However, since the displacements were considerably greater than the range of the sensor, it broke during trial runs. Thus the displacement profiles were obtained using numerical integration.

The integration can be performed both in the time and frequency domains.

Figure 14: Filtered acceleration signals from a 6 inch drop of the Shockwave seat.

Usually, numerical integration of acceleration signals in the time domain is done using the trapezoidal rule wherein the area under the acceleration curve is approximated by summing the infinitesimal elements [38]. Mathematically,

$$v(t) = \int a(t)dt \approx \sum_{i=1}^N \left[\frac{a_i + a_{i-1}}{2} \right] \Delta t \quad (17)$$

and

$$x(t) = \int v(t)dt \approx \sum_{i=1}^N \left[\frac{v_i + v_{i-1}}{2} \right] \Delta t \quad (18)$$

where:

$a(t)$ is the acceleration waveform in time

a_i, a_{i-1} is the i th and $(i-1)$ th samples of acceleration profile;

$v(t)$ is the derived velocity waveform in time;

$x(t)$ is the derived displacement waveform in time;

Δt is the sampling interval in time; and

N is the block size.

The trapezoidal rule for numerical integration is applied in MATLAB through the **cumtrapz.m** and **trapz.m** subroutines. However, the major drawback of using time domain double integration to obtain displacement is that any DC offset tends to dominate the result hence affecting the accuracy of estimation [[38], [28]].

The other way of deriving displacement from acceleration is by integrating the acceleration waveform in the frequency domain [39]. To illustrate this method, first consider the displacement as a cosine waveform for which

$$x(t) = A \sin(\omega t) \quad (19)$$

The velocity and acceleration waveforms are then derived respectively by differentiation as

$$v(t) = \frac{dx(t)}{dt} = A\omega \cos(\omega t) = A\omega \sin(\omega t + \frac{\pi}{2}) \quad (20)$$

and

$$a(t) = \frac{dv(t)}{dt} = -A\omega^2 \sin(\omega t) = A\omega^2 \sin(\omega t + \pi) \quad (21)$$

Thus it can be seen that displacement leads velocity by a phase difference of $\pi/2$ and acceleration by π .

To perform integration in the frequency domain, the discrete Fourier transform (DFT) relationship between the time-domain and frequency-domain waveforms are first considered, such that

$$\text{Acceleration : } \ddot{x}(t) = \int_{-\infty}^{+\infty} \ddot{X}(f) e^{2\pi i f t} df \quad (22)$$

$$\text{Velocity : } \dot{x}(t) = \int_{-\infty}^{+\infty} \dot{X}(f) e^{2\pi i f t} df \quad (23)$$

$$\text{Displacement : } x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi i f t} df \quad (24)$$

Thus, the waveforms in the time and frequency domains are interchangeable. The relationship between velocity and acceleration waveforms in the frequency domains is then derived as

$$a(t) = \ddot{x} = \frac{d\dot{x}(t)}{dt} \quad (25)$$

Substituting Equation (22) into the above equation, we obtain

$$\begin{aligned}\ddot{x} &= \frac{d}{dt} \left[\int_{-\infty}^{+\infty} \dot{X}(f) e^{2\pi i f t} df \right] \\ &= \int_{-\infty}^{+\infty} \dot{X}(f) \frac{d}{dt} [e^{2\pi i f t}] df\end{aligned}\quad (26)$$

Thus,

$$\ddot{x}(t) = \int_{-\infty}^{+\infty} \dot{X}(f) 2\pi i f e^{2\pi i f t} df \quad (27)$$

Comparing the integrand of the R.H.S in Equation (27) to that of the R.H.S in Equation (22), the relationship is obtained as

$$\ddot{X}(f) = \dot{X}(f) 2\pi i f \quad (28)$$

Knowing that $\omega = 2\pi f$, the velocity profile can be derived from the acceleration waveform using

$$\dot{X}(f) = \frac{\ddot{X}(f)}{2\pi i f} = \frac{\ddot{X}(f)}{i\omega} \quad (29)$$

By following a similar derivation for displacement, its waveform is obtained from acceleration as

$$X(f) = \frac{\ddot{X}(f)}{(2\pi i f)^2} = \frac{\ddot{X}(f)}{(i\omega)^2} \quad (30)$$

Thus, the same phase-difference relationships are obtained between acceleration, velocity, and displacement in the frequency domain as in the time-domain, where a 90° difference in the time domain translates to a shift by i in the frequency domain.

However, it is to be noted in Equations (29) and (30) that as f becomes very small (i.e. $f < 1$ Hz), $\dot{X}(f)$ and $X(f)$ tend to become mathematically indeterminate.

This is known as ‘ $\frac{1}{f}$ noise’. The problem can be addressed by zeroing $\ddot{X}(f)$ values below 1 Hz.

Thus, integration in the time domain corresponds to division by $i\omega$ in the frequency domain. An advantage of using the frequency domain direct integration technique is that it is least prone to errors below 10 Hz [39], and is thus more conducive to the analysis of whole-body vibrations since the critical frequencies of interest lie within this range. It has been determined that numerical integration of the acceleration waveform in the frequency domain is the more accurate technique of the two [[38], [39], [28]] and thus was the chosen method used in this derivation. The technique was implemented by coding it in MATLAB.

4.4 Frequency Response Function

The concept of the frequency response function (FRF) is of paramount importance to experimental modal analysis (EMA). Modelling of a structural system is usually achieved by measuring its frequency response to excitation. More often than not, EMA involves measurement of vibratory response at some degree of freedom when excited at the same or another location at some excitation frequency [40]. FRFs resulting a from combination of responses and excitations of different degrees of freedom are grouped together in a matrix called the FRF matrix.

Rarely, real world structures are modelled as single-DoF systems. However, the single DoF is important as any multi-DoF system can always be modelled as a linear superposition of characteristics resulting from single DoF discretization of elements [41].

Figure 15: Viscously damped SDoF system

If the mass in Figure 15 is excited by a harmonic function $F(t)$, the response $x(t)$ is also harmonic with the same frequency ω [40]. Letting the excitation function be

$$F(t) = F(\omega)e^{i\omega t}$$

and the response function be

$$x(t) = X(\omega)e^{i\omega t}$$

The equation of motion for this single DoF system is easily derived as

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \quad (31)$$

By substituting the expressions for the excitation and response functions defined before in the above equation, the following relationship is obtained

$$\frac{X(\omega)}{F(\omega)} = \frac{1}{k - \omega^2 m + i\omega c} \quad (32)$$

The above ratio of the response in frequency domain to the excitation in the frequency domain is called *receptance frequency response function* or simply receptance FRF,

$H(\omega)$, and is generally expressed as the following when normalized with respect to mass

$$H(\omega) = \frac{m^{-1}}{\omega_n^2 - \omega^2 + 2i\omega\omega_n\zeta} \quad (33)$$

where

ω_n is the natural frequency; and

ζ is the damping ratio.

The FRF is generally a complex function but becomes real in the absence of damping. It is easily seen that the FRF is a function of only the system parameters (k, m , and c) or (ω_n, ζ) when Equation (33) is considered. Thus intuitively, it can be inferred that measurement of the system's FRF enables deriving the system parameters, given the forcing function and the response.

For a multi DoF system, a generic equation known as the *generalized superposition equation of the FRF* can also be derived [28] and is expressed as

$$H_{pq}(j\omega) = \sum_{r=1}^N \left[\frac{A_{pqr}}{j\omega - s_r} + \frac{A_{pqr}^*}{j\omega - s_r^*} \right] \quad (34)$$

where:

N is the number of degrees of freedom;

A_{pqr} is the residue matrix;

A_{pqr}^* is the conjugate residue matrix; and

s_r and s_r^* are the poles and conjugate poles of the system. An important conclusion that can be implied from the above equation is called *Maxwell's reciprocity relation* stating that the FRF between two points is the same even when the excitation and

response points are interchanged. Mathematically,

$$H_{pq}(j\omega) = H_{qp}(j\omega) \quad (35)$$

FRFs are generally displayed as plots of magnitude versus frequency and phase versus frequency. There are various ways of measuring FRFs depending on whether the response is displacement, velocity, or acceleration when the structure is subjected to a given forcing function. They are listed in the following table.

Response Parameter <i>R</i>	FRF type: <i>R/F</i>
Displacement	Receptance
Velocity	Mobility
Acceleration	Accelerance/Inertance

Table 4: FRF terminology [41].

In EMA, FRFs can be obtained by various estimators depending on the extent to which noise is present in the input and output signals. All these estimators involve the discrete Fourier transform through the use of auto-correlation and cross-correlation functions [42] and their respective spectral densities.

$$\text{Auto - spectral density : } S_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{xx}(\tau) 2\pi i e^{-i\omega\tau} d\tau \quad (36)$$

$$\text{Cross - spectral density : } S_{xy}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_{xy}(\tau) 2\pi i e^{-i\omega\tau} d\tau \quad (37)$$

where:

$R_{xx}(\tau) = E[x(t)x(t + \tau)] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T R_{xx}x(t)x(t + \tau) dt$ is the auto-correlation function;

$R_{xy}(\tau) = E[x(t)y(t + \tau)] = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T R_{xy}x(t)y(t + \tau) dt$ is the cross-correlation

function;

T is the time-period of the wave; and

τ is the shift in time.

If the input signal is noise affected, the following H_1 estimator is used

$$H_1(\omega) = \frac{S_{xy}(\omega)}{S_{xx}(\omega)} \quad (38)$$

A second, H_2 , estimator is applied when the output signal is noise corrupted

$$H_2(\omega) = \frac{S_{xx}(\omega)}{S_{xy}(\omega)} \quad (39)$$

If both the signals have noise, then the following estimator is used

$$H_3(\omega) = \sqrt{\frac{S_{xx}}{S_{yy}} \cdot \frac{S_{xy}}{|S_{xy}|}} \quad (40)$$

Another way of obtaining the FRFs is derivation via the IRFs. As will be discussed in the next section, the FRF $H(\omega)$ and the IRF $h(t)$, are a Fourier transform pair and thus the FRF can be obtained by performing a discrete Fourier transform of the IRF such that

$$H(\omega) = DFT[h(t)] \quad (41)$$

4.5 Impulse Response Function

The impulse response function (IRF) is a powerful tool in modal analysis. It is the time-domain equivalent of the frequency response function, and is widely used to identify genuine modes of vibration and noise transmission paths [42]. Consider the following relationship between input and response for a system with transfer function

$H(s)$:

$$Y(s) = X(s)H(s) \quad (42)$$

The above equation in the Laplace domain, when represented in time domain, corresponds to the following convolution integral

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t - \tau) d\tau \quad (43)$$

where $h(t)$ is the inverse Laplace transform of the system's transfer function, and $H(s)$ is called the *impulse response* of the system [28]. The longer a system's impulse response is, the longer is the contribution of the excitation weighted together to produce the output at any instant of time [28]. A longer impulse response means higher vibrations and happens in systems with less damping. Mathematically, $h(t)$ is the output obtained from a system when it is subjected to a *Dirac delta impulse* function, $\delta(t)$. The Dirac delta impulse is an idealized mathematical function such that

$$\begin{aligned} \delta t &= 0 \quad \forall t \neq 0 \\ \int_{0^-}^{0^+} \delta(t) dt &= 1 \quad \forall t > 0 \end{aligned} \quad (44)$$

The above conditions imply that the Dirac impulse function has infinitesimal width and infinite magnitude such that the area under it on the time-axis is unity. It is practically impossible to generate such an impulse function as input to a system to obtain the impulse response. However, there are two ways of deriving the impulse response of a system.

The first method involves derivation from the FRF. For a *linear, time-invariant, and causal* system, let $H(\omega)$ and $h(t)$ be its FRF and IRF respectively. Considering the Dirac impulse as an input signal, $x(t) = \delta(t)$, such that its output is the impulse

response, $y(t) = h(t)$ [42], the Fourier transforms of $x(t)$ and $y(t)$ are

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \delta(t) e^{-i\omega t} dt = \frac{1}{2\pi} \quad (45)$$

$$Y(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} h(t) e^{-i\omega t} dt = \frac{1}{2\pi} \quad (46)$$

Since,

$$Y(\omega) = H(\omega)X(\omega) \quad (47)$$

Substituting Equations (47) and (48) into the above equation, yields

$$H(\omega) = \int_{-\infty}^{+\infty} h(t) e^{-i\omega t} dt \quad (48)$$

Performing the inverse Fourier transform on $H(\omega)$, results in

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\omega) e^{i\omega t} dt \quad (49)$$

Thus it is seen that the FRF and IRF are a Fourier transform pair and that the IRF can be obtained by simply performing the inverse Fourier transform on the FRF. The FRF can be obtained by using either of the techniques discussed in the foregoing section.

The second method [[43] [44]] makes use of the following input-output relationship for a linear system with a characteristic impulse response, $h(t)$, such that

$$y(t) = x(t)h(t) \quad (50)$$

If the signals are digitally sampled and the sampling time step is T with the number of measured samples being L , the IRF can be expressed as

$$h[n] = h(t)|_{t=nT} \quad \forall n = 0, 1, 2, \dots, L-1 \quad (51)$$

Letting the duration of the input signal, $x(t)$, be $M(< L)$, the following representation could be made

$$x[n] = x(t)|_{t=nT} \quad \forall n = 0, 1, 2, \dots, M-1 \quad (52)$$

Since the output is related to the input and the impulse response through a convolution integral of the latter two in the continuous time domain as represented in Equation (52), its discrete equivalence can be expressed as a convolution sum of the two. Thus,

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{+\infty} x[m]h[n-m] \quad (53)$$

Utilizing the symmetric property of convolution and the important implication that the discrete time-domain convolution operator, $*$, is essentially a Toeplitz matrix operator $\mathbf{X}$ [[43], [44]], constructed from the sampled input signal

$$\mathbf{X} = \begin{bmatrix} x[1] & 0 & \cdots & 0 \\ x[2] & x[1] & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ x[m] & x[m-1] & \cdots & x[1] \end{bmatrix} \quad (54)$$

Hence, Equation (55) can be recast as

$$y = \mathbf{X}h \quad (55)$$

A look at the above equation shows that the IRF vector, h , is derivable by matrix inversion, given vectors $\mathbf{X}$ and y are measured. Pre-multiplying both sides of Equation (57) by $\mathbf{X}^{-1}$, results in

$$h = \mathbf{X}^{-1}y \quad (56)$$

It is to be mentioned that rarely the Toeplitz convolution matrix, $\mathbf{X}$, is a square matrix and thus inversion is not possible. However, the problem can be overcome by utilizing the Moore-Penrose pseudo inverse principle [45] which essentially operates by performing a least squares solution of the intended matrix to be inverted. Using this approach, an estimation of the impulse response vector, $\tilde{h}$, can be obtained as

$$\tilde{h} = \tilde{\mathbf{X}}^{-1}y \quad (57)$$

where $\tilde{\mathbf{X}}^{-1} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T$ is the least squares estimated inverse matrix of the non-square Toeplitz matrix, $\mathbf{X}$.

A major advantage of using the second method is that the IRFs are readily obtained from the measured input and output signals, without having to find the FRFs using the discrete Fourier transform, hence reducing the likelihood of encountering spectral leakage which occurs when the signal energy within a designated frequency bin of the sampled waveform leaks out to the surrounding frequency bins due to aperiodic Fourier components in the discrete signal with respect to its fundamental frequency. Also, simulation conducted in literature [44] using a Dirac delta impulse input to a linear system, had proved that an IRF estimated using the second method is more accurate than the one estimated from the first method. As a result, in this project the second method was implemented in MATLAB and the Toeplitz matrix convolution operator was inverted using the **pinv.m** subroutine. The IRF has another useful application.

The IRF or free decay response can be used to determine the damping in a system by applying the *logarithmic decrement method*. Let the amplitudes of two successive peaks (x_1 and x_2) in the IRF be chosen such that the logarithmic decrement δ be expressed as:

$$\delta = \ln \left(\frac{x_1}{x_2} \right)$$

The damping ratio ζ is then calculated from the following equation [46]

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}} \quad (58)$$

The logarithmic decrement method becomes less accurate if the actual damping ratio in the system approaches 0.5 or more.

4.6 Mathematical Modelling of the Shock Mitigating Seat

A suitable mathematical model of a typical shock-mitigating seat was required to find out which degree(s) of freedom in the seat's structure needed sensor instrumentation for measuring the input-output signals. In particular, the seat-occupant system was considered for modelling so as to replicate the real world operating scenario as closely as possible. The modelling approach was also important since this would dictate generation of the right frequency response and impulse response functions to be used for the intended parameter extraction algorithm. Absolute coordinates were referenced to the carriage. From the design of a typical shock-mitigating seat and the dynamic deck-seat-occupant interaction, a simple two DoF model was considered to aid sensor placement and signal analysis. Figure 16 shows the 2-DoF model. The

Figure 16: Two DoF representation of the seat-occupant system

base in the model represents the carriage on the drop tower where the seats were fixed. In real conditions, the base/carriage represents the deck of a high-speed boat.

The annotations in the model represent the following

Base is the carriage on the drop tower;

m_1 is the mass of the seat-pan;

m_2 is the mass of the occupant;

k_1 is the equivalent linear spring stiffness of the seat's shock absorber;

k_2 is the equivalent linear spring stiffness of the seat foam;

c_1 is the viscous damping in the seat's shock absorber;

c_2 is the viscous damping in the seat's foam;

z_b is the absolute displacement of the base (carriage);

z'_1 is the absolute displacement of the seat-pan; and

z'_2 is the absolute displacement of the occupant mass.

The model includes viscous damping but neglects dry friction in the linear guides and the mechanical motion within the damper and suspension. This is especially

Figure 17: Free body diagrams of the seat's dynamics.

the case of the Shockwave seat where the seat-pan is guided through a well greased slider-rail system.

The model's dynamics is assumed to be *linear, time-invariant, and causal* since these are the precepts on which the entire analyses covered in this work are based. The dynamic interactions between different components in the model are shown in the Figure 17 depicting the free-body diagrams.

Kinematic variables are defined as follows

$\ddot{z}'_b$ be the absolute acceleration of the base;

$\dot{z}'_b$ be the absolute velocity of the base;

$\dot{z}'_1$ be the absolute velocity of the seat-pan;

$\dot{z}'_2$ be the absolute velocity of the occupant mass;

$\ddot{z}'_1$ be the absolute acceleration of the seat-pan; and

$\ddot{z}'_2$ be the absolute acceleration of the occupant mass.

Applying Newton's second law of motion to the seat-pan mass, results in

$$\sum_{m_1} F = m_1 \ddot{z}'_1$$

giving

$$-k_1(z'_1 - z_b) - c_1(\dot{z}'_1 - \dot{z}'_b) - k_2(z'_1 - z'_2) - c_2(\dot{z}'_1 - \dot{z}'_2) = m_1\ddot{z}'_1 \quad (59)$$

Applying Newton's second law to the occupant mass results in

$$\sum_{m_2} F = m_2\ddot{z}'_2$$

giving

$$k_2(z'_1 - z'_2) + c_2(\dot{z}'_1 - \dot{z}'_2) = m_2\ddot{z}'_2 \quad (60)$$

The following relative displacements are then considered by referencing all absolute coordinates with respect to that of the base

$$z_1 = z'_1 - z_b$$

$$z_2 = z'_2 - z_b$$

Substituting the new coordinates defined above into Equations (60) and (61), the equations of motions are recast so that the following matrix equation of motion is obtained

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -m_1\ddot{z}_b \\ -m_2\ddot{z}_b \end{bmatrix} \quad (61)$$

where:

$\ddot{z}_1$ is the acceleration of the seat-pan relative to the base;

$\ddot{z}_2$ is the acceleration of the occupant mass relative to the base;

$\dot{z}_1$ is the velocity of the seat-pan relative to the base;

$\dot{z}_2$ is the velocity of the occupant mass relative to the base;

z_1 is the displacement of the seat-pan relative to the base; and

z_2 is the displacement of the seat-pan relative to the base.

The forcing function column is defined as the vector

$$\begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} = \begin{bmatrix} -m_1 \ddot{z}_b \\ -m_2 \ddot{z}_b \end{bmatrix}$$

The redefined MIMO system can next be represented by employing a fixed support instead of a moving base as shown in Figure 18

Figure 18: Redefined MIMO representation (base-referenced coordinates)

4.6.1 Analytical FRFs from Model

Analytical expressions of the frequency response functions can now be obtained from the model's derived equations of motion. A useful way of deriving them is by working in the Laplace domain.

Taking the Laplace transform of Equation (62), results in

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} s^2 z_1(s) \\ s^2 z_2(s) \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} s z_1(s) \\ s z_2(s) \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} z_1(s) \\ z_2(s) \end{bmatrix} = \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \quad (62)$$

Simplifying the above equation, leads to

$$\begin{bmatrix} s^2 m_1 + (k_1 + k_2) + (c_1 + c_2)s & -k_2 - c_2 s \\ k_2 - c_2 s & s^2 m_2 + k_2 + c_2 s \end{bmatrix} \begin{bmatrix} z_1(s) \\ z_2(s) \end{bmatrix} = \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \quad (63)$$

which implies that

$$\begin{bmatrix} z_1(s) \\ z_2(s) \end{bmatrix} = \begin{bmatrix} s^2 m_1 + (k_1 + k_2) + (c_1 + c_2)s & -k_2 - c_2 s \\ k_2 - c_2 s & s^2 m_2 + k_2 + c_2 s \end{bmatrix}^{-1} \begin{bmatrix} F_1(s) \\ F_2(s) \end{bmatrix} \quad (64)$$

Since, a square 2×2 matrix is inverted as;

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = (ad - bc) \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (65)$$

after carrying out the matrix inversion, Equation (65) can be re-written as

$$\begin{Bmatrix} z_1(s) \\ z_2(s) \end{Bmatrix} = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \begin{Bmatrix} F_1(s) \\ F_2(s) \end{Bmatrix} \quad (66)$$

To express the above equation in the frequency domain, $s = j\omega$ is substituted, resulting in the following

$$\begin{Bmatrix} z_1(j\omega) \\ z_2(j\omega) \end{Bmatrix} = \begin{bmatrix} H_{11}(j\omega) & H_{12}(j\omega) \\ H_{21}(j\omega) & H_{22}(j\omega) \end{bmatrix} \begin{Bmatrix} F_1(j\omega) \\ F_2(j\omega) \end{Bmatrix} \quad (67)$$

Further compacting the equation results in

$$\begin{Bmatrix} z_1(j\omega) \\ z_2(j\omega) \end{Bmatrix} = [\mathbf{H}(j\omega)]_{2 \times 2} \begin{Bmatrix} F_1(j\omega) \\ F_2(j\omega) \end{Bmatrix} \quad (68)$$

where $[\mathbf{H}(j\omega)]$ is called the *frequency response function matrix* or simply, FRF matrix for all degrees of freedom in the system. The entries in this matrix are

$$H_{11}(j\omega) = \frac{1}{\Delta}(-m_2\omega^2 + k_2 + j\omega c_2)$$

$$H_{12}(j\omega) = \frac{1}{\Delta}(k_2 + j\omega c_2)$$

$$H_{21}(j\omega) = \frac{1}{\Delta}(k_2 + j\omega c_2)$$

$$H_{22}(j\omega) = \frac{1}{\Delta}(-m_1\omega^2 + k_1 + k_2 + j\omega c_1 + j\omega c_2)$$

where $\Delta = [(-\omega^2 m_1 + (k_1 + k_2) + j\omega(c_1 + c_2))(-\omega^2 m_2 + k_2 + j\omega c_2) - (-k_2 - j\omega c_2)(k_2 - j\omega c_2)]$

In modal analysis parlance, $H_{ij}(\omega)$ is the FRF obtained with response at the i th degree of freedom of a structure when it is excited at the j th degree of freedom. It is seen that $H_{12}(j\omega) = H_{21}(j\omega)$, which holds true for any FRF matrix owing to *Maxwell's reciprocity*. FRFs $H_{11}(j\omega)$ and $H_{22}(j\omega)$ are called *driving point FRFs*, while FRFs $H_{12}(j\omega)$ and $H_{21}(j\omega)$ are called *cross-point FRFs*. The resonant frequencies of a structure are always inferred from the plots of the driving point FRFs.

4.6.2 State-space Representation

The concept of state-space comes from modern control engineering and is important to time-varying MIMO systems. For the case of dynamic systems, *state* is defined as the least set of variables (or *state variables*) such that the behaviour of a system can be completely determined at any instant $t \geq t_0$ by knowing these variables at $t = t_0$ and the input function at $t \geq t_0$ [47]. The collection of the state variables in a vector which determines the behaviour of a system is called the *state vector*.

The n -dimensional space whose coordinate axes are made up of the n -state variables is known as *state space*. The basis of modelling using state space analysis lies in knowing three vectors, namely, input variables, output variables, and state variables. It needs to be mentioned that the state-space representation of a system is not unique; nonetheless, the number of state variables is always the same for any derived representation. Rewriting the derived equation of motion yields

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} +$$

$$\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} F_1(t) \\ F_2(t) \end{bmatrix} \quad (69)$$

Expanding the above matrix equation,

$$m_1 \ddot{z}_1 + (c_1 + c_2) \dot{z}_1 - c_2 \dot{z}_2 + (k_1 + k_2) z_1 - k_2 z_2 = F_1(t)$$

$$m_2 \ddot{z}_2 - c_2 \dot{z}_1 + c_2 \dot{z}_2 - k_2 z_1 + k_2 z_2 = F_2(t)$$

Solving for $\ddot{z}_1$ and $\ddot{z}_2$ in the above two equations, results in

$$\ddot{z}_1 = \frac{1}{m_1} [F_1(t) - (c_1 + c_2) \dot{z}_1 + c_2 \dot{z}_2 - (k_1 + k_2) z_1 + k_2 z_2] \quad (70)$$

and

$$\ddot{z}_2 = \frac{1}{m_2} [F_2(t) + c_2 \dot{z}_1 - c_2 \dot{z}_2 + k_2 z_1 - k_2 z_2] \quad (71)$$

To obtain the state-space equations, the above set of two second order equations are converted into four first order equations by letting the system's state-vector and output-vector be x and y respectively. The variables in the state vector are defined as given below

$$x_1 = z_1$$

$$x_2 = \dot{z}_1$$

$$x_3 = z_2$$

$$x_4 = \dot{z}_2$$

The following relationships between the states and their derivatives can be observed from the above definitions

$$\dot{z}_1 = x_2 = \dot{x}_1$$

$$\dot{z}_2 = x_4 = \dot{x}_3$$

$$\ddot{z}_1 = \dot{x}_2$$

$$\ddot{z}_2 = \dot{x}_4$$

Recasting Equations (71) and (72) in terms of the new coordinates, leads the following set of state-space equations

$$\dot{x}_1 = x_2 \quad (72)$$

$$\dot{x}_2 = \frac{1}{m_1}[F_1(t) - (c_1 + c_2)x_2 + c_2x_4 - (k_1 + k_2)x_1 + k_2x_3] \quad (73)$$

$$\dot{x}_3 = x_4 \quad (74)$$

$$\dot{x}_4 = \frac{1}{m_2}[F_2(t) + c_2x_2 - c_2x_4 + k_2x_1 - k_2x_3] \quad (75)$$

The above set of four first order state space equations can be compacted into a matrix form as shown below

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{1}{m_1}(k_1 + k_2) & -\frac{1}{m_1}(c_1 + c_2) & \frac{k_2}{m_1} & \frac{c_2}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} & -\frac{k_2}{m_2} & -\frac{c_2}{m_2} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{F_1(t)}{m_1} \\ 0 \\ \frac{F_2(t)}{m_2} \end{Bmatrix} (1) \quad (76)$$

The above equation is now written in the following form

$$\dot{x} = \mathbf{A}x + \mathbf{B}f \quad (77)$$

To account for the case where the output is not just the state but is some linear combination of the states, an output matrix $\mathbf{C}$ is defined to relate the output to the states and a transmission matrix $\mathbf{D}$ is defined for the outputs that are related to the inputs that bypass the state [48]. Thus,

$$\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (1) \quad (78)$$

where the output equation above is conveniently represented as

$$y = \mathbf{C}x + \mathbf{D}f \quad (79)$$

Equations (77) and (79) together form the state-space equations for the model. The state-space representation is important since determination of the system matrix, $\mathbf{A}$, is the objective of most modal parameter identification algorithms and is the matrix from which the system properties are computed. The state-space system, in general, is represented by the block diagram in Figure 19.

Figure 19: Block diagram of state-space representation [48]

Chapter 5

Modal Parameter Extraction from Drop Test Data

5.1 Introduction

The process of estimating the parameters of a dynamic system is called system identification. Depending on whether frequency response or impulse response data is used, system identification methods could be classified as frequency-domain or time-domain techniques.

More often than not, the system identification of dynamic systems makes use of modal analysis techniques to determine the inherent properties. Modal analysis has become an integral mechanism in the field of vibration and noise engineering to determine, and optimize the structural and dynamic characteristics of any system. It is the process of determining the inherent dynamic characteristics of a system in the forms of natural frequencies, damping factors, and mode shapes, and using them to formulate a mathematical model for its dynamic behaviour. The formulated mathematical model is referred to as the modal model of the system and the information for the characteristics are known as its modal data [40]. Numerical modelling

Frequency domain	Time domain
Peak-picking	LS complex exponential
Circle-fit	ERA
Polyreference frequency domain	

Table 5: Parameter estimation methods in different domains.

using computers to determine and fully define the dynamic characteristics could be insufficient in situations where damping, uncertainty in boundary conditions, and innate non-linearity in a system could not be totally described by traditional modelling approaches. In such cases, the advent of modern computational techniques like fast Fourier transform, data acquisition systems, coupled with the principles of modal analysis have proven to be highly effective in solving system identification problems related to dynamics.

In regards to the work dealt with in this chapter, a number of established modal analysis techniques were considered as a means of selecting the most appropriate method to be implemented towards identifying the modal properties of the representative shock mitigating seats.

5.2 Peak-picking Method

The ‘Peak-picking method’ is the simplest of all methods to analyse the FRF data in order to derive the mathematical model of a structure by dynamic testing. It also goes by the name of ‘half-power method’. The method makes a single degree of freedom (SDoF) assumption for the local resonant mode(s) and all the points in the proximity are assumed to be from a SDoF system [41].

The peak-picking method works well for a system with modes which are sufficiently well separated so as to be resolved for SDoF analysis. Thus structures which are medium-damped, that is neither too lightly nor too heavily damped, are conducive to this form of analysis. This is because accurate measurements at resonance are difficult to obtain with very lightly damped structures; and, in the case of heavily damped structures, the response at resonance also gets influenced by other modes.

Figure 20: Locating resonant modes of FRF for SDoF assumption [40].

The procedure for this method is as follows [41]

- (i) In the first step, the FRF is plotted on a magnitude versus frequency graph and a resonant mode in the form of a clear peak $|\alpha_r(\omega)|_{max}$ is identified (Figure 20). The corresponding frequency of the peak gives the natural frequency (ω_r) of that mode.
- (ii) The second step proceeds with the estimation of damping factor, as shown in Figure 21. For the estimation, the half-power points with magnitude $|\alpha_r(\omega)|_{max}/\sqrt{2}$

are located on the frequency axis as ω_a and ω_b . The damping loss factor η_r or the damping ratio ζ_r can then be found using the following formulae

$$\eta_r = \frac{\omega_b^2 - \omega_a^2}{2\omega_r^2} \quad (80)$$

$$\zeta_r = \frac{\omega_b^2 - \omega_a^2}{4\omega_r^2} \quad (81)$$

Figure 21: Identifying half-power frequencies of the peak [40].

(iii) Lastly, an estimate of the modal constant of the identified mode is calculated by assuming that the net response could be attributed to the contribution of this single term. Thus,

$$\alpha_{max} = \frac{A_r}{\eta_r \omega_r^2} \quad (82)$$

The modal constant can then be estimated as

$$A_r = \alpha_{max} \eta_r \omega_r^2 \quad (83)$$

In the case of viscous damping, the modal constant is estimated as

$$A_r = 2\alpha_{max} \zeta_r \omega_r^2 \quad (84)$$

Literature on this method [[40], [41]] note some reasonable limitations of using it. A study of the steps involved convey that the modal parameters draw on the accuracy of the FRF production from experimental data, and since the FRF is susceptible to extraneous noise as well as the instrumentation accuracy, this could be somewhat tricky for manipulation. Given this, error estimates are also denser near the resonant mode than any other region, especially for lightly-damped structures.

The other drawback is caused by the SDoF assumption of the method. Practical experience with this method has shown that even with well-separated modes, the neighbouring modes do have considerable contribution to the total response at resonance [40]. The effect is that an over-estimate of the damping value is obtained, thereby leading to a fallacious modal constant. The damping values are further made prone to error since, while using the half-power line on the peak FRF curve, a shape-preserving interpolation is performed in order to estimate the points and it is unlikely that the two correspond to the measured data points. Even though this method seems less useful from the point of view of application, its merit could be realized in complex cases when an initial estimate of structural parameters is needed to be applied to other curve-fitting routines such as the circle-fit, and inverse FRF straight-line fit methods, in an iterative manner to converge to accurate values.

5.3 Circle-fit Method

The circle-fit method is the most commonly used SDoF technique used in modal analysis. The method is based on an important property of the FRF in the vicinity of a resonant mode. It is generally known that the Nyquist plot of FRF for a SDoF system around the resonant mode is a circle. The completeness of the circle depends on the level of damping in a structural system.

The popular convention followed in selecting the proper FRF for modal analysis depends on the type of damping present in the structure. If the structure has structural damping (η) then receptance from of the FRF is used to produce the circle in a Nyquist plot. In the case where viscous damping is present, the mobility FRF is incorporated for Nyquist analysis [41].

Assuming structural damping in the system, the circularity of the Nyquist plot at resonance is observed as [40]

$$(Re(\alpha))^2 + \left(Im(\alpha) + \frac{1}{2h}\right)^2 = \left(\frac{1}{2h}\right)^2 \quad (85)$$

where h is the structural damping coefficient.

Figure 22: Nyquist circle and its shift by the complex constant term [40].

This unique property of circularity of the FRF around the resonant frequency is an effective technique of modal analysis, and although a legacy method, it still finds use in some industry applications. Unlike some other SDoF methods like the peak-picking technique, the circle-fit method does consider the contribution of modes that are in proximity to the resonant peak [41]. If the receptance FRF is considered, then its

expression is given by

$$\alpha_{jk}(\omega) = \sum_{r=1}^N \frac{{}_r A_{jk}}{\lambda_r^2 - \omega^2} \quad (86)$$

where ${}_r A_{jk}$ is the modal constant, λ_r is the eigenvalue, and ω is the angular frequency.

If the analysis is concentrated on the ' r 'th mode, then the FRF expression could be decomposed as

$$\alpha_{jk}(\omega) = \frac{{}_r A_{jk}}{\lambda_r^2 - \omega^2} + \sum_{s=1}^N \frac{{}_s A_{jk}}{\lambda_s^2 - \omega^2} \quad (87)$$

where $s \neq r$. The equation can be further expanded by assuming a constant complex term for the summation, that is

$$\alpha_{jk}(\omega) \approx \frac{{}_r A_{jk}}{\omega_r^2 - \omega^2 + j\eta_r \omega_r^2} + B_{jk} \quad (88)$$

The effect of introducing the complex constant term is that the circle is shifted away from the origin by a distance representative of the complex constant term in equation (89). The shift is pictorially shown in Figure 22. This is not considered since analysis is focused on the SDoF resonant frequency of the Nyquist circle.

The second step is to derive the natural frequency from the plotted circle. Figure 23 shows a typical diagram of the Nyquist circle on which further analyses are made. A Nyquist circle is first constructed by taking the data points around the natural frequency mark. Usually six to eight proximate points, including the natural frequency of the peak, are considered for curve fitting them to a circle. Once the circle is constructed, the natural frequency ω_r could be located at the point where the arc change is maximum on the circle. Analytically, this can be derived as

Figure 23: Derivation of natural frequency [40].

$$\tan(\theta/2) = \tan\left(\frac{\pi}{2} - \gamma\right) = \frac{\text{Re}(\alpha)}{\text{Im}(\alpha)} = \frac{\omega_r^2 - \omega^2}{\omega_r^2 \eta_r} \quad (89)$$

or

$$\omega^2 = \omega_r^2 \left(1 - \eta_r \tan\left(\frac{\theta}{2}\right)\right) \quad (90)$$

where θ and γ are angles in the coordinates of circle and origin respectively to locate the point. For deriving the natural frequency, the rate of change of theta with respect to $d\omega^2$ is equated to zero. Thus

$$\frac{d\theta}{d\omega^2} = \left(\frac{-2}{\omega_r^2 \eta_r}\right) \frac{\eta_r^2 \omega_r^4}{\eta_r^2 \omega_r^4 + (\omega_r^2 - \omega^2)^2} = 0 \quad (91)$$

Solving Equation (91), an expression for frequency in terms of the natural frequency could be obtained as

$$\omega^2 = \frac{1 + \sqrt{1 + 3(1 + \eta_r^2)}}{3} \omega_r^2 \quad (92)$$

When the modal damping factor is really small such that its effect could be neglected

at resonance in Equation (93) i.e, $\eta_r^2 \approx 0$, then it follows that

$$\omega_r = \omega \quad (93)$$

While analysing the FRF data, a usual practice is to increase the frequency resolution by performing an interpolation over the data points. Once the natural frequency is located on the circle, the damping loss factor could then be estimated as

$$\eta_r = \frac{\omega_r^2 - \omega_a^2}{\omega_r^2 \tan(\theta/2)} \quad (94)$$

In theory, the same value of damping loss factor could be obtained using any data point on the FRF, but for reasons of noise adulteration in the signals, system nonlinearities and measurement inaccuracies, this is not the typical case. Another way to obtain the damping estimate is to select two points on either sides of the natural frequency. Let the data points selected correspond to frequencies ω_a and ω_b as shown in Figure 24. Then from Equation (95), it follows that

$$\tan(\theta_a/2) = \frac{\omega_r^2 - \omega_a^2}{\omega_r^2 \eta_r} \quad (95)$$

and

$$\tan(\theta_b/2) = \frac{\omega_r^2 - \omega_b^2}{\omega_r^2 \eta_r} \quad (96)$$

Using Equations (96) and (97), the damping loss factor in Equation (95) can be alternatively expressed as (Figure 24)

$$\eta_r = \frac{\omega_a^2 - \omega_b^2}{\omega_r^2} \frac{1}{\tan(\theta_a/2) + \tan(\theta_b/2)} \quad (97)$$

An array of damping values could be obtained by analysing such data points on the FRF (Figure 25). A three-dimensional damping plot could be made by graphing

Figure 24: Estimating damping loss factor [40].

these values against the number of data points, which is a horizontal flat surface in the ideal case. In theory, the same value of damping loss factor could be derived using any data point on the FRF, but for reasons of noise adulteration in the signals, accuracy of the ω_r estimate, system non-linearities, and measurement inaccuracies, this is not the typical case. For an ideal case, this 3-D plot is a horizontal flat surface. It is a useful indicator for assessing the quality of the derived modal data. Figure 25 shows the three-dimensional damping plot with inaccurate and accurate ω_r estimates.

The last step of the circle-fit method is to estimate the modal constant using the following formula

$$|{}_rA_{jk}| = {}_rD_{jk}\omega_r^2\eta \quad (98)$$

where ${}_rD_{jk}$ is the diameter of the Nyquist circle and ${}_rA_{jk}$ is the modal constant whose phase is given by the location of the natural frequency.

Figure 25: 3-D plot of damping values with inaccurate and accurate ω_r value [40]

5.4 Polyreference Frequency Domain MIMO Method

The polyreference frequency domain method is a MIMO technique where the FRFs at all degrees of freedom are processed simultaneously to estimate a global set of modal parameters (frequencies, damping loss factors, and mode shapes). The technique was formulated by Zhang and Kanda [49].

The poly-frequency domain algorithm was laid out for a linear, and viscously damped MIMO mechanical system. However, it also holds true for the case with structural damping. For an experimental case, let there be m measurement coordinates, l reference locations, n effective modes in the frequency domain, and p frequency response function data points. The displacement and velocity impulse response matrices, $H(t)$ and $\dot{H}(t)$ respectively, are then:

$$[H(t)] = [\Phi]Q(t) \quad (99)$$

$$[\dot{H}(t)] = [\Phi]\Lambda Q(t) \quad (100)$$

where:

$Q(t) = [e^{\Lambda t}\Gamma]$ is a $(2n \times 1)$ matrix;

$[\Lambda]$ is a diagonal eigenvalue matrix; and

$[\Gamma]$ is the modal participation matrix.

Taking the Laplace transform of Equations (99) and (100) leads to

$$[H(s)] = [\Phi]Q(s) + R_1 \quad (101)$$

$$[\dot{H}(s)] = [\Phi]\Lambda Q(s) + R_2 \quad (102)$$

where Λ , Φ are $n \times n$ eigenvalue and $m \times n$ eigenvector matrices respectively; R_1 , R_2 are $m \times l$ residual matrices representative of the effect of conjugate poles and out-of-band modes. Substituting $j\omega$ for s to transform Equations (101) and (102) into the frequency domain leads to the following equation

$$\begin{bmatrix} [H_d(j\omega)] \\ [H_v(j\omega)] \end{bmatrix} = \begin{bmatrix} [\Phi] \\ [\Phi] [\Lambda] \end{bmatrix} Q(j\omega) \quad (103)$$

where the residuals have been neglected and $Q(j\omega) = [j\omega[I] - [\Lambda]]^{-1}[\Gamma]$. Since the number of measurement frequencies is greater than the number of system modes during practical testing situations, generally $m > l$. Therefore, the number of rows in the left hand side matrix of the above equation exceeds that of the columns by p . This implies the existence of a $m \times m$ non-trivial matrix $[A]$ such that

$$\begin{bmatrix} [A] & [I] \end{bmatrix} \begin{bmatrix} [\Phi] \\ [\Phi] [\Lambda] \end{bmatrix} = 0 \quad (104)$$

It can now be expressed as a standard eigenvalue problem where

$$A\Phi = \Phi\Lambda \quad (105)$$

and $[A]$ is called the *system matrix* of the structure and is represented in Cartesian coordinates. This matrix contains all the modal properties of the system. The modal frequencies (ω_r) and damping ratios (ζ_r) can be found from the eigenvalues of the system matrix $[A]$ as

$$\omega_r = \sqrt{[Re(\lambda_r)]^2 + [Im(\lambda_r)]^2} \quad (106)$$

$$\zeta_r = \frac{Re(\lambda_r)}{\omega_r} \quad (107)$$

The system matrix can be derived considering Equations (106) and (107) such that

$$AH(j\omega) = \dot{H}(j\omega) \quad (108)$$

Considering digital sampling in a testing situation where a sequence of frequency points ω_i ($i = 1, 2, \dots, m$) is available, the above equation can be re-written as

$$[A \ H(j\omega_1) : H(j\omega_m)] = [\dot{H}(j\omega_1) : \dot{H}(j\omega_m)] \quad (109)$$

or

$$AD = V \quad (110)$$

where $[D]$ is usually a singular matrix which can be solved by the singular value decomposition (SVD) method as

$$D = P\Sigma Q^H \quad (111)$$

where matrices P and Q are of dimensions $m \times m$ and $lp \times lp$ respectively, Σ is a singular-value diagonal matrix of dimension $m \times lp$, and superscript H is for the Hermitian of matrix Q .

$[A]$ is estimated by using the Moore-Penrose pseudo-inverse of $[D]$ as given by

$$D^+ = Q_1 \Sigma_1^+ P_1^H \quad (112)$$

where D^+ is the Moore-Penrose pseudo-inverse of D , Σ_1^+ is a $n \times n$ diagonal matrix of the reciprocals of singular values of the Σ matrix, and P_1 and Q_1 are constructed from the first N columns of P and Q respectively.

Literature [50] gives a useful final form of the polyreference frequency domain method where it is restated as

$$\underbrace{\begin{bmatrix} [A] & [\Phi][\Gamma] \end{bmatrix}}_{m \times (m+l)} \underbrace{\begin{bmatrix} [H(j\omega_1)] & \cdots & [H(j\omega_m)] \\ [I] & \cdots & [I] \end{bmatrix}}_{(m+l) \times lp} = \underbrace{\begin{bmatrix} [H(j\omega_1)] & \cdots & [H(j\omega_m)] \end{bmatrix}}_{m \times lp} \underbrace{\begin{bmatrix} j\omega_1 [I] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & j\omega_m [I] \end{bmatrix}}_{lp \times lp} \quad (113)$$

5.5 Least Squares Complex Exponential Method

The least squares complex exponential (LSCE) modal analysis method operates in the time-domain and is applicable to linear systems [51]. Since the method works in the time-domain, impulse response function (IRF) is used where it is then fitted to a polynomial function with the same coefficients for the global response. Although the method was initially developed as a single-input multi-output (SIMO) approach, it has also been adapted for multi-DoF systems through the *Polyreference LSCE* algorithm which would be discussed subsequently. LSCE is one of the most widely used modal analysis methods in industry.

As a first step, the IRF could be estimated from the FRF by using the inverse discrete Fourier transform (IDFT), that is,

$$h_{rs}(t) = IDFT[H_{rs}(\omega)] = 2Re\left[\sum_{k=1}^n A_{rs}^k e^{\lambda_k(j\Delta t)}\right] = 2Re\left[\sum_{k=1}^n A_{rs}^k V_k^j\right] \quad (114)$$

where $V_k^j = e^{\lambda_k(j\Delta t)}$ is the exponential term and Δt is the time-step of the sampling. There always exists a polynomial in V_k of order b with real coefficients β_j such that the following equation holds

$$\beta_0 + \beta_1 V_k + \dots + \beta_j V_k^j + \dots + \beta_b V_k^b = 0 \quad (115)$$

The natural frequencies and damping ratios for the system behaviour are then computed by a three step procedure. Firstly, the coefficients β_j are determined using the following equation

$$\sum_{j=0}^b \beta_j h_{rs}(j\Delta t) = 0 \quad (116)$$

$$\begin{bmatrix} h_{rs}(0) & h_{rs}(\Delta t) & \cdots & h_{rs}((b-1)\Delta t) \\ h_{rs}(\Delta t) & h_{rs}(2\Delta t) & \cdots & h_{rs}(b\Delta t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{rs}((b-1)\Delta t) & h_{rs}(b\Delta t) & \cdots & h_{rs}((2b-2)\Delta t) \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{b-1} \end{bmatrix} = \\
- \begin{bmatrix} h_{rs}(b\Delta t) \\ h_{rs}((b+1)\Delta t) \\ \vdots \\ h_{rs}((2b-1)\Delta t) \end{bmatrix} \quad (117)$$

In the case of noisy data, the data could be averaged out by considering a number of measured values n which is much greater than the model order b so that an over-determined system of equations exists to be solved

$$\begin{bmatrix} h_{rs}(0) & h_{rs}(\Delta t) & \cdots & h_{rs}((b-1)\Delta t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{rs}((n-1)\Delta t) & h_{rs}(n\Delta t) & \cdots & h_{rs}((n+b-2)\Delta t) \end{bmatrix} \begin{bmatrix} \beta_0 \\ \vdots \\ \beta_{b-1} \end{bmatrix} = \\
- \begin{bmatrix} h_{rs}(b\Delta t) \\ \vdots \\ h_{rs}((n+b-1)\Delta t) \end{bmatrix} \quad (118)$$

The third step is to determine V_k by finding the polynomial roots of Equation (115).

The system poles λ_k are then computed as

$$\lambda_k = \frac{1}{\Delta t}(\ln |V_k| \pm \iota \arg(V_k)) \quad (119)$$

The natural frequency and damping loss factors can then be readily extracted by considering the imaginary and real parts of the system poles.

LSCE can also be adapted to multi-DoF systems in a global way as was shown in reference [52]. The inputs to the method are the IRFs and the identification is carried out by the global estimation of poles and the modal participation factors.

Let the IRF matrix be represented by $[R_k]$, where k is the time lag between N_{op} responses and N_{in} inputs. The polyreference LSCE factorizes the IRF as a sum of decaying sinusoids such that

$$[R_k] = \sum_{r=1}^{N_{in}} \{\psi\}_r e^{\mu_r k \Delta t} \{L\}_r^T + \{\psi\}_r^* e^{\mu_r^* k \Delta t} \{L\}_r^{T*} \quad (120)$$

or

$$[R_k] = \sum_{r=1}^{N_{in}} \{\psi\}_r \lambda_r^k \{L\}_r^T + \{\psi\}_r^* \lambda_r^{k*} \{L\}_r^{T*} \quad (121)$$

where $\lambda_r = e^{\mu_r \Delta t}$ is the complex exponential with Δt being the sampling time-step, and $\{L\}_r$ is a column vector of N_{in} modal participation factors (i.e., the multipliers for all the response stations of the ' r 'th mode).

The products of the complex exponential terms and modal participation factors ($\lambda_r \{L\}_r^T$ and $\lambda_r^* \{L\}_r^{T*}$) are essentially solutions to the following finite difference

equation of order b

$$\lambda_r^k \{L\}_r^T [I] + \lambda_r^{k-1} \{L\}_r^T [W]_1 + \dots + \lambda_r^{k-b} \{L\}_r^T [W]_b = \{0\}^T \quad (122)$$

where $[W]_1, [W]_2, \dots, [W]_b$ are the coefficient matrices, each of dimension $N_{in} \times N_{in}$. In order to find $2N_m$ poles of the system, the following condition is necessary

$$b \geq \frac{2N_m}{N_{in}} \quad (123)$$

Since the IRFs are also linear combinations of the solutions to Equation (115), this implies that the IRF themselves are solutions to the same equation. Hence the following holds true

$$[R_k][I] + [R_{k-1}][W]_1 + \dots + [R_{k-b}][W]_b = [0] \quad (124)$$

Equation (124) enables a polyreference or global way to process measurements at all the degrees of freedom simultaneously. The next step proceeds by performing a least squares solution to estimate the coefficient matrices of Equation (122). Once the coefficient matrices are estimated, the system's poles could be derived by finding out the eigenvalues from Equation (122) from which the natural frequencies and damping loss factors are readily extracted.

The method suffers from the drawback that there is no provision to compute the global modal participation factors for a multi-DoF system and fails to identify the modal contributions of the actual modes [53]. Another major disadvantage of LSCE is that it does not always result in clear stabilization charts as was found out by Thomas et.al. [54]. This is shown in Figure 26.

Figure 26: LSCE frequency stabilization chart [54]

5.6 Eigensystem Realization Algorithm

The Eigensystem Realization Algorithm (ERA) is a powerful modal system identification technique that operates in the time-domain. It determines the linear differential equation of a structural system from its impulse response data. The method was first developed by Juang and Pappa at NASA's Langley Research Center, for system identification towards active control of flexible aero-structures, acoustic noise reduction, and jitter-induced vibration suppression [55].

The ERA method is broadly based on the realization techniques proposed by Ho and Kalman which use the transfer function matrix to solve the realization problem. They showed that the minimum realization problem is equivalent to a representation problem involving a sequence of real matrices known as Markov parameters (impulse response functions). Minimum realization means the smallest state-space dimensions among systems realized that has the same input-output

relations within a specified degree of accuracy. ERA uses this approach to solve problems involving noise in experimental data by realizing the minimum order first and then solving to extract the modal parameters of the system under consideration.

ERA makes use of the IRF matrices as input and operates on them in state-space form (discussed in Section 4.6) to estimate the modal parameters. An n DoF system with n excitation forces can be represented as

$$[M]_{n \times n} \{\ddot{x}\}_{n \times 1} + [C]_{n \times n} \{\dot{x}\}_{n \times 1} + [K]_{n \times n} \{x\}_{n \times 1} = \{F\}_{n \times 1} \quad (125)$$

The state-space representation of an n DoF system with p excitation forces and responses measured at l locations could be given as

$$\{\dot{X}\}_{2n \times 1} = [A]_{2n \times 2n} \{X\}_{2n \times 1} + [B]_{2n \times 2n} \{F\}_{2n \times 1} \quad (126)$$

$$\{Y\}_{l \times 1} = [C]_{l \times 2n} \{X\}_{2n \times 1} \quad (127)$$

For a discrete, linear, time invariant system sampled at a frequency f_s , the state-space equations can be written in the form of a difference equation as

$$X(k+1) = AX(k) + BF(k) \quad (128)$$

$$Y(k) = CX(k) \quad (129)$$

where k is the sample instant. The system matrices, A , B , and C , in state-space form for a continuous system are given as

$$A = \begin{bmatrix} 0_{n \times n} & I_{n \times n} \\ -M^{-1}K & -M^{-1}C_{damping} \end{bmatrix}$$

$$B = \begin{bmatrix} 0_{nXp} \\ -M^{-1}I_{nXp} \end{bmatrix}$$

$$C = \begin{bmatrix} I_{lXn} & I_{lXn} \end{bmatrix}$$

The impulse response matrix is constructed from the data measured at p input locations and l output locations of the structure, such that at the k th instant of time, the IRF matrix is

$$[H'(k)] = \begin{bmatrix} h_{11}(k) & h_{12}(k) & \cdots & h_{1p}(k) \\ h_{21}(k) & h_{22}(k) & \cdots & h_{2p}(k) \\ \vdots & \vdots & \ddots & \vdots \\ h_{l1}(k) & h_{l2}(k) & \cdots & h_{lp}(k) \end{bmatrix} \quad (130)$$

The next step of the ERA procedure is building a Hankel matrix using Markovs parameters (from the measured IRFs) as below [40]

$$[H(k-1)] =$$

$$\begin{bmatrix} H'(k) & H'(k+1) & \cdots & H'(k+t_{\beta-1}) \\ H'(j_1+k) & H'(j_1+k+t_1) & \cdots & H'(j_1+k+t_{\beta-1}) \\ \vdots & \vdots & \ddots & \vdots \\ H'(j_{\alpha-1}+k) & H'(j_{\alpha-1}+k+t_1) & \cdots & H'(j_{\alpha-1}+k+t_{\beta-1}) \end{bmatrix} \quad (131)$$

where j_r and t_s are arbitrary positive integers depending on the IRF length of the measurements.

Without the presence of noise, the above Hankel matrix has rank equal to the order of the system. With noise-adulterated signals, the rank varies until α and β reach certain values, which implies the rank reaching a constant and the dimension of the matrix is at a minimum.

Choosing j_r and t_s such that both equal $r\Delta t$ ($r= 1,2,\dots$), the above matrix becomes

$$[H(k-1)] =$$

$$\begin{bmatrix} H'(k) & H'(k+1) & \cdots & H'(k+\beta-1) \\ H'(k+1) & H'(k+2) & \cdots & H'(k+\beta) \\ \vdots & \vdots & \ddots & \vdots \\ H'(k+\alpha-1) & H'(k+\alpha) & \cdots & H'(k+\alpha+\beta-2) \end{bmatrix} \quad (132)$$

such that $l\alpha = p\beta$

The Hankel can be further generalized as

$$[H(k)] =$$

$$\begin{bmatrix} H'(k+1) & H'(k+2) & \cdots & H'(k+\beta) \\ H'(k+2) & H'(k+3) & \cdots & H'(k+\beta+1) \\ \vdots & \vdots & \ddots & \vdots \\ H'(k+\alpha) & H'(k+\alpha+1) & \cdots & H'(k+\alpha+\beta-1) \end{bmatrix} \quad (133)$$

The entries to the Hankel matrix are called *Markov's parameters* of the system.

Markov parameters can be defined by applying a Dirac delta impulse to the

state-space equation and obtaining a train of outputs as follows

$$H'(0) = Cx(0) + D(0) = D \quad \& \quad x(1) = Ax(0) + B(0) = B$$

$$H'(1) = Cx(1) + D(1) = CB \quad \& \quad x(2) = Ax(1) + B(1) = AB$$

$$H'(2) = Cx(2) + D(2) = CAB \quad \& \quad x(3) = Ax(2) + B(2) = A^2B$$

$$H'(3) = Cx(3) + D(3) = CA^2B \quad \& \quad x(4) = Ax(3) + B(3) = A^3B$$

Generalizing the correspondence,

$$H'(n) = Ax(n) + B\delta(n) = CA^{n-1}B \quad \forall \quad n > 0$$

$$H'(n) = D \quad \forall \quad n = 0$$

Further treatment of the realization process involves two fundamental concepts of control theory and their respective operators:

- *Observability*: the ability to obtain the response output which can contain a number of response characteristics starting with an initial response; and
- *Controllability*: the extent to which the input can excite a number of system response components, $x(t)$ by a train of excitations, $u(t)$.

The *observability operator* can be defined by first considering the state-space formulation, such that

$$X(k+1) = AX(k) + Bu(k)$$

$$Y(k) = CX(k)$$

If there were no subsequent excitations beyond the initial point of excitation (i.e., $x(0) \neq 0$) then the output y for discrete time steps is

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_p \end{bmatrix} = V_p x_0 \quad (134)$$

where the *observability matrix*, V_p , is expressed as

$$V_p = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{p-1} \end{bmatrix}_{lp \times n} \quad (135)$$

Further, a *controllability operator* can be defined by applying a train of excitations

such that the state variable vector x_r can be written as

$$\{x_r\} = A^r x_0 + W_r \begin{bmatrix} U_{r-1} \\ U_{r-2} \\ U_{r-2} \\ \vdots \\ U_0 \end{bmatrix} \quad (136)$$

where the *controllability matrix* is

$$W_r = \begin{bmatrix} B & AB & A^2B & \dots & A^{r-1}B \end{bmatrix}_{n \times mr} \quad (137)$$

The Hankel matrix, derived previously, can now be written in terms of these two operators

$$H_{pr}(k) = V_p A^k W_r \quad (138)$$

Assuming there exists a matrix $H^\#$ satisfying the relation

$$W_r H^\# V_p = I_n \quad (139)$$

The following relationship can be arrived at from Equations (138) and (139)

$$H_{pr}[0] H^\# H_{pr}[0] = V_p W_r H^\# V_p W_r = V_p W_r = H_{pr}[0] \quad (140)$$

from which it can be observed that $H^\#$ becomes the pseudo-inverse of $H_{pr}[0]$ such that

$$H^\# = \left[[H_{pr}[0]]^T [H_{pr}[0]]^{-1} [H_{pr}[0]] \right]^T \quad (141)$$

$H^\# [0]$ can be solved using singular value decomposition (SVD) on $H_{pr} [0]$. Thus,

$$H_{pr} [0] = P_p \Sigma_{pr} Q_r^T \quad (142)$$

where P_p & Q_r are unitary matrices; and Σ_{pr} is a diagonal matrix of singular values. By combining matrices P_p & Σ_{pr} , it follows that

$$H_{pr} [0] = [P\Sigma]_{pr} Q_r^T = P_p Q_r^T \quad (143)$$

Matrices P_p, Q_r^T, W_r, V_p^T are of rank n . For $k = 0$ in the SVD step

$$H_{pr} [0] = V_p W_r = P_p Q_r^T$$

To obtain the relationship between system matrices $[A]$, $[B]$ and, $[C]$ and those matrices derived from *SVD*, two new matrices are defined such that

$$[E_l]^T = [[I_l]_{lXl} \quad [0]_{lXl} \quad \cdots \quad [0]_{lXl}]_{lXl\alpha}$$

and

$$[E_p]^T = [[I_p]_{pXp} \quad [0]_{pXp} \quad \cdots \quad [0]_{pXp}]_{pXp\beta}$$

Thus,

$$\begin{aligned} H [(k+1)] &= [E_l]^T ([V] [W]) [H^\#] \left([V] [A]^k [W] \right) [H^\#] ([V] [W]) [E_p] \\ \Rightarrow H [(k+1)] &= [E_l]^T [H(0)] [H^\#] [H(1)]^k [H^\#] [H(0)] [E_p] \end{aligned}$$

Applying *SVD* on $[H(0)]$ and $[H^\#]$

$$H [(k+1)] = \left([E_l]^T [P]^T [\Sigma]^{1/2} \right) \left([\Sigma]^{-1/2} [P] [H(1)] [Q]^T [\Sigma]^{-1/2} \right)^k \left([\Sigma]^{1/2} [Q] [E_p] \right) \quad (144)$$

The system matrices can then be obtained by comparing Equation (144) to the derived Markov's parameters such that

$$[A] = [\Sigma]^{-1/2}[P][H(1)][Q]^T[\Sigma]^{-1/2} \quad (145)$$

$$[B] = [\Sigma]^{1/2}[Q][E_p] \quad (146)$$

$$[C] = [E_l]^T[P]^T[\Sigma]^{1/2} \quad (147)$$

The foregoing Equations (145), (146) and (147) are the final results of the ERA technique. To obtain the modal parameters of the seats, thus identifying the concerned system, only the system matrix A is needed, on which a subsequent eigenvalue decomposition is performed. This involves transformation of the system's coordinates from the state space to the modal domain. Poles of the system can then be derived from the eigenvalues, from which the modal parameters are readily estimated from the following considerations subsequent to the eigenvalue decomposition step

$$[\psi]^{-1}[A][\psi] = Z \quad (148)$$

where $[\psi]$ is the complex mode shape matrix, and Z is the diagonal matrix containing the eigenvalues, where its r th element is given as

$$Z_r = e^{\lambda_r \Delta t} = e^{(Re(\lambda_r) + Im(\lambda_r))\Delta t} \quad (149)$$

where Δt is the sampling time-step. Once the poles are extracted, the damping ratio (ζ), damped frequency (ω_d) and, natural frequency (ω_n) are readily obtained as given below

$$\zeta = -\frac{Re(\lambda_r)}{|\lambda_r|} \quad (150)$$

$$\omega_d = \text{Im}(\lambda_r) \quad (151)$$

$$\omega_n = \frac{\omega_d}{\sqrt{1 - \zeta^2}} \quad (152)$$

5.6.1 Modal Contribution: Mode Singular Value

The mode singular value (MSV) is an accuracy indicator to estimate the most significant contributing mode when multiple and closely-spaced modes (due to structural noise and nonlinearity) are derived. If a linear, discrete system with r inputs and m outputs is identified, it can be represented in modal coordinates in the following way [56]:

$$x_m(k+1) = \hat{\Lambda}x_m(k) + \hat{B}_m u(k)$$

$$y(k) = \hat{C}_m x_m(k) + Du(k)$$

where $\hat{\Lambda}$ is the identified eigenvalues diagonal matrix; and $\hat{B}_m$ and $\hat{C}_m$ are the input and output matrices in modal coordinates respectively. The map for excitation u to output y is fully contained in the Markov parameters matrix [57], such that

$$Y = \begin{bmatrix} D & \hat{C}_m \hat{B}_m & \hat{C}_m \hat{\Lambda} \hat{B}_m & \cdots & \hat{C}_m \hat{\Lambda}^{l-2} \hat{B}_m \end{bmatrix}$$

The input and output matrices can be partitioned as

$$\hat{B}_m = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} \quad \text{and} \quad \hat{C}_m = \begin{bmatrix} c_1 & c_2 & c_3 & \cdots & c_n \end{bmatrix} \quad (153)$$

Each Markov parameter is then expressed as a linear combination of n contributions from different modal coordinates, e.g.

$$\hat{Y}_2 = \hat{C}_m \hat{\Lambda} \hat{B}_m = \sum_{i=1}^n c_i \lambda_i b_i$$

Thus, each coordinate can be represented by a sequence of Markov parameters such that

$$Y(i) = \begin{bmatrix} c_i b_i & c_i \lambda_i b_i & \cdots & c_i \lambda_i^{l-2} b_i \end{bmatrix}$$

Recasting the sequence such that it's representative of the time history derived from the identified eigenvalues and input vector,

$$q_i = \begin{bmatrix} b_i & \lambda_i b_i & \cdots & \lambda_i^{l-2} b_i \end{bmatrix}$$

the total Markov parameter sequence can be written as

$$Y = \left[D \sum_{i=1}^n Y(i) \right] = \left[D \sum_{i=1}^n c_i q_i \right] \quad (154)$$

where q_i represents the identified modal amplitude time history of the i th mode since it represents the contribution of the i th mode associated with the output matrix c_i to the Markov parameter sequence Y . The quantitative representation of a point's

modal contribution to the impulse response can be expressed by its mode singular value, which is obtained by taking its maximum singular value

$$MSV_i = \sqrt{|c_i|(1 + |\lambda_i| + |\lambda_i^2| + \cdots + |\lambda_i^{l-2}|)|b_i|} \quad (155)$$

If $|\lambda_i| < 1$ and length l of the Markov parameters sequence is sufficiently large, then the geometric sequence can be summed up and an approximated MSV is derived as

$$MSV_i \approx \sqrt{\frac{|c_i||b_i|}{1 - |\lambda_i|}} \quad (156)$$

The MSV characterizes the modal contribution of each mode. Thus a mode with the highest MSV has the greatest contribution to the system's impulse response (and hence the FRF), thereby implying conformational identification by the ERA technique.

5.7 Stabilization Diagram

Not all estimated eigenvalues carry physical meaning in regards to the system's response. Many could be due to computational modes or structural noise. In experimental modal analysis, a *stabilization diagram* (or *consistency diagram*) is a plot of the derived pole (or damped frequency) estimates for different model sizes, and is a tool to help select the right mode(s). The stabilization diagram is considered the most applied tool to determine the physical poles [58]. Spurious modes jump around neighbouring points and hence do not tend to stabilize, while a genuine mode occurs at nearly identical locations, closer to the expected analytical model order [59] and subsequently stabilizing at higher model orders. A typical diagram, such as one seen in Figure 26, can contain upto 50 modes or a model order of 100. It has the following two major advantages lending to its wide use [60]

- It helps in determining the number of modes contained within a frequency band.
- By displaying consistent occurrences of a stable pole estimate with the least variability at several model orders, it confirms that the derived pole estimate is correct and hence represents the physical correspondence of that pole to the system's structural response.

5.8 Conclusion

Various system identification methods were explored for the purpose of extracting the modal parameters from test data. The peak-picking and circle-fit methods were not adopted considering the disadvantages associated with them. While the peak-picking method is simpler in form, it suffers from inaccuracies resulting from SDoF assumption, noise in signals, and interpolated FRF. The circle-fit method also has the disadvantages of being susceptible to noise, incorrect identification of resonant peaks in the FRF plots, and system non-linearities. A major disadvantage of LSCE is that it does not always result in clear stabilization charts. MIMO-based identification methods were looked at since the modal parameters can be estimated globally and have been proven to be more accurate than the previously discussed methods [61]. They can also identify closely-spaced modes; segregate pseudo-modes from the real ones, and also contribute to reduced computational time [61]. Also, the FRFs and IRFs obtained from MIMO methods tend to be more accurate and consistent since the excitation energy is uniformly distributed, hence resulting in a uniform response level [49]. An additional benefit of using global methods is the reduction in computational time. The polyreference frequency domain method, was considered initially, given its MIMO approach, but it failed to derive reasonable results and was found to be sensitive to spectral leakage and noise in the FRF matrix. The benefit of using ERA is that the singular value decomposition of the Hankel matrix significantly

reduces the model order to a minimum order realization problem. This filters out the bulk of uncorrelated noise and computational components in the Hankel matrix, thereby acting like a Wiener filter [40]. Stabilization diagrams obtained using ERA in the literature have been found to be better than comparable methods. Also, detection of the correct mode is made viable through the use of Mode Singular Values (MSVs) to determine modal contributions. The overall system identification procedure is illustrated in Figure 27.

Figure 27: Roadmap of the applied system identification process to derive modal parameters of the seats.

Chapter 6

Results and Discussion

6.1 Introduction

Drop tests were conducted using the drop rig presented in Chapter 3 and methodology presented in Chapter 5 from heights of 6 inch through 30 inch, in steps of 6 inch. Further, drops were done three times for each height. Both the Shockwave and Ullman Dynamics seats described in Section 2.5 were tested using the same procedure. The signals were captured at a sampling frequency of 2000 Hz by the block digram program implemented in Labview and were recorded in Excel files using Labview's TDMS logger function. Absolute acceleration signals of the seat and dummy mass were then subtracted from the carriage acceleration in the spreadsheet to obtain relative accelerations, as was required by the intended analyses. The signals were then imported into the MATLAB environment and were low-pass filtered with a cut-off frequency of 30 Hz. Relative displacement profiles were obtained by integrating accelerations in frequency domain. It is to be mentioned that some results have been presented in dual systems of units (imperial and metric).

IRFs were obtained to generate the Hankel matrix to be used in the ERA algorithm. This chapter deals with the results obtained and further explains

observations based on the results.

6.2 Acceleration, Velocity, and Displacement Profiles

Absolute vertical acceleration ($+Z$ direction) profiles for a typical drop test of the Shockwave seat are shown in Figure 28.

Figure 28: Absolute acceleration profiles measured during 6 inch drop of the Shockwave seat.

Table 8 gives the values of the absolute peak accelerations measured for different drop heights of the Shockwave seat. The negative values for g-reduction on the dummy mass are due to amplified loadings with respect to the accelerations of the carriage

Table 6: Peak accelerations (absolute) measured on the Shockwave seat.

Drop Ht. (in inch)	Base Ac. (in g)	Seat Ac. (in g)	Mass Ac. (in g)	g reduction (in %)
6	9.225	5.947	4.13	55.23%
12	13.46	6.187	6.63	50.74%
18	16.64	8.439	12.23	26.50%
24	19.33	9.298	14.490	25.03%
30	21.94	6.023	11.76	46.39%

Table 7: Peak accelerations (relative) measured on the Shockwave seat.

Drop Ht. (in inch)	Base Ac. (in g)	Seat Ac. (in g)	Mass Ac. (in g)
6	9.22	-6.22	-9.93
12	13.46	-7.32	-14.17
18	16.64	-13.69	-18.13
24	19.33	-10.95	-20.01
30	21.94	-19.88	-21.91

at higher drop heights.

Relative values of accelerations (referenced to the carriage) were also required for the system identification procedure. The acceleration profiles measured on the Shockwave seat relative to that of the carriage are shown in Figure 29 and their peak values are given in Table 7.

The absolute acceleration profiles (+Z direction) measured on the Ullman Dynamics

Figure 29: Relative acceleration profiles measured during 6 inch drop of the Shock-wave seat.

compact jockey seat are shown in Figure 30 and Table 8 gives the absolute acceleration values for all drop heights, while Figure 31 and Table 9 show the relative values when referenced to the carriage's acceleration.

Figure 30: Absolute acceleration profiles measured during 6 inch drop of the Ullman seat.

Waveforms representative of typical relative displacements and velocities for the Shockwave seat are depicted in Figure 32 for a 6 inch drop and values for all drop heights are recorded in Table 10. The relative displacement waveforms were required to obtain the IRFs, while the relative velocity profiles offer insights into the rapidity of compression during drop tests.

Table 8: Peak accelerations (absolute) measured on the Ullman seat.

Drop Ht. (in inch)	Base Ac. (in g)	Seat Ac. (in g)	Mass Ac. (in g)	g reduction (in %)
6	9.76	3.44	2.83	71%
12	13.43	7.67	8.225	38.75%
18	16.68	13.64	14.03	15.88%
24	19.67	16.05	20.43	-3.86%
30	22.48	15.62	25.41	-13.03%

Table 9: Peak accelerations (relative) measured on the Ullman seat.

Drop Ht. (in inch)	Base Ac. (in g)	Seat Ac. (in g)	Mass Ac. (in g)
6	9.76	-9.98	-9.82
12	13.43	-11.39	-13.21
18	16.68	-13.07	-16.31
24	19.67	-15.05	-18.9
30	22.48	-15.62	-21.98

Figure 31: Relative acceleration profiles measured during 6 inch drop of the Ullman seat.

Table 10: Peak relative displacements of the seat-pan and dummy mass on the Shockwave seat.

Drop Ht. (in inch)	Seat-pan displacement (in mm)	Dummy mass displacement (in mm)
6	-61.91	-67.36
12	-94.14	-102.36
18	-117.46	-130.48
24	-124.76	-173.56
30	-139.23	-168.12

Figure 32: Relative displacement profiles measured during a 12 inch drop of the Shockwave seat.

Table 11: Peak relative velocities of the seat-pan and dummy mass on the Shockwave seat.

Drop Ht. (in inch)	Seat-pan velocity (in m/s)	Dummy mass velocity (in m/s)
6	-2.22	-2.78
12	-2.83	-3.62
18	-3.85	-4.33
24	-4.24	-5.08
30	-4.89	-5.11

Figure 33: Relative velocity profiles measured during 12 inch drop of the Shockwave seat.

Plots depicting the relative displacement profiles obtained from a drop height of 12 inch for the Ullman seat are shown in Figure 34. Table 12 gives the average readings for all the drop heights. The high velocities as read from Table 11 indicate that compression of the airspring is rapid during the first impact.

Figure 34: Relative displacement profiles measured during 12 inch drop of the Ullman seat.

Table 12: Peak relative displacements of the seat-pan and dummy mass on the Ullman seat.

Drop Ht. (in inch)	Seat-pan displacement (in mm)	Dummy mass displacement (in mm)
6	-73.17	-70.23
12	-102.28	-111.93
18	-103.05	-129.40
24	-125.73	-157.26
30	-120.96	-160.60

Table 13: Peak relative velocities of the seat-pan and dummy mass on the Ullman seat.

Drop Ht. (in inch)	Seat-pan velocity (in m/s)	Dummy mass velocity (in m/s)
6	-2.31	-2.53
12	-3.01	-3.47
18	-3.34	-4.22
24	-3.85	-4.80
30	-4.06	-5.33

6.2.1 Discussion: Acceleration, Velocity, and Displacement Profiles

The pulse width of the carriage signals, as observed during the first impact, is around 50 milliseconds. Profiles of absolute accelerations, as seen in Figures 28 and 30 for the Shockwave and Ullman Dynamics seats respectively, demonstrate the characteristic behaviour of shock mitigating seats. In a typical slam impact load range of 9g–15g, the shock absorber in both cases reduces the number of gs on the seat-pan such that the seat-pan’s acceleration curve falls within the width of the first impact signal of the carriage. Moreover, the dummy mass’s acceleration curves showed positive shifts in time and response to the first impact is spread over two spikes such that their widths are more than that of the input carriage acceleration. This behaviour distributes energy and reduces the gs on the occupant, thus achieving the objective of a shock mitigating seat.

However, the waveforms of relative accelerations of the seat-pan and dummy mass were observed to be in the $-Z$ direction, as seen in Figures 29 and 31. This is

mainly due to the large values of the carriage acceleration which they are referenced to. Thus, both waveforms seemingly align to each other from the reference frame of the carriage, thereby resulting in minimal phase difference between the two. Also, no spreading out of the dummy-mass's signal is observed, unlike in the case of absolute coordinates.

Although velocities were not required for the intended parameter extraction process, they do provide information about the rapidity of compression/de-compression, especially in the case of air-spring suspension components where the high amplitude of excitation affects stiffness. The stiffness can increase if the velocity of compression/de-compression is high due to higher amplitude of vibration. It happens due to the increased likelihood of air getting choked at the orifice during volume transfer between the auxiliary chamber and the air-spring. [62]. It is seen from Tables 11 and 13 that the peak velocities recorded during the first impact are high for both seats.

Relative displacement profiles were obtained for both suspension seats to calculate receptance FRFs and their corresponding IRFs. Figure 32 and Figure 34 depict the displacement waveforms derived from the seat accelerations for a 12 inch drop for the Shockwave and Ullman Dynamics seats respectively. It can be seen that both plots have a first positive peak and a subsequent peak of higher magnitude in the negative direction. The first positive peak can be attributed to elongation in the spring elements of both seats due to an upward force on the dummy+seat mass which is under inertial acceleration of the carriage during the drop phase. The major first spike of higher magnitude in the negative direction results from the drop impact, and this region, representative of a slam event at sea, was used for our parameter extraction analysis. The maximum displacement peaks for the first impact are found

in Table 10 and Table 12 for the two seats respectively. For the Shockwave seat, the peak displacement value was 67.36 mm for a 6 inch drop. The negative sign means that the air spring gets compressed during the process. Between each of the drop heights up to 24 inch, the air-spring gets compressed by an average of 35.4 mm. The Ullman seat registered a compression of 70.33 mm during a 6 inch drop, and shows a decreasing trend of compression between each step, as is inferred from the table.

The displacement profiles of the seat-pan and dummy mass, in both cases, are almost aligned to each other with minimal phase difference, differing only slightly in magnitudes near the peaks. The small difference in magnitudes can be due to the seat foam getting squished under peak loads, the dummy-mass losing contact from the seat during impact, and the slight tilt of dummy mass under impact. Nonetheless, the phase difference between the two relative waveforms is almost zero. This can be seen in Figure 35 and Figure 36 providing zoomed up profiles of the compression in both cases. An important inference could be made from this observation that the assumed two-DoF system acts effectively as a single DoF system under impact loads when the absolute coordinates of the seat-pan and dummy mass are referenced to the carriage coordinate. A consequent implication is that the foam contributes at most very little towards shock mitigation, and that attenuation is primarily provided by the suspension component in the seats.

6.3 IRFs and FRFs

Usually, IRFs of a system are obtained by first finding its FRFs to a given excitation. However, in this work the IRFs were directly estimated from the inputs and outputs because of the given advantages cited in Section 3.5. The IRFs of the MIMO system were estimated at all degrees of freedom for the respective input and output at each

Figure 35: Magnified compression region in the displacement profile of the Shock-wave seat.

Figure 36: Magnified compression region in the displacement profile of the Ullman seat.

DoF. Figure 37 shows the IRFs at all degrees of freedom obtained for a 6 inch drop of the Shockwave seat.

Figure 37: A set of IRF plots for 6 inch drop of the Shockwave seat, showing: [a] IRF for response at DoF-1 to excitation at DoF-1; [b] IRF for response at DoF-1 to excitation at DoF-2; [c] IRF for response at DoF-2 to excitation at DoF-1; and [d] IRF for response at DoF-2 to excitation at DoF-2.

FRFs at the concerned degrees of freedom were then estimated by performing the discrete Fourier transform on the individual IRFs. FRFs obtained from a 6 inch drop of the Shockwave seat are shown in Figure 38. The FRFs were plotted upto the Nyquist frequency of 1000 Hz, but have been presented here up to 200 Hz in order to enhance resolution. The FRF plots reveal a dominant resonant peak at around 1 Hz. Peaks around the same frequency were also observed for other drop heights.

Figure 38: A set of FRF plots for 6 inch drop of the Shockwave seat, showing: [a] FRF for response at DoF-1 to excitation at DoF-1; [b] FRF for response at DoF-1 to excitation at DoF-2; [c] FRF for response at DoF-2 to excitation at DoF-1; and [d] FRF for response at DoF-2 to excitation at DoF-2.

A similar procedure was followed to derive the impulse and frequency responses on the Ullman seat. The IRF and FRF plots obtained for a 6 inch drop of the Ullman Dynamics seat are shown in Figures 39 and 40.

Figure 39: A set of IRF plots for 6 inch drop of the Ullman seat, showing: [a] IRF for response at DoF-1 to excitation at DoF-1; [b] IRF for response at DoF-1 to excitation at DoF-2; [c] IRF for response at DoF-2 to excitation at DoF-1; and [d] IRF for response at DoF-2 to excitation at DoF-2.

It is observed from the FRF plots for the Ullman dynamics seat that the dominant peak also lies around 1 Hz. The plot also does not have any repeated dominant peak modes throughout the frequency range below the Nyquist frequency. The serrated lines seen in the region beyond 15 Hz could be due to the effect of noise and leakage carried over by the discrete Fourier transform.

Figure 40: A set of FRF plots for 6 inch drop of the Ullman seat, showing: [a] FRF for response at DoF-1 to excitation at DoF-1; [b] FRF for response at DoF-1 to excitation at DoF-2; [c] FRF for response at DoF-2 to excitation at DoF-1; and [d] FRF for response at DoF-2 to excitation at DoF-2.

6.3.1 Discussion: IRFs and FRFs

By observing the IRF plots in Figures 37 and 39, it can be inferred that the impulse responses do not die out fast, but the impulse amplitude gradually decays to zero. This shows that suspensions on both the seats are lightly underdamped ($\zeta < 1$). The damping leads to dissipation of heat within the volume. The light damping in a shock mitigating seat distributes the vibration energy by reducing the amplitudes in steps within the comfort range of the occupant, which is the characteristic behaviour

of a typical shock mitigating seat.

Plots of the FRFs show a dominant resonant peak within the 1–2 Hz range for both representative seats (Figures 38 and 40). However, contrary to what is expected of the assumed two-DoF system, no significant second peak was observed in both FRF figures. An important implication can be drawn from this observation - that the seats behave dynamically as a single-DoF system, when the coordinates of the seat-pan and dummy mass are referenced to that of the carriage. Thus, only the prominent mode seen in the low-frequency region contributes significantly to the frequency response in the modal domain. This reinforces the single-DoF behaviour inference from the displacement profiles discussed in Section 6.2.1. Moreover, the combinations of individual IRFs and FRFs for all degrees of freedom have almost the same magnitude and phase behaviour, which is how a single-DoF system would respond. The saw-toothed variations seen after 15 Hz in the FRF plots are attributed to minor noise in the signals and signal leakage, considering that displacement profiles were obtained by the discrete Fourier transform of the relative acceleration waveforms. There is no loss of information when FRFs are obtained by the Fourier transform of the IRFs.

6.4 Modal Parameters Determined by the Eigen-system Realization Algorithm

This section presents the final results of obtaining the seats' modal parameters, which is the other major objective of the work carried out in this project. After determining the IRFs, Hankel matrices of free vibration responses (IRFs) at zero and unit time instances were formed as required by the ERA. A code was written

in MATLAB to do this job. Singular value decomposition (SVD) of the Hankel matrix was performed, from outputs of which the system matrix ($\mathbf{A}$) was formed for a given order. The process was repeated for every recorded drop, to a total of 15 drops for each seat. As discussed in Section 5.7, *stabilization diagrams* of model order versus damped frequencies were constructed for each of the drops to help determine the correct mode and to separate the genuine modes from computational ones. This procedure was applied to drops recorded on both the Shockwave and Ullman Dynamics shock mitigating seats.

Stabilization diagrams were obtained for the ERA method applied to the Shockwave drop test data for each drop to help zero in on the right damped frequency value. The first useful indicator is the FRF plot for those drop test data, which points to the approximate resonant frequency of the shock absorber, but cannot be solely relied upon because of the presence of closely-spaced modes and noise. Stabilization diagram gave all the modes of vibration for higher model orders including the close modes. Some stabilization diagrams constructed from the drop test data of the Shockwave seat are shown in Figure 42. It can be seen in Figure 42(a) that a clear stabilization is obtained at around 1 Hz frequency, while in Figures 42(b), and 42(d) a number of modes are seen. The modes occurring beyond 8 Hz are ignored since these occur at way higher model orders than the expected order of 4. Also, since the FRF plot points to a dominant mode in the 1–2 Hz frequency range, these account for insignificant modal contribution. This left only two closed modes at around 1 Hz and 2 Hz frequencies, between which the near 1 Hz mode occurs more consistently and nearer to the expected model order of four. Finally, the modal contribution of the modes were plotted for the computed MSVs against the derived damped frequencies (Figure 42). It is seen that the MSV of the near 1 Hz mode is significantly greater than that of the near 2 Hz mode. Taking account of the seat's single-DoF

behaviour, a further segregation could be made by examining the plot of acceleration versus displacement (Figure 43). The slope on the curve gives the specific modulus. It is seen that the plot is almost linear during compression and nonlinear during decompression. The higher modes seen can be attributed to nonlinearities in the system response and residual noise in the Hankel matrix. This coupled with the fact that the ERA, formulated for linear systems, gives the most-consistent parameters in the linear domain leads to the derived parameters given in Table 14.

Figure 41: A set of stabilization diagrams derived from ERA estimated parameters for the Shockwave seat, showing for drop heights of: [a] 6 inch; [b] 12 inch; [c] 18 inch ; and [d] 30 inch.

Figure 42: A set of plots showing MSVs versus ERA-derived damped frequencies for the Shockwave seat drops at [a] 6 inch; and [b] 12 inch.

Figure 43: Acceleration versus displacement for a 6 inch drop of the Shockwave seat.

Table 14: ERA derived modal parameters of the Shockwave seat.

Drop Ht. (in inch)	f_d (in Hz)	Avg. f_d (in Hz)	ζ (unitless)	Avg. ζ (unitless)	log. decrement (unitless)	f_n (in Hz)	Avg. f_n (in Hz)
6	1.185		0.283		0.2720	1.235	
	-	1.164	-	0.242	-	-	1.201
	1.144		0.202		0.2177	1.168	
12	1.260		0.369		0.3331	1.356	
	1.134	1.229	0.329	0.347	0.2568	1.200	1.311
	1.294		0.345		0.3455	1.378	
18	1.010		0.473		0.6320	1.146	
	0.992	1.104	0.520	0.445	0.1900	1.162	1.234
	1.310		0.343		0.1141	1.394	
24	1.106		0.412		0.2355	1.214	
	1.115	1.128	0.507	0.424	0.2460	1.293	1.237
	1.164		0.2355		0.3243	1.204	
30	1.108		0.505		0.2673	1.252	
	1.020	1.116	0.436	0.504	0.2978	1.133	1.291
	1.220		0.573		0.3066	1.488	

Using a similar analysis based upon stabilization diagrams (Figure 44), FRFs and the specific modulus plot (Figure 46), the derived modal parameters for the Ullman Dynamics seat are given in Table 15.

Figure 44: A set of stabilization diagrams derived from ERA estimated parameters for the Ullman Dynamics seat, showing for drop heights of: [a] 6 inch; [b] 12 inch; [c] 24 inch; and [d] 30 inch.

Figure 45: A set of plots showing MSVs versus ERA-derived damped frequencies for the Ullman Dynamics seat drops at [a] 12 inch; and [b] 24 inch

Figure 46: Acceleration versus displacement plot for a 6 inch drop of the Ullman Dynamics seat

Table 15: ERA derived modal parameters of the Ullman Dynamics seat.

Drop Ht. (in inch)	f_d (in Hz)	Avg. f_d (in Hz)	ζ (unitless)	Avg. ζ (unitless)	log. decrement (unitless)	f_n (in Hz)	Avg. f_n (in Hz)
	-		-		-	-	
6	1.087	1.132	0.314	0.430	0.3143	1.450	1.428
	1.178		0.547		0.2380	1.407	
12	1.314		0.561		0.7740	1.587	
	0.9506	1.132	0.390	0.475	-	1.031	1.309
	-		-		-	-	
18	0.734		0.810		-	1.251	
	0.910	0.822	0.840	0.825	0.3701	1.677	1.464
	-		-		-	-	
24	1.087		0.837		-	1.986	
	0.9005	1.060	0.752	0.773	-	1.365	1.700
	1.193		.732		-	1.751	
30	0.935		0.864		-	1.857	
	0.934	0.915	0.864	0.862	-	1.855	1.807
	0.875		0.860		-	1.709	

6.4.1 Discussion: Modal Parameters Determined by the Eigensystem Realization Algorithm

Multiple modes were derived from the ERA from the Hankel matrices, and a right mode was decided based on the FRF plots, stabilization charts, and MSVs. The rank of the block Hankel matrix was not found to be equal to the expected model order of the system (which was 4) due to residual noise in the data, nonlinearity in the system, and computational round off errors. The natural frequency for each recorded drop was calculated using the following equation for a single-DoF system:

$$f_n = \frac{f_d}{\sqrt{1 - \zeta^2}} \quad (157)$$

Shockwave Seat

For the Shockwave seat, no trend was observed for f_d and f_n with respect to the drop height. However, the damping ratio was found to increase with drop height (hence the rate of spring displacement). The damping ratios derived from the logarithmic decrement method were found to be very close to those ERA-derived values in the 0.2–0.35 range. There were considerable differences between the two for higher ERA-derived values since for actual damping in the system (i.e., ERA derived) approaching 0.5 or more, the logarithmic decrement method is less precise and yields erroneous values. The average value of f_d was 1.148 Hz and that of f_n was 1.255 Hz. The mean damping ratio was found to be 0.392. Knowing that the seat-pan+dummy mass was 132 kg, an average estimate of the air-spring's stiffness, k , was obtained by using the single-DoF expression:

$$k = \omega_n^2 m = (2\pi f_n)^2 m \quad (158)$$

Substituting the values of m and f_n into the above expression, the stiffness of the Shockwave seat's air spring was calculated to be 8207.70 N/m. For validation, the

Table 16: Stiffness values of shock absorber derived from MTS tests.

Frequency (in Hz)	Stiffness (in N/m)
1	7200
2	7100
3	7200
4	6800

result was compared to that obtained from quasi-static testing of the air-spring on an MTS machine which yielded the linearly curve fitted results shown in Table 16. These tests was conducted by another graduate student of the research group at Carleton University's Applied Dynamics Laboratory. The average value of stiffness obtained from the MTS tests was 7075 N/m. The mean stiffness of the Shockwave's air-spring derived from the ERA is within 16% of this value. The difference could be explained by first noting that the tests on the MTS machine were quasi-static (since the velocities were very low) and thus were not representative of the dynamic tests on the drop tower which results in rapidity of compression during drops and could slightly increase the stiffness value. Also, the air-spring is preloaded on the seat by shimming it against the mount using a hard rubber capsule during installation. This results in a net decrease in stroke length for the air to be compressed which could affect stiffness. However, the ERA-derived value is very close to the static-stiffness of the shock absorber. To compare the measured response with that from theory, an analytical second-order ordinary differential equation for a SDoF system was considered whose response was obtained from the drop test's input excitation (18 inch drop height) and the modal parameters derived from ERA. The second order SDoF equation considered was

$$\ddot{z}_{seat} + 2\zeta\omega_n\dot{z}_{seat} + \omega_n^2 z_{seat} = \omega_n^2 z_{base}$$

The solution to the above equation is [63]

$$z_{seat} = z_{base} \left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin \left(\sqrt{1-\zeta^2}\omega_n t + \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta} \right) \right]$$

The time domain simulation response comparison is shown in Figure 47. It is seen that the waveforms align well except that the measured signal lags the analytical profile near the impact response region. This can be explained by noting that the analytical model considered does not factor in the dry friction which is present in the Shockwave seat. Moreover, the experimental response showed multiple modes, whereas this fact is not taken into account in the simple SDoF model which only considers a constant ω_n and ζ . The multiple modes are interpreted as those arising due to nonlinearity in the system and vibration coming from other components of the seats not considered in the simplified model since it is essentially a lumped parameter model. Further, the damping ratios derived using ERA are over-estimated since dry friction was neglected; though the contribution can be significant at higher accelerations.

Figure 47: Analytical and measured responses for 18 inch drop of the Shockwave seat.

Ullman Dynamics Seat

For the Ullman Dynamics seat, the values of damping ratio increase with drop height. This can be observed from the IRF plot for a 24 inch drop height (Figure 48) where almost no peaks are observed after the first peak at zero time, which implies a higher damping behaviour at that height. It is for the same reason of high damping that the logarithmic decrement method could not be used for most of the data, except for one 6 inch drop test where it was very close to the ERA-derived damping ratio. A similar increasing trend is also observed for the natural frequency. Knowing that the magnitude of peak deck accelerations is in the range of 9g–12g, the mean natural frequency value was chosen to be the average of the values for the 6 inch and 12 inch drop heights, which is 1.368 Hz. The mean damping ratio for these two drop heights is calculated to be 0.452. Using the mean natural frequency, the effective vertical stiffness of the seat is computed to be 7979.13 N/m for a total sprung mass of 108 kg. Since the spring component of the Ullman Dynamics seat was an assembly of parallel composite boards and thus had no mechanical interaction, the damping ratios derived are believed to be close to the actual ones applicable during the drops.

6.5 Conclusion

The recorded signals were processed to obtain the IRFs and FRFs. The IRFs were fed to the ERA code to derive the modal parameters from state-space matrices. FRF plots of both seats indicated closely-spaced modes in the frequency range of 1 Hz–2 Hz. Damped modal frequencies were obtained using the ERA method and segregation to select the right damped frequency was performed on the basis of the FRF plots, consistency of selected modes in stabilization diagrams, and the MSVs for their individual modal contributions to the overall response. It was found that damping ratios in both cases increase with the drop heights.

Figure 48: Plot showing the inapplicability of the logarithmic decrement method due to the lack of peaks in the IRF for a 24 inch drop of the Ullman Dynamics seat.

Chapter 7

Conclusion

To assist DRDC-Atlantic in the selection of shock mitigating seats suited to various operating conditions, a drop tower was designed and manufactured by Carleton University's Applied Dynamics Laboratory (ADL). The drop tower was tested and confirmed to produce repeatability of excitation signals during testing. Two shock mitigating seats, supplied by DRDC-Atlantic, were tested and their signal data were recorded using a programmed interface in LabView software.

The seats were modelled as simple two-DoF systems from their underlying design in order to determine measurement locations, and a state-space representation was derived to generate the analytical system matrices so as to enable identification. The acceleration signals were integrated in the frequency domain to obtain accurate displacements so as to derive the IRFs and FRFs. To determine and generate quantifiable behaviour of seats for future work, an advanced system identification procedure, the Eigensystem Realization Algorithm, was applied to determine the modal properties of the seats. It was observed that, in relative coordinates, both seats behave effectively as single-DoF systems and that the seat-foam contributes little to the modal response of the seats to carriage excitation. Hankel matrices were generated from the IRFs and singular value decomposition was applied to generate

the system matrices. Damping ratios, as well as both damped and undamped natural frequencies were derived from the poles obtained from the system matrix. Further, the damping ratios were also derived directly from the impulse responses using the logarithmic decrement method to cross-check with the ones derived from the ERA. The damped frequencies were determined based on stabilization diagrams, MSVs, and FRFs.

For the Shockwave seat, it was found that the damped and undamped natural frequencies were almost the same for all drop heights and that the damping ratio increased with drop height. The results for the Shockwave seat were validated by comparing the single DoF stiffness from derived modal factors to the stiffness determined from testing the air spring on an MTS testing machine. Further analysis was carried out by comparing the response from a simple second-order two-DoF system, from which it was concluded that the seat's dry friction and the presence of multiple modes (interpreted as a result of nonlinearity in the system) contributed to phase lag between analytical and measured responses.

A similar process was followed to test and obtain the modal parameters of the Ullman Dynamics seat. The Ullman dynamics seat was found to produce consistent close-modes, and an accurate one was determined using similar methods to those applied to the Shockwave seat. An *ab initio* approach was taken to justify the results on the Ullman Dynamics seat since the same procedure was applied and validated on the Shockwave seat. It was found that both damping ratio and natural frequency increase significantly with drop height. However, averages of the values for drop heights of 6 inch and 12 inch were chosen to be representative since the magnitudes of excitation produced at those heights are similar to that of a typical slam event at sea.

Different types of seats can be tested using this procedure and the derived modal parameters from drop tests can be tabulated *vis-a-vis* performance indices such as the Dynamic Response Index (DRI), Vibration Dose Value (VDV), transmissibility, Seat Effective Amplitude Transmissibility (SEAT), and Impact Count Index (ICI) from at sea-trials. This creates a database which could be referenced in the future for procurement of seats with optimum dynamic parameters suited to different operating conditions at sea.

7.1 Contributions

The project developed a sound methodology to characterize the behaviour of various shock-mitigating seats by extracting the properties which define their shock attenuation characteristics such that they could be compared to any of the performance metrics discussed in Section 2.3. The work contributes by carving out a roadmap for digital signal processing of the acquired acceleration data from drop testing, and derivation of accurate displacement and impulse responses to further the study of the seats. The project furthers an understanding of the seats' dynamics by inferring an effectively SDoF behaviour of the seats (when referenced to the drop tower's carriage coordinates). The work is unique in that it also seeks to find out accurate modal parameters which describe the dynamics of the seats during the drop tests by using an advanced system identification technique, the ERA. The project is a time-effective alternative to individual component testing. To the best of the author's knowledge, no other work in the field of HSC shock-mitigating seats has determined the properties of the seats by applying advanced parameter extraction algorithms. The derived parameters are also intended to be used for detailed modelling of the seats by the Carleton University ADL.

A major achievement of the project is the realization of the drop tower, which provides a repeatable testing platform for the dynamic testing of seats. The tower is capable of simulating slam impact load events within the range of 8g–25g. Moreover, since the drop test rig was intended to be a monolithic testing base, it was designed as a free-standing stable structure, and can be disassembled conveniently to move it, if required. The scope of dynamic testing of seats on the drop tower can also be extended to similar severe events such as testing seats for armoured vehicles, earth-moving mining vehicles and helicopter seat crash simulation. Testing of two very different HSC seats showed the versatility of the drop rig.

Thirdly, sample acceleration, velocity, and displacement data derived from testing two representative seats can offer information towards understanding their shock mitigating characteristics at varying degrees of excitation, and also supporting detailed modelling of the seat.

7.2 Recommendations for Future Work

The following recommendations are made to build on the work dealt with in this thesis project

1. Use different impacting surfaces on the drop tower to increase the pulse width of the excitation signals to around 200 milliseconds from the current 50 milliseconds. Possible materials to experiment with could be elasto-polymer gels, honeycomb structure aluminum casings, and a heavy-duty hydraulic cylinder with connection to an auxiliary reservoir to control the pulse width.
2. Generate the Markov's parameters from input/output signals with an advanced

technique such as by using the *wavelets transform method*.

3. Use non-linear and human-seat interaction approaches to model the seat's behaviour in order to generate more accurate responses.
4. Factor dry friction into the simple mathematical model used in this work. This will help to determine Coulombic friction in seats with linearly-guided seat-pans.
5. Use other system identification techniques like the frequency domain PolyMAX and versions of the ERA like *NeXT-ERA* and *ERA/DC* to extract parameters from the recorded signals.

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Appendix A

Drop Tower Drawings

Figure 49: Complete assembly of the Carleton University Drop Tower

Figure 50: Foundation plate/ base

Figure 51: Upper base plate for I-beam

Figure 52: I-beam

Figure 53: Indexed plate of carriage

Figure 54: I-beam bracket

Figure 55: Corner piece of the base plate

Figure 56: Lower plate of the carriage

Figure 57: Bracket for PU impacting base

Figure 58: Seat-mounting carriage plate

Figure 59: Square plate for mounting post

Figure 60: Post for fixing safety curtain

Appendix B

MATLAB Code

```
% The code does data extraction, DSP, frequency domain integration,%
%Toeplitz convolution to obtain IRFs and FRFs, ERA method%
%stabilization diagram & MSVs for modal contribution%

% Load the excel files from LabView containing the experimental data,%
%de-mean and detrend them%
sys_ip1 = -(xlsread('june5.xlsx',8,'E2:E64000'));%system input on
DoF-1(seat-pan)%
sys_ip1=sys_ip1-mean(sys_ip1);
sys_ip1=detrend(sys_ip1);

sys_ip2 = -(xlsread('june5.xlsx',8,'E2:E64000'));%system input on
DoF-2(dummy mass)%
sys_ip2=sys_ip2-mean(sys_ip2);
sys_ip2=detrend(sys_ip2);

sys_op1 = (xlsread('june5.xlsx',8,'F2:F64000'));%system output on DoF-2%
sys_op1=sys_op1-mean(sys_op1);
```

```

sys_op1=detrend(sys_op1);

sys_op2 = xlsread('june5.xlsx',8,'G2:G64000');%system output on DoF-2%
sys_op2=sys_op2-mean(sys_op2);
sys_op2=detrend(sys_op2);

%Plot the raw signals%
f_sample = 2000;% Sampling frequency in Hz%
t=0:1/f_sample:(length(sys_ip1)-1)*(1/f_sample);
plot(t,sys_ip1,t,sys_op1, t,sys_op2)

Sampling specifications for the data acquired %

f_sample = 2000;

% This section of the code performs filtering%
f_cutoff = 30; % cutoff freq%
f_norm = f_cutoff/(f_sample/2); % normalized cut-off frequency %

[b_ip1,a_ip1] = butter(4,f_norm,'low');% Using 4th order butterworth filter %
sys_fil_ip1 = filtfilt(b_ip1,a_ip1,sys_ip1);

[b_ip2,a_ip2] = butter(4,f_norm,'low');% Using 4th order butterworth filter %
sys_fil_ip2 = filtfilt(b_ip2,a_ip2,sys_ip2);

[b_op1,a_op1] = butter(4,f_norm,'low');% Using 4th order butterworth filter %
sys_fil_op1 = filtfilt(b_op1,a_op1,sys_op1);

```

```

[b_op2,a_op2] = butter(4,f_norm,'low');% Using 4th order butterworth filter %
sys_fil_op2 = filtfilt(b_op2,a_op2,sys_op2);

n=1:length(sys_ip1);
t=0:1/f_sample:(length(sys_ip1)-1)*(1/f_sample);
plot(n,sys_fil_ip1,n,sys_fil_op1,n,sys_fil_op2)%plot sample no. vs.
    filtered signals%

%This section performs frequency domain integration to obtain dynamic
%displacement and velocity profiles%
y1=sys_fil_op1;

y2=sys_fil_op2;
N1=length(sys_fil_op1);%%
t_step=1/f_sample;
t1=0:t_step:(N1-1)*t_step;
f_res1 =1/(N1*t_step);
f1 = -0.5*f_sample:f_res1:0.5*f_sample-f_res1;
op1_fdom = fftshift(fft(9.81*y1));
dis1_fdom = zeros(size(op1_fdom));
vel1_fdom = zeros(size(op1_fdom));

op2_fdom = fftshift(fft(9.81*y2));
dis2_fdom = zeros(size(op2_fdom));
vel2_fdom = zeros(size(op2_fdom));
for i = 1 : N1

    if f1(i)>1||f1(i)<-1

```

```

        dis1_fdom(i) =op1_fdom(i)/((2*pi*sqrt(-1)*f1(i))^2);
        vel1_fdom(i)=op1_fdom(i)/((2*pi*sqrt(-1)*f1(i)));
        dis2_fdom(i) =op2_fdom(i)/((2*pi*sqrt(-1)*f1(i))^2);
        vel2_fdom(i)=op2_fdom(i)/((2*pi*sqrt(-1)*f1(i)));
    else

        f1(i) = 0;

        dis1_fdom(i)=0;
        dis2_fdom(i)=0;
        vel1_fdom(i)=0;
        vel2_fdom(i)=0;
    end
end

dis1_tdom = real(ifft(ifftshift(dis1_fdom)));
dis1_tdom = dis1_tdom(1:N1);%displacement profile of seat-pan%
vel1_tdom = real(ifft(ifftshift(vel1_fdom)));
vel1_tdom = vel1_tdom(1:N1);%velocity profile of seat-pan%

dis2_tdom = real(ifft(ifftshift(dis2_fdom)));
dis2_tdom = dis2_tdom(1:N1);%displacement profile of dummy-mass%
vel2_tdom = real(ifft(ifftshift(vel2_fdom)));
vel2_tdom = vel2_tdom(1:N1);%velocity profile of seat-pan%
figure (1);
n=1:length(dis1_tdom);
plot(n,dis1_tdom,n,dis2_tdom)
figure (2);
n=1:length(dis1_tdom);

```

```

plot(t,vel1_tdom,t,vel2_tdom)

%Set up data range for the Toeplitz convolution process to obtain the IRFs
%at all DoFs%

a=47320;%select the beginning sample no. for the first compression profile%
b=47570;%select the ending sample no. for the first compression profile%
sys1= 9.81*sys_fil_ip1(a:b);%multiply by 'g'%

N1=length(sys1);
sys2= 9.81*sys_fil_ip2(a:b);
dis1= dis1_tdom(a:a+5*N1-2);%output is of the same size as
the IRF,
typically 4 times the input%
dis2= dis2_tdom((a:a+5*N1-2));

n=1:length(dis1);
plot(n,dis1,n,dis2)%plot the selected primary compression region%

%This block performs the Toeplitz convolution process to obtain the IRFs at
%all DoFs from the input-output data%
x=sys1;
Nx = length(x); Nh = 4*Nx;
xc=[x; zeros(Nh-1, 1)];
xr=[x(1),zeros(1,Nh-1)];
X1=toeplitz(xc,xr);%%

% IRF_12 %

```

```

x=sys2;
Nx = length(x); Nh = 4*Nx;
xc=[x; zeros(Nh-1, 1)];
xr=[x(1),zeros(1,Nh-1)];
X2=toeplitz(xc,xr);

% This section computes the IRFs %
h_11=pinv(X1)*dis1;
h_12=pinv(X1)*dis2;
h_21=pinv(X2)*dis1;
h_22=pinv(X2)*dis2;%%

%Now plot the IRFs%
t1=0:t_step:(Nh-1)*t_step;
figure (3);
plot(t1,h_11)
xlabel('Time[sec]');
figure (4);
plot(t1,h_12)
xlabel('Time[sec]');
figure (5);
plot(t1,h_21)
xlabel('Time[sec]');
figure (6);
xlabel('Time[sec]');
plot(t1,h_22)
%Plot the FRFs from IRFs%

```

```

M=length(h_11);
NFFT = 2^nextpow2(M);
f=f_sample/2*linspace(0,1,NFFT/2+1);
frf1=fftshift(fft(h_11,NFFT));
frf1=frf1(NFFT/2:NFFT);
frf2=fftshift(fft(h_12,NFFT));
frf2=frf2(NFFT/2:NFFT);
frf3=fftshift(fft(h_21,NFFT));
frf3=frf3(NFFT/2:NFFT);
frf4=fftshift(fft(h_22,NFFT));
frf4=frf4(NFFT/2:NFFT);
figure (7);
plot(f,abs(frf1))
figure (8);
plot(f,abs(frf2))
figure (9);
plot(f,abs(frf3))
figure (10);
plot(f,abs(frf4))

%This block performs the ERA routine to extract modal parameters by
%plotting the stabilization diagram%
% Construction of Hankel matrices H0 and H1 %
h_11=h_11.';
h_12=h_12.';
h_21=h_21.';
h_22=h_22.';
b=length(h_11);

```



```

te=zeros(2,2);
M=zeros(2,2,b);

for i=1:b
    te=[h_11(1,i) h_12(1,i);h_21(1,i) h_22(1,i)];
    for j=1:b
        M(:, :, i)=te;
    end
end

H0=zeros(2*b,2*b);
Htem1=zeros(2*b,2*b);

for k=1:b
    Htem1(1:2,(2*k-1):2*k)=M(:, :, k);
end

H0=Htem1;

for m=1:b-1
    H0(2*m+1:2*m+2,1:2*b-2*m)=H0(2*m-1:2*m,3:2*b-2*m+2);
end

H1=zeros(size(H0));
H1(1:2*b-2,1:2*b)=H0(3:2*b,1:2*b);

[n1,n2]=size(H0);
[P,sigma,Q]=svd(H0);%singular value decomposition of Hankel matrix%
fd=zeros(1640,1);
ord=zeros(1640,1);

```

```

for j=1:40 % for-loop to plot stabilization diagram for
all model orders till 80%
A=sigma(1:2*j,1:2*j)^(-0.5)*P(:,1:2*j).'*H1*Q(:,1:2*j)*
sigma(1:2*j,1:2*j)^(-0.5);%state-space system
    matrix for a given model order%
[eig_vectors,eig_values]=eig(A);
Z=diag(eig_values);
lambda=log(Z).*f_sample;
damp_ratio(j^2-j+1:j^2+j)=-real(lambda)./abs(lambda);
damp_freq(j^2-j+1:j^2+j)=imag(lambda)./(2*pi);
ord(j^2-j+1:j^2+j)=2*j;
end%%%%
figure (11);%
plot(damp_freq,ord,'r.')
```

%plot stabilization first and
then only consider the positive part of the
plot because of conjugacy%

```

xlabel('Damped frequency (in Hz)');
ylabel('Model order (2:2:80)');

%To extract the stable damped frequency and damping ratio from
stabilization diagram inference%
n=30;%Choose a higher model order for stabilized poles%
A_new=sigma(1:n,1:n)^(-0.5)*P(:,1:n).'*H1*Q(:,1:n)*
sigma(1:n,1:n)^(-0.5);%state-space matrix
for the chosen model order%

[eig_vectors,eig_values]=eig(A_new); %transformation from state-space
to modal coordinates%
```

```

Z=diag(eig_values);
lambda=log(Z).*f_sample;%
damp_ratio=-real(lambda)./abs(lambda);
damp_freq=imag(lambda)./(2*pi); %%

Elt=[eye(1,1) zeros(1,(n1-1))];
Ept=[eye(1,1) zeros(1,(n2-1))];
B=sigma(1:n,1:n)^(0.5)*Q(1:n,1:n).'*Ept(:,1:n).';%%

C=Elt(:,1:n)*P(1:n,1:n).'*sigma(1:n,1:n);%%

% calculate mode singular values to determine
modal contribution for closely-spaced modes%
B1=eig_vectors\B;
C1=C*eig_vectors;
msv=zeros(size(Z));
for i=1:length(Z)
    c=C1(:,i);
    b=B1(i,:);
    msv(i)=sqrt((abs(c)*abs(b))/(1-abs(Z(i)))));
end%%

stem(damp_freq,msv)%plot MSVs against damped freq.%

%%Calculation of theoretical response%%
% Calculate the displacement profile of the carriage%
N1=length(sys_fil_ip1);
t_step=1/f_sample;

```

```

t1=0:t_step:(N1-1)*t_step;
f_res3 =1/(N1*t_step);
f3 = -0.5*f_sample:f_res3:0.5*f_sample-f_res3;
ax_fdom = fftshift(fft(sys_fil_ip1));
x_fdom = zeros(size(ax_fdom));
for i = 1 : NFFT1

    if f1(i)>1||f1(i)<-1
        x_fdom(i) =ax_fdom(i)/(-(2*pi*f1(i))^2);

    else

        f1(i) = 0;
        x_fdom(i)=0;

    end

end

x_tdom = real(ifft(ifftshift(x_fdom)));
x_tdom = x_tdom(1:N1); %displacement profile of carriage%
%Now enter the ERA-derived parameters%
zeta=.392;%%damp. ratio%%
fn=2*pi*1.25; %% natural freq from damped freq%%
resp1=zeros(size(x_tdom));
for i=1:length(x_tdom)    %calculate theoretical response%
    resp1(i)=x_tdom(i).*(1-(1/sqrt(1-zeta^2))*exp(-zeta*fn*t1(i))
    *sin(fn*t1(i)*sqrt(1-zeta^2)+1.23));
end

```

```
plot(t1,dis1_tdom,t1,resp)%compare experimental  
versus theoretical responses%
```

```
xlabel('time in sec')
```

```
ylabel('response')
```