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**EFFECT OF VIBRATION ON SHEARING
CHARACTERISTICS OF SOIL ENGAGING MACHINERY**

by

● **HANI MOHAMED SALEH KEIRA, B. Sc., Diploma of Higher Studies, M.Eng.**

A thesis submitted to the faculty of Graduate Studies
in partial fulfillment of the requirements for the
degree of Doctor of Philosophy

Faculty of Engineering
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August 1979

The undersigned recommend to the Faculty of Graduate Studies acceptance of the thesis "Effect of Vibration on Shearing Characteristics of Soil Engaging Machinery", submitted by Hani Mohamed Saleh Keira, B.Sc., Diploma of Higher Studies, M.Eng., in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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To the memory of my father,

To my mother,

To my wife,

and

To Nihal and Iman

Abstract

Vibratory techniques as a means of reducing the resistance of soil working devices have found applications in soil cutting, earth moving, pile driving and soil compaction.

In many vibratory soil working machines, such as vibratory soil cutting devices and vibratory soil compactors, the vibrational motion is perpendicular to the direction of shearing between the working elements and soil. A theoretical model has been developed for predicting the shearing characteristics of soil engaging elements subject to vertical vibration. It is found that the average value of the dynamic shearing force of the soil engaging element when subject to vertical vibrations may be less than that under static conditions. The reduction is mainly governed by the difference between the rate of development of the shearing force between the engaging element and the soil and the rate of increase of the ultimate shearing force (which is assumed directly proportional to the vertical force) in the loading part of the oscillation cycle.

The amplitude of dynamic vertical force, the frequency of excitation, the drawbar speed, the drawbar stiffness, the static normal load, the coefficient of interface friction, the rate of increase of shearing force with displacement and the time rate of change of dynamic vertical force are found to be the parameters which affect the average value of the dynamic shearing force of the soil engaging element.

To validate the mathematical model, a vibratory linear shear apparatus has been developed and assembled. It consists of a shear plate and a vibration excitor and is used to simulate the operation of a vibratory plate compactor. The amplitude and frequency of the dynamic load exerted on the plate can be varied. The horizontal shearing force and the dynamic load exerted on the plate are measured with appropriate transducers while the plate is being pulled along by a hydraulic ram. Thus the performance parameters of the model vibratory compactor are completely defined.

A series of tests have been carried out for determining the effects of the amplitude of exciting force, the drawbar speed, the frequency of excitation and the static normal

force on the shearing force developed on the plate-soil interface. It is shown that experimental results are in close agreement with the theoretical predictions, thus confirming the validity of the theoretical model.

Practical applications of the theoretical model to the analysis of the performance and the design of the vibratory plate compactor as well as to the evaluation of the effects of vibration on tracked vehicle performance are presented.

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LIST OF SYMBOLS

A_1	area between the tangent to a point on the loading part of a sinusoidal curve and the tangent to two successive minimum values of the curve for one cycle (see Fig. 6.1).
A_2	area between sinusoidal curve and the tangent to two successive minimum values of the curve for one cycle (see Fig. 6.1).
A_3	area defined in Fig. 6.1.
b	width of a track of tracked vehicle
B	width of friction plate
c	equivalent damping of soil in vertical direction
CVR	critical void ratio
d	distance between two road wheels of a tracked vehicle
D	difference in value of the forces in the two vertical transducers due to force in the horizontal transducer of the modified linear shear apparatus
e	voids ratio
e_{max}	maximum voids ratio
e_{min}	minimum voids ratio

f	frequency of excitation
F	vertical force
F_f	force in front transducer of the modified linear shear apparatus
F_r	force in rear transducer of the modified linear shear apparatus
F_{vs}	static vertical force
F_{vv}	measured amplitude of vertical force
F'_{vv}	theoretical amplitude of vertical force
g	acceleration of gravity
h	distance between upper plate and lower plate of the modified linear shear apparatus
$\bar{h}$	height of center of gravity of the apparatus from sand surface
h_1	height of the horizontal transducer from the soil surface
H	force in horizontal transducer of the modified linear shear apparatus or shearing force or gross tractive effort of a tracked vehicle

H_{av}	average value of shearing force per cycle of excitation
H_{ss}	static shearing force
H_{ult}	ultimate shearing force
H_{vs}	average value of dynamic shearing force
i	percentage of slip
K	spring stiffness (represents the drawbar stiffness)
K_s	equivalent stiffness to take into account the drawbar stiffness and the rate of increase of shearing force with shearing displacement
k_s	similar to K_s but defined in terms of stress/displacement
k_s'	rate of change of shear stress with distance under the track
K_v	equivalent stiffness models of soil in vertical direction
l	distance, in inches, between two successive peaks of vertical force on Oscillograph records
l_2	distance, in inches, between the maximum and minimum values of the vertical force

l_3	distance between minimum value of shearing force and the zero reference line of force
l_4	distance equal to A_1/l
L	length
m	mass of the vibrating machine
m_e	mass of the eccentric weights
M	moment acting on the frictional plate of the modified linear shear apparatus
M_x	moment about x axis
M_y	moment about y axis
n_r	number of wheels in a track of a tracked vehicle
N_y	bearing capacity coefficient
Q	ultimate bearing capacity
$R \text{ \& } R_1$	defined by equations 6.36. and 6.39. respectively
r	eccentricity of eccentric weights in inches
RD	relative density

s length defined in Fig. 6.2.
 S sensitivity of the accelerometer mv/g
 S_H scale of shearing force
 S_r speed of recording
 S_v scale of the vertical force
 t me
 t_1 time at which rate of increase of ultimate shearing force = $+\beta$
 T period of vertical dynamic force
 U_s work done by shear under no vibration/unit length
 U_v total work consumed by the vibratory soil compactor/unit length
 U_{vh} work done by shearing force/unit length
 U_{vv} work done by vertical vibration/unit length
 V actual speed of track vehicle
 $\bar{V}$ output voltage from the vibration pick-up preamplifier

v_j	speed of slip
v_t	theoretical speed
v_{sh}	drawbar speed
v_{vh}	dynamic component of horizontal velocity
v_{vv}	dynamic component of vertical velocity
w	angular velocity in rad/s
W	weight of a vehicle
x	horizontal displacement
$\dot{x}$	amplitude of velocity
$\ddot{x}$	horizontal acceleration
$\ddot{\bar{x}}$	amplitude of acceleration
α	rate of increase of shearing force with shearing displacement
β	rate of increase of shearing force with time
γ	unit weight of sand

ΔU_h	work done per cycle due to shearing
ΔU_v	work done per cycle of vertical vibration
ξ	damping coefficient
η	acceleration ratio
θ	phase difference between force and displacement
λ	defined as $1 - (A_1/A_2)$
μ	coefficient of friction = $\tan \phi$
σ_{vs}	mean pressure distribution under a tracked vehicle
σ_{max}	peak pressure distribution under a tracked vehicle
σ_{vv}	pressure defined by equation 6.31.
τ	shear stress
ϕ	angle of friction
$\phi_{critical}$	critical angle of friction
ψ	constant defined by equation 6.11.
ω	angular frequency rad/s

CHAPTER I

Introduction

Vibrations have been found to have a profound effect on the soil-structure and soil-machine interactions as shown by Mogami & Kubo (1953), Barkan (1962), Wong (1970), Youd (1970), Wong & Keira (1971) and Keira (1971). As the amount of large scale construction work grows, pressure for more efficient machinery for soil cutting, soil compaction and earthmoving is developing. The use of vibratory techniques as a mean of reducing the resistance of soil engaging devices and hence increasing the productivity has attracted a great deal of attention, and in recent years considerable effort has been spent on research and development in this area. It has been shown that the application of vibratory energy can considerably reduce the draft of cable plows (Bilotta, Boyd and Warren (1971)) and can increase the penetration rate of piles (Barkan (1957)). It has also been found that vibratory compactor is more efficient than an equivalent static one, mainly in compacting dry granular material, and that it

require much less towing force than conventional ones (Selig (1967)). A new system called VIBRA-LO for vibratory locomotion has been developed recently (Gaberson (1972)) as shown in Fig. 1.1.b. It is claimed that this vibratory locomotion can be used to propel certain types of construction machinery more efficiently.

On the other hand, the increase in resource exploration and exploitation in remote areas has created a need for more efficient off-road vehicles at increased operating speeds. Because of the unevenness of the terrain surface, high travelling speed will usually result in considerable vehicle vibration. Previous investigations by Wong (1971) have indicated that vehicle vibrations not only affect riding comfort but also may have a profound effect on vehicle performance, such as steerability and tractive performance.

Although the understanding of soil behaviour has rapidly improved through research, a general theory that adequately explains the effects of applying vibratory

energy to soil engaging devices has not yet been developed. To provide guiding principles for the rational design of vibratory soil-working machinery, it is necessary to reach a better understanding of the interaction between the vibratory soil working elements and soil.

Detailed investigations into the behaviour of foundation-soil systems subjected to vibration have been made. These studies were carried out under conditions quite dissimilar to those of off-road vehicles and other soil working machinery. A review of some of these investigations will be presented in Chapter II. The loading conditions of off-road vehicles and soil engaging machinery are shown in Figs. 1.1. to 1.4. For all of these loading conditions, the direction of vibration is normal to the direction of sliding of the working element, with the exception of the vibratory pile driving.

In this study, investigations were made to examine the shearing behaviour of a soil working element with vibration normal to its direction of motion. A theoretical

model is developed to elucidate the effects of vibration on the shear behaviour of a soil compactor. The details of the theoretical model are presented in Chapter III.

To examine experimentally the effect of vibration on the shearing behaviour of soil working elements, a vibratory linear shear apparatus was designed and built. This apparatus originally developed by Wong (1970) and used by Wong & Keira (1971) and Keira (1971) better simulates the shearing action of the vibrating soil engaging devices. Fig. 1.5. shows the details of the apparatus. The features and functions common to the vibratory linear shear apparatus and soil engaging elements are discussed in Chapter IV. In order to allow the direct measurements of the vertical exciting force and the shearing force acting on the soil-machine interface a special measuring device was designed, built and used in this research with the linear shear apparatus. An essential feature of this measuring device is the provision for torsional rigidity in the two planes perpendicular to the plane of motion while allowing the transducer to

monitor the force in the plane of motion without interference. This is achieved by using pairs of crossed flexure pivots. This arrangement permits free linear motion in the desired directions and offers high rigidity to torsion in the planes mentioned above. The detailed description of the measuring device, the evaluation of its performance and the method of data processing are presented in Chapter IV. The development of this measuring device was a major part of this research.

In order to confirm the validity of the theoretical model and to elucidate the mechanisms that cause changes in the behaviour of soil working machinery subjected to vibration, an experimental program was carried out and the effects of the following parameters on the performance of vibratory soil engaging elements were investigated:

1. The amplitude of exciting force.
2. The drawbar speed.
3. The frequency of excitation.
4. The static normal force.

The test results confirmed the validity of the theoretical model, and good agreement between the experimental and theoretical results is obtained. The experimental program, the results of the experimental and theoretical analysis and the discussion of the behaviour of the system are all presented in Chapter V.

The use of the model in evaluating the performance of vibratory plate compactors and off-road vehicles is presented in Chapter VI. A simple graphical method is developed for the prediction of the performance of such machines.

The model is used to estimate the energy consumed in shearing the soil by a vibratory soil compactor. This energy is important since it can be introduced as a measure of the pull required of vibratory soil compactors.

The model is also used to predict the tractive performance of vehicles with rigid track where the pressure distribution under the track is of multipeak sinusoidal form.

4
All the values are quoted in this thesis in the
imperial units with their equivalent in the international
system given in brackets.

CHAPTER II

Reviews of Studies on Vibratory Soil Engaging Machinery

2.1. Static Shear Behaviour of Sand

The shearing force-horizontal displacement relationship for sand under static normal load may take one of the forms shown in Fig. 2.1. The shear modulus of dense sand is greater than the shear modulus of loose sand. However, the shearing force becomes constant regardless of the initial condition after a large amount of displacement. This result has been reported, for the shear box test and triaxial test, by many investigators including: Tschebotarioff (1951), Roscoe et al (1958), Taylor (1967) and Lambe & Whitman (1969). It was confirmed by Keira (1971) using the Linear Shear Apparatus.

The curves in Fig. 2.2. show that as a granular system is sheared, there is an initial densification phase followed by a dilational phase and eventually a constant

volume phase. This behaviour was explained by Youd (1970) by assuming that shearing distortion simultaneously activates both the densification and dilational processes. Densification predominates during the intermediate phase, and the effects of the two processes counterbalance in the final phase.

Youd also mentioned that densification predominates over dilation at small shear strains because less strain is required to collapse a loose group of particles than is necessary to expand a dense group. In randomly packed systems both loose and dense groups are to be expected. This tendency toward densification is termed the "low strain densification bias". At larger strains, after the loose groups of particles have for the most part collapsed, the denser groups expand and dilation predominates. Dilation creates additional loose groups of particles which may eventually collapse. At some stage of strain the effects of expansion and collapse counterbalance and a constant volume state is reached, with a constant voids ratio called the critical void ratio (CVR).

The shearing force-displacement curves for sand in dense and loose states are shown in Fig. 2.3. According to the previous discussion, when the shear stress approaches a constant value with the increase of horizontal deformation, it is expected to be in the critical state with CVR, regardless of the initial condition of void ratio. If a vertical load is instantaneously applied, while the soil is fully mobilised, the shearing force will also increase. The shearing force-displacement curve may follow a curve similar to the CVR curve regardless of the initial condition before full mobilisation. As will be shown in Chapter V this statement was confirmed experimentally using the linear shear apparatus. In the same tests the vertical applied loads were released instantaneously with the result that the shearing force also decreased instantaneously in direct proportion to the change in vertical force.

It is concluded that the initial condition of void ratio has no effect on the response of the system to any step vertical load, as long as the soil is sheared beyond

its ultimate strength or reaches the critical void ratio.

Since the soil, in the model for vibratory soil engaging elements, will reach the CVR regardless of the initial void ratio, it can be assumed that the void ratio is equal to the critical void ratio. This simplifying assumption allows using a unique value for the rate of increase of shearing force with displacement α and the angle of internal friction ϕ .

2.2. Dynamic Shear Behaviour of Sand

There are many different explanations for the effects of vibration on soil properties. One of these explanations, proposed by Winter Korn (1953), is based on the analogy of the reversible transformation between the liquid and gas states caused by thermal energy. The medium was represented by what Winter called a "Macromeritic Liquid". Senator (1967) reported a similar approach given by Gumenskii and Komarov based on thixotropy, the reversible transformation between the solid and liquid states caused by a mechanical action such as

shaking. The thixotropic effects occur only in clay soils with the proper moisture content, and do not occur in sand and gravel.

Barkan's (1962) approach is to ignore the detailed chemical and physical properties of the soil and to measure experimentally the effects of vibrational parameters such as the amplitude and frequency of vibration on mechanical properties of soil. In his experiments the whole soil mass is vibrated.

Taking a similar approach to that of Barkan, other investigators have assumed that the behaviour of soil-foundation or soil-machine interaction is caused by changes of soil properties due to vibration. Among the many investigators are Mogami & Kubo (1953), Weissmann & Hart (1962), Hardin (1965), Parkash & Gupta (1967), Timmerman & Wu (1969), Atakol & Larew (1970), Drnevich & Richart (1970), Youd (1970 and 1972) and Hardin & Drnevich (1972).

Brumud & Leonards (1972) suggested that the parameter that governs the surface settlement of a vibrating footing on a granular mass is the steady-state transmitted energy. Youd (1970) followed an energy approach to illustrate the behaviour of sand subjected to horizontal vibration.

The following phenomena were reported by Barkan (1962) and confirmed by Youd (1970) under the loading condition shown in table 2.1. :

1. Vibration reduces the maximum coefficient of internal friction (Fig. 2.4.).
2. For acceleration ratio* η less than 1, the effect of vibration is not significant on either the initial slope of the shear force-displacement curve or the coefficient of internal friction.

In summary the effect of vibration on the angle of internal friction of soil has been extensively investigated previously and several different explanations exist of the mechanism by which vibration affects the angle of internal friction.

* Acceleration ratio = amplitude of acceleration of vibration/acceleration of gravity

The purpose of this research is to study the relationship between the shearing behaviour of soil engaging elements and the pertinent vibrational parameters. This study is limited to acceleration ratios of the vibrating elements in the range 0 and 1, within which the angle of friction can be assumed constant. However, it is believed that even if the amplitude of acceleration of the vibrating element exceeds $1g$ (or the vibrating element leaves from the ground), the soil acceleration can hardly exceed $1g$ in the unloading direction because of its inability to offer resistance to tension. Then the angle of friction of the soil will not be influenced by the acceleration of the element.

2.3. Models for Soil-Foundation and Soil-Machine Interaction

Previous studies of the foundations of structures or machines subjected to dynamic loading were made for vertical, rocking, torsional and sliding vibration modes. Solutions for foundation performance under dynamic conditions based on the modified earth pressure theory were introduced by Wallace (1961) and Prakash & Basavanna (1969).

Another fundamental approach is based on the mass-spring-dashpot analogy. In this approach the basic problem is to evaluate the equivalent spring constant, damping and soil mass vibrating in phase with the system. A variety of methods were proposed to evaluate these parameters including those presented by Pauw (1953-a), Rao & Nagaraj (1960), Hsieh (1962), Whitman (1965), Hall & Richart (1963), Barkan (1962), Converse (1953), Lorenz (1953) and Rainer (1975).

A third approach used for studying the vibration of footing-soil systems is usually referred to as the "elastic half space" theory. Most of the analysis assume that the distribution of stress on the contact area is the same as that for a statically loaded plate resting upon an elastic half-space. The first analysis of the elastic half-space theory was given in 1936 by Reissner followed by Quinlan (1953) and Sung (1953). Many investigators have followed that approach; among them are Pauw (1953-b), Richart (1962), Novak (1970) and Moore (1971).

The response of a foundation embedded in an elastic half-space was studied by Novak & Beredugo (1972). The nonlinear response of soil was taken into consideration by Funston & Hall (1967) and Ehrler (1968). The dynamic response of a foundation of arbitrary shape was considered by Elorduy, Nieto & Szekely (1967) and Chae (1969). Different modes of vibration were studied by Ratay (1971), Beredugo & Novak (1972) and Novak (1974).

Hsieh (1962) was probably the first to relate the elastic half-space theory to the equivalent mass-spring-dashpot system. By considering the propagation of divergent waves, a relationship was established between the externally applied periodic load on a rigid circular disk and its corresponding displacement and velocity. The equations of motion were expressed in a form similar to those of a system with single degree of freedom and viscous damping. Lysmer & Richart (1966), Richart & Whitman (1967), Whitman & Richart (1967) and Whitman (1969), among others extended and improved the approach developed by Hsieh and more simplified solutions were obtained.

A numerical procedure was applied by Hoeg & Rao (1970) to study the transient response of a footing on an elastic-perfectly plastic medium. The solution of nonlinear problems by the finite element method was introduced by Desai (1971). Krizek, Gupta & Parmelee (1972) used a two-dimensional, finite, lumped-parameter model to analyze the steady-state, coupled sliding and rocking response of an infinitely long dynamically loaded, rigid foundation embedded in elastic half-space.

In the area of foundation vibrations the shear mobilisation and horizontal displacement is very limited while in the area of soil-machine interaction, such as off-road vehicles, soil cutting tools, soil compactors,...etc, the soil experiences high degrees of mobilisation. The elastic half-space theory is probably not appropriate for this type of problems.

The approach developed by Blekhman and extended by Senator (1967), in their study of vibratory penetration of soils assumed that the soil properties were not influenced

by vibration parameters. They expressed the changes as reduction in bias force in the presence of vibration. Senator (1967) assumed that soils have a constant resistance to forward motion and a constant, but possibly different, resistance to backward motion (modified Coulomb friction). He compared various penetrating schemes based on their dynamic behaviour rather than on the properties of the soil. The loading conditions used by Senator differ from those under consideration here for the following reasons:

- a- The drawbar force is constant in Senator's model while it is varying in the case under consideration here.
- b- The dynamic force is the result of the rotation of one eccentric weight in Senator's model. The model under consideration here has an input of vertical dynamic component only.

Gaberson & Stone (1973) developed a new method for accomplishing land locomotion, which they call Vibratory Locomotion. The method is to vibrate a mass attached

to a skid, the mass oscillates in an inclined direction, upward and forward, downward and backward. On the up-forward stroke it lifts the skid and pulls it forward, on the down-backward stroke it pushes the skid into the ground and the increased friction inhibits backward motion. The main difference between the Vibratory Locomotion and the model under consideration is that the former is a self-propelled system while the latter requires an external propelling force to cause the forward motion.

The draft reduction by Vibratory Soil Cutting was studied by Sulatisky & Ukrainetz (1972). Their experimental model based on horizontal vibration of a blade moving with a constant speed. The result of their research showed that the vibrating earth cutters offered the possibility of reduced draught force at the expense of increased power.

Trapp, Abrahams & Reece (1974), performed more detailed study on this topic. They developed two models, one for the kinematic vibrator and the other for the

dynamic vibrator. The kinematic vibrator consisted of a mass whose forward velocity was unaffected by forces exerted on it by the cutting blade. A pure sinusoidal motion of the mass was superimposed onto the forward velocity. The dynamic vibrator consisted of an inertial drive system whose constant horizontal force was not affected by the rate of change of displacement. They claimed that the main reason for reduced draught force on a longitudinally vibrated cutter was the reduced time average effect and the reason for the increased power consumption was friction during the non-cutting parts of each cycle.

A simple lumped parameter model simulating soil cutting by a horizontally vibrating plow blade was developed by Boyd & Nalesny (1968). The cutting resistance of soil is simulated by Coulomb friction while the soil and blade elasticity is represented by a spring. The effects of velocity and inertia forces were neglected in the mathematical model of a spring with Coulomb friction. The carriage was assumed to move forward at a constant

velocity while the actuator vibrates the plow blade sinusoidally. It was found that the reduction in cutting force increases as the vibration frequency and amplitude and/or system stiffness increase. The differences between this model and the model under consideration are:

- a- The soil resistance is assumed constant in Boyd & Nalezny model while it is varying with the vertical force in the case under consideration here.
- b- The input vibration is in the same direction of the forward velocity in the Boyd & Nalezny model while it is normal to the direction of forward velocity in the case under consideration here.

Kofoed (1969) presented a kinematic model for the oscillating tillage tools for the evaluation of the power requirement. The movement is described by the path of a specific point on the tool using XYZ system of coordinates. Knowing the oscillation path, the velocity and acceleration can be evaluated. The total power can

be calculated from the integration along the time base of the product of the force $F(t)$ and velocity. The forcing function $F(t)$ was evaluated empirically.

The dynamic characteristics of a vibrating plate compactor was studied by Fu & Paul (1972). Their study was for a compactor with vertical vibration exceeding the acceleration of gravity and hence the compactor will leave the ground. They assumed that, during vibration, the compactor stays in contact with the soil during a very short time interval in which the impact energy of the compactor is delivered into the soil to do the work. They assumed that, for a plate compactor, the compaction process is a series of recurring impacts. At each blow, the vibrating plate impacts the soil surface, delivers part of its kinetic energy into the soil, and then rebounds from the soil surface. It was assumed that the compaction is directly related to the power input into the soil (energy delivery per blow multiplied by the blow frequency). Then the power input into the ground was introduced as a measure of the compaction effectiveness.

The model took into account the nonlinear features of the compaction process. During the contact period between the plate and soil, it is assumed that the sliding frictional coefficient is constant. If at the time of impact, the mass is moving in the forward direction, it will either continue sliding forward after impact or it will be brought to rest by friction. The dynamic force is the result of the rotation of one eccentric weight and no tractive force to move the system horizontally was considered in the study. The forward speed was only the result of the dynamic characteristics of the system. In general, it was found that the forward speed and power delivered to the soil are sensitive functions of the ratio of exciting force to dead weight.

2.4. Reviews of Previous Experimental Studies and Instrumentation

Barkan (1962), Terzaghi & Peck (1968), Youd (1970, 1972), Wong & Keira (1971), Keira (1971), Hardin & Drnevich (1972) and many others performed experimental studies on the effects of vibration on the behaviour of

soils and soil engaging elements. However, the loading conditions that are used in these studies are not similar. The discrepancies in the results obtained by different studies may have resulted from the use of different loading conditions and instrumentation in those investigations.

Modified triaxial devices for dynamic loadings were used by Whitman & Healy (1962), and modified direct shear devices were used by Mogami & Kubo (1953), Barkan & (1962) and Youd (1970). Peacock & Seed (1968) and Atakol & Larew (1970) used simple shear devices, while Hardin & Drnevich (1972) used resonant column apparatus to investigate the behaviour of soil subjected to vibrations. A summary of different loading conditions and instruments used in some of the previous research work are given in table 2.1.

The results obtained from all previous devices are suitable only for particular conditions. No device thus far is capable of solving general problems in the soil dynamics field. For example, triaxial compression and

Researcher	Apparatus and loading conditions	Results
Barkan (1962)	Shear Box	Displacement, Acceleration and frequency affect the angle of internal friction. Acceleration is found to be the main factor.
Youd (1970)	Shear Box	Displacement, Acceleration and frequency affect the angle of internal friction. Acceleration is found to be the main factor.
Mogami and Kubo (1953)	Shear Box	Vertical acceleration is found to be the main factor in reducing the angle of internal friction.
Youd (1972)	Simple Shear apparatus	Shear strain is found to be the primary factor for compaction
Hardin and Drnevich (1972)	Resonant Column	Strain amplitude, Effective mean principal stress and void ratio are main factors affecting the Shear Strength
Hertwig (1933)	Vibrating plate	Displacement and frequency affect the settlement
Wong (1970) Wong & Keira (1971) Keira (1971)	Vibratory Linear Shear	Ratio of dynamic load to static load, rate of shearing, static normal load and frequency affect shear behaviour.

Table 2.1
Summary of different loading conditions and Instruments used
in some of the previous research work in Soil Dynamics

direct shear tests both cause non-homogeneous stress and strain distributions in the specimen. The geometry of the specimen and rigidity of the boundaries dictate in advance the positions in which the shear strains and soil dilation are to be concentrated. This makes the interpretation of the results difficult. The inability to control and determine the stress conditions on the failure surfaces during vibration appears to be one of the greatest drawbacks in the use of triaxial or direct shear devices.

Generally, the loading conditions of modified triaxial, direct shear and similar devices for dynamic testing of soils do not simulate the actual operating conditions of vibratory soil engaging elements.

This is not only due to the previously mentioned limitations, but also to the use of very low rates of strain which are not suitable for soil engaging elements. Furthermore, these devices are suitable only for measurements up to the failure condition and they are not capable

of giving any measurements beyond failure. In most of the vibratory soil engaging elements, the performance of the element depends on the characteristics of soil beyond the failure condition.

The vibratory linear shear apparatus developed by Wong (1970) better simulates the shearing action of the vibrating soil engaging device. The features and functions common to the vibratory linear shear apparatus and soil engaging elements are:

1. The boundary conditions of the soil mass.
2. Specified excitation.
3. Similar shear rate to the soil engaging elements.
4. Equivalent loading condition and same conditions at contact areas.

There are, however, limitations in the original design of the vibratory linear shear apparatus. The shearing force developed on the vibrating soil engaging element could not be measured directly. Instead, the

drawbar pulling force was measured. The vertical force was estimated approximately but not measured, resulting in a further limitation.

Modification of the linear shear apparatus in order to overcome these limitations was found desirable in order to have an improved understanding of soil behaviour under these loading conditions.

CHAPTER III

Model for Shear Behaviour of a Vibratory Soil Engaging Element

3.1. Parameters for the Model for the Vibratory Soil Engaging Element

The preceding review suggested that the mechanism of soil-machine interaction during vibration affects the performance of the vibratory soil engaging elements. The dynamics involved seems to provide the answer to the draught reduction observed by many investigators, while changes in the physical properties of the soil due to vibration may also have some effects. In the research under consideration the values of vertical and horizontal accelerations will not exceed that due to gravity. The experimental results reported by Barkan or Youd show that for accelerations less than that due to gravity, the variation in angle of internal friction is not significant (Fig. 2.4.). It will be assumed in the following analysis that the changes in soil properties due to vibration are negligible as long as the acceleration is less than $1g$.

The response of the soil when its shear strength is fully mobilized will be represented by a Coulomb friction model with the frictional force proportional to the normal force and the angle of friction equal to the critical angle of friction of the sand. The critical angle of friction will be assumed to be independent of the shear rate (Keira 1971).

In the partially mobilized state, the shearing force will be assumed to be a function of the horizontal shear displacement. For simplicity, the shearing force-shear displacement relationship will be assumed to be linear. A unique value for the rate of increase of shearing force with displacement α will be used. The assumption that the shearing force-shear displacement relationship is linear and independent of all the other parameters seems to provide a reasonable basis for the development of a mathematical model.

The partially mobilized state will occur when the element starts moving from the rest condition or when

additional instantaneous downward vertical force is applied to the model while it is fully mobilized.

The fully mobilized state will occur when the shearing force reaches its ultimate value or when

$$H = \mu F$$

where H = shearing force
 F = vertical force
 μ = friction coefficient = $\tan \phi_{\text{critical}}$

If the vertical force F decreases instantaneously while the model is fully mobilized, the state of full mobilization will be maintained and the shearing force will decrease in proportion to the vertical force.

As will be shown later, the parameters which have a major influence on the shear behaviour of vibratory soil compactor are:

- a- The amplitude of the dynamic vertical force F_{vv}
- b- The frequency of excitation f

- c- The coefficient of friction μ
- d- The rate of increase of shearing force with displacement α
- e- The drawbar speed V_{sh}
- f- The drawbar stiffness K
- g- The rate of change of dynamic vertical force with time dF/dt (or the shape of the dynamic vertical force diagram)

The simplest model of vibratory soil engaging element with rigid drawbar will be investigated first (section 3.2.1.). Periodic loading of rectangular shape will be used with this simple model to eliminate the effect of rate of change of dynamic vertical force with time. This model will demonstrate the influence of F_{vv} , f , μ , α and V_{sh} .

To demonstrate the influence of the drawbar

stiffness, the response of the Coulomb frictional model with elastic drawbar will be presented in section 3.2.2. In the same section the combined influence of the drawbar stiffness K and soil stiffness α will be studied.

In section 3.3., the effect of the rate of change of dynamic vertical force with time will be studied by using periodic loading of triangular shape acting on the Coulomb frictional model. The triangular shape will give a constant value for dF/dt . The response of the vibratory soil engaging element with rigid and elastic drawbars under periodic loading of triangular shape will be also studied.

Finally the simplified model for the vibratory soil engaging element under harmonic loading will be discussed in section 3.4.

3.2. Soil Engaging Element Under Periodic Loading of Rectangular Shape

3.2.1. Rigid Drawbar

Assume a block with weight F_{vs} resting on a Coulomb frictional surface of constant friction coefficient μ . The block is moving on the surface with the aid of a rigid drawbar which moves with constant speed V_{sh} . The coefficient of friction will be assumed independent of the rate of shearing. The block is subjected to a vertical periodic loading of rectangular shape (Fig. 3.1.).

When D starts to move the plate will be immediately fully mobilized and the shearing force will be equal to

$$H = \mu F_{vs} \quad 3.1.$$

If a rectangular dynamic load excites the system vertically and the plate is sheared simultaneously, the shearing force will be as shown in Fig. 3.1. The value

of the shearing force for the loading part of the cycle is:

$$H = \mu (F_{vs} + F_{vv}) \quad 3.2.$$

The value of the shearing force for the unloading part of the cycle is:

$$H = \mu (F_{vs} - F_{vv}) \quad 3.3.$$

Since the shearing force under this loading condition is the ultimate value of shear, the shearing force diagram is also the ultimate shearing force diagram.

The average value of shearing force during a cycle for a periodic dynamic symmetrical loading such as the rectangular case can be proved to be equal to

$$H_{av} = \mu F_{vs}$$

That is, the average value of dynamic shearing force of a block mobilized on Coulomb frictional surface with rigid drawbar and subjected to vertical symmetrical rectangular periodic load is equal to the ultimate static

shearing force of the plate.

The model proposed to simulate the vibratory soil engaging element using rigid drawbar is similar to the previous model with the exception that the Coulomb frictional surface is replaced by soil surface as shown in Fig. 3.2. In the case of the Coulomb frictional surface, the shearing force-shearing displacement curve is of the rigid-plastic type which means that no displacement will occur until the force reaches the ultimate value. In the case of a soil the shearing force increases proportionally to the displacement until it reaches an ultimate value. At point 1 the system is stationary. When the rigid drawbar starts to move with constant speed V_{sh} a force αx will develop between the soil and the plate where α is the rate of increase of shearing force with displacement as discussed earlier. At point 2 the plate has moved a distance x_2 and developed a shearing force αx_2 . At point 3 the plate has moved to distance x_3 and the draft force becomes equal to the ultimate shearing force μF_{vs} . The soil is in its

critical voids ratio and the block will keep moving forward without any increase in force. At point 4 a vertical force F_{vv} is applied instantaneously to the block. The draft force will increase again in proportion to the displacement until $\alpha x_3 + \alpha (x_5 - x_4)$ is equal to $\mu (F_{vs} + F_{vv})$ at point 5. Beyond that the block will move again with constant draft force. When the vertical force F_{vv} is released at point 6 the shearing force will be reduced instantaneously to μF_{vs} .

The shearing force-time relationship and displacement-time relationship is shown in Fig. 3.3. The average value of the dynamic shearing force is less than the average value of the ultimate shearing force (or the static shearing force). The average value of the dynamic shearing force can be obtained from the equation:

$$H_{vs} = \frac{1}{T} \left\{ \mu (F_{vs} + F_{vv}) \frac{T}{2} - \frac{\mu^2 F_{vv}^2}{\alpha V_{sh}} + \mu F_{vv} + \mu (F_{vs} - F_{vv}) \frac{T}{2} \right\} \quad 3.4.$$

or

$$H_{vs} = \mu F_{vs} - \frac{2 f \mu^2 F_{vv}^2}{\alpha V_{sh}} \quad 3.5.$$

where T - Period of vertical dynamic force
 f - Frequency of excitation

Equation 3.5. shows that the reduction in average dynamic shearing force relative to the static shearing force is a function of:

- a- the amplitude of dynamic vertical force F_{vv}
- b- the frequency of excitation f
- c- the coefficient of friction μ
- d- the rate of increase of shearing force with horizontal displacement of sand α
- e- the drawbar speed V_{sh}

3.2.2. Elastic Drawbar

Assume first a block with weight F_{vs} resting on a Coulomb frictional surface of frictional coefficient μ . A spring with stiffness K is connected between the block and the drawbar which can be moved with constant speed V_{sh} as shown in Fig. 3.4. If the block is mobilized the coefficient of friction will be assumed to be independent of the rate of shearing. The inertia force of the block

also will be neglected. This assumption is verified experimentally in Chapter V. The block is subjected to a vertical periodic loading of rectangular shape. When the end D of the spring starts to move with constant velocity V_{sh} , the force will develop in the spring. The spring force will be balanced by the frictional force while the block remains stationary. When the spring force reaches the maximum shearing resistance of the plate μF_{vs} , the block will start to move. During the motion of the block the spring force will not increase and it will maintain the same value μF_{vs} as shown in Fig. 3.4. The time at which the motion of the block starts can be calculated from the equation:

$$\mu F_{vs} = k V_{sh} t_1$$

or

$$t_1 = \mu F_{vs} / k V_{sh} \quad 3.6.$$

At time t_2 another vertical force F_{vv} is instantaneously applied to the block. The shear resistance will increase to be $\mu(F_{vs} + F_{vv})$ while the spring force is still, at

time t_2 , equal to μF_{vs} . The block then remains stationary until the spring force becomes equal to the resisting force when it will move again. The motion will start again at time t_3 which can be obtained from the equation:

$$t_3 = t_2 + \frac{\mu F_{vv}}{k V_{sh}} \quad 3.7.$$

At time t_4 the force $2 F_{vv}$ is released, and the resisting force H decreases instantaneously from $\mu(F_{vs} + F_{vv})$ to $\mu(F_{vs} - F_{vv})$. For block equilibrium the spring force should also be reduced to $\mu(F_{vs} - F_{vv})$. Since the block is still fully mobilized, it will instantaneously move forward to allow the spring to contract a distance proportional to the reduction of spring force. The distance the block will move forward upon the release of vertical force $2 F_{vv}$ is:

$$\frac{2 \mu F_{vv}}{k_s}$$

Following this sudden movement, the block will proceed

with constant speed V_{sh} . At time t_5 another vertical force $2 F_{vv}$ is instantaneously applied and the plate stops moving until the force in the spring reaches the shear resistance, then

$$t_6 - t_5 = \frac{\mu^2 F_{vv}}{k V_{sh}} \quad 3.8.$$

The block will start to move again with constant speed V_{sh} until at time t_7 where an instantaneous release of force $2 F_{vv}$ occurs and the spring contracts instantaneously.

The time $(t_7 - t_4)$ represents the period of a rectangular vertical dynamic force acting on the block.

The response of this cycle will be repeated for subsequent cycles.

The average shearing force during one cycle is:

$$H_{vs} = \frac{1}{T} \left\{ \mu (F_{vs} + F_{vv}) \frac{T}{2} - \frac{\mu^2 F_{vv}}{k V_{sh}} 2 \mu F_{vv} + \mu (F_{vs} - F_{vv}) \frac{T}{2} \right\}$$

or

$$H_{vs} = \mu F_{vs} - \frac{2 f \mu^2 F_{vv}^2}{k V_{sh}} \quad 3.9.$$

Equation 3.9. shows that the reduction in average dynamic shearing force relative to the static shearing force is a function of:

- a- the amplitude of dynamic normal force F_{vv}
- b- the frequency f
- c- the coefficient of friction μ
- d- the drawbar spring constant k
- e- the drawbar speed V_{sh}

This reduction occurs because the shearing force does not reach its ultimate value instantaneously since the spring does not transmit the force immediately. The development of force in the spring depends on the velocity V_{sh} of the drawbar.

In order to investigate the validity of this model a test was carried out using the modified linear shear apparatus. The exciting motor and balancing weight were removed. A very soft spring was used to connect the plate while it was moving and the shearing force was recorded. The loads were released instantaneously and the

response was also recorded. Fig. 3.5. is a reproduction of the recording graph. It can be seen that, upon loading, the spring force developed at the rate $\beta = 1.74 \text{ lb/s}$ (7.74 N/s).

but $dH = k \cdot dx$

$$\frac{dH}{dt} = k \frac{dx}{dt}$$

or

$$\beta = k V_{sh} \quad 3.10.$$

then $k = \frac{\beta}{V_{sh}}$

The rate of shearing V_{sh} was 0.0427 in/s (1.0846 mm/s)

then $k = \frac{1.74}{0.0427} = 40.73 \text{ lb/in (7.133 N/mm)}$

which was the same as the calibrated value of the spring constant.

During the development of force in the spring no plate displacement was observed. Upon the release of the vertical load a sudden drop of the shearing force was shown and a sudden forward displacement was observed.

The shearing force-time relationship is shown in Fig. 3.6. for the vibratory soil engaging element with elastic drawbar and subjected to periodic loading of rectangular shape.

In the partially mobilized state, if point P moves distance x while point D moves a distance x' ($x' = V_{sh} t$), a shearing force will develop between the plate and sand and a spring force develops in the elastic drawbar.

$$\text{shearing force} = H = \alpha x$$

$$\text{drawbar force} = k (V_{sh} t - x)$$

but for equilibrium

$$k (V_{sh} t - x) = \alpha x$$

$$\text{or } x = \frac{k V_{sh} t}{\alpha + k}$$

$$\text{but } H = \alpha x = \frac{\alpha k V_{sh} t}{\alpha + k}$$

$$\text{or } \frac{H}{V_{sh} t} = \frac{\alpha k}{\alpha + k}$$

$$\frac{H}{x'} = \frac{1}{\frac{1}{k} + \frac{1}{\alpha}}$$

But $\frac{1}{\frac{1}{k} + \frac{1}{\alpha}}$ represents the equivalent spring constant of two springs in series with stiffnesses k and α .

The response of this model is similar to the Coulomb frictional model with elastic drawbar having an equivalent spring constant of

$$K_s = \frac{1}{\frac{1}{k} + \frac{1}{\alpha}}$$

The rate of change of the draft will be equal to:

$$\beta = K_s V_{sh} \quad 3.11.$$

The average value of shearing force can be shown to be

$$H_{vs} = \mu F_{vs} = \frac{2 \mu^2 F_{vv}^2 f}{V_{sh} K_s} \quad 3.12.$$

Equation 3.12 shows that the reduction of the average value of dynamic shearing force relative to the static shearing force is a function of μ , F_{vv} , f , V_{sh} , k and α .

3.3. Soil Engaging Element Under Periodic Loading of Triangular Shape

Assume a block with weight F_{vs} resting on a Coulomb frictional surface of frictional coefficient μ . The block is connected to the end P of a spring of stiffness k while the other end of the spring D can move with a constant velocity V_{sh} (Fig. 3.7.). The coefficient of friction will be assumed independent of the shear rate or any other factors. The inertial force will be neglected. This assumption was verified experimentally in Chapter V. The block is subjected to a vertical triangular periodic loading.

Two cases of drawbar speed V_{sh} will be considered, one high and the other low.

3.3.1. Model With High Drawbar Speed V_{sh}

Assume first one cycle of triangular loading with the initial force starting at zero (point 1) as shown in Fig. 3.7. The rate of change of the draft, as mentioned before, is $\beta = k V_{sh}$. If V_{sh} is high, then β will be high. It will be assumed in this case that β is higher than the rate of increase of the ultimate shearing force

$$\beta > \frac{d(\mu F)}{dt} \quad 3.13.$$

where F is the total vertical force.

Assume that the triangular loading is replaced by a step function as shown in Fig. 3.7-a. Following the same procedure taken in section 3.2.2., the shearing force will follow the path 1, 2, 3, 4, ...17.

If the interval length of the step function approaches zero, the shearing force diagram will coincide with the ultimate shearing force diagram as shown in

Fig. 3.7-b.

Three conclusions may be drawn from this model:

- a- If the rate of change of the draft β is higher than the rate of increase of the ultimate frictional force, the shearing force will be equal to the ultimate shearing force.
- b- For the unloading part of the cycle, the shearing force is equal to the ultimate shearing force.
- c- No reduction of average shearing force is obtained relative to the static.

If the model is subjected to static force F_{vs} and periodic loading of triangular shape with amplitude F_{vv} the response will be as shown in Fig. 3.8. In this case the average value of ultimate shearing force is equal to the average value of dynamic shearing force and no reduction is obtained.

3.3.2. Model With Low Drawbar Speed V_{sh}

Assume that the speed V_{sh} is small so that the rate of change of the draft β is smaller than the rate of increase of the ultimate shearing force

$$\beta < \frac{d(\mu F)}{dt} \quad 3.14.$$

Assume first one cycle of triangular loading starting from zero force at point 1, as shown in Fig. 3.9. The triangular loading is approximated by step loading. The spring force will keep increasing until it reaches the maximum frictional force at point 2. The block will start to move and the shearing force will be equal to the ultimate shearing force in the unloading part of the cycle as discussed before in the case of high speed V_{sh} .

If the length of the step function approaches zero, the shearing force in the unloading part will coincide with the triangular function in the remaining unloading

part of the cycle as shown in Fig. 3.9-b.

The conclusions to be drawn from this model are:

- a- If the rate of change of the draft β is less than the rate of increase of ultimate shearing force, the shearing force will be equal to the spring force.

$$\text{or} \quad H = H_0 + \beta (t - \bar{t}) \quad 3.15.$$

where H_0 is the value of shearing force at the time $\bar{t}$ when the plate stops and the spring starts to stretch.

- b- As soon as the spring force becomes equal to the ultimate shearing force in the unloading part of the cycle, the shearing force will remain equal to the ultimate shearing force for the rest of the unloading part of the cycle.
- c- The average value per cycle of the dynamic shearing force diagram is less than the average value per cycle of ultimate shearing force diagram.

The case of a model subjected to static force F_{vs} and periodic loading of triangular shape with amplitude F_{vv} is shown in Fig. 3.10. The average value of shearing force is shown to be less than the static shearing force (or the average value of ultimate shearing force). Shearing force can be obtained from the following relationships:

time t_0 to t_1 :

The plate is stationary, the force is increased proportional to the time and

$$H = k V_{sh} t$$

$$t_1 = \frac{\mu (F_{vs} + 2 F_{vv})}{k V_{sh} + \frac{4 \mu F_{vv}}{T}}$$

time t_1 to t_2 :

$$H = \mu (F_{vs} + F_{vv} - \frac{4 F_{vv}}{T} (t - \frac{T}{4}))$$

time t_2 to t_3 :

$$H = \mu (F_{vs} - F_{vv}) + k V_{sh} (t - t_2)$$

time t_3 can be obtained from the equation

$$t_3 = t_2 + \frac{1}{\frac{k V_{sh}}{4 \mu F_{vv}} + \frac{1}{T}}$$

and so on

The average value of shearing force can be obtained from the equation

$$H_{vs} = \frac{1}{T} \left\{ \mu (F_{vs} - F_{vv}) T + \frac{T}{2} k V_{sh} \frac{1}{\frac{k V_{sh}}{4 \mu F_{vv}} + \frac{1}{T}} \right\}$$

or

$$H_{vs} = \mu F_{vs} - \frac{4 \mu^2 F_{vv}^2 f - \mu F_{vv} k V_{sh}}{k V_{sh} + 4 \mu F_{vv} f} \quad 3.16.$$

It can be concluded from this equation that the average value of dynamic shearing force is less than the static shearing force if

$$4 \mu^2 F_{vv}^2 f > \mu F_{vv} k V_{sh}$$

or

$$4 \mu F_{vv} f > k V_{sh}$$

where $k V_{sh} = \beta$ = rate of change of the dynamic shearing force

For periodic loading of triangular shape, the rate of increase of the ultimate shearing force is:

$$\frac{d(\mu F)}{dt} = \mu \frac{2 F_{vv}}{(T/2)} = 4 \mu F_{vv} f$$

thus the condition for a reduction of average dynamic shearing force is satisfied if the rate of increase of ultimate shearing force is greater than the rate of change of the dynamic shearing force.

It can be concluded from equation 3.16. that the reduction in average value of the dynamic shearing force under this loading condition is a function of the amplitude of the dynamic force F_{vv} , the frequency f , the drawbar stiffness k , the speed of drawbar V_{sh} and the coefficient of friction μ .

The response of the vibratory soil engaging element with rigid drawbar and subjected to periodic loading of triangular shape is shown in Fig. 3.10. The average value of dynamic shearing force under this condition is found to be

$$H_{vs} = \mu F_{vs} = \frac{4 \mu^2 F_{vv}^2 f - \mu F_{vv} \alpha V_{sh}}{\alpha V_{sh} + 4 \mu F_{vv} f}$$

The reduction in the average value of the dynamic shearing force under this loading condition is a function of F_{vv} , f , V_{sh} , μ and α .

The shearing force-time relationship is shown in Fig. 3.10. for the vibratory soil engaging element with an elastic drawbar and subjected to periodic loading of triangular shape.

It can be proven that for a reduction in the average value of dynamic shearing force to occur, the following condition should be satisfied

$$\frac{d(\mu F)}{dt} > \beta$$

where

$$\beta = K_s V_{sh}$$

and

$$K_s = \frac{1}{\frac{1}{a} + \frac{1}{k}}$$

The average value of the dynamic shearing force can be shown to be

$$H_{vs} = \mu F_{vs} = \frac{4 \mu^2 F_{vv}^2 f}{V_{sh} K_s} - \mu F_{vv} V_{sh} K_s$$

3.4. Soil Engaging Element Under Harmonic Excitation

The ultimate shearing force diagram for a vertical harmonic load will also be harmonic. For the same amplitude of vertical force the harmonic load has values between those for the rectangular and triangular forms. It can be expected that for the same amplitude and frequency the value of the average shearing force under harmonic loading will have a value between those for the rectangular and the triangular loadings. In Fig. 3.11. the shaded areas should be deducted from the area under the ultimate shearing force-time curves to obtain the average value for the three cases. For the same rate of change of dynamic shearing force β , the shaded area of the rectangular loading is the largest, followed by the harmonic and the smallest is the triangular. This shows that the maximum reduction obtained for the rectangular shape loading and the smallest for the triangular shape loading.

The response of the Coulomb frictional model to harmonic loading is similar to the model response for the triangular loading except for the following:

1. The rate of change of ultimate shearing force will be harmonic.
2. In the case of harmonic loading the Coulomb frictional model will stop moving at the point where the tangent to the ultimate shearing force curve is equal to μ as shown in Figs. 3.11. and 3.12.

The response of the vibratory soil engaging element with rigid drawbar and subjected to periodic loading of harmonic shape is shown in Fig. 3.12. The average value of dynamic shearing force under this condition

is shown to be less than the average value of ultimate shearing force (or the static shearing force).

Similarly to the rectangular or triangular periodic loadings, the condition that a reduction in the average value of dynamic shearing force occurs under harmonic loading is

$$\frac{d \mu F}{d t} > \alpha V_{sh} \quad (\text{during the loading part of the cycle})$$

It can be also expected that the parameters that will influence the reduction in the average value of dynamic shearing force are:

$$P_{vv}, f, V_{sh}, \mu \text{ and } \alpha$$

The shearing force-time relationship is shown in Fig. 3.12. for the vibratory soil compactor with elastic drawbar and subjected to periodic loading of harmonic shape.

As demonstrated before, the condition for a reduction

in the average value of dynamic shearing force is

$$\frac{d(\mu F)}{dt} \quad \beta$$

where $\beta = K_s V_{sh}$

and $K_s = \frac{1}{\frac{1}{a} + \frac{1}{k}}$

if $F = F_{vs} + F_{vv} \sin \omega t$

$\frac{dF}{dt} = F_{vv} \omega \cos \omega t$

$\mu F_{vv} \omega \cos \omega t > K_s V_{sh}$

$\frac{\mu F_{vv} \omega \cos \omega t}{K_s} > V_{sh}$

The maximum slope of the ultimate shearing force curve as function of time will occur when $\omega t = 0, 2\pi, 4\pi, \dots$ etc. (or $\cos \omega t = 1$). Then the critical condition at which no reduction in the average value of dynamic shearing force will occur is when:

$\frac{\mu F_{vv} \omega}{K_s} = V_{sh} \text{ (critical)}$

or

$$\frac{2 \pi f \mu F_{vv}}{K_s} = V_{sh} \text{ (critical)}$$

For the same amplitude and frequency of dynamic force, and drawbar speed, the average value of dynamic shearing force for a vibratory soil compactor subjected to harmonic loading will have a value between the average values for the rectangular and triangular shape of loadings. The derivation of the equation for the average value of dynamic shearing force under harmonic loading is very complicated. A computer program and a graphical method will be presented in Chapter V for the evaluation of the average value of dynamic shearing force.

3.5. Summary

Different simplified cases for the vibratory soil engaging element model were presented. Each case explained the physical behaviour of one or more of the model parameters. The main parameters that influence the behaviour of the vibratory soil engaging element

are:

- a- the amplitude of dynamic vertical force F_{vv}
- b- the frequency of excitation f
- c- the coefficient of friction μ (or the angle of friction)
- d- the rate of increase of shearing force with displacement α
- e- the drawbar speed V_{sh}
- f- the drawbar stiffness K
- g- the rate of change of dynamic vertical force with time $d(F)/dt$ (or the shape of dynamic vertical force)
- h- the static normal force

The physical explanation for the reduction in the average value of the dynamic shearing force in the vibratory soil engaging element is that the rate of development of the shearing force between the compactor and the soil is slower than the rate of increase of the ultimate shearing force (which is directly proportional to the vertical force) in the loading part of the cycle.

This results from a partially mobilized soil condition, and/or a low rate of increase of drawbar force due to its elasticity throughout a large portion of the cycle. All the above mentioned parameters affect the rate of development of the dynamic shearing force relative to the rate of increase of the ultimate shearing force or the vertical force and hence the average value of the dynamic shearing force.

CHAPTER IV

The Vibratory Linear Shear Apparatus

4.1. Introduction

Proper laboratory testing of vibrating soil engaging devices is very important to reach a better understanding of the problem. It is also useful to confirm the validity of the mathematical modelling of the problem. A review of the devices previously used in soil mechanics and soil dynamics research shows that none is capable of simulating the real loading conditions of a vibrating soil engaging devices.

The vibratory linear shear apparatus shown in Fig. 1.5. better simulates the shearing action of vibrating soil engaging devices. However, it is desirable to measure directly the vertical exciting force and the horizontal shearing force acting on the soil as well as the vertical and horizontal acceleration, velocity or displacement of the frictional plate. For this reason, a new measuring

device was designed, made and used with the linear shear apparatus.

In this Chapter, the new design of the measuring device and the vibratory linear shear apparatus will be described. The evaluation of the performance of the vibratory linear shear apparatus with the new measuring device as well as its calibration will be reviewed.

4.2. Design of Instrumentation for Measuring the Vertical Exciting Force and the Shearing Force

The objectives of the new design are to better simulate the actual operating conditions of vibratory soil engaging elements and to measure all pertinent vibrational parameters, including those describing soil behaviour under dynamic conditions.

Basically, the new design consists of a special suspension connecting the frictional plate to the vibrating excitor and instrumentation which can measure the vertical and horizontal forces acting on the shear

plate directly. The mechanism used is shown in simplified form in Fig. 4.1.

The measuring device includes three piezoelectric quartz force transducers with fast response for measuring forces in either compression or tension (indicated by F_f , F_r and H in Figs. 4.1. and 4.2.). The transducers are of "Kistler" model 930 A series. They have a high natural frequency, and very high sensitivity. The sensing element is a rigid, linear, stable piezoelectric material. The transducers are connected to the frames by rod ends with self lubricating spherical bearings, 7, 8, 9, 10, 11 and 12 (Figs. 4.1. and 4.2.). The rod ends are high precision "Fafnir" model R A 3 M 4 - 2.

The vibration excitor is connected to frame I while the frictional plate is connected to frame II. Frame I and II are shown separately in Fig. 4.2. Any vertical force F applied to frame I will have 2 reactions $F(L - a)/L$ and $F a/L$ at supports 9 and 7 respectively.

A horizontal force H applied on frame I will have one reaction at support 12 only. The 2 hinged members 9-10 and 7-8 will transmit the vertical reaction through the vertical transducers to the supports 10 and 11 of frame II. The hinged member 11-12 will transmit the horizontal reaction through the horizontal transducer to frame II. Frame II is supported by the soil and its equilibrium will be maintained by the reaction from the soil. Frames I and II are made from aluminum for lightness of structure. The inertial force during vibration of frame II can thus be neglected and the horizontal shearing force between the frictional plate and the soil can be taken as being equal to the value measured by transducer H . Similarly the vertical reaction between the soil and the plate can be taken as being equal to the summation of the measured values of the two vertical transducers F_f and F_r . The horizontal transducer is located as close as possible to the soil surface to minimise the moment resulting from the drawbar pulling force (H).

With the use of high precision bearings and very

tight fittings, the slack in the system is insignificant thereby eliminating inaccuracies in the measurement of the forces. A second precaution to avoid slack is by using eccentric shafts to connect the transducer end bearings to the frames. The eccentric shafts permit the application of initial compression to the transducers. Fig. 4.3. shows the details of the eccentric shafts and the connections. When the eccentric shaft is in the position where it gives maximum tension or compression, the transducer and links are perfectly vertical for F_f and F_r and perfectly horizontal for H. If the eccentric shaft is in any other position, the transducer will be slightly inclined to the vertical or horizontal. It was found from the calibration that such inclinations have no noticeable effect on the measurement of the forces.

In order to ensure torsional rigidity in the X-Z and Y-Z planes, the three hinged frames 1-2-3 and 4-5-6 are used to connect frame I with frame II (Figs. 4.1. and 4.4.). Hinges 1, 3, 4 and 6 are made from 0.008 in. (0.2032 mm) thick strips of spring steel.

The three hinged frames will not carry any horizontal or vertical forces. Any eccentricity along the Z axis of the vertical force F or horizontal force H will result in a moment M_x or M_y respectively. The moment M_x will be transferred from frame I to the two vertical strips 3 V, to the rigid block, to the two other vertical strips 1 V and finally to frame II. Moment M_y will be transferred through the horizontal strips rather than the vertical ones. It is necessary to use this mechanism to eliminate bending moments.

All the bulldozing forces are carried by a guiding plate of the same dimensions as the frictional plate. The guiding plate is connected directly to the drawbar so that no bulldozing force will be transmitted to the horizontal transducer H (Fig. 4.5.).

The shear plate is connected to the bottom of frame II by two sets of screws so that it can be changed or replaced if desired. Three rectangular shear plates with approximately the same area (63 in.^2 or 40645 mm^2)

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were used with dimensions of 63.5×6.4 mm, 53.3×7.6 mm and 40.6×10.2 mm.

The plates were roughened by glueing on coarse sand corresponding to that used in the experiments.

Kistler model 503 M 1 charge amplifiers were used in conjunction with the force transducers. The proper selection of a feedback resistor with a feedback capacitor for the amplifier produces a time constant from a milli-second to days, thus making it possible to measure static as well as dynamic loads.

Two Brüel & Kjaer type 4338 peizoelectric accelerometers were mounted directly on frame II in the center line of the frictional plate. One was used to monitor the vertical vibration of the system. The other was used to monitor the horizontal vibration and was mounted on frame II as close as physically possible to the surface of the soil.

Two Brüel & Kjaer type 2625 vibration pick-up pre-amplifiers were used. They are high input impedance

amplifiers for use with piezoelectric accelerometers.

The preamplifiers are provided with integration networks for measurement of velocity and displacement in addition to acceleration.

The vibration pick-up preamplifiers were powered by 28 V D.C. external power supply, Brüel & Kjaer type 2805. The output from the preamplifiers consisted of a standing D.C. voltage (≈ 13 V) and an A.C. voltage which represents the acceleration, velocity or displacement. The recording instrument had very high sensitivity and the signals representing the acceleration, velocity or displacement are of a very low level (in the order of m.v.). Thus it was very important to increase the A.C. gain and to block out or balance the D.C. voltage.

To increase the signal level, the gain adjustment knob of the preamplifiers was adjusted to the position 20 dB which increased the voltage gain to 10. Attenuators were used to adjust the sensitivity of the signal and the

scale of recording. Two 15 micro-farad capacitors were used to block the 13 D.C. volts. Fig. 4.6. shows the details of the circuit.

In order to record the two vertical force components of F_f and F_r , as well as the resultant of the vertical forces, a circuit was built as shown in Fig. 4.7. The electronic circuits shown in Figs. 4.6. and 4.7. were built using the Electronic Associates Inc. model T R - 10 general purpose analog computer.

A Midwestern Instruments Model 800 oscillograph was used to record the output on eight-inch-wide, high speed, direct recording paper. The recording is effected by galvanometers of types 102 A-60 and 102 A-100 in these experiments.

4.3. The Vibratory Linear Shear Apparatus

The vibratory linear shear apparatus was used after replacing the original rigid connection between the vibration excitor and the frictional plate by the new

measuring device described in the previous section. The rest of the elements of the vibratory linear shear apparatus are used as they were originally developed. These elements include:

1. A vibration excitor which generates a sinusoidally varying force (Fig. 4.8.) was used to excite the shear plate in the vertical direction. The excitor is made up of a variable speed motor of $\frac{1}{8}$ H P (93.2 W) or $\frac{1}{4}$ H P (186.4 W) and counter-rotating eccentric masses. By regulating the motor speed with a speed control unit and changing the eccentric masses, various frequencies and amplitudes of the exciting force can be provided.
2. A concrete tank with inner dimensions 49 in. (1.25 m) long, 32½ in. (0.83 m) wide and 24 in. (0.61 m) deep was used to contain the sand as shown in Fig. 4.9. The tank was lined with thin layer of foam rubber to absorb any vibrations.
3. A hydraulic ram was used to pull the shear plate. The rate of shearing was regulated by a flow control valve.

4. The double parallelogram used in the vibratory linear shear apparatus was also used in some experiments conducted in this research. The purpose of the double parallelogram was to maintain the shear plate in a horizontal position while moving. A discussion about the function of the double parallelogram will be presented later in this Chapter.

4.4. Performance and Calibration of the Vibratory Linear Shear Apparatus

4.4.1. Performance of the Apparatus Under Static and Dynamic Loading

The modified vibratory linear shear apparatus was tested and calibrated under static conditions using a calibration device made especially for this purpose. The calibration was made by applying vertically and horizontally known static loads to the upper plate of frame I as shown in Fig. 4.10. It was found that the force measuring systems exhibited linear characteristics as shown in Fig. 4.12. If the horizontal force H is acting at height h from the

base of the frictional plate, the moment acting on the base will be

$$M = H \cdot h$$

This will result in an increase of force in the front transducer and decrease of force in the rear transducer. The difference in value between the forces in the two vertical transducers due to horizontal force H will be

$$D = 2 \frac{H \cdot h}{L}$$

where L = distance between the two vertical transducers. The results in Fig. 4.12. and table 4.1. confirm this relation.

Tests were made under another static loading condition in which the horizontal force was applied directly to the horizontal transducer as shown in Fig. 4.11. The results, as shown in Fig. 4.13., show complete independence in the response of the vertical and horizontal transducers despite the fact that there is actually a moment

$$M = H \cdot h_1$$

Table 4.1.

Measured and Calculated Values for the Difference in
Values Between the Forces in the Two Vertical
Transducers Due to Horizontal Force H
Under Static Loading Condition. (1)

H lb (N)	D measured lb (N)	D calculated lb (N)
10 (44)	14.7 (65)	14.0 (62)
20 (88)	29.4 (130)	28.8 (128)
30 (133)	44.1 (196)	44.0 (195)
40 (177)	58.8 (261)	58.6 (260)
50 (222)	73.5 (326)	72.0 (320)

h = 7.35 in. (186.7 mm)

L = 10.0 in. (254 mm)

Values between brackets are in Newtons

where h_1 = height of horizontal transducer above the ground surface. The reason for such independent response is due to the fact that frame I under this loading condition is not restrained horizontally and is free to rotate about the Z axis, consequently the horizontal transducer will transmit all the horizontal force to frame II only. Hence moment caused by the direct application of force to the horizontal transducer will not affect the reading of the two vertical transducers, and all the moment is balanced by the ground reaction under frame II.

The results of these experiments also show that the mechanism of the apparatus does not cause any cross-talk between the vertical and horizontal transducers under static loading.

A heavy steel I beam 34 in. (0.864 m) long with 1.3 in. (33 mm) thick stiffeners spaced at 8 in. (0.203 m) between centres was used as a base for the apparatus during the dynamic calibration. The beam was placed on the floor of the laboratory over a special rubber pad designed

to damp any vibrations. The beam was leveled carefully and clamped to the floor at each corner. A thin lead sheet was used on top of the beam to protect the frictional surface of the plate from damage due to vibration.

The first tests were conducted, as shown in Fig. 4.14., by applying a static horizontal force directly to the horizontal transducer and a vertical force of large amplitude (close to the weight of the apparatus) and with low frequencies (5 to 10 Hz). The results show very smooth sinusoidal response for the vertical forces while the horizontal forces are essentially static virtually without dynamic component. No cross-talk between the vertical and horizontal forces was noticed (Fig. 4.15.). It was noted that the two dynamic components of the vertical forces F_f and F_r were equal.

One of the purposes of this test was to ensure that the behaviour of the apparatus during loading and unloading would be similar. Since the frequency was very low, loading or unloading in this case can be taken as

quasistatic. The results proved that the behaviour is similar during loading and unloading.

Other tests were conducted under the same conditions using higher frequencies. The results are shown in Fig. 4.16. Considerable differences were noted between the dynamic and static components of the two vertical transducers. These differences reflect the static and dynamic imbalance of the system.

In conclusion, it is vital that the system be balanced statically and dynamically in order to control the static and dynamic input forces to the soil during the vibration process. This balance was achieved by adjusting the locations of the static and dynamic resultants of the vertical force to coincide with the geometric center of the frictional plate.

To achieve dynamic balance, the motor was adjusted to a position such that the center line of rotating gears and eccentric weights coincided with the geometric

center of the frictional plate. The center of gravity of the whole apparatus (including the motor) was adjusted to coincide with the geometric center of the frictional plate to assure static balance. The balance was accomplished by using a short symmetrical base of the vibration excitor instead of the overhanging plate and by counterbalancing the motor with a dead load.

The results of tests conducted after balancing the system show the dynamic components of vertical force in the two vertical transducers are similar for frequencies below 22 Hz and no cross-talk was observed. Above 22 Hz rocking was noticed which resulted with some difference between the vertical forces in the two vertical transducers as shown in Fig. 4.17.

The frequency response curves for the vertical force are shown in Fig. 4.18. Curve 3 is the response for the apparatus on the rigid base of the dynamic calibration device. F_{vv}/F_{vv}' is the magnification factor where F_{vv} is the measured amplitude of vertical force, while F_{vv}'

is the calculated value of the amplitude of vertical force

$$F_{vv} = m_e r \omega^2$$

m_e = mass of the eccentric weights

r = eccentricity of eccentric weights in inches

ω = angular velocity in rad/s

Curve 2 is the response of the apparatus on the sand base. The curve shows that the magnification factor increases almost linear with frequency in the case of a rigid base. In the case of vibration on sand it is linear up to a value of about 23 Hz.

Different tests were conducted using the double parallelogram. Results showed that at higher frequencies as soon as the plate was in full shear mobilization on the surface of sand, the dynamic components of the two vertical transducers became unequal.

The static and dynamic balancing of the apparatus and the testing of its performance on the rigid base or

on sand while the apparatus was stationary assumed that the dynamic characteristics of the system were not responsible for the differences between forces registered by the two vertical transducers at least when the frequency is below 23 Hz. The moment due to the drawbar pulling force was reduced to a minimum by having the horizontal transducer as close as possible to the surface of the sand. The calculated value of the moment due to the drawbar pulling force was compared with the estimated moment required to produce the measured difference between the vertical forces. The former moment was not significant compared to the latter.

Test results using the double parallelogram show that it was unable to withstand rocking moments and the values of the two vertical forces were unequal. The reasons for the ineffectiveness of the double parallelogram were thought to be due to slack in its hinges and low stiffness of its members.

After improving the design of the double parallelogram

to overcome the reasons thought to be responsible for its inefficient functioning, the disparity between vertical forces persisted.

The following discussion may explain why the double parallelogram, produces unequal forces in the vertical transducers. Assume the model shown in Fig. 4.19. The parallelogram is represented by two roller supports connected to the upper plate and having an inclination depending on the position of the parallelogram. The vertical and horizontal transducers are replaced by vertical and horizontal roller supports.

If a force H is acting directly on the horizontal transducer it will transfer directly to frame II. Frame II is supported by the soil. If the force H produces a moment M on the sand surface, that moment will be partially carried by the soil causing a non-uniform stress distribution and partially transferred to the two vertical transducers and thus to the parallelogram. The distribution of the moment depends on the ratio of the

stiffness of the soil to the stiffness of the parallelogram.

Thus, the vertical transducers will carry extra forces due to the moment developed by the horizontal force. This is an indeterminate condition to solve.

From this discussion it may be concluded that the results obtained in the presence of the double parallelogram could be misleading. Hence the parallelogram was disconnected and the experiments were run with the observations of a laboratory assistant to ensure that the apparatus was in the proper position. The adjustments needed by the laboratory assistant during the tests were rare and the reproducibility of the tests proved that such adjustments had little if any influence on the results.

Proper balancing of the apparatus and the application of the drawbar pulling force very close to the sand surface made it possible to carry out experiments without

the use of the double parallelogram. Rocking moments can develop due to the dynamic characteristics of the apparatus. Under these conditions the apparatus is not capable of preventing the transfer of such moments to the soil. However, the double parallelogram does not eliminate the problem. The presence of such rocking moments complicate the loading conditions and add another variable to the problem. For this reason, as shown in Chapter V, the values of the frequency and amplitude of excitation had to be limited to those which produced insignificant rocking moments.

4.4.2. Calibration of the Accelerometers, Transducers and Recording Instruments

To check the sensitivity given for the accelerometers by the manufacturers, the following tests were undertaken in the National Aeronautical Establishment of the National Research Council in Ottawa. The instrumentation shown in Fig. 4.20. was used.

A light washer was set on top of the accelerometer.

The vibrating table was operated under a specific constant frequency. The amplitude of acceleration was increased while the acceleration wave was observed on the oscilloscope. As soon as the acceleration exceeded the acceleration of gravity, the washer on top of the accelerometer started to vibrate and a noise was noticed on the acceleration wave. A voltmeter reading was then taken which represented the output voltage corresponding to an acceleration of 1g. The results showed good agreement with the values of sensitivity given by the manufacturer. The voltages representing the velocity and displacement were also measured which also confirmed the frequency response curves given by the manufacturer of the integrating networks (Instruction and Application Manual for Vibration Pick-Up Preamplifier Type 2625 B & K, Fig. 2.10.).

The acceleration signal resulting from the vibration of the linear shear apparatus was normally accompanied by noise of large amplitude and high frequency. The displacement signal always had a low voltage level and when amplified to a suitable voltage it was also

accompanied by noise of high magnitude. The voltage level for velocity was reasonable and the signal-noise ratio acceptable. On the selection knob of the integration networks, the position 1 Hz - 3 m/s was used in this experimental program.

Adjustment of Velocity Scale

If a body is vibrating sinusoidally with frequency f and acceleration of amplitude $\ddot{X}$, the vibrational amplitude of velocity will be:

$$\dot{X} = \frac{\ddot{X}}{2 \pi f}$$

If the sensitivity of the accelerometer is given equal to S millivolts/g, the output voltage due to acceleration of amplitude $\ddot{X}$ will be:

$$\bar{V} = \frac{S \ddot{X}}{g}$$

or

$$\bar{V} = \frac{S}{g} 2 \pi f \dot{X}$$

Since S , g and 2π are constants, then for the same frequency f the velocity is proportional to the output from the accelerometer. The output voltage from the accelerometer resulting from the motion of a body vibrating with an amplitude of velocity equal to 1 in./s will be:

$$V = \frac{\dot{V}}{\dot{X}} = \frac{2\pi S f}{g}$$

Table No. 4.2. was used for the routine calibration and scale adjustment of velocity.

Frequency Response of the Velocity Electronic Circuit

The instrumentation shown in Fig. 4.21. was used to test the frequency response of the velocity electronic circuit. A signal was introduced from a frequency generator through the circuit to an oscilloscope. Another signal was taken directly from the frequency generator to the second channel of the oscilloscope. No phase difference was noticed on the oscilloscope. It was found that the frequency response curve is constant as shown in Fig. 4.18.

Table 4.2.

Values of Voltage Required to Simulate the Acceleration
which Produces Amplitude of Velocity 1 in./s (25.4 mm/s)
for Different Frequencies

Frequency f Hz	Amplitude of Voltage for 1 in./s (25.4 mm/s) mv.	Amplitude of Voltage for 1 in./s (25.4 mm/s) (RMS) mv.
10	14.26	10.11
15	21.39	15.13
20	28.52	20.20
25	35.65	25.20
30	42.78	30.25

Accelerometer No. 393679

Sensitivity S = 87.7 mv./g

$$\frac{2 \pi S}{g} = 1.426$$

Frequency Response of the Oscillograph Galvanometers

The frequency responses of the galvanometers were checked by introducing a signal to each of them and comparing the response of each with the others. Knowing the deflection and sensitivity of the galvanometers and the input signal to the oscillograph the magnification factors were obtained. The galvanometers proved to be linear and working within the specifications. No phase shift between the galvanometers response was noticed.

Frequency Response of the Force Circuits

A test similar to that used to obtain the frequency response of the velocity circuit was used for the force circuits. The response also shows linearity and a constant magnification factor.

Phase Shift Between Force and Velocity Circuits

The set up shown in Fig. 4.22. was used to check the phase shift between the force and velocity circuits. Both the force transducer and the accelerometer were

vibrated simultaneously using a Mini-Shaker. The two signals were recorded, The result showed that there was no noticeable phase difference between force and displacement signals due to the circuits. The normal 90 degrees phase shift between force and velocity was recorded.

4.5. Summary

1. The new features of the modified linear shear apparatus are:
 - a- Capability of measuring directly the shearing force developed between the soil and the frictional plate.
 - b- Capability of measuring directly the vertical force acting on the soil surface.
 - c- Pairs of crossed flexure pivots are used to insure the torsional rigidity in the two planes perpendicular to the plane of motion.
 - d- Bulldozing forces are carried by a guiding plate connected directly to the drawbar.

- e- The shear plate can be replaced and a variety of shapes of shear plates can be used.
 - f- Horizontal and vertical accelerations can be measured.
 - g- Direct recording can be obtained for the total vertical force (or the two vertical reactions on the vertical transducers), shearing force, vertical vibrational velocity (or displacement or acceleration), and horizontal vibrational velocity (or displacement or acceleration). The recording is given as a function of time.
2. The performance of the modified vibratory linear shear apparatus was examined under static loading. It was found that the force measuring systems exhibited linear characteristics and the mechanism of the apparatus does not cause any cross-talk between the vertical and horizontal transducers under static loading.
 3. The performance of the modified vibratory linear shear apparatus was examined under dynamic loading and on

a rigid base. The following was found:

- a- At low frequencies (5 -10 Hz) no cross-talk between the vertical and horizontal forces was noticed and the two components of the vertical force are equal. The tests also showed that the behaviour of the apparatus in both loading and unloading conditions is similar.
- b- At frequencies higher than 10 Hz, it is important to balance the system statically and dynamically in order to control the static and dynamic input forces to the soil.
- c- After proper balancing of the system, no cross-talk was observed and the two components of the vertical force were found equal for frequency in the range 5 to 35 Hz.
- d- The frequency response curve of the apparatus on a rigid base shows a linear behaviour with magnification factor of 1.02 at 8 Hz increasing to 1.21 at 35 Hz.

4. The frequency response curve of the apparatus on sand shows a linear behaviour between 8 and 23 Hz and non linear behaviour over 23 Hz. The non linear behaviour suggested that above 23 Hz the apparatus is approaching the resonant frequency. In general, for frequencies over 23 Hz rocking moment was noticed and as a result cross-talk between vertical and horizontal forces resulted.
5. Results obtained in the presence of the double parallelogram could be misleading due to its structural indeterminacy.
6. Proper balancing of the apparatus and applying the drawbar pulling force very close to the sand surface made it possible to carry out experiments without the use of the double parallelogram.
7. No practical mechanism was found to prevent the development of rocking moment and hence the testing should be limited to the cases where rocking moment is insignificant.

8. A simple method was used to calibrate the accelerometers frequently. The accelerometers were always found to perform as specified by the manufacturer.
9. The frequency response of the velocity electronic circuit was found constant and linear. No phase shift was noticed between the input and output of the accelerometers.
10. The frequency response of the oscillograph galvanometers shows a linear relationship within the frequencies specified by the manufacturer. No phase shift between the input and output of the galvanometers was noticed.
11. The frequency responses of the force electronic circuits were found to be constant and linear.
12. No phase shift between force and displacement signals due to the structure of the electronic circuits was evident. The normal 90 degrees phase shift between force and velocity under sinusoidal loading was recorded.

CHAPTER V

Experimental Studies on the Effects of Vertical Vibration on the Shearing Characteristics of Soil Engaging Elements

5.1. Experimental Procedures

A series of tests were carried out for determining the effects of the amplitude of exciting force, the drawbar speed, the frequency of excitation, the static normal force and the shape of the frictional plate on the shear behaviour of dry sand. The basic principle of the experimental procedure is to apply a vertical sinusoidal force to a frictional plate resting on the sand surface and to drag the frictional plate horizontally with a drawbar. The end of the drawbar is controlled to move with constant forward velocity. Vertical force, horizontal force, vertical vibrational velocity and horizontal vibrational velocity are all measured and recorded simultaneously.

In order to have the same soil condition for each test, the sand was compacted from the surface using a plate compactor. The plate compactor consists of a Syntron Vibra-Flow Rotary Vibrator model RV 235 mounted on a wooden plate 27.5 in. (698.5 mm) × 24.0 in. (609.6 mm). The weight of the compactor is 62 lb (275.79 N) and it operates at 61 Hz. The soil was compacted for 60 seconds before each test. The soil surface was levelled before and after the compaction. The levelling was done by steel scraper, guided by a perfectly levelled steel frame mounted around the concrete tank.

The test procedure was as follows:

1. The apparatus was raised above the sand by a small winch.
2. The top layer of sand was disturbed.
3. The sand was compacted and levelled.
4. The apparatus was lowered to be as close as possible to the sand surface without touching it. The part of the drawbar attached to the apparatus

was supported at its free end. The zero reference of forces and velocities were set at this position on the recording instruments.

5. The required eccentric weights were fixed to the gears.
6. The flow control valve of the hydraulic pump was adjusted to the required rate of shearing.
7. The drawbar was carefully connected to the hydraulic piston so that no initial tension or compression was registered by the horizontal transducer.
8. The apparatus was lowered and set carefully on the sand surface.
9. The exciter was started and its speed of rotation was adjusted according to the readings shown by the stroboscope "Strobotac 153Y of General Radio Co.", until it reached the required frequency.

10. The apparatus was pulled by the hydraulic ram at a constant speed.
11. Recordings were taken before, during and after the shearing process. To economize in the use of recording paper, the speed of recording was set at 0.1 in./s (2.54 mm/s). In order to obtain data in suitable detail, sample recordings were taken with recording speeds of 20 in./s (508 mm/s) or 50 in./s (1270 mm/s) as shown in Fig. 5.1.
12. The vibration was stopped, the drawbar disconnected and the apparatus raised.
13. The sand was remoulded and compacted for the next test.

5.2. Soil Samples and Preparation

The soil used in the experiments is dry uniform Ottawa sand with ASTM designation C 190, in a dense condition. The properties of this sand are:

- Grain size passing sieve No. 20 and retained on 30 (U.S. Bureau of Standards Series), i.e. ranging between 0.0232 and 0.0331 in. (0.59 and 0.84 mm).
- Specific gravity = 2.66
- Critical angle of internal friction = 26.7 degree { after Taylor (1967) }
- $\tan \phi_{\text{critical}} = 0.5$
- Equivalent stiffness under dynamic vertical loading = 441000 lb/ft (6436 N/mm).
- Equivalent damping under dynamic vertical loading = 1085 lb.s/ft (15.8 N.s/mm).
- Ottawa sand is classified as spherical in nature.

Wu (1971) reported that, for foundations subjected to vertical dynamic force and resting on sand, the vertical deformation will increase with the number of cycles. If the sum of static and dynamic stresses is much less than the ultimate bearing capacity of the sand, the strain

increases with each cycle but approaches a limiting value. If the summation is near the ultimate bearing capacity, failure may occur after few cycles.

To avoid any complexities due to bearing capacity failure, the static and dynamic vertical forces were chosen such that their maximum combined value did not exceed the bearing capacity of the sand.

The estimated bearing capacity of the dry Ottawa sand was calculated as follows:

- The unit weight of the sand was measured using a cylinder of volume 5.522 in.³ (90489 mm³). The unit weight was found to be between 112-116 p.c.f. (17.6-18.2 kN/m³). The corresponding voids ratio can be estimated to range between 0.48-0.431
- The loosest state of the sand has a voids ratio
 $e_{\max} = 0.8-1.0$
- The densest state of the sand has a voids ratio
 $e_{\min} = 0.5-0.4$

- The relative density is given by

$$R.D. = (e_{max} - e)/(e_{max} - e_{min})$$
- According to the values of e , e_{max} and e_{min} obtained, the relative density of the sand when compacted for testing is between 87 and 95%

The equation of ultimate bearing capacity is

$$Q = \frac{1}{2} \gamma B N_{\gamma} (B L)$$

where N_{γ} can be obtained from Terzaghi bearing capacity charts while B and L are the width and length of the strip foundation.

Shearing plate No. 1 with $B = 2.5"$ (63.5 mm) and $L = 25.0"$ (635 mm), can be assumed as a strip foundation with $N_{\gamma} = 47$

Then the minimum value of ultimate bearing capacity load for this plate is

$$Q = 239 \text{ lb (1063 N)}$$

The ultimate bearing capacity of the shearing plate was also measured experimentally by loading with static weights. Load increase to 248 lb (1103 N) resulted in complete failure of the sand. This concludes that, for shearing plate No. 1, the theoretical value of the ultimate bearing capacity load is 239 lb (1063 N) and its value obtained experimentally is 248 lb (1103 N).

In the experimental program under consideration, the total weight of the apparatus plus the amplitude of the dynamic vertical force never exceeded 160 lb (712 N) which represents 65% of the ultimate bearing capacity load of the shearing plate.

5.3. Experimental Programs

The shear rate (or the drawbar speed) of soil in conventional soil engaging machinery is usually quite high compared with that used in the standard tests for soil strength. For example in off-road locomotion or vibratory soil compactors, a range of shear rate from 0.5 to 10 in./s (12.7 to 254 mm/s) is considered typical. In this

experimental investigation, tests with drawbar speeds of 0.2, 0.4, 0.6, 0.8 and 1.0 in./s (5.08, 10.16, 15.24, 20.32 and 25.4 mm/s) were carried out. Owing to the limitations of the apparatus, it was not possible to conduct tests at shear rates higher than 1 in./s (25.4 mm/s).

The amplitude of the vertical exciting force varied from 0.2 to 1.0 of the static force. This was achieved by using appropriate eccentric masses. If the ratio of dynamic to static force exceeds 1.0, the plate will separate from the soil during part of the cycle. Such a condition will result in an acceleration ratio η greater than 1. As mentioned in Chapter II this may cause a change of the critical angle of friction with acceleration. Since the purpose of this research is to study the influence of the input variables such as amplitude, frequency ...etc., the angle of friction is assumed constant. For this reason the study is limited to $\eta < 1$ or $F_{vv}/F_{vs} < 1$.

In existing vibratory soil engaging machinery, the

frequency of excitation is usually within the range ^a of 1-30 Hz. The frequency range used in this experimental investigation was from 10 to 30 Hz. Although it may be desirable to carry on the investigation below 10 Hz, it was not possible in these tests because the control of the speed of the motor in the vibration exciter was not adequate below 10 Hz. Since some of the natural frequencies of the apparatus were found to be within 20 to 30 Hz, the test results within this frequency range may be influenced by the dynamic characteristics of the apparatus.

The rigid drawbar used in this experimental investigation was made from two aluminum bars each of length 15.5 in. (393.7 mm) and with rectangular cross section 0.25 in. (6.35 mm) × 0.5 in. (12.7 mm) with stiffness 161300 lb/in. (28248 N/mm), connected to the hydraulic piston by a bracket of stiffness 108 800 lb/in. (19054 N/mm).

The stiffness of the hydraulic piston was measured experimentally under static loading and it was found to be 6250 lb/in. (1095 N/mm). Some attempts were made to measure

the dynamic stiffness of the piston but the results were not accurate. It is expected that the dynamic stiffness of the piston will be higher than the static stiffness.

The aluminum bars, the brackets and the hydraulic piston are the elements which represent the drawbar of the apparatus. The equivalent stiffness of the drawbar can be estimated from the stiffness of its three elements, assuming that they are three springs in series. The equivalent stiffness of drawbar was calculated under static loading to be

$$k = 5700 \text{ lb/in. (998 N/mm)}$$

Stiffness measured under static loading is expected to be less than that measured under dynamic loading.

5.4. Methods of Analysis of the Experimental Results

The following parameters were measured or evaluated for purposes of analysis:

1. The frequency f

The frequency was evaluated by measuring the distance L (Fig. 6.1.), in inches, between two successive peaks of the vertical force wave or the shearing force wave. If the speed of recording is S_r in./s, then the frequency

$$f = \frac{S_r}{L} \text{ Hz}$$

2. The amplitude of exciting force F_{vv}

The amplitude of the exciting force was evaluated by measuring the distance L_2 , in inches, between the maximum and minimum values of the vertical force wave. If the scale of the vertical force is S_v lb/in., then the amplitude is

$$F_{vv} = \frac{L_2 S_v}{2} \text{ lb}$$

This relation is valid for a sinusoidal shape. If the vertical force is not purely sinusoidal, the amplitude of the exciting force F_{vv} was measured

about (above & below) the static normal force line. In this case the load shape may not be symmetrical about the reference line.

3. The static normal force F_{vs}

The static normal force was calculated by summing the weights of the apparatus including the eccentric weights. As a check, the static normal force was also evaluated by

$$F_{vs} = \frac{\text{Area under complete vertical force cycle}}{\text{Distance between two successive peaks (L)}} \times S_v$$

Agreement between the two values within 1 to 2% was normally obtained.

4. The static shearing force H_{ss}

The static shearing force is calculated from the equation

$$H_{ss} = F_{vs} \cdot \tan \phi_{\text{critical}}$$

5. The average value of dynamic shearing force H_{vs}

The average value of dynamic shearing force was

evaluated using the equation

$$H_{vs} = \frac{\text{Area under complete shearing force cycle}}{\text{Distance between two successive peaks (L)}} \times S_H$$

where S_H is the scale of recording of shearing force in lb/in.

5.5. Theoretical Values of the Average Dynamic Shearing Force

A computer program based on the vibratory soil compactor model under harmonic loading, as presented in Chapter III, was developed. A printout of the program and a description of its use is given in Appendix I.

A graphical method can be also used. The details of the graphical method are presented in Chapter VI. Examples to use the graphical method in evaluating the theoretical values of the average dynamic shearing force are presented in Appendix II.

5.6. Experimental Results and Comparison with Theoretical Predictions

5.6.1. Measurement of the Rate of Increase of Shearing Force with Shearing Displacement α Using the Linear Shear Apparatus

Three tests were conducted on sand using the modified linear shear apparatus. One test using dense sand (Fig. 5.2.) and the other two using loose sand. In the three tests, the soil was fully mobilized and then an increasing step load was applied. The rate of increase of shearing force with time, β , due to the application of the step load was measured. The values of β obtained from the three tests were similar. Knowing β , V_{sh} and the drawbar stiffness, the value of α was calculated.

The value of α was found to range from 1150 to 1618 lb/in. (201 to 283 N/mm). For a drawbar of stiffness $k = 5700$ lb/in. (998 N/mm) (section 5.3.), the value of

$$K_s = 1/(1/K + 1/\alpha) = 957 \text{ lb/in. to } 1260 \text{ lb/in. (168 to 221 N/mm)}$$

Since the value of K_s is very close to the value of α , the

drawbar stiffness can be expected to have a minor effect on the average value of shearing force under dynamic conditions. The average measured value of K_s , 1100 lb/in. (193 N/mm), was used in the theoretical analysis.

To determine the effects of the different vibration parameters on the shearing characteristics of vibratory soil engaging elements, a series of tests, were conducted. The results of these tests will be presented in the rest of this Chapter. The test results are shown in Figs. 5.7 to 5.15, 5.21 to 5.23 and 5.29 to 5.30. It can be noticed through all of these tests that during the unloading part of the shearing force cycle::

$$H = F \times 0.5$$

$$\text{but since } \tan \phi = 0.5$$

$$\text{then } H = F \tan \phi$$

This indicates that during the unloading part of the cycle, the shearing force has a value of β reaching infinity and ideal Coulomb friction is a valid model in describing the vertical force-shearing force relationship.

On the other hand, during the loading part of the shearing force cycle

$$H < F \times 0.5$$

$$\text{or } H < F \tan \phi$$

This indicates that the plate did not reach full mobilization during the loading part of the cycle. The inclination of the curve with reference to time represents β . The values of β and K_s were measured from 50 test samples. The distribution of the number of samples for different values of K_s are shown in Fig. 5.3. The average value of K_s was found to be 1164 lb/in. (204 N/mm). The largest number of samples (40%) was found corresponding to $K_s = 1100$ lb/in. (193 N/mm).

The shearing force-shear displacement relationship was also investigated by direct measurement of displacement. The soil was sheared by the linear shear apparatus until it reached full mobilization. The direction of shear was then reversed and the soil was sheared to full mobilization in the backward direction, and the process was repeated a few times. The displacement was measured using the

"TOPP Industries Inc., rectilinear potentiometer, model LB 140". The shearing force-displacement curves were recorded on "Hewlett Packard 2 FA X-Y recorder". The outlines of the set up used are shown in Fig. 5.4.

The results of a test with drawbar forward speed = 0.04 in./s (1.02 mm/s) are shown in Fig. 5.5: The sand was compacted to a dense state before shearing. Curve 1 shows a hump followed by fully mobilization state which are the characteristics of the dense state of sand. Curve 1' is the result of reversing the direction of shear. The control of the drawbar speed in the backward direction was not accurate since the flow valve of the hydraulic ram can control the flow in the forward direction only. Then the speed of drawbar in the backward direction was controlled approximately and maintained at a low speed. The forward and backward shearing were repeated. Curves 2, 3 and 4 are the results of forward shearing while curves 2' and 3' are the results of backward shearing. The results shows that the shearing force-displacement curves are similar in both the forward and the backward directions.

The effects of drawbar speed and static vertical force on the shape of the shearing force-displacement curves were also investigated as shown in Fig. 5.6. The resulting curves were similar for different drawbar speeds and static vertical forces. The shearing force-shear displacement relationship before full mobilization is shown to be non linear in all the tested cases. In the theoretical model this relation was assumed linear. The slope of a straight line which averages the curved part of each test was measured to represent the value of K_s . Table 5.1. shows the results obtained from Figs. 5.5. and 5.6. The average value of K_s was found to be 1050 lb/in. (184 N/mm).

As a conclusion it seems that using $K_s = 1100$ lb/in. (193 N/mm) will give a good presentation for the range of loading condition used in this research.

5.6.2. Effect of the Amplitude of Exciting Force

In order to determine the effects of vibration on the shearing characteristics of vibratory soil compactors and to evaluate the validity of the theoretical model, a series

Measured Values of K_s from Shearing
Force-Shear Displacement Curves

Figure No.	Curve No.	Direction of motion	V_{sh} in./s (mm/s).	Vertical Force	Measured K_s lb/in. (N/mm)	Notes
5.5.	1	Forward	0.04 (1.02)		-	1
	1'	Backward			1050 (185)	
	2	Forward	0.04 (1.02)		866 (152)	
	2'	Backward			1192 (209)	
	3	Forward	0.04 (1.02)		900 (158)	
	3'	Backward			1192 (209)	
	4	Forward	0.04 (1.02)		994 (174)	
5.6.	1	Forward	0.04 (1.02)	Static weight	-	2
	1'	Backward		Static weight + 20 lb (88.96 N)	1267 (222)	
	2	Forward	0.04 (1.02)	Static weight + 20 lb (88.96 N)	1205 (211)	
	2'	Backward			-	
	3	Forward	1.5 (38.1)		940 (165)	3
	3'	Backward			1145 (201)	
	4	Forward	1.5 (38.1)		776 (136)	
	4'	Backward			1100 (193)	

Notes 1. Sand initially in dense state

2. Sand initially in dense state, 2 loads of 10 lb (44.48 N) were added after mobilization.

3. The 2 loads of 10 lb (44.48 N) were released during the partial mobilization.

of tests were conducted.

It was assumed in the model that the angle of friction at full mobilization is equal to the critical angle of internal friction of the sand. To confirm this, a series of tests measuring the shearing force under static conditions was first carried out. The results of these tests showed that the angle of friction ranges from 25 to 27 degrees. The value of the critical angle of internal friction quoted in the literature for Ottawa sand is 26.7 degrees. This suggests that the fully mobilized shear plate with a sand coated surface can be used to determine the friction component of the internal shear resistance of the same type of sand.

A series of tests for determining the shearing characteristics of vibratory soil compactors during vibration were then performed. Some of the oscillograph traces showing the vertical and shearing forces vs time at various amplitudes of excitation are shown in Figs. 5.7 to 5.15.

It is clearly demonstrated that the amplitude of vertical force has a profound effect on the shearing force developed between the soil and the plate. As mentioned before, the critical angle of friction $\phi_{\text{critical}} = 26.7$ degrees and $\tan \phi_{\text{critical}} = 0.5$. According to the theoretical model the ultimate dynamic shearing force is equal to

$$\begin{aligned} H_{\text{ult}}(t) &= (F_{\text{vs}} + F_{\text{vv}} \sin \omega t) \tan \phi_{\text{critical}} \\ &= (F_{\text{vs}} + F_{\text{vv}} \sin \omega t) \times 0.5 \end{aligned}$$

The scale of the horizontal shearing force is $\frac{1}{2}$ that of the vertical dynamic force, then the vertical dynamic force trace can also represent the ultimate dynamic shearing force curve with the same scale as the shearing force curve (10 lb/in. or 1.75 N/mm). It was shown in the theoretical model that the dynamic shearing force curve will coincide with the ultimate dynamic shearing force curve in a portion of the unloading part of the cycle and to a point where:

$$\beta < \frac{d(F(t) \tan \phi_{\text{critical}})}{d t}$$

This behaviour is clearly evident in all the test results.

To identify the effect of vertical vibration on the shearing force, and for purposes of comparison, the ratio H_{vs}/H_{ss} is plotted against the ratio F_{vv}/F_{vs} in Figs. 5.16 to 5.18.

Based on the experimental evidence, it can be concluded that, all other conditions being equal, the average shearing force reduces with increasing ratio of dynamic to static vertical forces.

In Figs. 5.16 to 5.18, the solid lines represent the theoretical values obtained in accordance with Chapter III. Fig. 5.16. is for very low shear rate 0.239 in./s (6.07 mm/s). Two curves are shown for frequencies 10 and 15 Hz. The band of variation for the theoretical or experimental values is very small while the curves are very steep of the same inclination which suggests that at the low values of V_{sh} the frequency has minor influence on the reduction of the average dynamic shearing force.

Fig. 5.17 shows the effect of intermediate value of $V_{sh} = 0.6$ in./s (15.24 mm/s). A wider band of variation and less steep curves were obtained, which shows that f , V_{sh} and F_{vv} have all a strong influence on the reduction of the shearing force. The third group of curves shown

Fig. 5.18 are for a high value of V_{sh} . The band of variation is much wider and the slopes are less steep than the case of $V_{sh} = 0.6$ in./s (15.24 mm/s) which indicate that the role of the shear rate in this case is smaller than the previous cases and the major cause of the reduction is the influence of frequency and F_{vv} .

The results show a good agreement between the experimental and theoretical results. The disagreement between theoretical and experimental values at higher frequencies can be explained by the fact that, as mentioned in Chapter IV, the natural frequency of the apparatus, resting on soil, is within the range 20 to 30 Hz. It was also noticed that within this range of frequencies the apparatus exhibited noticeable horizontal vibration even before starting the shear process. Furthermore there is

a slight deviation between the theoretical and experimental results for values of F_{vv}/F_{ss} greater than $0.8 F_{vs}$. Under these conditions excessive rocking of the apparatus occurred which resulted in distortions of vertical dynamic force shape compared with the harmonic sinusoidal input shape. As mentioned in Chapter IV, rocking could not be controlled.

In the cases plotted on Figs. 5.19. and 5.20. the frequency was maintained constant at 10 and 15 Hz respectively and the ratio F_{vv}/F_{vs} and speed V_{sh} were varied. Same observations made for Figs. 5.16. to 5.18. also apply and the effect of V_{sh} is clearly evident. The results show good agreement between the experimental and theoretical values. The effect of V_{sh} will be discussed in detail in the next section.

5.6.3. Effect of the Drawbar Speed

Under static conditions it was found experimentally that within the range of 0.2 to 1.0 in./s (5.08 to 25.4 mm/s), the drawbar speed has practically no effect on the shearing force. However, under vibratory conditions, it was found

that the drawbar speed has considerable effect on the development of shearing force. Some typical oscillograph traces showing the vertical and shearing forces against time at various shear rates are shown in Figs. 5.21 to 5.23. The figures show clearly the effect of drawbar speed on the shearing force developed between the soil and the plate. The effect of V_{sh} on the value of β (or the slope of the loading part of the shear cycle) is clearly demonstrated, where the increase of V_{sh} results in an increase in β . For all cases the unloading portion coincides with the ultimate shear curve. This behaviour matches that described by the mathematical model.

In Figs. 5.24 to 5.28, the ratio H_{vs}/H_{ss} is plotted against the drawbar speed V_{sh} . In Figs. 5.24 and 5.25, the ratio F_{vv}/F_{vs} was maintained constant at 0.25 and 0.5 respectively. In Figs. 5.26, 5.27 and 5.28, the frequency was maintained constant at 10, 15 and 20 Hz respectively.

From the results presented it is clear that, other

conditions being equal, the reduction in shearing force decreases with increasing drawbar speed. It is also interesting to note that as the drawbar speed approaches a critical value, the value of H_{vs}/H_{ss} become constant. This observation can be described by the theoretical model which indicates that no reduction in the average value of shearing force will occur when V_{sh} is greater than $V_{sh(critical)}$ where

$$V_{sh(critical)} = \frac{2 \pi f \mu F_{vv}}{K_s} \quad (\text{refer to section 3.4.})$$

Table 5.2. shows the values of $V_{sh(critical)}$ for the different curves shown in Figs. 5.24 to 5.28 and compares them with the measured values.

The theoretical values of the critical drawbar speed are confirmed experimentally. This indicates that the theoretical model well describes the effect of drawbar speed.

A good agreement between the experimental and

Comparison between the Theoretical and Experimental
Values of V_{sh} (critical)

fig.	K_s lb/in. (N/mm)	F_{vv} lb (N)	f Hz	$\frac{2 \pi f \mu F_{vv}}{K_s}$ $= V_{sh}$ (theo- retical) in./s (mm/s)	experimental value of V_{sh} (critical) in./s (mm/s)
5.25.	1100 (193)	16.75 (75)	10	0.48 (12.2)	0.50 (12.7)
			15	0.72 (18.3)	0.75 (19.1)
			20	0.96 (24.4)	0.96 (24.4)
5.26.	1100 (193)	34.00 (151)	10	0.97 (24.6)	1.0 (25.4)
			15	1.46 (37.1)	out of range
5.27.	1100 (193)	16.0 (71)	10	0.46 (11.7)	0.47 (11.9)
		35.50 (158)		1.01 (25.7)	1.00 (25.4)
		60.30 (268)		1.72 (43.7)	out of range
5.28.	1100 (193)	18.00 (80)	15	0.77 (19.6)	0.80 (20.3)
		27.25 (121)		1.17 (29.7)	out of range
		36.00 (160)		1.54 (39.1)	out of range
5.29.	1100 (193)	17.00 (76)	20	0.97 (24.6)	0.95 (24.1)
		26.50 (118)		1.51 (38.4)	out of range

theoretical values is shown especially at the higher values of drawbar speed V_{sh} . A slight deviation between experimental and theoretical values is noticed at lower shear rates. This results from the rocking moment effect which was also noticed to increase with the decrease of the drawbar speed.

5.6.4. Effect of the Frequency of Excitation

It was found that frequency of excitation has considerable effect on the development of shearing force. Some typical oscillograph traces showing the vertical and shearing forces against time at various frequencies of excitation are shown in Figs. 5.29 and 5.30.

In all the tests the unloading part of the shear cycle coincide with the unloading part of the ultimate shear cycle. The angle of friction during the unloading part of the cycle is equal to the critical angle of friction. The average value of dynamic shearing force increases with decreasing frequency. All these characteristics were demonstrated by the mathematical model.

The effect of frequency of excitation may also be observed in Figs. 5.16 to 5.18 , 5.24 and 5.25. From these curves it can be concluded that the average dynamic shearing force reduces with increasing frequency. Additional tests were conducted to show the effect of frequency. Fig. 5.31 shows the effect of frequency using shearing plate No. 1, $V_{sh} = 0.8$ in./s (20.3 mm/s) and $F_{vv}/F_{vs} = 0.5$. The theoretical values are in good agreement with the experimental ones. Shearing plate No. 2, was used for the tests of Fig. 5.32 with $V_{sh} = 0.8$ in./s (20.3 mm/s) and $F_{vv}/F_{vs} = 0.55$ and 0.80. In this case the experimental results deviate from the theoretical results at frequencies higher than 20 Hz for the reasons explained above. However, in general good agreement obtains in the lower frequency range.

A series of tests were conducted using 3 different shearing plates with different sizes (refer to section 4.2.). The results obtained using the 3 shearing plates are quite similar and all give good agreement with the theoretical values for frequencies below 20 Hz as shown in Fig. 5.33. This indicates that the shape of the frictional plate has no influence on the average value of dynamic shearing force.

5.6.5. Effect of the Static Normal Force

Under vibratory conditions, it was found that the static normal force has considerable effect on the development of shearing force. Some typical oscillograph traces showing the vertical and shearing forces against time for two static normal forces are shown in Fig. 5.34.

The experimental and theoretical results (Fig. 5.35) show that for the same ratio of dynamic to static normal force, frequency and drawbar speed, the ratio of average dynamic to static shearing force decreases with increasing static normal load. In both cases a good agreement between the experimental and theoretical results obtains except at frequencies exceeding 20 Hz. The results show that the static normal force influence the average value of shearing force.

5.6.6. Evaluation of the Longitudinal Inertia Forces

The presence of acceleration in the longitudinal direction will add an inertia force to the developed

shearing force between the soil and the plate. The evaluation of these inertia forces will depend on the complexity of the machine under consideration, whether it is plate compactor, vibratory roller, off-road vehicle ...etc.

In the research under consideration, the inertia forces were minimized in the laboratory testing and were ignored in the analysis in order to simplify the complexity of the problem and to have a clear understanding of the effects of the other parameters on the soil-machine interaction.

To evaluate the horizontal longitudinal vibrations, the dynamic component of horizontal velocity was recorded. Assuming that the system is a single degree of freedom with sinusoidal velocity-time relationship, the amplitude of acceleration can be estimated using the following equation:

$$A = \frac{2 \pi f}{12 \times 32.2} V$$

$$= 0.0163 f V$$

where A = amplitude of acceleration in g's
 V = amplitude of velocity in in./s

The amplitude of acceleration was found to vary between 0.035 g to 0.11 g in most of the tests conducted. The weight of the frictional plate is in the order of 1 lb (4.45 N) which will result with inertia force ranging between 0.035 lb to 0.11 lb (0.16 to 0.49 N). Then it can be concluded that the inertia force is not significant within the tested range.

Inertia forces were found with large values when the frequency approaches the resonance frequency of the system or when rocking moment is noticeable. An example for such condition is shown in Figs. 5.14(c) and 5.30(c) where the frequencies are 30 and 28.17 Hz respectively. Under such conditions no agreement is found between the theoretical and testing values of shearing force.

5.7. Summary

The experimental program was conducted to determine

the effects of the different vibrational parameters on the shear force developed between the soil and a soil engaging element. The results were also used to confirm the validity of the theoretical model for the vibratory soil engaging elements.

The ranges of variation of the parameters used in the experimental program were chosen to match the ranges of operating conditions of vibratory soil engaging elements. However, the dynamic characteristics of the modified linear shear apparatus presented limitations within the chosen ranges.

Based on the theoretical model and confirmed by the experimental evidence, it can be concluded that, other conditions being equal:

1. The average dynamic shearing force decreases with
 - a- increasing amplitude of the vertical loads
 - b- decreasing drawbar speed

c- increasing vibrating frequency

2. Other conditions being equal, the ratio of average dynamic to static shearing force decreases with increasing static normal load.
3. The shape of the frictional plate has no influence on the average value of dynamic shearing force.

The experimental results show good agreement with the theoretical values which confirms the validity of the theoretical model within the tested range. The theoretical model of the vibratory soil engaging element, as presented in Chapter III, can therefore be used to explain some effects of applying vibratory energy to soil engaging devices.

CHAPTER VI

Practical Application of the Model to the Prediction of the Performance of Vibratory Soil Working Machinery

6.1. Introduction

The model describing the vibratory soil engaging element provides a good representation of the shearing force developed between the vibrating element and the soil. The application of the model, as presented, is valid for vertical accelerations of amplitude less than that due to gravity, i.e. as long as the plate is continuously in complete contact with the ground. In spite of the model's limited range of use, it provides a good understanding of the relationship between machine-soil interaction and pertinent vibrational parameters.

The model has direct application in evaluating the performance of vibratory plate compactors as long as it is in contact with the ground. The performance

curves obtained are useful for estimating the drawbar pull required.

The performance curves, as will be described in this Chapter, will be mainly representing the energy consumed as a function of the dimensionless ratio F_{vv}/F_{vs} , frequency or drawbar speeds. The energy consumed with the operation of a vibratory soil compactor can be separated into two components:

- a- Energy consumed in vertical vibration
- b- Energy consumed in shearing the soil

The energy consumed in vertical vibration is important since it can be introduced as a measure of the compaction effectiveness (Fu & Paul 1972). The energy consumed in shearing the soil is important since it can be introduced as a measure of the tractive effort required.

A method for the evaluation of the two components of energy will be discussed. The performance curves of a vibratory soil compactor will also be presented.

Most of the theories which predict the tractive performance of vehicles are based on the simplifying assumption of a uniform pressure distribution over the contact area. Wills (1963) showed that the pressure distributions beneath a rigid track may take different shapes one of which is the multipeak sinusoidal distribution. He presented a method for predicting the track performance curve using the shear stress-deformation curve. As an alternative to Wills's method, a procedure based on the model for the shear behaviour of a vibratory soil engaging element will be presented.

6.2. Prediction of the Energy Consumption Due to Vertical Vibration of the Soil Engaging Element

Assume that the vibrating element and the soil can be represented by a system having a single degree of freedom with equivalent mass m , equivalent damping c and equivalent stiffness k . If the system is subjected to periodic force of the sinusoidal type, the equation of motion is:

$$m \ddot{x} + c \dot{x} + k x = F_{vv} \sin \omega t \quad 6.1.$$

where

x , $\dot{x}$ and $\ddot{x}$ are the displacement, velocity and acceleration of vibration

F_{vv} is the amplitude of vertical force

ω is the angular frequency

and t is the time

The solution of equation 6.1 is

$$x = X \sin (\omega t - \theta) \quad 6.2.$$

where

X = amplitude of displacement

$$= \frac{F_{vv}/k_v}{\sqrt{(1 - \frac{m \omega^2}{k_v})^2 + (\frac{c \omega}{k_v})^2}}$$

and

θ = the phase difference between the exciting force and displacement

$$\tan \theta = \frac{c \omega / k_v}{1 - m \omega^2 / k_v} \quad 6.3.$$

The equivalent mass m , equivalent damping C and equivalent stiffness K can all be estimated using the elastic half-space theory as reviewed by Keira (1971).

The work done per one cycle of vertical vibration can be defined by

$$\Delta U_v = \int_0^{2\pi/\omega} F(t) \cdot \dot{x} dt$$

$$\Delta U_v = F_{vv} \times \omega \frac{\pi}{\omega} \sin \theta = F_{vv} \times \pi \sin \theta \quad 6.4.$$

If the drawbar is infinitely rigid, the plate compactor including the drawbar will travel with a constant speed V_{sh} in./s. The time required by the compactor to travel 1 in. = $1/V_{sh}$ s

$$\text{the number of cycles of vibration/in.} = \frac{f}{V_{sh}} \text{ cycles} \quad 6.5.$$

$$\text{and the work done by vertical vibration/in.} = U_{vv} = \Delta U_v \frac{f}{V_{sh}}$$

$$U_{vv} = F_{vv} \times \pi \sin \theta \frac{f}{V_{sh}} \quad 6.6.$$

6.3. Prediction of Energy Consumption Due to Shearing of the Soil Engaging Element

6.3.1. Semi Graphical Method for Evaluation of Energy Consumption Due to Dynamic Shearing and the Prediction of the Average Values of Dynamic Shearing Force

In order to facilitate the method of calculation, a semi graphical method is presented. With this method it will be possible to evaluate both the average value of dynamic shearing force and the energy consumed due to shearing.

Average Value of Dynamic Shearing Force

As demonstrated in Chapter III, the time t_1 at which the rate of increase of ultimate shearing force is equal to β can be calculated from:

$$\beta = \mu F_{vv} \omega \cos \omega t_1$$

or

$$K_s V_{sh} = \mu F_{vv} \omega \cos \omega t_1$$

then

$$t_1 = \frac{1}{\omega} \cos^{-1} \frac{K_s V_{sh}}{\mu F_{vv} \omega} \quad 6.7.$$

For a sinusoidally shaped ultimate shearing force, the rate of increase of ultimate shearing force will become equal to $+\beta$ at time t_1 or $(T - t_1)$ as shown in Fig. 6.1. However the change of the shape of the shearing force from sinusoidal to linear will occur at $t = T - t_1$. If the areas A_1 and A_2 and the lengths L_3 and L_4 are defined as shown in Fig. 6.1, then

$$H_{vs} = L_4 + L_3$$

$$\frac{H_{vs}}{H_{ss}} = 1 - \frac{F_{vv}}{F_{vs}} \left(1 - \frac{A_1}{A_2} \right)$$

put $\lambda = \left(1 - \frac{A_1}{A_2} \right)$ 6.8.

then $\frac{H_{vs}}{H_{ss}} = 1 - \lambda \frac{F_{vv}}{F_{vs}}$ 6.9.

The ratio A_1/A_2 can be obtained by direct measurement of A_1 and A_2 . In order to simplify the method, the area A_1 and distance s were measured for different tangents as

shown in Fig. 6.2. The relation λ vs s/L was obtained and shown in Fig. 6.3. Comparing Fig. 6.1 with 6.2, it can be concluded that $s/L = t_1/T$

Dividing equation 6.7 by T gives

$$\frac{s}{L} = \frac{t_1}{T} = \left(\frac{1}{\omega} \cos^{-1} \frac{K_s V_{sh}}{\mu F_{vv} \omega} \right) \frac{\omega}{2\pi}$$

$$\text{or } \frac{s}{L} = \frac{t_1}{T} = \frac{1}{2\pi} \cos^{-1} \frac{K_s V_{sh}}{2\pi f F_{vv} \tan \phi_{\text{critical}}} \quad 6.10$$

where $\omega = 2\pi f$ and $\mu = \tan \phi_{\text{critical}}$

$$\text{put } \psi = \frac{K_s V_{sh}}{2\pi f F_{vv} \tan \phi_{\text{critical}}} \quad 6.11.$$

$$\text{then } \frac{t_1}{T} = \frac{1}{2\pi} \cos^{-1} \psi \quad 6.12.$$

Using the λ vs t_1/T curve and equation 6.12, the relation λ vs ψ is calculated and presented in Fig. 6.3.

To calculate H_{vs}/H_{ss} , the following procedure may be used:

- a- Evaluate ψ from equation 6.11.
- b- Use Fig. 6.3. to obtain λ
- c- Calculate the ratio F_{vv}/F_{vs}
- d- Substitute by λ and F_{vv}/F_{vs} in equation 6.9 to obtain H_{vs}/H_{ss}

Energy Consumed Due to Dynamic Shearing

The work done by the shearing force during one cycle of vibration can be defined by

$$\begin{aligned}\Delta U_h &= \int^{\text{cycle}} H(t) \cdot dx \\ &= \int_0^{2\pi/\omega} H(t) \cdot \frac{dx}{dt} dt\end{aligned}$$

If the drawbar is infinitely rigid, the plate compactor will travel with a constant speed V_{sh} and then

$$\frac{dx}{dt} = \text{constant} = V_{sh}$$

$$\begin{aligned}\Delta U_h &= \int_0^{2\pi/\omega} H(t) \cdot V_{sh} \cdot dt \\ &= V_{sh} \int_0^{2\pi/\omega} H(t) \cdot dt\end{aligned}\quad 6.13.$$

From Fig. 6.1

$$\int_0^{2\pi/\omega} H(t) \cdot dt = A_1 + A_3 \quad 6.14.$$

The relationship between A_1 and A_2 can be expressed by equation 6.8.

$$\lambda = 1 - \frac{A_1}{A_2}$$

or $A_1 = (1 - \lambda) A_2$

Referring to Fig. 6.1, it follows that:

$$A_2 = \frac{\mu F_{vv}}{f}$$

$$A_3 = \mu (F_{vs} - F_{vv}) \frac{1}{f} \quad 6.15.$$

then $A_1 = (1 - \lambda) \frac{\mu F_{vv}}{f} \quad 6.16.$

Substituting 6.14. and 6.16. in 6.14.

$$\begin{aligned} \int_0^{2\pi/\omega} H(t) \cdot dt &= (1 - \lambda) \frac{\mu F_{vv}}{f} + \mu (F_{vs} - F_{vv}) \frac{1}{f} \\ &= \frac{\mu}{f} (F_{vs} - \lambda F_{vv}) \end{aligned} \quad 6.17.$$

Substituting 6.17. in 6.13.

$$\Delta U_h = V_{sh} \frac{\mu}{f} (F_{vs} - \lambda F_{vv})$$

From equation 6.5., the number of cycles of

$$\text{vibration/in.} = \frac{f}{V_{sh}}$$

$$\text{then the work done by the shearing force/in.} = U_{vh} = \Delta U_h \frac{f}{V_{sh}}$$

$$\text{or} \quad U_{vh} = \mu (F_{vs} - \lambda F_{vv}) \quad 6.18.$$

6.3.2. Energy Consumption Due to Shearing with no Vibration

The work done by compactor under no vibration condition can be obtained from the equation

Work done = U_s

= $\mu \times \text{compactor's weight} \times \text{travelling distance}$

then for 1 in. travelling distance

$$U_s = \mu F_{vs}$$

6.19.

6.4. Prediction of the Performance of Vibratory Soil Compactor

The performance of the vibratory soil compactor will be presented in terms of the shearing force reduction, expressed as the ratio H_{vs}/H_{ss} where H_{vs} is the actual time averaged shearing force and H_{ss} is the shearing resistance of the soil, that is the value of shearing force obtained using a non-vibrating plate.

Using the equations presented in sections 6.2. and 6.3., it is possible to calculate for a given compactor (with frequency f , drawbar speed V_{sh} , dynamic vertical force F_{vv} and static weight F'_{vs}) the dimensionless ratios H_{vs}/H_{ss} . The required soil properties for these equations are the critical angle of friction $\phi_{critical}$, the rate of change of shearing force with horizontal displacement α

and the equivalent spring and damping constants of the soil in the vertical direction. In these equations, the drawbar is assumed to be rigid which is an acceptable assumption in most practical applications.

To calculate the ratio H_{vs}/H_{ss} , equation 6.19. can be used where

$$\frac{H_{vs}}{H_{ss}} = 1 - \lambda \frac{F_{vv}}{F_{vs}} \quad 6.20.$$

The total work consumed by the vibratory soil compactor is equal to the work consumed by vertical vibration plus the work consumed during shearing or

$$U_v = U_{vv} + U_{vh} \quad 6.21.$$

substituting equations 6.6. and 6.18. in 6.21.

$$U_v = F_{vv} \times \pi \sin \theta \frac{f}{V_{sh}} + \mu (F_{vs} - \lambda F_{vv}) \quad 6.22.$$

and the dimensionless work ratio can be obtained by

dividing equation 6.22. by equation 6.19.

$$\begin{aligned} \frac{U_v}{U_s} &= \frac{1}{\mu F_{vs}} \left\{ F_{vs} \times \pi \sin \theta \frac{f}{V_{sh}} + \mu (F_{vs} - \lambda F_{vv}) \right\} \\ &= \frac{\pi \times f \sin \theta}{\mu V_{sh}} \frac{F_{vv}}{F_{vs}} + (1 - \lambda) \frac{F_{vv}}{F_{vs}} \end{aligned} \quad 6.23.$$

The first term in equation 6.23. represents the ratio U_{vv}/U_s and the second term represents the ratio U_{vh}/U_s

Then

$$\frac{U_{vv}}{U_s} = \frac{\pi \times f \sin \theta}{\mu V_{sh}} \frac{F_{vv}}{F_{vs}} \quad 6.24.$$

$$\frac{U_{vh}}{U_s} = \left(1 - \lambda \frac{F_{vv}}{F_{vs}} \right) \quad 6.25.$$

comparing equations 6.20. with 6.25. then

$$\frac{U_{vh}}{U_s} = \frac{H_{vs}}{H_{ss}} \quad 6.26.$$

This means that the ratio of the work done by shear under vibration to the work done under no vibration is equal to the average shearing force under vibration to the shearing force under no vibration. This is because without vibration there is no energy consumed in the vertical direction and hence H_{ss} will be the force causing all the energy consumption. The work done by shear with or without vibration is equal to the average shearing force multiplied by the travelling distance, and since the travelling distance is the same with or without vibration, then equation 6.26 is valid.

Substituting 6.26 in 6.23,

$$\frac{U_v}{U_s} = \frac{\pi X f \sin \theta}{\mu V_{sh}} \frac{F_{vv}}{F_{vs}} + \frac{H_{vs}}{H_{ss}} \quad 6.27.$$

To simplify the calculation of H_{vs}/H_{ss} or U_{vh}/U_s , Fig. 6.4 is presented. In the figure, the relations λ vs ψ , as well as H_{vs}/H_{ss} (or U_{vh}/U_s) vs F_{vv}/F_{vs} (for constant values of λ) are plotted.

Table 6.1. can be used for the evaluation of the performance curves of vibratory plate compactors.

The performance curves of two typical vibratory plate compactors, compacting Ottawa sand will be presented next. The specifications of the compactors are shown in table 6.2.

Fig. 6.5. shows the effect of the amplitude of dynamic vertical force on the tractive effort and the energy consumed in compacting the soil with the previously mentioned compactors. The increase of the ratio F_{vv}/F_{vs} causes reduction of the tractive effort (H_{vs}/H_{ss}) and therefore the energy consumed in shearing the soil (U_{vh}/U_s). The energy consumed in compacting the soil (U_{vv}/U_s) increases with the increase of the ratio F_{vv}/F_{vs} . The most desirable condition is to compact soil in vertical direction with large amount of energy and to minimize the tractive effort. To achieve that the ratio U_{vv}/U_{vh} should be as high as possible. The ratio U_{vv}/U_{vh} was found to increase with F_{vv}/F_{vs} . As shown in Fig. 6.5., the values

Calculation of the Performance Curves

$F_{vs} =$

$K_s =$

$\mu =$

$c =$

$K_v =$

$m = F_{vs}/g =$

V_{sh}	f	ω	F_{vv}	$\frac{F_{vv}}{F_{vs}}$	X	θ	$\frac{U_{vv}}{U_s}$	ψ	λ	$\frac{H_{vs}}{H_{ss}}$	$\frac{U_v}{U_s}$	$\frac{U_{vv}}{U_{vh}}$

Value of λ as function of ψ can be obtained from Fig. 6.4.

$$\omega = 2\pi f$$

$$\psi = \frac{K_s V_{sh}}{2\pi f F_{vv} \mu}$$

$$X = \frac{F_{vv}/K_v}{\sqrt{(1 - \frac{m\omega^2}{K_v})^2 + (\frac{c\omega}{K_v})^2}}$$

$$\theta = \tan^{-1} \frac{c\omega/K_v}{1 - \frac{m\omega^2}{K_v}}$$

$$\frac{H_{vs}}{H_{ss}} = \frac{U_{vh}}{U_s} = 1 - \lambda \frac{F_{vv}}{F_{vs}}$$

$$\frac{U_{vv}}{U_s} = \frac{\pi X f \sin \theta}{\mu V_{sh}} \frac{F_{vv}}{F_{vs}}$$

$$\frac{U_v}{U_s} = \frac{U_{vv}}{U_s} + \frac{U_{vh}}{U_s}$$

Table 6.2.

Specifications of Two Commercial
Vibratory Plate Compactors

Compactor No.	Static Weight lb (N)	Maximum Speed of Travel in./s (m/s)	Maximum Frequency of Excitation Hz
1	860 (3826)	10.0 (0.254)	27
2	1675 (7451)	13.0 (0.33)	24

of U_{vv}/U_{vh} for compactor (1) is higher than that for compactor (2). This may conclude that under the operating conditions studied in Fig. 6.5., the compactor (1) is more efficient than compactor (2).

The effect of frequency on the draft reduction and the energy consumed in compacting the soil are shown in Fig. 6.6. for compactor (1). The energy consumed in the traction was found to decrease with increasing frequency. The energy delivered to compact the soil was found maximum at the natural frequency of the soil-compactor system. The ratio U_{vv}/U_{vh} was found also maximum at the natural frequency. This concludes that for compactor (1), the best performance can be obtained with frequency of excitation near to the natural frequency of the soil-compactor system.

The effect of the speed of travel V_{sh} is shown in Fig. 6.7. for compactor (1). It was found that the tractive effort increases with the increase of V_{sh} . The energy consumed in compacting the soil as well as the ratio

U_{vv}/U_{vh} decreases with the increase of V_{sh} . Then it is desirable to maintain the speed V_{sh} as low as possible.

The ratio representing the total energy U_v/U_s was found to decrease with increasing V_{sh} . When the speed reaches or exceeds 15.5 in./s (0.394 m/s), the value of U_v/U_s become almost constant. Above 15.5 in./s (0.394 m/s), the energy consumed in shearing the soil is larger than that compacting the soil. The constant value of the total energy consumed by the compactor travelling with speeds above 15.5 in./s (0.394 m/s) is due to increase in the energy consumed in shearing the soil and reduction in the energy of compaction of equal magnitude. Then for compactor (1) it is desirable to keep the speed of travel below 15.5 in./s (0.394 m/s) to benefit more from the energy in the compaction process.

6.5. Evaluation of the Performance of Vehicle with Rigid Track Travelling on Smooth Terrain with Multipeak Sinusoidal Normal Pressure Distribution Under the Track

Analysis of track vehicle has been studied by many investigators. A review of different methods of analysis is

given by Wong (1978). It has been found that, the normal pressure distribution is not uniform under the track (Wills 1963). Field measurements of the normal pressure under the track showed that one of the common forms is a multipeak sinusoidal pressure distribution, as shown in Fig. 6.8. The maximum pressure occurs under the wheels. If the actual speed of the tracked vehicle is V , the speed of the track with reference to the ground is V_j (speed of slip) and the theoretical speed is V_t , then by definition (Wong 1978):

$$i = 1 - \frac{V}{V_t} = \frac{V_j}{V_t}$$

$$\text{or } V = \frac{(1 - i) V_j}{i} \quad 6.28.$$

When the track moves on the soil surface it will shear it with a rate of shearing equal to V_j . When the vehicle is slipping, V_j will be in the direction opposite to that of vehicle motion.

Since the peak pressure occurs under the wheels then

the time between two peaks to act on the same soil particle is equal to

$$T = \frac{d}{V}$$

where d = distance between 2 wheels

The time T represent the period of a sinusoidal vertical pressure acting on each soil particle. Then the frequency of vertical pressure acting on each soil particle is

$$f = \frac{1}{T} = \frac{V}{d} = \frac{(1-i) V_i}{i d} \quad 6.29.$$

The mean value of the pressure distribution is

$$\sigma_{vs} = \frac{W}{b L}$$

where W = weight of the vehicle

b = width of the track

L = length of the track

It was assumed that the peak pressure is twice the mean pressure (Wong 1978) or

$$\sigma_{max} = \frac{2 W}{b L}$$

This is analogous to a track vehicle with static weight equivalent to

$$\sigma_{vs} = \frac{W}{bL} \quad 6.30.$$

and subjected to vertical dynamic force of amplitude

$$\sigma_{vv} = \frac{W}{bL} \quad 6.31.$$

In this case the ratio

$$\frac{F_{vv}}{F_{vs}} = \frac{\sigma_{vv}}{\sigma_{vs}} = 1 \quad 6.32.$$

The ultimate shear stress curve can be obtained by multiplying the normal pressure curve by $\tan \phi_{\text{critical}}$. For any soil particle the ultimate shear stress as function of time can be predicted as shown in Fig. 6.9. If the rate of change of shear stress with deformation is k_s and the shear rate of soil is V_j , then the actual shear stress developed between the track and any soil particle will follow the path shown in Fig. 6.9. The value of β is given by

$$\beta = k_s V_j$$

The shear stress curve under the track can be predicted by transferring the shear stress corresponding to each value of σ to its equivalent position under the track as shown in Fig. 6.10.

The slope of the shearing force can be calculated as

$$\beta = \frac{\tau_1 - \tau_2}{t}$$

and $k_s' = \frac{\tau_1 - \tau_2}{x}$

then $\beta t = k_s' x$

but $x = V t$

$$k_s' = \frac{\beta t}{x} = \frac{\beta t}{V t}$$

then $k_s = \frac{\beta}{V} = \frac{k_s' V_j^{-1}}{(1-i) V_j} = \frac{1}{1-i} k_s' \quad 6.33.$

The value of ψ as defined by equation 6.11. is

$$\psi = \frac{V_j^3 k_s'}{2\pi f \sigma_{vv} \tan \phi_{\text{critical}}}$$

substituting by f from equation 6.29., k_s' from equation 6.33 and σ_{vv} from equation 6.31 we get

$$\psi = \left(\frac{i}{1-i} \right)^2 \frac{k_s b d L}{2\pi W \tan \phi_{\text{critical}}} \quad 6.34.$$

Evaluation of the Tractive Effort

The tractive effort can be calculated from multiplying the track width by the integration of the shear stress along the track length. If the shear stress starts with peak value and decreases to zero in the first half cycle as shown in Fig. 6.9, then the shear stress for this part will be less than its ultimate value as described in Chapter III.

The shear stress of the first half cycle can be expressed by

$$\tau = \frac{W}{b L} \tan \phi \left(1 + \cos \frac{2\pi x}{d} \right)$$

where d is the distance between two wheels.

The area under the shear stress curve is composed of two areas A_1 and A_2 as shown in Fig. 6.10, where

$$A_1 = \frac{1}{2} x_1 \tau_1 = \frac{1}{2} \frac{x_1 W}{b L} \tan \phi \left(1 + \cos \frac{2\pi x_1}{d} \right)$$

the area A_2 can be calculated from

$$\begin{aligned} A_2 &= \int_{x_1}^{d/2} \tau dx = \int_{x_1}^{d/2} \frac{W}{b L} \tan \phi \left(1 + \cos \frac{2\pi x}{d} \right) dx \\ &= \frac{W}{b L} \tan \phi \left\{ \frac{d}{2} - x_1 - \frac{d}{2\pi} \sin \frac{2\pi x_1}{d} \right\} \end{aligned}$$

Then the tractive effort of the first half cycle is

$$\begin{aligned} H_1 &= b (A_1 + A_2) = b \frac{W}{b L} \tan \phi \left\{ \frac{x_1}{2} + \frac{x_1}{2} \cos \frac{2\pi x_1}{d} + \frac{d}{2} - x_1 - \frac{d}{2\pi} \sin \frac{2\pi x_1}{d} \right\} \\ &= \frac{W d}{L} \tan \phi R \end{aligned} \quad 6.35.$$

where

$$R = \left\{ \frac{1}{2} - \frac{x_1}{2d} + \frac{x_1}{2d} \cos \frac{2\pi x_1}{d} - \frac{1}{2\pi} \sin \frac{2\pi x_1}{d} \right\} \quad 6.36.$$

or R is function of $\frac{x_1}{2d}$

The value of x_1 depends on the values of k_s' , σ_{vv} and d .

$$\text{since } k_s' = \frac{\tau_1}{x_1} = \frac{1}{x_1} \left\{ \frac{W}{bL} \tan \phi \left(1 + \cos \frac{2\pi x_1}{d} \right) \right\}$$

but from equation 6.33,

$$k_s' = \frac{i}{1-i} k_s$$

equating the right hand side of the two equations it can be found that

$$k_s = \frac{2(1-i)W \tan \phi}{i b d L} R_1 \quad 6.37.$$

$$\text{where } R_1 = \frac{1 + \cos \left(\frac{2\pi x_1}{d} \right)}{\frac{2 x_1}{d}} \quad 6.38.$$

then R_1 also is function of $\frac{x_1}{2d}$

Then for any value of $x_1/2d$ the corresponding values of R and R_1 can be calculated. Fig. 6.11 shows the curve of R_1 vs R . The value of R_1 can be calculated from

equation 6.37

$$\text{or } R_1 = \frac{i b d L k_s}{2 (1 - i) W \tan \phi} \quad 6.39.$$

Knowing R_1 and using Fig. 6.11, the value of R can be obtained and hence H_1 can be evaluated.

For any intermediate complete cycle, starts and ends with zero stress, the value of ψ can be calculated from

$$\psi = \left(\frac{i}{1 - i} \right)^2 \frac{k_s b d L}{2 \pi W \tan \phi} \quad 6.40.$$

knowing ψ the value of H_{vs}/H_{ss} can be calculated as described in sec. 6.3.

Then for each intermediate cycle the tractive effort is

$$H_2 = \frac{W}{b L} \tan \phi \times b d \times \frac{H_{vs}}{H_{ss}}$$

$$H_2 = \frac{W d}{L} \tan \phi \frac{H_{vs}}{H_{ss}} \quad 6.41.$$

If the last half cycle is as shown in Fig. 6.10 then an approximate value of the tractive effort is

$$\begin{aligned}
 H_3 &= b \left\{ \frac{1}{2} \tau_e \frac{d}{2} \right\} \\
 &= b \left\{ \frac{1}{2} \left(k_s' \frac{d}{2} \right) \frac{d}{2} \right\} \\
 H_3 &= \frac{b d^2 i k_s}{8 (1 - i)} \quad 6.42.
 \end{aligned}$$

This equation is valid as long as the shear stress is less than its ultimate value or as long as $\psi < 1$

$$\text{if } \psi \geq 1 \text{ then } H_3 = \frac{W d}{2 L} \tan \phi \quad 6.43.$$

If the number of wheels in the track is n_r then the total tractive effort is

$$\begin{aligned}
 H &= H_1 + (n_r - 2) H_2 + H_3 \\
 H &= \frac{W d \tan \phi}{L} \left\{ R + (n_r - 2) \frac{H_{vs}}{H_{ss}} \right\} + \frac{b d^2 i k_s}{8 (1 - i)} \text{ for } \psi < 1 \quad 6.44.
 \end{aligned}$$

or

$$H = \frac{W d \tan \phi}{L} R + (n_r - 2) \frac{W d \tan \phi}{L} + \frac{W d}{2 L} \tan \phi \quad \text{for } \psi \geq 1$$

$$H = \frac{W d \tan \phi}{L} (R + n_r - 1.5) \quad \text{for } \psi \geq 1 \quad 6.45.$$

Procedure for the Prediction of the Performance of Track Vehicles with Sinusoidal Pressure Distribution

1. Calculate R_1 from equation 6.39.
2. Using Fig. 6.11, the corresponding value of R can be obtained.
3. Calculate ψ from equation 6.40.
4. Using Fig. 6.3, the corresponding value of λ can be obtained.
5. Calculate H_{vs}/H_{ss} , by substituting with $F_{vv}/F_{vs} = 1$ in equation 6.9 then

$$\frac{H_{vs}}{H_{ss}} = 1 - \lambda$$

6. Calculate the gross tractive effort from equation 6.44 or 6.45.

Wills (1963) presented a method for the prediction of the performance of track from a shear stress-deformation curve. He calculated as an example the gross tractive effort of a rigid track with the following specifications:

weight $W = 2950 \text{ lb (13.1 kN)}$

length of track $L = 50 \text{ in. (1.27 m)}$

width of track $b = 10 \text{ in. (0.254 m)}$

distance between wheels $d = 10 \text{ in. (0.254 m)}$

number of wheels $n_T = 6$

angle of internal friction of sand $= 33 \text{ degree}$

For that rigid track, the relation between the gross tractive effort and the slip percentage predicted by Wills and by using the procedure described before are shown in Fig. 6.12. The two curves show a great reduction of the tractive effort for percentage slip less than 20%. The tractive effort approaches its ultimate value ($W \tan \phi$)

for slip above 20%. The values predicted by Wills equation is noticed to be slightly less than that obtained by the procedure described before for slip greater than 16% and of larger values for slip less than 16%.

6.6. Summary

Assuming that the vibratory soil compactor is equivalent to a single degree of freedom system in the vertical direction, it was possible to calculate the work done in compacting the soil.

A graphical method is presented to evaluate the average value of shearing force and the work done due to shearing under vibrating condition. The method simplifies the evaluation of the ratios H_{vs}/H_{ss} and U_{vh}/U_s .

It was shown that the ratio

$$\frac{H_{vs}}{H_{ss}} = \frac{U_{vh}}{U_s}$$

This ratio is very important in the design of the vibratory soil compactor since it indicates the reduction in the average value of shearing force as well as the energy consumed in the shearing process.

The theoretical model can be used in evaluating the performance curves of the vibratory plate compactors. These performance curves are necessary in the optimization of the design and in providing guiding principles for the rational design of such machines.

The theoretical model can also be used in evaluating the tractive effort of vehicle with rigid track travelling on smooth terrain with multipeak sinusoidal normal pressure distribution under the track.

CHAPTER VII

Summary and Conclusions

Various mathematical models developed so far for solving foundation vibration problems and for describing soil-machine interaction are reviewed. It is found that none of these are directly applicable to vibratory soil engaging devices with vibration perpendicular to the direction of shearing. A new mathematical model is therefore developed.

It has been shown in previous investigations that vibration usually decreases the coefficient of internal friction of granular material. However the effect is insignificant for acceleration ratio η less than 1. The model for vibratory soil engaging devices described in this thesis has been developed for acceleration ratio η less than 1. The assumption that the angle of internal friction of soil is constant therefore seems acceptable.

In order to investigate the effect of different design and operation parameters on the behaviour of vibratory soil engaging devices, various simplified operating conditions were examined,

with particular reference to their effect on the average shearing force of the vibrating soil engaging element.

Study of a model with Coulomb friction and with rigid drawbar subjected simultaneously to rectangular, periodic, vertical loading and to translational motion has shown that the average value of the dynamic shearing force is equal to the ultimate static shearing force.

Another model with rigid drawbar using sand instead of the ideal Coulomb frictional surface gives values of the shearing force less than the ultimate value during the loading part of the cycle. In this case the average value of the dynamic shearing force is less than that of the Coulomb frictional model. This demonstrates the effect of the rate of increase of shearing force with displacement α on the reduction of the average value of dynamic shearing force.

A model with Coulomb friction and with elastic drawbar demonstrates that the drawbar force will be equal to the shearing force and less than the ultimate shearing force during the loading part of the cycle until the point where the spring force reaches the ultimate shearing force. For the rest of the cycle, the shearing force is equal to its ultimate value. Thus the average value of the dynamic shearing force decreases with the decrease of the drawbar stiffness.

Tests using a linear shear apparatus subjected to step load have confirmed the combined effect of the soil constant α and the drawbar spring stiffness.

To demonstrate the effect of the rate of change of vertical periodic loading on shear behaviour of the soil engaging element, a triangular shaped loading pattern was studied theoretically. It has been found that the reduction in the average value of dynamic shearing force is a

3

function of the relationship between the rate of increase of shearing force with time β and the rate of increase of the ultimate shearing force. The rate of increase of ultimate shearing force is a function of the frequency and amplitude of the dynamic vertical force. The rate of increase of shearing force with time β is a function of the parameter α , the drawbar stiffness and the drawbar speed.

The most common type of loading used for vibratory soil engaging elements is of harmonic type. The performance of the vibratory soil engaging element under harmonic loading was studied. It has been shown that the average value of dynamic shearing force is related to the ratio of the amplitude of dynamic force to the static weight by the following

relationship:

$$\frac{H_{vs}}{H_{ss}} = 1 - \lambda \frac{F_{vv}}{F_{vs}}$$

where λ is function of the ratio

$$\frac{K_s V_{sh}}{2\pi f F_{vv} \mu}$$

It has also been shown that the reduction of the average value of dynamic shearing force, as compared to the static shearing resistance, will occur if

$$\frac{K_s V_{sh}}{2\pi f F_{vv} \mu} < 1$$

The model demonstrates that, for a vibratory soil engaging element, the average value of dynamic shearing force will always be less than the static shearing force when the rate of development of shearing force between the element and soil is lower than the rate of increase of the ultimate shearing force. The above mentioned parameters affect the rate of development of shearing force and hence the average value of dynamic shearing force.

Previous experimental studies and the instrumentation used in studying the effects of vibration on the shearing behaviour of soil engaging devices are reviewed. None of the known equipment can simulate adequately the operating conditions of vibratory soil engaging devices with vibration perpendicular to the direction of shearing. A new apparatus was designed, manufactured and used, which allows the direct measurement of shearing and vertical forces on the soil engaging element. This apparatus is intended for validating the mathematical model developed for vibratory soil-engaging devices.

The apparatus is shown to perform satisfactorily under static and low frequency dynamic conditions. Tests at high frequencies indicate the importance of balancing the system statically and dynamically. Frequency response curves have been developed for the apparatus resting on a rigid base and on sand. At 30 Hz it appears that the system is close to resonance.

Experiments have been performed to validate the

proposed mathematical model. The experimental program includes the examination of the effects of drawbar speed, amplitude and frequency of the vertical exciting force and the static normal force.

The parameter α was measured experimentally for use in the prediction of the average value of dynamic shearing force. Although the approach of measuring α and the assumption that it is constant and independent of all the other parameters are a simplification, they seem to yield useful information. Predictions based on these show good agreement with experimental results.

Generally the test results confirm the validity of the theoretical model, and good agreement between the experimental and theoretical results is obtained for frequencies below 20 Hz. It has been shown that, other conditions being equal,

1. The average dynamic shearing force decreases with increasing ratio of dynamic to static force.

2. The average dynamic shearing force increases with increasing drawbar speed.
3. The average dynamic shearing force decreases with increasing frequency.
4. The average dynamic shearing force decreases with increasing static normal load.

The proposed theoretical model for the vibratory soil engaging element provides a basic understanding of the relationship between the shearing force developed on the soil engaging element and the pertinent vibrational parameters.

The theoretical model has been applied to the prediction of the performance of vibratory plate compactors. The energy consumed in compacting the soil is calculated by assuming the system having a single degree of freedom in the vertical direction. A graphical method is developed to evaluate the average value of dynamic shearing force and hence the energy consumed due to shearing U_{vh} .

It is shown that the ratio

$$\frac{H_{vs}}{H_{ss}} = \frac{U_{vh}}{U_s}$$

This ratio is very important in the design of a vibratory soil compactor since it relates the reduction in the average value of dynamic shearing force to the energy consumed by shearing.

The performance curves representing the energy consumed in compacting and in shearing the soil are invaluable in providing guiding principles for the optimum design of vibratory plate compactors.

The performance curves suggest that a reduction in draft of the compactor can only be realized at the expense of increased vibrational energy input. However, the compaction efficiency and performance are improved with increased vibrational energy. The theoretical model makes it possible to optimize the performance of

a vibratory soil compactor that would result in minimum total power consumption for a required degree of compaction effort.

The theoretical model for the vibratory soil engaging devices has also been applied to the evaluation of the tractive performance of a tracked vehicle with multi-peak sinusoidal normal pressure distribution.

6

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COMPUTER PROGRAM FOR THE ANALYSIS OF VIBRATORY
SOIL COMPACTOR UNDER HARMONIC EXCITATION

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FLAG :

~~★ FLAG VERSION DO1~~

★ AVAILABLE! MEMORY!

* PROGRAM & INITIALIZED VARIABLES = 6101 (WORDS)

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★ TOTAL = 20329 (WORDS)

PROGRAM FOR SOLVING A MODEL FOR THE SHEARING FORCE UNDER VERTIC

0002 C DYNAMIC LOADING

0003 C PV=STATIC WT. OF THE PLATE

~~EXAMPLE OF DYNAMIC VL FORCE~~

0005 C FREQUENCY OF VIB. DYNAMIC FORCE

AMU=COEFFICIENT OF FRICTION IN THE PLASTIC RANGE

0007 C-ALFA SHEARING FORCE/SHEARING DISPLACEMENT RATIO IN ELASTIC

RANGE

FN=NATURAL FREQUENCY IN HL. DIRECTION

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NOT NO. OF TIME INTERVALS / CYCLES

NC=NO. OF CYCLES REQUIRED TO BE CALCULATED

NEW-CYCLE NO. 0013

00014 C . FI=PHASE ANGLE TO INDICATE THE STARTING POINT OF THE VL. FORCE

DTIME INTERVAL C 0015 C

~~0016 C LONG. OF SMOOTHING OPERATIONS~~

[illegible]

00016 C HD=DRAW BAR PULL FORCE

0019. C Damping Coefficient

ZETA-DAMPING RATIO

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[illegible]

PRINT 2000

[illegible]

0034 0000 POKH (INT)
0035 PRINT 100; PR-VSH

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0036 100      FORMAT(3X,
0037      S  L, VEL,=0,2X,F8.3)
0038      PRINT 101,NDT,NC
0039 101      FORMAT(3X,=0, OF INTERVAL8/CYCLE=0,13,2X,=0, OF CYCLES=0,13)
0040      PRINT 102,ALFA,AMU,AK
0041 102      FORMAT(3X,=0,ALFA=0,F15.3,2X,=0,AMU=0,F8.4,2X,=0,AK=0,F15.3)
0042      PRINT 103,PVS,FVV,FI
0043 103      FORMAT(3X,=0,STATIC WT.=0,F8.3,2X,=0,AMP: OF DYNAMIC VL. FORCE=0,F8.3,
0044      2X,=0,PI(RAD.)=0,F8.4)
0045      OMIGA =2.*3.141592654*PR
0046      AMASS=PV8/(32.2*12.)
0047      VTOT=(NC*NDT)+1
0048      DT=1./(PR*FLOAT(NDT))
0049      OUTPUT DT
0050      EQUK=(1./AK)+(1./ALFA)
0051      RALPK=(ALFA/AK)+1.
0052      H(1)=0.
0053      X(1)=0.
0054      T(1)=0.
0055      PV(1)=FVS+(FVV*8IN(PI))
0056      DO 504 M=2,NTOT
0057      T(M)=T(M-1)+DT
0058      PV(M)=FVS+(FVV*8IN((OMIGA*T(M))+PI))
0059      H3=H(M-1)+((VSH*DT)/EQUK)
0060      H2=AMU*PV(M)
0061      IF(H3=H2)501,502,502
0062 501      H(M)=H3
0063      X(M)=X(M-1)+((VSH*DT)/RALPK)
0064      GO TO 503
0065 502      H(M)=H2
0066      X(M)=X(M-1)+VSH*DT=((H(M)-H(M-1))/AK)
0067      CONTINUE
0068      CONTINUE
0069      NDT1=NDT+1
0070      DO 600 K=NCN1,NCN2
0071      DO 600 I=1,NDT1
0072      J=I+NDT*(K-1)=5
0073      FV1(I)=FV(J)
0074      H1(I)=H(J)
0075      T1(I)=T(J)
0076      CONTINUE
0077      NDT2=NDT+9
0078      DO 602 I=1,NDT2

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0001 SUBROUTINE INTEG(Y,M,N,A)
0002 INTEGRATION BY SIMPSON'S RULE
0003 Y=COORDINATES OF FUNCTION
0004 H=INTERVAL LENGTH
0005 N=NO. OF POINTS, SHOULD BE ODD
0006 A=INTEGRATION RESULT
0007 DIMENSION Y(90)
0008 SUM=Y(1)-Y(N)
0009 N=N-1
0010 DO 4 I=2,N,N/2
0011 IF(I-(N-1))1,1,2
0012 1 SUM=SUM+4.*Y(I)+2.*Y(I+1)
0013 4 CONTINUE
0014 A=SUM*H/3.
0015 RETURN
0016 END

```

```

0001 SUBROUTINE DIFF(Y,N,DT,DFY)
0002 REF, JAMES SCARBOROUGH, NUMERICAL MATHEMATICAL ANALYSIS, THE JOHNS
0003 HOPKINS PRESS, 1966, P. 133.
0004 N=NO. OF POINTS REQUIRED TO DIFFERENTIATE
0005 DT=EQUAL SPACED INTERVAL LENGTH
0006 DIMENSION Y(90), DFY(90)
0007 N2=N/2
0008 DO 2 I=3,N2
0009 DFY(I)=(-Y(I+2)+8.*Y(I+1)-6.*Y(I-1)+Y(I-2))/(12.*DT)
0010 2 DFY(I-2)=DFY(I)
0011 RETURN
0012 END

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* ACTUAL PROGRAM SIZE:
* PROGRAM & INITIALIZED VARIABLES = 727 (WORDS)
* NON-INITIALIZED VARIABLES = 3491 (WORDS)
* TOTAL = 4218 (WORDS)

```

COMPILE TIME = 0001.08

FREQUENCY= 20.080 STATIC HL. VEL.= .800
 NO. OF INTERVALS/CYCLE= 36 NO. OF CYCLES= 10
 ALPHA= 1375.000 WJ= .5000 AK 5700.000
 STATIC WT.= 66.000 AMP. OF DYNAMIC VL. FORCE= 48.070 FI(RAD.)= .0000
 07-1383356E-03

N	FV	H	X	VHV	ACHV	T
1	29.1572	12.0614	.151570	.644492	.84503	.1922815
2	35.0784	14.0873	.152461	.644485	-.10045	.1936648
3	41.9385	15.3133	.153353	.644477	-.00519	.1950482
4	49.5302	16.5392	.154244	.644470	-.00517	.1964315
5	57.6223	17.7652	.155136	.644463	-.00520	.1978148
6	65.9689	18.9911	.156027	.644456	-.00522	.1991981
7	74.3165	20.2171	.156919	.644449	-.00870	.2005814
8	82.4108	21.4431	.157811	.644442	.01902	.2019647
9	90.0071	22.6690	.158702	.644435	.01898	.2033480
10	96.8740	23.8950	.159594	.644428	.00874	.2047313
11	102.8029	25.1209	.160485	.644421	-.00519	.2061146
12	107.6432	26.3469	.161377	.644414	-.00517	.2074979
13	111.1596	27.5728	.162268	.644407	-.00520	.2088812
14	113.3339	28.7988	.163160	.644400	-.00522	.2102645
15	114.9700	30.0247	.164051	.644393	-.00870	.2116478
16	113.3457	31.2507	.164943	.644386	.01902	.2130311
17	111.1829	32.4766	.165835	.644379	.01898	.2144144
18	107.6473	33.7026	.166726	.644372	-.00874	.2157977
19	102.8463	34.9285	.167618	.644365	.167675	.2171810
20	96.9257	36.1545	.168509	.644358	-21.79402	.2185643
21	90.0662	37.3804	.169401	.616543	34.26660	.2199476
22	82.4749	38.6064	.170292	.782794	229.53239	.2213309
23	74.3831	37.1915	.171647	1.182232	235.40550	.2227142
24	66.0365	33.0182	.173486	1.359685	37.10571	.2240975
25	57.6895	28.8448	.175325	1.323876	33.50795	.2254809
26	49.5943	24.7972	.177142	1.299985	-21.20903	.2268642
27	41.9976	20.9988	.178915	1.260873	33.51358	.2282475
28	35.1302	17.5651	.180624	1.207732	43.11905	.2296308
29	29.2006	14.6003	.182251	1.142189	-51.37677	.2310141
30	24.3896	12.1948	.183779	1.066302	-58.08122	.2323974
31	20.8423	10.4212	.185197	.982301	-63.02870	.2337807

32	18,6671	9,3335	186495	892791	-65,13325	2351640
33	17,9300	8,9650	187666	800395	-71,57108	2365473
34	18,6534	9,3267	188709	702243	-59,35098	2379306
35	20,8152	10,4076	189626	646011	-19,17825	2393139
36	24,3500	11,6336	190518	642959	2,74435	2406972
37	29,1502	12,0595	191409	644485	82470	2420805

H3= 22,6305 H33= 33,0005RH= 6864
 FVV= 48,0700FVS= 66,0010RF= 7283
 CYCLE NO.= 5

N	PV	H	X	VHV	ACHV	T
1	29,1502	12,0595	191409	644485	82470	2420805
2	35,0695	14,0855	192301	644477	09755	2434638
3	41,9291	15,3114	193192	644470	00517	2448471
4	49,5200	16,5374	194084	644463	00520	2462304
5	57,6116	17,7633	194975	644456	00522	2476137
6	65,9581	18,9893	195867	644449	00870	2489970
7	74,3051	20,2152	196759	644442	01902	2503803
8	82,4005	21,4412	197650	644435	01898	2517636
9	89,9977	22,6671	198542	644428	00874	2531469
10	96,8656	23,8931	199433	644421	00519	2545303
11	102,7954	25,1190	200325	644414	00517	2559136
12	107,6077	26,3450	201216	644407	00520	2572969
13	111,1558	27,5710	202108	644400	00522	2586802
14	115,3320	28,7969	203000	644393	00870	2600635
15	114,0700	30,0229	203891	644386	01902	2614468
16	113,3476	31,2488	204783	644379	01898	2628301
17	111,1866	32,4748	205674	644372	00874	2642134
18	107,6527	33,7007	206566	644365	00519	2655967
19	102,8533	34,9267	207457	644358	1,66983	2669800
20	96,9346	36,1526	208349	644351	-21,70760	2683633
21	90,0756	37,3786	209240	616650	34,05200	2697466
22	82,4851	38,6045	210132	782270	229,19321	2711299
23	74,3938	37,1969	211086	1,181650	235,71463	2725132

24	26,0481	33,0240	213324	1,359793	37,2839	.2738965
25	57,7002	28,8501	215163	1,323909	33,58310	.2752798
26	49,6046	24,8023	216980	1,300014	21,20816	.2766631
27	42,0070	21,0035	218753	1,260905	33,48991	.2780464
28	35,1391	17,5695	220462	1,207826	43,05948	.2794297
29	29,2081	14,6040	222089	1,142347	51,40054	.2808130
30	24,3950	12,1975	223618	1,066349	58,12982	.2821963
31	20,8460	10,4230	225036	982355	63,01836	.2835796
32	18,6690	9,3343	226334	892856	65,70584	.2849630
33	17,9300	8,9650	227505	800524	71,51442	.2863463
34	18,6515	9,3257	228549	702437	59,33755	.2877296
35	20,8115	10,4058	229466	646158	19,27054	.2891129
36	24,3446	11,6317	230357	642944	2,68358	.2904962
37	29,1428	12,8577	231249	644481	83896	.2918795

H3= 22.6503 H3= 33.0005RH= .6864

PV= 48.0700PV= 66.0010RF= .7283

CYCLE NO.= 6

N	FV	H	X	VHV	ACHV	T
1	29,1428	12,8577	231249	644481	.83896	.2918795
2	35,0612	14,0836	232140	644474	.09818	.2932628
3	41,9197	15,3096	233032	644467	.00521	.2946461
4	49,5098	16,5355	233924	644459	.00524	.2960294
5	57,6009	17,7615	234815	644452	.00872	.2974127
6	65,9465	18,9874	235707	644445	.01906	.2987960
7	74,2944	20,2134	236598	644495	.01903	.3001793
8	82,3903	21,4393	237490	644488	.00869	.3015626
9	89,9882	22,6653	238381	644481	.00526	.3029459
10	96,8568	23,8913	239273	644474	.00518	.3043292
11	102,7884	25,1172	240165	644467	.00521	.3057125
12	107,6023	26,3432	241056	644459	.00524	.3070958
13	111,1521	27,5691	241948	644452	.00872	.3084791
14	113,3299	28,7951	242839	644445	.01906	.3098624
15	114,0700	30,0210	243731	644495	.01903	.3112457

16	1.3,3495	31,2470	244622	644488	00869	3126290
17	111,1903	32,4729	245514	644481	00526	3140123
18	107,6582	33,6989	246405	644474	00518	3153957
19	102,8607	34,9248	247297	644467	1,66634	3167790
20	96,9429	36,1508	248189	644459	21,64180	3181623
21	90,0850	37,3767	249080	616704	33,78206	3195456
22	82,4954	38,6027	249972	781634	228,88177	3209289
23	74,4052	37,2026	251324	1,181072	236,07277	3223122
24	66,0590	33,0295	253163	1,359904	37,64803	3236955
25	57,7110	28,8555	255002	1,323995	33,65796	3250788
26	49,6148	24,8074	256819	1,300046	21,23303	3264621
27	42,0171	21,0085	258592	1,260937	33,46257	3278454
28	35,1474	17,5737	260301	1,207915	43,03903	3292287
29	29,2151	14,6075	261928	1,142437	51,37550	3306120
30	24,4005	12,2002	263457	1,066507	58,09489	3319953
31	20,8498	10,4249	264875	982520	63,04144	3333786
32	18,6710	9,3355	266173	892971	65,72353	3347619
33	17,9300	8,9650	267344	800650	71,49603	3361452
34	18,6496	9,3248	268388	702573	59,37480	3375285
35	20,8078	10,4039	269305	646190	19,33519	3389118
36	24,3388	11,6299	270197	642930	2,67319	3402951
37	29,1358	12,8558	271088	644474	84482	3416784

HS= 22,6501 HSS= 6864
 FVV= 48,0700 FVS= 66,0010 RFE= 7283
 CYCLE NO.= 7

N	PV	H	X	VHV	ACHV
1	29,1358	12,8558	271088	644474	84482
2	35,0529	14,0818	271980	644467	09865
3	41,9102	15,3077	272872	644459	00524
4	49,4988	16,5337	273763	644452	00872
5	57,5894	17,7596	274655	644445	01906
6	65,9356	18,9856	275546	644438	01903
7	74,2837	20,2115	276438	644430	00869

8	2,3801	21,4375	277329	644481	526	3513616
9	89,9782	22,6634	278221	644474	00518	3527449
10	96,8484	23,8894	279113	644467	00521	3541282
11	102,7814	25,1153	280004	644459	00524	3555115
12	107,5968	26,3413	280896	644452	00872	3568948
13	111,1481	27,5672	281787	644445	01906	3582781
14	113,3280	28,7932	282679	644495	01903	3596614
15	114,0699	30,0191	283570	644488	00869	3610447
16	113,3513	31,2451	284462	644481	00526	3624280
17	111,1940	32,4711	285354	644474	00518	3638113
18	107,6640	33,6970	286245	644467	00521	3651946
19	102,8677	34,9230	287137	644459	1,65936	3665779
20	96,9513	36,1489	288028	644452	21,55876	3679612
21	90,0945	37,3749	288920	616812	33,59526	3693445
22	82,5063	38,6008	289811	781167	228,54948	3707278
23	74,4159	37,2079	291162	1,180490	236,30246	3721111
24	66,0699	33,0349	293001	1,359898	37,88315	3734944
25	57,7217	28,8608	294840	1,323970	33,65364	3748778
26	49,6257	24,8129	296657	1,300132	21,18140	3762611
27	42,0265	21,0133	298430	1,261024	33,49382	3776444
28	35,1558	17,5779	300139	1,207944	43,06401	3790277
29	29,2221	14,6110	301767	1,142473	51,34396	3804110
30	24,4059	12,2030	303296	1,066604	58,07065	3817943
31	20,8537	10,4269	304714	982624	63,01146	3831776
32	18,6729	9,3365	306012	893140	65,71854	3845609
33	17,9301	8,9650	307184	800768	71,51379	3859442
34	18,6477	9,3239	308227	702702	59,38045	3873275
35	20,8038	10,4019	309145	646273	19,40158	3887108
36	24,5333	11,6279	310036	642905	2,65513	3900941
37	29,1288	12,8538	310928	644459	85583	3914774

HSB 22,6499 HSB 33,0005RH 6863
 FVV 48,0700FVS 66,0010RF 7283
 CYCLE NO. 8

N	FV	H	X	VHV	ACHV	T
1	29,1288	12,8538	310928	644459	.85583	.3914774
2	35,0446	14,0798	311820	644452	-.10281	.3928607
3	41,9002	15,3057	312711	644445	.01906	.3942440
4	49,4886	16,5317	313605	644495	.01903	.3956273
5	57,5787	17,7576	314494	644488	-.00869	.3970106
6	65,9247	18,9836	315386	644481	-.00526	.3983939
7	74,2729	20,2095	316277	644474	-.00518	.3997772
8	82,3692	21,4355	317169	644467	-.00521	.4011605
9	89,9687	22,6615	318061	644459	-.00524	.4025438
10	96,8401	23,8874	318952	644452	-.00872	.4039271
11	102,7744	25,1134	319844	644445	-.01906	.4053105
12	107,5910	26,3393	320735	644495	.01903	.4066938
13	111,1444	27,5653	321627	644488	-.00869	.4080771
14	113,3261	28,7912	322518	644481	-.00526	.4094604
15	114,0699	30,0172	323410	644474	-.00518	.4108437
16	113,3532	31,2431	324301	644467	-.00521	.4122270
17	111,1980	32,4691	325193	644459	-.00524	.4136103
18	107,6694	33,6950	326085	644452	-.00872	.4149936
19	102,8747	34,9210	326976	644445	1.68365	.4163769
20	96,9596	36,1469	327868	644495	-.21,49643	.4177602
21	90,1045	37,3729	328759	616855	33,31834	.4191435
22	82,5165	38,5988	329651	780578	228,29344	.4205268
23	74,4266	37,2133	331001	1,100017	236,66663	.4219101
24	66,0807	33,0404	332839	1,360055	38,05896	.4232934
25	57,7331	28,8666	334678	1,323937	33,75102	.4246767
26	49,6360	24,8180	336495	1,300157	21,12216	.4260600
27	42,0360	21,0180	338269	1,261106	33,47015	.4274433
28	35,1641	17,5820	339978	1,208034	43,06310	.4288266
29	29,2291	14,6145	341605	1,142567	51,34344	.4302099
30	24,4117	12,2059	343134	1,066705	58,03761	.4315932
31	20,8575	10,4287	344553	982789	63,00674	.4329765
32	18,6748	9,3374	345851	893255	65,73628	.4343598
33	17,9301	8,9650	347023	800894	71,52304	.4357432
34	18,6457	9,3229	348067	702781	59,41763	.4371265
35	20,0002	10,4001	348985	646305	19,43883	.4385098
36	24,3279	11,6260	349876	642890	2,62283	.4398931
37	29,1218	12,8520	350768	644456	.86323	.4412764

H3= 22.6496 H33= 33.0005RH= .6863
 FV= 48.0700FV3= 66.0010RF= .7283
 CYCLE NO. = 9

N	FV	H	X	VMV	ACHV	T
1	29.1218	12.8520	350768	.644456	.86323	.4412764
2	35.0357	14.0779	351659	.644449	-.10344	.4426597
3	41.0908	15.3039	352551	.644442	.01902	.4440430
4	49.4784	16.5298	353442	.644492	.01898	.4454263
5	57.5680	17.7558	354334	.644485	-.00874	.4468096
6	65.9138	18.9818	355226	.644477	-.00519	.4481929
7	74.2615	20.2077	356117	.644470	-.00517	.4495762
8	82.3589	21.4337	357009	.644463	-.00520	.4509595
9	89.9593	22.6596	357900	.644456	-.00522	.4523428
10	96.8317	23.8856	358792	.644449	-.00870	.4537261
11	102.7669	25.1115	359683	.644442	.01902	.4551094
12	107.5855	26.3375	360575	.644492	.01898	.4564927
13	111.1406	27.5634	361466	.644485	-.00874	.4578760
14	113.3242	28.7894	362358	.644477	-.00519	.4592593
15	114.0699	30.0153	363250	.644470	-.00517	.4606426
16	113.3552	31.2413	364141	.644463	-.00520	.4620259
17	111.2017	32.4672	365033	.644456	-.00522	.4634092
18	107.6748	33.6932	365924	.644449	-.00870	.4647925
19	102.8817	34.9191	366816	.644442	1.67672	.4661759
20	96.9685	36.1451	367707	.644492	-21.41341	.4675592
21	90.1139	37.3710	368599	.616966	33.13148	.4689425
22	82.5267	38.5970	369491	.780115	227.96082	.4703258
23	74.4373	37.2187	370383	1.179438	236.88936	.4717091
24	66.0923	33.0462	372678	1.360051	38.35146	.4730924
25	57.7438	28.8719	374517	1.324027	-33.77521	.4744757
26	49.6462	24.8231	376334	1.300186	-21.12802	.4758590
27	42.0454	21.0227	378107	1.261195	-33.43631	.4772423
28	35.1725	17.5862	379817	1.208127	-43.09171	.4786256
29	29.2366	14.6183	381444	1.142606	-51.34109	.4800089
30	24.4172	12.2086	382973	1.066805	-58.00412	.4813922
31	20.8612	10.4306	384392	.982894	-63.02910	.4827755
32	18.6767	9.3384	385690	.893312	-65.75903	.4841588
33	17.9301	8.9650	386662	.800962	-71.45772	.4855421
34	18.6438	9.3219	387906	.702972	-59.40591	.4869254

35	40,7964	10,3982	388824	646388	19,73616	4883087
36	24,3225	11,6242	389716	642869	2,58644	4896920
37	29,1144	12,8501	390607	644445	89831	4910753

HS= 22,6495 HSS= 33,0005RH= 6863
 FVV= 48,0700FVS= 66,0010RF= 7283
 CYCLE NO.= 10
 STOP

EXECUTE TIME = 0002.37

APPENDIX II

Examples to Illustrate the Use of the Graphical

Method to Predict the Ratio H_{vs}/H_{ss} Example 1

Consider the following parameters for the vibratory linear shear apparatus: static weight $F_{vs} = 70$ lb (311.4 N), frequency of excitation $f = 14$ Hz, drawbar speed $V_{sh} = 1.2$ in./s (30.5 mm/s) and the amplitude of vertical exciting force $F_{vv} = 60$ lb (266.9 N). The properties of the compacted soil are:

$$\tan \phi_{\text{critical}} = 0.5 \quad \alpha = 1100 \text{ lb/in. (192.6 N/mm)}$$

to evaluate H_{vs}/H_{ss} the following steps may be used:

a- Use equation 6.11. to evaluate ψ

$$\psi = \frac{K_s V_{sh}}{2\pi f F_{vv} \tan \phi_{\text{critical}}} = 0.5$$

Point 1 on Fig. A 1. represents $\psi = 0.5$

- b- Use $\lambda - \psi$ curve on Fig. A 1. to obtain λ (follow points 1 - 2 - 3) this will give $\lambda = 0.2$
- c- Calculate the ratio F_{vv}/F_{vs} , point 4 on Fig. A 1. represents

$$\frac{F_{vv}}{F_{vs}} = \frac{60}{70} = 0.857$$

- d- Use H_{vs}/H_{ss} vs F_{vv}/F_{vs} line (for $\lambda = 0.2$) to obtain H_{vs}/H_{ss} (follow points 4 - 5 - 6) this will give

$$\frac{H_{vs}}{H_{ss}} = 0.84$$

Example 2

In Example 1, if the drawbar speed $V_{sh} = 0.35$ in./s (8.89 mm/s) and the amplitude of vertical exciting force $F_{vv} = 42.1b$ (186.8 N), calculate the ratio H_{vs}/H_{ss}

The following steps may be used:

- a- Use equation 6.11. to evaluate ψ

$$\psi = \frac{K_s V_{sh}}{2\pi f F_{vv} \tan \phi_{critical}} = 0.21$$

point 7 on Fig. A 1. represents $\psi = 0.21$

- b- Use $\lambda - \psi$ curve on Fig. A 1. to obtain λ (follow points 7 - 8 - 9) this will give $\lambda = 0.55$

- c- Connect a line between point 0 and point 9, this will give H_{vs}/H_{ss} vs F_{vv}/F_{vs} line for $\lambda = 0.55$

- d- Calculate the ratio F_{vv}/F_{vs} , point 10 on Fig. A 1. represents

$$\frac{F_{vv}}{F_{vs}} = \frac{42}{70} = 0.6$$

- e- Use the curve H_{vs}/H_{ss} vs F_{vv}/F_{vs} line (for $\lambda = 0.55$) to obtain H_{vs}/H_{ss} (follow points 10 - 11 - 12) this will give

$$\frac{H_{vs}}{H_{ss}} = 0.675$$

Fig 1.1.a

VIBRATION EXCITED BY SURFACE IRREGULARITY

F-1

Fig 1.1.b **VIBRATORY LOCOMOTION**

Fig 1.1.

EFFECT OF VIBRATION ON VEHICLE THRUST

VIBRATION

Fig 1.2.a

VIBRATORY ROLLER

VIBRATION

Fig 1.2.b

VIBRATORY PLATE

Fig 1.2

VIBRATORY SOIL COMPACTION

Fig 1.3

VIBRATORY SOIL CUTTING

VIBRATION

Fig 1.4

VIBRATORY PILE DRIVING

Fig 1.5 THE VIBRATORY
 LINEAR SHEAR APPARATUS

Fig 2.2 Results of Simple Shear Tests on 1 mm steel balls with 20 psi normal pressure (after Roscoe, Schofield, and North)

curve 1 for dense sand $e_0 = 0.562$

curve 2 for loose sand $e_0 = 0.652$

Sample size - 3" square (76.2 mm)

Initial thickness - 0.41" (10.41 mm)

Speed of shearing displacement - 0.055" per min
 (1.4 mm/min)

Sample oven-dry

Fig. 2.3. Plots of Typical Direct Shear Tests (Gamm standard sand)

Fig 2.4 Effect of vibration on the shear behaviour of steel balls at 20 psi normal pressure (after Youd 1970)

Fig. 3.1. Response of Coulomb Frictional Model with Rigid Drawbar to Rectangular Periodic Loading

Fig. 3.3. Force and Displacement-Time Relationships for an Instrumented Soil Shearing Element Subjected to a Rigid Drawbar Under Periodic Loading of Rectangular Shape

Fig. 3.4. Response of Coulomb frictional model with elastic dashpot to rectangular periodic loading

Reference line for F_f and F_r ($F_f = F_r = 0$)

Vertical forces F_f and F_r
(Difference of values due to eccentric application of the additional vertical forces)

Shearing force

1.74 lb/s
(7.74 N/s)

Reference line for $H = 0$

Time

Start

End

Fig. 3.5. Measured Response of the Linear Shear Apparatus with Elastic Drawbar to Sudden Application or Release of Vertical Force During Shearing Ottawa Sand

Fig. 3.6. Response of Vibratory Soil Engaging Element with Elastic Drawbar to Rectangular Periodic Loading

Fig. 3.7. Response of Coulomb Frictioned Spring with Elastic Damper to Triangular Variable Loading Form of Step Coulomb Spring

Fig. 3.8. Response of Vibratory Soil Engaging Element with Elastic Drawbar to Triangular Periodic Loading (case of high drawbar speed)

Fig. 1.9.

Response of Constant Frictional Model with Elastic Damper to Triangular Shearing Loading (case of low damper ratio)

Fig. 3.10. Effect of Triangular Periodic Loading on the Shearing Force of:
 (a) Coulomb frictional model with elastic drawbar ($\delta - K V_{sh}$)
 (b) Vibratory Soil Engaging Element with rigid drawbar ($\delta - c V_{sh}$)
 (c) Vibratory Soil Engaging Element with elastic drawbar ($\delta - K_s V_{sh}$)

Fig. 3.11. Comparison Between the Response of the Coulomb Frictional Model to Triangular, Harmonic and Rectangular Loadings

Fig. 3.13. Effect of Harmonic Periodic Loading on the Dynamic Shearing Force of:
 (a) Coulomb frictional model with elastic drawbar ($\beta=KVs$)
 (b) Vibratory soil engaging element with rigid drawbar ($\beta=\alpha Vs$)
 (c) Vibratory soil engaging element with elastic drawbar ($\beta=KVs$)

Fig 4.1. Simplified Illustration for the mechanism of the measuring device

Fig 4.2. Simplified Illustration for the two main frames of the measuring device

Fig 4.4. Three hinged frame for torsional resistance

Fig 4.3 Details of the eccentric shaft

- 1 & 2 Vertical transducers F_f & F_r
 3 Horizontal transducer H
 4 Frictional plate
 5 Guiding plate
 6 Frame I
 7 Frame II
 8 & 9 Three hinged frames
 10, 11 & 12 End bearings on centric shafts
 13, 14 & 15 End bearings on eccentric shafts

Fig. 4.5. Details of the Apparatus for Measuring Normal and Shear Forces, Beneath a Vibrating Plate

Fig 4.6 Electronic circuit for blocking out the D.C. Voltage of the Vibration Pick-up Preamplifiers and for adjusting the scale of recording

Fig 4.7. Electronic circuit for adding the signals of the two vertical transducers

**Fig 4.8 Apparatus for Measuring Normal and
Shear Forces Beneath a Vibrating
Plate**

**Fig 4.9 General View of Modified Vibratory
-Linear Shear Apparatus**

Fig 4.10 Static Calibration Loading Condition (1).

Fig 4.11. Static Calibration Loading Condition (2)

Fig. 4.12 Results of static calibration test.
Loading condition (1)

Reference line for F_f & F_r
 $(F_f = F_r = 0)$

Electric signal was introduced here
 to show the presence of the two ver-
 tical transducers signals

Reading of F_f & F_r

Reading of H

Reference line for H
 $(H = 0)$

Time

Start

End

Fig 4.13 Results of static calibration test -
 Loading Condition (2)

Fig 4.14. Loading condition for calibration under vertical dynamic loading

Reference line, for F_r (F_f is shifted to be differentiated from F_r) ($F_r = 0$)

Fig 4.15. Results of horizontal force calibration, Apparatus subjected to vertical exciting force of low frequency and large amplitude

Fig. 4.15. (cont.) Results of horizontal force calibration, Apparatus subjected to vertical exciting force of low frequency and large amplitude

Fig 4.16. Response of the apparatus to vertical dynamic force with different frequencies.
Apparatus is not statically or dynamically balanced

Reference line for $F_r=0$ (F_f is shifted to be differentiated from F_r)

Fig 4.17 Response of the apparatus to vertical dynamic force with high frequency. Apparatus is statically and dynamically balanced

- 1 Response of horizontal velocity electric network
- 2 Response of apparatus to vertical vibration on sand base
- 3 Response of apparatus to vertical vibration on rigid base

Fig 4.10. Frequency response curves of the apparatus and electronic circuits

Fig 4.29. Simplified illustration for the loading condition of the Double Parallelogram connected to the upper plate of frame I

Fig 4.20. Instrumentation for the calibration of the Accelerometers in the Structural Laboratory of the National Research Council in Ottawa

Fig 4.21. Method of measuring the frequency response of the velocity electronic circuit.

Fig 4.22. Method of measuring the Phase Shift between force and velocity circuits

Scale 1 in = 20 lb (1 mm=3.5 N) for F
1 in = 10 lb (1 mm=1.75 N) for H

Time

End

Direction of time increase

Start

Fig. 5.1. Sample of the recording
 $F_{vs} = 63.97 \text{ lb (284.6 N)}$ $F_{vv} = 45 \text{ lb (200N)}$ $F = 10.26 \text{ Hz}$
 $V_{sh} = 0.8 \text{ in./s (20.3 mm/s)}$ Plate No 2
(Fig is reduced in the reproduction)

Fig 5.1. (cont.) Sample of recording

Fig 5.1. (cont.) Sample of recording

Vertical force increased by 20 lb (89 N)
Initial condition : loose sand

Shearing force

$\beta = 46 \text{ lb/s (204.6 N/s)}$
 $K_s = 1260.0 \text{ lb/in (220 N/mm)}$
 $\alpha = 1618.0 \text{ lb/in (283 N/mm)}$

Reference line for H=0

Time

End

Start

Shearing force

Vertical force increased by 30 lb (133 N)
Initial condition : dense sand

Fig 5.22 : Response of the Vibratory Linear Shear
Apparatus with rigid drawbar to sudden
application of vertical force during the
shearing of Ottawa sand

$\beta = 40.0 \text{ lb/s (178 N/s)}$
 $K_s = 1102.0 \text{ lb/in (193 N/mm)}$
 $\alpha = 1367.0 \text{ lb/in (239 N/mm)}$

Reference line for H=0

Time

End

Start

Shearing force

Vertical force increased by 30 lb (133 N)
Initial condition : loose sand

Summary :
Average value of $\alpha = 1378.0 \text{ lb/in (241 N/mm)}$
Average value of $K_s = 1106.0 \text{ lb/in (194 N/mm)}$

$\beta = 34.0 \text{ lb/s (156 N/s)}$
 $K_s = 957.0 \text{ lb/in (168 N/mm)}$
 $\alpha = 1150.0 \text{ lb/in (201 N/mm)}$

Reference line for H=0

Time

End

Start

Fig. 5.3. Frequency Diagram of the Values of K_s Measured from 50 Samples of Dynamic Shearing Force Curves

Fig. 5.4. Method of Measuring Horizontal Displacement for the Evaluation of K_s

Fig. 5.6. Shearing Force-Displacement Curves Resulting from Forward and Backward Shearing $V_{sh} = 0.04$ and 1.5 in./s (1.016 and 38.1 mm/s)

$$F_{vs} = 62 \text{ lb (275.8 N)}$$

Fig 5.7. Effect of amplitude of exciting force
 $f = 10 \text{ Hz}$ $V_{sh} = 0.239 \text{ in/s}$ (6 mm/s)
 $F_{vs} = 67 \text{ lb}$ Plate No 1
 (258 N)

(a) $F_{vv}/F_{vs} = 0.385$

(b) $F_{vv}/F_{vs} = 0.489$

(c) $F_{vv}/F_{vs} = 0.728$

Scale:

- 1 in=20 lb (1 mm=3.5 N) for F
- 1 in=10 lb (1 mm=1.75 N) for H
- 1 in=0.05 s (1 mm=0.002 s)
- 1 in=0.5 in/s (1 mm=0.5 mm/s)

Fig 5.8. Effect of amplitude of exciting force

(a) $F_{vv}/F_{vs} = 0.201$

Scale:

- 1 in=20 lb (1 mm=3.5 N) for F
- 1 in=10 lb (1 mm=1.75 N) for H
- 1 in=0.05 s (1 mm=0.002 s)
- 1 in=0.5 in/s (1 mm=0.5 mm/s)

(b) $F_{vv}/F_{vs} = 0.423$

(c) $F_{vv}/F_{vs} = 0.534$

Plate No. 1

$f = 15 \text{ Hz}$ $V_{sh} = 0.6 \text{ in/s}$ $F_{vs} \approx 67 \text{ lb}$ (15.24 mm/s) (298 N)

Time Reference line

(H = F = 0)

Fig 5.9. Effect of amplitude of exciting force

(a) $F_{VV}/F_{VS} = 0.329$

(b) $F_{VV}/F_{VS} = 0.435$

(c) $F_{VV}/F_{VS} = 0.81$

Scale:

- 1 in=20 lb (1 mm=3.5 N) for F
- 1 in=10 lb (1 mm=1.75 N) for H
- 1 in=0.05 s (1 mm=0.002 s)
- 1 in=0.5 in/s (1 mm=0.5 mm/s)

(25.4 mm/s) (298 N) Plate No. 1
 $F_{VS} = 1.0 \text{ in/s}$ $F_{VS} = 67 \text{ lb}$

$f = 10 \text{ Hz}$

Time Reference Line

($H = F = 0$)

Fig 5.10. Effect of amplitude of exciting force

Fig 5.11. Effect of Amplitude of Exciting Force

Fig 5.12. Effect of Amplitude of Exciting Force

(c) $F_{VV}/F_{VS} = 0.62$ (b) $F_{VV}/F_{VS} = 0.52$ (a) $F_{VV}/F_{VS} = 0.27$ Scale:

- 1 in=20 lb (1 mm=3.5 N) for F
- 1 in=10 lb (1 mm=1.75 N) for H
- 1 in=0.05 s (1 mm=0.002 s)
- 1 in=0.5 in/s (1 mm=0.5 mm/s)

Fig 5.13. Effect of Amplitude of Exciting Force

(a) $F_{VV}/F_{VS} = 0.197$

1 in=20 lb (1 mm=3.5 N) for F
 1 in=10 lb (1 mm=1.75 N) for H
 1 in=0.05 s (1 mm=0.002 s)
 1 in=0.5 in/s (1 mm=0.5 mm/s)

Scale:

(b) $F_{VV}/F_{VS} = 0.30$

$f = 30 \text{ Hz}$
 $V_{sh} = 1.0 \text{ in/s}$ (25.4 mm/s)
 $F_{VS} = 66 \text{ lb}$ (293.6 N)
 Plate No 1

(c) $F_{VV}/F_{VS} = 0.69$

Fig 5.14. Effect of Amplitude of Exciting Force

(a) $F_{vv}/F_{vs} = 0.672$

Scale:

- 1 in=20 lb (1 mm=3.5 N) for F
- 1 in=10 lb (1 mm=1.75 N) for H
- 1 in=0.05 s (1 mm=0.002 s)
- 1 in=0.5 in/s (1 mm=0.5 mm/s)

(b) $F_{vv}/F_{vs} = 0.781$

- f = 10 Hz
- V_{sh} = 0.8 in/s (20.32 mm/s)
- F_{vs} = 64 lb (284.7 N)

Plate No 2

(c) $F_{vv}/F_{vs} = 0.851$

Fig 5.15. Effect of Amplitude of Exciting Force

Time Reference Line ($H = F = 0$)

Time

theoretical ———

measured X $f = 10 \text{ Hz}$
O $f = 15 \text{ Hz}$

Fig. 5.16. Effect of Amplitude of Exciting Force and Frequency
 $F_{ss} = 67 \text{ lb (298 N)}$ $V_{ss} = 0.289 \text{ in/s (5 mm/s)}$
 $K_s = 1100 \text{ lb/in (200.5 N/mm)}$
 Plate No. 1

theoretical — — — — — measured

$\times$ $f = 10 \text{ Hz}$
 $\circ$ $f = 15 \text{ Hz}$
 Δ $f = 20 \text{ Hz}$

Fig. 5.27.

Effect of magnitude of exciting force
 on the ratio H_{vs}/H_{ss} for F_{vs}/F_{ss} from 0.2 to 1.0
 for $f = 10, 15, 20 \text{ Hz}$
 (See also Fig. 5.26)

theoretical ——— measured

$\times$ $f = 10$ Hz
 $\circ$ $f = 15$ Hz
 Δ $f = 20$ Hz
 $+$ $f = 25$ Hz
 $*$ $f = 30$ Hz

Fig. 5.10. Effect of Amplitude of Exciting Force and Frequency

$P_{ss} = 67$ lb (298 N) $\dot{\gamma}_{ss} = 5.0$ in/s (25.4 mm/s)

$E_s = 1.25 \times 10^6$ lb/in² (129.6 MPa)

Table No. 1

theoretical — measured

- $\times$ $V_{sh} = 1.0$ in/s (25.4 mm/s), Plate 1, $F_{vs} = 67$ lb (298 N)
 $\dagger$ $V_{sh} = 0.8$ in/s (20.32 mm/s), Plate 2, $F_{vs} = 64$ lb (284.7 N)
 $\circ$ $V_{sh} = 0.6$ in/s (15.24 mm/s), Plate 1, $F_{vs} = 67$ lb (298 N)
 Δ $V_{sh} = 0.239$ in/s (6 mm/s), Plate 1, $F_{vs} = 67$ lb (298 N)

Fig. 5.19. Effect of Shearing Force and Shear Speed
 $F = 40$ lb $\sigma_s = 1200$ lb/in (192.5 MPa)

theoretical ———— measured

- X $V_{sh} = 1.0 \text{ in/s (25.4 mm/s)}$
- O $V_{sh} = 0.6 \text{ in/s (15.24 mm/s)}$
- Δ $V_{sh} = 0.239 \text{ in/s (6.07 mm/s)}$

Fig. 8.22. Effect of Slitting Depth and Slitter Speed
 $V_{sh} = 1.0 \text{ in/s (25.4 mm/s)}$
 $V_{sh} = 0.6 \text{ in/s (15.24 mm/s)}$
 $V_{sh} = 0.239 \text{ in/s (6.07 mm/s)}$

(a) $V_{sh} = 0.194$ in/s (4.93 mm/s) (b) $V_{sh} = 0.305$ in/s (7.75 mm/s) (c) $V_{sh} = 0.91$ in/s (23.11 mm/s)

1 in=20 lb (1 mm=3.5 N) for F
 1 in=10 lb (1 mm=1.75 N) for H
 1 in=0.05 s (1 mm=0.002 s)
 1 in=0.5 in/s (1 mm=0.5 mm/s)

$f = 10$ Hz
 $F_{vs} = 68$ lb (302.5 N)

Plate No 1

Fig 5.21. Effect of Draw bar Speed ($f=10$ Hz)

Time Reference Line ($H=f=0$)

(c) $V_{sh} = 1.0 \text{ in/s} (25.4 \text{ mm/s})$

$f = 15 \text{ Hz}$

Plate No. 1

$F_{vv}/F_{vs} = 0.5$

$F_{vs} = 67 \text{ lb}^2$

(298 N)

(b) $V_{sh} = 0.8 \text{ in/s} (20.32 \text{ mm/s})$

$F_{vv}/F_{vs} = 0.5$

$F_{vs} = 67 \text{ lb}^2$

(298 N)

(a) $V_{sh} = 0.6 \text{ in/s} (15.24 \text{ mm/s})$

$f = 15 \text{ Hz}$

Plate No. 1

$F_{vv}/F_{vs} = 0.5$

$F_{vs} = 67 \text{ lb}^2$

(298 N)

Fig 5.22. Effect of Drawbar Speed ($f=15 \text{ Hz}$)

Time, Reference Line ($H=F=0$)

Time

(a) $V_{sh} = 0.239 \text{ in/s (6 mm/s)}$

$F_{vv}/F_{vs} = 0.36$
 $f = 20 \text{ Hz}$
 $F_{vs} = 66 \text{ lb (293.8 N)}$
 Plate No 1

(b) $V_{sh} = 0.6 \text{ in/s (15.24 mm/s)}$

Scale:

- 1 in=20 lb (1 mm=3.5 N) for F
- 1 in=10 lb (1 mm=1.75 N) for H
- 1 in=0.05 s (1 mm=0.002 s)
- 1 in=0.5 in/s (1 mm=0.5 mm/s)

(c) $V_{sh} = 1.0 \text{ in/s (25.4 mm/s)}$

Time Reference Line ($H=F=0$)

Fig 5.23. Effect of Draw bar Speed ($f=20 \text{ Hz}$)

- x $f = 10$ Hz
- o $f = 15$ Hz
- Δ $f = 20$ Hz

Fig. 2.26. Effect of draw bar speed and frequency
 $F_{ss} = 67.10$ $F_{vs}/F_{ss} = 0.25$ $t_s = 1000$ to 10000
 (mm/s) (mm/s) (mm/s) (mm/s)
 Phase No. 1

× $f = 10 \text{ Hz}$

○ $f = 15 \text{ Hz}$

Fig. 3-22. Effect of draw bar speed and frequency

$F_{vs} = 65 \text{ lbs}$

$F_{vv}/F_{vs} = 0.5$

$L = 1100 \text{ in}$

$\mu = 0.005$

$\sigma = 100,000 \text{ psi}$

theoretical ——— measured

$\times$ $F_{VV}/F_{VS}=0.24$ Plate No 1
 $\circ$ $F_{VV}/F_{VS}=0.53$ Plate No 1
 $\triangle$ $F_{VV}/F_{VS}=0.90$ Plate No 2

Fig. 5.25. Effect of drop bar length and magnitude of cutting force

$F_{VS} = 10 \text{ N}$ $F_{VV} = 10 \text{ N}$ $F_{VS} = 10 \text{ N}$
 $F_{VS} = 10 \text{ N}$ $F_{VV} = 10 \text{ N}$ $F_{VS} = 10 \text{ N}$

theoretical ————— measured

$\times$ $F_{vv}/F_{vs} = 0.27$
 $\triangle$ $F_{vv}/F_{vs} = 0.41$
 $\odot$ $F_{vv}/F_{vs} = 0.54$

4

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4

Abstract

$$F_{ww}/F_{vs} = 0.26$$
$$F_{WV}/E_{VS} = 0.40$$

Fig 5.29. Effect of Frequency of Excitation

(a) $f = 9.76 \text{ Hz}$

(b) $f = 11.91 \text{ Hz}$

(c) $f = 28.17 \text{ Hz}$

Scale:

- 1 in=20 lb (1 mm=3.5 N) for F
- 1 in=10 lb (1 mm=1.75 N) for H
- 1 in=0.05 s (1 mm=0.002 s)
- 1 in=0.5 in/s (1 mm=0.5 mm/s)

$$F_{VV}/F_{VS} = 0.9$$

$$V_{sh} = 0.8 \text{ in/s}$$

$$F_{VS} = 63 \text{ lb}(280.2 \text{ N})$$

Plate No 2

Fig 5.30. Effect of Frequency of Excitation

Fig 3.21. Effect of Frequency
 $f_{max} = 25 \text{ Hz}$ $f_{min} = 0 \text{ Hz}$ $f_{avg} = 12.5 \text{ Hz}$
 $\mu = 1.0 \text{ mm}$ $\lambda = 1.0 \text{ mm}$
 $\mu = 1.0 \text{ mm}$ $\lambda = 1.0 \text{ mm}$

theoretical

measured

●

P-69

$F_{VV}/F_{VS} = 0.55$

□

$F_{VV}/F_{VS} = 0.80$

Fig. 1.12. Effect of Frequency and Amplitude of

Exciting Force

$F_{VS} = 62 \text{ lb (274.2 N)}$ $F_{VV} = 0.8 \text{ lb (3.57 N)}$

$F_{VS} = 1100 \text{ lb (488.4 N)}$

theoretical measured Plate No. 1 Plate No. 2 Plate No. 3

Fig 3.32. Effect of Plate Shape and Frequency
 $P_{ss} = 61.5 \text{ lb (273.0 N)}$ $F_{vs}/F_{ss} = 0.25$
 $V_{ss} = 6.0 \text{ in/s (20.32 mm/s)}$

(a) $F_{vs} = 62.27 \text{ lb}$ (277 N).(b) $F_{vs} = 88.61 \text{ lb}$ (394.2 N).

Scale: 1 in=20 lb (1 mm=3.5 N) for F
 1 in=10 lb (1 mm=1.75 N) for H
 1 in=0.05 s (1 mm=0.002 s)
 1 in=0.5 in/s (1 mm=0.5 mm/s)

$$F_{vv}/F_{vs} = 0.65$$

$$V_{sh} = 0.8 \text{ in/s} (20.32 \text{ mm/s})$$

$$f = 14.29 \text{ Hz Plate No 2}$$

Reference Line (F=0)

Fig 5.34. Effect of Static Normal Force.

theoretical ——— measured

② $F_{vs} = 62$ lb (275.0 N) $F_{vs}/F_{vs0} = 0.55$ ③ $F_{vs} = 99$ lb (440.3 N) $F_{vs}/F_{vs0} = 0.65$

Fig 5.18. Effect of Static Normal Force and Frequency
 $V_{sh} = 0.8$ in/s (20.32 cm/s)

$E_s = 1100$ lb/in (122.5 N/mm)

Figure No. 2

Fig 6.1. Illustration of the relation between the area under the shearing force curve and the average value of shearing force

Fig 6.2. Typical surfaces used for measuring the areas A_1 and A_2

$$\psi = \frac{V_{sh} K_s}{2 \pi F_{vw} \tan \phi_{cr}}$$

$$\frac{H_{vs}}{H_{ss}} = 1 - \lambda \frac{F_{vv}}{F_{vs}}$$

Fig. 2.4. Graphical Nomogram for the evaluation of H_{vs}/H_{ss} vs F_{vv}/F_{vs}

Fig. 6.5. Performance Curves for the Vibratory Plate Compactors (1) and (2) Prescribed by the Curves U_{vh}/U_s , U_{vv}/U_s , and U_{vv}/U_{vh}

Compactor (1) $P_{ss} = 250$ lb (113.6 N) $F = 20$ Hz
 $U_{vh} = 25$ in/sec (635 mm/sec) $U_{vv} = 10$ in/sec (254 mm/sec)

Compactor (2) $P_{ss} = 1075$ lb (4811 N) $F = 20$ Hz
 $U_{vh} = 25$ in/sec (635 mm/sec) $U_{vv} = 10$ in/sec (254 mm/sec)

Fig. 6.6. Performance Curves for the Vibratory Plate Compactor (1) Presented by the Curves U_{vh}/U_s , U_{vv}/U_s and U_{vv}/U_{vh} vs f

Compactor (1) $F_{vs}=860$ lb (3826 N) $V_{sh}=10$ in./s (254 mm/s)

$F_{vv}/F_{vs}=1.0$

Compactor (1):

 $F_{vs} = 860 \text{ lb (3826 N)}$ $f = 20 \text{ Hz}$ $F_{vv}/F_{vs} = 1.0$

$\text{————— } U_{vh}/U_s \text{ or } H_{vs}/H_{ss}$
 $\text{————— } U_{vv}/U_s$
 $\text{----- } U_{vv}/U_{vh}$
 $\text{----- } U_v/U_s$

Fig. 6.7. Performance Curves for the Vibratory Plate Compactor (1) Presented by the Curves U_{vh}/U_s , U_{vv}/U_s , U_{vv}/U_{vh} and U_v/U_s vs V_{sh}

Fig. 6.8. Multipeak Sinusoidal Pressure Distribution Under a Rigid Track

Fig. 6.9. Shear Stress as Function of time for a Soil Particle Under the Track

Fig. 6.10. Shear Stress Distribution Under the Track

Fig. 6.12. Effect of λ Slip on Gross Tractive Performance of Tracked Vehicle
 $W=2950 \text{ lb (13.12 kN)}$ $l=50 \text{ in (1270 mm)}$
 $a_1=4$ $b=10 \text{ in (254 mm)}$ $d=10 \text{ in (254 mm)}$
 $\phi=33 \text{ degrees}$

Fig. 8-1. Example for the use of the

END

12 · 12 · 80

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