1

National Library
of Canada

Bibliothèque nationale
du Canada

Canadian Theses Service

Service des thèses canadiennes

Ottawa, Canada
K1A 0N4

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Previously copyrighted materials (journal articles, published tests, etc.) are not filmed.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

Les documents qui font déjà l'objet d'un droit d'auteur (articles de revue, tests publiés, etc.) ne sont pas microfilmés.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30.

EFFECTS OF PULSE SHAPING ON THE PERFORMANCE
OF MANCHESTER CODED DIGITAL FM
WITH DISCRIMINATOR DETECTION

by

CHARLES L.B. DESPINS (B.Eng.)

A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfilment of the requirements
for the degree of
Master of Engineering

Ottawa-Carleton Institute for Electrical Engineering
Faculty of Engineering
Department of Systems and Computer Engineering
Carleton University

July 1987

© copyright

1987, Charles L.B. Despins

Permission has been granted to the National Library of Canada to microfilm this thesis and to lend or sell copies of the film.

The author (copyright owner) has reserved other publication rights, and neither the thesis nor extensive extracts from it may be printed or otherwise reproduced without his/her written permission.

L'autorisation a été accordée à la Bibliothèque nationale du Canada de microfilmer cette thèse et de prêter ou de vendre des exemplaires du film.

L'auteur (titulaire du droit d'auteur) se réserve les autres droits de publication; ni la thèse ni de longs extraits de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation écrite.

ISBN 0-315-39419-6

The undersigned hereby recommends to the Faculty of Graduate
Studies and Research acceptance of the thesis,

"EFFECTS OF PULSE SHAPING ON THE PERFORMANCE OF
OF MANCHESTER CODED DIGITAL FM
WITH DISCRIMINATOR DETECTION"

submitted by Charles L.B. Despins, B.Eng., in partial fulfilment of
the requirements for the degree of Master of Engineering.

Dr. M. El-Tanany
Thesis Co-Supervisor

Prof. S.A. Mahmoud
Thesis Co-Supervisor
Chairman
Department of Systems
and Computer Engineering

Department of Systems and Computer Engineering
Faculty of Engineering
Carleton University
August 1987

ABSTRACT

The development of low rate (9.6 - 16 kb/s) speech coding devices, combined with the simplicity and availability of data encryption devices make it possible to transmit secure digitized speech over mobile radio channels. Manchester coded digital FM (MCDFM) with discriminator detection is a strong candidate for such an application in view of its robustness in the fading environment, its simple timing recovery and its compatibility with existing mobile radio transceivers. The only disadvantage of the MCDFM signal is that its power spectral density for modulation indices that are practically useful is too wide for data rates in the aforementioned range.

This thesis presents a comprehensive investigation of MCDFM performance in terms of the critical signal parameters (e.g. modulation index, pulse shapes) as well as the receive IF filter response which is considered the most crucial in the receiver hardware. This investigation is targeted at identifying the signal parameters and filter responses that yield nearly optimal performance under specific bandwidth and data rate constraints. Due to the nonlinearity inherent in all FM systems, the optimization procedure is mostly numerical in nature. For this purpose, two highly structured, numerically efficient and versatile software packages are developed. These are based on some existing algorithms in addition to analyses conducted as part of this work. It is shown that, for a data rate of 16 kb/s, the introduction of pulse shaping in a MCDFM system in order to satisfy a constraint of 99 percent signal power within a 25 kHz bandwidth yields a performance improvement of up to 3.75 dB over reduction of frequency deviation.

ACKNOWLEDGEMENTS

The support and guidance provided by Prof. S.A. Mahmoud and Dr. M. El-Tanany in the preparation of this thesis are gratefully acknowledged. As well, the author would like to thank the Natural Sciences and Engineering Research Council of Canada (NSERC) for its financial support.

Table of Contents

1	Introduction	1
1.1	Problem Description	1
1.2	Proposed Approach	4
1.3	Thesis Contribution	5
1.4	Thesis structure	6
2	System Description	8
2.1	Introduction	8
2.2	Digital FM Transmitter	8
2.2.1	Baseband Encoder	10
2.2.2	Premodulation Filter	11
2.2.3	Frequency Modulator	19
2.3	Channel	25
2.4	Digital FM Receiver	27
2.4.1	IF Filter	28
2.4.2	Limiter-Discriminator	31
2.4.3	Symbol timing recovery and MC-to-NRZ conversion	33
2.4.4	Integrate-and-dump Filter	34
2.5	Summary	37
3	Band Occupancy Analysis	39
3.1	Introduction	39
3.2	Background	39
3.3	Program Algorithm	44
3.4	Test Results	48
3.5	Summary	50

4 Bit Error Rate Analysis	56
4.1 Introduction	56
4.2 Signal Model and Distortion Analysis	56
4.3 Noise Statistics and BER Calculation	60
4.4 Summary of Assumptions and Restrictions	69
4.5 Program Algorithm	70
4.6 Test Results	72
4.7 Summary	73
5 Performance Optimization	78
5.1 Introduction	78
5.2 Optimization Procedure	78
5.3 Optimization results (16 kb/sec)	82
5.3.1 Modulation Index Optimization	83
5.3.2 IF Bandwidth Optimization	84
5.3.3 Final Results	85
5.4 Optimization results (9.6 kb/sec)	86
5.4.1 Modulation Index Optimization	86
5.4.2 IF Bandwidth Optimization	87
5.4.3 Final Results	87
5.5 Summary and Discussion	88
6 Conclusions and Recommendations	108
6.1 Conclusions	108
6.2 Recommendations for Future Research	109
6.2.1 Enhanced Detection Schemes	109
6.2.2 Optimized IF filter characteristics	109
6.2.3 Extension of analysis to M-ary signal formats	110
6.2.4 Use of phase-lock loops	110

6.2.5 <i>Use of trellis coded baseband waveforms</i>	110
References	111

List of Figures

2.1	Digital FM system model	9
2.2	Baseband encoder impulse response and encoded data sequence for a) Manchester signal format b) NRZ signal format	11
2.3	Noncausal analysis of premodulation filter convolution	13
2.4	Gaussian pulse shapes a)NRZ, $BT=0.2$ b)NRZ, $BT=0.4$ c)NRZ, $BT=\infty$ d)Manchester, $BT=0.2$ e)Manchester, $BT=0.4$ f)Manchester, $BT=\infty$	15
2.5	Raised Cosine Spectrum	16
2.6	SRP pulse shapes a)NRZ, $\beta=0.1$ b)NRZ, $\beta=0.5$ c)NRZ, $\beta=1.0$ d)Manchester, $\beta=0.1$ e)Manchester, $\beta=1.0$ f)Manchester, $\beta=1.0$	17
2.7	RC pulse shapes a)NRZ, $L=1$ b)NRZ, $L=2$ c)Manchester, $L=1$ d)Manchester, $L=2$	18
2.8	Frequency and phase pulses a)NRZ coded (no shaping) b)NRZ coded (with shaping) c)Manchester coded (no shaping) d)Manchester coded (with shaping)	21
2.9	NRZ coded transmitter waveforms a)Frequency pulse sequence b)Baseband message c)Phase pulse sequence d)Transmitted data phase e)Sine of data phase f)Cosine of data phase	22
2.10	Manchester coded transmitter waveforms a)Frequency pulse sequence b)Baseband message c)Phase pulse sequence d)Transmitted data phase e)Sine of data phase f)Cosine of data phase	24

2.11 IF filter low-pass equivalent magnitude transfer functions	
1) Gaussian 2) Second-order Butterworth 3) Ideal "brick-wall"	32
2.12 MC-to-NRZ conversion process	
a) MC phase derivative b) Clock signal	
c) NRZ converted phase derivative d) I and D filter output	35
2.13 Integrate-and-dump circuit	36
3.1 Rectangular NRZ coded pulse	51
3.2 Power spectral density of NRZ coded FSK	
(frequency pulse as in figure 3.1).	51
3.3 Duobinary pulse	52
3.4 Power spectral density of duobinary FSK	
(frequency pulse as in figure 3.3).	52
3.5 Modified duobinary pulse	53
3.6 Power spectral density of modified duobinary FSK	
(frequency pulse as in figure 3.5).	53
3.7 Shaped pulse $r(t)$ and its level-quantized approximation	54
3.8 Power spectral density of duobinary FM	
(frequency pulse as in figure 3.7)	54
3.9 Manchester coded rectangular pulse	55
3.10 Power spectral density of Manchester coded FSK	
(frequency pulse as in figure 3.9)	55
4.1 IF filtered noise a) autocorrelation function	
b) low-pass equivalent power spectral density	61
4.2 Composite output noise density for three IF signal-to-noise ratios	64

4.3	Probability density functions of the phase noise difference for NRZ and Manchester coded signals (constant SNR assumed throughout a bit period)	67
4.4	Probability distribution functions of the phase noise difference for NRZ and Manchester coded signals (constant SNR assumed throughout a bit period)	68
4.5	Comparison of BER vs $\frac{E_b}{N_0}$ with results in [9]; NRZ encoding, $h=0.5$, no pulse shaping, Gaussian IF filter	74
4.6	Comparison of BER vs $\frac{E_b}{N_0}$ with results in [9]; NRZ encoding, $h=0.7$, no pulse shaping, Gaussian IF filter	74
4.7	Comparison of BER vs $\frac{E_b}{N_0}$ with results in [9]; NRZ encoding, $h=1.0$, no pulse shaping, Gaussian IF filter	75
4.8	Comparison of BER vs $\frac{E_b}{N_0}$ with results in [9]; NRZ encoding, $h=0.707$, no pulse shaping, Gaussian or Butterworth IF filter ($B_R T=1.0$)	75
4.9	Comparison of BER vs $\frac{E_b}{N_0}$ with results in [17]; NRZ encoding, $h=0.5$, Gaussian pulse shaping, Gaussian IF filter	76
4.10	Comparison of BER vs $\frac{E_b}{N_0}$ with results in [15]; Manchester encoding, $h=1.30$, no pulse shaping, Gaussian IF filter	76
4.11	Comparison of BER vs $\frac{E_b}{N_0}$ with results in [15]; Manchester encoding, no pulse shaping Gaussian IF filter	77
5.1	Low-pass equivalent representation of a 25 kHz wide channel. a)data rate = 16 kb/s b)data rate = 9.6 kb/s	79

5.2 Out-of-band radiation vs modulation index (fixed pulse shape and data rate)	80
5.3 IF filter bandwidth optimization for three IF filter types (premodulation filter fixed, $h = h_{opt}$, $\frac{E_b}{N_0} = 15$ dB)	81
5.4 Comparison of BER vs $\frac{E_b}{N_0}$ relationships for a given shaping filter with the optimum no shaping case (broken line)	82
5.5 Power spectral densities of Manchester coded digital FM signals with 1)no shaping, $h=0.30$ 2)1RC shaping, $h=0.40$ 3)Gaussian shaping ($B_T T=0.75$), $h=0.54$ 4)Gaussian shaping ($B_T T=0.50$) $h=0.85$	92
5.6 IF filter bandwidth optimization for Gaussian pulse shaping with $B_T T=0.75$ and $h=0.54$	95
5.7 IF filter bandwidth optimization for Gaussian pulse shaping with $B_T T=0.50$ and $h=0.85$	96
5.8 IF filter bandwidth optimization for 1RC pulse shaping with $h=0.40$	97
5.9 Optimized error rate performance for Gaussian pulse shaping with $B_T T=0.50$ and $h=0.85$. The IF filter responses are 1)Gaussian 2)Butterworth 3)Ideal. The broken line indicates the performance obtained with no shaping	98
5.10 Optimized error rate performance for Gaussian pulse shaping with $B_T T=0.75$ and $h=0.54$. The IF filter responses are 1)Gaussian 2)Butterworth 3)Ideal. The broken line indicates the performance obtained with no shaping	99

- 5.11 Optimized error rate performance for Raised Cosine shaping with $L=1$ and $h=0.40$. The IF filter responses are
 1) Gaussian 2) Butterworth 3) Ideal. The broken line indicates the performance obtained with no shaping 100
- 5.12 Power spectral density of MCDFM signals with 1) no shaping, $h=1.10$ 2) Gaussian shaping ($B_T T=2.0$), $h=1.17$ 3) Raised Cosine shaping with $L=1$ and $h=1.25$. In all three cases, $h = h_{opt}$ 103
- 5.13 Optimized error rate performance for Gaussian pulse shaping with $B_T T=2.0$ and $h=1.17$. The IF filter responses are
 1) Gaussian 2) Butterworth 3) Ideal. The broken line indicates the performance obtained with no shaping 104
- 5.14 Optimized error rate performance for Raised Cosine shaping with $L=1$ and $h=1.25$. The IF filter responses are
 1) Gaussian 2) Butterworth 3) Ideal. The broken line indicates the performance obtained with no shaping 105
- 5.15 Optimized error rate performance for Gaussian pulse shaping and a second-order Butterworth IF filter. The modulation indices and IF filter bandwidths are:
 1) $h=1.21$, $B_T T=1.75$, $B_R T=1.90$ 2) $h=1.25$, $B_T T=1.50$, $B_R T=1.80$
 3) $h=1.25$, $B_T T=1.00$, $B_R T=2.25$ 4) $h=1.25$, $B_T T=0.75$, $B_R T=2.25$
 5) $h=1.25$, $B_T T=0.50$, $B_R T=2.25$. The broken line indicates the performance obtained with no pulse shaping. 107

List of Tables

4.1 Grouping through symmetry and selection of outer bits of the 32 5-bit patterns	59
5.1 Values of frequency deviation from carrier (normalized to the data rate) where the out-of-band radiation equals -20 dB.	93
5.2 Modulation index optimization (16 kb/s)	94
5.3 IF bandwidth optimization (16 kb/s)	94
5.4 Performance optimization (16 kb/s)	101
5.5 Modulation index optimization (9.6 kb/s)	102
5.6 IF bandwidth optimization (9.6 kb/s)	102
5.7 Performance optimization (9.6 kb/s)	106

List of Appendices

A	Gaussian filter 3 dB bandwidth	115
B	The Q function and its series approximation	116
C	Low-pass representation of band-pass signals	118
D	Frequency Discriminator	120
E	FM threshold effect	125
F	SPECTRUM program listing	128
G	SPECTRUM sample output	138
H	BER program listing	142
I	BER sample output	157

List of Symbols

- a_i Input bit stream
- $a(t)$ Time-varying signal envelope
- $b(t)$ Binary impulse stream
- B Double-sided 3 dB-down bandwidth
- $B_T T$ Premodulation filter time-bandwidth product
- $B_R T$ IF filter time-bandwidth product
- $c(t)$ Clock signal
- $d(t)$ Baseband encoder output
- DFM* Digital frequency modulation
- $E[\]$ Expectation operator
- E_b Energy per bit
- f_c Carrier frequency (Hz)
- f_d Peak frequency deviation (single-sided)
- f_o IF filter center frequency
- $g(t)$ Phase pulse
- h Modulation index
- $h_T(t)$ Premodulation filter impulse response
- $h_R(t)$ IF filter low-pass equivalent impulse response
- $h^*(t)$ IF filter (band-pass) impulse response
- $\bar{h}_R(t)$ Complex envelope of IF filter response
- $h_{Rc}(t)$ In-phase component of $h_R(t)$
- $h_{Rs}(t)$ Quadrature component of $h_R(t)$

- IF* Intermediate frequency
- ISI* Intersymbol interference
- L* Raised Cosine pulse shape duration (number of bit periods)
- MC* Manchester
- m(t)* Premodulation filter output
- N* Number of clicks in one bit period
- $\bar{N}$ Average number of clicks in one bit period
- NRZ* Non-return-to-zero
- N_0 White Gaussian noise single-sided power spectral density
- O.B.R.* Out-of-band radiation
- p(t)* Baseband encoder impulse response
- RC* Raised Cosine
- r(t)* Frequency pulse
- R(t)* IF filtered time-varying envelope (signal + noise)
- SNR* Signal-to-noise ratio
- SRC* Spectrally Raised Cosine
- s(t)* Frequency modulator output
- T* Bit period
- u(t)* Signal component of IF filter input
- u_c(t)* In-phase component of *u(t)*
- u_s(t)* Quadrature component of *u(t)*
- v(t)* Signal component of IF filter output
- w(t)* Noise component of IF filter input
- x(t)* IF filter input
- z(t)* IF filter output

$\eta(t)$ Phase noise

ω_c Radian carrier frequency

ω_o Radian IF filter center frequency

$\phi(t)$ IF filtered signal phase

$\Phi(t)$ IF filtered phase (signal + noise)

$\dot{\Phi}(t)$ First derivative of IF filtered phase

$\Phi_c(t)$ MC-to-NRZ converted phase

$\Delta\phi$ Signal phase variation within a bit period

$\Delta\Phi(t)$ Integrate-and-Dump filter output

Ψ Sum of phase variation due to clicks and phase noise within a bit period

$\rho(t)$ Time-varying SNR at IF filter output

σ^2 Variance of $\xi(t)$

$\Theta(t)$ Carrier phase

$\theta(t)$ Transmitted data phase

$\xi(t)$ Noise component of IF filter output

$\xi_c(t)$ In-phase component of $\xi(t)$

$\xi_s(t)$ Quadrature component of $\xi(t)$

Chapter 1

INTRODUCTION

1.1 Problem Description

Over the past twenty-five years, the performance of digital FM (DFM) with discriminator detection has been extensively studied [1-15]. Recently, there has been renewed interest in such systems, due to the emergence of applications such as frequency-hopped spread-spectrum systems that employ digital FM as the information bearing modulation [19-20], and narrowband mobile and portable radio systems in the UHF band [21]. The latter are now experiencing consistent and rapid growth, and thus will be forced to accept progressively narrower channel spacings. Although cellular systems which make use of low power transmitters and frequency re-use do alleviate the spectral congestion problem, nevertheless the requirement for a very narrowband signal still exists if these systems are to support an ever-increasing number of subscribers. In addition to spectral efficiency, modulation methods for digital mobile radio should have constant envelope and be easy to implement. The constant envelope requirement follows from the need to avoid the effects of hardware nonlinearities (such as with class C amplifiers) while maximizing power efficiency. Ease of implementation is desirable in order to realize the transceivers using large scale integrated (LSI) circuit technology at low cost.

Currently, cellular mobile radio systems use analog FM to transmit speech (typically in a 25 kHz channel bandwidth), while the signaling information is transmitted using the Frequency-Shift Keying (FSK) format. However, total digitalization is desirable not only in order to interface mobile and portable radio systems to Integrated Services Digital Networks (ISDN), but also in view of the development of low rate (9.6 - 16 kb/s) speech coding devices and the simplicity and availability of data encryption devices which make it possible to transmit secure digitized speech

over mobile radio channels. Manchester coded Digital FM (MCDFM) with discriminator detection is an attractive modulation scheme for such an application for the following reasons:

1. Discriminator detection is relatively robust in the mobile fading environment. On the other hand, coherent detection generally yields optimum static bit error rate (BER) performance, but fading creates problems for carrier recovery. Differential detection does not require carrier recovery, but is very sensitive to center frequency drifts. Discriminator detection, while not optimum, is the most robust and thus the most appealing scheme, because of its immunity against fast fading and center frequency drift, its applicability to arbitrary values of modulation index and its inherent simplicity.
2. Symbol timing recovery is simple with a Manchester coded signal as this format yields at least one transition (zero crossing) per bit period irrespective of the data sequence. Alternately, the possibility of a long sequence of 1's or 0's makes non-return to zero (NRZ) coded and bipolar coded signals difficult to synchronize.
3. The Manchester signal format suppresses the dc component (in the baseband signal) and has relatively insignificant low frequency components, which is advantageous to cellular systems feeding onto phone networks. Most important of all, this ac coupling capability makes a Manchester coded signal easily interfaceable to existing mobile radio transceivers.

Most of the previous studies on digital FM with discriminator detection have dealt with NRZ coded signals [1]-[12]. Such systems have been the subject of numerous analytical investigations stemming from the early work that studied the characteristics of analog transmission to the later calculations of bit error rates in the digital case. Studies of digital FM presented by Klapper [1], Mazo and Salz [2] and Schilling et al. [3] were based on the click theory of noise in FM, due to Rice [4] and Cohn [5]. These papers investigated bit error rate performance and assumed that the passband of any bandpass filter in the system is sufficiently broad

that there is no distortion in the FM signal. However, emphasis then shifted to the narrowband case where the previous assumption is not appropriate. Bennett and Salz [6] and Tjhung and Wittke [7] took into account the effects of distortion due to a bandpass filter, but their approach relied on computationally burdensome techniques which require Fourier series expansions of long signal patterns, numerical convolutions and multipattern averaging. This approach was simplified to some degree by Cartier [8] who noticed that for the IF time-bandwidth regime $BT \geq 1$, most of the relevant energy for bit detection is contained in the period of the bit being detected and the two adjacent bit periods (i.e. one on each side of the central bit). As a result, he was able to reduce the error calculation to an average over four bit patterns, again using Fourier series and numerical integration techniques. Pawula [9] introduced further simplifications in which analytical results were presented for the bit error rate requiring no numerical averaging over bit patterns. More recently, Yeo et al. [10] have studied the effects of premodulation pulse shaping. As the latter introduces more intersymbol interference, multipattern averaging could not, however, be avoided. In addition, recent work on NRZ coded digital FM has focused on the effects of duobinary precoding [11] and of soft decisions [12] on error rate performance.

The error rate performance of MCDFM signals with discriminator detection was first analyzed by Cartier [8] (as an extension of his work on NRZ coded digital FM) and Kwan et al. [13]. There are, however, discrepancies in their results. The paper by Kwan et al. was brief and did not indicate the method of decoding the Manchester coded signal. The paper by Cartier mentioned the method of decoding but, as pointed out by Korn [14], involved an incorrect expression for the phase noise difference at the integrate-and-dump filter output. In view of these disagreements, Tan et al. [15] then reinvestigated the problem and clearly defined the method of demodulation. However, these papers only considered rectangular pulse shapes and to the author's best knowledge, the effects of premodulation pulse shaping on the performance of Manchester coded digital FM with discriminator detection have not been examined in the open literature. This feature is deemed essential when using

MCDFM signals as their power spectral density (for modulation index values that are practically useful) is too wide for channel bandwidths smaller than 2.5 times the data rate. As mentioned previously, Yeo et al. [10] investigated the effects of pulse shaping on NRZ coded DFM signals. By applying premodulation shaping to a binary FM system (with modulation index = 0.7), they achieved the same spectral economy as by reducing the frequency deviation (to an index of 0.5) and simultaneously obtained a performance improvement of up to 1.5 dB in a channel bandwidth of 1.18 times the data rate. It is therefore of interest to determine if a similar conclusion can be drawn for a Manchester coded signal, albeit in a different channel bandwidth.

The objective of this thesis is thus to determine the combination of three parameters: 1) modulation index, 2) premodulation pulse shaping, and 3) IF filtering, which yields the optimum bit error rate (BER) performance in a Manchester coded Digital FM system (MCDFM) with discriminator detection under a bandwidth constraint of 25 kHz and at data rates of 9.6 and 16 kb/s. The bandwidth criterion is defined as 99 percent in-band power.¹

1.2 Proposed Approach

In order to determine the optimal combination of the aforementioned parameters, two approaches are considered. The first approach involves mathematical synthesis, i.e. from a description of the problem and constraints, one may attempt to obtain analytical expressions for the optimum pulse shape and optimum IF filter. Such an approach is likely to involve extremely difficult nonlinear analysis most probably resulting in a very uncommon pulse shape and IF filter transfer function, which, in turn, could prove very difficult to implement.

The second approach is based on computer-aided analysis and involves the following process:

¹ While regulatory agencies may impose a more stringent bandwidth occupancy criterion, much of the previous work [11,12] of this type has been based on a 99 percent in-band power criterion and so will this thesis

1. Define common types of pulse shapes and IF filters.
2. Develop software "packages" to compute:
 - Power Spectral Density of the transmitted DFM signals.
 - BER properties of these DFM signals with discriminator detection.
3. Use these two tools in a numerical search for the optimum combination of pulse shape, IF filter and modulation index.

The resulting combination may not be as "optimal" as the one obtained with the first approach but, in view of its advantages with regard to simplicity and concern for practical implementation, the second approach is selected over the first.

1.3 Thesis Contribution

This thesis extends Pawula's simplified approach [9] to Manchester coded digital FM signals and combines it with multipattern averaging in order to evaluate the effects (performance-wise) of premodulation pulse shaping for which no results are available in the open literature. In the process, the NRZ coded DFM results of Tjhung and Wittke [7], Cartier [8] and Pawula [9] are confirmed, as are those of Tan et al. [15] with respect to Manchester coded DFM with no pulse shaping. The power spectral density of digital FM signals is computed by a method due to Garrison [16], whose results are also confirmed. The aforementioned numerical search is then performed yielding new results for a variety of pulse shapes and IF filters in the Manchester coded case. Particular emphasis is placed upon the IF filter as the effects of varying its characteristics have been virtually ignored in previous publications ². A large number of performance curves for shaped MCDFM signals are presented and compared with the optimum, no-shaping case. A significant improvement in performance is observed at a data rate of 16 kb/sec.

² Previous studies of both NRZ and Manchester-coded digital FM [1-15,17,19-21,33] all assumed an IF filter with a Gaussian shape.

1.4 Thesis Structure

The remaining chapters of this thesis are organized as follows.

In chapter 2, the digital FM system under study is analyzed using a method based on Pawula [9] and Tan et al. [15]. The function of the various components are explained, typical waveforms at different stages of the system are provided and all variables affecting system performance are derived and defined. In addition, the various pulse shapes and filter types considered in the ensuing performance optimization are presented.

Chapter 3 outlines the development of a software package (SPECTRUM) computing the power spectral density of continuous phase digital FM signals for arbitrary pulse shapes and baseband encoding formats. First, some analytic background is presented (based on Garrison [16]), followed by a description of the program algorithm. Finally, the correctness of the program is demonstrated by comparing some of the data generated using SPECTRUM to previously published data.

Chapter 4 explains the development of a software package (BER) computing the bit error rate of both NRZ coded and Manchester coded digital FM signals with discriminator detection. The signal model is first defined taking into account the distortion effects of the shaping and IF filters examined in chapter 2. From the statistics of the relevant noise processes, the bit error rate calculation process and thus the program algorithm are defined. Finally, the BER program is found to generate results which match published data.

Chapter 5 deals with the performance optimization process outlined in the introductory chapter. The combinations of premodulation pulse shaping, modulation index and IF filter yielding the best error rate performances in a channel bandwidth of 25 kHz and at data rates of 9.6 and 16 kilobits per second are found for the Manchester coded digital FM system with discriminator detection described in chapter 2. The optimization procedure is divided into three steps, the first requiring the use of the SPECTRUM program and the last two involving the BER program. The

results at each step, for both the aforementioned data rate values, are presented and discussed.

Finally, chapter 6 summarizes the major conclusions from this work and provides recommendations for future research.

Chapter 2

SYSTEM DESCRIPTION

2.1 Introduction

In this chapter, the digital FM system under study will be analyzed using a method based on [9] and [15]. The function of the various components will be explained, typical waveforms at different stages of the system provided and all variables affecting system performance derived and defined. In addition, the various pulse shapes and filter types considered in the ensuing performance optimization will be presented. A functional block diagram of the complete digital FM system under consideration is depicted in figure 2.1.

2.2 Digital FM Transmitter

The transmitter model is made up of three components: a baseband coder, a premodulation filter and a frequency modulator. The latter, in practice, is usually followed by an up-converter translating the carrier frequency to the desired "radio" frequency (RF) level, and a high power amplifier (HPA) which feeds the transmitter output signal to an aerial. When used close to its maximum power level (generally called "saturation" level), a typical HPA introduces both amplitude and phase distortions in the transmitted signal. This further enhances the desirability of a constant envelope signal as this type of modulation scheme enables the HPA to be used close to saturation (increasing power efficiency) while avoiding signal distortions.

The binary data source $b(t)$ is represented as a train of random impulses occurring at the rate of one every T seconds, having weights of either $+1$ or -1 with equal

Figure 2-1: Digital FM system model

a) Transmitter b) Channel c) Receiver

probability; the weights correspond to the bits "1" and "0" respectively. Thus $b(t)$ may be written as

$$b(t) = \sum_{-\infty}^{+\infty} a_i \delta(t - iT) \quad (2.1)$$

2.2.1 Baseband Encoder

The baseband encoder converts the input binary impulse stream into a continuous baseband signal. If $p(t)$ denotes the impulse response of the baseband encoder (i.e. a single coded pulse), the resulting signal $d(t)$ is related to the input stream by the convolution

$$d(t) = p(t) * \sum_{-\infty}^{+\infty} a_i \delta(t - iT) \quad (2.2)$$

Using the rules of convolution with impulses then yields:

$$d(t) = \sum_{-\infty}^{+\infty} a_i p(t - iT) \quad (2.3)$$

There are several ways by which binary symbols may be represented by electrical signals, but this thesis deals primarily with the "Manchester" or "split-phase" format.¹ As shown in figure 2.2a, the symbol "1" is represented by two successive unit amplitude pulses (the first positive, the second negative) of duration $T/2$, where T is the symbol period. A "0" is represented similarly, but with the pulse polarities reversed. As mentioned in chapter 1, this type of baseband encoding suppresses the dc component and has relatively insignificant low-frequency components, regardless of the signal statistics, characteristics which make such signals easily interfaceable to existing mobile radio transceivers. In addition, the presence of at least one zero crossing per bit period (irrespective of the data sequence) simplifies considerably symbol timing recovery at the receiver.

Another baseband encoding format "non-return-to-zero" (NRZ), shown in figure 2.2 b, is obtained by representing the symbols "1" and "0" by pulses of unit positive and negative amplitudes respectively. NRZ coded signals occupy a narrower bandwidth than Manchester coded signals but they contain significant dc components

¹ Other types include on-off, return-to-zero, bipolar signaling.

**Figure 2-2: Baseband encoder impulse response and encoded data sequence for a) Manchester signal format
b) NRZ signal format**

and the occurrence of a long sequence of a long sequence of "1" or "0"'s creates problems for timing recovery. In this thesis, the interest in NRZ coded signals stems from the relative simplicity of the noise statistics analysis and of the detection process; these provide background for the more complex noise statistics, analysis and detection process of Manchester coded signals. In addition, the large number of error rate results for NRZ coded digital FM available in the open literature will serve as test data for the error rate software, whose development will be outlined in chapter 4.

2.2.2 Premodulation filter

The purpose of the premodulation filter is to shape the encoded input binary

data waveform before its application to the FM modulator so as to produce an RF output with well-suppressed out-of-band power. In the process, the pulses in the shaped waveform are smeared and thus overlap, causing intersymbol interference (ISI). Since a narrower premodulation filter bandwidth leads to an increase in the time dispersion of the pulses (and therefore more ISI), the error rate at the receiver will increase as the spectral occupancy of the transmitted RF signal is diminished in this fashion (with all other system parameters fixed).

Three common types of pulse shapes are considered in this thesis. The first shape, Gaussian, is obtained by using as a premodulation filter a Gaussian low-pass filter with transfer function [9]

$$H_T(f) = e^{-\pi f^2 / 2B^2} \quad (2.4)$$

where B is the 3 dB-down bandwidth.² The corresponding impulse response is

$$h_T(t) = \sqrt{2B} e^{-2\pi B^2 t^2} \quad (2.5)$$

The baseband signal $m(t)$, which serves as input to the frequency modulator then becomes

$$m(t) = h_T(t) * d(t) = \int_{-\infty}^{+\infty} h_T(t - \tau) d(\tau) d\tau \quad (2.6)$$

where $*$ denotes convolution.

Note that the preceding assumes a noncausal analysis (the causal case is handled by replacing the upper limit in equation 2.6 by "t"). In fact, figure 2.3 shows that the integration is carried out on both sides of $\tau = t$. A causal analysis complicates the notation as it requires a time shift in $h_T(t)$ in order that the integration window (in this case, only for $\tau \leq t$) contains the bulk of the area under the impulse response $h_T(t)$. Therefore, noncausality will be assumed throughout this thesis, unless indicated otherwise.

² As is commonly done, B denotes the approximate 3 dB (double-sided) bandwidth, with the exact being 0.9394B as shown in appendix A. This small difference is usually disregarded and will be done so here.

Figure 2-3: Noncausal analysis of premodulation filter convolution. The integration is carried out on both sides of $t=\tau$ (NRZ encoding is assumed for simplicity). In a causal case, the "peak" of the impulse response would be at $\tau \ll t$.

Using equation 2.3 where $d(t)$ is considered as a succession of pulses (such as in figure 2.2 a or b), equation 2.6 may be rewritten as

$$m(t) = \sum_{-\infty}^{+\infty} a_i r(t - iT) \quad (2.7)$$

where

$$r(t) = p(t) * h_T(t) \quad (2.8)$$

represents the response of the premodulation filter to a single Manchester or NRZ coded pulse. Therefore, $r(t)$ shall be referred to as "pulse shape" for simplicity. Using equations 2.5 and 2.8, it can then be shown [17] that the Gaussian pulse shape is

$$\begin{aligned} r(t) = & Q(\sqrt{2}\alpha T(-0.5 - t/T)) \\ & - Q(\sqrt{2}\alpha T(+0.5 - t/T)) \end{aligned} \quad (2.9)$$

in the case of NRZ encoding and

$$\begin{aligned}
 r(t) = & Q(\alpha(T/2)(-0.5 - 2t/T)) \\
 & - Q(\sqrt{2}\alpha(T/2)(+0.5 - 2t/T)) \\
 & - Q(\sqrt{2}\alpha(T/2)(-0.5 - (2(t - 0.5T)/T))) \\
 & + Q(\sqrt{2}\alpha(T/2)(+0.5 - (2(t - 0.5T)/T)))
 \end{aligned} \tag{2.10}$$

in the case of Manchester encoding. In the preceding,

$$\alpha T = 5.336BT \tag{2.11}$$

where B is the 3 dB-down bandwidth and $Q(x)$ is the Gaussian probability integral defined by (see Appendix B)

$$Q(x) = (1/\sqrt{2\pi}) \int_x^{+\infty} e^{-y^2/2} dy \tag{2.12}$$

The pulse shape $r(t)$ can be used to determine the extent of the time dispersion introduced by the premodulation filter. One observes from figure 2.4 that the significant intersymbol interference introduced by the Gaussian shaping filter extends to one adjacent bit (on each side of the central bit) for $BT > 0.4$ and to two adjacent bits for $0.2 \leq BT \leq 0.4$. These observations hold for both Manchester and NRZ encoding.

A second type of pulse shaping that has found wide use in digital transmission channels is one that has the Raised Cosine Spectral characteristic (SRC)

$$\begin{aligned}
 R(f) = T & \quad ; 0 \leq |f| \leq (1 - \beta) \\
 = (T/2)[1 - \sin(\pi T(f - (1/2T))/\beta)] & \tag{2.13} \\
 & ; (1 - \beta)/2T \leq f \leq (1 + \beta)/2T
 \end{aligned}$$

where β is called the rolloff parameter, as shown in figure 2.5. The pulse shape $r(t)$ having this spectrum is [18]

$$r(t) = \frac{(\sin(\pi t/T)\cos(\beta\pi t/T))}{(\pi t/T)(1 - 4\beta^2 t^2/T^2)} \tag{2.14}$$

Figure 2-4: Gaussian pulse shapes a) NRZ, $BT=0.2$
 b) NRZ, $BT=0.4$ c) NRZ, $BT=\infty$
 d) Manchester, $BT=0.2$ e) Manchester, $BT=0.4$
 f) Manchester, $BT=\infty$

Figure 2-5: Raised Cosine Spectrum

Note that $r(t)$ in equation 2.14 represents the response of a SRC shaping filter to a unit amplitude pulse of duration T ; i.e. it is the NRZ coded SRC pulse shape. In the case of Manchester encoding, the pulse shape becomes (as with the Gaussian shape), the sum of the responses to two successive unit amplitude pulses of duration $T/2$, of opposite polarity, and delayed with respect to each other by $0.5T$, i.e.

$$r(t) = \frac{\sin(2\pi t/T) \cos(2\beta\pi t/T)}{(2\pi t/T)(1 - 16\beta^2 t^2/T^2)} - \frac{\sin(2\pi(t - 0.5T)/T) \cos(2\beta\pi(t - 0.5T)/T)}{(2\pi(t - 0.5T)/T)(1 - 16\beta^2(t - 0.5T)^2/T^2)} \quad (2.15)$$

Thus, strictly speaking, the Manchester coded SRC pulse does not have a Raised Cosine spectral characteristic as defined in equation (2.13). Figure 2.5 shows that, for both Manchester and NRZ encoding, the significant ISI introduced by a SRC shaping filter extends to no more than two adjacent bits as long as $\beta \geq 0.5$.

Finally, the third type of shaping to be considered is one creating pulses with a Raised Cosine shape (RC pulses). The response of such a shaping filter to a unit amplitude pulse of duration T is defined as [22]

$$r(t) = (1/2LT)(1 - \cos(2\pi t/LT)) \quad ; 0 \leq t \leq T \\ 0 \quad ; \text{otherwise} \quad (2.16)$$

Figure 2-6: SRC pulse shapes a) NRZ, $\beta = 0.1$
 b) NRZ, $\beta = 0.5$ c) NRZ, $\beta = 1.0$ d) Manchester, $\beta = 0.1$
 e) Manchester, $\beta = 0.5$ f) Manchester, $\beta = 1.0$

Thus equation 2.16 defines the NRZ coded RC pulse shape of duration L symbol intervals. As opposed to the Gaussian and SRC pulse shapes, the RC pulse shapes are strictly time-limited to LT seconds; it is easily seen from figures 2.7a and 2.7b that no ISI is introduced by a NRZ coded RC pulse with $L=1$, and that such pulses with L larger than 1 (L integer) introduce ISI extending to $(L+1)$ adjacent bits.

The Manchester coded RC pulse shape is defined as the sum of a positive RC pulse of duration LT , and a negative RC pulse, also of duration LT , but delayed by $0.5T$ with respect to the first, i.e.

$$r(t) = r_1(t) - r_2(t) \quad (2.17a)$$

where

$$r_1(t) = \begin{cases} (1/2LT)(1 - \cos(2\pi t/LT)) & ; 0 \leq t \leq LT \\ 0 & ; \text{otherwise} \end{cases} \quad (2.17b)$$

and

$$r_2(t) = \begin{cases} (1/2LT)(1 - \cos(2\pi(t - 0.5T)/(LT))) & ; 0.5T \leq t \leq (L + 0.5)T \\ 0 & ; \text{otherwise} \end{cases} \quad (2.17c)$$

As shown in figures 2.7 c) and d), the total duration of Manchester encoded RC pulses is $(L+0.5)T$, implying that the the ISI introduced by these pulses extends to $(L+1)$ adjacent bits. It should also be noted that both Manchester and NRZ coded RC pulse shapes introduce ISI only on one side of the central bit.

2.2.3 Frequency Modulator

Frequency-modulated waves are readily generated by varying the frequency of a voltage controlled oscillator. Ideally, the oscillator frequency is varied linearly with the baseband signal $m(t)$, which creates an output signal representable as [23]

$$s(t) = \sqrt{2S} \cos[2\pi f_c t + 2\pi f_d \int_0^t m(t') dt' + \theta_0] \quad (2.18)$$

In this representation, $\sqrt{2S}$ is the carrier amplitude, f_c is the frequency of the oscillator in the absence of modulation, θ_0 is the initial phase of the carrier (i.e.

**Figure 2-7: RC pulse shapes a)NRZ, $L=1$ b)NRZ, $L=2$
c)Manchester, $L=1$ d)Manchester, $L=2$**

at $t=0$) and f_d is a conversion factor relating frequency displacement to baseband signal voltage. The argument of the sinusoidal wave is termed the instantaneous

phase, i.e.

$$\Theta(t) = 2\pi f_c t + \theta(t) \quad (2.19)$$

where

$$\theta(t) = 2\pi f_d \int_{-\infty}^t m(t') dt' \quad (2.20)$$

is the transmitted data phase and where the initial phase has been set to zero. The time derivative of the instantaneous phase is the instantaneous frequency. When the baseband waveform $m(t)$ is written as (equation 2.7) a succession of pulse shapes weighted by the binary data symbols $a_i = +1$ or -1 , this succession of pulse shapes is seen to be directly proportional to the instantaneous frequency of the FM wave $s(t)$. For this reason, the pulse shapes $r(t)$ are also called "frequency pulses". Similarly, one may define "phase pulses" $g(t)$ as

$$g(t) = \frac{\int_{-\infty}^t r(\tau) d\tau}{\int_{-\infty}^{+\infty} r(\tau) d\tau} \quad (2.21)$$

These represent the phase accumulation due to each individual bit as shown in figure 2.8. When the phase pulses are normalized as above, the constant f_d represents the peak (single-sided) frequency deviation and the modulation index "h" is defined as $h = 2f_d T$. The amplitude of the NRZ coded phase pulses then asymptotically increases towards one as $t \rightarrow \infty$. In contrast, the Manchester coded frequency pulses have a total area of zero, and therefore their corresponding phase pulses reach a peak value of 0.5 before asymptotically decreasing towards zero as $t \rightarrow \infty$. The transmitted data phase may now be written, using equations 2.7, 2.20 and 2.21, as a sum of delayed phase pulses weighted by the data sequence i.e.

$$\theta(t) = 2\pi f_d \sum_{i=-\infty}^{+\infty} a_i g(t - iT) \quad (2.22)$$

Substituting the preceding expression into equations 2.20 and 2.18 yields therefore the following representation for the transmitted signal

$$s(t) = \sqrt{2S} \cos[2\pi f_c t + 2\pi f_d \sum_{i=-\infty}^{+\infty} a_i g(t - iT)] \quad (2.23)$$

Figure 2-8: Frequency and phase pulses

- a) NRZ coded (no shaping)
- b) NRZ coded (with shaping)
- c) Manchester coded (no shaping)
- d) Manchester coded (with shaping)

As shown in figure 2.8, the phase pulses of a Manchester coded DFM signal with pulse shaping or those of any NRZ coded signal overlap into adjacent symbol periods, whereas the phase pulses in a Manchester coded signal with no shaping

Figure 2-9: NRZ coded transmitter waveforms

- a) Frequency pulse sequence
- b) Baseband message
- c) Phase pulse sequence
- d) Transmitted data phase
- e) Sine of data phase
- f) Cosine of data phase

are confined to a single symbol period. Therefore for the latter case, one may let $s(t; a_i)$ denote the non-shaped MCDFM waveform associated with the i^{th} symbol, resulting in [24]

$$s(t; a_i) = \sqrt{2S} \cos(2\pi f_c t + 2\pi f_d a_i g(t - iT)) \quad (2.24)$$

Using a common trigonometric identity, this may be expanded as

$$s(t; a_i) = s_c(t; a_i) \cos(2\pi f_c t) - s_s(t; a_i) \sin(2\pi f_c t) \quad (2.25)$$

where

$$s_c(t; a_i) = \sqrt{2S} \cos(2\pi f_d a_i g(t - iT)) \quad (2.26)$$

and

$$s_s(t; a_i) = \sqrt{2S} \sin(2\pi f_d a_i g(t - iT)) \quad (2.27)$$

are the "in-phase" and "quadrature" components of $s(t; a_i)$. Since only binary signaling is considered (i.e. $a_i = \pm 1$) and by restricting the phase arguments to the range $-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$, the FM waveform may be written as [24]

$$\begin{aligned} s(t; a_i) = & \sqrt{2S} \cos(2\pi f_c t) \cos(2\pi f_d g(t - iT)) \\ & - a_i \sqrt{2S} \sin(2\pi f_c t) \sin(2\pi f_d g(t - iT)) \end{aligned} \quad (2.28)$$

It is immediately obvious that the in-phase channel represents the deterministic component of the non-shaped MCDFM waveform which carries the bit timing information while the quadrature component carries the message bits. However, when pulse shaping is introduced in the MCDFM system, the phase pulses are no longer confined to one symbol period and the "message" is then carried on both the in-phase and quadrature channels [24]. Figures 2.9 and 2.10 show sequences of frequency and phase pulses, as well as the sine and cosine of the data phase, for NRZ coded and Manchester coded signals respectively.

Notice that the phase is assumed to be continuous. It is also possible to generate a digital FM wave without regard to phase continuity. This may be accomplished by having many oscillators (two in the binary case considered here) tuned to the

Figure 2-10: Manchester coded transmitter waveforms

- a) Frequency pulse sequence b) Baseband message
 c) Phase pulse sequence d) Transmitted data phase
 e) Sine of data phase f) Cosine of data phase

desired frequencies whose outputs are controlled by a digital baseband wave. This method gives rise to undesirable transients and a wider RF spectrum in addition to complicating the transmitter. For these reasons, it is rarely used in practice and will not be treated in this thesis.

2.3 Channel

The mobile communication channel is at best a difficult channel to accommodate because of inherent transmission and reception problems. In the first place, there is a great deal of multipath transmission. Because of reflections from buildings, lamp posts, utility wires and other obstructions, the signal may travel from the transmitter to the receiver by many different paths with different lengths. The problem is worse in cities and particularly in business districts, where the obstructions tend to be numerous.

When one end of the channel is moving, as is usually the case in mobile communications, the predominant feature of the multipath channel is fading. The fading results because, in some relative positions, phases of the signals arriving from the various paths interfere destructively, while in other positions the phases add constructively. Thus, as a vehicle moves, the received signal strength varies erratically and unpredictably.

Since the signal strength may go from one extreme to the other when the vehicle covers a distance on the order of one-half wavelength, these fades take place very rapidly when the transmission frequencies are high and the vehicle is traveling at very high speeds. The fading is often referred to as "Rayleigh" fading [25], the latter being the name of the probability distribution associated with these random amplitudes. A characteristic of Rayleigh fading is that it is very frequency-selective when the signal bandwidth exceeds the coherence bandwidth of the channel - that is, two signals that differ in frequency by an amount exceeding the coherence bandwidth of a given multipath channel will fade almost independently when they are transmitted and received over this channel. This is because the length of the

path measured in wavelengths is different at different frequencies.

Another phenomenon that takes place in the mobile radio channel is "log-normal" fading [25]. This kind of fading results from the signal being blocked by large structures or by hills and mountains. The resulting signal amplitude tends to follow a log-normal or Gaussian distribution when measured in decibels. The vehicle must move over a much greater distance to observe a change in average signal amplitude and these changes are not particularly frequency-sensitive. Thus the total signal strength observed at a point can be viewed as a Rayleigh fading signal with a random average value that is log-normally distributed.

Yet another set of problems that arise in the mobile channel is a result of the motion of the mobile user. The first, of course, is Doppler shift. At 900 MHz, for example, speeds in the range of 50 miles per hour may result in Doppler spreading of approximately ± 100 Hz [25]. In a frequency-modulated receiver, this may be heard as a low rumble known as random FM.

The second motion-related problem concerns cochannel and adjacent channel interference, the latter being interference from signals occupying adjacent channels while the former represents interference from signals occupying the same frequency band (the 25 kHz bandwidth assumed in this thesis is typical in current narrowband FM systems). Because of vehicle motion, a receiver and a transmitter operating on adjacent or nearly adjacent channels may be physically close together. The receiver selectivity must be extremely good to prevent substantial interference from the strong transmitter signal.

In a nonmobile communication system, this type of interference can be minimized by careful coordination between frequency assignments and physical location - a tactic that does not work so well with a noncellular mobile system. Also, in a cellular mobile communication system, where frequencies are reused in the same city area, cochannel interference will occur and cannot be reduced by improving receiver selectivity. This cochannel interference plays a predominant role in determining the capacity of any narrowband cellular system.

Another channel impairment is "impulsive" noise [25] originating from a variety of man-made sources. The noise here usually occurs in the form of randomly shaped pulses, with sometimes widely varying durations that may extend over many symbol periods, so that the noise pulses often appear as bursts of noise. When sufficiently severe, they may swamp the received data signal, causing errors in the detected data symbols, regardless of the modulation method and the detection process used.

Normally, the receiver treats all the aforementioned sources of additive noise and interfering signals as noise, and there is therefore a strict upper limit to the level of this interference that can be tolerated at the receiver without an unacceptable increase in the error rate of the detected symbols. Because of the differing natures of these various sources of noise, its characteristics may well vary from one location and/or time to another, so that no very detailed or precise model of the noise is entirely satisfactory, particularly if it is to be amenable to theoretical calculation. For these reasons, the error rate performance of the digital FM system considered in this thesis is evaluated on the basis of its tolerance to additive white Gaussian noise (AWGN) with (single-sided) power spectral density N_0 watts/Hz. Although not itself among the most important types of noise in land mobile radio systems, AWGN is relatively easy to analyze theoretically. It is also the least predictable of the different types of noise and is therefore, in this sense, the most random. The use of AWGN in this thesis is further justified by the fact that we are dealing with "narrowband" systems. The fading encountered in a practical mobile channel is then more likely to be flat (as opposed to frequency selective) and slow. The resulting SNR fluctuations within a bit period should then be quite small and as the receiver considered here performs "bit by bit" detection (i.e. decisions are not based on previously detected bits), there is no reason to believe that fading will significantly degrade the accuracy of the resulting static bit error rate results. The bit error rate over the fading channel can then be obtained by averaging the static BER curve over the distribution of the SNR in the fading environment.

2.4 Digital FM Receiver

As shown in figure 2.1, the receiver model consists of an intermediate frequency (IF) filter, a limiter-discriminator, a symbol timing recovery circuit (accompanied by a Manchester to NRZ conversion circuit if the transmitted signal was Manchester coded), an integrate-and-dump filter and finally a hard decision device. In practice, the IF filter is preceded by a low noise amplifier (LNA) and a down-converter translating the "radio" frequency (RF) to an "intermediate" frequency (IF).

2.4.1 IF Filter

The signal $x(t)$ at the input of the predetection IF filter can be written as [9]

$$x(t) = u(t) + w(t) \quad (2.29a)$$

where

$$u(t) = \sqrt{2S} \cos(\omega_o t + \theta(t)) \quad (2.29b)$$

and where $\omega_o = 2\pi f_o$ is the center IF filter radian frequency and $w(t)$ is a zero mean white Gaussian noise process with one-sided power spectral density N_o watts/Hz. The IF filter with low-pass equivalent transfer function $H_R(f)$ and corresponding low-pass equivalent impulse response $h_R(t) = F^{-1}(H_R(f))$ bandlimits $x(t)$ (ideally reducing the noise power as much as possible without significantly distorting the signal component). It produces an output signal $z(t)$

$$z(t) = v(t) + \xi(t) \quad (2.30)$$

where

$$v(t) = \sqrt{2S} \cos(\omega_o t + \theta(t)) * h_R^*(t) \quad (2.31)$$

and where

$$\xi(t) = w(t) * h_R^*(t) \quad (2.32)$$

In the preceding, $h_R^*(t)$ represents the IF filter band-pass impulse response. Use of the low-pass equivalent here tacitly assumes that $|H_R(f)|$ exhibits even symmetry

and $\angle H_R(f)$ odd symmetry; $h_R(t)$ is then purely real and so is the "complex envelope" $\tilde{h}_R(t)$ (see Appendix C) of the band-pass response $h_R^*(t)$, i.e.

$$\begin{aligned} h_R^*(t) &= \text{Re}[\tilde{h}_R \exp(j2\pi f_o t)] \\ &= h_{Rc}(t) \cos(\omega_o t) \\ &= h_R(t) \cos(\omega_o t) \end{aligned} \quad (2.33)$$

where $h_{Rc}(t)$ is the in-phase component of the complex envelope $\tilde{h}_R(t)$, the quadrature component $h_{Rs}(t)$ being equal to zero.

Using a well-known trigonometric identity, the expression for the filtered signal component $v(t)$ (equation 2.31) may be expanded as

$$\begin{aligned} v(t) &= [\sqrt{2S} \cos(\theta(t)) \cos(\omega_o t) - \sqrt{2S} \sin(\theta(t)) \sin(\omega_o t)] \\ &\quad \star h_R(t) \cos(\omega_o t) \end{aligned} \quad (2.34)$$

in which

$$u_c(t) = \sqrt{2S} \cos(\theta(t)) \quad (2.35)$$

and

$$u_s(t) = \sqrt{2S} \sin(\theta(t)) \quad (2.36)$$

represent respectively the in-phase and quadrature components of the band-pass signal $u(t)$. The canonical representation of $v(t)$ is then (using expressions from Appendix C)

$$\begin{aligned} v(t) &= \sqrt{2S} \int_{-\infty}^t h_R(t-\tau) \cos(\theta(\tau)) d\tau \cos(\omega_o t) \\ &\quad + \sqrt{2S} \int_{-\infty}^t h_R(t-\tau) \sin(\theta(\tau)) d\tau \sin(\omega_o t) \end{aligned} \quad (2.37)$$

This can be rewritten as

$$v(t) = \sqrt{2S} a(t) \cos(\omega_o t + \phi(t)) \quad (2.38)$$

where $a(t)$ is the filtered normalized time-varying signal envelope whose square is [9]

$$\begin{aligned} a^2(t) &= \left(\int_{-\infty}^t h_R(t-\tau) \cos(\theta(\tau)) d\tau \right)^2 \\ &\quad + \left(\int_{-\infty}^t h_R(t-\tau) \sin(\theta(\tau)) d\tau \right)^2 \end{aligned} \quad (2.39)$$

and $\phi(t)$ is the distorted signal phase given by [9]

$$\phi(t) = \tan^{-1} \frac{\int_{-\infty}^t h_R(t-\tau) \sin(\theta(\tau)) d\tau}{\int_{-\infty}^t h_R(t-\tau) \cos(\theta(\tau)) d\tau} \quad (2.40)$$

The signal-dependent noise term $\xi(t)$ can also be written in terms of its canonical representation

$$\begin{aligned} \xi(t) = & \xi_c(t) \cos(\omega_o t + \phi(t)) \\ & - \xi_s(t) \sin(\omega_o t + \phi(t)) \end{aligned} \quad (2.41)$$

where the in-phase and quadrature components ($\xi_c(t)$ and $\xi_s(t)$ respectively) are zero mean Gaussian random variables with variance³

$$\sigma^2 = N_o \int_{-\infty}^{+\infty} |H_R(f)|^2 df \quad (2.42)$$

Using equations 2.30, 2.38 and 2.41, the total (signal and noise) IF filter output becomes

$$z(t) = R(t) \cos(\omega_o t + \phi(t) + \eta(t)) \quad (2.43)$$

where the square of the envelope is now equal to

$$R^2(t) = (\sqrt{2S}a(t) + \xi_c(t))^2 + \xi_s(t)^2 \quad (2.44)$$

and the phase noise is

$$\eta(t) = \tan^{-1} \frac{\xi_s(t)}{\sqrt{2S}a(t) + \xi_c(t)} \quad (2.45)$$

In order to simplify the notation, the two quadrature components of the narrowband noise will be assumed to have unit variances in which case $\xi_c(t)$ and $\xi_s(t)$ must be normalized by σ (σ^2 is given by equation 2.42). The square of the time-varying signal envelope and the phase noise are then respectively expressed as [9]

$$R^2(t) = (\sqrt{2\rho(t)} + \xi_c(t))^2 + \xi_s^2(t) \quad (2.46)$$

and

$$\eta(t) = \tan^{-1} \frac{\xi_s(t)}{\sqrt{2\rho(t)} + \xi_c(t)} \quad (2.47)$$

³ Since $\xi(t)$ has zero mean, $\xi_c(t)$ and $\xi_s(t)$ also have zero mean and have the same variance as $\xi(t)$ [23].

where $\rho(t)$ is the time-varying signal-to-noise ratio given by [9]

$$\rho(t) = \frac{E_b}{N_o T} \frac{a^2(t)}{\int_{-\infty}^{+\infty} |H_R(f)|^2 df} \quad (2.48)$$

in which $E_b = ST$ is the signal energy per bit.

Three types of IF filters will be considered in the performance optimization of Chapter 5, notably a Gaussian filter, second-order Butterworth and Ideal "brick-wall" filter, which, of course, is optimal in terms of passband flatness and stopband attenuation. The low-pass equivalent transfer function and impulse response of a Gaussian filter are given by equations 2.4 and 2.5 respectively; the transfer function being purely real, the phase function of a Gaussian filter is linear. On the other hand, the low-pass equivalent transfer function of a second-order Butterworth filter is [9]

$$H_R(f) = \frac{2}{1 + (1 + j\sqrt{8}f/B)^2} \quad (2.49)$$

where B represents the double-sided 3 dB-down bandwidth. Its amplitude response is thus

$$|H_R(f)| = \frac{1}{\sqrt{1 + 16f^4/B^4}} \quad (2.50)$$

and its phase response equals

$$\angle H_R(f) = \tan^{-1} \frac{\sqrt{8}f/B}{(1 - 4f^2/B^2)} \quad (2.51)$$

which is nonlinear. Finding the inverse Fourier transform of equation 2.43 yields the low-pass equivalent impulse response

$$h_R(t) = \pi B \sqrt{2} e^{-\frac{\pi B t}{\sqrt{2}}} \sin\left(\frac{\pi B t}{\sqrt{2}}\right) \quad (2.52)$$

Finally, an Ideal "brick-wall" filter with low-pass equivalent transfer function

$$\begin{aligned} H_R(f) &= 1 && ; -B/2 \leq f \leq B/2 \\ &= 0 && ; \text{otherwise} \end{aligned} \quad (2.53)$$

has a corresponding impulse response given by

$$h_R(t) = \frac{\sin(\pi B t)}{\pi t} \quad (2.54)$$

The three amplitude responses are plotted in figure 2.11 from which it is seen that the second-order Butterworth filter is flatter in the passband than the Gaussian filter.

2.4.2 Limiter-discriminator

The main function of the limiter is to remove the amplitude variations in the IF filter output $z(t)$ so that the frequency-modulation detector, which is usually amplitude sensitive to some degree, may respond only to frequency variations. The limiter effectively passes only a narrow amplitude slice of the received signal centered about zero amplitude. Thus only the zero crossing information is preserved. If the carrier frequency is high compared to the data rate, the zero crossings of the FM waveform are sufficient to specify the instantaneous frequency of the received signal.

The discriminator is a frequency-selective network which creates an output signal whose amplitude variation is directly proportional to the frequency of the limited signal. As shown in appendix D, the ideal frequency-selective circuit necessary to recover the instantaneous frequency is a differentiator (or "slope circuit") followed by an envelope detector. However, in practical circuits, this is usually replaced by a tuned circuit with the "rest carrier frequency" placed in the center of an approximately linear portion of one side of the frequency response peak. As explained in appendix D, a second tuned circuit may also be used in order to remove a "bias" voltage produced by the first circuit. These two tuned circuits act in a "push-pull" fashion, yielding a net detected output which varies linearly with frequency over the band of interest. The discriminator output is thus the derivative $\dot{\Phi}(t)$ of the IF filtered phase given by

$$\Phi(t) = \phi(t) + \eta(t) \quad (2.55)$$

where $\phi(t)$ is the distorted signal phase (equation 2.40) and $\eta(t)$ is the phase noise (equation 2.47).

Figure 2-11: IF filter low-pass equivalent magnitude transfer functions 1)Gaussian 2)Second-order Butterworth 3)Ideal "brick-wall"

2.4.3 Symbol timing recovery and MC-to-NRZ conversion

The symbol timing recovery circuit in figure 2.1 uses the discriminator output $\dot{\Phi}(t)$ to recover the symbol timing. As mentioned previously, this can be difficult for NRZ coded signals when a long sequence of 1's or 0's occurs and leads to a long absence of zero crossings in $\dot{\Phi}(t)$. With this signal format, the discriminator output is fed directly to the integrate-and-dump filter; timing is only needed to indicate to the latter when it should sample and "dump" the accumulated voltage.

Timing recovery is much simpler with Manchester coded signals as one zero crossing occurs per bit period in $\dot{\Phi}(t)$ irrespective of the data sequence. However with this format, the recovery circuit (in addition to providing timing information to the integrate-and-dump filter) creates a clock signal $c(t)$, as shown in figure 2.12, of period T , which is fed to "Manchester-to-NRZ" conversion device [15]. The objective is to create a waveform $\dot{\Phi}_c(t)$ which has the same polarity throughout most of the bit period (assuming reasonable amounts of noise and intersymbol interference) in order that the integrate-and-dump filter may accumulate a significant voltage during the bit period. In the analysis that follows, it will be assumed that perfect symbol synchronization is available, and that the MC-to-NRZ conversion process amounts to multiplying the discriminator output $\dot{\Phi}(t)$ with the perfect clock waveform $c(t)$ resulting in the waveform $\dot{\Phi}_c(t)$ as shown in figure 2.12.

2.4.4 Integrate-and-Dump Filter

Figure 2.12 also illustrates the process of converting the phase derivative ($\dot{\Phi}(t)$ for NRZ coded signals or $\dot{\Phi}_c(t)$ for Manchester coded signals) into the phase difference $\Delta\Phi(t)$ by an integrate-and-dump filter, a diagram of which is shown in figure 2.13. The parallel switch closes briefly at time $t_o = (n - 1)T$ (n integer) dumping any residual energy in the filter. This ensures that the signal energy received before $t_o = (n - 1)T$ does not contribute to the output at time $t_o = nT$. The series switch then closes briefly at time $t_o = nT$, thereby sampling the filter output at the end of the bit period.

Figure 2-12: MC-to-NRZ conversion process
a)MC phase derivative b)clock signal
c)NRZ converted phase derivative
d) I and D filter output

Figure 2-13: Integrate-and-dump circuit

In the case of NRZ coded signals, the integrate-and-dump filter "re-integrates" the discriminator output $\dot{\Phi}(t)$ producing a phase difference $\Delta\Phi(t)$ [9]

$$\Delta\Phi(t_o) = \int_{t_o-T}^{t_o} \dot{\Phi}(t) dt \quad (2.56)$$

whereas in the case of Manchester coded signals, it "re-integrates" the MC-to-NRZ convertor output $\dot{\Phi}_c(t)$ yielding

$$\Delta\Phi(t_o) = \int_{t_o-T}^{t_o} \dot{\Phi}_c(t) dt \quad (2.57)$$

Since $\dot{\Phi}(t) = \dot{\Phi}_c(t)$ in the first half of the bit period and $\dot{\Phi}(t) = -\dot{\Phi}_c(t)$ in the second half of the bit period, equation 2.57 can be rewritten in terms of the initial discriminator output as [15]

$$\Delta\Phi(t_o) = \int_{t_o-T}^{t_o-T/2} \dot{\Phi}(t) dt - \int_{t_o-T/2}^{t_o} \dot{\Phi}(t) dt \quad (2.58)$$

For both encoding formats, samples are taken at $t_o = nT$ (i.e. at the end of a given bit period) and the resulting phase difference ⁴ can be written as [9,15]

$$\Delta\Phi = \Delta\phi + \Delta\eta + 2\pi N \quad (2.59)$$

⁴ Although all other phases can be regarded as modulo 2π , $\Delta\Phi$ lies in the interval $(-\infty, +\infty)$ because it is the output voltage of an integrator.

In the above equation, $\Delta\phi$ represents the signal phase component equal to [9]

$$\Delta\phi = \phi(nT) - \phi((n-1)T) \quad (2.60)$$

for NRZ encoding and equal to [15]

$$\Delta\phi = 2\phi((n-0.5)T) - \phi((n-1)T) - \phi(nT) \quad (2.61)$$

in the case of Manchester encoding. Similarly, $\Delta\eta$ represents the phase noise component which is [9]

$$\Delta\eta = \eta(nT) - \eta((n-1)T) \quad (2.62)$$

for NRZ encoding and [15]

$$\Delta\eta = 2\eta((n-0.5)T) - \eta((n-1)T) - \eta(nT) \quad (2.63)$$

for Manchester encoding ⁵.

Finally, N denotes [9], in the case of NRZ encoding, the number of "clicks" (see Appendix E) in the interval $[(n-1)T, nT]$ whereas [15]

$$N = N_1 - N_2 \quad (2.64)$$

in the case of Manchester encoding with N_1 denoting the number of "clicks" in the interval $[(n-1)T, (n-0.5)T]$ and N_2 the number of clicks in the interval $[(n-0.5)T, nT]$.

The sampled output of the integrate-and-dump filter is presented to the hard decision device, which decides that a "1" was sent if $\Delta\Phi \geq 0$ and a "0" otherwise. In the preceding, the decisions times have been assumed to be at integer multiples of the bit time T . However, these may be shifted from the bit times due to the group delay of the IF filter and also the phase distortion due to the filter. This results, as

⁵ It should be noted that since the individual phases ϕ and η are regarded as modulo 2π (i.e. restricted to an interval $-\pi$ to π), the composite phases $\Delta\phi$ and $\Delta\eta$ have the range $(-2\pi$ to $2\pi)$ in the case of NRZ encoding and the range $(-4\pi$ to $4\pi)$ in the case of Manchester encoding.

shown in figures 2.9 and 2.10, in shifts of the peaks (NRZ case) or the zero crossings (Manchester coded case) of the filtered phase. The ability of the decision circuit to make correct decisions, and thus the error rates, then depend solely upon the statistics of the noise processes $\Delta\eta$ and $2\pi N$.

2.5 Summary

This chapter has presented a detailed analysis of a digital FM system model for both NRZ and Manchester signal formats. This description serves, first and foremost, as background material for the bit error rate (BER) analysis of Chapter 4 which is based on the statistics of the noise processes $\Delta\eta$ and $2\pi N$. Prior to this, the following chapter examines a method of computing the power spectral density and band occupancy characteristics of the transmitted FM waveform described in section 2.1.3.

Chapter 3

BAND OCCUPANCY ANALYSIS

3.1 Introduction

An important descriptive feature of any information-carrying system is the power spectral density function that results from the combination of the characteristics of the information signal and the technique of modulation. In this thesis, estimates of bandwidth occupancy are required to determine the optimum combination of digital FM system parameters (under data rate and bandwidth constraints); knowledge of the transmitted DFM signals' power spectral density is thus imperative.

This chapter outlines the development of a software package (SPECTRUM) computing the power spectral density of continuous phase digital FM signals for arbitrary pulse shapes and baseband encoding formats. First, some background relating to the computation process is given, followed by a description of the program algorithm. Finally, the correctness of the program is demonstrated by comparing some of the data generated using SPECTRUM to previously published data.

3.2 Background

The power spectral density of binary Frequency-Shift Keying (FSK) was first studied by Bennett and Rice [25], for both continuous and discontinuous phase signals. Anderson and Salz [26] then extended the continuous phase results to include multilevel signaling. In both papers, the method of attack on the problem is similar and direct. One calculates the segmented Fourier transform of the transmitted FSK waveform, obtains its magnitude squared, averages over all random variables,

divides by the length of the segment and then evaluates the limit as the length increases without bound. If one lets

$$\begin{aligned} s_N(t) &= s(t) && ; 0 \leq t \leq NT \\ &= 0 && ; \text{elsewhere} \end{aligned} \quad (3.1)$$

where $s(t)$ denotes the FSK waveform, then, by definition [25,26]

$$S(f) = \lim_{N \rightarrow \infty} \frac{1}{NT} E[|S_N(f)|^2] \quad (3.2)$$

where $S_N(f)$ and $S(f)$ denote the Fourier transforms of $s_N(t)$ and $s(t)$ respectively, while "E" represents expectation.

However, the numerical computation of $S(f)$ using the results of [25,26] is a nontrivial task, primarily due to the nonlinearity inherent in the FM signal. For digital modulation signals, the analysis and computation are found to be particularly awkward as soon as one deviates from the FSK case [25,26] that employs uncurbed rectangular frequency pulses and attempts to consider an information signal with premodulation filtering. Garrison [16] has developed an analysis that allows inclusion of premodulation filtering for both binary and multilevel signal sets (in this thesis, our interest lies only in binary digital FM). His results are exact, to within machine-computation accuracy, for those signal sets that fall within the FSK definition and yield an excellent approximation for the general FM case arranged with premodulation filtering.

The transmitted FM waveform $s(t)$ and the information-carrying baseband signal $m(t)$ are given by equations 2.18 and 2.7 respectively, repeated here for convenience

$$s(t) = \sqrt{2S} \cos[2\pi f_c t + 2\pi f_d \int_0^t m(t') dt' + \theta_0] \quad (3.3)$$

$$m(t) = \sum_{i=-\infty}^{\infty} a_i r(t - iT) \quad (3.4)$$

where $r(t)$ is a frequency pulse and for binary transmission at a rate of $1/T$, the a_i 's are independent binary random variables. Garrison's approach [16] is based on recognition that substitution of 3.4 into 3.3 is greatly facilitated if $r(t)$ in 3.4 is

replaced by a duration-limited approximation $r_a(t)$ constrained within "d" signaling intervals. Further, if one subdivides each signaling interval T into "Q" subintervals (Q being even for computational convenience) and quantizes the level of $r_a(t)$ within each interval, then the phase accumulation resulting from the integration of $m(t)$ in 3.3 becomes linear in t and the analysis for the power spectral density becomes readily amenable to machine computation. Thus, one may establish the interval-centered approximation [16]

$$\begin{aligned} r(t) &\simeq r_a(t) && ; -(d-1)T/2 \leq t \leq (d+1)T/2 \\ &= 0 && ; \text{elsewhere} \end{aligned} \quad (3.5)$$

where $r_a(t)$ is defined as

$$r_a(t) = \sum_v h_v f(t - vT/Q) \quad (3.6)$$

Here, the h_v define the assigned quantization level within a specific v^{th} subinterval and $f(t)$ is described by a unity amplitude rectangular pulse in the interval $0 \leq t \leq T/Q$.

Garrison's computation of the power spectral density then follows closely that of Bennett and Rice [25] and of Anderson and Salz [26]. The truncated Fourier transform of equation 3.3 is first computed as well as its magnitude squared. Next one performs an ensemble average of the squared Fourier transform consistent with the binary (or M-ary in Garrison's work [16]) form of $m(t)$. After dividing by the length of the Fourier transform segment, the limit is evaluated as the length tends to infinity. The results are then arranged in a form suitable for machine computation in accordance with the approximation for $r(t)$ given by equations 3.5 and 3.6.

The reader is referred to Garrison's paper [16] for the analytical development of the aforementioned procedure. Thus, only the final result will be presented here. The total single-sided power spectral density can be expressed as [16]

$$S(f) = (1/T)\psi_{k=0} + (2/T) \sum_{k=1}^{\mu-1} \psi_k + (2/T)\phi_\mu \quad (3.7)$$

where

$$\begin{aligned}\mu &= d && ; d \text{ odd} \\ &= d + 1 && ; d \text{ even}\end{aligned}\quad (3.8)$$

and [16]

$$\begin{aligned}\psi_k &= \frac{1}{2^{c(k+\mu)-3}} \sum_{l=1}^Q \sum_{m=1}^Q \sum_{p=1}^{2^{c(k+\mu)}} \\ &\quad \frac{\sin(\omega - \omega_c + R_{l,m,k}^{(p)}(T/2Q)) \sin(\omega - \omega_c - G_{l,m,k}^{(p)}(T/2Q))}{(\omega - \omega_c + R_{l,m,k}^{(p)})(\omega - \omega_c - G_{l,m,k}^{(p)})} \\ &\quad \cos[2(kQ + l - m)(\omega - \omega_c) - 2H_{l,m,k}^{(p)} - (2m - 1)R_{l,m,k}^{(p)} - (2l - 1)G_{l,m,k}^{(p)}]T/2Q \\ &\quad 0 \leq k < \mu\end{aligned}\quad (3.9)$$

In the preceding "c" represents $\log_2(M)$ where "M" is the number of signaling levels (in this thesis, we consider only binary signaling, i.e. $c=1$). The parameters "H", "R" and "G" are derived, as described in Garrison's Appendix [16], from a matrix formulation of the h_v components specifying the pulse approximation $r_a(t)$. This matrix is referred to as the $\delta_{l,m,k}$ matrix and, as the subscript indicates, each combination of $l=1, Q$, $m=1, Q$ and $k=0, \mu$ corresponds to a different "δ" matrix, of dimension $(k+\mu) \times 3$ (see equations 38, 43, 44, 47, 48 in Garrison's Appendix [16]). $H_{l,m,k}^{(p)}$, $R_{l,m,k}^{(p)}$ and $G_{l,m,k}^{(p)}$ then correspond to the summation of the first, second and third column elements of $\delta_{l,m,k}$ respectively, with the notation $[\]^{(p)}$ indicating a specific (+/-) polarity permutation of the matrix column elements forming H, R, and G. For each matrix $\delta_{l,m,k}$, there are $2^{k+\mu}$ different permutations and these can be "stored" in a matrix P of dimension $(2^{k+\mu}) \times (k+\mu)$, whose elements take on either one of the values +1 or -1. H, R and G are then the sum of products of matrix elements, i.e. [16]

$$H_{l,m,k}^{(p)} = \sum_{i=1}^{k+\mu} \delta_{l,m,k}[i, 1] P[(p), i] \quad (3.10)$$

$$R_{l,m,k}^{(p)} = \sum_{i=1}^{k+\mu} \delta_{l,m,k}[i, 2] P[(p), i] \quad (3.11)$$

$$G_{l,m,k}^{(p)} = \sum_{i=1}^{k+\mu} \delta_{l,m,k}[i,3]P[(p),i] \quad (3.12)$$

where (p) ranges from 1 to $2^{(k+\mu)}$.

An interesting feature of the digital FM power spectral density is the presence of sharp peaks or under specific conditions, the presence of discrete spectral components ("delta" functions). It has been shown [16,25,26] that discrete spectra appear under two conditions:

1. $h = \frac{\epsilon}{\alpha}$ ($\epsilon = 1, 2, 3, \dots$) where "h" is the modulation index and $\alpha (\neq 0)$ represents the area under the frequency pulse $r(t)$. In this case, the spectral impulses appear at integer multiples of half the data rate from the carrier frequency. For normalized NRZ pulses ($\alpha=1$), this implies that discrete spectra occur when the modulation index has an integer value ($h=1.0, 2.0, 3.0, \dots$).
2. When $\alpha=0$ (Manchester coded pulses), discrete spectral components are always present and occur at integer multiples of the data rate from (and including) the carrier frequency.

Thus, when one of the above conditions is met, discrete components are present and these are expressed by the ϕ_μ component of equation 3.7, i.e. [16]

$$\begin{aligned} \phi_\mu = & \frac{1}{2^{2c\mu-3}} \sum_{l=1}^Q \sum_{m=1}^Q \sum_{p=1}^{2^{c\mu}} \\ & \frac{\sin(\omega - \omega_c + R_{l,m,\mu}^{(p)})(T/2Q) \sin(\omega - \omega_c - G_{l,m,\mu}^{(p)})(T/2Q)}{(\omega - \omega_c + R_{l,m,\mu}^{(p)})(\omega - \omega_c - G_{l,m,\mu}^{(p)})} \\ & [\cos(2(\mu Q + l - m)(\omega - \omega_c) - 2QH_{l,m,\mu}^{(p)} - (2m - 1)R_{l,m,\mu}^{(p)}) \\ & - (2l - 1)G_{l,m,\mu}^{(p)}](T/2Q)] \left[\pi \sum_{v=0}^{\infty} \delta((\omega - \omega_c)T - 2\pi v - i\pi) + 0.5 \right] \\ & - 0.5 \cot(\omega - \omega_c)(T/2) \\ & \sin[2(\mu Q + l - m)(\omega - \omega_c) \\ & - 2QH_{l,m,\mu}^{(p)} - (2m - 1)R_{l,m,\mu}^{(p)} - (2l - 1)G_{l,m,\mu}^{(p)}](T/2Q) \end{aligned} \quad (3.13)$$

If none of the aforementioned conditions are met, there are no discrete spectral

components and ϕ_μ is [16]

$$\phi_\mu = \frac{1}{2^{2c\mu-3}} (1/\eta) \sum_{l=1}^Q \sum_{m=1}^Q \sum_{p=1}^{2^{2c\mu}} \frac{\sin(\omega - \omega_c + R_{l,m,\mu}^{(p)})(T/2Q) \sin(\omega - \omega_c - G_{l,m,\mu}^{(p)})(T/2Q)}{(\omega - \omega_c + R_{l,m,\mu}^{(p)})(\omega - \omega_c - G_{l,m,\mu}^{(p)})} \quad (3.14)$$

$$[\cos(2(\mu Q + l - m)(\omega - \omega_c) - 2QH_{l,m,\mu}^{(p)} - (2m - 1)R_{l,m,\mu}^{(p)} - (2l - 1)G_{l,m,\mu}^{(p)})(T/2Q) - \Lambda_k \cos[2((\mu - 1)Q + l - m)(\omega - \omega_c) - 2QH_{l,m,\mu}^{(p)} - (2m - 1)R_{l,m,\mu}^{(p)} - (2l - 1)G_{l,m,\mu}^{(p)}](T/2Q)]$$

where

$$\eta = 1 - 2\Lambda_k \cos((\omega - \omega_c)T) + \Lambda_k^2 \quad (3.15)$$

and

$$\Lambda_k = \prod_{\gamma=0}^{c-1} \cos\left(\frac{2^\gamma \alpha \omega_d T}{(M-1)}\right) \quad (3.16)$$

In the case of binary signaling, $M=2$, $c=1$ and Λ_k reduces to

$$\Lambda_k = \cos(\alpha \omega_d T) \quad (3.17)$$

where $\omega_d = 2\pi f_d$, f_d representing the peak frequency deviation.

3.3 Program Algorithm

The SPECTRUM program source listing (written in FORTRAN) is shown in Appendix F. It was developed and used on an IBM PC-AT. Six subroutines accompany the main program; one computes the permutation matrix, another the various "δ" matrices, three others implement equations 3.9, 3.13 and 3.14, and a sixth is used to interpolate around a singularity in the cotangent function of equation 3.13 (when the cotangent becomes infinite). The main program algorithm can be described as follows:

-Input prompt sequence, preliminary computations and output headers.

-Compute permutation matrix using the subroutine "PERMUT".


```

-Do k=0,mu
  Do l=1,Q
    Do m=1,Q
      -Compute delta matrix using the subroutine "DELTA".

-Main loop: at each frequency step, initialize power and tone power
            to zero.

Do k=0,mu
  Do l=1,Q
    Do m=1,Q
      Do i=1,2**(k+mu)
        -Compute H, R and G.
        -If k < mu
          -Compute psi using the subroutine "PEPSI"
          -If k > 0; power = power + (2 psi)
          -If k = 0; power = power + psi
        -If k = mu
          -If discrete components are present,
            compute phi using the subroutine "PHIV".
          -If singularity in phi, interpolate
            using the subroutine "INTERPLT".
          -If there are no discrete components,
            compute phi using the subroutine "PHIU".

        -power = power + ( 2 phi)

-Output frequency deviation from carrier (normalized to the data
rate), continuous part and discrete components (if present) of
power spectral density.

```

-If not last frequency point, go to top of main loop.

Else

Compute using Simpson's rule and output out-of-band radiation
(if desired).

-End

As reflected in the sample output of Appendix G, the program first prompts the user for the number of subintervals "Q" per symbol period, the sampled filter response and the modulation index. The sampled premodulation filter impulse response contains $Q\mu$ components where μ is related to the pulse duration "d" by equation 3.8. After entering the area under the filter response, the user decides whether or not the out-of-band radiation results are desired. If not, there is no restriction on the values of the frequency start and end points or the frequency step, other than to limit the total number of points to 4000. If however, the out-of-band radiation is desired, the program will only accept a start frequency of zero (in terms of deviation from the carrier normalized to the data rate) and a step no larger than 0.01. Simpson's rule (stated as " $area = (\Delta/6)(f(x_0) + 4f(x_1) + f(x_2))$ " where $\Delta = x_2 - x_0$) is used to approximate the area under the power spectral density curve and if the frequency step is too large, the results of the integral will be inaccurate whenever the spectral curve has a large slope. Thus, if the step were larger than 0.01, the out-of-band radiation could become inaccurately large as the frequency deviation from the carrier increases.

After determining if discrete spectral components are present and setting output headers, the two-dimensional ($2^{2\mu} \times 2\mu$) permutation matrix is computed. If μ is no larger than 3, the matrix is stored in the array "P". If μ is larger than 3, the matrix must be stored on the PC hard disk in order to avoid exceeding the 65536 kilobyte limit per addressable area. The various $\delta_{l,m,k}$ matrices are then computed using the "DELTA" subroutine and are stored in the three-dimensional array "DEL" whose first index, referred to as "PTR", identifies each separate matrix according to the values of K, L and M.

The permutation matrix and the $\delta_{l,m,k}$ matrices are used at every frequency step; precomputing them in this fashion thus expends memory space in order to shorten the computation time. However, since external file (hard disk) accesses are quite time-consuming, the advantage of precomputation is lost when the values of μ become so large as to require hard disk storage. The size of the "DEL" array is such that the program will accept up to 8 subintervals per symbol period as long as the pulse duration (and thus μ) does not exceed 3 symbol periods. On the other hand, if the number of subintervals per symbol period is no larger than 4, the pulse duration (and thus μ) may be up to 5 symbol periods. Note that the size of μ has the largest influence on memory requirements; in fact, the size of the two-dimensional array "P" only depends on μ . Memory has been allocated such that for μ smaller than 3, hard disk storage is not necessary. With 4 bytes per element, the array P(64,6) results in 1536 bytes, DEL(384,10,3) results in 46080 bytes and the array containing the spectral density values at each step, PO(4000), yields 16000 bytes for a total of close to 64 kilobytes. A μ of 5 would require larger "P" and "DEL" arrays, e.g. the permutation matrix array would become P(1024,10) or 40 kilobytes, thus a total of 85 kilobytes (which exceeds the 65 kilobyte limit), without considering the increased size of "DEL". The execution time per frequency point increases quite significantly as the values of μ and Q increase; 20 points of a $Q=2$, $\mu=1$ case are computed in a matter of seconds whereas 20 points of $Q=8$, $\mu=3$ are computed in less than 10 minutes (still reasonable). However, with $\mu=5$, the hard disk accesses extend the computation time to an hour, even with a small "Q".

In view of the approximation format for the FM analysis, use of this method leads to a trade-off between computation time and the asymptotic accuracy of the resulting power spectral density. Clearly, with coarse quantization (Q small), the asymptotic accuracy of the baseband spectra will be poor and in turn, the asymptotic accuracy of the FM spectra will be limited. However, Garrison [16] has shown that, for most pulses of interest (such as those used for test purposes in the next section), the principal portion of the FM spectrum (i.e. within a frequency deviation from the carrier equal to the data rate) is accurately reproduced even with

coarse quantization. Further, the method yields a useful estimate of the spectrum excursions between one and twice the data rate and significant degradation only appears when exceeding twice the data rate as frequency deviation from the carrier. If exact results are desired in the latter region, one may then turn to Baker's method [27]. In addition, Aulin and Sundberg [28] have developed a method yielding greater accuracy than Garrison's method for intermediate (between once and twice the data rate) frequency deviations from the carrier.

As indicated in Appendix G, the double-sided power spectral density appears in the output file, consistent with the fact that the RF spectrum is of interest in this thesis. Thus, the results of equation 3.7 have been divided by 2. Similarly, in computing the out-of-band radiation with respect to both sides of the carrier, both the (double-sided) continuous and discrete spectral components have been multiplied by two to subtract the area on both sides of the carrier. The out-of-band power is expressed relative to 0 dB; the frequency deviation from the carrier yielding 99 percent bandwidth occupancy thus corresponds to the point at which the out-of-band power equals -20 dB.

The "SPECTRUM" program is intended for binary (two-level) signaling. The extension to M-ary signaling would require even more memory space, in view of the expanded permutation and δ matrices; e.g. for quaternary signaling, the δ matrix would now contain all permutations of $(-3, -1, 1, 3)$ instead of simply $(-1, 1)$ for binary signaling.

3.4 Test Results

The SPECTRUM program was tested thoroughly using results from Tey and Tjhung [24] (Manchester coded FSK) and from Garrison [16]¹. The results obtained with the SPECTRUM program are compared with those obtained in the aforementioned publications in figures 3.2, 3.4, 3.6, 3.8 and 3.10, with the corresponding frequency pulses shown in figures 3.1, 3.3, 3.5, 3.7 and 3.9 respectively.

¹ Recall that FSK and FM refer to the absence and presence of pulse shaping respectively.

In all cases, there is excellent agreement between current and previous results.

The test data for figures 3.2, 3.4, 3.6 and 3.8 was obtained from Garrison's paper [16]; the curves generated using SPECTRUM closely match Garrison's results, particularly for pulses of short duration ($d = 1$ or 2) and with coarse quantization ($Q=2$). Very small discrepancies occur when the pulse duration increases ($d=3$, figure 3.6 and 3.8) or the quantization becomes finer² ($Q=4$). This can be attributed to the finite (single) precision used in the computation; larger values of d and Q increase the number of terms in the sums of equations 3.9 through 3.14, thus causing more error propagation. However, the differences are minute (if not negligible) and one may therefore conclude that SPECTRUM accurately reproduces Garrison's technique.

The test data for figure 3.10 was obtained from Tey and Tjhung [24] who used closed form expressions specific to Manchester coded FSK (i.e. with no pulse shaping) to compute the power spectral density of such signals. The results of SPECTRUM are seen to match the test data very closely; small discrepancies occur at high frequency excursions from the carrier (i.e. larger than twice the data rate) where the accuracy of Garrison's technique is limited. Therefore, SPECTRUM should yield accurate results for arbitrary pulse shapes, as long as the frequency excursion from the carrier is not too large.

Finally, note that as Garrison [16] does not mention the method of computing the out-of-band radiation (O.B.R.), it is difficult to interpret the close match between the O.B.R. results of SPECTRUM and those of Garrison. However, assuming that the latter are correct³, the aforementioned fixed integration step of 0.01 times the data rate used by SPECTRUM should suffice to compute the 99 percent bandwidth points for digital FM spectra obtained using arbitrary pulse shapes.

² The reader should keep in mind that in such a case the results are a more accurate representation of the actual spectrum.

³ Tey and Tjhung [24] do not provide 99 percent bandwidth points.

3.5 Summary

This chapter has presented the algorithm development and verification of a FORTRAN software package computing the power spectral density and out-of-band radiation of digital FM signals with arbitrary modulating pulse shapes. These are specified by a duration-limited, level-quantized approximation.

As will be shown in chapter 5, the performance optimization process involves determining combinations of pulse shapes and modulation indices yielding 99 percent in-band power at frequency deviations from the carrier equal to 0.78 and 1.3 times the data rate for the 16 and 9.6 kb/sec cases respectively. Therefore, there is no need to compute the power spectral density at frequency deviations exceeding 1.5 times the data rate, well within the accuracy range of Garrison's method. The use of the SPECTRUM package in this thesis is thus justified. The second tool required for the optimization, i.e. one computing the error rate performance of these signals with discriminator detection, will thus be presented in the next chapter.

Figure 3-1: Rectangular NRZ coded pulse ($d=1$, $Q=2$, $\alpha=1$).

Figure 3-2: Power Spectral Density of NRZ coded binary FSK (frequency pulse as in figure 3.1)
 $h=0.775$. * = 99 percent bandwidth.
 Circles represent Garrison's results [16].

Figure 3-3: Duobinary pulse ($d=2$, $Q=2$, $\alpha=1$).

Figure 3-4: Power Spectral Density of duobinary FSK (frequency pulse as in figure 3.3).

★ = 99 percent bandwidth.

Circles represent Garrison's results [16].

1: $h=0.75$ 2: $h=0.50$ 3: $h=0.25$

Figure 3-5: Modified duobinary pulse ($d=3$, $Q=2$, $\alpha = 0$).

Figure 3-6: Power Spectral Density of modified duobinary FSK (frequency pulse as in figure 3.5).

* = 99 percent bandwidth.

Discrete spectral components not shown.

Circles represent Garrison's results [16].

1: $h=0.75$ 2: $h=0.50$ 3: $h=0.25$

Figure 3-7: Shaped pulse $r(t)$ and its level-quantized approximation, $r_a(t)$ ($d=3$, $Q=4$, $\alpha = 1$).

Figure 3-8: Power Spectral Density of duobinary FM (frequency pulse as in figure 3.7).

* = 99 percent bandwidth.

Circles represent Garrison's results [16].

1: $h=0.75$ 2: $h=0.50$ 3: $h=0.25$

Figure 3-9: Manchester coded rectangular pulse

($d=1$, $Q=2$, $\alpha=0$).

Figure 3-10: Power Spectral Density of Manchester coded FSK
(frequency pulse as in figure 3.9).

* = 99 percent bandwidth.

Discrete spectral components not shown.

1: $h=1.43$ 2: $h=1.00$ 3: $h=0.75$

Chapter 4

BIT ERROR RATE ANALYSIS

4.1 Introduction

This chapter explains the development of a software package (named "BER" for obvious reasons) computing the bit error rate (BER) of both NRZ coded and Manchester coded digital FM signals with discriminator detection. First, the signal model is defined taking into account the distortion effects of the shaping and IF filters examined in Chapter 2. From the statistics of the pertinent noise processes, the bit error rate calculation and thus the program algorithm are defined. Finally, the BER program is found to generate results which match published data.

4.2 Signal Model and Distortion Analysis

The signal term $\Delta\phi$ of equation 2.59, a major component of the input to the decision device, includes the band-restrictive (and thus time-dispersive) effects of both the IF filter and the premodulation filter. In order to compute the correct bit error probability, the data sequences used as input to the simulated system must therefore reflect these distortions. Specifically, it was shown in Chapter 2 that imposing the restrictions $B_T T \geq 0.2$, $\beta \leq 0.5$ and $L \leq 2$ on the Gaussian, Spectrally Raised Cosine (SRC) and Raised Cosine (RC) shaping filters respectively implies that significant intersymbol interference (ISI) effects arise from only the two symbols prior to and the two symbols following the symbol under consideration. In addition, since the shape of the transmitted pulses is rather smooth, one can assume that the additional time dispersion introduced by the receiver IF filter is negligible as long as its bandwidth (specified by $B_R T$) is relatively large compared to the shaping filter bandwidth. As the test data will show, the combination of the aforementioned shaping filter restrictions and $B_R T \geq 1.0$ leads to ISI effects

extending to no more than 5 bits (two bits on each side of the central bit) and therefore the use of bit patterns of length "5" under these conditions is sufficient to provide correct results. The bit error rates can then be computed as follows:

1. Compute the error probability for the central bit in every possible 5-bit pattern, of which there are $2^5=32$.
2. The overall bit error rate is obtained by averaging over all these 32 individual error rates.

Two procedures can be considered in order to compute these 32 error rates. The first involves using a 32-bit pseudorandom sequence as input signal $b(t)$ as shown below [7,15]

$$b(t) = \sum_{i=1}^{32} a_i \delta(t - iT) \quad (4.1)$$

where

$$\begin{aligned} a_i = \{ & -1 -1 -1 -1 +1 -1 +1 -1 +1 +1 +1 -1 \\ & +1 +1 -1 -1 -1 +1 +1 +1 +1 +1 +1 \\ & -1 -1 +1 +1 -1 +1 -1 -1 +1 -1 \} \end{aligned} \quad (4.2)$$

in which the values +1 and -1 represent the bits "1" and "0" respectively. It is seen that in scanning this sequence from left to right, all 32 possible words of length 5 occur once. Hence, the final bit error probability can be evaluated by calculating the BER for each bit in the sequence and then averaging over all 32 error rates (with weights equal to 1/32). Note that in order to perform the simulation properly, it is necessary to "pad" the 32-bit sequence with identical sequences (or at least part of them) at each end so that the error rate calculation for the first two and the last two bits do take into account the ISI extending over five bits. In addition, since the bits "1" and "0" are assumed equiprobable and from a symmetry argument, it is only necessary to compute the overall BER based on the 16 "1" bits in the sequence (i.e. compute and average over 16 individual error rates).

The second method is based on an extension of this symmetry argument. Table 1 shows the 32 5-bit binary patterns matched by pair with their opposite polarity counterpart yielding the same ISI effects, e.g. a pattern (-1+1+1+1+1) is matched

with the pattern (+1-1-1-1-1). However, this argument can be pushed further [10] as (+1+1+1+1-1) will also produce similar ISI effects; grouping the patterns in this manner then results in only 10 5-bit patterns for which the central bit error probability must be computed. Furthermore, by combining some of these 5-bit patterns and appropriately selecting the bits outside a given segment, it is possible to consider only six bit sequences.¹ These are "all ones", alternating (+1-1), alternating (-1-1-1-1)(+1+1+1+1), and repetitions of (-1-1+1+1+1-1-1+1), (+1+1-1-1-1+1+1-1) and (+1-1-1-1+1+1+1-1). The bits which are underlined in each of the sequences are those "central" bits for which the error performance must be calculated. Using this method, the overall bit error probability is obtained by averaging (with weights of either 1/8 or 1/16 as indicated below) over the error rates for the ten underlined bits contained in the six sequences or "composite" patterns, i.e. [10].

$$BER = \frac{1}{16}[r_1 + r_2 + r_5 + r_9] + \frac{1}{8}[r_3 + r_4 + r_6 + r_7 + r_8 + r_{10}] \quad (4.3)$$

where r_i represents the error rate corresponding to bit a_i ($i=1,\dots,10$) as shown in Table 1.

Note that one of the six sequences is constant, one has a period of two bits and four have a period of eight bits. Periodicity of the two sequences is only essential if Fourier series analysis is used [7,10] to determine the IF filtered signal envelope and phase (equations 2.39 and 2.40 respectively). However, in this thesis, the calculation will simply be based on numerical convolution and therefore periodic sequences are not strictly necessary. In any case, the final three sequences (out of six) must remain intact since the addition of outer bits was done to combine two or three of the previous ten 5-bit patterns and not only for the sake of periodicity (i.e. these three 8-bit sequences or "composite" patterns contain more than one bit for which the BER must be calculated); on the other hand, the initial 5-bit

¹ A bit-pattern is here considered as one particular type of time dispersion characteristic with its "central" bit for which the BER must be evaluated (there are ten different such patterns for a string of 5 bits) whereas a sequence is considered as a "composite" pattern combining two or more patterns and thus containing possibly more than one bit for which the BER must be evaluated.

Table 4-1: Grouping through symmetry and selection of outer bits of the 32 5-bit patterns [10].

Binary Patterns	Grouping by symmetry	Probability	Grouping through selection of outer bits	
$+1+1+1+1+1$ $-1-1-1-1-1$ $+1-1+1-1+1$ $-1+1-1+1-1$		1/16 1/16	$+1+1+1+1+1$ $+1-1+1-1+1$	1 2
$-1+1+1+1+1$ $+1-1-1-1-1$ $+1+1+1+1-1$ $-1-1-1-1+1$		1/8	$+1+1+1+1-1$ $-1-1-1-1+1$	3
$-1-1+1-1-1$ $+1+1-1+1+1$ $-1-1+1+1+1$ $+1+1-1-1-1$ $+1+1+1-1-1$ $-1-1-1+1+1$		1/16 1/8	$-1-1+1-1-1$ $+1+1-1-1-1$	4 5
$-1-1+1+1-1$ $+1+1-1-1+1$ $-1+1+1-1-1$ $+1-1-1+1+1$		1/8	$-1-1+1+1-1$ $+1+1-1-1+1$	6 7
$-1+1+1+1-1$ $+1-1-1-1+1$ $+1-1+1+1+1$ $-1+1-1-1-1$		1/8	$-1+1+1+1-1$ $+1-1-1-1+1$	8 9 10
$-1-1+1-1+1$ $+1+1-1+1-1$ $+1-1+1-1-1$ $-1+1-1+1+1$		1/8	$-1-1+1-1+1$ $+1+1-1+1-1$	

patterns can be used for the first three sequences as the outer bits are only present for periodicity purposes.

As will be shown in the following sections of this chapter, evaluating a probability of error for a given bit is by far the greatest concern in terms of computation time as opposed to the number of sequences used as input. Since the first of the two methods just described involves using one 32-bit sequence and computing 16

probabilities of error, whereas the second requires six 8-bit patterns with 10 error probabilities to compute, the latter procedure was selected. In the actual BER program (whose listing appears in Appendix F), four 8-bit periods of the six sequences are used as input in the simulation; the actual calculation of the ten probabilities of error are performed in the third period, with the first, second and fourth periods acting as (more than necessary) "padding" to ensure that the bits under consideration are affected by the desired time dispersion characteristics.

4.3 Noise Statistics and BER Calculation

As explained in Chapter 2, the ability of the receiver to make correct decisions depends solely upon the statistics of the noise processes $\Delta\eta$ and $2\pi N$. In addition, it has just been shown that the overall bit error probability can be computed from the probabilities of error associated with the transmission of ten positive bits² within six bit sequences. With the I and D filter's output phase difference $\Delta\Phi$ as given by equation 2.59, each of these ten individual error probabilities can then be evaluated as

$$\begin{aligned} Pr(\text{Error} | "1" \text{ sent}) &= Pr(\Delta\Phi \leq 0 | "1" \text{ sent}) \\ &= Pr(\Psi \leq -\Delta\phi) \end{aligned} \quad (4.4)$$

where Ψ is the sum of the phase noise and the clicks, i.e [9]

$$\Psi = \Delta\eta + 2\pi N \quad (4.5)$$

In order to compute this probability of error, one must first determine the probability density function of the composite process $(\Delta\eta + 2\pi N)$. Obtaining this p.d.f. can pose great analytical difficulties unless one assumes that all the random variables which make up $(\Delta\eta + 2\pi N)$ are independent. This statement is based on two assumptions [2], one having to do with bandwidth and the other with signal-to-noise ratio.

² A positive bit represents $a_i = +1$ (equation 2.1) corresponding to a "1" bit while a negative bit represents $a_i = -1$ corresponding to a "0" bit.

Figure 4-1: IF filtered noise

a) Autocorrelation function

b) Low-pass equivalent power spectral density.

1. The first assumption states that the components of $\Delta\eta$ are independent; from equations 2.62 and 2.63, this means that phase noise samples one bit duration apart in the case of NRZ encoding and phase noise samples one half bit duration ($T/2$) apart in the case of Manchester encoding are independent. If the IF filtered noise power spectral density is as in figure 4.1 (i.e. the IF filter has a very sharp cut-off), this is a good approximation if $B_R T \geq 1$ for NRZ encoding and if $B_R T \geq 2$ for Manchester encoding, where B_R^3 is the (double-sided) bandwidth of the IF filter and thus approximately of the filtered noise. Since the autocorrelation function of the filtered noise has its first zero at $\tau = 1/B_R$, the motivation for this assumption is clear.
2. The second commonly used [2,3,6,7,8,9] assumption states that the clicks are independent from one another and also from the phase noise samples. This is clearly an assumption expressing an intuitive feeling that the clicks occur rarely, which is reasonable if the signal-to-noise ratio is large.

³ Throughout this thesis, B_R and B_{IF} are used interchangeably and both refer to the double-sided bandwidth of the IF filter.

The probability density function of each random variable η corresponding to a sample at $t=t_o$ of the phase noise $\eta(t)$ (i.e. $\eta(t_o)$) is given by the well-known result [29]

$$p(\eta) = \int_0^\infty dx \frac{x}{\pi} \exp[-(x^2 + \rho(t_o) - 2x\sqrt{\rho(t_o)}\cos(\eta))] \quad |\eta| \leq \pi \quad (4.6)$$

where $\rho(t_o)$ is the value of the signal-to-noise ratio (given by equation 2.48) at time $t = t_o$.

In addition, the random variable N , which denotes the number of clicks in the interval $(t_o - T, t_o)$ for NRZ encoding, is replaced by $N_1 + N_2$ in the case of Manchester encoding where N_1 and N_2 denote the number of clicks in the intervals $(t_o - T, t_o - 0.5T)$ and $(t_o - 0.5T, t_o)$ respectively. N , N_1 and N_2 are all discrete random variables with Poisson distribution [9]

$$Pr[N = k] = \frac{e^{-\bar{N}} \bar{N}^k}{k!} \quad k = 0, 1, \dots \quad (4.7)$$

where $\bar{N}$ is the average number of clicks in $((n-1)T, nT)$ and is given by [9]

$$\bar{N} = \int_{t_o-T}^{t_o} \frac{\dot{\phi}(\tau) e^{-\rho(\tau)}}{2\pi} d\tau \quad (4.8)$$

in the case of NRZ encoding and by [15]

$$\bar{N} = \int_{t_o-T}^{t_o-0.5T} \frac{\dot{\phi}(\tau) e^{-\rho(\tau)}}{2\pi} d\tau + \int_{t_o-0.5T}^{t_o} \frac{\dot{\phi}(\tau) e^{-\rho(\tau)}}{2\pi} d\tau \quad (4.9)$$

in the case of Manchester encoding ($\dot{\phi}(t)$ is the derivative of the IF filtered phase $\phi(t)$ given by equation 2.40). These two expressions follow from the modification of Rice [4, equation 68] to include the filtered modulation and time-varying SNR and then evaluation for large SNR.

Note that in the case of large SNR, for a positive signal bit ($f_d \geq 0$), the majority of clicks are "negative" clicks (i.e. decreasing the I and D output phase difference $\Delta\Phi$ by 2π) and the "positive" clicks (those increasing $\Delta\Phi$ by 2π radians) may be neglected for purposes of computing the bit error rate [2, p.1521]. This implies that $Pr(N > 0) = 0$. However, for notational convenience, it is simpler to use $N \geq 0$ and $\bar{N} \geq 0$ and equation 4.5 is rewritten as

$$\Psi = \Delta\eta - 2\pi N \quad (4.10)$$

The "negative" clicks may thus be considered as "positive" clicks and $Pr(N < 0) = 0$.

Since the probability density function of a sum of independent random variables is equal to the convolution of the individual density functions, the second of the aforementioned assumptions (i.e. large SNR implying independence of phase noise samples and clicks) can now be used to determine the probability density function of the composite noise process $\Psi(t)$ as [9]

$$\begin{aligned} p(\Psi) = & p(\Delta\eta)Pr(N = 0) + p(\Delta\eta + 2\pi)Pr(N = 1) \\ & + p(\Delta\eta + 4\pi)Pr(N = 2) + \dots \end{aligned} \quad (4.11)$$

where $p(\Delta\eta)$ is the probability density function of the phase difference defined by equations 2.62 or 2.63.

As shown in figure 4.2, each of the terms $p(\Delta\eta + 2\pi k)$ is a lobe of width 2π centered about $-2\pi k$. Therefore, from equation 4.4, the probability of error given that a positive bit was sent is found by integrating equation 4.11 from $-\infty$ to $\Delta\phi$, i.e.

$$\begin{aligned} Pr(\Psi \leq -\Delta\phi) = & Pr(N = 0) \int_{-\pi}^{-\Delta\phi} p(\Delta\eta) d\Delta\eta \\ & + Pr(N = 1) + Pr(N = 2) + \dots \end{aligned} \quad (4.12)$$

Since $Pr(N < 0) = 0$, equation 4.12 can be rewritten as

$$Pr(\Psi \leq -\Delta\phi) = Pr(N = 0) \int_{-\pi}^{-\Delta\phi} p(\Delta\eta) d\Delta\eta + (1 - Pr(N = 0)) \quad (4.13)$$

At high SNR, $Pr(N = 0)$ is very close to (but just smaller than) 1.0 and thus, using a symmetry argument, the preceding expression can be approximated by [9]

$$Pr(\Psi \leq -\Delta\phi) \approx \int_{\Delta\phi}^{\pi} p(\Delta\eta) d\Delta\eta + \bar{N} \quad (4.14)$$

Note that since the left-hand side of equation 4.13 is a small positive number for high SNR, so must also be the right-hand side, and since the right-hand side is the sum of two positive numbers, each must then be a small number. Consequently, 4.14 follows from 4.13 with virtually no error in the high SNR region of interest.

Figure 4-2: Composite output noise density for three IF signal-to-noise ratios [a) $\leq$ b) $\leq$ c)]. Shaded area represents probability of error.

Using equation 4.14 to determine error rates requires an analytical expression for the probability density function of the phase noise difference; since $\Delta\eta$ is defined differently for NRZ and Manchester encoding (equations 2.62 and 2.63), so will the p.d.f. $p(\Delta\eta)$. Using the first of the two assumptions (independence of the phase noise samples), the probability density functions of $\Delta\eta$ for NRZ and Manchester

encoding, $p_{NRZ}(\Delta\eta)$ and $p_{MC}(\Delta\eta)$ respectively, can therefore be written as

$$p_{NRZ}(\Delta\eta) = p_1(\eta) * p_2(\eta) \quad (4.15)$$

and as

$$\begin{aligned} p_{MC}(\Delta\eta) &= p_1(\eta) * p_2(\eta) * p_3(2\eta) \\ &= p_{NRZ}(\Delta\eta) * p_3(2\eta) \end{aligned} \quad (4.16)$$

where the asterisk denotes convolution. In the preceding, the subscripts 1, 2 and 3 refer to the time-dependence of $p(\eta)$ (equation 4.6) included in the SNR $\rho(t)$, i.e.

$$p_1(\eta) = p(\eta) \quad \text{with } t = t_o \quad (4.17)$$

$$p_2(\eta) = p(\eta) \quad \text{with } t = t_o - T \quad (4.18)$$

$$p_3(\eta) = p(\eta) \quad \text{with } t = t_o - 0.5T \quad (4.19)$$

where t_o refers to the I and D output sampling time.

The function $p_{NRZ}(\Delta\eta)$ has been derived by Pawula [9] as a generalization of the equal SNR case given by Fleck and Trabka [30], i.e.

$$\begin{aligned} p_{NRZ}(\Delta\eta) &= \frac{e^{-U}}{2\pi} [\cosh(V) + 0.5 \int_0^\pi d\alpha] \\ &\quad [(U \sin(\alpha) + W \cos(\Delta\eta)) \cosh(V \cos(\alpha)) e^{W \sin(\alpha) \cos(\Delta\eta)}] \\ &\quad |\Delta\eta| \leq \pi \end{aligned} \quad (4.20)$$

in which

$$U = 0.5[\rho(t_o) + \rho(t_o - T)] \quad (4.21)$$

$$V = 0.5[\rho(t_o) - \rho(t_o - T)] \quad (4.22)$$

and

$$W = \sqrt{U^2 - V^2} = \sqrt{\rho(t_o)\rho(t_o - T)} \quad (4.23)$$

Using equation 4.16 then yields

$$p_{MC}(\Delta\eta) = \int_{-\pi}^{\pi} 0.5 p_3(0.5(\Delta\eta - \eta')) p_{NRZ}(\eta') d\eta' \quad |\Delta\eta| \leq \pi \quad (4.24)$$

⁴ where η' is a dummy variable of integration and where the p.d.f. of twice the phase noise sample at $t = t_o - 0.5T$ has been rewritten as⁵

$$p_3(2\eta) = 0.5p_3(0.5\chi) \quad (4.25)$$

in which $\chi = 2\eta$. In this form, equation 4.6 can thus be used to compute the p.d.f. $p_3(2\eta)$.

The probability density functions $p_{NRZ}(\Delta\eta)$ and $p_{MC}(\Delta\eta)$ are plotted in figure 4.3 for signal-to-noise ratios (for convenience assumed constant throughout the bit period) of -3 dB, 3 dB and 10 dB. It is immediately seen that both density functions are quite peaked for large SNR and, in this case, can justifiably be assumed to be equal to zero for $|\Delta\eta| \geq \pi$. However, the NRZ density function has a sharper peak than its Manchester coded counterpart; this is reflected in the corresponding probability distribution functions,

$$Pr_{NRZ}(\Delta\eta \geq \Delta\phi) = \int_{\Delta\phi}^{\pi} p_{NRZ}(\Delta\eta) d\Delta\eta \quad (4.26)$$

and

$$Pr_{MC}(\Delta\eta \geq \Delta\phi) = \int_{\Delta\phi}^{\pi} p_{MC}(\Delta\eta) d\Delta\eta \quad (4.27)$$

illustrated in figure 4.4 for an SNR of 10 dB, i.e. increased sharpness of the density function implies a lower error rate for a given value of $\Delta\phi$.

It should be emphasized again that the density functions given by equations 4.20 and 4.24 are only valid under the assumption that the phase noise samples involved in the computation of $\Delta\eta$ are statistically independent, i.e. $B_R T \geq 1.0$ for NRZ coded signals and $B_R T \geq 2.0$ for Manchester coded signals. If the phase noise samples are correlated, one may refer to Pawula et-al [32] who have developed expressions (albeit complex and divided into a number of special cases) for the

⁴ With large SNR, the density functions in 4.20 and 4.24 can be assumed to be equal to zero for $|\Delta\eta| \geq \pi$.

⁵ It can be shown that, if two random variables X and Y are related by $Y = aX + b$ (a, b being constants), their probability density functions $p_X(x)$ and $p_Y(y)$ are related by $p_Y(y) = \frac{1}{|a|} p_X(\frac{y-b}{a})$ [31].

Figure 4-3: Probability density functions of phase noise $\Delta\eta$ for NRZ and Manchester coded signals (constant SNR assumed throughout a bit period).

distribution and density functions of the phase noise difference for NRZ coded signals. However, no closed form expressions have yet been found [32, p.1838] in the Manchester coded case which is the main interest of this thesis.

It should also be noted that in the performance optimization of chapter 5, we

Figure 4-4: Probability distribution functions of the phase noise difference $\Delta\eta$ for NRZ and Manchester coded signals (constant SNR = 10 dB is assumed throughout a bit period.)

will actually be using values of $B_R T$ as low as 1.8; however, the test data will show that the results do not show significant discrepancies with previous studies. In addition, the $B_R T$ values of interest (optimum in terms of BER) are found to be larger than 2.0.

4.4 Summary of Assumptions and Restrictions

The restrictions on the various digital FM system parameters and other assumptions can thus be summarized as follows:

1. Premodulation filter:

- Gaussian: $B_T T \geq 0.2$
- Raised Cosine: $L = 1$ or 2 .

This is to ensure that the intersymbol interference introduced by this filter does not extend to more than two bits on each side of the central bit.

2. The channel is assumed to introduce white Gaussian noise with (one-sided) power spectral density N_o watts/Hz.

3. IF filter:

- NRZ signal format: $B_R T \geq 1.0$
- Manchester signal format: $B_R T \geq 2.0$.

This is to ensure that phase noise samples separated by one bit period in the NRZ case and by one-half bit period in the Manchester case, are uncorrelated. Also, for both signal formats, $B_R T \geq 1.0$ ensures that no additional intersymbol interference is introduced by the IF filter ⁶.

4. "Large" SNR is assumed in order to ensure the independence of the clicks and the phase noise samples.

⁶ It is also assumed that no additional ISI is introduced by the I and D postdetection filter.

4.5 Program Algorithm

The BER program source listing (written in FORTRAN) is shown in Appendix H followed by a sample output in Appendix I. The software package was developed and used on an IBM PC-AT. The main program and the accompanying ten sub-routines are documented in detail and are self-explanatory. We shall thus simply describe the error rate computation procedure and point out some salient features of the code. The BER algorithm can be summarized as follows:

1. The user inputs:
 - The modulation index (h).
 - The type of baseband encoding (NRZ or Manchester).
 - The type of premodulation filter (Gaussian, RC, SRC or none); if one of the first three, the user also selects the filter bandwidth ($B_T T$, β or L), the pulse response duration (in terms of bit periods) and the area under this rectified response.
 - The type of IF filter (Gaussian, second-order Butterworth or Ideal "brick-wall"), the corresponding value of $B_R T$ and the impulse response duration (again in terms of bit periods).
 - The increment and range of values of $\frac{E_b}{N_0}$ for which the error rate must be computed.
2. For each of the final six bit sequences in Table 1, the transmitted data phase $\theta(t)$ is computed as the sum of normalized phase pulses weighted by the data sequence (equation 2.22).
3. The IF filtered phase $\phi(t)$ and the time-varying signal-to-noise ratio $\rho(t)$ are computed using equations 2.40 and 2.48 respectively.
4. The peak of the phase variation within a time span equal to a bit period is identified for each of the ten bits for which the error rate must be computed. These peaks may have been shifted from the "bit times" nT (n integer) due to

the group delay and possible phase distortion of the IF filter.

5. For each of the ten bits, the error rate is computed using equation 4.14, and the overall error rate is obtained by averaging over all bits as in equation 4.3.

Four periods (of eight bits) of the six sequences of Table 1 are input to the simulation and the phase variation peaks are identified within the third period to ensure that no initial transients affect the computation and that the ISI effects of both "sides" of a given bit are taken into account. The transmitted data phase is computed at 16 subintervals within each bit period; the convolution integrals involved in step 3 are then evaluated using Simpson's rule yielding values of $\phi(t)$ and $\rho(t)$ at 8 subintervals per bit period (this size of integration step was found to be small enough to account for any rapid and large variations in the integrand). It should also be noted that, in order to save on computation time, $\phi(t)$ and $\rho(t)$ were not computed for all four sequence periods, but only in the areas of interest, i.e. as of a few bit periods prior to, and up to a few bit periods following each of the ten bits under consideration. In addition, the computation time for Manchester coded signals is longer than for NRZ coded signals⁷.

Finally, the evaluation of the the bit error rates for digital FM systems with different types of premodulation or IF filters can be easily performed by adding a subroutine to compute the desired pulse response (and adjusting the "PREMOD" subroutine⁸ and the input sequence properly) or by including the IF filter impulse response within the "RCFTR" subroutine⁹ (and adjusting the input sequence accordingly).

⁷ A closed form expression is available for the probability density function of $\Delta\eta$ for NRZ coded signals (equation 4.20) whereas numerical convolution must be used to determine this function for Manchester coded signals (equation 4.24).

⁸ This subroutine computes phase pulses for a variety of pulse shapes (refer to Appendix H).

⁹ This subroutine computes the IF filter impulse response specified in the input sequence to the BER program (refer to Appendix H).

4.6 Test Results

The BER program was tested quite thoroughly using results from Pawula [9] (NRZ encoding, no premodulation shaping), Simon and Wang [17] (NRZ encoding, Gaussian shaping) and Tan et-al [15] (Manchester encoding, no shaping). The results obtained with the BER program are compared with those obtained in the three aforementioned publications in figures 4.5 to 4.11. Figures 4.5, 4.6 and 4.7 deal with the case of NRZ encoding, no pulse shaping and a Gaussian IF filter. An excellent match is observed between the BER results and those of Pawula [9] throughout the allowable range of the IF filter time-bandwidth product (i.e. $B_R T \geq 1.0$). As shown in figure 4.8, Pawula [9] obtained identical performance with a Gaussian and a second-order Butterworth IF filter, whereas the BER results show a slight improvement in performance with the Butterworth filter. The two curves in this figure are nonetheless very close and thus confirm the near-equivalence (performance wise) of Gaussian and second-order Butterworth IF filters in the case of NRZ encoding.

The results generated by the BER program when pulse shaping is introduced in a NRZ coded system, are shown in figure 4.9 for a variety of Gaussian shaping filter BT products ranging from $BT=10.0$ (weak shaping) to $BT=0.25$ (strong shaping) thus covering the range allowable in terms of intersymbol interference (i.e. $BT \geq 0.2$). These results are found to be in excellent agreement with the data of Simon and Wang [17] which was obtained with a (one-bit) differential detector. However, it has been shown [19] that, for $h \leq 0.5$, the latter type of detector performs identically to a limiter-discriminator detector, thus justifying the use of these results as test data.

Finally, the BER results for Manchester encoding (no shaping) are shown to match those of Tan et-al [15] in figures 4.10 and 4.11. It should be noted that the BER program still yields accurate results with an IF filter time-bandwidth product of 1.8 (slightly below the lower bound of 2.0 for Manchester encoding).

In all cases tested (1.NRZ, no shaping 2.NRZ, with shaping 3.Manchester, no

shaping), there is excellent agreement between current and previous results over the allowable ranges of shaping filter and IF filter time-bandwidth products. Any (very small) discrepancies tend to occur at high values of $\frac{E_b}{N_0}$ or for low IF filter time-bandwidth products. In these cases, the time-varying SNR $\rho(t)$ (equation 2.48) becomes increasingly large; the probability density function of the phase noise is then much sharper and thus numerical integration (carried out with a fixed integration step) may be slightly less precise. Simultaneously, the corresponding distribution function has a much stronger slope; a small discrepancy in the value of the signal phase variation $\Delta\phi$ will then cause a larger discrepancy in the error rate result. On the other hand, if $\rho(t)$ is too small, the "large SNR" assumption may no longer be valid. However, the test curves reveal that this does not occur within the range of interest for $\frac{E_b}{N_0}$ (i.e. ≥ 5 dB). The BER program should therefore generate reliable bit error rate data in the case of Manchester coded digital FM with pulse shaping for which no results are available in the open literature.

4.7 Summary

This chapter has presented the algorithm development and testing of a FORTRAN software package computing the bit error probabilities of NRZ or Manchester coded digital FM signals with limiter-discriminator detection. A variety of premodulation and IF filter types are already included and others may be added by simply introducing the new filter response in an appropriate subroutine.

Using this BER package, results have been generated for NRZ coded digital FM signals with and without pulse shaping, and also for Manchester coded digital FM signals without pulse shaping. This data was found to match previously published results over a wide range of shaping filter and IF filter time-bandwidth products. In the following chapter, the BER program will therefore be used to generate new bit error rate data for Manchester coded digital FM with pulse shaping. In combination with the power spectral density package presented in chapter 3, the performance optimization outlined in the introductory chapter will also be presented.

Figure 4-5: Comparison of BER vs. $\frac{E_b}{N_0}$ with results in [9] (circles); NRZ encoding, $h=0.5$, no pulse shaping, Gaussian IF filter.

Figure 4-6: Comparison of BER vs. $\frac{E_b}{N_0}$ with results in [9] (circles); NRZ encoding, $h=0.7$, no pulse shaping, Gaussian IF filter.

Figure 4-7: Comparison of BER vs. $\frac{E_b}{N_0}$ with results in [9] (circles); NRZ encoding, $h=1.0$, no pulse shaping, Gaussian IF filter.

Figure 4-8: Comparison of BER vs. $\frac{E_b}{N_0}$ with results in [9] (circles); NRZ encoding, $h=0.707$, no shaping, Gaussian or Butterworth IF filter ($B_{RT} = 1.0$).

Figure 4-9: Comparison of BER vs. $\frac{E_b}{N_0}$ with results in [17] (circles); NRZ encoding, $h=0.5$, Gaussian pulse shaping, Gaussian IF filter.

Figure 4-10: Comparison of BER vs. $\frac{E_b}{N_0}$ with results in [15] (circles); Manchester encoding $h=1.30$, no shaping, Gaussian IF filter.

Figure 4-11: Comparison of BER vs. $\frac{E_b}{N_0}$ with results in [15] (circles); Manchester encoding, no pulse shaping, Gaussian IF filter ($B_R T=1.8$)

Chapter 5

PERFORMANCE OPTIMIZATION

5.1 Introduction

This chapter deals with the performance optimization process outlined in the introductory chapter. The main focus will be on obtaining the combinations of premodulation pulse shaping, modulation index and IF filter yielding the best error rate performances in a channel bandwidth of 25 kHz and at data rates of 9.6 and 16 kilobits per second for the Manchester encoded digital FM system with discriminator detection described in chapter 2. The underlying methodology will be shown to consist of three steps. The first step requires the use of the SPECTRUM program and the last two steps involve the BER program. The results at each step, for both of the aforementioned data rate values, will then be presented and discussed.

5.2 Optimization Procedure

In describing the optimization process, all frequencies are expressed as frequency deviations from the carrier normalized to the data rate. Therefore, with a rate of 16 kbs/sec, the 25 kHz bandwidth corresponds to $(f - f_c)T = 25/16 = 1.56$; at 9.6 kbs/sec, it corresponds to $(f - f_c)T = 25/9.6 = 2.60$. As illustrated in figure 5.1, the channel edge to carrier separation is then equal to 0.78 and 1.30 times the data rate when the latter equals 16 and 9.6 kb/sec respectively.

The optimization process is carried out, for a given data rate, on a number of different premodulation filter types and bandwidths, among the common classes defined in chapter 2. For each of these pulse shapes, the optimum combinations of modulation index and IF filter are then determined using the three step approach described below:

2 of/de 2

MICRO

Figure 5-1: Low-pass equivalent representation of a 25 kHz wide channel.

a) data rate = 16 kb/sec

b) data rate = 9.6 kb/sec.

1. Optimize the modulation index. This step involves determining the admissible range of modulation indices and then optimizing with respect to error rate within this range.
 - 1a) Determine the highest admissible modulation index, " h_{max} ", yielding 99 percent in-band power. The SPECTRUM program is used to find the modulation index producing out-of-band radiation equal to -20 dB at $(f - f_c)T = 0.78$ or 1.30 for data rates of 16 or 9.6 kbs/sec respectively. As illustrated in figure 5.2, $h \geq h_{max}$ leads to less than 99 percent in-band power (out-of-band radiation ≥ -20 dB) and asymptotically tends to 0 percent in-band power (O.B.R. = 0 dB) as h tends to ∞ .
 - 1b) Determine the optimum (in terms of bit error rate) modulation index, " h_{opt} ", within the admissible range, i.e. $h_{opt} \leq h_{max}$. The BER program is used to find the modulation index ($\leq h_{max}$) producing the smallest error

Figure 5-2: Out-of-band radiation vs modulation index
(fixed pulse shape and data rate)

rate. The value of $\frac{E_b}{N_0}$ is arbitrarily set at 15 dB¹ and the second-order Butterworth response is used as IF filter². The latter's BT product must be simultaneously optimized with the modulation index to ensure the determination of the "true" h_{opt} . For each value of "h" considered, the IF filter BT product is varied from 1.50 to 3.50³ in steps of 0.25 and h_{opt} is selected as that value of "h" yielding the best error rate among all these values of $B_R T$. If this is not done, the resulting h_{opt} will depend on the particular value of $B_R T$ chosen for the optimization.

It should be noted that the premodulation filters yielding the best error rates are generally those for which $h_{opt} = h_{max}$. If $h_{opt} < h_{max}$, the bandwidth constraint is being exceeded (i.e. more than 99 percent in-band power); the

¹ $\frac{E_b}{N_0} = 15$ dB produces error rates well below 10^{-4} in the optimum no shaping cases [15] and should still yield "reasonable" error rates in our bandwidth-constrained cases.

² The second-order Butterworth filter is selected as, for the pulse shapes considered, its bit error rate performance (at $\frac{E_b}{N_0} = 15$ dB) is superior to the performance obtained with a Gaussian or Ideal "brick-wall" filter. The choice is however arbitrary.

³ The no shaping optimum BT products have been found to vary between 1.80 and 2.30 [15].

Figure 5-3: IF filter bandwidth optimization for three IF filter types (premodulation filter fixed, $h = h_{opt}$, $\frac{E_b}{N_0} = 15$ dB).

shaping is thus too strong and the resulting superfluous intersymbol interference will degrade the error rate performance beyond what is necessary.

2. Determine the optimum (in terms of bit error rate) IF filter BT products for all three IF filter types. The modulation index is h_{opt} and, once more, $\frac{E_b}{N_0}$ is set at 15 dB. In fact, this step involves the determination of only the Gaussian and Ideal filter optimum bandwidths, the second-order Butterworth filter's optimum bandwidth having been found in step 2. The BT products are again varied in the range 1.50 to 3.50, yielding curves such as in figure 5.3.
3. Evaluate the BER vs. $\frac{E_b}{N_0}$ relationships for the three IF filter types and their corresponding optimum bandwidths (using again $h = h_{opt}$). As shown in figure 5.4, these three curves may be compared with the optimum no shaping case in order to evaluate the performance improvement (if any) obtained by the

Figure 5-4: Comparison of BER vs. $\frac{E_b}{N_0}$ relationships for a given shaping filter with the optimum no shaping case (broken line).

introduction of pulse shaping. The no shaping case is optimized using this same 3-step procedure and the BER vs $\frac{E_b}{N_0}$ comparison curve is the optimum among the three IF filter types.

This optimization process may be repeated for a number of premodulation filter types and bandwidths, as will be done in the next two sections, for data rates of 16 and 9.6 kilobits per second respectively.

5.3 Optimization results (16 kb/sec)

Six different pulse shapes will be considered initially; two Raised Cosine shapes, 1RC and 2RC ⁴, and four Gaussian shapes with $B_T T = 0.25$, 0.50, 0.75 and 1.0.

⁴ 1RC and 2RC denote Raised Cosine shapes with $L=1$ and $L=2$ respectively (refer to chapter 2).

This selection respects the constraints of the BER program ($L \leq 2$ for RC filters and $B_T T \geq 0.2$ for Gaussian filters) and covers a reasonable range of time dispersion effects (Gaussian shaping with $B_T T = 1.0$ is quite mild). Spectrally Raised Cosine (SRC) shapes are not considered here; as shown in chapter 2, these exhibit tails that decay in an oscillatory fashion as opposed to the rapid and steady decay of the Gaussian tails and the total absence of tails for RC shapes. It is thus expected that SRC shaping would not yield a BER improvement over RC or Gaussian shaping.

5.3.1 Modulation Index Optimization

The results of the first step (part 1a) of the optimization process are summarized in Table 5.1⁵ which gives, for different modulation indices "h", the value of frequency deviation from the carrier (normalized to the data rate) within which 99 percent of the signal power is contained. As this is the only step involving the SPECTRUM program, these results will also be used in the 9.6 kb/sec analysis (next section) and thus cover at least the range $(f - f_c)T = 0.78$ to $(f - f_c)T = 1.30$. The spectral occupancy of many index-shaping combinations expected to yield values of $(f - f_c)T$ outside the aforementioned range was not computed as it is of no interest (indicated by a "-" in Table 5.1). One notes that in terms of 99 percent spectral occupancy, the 1RC and $B_T T = 1.0$ shapes are nearly equivalent for most values of "h". In addition, the recurring value of $(f - f_c)T = 1.0$ is due to the presence of a relatively large discrete spectral component at this frequency deviation from the carrier which causes a large "impulsive" decrease in the out-of-band radiation.

The computations were actually carried out with a much finer increment of "h" than 0.25 (i.e. 0.05 or less in the regions of interest); the resulting values of h_{max} (i.e. yielding 99 percent signal power at $(f - f_c)T = 0.78$) are shown in the first row of Table 5.2. The fact that $h_{max} = 0.30$ in the no shaping case indicates that, at this data rate of 16 kb/sec, pulse shaping can be used to raise the highest admissible

⁵ Tables 5.1 to 5.7 and figures 5.5 to 5.15 can be found at the end of this chapter.

modulation index closer to the optimum (in terms of bit error rate) range of $h = 1.20$ to 1.40 ⁶. However, as the results of the modulation index optimization (carried out with a second-order Butterworth IF filter) show, there is a limit to how much shaping may be applied before it significantly degrades the error rate, irrespective of the modulation index used. With 2RC shaping, $h_{opt} = h_{max} = 1.25$ and the best error rate achievable is still worse than in the no shaping case ⁷. The same can be said of the Gaussian $B_T T = 0.25$ case where the shaping is obviously too strong ($h_{max} = 2h_{opt}$). On the other hand, Gaussian shaping with $B_T T = 1.0$ is too mild as it does not permit an increase in the modulation index sufficient to improve the error rate as significantly as with $B_T T = 0.50$ and 0.75 .

Therefore, the optimization process is pursued only with the most promising cases, i.e. Gaussian shaping with $B_T T = 0.50$ and 0.75 . Although $B_T T = 1.0$ also yields an error rate improvement over the no shaping case, it is certainly not as significant. 1RC shaping also yields a small improvement, but this case is also pursued in order to compare the two types of shaping. The power spectral densities of the MCDFM signals with these three forms of shaping and also of the signal with no shaping are shown in figure 5.5 for frequency deviations within the band.

It seems, at this stage, that RC shaping requires a smaller IF bandwidth than Gaussian shaping. Finally, it should be noted that the strongest shaping cases (Gaussian $B_T T = 0.25$ and 2RC) yielded error rates, which although relatively high, were not as sensitive to the IF filter bandwidth as were the cases with milder shaping.

5.3.2 IF Bandwidth Optimization

The next step in the optimization process involves the determination of the optimum (in terms of bit error rate at $\frac{E_b}{N_0} = 15$ dB) Gaussian and Ideal "brick-wall" IF filter bandwidths, the second-order Butterworth IF filter bandwidth having

⁶ Tan et-al. [15] have shown that the exact optimum index depends on the bandwidth of the IF filter (optimum range: $B_R T = 1.80$ to 2.20 for a Gaussian IF filter).

⁷ Even if $h \geq 1.25$ for 2RC shaping, the error rate cannot be improved.

been optimized simultaneously with the modulation index. Figures 5.6, 5.7 and 5.8 illustrate this process for the aforementioned three most promising pulse shapes and the results are summarized in Table 5.3.

It is seen from Table 5.3 that the second-order Butterworth filter yields the best error rate (at the optimum bandwidth) and the Gaussian filter the worst error rate for all three types of shaping. In addition, the figures demonstrate that increased sharpness of the IF filter cut-off (the Ideal and the Gaussian being the most and least sharp respectively) leads to an increased sensitivity of the error rate to changes in the filter bandwidth, with the Butterworth filter yielding an error rate performance superior to the two other types for any given bandwidth and for all three pulse shapes considered.

5.3.3 Final Results

The final step involves the evaluation of the BER vs $\frac{E_b}{N_0}$ relationships. Figures 5.9, 5.10 and 5.11 show these error rate curves for the various shaping cases. Each figure contains curves for all three IF filter types and their corresponding optimum bandwidths as determined in the previous step; the best achievable performance in the "no shaping" case, i.e. with a second-order Butterworth IF filter ($B_R T = 1.80$) is also included for comparison purposes. As summarized in Table 5.4, use of a Gaussian ($B_T T = 0.75$) pulse shape with $h = 0.54$ and a Butterworth or Ideal IF filter is seen to yield a performance improvement (at $\text{BER} = 10^{-4}$)⁸ of 3.75 dB over the no shaping case. Similarly, use of a Gaussian ($B_T T = 0.50$) pulse shape with $h = 0.85$ also yields an improvement of 3.75 dB but only with an Ideal IF filter, the Butterworth IF filter leading to an improvement of 3 dB in this case. 1RC shaping with $h = 0.40$ shows an improvement of 2.75 dB with an Ideal IF filter.

It should also be noted that for all types of shaping considered, a Gaussian IF filter led to a much smaller performance improvement (no more than 1 dB) compared to the Butterworth or Ideal IF filters. On the other hand, Gaussian and

⁸ This error rate is often used for comparison purposes as it represents in most cases, the minimum performance requirement.

second-order Butterworth IF filters performed comparably in a NRZ coded digital FM system [9]. For the Manchester coded system, the Ideal filter (sharpest cut-off) is increasingly superior to the second-order Butterworth as $\frac{E_b}{N_0}$ increases above 20 dB.

5.4 Optimization results (9.6 kb/sec)

The data rate is now smaller implying that for a given pulse shape, the highest admissible modulation index (for 99 percent in-band power) is now larger. Therefore, milder pulse shaping is required to achieve optimum error rate performance. It was shown in the 16 kb/sec analysis that 2RC and Gaussian ($B_T T = 0.25$) shaping both degraded the error rate and that the latter case was too strong as it yielded $h_{max} \gg 1.25$, the optimum modulation index in the unconstrained "no shaping" case. Only 1RC shaping will thus be considered along with Gaussian shaping with $B_T T \geq 0.50$.

5.4.1 Modulation index optimization

In addition to the "no shaping" case and the 1RC pulse shape, only three Gaussian shapes ($B_T T = 0.50, 0.75$ and 1.0) were initially considered. The spectral occupancy results are shown in Table 5.1 and the exact values of h_{max} appear in the first row of Table 5.5. However, after determining h_{opt} (optimum in terms of error rate at $\frac{E_b}{N_0} = 15$ dB) and simultaneously optimizing the Butterworth IF filter bandwidth, it became evident that Gaussian shaping with $B_T T = 1.0$ was still too strong as $h_{opt} \leq h_{max}$ (see Table 5.5). Milder Gaussian shaping responses ($B_T T = 1.50, 1.75$ and 2.00) were thus also considered in order to achieve $h_{opt} = h_{max}$.

The "no shaping" case has $h_{max} = h_{opt} = 1.10$ which is very close to the optimum (without bandwidth constraints). It is thus already evident that pulse shaping will not yield a great error rate improvement as there is not much need to increase h_{max} and thus not much possibility to increase h_{opt} . In fact, only very mild shaping (Gaussian with $B_T T = 2.0$) yields an error rate improvement over the "no shaping"

case and this improvement is negligible. The error rate consistently worsens as the shaping is made stronger (i.e. as $B_T T$ is reduced). On the other hand, 1RC is the mildest Raised Cosine shape available and it is also too strong. One should also note that as the Gaussian shaping becomes milder, the optimum Butterworth IF bandwidth seems to decrease.

Considering these initial results, the optimization process will thus be pursued only with the Gaussian $B_T T=2.0$ and 1RC pulse shapes. The power spectral density within the 25 kHz band of MCDFM signals with these two pulse shapes and with no shaping are shown in figure 5.12 ($h=h_{opt}$). Note that the 1RC shaped signal has $h_{opt} < h_{max}$ and thus has more than 99 percent power within the band.

5.4.2 IF bandwidth optimization

The results of the Gaussian and Ideal IF filter bandwidth optimization are shown in Table 5.6 along with those of the Butterworth filter, repeated for convenience. The optimization curves are not shown here as they have approximately the same shape as those in the 16 kb/sec analysis. The Gaussian IF filter still performs badly compared to the other two; the Butterworth filter is the best in the case of the Gaussian ($B_T T=2.0$) pulse shape whereas the Ideal filter performs best with 1RC shaping. In addition, note that the optimum bandwidths for 1RC shaping are all slightly larger than those for Gaussian shaping whereas the reverse was observed in the 16 kb/sec analysis ⁹.

5.4.3 Final results

The BER vs $\frac{E_b}{N_0}$ relationships were evaluated and figures 5.13 and 5.14 show these error rate curves for Gaussian ($B_T T=2.0$) and 1RC pulse shapes respectively. As in the 16 kb/sec analysis, each figure contains curves for all three IF filter types and their corresponding optimum bandwidths; the optimum "no shaping" case is also included for comparison purposes. As summarized in Table 5.7, use of the

⁹ The observed differences are however quite small and one may thus question the significance of these discrepancies.

Gaussian ($B_T T = 2.0$) pulse shape with $h = 1.17$ and a Butterworth IF filter yields a negligible performance improvement of 0.2 dB over the "no shaping" case. One may thus conclude that, at a data rate of 9.6 kb/sec, pulse shaping cannot yield a performance improvement under a constraint of 99 percent signal power within a 25 kHz bandwidth; the rate-normalized bandwidth requirement is just not stringent enough to warrant use of shaping. In fact, with the 1RC pulse shape (and $h = 1.25$ and an Ideal IF filter), one cannot even match the performance of the "no shaping" case.

The degradation in error rate performance occurring with a strengthening of Gaussian pulse shaping is illustrated in figure 5.15. For $B_T T = 1.75$ and 1.50, $h_{opt} \simeq h_{max}$ and the performance of the "no shaping" case is more or less matched. However, for $B_T T \leq 1.0$, the shaping is much stronger and severe degradation results.

5.5 Summary and Discussion

The effect of premodulation pulse shaping on the bit error rate performance of Manchester coded digital FM with discriminator detection was investigated in this chapter. It has been shown that under a constraint of 99 percent signal power within a 25 kHz bandwidth, and with the appropriate selection of modulation index and IF filter, the following results can be obtained:

- At a data rate of 16 kb/sec, the use of pulse shaping leads to a performance improvement of up to 3.75 dB over reduction of frequency deviation alone.
- At a data rate of 9.6 kb/sec, no noticeable improvement is observed. The performance is comparable to the one obtained using frequency deviation alone.

It has been found that Gaussian shaping leads to an error rate performance superior to the one obtained with Raised Cosine shaping (i.e. a 1 dB advantage in the 16 kb/sec case). For the latter, the pulse duration can be changed only in discrete steps (unless one modifies the original definition), which makes it difficult to "fine tune" h_{max} and thus h_{opt} . On the other hand, the Gaussian BT product can

be varied continuously and it may be possible to improve (at least slightly) upon the performance of the $B_T T = 0.75$ and $B_T T = 0.50$ pulse shapes by considering $0.50 < B_T T < 0.75$.

The advantage of Gaussian over Raised Cosine shaping may be attributed to two factors: reduced Intersymbol Interference (ISI) and reduced noise power. As can be seen from Table 5.4, the optimum IF bandwidths are nearly identical for both types of shaping, implying that the noise bandwidths are also quite similar. Thus the performance advantage of Gaussian shaping is most likely due ISI reduction. This also lends credence to the argument that Spectrally Raised Cosine (SRC) shaping will not perform as well because of its oscillatory decaying tails which produce, for a given bandwidth, more intersymbol interference than Gaussian or RC shaping.

The results of the 16 kb/sec analysis showed that as the shaping became very strong (e.g. 2RC or Gaussian ($B_T T = 0.25$) shaping), the error rates tend to level off and do not yield an improvement over the no shaping case even though h_{max} , and therefore h_{opt} , are within the range of the non-constrained optimum modulation index (≈ 1.25). In fact, 2RC shaping with $h = 1.25$ yields an error rate twice as large as the one obtained with no pulse shaping and $h = 0.30$; as seen in Table 5.2, Gaussian shaping with $B_T T = 0.50$ yields no improvement over $B_T T = 0.75$ although the modulation index is closer to 1.25. Judging from the results for $B_T T = 0.25$, the Gaussian shape (probably $B_T T = 0.35$ to 0.40) yielding $h_{max} = h_{opt} = 1.25$, would result in an error rate degradation from the $B_T T = 0.50$ and $B_T T = 0.75$ cases. It seems that the "eye pattern" becomes so closed that any increase in modulation index towards 1.25 cannot improve the error rate, degraded by excessive intersymbol interference.

As ISI reduction is the dominant cause of error rate improvement, the latter cases are good candidates for the use of a (decision-feedback) equalizer that would suppress intersymbol interference sufficiently to produce an improvement over the no shaping case. Note that an equalizer will always improve the bit error rate over the AWGN channel, but it is most useful at "high" data rates (≥ 16 kb/sec) where

strong shaping is required in order to attain a sufficiently high modulation index.

In a practical implementation, the bandwidth constraint may very well be enforced by the presence of a transmit filter eliminating all RF signal power outside of the (25 kHz-wide) channel, i.e. 1.56 and 2.60 times the data rate of 16 and 9.6 kb/sec respectively. In the 16 kb/sec case, the optimum IF filter time-bandwidth products were found to vary from 1.80 to 2.55, i.e. much larger than the channel bandwidth of 1.56. If such an "optimum" IF filter were used, a transmit filter would then lead to a degradation of the error rate performance as, outside the channel bandwidth, only adjacent channel interference (and no signal component) would be introduced in the detection process. It is thus desirable to use an IF filter whose bandwidth can be reduced from its optimum value without severely affecting the error rate. Figures 5.6 and 5.7 illustrate the sensitivity of the error rate to the IF bandwidth for the two shaping cases (Gaussian $B_T T = 0.75$ and 0.50) yielding the best performance at 16 kb/sec. It is seen that the second-order Butterworth filter is clearly a superior choice to the Ideal filter as reduction of the latter's optimum bandwidth to 1.56 causes a 15 to twenty-fold increase in the error rate as opposed to a smaller than twofold increase with the Butterworth filter. Since the Gaussian $B_T T = 0.50$ shaping case is only optimal with an Ideal IF filter (see Table 5.4), the best system parameter combination at 16 kb/sec thus includes Gaussian ($B_T = 0.75$) shaping, $h = 0.54$ and a second-order Butterworth IF filter whose time-bandwidth product may have to be reduced from its optimum value of 2.30. Note that at 9.6 kb/sec, the optimum IF bandwidths are all smaller than the channel bandwidth of 2.60 and thus the presence of a transmit filter is really of no concern in terms of excessive adjacent channel interference.

The results for both data rates indicate that, for Manchester encoding, the Gaussian IF filter is clearly inferior (by 2 to 3 dB) to the second-order Butterworth and Ideal IF filters which perform comparably. In addition, the 16 kb/sec results show that the Ideal filter (sharpest) becomes superior to the Butterworth at very high $\frac{E_b}{N_0}$ (≥ 20 dB). In contrast, Pawula [9] has shown that Gaussian and second-

order Butterworth IF filters perform similarly in the case of NRZ encoding (see figure 4.8). This seems to indicate that the Manchester signal format requires IF filters with sharper cut-off than the NRZ format. The spectral shapes of the transmitted RF signals tend to support this conclusion. In fact, the spectral lobes of Manchester coded digital FM (figure 3.10) exhibit a much sharper roll-off than those of NRZ coded digital FM (figure 3.2). As optimum performance is expected with an IF filter frequency response shape that "matches" the shape of the signal spectrum, the use of IF filters with sharper cut-off is justified for MCDFM. It should also be noted that, for both NRZ-DFM and MCDFM, the nonlinear phase characteristic of the Butterworth filter has no detrimental effect on the error rate performance.

Finally, the Butterworth filter exhibits (with only one exception in the 16 kb/sec case) an optimum bandwidth lower or equal to that of the Ideal filter. It is thus fair to say that the second-order Butterworth IF filter is a good compromise between the sharpness of the cut-off (necessary to achieve low error rates) and the optimal size of the BT product, desired small at high data rates (≥ 16 kb/sec) if a transmit filter is imposed.

Figure 5-5: Power spectral densities of Manchester coded digital FM signals with 1)no shaping, $h=0.30$
 2)1RC shaping, $h=0.40$
 3)Gaussian shaping ($B_T T=0.75$), $h=0.54$
 4)Gaussian shaping ($B_T T=0.50$), $h=0.85$
 Discrete components not shown.

Table 5-1: Values of frequency deviation from the carrier (normalized to the data rate) where the out-of-band radiation equals -20 dB ("-" indicates not computed).

shaping filter h	Gaussian BT = 1.0	Gaussian BT = 0.75	Gaussian BT = 0.50	Gaussian BT = 0.25	1RC	2RC	no shaping
0.25	0.54	-	-	-	0.56	-	0.66
0.50	0.38	0.76	0.58	-	0.92	-	1.00
0.75	1.00	1.00	0.72	-	1.00	-	1.06
1.00	1.00	1.00	1.00	-	1.00	-	1.26
1.25	1.00	1.00	1.00	0.48	1.00	0.76	1.36
1.50	1.20	1.00	1.00	0.54	1.40	0.96	-
1.75	1.50	1.44	1.00	0.60	-	-	-
2.00	-	1.64	1.26	0.64	-	-	-
2.25	-	-	1.40	0.70	-	-	-
2.50	-	-	-	0.78	-	-	-

Table 5-2: Modulation index optimization (16 kb/sec).

Pulse shape	Gaussian BT = 1.0	Gaussian BT = 0.75	Gaussian BT = 0.50	Gaussian BT = 0.25	1RC	2RC	no shaping
$b_{\max}$	0.40	0.54	0.85	2.50	0.40	1.25	0.30
b_{opt}	0.40	0.54	0.85	1.25	0.40	1.25	0.30
BER (15 dB)	1.7E-02	4.9E-03	4.1E-03	3.0E-02	1.3E-02	9.5E-02	5.3E-02
$BT_{\text{if opt}}$	2.25	2.30	2.55	2.85	1.30	1.80	1.80

Table 5-3: IF bandwidth optimization (16 kb/sec)

Pulse shape IF filter		Gaussian BT = 0.50	Gaussian BT = 0.75	1 RC
Gaussian:	BT_{if}	2.45	2.25	2.05
	BER (15 dB)	2.4 E-02	2.0 E-02	3.3 E-02
Butterworth:	BT_{if}	2.55	2.30	1.80
	BER (15 dB)	4.1 E-03	4.9 E-03	1.3 E-02
Ideal:	BT_{if}	2.35	2.35	2.30
	BER (15 dB)	7.1 E-03	7.0 E-03	1.6 E-02

Figure 5-6: IF filter bandwidth optimization for Gaussian pulse shaping with $B_T T = 0.75$ and $h = 0.54$.

Figure 5-7: IF filter bandwidth optimization for Gaussian pulse shaping with $B_T T = 0.50$ and $h = 0.85$.

Figure 5-8: IF filter bandwidth optimization for 1RC pulse shaping with $h=0.40$.

Figure 5-9: Optimized error rate performance for Gaussian pulse shaping with $B_T T=0.50$, $h=0.85$.

The IF filter responses are

(1) Gaussian (2) Butterworth (3) Ideal.

Curve 4 represents the performance obtained with no shaping.

Figure 5-10: Optimized error rate performance for Gaussian pulse shaping with $B_T T = 0.75$ and $h = 0.54$. The IF filter responses are (1) Gaussian (2) Butterworth (3) Ideal. Curve 4 represents the performance obtained with no shaping.

Figure 5-11: Optimized error rate performance for Raised Cosine pulse shaping with $L=1$ and $h=0.40$. The IF filter responses are (1) Gaussian (2) Butterworth (3) Ideal. Curve 4 represents the performance obtained with no shaping.

Table 5-4: Performance optimization (16 kb/sec)

Pulse shape	Gaussian BT=0.75	Gaussian BT=0.50	IRC	no shaping
η	0.54	0.85	0.40	0.00
IF filter	Butterworth (BT=2.30) Ideal (BT=2.35)	Ideal (BT=2.35)	Ideal (BT=2.30)	Butterworth (BT=1.80)
E_b / N_0 (dB) (for BER=1.0E-04)	19.75	19.75	20.75	23.50

Table 5-5: Modulation index optimization (9.6 kb/sec)

Pulse shape	Gaussian BT=0.50	Gaussian BT=0.75	Gaussian BT=1.0	Gaussian BT=1.50	Gaussian BT=1.75	Gaussian BT=2.0	1RC	no shaping
$\eta_{\max}$	2.05	1.68	1.51	1.28	1.21	1.17	1.40	1.10
η_{opt}	1.25	1.25	1.25	1.25	1.21	1.17	1.25	1.10
BER(15 dB)	3.5E-04	2.0E-05	9.7E-06	4.5E-06	3.1E-06	2.8E-06	7.1E-06	3.1E-06
$\sigma_{T_{\text{if opt}}}$	2.50	2.25	2.25	2.15	1.90	1.90	2.05	2.25

Table 5-6: IF bandwidth optimization (9.6 kb/sec)

IF filter \ Pulse shape	Gaussian BT=2.0	1 RC
Gaussian: BT_{if}	1.90	2.05
BER (15 dB)	1.5 E-05	1.9 E-05
Butterworth: BT_{if}	1.90	2.05
BER (15 dB)	2.8 E-06	7.1 E-06
Ideal: BT_{if}	2.20	2.25
BER (15 dB)	3.7 E-06	4.3 E-06

Figure 5-12: Power spectral density of MCDFM signals with

- (1) no shaping, $h=1.10$
- (2) Gaussian shaping ($B_T T=2.0$), $h=1.17$
- (3) Raised Cosine shaping with $L=1$, $h=1.25$.

In all three cases, $h = h_{opt}$.
Discrete components not shown.

Figure 5-13: Optimized error rate performance for Gaussian pulse shaping with $B_T T=2.0$ and $h=1.17$. The IF filter responses are (1) Gaussian (2) Butterworth (3) Ideal. Curve 4 represents the performance obtained with no shaping.

Figure 5-14: Optimized error rate performance for Raised Cosine pulse shaping with $L=1$ and $h=1.25$. The IF filter responses are (1) Gaussian (2) Butterworth (3) Ideal. Curve 4 represents the performance obtained with no pulse shaping.

Table 5-7: Performance optimization (9.6 kb/sec).

Pulse shape	Gaussian BT=2.0	1RC	no shaping
h	1.17	1.25	1.10
IF filter	Butterworth (BT=1.90)	Ideal (BT=2.25)	Butterworth (BT=2.25)
E_b / N_0 (dB) (for BER=1.0E-04)	13.3	13.7	13.5

Figure 5-15: Optimized error rate performance for Gaussian pulse shaping and a second-order Butterworth IF filter. The modulation indices and the shaping and IF filter bandwidths are

(1) $h=1.21$, $B_T T=1.75$, $B_R T=1.90$
 (2) $h=1.25$, $B_T T=1.50$, $B_R T=1.80$ (3) $h=1.25$
 $B_T T=1.00$, $B_R T=2.25$ (4) $h=1.25$, $B_T T=0.75$,
 $B_R T=2.25$ (5) $h=1.25$, $B_T T=0.50$, $B_R T=2.25$.
 Curve 6 represents the performance obtained with no shaping.

Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The algorithm development and verification of two FORTRAN software packages and the results generated by these with respect to the power spectral density of Manchester coded digital FM signals and their bit error rate performance with discriminator detection, have been presented in this thesis. The SPECTRUM package computes the power spectral density of digital FM signals with arbitrary modulating pulse shapes. These are specified by a duration-limited, level-quantized approximation. Results obtained using SPECTRUM have been shown to match those of Garrison [16], on which the algorithm is based.

The BER package computes the bit error probabilities of NRZ and Manchester coded digital FM signals with limiter-discriminator detection. A variety of premodulation and IF filter types are included and others may be added by simply introducing the new filter response in the appropriate subroutine. For verification purposes, results were generated for NRZ coded digital FM signals with and without pulse shaping and also for Manchester coded digital FM signals without pulse shaping. This data was found to match previously published results [9,15,19] over a wide range of shaping filter and IF filter time-bandwidth products.

Using these two packages, the combinations of pulse shaping, modulation index and IF filtering yielding the best error rates in a Manchester coded digital FM system with discriminator detection, under a constraint of 99 percent signal power within a 25 kHz bandwidth, were then determined for data rates of 16 and 9.6 kb/sec. The calculated results reveal that, at 16 kb/sec (i.e. for a bandwidth corresponding to 1.56 times the data rate), introducing Gaussian premodulation pulse shaping in order to satisfy the band occupancy constraint leads to a performance

improvement of 3.75 dB over frequency deviation reduction. Raised Cosine shaping yields an improvement of 2.5 dB. On the other hand, at 9.6 kb/sec (i.e. for a bandwidth corresponding to 2.60 times the data rate), the introduction of pulse shaping yields a negligible (0.2 dB) improvement over reduction of frequency deviation.

It was also found that second-order Butterworth and Ideal IF filters are superior to a Gaussian IF filter, whereas the Butterworth and Gaussian filters performed comparably in a NRZ coded digital FM system. In addition, at high (≥ 20 dB) values of $\frac{E_b}{N_0}$, the Ideal filter becomes superior to the second-order Butterworth.

At a data rate of 16 kb/sec, the optimum rate-normalized IF bandwidths are larger than the rate-normalized channel bandwidth (1.56); it is therefore preferable in this case to use a (second-order) Butterworth IF filter instead of an Ideal IF filter as the error rate performance obtained with the former is much less sensitive to a reduction in the IF filter bandwidth.

6.2 Recommendations for Future Research

Some of the findings presented in this thesis have indicated the need for further research in the areas described below.

6.2.1 Enhanced detection schemes

It is important to note that the error rate performances reported herein are based on the simple data detector shown in Chapter 2. It is anticipated that the use of a more sophisticated detection scheme; e.g. a Viterbi algorithm and/or as suggested in Chapter 5, a decision feedback equalizer would yield further performance improvement, however at the expense of complexity. Such a detector may prove that the more spectrally efficient pulse shapes (Gaussian with $B_T T = 0.25$ or 2RC) are a viable choice for data rates higher than 16 kb/sec.

6.2.2 Optimized IF filter characteristics

The IF filter responses considered in this thesis were selected among common types of filter characteristics. However, through formal mathematical optimization

(which may involve difficult nonlinear analysis), it should be possible to determine non-conventional IF filter responses which would reduce the $\frac{E_b}{N_0}$ margin between NRZ-DFM and MCDFM signals. As optimum performance should be expected with an IF filter matching the received signal's spectral characteristics, an IF filter consisting of two tuned filters spaced by the data rate (i.e. not maximally flat) would be a good candidate for Manchester coded digital FM. The "Q" and frequency spacing of these filters could then be varied to optimize performance.

6.2.3 Extension of analysis to M-ary signal formats

The data rate may be increased through the use of multilevel signaling at the expense of more power to achieve the same error rate performance. As discussed in chapter 3, the M-ary case can be handled by the SPECTRUM program if the various matrix dimensions are expanded appropriately. The error rate analysis is quite similar for M-ary and binary signaling except for the multipattern averaging process whose complexity is proportional to the number of signal levels.

6.2.4 Use of phase-lock loops

As phase-lock loops have a lower threshold (see Appendix E) than discriminators, they perform adequately over a wider range of signal-to-noise ratios. The analysis of digital FM performance using phase-lock loops is thus warranted.

6.2.5 Use of trellis coded baseband waveforms

When selecting signal points in a constellation, it is generally desired to maximize the minimum distance between points. This can be achieved through the use of trellis coded baseband waveforms. The performance evaluation of digital FM with discriminator detection, in conjunction with such a scheme, is thus justified.

REFERENCES

1. J. Klapper, "Demodulator threshold performances and error rates in angle modulated digital signals", RCA Rev., pp. 226-244, June 1966.
2. J.E. Mazo and J. Salz, "Theory of error rates for digital FM", Bell Sys. Tech. J., vol. 45, pp. 1511-1535, November 1966.
3. D.L. Schilling, E. Hoffman and E.A. Nelson, "Error rates for digital signals demodulated by an FM discriminator", IEEE Trans. Communication Technology, vol. COM-15, pp. 507-517, August 1967.
4. S.O. Rice, "Noise in FM receivers", Time series analysis, M. Rosenblatt, Ed. New York: Wiley, 1963, ch. 25.
5. J. Cohn, "A new approach to the analysis of FM threshold extension", Proc. 1956 NEC, vol. 12, pp. 221-236, 1956.
6. W.R. Bennett and J. Salz, "Binary data transmission by FM over a real channel", Bell Sys. Tech. J., vol. 42, pp. 2387-2426, September 1963.
7. T.T. Tjhung and P.H. Wittke, "Carrier transmission of binary data in a restricted band", IEEE Trans. Communication Technology, vol. COM-18, pp. 295-304, August 1970.
8. D.E. Cartier, "Limiter-discriminator detection performance of Manchester and NRZ coded FSK", IEEE Trans. Aerosp. Electron. Sys., vol. AES-13, pp. 62-70, January 1977.
9. R.F. Pawula, "On the theory of error rates for narrow-band digital FM", IEEE Trans. on Communications, Vol. COM-29, pp. 1634-1643, November 1981.
10. K.K. Yeo and T.T. Tjhung, "Effects of premodulation pulse shaping on digital FM performance", Proc. GLOBECOM 1984, 32.6.
11. T.T. Tjhung, K.K. Yeo and P.H. Wittke, "Error performance analysis for

- narrow-band duobinary FM with discriminator detection", IEEE Trans. on Communications, Vol. COM-33, pp. 399-408, May 1985.
12. T.T. Tjhung, K.K. Yeo and P.H. Wittke, "Effects of pulse shaping and soft decisions on the performance of digital FM with discriminator detection", IEEE Trans. on Communications, Vol. COM-34, pp. 1116-1122, November 1986.
 13. C.Y. Kwan, R.S. Simpson and B.H. Batson, "Performance of biphase digital FM transmission", IEEE Trans. Communication Tech., Vol. COM-25, p. 564, May 1977.
 14. I. Korn, "Comments on limiter-discriminator detection performance of Manchester and NRZ coded FSK", IEEE Trans. Aerosp. Electr. Sys., Vol. AES-16, p. 415, 1980.
 15. C.H. Tan, T.T. Tjhung and H. Singh, "Error rates for narrowband Manchester coded digital FM", Proc. National Telecommunications Conference 1981, D.8.4.
 16. G.J. Garrison, "A power spectral density analysis for Digital FM", IEEE Trans. Communication Tech., Vol. COM-23, pp. 1228-1243, November 1975.
 17. M.K. Simon and C.C. Wang, "Differential detection of Gaussian MSK in a mobile radio environment", IEEE Trans. Vehicular Tech., Vol. VT-33, pp. 307-320, November 1984.
 18. J.G. Proakis, "Digital Communications", McGraw-Hill Book Company, New York, 1983.
 19. M.K. Simon and C.C. Wang, "Differential versus limiter-discriminator detection of narrow-band FM", IEEE Trans. on Communications, pp. 1227-1234, November 1983.
 20. -, "Limiter-discriminator detection of narrowband FM in jamming and multipath environments", Proc. MILCOM, Boston, MA, October 17-20, 1982, pp. 28.1-1 - 28.1-7.
 21. Y. Akaiwa, I. Takase, S. Kojima, M. Ikoma and N. Saegusa, "Performance of baseband-bandlimited multilevel FM with discriminator detection for digital

- mobile telephony", Trans. IECE Japan, vol. E64, pp.463-469, July 1981.
22. T. Aulin, N. Rydbeck and C.W. Sundberg, "Continuous phase modulation - Part 2: Partial response signaling", IEEE Trans. Communications, Vol. COM-29, pp. 210-225, March 1981.
 23. S. Haykin, "Communication Systems", John Wiley and Sons Inc., New York, 1983.
 24. W.M. Tey and T.T. Tjhung, "Characteristics of Manchester-coded FSK", IEEE Trans. Communications, Vol. COM-27, pp. 209-216, January 1979.
 25. W.C. Jakes, "Microwave Mobile Communications", John Wiley and Sons Inc., New York, 1974.
 26. R.R. Anderson and J. Salz, "Spectra of digital FM", Bell Sys. Tech. J., Vol. 44, pp. 1165-1189, July-August 1965.
 27. T.J. Baker, "Asymptotic behavior of digital FM spectra", IEEE Trans. Communication Tech., Vol. COM-22, pp. 1585-1594, October 1974.
 28. T. Aulin and C.W. Sundberg, "Numerical calculation of spectra for digital FM signals", Proc. National Telecommunications Conference 1981, D.8.3.
 29. W.R. Bennett, "Methods of solving noise problems", Proc. IRE, Vol. 44, pp. 609-638, 1956.
 30. J.T. Fleck and E.A. Trabka, "Error probabilities of multiple-state differentially coherent phase-shift keyed systems in the presence of white Gaussian noise", Detect. Memo. 2A, in "Investigation of digital data communication systems", J.G. Lawton, Ed., Cornell Aeronaut. Lab., Inc., Buffalo, N.Y., Rep. UA-1420-S-1, January 1961. Available as NTIS Doc. AD256584.
 31. A. Papoulis, "Probability, random variables and stochastic processes", McGraw-Hill Book Company, New York, 1984.
 32. R.F. Pawula, S.O. Rice and J.H. Roberts, "Distribution of the phase angle between two vectors perturbed by Gaussian noise", IEEE Trans. Communications, Vol. COM-30, pp. 1828-1841, August 1982.

33. M. Ishizuka and K. Hirade, "Optimum Gaussian filter and deviated-frequency-locking scheme for coherent detection of MSK", IEEE Trans. Communications, Vol. COM-28, pp. 850-854, June 1980.
34. I. Stegun and M. Abramowitz, "Handbook of mathematical functions", Dover Publications, New York, 1964.

Appendix A

Gaussian filter 3 dB bandwidth

The low-pass equivalent transfer function $H(f)$ and the corresponding impulse response of a Gaussian filter are generally given by [33]

$$H(f) = e^{-\alpha^2 f^2} \quad (A.1)$$

$$h(t) = \frac{\sqrt{\pi}}{\alpha} e^{-\frac{\pi^2 t^2}{\alpha^2}} \quad (A.2)$$

where α is a coefficient associated with the filter bandwidth as described below. The 3 dB-down bandwidth B and the equivalent noise bandwidth B_n are defined respectively by

$$|H(\frac{B}{2})|^2 = \frac{1}{2} \quad (A.3)$$

and

$$B_n = \int_{-\infty}^{+\infty} |H(f)|^2 df \quad (A.4)$$

Substituting A.1 into A.3 and A.4 yields

$$B\alpha = \sqrt{2\ln(2)} = 1.1774 \quad (A.5)$$

and

$$B_n\alpha = \sqrt{\frac{\pi}{2}} = 1.2533 \quad (A.6)$$

Therefore, the following relationship holds between B and B_n

$$\frac{B}{B_n} = 2\sqrt{\frac{\ln(2)}{\pi}} = 0.9386 \quad (A.7)$$

Substituting A.6 into A.1 yields the Gaussian low-pass transfer function used in this thesis, i.e.

$$H(f) = e^{-\frac{\pi f^2}{2B^2}} \quad (A.8)$$

where B_n has been approximated by B . Equation A.7 shows that the difference between these two quantities is small and, for practical purposes, can thus be neglected.

Appendix B

The Q function and its series approximation

The "Q" function, otherwise known as the Gaussian probability integral, is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} dy \quad (B.1)$$

Related functions are the error function and the complementary error function defined respectively as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy \quad (B.2)$$

and

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2} dy \quad (B.3)$$

A simple change of variables ($y' = \frac{y}{\sqrt{2}}$) shows that

$$Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right) \quad (B.4)$$

The complementary ~~error~~ function can be approximated by a finite series as follows [34]

$$\text{erfc}(x) = (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) e^{-x^2} \quad (B.5)$$

where

$$t = \frac{1}{(1 + px)} \quad (B.6)$$

and where

$$a_1 = 0.254829592$$

$$a_2 = -0.284496736$$

$$a_3 = 1.421413741$$

$$a_4 = -1.453152027$$

$$a_5 = 1.061405429 \text{ and}$$

$$p = 0.3275911$$

Using equation B.4, the Q function can thus be approximated as

$$Q(x) = \frac{1}{2}(a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5)e^{-\frac{x^2}{2}} \quad (B.7)$$

where

$$t = \frac{1}{(1 + \frac{px}{\sqrt{2}})} \quad (B.8)$$

and where a_1 , a_2 , a_3 , a_4 and a_5 have the aforementioned values.

Appendix C

Low-pass representation of band-pass signals

A real signal $x(t)$ is a band-pass signal if its Fourier transform $X(f)$ is non-negligible only in a band of frequencies of total extent B , say, centered about some frequency $\pm f_c$ generally referred to as the carrier frequency. In the majority of communication signals, the bandwidth B is small compared to f_c , and so such a signal is also referred to as a narrow-band signal.

One defines the "complex envelope" $\tilde{x}(t)$ as follows [23]

$$x(t) = \text{Re}[\tilde{x}(t)\exp(j2\pi f_c t)] \quad (C.1)$$

In general, $\tilde{x}(t)$ is a complex-valued quantity and so it may be expressed as

$$\tilde{x}(t) = x_c(t) + jx_s(t) \quad (C.2)$$

where $x_c(t)$ and $x_s(t)$ are both real-valued low-pass functions. The "canonical" form of the band-pass signal $x(t)$ is then

$$x(t) = x_c(t)\cos(2\pi f_c t) - x_s(t)\sin(2\pi f_c t) \quad (C.3)$$

One refers to $x_c(t)$ as the "in-phase" component of the band-pass signal and to $x_s(t)$ as the "quadrature" component of the signal, both with respect to the carrier wave $\cos(2\pi f_c t)$. The complex envelope $\tilde{x}(t)$ may thus be pictured as a time-varying phasor at the origin of the $x_c - x_s$ plane. The end of the phasor moves about in the plane, and at the same time rotates with an angular velocity equal to $2\pi f_c$ radians per second. The given signal $x(t)$ is the projection of this time-varying phasor on a fixed line.

Now, let the signal $x(t)$ be applied to a linear time-invariant band-pass system with impulse response $h(t)$. This band-pass response may also be written in the canonical form

$$h(t) = h_c(t)\cos(2\pi f_c t) - h_s(t)\sin(2\pi f_c t) \quad (C.4)$$

where the in-phase and quadrature components $h_c(t)$ and $h_s(t)$ respectively, are related to the complex impulse response $\tilde{h}(t)$ by

$$\tilde{h}(t) = h_c(t) + j h_s(t) \quad (C.5)$$

Therefore, as with the signal $x(t)$

$$h(t) = \text{Re}[\tilde{h}(t) \exp(j\pi f_c t)] \quad (C.6)$$

Now, if the output of the system, $y(t)$, is given by

$$y(t) = h(t) * x(t) \quad (C.7)$$

where the asterisk denotes convolution, then the complex envelope $\tilde{y}(t)$ is [22]

$$\tilde{y}(t) = \tilde{h}(t) * \tilde{x}(t) \quad (C.8)$$

Substituting equations C.2 and C.5 into the preceding yields

$$\tilde{y}(t) = [h_c(t) + j h_s(t)] * [x_c(t) + j x_s(t)] \quad (C.9)$$

Because convolution is distributive, this may be rewritten as

$$\tilde{y}(t) = [h_c(t) * x_c(t) - h_s(t) * x_s(t)] + j [h_s(t) * x_c(t) + h_c(t) * x_s(t)] \quad (C.10)$$

Defining the complex envelope $\tilde{y}(t)$ in terms of its in-phase and quadrature components as

$$\tilde{y}(t) = y_c(t) + j y_s(t) \quad (C.11)$$

yields

$$y_c(t) = h_c(t) * x_c(t) - h_s(t) * x_s(t) \quad (C.12)$$

and

$$y_s(t) = h_s(t) * x_c(t) + h_c(t) * x_s(t) \quad (C.13)$$

It should be noted that if one assumes a purely real low-pass equivalent impulse response, the quadrature component $h_s(t)$ is equal to zero. Such is the case in section 2.3.1 of this thesis where, for notational convenience, the low-pass equivalent response is named $h(t)$ and the band-pass response $h^*(t)$ (instead of $h(t)$ in this appendix).

Appendix D

Frequency Discriminator

Basically, a frequency discriminator consists of a "slope circuit" followed by an envelope detector. An ideal slope circuit is characterized by a transfer function that is purely imaginary, varying linearly with frequency inside a prescribed frequency interval. Consider the transfer function [23]

$$\begin{aligned}
 H_1(f) &= j2\pi a(f - f_c + \frac{B_f}{2}) & ; f_c - \frac{B_f}{2} \leq f \leq f_c + \frac{B_f}{2} \\
 &= j2\pi a(f + f_c - \frac{B_f}{2}) & ; -f_c - \frac{B_f}{2} \leq f \leq -f_c + \frac{B_f}{2} \\
 &= 0 & ; \text{otherwise}
 \end{aligned} \tag{D.1}$$

where "a" is a constant. It is desired to evaluate the response of this slope circuit, denoted by $s_1(t)$, which is produced by an FM wave $s(t)$ of carrier frequency f_c and transmission bandwidth B_f (i.e. the bandwidth within which the side frequencies of the FM wave are significant). It is assumed that the spectrum of the incoming FM wave $s(t)$ is essentially zero outside the frequency interval $f_c - B_f/2 \leq |f| \leq f_c + B_f/2$. For evaluation of the response $s_1(t)$, it is convenient to replace the slope circuit with an equivalent low-pass filter and to drive this filter with the complex envelope of $s(t)$.

Let $\tilde{H}_1(f)$ denote the complex transfer function of the slope circuit defined by equation D.1. This complex transfer function is related to $H_1(f)$ by

$$\tilde{H}_1(f - f_c) = H_1(f), \quad f \geq 0 \tag{D.2}$$

Hence using D.1 and D.2, one gets

$$\begin{aligned}
 \tilde{H}_1(f) &= j2\pi a(f + B_f/2) & ; -B_f \leq f \leq B_f/2 \\
 &= 0 & ; \text{otherwise}
 \end{aligned} \tag{D.3}$$

The incoming FM wave $s(t)$ is defined by

$$s(t) = \sqrt{2S} \cos[2\pi f_c t + 2\pi f_d \int_0^t m(t') dt'] \quad (D.4)$$

and the complex envelope of this FM wave is

$$\tilde{s}(t) = \sqrt{2S} \exp[j2\pi f_d \int_0^t m(T') dt'] \quad (D.5)$$

Let $\tilde{s}_1(t)$ denote the complex envelope of the response of the slope circuit. The Fourier transform of $\tilde{s}_1(t)$ is then expressed as follows

$$\begin{aligned} \tilde{S}_1(f) &= j2\pi a(f + B_f/2)\tilde{S}(f) & ; -B_f/2 \leq B_f \leq B_f/2 \\ &= 0 & ; \text{otherwise} \end{aligned} \quad (D.6)$$

where $\tilde{S}(f)$ is the Fourier transform of $\tilde{s}(t)$. Now, since the multiplication of the Fourier transform of a signal by the factor $j2\pi f$ is equivalent to differentiation in the time domain, it follows that

$$\tilde{s}_1(t) = a \left[\frac{d\tilde{s}(t)}{dt} + j\pi B_f \tilde{s}(t) \right] \quad (D.7)$$

Substituting D.5 into D.7 yields

$$\tilde{s}_1(t) = j\pi B_f a \sqrt{2S} [1 + 2f_d m(t)/B_f] \exp[j2\pi f_d \int_0^t m(t') dt'] \quad (D.8)$$

The desired response of the slope circuit is therefore

$$\begin{aligned} s_1(t) &= \text{Re}[\tilde{s}_1(t) \exp(j2\pi f_c t)] \\ &= \pi B_f a \sqrt{2S} [1 + 2f_d m(t)/B_f] \cos[2\pi f_c t + 2\pi f_d \int_0^t m(t') dt' + \frac{\pi}{2}] \end{aligned} \quad (D.9)$$

The signal $s(t)$ is a hybrid-modulated wave in which both the amplitude and frequency of the carrier wave vary with the baseband signal $m(t)$. However, provided that one chooses

$$|2f_d m(t)/B_f| \leq 1 \quad (D.10)$$

for all t , then one may use an envelope detector to recover the amplitude variations, and thus, except for a bias term, obtain the original baseband signal. The resulting envelope detector output is therefore

$$|\tilde{s}_1(t)| = \pi B_f a \sqrt{2S} [1 + 2f_d m(t)/B_f] \quad (D.11)$$

Figure D-1: Idealized model of balanced frequency discriminator [23].

The bias term $\pi B_f a \sqrt{2S}$ in the right-hand side of equation D.11 is proportional to the slope "a" of the transfer function of the slope circuit. This suggests that the bias may be removed by subtracting from the envelope detector output $\tilde{s}_1(t)$ the output of a second envelope detector preceded by the complementary slope circuit with a transfer function $H_2(f)$ as described in figure D.1. That is, the respective complex transfer functions of the two slope circuits are related by

$$\tilde{H}_2(f) = \tilde{H}_1(-f) \quad (D.12)$$

Let $s_2(t)$ denote the response of the complementary slope circuit produced by the incoming FM wave $s(t)$. Then, following a procedure similar to that described above, one finds that

$$|\tilde{s}_2(t)| = \pi B_f a \sqrt{2S} [1 - 2f_d m(t)/B_f] \quad (D.13)$$

where $\tilde{s}_2(t)$ is the complex envelope of the signal $s_2(t)$. The differences between the two envelopes in equations D.11 and D.13 is

$$s_0(t) = |\tilde{s}_1(t)| - |\tilde{s}_2(t)| = 4\pi f_d a \sqrt{2S} m(t) \quad (D.14)$$

which is free from bias.

Figure D-2: Balanced frequency discriminator [23]

a) Circuit diagram b) Frequency response

An "ideal frequency discriminator" may thus be modeled as a pair of slope circuits with their complex transfer functions related by equation D.12, followed by envelope detectors and a summer, as in figure D.1. This scheme is called a "balanced frequency discriminator".

The idealized scheme of figure D.1 can be closely realized using the circuits shown in figure D.2a. The upper and lower resonant filter sections of this circuit are tuned to frequencies above and below the unmodulated carrier frequency f_c . In figure D.2b, the amplitude responses of these two tuned filters, together with their total response, have been plotted, assuming that both filters have a high "Q" factor. The linearity of the useful portion of this total response, centered at f_c ,

is determined by the separation of the two resonant frequencies. As illustrated in figure D.2b, a frequency separation of $3B$ gives satisfactory results, where $2B$ is the 3-dB bandwidth of either filter. However, there will be distortion in the output of this frequency discriminator due to the following factors

1. The spectrum of the input FM wave $s(t)$ is not exactly zero for frequencies outside the range $f_c - B_f/2 \leq f \leq f_c + B_f/2$.
2. The tuned filter outputs are not strictly band-limited, and so some distortion is introduced by the low-pass RC filters following the diodes in the envelope detectors.
3. The tuned filter characteristics are not linear over the whole frequency band of the input FM wave $s(t)$.

Nevertheless, by proper design, it is possible to maintain the FM distortion produced by these factors within tolerable limits.

Appendix E

FM Threshold Effect

It is found experimentally that as the input noise is increased so that the carrier-to-noise ratio is decreased, the FM receiver (discriminator) "breaks". At first, individual clicks are heard in the receiver output, and as the carrier-to-noise ratio decreases still further, the clicks rapidly merge into a crackling or sputtering sound. Near the breaking point, expressions yielding the output signal-to-noise ratio under conditions of small noise, begin to fail by predicting values larger than the actual ones. This phenomenon is known as the "threshold effect".

For a qualitative discussion of the FM threshold effect, consider the case when there is no signal present, so that the carrier wave is unmodulated [23]. Then the composite signal at the discriminator input is

$$y(t) = [\sqrt{2S} + \xi_c(t)]\cos(2\pi f_c t) - \xi_s(t)\sin(2\pi f_c t) \quad (E.1)$$

where $\xi_c(t)$ and $\xi_s(t)$ are the in-phase and quadrature components of the narrow-band noise $\xi(t)$ with respect to the carrier wave $\cos(2\pi f_c t)$. The phasor diagram of figure E.1 shows the phase relations between the various components of $y(t)$ in equation E.1. As the amplitudes and phases of $\xi_c(t)$ and $\xi_s(t)$ change with time in a random manner, the point P wanders around the point Q. When the carrier-to-noise ratio is large, $\xi_c(t)$ and $\xi_s(t)$ are usually much smaller than the carrier amplitude $\sqrt{2S}$, and the wandering point P in figure E.1 spends most of its time near the point Q. Thus the angle $\Theta(t)$ is approximately $\frac{\xi_c(t)}{\sqrt{2S}}$ to within a multiple of 2π . The wandering point P occasionally sweeps around the origin and $\Theta(t)$ increases or decreases by 2π radians. Figure E.2 illustrates how, in a rough way, these excursions in $\Theta(t)$ produce impulse-like components in $\dot{\Theta}(t) = \frac{d\Theta}{dt}$. The discriminator output is directly proportional to $\dot{\Theta}(t)$. These impulse-like components have different heights depending on how close the wandering point P comes to the origin O, but all have

Figure E-1: A phasor diagram interpretation [23] of equation E.1.

Figure E-2: Sketch showing impulse-like components in $\dot{\Theta}(t) = \frac{d\Theta}{dt}$ produced by changes of 2π in $\Theta(t)$.

areas nearly equal to $\pm 2\pi$ radians. When the signal shown in figure E.2b is passed through the post-detection integrate-and-dump (low-pass) filter, corresponding but wider impulse-like components are excited in the receiver output and heard as clicks. The clicks are produced only when $\Theta(t)$ changes by $\pm 2\pi$ radians.

From the phasor diagram of figure E.1, one may deduce the conditions re-

quired for clicks to occur. A positive-going click occurs when the envelope $r(t) = \sqrt{\xi_c^2(t) + \xi_s^2(t)}$ and the phase $\psi(t) = \tan^{-1} \frac{\xi_c(t)}{\xi_s(t)}$ of the narrow-band noise $\xi(t)$ satisfy the following conditions

$$r(t) > \sqrt{2S} \quad (E.2)$$

$$\psi(t) < \pi < \psi(t) + d\psi(t) \quad (E.3)$$

$$\frac{d\psi(t)}{dt} > 0 \quad (E.4)$$

These conditions ensure that the phase $\Theta(t)$ of the resultant phasor $y(t)$ changes by 2π radians in the time increment dt , during which the phase of the narrow-band noise increases by the incremental amount $d\psi(t)$. Similarly, the conditions for a negative-going click to occur are as follows

$$r(t) > \sqrt{2S} \quad (E.5)$$

$$\psi(t) > -\pi > \psi(t) + d\psi(t) \quad (E.6)$$

$$\frac{d\psi(t)}{dt} < 0 \quad (E.7)$$

These conditions ensure that $\Theta(t)$ changes by -2π radians during the time-increment dt .

As the carrier-to-noise ratio is decreased, the average number of clicks per unit time increases. When this number becomes appreciably large, the threshold is said to occur.

Appendix F
SPECTRUM program listing

```

* ..... AUTHOR. CHARLES DESPINS (05/86 - 07/86) .....
* ..... SPECTRUM .....
* THIS PROGRAM COMPUTES THE DOUBLE-SIDED POWER SPECTRAL DENSITY AND
* THE AMOUNT OF OUT-OF-BAND RADIATION (ON BOTH SIDES OF THE CARRIER)
* OF DURATION-LIMITED, LEVEL-QUANTIZED, CONTINUOUS-PHASE,
* BINARY (TWO-LEVEL) DIGITAL FM SIGNALS.
*
* REFERENCE. G. J. GARRISON, "A POWER SPECTRAL DENSITY ANALYSIS FOR
* DIGITAL FM", IEEE TRANS. ON COMM.,
* NOVEMBER 1975, PP. 1228-1243.
*
* COMMENTS.
* -POWER = CONTINUOUS COMPONENT OF POWER SPECTRAL DENSITY.
* -TONE = DISCRETE COMPONENT OF THE POWER SPECTRAL DENSITY.
* -NUMBER OF SUBINTERVALS PER SYMBOL PERIOD = IQ (EVEN INTEGER).
* -PULSE DURATION = IR
* -OVERLAP FACTOR = IU = IR (IR ODD)
* IR+1 (IR EVEN)
* -MODULATION INDEX (DOUBLE SIDED) = INDEX
* -AREA UNDER PREMODULATION FILTER IMPULSE RESPONSE = AREA
* -THE ACTUAL QUANTIZATION LEVELS OF A COMPLETE SIGNALING WAVEFORM
* ARE STORED IN A ONE-DIMENSIONAL ARRAY Q.
* -THE PROGRAM WILL ACCEPT UP TO 8 SUBINTERVALS PER SYMBOL PERIOD
* AS LONG AS THE PULSE DURATION DOES NOT EXCEED 3 SYMBOL PERIODS
* HOWEVER, IF THE NUMBER OF SUBINTERVALS PER SYMBOL PERIOD IS
* NO LARGER THAN 4, THE PULSE DURATION MAY BE UP TO 5 SYMBOL
* PERIODS.
* -THE RESULTS ARE STORED IN THE FILE "RESULTS"
* -THE FREQUENCY VALUES ARE EXPRESSED IN TERMS OF THE FREQUENCY
* DEPARTURE FROM THE CARRIER, NORMALIZED TO THE SIGNALING RATE:  $f$ 
* -C, M AND K CORRESPOND TO THE INDICES IN GARRISON'S PAPER
* -IC, IQ AND IU CORRESPOND TO C, Q AND U IN GARRISON'S PAPER
* -H, R AND G CORRESPOND TO THE DELTA MATRIX COLUMN SUMMATIONS
* AS DEFINED IN GARRISON'S APPENDIX.
* -WD SPECIFIES THE FREQUENCY DEVIATION SENSITIVITY FROM THE
* APPLIED BASEBAND SIGNALING VOLTAGE AND IS EQUAL TO THE PEAK
* FREQUENCY DEVIATION (HALF THE DOUBLE-SIDED MODULATION INDEX,
* MULTIPLIED BY  $2\pi$ )
* -FT REPRESENTS THE NORMALIZED FREQUENCY (CW IN RADIANS)
* -PTR IS THE POINTER FOR EACH SEPARATE DELTA MATRIX IN THE ARRAY
* -DEL, WHILE RECORD SPECIFIES THE EXTERNAL FILE RECORD WHERE A
* GIVEN PERMUTATION MATRIX ELEMENT IS STORED.
* .....
* DECLARATIONS:
* DIMENSION DEL(384,10,3),Q(40),PO(400),POTONE(20)
* INTEGER P(64,6),PERM
* COMMON L,M,K,IC,IQ,IU,R,G,H,DW,AREA,WD,DDW
* REAL INDEX
* INTEGER PTR,LXM,RECORD,E1,E2,E3,E4,E5,ITONE(20)
* .....
* INPUT PROMPT SEQUENCE:
* WRITE(*,140)
* 140 FORMAT(25X,"** P.S.D. OF DIGITAL FM **",//)
* PRINT,"Input # of subintervals per symbol period--"
* READ(*,*)IQ
* PRINT,"Input pulse duration (* of symbol periods)--"
* READ(*,*)IR
* IF(MOD(IR,2).EQ.0)THEN
* IU=IR+1
* WRITE(*,130)IQ
* 130 FORMAT("Input sampled filter response (pad with .13,
* 3 zeros)--")
* ELSE
* IU=IR
* PRINT,"Input sampled filter response--"
* ENDIF
* IMAX=IQ*IU
* ITOP=IQ*IR
* READ(*,*)(Q(I),I=1,IMAX)
* PRINT,"Input modulation index--"
* READ(*,*)INDEX
* PRINT,"Input area under filter response--"
* READ(*,*)AREA
* PRINT,"Are out-of-band radiation results desired?"
* PRINT,"Enter 1 if yes, 0 if no--"

```

```

      READ(*,*)E3
      IF(E3 EQ 1) THEN
        E4=0
      ELSE
        E4=1
      ENDIF
      IF(E3 EQ 1) THEN
        PRINT, 'To obtain accurate out-of-band radiation results, the
        & start frequency should be zero and the frequency step should
        & not exceed 0.01, normalized to the symbol rate'
      ENDIF
10    PRINT, 'Input start freq normalized to symbol rate->'
      READ(*,*)FT
      PRINT, 'Input stop freq normalized to symbol rate->'
      READ(*,*)FTM
      PRINT, 'Input freq step normalized to symbol rate->'
      READ(*,*)DFT
      IF((E3 EQ 1) AND ((DFT GT 0.01) OR (FT.NE.0))) THEN
        PRINT, 'WARNING: Frequency step too small or start frequency
        & not equal to 0 Out-of-band radiation results not reliable'
        GO TO 10
      ENDIF
      IF(((FTM-FT)/DFT) GT 4000.0) THEN
        PRINT, 'No more than 4000 points may be computed'
        GO TO 10
      ENDIF
      IF(E3 EQ 1) THEN
        PRINT, 'Out-of-band radiation print increment?'
        PRINT, '1 Same as frequency step'
        PRINT, '2 Every 10 values'
        PRINT, '3 Every 100 values'
        PRINT, 'Enter number ->'
        READ(*,*)E5
        E5=10**(E5-1)
      ENDIF
*****
* PRELIMINARY COMPUTATIONS AND OUTPUT HEADERS
*****
      IC=1
      LXM=10**2
      IUP=10+1
      T=1.00000
      PI=4.0*ATAN(1.0)
      DW=2.0*PI*FT
      DDW=2.0*PI*DFT
      DWM=2.0*PI*FTM
      WD=2.0*PI*INDEX/2.0
      EPSLN=AREA*WD*T/((2**IC-1.0)/PI)
      IF(EPSLN.EQ.0.0 OR EPSLN.EQ.1.0 OR EPSLN.EQ.2.0 OR
3      EPSLN.EQ.3.0 OR EPSLN.EQ.4.0 OR EPSLN.EQ.5.0) E1=1
      OPEN(7, FILE='RESULTS', ACCESS='SEQUENTIAL', STATUS='UNKNOWN')
      WRITE(*,700)
      WRITE(7,700)
700    FORMAT(20X, '*** DIGITAL FM POWER SPECTRAL DENSITY ***', //)
      $30X, '(Binary signaling)', //)
      WRITE(7,800) IQ, IR, INDEX, AREA, DFT
      WRITE(*,800) IQ, IR, INDEX, AREA, DFT
800    FORMAT('  # of subintervals/symbol period= .I3.//
3      Pulse duration= .I3.// Modulation index= .F7.4.//
3      Area under filter response= .F5.2.//
3      Frequency step used for computation= .F7.4.//
3      Filter response=')
      DO 810 I=1, (IUP/5)+1
        J=5*(I-1)
        WRITE(*,820) Q(J+1), Q(J+2), Q(J+3), Q(J+4), Q(J+5)
        WRITE(7,820) Q(J+1), Q(J+2), Q(J+3), Q(J+4), Q(J+5)
820    FORMAT(5(1X, F6.3))
810    CONTINUE
      IF(E1 NE 1.0) THEN
        WRITE(*,875)
        WRITE(7,875)
        375    FORMAT('No discrete spectral components', //)
      ENDIF
      IF(E4 EQ 1) THEN
        IF(E1 EQ 1) THEN
          WRITE(*,900)

```

```

      WRITE(7,900)
300  FORMAT(////, FREQUENCY ( (f-fc)*T ) ,15X, POWER (dB) ,10X,
      &TONE POWER (dB) ,10)
      ELSE
      WRITE(*,901)
      WRITE(7,901)
901  FORMAT(////, FREQUENCY ( (f-fc)*T ) ,15X, POWER (dB) ,10X,
      &TONE POWER (dB) ,10)
      ENDIF
      ENDIF
      LAB=1
      LABM=IFIX(FTM/OFT)
      DO 63 I=1,20
      POTONE(I)=0
      ITONE(I)=0
63  CONTINUE
.....
* PRIOR TO THE MAIN LOOP, THE PERMUTATION MATRIX, OF DIMENSION
* 2*(2*IU), 2*IU IS COMPUTED ALSO, THE 911 LOOP COMPUTES THE
* VARIOUS DELTA MATRICES, ONE FOR EACH COMBINATION OF K=0,10, L=1
* AND M=1,10, THEIR DIMENSION IS (K*IU),3 (SEE GARRISON'S APPENDIX)
.....
      CALL PERMUT(IU,P)
      DO 911 KP=1,IUP
      K=KP-1
      DO 911 L=1,10
      DO 911 M=1,10
      CALL DELTA(L,M,K,10,IU,AREA,Q,DEL)
911  CONTINUE
.....
* THE DO 99 LOOP IS THE MAIN LOOP OF THE PROGRAM, IT IMPLEMENTS
* EQUATION 27, P 1231, IN GARRISON'S PAPER AND IS EXECUTED
* FOR EACH FREQUENCY STEP AFTER THE RESULTS AT A GIVEN FREQUENCY
* ARE PRINTED, THE PROGRAM LOOPS BACK TO LABEL 44 IN ORDER TO
* INITIALIZE VARIABLES PRIOR TO COMPUTING THE NEXT POINT
* THE DO 99 LOOP COMPUTES THE SINGLE-SIDED SPECTRUM, IN ORDER TO GET
* THE DOUBLE-SIDED SPECTRUM, THE RESULTS, BOTH CONTINUOUS POWER
* AND DISCRETE (TONE), ARE DIVIDED BY A FACTOR OF 2.
.....
44  IKTO=1
      POWER=0.0
      TONE=0.0
      E2=0.0
      DO 99 KP=1,IUP
      K=KP-1
      KU=K+IU
      LKU=2*KU
      DO 99 L=1,10
      DO 99 M=1,10
      PTR=(K*LXM)+(10*(L-1))+M
      DO 99 IP=1,LKU
      H=0.0
      R=0.0
      G=0.0
      DO 77 I=1,KU
      IF(IU.GT.3)THEN
      RECORD=((IP-1)*10)+1
      READ(UNIT=4,REC=RECORD,FMT=(12))PERM
      H=H+DEL(PTR,1,1)*PERM
      R=R+DEL(PTR,1,2)*PERM
      G=G+DEL(PTR,1,3)*PERM
      ELSE
      H=H+DEL(PTR,1,1)*P(IP,1)
      R=R+DEL(PTR,1,2)*P(IP,1)
      G=G+DEL(PTR,1,3)*P(IP,1)
      ENDIF
77  CONTINUE
      H=H*WD/10
      R=R*WD
      G=G*WD
      IF(K.NE.10)THEN
      CALL PEPS1(EPS1)
      IF(K.NE.0)POWER=POWER+2.0*EPS1
      IF(K.EQ.0)POWER=POWER+EPS1
      ENDIF
      IF(K.EQ.10)THEN
      IF(E1.NE.1)CALL PH1U(PH1)

```



```

      IF(E1.EQ.1)THEN
        DWS=(DW+AREA*WD)/(2*PI)
        DEC=ABS(DWS-AINT(DWS))
        IF(DEC.LT.(0.1E-4))THEN
          CALL INTERPLT(PHI,DTONE)
        ELSE
          CALL PHIV(PHI,DTONE)
        ENDIF
      ENDIF
      POWER=POWER+PHI*2.0
      TONE=TONE+2.0*DTONE
    ENDIF
39  CONTINUE
      POWER=POWER/2.0
      TONE=TONE/2.0
*****
* OUTPUT POWER SPECTRAL DENSITY AT CURRENT FREQUENCY VALUE
* WHEN DISCRETE SPECTRAL COMPONENTS ARE PRESENT, THEY ARE LOCATED AT
* ODD INTEGER MULTIPLES OF HALF THE DATA RATE FROM THE CARRIER
* FREQUENCY; HOWEVER, ONLY THE COMPONENT AT  $(f-f_c)T = 0.5$  IS
* SIGNIFICANT WHEREAS THOSE AT 1.5, 2.5, 3.5, ... ARE NEGLIGIBLE
* (-70 dB AND LESS). ALTERNATIVELY, WITH AN "AC" PULSE
* FORMAT (AREA=0) THE SPECTRAL LINES APPEAR AT THE CARRIER
* FREQUENCY AND AT INTEGER MULTIPLES OF THE CARRIER FREQUENCY
      PO(LAB)=POWER
      IF(POWER.GT.0.0)THEN
        DPOW=10.0*ALOG10(POWER)
      ENDIF
      IF(E1.EQ.1)THEN
        IF((AREA.EQ.0.0).AND.((ABS(FT).LT.(0.1E-4))OR.
3      (ABS(FT-1).LT.(0.5E-4))OR.(ABS(FT-2).LT.(0.5E-4))OR.
3      (ABS(FT-3).LT.(0.5E-4))OR.(ABS(FT-4).LT.(0.5E-4))OR.
3      (ABS(FT-5).LT.(0.5E-4))OR.(ABS(FT-6).LT.(0.5E-4))))E2=1
        IF((AREA.NE.(0.0)).AND.(ABS(FT-0.5).LT.(0.5E-4)))E2=1
      ENDIF
      IF((E1.EQ.1).AND.(E2.EQ.1))THEN
        IF(LAB.NE.1)PO(LAB)=PO(LAB-1)
        POTONE(IKTO)=TONE
        ITONE(IKTO)=LAB
        IKTO=IKTO+1
      ENDIF
      IF(E4.EQ.1)THEN
        IF((E1.EQ.1).AND.(E2.EQ.1))THEN
          IF(TONE.GT.(0.1E-9))DTONE=10.0*ALOG10(TONE)
          WRITE(7,300)FT,DPOW,DTONE
          WRITE(*,300)FT,DPOW,DTONE
        ELSE
          WRITE(*,301)FT,DPOW
          WRITE(7,301)FT,DPOW
        ENDIF
      ENDIF
300  FORMAT(6X,E11.4,21X,E11.4,11X,E11.4)
301  FORMAT(6X,E11.4,21X,E11.4)
      FT=FT+DFT
      DW=DW+DDW
      LAB=LAB+1
      IF(DW.LE.DWM)GO TO 44
*****
* COMPUTE AND OUTPUT OUT-OF-BAND RADIATION AT CURRENT FREQUENCY
* VALUE. SIMPSON'S RULE IS USED. IN ORDER TO OBTAIN THE O.B.R. WITH
* RESPECT TO BOTH SIDES OF THE CARRIER, BOTH THE CONTINUOUS AND
* DISCRETE COMPONENTS (DOUBLE-SIDED) ARE MULTIPLIED BY 2 TO SUBTRACT
* THE AREA ON BOTH SIDES OF THE CARRIER.
      IF(E3.EQ.1)THEN
        WRITE(*,600)
        WRITE(7,600)
600  FORMAT(///,35X, RESULTS: ///, Powers are double-sided (half
      of their single-sided value). The out-of-band radiation is give
      n with respect to both sides of the carrier. ///)
        IF(E1.EQ.0)THEN
          WRITE(*,601)
          WRITE(7,601)
601  FORMAT(3X, FREQUENCY,5X, POWER (dB),4X, OUT-OF-BAND POWER
      & RELATIVE TO 0 dB)
        ELSE

```



```

NP=N-1
DO 5 M=1,KU
IR=NP-NP/2*2
NP=NP/2
IF(IR.NE.0)P(N,M)=1
IF(NP.EQ.0)GO TO 3
CONTINUE
CONTINUE
ELSE
OPEN(4,STATUS='SCRATCH',ACCESS= DIRECT,FORM= FORMATTED
RECL=4)
NEGONE=-1
ONE=1
DO 11 IP=1,LKU
DO 11 I=1,KU
RECORD=((IP-1)*10)+1
WRITE(UNIT=4,REC=RECORD,FMT=(12) )NEGONE
CONTINUE
DO 13 IP=1,LKU
IPP=IP-1
DO 15 I=1,KU
IR=IPP-IPP/2*2
IPP=IPP/2
IF(IR.NE.0)THEN
RECORD=((IP-1)*10)+1
WRITE(UNIT=4,REC=RECORD,FMT=(12) )ONE
ENDIF
IF(IPP.EQ.0)GO TO 13
CONTINUE
CONTINUE
ENDIF
RETURN
END

```

```

.....
SUBROUTINE PEPSI(EPSI)
.....
* THIS SUBROUTINE IMPLEMENTS EQUATION 18, P.1230, IN GARRISON'S
* PAPER. EPSI IS USED IN THE MAIN PROGRAM FOR VALUES OF X RANGING
* FROM 0 TO (K-1). NOTE THAT WHEN THE ARGUMENT OF A SINE FUNCTION
* IS LESS THAN 0.1E-3, THE APPROXIMATION SIN(X)=X IS MADE.
.....
COMMON L,M,K,IC,IQ,IU,R,G,H,DW
REAL LKU
T=1.0
LKU=2.0**((IC*(K+IU)-3.0)
DWS=DW
PI=4.0*ATAN(1.0)
DWMIN=2.0*PI*0.001
IF(DWS.LT.DWMIN)DW=DWMIN
X1=(DW+R)*T/2.0/IQ
X2=(DW-G)*T/2.0/IQ
Y1=2.0*(K*IQ+L-M)*DW-2.0*IQ*H
Y2=(2.0*M-1)*R+(2.0*L-1.0)*G
Y=(Y1-Y2)*T/2.0/IQ
DWR=ABS(X1)
DWG=ABS(X2)
THR=0.1E-3
IF(DWR.LT.THR.OR.DWG.LT.THR)THEN
IF(DWR.LT.THR.AND.DWG.LT.THR)EPSI1=(T/2/IQ)**2
IF(DWR.LT.THR.AND.DWG.GE.THR)EPSI1=SIN(X2)/(DW-G)*T/2/IQ
IF(DWR.GE.THR.AND.DWG.LT.THR)EPSI1=SIN(X1)/(DW+R)*T/2/IQ
ELSE
EPSI1=SIN(X1)*SIN(X2)/(DW+R)/(DW-G)
ENDIF
EPSI=EPSI1*COS(Y)/LKU
DW=DWS
RETURN
END

```

```

.....
SUBROUTINE PHIU(PHI)
.....

```

```

*****
* THIS SUBROUTINE IMPLEMENTS EQUATION 24, P.1231, IN GARRISON'S
* PAPER. THE RETURNED VALUE IS USED IN THE MAIN PROGRAM WHEN
* BOTH K=IU AND NO DISCRETE SPECTRAL COMPONENTS ARE PRESENT (I.E.
* THE SERIES IN EQUATION 20, P.1230, IS CONVERGENT) AGAIN, WHEN
* THE ARGUMENT OF A SINE FUNCTION IS VERY SMALL, THE APPROXIMATION
* SIN(X)*X IS MADE.
*****

```

```

COMMON L,M,K,IC,IQ,IU,R,G,H,DW,AREA,WD
REAL A,LKU
T=1.0
LKU=2.0**((2.0*IC*IU-3.0))
X1=(DW+R)*T/2/IQ
X2=(DW-G)*T/2/IQ
Y1=2.0*(IU*IQ+L-M)*DW-2.0*IQ*H
Y2=(2.0*M-1.0)*R*(2*L-1)*G
Z1=2.0*((IU-1)*IQ+L-M)*DW-2.0*IQ*H
Z2=(2*M-1)*R*(2.0*L-1)*G
Y=(Y1-Y2)*T/2/IQ
Z=(Z1-Z2)*T/2/IQ
ANEW=1.0
DO 8 I=1,IC
  A=(2**((I-1)))/(2**IC-1)*AREA*WD
  ANEW=ANEW*COS(A)
8 CONTINUE
  IF((DW+R).EQ.0 OR (DW-G).EQ.0) THEN
    IF((DW+R).EQ.0 AND (DW-G).EQ.0) PH11=(T/2/IQ)**2
    IF((DW+R).EQ.0 AND (DW-G).NE.0) PH11=SIN(X2)/(DW-G)*T/2/IQ
    IF((DW+R).NE.0 AND (DW-G).EQ.0) PH11=SIN(X1)/(DW+R)*T/2/IQ
  ELSE
    PH11=SIN(X1)*SIN(X2)/(DW+R)/(DW-G)
  ENDIF
  PH12=COS(Y)-ANEW*COS(Z)
  ETA=1.0-2*ANEW*COS(DW*T)-ANEW**2
  PHI=PH11*PH12/ETA/LKU
  RETURN
END

```

```

*****
SUBROUTINE DELTA(L,M,K,IQ,IU,AREA,C,DEL)
*****

```

```

* THIS SUBROUTINE COMPUTES THE DELTA MATRICES AS EXPLAINED IN THE
* APPENDIX OF GARRISON'S PAPER. THESE (2*IU)*3 MATRICES ARE STORED
* IN THE THREE-DIMENSIONAL ARRAY DEL ACCORDING TO THE INDEX PTR.
* SINCE THERE IS ONE DELTA MATRIX FOR EACH VALUE OF K, L AND M

```

```

DIMENSION BETA(10,3),GAMA(10,3),EXI(10,3),Q(40),DEL(384,10,3)
LXM=IQ**2
PTR=(K*LXM)+(IQ*(L-1))+M
KU=K+IU
DO 1 I=1,KU
  DO 1 J=1,3
    BETA(I,J)=0.0
    GAMA(I,J)=0.0
    EXI(I,J)=0.0
    DEL(PTR,I,J)=0.0
1 CONTINUE
  DO 333 LR=1,IU
    LINK=(IU-LR)*IQ+L-1
    BETA(LR,K,3)=Q(LINK+1)
    IF(LINK.EQ.0) GO TO 333
    BETA(LR,K,1)=-Q(LINK+1)*(L-1)
    DO 3 I=1,LINK
      BETA(LR,K,I)=BETA(LR,K,I)+Q(I)
3 CONTINUE
333 CONTINUE
  DO 555 MR=1,IU
    LINK=(IU-MR)*IQ+M-1
    GAMA(MR,2)=Q(LINK+1)
    IF(LINK.EQ.0) GO TO 555
    GAMA(MR,1)=-Q(LINK+1)*(M-1)
    DO 5 J=1,LINK
      GAMA(MR,1)=GAMA(MR,1)+Q(J)
5 CONTINUE

```

```

555  CONTINUE
      IF (K.NE.0) THEN
        DO 7 I=1,K
          EXI(I,1)=IQ*AREA
        CONTINUE
      ENDIF
      DO 9 I=1,KU
        DO 9 J=1,3
          DEL(PTR,I,J)=EXI(I,J)+BETA(I,J)-GAMA(I,J)
        CONTINUE
      RETURN
      END
.....
      SUBROUTINE PHIV(PHI,PHI1)
.....
      * THIS SUBROUTINE IMPLEMENTS EQUATION 25, P.1231, IN GARRISON'S
      * PAPER. THE RETURNED VALUES ARE USED IN THE MAIN PROGRAM WHEN
      * BOTH K=IU AND DISCRETE SPECTRAL COMPONENTS ARE PRESENT (I.E.
      * THE SERIES IN EQUATION 20, P.1230, IS DIVERGENT). PHI1
      * REPRESENTS THE IMPULSIVE COMPONENT. AGAIN, WHEN THE ARGUMENT
      * OF A SINE FUNCTION IS SMALL, THE APPROXIMATION SIN(X)=X IS MADE
      *
      COMMON L,M,K,IC,IQ,IU,R,G,H,DW,AREA,WD,DDW
      REAL LKU
      T=1.0
      P1=4.0*ATAN(1.0)
      LKU=2.0*(2.0*IC*IU-3.0)
      DWS=DW+AREA*WD
      X1=(DW+R)*T/2/IQ
      X2=(DW-G)*T/2/IQ
      Y1=2.0*((1+IU-0)*IQ+L-M)*DW-2.0*IQ*H
      Y2=(2.0*M-1)*R+(2.0*L-1)*G
      Z1=2.0*((1+IU-0)*IQ+L-M)*DW-2.0*IQ*H
      Z2=(2.0*M-1)*R+(2.0*L-1)*G
      Y=Y1-Y2)*T/2.0/IQ
      Z=(Z1-Z2)*T/2.0/IQ
      DWR=ABS(DW+R)
      DWG=ABS(DW-G)
      IF(DWR.LT.0.1E-5 OR DWG.LT.0.1E-5) THEN
        IF(DWR.LT.0.1E-5 AND DWG.LT.0.1E-5) X=(T/2.0/IQ)**2
        IF(DWR.LT.0.1E-5 AND DWG.GT.0.1E-5) X=SIN(X2)/(DW-G)*T/2.0/IQ
        IF(DWR.GT.0.1E-5 AND DWG.LT.0.1E-5) X=SIN(X1)/(DW+R)*T/2.0/IQ
      ELSE
        X=SIN(X1)*SIN(X2)/(DW+R)/(DW-G)
      ENDIF
      Y=COS(Y)
      Z=SIN(Z)
      * WHEN DWS IS AN INTEGER MULTIPLE OF 2*PI, THE NEXT STATEMENT
      * CAUSES A SINGULARITY, I.E PHI BECOMES INFINITE
      PHI=+0.5*Y-0.5*Z*(COS(DWS*T/2.))/(SIN(DWS*T/2.))
      PHI=PHI*X/LKU
      PHI1=0.0
      PHI1=X*Y*PI/LKU/2.0/PI
      RETURN
      END
.....
      SUBROUTINE INTERPLT(PHI,PHI1)
.....
      * WHEN, IN THE PHIV SUBROUTINE, DWS IS A MULTIPLE OF 2*PI, THE VALUE
      * OF PHI BECOMES INFINITE. THIS SUBROUTINE IS THEN CALLED BY THE
      * MAIN PROGRAM TO INTERPOLATE PHI AROUND THIS SINGULARITY. THE
      * INTERPOLATION STEP IS 0.01 TIMES THE FREQUENCY STEP
      *
      COMMON L,M,K,IC,IQ,IU,R,G,H,DW,AREA,WD,DDW
      IF(DW.NE.0.0) THEN
        DW=DW+(0.01*DDW)
        CALL PHIV(PHI1P,PHI1TP)
        DW=DW-(0.02*DDW)
        CALL PHIV(PHI,PHI1)
        PHI=(PHI+PHI1P)/2.0
      ENDIF

```

```
PH11=(PH11+PH11TP)/2.0  
DW=DW+(0.01*DDW)  
ELSE  
DW=0.001*DDW  
CALL PHIV(PH1,PH11,  
DW=0.0  
ENDIF  
RETURN  
END
```

```
*****  
*****  
*****  
*****  
*****
```

Appendix G

SPECTRUM sample output

*** DIGITAL FM POWER SPECTRAL DENSITY ***
 (Binary signaling)

of subintervals/symbol period= 2
 Pulse duration= 1
 Modulation index= 0.7500
 Area under filter response= 0.00
 Frequency step used for computation= 0.0100
 Filter response=
 1.000 -1.000 0.000 0.000 0.000 0.000

RESULTS

Powers are double-sided (half of their single-sided value). The out-of-band radiation is given with respect to both sides of the carrier.

FREQUENCY	POWER (dB)	TONE POWER (dB)	OUT-OF-BAND POWER REL TO 0 dB
0.0000E+00	-0.5615E+01	-0.2111E+01	0.0000E+00
0.2000E-01	-0.5617E+01		-0.6181E+01
0.4000E-01	-0.5626E+01		-0.6383E+01
0.6000E-01	-0.5641E+01		-0.6594E+01
0.8000E-01	-0.5663E+01		-0.6816E+01
0.1000E+00	-0.5690E+01		-0.7048E+01
0.1200E+00	-0.5724E+01		-0.7291E+01
0.1400E+00	-0.5764E+01		-0.7546E+01
0.1600E+00	-0.5810E+01		-0.7815E+01
0.1800E+00	-0.5863E+01		-0.8098E+01
0.2000E+00	-0.5922E+01		-0.8397E+01
0.2200E+00	-0.5987E+01		-0.8713E+01
0.2400E+00	-0.6058E+01		-0.9048E+01
0.2600E+00	-0.6136E+01		-0.9406E+01
0.2800E+00	-0.6220E+01		-0.9787E+01
0.3000E+00	-0.6310E+01		-0.1020E+02
0.3200E+00	-0.6407E+01		-0.1064E+02
0.3400E+00	-0.6511E+01		-0.1112E+02
0.3600E+00	-0.6621E+01		-0.1165E+02
0.3800E+00	-0.6738E+01		-0.1223E+02
0.4000E+00	-0.6862E+01		-0.1288E+02
0.4200E+00	-0.6992E+01		-0.1363E+02
0.4400E+00	-0.7130E+01		-0.1450E+02
0.4600E+00	-0.7274E+01		-0.1554E+02
0.4800E+00	-0.7425E+01		-0.1688E+02
0.5000E+00	-0.7584E+01		-0.1872E+02
0.5200E+00	-0.7750E+01		-0.2182E+02
0.5400E+00	-0.7923E+01		-0.2510E+02
0.5600E+00	-0.8104E+01		-0.2820E+02
0.5800E+00	-0.8292E+01		-0.3594E+02
0.6000E+00	-0.8488E+01		-0.2256E+02
0.6200E+00	-0.8692E+01		-0.6233E+02
0.6400E+00	-0.8903E+01		-0.2278E+02
0.6600E+00	-0.9123E+01		-0.3602E+02
0.6800E+00	-0.9352E+01		-0.2345E+02
0.7000E+00	-0.9589E+01		-0.5842E+02
0.7200E+00	-0.9834E+01		-0.2369E+02
0.7400E+00	-0.1009E+02		-0.3624E+02
0.7600E+00	-0.1035E+02		-0.2448E+02
0.7800E+00	-0.1063E+02		-0.5000E+02
0.8000E+00	-0.1091E+02		-0.2476E+02
0.8200E+00	-0.1120E+02		-0.3689E+02
0.8400E+00	-0.1150E+02		-0.2564E+02
0.8600E+00	-0.1182E+02		-0.5367E+02
0.8800E+00	-0.1214E+02		-0.2596E+02
0.9000E+00	-0.1247E+02		-0.3742E+02
0.9200E+00	-0.1282E+02		-0.2700E+02
0.9400E+00	-0.1318E+02		-0.4939E+02
0.9600E+00	-0.1355E+02		-0.2737E+02
0.9800E+00	-0.1393E+02		-0.3848E+02
0.1000E+01	-0.1413E+02	-0.1783E+02	-0.4822E+02
0.1020E+01	-0.1474E+02		-0.4822E+02
0.1040E+01	-0.1517E+02		-0.4999E+02

1360E+01	-0	1561E+02
1380E+01	-0	1607E+02
1400E+01	-0	1654E+02
1420E+01	-0	1704E+02
1440E+01	-0	1755E+02
1460E+01	-0	1808E+02
1480E+01	-0	1864E+02
1500E+01	-0	1921E+02
1520E+01	-0	1982E+02
1540E+01	-0	2045E+02
1560E+01	-0	2111E+02
1580E+01	-0	2180E+02
1600E+01	-0	2253E+02
1620E+01	-0	2329E+02
1640E+01	-0	2410E+02
1660E+01	-0	2496E+02
1680E+01	-0	2588E+02
1700E+01	-0	2686E+02
1720E+01	-0	2792E+02
1740E+01	-0	2907E+02
1760E+01	-0	3033E+02
1780E+01	-0	3172E+02
1800E+01	-0	3329E+02
1820E+01	-0	3509E+02
1840E+01	-0	3722E+02
1860E+01	-0	3986E+02
1880E+01	-0	4336E+02
1900E+01	-0	4879E+02
1920E+01	-0	6306E+02
1940E+01	-0	5389E+02
1960E+01	-0	4689E+02
1980E+01	-0	4333E+02
2000E+01	-0	4101E+02
2020E+01	-0	3934E+02
2040E+01	-0	3807E+02
2060E+01	-0	3709E+02
2080E+01	-0	3631E+02
2100E+01	-0	3569E+02
2120E+01	-0	3520E+02
2140E+01	-0	3482E+02
2160E+01	-0	3453E+02
2180E+01	-0	3431E+02
2200E+01	-0	3417E+02
2220E+01	-0	3410E+02
2240E+01	-0	3409E+02
2260E+01	-0	3413E+02
2280E+01	-0	3423E+02
2300E+01	-0	3430E+02
2320E+01	-0	3459E+02
2340E+01	-0	3485E+02
2360E+01	-0	3517E+02
2380E+01	-0	3554E+02
2400E+01	-0	3598E+02
2420E+01	-0	3648E+02
2440E+01	-0	3705E+02
2460E+01	-0	3771E+02
2480E+01	-0	3845E+02
2500E+01	-0	3931E+02
2520E+01	-0	4029E+02
2540E+01	-0	4144E+02
2560E+01	-0	4279E+02
2580E+01	-0	4442E+02
2600E+01	-0	4646E+02
2620E+01	-0	4915E+02
2640E+01	-0	5309E+02
2660E+01	-0	6047E+02
2680E+01	-0	7006E+02
2700E+01	-0	5611E+02
2720E+01	-0	5106E+02
2740E+01	-0	4794E+02
2760E+01	-0	4568E+02
2780E+01	-0	4394E+02
2800E+01	-0	4253E+02
2820E+01	-0	4135E+02
2840E+01	-0	4035E+02
2860E+01	-0	3949E+02

-0 3088E+02

[illegible]

0	2580E+01	-0	3874E+02	-0	1687E+02
0	2600E+01	-0	3808E+02	-0	1688E+02
0	2620E+01	-0	3750E+02	-0	1688E+02
0	2640E+01	-0	3700E+02	-0	1688E+02
0	2660E+01	-0	3655E+02	-0	1688E+02
0	2680E+01	-0	3615E+02	-0	1688E+02
0	2700E+01	-0	3581E+02	-0	1688E+02
0	2720E+01	-0	3551E+02	-0	1689E+02
0	2740E+01	-0	3524E+02	-0	1689E+02
0	2760E+01	-0	3502E+02	-0	1689E+02
0	2780E+01	-0	3483E+02	-0	1689E+02
0	2800E+01	-0	3467E+02	-0	1690E+02
0	2820E+01	-0	3454E+02	-0	1690E+02
0	2840E+01	-0	3444E+02	-0	1690E+02
0	2860E+01	-0	3437E+02	-0	1691E+02
0	2880E+01	-0	3433E+02	-0	1691E+02
0	2900E+01	-0	3431E+02	-0	1691E+02
0	2920E+01	-0	3432E+02	-0	1692E+02
0	2940E+01	-0	3435E+02	-0	1692E+02
0	2960E+01	-0	3441E+02	-0	1692E+02
0	2980E+01	-0	3449E+02	-0	1693E+02
0	3000E+01	-0	3454E+02	-0	1693E+02
0	3020E+01	-0	3473E+02	-0	1700E+02
0	3040E+01	-0	3488E+02	-0	1700E+02
0	3060E+01	-0	3506E+02	-0	1700E+02
0	3080E+01	-0	3526E+02	-0	1701E+02

-0 3810E+02

Appendix H

BER program listing

AUTHOR: CHARLES DESPINS (07/86 - 12/86)

 ***** BER *****
 THIS PROGRAM COMPUTES THE BIT ERROR RATES OF DIGITAL FM SIGNALS.

REFERENCES:

1. R.F. PAWULA
 On the theory of error rates for narrow-band digital fm. IEEE tr. comm., November 1981, pp. 1634-1643.
2. K. YEO and T.T. TJHUNG.
 Effects of premodulation pulse shaping on digital fm performance. Globecom 84, pp. 32.6.1-32.6.5
3. M. SIMON and C.C. WANG.
 Differential detection of gaussian MSK in a mobile radio environment. IEEE tr. on Veh. Tech., November 1981, pp. 307-320
4. R.L. BURDEN.
 Numerical analysis. Prentice, Weber & Schmidt, Boston, Mass., 1981
5. M. ISHIZUKA and K. HIRADE.
 Optimum gaussian filter and deviated-frequency-locking scheme for coherent detection of MSK. IEEE tr. comm., June 1980, pp. 850-857
6. T. AULIN, N. RYDBECK and C. SUNDBERG.
 Continuous phase modulation-Part 1. Full response signaling. IEEE tr. comm., March 1981, pp. 196-209.
7. T. AULIN, N. RYDBECK and C. SUNDBERG.
 Continuous phase modulation-Part 2. Partial response signaling. IEEE tr. comm., March 1981, pp. 210-225.
8. J.G. PROAKIS.
 Digital communications. McGraw-Hill, 1983.
9. M. ABRAWOWITZ and I. STEGUN.
 Handbook of mathematical functions. Dover publications, New York.
10. C.H. TAN, T.T. TJHUNG and H. SINGH.
 Error rates for narrowband Manchester coded digital FM. NTS 1981, pp. D8.4.1-D8.4.5

THE USER SELECTS THE FOLLOWING SYSTEM CHARACTERISTICS:

- MODULATION INDEX
- BASEBAND ENCODING.
 1. NON-RETURN TO ZERO (NRZ)
 2. MANCHESTER
- PREMODULATION PULSE SHAPING:
 1. NONE
 2. GAUSSIAN
 3. SPECTRALLY RAISED COSINE (SRC)
 4. RAISED COSINE (RC)
- PREMODULATION PULSE RESPONSE DURATION
- I.F. PRE-DETECTION RECEIVE FILTER:
 1. GAUSSIAN
 2. BUTTERWORTH
 3. IDEAL "BRICK WALL"
- I.F. FILTER IMPULSE RESPONSE DURATION

THE BIT ERROR RATES ARE COMPUTED FROM EQUATION 29, REF. 1. AVERAGED OVER ALL POSSIBLE BIT PATTERNS. HOWEVER, SINCE PREMODULATION PULSE SHAPING IS CONSIDERED HERE, FIVE-BIT PATTERNS MUST BE USED (INSTEAD OF 3 AS IN REF. 1) IF THE GAUSSIAN PREMODULATION FILTER'S "BT PRODUCT" IS GREATER THAN 0.2; LONGER BIT-PATTERNS WOULD BE NECESSARY IF THIS BT PRODUCT WERE SMALLER THAN 0.2 (REF. 8, P. 311). FOR THE RAISED COSINE OR SPECTRALLY RAISED COSINE PREMODULATION FILTERS, THE USER MUST ENSURE THAT THEIR PULSE RESPONSES DO NOT EXCEED 5 BIT PERIODS WHEN SELECTING THE FILTER PARAMETERS. IN ADDITION, THE I.F. FILTER'S BT PRODUCT MUST BE GREATER THAN 1.0 IF THE ASSUMPTION THAT NOISE SAMPLES ONE BIT PERIOD APART ARE UNCORRELATED IS TO REMAIN VALID (REF. 2, P. 2). THROUGHOUT THE PROGRAM, A NON-CAUSAL ANALYSIS IS PERFORMED (EXCEPT FOR THE BUTTERWORTH "I.F." FILTER) AND SIMPSON'S RULE IS USED TO APPROXIMATE ALL INTEGRALS. THE ANALYSIS IS VALID FOR "NARROWBAND" FM, I.E. WHERE THE I.F. FILTERED PHASE VARIATION OVER ONE BIT PERIOD IS AT MOST 180 DEGREES (SEE REF. 1) WHENEVER

[illegible]

```

PRINT, 4, Raised cosine (RC)
PRINT, Enter number ->
READ(*,*)PS
IF(PS.NE.1)THEN
IF(PS.EQ.2)THEN
931 PRINT, Enter premodulation filter's BT product ->
READ(*,*)BWT
IF((BWT.LT.(0.2)) AND (BBEC.EQ.1))THEN
PRINT, Premodulation BT product must be greater than 0.2
PRINT, Please re-enter ->
GO TO 931
ENDIF
PARM=BWT
ENDIF
IF(PS.EQ.3)THEN
PRINT, Enter BETA (from 0 to 1) ->
READ(*,*)PARM
ENDIF
IF(PS.EQ.4)THEN
PRINT, Enter limit in terms of bit periods ->
ELSE
PRINT, Enter premodulation filter impulse response duration
PRINT, (one-sided) ->
ENDIF
READ(*,*)DUR1
IF(PS.EQ.4)PARM=DUR1
PRINT, Enter area under (rectified) pulse response ->
READ(*,*)AREA
ENDIF

PRINT, Choose IF receive filter
PRINT, 1. Gaussian
PRINT, 2. Butterworth
PRINT, 3. Ideal
PRINT, Enter number ->
READ(*,*)RC
932 PRINT, Enter IF receive filter's BT product ->
READ(*,*)BWR
IF(BWR.LT.(1.0))THEN
PRINT, IF filter BT product must be greater than 1.0
PRINT, Please re-enter ->
GO TO 932
ENDIF
PRINT, Enter IF filter impulse response duration (one sided)
IF(RC.EQ.2)THEN
PRINT, N.B.: Duration should not exceed 2 bit periods for
PRINT, reliable results.
ENDIF
PRINT, (number of bit periods) ->
READ(*,*)DUR2

PRINT, Enter start value of Eb/No ->
READ(*,*)ENST
PRINT, Enter end value of Eb/No->
READ(*,*)ENFN
PRINT, Enter increment for Eb/No->
READ(*,*)ENIC

*****
* OUTPUT HEADERS.
*****

WRITE(*,922)
WRITE(7,922)
922 FORMAT(20X, '*** DIGITAL FM BIT ERROR RATES ***',
$30X, '(Binary signaling)')
WRITE(*,923)INDEX,BSENC(BBEC),PSNAME(PS)
WRITE(7,923)INDEX,BSENC(BBEC),PSNAME(PS)
923 FORMAT(' Modulation index = .F5 2.//, Baseband encoding=
$A28.//, Premodulation filter= .A28).
IF(PS.NE.1)THEN
IF(PS.EQ.2)THEN
WRITE(*,924)BWT
WRITE(7,924)BWT
924 FORMAT(' Premodulation filter BT product = .F5 2)
ENDIF
WRITE(*,939)DUR1

```

```

WRITE(7,939)DUR1
939 FORMAT(' Premodulation filter pulse response duration= ,F5.2)
IF(PS.EQ.3)THEN
WRITE(*,929)PARM
WRITE(7,929)PARM
929 FORMAT(' Beta = ,F5.2)
ENDIF
WRITE(*,949)AREA
WRITE(7,949)AREA
949 FORMAT(' Area under rectified filter response = ,F5.2)
ENDIF
WRITE(*,925)IFNAME(RC),BWR,DUR2
WRITE(7,925)IFNAME(RC),BWR,DUR2
925 FORMAT(' IF receive filter = ,A15.1)
3 IF receive filter BT product = ,F5.2)
3 IF receive filter impulse response duration = ,F5.2)
PRINT, Computing... Please wait...
*****
* COMPUTE ALL THE NECESSARY PREMODULATION PHASE PULSE VALUES IN
* ORDER TO AVOID REPEATED SUBROUTINE CALLS IN CALCULATING THETA
*****
DO 131 J=-120,120
PSHAP(J)=0.0
131 CONTINUE

IDUR1=INT(DUR1)
IF(PS.NE.1)THEN
IF(PS.NE.4)THEN
JS=-16*(IDUR1+1)
ELSE
JS=0
ENDIF
JF=16*(IDUR1+1)

DO 132 J=JS,120
JT=J/16.0
IF(J.GT.JF)THEN
IF(BBEC.EQ.1)PSHAP(J)=1.0
IF(BBEC.EQ.2)PSHAP(J)=0.0
ELSE
PSHAP(J)=PREMOD(J,BBEC,PS,PARM,AREA)
ENDIF
132 CONTINUE
ENDIF

*****
* COMPUTE TRANSMITTED DATA PHASE, THETA.
* IF "PS" EQUALS 1, NO PREMODULATION SHAPING:
* IN THIS CASE, THE INDEX I CORRESPONDS TO EACH OF THE 6 SEQUENCES
* (BIT PATTERNS). J TO ALL 512 VALUES OF THETA COMPUTED FOR EACH
* SEQUENCE, I.E. 4 PERIODS X 8 BITS/PERIOD X 16 SUBINTERVALS/BIT
* THE VARIABLE "K" DETERMINES THE BIT (WITHIN AN 8-BIT PATTERN) TO
* WHICH THE INDEX "J" POINTS. "K" IS FIRST SET TO A VALUE BETWEEN
* 1 AND 128 (ONE PERIOD); THEN, SINCE EACH BIT CONTAINS 16
* SUBINTERVALS, "K" IS DIVIDED BY 16 TO DETERMINE THE APPROPRIATE
* BIT. "THETA" IS COMPUTED AT T/16, 2T/16, 3T/16, ...
* IFOL IS USED TO INVERT THE POLARITY OF THE DATA SEQUENCE IN
* THE SECOND HALF OF THE BIT PERIOD IF MANCHESTER ENCODING IS
* CHOSEN.

* IF "PS" IS NOT EQUAL TO ONE, THE TRANSMITTED DATA PHASE IS
* COMPUTED (AGAIN AT INTEGER MULTIPLES OF "T/16") AS A SUM OF
* DELAYED PHASE PULSES, WEIGHTED BY THE DATA SEQUENCE (REF. 6,
* PP 196-197). THE 212 LOOP COMPUTES THE CONTRIBUTIONS OF THE
* VARIOUS OVERLAPPING PHASE PULSES AT A GIVEN TIME "J/16". EACH
* PULSE IS ASSUMED TO VARY FROM ZERO TO ONE OVER A RANGE OF
* (2*DUR1)+1 BIT PERIODS (THE "+1" FACTOR REPRESENTS THE CENTRAL
* BIT). "JL" AND "K" DETERMINE THE POLARITY (WEIGHT) OF EACH PULSE.
* WHILE "JT" DETERMINES THE "PULSE TIME" ZERO REFERENCED AT THE
* CENTER OF THE PULSE (EXCEPT FOR THE "RC" PULSE WHERE "TIME ZERO
* CORRESPONDS TO THE BEGINNING OF THE PULSE); HENCE THE "JT-8.0"
* OPERATION (EXCEPT FOR THE "RC" PULSE) SINCE FOR THE DATA SEQUENCE,
* "TIME ZERO" IS A BIT TRANSITION POINT. SINCE THE 212 LOOP DOES NOT
* TAKE INTO ACCOUNT THOSE PULSES WHICH HAVE REACHED ONE (I.E. THOSE
* DUR1+1 OR MORE BIT PERIODS AHEAD OF THE CURRENT TIME "J"), THE
* PHASE IS ACCUMULATED IN "SUM", WITH "THBAK" REPRESENTING THE PHASE

```

* ACCUMULATION IN THE PREVIOUS INTERVAL OF $T/16$. "SPEC1" AND
 * "SPEC2" ARE USED AT THE BIT TRANSITION POINTS WHERE A PHASE PULSE
 * HAVING REACHED "ONE" IS DROPPED FROM THE 212 LOOP IN FAVOR OF
 * ANOTHER PULSE JUST STARTING TO RISE FROM ZERO. NOTE THAT WHEN
 * PREMODULATION PULSE SHAPING IS APPLIED, MANCHESTER ENCODING (IF
 * CHOSEN) IS PERFORMED IN THE "PREMOD" SUBROUTINE.

```

      IF(BBEC.EQ.1)IPOL=1
      IF(BBEC.EQ.2)IPOL=-1
      IF(PS.EQ.1)THEN
        RH=(T/32.0)/3.0
        DO 100 I=1,6
          SUM=0.0
          DO 101 J=1,512
            KP=J-(128*INT((J-1)/128))
            K=INT((KP-1)/16)+1
            KPBIT=MOD(KP,16)
            IF(KPBIT.EQ.0)KPBIT=16
            IF((KP.EQ.17).OR.(KP.EQ.33).OR.(KP.EQ.49).OR.(KP.EQ.65).OR.
              (KP.EQ.81).OR.(KP.EQ.97).OR.(KP.EQ.113))THEN
              3 THETA(I,J)=(RH*(IPOL*SEQ(I,K-1)+5*SEQ(I,K)))+SUM
            ELSE
              IF(KP.EQ.1)THEN
                THETA(I,J)=(RH*(IPOL*SEQ(I,8)+5*SEQ(I,1)))+SUM
              ELSE
                IF(KPBIT.EQ.9)THEN
                  THETA(I,J)=(RH*(SEQ(I,K)+5*IPOL*SEQ(I,K)))+SUM
                ELSE
                  IF(KPBIT.GE.10)THEN
                    THETA(I,J)=(RH*(6*IPOL*SEQ(I,K)))+SUM
                  ELSE
                    THETA(I,J)=(RH*6*SEQ(I,K))+SUM
                  ENDIF
                ENDIF
              ENDIF
            ENDIF
          SUM=THETA(I,J)
          THETA(I,J)=2*PI*FD*THETA(I,J)
          101 CONTINUE
          100 CONTINUE
        ELSE
          DO 110 I=1,6
            SUM=0.0
            THBAK=0.0
            DO 210 J=1,512
              THETA(I,J)=0.0
              LOOP=0
              DO 212 JJ=-IDUR1,IDUR1
                JL=J+(16*JJ)
                JTT=J-(16*JJ)-(16*INT((J-1)/16))
                IF(JL.GT.0)THEN
                  KP=JL-(128*INT((JL-1)/128))
                  K=INT((KP-1)/16)+1
                  IF(PS.NE.4)JTT=JTT-8
                  TERM=PSHAP(JTT)*SEQ(I,K)
                  THETA(I,J)=THETA(I,J)+TERM
                  LOOP=LOOP+1
                ENDIF
                IF((LOOP.EQ.1).AND.(MOD(J,16).EQ.0))SPEC1=TERM
                IF((JJ.EQ.IDUR1).AND.(MOD(J,16).EQ.1))SPEC2=TERM
                212 CONTINUE
                TEMP=THETA(I,J)
                IF(MOD(J,16).EQ.1)THEN
                  IF(J.LT.((IDUR1+1)*16))THBAK=THBAK+SPEC2
                  IF(J.GT.((IDUR1+1)*16))THBAK=THBAK+SPEC2-SPEC1
                ENDIF
                THETA(I,J)=SUM+THETA(I,J)-THBAK
                THBAK=TEMP
                SUM=THETA(I,J)
                THETA(I,J)=2.0*PI*FD*THETA(I,J)
                210 CONTINUE
                110 CONTINUE
              ENDIF

```


PRINT, Transmit stage computed .

 COMPUTE THE CONVOLUTION OF $H_r(t)$ AND $\sin(\theta(t))$, AND THE
 CONVOLUTION OF $H_r(t)$ AND $\cos(\theta(t))$, AS $HRSIN$ AND $HRCOS$
 RESPECTIVELY. THEN, FIND THE TIME-VARYING SIGNAL-TO-NOISE RATIO
 $RO(t)$ AND THE IF FILTERED DATA PHASE $\phi(t)$ ACCORDING TO EQU.
 4 AND 6 IN REFERENCE 1. THE 200 LOOP COMPUTES 42 VALUES OF
 $\phi(t)$ AND $RO(t)$ FOR EACH OF THE TEN CENTRAL BITS; 2.25 BIT
 PERIODS (18 SUBINTERVALS) PRIOR TO THE "BIT TIME" AND THREE BIT
 PERIODS (24 SUBINTERVALS) FOLLOWING IT. THE TWO EXTRA SUBINTERVALS
 ARE INCLUDED SO THAT THE SUBROUTINE "PHIDOT" HAS ALL THE ϕ
 VALUES IT NEEDS.
 SINCE THE BUTTERWORTH FILTER'S IMPULSE RESPONSE CANNOT BE DEFINED
 IN A NON-CAUSAL FASHION, ITS TIME ARGUMENT IS SHIFTED BY HALF THE
 RESPONSE LENGTH IN ORDER TO OBTAIN A CAUSAL ANALYSIS (WITH THE
 GAUSSIAN OR IDEAL FILTERS, THE COMPUTATION IS DONE NON-CAUSALLY).
 ϕ AND RO ARE COMPUTED AT $t=T/16$, $t=3T/16$, ETC. WITH THE VALUE
 OF t FIXED, THE 202 LOOP COMPUTES $\phi(t)$ AND $RO(t)$ BY INCREMENTING
 THE VARIABLE OF INTEGRATION "TAU" AND USING SIMPSON'S RULE. E.G.
 FOR THE SECTION BETWEEN $TAU=T/16$ AND $TAU=3T/16$, GAUSS AND θ
 ARE REQUIRED AT $T/16$, $2T/16$ AND $3T/16$. THUS THE INDEXING SCHEME
 THAT CONVERTS THE 8 SUBINTERVALS/BIT INDICES J, JJ, JS , AND JF TO
 THE 16 SUBINTERVALS/BIT INDICES JT AND TAU .
 SINCE THE FORTRAN FUNCTION "ATAN" RETURNS ANGLE VALUES IN THE
 RANGE $-\pi/2$ TO $\pi/2$, SPECIAL CARE MUST BE TAKEN TO ENSURE THAT
 "PHI" CORRESPONDS TO THE DESIRED PHASE ACCUMULATION. THE INTEGER
 VARIABLE "CYCLE" DETERMINES IN WHICH SEGMENT OF LENGTH π THE
 ACCUMULATED PHASE LIES. NOTE THAT THE CHANGE OF SEGMENT IS
 DETERMINED BY AN ABNORMALLY HIGH PHASE CHANGE, I.E. THE VALUE OF
 "ATAN" GOES BACK TO ABOUT $-\pi/2$ AFTER HAVING BEEN CLOSE TO
 $\pi/2$ OR VICE-VERSA.

```

      RH=(T/16.0)/3.0
      DO 200 L=1,10
        BSO=BIT(L,2)
        CYCLE=0
        PHIBAK=0.0
        DO 200 K=1,42
          J=BIT(L,1)-18+(K-1)
          JS=J-(8*DUR2)
          JF=J+(8*DUR2)
          HRSIN=0.0
          HRCOS=0.0
          DO 202 JJ=JS,JF
            JT=FLOAT((2*J)-1)
            ITAU=(2*JJ)-1
            ITAU1=ITAU-1
            ITAU2=ITAU-2
            TAU=FLOAT(ITAU)
            TAU1=FLOAT(ITAU1)
            TAU2=FLOAT(ITAU2)
            IF(RC.EQ.2) JT=JT+(16*DUR2)
            HRSIN=RH*
              (RCFTR((JT-TAU2)/16,BWR,RC)*SIN(THETA(BSQ,ITAU2)) +
              4*RCFTR((JT-TAU1)/16,BWR,RC)*SIN(THETA(BSQ,ITAU1)) +
              RCFTR((JT-TAU)/16,BWR,RC)*SIN(THETA(BSQ,ITAU)))
              + HRSIN
            HRCOS=RH*
              (RCFTR((JT-TAU2)/16,BWR,RC)*COS(THETA(BSQ,ITAU2)) +
              4*RCFTR((JT-TAU1)/16,BWR,RC)*COS(THETA(BSQ,ITAU1)) +
              RCFTR((JT-TAU)/16,BWR,RC)*COS(THETA(BSQ,ITAU)))
              + HRCOS
          202 CONTINUE
          PHI(L,K)=ATAN(HRSIN/HRCOS)
          DELTA=PHI(L,K)-PHIBAK
          IF((ABS(DELTA).GT.((2*PI*FD*T/8)*(PI/16))) AND (K.NE.1)) THEN
            IF(DELTA.LT.(0.0)) CYCLE=CYCLE+1
            IF(DELTA.GT.(0.0)) CYCLE=CYCLE-1
          ENDIF
          PHIBAK=PHI(L,K)
          PHI(L,K)=(CYCLE*PI)+PHI(L,K)
          RO(L,K)=((HRSIN**2)+(HRCOS**2))/(T*BWR)
        200 CONTINUE

```

```

*****
* THE 400 AND 300 LOOPS COMPUTE THE BIT ERROR RATE FOR VALUES OF
* Eb/No SPECIFIED IN THE INPUT SEQUENCE.
* THE DECISION TIMES ARE AT MULTIPLES OF THE BIT TIME 'T', BUT MAY
* BE SHIFTED FROM THE BIT TIMES DUE TO BOTH THE GROUP DELAY OF THE
* IF FILTER AND ALSO THE PHASE DISTORTION DUE TO THE FILTER, WHICH
* RESULTS IN SHIFTS OF THE PEAKS OF THE FILTERED PHASE.  THUS THE
* 865 LOOP SELECTS AS "DELPHI" THE LARGEST VARIATION (OVER ONE BIT
* PERIOD) OF PHI IN A TIME FRAME ABOUT THE CORRESPONDING 'BIT TIME'
* FOR THE GAUSSIAN AND IDEAL FILTERS (WHICH CAN BOTH BE DEFINED
* NON-CAUSALLY).  THIS IS DONE IN THE TWO BIT PERIODS FOLLOWING THE
* 'BIT TIME'
* HOWEVER, THE BUTTERWORTH FILTER CAUSES LARGER SHIFTS (IT IS ONLY
* DEFINED CAUSALLY-> SHIFT TRUNCATED RESPONSE BY HALF THE TRUNCATION
* LENGTH -> SHIFT IN DATA PHASE PEAK) AND THUS THE SEARCH FOR THE
* DATA PHASE PEAK IS PERFORMED ONE BIT PERIOD PRIOR TO AND THREE
* BIT PERIODS FOLLOWING THE 'BIT TIME'.  TESTS HAVE SHOWN THAT IF
* THIS WERE DONE FOR THE GAUSSIAN FILTER, THE RESULTS WOULD BE
* INACCURATE.
*
  WRITE(*,941)
  WRITE(7,941)
941  FORMAT(///, Eb/No (dB), 15X, PROB. OF BIT ERROR,/)
  IE=1
400  EBNO=10.0*((ENST)/10.0)
  EBNOX=10.0*((ENST-ENIC)/10.0)
  DO 277 L=1,10
  DO 277 K=1,42
    RO(L,K)=EBNO*RO(L,K)
    IF(IE.NE.1) THEN
      RO(L,K)=RO(L,K)/EBNOX
    ENDIF
277  CONTINUE
  BERR(IE)=0.0
  DO 300 L=1,10
  IF((RC.EQ.1).OR.(RC.EQ.3)) BTM=19
  IF(RC.EQ.2) BTM=11
  IF(BBEC.EQ.1) THEN
    DELPHI=PHI(L,BTM) - PHI(L,BTM-8)
  ELSE
    DELPHI=PHI(L,BTM-4)-PHI(L,BTM-8) - (PHI(L,BTM)-PHI(L,BTM-4))
  ENDIF
  BTMX=BTM
  IF((RC.EQ.1).OR.(RC.EQ.3)) AND.(BBEC.EQ.1) KF=14
  IF((RC.EQ.1).OR.(RC.EQ.3)) AND.(BBEC.EQ.2) KF=6
  IF(RC.EQ.2) KF=30
  DO 865 I=1,KF
    BTMX=BTMX+1
    IF(BBEC.EQ.1) THEN
      DELX=PHI(L,BTMX)-PHI(L,BTMX-8)
    ELSE
      DELX=PHI(L,BTMX-4)-PHI(L,BTMX-8) - (PHI(L,BTMX)-PHI(L,BTMX-4))
    ENDIF
    IF(DELX.GT.DELPHI) THEN
      DELPHI=DELX
      BTM=BTMX
    ENDIF
865  CONTINUE
  U=(RO(L,BTM)+RO(L,BTM-8))/2.0
  V=(RO(L,BTM)-RO(L,BTM-8))/2.0
  W=SQRT(RO(L,BTM)*RO(L,BTM-8))

* COMPUTE THE CONTINUOUS PART OF THE ERROR RATE, FOR THE NRZ CASE.
* USE EQ.A14, REF.1, WHICH IS THE PROBABILITY DISTRIBUTION FUNCTION
* OF THE "IF" FILTERED PHASE NOISE DIFFERENCE (SEE EQ.15, REF.1
* IN THE MANCHESTER CASE, NO EXPRESSION IS AVAILABLE FOR THE
* DENSITY FUNCTION OF THE PHASE NOISE DIFFERENCE AND THUS ITS
* DENSITY FUNCTION MUST BE INTEGRATED.  NOTE THAT THE COMPUTATION IS
* QUITE PRECISE FOR ABS(DELPHI) < PI, OUTSIDE OF THIS RANGE, THE
* APPROXIMATION IMPROVES AS THE SIGNAL-TO-NOISE RATIO "RO"
* INCREASES (THE DENSITY FUNCTIONS ARE ASSUMED TO VANISH OUTSIDE OF
* THE "-PI TO PI" RANGE.
*
  IF(BBEC.EQ.1) THEN
*
  IF(DELPHI.GT.PI) THEN

```

```

FLAG=1
CONTERR=0.0
ELSE
IF (DELPHI.LT. (0.01)) THEN
CONTERR=0.5
ELSE
STEP=PI/64.0
RH=STEP/3.0
CONTERR=0.0
ALPHA=0.0
477 TERM=RH*(PROB(ALPHA,U,V,W,DELPHI)
3      + (4*PROB(ALPHA+STEP,U,V,W,DELPHI)
3      + PROB(ALPHA+2*STEP,U,V,W,DELPHI))
CONTERR=CONTERR + TERM
ALPHA=ALPHA+(2*STEP)
IF (ALPHA.LT.PI) GO TO 477
CONTERR = W*SIN(DELPHI)*CONTERR/(4.0*PI)
IF (DELPHI.LT.(0.0)) THEN
CONTERR=CONTERR+1.0
ENDIF
ENDIF
ENDIF
ELSE
IDONE=0
IF (L.NE.1) THEN
DO 611 LL=1,L-1
T1=ABS(DELPHI-COMP(LL,1))
T2=ABS(RO(L,BTM)-COMP(LL,2))
T3=ABS(RO(L,BTM-8)-COMP(LL,3))
T4=ABS(RO(L,BTM-4)-COMP(LL,4))
T11=0.01*DELPHI
T21=0.01*RO(L,BTM)
T31=0.01*RO(L,BTM-8)
T41=0.01*RO(L,BTM-4)
IF ((T1.LT.T11).AND.(T2.LT.T21).AND.(T3.LT.T31).AND.
3  (T4.LT.T41)) THEN
CONTERR=COMP(LL,5)
IDONE=1
ENDIF
611 CONTINUE
ENDIF

IF (IDONE.EQ.0) THEN
STEP=PI/16.0
RH=STEP/3.0
IF (DELPHI.LT.(0.01)) THEN
CONTERR=0.5
ELSE
CONTERR=0.0
PSI=DELPHI
TERM1=PDENS2(RO(L,BTM),RO(L,BTM-8),RO(L,BTM-4),PSI)
GO TO 479
478 TERM1=TERM3
479 TERM2=PDENS2(RO(L,BTM),RO(L,BTM-8),RO(L,BTM-4),PSI+STEP)
TERM3=PDENS2(RO(L,BTM),RO(L,BTM-8),RO(L,BTM-4),PSI+2*STEP)
SUM=RH*(TERM1+4*TERM2+TERM3)
CONTERR=CONTERR + SUM
PSI=PSI+(2*STEP)
IF ((PSI.LT.PI).AND.(SUM.GE.(0.0001*CONTERR))) GO TO 478
ENDIF
ENDIF
COMP(L,1)=DELPHI
COMP(L,2)=RO(L,BTM)
COMP(L,3)=RO(L,BTM-8)
COMP(L,4)=RO(L,BTM-4)
COMP(L,5)=CONTERR
ENDIF

* COMPUTE NBAR, THE CONTRIBUTION OF 'CLICKS' TO THE ERROR RATE.
* A STEP OF T/8 IS USED SINCE 'PHI' AND 'RO' ARE ONLY AVAILABLE AT
* INTEGER MULTIPLES OF T/8. ARITHMETIC UNDERFLOW CHECKS ARE PRESENT

STEP=T/8
RH=STEP/3.0

```

```

        NBAR=0.0
        ITAU=BTM-8
577 IF(RO(L,ITAU).GT.(70.0))THEN
        TERM1=0.0
        ELSE
        TERM1=PHIDOT(PHI,L,ITAU,BBEC)*EXP(-RO(L,ITAU))
        ENDIF
        IF(RO(L,ITAU+1).GT.(70.0))THEN
        TERM2=0.0
        ELSE
        TERM2=PHIDOT(PHI,L,ITAU+1,BBEC)*EXP(-RO(L,ITAU+1))
        ENDIF
        IF(RO(L,ITAU+2).GT.(70.0))THEN
        TERM3=0.0
        ELSE
        TERM3=PHIDOT(PHI,L,ITAU+2,BBEC)*EXP(-RO(L,ITAU+2))
        ENDIF
        TERM=RH*(TERM1 + (4*TERM2) + TERM3)
        NBAR=NBAR+TERM
        ITAU=ITAU+2
        IF(ITAU.LT.BTM)GO TO 577
        NBAR=NBAR/(2.0*PI)
*
* ADD CONTINUOUS AND CLICK COMPONENTS OF THE ERROR RATE
*
        BERR(IE)=((NBAR+CONTERR)*BIT(L,3)/16) + BEPR(IE)
300 CONTINUE
        WRITE(*,678)ENST,BERR(IE)
        WRITE(7,678)ENST,BERR(IE)
578 FORMAT(1X,F7.4,21X,E11.4)
        IE=IE+1
        ENST=ENST+ENIC
        IF(ENST.LE.ENFN)GO TO 400
*
        IF(FLAG.EQ.1)THEN
        WRITE(*,705)
705 FORMAT('////',WARNING: Phase variations larger than PI ...
        'Results may be inaccurate')
        ENDIF
*
*****
        STOP
        END
*****
        FUNCTION PREMOD(TT,BBEC,PS,PARM,AREA)
*****
* THIS FUNCTION COMPUTES NORMALIZED PHASE PULSES BY INTEGRATING
* FREQUENCY PULSES FROM "- INFINITY" TO "TT". THE INTEGRATION IS
* CARRIED OUT FROM "TT" DOWNWARDS USING SIMPSON'S RULE. BBEC
* DETERMINES WHETHER "NRZ" OR "MANCHESTER" BASEBAND ENCODING IS
* USED.
        REAL TT,BW,PARM,AREA,RH,STEP
        INTEGER BBEC,PS
        STEP=1.0/64.0
        RH=STEP/3.0
        PREMOD=0.0
        TIME=TT
977 IF(BBEC.EQ.1)THEN
        IF(PS.EQ.2)THEN
        TERM1=GSPUL(TIME,PARM,BBEC)
        TERM2=GSPUL(TIME-STEP,PARM,BBEC)
        TERM3=GSPUL(TIME-2*STEP,PARM,BBEC)
        ENDIF
        IF(PS.EQ.3)THEN
        TERM1=RACOS1(TIME,PARM,BBEC)
        TERM2=RACOS1(TIME-STEP,PARM,BBEC)
        TERM3=RACOS1(TIME-2*STEP,PARM,BBEC)
        ENDIF
        IF(PS.EQ.4)THEN
        TERM1=RACOS2(TIME,PARM)

```

```

TERM2=RACOS2(TIME-STEP,PARM)
TERM3=RACOS2(TIME-2*STEP,PARM)
ENDIF

ELSE

TIME1=TIME+0.25
TIME2=TIME-0.25
TIME3=TIME-0.5

IF(PS.EQ.2)THEN
TERM1=GSPUL(TIME1,PARM,BBEC) - GSPUL(TIME2,PARM,BBEC)
TERM2=GSPUL(TIME1-STEP,PARM,BBEC)
- GSPUL(TIME2-STEP,PARM,BBEC)
TERM3=GSPUL(TIME1-2*STEP,PARM,BBEC)
- GSPUL(TIME2-2*STEP,PARM,BBEC)
ENDIF

IF(PS.EQ.3)THEN
TERM1=RACOS1(TIME1,PARM,BBEC) - RACOS1(TIME2,PARM,BBEC)
TERM2=RACOS1(TIME1-STEP,PARM,BBEC)
- RACOS1(TIME2-STEP,PARM,BBEC)
TERM3=RACOS1(TIME1-2*STEP,PARM,BBEC)
- RACOS1(TIME2-2*STEP,PARM,BBEC)
ENDIF

IF(PS.EQ.4)THEN
TERM1=RACOS2(TIME,PARM) - RACOS2(TIME3,PARM)
TERM2=RACOS2(TIME-STEP,PARM) - RACOS2(TIME3-STEP,PARM)
TERM3=RACOS2(TIME-2*STEP,PARM) - RACOS2(TIME3-2*STEP,PARM)
ENDIF
ENDIF

```

```

TERM=RH*(TERM1+(4.0*TERM2)+TERM3)
PREMOD=PREMOD+TERM
TIME=TIME-(2.0*STEP)
IF((TERM.GT.(0.1E-6)) OR.(TIME.GT.(-1.0)))GO TO 977
PREMOD=PREMOD/AREA

```

```

RETURN
END

```

```

*****
FUNCTION RCFTR(TT,BW,RC)
*****

```

```

* THIS FUNCTION COMPUTES THE IMPULSE RESPONSE OF THE 'IF' FILTER,
* WHOSE TYPE IS DETERMINED BY 'RC'. FOR THE GAUSSIAN FILTER, EQ. 1,
* REF 5 WAS USED TO TRANSFORM EQ.41, REF.1. THE BUTTERWORTH FILTER
* IMPULSE RESPONSE WAS OBTAINED BY INVERSE FOURIER TRANSFORMING EQ
* 42. REF.1. THE IDEAL FILTER RESPONSE IS SIMPLY A 'SINC' FUNCTION

```

```

REAL T,TT,BW,PI,XO
INTEGER RC
PI=4.0*ATAN(1.0)

```

```

* GAUSSIAN FILTER:

```

```

IF(RC.EQ.1)THEN
XO=2.0*PI*((BW*TT)**2)
IF(XO.GT.(70.0))THEN
RCFTR=0.0
ELSE
RCFTR=(SORT(2.0))*BW*(EXP(-XO))
ENDIF
ENDIF

```

```

* BUTTERWORTH FILTER:

```

```

IF(RC.EQ.2)THEN
XO=(SORT(2.0)/2.0)*PI*BW*TT
IF((XO.GT.(70.0))OR.(XO.LT.(0.0)))THEN
RCFTR=0.0
ELSE
RCFTR=(PI*BW*SORT(2.0))*(EXP(-XO))*SIN(XO)
ENDIF
ENDIF

```

```

* IDEAL BRICK WALL FILTER:

```

```

IF(RC.EQ.3)THEN

```

```

      IF(TT.EQ.(0.0))THEN
        RCFTF=BW
      ELSE
        RCFTF=(SIN(PI*BW*TT))/(PI*TT)
      ENDIF
    ENDIF

    RETURN
  END

*****
  FUNCTION PROB(ALPHA,U,V,W,DELPHI)
*****
  * THIS FUNCTION COMPUTES THE INTEGRAND OF EQ. A14, REF. 1, WHICH IS
  * THE PROBABILITY DISTRIBUTION FUNCTION OF THE IF FILTERED PHASE
  * NOISE DIFFERENCE FOR AN NRZ CODED SYSTEM.
  *
    REAL ALPHA,U,V,W,DELPHI,PROB
    VCOS=V*COS(ALPHA)
    WSIC=W*COS(DELPHI)*SIN(ALPHA)
    TERM=U-VCOS-WSIC
    IF(TERM.GT.(70.0))THEN
      PROB=0.0
    ELSE
      PROB=(EXP(-TERM))/TERM
    ENDIF
    RETURN
  END

*****
  FUNCTION PDENS1(RO,ETA)
*****
  * THIS FUNCTION COMPUTES THE PROBABILITY DENSITY FUNCTION OF THE
  * PHASE NOISE AS GIVEN BY EQ. 8, REF. 1
  *
    REAL RO,ETA,X,PI,STEP,RH,TERM
    PI=4.0*ATAN(1.0)
    X=0.0
    PDENS1=0.0
    IF(ABS(ETA).GT.PI)GO TO 20
    STEP=1.0/4.0
    RH=STEP/3.0
10   X1=X+STEP
    X2=X+(2*STEP)
    TERM=(X**2)+RO-(2*X*SQRT(RO)*COS(ETA))
    TERM1=(X1**2)+RO-(2*X1*SQRT(RO)*COS(ETA))
    TERM2=(X2**2)+RO-(2*X2*SQRT(RO)*COS(ETA))
    IF(TERM.GT.(70.0))THEN
      SUM=0.0
    ELSE
      SUM=X*EXP(-TERM)
    ENDIF
    IF(TERM1.GT.(70.0))THEN
      SUM1=0.0
    ELSE
      SUM1=X1*EXP(-TERM1)
    ENDIF
    IF(TERM2.GT.(70.0))THEN
      SUM2=0.0
    ELSE
      SUM2=X2*EXP(-TERM2)
    ENDIF
    PDENS1=PDENS1 + RH*(SUM+(4*SUM1)+SUM2)
    X=X+(2*STEP)
    IF(X.LT.(50.0))GO TO 10
    PDENS1=PDENS1/PI
    IF(PDENS1.LT.(0.1E-15))PDENS1=0.0
20   RETURN
  END

*****
  FUNCTION DSNRZ(RO1,RO2,PSI)
*****
  * THIS FUNCTION COMPUTES THE PROBABILITY DENSITY FUNCTION OF THE
  * PHASE NOISE DIFFERENCE FOR NRZ DATA (REF. 1, EQ. A3)
  *
    REAL PSI,U,V,W,RO1,RO2
    PI=4.0*ATAN(1.0)
    U=(RO1+RO2)/2.0

```

```

V=(R01-R02)/2.0
W=SQRT(R01*R02)
STEP=PI/32.0
RH=STEP/3.0
SUM=0.0
10 ALPHA=0.0
ALPHA1=ALPHA+STEP
ALPHA2=ALPHA+2*STEP
ET1=W*SIN(ALPHA)*COS(PS1)
ET2=W*SIN(ALPHA1)*COS(PS1)
ET3=W*SIN(ALPHA2)*COS(PS1)
IF(ET1.LT.(-50.0))THEN
TERM1=0.0
ELSE
TERM1=(U*SIN(ALPHA)+W*COS(PS1))*COSH(V*COS(ALPHA))*
EXP(W*SIN(ALPHA)*COS(PS1))
ENDIF
IF(ET2.LT.(-50.0))THEN
TERM2=0.0
ELSE
TERM2=4*(U*SIN(ALPHA1)+W*COS(PS1))*
COSH(V*COS(ALPHA1))*EXP(W*SIN(ALPHA1)*COS(PS1))
ENDIF
IF(ET3.LT.(-50.0))THEN
TERM3=0.0
ELSE
TERM3=(U*SIN(ALPHA2)+W*COS(PS1))*COSH(V*COS(ALPHA2))*
EXP(W*SIN(ALPHA2)*COS(PS1))
ENDIF
SUM=SUM+(RH*(TERM1+TERM2+TERM3))
ALPHA=ALPHA+2*STEP
IF(ALPHA.LT.PI)GO TO 10
DSNRZ=(EXP(-U))*(COSH(V)+0.5*SUM)/(2.0*PI)
IF(DSNRZ.LT.(0.1E-15))DSNRZ=0.0
RETURN
END
*****
FUNCTION PDENS2(R01,R02,R03,PS1)
*****
* THIS FUNCTION COMPUTES THE PROBABILITY DENSITY FUNCTION OF THE
* "IF" FILTERED PHASE NOISE DIFFERENCE FOR A MANCHESTER CODED
* SYSTEM. THIS IS DONE BY CONVOLVING THE THREE INDEPENDENT PHASE
* NOISE COMPONENTS AS SPECIFIED BY EQ.10, REF.10. NOTE THAT THE
* CONVOLUTION OF TWO OF THESE NOISE COMPONENTS IS THE PROBABILITY
* DENSITY FUNCTION OF THE PHASE NOISE DIFFERENCE FOR AN NRZ SYSTEM
*
REAL R01,R02,R03,PS1,STEP,RH,ETAPR,PI
PI=4.0*ATAN(1.0)
STEP=PI/32.0
RH=STEP/3.0
ETAPR=-PI
PDENS2=0.0
TERM1=0.5*PDENS1(R03,0.5*(PS1-ETAPR))*DSNRZ(R01,R02,ETAPR)
GO TO 11
10 TERM1=TERM3
11 TERM2=0.5*PDENS1(R03,0.5*(PS1-ETAPR-STEP))*
DSNRZ(R01,R02,ETAPR+STEP)
3 TERM3=0.5*PDENS1(R03,0.5*(PS1-ETAPR-2*STEP))*
DSNRZ(R01,R02,ETAPR+2*STEP)
3 PDENS2=PDENS2 + RH*(TERM1+4*TERM2+TERM3)
ETAPR=ETAPR+(2*STEP)
IF(ETAPR.LT.PI)GO TO 10
RETURN
END
*****
FUNCTION PHIDOT(PHI,L,K,BBEC)
*****
* THIS FUNCTION COMPUTES THE FIRST DERIVATIVE OF PHI(.) USING A
* FIVE-POINT FORMULA; REF.4, P.130, EQ. 4.22 (FOR THE NRZ CASE).
* FOR THE MANCHESTER CASE, THE PHASE DERIVATIVE IS COMPUTED
* SEPARATELY FOR BOTH HALVES OF THE BIT PERIOD USING A SIMPLE
* TWO-POINT FORMULA. THE PHASE DERIVATIVE IS ALWAYS MADE POSITIVE
* SINCE WE ARE SEARCHING FOR A "1" BIT (POSITIVE PHASE DERIVATIVE)
*
REAL PHI(10.42),PHIDOT,RH,T

```

```

      INTEGER L,K,BBEC
      T=1.0
      RH=T/8
      IF(BBEC.EQ.1)THEN
3        PHIDOT = (PHI(L,K-2) - 8*PHI(L,K-1) + 8*PHI(L,K+1)
                  - PHI(L,K+2)) / (12*RH)
      ELSE
        IF(PHI(L,K+1) LT PHI(L,K))TERM=(PHI(L,K)-PHI(L,K+1))/RH
        IF(PHI(L,K) GE PHI(L,K-1))TERM=(PHI(L,K)-PHI(L,K-1))/RH
        PHIDOT=TERM
      ENDIF
      RETURN
      END
*****
      FUNCTION GSPUL(TT,BW,BBEC)
*****
      * THIS FUNCTION COMPUTES THE RESPONSE OF A GAUSSIAN FILTER TO A UNIT
      * AMPLITUDE RECTANGULAR PULSE OF DURATION "T" FOR NRZ ENCODING, AND
      * OF DURATION "T/2" FOR MANCHESTER ENCODING. EQ 18, REF 3 IS USED
      * THE "Q" FUNCTION IS APPROXIMATED BY EQ 7 1 26, P 299, REF 9
      *
      REAL T,TT,BW,X,XX,P,PT
      DIMENSION A(5),ARG(2),QFCT(2)
      INTEGER BBEC
      *
      T=1.0/BBEC
      ARG(1)=SQRT(2.0)*5.336*BW*T*(-0.5-TT/T)
      ARG(2)=SQRT(2.0)*5.336*BW*T*(0.5-TT/T)
      *
      A(1)=0.254829592
      A(2)=-0.284496736
      A(3)=1.421413741
      A(4)=-1.453152027
      A(5)=1.061405429
      P=0.3275911
      *
      DO 100 J=1,2
        X=ARG(J)/SQRT(2.0)
        XX=ABS(X)
        PT=1.0/(1.0+(P*XX))
        QFCT(J)=0.0
        DO 101 I=1,5
          IF(PT.LT.(10.0**(-30.0/I)))THEN
            TERM=0.0
          ELSE
            TERM=A(I)*(PT**I)
          ENDIF
          QFCT(J)=QFCT(J)+TERM
101        CONTINUE
          IF(XX.GT.(8.4))THEN
            TERM=0.0
          ELSE
            TERM=EXP(-(XX**2))
          ENDIF
          QFCT(J)=QFCT(J)*TERM
          IF(X.LT.(0.0))QFCT(J)=2.0-QFCT(J)
          QFCT(J)=0.5*QFCT(J)
100        CONTINUE
      *
      GSPUL=QFCT(1)-QFCT(2)
      *
      RETURN
      END
*****
      FUNCTION RACOS1(TT,BETA,BBEC)
*****
      * THIS FUNCTION COMPUTES THE RESPONSE OF A SPECTRALLY RAISED COSINE
      * FILTER TO A UNIT AMPLITUDE PULSE OF DURATION "T" FOR NRZ ENCODING,
      * AND OF DURATION "T/2" FOR MANCHESTER ENCODING. REF.8, P 339
      *
      REAL TT,BETA,T
      INTEGER BBEC
      PI=4.0*ATAN(1.0)
      T=1.0/BBEC
      IF((ABS(TT) EQ.(T/2.0*BETA))) OR (TT.EQ.(0.0))TT=TT+0.0001
      RACOS1=(SIN(PI*TT/T))*(COS(BETA*PI*TT/T))

```



```

3      RETURN / ((PI*TT/T)*(1-((2*BETA*TT/T)**2)))
      END
.....
      FUNCTION RACOS2(TT,LENGTH)
.....
      * THIS FUNCTION COMPUTES A RAISED COSINE PULSE RESPONSE OF DURATION
      * LENGTH*T
      *
      REAL TT,LENGTH,PI
      PI=4.0*ATAN(1.0)
      T=1.0
      IF (TT LT (0.0)) OR (TT GT LENGTH) THEN
      RACOS2=0.0
      ELSE
      RACOS2=(1-COS(2.0*PI*TT/(LENGTH*T)))/(2*LENGTH*T)
      ENDIF
      RETURN
      END
.....

```

Appendix I

BER sample output

*** DIGITAL FM BIT ERROR RATES ***
 (Binary signaling)

Modulation index = 0.50
 Baseband encoding = NON-RETURN TO ZERO (NRZ)
 Premodulation filter = GAUSSIAN
 Premodulation filter BT product = 0.50
 Premodulation filter pulse response duration = 2.00
 Area under rectified filter response = 1.00
 IF receive filter = GAUSSIAN
 IF receive filter BT product = 1.10
 IF receive filter impulse response duration = 2.00

Eb/No (dB)	PROB. OF BIT ERROR
0.0000	0.2643E+00
1.0000	0.2258E+00
2.0000	0.1862E+00
3.0000	0.1473E+00
4.0000	0.1113E+00
5.0000	0.8002E-01
6.0000	0.5488E-01
7.0000	0.3614E-01
8.0000	0.2309E-01
9.0000	0.1438E-01
10.0000	0.8631E-02
11.0000	0.4860E-02
12.0000	0.2473E-02
13.0000	0.1091E-02
14.0000	0.3998E-03
15.0000	0.1156E-03
16.0000	0.2484E-04
17.0000	0.3672E-05
18.0000	0.3394E-06
19.0000	0.1738E-07
20.0000	0.4234E-09

END

22.06.88

FIN