1

ALCANTARA
ALCANTARA
SINCE 1920

**National Library
of Canada**

**Bibliothèque nationale
du Canada**

Canadian Theses Service

Service des thèses canadiennes

**Ottawa, Canada
K1A 0N4**

NOTICE

The quality of this microform is heavily dependent upon the quality of the original thesis submitted for microfilming. Every effort has been made to ensure the highest quality of reproduction possible.

If pages are missing, contact the university which granted the degree.

Some pages may have indistinct print especially if the original pages were typed with a poor typewriter ribbon or if the university sent us an inferior photocopy.

Reproduction in full or in part of this microform is governed by the Canadian Copyright Act, R.S.C. 1970, c. C-30, and subsequent amendments.

AVIS

La qualité de cette microforme dépend grandement de la qualité de la thèse soumise au microfilmage. Nous avons tout fait pour assurer une qualité supérieure de reproduction.

S'il manque des pages, veuillez communiquer avec l'université qui a conféré le grade.

La qualité d'impression de certaines pages peut laisser à désirer, surtout si les pages originales ont été dactylographiées à l'aide d'un ruban usé ou si l'université nous a fait parvenir une photocopie de qualité inférieure.

La reproduction, même partielle, de cette microforme est soumise à la Loi canadienne sur le droit d'auteur, SRC 1970, c. C-30, et ses amendements subséquents.

**Tire Modelling using a Three-Dimensional
Finite Element Method**

by

Lei Wang

B.Eng., Jilin University of Technology, 1985

**A thesis submitted to the Faculty of Graduate
Studies and Research in partial fulfillment
of the requirements for the Degree of**

Master of Engineering

**Department of
Mechanical and Aerospace Engineering
Carleton University
Ottawa, Ontario
Canada**

March, 1990

**© copyright
1990, L. Wang**

**National Library
of Canada**

**Bibliothèque nationale
du Canada**

Canadian Theses Service Service des thèses canadiennes

**Ottawa, Canada
K1A 0N4**

The author has granted an irrevocable non-exclusive licence allowing the National Library of Canada to reproduce, loan, distribute or sell copies of his/her thesis by any means and in any form or format, making this thesis available to interested persons.

The author retains ownership of the copyright in his/her thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without his/her permission.

L'auteur a accordé une licence irrévocable et non exclusive permettant à la Bibliothèque nationale du Canada de reproduire, prêter, distribuer ou vendre des copies de sa thèse de quelque manière et sous quelque forme que ce soit pour mettre des exemplaires de cette thèse à la disposition des personnes intéressées.

L'auteur conserve la propriété du droit d'auteur qui protège sa thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

ISBN 0-315-59031-9

Acceptance Sheet

The undersigned recommends to the Faculty of Graduate Studies and Research, acceptance of the thesis: *Tire Modelling using a Three-Dimensional Finite Element Method*, submitted by Lei Wang, in partial fulfillment of the Degree of Master of Engineering.

Thesis Supervisor

Chairman
Department of Mechanical
& Aerospace Engineering

Abstract

As a first step in the detailed study of tire-terrain interaction, a three dimensional (3-D) finite element model of a tire designed for off-road operation is developed. The tire is represented by 876 3-D finite elements, and is first 'inflated' to operating pressure and subsequently loaded against a frictionless hard surface. To simplify the analysis the 'carcass' is assumed to be composed of a homogeneous, isotropic elastic material with a Poisson ratio of 0.48. The other parameter, Young's modulus, is derived from the measured load-deflection curve of the actual tire. The tire-ground geometry predicted using the 3-D finite element model is compared with the measured one. The influence of friction on the geometry of the contact area is examined. The stress distributions on the contact patch under different loadings are also analyzed.

It is shown that the tire-ground contact geometry predicted using the model is in reasonable agreement with that found in the Generalized Deflection Chart provided by the tire manufacturer. Thus it would appear that the tire model presented in this thesis could provide an improved framework for the detailed analysis of tire-terrain interaction.

Acknowledgments

I would like to take this opportunity to express my sincere appreciation to Prof. J. Y. Wong, my thesis supervisor, for his guidance, encouragement, and valuable suggestions throughout the course of this work, as well as in the preparation of this thesis.

I would also like to thank Dan Downey for his invaluable assistance, especially in generating the 3-D meshes of the model tire; and Wei Ping Wang, Song Tao Chen and Hong Lin Yang for their help in overcoming certain problems that I faced during the course of my research.

Finally I would like to thank Prof. John Goldak for permitting me to use his well-developed finite element software packages. Without them I would not have been able to complete my research.

Table of Contents

Acceptance Sheet	ii
Abstract	iii
Acknowledgments	iv
Table of Contents	v
List of Figures	viii
List of Tables	xi
List of Appendices	xii
List of Symbols	xiii
1 Introduction	1
1.1 Review of Previous Research	1
1.1.1 Bekker's Model	2
1.1.2 Fujimoto's Model	3
1.1.3 Baladi's Model	3
1.1.4 Yong's Model	5
1.1.5 Chen's Model	10
1.1.6 Boonsinsuk's Model	12
1.1.7 Qun's Model	13
1.1.8 Review of Currently Available Computer Based Models	13
1.2 Overview of the Thesis	16
1.2.1 Objective of Thesis	16
1.2.2 The General Approach of the Thesis	16
1.2.3 Organization of the Thesis	17
2 Modelling the Geometry of the Tire	19
2.1 The Finite Element Representation of a Tire	19
2.1.1 The Tire to Be Modelled	19

2.1.2	<i>The Tire Mesh</i>	19
2.1.3	<i>Creating the Input File to ASGARD</i>	27
2.2	<i>The Model Tire Under Inflation</i>	31
2.2.1	<i>Introduction</i>	31
2.2.2	<i>Results</i>	32
3	Modelling the Material Properties of the Model Tire	36
3.1	Assumptions	36
3.2	Loading	36
3.3	The Incremental-Iterative Process	37
3.4	The Material Properties	37
3.5	Algorithm for Predicting the Contact Geometry	38
3.6	Overview of the Computer Programs used to Predict Tire Behaviour under Normal Load	41
4	Tire Behaviour on a Frictionless Surface	51
4.1	Contact Length and Contact Width	51
4.2	Reaction from the Ground and Young's Modulus	57
4.3	Deformed Tire Geometry	76
4.4	Footprints	76
4.5	Trace of Contact Nodes	76
5	Effects of Surface Friction on Tire Behaviour	102
5.1	Introduction	102
5.2	Results	103
6	Conclusions and Suggestions for Future Work	112
6.1	Conclusions	112
6.2	Suggestions for Future Work	113
	References	114

Appendices	117
-----------------------------	------------

List of Figures

<u>Figure #</u>	<u>Description</u>	<u>Page</u>
1-1	Contact geometry in Fujimoto's model	4
1-2	Equivalent spring constant for tire-soil system in Baladi's model	6
1-3	Contact geometry and normal stress distribution in Baladi's model	7
1-4	Finite element mesh of Yong's model	8
1-5	Tire-terrain configuration according to Yong's model	9
1-6	Finite element mesh of Chen's model	11
1-7	Other tire models	15
2-1	Definitions of tire parameters	21
2-2	Cross sectional view of the real tire when uninflated	22
2-3	Generalized Deflection Chart (GDC)	23
2-4	The general shape of the uninflated tire	25
2-5	A cross-sectional view of the uninflated tire	26
2-6	The mesh of the tire model	29
2-7	Interface between the 8-node and 10-node elements	30
2-8	The tire model before and after inflation (displacement enlarged two times)(exterior view)	33
2-9	The tire model before and after inflation (displacement enlarged two times) (interior view)	34
2-10	Magnified view (10 times) of tire displacement upon inflation	35
3-1	Flowchart of the contact algorithm	39
3-2	Illustration of the algorithm for deriving the contact geometry	40
3-3	Procedure for modelling tire behaviour under vertical loading	43
3-4	Flowchart of ASGARD program	44
3-5	Structure of ASGARD program	45
3-6	Flowchart of program UPDATE.FTN	47
3-7	Flowchart of program RESULT.FTN	48
3-8	Interrelationship of files used in tire analysis	49

<u>Figure #</u>	<u>Description</u>	<u>Page</u>
4-1	Comparison of the contact lengths generated by 3-D FEM, the measured and Chen's 2-D FEM	52
4-2	Comparison of the contact widths generated by 3-D FEM, the measured and Chen's 2-D FEM	53
4-3	Potential contact nodes in the contact length direction	55
4-4	Potential contact nodes in the contact width direction	56
4-5	The relationship between Young's modulus and deflection	60
4-6	The relationship between Young's modulus and load	61
4-7	Incremental deflection vs. total tire deflection	63
4-8	Incremental load vs. deflection (from GDC)	64
4-9	Incremental reaction from the ground vs. deflection	65
4-10	Young's modulus vs. deflection for all incremental steps	66
4-11	Distribution of normal reaction from the ground when deflection is 4 cm	68
4-12	Distribution of normal reaction from the ground when deflection is 8 cm	69
4-13	Distribution of normal reaction from the ground when deflection is 12 cm	70
4-14	Distribution of normal reaction from the ground when deflection is 16 cm	71
4-15	Tire deformation when deflection is 4 cm (side view)	77
4-16	Tire deformation when deflection is 8 cm (side view)	78
4-17	Tire deformation when deflection is 12 cm (side view)	79
4-18	Tire deformation when deflection is 16 cm (side view)	80
4-19	Tire deformation when deflection is 4 cm (axial view)	81
4-20	Tire deformation when deflection is 8 cm (axial view)	82
4-21	Tire deformation when deflection is 12 cm (axial view)	83
4-22	Tire deformation when deflection is 16 cm (axial view)	84
4-23	The geometry of the model tire when deflection is 16 cm (axial view)	85
4-24	The geometry of the model tire when deflection is 16 cm (side view)	86
4-25	The geometry of the model tire when deflection is 16 cm (oblique view)	87
4-26	The geometry of the model tire when deflection is 16 cm (bottom view)	88

<u>Figure #</u>	<u>Description</u>	<u>Page</u>
4-27	Predicted and measured footprints when deflection is 4 cm	89
4-28	Predicted and measured footprints when deflection is 8 cm	90
4-29	Predicted and measured footprints when deflection is 12 cm	91
4-30	Predicted and measured footprints when deflection is 16 cm	92
4-31	The traces of outermost contact nodes along contact length direction	97
4-32	The traces of outermost contact nodes along contact width direction	98
5-1	Procedure for modelling tire behaviour on a no-slip surface	104
5-2	A comparison of contact length vs. deflection for tire in contact with frictionless and no-slip surface	105
5-3	A comparison of contact width vs. deflection for tire in contact with frictionless and no-slip surface	106
5-4	Comparison of traces of outermost contact nodes along contact length direction on a frictionless and on a no-slip surface	108
5-5	Comparison of traces of outermost contact nodes along contact width direction on a frictionless and on a no-slip surface	109
A2-1	Local coordinate system	118
A2-2	Gauss integration scheme	120

List of Tables

<u>Table #</u>	<u>Description</u>	<u>Page</u>
2-1	Specifications of a 29.5-25 SHRL DL L-3 tire when inflated	20
2-2	Sample points from Generalized Deflection Chart	24
2-3	Input file for INGRID program	28
3-1	Description of main ASGAR subroutines	46
4-1	Calculated contact length and width of the 3-D model tire on a frictionless surface	54
4-2	Relationship between deflection, load and Young's modulus	62
4-3	Comparison of calculated Young's modulus with Young's modulus used in other research	59
4-4	Data required to calculate Young's modulus	67
4-5	Distribution of normal reaction from the ground with deflection is 4 cm	72
4-6	Distribution of normal reaction from the ground with deflection is 8 cm	73
4-7	Distribution of normal reaction from the ground with deflection is 12 cm	74
4-8	Distribution of normal reaction from the ground with deflection is 16 cm	75
4-9	Estimate the actual shape of the contact patch from the measured data using $(x/a)^n + (y/b)^n = 1$	93
4-10	Calculated contact width, length at selected deflection	94
4-11	Calculated positions of outermost contact nodes	95
4-12	Positions of outermost contact nodes along contact length direction	99
4-13	Positions of outermost contact nodes along contact width direction	100
4-14	Initial and final positions of outermost contact nodes in contact length and width directions	101
5-1	Numerical results of the 3-D model tire on a no-slip surface	107
5-2	Calculated positions for outermost contact nodes with no slip	110
5-3	Calculated positions for outermost contact nodes with no slip	111

List of Appendices

<u>Appendix</u>	<u>Description</u>	<u>Page</u>
A. 2-1	The Theory of Adding Surface Load	117
A. 4-1	Method of Estimating the Shape of the Actual Contact Patch from the Measured Data	121

List of Symbols

- ξ, η, ζ - element natural coordinates for hexahedra
 h - surface force vector (per unit surface area)
 f^{surf} - element surface force vector
 N_a - shape function
 j_s - surface Jacobian
 x - position vector
 n - unit normal vector to a surface
 S - parent domain of a quadrilateral
 e_ξ - unit tangent vector in the ξ -coordinate direction
 e_η - unit tangent vector in the η -coordinate direction
 E - Young's modulus
 ν - Poisson's ratio
 a - major axis of a hyper-ellipse
 b - minor axis of a hyper-ellipse
 $Area$ - area enclosed within a hyper-ellipse
 n - exponent of a hyper-ellipse
 α - weight of Gauss integration
 γ - Gauss integration point

Chapter 1

Introduction

The objective of the study of tire-soil interaction is to establish the interrelationship between the performance of a tire, its design parameters, and terrain conditions^[1]. The prime purpose is to provide guiding principles for the rational design and selection of a tire to suit particular operational requirements^[1]. In this regard, the designer of wheeled vehicles has a great deal of interest in the accurate modelling of the tire-soil interaction system.

Various approaches to the prediction of tire performance over unprepared terrain have been developed, ranging from theoretical to empirical. In recent years, considerable effort has been expended in developing an analytical framework for predicting tire performance, in light of a better understanding of the mechanics of tire-soil interaction and the availability of sophisticated numerical methods and computer technology^[2]. A brief survey of these approaches is given below.

1.1 Review of Previous Research

A central issue in the study of tire-soil interaction is the development of a mathematical model for predicting the geometry of the contact patch and the interacting forces on the tire-soil interface, in terms of the design parameters of the tire and the mechanical properties of the soil^[3]. Over the years, a number of models for tire-terrain interaction have been developed^[4,5,6,7,8,9,10]. These have been reviewed by various authors (see Ref.[1,2]). Song-Tao Chen, as part of his Master's thesis^[4] (1988), provided a comprehensive review of the state of the art in the study of tire-terrain interaction. This thesis, therefore, will not attempt to provide such an exhaustive review of previous work, but rather will focus on

the main features of some of the better known models, as well as their respective advantages and disadvantages with regard to their ability to predict the contact geometry, the stress distribution on the contact area and the tractive performance of a tire (i. e., its thrust, motion resistance, drawbar pull, etc). However, recent research not included in Chen's review will be examined in greater detail.

1.1.1 Bekker's Model

In the 1960's Bekker pioneered a simple method of evaluating tire-terrain interaction^[5]. With regard to contact geometry, Bekker assumed that the tire would remain undeformed like a rigid wheel, if the sum of the inflation pressure and the pressure due to tire carcass stiffness was greater than or equal to the maximum pressure exerted by the terrain at the lowest point of the tire circumference. On the other hand, if the terrain was firm enough, the lower part of the tire would deform, with the contact length corresponding to a chord of the circle representing the shape of the tire. As for stress distribution, the normal pressure on the contact patch was calculated from Bekker's pressure-sinkage relationship. The pressure varied from zero at the front edge of the contact patch to a constant value along the flat portion of the contact patch for a deformed tire. However, no consideration was given to tangential stresses on the tire-terrain interface in his model. Hence, Bekker's analysis can only be used to predict the motion resistance of the tire due to soil compaction, and the internal resistance resulting from the hysteresis of tire material.

Obviously there are some fundamental weaknesses in Bekker's approach. First, the contact geometry will, in reality, be somewhere between the two extreme cases considered by Bekker. Second, he completely ignored the presence of tangential stresses on the tire-terrain interface. Consequently, the model is not suited for predicting the thrust and drawbar pull.

1.1.2 Fujimoto's Model

In 1977, Fujimoto developed another model of tire-terrain interaction^[6]. It treated an elastic tire as a rigid wheel of larger diameter (see Fig.1-1). He assumed that the normal stress along the contact patch was constant. The tangential stress was expressed as an exponential function of the shear strain of the soil. The front and back contact points, A and B, were defined by the angles θ_1 and θ_2 . The latter were calculated as a function of the ratio of tire stiffness and soil strength. The radius of the larger rigid wheel was geometrically determined (see Fig.1-1). By integrating the stresses along the contact patch AB, the drawbar pull, motion resistance, and thrust were calculated.

In contrast to Bekker's model, this model took into account the effects of tangential stress, permitting the prediction of torque, thrust and drawbar pull. It also provided a more realistic contact geometry. However, one major criticism of Fujimoto's model is the assumption of constant normal stress along the contact patch. This is not usually realistic. Furthermore, the prediction procedure is much more complex than Bekker's.

1.1.3 Baladi's Model

In 1984, Baladi et al developed a mathematical model in which the entire tire-soil interaction process was treated as two springs in series, one describing the flexibility of the tire and the other the elastic-plastic deformation of the soil (see Fig.1-2)^[7]. Mathematical expressions were derived for the two springs' constants in terms of the load deflection characteristics of the tire, the undeflected geometry of the tire and the mechanical properties of the soil. Like the Fujimoto model, the contact geometry was assumed to be part of an equivalent rigid wheel (see Fig.1-3). The normal stress distribution was calculated from the equivalent spring system, and was assumed to be symmetric along the contact patch, varying from

Figure 1-1: Contact geometry in Fujimoto's model

zero at its front and rear contact points to a maximum at its center (see Fig.1-3). The tangential stress distribution was a function of several soil parameters. The position of the contact patch was defined by θ_1 and θ_2 which in turn were functions of sinkage, tire deflection and the radius of the wheel.

As in the Fujimoto model, the diameter of the equivalent rigid wheel was geometrically determined. By integrating both the normal and tangential stress distributions, the motion resistance, drawbar pull, torque and thrust for a tire travelling over a deformable surface was determined.

One strength of Baladi's model vis-a-vis Fujimoto's is that it is comparatively free of excessive empiricism. Another important advantage relates to the dropping of Fujimoto's rather artificial assumption of a constant normal stress distribution along the contact patch. Moreover, Baladi et al showed that there was reasonably good agreement between tire performance predicted using their model and laboratory test data. On the other hand, the assumption that the maximum normal pressure always occurs at the mid-point of the contact patch is not totally realistic. Experimental data suggest that the location of the maximum normal pressure varies with the slip of the tire^[3].

1.1.4 Yong's Model

In 1980, Yong et al developed another model to analyze tire-soil interaction^[8]. They assumed the tire to be a linear-elastic, cylinder-shaped body, while the terrain was treated as another cylindrical body with an infinite radius and as a piecewise linear elastic material. The terrain was discretized by finite elements (see Fig.1-4). The tire-terrain system is illustrated in Fig.1-5.

The normal and shear stress distributions on the contact surface were taken from experimental data and used as inputs to the finite element analysis. The contact geometry was calculated from the Hertz contact theory.

It should be pointed out that, if the normal and tangential stress distributions

A. SPRING CONSTANT FOR FLEXIBLE TIRE (k_t)

B. SPRING CONSTANT FOR SOIL (k_s)

C. SPRING CONSTANT FOR SOIL-TIRE SYSTEM $\frac{k_s k_t}{k_s + k_t}$

**Figure 1-2: Equivalent spring constant for soil-tire system
in Baladi's model**

**Figure 1-3: Contact geometry and normal stress distribution
in Baladi's model**

Figure 1-4: Finite element mesh of Yong's model

Figure 1-5: Tire-soil configuration according to Yong's model

and contact geometry can be specified at the outset, there is no need to use finite element analysis to study the tire-soil interaction, as far as the prediction of tire performance is concerned. The Yong method is really only suitable for studying the strain and stress fields in the terrain itself. Moreover, it should be noted that most unprepared terrains under tire load undergo large yielding and plastic deformations; they do not behave elastically. Therefore, the underlying assumption in this model that the terrain is piecewise elastic is unrealistic^[3].

1.1.5 Chen's Model

In 1988, Chen developed a method for analyzing tire-terrain interaction in which the tire is modelled by finite elements, while the terrain is represented by a group of semi-empirical equations describing the pressure-sinkage relationship, the shear stress and shear displacement relationships, and the response of the terrain to repetitive loadings^[4]. He used a simple finite element model to represent the real tire. The model tire was assumed to be a smooth homogeneous cylinder with the same overall dimension as the real tire to be modelled.

A Poisson ratio and Young's modulus were selected to reflect the behaviour of the carcass and air pressure inside the tire. The finite element mesh of the tire model is shown in Fig.1-6. The elastic properties of the tire model were derived in such a way that under the same load, it would produce similar contact length, width and area, and deflection of the real tire as described in the *Generalized Deflection Chart* (GDC) (see Fig.2-3) provided by tire manufacturer. Once a reasonably good fit with the empirical curves was achieved, Chen then used his tire model, in conjunction with the semi-empirical equations describing the properties of the terrain, to determine the contact geometry and pressure distribution at the tire-terrain interface. Based on these, the rolling resistance, sinkage, draw-bar pull and torque were then predicted.

In contrast to Yong's methodology, where the terrain was discretized by finite

Figure 1-6: Finite element mesh of Chen's model

elements, Chen approached the problem from the opposite direction, that is, using finite elements to represent the tire itself. Furthermore, Chen's model does not require the stress distributions on the tire-terrain interface as inputs.

One of the shortcomings of Chen's model was his improper assumption of plane stress, which would only apply when one of the three dimensions (in this case the width of the tire) was very small in relation to the other two dimensions. Another limitation is that his model, being two-dimensional (2-D), can predict the contact length but not the width (or area) of the contact patch. To circumvent this problem, Chen made the assumption that the contact patch was rectangular. Knowing the contact area under various load conditions as found in the GDC permitted Chen to deduce what the contact width should be. Still another limitation relates to the long rectangular shape of the finite elements comprising the model tire's meshes, which are apt to produce major numerical errors. To minimize this numerical problem, a mesh made up of square elements would have been preferable.

In the 9th International Conference of the International Society for Terrain-Vehicle Systems held in Barcelona, Spain in 1987, Boonsinsuk et al, and Qun et al presented papers^[9,10] dealing with tire-soil interaction. Like Chen, both papers derived values of equivalent Young's modulus of a tire, and subsequently used these in their analysis of tire-terrain interaction.

1.1.6 Boonsinsuk's Model

In Boonsinsuk's model^[9], measurements of the load/contact area relationship were taken on a hard, flat surface to derive, using the Hertz contact theory, the equivalent modulus of elasticity (i. e. Young's modulus), which reflected inflation pressure, tire carcass stiffness, and the effect of the tread pattern. Once the modulus of elasticity is known, in conjunction with the assumption that the terrain

is elastic, then the contact area of the tire with the soil can be calculated.

One criticism of this model pertains to the unrealistic assumption of the soil as an elastic material. In fact most soils are closer to being plastic than to elastic. Another problem is that the Hertz contact theory is only applicable in a linear, small deformation situation, whereas the deformation that a tire undergoes is normally quite significant, implying a non-linear contact relationship between the tire and the terrain.

1.1.7 Qun's Model

This model describes an equation which expresses the equivalent modulus of elasticity. The modulus of elasticity is a function of tire pressure and load^[10].

The interface shape of the tire-soil interaction is simplified by limiting the analysis to two kinds of essential interface shapes. First, when the soil is harder than the tire, the deformation of the tire is larger and the interface is by-and-large flat. The interface is supposed to be formed by a plane connected with a spiral surface. Second, when the tire is harder than the soil, the tire deformation is smaller and the interface has a convex shape.

The soil is modelled as a fully plastic body. Equations are developed for the tire load, net traction and input torque as a function of interface stresses and contact geometry. The equations were solved by computer using an iterative process. Inputs were tire load, equivalent Young's modulus, CI (Cone Index) and slip. The results of the model were claimed to correlate well with soil bin measurements.

One of the shortcomings of Qun's approach is that only two interface shapes were assumed when in fact innumerable contact profiles are likely to arise.

1.1.8 Review of Currently Available Computer Based Models

Aside from the tire models encountered in the study of tire-terrain interaction, numerous other models have been proposed for other purposes such as in thermal

and fatigue analysis, etc. They have varying degrees of sophistication. In 1985, NASA and the U.S. tire industry jointly sponsored the National Tire Modelling Program (NTMP). A paper entitled 'Advances and Trends in the Development of Computational Models for Tires'^[11] reviewed the different tire models that had been developed, and discussed their merits as well as their deficiencies. The models can be grouped into six categories(see Fig.1-7) as follows.

Early tire models These are characterized by their simplicity. These models considered the tread to be a prestressed string or a ring, and the sidewalls as elastic(or viscoelastic) foundations supported by the tread structure. Their major limitations are: (1) they require extensive experiments to evaluate the equivalent properties; and (2) their accuracy and range of validity are not known in advance.

Cord-network models These are sometimes referred to as netting analysis, wherein inflation pressure is assumed to be carried exclusively by the cords. These models have the limitation of neglecting both the bending of the tire and stiffening effects of the rubber surrounding the cords.

Membrane models These are based on the use of a linear or nonlinear momentless theory of shells wherein the effects of bending on shell deformation are neglected. Nonlinear momentless analysis can be useful in calculating the profiles but they cannot handle discontinuities in loading, geometry, or material properties.

2-D axisymmetric models These are limited to axisymmetric loading.

3-D continuum models Two approaches have been proposed for formulating these models. The first approach is based on using semi-analytic techniques to reduce the dimensionality of the problem (e.g. Fourier expansions in the circumferential direction.). The second approach is based on using 3-D isoparametric solid finite elements.

2-D thin and thick shell models Thin shell models are based on Kirchhoff-

Figure 1-7: Other tire models

Love shell theories which neglect transverse shear deformation and their use for modelling tires is therefore questionable. Anisotropy results in increasing the complexity of the model and consequently many investigators neglect its effect by using an orthotropic model.

Of the aforementioned six groups of tire models, the 3-D continuum models and the 2-D shell models are the most commonly used at the present time. This is particularly true when the finite element method is used in modelling the tire.

1.2 Overview of the Thesis

1.2.1 Objective of Thesis

The purpose of this thesis is to develop a 3-D finite element tire model for predicting the behaviour of off-road tires under load on a hard surface. This will provide a basis for the detailed analysis of tire-terrain interaction and for the development of improved methods for predicting the tractive performance of off-road tires.

1.2.2 The General Approach of the Thesis

Tire manufacturers normally provide data and specifications relating to the behaviour of tires. This information can be divided into two groups. One group concerns the geometry of the tire before and after inflation. The other, found in the GDC, describes the relationships between the applied vertical load and the contact area, length and width on a hard surface.

In this thesis, the geometry of the model tire is represented by a large number of 3-D finite elements. However, while the real tire is made up of complex and heterogeneous materials, the carcass of the model tire is assumed to be composed of only one homogeneous isotropic elastic material.

The general thrust of the thesis is to determine the material properties of the carcass of the model tire that will produce a GDC similar to that provided by the tire manufacturer for a real tire. The model tire can then be used as a digital representation of a real tire for the detailed study of tire-terrain interaction.

1.2.3 Organization of the Thesis

- Chapter 2 - Modelling the geometry of the tire with finite elements. This involves discretizing the structure of the real uninflated tire by 3-D finite element meshes. The model tire is subsequently 'inflated' to operating pressure so that the geometry of the model tire corresponds to that of the real tire.
- Chapter 3 - Modelling the material properties of the model tire. Starting with the inflated tire modelled in Chapter 2, vertical load is applied to the tire in an incremental fashion. Given sufficiently small increments, the behaviour of the model tire can be considered as linear, permitting derivation of the values of Young's modulus. The value of the other material property, Poisson's ratio, is set to 0.48, as has been the convention in tire research. The method of determining the size of the incremental load, and the flowcharts of the computer programs used to model the material properties, are described in this chapter.
- Chapter 4 - Results on a frictionless surface. The results of the analysis when the model tire is in contact with a perfectly smooth, frictionless surface are described. The results include predictions of the contact patch, the reaction from the ground and Young's modulus, and graphical representations of the tire deformation.
- Chapter 5 - The effects of friction on tire behaviour at the tire-ground interface. The effect of friction on the behaviour of the tire is studied by loading the tire against a no-slip hard surface. The results are

then compared with those obtained when the surface was assumed frictionless.

- Chapter 6 - Conclusions and suggestions for future work are given.

Chapter 2

Modelling the Geometry of the Tire

2.1 The Finite Element Representation of a Tire

2.1.1 The Tire to Be Modelled

The real tire upon which the model tire is based is the *29.5-25 Super Hard Rock Lug DL L-3* tire manufactured by the *Goodyear Tire & Rubber Company*. Table 2-1 contains the manufacturer's specifications of the tire under study. Fig.2-1 illustrates the definitions of tire parameters. A cross-sectional view of the tire is illustrated in Fig.2-2. The Generalized Deflection Chart is given in Fig.2-3. Table 2-2 gives the sample data points of the Generalized Deflection Chart for the tire. The general shape of the uninflated tire represented by finite elements is shown in Fig.2-4. Fig.2-5 offers a cross-sectional view of the uninflated tire.

It should be noted that while the finite element representation of a specific tire is given in this thesis, the general approach to the tire modelling described here can be applied to other tires.

2.1.2 The Tire Mesh

The tire manufacturer normally provides all the necessary data on tire dimensions. With this information, an input file for the INGRID¹ program was prepared (see Table 2-3). The INGRID program generated 3-D meshes along with their topology and nodal coordinates.

¹ INGRID: a 3-D mesh generator for modelling non-linear systems

**Table 2-1: Specifications of a 29.5-25 SHRL DL L-3 tire
when inflated**

Diameter	183.9cm
Tread Width	69.1 cm
Section Height	51.3 cm
Lug Area/ Carcass Area	.67
Lug Height	4.4 cm
Lug Width	17.5 cm
Inflation Pressure	206.8 kPa
Ground Pressure	316.2 kPa
Tire Construction	Bias-ply

Figure 2-1: Definitions of tire parameters

29.5-25 Super Hard Rock Lug DL L-3 tire
Goodyear Tire and Rubber Company, Akron, Ohio

Unit: cm

**Figure 2-2: Cross sectional view of the real tire
when uninflated**

29.5-25 Super Hard Rock Log DL-3 tire
Goodyear Tire and Rubber Company, Akron, Ohio
Lab data 4-28-82

Figure 2-3: Generalized Deflection Chart

Table 2-2: Sample points from Generalized Deflection Chart

Deflection cm	Load [*] KN	Contact Width cm	Contact Length cm	Area cm ²
1.27		30.48	22.10	25.80
2.54		38.1	27.94	658.06
3.81		45.72	33.78	1290.32
5.08		51.31	39.8	1741.93
6.35	62.23	54.84	44.20	2129.03
7.62		57.15	48.26	2516.12
8.89		59.94	52.83	2903.20
10.16		61.98	57.66	3290.30
11.43		64.77	60.45	3612.90
12.7	128.9	67.06	64.5	3967.70
13.97		68.83	69.60	4354.80
15.24		70.6	73.79	4645.15
16.51		71.88	79.25	5096.76

^{*} Load = 1049.92 * Deflection - 4.44

in this formula, m is the unit of deflection; KN is the unit of load

Figure 2-4: The general shape of the uninflated tire

Figure 2-5: A cross-sectional view of the uninflated tire

Given the inherent symmetry associated with the geometry of the tire subject to static normal load, only a quarter of the complete tire need be studied. Moreover, the mesh configuration required for the finite element model depends on the deformation characteristics of the object being studied. In the case of a tire under normal load, deformation is by and large confined to the area in and around the contact area, with the shape of the rest of the tire largely left unchanged. It follows that it is the behaviour of the tire contact area under various loading conditions that deserves the most attention. Thus a finer mesh was used in and around the contact patch, whereas a coarser mesh was used in areas further away from the contact patch (see Fig.2-6)^[12,13,14].

The model tire used in this analysis is divided into 864 8-node cube elements, and 12 transitional elements located between the fine and coarse meshes that contain 10 nodes each (see Fig.2-7). In total the tire model consists of 1455 nodes and 876 elements. The number of elements chosen falls generally within the range found in other research.

2.1.3 Creating the Input File to ASGARD

When creating the input file, strict adherence to the instructions set out in the *Documentation for the ASGARD program*^[15] is required. Much of the input is generated by the INGRID program, such as the topology and nodal coordinates. Other parameters such as the material properties, the initial boundary conditions, etc, must be inputted separately.

It is important to note that because only a quarter of the tire is actually being modelled, the nodes along the 'cut' edges of the tire cannot leave the surface of the plane. This is a reasonable restriction in light of the symmetry of the tire^[14,16,17]. Furthermore, the inner circumference of the tire is assumed to be fixed to the rim^[12,13,16,17].

Table 2-3: Input file for INGRID program

SECOND ATTEMPT AT TIRE ASGARD MAT 1 1 RO 1.0 E 3E7 PR 0.3 ENDMAT ENDC COOR 1 CSCA 0.01 ; LEV 1 REPE 1 ; ; PSLV 1 START 1 3 5 7 9 13 15 17 19 21 23 25 ; 1 3 ; 1 2 ; 0 0 0 0 0 0 0 0 0 0 0 0 83.2 90.15 0.0 0.0 PA 2 2 0 XY 9 91.65 PA 2 1 0 XY 9 82.75 PA 3 2 0 XY 18 90.55 PA 3 1 0 XY 15 81.95 PA 4 2 0 XY 27 88.65 PA 4 1 0 XY 20 79.95 PA 5 1 0 XY 25.67 76.28 PA 5 2 0 XY 35 80.65 PA 6 2 0 XY 38.5 60.45 PA 6 1 0 XY 30 59.28 PA 7 2 0 XY 38.65 55.55 PA 7 1 0 XY 34.0 52.65 PA 8 2 0 XY 38.65 48.75 PA 8 1 0 XY 33.3 48.65 PA 9 2 0 XY 36.3 43.65 PA 9 1 0 XY 32 46.35 PA 10 2 0 XY 34.3 40.95 PA 10 1 0 XY 30.67 43.95 PA 11 2 0 XY 31.8 37.07 PA 11 1 0 XY 26.67 40.28 PA 12 2 0 XY 31.8 31.88 PA 12 1 0 XY 24.3 31.88 SF 4 2 0 5 2 0 TS 0 0 0 1 0 0 76.85 23.6 12.28 SF 5 1 0 6 1 0 TS 0 0 0 1 0 0 55.05 2.0 31.8 SF 3 1 0 5 1 0 TS 0 0 0 1 0 0 64.35 12.5 17.78 SF 10 1 0 12 1 0 TS 0 0 0 1 0 0 31.88 40.2 15.9 SF 10 2 0 11 2 0 TS 0 0 0 1 0 0 36.48 37.8 5.68 SF 8 2 0 10 2 0 TS 0 0 0 1 0 0 52.37 24.3 15.18 SF 7 1 0 10 1 0 TS 0 0 0 1 0 0 52.37 24.3 9.7 RR 0 0 2 0 0 2 RX -10 ; B 12 0 0 12 0 0 111111 COOR 14 RX 10 ; RX 20 ; RX 30 ; RX 40 ; RX 50 ; RX 60 ; RX 70 ; RX 80 ; RX 90 ; RX 100 ; RX 110 ; RX 120 ; RX 130 ; RX 140 ; LREP 1 2 3 4 5 6 7 8 9 10 11 12 13 14 ;	MATE 1 END START 1 3 5 7 9 13 15 17 19 21 23 25 ; 1 3 ; 1 2 3 4 5 ; 0 0 0 0 0 0 0 0 0 0 0 0 83.2 91.95 0.0 0.0 0.0 0.0 0.0 0.0 PA 2 2 0 XY 9 91.65 PA 2 1 0 XY 9 82.75 PA 3 2 0 XY 18 90.55 PA 3 1 0 XY 15 81.95 PA 4 2 0 XY 27 88.65 PA 4 1 0 XY 20 79.95 PA 5 1 0 XY 25.67 76.28 PA 5 2 0 XY 35 80.65 PA 6 2 0 XY 38.5 60.45 PA 6 1 0 XY 30 59.28 PA 7 2 0 XY 38.65 55.55 PA 7 1 0 XY 34.0 52.65 PA 8 2 0 XY 38.65 48.75 PA 8 1 0 XY 33.3 48.65 PA 9 2 0 XY 36.3 43.65 PA 9 1 0 XY 32 46.35 PA 10 2 0 XY 34.3 40.95 PA 10 1 0 XY 30.67 43.95 PA 11 2 0 XY 31.8 37.07 PA 11 1 0 XY 26.67 40.28 PA 12 2 0 XY 31.8 31.88 PA 12 1 0 XY 24.3 31.87 SF 4 2 0 5 2 0 TS 0 0 0 1 0 0 76.85 23.6 12.28 SF 5 1 0 6 1 0 TS 0 0 0 1 0 0 55.05 2.0 31.8 SF 3 1 0 5 1 0 TS 0 0 0 1 0 0 64.35 12.5 17.78 SF 10 1 0 12 1 0 TS 0 0 0 1 0 0 31.88 40.2 15.9 SF 10 2 0 11 2 0 TS 0 0 0 1 0 0 36.48 37.8 5.68 SF 8 2 0 10 2 0 TS 0 0 0 1 0 0 52.37 24.3 15.18 SF 7 1 0 10 1 0 TS 0 0 0 1 0 0 52.37 24.3 9.7 RR 0 0 2 0 0 2 RX -2.5 ; RR 0 0 3 0 0 3 RX -5 ; RR 0 0 4 0 0 4 RX -7.5 ; RR 0 0 5 0 0 5 RX -10.0 ; B 12 0 0 12 0 0 111111 COOR 4 RX 150 ; RX 160 ; RX 170 ; RX 180 ; LREP 1 2 3 4 ; MATE 1 END pplv END
---	---

Figure 2-6: The mesh of the tire model

Figure 2-7: Interface between the 8-node and 10-node elements

2.2 The Model Tire Under Inflation

2.2.1 Introduction

The process of applying normal load to the tire is divided into two stages. During the first stage, an inflation pressure of 206.8 MPa is applied in four equal increments. In the second stage, the rim of a normally inflated tire is displaced incrementally to model tire behaviour under normal load^[12,14,18,19].

The assumptions that simplify the analysis include:

- The boundary in the bead region is assumed to be rigidly fixed.
- The model tire has a completely smooth tread.
- The material of the carcass of the model tire is homogeneous and linear elastic.
- The weight of the tire carcass itself is neglected.

The uninflated tire is mounted onto the rim whose diameter is fixed. The diameter of the rim is deduced from the data given in Table 2-1, that is, tire diameter less twice the section height (see Fig.2-1). The rim diameter is therefore 81.3 cm. Mounting the uninflated tire, whose section height is given in Fig.2-2, onto the rim therefore gives the overall diameter of the uninflated tire. The difference between the diameter of the uninflated and inflated tire is 2.6 cm. The next step is to determine the value of Young's modulus (when the Poisson ratio is assigned a value of 0.48) that will result in the model tire inflated to the proper size when the inflation pressure is 206.8 KPa.

ASGARD was not capable of applying pressure to the transitional elements. Therefore the author had to modify an ASGARD subroutine, Etpl3d.ftn, which calculates the Element Stiffness Matrix and Nodal Force Vectors.

There are several steps in the procedure for applying inflation pressure to the tire. First, determine the element numbers for the inner wall elements of the tire

from the ASGARD input file described in the previous section. Second, determine which side of the elements on the inner wall are subjected to surface load (i. e. inflation). This information is then inputed into ASGARD subroutine Etpl3d.ftn. Third, transform the surface load to equivalent nodal force by the virtual work principle. The detailed theory of how to apply surface load is given in Appendix 2-1.

2.2.2 Results

The shapes of the tire before and after inflation are displayed in Fig.2-8 and 2-9 ($2\times$ displacement). They show the shape of the inflated tire superimposed on the uninflated tire. The outer mesh represents the tire after inflation. Fig.2-10 is a ten-fold graphical representation of the displacement of the tire after inflation. The symmetry of the displacement reveals that the effect of the inflation pressure was correctly assessed.

**Figure 2-8: The tire model before and after inflation
(tire displacement enlarged two times)(exterior view)**

**Figure 2-9: The tire model before and after inflation
(tire displacement enlarged two times)(interior view)**

Figure 2-10: Magnified view (10 times) of tire displacement upon inflation

Chapter 3

Modelling the Material Properties of the Model Tire

Once the geometry of the tire has been modelled with finite elements, the next step is to determine the values of the material properties for the model tire, so that its behaviour will be similar to that of the real tire under similar loading conditions.

3.1 Assumptions

A number of simplifying assumptions were made when modelling the tire under normal load. Most of these assumptions are the same as those that were made when it was subject to inflation pressure but now the tire is loaded on a frictionless hard surface. The changes in the inflation pressure caused by tire deformation is, however, neglected.

3.2 Loading

The process of applying normal load to the model tire is similar to that used by other researchers. Rather than explicitly applying a load to the tire and measuring the resulting hub displacement towards the hard surface, load is applied by fixing the hub and vertically raising the hard surface towards the hub^[12,14,16,19]. The relative displacement, or tire deflection, in either case is the same. The load-deflection relationships for the tire in both two cases should, therefore, be identical.

3.3 The Incremental-Iterative Process

During the loading process, the deflection of the tire is increased to about 16 centimeters under the maximum load allowed for the tire. Such a significant deflection implies that the tire response to loading will be geometrically nonlinear. To overcome the nonlinear problem, the final deformed shape of the tire is determined by passing incrementally through a sequence of equilibrium configurations, each of which is obtained from the previous one by the application of linear theory^[16,18]

The incremental loading or deflection is not the same for each step. The incremental deflection is determined by the need to avoid having nodal points penetrate the hard surface. Furthermore, at each incremental loading the boundary conditions are updated to reflect the changing size of the contact patch. Details of how the incremental deflection is determined, and the boundary conditions changed, are given in section 3.5 in this chapter. The *Modified Newton-Raphson Method* is used during the iteration process.

The analysis of the complete loading process consists of 90 incremental steps in total. Therefore, on average, the incremental deflection is about 2 millimeters. After the tire deflection reaches 8 cm, the process of iteration begins with two iterations for each incremental step. The displacement norm residual must be less than 10^{-3} for convergence to occur.

3.4 The Material Properties

Similar to that in previous research, the Poisson ratio (ν), which is one of the two material properties of the model tire, is set to a value of 0.48^[12,18]. The Poisson ratio is assumed constant over the range of loading.

The other material property, Young's modulus (E), is derived by assuming that at each incremental step the response of the tire to the incremental load is

linear. For a given value of Young's modulus, the value of the reaction from the hard surface is calculated. Because the relationship is assumed to be linear, it is possible to determine the value of *average* Young's modulus that will produce the same value of reaction at a given deflection as that given in the GDC.

3.5 Algorithm for Predicting the Contact Geometry

The *contact algorithm* was developed to determine the size of the incremental deflection. The response of the tire to loading is inherently non-linear. This being the case, if the incremental deflection is too large, several iterations may be required. The iterations themselves are not problematic. The problem is that during the iteration process, if the boundary conditions are not updated, the size of the contact patch will not adjust to the higher incremental load, causing the nodal points adjacent to the contact patch to penetrate unrealistically below the hard surface.

To avoid this penetration, the boundary conditions must be updated, and sufficiently small increments of deflection are required. On the other hand, to minimize computation, larger increments are desirable. The objective is, therefore, to maximize the size of the increments while avoiding the penetration problem.

In addition to determining the size of the incremental deflection, that is, the vertical displacement of the contact patch, the algorithm also decides which nodal points should be considered to be on the contact patch at each incremental step.

The Algorithm is summarized below and its flowchart is shown in Fig.3-1.

a) A vertical displacement is applied at the nodal point A of the element at the bottom of the tire model (see Fig.3-2). The displacement is measured by the difference in the vertical coordinates of node A and the immediately adjacent node B. This displacement is used as prescribed displacement for the boundary

Figure 3-1: Flowchart of the contact algorithm

Figure 3-2: Illustration of the algorithm for deriving the contact geometry

conditions of the input file used in the execution of the ASGARD program.

b) Subsequent to the execution of the ASGARD program, the adjacent nodal point B in Fig.3-2(a) may still be above the bottom point A due to the deformation of the whole tire structure. Nevertheless, the vertical displacement between the two points is reduced. The program UPDATE.FTN checks whether nodal point B is within a certain distance, in the vertical direction, of point A. If the distance is too great, (see Fig.3-2(b)), step a) is repeated. However, at some point the difference in height between nodes A and B will be sufficiently small, (see Fig.3-2(c)), such that node B is considered to lie on the same horizontal surface as node A.

c) As illustrated in Fig.3-2(d) nodes A and B are now both on the same surface. This was accomplished by giving a slightly greater vertical displacement to node A than to node B to compensate for the fact that the difference in height between node A and node C was larger than that between node B and node C. Node C can now be considered the adjacent point used to determine the amount of vertical displacement to be applied to the line defined by nodal points A and B. Thus point C performs the role that point B performed in the previous steps of the incremental process, while the line AB now substitutes for the initial point A.

Step a) through c) are repeated using new points adjacent to the contact patch, until the pre-assigned maximum tire deflection is reached.

3.6 Overview of the Computer Programs used to Predict Tire Behaviour under Normal Load

Three Fortran programs are used to model the tire behaviour under vertical loading. The relationship between these three programs is outlined in Fig.3-3. ASGARD is a general purpose, nonlinear, transient finite element package developed at Carleton University. The flowchart of ASGARD is given in Fig.3-4. The main structure of ASGARD is given in Fig.3-5. The main ASGARD subroutines

and their descriptions are given in Table 3-1. UPDATE.FTN is a fortran program written by the author which updates the input file for ASGARD at each incremental step. The flowchart of UPDATE.FTN is given in Fig.3-6. RESULT.FTN, also written by the author, receives the output from ASGARD, then organizes and prints the results. The flowchart of RESULT.FTN is given in Fig.3-7. Fig.3-8 provides a more detailed flowchart describing the interrelationship between the three programs.

Given the fact that ASGARD is a general purpose package, some modifications to the program were necessary in order to be able to apply it to the specific case of tire modelling. For instance, modifications were required so that the boundary conditions could be updated after each incremental increase in the deflection of the tire, as the contact patch increases in size concomitantly with the load. Moreover, the tire modelling requires that the dimensions of the contact patch, and the force exerted on it by the hard surface be printed and saved for subsequent analysis. Therefore three subroutines in ASGARD were modified by the author: Input.ftn, Updacc.ftn and Conout.ftn (see Fig.3-5). These modified subroutines produce two output files Coord.0 and Freact (see Fig.3-8). Coord.0 stores updated nodal coordinates and incremental tire deflection, while Freact stores the values of the reactions at all constrained points.

As mentioned previously, RESULT.FTN was developed to organize the results pertaining to total deflection, contact length, width, Young's modulus, and reaction from the contact patch, and also to print out the data for drawing the footprint of the tire at different incremental steps. RESULT.FTN reads three files (see Fig.3-8): Coord.0 and Freact, which are both generated by the ASGARD program, and Reaction.0. RESULT.FTN writes six files: Coord.1, Resul1.dat, Resul2.dat, Resul3.dat, Footpr.plt and Reaction.0. Coord.1 stores updated nodal coordinates and is also used as input to UPDATE.FTN. Resul1.dat stores the results such as the total deflection, and the contact length and width. Resul2.dat

Figure 3-3: Procedure for modelling the tire's behaviour under vertical loading

(1): loops over iterations

(2): loops over increments

Figure 3-4: Flowchart of ASGARD program

(1) loops over iterations

(2) loops over increments

(3) loops over elements

* modified for the tire analysis

Figure 3-5: Structure of ASGARD program

Table 3-1: Description of main ASGARD subroutines

Subroutine	Description
START	records start time of the run.
NURSE	reads, sets and checks the first line i.e. global parameters in the input file.
INPUT	reads the remaining input data cards.
MATRON	diagnoses all the errors in the input file, and computes a few more dimensions needed by subroutine FRONTN.
GETMEM	sets the size of run unit for dynamic memory management.
NEWLOD	reads and sets user defined incremental flags such as convergence norm and tolerance for the norm.
ALGOR	selects resolution scheme for current load step. e.g. the tangential stiffness method.
DOCTOR	diagnoses errors.
ELFILE	computes element stiffness matrix and load vector, if first iteration.
ETPL3D	calculates stiffness matrix and load vectors at the element level for solid rectangular cubed elements having from 8 to 26 nodes.
FRONTN	assembles global stiffness matrix, and solves the $KX=R$ equations.
POSPRO	post processing (stresses, strains, residuals, etc.).
UPDACC	updates, saves and outputs the displacement, etc.
RESIPL	calculates stresses, strains, residual loads.
CONOUT	checks convergence and outputs results for nonlinear stress problems.

Figure 3-6: Flowchart of program UPDATE.FTN

Figure 3-7: Flowchart of program RESULT.FTN

Figure 3-8: Interrelationship of files used in tire analysis

stores Young's modulus values, and total reaction from the contact patch. Result3.dat stores the reaction distribution at different deflections. Footpr.plt stores all the data required to draw the footprint. Reaction.0 is used to update the reaction distribution.

UPDATE.FTN updates the input file for ASGARD at each incremental step. UPDATE.FTN reads both Coord.1, and Tire.in0 which is the ASGARD input file from the last incremental step (see Fig.3-8). The nodal coordinates found in Tire.in0 are replaced by the updated values found in Coord.1. Moreover, the boundary conditions which change in accordance with the changing size and shape of the contact patch are also updated. The method to update the boundary conditions is given in section 3.5 in this chapter. Thus a new Tire.in0 is written for subsequent input to ASGARD.

Chapter 4

Tire Behaviour on a Frictionless Surface

This chapter describes the results of the predicted tire behaviour obtained using the model on a perfectly smooth frictionless surface. The results include the following at various deflections:

- The contact length and width.
- The reaction from the ground and Young's modulus.
- A series of graphical representations of the tire deformation at various normal loads.
- The footprint.
- The movements of the nodes in contact with the surface.

4.1 Contact Length and Contact Width

With an assumed value of the Poisson ratio of 0.48, Fig.4-1 and Fig.4-2 illustrate the predicted relationships between tire deflection and the contact length and width respectively, obtained using the procedure described previously. Table 4-1 contains the data used to plot Fig.4-1 and Fig.4-2. These are juxtaposed with the measured length and width of the real tire as described in the GDC. Moreover, the table includes a measure of error between the real and model tire results, and a listing of the outermost nodes in the longitudinal and lateral directions. Fig.4-3 and Fig.4-4 illustrate the position of the nodes (from output of INGRID program), identified in Table 4-1, on the contact patch.

For purposes of comparison, the results obtained by Chen^[4] are also included in the figures. As can be seen, the 3D F.E.M. model employed in this thesis generates

Figure 4-1: Comparison of the contact lengths generated by 3-D FEM, the measured and Chen's 2-D FEM

Figure 4-2: Comparison of the contact widths generated by 3-D FEM, the measured and Chen's 2-D FEM

Table 4-1: Calculated contact length and width of the 3-D model tire on a frictionless surface

Calculated contact length vs. deflection of the tire

Deflection (cm)	Length1 (cm)	Node	Length2 (cm)	Error(%)
2.27	23.	1378	26.69	13.8
4.56	31.	1364	37.33	16.9
6.45	38.74	1330	44.52	12.9
8.81	46.13	1287	52.54	12.2
11.56	53.42	1146	60.88	12.2
14.36	60.68	1095	70.88	14.3
16.12	65.24	1042	77.60	15.9

Length1 is calculated contact length

Length2 is measured contact length

Calculated contact width vs. deflection of the tire

Deflection (cm)	Width1 (cm)	Node	Width2 (cm)	Error%
0.5	17.97	1396	25.00	28.1
2.27	26.69	1405	36.48	26.8
3.01	35.52	1411	40.92	13.1
4.93	44.24	1437	50.64	12.6
5.79	53.21	1439	53.78	1.0
15.65	64.21	1443	71.01	9.5
16.76	64.19	1441	72.13	11

Width1 is calculated contact width

Width2 is measured contact width

Figure 4-3: Potential contact nodes in contact length direction

Figure 4-4: Potential contact nodes in contact width direction

a contact length and width that more closely correspond to measurements of the real tire than does Chen's 2D F.E.M.. The greater realism of the 3D model is partly explained by the fact that in Chen's model the tire is represented by a solid body which fails to differentiate the influences of the carcass of the tire and the inflation pressure inside it. The 3D model, on the other hand, does make this differentiation, and is in this sense much more representative of the structure of the real tire. The 3D model performs particularly well under normal loading conditions (i. e., between a deflection of 5 and 15 cms). The error in the length direction is about 14% and in the width direction from 1% to 10% (see Table 4-1).

There are several possible reasons why the present tire model underestimates the size of the contact patch. First, the structure of the tire material may have been over-simplified by using only one Poisson's ratio. Second, the real tire has a tread while the model tire possesses a smooth surface. This difference may be especially large when contact patch is small. Third, the results described in the GDC were derived from a tire in contact with a surface characterized by friction. The effects of friction on the tire behaviour predicted using the tire model will be examined in the next chapter.

4.2 Reaction from the Ground and Young's Modulus

Fig.4-5 illustrates the relationship between Young's modulus and deflection. The value of Young's modulus declines under increasing load. Starting with a value of 50 MPa at zero load, Young's modulus declines at a decreasing rate until the deflection attains 8 cm. Between this load and the maximum load, (at 16 cm deflection), the value of Young's modulus remains relatively steady at about 10 MPa. Fig.4-6 shows the relationship between Young's modulus and load. This is derived from the GDC using the relationship between deflection and load. Table 4-2 contains the data used to plot Figs.4-5 and 4-6. Table 4-3 compares Young's

modulus used by other researchers with that calculated by the tire model described previously. The calculated Young's modulus falls in the range used or found in other research.

Following is the procedure for deriving and graphing the relationship between Young's modulus and load. The response of the tire to incremental load within an incremental step is approximately linear when the incremental load is small, so that

$$\frac{\Delta R_1}{\Delta R_2} = \frac{E_1}{E_2}$$

exists. E_1 is Young's modulus which is assigned to the model tire. Its value is 3 MPa. ΔR_1 represents the incremental reaction from the ground when Young's modulus is the above value. ΔR_2 is the incremental reaction from the ground found in GDC given the incremental and total deflection. Knowing the values of ΔR_1 , ΔR_2 , and E_1 permits the derivation of E_2 . Table 4-4 contains the values of ΔR_1 , ΔR_2 , E_1 and E_2 for all incremental steps. Fig.4-7 plots incremental deflection against total deflection. Note that the range of incremental deflection is between 1.5 mm and 3.4 mm. Any incremental deflection less than 1.5 mm would be considered on the surface. The bottom of the valleys represents a stage when a nodal point makes contact with the surface. The decline towards the bottom point signifies that a nodal point is approaching the hard surface.

Fig.4-8 is derived from both the total and incremental deflections described in the GDC which shows the relationship between load and deflection. Note the close relationship between Figs.4-7 and 4-8. This relates to the fact that there is an approximately linear relationship between load and deflection (see GDC).

Fig.4-9 shows the relationship between incremental reaction and tire deflection. With all of the above data it is possible to derive the relationship between Young's modulus and deflection. As can be seen in Fig.4-10 the values of Young's modulus gyrated widely during the initial stages of deflection but became increasingly more stable as deflection increased. This is because Young's modulus is a function

of deflection in the model. Each incremental step corresponds to a number of Young's modulus values. Thus in order to get a single value of Young's modulus the incremental deflection should be very small. Not all Young's modulus values shown in Fig.4-10 therefore qualify as legitimate values. Only those values which have resulted from small incremental deflection of around 1.5 mm qualify. Fig.4-5 shows the same relationship between Young's modulus and deflection when the latter restriction is applied.

**Table 4-3: Comparison of calculated Young's modulus with
Young's modulus used in other research**

(3-D FEM model tire) (for normal load condition) whole carcass	(from 50 KN to 160 KN) from 8 MPa to 12 MPa
(see ref 19) whole carcass	5.9 MPa
(see ref 11) whole carcass	26.2 MPa

Figs.4-11,12,13,14 display the distribution of nodal force at the contacting nodes at selected values of deflection. The contact forces can be converted to a statically-equivalent distributed pressure using virtual work principles (see Appendix 2-1). The computed pressure distributions indicate that a small tire deflection produces a parabolic normal pressure distribution; whereas a large tire deflection leads to a saddle-shaped distribution where high concentrations at the edges in the width direction of the footprint, and very low pressure at its center, are observed. These results are similar to those obtained by other researchers (see Ref. [12,17]). The data shown in Table 4-5 to Table 4-8 are the basis for plotting Figs.4-11,12,13,14. As can be seen in Fig.4-14 the general pattern of pressure distribution are similar for a real tire and the model tire.

Figure 4-5: The relationship between Young's modulus and deflection

Figure 4-6: The relationship between Young's modulus and load

**Table 4-2: Relationship between deflection, load
and Young's modulus**

deflection (cm)	load (KN)	ΔD (mm)	Young's modulus (MPa)
0.237	0.001	2.37	50.55
0.964	5.681	1.89	25.73
2.065	17.241	2.43	23.94
2.826	25.231	1.71	21.97
3.637	33.746	1.98	18.10
4.214	39.804	1.79	17.19
5.594	54.293	1.62	14.54
6.451	63.290	1.73	14.30
7.437	73.643	1.61	11.36
8.471	84.499	1.77	10.44
9.424	94.504	1.76	9.99
10.385	104.59	1.55	9.80
11.240	113.57	1.80	9.49
11.973	121.26	1.70	9.13
12.825	130.21	1.63	8.97
14.040	142.96	1.75	8.53
15.481	158.09	1.77	8.16

Figure 4-7: Incremental deflection vs. total tire deflection

Figure 4-8: Incremental load vs deflection (from GDC)

**Figure 4-9: Incremental reaction from the ground
vs. deflection**

Figure 4-10: Young's modulus vs. deflection for all incremental steps and incremental deflection (enlarged 4 times) vs. deflection

Table 4-4: Data required to calculate Young's modulus

Deflection (cm)	ΔD (mm)	ΔR_2 (KN)	ΔR_1 (KN)	E_2 (MPa)	Deflection (cm)	ΔD (mm)	ΔR_2 (KN)	ΔR_1 (KN)	E_2 (MPa)
0.237	2.37	2.491	0.0369	50.55	9.047	2.34	2.463	0.1734	10.68
0.501	2.64	2.778	0.0437	47.55	9.348	2.00	2.106	0.1543	10.23
0.775	2.73	2.869	0.0530	40.55	9.424	1.76	1.852	0.1389	9.99
0.964	1.89	1.989	0.0579	25.73	9.581	1.56	1.643	0.1237	10.04
1.137	1.72	1.810	0.0624	25.88	9.762	1.81	1.906	0.1247	11.45
1.294	1.56	1.647	0.0474	26.02	9.923	1.60	1.681	0.1104	11.41
1.481	1.87	1.965	0.0437	33.66	10.080	1.57	1.651	0.1039	11.91
1.821	3.40	3.575	0.0922	29.08	10.230	1.50	1.576	0.1268	9.31
2.065	2.43	2.559	0.0801	23.94	10.386	1.55	1.628	0.1246	9.80
2.268	2.02	2.126	0.0860	24.14	10.650	2.65	2.785	0.2155	9.69
2.436	1.68	1.768	0.0396	33.42	10.863	2.13	2.237	0.1786	9.39
2.654	2.17	2.283	0.0609	28.10	11.059	1.96	2.059	0.1635	9.44
2.826	1.71	1.805	0.0616	21.97	11.240	1.80	1.895	0.1497	9.49
3.005	1.79	1.888	0.0459	30.83	11.406	1.66	1.745	0.1372	9.54
3.176	1.70	1.788	0.0341	39.25	11.557	1.51	1.587	0.1243	9.58
3.439	2.62	2.759	0.0101	20.67	11.803	2.48	2.575	0.1617	11.94
3.637	1.98	2.079	0.0861	18.10	11.973	1.70	1.790	0.1471	9.13
3.801	1.64	1.726	0.0710	18.23	12.125	1.51	1.595	0.1341	8.91
4.035	2.33	2.450	0.0819	22.44	12.315	1.89	1.991	0.1542	9.69
4.214	1.79	1.888	0.0823	17.19	12.492	1.76	1.854	0.1563	8.89
4.377	1.62	1.703	0.0738	17.30	12.661	1.69	1.783	0.1497	8.93
4.563	1.86	1.956	0.0899	20.97	12.825	1.63	1.716	0.1434	8.97
4.758	1.95	2.047	0.0741	20.72	12.982	1.57	1.650	0.1374	9.01
4.930	1.71	1.804	0.0586	23.05	13.133	1.51	1.588	0.1317	9.04
5.088	1.58	1.659	0.0663	18.77	13.297	1.63	1.719	0.1283	10.04
5.239	1.50	1.584	0.0591	20.08	13.468	1.70	1.789	0.1244	10.78
5.432	1.92	2.025	0.0973	15.60	13.677	2.08	2.193	0.1803	9.12
5.595	1.62	1.709	0.0881	14.54	13.864	1.87	1.972	0.1740	8.50
5.792	1.96	2.068	0.0853	18.18	14.040	1.75	1.840	0.1616	8.53
5.966	1.74	1.833	0.0847	16.22	14.203	1.63	1.718	0.1502	8.57
6.125	1.58	1.663	0.0720	17.32	14.356	1.52	1.604	0.1397	8.60
6.278	1.53	1.607	0.0830	14.52	14.543	1.87	1.963	0.1609	9.15
6.451	1.73	1.819	0.0953	14.30	14.701	1.57	1.654	0.1438	8.62
6.703	2.52	2.649	0.1200	16.55	14.929	2.28	2.394	0.1909	9.40
6.915	2.11	2.218	0.1447	11.49	15.116	1.86	1.961	0.1709	9.60
7.099	1.84	1.939	0.1293	11.24	15.303	1.87	1.964	0.1408	10.01
7.275	1.75	1.846	0.1224	11.30	15.481	1.77	1.866	0.1714	8.16
7.437	1.61	1.700	0.1122	11.38	15.649	1.68	1.768	0.1618	8.19
7.597	1.59	1.676	0.8446	14.86	15.812	1.63	1.712	0.1295	9.09
7.748	1.51	1.588	0.1108	10.77	15.965	1.52	1.605	0.1295	8.85
7.904	1.55	1.632	0.1080	11.55	16.121	1.55	1.632	0.1389	8.81
8.071	1.67	1.757	0.9227	14.20	16.288	2.67	2.807	0.2514	8.37
8.293	2.21	2.330	0.1592	10.97	16.558	1.99	2.095	0.2089	7.53
8.471	1.77	1.866	0.1340	10.44					
8.650	1.79	1.884	0.1067	13.24					
8.812	1.62	1.702	0.1282	9.95

Figure 4-11: Distribution of normal reaction from the ground
when deflection is 4 cm

**Figure 4-12: Distribution of normal reaction from the ground
when deflection is 8 cm**

**Figure 4-13: Distribution of normal reaction from the ground
when deflection is 12 cm**

a conventional tire, size 11.00-20, without tread

(a) Distribution of normal reaction from the ground for a real tire^[23]

Figure 4-14: (b) Distribution of normal reaction from the ground for model tire when deflection is 16 cm

**Table 4-5: Distribution of normal reaction from the ground
when deflection is 4 cm**

x (cm)	s (cm)	R_y (KN)
08.79	11.73	01.558
00.00	11.73	00.507
04.39	11.70	-6.103
00.00	07.80	03.627
04.38	07.78	0.568
08.78	07.80	09.001
13.23	07.78	03.264
00.00	03.89	15.460
00.00	00.00	09.359
04.36	00.00	02.189
04.37	03.88	06.071
08.75	00.00	05.923
08.76	03.89	01.041
13.19	03.88	03.852
13.18	00.00	01.707
17.69	07.77	-3.240
17.66	03.88	01.863
17.65	00.00	00.712

**Table 4-6: Distribution of normal reactions from the ground
when deflection is 8 cm**

x (cm)	s (cm)	R_y (KN)	x (cm)	s (cm)	R_y (KN)
17.55	19.14	-2.23	08.63	07.61	8.86
13.12	19.15	0.00	12.96	07.58	-1.08
08.72	19.22	-0.28	00.00	03.80	14.53
00.00	19.24	-0.03	00.00	00.00	8.88
04.35	19.21	-2.10	04.30	00.00	1.13
22.01	15.20	-2.33	04.30	03.79	3.98
17.45	15.27	2.93	08.61	00.00	4.85
13.06	15.27	0.68	08.62	03.80	8.45
00.00	15.34	1.22	12.94	03.78	2.81
04.34	15.31	0.06	12.94	00.00	0.96
08.69	15.32	3.79	17.32	07.58	7.03
26.49	11.35	-0.17	21.84	07.56	2.32
21.91	11.37	0.66	26.42	07.56	2.70
17.37	11.41	4.84	17.29	03.78	7.70
13.00	11.41	1.20	17.28	00.00	3.16
08.65	11.45	6.13	21.80	03.77	2.61
00.00	11.46	1.91	21.79	00.00	1.61
04.32	11.45	-3.47	26.37	03.77	4.43
00.00	07.62	2.98	26.36	00.00	2.64
04.31	07.61	0.14			

**Table 4-7: Distribution of normal reaction from the ground
when deflection is 12 cm**

x (cm)	z (cm)	R_g (KN)	x (cm)	z (cm)	R_g (KN)
17.4	26.6	00.000	17.1	11.2	04.148
08.7	26.6	-1.602	12.8	11.2	00.175
00.0	26.6	-0.868	08.5	11.2	08.271
26.3	22.5	-0.186	00.0	11.2	01.566
21.8	22.5	-1.411	04.2	11.2	-4.203
17.3	22.6	01.368	00.0	07.5	02.677
13.0	22.6	-0.204	04.2	07.5	-0.546
08.6	22.7	01.921	08.5	07.5	07.987
00.0	22.8	00.596	12.8	07.4	-2.177
04.3	22.7	-0.276	00.0	03.7	14.260
26.2	18.7	05.096	00.0	00.0	08.760
21.7	18.7	01.469	04.2	00.0	00.856
17.2	18.8	03.270	04.2	03.7	03.384
12.9	18.8	00.675	08.5	00.0	04.459
08.6	18.8	02.329	08.5	03.7	07.633
00.0	18.9	01.068	12.8	03.7	01.775
04.3	18.9	-3.518	12.8	00.0	00.461
26.1	14.9	07.459	17.1	07.4	05.915
21.6	14.9	01.458	21.5	07.4	01.959
17.2	14.9	04.329	25.9	07.4	10.250
12.9	15.0	00.483	17.1	03.7	06.522
00.0	15.0	00.990	17.1	00.0	02.573
04.3	15.0	-0.472	21.5	03.7	02.013
08.6	15.0	13.557	21.5	00.0	01.288
26.0	11.1	09.283	25.9	03.7	11.340
21.5	11.1	01.488	25.9	00.0	06.016

**Table 4-8: Distribution of normal reaction from the ground
when deflection is 16 cm**

x (cm)	s (cm)	R_y (KN)	x (cm)	s (cm)	R_y (KN)
26.33	29.98	0.184	04.28	14.98	-1.044
21.79	29.94	0.000	08.56	14.91	2.900
17.37	30.06	-0.501	25.64	11.12	14.960
13.02	30.09	0.000	21.31	11.10	1.037
08.67	30.19	-0.399	17.00	11.12	3.236
00.00	30.24	-0.616	12.79	11.14	-0.611
26.14	26.16	6.471	08.56	11.17	4.659
21.68	26.11	0.955	00.00	11.19	1.381
17.26	26.19	2.186	04.28	11.18	-4.722
12.96	26.23	-0.166	00.00	07.45	2.520
08.63	26.32	1.466	04.28	07.44	-0.922
00.00	26.37	0.603	08.54	07.44	7.431
04.32	26.32	-0.514	12.78	07.42	-2.904
25.98	22.36	9.827	00.00	03.72	14.110
21.56	22.31	1.204	00.00	00.00	8.694
17.17	22.37	2.817	04.28	00.00	0.702
12.90	22.41	0.355	04.28	03.71	3.059
08.60	22.48	2.608	08.54	00.00	4.211
00.00	22.52	0.924	08.54	03.71	7.123
04.30	22.49	-0.031	12.77	03.70	1.097
25.84	18.59	11.970	12.77	00.00	0.131
21.46	18.54	1.475	31.61	07.33	-1.585
17.10	18.59	2.606	16.97	07.41	5.026
12.86	18.62	-0.106	21.26	07.39	1.490
08.58	18.68	1.759	25.58	07.40	15.630
00.00	18.71	0.876	16.96	03.70	5.686
04.29	18.69	-0.618	16.96	00.00	2.163
25.73	14.84	13.400	21.23	03.69	1.600
21.37	14.81	1.049	21.23	00.00	1.083
17.04	14.85	3.415	25.54	03.69	16.820
12.82	14.87	-0.369	31.57	00.00	-0.196
00.00	14.94	0.774	31.58	03.66	-0.686
25.54	00.00	8.742			

4.3 Deformed Tire Geometry

Fig.4-15 to Fig.4-22 contain two series of graphic representations of the model tire from two different perspectives. These series illustrate the extent and nature of the tire deformation at selected intervals of deflection of approximately 4 cm. As can be seen that the upper half of the tire remains largely unchanged, whereas the lower half undergoes pronounced deformation with loading.

Fig.4-23 to Fig.4-26 show the complete tire when the deflection is 16 cm.

4.4 Footprints

Fig.4-26 to Fig.4-29 have been selected to show the relationship between the size and shape of the contact patches of the model tire and the real tire under progressively larger deflection. The data pertaining to the model tire can be found in Table 4-9 to Table 4-11 and Appendix 4-1. As discussed in the following section, the dimensions of the contact patch of the real tire was derived mathematically from the information provided in the GDC.

As can be seen from Fig.4-27 to Fig.4-30, the contact patch generated by the 3-D FEM model consistently lies within the estimated contact patch of the real tire for a given deflection. It should be noted, however, that the points tracing the contact patch of the model tire represent the lower limits of contact since they represent the outer nodal coordinates of only those elements on the periphery which are in full contact with the hard surface. Therefore the size of the modelled contact patch has, to a small extent, been underestimated.

4.5 Trace of Contact Nodes

Fig.4-31 to Fig.4-32 trace the position of the outermost contact nodes in the longitudinal and lateral directions as the tire is subjected to increasing vertical

Figure 4-15: Tire deformation when deflection is 4 cm
(side view)

Figure 4-16: Tire deformation when deflection is 8 cm
(side view)

Figure 4-17: Tire deformation when deflection is 12 cm
(side view)

Figure 4-18: Tire deformation when deflection is 16 cm
(side view)

**Figure 4-19: Tire deformation when deflection is 4 cm
(axial view)**

**Figure 4-20: Tire deformation when deflection is 8 cm
(axial view)**

2

OF/DE

2

Microl

Figure 4-21: Tire deformation when deflection is 12 cm
(axial view)

**Figure 4-22: Tire deformation when deflection is 16 cm
(axial view)**

Figure 4-23: The geometry of the model tire
when the deflection is 16 cm (axial view)

**Figure 4-24: The geometry of the model tire
when the deflection is 16 cm (side view)**

Figure 4-25: The geometry of the model tire
when the deflection is 16 cm (oblique view)

Figure 4-26: The geometry of the model tire
when the deflection is 16 cm (bottom view)

**Figure 4-27: Predicted and measured footprints
when deflection is 4 cm**

**Figure 4-28: Predicted and measured footprints
when deflection is 8 cm**

**Figure 4-29: Predicted and measured footprints
when deflection is 12 cm**

**Figure 4-30: Predicted and measured footprints
when deflection is 16 cm**

Table 4-9: Estimate the actual shape of the contact patch from the measured data using $(x/a)^n + (y/b)^n = 1$

Deflection cm	Contact Width cm	Contact Length cm	Contact Area cm ²	n	Error of Area %
4.04	46.73	34.84	1372.09	4.33	4.
8.07	58.14	49.89	2653.26	9.32	5.99
12.13	66.03	62.69	3808.48	25	6.35
16.12	71.49	77.57	4958.08	7.89	4.

Note : Deflection, contact width, length and area are from Generalized Deflection Chart,
 $a = (\text{contact width})/2$, $b = (\text{contact length})/2$

Table 4-10: Calculated contact width, length at selected deflection

Deflection cm	Contact Width cm	Contact Length cm
4.04	35.36	23.46
8.07	52.96	38.48
12.13	52.78	53.38
16.12	63.26	60

Table 4-11: Calculated positions of outermost contact nodes

Deflection is 4.04 cm		
Outermost node no	X coordinate	Y coordinate
1364	0.0	14.59
1363	8.79	11.73
1388	13.23	7.78
1427	17.68	7.77
1411	20.1	0.0
Deflection is 8.07 cm		
Outermost node no	X coordinate	Y coordinate
1287	0.0	21.9
1284	8.72	19.22
1280	13.12	19.15
1276	17.55	19.14
1318	22.00	15.20
1347	26.49	11.35
1439	27.96	0.0
Deflection is 12.13 cm		
Outermost node no	X coordinate	Y coordinate
1095	0.0	27.5
1091	8.71	26.64
1085	17.48	26.52
1133	21.97	22.56
1130	26.39	22.56
1439	30.14	0.0
Deflection is 16.12 cm		
Outermost node no	X coordinate	Y coordinate
1042	0.0	32.62
1043	4.34	30.18
1040	8.69	30.18
1036	13.02	30.07
1034	17.35	30.03
1027	26.29	29.97
1343	31.64	10.99
1441	32.1	0.0

load.

Their positions are tracked because it is of interest to determine the direction of movement of the nodes on the contact patch as the load is increased. This will be helpful later on when analyzing the influence of friction on the behaviour of the model tire.

Note that the contact width remains relatively constant over the deflection range of 5 to 15 cm, whereas the contact length increases steadily with load. This is similar to the behaviour of a real tire. Table 4-12 to Table 4-14 contain the data for plotting Fig.4-31 and Fig.4-32.

Figure 4-31: The traces of outermost contact nodes along contact length direction

Figure 4-32: The traces of outermost contact nodes along contact width direction

**Table 4-12: Positions of outermost contact nodes
along contact length direction**

Deflection (m)	Contact Length (m)	Deflection (m)	Contact Length (m)	Deflection (m)	Contact Length (m)
(for node 1378)		(for node 1267)		(for node 1066)	
0.0326	0.2373	0.0881	0.4612	0.1436	0.6068
0.0248	0.2371	0.0904	0.4613	0.1454	0.6068
0.0266	0.2367	0.0924	0.4610	0.1470	0.6070
0.0282	0.2364	0.0942	0.4607	0.1492	0.6066
0.0300	0.2361	0.0958	0.4604	0.1511	0.6063
0.0317	0.2359	0.0976	0.4600	0.1530	0.6069
0.0343	0.2355	0.0992	0.4597	0.1548	0.6065
0.0363	0.2352	0.1008	0.4594	0.1564	0.6063
0.0380	0.2349	0.1023	0.4591	0.1581	0.6060
0.0408	0.2346	0.1038	0.4589	0.1596	0.6047
0.0421	0.2343	0.1065	0.4584	(for node 1042)	
0.0437	0.2341	0.1086	0.4581		0.6544
		0.1105	0.4578		0.6540
(for node 1364)		0.1124	0.4575		0.6536
0.0456	0.2124	0.1140	0.4572	0.1676	0.6533
0.0475	0.2121	(for node 1146)			
0.0493	0.2117				
0.0508	0.2115		0.5341		
0.0523	0.2112		0.5343		
0.0543	0.2109	0.1155	0.5341		
0.0559	0.2107	0.1180	0.5341		
0.0579	0.2104	0.1197	0.5338		
0.0596	0.2101	0.1212	0.5334		
0.0612	0.2098	0.1231	0.5331		
0.0627	0.2095	0.1249	0.5328		
		0.1266	0.5325		
(for node 1330)		0.1282	0.5322		
0.0645	0.2673	0.1298	0.5320		
0.0670	0.2671	0.1313	0.5317		
0.0691	0.2668	0.1329	0.5314		
0.0709	0.2665	0.1346	0.5310		
0.0727	0.2663	0.1367	0.5307		
0.0743	0.2659	0.1386	0.5304		
0.0759	0.2656	0.1404	0.5301		
0.0774	0.2654	0.1420			
0.0790	0.2652				
0.0807	0.2648				
0.0829	0.2644				
0.0847	0.2641				
0.0865	0.2636				

**Table 4-13: Positions of outermost contact nodes
along contact width direction**

Deflection (m)	Contact Width (m)	Deflection (m)	Contact Width (m)	Deflection (m)	Contact Width (m)
(for node 1396)		(for node 1436)		(for node 1456)	(cont)
0.0060	0.1797	0.0579	0.5321	0.1282	0.5264
0.0077	0.1794	0.0596	0.5320	0.1298	0.5260
0.0096	0.1791	0.0612	0.5318	0.1313	0.5257
0.0113	0.1789	0.0627	0.5314	0.1329	0.5255
0.0129	0.1787	0.0645	0.5313	0.1346	0.5254
0.0148	0.1785	0.0670	0.5312	0.1367	0.5274
0.0182	0.1781	0.0691	0.5308	0.1386	0.5270
0.0206	0.1777	0.0709	0.5304	0.1404	0.5267
		0.0727	0.5300	0.1420	0.5264
(for node 1406)		0.0743	0.5306	0.1435	0.5261
0.0226	0.2669	0.0759	0.5306	0.1454	0.5257
0.0243	0.2667	0.0774	0.5302	0.1470	0.5254
0.0266	0.2664	0.0790	0.5299	0.1492	0.5250
0.0282	0.2661	0.0807	0.5297	0.1511	0.5251
(for node 1411)		0.0829	0.5292	0.1530	0.5251
0.0300	0.35523	0.0847	0.5300	0.1548	0.5276
0.0317	0.35519	0.0865	0.5301		
0.0343	0.35466	0.0881	0.5298	(for node 1443)	
0.0363	0.35427	0.0894	0.5293	0.1564	0.6321
0.0380	0.35408	0.0904	0.5289	0.1581	0.6323
0.0403	0.35388	0.0924	0.5283	0.1596	0.6321
0.0421	0.35344	0.0958	0.5280	0.1612	0.6327
0.0437	0.35304	0.0976	0.5293	0.1638	0.6324
0.0456	0.35274	0.0992	0.5293	0.1658	0.6321
0.0475	0.35299	0.1008	0.5293	0.1676	0.6319
(for node 1437)		0.1023	0.5290		
0.0493	0.4423	0.1038	0.5287		
0.0508	0.4424	0.1055	0.5283		
0.0523	0.4423	0.1066	0.5278		
0.0543	0.4417	0.1106	0.5274		
0.0559	0.4417	0.1124	0.5271		
		0.1140	0.5267		
		0.1155	0.5269		
		0.1180	0.5283		
		0.1197	0.5280		
		0.1212	0.5277		
		0.1231	0.5274		
		0.1249	0.5271		
		0.1266	0.5267		

**Table 4-14: Initial and final positions of outermost contact nodes
in contact length and width directions**

Outermost contact nodes in contact length direction

Node No	Start Point	End Point	Linear Relationship
1378	(2.27, 23.73)	(4.38, 23.41)	$y = 24.08 - 0.15x$
1364	(4.56, 31.24)	(6.28, 30.96)	$y = 31.98 - 0.16x$
1330	(6.45, 38.74)	(8.65, 38.39)	$y = 39.84 - 0.17x$
1287	(8.81, 46.13)	(11.41, 45.72)	$y = 47.66 - 0.17x$
1146	(11.56, 53.42)	(14.20, 53.02)	$y = 55.45 - 0.17x$
1095	(14.36, 60.68)	(15.97, 60.48)	$y = 62.65 - 0.13x$
1042	(16.12, 60.44)	(16.76, 65.33)	$y = 68.15 - 0.17x$

Outermost contact nodes in contact width direction

Node No	Start Point	End Point	Linear Relationship
1396	(0.5, 17.97)	(2.07, 17.78)	$y = 18.03 - 0.12x$
1405	(2.27, 26.69)	(2.83, 26.62)	$y = 26.99 - 0.13x$
1411	(3.01, 35.53)	(4.76, 35.30)	$y = 35.98 - 0.15x$
1437	(4.93, 44.23)	(5.60, 44.18)	$y = 44.82 - 0.11x$
1439	(5.79, 53.21)	(15.48, 52.77)	$y = 53.45 - 0.06x$
1443	(15.65, 63.21)	(16.76, 63.19)	$y = 63.08 - 0.01x$

Chapter 5

Effects of Surface Friction on Tire Behaviour

5.1 Introduction

In the analysis of tire-ground interaction described previously, the supporting hard surface is assumed to be perfectly smooth and without friction. However, the data given in the GDC relates to a tire under load in contact with a surface with friction.

Fig.4-31 shows the relationship between tire deflection and the length of the contact patch when friction is ignored. As the tire deflection increases so naturally does the contact length. The lines in the figure describe the position of the successive outermost nodes of the contact length. Note that as the tire deflection increases, there is a tendency for the nodes to contract towards the center of the tire (as indicated by the slight downward slope of the lines). As shown in Fig.4-32, which relates tire deflection and the contact width, a similar contractionary effect is observed with regard to the outermost nodes of the contact width. The tendency for there to be contraction applies to all nodes on the contact patch and in all directions, not just in the directions of length and width.

In the presence of friction on the tire-ground interface, this contractionary behavior will be somewhat mitigated. The extreme case was studied where friction was such as to prevent any contraction of the nodes on the contact patch, i. e. the no-slip case. The assumptions made were the same as those outlined at the beginning of chapter 4. This showed the maximum influence of friction on the contact geometry. Fig.5-1 is the flowchart for modelling the tire on a no-slip surface.

UPDATFIX.FTN is similar to UPDATE.FTN which updates the input file on a smooth surface. Updatfix.ftn, on the other hand, updates the input file on a no-slip surface, i. e., once a node comes into contact with the hard surface, it is fixed on the contact patch. In other words, it is not allowed to move in the horizontal direction on the tire ground interface.

5.2 Results

Fig.5-2 and Fig.5-3 show the relationships between the contact length and width and the deflection of the tire on a no-slip surface, while table 5-1 contains the data used to plot Fig.5-2 and Fig.5-3. With the same deflection the contact length and width are slightly smaller when friction is added to the model. It seems safe to assume that these same tendencies will also apply in other directions along the contact patch. Friction therefore reduces the size of the contact patch slightly in the model.

Fig.5-4 and Fig.5-5 trace the positions of the outermost nodes on the contact patch under increasing deflection when surface friction is present. For purposes of comparison, the traces of the same nodes (as identified in Fig.4-3 and Fig.4-4), when no friction is assumed are also included in the figure. Note that the process of fixing the nodes on the contact surface produces perfectly horizontal traces in contrast to the slightly sloping lines when no friction is assumed. Tables 5-2 and 5-3 contain the data for plotting Fig.5-4 and Fig.5-5, respectively.

Figure 5-1: Procedure for modelling the tire's behaviour on a no-slip surface

Figure 5-2: A comparison of Contact length vs. deflection for tire in contact with frictionless and no-slip surface

Figure 5-3: A comparison of contact width vs. deflection for tire in contact with frictionless and no-slip surface

**Table 5-1: Numerical results of the 3-D model tire on
a no-slip surface**

Deflection (cm)	Contact lenth (cm)	Node	Deflection (cm)	Contact width (cm)	Node
2.23	23.87	1370	0.50	17.97	1396
4.79	31.59	1364	2.07	26.88	1405
7.18	39.13	1330	3.03	35.76	1411
10.07	46.69	1287	5.18	44.78	1437
12.80	54.21	1146	6.20	53.82	1439
15.87	61.33	1995	17.68	65.58	1443
16.85	61.71	1042

Figure 5-4: Comparison of traces of outermost contact nodes along contact length direction on a frictionless and on a no-slip surface

Figure 5-5: Comparison of traces of outermost contact nodes along contact width direction on a frictionless and on a no-slip surface

Deflection (m)	Contact Length (m)	Deflection (m)	Contact Length (m)	Deflection (m)	Contact Length (m)
(for node 1370)		(for node 1387)		(for node 1096)	
0.0223	0.2387	0.1007	0.4669	0.1587	0.6133
0.0249	0.2387	0.1022	0.4669	0.1602	0.6169
0.0270	0.2387	0.1038	0.4675	0.1622	0.6168
0.0288	0.2387	0.1054	0.4675	0.1638	0.6166
0.0303	0.2387	0.1075	0.4675	0.1654	0.6168
0.0320	0.2387	0.1094	0.4675	0.1670	0.6168
0.0351	0.2387	0.1112	0.4675		
0.0372	0.2387	0.1129	0.4675	(for node 1042)	
0.0399	0.2387	0.1146	0.4675	0.1686	0.6771
0.0411	0.2387	0.1161	0.4675	0.1704	0.6771
0.0430	0.2387	0.1178	0.4675	0.1721	0.6771
0.0448	0.2387	0.1193	0.4675	0.1737	0.6771
0.0463	0.2387	0.1210	0.4675	0.1753	0.6771
		0.1230	0.4675		
(for node 1364)		0.1247	0.4675		
0.0479	0.3159	0.1264	0.4675		
0.0499	0.3159	(for node 1146)			
0.0518	0.3159	0.1280	0.5421		
0.0535	0.3159	0.1297	0.5421		
0.0551	0.3159	0.1314	0.5421		
0.0567	0.3159	0.1330	0.5421		
0.0582	0.3159	0.1345	0.5425		
0.0604	0.3159	0.1366	0.5425		
0.0620	0.3159	0.1385	0.5425		
0.0636	0.3159	0.1400	0.5425		
0.0652	0.3159	0.1418	0.5425		
0.0667	0.3159	0.1434	0.5425		
0.0684	0.3159	0.1449	0.5425		
0.0702	0.3159	0.1467	0.5425		
(for node 1330)		0.1483	0.5425		
0.07189	0.3913	0.1499	0.5425		
0.07351	0.3920	0.1514	0.5425		
0.07576	0.3920	0.1536	0.5425		
0.07751	0.3920	0.1554	0.5425		
0.07958	0.3920	0.1571	0.5425		
0.08139	0.3920				
0.08296	0.3920				
0.08467	0.3920				
0.08632	0.3920				
0.08790	0.3920				
0.08948	0.3920				
0.09198	0.3920				
0.09398	0.3920				
0.09586	0.3920				
0.09759	0.3920				
0.09921	0.3920				

**Table 5-3: Positions of outermost contact nodes
along contact width direction on no-slip surface**

Deflection (m)	Contact Width (m)	Deflection (m)	Contact Width (m)	Deflection (m)	Contact Width (m)
(for node 1398)		(for node 1439)		(for node 1439	(cont))
0.0060	0.1797	0.0620	0.5382	0.1230	0.5382
0.0073	0.1797	0.0636	0.5382	0.1247	0.5382
0.0098	0.1797	0.0662	0.5382	0.1264	0.5382
0.0110	0.1797	0.0667	0.5382	0.1280	0.5382
0.0126	0.1797	0.0684	0.5382	0.1297	0.5382
0.0146	0.1797	0.0702	0.5382	0.1314	0.5382
0.0169	0.1797	0.0718	0.5382	0.1330	0.5382
0.0189	0.1797	0.0736	0.5382	0.1346	0.5382
		0.0757	0.5382	0.1366	0.5382
(for node 1406)		0.0776	0.5382	0.1386	0.5382
0.0207	0.2688	0.0796	0.5382	0.1400	0.5382
0.0223	0.2688	0.0813	0.5382	0.1418	0.5382
0.0249	0.2688	0.0829	0.5382	0.1434	0.5382
0.0270	0.2688	0.0846	0.5382	0.1449	0.5382
0.0288	0.2688	0.0863	0.5382	0.1467	0.5382
(for node 1411)		0.0879	0.5382	0.1483	0.5382
0.0303	0.3576	0.0894	0.5382	0.1499	0.5382
0.0320	0.3576	0.0919	0.5382	0.1514	0.5382
0.0351	0.3576	0.0939	0.5382	0.1536	0.5382
0.0372	0.3576	0.0968	0.5382	0.1554	0.5382
0.0389	0.3576	0.0976	0.5382	0.1571	0.5382
0.0411	0.3576	0.0992	0.5382	0.1587	0.5382
0.0430	0.3576	0.1007	0.5382	0.1602	0.5382
0.0448	0.3576	0.1022	0.5382	0.1622	0.5382
0.0463	0.3576	0.1038	0.5382	0.1638	0.5382
0.0479	0.3576	0.1064	0.5382	0.1654	0.5382
0.0499	0.3576	0.1076	0.5382	0.1670	0.5382
(for node 1437)		0.1094	0.5382	0.1686	0.5382
0.0518	0.4478	0.1112	0.5382	0.1704	0.5382
0.0536	0.4478	0.1129	0.5382	0.1721	0.5382
0.0551	0.4478	0.1146	0.5382	0.1737	0.5382
0.0567	0.4478	0.1161	0.5382	0.1753	0.5382
0.0583	0.4478	0.1178	0.5382		
0.0604	0.4478	0.1193	0.5382		
		0.1210	0.5382		

Chapter 6

Conclusions and Suggestions for Future Work

6.1 Conclusions

- The 3-D tire model developed in this thesis has the advantage of being able to model the behaviour of the tire in a more detailed and realistic manner than a 2-D model. Tire contact lengths and widths predicted using the 3-D tire model are in better agreement with those measured, in comparison with that predicted using the 2-D tire model, particularly in the range of tire deflection between 5 and 15 cm.
- Knowing both the geometry of the real tire, such as its diameter and cross-sectional dimensions, and the tire behaviour under normal load, it is possible to develop a 3-D finite element model that can duplicate reasonably well the measured results described in the Generalized Deflection Chart for the real tire provided by the tire manufacturer.
- It appears that assigning a value of 0.48 to the Poisson ratio generates good geometric results over the range of deflection considered in this study. The contact lengths and widths predicted by the model at the different stages of deflection are in reasonably good agreement with those measured for the real tire. While the contact width quickly approached its maximum value during the early stages of deflection the contact length increased more gradually. As in the case of the real tire, the general shape of the contact patch of the model tire gradually changes from elliptical to rectangular as the load increases.

The behaviour of the model tire, that is the manner in which it deforms, also closely resembles that of the real tire. The deformation by and large occurs in and around the contact patch.

- The values of the equivalent Young's modulus for the tire carcass, derived from the 3-D tire model within the normal loading range, are in general agreement with those reported in the literature.
- Within the context of the present analysis, the influence of friction on the behaviour of the tire was negligible. When the tire was subjected to vertical load the contact patch was slightly smaller with friction than without.
- It appears that the 3-D finite element model developed in this thesis can provide a reasonable basis for the detailed study of the interaction between a pneumatic tire and unprepared terrain.

6.2 Suggestions for Future Work

- The establishment of reliable and efficient finite element models for the nonlinear analysis of shell structures has recently become the focus of intense research effort. It would be interesting to examine the possibility of using shell elements in the modelling of pneumatic tires.
- Instead of assuming the pneumatic tire as a homogeneous body, it would be preferable to model the tire taking into account the design and construction features of its carcass structure.
- Since Young's modulus is a function of deflection and declines at a decreasing rate until the deflection is in the vicinity of 8 cm, it is recommended that during the initial stages of loading the incremental load be made smaller so as to permit a more accurate evaluation of Young's modulus.

References

- [1] J. Y. Wong, *On the Study of Wheel-Soil Interaction*, **Journal of Terramechanics**, Vol.21, No.2, pp. 117-131, 1984.
- [2] E. B. MaClaurin, *Soil-Vehicle Interaction*, **Journal of Terramechanics**, Vol.24, No.4, pp. 281-294, 1987.
- [3] J. Y. Wong, *An Introduction to terramechanics*, **Proceedings of the Second European Conference of ISTVS, Ferrara, Italy 1983**, and **Journal of Terramechanics**, Vol.21, No.1, pp. 5-17, 1984.
- [4] Song Tao Chen, *Study of Tire-Terrain Interaction Using the Finite Element Method*, Master's thesis, Mechanical and Aeronautical Engineering, Carleton University, Ottawa, 1988.
- [5] Bekker, M. G., *Introduction to Terrain- Vehicle Systems*, University of Michigan, pp. 436-452, 1969.
- [6] Fujimoto, Y., *Performance of Elastic Wheel on Yielding Cohesive Soils*, **Journal of Terramechanics**, Vol.14, No.4, pp. 191-210, 1977.
- [7] George Y. Baladi and Behzad Rohani, *Development of a Soil- Wheel Interaction Model*, **Proceeding of the Eight International Conference of ISTVS, Cambridge, U.K.**, pp. 33-57, 1984.
- [8] Yong, R. N., Fattah, E. A. & Boonsinsuk, P., *Analysis and Prediction of Tire-Soil Interaction Using Finite Elements*, **Journal of Terramechanics**, Vol.15, No.1, pp. 43-46, 1978.
- [9] Boonsinsuk, P., Irwin, G.J., Yong, R. N. & Ishikawa, F., *Characterization of Tires in Studies of Tire-Soil Interaction*, **9th International conference of the international society for terrain-vehicle systems, Barcelona**,

Spain, Vol.1, pp. 249-256, 31 August-4 September 1987.

- [10] Yu Qun, Gong Sunrong & Yu Guyuan, *On the Modelling and Simulation of Tire-Soil Systems*, 9th International conference of the international society for terrain-vehicle systems, Barcelona, Spain, Vol.1, pp. 257-266, 31 August-4 September 1987.
- [11] Ahmed K. Noor, & John A. Tanner, *Tire Modelling and Contact Problems, Advances and Trends in the Development of Computational Models for Tire, Computer & Structures*, Vol.20, No.1-3, pp. 517-533, 1985.
- [12] Ridha, R. A., Satyamurthy, K. & Hirschfeld, L. R., *Finite Element Modelling of a Homogeneous Pneumatic Tire Subjected to Footprint Loading*, Tire Science and Technology, TSTCA, Vol.13, No.2, pp. 91-110, April-June 1985.
- [13] Chang, J. P., Satymurthy, K. & Tsing, N. T., *An Efficient Approach for the Three-Dimensional Finite Element Analysis of tires*, Tire Science and Technology, TSTCA, Vol.16, No.4, pp. 249-273, October-December 1988.
- [14] Rothert, H & Gall, L., *On the Three Dimensional Computation of Steel-Belted tire*, Tire Science and Technology, TSTCA, Vol.15, No.14, pp. 116-124, April-June 1986.
- [15] John Goldak, *Documentation for ASGARD*, CASCADE Room, Department of Mechanical and Aeronautical Engineering, Carleton University, Ottawa, 1989.
- [16] Sarkar, K., Kwon, Y. D., & Prevorsek, D. C., *A New Approach for the Thermomechanical Analysis of Tires by the Finite Element Method*, Tire Science and Technology, TSTCA, Vol.15, No.14, pp. 261-275. October-December 1987.
- [17] Trinko, M. J., *Ply and Rubber Stresses and Contact Forces for a Loaded Radial Tire*, Tire Science and Technology, TSTCA, Vol.11, No.s1-4, pp. 20-38, January-December 1984.

- [18] Kaga, H., Okamoto, K. & Tozawa, Y., *Stress Analysis of a Tire Under Vertical Load By a Finite Element Method*, Tire Science and Technology, TSTCA, Vol.5, No.2, pp. 102-118, May 1977.
- [19] B. Kao, M. Riesner & P. Surulinarayanasami, *Model Analysis of a Tire and Wheel and its Application for Vehicle Ride Evaluation*, SAE paper 860826 pp. 4.327-4.333, 1986.
- [20] Rothert, H., Idelberger, H., Jacobi, W. & Laging, G., *On the Contact Problem of Tires, Including Friction*, Tire Science and Technology, TSTCA, Vol.13, No.2, pp. 111-123, April-June 1985.
- [21] DeEskinazi, J., Soedel, W. & Yang, T. Y., *Contact of an Inflated Toroidal Membrane with a Flat Surface as an Approach to the Tire Deflection Problem*, Tire Science and Technology, TSTCA, Vol.3, No.1, pp. 43-61, Feb. 1975.
- [22] Thomas J. R. Hughes, *The Finite Element Method*, Englewood Cliffs, NJ., Prentice-Hall, pp. 397, 1987.
- [23] G. Bode, *Automobiltechn. Z.* 64, 300 (1962).

Appendix 2-1: The Theory of Adding Surface Load

The surface force vector[22] of an element is defined by

$$\mathbf{f}^{surf} = \{ \mathbf{f}_a^{surf} \} \quad (2.1)$$

$$\mathbf{f}_a^{surf} = \int_S \mathbf{N}_a^T \mathbf{h} j_s dS, \quad \zeta = \begin{cases} +1, & \text{top} \\ -1, & \text{bottom} \end{cases} \quad (2.2)$$

where a represents nodal points on the surface, where

$$j_s = \| \mathbf{x}_{,\xi} \times \mathbf{x}_{,\eta} \| \quad (\text{surface Jacobian}) \quad (2.3)$$

and $\mathbf{h}$ is the surface force vector (per unit area).

In the tire case considered, surface force is strictly inflation, therefore

$$\mathbf{h} = -\zeta p \mathbf{n}, \quad (\zeta = +1 \text{ or } -1) \quad (2.4)$$

where p is the pressure and $\mathbf{n}$ is the unit normal vector to the surface.

See Fig.A2-1,

$$\mathbf{n} = \frac{\mathbf{e}_{,\xi} \times \mathbf{e}_{,\eta}}{\| \mathbf{e}_{,\xi} \times \mathbf{e}_{,\eta} \|} \quad (2.5)$$

$$\begin{aligned} \mathbf{e}_\xi &= \frac{\mathbf{x}_{,\xi}}{\| \mathbf{x}_{,\eta} \|} \\ \mathbf{e}_\eta &= \frac{\mathbf{x}_{,\eta}}{\| \mathbf{x}_{,\eta} \|} \end{aligned} \quad (2.6)$$

Since

$$\mathbf{n} \cdot \mathbf{j}_s = \mathbf{x}_{,\xi} \times \mathbf{x}_{,\eta} \quad (2.7)$$

Figure A2-1: Local coordinate system

therefore,

$$\mathbf{f}_a^{surf} = \int_S -\mathbf{N}_a^T \zeta p(\mathbf{x}_{,\xi} \times \mathbf{x}_{,\eta}) dS \quad (2.8)$$

The value of $\mathbf{f}_a^{surf}$ is calculated using Gauss integration. There are four nodes on the surface in the case of the 8-node elements and a 2×2 Gaussian integration scheme is used. In the case of the 10-node transition elements, there are 5 nodes that are subject to surface load and hence a 2×3 Gaussian integration scheme is applied in the following manner. As illustrated in Fig.A2-2, the element is divided into two sub-elements with the dividing line intersecting the extra node. In the ξ direction one Gauss point is used and in the η direction three Gauss points are used in each sub-element.

Figure A2-2: Gauss integration scheme

Appendix 4-1: Method of Estimating the Shape of the Actual Contact Patch

The tire manufacturer provides information pertaining to the length, width and area of the contact patch; however no information is given with regard to the actual shape of the contact patch and so it must be estimated. Following is a mathematical description of the method used to estimate its shape. The mathematical function describes a geometrical shape somewhere in between a rectangle and an ellipse with the major and minor axes equal to half the measured contact length and width. The exponent, n , is adjusted (see equation 4.1) so that the area enclosed within the hyper-ellipse approaches the measured area.

The mathematical function is:

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1 \quad (-a \leq x \leq a, -b \leq y \leq b) \quad (4.1)$$

Where n is a real number, $a = (\text{contact width})/2$, $b = (\text{contact length})/2$.

From (4.1), we get:

$$y = b\left(1 - \left(\frac{x}{a}\right)^n\right)^{\frac{1}{n}} \quad (4.2)$$

The area is calculated by integrating y :

$$\begin{aligned} \text{Area} &= 4 \int_0^a y dx \\ &= 4 \int_0^1 ab\left(1 - \left(\frac{x}{a}\right)^n\right)^{\frac{1}{n}} d\left(\frac{x}{a}\right) \\ &= 4ab \int_0^1 (1 - t^n)^{\frac{1}{n}} dt \end{aligned} \quad (4.3)$$

Let

$$\begin{aligned} Area_1 &= \int_0^1 (1-t^n)^{(\frac{1}{n})} dt \\ &= \int_0^{0.5} (1-t^n)^{(\frac{1}{n})} dt + \int_{0.5}^1 (1-t^n)^{(\frac{1}{n})} dt \end{aligned} \quad (4.4)$$

Using Gauss numerical integration, for 3 sample points, we have

$$\int_{-1}^1 F(\gamma) d\gamma = \alpha_1 F(\gamma_1) + \alpha_2 F(\gamma_2) + \alpha_3 F(\gamma_3) \quad (4.5)$$

The sampling points and weights in Gauss numerical integration (interval -1 to +1) are given in the following table:

n	γ_i	α_i
3	± 0.7746	0.5556
	0	0.8889

The corresponding sample points and weights in the integration from x_1 to x_2 are:

$$\frac{(x_1 + x_2)}{2} + \frac{(x_2 - x_1)}{2} \gamma_i \quad \text{and} \quad \frac{(x_2 - x_1)}{2} \alpha_i \quad (4.6)$$

Using (4.6), we get:

From 0 to 0.5

$$\begin{aligned} \gamma_1 &= 0.056 & \alpha_1 &= 0.139 \\ \gamma_2 &= 0.4438 & \alpha_2 &= 0.139 \\ \gamma_3 &= 0.25 & \alpha_3 &= 0.223 \end{aligned} \quad (4.7)$$

From 0.5 to 1

$$\begin{aligned} \gamma_1 &= 0.556 & \alpha_1 &= 0.139 \\ \gamma_2 &= 0.944 & \alpha_2 &= 0.139 \\ \gamma_3 &= 0.75 & \alpha_3 &= 0.2223 \end{aligned} \quad (4.8)$$

Therefore:

$$Area_1 = 0.139F_1 + 0.139F_2 + 0.223F_3 + 0.139F_4 + 0.139F_5 + 0.2223F_6 \quad (4.9)$$

where:

$$\begin{aligned} F_1 &= (1 - 0.056^n)^{\frac{1}{n}} \\ F_2 &= (1 - 0.4438^n)^{\frac{1}{n}} \\ F_3 &= (1 - 0.25^n)^{\frac{1}{n}} \\ F_4 &= (1 - 0.556^n)^{\frac{1}{n}} \\ F_5 &= (1 - 0.944^n)^{\frac{1}{n}} \\ F_6 &= (1 - 0.75^n)^{\frac{1}{n}} \end{aligned} \quad (4.10)$$

Therefore,

$$Area = 4abArea_1 \quad (4.11)$$

Given the contact width, length and area, the next step is to find the value of n so that $(\frac{z}{a})^n + (\frac{y}{b})^n = 1$ gives the same area as the measured one.

END

1 9/0 2/9 1

FIN