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Modeling Nonuniform Interconnects

by

K. Mark Cannon

**A thesis submitted to the
Faculty of Graduate Studies and Research
in partial fulfillment of the requirements
for the degree of
Master of Applied Science**

**Ottawa-Carleton Institute for Electrical Engineering,
Department of Electronics,
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December 2001**

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acceptance of this thesis**

Modeling Nonuniform Interconnects

**Submitted by
Mark Cannon, B.Math.**

**In partial fulfillment of the requirements
for the degree of
Master of Applied Science**

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Abstract

A method to model nonuniform transmission line networks is presented. These networks can be composed of both nonuniform and uniform interconnects, with lossy, coupled and frequency dependent properties. The resulting model can be used in general circuit simulators such as SPICE and used for frequency or time domain simulation. The method segments the nonuniform transmission lines into uniform transmission lines that are modelled using a high order Padé approximation resulting in an accurate passive network which can then be reduced using a Krylov space model reduction technique without significant loss of accuracy.

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List of symbols

λ	Wavelength
v	Propagation velocity
γ	Propagation constant
Ψ	MNA matrix
b	Constant vector determined by independent voltage and current sources
C_Ψ	Lumped memory elements of MNA matrix
C	Per Unit length capacitance matrix of transmission line
d	Length of transmission line
f	Frequency
G_Ψ	Lumped memoryless elements of MNA matrix
G	Per Unit length conductance matrix of transmission line
$H(s)$	Transfer function of the system
$h(t)$	Time-domain impulse response
$I(0,s), I(d,s)$	Terminal current vectors of transmission lines
$I(x,s)$	Current vector
L	Per Unit length inductance matrix of transmission line
M	Order of $Q_{MN}(Z)$ polynomial
N	Order of $P_{NM}(Z)$ polynomial

$\mathbf{P}_{NM}(Z)$	Polynomial matrix of Padé expression given by the denominator
$\mathbf{Q}_{MN}(Z)$	Polynomial matrix of Padé expression given by the numerator
$\mathbf{Q}$	Orthogonal matrix used to reduce original system
$\mathbf{R}$	Per Unit length resistance matrix of transmission line
s	Laplace variable ($j\omega$)
t	Time
$\mathbf{U}$	Unity matrix
$\mathbf{V}(0,s), \mathbf{V}(d,s)$	Terminal voltage vectors of transmission lines
$\mathbf{V}(x,s)$	Voltage vector
$\mathbf{v}(t)$	Vector of node voltages
x	Position variable
$\mathbf{Z}$	Matrix of transmission line parameters
$\mathbf{Z}_0$	Characteristic Impedance

Abbreviations

AWE	Asymptotic waveform evaluation.
CAD	Computer Automated Design
CAE	Computer Automated Electronic analysis
CFH	Complex frequency hopping.
EM	Electromagnetic (field).
FFT	Fast Fourier transform.
IFFT	Inverse fast Fourier transform.
IWR	Iterative waveform relaxation.
LMS	Linear multi-step.
MCM	Multi-chip module.
MMT	Moment matching techniques
MNA	Modified nodal admittance.
MRF	Matrix Rational Form
NILT	Numerical Inversion of the Laplace Transform
PCB	Printed circuit board.
RC	Resistor-capacitor (network).
RLC	Resistor-inductor-capacitor (network).
TEM	Transverse electromagnetic

Chapter 1

Introduction

1.1 Background and Motivation

Three of the most important considerations for any electrical product are power, area and cost. The motivation for transferring more and more functionality onto integrated chips (IC) arises from IC's capability to reduce power consumption, area and cost. As chip features reduce in size, the lower the power consumption, the more functionality for a given area, and the more chips per wafer.

Another factor spurring electronic products is the demand for increased functionality. Electronic products are becoming dominant in more consumer areas, from toys to electronic books, requiring more versatile building blocks, and thus electrically faster components.

These components can be interconnected by optical or electrical interconnects. The most common is the electrical interconnect. Electrical signals travelling between compo-

nents carry information that, if corrupted, will cause the electronic products to fail. These interconnects affect the quality of the signal causing ringing, delay, distortion, and interference between interconnects (crosstalk). As the operating frequency increases, the length of the interconnect becomes a significant fraction of the operating wavelength and the geometry of the interconnect becomes more dominant in delaying and distorting signals. Most interconnects are consistent in shape, varying only in length. If the shape of the interconnect varies along the length (whether physically or electrically), the effect of the interconnect on signal quality increases dramatically. These interconnects are known as nonuniform interconnects.

Many factors are forcing electronic designers to start using nonuniform interconnects. With this reduction in component size, and increase in operating speeds, users must still be able to manipulate or control these products. In essence, handheld or human operated devices must be operated using finger size buttons. In larger systems, congestion of components require interconnects to route around obstacles, or go between them. With more functionality being handled in a smaller area, this congestion is increasing. Thus designers are being forced to use simple nonuniform interconnects to shrink interconnects down to component sizes.

There are many different nonuniform transmission lines, some used intentionally to modify signals, and thus acting as components themselves. The focus of this thesis is on the use of nonuniform transmission lines in digital high speed electronic products to resolve size issues caused by area requirements and the interconnections of IC's to other parts of a system. The techniques used in this thesis for modeling nonuniform interconnects can also be applied to any application of nonuniform interconnects, but not all applications will be explored.

Since the early 1980's dramatic improvements in modeling interconnects have been made. Circuit simulators such as SPICE provide both simple lumped models and more complex interconnect models. Developers of these models have focused on modeling linear interconnects in an efficient and accurate manner and the simulation of these interconnects within the context of nonlinear elements. Given the complexity of designing with nonuniform interconnects, designers of electrical products have been avoiding using them unless it is necessary. This shows up in the research of nonuniform interconnects as a lack of papers on this subject[9] [15][16][33] as compared to uniform interconnects.

From a practical viewpoint, the lumped element model is the easiest way of modeling nonuniform interconnects. The division of the nonuniform interconnect into subsections provides an easy solution to the problem of mixed frequency/time simulation [31]. Unfortunately, the complexity of nonuniform interconnect and high frequency requirements require large numbers of segments, leading to extremely large circuit matrices, making this approach expensive in terms of simulation time.

1.2 Objective

The objective of this thesis is to apply the many advances in uniform interconnect modeling to the nonuniform case. The approach will be to continue segmenting the nonuniform line into sub-segments, but instead of modelling the sub-segments using lumped elements, model them as uniform transmission lines. These uniform transmission lines can be modelled using a macromodelling technique resulting in a linear lumped network. Then a model reduction technique can be applied to this network, resulting in a smaller network that is accurate and numerically efficient.

1.3 Contributions

The main contributions made by this thesis are:

1. Different methods of segmenting the nonuniform interconnect are explored providing guidance on which method may provide the best solution.
2. The use of transmission line macromodels instead of lumped models improve accuracy and efficiency of the simulation. Models that address stability and passivity can be used to address numerical issues that arise during simulation.
3. Applying model reduction techniques to the resulting linear network allows the user to use the most accurate macromodel to create the linear network, knowing that the model reduction that follows will reduce the computational time required during simulation.

1.4 Organization of the thesis

Chapter 2 reviews how distributed transmission lines are modelled, and the difficulties encountered with nonlinear circuit simulators. A variety of techniques to model uniform transmission lines are discussed. Nonuniform transmission lines are introduced and some examples are given. A macromodel approach using the Padé Approximation technique is reviewed in Chapter 3. The theory of Padé approximation is discussed, followed by the implementation of a specific model covering stability, passivity, and frequency dependent parameters. Chapter 4 reviews Model Reduction Techniques. Explicit and implicit moment matching techniques are reviewed and compared. In Chapter 5, the proposed method and examples are presented. A brief discussion on segmenting nonuniform interconnects with examples, is followed by simulations of nonuniform transmission lines using the proposed method. Conclusions and suggestions for further research are presented in Chapter 6.

Chapter 2

Transmission Lines

2.1 Introduction

High speed signals traverse many levels of an electronic product. Originating on a chip, travelling through the chip package, onto a Printed Circuit Board (PCB), and through a Backplane to other PCB's, signals travel through many types of physical interconnect (see Figure 2.1.) Simulation of this interconnect allows designers to understand how the ringing, reflection and other parasitic effects are changing the original signal.

The interconnects are modelled using many different techniques, some that are very accurate at the expense of long simulation times, and others that have relatively short simulation times but are less accurate.

Figure 2.1 System interconnections

2.2 Interconnect Models

Transverse electromagnetic (TEM) waves exist for interconnects with homogeneous media and perfect conductors[6]. TEM implies that interconnects produce electric and magnetic fields that are transverse or perpendicular to one another and to the direction of propagation. In reality, products being manufactured have heterogeneous mediums and imperfect conductors. This results in electromagnetic waves with many velocities and electric fields along surface conductors. In the majority of cases, these small violations of the TEM assumption, still result in a field structure similar to the TEM structure. These field structures can be approximated using TEM waves, and are referred to as quasi-TEM waves [6]. Most interconnect models make use of the quasi-TEM assumption due to the enormous computational expense required for Full-Wave models.

2.2.1 Lumped Models

In today's standard simulators, such as Spice, the lumped model approach is the easiest one to implement. The interconnect is converted into a series of RC or RLC circuits by dividing the interconnect into subsections. For example, Figure 2.2 shows one subsection of a lumped transmission line model. The number of subsections depends on the highest frequency of interest. Typically, the length of each subsection must be much less than the wavelength of the highest frequency of interest (i.e. $d/n \ll \lambda$ where d is the length, n is the number of sections, $\lambda = v/f$, v is the propagation velocity and f is the frequency of interest).

If the interconnect is long compared to the frequency of interest, then a large number of lumped elements are required. This leads to large circuit matrices and long simulation times.

Figure 2.2 Lumped transmission model

2.2.2 Distributed Models

As signal frequency increases, the electrical length of the interconnect becomes a significant portion of the wavelength. This causes effects that are not present at lower frequencies including frequency dependent losses. Distributed transmission line models with per unit length parameters are effectively delay lines, but can include per unit length loss (conductor and dielectric) or resistive, capacitive, or inductive coupling between adjacent conductors. Figure 2.3 shows a distributed transmission line model.

Figure 2.3 Distributed transmission line model

2.2.3 Full-Wave Models

When the cross-sectional dimensions of the interconnect become a significant fraction of the circuit's operating wavelength, it is no longer possible to use the quasi-TEM assumption. Full wave-models account for all possible field components and satisfy all boundary conditions, resulting in an accurate model of the high frequency effects of interconnects.

Though full-wave models provide accurate results, these models are not used by circuit simulators due to the computational requirements. Each frequency point analysis using a full-wave model requires a large amount of computation. Since most high speed designs have many interconnects requiring analysis at thousands of frequency points the method is not practical for most cases. Current simulators call for parameters in terms of voltages, currents and impedances. Full-wave models provide results in terms of field parameters such as propagation constants, characteristic impedances, current eigenvectors, etc. This requires significant analysis to link the results of the full-wave analysis into the circuit simulator.

2.3 Interconnect Simulation

In this section, a brief summary of circuit simulation with lumped elements is given, followed by the development of the distributed transmission line stamp. Then the difficulty of implementing this stamp within standard simulators is shown. Different methods of performing time domain simulation of transmission lines are reviewed. There are two main approaches to time domain simulation: the macromodel approach, where individual transmission lines are approximated with ODE's, and the model reduction approach, where the linear portion of the network is reduced to a single transfer function.

2.3.1 Background

Consider a linear network $\mathbf{p}$ containing linear lumped components which may be described by equations in either time-domain or frequency-domain. The system equations of the network, known as the Modified Nodal Admittance (MNA) matrix[1], $\mathbf{p}$ can be formulated as:

$$\mathbf{C}_\Psi \frac{\partial}{\partial t} \mathbf{v}(t) + \mathbf{G}_\Psi \mathbf{v}(t) - \mathbf{b}(t) = 0 \quad (2.1)$$

where

- $\mathbf{v}(t) \in \mathbb{R}^N$ is the vector of node voltage waveforms appended by independent voltage source current, linear inductor current, nonlinear capacitor charge and nonlinear inductor flux waveforms,
- $\mathbf{C}_\Psi \in \mathbb{R}^{N \times N}$ and $\mathbf{G}_\Psi \in \mathbb{R}^{N \times N}$ are constant matrices describing the lumped memory and memoryless elements of network $\mathbf{p}$, respectively,
- $\mathbf{b} \in \mathbb{R}^N$ is a constant vector with entries determined by the independent voltage and current sources,
- N is the total number of variables in the MNA formulation.

The first three terms in (2.1) cover the network's lumped components and independent sources, respectively. The Laplace transform of (2.1) assuming zero initial conditions can be written as

$$[\mathbf{G}_\Psi + s\mathbf{C}_\Psi] \mathbf{V}(s) - \mathbf{b}(s) = 0 \quad (2.2)$$

where the matrix represented by $[\mathbf{G}_\Psi + s\mathbf{C}_\Psi]$ is referred to as the MNA matrix.

Figure 2.4 Simple Circuit

For example, the MNA equations in the frequency domain for the simple circuit in Figure 2.4 are

$$\left(\begin{bmatrix} 0 & 0 & 1 \\ 0 & G_1 & -1 \\ 1 & -1 & 0 \end{bmatrix} + s \begin{bmatrix} 0 & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & -L \end{bmatrix} \right) V(s) = \begin{bmatrix} J(s) \\ 0 \\ 0 \end{bmatrix} \quad (2.3)$$

2.3.2 Distributed Transmission Line Equations

Distributed transmission lines under the assumption of Quasi-TEM are described by Telegrapher's Equations

$$\begin{aligned}\frac{\partial}{\partial x}v(x, t) &= -\mathbf{R}i(x, t) - \mathbf{L}\frac{\partial}{\partial t}i(x, t) \\ \frac{\partial}{\partial x}i(x, t) &= -\mathbf{G}v(x, t) - \mathbf{C}\frac{\partial}{\partial t}v(x, t)\end{aligned}\tag{2.4}$$

where $\mathbf{R} \in \mathbb{R}^{N \times N}$, $\mathbf{L} \in \mathbb{R}^{N \times N}$, $\mathbf{C} \in \mathbb{R}^{N \times N}$, and $\mathbf{G} \in \mathbb{R}^{N \times N}$ are the per-unit-length parameters of the interconnect. $v(x, t) \in \mathbb{R}^N$ and $i(x, t) \in \mathbb{R}^N$ represent the voltage and current vectors as a function of position x and time t . N is the number of coupled lines.

In the frequency domain, equations (2.4) can be written as:

$$\begin{aligned}\frac{\partial}{\partial x}V(x, s) &= -(\mathbf{R} + s\mathbf{L})I(x, s) \\ \frac{\partial}{\partial x}I(x, s) &= -(\mathbf{G} + s\mathbf{C})V(x, s)\end{aligned}\tag{2.5}$$

Equation (2.5) can be written in hybrid form as:

$$\frac{\partial}{\partial x} \begin{bmatrix} V(x, s) \\ I(x, s) \end{bmatrix} = (\mathbf{D} + s\mathbf{E}) \begin{bmatrix} V(x, s) \\ I(x, s) \end{bmatrix}\tag{2.6}$$

where $\mathbf{D} = \begin{bmatrix} 0 & -\mathbf{R} \\ -\mathbf{G} & 0 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} 0 & -\mathbf{L} \\ -\mathbf{C} & 0 \end{bmatrix}$.

When R , G , L , and C are constant with respect to x , that is the interconnect is uniform, the solution for (2.6) can be written as:

$$\begin{bmatrix} V(d, s) \\ I(d, s) \end{bmatrix} = e^{(\mathbf{D} + s\mathbf{E})d} \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix}\tag{2.7}$$

For convenience, this solution in the hybrid parameter form can be mapped back into the y-parameter form:

Define $T(s)$ as

$$T(s) = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = e^{(D+sE)d} \quad (2.8)$$

Using algebraic manipulations it can be shown that in y-parameter form, the solution can be expressed as:

$$\begin{bmatrix} I(0) \\ I(d) \end{bmatrix} = \begin{bmatrix} -(T_{12})^{-1}T_{11} & (T_{12})^{-1} \\ T_{21} - T_{22}(T_{12})^{-1}T_{11} & T_{22}(T_{12})^{-1} \end{bmatrix} \begin{bmatrix} V(0) \\ V(d) \end{bmatrix} \quad (2.9)$$

where $(T_{12})^{-1}$ represents the inverse of the sub-matrix T_{12} . This formulation can be easily mapped into the MNA matrix.

Figure 2.5 Simple network with transmission line

Figure 2.5 shows a simple network with a single transmission line. If in the frequency domain, the y-parameter solution for the transmission line is

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \quad (2.10)$$

then the MNA equations in the frequency domain can be written as

$$\left(\begin{bmatrix} Y_{11} & Y_{12} & 0 & 0 \\ Y_{21} & Y_{22} & 0 & 1 \\ 0 & 0 & G & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} + s \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & -L \end{bmatrix} \right) \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ I \end{bmatrix} = \begin{bmatrix} J(s) \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (2.11)$$

This set of equations is solved at each frequency point of interest for the unknowns V_1 , V_2 , V_3 and I .

2.3.3 Mixed Frequency - Time Domain Problem

Though high-frequency models such as distributed transmission lines are described in terms of partial differential equations, they are best represented in the frequency domain and do not have a direct representation in the time-domain. Nonlinear devices do not have a frequency domain representation, and can only be described in the time domain. These contradictory requirements make it difficult for standard ordinary differential equation solvers like SPICE.

2.3.4 Stability and Passivity

Most interconnects are passive structures that do not generate more energy than they absorb. Thus models for transmission lines must be stable and passive. To be passive the interconnect model cannot generate more energy than it absorbs, and no passive termination of the network will cause the system to go unstable [4]. An example of a stable but nonpassive model would be an amplifier circuit. A circuit composed solely of resistors, capacitors, and inductors is both stable and passive.

2.4 Macromodel Simulation Techniques

Many methods in the literature have attempted to solve systems of simultaneous equations that have both frequency and time equations with varying success. Two methods will be discussed in the following sections.

2.4.1 Method of Characteristics

In 1967 Branin developed the Method of Characteristics[7]. This method provides a frequency domain solution that can be represented in the time domain as a pure delay, i.e. a lossless delay line. For a single reference transmission line the Laplace Domain solution of (2.5) is [8]

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{Z_0(1 - e^{-2\gamma d})} \begin{bmatrix} 1 + e^{-2\gamma d} & -2e^{-\gamma d} \\ -2e^{-\gamma d} & 1 + e^{-2\gamma d} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (2.12)$$

$$\gamma = \sqrt{(R + sL)(G + sC)}$$

$$Z_0 = \sqrt{\frac{(R + sL)}{(G + sC)}}$$

where γ is the propagation constant and Z_0 is the characteristic impedance. V_1, I_1 are the near end terminal voltage and current and V_2, I_2 are the far end terminal voltage and current as shown in Figure 2.6.

Figure 2.6 Y parameters of a single reference transmission line

The Y parameters of the transmission line are complex functions of s and there is no general solution that can directly transform them into ordinary differential equations in the time domain. If the transmission line is lossless, that is, there are no Resistance or Conductance effects, then γ and Z_0 can be reduced to

$$\gamma = s\sqrt{LC} \quad Z_0 = \sqrt{\frac{L}{C}} \quad (2.13)$$

This makes γ purely imaginary and Z_0 a real constant. Taking the inverse Laplace transformation of (2.12) we get

$$Z_0 \left[\begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} - \begin{bmatrix} i_1(t-2\gamma d) \\ i_2(t-2\gamma d) \end{bmatrix} \right] = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} + \begin{bmatrix} v_1(t-2\gamma d) - 2v_2(t-\gamma d) \\ -2v_1(t-\gamma d) + v_2(t-2\gamma d) \end{bmatrix} \quad (2.14)$$

where the inverse Laplace transform replaces $e^{-\gamma d}$ and $e^{-2\gamma d}$ with the time shifts or delays of $t - \gamma d$ and $t - 2\gamma d$ respectively.

This time domain representation has been used successfully in nonlinear circuit simulators as the stamp of a lossless transmission line. Time domain stamps for Multiconductor lossless lines have been developed as well [6]. The major issues with this algorithm are the numerical stability of the integration formula due to the effect of the delay, and the difficulty of extending this technique to incorporate lossy effects. Both issues require a small step size, increasing the CPU time.

2.4.2 Compact Finite-Differences Based Approximation

Compact finite-differences based approximations can be used to convert Telegraph's equations into ordinary-differential equations [10]. This technique can achieve better accuracy with fewer variables than direct lumped RLC segmentation. It is also passive by construction.

2.4.3 Integrated Congruence Transform

The Integrated Congruence Transform based approximation is another approach that can be used to convert transmission line equations into ordinary differential equations. This method has been shown to preserve passivity [11]

2.5 Model-Reduction Simulation Techniques

In general, interconnect networks have an infinite number of poles covering a wide frequency range. Most of these poles do not have a significant effect on simulation results. The poles that have a major influence on the simulation, referred to as the dominant poles, are located close to the imaginary axis and influence both time and frequency characteristics of the system. This is illustrated in Figure 2.7

Figure 2.7 Concept of dominant poles

Model-Reduction techniques identify the dominant poles, and remove poles that have minimal effects on the simulation results. By simulating using only dominant poles, the numerical computations are reduced.

2.6 Non-Uniform Transmission Lines

Nonuniform transmission lines have been studied since the early 1930's [9]. Initially, research was focused on special applications of nonuniform lines such as impedance transformers, resonators, filters, directional couplers and other microwave components. Requirements of high-speed interconnects have renewed interest in nonuniform transmission lines.

As described previously, interconnects are modeled in the time domain by partial differential equations in the time domain, known as the Telegrapher Equations (2.4). Parameters of the transmission line equations are the per unit length resistive and dielectric loss matrices, R and G respectively, and the per unit length inductance and capacitance matrices, L and C respectively. Uniform transmission lines are characterized by constant R , L , C , and G parameters. When the R , L , C , and G parameters vary along the length of the lines they represent nonuniform transmission lines. The differential equations for multi conductor transmission lines will be non constant-coefficient differential equations. These equations are linear, if the surrounding dielectric is linear, but are just as difficult to solve as nonlinear equations.

2.7 Examples of Nonuniform lines

Figure 2.8 Nonuniform two line coupler

Nonuniform transmission lines have long been used in microwave applications for directional couplers and filters. These couplers and filters can vary drastically over the length of the interconnect. A much simpler nonuniform interconnect is the tapered line. Tapered interconnects have been used in applications where interconnections between a larger perimeter must be made to a smaller inner perimeter within a short distance. Good examples are chip packages and MCM applications.

Figure 2.9 Tapered Lines

2.8 Published techniques

2.8.1 Segmentation - Lumped Model

The simplest way of modelling non-uniform transmission lines is to segment it into uniform transmission lines, and then approximate each transmission line using the lumped model approach. As with uniform transmission lines, the number of segments per uniform section of the nonuniform transmission line is frequency dependent. The number of uniform transmission lines that a non-uniform transmission line is divided into is dependent on the non-uniformity of the original line.

This technique is easy to implement and reasonably accurate at low frequencies. It suffers from the same problems that occur with regular uniform lines but to a greater extent. The more the line is segmented, the larger the MNA matrix, and thus the longer the simulation time.

2.8.2 Segmentation - Perturbation Approach

M. A. Mehalic and Raj Mittra use a frequency domain scattering parameter approach in [15]. This technique makes no assumption other than the physical dimensions of the interconnects. It accounts for the dielectric frequency effects. Nonuniform lines are approximated by a cascade of uniform sections. It involves the calculation of S parameters for each uniform section by determining Green's functions and potentials. The integral equations of charge and current distributions are solved for each section. The S parameters of each section are then cascaded to give an overall S parameter representation in the frequency domain. The time domain response is obtained by doing an inverse Fast Fourier transform of S parameters. The time domain response is obtained by convolving the I-V characteristics of the terminations with this transform of the S parameters.

2.8.3 Numerical Inversion of the Laplace Transform

S. Manney for his Master Thesis [33] proposed solving the transmission line equations in the frequency domain using the Runge-Kutta numerical integration technique. He then uses the Numerical inversion of the Laplace transform (NILT) technique to generate the time domain solution. This avoids the aliasing errors typically encountered when converting from the frequency domain to the time domain using Fourier analysis techniques.

2.8.4 Method of Moment in Frequency Domain

N. Boulejfen, A. B. Kouki, F. M. Ghannouchi propose two approaches for frequency and transient analysis of multiple-coupled nonuniform transmission lines [16].

For frequency domain analysis the method of moments is used to compute the scattering parameters of the nonuniform structure under an arbitrary reference impedance system. It uses basis functions derived from the structure's own propagation characteristics and thus any frequency dependence. In the time domain the frequency domain data is transformed into time domain scattering parameters by a Fourier transform and a convolution algorithm is used to obtain the transient behavior of the structure. Again, since the frequency dependent effects, including loss, dispersion, and discontinuity, are included in the frequency domain S parameters, the transient response also includes these effects.

2.9 Proposed Approach

The proposed approach in this thesis is to segment nonuniform transmission lines into uniform transmission lines. Then, model each transmission line segment as an uniform transmission line using Padé approximation. The final step is to reduce the resulting model using Krylov-subspace model reduction techniques.

The key advantage of this approach is that very accurate transmission line models can be used for the uniform transmission lines without the penalty of high cpu expense, because the resulting network is scaled down in size by model reduction techniques. The accuracy of the final netlist is maintained and is still substantially smaller than the original netlist.

The next two chapters will explain in detail the Padé Approximation technique and Model Reduction techniques.

Chapter 3

Padé Approximation

3.1 Introduction

This chapter shows how an interconnect model based on a closed-form Padé rational function approximation is developed. This includes the time domain macromodel, the ability to handle frequency effects, and proof of passivity.

3.2 Theory of Padé Approximates

To estimate a function using Padé approximates is equivalent to the set of approximate derived from an infinite set of continued fractions. The key difference is that a power series determines the set of approximates that are usually unique, whereas continued fractions can be written in many ways[3].

The (N,M) Padé approximate to $f(s)$ defined from a series $K(s)$ is a rational function:

$$f_{N, M}(s) = \frac{\sum_{m=0}^M a_m s^m}{\sum_{n=0}^N b_n s^n} \quad (3.1)$$

and the coefficients in the numerator and the denominator are defined by the formal identity:

$$\left(\sum_{n=0}^N b_n s^n \right) \left(\sum_{r=0}^{\infty} c_r s^r \right) = \sum_{m=0}^M a_m s^m + o(s^{M+N}) \quad (3.2)$$

By definition, the notation $o(s^{M+N})$ and the coefficients of $1, s, s^2, \dots, s^{M+N}$ on both sides of Equation (3.2) are equated. It has been proved that the coefficients a_m, b_n always exist and the rational fraction $f_{N, M}$ is unique [2]

Since Padé's original work was based on $f(s) = e^s$, and coefficients of the Maclaurin expansion of $f(s)$ are sufficiently simple, then explicit forms of the numerator and denominator of the approximates can be found. This fact will be used in the following section in order to model transmission lines.

3.3 Padé Approximate Model of Transmission lines

In this section a closed-form Padé approximate is used to replace the exponential matrix derived from Telegraphers equation.

3.3.1 Interconnect Model

As described in chapter 2, interconnects can be described by a set of partial differential equations (reference equation in chap. 2), which in the frequency domain are:

$$\frac{\partial}{\partial x} V(x, s) = -(\mathbf{R} + s\mathbf{L})I(x, s) \quad (3.3a)$$

$$\frac{\partial}{\partial x} I(x, s) = -(\mathbf{G} + s\mathbf{C})V(x, s) \quad (3.3b)$$

For Uniform Transmission lines, the solution of (3.3) can be expressed as an exponential matrix

$$\begin{bmatrix} V(d, s) \\ I(d, s) \end{bmatrix} = e^{\mathbf{Z}} \begin{bmatrix} V(0, s) \\ I(0, s) \end{bmatrix} \quad (3.4)$$

where

$$\mathbf{Z} = (\mathbf{D} + s\mathbf{E})d; \mathbf{D} = \begin{bmatrix} 0 & -\mathbf{R} \\ -\mathbf{G} & 0 \end{bmatrix}; \mathbf{E} = \begin{bmatrix} 0 & -\mathbf{L} \\ -\mathbf{C} & 0 \end{bmatrix} \quad (3.5)$$

V, I represent the frequency domain terminal voltage and current vectors of the multiconductor transmission line and d is the length of the line.

Using the Padé approximate for $e^{\mathbf{Z}}$, Eq. (3.4) can be represented as ordinary differential equations and retain the passivity of the structure. The matrix exponential of $e^{\mathbf{Z}}$ can be approximated as

$$P_{N, M}(\mathbf{Z})e^{\mathbf{Z}} \approx Q_{M, N}(\mathbf{Z}) \quad (3.6)$$

where $P_{N,M}$, $Q_{M,N}$ are polynomial matrices representing the numerator and denominator of the Padé rational functions. The coefficients of $P_{N,M}$ and $Q_{M,N}$ have a closed form representation of [1]

$$P_{N,M}(Z) = \sum_{i=0}^N \frac{(M+N-i)!N!}{(M+N)!i!(N-i)!} (-Z)^i \quad (3.7a)$$

$$Q_{M,N}(Z) = \sum_{i=0}^M \frac{(M+N-i)!M!}{(M+N)!i!(M-i)!} (-Z)^i \quad (3.7b)$$

This closed form relation is computationally less expensive than other methods that use a form of numerical optimization.

3.3.2 Time Domain Macromodel

In order to use this interconnect model in non-linear simulators, the model must be expressed as ordinary differential equations. In this section, a reasonably straight forward approach is used to show the Time Domain realization of the macromodel.

Equation (3.6) can also be represented as:

$$\begin{bmatrix} P_{N,M_{11}} & P_{N,M_{12}} \\ P_{N,M_{21}} & P_{N,M_{22}} \end{bmatrix} e^Z \cong \begin{bmatrix} Q_{M,N_{11}} & Q_{M,N_{12}} \\ Q_{M,N_{21}} & Q_{M,N_{22}} \end{bmatrix} \quad (3.8)$$

where the coefficients of (3.8) are expressed as

$$P_{N, M_{11}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 + (-1)^i) (ab)^{\frac{1}{2}} \right] \quad (3.9a)$$

$$P_{N, M_{12}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 - (-1)^i) (ab)^{\frac{i-1}{2}} a \right] \quad (3.9b)$$

$$P_{N, M_{21}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 - (-1)^i) (ba)^{\frac{i-1}{2}} b \right] \quad (3.9c)$$

$$P_{N, M_{22}} = \sum_{i=0}^N \frac{(M+N-i)!}{(M+N)!} \binom{M}{i} \left[\frac{1}{2} (1 + (-1)^i) (ba)^{\frac{i}{2}} \right] \quad (3.9d)$$

$$Q_{M, N_{11}} = \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 + (-1)^i) (ab)^{\frac{i}{2}} \right] \quad (3.9e)$$

$$Q_{M, N_{12}} = \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 - (-1)^i) (ab)^{\frac{i-1}{2}} ab \right] \quad (3.9f)$$

$$Q_{M, N_{21}} = \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 - (-1)^i) (ba)^{\frac{i-1}{2}} b \right] \quad (3.9g)$$

$$Q_{M, N_{22}} = \sum_{i=0}^M \frac{(M+N-i)!}{(M+N)!} \binom{N}{i} \left[\frac{1}{2} (1 + (-1)^i) (ba)^{\frac{i}{2}} \right] \quad (3.9h)$$

where $a = (R + sL)d$ and $b = (G + sC)d$. There also exists several recursive relationships for $P_{N,M}(Z)$ and $Q_{M,N}(Z)$, which can be used to reduce computational effort for these relations.

When $N=M=n$, the polynomials Equation (3.8) can be expressed as

$$\begin{bmatrix} P_{N,M_{11}} & P_{N,M_{12}} \\ P_{N,M_{21}} & P_{N,M_{22}} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^n F_i s^i & \sum_{i=0}^n H_i s^i \\ \sum_{i=0}^n J_i s^i & \sum_{i=0}^n F_i s^i \end{bmatrix} \quad (3.10a)$$

$$\begin{bmatrix} Q_{M,N_{11}} & Q_{M,N_{12}} \\ Q_{M,N_{21}} & Q_{M,N_{22}} \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^n F_i s^i & -\sum_{i=0}^n H_i s^i \\ -\sum_{i=0}^n J_i s^i & \sum_{i=0}^n F_i^T s^i \end{bmatrix} \quad (3.10b)$$

Using (3.4), (3.10a) and (3.10b), the time domain macromodel of the interconnect becomes

$$[[p_a][q_a]] \begin{bmatrix} \dot{x}_2 \\ \dot{x}_1 \end{bmatrix} + [[p_b][q_b]] \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = 0 \quad (3.11)$$

The matrices of (3.11) are defined as

$$\begin{aligned}
 [p_a] &= \begin{bmatrix} U_x \\ 0 \\ M \end{bmatrix}; [p_b] = \begin{bmatrix} U_r \\ 0 \\ S \end{bmatrix} \\
 [q_a] &= \begin{bmatrix} 0 \\ U_x \\ N \end{bmatrix}; [q_b] = \begin{bmatrix} 0 \\ U_r \\ T \end{bmatrix}
 \end{aligned} \tag{3.12}$$

$$x_2 = [v_2, i_2, \gamma_2^{0\ 0} \dots \gamma_2^{N-1\ N-1}]^t; x_1 = [v_1, i_1, \gamma_1^{0\ 0} \dots \gamma_1^{N-1\ N-1}]^t$$

where $[\gamma_2^{0\ 0} \dots \gamma_2^{N-1\ N-1}]$ and $[\gamma_1^{0\ 0} \dots \gamma_1^{N-1\ N-1}]$ are extra variables

needed for the realization. The variables M, N, S, T, U_X and U_Y are defined as

$$M = \begin{bmatrix} F_1 H_1 F_2 H_2 \dots F_{n-1} H_{n-1} F_n H_n \\ J_1 F_1^T J_2 F_2^T \dots J_{n-1} F_{n-1}^T J_n F_n^T \end{bmatrix} \tag{3.13a}$$

$$N = \begin{bmatrix} -F_1 H_1 - F_2 H_2 \dots -F_{n-1} H_{n-1} - F_n H_n \\ J_1 - F_1^T J_2 - F_2^T \dots J_{n-1} - F_{n-1}^T J_n - F_n^T \end{bmatrix} \tag{3.13b}$$

$$S = \begin{bmatrix} F_0 & H_0 & 0 & 0 & \dots & 0 & 0 \\ J_0 & F_0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{3.13c}$$

$$T = \begin{bmatrix} -F_0 & H_0 & 0 & 0 & \dots & 0 & 0 \\ J_0 & -F_0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{3.13d}$$

$$U_X = \begin{bmatrix} U & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & U & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & U & \dots & 0 & 0 & 0 & 0 \\ \\ 0 & 0 & 0 & 0 & \dots & U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & U & 0 & 0 \end{bmatrix} \quad (3.13e)$$

$$U_Y = 0 \begin{bmatrix} 0 & 0 & -U & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -U & \dots & 0 & 0 & 0 & 0 \\ & & & & \dots & & & & \\ 0 & 0 & 0 & 0 & \dots & -U & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & -U & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & -U & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & -U \end{bmatrix} \quad (3.13f)$$

The time domain macromodel described by (3.11) is fairly easy to implement as the polynomial coefficients of (3.10a) and (3.10b) can be directly stencilled in the differential equations. The number of variables required to accomplish (3.11) is $2\psi(2n - 1)$, where ψ is the number of coupled interconnects.

3.3.3 Time Domain Realization in Terms of Circuit Elements

The time domain macromodel described in the previous section can also be represented in terms of RLC circuit elements. This is done by converting (3.11) into Y-Parameter form and deriving a circuit equivalent model composed of resistors, capacitors and inductors.

Various other methods have been developed to represent the time domain macromodel in RLC circuit elements, some more efficient than others.

3.3.4 Passivity of Transmission Line Model

Interconnect models must preserve passivity of the final network. An interconnect model may be stable, yet when combined with a passive termination will cause the resultant system to become unstable[4]. To be passive, the macromodel must remain asymptotically stable for any passive termination.

It has been shown that the Padé macromodel is passive when the order of the numerator and denominator of (3.6) are equal.

3.3.5 Order of Approximation

Since the elements in (3.6) are computed in a closed-form manner, the order of the approximation (N,M) can be estimated using the following error criterion

$$\|e^Z - (P_{N,M}^{-1}(Z)Q_{N,M}(Z))\| < \varepsilon \|e^Z\| \quad (3.14)$$

3.4 Frequency Dependent Parameters

3.4.1 Frequency Dependent Interconnect

For high speed applications it is important that interconnect modes incorporate the effects of frequency dependent parameters. As the frequency of the signal increases, an electric field is induced which changes the current distribution. These changes are classified as proximity, edge and skin effect.

At low frequencies, the per unit length resistance and inductance are, for practical purposes, constant. As the frequency increases, the edge effect causes current to concentrate at the sharp edges of the conductor, increasing the resistance. This occurs in both the signal and ground conductors, but has a greater impact in the signal conductor.

As the frequency of the signal increases, the current concentrates in the sections of the ground plane that are near the signal conductors. This is known as the proximity effect. This causes the inductance to drop because the magnetic field between the ground and signal conductor is reduced. It also increases the resistance of the interconnect as more current is confined in the ground plane near the signal conductors.

Both edge and proximity effects are apparent at the medium frequency range.

Increasing the frequency of a signal concentrates the current in a thin layer at the conductor surface. This is referred to as the Skin Effect. This occurs at high frequencies, and affects both ground and signal conductors. As the frequency increases, the thickness of this layer of current decreases, and thus increases resistance. The inductance of the interconnect drops as the frequency increases, as the magnetic fields inside the ground and signal conductors are reduced by this effect. At some point, the magnetic fields inside the ground and signal conductors become negligible and inductance becomes constant.

3.4.2 Modelling of Frequency Dependent Parameters

When incorporating frequency dependent effects into an interconnect model, the line parameters can be modelled analytically or by curve fitting techniques [12],[13]. It is important that these models generate passive circuit networks ensuring that the interconnect model maintains its passivity.

Analytical methods are limited in that they only model specific characteristics. For example, T. Vu Dinh, B. Cabon and J. Chilo [14] models the skin effect as a square root of frequency, but does not account for other effects such as the edge and proximity effects.

Measured or simulated data provides better models for frequency dependent parameters than closed form functions [13].

Chapter 4

Model Reduction Techniques

4.1 Model-Reduction

In general, interconnect networks have a large number of poles covering a wide frequency range. Most of these poles do not have a significant effect on simulation results. The poles that have a major influence on the simulation, referred to as the dominant poles, are located close to the imaginary axis and influence both time and frequency characteristics of the system. This is illustrated in Figure 4.1

Figure 4.1 Concept of dominant poles

Model-Reduction techniques identify the dominant poles, and remove poles that have minimal effects on the simulation results. By simulating using only dominant poles, the simulator becomes more CPU efficient.

4.1.1 Moment Matching Techniques

A simple demonstration of the concepts of moment-matching techniques will be given as a basis for more detailed explanation.

For a single input, single output system, $H(s)$ is the transfer function. In a rational form, $H(s)$ can be represented as

$$H(s) = \frac{P(s)}{Q(s)} \quad (4.1)$$

where $P(s)$ and $Q(s)$ are polynomials in s . This can be re-written as

$$H(s) = c + \sum_{i=1}^{N_p} \frac{k_i}{s - p_i} \quad (4.2)$$

with p_i and k_i are the i^{th} pole-residue pair, N_p is the total number of system poles and c is the direct coupling constant. By applying the inverse Laplace transform, the time-domain impulse response can be computed in the closed form

$$h(t) = c\delta t + \sum_{i=1}^{N_p} k_i e^{p_i t} \quad (4.3)$$

In large networks the total number of poles, N_p , would be in the order of thousands. This would be computationally impractical to calculate completely. Model-reduction tech-

niques attempt to derive a reduced-order approximation in terms of dominant poles, instead of calculating all poles. If only L poles are extracted, (4.2) and (4.3) can be re-written as

$$H(s) \approx \hat{H}(s) = \hat{c} + \sum_{i=1}^L \frac{\hat{k}_i}{s - \hat{p}_i} \quad (4.4)$$

$$h(t) \approx \hat{h}(t) = \hat{c}\delta t + \sum_{i=1}^L \hat{k}_i e^{\hat{p}_i t} \quad (4.5)$$

The coefficients of the Taylor series expansions are generally referred to as *moments*. To demonstrate why they are called *moments*, consider the Taylor series expansion of a given transfer-function, $H(s)$, at point, $s=0$,

$$H(s) \approx \hat{H}(s) = m_0 + m_1 s + m_2 s^2 + \dots + m_n s^n = \sum_{i=0}^n s^i m_i; \quad (4.6)$$

$$m_i = \frac{H(s)^{(i)}}{i!}$$

where the super-script (i) denotes the i^{th} derivative. By taking the Laplace transform of $h(t)$ one can see that the coefficients of the Taylor series expansion, (m_i) are identical to the *time-domain moments* of the impulse response $h(t)$.

$$\begin{aligned}
H(s) &= \int_0^{\infty} h(t) e^{-st} dt = \int_0^{\infty} h(t) \left[1 - st + \frac{s^2 t^2}{2!} \dots \right] dt \\
&= \int_0^{\infty} h(t) dt + s \int_0^{\infty} (-1) t h(t) dt + s^2 \int_0^{\infty} \frac{t^2 h(t)}{2!} dt + \dots \\
&= \sum_{i=0}^{\infty} s^i \left(\frac{(-1)^i}{i!} \int_0^{\infty} t^i h(t) dt \right)
\end{aligned} \tag{4.7}$$

References [18] and [19] show that the moments provide an estimation of delay and rise times. Elmore delay [18] matches the first moment of the response. To get an accurate prediction of interconnect effects, a reduced-order model should preserve as many moments as possible.

Techniques used to reduce large interconnect subnetworks can be classified as either Explicit Moment-Matching or Implicit Moment-Matching. The following sections briefly cover these approaches.

4.2 Explicit Moment-Matching Techniques

Padé approximation, based on explicit moment-matching, is used to determine the dominant poles and residues of a system [20]. This approach was discussed in Chapter 3 with respect to generating transmission line stamps. In this section, the method for Padé based circuit reduction will be summarized and various improvements will be reviewed.

4.2.1 Summary of MMT algorithm

Given the MNA circuit equations containing MTL stamps:

$$\begin{bmatrix} \Psi(s) \end{bmatrix} \begin{bmatrix} X(s) \end{bmatrix} = \begin{bmatrix} b(s) \end{bmatrix} \quad (4.8)$$

The computational relationship that provide moments for a system of MNA equations containing MTL stamps can be derived as follows [20][21]. Expanding (4.8) using a Taylor's series at an expansion point $s = \alpha$, we get

$$\begin{bmatrix} \psi(\alpha) + \frac{\psi^{(1)}|_{s=\alpha}}{1!}(s-\alpha) + \dots + \frac{\psi^{(n)}|_{s=\alpha}}{n!}(s-\alpha)^n \end{bmatrix} \\ [M_0 + M_1(s-\alpha) + \dots + M_n(s-\alpha)^n] = [b] \quad (4.9)$$

where $\psi^{(n)}$ denotes the n^{th} derivative of $\psi(s)$ and M^n denotes the n^{th} moment of $X(s)$ at $s = \alpha$. Equating coefficients of similar powers of $s = \alpha$ on both sides of (4.9), we get

$$[\psi]M_0 = b \quad (4.10)$$

$$[\psi]M_1 + \frac{\psi^{(1)}}{1!}M_0 = 0 \Rightarrow [\psi]M_1 = -\frac{\psi^{(1)}}{1!}M_0$$

$$[\psi]M_2 + \frac{\psi^{(1)}}{1!}M_1 + \frac{\psi^{(2)}}{2!}M_0 = 0$$

$$\Rightarrow [\psi]M_2 = -\sum_{r=1}^2 \frac{(\psi^{(r)})M_{2-r}}{r!}$$

Equations (4.10) can be generalized to a recursive relation

$$[\psi(\alpha)]M_0 = b \quad (4.11)$$

$$[\psi(\alpha)]M_n = -\sum_{r=1}^n \frac{(\psi^{(r)}|_{s=\alpha})M_{n-r}}{r!}$$

This equation requires the calculation of the derivatives of ψ , which can be obtained from [21]

$$[\psi]^{(1)} = \begin{bmatrix} W_\pi & 0 \\ Y_d^{(1)} L_d^t & 0 \end{bmatrix}; \quad [\psi]^{(r)} = \begin{bmatrix} W_\pi & 0 \\ Y_d^{(r)} L_d^t & 0 \end{bmatrix}; \quad (r \geq 2) \quad (4.12)$$

The derivatives $Y_d^{(r)}$ are a function of the derivatives of the entries in the right hand side of (2.9) and the use of the Leibnitz theorem. Once the Moments of the MNA matrix are

determined, the System Moments can be calculated by doing one L/U decomposition and forward/backward substitutions during the recursive computation of the higher order moments.

$$X(s) = M_0 + M_1s + M_2s^2 + \dots \quad (4.13)$$

Using Padé approximation, the rational transfer function for a particular element of $X(s)$ is calculated

$$X_{out}(s) = \frac{P(s)}{Q(s)} \quad (4.14)$$

Finally the Poles and residues are determined

$$X_{out}(s) = \sum_i \frac{k_i}{s - p_i} \quad (4.15)$$

Using a single Padé expansion is commonly referred to as *Asymptotic Waveform Evaluation (AWE)*. It often gives inaccurate results due to its limitations, including:

- Ill-conditioning of large expansions, limiting the number of accurate poles available from a single expansion
- AWE's accuracy reduces as you get farther away from the expansion point, so that complex systems with many dominant poles can not be adequately modelled
- Passivity of the approximation for the system is not guaranteed (whereas, as shown in chapter 3 individual distributed element passivity is assured).

These limitations of *AWE* have led to the development of multi-point expansion techniques.

4.2.2 Multi-Point Expansion Techniques

Given the limitations of the single expansion approach, it follows that using multiple expansion points would overcome many of the problems. The goal is to determine a minimum number of frequency point expansions that will generate an approximate transfer function that matches the original function within a required frequency range.

Figure 4.2 Illustration of Multi-Point Expansion MMT

One such approach is *Complex Frequency Hopping (CFH)*. CFH uses multiple expansion points (or hops) in the complex plane near or on the imaginary axis. Since Padé approximation is accurate near the point of expansion, and decreases as you get farther away, at least two expansion points are necessary. The accuracy of the two expansion points can be validated by matching two poles generated by both expansions. This is referred to as the “*pole-matching based approach*”. A second means of validating the two poles is to compare the value of the transfer functions calculated at the mid-point between them. This is referred to as the “*transfer-function based approach*”. CFH uses a binary search algorithm to determine the expansion points and thus minimize the number of expansions.

Figure 4.3 Expansion point selection process

The transfer-function based approach collects a set of transfer functions which accurately matches the frequency response up to the highest frequency of interest. The pole-matching approach collects all the dominant poles accurately up to the highest frequency of interest.

For a multiport system, the CFH algorithm calculates an accurate frequency-response for each of the entries in the Y-parameter formulation of the system using the transfer-function approach. Then using the pole matching approach an accurate pole-set is obtained as the union of driving point impedances. With these poles and frequency responses, residues for each Y_{ij} are obtained using the residue computation algorithm [22][21].

It has been shown that these reduced-order models can be linked to nonlinear simulators that are similar to SPICE [21]. Figure 4.4 shows how model reduction is used with nonlinear simulators.

Figure 4.4 Macro model synthesis for nonlinear simulation

The linear subnetwork is extracted from the complete network. The reduced-order model is obtained and the differential equations are derived. This process is known as macro model synthesis. Since the differential equations are described in the time-domain, they can be easily linked to nonlinear simulators either directly or, if the simulator does not allow this, they can be converted to an equivalent subcircuit and used.

4.3 Implicit Moment-Matching Techniques

Problems with Direct moment-matching techniques such as AWE, previously discussed, led to the development of a different set of algorithms which have been classified as *indirect moment-matching techniques* [23][24].

Krylov-subspace formulation and *Congruent transformation* form the foundation for these algorithms. Whereas, the CFH technique previously discussed is based on the extraction of the dominant poles of a given system, Krylov-subspace techniques construct a reduced model based on the extraction of eigenvalues with the largest magnitude (*leading eigenvalues*).

4.3.1 Model Reduction based on Krylov-subspace Techniques

A more general for of the Time-domain MNA equations from section 2.3 are:

$$\begin{aligned} C_{\Psi}\dot{x}(t) + G_{\Psi}x(t) &= Bu(t); & C_{\Psi}, G_{\Psi} &\in \mathbb{R}^{n \times n}; B, x \in \mathbb{R}^{n \times 1} \\ w &= L^T x(t); & L &\in \mathbb{R}^{n \times 1} \end{aligned} \quad (4.16)$$

where n is the total number of MNA variables. By multiplying both sides by the inverse of G_{Ψ}

$$\begin{aligned} A\dot{x}(t) &= x(t) - Ru(t); & A &= -G_{\Psi}^{-1}C; & R &= G_{\Psi}^{-1}B \\ w &= L^T x(t) \end{aligned} \quad (4.17)$$

Applying the Laplace transform to (4.17)

$$\begin{aligned} sAX(s) &= X(s) - RU(s) \\ W(s) &= L^T X(s) \end{aligned} \quad (4.18)$$

The transfer function $Y(s)$ of the system is then

$$Y(s) = \frac{W(s)}{U(s)} = L^T (I - sA)^{-1} R; \quad I = \text{Identity} \quad (4.19)$$

It has been shown that the eigenvalues of A are the reciprocal of the poles of the system, where the leading eigenvalues correspond to the poles closer to the origin. Furthermore, the transfer function can be obtained once the eigenvalues and eigenvectors are available [21]. The calculation of the eigenvalues and eigenvectors of a given matrix A is computationally expensive as the size of A becomes larger than a hundred or so. To avoid the expense a smaller matrix, $\hat{A}$, is found that has leading eigenvalues approximately the same as A . Since $\hat{A}$ is smaller than A , the eigenvalue calculations will be less numerically expensive.

4.3.2 Model Reduction

Given a circuit, a change of variables can be applied to map the vector x of dimension n into a smaller vector $\ddot{x}$ of dimension q , where q is much smaller than n using the orthogonal matrix Q [10],[21]-[27]:

$$x = Q\ddot{x} \quad (4.20)$$

Substituting (4.20) into (4.18)

$$\begin{aligned} sAQ\hat{X}(s) &= Q\hat{X}(s) - RU(s) \\ W(s) &= L^T Q\hat{X}(s) \end{aligned} \quad (4.21)$$

Multiply both sides of (4.21) by Q^T and since by definition, to be an orthogonal matrix Q must satisfy the relation $QQ^T = Q^T Q = I$ simplify to get

$$\begin{aligned} sQ^T A Q \hat{X}(s) &= \hat{X}(s) - Q^T R U(s) \Rightarrow \hat{X}(s) = (I - sQ^T A Q)^{-1} Q^T R U(s) \\ W(s) &= L^T Q \hat{X}(s) \Rightarrow W(s) = L^T Q (I - sQ^T A Q)^{-1} Q^T R U(s) \end{aligned} \quad (4.22)$$

Thus the transfer-function of the reduced system is

$$\hat{Y} = \frac{W(s)}{U(s)} = L^T Q(I - sH)^{-1} Q^T R; \quad H = Q^T A Q \quad (4.23)$$

There are a number of ways to calculate Q , the most popular being Arnoldi's algorithm [23],[21] and Lanczos algorithm [28]. The key advantage of the Arnoldi's algorithm over the Lanczos's algorithm is that with minor modifications to the approach, Arnoldi's algorithm can, with certain conditions, preserve passivity. Specifically, unlike the above formulation, reduce C and G directly in (4.18) to get:

$$\begin{aligned} C_\Psi Q \hat{X}(s) + G_\Psi Q \hat{X}(s) &= B U(s) \\ W(s) &= L^T Q \hat{X}(s) \end{aligned} \quad (4.24)$$

Again, multiply both sides by Q^T and simplifying

$$\begin{aligned} \hat{X}(s) &= (\hat{G} + s\hat{C})^{-1} \hat{B} U(s) \\ W(s) &= \hat{L}^T \hat{X}(s) \end{aligned} \quad (4.25)$$

where

$$\begin{aligned} \hat{C} &= Q^T C_\Psi Q & \hat{G} &= Q^T G_\Psi Q \\ \hat{B} &= Q^T B & \hat{L} &= Q^T L \end{aligned} \quad (4.26)$$

The transformations (4.26) are known as *congruence transformations*. Furthermore, $W(s)$ can be re-written as

$$W(s) = [\hat{L}^T (\hat{G} + s\hat{C})^{-1} \hat{B}] U(s) \quad (4.27)$$

and thus the transfer function is

$$\hat{Y}(s) = \hat{L}^T (\hat{G} + s\hat{C})^{-1} \hat{B} \quad (4.28)$$

It has been proved that the reduced model given by (4.28) is passive [29].

4.3.3 Krylov-Subspace method for Iterative Computation of Eigenvalues

In this section the calculation of the matrix Q will be explained. First, two basic matrix forms will be explained.

QR decomposition: Let K be a m by n matrix with $m > n$. If K has a full column rank, then there exists a unique $m \times n$ orthogonal matrix Q and unique upper-triangular matrix R_u with positive diagonals ($r_{ii} > 0$) such that $K=QR_u$.

Upper-Hessenberg Matrix: A matrix H is called Upper-Hessenberg if $H_{ij}=0$ for $(i>j+1)$. The companion form of a upper Hessenberg matrix of order n would be:

$$H = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & -c_1 \\ 1 & 0 & 0 & \dots & 0 & -c_2 \\ 0 & 1 & 0 & \dots & & \\ & 0 & & \dots & & \\ & & & \dots & 0 & \\ & & & & \dots & 1 & -c_n \end{bmatrix} \quad (4.29)$$

The advantage of the companion form is that the characteristic polynomial, $p(x)$, can be computed by

$$p(x) = \sum_{i=1}^n c_i x^{i-1} \quad (4.30)$$

The eigenvalues of H are the roots of $p(x)$.

Starting with equation (4.13) and define K such that $K = [R \ AR \dots A^{n-1}R]$, to create a similarity transformation $AK = KH_n$, with H_n having the upper-Hessenberg companion form as defined above. This leads us to the relation

$$H_n = K^{-1}AK \quad (4.31)$$

Since H_n is related to the matrix A by a similarity transformation, it has the same eigenvalues as of A . Unfortunately, to calculate H_n , one needs the inverse of K , which is a dense matrix, thus a numerically expensive task. Also, since the columns of K are formed on the sequence of $A_i R$, it is likely to be ill-conditioned [21].

To address these limitations substitute for K the orthogonal matrix Q such that for all n , the leading columns of K and Q span the same space. This space is called a Krylov subspace and is denoted by $\kappa(A, R, n)$. Therefore, any vector that is a linear combination of the leading n columns of K can be expressed as a linear combination of the leading n columns of Q .

If we express the matrix K using QR decomposition $K = QR_u$ and modify equation (4.31) as

$$\begin{aligned}
AK &= KH_n \\
&= (QR_u)(QR_u)^{-1}A(QR_u) \\
&= (QR_u)(R_u^{-1}Q^T)A(QR_u) \\
&\Rightarrow Q^T A Q = R_u H_n R_u^{-1} = H
\end{aligned} \tag{4.32}$$

H , the new matrix, is also upper Hessenberg since R_u and R_u^{-1} are upper triangular and H_n is upper Hessenberg.

Q and H are the matrices referenced in equation (4.23). Equation (4.32) implies that if we use only the first leading q columns, $q < n$ of Q it is possible to reduce the matrix A to a smaller upper Hessenberg matrix H , and still retain approximations of the first q leading eigenvalues of the larger system represented by A .

4.3.4 Arnoldi Algorithm

The Arnoldi Algorithm can be used to do a partial reduction of a large matrix to a smaller upper Hessenberg matrix using Q . The version that will be described is appropriate for the two port case.

Represent Q as a vector of columns, i.e. $Q = [q_1 \ q_2 \ \dots \ q_k]$, where each q_i represents the i^{th} column of the matrix Q . We know from (4.32) that

$$AQ = QH \quad (4.33)$$

$$A \begin{bmatrix} q_1 & q_2 & q_3 & \dots & q_k \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & \dots & q_k \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} & \dots \\ h_{21} & h_{22} & h_{23} & \dots \end{bmatrix}$$

All the columns or rows of orthogonal matrices have $\|q_i\|_2 = 1$ and are orthogonal to one another. This means that $q_i^T q_i = 1$ and $q_i^T q_j = 0$. To compute q_1 , remember that $\|q_1\|_2 = 1$ and R , from (4.17), by its magnitude $\|R\|_2$. This will provide a unit vector in the direction of R . To compute q_2 and the first column of H , we multiply A by the first column of Q . With this we now have Aq_1 , the first column of the left hand side of (4.33). Equate this to the first column of the right hand side and multiply by q_1^T to get

$$\begin{aligned} Aq_1 &= h_{11}q_1 + h_{21}q_2 \\ q_1^T Aq_1 &= h_{11}q_1^T q_1 + h_{21}q_1^T q_2 \\ q_1^T Aq_1 &= h_{11} \end{aligned} \quad (4.34)$$

Since we know that $\|q_2\|_2 = 1$ and h_{11} , we can compute h_{21} from (4.34)

$$h_{21} = \|Aq_1 - h_{11}q_1\| \quad (4.35)$$

The direction for q_2 can be then calculated

$$q_2 = \frac{Aq_1 - h_{11}q_1}{h_{21}} \quad (4.36)$$

By generalizing these steps, the rest of the columns of the Q and H matrices can be obtained.

Chapter 5

Proposed Method and Examples

5.1 Proposed Method

As mentioned previously, the proposed approach of this thesis is to segment all non-uniform lines into uniform transmission lines. Then model each uniform transmission line using Padé approximation, as described in chapter four. Model reduction is applied to the resulting interconnect to reduce complexity of the interconnect and improve speed of simulation.

5.2 Discussion on Segmentation

Dividing a non-uniform transmission line into shorter transmission lines is referred to as segmentation. These segments can be treated as uniform transmission lines. To get accurate results, the number of sub-segments depends on the rate of change along x of the R , L , G and C matrices for the non-uniform line. If these transmission lines are then modelled

using the lumped model approach, then further sub-division may be required depending on the frequency of interest. It is possible that the non-uniformity of the original interconnect may sub-divide the line such that no further division is required for the lumped model approach.

Figure 5.1 Extreme non-uniformity compared to near-linear non-uniformity

The non-uniformity of example 5.1 (A) requires a large number of segments independent of modeling using the lumped modeling of transmission lines. For 5.1 (B), depending on the frequency of interest, the number segments required for non-uniformity may be less than what is required for modeling transmission lines using lumped models.

Physically, if the non-uniform line varies in a linear or near linear manner then dividing the non-uniform line in equal segments is the easiest approach. In some cases, non-uniform segmentation may be appropriate.

Figure 5.2 Non-uniform segmentation

In Figure 5.2 (B), the rate of change in width of the non-uniform line is faster during the first portion of the line. More detail in this portion of the line is required to adequately model it.

It is important to note that geometric factors other than segment width can affect the R , G , L , and C parameters of an interconnect to behave in a non-uniform manner. For example, holes in the ground plane beneath an interconnect may change the inductance and capacitance of the line dramatically.

5.2.1 Examples of Segmentation

Figure 5.3 shows a simple nonuniform transmission line. It is a tapered segment that varies from 8 mils down to 1 mil.

Figure 5.3 Tapered line evenly sub-divided into uniform transmission lines

A number of simulations are run in the frequency domain, using Matlabs Ordinary Differential Equation Solver (ODE), *ode45*, which is an ODE solver based on the explicit Runge-Kutta method, compared to segmenting the non-uniform line and modelling each sub-segment as a uniform transmission line. Numerical integration is done using an explicit Runge-Kutta method, fourth and fifth order. The Runge-Kutta method evaluates the solution at several different values, and combines the results to approximate the solution at the next step [17]. Each uniform transmission line is solved using the hybrid parameter form, where $e^{(D + sE)d}$ is solved using Matlab's *expm* function[30].

Figure 5.4 ODE vs Segmentation into 10 equal length transmission lines

Figure 5.5 ODE vs Segmentation into 5 equal length transmission lines

Figure 5.6 ODE vs Segmentation into 2 equal length transmission lines

Figure 5.7 ODE vs a single uniform transmission line

Figure 5.4 to Figure 5.7 compare results when the non-uniform transmission line is segmented into equal length uniform transmission lines, each representing a portion of the non-uniform transmission line. The figures show that the larger the number of segments one uses to model the nonuniform transmission line the more accurate the frequency response is in higher frequencies. Figure 5.7 shows that modeling a tapered transmission line as a uniform transmission line is not an accurate representation of the non-uniform transmission line.

Another approach is to determine the segmentation of the nonuniform transmission line by breaking the line up whenever a significant change in characteristic impedance (Z_0) occurs. For these simulations, a sub-segment is created whenever Z_0 changes by a prescribed percentage.

Figure 5.8 Tapered line sub-divided by change in Z_0 into uniform transmission lines

Table 2.1: End point of sub-segments for different Delta Z_0

Change in Z_0 (%)	# of segments	Sub-Division (m)
6	10	0.0170 0.0320 0.0450 0.0570 0.0670 0.0760 0.0840 0.0910 0.0970 0.1000
15	5	0.0370 0.0650 0.0850 0.0980 0.1000
35	2	0.0670 0.1000

Figure 5.9 ODE vs 10 Segments Delta $Z_o=6\%$

Figure 5.10 ODE vs 5 Segments Delta $Z_o=15\%$

Figure 5.11 ODE vs 2 Segments Delta $Z_0=35\%$

Comparing the results of equally sub-dividing the nonuniform transmission line with using change in characteristic impedance shows that the first method is more accurate up to relatively higher frequencies.

Next, a more complex example of a nonuniform line is examined. Using the same circuit as Figure 5.8, the tapered transmission line is replaced with one like Figure 5.1 (B).

Table 2.2: Nonconstant change in width with x

x (m)	Width (mm)	C (pF/m)	L (uH/m)	R (ohms/m)
0.00	0.2048	64.28	0.5434	2.3161
0.01	0.1760	60.41	0.5732	2.6944
0.02	0.1473	56.36	0.6084	3.2206
0.03	0.1221	52.61	0.6456	3.8841
0.04	0.1006	49.19	0.6843	4.7169
0.05	0.0790	45.49	0.7324	6.0042

Table 2.2: Nonconstant change in width with x

x (m)	Width (mm)	C (pF/m)	L (uH/m)	R (ohms/m)
0.06	0.0646	42.80	0.7724	7.3389
0.07	0.0503	39.83	0.8226	9.4368
0.08	0.0396	37.36	0.8704	11.9829
0.09	0.0326	35.57	0.9092	14.5536
0.10	0.0256	33.56	0.9575	18.5285

In this case, the width of the tapered segment varies in a step like fashion. In the first 2.5 centimeters, the width changes 40% of the total change; at 5 cm 30% more, then at 7.5 cm 20% more, reaching the final width at 10 cm.

Figure 5.12 Segmented into 4 uniform segments

Figure 5.13 Segmented into 4 based on 20% change in Z_o

Figure 5.12 and Figure 5.13 compare the two approaches of segmentation of a tapered interconnect with a varying change in width. Both simulations divided the tapered line into 4 segments. Again, the uniform division of the tapered segment provided a more accurate result than using Z_o as the means of deciding on segment division.

From experimentation it has been shown that dividing the nonuniform line in equal parts results in better accuracy to a higher frequency than segmenting according to change in Z_o .

5.3 Use of Matrix Rational Padé Approximation

Once the nonuniform interconnect has been divided into segments, each segment can be modelled as a uniform transmission line. The following examples compare the accuracy of the Matrix Rational Form approximation of the transmission line to an exact frequency domain solution using an ODE solver, and lumped model approximation. For these examples, a reasonably high order of approximation is used. This results in a more accurate model at the expense of cpu time during simulation. Since the proposed method assumes that model reduction will be done on the resulting MRF system, the cost of high accuracy will be reduced considerably.

5.3.1 Single Tapered Interconnect

A tapered interconnect, Figure 5.14, is used to show the accuracy of the proposed approach. Physically, it represents a short trace on a Printed Circuit Board, where the track is forced to narrow to go between two obstacles. For example, this may occur on a backplane, where a track must negotiate a connector before reaching its destination. The figure shows metric measurements, which equate to a ~3 inch long track initially 8 mils wide, narrowing to 2 mils, and then widening back to 8 mils. The thickness is constant at about 1.4 mils.

Figure 5.14 Tapered Interconnect

Table 2.3: shows the R, C, & L parameters for the tapered interconnect.

Table 2.3: Parameters for tapered interconnect

x (m)	C (pF/m)	L (uH/m)	R (ohms/m)
0.0-0.02	64.3	0.55	2.32
0.022	55.8	0.62	3.30
0.023	51.1	0.66	4.21
0.024	46.0	0.72	5.79
0.025-0.045	40.0	0.82	9.26
0.046	46.0	0.72	5.79
0.047	51.1	0.66	4.21
0.048	55.8	0.62	3.30
0.050-0.070	64.3	0.55	2.32

Figure 5.15 Tapered interconnect frequency response to 10 GHz

The results shown in Figure 5.15 compare the “exact” frequency response solved using an ODE solver, modeling the five sections as uniform transmission lines (Uniform), and finally approximating the tapered lines by segmenting them into four parts each and modeling all transmission lines using Padé approximation (MRF). The MRF approach is accurate up to 8 GHz, whereas the uniform approach is accurate to 5 GHz. Of the three approaches, MRF is the least computationally expensive.

Figure 5.16 ODE vs. Order 10/10 Padé Approximation using Spice

Figure 5.16 compares the ODE frequency domain solution, to an order 10/10 Padé approximation SPICE circuit.

5.3.2 Coupled Tapered Interconnects

In this example, two tapered interconnects are coupled (Figure 5.17.) Physically, this could occur when two signals are forced to go between the same obstructions, for example connector pins.

Figure 5.17 Coupled tapered interconnects

The dimensions of the tapered interconnects are the same as in example 1. For the simulation, only capacitive and inductive coupling were analyzed.

Table 2.4: Parameters for Coupled Tapered Interconnect

x (m)	C11=C22 (pF/m)	C12=C21 (pF/m)	L11=L22 (uH/m)	L12=L21 (uH/m)	R (ohms/m)
0.0	64.3	-8.76e-5	0.543	5.90e-8	2.32
0.02	870	-806	0.543	0.542	2.32
0.025	373	-333	0.819	0.542	9.26
0.045	373	-333	0.819	0.542	9.26
0.05	870	-806	0.543	0.542	2.32
0.07	64.3	-8.76e-5	0.543	5.90e-8	2.32

Figure 5.18 Coupled Tapered interconnect frequency response to 10 GHz
(MRF Order 4/4 13 sections)

Figure 5.19 Coupled Tapered interconnect frequency response to 10 GHz
(MRF Order 5/5 16 sections)

Figure 5.20 110 Lumped PI segments vs ODE

Figure 5.21 220 Lumped PI segments vs ODE

Figure 5.22 400 Lumped PI segments vs ODE

Figure 5.23 ODE vs HSpice Padé Order 10/10

Figure 5.24 HSpice Padé Order 10/10 vs IFFT of ODE solution

Figures 5.18 and 5.19 compares the ODE solution to the a solution assuming only uniform transmission lines, and to the MRF solution of order 4/4 with 13 segments and order 5/5 with 16 segments. For the uniform case, the tapered interconnects are replaced with a single uniform line using the average width of the tapered sections. This results in a sudden change in the characteristic impedance leading to the dramatic difference between the actual solution and the uniform solution. This emphasizes the importance of modeling the tapered sections appropriately.

Figures 5.20 to 5.22 compare the ODE solution to lumped model simulations. Using the lumped segmentation approach, 400 segments are required to match the MRF approach of 13 segments, and order 5/5. The MRF system matrix is about five times smaller than the segmentation approach, leading to a much more efficient solution.

Figures 5.23 and 5.24 compare the ODE solution to MRF models used within HSpice. The MRF solution compares very favorably to the ODE solution.

5.4 Use of Model Reduction

Once the MRF macro model has been generated, the resulting netlist can be reduced using the Arnoldi algorithm to find a matrix $\mathbf{Q}$ that can be used as part of the congruent transformation, equation (4.26), to approximate the original system. The original coupled tapered line example results in a system matrix of size 768 by 768.

Figure 5.25 Order 25 resulting in a Q matrix of size 100 by 100

Figure 5.26 Order 20 resulting in a Q matrix of size 80 by 80

Figure 5.27 Time Domain, MRF compared to Arnoldi Order 20 reduction

Figure 5.25 and Figure 5.26 compare the MRF results to approximations using Arnoldi's algorithm with orders 25 and 20 respectively. The results show that the network generated by the MRF benefits significantly with a minimum loss of accuracy from applying model reduction. Figure 5.27 shows that the reduced model is almost identical to the original MRF approximation.

Table 2.5: Performance Comparison

	Lumped Segmentation (400 sections)	Matrix Rational Form Approximation (order 10/10, 13 sections)	Arnoldi (order 20) applied to MRF 10/10
Matrix Size	2406	786	80
Number of Poles	1602	520	80
HSpipe AC Analysis (s)	35	11	2.0
HSpipe Transient Analysis (s)	15	5	4.6
NAG Transient Analysis (s)	14	4.6	0.9

Table 2.5: compares the performance of lumped segmentation, MRF and the reduced MRF system. As seen in Figure 5.23, MRF 10/10 with 13 sections has better accuracy than the 400 section lumped model. The transient analysis time is also significantly faster.

The MRF 10/10 model was then reduced using the Arnoldi algorithm and the resulting system was converted to a reduced order spice compatible circuit by applying an eigen decomposition[35]. The reduced MRF 10/10 model does show that we can maintain high accuracy but the transient analysis performance in HSpice did not improve as expected. It is suspected that the form of the reduced order circuit is suboptimal for HSpice. Further tests were done by integrating the MNA equations directly using Numerical Algorithms Group (NAG) Fortran 77 library running under Windows 2000 on a Pentium III 850 MHz PC. The results show that the reduced model runs about five times faster than the original MRF form.

5.4.1 Series of coupled tapered lines

In this example Figure 5.28, a series of coupled tapered interconnects are simulated. Again, this example may occur on a backplane where a number of connectors must be negotiated before the signals reach their destination. Each coupled tapered interconnect is the same as the one used in Example 2.

Figure 5.28 Series of Coupled Tapered interconnects

Figure 5.29 ODE vs MRF 10/10 42 Segments

Figure 5.30 ODE vs Reduced (order 64) MRF

Figure 5.31 Time domain MRF vs Reduced MRF

Figure 5.29 shows that the MRF of order 10/10 is quite accurate, as compared to the ODE solution. Reduced MRF simulation in Figure 5.30 shows that it is almost as accurate as the MRF solution in the frequency domain. Finally, Figure 5.31 shows that the reduced model follows the MRF solution fairly well in the time domain.

The MRF macro model can be reduced significantly by using the Arnoldi algorithm to derive a reduced system. This results in a dramatic improvement in numerical computational cost. It has been shown that these improvements are not as dramatic for networks composed solely of uniform transmission lines[34]. It is surmised that since the segmentation of the nonuniform transmission lines result in many uniform transmission lines with similar poles, the reduction algorithm is able to filter out less significant poles more efficiently.

Table 2.6: Performance Comparison

	Matrix Rational Form Approximation (order 10/10, 42 sections)	Arnoldi (order 64) applied to MRF 10/10
Matrix Size	2526	256
Number of Poles	1680	256
HSpice AC Analysis (s)	34	7
HSpice Transient Analysis (s)	16	16
NAG Transient Analysis (s)	7.4	1.8

Table 2.6: compares the MRF netlist with its reduced version. AC analysis is almost five times faster after model reduction. As in the previous example, the transient performance of the reduced MRF model in HSpice was not as expected. Solving using the NAG library shows that the reduced model is 4 times faster than the MRF model.

It was noticed that the reduced model loads much more quickly into HSpice than the MRF or lumped model (0.8, 6 and 9.5 seconds respectively). This would make a significant reduction in total simulation time when doing optimization or sensitivity analysis of a circuit.

These results show that segmenting the nonuniform interconnects and applying Padé approximation techniques leads to fast and accurate results. They also show that very high order Padé approximations can be used for each section, and then using model reduction techniques, the resulting system can be reduced to create a smaller macromodel. This macromodel maintains the high accuracy of the Padé approximation and further improves overall simulation time.

Chapter 6

Conclusions and Future Research

6.1 Conclusions

A method to model nonuniform interconnect networks has been presented that is suitable for use in general purpose simulators, such as SPICE. The proposed method segments the nonuniform lines into uniform subsegments that are then modeled using a high-order Padé approximation. This network is then reduced using a Krylov subspace reduction technique, resulting in an accurate and numerically efficient model.

In this thesis, the advantages of the proposed method have been shown to be:

1. Guidance is given for segmenting the nonuniform transmission lines within the network.
2. Using matrix rational form to model the individual transmission lines results in a passive model that is very accurate. High-order Padé approximates can be used to ensure accuracy.

3. Applying Krylov sub-space reduction techniques to the system reduce large complex models without a significant loss in accuracy. This is because the segmented nonuniform interconnects result in a network with many poles that are close in value but are not identical. This is an optimal situation for model reduction.

6.2 Future Research

There are many opportunities for research in the area of nonuniform transmission lines. Some improvements in the current proposal include

1. More rigorous approach to the segmenting of the nonuniform transmission line would result in less segmentation than is currently proposed. Since the final model is reduced, the extra work currently done is not costly.
2. For this thesis, the PRIMA reduction algorithm was used. More recent approaches may result in a better reduction [36].
3. Though the algorithm to convert the reduced system matrix into a SPICE equivalent circuit works, improvements in it's resulting netlist would result in faster simulation times within HSpice.

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