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Modeling Open Economy with Imperfect Capital Flows

by

Jafar Ahmed Khondaker

**A thesis submitted to the Faculty of Graduate Studies
in partial fulfilment of the requirements for the degree of**

Doctor of Philosophy

**Carleton University
Ottawa, Ontario**

April 2000

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submitted by

Jafar Khondaker, B.Sc., M.Sc., M.A.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

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Abstract

This dissertation analyzes the implications of international parity conditions for open-economy modeling, and uses the insights from this analysis to model an open economy with imperfect capital flows. The principal focus in modeling the open economy is on the dynamic interactions of the real exchange rate, the capital stock and debt accumulation. Chapters 1 through 3 supply the motivating background for modeling the open economy with imperfect capital flows. Chapter 2 examines the nature of failures of two international parity conditions – *uncovered interest rate parity* and *purchasing power parity*. The analysis in this chapter leads to two conclusions. First, the risk premium explanation for the failure of uncovered interest rate parity is incomplete in that existing models of risk premium ignore country-specific risk factors. Secondly, the evidence of a painfully slow mean-reversion in the real exchange rate is inadequate to give *purchasing power parity* a pivotal role in the modeling of exchange rates. Chapter 3 empirically examines, using Johansen's cointegration methodology, whether there is any tendency for real interest rates across countries to converge over time. While real interest rates are found to be cointegrated in the majority of the cases examined, the empirical evidence does not support the real interest rate parity that can justify setting, in open economy modeling, the domestic real interest rate equal to the foreign real interest rate. Chapter 4 synthesizes the work of Dooley and Isard (1991), Branson (1988) and Stein (1994) to formulate a simple model of an open economy that emphasizes the dynamics of the real exchange rate, capital accumulation, and debt within an imperfect international capital market framework. Chapter 5 extends the base-line intertemporal optimizing macro-economic framework by incorporating less-than-perfectly-elastic capital flows and treating investment as composite of both domestic and imported goods. The resulting three-good small open-economy model is calibrated to the Canadian data; solved and analyzed with tools borrowed from the real business cycle literature. The model's ability to mimic some empirical characteristics of the Canadian economy is promising. Some degree of sensitivity of the real interest rate to the stock of foreign debt gives the model stability, and allows the model to better fit the data.

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1 Modelling Open Economy with Imperfect Capital Flows: Introduction and an overview

1.1 The Environment

About a decade and a half ago, Mishkin (1984) investigated and rejected empirically three international parity conditions (real interest parity, uncovered interest parity, and *ex ante* purchasing power parity). His conclusion was that it was 'worth studying open economy macroeconomic models which allow domestic real interest rates to differ from world rates' (p. 1356). A decade later, when barriers to international capital mobility were largely eliminated and international capital markets had been dramatically integrated, Marston (1995) reinvestigated the real interest parity condition and again found economically significant differentials that persisted for several years at a time.¹ A large volume of work both before and after Mishkin's work has also failed to find much support for two international parity conditions – *uncovered interest rate parity* and *purchasing power parity* – that form the building blocks for *real interest rate parity*. Despite this, much open economy modeling continues to use the assumption of real interest rate parity.

¹Mishkin (1984) analyzes real interest rates in the euro deposit market in the 1967:2-1979:2 period. The decade following this period is marked by dramatic increase in cross-border transactions in bonds and equities. Thus, as Marston (1995) reports from the 62nd Annual Report of the Bank of International Settlements, cross-border transactions between U.S. and other countries increased from 9.3 percent of GDP in 1980 to 92.5 percent in 1990. The corresponding numbers for Japan and Germany over the same period are, respectively, from 7 percent to 118.6 percent and from 7.5 percent to 57.5 percent. For France the growth is from 8.4 percent in 1982 to 53.3 percent in 1990.

1.2 The Objective

The present research has two objectives. Firstly, it extends our understanding of the implications of international parity conditions for open-economy modeling. In meeting this objective, we first analyze the parity conditions empirically. Second, we use the insights from that analysis to model an open economy with imperfect capital flows in order to show the dynamic interactions of the real exchange rate, the capital stock and debt accumulation. To meet the second objective, we develop two models. The first is a simple dynamic model that integrates the equilibrium condition of an imperfect asset market with the fundamentals of demand and supply shocks affecting goods markets. The second model shows more rigorously the dynamics of the real exchange rate and the current account within a stochastic optimizing model of capital accumulation in the context of imperfect international capital markets. The second model is then parameterized using Canadian data and solved with numerical solution tools borrowed from real business cycle modelers. The motivation for this research comes from the following stylized facts.

1.3 Some Stylized Facts

First, despite a significant lowering in the recent decades of the barriers to international capital flows in the major industrialized countries, the real rates of return on comparable securities denominated in different currencies continue to differ. There is now ample evidence that is unfavourable to the uncovered interest parity condition.² Figure 1.1 below

²See, for example, Cumby and Obstfeld (1984), Cumby (1988), Froot and Thaler (1990), Backus et al (1993), Hodrick (1987), and Lewis (1995).

plots real interest differentials between Canada and the U.S., and between U.K. and the U.S. respectively.³ The interest rates are three month treasury bill rates and the yields on medium-term (three to five-years) government bonds. The data come from the International Monetary Fund's database *International Financial Statistics*, and is for the period 1957 quarter one (1957:1 for short) to 1999:2.⁴ Both of these plots summarize the same story: considerable differentials remain among these countries that cannot be explained by expected real exchange rate movements. One can clearly identify a positive trend in the Canada-U.S. real interest differentials over the period 1983-1993. Virtually the same pattern can be documented using long-term interest rates.⁵ This pattern of difference is difficult to explain using the uncovered interest parity condition. One regularity that can be linked to such persistent real interest differentials is that international investors hold only a modest number of securities denominated in a foreign currency in their portfolios, much less than they would if they followed a world market index approach in deciding on the amount of financial wealth to allocate to foreign securities.⁶

³The real interest series are generated by subtracting realized annual rate of inflation from the nominal interest rate (expressed as percent per annum) in the *International Financial Statistics* database. The calculations are detailed in chapter 3 (section 3.4.1).

⁴All data used in the present and the next two chapters are from the IMF database unless otherwise indicated. Series codes are provided in Appendix 5D.

⁵Chapter 3 presents an extensive empirical analysis of real interest parity between a group of five industrialised countries including Canada, U.S., U.K., Japan and Germany over last four decades for both medium-term and long-term government bond yields. We also show there (see section 3.2) how uncovered interest parity and *ex ante* purchasing power parity form building blocks for real interest rate parity.

⁶See Dumas (1994) and references therein relating to this strong home securities preference. Lewis (1998) is the most recent and a comprehensive analysis of this home bias.

Figure 1.1 Canada-U.S. and U.K.-U.S. Real Interest Rate Differentials, 1957:1-1999:2

A second common empirical observation about exchange rates is that both nominal and real exchange rates between the major currencies are generally well described as random walks in which month-to-month and quarter-to-quarter changes are almost entirely unpredictable.

Third, following from the random walk nature of the real exchange rates, there is almost universal agreement that purchasing power parity does not hold in the short run, although long-run purchasing power parity has not yet been rejected convincingly by statistical tests.⁷ Empirical evidence suggests that monthly and quarterly changes in exchange rates are often not closely related to differentials in national inflation rates. Thus, most short-run changes in nominal exchange rates correspond to changes in real exchange rates. Fluctuations in exchange rates have been much higher than the fluctuations in the national price levels implying that changes in real and nominal exchange rates are very highly correlated and have similar variances. This observation along with the statistical evidence suggesting that changes in real exchange rates tend to be nearly permanent (on average), or to persist for very long periods of time, leads some “equilibrium” economists (Stockman, 1987, for example) to claim that most changes in nominal exchange rates -- even very short-term day-to-day changes are largely due to permanent real disturbances. While the view that everyday fluctuations in nominal exchange rates are caused by real disturbances seems extreme, there is universal agreement that real events can change equilibrium exchange rates, which are after all relative prices like any others. Economic theory predicts that real disturbances to supplies of goods or demands for goods cause changes in relative prices, including the real exchange rate. It is likely that in many circumstances, these changes in the real exchange rate will be accomplished through changes in nominal exchange rates.

⁷In its most common form, purchasing power parity (PPP) asserts that the exchange rate change between two currencies is determined by the change in the two countries' relative price levels. See Dornbusch (1992) for a good review of the theory. Breuer (1994) gives an excellent recent survey on the empirical evidence.

Figure 1.2 Canada-U.S., U.K.-U.S., Germany-U.S., and Japan-U.S. Real Exchange Rates, 1957:1-1999:2 (1998:1=1.00)

A fourth empirical observation is the high degree of persistence and *regime-dependent variability* of real exchange rates. Figure 1.2 above plots the data for Canada-U.S. and U.K.-U.S., Japan-U.S., and Germany-U.S. real exchange rates for the period 1957:1 through

1999:2.⁸ In all cases, the real exchange rates have exhibited very long cycles and long-term drifts. Thus, the real value of the Canadian dollar appreciated against U.S. dollar persistently at an annual average rate of 1.45 per cent over the eight-year period between 1962:3 -1974:2. Then over the five-year period between 1976:3 -1981:3, Canadian dollar depreciated at an annual average rate of 4.89 per cent. The five and a half year period between 1986:1-1991:3 then exhibits persistent appreciation at an annual average rate of 3.78 per cent to be followed by a four-year period of real depreciation of similar magnitude. The U.K.-U.S. real exchange rate indicates the same pattern. The data for the Japan-U.S., and the Germany-U.S. exchange rates exhibit long-term drift. Thus, over the twenty-year period between 1957:2 and 1978:3, the real yen-U.S. dollar rate declined by more than 68 per cent implying real depreciation of the US dollar against yen at an annual average rate of 3.43 per cent. The Germany-U.S. real exchange rate also presents a similar picture.

Another feature present in the data has been the regime dependent variability of real exchange rates. Table 1.1 below reports the standard deviation as a fraction of the mean quarterly percentage changes in the real exchange rates over two time periods, 1957:1-1973:1 and 1973:2-1999:1.⁹ Evidently, with the exception of the Canada-U.S. rate, the real exchange rates of industrialized countries have become far more volatile in the post-Bretton Woods

⁸The real exchange rate is measured here as the ratio of foreign to domestic aggregate price indexes, both measured in units of domestic currency.

⁹The second time period in the Germany-U.S. case covers 1973:2-1998:3. For Canada-U.S. real exchange rate variability, the first time period covers 1957:1 through 1971:2.

Table 1.1 Variability of Real Exchange Rates

Real Exchange Rates	1957:1-1973:1	1973:2-1999:1
Canada-U.S.	8.05	6.18
U.K.-U.S.	11.51	206.84
Japan-U.S.	1.98	23.28
Germany-U.S.	2.98	30.64

Note: The variability is measured by the absolute value of coefficient of variation.

era than they had been in the Bretton Woods era.¹⁰ This finding has been amply demonstrated by Mussa (1986) and Stockman (1983, 1987). Grilli and Kaminsky (1991) examine a one-century long data set (1885-1986) composed of monthly observations on the real exchange rate between the U.S. dollar and the British pound. Their research demonstrates that the behavior of real exchange rates varies substantially across historical periods, but not necessarily across nominal exchange rate arrangements. But what we see in the data is also real: something seems to have happened in the post-Bretton Woods era that increased the variability of the U.K.-U.S. real exchange rate by more than 18 times, Germany-U.S. real exchange rate by more than 10 times, and Japan-U.S. rate by about 12 times. Increased variability is most likely the result of increased variability in equilibrium real exchange rates. If the underlying fundamentals, specifically taste and technology and possibly either policy changes or expectations of policy change frequently, then it becomes conceivable that all increased variability is merely a reflection of these changes. Since asset markets adjust faster

¹⁰Canadian dollar has been floating against US dollar over our entire sample period except between May 1962-May 1970. Besides, given the degree of integration of goods and financial markets between the two countries, and the similarity of macroeconomic policies over our sample period, it is not surprising that the variability of the real exchange rate has not increased with the recent float.

than the goods markets, the effects of changes in the fundamentals are reflected faster in the nominal exchange rates. This can explain the regime dependence of the variability of the real exchange rate.

1.4 An Overview

The next two chapters present an empirical analysis of real interest rate parity condition and its two building-blocks – *uncovered interest rate parity* and *purchasing power parity*. In chapter 2, we first try to understand the nature of the failures of the above-mentioned two building blocks. Given the behavior of these two parity conditions, we attempt to examine the behavior of real interest rate differential with a less restrictive assumption in chapter 3. We argue that in a world where countries experience different shocks to their goods and assets markets, real interest parity in the short-run is too strong a requirement. Full parity in the short-run would require fast adjustment not only in assets markets among countries, but also in their goods markets. A much less restrictive assumption is real interest parity as a long-run equilibrium condition, so that short-run parity describes a tendency to react toward long-run equality. This implies that the parity might never be satisfied at any given point in time. All that is required is that when either prices or nominal interest rates diverge substantially, adjustment forces are evoked that restore equilibrium in the long-run. Evidence from Mishkin (1984) and Marston (1995) does not rule out the tendency for real rates across countries to equalize over time. We use Johansen's maximum-likelihood-based cointegration methodology (Johansen and Juselius, 1990; Johansen, 1995) in chapter 3 to examine the above possibility. The empirical analysis with data spanning over four decades

demonstrates that the data is consistent with interpreting real interest differentials as a risk premium. This analysis then provides the motivation for modeling open economy in an imperfect capital market framework.

One can identify two approaches to modeling the long-run equilibrium real exchange rate in the literature. One is based on PPP that maintains that the relative price of two national currencies should bear some predictable relationship to the relative value aggregates of their prices for current goods and services. According to this view, the long-run real exchange rate can be assumed to be time-invariant. The second approach is based on the belief that the current account balance, which affects the country's stock of net claims on, or indebtedness to the rest of the world, imposes a boundary condition that pins down the expected long-run level of the real exchange rate. It is universally accepted that large and persistent current account imbalances are unsustainable and that changes in real exchange rates help current account adjustment. Thus given the time paths of other fundamental variables that influence the current account, the real exchange rate cannot be sustained at a level that would generate large and persistent current account imbalances. Accordingly, developments that permanently shift the current account, other things remaining the same, will affect the equilibrium level of the real exchange rate.

The model in chapter 4 follows the second approach above, but extends the argument to include shocks that originate in the asset market. The balance of payment constraint implies that changes in preferences for holding assets in different countries – or for holding claims against the residents of different countries – cannot automatically give rise to net international capital flows; rather, a change in net international creditor positions requires

a current account imbalance. A rise in the net capital flow into a country requires an equal reduction in the current account surplus, or a rise in the current account deficit. Moreover, to change their current account balance, countries must change their production of goods and services relative to their absorption of consumption and investment goods. Thus, the real exchange rate changes to give rise to a current account deficit equal to the rate at which foreigners will acquire claims on the domestic country. Dooley and Isard have extensively pursued this idea in a number of papers. In support of this view, Dooley and Isard (1991) point to a number of debt-burdened countries during the 1980s that experienced substantial depreciations of their real exchange rate around the time of the outbreak of the international debt crises in 1982. Dooley and Isard (1991) argue that these depreciations can be attributed to events that considerably reduced the attractiveness of owning assets located in the debt-burdened countries. We shall formulate Dooley and Isard's ideas in the model in chapter 4.

The model developed in chapter 4 is a model where a forward-looking variable, the real exchange rate, interacts with two state variables, capital stock and foreign debt. The basic framework developed in chapter 4 is similar in some respects to Branson (1988), and Stein (1994). Branson (1988) uses a simple model to analyse the impact of fiscal expansion over time on exchange rate and foreign debt. The model introduces an explicit risk premium into the dynamic analysis, and integrates goods market equilibrium condition with the financial market equilibrium.

Stein (1994) analyses what he defines as the *natural real exchange rate* of the US dollar with the G-10 currencies in a model that takes into account the effect of domestic capital

formation and foreign debt on the real exchange rate.¹¹ The natural real exchange rate is defined to be the exchange rate that would prevail if speculative and cyclical factors could be removed while unemployment is at its natural rate. In this model, equilibrium foreign debt is an endogenous variable whose steady-state value, like that of the capital stock, is determined by the fundamentals of social time preference and productivity. The pattern of exchange rate adjustment during the transition to external balance depends on the type of shock that initially disturbed the external balance. A current account deficit that finances consumption spending and a deficit that finances productive investment lead to different exchange rate paths. Stein shows that a consumption-led external deficit will produce a real depreciation in the long run, while the long-run effect of an investment-led deficit on the real exchange rate is ambiguous.

In formulating the model in chapter 4, we follow Branson (1988) and include a risk premium in the uncovered interest parity condition. In the light of the widespread violations of the interest rate parity conditions, this is a useful and realistic extension. The combination of persistent differentials in nominal and real interest rates, and limited international portfolio diversification by investors resident in different countries suggests that one or more factors deter investors from moving funds internationally to take advantage of higher returns abroad. At least a part of this differential can be explained by risks unique to crossing national borders, namely: uncertainty about changes in exchange rates (*currency risk*), and uncertainty about the political and economic conditions of the countries that have issued the portfolio

¹¹See also Stein, et al (1995). In this collection, Jerome Stein and Guay C. Lim applied the Stein (1994) framework to Australia. Liliane Crouhy-Veyrac and Michele Saint Marc applied it to analyze the natural real exchange rate between the French Franc and the Deutschmark, while Michael Connolly and John Devereux used it for evidence for Latin America. The results appear supportive of the framework.

claims (*country risk*). In the model in chapter 4, we implicitly emphasize that *currency risks* and *country risks* are interrelated, and that the magnitudes of these risks reflect the relative macroeconomic and political conditions of the countries issuing debt. By making the risk premium a function of foreign debt and other factors that influence perceptions of future returns on foreign portfolio investment, we emphasize joint determination of country and currency preferences in the model.

In chapter 5, we develop a three-sector stochastic intertemporal optimizing model of a small open economy to analyse the dynamics of the real exchange rate, the current account and debt accumulation with an imperfect world capital market. The imperfection in the world capital market is modeled by assuming that the real rate of interest paid on foreign debt is increasing in the stock of the country's debt. The model is solved, simulated, and analysed using Canadian data with tools borrowed from the real business cycle modelers. The model's equilibrium stochastic processes indicate that 'real' shocks (to technology and terms of trade) can adequately explain the behaviour of the real exchange rate in terms of volatility and persistence. Thus, the model's impulse response functions show that a shock to technology in the export-good sector that raises productivity one percent above its steady-state level leads immediately to an appreciation to be followed by a depreciating path for more than forty years after the shock. The real exchange rate exhibits similar behaviour in response to shocks to technology in the non-traded sector, as well as to terms-of-trade. Given the persistence of these shocks, the behaviour of the real exchange rate can be easily inferred.

The dissertation is concluded in chapter 6 with the summary of the work and some directions for future research.

2 Two International Parity Conditions

2.1 Introduction

Two international parity conditions, *uncovered interest rate parity* and *purchasing power parity* (PPP), have fundamental implications for the modeling of open economy and the real exchange rate. These two parity conditions also are the building blocks for real interest rate parity.¹ It is popularly believed that in a world with continuous arbitrage in the goods and asset markets, these parity conditions either must hold, or at least, have a tendency to hold. Empirically, on the other hand, there is virtual agreement that the uncovered interest rate parity condition does not hold. However, at present there is little agreement as to what causes this disparity.

Purchasing power parity is fundamental in a variety of theoretical models, ranging from simple open-economy versions of the quantity theory of money to the two-country, cash-in-advance model of Lucas (1982). In these basic models, PPP holds continuously. Enshrined in the celebrated Mundell-Fleming-Dornbusch model, another class of models posit that prices are sticky in the short-run and that monetary shocks are the major sources of disturbances (Dornbusch (1976)). Monetary policy shocks then result in transient divergences of nominal exchange rates from PPP, and, as a result, of actual real exchange rates from their equilibrium values. According to this class of models, PPP tends to be established over the

¹The meaning of real interest rate parity and its building blocks is discussed later in section 3.2 in chapter 3. A strong form of such parity requires that both uncovered interest rate parity and *ex ante* purchasing power parity hold. However, if real interest rate parity is used to imply the equality of the real interest cost of borrowing by a firm in its domestic and foreign markets, then the uncovered interest rate parity condition must hold.

longer run as sticky prices adjust. For years, the empirical evidence has indicated that the real exchange rate has a unit-root, implying that PPP does not hold even in the long run (e.g. Enders (1988), Taylor (1988)). A sizable body of recent empirical research using very long data series, however, finds that real exchange rates contain considerable mean-reverting components, although this process of mean reversion is extremely slow. It typically takes 70 or more years data to identify the mean-reversion tendency.² A natural question then arises: do we take this recent mild evidence of mean-reversion in the real exchange rate as supportive of the Purchasing Power Parity theory and sticky-price-based models of the real exchange rate? Or, is the evidence more consistent with another major class of models that regards disturbances as predominantly real in origin and permanent in their impact (e.g. Stockman (1987))? Shocks to productivity and terms-of-trade, and differential rates of productivity growth in the traded and nontraded sectors are factors that are often cited as sources of permanent shifts in the real exchange rate.

This chapter reviews the theoretical and empirical literature on uncovered interest rate parity and purchasing power parity in an attempt to determine their relevance for open economy modeling. We also review the usual explanations given for the poor empirical performance of these two conditions. Since the model developed in chapter 4 includes a risk premium in the asset market equilibrium condition, more than proportionate space is devoted to risk premium explanations for the failure of *uncovered interest rate parity*.

²See Abuaf and Jorion (1990), Glen (1992), Frankel (1986), Grilli and Kaminsky (1991), Lothian (1997) and Lothian and Taylor (1996, 1997) for relevant evidence. Breuer (1994) is an excellent survey summarizing many such studies.

2.2 The Uncovered Interest Rate Parity

The uncovered interest parity condition has become one of the basic building blocks of open economy macroeconomic analysis. The theory was given prominence by Keynes (e.g., 1923: 115-139) who regarded the interest rate differential as the “most fundamental” determinant of the ‘market’s preference for holding funds in one financial center rather than another’ (Keynes 1923: 124). It provides a simple relationship between interest differentials between countries’ and the expected rate of change in the spot exchange rate by generalizing the simple proposition that two assets with the same risk characteristics should have the same real return. The condition then asserts that international asset market arbitrage ensures that the following will hold:

$$(1 + i_{t,1}) = (1 + i_{t,1}^*) \frac{\mathcal{E}_t S_{t+1}}{S_t}, \quad (2.1)$$

where, $i_{t,1}$ is the nominal rate of interest on a one-period domestic bond, S_t is the nominal exchange rate, expressed as the domestic currency price of a unit of foreign currency, and $\mathcal{E}_t$ is an expectations operator with expectations formed based on all information available at date t .³ The condition in (2.1) can be approximated by the following convenient condition:

$$i_{t,1} \approx i_{t,1}^* + \mathcal{E}_t \hat{S}_{t,1} \quad (2.2)$$

Here $\mathcal{E}_t \hat{S}_{t,1}$ is the expected rate of depreciation of domestic currency. This gives a relation between the domestic interest rate, the foreign interest rate, and the expected rate of

³ An asterisk as superscript is used to indicate the foreign counterpart of any variable. The natural logarithm of any variable X is indicated by x , and, thus, $\hat{x}_{t,r}$ indicates the percentage change in the variable X between dates t and T . Appendix 2 lists for easy reference the notations that will be used frequently in chapters 2 through 4.

depreciation. If at any given moment of time, the left-hand side of (2.2) exceeds the right-hand side, investors will sell foreign bonds for domestic bonds putting an upward pressure on the value of domestic currency (lowering S_t and thus $\mathcal{E}_t \hat{S}_{t+1}$) until the two sides are equal again.

2.2.1 Tests of Uncovered Interest Rate Parity

Empirical testing of the uncovered interest rate parity condition is made difficult because of the absence of recorded data on expected future exchange rates. A common approach in testing this parity condition, then, has been to test the joint hypothesis of uncovered interest parity together with unbiased exchange rate expectations. Given that the empirical evidence on *covered interest parity* has been quite favourable, at least when the interest rates involved are Eurocurrency or other offshore rates, the test of uncovered interest parity is often conducted by estimating the following equation:⁴

$$\hat{S}_{t,T} = \beta_0 + \beta_1 (f_{t,T} - S_t) + \varepsilon_{t,T} \quad (2.3)$$

where $\varepsilon_{t,T}$ is the regression error term, and $f_{t,T}$ is the natural logarithm of the forward exchange rate at time t for delivery at time T . Thus, essentially the test becomes a test of unbiasedness of the forward rate as a predictor of a future spot exchange rate. Confirmation of the hypothesis requires finding an estimated slope coefficient of one and an intercept of zero. But numerous studies over last 20 years have found consistently point estimates for

⁴When $\mathcal{E}_t \hat{S}_{t+1}$ in equation (2.2) is replaced by forward discount $fd = f_{t,1} - S_t$, the result is the covered interest rate parity condition. See Frenkel and Levich (1975, 1977), Marston (1976), McCormick (1979), Levich (1985), Clinton (1988), Taylor (1987, 1989) for evidence on covered interest rate parity.

β_1 that are statistically significantly negative and often more than two standard errors from zero.⁵ Froot and Thaler (1990) report that the average coefficient across some 75 published estimates is -0.88.

To see whether these results are still valid with updated data, we examine Canada-U.S., U.K.-U.S., and Germany-U.S. data on exchange rate depreciations and three-month treasury bill rate differentials, and estimate equation (2.3) above by replacing forward rate discount on the right-hand side with interest rate differentials. The data are quarterly, and cover the period 1973:1 through 1999:1, except for Germany, in which case the sample period is 1975:4-1999:1. Table 2.1 reports the Ordinary Least-Squares (OLS) estimation results.

Table 2.1 OLS Estimation of Uncovered Interest Rate Parity Equation, 1973:1-1999:1^a

Interest Rate Differential / Constant	Estimate	<i>t</i> -ratio (<i>p</i> -value)	$\bar{R}^2$
Canada-U.S.	-0.167	-1.83 (0.070)	0.022
Constant	0.683	2.97 (0.003)	
U.K.-U.S.	-0.457	-2.124 (0.036)	0.032
Constant	1.782	2.308 (0.023)	
Germany-U.S.	-0.239	-1.168 (0.246)	0.003
Constant	-0.685	-1.177 (0.242)	

Note: ^aThe sample period for Germany is 1975:4-1999:1. The dependent variable in all cases is the percentage depreciation in the exchange rate. *P*-values of the *t*-ratios associated with the estimates are reported in parentheses.

⁵A partial list of studies reporting the relevant evidence includes Baillie, Lippens, and McMahon (1983), Bilson (1981), Boothe and Longworth (1986), Cumby and Obstfeld (1984), Fama (1984), Gregory and McCurdy (1984), Hansen and Hodrick (1983), Hodrick and Srivastava (1984), Cumby (1988), Cavaglia et al (1994). Hodrick (1987) provides a very comprehensive review of the vast literature on unbiased forward rate tests and market efficiency.

Table 2.2 ADF τ Tests
Exchange Rates and three-month Treasury Bill Rates Differentials
(1973:1-1999:1)

Country pair	Exchange Rate Depreciation	Interest Rate Differential
Canada-U.S.A.	-3.529 (2.889)	-2.525 (2.889)
U.K.-U.S.A.	-4.795 (2.889)	-2.554 (2.889)
Germany-U.S.A.	-4.193 (2.892)	-1.528 (2.892)

Notes: 'The sample period for Germany is 1975:4-1999:1. Numbers in parentheses are 5% MacKinnon critical values for rejection of hypothesis of a unit root.

Table 2.3 Johansen Trace Tests
Uncovered Interest Parity Cointegration, (1973:1-1999:1)

Country Pair	g	Trace Statistic	Estimated Vector [1 $-\beta_0$ $-\beta_1$]
Canada-U.S.	0 1	20.0864** 6.05	[1 -0.555 0.1047]
U.K.-U.S.	0 1	34.899** 8.407*	[1 -0.240 -0.105]
Germany-U.S.	0 1	18.039* 2.048	[1 0.392 -0.0046]

Note: 'The sample period for Germany is 1975:4-1999:1. g denotes the number of cointegrating vectors under the null hypothesis. See Appendix 3B for the test statistic and its derivation. 5% and 10% critical values of the trace statistic under the null hypothesis of zero cointegrating vector are, respectively, 19.96 and 17.85. Corresponding critical values under the null hypothesis of 1 cointegrating vector (against the alternative hypothesis of more than 1 cointegrating vector) are, respectively, 9.24 and 7.52. (**) and (*) denote statistical significance at 5% and 10% levels, respectively. Critical values are taken from Osterwald-Lenum (1992) Table 1*.

Evidently, the data show little support for a linear relationship between exchange rate depreciation and interest rate differential. While the coefficient estimates in the Germany-

U.S. equation are not statistically significant at any reasonable level, estimated coefficients of interest rate differentials in the Canada-U.S. and U.K.-U.S. equations are statistically significant at a 7% and 3.6% level, respectively. The estimated intercepts in both cases are (statistically) significantly different from zero. We find that the results are broadly consistent with the previous results.

In order to get some further insight, we use the Augmented Dickey-Fuller *tau* (ADF τ) test to determine the stationarity properties of the variables. Table 2.2 reports the results of this test. It is evident that the null hypothesis of a unit root in the percentage change in the exchange rate can be rejected at a 5% level of significance for all three country pairs. However, for three-month treasury bill rate differentials the null hypothesis of a unit root cannot be rejected at a 5% level of significance in any of the three cases. Given the nonstationarity of the interest rate differentials, one can question the validity of the results reported in Table 2.1.⁶ For analytical completeness, let us assume that the variables on the both sides of (2.3) are nonstationary, and estimate the equation using Johansen's maximum-likelihood cointegration methodology. Dickey and Rossana (1994; p. 347) indicate that the canonical-correlations-based Johansen cointegration tests may still be used if some series are stationary. Table 2.3 reports the results of Johansen's *trace test*.⁷ Trace tests indicate that Canada-U.S., and U.K.-U.S. exchange rate depreciations and the corresponding interest rate

⁶Of course, given the generally recognized low power of the ADF tests to reject a unit root null hypothesis, one may not be much inclined to accept the nonstationarity of the interest rate differentials.

⁷Appendix 3B presents a brief discussion of the Johansen methodology and the *trace test*. For an exhaustive presentation of this methodology, see Johansen (1995). See Dickey and Rossana (1994) for a very useful guide to estimation and hypothesis testing using this method. Johansen and Juselius (1990, 1992), Juselius (1995), Cushman et. al (1996) are also useful references.

differentials are cointegrated at a 5% level of significance.⁸ The Germany-U.S. relationship holds at a 10% level of significance. Although coefficient estimates in the cointegrating vectors are marginally different from the OLS estimates reported in Table 2.1, the results are broadly consistent with the previous results – the estimated value of β_1 is nowhere close to its expected value of 1.

2.2.2 Possible Explanations for the Uncovered Interest Rate Differentials

Various explanations have been suggested for the rejection of the unbiasedness hypothesis. Some of these explanations are based on the failure of the expectations component of the joint hypothesis. This category of explanations includes the so-called *peso problem*, *learning about regime shifts*, *rational bubbles*, and *inefficient information processing*. The peso problem phenomenon, initially suggested by Rogoff (1980), originated from an episode in which Mexican peso sold at a forward discount for a prolonged period of time prior to its widely-anticipated devaluation in 1976. Thus the problem refers to a situation where market participants repeatedly expect the spot rate to be affected by a policy action or some other event that does not occur in a particular sample period. Thus even if uncovered interest rate parity is valid and agents form their expectations rationally, the non-realization of expectations within finite data samples can generate evidence of non-zero

⁸In case of the U.K.-U.S. relationship, the null hypothesis of zero cointegrating vector can be rejected at a 1% level of significance. As a matter of fact, the trace test indicates the presence of two statistically significant cointegrating vectors in that case.

excess returns from forward speculation.⁹ A similar explanation suggested by Lewis (1989) is that agents must learn about their environment only after various regime shifts and may be unable to exploit fully arbitrage opportunities which are apparent in the data *ex post*. Thus, exchange rate forecast errors may be systematically related during learning. One problem with accepting the peso problem or the rational learning hypothesis is the persistence in the direction of the bias. Thus Lewis (1989) observes a persistence in the forward rate forecast errors which, in itself, is evidence against rational learning- agents cannot be learning forever about a once-for-all regime shift. This has led some researchers (Bilson (1981), for example) to interpret rejection of the unbiasedness hypothesis as evidence that agents do not process information efficiently.

The unbiasedness hypothesis is the joint hypothesis of a zero risk premium and market efficiency or rational expectations. There are some survey-based evidence suggesting violations of the rational expectations hypothesis. Frankel and Froot (1987), for example, use survey data to show that market participants systematically underpredicted the strength of the US dollar during the early 1980s. Cavaglia *et al.* (1993) also find that exchange rate forecasts are not rational in the sense that agents do not use all available information in an efficient manner. Their finding applies to the post-1986 period. Liu and Maddala (1992), however, employ the cointegration theory to analyse the same survey data as used by Frankel and Froot (1987) and find that the data in general satisfy the rational expectations hypothesis. Finally, Cavaglia *et al.* (1994) use survey data for a set of bilateral rates relative to the U.S.

⁹See Froot and Thaler (1990) for a concise discussion of the peso problem; Engel and Hamilton (1990) for evidence of similar regime shifts experienced by the US dollar. Krasker (1980) and Kaminsky (1993) also provide similar peso problem based explanations.

dollar and relative to the deutschmark over the 1985-90 period. Their evidence suggests that the bias in the forward discount for these and other currencies is attributable to *both* the failure of rational expectations and the existence of a time varying risk premium.

Despite this evidence, many economists do not find the survey evidence against rational expectations or market efficiency convincing, in part, because surveys may not reflect true expectations of investors in the market. In addition, rejection of the notion of market efficiency would break an important article of faith for many economists – the rationality of economic agents – which they would be reluctant to sacrifice. Since market efficiency is consistent with the holders of a "risky" currency requiring a premium on forward contracts, this interpretation will continue to remain prominent for some time. Given the sheer volume of trading in the foreign exchange market relative to the trade in real goods and services, and the fact that the foreign exchange market is among the most active of all financial markets, it is natural to assume that foreign exchange markets would be highly liquid and efficient. Investors have a great incentive not to make systematic mistakes in estimating future exchange rates, since failure to do so can lead to large losses. It is then hard to explain how rational economic agents could survive making systematic expectational errors in such a highly liquid market.

Given the inherent plausibility of rational expectations in the foreign exchange market, many economists are reluctant to discard it. However, the empirical evidence does suggest that returns from speculation in forward and spot currency markets are predictable and that the predictable components vary over time. This has prompted many economists to model the predictable returns from currency speculation as time-varying risk premiums. We devote

the next sub-section to review this explanation of the biasedness of forward exchange rates.

2.2.3 Foreign Exchange Risk Premium

If foreign exchange market participants are risk-averse, the uncovered interest parity condition is distorted by a risk premium. That is, (2.1) becomes

$$(i_{t,t} - i_{t,t}^*) = \mathcal{E}_t \hat{s}_{t,t} + RP_t \quad (2.4)$$

where RP_t is the risk premium. If the risk premium is time varying and correlated with the forward premium or the interest rate differential, the forward exchange market efficiency test results reported in section 2.2.1 can be explained. In order to see this, let us assume that the market participants have rational expectations. Exchange rate forecast errors $\eta_t = (\hat{s}_{t,T} - \mathcal{E}_t \hat{s}_{t,T})$ are then white noise. Now the coefficient estimate $\hat{\beta}_1$ of β_1 in (2.3) can be written as:

$$\hat{\beta}_1 = \frac{COV(\hat{s}_{t,T}, \delta_t)}{VAR(\delta_t)} = \frac{COV(\mathcal{E}_t \hat{s}_{t,T} + \eta_t, \mathcal{E}_t \hat{s}_{t,T} + RP_t)}{VAR(\mathcal{E}_t \hat{s}_{t,T} + RP_t)} \quad (2.5)$$

where we have used $\delta_t = (i_{t,T} - i_{t,T}^*)$ instead of the forward discount. Fama (1984) used this expression to interpret the negative coefficient estimate observed in the forward market efficiency tests. That is, (2.5) can be manipulated to show that $\hat{\beta}_1 < 0$ implies that $VAR(RP_t) > VAR(\mathcal{E}_t \hat{s}_{t,T})$. It then follows that if the variance of the risk premium is greater than that of the expected change in the spot exchange rate, then the risk premium is not only of theoretical interest, it becomes quantitatively more important than expected depreciation

for explaining changes in the exchange rates. This has led to a search for useful empirical models of the risk premium under rational expectations.

The search for a risk premium in the foreign exchange market has been pursued along two main lines. One line of research uses consumption-based intertemporal asset pricing models such as those of Lucas (1978, 1982), Brock (1982), and others. In this class of models, a risk premium is generated by exploiting intertemporal Euler equation in an international environment. The other line of research is based on the 'portfolio balance model'.

2.2.3.1 Consumption-Based Capital Asset Pricing Models of Risk Premium

The International Consumption-based Capital Asset Pricing Model (ICCAPM) builds on the intertemporal efficiency condition for a representative consumer-investor who maximizes expected discounted utility defined on future levels of consumption subject to sequential budget constraints. The model, therefore, has the same roots as the class of equilibrium asset pricing models constructed by Rubinstein (1976), Brock (1982), Breeden (1979), Lucas (1978, 1982), Cox *et al.* (1985) and others.¹⁰ In this type of individual utility maximization model, an intertemporal Euler equation dictates consumption-investment decisions. The equilibrium price of an asset is found by equating the forgone marginal utility from purchasing the asset and thus sacrificing present consumption to the conditional expectation of the present discounted value of the marginal utility of return from holding the asset.

¹⁰This is basically an application of the intertemporal substitution hypothesis which has been so prominent in the business cycle literature, e.g., Lucas and Rapping (1969), Barro (1976), and Long and Plosser (1983).

Let C_t be consumption at date t , $U'(C_t)$ be the marginal utility of consumption at t , and π_t be the purchasing power of a dollar at t .¹¹ The intertemporal marginal rate of substitution of money between dates t and $t+1$ can then be defined as:

$$M_{t,1} = \frac{\partial \mathcal{E}_t [u'(C_{t+1} \pi_{t+1})]}{u'(C_t) \pi_t} \quad (2.6)$$

where $\mathcal{E}$ is the agent's discount factor. Let $R'_{t,1}$ be one plus the nominal rate of return on a nominally riskless bond denominated in currency j . If R_t is the dollar return at $t+1$ on a dollar invested at t , then equating the marginal rate of substitution to the marginal rate of transformation, one obtains the intertemporal Euler equation:

$$\mathcal{E}_t (M_{t,1} R'_{t,1}) = 1 \quad (2.7)$$

where $\mathcal{E}_t$ is the expectations operator defined earlier (conditional on available information at date t). It is now straightforward to use (2.7) to price forward exchange rate by considering the covered and uncovered dollar returns from investing in a nominally riskless foreign currency denominated bond. A covered position in the investment of such a foreign bond gives the equilibrium condition:

$$\mathcal{E}_t \left(M_{t,1} R'_{t,1} \frac{F'_{t,1}}{S'_t} \right) = 1 \quad (2.8)$$

where S'_t and $F'_{t,1}$ are, respectively, the spot and one-period forward exchange rates of

¹¹ $U'(C_t) \pi_t$ can be interpreted as the utility denominated value of a dollar. Here, π_t equals $1/P_t$, where P_t is the domestic currency price of the consumption good.

domestic currency per unit of foreign currency j observed at t .

An uncovered position, on the other hand, gives the equilibrium condition:

$$\mathcal{E}_t \left(M_{t,1} R'_{t,1} \frac{S'_{t+1}}{S'_t} \right) = 1 \quad (2.9)$$

Noting that $R'_{t,1}$ can be observed at time t , we can divide (2.8) and (2.9) by $R'_{t,1}$ and then subtract (2.8) from (2.9) to obtain the fundamental restriction of the model:

$$\mathcal{E}_t \left(M_{t,1} RP'_{t,1} \right) = 0 \quad (2.10)$$

where $RP'_{t,1} = (S'_{t+1} - F'_{t,1}) / S'_t$ is the *ex-post* profit on a long position in the forward market, with profit standardized by S'_t in order to induce stationarity in the series. Now expanding (2.10), the risk premium, defined as the expected profit on forward foreign exchange speculation, can be expressed as:

$$RP_t = \mathcal{E}_t(RP'_{t,1}) = - \frac{COV_t(M_{t,1}, RP'_{t,1})}{\mathcal{E}_t(M_{t,1})} \quad (2.11)$$

where COV_t denotes the covariance conditional on the date t information set, I_t . Since the conditionally anticipated marginal rate of substitution of money is always positive, the risk premium will be non-zero when the marginal rate of substitution is conditionally correlated with the profits from forward foreign exchange speculation. Both the numerator and the denominator on the right-hand side in (2.11) depend on sample information and hence need not be time invariant. Thus, the risk premium can fluctuate over time.

One implication of (2.11) that can be noted immediately is that risk neutrality is not enough for the absence of risk-premium in the forward contract. In the event the representative aggregate consumer-investor is risk-neutral the marginal utility of consumption is constant. Thus, the intertemporal marginal rate of substitution given in equation (2.6) becomes $M_{t,1} = \partial P_t / P_{t-1}$.¹² The implication, as Frenkel and Razin (1980) and Engel (1984) point out, is that a non-zero covariance of inflation and the *ex post* profit on a long position in the forward exchange market will lead to a risk premium in the forward contract.

Given the standard assumption of a concave utility function of the consumer-investor, we can see the intuition behind (2.11). The consumer-investor values an asset that has a high payoff when consumption is low (and marginal utility is high) more than an asset that has a high payoff when consumption is high (and thus, marginal utility is low). Now suppose the return from contracting in the forward market covaries negatively with the marginal rate of substitution.¹³ In this case, the expected gain from contracting in the forward market, or equivalently the risk premium RP_t , must be positive. The agent considers his low-consumption state a "bad state". If he is making a loss on his forward contract in a bad state, then by entering into forward contract he is making his consumption more variable than it would be otherwise. Consequently, the agent requires a risk premium on the forward contract in order to induce him to enter into it. If, on the other hand, the return $RP'_{t,1}$ covaries

¹²See footnote 10.

¹³Note that (see footnote 10 and equation 2.6) the marginal rate of substitution can be interpreted as the ratio of the expected utility denominated value of a dollar at date $t+1$ to that at date t . Consequently, if the agent is in a bad state at date $t+1$ so that consumption is low and marginal utility is high, the marginal rate of substitution must also be high.

positively, the risk premium must be negative, given that no investment on a forward contract at date t is required for delivery at date $t+1$. The equilibrium foreign exchange prices reflect the insurance role of exchange markets.

There are now a large number of implementors of this approach including: Hansen and Hodrick (1983), Hodrick and Srivastava (1984), Cumby (1988), Kaminsky and Peruga (1990), and Bailee and Bollerslev (1990). However, to date this line of research has not been able to explain fully the *ex post* returns from currency speculation as risk premia. For example, Hansen and Hodrick use the intertemporal optimizing risk premium equation (2.10) to develop three linear econometric models – a log-normal model with implications of a constant risk premium; a model investigating whether time variation in the nominal return on one-month U.S. treasury bills can explain the time variation in the risk premia on one-month forward contracts; and a latent variable model to explain the risk premium using an unobservable variable. Forward rate forecast errors are used as instruments in this third model. Although they cannot reject the restrictions of this third model in the case of two of the five currencies studied, Hodrick and Srivastava (1984) reject these restrictions using a larger data set and forward premia rather than past forecast errors as instruments.

Mark (1985) estimates equation (2.9) assuming a constant relative risk aversion utility function to obtain an estimating equation of the form:

$$\mathcal{E}_t \left[\left(C_t / C_{t-1} \right)^{-\gamma} \left(P_t / P_{t+1} \right) R P_{t+1} \right] = 0$$

where γ is the parameter of relative risk aversion. Under the null hypothesis, the composite variable inside the conditional expectation in the above equation is orthogonal to the time

t information set. Mark's empirical results using GMM estimation with six sets of instruments including the lagged consumption ratio, and either k lags, $k=0, 1, 2$, of four normalized profits or four forward premiums do not lend much support for the model.¹⁴

Cumby (1988) implements a version of the Consumption CAPM risk premium model due to Stulz (1981). He finds that *ex ante* returns to forward speculations exhibit considerable fluctuation over time and often change from positive to negative. But the evidence also shows that while the comovement of the estimated *ex ante* returns to forward speculation and the estimated conditional covariance between these returns and consumption growth is broadly consistent with the predictions of the consumption-based model, the model as a whole does not provide adequate description of returns to forward speculation.

Kaminsky and Peruga (1990) use the ARCH-in-Mean (ARCH-M) approach along with the assumption of a constant relative risk aversion (CRRA), and reject the restrictions of the intertemporal model. One potential problem associated with the Kaminsky and Peruga study is their use of monthly data which typically do not possess ARCH type behaviour. To circumvent this problem, Baillie and Bollerslev (1990) use weekly data, which are potentially rich in ARCH effects. They parameterize the conditional covariance matrix of their model assuming that each conditional variance term follows a GARCH(1,1) process. Their empirical results suggest that a time varying risk premium can explain between 14 and 19 percent of the total variability of returns to forward speculation.

Backus, Gregory and Telmer (1993) add a new dimension to the search for a risk premium in forward exchange contracts by introducing non-separability into the

¹⁴Mark finds support when current and past speculative profits are used as instruments.

representative agent's preferences. Their simulation exercise indicates that habit persistence can account for a great deal more of the variability of expected returns from currency speculation than can time-additive preferences. On the other hand, the model cannot account for the strong autocorrelation of the forward premium, nor can it account for more than half of the estimated standard deviation of expected returns for any plausible range of preference parameters.

2.2.3.2 The Mean-Variance Capital Asset Pricing Model of Risk Premium

The international mean-variance capital asset pricing model (IMVCAP) builds on the hypothesis that investors diversify their portfolios by holding a variety of domestic and foreign assets with the aim of maximizing the return on their portfolios while taking into account the riskiness of these assets. Due to their different currency denominations, domestic and foreign assets of even identical default and political risk are viewed as imperfect substitutes. This imperfect substitutability drives a wedge between expected rates of return on domestic and foreign assets and gives rise to the risk premium. These risk premia are endogenously determined prices that match demands of asset stocks to their supply. Variability in the relative supply of assets denominated in different currencies then determines the change in the foreign exchange risk premia. Major work along this line has been done, among others, by Frankel (1982, 1984), Dooley and Isard (1983) and Lewis (1988b). But attempts to find risk premia that respond in the expected way to changes in the stocks of domestic and foreign currency assets have been largely unsuccessful.

One refined version of the portfolio balance approach derives the asset demand equations from a utility maximizing framework. In this kind of model, investors optimize over the mean and variance of returns. Major papers that employ this framework include Frankel (1982), Frankel and Engel (1984), Lewis (1988a), Engel and Rodrigues (1989), and Giovannini and Jorion (1989). The outcome of this line of research, however, is also not encouraging. Frankel (1982) was unable to reject the hypothesis that investors were risk-neutral and that a risk premium did not exist. A general conclusion of the later work using the mean-variance optimization framework can be stated in the following words of Frankel and Engel (1984): “investors are either not sophisticated enough to maximize a function of the mean and variance of end-of-period wealth, or else are too sophisticated to do so” (Frankel and Engel (1984)).

2.2.4 *Is Risk Premium still the answer?*

There is no doubt that the theoretical rationale for the existence of a risk premium in the foreign exchange market is strong, no matter whether it is sought in consumption-based intertemporal optimizing models or in mean-variance optimizing capital asset pricing models. The summary presented above, however, indicates that neither set of models perform well in empirical implementation.¹⁵ One may wonder if it is really the risk premium that is responsible for the failure of the *uncovered interest rate parity*. We argue that the search for a risk premium in these models is incomplete.

¹⁵The review of the empirical models of risk premium presented above is not exhaustive. See Lewis (1995) for a comprehensive review of the empirical models of risk premium.

The current search for a risk premium in the forward exchange market searches for a *currency risk premium* but ignores a possible *country risk premium*.¹⁶ That is, the unbiased forward rate hypothesis (equation 2.3) becomes a test of uncovered interest rate parity only given the assumption of *the covered interest rate parity*. Virtually all tests of the covered interest rate parity use interest rate data on euro-deposits. These deposits, denominated in different countries' currencies, are offshore securities issued by the same bank, having similar default risk. It is, therefore, not difficult to see why *covered interest parity* holds in the euro-currency market.

In the mean-variance capital asset pricing models, the risk premium arises because of different currency denominations of different assets, not because of any country related risk. In these models, two assets, one issued by Canada and the other by Mexico, will be considered perfect substitutes in the international investors' portfolios when the two assets are denominated in the same currency. Dooley (1982) points out the difficulty in defending a search for a risk premium in models that distinguish assets only by currency denomination when countries with relatively high credit risks usually do not denominate their international borrowing in their own currencies. It is, therefore, not surprising that these models can explain at most a small portion of the variation over time in the exchange risk premium.

A central banker's characterization of the origin of a risk premium can be considered an important piece of insider information. Gordon Thiessen (1995; p. 8), the Governor of the Bank of Canada, indicates that risk premium in interest rates reflect such factors as the

¹⁶While the currency risk premium arises due to the risk associated with the currency denomination of an asset, the country risk premium is associated with the country issuing the asset. The country premium covers for transactions costs, information costs, risk of possible future capital controls, tax laws that discriminate by country of residence, and default risk.

expected path of fiscal policy and political developments in the home country. Clinton (1998; p. 31-32) reports evidence from a number of recent empirical studies that support the hypothesis that Canada-U.S. yield spread is a function of outstanding stocks of government debts in the two countries.¹⁷ Such evidence motivates Macklem et al. (1995) to incorporate the impact of the debt and deficits on risk premiums by calibrating an increase in the risk premium by 1.7 basis points for one percentage point increase in the debt-to-GDP ratio, when the ratio is above 50 percent.¹⁸

It is not difficult to see that the risk premiums reported in the above-mentioned papers are essentially country risk premiums originating from the declining creditworthiness of the borrowing country.¹⁹ An interesting recent work by Bayoumi et al. (1995) can shed important light in this respect. The work draws on a unique set of survey data on municipal bond yields for U.S. states to identify the nonlinear debt supply curve faced by risky sovereign borrowers. The empirical evidence indicates that municipal bond yields rise gradually at first, but eventually yield spreads rise in a steep, nonlinear way with the level of borrowing. When the implication of such evidence in the domestic capital market is extended in the international context, one must consider additional premiums for higher information and transaction costs,

¹⁷These studies include Fenton and Paquet (1997), Fillion (1996), Djoudad (1998), and Orr et al. (1995). The simulation experiments by Fillion (1996) suggest that the increase, of about 30 percentage points, in Canada's government debt to GDP ratio in the 1990s would have raised Canadian long-term yields by about 100 basis points.

¹⁸In calibrating the effect of government debt on risk premium, Macklem et al. (1995) also use the evidence reported by Alesina et al. (1993) for 12 OECD countries for the period 1979-1989.

¹⁹While default is extremely unlikely on bonds issued by a country like Canada, U.S., or Japan, credit-rating agencies often consider an increasing ratio of government debt to GDP beyond some point as an indicator of declining creditworthiness. Thus, for example, international credit rating agency Moody's has recently put Japan's government debt under review for a possible downgrade on the ground that the debt-to-GDP ratio is likely to increase over several years. Some credit rating agencies downgraded Government of Canada foreign currency debt in 1993. It is very likely that such downgrading of government debts will be followed by a rise in the risk premiums.

as well as for the risks of future capital controls and tax laws that discriminate by country of residence.²⁰

The poor empirical performance of the existing conceptual models of risk premiums in the foreign exchange market and the recent evidence supporting that bond yields are increasing functions of outstanding stocks of debt suggest the need for a framework that considers both exchange and country risks simultaneously. We consider such a framework in chapter 4.

2.3 The Purchasing Power Parity Theory²¹

The *purchasing power parity theory* (PPP) is one of the oldest and simplest exchange rate theories (attributed to Gustav Cassel) that asserts that the nominal exchange rate should reflect the purchasing power of one currency against another. The parity, in its most common form, prevails when:

$$s_t = p_t - p_t^* \quad (2.12)$$

where s_t , p_t , and p_t^* are, respectively, the nominal exchange rate, and the domestic and foreign price levels, all in logarithms. A more relaxed relative version of (2.12) accommodates differences in weights in price indexes and the fact that due to the existence

²⁰Hermalin and Rose (1999) provide an exhaustive analysis of how risks present in the domestic capital markets tend to be exacerbated in the international context.

²¹The purpose of this very brief review is to understand the implications of the current state of research for the modeling of the real exchange rate. For a more complete review of this vast literature, see Dornbusch (1992), MacDonald (1995), Breuer (1994), Froot and Rogoff (1994), and Rogoff (1996).

of transport costs and other impediments of trade, such as tariffs, quotas, etc., the price of a commodity will not be the same in different locations. If the above factors stay constant over time, the *relative* version of PPP holds when an increase in the home price level relative to the price level abroad results in an equiproportionate depreciation of the home currency:

$$\hat{s}_t = \hat{p}_t - \hat{p}_t^* \quad (2.13)$$

where a hat denotes a percentage change.

2.3.1 Empirical Performance of Purchasing Power Parity

2.3.1.1 A quick look at diagrams

Let us start by looking at the performance of PPP in diagrams. Figure 2.1 plots actual percentage change in the Canada-U.S., U.K.-U.S., and Germany-U.S. spot exchange rates along with the percentage changes as predicted by the *relative* PPP in (2.13) above. The time-period covered is 1957:1-1999:1. It is immediately evident that actual exchange rate movements are much larger than the PPP-consistent exchange rates. Thus, most changes in the nominal exchange rate reflect changes in the real exchange rate. A second feature present in data is that exchange rate changes in the post- Bretton-Woods era are much larger than the changes during the Bretton-Woods era. A third feature is that quite often the actual rate moves in the opposite direction of the PPP-consistent rate for long periods in a row. This

Figure 2.1 Canada-U.S., U.K.-U.S., and Germany-U.S. Exchange Rates, 1957:1-1999:2

is especially evident in case of the Canada-U.S. dollar exchange rate.²² Given the volume of trade in goods (and also in assets) between the two countries and the geographical proximity, which should significantly reduce transactions and transportation costs, the behaviour of exchange rates has important implications for the PPP theory.

2.3.1.2 Tests of Purchasing Power Parity

In order to understand the empirical tests of the PPP theory, let us re-write equation (2.12) in the following regression equation form:

$$s_t = \alpha + \beta_1 p_t + \beta_2 p_t^* + u_t \quad (2.13)$$

where u_t is an error term. Prior to mid-1980s, tests of PPP were conducted using ordinary and generalized least squares estimation of (2.13). Absolute purchasing power parity would be confirmed if $\alpha = 0$, and the beta coefficients satisfied two conditions: (1) *symmetry* – the coefficients were of the same magnitude but opposite signs, and (2) *proportionality* – the coefficients were [1 -1]. A less strict version of the theory would permit α to be nonzero in order to accommodate transactions and transportation costs as well as differing goods and weights thereof in the price indexes of the domestic and foreign countries. With the advances in econometrics for nonstationary time series, researchers now increasingly realize that classical hypothesis testing of equation (2.13) is inappropriate in the presence of nonstationary regressors.

²²Notice the behavior of Canada-U.S. exchange rates between 1986-1990, and then between 1992-1995. Interestingly, the (absolute) lowest value of the ADF τ statistic reported later in Table 2.3 provides a statistical evidence supporting the above observation of the extent of PPP violations in the Canada-U.S. exchange rate.

The more recent tests of purchasing power parity involve a looser version of equation (2.13). Since mid-1980s, researchers have been seeking either a cointegration among the variables in (2.13), or the absence of a unit root in the real exchange rate. The focus has thus shifted to the search for “long-run” purchasing power parity. This concept of PPP does not require the real exchange rate to be constant; it admits fluctuations in the real exchange rate about some fixed mean. Tests of coefficient restrictions (symmetry and proportionality) are often absent in this search. However, as the 1980s drew to a close, PPP and exchange rate models that relied on it stood in near total discredit as an increasing number of economists found that their ability to explain movements in the real exchange rate was not significantly better than a simple random walk process. Economists have increasingly rejected the purchasing power parity hypothesis on this account. The logic is simple: if the real exchange rate follows a random walk process, PPP cannot hold even in the long-run since deviations from it are permanent in nature. The random walk characterization of real exchange rates fares well in simple empirical tests. Table 2.3 below presents ADF τ test on Canada-U.S., U.K.-U.S., Japan-U.S. and Germany-U.S. real exchange rates over two time periods, 1957:1 through 1999:1, and 1973:1 through 1999:1. Evidently, the hypothesis of a unit root is not rejected over both time periods by the Augmented Dickey-Fuller test. If the real exchange rates are indeed random walk processes as the evidence above shows, then there is no long-run equilibrium for real exchange rates. Monte Carlo simulations have shown, however, that the power of the Dickey-Fuller test to reject a false null hypothesis is very low – these

Table 2.3 ADF τ Unit Root TestsCanada-US, UK-US, Japan-U.S. and Germany-US Real Exchange Rates, 1957:1-1999:1¹

Country Pair	1957:1-1999:1	1973:1-1999:1
Canada-U.S.	-0.579	-0.764
U.K.-U.S.	-2.445	-2.645
Germany-U.S.	-2.186	-2.641
Japan-U.S.	-1.856	-2.443

Notes: ¹ For Germany, the sample period is 1957:1-1998:4. 5% MacKinnon critical value for rejection of the hypothesis of a unit root is -2.8791 for the period 1957:1-1999:1. The 5% critical value for the period 1973:1-1999:1 is -2.889.

tests do not have the power to distinguish between a unit root and near unit root process.²³

Thus unit root tests too often indicate that a series contains a unit root. Moreover, they have little power to distinguish between trend stationary and drifting processes.²⁴ In finite samples, any trend stationary process can be arbitrarily well approximated by a unit root process, and similarly a unit root process can be well approximated by a trend stationary process.

On the other hand, small departures from random walk behavior change the economics of the real exchange rate radically. To see this, let us characterize the real exchange rate process by the following *autoregressive process*:

$$e_{t+1} = \alpha_0 + \alpha_1 e_t + v_t \quad (2.15)$$

²³Lothian and Taylor (1997) use random simulations with artificial data with identical sample characteristics to the long-sample exchange rate data employed in their 1996 paper (mentioned in footnote 2) to show that the probability of rejecting the null hypothesis of a unit root when it is false is extremely low with even 50 years of data. The probability reaches an acceptable level over much longer spans.

²⁴See De Jong et al (1992) for relevant evidence.

This process is stationary if α_1 is less than unity, in which case real exchange rate is a *mean-reverting process* that adjusts to its long-run equilibrium value, $\alpha_0/(1 - \alpha_1)$, over time. The speed of adjustment depends on α_1 – the smaller the value of α_1 , the greater the adjustment speed.²⁵ Thus it has been recognized that in a world of sluggish price level adjustment, tests of whether changes in real exchange rates persist will fail to reject the random walk hypothesis unless applied to long data series.

Evidence from the studies of PPP published in the 1990s is considered favorable for the theory. Frankel (1990) points out that if purchasing power parity deviations damp sufficiently slowly, then it may require many decades of data for one to be able to reliably reject the existence of a unit root in real exchange rates. Using very long data series, researchers are now increasingly able to reject the random walk hypothesis. Breuer (1994) gives an average estimate of 70 years data needed to identify this mean-reversion. Thus, Frankel employs annual data for the dollar/pound exchange rate for the period 1869-1984, and rejects a unit root hypothesis. He finds a half-life for PPP deviations of 4.6 years. Abuaf and Jorion (1990) use 1901-1972 data for eight currencies and find strong evidence against a unit root in real exchange rates. They estimate a half-life for PPP deviations of 3.3 years. Lothian and Taylor (1996) from their examination of dollar-pound exchange rate data spanning over 1791-1990 and franc-pound exchange rate data for the period 1803-1990 estimate a half-life deviation of 4.7 years for the dollar-pound and 2.5 years for the franc-pound. Frankel and Rose (1995) examine a panel data set for 150 countries for the period 1948-1992 and estimate a half-life

²⁵When the half-life of the deviation is defined as the number of years required for the real exchange rate to return half-way to its equilibrium value, a coefficient estimate of $\alpha_1 = 0.96$ in a regression with annual data implies a half-life of 17 years. See Abuaf and Jorion (1990) and Frankel (1993) for these estimates.

deviation of about 4 years.

It, therefore, appears that a considerable volume of recent research identifies sizable mean-reverting components in real exchange rates. But the process of mean-reversion is quite slow. The question now is: do we take the evidence of the mean-reversion in the real exchange rate as supportive of the PPP theory and the exchange rate models that rely on it? Does this evidence point to the conclusion that the long-run equilibrium real exchange rate is fixed?

2.3.2 Implications for the Modeling of the Real Exchange Rate

There is little disagreement that purchasing power parity does not hold in the short- and medium-run. Evidence of mean reversion in real exchange rates over long horizons usefully puts to rest the highly implausible suggestion that the forces of international trade and competition are too weak to prevent real exchange rates from rising or falling indefinitely. However, this does not imply that real exchange rates have time-invariant means. Notice that the recent tests of PPP often test a loose version of the theory. The studies that confirm a stationary relationship between the exchange rate and domestic and foreign price series do not impose the symmetry and proportionality restrictions on the coefficients. Without these restrictions, these studies are not truly testing long-run purchasing power parity. What must be explained is why the adjustment of real exchange rates to equilibrium takes so long. These studies have failed to undermine the theoretical arguments of Balassa (1964), and Samuelson (1964) suggesting that traded- nontraded goods productivity differentials will affect the real exchange rate. Change in growth differentials between countries, shift in

technology and taste, change in similar structural factors will continue to contribute to the trend deviations from PPP.²⁶ The evidence we see in the recent tests of PPP is rather more supportive of the models that consider disturbances as predominantly real in origin and permanent in their impact. The high degree of persistence in the PPP deviations suggests that sticky price explanation of the behaviour of the real exchange rate is not holding.

2.4 Conclusions

The review of the theoretical and empirical literature presented in this chapter leads to two conclusions. Firstly, the risk premium explanation for the failure of uncovered interest rate parity is incomplete in that existing models of risk-premium ignore country-specific risk factors.²⁷ What is needed is a model that integrates both *country-* and *currency risks* associated with international investments. Secondly, the evidence of a painfully slow mean-reversion in the real exchange rate is inadequate to give PPP a pivotal role in the modeling of exchange rates. Pursuit of an equilibrium real exchange rate that emerges in the “ultra-long” run may be misguided if there are important real factors that are driving the equilibrium rate away from its PPP level for lengthy periods of time. We take account of this evidence in modeling open economy in chapters 4 and 5.

²⁶This may seem contradictory. To reconcile, given the statistical evidence, the PPP should be viewed to hold in the “ultra-long run” (the term is borrowed from Breuer, 1994) when all real shocks have jointly negated each other. See Breuer (1994) for this interesting but informal explanation.

²⁷The significance of the country risk premium will be discussed further in chapter 3.

Appendix 2

A Note on Notations

We list here for easy reference the notations that are used frequently in chapters 2 through 4. The notations used in chapter 5 have been provided there.

- S_t The nominal exchange rate, expressed as the domestic currency price of a unit of foreign currency.
- P_t The domestic price level at time t .
- $F_{t,T}$ The forward exchange rate at time t for delivery at time T , (again in units of domestic currency per unit of foreign currency), $T > t$.
- $i_{t,T}$ The continuously compounded nominal interest rate between time t and T on a nominally riskless domestic bond, denominated in domestic currency, maturing at date T .
- $\mathcal{E}_t$ The expectations operator with expectations formed based on all information available at date t .
- x The natural logarithm of a variable X .
- X^* The foreign counterpart of the variable X .
- E_t The real exchange rate defined as, $S_t P_t^* / P_t$, the ratio of two countries' price levels in a common currency.
- $\mathcal{E}_t \hat{x}_{t,T}$ The expected percentage change, $(\mathcal{E}_t x_T - x_t)$, in the variable X between dates t and T .
- $\hat{x}_{t,T}$ The actual percentage change, $(x_T - x_t)$, in the variable X between dates t and T .
- $\mathcal{E}_t r_{t,T}$ The *ex ante* real rate of interest, $(i_{t,T} - \mathcal{E}_t \hat{p}_{t,T})$, between dates t and T on a domestic asset.

$r_{t,T}$ The actual (*ex post*) real rate of interest between dates t and T on a domestic asset.

3 Real Interest Rate Parity and the Modeling of the Real Exchange Rate

3.1 Introduction

It is often suggested that perfect arbitrage in both the goods and capital markets will lead to real interest rate parity. The argument is that with the increasing integration of countries and gradual elimination of barriers to trade in goods and assets, *uncovered interest rate parity* and *purchasing power parity* – the building blocks for *real interest rate parity* – will hold in the long-run. The present chapter examines this possibility.

We examine real interest rate parity as a long-run equilibrium relationship rather than as a parity condition that holds continuously. The intuition is straightforward. In a world where countries experience different shocks to their goods and assets markets, real interest parity in the short run is a too-strong requirement. Such parity in the short run would require fast adjustment not only in assets markets among countries, but also in their goods markets. A more reasonable assumption is that in the short run, parity describes a tendency in the markets to react in the direction of equilibrium, implying that attaining equilibrium might actually never be satisfied. What can be reasonably hypothesized is that neither prices nor nominal interest rates between countries diverge substantially without evoking the adjustment forces that tend to restore equilibrium in the long-run. We use Johansen's full-information maximum-likelihood cointegration methodology to examine whether the existence of such adjustments is consistent with the data. Given the nonstationarity of real

interest rates, cointegration methodology is the appropriate and possibly the only methodology to examine the long-run behaviour of real interest rates.¹

This chapter is arranged as follows. The next section provides an understanding of the true meaning of the *real interest rate parity* condition. It will be shown that the assumption that is so often used in open economy modeling is restrictive and demanding. We then present in section 3.3 a simple accounting framework that shows that in the presence of both the *currency-* and *country-risk* premiums, the parity condition cannot hold even in the long run. In section 3.4, the hypothesis of real interest rate parity holding in the long run is empirically investigated. The evidence suggests that a statistically significant differential remains even in the long run.

3.2 What really is Real Interest Parity?

The meaning of real interest parity condition is not always made clear, although it is frequently used in open economy modeling. Parity can be seen in at least two ways. First, let $\mathcal{E}_t r_d = (i_{t,T} - \mathcal{E}_t \hat{p}_{t,T})$ be the *ex ante* real interest cost of borrowing by a domestic firm in the domestic market. The cost is $\mathcal{E}_t r_d^* = (i_{t,T}^* + \mathcal{E}_t \hat{s}_{t,T} - \mathcal{E}_t \hat{p}_{t,T})$ if the firm borrows in the foreign market. A foreign firm, on the other hand, faces the *ex ante* real cost $\mathcal{E}_t r_f^* = (i_{t,T}^* - \mathcal{E}_t \hat{p}_{t,T})$ when it borrows in its own market. With these definitions, the term *real interest rate parity* is sometimes taken to refer to the following condition:

¹Standard econometric techniques neglecting the time-series properties of the data is likely to invalidate the analysis, and may, in fact, find “spurious” correlations. We will see shortly that the medium- and long-term real interest rates appear to follow unit root processes. The advantage of the Johansen methodology is that it can be used even when some variables in a relationship do not have unit-roots. See Dickey and Rossana (1994: p. 347).

$$\mathcal{E}_t r_d = \mathcal{E}_t r_f^* \quad (3.1)$$

Marston (1995, 1997) observes that there is no sound theoretical rationale behind such parity, as there is no set of agents willing and able to take advantage of deviations from the above parity. This is especially true when the domestic and foreign countries produce and consume different baskets of goods. Equation (3.1) can, however, be shown to hold when uncovered interest parity and *ex ante* purchasing power parity conditions hold. To see this, let us decompose the differential between the two rates in (3.1) as follows:

$$\mathcal{E}_t r_d - \mathcal{E}_t r_f^* = (i_{t,T} - i_{t,T}^* - \mathcal{E}_t \hat{s}_t) - (\mathcal{E}_t \hat{p}_{t,T} - \mathcal{E}_t \hat{p}_{t,T}^* - \mathcal{E}_t \hat{s}_t) \quad (3.2)$$

The first term on the right-hand side of equation (3.2) above is the uncovered interest rate differential and the second term is the *ex ante* purchasing power parity differential. Therefore, real interest parity condition (3.1) holds not as the result of any arbitrage activity by the domestic and foreign firms involved but as the byproduct of two parity conditions simultaneously holding – uncovered interest rate parity and *ex ante* purchasing power parity.

Second, real interest rate parity is often used to imply the equality of the real interest cost of borrowing by a firm in the domestic and foreign capital markets:

$$\mathcal{E}_t r_d = \mathcal{E}_t r_d^* \quad (3.3)$$

It is easy to see that $(\mathcal{E}_t r_d - \mathcal{E}_t r_d^*) = (i_{t,T} - i_{t,T}^* - \mathcal{E}_t \hat{s}_t)$, the uncovered interest rate differential. Thus, the parity condition in (3.3) will hold as long as the uncovered interest rate parity condition holds. In this case, financial conditions alone determine the real interest rate

parity condition, and the relative prices of domestic and foreign goods do not matter.²

The real interest differential in (3.2) has important implications for modeling real exchange rates and open economy. Even if one assumes a very high degree of international capital market integration so that asset markets between countries adjust quickly to eliminate any uncovered interest differential, the second term on the right hand side of (3.2) vanishes only when *ex ante* PPP holds, implying that real exchange rates follow a random walk process. Thus, an open economy model that starts with an assumption of real interest parity as specified in (3.1) essentially imposes the implicit condition that the most important relative price variable in an open economy, the real exchange rate, does not follow any long-run equilibrium path.

3.3 Real Interest Differential and the Risk Premium: An Accounting Framework

Let us now use some definitions to rewrite (3.2) in a more useful way. We know that the exchange risk premium $RP_{t,T}$ is defined as:

$$RP_{t,T} \equiv (f_{t,T} - \mathcal{E}_t s_{t-T}) \quad (3.4)$$

while the covered interest differential is defined as the country premium, $CP_{t,T}$:

$$CP_{t,T} \equiv (i_{t,T} - i_{t,T}^*) + s_t - f_{t,T} \quad (3.5)$$

The above is called the country premium because it captures all barriers to integration of financial markets across national boundaries: transactions costs, information costs, tax laws

²These results are robust to the inclusion of domestic as well as imported goods prices in the price index.

that discriminate by country of residence, default risk, and the risk of future capital controls. Now substituting (3.4) and (3.5) into (3.2), and noting that the second term on the right hand side of (3.2) can be expressed as expected real depreciation, we obtain,

$$\mathcal{E}_t r_d - \mathcal{E}_t r_f^* = RP_{t,T} + CP_{t,T} + \mathcal{E}_t \hat{e}_{t,T} \quad (3.6)$$

Equation (3.6) states that the real interest differential is by definition equal to the sum of the expected real depreciation, and exchange and country risk premia. This accounting framework has been the source of a number of empirical models.³ We can rewrite it as:

$$e_t = \mathcal{E}_t e_{t+T} + \mathcal{E}_t (r_{t,T}^* - r_{t,T}) + RP_{t,T} + CP_{t,T} \quad (3.7)$$

The relationship is valid for any time horizon. Suppose T is specified as a horizon over which long-run equilibrium is reached. Then the first-differenced form of equation (3.7) implies that any observed change in the real exchange rate over a short period of time is the sum of any change in the long-term real interest differential, any revision in expectations about the long run real exchange rate, and any change in the exchange risk premium or the country premium over the time horizon. This is a relationship between endogenous variables, and not a fully specified model of exchange rate determination. It is an equation that holds by definition, even in cases when the market exhibits either irrationality or rational bubbles. This accounting framework, together with the stock of our knowledge about the behaviour of exchange rates empirically, provides perspectives on where it may be fruitful to

³Korajczyk (1985), Levine (1989, 1991), and Gokey (1994) used the framework to estimate exchange risk premium. Frankel (1993), and Frankel and MacArthur (1988) present detailed empirical analysis of the component parts of the identity.

concentrate attention in seeking systematic explanations of exchange rate variability.

It is now apparent from (3.6) that real interest parity will not hold, even in the long-run, unless the two risk premium terms vanish. With this background, we now explore the possibility of the real interest rate parity as a long-run equilibrium condition.

3.4 Real Interest Rate Parity in the Long-Run?

In a world of perfect capital flows real interest rate parity should hold not only for short-term interest rates but also for both the medium- and long-term real interest rates. All of the tests of parity previously mentioned, however, examine parity for short-term interest rates alone. It may be interesting to see if the medium-term and long-term rates show a different tendency that can support the real interest rate parity as a long-run equilibrium condition.

3.4.1 Parity in the medium- and long-term rates: An Empirical Investigation

In this section, we consider the parity of medium- and long-term real interest rates between five industrialized countries over last forty years.⁴ The countries in the group are Canada, U.S., U.K., Japan, and Germany, and the interest rates are the returns on three to five year government bonds for the medium-term, and the yields on ten to fifteen-year

⁴The sample used here covers the period 1957 Quarter 1 (1957:1) through 1999 Quarter 2 (1999:2) in all cases except for Japan and Germany. The sample period for Japan is 1967:4-1999:2. In case of the medium term real interest rate for Germany, the period covered is 1977:3-1999:2. The last four data points are, however, used up by way of computing real interest rates by subtracting *ex post* rate of inflation from the nominal rate of interest. The empirical analysis is done over the entire sample periods as well as over the shorter period 1980:1-1999:2. The shorter sample period can be characterized as the period of high capital mobility, and it is possible that the behaviour of real interest rates in this period is different from that in the earlier period of capital controls.

government bonds for the long-term.

The computation of *ex ante* real interest rates for the medium- and long-term bonds is difficult because of the difficulty in forecasting inflation for the terms of these bonds. An alternative to computing a measure of *ex ante* rates is to compute *ex post* rates. The *ex post* rate differs from the *ex ante* rate by the forecast error for the rate of inflation. If expectations are rational, however, the forecast error is a white noise process and, therefore, if parity holds for *ex ante* rates, it must also hold for *ex post* rates. But a difficulty still remains. The true measure of *ex post* real rate of interest should use the rate of inflation over the term of the relevant bond. If this is followed, with only forty years of data, we have only eight to thirteen non-overlapping observations for the medium-term interest rate, and three or four observations for the long-term real interest rate.

We follow both Meese and Rogoff (1988) and Baxter (1994) by generating the real interest rate series, by subtracting realized annual inflation from the nominal interest rate (expressed as percent per annum in the IFS database). Given our objective of searching for long-run parity conditions through co-integrating relationships between real interest rates, the above method of generating real interest rates should not affect our empirical results in that we use the same method for all countries. In calculating inflation, we use CPI data rather than Wholesale or Producer Price Index data. This choice is appropriate given our objective in the model in chapter 5; i.e., we are interested in the intertemporal consumption-investment decision of a representative consumer-investor.

3.4.2 The Formulation of Hypotheses

Let us consider a bivariate real interest rate system $r_t = [r_{(i)t} \quad r_{(j)t}]'$, where i and j refer to countries i and j respectively.⁵ If real interest rates follow independent univariate time series processes with a unit root, then random shocks would change these rates permanently so that they would drift apart over time. There would be no tendency for real interest rates between countries to maintain any parity relationship even in the long-run.⁶ If, however, real interest rates are cointegrated, even two non-stationary processes will maintain a long-run relationship. Under the strong real interest rate parity hypothesis, $r_{(i)t} = r_{(j)t}$, for all country pairs, i and j , we can write the long-run equilibrium condition as the following hypothesis:

$$H1: \beta'_0 r_t = 0 \quad (3.8)$$

where $\beta'_0 = [1 \quad -1]$. In a time series framework, we expect that real interest rates must be cointegrated with the cointegrating vector β'_0 . Notice the strong requirement for the parity condition. The parity requires satisfaction of two conditions: (1) *symmetry* – real interest rate coefficients of the same magnitude but opposite sign – and, (2) *proportionality* – real interest rate coefficients restricted to be $[1 \quad -1]$. A finding in favor of cointegration without satisfying these two conditions merely states that the two interest rates follow a stable long-run relationship. In this case, interest differential does not wander aimlessly. However, the evidence cannot be considered to be supportive of the strong form of real interest rate parity.

⁵In the empirical analysis that follows, we use the subscript 1 for Canada, 2 for the U.S., 3 for the U.K., 4 for Germany, and 5 for Japan.

⁶It is the fundamental property of variables that are integrated of order 1 that they tend to diverge from each other as $t \rightarrow \infty$, because their unconditional variances are proportional to t .

Let us now postulate a more general long-run equilibrium condition as:

$$H2: \beta'_0 r_t = c. \quad (3.9)$$

We want to see if the relationship $(r_{(t)} - c - \alpha r_{(t)})$ is a stationary process. Thus we start by searching for a cointegrating vector $\beta'_1 = [1 - c - \alpha]$ in the cointegration space, if one exists. We can then examine if the space contains the restricted vector $\beta'_0 = [1 - 1]$, which is only obtained under the strict real interest rate parity condition. This is equivalent of a restriction $c = 0$ and $\alpha = 1$ on the vector β'_1 .

Now suppose we find a cointegrating vector $\beta'_1 = [1 - c - 1]$. Evidently, the vector satisfies both the symmetry and proportionality conditions for the strong form of real interest rate parity with $c \neq 0$. The result clearly indicates that the two real interest rates bear a long-run parity relationship in that their differential fluctuates about some fixed mean. Given the accounting framework in section 3.3, at least a part of this mean differential can be attributed to risk premium. In any case, the finding is not consistent with the parity that justifies modeling open economy using a one-asset-world framework.

3.4.3 The Johansen Estimation Approach

According to the Johansen approach, we start by assuming that our two dimensional vector r_t follows a vector autoregression of order k , $VAR(k)$, with Gaussian errors:

$$r_t = \mu_t + \pi_1 r_{t-1} + \dots + \pi_k r_{t-k} + \varepsilon_t \quad (3.10)$$

where μ_t is a 2×1 vector of constants, π_t is a two dimensional square matrix of parameters,

and ε_t are i.i.d. $N_2(0, \Psi)$. The plots of the long-term real interest rates in Figure 3A.1 in Appendix 3A do not indicate the presence of any deterministic trend in the data. We, therefore, do not allow for a linear trend in the non-stationary part of the process. Consistent with the hypothesis *H2*, however, we allow for a constant in the cointegrating relationship. This, combined with the assumption of possible cointegration, enables us to write the model in (3.10) in the following error correction form:

$$\Delta r_t = \Gamma_1 \Delta r_{t-1} + \dots + \Gamma_k \Delta r_{t-k+1} + \alpha \beta' r_{t-k}^* + \varepsilon_t \quad (3.11)$$

where $r_{t-k}^* = \begin{bmatrix} r_{(i)t-k} & 1 & r_{(j)t-k} \end{bmatrix}'$, β' is the $g \times 3$ matrix of the cointegrating vectors, and g is the number of possible cointegrating vectors. α is the $3 \times g$ matrix of adjustment or error correction coefficients. Estimation procedure relating to α and β' , as well as the test statistics used below is presented in Appendix 3B in some detail.

3.4.4 The Empirical Analysis

3.4.4.1 Unit Root Tests

The logical place to start the empirical analysis is the examination of the time series properties of the real interest rates. We use the augmented Dickey-Fuller τ (ADF τ) test and an F-type statistic Φ_2 to determine the unit roots in the real interest rates in levels as well as in first differences.⁷ The Φ_2 statistic tests the joint hypothesis of a unit root with no constant or deterministic trend. A brief discussion of the testing procedure as well as the reason for putting so much emphasis on the non-inclusion of the deterministic trend term is given in the

⁷The F-type statistic Φ_2 along with a few other similar tests have been suggested by Dickey and Fuller (1981).

Appendix 3A. The key point is that the inclusion of additional regressors that are not present in the data-generating process not only reduces the degrees of freedom but also the power of the test. For analytical completeness, however, we repeat the unit root tests including both intercept and trend terms in the unit root regressions.⁸ The test results are reported in Tables 3A.1 through 3A.3 in the Appendix 3A. The results indicate that the real interest rates of all countries in the sample, except for the medium-term rate of Germany and both the long-term as well as the medium-term rate of the U.S., have a unit root at a 5% level of significance over the entire sample periods, as the cases may be. The null hypothesis of a unit root in the U.S. medium-term and long-term real interest rates as well as the medium-term real interest rate of Germany can be rejected a 5% level of significance by the ADF τ test, although not by the Φ_2 test.⁹ When a deterministic trend term is included in the unit root regression, real interest rates of all countries in the sample seem to have a unit root by the ADF τ test (see Table 3A.1 in Appendix 3A). However, there is no *a priori* reason to include a trend term in the unit root regression of the real rate of interest.¹⁰ Both the ADF τ test as well as the Φ_2 test on the first-differences of the real interest rates indicate that the presence of more than one unit root in the real interest rates can be rejected at a 5% level of significance in all

⁸An intercept term is included in all regressions because the initial values of the real interest rates are positive in all cases. See Dickey and Rossana (1994, p.338-339) for an analysis of the importance of the intercept term in unit root regressions.

⁹As already mentioned, the sample covers the period 1957:1-1998:2 in all cases except for Japan, in which case the sample period is 1966:4-1998:2.

¹⁰We plot the long-term real interest rates of the countries in our sample in Figure 3A.1 in Appendix 3A. The plots do not indicate the presence of any deterministic trend in any of the five real interest rates.

cases.¹¹

3.4.4.2 VAR order Selection

Our next task is the determination of the order of the VAR in (3.10). A carefully determined lag length k is very important for the Johansen methodology as this has implications for not only the number of estimated cointegrating vectors and their coefficient estimates, but also for the tests of hypotheses. In particular, the lag length k must be chosen so that the estimated residuals from the unrestricted VAR have no serial correlation, no conditional heteroscedasticity, and thus do not deviate very far from Gaussian white noise. Johansen (1995), however, indicates that although the methods derived are based upon the Gaussian likelihood, the asymptotic properties of the methods only depend on the i.i.d. assumption of the errors. Thus although ARCH effects have significant implications, the normality assumption is not so serious for the conclusions.

Various lag selection criteria available are not often useful when the objective is to whiten the error. Gonzalo (1990) finds that the loss of efficiency in the Johansen procedure from choosing too long a lag is small, but the method performs poorly when too short a lag length is used.¹² In selecting lag length, we, therefore, emphasize selection of adequate lags so as to eliminate autocorrelation and ARCH effects without, of course, losing too many degrees of freedom. Given that we have a model with a two-dimensional VAR, a few additional lags are unlikely to cause any significant loss in terms of degrees of freedom. We

¹¹ As a matter of fact, the null hypothesis of the presence of more than one unit root in all the real interest rates, except for the medium-term real interest rate of Germany, can be rejected at even a 1% level of significance. The absence of more than one unit root is important for the application of the Johansen methodology. As we already mentioned, unlike the Engle-Granger method, the Johansen method can be used even when some variables in a cointegration equation are stationary.

¹² Banerjee et al (1993) p.286.

use the likelihood ratio test statistic recommended by Sims (1980) supplemented by a multivariate Akaike Information Criteria (AIC) for a first guess about the optimal lag length.¹³ Based on this approach, we select a VAR of order between five and eight. The univariate diagnostic statistics from the regressions using the above selected lag lengths are reported in Tables 3C.1-3C.4 in Appendix 3C. We use Jarque-Bera (JB) test for normality of residuals, and Ljung-Box Q statistic and Breusch-Godfrey Lagrange Multiplier tests for the autocorrelation of the residuals. The JB test statistic has an asymptotic χ^2 distribution with 2 degrees of freedom under the null hypothesis of normality. The Ljung-Box $Q(l)$ tests the joint hypothesis that all autocorrelation coefficients of up to order l are simultaneously equal to zero. Under the null hypothesis the statistic is distributed as χ^2 with degrees of freedom equal to the maximum order of the autocorrelation coefficient selected. We use the ARCH test suggested by Engle (1982) to test for both first- and second-order ARCH effects in the residuals. These statistics are distributed as χ^2 with degrees of freedom equal to the highest order of ARCH being tested for. Finally, the Breusch-Godfrey LM test statistic is also distributed as χ^2 with degrees of freedom equal to the highest order of autocorrelation being tested for. We use this statistic to test for autocorrelation of up to order four.

The results reported in Tables 3C.1-3C.4 indicate that our approach in eliminating ARCH effects and autocorrelation has been considerably successful in the period of high capital mobility (1980:1-1998:2), but not in the entire period (1957:1-1998:2) including the earlier

¹³Sims (1980) suggests using the statistic, $(T - c)[\ln|\Sigma_l| - \ln|\Sigma_{l+k}|]$ which is distributed as a χ^2 with degrees of freedom equal to the number of restrictions under the null hypothesis of l lags (as opposed to $(l+k)$ lags), in order to choose between l and $(l+k)$ lags. Here T is the number of observations, c is the number of parameters in the unrestricted model with $(l+k)$ lags, and $\ln|\Sigma_l|$ is the natural logarithm of the variance-covariance matrix of the residuals from the model with l lags.

period of capital controls. In particular, the only problems that remain in the long-term real interest rate equations over the shorter sample period are deviations from normality of residuals in the cases where Germany is the partner country in the vector autoregressions. Moderate second-order ARCH effects (at a 10% level of significance) are also evident in the Canada-U.S. and Canada-U.K. equations. Although the LM test reports the presence of autocorrelation at a 10% level of significance in the cases of the Canada-U.S., U.S.-U.K., U.K.-Germany, and Germany-Japan equations, both the fourth-order as well as the sixteenth-order Q statistics do not indicate any autocorrelation. In case of medium-term real interest rates over the shorter sample period (1980:1-1998:2), significant second-order ARCH effect remains in all the equations. Deviations from normality of residuals in the Canada-U.S. and Canada-U.K. equations also remain. In the cases of the Canada-U.K., and U.S.-U.K. equations, the LM test statistic indicates presence of autocorrelation at 1% and 10% levels of significance, respectively. But, as before, both fourth-order as well as sixteenth-order Q statistics do not indicate any autocorrelation. Over the entire-sample periods, as the cases may be, the deviation from normality of the residuals is a general problem in virtually all equations for both medium-term as well as long-term real interest rates. Except for the residuals from the U.S.-Germany, U.K.-Japan, and Germany-Japan equations for long-term real interest rates and the Canada-U.K. and Canada-Germany equations for medium-term real interest rates, the residuals from all other equations indicate some violations of the assumptions underlying the model.

In cases where ARCH, JB, and LM or Q statistics are still significant, it is not possible to whiten these residuals any further no matter how long or how short a lag length is selected.

In fact, this is an indirect test of the degree of capital mobility between the countries in our sample. When the lags of the real interest rate of a partner country cannot eliminate the ARCH and autocorrelation of the residuals, this must imply that the country's real interest rate does not have much of any parity relationship with that of the partner country. As we have already mentioned, the behaviour of the residuals is much better over the shorter sample period (1980:1-1998:2) of high capital mobility. What remains to be seen is whether this high capital mobility has been high enough to lead to long run real interest parity in any form. This is done in the next section.

3.4.4.3 Cointegration results

We start our cointegration analysis by applying unit root tests on both the medium-term and the long-term real interest differentials. Although such an exercise cannot determine the structure of the relationship, it can be useful as a first approximation. In Tables 3D.1-3D.2 in the appendix 3D, we report the unit root results for the two types of interest rates over the entire sample periods. ADF τ test indicates cointegration of Canada-U.S., Canada-U.K., and U.S.-U.K. long-term real interest rates at a 5% level of significance. Canada-U.S. and Canada-U.K. medium-term real interest rates also appear to be cointegrated at a 5% level of significance over 1957:1-1998:2. Now let us see how these results compare with those from the Johansen cointegration tests. Tables 3.4.1-3.4.4 report the Johansen cointegration results.

Long-term Real Interest Rates over the Entire Sample Periods

Formal tests indicate that Canada-U.K., Canada-Germany., U.S.-Germany, and U.K.-Germany long-term real interest rates are cointegrated at a 5% level of significance by both

Table 3.4.1 Johansen's Cointegration Tests

Long-Term (10 years and above) Real Interest Rates, 1957:1-1998:2'

Interest Rates	g	η_g	ζ_g	Estimated Vector (β_1)	χ^2 on [1 0 -1]	χ^2 on [1 -c -1]
Canada-U.S.	0	19.49*	15.00*	[1 -0.17 -1.28]	7.73**	2.54
	1	4.48	4.48			
Canada-U.K.	0	29.66***	25.46***	[1 -1.19 -1.00]	11.92***	0
	1	4.20	4.20			
Canada-Germany	0	22.00**	17.93**	[1 10.55 -3.52]	12.03***	11.96***
	1	4.06	4.06			
Canada-Japan	0	16.07	12.61	[1 -2.10 -1.12]	6.94**	0.18
	1	3.45	3.45			
U.S.-U.K.	0	21.17**	15.39*	[1 -0.51 -0.91]	0.66	0.22
	1	5.77				
U.S.-Germany	0	25.27***	18.64**	[1 9.78 -3.09]	13.09***	10.46***
	1	6.63	6.63			
U.S.-Japan	0	14.04	8.65	[1 -1.22 -1.04]		
	1	5.39	5.39			
U.K.-Germany	0	26.54***	19.94**	[1 10.24 -3.14]	12.66***	9.22***
	1	6.59	6.59			
U.K.-Japan	0	16.84	11.38	[1 -1.13 -0.79]	2.45	0.82
	1	5.45	5.45			
Germany-Japan	0	13.28	8.97	[1 -3.60 -0.22]		
	1	4.29	4.29			

Notes: ' The time period for Japan is 1966:4-1998:2.

η_g and ζ_g are, respectively, trace and maximal eigenvalue statistics; g denotes the number of cointegrating vectors under the null-hypothesis. See the notes under Table 3.4.3 for the critical values and other details.

Table 3.4.2 Johansen's Cointegration Tests
Long-Term (10 years and above) Real Interest Rates, 1980:1-1998:2

Interest Rates	g	η_g	ζ_g	Estimated Vector (β_1)	χ^2 on [1 0 -1]	χ^2 on [1 -c -1]
Canada-U.S.	0	18.26*	12.99	[1 -8.27 0.42]	11.63***	9.41***
	1	5.27	5.27			
Canada-U.K.	0	22.90**	18.33**	[1 0.64 -1.42]	17.73***	4.16**
	1	4.57	4.57			
Canada-Germany	0	14.70	8.36	[1 6.48 -2.67]		
	1	6.33	6.33			
Canada-Japan	0	13.08	10.31	[1 -6.56 0.11]		
	1	2.77	2.77			
U.S.-U.K.	0	19.84*	15.28*	[1 14.2 -3.98]	13.05***	12.97***
	1	4.55	4.55			
U.S.-Germany	0	21.71**	13.87*	[1 8.20 -2.78]	7.97**	7.53***
	1	7.83*	7.83			
U.S.-Japan	0	11.16	9.28	[1 -3.86 -0.34]		
	1	1.88	1.88			
U.K.-Germany	0	25.20***	17.39**	[1 -2.18 -0.55]	1.245	1.204
	1	7.80*	7.80			
U.K.-Japan	0	12.19	11.14	[1 -4.65 -0.04]		
	1	1.05	1.05			
Germany-Japan	0	11.08	8.35	[1 -4.45 -0.04]		
	1	2.73	2.73			

Notes: η_g and ζ_g are, respectively, trace and maximal eigenvalue statistics; g denotes the number of cointegrating vectors under the null-hypothesis. See the notes under Table 3.4.3 for critical values and other details.

the trace and maximal eigenvalue tests.¹⁴ At a 10% level, Canada-U.S. rates also become cointegrated over the entire sample period by both tests. U.S.-U.K. long-term real interest rates are cointegrated at a 5% level of significance by the trace test and at a 10% level of significance by the maximal eigenvalue test. At a 20% level of significance, Canada-Japan real interest rates become cointegrated by both tests, and U.K.-Japan rates become cointegrated by the trace test. Thus, if we allow a 20% probability in the critical right-hand tail of the probability distributions of the above-mentioned statistics, all real interest rate pairs, except the U.S.-Japan and Germany-Japan pairs appear to be cointegrated. Two features can be clearly noted over this sample period for long-term rates. Firstly, estimated cointegrating vectors indicate that the signs of the coefficients are consistent with some kind of parity relationships between the real interest rates. Second, all estimated vectors contain a non-zero intercept term. The intercept term is often quite large. But the χ^2 test of restrictions in all cases indicate that the strong form of the parity restriction $[1 \ 0 \ -1]$ is rejected in all but two cases-- the U.S.-U.K. and U.K.-Japan cointegrating equations. In all but three cases -- Canada-US, Canada-U.K., and Canada-Japan -- the restriction $[1 \ c \ -1]$ on the cointegrating vector is also rejected. The key question now is how do we interpret the results when either both restrictions are rejected or both are not rejected. Notice that in cases where both restrictions are rejected, the estimated slope coefficient (in absolute value) is considerably greater than 1. Given the sizes of the non-zero intercept and the slope coefficient, it is easy to see why the two restrictions are rejected. In cases where both restrictions are not rejected, the estimated slope coefficients are not far away from unity,

¹⁴The trace test used here includes a finite-sample correction mentioned in Gregory (1994).

while the intercepts are considerably greater than zero. Evidently, our χ^2 statistic in this case is not powerful enough to distinguish between the two restrictions. However, whatever evidence we see in the results is not consistent with any strong form of real interest rate parity. The evidence is rather more consistent with a long-run relationship that includes a mean differential between the real interest rates.¹⁵

Long-term Real Interest Rates over the period 1980:1-1998:2

Over the shorter sample period, when the behavior of the residuals is much better, Canada-U.K. and U.K.-Germany real interest rates are cointegrated at a 5% level of significance by both the trace and eigenvalue tests. U.S.-Germany rates are cointegrated by the trace test at a 5% level of significance, and by the eigenvalue test at a 10% level of significance. Notice that the trace test indicates the presence of a second cointegrating vector in both the U.S.-Germany and U.K.-Germany rates at a 10% level of significance. The Canada-U.S. and U.S.-U.K. real interest rates appear to be cointegrated at a 10% level of significance – the former by the trace test only and the latter by both tests. Notice the sign of the slope coefficient in the Canada-U.S. equation. The positive sign of the coefficient is obviously not consistent with a behaviour of the real interest rates that shows convergence towards a parity relationship following some disturbance. The estimated cointegrating vectors in all other cases (where there is evidence of some cointegration) indicate that both the symmetry and proportionality conditions for a strong form of parity are violated, and both types of restrictions on the estimated cointegrating vectors are rejected at a 5% level of

¹⁵It may be useful to remind the reader about the behaviour of the residuals over the long sample period. Given the behaviour of the residuals, the validity of our tests can be questioned. However, the general pattern that emerges in the results is worth noting. The data indicate some kind of long-run relationship between the real interest rates in the sample with some mean differential.

significance in all but one cases. Both restrictions are not rejected in the case of U.K.-Germany. Notice, as before, that all the estimated vectors often contain a large non-zero constant term. Moreover, the slope coefficient is also often considerably different from unity.

Medium-term Real Interest Rates

Medium-term real interest rate cointegration results are reported in Tables 3.4.3 and 3.4.4. The results indicate that, except for the Canada-U.K. real interest rates over the shorter sample period (1980:1-1998:2), all interest rate pairs are cointegrated at a 20% level of significance. The Canada-U.K. pair is cointegrated at a 1% level of significance by both the trace and maximal eigenvalue tests over the period 1957:1-1998:2. U.S.-U.K. rates are cointegrated over the same period by the trace test at about 10% level of significance. U.K.-Germany medium-term rates are cointegrated by both the tests over the sample period 1977:3-1998:2 at a 5% level of significance. Notice, however, in Table 3.4.3 that even though the evidence in favor of a cointegration is strong, the sign of the slope coefficient in the estimated cointegrating vector for the U.K.-Germany pair does not indicate any convergent parity relationship. Given the large size of the estimated intercept and the small size of the slope coefficient (far away from the hypothesized unity), it is not surprising that both restrictions on the estimated vectors are violated. As a matter of fact, the two restrictions are violated for all medium-term real interest rate pairs over the longer as well as the shorter sample periods.

Over the shorter sample period (1980:1-1998:2), Canada-U.S. and U.S.-U.K. rates appear to be cointegrated at a 10% level of significance. The positive sign of the slope coefficient

Table 3.4.3 Johansen's Cointegration TestsMedium-Term (3-5 Years) Real Interest Rates, 1957:1-1998:2¹

Interest Rates	g	η_g	ζ_g	Estimated Vector (β_1)	χ^2 on [1 0 -1]	χ^2 on [1 -c -1]
Canada-U.S.	0	15.97	13.07	[1 0.08 -1.34]	5.42*	2.74*
	1	2.90	2.90			
Canada-U.K.	0	25.76***	22.47***	[1 -1.57 -0.77]	10.76***	4.35**
	1	3.28	3.28			
Canada-Germany	0	17.50	11.03	[1 22.10 -3.30]	13.75***	5.63**
	1	6.47	6.47			
U.S.-U.K.	0	19.51*	14.76	[1 -1.17 -0.62]	4.68*	4.24**
	1	4.75	4.74			
U.S.-Germany	0	16.02	13.38	[1 -26.16 2.60]	16.61***	13.32***
	1	2.63	2.63			
U.K.-Germany	0	22.98**	16.7**	[1 -5.93 0.12]	20.60***	5.77**
	1	6.29	6.29			

Notes: ¹ The time period for Germany is 1977:3-1998:2.

η_g and ζ_g are, respectively, trace and maximal eigenvalue statistics; g denotes the number of cointegrating vectors under the null-hypothesis. The trace statistic tests for the null hypothesis of g or fewer cointegrating vectors against the alternative of more than g vectors. The maximal eigenvalue statistic is a test of g cointegrating vectors against the alternative of $(g+1)$ cointegrating vectors. See Appendix 3B for the test statistics and their derivations. 1%, 5%, 10%, and 20% critical values of the trace test statistic under the null hypothesis of zero cointegrating vector are, respectively, 24.60, 19.96, 17.85, and 15.25. Corresponding critical values of the maximal eigenvalue statistic are, respectively, 20.20, 15.67, 13.75, and 11.54 under the null hypothesis of zero cointegrating vector against the alternative of 1 cointegrating vector. 1%, 5%, 10%, and 20% critical values of the trace test statistic under the null hypothesis of 1 cointegrating vector are, respectively, 12.97, 9.24, 7.52, and 5.91. (***), (**) and (*) denote statistical significance at 1%, 5%, and 10% levels, respectively. Critical values are taken from Osterwald-Lenum (1992) Table 1*.

Table 3.4.4 Johansen's Cointegration Tests

Medium-Term (3-5 Years) Real Interest Rates, 1980:1-1998:2

Interest Rates	g	η_g	ζ_g	Estimated Vector (β_1)	χ^2 on [1 0 -1]	χ^2 on [1 -c -1]
Canada-U.S.	0	18.85*	13.95*	[1 -6.33 0.16]	12.78***	11.19***
	1	4.89	4.89			
Canada-U.K.	0	14.08	9.54	[1 -138.9 26.15]	6.84**	6.16**
	1	4.54	4.54			
U.S.-U.K.	0	17.71	14.17*	[1 15.76 -4.07]	14.35***	13.36***
	1	3.54	3.54			

Notes: See the notes under Table 3.4.1.

in the Canada-U.S. equation, however, indicates that while a long-run relationship between the two real interest rates exists, the relationship is not consistent with the strong form of parity. The signs of the elements in the estimated U.S.-U.K. cointegration vector are consistent with a convergent parity relationship. But, once again, the symmetry and proportionality conditions for a strong form of real interest rate parity are not satisfied. As already mentioned, both restrictions on the estimated cointegrating vectors are also rejected.

3.5 Summary and Conclusions

This chapter started by providing a clear understanding of the meaning of *real interest rate parity*. It then presented an accounting framework to argue that real interest rates cannot equalize even in the long-run in the presence of *country-* and *currency-premium*. We then examined empirically whether there is any tendency for real interest rates across

countries to converge over time. We used Johansen's cointegration methodology to examine data spanning over four decades from five industrialized countries to examine whether the data supported the *symmetry* and *proportionality* conditions that must be obtained for real interest rate parity. In general, we found little evidence in the data to support the above-mentioned conditions, although we found that most of the real interest rate pairs in our sample followed cointegrating relationships over the sample periods 1957:1-1998:2 and 1980:1-1998:2.¹⁶ We also examined to see whether the data supported a parity relationship such that the differential between two real interest rates fluctuates about some fixed mean. We found some evidence (in case of the Canada-U.S., Canada-U.K., and Canada-Japan long-term real interest rates over the period 1957:1-1998:2) supporting such a parity relationship. But in most cases, the data did not support any parity with or without a mean-differential. We, however, found that in most cases estimated cointegrating vectors contained a nonzero intercept as well as a slope coefficient that is significantly different from unity. Thus, a linear combination of two interest rates in the majority of the cases is found to be stationary. Obviously, this is a statistical relationship that has very little bearing on the strong form of real interest rate parity that we have in mind. We can, however, conclude that whatever evidence was uncovered using Johansen's maximum likelihood-based cointegration tests is consistent with the hypothesis that even in the long run real interest rates between the five industrialized countries in the sample do not become equal. The evidence is rather consistent

¹⁶The sample period for Japan is 1966:4-1998:2. For Germany, in case of medium-term real interest rate, the period covered is 1977:3-1998:2.

with a parity relationship that may include a risk premium.¹⁷ This finding is then our motivation for modeling the differential as risk premium in an open economy in chapter 4.

¹⁷Marston (1997) develops joint tests of three parity conditions: uncovered interest parity, purchasing power parity, and real interest parity. These tests find a variety of cases where deviations from some or all of the parity conditions can be systematically related to variables currently known to investors and borrowers. One can take this evidence as yet another justification to model the real interest rate differential as risk premium.

Appendix 3A

Testing for Unit Roots in Real Interest Rates¹⁸

In testing for unit roots in the real interest rate of the i th country, $i=1,...,5$, we consider the “augmented Dickey-Fuller” regression equation:

$$\Delta r_{(i)t} = \alpha_0 + \alpha_1 r_{(i)t-1} + \alpha_2 t + \sum_{j=1}^p \gamma_j \Delta r_{(i)t-j} + \varepsilon_t \quad (3A.1)$$

where t is a trend term and the error term ε_t is assumed to be Gaussian white noise. The number of lagged terms, p , is chosen to ensure that the residuals satisfy the underlying model assumption. The augmented Dickey-Fuller (ADF) τ statistic tests the null hypothesis that $\alpha_1 = 0$ against the one-sided alternative that $\alpha_1 < 0$. The probability values (p -values) reported in the text are asymptotic and based on MacKinnon (1994). An alternative to the inclusion of p lagged terms is to use a non-parametric correction of the test statistic as suggested by Phillips and Perron (1988) so that serial correlation does not affect the asymptotic distribution of the test statistic. Campbell and Perron (1991, p.155-156) report several simulation studies that show that Phillips-Perron tests have serious size distortions in finite samples so that it is potentially dangerous to rely on these statistics. Davidson and Mackinnon (1993, p.713) also discuss this potential danger. We, therefore, depend mostly on the generally preferable ADF tests (particularly the ADF τ test).

¹⁸See section 3.4.1 for calculation of real interest rates.

In testing for unit roots, we start with the least restrictive of the plausible models with intercept and trend terms as in (3A.1). The ADF τ test results are reported in Table 3A.1 below. The null hypothesis of a unit root in the levels of both types of real interest rates cannot be rejected at any reasonable level of significance.

Table 3A.1 ADF τ Tests on Real Interest Rates: 1957:1-1998:2'

Real Interest Rate	Canada	US	UK	Germany	Japan
Long-Term	-2.4265	-3.2547	-2.2583	-3.0103	-1.9462
Medium-Term	-2.3195	-3.3503	-2.1199	-2.8167	

Notes: 'The time period for Japan is 1966:4-1998:2. In case of the medium-term real interest rate for Germany, the time period is 1977:3-1998:2. 1% and 5% critical values for the ADF τ statistic are, respectively, -3.96 and -3.41. The ADF τ unit root regression includes both a constant and a trend.

We have already mentioned that unit root tests have low power to reject a false null hypothesis of a unit root, and that these tests often fail to distinguish between trend stationary and drifting processes. Campbell and Perron (1991) report that the inclusion of additional regressors that are not present in the data-generating process not only reduces the degrees of freedom but also the power of the test. Figure 3A.1 below plots long-term real interest rates for Canada, U.S., U.K., Japan, and Germany over last forty years (thirty years in case of Japan). It is hard to see any deterministic trend in these plots. It is, therefore, possible that our tests reported in Table 3A.1 have low powers.

Tables 3A.2-3A.3 report unit root test results from regressions that do not contain a trend

Figure 3A.1 Long-term Real Interest Rates, 1957:1-1998:2
(For Japan, the period is 1966:4-1998:2)

Table 3A.2
Unit Root Tests of Long-Term Real Interest Rates, 1957:1-1998:2^a

Interest Rates		ADF τ Statistic	Φ_2
Canada	Level	-2.1274	1.9733
	First Difference	-4.2001	5.8586
US	Level	-3.0052	3.5314
	First Difference	-4.1229	5.6430
UK	Level	-1.8441	1.7805
	First Difference	-5.1845	8.9488
Japan	Level	-1.7927	1.3208
	First Difference	-3.8622	4.9612
Germany	Level	-2.7629	3.0418
	First Difference	-4.1153	5.6345

Notes: ^a The time period for Japan is 1966:4-1998:2.

1% and 5% critical values for the ADF τ statistic are, respectively, -3.43 and -2.86. The corresponding critical values for the Φ_2 statistic are, respectively, 6.09 and 4.68. Φ_2 is an F-type statistic that tests the joint null hypothesis of unit root with no trend or constant. The ADF τ unit root regression includes a constant, but no trend.

term. We also report an F-type statistic Φ_2 suggested by Dickey and Fuller (1981) that tests the joint null hypothesis of a unit root with no trend or constant. Evidently, when a trend term is not included in the unit-root regression, the null hypothesis of unit roots in levels of real interest rates cannot be rejected at a 5% level of significance for any of the real interest rates in our sample using the Φ_2 test. However, using the ADF τ test, the null hypothesis can be rejected for German medium-term real interest rate and the U.S. medium-term as well as long-term real interest rate. ADF τ test on first differences of real interest rates indicate that

Table 3A.3

Unit Root Tests of Medium-term Real Interest Rates, 1957:1-1998:2'

Interest Rates		ADF τ Statistic	Φ_2
Canada	Level	-2.1028	1.8068
	First Difference	-4.5433	6.8438
US	Level	-3.2167	3.7519
	First Difference	-4.9822	8.2227
UK	Level	-1.7746	1.5610
	First Difference	-4.6281	7.1390
Germany	Level	-3.0117	3.2720
	First Difference	-3.1062	-3.5022

Notes: 'The time period for Germany is 1977:3-1998:2.

1% and 5% critical values for the ADF τ statistic are, respectively, -3.43 and -2.86. The corresponding critical values for the Φ_2 statistic are, respectively, 6.09 and 4.68. Φ_2 is an F-type statistic that tests the joint null hypothesis of unit root with no trend or constant. The ADF τ unit root regression includes a constant, but no trend.

none of the real interest rates have more than one unit root. This result is used in formulating our Johansen cointegration model.

Appendix 3B

Johansen's Maximum Likelihood Estimation of Co-integrating Vectors in Systems of Equations

The purpose of this appendix is to provide a reasonably complete presentation of the Johansen methodology and the test statistics used in section 3.4. The brief presentation here is intended to help understand this relatively new methodology as we have applied for our purpose. A thorough treatment can be found in Johansen (1995).

Let a p -dimensional vector z follow a vector autoregression of order k , VAR(k):

$$z_t = \mu + \pi_1 z_{t-1} + \dots + \pi_k z_{t-k} + \varepsilon_t \quad (3B.1)$$

$$t = 1..T$$

where ε_t are iid as $N_p(0, \Psi)$ and μ is a constant term. One can add centered seasonal dummies, deterministic time trends, and other exogenous conditioning variables to the system without loss of generality. Using $\Delta = 1 - L$, where L is a lag operator, the model can be written in the form:

$$\Delta z_t = \mu + \Gamma_1 \Delta z_{t-1} + \dots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-k} + \varepsilon_t \quad (3B.2)$$

where $\Gamma_i = -\mathbf{I} + \pi_1 + \dots + \pi_i$, $i = 1, 2, \dots, (k-1)$, and $\Pi = -(\mathbf{I} - \pi_1 - \dots - \pi_k)$. The coefficient matrix Π contains information about long-run relationships between the variables in z_t ; the rank of this matrix is equal to the number of co-integrating relationships denoted by g . Three situations now arise related to the rank of Π . Firstly, if $\text{rank}(\Pi) = 0$, the matrix is null and (3B.2) is the usual VAR model in first differences. If Π has full rank p , then the vector process must be stationary. If, on the other hand, the rank of Π is equal to $g < p$, then there

are two matrices α and β of dimension $(p \times g)$ such that $\Pi = \alpha\beta'$. In this case, the matrix β defines the cointegrating space, and the matrix α is the matrix of weights with which each cointegrating vector enters the p equations of the VAR. The matrix α is also interpreted as the 'adjustment' factors. The cointegration vector has the property that even though z_t itself is non-stationary, $\beta'z_t$ is stationary, and thus (3B.2) has the interpretation of an error correction model.

For estimation purposes, the model (3B.2) can be written in the following compact form:

$$Z_{0t} = \Gamma Z_{1t} + \Pi Z_{kt} + \varepsilon_t$$

where $Z_{0t} = \Delta z_t$, $Z_{1t} = [\Delta z_{t-1}, \dots, \Delta z_{t-k-1}, 1]$ and $Z_{kt} = z_{t-k}$. It is then convenient to first partial out the effect of Z_{1t} on both Z_{0t} and Z_{kt} , and then estimate Π by regressing the adjusted Z_{0t} on Z_{kt} . It can be noted that these adjusted Z_{0t} and Z_{kt} are nothing but the residuals of two multivariate regressions on Z_{1t} :

$$R_0 = Z_0 - Z_1(Z_1'Z_1)^{-1}Z_1'Z_0, \text{ and}$$

$$R_k = Z_k - Z_1(Z_1'Z_1)^{-1}Z_1'Z_k$$

where Z_0 , Z_k , R_0 , and R_k are of dimension $T \times p$, and Z_1 is $T \times [p(k-1) + 1]$. The likelihood function of the model now depends only on R_0 and R_k . Let us now define residual product moment matrices of all these residuals and their cross-products:

$$S_{ij} = T^{-1}R_i'R_j, \quad i, j = 0, k$$

An estimate of Π , $\hat{\Pi} = S_{0k}S_{kk}^{-1}$, can now be obtained from the regression:

$$R_0 = \Pi R_k + \varepsilon_t \quad (3B.3)$$

If, however, the system is characterized by g cointegrating relations, the coefficient matrix Π is of reduced rank. We thus have a problem of reduced rank regression. Johansen (1995) uses the technique developed by Anderson (1951) to reduce the problem of maximizing the concentrated likelihood function:

$$|\Psi|^{-T/2} \exp \left\{ -\frac{1}{2} (R_0 - R_k \Pi')' \Psi^{-1} (R_0 - R_k \Pi') \right\}$$

to a problem of solving the eigenvalue problem:

$$|\lambda S_{kk} - S_{k0} S_{00}^{-1} S_{0k}| = 0. \quad (3B.4)$$

Johansen shows that the first g of the estimated eigenvalues $1 > \hat{\lambda}_1 > \dots > \hat{\lambda}_g > 0$ can be used to give the maximum of the likelihood function:

$$L_{\max}^{-2/T} = |S_{00}| \prod_{i=1}^g (1 - \hat{\lambda}_i) \quad (3B.5)$$

The likelihood function in (3B.5) defines g cointegrating vectors.

The space $\hat{V} = (\hat{v}_1, \dots, \hat{v}_g, \dots, \hat{v}_p)$ spanned by all the eigenvectors is normalized such that $\hat{V}' S_{kk} \hat{V} = I$. The maximum likelihood estimator of β is thus:

$$\hat{\beta} = (\hat{v}_1, \dots, \hat{v}_g).$$

The estimates of the adjustment factors are then given by:

$$\hat{\alpha} = S_{0k} \hat{\beta} = (\hat{w}_1, \dots, \hat{w}_g).$$

An important reason behind the popularity of the Johansen methodology is that it allows a whole battery of likelihood-based tests of hypotheses. Although the decomposition of $\Pi = \alpha\beta'$ is not unique, the space spanned by the columns of β can be estimated. Thus, a testable hypothesis is whether β can be spanned by at least g vectors of dimension p . Additionally, tests can be performed to determine whether a specific vector is contained in the cointegration space.

The η_g and the ζ_g statistics, called trace and maximal eigenvalue statistics respectively, which are used in the text, can be easily obtained by dividing the maximized likelihood (3B.5) for g with the corresponding expression for p or $(g+1)$, and then taking the logarithm of the resulting expression:

$$\eta_g = \frac{|S_{00}| \prod_{i=1}^g (1 - \hat{\lambda}_i)}{|S_{00}| \prod_{i=1}^p (1 - \hat{\lambda}_i)} = -T \sum_{i=g+1}^p \ln(1 - \hat{\lambda}_i) \quad (3B.6)$$

$$\zeta_g = -T \ln(1 - \hat{\lambda}_{g+1})$$

The trace test provides a test of the null hypothesis of g_0 or fewer cointegrating vectors (versus more than g_0), $H_0: g \leq g_0$ against $H_A: g > g_0$. The maximal eigenvalue test, on the other hand, has more specific null and alternative hypotheses; it is a test of g_0 cointegrating vectors against the alternative of $(g_0 + 1)$ cointegrating vectors: $H_0: g = g_0$ against $H_A: g = g_0 + 1$. Johansen and Juselius (1990) suggest that the maximal eigenvalue test has

greater power than the trace test. These two statistics have non-standard distribution and the critical values are tabulated by simulation. Osterwald-Lenum (1992) gives tables for p - $g=1..10$. The Johansen procedure also allows tests of various coefficient restrictions on β by comparing the eigenvalues with and without restrictions imposed. Given c restrictions the eigenvalues are re-estimated under the constraint that:

$$\beta = H\Phi$$

where H is $p \times (p - c)$, and Φ is $(p - c) \times g$. If the restriction fits the data, the new estimated eigenvalues under the restrictions, λ_i^* , should not be significantly different from the eigenvalues λ_i estimated without restriction. This leads to a likelihood ratio test of the restrictions:

$$Q(H_c|H_u) = T \sum_{i=1}^g \ln \left[(1 - \lambda_i^*) / (1 - \lambda_i) \right] \quad (3B.8)$$

This statistic is distributed asymptotically as χ^2 with $g \times c$ degrees of freedom.

The Johansen methodology is exhaustively presented in Johansen (1995). Johansen and Juselius (1990, 1992) are also very useful. Dickey and Rossana (1994) carefully demonstrate the empirical implementation of the methodology.

Appendix 3C

Table 3C.1 Residual Misspecification Tests
Long-Term Real Interest Rate Cointegration Equations, 1957:1-1998:2

Differential	r	ARCH(1)	ARCH(2)	JB	Q(4)	Q(16)	LM(4)
Canada-US	r_1	3.367*	5.217*	3.140	0.042	4.074	3.775
	r_2	0.719	6.447**	8.567**	0.220	4.768	3.654
Canada-UK	r_1	6.557**	9.327***	1.937	0.0315	4.625	2.471
	r_3	2.552	2.704	48.120***	0.283	11.267	1.994
Canada-Germany	r_1	2.845*	13.890***	3.534	0.083	3.867	1.912
	r_4	0.005	0.045	137.108***	0.519	18.960	9.996*
Canada-Japan	r_1	4.032*	12.049***	1.380	0.868	10.894	5.837
	r_5	0.312	0.334	144.054***	2.391	16.475	11.757**
U.S.-U.K.	r_2	2.186	5.342*	10.213***	0.079	5.209	1.951
	r_3	3.871**	3.941	59.612***	0.345	8.013	3.086
U.S.-Germany	r_2	0.212	3.632	15.524***	0.160	3.691	2.388
	r_4	0.148	0.167	96.610***	0.396	22.612	9.452*
U.S.-Japan	r_2	0.004	1.886	13.922***	0.306	5.212	2.217
	r_5	0.253	0.240	254.027***	2.970	19.631	11.872**
U.K.-Germany	r_3	2.347	2.587	110.340***	0.538	7.906	7.966*
	r_4	0.036	0.429	79.444***	0.295	15.304	11.012**
U.K.-Japan	r_3	1.303	1.736	124.408***	0.841	8.146	4.459
	r_5	1.127	1.111	206.053***	1.430	14.616	6.401
Germany-Japan	r_4	0.0006	0.053	104.810***	0.646	14.340	7.474
	r_5	0.602	0.615	285.626***	1.786	16.862	8.761*

Notes: The table reports some important tests on the residuals from the unrestricted vector autoregressions for the Johansen procedure. r_i denotes the real interest rate of the i th country; with $i=1$ for Canada, 2 for U.S., 3 for U.K., 4 for Germany, and 5 for Japan. The row corresponding to r_i reports the tests based on the residuals from the regression of r_i . ARCH(j), $j=1,2$; tests for the j th order ARCH effects in the residuals, and JB tests for the normality of residuals. Q(l), $l=4$ or 16, is the Ljung-Box Q statistics for residual autocorrelation of up to order l .

Continued ... in the notes under Table 3C.2.

Table 3C.2 Residual Misspecification Tests

Long-Term Real Interest Rate Cointegration Equations, 1980:1-1998:2

Differential	r	ARCH(1)	ARCH(2)	JB	Q(4)	Q(16)	LM(4)
Canada-US	r_1	0.659	2.357	1.337	0.5178	9.056	7.727*
	r_2	0.238	5.028*	2.861	0.463	14.040	3.361
Canada-UK	r_1	0.179	0.886	0.693	0.269	3.454	1.000
	r_3	1.342	3.721	2.492	2.369	9.621	3.237
Canada-Germany	r_1	1.504	4.947*	0.114	0.387	4.977	3.254
	r_4	0.163	0.519	70.721***	0.485	12.290	5.369
Canada-Japan	r_1	2.728	3.694	1.674	0.202	1.034	2.318
	r_5	1.157	3.246	1.702	0.992	10.051	5.251
U.S.-U.K.	r_2	0.197	0.230	0.218	0.234	9.359	1.180
	r_3	0.011	0.052	0.633	2.905	11.032	8.113*
U.S.-Germany	r_2	0.318	1.749	2.516	0.321	10.791	0.736
	r_4	0.134	0.600	59.84***	1.392	17.313	7.753*
U.S.-Japan	r_2	0.888	1.538	1.900	0.315	10.319	3.622
	r_5	1.700	2.015	1.773	0.826	16.889	4.680
U.K.-Germany	r_3	0.641	0.849	0.546	2.215	11.534	7.720*
	r_4	1.102	1.267	13.793***	1.303	15.672	7.733*
U.K.-Japan	r_3	1.953	1.985	0.627	2.569	14.144	3.950
	r_5	0.731	0.816	1.652	0.903	12.135	6.299
Germany-Japan	r_4	0.138	1.149	22.071***	0.979	13.958	9.049*
	r_5	0.006	0.287	1.285	1.222	5.960	3.235

Notes: LM(4) is the Breusch-Godfrey Lagrange Multiplier test for autocorrelation of order 4. (***), (**), and (*) denote statistical significance at 1%, 5%, and 10% levels respectively. 1%, 5% and 10% critical values for ARCH(1) test are, respectively, 6.63, 3.841, and 2.705. The corresponding critical values for ARCH(2) and JB tests are 6.63, 3.841, and 2.705, respectively. 1%, 5%, and 10% critical values for Q(4) and LM(4) are 13.276, 9.487, and 7.779, respectively. For Q(16), the corresponding critical values are 31.999, 26.296, and 23.541, respectively.

Table 3C.3 Residual Misspecification Tests
Medium-Term Real Interest Rate Cointegration Equations, 1957:1-1998:2

Differential	r	ARCH(1)	ARCH(2)	JB	Q(4)	Q(16)	LM(4)
Canada-US	r_1	2.483	5.059*	6.916**	0.118	5.614	1.076
	r_2	0.392	19.107***	10.465***	0.138	4.591	8.969*
Canada-UK	r_1	0.312	0.366	2.062	0.232	3.716	4.634
	r_3	0.297	0.506	11.404	0.707	14.839	3.253
Canada-Germany	r_1	0.313	0.366	2.062	0.232	3.716	4.634
	r_4	0.297	0.506	11.404***	0.707	14.839	3.253
U.S.-U.K.	r_2	4.199**	13.883***	16.047***	0.333	4.389	3.937
	r_3	9.016***	9.017**	43.968***	0.023	7.442	4.365
U.S.-Germany	r_2	0.433	5.771*	5.665*	0.272	8.395	1.518
	r_4	0.270	0.241	16.684***	1.846	24.184*	11.958**
U.K.-Germany	r_3	0.207	0.279	0.513	1.483	11.627	2.870
	r_4	0.283	0.258	4.237*	2.806	28.991**	23.494***

Notes: See the notes under Tables 3C.1 and 3C.2.

Table 3C.4 Residual Misspecification Tests
Medium-Term Real Interest Rate Cointegration Equations, 1980:1-1998:2

Differential	r	ARCH(1)	ARCH(2)	JB	Q(4)	Q(16)	LM(4)
Canada-US	r_1	0.684	3.943	14.476***	0.379	10.547	7.199
	r_2	0.044	14.898***	10.070***	1.426	16.418	4.547
Canada-UK	r_1	0.009	5.202*	11.951***	0.211	7.296	1.631
	r_3	0.234	0.574	1.217	3.371	14.936	13.423***
U.S.-U.K.	r_2	0.980	5.227*	1.031	0.101	11.088	2.783
	r_3	0.527	1.048	1.653	4.535	15.943	9.161*

Notes: See the notes under Tables 3C.1 and 3C.2.

Appendix 3D

Table 3D.1
Unit Root Tests of Long-Term Real Interest Rate Differentials, 1957:1-1998:2'

Interest Rate Differentials	ADF τ Statistic	Φ_2
Canada-U.S.	-3.1824	3.7218
Canada-U.K.	-4.1995	6.1061
Canada-Germany	-2.0896	2.3130
Canada-Japan	-1.8340	1.1406
U.S.-U.K.	-2.9417	2.9782
U.S.-Germany	-2.6490	2.7882
U.S.-Japan	-1.5152	0.78709
U.K.-Japan	-1.3701	0.68753
Germany-Japan	-1.5978	0.84862
U.K.-Germany	-2.2862	2.3318

Notes: 'The time period for Japan is 1966:4-1998:2.

1% and 5% critical values for the ADF τ statistic are, respectively, -3.43 and -2.86. The corresponding critical values for the Φ_2 statistic are, respectively, 6.09 and 4.68. Φ_2 is an F-type statistic that tests the joint null hypothesis of unit root with no trend or constant. The ADF τ unit root regression includes a constant, but no trend.

Table 3D.2

Unit Root Tests of Medium-Term Real Interest Rate Differentials, 1957:1-1998:2'

Interest Rate Differentials	ADF τ Statistic	Φ_2
Canada-U.S.	-3.0543	3.6306
Canada-U.K.	-3.2357	3.5092
Canada-Germany	-2.1924	2.5626
U.S.-U.K.	-2.4241	2.3590
U.S.-Germany	-2.0415	3.4670
U.K.-Germany	-2.4931	3.1219

Notes: 'The time period for Germany is 1977:3-1998:2.

1% and 5% critical values for the ADF τ statistic are, respectively, -3.43 and -2.86. The corresponding critical values for the Φ_2 statistic are, respectively, 6.09 and 4.68. Φ_2 is an F-type statistic that tests the joint null hypothesis of unit root with no trend or constant. The ADF τ unit root regression includes a constant, but no trend.

4 A Simple Dynamic Model for a Small Open Economy

4.1 Introduction

The evidence presented in the last three chapters clearly demonstrates that open economy modeling in an implicit one-asset framework with the assumption of perfect world capital market is inadequate. In this chapter, we formulate a simple model of an open economy that emphasizes the dynamics of the real exchange rate, capital accumulation, and debt within an imperfect international capital market framework. This work can be considered a synthesis of the work by Dooley and Isard (1991), Branson (1988) and Stein (1994) mentioned in the introduction of this thesis. Our objective in this synthesis is to explain the stylized fact that real exchange rates of the industrialised countries have become far more volatile in the post-Bretton-Woods era than they had been in the Bretton-Woods era.

We want to argue that the increased variability of real exchange rates in this era is merely a reflection of the fact that the fast adjusting asset market now plays a more prominent role that was absent in the Bretton-Woods era. We know that international capital markets have been dramatically integrated and barriers to international capital mobility have been largely eliminated in the post-Bretton-Woods era. Increased variability in nominal exchange rates is most likely the result of increased variability in equilibrium real exchange rates. If the underlying fundamentals, specifically tastes and technology and possibly either policy changes or expectations of policy change frequently, then it becomes conceivable that all increased variability is merely a reflection of these changes. Since asset markets adjust faster

than the goods markets, the effects of changes in the fundamentals are reflected faster in the nominal exchange rates. With unchanged prices in the short run, the effects are also reflected as changes in real exchange rates. Under the fixed exchange rate system, nominal exchange rates could not reflect the effects of these perceived changes in the fundamentals.

We integrate the asset market equilibrium conditions with the fundamentals of demand and supply shocks in the goods markets to explain movements in the real exchange rate. We focus on the implications for the real exchange rate of changes in investment, saving, and long-term net capital flows and consider the effects of shocks originating in the asset market on the real exchange rate. More specifically, we argue that in a world with imperfect world capital markets, a shock originating in the asset market can also affect the equilibrium real exchange rate significantly. To incorporate an imperfect world capital market, we include a risk premium term in the asset market equilibrium condition to capture the difference in the relative attractiveness of assets issued by different countries.¹ The risk premium is intended to account for both the *currency* and the *country* related factors.

4.2 Modeling the Risk Premium

The implication of the one-asset framework employed by traditional open economy models is that domestic and foreign assets are perfect substitutes in the portfolio of the international investor. In this case, a country can issue any quantity of debt in the

¹Dooley and Isard (1980, 1987, 1991) emphasize the role of risk premium arising from the difference in the relative attractiveness of assets issued by different countries. In their 1987 paper, they formulate a model with the risk premium originating from the variations over time in the prospective income streams on physical capital in different countries. The model is then used to explain nominal exchange rate movements in the presence of the purchasing power parity.

international capital market without suffering any deterioration in its credit rating and an increase in the cost of borrowing.² In the light of the failure of the uncovered interest parity condition and the dominance of home country assets in the international investor's portfolios, the one-asset framework is inadequate. International investors consider the risks associated with the investment in foreign assets.³ Keynes (1923:126-127), for example, emphasized that investor choices between foreign and domestic assets depend on more than interest and exchange rates alone:

..the various uncertainties of financial and political risk..introduce a further element which sometimes quite transcends the factor of relative interest. The possibility of financial trouble or political disturbance, and the quite appreciable probability of a moratorium in the event of any difficulties arising, or of the sudden introduction of exchange regulations which would interfere with the movement of balances out of the country, and even sometimes the contingency of drastic demonetisation, – all these factors deter..[market participants], even when the exchange risk proper is eliminated, from maintaining large..balances at certain foreign centres.

The accounting framework developed earlier and given again in equation (4.1) illustrates the importance of both the exchange risk and the country risk in affecting international asset preferences.⁴ In the past, economists made a great deal of efforts to model the risk premium

²Thus, for example, in the models developed by Kouri (1976), Dornbusch and Fischer (1980), and Mussa (1984), current account imbalances are "settled" with a single type of asset, and conditions are directly or indirectly imposed on the terminal net stock of that asset. In the long-run equilibrium, current account imbalances must accumulate to that terminal net stock. Faruquee (1995) and MacDonald (1995) are two recent papers based on Mussa (1984).

³There are international institutions that help investors with international risk analysis. Thus, for example, *Institutional Investor* magazine surveys international bankers twice a year seeking their evaluation of country creditworthiness. In their evaluations, bankers put significant weights on current account and net foreign debt of the countries.

⁴Recall that e_t is the logarithm of the real exchange rate at date t , $\mathcal{E}_t$ is an expectations operator, $RP_{t,T}$ and $CP_{t,T}$ are, respectively, exchange and country risk premia, and r_t is the domestic real interest rate (* indicates its foreign counterpart).

ignoring possible country-related risk. The earliest conceptual models of exchange risk were based on portfolio balance where financial claims are distinguished by their currency of

$$e_t = \mathcal{E}_t e_{t+T} + \mathcal{E}_t (r_{t,T}^* - r_{t,T}) + RP_{t,T} + CP_{t,T} \quad (4.1)$$

denomination but not by the country obligated to meet the claims (see, for example, Dooley and Isard (1983)). As we already mentioned, Dooley (1982) points out that countries with relatively high credit risks usually do not denominate their international borrowing in their own currencies.

The question of optimal international portfolio diversification has been examined in a number of partial equilibrium and general equilibrium models.⁵ In both class of models, a demand for foreign assets can arise either because of the transactions role of different currency units in different countries, or because assets denominated in particular currencies enter the utility function directly as inputs in the production of consumption services. In the general equilibrium model of Lucas (1982), for example, households across different countries are highly similar and differ only in the currency that they wish to hold. This simplification allows an equilibrium in which, except for money, cross country households' portfolios are identical.

While these models have enhanced our understanding of risks in international capital markets, evidence suggests that there is more to an adequate explanation of asset preference.

⁵For examples of partial equilibrium models, see de Macedo et al (1984), Dornbusch (1983), Frankel (1982), Frankel and Engel (1984), and Engel and Rodrigues (1989). These models are based on the classic mean-variance framework of Tobin (1969), and derive asset demands from an explicit utility maximizing framework. Also see Branson and Henderson (1985) for a survey of this branch of literature. See Lucas (1982) and Stulz (1984), and the references cited by Obstfeld and Stockman (1985: 964-972) for general equilibrium models.

What seems to be needed is a framework that considers both exchange and country risk simultaneously. Dooley and Isard (1987) consider such a joint determination of country and exchange risk premium in a model incorporating country-specific returns to physical capital.⁶ We follow them closely in specifying the risk premium.

We assume that the differential yield or the risk premium required on home country assets depends on the relative "exposure" of investment portfolios to the home country. It is also affected by other variables that influence perceptions of the future returns per unit of this exposure. The exposure can be conveniently described in terms of the net foreign debt position, F , of the home country. This captures both exchange rate risk as well as country risk. The higher the quantity of foreign debt in the international portfolio, the greater the exposure to the risk associated with exchange rate fluctuations as well as to the country-related risks – default risks, risks of future capital controls, tax laws that discriminate by country of residence, etc. The size of the differential yield demanded on home country assets will, in general, be influenced by other factors that influence perceptions of the future returns per unit of the exposure. We assume that an increase in productivity or growth will positively influence perceptions of the future returns. We therefore specify the risk premium as:

$$\rho_t = \eta(f_t, \mathcal{G}_t), \text{ with } \eta_f > 0, \eta_{\mathcal{G}} < 0. \quad (4.2)$$

where ρ_t is the risk premium, f is the logarithm of the stock of foreign debt and $\mathcal{G}$ summarizes the effect of factors including the economic and the political outlooks that

⁶In the Dooley and Isard (1987) model, risk premium originates from the variations over time in the prospective income streams on physical capital in different countries. In their 1991 paper, Dooley and Isard provide some evidence attributing the sharp depreciations in the real exchange rates of a number of debt-burdened developing countries in the early eighties to the substantial deterioration in the attractiveness of owning assets in those countries.

may influence perceptions of future returns. θ can also take account of external factors that affects relative attractiveness of domestic assets to foreign assets. Thus political instability, for example, in a foreign country can make domestic assets more attractive relative to foreign assets. This will be reflected in a lower θ : $\eta_\theta < 0$. Other things equal, the larger is the net external indebtedness of the home country the larger will be the premium that is required at the margin to hold an additional unit of the domestic asset in the international portfolio.

4.3 The Model

The model we formulate here is a model with two stock variables, physical capital and foreign debt, interacting with a forward-looking variable, the real exchange rate. Although government spending is included as an expenditure component in the goods market equilibrium condition, government is not explicitly modelled as a separate economic agent in the present setup. For analytical simplicity, we combine households and the government into one economic entity, whose spending depends on its wealth – the stock of capital net of foreign debt. Stein (1994) shows that such a formulation ensures convergence. The key to such convergence is that increased debt induces agents to lower spending, and this keeps debt from growing unboundedly. Masson et al (1994) examine the convergence of net foreign assets in the USA, Japan, and Germany empirically. Their evidence suggests that domestic absorption responds in a manner that is conducive to the smooth adjustment of net foreign assets to equilibrium.

4.3.1 Goods Market

Agents use two goods, a domestic good and a foreign good, for consumption and capital accumulation. We model the net demand for the domestic good as the sum of absorption including government spending and trade balance. Wealth is defined as the real value of capital stock net of foreign debt. Wealth and the real rate of interest, given the exogenous rate of social time preference and the productivity of capital, determine the level of absorption in terms of the private sector consumption spending, and the real exchange rate determines its distribution between the domestic and foreign goods. The small economy faces a given foreign currency price for its imports that does not depend on the level of domestic demand. However, the economy is assumed to be "large" in the market for its export good, facing a less-than-perfectly elastic demand for its exports. Its export demand then depends on its real exchange rate; a depreciation switches absorption from the imported foreign goods to the domestic goods, and it also raises export demand. A real exchange rate depreciation then both raises aggregate demand and improves the trade balance.

The *goods market equilibrium condition* is the familiar national income identity; namely, income is equal to spending:

$$y(k_t; \theta) = A(k_t, f_t, r_t; g) + i(k_t, r_t; \theta) + tb(e_t) \quad (4.3)$$

where subscripts are used to indicate partial derivatives, e.g., $tb_e = \partial tb / \partial e$; and where $y_k > 0$, $y_{kk} < 0$, $t_e > 0$, $A_f < 0$, $A_k > 0$, $i_k < 0$, $i_r < 0$, and $i_\theta > 0$. The variables are: $y(k; \theta)$ = GDP, A = private sector consumption spending plus government spending; f = the stock of foreign debt, $tb(e)$ is the trade balance, and k and i are, respectively, the capital stock

and investment.⁷ $Z(g, \theta)$ is a vector of exogenous disturbances, where θ is a productivity parameter and g is a parameter of social time preference. The goods market equilibrium equation (4.3) specifies equilibrium at full-employment.

4.3.2 *The Financial Asset Market*

The uncovered interest rate parity condition imposes a relationship between the real interest rate and the real exchange rate.⁸ We augment the parity condition by the risk premium on domestic financial assets defined in (4.2). This gives the financial asset market equilibrium condition:

$$r_t = r^* + \mathcal{E}_t \hat{e}_{t,T} + \eta(f_t, \vartheta) \quad (4.4)$$

In this equation, $\mathcal{E}_t \hat{e}_{t,T} = (\mathcal{E}_t e_T - e_t)$, which is the expected rate of real depreciation over the period t to T , $\mathcal{E}_t$ is an expectations operator with expectations formed based on all information available at date t , and $\eta(f_t, \vartheta)$ is the risk premium on domestic assets.

4.3.3 *Growth in Foreign Debt and Capital Accumulation*

The rate of change of foreign debt is given by the current account deficit, which, by the national income identity, is equal to investment less saving:

$$\dot{f} \equiv i_t - s_t \equiv r_t f_t - tb(e_t) \quad (4.5)$$

⁷Here all variables, except the real interest rate, are in natural logarithms. This is done with the objective of solving goods and assets markets equilibrium conditions for the equilibrium real exchange rate and the real interest rate later.

⁸Note from section 3.3 in chapter 3 that the uncovered interest rate parity can be written in this form.

where $\dot{f}$ is the time derivative of f . Although it is an identity, (4.5) is a generally acceptable proposition about the current account. As with any identity, we cannot, however, deduce any causation from it. A nation with a current-account deficit will have a capital-account surplus. Whether this situation is 'caused' by developments in goods markets or in financial markets (either at home or abroad) cannot be determined without modeling the behavior of its components.⁹ The identity does, however, highlight the linkage between the consumption, savings and investment behavior in one country and its payments balance versus the rest of the world. It can be transformed to a market-clearing condition when its components are described by independent behavioural equations and an equilibrating process is specified.

We assume that social saving (private sector plus the government saving) is a decreasing function of wealth and hence an increasing function of debt. This is both a traditional assumption, and, one supported by the evidence in Masson et al (1994). This assumption ensures the convergence of the system to a stable long run equilibrium.

Capital stock is assumed to depreciate at a constant rate δ . Investment in the economy is an increasing function of the q -ratio: the capital value of an asset relative to its supply price. Thus, the rate of change in capital intensity is given by:

$$\dot{k} = i_t(k_t, r_t; \theta) - \delta k_t \quad (4.6)$$

⁹ In the *natural real exchange rate* model of Stein and others, (4.5) is interpreted as the equilibrium condition for the balance of payments, as well as for the goods market in the medium-run when the economy is at the capacity output. In this medium-run setting, investment over saving is interpreted as describing the desired net long-term capital inflows. The problem with such a claim is that the components of (4.5) are jointly determined, and can be the outcomes of developments in goods markets or financial markets at home or abroad. This further reiterates the importance of including asset market equilibrium condition in any modeling of the exchange rate.

4.3.4 *Summary of the Model*

Equations (4.2) through (4.6) define the simple dynamic model. At each point of time, following some shock, the real exchange rate is determined in the asset market in a way that satisfies equation (4.4). The exchange rate and interest rate then adjust to clear goods and asset markets simultaneously at predetermined levels of capital stock and foreign debt. Suppose there is a sudden rise in government spending that creates excess demand in the goods market. This will immediately affect the asset market and determine a new real exchange rate and a real interest rate. The exchange rate and interest rate will soon adjust in a way that generates adequate import surplus and fall in investment spending to offset the excess demand in the goods market. Short-run equilibrium levels of exchange rate and interest rate set the paths of capital accumulation (4.6) and the growth in foreign debt (4.5). Changes in the stocks of foreign debt and capital will affect exchange rate expectations and determine a new short-run equilibrium. This process continues until the system settles at the steady state. The assumptions that absorption depends on wealth and that $y_{kk} < 0$, $i_k < 0$, guarantee that the system will converge to such a state.

4.4 Short-run Equilibrium and Dynamics

4.4.1 *Equilibrium in the goods and assets markets*

Given the existing stock of capital and foreign debt, and exchange rate expectations, short-run equilibrium real exchange rate and real interest rate are determined jointly by the

goods and asset market equilibrium conditions.¹⁰ The goods market equilibrium equation (4.3) and the asset market equation (4.4) are depicted in Figure 4.1 as GG and AA curves respectively. The GG curve depicts combinations of real exchange rate and real interest rate such that goods market is in equilibrium. The slope of this curve is given by:

$$\left. \frac{dr_r}{de_t} \right|_{GG} = -\frac{tb_e}{(A_r + i_r)} > 0.$$

Here we assume that the Marshall-Lerner condition holds ($tb_e > 0$) and that a rise in the real interest rate lowers domestic absorption ($A_r < 0$, $I_r < 0$). The schedule GG is positively sloped for the following reason. A rise in the domestic rate of interest reduces aggregate demand relative to capacity GDP. To equilibrate the goods market, e must rise: the domestic currency must depreciate in real terms, so as to increase net exports.

Figure 4.1 Short-Run Equilibrium

¹⁰Short-run equilibrium is established with fixed capital stock and foreign debt. In long-run equilibrium, these two state variables have already converged to their steady state levels.

The asset market equilibrium schedule AA , on the other hand, is negatively sloped:

$$\left. \frac{dr_t}{de_t} \right|_{AA} = -1$$

This condition states that for given values of the stock of foreign debt, the foreign real interest rate, and the expected rate of depreciation, an increase in the real interest rate requires an appreciation of domestic currency. In terms of how the market works, the rise in the domestic real interest rate causes portfolio shifts towards domestic assets. The sales of foreign assets and the purchases of domestic assets, in turn, appreciates the domestic currency until equilibrium is re-established.

The downward sloping asset market equilibrium schedule AA intersecting with the upward sloping goods market equilibrium schedule GG determines the unique levels of the equilibrium real exchange rate and real interest rate, e_0 and r_0 respectively.

4.4.2 *Continual Movements in the Equilibrium Real Exchange Rate*

In the simple framework specified above, the real exchange rate responds continually to changes in the fundamentals. The short-run equilibrium is defined contingent upon the exchange rate expectation, and the stocks of capital and foreign debt. Associated with the short-run equilibrium of (4.3) and (4.4) are a rate of growth in the capital stock, $\dot{k}$, from (4.6), and a current account surplus or deficit, $\dot{f}$, from (4.5). The latter two equations describe how the capital stock and the stock of foreign debt held evolve over time. Economic agents use the current information on foreign debt, capital stock and exchange rate to form expectations on the long run exchange rate, $\mathcal{E}_t e_T$. As these variables change, short-run

equilibrium determines a new set of e and r for the new values of k, f and $\mathcal{L}e_T$ and induces further movements in k, f , and $\mathcal{L}e_T$. The dynamic process continues until the system finally settles at a long run steady state, assuming that one exists.

A shock originating either in the goods market or in the assets market disturbs the short-run equilibrium, and sets in a series of adjustments in k, f , and $\mathcal{L}e_T$. Let us now see how the real exchange rate responds to these shocks in its dynamic adjustment process. Differentiating equations (4.3) and (4.4) totally, we can obtain the following system:

$$\begin{bmatrix} tb_e & (A_r + i_r) \\ 1 & 1 \end{bmatrix} \begin{bmatrix} de_t \\ dr_t \end{bmatrix} = \begin{bmatrix} (y_k - A_k - i_k)dk_t - A_f df_t - A_g dg_t + (y_\theta - i_\theta)d\theta \\ dr^* + \eta_f df_t + \eta_g dg_t + d\mathcal{L}e_T \end{bmatrix}$$

This can be solved for:

$$de_t = \frac{D_1}{D} \quad (4.7)$$

where $D = [tb_e - (A_r + i_r)] > 0$, and:

$$\begin{aligned} D_1 = & (y_k - A_k - i_k)dk_t + (y_\theta - i_\theta)d\theta - A_f df_t - A_g dg_t \\ & - (A_r + i_r)[dr^* + \eta_g dg_t + \eta_f df_t + d\mathcal{L}e_T]. \end{aligned}$$

The above gives us an equation for how the real exchange rate is influenced by all the fundamentals of the model as well as exchange rate expectations, and implies (by integration) the following relationship:

$$e_t = \phi(k_t, f_t, \mathcal{L}e_T, r^*, g_t, \theta) \quad (4.8)$$

The partial derivatives from (4.8) are:

$$\begin{aligned}
\phi_\theta &= [(y_\theta - i_\theta)/D] < 0, \quad \phi_g = (-A_g/D) < 0, \quad \phi_g = [-\eta_g(A_r + i_r)/D] < 0 \\
\phi_k &= [(y_k - A_k - i_k)/D] > 0, \quad \phi_f = [-\eta_f(A_r + i_r) - A_f/D] > 0 \\
\phi_{r^*} &= [-(A_r + i_r)/D] > 0, \quad \phi_{\mathcal{L}_t e_{t-\tau}} = [-(A_r + i_r)/D] > 0.
\end{aligned}$$

In signing ϕ_θ , we assume that investment's response to productivity is greater than the response of output to productivity: $i_\theta > y_\theta$. The impulse response results in the next chapter provides evidence for this assumption. In signing ϕ_k , we assume that $|(y_k - i_k)| > |A_k|$. Note that $i_k < 0$ and $y_k > 0$. In this case, the effect of capital stock on further accumulation is added to the magnitude of the marginal product of capital on the left-hand side of the inequality in question. The right-hand side is the marginal propensity to spend out of wealth (in this case, fixed capital less foreign debt). The signing of the remaining partial derivatives is straightforward, given the assumptions about the behavior of absorption (see equation 4.3 and its associated partial derivatives) and that $D > 0$ (see equation 4.7).

The equation for the real interest rate can be obtained in a similar manner by integration:

$$r_t = \varphi(k_t, f_t, \mathcal{L}_t e_{t-\tau}, r^*, g_t, \theta) \quad (4.9)$$

with the following partial derivatives:

$$\begin{aligned}
\varphi_k &= -[(y_k - A_k - i_k)/D] < 0, \quad \varphi_\theta = -[y_\theta - i_\theta/D] > 0 \\
\varphi_f &= [(tb_e \eta_f + A_f)/D] = ?, \quad \varphi_g = (\eta_g tb_e/D) < 0 \\
\varphi_{\mathcal{L}_t e_{t-\tau}} &= (tb_e/D) > 0, \quad \varphi_{r^*} = (tb_e/D) > 0, \quad \varphi_g = (A_g/D) > 0.
\end{aligned}$$

Note that the signs of φ_k and φ_θ follow straightforwardly from the signs of ϕ_k and ϕ_θ . The

remaining partial derivatives use the assumptions about the Marshall-Lerner condition and the behavior of absorption.

We note the asset price nature of the real exchange rate means that the current exchange rate depends on its long run expected value.¹¹ Mussa (1990), for example, presents a simple model to show how the current exchange rate can be linked to these future economic conditions that affect the demand and supply in the foreign exchange market in future periods. This then explains why observed changes in exchange rates often appear to be random.

Although the real exchange rate has an asset price characteristic because its current value is affected by its long-run expected value, one should not take it simply as the price of a traded asset. The current equilibrium real exchange rate should instead be considered as a price that clears goods markets (see equation 4.3) consistent with asset market equilibrium. The role of the expectations term is to pull forward into the present some of the future changes in the real exchange rate. Since the agents look ahead to the exchange rate consequences of current disturbances in the goods- or asset market, the exchange rate will jump at the time when the consequences of that change are recognized.

¹¹Mussa (1984;18) provides an illustration of the behavior of asset prices. The current price of any asset that may be held in inventory at a relatively small storage cost and bought and sold with a relatively small transaction cost will be reasonably closely linked to the price that is expected at some day in the near future. The reason for this linkage is that if there were a substantial expected rise in the price of the asset, agents would have a strong incentive to acquire and hold the asset, putting upward pressure on its current price and downward pressure on its expected future price, until the difference between these two prices was brought within the limits implied by storage and transactions costs.

4.5 Long-Run Equilibrium and Stability

4.5.1 Steady State Equilibrium

The steady state of the economy in this simple model is reached when both the capital stock and the foreign debt position cease to change, and when exchange rate expectations are fulfilled. With a constant rate of depreciation of capital, steady-state investment will just be replacement investment, and with exchange rate expectations realized, the domestic real interest rate will exceed the foreign real rate by the amount of risk premium on domestic assets. The equilibrium is associated with a trade balance deficit (surplus) and a corresponding service account surplus (deficit). Letting a tilde over a variable denote the steady state value of that variable, the long run equilibrium values of the exchange rate, the capital stock, and the foreign debt must satisfy the following conditions:

$$\begin{aligned}
 y(\tilde{k}; \theta) &= A(\tilde{k}, \tilde{f}, \tilde{r}; g) + i(\tilde{k}, \tilde{f}; \theta) + \iota b(\tilde{e}) \\
 \tilde{i} &= \delta \tilde{k} \\
 \iota b(\tilde{e}) &= \tilde{r} \tilde{f} \\
 \tilde{r} &= r^* + \eta(\tilde{f}, \vartheta)
 \end{aligned} \tag{4.10}$$

Steady state equilibrium values of the variables depend on the exogenous variables:

$$\begin{aligned}
 \tilde{e} &= \tilde{e}(g, \theta, \vartheta, r^*) \\
 \tilde{r} &= \tilde{r}(g, \theta, \vartheta, r^*) \\
 \tilde{k} &= \tilde{k}(g, \theta, \vartheta, r^*) \\
 \tilde{f} &= \tilde{f}(g, \theta, \vartheta, r^*)
 \end{aligned} \tag{4.11}$$

In general, the comparative static analysis of the variables in the present form is extremely

complicated. We, therefore, adopt the alternative strategy of first developing the stability properties of the model. Then by imposing the stability condition, we can obtain the analytical solution that describes the dynamic behavior of the economy from an initial steady state to the final steady state.

4.5.2 Stability and the Long-Run Dynamics

The dynamics in the present model are governed by exchange rate expectations and two endogenous fundamentals: the capital stock and the stock of foreign debt. The dynamics governing the latter two variables are given by equations (4.6) and (4.5), respectively. Capital stock and foreign debt determine not only the real exchange rate in the medium run, but also the expected level of the long run real exchange rate. Economic agents consider the effects of current shocks on the capital stock and foreign debt in the long run, and accordingly form their expectation about the long run equilibrium real exchange rate, $\mathcal{E}_t e_T$. Noting that investment depends on the Keynes-Tobin q -ratio, and social saving on interest rate and wealth, we can express (4.5) and (4.6), using (4.9), as:¹²

$$\begin{aligned}\dot{f} &= \gamma(k_t, f_t, \mathcal{E}_t e_T, g, \theta, \vartheta, r^*) \\ \dot{k} &= \psi(k_t, f_t, \mathcal{E}_t e_T, g, \theta, \vartheta, r^*)\end{aligned}\tag{4.12}$$

Using equation (4.4), the expected long-run equilibrium level of the real exchange rate can be expressed as:

¹²Note that the q -ratio is inversely related to the interest rate and the capital stock-in the latter case through the diminishing marginal productivity of capital.

$$\mathcal{E}_t e_T = \Phi(r_t, e_t, r^*, f, \vartheta) = r_t + e_t - r^* - \eta(f, \vartheta) \quad (4.13)$$

with $\Phi_k = (\varphi_k + \phi_k - \eta_\vartheta \vartheta_k)$, and $\Phi_f = (\varphi_f + \phi_f - \eta_f)$. Recall that higher capital stock is expected to lower the country-specific risk premium and thus the term $\eta_\vartheta \vartheta_k$ is expected to be negative. It appears that given the previously obtained derivatives, $\phi_k > 0$, $\varphi_k < 0$, $\phi_f > 0$, $\varphi_f = ?$, the derivatives Φ_k and Φ_f have ambiguous signs. In an attempt to perform a state-space analysis, let us, however, linearize the system in (4.12) around the steady-state equilibrium. The linearized system can be presented as:

$$\begin{bmatrix} \dot{k} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} \psi_k & \psi_f \\ \gamma_k & \gamma_f \end{bmatrix} \begin{bmatrix} dk \\ df \end{bmatrix} + \begin{bmatrix} \psi_\theta & \psi_g & \psi_{r^*} & \psi_\vartheta \\ \gamma_\theta & \gamma_g & \gamma_{r^*} & \gamma_\vartheta \end{bmatrix} \begin{bmatrix} d\theta \\ dg \\ dr^* \\ d\vartheta \end{bmatrix} \quad (4.14)$$

where the derivatives (dz) are expressed as deviations from steady-state values. The stability of the model depends on the signs of the trace, $(\psi_k + \gamma_f)$, and of the determinant, $(\psi_k \gamma_f - \gamma_k \psi_f)$, of the characteristic equation associated with (4.14). The system would be dynamically stable if the trace were negative and the determinant were positive.¹³ One can derive the elements of the characteristic matrix in an attempt to sign these elements. It is relatively straightforward to obtain the following expressions for these elements:

¹³ Recall that the trace of a matrix is equal to the sum of its characteristic roots, and its determinant is equal to the product of its roots.

$$\begin{aligned}
\psi_k &= (i_k - \delta + i_r \varphi_{\mathcal{L}_t e_T} \Phi_k) + i_r \varphi_k \\
\psi_f &= (i_r \varphi_f + i_r \varphi_{\mathcal{L}_t e_T} \Phi_f) \\
\gamma_k &= (\varphi_k + \varphi_{\mathcal{L}_t e_T} \Phi_k) f - t b_e \phi_k \\
\gamma_f &= r + (\varphi_f + \varphi_{\mathcal{L}_t e_T} \Phi_f) f - t b_e \phi_f .
\end{aligned} \tag{4.15}$$

As it appears, all elements in the characteristic matrix (the first matrix on the right-hand side) associated with the state-space representation in (4.14) are of ambiguous signs. We thus arrive at a point that is inherent in this kind of analysis: it is very difficult to evaluate the signs of the *trace* or of the *determinant* of the characteristic matrix in our state-space analysis. However, if we assume that higher debt lowers the accumulation of further debt, $\gamma_f < 0$, and that higher capital stock reduces the rate of further capital accumulation, $\psi_k < 0$, the trace of the characteristic matrix can be signed to be negative. Now the stability of the model depends on the positive sign of the determinant.

Given the assumption that the risk premium is increasing in the stock of foreign debt, higher debt increases the finance cost of investment and thus $\psi_f < 0$. It can be reasonably assumed that $\gamma_k < 0$ – higher stock of capital is expected to improve the current account. Higher capital stock increases output and lowers investment. Although, it will also lead to a rise in consumption, the marginal propensity to import is unlikely to exceed the marginal propensity to spend on domestic goods. The sign of the determinant now depends on the following inequality:

$$\left| \psi_k \gamma_f \right| > \left| \gamma_k \psi_f \right|. \tag{4.16}$$

Once again, it is difficult to evaluate (4.16). In order to get some further insights into the stability properties of the model, let us consider the characteristic roots associated with the system in (4.14). The negative trace of the characteristic matrix ensures that the system has at least one negative characteristic root. Let λ_1 be this stable root, which is given by:

$$\lambda_1 = \frac{1}{2} \left[(\psi_k + \gamma_f) - \left\{ \sqrt{(\psi_k - \gamma_f)^2 + 4\psi_f \gamma_k} \right\} \right] \quad (4.17)$$

The other root of the present system, let it be λ_2 , is then given by:

$$\lambda_2 = \frac{1}{2} \left[(\psi_k + \gamma_f) + \left\{ \sqrt{(\psi_k - \gamma_f)^2 + 4\psi_f \gamma_k} \right\} \right] \quad (4.18)$$

It is thus possible that λ_2 is positive. A close look at (4.18) will, however, reveal that λ_2 can be negative for a wide range of values of the partial derivatives in that equation. It can be shown that as long as the absolute value of γ_f , the effect of foreign debt on current account, is greater than γ_k , the effect of capital stock on foreign debt, λ_2 will be negative. As a matter of fact, the effect of capital stock on debt accumulation is likely to be small. Capital accumulation will slow down investment, and can eventually increase saving relative to investment. But a rise in capital stock through its effect on wealth can adversely affect the current account. It then follows that the dynamic stability condition may in fact hold.

Figure 4.2 presents the phase diagram of the stable system.¹⁴ The curve $\dot{f} = 0$ gives the locus of pairs of (k, f) along which foreign debt remains constant, while the $\dot{k} = 0$ curve plots the (k, f) pairs for which there is no capital accumulation in the economy. The directions of

¹⁴It is straightforward to derive the slope of the $\dot{k} = 0$ and the $\dot{f} = 0$ curves from (3.12). It can be noted from the determinant condition above that when the system is stable, the $\dot{f} = 0$ curve is steeper than the $\dot{k} = 0$ curve.

Figure 4.2 *Steady-State Equilibrium*

the arrows indicate that if the determinant were positive, the economy would move to its steady-state point (*ss*) with capital stock k_s and debt f_s regardless of the initial conditions.

In the event the determinant is negative, the steady-state equilibrium at which $\dot{k} = \dot{f} = 0$ will be a saddle point. As is common in this kind of analysis, let us assume that the system has one stable root. We explore the system's transitional dynamics and the behavior of the real exchange rate along the stable adjustment path in the next section.

4.5.3 Transitional Dynamics and the Real Exchange Rate

The general solution of the system of equations in (4.14) for constant levels of the exogenous variables are given by:

$$\begin{bmatrix} dk \\ df \end{bmatrix} = c_1 \begin{bmatrix} \omega_{11} \exp(\lambda_1 t) \\ \omega_{12} \exp(\lambda_1 t) \end{bmatrix} + c_2 \begin{bmatrix} \omega_{21} \exp(\lambda_2 t) \\ \omega_{22} \exp(\lambda_2 t) \end{bmatrix} \quad (4.19)$$

where $\omega_1 = \{\omega_{11}, \omega_{12}\}$ and $\omega_2 = \{\omega_{21}, \omega_{22}\}$ are the characteristic vectors corresponding to

the two characteristic roots, λ_1 and λ_2 , respectively, and c_1 and c_2 are two arbitrary constants determined by the initial values of k and f respectively. If both characteristic roots were negative, it follows from (4.16) that both capital stock and foreign debt would approach their steady state values ($dk \rightarrow 0$, $df \rightarrow 0$) as $t \rightarrow \infty$ (regardless of the initial conditions). Let us assume that λ_1 is the stable root. For analytical convenience, let us also assume that the initial level of foreign debt is such that $c_2 = 0$. Accordingly, the solution trajectory (dk , df) will move along the straight line:

$$dk = \left(\frac{\omega_{11}}{\omega_{12}} \right) df. \quad (4.20)$$

Now if we differentiate (4.8) totally, and assume that economic agents have perfect foresight expectations along the adjustment path defined by (4.20), we have:

$$de = \left[\phi_k \left(\frac{\omega_{11}}{\omega_{12}} \right) + \phi_f \right] df \quad (4.21)$$

It is also straightforward to derive from the eigen vectors associated with our assumed stable root that:

$$\frac{\omega_{11}}{\omega_{12}} = - \left(\frac{\psi_f}{\psi_k - \lambda_1} \right).$$

Once again, it is difficult to evaluate the sign of the above expression, even with our assumed stable root. One can, however, note that a negative value of the above ratio would imply that a rising foreign debt along the stable adjustment path is associated with an appreciating real value of domestic currency. Such a result is clearly contrary to any reasonable prediction: one would rather expect that for a stable adjustment, a rising debt must depreciate the currency

in order to increase exports to service the higher debt. Thus, the ratio must be positive. A straightforward rearrangement of (4.19) and (4.20) will also show that along the same adjustment path capital accumulation also accompanies real depreciation.

4.6 Dynamic Adjustments to Long-Run Equilibrium

The dynamic behavior of the real exchange rate, capital and debt accumulation link the short-run equilibrium to the long-run steady state. Assuming the stability of the system, let us now use the results from section 4.4.2 to analyse how the equilibrium values of e and r evolve over time in response to a number of shocks below.

4.6.1 *A Rise in the Productivity of Capital*

A permanent rise in the productivity of capital increases the q -ratio, and thus the demand for investment. The adjustments are illustrated in Figure 4.3 below. In the short-run, the goods market equilibrium changes requiring a shift in the GG curve to $G'G'$. Higher domestic interest rate causes portfolio shifts toward domestic assets appreciating domestic currency, which eventually deteriorates trade balance of the country. This deterioration in the trade balance is required for the goods market equilibrium (in equation (4.3)) given our assumption that increased investment induced by the positive productivity shock exceeds the output gain on impact. Thus at point (b) , the country is experiencing a negative trade balance, and, possibly, a current account deficit.¹⁵ As the current account deficit adds net domestic assets to international investors portfolios, international investors demand a differential yield or risk premium on domestic debt. Thus, the asset market equilibrium schedule AA shifts to

¹⁵The country will experience current account deficit if we assume that trade was balanced before this shock.

Figure 4.3 *Adjustments to a Productivity shock*

the right to $A'A'$. The country's positive economic outlook from the increased productivity can, however, reduce the risk premium to some extent. In any case, the domestic currency depreciates in real terms. Increased investment will over time increase the economy's capital stock and capacity output. Increased capital stock will eventually decrease the marginal productivity of capital, which in turn, will reduce investment. Goods market equilibrium in the face of increased domestic output will now require a real depreciation. The goods market equilibrium curve ($G'G'$) starts shifting back towards $G''G''$ in order to accommodate the depreciation. As the domestic currency depreciates, the trade balance starts improving. Foreign debt gradually starts falling with the consequent effect of a lower risk premium on the domestic asset. The asset market equilibrium curve now starts shifting towards $A''A''$ in response to the lower risk premium. Eventually, the economy returns to the long-run equilibrium where the current account is again balanced; capital stock stabilizes at a higher level with foreign debt at a lower level. It is also possible that the country is now a creditor

in its net foreign asset position. In its new long-run equilibrium, the country can be at a point such as *(c)* or *(d)*. The country may experience a real appreciation of its currency because its positive investment income account must be offset by a negative trade balance account in the long-run equilibrium. On the other hand, the rise in capital stock of the economy will increase output relative to its absorption which will have a tendency to worsen its terms of trade. In general, exactly how a rise in productivity will affect the real exchange rate in the long run will also depend on income and price elasticities of the demand for exports and for imports.¹⁶

4.6.2 *A Rise in Government Spending*

Suppose there is a rise in desired government spending.¹⁷ This case is illustrated in Figure 4.4 below. Excess demand in the goods market at *(a)* causes the GG curve to shift up to $G'G'$ (see the results in equations 4.8 and 4.9). The rising domestic real interest rate invites portfolio shifts toward the domestic asset. Increased demand for domestic assets, in turn, appreciates the value of domestic currency, which deteriorates the current account. Thus, assuming that in its initial equilibrium the country had zero foreign debt, at *(b)* the country is running a current account deficit; foreign debt is now gradually rising. The higher rate of interest, on the other hand, will reduce the rate of investment, and eventually, the economy's capital stock. Initial excess demand in the goods market is thus offset partly through a fall

¹⁶Johnson (1958) pointed out that economic growth is likely to require secular changes in real exchange rate. This would then mean that a country like Japan would have experienced continuous real depreciation of its currency. Krugman (1990) provides a nice empirical analysis of why this has not been the case showing that the differences in underlying growth rates of countries are somehow systematically related to the differences in income elasticities in a way that Japan did not have to experience continuous real depreciation of its currency.

¹⁷We assume here that the increased government spending is to finance present consumption, not investment.

Figure 4.4 *Adjustments to a rise in Government Spending*

in investment, and partly through a decrease in the trade balance. To induce foreigners to accumulate domestic assets, a risk premium is now required; the AA curve shifts to $A'A'$ to accommodate the risk premium. The country now experiences a decrease in domestic absorption in response to the decrease in real wealth. The $G'G'$ curve will now start shifting back to absorb this fall in domestic absorption as well as to offset the increased servicing of the foreign debt. In the new long-run equilibrium, at a point such as (c) , the country will definitely experience a depreciation of its currency in real terms so as to generate enough trade surplus in order to service the new higher level of foreign debt.¹⁸

4.6.3 A Fall in Risk Premium

Let us now use the model to explain the so-called "*Safe Haven Effect*". Suppose there is

¹⁸Clarida and Prendergast (1999) use an 'exactly identified' structural VAR model to examine the dynamic response of the real exchange rate to fiscal shocks in the US, Japan, and Germany, and find evidence that supports our analysis. They find that the real exchange rate appreciates on impact, and for several years thereafter, in response to a fiscal shock. However, eventually, the process is reversed: the real exchange rate overshoots and actually depreciates relative to its initial level prevailing before the fiscal shock. Thus, while our results are not unique, our model is more realistic, in that it integrates imperfect capital flows.

Figure 4.5 *Adjustments to a Safe-Haven Effect*

a decrease in the risk premium on domestic assets. This can result from a change in the economic or political outlook favouring the home country against the foreign country. It will mean a lower θ in the risk premium term. The case is illustrated in Figure 4.5. The initial equilibrium at (a) now represents a risk-adjusted return on domestic assets that exceeds the risk adjusted return on foreign assets. As seen in the results in equations (4.8) and (4.9), the asset market equilibrium curve AA shifts down to $A'A'$. At (a), the country now experiences portfolio shifts toward domestic assets, appreciating domestic currency, and lowering real interest rate. Lower interest rate increases capital accumulation, and, in turn, the capacity output of the country. The appreciating currency, on the other hand, gradually deteriorates the current account, and it is possible that at (b) the country now has a positive foreign debt. Domestic currency now must depreciate in real terms not only to offset the servicing of the foreign debt, but also to eliminate the surplus in the goods market. The GG curve now shifts to the right to $G'G'$. Asset market equilibrium curve $A'A'$ will also now start shifting to the right to offset the risk premium. Depending on the amount of foreign debt (and thus the

position of the AA curve), it is possible that the country's real interest rate in the long run exceeds the initial rate r_0 . But the country will undoubtedly experience real depreciation in the long run.

4.6.4 A Permanent Rise in the Foreign Interest Rate

With a rise in the foreign real interest rate, asset market equilibrium curve AA in Figure 4.6 shifts to $A'A'$. At (a) the country now experiences a portfolio shift toward foreign assets leading to a depreciation of domestic currency. There is a decrease in the price of domestic

Figure 4.6 Adjustments to a Rise in Foreign Interest Rate

assets thus raising the domestic interest rate, and eventually, decreasing capital accumulation. Assuming that in initial equilibrium the country had zero foreign debt, at (b) it has a positive net foreign asset position with lower capital stock. Over time, excess demand in the goods market will cause a leftward shift in the GG curve and a fall in the risk premium (due to a positive net foreign asset position) will cause $A'A'$ curve start shifting back. It is possible that in the new long run equilibrium, at a point such as (c), the country experiences a real

appreciation of currency to offset its positive investment income account by a negative trade balance.

4.7 Concluding Comments

In this chapter, we use goods and assets market equilibrium conditions to formulate a simple open economy model that incorporates imperfect capital flows. We use the model to analyse the dynamics of the real exchange rate, the current account and foreign debt. The objective behind this model is to give the asset market a key role in dynamic models. We have attempted to implement the idea of Dooley and Isard (1991) that a change in net international creditor position requires a current account imbalance. Thus, for example, a rise in the net capital flow into a country requires an equal reduction in the current account surplus, or a rise in the current account deficit of the country. The role of the real exchange rate in this framework is to accommodate this current account deficit (or result in a reduction in current account surplus). We have used our model to show that a rise in a current account deficit with the resulting gradual rise in foreign debt with an eventual real exchange rate depreciation can initially be triggered by a disturbance in the asset market.¹⁹ Such a disturbance can initially appreciate the real value of domestic currency (see section 4.6.3). Thus in contrast with many real exchange rate models (e.g. Stein (1994)) that ignore the asset market, the present model shows that an asset market disturbance, even one that originates in the foreign country, can affect the real exchange rate of the country. In deriving the key results in the present model, we emphasize an imperfect international capital market by

¹⁹For example, a fall in the risk premium.

allowing for a risk premium in the asset market equilibrium condition. We have used our framework to analyse the effects on the real exchange rate of a number of other demand and supply shocks.

The difficulty we faced in the present model in deriving the results is a common one in this kind of model. We have to assume the stability of the model to derive its predictions, and it is difficult to validate empirically such assumptions. This leads us to a promising research methodology that has been popularised by the real business cycle modelers. In the next chapter, we borrow the numerical solution methods from the real business cycle theory to empirically implement a more rigorously grounded model that encompasses a more complete structure of an open economy with imperfect capital flows.

5 The Real Exchange Rate, the Current Account, and Foreign Debt in a Stochastic Optimizing Model with Imperfect Capital Flows

5.1 Introduction

The last two decades have witnessed a large volume of work directed at modeling the dynamics of the real exchange rate, the current account and asset accumulation, and long-run external indebtedness in response to various demand and supply shocks in an intertemporal optimizing setting. A partial list of this work includes Buiter (1987), Brock (1988), Frenkel and Razin (1987), Matsuyama (1987), Obstfeld (1982, 1989), Gavin (1992), Sen and Turnovsky (1989), Devereux and Shi (1991), Murphy (1989), Karayalcin (1994), and Serven (1995). While the disturbances studied in these papers are different, all of these papers share a common methodology: they derive the underlying behavioural relationships from intertemporal optimization by a representative agent.¹ These models also assume the existence of a perfect world capital market for debt so that a country can borrow or lend as much as desired at the given world interest rate.² This implies that a steady state will be attained if and only if the rate of time preference becomes equal to the parametrically given

¹Frenkel and Razin (1987), Buiter (1987), Brock (1988), Obstfeld (1989), Murphy (1989), Karayalcin (1994), Serven (1995), Devereux and Shi (1991) examine various fiscal shocks; Gavin considers monetary shocks; Matsuyama (1987) considers a shock in the form of a permanent change in the price of imported intermediate goods, and Sen and Turnovsky (1989) study the effects of tariffs. Devereux and Shi and Karayalcin also examine the effects of productivity shocks.

²There is a class of models that assumes an imperfect capital market for debts in analyzing growth and debt problems of highly indebted countries. See, for example, Bardhan (1967) for an early contribution in this area. A more recent example is Bhandari, Haque, and Turnovsky (1990). These authors, however, do not consider the dynamic interactions of the real exchange rate, the current account and foreign debt that is the main focus of the present work.

world interest rate. If the two rates are initially different, the system has no equilibrium unless one or both rates are endogenous. Some authors simply assume that the two rates are always equal.³ However, for any given set of initial conditions determining the strength of time preference and the world interest rate, this outcome would be an improbable accident. Devereux and Shi (1991), and Karayalcin (1994) used recursive preferences to circumvent this problem by allowing the implied rate of time preference to be endogenous. This requires the rate of time preference to increase with the level of utility, and therefore, with the level of consumption. Such an assumption is not particularly appealing and has been subject to criticism.⁴ A second way of bringing the two rates into equality is to follow Yaari (1965) and Blanchard (1985) and assume a finite probability of death.⁵

Given the evidence of chapters 2 and 3 relating to the violations of real interest rate parity, we adopt a relatively new and more realistic approach in this chapter. We develop an optimizing model of a small open economy to analyse the dynamics of the real exchange rate, the current account and debt accumulation with constrained access to the world capital market.⁶ In this structure, the real rate of interest paid on foreign debt is increasing in the stock of foreign debt.⁷ Our objective is to examine the impact of technology and terms-of-

³See, for example, Froot and Rogoff (1991), Turnovsky (1993), Murphy (1989) and Serven (1995).

⁴See, for example, Turnovsky (1997, p. 39).

⁵Cardia (1991) and Macklem (1993) adopt this approach.

⁶Pitchford (1995) develops a simple one-sector open economy model of intertemporal choice assuming that real interest rate is an increasing function of total borrowing. Harjes (1997) applies a one-sector open economy real business cycle model to German data, and assumes that the rate of return on foreign bonds is decreasing in the stock of foreign bonds in Germany. Although an essential analytical tool of aggregate economics, the one-sector framework is of limited use in an international environment where exogenous shocks affect different sectors of the economy differently.

⁷This assumption may not at all seem unrealistic given the evidence reported by Bayoumi et al. (1995) from risky U.S. states facing the non-linear debt supply curve. Hermalin and Rose (1999) provide an exhaustive analysis of how risks present in the domestic capital markets tend to be exacerbated in the international context. In their

trade shocks on the real exchange rate and the current account. In this model, the intertemporal efficiency conditions place restrictions on the rate of growth of consumption and the consumption rate of interest. Given that the rate of interest on international borrowing is increasing in the stock of foreign debt, these efficiency conditions govern the rate of change of savings and investment, and, therefore, the size of the current account.

To compute the equilibrium stochastic processes of a three-sector intertemporal model, we borrow numerical solution methods from real business cycle theory. The model developed, however, is distinct from a typical real business cycle model in a number of important respects. Charles Plosser (1989) describes real business cycle models as asking the question: “how do rational maximizing individuals respond over time to changes in the economic environment and what implications do those responses have for the equilibrium outcomes of aggregate variables?” The aggregate variables are then viewed as being the outcomes of decisions made by many individual agents maximizing utility subject to their production and resource constraints. For more than a decade and a half now, particularly since the publications of Kydland and Prescott (1982) and Long and Plosser (1983), this school of thought has been the focus of attention of a significant portion of the research on dynamic macroeconomics. More recently, however, Gregory Mankiw (1989), while acknowledging that ‘real business cycle theory has served the important function of stimulating and provoking scientific debate’, predicts that it will ‘ultimately be discarded as

application of the Bank’s Quarterly Projection Model (QPM), economists at the Bank of Canada (see Macklem et al. 1995) have recently incorporated the impact of the debt and deficits on risk premiums by calibrating an increase in the risk premium by 1.7 basis points for one percentage point increase in the debt-to-GDP ratio, when the debt-to-GDP ratio is above 50 percent. This calibration is based on the evidence reported by Alesina et al. (1993) for 12 OECD countries for the period 1979 to 1989. Although Macklem et al. analysis applies to government debt, they also show that Canada’s foreign debt is largely driven by the need to finance the government debt. Indeed, the authors show that, if there were no government debt, Canada would be a net foreign creditor.

an explanation of observed fluctuations'. According to Mankiw, the two fundamental weaknesses of this approach are its reliance on the intertemporal substitution of leisure to explain changes in employment and its heavy reliance on technological disturbances as the primary cause of economic fluctuations.

While our model assumes that productivity disturbances motivate rational individuals to adjust savings and investment in order to smooth consumption over time, it does not rely on intertemporal substitution of leisure to explain economic fluctuations. In our model, economic fluctuations arise from productivity and terms of trade disturbances. McCallum (1989) observes that when business cycle models do not include a foreign sector and ignore the terms-of-trade shocks that affect the real price of imported raw materials, these shocks may appear to be productivity shocks. It is now widely believed that large and recurrent fluctuation in the terms of trade is an important driving force of recent economic fluctuations.

The paper that is closest to what we are doing is that of Mendoza (1995). His analysis is perhaps the most effective analysis of open economy real business cycles that encompass evidence for the group of seven largest industrialized countries (G-7) and 23 developing countries. While Mendoza's is not the first real business cycle analysis of the open economy, it is more realistic in that it does not impose purchasing power parity and real interest rate parity to induce near-perfect correlation between consumption and GDP.⁸ We begin from

⁸Mendoza (1991) examines the effects of terms-of-trade shocks in the real business cycle model of a small open economy with no non-traded goods assuming the existence of a perfect international capital market. Correia et al (1995) use a similar one-sector open economy model. Without a non-traded goods sector, these models fail to account for the substitution effects induced by changes in intra- and intertemporal relative prices of non-traded goods. Macklem (1993) applies a small open economy overlapping generations model to Canadian data to explore the steady state and dynamic effects of terms-of-trade disturbances and government deficits. Stockman and Tesar (1995) is a two-country real business cycle model with non-traded goods. Backus, et al (1994, 1995), and, Baxter and Crucini (1995) use two-country models without a non-traded goods sector.

the same three-sector framework as Mendoza but make some key departures. Firstly, while Mendoza (1995) assumes that agents in the small open economy have access to a perfect world capital market and uses preferences to endogenously bring the rate of time preference into line, we assume that the rate of time preference is fixed and that the real rate of interest, which is increasing in the stock of foreign debt, comes into line.⁹ Secondly, as already mentioned, we do not use intertemporal substitution of labor to propagate economic fluctuations through time. Finally, while Mendoza assumes that capital stock is composed entirely of imports, we assume that investment is a composite commodity with inputs of domestic and foreign goods. This is important in view of the fact that the response of investment to a terms-of-trade shock is likely to be proportionate to the share of imports in total investment expenditure.

The resulting model presents the following general approach. Rational optimizing agents smooth consumption intertemporally and so adjust savings and investment decisions to both productivity and terms-of-trade shocks. All key variables, including the real exchange rate, are thus the outcome of such optimizing decisions. To determine whether the model can mimic the stylized facts of an actual economy and hence become relevant for policy analysis, we parameterize, simulate and calibrate the model to Canadian data.

In the next section, the production technologies, capital accumulation equations, and stochastic processes are laid out together with the preferences of an optimizing consumer-investor. The model's *intra-* and *intertemporal* optimizing equilibrium conditions are then derived. The model is calibrated, parameterized and solved in section 5.3. In section 5.4, we

⁹Note that in Mendoza's model, purchasing power and real interest rate non-parities arise from the inclusion of non-traded goods even in the presence of a perfect world capital market.

present the results obtained from the parameterized model and examine its impulse response functions and sensitivities to variations in key structural parameters. Its simulated moments are then compared with Canadian data to see if the model mimics the stylized facts of the Canadian economy. Section 5.5 is devoted to an analysis of this chapter's key contribution. It shows how dropping the assumption of a perfect world capital market affects the results of a typical dynamic model of an open economy. Section 5.6 presents the conclusions.

5.2 The Model

We consider an economy with three goods: an import good, an export good, and a nontraded good. The country has a fixed labor force of size N that is mobile between its nontraded and export sectors. The representative consumer-investor in this economy is an infinitely-lived agent who owns a $(1/N)$ th share in all firms, consumes all three goods, and inelastically supplies her entire labor endowment. Although labor is costlessly mobile between the two production sectors, capital is sector-specific. To accumulate capital, both sectors are assumed to need both exports and imports.¹⁰ We assume that the country is initially a net-debtor in the international capital market, and can borrow further at a rate of interest that is increasing in its current stock of debt. While the price of the nontraded good varies endogenously in response to local fluctuations in supply and demand, the small open economy faces exogenously determined terms-of-trade. The source of sustained growth in this economy is labour-augmenting technological change that grows at a constant rate (to be

¹⁰A more complete analysis of capital formation would include nontraded goods as inputs as well. We, however, exclude such goods here to simplify our analysis.

specified shortly). It also experiences temporary shocks to total factor productivity in both production sectors.

5.2.1 Technologies and Shock Processes

The technologies used by the two production sectors are given by:¹¹

$$\begin{aligned} Y_{ht} &= Z_{ht} K_{ht}^{\rho} (X_t N_{ht})^{1-\rho} \\ Y_{xt} &= Z_{xt} K_{xt}^{\rho} (X_t N_{xt})^{1-\rho}, \end{aligned} \tag{5.1}$$

where all non-price variables are in per-capita terms.¹² Y_{ht} and Y_{xt} are, respectively, the outputs of the nontraded and export goods, X_t is the efficiency level of labour, subject to labor-augmenting technological change, Z_{it} is the stochastic parameter of the production process, K_{it} is per capita capital stock and N_{it} is the fraction of the labor-force employed in sector i , ($i = x, h$). Both production functions are concave, twice continuously differentiable, increasing in their arguments, and both display constant returns to scale in the inputs capital and effective labour, i.e. K and XN . In both production functions, $0 < \rho \leq 1$. The stochastic parameter is assumed to be observed at the beginning of the period. It has been amply documented that the parameter displays considerable serial correlation with the first differences nearly serially uncorrelated. To reflect this high persistence, we assume that the stochastic technology parameter evolves exogenously according to the following law of

¹¹We use the letters h , x , and m as subscripts to relate a variable to the nontraded good, the export good, and the import good, respectively. The subscript t denotes the time period. The absence of any such time subscript indicates the steady state level of the variable.

¹²In what follows, all variables, except, of course, interest rate and the two prices, are measured in per-capita terms, unless otherwise indicated.

motion:¹³

$$\begin{aligned}\log Z_{it} &= (1 - \psi_i) \log Z_i + \psi_i \log Z_{it-1} + \varepsilon_{it}, \\ \varepsilon_{it} &\sim i.i.d. \mathcal{N}(0; \sigma_i^2),\end{aligned}\tag{5.2}$$

In the above specification, $\log(Z_i)$ is the unconditional mean of the process, and $0 < \psi_i < 1$, ($i = x, h$). On the other hand, labor-augmenting technological progress takes place at a constant gross rate γ so that

$$X_t = \gamma X_{t-1} = \gamma^t X_0 \tag{5.3}$$

where X_0 is the initial level of the labor-augmenting technology.

The remaining source of uncertainty in this economy is its exogenously determined terms-of-trade. Like the shocks to productivity, terms-of-trade shocks are also highly persistent. They are also assumed to be temporary, evolving according to the following stochastic process:

$$\begin{aligned}\log p_{mt} &= (1 - \psi_m) \log p_m + \psi_m \log p_{mt-1} + \varepsilon_{mt}, \\ \varepsilon_{mt} &\sim i.i.d. \mathcal{N}(0; \sigma_m^2), \quad 0 < \psi_m < 1\end{aligned}$$

where p_{mt} is the terms-of-trade at time t , and p_m is the steady-state level of p_{mt} , and ε_{mt} is an independently and identically distributed period-specific innovation in p_{mt} . The terms-of-trade is defined here as the relative price of the import good to the export good.

¹³Note that this specification ensures that the technology components converge to their mean levels over time.

5.2.2 Capital Accumulation

We want to model the capital accumulation process in a realistic manner so that the model can generate appropriate responses of its production sectors to exogenous shocks. The assumption that all factors of production are perfectly mobile across production sectors has been fundamental for many classic results in economics.¹⁴ While the assumption that in the long-run all factors are effectively mobile is not unreasonable, the ex-post malleability of capital is unrealistic for the shorter run. Mussa (1978) examines the adjustment process through which capital can be reallocated across sectors. Neary (1978) notes that reallocation of capital rarely takes the form of physical movement of installed capital from one sector's use to another. In reality, this is accomplished by investment in new capital goods in one sector and the depreciation of old capital goods in the other. This is a long-run process.

We assume that capital is sector-specific and both sectors accumulate capital using a *homogeneous of degree one* investment technology that combines exports and imports into investment goods:¹⁵

$$I_{it} = j_{ixt}^{\theta_i} j_{imt}^{(1-\theta_i)}, \quad i = h, x, \quad (5.4)$$

where j_{ixt} and j_{imt} , are, respectively, the quantities of the export-good and the import-good used as inputs into the investment process of the i th sector, and $0 < \theta_i < 1$ is the share of

¹⁴Thus, many of the classic results from the Heckscher-Ohlin-Samuelson model rely on this assumption. Based on similar assumptions, the Balassa-Samuelson model [Balassa (1964), Samuelson (1964)] ends up with the conclusion that demand factors have no effects on the relative price of the nontraded good. With sector-specific capital, demand factors can affect the relative price of the nontraded good. Rogoff (1992) derives such results with fixed stocks of capital and labor in the traded and the nontraded sectors.

¹⁵Gavin (1992) and Serven (1995) following the analysis of Hayashi and Inoue (1991) also consider imported investment good as an input in more general forms into the investment process in one-sector Solow-type models with analysis restricted in the traditional state-space diagrams. Little is known about the quantitative implications of this approach.

exports in I_{it} , the investment in the i th sector. Note that although the two sectors share the same technology for combining exports and imports into capital, we allow the share of exports in the capital of the two sectors to differ.

The optimization problem of the representative consumer-investor with respect to capital accumulation can now be viewed as a “two-stage” process. The first stage determines the optimal time paths for investments, I_{it} ; the second stage determines the cost-minimizing quantities of j_{ixt} and j_{imt} :

$$\begin{aligned} j_{imt}^* &= \left(\frac{1 - \vartheta_i}{\vartheta_i p_{mt}} \right)^{\vartheta_i} I_{it} \\ j_{ixt}^* &= \left(\frac{\vartheta_i p_{mt}}{1 - \vartheta_i} \right)^{1 - \vartheta_i} I_{it} \end{aligned} \quad (5.5)$$

The associated price q_{it} of investment in the i th sector can be found as the minimized unit-cost function:¹⁶

$$\begin{aligned} q_{it} &= \Phi_i p_{mt}^{1 - \vartheta_i} ; \\ \Phi_i &= \left[\left((1 - \vartheta_i) / \vartheta_i \right)^{\vartheta_i} + \left(\vartheta_i / (1 - \vartheta_i) \right)^{1 - \vartheta_i} \right]. \end{aligned} \quad (5.6)$$

Once capital goods are procured, additional costs are assumed to be incurred for the installation of capital. Absent these installation costs, investment would become passive, as whatever is saved could be added to the capital stock (if not used to pay off debt) at no cost, to equate capital's marginal product to the rate of interest that the country faces. Capital installation costs reflect the facts that capital stocks move sluggishly and that protracted

¹⁶The minimized unit-cost function in (5.6) is derived in Appendix 5A.

deviations of capital's marginal product from the real rate of interest are possible. We assume here, for analytical simplicity, that adjustment costs are incurred only when investment is above or below its steady-state level:

$$g_{it} = \frac{\phi_i q_{it}}{2} \left[\frac{I_{it}}{K_{it}} - \frac{I_i}{K_i} \right]^2 K_{it} \quad (5.7)$$

where g_{it} is the installation cost. The parameter $\phi_i > 1$ makes the installation cost increasing in investment relative to the existing capital stock. Investment at time t becomes productive capital at time $t+1$. In particular, the capital stock in the i th sector evolves according to the law of motion:

$$K_{it+1} = K_{it} + I_{it} - \delta_i K_{it}, \quad 0 < \delta_i < 1, \quad (5.8)$$

where δ_i is the rate of depreciation.

5.2.3 Preferences

The infinitely-lived representative consumer-investor inelastically supplies her entire labor endowment and owns a per-capita share in all the firms in the economy, which entitles her to a flow of profits accruing to each sector. The agent's objective is to maximize discounted expected lifetime utility given by:

$$U(C) = E_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right], \quad 0 < \beta < 1, \quad (5.9)$$

where u is concave, strictly increasing, and twice continuously differentiable in consumption

in period t , C_t with $t = 0, 1, 2, \dots, \infty$. Here β is the agent's discount factor and C_t is a real consumption bundle (to be specified below). Given that consumers own all factor shares, the representative consumer faces a difficult problem of choosing the paths of consumption and investment in an environment with uncertainty about future productivity and terms-of-trade so as to maximize (5.9). These choices are based on expectations over future prices, and are subject to the laws of motion for capital stocks in the two sectors (5.8) and sequences of budget constraints:¹⁷

$$\begin{aligned}
 F_t = & R_t F_{t-1} + P_t C_t + q_{xt} \left[I_{xt} + \frac{\phi_x}{2} \left(\frac{I_{xt}}{K_{xt}} - \frac{I_x}{K_x} \right)^2 K_{xt} \right] - Z_{xt} K_{xt}^\rho (X_t N_{xt})^{1-\rho} \\
 & + q_{ht} \left[I_{ht} + \frac{\phi_h}{2} \left(\frac{I_{ht}}{K_{ht}} - \frac{I_h}{K_h} \right)^2 K_{ht} \right] - p_{ht} Z_{ht} K_{ht}^\rho (X_t N_{ht})^{1-\rho}
 \end{aligned} \tag{5.10}$$

given initial stocks F_0 , K_{h0} , and K_{x0} , and the transversality conditions¹⁸:

$$\begin{aligned}
 \lim_{T \rightarrow \infty} E_t \prod_{t=0}^T R_t^{-1} F_t &= 0 \\
 \lim_{T \rightarrow \infty} E_t \prod_{t=0}^T R_t^{-1} q_{it} K_{it} &= 0.
 \end{aligned} \tag{5.11}$$

In the above specification, F_t is the representative agent's debt, which grows according to the discrepancy between the value of absorption and of income, P_t is the consumption-based price index, p_{ht} is the relative price of the nontraded good in terms of the export good, and $R_t = [1 + r(F_{t-1})]$ is the gross real rate of interest on foreign debt. Note that with the absence of

¹⁷One can re-write the constraint in (5.10) to express the current account deficit as absorption (in terms of consumption and investment) over income or output.

¹⁸The conditions in (5.11) rule out the possibility of explosive solutions.

any government, the consumers' debt is identical to the country's net foreign indebtedness, and the growth of that indebtedness is the current account deficit of the country.¹⁹

The representative agent's contemporaneous utility is given by:

$$u(C_t) = \frac{1}{1-\eta} C_t^{1-\eta} \quad (5.12)$$

where $\eta > 0$ is the coefficient of relative risk aversion. The real consumption bundle C_t is given by:

$$C_t = \left[C_{nt}^{-\mu} + (C_{xt}^\theta C_{mt}^{1-\theta})^{-\mu} \right]^{-\frac{1}{\mu}} \quad (5.13)$$

where C_{nt} , C_{xt} , and C_{mt} are, respectively, the consumption of nontradables, exportables, and importables.²⁰ In the above specification, tradables and nontradables are represented in constant-elasticity-of-substitution (CES) form, with $[1/(1+\mu)]$ being the elasticity of substitution, and tradables are expressed in Cobb-Douglas unitary-elasticity form, with θ being the share of exportables in tradables expenditure. The intertemporal elasticity of substitution in aggregate consumption is given by $1/\eta$. Given any optimal total consumption level C_t obtained by solving the intertemporal allocation problem specified above, demands for the three goods are obtained by solving a static problem of *intratemporal* substitution to minimize the consumption cost in terms of exportables.²¹ This static optimization gives:²²

¹⁹The inclusion of a government in the model with government debt complicates the analysis without adding insight into the impact of productivity and terms-of-trade shocks.

²⁰We ignore the share parameters of traded and nontraded goods in the consumption bundle in (5.13) without loss of generality.

²¹See Obstfeld and Rogoff (1995, p. 1752-1755) for derivation of similar *inter*- and *intra*-temporally optimizing consumption levels in a two-good closed-economy model.

²²The static optimization problem is solved in Appendix 5B.

$$\begin{aligned}
C_{ht} &= \left[p_{ht} \theta^{-\mu\theta} (1-\theta)^{1+\mu\theta} \right]^{\frac{-1}{1+\mu}} p_{mt}^{\frac{1-\mu\theta}{1+\mu}} C_{mt} \\
C_{xt} &= \left(\frac{\theta}{1+\theta} \right) p_{mt} C_{mt} \\
C_{mt} &= C_t \left[\left(\frac{\theta p_{mt}}{1-\theta} \right)^{-\mu\theta} + \left\{ p_{ht} \theta^{-\mu\theta} (1-\theta)^{1+\mu\theta} \right\}^{\frac{\mu}{1+\mu}} p_{mt}^{-\mu \left(\frac{1-\mu\theta}{1+\mu} \right)} \right]^{1/\mu}
\end{aligned} \tag{5.14}$$

As already mentioned, p_{ht} and p_{mt} are, respectively, the current relative prices of nontraded and import goods in terms of exports.

The consumption-based price index P_t measures the least expenditure needed to buy a unit of the consumption index, on which instantaneous utility depends. The index is given by:

$$P_t = \left[\left\{ \kappa p_{mt}^{1-\theta} \right\}^{\mu/(1+\mu)} + p_{ht}^{\mu/(1+\mu)} \right]^{(1+\mu)/\mu} \tag{5.15}$$

where $\kappa = \theta^{-\theta} (1-\theta)^{-(1-\theta)}$.

5.2.4 The Non-traded Goods Market Equilibrium

Equilibrium in the non-traded goods market requires that the domestic demand for such goods equals its supply. Using the expression for P_t in (5.15), the consumption demand for the imported good can be written as:

$$C_{mt} = \theta^{\frac{-\mu^2\theta}{1+\mu}} \left(\frac{p_{mt}}{1-\theta} \right)^{\frac{-\mu(1-\mu\theta)}{1+\mu}} P_t^{\frac{\mu}{(1+\mu)}} C_t$$

Now substituting this expression into the first equation in (5.14), the non-traded goods market equilibrium condition can be obtained as:²³

$$\begin{aligned} & \left[p_{ht} \theta^{-\mu\theta} (1-\theta)^{1+\mu\theta} \right]^{-1/(1-\mu)} p_{mt}^{\frac{1+\mu\theta}{1+\mu}} \theta^{\frac{-\mu^2\theta}{1+\mu}} \left(\frac{p_{mt}}{1-\theta} \right)^{\frac{-\mu(1+\mu\theta)}{1+\mu}} P_t^{\frac{\mu}{1+\mu}} c_t \\ & = Z_{ht} k_{ht}^\rho N_{ht}^{1-\rho} \end{aligned} \quad (5.16)$$

Labor demand in the non-traded production is given by:

$$N_{ht} = \left[\frac{(1-\rho) p_{ht} Z_{ht} k_{ht}^\rho}{W_t} \right]^{1/\rho} \quad (5.17)$$

where W_t is the economy-wide wage rate in terms of the export good. The assumption that labor can migrate instantaneously between sectors ensures a unique economy-wide wage rate:

$$W_t = (1-\rho) \left[\left(Z_{xt} k_{xt}^\rho \right)^{1/\rho} + \left(p_{ht} Z_{ht} k_{ht}^\rho \right)^{1/\rho} \right]^\rho \quad (5.18)$$

Now one can substitute (5.18) into (5.17), and use the resulting equation along with (5.15) and (5.16) to solve for the equilibrium price index, the relative price of non-traded goods, and the wage rate. With the intratemporal allocation rules in (5.5) and (5.14), the consumer-investor's *intertemporal* decision can be analyzed entirely in terms of composite consumption, C_t , investments in the two production sectors, I_{it} , ($i = x, h$), and the price-index, P_t . The behavior of the *intra-* and *inter-temporal* optimizing price index P_t can be compared

²³Here we de-trend consumption and capital stock variables by the labor-augmenting technology component, and use lower-case letters to denote the de-trended per-capita variables. This is done here for convenience ahead of time for later use. See Appendix 5B for intra-temporal optimization and labor market equilibrium conditions.

with the behavior of the actual real exchange rate.²⁴ Real exchange rate's key intertemporal link in the context of this small open economy is as follows. Shocks to technology or to terms-of-trade cause fluctuations in the relative price of the nontraded good and, thus, in the consumption-based price index. This, in turn, affects the consumer's "effective" real interest rate. The representative consumer-investor then adjusts her consumption-investment decisions in order to smooth consumption intertemporally.

We now have all the inputs needed to explore the model's intertemporal optimization conditions. The next step is to solve the model to obtain its recursive equilibrium laws of motion. These will be used to examine the implications of both terms-of-trade and technology shocks for the real exchange rate and other key macro variables.

5.2.5 Intertemporal Optimization

The consumer-investor's intertemporal optimization problem involves the maximization of the following Lagrangian by choosing C_t , K_{xt-1} , K_{ht-1} , F_t , I_{xt} , and I_{ht} :²⁵

²⁴In empirical studies, the real exchange rate is typically defined as the ratio of a domestic and foreign aggregate price index both measured in units of the same currency. Given the absence of any nominal exchange rate in the present model, we can define an analogous measure of the real exchange rate: $e_t = p_{mt} P_t^* / P_t$, where P^* is the foreign counterpart of P , the consumption-based price index. Notice that e_t gives the price of a unit of the foreign consumption index in terms of units of the domestic consumption index.

²⁵We continue to follow the convention that the time subscript t indicates the level of a variable at time t , while the absence of any such time subscript indicates the steady state level of the variable.

$$\mathcal{L}_t = E_t \sum_{t=0}^{\infty} \beta^t \left[\begin{aligned} & \left(\frac{1}{1-\eta} C_t^{1-\eta} \right) \\ & - \lambda_t \left\{ \begin{aligned} & P_t C_t + q_{xt} I_{xt} + \frac{\phi_x q_{xt}}{2} \left(\frac{I_{xt}}{K_{xt}} - \frac{I_x}{K_x} \right)^2 K_{xt} - F_t - Z_{xt} K_{xt}^\rho (X_t N_{xt})^{1-\rho} \\ & + R_t F_{t-1} + q_{ht} I_{ht} + \frac{\phi_h q_{ht}}{2} \left(\frac{I_{ht}}{K_{ht}} - \frac{I_h}{K_h} \right)^2 K_{ht} - p_{ht} Z_{ht} K_{ht}^\rho (X_t N_{ht})^{1-\rho} \end{aligned} \right\} \\ & - Q_{xt} \{ K_{xt+1} - I_{xt} - (1-\delta_x) K_{xt} \} \\ & - Q_{ht} \{ K_{ht+1} - I_{ht} - (1-\delta_h) K_{ht} \} \end{aligned} \right] \end{aligned}$$

The specification of labor-augmenting technological progress at the gross rate γ implies a mean steady-state growth at the same rate. It will therefore be convenient to transform the variables into stationary ones by rescaling them as $c = C/X$, $k = K/X$, $i = I/X$, etc. With the rescaled variables, the Lagrangian can be re-written as:

$$\mathcal{L}_t = E_t \left[\beta^{*t} \sum_{t=0}^{\infty} \left\{ -\lambda_t \left\{ \begin{aligned} & X_0^{1-\eta} \left(\frac{1}{1-\eta} c_t^{1-\eta} \right) \\ & P_t c_t + q_{xt} i_{xt} + \frac{\phi_x q_{xt}}{2} \left(\frac{i_{xt}}{k_{xt}} - \frac{i_x}{k_x} \right)^2 k_{xt} + (R_t/\gamma) f_{t-1} - f_t \\ & - Z_{xt} k_{xt}^\rho - p_{ht} Z_{ht} k_{ht}^\rho + q_{ht} i_{ht} + \frac{\phi_h q_{ht}}{2} \left(\frac{i_{ht}}{k_{xt}} - \frac{i_h}{k_h} \right)^2 k_{ht} \end{aligned} \right\} \right. \right. \quad (5.19) \\ \left. \left. - Q_{xt} (\gamma k_{xt+1} - (1-\delta_x) k_{xt} - i_{xt}) - Q_{ht} (\gamma k_{ht+1} - (1-\delta_h) k_{ht} - i_{ht}) \right\} \right]$$

where γ is the gross rate of growth in the labor-augmenting technological progress as defined in (5.3), and $\beta^* = \beta\gamma^{1-\eta}$ is the consumer's effective discount factor.²⁶

²⁶Note that in above formulation we assume that the consumer-investor takes the intratemporally optimal fractions of the labour force employed in the two production sectors as given, and exclude them from the intertemporal optimization problem. Given that we do not rely on the intertemporal substitution of labour to explain economic fluctuations, inclusion of labour is not central to any intertemporal adjustment process in the present

The first-order efficiency conditions are obtained by differentiating the Lagrangian with respect to the choice variables c_t , f_t , i_{xt} , i_{ht} , k_{xt-1} , k_{ht-1} and the multipliers, λ_t , Q_{xt} and Q_{ht} at each time t . This is done fully in Appendix 5C. It is, however, important to note here that although the small open economy faces a real rate of interest that is increasing in the stock of its foreign debt, the representative agent, in deriving her optimality conditions, takes the real rate of interest as given. The agent is “small” relative to the economy, and, in making her decisions, she assumes that she is unable to influence the real rate of interest.

5.3 The Solution of the Model

5.3.1 Log-linearization

A look at the first-order conditions associated with the intertemporal optimization problem derived in Appendix 5C indicates that these conditions are nonlinear in the decision variables, and, in general, do not lend themselves to an exact solution. A standard approximate solution procedure is to compute the nonstochastic steady state values of the variables of the system, and then log-linearize the first-order conditions, as well as other equations, around the nonstochastic steady state values. The model then becomes a system of log-linear expectational difference equations in the capital stock, the foreign debt, and the exogenous variables driving the economy. The resulting system of equations can be solved using *the method of undetermined coefficients* for the laws of motion for the state variables k_{xt} , k_{ht} , and f_t , as well as linear relations connecting other variables such as P_t , p_{ht} , c_t , etc.²⁷

model.

²⁷The method has been presented in extensive details by Uhlig (1997), Christiano (1998), and Campbell (1994). There are some alternative methods as well. Taylor and Uhlig (1990) provide a good review and comparison of these alternative methods.

Box 5.1 The Log-linearized Model

- 1)
$$0 = y_x \tilde{y}_{xt} + (p_h y_h) \tilde{p}_{ht} + (p_h y_h) \tilde{y}_{ht} - (Pc) \tilde{p}_t - (Pc) \tilde{c}_t - (\alpha_0 + \alpha_1) \tilde{p}_{mt} \\ - i_x q_x \tilde{i}_{xt} - i_h q_h \tilde{i}_{ht} - (Rf/\gamma)(\tilde{r}_t + \tilde{f}_{t-1}) + f \tilde{f}_t$$
- 2)
$$0 = \tilde{y}_{xt} - \tilde{z}_{xt} - \rho \tilde{k}_{xt}$$
- 3)
$$0 = \tilde{y}_{ht} - \tilde{z}_{ht} - \rho \tilde{k}_{ht}$$
- 4)
$$0 = \gamma \tilde{k}_{xt-1} - (1 - \delta_x) \tilde{k}_{xt} - (\gamma - 1 + \delta_x) \tilde{i}_{xt}$$
- 5)
$$0 = \gamma \tilde{k}_{ht-1} - (1 - \delta_h) \tilde{k}_{ht} - (\gamma - 1 + \delta_h) \tilde{i}_{ht}$$
- 6)
$$0 = P^{\mu/(1-\mu)} \tilde{p}_t - \alpha_2 \tilde{p}_{mt} - p_h^{\mu/(1+\mu)} \tilde{p}_{ht}$$
- 7)
$$0 = (1 - \mu) a_1 \tilde{p}_{mt} - (a_0 + a_3) \tilde{p}_{ht} + a_6 \tilde{p}_t + \tilde{c}_t \\ + a_2 \tilde{k}_{xt} - a_4 \tilde{z}_{ht} - \rho a_4 \tilde{k}_{ht} + a_5 \tilde{z}_{xt}$$
- 8)
$$0 = \tilde{r}_t - \omega \tilde{f}_{t-1}$$
- 9)
$$0 = E_t [\eta \tilde{c}_t - \eta \tilde{c}_{t-1} + \tilde{p}_t - \tilde{p}_{t+1} + \tilde{r}_{t-1}]$$
- 10)
$$0 = E_t \left[\gamma q_h \tilde{\lambda}_{t+1} - \gamma q_h \tilde{\lambda}_t + \beta^* \alpha_3 \tilde{i}_{ht+1} + \beta^* \alpha_4 \tilde{k}_{ht+1} + \beta^* \alpha_5 \tilde{z}_{ht+1} + \beta^* \alpha_5 \tilde{p}_{ht+1} \right. \\ \left. + \beta^* a_8 \tilde{p}_{mt+1} - \gamma a_7 \tilde{p}_{mt} + \alpha_3 (\tilde{k}_{ht} - \tilde{i}_{ht}) \right]$$
- 11)
$$0 = E_t \left[\gamma q_x \tilde{\lambda}_{t+1} - \gamma q_x \tilde{\lambda}_t + \beta^* \alpha_6 \tilde{i}_{xt+1} + \beta^* \alpha_7 \tilde{k}_{xt+1} + \beta^* \alpha_8 \tilde{z}_{xt+1} + \beta^* a_{10} \tilde{p}_{mt+1} \right. \\ \left. - \gamma a_9 \tilde{p}_{mt} + \alpha_6 (\tilde{k}_{xt} - \tilde{i}_{xt}) \right]$$
- 12)
$$0 = E_t \left[\alpha_7 \tilde{k}_{xt+1} + (R\alpha_6/\gamma)(\tilde{k}_{xt} - \tilde{i}_{xt}) + \alpha_6 \tilde{i}_{xt+1} + \alpha_8 \tilde{z}_{xt+1} + a_{10} \tilde{p}_{mt+1} - R a_9 \tilde{p}_{mt} \right. \\ \left. - R q_x \tilde{r}_{t+1} \right]$$
- 13)
$$\tilde{z}_{xt} = \psi_x \tilde{z}_{xt-1} + \varepsilon_{xt}$$
- 14)
$$\tilde{z}_{ht} = \psi_h \tilde{z}_{ht-1} + \varepsilon_{ht}$$
- 15)
$$\tilde{p}_{mt} = \psi_m \tilde{p}_{mt-1} + \varepsilon_{mt}$$

The model is log-linearized in some detail in Appendix 5C. Here we just present the results of this exercise in Box 5.1 in an order that is convenient for the application of the method of undetermined coefficients. In log-linearizing the model, we use a short-cut

proposed by Uhlig (1997) that must be noted here.²⁸ Let $\tilde{x}_t$ denote the percentage deviation of any variable x_t from its steady-state value x : $\tilde{x}_t = \log(x_t) - \log(x)$. This can be usefully re-written as $x_t = xe^{\tilde{x}_t} \approx x(1 + \tilde{x}_t) = x + x\tilde{x}_t$. Thus, if $\tilde{x}_t = 0.04$, then x_t is approximately 4 percent above its steady state value.

In Box 5.1, equation (1) presents the feasibility constraint facing the representative consumer-investor. Equations (2) through (5) represent the production functions and the capital accumulation equations for the export good and the nontraded good sectors, respectively. Equations (6) through (8), respectively, capture the price index, the non-traded goods market equilibrium condition consistent with labor market equilibrium, and the real interest rate.²⁹ Notice that, although the optimality conditions of the representative consumer-investor are obtained assuming a given real rate of interest, equation (8) introduces the effect on the rate of interest of any change in debt from its steady-state level. Given that we analyze the dynamics of the model-economy using the model log-linearized around its steady-state, this is the ideal place to introduce the effect of debt on the real rate of interest. The expectational equations (9) through (11) in Box (5.1) represent the Euler equations for first consumption and then investment for nontraded and export good sectors. Equation (12) is derived by equating the gross rate of return on investment in the export good sector to

²⁸Note that we use lowercase letters to indicate per-capita levels of variables de-trended by the growth component X_t . Now tildes are used to denote percentage deviations of these de-trended per-capita variables from their steady state levels. There are, however, some exceptions. We use P_t , p_{tr} and p_{nt} , respectively, for the price index, the relative price of non-traded to export goods, and the *terms-of-trade*. The absence of time subscript with any variable is used to indicate the steady state level of the variable. We use r_t for the real rate of interest, while $\tilde{r}_t$ is used to denote the logarithmic deviation of the gross rate of interest from its steady state value: $\tilde{r}_t = \log R_t - \log R$.

²⁹The two ratios τ_0 and $\tau_1 (= 1 - \tau_0)$ that appear in the coefficients a_2 , a_3 , and a_4 (see Box 5.2) in equation (7) in Box 5.1 depend on the steady-state output in the export- and the non-traded good sectors. They are defined in equation (5C.15) in Appendix 5C.

Box 5.2 The Log-linearized Model: Coefficients

$$\begin{aligned}
 \alpha_0 &= i_h(1 - \vartheta_h)q_h, & \alpha_1 &= i_x(1 - \vartheta_x)q_x, & \alpha_2 &= (1 - \theta)(\kappa p_m^{1-\theta})^{\mu/(1-\mu)}, \\
 \kappa &= \theta^{-\theta}(1 - \theta)^{-(1-\theta)}, & \alpha_3 &= \gamma q_h \phi_h(\gamma - 1 + \delta_h), & \alpha_4 &= \{\rho(\rho - 1)(p_h y_h/k_h) - \alpha_3\}, \\
 \alpha_5 &= (\rho p_h y_h/k_h), & \alpha_6 &= \gamma q_x \phi_x(\gamma - 1 + \delta_x), & \alpha_7 &= \{\rho(\rho - 1)(y_x/k_x) - \alpha_6\}, \\
 \alpha_8 &= (\rho y_x/k_x), & \beta^* &= \beta \gamma^{1-\eta}, & a_0 &= 1/(1 + \mu), & a_1 &= a_0(1 + \mu\theta), \\
 a_2 &= (1 - \rho)\tau_0, & a_3 &= (1 - \tau_1)(1 - \rho)/\rho, & a_4 &= (1 - \tau_1 + \rho\tau_1)/\rho, & a_5 &= \tau_0(1 - \rho)/\rho, \\
 a_6 &= \mu a_0, & a_7 &= (\alpha_0/i_h), & a_8 &= a_7(1 - \delta_h), & a_9 &= (\alpha_1/i_x), & a_{10} &= a_9(1 - \delta_x), \\
 \tau_0 &= \frac{(Z_x k_x^\rho)^{1/\rho}}{\left[(Z_x k_x^\rho)^{1/\rho} + (p_h Z_h k_h^\rho)^{1/\rho}\right]}, & \tau_1 &= \frac{(p_h Z_h k_h^\rho)^{1/\rho}}{\left[(Z_x k_x^\rho)^{1/\rho} + (p_h Z_h k_h^\rho)^{1/\rho}\right]}.
 \end{aligned}$$

the gross finance cost of investment.³⁰ Equations (13) through (15), respectively, give the model's exogenous shock processes with respect to technology shocks in the traded, and nontraded sectors, and terms-of-trade shocks. Box 5.2 presents the coefficients associated with the variables in Box 5.1.

With the model log-linearized, we now have a system with an endogenous *state* variable vector $\tilde{S}_t$ of size 4, a vector $\tilde{E}_t$ of 8 other endogenous variables, and a vector $\tilde{Z}_t$ of 3 exogenous stochastic processes.³¹ The objective in applying the *method of undetermined coefficients* is to obtain the matrices P , V , A and B for the system's recursive equilibrium laws of motion:

³⁰Note that the gross rate of return includes the marginal product of capital in production and the marginal reduction in the cost of installing a given flow of capital. See equation (5C.7) in Appendix 5C (and the associated discussion) for the gross return on a one-period investment and the finance cost of investment in terms of the export good.

³¹In solving the model, we take the *co-state* variable λ_t as a state variable in addition to $\tilde{k}_t$, $\tilde{k}_h$ and $\tilde{f}_t$.

$$\begin{aligned}\tilde{S}_t &= P\tilde{S}_{t-1} + V\tilde{Z}_t \\ \tilde{E}_t &= A\tilde{S}_{t-1} + B\tilde{Z}_t\end{aligned}\tag{5.20}$$

The stability of the system requires that the equilibrium described by the rules in (5.20) is stable. As it turns out, the equilibrium will be stable if and only if all eigenvalues of P are smaller than unity in absolute value.³² The solution process requires solving numerically a matrix quadratic equation. The results from this process are described in Section 5.4. Before these results can be obtained, however, we must parameterize the model.

5.3.2 Parameterization of the Model

5.3.2.1 Steady state in the model and stochastic trend in the data

One must exercise caution when parameterizing a model that requires log-linearization around the steady state of the economy. In the parameterization of dynamic models, researchers often use moments calculated with historical data. In the models calibrated to the U.S. data, parameter values are often so set that the models' steady states match the average postwar behavior of the U.S. economy. Thus, King et al. (1988), and Baxter and Crucini (1995), for example, assign the value for β , the discount factor, to yield the steady state annual real interest rate of 6.5 percent, which is the average real return to equity from 1948-1981. The use of long historical moments to characterize economy's steady-state and then viewing the behavior of aggregates as deviations from their steady-state value implies a

³²See Uhlig (1997, section 6) for the stability conditions and the solution procedure. Uhlig shows that while his stability conditions are more straightforward than the usual Blanchard-Kahn conditions, the two approaches are fundamentally equivalent.

steady state that has remained unchanged for a long period, even in the face of constant changes in tastes and technology, and in the structure of the economy. The average real rate of return on long-term government bonds in Canada was 6.2 percent for the period 1948-1981.³³ The average is 7.3 percent for the period 1948-1998, and 9.8 percent for 1982-1998. Figure 5.1 below plots percentage contributions of goods and services producing industries to Canadian GDP for the period 1961-1999. Between the period 1965-1998, the contribution of services producing industries to GDP increased by eight percent while the contribution of the goods producing industries declined by the same magnitude. Note (see equations in 5C.6 in Appendix 5C) that steady-state equilibrium requires $\beta R = \gamma^\eta$. Evidently, with given preference parameters β and η , steady-state equilibrium cannot be consistent simultaneously

Figure 5.1 *Structural Changes in Canadian Economy, 1961-1999*
Contributions as a percentage of GDP of Goods and Services producing industries

³³All data, except the *terms-of-trade* series, are from Statistics Canada's *CANSIM* database. Export and import price deflators used for calculation of the *terms-of-trade* series are obtained from International Monetary Fund's *International Financial Statistics (IFS)*. The choice of *IFS* over *CANSIM* is mainly for convenience of not having to adjust the data due to the change in the base year, or discontinuation of a particular series. The standard unit prices of imports and exports reported in the *IFS* database are frequently used (see, for example, Stockman and Tesar (1995), Mendoza (1995)) to calculate *terms-of-trade* shocks. Appendix 5D provides complete series codes for all data used here.

with the three historical averages of the real rate of interest given above. All this suggests that results based on parameterization of dynamic models using moments from long historical data may be unreliable, especially when the model requires log-linearization around the steady-state of the economy.

In the parameterization of our model, we use annual data for the period 1987-1998, for which dis-aggregated sectoral data on output and employment are available, in order to calculate productivity shocks.³⁴ Since many macroeconomic time series possess a *stochastic* trend, the researcher must remove such a trend from the data before meaningful moments can be computed. The real business cycle modelers almost universally use the linear filter advocated by Hodrick and Prescott (1997) to remove the stochastic trend from the data, and then compare this filtered data with the moments produced by the model.³⁵ The Hodrick-Prescott filter (HP-filter) renders stationary any time series that is integrated of order four or less. Technically, the Hodrick-Prescott filter computes the smoothed series s of any series y by minimizing the variance of y around s , subject to a penalty that constrains the second difference of s :

$$\min_{\{s_t\}} \sum_{t=1}^T (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} ((s_{t-1} - s_t) - (s_t - s_{t-1}))^2$$

The penalty parameter λ controls the smoothness of the series s_t , such that the larger the λ , the smoother the series. In the calibration of the present model, we first take natural

³⁴Note that although disaggregated data on output, capital stock, and investment are available for the period since 1955, similar employment data are available only since 1987. While a longer time period may be desirable, given the above discussion, it may not be particularly appealing for the parameterization of a dynamic model, especially when the model needs log-linearization around its steady-state.

³⁵Hodrick and Prescott first used this filter in a working paper circulated in the early 1980's. The paper was published in 1997. King and Rebelo (1993) provide a detailed discussion of the Hodrick-Prescott filter.

logarithms of all the variables, including the forcing variables. Then all the variables, except the forcing variables, are de-trended applying the HP-filter with the smoothing parameter $\lambda = 10$, although the literature standard is to set $\lambda = 100$ in case of annual data. The reasons for our particular choices are stated below.

King and Rebelo (1993) emphasize that the mechanical application of the HP-filter as a detrending procedure removes important time series components that have traditionally been regarded as representing business cycle phenomena. In particular, in their application of the filter to the U.S. time series, the authors find that HP filtering alters the moment implications of the model in a quantitatively important manner, and this influence is not constant across series. We use the “Solow residuals” in the Canadian goods and services producing industries to calculate the shocks to productivity in the export- and the non-traded goods sectors, respectively.³⁶ Augmented Dickey-Fuller unit root tests applied to these residuals as well as to the terms-of-trade indicate that none of the three forcing variables of our model contains a unit root over the sample period.³⁷ It is, therefore, not necessary to pre-filter these series. Moreover, as Baxter (1995) notes, it is incorrect to pre-filter variables to obtain stationary forcing processes, since the low-frequency and non-stationary components of the shock processes will generally have implications for the behavior of the endogenous variables at all frequencies.

The most common approach to model evaluation in the real business cycle literature has been the comparison of the model’s central moment predictions with corresponding moments

³⁶Section 5.3.3 below describes the measurement of these Solow residuals.

³⁷Calculated test statistics for the Solow residuals in the export- and the non-traded goods sectors, and for the terms-of-trade, respectively, are -6.64, -6.52 and -3.94, while the 5% critical value is -3.22 in all the three cases.

computed from the data. We borrow this approach to evaluate the predictions of the present model. It is, therefore, necessary to remove any non-stationarity present in the variables of the model. As mentioned before, we use the HP-filter with the smoothing parameter $\lambda = 10$. This follows Baxter (1995), who observes that higher values of λ in case of annual data remove too much of the low-frequency variation in a series. Table 5.1 below presents the values of the augmented Dickey-Fuller test statistics applied to the raw (unfiltered) data (all in natural logarithms) as well as to the HP-filtered data with $\lambda = 10$. In the table, all variables, except price and interest rate, are in per-capita terms. In deriving the test statistics presented in Table 5.1, we include a constant and a trend in all equations, except in the equation for the gross rate of interest, in which case only a constant is included. Evidently, the smoothing parameter $\lambda = 10$ removes nonstationarity in all cases, except for consumption and the gross real rate of interest. One can, however, note that the HP-filtering increases

Table 5.1
Augmented Dickey-Fuller Unit-Root Tests of Variables

Variable	Unfiltered	Filtered	Variable	Unfiltered	Filtered
Y_{xt}	-2.48	-8.17	K_{ht}	-2.53	-4.67
Y_{ht}	-1.31	-3.99	P_t	-1.8	-8.93
I_{xt}	-1.79	-5.76	F_t	-2.65	-5.65
I_{ht}	-1.12	-16.21	C_t	-1.84	-3.1
K_{xt}	-3.76	-6.61	R_t	-0.08	-0.43

Notes: Price (P_t) is CPI. The gross rate of interest (R_t) is *one* plus the real rate of interest. Filtered series are HP-filtered with smoothing parameter $\lambda = 10$. The 5% critical value for the test statistic is -3.99 in all cases, but the gross rate of interest, in which case the critical value is -4.08. Evidently, HP $\lambda = 10$ removes nonstationarity in all cases, except for consumption and the rate of interest.

(in absolute value) the Dickey-Fuller test statistic for consumption and the real rate of interest. Given our short sample period and the generally recognized low power of the unit root test to reject a false null hypothesis, we continue using the smoothing parameter $\lambda = 10$, and report in Table 5.2 below (see section 5.4.3) the sensitivity of results when both the actual and the simulated data are HP-filtered with $\lambda = 100$.

5.3.2.2 Estimation and Calibration of Parameters

Recently some researchers (most notably, Christiano and Eichenbaum (1992)) have advocated the estimation of the structural parameters in dynamic macroeconomic models using the Generalized Method of Moments (GMM). We use the GMM in an attempt to estimate depreciation and preference parameters. When formal estimation techniques fail to produce meaningful parameter values, we use the model's restrictions to calculate parameters from actual data.

In estimating the depreciation rates, δ_i ($i = x, h$), we follow Christiano and Eichenbaum (1992) and impose the following restriction on the laws of motion of capital stocks in the two production sectors:³⁸

$$E \left[\delta_i - \left(1 - \frac{K_{it-1}}{K_{it}} + \frac{I_{it}}{K_{it}} \right) \right] = 0 \quad (5.21)$$

For the export good sector, the estimate is $\delta_x = 0.0986$, with a t -ratio of 125.27 and a p -value of 0.00 for the t -ratio. The corresponding estimate for the nontraded sector is

³⁸This basically re-writes equation (5.8) as an expectational difference equation.

$\delta_h = 0.079$, with a t -ratio of 58.9 and a p -value of 0.00 for the t -ratio.³⁹

The growth rate of labor-augmenting technological change, γ , is usually set equal to the rate of growth of real per-capita output.⁴⁰ We set $\gamma = 1.0095$, which is the (gross) rate of growth of real per-capita output for the period 1987-1998. The value of $\mu = 0.35$ is borrowed from Mendoza (1995) so that the elasticity of substitution between traded and nontraded goods, $1/(1 + \mu)$, equals 0.74.⁴¹ θ , the share of domestically produced goods in the consumer's bundle of traded goods, is set at 0.3 in the benchmark model.⁴² With these parameter values set, we attempt to estimate the following consumption Euler equation by GMM for the parameters η and β :

$$E_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\eta} \left(\frac{P_t}{P_{t+1}} \right) R_{t+1} - \gamma^\eta \right] = 0 \quad (5.22)$$

³⁹We use the J -statistics from the above two GMM estimations to test the validity of over-identifying restrictions. The J -statistic times the number of regression observations is asymptotically χ^2 with degrees of freedom equal to the number of over-identifying restrictions. We use five instruments including GDP and a constant to estimate the depreciation parameters in both sectors, and thus there are four over-identifying restrictions in each equation. The estimates for our χ^2 statistics are 0.097 and 1.362 for the parameters in the export good and the nontraded good sectors, respectively, with p -values of 0.998 and 0.850. Thus, the over-identifying restrictions are satisfied in both cases.

⁴⁰See, for example, Correia et. al (1995), Baxter and Crucini (1995), Harjes (1997).

⁴¹Mendoza estimated μ by running a regression of relative expenditures on relative prices and per-capita GDP, all in logarithms, as suggested by Stockman and Tesar (1995). The estimate is representative of the G-7 countries. Although we use Mendoza's estimate for the G-7 in our benchmark model, we examine the model's sensitivity to variations in this parameter later in sub-section 5.4.4.2.

⁴²Although this is done informally, the underlying reason for using this value is as follows. Stockman and Tesar (1995) set $\theta = 0.5$ in their model calibrated to the G-7 data. Macklem (1993) puts exportable consumption at about 22 percent of tradable consumption expenditure. This parameterization is based on evidence from the final demand tables in 1986. To account for the possibility of an increasing contribution of the import-competing domestic non-resource-based manufacturing sector, we set $\theta = 0.3$ as an approximation. We, however, examine the model's sensitivity to variations in θ in the range between $\theta = 0.25$ and $\theta = 0.4$. Except for the impact effects of a terms-of-trade shock on consumption and the two prices, no considerable change in the results in response to the variations in θ can be noted. We find that the higher the values of θ are assigned, the lower are the impact effects of terms-of-trade shocks on consumption and the price index, but the higher are the effects on the relative price of the nontraded good.

Our estimation of (5.22) with annual data for the period 1987-1998 gives some strange results that are not worth reporting. In order to obtain some guidance, however, we try to estimate the equation using quarterly data covering the period 1961:1-1998:4. This gives estimates of $\beta = 0.979$ and $\eta = -1.967$ with t -ratios 12.38 (p -value=0.00) and -0.25 (p -value = 0.8), respectively. When estimated over the shorter period 1987:1-1998:4, using the same quarterly data, the estimates of β and η are, 0.982 and -1.336, respectively, with t -ratios 14.83 (p -value= 0.00) and -0.156 (p -value = 0.87), respectively. While the estimate of β using quarterly data (over both time periods) appears reasonable, it is difficult to defend the estimate of η .⁴³ Given the nature of the GMM estimation methodology, one cannot accept either estimate with any confidence. We, therefore, resort to the alternative calibration methodology.

Prescott (1986) cites a number of studies including Hansen and Singleton (1983) that point to the conclusion that η is not too far from 1. We set $\eta = 1.01$ in our benchmark model. With $\gamma = 1.00955$, all one needs to obtain is the estimate of either the parameter β or the steady-state gross rate of interest R to satisfy the steady-state equilibrium condition $\beta R = \gamma^\eta$. We use the guidance from the GMM estimate above, set $\beta = 0.94$ in the benchmark model, and let R be determined residually so as to satisfy the steady-state equilibrium condition.⁴⁴

⁴³Note that a negative value for η implies a negative intertemporal elasticity of substitution of consumption! Of course, based on the t -ratio, one will not be inclined to accept the estimate for η . But, interestingly, the model's over-identifying restrictions are satisfied in that the J -statistic-based χ^2 value is 0.03 with a p -value of 0.998 (for estimation over the shorter period).

⁴⁴Note that R is endogenous in the present model. We, therefore, do not set it arbitrarily at some historical mean rate of return to equity or some other arbitrarily chosen asset. In choosing the value of the parameter β , however, we note that, given other parameter values and the mean gross rate of interest (1.088) on long-term government bonds for the period 1987-1998, β is a reasonable approximation.

The production function parameter ρ as well as other steady-state ratios are set using actual data for the period 1987-1998. Thus, $1 - \rho$ is set equal to 0.7135, which is average labor income as a fraction of the average net domestic income at factor cost for the period 1987-1998. The steady-state ratio τ_0 is set at 0.44 using average outputs of the goods and services producing industries for the period 1987-1998.⁴⁵ Steady-state debt as a percentage of GDP is set at 0.39, which is the average foreign debt as a percentage of GDP for above-mentioned period. We set the value of ω , the elasticity of the real interest rate with respect to foreign debt, equal to 0.01 in the benchmark model, and then examine the sensitivities of results to variations in the values of this parameter. With all other parameter values set, we assign the values $\phi_x = 1.1$ and $\phi_h = 1.3$ for the capital adjustment cost parameters in the export good and the nontraded good production sectors of the economy, respectively, so as to reproduce the observed volatilities of investment series as closely as possible.⁴⁶

Box 5.3 below summarizes our calibrated preference and technology parameters, various steady-state ratios and factor shares, and the parameters associated with the shock processes.

Box 5.3 Benchmark Parameters				
$\beta = 0.94$	$\eta = 1.01$	$\mu = 0.35$	$\delta_x = 0.098$	$\delta_h = 0.0788$
$\rho = 0.2865$	$\phi_x = 1.1$	$\phi_h = 1.3$	$\gamma = 1.00955$	$\theta = 0.3$
$\frac{f}{y} = 0.39$	$\tau_0 = 0.44$	$\omega = 0.01$	$Z_x = 1$	$Z_h = 1$
$p_m = 1$	$\psi_x = 0.696$	$\psi_h = 0.726$	$\psi_m = 0.469$	$\sigma_x = 0.086$
$\sigma_h = 0.075$	$\sigma_m = 0.0237$	$\sigma_{x,h} = 0.903$	$\sigma_{x,m} = 0.714$	$\sigma_{h,m} = 0.625$
$\vartheta_x = 0.3$	$\vartheta_h = 0.4$			

⁴⁵The ratios τ_0 and τ_1 are defined in (5C.15) in Appendix 5C.

⁴⁶This follows the approach adopted by Cardia (1991).

In the box, ψ_i and σ_i are, respectively, the estimated autocorrelation coefficient and the standard deviation of the i th shock process, and σ_{ij} is the correlation coefficient between the i th and the j th shock processes, for $i, j =$ shocks to productivity in the export and non-traded good sectors, and to the terms-of-trade.

5.3.2.3 Estimation of Shocks to Productivity and Terms of Trade

In the real business cycle literature, productivity shocks are frequently measured from the “Solow residuals”.⁴⁷ These residuals are measured as the difference between changes in output and the changes in measured inputs (labor and capital) times their shares. We follow the same approach here. Using our symbols, the productivity changes in the export- and the non-traded goods sectors are given by:⁴⁸

$$\begin{aligned}\ln Z_{xt} &= \ln Y_{xt} - \rho \ln K_{xt} - (1 - \rho) \ln N_{xt} - t(1 - \rho) \ln \gamma \\ \ln Z_{ht} &= \ln Y_{ht} - \rho \ln K_{ht} - (1 - \rho) \ln N_{ht} - t(1 - \rho) \ln \gamma\end{aligned}\tag{5.23}$$

One exception to the definitions provided earlier must be noted here. In (5.23), Y_t , K_t , and N_t are, respectively, aggregate output, capital stock, and employment in the i th sector. We use employment and output in the goods and services producing industries for the employment and output of the traded and non-traded sectors, respectively, of the model. However, such aggregated data on capital stocks in the goods and services producing industries are not

⁴⁷In virtually all the papers mentioned so far that consider productivity shocks, these shocks have been measured from the “Solow residuals”. Baxter and Crucini (1995) point out some potential problems when employment rather than total hours worked are used for labour input. However, due to the paucity of reliable data on hours worked, researchers often end up using employment in place of total hours worked. In the end, Baxter and Crucini (1995) also use employment for labour input.

⁴⁸The last term arises from specification of the growth in labor-augmenting technology: $X_t = \gamma^t X_0$, with the assumption that $X_0 = 1$.

available, although disaggregated data by industry categories are available. We obtain the capital stock in the nontraded sector by aggregating the capital stocks of the service-oriented industries.⁴⁹ The capital stock in the export good sector is then taken to be the residual from the total capital stock in the economy.

Our model's shock processes are characterized fully by the auto-correlation coefficients and standard deviations of the terms-of-trade and the "Solow residuals" described above. These statistics are reported in Box 5.3 along with all other benchmark parameters for the calibration of the model.

At this point, it may be useful to remind the reader that we are working with an approximate solution of a highly nonlinear model. The solution is obtained by log-linearizing the first-order conditions of the model in its steady-state equilibrium.⁵⁰ All variables in the log-linearized system are thus expressed as log-deviations from their steady-state values. The linearized model is parameterized and calibrated to Canadian data for the period 1987-1998. Thus, an essential implicit assumption in the calibration process of the model is that the model economy is in its steady-state equilibrium when any shock occurs.

5.4 Results

5.4.1 *Recursive Equilibrium Laws of Motion*

The solution of the model with the method of undetermined coefficients gives the *linear recursive laws of motion* of the general form in system (5.20), reproduced here for

⁴⁹The industries considered to be service-oriented are: construction, transportation, and storage, communication and other utilities industries, wholesale and retail trade, finance and insurance, health and social service, government, business, and other service. Detailed CANSIM series codes are provided in Appendix 5D.

⁵⁰The steady-state equilibrium conditions used in the process are presented in (5C.7) in the Appendix 5C.

convenience:

$$\begin{aligned}\tilde{S}_t &= P\tilde{S}_{t-1} + V\tilde{Z}_t \\ \tilde{E}_t &= A\tilde{S}_{t-1} + B\tilde{Z}_t\end{aligned}\tag{5.24}$$

where $\tilde{E}_t$, $\tilde{S}_t$ and $\tilde{Z}_t$ are, respectively, the vectors of endogenous variables, state variables, and exogenous state or “forcing” variables, and, A , B , P and V are the matrices of coefficients. Since all variables in the transformed system are defined as log-deviations from their steady-state values, the coefficients can be interpreted as elasticities. The estimated linear laws of motion for our model’s endogenous variables from the solution of the model using the benchmark parameters are presented below:⁵¹

$$\begin{bmatrix} \tilde{k}_{ht} \\ \tilde{k}_{xt} \\ \tilde{f}_t \\ \tilde{c}_t \\ \tilde{y}_{ht} \\ \tilde{y}_{xt} \\ \tilde{i}_{ht} \\ \tilde{i}_{xt} \\ \tilde{p}_t \\ \tilde{p}_{ht} \end{bmatrix} = \begin{bmatrix} 0.7596 & 0.0517 & -0.0216 & 0.1235 & 0.0123 & -0.2882 \\ 0.0086 & 0.7597 & -0.0149 & 0.1732 & 0.0016 & -0.3994 \\ -0.2306 & -0.4061 & 0.8525 & -0.6595 & -0.0408 & -1.2944 \\ 0.2310 & 0.0658 & -0.0688 & 0.0015 & 0.5486 & -0.4846 \\ 0.2865 & 0.0000 & 0.0000 & 0.0000 & 1.0000 & 0.0000 \\ 0.0000 & 0.2865 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\ -1.7466 & 0.5908 & -0.2468 & 1.4114 & 0.1404 & -3.2932 \\ 0.0804 & -1.2433 & -0.1388 & 1.6170 & 0.0150 & -3.7285 \\ -0.1272 & 0.1308 & -0.0237 & 0.3779 & -0.5328 & 0.5351 \\ -0.2191 & 0.2253 & -0.0408 & 0.6509 & -0.9177 & 0.1014 \end{bmatrix} * \begin{bmatrix} \tilde{k}_{ht-1} \\ \tilde{k}_{xt-1} \\ \tilde{f}_{t-1} \\ \tilde{z}_{xt} \\ \tilde{z}_{ht} \\ \tilde{p}_{mt} \end{bmatrix}\tag{5.25}$$

Let us first consider the estimated law of motion for the stock of capital in the nontraded

⁵¹The model as calibrated with the parameters in Box 5.3 will be called the Benchmark model 2 to distinguish it from the Benchmark model 1, in which ω , the elasticity of the real rate of interest with respect to foreign debt, is set equal to zero. As it appears, the calibration of the latter model gives solutions that are not stable in that the matrix P (see equations in 5.20) is found to contain a unit root. The Benchmark model 2, however, gives convergent solutions. The results reported in (5.25) as well as the sensitivity and the impulse response analyses reported later are, therefore, based on the Benchmark model 2. The reason for the instability in the Benchmark model 1 is briefly discussed later in section 5.4.4.1.

sector. The results presented in (5.25) indicates that when capital stock in the nontraded good sector at time $t-1$ is 1 percent above its steady-state level, it is only about 0.76 percent above the steady-state level at time t . This behavior is consistent with the model's convergence and stability properties.⁵² Notice that when debt at time $t-1$ is 1 percent above its steady-state level, investment in the nontraded and traded sectors at time t are, respectively, 0.2468 and 0.139 percent below their steady-state levels. This follows from our assumption that the real interest rate on foreign debt paid at time t depends on the stock of debt at time $t-1$, and that higher rate of interest means higher finance cost of investment. The most notable property of the results reported in (5.25) is the considerable volatility of investment to terms-of-trade shocks. Thus, a terms-of-trade shock that raises the price of imports relative to exports by one percent above its steady-state level, lowers investment in the export good sector by 3.73 percent below its steady state level. The shock causes the investment in the nontraded good sector to fall 3.29 percent below its steady-state level. The reason for this behavior of investment will be discussed in subsection 5.4.2.3.

5.4.2 Impulse Response Functions

The interpretation of the results from the solution of the model as well as the understanding of the dynamics of the model can be made easier by looking at *the impulse response functions* that represent the response, over time, of the elements of $\tilde{E}_t$ as well those of $\tilde{S}_t$ to a shock to one of the forcing variables. These functions provide information on the

⁵²Notice the behavior of the other two state variables. When capital stock in the export-good sector at time $t-1$ is 1 percent above its steady-state level, it is only about 0.76 percent above its steady-state level at time t . Debt also shows a convergent behavior in that if it rises 1 percent above its steady-state level at time $t-1$, it is 0.85 percent above its steady-state level at time t .

system's average conditional response in period $t+l$ ($l=0, 1, 2, 3, \dots$) to a shock at date t , given the posited stochastic processes. Figures 5.2 through 5.4 below present the impulse response functions for the key variables of particular interest in this model. In each case, the shock to a forcing variable is one percent in size, and in calculating impulse responses to the shock to a given variable all others are kept at zero in the first period.

5.4.2.1 Productivity shock in the export-good sector

Consider a technology shock in the export good sector that increases productivity by 1 percent above its steady-state level. The impulse response functions to this shock are presented in Figure 5.2. Exportable output rises by 1 percent on impact. Increased marginal productivity of capital quite naturally stimulates an investment boom as evident in the rise in investment in the export good sector by more than 1.61 percent above its steady-state level. The nontraded good sector also experiences a “spill-over” effect in that investment in that sector rises on impact by 1.41 percent. In order to understand the strong positive effect on investment in the two sectors of the economy, notice the behavior of debt in the second diagram in Figure 5.2. The effect of the higher exportable output following the shock is to lower debt by 0.66 percent immediately. This leads to a fall in the real rate of interest, and thus, in the finance cost of investment.⁵³ The debt takes a falling path for more than five years after the shock before the effect starts to dissipate.

The significant rise in investment and the current account reflects an attempt by our consumer-investor to shift consumption intertemporally away from the present into the

⁵³Note that the real rate of interest in the present parameterization is increasing in the stock of debt -- ($\omega = 0.01$).

Figure 5.2 Impulse response functions of a productivity shock in the export good sector

future. Of course, the positive wealth shock arising from the productivity gain induces the consumer to raise consumption by 0.0015 percent on impact. Notice the consumption-smoothing behavior of the representative consumer-investor – the impact effect on consumption is much less than proportionate to the rise in output. The effect of the productivity-driven wealth shock is also felt in the market for the non-traded good: increased demand increases its relative price by about 0.65 percent above its steady-state level and it pulls the price-index up by about 0.38 percent. The immediate effect of the shock on the real exchange rate is thus appreciation, to be followed by a depreciating path that persists for over 40 years after the shock.⁵⁴ Notice in the first diagram in Figure 5.2 that consumption takes a rising path while prices take falling paths, implying a falling path for our consumer's “effective” real interest rate.⁵⁵ Evidently, consumption continues to rise for more than 7 years after the shock before the effect starts to dissipate. As the Euler equation (5C.3) in Appendix 5C indicates, this is exactly how the consumer is expected to behave: the falling path of prices, and, consequently, the falling path of the “effective” real interest rate, induces the consumer-investor to substitute consumption intertemporally away from the present into the future.

⁵⁴The horizon in the presentation of the impulse response functions is chosen to be 20 years for the clarity of the graphs. It should, however, be noted that the effects on consumption, prices, investment, and debt following a productivity shock in the export-good sector persist for long periods of time. Thus, it takes about forty years for the effects on prices, consumption and investment to dissipate completely. Debt takes more than sixty years after the initial shock to return to its steady-state level.

⁵⁵One can recognize in the Euler equation (5C.3) in Appendix 5C that the “effective” real interest rate is: $R_{t+1}(P_t/P_{t+1})$.

5.4.2.2 Productivity shock in the non-traded good sector

Let us now consider a technology shock in the non-traded sector that raises productivity by 1 percent above its steady state level. The impulse response functions are presented in Figure 5.3. The relative price of the non-traded good goes down on impact by about 0.92 percent, and, consequently, the price index falls about 0.53 percent below its steady-state level, while consumption rises about 0.55 percent above its steady-state level. The sharp increase in consumption following the productivity gain now reflects the fact that the nontraded good cannot be *intra*- or *inter*-temporally transferred. The consumption behavior can also be rationalized by the rising path taken by the “effective” real interest rate. The real exchange rate depreciates on impact following the shock, and then takes an appreciating path as prices rise. Notice that the response of investment to the productivity gain in the nontraded sector is not as large as before – investment in the nontraded and traded sectors rise by 0.14 percent and 0.015 percent, respectively. Even these small effects dissipate almost completely in a little over 5 years after the shock. This small and short-lived response of investment can be explained by the insignificant (about 0.04 percent) improvement in the debt situation following the shock. The small decrease in debt below its steady-state level in the face of higher investment and the virtually unchanged output of the export good, indicates some *intra*-temporal substitution of the nontraded good for traded goods.⁵⁶ It takes about 15 years after the initial shock for the effects on all variables except debt to dissipate completely.

⁵⁶Note that investment in both sectors of the present model is a composite good that uses as inputs both exportables and imports.

Figure 5.3 Impulse response functions of a productivity shock in the nontraded good sector

5.4.2.3 A terms-of-trade shock

Consider a terms of trade shock that raises p_{mt} by 1 percent above its steady-state value. Our consumer-investor now faces a decline in her income in terms of imports. This will have both *substitution* as well as *income effects*. With lower income, the consumer will decrease demand for all consumption goods including the non-traded good. This is the income effect of the terms of trade shock. On the other hand, for a given relative price of the non-traded good in terms of the export good, p_{hr} , the deterioration in the terms of trade implies that the non-traded good has now become cheaper relative to the imported good. Thus, the consumer now may substitute away from the import good into the non-traded good. This is the substitution effect of the terms of trade shock. If the intra-temporal elasticity of substitution, $1/(1 + \mu)$, is so low that the income effect dominates the substitution effect, the deterioration in the terms of trade can lead to a fall in p_{hr} . Notice in equation (5.15) that when θ , the share of the export good in the tradables expenditure, is small, it is *remotely* possible that the deterioration in the terms-of-trade lowers the consumption-based price index.⁵⁷ This fall in the price index combined with the rise in p_{mt} can lead to a depreciation of the real exchange rate. If, on the other hand, the substitution effect outweighs the income effect, the terms-of-trade shock will lead to a rise in the relative price of non-traded goods, and, consequently, in the consumption-based price index. The effect on the real exchange rate now depends on the relative size of the terms-of-trade shock and its effect on the price index. For plausible parameter values and the size of the effect of the shock on the relative price of

⁵⁷The ability of a terms-of-trade shock to lower the consumption-based price index is only a *remote* possibility in that it requires a considerable decrease in the relative price of the nontraded good, and thus a strong income effect of the shock. It also requires that the share of export goods in the tradables expenditure is very small. With a plausible θ and the size of the income effect, a terms-of-trade shock cannot lower the price index.

Figure 5.4 Impulse response functions of a shock to the terms-of-trade

non-traded goods, it is, however, very likely that the rise in the price index is less than proportionate to the rise in p_m . The effect of the terms-of-trade shock on the real exchange rate can thus be depreciation.⁵⁸

Impulse response functions to a terms-of-trade shock are presented in Figure 5.4. Evidently, in the present parameterization, the substitution effect dominates the income effect, and thus the terms-of-trade shock causes a rise in the relative price of the non-traded good by 0.10 percent on impact. This effect, however, starts to dissipate immediately. The price-index rises by about 0.54 percent on impact when consumption goes about 0.48 percent below its steady-state level. Thus, the real exchange rate appreciates on impact. Thereafter, while consumption takes a quickly rising path, the price-index takes a steeply declining path, along which the real exchange rate depreciates continuously. The effects of the terms-of-trade shock on all the variables except debt dissipate completely in fifteen years after the shock. It takes about 50 years after the shock for the effect on debt to dissipate completely.

The terms-of-trade shock lowers investments in both the export good and the nontraded good sectors by 3.73 percent and 3.29 percent respectively below their steady-state levels.⁵⁹ With lower investment and lower consumption, debt naturally goes down below its steady-

⁵⁸Thus, for example, given the parameter values in the Box 5.3 and assuming that a 10 percent rise in p_m causes a 10 percent rise in p_n , the effect on the price index can be calculated to be an 8.4 percent rise. This implies real depreciation according to our definition of the real exchange rate. See footnote 23 on page 134.

⁵⁹Note that the differential response of investments in the two sectors is explained by the different sets of parameters used to characterize capital accumulation in the two sectors. Consider, for example, the gross rate of return on a one-period investment in the export-good sector. This return equals the marginal product of capital in production plus the value of the undepreciated capital. (See equation (5C.7) and the associated discussion in Appendix 5C). Evidently, the return depends on two parameters: (1) the rate of depreciation of capital, and (2) the price of a unit of capital. When a longer horizon is considered, and capital adjustment costs exist, a third component – the marginal reduction in the cost of installing a given flow of capital – is added to the gross rate of return. The value of this third component depends on ϕ_x , the adjustment cost parameter. The finance cost of investment, on the other hand, is affected by the rate of interest paid on debt as well as the price of capital that, in turn, is affected by the terms-of-trade (see Appendix 5A). Optimal capital stock and thus the rates of investment in the two sectors are determined by equating the finance cost to the gross rate of return.

state level by 1.29 percent on impact. It takes only about a year before it starts moving towards its steady-state level. Investments in the two sectors, on the other hand, take steeply rising paths after the initial drops following the shock, both overshoot their steady-state levels in about two years, and then gradually start moving back to their long-run levels. These paths of convergence are evidently affected by the behavior of debt with its consequent effect on the real interest rate.

5.4.3 Some Second-Moment Properties

It is a commonplace in the real business cycle literature to compare model second moments with those in the data as a first cut for evaluating the fit of the model to the data. When using this approach for the evaluation of the present model's performance, one must bear in mind that consumption, investment, and capital stock in the model are composite goods, and that price and debt are in terms of the export-good. Table 5.2 below presents model standard deviations along with those calculated from the data. It is also worth noting at the outset that the model standard deviations relate to the standard deviations of variables expressed as log-deviations from a steady state. The moments reported in Table 5.2 are calculated from HP-filtered series both in the model and in the data. In calculating the model standard deviations, we simulate the model 50 times, with each simulation being 12 periods long.⁶⁰ This is done to match the number of observations underlying the calculations of the standard deviations from the data. Evidently, the model's performance in its present parameterization in matching the moments in the data is good. In the benchmark model 2,

⁶⁰A simulation of the model 500 times was done with no noticeable change in results.

Table 5.2 Standard deviations of variables in data compared with those in the model that in the data.

Variable (z)	Data (HP $\lambda = 10$)		Data (HP $\lambda = 100$)		Benchmark 1 (HP $\lambda = 10$)		Benchmark 2 (HP $\lambda = 10$)		Benchmark 3 (HP $\lambda = 100$)	
	σ_z	σ_z/σ_y	σ_z	σ_z/σ_y	σ_z	σ_z/σ_y	σ_z	σ_z/σ_y	σ_z	σ_z/σ_y
Capital Stock (h)	0.0523	2.3450	0.0503	3.3090	0.0039	0.0710	0.0058	0.1048	0.010	0.1490
Capital Stock (x)	0.0190	0.8520	0.0106	0.6970	0.0057	0.1050	0.0072	0.1300	0.011	0.1780
Foreign Debt	0.0706	3.1650	0.0561	3.6900	0.0718	1.324	0.0562	1.0160	0.088	1.3180
Consumption	0.0210	0.9420	0.0206	1.3550	0.0131	0.2410	0.0200	0.3650	0.028	0.4290
Nontraded good	0.0417	1.8690	0.0416	2.7360	0.0462	0.8520	0.0450	0.8280	0.059	0.8880
Export good (y)	0.0223	1.0000	0.0152	1.0000	0.0542	1.0000	0.0550	1.0000	0.067	1.0000
Investment (h)	0.0859	3.8520	0.0835	5.4930	0.0441	0.8130	0.0600	1.0850	0.079	1.1850
Investment (x)	0.0633	2.8380	0.0285	1.8750	0.0544	1.0030	0.0640	1.1610	0.082	1.2230
Price Index	0.0926	4.1520	0.0907	5.9670	0.0148	0.2730	0.0136	0.2460	0.016	0.2380

Note: The 'smile' symbol is used to distinguish moments produced by the simulated data from those calculated using actual data. The letter y is used for the output of the export good. h and x are used to relate a variable to the nontraded and export good sectors, respectively. Benchmark model 2 differs from model 1 in only one respect: while the former sets $\omega = 0.01$, the latter sets it virtually equal to zero ($\omega = 0.000001$). In what follows, $\omega = 0$ is used to imply the above-chosen $\omega = 0.000001$, for convenience. Benchmark model 3 (in which $\lambda = 100$) is also simulated with ω set equal to 0.01. Evidently, a small sensitivity of the real rate of interest to the stock of debt brings the variability of most of the variables in the model closer to that in the data.

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change with different values for the preference parameters that change both the *inter*- and *intra*-temporal elasticities of substitution. But before doing this, we want to complete the discussion that started in Table 5.2 on the role of the parameter ω .

5.4.4.1 More on the role of the elasticity of the real interest rate

We reported earlier that a small sensitivity (ω) of the real interest rate to the stock of debt is necessary for the stability of the model. This is, however, not surprising. When a small open economy can borrow as much as desired at a parametrically given world interest rate, consumers will borrow indefinitely whenever the interest rate in the international capital market is below the agents' rate of time preference. Domestic profit-maximizing firms will also be inclined to borrow as much as desired in order to adjust capital to its profit-maximizing level whenever productivity or terms-of-trade shocks make investment profitable. In our case, constrained access to the world capital market can serve as a brake to give stability of the model.

Table 5.3 below presents the sensitivity of the model in terms of the effects on the second moments of the variables to variations in ω . We use HP $\lambda = 10$ in all the cases reported. We note that the standard deviation of the variables relative to that of the output of the export good comes closer to the data as the value of ω goes up. One can, however, note that the Benchmark model 2 with $\omega = 0.01$ gives the best fit in terms matching the standard deviations in the data. This can be taken as indicative of the fact that the supply schedule for international borrowing that our model economy faces is quite flat, although not horizontal.

Table 5.3
Sensitivity Analysis: variations in ω

Variable ($\bar{x}$)	σ_x (Data)	σ_x/σ_y (Data)	σ_x ($\omega = 0.05$)	σ_x/σ_y ($\omega = 0.05$)	σ_x ($\omega = 0.1$)	σ_x/σ_y ($\omega = 0.1$)	σ_x ($\omega = 0.5$)	σ_x/σ_y ($\omega = 0.5$)
Capital Stock (h)	0.052	2.345	0.0086	0.149	0.0087	0.169	0.0116	0.206
Capital Stock (x)	0.019	0.852	0.01	0.174	0.0099	0.192	0.013	0.231
Foreign Debt	0.071	3.165	0.0344	0.599	0.0221	0.429	0.008	0.149
Consumption	0.021	0.942	0.0266	0.463	0.0259	0.503	0.0321	0.57
Nontraded good	0.042	1.869	0.0494	0.86	0.0444	0.862	0.0507	0.9
Export good	0.022	1	0.0574	1	0.0515	1	0.0563	1
Investment (h)	0.086	3.852	0.0874	1.522	0.0914	1.775	0.1207	2.143
Investment (x)	0.063	2.838	0.0846	1.474	0.0874	1.697	0.1125	1.998
Price	0.093	4.152	0.0136	0.237	0.0139	0.269	0.0148	0.262

Note: HP $\lambda = 10$ in all cases. The 'smile' symbol is used to distinguish moments produced by the simulated data from those calculated using actual data. The letter y is used for the output of the export good. h and x are used to relate a variable to the nontraded and export good sectors, respectively.

5.4.4.2 Sensitivities to the inter- and the intra-temporal elasticities of substitution

As we mentioned in section 5.4.3, when variations in the structural parameters affect the standard deviations of the variables, the performance of the model can be better understood by looking at its sensitivities to variations in important structural parameters. The magnitude of the effects of shocks to productivity or terms-of-trade on the key variables of the model is determined by the degree of *inter-temporal* substitution of consumption as well as the *intra-temporal* substitution between the traded and nontraded goods. A high degree of *inter-temporal* substitutability in consumption for any given elasticity of *intra-temporal* substitution induces the consumer to shift consumption away from periods when consumption is relatively expensive, and towards periods when it is relatively cheap. A high degree of *intra-temporal* substitution between traded and nontraded goods for any given elasticity of *inter-temporal* substitution in consumption, on the other hand, makes it possible for the consumer to substitute the good that is relatively cheap for one that is relatively expensive.

Let us first examine the sensitivity of our model to variations in μ .⁶¹ In Table 5.4 below we report the model's simulated standard deviations for four different values of the parameter μ . Evidently, the model does not exhibit any considerable sensitivity to a change in μ from $\mu = 0.25$ to $\mu = 0.35$. However, its overall performance in terms of producing a match with the standard deviations in the data diminishes for larger values of the parameter.

⁶¹It may be useful to remind the reader that $1/(1 + \mu)$ gives the elasticity of substitution between traded and nontraded goods in consumption.

Table 5.4
Sensitivity Analysis: variations in μ

Variable (z)	Data	Benchmark Model 2			
	σ_z	σ_z ($\mu = 0.25$)	σ_z ($\mu = 0.35$)	σ_z ($\mu = 0.55$)	σ_z ($\mu = 0.75$)
Capital Stock (h)	0.052	0.0061	0.0061	0.0059	0.0063
Capital Stock (x)	0.019	0.0073	0.0073	0.0073	0.0079
Foreign Debt	0.071	0.0548	0.0543	0.0547	0.0539
Consumption	0.021	0.0216	0.0215	0.0219	0.0235
Nontraded good	0.042	0.0475	0.0475	0.0482	0.0511
Export good (y)	0.022	0.0535	0.0535	0.0545	0.0568
Investment (h)	0.086	0.0598	0.0595	0.0628	0.0666
Investment (x)	0.063	0.0634	0.0634	0.0675	0.0715
Price Index	0.093	0.0134	0.0129	0.0129	0.0124

Note: The letters h and x are used to relate a variable to the nontraded and export good sectors, respectively.

Tables 5.5 through 5.7 below present the sensitivities of the price index, debt, consumption, and investment in the two sectors of the economy to variations in the *inter-temporal* elasticities of substitution with μ set equal to 0.35. Notice that the sensitivities reported are in terms of changes in the impact effects following shocks to our model's forcing variables. It may be useful to remind the reader at this point that all variables in our transformed model are defined as log-deviations from their steady-state values. Hence, the sensitivities reported in the tables below can be interpreted as elasticities with respect to shocks to the forcing variables of the model.

Productivity shock in the export-good sector

Notice in Table 5.5 below that the lower is the intertemporal elasticity of substitution in consumption, the higher are the elasticities of consumption, investment and the price index with respect to productivity shocks in the export-good sector, and the lower is the elasticity of debt. Note that the debt situation improves and the price-index rises on impact following a positive productivity shock in the export-good sector (see the discussion in sub-section 5.4.2.1).

In order to understand the above-mentioned sensitivities, let us consider two states – a *high* intertemporal elasticity state and a *low* intertemporal elasticity state. In the high-elasticity state, the consumer is most-able to shift consumption away from periods when consumption is relatively expensive, and towards periods when it is relatively cheap. In the low-elasticity state the consumer is least-able to make such intertemporal transfers. A productivity shock in the export-good sector that raises the price-index is likely to induce the consumer to shift consumption away from the present into the future. It is easy to see that in the high-elasticity state the impact effect on consumption will be small, and, accordingly, the effect on the price-index will also be small. It also follows that in such a state the current account gain following the productivity shock will be large. On the other hand, in the low-elasticity state current consumption following the productivity shock will be high, as the consumer is least-able to smooth consumption intertemporally. Accordingly, the pressure on the price-index will also be high and the effect on the current account will be small. The behavior of investment in the above-mentioned two states can be explained by the relative behavior of the price-index in the two states.

Table 5.5

Elasticities of some key variables with respect to productivity shocks in the export good sector

$\omega = 0.01,$ $\varepsilon = 0.74$	$1/\eta = 0.99$	$1/\eta = 0.67$	$1/\eta = 0.57$	$1/\eta = 0.5$	$1/\eta = 0.33$
Price Index	0.378	0.392	0.397	0.401	0.412
Consumption	0.0015	0.0424	0.0567	0.0683	0.1007
Investment (h)	1.4114	1.4146	1.419	1.4246	1.4532
Investment (x)	1.617	1.6208	1.6251	1.6302	1.6547
Foreign Debt	-0.6595	-0.593	-0.5707	-0.5529	-0.5066

Note: $\varepsilon = 1/(1 + \mu)$ is the elasticity of substitution between traded and non-traded goods, ω is the elasticity of the real rate of interest with respect to foreign debt, when evaluated at the steady state, and $1/\eta$ is the intertemporal elasticity of substitution. The intertemporal elasticity of substitution declines as one goes across the shaded blocks.

Productivity shock in the nontraded good sector

The sensitivities of key variables to variations in the intertemporal elasticity of substitution are reported in Table 5.6 below. It is evident that the lower is the inter-temporal elasticity of substitution in consumption, the higher are the elasticities of the price-index, foreign debt and investment in both sectors of the economy with respect to productivity shocks in the nontraded sector, and the lower is the elasticity of consumption. A productivity shock in the non-traded sector lowers the price-index and debt, and increases consumption and investment in both sectors (see the discussion in sub-section 5.4.2.2). The productivity

Table 5.6

Elasticities of some key variables with respect to productivity shocks in the nontraded good sector

$\omega = 0.01,$ $\varepsilon = 0.74$	$1/\eta = 0.99$	$1/\eta = 0.67$	$1/\eta = 0.57$	$1/\eta = 0.5$	$1/\eta = 0.33$
Price Index	-0.5328	-0.5654	-0.5765	-0.5853	-0.6072
Consumption	0.5486	0.4541	0.422	0.3965	0.3328
Investment (h)	0.1404	0.1678	0.1767	0.1836	0.2
Investment (x)	0.015	0.0732	0.0934	0.1095	0.1494
Foreign Debt	-0.0408	-0.2129	-0.2727	-0.3209	-0.4454

Note: See the notes under Table 5.5 for definitions.

Table 5.7

Elasticities of some key variables with respect to terms-of-trade shocks

$\omega = 0.01,$ $\varepsilon = 0.74$	$1/\eta = 0.99$	$1/\eta = 0.67$	$1/\eta = 0.57$	$1/\eta = 0.5$	$1/\eta = 0.33$
Price Index	0.5351	0.5688	0.5801	0.5891	0.6115
Consumption	-0.4846	-0.3867	-0.3538	-0.3277	-0.2627
Investment (h)	-3.2932	-3.3326	-3.3453	-3.3551	-3.3779
Investment (x)	-3.7285	-3.7718	-3.7867	-3.7985	-3.8273
Foreign Debt	-1.2944	-1.0773	-0.9973	-0.93	-0.7351

gain lowers the relative price of non-traded goods, which pulls the price-index down. The effect on price would be lower, if the intertemporal elasticity of substitution were higher, because the consumer would intertemporally shift consumption away from the future into the present. Sensitivities in terms of the elasticities of consumption, investment, and debt follow straightforwardly from the behavior of price and the resulting behavior of the “effective” real interest rate.

A negative terms-of-trade shock

The lower the *inter-temporal* elasticity of substitution, the higher are the elasticities of the price-index and investment in both sectors of the economy with respect to terms-of-trade shocks, and the lower are the elasticities of consumption and debt. In this case, as explained in sub-section 5.4.2.3, although the relative price of the non-traded good falls following a terms-of-trade shock, the price-index rises, and the current account improves as both consumption and investment fall. Again, as in the previous cases, the links are through the effects on price and, consequently, on the “effective” real rate of interest.

5.5 Impact and Dynamic Effects of Shocks: the role of ω and ϑ

The purpose of this section is to explore the role of the parameters ϑ , which is the share of exports in investment and ω , which is the elasticity of real interest rate with respect to foreign debt, when evaluated at the steady-state. In modeling a small open economy, researchers often assume that capital is formed either from imports or from domestic goods

entirely.⁶² This seems an overly stark characterization of capital formation in any country. Stockman and Tesar (1995, p.183) indicate that the unsatisfactory performance of their model in matching the correlation between the trade balance and output is likely to be a result of their assumption that the traded-good industry in each country uses only domestically produced capital in production. Hence, one of the key contributions of the present chapter is to model investment as a composite good that uses as inputs both exportables and imports. This implies that the response of investment to terms-of-trade shocks will be proportionate to the share of imports in total investment expenditure. In a dynamic optimizing setting, where the representative consumer-investor adjusts savings and investment decisions in order to smooth consumption intertemporally, this share of imports can, in turn, influence the entire path of consumption, investment, output, and debt.

The second but fundamental contribution of the present chapter is to allow the schedule for international borrowing to be upward sloping. We have already seen that a small degree of sensitivity of the real interest rate to the stock of foreign debt ($\omega > 0$) allows the model to fit the data better in terms of coming closer to the standard deviations in the data. In addition, as we saw earlier (see section 5.4.1), this sensitivity is necessary for the convergence and stability properties of the model. What remains to be seen is how the two parameters (ϑ and ω) change the impact and the dynamic effects on the key variables of the model arising from shocks to the model's forcing variables. To that end, we first compare our results with those that would arise if $\omega = \vartheta = 0$ as typified by the works of Macklem

⁶²Thus, Macklem (1993) and Mendoza (1995), for example, assume that capital is formed entirely from imports. Mendoza (1991) assumes that investment is composed of domestic goods.

(1993) and Mendoza (1995).⁶³ This allows us to show how the inclusion of our two parameters ω and ϑ affects the incidence and persistence properties of a typical computable dynamic model.

5.5.1 A comparison with the results of Macklem (1993) and Mendoza (1995)

Table 5.8 below reports the effects that follow shocks to the terms-of-trade and sector productivity for a variety of ω , ϑ values.⁶⁴ The corresponding dynamic effects are presented in Figures 5.5 through 5.8, and in Figures 5E.1-5E.3 in Appendix 5E.⁶⁵ As before, the shock to a forcing variable will be one percent in size in all the cases reported. To obtain these results, we parameterize the model in a way that the results can be compared with those obtained by Macklem (1993) and Mendoza (1995). Macklem (1993) adopts the Blanchard-Weil overlapping generations structure for a three-good economy (Blanchard (1985), Weil (1989)). The model is calibrated to the Canadian data to examine the effects of terms-of-trade disturbances and fiscal shocks. Mendoza (1995) considers both productivity and terms-of-trade shocks. As already mentioned, the structure of our model is similar to Mendoza's

⁶³It may be useful to remind the reader that, for this case, we select a value of ω that is arbitrarily close to zero – ($\omega = 0.000001$). This is necessary to ensure the stability of the model. Thus the comparison presented above is between the results from two stable models – one with virtually perfectly-elastic capital flows and the other with less-than-perfectly-elastic capital flows.

⁶⁴We consider three sets of parameter values: $\omega = \vartheta = 0$, $\omega = 0, \vartheta = 0.5$, and $\omega = 0.1, \vartheta = 0.5$. An $\omega = 0.1$ implies a 0.1 percent rise in the real interest rate for every 1-percent rise in foreign debt in the steady-state. While this sensitivity of interest rate might be thought to be high, its use is convenient for illustration. $\vartheta = 0.5$ is also arbitrarily chosen for convenience of illustration. The $\omega = \vartheta = 0$ case can be compared with Macklem (1993) and Mendoza (1995) in that both authors assume perfect capital flows, and both assume that investment is composed of imports entirely.

⁶⁵Unless otherwise indicated, the solid-thick line in these figures indicates the dynamic effects when $\omega = 0.1, \vartheta = 0.5$. The thin line indicates the dynamic effects when $\omega = 0, \vartheta = 0.5$, and the dashed line is used to indicate the dynamic effects when $\omega = \vartheta = 0$.

Table 5.8
Impact effects of various shocks on the key variables: the role of ω and ϑ

Variables	Shock to terms-of-trade		Shock to technology (Export good)		Shock to technology (Nontraded good)	
	$\omega = 0$ $\vartheta = 0$	$\omega = 0$ $\vartheta = 0.5$	$\omega = 0.1$ $\vartheta = 0.5$	$\omega = 0$ $\vartheta = 0$	$\omega = 0.1$ $\vartheta = 0.5$	$\omega = 0$ $\vartheta = 0.5$
Consumption	-0.3985	-0.393	-0.327	-0.093	-0.093	0.418
Investment	-5.697	-2.848	-2.128	1.286	1.286	0
Debt	-2.251	-1.037	-0.539	-0.984	-0.984	-0.321
Export Good	0	0	0	1	1	0
Price(Nontraded)	0.1524	0.156	0.194	0.595	0.595	-0.995
Price Index	0.564	0.567	0.589	0.345	0.345	-0.578
						-0.559

Note: Investment considered here is the investment in the export-good sector. ω is the elasticity of the real interest rate with respect to foreign debt, when evaluated at the steady-state. ϑ is the share of exports in investment. The effects reported above are impacts following the shocks. See Figures 5.5 through 5.8 (and 5E.1-5E.3 in Appendix 5E) for dynamic effects. To derive these results, the model is parameterized so that the results are comparable with those obtained from similar calibrated dynamic models for Canada (see, for example, Macklem (1993), Mendoza (1991). See also Mendoza (1995)). In particular, the key parameter values are assigned as follows. $\delta = 0.1$, $\beta = 0.96$, and $\eta = 1.5$.

(1995), with some key modifications.⁶⁶ We can compare our results to those obtained by Mendoza when using the G-7 as his benchmark.⁶⁷ Let us first consider a negative terms-of-trade shock. For the first set of parameter values, $\omega = \beta = 0$, both the impact and the dynamic effects on all the variables, except debt and the relative price of the nontraded good (p_{nt}), are qualitatively similar to those obtained by Macklem (1993) and Mendoza (1995).⁶⁸ Both find that the relative price of the nontraded good falls below its steady-state level following the shock, and stays below that level for more than ten years after the shock.⁶⁹ Note that while we use the export good as the numeraire, both Mendoza (1995) and Macklem (1993) use importables as the numeraire. In our case, although p_{nt} rises above its steady-state level on impact, it soon starts falling, and goes below the steady-state level after about thirteen years following the shock.⁷⁰ It then stays below that level for more than eighteen years after the shock. As explained in sub-section 5.4.2.3, the response of p_{nt} on impact depends on the relative strength of the *income* and *substitution* effects in consumption of the shock. The magnitude of the income effect of the shock depends partly on θ , the share of exports in tradables expenditure.

⁶⁶As mentioned in Section 5.1, while Mendoza allows for perfect capital flows, we assume imperfect capital flows. We also ignore the intertemporal substitution of leisure as an explanation for economic fluctuations.

⁶⁷Mendoza's (1991) one-sector small open economy model is also calibrated to the Canadian data. The work is, however, an attempt to rationalize the observed pattern of postwar Canadian business fluctuations within a real-business-cycle model. Moreover, Mendoza (1991) does not report impact and dynamic effects of shocks. The results are, therefore, not directly comparable with our's in this chapter.

⁶⁸One may not expect quantitative similarity of results, as these models and their solution procedures are different. Note that Macklem (1993) and Mendoza (1995) are real-business-cycle models, and we ignore intertemporal substitution of leisure explanation for fluctuations in economic activity.

⁶⁹Note that Mendoza (1995) considers a positive terms-of-trade shock. Hence, in comparing our results with those of Mendoza, we consider the opposite of his results reported for a positive shock.

⁷⁰Note that the rise in the relative price on impact is less than one-sixth of a percent.

Figure 5.5 The dynamic effects of a *terms-of-trade* shock on debt and the relative price of the nontraded good

Macklem (1993) finds that a temporary terms-of-trade shock leads to a current account deterioration within the first year after the shock while Mendoza (1995) finds that the trade balance initially deteriorates, but later improves (about three years following the shock). Our results indicate that current account initially improves (debt declines), but then deteriorates after about eight years following the shock. By that time, investments in both production sectors have overshoot their long-run levels (see Figures 5.7 and 5E.1), although both consumption and the output of the export good are still below their steady-state levels. Thus, the growth in debt is essentially reflecting the differences between saving and investment.

Given the behavior of debt that our results above show, a natural question arises: is the behavior consistent with the classic analysis of Harberger (1950) and Laursen and Metzler (1950)? The Harberger-Laursen-Metzler (HLM) effect implies that an worsening of the terms-of-trade leads to a decline in net exports and saving. This follows from the fact that the reduction in income caused by a fall in the purchasing power of exports does not reduce consumption proportionately, because marginal propensities to consume and save are less than unitary. Although we see a HLM-effect kind of result, it happens about eight years after the shock.

For about two decades now, initiated first by Obstfeld (1982a), the Harberger-Laursen-Metzler proposition has been the subject of considerable scrutiny in a range of stochastic and deterministic models.⁷¹ One conclusion that emerges from this scrutiny is that the relationship between the trade balance and the terms-of-trade depends critically on two

⁷¹A partial list of the major works that analyze the validity or otherwise of the proposition includes Obstfeld (1982 a,b), Svensson and Razin (1983), Persson and Svensson (1985), Stockman and Svensson (1987), Backus et al. (1994), and Karayalcin (1995).

factors: the persistence of the shock and the form of dependence of the discount factor, or the rate of time preference, on future utility. In general, with a fixed rate of time preference, transitory shocks result in the HLM effect, as agents borrow from abroad to smooth consumption, but permanent changes tend to leave the trade balance unaffected. For example, Obstfeld (1982b) considers a simple intertemporal optimization model in which representative agents have a constant rate of time preference. In that model, a temporary deterioration leads to a current account deficit for the duration of the adverse movement in the terms-of-trade. The current account moves into surplus when the terms of trade revert to their initial level, and the economy eventually returns to its original level of external indebtedness. Thus, by running current account deficit when times are bad and surplus after the terms of trade have improved, the household softens the immediate impact of the shock at the cost of reducing consumption in the new long-run equilibrium below the level that would have been feasible if no current account deficit had been run. Obstfeld (1982b), however, shows that in the face of a permanent unanticipated terms-of-trade deterioration, agents cut their consumption by the full amount of the fall in their real income. Consequently, no current imbalance ensues.

On the other hand, Obstfeld (1982a) shows that with an endogenous rate of time preference, the representative household saves more and runs a current account surplus following the terms-of-trade deterioration. Evidently, this result contradicts the HLM proposition. Backus, et al. (1994), however, find that the relation between the trade balance and the terms-of-trade is determined by the elasticity of substitution, regardless of the persistence of shocks.

Notice that the last result above has implications for our results. The static and income-based HLM effect assumes that the home country and the rest of the world produce the same good, and that this good is tradable across countries. Accordingly, the possibility of the *intratemporal* as well as *intertemporal* substitution, present in our three-good model, is absent in the HLM framework. The substitution effect on consumption and saving decisions arising from a temporary terms-of-trade shock in our framework depends on the elasticities of the *intertemporal* as well as *intratemporal* substitution. The elasticity parameters determine the ease with which the representative consumer-investor can switch between current and future consumption in response to a shift in the consumption-based price index induced by a shock (intertemporal substitution), and the ease with which the agent can switch between importables and nontradables (intratemporal substitution). An adverse temporary terms-of-trade shock can, depending, of course, on the sizes of the elasticity parameters, lower current consumption and improve the current account. This effect on the current account is augmented in the present model by the decrease in investment in the two sectors of the economy.⁷²

We have seen in section 5.4.2.3 that a terms-of-trade shock induces the consumer-investor to lower consumption. The shock also lowers investment in both production sectors by raising the price of capital. The rise in the relative price of nontraded good on impact indicates that the consumer-investor attempts to substitute for the more expensive imported good (see Figure 5.6). We also find that the output of the export good remains unchanged on impact although it gradually decreases following the terms-of-trade loss. The result of all this

⁷²Note that investment is a composite of both domestic and imported goods in our model, and an adverse terms-of-trade shock lowers investment by raising the price of investment.

is an improvement in the current account on impact following the shock. The improved debt situation along with the reversing behavior of the temporary shock soon initiates a steeply rising path of investment. Consumption also starts rising almost immediately after the initial negative effect. The output of the export good reaches its lowest level in about two years following the shock, when investments in both sectors have almost returned back to their steady-state levels. The deterioration in the current account after about eight years following the shock is the outcome of investment in both sectors exceeding their steady-state levels when the output of the export good is still below its steady-state level.

We now consider a productivity shock in the export good sector. The case with $\omega = \vartheta = 0$ can be compared with Mendoza (1995). Both the impact and the dynamic effects on all the variables, except the relative price of the nontraded good (p_n), are qualitatively similar to those obtained by Mendoza (1995), who considers a 1-percent positive productivity shock uniform across sectors. Interestingly, except for the trade balance and the relative price, even the quantitative effects are largely similar. Mendoza finds that the relative price of nontradables falls by about 0.5 percent below its steady-state level on impact. The price, however, starts rising immediately, and thus exceeds its steady-state level within the first year of the shock. It then stays above that level by about 0.1 percent for more than 12 years after the shock. Mendoza's results indicate a strong trade balance effect because it increases by more than 2 percent on impact. It, however, starts falling immediately and stays below the steady-state level by about 0.25 percent for more than 12 years after the shock.

Figure 5.6 The dynamic effects of a productivity shock in the export good sector on debt and the relative price of the nontraded good

We find that the wealth effect caused by the productivity gain in the export good sector raises (see Figure 5.6) the relative price of the nontraded good by about 0.6 percent on impact. It, however, starts falling immediately. The price stays above its steady-state level by about 0.05 percent even after eighteen years following the shock. Debt in our case (see Figure 5.6) drops by 0.98 percent on impact. It then takes an ever-falling path, showing little tendency even after eighteen years following the shock to return to its steady-state level. We reported earlier that with $\omega = 0$, our model displays a unit-root characteristic. The non-convergent behavior of debt is evidently consistent with that unit-root.

5.5.2 The potential role of ω and ϑ in a dynamic model

Case 1: $\omega = 0, \vartheta = 0.5$

Let us now first consider the role of domestic goods in the investment expenditure. Evidently, investment and debt now show significant changes in response to a terms-of-trade shock. With exportables accounting for 50 percent of the investment expenditure (i.e., $\vartheta = 0.5$), the rise in the price of investment is now smaller than before and, consequently, the reduction in investment in the export good sector is now by 2.85 percent, as opposed to the 5.697 percent in the case where capital is formed entirely from imports (see Appendix 5A).⁷³ It also follows that our representative consumer-investor's income loss due to the terms-of-trade shock is not as large as before. This is reflected in the mild gain in terms of consumption.⁷⁴ It is, therefore, natural that when both investment and consumption are not

⁷³See in Figure 5E.1 in Appendix 5E that investment in the nontraded sector also shows similar behavior.

⁷⁴Consumption is now falling by 0.393 percent as opposed to the 0.3985 percent before.

Figure 5.7 The dynamic effects of a *terms-of-trade* shock on output and investment in the export good sector

Figure 5.8 The dynamic effects of a productivity shock in the export good sector on consumption and investment in the export good sector

falling by as much as before, the current account is not increasing as much. Debt is now falling on impact by 1.037 percent, compared with the 2.25 percent decline before. The mild price responses evident in Table 5.8 can be rationalized by the smaller adverse effect on income due to the terms-of-trade shock. Notice in Figure 5.7 that the reduction in the output of the export-good after the shock to the terms-of-trade is also not as large as before. This can be accounted by the smaller negative effect on investment. It is evident, as one will expect, that the share of imports in the investment expenditure does not make any considerable change in the responses of the variables to productivity shocks.

Case 2: $\omega = 0.1, \vartheta = 0.5$

We now consider the role of ω , our most significant contribution in this chapter. One can immediately note the convergent behavior of debt in Figures 5.6, 5.7, and 5E.2 in Appendix 5E, following shocks, respectively, to terms-of-trade and productivity in the export and nontraded good sectors. This verifies the fact first reported in section 5.4.1 that with $\omega = 0$ our model exhibits a unit root property, and that some sensitivity of the real rate of interest to the stock of foreign debt gives convergent solutions. This reiterates the fact that when a small open economy can borrow as much as desired at a parametrically given world interest rate, the economy will borrow indefinitely whenever the interest rate in the international capital market is below the representative consumer-investor's rate of time preference. It is, therefore, not surprising that $\omega > 0$ gives our model stability. We examined the sensitivity of the model to variations in ω in sub-section 5.4.4.1 and noted that $\omega = 0.01$ gave our model the best fit in terms of matching the standard deviations of data.

Let us now see how ω changes the impact effects. Table 5.8 illustrates that all the variables, except the output of the export good, show significant changes in response to shocks to productivity and terms of trade. Thus, in response to a positive productivity shock in the export good sector, investment rises on impact by 1.94 percent when $\omega = 0.1$, as opposed to the 1.286 percent rise when $\omega = 0$. Notice also the changes in the negative response of investment to a terms-of-trade shock and in the response of consumption to shocks to productivity in both sectors as well as to the terms-of-trade. In all cases, the key to understanding the changes is that changes in the stock of foreign debt affect the cost of borrowing in the international capital market. We have already seen that positive productivity shocks in the two sectors of the economy (as well as a terms-of-trade loss) led to a reduction in foreign debt. Thus, in addition to the wealth gain of a positive productivity shock, we now have the lower cost of servicing foreign debt.

In terms of the dynamic effects, the most significant change is now the convergent behavior of foreign debt in particular, but also of the behavior of consumption to a shock to productivity in the export good sector. It is also evident that the sensitivity of the real interest rate in the stock of foreign debt, when evaluated at the steady-state, realistically lowers the impact and dynamic responses of all the endogenous variables in the model to shocks to its forcing variables. Although we have used a large value of ω ($= 0.1$) in this section for convenience, we consider in Figure 5.9 below, for analytical completeness, the dynamic behavior of consumption, investment and debt to a productivity shock in the export good sector when $\omega = 0.01$, the value that gave our model its best fit. We now consider a 50-year

Figure 5.9 Dynamic responses of a productivity shock in the export good sector ($\omega = 0$ vs. $\omega = 0.01$)

time horizon. A comparison of the behavior of debt in the above diagram with that in Figure 5.6 indicates that the smaller value of ω increases the impact effect of the productivity shock on debt and slows down the convergence of debt. What is important to note is that even a small sensitivity of the real interest rate to the stock of debt gives the model stability and realistically brings the behavior of debt in the model in line with what we expect in the real world. In this respect, the present model can be considered more useful than one that is not stable to analyze the behavior of a small open economy to temporary exogenous shocks.

5.6 Conclusions

This chapter develops a three-good small open-economy model with intertemporal optimizing agents. The purpose of this model is to analyze the dynamics of the real exchange rate, the current account and foreign debt following shocks to productivity and terms-of-trade. The model extends the base-line intertemporal optimizing macro-economic framework by incorporating less-than-perfectly-elastic capital flows and treating investment as composite of both domestic and imported goods. Modeling a small open economy with imperfect capital flows is necessary given the evidence reported in chapters 2 and 3 pertaining to the violations of real interest rate parity. Making investment a composite good is useful because investment's response to terms-of-trade shocks is proportionate to the share of the imported good in the investment composite. Stockman and Tesar (1995, p.183) attribute the unsatisfactory performance of their model in matching the correlation between the trade balance and output to their assumption that the traded-good industry in each country uses only domestically produced capital in production. In this regard, the modeling of investment as a composite good is useful and realistic. Quadratic capital installation costs are included in the present model to emphasize that capital stocks move sluggishly, and protracted deviations of capital's marginal product from the real rate of interest are then possible. We have also realistically made capital stocks sector-specific so that the model can generate appropriate responses of its production sectors to exogenous shocks. The model is then calibrated to the Canadian data for the period 1987-1998, solved, simulated, and analyzed with tools borrowed from the real business cycle literature. Not only that the model's shock processes are estimated from data for the above specific period, we attempt

GMM estimation of key structural parameters in the model's parameterization process. When formal estimation methods fail to produce reasonable parameter values, we use the model's restrictions to 'calibrate' parameters and other steady-state ratios, again with the same data as above. Thus, the parameterization process is made as formal as possible.

The model's performance is reasonably good, given the limitations of data. We find that some degree of sensitivity of the real interest rate to the stock of foreign debt gives the model stability, and allows the model to better fit the data in terms of producing a match with the standard deviations in the data. However, while the use of non-stationary data would make predictions unreliable, the use of filters to remove the non-stationarity may have removed too much information contained in the data (see King and Rebelo, 1993). Even then the ability of the model to mimic some empirical characteristics of the Canadian economy is promising. More promising is its ability to incorporate open economy considerations within a more complete model of capital accumulation. As Dornbusch (1983) acknowledges, at the conclusion of his two-good open-economy model, a model without capital accumulation is oversimplified to study the "interaction of saving and investment on relative prices and the balance of trade." Over the years, a large number of economists developed models using the Euler equation of an optimizing consumer, ignoring the implications of production and capital accumulation for key relative prices and other macroeconomic variables.⁷⁵ In this respect, the present framework can perform well in that it encompasses a more complete and realistic structure of an open-economy with imperfect capital flows.

⁷⁵A partial list includes Froot and Rogoff (1991), Rogoff (1992), Lin (1996), Chinn (1997), Chinn and Johnston (1996).

Appendix 5A

Minimization of Investment Expenditure

The representative consumer-investor aggregates exportables and importables into a capital good according to a *homogenous-of-degree-one* technology:⁷⁶

$$I_t = j_{xt}^{\vartheta} j_{mt}^{(1-\vartheta)} \quad (5A.1)$$

where j_{xt} and j_{mt} are, respectively, the quantities of exportables and importables used as inputs into the investment process, and ϑ is the share of export goods in the investment composite. In order to minimize investment cost, the consumer-investor solves the static problem:

$$\begin{aligned} \min & (j_{xt} + p_{mt} j_{mt}) \\ \text{subject to } & I_{xt} = j_{xt}^{\vartheta} j_{mt}^{(1-\vartheta)} \end{aligned} \quad (5A.2)$$

The static problem solves for the cost minimizing choices:

$$\begin{aligned} j_{mt}^* &= \left(\frac{1-\vartheta}{\vartheta p_{mt}} \right)^{\vartheta} I_{xt} \\ j_{xt}^* &= \left(\frac{\vartheta p_{mt}}{1-\vartheta} \right)^{1-\vartheta} I_{xt} \end{aligned} \quad (5A.3)$$

The minimized expenditure is thus given by:

$$q_t I_{xt} = \Phi p_{mt}^{1-\vartheta} I_{xt} \quad (5A.4)$$

where $q_t = \Phi p_{mt}^{1-\vartheta}$; $\Phi = \left[((1-\vartheta)/\vartheta)^{\vartheta} + (\vartheta/(1-\vartheta))^{1-\vartheta} \right]$, implicitly defines the price of a

⁷⁶The sectoral subscript is ignored here for convenience.

new unit of capital. Once exportables and importables are aggregated into the composite investment good, the consumer-investor now must incur additional costs for installation of the newly acquired investment goods.

Appendix 5B

Intra-temporal Optimization, the derivation of the Price Index, and Equilibrium in the Labour Market

Intra-temporal Optimization and the derivation of the Price Index

The consumption-based price index P_t is the minimum expenditure, in terms of exportables, $z_t = p_{ht} C_{ht} + C_{xt} + p_{mt} C_{mt}$, such that $C_t = \left[C_{ht}^{-\mu} + (C_{xt}^\theta C_{mt}^{1-\theta})^{-\mu} \right]^{-1/\mu} = 1$. Given any level of consumption C_t^0 derived from the intertemporal optimization, the consumer chooses C_{ht} , C_{xt} , and C_{mt} so as to minimize expenditure z_t in terms of exportables. This problem can be expressed as a problem of minimizing the following Lagrangian for choices of C_{ht} , C_{xt} , C_{mt} and λ :

$$\mathcal{L}_t = (p_{ht} C_{ht} + C_{xt} + p_{mt} C_{mt}) + \lambda \left[C_t^0 - \left\{ C_{ht}^{-\mu} + (C_{xt}^\theta C_{mt}^{1-\theta})^{-\mu} \right\}^{-1/\mu} \right] \quad (5B.1)$$

The first-order conditions associated with the minimization of (5B.1) for choices of C_{ht} , C_{xt} , C_{mt} and λ gives, after little manipulations, the following three conditions:

$$\begin{aligned} C_{ht} &= \left[p_{ht} \theta^{-\mu\theta} (1-\theta)^{1+\mu\theta} \right]^{\frac{-1}{1-\mu}} p_{mt}^{\frac{1+\mu\theta}{1-\mu}} C_{mt} \\ C_{xt} &= \left(\frac{\theta}{1+\theta} \right) p_{mt} C_{mt} \\ C_t^0 &= \left[C_h^{-\mu} + (C_x^\theta C_m^{1-\theta})^{-\mu} \right]^{-1/\mu} \end{aligned} \quad (5B.2)$$

We now substitute for C_{ht} and C_{xt} from the first two equations in (5B.2) into the third equation, and solve it for C_{mt} to obtain:

$$C_{mt} = C_t^0 \left[\left(\frac{\theta p_{mt}}{1-\theta} \right)^{-\mu\theta} + \left\{ p_{ht} \theta^{-\mu\theta} (1-\theta)^{1+\mu\theta} \right\}^{\frac{\mu}{1-\mu}} p_{mt}^{-\mu \left(\frac{1+\mu\theta}{1-\mu} \right)} \right]^{1/\mu} \quad (5B.3)$$

Given the *inter-temporally* optimized level of consumption C_t^0 and the preference parameters, *intra-temporally* optimized quantity of the import good in the consumption bundle is determined by the exogenously given *terms-of-trade*, p_{mr} , and the endogenously determined (from the non-traded goods market equilibrium condition consistent with the labor market equilibrium) relative price of non-traded to export goods, p_{ht} . With (5B.3) derived, consumption demands for non-traded and export goods in (5B.2) can be similarly written as functions of p_{mt} and p_{ht} .

The minimized expenditure $z_t = p_{ht} C_{ht} + C_{xt} + p_{mt} C_{mt}$ in terms of exportables is then given by:

$$z_t = \left\{ \frac{p_{mt}}{1-\theta} + p_{ht} \left[p_{ht} \theta^{-\mu\theta} (1-\theta)^{1+\mu\theta} \right]^{-1/(1-\mu)} p_{mt}^{\frac{1+\mu\theta}{1-\mu}} \right\} \Omega(p_{mt}, p_{ht}) C_t^0;$$

$$\Omega(p_{mt}, p_{ht}) = \left[\theta^{-\mu\theta} \left(\frac{p_{mt}}{1-\theta} \right)^{-(1+\mu\theta)} \left\{ \frac{p_{mt}}{1-\theta} + p_{ht}^{\frac{\mu}{1-\mu}} \theta^{\frac{\mu\theta}{1+\mu}} (1-\theta)^{\frac{-(1+\mu\theta)}{(1-\mu)}} p_{mt}^{\frac{(1-\mu\theta)}{(1-\mu)}} \right\} \right]^{1/\mu}.$$

After some lengthy manipulations, one can re-write the minimized expenditure as:

$$z_t = \left[\left\{ \kappa p_{mt}^{1-\theta} \right\}^{\mu/(1-\mu)} + p_{ht}^{\mu/(1-\mu)} \right]^{(1-\mu)/\mu} C_t^0 \quad (5B.4)$$

where $\kappa = \theta^{-\theta} (1-\theta)^{-(1-\theta)}$. Now noting that the consumption-based price index P_t is the minimum expenditure z_t such that $C_t = 1$, we have:

$$P_t = \left[\left\{ \kappa p_{mt}^{1-\theta} \right\}^{\mu/(1-\mu)} + p_{ht}^{\mu/(1-\mu)} \right]^{(1-\mu)/\mu} \quad (5B.5)$$

which is equation (5.15) of the model.

Labor Market Equilibrium

Labor market clearing requires that $N_{xt} + N_{ht} = 1$. Note that we can de-trend per-capita output by the labor-augmenting productivity component X_t , and write the production functions per efficiency unit of labor as:

$$\begin{aligned} y_{ht} &= Z_{ht} k_{ht}^\rho N_{ht}^{1-\rho} \\ y_{xt} &= Z_{xt} k_{xt}^\rho N_{xt}^{1-\rho} \end{aligned}$$

It is relatively easy to obtain marginal products of labor from the two production functions. Equating the marginal products with real wage, which is in terms of the exportables, we obtain, after a little manipulation, the labor demand functions in the two sectors:⁷⁷

$$\begin{aligned} N_{ht}^d &= \left[\frac{(1-\rho) p_{ht} Z_{ht} k_{ht}^\rho}{W_t} \right]^{1/\rho} \\ N_{xt}^d &= \left[\frac{(1-\rho) Z_{xt} k_{xt}^\rho}{W_t} \right]^{1/\rho} \end{aligned}$$

The equilibrium real wage W_t is obtained from the labor market clearing condition:

$$\left[\frac{(1-\rho) p_{ht} Z_{ht} k_{ht}^\rho}{W_t} \right]^{1/\rho} + \left[\frac{(1-\rho) Z_{xt} k_{xt}^\rho}{W_t} \right]^{1/\rho} = 1$$

Thus we obtain equation (5.18) of the model:

$$W_t = (1-\rho) \left[(Z_{xt} k_{xt}^\rho)^{1/\rho} + (p_{ht} Z_{ht} k_{ht}^\rho)^{1/\rho} \right]^\rho, \quad (5B.6)$$

⁷⁷We use a superscript d to denote labor demands.

Appendix 5C

Intertemporal Optimization and the Log-linearization of the Model

Intertemporal Optimization

We set up the consumer-investor's optimization problem in terms of the maximization of the following Lagrangian function from (5.19) for choices of $c_t, f_t, k_{ht-1}, k_{xt-1}, i_{ht}$ and i_{xt} :

$$\mathcal{L}_t = E_t \left[\beta^{*t} \sum_{t=0}^{\infty} \left\{ -\lambda_t \left\{ \begin{aligned} & X_0^{1-\eta} \left(\frac{1}{1-\eta} c_t^{1-\eta} \right) \\ & P_t c_t + q_{xt} i_{xt} + \frac{\phi_x q_{xt}}{2} \left(\frac{i_{xt}}{k_{xt}} - \frac{i_x}{k_x} \right)^2 k_{xt} + (R_t/\gamma) f_{t-1} - f_t \\ & -Z_{xt} k_{xt}^\rho - p_{ht} Z_{ht} k_{ht}^\rho + q_{ht} i_{ht} + \frac{\phi_h q_{ht}}{2} \left(\frac{i_{ht}}{k_{xt}} - \frac{i_h}{k_h} \right)^2 k_{ht} \\ & -Q_{xt} (\gamma k_{xt-1} - (1-\delta_x) k_{xt} - i_{xt}) - Q_{ht} (\gamma k_{ht-1} - (1-\delta_h) k_{ht} - i_{ht}) \end{aligned} \right\} \right\} \right] \quad (5C.1)$$

where, as stated before, γ is the gross rate of labor augmenting technological progress such that $X_t = \gamma X_{t-1} = \gamma^t X_0$, and $\beta^* = \beta \gamma^{1-\eta}$ is the consumer's effective discount factor. The first-order efficiency conditions are obtained by differentiating the Lagrangian with respect to the choice variables stated above and the multipliers, λ_t , Q_{xt} and Q_{ht} at each time t :

$$\begin{aligned}
X_0^{1-\eta} c_t^{-\eta} - \lambda_t P_t &= 0 \\
E_t[\lambda_t - \beta^* \lambda_{t+1} (R_{t+1}/\gamma)] &= 0 \\
Q_{xt} &= \lambda_t q_{xt} \left[1 + \phi_x \left(\frac{i_{xt}}{k_{xt}} - \frac{i_x}{k_x} \right) \right] \\
Q_{ht} &= \lambda_t q_{ht} \left[1 + \phi_h \left(\frac{i_{ht}}{k_{ht}} - \frac{i_h}{k_h} \right) \right] \\
\beta^* E_t \left[\lambda_{t+1} \left\{ \begin{aligned} &\phi_x q_{xt+1} \left(\frac{i_{xt+1}}{k_{xt+1}} - \frac{i_x}{k_x} \right) \left(\frac{i_{xt+1}}{k_{xt+1}} \right) \\ &- \frac{\phi_x q_{xt+1}}{2} \left(\frac{i_{xt+1}}{k_{xt+1}} - \frac{i_x}{k_x} \right)^2 + \rho Z_{xt+1} k_{xt+1}^{\rho-1} \end{aligned} \right\} + (1 - \delta_x) Q_{xt+1} \right] &= \gamma Q_{xt} \quad (5C.2) \\
\beta^* E_t \left[\lambda_{t+1} \left\{ \begin{aligned} &\phi_h q_{ht+1} \left(\frac{i_{ht+1}}{k_{ht+1}} - \frac{i_h}{k_h} \right) \left(\frac{i_{ht+1}}{k_{ht+1}} \right) \\ &- \frac{\phi_h q_{ht+1}}{2} \left(\frac{i_{ht+1}}{k_{ht+1}} - \frac{i_h}{k_h} \right)^2 + \rho P_{ht+1} Z_{ht+1} k_{ht+1}^{\rho-1} \end{aligned} \right\} + (1 - \delta_h) Q_{ht+1} \right] &= \gamma Q_{ht}
\end{aligned}$$

The first two equations in (5C.2) above can be written as:

$$E_t \left[\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\eta} \left(\frac{P_t}{P_{t+1}} \right) R_{t+1} \right] = \gamma^\eta \quad (5C.3)$$

Substituting the third equation in (5C.2) into the fourth we obtain:

$$\begin{aligned}
&\beta^* E_t \left[\phi_x \lambda_{t+1} q_{xt+1} \left(\frac{i_{xt+1}}{k_{xt+1}} - \frac{i_x}{k_x} \right) \left\{ \frac{1}{2} \left(\frac{i_{xt+1}}{k_{xt+1}} + \frac{i_x}{k_x} \right) + (1 - \delta_x) \right\} \right. \\
&\quad \left. + \lambda_{t+1} \left\{ q_{xt+1} (1 - \delta_x) + \rho Z_{xt+1} k_{xt+1}^{\rho-1} \right\} \right] \\
&= \gamma q_{xt} \lambda_t \left\{ 1 + \phi_x \left(\frac{i_{xt}}{k_{xt}} - \frac{i_x}{k_x} \right) \right\} \quad (5C.4)
\end{aligned}$$

One can recognize that the equation in (5C.4) is indeed the investment Euler equation for the export good sector. Using the fourth and sixth equations in (5C.2), one obtains the investment Euler equation for the nontraded good sector:

$$\begin{aligned} \beta \cdot E_t \left[\phi_h \lambda_{t+1} q_{ht+1} \left(\frac{i_{ht+1}}{k_{ht+1}} - \frac{i_h}{k_h} \right) \left\{ \frac{1}{2} \left(\frac{i_{ht+1}}{k_{ht+1}} + \frac{i_h}{k_h} \right) + (1 - \delta_h) \right\} \right. \\ \left. + \lambda_{t+1} \{ q_{ht+1} (1 - \delta_h) + \rho p_{ht+1} Z_{ht+1} k_{ht+1}^{\rho-1} \} \right] \\ = \gamma q_{ht} \lambda_t \left\{ 1 + \phi_h \left(\frac{i_{ht}}{k_{ht}} - \frac{i_h}{k_h} \right) \right\} \end{aligned} \quad (5C.5)$$

Three remaining efficiency conditions are:

$$\begin{aligned} P_t c_t + q_{xt} i_{xt} + \frac{\phi_x q_{xt}}{2} \left(\frac{i_{xt}}{k_{xt}} - \frac{i_x}{k_x} \right)^2 k_{xt} + (R_t / \gamma) f_{t-1} - f_t \\ - Z_{xt} k_{xt}^\rho - p_{ht} Z_{ht} k_{ht}^\rho + q_{ht} i_{ht} + \frac{\phi_h q_{ht}}{2} \left(\frac{i_{ht}}{k_{ht}} - \frac{i_h}{k_h} \right)^2 k_{ht} = 0 \\ \gamma k_{xt+1} - (1 - \delta_x) k_{xt} - i_{xt} = 0 \\ \gamma k_{ht+1} - (1 - \delta_h) k_{ht} - i_{ht} = 0 \end{aligned} \quad (5C.6)$$

Log-Linearization of the Efficiency and other Conditions

The principle of log-linearization is the use of Taylor approximations around the steady state of the model economy to replace all equations by their linear approximations. We note from the above equations that in the steady state of the model the following relationships hold:

$$\begin{aligned}
Pc &= (y_x + p_h y_h) - q_x i_x - q_h i_h - [(R/\gamma) - 1]f \quad (\text{from 5C.6}) \\
\beta^* R &= \beta \gamma^{1-\eta} R = \gamma \Leftrightarrow \beta R = \gamma^\eta \quad (\text{from 5C.2}) \\
\rho Z_x k_x^{\rho-1} + q_x (1 - \delta_x) &= q_x \gamma / \beta^* = q_x R \quad (\text{from 5C.4}) \\
\rho p_h Z_h k_h^{\rho-1} + q_h (1 - \delta_h) &= q_h \gamma / \beta^* = q_h R \quad (\text{from 5C.5}) \\
k_x &= \left(\frac{q_x (R - 1 + \delta_x)}{\rho Z_x} \right)^{1/(\rho-1)} \quad (5C.7) \\
k_h &= \left(\frac{q_h (R - 1 + \delta_h)}{\rho p_h Z_h} \right)^{1/(\rho-1)} \\
i_x / k_x &= (\gamma - 1 + \delta_x) \quad (\text{from 5C.6}) \\
i_h / k_h &= (\gamma - 1 + \delta_h) \quad (\text{from 5C.6})
\end{aligned}$$

The third and fourth expressions in (5C.7) equate the gross finance cost of installing a unit of capital to its gross return. Note that in the present setup, there is no installation cost in the steady state. Thus, the left hand sides in the above-mentioned two expressions give the gross rates of return on a one-period investment in capital, which equal the marginal product of capital in production plus the value of the undepreciated capital. The right hand side gives the gross finance cost in terms of exportables.

In order to log-linearize the first-order conditions above, we use a short-cut proposed by Uhlig (1997). Let $\tilde{x}_t$ denote the logarithmic deviation of any variable x_t from its steady-state value x . Formally:

$$\tilde{x}_t = \log(x_t) - \log(x)$$

Thus, if $\tilde{x}_t = 0.04$, then x_t is approximately 4 percent above its steady state value.⁷⁸ It is useful to re-write the above expression as:

$$x_t = x e^{\tilde{x}_t} \approx x(1 + \tilde{x}_t) = x + x\tilde{x}_t \quad (5C.8)$$

Let us first consider log-linearization of the feasibility constraint, the first equation in system (5C.6). Using (5C.8), one can obtain:

$$\begin{aligned} y_x \tilde{y}_{xt} + (p_h y_h) \tilde{p}_{ht} + (p_h y_h) \tilde{y}_{ht} - P c \tilde{p}_t - P c \tilde{c}_t - i_x q_x \tilde{i}_{xt} - i_h q_h \tilde{i}_{ht} \\ - i_x q_x (1 - \vartheta_x) \tilde{p}_{mt} - i_h q_h (1 - \vartheta_h) \tilde{p}_{mt} - (Rf/\gamma)(\tilde{r}_t + \tilde{f}_{t-1}) + f \tilde{f}_t = 0 \end{aligned} \quad (5C.9)$$

Given the assumption that the real rate of interest at time t is an increasing function of the stock of foreign debt at time $(t-1)$, the logarithmic deviation of R_t from its steady state value R is given by:

$$\tilde{r}_t = \omega \tilde{f}_{t-1} \quad (5C.10)$$

where $\omega \equiv R'(f)f/R(f)$ is the elasticity of the real rate of interest with respect to foreign debt, when evaluated at the steady state.

Let us now apply Uhlig's method to the Euler equation in (5C.3) to obtain the following linear approximation:

⁷⁸It may be useful to remind the reader that lowercase letters are used to indicate per-capita levels of variables de-trended by the labour-augmenting growth component X_t , and tildes are added to denote percentage deviations of these de-trended per-capita variables from their steady state levels. The exceptions, however, are the following. We use P_t and p_{ht} , and p_{mt} , respectively, for the price index, the relative price of non-traded to export goods, and the *terms-of-trade*. The absence of time subscript with any variable indicates the steady state level of the variable. We use r_t for the real rate of interest, while $\tilde{r}_t$ is used to denote the logarithmic deviation of the gross rate of return from its steady state value: $\tilde{r}_t = \log R_t - \log R$.

$$E_t[\eta\tilde{c}_t - \eta\tilde{c}_{t+1} + \tilde{p}_t - \tilde{p}_{t-1} + \tilde{r}_{t-1}] = 0 \quad (5C.11)$$

For the price level, capital accumulation, and the outputs in the two sectors of the economy, we have:

$$\begin{aligned} P^{\mu/(1-\mu)}\tilde{p}_t &= (1-\theta)(\kappa P_m^{1-\theta})^{\mu/(1-\mu)}\tilde{p}_{mt} + P_h^{\mu/(1+\mu)}\tilde{p}_{ht} \\ \gamma\tilde{k}_{xt+1} - (1-\delta_x)\tilde{k}_{xt} - (\gamma-1+\delta_x)\tilde{i}_{xt} &= 0 \\ \gamma\tilde{k}_{ht+1} - (1-\delta_h)\tilde{k}_{ht} - (\gamma-1+\delta_h)\tilde{i}_{ht} &= 0 \\ \tilde{y}_{xt} - \tilde{z}_{xt} - \rho\tilde{k}_{xt} &= 0 \\ \tilde{y}_{ht} - \tilde{z}_{ht} - \rho\tilde{k}_{ht} &= 0 \end{aligned} \quad (5C.12)$$

In the capital accumulation equations (the second and third equations in (5C.12)), we use the steady state ratios of investment to capital stock given by $(\gamma+1-\delta_i)$, $(i=x, h)$ (see 5C.7).

The log-linearization of the formidable looking equation (5C.4) can be made a bit less difficult by utilizing the steady state conditions given in (5C.7). One can obtain:

$$E_t \left[\begin{aligned} &\gamma q_x \tilde{\lambda}_{t-1} - \gamma q_x \tilde{\lambda}_t + \beta^* \{ \rho(\rho-1)(y_x/k_x) - q_x \gamma \phi_x(i_x/k_x) \} \tilde{k}_{xt+1} \\ &+ \rho \beta^* (y_x/k_x) \tilde{z}_{xt+1} + q_x (1-\vartheta_x) \{ \beta^* (1-\delta_x) \tilde{p}_{mt+1} - \gamma \tilde{p}_{mt} \} \\ &+ \beta^* q_x \phi_x \gamma (i_x/k_x) \tilde{i}_{xt-1} + \gamma q_x \phi_x (i_x/k_x) \{ \tilde{k}_{xt} - \tilde{i}_{xt} \} \end{aligned} \right] = 0$$

Substituting for $\gamma\lambda_t = \beta^* E(\lambda_{t+1} R_{t+1})$ from (5C.2) into (5C.4) one obtains an expression that links the marginal productivity of capital to the gross rate of interest. We use results from the above log-linearization to obtain:

$$E_t \left[\begin{aligned} & \left\{ \rho(\rho-1)(y_x/k_x) - q_x \gamma \phi_x(i_x/k_x) \right\} \tilde{k}_{xt+1} + q_x \gamma \phi_x(i_x/k_x) \tilde{i}_{xt+1} \\ & + q_x R \phi_x(i_x/k_x) \left\{ \tilde{k}_{xt} - \tilde{i}_{xt} \right\} + q_x (1 - \vartheta_x) \left[(1 - \delta_x) \tilde{p}_{mt+1} \right] \\ & + (\rho y_x/k_x) \tilde{z}_{xt+1} - q_x R \tilde{r}_{t+1} - R \tilde{p}_{mt} \end{aligned} \right] = 0$$

It is now straightforward to log-linearize the investment Euler equation for the nontraded sector:

$$E_t \left[\begin{aligned} & \gamma q_h \tilde{\lambda}_{t-1} - \gamma q_h \tilde{\lambda}_t + \beta^* q_h \phi_h(i_h/k_h) \tilde{i}_{ht+1} + \rho \beta^* (p_h y_h/k_h) \{ \tilde{z}_{ht+1} + \tilde{p}_{ht+1} \} \\ & + \beta^* \left\{ \rho(\rho-1)(p_h y_h/k_h) - q_h \gamma \phi_h(i_h/k_h) \right\} \tilde{k}_{ht+1} \\ & + q_h (1 - \vartheta_h) \left\{ \beta^* (1 - \delta_h) \tilde{p}_{mt+1} - \tilde{p}_{mt} \right\} + \gamma q_h \phi_h(i_h/k_h) \left\{ \tilde{k}_{ht} - \tilde{i}_{ht} \right\} \end{aligned} \right] = 0$$

In order to obtain a linearized expression for the relative price of non-traded to export goods, we first note the equilibrium condition in the market for non-traded goods: domestic consumption demand for such goods must equal its supply in equilibrium:

$$\begin{aligned} & \left[p_{ht} \theta^{-\mu\theta} (1-\theta)^{1+\mu\theta} \right]^{-1/(1-\mu)} p_{mt}^{\frac{1+\mu\theta}{1-\mu}} \theta^{\frac{-\mu^2\theta}{1-\mu}} \left(\frac{p_{mt}}{1-\theta} \right)^{\frac{-\mu(1-\mu\theta)}{1-\mu}} P_t^{\frac{\mu}{(1-\mu)}} c_t \\ & = Z_{ht} k_{ht}^\rho \left[(1-\rho) p_{ht} Z_{ht} k_{ht}^\rho / W_t \right]^{\frac{1-\rho}{\rho}} \end{aligned} \quad (5C.13)$$

where W_t is the economy-wide wage rate (see equation 5B.6 in Appendix 5B):

$$W_t = (1-\rho) \left[\left(Z_{xt} k_{xt}^\rho \right)^{1/\rho} + \left(p_{ht} Z_{ht} k_{ht}^\rho \right)^{1/\rho} \right]^\rho$$

Evidently, an explicit solution for the relative price of non-traded to export goods is made

difficult by the extreme non-linear nature of the equilibrium condition in (5C.13). We, therefore, first, log-linearize the two equations, and then, combine the resulting linear expressions to obtain a linear expression for the relative price.

Log-linearization of the wage rate equation is straightforward:

$$\tilde{w}_t = \tau_0 \tilde{z}_{xt} + \rho \tau_0 \tilde{k}_{xt} + \tau_1 \tilde{p}_{ht} + \tau_1 \tilde{z}_{ht} + \rho \tau_1 \tilde{k}_{ht} \quad (5C.14)$$

Here τ_0 and τ_1 are, as defined below in (5C.16), parameters determined by the steady state levels of output of the two sectors. The right-hand side of the equation in (5C.13) can also be straightforwardly log-linearized:

$$\tilde{y}_{ht} = (1/\rho) \tilde{z}_{ht} + \left(\frac{1-\rho}{\rho} \right) \tilde{p}_{ht} - \left(\frac{1-\rho}{\rho} \right) \tilde{w}_t + \tilde{k}_{ht} \quad (5C.15)$$

Let us now log-linearize the left-hand side (the demand for nontraded goods) of the expression in (5C.13) and equate the resulting expression to (5C.15), which gives the output of the nontraded good, with substitution for $\tilde{w}_t$ from (5C.14), to obtain our desired linearized expression for the equilibrium condition for the non-traded goods market (consistent with the labour market equilibrium):

$$\begin{aligned} & (1-\mu)a_1 \tilde{p}_{mt} - (a_0 + a_3) \tilde{p}_{ht} + a_6 \tilde{p}_t + \tilde{c}_t \\ & + a_2 \tilde{k}_{xt} - a_4 \tilde{z}_{ht} - \rho a_4 \tilde{k}_{ht} + a_5 \tilde{z}_{xt} = 0 \end{aligned} \quad (5C.16)$$

where:

$$\begin{aligned}
a_0 &= 1/(1 + \mu), \quad a_1 = a_0(1 + \mu\theta), \quad a_2 = (1 - \rho)\tau_0, \quad a_3 = (1 - \tau_1)(1 - \rho)/\rho \\
a_4 &= (1 - \tau_1 + \rho\tau_1)/\rho, \quad a_5 = \tau_0(1 - \rho)/\rho, \quad a_6 = \mu a_0 \\
\tau_0 &= \frac{(Z_x k_x^\rho)^{1/\rho}}{\left[(Z_x k_x^\rho)^{1/\rho} + (p_h Z_h k_h^\rho)^{1/\rho} \right]}, \quad \tau_1 = \frac{(p_h Z_h k_h^\rho)^{1/\rho}}{\left[(Z_x k_x^\rho)^{1/\rho} + (p_h Z_h k_h^\rho)^{1/\rho} \right]}
\end{aligned}$$

We complete our log-linearization of the model by approximating the exogenous state variables. Let us first consider the process of evolution of the total factor productivity in the export good sector:

$$\log Z_{xt} = (1 - \psi) \log Z_x + \psi \log Z_{xt-1} + \varepsilon_{xt}$$

Using Uhlig's short-cut, we can write:

$$\log(Z_x e^{\tilde{z}_{xt}}) = (1 - \psi) \log Z_x + \psi \log(Z_x e^{\tilde{z}_{xt-1}}) + \varepsilon_{xt}$$

Log-linearized form of the above equation is:

$$\tilde{z}_{xt} = \psi \tilde{z}_{xt-1} + \varepsilon_{xt} \quad (5C.16)$$

Following the same method for the total factor productivity in the non-traded sector as well as for the terms of trade, we obtain:

$$\tilde{z}_{ht} = \psi_h \tilde{z}_{ht-1} + \varepsilon_{ht} \quad (5C.17)$$

$$\tilde{p}_{mt} = \psi_m \tilde{p}_{mt-1} + \varepsilon_{mt} \quad (5C.18)$$

We collect in Box 5.1 the linearized equations characterizing the complete model in a convenient order for application of the method of undetermined coefficients.

Appendix 5D

Data

Data used in this thesis come from two sources: International Monetary Fund's *International Financial Statistics* (IFS), and Statistics Canada's *CANSIM* database. All data on investment and capital stock are obtained from *CANSIM*. Data on capital stock and investment in the nontraded sector are derived by aggregating over the first ten industries below. Data for the export good sector are then obtained as residuals.

<i>Industry</i>	<i>Series Code</i>	
	Capital	Investment
Construction	D994557	D994533
Transportation and Storage	D994593	D994569
Communications and other utility	D995097	D995073
Wholesale Trade	D995484	D995460
Retail Trade	D995520	D995496
Finance and Insurance	D995550	D995532
Business Services	D995768	D995750
Government Services	D995801	D995777
Health and Social Services	D996074	D996056
Other Services	D996236	D996218
Total Economy	D993333	D993309

All other series codes along with their sources are provided below.

<i>Series</i>	<i>Source</i>	<i>Series Code</i>
Exchange rate (market rate)	IFS	156..RF..ZF*
Treasury Bill Rate	IFS	15660C..ZF
Government Bond Yield (3-5 years)	IFS	15661A..ZF
Government Bond Yield (10-15 years)	IFS	15661..ZF
Consumer Prices	IFS	15664..ZF
Export Prices	IFS	15674..ZF
Import Prices	IFS	15675..ZF
GDP at factor cost, Goods producing industries	CANSIM	I53008
GDP at factor cost, Services producing industries	CANSIM	I53009
Employment: Goods producing industries	CANSIM	D968567
Employment: Services producing industries	CANSIM	D968578
Population	CANSIM	D1
Household Consumption Expenditure	IFS	15696F.CZF
Government Consumption Expenditure	IFS	15691F.CZF
Wages, Salaries, and Supplementary Labour Income	CANSIM	D14805
Net Domestic Income at factor cost	CANSIM	D14812
GDP at 1992 prices	CANSIM	D14872
Net International Investment Position	CANSIM	D65219

Note: 156 is the country code for Canada in the IFS database. The corresponding country codes for the US, UK, Germany, and Japan are, respectively, 111, 112, 134, and 158.

Appendix 5E

Dynamic Effects to Productivity and Terms-of-Trade Shocks: the role of ω and ϑ

Figure 5E.1 The dynamic effects of a *terms-of-trade* shock on consumption and investment in the nontraded sector

Figure 5E.2 The dynamic effects of a productivity shock in the nontraded sector on consumption and debt

6 Summary and Conclusions

The research contained in this thesis has two objectives. First, it extends our understanding of the implications of international parity conditions for the purposes of open economy modeling. The second objective is to use the insights provided by the nature of disparities in the above conditions to model an open economy with imperfect capital flows. In modeling the open economy, our principal focus is on the dynamic interactions among the real exchange rate, the current account, and foreign debt.

Chapters 2 and 3 are devoted to the first objective. While we did not expect to unearth any new evidence relating to the violations of uncovered interest rate parity in our analysis, in chapter 2, the analysis emphasized the existence of a time-varying risk premium. We found that while there has been an extensive search for a risk premium in the foreign exchange market, the search has been restricted essentially to a search for a *currency* premium. Little has been done in this search to account for a *country* premium. It is argued that what is needed in this regard is a framework along the lines of Dooley and Isard (1987, 1991) that considers both exchange and country risks simultaneously.

Another key objective in chapter 2 is to examine the implications of the recent evidence of mean reversion in real exchange rates for the modeling of *open economy* and *the real exchange rate*. As the 1980s drew to a close, purchasing power parity (PPP) theory and exchange rate models that relied on PPP stood in near total discredit as an increasing number of economists found that the real exchange rate followed a random walk process. However, using very long data series (spanning over 100 years or so), researchers in the 1990s are now

increasingly able to find sizeable mean-reverting components in real exchange rates. The common conclusion of this research is that it takes 70 or more years' data to capture the mean-reverting tendency in the real exchange rate. We argue that the recent evidence of the painfully slow mean-reversion in the real exchange rate is inadequate to give PPP a pivotal role in the modeling of exchange rates. Pursuit of an equilibrium real exchange rate that emerges in the "ultra-long" run may be misguided if there are important real factors that are driving the equilibrium rate away from its PPP level for lengthy periods of time. We find that the evidence is rather more supportive of the models that consider disturbances as predominantly real in origin and permanent in their impact.

Chapter 3 is devoted to an empirical investigation of real interest rate parity as a long-run equilibrium condition. We first note that one form of real interest rate parity that requires equality of the interest cost of borrowing by domestic and foreign firms, in their respective markets, can hold only when both uncovered interest rate parity and *ex ante* purchasing power parity hold. However, the equality of the real interest cost of borrowing by a firm, in its domestic and foreign markets, can hold when uncovered interest rate parity holds. We note that in a world where countries experience different shocks to their goods and assets markets, real interest rate parity in the short-run is too strong a requirement. In chapter 3, we examined a less restrictive hypothesis for the parity. The hypothesis does not require parity to hold at any given point in time. All that is required is when prices or nominal interest rates diverge substantially, adjustment forces are evoked that tend to restore equilibrium in the long-run. However, we note that the real interest rate parity that justifies setting the domestic real interest rate equal to the foreign rate, in open-economy modeling, requires two

coefficient restrictions. The first restriction is one of *symmetry* that requires that the real interest rate coefficients in a cointegrating equation must be of the same magnitude but of opposite sign. The second is a *proportionality* restriction that requires the cointegrating vector to be $[1 \ -1]$. A mere finding of cointegration without satisfying these two conditions merely states that the two interest rates follow a stable long-run relationship. But the evidence cannot be considered supportive of the strong form of parity that can justify modeling open economy in a one-world-asset framework. We also examined to see whether the data supported a parity relationship such that the differential between two real interest rates fluctuates about some fixed mean. We used Johansen's Cointegration methodology to analyze data spanning over four decades from five industrialized countries – Canada, U.S.A., U.K., Japan, and Germany – to see if the behavior of interest rates and prices was consistent with any form of real interest rate parity. Although we found that most of the real interest rate pairs in our sample followed cointegrating relationships over the sample period 1957:1-1998:2, coefficient restrictions on the cointegrating vectors in most cases did not support any parity with or without a mean-differential. We found that in most cases estimated cointegrating vectors contained a nonzero intercept as well as a slope coefficient that is significantly different from unity. Thus, a linear combination of two interest rates in the majority of the cases was found to be stationary. Obviously, this is a statistical relationship that has little bearing on the strong form of parity that justifies setting the domestic real interest rate equal to the foreign rate in open economy modeling. All we can conclude from our empirical analysis in chapter 3 is that there is considerable evidence supportive of a parity relationship that may include a risk premium.

Chapters 4 and 5 are devoted to the modeling of open economy by assuming that the above-mentioned real interest rate differential is a risk premium. The model developed in chapter 4 is a simple one that emphasizes the dynamic interactions of the forward-looking real exchange rate with two state variables, the capital stock and the foreign debt. It takes the approach based on the belief that the current account balance, which affects the country's stock of net claims on, or indebtedness to the rest of the world, imposes a boundary condition that pins down the expected long-run level of the real exchange rate. It is universally accepted that large and persistent current account imbalances are unsustainable, and that changes in real exchange rates facilitate current account adjustment. Thus given the time paths of other fundamental variables that influence the current account, the real exchange rate cannot be sustained at a level that would generate large and persistent current account imbalances. Accordingly, *ceteris paribus*, developments that permanently shift the current account, will affect the equilibrium level of the real exchange rate.

The model in chapter 4 extends the above argument to include shocks that originate in the asset market. The balance of payment constraint implies that changes in preferences for holding assets in different countries- or for holding claims against the residents of different countries - cannot automatically give rise to net international capital flows; rather, a change in net international creditor positions requires a current account imbalance. A rise in the net capital flow into a country requires an equal reduction in the current account surplus, or a rise in the current account deficit. Moreover, to change their current account balance, countries must change their production of goods and services relative to their absorption. Thus, the real exchange rate changes to give rise to a current account deficit equal to the rate at which

foreigners will acquire claims on the domestic country.

In the model in chapter 4, we integrated goods market equilibrium condition with the financial market equilibrium, and included a risk premium in the financial market equilibrium condition. We emphasized that *currency risks* and *country risks* are interrelated, and that the magnitudes of these risks reflect the relative macroeconomic and political conditions of the countries issuing debt. By making the risk premium a function of foreign debt and other factors that influence perceptions of future returns on foreign portfolio investment, we emphasized joint determination of country and currency preferences in the model.

The nicety of the framework in chapter 4 is that we could use it to examine the behavior of the real exchange rate in response to shocks that originated in both the goods as well as asset markets. The stylized fact that real exchange rates of the industrialised countries have become far more volatile in the post-Bretton-Woods era than they had been in the Bretton-Woods era and the recent experience of the Asian countries point to the need for a more prominent role for asset market in dynamic macroeconomic models. Our implicit argument in chapter 4 was that the increased variability of exchange rates in this era is merely a reflection of the fact that the fast adjusting asset market now plays a prominent role that was absent in the Bretton-Woods era. We used our model to show that a rise in a current account deficit with the resulting gradual rise in foreign debt with eventual real exchange rate depreciation could initially be triggered by a disturbance in the asset market.¹

¹For example, a fall in the risk premium.

In chapter 5, we developed a more rigorously grounded model to examine the dynamic interactions of the real exchange rate, the current account and foreign debt within a framework that emphasizes imperfect capital flows. Our objective was to examine the impact of technology and terms-of-trade shocks in an intertemporal optimizing framework on the real exchange rate and the current account. The emphasis on the real factors in the determination of the real exchange rate was motivated by the recent evidence mentioned above from the tests of purchasing power parity. We allowed for imperfection in the world capital market by assuming that the real rate of interest paid on foreign debt was increasing in the stock of the country's debt. The model's capital accumulation process was formulated in a realistic manner so that the model could generate appropriate responses of its production sectors to exogenous shocks. More specifically, we allowed for sector-specific capital accumulation in the traded and nontraded sectors of the economy. The model was solved and simulated, and analysed using Canadian data for the period 1987-1998 with tools borrowed from the real business cycle modelers. Equilibrium stochastic processes of the model indicate that 'real' shocks (to technology and terms of trade) can adequately explain the behavior of the real exchange rate in terms of volatility and persistence. The model's impulse response functions show that a shock to technology in the export-good sector that raises productivity one percent above the steady-state level leads immediately to an appreciation to be followed by a depreciating path for more than forty years after the shock. The real exchange rate exhibits similar behavior in response to shocks to technology in the non-traded sector, as well as to terms-of-trade. One interesting contribution of the inclusion of some sensitivity of the real interest rate to the stock of foreign debt was to make the model's predictions more

reliable in that it gave the model stability. We also found that the sensitivity of the real interest rate to the stock of debt realistically lowered the impact and dynamic responses of the endogenous variables in the model. It thus allowed the model to fit the data better in terms of producing a match with the standard deviations in the data.

The model in chapter 5 can be used to analyze the effects of a broad range of exogenous shocks. In this regard, it can analyze the impact and dynamic effects of shocks to *taste*, *government spending*, *the foreign interest rate* etc. One interesting aspect of the results in chapter 5 was that while the model's forcing variables were found to be stationary, its endogenous variables were non-stationary. It is necessary to explain this result. It probably directs us to the inclusion of other possible exogenous shocks in the model.

We found in chapter 5 that a shock to the terms-of-trade or technology could cause the real exchange rate and other endogenous variables to stay above or below their steady-state levels for more than 10 years after the shock. Our focus in the model was the behavior of the variables around the steady state of the economy and we did not consider structural adjustments in the economy following shocks. It is quite reasonable to think that shocks to terms-of-trade and technology can induce structural adjustments in the economy. Further research may usefully integrate such adjustments into the present framework in order to explain further, and better understand, the behavior of the real exchange rate and other key macroeconomic variables in response to various types of macroeconomic shocks.

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