Exploring baryon asymmetry and naturalness with the current experimental data and observations

by

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Abstract

This thesis explores some extensions of the Standard Model in search of new physics. The research presented here concerns aspects of cosmology and collider physics. The research includes a combination of model building, constraints imposed by the current experimental data, and prospects of future experimental bounds. There is an aggregate of three main research projects composing this thesis.

Firstly, the possibility of multiple hidden sectors to accommodate a successful framework to explain the abundance of matter over anti-matter and study prospects of a viable dark matter candidate. The evolution of cosmological history and the baryon asymmetry is studied thoroughly. The baryon asymmetry and the dark matter relic abundance are checked in light of the current cosmological data and a potential parameter space is computed. The analysis conducted reveals that a viable mechanism for the origin of matter and a feasible model for dark matter can be constructed considering the multiple hidden sectors framework.

Secondly, we study the phenomenology of an extension of the Standard Model with fermionic top partners. The introduction of such new particles gives rise to rich collider phenomenology which is examined given the copious amount of data collected at the Large Hadron Collider. The discovery potential and the constrained parameter space for the proposed model are derived and discussed in great detail. The various available collider searches for top partners suggest that the limits obtained on the fully hadronic decay channels of the proposed model are less stringent compared to the traditional decay modes and the effects of future dedicated searches are outlined.

Finally, a dedicated study analyzing some aspects of supersymmetric quantum chromodynamics (SQCD) like gauge theories is conducted. In particular, exploring the behaviour of a sub-category of SQCD theories which confines without chiral sym-
metry breaking for three quark flavours. The computations of the SQCD with the anomaly-mediated supersymmetry breaking effects lead to the conclusion that for the three flavour scenario, the scalar potential does not have a minimum away from the origin.
Acknowledgments

This thesis would not have been possible without the many different people’s considerable and generous support throughout my graduate studies. First and foremost, I would like to express my utmost gratitude to my supervisors Dr. Thomas Gregoire and Dr. Daniel Stolarski. I have learned and benefitted immensely from their expertise and profound understanding of elementary particle physics. Their patience and willingness to constantly guide me throughout my research is inspiring. They have tolerated all of my naive questions and answered them patiently in great detail. I greatly appreciate all the time and effort that Dr. Thomas Gregoire and Dr. Daniel Stolarski dedicated to my thesis research and writing.

Next, I would like to thank the other faculty members of the theory group at Carleton: Dr. Bruce Campbell, Dr. Heather Logan, and Dr. Stephen Godfrey. They have always been kind, and approachable and their lectures on quantum field theory and cosmology were instrumental in my studies. Furthermore, I would like to express my gratitude to Dr. Catarina Cosme. Dr. Catarina Cosme was an excellent collaborator with a deep understanding of modern cosmology and I am indebted to her for sharing her knowledge.

Finally, I would like to extend my sincere thanks to my family and close friends for their immense support throughout this process.
Statement of Originality

The materials in chapter 2, 3 are reviews of the Standard Model and cosmology respectively. The contents of these chapters are well established in the pre-existing literature and numerous textbooks on particle physics and modern cosmology. The beginning sections of chapter 6 are background information on supersymmetry with the relevant references provided in this thesis. Chapter 4, 5, and 6 contain original research materials. Each of these chapters is based on publications except chapter 6 and each of these projects was completed in collaboration with other physicists. Below, we list the chapters with the corresponding publication as well as the specific contributions of the collaborators.

- The results presented in chapter 4 is published in [1]. This project was done in collaboration with Dr. Thomas Gregoire, Dr. Daniel Stolarski and Dr. Catarina Cosme.
  - The results obtained for the cosmological evolution of the energy densities and temperatures were done by Dr. Catarina Cosme and independently verified by the author.
  - The rest of the calculations, plots, and the codes required for the results were produced by the author.

- The results presented in chapter 5 is published in [2]. This project was done in collaboration with Dr. Thomas Gregoire and Dr. Daniel Stolarski. All the results in this chapter were derived by the author including the various codes/scripts used to recast the different experimental searches.

- The results presented in Section 6.5 of chapter 6 are yet to be published. This
project was done in collaboration with Dr. Thomas Gregoire and Dr. Daniel Stolarski. All the results in this chapter were derived by the author and verified by Dr. Daniel Stolarski.

The initial drafts of the various published projects were first written by the author and subsequently modified/appended by the collaborators on the respective research projects. To the best of my knowledge, the content of this thesis is my own work and all the assistance received in preparing this thesis and the sources used have been properly acknowledged.
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List of Acronyms, Constants, and Symbols

Throughout this thesis, we will work in Natural units ($\hbar = c = 1$). Any lowercase Greek indices ($\mu, \nu, ...$) run from 0 to 3 while the Latin indices ($i, j, ...$) will run from 1 to $n$. Furthermore, we will be employing Einstein summation convention where repeated indices are summed over ($a_i x^i = \sum_{i=1}^{n} a_i x^i$). Here are a list of common constants (some of the values for the constants are taken from Particle Data Group):

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced Planck mass</td>
<td>$M_P$</td>
<td>$2.43 \times 10^{18}$ GeV</td>
</tr>
<tr>
<td>Scaling factor for Hubble expansion rate</td>
<td>$h$</td>
<td>0.674</td>
</tr>
<tr>
<td>Present day CMB temperature</td>
<td>$T_0$</td>
<td>2.7255 K</td>
</tr>
<tr>
<td>Critical density of the Universe</td>
<td>$\rho_{c,0}$</td>
<td>$3.995 \times 10^{-47}$ GeV$^4$</td>
</tr>
<tr>
<td>Baryon density of the Universe</td>
<td>$\Omega_b = \frac{\rho_b}{\rho_{c,0}}$</td>
<td>0.0493</td>
</tr>
<tr>
<td>Cold dark matter density of the Universe</td>
<td>$\Omega_b = \frac{\rho_{DM}}{\rho_{c,0}}$</td>
<td>0.265</td>
</tr>
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<td>Electron mass</td>
<td>$m_e$</td>
<td>$5.11 \times 10^{-4}$ GeV</td>
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<tr>
<td>Muon mass</td>
<td>$m_\mu$</td>
<td>$1.056 \times 10^{-1}$ GeV</td>
</tr>
<tr>
<td>Tau mass</td>
<td>$m_\tau$</td>
<td>$1.777 \times 10^{0}$ GeV</td>
</tr>
<tr>
<td>Proton mass</td>
<td>$m_p$</td>
<td>$9.383 \times 10^{-1}$ GeV</td>
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<tr>
<td>Neutron mass</td>
<td>$m_n$</td>
<td>$9.396 \times 10^{-1}$ GeV</td>
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<td>Top quark mass</td>
<td>$m_t$</td>
<td>172.9 GeV</td>
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<td>W boson mass</td>
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<td>Z boson mass</td>
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<tr>
<td>Higgs boson mass</td>
<td>$m_H$</td>
<td>125.10 GeV</td>
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A list of the most used Acronyms:

<table>
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<th>Acronym</th>
<th>Description</th>
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<td>SM</td>
<td>Standard Model</td>
</tr>
<tr>
<td>BSM</td>
<td>Beyond Standard Model</td>
</tr>
<tr>
<td>NP</td>
<td>New Physics</td>
</tr>
<tr>
<td>EW</td>
<td>Electroweak</td>
</tr>
<tr>
<td>EWSB</td>
<td>Electroweak Symmetry Breaking</td>
</tr>
<tr>
<td>vev</td>
<td>Vacuum Expectation Value</td>
</tr>
<tr>
<td>CMB</td>
<td>Cosmic Microwave Background</td>
</tr>
<tr>
<td>BBN</td>
<td>Big Bang Nucleosynthesis</td>
</tr>
<tr>
<td>DM</td>
<td>Dark Matter</td>
</tr>
<tr>
<td>LHC</td>
<td>Large Hadron Collider</td>
</tr>
<tr>
<td>ATLAS</td>
<td>A Toroidal LHC ApparatuS</td>
</tr>
<tr>
<td>CMS</td>
<td>Compact Muon Solenoid</td>
</tr>
<tr>
<td>CERN</td>
<td>European Organization for Nuclear Research</td>
</tr>
<tr>
<td>QCD</td>
<td>Quantum Chromodynamics</td>
</tr>
<tr>
<td>SUSY</td>
<td>Supersymmetry</td>
</tr>
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<td>Anomaly Mediated Supersymmetry Breaking</td>
</tr>
<tr>
<td>BAU</td>
<td>Baryon Asymmetry of the Universe</td>
</tr>
<tr>
<td>CP</td>
<td>Charge Plus Parity</td>
</tr>
<tr>
<td>GUT</td>
<td>Grand Unified Theory</td>
</tr>
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Chapter 1

Introduction

Our Universe has existed for approximately 13.8 billion years and will exist long after our species have visited Alpha Centauri. The pursuit of understanding the physical world and natural phenomena around us has steered civilizations toward progress in every aspect of our modern lives. Throughout history, there has been great progress toward grasping the fundamental blocks composing our reality. Although nature has remained reserved about revealing some of its aspects, there have been tremendous strides at the forefront of elementary particle physics and cosmology. The basis of almost all fundamental forces of nature are encoded in the Standard Model (SM) and its predictions agree incredibly well with experimental observations. However, there are some aspects where the SM falls short and new physics (NP) is required to accurately model/describe the observed phenomena which we will discuss later in some detail.

The history and investigation of the SM are quite rich and with the discovery of the Higgs boson [3,4] at the Large Hadron Collider (LHC), the SM is complete. A schematic diagram of the SM is displayed in Figure 1.1. Firstly, although the most familiar, the gravitational force is not part of the SM mathematical framework. The
combination of electromagnetism, mediated by the photon $\gamma$, and the weak interaction into a single framework called electroweak theory was proposed in the late sixties by Glashow, Salam and Weinberg [5–8]. The weak interactions are responsible for various radioactive decays and they are mediated by the $W^\pm$ and $Z$ bosons, called vector gauge bosons. In addition, the strong interaction or quantum electrodynamics (QCD) binds quarks into mesons (such as pions, kaons,...) and baryons (such as protons, neutrons,...) via gluons. Finally, an essential ingredient of the Standard Model is a complex scalar field that is responsible for generating masses for the various fermions and vector bosons through the so-called Higgs mechanism [9–11]. The discovery and the experimental verification of the various properties including the masses of vector bosons, Higgs boson, and electroweak parameters reaffirm the Standard Model framework. Nevertheless, the SM has several deficiencies and there have been numerous attempts at its extension.

Currently, there are several problems plaguing our present understanding of physics and they are both theoretical and practical in nature. On the theoretical side, the smallness of the QCD angle, i.e. the strong CP problem, desperately seeks
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an explanation [12–15]. Another open question is, why is the cosmological constant so small? Furthermore, why is there such a large hierarchy between the electroweak scale, around $\mathcal{O}(100)$ GeV and the Planck scale, $\mathcal{O}(10^{19})$ GeV, where gravitational interactions become important? This is essentially the hierarchy problem and the SM lacks any explanation and instead one must rely on some miraculous cancellation between various parameters of the theory. On the practical side, we know from neutrino oscillation experiments, such as Super-Kamiokande and Sudbury Neutrino Observatory, that neutrinos have tiny but non-vanishing masses [16, 17], but we don’t know why they are so small and whether neutrinos are their own anti-particles. Moreover, the precise nature of dark matter (DM) remains a mystery. Dark matter appears to be a form of non-baryonic matter that makes up approximately 26% of the energy density budget of the Universe [18]. One other interesting problem concerns the very existence of matter at all. Why is the Universe devoid of anti-matter? The mechanism generating an asymmetry of matter over anti-matter, i.e. baryogenesis, remains one the most intriguing and highly explored questions in modern particle physics and cosmology.

Many of the problems mentioned above will not be examined in this thesis. However, our studies in Chapter 4 are primarily focused on the exploration of the excess of matter over anti-matter and the nature of dark matter. Admittedly, there exist numerous possible solutions to such problems but the experiments have yet to conclusively steer us toward a particular direction. Some of the most prominent proposed solutions are as follows: GUT baryogenesis which produces the baryon asymmetry via the out of equilibrium decays of the heavy bosons in Grand Unified Theories [19–29]. GUT baryogenesis has a lower limit on the decaying boson due to the non-observations of the proton decay. This lower mass limit in turn translates
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into a constraint on the reheating temperature after inflation. Leptogenesis was first introduced by Fukugita and Yanagida [30] and includes right-handed neutrinos that subsequently decay to the SM leptons which can then generate baryon asymmetry through Standard Model sphaleron processes [31]. Leptogenesis occurs at a very high scale that is beyond any current experimental verification. Electroweak baryogenesis occurs during the electroweak phase transition [32, 33]. The phase transition is required to be first-order for successful Electroweak baryogenesis; otherwise, sphalerons would wash out any generated baryon asymmetry. Finally, there is an Affleck-Dine mechanism in which a complex scalar field carrying a baryon number spontaneously produces baryon asymmetry as it is displaced from the minimum of the potential in the very early Universe [34, 35]. Ascertaining an exact mechanism behind baryogenesis remains a mystery and so does determining the apparent coincidence of visible matter and dark matter abundances today. The present-day visible matter density and dark matter appears to be closely related \( \frac{\Omega_{DM}}{\Omega_{\text{Visible}}} \simeq 5 \) where \( \Omega \) is the energy density divided by the critical energy density [18]. Consequently, exploring the hypothesis which gives rise to the connection between the mass density of visible and dark matter rather than a mere cosmological coincidence would be fascinating and worthwhile to study.

Observations suggest that DM behaves just like ordinary baryonic matter gravitationally; however, it interacts very dissimilarly otherwise. For starters, DM does not interact electromagnetically so it does not emit/absorb any photons. The precise particle candidate(s) for DM has yet to be identified. Even though the DM particle(s) have evaded detections, there is a stack of indirect astrophysical evidence for its existence [18, 36–39]. DM has eluded direct and indirect searches [40–45] which means that the interaction of SM and DM must be very feeble.
Setting aside the cosmological problem of invisible matter, we turn our attention to a more theoretical issue concerning the SM electroweak scale. The question of why is there such a large hierarchy between the electroweak scale, around $O(100) \text{ GeV}$ and the Planck scale, $O(10^{19}) \text{ GeV}$, where the gravitational interactions become non-negligible? Stated otherwise, why is the SM Higgs boson so much lighter than the Planck scale where the radiative corrections to the Higgs mass are expected to suffer from a huge contribution, primarily from the top quark, proportional to the Planck scale?

The quadratic sensitivity of the Higgs mass to high energy scales due to radiative correction represented by loop diagrams destabilizes our current paradigm. It is feasible that due to the limitation of our experiments, we have yet to cross the energy threshold where new physics unfolds. However, this is highly speculative and it is doubtful that the EW scale is accidentally highly fine-tuned. Thus it is crucial to study the behaviour of beyond Standard Model (BSM) theories especially those introducing new particles near the electroweak scale and such models are highly sought after.

There are numerous BSM theories that present solutions to various issues raised in the Standard Model and we will not include such an exhaustive list here. However, we will briefly discuss a few prominent candidates. Firstly, supersymmetry (SUSY) is spacetime symmetry that relates fermions (half-integer spin particles) and bosons (integer spin particles) [46]. SUSY associates a fermion with its superpartner which is a boson and vice versa so in its minimal incarnation doubles the number of particles. The superpartners are charged under the same SM gauge group ($SU(3) \times SU(2) \times U(1)$) with their gauge charges identical to their corresponding SM counterpart. The resolution to the hierarchy problem requires that the masses of the superpartners
must reside close to the electroweak scale. Furthermore, SUSY allows the possibility of gauge coupling unification at a very high scale of approximately $\sim 10^{16}$ GeV. One more compelling aspect of SUSY is that it can contain a natural dark matter (DM) candidate, which is the lightest supersymmetric partner (LSP). Imposing a symmetry called R-parity, a $\mathbb{Z}_2$ symmetry under which SM particles are even and superpartners are odd, on the model leads to a stable LSP and it implies that SUSY particles are produced in pairs. Despite the theoretical success of supersymmetry with rich predictions for experiments to explore, the current null experimental findings, particularly at the Large Hadron Collider, have been disappointing. So alternative approaches such as $N$naturalness [47] have been considered to alleviate the hierarchy problem.

The mechanism of $N$naturalness solves the hierarchy problem by introducing $N$-copies of the SM with identical particle content, gauge group and Yukawa structure. The Higgs mass parameters are allowed to take values distributed between $(-\Lambda_H^2, \Lambda_H^2)$ with $\Lambda_H$ being the typical cutoff scale for the quadratic divergences. If the Higgs mass parameters distribution takes on a random value, some of the sectors should be accidentally tuned at the $1/N$ level. The sector corresponding to the smallest non-vanishing vacuum expectation value (vev) is identified as our “SM”. It is crucial to mention that this model does not predict any new particles for the LHC. However, our interest lies in determining if it would be viable to accommodate a possible DM candidate and resolve the matter anti-matter asymmetry problem within the $N$naturalness framework. The mechanism for baryogenesis and dark matter embedded in the $N$naturalness framework is the subject of Chapter 4 presented in this thesis. By studying the cosmological evolution of the baryon asymmetry, lepton asymmetry, and relic abundance in the various copies of the Standard Model, we obtain various
constraints on such multiple hidden sectors.

Another well-motivated solution to the hierarchy problem poses the existence of fermionic top partners, fermions with the same quantum numbers as the top quark whose contributions to the Higgs mass parameter cancel those of the top quark. Such particles can appear in composite Higgs models [48–52] and Little Higgs models [53, 54]. The second project of this thesis concerns fermionic top partners, presented in Chapter 5, where we consider its previously unexplored decay channels and recast ATLAS and CMS searches for such particles. In particular, we consider the top partners decaying into a light flavour quark ($j$) and a new scalar $\eta$, and that scalar decays to two light flavour jets resulting in three light jets. We also consider the possibility of the scalar particle ($\eta$) decaying into two bottom jets instead of light jets, as might be expected from a Higgs-like scalar. Novel constraints are derived from such scenarios, by recasting recent ATLAS and CMS searches that require at least six jets without any leptons, photons, or missing energy.

Now we move into a completely different territory of supersymmetric QCD-like (SQCD) theories. Recently, there have been quite a lot of attention drawn toward non-abelian $SU(N)$ supersymmetric gauge theories with a various number of massless quark flavours and colours perturbed by anomaly-mediated supersymmetry breaking (AMSB) [55–58]. The main motivation for such strongly coupled theories is to understand the dynamics of chiral gauge and QCD-like theories. An SQCD theory could shed some light on the features of Standard Model QCD such as chiral symmetry breaking and confinement. A connection between non-supersymmetric QCD and SQCD can be explored by introducing soft supersymmetry breaking to SQCD to imitate non-supersymmetric QCD. The anomaly-mediated supersymmetry breaking has a property known as ultraviolet (UV) insensitivity [59, 60] which permits us to
compute soft terms that are completely under control at any scale. A detailed study of SQCD with AMSB for three massless flavours is conducted in Chapter 6.

This thesis is organized as follows. A brief overview of the Standard Model and cosmology are provided in Chapter 2 and Chapter 3. Chapter 4 covers a mechanism for producing the baryon asymmetry of the Universe and dark matter phenomenology in multiple hidden sectors. In Chapter 5, we consider experimental limits on colour triplet fermions that decay dominantly to three jets via a scalar mediator that can be on- or off-shell. A review of the background information about supersymmetry and the SQCD for three flavours and two colours with anomaly-mediated SUSY breaking makes up Chapter 6. Finally, the results of all the research projects are summarized in Chapter 7.
Chapter 2

The Standard Model and Beyond

The Standard Model of elementary particle physics is a Quantum Field Theory (QFT) developed in the late sixties which describes a significant portion of the natural world with excellent precision [5–11, 61–63]. The SM is based on the non-abelian gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ where $C$ denotes colour, $L$ denotes left, and $Y$ denotes hypercharge. In addition to the fermions listed in Table 2.1, the SM contains a scalar particle as well. The scalar obtains a vacuum expectation value ($vev$) that breaks the gauge symmetry as $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}$ where EM stands for electromagnetic. The electroweak symmetry breaking (EWSB) leaves the gauge group $SU(3)_C$ unchanged, so the corresponding gauge bosons, called gluons, remain massless. However, three linear combinations of $SU(2)_L \times U(1)_Y$ gauge bosons become massive due to the spontaneous EWSB which are identified as $W^\pm$ and $Z$ while one remains massless that corresponds to the photon, $\gamma$.

As we will show in detail, chiral fermions also acquire masses after the electroweak symmetry breaking. The SM is composed of two different types of fermions: leptons and quarks. Quarks transform under the $SU(3)_C$ gauge group while leptons transform as a singlet. There are six types of leptons, three neutral leptons named
the electron neutrino $\nu_e$, the muon neutrino $\nu_\mu$, and the tau neutrino $\nu_\tau$, and there are three charged leptons called the electron $e$, the muon $\mu$, and the tau $\tau$. Similarly, there are six types of quarks categorized as up-type and down-type quarks. The three up-type quarks are called up $u$, charm $c$, and top $t$, while the down-type are called down $d$, strange $s$, and bottom $b$. The final ingredient of the SM is a complex scalar Higgs doublet which after EWSB leaves a single real scalar called the Higgs boson, while the three other components constitute longitudinal modes of the massive gauge bosons. We will provide the necessary background QFT and SM where we rely heavily on the source materials in textbooks by Peskin and Schroeder [64] and Matthew D. Schwartz [65].

**Table 2.1:** Field content of the Standard Model. The index $i$(flavour), $j$(SU(2)$_L$ gauge) goes from 1 to 3, and $a$ from 1 to 8. The $L$ and $R$ next to a spin 1/2 field represents its chirality.

<table>
<thead>
<tr>
<th>Field</th>
<th>SU(3)$_C$</th>
<th>SU(2)$_L$</th>
<th>U(1)$_Y$</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$</td>
<td>1</td>
<td>2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>$Q^i_L$</td>
<td>3</td>
<td>2</td>
<td>1/6</td>
<td>1/2</td>
</tr>
<tr>
<td>$u^i_R$</td>
<td>3</td>
<td>1</td>
<td>2/3</td>
<td>1/2</td>
</tr>
<tr>
<td>$d^i_R$</td>
<td>3</td>
<td>1</td>
<td>$-1/3$</td>
<td>1/2</td>
</tr>
<tr>
<td>$L^i_L$</td>
<td>1</td>
<td>2</td>
<td>$-1/2$</td>
<td>1/2</td>
</tr>
<tr>
<td>$e^i_R$</td>
<td>1</td>
<td>1</td>
<td>$-1$</td>
<td>1/2</td>
</tr>
<tr>
<td>$B_\mu$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$W^j_\mu$</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$G^a_\mu$</td>
<td>8</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>


## 2.1 Gauge Invariance

Physicists are frequently interested in a particular class of quantum field theories called gauge theories which demands that the laws of physics, in particular the Lagrangian, be invariant under a set of local symmetry transformations. This set of local symmetry transformations forms a Lie group and most often deploys its corresponding Lie algebra. We will present some examples to illustrate the general idea in practice for some well-established particle physics toy models. Each group generator has a corresponding gauge boson and gauge invariance implies that these gauge bosons must be massless. As an example, let’s consider the following transformation

\[
\phi \rightarrow \phi' = U \phi ,
\]

where \( U \) is a general group transformation. In the simplest case \( U = e^{i\theta} \), where \( \theta \) is a real transformation parameter. It is straightforward to deduce that for a free scalar particle of mass \( m \), the following Lagrangian is invariant under this global transformation

\[
\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi .
\]

(2.2)

On the other hand, the mass term remains invariant but the kinetic term of the Lagrangian is no longer invariant under local gauge transformation i.e. \( \theta \rightarrow \theta(x) \). The typical derivative of the field does not have the same transformation at every spacetime point

\[
\partial^\mu \phi \rightarrow \partial^\mu \phi' = U \cdot [\partial_\mu + i\partial_\mu \theta(x)] \cdot \phi \neq U \cdot \partial_\mu \phi .
\]

(2.3)
To preserve gauge invariance, the standard derivative must be replaced by the covariant derivative which compensates for the additional terms introduced from the local transformation of the field. For $U(1)$, the covariant derivative is of the form,

\[ \partial_\mu \rightarrow D_\mu \equiv \partial_\mu - iB_\mu , \]  

(2.4)

where $B_\mu$ is a new field called the gauge field, sometimes also referred to as the connection, which has the following transformation:

\[ B_\mu \rightarrow B'_\mu = B_\mu + \partial_\mu \theta(x) . \]  

(2.5)

We can explicitly check that the covariant derivative transforms similarly to the field; consequently, it makes the kinetic term invariant under local $U(1)$ symmetry transformation:

\[ D_\mu \phi \rightarrow (D_\mu \phi)' = [\partial_\mu - i (B_\mu + \partial_\mu \theta(x))] e^{i\theta(x)} \phi = U \cdot D_\mu \phi . \]  

(2.6)

Finally, it is crucial to state that gauge invariance forbids mass terms for gauge bosons. Going back to our previous example, a mass term of the form $m_B B_\mu B^\mu$ transforms as

\[ m_B B_\mu B^\mu \rightarrow m_B (B_\mu B^\mu)' = m_B [B_\mu + \partial_\mu \theta(x)] [B^\mu + \partial^\mu \theta(x)] \neq m_B B_\mu B^\mu . \]  

(2.7)

All of our previous discussions can be generalized to $U(N)$ by replacing the $\theta$ parameter with $N \times N$ unitary matrices.
2.2 The Higgs Mechanism

A complex scalar field, $\Phi$, that transforms as a doublet under $SU(2)_L$ and as a singlet, under $SU(3)_C$, with hypercharge $Y = +1/2$, is described by following most general renormalizable Lagrangian

$$\mathcal{L} = (\partial_{\mu} \Phi)^\dagger \partial^{\mu} \Phi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2.$$  \hspace{1cm} (2.8)

Minimizing the scalar scalar potential $V(\Phi)^{SM}_{\text{Higgs}} = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$ of this theory,

$$\frac{\partial V(\Phi)}{\partial \Phi} \bigg|_{\Phi_0} = 0 \Rightarrow \Phi_0^\dagger \Phi_0 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2},$$  \hspace{1cm} (2.9)

where $v$ is the vev of the Higgs boson. The parameter $\lambda > 0$; if one sets $\lambda < 0$ then there is no stable vacuum state since the scalar potential is unbounded from below. There are two possibilities for $\mu^2$:

**Case 1, $\mu^2 \geq 0$:**
The potential energy is minimized at the origin $\langle |\Phi_0| \equiv \sqrt{\Phi_0^\dagger \Phi_0} \rangle = 0$, as illustrated in Figure 2.1 (a). A gauge transformation acting on the vacuum state $\Phi_0 = 0$ leaves it invariant; hence, the electroweak symmetry is unbroken in the vacuum.

**Case 2, $\mu^2 < 0$:**
The potential energy is minimized away from the origin $|\Phi| = 0$ as illustrated in Figure 2.1 (b). The electroweak symmetry $SU(2)_L \times U(1)_Y$ acting on the vacuum state $\Phi_0 = 0$ changes the vacuum; hence, the electroweak symmetry is broken by the vacuum and the fermions and gauge bosons acquire masses. The minimum of the potential occurs at $|\Phi_0| = \sqrt{\frac{\mu^2}{2\lambda}}$ and is colloquially knowns as the vacuum expectation
The Higgs field $\Phi$ is a complex doublet which can be expanded in terms of four real scalar fields as

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}.$$  \hspace{1cm} (2.10)

Then clearly the expression in Eq. (2.9) is a fixed-length four dimension surface and one has the freedom to work in any basis as long as the total length of the vector satisfies the relation $\Phi_0^\dagger \Phi_0 = \mu^2/2\lambda$. This is the relation for the 4-dimensional sphere which is invariant under the three-dimensional rotations (or the scalar potential is invariant under the gauge transformation $SU(2) \times U(1)$) and the rotations cost zero energy since the potential is flat along such directions. Furthermore, typically in gauge theories, there are redundant degrees of freedom. To deal with such an issue one typically fixes the gauge which is analogous to choosing a specific coordinate system. Generically, one convenient choice of gauge, known as a unitary gauge, is to
set all the fields to zero except one of them. For example\footnote{Another way is to parametrize the Higgs field as
\[ \Phi = \frac{U(x)}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} , \]
where \( U(x) = e^{i\alpha_j \sigma^j} \) represent a general \( SU(2)_L \) local transformation.},
\[
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} , \quad \langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_4 \rangle = 0 , \quad \langle \phi_3 \rangle \equiv v ,
\] (2.11)
where \( H \) is the Higgs boson with a vanishing vev. So we have gauged away the fields \( \phi_1, \phi_2 \) and \( \phi_4 \) with the three fields completely eliminated from the Lagrangian via gauge transformation. Expanding the potential energy in terms of the real scalar fields, we find a mass term for the Higgs boson
\[
V(\Phi) \supset \frac{3}{2} \lambda v^2 H^2 + \frac{1}{2} \mu^2 H^2
\]
resulting in \( m_H = \sqrt{2\lambda v^2} \) while the other real fields are massless.

Recall, the invariance of a field under a transformation implies
\[ e^{\frac{i}{2} \alpha_j \Omega_j} \phi = \phi \]
and expanding this equation to first order in \( \alpha_j \) implies \( \alpha_j \Omega_j \phi = 0 \). Under the action of the \( SU(2)_L \times U(1)_Y \) gauge transformation, the vacuum transforms as
\[
U(1)_Y \times SU(2)_L : \frac{1}{2} [Y + \alpha_1 \sigma_1 + \alpha_2 \sigma_2 + \alpha_3 \sigma_3] \Phi_0 = \frac{1}{2\sqrt{2}} \begin{pmatrix} Y + \alpha_3 & \alpha_1 - i\alpha_2 \\ \alpha_1 + i\alpha_2 & Y - \alpha_3 \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \\
= \frac{v + H}{2\sqrt{2}} \begin{pmatrix} \alpha_1 - i\alpha_2 \\ Y - \alpha_3 \end{pmatrix} .
\] (2.12)
Consequently, the electroweak symmetry is spontaneously broken and the above relation reveals that out of the four symmetry generators, only one linear combination with \( \alpha_1 = \alpha_2 = 0 \) and \( Y = \alpha_3 \) remains unbroken. This unbroken symmetry generator
is identified with the familiar electromagnetic charge $Q$ which generates the $U(1)_{\text{EM}}$. Generically the generator is expressed in terms of the diagonal generator of $SU(2)_L$ and the weak hypercharge $Y$ as

$$Q = T_3 + Y .$$  \hspace{1cm} (2.13)

Hence, the three broken generators correspond to the three massless Goldstone modes as expected from Goldstone’s theorem.

### 2.2.1 Gauge Boson Masses

Before discussing how the gauge bosons acquire masses via the Higgs mechanism, we would be remiss not to mention Goldstone’s theorem which concerns massless modes due to spontaneous symmetry breaking [66–68]. Goldstone’s theorem states that for every spontaneous symmetry breaking, there is a corresponding massless boson called the Nambu-Goldstone boson. Goldstone’s theorem applies to continuous global symmetries and if a symmetry is gauged then the corresponding massless mode is eaten by the gauge bosons corresponding to the broken generators and they acquire non-zero masses. Phenomenological observations require the gauge bosons ($W^\pm$ and $Z$) to be massive and the theoretically generated masses must be consistent with the experimental measurements. So the electroweak symmetry must be broken in the SM to satisfy experimental constraints. For a gauge-invariant theory, the usual derivative in the Lagrangian for the $SU(2)_L \times U(1)_Y$ theory must be promoted to a covariant derivative. As such, the covariant derivative and the corresponding infinitesimal gauge
fields transformation are \([64, 65]\)

\[
D_\mu = \partial_\mu - igYB_\mu - ig_2W_\mu^j T^j,
\]
\[
B_\mu \rightarrow B'_\mu = B_\mu + \frac{1}{g}\partial_\mu \beta(x),
\]
\[
W_\mu^j \rightarrow (W_\mu^j)' = W_\mu^j + \frac{1}{g_2}\partial_\mu \alpha^j(x) + \epsilon^{jkl} W_\mu^k \alpha^l(x),
\]

where \(T^j = \frac{1}{2}\sigma^j\) are the \(SU(2)_L\) generators (the full gauge transformation of a field \(\psi\) under \(SU(2)_L \times U(1)_Y\) is \(\psi \rightarrow \exp[i\beta(x)Y] \exp[i\alpha(x)^j T^j] \psi\), \(g\) is the \(U(1)_Y\) gauge coupling, \(g_2\) is the \(SU(2)_L\) coupling, \(B_\mu\) and \(W_\mu^j\) are the \(U(1)_Y\) and \(SU(2)_L\) gauge bosons respectively. The relevant terms for the gauge bosons masses are obtained by subbing in the Higgs vev, \(\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}\), in the gauge kinetic term,

\[
\Delta \mathcal{L} \supset (D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{v^2}{8} \left\{ g_2^2 \left( W_\mu^1 - iW_\mu^2 \right) \left( W^{1\mu} + iW^{2\mu} \right) + \left( g_2 W_\mu^3 - gYB_\mu \right)^2 \right\},
\]

where the linear combinations in the first term correspond to physical gauge bosons

\[
W^{\mp}_\mu \equiv \frac{1}{\sqrt{2}} \left[ W^1_\mu \pm iW^2_\mu \right] , \quad \text{with mass } m_W = \frac{gv}{2}.
\]

The second term in Eq. \((2.15)\) can be diagonalized, \(A \cdot V = \lambda V\), in the basis vector \(V = \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}^T\) as follow

\[
A_\mu = \frac{1}{\sqrt{g^2 + g_2^2}} \left( gW^3_\mu + g_2B_\mu \right) , \quad \text{with mass } M_A = 0 ,
\]
\[
Z_\mu = \frac{1}{\sqrt{g^2 + g_2^2}} \left( g_2W^3_\mu - gB_\mu \right) , \quad \text{with mass } M_Z = \frac{\sqrt{g^2 + g_2^2}}{2} .
\]
Consequently, all three of the weak gauge bosons become massive while the photon, $A_\mu$, remains massless. Colloquially this way of generating masses for the gauge boson is known as the Higgs mechanism and it forms the basis of the SM.

## 2.3 Quark Sector

In this section, we lay out the foundation for the quark sector including how they obtain their masses through electroweak symmetry breaking. In the standard Dirac representation, fermionic fields $\psi$ are four-component spinors but in the Weyl representation they are composed of two-component bi-spinors and one can write

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}, \quad (2.18)$$

where $\psi_L = P_L \psi = \frac{1}{2}(1 - \gamma^5)\psi$ and $\psi_R = P_R \psi = \frac{1}{2}(1 + \gamma^5)\psi$ are two components object known as the left and right handed Weyl spinors respectively. In order to construct a viable Lagrangian, every term must be Lorentz invariant and the simplest Lorentz invariant object is $\bar{\psi}\psi$ with $\bar{\psi} \equiv \psi^{\dagger} \gamma^0$. Furthermore, it is well established experimentally that the left and right handed fermions states have different charges under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. For example, the right handed SM fermions are gauge singlet under $SU(2)_L$ transformation, i.e: $\psi_R \overset{SU(2)_L}{\longrightarrow} \psi_R$, while the left handed fermions are doublets under $SU(2)_L$ transformation, i.e: $\psi_L \overset{SU(2)_L}{\longrightarrow} \psi'_L = e^{i\alpha_j T_j} \psi_L \simeq [1 + i\alpha_j T_j] \psi_L$.

In the SM the Higgs field $\Phi$ is an $SU(2)_L$ doublet, then gauge invariance demands there must be another $SU(2)_L$ doublet and a re-normalizable term in the

\[\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}.\]
Lagrangian have to contain one $SU(2)_L$ singlet. So the Yukawa interaction for the
first generation quarks can be written as

$$
-\mathcal{L}_Y \supset \lambda_u \bar{u} R \hat{\Phi}^\dagger Q_L + \lambda_u^* \bar{Q}_L \hat{\Phi} u_R + \lambda_d \bar{d} R \Phi \bar{d}_L + \lambda_d^* \bar{Q}_L \Phi d_R \\
= \frac{1}{\sqrt{2}} (v + H) \left[ \lambda_u \bar{u} u_L + \lambda_u^* \bar{u} u_R + \lambda_d \bar{d} d_L + \lambda_d^* \bar{d} d_R \right] \\
= \frac{1}{\sqrt{2}} (v + H) \left[ \lambda_u \bar{u} u_L + \lambda_d \bar{d} d_R \right],
$$

(2.19)

where in the last step Yukawa couplings are assumed to be real, $\Phi \equiv i\sigma^2 \Phi^*$ is the
Higgs anti-doublet and $Q_L^j = \begin{pmatrix} u^j \\ d^j \end{pmatrix}_L$ is the left handed quark doublet. The masses
of the up and down type quarks from Eq. (2.19) are $m_u = \frac{1}{\sqrt{2}} \lambda_u v$ and $m_d = \frac{1}{\sqrt{2}} \lambda_d v$
respectively. In general, considering all three families, the Yukawa couplings $\lambda_u,d$ are
replaced by $3 \times 3$ complex matrices. Therefore, the Yukawa structure for the quark
sector is

$$
-\mathcal{L}_{Y,Quarks}^{SM} \supset \lambda_{uj}^k \bar{u} R \hat{\Phi}^\dagger Q_L^k + \lambda_{dj}^k \bar{d} R \Phi \bar{d}_L^k + \text{h.c.},
$$

(2.20)

where the prime denotes the quarks being in the weak eigenstate or flavour basis. To
rotate to the mass basis, we need to diagonalize the Yukawa matrices by performing
a unitary transformation

$$
u_{L,R}^j = U_{L,R}^{jk} u_{L,R}^k, \quad \nu_{L,R} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R},
$$

(2.21)

$$
d_{L,R}^j = D_{L,R}^{jk} d_{L,R}^k, \quad \nu_{L,R} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R}.
$$
where \( U_{L,R}^\dagger U_{L,R} = U_{L,R}^\dagger U_{L,R} = 1 \) and \( D_{L,R}^\dagger D_{L,R} = D_{L,R}^\dagger D_{L,R} = 1 \). Hence the diagonal quark mass matrices are given by

\[
M_u^\text{diag} = \frac{v}{\sqrt{2}} U_R^\dagger \lambda_u U_L = \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_c & 0 \\
0 & 0 & m_t 
\end{pmatrix}, \quad M_d^\text{diag} = \frac{v}{\sqrt{2}} D_R^\dagger \lambda_d D_L = \begin{pmatrix}
m_d & 0 & 0 \\
0 & m_s & 0 \\
0 & 0 & m_b 
\end{pmatrix}.
\]

(2.22)

As a consequence of these rotations, the mass eigenstates for the up-type quarks are no longer matched with the down-type quarks within the same doublet which further enriches the SM allowing flavour-changing interactions to occur via charged-current interactions \[64\] which will be discussed in the next section.

Finally, the SM leptons have a similar Yukawa structure and it is described by the following Lagrangian

\[
-\mathcal{L}^\text{SM}_{Y,\text{Leptons}} \ni \lambda_i^{jk} \bar{L}_L^j \Phi e_R^k + \text{h.c.}, \quad L_L^i = \begin{pmatrix}
\nu_i \\
\nu^i
\end{pmatrix}_L.
\]

(2.23)

where the charged SM leptons, \( e_L^i \), receive masses proportional to the Higgs vev and the Yukawa coupling while the neutrinos, \( \nu^i_L \), remain massless.

### 2.4 Gauge Structure

The cumulation of all the essential details in the previous section leads us to the final ingredient of the SM, gauge kinetic terms. The entire structure of the SM is based on the gauge group \( SU(3)_C \times SU(2)_L \times U(1)_Y \) and it is spontaneously broken down to \( SU(3)_C \times U(1)_\text{EM} \), as we explored in Section 2.2. Let’s first review some of the most frequently encountered Lie groups in particle physics which are the unitary \( U(N) \),
orthogonal $O(N)$ and symplectic $Sp(N)$ groups.

- **$SU(N)$**: A unitary group of degree $N$ is a group of $N \times N$ unitary matrices, $U^\dagger U = 1 = UU^\dagger$, with the group operations being the matrix multiplications. Unitary matrices with determinant +1 forms the special unitary group $SU(N)$. Moreover, the lie algebra $\mathfrak{su}(N)$ of $SU(N)$ is composed of traceless $N \times N$ hermitian matrices ($A^\dagger = A$)\textsuperscript{3}.

- **$SO(N)$**: An orthogonal group of degree $N$ is a group of $N \times N$ orthogonal matrices, $O^T O = 1 = OO^T$, with the group operations being the matrix multiplications. Orthogonal matrices with determinant +1 forms the special orthogonal group $SO(N)$. Moreover, the lie algebra $\mathfrak{so}(N)$ of $SO(N)$ is composed of traceless $N \times N$ skew-symmetric matrices ($A^T = -A$).

- **$Sp(N)$**: A symplectic group of degree $N$ is a group of $N \times N$ quaternionic matrices, $M$, which satisfies

$$\Omega = M\Omega M^T, \quad \text{with} \quad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (2.24)$$

The explicit form of the $\mathfrak{su}(3)$ Lie algebra generators are the well known Gell-\textsuperscript{3}The generators in the physicist's convention are multiplied by $i$.
Mann matrices [69]:

\[
\lambda_1 = \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad \lambda_2 = \begin{pmatrix}
0 & -i & 0 \\
i & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}, \quad \lambda_3 = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

\[
\lambda_4 = \begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{pmatrix}, \quad \lambda_5 = \begin{pmatrix}
0 & 0 & -i \\
0 & 0 & 0 \\
i & 0 & 0
\end{pmatrix}, \quad \lambda_6 = \begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 1 & 0
\end{pmatrix}, \quad (2.25)
\]

\[
\lambda_7 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & -i \\
0 & i & 0
\end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -2
\end{pmatrix}.
\]

The Gell-Mann matrices are traceless hermitian with the first three analogous to the Pauli matrices of \(\mathfrak{su}(2)\) making it a sub-algebra of \(\mathfrak{su}(3)\). The Gell-Mann matrices satisfy the usual commutation relation

\[
[\lambda_a, \lambda_b] = 2i f_{abc} \lambda_c, \quad a, b, c = 1, 2, \ldots, 8, \quad (2.26)
\]

where \(f_{abc}\) are the structure constants which are fully antisymmetric under the interchange of any pair of indices. It is interesting to point out that the structure constant themselves form complex conjugates or adjoint representations. The adjoint representations are useful because they define how the gauge fields transform in the SM. Since the number generators of \(SU(N)\) are \(N^2 - 1\), the adjoint representation is \(N^2 - 1\) dimensional.

The special unitary group \(SU(N)\) has \(N^2 - 1\) generators or its lie algebra has \(N^2 - 1\) dimensions. Hence, the gauge fields corresponding to the SM gauge groups
are $G^a_{\mu\nu}$, $W^{ij}_{\mu\nu}$ and $B_{\mu\nu}$ respectively, with $a = 1, 2, .., 8$ and $j = 1, 2, 3$. Respectively, the associated field strength tensors labelled by $G^a_{\mu\nu}$, $W^{ij}_{\mu\nu}$ and $B_{\mu\nu}$ are:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu,$$

$$W^{ij}_{\mu\nu} = \partial_\mu W^{ij}_\nu - \partial_\nu W^{ij}_\mu + g_2 \epsilon^{ijkl} W^k_\mu G^l_\nu,$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.$$ 

(2.27)

So the pure gauge dynamics, expressed in terms of field strength tensors, are encoded in the following Lagrangian

$$-\mathcal{L}_{\text{Gauge}} = \frac{1}{4} \left[ G^a_{\mu\nu} G^a_{\mu\nu} + W^{ij}_{\mu\nu} W^{ij}_{\mu\nu} + B_{\mu\nu} B_{\mu\nu} \right],$$

(2.28)

where we are working in the Einstein summation convention. Moreover using the Feynman slash notation, i.e: $\bar{A} \equiv \gamma^\mu A_\mu$, the gauge-invariant kinetic energy terms for all the SM fermions can be compactly written as

$$\mathcal{L}_{\text{K}} = i \sum_{\psi = E_L, e_R, Q_L, u_R, d_R} \bar{\psi} D\psi,$$

(2.29)

where the covariant derivative is

$$D_\mu = \partial_\mu - \frac{i}{2} g_s G^a_\mu \lambda^a - \frac{i}{2} g_2 W^{ij}_\mu \sigma^j - igY B_\mu.$$ 

(2.30)

In Eq. (2.30), the group generators are explicitly expressed in terms of Gell-Mann $\lambda^a$ and Pauli matrices $\sigma^j$. There is an important implication of gauge kinetic term especially for the quark sector, where rotating into the quark mass eigenstates, the weak neutral-current interactions are unaffected; however, the charged-current inter-
actions are modified. The interactions between quarks and $W^\pm$, charged-current, are described by (I will only focus on one term involving $u_L$ and $d_L$)

$$\mathcal{L}_{c.c}^{SM} \supset \frac{g_2}{\sqrt{2}} u^j_L \gamma^\mu W^+_\mu d^j_L \xrightarrow{\text{Rotate}} \frac{g_2}{\sqrt{2}} u^j_L V^{jk} \gamma^\mu W^+_\mu d^k_L ,$$

(2.31)

where $V \equiv U^D_L$ is the well known unitary ($V^\dagger V = VV^\dagger = 1$) Cabibbo-Kobayashi-Maskawa (CKM) matrix [70, 71]. The CKM matrix induces flavour-changing transitions via the exchange of $W^\pm$ across various quark generations. The generic form of the CKM matrix involves three angels and a single phase since the chiral rotations of fermions, $u^j_L$ and $d^j_L$, allow us to remove 5 phases. The presence of the single complex phase is the only known source of charge-parity (CP) violation in the SM. Hence, the full Standard Model Lagrangian is:

$$\mathcal{L}^{SM} = \mathcal{L}^{SM}_{\text{Gauge}} + \mathcal{L}^{SM}_K + \mathcal{L}^{SM}_{\text{Higgs}} + \mathcal{L}^{SM}_{Y, \text{Leptons}} + \mathcal{L}^{SM}_{Y, \text{Quarks}} ,$$

(2.32)

where $\mathcal{L}^{SM}_{\text{Higgs}}$ is precisely the Lagrangian presented in Eq. (2.8) except the regular derivative needs to be promoted to the covariant derivative for the Higgs field and the entire field content of the SM is neatly given in table 2.1 with various charges.

Although it is not the subject of this thesis, for the sake of completeness we will append a small discussion regarding neutrino masses. It is well documented from neutrino oscillation experiments that the neutrinos have extremely small but non-zero masses [72–74]. However, the SM framework does not allow for non-vanishing neutrino masses. Neutrinos are singlet under $SU(3)_C \times U(1)_{\text{EM}}$ as they have neither strong nor electromagnetic interactions. The masses of the other fermions (charged fermions) arise, after EWSB, from Yukawa-type interactions of the Higgs doublet, the left-handed fermion doublet and a right-handed lepton. But the absence of the right-
handed neutrino leaves the SM neutrinos to be massless. Furthermore, the other possibility one could contemplate is a Majorana mass term but it violates $SU(2)_L$ gauge symmetry and the lepton number by two units. Similar to quarks, the introduction of the three right-handed neutrino fields, the “flavour eigenstates” of the neutrinos can be related to their respective ”mass eigenstates” by an analogous type CKM matrix, commonly known as the Maki-Nakagawa-Sakata-Pontecorvo (PMNS) matrix [75].

2.5 Quantum Chromodynamics

Quantum chromodynamics (QCD) describes the interaction between quarks and gluons. All the quarks and gluons carry a colour charge, which determines their interactions under QCD or the $SU(3)_C$ gauge group. At high energy, the interactions are well described by perturbation theory with quarks and gluons as the fundamental degrees of freedom. An interesting aspect of QCD is a strange property known as colour confinement, which means that we can only observe colour singlet and no single colour charged particle can be directly experimentally observed. Various types of quarks combine to form baryons (three quarks bound states) and mesons (quark and anti-quark bound states). The strength of the strong force increases with increasing distance between quarks. Moreover, due to confinement, supplying partons (quarks and gluons) with enough energy one can create a copious amount of bound states. In collider phenomenology, this process is referred to as hadronization and most often these hadrons travel collinearly in the detector and are referred to as jets. Although confinement is difficult to show analytically, there have been some attempts in the literature for QCD-like theories with massless quarks to generalize to SM-QCD.

At low energies, the fundamental degrees of freedom are trapped inside bound
states at some scale \( \Lambda \) and the dynamics are described by non-perturbative methods such as lattice QCD. The strong interaction is scale-dependent, which means the strength of the interaction varies with energy scales. For example, the coupling of quarks and gluons decreases as the energy increases and vice versa. This behaviour of the theory is known as asymptotic freedom. So to accurately determine the behaviour of the QCD interactions, one studies the beta-function of the running coupling constant \( \alpha_s \) [64] which encodes the strength of the strong interaction,

\[
\beta(g_s) = \frac{dg_s}{d \log \Lambda},
\]  

where \( \Lambda \) is the energy scale. For QCD, with \( n_f \) number of approximately massless quarks at the energy scale and with three colours, the one-loop \( \beta \) function can be expressed as:

\[
\beta(g_s) = -\left(11 - \frac{2}{3}n_f\right) \frac{g_s^3}{(4\pi)^2} = -\frac{b}{3} \frac{g_s^3}{(4\pi)^2},
\]

where \( b = 11 - \frac{2}{3}n_f \). The result in Eq. (2.33) can be written in terms of the coupling constant \( \alpha_s = \frac{g^2}{4\pi} \):

\[
\frac{d\alpha_s}{d \log \Lambda} = -\frac{b}{2\pi} \alpha_s^2.
\]

So the solution to the differential equation in Eq. (2.35) is found to be:

\[
\alpha_s(\Lambda) = \frac{2\pi}{b \log \left(\frac{\Lambda}{\Lambda_{QCD}}\right)},
\]

where \( \Lambda_{QCD} \) is the integration constant. It evident from the result in Eq. (2.36) that the coupling strength decreases with increasing energy scale \( \Lambda \) and perturbation fails when \( \Lambda = \Lambda_{QCD} \) which is colloquially referred to as the QCD scale (\( \Lambda_{QCD} \simeq 332 \) MeV).
2.6 Hierarchy Problem

Experimental observations at the LHC determined the physical mass of the Higgs bosons to be $m_H \simeq 125.1$ GeV [76]. In contrast to the gauge bosons and fermions masses which are protected by gauge symmetry, the Higgs boson mass is not protected by any symmetry. The Yukawa couplings of the scalar Higgs boson are proportional to the mass of the particle to which it couples; consequently, it couples most strongly to the top quark. The Higgs boson mass is unstable under one-loop correction and the primary contributions are from the top quark, Higgs boson, and gauge bosons running in the loop as shown in Feynman diagrams given in Figure 2.2. The contributions go as $\Lambda^2$, where $\Lambda$ is the cutoff scale of new physics and this is the quadratic divergence of the Higgs mass parameter. It is widely believed, and hinted by the various challenges for the SM, that the SM is an effective theory of a more general high energy theory, which is valid at an arbitrary energy scale such as the Planck scale where we know the gravitational interactions become important. So effectively, the Planck scale is the physical cutoff scale for the Standard Model. An approximate relation for the first loop diagram is

$$\delta m_H^2 \simeq (-1)^3 \left( -\frac{iy_{t}}{\sqrt{2}} \right)^2 \int \frac{d^4k}{(2\pi)^4} \text{tr} \left( \frac{i}{k - m_H} \frac{i}{k + q - m_H} \right),$$

(2.37)

where $q$ is the incoming momentum and $k$ is the momentum in the loop. Then computing the trace, combining the denominator by introducing Feynman’s parameter,
and cutting off the integral at momentum $\Lambda$ leads to

$$\delta m_H^2 \simeq -\frac{6y_t^2}{16\pi^2} \Lambda^2. \quad (2.38)$$

A similar computation can be done for the other diagrams and in fact, this is true for any scalar coupled to fermions or gauge bosons. So the total contribution for all three Feynman’s diagrams in Figure 2.2 is:

$$\delta m_H^2 \simeq \left[ -6 y_t^2 + 6 \lambda + \frac{3}{4} (3 g_2^2 + g^2) \right] \frac{\Lambda^2}{16\pi^2}, \quad (2.39)$$

where $\lambda$ is the Higgs self-coupling. Consequently, the physical Higgs mass with the bare mass term included is

$$m_{H,\text{phys}}^2 \simeq m_{H,\text{bare}}^2 + \delta m_H^2, \quad (2.40)$$

where $m_{H,\text{phys}}^2$ is the experimentally observed Higgs boson mass and $m_{H,\text{bare}}^2 = 2\mu^2$ with $\mu^2$ being the Higgs mass squared parameter appearing in the Higgs potential in Eq. (2.8). We can see from Eq. (2.40) that depending on the scale of the new physics, there will be some fine-tuning required to reproduce the experimentally observed Higgs mass. Pushing the scale $\Lambda$ away from the EW scale worsens the fine-tuning problem. Furthermore, this fine-tuning has a ripple affects across all massive SM mass spectra since the parameter $\mu^2$ is intricately connected to all particles via the EW symmetry breaking. For example, $m_{Z,W}^2 \propto \mu^2$ and $m_f^2 \propto \mu^2$ where $f$ denotes any massive SM fermion. Any potential solution to the hierarchy problem must cancel or match such quadratically divergent contribution. The most notable possibility that

---

4In the above integral, there are divergent terms proportional to a constant and $q^2$ which are dropped.
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(a) Top Quark  (b) Scalar Higgs  (c) Gauge Bosons

Figure 2.2: The quadratically divergent contribution to the Higgs mass from the top, Higgs, and gauge bosons loop.

tackles the hierarchy problem is supersymmetry where the superpartner of the top quark called stop gives a similar contribution with an opposite sign due to the stop being a scalar, unlike the SM top quark [46].

2.7 Anomalies

Baryon number, $B$, in the SM is an accidental symmetry of the Lagrangian corresponding to a $U(1)_B$ symmetry where all lepton carries zero $B$ while quarks carry $B = 1/3$. Experimental observations of the stability of the proton, with a lifetime greater than $\tau_p > 1.6 \times 10^{34}$ years [77], strongly indicate that after QCD confinement baryon number is conserved. However, due to the significant lack of anti-matter, there must be baryon violation in the early epoch. The baryon and lepton numbers are anomalous in the SM [14]. The transition between various vacuum states in the $SU(2)_L$ gauge theory violates $B$ by non-perturbative transitions at high temperatures. The transition rate at low temperature is suppressed since it proceeds by quantum tunnelling. An anomaly is the breaking of a classical symmetry by quantum effects. Noether’s theorem tells us that the invariance of the action under a global symmetry transformation corresponds to conserved currents i.e. $\partial_\mu J^\mu = 0$; however, when a symmetry is anomalous such condition is violated by quantum mechanical effects.

To be clear, any viable theory must be anomaly free or free of anomalies with
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respect to gauged symmetries. Global anomalies are fine since they do not lead to any inconsistencies in the theory. For example in the SM, the quantum electrodynamics (QED) Lagrangian, $\mathcal{L} \supset \bar{\psi} \slashed{D} \psi - m \bar{\psi} \psi$, is invariant under the following two global symmetries (in the limit $m \to 0$):

$$\psi \to \psi' = e^{i \alpha} \psi, \quad \psi \to \psi' = e^{i \beta \gamma^5} \psi,$$

(2.41)

with the corresponding Noether currents are given by

$$j^\mu = \bar{\psi} \gamma^\mu \psi, \quad j^{\mu 5} = \bar{\psi} \gamma^\mu \gamma^5 \psi.$$

(2.42)

Applying the equation of motion, we obtain $\partial_\mu J^\mu = \partial_\mu J^{\mu 5} = 0$. However, the mass term is not invariant under a chiral transformation (second transformation in Eq. (2.42)) with the divergence of the current given by $\partial_\mu J^{\mu 5} = 2i m \bar{\psi} \gamma^5 \psi$. So classically, only the vector current $j^\mu$ is conserved exactly but the axial current $J^{\mu 5}$ is conserved in the massless limit $m \to 0$. However, computing the action of the axial rotation using the path integral for massless QED gives [64,65]:

$$\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2} e^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma},$$

(2.43)

where $F_{\mu\nu}$ is the $U(1)$ field strength tensor. This is known as the abelian anomaly. A similar exercise for massless QCD, 2 flavours, under the symmetry group $SU(2) \times SU(2)$ leads to

$$j^\mu = \overline{Q} \gamma^\mu Q, \quad j^{\mu i} = \overline{Q} \gamma^\mu \tau^i Q,$$

(2.44)
where $\tau^i$ are the generators of the special unitary group $SU(2)$ and $Q$ is the quark doublet. The difference between the left and right currents gives the axial vector currents

$$j^\mu_5 = \bar{Q}\gamma^\mu\gamma^5 Q, \quad j^{\mu 5i} = \bar{Q}\gamma^\mu\gamma^5 \tau^i Q.$$  \hspace{1cm} (2.45)

Then the corresponding divergence is

$$\partial_\mu j^{\mu 5i} = -\frac{g_s^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} G^c_{\alpha\beta} G^d_{\mu\nu} \text{tr}[\tau^i t^c t^d],$$  \hspace{1cm} (2.46)

where $\tau^i$ are the isospin matrices, $t^c$ are the generators of $SU(3)$, and $G^c_{\mu\nu}$ is the gluon field strength. Using the traceless property of the $SU(N)$ generators, and using $\text{tr}(t^c t^d) = \frac{1}{2} \delta^{cd}$, we can obtain

$$\text{tr}[\tau^i t^c t^d] = \text{tr}[\tau^i] \text{tr}[t^c t^d] = 0.$$  \hspace{1cm} (2.47)

Therefore, the axial isospin current leads to vanishing divergence implying the conservation of the axial isospin current. On the other hand, the isospin singlet axial current leads to a non-vanishing divergence

$$\partial_\mu j^\mu_5 = -\frac{g_s^2 n_f}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} G^a_{\alpha\beta} G^a_{\mu\nu},$$  \hspace{1cm} (2.48)

where $\tau^i$ is replaced by the identity and $\text{tr}(t^c t^d) = \frac{n_f}{2} \delta^{cd}$ for $n_f$ flavours. Consequently in QCD, the isospin singlet axial current is not conserved.

The discussion of anomalies plays an important role in physics. Firstly, if a symmetry is anomalous then the corresponding currents will not be conserved. In
Yang-Mills theory, the coupling of the current to massless spin-1 particles, with gauge symmetry, leads to disastrous consequences since the Ward identity and unitarity will be violated. So a consistent quantum field theory with gauge symmetry must be anomaly free. More precisely, a theory must be gauge anomalies-free (anomalies of symmetries associated with gauge bosons). Anomalies are infrared (IR) effects due to the presence of massless particles. So the global anomalies of the ultraviolet theory must be equal to that of the low energy effective theory. This is known as the 't Hooft anomaly matching [78]. In general for a non-abelian gauge theory, the associated current with gauge fields are

\[ J^{\mu p} = \overline{\psi}_i T^p_{ij} \gamma^\mu \psi_j , \]  

(2.49)

where \( T^p_{ij} \) are the group generators in a given representation. Then the divergence of the current, from the triangle diagram with the current at one vertex and gauge fields on the other two vertices, is proportional to \( \text{tr}(T^a \{ T^b, T^c \}) \). The list of possible anomalies in \( SU(3)_C \times SU(2)_L \times U(1) \) are

- \( U(1)_Y^3 \): The divergence is proportional to

\[ \sum_{\text{left}} Y^3_i - \sum_{\text{right}} Y^3_r = n_f \left[ 2Y^3_{L_L} - Y^3_{e_R} \right] + n_f \left[ 2Y^3_{Q_L} - Y^3_{u_R} - Y^3_{d_R} \right] = 0 . \]  

(2.50)

- \( SU(3)_C^2 U(1)_Y \): The divergence is proportional to

\[ \text{tr}(T^a \{ T^b, Y \}) = \delta^{ab} \left[ 6Y_{Q_L} - 3Y_{u_R} - 3Y_{d_R} \right] = 0 . \]  

(2.51)
• $SU(2)^2_L U(1)_Y$: The contribution is of the form

$$
\text{tr}(\tau^a\{\tau^b, Y^c\}) = \delta^{ab}[6Y_{Q_L} + 2Y_{L_i}] = 0. \quad (2.52)
$$

• There are no $SU(3)^3_C$ anomalies since the SM QCD is non-chiral. Furthermore, an anomalies of the form $SU(N)U(1)^2$ (anomalies with exactly one factor of $SU(2)$ or $SU(3)$) are proportional to $\text{tr}(T^a\{1, 1\}) = 2\text{tr}(T^a) = 0$.

Therefore, all anomalies in the Standard Model cancel exactly.

Finally, we bring our attention to global anomalies in the Standard Model. In particular, our interest lies in the global symmetries of the baryon ($B$) and lepton ($L$) numbers. The non-vanishing contribution to the anomalies is due to $SU(2)^2U(1)_B$ and $SU(2)^2U(1)_L$ with divergence\(^5\)

$$
\partial_\mu j_B^\mu = \partial_\mu j_L^\mu = \frac{g^2 n_g}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} W^\alpha_{\alpha\beta} W^{\alpha\mu}_{\mu\nu}, \quad (2.54)
$$

where $n_g$ is the number generations and $W^a_{\mu\nu}$ is the $SU(2)$ field strength. Hence, the baryon and lepton numbers are anomalous in the SM. However, the expression in Eq. (2.54) suggests that $B - L$ is non-anomalous which is important for our discussion of baryon asymmetry generation.

\(^5\)The currents associated with $B$ and $L$ are

$$
\begin{align*}
\tilde{j}_B^\mu &= \frac{1}{3} \sum_i [\bar{Q}_L, \gamma^\mu Q_L, - \bar{u}_R, \gamma^\mu u_R, - \bar{d}_R, \gamma^\mu d_R], \\
\tilde{j}_L^\mu &= \sum_i [\bar{L}_L, \gamma^\mu L_L, - \bar{e}_R, \gamma^\mu e_R].
\end{align*} \quad (2.53)
$$
2.8 Sphaleron Transitions

As we have stated in the previous section, the baryon and lepton numbers are anomalous or violated by the triangle anomaly. The B-violation is associated with the vacuum structure of the spontaneously broken non-abelian $SU(N)$ gauge theory. To quantitively describe sphalerons, we need to first introduce the Chern-Simons number defined as [79]

$$n_{CS} = \int d^3 x K^0, \quad (2.55)$$

where the current $K^\mu$ is

$$K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} \left[ W^a_{\nu} A^a_\alpha - \frac{g}{3} \epsilon_{abc} A^b_\nu A^c_\alpha A^a_\beta \right]. \quad (2.56)$$

The current satisfies

$$\partial_\mu K^\mu = \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\alpha\beta} W^a_{\mu\nu} W^a_{\alpha\beta}. \quad (2.57)$$

Then for some field configurations that are pure gauge\(^6\), at some initial and final times $t_i$ and $t_f$

$$n_{CS}(t_f) - n_{CS}(t_i) = \int_{t_i}^{t_f} dt \int d^3 x \partial_\mu K^\mu = \nu, \quad (2.59)$$

is an integer $\nu$ which is a winding number. So there are infinitely many degenerate ground states labelled by the winding number that are separated by an energy barrier

---

\(^6\)Pure gauge refers to gauge-transform of zero or a gauge transformation on the null-field configuration. In non-abelian gauge theory, one can take the vacuum (the state with a minimum amount of energy) where the vector potential is zero $A_\mu = 0$. However, that’s not the only potential configuration with minimum (zero) energy. From gauge transformation $U$

$$A_\mu \rightarrow A'_\mu = U A_\mu U^\dagger - i g U \partial_\mu U^\dagger \Rightarrow A^\text{pure gauge}_\mu = - \frac{i}{g} U \partial_\mu U^\dagger. \quad (2.58)$$
with each vacuum labelled by a different Chern-Simons number $n_{CS}$. The height of the energy barrier is [80–83]

$$E_{\text{Sp}} \simeq 4 \frac{m_W(T)}{\alpha_W}.$$ \hspace{1cm} (2.60)

The non-trivial topological structure of the $SU(N)$ gauge theories was first discussed by Faddeev [84] and Jackiw and Rebbi [85].

The transition between the various configurations is referred to as sphaleron transition. A schematic illustration of the potential barrier as a function of the topological/winding number is presented in Figure 2.3. The sphaleron transition is exponentially suppressed at temperatures below the EW phase transition, occurring via quantum tunnelling; however, at high temperatures the transition becomes unsuppressed and the thermal energy allows it to hop over the barrier. This is important because such transitions leads to $B$ and $L$ violating process due to the relation

$$n_g \partial_\mu K^\mu = \partial_\mu J^\mu_{B+L}.$$ So each transition violates $B$ and $L$ by three units each and at zero or temperatures much lower than the EW scale, the $B$ violating processes are highly suppressed (proceeding via quantum tunnelling with the transition amplitude proportional to $e^{-8\pi^2/g^2} \sim 10^{-173}$). At temperatures above the EW scale, the baryon and lepton numbers can be significantly violated. Since the baryon and lepton number changes by a multiple of three when transitioning from each different vacuum configuration, the anomaly induces an operator involving 9 quarks and three leptons (all left-handed) of the form $\sum_{i=1}^{3} Q_{L_i} Q_{L_i} Q_{L_i} L_{L_i}$, where $i$ is the generation index. Hence such processes play a crucial role in generating baryon asymmetry and the rate of baryon violation occurring whenever transition between the various vacua takes place is an essential ingredient for a given model. As it will be discussed in Chapter 4 in detail, the sphaleron rate is very sensitive to the temperature and the Higgs vacuum expectation value.
2.9 Beyond the Standard Model

The Standard Model has survived extensive experimental searches over the last few decades and it is widely accepted as a solid foundation for elementary particle physics. However, as we have aforementioned at the beginning of this thesis, the SM is not perfect due to various challenges such as the hierarchy problem, DM, and baryogenesis. Such problems are the main motivations for developing theories beyond the Standard Model. Following the establishment of the SM, there have been numerous BSM models explored over the last few decades addressing one or several issues simultaneously and quite a significant number of them introduce new interactions/particles. Although some models are more highly motivated than others, they all have one common feature; evading current detection limits. In this thesis, our discussion will be limited to Nnaturalness, top partners, and supersymmetry. The latter will be discussed in detail in Chapter 6 while the primary motivation for the Nnaturalness and top partners is the hierarchy problem as described in Section 2.6.

2.9.1 Nnaturalness

The Nnaturalness idea was presented by N. Arkani-Hamed et al in [47]. It addresses the hierarchy problem by introducing $N$ copies of the Standard Model. The precise
structure of the additional sectors is irrelevant with the exception of the SM which corresponds to the visible Universe (same particle content, gauge group structure, Yukawa structure, etc). In these $N$ additional sectors, the Higgs mass squared parameter takes on random values from $-\Lambda_H^2$ to $\Lambda_H^2$ where $\Lambda_H$ is the cutoff scale of the theory. For a random distribution for $m_H^2$, we should expect that some of the sectors would be accidentally tuned at the $1/N$ level corresponding to $|m_H^2|_{\text{min}} \sim \Lambda_H^2/N$. It is noteworthy to mention that one of the $N$ copies must correspond to the SM to describe our observable Universe and one identifies the sector with the smallest non-zero vev as our SM.

The Higgs mass squared parameter in the SM is negative; however, to populate small values of $m_H^2$, the Higgs mass squared parameter should be allowed to take positive values. This leads to quite an intriguing feature which we explore briefly here. For example, suppose $m_H^2$ take on a uniform distribution as

$$\left( m_H^2 \right)_i = \frac{-\Lambda_H^2}{N} (2i + r), \quad -\frac{N}{2} \leq i \leq \frac{N}{2}, \quad (2.61)$$

where $i = 0$ denotes our sector with $(m_H^2)_0 = -r \Lambda_H^2/N \simeq -(88.4 \text{ GeV})^2$ and $r$ is a fine-tuning parameter. In the Nnaturalness framework, there are two distinct groups of sectors: $m_H^2 < 0$ called the standard sectors which are quite similar to the SM except for the heavier particle spectrum and $m_H^2 > 0$ referred to as exotic sectors which are drastically different because they have vanishing vev and the electroweak symmetry is being triggered by the QCD phase transition. The particles in the exotic sectors are very light as discussed in Section A.1.

One important constraint on introducing any additional sector is the contribution to the number of effective neutrinos $N_{\text{eff}}$ which relates the fraction of the energy
density of the Universe stored in neutrino $\rho_\nu$ to radiation $\rho_\gamma$

$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}}.$$  \hspace{1cm} (2.62)

In the SM, the expected value of $N_{\text{eff}}^{\text{SM}} = 3.045$ considering neutrino oscillations and the effects of neutrinos not being fully decoupled during electron-positron annihilation [18]. The experimental measurements impose very strong limits on the number of effective neutrinos with $\Delta N_{\text{eff}} < 0.4$, where $\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}$, at 95% confidence level [18]. So in the naive scenario, if all the sectors have the same temperature then that would contribute $\Delta N_{\text{eff}} \sim N$ which would be instantly excluded. Thus the various sectors must be at different temperatures with our sector being predominantly reheated. How does one ensure that the SM sector is reheated predominantly compared to all the others? Enters a new field called “reheaton”: The reheaton field is responsible for reheating the Universe through its decay and the branching ratios are such that the reheaton deposits as much energy as possible into the sector with the smallest Higgs vev. The reheaton can be a fermion or scalar which are analyzed in the original Nnaturalness paper in extensive detail. For example in the minimal case, the Lagrangian for a fermionic reheaton is:

$$-L_l \supset \lambda S^c \sum_i H_i L_i + m_S S S^c,$$  \hspace{1cm} (2.63)

where $L_i$ and $H_i$ are the usual left-handed lepton and Higgs doublets, respectively. Then for a relatively light reheaton compared to the Higgs and the electroweak gauge bosons, the leading order operators describing the reheaton decays are given by an
effective Lagrangian \[ [47] \] (this is obtained by integrating out \( H_i \) and the gauge bosons)

\[
\mathcal{L}_{l}^{(H) \neq 0} \supset C_1 \frac{\lambda}{m_Z^2 m_S} \nu^\dagger \sigma^\mu S^c f^\dagger \sigma_\mu f ,
\]

\[
\mathcal{L}_{l}^{(H) = 0} \supset C_2 \frac{\lambda}{m_H^2} S l Q_{3}^3 u_{3}^c , \tag{2.64}
\]

where \( C_i \) are numerical coefficients and \( f \) represent any charged fermion. From the effective Lagrangian, one can conclude that the decay widths to the various sectors scale as follow: for the standard sectors \( m_H^2 < 0 \), the reheaton decay width goes as \( \Gamma_{m_H^2 < 0} \sim 1/m_H^2 \), with \( m_{H_i} \) being the physical Higgs mass in the \( i^{th} \) sector; on the other hand in the exotic sectors, the reheaton’s partial decay width is \( \Gamma_{m_H^2 > 0} \sim 1/m_{H_i}^4 \).

Henceforth, the decays of \( S \) preferentially reheat sectors with the lightest Higgs bosons with substantial energy primarily in the \( i = 0 \) sector. In our presented model for baryogenesis, the reheaton is a Dirac fermion \( S \) responsible for reheating the various sectors. Moreover, to accomplish the desired branching ratio required for successful dark matter and baryogenesis in our model, there are also fourth-generation vector-like leptons which give rise to Feynman diagrams shown in Figure 2.4 and that discussion is left for Chapter 4.
Chapter 2. The Standard Model and Beyond

2.9.2 Top Partners

Another non-supersymmetric extension dealing with the fine-tuning issue is the introduction of top partners. Many extensions of the SM trying to address the hierarchy problem involve fermionic top partners $T$ [54, 86] which are expected to be around the TeV scale to avoid fine-tuning. Generally, one of the top partners’ is an $SU(2)_L$ singlet and one doublet, colour triplets, and has an electric charge of $+2/3$. The new fermion typically couples to the top quark and the SM Higgs cancelling the quadratic divergences that appear due to the SM top quark by one-loop contributions such as the one in Figure 2.5 [87]. Since the top partners are non-chiral, they can get masses independent of their coupling to the Higgs field. Such heavy vector-like particles (VLQs), with right and left components transforming similarly under the SM gauge group, are studied extensively in colliders leading to rich phenomenology which needs to be explored thoroughly. The LHC searches, which will be discussed in detail in Chapter 5, put stringent limits on the masses of the coloured top partners for some generic decays such as top quark and a Higgs or gauge bosons. However, we investigate previously unexplored decay topologies leading to new signals and weaker constraints from the current experimental searches compared to the traditional decay modes.

\textbf{Figure 2.5:} The quadratically divergent contribution to the Higgs mass from the top partner at one loop cancelling the SM Higgs quadratic divergences from the top Yukawa coupling.
Chapter 3

Introduction to Cosmology

In the present Universe, the precise origin of the excess of matter over anti-matter (baryon asymmetry) remains one of the biggest puzzles in elementary particle physics and modern cosmology. At the beginning of our cosmos, the Big Bang must have produced an equal amount of matter and anti-matter. However, we know from observations at various scales that the Universe is devoid of anti-matter by many orders of magnitude. There is an abundance of cosmological and everyday evidence that our world is entirely constructed of matter from the smallest scale to the largest scale observable. The absence of proton anti-proton annihilations in our daily activities presents very compelling evidence that the world is constituted out of matter. Furthermore, at the scale of our solar system, various artificial satellites and probes indicate a considerable lack of anti-matter. At galactic scales, experimental observation of anti-proton in cosmic rays is of the $\mathcal{O}(10^{-4})$ compared to protons but this expected from processes such as $p + p \rightarrow 3p + \overline{p}$. At a much grander scale such as in clusters of galaxies, there is no detectable background of gamma radiation which would be expected from nucleon and anti-nucleon annihilation. Hence, a mechanism for generating the observed baryon asymmetry of the Universe (BAU) is required to
describe the observable Universe.

### 3.1 Evolution of the Universe

To a very high degree, the Universe appears to be homogeneous and isotropic at large scales. To describe the standard cosmological evolution of the Universe, we can start with the metric, called the Robertson-Walker metric, expressed as

\[
\text{d}s^2 = -\text{d}t^2 + a(t)^2 \left[ \text{d}r^2 + S_\kappa(r)^2 \text{d}\Omega^2 \right],
\]

(3.1)

where \( \text{d}\Omega^2 = \text{d}\theta^2 + \sin^2 \theta \text{d}\phi \) and

\[
S_\kappa(r) = \begin{cases} 
\sin(r), & \text{for } \kappa = +1 \\
r, & \text{for } \kappa = 0 \\
\sinh(r), & \text{for } \kappa = -1 
\end{cases}
\]

(3.2)

where \( \kappa = +1, \kappa = 0 \) and \( \kappa = -1 \) corresponds to positively, flat, and negatively curved space. The variable \( t \) in the Robertson-Walker metric is the cosmological proper time and it is measured from the frame of an observer with the Universe expanding around him/her while the spatial coordinates \( (r, \theta, \phi) \) are known as the comoving coordinates. The expression in Eq. (3.1) can be written in many forms, and it is most commonly presented as (with a change of variable \( r \to S_\kappa(r) \))

\[
\text{d}s^2 = -\text{d}t^2 + a(t)^2 \left[ \frac{dr^2}{1 - \kappa r^2} + r^2 \text{d}\Omega^2 \right].
\]

(3.3)

Then applying the Einstein’s equations on the above metric assuming a perfect fluid form for the energy-momentum tensor, the resultant equations are [88,89]: the Fried-
Chapter 3. Introduction to Cosmology

The cosmological equation

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{\kappa}{a^2}, \quad (3.4) \]

and the acceleration equation

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \sum_i (\rho_i + 3P_i), \quad (3.5) \]

where \( G \) is the Newton’s constant\(^1\), \( \rho_i \) is the energy density and \( P_i \) is the pressure of a given species. In cosmology, we often deal with dilute gases and in such cases, pressure and energy density are related as \( P = \sum_i \omega_i \rho_i \) where \( \omega_i \) is a dimensionless parameter. The first term on the right-hand side in Eq. (3.4) is due to the various contributions of matter/radiation while the second term is purely due to the inherent curvature of the Universe. Current experimental observation strongly supports the hypothesis of a spatially flat, \( \kappa = 0 \), Universe \([90,91]\). Furthermore, it is important to mention that in the Friedmann’s equation (and in the acceleration equation), we have neglected the cosmological constant term \(+\Lambda/3\) with \( \Lambda \simeq 4.6 \times 10^{-84} \) GeV\(^2\) appearing on the right hand side of Eq. (3.5). Although \( \Lambda \) is very small, the scale factor today is entirely determined by the cosmological constant term. Unlike the other components of energy densities, vacuum energy does not dilute at all as the Universe expands over time.

There are two types of contributions to the energy density: Firstly the matter component, \( \omega = 0 \), corresponds to non-relativistic particles. Secondly, the radiation component, \( \omega = +1/3 \), corresponds to relativistic species. The behaviour of the scale

---
\(^1\)In terms of the reduced Planck mass \( M_P \simeq \left( \frac{1}{8\pi G} \right)^{1/2} \simeq 2.43 \times 10^{18} \) GeV.
factor for the two types of contribution is:

$$\rho \propto \begin{cases} a^{-3}, & \text{Matter} \\ a^{-4}, & \text{Radiation} \end{cases}$$

(3.6)

In early times, the energy density of the Universe is dominated by radiation; however, eventually, matter will take over since the radiation dilutes faster compared to matter. The acceleration equation essentially tells us the eventual fate of our Universe since it relates the acceleration of the scale factor to the total energy density and pressure of the Universe. For example, in the case where the cosmological constant term is set to zero, the Universe will eventually stop expanding and collapse back on itself. But once the cosmological constant dominates, the expansion of the Universe will continue to accelerate, ultimately reaching the state of maximum entropy.

### 3.2 Boltzmann Equations

In this section, we will lay out some of the necessary quantities employed throughout this thesis. The number density, energy density, and pressure of particles are respectively given by [89]

$$n = \frac{g}{(2\pi)^3} \int \! d^3 p \, f(p) ,$$

$$\rho = \frac{g}{(2\pi)^3} \int \! d^3 p \, f(p) \cdot E(p) ,$$

$$P = \frac{g}{(2\pi)^3} \int \! d^3 p \, f(p) \cdot \frac{p^2}{3E(p)} ,$$

(3.7)
where \( g \) is the number of internal degrees of freedom, \( E(p) = \sqrt{p^2 + m^2} \), \( f(p) \) are the Fermi-Dirac (+ sign) and Bose-Einstein distributions (− sign)

\[
f(p) = \frac{1}{e^{\frac{E(p) - \mu}{T}} \pm 1}, \quad (3.8)
\]

where \( \mu \) is the chemical potential. In general, every particle species carries a distinct distribution function with distinct chemical potentials \( \mu \) and temperature \( T \); hence, there are unique \( n, \rho \) and \( P \) corresponding to each species. Since the number of photons is not a conserved quantity, then \( \mu_\gamma = 0 \) which implies that chemical potential for particles and anti-particles are related as \( \mu_X = -\mu_{\bar{X}} \). We will work within the limit where one can neglect the chemical potentials and we will not discuss the degenerate case (\( \mu \gg T \)). One can obtain approximate analytic expressions for the set of equations in Eq. (3.7) in the relativistic and non-relativistic regimes. In the relativistic limit \( T \gg m \),

\[
\begin{align*}
  n &\simeq \begin{cases} 
    \frac{\zeta(3)}{\pi^2} g T^3, & \text{Bosons} \\
    \frac{3\zeta(3)}{4\pi^2} g T^3, & \text{Fermions}
  \end{cases} \\
  \rho &\simeq \begin{cases} 
    \frac{\pi^2}{30} g T^4, & \text{Bosons} \\
    \frac{7\pi^2}{240} g T^4, & \text{Fermions}
  \end{cases} \\
  P &= \frac{1}{3} \rho \simeq \begin{cases} 
    \frac{\pi^2}{90} g T^4, & \text{Bosons} \\
    \frac{7\pi^2}{720} g T^4, & \text{Fermions}
  \end{cases}
\end{align*}
\]

(3.9)
However, in the non-relativistic regime \( T \ll m \), the expressions for fermions and bosons take the same form, which are

\[
\begin{align*}
n &\simeq g \left( \frac{m T}{2\pi} \right)^{3/2} e^{-\frac{m}{T}}, \\
\rho &= m n + \frac{3}{2} n T \simeq \left( m + \frac{3}{2} T \right) \cdot g \cdot \left( \frac{m T}{2\pi} \right)^{3/2} e^{-\frac{m}{T}}, \\
P &= n T \simeq g T \left( \frac{m T}{2\pi} \right)^{3/2} e^{-\frac{m}{T}}.
\end{align*}
\]

(3.10)

However for \( \mu \neq 0 \), \( n = n_{eq} \exp \left\{ \frac{\mu}{T} \right\} \). One prominent feature of the quantities in the non-relativistic regime is the exponential decrease, Boltzmann suppression, in the number density, energy density and pressure. The Boltzmann suppression occurs when the temperature falls below the particle mass or when the interaction rate, which is proportional to the particle number density, becomes smaller than the Hubble expansion. In general, we are dealing with more than one particle species in which case the total energy density and pressure are summed over the various species. However, one has to be careful about the summation since it is possible that a given particle type might be relativistic yet decoupled from a thermal bath at temperature \( T \). For example in the SM, neutrinos decouple from the plasma around \( T \sim 0.8 \text{ MeV} \) while still relativistic and they end up with a different temperature compared to photons. In many of the BSM models, new particles are introduced and in general, they are thermalized with the SM bath. In such a case, the total energy contribution can be written as

\[
\rho = \sum_i \rho_i = \frac{\pi^2}{30} g_\ast \left( T \right) T^4,
\]

(3.11)
where \( g_* (T) \) is total effective number of relativistic degrees of freedom

\[
g_* (T) = \begin{cases} 
\sum_{\text{Bosons},b} g_b + \frac{7}{8} \sum_{\text{Fermions},f} g_f , & \text{For } T_i = T \gg m_i \\
\sum_{\text{Bosons},b} g_b \left( \frac{T_i}{T} \right)^{4} + \frac{7}{8} \sum_{\text{Fermions},f} g_f \left( \frac{T_i}{T} \right)^{4} , & \text{For } T_i \neq T \gg m_i
\end{cases}
\tag{3.12}
\]

In the summation, we need to remove any species that becomes non-relativistic \( T < m_i \). Hence, over time as the Universe expands, the temperature of the Universe decreases and various particles will decouple reducing the overall effective degrees of freedom. For example in the SM, all the particles are relativistic at a temperature above 100 GeV and the total internal degrees of freedom are

\[
g_{\text{Bosons}} = 1[H] + 2 (\text{spin}) [\gamma] + 8 \times 2 (\text{colour} \times \text{spin}) [g] + 3 \times 3 (\text{spin}) [W^\pm, Z^0] = 28 ,
\]

\[
g_{\text{Fermions}} = 3 \times 2 (\text{flavour} \times \text{spin}) [\nu_i] + 3 \times 2 \times 2 (\text{flavour} \times \text{particle anti-particle} \times \text{spin}) [e_i] + 6 \times 2 \times 3 \times 2 (\text{flavour} \times \text{particle anti-particle} \times \text{colour} \times \text{spin}) [q_i] = 90 ,
\tag{3.13}
\]

where the parentheses specify the contribution to the internal degrees of freedom due to spin/colour/particle-antiparticle/flavour and the square bracket labels the type of particles. Hence, before the electroweak symmetry breaking, the total effective degrees of freedom is \( g_* = 28 + \frac{7}{8} \cdot 90 = 106.75 \). It is noteworthy to mention that in describing the evolution of the Universe, we track conserved quantities and such quantities include entropy density \( s \) and comoving number density \( Y \equiv \frac{n}{s} \). The entropy density can be written as (neglecting the chemical potential contribution \(- \sum_i \mu_i n_i \) in the numerator)

\[
s = \frac{\rho + P}{T} = \frac{2\pi^2}{45} g_* s (T) T^3 ,
\tag{3.14}
\]

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where \( g_{\ast S}(T) \) is effective number of degrees of freedom in entropy

\[
g_{\ast S}(T) = \begin{cases} 
\sum_{\text{Bosons,b}} g_b + \frac{7}{8} \sum_{\text{Fermions,f}} g_f, & \text{For } T_i = T \gg m_i \\
\sum_{\text{Bosons,b}} g_b \left( \frac{T_i}{T} \right)^3 + \frac{7}{8} \sum_{\text{Fermions,f}} g_f \left( \frac{T_i}{T} \right)^3, & \text{For } T_i \neq T \gg m_i
\end{cases}
\]  

(3.15)

A set of particle species, two or more, are considered in chemical equilibrium if the rate of forward conversion is equal to the reverse reactions. Moreover, two or more species are in thermal equilibrium if they all have the same temperature and they are in kinetic equilibrium (described by Bose-Einstein, Fermi-Dirac, or Maxwell-Boltzmann distribution). However, throughout the Universe’s expansion, the various thermodynamics quantities will go out of equilibrium and their evolution is described by the Boltzmann equation (BZE). In general, from the definition of the number density, the Boltzmann equation can be expressed as [92]

\[
\frac{dn}{dt} + 3 \frac{\dot{a}}{a} n = \frac{g}{(2\pi)^3} \int C[f] \frac{d^3 p}{E},
\]  

(3.16)

where \( g \) is the internal degrees of freedom and \( C[f] \) is known as the collision operator which depends on the phase space density. The collision term for a process such as
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\[ \Psi + a + b + \cdots \leftrightarrow i + j + \cdots \] is given by (for the evolution of a species $\Psi$)

\[
\frac{g}{(2\pi)^3} \int \mathcal{C}[f] \frac{d^3 p}{E_{\Psi}} = - \int d\Pi_{\Psi} d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots 
\times (2\pi)^4 \delta^4 (p_{\Psi} + p_a + p_b + \cdots - p_i - p_j \cdots) 
\times \left[ |\mathcal{M}|^2_{\Psi + a + b + \cdots \leftrightarrow i + j + \cdots} f_a f_b \cdots f_{\Psi} (1 \pm f_i) (1 \pm f_j) \cdots 
- |\mathcal{M}|^2_{i + j + \cdots \leftrightarrow \Psi + a + b + \cdots} f_i f_j \cdots (1 \pm f_{\Psi}) (1 \pm f_a) (1 \pm f_b) \cdots \right],
\]

(3.17)

where $f_i$, $f_i$, $f_a$, $f_b$, $\cdots$ are the phase densities of the respective species labeled by the subscripts, the “+$” and “−” sign denotes bosons and fermions respectively. Moreover, the matrix element squared is averaged over initial, summed over final state spins and the Lorentz invariant phase is

\[ d\Pi \equiv \frac{g}{(2\pi)^3} \frac{d^3 p}{2E}. \]

(3.18)

Although these expressions can be quite complex in general, certain approximations can greatly reduce their complexity. Firstly, one can impose that the interaction respects time-reversal symmetry

\[ |\mathcal{M}|^2 \equiv |\mathcal{M}|^2_{\Psi + a + b + \cdots \leftrightarrow i + j + \cdots} = |\mathcal{M}|^2_{i + j + \cdots \leftrightarrow \Psi + a + b + \cdots}. \]

(3.19)

Secondly, one can simplify the phase space densities by replacing them with the Maxwell-Boltzmann distribution for any species. In such approximation, we can
write for any particle species

\[ 1 \pm f_i \simeq 1, \quad f_i = e^{-\left(\frac{E_i - \mu_i}{T}\right)}. \]  

(3.20)

Consequently, Eq. (3.17) simplifies with \( \dot{H} \equiv \frac{\dot{a}}{a} \)

\[ \frac{dn_\Psi}{dt} + 3 H n_\Psi = - \int d\Pi_\Psi d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots (2\pi)^4 \delta^4 (p_\Psi + p_a + p_b + \cdots - p_i - p_j \cdots) |M|^2 \left[ f_a f_b \cdots f_\Psi - f_i f_j \cdots \right]. \]

(3.21)

The last term in \([\cdots]\) can be rewritten by applying the conservation of energy \( E_\Psi + E_a + E_b + \cdots = E_i + E_j + \cdots \), enforced by the delta function, and the expression \( n/n_{eq} = \exp\left\{\frac{\mu}{T}\right\} \), we have

\[ \left[ f_a f_b \cdots f_\Psi - f_i f_j \cdots \right] = \left[ e^{\frac{\mu_\Psi + \mu_a + \mu_b + \cdots}{T}} - e^{\frac{\mu_i + \mu_j + \cdots}{T}} \right] e^{-\frac{E_\Psi + E_a + E_b + \cdots}{T}}, \]

\[ = \left[ \frac{n_\Psi n_a n_b \cdots}{n_\Psi, eq n_a, eq n_b, eq \cdots} - \frac{n_i n_j \cdots}{n_i, eq n_j, eq \cdots} \right] \tilde{f}_\Psi, eq \tilde{f}_a, eq \tilde{f}_b, eq \cdots. \]

(3.22)
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So Eq. (3.21) can be expressed as:

\[
\frac{dn_\Psi}{dt} + 3 H n_\Psi = -\int d\Pi_\Psi d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots (2\pi)^4 \delta^4 (p_\Psi + p_a + p_b + \cdots - p_i - p_j \cdots) |M|^2 \left[ \frac{n_\Psi n_a n_b \cdots}{n_\Psi, eq n_a, eq n_b, eq \cdots} - \frac{n_i n_j \cdots}{n_i, eq n_j, eq \cdots} \right] f_{\Psi, eq} f_{a, eq} f_{b, eq} \cdots .
\]

(3.23)

where we defined

\[
Q \equiv \frac{1}{n_\Psi, eq n_a, eq n_b, eq \cdots} \int d\Pi_\Psi d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots (2\pi)^4 \delta^4 (p_\Psi + p_a + p_b + \cdots - p_i - p_j \cdots) |M|^2 f_{\Psi, eq} f_{a, eq} f_{b, eq} \cdots .
\]

(3.24)

In one of the simplest cases of a single particle decaying, the expression for \( Q \) simplifies to

\[
Q = \frac{1}{n_\Psi, eq} \int d\Pi_\Psi d\Pi_i d\Pi_j \cdots (2\pi)^4 \delta^4 (p_\Psi - p_i - p_j \cdots) |M|^2 f_{\Psi, eq} \equiv \langle \Gamma_\Psi \rightarrow a + b + \cdots \rangle ,
\]

(3.25)

where \( \langle \Gamma_\Psi \rightarrow a + b + \cdots \rangle \) is the thermally averaged decay width. Finally, for the two initial state particles, the expression for \( Q \) reduces to

\[
Q = \frac{1}{n_\Psi, eq n_a, eq} \int d\Pi_\Psi d\Pi_a d\Pi_b d\Pi_j \cdots (2\pi)^4 \delta^4 (p_\Psi + p_a - p_i - p_j \cdots) |M|^2 f_{\Psi, eq} f_{a, eq} f_{b, eq} \cdots Q \equiv \langle \sigma_\Psi \rightarrow a + b + \cdots v \rangle ,
\]

(3.26)
with \( v = |v_\Psi - v_a| \) the relative velocity and \( \langle \sigma_{\Psi \rightarrow a+b+... v} \rangle \) being the thermally averaged cross section times relative velocity.

### 3.3 Thermal History

The origin and evolution of some aspects of the Universe are highly speculative and we will briefly comment on some important epochs in the vast history of the Universe. In the very early times, spacetime starts to expand exponentially rapidly at an accelerated rate and the widely accepted mechanism to describe it is known as inflation [93]. The alluring aspect of inflation is that it solves two main problems: the horizon and flatness problem. Roughly speaking, the horizon problem concerns the isotropy of the observable Universe with the Cosmic Microwave Background temperature relative fluctuations of \( O(10^{-5}) \). This has a remarkable implication, the whole observable Universe was in casual contact which is in stark contradiction with the Hubble expansion observed today which would suggest a very small portion to have been in casual contact in the past. On the other hand, the flatness problem concerns the amount of matter which has just sufficient energy density to lead to a flat Universe. The current experimental measurements suggest with very high precision that the observable Universe is spatially flat [18]. Inflation resolves the flatness and horizon issues by putting the early Universe in casual contact and the accelerated expansion flattens out the Universe and smoothens the temperature fluctuations.

The thermal history of the Universe depends strongly on the ratio of interactions between the particles present at a given time, generally denoted by \( \Gamma = n\sigma|v| \) where \( n \) is the target particle number density and \( \sigma|v| \) is the interaction cross section times the relative velocity, and the expansion of the Universe which is determined by the Hubble parameter \( H \). If \( \Gamma \gtrsim H \), the particles maintain thermal equilibrium or they
are coupled and if $\Gamma \lesssim H$, the particles decouple and they freeze out. In the SM, all the particles are in thermal equilibrium at the very early epoch and at various stages, different particles will fall out of equilibrium and decouple as we mentioned in the previous section. After inflation, the Universe is expected to produce the observed baryon asymmetry sometime before Big Bang nucleosynthesis (BBN) and we will discuss that in greater detail later. In the SM, the EW phase transition occurs roughly at around 160 GeV. Before, EW was unbroken and all the SM particles (except the Higgs boson) were massless and in equilibrium with each other. The second significant era is marked by the QCD phase transition at around $T \sim 170$ MeV where the asymptotically free quarks and gluons are confined in hadrons.

Moreover, below the EW phase transition ($T \ll M_{W,Z}$), one can roughly estimate the weak interaction cross section mediated by the SM weak gauge bosons as $\sigma \sim G_F^2 T^2$ where $G_F = \frac{\sqrt{2} g^2}{8 \pi m_W}$. So

$$\frac{\Gamma}{H} \sim \frac{g_2^4 M_P}{m_W^4} T^3 \sim \left( \frac{T}{1 \text{ MeV}} \right)^3.$$  \hfill (3.27)

This has very important implications since it essentially marks the period where all the particles interacting weakly with the plasma decouples. Neutrinos belong to the weak interaction category, so all the SM neutrinos decouple from the primordial plasma around the MeV scale and afterward they stream freely and form the cosmic neutrino background. Shortly after the neutrinos decouple from the primordial plasma, the temperature of the Universe falls below the electron mass and we have a period of electron-positron annihilation $e^+ e^- \rightarrow \gamma + \gamma$. This is significant because the annihilation process releases energy and entropy which is transferred to the plasma but not to the neutrinos. Consequently, the photons are heated compared to the decoupled neutrinos. Quantitatively, we can compute the relative temperature dif-
ference as follow: the effective degrees of freedom before and after electron-positron annihilation is

\[ g_* = \begin{cases} 2 + \frac{7}{8} \cdot 4, & T \gtrsim m_e \\ 2, & T < m_e \end{cases} \]  

(3.28)

Then applying the conservation of entropy, since entropy is conserved for the decoupled species and thermal baths separately, \( S(T < m_e) = S(T > m_e) \) leads to

\[ T_\nu = \left( \frac{4}{11} \right)^{1/3} T_\gamma. \]  

(3.29)

The electron-positron annihilation is followed by Big Bang nucleosynthesis, where the formation of the light elements, such as hydrogen, helium, lithium etc, abundance occurs at around \( T \sim O(100 \text{ keV}) \). Subsequently, the Universe undergoes recombination with the production of neutral hydrogen via \( e^- + p^+ \rightarrow H + \gamma \) and as the temperature reduces the reverse reactions become highly energetically disfavoured. Finally, we enter the photon decoupling epoch where the photons stream freely through the Universe at around \( T \sim 0.32 \text{ eV} \). These photons are what is observed today as the cosmic microwave background (CMB) having a temperature of \( T \simeq 2.725 \text{ K} \).

### 3.4 Introduction to Baryon Asymmetry of the Universe

The primordial abundance of light elements, such as hydrogen, helium, lithium and deuterium, and their prediction from standard cosmology gives a measure of the net baryon number of the Universe. The cosmic microwave background and the Big Bang
Chapter 3. Introduction to Cosmology

nucleosynthesis measurements yield the baryon asymmetry of the Universe (BAU) to be [94]

\[ \eta \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq (8.6 \pm 0.01) \times 10^{-11}, \quad (3.30) \]

where \( n_B(\bar{B}) \) is (anti)baryon number density and \( s \) is the entropy density of the Universe. Any possible model that could explain the asymmetry must fulfill the three Sakharov conditions [95]:

(i) Baryon number violation,

(ii) Charge \( C \) and Charge-Parity (\( CP \)) symmetry violation,

(iii) Out of thermal equilibrium processes.

The first condition is rather self-evident since any initial baryon symmetric state will evolve symmetrically and the total number of baryons will be constant. For a baryon symmetric Universe to end up in an asymmetric state, there must be processes that violate the baryon number. Secondly, charge and charge-parity violation is necessary to ensure that the rate of interaction producing an excess of baryons is different than that of the complementary process resulting in anti-baryons (\( \Gamma(X \to A+B) \neq \Gamma(\bar{X} \to \bar{A} + \bar{B}) \)). Finally, the third criterion concerns the fact the total baryon asymmetry generated must not be washed out by the inverse processes.

Although the SM fulfills the three Sakharov conditions, the amount of CP violation is insufficient to account for the total observed baryon asymmetry of the Universe. CP violation in the SM appears in the quark sector due to the complex phase in the CKM matrix. An invariant measure of the amount of CP-violation in
weak interaction can be written as [96–98]:

\[ J_{CP} = \prod_{i>j} (m_i^2 - m_j^2) \prod_{i>j} (m_i^2 - m_j^2) \text{Im} V , \tag{3.31} \]

where \( \text{Im} V \) is the imaginary part of the 4 CKM elements (phase invariant convention) and the Jarlskog invariant, \( J \), is given by (various combinations):

\[ \text{Im} V = \text{Im} (V_{ij}V_{kl}V_{ij}^*V_{kl}^*) = J \sum_{m,n=1}^3 \epsilon_{ikm} \epsilon_{jln} . \tag{3.32} \]

Conventionally, one chooses to work with [76]:

\[ J = \text{Im} (V_{ud}V_{cs}V_{us}^*V_{cd}^*) = c_{12}c_{23}c_{13}^2 s_{12} s_{23} s_{13} \sin \delta \simeq 3.2 \times 10^{-5} , \tag{3.33} \]

where \( c_{ij} = \cos \theta_{ij} \) and \( c_{ij} = \sin \theta_{ij} \) for \( i < j = 1, 2, 3 \). So the total amount of CP violation can be expressed in terms of a dimensionless quantity as (in the broken phase since it vanishes in the symmetric phase):

\[ \Delta_{CP} = \frac{J_{CP}}{T^{1/2}} \simeq 1.2 \times 10^{-19} , \quad T = 100 \text{ GeV} . \tag{3.34} \]

Consequently, new sources of CP asymmetry are required to generate the necessary baryon asymmetry. One prominent approach is leptogenesis where one introduces weakly coupled heavy fermions or right-handed neutrinos [30,99]. The CP violation is determined from the interference of the tree and one loop decays of the right-handed neutrinos which is eventually partially converted into a baryon asymmetry by sphaleron processes. A viable mechanism for the baryogenesis with leptogenesis occurring at the early epoch is discussed in Chapter 4. Finally, there is another
mysterious aspect of the Universe that requires some special attention which is dark matter.

### 3.5 Prelude to Dark Matter

One of the most unexpected revelations in the early 1930s was that the dominant component of matter in the Universe is not ordinary baryonic matter that makes up protons and neutrons. Instead, there is a rather bizarre form of matter dubbed “dark matter” that is approximately five times more abundant than ordinary baryonic matter \( \Omega_{\text{DM}} \simeq 5\Omega_{\text{Visible}} \) [18]. The observation supporting the dark matter hypothesis was first made by J. H. Oort whose studies of the motion of stars in the Milky Way suggested far more galactic mass than previously predicted by anyone from visible matter alone [100,101]. Around the same time, Swiss astronomer F. Zwicky made a similar discovery by investigating the Coma cluster where he noticed that members of the Coma cluster were moving quite faster than simply tracing the gravitational potential of the visible cluster mass [102]. Employing classical mechanics by setting the centripetal force to the gravitational force, the rotational speed for an object at a radius \( r \) in circular orbits can be written as

\[
v(r) = \sqrt{G \frac{m(r)}{r}},
\]

where \( G \) is the gravitational constant and \( m(r) \) is the total mass contained at radius \( r \).

So assuming the orbits of stars within a galaxy mimic a similar behaviour, one would expect \( v(r) \propto 1/\sqrt{r} \). However, the observational data collected from the stars in the Milky Way and the rotation curves of other spiral galaxies show a stark deviation from what is expected only from luminous matter [103,104]. For most galaxies, the
rotational velocity curves far away from the center of the galaxy are approximately flat.

Furthermore, another piece of very compelling evidence for dark matter is presented by the mass distribution of the well-known “Bullet Cluster” (1E0657-56) which is a system of two merging clusters. The total mass distribution of the cluster has been deduced from gravitational weak lensing, the distortion of the apparent shape of a luminous object by the gravitational potential of the massive object located in the line of sight of an observer. The luminous matter distribution is determined using X-ray emission from the hot gas dispersed within the cluster [39]. The collision of the two clusters exhibits a peculiar behaviour where the gravitational lensing map displays a significant dark matter distribution that is missing from the X-ray gas map. Additionally, the dark matter halos pass right through each other after the collision almost completely unaltered, unlike the gas clouds which exert friction on each other.

Finally, the CMB anisotropy map conducted by the Planck collaboration also suggests the presence of dark matter [18]. Admittedly, there have been numerous attempts to tackle the dark matter problem and we will not bother with such an exhaustive list here. However, we will give a brief overview of the so-called “asymmetric dark matter (ADM)” [105–108]. The ADM hypothesis states that present-day dark matter density is due to a dark matter particle-antiparticle asymmetry in the early cosmological epoch due to certain rapidly occurring processes which are later decoupled. The primary motivation for ADM is the striking relations between the present-day mass density of dark matter and visible baryons $\Omega_{DM} \simeq 5\Omega_B$. This perhaps suggests that DM and baryon asymmetries might have a common origin; however, it is entirely possible that this is an accident and these quantities are not related to each other at all. In ADM models, the baryon and dark matter number
densities can be naturally connected if an asymmetry is created in the visible and/or dark sectors by some mechanism (which could be leptogenesis, baryogenesis, etc.), and this asymmetry is then transferred to the other sector. The process responsible for communicating the asymmetry between the visible/dark sectors decouples as the Universe evolves; hence, freeze-in the asymmetry. In Chapter 4, we will present a plausible ADM model that produces the correct relic abundance required by the present observational evidence.
Chapter 4

Baryogenesis and Dark Matter in Multiple Hidden Sectors

One of the most intriguing outstanding mysteries is the origin of the matter over-abundance over anti-matter. Starting with a baryon symmetric universe, how do we end up in a present asymmetric state? Well, in order to for the present Universe to arise full of the wonderful cluster of galaxies, blackholes, stars, planets and the delicious coffee we enjoy daily, the baryon asymmetry needs to be generated via some mechanism. The SM framework allows baryogenesis to occur; however, the net baryons produced are not nearly sufficient to bring into existence the observable cosmos. We will allocate a significant portion of the thesis to tackle not only a successful baryogenesis mechanism but dark matter as well.

4.1 Introduction

The existence of a large number of Standard Model-like hidden sectors [109], and the Nnaturalness [47] paradigm in particular provide a novel and interesting solution to
the hierarchy problem. If there are a large number $N$ of sectors where the Higgs mass parameter takes on random values, then one expects one sector to have a value of order $\Lambda^2 / N$ where $\Lambda$ is the cutoff of the theory. If such a sector is identified with the Standard Model (SM), this setup can explain why the Higgs mass is parametrically smaller than the cutoff as long as $N \gg 1$.

In order for such a scenario to describe our Universe, a novel cosmological history is required that naturally allows most of the energy of the Universe to be in the SM sector with relatively little in the others. In [47], this was accomplished by a “reheaton” field which carries all of the energy of the Universe at early times. The reheaton field has a weak scale mass and democratic coupling to the Higgs in all the sectors. Because of its weak scale mass, it can easily decay into the SM sector, but decays to sectors with a heavier Higgs are kinematically suppressed naturally giving the SM the dominant fraction of the energy of the Universe after the reheaton decay. The coupling of the reheaton to all sectors is required to be very small in order to prevent loop-level interactions across the sectors, and this small coupling ensures that the reheaton width is orders of magnitude smaller than its mass.

Because of the unique cosmology, it remains an open question of how the baryon asymmetry of the Universe (BAU) and the dark matter abundance are generated in such a framework (see [110] for a dark matter review and [111] for a review of models that relate dark matter to the BAU). Obviously, BAU and dark matter are necessary ingredients in any realistic model, and in this work, we build a model that addresses both of these problems. The reheaton field is a fermion that carries a lepton number. At very early times, it lives in a thermal bath of a “reheaton sector” that does not contain any SM-like fields. Such a sector has similar matter content as traditional leptogenesis models [30], and the dynamics of that sector create an
asymmetric population of reheatons.

When the reheaton decays, it transfers its asymmetry to the lepton number in each of the various sectors. By the same kinematic mechanism described in [47], the dominant decay of the reheaton is to the SM sector, and thus most of the asymmetry is transferred to the lepton number of the SM sector. The lepton asymmetry can then be transferred to a baryon asymmetry by the SM sphaleron process [31, 80, 112, 113]. In order for the reheaton to dominantly decay its energy into the SM sector, the reheating temperature of the reheaton decay must be of order the weak scale, which in turn sets an upper bound of the width of the reheaton [47]. If we naively assume that reheating is instantaneous, then the temperature of the SM is always around or below the weak scale, and the sphaleron rate is exponentially suppressed for nearly all time. Doing a more careful calculation of the thermal history, taking into account the early decays of the reheaton, shows that the SM sector in fact reaches temperatures much higher than the naive reheating temperature [114–118], and the SM sphaleron can be unsuppressed for a sufficiently long time to generate the observed BAU.

The subdominant decays of the reheaton will populate the other sectors, with the temperature of the other sectors decreasing as the Higgs mass increases. The lepton asymmetry of the reheaton will also be transferred to other sectors. If the sphaleron in other sectors is active, then the dark baryon asymmetry can serve as an asymmetric dark matter candidate [105–108]. Because constraints on extra relativistic degrees of freedom [18] require that the vast majority of the energy of the reheaton decay to the SM sector, one also expects that the dark baryon asymmetry is significantly smaller than the SM baryon asymmetry. Furthermore, because the Higgs mass in the dark sectors is larger than in the SM, the sphaleron decouples earlier further suppressing the dark baryon asymmetry.
The above analysis, however, changes qualitatively if one considers dark sectors with positive Higgs mass squared parameter. Such sectors were dubbed “exotic” in [119] and shown to produce interesting gravitational wave phenomenology. Because the Higgs mass squared is positive, the Higgs does not break electroweak symmetry, and $SU(2) \times U(1)$ is a good symmetry until the QCD phase transition at much lower temperatures. Therefore, even though the lepton asymmetry transferred to exotic sectors is smaller than that of the SM sector, the sphaleron process is significantly more efficient, and this can accommodate the observed dark matter abundance of $\Omega_{\text{DM}} \sim 5\Omega_B$. The requirement that dark matter is in one of the exotic sectors also gives a lower bound on the effective relativistic degrees of freedom, $\Delta N_{\text{eff}} \gtrsim 0.05$, which could be observable at CMB-S4 [120].

We note that it was previously argued that sectors with a positive Higgs mass parameter cannot have a baryon asymmetry [121]. The argument was that after QCD and electroweak phase transition, even though the sphaleron is exponentially suppressed, it is still faster than Hubble (which is Planck suppressed), and thus all the baryon asymmetry is washed out. Because of the exponential suppression of the sphaleron, this conclusion depends very sensitively on the phase transition temperature compared to the electroweak boson mass. While that ratio can be computed on the lattice in an SM-like setup, QCD in the exotic sector has six massless flavours, so the phase transition is expected to be first order [122] and qualitatively different than the SM. Furthermore, the electroweak boson mass depends on the $SU(2)$ gauge coupling, $g$. While the minimal Nnaturalness setup has $g$ being the same in the exotic and SM sector, this need not be the case to solve the hierarchy problem. Therefore, we find that a baryon asymmetry in the dark sector is indeed possible, and asymmetric dark matter can be accommodated.
The dark matter will be dominated by the neutrons in the exotic sector with the lightest Higgs mass. Furthermore, the pions in these sectors are much lighter than the SM pions, so these neutrons can form large nuclei [123, 124]. In fact, [124] showed that such an agglomeration of neutrons is generic and the exotic nuclei can be expected to grow quite large. This means that dark matter self-interaction is suppressed and the bounds from the Bullet Cluster [125] can be evaded.

This paper is organized as follows. In Section 4.2, we give a qualitative overview of the model including the field content and the mechanisms employed to generate the BAU and asymmetric dark matter. Section 4.3 quantitatively presents the details of the reheaton sector and how its asymmetry is generated. The reheating of the various sectors via the decays of the reheaton field and the constraints imposed by cosmological observations are analyzed in Section 4.4. In Section 4.5, we discuss the baryon asymmetry across the various sectors and the relevant parameter space consistent with observations, while Section 4.6 focuses on dark matter phenomenology. Section 4.7 contains our conclusion and a summary of the baryon asymmetry and dark matter within the naturalness framework. Various technical details are given in the appendices.

4.2 Model Overview

A schematic diagram of the cosmological history of our model is shown in Figure 4.1. We begin our story after inflation when the inflaton decays and deposits all of its energy into a reheating sector which contains a thermal bath that includes:

- At least two heavy Majorana fermions $N_i$.
- A complex scalar $\phi$.
Chapter 4. Baryogenesis and Dark Matter in Multiple Hidden Sectors

- A Dirac fermion $S$ which we refer to as the reheaton.

The $S$ and $N$ carry an equal and opposite quantum number which we will assign to the lepton number. This sector is constructed to mimic the standard leptogenesis setup [30], although we stress that these fields are neutral under all gauge symmetries. As will be detailed in Section 4.3.1, out of equilibrium decays of the $N_i$ fields will eventually impart a lepton number asymmetry on the thermal bath, and the $\phi$ scalar will decay away. There will thus be an era where the Universe is dominated by an asymmetric population of reheatons. The asymmetry of the reheatons will be partially transferred to SM baryons, explaining the baryon asymmetry of the Universe, and partially to an exotic sector, giving rise to asymmetric dark matter.

**Figure 4.1:** A schematic of the cosmological evolution of our model. After inflation, there is a thermal bath composed of Majorana fermions, $N_i$, scalars $\phi$ and reheatons, $S$. The decays of the heavy $N_i$ generate the lepton number asymmetry, which will be carried by the reheaton. Since $\phi$ can only decay into $S$, at some point, the energy density of the Universe is dominated by $S$. Finally, the reheaton decays into the various sectors and populates them. This process leads to a baryon asymmetry in the SM $i = 0$ sector, as well as an asymmetry in the first exotic $i = -1$ sector which is the dominant source of dark matter in the Universe.

In addition to the reheating sector containing $\{N_i, \phi, S\}$, there are also $N$ nearly
decoupled SM-like sectors each containing a copy of the SM field content plus a vector-
like fourth generation of leptons \((L_4, L_4^c)\) with charges of \(2_{\pm 1/2}\) under a given sector’s
\(SU(2) \times U(1)\) gauge symmetry. The dimensionless parameters: gauge, Yukawa, and
quartic couplings, of each sector are (approximately) equal to those of the SM. The
dimensionful Higgs mass parameter takes random values in each sector which is the
way this setup solves the hierarchy problem [47]. For simplicity, we parameterize the
Higgs mass squared parameter in the \(i\)th sector by

\[
(m_H^2)_i = -\frac{\Lambda^2}{N} (2i + r) , \quad -\frac{N}{2} \leq i \leq \frac{N}{2} ,
\]

and we can identify the \(i = 0\) sector, the one with a negative mass parameter with
the smallest magnitude, as the SM sector. Sectors with \(i > 0\) are dubbed “standard
sectors” and have a similar structure to the SM but with a larger Higgs vacuum
expectation value (vev). Those with \(i < 0\) are “exotic sectors” in which the Higgs
does not acquire a vev at all and can be integrated out in an \(SU(3) \times SU(2) \times U(1)\)
symmetric theory. These exotic sectors will be the dominant source of dark matter
(DM) in the Universe. The parameter \(r\) indicates the spacing between sectors, with
\(r = 1\) corresponding to uniform spacing and \(r < 1\) corresponding to a large splitting
between our sector and the next one [47]. More details on the mass spectrum of each
sector are given in Section A.1.

Following [47], the vector-like fourth generation \(L_4\) and \(L_4^c\) serve as the portal
to the reheaton. This is given by the following interaction Lagrangian:

\[
-\mathcal{L}_{L_4} = \sum_i [\lambda S^c (L_4 H)_i + \mu_L (l L_4^c)_i + M_L (L_4^c L_4)_i] + m_{SS} S^c + \text{ h.c.} ,
\]

where \(\mu_L\) is the bilinear coupling and \(l_i\) denotes the SM left-handed lepton doublet,
and as before the $i$ index labels the sectors. For simplicity, we also take the dimensionful parameters $\mu_L$ and $M_L$ to be the same in all sectors. For this model to reproduce the observation that much of the structure of the Universe is governed by SM fields, the reheaton must preferentially decay into the SM sector. Since $S$ couples democratically (with the same $\lambda$) to all sectors, kinematic effects are required to achieve our observable Universe with most of the energy in the visible sector. This can be accomplished if the reheaton’s mass is comparable to the SM Higgs mass, such that its decays into sectors with larger Higgs masses are suppressed. This way, the reheaton dynamically selects the sector with the smallest Higgs mass (defined as the Standard Model sector, with $i = 0$) and populates it preferentially. Since the population of $S$ is asymmetric, its decay will transfer the asymmetry into all sectors, with the quantum number being a linear combination of the lepton number of each sector. The inclusion of $L_4$ is necessary to control the reheaton’s branching ratio into the exotic sectors and in particular, generate the requisite dark baryon density for DM.

In the SM sector, the lepton asymmetry transferred by the reheaton is converted into baryon asymmetry via SM sphalerons [31, 80, 112, 113]. The baryon asymmetry depends on the evolution of the temperature of the SM thermal bath, as the sphaleron rate becomes exponentially suppressed after the Electroweak phase transition. A first pass-estimate of this effect would be to assume an instantaneous decay of the reheaton, that is, assuming that the energy density stored in the reheaton is suddenly deposited in the other sectors when $\Gamma_S \sim H$, where $\Gamma_S$ is the width of the reheaton and $H$ is the Hubble parameter, which would imply a reheating temperature $T_{\text{RH}} \sim \sqrt{\Gamma_S M_P}$, with $M_P$ the reduced Planck mass. Nevertheless, this estimate does not tell us how the temperature of the Universe evolves before reheating, as $T_{\text{RH}}$ only depends on the
duration of the reheating period, \( \tau_S = \frac{1}{\Gamma_S} \). Hence, we need to track the evolution of the temperature by solving the Boltzmann equations for the evolution of the energy density of each component of the Universe, assuming a non-instantaneous reheating (see Section 4.4). As we will see in Section 4.4, the temperature of the Universe is larger than \( T_{\text{RH}} \) for \( t < \frac{1}{\Gamma_S} \) and therefore, during that period, the sphaleron process can occur very efficiently. Although the decays of the reheaton do not heat up the Universe, they make it cool more slowly due to entropy release [114–118]. Thus, the temperature at the end of reheaton decays - the so-called reheating temperature - is not the largest temperature achieved by the thermal bath, but the one at which the entropy density levels off [114]. The reheating process from the reheaton and its consequences for the BAU are explored in detail in Section 4.4 and Section 4.5, respectively.

Qualitatively, the process for the generation of the baryon asymmetry in the other sectors is essentially the same as described above, but the different Higgs masses and vev’s of those sectors lead to different baryon asymmetries. In the case of the standard sectors \((i > 0)\), the corresponding Higgs vev is larger than in the usual SM sector, which suppresses the associated baryon asymmetry in two different ways: 1) the reheaton decay is kinematically suppressed, implying that there is less total energy in the standard sector compared to the SM sector, which allows us to satisfy the bounds coming from \( \Delta N_{\text{eff}} \), and 2) in these sectors, the electroweak phase transition happens at higher temperatures and, therefore, the sphaleron freezes out earlier. In the exotic sectors \((i < 0)\), electroweak symmetry breaking is not due to the Higgs, but due to the confinement of the SU(3) colour group instead [126]. Hence, the electroweak scale is \( \sim 100 \text{ MeV} \), allowing the sphaleron to be active at much lower temperatures. Therefore, even though the energy density of the exotic sectors is lower
than in the SM sector, the corresponding baryon asymmetry can be larger than the one of the SM. This way, the lightest baryon in the exotic sector, the neutron, can be a DM candidate, and one can easily accommodate $\Omega_{\text{DM}} \sim 5\Omega_B$. This DM dynamics is described quantitatively in Section 4.6.

### 4.3 Origin of Lepton Number Asymmetry

In this section, we show that the lepton number asymmetry in our framework can be generated with a leptogenesis-like scenario in the early Universe. As succinctly explained in Section 4.2, we assume that after inflation there is a thermal bath composed of at least two Majorana fermions, $N_i$, the fermionic reheaton, $S$, and a complex scalar, $\phi$. The lepton number asymmetry originates from the decay of the lightest $N_i$ into the reheaton, analogous to the standard leptogenesis mechanism [30]\(^1\). In particular, one can make an analogy with the $N$ being the right-handed neutrino, the $\phi$ being the Higgs, and the $S$ the lepton doublet, but we stress that this is just an analogy and all the fields in the reheaton sector are singlets under all gauge symmetries.

The decays of the $N_1$ imprint an asymmetry on the remnant population of $S$ which dominates the Universe. Later, this asymmetry is transmitted to a lepton number asymmetry in the various sectors through the reheaton decay and, eventually, it is converted into baryon asymmetry due to sphaleron processes. In what follows, we build a model to explain the production of the lepton asymmetry within our setup.
Table 4.1: Matter content and the correspondent charges of the leptogenesis sector: the fermionic reheaton \((S, S^c)\), at least two Majorana fermions \((N_i)\) with different masses, and a scalar \(\phi\). Fermions are written in terms of two component (Weyl) spinors.

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<td>(S)</td>
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<td>(S^c)</td>
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\[\text{Figure 4.2: Feynman diagrams for the tree level decays in our setup.}\]

4.3.1 Particle Physics Model

We consider a simple sector that contains at least two Majorana fermions, \(N_i\), and, for simplicity, we take their masses to be hierarchical, \(M_{i-1} \ll M_i\). The reheaton, \(S\), is a Dirac fermion that couples to the \(N_i\) via a Yukawa coupling to a complex scalar \(\phi\). The field content and the corresponding charges are given in Table 4.1. The interactions in this setup are given by

\[\mathcal{L} = \frac{1}{2} M_i N_i^c N_i + y_i \phi S^c N_i + \kappa \phi S^c S + \text{h.c.}, \quad (4.3)\]

where the subscript \(i \geq 2\) denotes the number of Majorana fermions with the Majorana mass term breaking the \(U(1)\) global symmetry.

1Unlike the “Affleck-Dine” inflation type mechanism where the complex scalar (inflaton) carrying lepton number is responsible for leptogenesis \([34, 127, 128]\).
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We take the mass hierarchy to be $M_1 \gg M_\phi \gg m_S$, with $M_1$ the mass of the $N_1$, the lightest $N$ state, so that the decay processes $N_i \rightarrow S\phi$ and $\phi \rightarrow SS^c$ shown in Figure 4.2 are kinematically allowed. At late times compared to the lifetime of the $N_i$, all that remains is an asymmetric population of $S$ and $S^c$. This asymmetry arises due to the interference in the decay of the $N_i$ between the loop-level diagrams shown in Figure 4.3 and the tree-level decay. The standard $CP$-asymmetry is defined as:

$$\epsilon \equiv \frac{\Gamma (N_i \rightarrow \phi^+ + S) - \Gamma (N_i \rightarrow \phi + S^c)}{\Gamma (N_i \rightarrow \phi^+ + S) + \Gamma (N_i \rightarrow \phi + S^c)}. \quad (4.4)$$

The $CP$-violating parameter $\epsilon$ is given by $[30, 99, 129–131]^2$:

$$\epsilon = \frac{1}{8\pi} \sum_{k \neq 1} \frac{\text{Im} (y_k^* y_k)}{y_1^2} \left( f(x_k) - \frac{M_1 M_k}{M_k^2 - M_1^2} \right) \sim -\frac{3}{16\pi} \sum_{k \neq 1} \frac{1}{x_k} I_{1k}, \quad (4.5)$$

where $x_k = \left( \frac{M_k}{M_1} \right)^2$, $I_{1k} = \text{Im}(y_k^* y_k) (y^* y)_{11}$ and

$$f(x) = \sqrt{x} \left[ 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right] \xrightarrow{x \gg 1} -\frac{1}{2\sqrt{x}}. \quad (4.6)$$

The generation and evolution of the lepton asymmetry can be described by the usual Boltzmann equations for leptogenesis. In our model, we take into account the

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$^2$In our setup, there is an additional one-loop diagram for the $N_i$ decays. Since its matrix element is $\mathcal{M} \propto |\kappa|^2$, it does not contribute to the $CP$ asymmetry $\epsilon$.  

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decays and inverse decays involving \( N_i \), neglecting \( 2 \rightarrow 2 \) scattering processes as a first approximation. In the next section, we show an analytical approximation for the lepton number asymmetry that will be transferred to the reheaton.

### 4.3.2 Lepton Number Asymmetry

In this section, we first analyze the amount of entropy injected into the thermal bath due to the out-of-equilibrium decay of a massive particle such as the Majorana fermion \( N_1 \) and the reheaton \( S \). The entropy release will dilute the initial lepton asymmetry, affecting the eventual production of baryon asymmetry. For simplicity, suppose that we have a massive unstable particle, \( X \), which is non-relativistic and long-lived. In our scenario, the unstable particle \( X \) could be the Majorana fermions \( N_i \) or the reheaton \( S \), and their subsequent decays will produce considerable entropy in our Universe. If \( X \) is sufficiently long-lived, it decays while dominating the energy density of the Universe, since \( \rho_X \sim a^{-3} \), whereas the energy density of radiation, \( \rho_R \), scales with \( a^{-4} \), with \( a \) being the scale factor of the Universe. Assuming an instantaneous decay, we may estimate the entropy released during the process. The particle \( X \) decays when its decay width is of the order of the Hubble parameter, \( \Gamma_X \sim H \). The temperature of the thermal bath immediately prior to the decay of \( X \), \( T_i \), is given by [92]:

\[
\Gamma_X^2 \sim H^2 = \frac{\rho_X}{3M_P^2} \sim \frac{M_X s_i Y_i}{3 M_P^2} \Rightarrow T_i^3 \simeq \left[ \frac{135}{2\pi^2} \frac{M_X^2 \Gamma_X^2}{M_P^2 g_*} \right],
\]

(4.7)

where \( Y_i \equiv \frac{n_i}{s_i} \) is the initial comoving number density of \( X \), with \( s_i = \frac{2\pi^2}{45} g_* T_i^3 \).

Supposing that \( X \) decays into relativistic particles that quickly thermalize, the cor-
responding radiation energy density, $\rho_f$, and temperature, $T_f$, after the $X$ decay are:

$$\rho_f \simeq \frac{\pi^2}{30} g_* T_f^4 = 3 M_P^2 \Gamma_X^2 \Rightarrow T_f^3 \simeq \left[\frac{90 M_P^2 \Gamma_X^2}{\pi^2 g_*}\right]^{3/4},$$  

(4.8)

where we used the conservation of energy. Hence, the ratio of total comoving entropies is:

$$\delta \equiv \frac{S_f}{S_i} \simeq \frac{T_f^3}{T_i^3} \simeq 0.78 \frac{M_X Y_i g_*^{1/4}}{(M_P \Gamma_X)^{1/2}}. \tag{4.9}$$

We conducted a careful analysis, by tracking the energies densities for $X$ and radiation with the Boltzmann equations and solving them using numerical methods, which demonstrated that the rough estimate in Eq. (4.9) is good, with a difference of $O(1)$ compared to the numerical value.

We now turn to the details of our model. We assume that after inflation there is a thermal bath that includes relativistic $N_1$, $\phi$ and $S$, so the temperature must satisfy $T \gg M_1$. The total energy density is given by $\rho_I$ and the scale factor at that moment is $a_I = 1$. In addition, we assume that the $N$ SM-like sectors carry negligible energy. In our analysis, we do track the evolution of $\phi$; however, $\phi$ is short-lived and does not play any relevant role in our analysis. We then numerically solve the full set of Boltzmann equations for our setup, and the results are illustrated in Figure 4.4.

The left upper panel in Figure 4.4 shows the energy density evolution of the various particle species present in our Universe. At early times, $T \gg M_1$, the Universe is dominated by a relativistic admixture of $N$, $\phi$, and $S$ with $\rho \propto a^{-4}$. When $T < M_1$, $N_1$ becomes non-relativistic, behaving like matter ($\rho_N \propto a^{-3}$) and dominating the energy density of the Universe for some period. After the $N_1$ decays are complete, the energy density of the system is stored in the reheatsons that take over the evolution.
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\[ \rho_{N}, \rho_{S}, \rho_{R,SM} \]

\[ T = M_1 \]

\[ T = m_S \]

\[ 10^4, 10^8, 10^{12}, 10^{16}, 10^{20} \]

\[ 10^{-8}, 10^{0}, 10^{12}, 10^{22} \]

\[ 10^{-2}, 10^{11}, 10^{-15} \]

\[ 5 \times 10^2 \]

\[ a_I = 1 \]

\[ \frac{\Gamma_N}{H(a)} \]

**Figure 4.4:** Left upper panel: energy densities of the Majorana fermions, \( \rho_N \), in solid blue, reheatons, \( \rho_S \), in dashed magenta, and the SM thermal bath, \( \rho_{R,SM} \), in dashed green. Right upper panel: the evolution of the entropy per comoving volume as a function of the cosmic scale factor. Bottom panel: the ratio of the \( N \) decay width to the Hubble parameter as a function of the scale factor. In all panels, the vertical dashed black line corresponds to the time when \( T = M_1 \), whereas the black dotted line corresponds to the time when \( T = m_S \). The benchmark parameters for the three panels are: \( \Gamma_N = 10^{-2} \) GeV, \( M_1 = 10^{11} \) GeV, \( \Gamma_S = 10^{-15} \) GeV, \( m_S = 5 \times 10^2 \) GeV and \( a_I = 1 \).

As we can see from the right upper panel in **Figure 4.4**, the decay of \( N_1 \) and the \( S \) will inject entropy into the SM sector. The first increase in the entropy density occurs when \( N_1 \) decays, while the second one corresponds to the decay of \( S \). In the bottom panel, we can observe when \( N_1 \) departs from thermal equilibrium when \( \Gamma_N \lesssim H \).
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The tree level decay widths of the $N_i$ and the complex scalar, $\phi$, are given by:

$$\Gamma_{N_i} = \frac{y^2}{8\pi} M_i \left(1 + \frac{m_S^2 - m_\phi^2 - 2m_SM_i}{M_i^2}\right) \left[\left(1 - \frac{m_S^2 + m_\phi^2}{M_i^2}\right)^2 - \frac{4m_S^2m_\phi^2}{M_i^4}\right]^{1/2},$$

$$\Gamma_\phi = \frac{\kappa^2}{8\pi} m_\phi \left(1 - \frac{4m_S^2}{m_\phi^2}\right)^{3/2}.$$

(4.10)

To quantify departure from the thermal equilibrium of $N_1$, we define the ratio $K$:

$$K = \frac{\Gamma_{N_1}}{H(M_1)},$$

(4.11)

where we compare the decay width of the lightest Majorana fermion, $\Gamma_{N_1}$, with the expansion rate of the Universe, given by the Hubble parameter, $H(T)$, at a temperature $T \sim M_1$. The decay rate of $N_1$ is given in Eq. (4.10), whereas the Hubble parameter for the radiation domination era at $T = M_1$ is:

$$H(T_1 = M_1) = \sqrt{\frac{\pi^2 g_*(T_1)}{90}} \frac{M_1^2}{M_P}.$$

(4.12)

As we will see, our model requires a large lepton asymmetry, so we want to be in the weak washout regime, where $K \ll 1$. In order to agree with observations, the Hubble parameter at the end of inflation, $H_I$, must satisfy $H_I \lesssim 4 \times 10^{13} \left(\frac{r}{0.032}\right)^{1/2}$ GeV [132], with $r$ being the tensor-to-scalar ratio. This imposes an upper bound on the maximum $M_1$ that can be attained. Since $H(T) \lesssim H_I$, $T \lesssim 10^{16}$ GeV and, therefore, the largest mass for the $N_1$ is $M_1 \sim 10^{16}$ GeV.

The evolution of the yield of $N_1$, $Y_{N_1} = \frac{n_{N_1}}{s}$, where $n_{N_1}$ is the number density of $N_1$, can be tracked by the usual Boltzmann equations [133]. By defining $z = \frac{M_1}{T}$,
the equilibrium $N_1$ yield is (with $g_*(T) = g_{*,s}(T)$):

$$Y_{N_1}^{eq}(z) = \frac{90}{7\pi^4} z^2 K_2(z),$$

where $K_n(z)$ is the $n$-th order modified Bessel function of the second kind. In the weak washout regime, $K \ll 1$, the generated lepton asymmetry, $\mathcal{L}_I \equiv Y_L - Y_{L\bar{\nu}}$, is [133]:

$$\mathcal{L}_I \simeq \epsilon Y_{N_1}^{eq}(0) \delta_N^{-1} \simeq -\frac{3}{32\pi} \left(\frac{81}{256}\right)^{1/4} \Delta_1 K^{1/2} y^2,$$

assuming that $N_1$ has an initial thermal abundance, that is, $Y_{N_1}(z_i) = Y_{N_1}^{eq}(0)$, where $z_i = \frac{M_1}{T_i}$ and $T_i$ the initial temperature, with $z_i \sim 0$ corresponding to early times. $\Delta_1 \equiv M_1/M_2 \ll 1$ and $\delta_N^{-1}$ is the entropy dilution factor in Eq. (4.9) for $N_1$ decays. For simplicity, we consider two Majorana fermions and assume that the imaginary and real parts of the Yukawa type coupling $y_i$ are of the same order, such that

$$y^2 \equiv \text{Im}(y_1^i y_2) \approx \text{Re}(y_1^i y_2) \Rightarrow \frac{\text{Im}((y_1^i y_k)^2)}{(y_1^i y_{11})^2} = \frac{y^2}{2}.$$

In this work, we focus on the weak washout regime, since it renders a larger lepton asymmetry compared to other regimes.

### 4.3.3 Favourite Parameter Region

We now determine the favourite region of parameter space that satisfies all the observational constraints and allows the model to attain the observed value of the SM baryon asymmetry. Since in our scenario there is no direct coupling of the $N_i$ to the SM neutrinos, there are no bounds on their masses $M_i$ (for instance, the Davidson-Ibarra bound [134] does not apply here). Nevertheless, since our ultimate goal is to
explain baryogenesis in the SM sector, we have to take into account the initial lepton number asymmetry of the reheaton that is required for successful leptogenesis and the corresponding constraints that it can impose on our model.

To a first approximation, all the asymmetry in the reheaton sector is transferred to the SM, which we can use to set an upper bound on the lepton asymmetry $\mathcal{L}_I$, assuming that the sphaleron is very inefficient and that a small fraction of the lepton number is converted to baryon number. To get a lower bound on $\mathcal{L}_I$, we assume that SM sphaleron is maximally efficient, converting roughly $1/3$ of the lepton asymmetry into baryon asymmetry (see Section 4.5) and then requiring that all the SM baryon asymmetry comes from this mechanism. The upper bound on $\mathcal{L}_I$ can be obtained using Eq. (4.14). We can take $K = \Delta_1 = 10^{-1}, M_1 = 1 \times 10^{16}$ GeV as the largest values of those parameters. Using Eq. (4.10) and Eq. (4.11) for the width of $N_1$, this leads to an upper limit of $y \simeq 0.06$. Plugging those values in Eq. (4.14), we get:

$$3 \times 10^{-11} \lesssim |\mathcal{L}_I| \lesssim 3 \times 10^{-6}.$$  (4.16)

We can also express the weak washout bound ($K \ll 1$) in terms of the particle physics parameters using Eq. (4.11), such that the lower bound on $M_1$ is:

$$M_1 \gtrsim (3 \times 10^{11} \text{ GeV}) \left(\frac{y_1^2}{10^{-6}}\right) g_*^{-1/2},$$  (4.17)

where $y_1^2 = y_1^\dagger y_1$. In addition, we must ensure that the $N_1$ mass is larger than the sphaleron freeze-out temperature, $T_{sp} \sim 100$ GeV [135]. In what follows, we will explore how the various sectors are reheated.
4.4 Reheating N Sectors

In the previous section, we showed that the initial lepton number asymmetry carried by the reheaton can be generated by a leptogenesis-like setup. Eventually, the decays of $N_1$ and $\phi$ into $S$ (see Figure 4.2) will lead to a Universe dominated by reheatons. According to the $N_{\text{naturalness}}$ framework, the reheaton should couple universally to each sector [47]. Nevertheless, a large fraction of the energy density of the Universe must be transferred into the SM to reproduce observations. This can be achieved if the width of the reheaton into each sector decreases as $|m_{\tilde{H}}^2|$ grows, which can be accomplished with the introduction of a 4-th generation of vector-like leptons, as explained in Section 4.2. In this section, we explain how the reheating of the several sectors proceeds and show how the temperature evolves in these sectors, which will be important for computing the baryon asymmetries, as we discuss in Section 4.5.

4.4.1 Cosmological Evolution

The importance of the introduction of the vector-like leptons, as discussed in Section 4.2, is twofold. Firstly, the direct coupling of the reheaton to the neutrinos causes mixing in the standard sectors after electroweak symmetry breaking. The mixing can generate a freeze-in abundance of neutrinos in the standard sectors from the process $\nu\nu \to \nu_i\nu$, where $\nu$ are SM neutrinos and $\nu_i$ are neutrinos in the $i$-th-standard sector. This process would generate extra dark radiation, which puts an upper bound on the number of sectors (for further details see Section A.2). Secondly, the introduction of $L_4$ is necessary to control the reheaton’s branching ratio and give rise to enough matter content in the SM and exotic sectors. This in turn will provide a consistent picture of baryon asymmetry and dark matter in the Universe.
We assume that the vector-like leptons are heavier than the $S^3$ and compute the partial width of the $S$ to different sectors by integrating out the vector-like leptons. This generates an effective Lagrangian:

$$\mathcal{L}_{(H)\neq 0} \supset \frac{g}{\sqrt{2}} \frac{\lambda_{LL} v_i}{M_L^2} W^\mu_i \gamma^\mu P_L S^c,$$

$$\mathcal{L}_{(H)=0} \supset \frac{\lambda_{LR} M_L}{M} \tilde{H} P_R S^c,$$

where $P_L$ and $P_R$ are the left and right-handed projection operators, respectively. Therefore for the SM and standard sectors, the leading decay of the reheaton is given by $S \rightarrow W e$, while for the exotic sectors the leading decay is $S \rightarrow H e$. The partial decay widths are given by:

$$\Gamma_{S \rightarrow W e} = \frac{\lambda^2}{32\pi} \left( \frac{M^4_L}{m^4_S} \right) \left( 1 - \frac{m^2_W}{m^2_S} \right) \left( 1 + 2 \frac{m^2_W}{m^2_S} \right),$$

$$\Gamma_{S \rightarrow H_j e_j} = \frac{\lambda^2}{32\pi} \left( \frac{M^4_L}{m^4_S} \right) \left( 1 - \frac{M^2_H_J}{m^2_S} \right) \left( 1 - \frac{m^2_W}{m^2_S} \right),$$

$$\Gamma_{S \rightarrow W_k e_k} = \frac{\lambda^2}{32\pi} \left( \frac{M^4_L}{m^4_S} \right) \left( 1 - \frac{m^2_W}{m^2_S} \right) \left( 1 + 2 \frac{m^2_W}{m^2_S} \right),$$

where $m_W$ is the mass of the W-boson in the SM sector, and the indexes $j$ and $k$ label the exotic and standard sectors, respectively. Thus, the branching ratios into the $j$th exotic sector, $\beta_j$, and into the $k$th standard sector, $\gamma_k$, are:

$$\beta_j \simeq \frac{\Gamma_{S \rightarrow H_j e_j}}{\Gamma_{S \rightarrow W e}} = \left( \frac{M^2_L}{m^2_S} \right) \left( 1 - \frac{m^2_W}{m^2_S} \right) \left( 1 + 2 \frac{m^2_W}{m^2_S} \right),$$

$$\gamma_k \simeq \frac{\Gamma_{S \rightarrow W_k e_k}}{\Gamma_{S \rightarrow W e}} = \left( 1 - \frac{m^2_W}{m^2_S} \right) \left( 1 + 2 \frac{m^2_W}{m^2_S} \right) \left( 1 - \frac{m^2_W}{m^2_S} \right).$$

(4.19)
Here we have used the fact that the decay $S \rightarrow We$ into the SM sector dominates over all others, which implies that $\beta_j, \gamma_k \ll 1$.

In Figure 4.5, we illustrate the reheaton’s branching ratio for some benchmark reheaton’s masses for the exotic ($i = -1$, left panel) and standard ($i = +1$, right panel) sectors, as a function of the Higgs and the $W$-boson mass, respectively, for different $m_S$. In Section 4.5.3, we will show that the region of interest for the branching ratio into the first exotic sector is $10^{-2} \lesssim \beta_1 \lesssim 8 \times 10^{-2}$, with lower bound based on reproducing the correct relic abundance of dark matter and the upper bound imposed by $\Delta N_{\text{eff}} \lesssim 0.4$. This range of branching ratios requires that the masses of the Higgs in the $j = -1$ sector and the reheaton be numerically very close to each other as shown in the left panel of Figure 4.5. Similarly, branching ratios in the standard sectors need to satisfy $\gamma \lesssim 8 \times 10^{-2}$. Thus, the lower bound on the $W$ masses in the standard sectors is similar to the reheaton mass.

If the $W$ or $H$ is heavier than the reheaton, then the decay proceeds via a 3-
body process, which is not shown in Figure 4.5. For the lowest exotic sector \((i = -1)\), three body decays would give a branching ratio too small to give the right dark matter abundance. For sectors with heavier Higgs masses, three body decays are expected, and the branching ratio of the reheaton will be \(\lesssim 10^{-3}\). Similarly, for the standard sectors, they are all expected to be in the 3-body regime except the one with the lowest Higgs mass \((i = 1)\).

Let us consider the simplest case where the reheaton decays into two sectors only: the SM one and a hidden sector (that could be an “exotic sector”, with \(i < 0\), or a “standard sector”, with \(i > 0\)). The generalization for \(N\) sectors can be accomplished by replacing \(\beta \rightarrow \beta_i\) for the exotic sectors and \(\beta \rightarrow \gamma_i\) for standard sectors. The evolution of the energy density of the Universe is given by the following set of differential equations:

\[
\begin{align*}
\dot{\rho}_S + 3 (1 + \omega) H \rho_S &= -\Gamma_S \rho_S , \\
\dot{\rho}_{R,h} + 4H \rho_{R,h} &= \beta \Gamma_S \rho_S , \\
\dot{\rho}_{R,SM} + 4H \rho_{R,SM} &= (1 - \beta) \Gamma_S \rho_S , 
\end{align*}
\]

where \(\beta\) is the fraction of the reheaton that decays into the hidden sector, \(\Gamma_S\) is the total reheaton’s decay width, \(\rho_S, \rho_{R,SM}\) and \(\rho_{R,h}\) are the energy density of the reheaton, SM and hidden sectors, respectively. The equation of state parameter, \(\omega\), is defined by the ratio between the pressure and the energy density of the species (in this case, the reheaton), \(p = \omega \rho\), where \(\omega = 0\) if it is non-relativistic \((T \ll m)\), and \(\omega = 1/3\) if it is relativistic \((T \gg m)\). The Hubble rate, \(H\), can be written as:

\[
H^2 = \frac{1}{3M_P^2} (\rho_S + \rho_{R,SM} + \rho_{R,h}) ,
\]
with $M_P \simeq 2.43 \times 10^{18}$ GeV. The system of equations in Eq. (4.21) is solved numerically with the following set of initial conditions:

$$\rho_S (t_I) = 3 M_P^2 \Gamma_N^2 , \quad (4.23)$$

$$\rho_{SM} (t_I) = 0, \quad \rho_h (t_I) = 0 , \quad (4.24)$$

where $t_I$ corresponds to the time where the Majorana fermion $N_1$ has completely decayed. We assume that, at this stage, the energy density of the Universe is dominated by the reheaton, $S$, and $\phi$ and $N_i$ have decayed away. As we can see in the numerical analysis shown in Figure 4.4, Eq. (4.23) and Eq. (4.24) are relatively accurate estimates for the initial conditions for our system when $S$ starts to dominate, since $\rho_{SM} \ll \rho_S$.

The temperature of each sector can be tracked using the relation with the radiation energy density:

$$\rho_{R,i} = \frac{\pi^2}{30} g_{*,i} (T_i) T_i^4 \Rightarrow T_i = \left( \frac{30}{\pi^2 g_{*,i} (T_i)} \right)^{1/4} \rho_{R,i}^{1/4} . \quad (4.25)$$

These formulas will also apply to the reheaton as long as $T_S > m_S$. Once the temperature of the reheaton equals its mass, we model the equation of state as dropping instantaneously from $1/3$ to $0$. The decay of the reheaton is complete when its decay width is comparable to the Hubble parameter, $H \sim \Gamma_S$. Here, the radiation energy density starts to dominate the Universe, and the reheating temperature in each sector can be estimated as:

$$T_{RH,i} = \left( \frac{90}{\pi^2 g_* (T)} \right)^{1/4} \sqrt{\text{BR}(S \to i) \Gamma_S M_P} . \quad (4.26)$$
The reheating temperature is generally defined as the temperature of the thermal bath assuming an instantaneous transfer of the reheaton’s energy density into radiation, initiating the radiation-dominated phase of the Universe below which the Universe expands with $T \sim a^{-1}$. However, the reheating phase is a non-instantaneous process where the expansion of the Universe is faster compared to the radiation domination and it could attain a temperature much larger than $T_{RH}$. During such period, $t < \Gamma_S^{-1}$, the decays of the reheaton provide a continuous supply of entropy [116]. For simplicity, in this section, we treat the masses and branching ratios as constant and ignore thermal effects which are important at early times. However, since the reheaton decays mostly after the electroweak scale, these thermal effects which are presented in details in Section A.3 do not change our final result.

In the SM, there is a lower bound on the reheating temperature of $T_{RH,SM} > 4.1$ MeV at 95% confidence level imposed by BBN [137]. This places a lower limit on the reheaton’s width to be roughly $\Gamma_S \gtrsim 7 \times 10^{-24}$ GeV. This turns out to be a much weaker constraint than the one imposed by requiring the correct baryon asymmetry. Consideration of the latest cosmological bounds on $\Delta N_{\text{eff}}$ leads to an upper limit on the reheaton’s width, see Section 4.5 for a more detailed discussion.

In order to facilitate numerical computations, we express Eq. (4.21) in terms of dimensionless variables and replace time derivatives to derivatives with respect to the scale factor $a$ (using the definition of the Hubble parameter, $da = H a dt$) [116]. We can choose then:

\[
S \equiv \begin{cases} 
\rho_S a^4, & \text{if } \omega = \frac{1}{3} \\
\rho_S a^3, & \text{if } \omega = 0
\end{cases}, \tag{4.27}
\]

\[
R_{SM/\hbar} = \rho_{R_{SM/\hbar}} a^4, \tag{4.28}
\]
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\[ A \equiv \frac{a}{a_I} = a \Lambda, \quad (4.29) \]

where \( a_I \) is the initial value of the scale factor for the numerical integration. Since the results cannot depend on the choice of the scale \( a_I \), we assume that \( a_I^{-1} \) is equal to some \( \Lambda \), which we set to be \( \Lambda = 1 \) GeV in our numerical computation. Thus, with this change of variables, the initial conditions in Eq. (4.23) and Eq. (4.24) become:

\[ S(A_I) \equiv S_I = 3 M_P^2 \frac{\Gamma_N^2}{\Lambda^4} A_I^3, \quad \text{with} \quad A_I = 1, \]
\[ R_{SM}(A_I) = 0, \]
\[ R_h(A_I) = 0. \quad (4.30) \]

As noted above, we model the equation of state \( \omega \) as a step function going from \( 1/3 \) to 0 when \( T_S = m_S \). In Figure 4.6, we show the temperature evolution of the SM sector (solid line) and a hidden sector (dashed line), as a function of the scale factor, for a fixed reheaton decay width and \( \beta \).

It is possible to perform an analytical estimate for \( T_{\text{max}} \) and the dependence of the temperature on the scale factor in the range \( T_{\text{RH}} < T < T_{\text{max}} \). At early times \( (H \gg \Gamma_S) \), we can approximate the Boltzmann equation for \( R_{SM/h} \) by assuming that a large fraction of the energy density is still retained by the reheaton, that is \( S \sim S_I \). Thus, the equation for \( R_{SM/h} \) can be solved analytically and we are left with:

\[ R_{\text{i}} \simeq \sqrt{3} M_P \frac{\Gamma_S}{\Lambda^2} \text{BR}(S \rightarrow i) S_I^{1/2} \cdot \begin{cases} \frac{1}{2} \left( A^2 - A_I^2 \right), & \text{if } \omega = \frac{1}{3} \\ \frac{2}{5} \left( A^{5/2} - A_I^{5/2} \right), & \text{if } \omega = 0 \end{cases}, \quad (4.31) \]

where \( A_I \) and \( S_I \) in the second line are the scale factor and scaled energy density of the \( S \) when the \( S \) becomes non-relativistic. By substituting Eq. (4.31) in the expression
for the temperature in Eq. (4.25), we can see that

\[
T \sim \begin{cases} 
  a^{-1/2}, & \text{if } \omega = \frac{1}{3}, \\
  a^{-3/8}, & \text{if } \omega = 0 
\end{cases},
\]  

(4.32)

for both SM and hidden sectors, whereas the maximum temperature achieved is:

\[
T_{\text{max},i} \sim \left( M_i^2 \Gamma_N \text{BR}(S \to i) \Gamma_S \right)^{1/4},
\]

(4.33)

where \( T_{\text{max}} \gtrsim \mathcal{O}(10^2 \text{ GeV}) \). This behaviour is confirmed in Figure 4.6. At very early times when the reheaton is relativistic, after attaining a maximum, the temperature decreases with scale factor as \( T \sim a^{-1/2} \). Once the reheaton becomes non-relativistic, the temperature scales as \( a^{-3/8} \). At \( T_{\text{RH}} \), which marks the change of the slope in

\[
\begin{array}{c}
  \text{Figure 4.6: The evolution of the temperature in the SM sector (solid line) and a} \\
  \text{hidden sector (dashed line), as a function of the scale factor, } a, \text{ with } \Gamma_S = 10^{-16} \text{ GeV}, \\
  \beta = 0.01, \text{ and } \Gamma_N = 10^{-2} \text{ GeV. The green horizontal line corresponds to } T = 100 \\
  \text{GeV}. \text{On the top of the lines, we write the relation between temperature and the scale} \\
  \text{factor in each portion of the plot.}
\end{array}
\]
the plot, the decay of the reheaton is complete and there is no more entropy to be injected. From that point on, the usual radiation era commences and the temperature follows \( T \sim a^{-1} \) as one would expect. We also see that throughout the evolution, the ratio of the temperature of the SM to the hidden sector remains approximately constant.

In the next section, we will make use of the evolution of the temperature to compute the baryon asymmetry generated in the SM sector but before that let us examine some possible constraints on our model.

### 4.4.2 \( \Delta N_{\text{eff}} \) Constraints

The existence of \( N \) sectors leads to a large number of nearly massless degrees of freedom, given that all sectors contain photons and neutrinos. The presence of these extra relativistic degrees of freedom can have observational effects as they modify the expansion of the Universe, which can lead to changes in predictions for light elements or the CMB. These contributions are encoded in the effective number of neutrino species, \( N_{\text{eff}} \), which, for a completely decoupled hidden sector can be written as [138,139]:

\[
N_{\text{eff}} = N_{\text{eff}}^{\text{SM}} + \Delta N_{\text{eff}} = N_{\text{eff}}^{\text{SM}} + \frac{4}{7} \left( \frac{11}{4} \right)^{4/3} g_h \left( \frac{T_h}{T_{\gamma}} \right)^4,
\]

where \( T_h \) and \( T_{\gamma} \) are the hidden and SM photon temperatures, respectively. In Eq. (4.34), \( g_h \) is the number of effective degrees of freedom (d.o.f) in the hidden sector and \( N_{\text{eff}}^{\text{SM}} \) is the effective d.o.f of the SM neutrinos, \( N_{\text{eff}}^{\text{SM}} = 3.046 \) [140]. The current 95% confidence level constraints from 2018 Planck data, considering the Cosmic Microwave Background (CMB) and the baryon acoustic oscillations (BAO) measurements, lead to \( N_{\text{eff}} = 2.99^{+0.34}_{-0.33} \) [18]. In what follows, the hidden sector parameters (that can be
“standard sectors” with $i > 0$, or “exotic sectors” with $i < 0$) are denoted by the subscript $h$. In order to determine the precise contribution to $\Delta N_{\text{eff}}$, we need to track the ratio of temperatures between the hidden and the SM sectors, $\xi \equiv \frac{T_h}{T_\gamma}$. As we described in Section 4.4.1, the temperatures of the SM and hidden sectors are, in general, different. Since after $T_{\text{RH}}$ the entropy in each sector is conserved, the following expression holds:

$$\frac{T_i^{\text{Dec}}}{T_i^{\text{RH}}} = \left[ \frac{g^h_i(T_{\text{RH}})}{g^i(T_{\text{Dec}})} \right]^{1/3} \frac{a(T_{\text{RH}})}{a(T_{\text{Dec}})},$$

(4.35)

where $T_{\text{Dec}}^h$ stands for the temperature of the hidden sector at the time of the CMB. Hence, the ratio between the temperatures of the photon decoupling in the hidden and SM sectors is given by:

$$\frac{T_{h\text{Dec}}}{T_{\text{SMDec}}} = \left( \frac{T_{\text{RH}}}{T_{\text{RH}}} \right)^4 \left( \frac{g^h_i(T_{\text{RH}})}{g^i(T_{\text{Dec}})} \right)^{1/3} \left( \frac{g^\text{SM}(T_{\text{SM}})}{g^h_i(T_{\text{Dec}})} \right)^{1/3}.$$  

(4.36)

As we have discussed in Section 4.4.1, the reheaton deposits its energy into the various sectors. The ratio between the energy densities of the hidden and the SM sectors is, then:

$$\frac{\rho_h}{\rho_{\text{SM}}} = \frac{g^h_i(T_{\text{RH}})}{g^i(T_{\text{Dec}})} \left( \frac{T_{\text{RH}}}{T_{\text{RH}}} \right)^4 \left( \frac{g^\text{SM}(T_{\text{SM}})}{g^h_i(T_{\text{Dec}})} \right)^{1/3} \frac{\Gamma(S \rightarrow \text{Hidden})}{\Gamma(S \rightarrow \text{SM})}.$$  

(4.37)

Using the relations obtained in Eq. (4.36) and Eq. (4.37), we get:

$$\left( \frac{T_{h\text{Dec}}}{T_{\text{SMDec}}} \right)^4 = \left( \frac{g^h_i(T_{\text{RH}})}{g^i(T_{\text{Dec}})} \right)^{1/3} \left( \frac{g^\text{SM}(T_{\text{SM}})}{g^h_i(T_{\text{Dec}})} \right)^{4/3} \frac{\Gamma(S \rightarrow \text{Hidden})}{\Gamma(S \rightarrow \text{SM})}.$$  

(4.38)

\(^4\)The hidden sector quantities are computed at the time of the SM. For example, $T_{h\text{Dec}}$ is the temperature of the hidden sector at the $T_{\text{Dec}}^\text{SM}$.\]
Combining Eq. (4.38) with the expression for $\Delta N_{\text{eff}}$ from Eq. (4.34), the contribution from the additional sectors is:

$$\Delta N_{\text{eff}} = \frac{4}{7} \left( \frac{11}{4} \right)^{4/3} g_\star^h(T_{\text{Dec}}^h) \left[ \frac{g_\star^h(T_{\text{RH}}^h)}{g_\star^\text{SM}(T_{\text{RH}}^\text{SM})} \right]^{1/3} \left[ \frac{g_\star^\text{SM}(T_{\text{Dec}}^\text{SM})}{g_\star^h(T_{\text{Dec}}^h)} \right]^{4/3} \frac{\Gamma(S \to \text{Hidden})}{\Gamma(S \to \text{SM})}. $$

(4.39)

The computations of $\Delta N_{\text{eff}}$ depend on the particle content of the various sectors and their respective mass spectrum, see Section A.1. Consequently, the contribution to $\Delta N_{\text{eff}}$ from any given exotic sector ($j < 0$) due to the reheaton’s decay can be expressed as

$$\Delta N_{\text{eff,}\,j}^{\text{Decay}} \simeq 7.4 \left[ \frac{g_\star^j(T_{\text{RH}}^j)}{g_\star^\text{SM}(T_{\text{RH}}^\text{SM})} \right]^{1/3} \left( \frac{\beta_j}{1 - \beta - \gamma} \right), $$

(4.40)

where $\beta_j$ denotes the branching ratio into the exotic sector $j$, $\beta = \sum_j \beta_j$ is the total reheaton’s branching ratio into the exotic sectors, and $\gamma = \sum_k \gamma_k$ parameterizes the reheaton’s total branching ratio into the standard sectors, $g_\star^j(T_{\text{RH}}^j) \sim 102.75$, and we took $g_\star^\text{SM}(T_{\text{Dec}}^\text{SM}) \sim 3.36$, and $g_\star^j(T_{\text{Dec}}^j) \sim 17.75$.

The $\Delta N_{\text{eff}}$ contribution is plotted in Figure 4.7 as a function of the fraction of reheatons decaying into the lowest exotic sector, $\beta_{-1}$. We may conclude that $\Delta N_{\text{eff}} \lesssim 0.4$ can be obtained for a branching fraction $\beta_{-1} \lesssim 0.08$, and that changing the reheaton’s width for various $\beta$ values alter $\Delta N_{\text{eff}}$ very slightly. Sectors with larger Higgs mass parameters have significantly smaller contributions to $\Delta N_{\text{eff}}$ because the reheaton partial width scales as $\Gamma_{m_H^2 > 0} \sim 1/m_{h_i}^4$. For larger $i$, the decay is both three-body and loop suppressed. Hence, we just need to compute the effects coming from the sectors closest to the SM.
Figure 4.7: The $\Delta N_{\text{eff}}$ as a function of the branching ratio to the lowest exotic sector for two different reheaton widths: $\Gamma_S = 10^{-12}$ GeV (dashed cyan curve) and $\Gamma_S = 10^{-17}$ GeV (solid magenta curve). $\Delta N_{\text{eff}}$ changes little when varying the reheaton’s decay width.

It is interesting to explore the $\Delta N_{\text{eff}}$ constraints from the standard sectors as well. In particular, one needs to consider two main contributions. Firstly, the decay of the reheatons into the standard sector, following Eq. (4.39), is

$$
\Delta N_{\text{eff,k}}^{\text{Decay}} \simeq 7.4 \left[ \frac{g^k(T^k_{\text{Dec}})}{g^\text{SM}(T^\text{SM}_{\text{RH}})} \right]^{1/3} \left( \frac{\gamma_k}{1 - \beta - \gamma} \right),
$$

where $g^k(T^k_{\text{Dec}}) \sim 3.36$, which also gives $\gamma_1 \lesssim 0.08$. Analogous to the exotic sectors case, the contribution to $\Delta N_{\text{eff}}$ is dominated by the sector whose Higgs mass squared parameter has the smallest absolute value, as the width scales as $\Gamma_{m_h^2 < 0} \sim 1/m_h^2$ for the first few sectors, while for larger $i$ the decay is both three body and loop suppressed.

In the standard sectors, the Higgs vev induces a small mixing between the neutrinos in the hidden sector and the SM neutrinos. This can lead to freeze-in (FI) production of hidden sector neutrinos in the early Universe. The process $\nu \nu \rightarrow \nu_i \nu$
also requires the mixing of SM neutrinos with the heavy vector-like neutrino, and thus it is very suppressed. It can be approximated as

\[ \Delta N_{\text{eff},k} \simeq (3.7 \times 10^{-13}) \left( \frac{\beta}{10^{-2}} \right)^{99/12} \left( \frac{10^{-9} \text{ GeV}}{m_k} \right)^{8/3} \left( \frac{10^3 \text{ GeV}}{M_L} \right)^{16/3} \left( \frac{2k + r}{r} \right)^{4/3}, \]

(4.42)

where \( m_k \) is the mass of the standard sector neutrinos and this assumes that both the observed dark matter relic abundance and the baryon asymmetry are attained with the detailed computation shown in Section A.2. This is of course negligible compared to current and future bounds, but in Section A.2 we also show that a model where the reheaton couples directly to the SM lepton doublets is excluded by this process.

### 4.5 Baryon Asymmetry

In this section, we describe how the baryon asymmetry is generated in various sectors. In Section 4.3, we presented a framework of how the lepton asymmetry can be produced in our model, and in Section 4.4 we showed how that lepton asymmetry is distributed to the different sectors. That lepton asymmetry is then reprocessed by the electroweak sphalerons [80, 112, 113]. In the SM, classically \( U(1)_{B+L} \) is an accidental global symmetry but it is violated quantum mechanically due to quantum anomalies and \( SU(2)_L \) field configurations [12, 141, 142]. The \( (B + L) \) violation allows the electroweak sphaleron to partially convert the lepton asymmetry, carried by the reheaton, into the baryon asymmetry distributed across the various sectors since anomalous electroweak processes conserve the difference between baryon (B) and lepton numbers (L). For the SM field content, assuming the sphaleron interactions are rapid and chemical equilibrium is maintained, the baryon number can be expressed as \( B = \frac{28}{79} (B - L) \) [143]. However, around the electroweak symmetry breaking phase
transition, the sphaleron rate becomes suppressed, complicating the relationship be-
tween the baryon number and the conserved quantity \((B - L)\) in a manner that will
be described here.

### 4.5.1 Standard Sectors

Mathematically, the sphaleron is a static saddle point solution of the field equations
and the *sphaleron rate* per unit time per unit volume, \(\Gamma_{\text{diff}}\), which is half the Chern-
Simons diffusion rate.\(^5\) At high temperatures well above the electroweak scale, the
sphaleron occurs via thermal hopping and there is no suppression. The rate can be
estimated from lattice simulations \([144,145]\):

\[
\Gamma_{\text{diff}}(T) = (25.4 \pm 2.0) \alpha_w^5 T^4.
\]

and will always be larger than the Hubble parameter.

At low temperatures, \(T \ll 100 \text{ GeV}\), the transition amplitude between the
various vacua is highly suppressed (being proportional to a quantum-tunneling factor
\(e^{-8\pi^2/g^2} \sim 10^{-173}\), where \(g\) is the \(SU(2)_L\) gauge coupling). Here we are interested in
the intermediate regime near the temperature of the electroweak phase transition. In
the SM, the phase transition is a crossover \([146,147]\), and the Higgs vev is continuous in
time/temperature. Defining \(T_c\) as the temperature where the Higgs vev first becomes

\(^5\)The Chern-Simons diffusion rate is defined in terms of the winding number as:

\[
\Gamma_{\text{diff}} \equiv \lim_{V, t \to \infty} \frac{\langle (N_{\text{CS}}(t) - N_{\text{CS}}(0))^2 \rangle}{V t},
\]

where \(N_{\text{CS}}(t) - N_{\text{CS}}(0) = \frac{1}{n_G} (B(t) - B(0)) = L_i(t) - L_i(0)\) and \(V\) is the volume of space.
non-zero, the Higgs vev and $W$ mass at temperatures below $T_c$ can be written as [148]:

\[
v^2(T) = v^2(0) - \left[ \frac{1}{2} + \frac{3g^2}{16\lambda} \right] T^2,
\]

\[
m_W(T) = m_W(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^{1/2},
\]

(4.45)

where $\lambda$ is the Higgs quartic coupling and $m_W(0)$ is the $W$ boson mass at zero temperature. For the SM, $T_c \sim 159$ GeV [135], and the formulas are the same for the standard sector with larger values of $v(0)$ and $T_c$.

The sphaleron rate for $T < T_{c,i}$ can be estimated as [148]:

\[
\Gamma_{\text{diff}}(T) = 4T^4 \frac{\omega_\perp}{g v} \left( \frac{\alpha_w}{4\pi} \right)^4 \left( \frac{4\pi v}{gT} \right)^7 N_{\text{tr}}(N\chi)_{\text{rot}} \kappa \exp \left( -\frac{E_{\text{Sp}}}{T} \right),
\]

(4.46)

where the sphaleron energy $E_{\text{Sp}} \simeq 4 m_W(T)/\alpha_w$ [80–83], $\omega_\perp$ is called the dynamical prefactor, $|\omega^2| \simeq 2.3 m_W^2$ [149], $N_{\text{tr}}(N\chi)_{\text{rot}} \sim 26 \times 5.3 \times 10^3$ are the normalization factors related to the zero-modes of the fluctuation operator around the sphaleron solution [148,150,151] and $\kappa$ is an $O(1)$ numerical factor [83,152]. This approximation is valid in the region $m_{W,i}(T) \ll T \lesssim T_{c,i}$. A simple comparison of the sphaleron rate with the Hubble parameter ($\Gamma_{\text{Sp}} \sim H$) shows that the sphaleron freezes out around $T \sim 131$ GeV and, below this, leptons are no longer converted into baryons. A rapid decrease in the sphaleron rate occurs around the EW phase transition as it becomes exponentially suppressed when the Higgs acquires a vev.

Defining the conserved global charge as

\[
X \equiv B - L,
\]

(4.47)

where $L$ is the total lepton and $B$ is the baryon number ($B \equiv \frac{n_B - n_{\bar{B}}}{s}$, $L \equiv \frac{n_l - n_l}{s}$). For
the sphaleron in chemical equilibrium at a temperature $T$, these numbers are \([153]\):

$$B_{\text{eq}} \equiv \chi \left( \frac{v}{T} \right) X, \quad L_{\text{eq}} \equiv B_{\text{eq}} - X,$$

$$\chi(x) = \frac{4 \left[ 5 + 12n_G + 4n_G^2 + (9 + 6n_G) x^2 \right]}{65 + 136n_G + 44n_G^2 + (117 + 72n_G) x^2},$$

where $n_G = 3$ is the number of generations. In the case of a deviation from the chemical equilibrium, with a given source for leptons $F(t)$, the total lepton and baryon numbers evolve as \([31,32,153,154]\):

$$\dot{B}(t) = -\Upsilon(t) \left[ B(t) + \eta(t)L(t) \right],$$

$$\dot{L}(t) = -\Upsilon(t) \left[ B(t) + \eta(t)L(t) \right] + F(t),$$

where

$$\Upsilon \equiv n_G^2 \rho \left( \frac{v}{T} \right) \left[ 1 - \chi \left( \frac{v}{T} \right) \right] \frac{\Gamma_{\text{diff}}(T)}{T^3}, \quad \eta \equiv \frac{\chi \left( \frac{v}{T} \right)}{1 - \chi \left( \frac{v}{T} \right)},$$

$$\rho(x) = \frac{3 \left[ 65 + 136n_G + 44n_G^2 + (117 + 72n_G) x^2 \right]}{2n_G \left[ 30 + 62n_G + 20n_G^2 + (54 + 33n_G) x^2 \right]}.$$

To obtain the baryon asymmetry in the various sectors, we need to know the lepton source function $F(t)$. In our model, the total lepton asymmetry is carried by the reheaton (S) at early times and can be tracked using Boltzmann equations, assuming the decay of the reheaton into two particles that rapidly reach thermal equilibrium with each other. The decay products of the reheaton are SM particles and we have explored the branching ratios in Section 4.4. At an early epoch, reheatons will be in thermal equilibrium with each other, so the evolution of their number density
can be written as:

\[
\begin{align*}
\dot{n}_S + 3Hn_S &= -\langle \Gamma_S \rangle (n_S - n_S^{eq}) , \\
\frac{dY_S}{dt} &= -\langle \Gamma_S \rangle (Y_S - Y_S^{eq}) ,
\end{align*}
\] (4.51)

where \( Y_S \equiv \frac{n_S}{s} \) is the comoving number density, \( \langle \Gamma_S \rangle = \frac{K_1(m_S/T)}{K_2(m_S/T)}\Gamma_S \), \( K_1(m_S/T) \) and \( K_2(m_S/T) \) are the modified Bessel functions of order 1 and 2 respectively. Defining \( \mathcal{L} \equiv Y_S - \bar{Y}_S \), we can subtract the equations for \( S \) and \( S^\bar{} \)

\[
\frac{d\mathcal{L}}{dt} = -\langle \Gamma_S \rangle \mathcal{L} ,
\] (4.52)

with the boundary condition \( \mathcal{L}(t_I) \equiv \mathcal{L}_I \), and \( \mathcal{L}_I \) satisfying the bound in Eq. (4.14). Thus, the lepton number source function can be deduced from Eq. (4.49) and Eq. (4.52), yielding:

\[
\begin{align*}
F(t)_0 &= -\langle \Gamma_S \rangle (1-\beta-\gamma) \mathcal{L} , \\
F(t)_j &= -\langle \Gamma_S \rangle \beta_j \mathcal{L} , \\
F(t)_k &= -\langle \Gamma_S \rangle \gamma_k \mathcal{L} .
\end{align*}
\] (4.53)

As before \( \beta (\gamma) \) is the sum of the branching ratios to all exotic (standard) sectors. Using the source function in Eq. (4.53), one can compute the baryon asymmetry as a function of time for a given reheaton width. We are interested in the late time behaviour of the baryon asymmetry. In Figure 4.8, we represent the BAU as a function of the reheaton width, for the SM \((i=0)\) and \(i=1\) sectors, displayed in the left and right panels, respectively, for different branching ratios \((\beta_{-1} = 10^{-3}\) in dashed green, \(\beta_{-1} = 10^{-2}\) in solid red, and \(\beta_{-1} = 10^{-1}\) in dashed magenta). The horizontal dot-dashed lines corresponds to the lowest exotic sector \((i = -1)\) with magenta, red, and green denoting \(\beta_{-1} = 10^{-1}, \beta_{-1} = 10^{-2}, \) and \(\beta_{-1} = 10^{-3}\) respectively. The SM
curve is proportional to $1 - \beta - \gamma \approx 1$, while the $i = 1$ curves are proportional to $\gamma_1$. We can clearly observe that the conversion of lepton into baryon asymmetry is less efficient for sectors $i \geq 1$ rather than for $i = 0$. This is primarily due to the fact that $m_{W,i} > m_{W,0}$, $v_i > v_0$, and $T_{RH,i} < T_{RH,0}$ for $i \geq 1$. The modification in these parameters change the sphaleron rate which, in turn, alter the baryon asymmetry that is produced.

### 4.5.2 Exotic Sectors

The baryon asymmetry generated in the exotic sectors, $i < 0$, are dramatically different from the $i > 0$ sectors due to a combination of factors: 1) electroweak symmetry will be broken by the QCD confinement at a much lower temperature around $T \approx 89$
MeV [126]; 2) the reheating temperatures in those sectors are large compared to the masses of the particles that live there; 3) the baryon asymmetry is generated very efficiently by the sphalerons due to the low electroweak scale, permitting the sphalerons to be active much longer. In these sectors, the phase transition is also qualitatively different because there are six massless quarks, so it is expected to be first order [122,155]. Therefore we model the phase transition as a step function:

\[ v \to \Lambda_{\text{QCD}} = \begin{cases} 0, & T > T_c \\ 89 \text{ MeV}, & T \lesssim T_c \end{cases} \]

(4.54)

The calculated baryon asymmetry in the exotic sectors does not depend on how exactly the phase transition is modelled.

One may worry that there will be baryon or lepton number violation during this first order phase transition. This turns out not to be a problem because there is not enough $CP$-violation for such processes. The $CP$ violation can be written in terms of the Jarlskog invariant [96–98] and goes like $\sim \prod \Delta m^2 / T_c^{12}$. Since the quarks are much lighter than in the SM sector, this dimensionless quantity is $\sim 10^{-103}$, so these processes during the phase transition are negligible.

Because the phase transition in the exotic sector is much lower than the reheating temperature from reheaton decays, the sphaleron reaches equilibrium and the baryon asymmetry can be parametrically larger than in the SM sector, even though the reheating temperature is lower. The behaviour of the baryon asymmetry in the exotic sector is illustrated in the left panel in Figure 4.8 by the horizontal dot-dashed lines, for different branching ratios ($\beta_{-1} = 10^{-3}$ in green, $\beta_{-1} = 10^{-2}$ in red and $\beta_{-1} = 10^{-1}$ in magenta). We can see that in the $i = -1$ sector the baryon asymmetry is much larger compared to the $i = 1$ case, as shown in the right panel of Figure
4.8, in particular, for smaller reheaton’s widths. This enhancement of the baryon asymmetry in the exotic sector is due to the sphaleron being active well below the SM electroweak scale for a longer duration. Thus in this model, the dark matter can be the lightest baryon of the exotic sectors, dominated by \( i = -1 \). The baryon asymmetry in the \( i = -1 \) sector is independent of the reheaton decay width because the reheating temperature is well above the sphaleron freeze-out temperature so the sphaleron reaches equilibrium for all the allowed values of \( \Gamma_S \).

We can give an analytic estimate of the behaviour of the baryon symmetry across sectors shown in Figure 4.8 (a) and Figure 4.8 (b). This can be obtained from Eq. (4.49) with the initial conditions \( B(t_I) = L(t_I) = 0 \). By looking at Eq. (4.48) and Eq. (4.50), we can state that to a reasonable approximation \( \eta(t) \) is essentially constant over time. By assuming a step function for \( \Upsilon(t) \), the approximate solution is:

\[
B(t) \sim -\frac{\eta}{1+\eta} \int F(t) dt ,
\]

where \( F(t) \) is given in Eq. (4.53), \( \frac{n}{1+n} \approx 0.342 \) and \( \int_{t_I}^{t_f} F(t) dt = \mathcal{L}(t') - \mathcal{L}(t_I) \). Finally, there must also be a factor of either \( \beta, \gamma \), or \( (1 - \beta - \gamma) \) in numerator depending on the sector under consideration. Consequently, the baryon asymmetry in the standard and the exotic sectors, in the relevant region of the parameter space, can be approximately parameterized as

\[
\frac{B_j}{\mathcal{L}_I} \approx \begin{cases} 
\left( \frac{\Gamma_S}{5.4 \times 10^{-14} \text{ GeV}} \right)^2 \frac{\eta}{1+\eta} \frac{\gamma_1}{\delta_S^{-1}}, & \text{For } j = 1 \\
\left( \frac{\Gamma_S}{4 \times 10^{-14} \text{ GeV}} \right)^2 \frac{\eta}{1+\eta} (1 - \beta - \gamma) \frac{\delta_S^{-1}}, & \text{For } j = 0 \\
\frac{\eta}{1+\eta} \beta_1 \frac{\delta_S^{-1}}, & \text{For } j = -1
\end{cases}
\]

where \( \mathcal{L}_I \) is the initial lepton asymmetry with \( |\mathcal{L}_I| \lesssim 3 \times 10^{-6} \), as obtained in Eq. (4.16)
and $\delta_{S}^{-1}$ is the entropy dilution factor in Eq. (4.9) from the reheaton decay. The baryon asymmetry in the exotic sector is independent of $\Gamma_{S}$ because sphalerons have enough time to reach chemical equilibrium in those sectors unlike in the $i > 0$ sectors. The upper bound on the initial lepton asymmetry places a lower bound on the reheaton width while using the current $\Delta N_{\text{eff}} \simeq 0.4$ measurements [18] imposes an upper bound on the width. Thus, the reheaton’s width lies in the following window,

$$
6.2 \times 10^{-16} \text{ GeV} \lesssim \Gamma_{S} \lesssim 1.4 \times 10^{-14} \text{ GeV}.
$$

(4.57)

It was argued in [121] that in exotic sectors there will be no late time baryon asymmetry. This is because, after the QCD/EW phase transition, the lightest baryon is much heavier than the temperature, so the sphaleron reaction is strongly biased toward baryons’ destruction. The rate of B-violation, in Eq. (4.48), can be approximated as:

$$
\frac{1}{B} \frac{dB}{dt} \simeq -\Gamma_{Sp}(T),
$$

(4.58)

where $\Gamma_{Sp}(T) \sim \Gamma_{\text{diff}}/T^3$ from Eq. (4.46), with $m_{W,i}^2(T_i) = \left(\frac{g}{2}\right)^2 f_{\pi}^2$ [126] with $f_{\pi}$ being the pion decay constant, and the vev replaced as in Eq. (4.54). Right after the phase transition, the sphaleron is exponentially suppressed, $\Gamma_{Sp} \sim e^{-f_{\pi}/(gT_{c})}$, but the question remains if it is smaller than the Hubble rate which is Planck suppressed. One could estimate $T_{c}$ by rescaling the SM value by the ratio $\Lambda_{\text{QCD},i}/\Lambda_{\text{QCD},0}$ to compute the sphaleron, and by that method, one confirms the conclusions of [121] that the sphaleron is fast enough to wash out any baryon asymmetry. We note, however, that the phase transition of the exotic sector is qualitatively different from that of the SM, and there is no reason to expect the non-perturbative parameters of $f_{\pi}$ and $T_{c}$ to be the same as the SM. Furthermore, the sphaleron is exponentially sensitive to
Figure 4.9: The $SU(2)_L$ gauge coupling constant, $g$, as a function of the ratio of pion decay constant over temperature, $f_\pi / T_c$, with contours corresponding to $\Gamma_{\text{Sp}}(T_c) = H(T_c)$ for some fixed pion decay constants ($f_\pi = 93$ MeV in solid blue, $f_\pi = 25$ MeV in solid orange). The black star represents the SM value (with $g \simeq 0.65$ and $f_{\pi,0} / T_{c,0} \approx 0.54$). In the region below the curves, baryon washout is frozen out and the hidden sector is a viable dark matter model.

that ratio, so small changes can alter the qualitative picture. If $f_\pi / T_c \gtrsim 1.5$, then the sphaleron rate after the phase transition is always smaller than Hubble and there is no washout. This ratio can ultimately only be calculated with non-perturbative methods such as the lattice. The critical temperature of the QCD phase transition of the SM (considering two flavours only), determined using QCD lattice simulations, is $T_{c,0} \simeq (172 \pm 3)$ MeV [156,157]. However, its precise value for a larger number of flavours is fairly ambiguous. In the literature, the value varies significantly so there is a large uncertainty associated with the six flavours $T_c$ [158–162].

We also note that if $SU(2)_L$ gauge coupling $g$ in the $i = -1$ sector is smaller than the SM value of $g$, the sphaleron rate is also suppressed. While the simplest
Nnaturalness setup has $g$ equal in all sectors, this is not necessary for the solution to the hierarchy problem. Therefore, the exotic sectors can more easily suppress this freeze-out process and accommodate dark matter if $g$ is reduced. These results are summarized in Figure 4.9 where we show contours for $\Gamma_{sp}(T_c) = H(T_c)$ in the $f_\pi/T_c$ vs. $g$ plane. The region below the curve will not have any baryon washout.

### 4.5.3 Dark Baryon Relic Abundance

The exotic sectors can have large baryon asymmetries, so the lightest baryon in the exotic sectors is a stable dark matter candidate which can provide the observed relic abundance. From observations [18], the current DM relic abundance is:

$$\Omega h^2 = 0.120 \pm 0.001 \text{ , and } h = 0.673.$$  \hspace{1cm} (4.59)

We focus on the dark neutron of the $i = -1$ to serve as a DM candidate. The masses of the nucleons in these sectors, $m_{N,i}$, are linearly proportional to the confinement scale (neglecting the constituent mass effects) [163] and can be written as:

$$m_{N,i} \simeq m_{N,0} \frac{\Lambda_{QCD,i}}{\Lambda_{QCD,0}},$$  \hspace{1cm} (4.60)

where the subscript 0 denotes the usual SM sector. Using $B_i = \frac{n_{b_{i}} - n_{\bar{b}_{i}}}{s_i}$ as the dark neutron yield, the dark matter abundance is given by:

$$\Omega h^2 \equiv \frac{\rho_i}{\rho_{c,0}} ,$$

$$\simeq \frac{m_{N,i} B_i s_0}{\rho_{c,0}/h^2} \mathcal{R} ,$$  \hspace{1cm} (4.61)
Figure 4.10: The relic density of the DM as function of the reheaton’s width for various $\beta$ in the $i = -1$ (left panel) and $i = 1$ sector (right panel). The black dotted horizontal line represents the experimentally observed dark matter relic abundance and the black dotted vertical line indicates the decay width value taking into account the upper bound on initial lepton asymmetry in Eq. (4.16).

where $\rho_c/h^2 = 8.09 \times 10^{-47}$ GeV$^4$ is the critical density of the Universe, $s_0$ is the entropy density of the SM sector today and the parameter $R$ is the ratio of the entropy densities of $i = -1$ over $i = 0$ sector at late times ($t_f$):

$$R \equiv \frac{s_{i=-1}}{s_{i=0}} = \frac{g_{*,i=-1}(t_f)}{g_{*,i=0}(t_f)} \left[ \frac{T_{i=-1}(t_f)}{T_{i=0}(t_f)} \right]^3. \quad (4.62)$$

Using Eq. (4.56), Eq. (4.60), Eq. (4.62), and the relation $\frac{T_{-1}}{T_0} \simeq \left( \frac{\beta_{-1}}{1 - \beta - \gamma} \right)^{1/4}$, the DM relic abundance can be written as:

$$\Omega h^2 \simeq 0.12 \left( \frac{1}{1 - \beta - \gamma} \right)^{7/4} \left( \frac{\beta_{-1}}{10^{-2}} \right)^{7/4} \left( \frac{B_0}{8.59 \times 10^{-11}} \right) \left( \frac{2.8 \times 10^{-16} \text{ GeV}}{\Gamma_S} \right)^2. \quad (4.63)$$

In Figure 4.10, we show the parameter space for DM in two sectors: $i = -1$
(left panel) and $i = +1$ (right panel). In both panels, the horizontal black dashed line is the observed DM abundance, whereas the vertical black dotted line corresponds to the upper bound on the initial lepton asymmetry in Eq. (4.16). The upper bound on the initial lepton asymmetry gives a lower bound on $\Gamma_S$ using the relation in Eq. (4.56) and requiring the observed baryon asymmetry of the Universe (and the correct relic abundance for DM). In the left panel, we represent the DM relic abundance provided by our model, for three different branching ratios: $\beta_{-1} = 10^{-3}$ in dashed green, $\beta_{-1} = 10^{-2}$ in solid red and $\beta_{-1} = 10^{-1}$ in dashed magenta. We can see that, in the $i = -1$ sector, there is a region of the parameter space that yields the observed DM abundance. In the right panel, we plot the DM abundance in the $i = +1$ sector for three different branching ratios as well: $\gamma_1 = 10^{-3}$ in dashed green, $\gamma_1 = 10^{-2}$ in solid red, and $\gamma_1 = 10^{-1}$ in dashed magenta in the right panel. As expected, we can see that there are not enough baryons in the standard sectors ($i > 0$) to accommodate a DM candidate.

In Figure 4.11, we represent the viable parameter space of our model (solid magenta curve) that can account for the observed DM relic abundance (in the $i = -1$ sector) and the observed baryon asymmetry (in the $i = 0$ sector), and it includes the theoretical upper bound on the initial lepton asymmetry $|L_I| \leq 3 \times 10^{-6}$, in the plane of the branching ratio into the exotic sector, $\beta$, as a function of the reheaton width, $\Gamma_S$. The horizontal axis only shows the allowed range for $\Gamma_S$, as computed in Eq. (4.57). The dashed lines correspond to different contours of $\Delta N_{\text{eff}}$, with $\Delta N_{\text{eff}} = 10^{-2}$ in orange, $\Delta N_{\text{eff}} = 5 \times 10^{-2}$ in green and $\Delta N_{\text{eff}} = 0.4$ in red. The minimum values for our parameters in $\Gamma_S - \beta$ plane are approximately $\beta = 0.011$ and $\Gamma_S = 6.2 \times 10^{-16}$ GeV, yielding $\Delta N_{\text{eff}} \simeq 0.048$. CMB-S4 is expected to constrain $\Delta N_{\text{eff}} \leq 0.06$ at the 95% confidence level [120], allowing our model to be potentially probed by the future
Figure 4.11: Contours of $\Delta N_{\text{eff}}$ in $\Gamma_S - \beta$ plane, with the solid magenta curve providing the observed DM abundance, the observed baryon asymmetry and satisfying the theoretical upper bound on the initial lepton asymmetry ($|L_I| \leq 3 \times 10^{-6}$, computed in Eq. (4.16)). The horizontal axis only represent the allowed range for $\Gamma_S$, as given in Eq. (4.57).

4.6 Dark Matter Phenomenology

In this section, we study the possibility of large DM bound states. Dark matter is made up of dark neutrons in the $i = -1$ exotic sector. SM neutrons have significant self-interactions, and these are potentially even larger in the exotic sector since the pions are lighter than those of the SM. The dark neutrons can also form dark nuclei [123,124], which turns out to be the dominant state of dark matter in this model.

The neutron is expected to be lighter than the proton in these sectors, making the proton unstable. In the SM, the proton is lighter than the neutron mainly due to the down quark outweighing the up quark. However, in the exotic sectors, the $u$ and $d$ (and the electron) are nearly massless, and the dominant contribution to the proton-neutron mass splitting is the electromagnetic self energy which raises the mass of the proton. The decay width of the dark proton $\Gamma \propto G_{F,i}^2 |m_{n,i} - m_{p,i}|^5$ similar to the SM beta-decay. The lifetime of the dark proton in exotic sectors will be much shorter than that of the SM neutron because $G_{F,i} \gg G_{F,\text{SM}}$ and because there is no phase space suppression due the electron mass.
This allows constraints from self-interacting dark matter to be evaded. Finally, we briefly comment on the possibility of dark quark nuggets [164], and show that these nuggets evaporate rapidly.

### 4.6.1 Dark Neutrons Self-Interaction

Before considering dark matter composite states, we first consider observational constraints on free dark neutrons if they are all of the dark matter. The limit on dark matter self-interaction cross section coming from the Bullet Cluster is [125]:

\[
\frac{\sigma_N}{m_N} < 1.25 \text{ cm}^2 \text{ g}^{-1}. \tag{4.64}
\]

For a 1 GeV DM neutron, this bound limits the cross-section to be:

\[
\sigma_N < 2.2 \times 10^{-24} \text{ cm}^2. \tag{4.65}
\]

In the SM, the total cross-section of neutrons scattered by protons at low energy is \(\sigma_{pn} \sim 2.04 \times 10^{-23} \text{ cm}^2\) [165–167] and, by isospin symmetry, (neglecting the \(p\) and \(n\) mass difference) the following approximation is valid: \(\sigma_{nn} \simeq \sigma_{pn}\). Applying naive dimensional analysis, the scattering cross-section in the exotic sector is:

\[
\sigma \sim \sigma_{nn} \left(\frac{\Lambda_{\text{QCD},0}}{\Lambda_{\text{QCD},i}}\right)^2 \simeq 2.84 \times 10^{-22} \text{ cm}^2. \tag{4.66}
\]

From Eq. (4.66), we may conclude that the 1 GeV neutron DM is inconsistent with the stringent bound from the Bullet Cluster given in Eq. (4.65). In addition, the mass of the neutron in the exotic sector is smaller than the that of the SM one by a factor of \(\Lambda_{\text{QCD},i}/\Lambda_{\text{QCD},0} \sim 0.27\) which makes the upper bound even stronger on the self-interaction. In order to satisfy Eq. (4.65), the DM mass is required to be
$m_{\text{DM}} \gtrsim 1.3 \times 10^2 \text{ GeV}$. An alternative way to evade the Bullet Cluster bound is raising the confinement scale of the exotic sectors. Assuming the mass and cross section can be scaled using naive dimensional analysis, the confinement scale must be:

$$\frac{\sigma_{nn}}{m_{N,0}} \left( \frac{\Lambda_{\text{QCD},0}}{\Lambda_{\text{QCD},i}} \right)^3 < 1.25 \text{ cm}^2 \text{ g}^{-1}$$

$$\Rightarrow \Lambda_{\text{QCD},i} \gtrsim 700 \text{ MeV}.$$ (4.67)

This can be achieved by assuming the $SU(3)$ gauge coupling of the $i = -1$ sector is a bit larger than the SM at high energy. As noted in Section 4.5, different gauge couplings in different sectors are consistent with the Nnaturalness paradigm. As we will next show, most of the dark matter is in bound states, so it is not necessary to alter the particle physics parameters to satisfy the bounds.

### 4.6.2 Nuclear Dark Matter

In this section, we argue that composite DM, made of stable dark nuclei (DN) of large dark nucleon number, can evade the constraints from the Bullet Cluster on the DM self-interactions and provide a viable DM candidate [123, 124]. As the detailed mechanism is explained in [124], in this work, we will just point out some of the main steps to get the observed DM abundance and the maximum size of these DN. In the SM, the main bottleneck that prohibits the synthesis of nuclei is the substantial binding energy per nucleon of Helium-4, $\sim 7$ MeV, relative to the following smaller nuclei.\footnote{During SM BBN, almost all the nucleons present wind up in Hydrogen and Helium-4 while a small fraction leads to the synthesis $4 < A < 8$. There are subsequent bottlenecks post Helium-4, such as $^{12}\text{C}$, where the binding energy per nucleon exceeds Helium-4. However, there are no bottlenecks in the formation of large dark nuclei in the exotic sectors.} As the Coulomb repulsion term is absent in this scenario, unlike in the
SM, there can be stable DN up to a large dark nucleon number. DM will thus be produced at low temperatures given that, in this case, the energy term dominates over the binding energy, which favours the generation of bound states, and it will be built up by aggregation, as fusion processes dominate over dissociations and fissions in the low temperature regime [124].

Our composite Dark nucleons can be made to satisfy all the assumptions stated in [124] that are necessary for the formation of the large Dark nucleon (A-DN). The cross-section of the A-DN over its mass can be expressed as:

\[
\frac{\sigma_A}{m_A} = \left( \frac{4\pi}{m_1 A} \right)^{1/3} \left( \frac{3}{\rho_i} \right)^{2/3} = 0.043 \text{ cm}^2 \text{ g}^{-1} \left( \frac{252 \text{ MeV}}{A m_1} \right)^{1/3} \left( \frac{\text{GeV fm}^{-3}}{\rho_0} \right)^{2/3} \left( 1 - \beta - \gamma \right)^{2/3},
\]

(4.68)

where \( A \) is a dark nucleon number (analogous to the mass number in usual atoms), \( m_1 \) is the mass of a single nucleon, and \( \rho_i \) is the internal mass-energy density. From the expression above, we can conclude that DN can easily evade the bounds of the Bullet Cluster. Also, due to the slight differences in our parameters compared to the benchmark points employed in [124], our DN could be orders of magnitude larger.

To finish this section, we can determine the maximum size of the DN due to the aggregation process. We should consider two regimes: the case where the last fusions to freeze-out are those between large DN ("large" + "large" fusions) and the scenario where the last fusions to freeze-out are between small and large DN ("small" + "large" fusions) [124]. In the case of "large" + "large" fusions, we assume that there is a ‘peaked’ DNs mass distribution where almost all of the mass lies in the large A-DN. We can estimate when the freeze-out occurs by comparing the rate of the fusion, thermally average cross sections of fusions times the A-DN number density,
with the Hubble parameter. The rate of the fusion over the Hubble parameter can be expressed as:

$$\frac{\Gamma}{H} = \frac{\langle \sigma v \rangle n_A}{H} \simeq 5.7 \times 10^7 \left( \frac{1 \text{ GeV fm}^{-3}}{\rho_0} \right)^{2/3} \left( \frac{g_*}{10.75} \right)^{1/2} \left( \frac{m_1}{252 \text{ MeV}} \right)^{-5/6} \left( \frac{T}{1 \text{ MeV}} \right)^{3/2} \left( \frac{1}{1 - \beta - \gamma} \right)^{-13/24} A^{-5/6}, \quad (4.69)$$

where we have used $m_{N,i} = m_{N,0} \frac{\Lambda_{QCD,i}}{\Lambda_{QCD,0}} \simeq 251.9$ MeV with $\Lambda_{QCD,i} = 89$ MeV, $\Lambda_{QCD,0} = 332$ MeV and $m_{N,0} = 939.6$ MeV, and the approximations $\frac{T_0}{T_i} \simeq \left( \frac{1 - \beta - \gamma}{\beta} \right)^{1/4} \beta = \gamma = 0.01 \rightarrow 3.15$ and $\frac{\rho_0}{\rho_i} \simeq \left( \frac{1 - \beta - \gamma}{\beta} \right)^{\beta = \gamma = 0.01} 98$. Thus, the maximum mass obtained from Eq. (4.69) is:

$$M_{\text{max}} = A_{\text{max}} m_1 \simeq \left( 5.7 \times 10^7 \right)^{6/5} \left( \frac{1}{1 - \beta - \gamma} \right)^{-13/20} m_1 \simeq 10^{10} \text{ GeV} . \quad (4.70)$$

In the “small” + “large” fusions regime, the number density of small DN, $k$-DN, with $k < A$, may be larger than in the “large” + “large” scenario, implying that the number of fusion processes can be larger as well. In addition, the velocities of “$k$-DN” in thermal equilibrium are smaller than the ones of “$A$-DN”, which contributes to a size enhancement. Assuming there is a sufficient population of small DNs, and it remains long enough, we can compute their rate of fusion to be:

$$\Gamma \sim \langle \sigma v \rangle_{k+A} n_k \frac{k}{A}$$
\[ \sim \frac{1}{4} \delta \sigma_1 v_1 k^{-1/2} A^{-1/3} n_0, \]  

(4.71)

where \( v_1 \) is the velocity of one nucleon, \( n_0 \) is the total DN number density, \( \langle \sigma v \rangle_{k+A} = \frac{\delta}{4} \sigma_1 v_1 k^{-1/2} A^{2/3} \) [124] and \( \frac{1}{4} \delta \) is a suppression factor for “small” + “large” cross section relative to the geometric limit. This leads to a similar result as before but with \( A^{-5/6} \rightarrow \frac{1}{4} \delta k^{-1/2} A^{-1/3} \). Thus, the fusion rate over the Hubble parameter for the “small” + “large” regime is:

\[ \frac{\Gamma}{H} \simeq 5.7 \times 10^7 \left( \frac{1 \text{ GeV fm}^{-3}}{\rho_0} \right)^{2/3} \left( \frac{g_*}{10.75} \right)^{1/2} \left( \frac{m_1}{252 \text{ MeV}} \right)^{-5/6} \left( \frac{T}{1 \text{ MeV}} \right)^{3/2} \left( \frac{1}{4} \delta k^{-1/2} A^{-1/3} \right). \]  

(4.72)

Similarly, the maximum DN synthesized would be, assuming that there is no suppression:

\[ M_{\text{max}} = A_{\text{max}} m_1 \]

\[ \simeq \left( \frac{5.656 \times 10^7}{4} \right)^3 \left( \frac{m_1}{252 \text{ MeV}} \right)^{-3/2} \left( \frac{\beta_{-1}}{1 - \beta - \gamma} \right)^{-13/8} \]

\[ \simeq 1.2 \times 10^{24} \text{ GeV}, \]  

(4.73)

where in the last step we set \( \beta = \gamma = 0.01 \).

### 4.6.3 Comment on the formation of Dark Quark Nuggets

In addition, we study the possible formation of quark nuggets in the exotic sectors. The hypothesis of quark nuggets proposed by Witten [164] is that after the QCD phase transition, there will be stable nuggets that remain in the quark phase and have very high baryon density. The formation of such objects requires a first order
QCD phase transition, which does occur in exotic sectors [122] with six nearly massless quark flavours. Studies of the six-flavour quark matter (6FQM) quark nuggets have been conducted in [168, 169]. One process not considered in those scenarios is the evaporation of pion and a neutrino from the quark nugget. Such a process conserves all gauge quantum numbers as well as \( B - L \), but it can decrease the lepton number of the nugget. In Section A.4, we show that such processes are quite fast and thus render the nugget lifetime much too short to play any role in dark matter phenomenology.

4.7 Summary

Models with multiple SM-like hidden sectors may have interesting phenomenology. On the one hand, if the Higgs mass varies across sectors, these models can potentially solve the electroweak hierarchy problem [47, 109]. On the other hand, if the other sectors have similar temperatures as the Standard Model, then such a scenario is immediately excluded by cosmological observations such as \( \Delta N_{\text{eff}} \), and novel cosmological histories are required.

Focusing on the reheaton scenario of [47], the cosmological history requires a reheating temperature of the order of the weak scale. In such a setup, the creation of the baryon asymmetry of the Universe and dark matter are open problems. In this work, we solve them. Although the strong self-interaction rules out dark neutrons of the exotic sectors as the dark matter candidate, the unique structure of the exotic sectors naturally allows the formation of large dark nuclei [123, 124], which easily evade the self-interaction bounds from the Bullet Cluster. Dark matter will then form large neutral dark nuclei, which can have interesting observable consequences. We assume that the reheaton, which dominates the energy of the Universe at early times, is a fermion and that the population of reheatons develops an asymmetry from
out-of-equilibrium decays, analogous to leptogenesis. The decay of the asymmetric population of reheatons reheats the SM and hidden sectors and imparts the asymmetry into the lepton number of those sectors. Part of the lepton number asymmetry is then processed into a baryon number asymmetry via the electroweak sphaleron.

The constraint coming from $\Delta N_{\text{eff}}$ requires the temperatures of all the hidden sectors to be significantly lower than the temperature of the SM sector. Naively, this would mean that the energy density of dark baryons should be less than that of visible baryons, in contradiction with observation. This is resolved by the fact that the dark matter lives in an exotic sector where the electroweak symmetry is not broken by a Higgs. In such a sector, the QCD confinement at a scale around 100 MeV leads to electroweak symmetry breaking, which in turn means that the electroweak sphaleron is active down to a much lower temperature. These sectors have light photons and neutrinos, and the requirement of achieving the correct baryon and dark matter density predicts $\Delta N_{\text{eff}} \gtrsim 0.05$, which may be detectable in the next generation of CMB experiments [120]. We must stress that the standard sectors have a dark baryon density substantially smaller than that of the visible sector, which precludes the possibility of having a dark matter candidate in those sectors.
Chapter 5

Three Jet Resonances

One of the non-supersymmetric approaches to solving the hierarchy problem involves the introduction of a new fermionic partner to the SM top quark called the “top partner” and we are interested in such an approach. As a matter of fact, top partners have been studied at colliders and they typically involve coupling to the third-generation quarks. The generic top partner decay topologies studied at the LHC are: $T \rightarrow th$, $T \rightarrow tZ$, $T \rightarrow bW$. But there are strong constraints on these topologies and the typical lower mass limit on the top partner is around $m_T \geq 1.3–1.66$ TeV depending on the branching ratios, single or pair production of top partners [170–190]. However, we study alternative decay modes that leads to less severe experimental constraints and some of the implementation tools employed are described in Section B.5.

5.1 Introduction

With the discovery of the Higgs boson [3, 4] at the Large Hadron Collider (LHC), the Standard Model of particle physics (SM) is complete. The confirmation of the
properties of the Higgs being SM-like and the lack of discovery of new physics at the TeV scale exacerbates the hierarchy problem: what cuts off the quantum corrections to the Higgs mass? One well known solution to the hierarchy problem poses the existence of fermionic top partners, fermions with the same quantum numbers as the top quark whose contributions to the Higgs mass parameter cancel those of the top quark. These can appear in composite Higgs models [48–52] and Little Higgs models [53,54].

In these models, top partners typically decay to a top quark and a Higgs or $Z$, or to a bottom quark and $W$. LHC searches for top partners in these modes are extensive, both in pair production [170–179] and in single production [180–190], with limits $\approx 1.3–1.66$ TeV on the mass of the top partners from the various searches depending on their branching ratios. Due to the lack of discovery, it is critical to explore alternative models, particularly those with different decay modes for the top partners. One could imagine, for example, decays involving a charged or neutral scalar [191–193], decays involving a top quark in association with a gluon or a photon [194,195], a neutral boson that subsequently decays to two photons [196], a dark photon or a dark Higgs [197], or a pseudo-scalar which promptly decays to a pair of gluons or $b$-quarks [198–200]. In this work we consider the particularly challenging possibility of a final state with only hadronic activity and no leptons or missing energy and study the limits on the masses of such top partners.

We consider a simplified model for our top partners ($T$) which contains the following processes: $T$ is pair produced via the strong interactions, $pp \rightarrow T\bar{T}$. It then decays to a light flavour quark ($j$) and a new scalar $\eta$, and that scalar decays to two light flavour jets. The full process is

$$pp \rightarrow T\bar{T} \rightarrow jj\eta \rightarrow 6j,$$  \hspace{1cm} (5.1)
with a representative Feynman diagram shown in the right panel of figure 5.1. In our studies, we consider the decay channel $T \rightarrow jjj$ where $j$ can be associated with any light quark. However, if one were to focus mainly on the decay channel $T \rightarrow cjj$, the limits obtained from considering charm tagging will not significantly improve the sensitivity of the signal or alter the limits obtained, mainly because charm tagging is notoriously difficult and the efficiency is significantly worse than $b$-tagging. For example, the $c$-tagging efficiency is approximately 20% for about 1% light jet misidentification rate, whereas the $b$-tagging efficiency can be approximately 70% at about 1% misidentification rate [201–204]. Note the tagging efficiencies depend on jet $p_T$ and $\eta$. An explicit model which gives rise to this signature (without associated top and bottom signatures) and solves the hierarchy problem is given in [205]. This scenario can be thought of as the fermionic analogue of hadronic $R$-parity violation (RPV) [206–208] in Supersymmetry where the top squark can decay to two light jets.

This six-jet final state is experimentally very challenging as the QCD multijet background is very large and difficult to determine. While one might expect that the limits for this model are significantly weaker than for the traditional decay modes, we will show that these models are also strongly constrained, with the best limits coming from recasting searches for RPV gluino searches from CMS [209] and ATLAS [210].
The RPV gluino has the same signal topology as Eq. (5.1), but the cross section for a colour octet is larger than for a colour triplet top partner. In this work we will study various qualitatively different regions of parameter space including:

- Off-shell $\eta$: $m_\eta \gg m_T$,
- Bulk on-shell region: $m_\eta \lesssim m_T$,
- Very light $\eta$: $m_\eta \ll m_T$,
- Degenerate region: $m_\eta \sim m_T$.

We will show that all of these regions are constrained up to a $T$ mass of about 700 to 900 GeV.

We also consider the possibility that the scalar particle ($\eta$) decays to two bottom jets instead of light jets, as might be expected from a Higgs-like scalar. The complete process for this particular decay mode is:

$$pp \to T\bar{T} \to jj\eta \eta \to 2j4b.$$  \hspace{1cm} (5.2)

From the presence of the two $b$-jets in the final state, one might expect that the corresponding limit on $T$ would be stronger than in the light jets case, but in fact the constraints are very similar. Adding $b$-tagging to current search strategies can significantly improve limits on this scenario.

This paper is structured as follows. In section 5.2 we present the bounds coming from the latest LHC searches for top partners decaying exclusively to light jets. In particular, we consider searches looking for pairs of resonances decaying to three jets, and searches looking for pairs of di-jet resonances. In section 5.3 we repeat this exercise for final state containing $b$-jets. In section 5.4 we give a brief summary of the
results. This work is augmented by four appendices: in appendix B.1 we give some
details regarding the three-jet CMS resonance search performed at $\sqrt{s} = 13$ TeV, in
appendix B.2 we present all the selection requirements for the di-jet ATLAS search
applied to our model, in appendix B.3 we discuss the three-jets CMS search conducted
at $\sqrt{s} = 8$ TeV, and in appendix B.4 we give details on how QCD background events
are simulated.

5.2 Bounds from LHC Searches

The topologies we consider here consist of resonances that ultimately decay fully
hadronically leading to a six-jet final state, not counting initial and final state radi-
ation. Furthermore, we assume that the scalar ($\eta$) decays promptly; hence, resulting
in the absence of any displaced vertices. This multijet final state narrows down the
list of possible searches sensitive to this model. The pertinent searches to consider
are the ones looking for multiple jets but no missing energy, leptons, or photons.

In order to recast existing searches, we use a few publicly available software
packages/tools. The model file for our model was created using the Mathematica
package *FeynRules* [211] which was then supplied as an input to *MadGraph5* [212]
for Monte Carlo (MC) event generation. Next, the events were passed to *PYTHIA 8*
for showering and hadronization [213]; subsequently, *DELPHES 3* [214] was used for
fast detector simulation and *FastJet* [215] was deployed to reconstruct jets.

The top partner pair production cross section was computed at next-to-leading
order (NLO) using *MadGraph5* by setting the top quark mass to $m_T$ and its behaviour
as a function of the top partner mass is shown in figure 5.2. The theoretical cross
sections were also computed with the *TOP++2.0* program [216] for comparison and
were found to be consistent. The benchmark points displayed in figure 5.2 contain
5.2.1 CMS Pair-Produced Three-jet Resonances

Searches that explore the fully hadronic decay channels with at least six jets (light) jets in the final state have been conducted at the LHC. Older searches include CMS [217] and ATLAS [218,219] searches at $\sqrt{s} = 7, 8$ TeV that place no constraint on our signal.
A similar conclusion was reached by the authors for the double trijet resonances for the composite models in [220] which considers the CMS search [217]. However, their topology is slightly different since it involves a color octet rather than a singlet $\eta$.

The latest multijet search that matches our desired search criteria was conducted by CMS using 35.9 fb$^{-1}$ of data collected at a 13 TeV center-of-mass energy [209]. The search is designed to look for a pair of particles each decaying to three jets. The analysis interprets the results in the framework of an R-parity violating (RPV) SUSY model where gluinos are pair produced and each decay to three quarks, resulting in a six-jet final state. The search explores a gluino mass range from 200 to 2000 GeV and excludes gluino masses below 1500 GeV at 95% confidence level. This dedicated analysis focuses on three-jet resonances and takes advantage of Dalitz variables [221] to enhance signal sensitivity. A distance parameter, sensitive to the symmetry of the jets inside a triplet is defined as:

$$D_{[3,2]}^2 = \sum_{i>j} \left( \hat{m}(3, 2)_{ij} - \frac{1}{\sqrt{3}} \right)^2 ,$$

where $\hat{m}(3, 2)_{ij}$ is the normalized di-jet invariant masses and is defined as:

$$\hat{m}(3, 2)_{ij}^2 = \frac{m_{ij}^2}{m_i^2 + m_j^2 + m_k^2 + m_{ijk}^2} , \text{ where } i, j, k \in \{1, 2, 3\} .$$

Here, $m_{ijk}$ is the triplet invariant mass and $m_i$ are the constituent jet masses of the triplet. The complete list of selection criteria used in this particular search are given in table B.1 in the appendix. To set bounds, the QCD and combinatorial backgrounds are modelled with a monotonically decreasing function, which is optimized in four mass regions. A statistically significant signal-like “bump”, parametrized by a double Gaussian, is then looked for on top of this background.
In order to recast this search and obtain bounds on the parameter space of our model, we first simulated the RPV SUSY topology given in [209] with all superpartners except the gluino decoupled. We then simulated our particular model for the corresponding $m_T (m_T = m_{\tilde{g}})$ for a fixed $m_\eta$. We computed the acceptance, defined as the number of correct triplets passing all the selection criteria given in table B.1 divided by the total number of events generated, for both the original RPV topology and our model. The correct triplets are the ones constructed from the three jets associated with the decay of a gluino (for the CMS topology) or a top partner (for our topology). They have an invariant mass distribution peaked around the resonance mass.\footnote{In our simulation we found the invariant distribution to be slightly skewed towards lower masses, see appendix B.1 for more details of this discrepancy.} In order to identify these correct triplets, we require the parton level decay products to be within $\Delta R = 0.3$ from the detector level jet axis. We then

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5_3}
\caption{The ratio of acceptance in the recasted CMS search [209] for our model over the RPV benchmark model as a function of the scalar mass for a few top partner masses.}
\end{figure}
rescale the pair production cross section for $\bar{T}T$ shown in figure 5.2 by the ratio of the acceptances as:

$$\sigma_{\text{rescaled}} = \sigma(pp \to \bar{T}T) \times \frac{A_{\text{our}}}{A_{\text{RPV}}},$$  \tag{5.5}$$

where $A_{\text{our}}$ is the acceptance for our topology and $A_{\text{RPV}}$ is the acceptance obtained by simulating the RPV benchmark model in [209]. Because the invariant mass distribution of correct triplets for our topology, including cases where $\eta$ is on-shell, is very similar to the RPV topology, the number of events that our model would produce in an invariant mass peak distinguishable from background is given by $\sigma_{\text{rescaled}} \times A_{\text{RPV}}$. So a mass point in the $m_T - m_\eta$ plane is excluded if $\sigma_{\text{rescaled}}$ is greater than the observed 95% upper bound on the cross section $pp \to \tilde{g}\tilde{g}$ for a given value of $m_T$ ($m_T = m_{\tilde{g}}$) obtained by the search.

To understand the main features of the exclusion regions we obtained, it is instructive to look first at the ratio of acceptance $\frac{A_{\text{our}}}{A_{\text{RPV}}}$, which is shown in figure 5.3 as a function of the scalar mass for various fixed top partner masses. If $m_\eta \gg m_T$, then the topology of our model is the same as the RPV gluino decaying to three SM jets and we would expect $\frac{A_{\text{our}}}{A_{\text{RPV}}} \sim 1$. In this case the bound on $m_T$ can be found by simply comparing $\sigma(pp \to \bar{T}T)$ with the limit obtained by the CMS Collaboration [209]. However, if $m_\eta \lesssim m_T$, the scalar $\eta$ is on-shell and the topology is different from the RPV gluino. In particular, the distribution of the $D^2_{[3,2]}$ variable changes significantly as $\eta$ goes on-shell and becomes strongly dependent on the mass difference $m_T - m_\eta$. This is shown in figure 5.4 where distributions of the $D^2_{[3,2]}$ variable is shown for a fixed $m_T = 900$ GeV and several scalar masses.\footnote{Other $m_T$ values show similar behaviour.} We find that even with such different distributions, the efficiency of the search remains high in most of the on-shell $\eta$ parameter space because the cut applied on $D^2_{[3,2]}$ is relatively high, with the
Figure 5.4: The \( D_{[3,2]}^2 \) variable distributions for signal triplets with top partner mass of 900 GeV and various scalar masses. The black dashed line represent the cut placed on \( D_{[3,2]}^2 \) (accepting events below this value) for this particular top partner mass.

Exceptions being for \( m_\eta \approx m_T \) and \( m_T \gg m_\eta \). The \( D_{[3,2]}^2 \) distributions for the RPV signal are extremely similar to case where \( m_\eta \gg m_T \) as one would expect which we
have alluded to before during the discussion of figure 5.3.

These regions of low efficiencies can be understood as follows: when the mass splitting between $T$ and $\eta$ is small, the jet from the $T \to j\eta$ decay is soft, resulting in a decrease in $A_{our}$ from the requirement of six hard jets. This topology however still has a di-jet signature, and can be probed by the di-jet search discussed in the next section. If $m_T \gg m_\eta$, the scalars are produced with very high boost and the jets resulting from their decay $\eta \to jj$ will often merge into a single jet leading once again to reduced sensitivity, but also having a di-jet like topology.

![Diagram](image.png)

**Figure 5.5:** The CMS and ATLAS bounds on the fully hadronic decay mode (to light jets) of the top partner. The shaded regions are excluded by the current searches conducted at the LHC [209,210].

Performing a grid search for all mass points in the $m_T - m_\eta$ plane with increments of 25 GeV, we obtain the exclusion regions shown by the blue shaded area of figure 5.5. We see that the dips in figure 5.3 translate to holes in the sensitivity of the CMS search for $m_T \sim m_\eta$ and for $m_\eta \ll m_T$. We also see that the bounds in the bulk on-shell region $m_T \gtrsim m_\eta$ are stronger than the off-shell region $m_T \lesssim m_\eta$. The
isolated exclusion near $m_T \sim 800$ GeV in the off-shell region can be attributed to a downward fluctuation of the background in that region.

### 5.2.2 ATLAS Pair-Produced Di-Jet Resonances

As mentioned in the previous section, searches that look for pair-produced di-jet resonances can constrain this scenario in the region where $\eta$ is on-shell. It is especially useful when $T$ and $\eta$ are close in mass or the scalar is very light as the three-jet search is not sensitive in those regions. Such a search was conducted by ATLAS at $\sqrt{s} = 13$ TeV with an integrated luminosity of 36.7 fb$^{-1}$ [210]. It explores coloured resonances that are pair-produced and that each decay to two jets, giving rise to a four-jet final state. The results of the analysis are interpreted in a simplified R-parity violating SUSY model where the top squark is the lightest supersymmetric particle and decays promptly into two quarks ($\tilde{t} \rightarrow q \bar{q}_k$). The search explores the region $100$ GeV $< m_{\tilde{t}} < 800$ GeV and excludes top squark masses in the range $100$ GeV $< m_{\tilde{t}} < 410$ GeV at 95% CL. The list of selection criteria is given in table B.2 in the appendix. Since it is expected that the resonances are produced with high transverse momentum, their decay products will be located close to each other. As such, the four leading jets are paired using an angular distance:

$$\Delta R_{min} = \min \left\{ \sum_{i=1}^{2} |\Delta R_i - 1| \right\},$$

(5.6)

where $\Delta R_i = \sqrt{\Delta \phi_i^2 + \Delta \eta_i^2}$ is the distance between the two jets in $i^{th}$ pair. The two jet pairs selected must minimize $\Delta R_{min}$ and satisfy the $\Delta R_{min}$ cut. Signal jets are expected to be produced in the central region so putting a cut on $|\cos \theta^*|$, where $\theta^*$ is the angle that either of the resonances makes with the beamline in the center-of-mass.

---

3We will show later that this search also has significant sensitivity in the off-shell region.
Chapter 5. Three Jet Resonances

frame, is beneficial. Finally, the masses of the resonances are expected to be equal; hence, their invariant mass differences would be an ideal discriminant between signal and background. As such, the mass asymmetry ($A_m$), defined as:

$$A_m = \frac{|m_1 - m_2|}{m_1 + m_2},$$  \hspace{1cm} (5.7)

where $m_1$ and $m_2$ are the invariant masses of the two reconstructed di-jet pairs is required to be small. To set bounds, the ATLAS collaboration employed a modified frequentist approach using the CL$_s$ [222–224] technique and a profile likelihood ratio as the test statistics. For each mass hypothesis, a counting experiment is performed in a window around the average mass of the two reconstructed resonances. The dominant source of background comes from QCD multijet production and is estimated directly from the data with a method that predicts both the normalization and the shape of the average di-jet mass distribution [210].

In order to recast the search, we simulated our particular model for the corresponding $m_T$ ($m_T = m_i$) for a fixed $m_\eta$. We computed the efficiency of the search for our model ($\epsilon_{\text{our}}$), defined as the number of events satisfying all the cuts in table B.2 plus an invariant mass window cut over the total number of simulated events. The number of signal events is then given as follows:

$$N_s = \mathcal{L}_{\text{luminosity}} \times \sigma(pp \rightarrow \bar{T}T) \times \epsilon_{\text{our}}.$$  \hspace{1cm} (5.8)

We computed this number for various window mass cuts, corresponding to different stop masses considered in [210]. Taking the number of background and observed events and their uncertainties in the different windows from tables 3 and 5 of [210],
Figure 5.6: The $A_m$ spectrum passing the various selection criteria as highlighted in the ATLAS search [210] for $m_T = 500$ GeV. The distributions of the (a) are for events passing only the $p_T$ requirement, the (b) satisfies both $p_T$ and $\Delta R_{\text{min}}$ requirements, and the (c) have all except $A_m$ cuts applied.

we computed the confidence level:

$$CL_s = \frac{P_{s+b}(X \leq X_{\text{obs}})}{P_b(X \leq X_{\text{obs}})} = \frac{CL_{s+b}}{CL_b}.$$  

(5.9)

Here $CL_{s+b}$ is the confidence level for excluding the possibility of simultaneous presence of signal and background while $CL_b$ is the probability that the test statistic is less than or equal to that observed in the data (assuming only the presence of
background) [222–224]. Then a mass point is excluded at 95% confidence level if 
\((1 - CLs) \times 100\%\) is greater than 95%. The parameter space excluded by the ATLAS search is given by the red shaded region of figure 5.5. It extends up to top partner masses of \(~ 750 \text{ GeV}\).

The di-jet search allows us to close some of the gaps that remain in the three-jet resonance search, namely the regions where \(m_\eta \approx m_T\) or \(m_\eta \ll m_T\). The gap in di-jet exclusion curve for \(400 \text{ GeV} \leq m_T \leq 425 \text{ GeV}\) and \(m_T < m_\eta\) can be explained by a downward fluctuation in the background in that region as illustrated in figure 9 (a) of the ATLAS search [210].

While the ATLAS search is primarily designed for di-jet topologies we also find reasonably good efficiency in the off-shell region where there is in fact no di-jet resonance, placing an exclusion up to \(m_T \leq 525 \text{ GeV}\) in that region. In particular, one would expect the \(A_m\) cut in Eq. 5.7 would be very inefficient when there is no 2-jet resonance. However, we find that the \(\Delta R_{\text{min}}\) and \(|\cos(\theta^*)|\) selection criteria sculpt the \(A_m\) distribution which becomes similar for the on-shell and off-shell cases. This is shown in figure 5.6 where \(A_m\) is plotted after various cuts.

For very light top partner masses \(m_T \lesssim 200 \text{ GeV}\), searches at previous experiments may be sensitive. For example, a three-jet hadronic resonance search was performed at \(\sqrt{s} = 1.96 \text{ GeV}\) at the Collider Detector at Fermilab (CDF) [225] and excluded gluinos below 144 GeV. Multijet searches performed at LEP has excluded the neutralino decaying to three jets for masses up to about 100 GeV [226,227]. There does remain a gap for \(m_T \simeq m_t\) near the top quark. It may be possible to exclude this region of parameter space using the measurement of the all hadronic top quark decay [228–232], but a detailed analysis is beyond the scope of this work.
5.3 Heavy Flavour Scenario

In this section we study the case where the scalar $\eta$ decays to two $b$-jets: $pp \rightarrow T\bar{T} \rightarrow jj\eta \rightarrow 2j4b$. This decay topology is what one would expect if, for example, the $\eta$ coupled to fermions proportionally to their masses and was lighter than twice the top mass. In principle such a final state allows for better discrimination from QCD multijet events. However, we find that existing searches do not give much stronger constraints. We considered the fully hadronic ATLAS R-parity-violating multijet searches [219] and [233] performed at $\sqrt{s} = 8$ TeV and 13 TeV respectively, the 13 TeV ATLAS di-jet search [210] and the heavy flavour three-jet resonance CMS search [234] conducted at $\sqrt{s} = 8$ TeV. All of these searches have a $b$-tagging requirement in some of their signal regions, but only the CMS three-jet resonance and the ATLAS di-jet searches were found to place a limit on this scenario. The selection criteria for the heavy flavour three-jet resonance CMS search\(^4\) are presented in table B.3 (in the appendix) and the limits on the parameter space are computed using the same method we used to recast [209] (see equation 5.5). The region of parameter space excluded at 95% confidence level by this particular CMS search is presented by the blue shaded region of figure 5.7.

We also recasted the ATLAS di-jet search [210], which requires at least two $b$-tagged jets in addition to the window cut around the average invariant mass of the di-jets with additional selection criteria given in table B.2 (in the appendix). Computing the exclusion limits using the CL\(_s\) method [222–224], we obtain the red shaded region in figure 5.7. The presence of at least two $b$-jet in the reconstructed resonances reduces the combinatoric background coming from the two jet pairing. Hence, the search often selects the di-jet pairs corresponding to the scalar resonance,

\(^4\)The search also requires at least 6 jets and at least one $b$-jet.
making the search more effective in both on-shell and off-shell regions in comparison to the light jet scenario.

The searches considered in section 5.2 do not apply $b$-tags, but they also do not veto events with $b$-jets, so the exclusion regions shown in figure 5.5 also apply to the $b$-rich scenario considered in this section. Therefore, comparing figures 5.5 and 5.7, we see that adding $b$-tagging only provides new exclusions for $m_T$ between about 700 GeV and 800 GeV and some values of $m_\eta$. The ATLAS search [210], however, only investigates $m_\tilde{t}$ up to 800 GeV which is almost entirely excluded.\footnote{With the exception of the very light scalars for $m_T \leq 150$ GeV and low scalar masses for $m_T \geq 775$ GeV.} So in principle, the ATLAS di-jet search could have excluded larger $m_T$. Due to the steeply decreasing cross section, shown in figure 5.2, it is unlikely that extending the search would have resulted in an exclusion region significantly above 800 GeV.

The $\sqrt{s} = 13$ TeV three light jets resonance search from CMS analyzed in
section 5.2.1 places the strongest constraints on three-jet resonances. If this search included a signal region with $b$-tagging, it could also improve constraints on the $b$-rich scenario. Here we give a rough appraisal of the potential improvement. First, we estimate the total number of QCD multijet events by simulating $pp \rightarrow jjjj$ using MadGraph5 interfaced with Pythia8 and Delphes (more detail on the simulation is given in appendix B.4). The acceptance ($A_{QCD}$) is obtained by applying all the selection criteria for the three-jet CMS search and we approximate the number of background events to be: $b = \mathcal{L}_{\text{luminosity}} \times \sigma(pp \rightarrow jjjj) \times A_{QCD}$. We set the minimum parton level $p_T$ for the simulated QCD events to be 100 GeV in order to have enough events for our rough approximation. In a similar way we compute $s$, the number of signal events using simulation of our model. We then compare the $s/\sqrt{b}$ values obtained without $b$-tagging to the case where with 2 $b$-tags included in the cuts, and find that $s/\sqrt{b}$ is roughly a factor of three larger. The three-jet CMS search excludes $m_T \lesssim 900$ GeV for the light jet case, so our rough estimate is that including a $b$-tag requirement could exclude the $b$-jet topology up to $m_T \sim 1050$ GeV as the ratio of top partner cross section for 900 GeV and 1050 GeV top partner masses is also $\sim 3$. Requiring at least 4 $b$-tags could further increase the expected exclusion to $\sim 1300$ GeV. We summarize the improvements for different numbers of $b$-tags in table 5.1.

In a similar manner, we can roughly estimate the effect of extending the ATLAS di-jet search to higher masses by computing $R \equiv \sigma_{\text{rescaled}}/\sigma_{\text{obs}}$ at the cut off region ($m_T = 800$ GeV) where $\sigma_{\text{obs}}$ is the observed 95% CL from ATLAS di-jet search for a given mass and $\sigma_{\text{rescaled}}$ is the rescaled cross section as defined in equation 5.5. The corresponding value for $R$ is approximately 3 which points to a limit of $\sim m_T = 950$ GeV as the ratio of cross section for top partner masses of 950 GeV and 800 GeV is
Table 5.1: The approximate improvement in $s/\sqrt{b}$ and mass reach as a function of increasing number of $b$-tags, $N_b$. The first line is the actual limit, while the subsequent lines are estimated potential improvements. The $s/\sqrt{b}$ values are computed at the cut off region of $m_T = 900$ GeV for the three-jet CMS search [209] with $35.9$ fb$^{-1}$.

<table>
<thead>
<tr>
<th>B-tagging requirement</th>
<th>$\frac{s}{\sqrt{b}}$ Improvement</th>
<th>Mass Sensitivity [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_b \geq 0$</td>
<td>0.31</td>
<td>900</td>
</tr>
<tr>
<td>$N_b \geq 1$</td>
<td>0.58</td>
<td>1000</td>
</tr>
<tr>
<td>$N_b \geq 2$</td>
<td>1.04</td>
<td>1050</td>
</tr>
<tr>
<td>$N_b \geq 3$</td>
<td>2.30</td>
<td>1200</td>
</tr>
<tr>
<td>$N_b \geq 4$</td>
<td>4.66</td>
<td>1300</td>
</tr>
</tbody>
</table>

5.4 Conclusion

Although there has been a significant experimental search program for top partner pair production with $th$, $bW$ and $tZ$ decay modes, to our knowledge there has not been any studies to explore the all light jet decay mode. In this work we have recasted the latest available LHC searches that can impose significant constraints on the parameter space of models where the top partner decays to light jets. Our results are shown in figures 5.5 and 5.7 for models with final states containing only light jets and for final states containing 4 $b$-jets respectively. Top partner masses are generally excluded up to $m_T \sim 900$ GeV, but there are a few gaps in the $m_T - m_\eta$ plane for lighter $m_T$. Because the three-jet resonance search we recast focused on a resonance that decays through an off-shell scalar, it might be possible to obtain better limit in the on-shell $\eta$ region by designing a search specifically for that topology. Furthermore we found that existing searches do not provide significantly better constraints for the case where the final states contain $b$-jet, but those limits could be improved by adding $b$-tagging requirements to the 13 TeV three-jet resonance search.
Chapter 6

Supersymmetry

Here I would explain the general picture: in the 90's, Seiberg was able to understand the non-perturbative dynamics of many supersymmetric quantum chromodynamics (SQCD) theories, using symmetry arguments, the holomorphy of the superpotential and anomaly matching [235]. In theoretical physics, understanding of strongly correlated systems are highly sought after. For example, the behaviour of superconductors in condensed matter and the strong interactions in particle physics are an essential part of the SM. We are primarily interested in strong interaction dynamics and how to properly parametrize them at various scales. At very short distances (high energy regime), the fundamental degrees of freedom in strong interaction are quarks and gluons. Their interactions are described by quantum chromodynamics (QCD) and perturbation theory can be applied to study their behaviour very precisely. However, the computations become quite complicated if one analyzes the system at long distances (low energy regime). At long distances, confinement occurs and the fundamental degrees of freedom are trapped inside bounds states such as mesons and baryons. In that regime, perturbation theory fails and one must resort to the non-perturbative physics which is computationally expensive and the results are
not sufficiently precise. Also, thus far any attempts to show confinement analytically have proven to be challenging. Hence, it would be highly desirable if one can explore strong dynamics with an analytical approach. In such a pursuit, we would like to study SQCD-like theories and analyze their behaviour.

## 6.1 Overview

In this section, we review supersymmetry and most of the details follow from a supersymmetry primer by Stephen P. Martin [46]. Supersymmetry (SUSY) is a space-time symmetry that relates fermions and bosons. SUSY transformations convert a fermionic state into a bosonic and vice versa. An operator $Q$ that generates SUSY transformation is fermionic, so it must be an anti-commuting spinor such that

$$Q |\text{Fermion}\rangle = |\text{Boson}\rangle , \quad Q |\text{Boson}\rangle = |\text{Fermion}\rangle . \quad (6.1)$$

Since $Q$ (and its Hermitian conjugate $Q^\dagger$) carry spin angular momentum $1/2$, it is clear that supersymmetry must be spacetime symmetry. The generators $Q$ and $Q^\dagger$, by Haag-Lopuszanski-Sohnius theorem, satisfy the following commutation and anti-commutation relations [236]

$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0 ,$$

$$\{Q, Q^\dagger\} = P^\mu , \quad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0 , \quad (6.2)$$

where $P^\mu$ is the four-momentum generator for the spacetime translations. In supersymmetry, particles are put into an irreducible representation of the SUSY algebra called supermultiplets and they hold both fermions and bosons. It is interesting to point out that since the squared mass operator $-P^2$ commutes with the supersymme-
try generators $Q$ and $Q^\dagger$, and with all the rotations and translation operators, so the
eigenvalues of $-P^2$ must be equal which implies particles within a given supermul-
tiplet must have identical masses. Moreover, for any supermultiplet, the number of
fermionic $n_F$ and bosonic degrees of freedom $n_B$ must be equal. For example, in the
simplest case of a scalar supermultiplet with a two-component Weyl fermion, there
must be a corresponding complex scalar field to satisfy $n_F = n_B$.

In general, various types of supermultiplets can be constructed; however, they
fall into two main categories: gauge or vector and chiral supermultiplets. Gauge su-
permultiplets are composed of massless spin $-1$ vector bosons and their corresponding
spin $-1/2$ Weyl fermion superpartners. The naming convention for the fermionic su-
perpartners is the suffix ”-ino”, for example $W$'s have superpartners called winos $\tilde{W}$.
On the other hand, chiral supermultiplets contain a Weyl fermion and a complex
scalar field or two real scalars. The scalar superpartners are denoted by the prefix
”s-”, for example the top quark $t$ superpartner is the stop $\tilde{t}$.

In the SM, the Higgs is a complex scalar (spin 0) so it must reside in a chiral
supermultiplet. However, a single Higgs multiplet is not sufficient because it will
lead to gauge anomalies. One of the gauge anomaly cancellations requires $\text{tr}(T_3^2Y) = \text{tr}(Y^3) = 0$ where $T_3$ is the third component of the weak isospin and $Y$ is the weak
hypercharge. The fermionic superpartner of the Higgs must be an isodoublet with
weak hypercharge $Y = \pm 1/2$, so such a fermion leads to a non-zero trace unless
there are two Higgs supermultiplets. Furthermore, due to the structure imposed
by supersymmetry, the $Y = +1/2$ gives masses only to the up-type quarks while
the $Y = -1/2$ Higgs chiral multiplet gives masses to the down type quarks. So
the complete field content of the Minimal Supersymmetric Standard Model (MSSM)
comprises doubling the SM particles with two scalar Higgs doublets (Higgsinos).
6.2 Supersymmetry Breaking Mechanism

In the scenario of exact supersymmetry, all the Standard Model particles will be degenerate with their superpartners, so all the predicted particles would have identical masses to their SM counterparts. The lack of discovery of any of the superpartners implies that supersymmetry must be broken in nature. In particle physics, we can either have explicit or spontaneous symmetry breaking. In the latter case, the Lagrangian is supersymmetrically invariant while the vacuum state is not invariant under a supersymmetry transformation. Furthermore, one of the main motivations of SUSY is to resolve the hierarchy problem despite supersymmetry breaking, the breaking must be "soft" (SUSY breaking terms that do not reintroduce quadratically divergent contributions to the scalar masses). In general, the possible soft SUSY breaking terms take the following form:

\[-\mathcal{L}_{\text{soft}} = \frac{1}{2} M_a \lambda^a \lambda^a + m_{ij} \phi^i \phi^j + \frac{b_{ij}}{2} \phi^i \phi^j + \frac{a_{ijk}}{3!} \phi^i \phi^j \phi^k + \text{h.c.} , \quad (6.3)\]

where the index $a$ labels the gauge group, $\lambda$ is a fermionic field called a gaugino, and $\phi$ is a complex scalar field. The first, second, third, and fourth terms are the soft gaugino masses, soft scalar mass-squares, soft bilinear scalar interactions, and soft trilinear scalar interactions respectively. Note that we have not included the tadpole term $t^i \phi_i$ since it requires $\phi_i$ to be a gauge singlet which does not exist in MSSM.

A very useful parametrization for constructing a supersymmetric Lagrangian is the quantity called superpotential $W$ which is a holomorphic function of the chiral superfields not their complex conjugates with dimensions $\leq 3$.\textsuperscript{1} In general, the superpotential, in terms of superfields $\Phi_i$ which contains all the bosonic, fermionic, and

\textsuperscript{1}We can have a non-renormalizable superpotential with dimensions $> 3$. 
auxiliary fields (fields without kinetic terms), can be expressed as:

\[ W = W(\Phi_i) = \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{3!} y^{ijk} \Phi_i \Phi_j \Phi_k , \tag{6.4} \]

where we have not included linear terms \( L^i \Phi_i \) since the MSSM does not contain any gauge singlet chiral supermultiplets \( \Phi_i \). Then the scalar potential in general is composed of the two pieces called the F-term and D-terms (for gauge theories). However, our interest mainly lies in non-gauge theories without D-terms, so only F-terms contribute and the scalar potential takes the form

\[ V \supset \sum_i F_i^\dagger F_i = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 , \tag{6.5} \]

where \( \phi_i \) is a complex scalar field. It is evident from the form of the scalar potential in Eq. (6.5), since it is a sum of squares, that is \( V \geq 0 \). It can be shown that spontaneous symmetry breaking is guaranteed in models where the equations \( F_i = 0 \) cannot be simultaneously solved for any values of the fields [46].

There are three main types of supersymmetry breaking mediation mechanisms: gravity mediation, gauge mediation, and anomaly mediation [59, 60, 237–242]. We only focus on the latter case of anomaly mediation which is explored in the next section.

### 6.3 Anomaly Mediation and Supersymmetric QCD

In this section, we explore the anomaly mediation mechanism [59, 60]. Suppose a chiral superfield \( X \) with F-term SUSY breaking occurs in a hidden sector and it is communicated to the visible sector with some operators. Anomaly mediation requires supergravity and local supersymmetry as a messenger of SUSY breaking. The
mediation is essentially due to the presence of the F-component vev of the Weyl compensator (a field that can be gauged away), $\Phi = 1 + \theta^2 m$, with $m$ being the anomaly mediation SUSY breaking parameter and $\theta$ is complex two-component anticommuting spinor. The SUSY breaking is related to the conformal (scale) anomaly manifested by the running of the coupling constants. The mediation of supersymmetry breaking effects is entirely due to supergravity and it results in anomaly-mediated supersymmetry breaking (AMSB). The local superconformal invariance (extension of scale invariance) is spontaneously broken with vev $\langle \Phi \rangle = 1$. Furthermore, the auxiliary field component also obtains a vev, for spontaneously broken supersymmetry $\langle F \rangle \neq 0$, we have

$$\langle F_\Phi \rangle \sim \frac{\langle F \rangle}{M_P} \sim m,$$  

(6.6)

where $M_P$ is the reduced Planck mass and $m$ (generally denoted by $m_{3/2}$) is the gravitino mass. Anomaly-mediated SUSY breaking has an interesting property called ultraviolet insensitivity, meaning that SUSY breaking terms are insensitive to physics at high energy scales and can be computed at any scale in terms of the parameters of the theory at that scale \cite{59,60}. The relation in Eq. (6.6) can be explained purely by dimensional grounds as follow: the gravitino mass must vanish in the absence of SUSY breaking ($\langle \Phi \rangle = 1$) and in the limit when gravitational interaction interactions can be ignored ($M_P \to \infty$). Furthermore, Anomaly mediation appears because the conformal compensator field, $\Phi$ couples to all sources of scale invariance violation and the soft symmetry breaking terms can be computed using the renormalization group
equation. The contributions are:

\[ M_a = mβ_{g_a} \gamma_a \]

\[
(m^2)^i_j = \frac{1}{2} m^2 \frac{d}{dt} \gamma^i_j = \frac{1}{2} m^2 \left[ β_{g_a} \frac{∂}{∂g_a} + β_{y_{kmn}} \frac{∂}{∂y_{kmn}} + β_{y^*_{kmn}} \frac{∂}{∂y^*_{kmn}} \right] \gamma^i_j, \quad (6.7)
\]

\[
a^{ijk} = -mβ_{y^{ijk}},
\]

where the index \( a \) runs over the adjoint representation of the gauge group, \( g_a \) is the gauge coupling, \( M_a \) are the gaugino masses, \( \gamma^i_j \) are anomalous dimensions, \( β_{g_a} \) are the renormalization of gauge couplings (with \( t = \log(Q/Q_0) \) where \( Q \) and \( Q_0 \) are the renormalization and reference scale respectively) and

\[ β_{y^{ijk}} = \frac{d}{dt} y^{ijk} = \gamma^i_n y^{njk} + \gamma^j_n y^{ink} + \gamma^k_n y^{ijn}. \quad (6.8)\]

In general, at one-loop order, the anomalous dimensions and the renormalization of gauge couplings are:

\[ \gamma^i_j = \frac{1}{16\pi^2} \left[ \frac{1}{2} y^{inm} y_{jm}^* - 2g_a^2 C_a(i)δ^i_j \right], \]

\[ β_{g_a} = \frac{d}{dt} g_a = \frac{1}{16\pi^2} g_a^3 \left[ \sum_i I_a(i) - 3C_a(G) \right], \quad (6.9)\]

where \( I_a(i) \) is the Dynkin index of the chiral supermultiplet \( φ_i \), and \( C_a(i)^2 \) is the quadratic Casimir group theory invariants for a given superfield and they can be expressed in terms of the Lie algebra generators as

\[ (T^a T^a)_i^j = C_a(i) δ^j_i. \quad (6.10)\]

\(^2C_a(G)\) is 0 for \( U(1) \) and \( N \) for \( SU(N) \). The index \( a \) here labels the gauge group.
The soft SUSY breaking terms are obtained from the following Lagrangian

\[ \mathcal{L} = \int d^4 \theta \Phi^* \Phi K + \int d^2 \theta \Phi^3 W + \text{c.c.}, \tag{6.11} \]

where \( K \) is called the Kähler potential. Rescaling the fields \( \phi_i \rightarrow \frac{\phi_i}{\mathcal{F}} \), the tree-level supersymmetry breaking contribution from the violation of conformal invariance is

\[ \mathcal{L}_{\text{tree}} = m \left( \phi_i \frac{\partial W}{\partial \phi_i} - 3W \right) + \text{c.c.}. \tag{6.12} \]

Note that since conformal invariance holds classically for dimensionless coupling constants, at tree-level there are no supersymmetry breaking contributions. Finally, at the one-loop level, the SUSY breaking terms are [57, 59, 60]:

\[ \mathcal{L}_{\text{loop}} = -\frac{1}{4} \gamma_i m^* m \phi_i^* \phi_i + \frac{1}{2} (\gamma_i + \gamma_j + \gamma_k) m \lambda_{ijk} \phi_i \phi_j \phi_k + \text{c.c.}. \tag{6.13} \]

In the next section, we apply these relations to compute the soft supersymmetry contribution.

### 6.4 Supersymmetric Quantum Chromodynamics

In a series of groundbreaking papers, Seiberg and collaborators were able to understand the dynamics of supersymmetric QCD-like theories. Even at strong coupling. This understanding is in general possible only due to the strong constraints that supersymmetry put on a theory. More recently it was realized that, due to its UV insensitivity, AMSB can be used to make progress in our understanding of non-supersymmetric theories as well. Some of the questions to explore in such theories are, is there a phase transition as one varies the size of the SUSY breaking scale,
the number of colours and the number of flavours? What are the symmetry-breaking patterns for any given number of flavours/colours?

As a first step, we will describe some of the results that were first presented by Seiberg and his collaborators in a series of groundbreaking works particularly the $SU(N_c)$ supersymmetric gauge theory with $N_f$ matter supermultiplets in the fundamental and anti-fundamental representations [243–245]. In SQCD, quarks and squarks are grouped into chiral superfields in the $N_c$ or $\overline{N}_c$ representations of the $SU(N_c)$ Yang-Mills theory. For clarity, we will represent the superfields by $Q^a_i$ and $\overline{Q}^i_a$ where $a = 1, \ldots, N_c$ denotes the color index and $i = 1, \ldots, N_f$ labels the flavour index. Unless explicitly stated, we will assume the quark superfields are massless; hence, the supersymmetry theory has the following global symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_R \times U(1)_B ,$$

(6.14)

where $B$ refers to the baryon number and $R$ denotes the R-symmetry. To understand the behaviour of a given theory below the confinement scale, we need to construct an effective superpotential in terms of gauge-invariant chiral superfields. To such end, one of the gauge-invariant objects is the meson field

$$M^{ij} = Q^i \overline{Q}^j , \quad i, j = 1, \ldots, N_f .$$

(6.16)

However, as the number of flavours is increased, there are other gauge-invariant chiral

---

$^3$R-symmetry is a global $U(1)$ symmetry with the anti-commuting coordinates $\theta$ and $\theta^\dagger$ carrying $R$ charges of +1 and −1 respectively,

$$\theta \rightarrow e^{i\alpha} \theta , \quad \theta^\dagger \rightarrow e^{-i\alpha} \theta^\dagger .$$

(6.15)

where $\alpha$ parametrizes the transformation. A superpotential $W$ have $R$ charge of +2.
fields (for $N_f \geq N_c$) with the quantum number of baryons

$$B_{i_1...i_{N_f-N_c}} = \epsilon_{j_1...j_{N_c} i_1...i_{N_f-N_c}} \epsilon^{k_1...k_{N_c}} Q^j_{k_1} \ldots Q^{2N_c}_{k_{N_c}}.$$  \hspace{1cm} (6.17)

For $N_f < N_c$: the dynamics are described by the non-perturbative Affleck–Dine–Seiberg (ADS) superpotential in terms of mesons and there are no baryon operators

$$W = (N_c - N_f) \left( \frac{\Lambda^{3N_c-N_f}}{\det M} \right)^{\frac{1}{N_c-N_f}},$$  \hspace{1cm} (6.18)

where $\Lambda$ is called the intrinsic scale of the non-Abelian gauge theory. This result is obtained by using symmetry arguments and holomorphy. This particular case has no ground states since it has run-away potential which can be deduced from the following potential

$$V = \left| \frac{\partial W}{\partial M} \right|^2 \sim |M|^{-\frac{2N_c}{N_c-N_f}}, \text{ with } \det M \sim M^{N_f}.\hspace{1cm} (6.19)$$

For $N_f = N_c$: the dynamics and the quantum moduli space (space of vevs where the scalar potential vanishes), the manifold of supersymmetric ground states, are described by

$$W = X \left( \det M - \bar{B}B - \Lambda^{2N_c} \right),$$  \hspace{1cm} (6.20)

where $X$ is a Lagrange’s multiplier field, an auxiliary chiral superfield, of the theory that enforces the quantum modified constraints. The superpotential can be re-
expressed, by going to canonical normalization of the fields, as

\[ W = X \left( \lambda \frac{\det M}{\Lambda^{N_c-2}} - \kappa BB - \Lambda^2 \right), \tag{6.21} \]

with \( \lambda \) and \( \kappa \) being the dimensionless coupling constants and \( W \) with energy dimensions of 3. There is a manifold of ground states satisfying the relation \( \left( \frac{\partial W}{\partial X} = 0 \right) \)

\[ \lambda \frac{\det M}{\Lambda^{N_c-2}} - \kappa BB = \Lambda^2. \tag{6.22} \]

All of the above results are taken from the literature and more details can be found in [57, 235, 246]. In Refs. [57, 246], it was shown that SQCD perturbed with AMSB could be used to determine the chiral symmetry breaking pattern by minimizing the total scalar potential for various numbers of flavours and colours along a particular meson superfield and there are well-defined non-trivial minima for SQCD with AMSB (for \( N_c \geq 3 \)). In the next section, we will explore the most interesting scenario of our research the \( N_f = N_c + 1 \) and show our main result, which has not yet been explored in the literature, for \( N_f = 3 \) and \( N_c = 2 \) with AMSB perturbation.

### 6.5 S-Confinement

The theory for \( N_f = N_c + 1 \) is \textit{s-confining}. S-confinement means a theory with a non-vanishing confining superpotential and smooth confinement without chiral symmetry breaking. A theory is confining when its low energy physics can be completely described by gauge-invariant composites and their interactions. The origin of the classical moduli space (space of inequivalent ground states of the classical theory) remains the vacuum in the quantum theory and there is no spontaneous symmetry breaking (no spontaneous chiral symmetry breaking). The dynamical superpotential
for $N_f = N_c + 1$ is given in terms of the baryon $B$ and meson $M$ composites as (for $N_c > 2$)

$$W = \frac{1}{\Lambda^{2N_c-1}} (\det M - B M \overline{B}) .$$  \hspace{1cm} (6.23)

The theory respects the full global symmetry $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$. We can apply the equations of motion, F-flatness conditions\(^4\), to obtain the following set of relations:

$$\frac{\partial W}{\partial B} = 0, \quad \frac{\partial W}{\partial \overline{B}} = 0 \Rightarrow M \overline{B} = 0, \quad B M = 0, \quad \frac{\partial W}{\partial M} = 0 \Rightarrow M^{-1} \det M - B \overline{B} = 0 .$$  \hspace{1cm} (6.25)

We see that the origin $B = \overline{B} = M = 0$ satisfies the above restrictions; hence, it is a point on the moduli of vacuum states and the full global symmetry remains unbroken. Expressing the superpotential by going into canonical normalization

$$W = \lambda \frac{\Lambda^{N_f - 3}}{\Lambda^{N_f - 3}} \det M - \kappa B M \overline{B} ,$$  \hspace{1cm} (6.26)

where $\lambda$ and $\kappa$ are dimensionless parameters. It is important to emphasize that all the previous computations are for the unbroken SUSY, and SUSY breaking perturbations are considered in the subsequent calculations. Then the tree-level AMSB potential

\(^4\)The relations $F_j = 0$ known as the "F-flatness" condition where

$$F_j = \frac{\partial W}{\partial \Phi_j} = 0 ,$$  \hspace{1cm} (6.24)

and satisfying it leads to a supersymmetric vacuum state.
can be written as

\[ V_{\text{AMSB}} = -m (N_f - 3) \frac{\lambda}{\Lambda^{N_f-3}} \det M + \text{h.c.} \]  \hspace{1cm} (6.27) \]

So the total scalar potential with AMSB (tree level) along the direction \( M^{ij} = \phi \delta^{ij} \) is [57]:

\[ V = \sum_i \left| \frac{\partial W}{\partial \phi^i} \right| + V_{\text{AMSB}} \]

\[ V = N_f \frac{\lambda^2}{\Lambda^{2N_f-2}} |\phi|^{2N_f-2} - m \frac{\lambda}{\Lambda^{N_f-3}} (N_f - 3) \phi^{N_f} + \text{h.c.} \]  \hspace{1cm} (6.28) \]

The term proportional to \( m \) is due to AMSB. The potential has a minimum \((\frac{dV}{d\phi} = 0, \frac{d^2V}{d\phi^2} > 0)\) at

\[ \phi = \Lambda \left[ \frac{m (N_f - 3)}{\lambda \Lambda (N_f - 1)} \right]^{\frac{1}{N_f-2}}. \]  \hspace{1cm} (6.29) \]

The above computation holds for \( N_f > 3 \) and it is thoroughly studied in [57]. Note that the tree-level AMSB contribution in Eq. (6.27) vanishes \( V_{\text{AMSB}} = 0 \) for \( N_f = 3 \), so we need to compute the one-loop contributions and that will be our main result.

Furthermore, the \( N_f = N_c + 1 = 3 \) when \( N_c = 2 \), theory has a different superpotential with the determinant replaced by the Pfaffian. There is no difference between quarks and anti-quarks, \( Q = \bar{Q} \), with a total of \( 3Q + 3\bar{Q} = 6Q \) quarks and the global flavour symmetry \( SU(6) \). The mesons \( M^{ij} = Q^i \bar{Q}^j \) are in the anti-symmetric representation of \( SU(6) \) and there is no baryons operator in the superpotential. The superpotential,

\[ T_a = \sigma_a / 2. \]

The fundamental and complex representations are equivalent because \( \sigma_a = -\sigma_2 \sigma_a^* \sigma_2 \) and the representation is pseudoreal since the transformation matrix \( \sigma_2 \) is anti-symmetric.
Chapter 6. Supersymmetry

at low energies, in terms of the meson operator is

\[
W = -\frac{1}{\Lambda^3} \text{PfM} \equiv \kappa \text{PfM}
\]

\[
W = \frac{\kappa}{2^3 \cdot 3!} \epsilon_{i_1j_1i_2j_2i_3j_3} M^{i_1j_1} M^{i_2j_2} M^{i_3j_3}.
\]  

(6.30)

Starting with the anomalous dimensions using Eq. (6.9)

\[
\gamma_i = \frac{1}{16\pi^2} \left[ \frac{1}{2} y^{imn} y^{*}_{imn} \right] = \frac{1}{16\pi^2} (3\kappa \kappa^*) = \frac{3|\kappa|^2}{16\pi^2},
\]

(6.31)

where \(i, m, n = 1, 2, 3\) are the flavour indices and

\[
\frac{d\kappa}{dt} = 3 \gamma_i \cdot \kappa = \frac{3(3\kappa \kappa^*)}{16\pi^2} \kappa = \frac{9\kappa^3}{16\pi^2}.
\]  

(6.32)

Consequently,

\[
\dot{\gamma} = \frac{d\gamma}{dt} = \frac{3}{16\pi^2} \left(2\kappa \frac{d\kappa}{dt}\right)
\]

\[
\dot{\gamma} = 6 \left(\frac{3}{16\pi^2}\right)^2 |\kappa|^4 = \frac{27|\kappa|^4}{128\pi^4}.
\]  

(6.33)
Hence, the scalar potential is:

\[ V = \sum \left| \frac{\partial W}{\partial \Phi_i} \right| + \frac{1}{4} \gamma m^* m \phi_i^* \phi_i - \frac{1}{2} (\gamma_i + \gamma_j + \gamma_k) m \lambda_{ijk} \phi_i \phi_j \phi_k + \text{h.c.} \]

\[ V = \kappa^2 \left[ \frac{\partial \text{Pf}(M)}{\partial M_{ij}} \right] \left[ \frac{\partial \text{Pf}(M^*)}{\partial M^*_{ij}} \right] + \frac{1}{4} \left[ \frac{27 \kappa^4}{128 \pi^4} \right] m^2 \frac{\text{tr} (M^\dagger M)}{2} - \frac{1}{2} 3 \left[ \frac{3 \kappa^2}{16 \pi^2} \right] m \kappa \text{Pf} M + \text{h.c.} \]  

Suppose we choose the vacuum configuration to be a skew-symmetric tridiagonal matrix

\[ M = i \phi \begin{pmatrix} \sigma_2 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_2 \end{pmatrix}, \]  

where \( \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \) is the 2 \( \times \) 2 Pauli matrix. The main reason to pick such vacuum direction is due to the presence of the Pfaffian, which is non-vanishing for even dimensional skew-symmetric matrices, in the superpotential. If \( \phi \neq 0 \), the
symmetry breaking pattern would be $SU(6) \to Sp(6)$. Then
\[
\sum_{i,j} \left[ \frac{\partial \text{Pf}(M)}{\partial M_{ij}} \right] \left[ \frac{\partial \text{Pf}(M^*)}{\partial M_{ij}^*} \right] = +3 \phi^4,
\]
\[
Pf(M) = +\phi^3,
\]
\[
\text{tr}(M^\dagger M) = +6 \phi^2.
\]
(6.36)

For this particular scenario,
\[
V = +3 \kappa^2 \phi^4 + \frac{81}{512 \pi^4} m^2 \kappa^4 \phi^2 - \frac{9}{16 \pi^2} \kappa^3 m \phi^3.
\]
(6.37)

The scalar potential in Eq. (6.37) has no real roots besides zero; therefore, the potential has no minimum away from the origin. So the theory has no global symmetry breaking and the full global symmetry, $SU(6)$, remains intact with $M_{ij} = 0$ being the global vacuum. Unlike the $N_f > 3$ case with the scalar potential in Eq. (6.28), which does have a non-trivial global vacuum with spontaneous symmetry breaking.

All of the computations so far work in the $N_f = 3$ massless flavour regime. How is the theory transformed if we add a mass term to the superpotential for the $N_f$-th flavour? In such realization, the superpotential for the massive theory takes the form
\[
W = \kappa \text{Pf}(M) \to W = \kappa \text{Pf}(M) + \mu \Omega \Lambda,
\]
(6.38)

where $\mu$ is the mass of the $N_f$-th flavour, $M'$ is $4 \times 4$, $Z$ and $Y$ are $4 \times 2$ and $2 \times 4$.
matrices respectively. Apply the F-flatness conditions (equations of motion),

\[
\frac{\partial W}{\partial Y} = 0 = \kappa \frac{1}{2} (\det M)^{-1/2} \frac{\partial \det M}{\partial Y} = \frac{\kappa}{2} \frac{1}{\text{Pf}(M)} \text{cof}(Y),
\]

\[
\frac{\partial W}{\partial Z} = 0 = \frac{\kappa}{2} \frac{1}{\text{Pf}(M)} \text{cof}(Z),
\]

\[
\frac{\partial W}{\partial \Omega} = 0 = \mu \Lambda + \frac{\kappa}{2} \frac{1}{\text{Pf}(M)} (\det M') \ 2\Omega = \mu \Lambda + \kappa \text{Pf}(M'),
\]

where \( \text{cof} Y (\text{cof} M_{i,j} = (-1)^{i+j} \det M_{i,j} \text{ where } M_{i,j} \text{ is the matrix } M \text{ with } i^{th} \text{ row and } j^{th} \text{ column removed}) \) is the cofactor of the matrix \( Y \). From these relations, we obtain the following set of constraints

\[
Y = Z = 0,
\]

\[
\text{Pf}(M') = -\frac{\mu \Lambda}{\kappa}.
\]

Hence the superpotential becomes,

\[
W = \kappa \text{Pf}(M) + \mu \Lambda \Omega = \kappa \Omega \left[ \text{Pf}(M') + \frac{\mu \Lambda}{\kappa} \right].
\]

Here \( \Omega \) serves as a Lagrange’s multiplier field of the parent theory \((N_f = 3)\) that enforces the quantum modified constraints. The superpotential takes on a similar form as the one in Eq. (6.21) for \( N_f = N_c \) massless flavours. In fact, one could start with the superpotential for \( SU(N_c) \) theory with \( N_f \) massless flavours and add a mass term for the \( N_f^{th} \) flavour, then apply the F-flatness conditions to this superpotential. Next, match the running gauge coupling of the low energy theory onto the high energy theory at the scale \( Q \), at the mass of the \( N_f^{th} \) flavour, to obtain the superpotential
for the $N_f - 1$ massless theory. Explicitly the Pfaffian can be written as,
\begin{equation}
\text{Pf}(M') = \frac{1}{8} \epsilon_{i_1 j_1 i_2 j_2} M^{i_1 j_1} M^{i_2 j_2} = \left[ M^{12} M^{34} - M^{13} M^{24} + M^{14} M^{23} \right].
\end{equation}

Applying the F-flatness conditions to the above superpotential, one arrives at
\begin{align}
\frac{\partial W}{\partial M_{ij}} &= \frac{\kappa}{4} \Omega \epsilon_{ij j_2} M^{i j_2} = 0, \\
\frac{\partial W}{\partial \Omega} &= \kappa \text{Pf}(M') + \mu \Lambda = 0.
\end{align}

Working along a particular vacuum direction for $M^{13} = M^{24} = \phi$, then the above conditions in Eq. (6.43) reduces to
\begin{equation}
-2\kappa \phi \Omega = 0, \quad -\kappa \phi^2 + \mu \Lambda = 0 \Rightarrow \phi_0^2 = +\frac{\mu \Lambda}{\kappa}, \quad \Omega_0 = 0.
\end{equation}

Next up, let’s attempt to include AMSB contributions, $m$, by doing perturbation around these known set of solutions, $\phi_0$, computed above. One can write the perturbation as,
\begin{equation}
\phi = \phi_0 + a \, m, \\
\Omega = \Omega_0 + b \, m.
\end{equation}

Then the total scalar potential can be obtained as follow:
\begin{equation}
V = \left| \frac{\partial W}{\partial M_{ij}} \right|^2 + \left| \frac{\partial W}{\partial \Omega} \right|^2 + V_{\text{AMSB}},
\end{equation}
where

\[ V_{\text{AMSB}} = +2 \left[ m \mu \Lambda \Omega + m^* \mu^* \Lambda \Omega^* \right], \]

\[
\left| \frac{\partial W}{\partial M^{ij}} \right|^2 + \left| \frac{\partial W}{\partial \Omega} \right|^2 = \kappa^2 \Omega \Omega^* \sum_{i_2 > j_2} M^{i_2 j_2} M^{i_1 j_1} + \kappa^2 \left[ \text{Pf}(M') + \frac{\mu \Lambda}{\kappa} \right] \cdot \left[ \text{Pf}(M') + \frac{\mu \Lambda}{\kappa} \right]^*.
\]

(6.47)

Thus, the total scalar potential is of the form,

\[
V = +2 \left[ m \mu \Lambda \Omega + m^* \mu^* \Lambda \Omega^* \right] + \kappa^2 \Omega \Omega^* \sum_{i_2 > j_2} M^{i_2 j_2} M^{i_1 j_1} + \kappa^2 \left[ \text{Pf}(M') + \frac{\mu \Lambda}{\kappa} \right] \cdot \left[ \text{Pf}(M') + \frac{\mu \Lambda}{\kappa} \right]^*.
\]

(6.48)

Keeping terms only up to \( O(m^2) \), we can compute the parameters \( a \) and \( b \) by minimizing the scalar potential, \( \frac{\partial V}{\partial a} = \frac{\partial V}{\partial b} = 0 \), implies \( a = 0 \) and \( b = -\frac{1}{\kappa} \).

As we have stated earlier, the superpotential term \( W \supset \kappa \text{Pf}(M) \) respects \( SU(6) \) symmetry. While the explicit breaking mass term \( W \supset \mu \Omega \Lambda = \text{tr} (A \cdot M) \) breaks the
symmetry explicitly as $SU(6) \rightarrow SU(4) \times SU(2)$ where $A$ is:

$$A = \frac{1}{2} \mu A \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}. \tag{6.49}$$

Consequently, there are a total of $15 + 3 = 18$ unbroken generators corresponding to the $SU(4) \times SU(2)$ symmetry. Applying the generators of $su(6)$ to the vacuum configuration

$$\langle M \rangle = \begin{pmatrix}
0 & 0 & \phi_0 & 0 & 0 \\
0 & 0 & 0 & \phi_0 & 0 \\
-\phi_0 & 0 & 0 & 0 & 0 \\
0 & -\phi_0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \Omega_0 \\
0 & 0 & 0 & 0 & -\Omega_0 \\
\end{pmatrix}, \tag{6.50}$$

where $\Omega_0 = -\frac{m}{\kappa}$ and $\phi_0^2 = \frac{\mu A}{\kappa}$, the residual symmetry (unbroken generators) is $Sp(4) \times SU(2)$. In this case, the $SU(6)$ symmetry is spontaneously broken to $Sp(4) \times SU(2)$ by the vacuum configuration while the superpotential respects $SU(4) \times SU(2)$. From Goldstone’s theorem, $SU(4) \times SU(2) \rightarrow Sp(4) \times SU(2)$ would imply a total of $18 - 13 = 5$ massless Nambu–Goldstone bosons corresponding to the broken generators.

For the massless flavour scenario ($N_f = 3$ and $N_c = 2$), we have computed the non-vanishing AMSB contribution at one-loop to the scalar potential. For this
particular theory, minimizing the full scalar potential along $M^{ij} = \phi \delta^{ij}$ leads to no real roots suggesting there are no vacua besides the origin. This is qualitatively different from the other s-confining $SU(N)$ theories, which do have non-trivial vacua away from the origin, making it interesting to explore.

On of the most intriguing aspect of the SM QCD is quark confinement. In the SM, due to asymptotic freedom, quarks confine into hadrons around $\Lambda_{QCD}$ where the quark condensate forms at the critical temperature $T_C$. Below the critical temperature, the vacuum state contains quark-antiquark pairs with non-zero vacuum expectation value $\langle \bar{\psi} \psi \rangle \neq 0$, with $\psi$ being the 4-component Dirac spinor field. So the ground state of the system is formed away from the origin unlike our supersymmetric scenario with 2 colours and 3 flavours which does not generate a non-zero vacuum. Moreover, our SUSY model (for $N_f = N_c + 1$) is s-confining in the IR which differs from SM QCD where confinement occurs with chiral symmetry breaking. In general, the expectation is that when the SUSY breaking parameter becomes large, relative to the dynamical scale of the gauge theory, the non-supersymmetric theory can be recovered [57, 247].
Chapter 7

Conclusion

Mostly guided by the nature of the electroweak scale in the SM and the origin of the matter and anti-matter asymmetry conundrum, we explored the genesis of a matter and anti-matter asymmetry and the source of dark matter in the context of the naturalness framework. The research projects included a focus on the potential signals and limits imposed by the current LHC data on the top partner decay models specifically for the multijet topologies.

Chapter 4 investigated a model for the baryon asymmetry and dark matter. The mechanism for baryogenesis is considered in the naturalness model where the inclusion of Majorana fermions allows us to generate CP-asymmetry due to the interference of their tree and one-loop decays. The lepton asymmetry is transferred to the reheatons which via their decay is partially transferred into the baryon asymmetry into the various $N$ sectors by the electroweak sphaleron. It is imperative for the reheatons to predominantly decay into the SM sector to avoid instant exclusion from cosmological constraints; hence, most of the lepton asymmetry is transferred into our sector. Although, the decays of reheatons are subdominant to the other sectors; the distinct nature of the sectors with a positive Higgs mass squared pa-
rameter, $m^2_H > 0$, allows the dark baryon asymmetry to be sufficiently large which facilitates a viable dark matter model. The dark baryons of the exotic sectors serve as an asymmetric DM candidate. On the other hand, sectors with a negative Higgs mass squared parameter, $m^2_H < 0$, are less efficient in converting lepton into baryon asymmetry. Thus they are found to lack the sufficient amount of relic abundance for dark matter required by observations. We performed a detailed analysis of standard and exotic sectors considering the current experimental constraints on $N_{\text{eff}}$, observed baryon asymmetry, and the dark matter relic abundance. The summary of all the results is presented in Figure 4.11.

Chapter 5 analyzes a top partner model. We consider a simplified scenario investigating the pair production of top partners that decays to multiple hadronic jets. The final state topologies contain no leptons or missing energy and we studied the latest LHC constraints on such top partners. In particular, we examined two main decay channels: firstly, the top partner decaying into a light jet and a scalar which subsequently decays into two light flavour jets. Secondly, we consider the top partner decays into a light flavour jet with the scalar decaying into two bottom jets. Although one might expect the limits in the presence of b-jets to be stronger relative to the light jet scenario, we found the constraints to be quite similar. The excluded regions for our model are illustrated in Figure 5.5 and Figure 5.7. The resonances which eventually decay fully hadronically resulting in the final state with six jets are extremely difficult to find since they are prone to the considerable QCD multijet background and one might expect the limits to be substantially weaker than the traditional decay modes. In spite of such challenges, the current LHC searches can also impose strong limits on our model.

Finally, in Chapter 6, we consider supersymmetric QCD for a fixed number of
flavours and colours. In particular, we examine the behaviour of s-confining theories (without chiral symmetry breaking) for $N_f = 3$ and $N_c = 2$. We perturb the theory with the soft SUSY-breaking terms due to anomaly mediation. Studying the behaviour of the scalar potential reveals that there is no minimum away from the origin—including the soft SUSY-breaking contribution.
Appendix A

Matter Anti-matter Asymmetry

The materials contained in these appendices, with the exception of Section A.5, are published in [1] and they expand on some of the concepts/computations in Chapter 4.

A.1 Mass Spectrum of \( N \) Sectors

In this section, we study the mass spectrum of the \( N \) sectors. The “standard sectors”, with \( m_{H,k}^2 < 0 \), corresponding to \( k \geq 0 \), exhibit electroweak symmetry breaking just like the SM case, with the exception that the vacuum expectation value (vev) of the Higgs is given by:

\[
v_k^2 = -\frac{(m_{H}^2)_k}{\lambda} = v_0^2 \mathcal{C}_k, \tag{A.1}
\]

where \( \mathcal{C}_k = \frac{2k + r}{r} \) and the parameter \( r \) indicates the spacing between sectors, with \( r = 1 \) corresponding to uniform spacing and \( r < 1 \) corresponding to a large splitting between our sector and the next one [47]. The sector with the smallest absolute value of \( m_{H,k}^2 \), \( k = 0 \), corresponds to our SM sector, with \( v_0 = 246 \) GeV and \( (m_{H}^2)_0 = -\frac{\Lambda^2}{N} r \simeq -(88.4 \text{ GeV})^2 \). In these sectors, the electroweak symmetry is broken by the Higgs vev given in Eq. (A.1) and, consequently, the masses of the particles (both
Appendix A. Matter Anti-matter Asymmetry

fermions and gauge bosons) will increase proportionally to $\sqrt{k}$. In particular, for $k > 10^8$, the quarks are heavier than their corresponding QCD scales, meaning that those heavy sectors will not feature baryons [47].

On the other hand, “exotic” sectors, with $m_H^2 > 0$ and corresponding to $i < 0$, are radically different from the SM [126,248,249]. Since $m_H^2 > 0$, these sectors do not acquire a vev for the Higgs and the electroweak symmetry is broken at low scales by the phase transition from free quarks to confinement at the QCD scale, $\Lambda_{\text{QCD}}$ [126]. In these sectors, fermions masses, $m_{f,i}$, are obtained from the four-fermion interactions with the Higgs being integrated out:

$$m_{f,j} \sim y_f y_t \frac{\Lambda_{\text{QCD}}^3}{(m_H^2)_{j}} = -\frac{\sqrt{2}y_t}{\lambda} \left( \frac{\Lambda_{\text{QCD}}}{v_0} \right)^3 \frac{m_{f,0}}{C_i} \lesssim 100 \text{ eV ,} \quad (A.2)$$

where $y_t$ and $y_f$ are the top and the fermion $f$ Yukawa couplings, respectively, and $m_{f,0}$ is the mass of the given fermion in the SM. The gauge bosons receive masses when $SU(3)$ confines and their mass is given by [126]:

$$m_W^2 = \left( \frac{g}{2} \right)^2 f_\pi^2, \quad m_Z^2 = \left( \frac{g + g'}{2} \right)^2 f_\pi^2 , \quad (A.3)$$

where $f_\pi$ is the pion decay constant, $g$ and $g'$ are the $SU(2)$ and $U(1)$ gauge coupling, respectively. As all six flavour quarks are lighter than the $SU(3)$ confinement, there will be many more light hadrons than in the SM [248]. The spontaneous breaking of $SU(6) \times SU(6) \rightarrow SU(6)$ results in 35 pseudo-Goldstone bosons, three of which are absorbed to become the longitudinal polarization components of the $W$ and $Z$ bosons, and the remaining ones are analogous to the SM pions. The masses of the pions in the various sectors through the QCD phase transition can be obtained by
applying the well known Gell-Mann–Oakes–Renner relation \([65,250]\):

\[
m_\pi^2 = \frac{V^3}{f_\pi^2} (m_u + m_d),
\]  

(A.4)

where \(V \sim \Lambda_{\text{QCD}}\). Assuming \(V^3/f_\pi^2 \sim V_i^3/f_{\pi,i}^2\), the pion’s masses in the “standard” and “exotic” sectors are:

\[
m_{\pi,i}^2 \simeq \begin{cases} 
\frac{C_i^{1/2} m_\pi^2}{(m_a,i + m_b,i)} m_\pi^2, & \text{Standard} \\
\frac{(m_a,i + m_b,i)}{(m_u + m_d)} m_\pi^2, & \text{Exotic}
\end{cases}
\]  

(A.5)

where \(a\) and \(b\) denote the flavour of the component quark given in Eq. (A.2), and \(m_\pi\) is the experimentally measured pion mass. We have ignored corrections due to the changes in running couplings induced by different quark masses, although those effects are detailed in [119].

With the spectrum, we can compute the effective number of relativistic degrees of freedom (d.o.f), \(g_\star(T)\), for each sector. The \(i = 0\) sector has \(g_\star = 106.75\) at \(T \gtrsim 100\) GeV. As the temperature decreases, the various particle species become non-relativistic and they need to be removed from the total \(g_\star\) value. The top quark is the first particle to decouple at \(T \sim \frac{1}{6} m_t\), reducing the number of relativistic d.o.f to \(g_\star = 96.25\) and, similarly, the rest of the particles above QCD scale follows. There is a significant drop in \(g_\star\) when the QCD phase transition occurs, at \(T \sim 200\) MeV, with quarks and gluons confined into hadrons, and the only particles being left are three pions, electrons, muons, neutrinos, and photons, resulting in total \(g_\star = 17.25\). These particles, with the exception of photons, will also eventually become non-relativistic and decouple as the temperature drops. The story of the relativistic d.o.f is analogous in the \(i > 0\) sectors with different particle species removed from the total
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$g_*(T)$ counting at different times due to the distinct mass spectrum and temperature of those sectors. In the SM sector, photons decouple and form the cosmic microwave background at $T_0 \sim 0.32$ eV, so that the total relativistic d.o.f can be computed as $g^0_{*,\text{Dec}} = 2 + \frac{7}{8} (2 \times 3) \times \left(\frac{4}{11}\right)^{4/3} \approx 3.36$ where the last factor is due to photons being reheated relatively to neutrinos. The standard sectors are colder than $i = 0$, so like the SM the only relativistic particles are only the photons and neutrinos giving $g^k_{*,\text{Dec}} \approx 3.36$.

The number of relativistic degrees of freedom in the exotic sectors is of course different because of the altered spectrum. Firstly, EWSB is triggered by the QCD scale at $T \approx 89$ MeV, above which $g_* = 102.75$, with only the complex Higgs doublet integrated out. After the QCD phase transition, we need to remove the contribution due to all the quarks and gluons being trapped inside hadrons. Moreover, we need to take into account the presence of the 35 pseudo-Goldstone bosons from the spontaneous breaking of $SU(6) \times SU(6) \rightarrow SU(6)$ symmetry, thus reducing the total d.o.f to $g_* = 58.75$. Next, at a temperature approximately $T \sim \frac{1}{6} m_{W,i}$, gauge bosons $W^\pm, Z$ annihilates so $g_* = 58.75 - 9 = 49.75$. The rest of the particle species annihilate analogously to the SM as the temperature drops over time. The temperature of the exotic exotic sector at the time of the photon decoupling can be computed using the relation $T_j/T_0 \simeq \left(\frac{\beta}{1 - \beta - \gamma}\right)^{1/4}$. Using $T^0_{\text{Dec}} \sim 0.32$ eV and the most optimistic value of $\beta \sim 10^{-2}$ leads to $T^j_{\text{Dec}} \sim 0.1$ eV, which is larger than $\frac{1}{6} m_{\tau, -1} \approx 0.03$ eV. Therefore, the total number of relativistic d.o.f, at the time of the CMB, in the first exotic sector is $g^{-1}_{*,\text{Dec}} = 17.75$. 
A.2 Dark Neutrino Freeze-in

As mentioned in Section 4.4.2, electroweak symmetry breaking induces mixing between the SM neutrinos and hidden sector neutrinos in standard sectors. This in turn can mediate a freeze-in production of hidden sector neutrinos which contribute to $\Delta N_{\text{eff}}$. This process actually places strong constraints on a model where the reheaton couples directly to the lepton doublet [47], and we begin by computing that effect.

In that model, the reheaton couples to the $N$ sectors via the coupling

$$-\mathcal{L}_l \supset \lambda S^c \sum_i l_i H_i$$

(A.6)

where $l_i$ is one of the SM-like lepton doublets in the $i$th sector. The dominant process for freeze-in production of neutrinos is then

$$\nu \nu \rightarrow \nu_i \nu_i$$

(A.7)

where $\nu$ corresponds to the SM neutrinos and $\nu_i$ to the neutrinos in the $i$th sector. The leading order diagram for this process is shown in Figure A.1.

Because this process proceeds through a heavy mediator, the dominant freeze-in occurs at high energy. The cross section in the high energy limit ($E \gg M, m_S$), is given by

$$\sigma \simeq \frac{1}{32\pi} \left[ \frac{\lambda^2 v_0 v_i a^2}{m_S (m_i^2 - m_0^2)} \right]^2 m_i^2 \left\{ \frac{4}{M^2} + \frac{1}{E^2} \left[ \frac{1}{2} + \ln \left( \frac{M^2}{4E^2} \right) \right] \right\},$$

(A.8)

where $E$ is SM neutrino’s energy in the CM-frame, $a \equiv \frac{g}{2\cos(\theta_W)}$, with $g$ being the

\(^{1}\)SM flavour indices are ignored for simplicity.
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$SU(2)$ gauge coupling and $\theta_w$ the Weinberg angle, $M \equiv M_Z$ is the $Z$ boson mass, $m_i \equiv m_{\nu_i}$ is the $i$th sector neutrinos mass, $m_0 \equiv m_{\nu_{us}}$ is the mass of the SM neutrinos, $v_0 \equiv v_{us}$ is the SM Higgs vev and $v_i = v_0 \left(\frac{2i+r}{r}\right)^{1/2}$ is the Higgs vev in the $i$th sector.

We will assume a mechanism similar to the well-known see-saw mechanism, such that the neutrinos masses in the $i > 0$ sector scale as $m_i = m_0 \left(\frac{2i+r}{r}\right)$.

We compute the total contributions to $\Delta N_{\text{eff}}$ considering only the freeze-in process and the results are displayed in Figure A.2. In these curves, we set the masses of the neutrino in the SM sector, $m_0$, to $m_0 = 10^{-9}$ eV while the masses of the neutrinos in the lowest standard sector, $m_{+1}$, are larger by one (magenta) or two (blue) orders of magnitude compared to $m_0$. This shows that the $l$-model is entirely excluded, in the relevant region for BAU and DM, for $\beta \gtrsim 10^{-2}$ and $m_{+1} = 10^2 m_0$ for even a single standard sector. By increasing the number of sectors, the freeze-in contribution to $\Delta N_{\text{eff}}$ can be quite large even for $m_{+1} = 10^3 m_0$, so this setup cannot match current observations. Therefore, we include a fourth-generation vector-like lepton to alleviate the experimental limits and resolve the freeze-in issue.

In the model with the fourth-generation vector-like lepton, the freeze-in process can still occur, but it must also pass through the mixing of ordinary neutrinos with the heavy fourth generation. In that case, the cross section in the high energy limit...
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Figure A.2: $\Delta N_{\text{eff}}$ as a function of the reheaton’s width due to the freeze-in process from just one sector. We consider two branching ratios and two different neutrino masses for the lowest standard sector: the solid (dashed) magenta corresponds to $\beta^{-1} = 10^{-2}$ and $m_{+1} = 10^{+1} m_0$ ($m_{+1} = 10^{+2} m_0$) and solid (dashed) blue corresponds to $\beta^{-1} = 10^{-3}$ and $m_{+1} = 10^{+1} (m_{+1} = 10^{+2} m_0)$. The black dotted horizontal lines marks $\Delta N_{\text{eff}} = 0.4$ whereas the shaded region corresponds to the viable region for DM and BAU.

$(E \gg M, m_S)$ is given by

$$\sigma \simeq \frac{1}{32\pi} \left[ \frac{\lambda^2 v_0 v_i a^2}{m_S (m_i^2 - m_0^2)} \right]^2 m_i^2 \left( \frac{\mu_L}{M_L} \right)^4 \left\{ \frac{4}{M^2} + \frac{1}{E^2} \left[ \frac{1}{2} + \ln \left( \frac{M^2}{4E^2} \right) \right] \right\} , \quad (A.9)$$

which is of the same form as Eq. (A.8) with an additional suppression of $(\mu_L/M_L)^4 \sim 10^{-12}$. The evolution of the neutrino number density can be described by the Boltzmann equations as

$$\dot{n}_i + 3 \, H \, n_i = + \langle \sigma_{\nu \nu \rightarrow \nu \nu} | v \rangle \left[ n_{0,\text{EQ}}^2 - n_i^2 \right] , \quad (A.10)$$

where $\langle \sigma_{\nu \nu \rightarrow \nu \nu} | v \rangle$ is the thermally averaged cross section times velocity, $n_i$ is the number density of neutrino in the $i$th sector, and $n_{0,\text{EQ}}$ is the equilibrium number
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density in the SM sector. By defining

\[ x = \frac{m_i}{T}, \quad Y_i = \frac{n_i}{s}, \quad (A.11) \]

where \( s = \frac{2\pi^2}{45} g_{*s} \left( \frac{m_i}{T} \right)^3 \) and \( g_\ast \) is the effective d.o.f of the SM sector, and rewriting

the Hubble parameter as

\[ H(x) = H(m_i) \left( \frac{T}{m_i} \right)^2 = \frac{H(m_i)}{x^2}, \quad (A.12) \]

the Boltzmann equation that tracks the neutrinos' yield in the \( i \)th sector is (assuming

that its initial abundance is negligible, that is, \( Y_i(x_i) = 0 \)):

\[ \frac{dY_i}{dx} = + \frac{\langle \sigma_{\nu\nu \rightarrow \nu_i\nu} |v| \rangle}{H(m_i)} x s Y_i^2_{0, EQ}, \quad (A.13) \]

where

\[ Y_{0, EQ} = \frac{135}{7\pi^4} \frac{g_\ast}{g_{*s}} \zeta(3). \quad (A.14) \]

Eq. (A.13) can be solved analytically, given the cross section in Eq. (A.9):

\[ Y(x_f) \approx \left[ \frac{\lambda^2 \nu_0 a_s^2 m_L^2}{m_S M_L^2 (m_i^2 - m_0^2)} \right]^2 \left\{ \frac{m_i^5}{M^2} \right\} \frac{Y_{0, EQ}^2}{H(m_i)} \frac{\pi}{180 g_{*s}} \left( \frac{1}{x_i} - \frac{1}{x_f} \right), \quad (A.15) \]

where we used \( E \sim \frac{1}{2} T \). The behaviour of the yield is completely determined by the

high energy regime and as such, only the high energy limit of the freeze-in neutrino

number density is crucial for our analysis.

The estimates of the comoving number density can be employed to compute an

approximate analytical expression for \( \Delta N_{\text{eff}, i}^{\text{FI}} \). Assuming that the dominant contribu-
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The contribution to $\Delta N_{\text{eff},i}$ for the $i > 0$ sector is from the freeze-in process, one obtains:

$$\Delta N_{\text{eff},i}^\text{FI} = \frac{4}{7} \left( \frac{11}{4} \right)^{4/3} g_i \left( \frac{T_i}{T_{\gamma,0}} \right)^4 \approx (3.7 \times 10^{-13}) \left( \frac{\beta}{10^{-2}} \right)^{99/12} \left( \frac{10^{-9} \text{ GeV}}{m_i} \right)^{8/3} \left( \frac{10^3 \text{ GeV}}{M_L} \right)^{16/3} \left( \frac{2i + r}{r} \right)^{4/3},$$

(A.16)

where we used $T_i \propto n_i^{1/3}$ and

$$\Omega h^2 \approx 1.59 \times 10^8 \left( \frac{\beta}{1 - \beta - \gamma} \right)^{3/4} \left( \frac{\eta}{1 + \eta} \right) \frac{B_0}{1.7 \times 10^{26} \Gamma_S^2 (1 - \beta) \beta},$$

(A.17)

with $\Omega h^2 \approx 0.12$, $\eta \approx 0.52$ (as in Eq. (4.50)) and $B_0 = \eta_{\text{obs}} = 8.59 \times 10^{-11}$. Hence, for our model, the freeze-in contribution to $N_{\text{eff}}$ is extremely small for the region of parameter space of interest and this effect can be safely ignored.

### A.3 Effects of Thermal Corrections

In this section, we study the thermal effects on the various quantities relevant to baryogenesis and dark matter in the Nnaturalness framework, in particular, whether the thermal corrections to the Higgs and $W$ boson masses of the different sectors are relevant in the partial decay widths in Eq. (4.19). The temperature-dependent masses are [148,251]:

$$m_{W,i}^2 (T_i) = \begin{cases} M_{H,i}^2 (T_i) = \frac{y_i^2}{4} \left( T_i^2 - T_{c,i}^2 \right), & \text{for } T_i > T_{c,i}, \\ m_{W,i}^2 (0) \left[ 1 - \left( \frac{T_i}{T_{c,i}} \right)^2 \right], & \text{for } T_i < T_{c,i}, \end{cases}$$

(A.18)

$$M_{H,j}^2 (T_j) = M_{H,j}^2 (0) + \frac{y_j^2}{4} T_j^2, \quad \forall T_j,$$
where $y_t$ is the top Yukawa coupling and $T_{c,i}$ is the phase transition temperature of the $i$th sector, $M_{H,j}(0)$ and $m_{W,i}(0)$ are the zero temperature Higgs and $W$ boson masses respectively. If thermal masses are sufficiently large that the two body decays ($S \rightarrow H_j e_j$ and $S \rightarrow W_k e_k$) are kinematically forbidden, the reheaton’s decay is three body. The decay $S \rightarrow t \bar{b} e$, in the limit of $m_H \gg m_S$ is given as

$$\Gamma_{S \rightarrow t \bar{b} e} \simeq \left( \frac{N_c y_b^2 \lambda^2}{3072 \pi^3} \right) \frac{\mu_L^2 m_S^5}{M_H^2 m_H^4},$$

(A.19)

where $N_c = 3$ is the number of colours and $y_b$ is the bottom quark Yukawa coupling.

Taking into account the temperature dependence of the various particle masses, the evolution of the energy densities in the different sectors can be determined by solving Eq. (4.21) with the appropriate temperature-dependent widths and branching ratios.

The behaviour of the energy densities and the partial decay widths with thermal corrections taken into account are shown in Figure A.3 as a function of time. We can compare this to the results in the left panel of Figure 4.4 which shows the dynamics ignoring thermal effects. In panel (a) of Figure A.3, we can observe that, as before, the energy density of the Universe is dominated by the reheaton for times $t \lesssim \Gamma_S^{-1}$.

The initial condition is that the $N$ sectors are not populated, so early decays of the reheaton quickly heat these sectors up. Once these sectors reach the electroweak temperature, they undergo a phase transition from the broken to the unbroken phase. This in turn reduces the reheaton width which slows down the decays of the reheaton. This can be seen as the plateau in the top left panel of Figure A.3. It can also be seen as the decrease of partial widths in the bottom left panel. The expansion of the Universe then cools the sectors and they undergo another phase transition back to

\footnote{The evolution here begins after the $N_1$ has fully decayed.}
the electroweak broken phase, and the width increases again. This interesting double phase transition behaviour could potentially have interesting observable consequences which we leave to future work.

In the top right panel of Figure A.3, we show the ratio of the energy density of
the hidden sector to the SM sector as a function of time. Without thermal effects, this ratio will be equal to the ratio of branching ratios, and that is the result before the first phase transition. After the first phase transition, the $W$ mass in the $i = 1$ reduces and the branching ratio increases, so the energy density in that sector increases relative to the others. After the second phase transition, the ratio of energy densities is restored to the naive branching ratio prediction. This shows that thermal effects are unimportant, and this is ultimately because most of the decays of the $S$ happen at late times, so the complicated dynamics at earlier times do not make a large impact.

This conclusion can be confirmed in the bottom right panel of Figure A.3, which shows the ratio of late time baryon asymmetry with and without thermal effects. We see that this ratio is very close to one as long as $\beta$ or $\Gamma_S$ are not too large, which is the case in the allowed region of $6.2 \times 10^{-16} \text{GeV} \lesssim \Gamma_S \lesssim 1.4 \times 10^{-14} \text{GeV}$ and $0.01 \lesssim \beta \lesssim 0.08$.

### A.4 Dark Quark Nuggets

In this section, we will lay out some requisite details on the possible formation of Quark Nuggets in the exotic sector ($i = -1$) of our model. The formation of Dark Quark Nuggets can be another interesting possibility in the scope of macroscopic Dark Matter. These objects are primarily composed of quarks and their formation requires first-order QCD phase transition. This hypothesis was first proposed by Witten in Ref. [164], and has gained some attention lately [168, 169].

Lattice studies have shown that the SM QCD transition is a continuous crossover, meaning that SM physics alone cannot form quark nuggets in the early Universe [157]. However, it was also shown that the phase transition in QCD-like gauge theories (that are not the SM one) is first order if the number of light quarks below the confinement
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scale is $N_f \geq 3$ [122,155]. In [168,169], the authors study the formation of six quark flavour matter (6FQM) quark nuggets, arguing that, in addition to a first-order phase transition, it is also necessary for a non-zero baryon number in the dark sector. This is exactly the setup of the exotic sectors, with the $i = -1$ sector having the largest baryon asymmetry.

Here we study a possible evaporation process that was not discussed in [168,169], which is the emission of a pion and a neutrino, or in terms of $SU(2) \times U(1)$ states, $q_L \bar{u}_R \bar{\ell}_L$ and $d_R \bar{u}_R \bar{e}_R$. This emission conserves all gauge quantum numbers, but it does change the lepton number of the quark nugget.

The total number of leptons, in the 6FQM, contained in the nugget is [168]

$$N_{\text{Initial} \, L} \equiv n_L \, V_{\text{QN}} = -\frac{55}{2\pi^2} \left( \frac{8\pi^2}{415} B \right)^{3/4} \frac{4}{3} \pi \, R_{\text{QN}}^3,$$  \hspace{1cm} (A.20)

where $B$ is the MIT Bag constant, $V_{\text{QN}}$ is the quark nugget volume and $R_{\text{QN}}$ is its radius. The MIT Bag constant is not well known; nevertheless, our results for the nugget’s lifetime will be relatively insensitive to any variation in $B$. Then, the rate of the nugget’s evaporation can be estimated following the technique in [252], as:

$$\frac{dN_\nu}{dt} = n_\nu \cdot v_\nu \cdot A$$

$$= \left( \frac{g_\nu}{6\pi^2} \frac{T^3}{T} \right) \left[ \left( \frac{\mu_\nu}{T} \right)^3 + \pi^2 \frac{\mu_\nu}{T} \right] 4\pi \, R_{\text{QN}}^2,$$  \hspace{1cm} (A.21)

assuming that it is proportional to the flux of neutrinos from the nugget, $n_\nu v_\nu$, times the nugget’s surface area, $A$. In Eq. (A.21), $\mu_\nu$ is the chemical potential of neutrinos, and we assumed that the neutrinos are relativistic, i.e., $v_\nu \sim c$. Using $\mu_\nu = -3 \left( \frac{8\pi^2}{415} B \right)^{1/4}$, which is obtained by assuming an equilibrium is maintained between the QN and the surrounding $\Delta P_{\text{Particles}} = \Delta P_{\text{Vacuum}}$, the expression above
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becomes:

\[
\frac{dN^\nu}{dt} = 2g^\nu T^2 \left( \frac{\pi}{36} \right)^{2/3} \left( \frac{8\pi^2}{415} \right)^{-1/4} B^{-1/4} \left[ \pi + \frac{9}{T^2} \left( \frac{8}{415} \right)^{1/2} B^{1/2} \right] N_\nu^{+2/3}. \quad (A.22)
\]

By writing \( \frac{dN^\nu}{dt} \) in terms of time using \( T^2 \approx \left( \frac{90}{\pi^2 g^*} \right)^{1/2} \frac{M_P}{t} \), and integrating Eq. (A.22) over time, we get:

\[
\Delta N(t)^{1/3} = \frac{2g^\nu}{3} \left( \frac{\pi}{36} \right)^{2/3} \left( \frac{8\pi^2}{415} \right)^{-1/4} B^{-1/4} \left[ \left( \frac{45}{2g^*} \right)^{1/2} M_P \ln \left( \frac{t}{t_C} \right) + 9 \left( \frac{8}{415} \right)^{1/2} B^{1/2} (t - t_C) \right], \quad (A.23)
\]

where \( \Delta N^{1/3}(t) \equiv N(t)^{1/3} - N(t_C)^{1/3} \) and \( t_C \) is the time of the phase transition where the nugget forms \( (T \approx \Lambda_{QCD, i} \approx 89 \text{ MeV}) \). A plot of the nugget’s lifetime as a function of \( N(t) \) is shown in Figure A.4. We can see that for a nugget with a lifetime of about 1 second requires \( N \sim 10^{63} \). To understand the order magnitude of the lepton number computed in Eq. (A.23), one might inquire, what is the typical number of baryons contained in one Hubble patch?

\[
N_B(T) = \frac{4}{3} \pi R^3 s(t) \eta_{\text{obs}} = \frac{8}{135} (90)^{3/2} \frac{\eta_{\text{obs}}}{g^{1/2}} \left( \frac{M_P}{T} \right)^3 \quad (A.24)
\]

\[
N_B(T) \approx 9 \times 10^{48} \left( \frac{100 \text{ MeV}}{T} \right)^3,
\]

where we used \( R = \frac{1}{H} \), \( \eta_{\text{obs}} = 8.59 \times 10^{-11} \) and \( g^* = 49.75 \). Consequently, assuming the lepton number to be the same order magnitude as the baryon number, the maximum lepton number in a single patch can be approximately \( \sim 10^{69} \) which corresponds to the nugget’s lifetime about 100 nanoseconds and it is represented by the vertical dashed black line in Figure A.4.
Figure A.4: The lifetime of the nugget as a function of the lepton number, $N(t)$, for $B^{1/4} = 150$ MeV, $g_* = 49.75$, and $g_\nu = 6$. As an example, for a nugget with a lifetime of approximately 1 ns corresponds to $N \sim 10^{43}$. This does not take into account (re)absorption. The vertical dashed black line corresponds to $N = 10^{49}$ which is the maximum number of leptons in a nugget determined by the simple Hubble patch argument.

Another approach to model the nugget dynamics is to treat it as a blackbody. We compute the total energy loss of the quark nugget and compare it to the total energy contained in the nugget. Inside the nuggets, electroweak symmetry is unbroken and, therefore, leptons and quarks are massless. Considering that the quark nugget behaves like a blackbody is reasonable since the mean free path of the neutrino is small compared to the typical $R_{QN}$. This way, in principle, all the neutrinos falling on it will be absorbed and the quark nugget will also emit neutrinos with a thermal spectrum. The typical power for a given surface area can thus be written using Stefan-Boltzmann Law:

$$P = 4\pi \sigma \epsilon R^2 T^4,$$

where $\epsilon$ is the emissivity ($\epsilon = 1$ for idealized blackbody) and the Stefan-Boltzmann constant is $\sigma = 5.6705 \times 10^{-8} \, W/m^2K^4 = \frac{\pi^2}{60}$ (in terms of fundamental constants).
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Figure A.5: Energy loss due to neutrino emission and the total mass of the quark nugget as a function of its radius in the blackbody treatment of neutrino evaporation. In the above curves, we set $B^{1/4} = 150$ MeV, $T_i = 100$ MeV, and $T_f = 1$ MeV.

Then, the total amount of energy expended over a period $\Delta t = t_f - t_i$ is:

$$\Delta E_{Bb} = \int_{t_i}^{t_f} P \, dt = (360)^{1/2} \, M_P \, \sigma \, \epsilon \, R_{QN}^2 \left[ \frac{T_i^{1/2}}{g_{*i}} - \frac{T_f^{1/2}}{g_{*f}} \right], \quad (A.26)$$

On the other hand, using the MIT bag model, the energy density of the quark nugget can be expressed in terms of the Bag constant $B$ as (in the massless particle regime):

$$\rho_{QN} = \sum_i \rho_i + B = 3 \sum_i P_i + B = 4B. \quad (A.27)$$

Hence, the total energy contained inside the QN can be expressed as:

$$M_{QN} = \frac{16}{3} \pi R_{QN}^3 B. \quad (A.28)$$

In the SM, the typical value for the Bag parameter is $B^{1/4} \simeq 150$ MeV. In Figure A.5, we compare the total energy of the nugget with the energy loss due to neutrino emissions, approximating the nugget as a blackbody, and we see that the
nugget is completely depleted by evaporation. In our analysis, we have neglected any (re)absorption of leptons from the environment.

To conclude, we may estimate the consequences of the shut off of the sphaleron process once the size of the nugget becomes smaller than the size of the sphaleron. For the classical Yang-Mills theory, the sphaleron size is \cite{253}:

\begin{equation}
R_{Sp} \sim \frac{5}{g^2 T} \simeq \begin{cases}
0.12 \text{ GeV}^{-1}, & \text{For } T = 100 \text{ GeV} \\
120 \text{ GeV}^{-1}, & \text{For } T = 100 \text{ MeV}
\end{cases}
\end{equation}

\hspace{1cm}(A.29)

whereas its energy is:

\begin{equation}
E_{Sp} \sim \frac{4 m_W}{\alpha_W} \simeq \begin{cases}
9.6 \text{ TeV}, & \text{For } m_W = 80.4 \text{ GeV} \\
3.6 \text{ GeV}, & \text{For } m_W = 30.3 \text{ MeV}.
\end{cases}
\end{equation}

\hspace{1cm}(A.30)

We may then conclude that the sphaleron interactions in the nugget never shut off since $R_{QN} \gg R_{Sp}$ (for a typical value $R_{QN} = 1 \text{ mm} = 5.06 \times 10^{11} \text{ GeV}^{-1}$).

From these estimates, we see that if the dark quark nugget does form in this scenario, its lifetime will be much too small to affect the dark matter phenomenology.
A.5 Hidden Sectors Decoupling

We can estimate the temperature at the photon decoupling from the Saha equation for the hidden sectors. The Saha equation gives the temperature of a photon at the recombination era but one could assume that it is the same as the photon decoupling epoch. The Saha equation takes the form \[92\]:

\[
X_{e,h} + X_{e,h}^2 \left[ \frac{2\zeta(3)}{\pi^2} \eta_B \right] \left[ \frac{2\pi T}{m_{e,h}} \right]^{3/2} e^{\Delta_{H,h}/T} = 1 , \tag{A.31}
\]

where \(\eta_B \equiv \frac{n_B}{n_\gamma} \approx 5.5 \times 10^{-10}\) is the baryon to photon ratio, \(\Delta_{H,h} = m_{e,h} + m_{p,h} - m_{H,h}\) is the binding energy of hydrogen and \(X_{e,h} \equiv \frac{n_{e,h}}{n_{p,h} + n_{H,h}}\) is the fractional ionization of the baryonic content of the Universe or the number of free electrons per hydrogen nucleus (\(X = 1\) corresponds to the baryonic content being fully ionized) and \(\zeta(3) \approx 1.2\) is the Riemann zeta function. The relation in Eq. (A.31) can be rewritten as:

\[
\frac{1 - X_{e,h}}{X_{e,h}^2} = \left[ \frac{2\zeta(3)}{\pi^2} \eta_B \right] \left[ \frac{2\pi T}{m_{e,h}} \right]^{3/2} e^{\Delta_{H,h}/T} . \tag{A.32}
\]

Then we can extract the recombination temperature by defining the point where \(X = 1/2\), which for SM leads \(T_{\text{Dec, SM}} \approx 0.323\) eV. Plugging in numbers in the expression for the fermion masses in Eq. (A.2) with \(\Lambda_{\text{QCD},j<0} = 89\) MeV, \(v_0 = 246\) GeV, \(\lambda = 0.129\) and \(y_t \approx 1\), leads to \(m_{f,j} \sim -5.19 \times 10^{-10} m_{f,0} \frac{r}{2i+r} \). So the mass of the electrons and hydrogen are rescaled in a similar manner. Consequently, the photon decoupling temperature in the hidden sector is:

\[
T_{\text{Dec, h}} = -5.19 \times 10^{-10} \cdot \frac{r}{2i+r} \cdot T_{\text{Dec, SM}} \approx -5.59 \times 10^{-11} \cdot \frac{r}{2i+r} \text{ eV} . \tag{A.33}
\]

For a very rough estimate of neutrino decoupling temperature, we can approx-
imate it by $\Gamma_\nu = H$. Recall:

$$H(T) = \frac{\rho_R(T)}{3M_P^2}, \quad \rho_R(T) = g^{\pi^2}_{*R}T^4, \quad \Gamma_\nu = n_\nu \langle \sigma \cdot v \rangle, \quad n_\nu = \frac{\zeta(3)}{\pi^2}g_{*\nu}T^3. \quad (A.34)$$

Then using $\sigma \sim G_F^2 T^2 \Rightarrow \Gamma_\nu \sim g_{*\nu}G_F^2 T^5$, where we are only keeping the relevant factors. Thus one obtains $^3$:

$$T^3 \sim \frac{\sqrt{g^{*}_{R}}}{g_{*\nu}}G_F^{-2}M_P^{-1} \approx \frac{\sqrt{g^{*}_{R}}}{g_{*\nu}} \cdot 10^{-9} \text{ GeV}^3$$

$$T \sim \left( \frac{\sqrt{g^{*}_{R}}}{g_{*\nu}} \right)^{1/3} \text{ MeV}. \quad (A.35)$$

Since in the sectors with $m_H^2 > 0$, the EW symmetry is preserved below the Higgs mass and broken by the QCD confinement, the mass of the W-boson is given by $m_W^2 = \left( \frac{g}{\sqrt{2}} \right)^2 f_\pi^2 \approx (30.34 \text{ MeV})^2$ which result in $G_{F,h} \approx 81.76 \text{ GeV}^{-2}$. So the hidden sector neutrino decoupling temperature according to Eq. (A.35) is:

$$\left(T^{h}_{\nu,\text{Dec}}\right)^3 \sim \frac{\sqrt{g^{*}_{R,h}}}{g_{*\nu,h}}G_F^{-2}M_P^{-1} \approx \frac{\sqrt{g^{*}_{R,h}}}{g_{*\nu,h}} \cdot 6.1 \times 10^{-23} \text{ GeV}^3,$$

$$T^{h}_{\nu,\text{Dec}} \sim \left( \frac{\sqrt{g^{*}_{R,h}}}{g_{*\nu,h}} \right)^{1/3} 3.9 \times 10^{-5} \text{ MeV}. \quad (A.36)$$

We observe that at that time all the pions would be non-relativistic but all the leptons are still relativistic (the $\tau$ mass is about 1-2 order of magnitude smaller than the temperature).

$^3$The Fermi constant $G_F = \frac{\sqrt{2}}{8} \frac{g_2^2}{m_W^2}$, so it is much larger for the hidden sectors $i < 0$. The Fermi constant, for the SM, is $G_F \approx 1.16 \times 10^{-5} \text{ GeV}^{-2}$ with $m_W \approx 80.38 \text{ GeV}$ and $g_2 \approx 0.65$. 

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The structure of the $i < 0$ sectors is quite fascinatingly rich in addition to the plethora of hadrons compared to the Standard Model sector. The strength of the weak interactions is significantly enhanced (since $G_{F,i<0}/G_{F,i=0} \sim 7 \times 10^6$) and in the spirit comparable to the residual SM strong interaction which changes the phenomenology of these sectors quite a bit. For example, the lifetime of the nucleon such as beta decays can be very short relative to the SM scenario mainly due to $\Gamma \propto G_{F,i}^2$. Furthermore, the formation of atoms would be quite difficult due to the very light nature of the electrons or even tau leptons. For such a process to occur, the Universe would have to cool for a very long period to even permit the formation of atoms due to the order of magnitude smaller binding energy of the atoms ($E_{\text{binding}} \propto m_{f,i}$) and the atoms produced would be significantly larger.
Appendix B

Three Jets

The information presented in the following appendices, with the exception of Section B.5, are published in [2] and they go with Chapter 5. The additional material provides some necessary background and expand on the three jet resonance project explored in Chapter 5.

B.1 CMS Three Jet Resonance Search

The 13 TeV CMS search [209] requires an event to contain at least six jets with $|\eta| < 2.4$. The jet reconstruction is performed using the anti-$k_t$ algorithm [254] with a radius parameter of $R = 0.4$. The list of all the selection criteria used in this particular search is given in table B.1. The analysis employs the jet-ensemble technique [217, 225], which takes the six highest $p_T$ jets in a given event and group them into 20 unique triplets. For signal, at most 2 of these triplets per event corresponds to the pair produced gluino decay while the rest contributes to the combinatoric background which is referred to as “incorrect” triplets. Consequently, the acceptance is defined as the ratio of the correct triplet over the total number of triplets (20) in the event.
Furthermore, an event-level variable $D_{[(6,3)+(3,2)]}^2$ is defined in order to characterize the angular spread of the six constituent jets inside a pair of triplets. The six-jet distance measure is defined as:

$$D_{[(6,3)+(3,2)]}^2 = \sum_{i<j<k} \left( \sqrt{\hat{m}(6,3)^2_{ijk}} + D_{[3,2],ijk}^2 - \frac{1}{\sqrt{20}} \right)^2,$$

where $\hat{m}(6,3)^2_{ijk} = \frac{m^2_{ijklmn}}{4m^2_{ijklmn} + 6 \sum_i m^2_T}$ with $i, j, k, l, m, n \in \{1, 2, ..., 6\}$ and $m_{ijklmn}$ the invariant mass of the six highest $p_T$ jets. For a new particle decaying to three-jets, the jets produced would be uniformly distributed in a detector resulting in $\hat{m}(6,3)^2_{ijk}$ approximately $1/20$. While the jets from the QCD are usually grouped together giving $\hat{m}(6,3)^2_{ijk}$ close to zero or one.

Table B.1: The list of selection criteria with the direction of the cuts and the mass ranges analyzed by the CMS search [209].

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>200-400</td>
<td>&gt; 30</td>
<td>&gt; 650</td>
<td>&gt; 40</td>
<td>&lt; 1.25</td>
<td>&lt; 0.25</td>
<td>&gt; 250</td>
<td>&lt; 0.05</td>
</tr>
<tr>
<td>400-700</td>
<td>&gt; 30</td>
<td>&gt; 650</td>
<td>&gt; 50</td>
<td>&lt; 1.00</td>
<td>&lt; 0.175</td>
<td>&gt; 180</td>
<td>&lt; 0.175</td>
</tr>
<tr>
<td>700-1200</td>
<td>&gt; 50</td>
<td>&gt; 900</td>
<td>&gt; 125</td>
<td>&lt; 0.9</td>
<td>&lt; 0.15</td>
<td>&gt; 20</td>
<td>&lt; 0.2</td>
</tr>
<tr>
<td>1200-2000</td>
<td>&gt; 50</td>
<td>&gt; 900</td>
<td>&gt; 175</td>
<td>&lt; 0.75</td>
<td>&lt; 0.15</td>
<td>&gt; −120</td>
<td>&lt; 0.25</td>
</tr>
</tbody>
</table>

Furthermore one of the most efficient cuts for the three-jet resonance is the "Delta cut" defined as:

$$M_{jjj} < \sum_{i=1}^3 p^i_T - \Delta,$$

where $M_{jjj}$ is the invariant mass of the triplet and $\Delta$ is an adjustable parameter. The parameter $\Delta$ is determined in each signal region by optimizing the signal significance $\alpha = s/\sqrt{s+b}$. This particular selection criteria can be understood due to the observation of the linear correlation of the triplet invariant mass with a scalar sum of the transverse momentum for the QCD background. While the triplet invariant mass of the correctly combined signal triplets is unchanged by varying $p_T$ since $M_{jjj}$
Appendix B. Three Jets

is fixed. Consequently, this not only reduces the QCD multijet background but the combinatoric background raising from the incorrectly combined signal triplets as well. Finally, the mass asymmetry variable is defined as:

\[
A_m = \frac{|m_{ijk} - m_{lmn}|}{m_{ijk} + m_{lmn}},
\]

(B.3)

where \(m_{ijk}\) is invariant mass of the triplet. This variable has discriminating power

Figure B.1: Mass distributions for two of the mass regions. The distributions in figure (a) and (b) are produced by CMS [209] while the bottom figures are the corresponding ones for \(m_\eta = 5\) TeV.
between signal and background since the signal triplets are expected to be close each other in mass but not the background.

When trying to reproduce the 13 TeV CMS search [209] we encountered some difficulties. The CMS collaboration paper contains $M_{jjj}$ distributions for their signal topologies reproduced in the top panels of figure B.1. The shapes of the signals appear as perfect Gaussians centered around the gluino mass. However, our simulations of the RPV model result in an invariant mass peak that is slightly shifted below the true mass points and is asymmetric about the peak with a longer tail at a lower invariant mass. We show the signal distributions for two different gluino masses in the bottom panel of figure B.1 both with and without detector simulation. Furthermore, it is stated in [209] that the invariant mass distribution of the incorrectly combined signal triplets (the combinatoric background) is similar to the multijet background; however, we find them to be different. There is also an ambiguity in the definition of the acceptance in the case where more than two triplets in an event satisfy all the selection criteria. Finally, [209] refers to Monte Carlo simulations of the QCD background, but the work does not specify how the QCD samples are generated. More details about the procedures for computing the signal efficiency and simulating the background would be helpful for future studies and recasts.

B.2 ATLAS Di-Jet Resonance Search

Similar to the three-jet CMS search, this di-jet search [210] also reconstructs the jet candidates using an anti-$k_t$ algorithm with a radius parameter of 0.4. The complete list of cuts is displayed in table B.2. The average mass of the two reconstructed resonances is expected to peak around the mass of the resonance being searched for.
Appendix B. Three Jets

The average mass,

\[ m_{\text{avg}} = \frac{1}{2} (m_1 + m_2) \text{,} \tag{B.4} \]

is thus required to be inside of a window around the searched for mass, with the width of the window varying from 10 to 100 GeV and is given in tables 3 and 5 of [210]. In order to recast this particular search, the RPV top squarks were pair produced with radiation of up to two additional partons. The merging with parton shower was done using the MLM [255] prescription with a merging scale set to 1/4 of the top squark mass. In addition, all the SUSY particles except the top squark were decoupled by setting their masses to 5 TeV.

Table B.2: The list of selection criteria with the direction of the cut for the ATLAS di-jet search [210].

| Jet $p_T$ [GeV] | $A_m$ | $|\cos(\theta^*)|\text{,}$ | $\Delta R_{\text{min}}$ |
|----------------|--------|-----------------|------------------|
| $> 120$        | $< 0.05$ | $< 0.3$       | $< -0.002 \cdot \left( \frac{m_{\text{avg}}}{\text{GeV}} - 225 \right) + 0.72$, if $m_{\text{avg}} \leq 225 \text{ GeV}$ |
|                |        |                 | $< +0.0013 \cdot \left( \frac{m_{\text{avg}}}{\text{GeV}} - 225 \right) + 0.72$, if $m_{\text{avg}} > 225 \text{ GeV}$ |

B.3 CMS Three Jet Resonance Search at $\sqrt{s} = 8$ TeV

The work of [234] is an earlier version of the three-jet CMS search [209] performed at the center-of-mass energy of 8 TeV. The jet candidates are constructed using an anti-$k_t$ algorithm with a radius parameter of 0.5. The search considers two scenarios, first when the gluino decays into light flavour jets and secondly when it decays to a $b$-jet and two light flavour jets. The latter case requires the existence of at least one bottom quark jet in the resonance decay products. Besides the usual $p_T$ and $\Delta$ variables requirements described in appendix B.1, event shape information is exploited. Typically in the high mass region, the signal events have a more spherical...
shape than the background (which generally contains back-to-back jets thus a more linear shape) [234]. Consequently, the sphericity variable is defined as,

\[ S = \frac{3}{2} (\lambda_2 + \lambda_3) , \quad \lambda_1 \geq \lambda_2 \geq \lambda_3 , \]  

(B.5)

where \( \lambda \)'s are the eigenvalues of the sphericity tensor,

\[ S_{\alpha \beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i |p_i|^2} , \quad \alpha , \beta = x , y , z , \]  

(B.6)

where \( \alpha \) and \( \beta \) label separate jets, and the sphericity \( S \) is calculated using all jets in each event. The complete list of selection criteria are shown in table B.3.

**Table B.3:** The selection criteria with the direction of the cut for the CMS heavy flavour search performed at \( \sqrt{s} = 8 \) TeV [234].

<table>
<thead>
<tr>
<th>Mass Range [GeV]</th>
<th>( \Delta ) [GeV]</th>
<th>( p_{T,j}^{4th} ) [GeV]</th>
<th>( p_{T,j}^{6th} ) [GeV]</th>
<th>Sphericity</th>
</tr>
</thead>
<tbody>
<tr>
<td>200-600</td>
<td>&gt; 110</td>
<td>&gt; 80</td>
<td>&gt; 60</td>
<td>—</td>
</tr>
<tr>
<td>600-1500</td>
<td>&gt; 110</td>
<td>&gt; 110</td>
<td>&gt; 110</td>
<td>&gt; 0.4</td>
</tr>
</tbody>
</table>

**B.4 Multijet Background**

The principal background for our signal arises from the QCD multijet events. Other SM processes have negligible contributions, and we have performed simulations of \( \bar{t}t \) events to confirm that their rates are indeed very small. The QCD multijet background is very large as one can observe from the crude cross section estimates shown in table B.4 (similar results were obtained using Sherpa [256]). The QCD multijet events were obtained by simulating \( pp \rightarrow j jjj \) using MadGraph5 interfaced with Pythia8 and Delphes. The cross sections are orders of magnitude larger than the pair production cross section for the top partner as displayed in figure 5.2. In our
Appendix B. Three Jets

simulations, we require each of the four partons to have $p_T > p_{T,min}(j) = 100$ GeV in order to make sure enough events satisfy all the selection requirements for our recasted searches. The minimum parton level $p_T$ is well below the detector level jet $p_T$ requirement of 125 GeV, given in table B.1, at the cut off region ($m_T = 900$ GeV), so it does not affect our analysis.

Table B.4: Four partonic hard jets production cross section using MadGraph5 at $\sqrt{s} = 13$ TeV with various minimum parton level cut.

<table>
<thead>
<tr>
<th>$p_{T,min}(j)$ Generator Level [GeV]</th>
<th>$\sigma_{4j}$ [pb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>$1.79 \times 10^4$</td>
</tr>
<tr>
<td>60</td>
<td>$7.64 \times 10^4$</td>
</tr>
<tr>
<td>100</td>
<td>$4.68 \times 10^3$</td>
</tr>
<tr>
<td>200</td>
<td>$7.216 \times 10^4$</td>
</tr>
</tbody>
</table>

B.5 Implementation Tools

B.5.1 Implementation Methodology

To analyze our top partner model and study its phenomenology, we use several publicly available software packages. Firstly, FeynRules which is a Mathematica based package that can be utilized to compute Feynman rules for any BSM model [257]. The package requires inputs regarding the various quantum fields, the parameters of the model and the full BSM Lagrangian. Finally, FeynRules returns an output appropriate for many users’ desired Monte Carlo event generation programs. Consequently, FeynRules facilitate the initial step in constructing a new theoretical model and paves the way toward comparing a theory of interest with various experimental observations. After the implementation of the model, the primary goal of any BSM model is to establish any constraints on the proposed model which require simulating collider events.
The most notable software tool capable of simulating events according to a user-specified model is *MadGraph5aMC@NLO*, which is a Monte Carlo event generator that produces Feynman diagrams for the desired process [258]. *MadGraph5aMC@NLO* has the capability of computing tree-level and next-to-leading order cross sections and their matching to parton shower simulations. Once the events are generated, they are then passed to *PYTHIA8*, which is an event showering and hadronization program [259]. The resulting events are eventually processed by *DELPHES*, which is a detector simulator that recreates physics objects (e$, \mu$, and jets, etc ...) from a simulated detector response [260]. In the end, the output is Monte Carlo events that can be analyzed to give physicists an idea of what to expect at colliders and compare the results to the current observations. Some of the statistical analysis required for any large data set will be explored next.

### B.5.2 Statistical Methodology

In collider phenomenology, a large amount of data needs to be analyzed so we require a method to differentiate signal-like events from a background like. Quite often, physicists are interested in how significant a particular observation is and when can one declare a discovery in favour of a new physics? Therefore, various statistical techniques are essential to answer some of these questions. One of the primary goals of an experimentalist is to perform a statistical test to make a statement about how well the predicted probability stands in comparison to the observed data. Generally the default hypothesis under study is called the *null hypothesis* $H_0$, which one often tries to nullify and there is counter test denoted by the *alternative hypothesis* $H_a$. To test the compatibility between data and a given hypothesis, one needs to construct a selection criterion on which a decision for the hypothesis is based. It is called the
Given the binary decision from the test statistic, it is feasible to incorrectly reject the null hypothesis when it is true. This is known as a type I error and it is denoted by $\alpha$ (significance level) [261]. For any given model with a probability density function (f), and a test statistic (T), we have:

$$\alpha = \int_{T_d}^{\infty} f(T|H_0) \, dT ,$$

(B.7)

where $T_d$ is the cut-off value of the test statistic above which the null hypothesis is rejected. However, it is also possible to accept the null hypothesis even when it is not true. This is referred to as a type II error and is denoted by $\beta$. Hence we have,

$$\beta = \int_{-\infty}^{T_d} f(T|H_a) \, dT .$$

(B.8)

As we have stated earlier, one is quite often interested in how compatible the null hypothesis is with the observed data set; it is known as the goodness of fit and the significance of such a test is represented by the $P$-value. The $P$-value is essentially the probability of observing an event that is at least as contradictory as the one observed under the assumption that the null hypothesis is true [261] defined as

$$P - value = P(n \geq n_{obs}) .$$

(B.9)

For simple counting experiments, $n$ would be the number of events and one rejects the null hypothesis, $H_0$, only if $P - value \leq \alpha$. Stated otherwise, the $P$-value is the minimum observed significance level at which the null hypothesis can be rejected.

In addition, we define the expectation value, $E(x)$, which is essentially an average
value for a random variable distributed according to a probability distribution $f(x)$ and it is defined as:

$$E(x) = \mu \equiv \int_{-\infty}^{\infty} x f(x) \, dx.$$ \hfill (B.10)

This is generically called the first algebraic moment and another related quantity that is often used is the second central moment,

$$E[(x - E(x))^2] = V(x) \equiv \int_{-\infty}^{\infty} (x - \mu)^2 f(x) \, dx = \sigma^2,$$ \hfill (B.11)

where $\sigma$ is referred to as the standard deviation.

Besides the significance level $\alpha$, one more term that is often uttered is the confidence level, $100\%(1 - \alpha)$, which is in essence is the probability for the occurrence of the true value of the statistics within the obtained interval using the applied method. Our concern lies with the confidence level of the signal but in actual experiments, our signal is almost always accompanied by background. Therefore, we have to deal with the inevitable background and possible errors associated with it. We use the so-called modified frequentist confidence level, which is defined as [222]:

$$CL_s = \frac{CL_{s+b}}{CL_b},$$ \hfill (B.12)

where $CL_{s+b}$ is the confidence level for excluding a possible signal and background hypothesis,

$$CL_{s+b} = P_{s+b}(X \leq X_{\text{Obs}}),$$ \hfill (B.13)

with $CL_b$ being the probability for background only hypothesis of observing events.
less than or equal to the events observed given by

\[ CL_b = P_b(X \leq X_{\text{Obs}}). \quad (B.14) \]

Finally, the confidence level is \( CL = (1 - CL_s) \times 100\% \). In actual experiments we are dealing with systematic uncertainties associated with background and signal, so we need to include its effects on the confidence intervals. These systematic uncertainties are encoded in variables called \textit{nuisance parameters} which modifies the confidence interval but they are not the primary observable of interest to us. So for a counting experiment, with the gaussian error in the background, the confidence level is constructed as follows:

\[
CL_b(n, b, \sigma_b) = \frac{\sum_{i=0}^{i=n} \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_b} e^{-\frac{(b-b')^2}{2\sigma_b^2}} \frac{b'e^{-b'}}{i!} \, db'}{\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_b} e^{-\frac{(b-b')^2}{2\sigma_b^2}} \, db'}, \]

\[
CL_{s+b}(n, s, b, \sigma_b) = \frac{\sum_{i=0}^{i=n} \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_b} e^{-\frac{(b-b')^2}{2\sigma_b^2}} (b'+s)^i e^{-(b'+s)} \frac{b'e^{-b'}}{i!} \, db'}{\int_{0}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_b} e^{-\frac{(b-b')^2}{2\sigma_b^2}} \, db'}.
\]
Appendix C

Supersymmetry Essentials

In this section, we will include some of the background formalism/conventions employed in supersymmetry theories such as those presented in Chapter 6. Beginning with the convenient formalism of superspace and superfields that expresses the supersymmetry structure in an elegant form. Superspace is essentially the extension of the usual bosonic spacetime dimensions \( x^\mu = (t, x, y, z) \) with anti-commuting dimensions labelled by Grassmann numbers. So points in superspace are denoted by the coordinates \( (x^\mu, \theta^\alpha, \theta^{\dot{\alpha}}) \) with \( \theta^\alpha \) and \( \theta^{\dot{\alpha}} \) being complex two-component anticommuting spinors, with the spinor indices \( \alpha = 1, 2 \) and \( \dot{\alpha} = 1, 2 \). Let’s review some of the properties for anticommuting numbers (Grassmann numbers):

- Any two anticommuting numbers \( \Theta \) and \( \eta \) satisfies, \( \Theta \eta = -\eta \Theta \). The product \( \Theta \eta \) commutes with any other Grassmann numbers.

- From anticommutative property: \( \Theta^2 = \eta^2 = 0 \).

- Differentiation: the important point to remember is that since the square of any Grassmann number is vanishes, a series expansion of a function in terms of anticommuting numbers always terminates. Hence, a general function is linear:
Appendix C. Supersymmetry Essentials

\( f(\Theta) = f_0 + \Theta f_1 \), where \( f_0 \) and \( f_1 \) are independent of \( \Theta \). Then the differential operator \( \frac{d}{d\eta} \) anticommutes with other Grassmann numbers and we have:

\[
\frac{df}{d\eta} = f_1, \quad \frac{d(\Theta \eta)}{d\eta} = -\frac{d(\eta \Theta)}{d\eta} = -\Theta. \tag{C.1}
\]

- Integration: the integral of a general function \( f \) must be linear in Grassmann numbers and the integration should be invariant under translation. So

\[
\int f(\Theta) d\Theta = \int [f_0 + \Theta f_1] d\Theta \xrightarrow{\Theta = \Theta + \eta} \int [f_0 + (\Theta + \eta) f_1] d\Theta,
\]

\[
\Rightarrow \int d\Theta = 0. \tag{C.2}
\]

Using the definition by Berezin for Grassmann’s variables \([262]\)

\[
\int \Theta d\Theta = 1. \tag{C.3}
\]

So for Grassmann’s variables, integration and differentiation are the same things.

In the superspace notations, the various component fields of a supermultiplet are combined into a single object called the superfield, \( S \), which in general can be expressed as

\[
S(x, \theta, \theta^\dagger) = a + \theta \xi + \theta^\dagger \chi^\dagger + b \theta \theta + c \theta^\dagger \theta^\dagger + \theta^\dagger \sigma^\mu \theta v_\mu + \theta^\dagger \theta^\dagger \eta + \theta \theta \theta^\dagger \zeta^\dagger + d \theta \theta \theta^\dagger \theta^\dagger, \tag{C.4}
\]

where \( \sigma^\mu = (1, \sigma), \ \overline{\sigma}^\mu = (1, -\sigma) \), there are at most two \( \theta \)'s and \( \theta^\dagger \)'s (two component left-handed spinors) with \( \theta_\alpha \theta_\beta = \frac{1}{2} \epsilon_{\alpha \beta} \theta \theta \) and \( \theta^\dagger_\alpha \theta^\dagger_\beta = \frac{1}{2} \epsilon_{\alpha \beta} \theta^\dagger \theta^\dagger \). These are obtained
using:

\[ \theta \theta \equiv \theta^a \theta_a , \quad \theta^a = \epsilon^{a \beta} \theta_\beta , \quad \epsilon_{ij} \epsilon^{mn} = \delta^m_i \delta^n_j - \delta^m_j \delta^n_i , \]

(C.5)

where \( i, j, m, n = 1, 2 \). The field \( \xi \) is called a “left-handed Weyl spinor” and \( \chi^\dagger \) is a “right-handed Weyl spinor”. The fields \( a, b, c, d, v_\mu \) denotes bosonic fields while \( \zeta^\dagger, \eta \) labels 2 component fermionic fields. Furthermore, a supersymmetry transformation converts a scalar field \( \phi \) into a fermionic field \( \psi \) and vice versa:

\[ \delta \phi = \epsilon \psi , \quad \delta \phi^\star = \epsilon^\dagger \psi^\dagger , \]
\[ \delta \psi_\alpha = -i (\sigma^\mu)^\dagger_\alpha \partial_\mu \phi + \epsilon_\alpha F , \quad \delta \psi^\dagger_\alpha = i (\epsilon \sigma^\mu)_\alpha \partial_\mu \psi^\dagger + \epsilon_\alpha^\dagger F^\star , \]
\[ \delta F = -i \epsilon^\dagger \sigma^\mu \partial_\mu \psi , \quad \delta F^\star = i \partial_\mu \psi^\dagger \sigma^\mu \epsilon , \]

(C.6)

where \( F \) is a complex scalar field (called an auxiliary field with dimension \([E]^2\)) and \( \epsilon \) is an infinitesimal, anti-commuting, two-component Weyl fermion labelling the supersymmetry transformation (with dimension \([E]^{-1/2}\)). Then for a superfield, an infinitesimal supersymmetry transformation is:

\[ \sqrt{2} \delta S = S \left( x^\mu + i \epsilon \sigma^\mu \theta^\dagger + i \epsilon^\dagger \sigma^\mu \theta, \theta + \epsilon, \theta^\dagger + \epsilon^\dagger \right) - S \left( x^\mu, \theta, \theta^\dagger \right) . \]

(C.7)

Moreover, there are two ways to obtain supersymmetrically invariant actions, one way is to integrate a function over \( d^2 \theta \ d^2 \theta^\dagger \) while the other way is to integrate a holomorphic function over \( d^2 \theta \) and it’s respective hermitian conjugate. Note that the general superfield in Eq. (C.4) is not a holomorphic function since it also depends on \( \theta^\dagger \). So using the previous integration relations for Grassmann variables \( \int \theta \theta \ d^2 \theta = 1 = \int \theta^\dagger \theta^\dagger \ d^2 \theta^\dagger \), the respective coefficients of the various terms can be computed from
Appendix C. Supersymmetry Essentials

a general superfield as

\[
\int S(x, \theta, \theta^\dagger) \, d^2 \theta = b(x) + \theta^\dagger \zeta^\dagger(x) + d(x) \theta^\dagger \theta^\dagger ,
\]

\[
\int S(x, \theta, \theta^\dagger) \, d^2 \theta^\dagger = c(x) + \theta \eta(x) + d(x) \theta\theta , \tag{C.8}
\]

\[
\int S(x, \theta, \theta^\dagger) \, d^2 \theta \, d^2 \theta^\dagger = d(x) .
\]

These are often employed in SUSY computations for calculating various terms in the Lagrangians.
References


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References


References


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