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ITERATIVE ADAPTATION OF PREDICTIVE QUANTIZERS

by

Joseph Nicholas Maksym, B.A.Sc., M.Eng.

A Thesis submitted to the Faculty
of Graduate Studies in partial fulfilment
of the requirements for the degree
of Doctor of Philosophy

Faculty of Engineering

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ABSTRACT

This thesis is concerned with the source encoding properties of quantizers employing linear prediction to reduce the variance of the quantizer input. A class of algorithms for iterative adjustment of the coefficients in the linear predictor is developed and studied from the point of view of dynamic and steady state behavior. Simulation results are presented to show that the signal-to-quantization noise ratio in the case of Gaussian Markov input processes approaches very closely the performance given in the literature for previous, more complex, systems.

The effect of channel errors on a complete digital transmission system which employs an adaptive predictor in both the transmitter and receiver is examined, and the effect of parameter mismatch on the system performance is determined.

A study of the speech encoding properties of an iteratively adapted predictive quantizer is made by means of an experimental speech waveform-to-binary encoder. A fundamental difference between speech encoding and Gaussian Markov source encoding is demonstrated; whereas a stationary, or slowly varying, Gaussian Markov source produces a prediction error waveform which is at least quasi-stationary, the prediction

error for voiced speech is highly nonstationary. The error obtained by the experimental encoder becomes as great as the signal itself for a short interval which occurs once per pitch period. The results of subjective comparison tests indicate that a binary version of the adaptive-predictive encoder is slightly inferior for speech source encoding to delta modulation with syllabic companding.

Finally, it is shown how the nonstationary prediction error waveform may be used as a basis for a readily implemented real time pitch period extractor.

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CHAPTER 1

INTRODUCTION

1.1 SOURCE ENCODING

Digital transmission of analogue messages is now widely accepted and promises to supplant many of the analogue methods presently in use. Various factors are responsible for the rather late introduction of digital transmission, with its advantages of greater reliability, freedom from interference and potential economy. Turning aside the obvious lag in digital technology there is a more fundamental reason, namely a lag in the development of an adequate theory of source encoding for analogue messages. One consequence of this lag is that digital systems built to carry analogue information often use many times the bandwidth occupied by the system they are designed to replace.

In his now classic paper Shannon [1] obtained bounds on the performance of digital transmission systems. In particular he showed that one can obtain for most channels a channel capacity C , and that channel encoding methods exist, at least in theory, which allow error free information transmission at rates $\leq C$. Since then much research has been done toward finding implementable channel codes allowing transmission at rates approaching the channel capacity bound. Because of this work in channel coding, one can be reasonably assured that reliable digital information transmission systems can be constructed. This leaves the source coding problem; that is, the conversion of a continuous, analogue valued message waveform into a minimal sequence of digital symbols.

A less well known result of Shannon concerns the minimum information rate $R(D)$ at which source information can be encoded in order that the source be reproducible with average distortion D . Unfortunately this quantity, called the Rate-Distortion function of the source, is usually much more difficult to calculate than the channel capacity C . One problem is a suitable mathematical definition for the distortion measure. For example, no mathematically tractable measure for intelligibility of speech has yet been found. Even for distortion measures such as mean-square-error, which are amenable to mathematical analysis, computation of $R(D)$ for any but the simplest sources is a major task. Where such computations exist, however, they serve as useful bounds on the performance of source encoding systems. For speech sources, in particular, bounds derived by indirect methods [2] demonstrate that present systems of source encoding fall far short of the theoretically possible performance. This includes not only pulse-code-modulation (PCM), but differential pulse-code-modulation (DPCM), delta-modulation (DM), as well as adaptive versions of these techniques, so far developed. Nor does entropy coding¹ of the output data from these devices attain the bounds.

The deficiency of current source encoding systems can be attributed partly to an inadequate model of the source process, and partly to the difficulties inherent in definition of an appropriate distortion measure. Both these factors are clearly demonstrated in speech communi-

¹ Entropy coding is a technique which assigns the shortest code words to the most likely source symbols, or sequences of symbols, thus assuring the smallest average rate of generation of code letters. The technique is due to Huffman [3].

cation where, in the past, various simple models of the speech signal have been used. The simplest model assumes independent speech samples uniformly distributed over a finite amplitude range. Optimization based on this model leads to linear PCM with appropriately chosen quantizer steps so that saturation error and quantization error are balanced for minimum distortion.

1.2 PULSE CODE MODULATION

Optimum source encoding is predicated on a complete a priori knowledge of the physical source process, and an encoder optimized for one source may perform poorly on another whose physical process differs from the first. In some transmission systems intended for general use it is preferable to use a source independent quantization system such as linear pulse-code-modulation. Although this system is not optimized for the sources that might be used, a reasonable level of performance is attained for a wide variety of sources provided only that input amplitude is matched to the range of the quantizer.

Historically, improvements to PCM are based on successively more restrictive assumptions about the input signal process. When the first order probability density of the input signal amplitude is given the optimum quantizer for minimum distortion is the non-uniform one derived by Max [4]. Adding such knowledge as the signal spectrum suggests that in general an optimum PCM system would contain such operations as filtering prior to sampling and quantization, and again for reconstruction. Many papers have been published on various aspects of optimization of this general PCM system. The most recent work, by Chan and Donaldson [5] encompasses most aspects of the optimization, and refers to previous works.

The nature of optimum sampling, filtering, quantization, and reconstruction operations is suggested by the work of Huang and Schultheiss [6], and Wintz and Kurtenbach [7] when viewed in the light of rate distortion theory. Digital transmission of a T-second record of an analogue signal at minimum rate for a given distortion limit can be accomplished by the following operations. The waveform must be decomposed into a sequence of independent components. This is easily accomplished for Gaussian processes, as the well known Karhunen-Loève expansion yields uncorrelated coefficients which are independent as a consequence of the Gaussian distribution. Each statistically independent component can then be encoded separately for transmission. Again in the Gaussian situation the results of rate distortion theory suggest that optimum transmission of the waveform with a given distortion limit is accomplished by encoding only those components with variance greater than a certain value and ignoring the rest. As shown by Wintz and Kurtenbach, one need not use the Karhunen-Loève procedure to accomplish the desired results for Gaussian processes. Simple time sampling followed by a digital filtering operation which orthogonalizes the covariance matrix of the samples accomplishes very nearly the same result.

The preceding discussion introduces the source encoding philosophy of this thesis, that of operating on the signal in such a way as to produce a sequence of uncorrelated, or in the ideal case independent, components which are then encoded for transmission. What is transmitted may be called new information, and is that part of the signal process that cannot be predicted from a priori knowledge about the physical source process and data received in the past.

1.3 DIFFERENTIAL PULSE CODE MODULATION

A waveform such as band-limited speech sampled at a rate greater than twice the bandwidth displays high correlation between adjacent samples. Delta modulation, introduced by Deloraine [8], and by De Jaeger [9], is a simple application of this fact to binary quantization of an analogue source. Use is made of the fact that, for correlated samples, the difference between adjacent samples has a smaller variance than the signal itself. This difference is estimated by forming a crude prediction of the next sample value by integrating the binary output of the system, and forming the difference between the actual input and the prediction. This difference is subsequently quantized to form the binary output of the system.

Numerous articles dealing with various design and performance aspects of delta modulation have appeared in the literature, but the application of the system to source quantization has been limited primarily because of the high data rates necessary for good quality reproduction.

A simple extension of the basic technique to multilevel quantization was introduced by Cutler [10] who used the term differential pulse-code modulation (DPCM) to describe the system.

1.4 PREDICTIVE QUANTIZATION

Delta modulation and DPCM employ a simple form of predictor in their operation. Many authors, among them Elias [11], O'Neill [12], and Graham [13] introduced the theory of linear prediction to the design of DPCM systems. The basic structure of the resulting predictive quantiza-

tion system is given in Fig. 1.1. Various configurations for the predictor structure are possible, but in general it may be assumed that the predictor takes the form of a linear digital filter which produces the estimate $\tilde{x}_n$ of the next signal sample x_n . Under the assumption that the channel is error-free, and that an identical predictor is used at the receiver the following relations hold:

$$y_n = z_n + e_n \quad (1.1)$$

where e_n is the quantization error, and the reconstructed signal at the decoder output is

$$\hat{x}_n = y_n + \tilde{x}_n = x_n + e_n \quad (1.2)$$

Many fundamental studies of the digital communication system of Fig. 1.1 have been conducted with the intent of optimizing its performance. Fine developed a general procedure [14] which, theoretically at least, specifies the jointly optimum forms for the quantizer, predictor, the channel encoding procedure, and the receiver design. Subsequently, Bello, Lincoln and Gish [15] applied Fine's theoretical results toward the optimum design of a predictive binary quantization system by the use of a Monte-Carlo approach which essentially measures the statistical properties of the signal process. More recently, Curry [16], using non-linear estimation theory for dynamical systems described by state variables, and observed by quantized measurements, derived forms for nearly optimum predictive quantization systems.

FIGURE 1.1 DIGITAL COMMUNICATION SYSTEM WITH PREDICTIVE QUANTIZER

The preceeding results will be discussed in further detail in Chapter 2 since they represent fundamental work in predictive source encoding, and serve as the basis for the further developments introduced in this thesis.

1.5 ADAPTIVE SYSTEMS

The optimization procedures mentioned previously are restricted to stationary signal processes. It can readily be appreciated that the statistics of a speech signal, for example, vary with the syllable being spoken. Of course, if the observation time is sufficiently long to encompass statistically significant samples of the various types of waveform present in speech, stationary statistics can be obtained. A fixed source encoder optimized for these statistics will be optimum with respect to the class of fixed encoders, but may perform poorly with respect to some time-varying encoder based upon short term, nonstationary, statistics. This observation, then, motivates the search for adaptive source encoders for signals such as speech.

Much of the work on adaptive source encoding began with efforts to incorporate an adaptive capability in the basic delta-modulation system. Simple delta-modulation forms its prediction by integrating the binary output data, producing a staircase-like waveform with steps of uniform height. It is easy to see that an accurate reproduction of the input waveform is limited for low level inputs by the graininess of the staircase, while for high level signals the reconstruction may not be able to follow a steeply rising waveform since the staircase has a finite

maximum slope, attained when the data pulses are all of the same sign. Winkler [17] obtained improved slope following capability by replacing the integrator with exponential function generators. Thus if a large amplitude transient is encountered in the input signal the successive output pulses which are produced trigger the exponential function generators thereby creating a reconstructed waveform which is a sequence of exponential line segments.

Various systems for adaptive delta modulation have been proposed, all having the capability of altering the step size in response to estimates of the coders overload condition or input signal slope. The actual techniques for accomplishing the adjustment of step size accounts for the differences among the systems. As these are numerous, the interested reader is referred to an article by Schindler [18] for descriptions and further references.

More general adaptive systems for source quantization are described in a report by Stroh and Boorstyn [19]. They develop a differential quantization system involving linear prediction, with prediction coefficients adaptively adjusted to minimize fitting error; that is, the average squared prediction error that would have resulted had the present coefficient values been used over an interval which ended with the current signal sample. Since the signal source is not expected to vary significantly, the predictor coefficients may be expected to be nearly optimum for prediction of the next signal sample.

The main difficulty in this approach to adaptation is one of implementation. Considerable storage and computation are necessary to

obtain the predictor coefficients. Since Stroh and Boorstyn restrict their study to simple Markov source models, performance of their adaptive system for speech sources is not known.

Efficient encoding of speech based on linear prediction of the next signal sample has been demonstrated by Atal and Schroeder [20], and Atal and Hanauer [21]. A model of the speech source based more soundly on the physical processes involved was used to specify a suitable predictor structure. In the model the vocal tract was represented by a time varying linear system driven by a quasi-periodic impulse train for voiced sounds, and by a sequence of independent random variables for unvoiced sounds.

The resulting source encoder is an analysis-synthesis procedure rather than a differential quantizer. The analyzer, which takes the form of a general purpose digital computer, obtains the coefficient values for the predictor, extracts the period of the impulse train, decides whether the speech is voiced or unvoiced, and encodes this information, together with a signal power parameter, for transmission to the synthesizer.

Although implementation of the analysis-synthesis procedure is complex, significant reduction in digital transmission rate is possible compared to adaptive delta modulation and differential pulse code modulation systems.

1.6 AIMS, SCOPE, AND CONTRIBUTIONS OF THE THESIS

1.6.1 Aims

The foregoing discussion indicates that considerable improvement in systems for source encoding is possible through the application of adaptive prediction. The complexity of the resulting system, however, has thus far precluded its practical application. It is the primary aim of this thesis to investigate the feasibility of adaptive-predictive source encoding techniques which are simple to implement. Such techniques will be shown to arise naturally from the philosophy of source encoding assumed by the thesis; that of operating on the input signal in such a way as to reduce the input signal into a sequence of uncorrelated, or in the ideal case independent, components, which are then encoded for transmission.

It will be shown that linear prediction with predictor coefficients adaptively adjusted by an iterative algorithm introduced in this thesis can perform the above decomposition into uncorrelated components reasonably well for most sources including speech. All that seems to be required is that the present signal sample be a reasonably linear function of the immediate past. To the author's knowledge the iterative algorithms suggested in this thesis for adaptive-predictive source encoding have not previously been used for this purpose, although similar techniques have found application in adaptive equalizers for dispersive data channels. The application here is sufficiently different in nature that little of the adaptive equalizer literature is applicable. One major difference in the source encoding problem that does not exist with the equalization problem is the necessity of adaptation of the de-

coder predictor coefficients from data transmitted over the channel, which may have digital errors. This introduces a coefficient tracking problem not present in the adaptive equalizer. This thesis aims to provide some reasonable solutions in the form of algorithms which tolerate channel errors.

Previous to the papers by Atal and Schroeder [20], and Atal and Hanauer [21] very little work on speech encoding from a time domain point of view had been done. Most previous analyses were concerned with spectral properties. This thesis investigates the performance of simpler adaptive-predictive structures than those used by Atal et al. The results point out one of the major difficulties in the source encoding of speech; that the information content of speech occurs in bursts whereas a channel transmits information usually at a constant rate. Thus a source encoder without some capability of smoothing the information rate will be suboptimal. This will be demonstrated by means of an experimental system constructed to encode speech into binary form through the use of adaptive-prediction.

1.6.2 Scope

It is worth mentioning here some limitations on the study reported in this thesis due to limitations on time and equipment.

Ideally both the predictor and the quantizer elements of the source encoder should be made adaptive. This is required because of dynamic range of the input, and also because of the nonconstant rate at which information is generated for sources such as speech. In this thesis the adaptive quantizer problem was not considered in detail.

While multilevel quantization is investigated for Gaussian-Markov sources no results were obtained for speech. Our results for Markov signals indicate that eight-level (3 bit) quantization can lead to improved performance at equivalent transmission rate due mainly to the improved prediction made possible by finer quantization. It is likely that the same may be true with speech sources.

This thesis is concerned mainly in implementational simplicity of the predictor structures and adaptation algorithm. It is likely that modification to improve prediction or adaptation rate may lead to significant improvement at the cost of some additional complexity. In this respect it might be mentioned that reduced sensitivity to channel errors may be obtained through the use of more complex algorithms than those considered in the thesis.

1.6.3 Contributions

Simply implemented iterative techniques for the adjustment of predictive-quantization systems are presented and their performance studied both by simulation and the construction of an experimental binary encoder for speech. To the author's knowledge this thesis represents the first such study of adaptive source encoding through the use of these iterative methods.

The problem of channel errors is investigated and expressions for predicting the performance in the presence of channel errors are derived. Results on the sensitivity to coefficient mismatch between encoder and decoder are also given.

A somewhat unexpected but useful result of the study of speech encoding by means of the experimental binary coder is the development of a new system for real time extraction of fundamental frequency (pitch) information in voiced speech. The operation of this system is simply explained by noting that between vocal cord excitation pulses the speech waveform is simply the free response of a slowly varying linear system, the vocal tract. The adaptive-predictor optimizes its coefficients between vocal cord inputs thereby maintaining a small prediction error. When the vocal tract response is forced by the vocal cord excitation the waveform cannot be accurately predicted, and a short error burst occurs. Filtering and detection of these prediction error bursts offers a simple and reliable method of pitch period extraction. The simplicity and potential economy of the new pitch extractor makes its use attractive not only for speech processing and communication but also for application to linguistics and speech pathology.

1.7 STRUCTURE OF THE THESIS

The thesis is organized into seven chapters and five appendices. Chapter 1 introduces the source encoding problem, and outlines the approach and contributions of this thesis.

Chapter 2 is a summary of basic results concerning digital encoding of analogue sources.

Iterative algorithms for the adaptive adjustment of predictive quantization systems are defined in Chapter 3.

Chapter 4 presents the results of digital computer simulations of the adaptive-predictive quantization systems introduced in Chapter 3.

Operation of an encoder-decoder pair over a channel with digital errors is studied in Chapter 5, and design considerations for this situation are discussed.

An experimental binary encoder for speech is described in Chapter 6, and its properties are investigated.

Fundamental frequency (pitch) determination by adaptive prediction of the speech waveform is demonstrated in Chapter 7 via the experimental binary encoder's prediction error waveform.

The basic results of the thesis are summarized, and problems for future research are suggested in Chapter 8.

CHAPTER 2

OPTIMUM AND ADAPTIVE SOURCE ENCODING

In this chapter are presented some fundamental results necessary for the understanding of the source encoding process. Proofs are not given, as they may be found in the references cited.

2.1 OPTIMUM DIGITAL SYSTEMS

The first general formulation of the problem of digital transmission of analogue data is given by Fine [14]. His model of the digital communication system is shown in Fig. 2.1. Input signal samples represented by the vector

$$\vec{X}_n = (x_n, x_{n-1}, \dots, x_{n-i})^T$$

are transformed by a mapping M_1 into a sequence of data samples

$$\vec{Y}_n = (y_n, y_{n-1}, \dots, y_{n-i})^T$$

where, each y value comes from an M -ary alphabet $\{a_1, a_2, \dots, a_M\}$. The channel mapping, which may be deterministic or stochastic, further transforms the data vector $\vec{Y}_n$ into a received vector $\vec{V}_n$ whose components in general come from some N -ary alphabet $\{b_1, \dots, b_N\}$. Finally the receiver mapping M_3 forms the reconstruction

$$\vec{R}_n = (r_n, r_{n-1}, \dots, r_{n-i})^T$$

FIGURE 2.1 DIGITAL TRANSMISSION SYSTEM MODEL

FIGURE 2.2 MODEL FOR DPCM SYSTEM OPTIMIZATION

There is sufficient generality in the model to allow for the above vectors to be of infinite dimension, implying a constantly increasing memory in the mappings.

Optimality is defined by the value of the general error criterion

$$\varepsilon [\phi(x_{n+k}, r_n)] \quad (2.1)$$

where, $\phi(x, y)$ is a cost function with the following properties.

1. $\phi(x, y)$ is continuous in x and y .
2. $\phi(x, y) \geq 0$
3. $\phi(x, x) = 0$
4. For each y , there are at most countably many solutions of $\phi(x, y) = 0$.
5. For $y_1 \neq y_2$, there are at most countably many solutions of $\phi(x, y_1) = \phi(x, y_2)$.

The commonly used difference distortion measures $\phi(x, y) = |x - y|^N$ are, of course, included by the above definition. The error criterion, Eqn. (2.1) allows formulation of the optimum digital system which performs smoothing ($k < 0$), prediction ($k > 0$), or synchronous reconstruction ($k = 0$). Since prediction is not of interest in a communication system, only synchronous reconstruction and smoothing (reconstruction with delay) are dealt with here. A difference distortion measure such as squared error is assumed.

The optimum mapping M_1 , for M_2 and M_3 given, is found by noting that at time t_{n+k} the input samples $\{x_{n+k-1}, x_{n+k-2}, \dots\}$ have been en-

coded, and the mapping M_1 is sought which assigns to the sample value x_{n+k} that symbol a_i from the alphabet $\{a_1, a_2, \dots, a_M\}$ such that

$$\begin{aligned} \varepsilon\{\phi(x_{n+k} - r_n) | \bar{x}_{n+k}, \bar{x}_{n+k-1}, y_{n+k} = a_i\} &\leq \\ \varepsilon\{\phi(x_{n+k} - r_n) | \bar{x}_{n+k}, \bar{x}_{n+k-1}, y_{n+k} = a_j\} ; j \neq i \end{aligned} \quad (2.2)$$

The inequality (2.2) specifies point sets on the real line which map into symbols of the output alphabet and yield minimum average distortion with the given M_2 and M_3 mappings. A necessary condition for L_{n+k} to be on the boundary of two adjacent point sets mapping into a_i and a_j is

$$\begin{aligned} \varepsilon\{\phi(L_{n+k} - r_n) | x_{n+k} = L_{n+k}, \bar{x}_{n+k-1}, y_{n+k} = a_i\} &= \\ \varepsilon\{\phi(L_{n+k} - r_n) | x_{n+k} = L_{n+k}, \bar{x}_{n+k-1}, y_{n+k} = a_j\} \end{aligned} \quad (2.3)$$

Now the data vector $\bar{y}_{n+k}$ is a sufficient statistic for the reconstruction R_n with respect to the signal vector $\bar{x}_{n+k}$, a result stated by Fine [14] and proved by McMillan [22]. Hence a simpler statement of Eqn. (2.3) is

$$\begin{aligned} \varepsilon\{\phi(L_{n+k} - r_n) | y_{n+k} = a_i, \bar{y}_{n+k-1}\} &= \\ \varepsilon\{\phi(L_{n+k} - r_n) | y_{n+k} = a_j, \bar{y}_{n+k-1}\} \end{aligned} \quad (2.4)$$

Step 2 of the optimization begins by observing that, with M_1 and M_2 specified, the received data vector $\bar{y}_{n+k}$ is uniquely determined by the input sequence $\bar{x}_{n+k}$. The receiver mapping M_3 is chosen to minimize

$$\varepsilon\{\phi(x_{n+k} - r_n) | \vec{v}_{n+k}\} \quad (2.5)$$

A necessary condition for a minimum is that

$$\frac{d}{dr_n} [\varepsilon\{\phi(x_{n+k} - r_n) | \vec{v}_{n+k}\}] = 0 \quad (2.6)$$

Under conditions of continuous differentiability of ϕ , continuity of the conditional probability density $p(x_{n+k} | \vec{v}_{n+k})$, and uniform convergence of the expectation, Eqn. (2.6) may be replaced by

$$\varepsilon \left\{ \frac{d}{dr_n} \phi(x_{n+k} - r_n) | \vec{v}_{n+k} \right\} = 0 \quad (2.7)$$

Assuming synchronous reproduction ($k = 0$), and a mean square error performance criterion, Eqns. (2.4) and (2.7) may be used to write

$$L_n(a_i, a_j) = \frac{\varepsilon\{r_n^2 | y_n = a_j, \vec{v}_{n-1}\} - \varepsilon\{r_n^2 | y_n = a_i, \vec{v}_{n-1}\}}{2\varepsilon\{r_n | y_n = a_j, \vec{v}_{n-1}\} - 2\varepsilon\{r_n | y_n = a_i, \vec{v}_{n-1}\}} \quad (2.8)$$

and

$$r_n = \varepsilon\{x_n | \vec{v}_n\} \quad (2.9)$$

In application the solution of Eqns. (2.3) and (2.9), or (2.4) and (2.7) must be examined by resorting to Eqn. (2.2) to establish which of the solutions are true minima and not just stationary points.

Many previously reported results for quantization and PCM may be derived from Fine's model. For example the nonuniform quantizer for

independent random variables with known amplitude probability density is readily obtained from Eqns. (2.8) and (2.9). Since the signal samples are independent, previous data outputs do not affect the quantizer design which becomes

$$L_n(a_i, a_j) = \frac{r_j + r_i}{2} \quad (2.10)$$

$$r_i = \varepsilon\{x_n | x_n \in (a_i, a_j)\}$$

These are identical to expressions obtained by Max [4].

By the use of the system model shown in Fig. 2.2 optimum delta-modulation and DPCM systems may be defined. For a noiseless channel $\vec{V}_n = \vec{Y}_n$, and the optimization expressions become

$$\varepsilon\{\phi(L_n - r_n) | y_n = a_i, \vec{Y}_{n-1}\} = \varepsilon\{\phi(L_n - r_n) | y_n = a_j, \vec{Y}_{n-1}\} \quad (2.11)$$

$$\varepsilon \left\{ \frac{d\phi(x_n - r_n)}{d r_n} \middle| \vec{Y}_n \right\} = 0 \quad (2.12)$$

In delta modulation the output alphabet has two symbols, $a_1 = 1$ and $a_2 = -1$ and the optimum feedback function is given by

$$L_r = \frac{1}{2} r_n(a_1) + \frac{1}{2} r_n(a_2) \quad (2.13)$$

while the optimum reconstruction satisfies

$$\epsilon \left\{ \frac{d}{dr_n} [\phi(x_n - r_n)] \vec{Y}_n \right\} = 0 \quad (2.14)$$

The preceeding expressions for the optimum differential quantizer indicate that the feedback function and the receiver will vary with time according to the sequence of outputs already encoded. A result of importance to later development in this thesis is the implications of Eqn. (2.13) when the receiver is constrained to be a linear operation on the data, i.e.,

$$r_n(a_j) = \sum_{i=1}^P h_i(n) y_{n-i} + h_o(n) a_j; \quad j = 1, 2 \quad (2.15)$$

Substitution in Eqn. (2.13) yields a related linear operation for the feedback function.

$$L_n = \sum_{i=1}^P h_i(n) y_{n-i} \quad (2.16)$$

A rigorous mathematical study of the digital transmission system shown in Fig. 2.1 is reported by McMillan [22]. Restricting the analysis to error free channels he presents proofs for certain properties stated by Fine. Specifically it is proved that an optimum encoder need only use previously encoded data rather than the actual analogue input for its encoding scheme. Further, if the receiver is optimized under a limited memory constraint, the transmitter need only have the same amount of memory.

Here, three theorems giving basic properties of optimum digital transmission systems will be stated. Proofs of these theorems are lengthy and will not be given. First it is necessary to present a definition.

Definition: A digital system described by the sequence $\{x_n, y_n, r_n\}$ will be called $\{\phi_n, d\}$ - optimal if for each n , $\varepsilon\{|\phi_n(x_{n-d}, r_n)|\} < \infty$, and for every other system $\{x_n, y'_n, r'_n\}$, $\varepsilon\{\phi_n(x_{n-d}, r_n)\} \leq \varepsilon\{\phi_n(x_{n-d}, r'_n)\}$ where, d is a delay ≥ 0 .

The cost function ϕ_n may vary with time but, other than that, is defined by Eqn. (2.1).

McMillan's theorems may now be stated.

Theorem 1: If the system $\{x_n, y_n, r_n\}$ is $\{\phi_n, 0\}$ - optimal then, each y_n is equal with probability one to a random variable measurable on the sample space of $\{x_n, y_{n-1}, y_{n-2}, \dots\}$ provided the signal samples $\{x_n\}$ are subject to the following restriction.

Restriction: $\{x_n\}$ consists of those processes for which

$$\Pr(z = x_n) = 0 \quad (2.17)$$

where, z is a random variable measurable on the space of $\{x_{n-1}, x_{n-2}, \dots\}$. McMillan gives the designation CCD (continuous conditional density) to such processes. In theorems 2 and 3 a class of processes called CCDf, implying that z is determined by events of a finite Borel field A , or the completion of A , as well as $\{x_{n-1}, x_{n-2}, \dots\}$, is

assumed. The above implies of course that the estimation problem for $\{x_n\}$ is nonsingular with probability one.

Theorem 2: Let the system $\{x_n, y_n, r_n\}$ be $\{\phi_n, d\}$ - optimal and $\{x_n\} \in \text{CCDf}$, then each y_n is equal with probability one to a random variable measurable on the sample space of $\{x_{n-d}, y_{n-1}, y_{n-2}, \dots\}$. Further defining $y'_n = y_{n+d}$ and $r'_n = r_{n+d}$ the system $\{x_n, y'_n, r'_n\}$ is $\{\phi'_n, 0\}$ - optimal where $\phi'_n = \phi_{n+d}$.

The implications of Theorem 2 are that a delay in reconstruction is of no advantage since the signal samples $x_{n-d+1}, \dots, x_n$ are unavailable to the receiver.

In a practical system it is often desired to constrain the memory to some finite number of samples m , in which case a $\{\phi_n, d, m\}$ - optimal system may be defined within the memory constraint.

Theorem 3: In the system $\{x_n, y_n, r_n\}$ which is $\{\phi_n, d, m\}$ optimal for $m < \infty$, each y_n is equal with probability one to a random variable measurable on the sample space of $\{x_{n-d}, y_{n-1}, \dots, y_{n-m}\}$. Further defining $y'_n = y_{n+d}$ and $r'_n = r_{n+d}$ the system $\{x_n, y'_n, r'_n\}$ is $\{\phi_n, 0, m\}$ - optimal where $\phi'_n = \phi_{n+d}$. Also if $d = 0$ then it suffices that $\{x_n\} \in \text{CCD}$ and the restriction $m < \infty$ may be removed. Then if $m < \infty$, $\{x_n\} \in \text{CCDf}$ may be replaced by the simpler requirement that if z is a random variable that takes on only finitely many values, then $P_r(x_n = z) = 0$.

In applications the rigorous measure theoretic language may be dropped. Such statements as "equal with probability one to a random variable

measurable on the sample space of $\{x_n, y_{n-1}, \dots\}$," are equivalent in practice to "functionally dependent only upon $\{x_n, y_{n-1}, \dots\}$." A deficiency of the theory, of course, is that the form of the functional dependence is unspecified. The overall block structure of the optimum system is given by McMillan's theory and the following lemma, restated from Mc Millan [22].

Lemma: An M-discrete random variable y is given, together with a Borel field G . If r is a random variable measurable on the Borel field determined by G and the sample space of y , then there exist random variables $\{z_p: p \in a_1, a_2, \dots, a_M\}$ such that

- (i) each z_p is measurable on G , and
- (ii) for each $\omega \in \Omega$, if $y(\omega) = a_p$, then $r(\omega) = z_p(\omega)$.

The resulting optimum system is given by Fig. 2.3 for the situation of zero-delay encoding described in Theorem 1. Rigorous derivation of this structure is given in [22]. The operation may be described in simple terms. Just before x_n is received the functions $z_{in}: i = 1, 2, \dots, M$ are formed from the stored values of $y_{n-1}, y_{n-2}, \dots$. These functions can in general vary with time. When x_n is known the cost functions $\phi(x_n, z_{in})$; $i = 1, 2, \dots, M$ are formed. The output symbol $y = a_j$ is selected corresponding to the index j which produced the minimum $\phi(x_n, z_{jn})$.

One might ask whether the optimum encoder may be realized by a differential quantizer such as that illustrated in Fig. 2.4. To show this a time varying quantizer with output levels $a_{1,n}, a_{2,n}, \dots, a_{M,n}$ is assumed, where each $a_{i,n}$ is a function of the past data $y_{n-1}, y_{n-2}, \dots$.

FIGURE 2.3 STRUCTURE OF THE OPTIMUM ENCODER
(from [22], Fig. 1)

FIGURE 2.4 ALTERNATIVE STRUCTURE FOR A RESTRICTED CLASS
OF COST FUNCTIONS

Also define a function of the past data, $\tilde{x}_n$, independent of the index i .

Then let

$$\begin{aligned} z_{1n} &= \tilde{x}_n + a_{1,n} \\ z_{2n} &= \tilde{x}_n + a_{2,n} \\ &\vdots \\ z_{mn} &= \tilde{x}_n + a_{m,n} \end{aligned} \quad (2.18)$$

The optimum encoder of Fig. 2.3 transmits a_i if

$$\phi(x_n - z_{in}) < \phi(x_n - z_{jn}); \quad \forall j \neq i \quad (2.19)$$

Using Eqn. (2.18) and the fact that $z_n = x_n - \tilde{x}_n$, Eqn. (2.19) may be written

$$\phi(z_n - a_{i,n}) < \phi(z_n - a_{j,n}); \quad \forall j \neq i \quad (2.20)$$

This comparison operation may be performed by a quantizer for a restricted class of cost functions $\phi(x,y)$ which are continuous strictly monotone functions of $|x-y|$, a class which includes the commonly used squared-difference function $\phi(x,y) = (x-y)^2$. The system of Fig. 2.4 may then be realized through the use of a time varying quantizer with outputs $a_1, a_2, \dots, a_m$. The quantizer input crossover points are given by

$$z_i = \frac{a_{i,n} + a_{i+1,n}}{2} \quad (2.21)$$

The receiver forms the function $\tilde{x}_n$ and the $a_{i,n}$ corresponding to the data $y_n = a_i$ currently received. These are added to form the reconstruction

$$r_n = \tilde{x}_n + a_{i,n} \quad (2.22)$$

It should be noted that the system of Fig. 2.3 could also be formulated as a time varying quantizer. Thus no essential differences exist in the operations which must be performed in an optimum system whether it be realized according to Fig. 2.3 or Fig. 2.4. Where a suboptimum system is to be used to simplify physical implementation, a form similar to Fig. 2.4 will be used in this thesis.

2.2 STATISTICAL DELTA MODULATION

In the preceeding section two different approaches to a general theory of digital transmission systems are described. Here an application of Fine's theory [14] to the optimum design of binary digital transmission systems, by Bello, Lincoln, and Gish [15], is summarized. Since the system design is determined by the statistical properties of the signals, the approach is called statistical Delta Modulation. The design begins by noting that the reconstructed value r_n is solely determined by the past data samples according to the, yet to be determined, mapping

$$r_n = f_n(y_n, y_{n-1}, y_{n-2}, \dots, y_{n-m}) \quad (2.23)$$

This statement, as previously noted in Section 2.1, is proved by McMillan [22]. In Eqn. (2.23) a limited memory is assumed. The first step of the optimum system design is to assume a fixed receiver and design a binary quantizer producing +1 and -1 outputs by comparing the input sample x_n with

$$L_n = \frac{f_n(-1, y_{n-1}, y_{n-2}, \dots) + f_n(+1, y_{n-1}, y_{n-2}, \dots)}{2} \quad (2.24)$$

This follows from Eqn. (2.8) directly by assuming that the receiver is fixed. Having determined the optimum sequence $\{L_n\}$, subject, of course, to the fixed receiver constraint, the next step of the optimization procedure is carried out. For a mean-square cost function the optimum receiver is given by Eqn. (2.9), which is restated below.

$$r_n = \varepsilon\{x_n | y_n, y_{n-1}, \dots, y_{n-m}\} \quad (2.9a)$$

Combining Eqns. (2.24) and (2.9a), the condition for an overall optimum system is

$$L_n = \frac{\varepsilon\{x_n | y_n = 1, y_{n-1}, \dots\} + \varepsilon\{x_n | y_n = -1, y_{n-1}, \dots\}}{2} \quad (2.25)$$

Unfortunately the conditioning on $y_{n-1}, y_{n-2}, \dots, y_{n-m}$ involves dependence on prior values of $\{L_n\}$. This difficulty is resolved in [15] by an iterative procedure as follows. An initial receiver mapping

$$r_{on} = f_{on}(y_n, y_{n-1}, \dots, y_{n-m}) \quad (2.26)$$

is first assumed between all possible binary words of length $m+1$ and the 2^{m+1} reconstruction values. A set of 2^m values of L_0 are obtained from Eqn. (2.24), each corresponding to a possible binary word of length m . i.e.,

$$L_o = L_o(y_{n-1}, y_{n-2}, \dots, y_{n-m}) \quad (2.27)$$

This set of L_o values is then used to encode a sufficiently long sequence of input samples in order that a new receiver mapping r_{1n} may be formed by estimating the conditional means in Eqn. (2.9a). A new set of levels L_1 is then obtained, and the iteration is repeated until little change results in subsequent iterations. The procedure requires a large number of samples of the input process for accurate estimates since each $y_n, y_{n-1}, \dots, y_{n-m}$ sequence occurs infrequently, and only those x_n values associated with each particular sequence are useful in forming each conditional mean estimate.

The iterative design procedure just described was performed by Bello, Lincoln, and Gish for the second order Gauss-Markov signal process generated by the recursive equation

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + b_1 w_n + b_2 w_{n-1} \quad (2.28)$$

where, $\{w_n\}$ is a sequence of zero-mean, unit variance, independent, Gaussian random variables. The coefficients a_1, a_2, b_1 , and b_2 are suitably chosen to correspond to a system transfer function

$$H(s) = \frac{A}{(1 + \tau s)^2} \quad (2.29)$$

where, A is selected to obtain a unit variance process. The coefficients, as well as initial values x_0 and x_1 for the process to appear stationary

from the start, are calculated (see Appendix A) by a method given by Levin [23]. The autocorrelation function corresponding to the above signal is given by

$$\phi(n) = \left(1 + \frac{|n|}{R}\right) \exp\left(-\frac{|n|}{R}\right) \quad (2.30)$$

where, $R = \frac{T}{T_s} =$ number of samples per time constant
and, $T_s =$ sample period.

By adjusting R , sequences with various values of adjacent sample correlation are produced.

It will be useful, for later comparison with systems developed in this thesis, to summarize the performance of Statistical Delta Modulation. Signal-to-noise ratio as a function of memory length m for $R = 2.5$ is shown in Fig. 2.5 (redrawn from [15], Fig. 4). It is interesting to note that for the given R value the SNR curve begins to saturate at $m = 8$. On the other hand, for zero memory one would expect that Statistical Delta Modulation would produce an optimum quantizer in the sense of Max [4]. The SNR calculated on this basis (4.405 dB) is very close to the experimental result.

As R is increased one would expect performance to improve. This is illustrated in Fig. 2.6 (from [15], Fig. 5).

The conclusion obtained by Bello, Lincoln and Gish that Statistical Delta Modulation with linear mapping constraints provides essentially the same mean squared reconstruction error, at least for the signal process given by Eqn. (2.28), is important in later sections of this thesis. There, quantizers with linear feedback optimized by iterative techniques fundamentally different to those just described are introduced.

FIGURE 2.5 STATISTICAL DELTA MODULATION PERFORMANCE FROM ([15], FIGURE 4)

FIGURE 2.6 STATISTICAL DELTA MODULATION PERFORMANCE (FROM [15] FIGURE 5)

2.3 ESTIMATION THEORY APPROACH TO PREDICTIVE QUANTIZATION

The preceeding discussion has illustrated the central role played by the source model in determining the parameters of the optimum digital transmission system. It is natural, then, to consider state space modelling, and the well known theory of estimation for such models for the design of optimum digital systems. The (linear, sampled-data) state variable model of the dynamical process generating the observed signal sequence may be written

$$\begin{aligned}\vec{X}_{n+1} &= \vec{\Phi}_n \vec{X}_n + \vec{W}_n \\ \vec{Z}_n &= \vec{H}_n \vec{X}_n + \vec{V}_n\end{aligned}\quad (2.31)$$

where, $\vec{X}_n$ is an n -vector
 $\vec{\Phi}_n$ is a $(n \times n)$ - dimensional transition matrix
 $\vec{W}_n$ is an n -vector of random variables
 $\vec{H}_n$ is a $(r \times n)$ -dimensional observation matrix
 $\vec{V}_n$ is an r -vector of random variables.

In most state variable models the following assumptions are commonly made.

$$\begin{aligned}\vec{W}_n \text{ and } \vec{V}_n &\text{ are normally distributed} \\ \varepsilon\{\vec{W}_n\} &= \varepsilon(\vec{V}_n) = 0 \\ \varepsilon\{\vec{X}_0\} &= 0 \\ \text{Cov}\{\vec{X}_0\} &= \vec{P}_0 \\ \varepsilon\{\vec{W}_n \vec{W}_m^T\} &= \vec{Q}_n \delta_{mn} \\ \varepsilon\{\vec{V}_n \vec{V}_m^T\} &= \vec{R}_n \delta_{mn} \\ \{\vec{W}_n\} \text{ and } \{\vec{V}_n\} &\text{ are mutually independent and independent of } \vec{X}_0.\end{aligned}\quad (2.32)$$

Algorithms for filtering, smoothing and prediction for the linear model of Eqn. (2.31) under the assumptions stated in Eqn. (2.32) are available. These derive from the work of Kalman [24], and later papers by other authors too numerous to list here.

Unfortunately the process of quantization in the digital transmission system introduces a nonlinear aspect to the overall system model. Two approaches are possible.

1. Make prior assumptions about the nature of the quantization error process in order to formulate the problem as a linear one.
2. Obtain algorithms for the nonlinear estimation problem involving quantized measurements.

The linearization approach, first reported by Irwin and O'Neill [25], is described below. A scalar signal process observed with additive quantization noise is assumed according to Fig. 2.7a, resulting in the state variable model.

$$\begin{aligned} x_{n+1} &= \alpha x_n + w_n \\ \hat{x}_n &= x_n + \varepsilon_n \end{aligned} \quad (2.33)$$

where, ε_n , the quantization error samples, are assumed mutually uncorrelated and normally distributed. The Kalman filter algorithm for prediction of x_n based on past observations $\hat{x}_{n-1}, \hat{x}_{n-2}, \dots$ is given by

$$\tilde{x}_n = \alpha \tilde{x}_{n-1} + \alpha K_n [\hat{x}_{n-1} - \tilde{x}_{n-1}] \quad (2.34)$$

FIGURE 2.7a MODEL OF DPCM SYSTEM

FIGURE 2.7b DPCM SYSTEM OPTIMIZED VIA KALMAN PREDICTION

where, K_n is obtained via the recursive equations

$$\begin{aligned} K_n &= (D_n + \sigma_\epsilon^2)^{-1} D_n \\ D_n &= \alpha^2 E_{n-1} + \sigma_w^2 \\ E_n &= D_n (1 - K_n) \end{aligned} \quad (2.35)$$

By noting that

$$\hat{x}_{n-1} = x_{n-1} + \epsilon_{n-1} \quad (2.36)$$

the prediction equation can be written

$$\tilde{x}_n = \alpha \tilde{x}_{n-1} + K_n y_{n-1} \quad (2.37)$$

This leads to the optimum structure shown in Fig. 2.7b. Not shown is the recursive filter that determines the time varying gain K_n . Initial values $\tilde{x}_0$ and E_0 are required at the beginning because of the recursive nature of the system. For the stationary signal model in Eqn. (2.33) the gain K_n will approach a constant.

The best example of nonlinear estimation with quantized measurements is reported in Curry [26], and Curry, Vander Velde, and Potter [16]. The nonlinear estimator is derived from the linear one given by the recursive equations

$$\hat{\tilde{x}}_{n|n} = \hat{\tilde{x}}_{n|n-1} + \hat{K}_n [\hat{Z}_n - \hat{H}_n \hat{\tilde{x}}_{n|n-1}] \quad (2.38)$$

$$\hat{\vec{X}}_{n+1|n} = \hat{\vec{\phi}}_n \hat{\vec{X}}_{n|n} \quad (2.39)$$

$$\hat{\vec{K}}_n = \hat{\vec{M}}_n \hat{\vec{H}}_n^T (\hat{\vec{H}}_n \hat{\vec{M}}_n \hat{\vec{H}}_n^T + \hat{\vec{R}}_n)^{-1} \quad (2.40)$$

$$\hat{\vec{M}}_{n+1} = \hat{\vec{\phi}}_n \hat{\vec{P}}_n \hat{\vec{\phi}}_n^T + \hat{\vec{Q}}_n \quad (2.41)$$

$$\hat{\vec{P}}_n = \hat{\vec{M}}_n - \hat{\vec{M}}_n \hat{\vec{H}}_n^T (\hat{\vec{H}}_n \hat{\vec{M}}_n \hat{\vec{H}}_n^T + \hat{\vec{R}}_n)^{-1} \hat{\vec{H}}_n \hat{\vec{M}}_n \quad (2.42)$$

These are known as the Kalman Filtering equations for the state vector $\hat{\vec{X}}_n$ given by the system, Eqn. (2.31) under the assumptions in Eqn. (2.32). Under these assumptions

$$\hat{\vec{X}}_{n|n} = \varepsilon(\hat{\vec{X}}_n | \hat{\vec{Z}}_n, \hat{\vec{Z}}_{n-1}, \dots) \quad (2.43)$$

$$\hat{\vec{P}}_n = \text{Cov}(\hat{\vec{X}}_n | \hat{\vec{Z}}_n, \hat{\vec{Z}}_{n-1}, \dots) \quad (2.44)$$

Curry [16], [26] shows that an extension of the above equations may be made by an application of a well known result for iterated conditional expectations, Doob [27], or Papoulis [28], for example. Applied to a quantized observation the iterated expectation formula states that if $f(x)$ is a function of x , then

$$\varepsilon\{f(x) | Z \in A\} = \varepsilon\{\varepsilon[f(x) | Z] | Z \in A\} \quad (2.45)$$

An approximate nonlinear estimation procedure based on the Kalman Filter equations is derived by Curry as follows. The distribution of $\hat{\vec{X}}_n$ con-

ditioned on the quantized measurements

$$\vec{Z}_{n-1} \in A_{n-1}, \vec{Z}_{n-2} \in A_{n-2}, \dots \quad (2.46)$$

is modelled as if it were Gaussian; that is, the algorithm transmits the conditional mean, and covariance of $\vec{X}_n$ under a Gaussian assumption. The above Gaussian fit assumption together with Eqn. (2.45) allows the, approximate, nonlinear estimations to be derived. These are stated below without proof.

$$\vec{X}_{n|n} = \vec{X}_{n|n-1} + \vec{K}_n [\epsilon(\vec{Z}_n | \vec{Z}_n \in A_n) - \vec{H}_n \vec{X}_{n|n-1}] \quad (2.47)$$

$$\vec{X}_{n+1|n} = \vec{\phi}_n \vec{X}_{n|n} \quad (2.48)$$

$$\vec{K}_n = \vec{M}_n \vec{H}_n^T (\vec{H}_n \vec{M}_n \vec{H}_n^T + \vec{R}_n)^{-1} \quad (2.49)$$

$$\vec{P}_n = \vec{M}_n - \vec{M}_n \vec{H}_n^T (\vec{H}_n \vec{M}_n \vec{H}_n^T + \vec{R}_n)^{-1} \vec{H}_n \vec{M}_n \quad (2.50)$$

$$\vec{E}_n = \vec{P}_n + \vec{K}_n \text{cov}(\vec{Z}_n | \vec{Z}_n \in A_n) \vec{K}_n^T \quad (2.51)$$

$$\vec{M}_{n+1} = \vec{\phi}_n \vec{E}_n \vec{\phi}_n^T + \vec{Q}_n \quad (2.52)$$

The form of the expressions is identical to the linear measurement case.

The main differences are that the input is now the conditional mean

$\epsilon(\vec{Z}_n | \vec{Z}_n \in A_n)$, and the covariance equation is driven by an additional term $\vec{K}_n \text{cov}(\vec{Z}_n | \vec{Z}_n \in A_n) \vec{K}_n^T$.

Apart from the necessity for justification of the Gaussian assumption, there are further difficulties since $\varepsilon\{\vec{Z}_n | \vec{Z}_n \in A_n\}$ and $\text{cov}(\vec{Z}_n | \vec{Z}_n \in A_n)$ are not directly available as was $\vec{Z}_n$. They must be computed under an assumption such as

$$\vec{Z}_n \sim N(\vec{H}_n \vec{X}_n |_{n-1}, \vec{H}_n \vec{M}_n \vec{H}_n^T + \vec{R}_n) \quad (2.53)$$

Optimization of predictive quantization systems by means of the Gaussian fit algorithm proceeds as follows. Consider the system shown in Fig. 2.8. The sequence $\{z_n\}$ to be quantized is scalar, but is related to the state variable vector $\{\vec{X}_n\}$ through the measurement matrix $\vec{H}_n$. The conditional mean receiver structure of Fig. 2.8 is a direct consequence of the Gaussian fit algorithm provided the Gaussian assumption is true. The design procedure reduces to specifying an optimum quantizer and predictor function L_n^* such that

$$\begin{aligned} & \varepsilon\{\text{cov}(\vec{X}_n | L_n^*, y_n, y_{n-1}, \dots) | y_{n-1}, y_{n-2}, \dots\} \\ & - \varepsilon\{\text{cov}(\vec{X}_n | L_n, y_n, y_{n-1}, \dots) | y_{n-1}, y_{n-2}, \dots\} \geq 0 \end{aligned} \quad (2.54)$$

Such a specification assures a minimum variance estimate for any linear combination of state variables including, of course, $Z_n = \vec{H}_n \vec{X}_n$. The covariance in Eqn. (2.54) is generated during the operation of the Gaussian fit algorithm. Recall that in Eqn. (2.51)

$$\vec{E}_n = \text{cov}(\vec{X}_n | \vec{Z}_n \in A_n, \vec{Z}_{n-1} \in A_{n-1}, \dots) \quad (2.55)$$

FIGURE 2.8 NOISELESS CHANNEL PREDICTIVE QUANTIZATION SYSTEM
(from Curry [26])

Now note that the conditioning on $y_{n-1}, y_{n-2}, \dots$ in Eqn. (2.54) implies that L_n is not a random variable so that in Eqn. (2.57) z_n may be replaced by u_n . Thus, in Eqn. (2.54) the covariances are given by

$$\hat{E}_n = \hat{P}_n + \hat{K}_n \text{cov}(u_n | y_n) \hat{K}_n^T \quad (2.56)$$

Now let the quantizer have m outputs so that $y_n \in \{A^1, A^2, \dots, A^m\}$, where superscripts are used to distinguish the different outputs. The optimality condition, Eqn. (2.54), may be written

$$\sum_{i=1}^m \text{cov}(u_n | y_n = A^i) P_r(y_n = A^i) \geq \sum_{i=1}^m \text{cov}(u_n | y_n^* = A_i^*) P_r(y_n^* = A_i^*) \quad (2.57)$$

An assumption such as Eqn. (2.53) allows computation of the quantities in Eqn. (2.57). Expressions for $\varepsilon(z | z \in A)$ and $\text{cov}(z | z \in A)$ are given by Curry [26] for this purpose.

One further assumption [26], that

$$L_n^* = \varepsilon(z_n | y_{n-1}, y_{n-2}, \dots) = \hat{L}_n \hat{X}_{n|n-1} \quad (2.58)$$

completes the predictive quantization system design.

The general technique may be illustrated by application to binary predictive quantization of a stationary signal process. Because of the fact that

$$\text{cov}[z_n | z_n \in (0, \infty)] = \text{cov}[z_n | z_n \in (-\infty, 0)] \quad (2.59)$$

Eqns. (2.49) through (2.52) are predetermined functions of time, independent of the particular data sequence. Furthermore, for stationary signal processes asymptotic values $\vec{K}_\infty$, $\vec{E}_\infty$, and $\vec{M}_\infty$ will be approached as more samples are processed. This asymptotic solution is given by [26],

$$\vec{X}_{n|n-1} = \vec{\phi} \vec{X}_{n-1|n-1} \quad (2.60)$$

$$L_n^* = \vec{H} \vec{X}_{n|n-1} \quad (2.61)$$

$$\vec{X}_{n|n} = \vec{X}_{n|n-1} + \vec{K}_\infty \sqrt{\frac{2}{\pi}} \sigma_{z^\infty} \text{sgn}(z_n - L_n^*) \quad (2.62)$$

$$\sigma_{z^\infty}^2 = \vec{H} \vec{M}_\infty \vec{H}^T + \vec{R} \quad (2.63)$$

$$\vec{K}_\infty = \vec{M}_\infty \vec{H}^T (\vec{H} \vec{M}_\infty \vec{H}^T + \vec{R})^{-1} \quad (2.64)$$

Application to the scalar signal model

$$\begin{aligned} x_{n+1} &= \alpha x_n + w_n \\ z_n &= x_n \end{aligned} \quad (2.65)$$

yields a structure identical to Fig. 2.7b, the only difference being in the method of calculation of the gain $\vec{K}_n$, and its exact value.

This difference is, of course, a consequence of different assumptions about the quantization process.

The asymptotic mean square error for PCM, DPCM, and Data Compression systems using the algorithms just described has been derived in [16] and [26], both analytically and by digital computer simulation for the second order Gaussian-Markov process defined by Eqns. (2.28) and (2.29). The results are reproduced in Fig. 2.9a and Fig. 2.9b for the DPCM system for later comparison with the systems developed in this thesis. It is interesting to compare these results with the Statistical Delta Modulation results given in Fig. 2.5 and Fig. 2.6. Performance is virtually identical for two conditions:

1. $R = 2.5$ and long memory SDM.
2. $R < 2$ and $m = 5$ in SDM.

These conditions define the region of near-optimum performance of Statistical Delta Modulation; that is, sufficient memory for a given degree of correlation between samples.

2.4 SOURCE ENCODERS USING LINEAR PREDICTION

The theory of linear prediction is, of course, well known, and source encoders employing linear prediction are well documented in the literature [11], [12], [13]. Since the linear transversal filter is the basic component of the adaptive systems studied in this thesis, it will be useful to review some of the fundamental concepts of linear prediction theory, however.

Linear prediction of sample values of a stationary sequence $\{x_n\}$ is based on knowledge of the autocorrelation function

$$R_{xx}(k) = E(x_n x_{n-k}) \quad (2.66)$$

FIGURE 2.9a PREDICTIVE QUANTIZATION VIA GAUSSIAN FIT ALGORITHM (FROM CURRY [26])

FIGURE 2.9b PREDICTIVE QUANTIZATION VIA GAUSSIAN FIT ALGORITHM (FROM CURRY [26])

The linear finite memory predictor given by

$$\tilde{x}_n = \sum_{i=1}^M C_i x_{n-i} \quad (2.67)$$

is optimized in the sense of minimum mean-squared error if the coefficient set $\{C_i: i = 1, \dots, M\}$ consists of solutions to the linear equations

$$\sum_{i=1}^M C_i R_{xx}(j-i) = R_{xx}(j) \quad (2.68)$$

where, $j = 1, 2, \dots, M$.

It is well known that the mean square prediction error is a quadratic form in the coefficient values with an unique minimum given by the solutions to Eqn. (2.68). These equations, then, represent necessary and sufficient conditions for minimum mean-square-error prediction subject, of course, to the memory and linearity constraints imposed by Eqn. (2.67).

Of interest in adaptive systems which use linear prediction are methods of obtaining the optimum coefficients without solving Eqn. (2.68) directly, a procedure which is highly sensitive to small errors in the measurement of $R_{xx}(j-i)$ and $R_{xx}(j)$ [29]. One such method, suggested for predictive quantization systems by Stroh and Boorstyn [19], is a gradient following technique which measures the gradient of the mean-squared-prediction error and adjusts the coefficient set in a direction opposite to the maximum gradient. By a sequence of such steps the, unique, optimum coefficient set is approached. As will be shown later, in Section 2.6, the approach of Stroh and Boorstyn still represents considerable difficulty for real-time hardware implementation. A modified iterative method ideally suited for real-time implementation in predictive quantization

systems will be introduced in this thesis. The iterative method is based upon the following orthogonality property of optimum linear prediction [28].

$$E \left\{ \left(x_n - \sum_{i=1}^M C_i x_{n-i} \right) x_{n-j} \right\} = 0; \quad j = 1, 2, \dots, M \quad (2.69)$$

It should be noted that this is equivalent to the linear Eqn. (2.59). The condition, Eqn. (2.69), states that for optimally adjusted C_i values the prediction error is uncorrelated with each sample in the memory of the predictor. If, however, one or more of the C_i values are in error this is no longer true and some correlation will exist. It will be shown in Chapter 4 that an efficient adaptive procedure may be realized by adjusting each C_j independently according to the sign of the stochastic variable

$$e_n x_{n-j} = \left(x_n - \sum_{i=1}^M C_i x_{n-i} \right) x_{n-j}; \quad j = 1, \dots, M \quad (2.70)$$

by a small amount at each sampling instant. Since the above quantity is readily available in predictive quantization systems, physical implementation is simple.

Of particular interest in this thesis are signal processes which are generated by linear systems driven by stochastic inputs. Such systems, for example, provide adequate models of the speech process for predictive encoder design and analysis-synthesis [20], [21]. In general a time-varying linear system as well as a non-Gaussian stochastic input will be necessary for a good model. Optimum prediction will not be ob-

tained by the use of Eqn. (2.67) and Eqn. (2.68) in such a situation. Nevertheless, if the linear filter in the source is slowly varying, an adaptive linear predictor will closely approximate optimum performance. The stochastic driving process presents more of a problem. In some situations, such as unvoiced speech, for example, an uncorrelated Gaussian sequence provides a reasonable approximation [20], [21]. In most situations our assumptions about the driving process must be limited by complexity constraints of the source encoder-decoder system. A simple illustration of this is provided by voiced speech, modelled by a time-varying linear system driven by a quasi-periodic pulse train. What we define as stochastic is dependent partly on our knowledge of the physical source process and partly on the complexity allowable in the encoder. For example, each pulse in the quasi-periodic driving signal may be considered as a random input. Alternatively the quasi-periodic pulse train may itself be modelled as a time-varying linear system driven by some, as yet undefined, stochastic process. With this new model, the source encoder may be more efficient, but its complexity will increase. Furthermore, the adaptive structure necessary to implement the encoder will introduce errors which will offset some of the improvement.

In the preceeding some intuitive justification for the use of time-varying linear systems with stochastic excitation as models for sources was given. A properly chosen model, ideally, would be driven by a sequence of independent random variables which provide new information to a dynamical system actually producing the source output. When the dynamical system varies slowly with time an adaptive linear predictor is

suggested for the source encoder. Here we present some properties of signal sources which may be modelled by time-invariant, stable, linear systems driven by a white random process. (These properties will be approximately true for slowly time-varying models.) The sampled power spectrum is given by

$$G_x(z) = \sum_{n=-\infty}^{\infty} R_{xx}(n) z^{-n} \quad (2.71)$$

where, $G_x(z)$ is a rational function. Signal source models may be realized in various forms. For example, the finite mixed moving-average form

$$\sum_{i=0}^N a_i x_{n-i} = \sum_{i=0}^M b_i w_{n-i} \quad (2.72)$$

where, $\{w_n\}$ is a white noise process. Another is the impulse response form

$$x_n = \sum_{i=0}^{\infty} h_i w_{n-i} \quad (2.73)$$

where, $\{h_i\}$ is the sampled data impulse response. The power spectrum of the rational spectrum source takes the form

$$G_x(z) = H(z)H(z^{-1}) \sigma_w^2 \quad (2.74)$$

where,

$$H(z) = \frac{1 + a_1 z^{-1} + \dots + a_m z^{-m}}{1 + b_1 z^{-1} + \dots + b_N z^{-N}} \quad (2.75)$$

For convenience, Eqn. (2.75) is normalized so that $a_0 = b_0 = 1$. In general this may involve a constant factor b_0/a_0 outside the expression.

Two equivalent structures for linear prediction of the above process may be obtained. One of these, restricted to finite memory, is given by Eqn. (2.67). The general form for infinite memory is

$$\tilde{x}_n = \sum_{i=1}^{\infty} c_i x_{n-i} \quad (2.76)$$

Applying Eqn. (2.73) an equivalent predictor may be written

$$\tilde{x}_n = \sum_{j=1}^{\infty} d_j w_{n-j} \quad (2.77)$$

where,

$$d_j = \sum_{i=1}^{\infty} c_i h_{j-i}$$

Thus, the predictor may use as data either the past values of the signal or past values of the excitation sequence of the source. In terms of z-transforms, Eqn. (2.76) may be written

$$\tilde{X}(z) = C_1(z) X(z) \quad (2.78)$$

and Eqn. (2.77) may be written

$$\tilde{X}(z) = D_1(z) W(z) \quad (2.79)$$

where

$$D_1(z) = C_1(z) H_0(z) \quad (2.80)$$

The subscripts in the above z-transforms indicate the range of the corresponding summation forms. The preceding discussions suggest the equivalent or "dual" predictor structures shown in Fig. 2.10 (a) and Fig. 2.10(b). Minimum mean squared error results if

$$D_1(z) = H_0(z) \quad (2.81)$$

then

$$D_1(z) = \frac{C_1(z)}{1 - C_1(z)} \quad (2.82)$$

The relation, Eqn. (2.82) for equivalent predictor is maintained even if Eqn. (2.81) is not satisfied; that is, for non-minimum mean squared error.

We are now in a position to specify predictive quantization systems based on the structures shown in Fig. 2.10. In practice finite memory approximations must be made, and in general one would expect more storage to be necessary in the nonrecursive form shown in Fig. 2.10(b). To avoid excessive storage a hybrid form involving an integrator, or other linear filtering operation, in the feedback path is suggested as well. For the sake of easy reference these systems are labelled as Type I, Type II and Type III, and are shown in Fig. 2.11 (a), (b), and (c) respectively.

Because of the quantization, and the finite memory constraint these structures cannot perform optimally. We are reasonably assured, however, by the "organic property" of adaptive systems defined by

(a)

(b)

FIGURE 2.10 EQUIVALENT PREDICTOR STRUCTURES

(a) Type I

(b) Type II

(c) Type III

FIGURE 2.11 STRUCTURES FOR PREDICTIVE QUANTIZATION

Gersho [30], that if some subset of coefficients is fixed, the rest will adapt optimally within that constraint. The "organic property" also assures a form of optimality for the type III system as the adaptive algorithm will always act in such a way as to reduce correlations in the error sequence. This intuitive statement will be substantiated later when performance results of system simulations are given.

2.5 MEAN-SQUARED ERROR IN PREDICTIVE QUANTIZATION

The predictive quantization structures of Fig. 2.11 may be compared on the basis of mean-squared prediction error. Unfortunately this calculation is not trivial because of the presence of a nonlinear element, the quantizer, in the feedback loop. In this chapter the random-input describing function approximation

$$y_n = q z_n + \varepsilon_n \quad (2.83)$$

is used to model the quantizer [26], [31]. In Eqn. (2.83) the gain q is selected in such a way that the resulting error sequence is as white as possible and uncorrelated with the input z_n . The theory of random-input describing functions may be found in Gelb and Vander Velde [32].

In what follows the theory of z -transforms is used to derive mean-squared prediction error expressions for each system shown in Fig. 2.11. Because of the tedious computations involved the results are restricted to the simple Markov source

$$x_{n+1} = \alpha x_n + w_n \quad (2.84)$$

where, $w_n \sim \text{NID}(0,1)$

and one predictor coefficient. Some fundamental properties of the systems are, nevertheless, illustrated by the analysis.

The structure in Fig. 2.11(a) is considered first. The z-transform of the prediction may be written

$$\tilde{X}(z) = C(z) \hat{X}(z) \quad (2.85)$$

where, $C(z)$ is the transform of the prediction filter. But,

$$\hat{X}(z) = Y(z) + \tilde{X}(z) \quad (2.86)$$

and from Eqn. (2.83),

$$Y(z) = q Z(z) + E(z) \quad (2.87)$$

Thus, $\tilde{X}(z)$ may be expressed by

$$\tilde{X}(z) = \frac{q C(z) Z(z)}{1 - C(z)} + \frac{C(z) E(z)}{1 - C(z)} \quad (2.88)$$

Since $z_n = x_n - \tilde{x}_n$, Eqn. (2.88) may be written as

$$X(z) = \frac{q C(z) \tilde{X}(z)}{1 - C(z) + q C(z)} + \frac{C(z) E(z)}{1 - C(z) + q C(z)} \quad (2.89)$$

This last expression gives some insight into the nature of the system since it decomposes the prediction into two parts: a recursive digital

filter operating only on the signal, and another recursive digital filter operating only upon quantization error. Except for the gain factor q , the filters are identical functions of $C(z)$, indicating that the optimum choice of $C(z)$ must necessarily be a compromise. The z -transform of the prediction error may be written in similar form as

$$Z(z) = \frac{1 - C(z)}{1 - C(z) + q C(z)} \cdot X(z) - \frac{C(z) E(z)}{1 - C(z) + q C(z)} \quad (2.90)$$

A well known expression for evaluating the mean-squared prediction error is given by [33]

$$\epsilon(z^2) = \frac{1}{2\pi j} \oint Z(z) Z\left(\frac{1}{z}\right) \frac{dz}{z} \quad (2.91)$$

where, the contour encloses poles within the unit circle.

Applying the transformation $z = \frac{1+w}{1-w}$, the above integral may be evaluated as

$$\epsilon(z^2) = \frac{1}{\pi j} \int_{-j\infty}^{j\infty} Z(w) Z(-w) \frac{dw}{1-w^2} \quad (2.92)$$

provided the integrand vanishes at $w = \infty$. The integral may be evaluated using tabulated values, such as those given by Newton, Gould, and Kaiser [34], if it can be put into the form

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{p(w)q(-w)}{q(w)q(-w)} dw \quad (2.93)$$

where, $p(w)$ and $q(w)$ are polynomials in w .

Evaluation of the mean-squared prediction error from Eqn. (2.90) by means of Eqn. (2.91) is simplified if it can be done in two parts under the assumption that the error sequence $\{\epsilon_n\}$ is uncorrelated with the signal sequence $\{x_n\}$. Such a procedure has been used by McDonald [35], but is not justified there since he used a quantizer representation with $q = 1$. Thus he neglects a correlation between the signal and error which is significant in relation to the total error variance. That appropriate choice of q does indeed make the error-signal correlation small is demonstrated in Table 2.1, in which the results for binary predictive quantization systems with input given by Eqn.(2.84) are tabulated. These results were obtained by simulation over 10,000 samples. It is interesting to compare the values of q with those in Table 2.2 for an eight level quantizer.

For convenience, Eqn. (2.90) will be written

$$Z(z) = H(z)X(z) + G(z)E(z) \quad (2.94)$$

The mean-squared prediction error may then be written, under the assumption that $\{\epsilon_n\}$ and $\{x_n\}$ are uncorrelated, as

$$\begin{aligned} \epsilon(z^2) &= \frac{1}{2\pi j} \oint H(z)H\left(\frac{1}{z}\right) \phi_{xx}(z) \frac{dz}{z} + \\ &+ \frac{1}{2\pi j} \oint G(z)G\left(\frac{1}{z}\right) \phi_{\epsilon\epsilon}(z) \frac{dz}{z} \end{aligned} \quad (2.95)$$

q	$\overline{\epsilon_n x_{n-2}}$	$\overline{\epsilon_n x_{n-1}}$	$\overline{\epsilon_n x_n}$	$\overline{\epsilon_n x_{n+1}}$	$\overline{\epsilon_n x_{n+2}}$
0.7	-0.072	-0.145	-0.277	-0.249	-0.227
0.6	-0.044	-0.081	-0.122	-0.110	-0.103
0.5	-0.016	-0.016	-0.034	-0.029	-0.021
0.4	0.013	0.048	0.189	0.168	0.144

(a) Type I system with coefficient = 0.9

q	$\overline{\epsilon_n x_{n-2}}$	$\overline{\epsilon_n x_{n-1}}$	$\overline{\epsilon_n x_n}$	$\overline{\epsilon_n x_{n+1}}$	$\overline{\epsilon_n x_{n+2}}$
0.5	-0.391	-0.436	-0.528	-0.471	-0.417
0.4	-0.123	-0.155	-0.179	-0.159	-0.138
0.35	0.011	-0.015	-0.004	-0.003	0.002
0.3	0.145	0.126	0.171	0.153	0.142

(b) Type II System with coefficient = 1.3

q	$\overline{\epsilon_n x_{n-2}}$	$\overline{\epsilon_n x_{n-1}}$	$\overline{\epsilon_n x_n}$	$\overline{\epsilon_n x_{n+1}}$	$\overline{\epsilon_n x_{n+2}}$
0.7	0.073	-0.028	-0.189	-0.165	-0.146
0.6	0.030	-0.026	-0.090	-0.078	-0.069
0.5	-0.012	-0.024	-0.009	0.010	0.008
0.4	-0.055	-0.022	0.108	0.098	0.085

(c) Type III system with coefficient = 0.9

TABLE 2.1 SIGNAL ERROR CORRELATION IN A BINARY PREDICTIVE QUANTIZER

q	$\overline{\epsilon_n x_{n-2}}$	$\overline{\epsilon_n x_{n-1}}$	$\overline{\epsilon_n x_n}$	$\overline{\epsilon_n x_{n+1}}$	$\overline{\epsilon_n x_{n+2}}$
1.1	-0.006	-0.007	-0.135	-0.120	-0.105
1.0	-0.007	-0.006	-0.037	-0.033	-0.027
0.9	-0.008	-0.006	-0.062	0.055	0.052
0.8	-0.010	-0.005	+0.161	0.143	0.130

(a) Type I system with coefficient = 0.9

q	$\overline{\epsilon_n x_{n-2}}$	$\overline{\epsilon_n x_{n-1}}$	$\overline{\epsilon_n x_n}$	$\overline{\epsilon_n x_{n+1}}$	$\overline{\epsilon_n x_{n+2}}$
0.8	-0.163	-0.172	-0.407	-0.375	-0.330
0.7	0.041	0.031	-0.129	-0.123	-0.108
0.68	0.082	0.072	-0.073	-0.073	-0.064
0.6	0.246	0.234	0.149	0.128	0.114

(b) Type II system with coefficient = 1.3

q	$\overline{\epsilon_n x_{n-2}}$	$\overline{\epsilon_n x_{n-1}}$	$\overline{\epsilon_n x_n}$	$\overline{\epsilon_n x_{n+1}}$	$\overline{\epsilon_n x_{n+2}}$
1.1	0.079	0.067	-0.076	-0.066	-0.061
1.0	0.027	0.023	-0.017	-0.014	-0.015
0.9	-0.026	-0.021	0.041	0.037	0.032
0.8	-0.078	-0.066	0.099	0.089	0.078

(c) Type III system with coefficient = 0.9

TABLE 2.2 SIGNAL-ERROR CORRELATION IN AN 8-LEVEL MAX-TYPE PREDICTIVE QUANTIZER

A further assumption, also justified by the use of the quantizer model, Eqn. (2.87), and demonstrated by simulation results in Table 2.1, allows the simplification

$$\phi_{\varepsilon\varepsilon}(z) = \sigma_{\varepsilon}^2 \quad (2.96)$$

For simple Markov source, Eqn. (2.84) and a one coefficient predictor given by

$$C(z) = C z^{-1} \quad (2.97)$$

the mean-squared prediction error, evaluated in Appendix B, is given by

$$\begin{aligned} \varepsilon(z_n^2) &= \frac{1 + \alpha C(1-q) - 2C\alpha - 2C^2(1-q) + C^2 + C^3 \alpha(1-\alpha)}{(1-\alpha^2)[1 - C^2(1-q)^2][1 - \alpha C(1-q)]} \\ &+ \frac{C^2 \sigma_{\varepsilon}^2}{1 - C^2(1-q)^2} \end{aligned} \quad (2.98)$$

When q may be assumed unity, as is the situation with quantizers employing eight or more levels, the above expression simplifies to

$$\varepsilon(z_n^2) = \frac{1 - 2C\alpha + C^2}{1 - \alpha^2} + C^2 \sigma_{\varepsilon}^2 \quad (2.99)$$

It should be noted that the value of q necessary in the model of the quantizer is dependent on the relative magnitudes of the input signal standard deviation and the output levels. For example $q = 0.64$ for a Max optimum binary quantizer (± 0.798 output levels) with unit variance

Gaussian input. Increasing the output levels $\pm D$ increases the value of q according to the expression [32]

$$q = \sqrt{\frac{2}{\pi}} \frac{D}{\sigma} \quad (2.100)$$

for an input process which is Gaussian with variance σ^2 . In the binary predictive quantization system it is not expected that the variance will be unity because of the errors introduced by the quantization process, indeed the quantizer input may not be adequately represented by a Gaussian distribution.

Similar z -transform expressions are now developed for the structure of Fig. 2.11(b). The derivations will not be repeated as they are similar. In this case

$$\tilde{X}(z) = \frac{q D(z) X(z)}{1 + q D(z)} + \frac{D(z) E(z)}{1 + q D(z)} \quad (2.101)$$

and,

$$Z(z) = \frac{1}{1 + q D(z)} X(z) - \frac{D(z) E(z)}{1 + q D(z)} \quad (2.102)$$

where, $D(z)$ is the z -transform of the prediction filter.

With a one tap predictor $D(z) = dz^{-1}$ (which it should be observed is rather suboptimal) and the simple Gaussian-Markov input, Appendix B gives the mean squared prediction error

$$\begin{aligned} \epsilon(z_n^2) &= \frac{1 - qd\alpha}{1 + qd\alpha + q^2d^2\alpha^2 + q^3d^3\alpha^3 - \alpha^2 - qd\alpha^3 - q^2d^2 - q^3d^3\alpha} + \\ &+ \frac{d^2 \sigma_\epsilon^2}{1 - q^2d^2} \end{aligned} \quad (2.103)$$

If $q = 1$ may be assumed

$$\epsilon(z_n^2) = \frac{1 - d\alpha}{(1 + d\alpha)(1 - d^2)(1 - \alpha^2)} + \frac{\sigma_\epsilon^2 d^2}{1 - d^2} \quad (2.104)$$

Finally repeating the same derivations for the system of Fig. 2.11(c).

$$\tilde{X}(z) = \frac{q A(z)}{1 - z^{-1} + q A(z)} X(z) + \frac{A(z) E(z)}{1 - z^{-1} + q A(z)} \quad (2.105)$$

$$Z(z) = \frac{1 - z^{-1}}{1 - z^{-1} + q A(z)} X(z) - \frac{A(z) E(z)}{1 - z^{-1} + q A(z)} \quad (2.106)$$

where, $A(z)$ is the transform of the digital part of the predictor. For a single tap ($A(z) = a z^{-1}$) and simple Gaussian-Markov input the mean-squared prediction error, derived in Appendix B, may be written

$$\epsilon(z^2) = \frac{2}{(1 - \alpha + \alpha a q)(2 - a q)(1 + \alpha)} + \frac{a \sigma_\epsilon^2}{q(2 - a q)} \quad (2.107)$$

Finally if $q = 1$ may be assumed,

$$\varepsilon(z^2) = \frac{2}{(2-a)(1+\alpha)(1-\alpha+a\alpha)} + \frac{a\sigma_\varepsilon^2}{2-a} \quad (2.108)$$

The complicated nature of the expressions even for a simple one coefficient system, and a simple first order Markov source limits the utility of this method of analysis. For multi-level quantization $q = 1$ may be assumed and the simpler expressions, Eqns. (2.99), (2.104), and (2.108) may be used. In this case the error ε_n is equivalent to the reconstruction error of the system as assumed by McDonald [35]. The mean square error $\varepsilon\{\varepsilon_n^2\}$ may be expressed in terms of $\varepsilon\{z_n^2\}$ for a quantizer of known transfer function, and the expressions, Eqns. (2.99), (2.104), and (2.108) may be used to find the optimum value of the predictor coefficient.

The situation is somewhat more difficult for binary quantization since the value of q may be significantly less than unity. The expressions, Eqns. (2.98), (2.103), and (2.107) are, nevertheless, useful for demonstrating the system behavior as a function of the parameters. Some interesting observations include the following. For all systems a zero coefficient value yields the result

$$\varepsilon(z_n^2) = \frac{1}{1-\alpha^2} \quad (2.109)$$

This is, of course, just the variance of the input process as might be expected since the prediction is zero. Another result is that in a type III configuration the mean square error becomes infinite as $a \rightarrow 2/q$ and also as $a \rightarrow -\left(\frac{1-\alpha}{\alpha q}\right)$. Only between these values is the system stable and attains minimum mean-squared prediction error.

2.6 ADAPTIVE SOURCE QUANTIZATION

In this section a brief outline of prior work on adaptive methods of source quantization are presented. There will be no attempt to provide a comprehensive survey, but rather those developments which lead to the work in this thesis will be described.

So far, this thesis has presented a tutorial discussion of several techniques of optimization of digital transmission systems restricted to stationary source models. In theory, Fine's technique, and Curry's Gaussian-fit algorithm may be used for the encoding of non-stationary sources but there are rather difficult implementation problems. As is noted in Section 2.2, implementation of statistical Delta Modulation involves an iterative design technique which is not readily applicable for time varying sources. The Gaussian-fit algorithm likewise is inadequate for general time-varying signal sources since the algorithms assume a priori knowledge of the evolution of the source parameters with time; specifically the source transition matrix $\vec{\Phi}_n$ and the observation matrix $\vec{H}_n$ are assumed known functions of the time parameter n . The possibility of estimating these source parameters exists, of course, but this is a rather difficult nonlinear estimation problem and will not be considered in this thesis. Instead we will present computationally simpler methods of adaptive adjustment of the systems shown in Fig. 2.11, and demonstrate by simulation and construction of a working system that the performance of these systems approaches the results for the Gaussian-fit algorithm for stationary signals, and performs well with nonstationary sources such as speech.

Historically, the first attempts at adaptive source encoding resulted from a desire to improve the performance of simple delta modulation by sensing slope-overload conditions and adjusting the step size. The techniques mentioned in Section 1.5 are examples of this approach. The latest such technique is described in a paper by Jayant [36]. It is described here as there is a point of similarity to the systems of this thesis, and will be used for performance comparison. The methods in this thesis are applicable, however, to more general prediction structures than that used by Jayant.

Jayant's one-bit memory adaptive delta modulator bases its step size adjustment on the signs of the two most recent digital outputs. If these are equal the previous step size is multiplied by a factor P , typically 1.5, and by the factor $Q = 1/P$ if the signs are opposite. In practice minimum and maximum step sizes must be specified such that the minimum size is small enough to provide an acceptable level of noise during zero input conditions. Unlike simple delta-modulation Jayant's system does not revert to the 1010101... sequence during zero signal but produces an output which resembles a random binary sequence, which, unfortunately, contains frequency components within the signal bandwidth.

For later reference, Fig. 2.12 presents results obtained by Jayant for the signal-to-noise ratio of his adaptive system with the speech sample "Have you seen Bill" as input. The voice signal was bandlimited to 3.1 kHz and sampled at three different rates: 20, 40, and 60 kHz. The samples were initially quantized by linear analogue to digital conversion employing 12 bits. The smallest step size for the

FIGURE 2.12 PERFORMANCE OF THE ADAPTIVE DELTA MODULATION SYSTEM OF JAYANT COMPARED WITH LOGARITHMICALLY COMPANDED PCM (FROM JAYANT [36])

adaptive system was equal to one least significant bit of the total 4096 level range. Jayant found that his adaptive system produced a signal-to-quantization noise ratio higher than that for a logarithmically compressed PCM system [37] at binary transmission rates below 42 kilobits.

Adaptive operation of more general predictors of the type discussed in Section 2.4 is considered by Stroh and Boorstyn [19]. They use a method proposed by Balakrishnan [38] which assumes that a predictor which minimizes an error criterion over an interval immediately preceding the sample to be predicted will, at least approximately, be optimum for prediction of that sample value. Consider a linear predictor of the form

$$\tilde{x}_n = \sum_{j=1}^m c_{j,n} x_{n-j} \quad (2.110)$$

According to the above criterion the coefficients $c_{j,n}$ are chosen to minimize

$$F_n^2 = \frac{1}{L} \sum_{i=1}^L \left(x_{n-i} - \sum_{j=1}^m c_{j,n} x_{n-i-j} \right)^2 \quad (2.111)$$

Here L is called the learning period and F_n^2 is called the fitting error.

F_n^2 is a quadratic surface in the coefficient values and consequently has a unique minimum which may be determined by equating derivatives of F_n^2 with respect to each $c_{j,n}$ to zero. This leads to a system of linear equations

$$\sum_{j=1}^m C_{j,n} \left[\sum_{i=1}^L \frac{1}{L} x_{n-i-j} x_{n-i-l} \right] = \frac{1}{L} \sum_{i=1}^L x_{n-i} x_{n-i-l} \quad (2.112)$$

where $j = 1, 2, \dots, m$
 $l = 1, 2, \dots, m$

It is readily apparent that the above is simply a discrete time Wiener-Hopf equation with the correlation functions $R_{xx}(\ell-j)$ and $R_{xx}(\ell)$ replaced by their short term estimates. Direct solution for the coefficients $C_{j,n}$ has the disadvantage not only of relatively great implementational complexity, but is extremely sensitive to errors in the correlation estimates [29].

2.6.1 Gradient Methods

The fitting error F_n^2 is a unimodal function of the coefficients $C_{j,n}$ so that standard gradient following methods may be used. Define

$$\begin{aligned} \vec{C}_n &= (C_{1,n}, C_{2,n}, \dots, C_{m,n})^T \\ \vec{X}_n &= (x_{n-1}, x_{n-2}, \dots, x_{n-m})^T \end{aligned} \quad (2.113)$$

Then,

$$F_n^2 = \frac{1}{L} \sum_{i=1}^L (x_{n-i} - \vec{C}_n^T \vec{X}_{n-i})^2 \quad (2.114)$$

The gradient of F_n^2 is given by

$$\nabla F_n^2 = -\frac{2}{L} \sum_{i=1}^L (x_{n-i} - \vec{C}_n^T \vec{X}_{n-i}) \vec{X}_{n-i} \quad (2.115)$$

Since ∇F_n^2 is a vector whose direction is that of the greatest slope on the F_n^2 surface a small increment in the opposite direction will reduce F_n^2 provided the increment is not too large. An iterative search for the optimum coefficient vector may be expressed as

$$\vec{C}_n(K+1) = \vec{C}_n(K) + \mu_n(K) \nabla F_n^2 \quad (2.116)$$

where, $\mu_n(K)$ is the step size at iteration K .

In the present form, Eqn. (2.116) the gradient search procedure is also difficult to implement. Relatively complex computations are required to find $\mu_n(K)$ at each iteration [19]. Stroh and Boorstyn propose a simplified expression for the optimum fixed step size at each n [19].

$$\mu_n = -\frac{m+L-1}{m} \left[\sum_{i=2}^{M+L} x_{n-i}^2 \right]^{-1} \quad (2.117)$$

Even with this simplification there remain rather great implementational difficulties. For example, at a given time n , several iterations are necessary, implying that ∇F_n^2 must be recomputed according to Eqn. (2.115) at each iteration. This is in contrast with the iterative methods of this thesis where only one step is taken at each time n , and the gradient is readily available by simple logical operations on stored values of the digital output sequence.

2.6.2 Application to Predictive Quantization

In a predictive quantization system the prediction is based on quantized data. For example $\{x_n + \varepsilon_n\}$ rather than $\{x_n\}$. If the decoder

is also to adapt its coefficients from data transmitted over the channel the fitting error, Eqn. (2.111) must be approximated by

$$F_n^2 = \frac{1}{L} \sum_{i=1}^L [\bar{X}_n + \vec{E}_n - C_n^T (\vec{X}_{n-i} + \vec{E}_{n-i})]^2 \quad (2.118)$$

where,

$$\begin{aligned} \vec{X}_n + \vec{E}_n &= (x_{n-1} + \epsilon_{n-1}, \dots, x_{n-m} + \epsilon_{n-m})^T \\ &= (\hat{x}_{n-1}, \dots, \hat{x}_{n-m})^T \end{aligned}$$

2.7 ANALYSIS-SYNTHESIS PROCEDURE FOR SPEECH

This section on adaptive systems will be concluded by a description of a somewhat different approach to source encoding which is better suited to the physical nature of sources such as speech. It is well known that a good model for voiced speech consists of a time varying linear system representing the vocal tract which is excited by an almost periodic pulse train generated by the vocal cords. For unvoiced speech the pulse train is replaced by random noise which in the physical speech process is generated by turbulent air flow through constricted passages.

It is shown by Atal and Hanauer [21] that the time varying linear system representing the vocal tract can be adequately represented by a recursive linear filter of the form

$$x_n = \sum_{i=1}^p a_i(n) x_{n-i} + u_n \quad (2.119)$$

where, x_n is the sample of speech at time n , and u_n represents the excitation, either impulsive or random. They show that the number of coefficients p should be approximately 12 for an adequate representation of the waveform at a sampling frequency of 10 kHz. The speech model described by Eqn. (2.119) forms the basis for the analysis-synthesis speech encoding method described by Atal and Hanauer.

Between vocal cord pulses the next speech sample is determined solely by the coefficients $a_i(n)$ and past samples of the waveform. Consequently, if the speech waveform were analyzed between two successive vocal cord pulses to estimate the coefficients $\{\hat{a}_i(n)\}$ sufficiently close to those describing the actual physical speech process, the resulting recursive filter

$$\hat{x}_n = \sum_{i=1}^p \hat{a}_i(n) \hat{x}_{n-i} + \delta_m \quad (2.120)$$

could be used to re-synthesize that portion of the waveform from time $n = m$ to the next occurrence of a vocal cord pulse merely by excitation of the filter by the impulse δ_m at the beginning of the pitch period. When the next vocal cord pulse occurs the waveform described by Eqn. (2.120) will not follow the resulting speech waveform, which is now forced by the vocal cord pulse. The resulting error impulse identifies a new pitch period, and the initiation of a new analysis cycle.

The parameters of the filter described by Eqn. (2.120) are obtained by minimization of the mean-squared fitting error over a pitch period through the solution of a system of equations similar to Eqn. (2.68).

The results of the work of Atal and Hanauer have interesting implications for the adaptive techniques of this thesis. Whereas a medium scale digital computer (GE-635) is required for the speech analysis of Atal et al, and runs at some thirteen times real time, the iterative algorithms of this thesis can be used for pitch period determination, and voiced-unvoiced decisions in real time operation. Although not investigated in this thesis, it may also be possible to determine the filter coefficients in real time by similar iterative methods.

Further elaboration of this aspect of the thesis is left to Chapter 7.

CHAPTER 3

ITERATIVE ADAPTATION OF SOURCE ENCODERS

3.1 ENCODER STRUCTURE

The design philosophy adopted in this thesis is motivated by two conflicting considerations: maintaining simplicity of implementation, while at the same time achieving an encoder which can adapt to a nonstationary source. The conventional approach to this problem is to formulate the optimum encoder for nonstationary signal source using the framework of estimation theory. The optimum encoder is then examined, and approximations are made for the purpose of simplifying its implementation. It is readily apparent, however, that optimum encoder structures as described in Chapter 2 do not lend themselves to such simplification even for stationary signals. Appending state variables to the model given by Eqn. (2.31) to allow for time varying parameters at once generates a nonlinear estimation problem whose optimum solution is even less amenable to simple implementation, and poses additional questions as to the validity of any assumptions such as Eqn. (2.32) necessary for the solution.

Rather than attempt to derive the optimum structure for a particular source, as there is some doubt that this would lead to a readily implementable structure, this thesis considers the knowledgeable use of simple structures in the synthesis of efficient encoding systems for sources such as speech. In some respects this approach is more difficult than the optimum nonlinear estimator method in that a great

deal of study is necessary to specify an encoder structure and to demonstrate its value. At the same time rigid assumptions about the source need not be made to ensure mathematical tractability, as in the estimation problem and effective use may be made of all aspects of the physical process that generates the source waveform.

This thesis, therefore, began with a study of the current state of knowledge about optimum digital systems, current source encoding methods, and fundamental information theoretic properties of sources. From this, several important conclusions about source encoding may be derived. The theoretical work of McMillan [22] suggests the basic structure for the optimum encoder shown in Fig. 2.3. We then demonstrate that for a restricted but practically useful class of distortion criteria the optimum structure may be realized by the predictive quantization system illustrated by Fig. 2.4. McMillan's theoretical results show that only the past data output is necessary to specify the optimum prediction and quantizer structure; that is, there exist mappings between the possible output sequences, the value of the current prediction, and the level structure of the quantizer. It is the primary aim of this thesis to suggest readily implemented methods of determining these mappings, or approximations thereto.

Statistical Delta Modulation as described by Bello et al [15] determines the optimum mappings for the special case of binary quantization of a Gaussian-Markov Process by direct measurement of statistics which specify the mapping. As is noted in Section 2.2 the procedure is not suited for real-time adaptive operation. One conclusion important

to this thesis is made, however. The results of Statistical Delta Modulation experiments with mappings constrained to be linear show little degradation in performance compared with mappings not so constrained [15]. Since a linear mapping may be realized by a linear digital filter operating on the data sequence we conclude that the predictive quantizer structures of Fig. 2.11 may serve as reasonable approximations to the optimal source quantizer.

From the implementation point of view the type II structure is simpler for an equivalent number of coefficients as it uses a digital input. It was demonstrated in Section 2.4 that, for the same input sequence, such a system requires a larger number of coefficients. Consequently we introduce the system in Fig. 2.11(c) which preserves the simplicity of the digital filter while requiring a shorter memory than the type II system.

3.2 STOCHASTIC APPROXIMATION METHODS

In the remainder of Chapter 3, simply implemented procedures for adaptively adjusting the predictive quantizers of Fig. 2.11 are introduced. The techniques are similar to stochastic approximation procedures, and may perhaps be more fully understood by reference to them. A short introduction to stochastic approximation, restricted to material of direct relevance to this thesis, will now be presented.

Stochastic approximation is a procedure for iteratively adjusting a parameter (or set of parameters) such that a regression function on the parameters either takes on some assigned value, or takes on a

maximum or minimum value. In this thesis we are concerned with adjusting the coefficient set for the prediction filter in a predictive quantization system in such a way as to minimize the mean-squared error function. At first consideration it would seem that the second procedure, that of finding a minimum, should be applied directly. It will be shown, however, that in predictive quantization an estimate of the gradient of the mean square error is directly available and it is more natural to consider the gradient as a regression function and determine the parameter set for which the gradient is zero.

It is instructive to consider first the scalar problem illustrated by Fig. 3.1. The approach is to determine the value x_0 at which the function $y(x) = 0$ by performing a suitable sequence of measurements at a series of parameter levels $\{x_n\}$. A solution is given by the Robbins-Monro stochastic approximation procedure [46]. Subject to the restrictions

$$\begin{aligned} \varepsilon\{[z(x_n) - y(x_n)]^2\} &< \infty \\ \varepsilon\{z(x_n)\} &= y(x_n) \\ |y(x_n)| &< A|x_n - x_0| + B < \infty \end{aligned} \quad (3.1)$$

where,

$$0 < A < \infty$$

and, $0 < B < \infty$

the procedure for choosing a suitable sequence $\{x_n\}$ may be written

$$x_{n+1} = x_n - a_n z(x_n) \quad (3.2)$$

FIGURE 3.1 ROBBINS-MONRO PROBLEM

FIGURE 3.2 CONDITIONS ON DETERMINISTIC PART

where, a_n satisfies the following conditions.

$$\begin{aligned} \lim_{n \rightarrow \infty} a_n &= 0 \\ \sum_{i=1}^{\infty} a_n &= \infty \\ \sum_{i=1}^{\infty} a_n^2 &< \infty \end{aligned} \quad (3.3)$$

For the example in Fig. 3.1 it is readily seen that with $x_n > x_0$ and the restrictions, Eqn. (3.1) $z(x_n)$ will be positive more often than negative. Thus in the stochastic approximation procedure, Eqn. (3.2) it is more likely that the next value x_{n+1} will be nearer x_0 . The conditions on $\{a_n\}$ given by Eqn. (3.3) are strong enough to ensure convergence to x_0 with probability one [47].

It is instructive to view the stochastic approximation procedure as the sum of a deterministic function of x_n and a random error as follows

$$\begin{aligned} x_{n+1} &= [x_n - a_n y(x_n)] - a_n \hat{\delta}_n \\ &= D(x_n) + R_n \end{aligned} \quad (3.4)$$

In general one must take care that the partitioning produces an error term which is unbiased. This is, of course, true for the Robbins-Monro procedure as a consequence of Eqn. (3.1). Sufficient conditions on R_n for convergence of Eqn. (3.4) are given by Dvoretzky's conditions [48],

$$\varepsilon\{R_n\} = 0$$

$$\sum_{n=1}^{\infty} \varepsilon\{R_n\} < \infty \quad (3.5)$$

Intuitively, these imply that on average the corrections will be in the proper direction, and that at a given iteration the variance of the error due to the random component for the remaining steps decreases as $m \rightarrow \infty$.

Convergence proofs also require restrictions on the deterministic component of Eqn. (3.4). The nature of these restrictions is illustrated by Fig. 3.2. In (a) an overshoot is not possible and x_{n+1} is desired as close as possible to x_0 . A statement of this requirement is that

$$|D(x_n) - x_0| \leq |x_n - x_0| - \gamma_n \quad (3.6)$$

with, $\sum_{n=1}^{\infty} \gamma_n = \infty$

In Fig. 3.2(b) an overshoot is possible and placing x_{n+1} close to x_0 requires that

$$|D(x_n) - x_0| \leq \alpha_n \quad (3.7)$$

with, $\lim_{n \rightarrow \infty} \alpha_n = 0$

The Robbins-Monro procedure satisfies these conditions as a direct consequence of Eqns. (3.1) and (3.3).

In this thesis the vector valued version of the Robbins-Monro procedure is required. The multidimensional theory is due to Blum [49]. A condensed statement of the procedure and conditions for probability one convergence, as given by Sakrison [49], is sufficient for our purpose.

Let $\vec{X}_n$ be the vector of parameters which is to be iteratively adjusted, and let $\vec{Y}(x_n)$ be the corresponding value of the regression function, which may be assumed to be the gradient mentioned at the beginning of this section. The observation is $\vec{Z}(\vec{X}_n)$, a measurement of $\vec{Y}(\vec{X}_n)$ perturbed by a noise vector $\vec{\Delta}_n$. A vector valued version of Eqn. (3.2) may be written

$$\vec{X}_{n+1} = \vec{X}_n - a_n \vec{Z}(\vec{X}_n) \quad (3.8)$$

As before this is partitioned into deterministic and random components

$$\begin{aligned} \vec{X}_{n+1} &= \vec{X}_n - a_n \vec{Y}(\vec{X}_n) - a_n [\vec{Z}(\vec{X}_n) - \vec{Y}(\vec{X}_n)] \\ &= \vec{D}(\vec{X}_n) + \vec{R}_n \end{aligned} \quad (3.9)$$

Convergence with probability one is assured by the conditions [49].

$$\begin{aligned} \varepsilon\{\vec{R}_n\} &= 0 \\ \varepsilon\{|\vec{R}_n|^2 | \vec{X}_n\} &< \infty, \quad \forall \vec{X}_n \end{aligned} \quad (3.10)$$

for $k_0, k_1 > 0$ and for every $\vec{X}_n$

$$\begin{aligned} k_0 \|\vec{X}_n - \vec{X}_0\|^2 &< (\vec{X}_n - \vec{X}_0)^T a_n \vec{Y}(\vec{X}_n) < k_1 \|\vec{X}_n - \vec{X}_0\|^2 \\ \|\vec{D}(\vec{X}_n) - \vec{X}_0\|^2 &\leq \alpha_n \end{aligned}$$

where, $\lim_{n \rightarrow \infty} \alpha_n = 0$

$$\|\vec{D}(\vec{X}_n) - \vec{X}_0\|^2 \leq \|\vec{X}_n - \vec{X}_0\|^2 - \gamma_n$$

where, $\sum_{n=1}^{\infty} \gamma_n = \infty$

(3.10)

These requirements are analogous to the conditions previously stated for the scalar problem. Although these conditions are sufficient for convergence with probability one they may not be necessary. For example, convergence with probability one holds if the first condition is weakened to

$$\sum_{i=1}^{\infty} \vec{R}_n < \infty \quad (3.11)$$

This implies that the bias in $\vec{R}_n$ must tend to zero as n increases.

In the standard Robbins-Monro stochastic approximation procedures given by Eqns. (3.2) and (3.8), the regression function is time invariant. Consequently it is reasonable to consider a decreasing step size parameter a_n such that as $n \rightarrow \infty$ the process converges with probability one to the root of the regression function x_0 such that $Y(x_0) = 0$. In this thesis, however, the primary interest is in adaptive source encoders which are able to follow time varying source models. Thus, procedures

with a_n decreasing to zero are unsuitable, and probability one convergence is no longer an appropriate criterion.

Very few results are available for the time varying situation. Dupac [51] considers a modified procedure

$$x_{n+1} = \left(1 + \frac{1}{n}\right) x_n - a_n z_n \quad (3.12)$$

where, the root x_0 varies by the multiplier $(1 + 1/n)$. He was able to establish convergence in mean square for this special example. Even this work has little application to the thesis since the time variation assumed in Eqn. (3.12) decreases asymptotically to zero as $n \rightarrow \infty$.

Daniell [52] considers the algorithm, Eqn. (3.8) with $a_n = a$, a constant, and shows that for every $\epsilon > 0$ there exists an a_1 such that for $0 < a < a_1$,

$$\lim_{n \rightarrow \infty} \sup \epsilon ||\vec{X}_n - \vec{X}_0||^2 < \epsilon \quad (3.13)$$

Daniell's proof relies on stationarity of the regression function $\vec{Y}(\vec{X}_n)$, and the result may not be extended to a general time varying situation. It seems likely that a condition $\epsilon < \epsilon_1$ might hold for $a < a_1$ under mild restrictions on the nonstationary nature of $\vec{Y}(\vec{X}_n)$ such as slope constraints and power constraints on the root $\vec{X}_0$. It is not the purpose of this thesis to investigate these matters, however. For the algorithms introduced in the next section this form of convergent behavior will be amply demonstrated through simulation and experimental evidence.

One further comment of relevance to this thesis may be added. It is of greater importance to determine mean-squared prediction error than some mean-square convergence property of the predictor parameters as errors in parameters may not relate easily to mean-squared prediction error.

Further discussion of mean-squared prediction error is deferred to Section 3.6.

3.3 ITERATIVE ADAPTATION OF PREDICTIVE QUANTIZATION SYSTEMS

Consider the predictive quantization system illustrated in Fig. 2.11(b). A method for iteratively optimizing the predictor coefficients may be developed as follows. The predictor output may be written

$$\tilde{x}_n = \sum_{i=1}^m d_i y_{n-i} = \vec{D}^T \vec{Y}_n \quad (3.14)$$

where,

$$\vec{D} = (d_1, d_2, \dots, d_m)^T$$

and,

$$\vec{Y}_n = (y_{n-1}, y_{n-2}, \dots, y_{n-m})^T$$

The mean-squared prediction error associated with the above predictor is given by

$$\begin{aligned} \text{mse} &= E\{(x_n - \tilde{x}_n)^2\} \\ &= E\{x_n^2 - 2x_n \vec{D}^T \vec{Y}_n + \vec{D}^T \vec{Y}_n \vec{Y}_n^T \vec{D}\} \end{aligned} \quad (3.15)$$

Evaluating the gradient,

$$\begin{aligned} V_{mse} &= \varepsilon\{-2x_n \vec{Y}_n + 2\vec{Y}_n \vec{Y}_n^T \vec{D}\} \\ &= -2\varepsilon\{z_n \vec{Y}_n\} \end{aligned} \quad (3.16)$$

where,

$$z_n = x_n - \vec{D}^T \vec{Y}_n = x_n - \vec{Y}_n^T \vec{D}.$$

The mean-squared prediction error, as given by Eqn. (3.15), is recognized as a downward convex quadratic hypersurface in the space of the coefficients $d_1, d_2, \dots, d_m$, and consequently has a unique minimum given by

$$V_{mse} = -2\varepsilon\{z_n \vec{Y}_n\} = 0 \quad (3.17)$$

Provided good estimates for $\varepsilon\{x_n \vec{Y}_n\}$ and $\varepsilon\{\vec{Y}_n \vec{Y}_n^T\}$ are obtained, solution of Eqn. (3.17) for the optimum coefficients is possible. Implementation is difficult, however.

A much simpler method may be obtained by recognizing the gradient as a regression function whose root is the optimum vector of coefficients. The preceeding section immediately suggests the following algorithm for iteratively adjusting the coefficient vector.

$$\vec{D}_{n+1} = \vec{D}_n + \gamma_n z_n \vec{Y}_n \quad (3.18)$$

where, γ_n is a suitably chosen sequence of positive constants.

This algorithm is not suitable for use in the adaptive systems of this thesis for the following reasons. First, the coefficients must adjust to follow a nonstationary signal source, a form of operation not considered in standard stochastic approximation techniques. Second, the receiver is to be realized through the same predictor structure, which cannot use the above algorithm, as z_n is available only at the encoder.

A suitable solution to both problems is the modified algorithm

$$\vec{D}_{n+1} = \vec{D}_n + \gamma y_n \vec{Y}_n \quad (3.19)$$

where, γ is a small positive constant. Of course convergence with probability one to the optimum coefficient vector is not possible with the modified algorithm because of the constant step size. Exact convergence, fortunately, is not necessary since the mean-squared prediction error is not very sensitive to small variation of predictor coefficients. For a stationary situation the results of Daniell [52] apply, provided it can be shown that

$$\lim_{\substack{D \rightarrow D \\ n}} [\epsilon(y_n \vec{Y}_n) - \epsilon(z_n \vec{Y}_n)] = 0 \quad (3.20)$$

where,

$$\epsilon\{z_n \vec{Y}_n | \vec{D}\} = 0.$$

The situation is illustrated in Fig. 3.3, where estimates of the quantities in Eqn. (3.20) over 10,000 samples are plotted for a one coefficient

FIGURE 3.3 $Z_n Y_{n-1}$ AND $Y_n Y_{n-1}$ IN A TYPE II PREDICTIVE QUANTIZER

type II predictive quantizer structure. The quantizer was binary with ± 1 output levels. It appears from Fig. 3.3 that Eqn. (3.20) is at least approximately true. The results of the identical experiment with a type III structure are given in Fig. 3.4. Here it appears that Eqn. (3.20) is approximately satisfied. However a look at the mean-squared prediction error curve indicates that little difference would result with convergence to the root of $\varepsilon(y_n y_{n-1})$ rather than $\varepsilon(z_n y_{n-1})$. From these experimental results the use of Eqn. (3.19) seems justified. In any case the results of simulation studies described in the next chapter will serve to justify the algorithms that are proposed here.

Similar iterative algorithms may be developed for the other predictive quantizer structures. Let

$$\tilde{x}_n = \sum_{i=1}^m c_i \hat{x}_{n-i} = \tilde{C}^T \tilde{V}_n \quad (3.21)$$

where,

$$\tilde{V}_n = (\hat{x}_{n-1}, \hat{x}_{n-2}, \dots, \hat{x}_{n-m})^T$$

By similar reasoning to that used to obtain Eqn. (3.19), the following algorithm is obtained:

$$\tilde{C}_{n+1} = \tilde{C}_n + \gamma y_n \tilde{V}_n \quad (3.22)$$

It should be noted that implementation of this predictive quantizer implies storage of $\tilde{V}_n$, a vector with analogue valued components. The iterative algorithm, Eqn. (3.22) may be simplified for the sake of implementation by the use of $\gamma \text{sign}(y_n \hat{x}_{n-i})$; $i = 1, 2, \dots, m$ as the adjust-

FIGURE 3.4 $Z_n Y_{n-1}$ AND $Y_n Y_{n-1}$ IN A TYPE III PREDICTIVE QUANTIZER

ment to each coefficient. It will be shown by simulation in the next chapter that this is in fact a valid procedure.

An iterative adjustment procedure will now be developed for the type III structure shown in Fig. 2.11(c). Here, the predictor output is given by

$$\tilde{x}_n = \tilde{x}_{n-1} + \vec{A}^T \vec{Y}_n \quad (3.23)$$

where, $\vec{A} = (a_1, a_2, \dots, a_m)^T$

Let $\vec{A}(n)$ be the coefficient vector at time n . Then the predicted value is given by

$$\tilde{x}_n = \tilde{x}_0 + \vec{A}^T(0)\vec{Y}_0 + \vec{A}^T(1)\vec{Y}_1 + \dots + \vec{A}^T(n)\vec{Y}_n \quad (3.24)$$

Consider first a stationary situation. Then an approximation to Eqn.

(3.24) may be written

$$\begin{aligned} \tilde{x}_n &= \tilde{x}_0 + \vec{A}^T(n) [\vec{Y}_0 + \vec{Y}_1 + \dots + \vec{Y}_n] \\ &= \tilde{x}_0 + \vec{A}^T(n) \vec{W}_n \end{aligned} \quad (3.25)$$

where, $\vec{W}_n$ now represents the arithmetic average of all the past data vectors. Now, if n is large the initial value $\tilde{x}_0$ may be neglected, and

$$\tilde{x}_n = \vec{A}^T(n) \vec{W}_n \quad (3.26)$$

An argument similar to that used for the derivation of Eqn. (3.19) gives the iterative algorithm

$$\vec{A}_{n+1} = \vec{A}_n + \gamma y_n \vec{W}_n \quad (3.27)$$

This algorithm has the disadvantage that it was derived on the basis of a stationary assumption. It may be argued that in a nonstationary situation past values of $\vec{Y}_n$ do not usefully contribute toward determination of the prediction coefficients. Consequently the following algorithm, identical in form to Eqn. (3.19), is proposed,

$$\vec{A}_{n+1} = \vec{A}_n + \gamma y_n \vec{Y}_n \quad (3.28)$$

Simulation results are given in Chapter 4 which indicate the validity of Eqn. (3.28). This is fortunate as Eqn. (3.28) is implementationally simpler than Eqn. (3.27). A heuristic explanation for this may be advanced as follows. While it is true that the present value of the predictor outputs depends on the past data vectors as a result of the integration, adjustment of the coefficients on the basis of the sum of the past data vectors implies a delay between the adjustment and its effect. As with any system with delay, it is prone to unstable behavior. With the algorithm, Eqn. (3.28), the adjustment is more immediate and less prone to instability.

FIGURE 3.5 FIRST NINE WALSH FUNCTIONS

FIGURE 3.6 PREDICTOR IMPLEMENTATION WITH WALSH FUNCTIONS

3.3.1 Predictor Synthesized by Walsh Functions

The large number of predictor coefficients required for the structure in Fig. 2.11(b) can sometimes be reduced by the use of orthogonal functions. Consider the encoding of a source bandlimited to W kHz at a sampling rate of R kHz, where R is greater than $2W$, as occurs, for example, in delta modulation. If the predictor has a memory containing M past samples the memory time is given by

$$T = \frac{M}{R} \text{ seconds} \quad (3.29)$$

It is reasonable to expect that the predictor weighting functions may be represented by T second portions of waveforms of bandwidth not significantly greater than W Hz. Thus the maximum number of coefficients necessary is given by

$$p = 2WT = \frac{2WM}{R} \quad (3.30)$$

This is just a statement of the well known fact that the weighting function may be represented as a point in a $2WT$ dimensional linear space whose basis consists of a set of orthogonal functions and the coordinates are just the coefficients in the series expansion in terms of these functions. From the point of view of implementation the Walsh Functions, the first nine of which are shown in Fig. 3.5, are ideally suited for synthesis of the predictor. The binary nature of the Walsh functions implies that only simple operations on the data samples are necessary to implement the predictor.

As an example of the reduction in number of necessary coefficients consider binary encoding of a 3 kHz lowpass waveform at a 40 kHz sampling rate. Further, assume that a memory time of 5 milliseconds is required. For the structure of Fig. 2.11(b) the number of coefficients is given by

$$M = RT = 200 \quad (3.31)$$

With Walsh functions this may be reduced to

$$p = 2WT = 30 \quad (3.32)$$

Implementation of the predictor employing Walsh functions is shown in Fig. 3.6 for a binary encoder. Appropriate binary data bits or their complements are summed to form the quantities $W_1, W_2, \dots, W_p$. The predicted value is formed as a weighted sum

$$\tilde{x}_n = \sum_{i=1}^p a_i W_{i,n} = \vec{A}^T \vec{U}_n \quad (3.33)$$

where, $\vec{A} = (a_1, \dots, a_p)^T$
 and, $\vec{U} = (W_{1,n}, \dots, W_{p,n})^T$

Consideration is now given to adaptively adjusting the coefficients a_i in Eqn. (3.33) to minimize the mean-squared prediction error. As before this will be done by a stochastic approximation procedure to find the root of the gradient of the error hypersurface. Thus

$$\begin{aligned}
\text{mse} &= \varepsilon\{(\mathbf{x}_n - \tilde{\mathbf{x}}_n)^2\} \\
&= \varepsilon\{\mathbf{x}_n^2 - 2\mathbf{x}_n \tilde{\mathbf{A}}^T \tilde{\mathbf{U}}_n + \tilde{\mathbf{A}}^T \tilde{\mathbf{U}}_n \tilde{\mathbf{U}}_n^T \tilde{\mathbf{A}}\}
\end{aligned} \tag{3.34}$$

The gradient is given by

$$\begin{aligned}
\nabla \text{mse} &= -2\varepsilon(\mathbf{x}_n \tilde{\mathbf{U}}_n) + 2\varepsilon(\tilde{\mathbf{U}}_n \tilde{\mathbf{U}}_n^T \tilde{\mathbf{A}}) \\
&= -2\varepsilon(z_n \tilde{\mathbf{U}}_n)
\end{aligned} \tag{3.35}$$

where, $z_n = \mathbf{x}_n - \tilde{\mathbf{x}}_n$ = prediction error.

As before, Eqn. (3.35) is treated as a regression function, and z_n is replaced with its quantized version y_n for the sake of implementation at the receiver. This results in the algorithm

$$\tilde{\mathbf{A}}_{n+1} = \tilde{\mathbf{A}}_n + \gamma y_n \tilde{\mathbf{U}}_n \tag{3.36}$$

3.4 DIGITAL COMMUNICATION SYSTEM EMPLOYING ADAPTIVE-PREDICTIVE SOURCE ENCODING

So far in this chapter only the adaptive adjustment of the predictive source encoder has been examined. A complete communication system requires that a reconstruction of the original source be presented to the user at a remote location through the use of a suitable source decoder such as that shown in Fig. 1.1. This requires that the decoder be implemented, as shown in Fig. 3.7, by an identical predictor structure to that in the source encoder so that the reconstructed output sample is given by

FIGURE 3.7 COMPLETE DIGITAL COMMUNICATION SYSTEM USING ADAPTIVE-PREDICTIVE SOURCE QUANTIZATION

$$\hat{x}_n = \tilde{x}_n + y_n = x_n + \varepsilon_n \quad (3.37)$$

where, $\varepsilon_n = y_n - z_n$ is the quantization error. Such operation requires, of course, that the coefficient values of the predictor be identical at both the encoder and decoder. There are two ways this can be accomplished: by transmission of the coefficient values over a side channel, for example by multiplexing techniques, or by adaptively adjusting the decoder from the encoder output data sequence. The second method is considered in this thesis since it uses no additional channel capacity. This is readily accomplished for systems of type II and type III since the adaptive algorithms for predictors require only the digital output of the source encoder.

Several difficulties are inherent in the implementation of the system shown in Fig. 3.7. The predictor parameters at both the encoder and decoder must be equal at all times. Initial adjustment for equality presents only a small implementation problem, but the effect of digital errors in the channel is more serious. This problem is considered in Chapter 5, and modification to the adaptation algorithms suitable for channels with errors are suggested there. Initial simulation studies will assume an error free channel, however.

Another potential difficulty concerns the quantizer. Ideally this should be adaptive as well. A predictor, even if optimized, can only follow the predictable component of the input signal. Consequently the error z_n will have a variance which changes with signal source model and with signal power. The quantization error for a fixed quantizer is

minimized only at a fixed input variance and amplitude probability density, and any change will increase the error. The ideal adaptive quantizer would alter its parameters accordingly. There is some justification, however, for a Gaussian assumption for the amplitude probability density [19], and in any case, an adaptive algorithm would be extremely difficult to implement. In Chapter 4 an adaptive 4-level quantizer which changes its input and output gain according to the quantizer output statistics is simulated, and results in a large increase in the dynamic range.

Another approach to the adaptive quantizer problem may be possible in systems which do not require transmission of the absolute signal power level. An automatic gain control which operates on the input signal according to the power in the sequence $\{z_n\}$ may be adequate. This would ensure that the quantizer input variance is close to the optimum value even though the input signal power changes radically. The reconstructed signal power would not then be identical to the input signal power, but for some systems such as speech transmission this may not be a disadvantage since intelligibility would not be lost provided the automatic gain control had a long enough time constant. It might be noted that this is precisely the mode of operation of automatic recording level devices in some types of cassette recorders intended for dictation work.

3.5 ANALYSIS-SYNTHESIS COMMUNICATION SYSTEM FOR SPEECH SOURCES

The adaptive-prediction system with the iterative algorithms described in this chapter is suitable for speech analysis work. This will be further examined in Chapter 7, where a simple binary adaptive-predictive quantization system is used to extract pitch period information from a speech waveform. As noted in Chapter 2, the rate at which a speech source generates information is not constant. Consequently an adaptive-predictor source encoder which transmits digital symbols at a constant rate produces a quantization error variance which is not constant with time. Obviously further coding must be done to match the source information rate to the channel. At present the most efficient method seems to be the analysis-synthesis technique suggested by Atal and Hanauer [21]. The stochastic approximation techniques examined in this thesis are suggested as a readily implemented means of accomplishing the analysis of speech. Their use for pitch period detection and voiced-unvoiced decisions will be demonstrated, as noted above, in Chapter 7.

It is likely that a simple iterative method similar to that in this thesis can be used to obtain the prediction coefficients described in Section 2.7. This problem, however, is considered outside the scope of the current study.

3.6 MEAN-SQUARED PREDICTION ERROR

In this thesis the performance of adaptive-predictive quantizers is investigated largely by simulation and experiment. In Chapter 4 the signal-to-quantization noise ratio will be examined, both during the transient as the coefficients adapt from zero initial values, and in steady state. From a knowledge of the performance in these modes of operation reasonable design parameters for the experimental speech encoder, to be described in Chapter 6, are obtained.

In what follows an analytical expression obtained by Davisson [53] for the steady-state error in pure prediction of a stationary process is compared with simulation results for adaptive-predictive quantization. Consider prediction of the sequence $\{x_n\}$ according to

$$\tilde{x}_n = \sum_{i=1}^m a_i(n) x_{n-i} = \vec{A}^T(n) \vec{X}_n \quad (3.38)$$

where, the coefficient vector is adapted by

$$\vec{A}(n+1) = \vec{A}(n) + \frac{\gamma}{\lambda} z \vec{X}_n \quad (3.39)$$

where, $\lambda = E\{x_n^2\}$.

and, $z_n = x_n - \tilde{x}_n$.

Under certain conditions on the sequence $\{x_n\}$ such as would be satisfied by a Gaussian-Markov process of order m , for example [53], the actual steady-state mean-squared prediction error $\sigma^2(m, \gamma)$ for adaptive coefficients is related to $\sigma^2(m)$, the error for an optimum linear fixed predictor by

$$\sigma^2(m, \gamma) \cong \sigma^2(m) \left[1 + \frac{\gamma m}{2} \right] \quad (3.40)$$

For predictive quantization systems two complications exist:

1. The observation vector $\vec{Y}_n$ consists of quantized data, which depend upon past coefficient values.
2. The quantized error y_n , rather than z_n is used in the algorithm.

Thus, if, as in Davisson's analysis, one defines an optimum coefficient vector $\vec{A}$ such that an optimum error sequence is produced, according to

$$z_n^* = x_n - \vec{A}^T \vec{Y}_n \quad (3.41)$$

the observations $\vec{Y}_n$ will in general be different from those defined by

$$z_n = x_n - \vec{A}^T(n) \vec{Y}_n \quad (3.42)$$

where, $\vec{A}(n+1) = \vec{A}(n) + \gamma y_n \vec{Y}_n$.

It does not seem possible to carry out the derivation taking the dependence above into account. One might conjecture that for small deviations from optimum the statistical properties of $\vec{Y}_n$ in Eqns. (3.41) and (3.42) will be similar so that Eqn. (3.40) may hold in an approximate form.

Table 3.1 illustrates the results of simulating several predictive quantization systems with a second order Gaussian-Markov source as input (see Appendix A, and Chapter 4). The results in the table are averages over an ensemble of twenty realizations, each comprising 1000 samples, beginning after the steady state is reached at sample number 1000.

TABLE 3.1
Mean-Squared Prediction Error vs γ
For Second-Order Gaussian-Markov Input

γ	Observed mse	$1 + \frac{m\gamma}{2}$	calculated mse
0.005	2.95	1.025	2.95
0.01	3.00	1.05	3.02
0.02	3.25	1.1	3.16
0.05	4.03	1.25	3.60
0.1	5.50	1.5	5.30

(a) Type II structure, $m = 10$, Binary ± 1 quantizer, $R = 2.5$

0.005	0.83	1.025	0.83
0.01	0.98	1.05	0.85
0.02	1.25	1.1	0.89
0.05	2.28	1.25	1.01
0.1	4.71	1.5	1.21

(b) Type II structure, $m = 10$, Eight-level Max type quantizer,
 $R = 2.5$

0.01	2.44	1.04	2.44
0.04	2.70	1.16	2.72
0.1	3.66	1.4	3.29

(c) Type III structure, $m = 8$, Binary ± 0.798 quantizer,
 $R = 7.0$

For binary quantization Eqn. (3.40) gives a reasonable estimate of the error for increasing γ . For the eight-level Max-type quantizer, which was designed for $N(0,1)$ input distribution, the mean-squared prediction error grows much more rapidly with γ . A possible reason may be suboptimality with larger values of input variance, an effect not present in the binary system.

CHAPTER 4

SIMULATION STUDIES

It was pointed out in Chapter 3 that analytical derivation of performance of predictive quantization systems employing the stochastic approximation algorithms introduced there is difficult and involves approximations and assumptions that may not be justified. For nonstationary signal models, moreover, an analytical derivation of performance does not exist. In this chapter a series of system simulations was carried out, first to show the validity of the iterative adjustment techniques, and second, to establish design parameters for the experimental binary encoder for speech described in Chapter 6.

The details of the digital computer programs will not be given as they are obvious implementations of the systems shown in Fig. 2.11 with the adaptive algorithms given in Chapter 3. Four different signal inputs were used.

1. The simple Gaussian-Markov source given by Eqn. (4.1).
2. The second order Gaussian-Markov source of Appendix A.
3. Sinusoidal waveforms.
4. A sample of a speech waveform.

All the digital simulations were run on the XDS-SIGMA 7 computer using the random number generator routine in the XDS-SIGMA 7 scientific programming library. Normally distributed random numbers were obtained by summing twelve consecutive outputs from the random number routine.

4.1 NUMBER OF PREDICTOR COEFFICIENTS

The first question which should be answered by simulations is that of convergence of the adaptive algorithm. In the many simulation runs that were carried out, not one exhibited divergent behavior in the sense that the prediction error variance increased with time. Further, the prediction error variance tends to degrade gracefully with increasing step size γ , as is demonstrated in Section 4.2.

In this section the intent is to compare the type I, II and III structures of Fig. 2.11 as a function of the number of coefficients in the predictor. Binary quantization with ± 1 output levels is used. The input sequence is the simple Gaussian-Markov process

$$x_{n+1} = 0.9 x_n + w_n \quad (4.1)$$

where, $w_n \sim \text{NID}(0,1)$.

The performance criterion which will be used for this, and subsequent simulations, is the signal-to-noise ratio defined as the ratio of input signal variance to quantizer error variance. As the quantizer error is the same as the reconstruction error in a communication system employing predictive quantization, this definition is consistent with that used by Bello et al [15] and Curry [16], and allows direct comparison of performance.

One word of caution is necessary about the mode of operation. Although the predictor is adaptive the quantizer itself is not. Consequently the system can adapt to changes in the system model, but not to

a change in signal variance due to a gain parameter. The effect of such a gain is, of course, a suboptimum input variance to the quantizer. Ideally the quantizer should be adaptive as well, and this aspect will be investigated in this chapter. Here, a suitable input level is obtained by scaling the input signal. It might be added that for systems in which the absolute signal levels are not important, scaling the input to the encoder by a suitable automatic gain control can avoid the difficult problem of implementation of an adaptive quantizer.

The intent of the simulations in this section is to demonstrate the effect of the number of adaptive predictor coefficients, or taps, on the asymptotic or steady state signal-to-noise ratio reached after the parameters have converged from an initial zero setting. The results in Fig. 4.1, 4.2 and 4.3 represent an average over the last 1000 samples of a 3000 sample run. Only a single realization of the signal process was used as the primary purpose was to indicate the differences in performance between numbers of predictor coefficients and between system types.

Figure 4.1 shows the simulation results for a type I system employing the following adaptation algorithm

$$C_i(n+1) = C_i(n) + \gamma \text{sign} (y_n \hat{x}_{n-i}) \quad (4.2)$$

where, $i = 1, 2, \dots, m$

and, $\gamma = 0.01$.

This algorithm was used as it is implementationally simpler, and results in the same step size for all three systems. It will be demonstrated

FIGURE 4.1 STEADY STATE SNR VERSUS NUMBER OF ADAPTIVE PREDICTOR COEFFICIENTS

FIGURE 4.2 STADY STATE SNR VERSUS NUMBER OF ADAPTIVE PREDICTOR COEFFICIENTS

FIGURE 4.3 STEADY STATE SNR VERSUS NUMBER OF ADAPTIVE PREDICTOR COEFFICIENTS

that the effect of Eqn. (4.2) on asymptotic performance is negligible, although transient behavior is slightly slower than that with the step $\gamma y_n \hat{x}_{n-i}$. As expected from the discussion in Chapter 2, the performance is best with a small number of taps. Somewhat unexpectedly, little difference is obtained with one, two, or three taps while only 1 dB degradation results from using fifteen taps. This is pleasing from the point of view of adaptive operation since a multi-tap filter can adapt to a more complex signal source, yet suffer little degradation when the source is a simple one requiring, in theory, only one tap for optimum performance.

Figure 4.2 illustrates simulation results for a type II structure using the algorithm

$$d_i(n+1) = d_i(n) + \gamma y_n y_{n-i} \quad (4.3)$$

where, $i = 1, 2, \dots, m$

and, $\gamma = 0.01$.

Optimum performance is obtained with approximately ten taps, and approximately 1 dB degradation results from using fifty adaptive taps. In theory one would expect that an optimum fixed predictor would steadily improve as the number of taps increased. The adaptive system degrades, however, as soon as the fixed tap size is of the same order of magnitude as the random noise introduced by the adaptive algorithm. It may therefore be conjectured that beyond the fifteenth tap useful contributions to the prediction output are not being made.

Results of the simulation of a type III system are shown in Fig. 4.3. The adaptation algorithm is identical to Eqn. (4.3). The best performance is obtained with one tap, but there is less than 1 dB degradation with a larger number of taps up to about ten.

The conclusion that may be drawn from these results is that although the simple Markov nature of the source theoretically requires only a small number of taps in the fixed systems of type I and III, little degradation is incurred by the use of a somewhat larger number of taps in the adaptive system. This allows adaptation to more complex signals, but at the same time allows near optimum performance for simple signals. In many nonstationary situations, signals of various degrees of complexity will be encountered, as, for example, in speech encoding.

4.2 PERFORMANCE AS A FUNCTION OF ADAPTIVE STEP SIZE

There are two important considerations in the selection of step size for an iterative adjustment algorithm: dynamic behavior and asymptotic performance for a stationary source signal. In this section a type II system is simulated with several different γ values. In all cases the input is the first order Gaussian-Markov process given by Eqn. (4.1).

In Fig. 4.4 the tap values are initially zero and the transient behavior of the signal-to-noise ratio is plotted as a function of the number of samples already encoded. The results shown are the arithmetic averages of signal-to-noise ratio in dB over twenty different realizations of the Gaussian-Markov input process. It should be noted that these curves will have rather large error variance since few samples were avail-

FIGURE 4.4 TRANSIENT BEHAVIOR FROM ZERO INITIAL TAP SETTINGS

able to make the signal-to-noise ratio estimates, i.e., ten in the region below sample 100, and 25 from sample 100 to 300. The standard deviation on the signal-to-noise ratios for the twenty runs ranged from approximately 1.5 dB for $\gamma = 0.002$, 2.5 dB for $\gamma = 0.01$, to 3.5 dB for $\gamma = 0.1$ over the first 100 samples and were not significantly better from sample 100 to sample 300. The results do indicate however that convergence rate increases with γ , although performance can become erratic if γ is too large.

Figure 4.5 illustrates the steady state signal-to-noise ratio as a function of γ . Twenty realizations of the Markov input process were used, and signal-to-noise ratio for each individual run was estimated from measurements taken over 1000 samples, beginning at sample 1000. The standard deviation for the twenty individual SNR measurements is also indicated on the figure. The most notable result of these simulations is the rather graceful degradation of average performance with increasing step size. Also of interest, is the observation that step sizes below about 0.01 offer little improvement, a fact that can be of value in the choice of γ for a slowly varying signal model.

The effect of multi-level quantization is given by Fig. 4.6. The binary quantizer with ± 1 output levels is replaced by an eight level minimum distortion Max [4] type quantizer for normally distributed input samples with unit variance. This choice is in keeping with the discussion of linear prediction in Chapter 2. With optimum prediction it is expected that the prediction error variance will be close to unity. The results of the simulations show this to be approximately true (the measured error variance over 1000 samples was approximately 1.05 for

FIGURE 4.5 ASYMPTOTIC SIGNAL-TO-NOISE RATIO VERSUS STEP SIZE

$\gamma \leq 0.005$). The results shown in Fig. 4.6 show a somewhat steeper decline of signal-to-noise ratio with increasing γ than that in Fig. 4.5. Below $\gamma = 0.01$ the performance is identical for any value of γ down to 0.001, the smallest value used in the simulation.

4.3 SECOND ORDER GAUSSIAN MARKOV INPUT

Simulations with the second order Gaussian-Markov input process described in Chapter 2 were carried out. This allows performance comparison with Statistical Delta Modulation and the Gaussian-fit estimation algorithm.

Before giving simulation results some comment about the quantizer is appropriate. In what follows the systems will use a Max type [4] quantizer which yields minimum mean-squared quantization error for normally distributed inputs with unit variance. As the simulations are intended to demonstrate the best performance attainable with a given adaptive predictor structure, some effort is made to adjust the input signal variance so that, in operation, the prediction error variance is approximately unity. In practice this is very difficult to accomplish since the prediction error variance is a function of such parameters as step size and number of coefficients in the adaptive predictor, and is not linearly related to the input signal variance. Some of the simulations, therefore, represent performance somewhat lower than would have been attained by a better adjustment of input signal level. An indication of this is obtained by noting the measured value of prediction error variance.

FIGURE 4.6 ASYMPTOTIC SIGNAL-TO-NOISE RATIO VERSUS STEP SIZE

Figure 4.7 is the result of simulation of a 5 tap, type I structure with the adaptation algorithm given by Eqn. (4.2). The input process was the second order Gaussian-Markov process described in Appendix A. An ensemble of twenty realizations of the input process was used, and the signal-to-noise ratio in dB for individual runs were averaged. The measured prediction error variance over the 1000 samples between sample 1000 and sample 2000 was 0.92 with standard deviation 0.06 for 8-levels, and 1.32 with standard deviation 0.17 for the binary quantization system. Consequently the results in Fig. 4.7 are slightly suboptimum. A comparison with the Gaussian-fit algorithm result given in Fig. 2.9 shows that both results in Fig. 4.7 are approximately 2 dB lower than the Gaussian-fit algorithm. By comparison, the results in Fig. 4.7 show a 4 dB and a 10 dB improvement respectively for the binary and 8-level adaptive predictive systems over optimum PCM as given by Curry [16]. It appears, therefore, that adaptive prediction offers improved performance when associated with multi-level quantization. The question that must be answered, of course, is whether a tripling of digital transmission rate is justified by the 15 dB increase in signal-to-noise ratio offered by the 8-level system in Fig. 4.7. This is readily answered by considering binary quantization for a signal process with $R = 3 \times 2.5 = 7.5$. The optimum SNR for this process, obtained by the Gaussian-fit-algorithm is about 20 dB. Since our simulation results for binary quantization of a signal process with $R = 7.0$ yield a somewhat lower SNR, the conclusion is that an eight-level quantizer is justified.

FIGURE 4.7 COMPARISON OF BINARY AND MULTI-LEVEL ADAPTIVE-PREDICTIVE QUANTIZATION

Figure 4.8 demonstrates the effect of increasing the normalized sampling rate R for a predictive binary quantization in a type II configuration. Notice that the steady state signal-to-noise ratio at $R = 1.0$ and $R = 2.5$ is close to the values obtained for Statistical Delta Modulation and Gaussian-fit algorithm as given in Chapter 2. For higher sampling rate; that is, more correlation between samples, the steady-state signal-to-noise ratio falls quite a bit short.

Measured performance of a type I predictive-quantizer is illustrated in Fig. 4.9. An eight-level nonuniform Max type quantizer optimum for zero-mean unity variance input was used. The algorithm used for coefficient adaptation is given by Eqn. (4.2). The rate of adaptation from the initial zero coefficient setting used in the simulation is seen to be a function of the number of coefficients. The five coefficient system responded more slowly than those with one or two coefficients. The best performance was attained by the two coefficient adaptive system both in terms of initial adaptation rate and asymptotic performance.

Finally in Fig. 4.10 the performance of a binary type III system is illustrated. Its best signal-to-noise ratio of 18 dB is somewhat larger than that attained by the type II system of Fig. 4.8. A comparison with the results of Curry, given in Chapter 2, shows that the asymptotic performance of the type III adaptive-predictive quantization system is within 1 dB of the Gaussian-fit algorithm.

 FIGURE 4.8 SIGNAL-TO-NOISE RATIO VERSUS NORMALIZED SAMPLING RATE R

FIGURE 4.9 TYPE I SYSTEM PERFORMANCE

FIGURE 4.10 TYPE III SYSTEM PERFORMANCE

4.4 SIMULATION OF A SYSTEM USING WALSH FUNCTIONS

It was demonstrated in Chapter 3 that the large number of adaptive coefficients in systems of the type II structure may often be reduced by forming the prediction as a sum of P Walsh functions which themselves weight the M past data outputs. It was conjectured that the number P of Walsh function coefficients which must be adaptively adjusted can be fewer than M , the samples of stored data. Here that statement will be substantiated by the results of simulation.

A binary predictive quantizer with a predictor synthesized by means of Walsh functions is simulated for a data storage length $M = 16$, and $P = 1, 3, 5$, and 9 coefficients. The source used is the simple Gaussian-Markov process given by Eqn. (4.1).

The asymptotic signal-to-noise ratio, measured over the 1000 sample interval beginning at sample 1000 is shown in Fig. 4.11. From these results it is concluded first, that the performance with $P = 9$ is as good or better than any of those given previously in Section 4.1. Also it is apparent that $P = 5$ gives almost identical performance. Thus it appears that a reduction in the number of coefficients from sixteen, down to five is feasible by this method.

Figure 4.12 gives the transient performance from an initial zero tap setting for $P = 5$. The rate of adaptation appears comparable to the systems not using the Walsh function prediction. However, it appears that much lower step size is necessary for good asymptotic signal-to-noise ratio.

FIGURE 4.11 ASYMPTOTIC SIGNAL-TO-NOISE RATIO FOR ADAPTIVE-PREDICTIVE BINARY QUANTIZATION WITH PREDICTOR SYNTHESIZED USING WALSH FUNCTIONS

FIGURE 4.12 ADAPTION RATE FROM ZERO INITIAL COEFFICIENT VALUES

It is interesting to note that the Walsh function predictor with $P = 1$ is equivalent to a finite time average over the last M data samples. Its behavior is somewhat similar, but not identical, to delta-modulation with syllabically varying gain. The ability to generate more complex weightings on the past data imparted by more than one Walsh function is shown in the simulation results to yield better signal-to-noise ratio.

4.5 ADAPTIVE ADJUSTMENT OF QUANTIZER PARAMETERS

For optimum performance over a wide range of input variance the quantizer as well as the predictor must be adaptive. One way to accomplish this is shown in Fig. 4.13. An adaptively adjustable gain g_n precedes the quantizer, while the inverse gain $1/g_n$ precedes the predictor. In this way the quantizer input variance is kept reasonably constant. Since the adaptive procedure does not alter the quantizer parameters except in terms of a multiplicative constant, this arrangement cannot adapt to a change in the amplitude probability distribution at the quantizer input. It has been pointed out, however, by Strohn and Boorstyn [19] that a normal distribution of prediction error is a reasonable assumption at least for multi-level quantization.

In this thesis the question of adaptation of the quantizer will not be considered in detail. Rather, a simulation result will be given which indicates the feasibility of employing an adaptive quantizer gain together with the adaptive prediction systems of Chapter 3.

FIGURE 4.13 DIGITAL COMMUNICATION SYSTEM EMPLOYING ADAPTIVE PREDICTOR AND ADAPTIVE QUANTIZER PARAMETERS

There are many ways of implementing an adaptive gain. The method used for the simulation operates as follows. Assuming that the probability distribution of the prediction error is Gaussian, a four level minimum mean-square error quantizer is designed by the method of Max [4] for a nominal variance of unity. Thus its input and output parameters are given by

$$\begin{aligned}
 x_1 &= -\infty & , & & y_1 &= -1.510 \\
 x_2 &= -0.9816 & , & & y_2 &= -0.4528 \\
 x_3 &= 0.0 & , & & y_3 &= 0.4528 \\
 x_4 &= 0.9816 & , & & y_4 &= 1.510 \\
 x_5 &= +\infty & , & & &
 \end{aligned} \tag{4.4}$$

where, for example, an input amplitude between $-\infty$ and -0.9816 is quantized into the output level -1.510 . When a large number of unit variance normally distributed random variables are quantized the fraction of the total number of output samples quantized into the values ± 1.51 is 0.23. For a smaller variance this fraction will decrease, while for a larger variance it will increase. This is the basis for the adaptive technique used in the simulation.

The basic system used is a type II structure with twenty coefficients in the predictor. The adaptation algorithm for the predictor coefficients is given by

$$a_i(n+1) = a_i(n) + 0.01 \operatorname{sign}(y_n y_{n-i}) \tag{4.5}$$

Thus it is immaterial whether $y_n y_{n-i}$ or $y'_n y'_{n-i}$ is used in Eqn. (4.5).

The algorithm used to adapt g_n operates as follows. Let p be the number of past data outputs, out of twenty, equal to ± 1.510 , and Q be the number equal to ± 0.4528 .

For a unit variance input, on the average,

$$Q - P \approx 10 \quad (4.6)$$

For very small variance $Q - P \approx 20$ and for very large variance $Q - P \approx -20$.

It was found by experiment that a reasonable algorithm for g_n is given by

$$\begin{aligned} g_{n+1} &= 1.01 g_n \quad \text{if } Q - P > 11 \\ g_{n+1} &= 0.99 g_n \quad \text{if } Q - P < 3 \end{aligned} \quad (4.7)$$

Figure 4.14 shows the steady-state signal-to-noise ratio as a function of input signal variance. Note that the system performance is within about 2 dB of the optimum given by the Gaussian fit algorithm over a wide range of input levels. The degradation at -50 dB is due primarily to the fact that $\gamma = 0.01$ is too large at this input level.

Transient behavior from $g_0 = 1$ and $a_i(0) = 0$, $i = 1, 2, \dots, 20$ was reasonably fast in that an output signal-to-noise ratio of greater than 16 dB was reached on the average after 400 samples over the input variance range +10 dB to -40 dB.

FIGURE 4.14 PREDICTOR AND QUANTIZER BOTH ADAPTIVE

4.6 SINUSOIDAL SOURCE INPUT

One is not always interested in Gaussian-Markov source inputs. In practical situations such as source encoding of speech the waveform may often be adequately modelled as a sum of a number of sinusoids. Figure 4.15 illustrates comparative results of a type III adaptive-predictive system with simple delta modulation. In both cases the input level was adjusted to produce a quantizer input variance close to unity; it was measured at approximately 0.6 in the simulation so that the results presented are somewhat less than optimum. However increasing the input level to the delta-modulation system by 1.4 reduced the signal-to-noise ratio to about 15 dB, indicating extreme sensitivity to input level.

It should be noted that the signal-to-noise ratio in Fig. 4.15 is measured on the basis of the input samples x_n and the quantization error samples ε_n . Consequently it includes a good deal of aliasing, or foldover, of the quantization error spectrum. Thus the signal-to-noise ratio results of Fig. 4.15 are conservative. The improvement obtained by the use of adaptive prediction has been demonstrated however.

Figures 4.16 and 4.17 illustrate the actual input $x(t)$ and reconstructed waveform $\hat{x}_n$ before lowpass filtering for the delta modulator and an eight tap adaptive-prediction encoder with $\gamma = 0.04$. Note the initial startup transient in Fig. 4.17 which results from an initial zero tap setting. For $\gamma = 0.04$ this transient lasts for only 40 samples after which tracking of the waveform is excellent. The signal $x(t)$ is equivalent to the sum of a 400 Hz and a 800 Hz sinusoid sampled at 40 kHz.

FIGURE 4.15 TYPE III SYSTEM PERFORMANCE WITH SINUSOIDAL INPUT VERSUS DELTA MODULATION PERFORMANCE AT OPTIMUM INPUT LEVEL

FIGURE 4.16 DELTA MODULATION ENCODING OF SUM OF TWO SINUSOIDS

FIGURE 4.17 TYPE III ADAPTIVE-PREDICTIVE BINARY ENCODING OF SUM OF TWO SINUSOIDS

4.7 SPEECH WAVEFORM INPUT

Finally, a short sample of speech is used as input. A portion of the vowel a in "Have", bandlimited with an 8-pole Butterworth lowpass filter with cutoff frequency of 3.1 kHz, sampled at 40 kHz rate is used as input to three different systems: simple delta modulation, a type III adaptive-predictive binary system with 3 taps, and the same system with 8 taps. The results are shown in Figs. 4.18, 4.19 and 4.20. The initial startup transient is not shown, the results starting at sample 128. The adaptation algorithms in Figs. 4.19 and 4.20 are given by Eqn. (4.3).

The reconstructed samples $\hat{x}_n = \tilde{x}_n + y_n$ are lowpass filtered by means of a fast Fourier transform program involving 256 samples of $\hat{x}_n$. The 256 resulting spectral components are ideally filtered by setting components numbered 26 through 232 to zero, and then taking the inverse transform. Theoretically this procedure amounts to ideal lowpass filtering with a bandwidth equal to approximately 3 kHz. Some "smearing" of the spectrum results from this simple procedure, however, as no weighting window on the initial samples was used. Figures 4.18 through 4.20 show a reasonable filtered result, and may be used for visual comparison of the performance.

The simple delta-modulator is somewhat overloaded in Fig. 4.18, and cannot follow the waveform. Both the 3 tap and 8 tap adaptive-predictive systems perform reasonably well.

A comparison with the one-bit memory adaptive delta modulator described by Jayant [36] and simulated here (see Fig. 4.21) shows as good

FIGURE 4.18 SPEECH ENCODING BY DELTA MODULATION

FIGURE 4.19 SPEECH ENCODING BY ADAPTIVE SYSTEM WITH THREE ADAPTIVE COEFFICIENTS

FIGURE 4.20 SPEECH ENCODING BY TYPE III SYSTEM WITH EIGHT ADAPTIVE COEFFICIENTS

FIGURE 4.21 SPEECH ENCODING BY THE METHOD OF JAYANT

if not better performance for the particular sample of speech used. The Jayant system operated with parameters $P = 1.5$ and $Q = 0.66$ (see Chapter 2) and used an identical ± 0.798 output level binary quantizer.

In each of the preceeding diagrams the encoded binary output is shown below the waveforms. The random appearance of the output in the adaptive systems should be compared with that for the delta-modulation.

CHAPTER 5

EFFECT OF CHANNEL AND PARAMETER ERRORS

5.1 THE EFFECT OF CHANNEL ERRORS

Channel errors will introduce mistracking of coefficient values in the simple algorithms

$$\begin{aligned}a_i(n+1) &= a_i(n) + \gamma y_n y_{n-i} \\i &= 1, 2, \dots, m \\b_i(n+1) &= b_i(n) + \gamma r_n r_{n-i}\end{aligned}\tag{5.1}$$

where, $a_i(n)$ and $b_i(n)$ are the corresponding values at the encoder and decoder respectively. For convenience the data will be assumed binary with ± 1 values. Extensions to multi-level systems is straightforward, but makes statement of results more complex.

The received data differ from the encoder output by the channel error d_n given by

$$d_n = r_n - y_n\tag{5.2}$$

The random variable d_n is assumed to have the properties

1. d_n is independent of d_m if $m \neq n$.
 2. $d_n = 2$, with probability $p/2$
 3. $d_n = -2$, with probability $p/2$
 4. $d_n = 0$, with probability $1-p$
- (5.3)

Implicit in the above is the assumption that +1 and -1 occur with equal probability in the data, and the error channel is symmetric. Defining the difference between coefficients as $e_i(n)$ results in the equation

$$e_i(n+1) = e_i(n) + \theta_i(n) \quad (5.4)$$

The driving term $\theta_i(n)$ is given by

$$\begin{aligned} \theta_i(n) &= \gamma(y_n + d_n)(y_{n-i} + d_{n-i}) - \gamma y_n y_{n-i} \\ &= \gamma y_n d_{n-i} + \gamma d_n y_{n-i} + \gamma d_n d_{n-i} \end{aligned} \quad (5.5)$$

The last term $\gamma d_n d_{n-i}$ will be nonzero with probability p^2 and may be neglected in a first order analysis. A little reflection shows that $\theta_i(n)$ consists of a pair of random variables of value $\pm 2\gamma$ separated by i sample periods, one such pair occurring for each channel error. The net effect on the encoder coefficient is either $\pm 4\gamma$ or zero. It is therefore possible to define a new error process as an approximation to Eqn. (5.4).

$$e_i(n+1) = e_i(n) + \xi_i(n) \quad (5.6)$$

where, $\xi_i(n)$ is a time discrete random variable defined by the following.

1. $\xi_i(n)$ is independent of $\xi_i(m)$ if $m \neq n$.
2. $\xi_i(n) = 4\gamma$, with probability $p/4$.
3. $\xi_i(n) = -4\gamma$, with probability $p/4$.
4. $\xi_i(n) = 0$, with probability $1-p/2$.

(5.7)

The assumptions in Eqn. (5.7) are a result of independence of channel errors, and the fact that the predictive encoder produces a data sequence with uncorrelated samples.

It is obvious that the error process, Eqn. (5.6), describes a random walk whose variance increases with time. Modifications to the simple adaptive algorithm which lead to a stationary error variance will now be considered.

5.2 ITERATIVE ALGORITHM WITH EXPONENTIAL DECAY

Perhaps the simplest means to assure strictly stable dynamics for the error process is the modified algorithms

$$\begin{aligned} a_i(n+1) &= \alpha a_i(n) + \gamma y_n y_{n-i} \\ b_i(n+1) &= \alpha b_i(n) + \gamma r_n r_{n-i} \end{aligned} \quad \begin{matrix} i = 1, 2, \dots, m \\ (5.8) \end{matrix}$$

where, $0 < \alpha < 1$.

The resulting error process is given by

$$e_i(n+1) = \gamma e_i(n) + \xi_i(n) \quad (5.9)$$

The relationship between the true coefficient error sequence resulting from the use of Eqn. (5.4), and the approximation given by Eqn. (5.6) is illustrated in Fig. 5.1. In the figure channel errors occur at times k, l, m and n . Note that if i is small compared to the average distance

$$(a) \quad e_i(n+1) = e_i(n) + \theta_i(n)$$

$$(b) \quad e_i(n+1) = e_i(n) + \xi_i(n)$$

$$(c) \quad e_i(n+1) = \alpha e_i(n) + \theta_i(n)$$

$$(c) \quad e_i(n+1) = \alpha e_i(n) + \xi_i(n)$$

FIGURE 5.1 APPROXIMATION OF COEFFICIENT ERROR PROCESS

between channel errors Figs. 5.1(a) and 5.1(b) will be different for only a small fraction of the time. This will also be true of Figs. 5.1(c) and (d) provided α is not too small. It is readily shown that the above process has a finite steady state variance given by

$$\begin{aligned}\varepsilon\{e^2(n)\} &= \frac{\varepsilon\{\xi^2(n)\}}{1 - \alpha^2} \\ &= \frac{8 \gamma^2 p}{1 - \alpha^2}\end{aligned}\quad (5.10)$$

Equation (5.10) suggests that α should be small to minimize the effect of channel errors on the decoder prediction coefficients. This is not possible without penalty, however. For a given coefficient to remain constant the decay must be offset by a bias in $\varepsilon\{y_n y_{n-i}\}$ given by

$$\varepsilon\{y_n y_{n-i}\} = \frac{a_i(1-\alpha)}{\gamma} \quad (5.11)$$

where, a_i is the coefficient value. In Section 5.8 a method to avoid this bias is discussed.

The increase in mean-squared prediction error at the decoder over that already present at the encoder will now be computed. For a type II predictor structure the predicted values at encoder and decoder are given by

$$\begin{aligned}\tilde{x}_n &= \sum_{i=1}^m a_i(n) y_{n-i} \\ v_n &= \sum_{i=1}^m b_i(n) r_{n-i}\end{aligned}\quad (5.12)$$

The decoder prediction may be expressed in terms of coefficient and channel errors as follows:

$$\begin{aligned} v_n = & \sum_{i=1}^m a_i(n) y_{n-i} + \sum_{i=1}^m e_i(n) y_{n-i} + \\ & + \sum_{i=1}^m a_i(n) d_{n-i} + \sum_{i=1}^m e_i(n) d_{n-i} \end{aligned} \quad (5.13)$$

This may be written in the form

$$v_n = \tilde{x}_n + E_1 + E_2 + E_3 \quad (5.14)$$

The mean-squared prediction error at the decoder is therefore given by

$$\text{mse}(D) = \epsilon\{(x_n - \tilde{x}_n - E_1 - E_2 - E_3)^2\} \quad (5.15)$$

A detailed calculation of the various components of Eqn. (5.15) is carried out in Appendix D. The somewhat complex result (D39) may be stated in the approximate form

$$\text{mse}(D) = \text{mse}(E) + \frac{8\gamma^2 p m}{1 - \alpha^2} + 4p \sum_{i=1}^m \epsilon\{a_i^2(n)\}. \quad (5.16)$$

where, it is assumed that p is small, and α is reasonably close to unity so that the above quantities predominate. This is equivalent to the assumption that only the terms $\epsilon\{E_1^2\}$ and $\epsilon\{E_2^2\}$ are significant.

In the simple form given by Eqn. (5.16) the total mean-squared error at the decoder consists of three components; that produced by the encoder, that resulting from the coefficient error process, and that which results from channel errors for a predictor with coefficients equivalent to those in the encoder. The second component is, therefore, a consequence of adaptive operation, while the last is an irreducible error which is present even for a fixed decoder. It should be added that one result of decreasing α is an increase of $\text{mse}(E)$, which precludes overall optimization by the simple process of reducing α .

5.3 SIMULATION RESULTS FOR A TYPE II SYSTEM

To test the validity of the expression, Eqn. (5.13), digital simulations were carried out. A binary, ± 1 output level, predictive quantizer with a type II structure was simulated with Gaussian-Markov input as given by Eqn. (4.1). Channel errors with specified probability were generated by comparing uniformly distributed random numbers with an appropriate threshold. The adaptive algorithms given by Eqn. (5.8) were used with $\dot{\gamma} = 0.05$. Each experiment consisted of a run of 12,000 samples; the first 2000 samples to take care of transients, and the last 10,000 samples to allow a reasonably good estimate of mean-squared error.

Figure 5.2 illustrates measured values of the increase in mean-squared prediction error between encoder and decoder as a function of the decay parameter α . Two independent runs, using different error and input sequences, were carried out at each value. Except at small α the agreement with values computed by Eqn. (5.16) is good. For the lower α values the approximations inherent in Eqn. (5.16) do not hold.

FIGURE 5.2 MEAN-SQUARED PREDICTION ERROR INCREASE VERSUS α

The agreement between measured and computed values of the mean square error increase as a function of error probability p is shown in Fig. 5.3. The value $\alpha = 0.995$ is sufficiently large for reasonable accuracy of Eqn. (5.16).

5.4 EXTENSION TO A TYPE III SYSTEM

In the type III system both the predicted value and the predictor coefficients are recursively defined. The predicted value may be stabilized with respect to channel errors by the so-called "leaky integrator" method described by

$$\tilde{x}_n = \beta \tilde{x}_{n-1} + \sum_{i=1}^m a_i(n) y_{n-i} \quad (5.17)$$

where, $0 < \beta < 1$. Expressing the difference between the predicted value v_n at the decoder and the predicted value $\tilde{x}_n$ at the encoder by

$$u_n = v_n - \tilde{x}_n \quad (5.18)$$

a recursive expression for u_n may be obtained, given by

$$\begin{aligned} u_{n+1} &= \beta u_n + E_1(n) + E_2(n) + E_3(n) \\ &= \beta u_n + E(n) \end{aligned} \quad (5.19)$$

where, $E_1(n)$, $E_2(n)$, $E_3(n)$ are as defined previously by Eqn. (5.14).

The mean-squared prediction error at the decoder may be expressed in terms of u_n as follows:

FIGURE 5.3 MEAN-SQUARED PREDICTION ERROR INCREASE VS CHANNEL ERROR PROBABILITY

$$\begin{aligned}
\text{mse}(D) &= \epsilon\{(x_n - \tilde{x}_n + \tilde{x}_n - v_n)^2\} \\
&= \epsilon\{z_n^2\} + \epsilon\{u_n^2\} + 2\epsilon\{z_n u_n\}
\end{aligned} \tag{5.20}$$

It is demonstrated in Appendix D.2 that $\epsilon\{z_n u_n\}$ is small. Computation of the increase in mean-squared error therefore reduces to

$$\epsilon\{u_n^2\} = \sum_r \sum_s \beta^{n-r-1} \beta^{n-s-1} \epsilon\{E(r)E(s)\} \tag{5.21}$$

where, $r = n-1, n-2, \dots$

and, $s = n-1, n-2, \dots$

Unfortunately it is difficult to simplify this expression as it is not obvious that, for $r \neq s$,

$$\epsilon\{E(r)E(s)\} = 0 \tag{5.22}$$

Consequently it appears that an accurate expression is possible only by computing each term by the methods of Appendix D.1.

An approximate expression for the increase in mean-squared prediction error at the decoder may be obtained by considering the system in Fig. 5.4. The signal sequence $\{y_n\}$ plus an independent error sequence $\{d_n\}$, defined by Eqns. (5.2) and (5.3), is applied to a cascade of two linear systems. There will be several contributions to the error $u_n = v_n - \tilde{x}_n$. It is now assumed that these contributions may be defined as follows:

FIGURE 5.4 ERROR GENERATION AT THE DECODER

1. Assuming the $a_i(n)$ are error-free, there will be an error due to the sequence $\{d_n\}$ acting on the cascade.
2. Secondly, the sequence $\{y_n\}$ operating on the coefficient errors $\{e_i(n)\}$ forms the second major error contribution.
3. Finally, a small term, to be assumed negligible in the analysis, is present due to the sequence $\{d_n\}$ operating on the coefficient errors.

An approximate expression for the mean-squared error on w_n may be obtained from Eqn. (D7) and (D8), and is given by

$$\text{mse}(w_n) \cong 4p \sum_{i=1}^m \epsilon\{a_i^2(n)\} + \frac{8 \gamma^2 p m}{1 - \alpha^2} \quad (5.23)$$

If the errors on w_n were white it would be a simple matter to compute the mean-squared error at v_n . It would be simply

$$\text{mse}(v_n) \cong \frac{4p}{1-\beta^2} \sum_{i=1}^m \epsilon\{a_i^2(n)\} + \frac{8 \gamma^2 p m}{(1-\alpha^2)(1-\beta^2)} \quad (5.24)$$

Note that the error processes are always assumed independent of the desired signal process $\tilde{x}_n$.

The problem of colored errors is somewhat lessened in a type III system since it often happens that a_1 is larger than the other coefficients, whose values are small. The errors arising from the filter involving $a_i(n)$ in Fig. 5.4 then tend to be white. This cannot be said

for the other term, however, as $e_i(n)$, $i = 1, \dots, m$ will all have about the same degree of random fluctuation. This randomness makes it difficult to estimate the effect on Eqn. (5.24) except possibly by the involved procedure suggested in connection with Eqn. (5.21). In what follows the approximate result, Eqn. (5.24) based upon the white assumption is compared with mean-squared prediction error measurements obtained by simulation.

5.5 SIMULATION RESULTS FOR A TYPE III SYSTEM

In Fig. 5.5 encoder and decoder prediction error measured from a simulation similar to that described in Section 5.3 are given. The system simulated was a type III configuration with adaptive algorithm given by Eqn. (5.8) with $\alpha = 0.995$, and prediction given by Eqn. (5.17). All other parameters were identical to the simulation in Section 5.3. The channel error probability was 0.005 in the simulation.

In Fig. 5.5 the lower points are the measured encoder mean-squared prediction error while the upper points represent the measured decoder mean-squared prediction error. The curve represents the decoder mean-squared prediction error obtained by adding the result of Eqn. (5.24) to the encoder mean-squared prediction error. These results seem to indicate that Eqn. (5.24) is at least a reasonable approximation to the increase in prediction error for those β values of practical significance.

Figure 5.6 shows how the difference between decoder and encoder mean-squared error grows with channel error probability. Reason-

FIGURE 5.5 MEAN-SQUARED PREDICTION ERROR VERSUS β

FIGURE 5.6 INCREASE IN MEAN-SQUARED PREDICTION ERROR

able agreement between measured results and those computed by Eqn.

(5.24) was again obtained.

5.6 SENSITIVITY TO PARAMETER MISMATCH

In physical implementations of adaptive systems such as those described in this thesis there will be matching errors in the parameters α, β , and γ between the encoder and decoder. These errors result unavoidably from component tolerances and aging effects in analogue systems, and from quantization in digital systems.

The effect of an error in the parameter α may be evaluated as follows. Set

$$\begin{aligned} a_i(n+1) &= (\alpha + \delta) a_i(n) + \gamma y_n y_{n-i} \\ i &= 1, \dots, m \\ b_i(n+1) &= \alpha b_i(n) + \gamma y_n y_{n-i} \end{aligned} \quad (5.25)$$

where, channel errors are neglected for the purpose of the discussion.

A recursive relation for the coefficient errors is given by

$$e_i(n+1) = \alpha e_i(n) - \delta a_i(n) \quad (5.26)$$

The term $-\delta a_i(n)$ may be expected to result in a bias in $e_i(n)$. Assume $\epsilon\{a_i(n)\} = a_i$, a constant

$$\begin{aligned} e_i(n) &= \alpha^n e_i(o) - \delta a_i(n-1) - \delta \alpha a_i(n-2) + \dots \\ &\quad + \delta \alpha^{n-1} a_i(o) \end{aligned} \quad (5.27)$$

Now assuming large n and taking expectations leads to the following expression for the fractional bias.

$$\frac{\varepsilon\{e_i(n)\}}{a_i} = -\frac{\delta}{1-\alpha} \quad (5.28)$$

If α is close to unity, δ must be small to avoid a large bias.

5.7 MISMATCH IN ALGORITHM STEP SIZE

Let the encoder and decoder algorithms be given by

$$\begin{aligned} a_i(n+1) &= \alpha a_i(n) + (\gamma - \eta) y_n y_{n-i} \\ b_i(n+1) &= \alpha b_i(n) + \gamma y_n y_{n-i} \end{aligned} \quad (5.29)$$

Since the sequence $\{-\eta y_n y_{n-i}\}$ is proportional to the adaptive adjustment steps taken by the encoder the effect is one of gain difference given by

$$\frac{b_i(n)}{a_i(n)} = \frac{\gamma}{\gamma - \eta} \quad (5.30)$$

As such this is not serious if $\eta \ll \gamma$.

A more serious problem is an asymmetry between γ_1 , for positive steps, and γ_2 , for negative steps which is not compensated by an identical asymmetry at the decoder. Let

$$\Delta\gamma = \gamma_1(E) - \gamma_1(D) + \gamma_2(D) - \gamma_2(E) \quad (5.31)$$

where, the labels D and E indicate decoder and encoder respectively.

Over a large number of steps coefficient stability at the encoder demands a roughly equal number of positive and negative adjustments. Hence the resulting bias due to asymmetry in γ is given by

$$|\epsilon\{e_i(n)\}| \leq \left| \frac{\Delta\gamma}{1-\alpha} \right| \quad (5.32)$$

This can be large if α is close to unity. It may be noted that these problems with γ mismatch do not occur in a digital implementation, while in analogue implementations a self correcting system can likely be devised.

5.8 MISMATCH IN INTEGRATOR DECAY PARAMETER

A somewhat different approach to β mismatch must be taken.

Let

$$\begin{aligned} \tilde{x}_n &= (\beta - \mu) \tilde{x}_{n-1} + \sum_{i=1}^m a_i(n) y_{n-i} \\ v_n &= \beta v_{n-1} + \sum_{i=1}^m a_i(n) y_{n-i} \end{aligned} \quad (5.33)$$

Then a recursive expression for $u_n = v_n - \tilde{x}_n$ may be written

$$u_n = \beta u_{n-1} + u_n \tilde{x}_{n-1} \quad (5.34)$$

The mean-squared prediction error may be expressed in terms of u_n as follows

$$\begin{aligned}
\text{mse}(D) &= \varepsilon\{(x_n - \tilde{x}_n + \tilde{x}_n - v_n)^2\} \\
&= \text{mse}(E) + \varepsilon\{u_n^2\} + 2\varepsilon\{z_n u_n\}
\end{aligned} \tag{5.35}$$

where, $z_n = x_n - \tilde{x}_n$.

By Eqn. (5.34) u_n is a linear combination of $\tilde{x}_{n-1}, \tilde{x}_{n-2}, \dots$, hence it also a linear combination of the past data $y_{n-2}, y_{n-3}, \dots$. Assuming that z_n is the resulting error of nearly optimum linear prediction based upon the data $y_{n-1}, y_{n-2}, \dots$, it follows from the orthogonality relation for optimum linear prediction that

$$\varepsilon\{z_n u_n\} = 0 \tag{5.36}$$

Proceeding to evaluate the rest of the expression

$$\begin{aligned}
u_n &= \beta^n u_0 + \mu\beta^{n-1} \sum_{i=1}^m a_i(o) y_{-i} + \dots + \\
&+ \mu\beta^0 \sum_{i=1}^m a_i(n) y_{n-i}
\end{aligned} \tag{5.37}$$

We now proceed to find an upper bound by assuming that $a_i(n) = a$ for every n and i . Then Eqn. (5.37) may be written

$$\begin{aligned}
u_n &= \beta^n u_0 + \mu y_{n-1}^a \\
&+ \mu y_{n-2}^a (1+\beta) \\
&+ \dots \\
&+ \mu y_{n-m}^a (1+\beta+\dots+\beta^{m-1}) \\
&+ \dots \\
&+ \mu y_{n-k}^a (\beta^{k-m+1}+\dots+\beta^{k-1}) \\
&+ \dots
\end{aligned} \tag{5.38}$$

where, $k > m$

The data sequence $\{y_n\}$ is the output of a predictive quantization system and can therefore be assumed reasonably white. Consequently

$$\begin{aligned}
\varepsilon(u_n^2) &\leq \mu^2 a^2 [1 + (1+\beta)^2 + \dots \\
&\quad (1 + \beta + \dots + \beta^{m-1})^2 + \dots \\
&\quad (\beta^{k-m+1} + \dots + \beta^{k-1})^2 + \dots]
\end{aligned} \tag{5.39}$$

Then, defining the quantity within the brackets by s ,

$$\varepsilon(u_n^2) \leq \mu^2 a^2 s \tag{5.40}$$

For $0 < \beta < 1$ the sequence for s is convergent and may be readily evaluated by digital computation. Some results are given in Table 5.1. Finally the mean-square error in the decoder prediction is bounded by

$$\text{mse}(D) \leq \text{mse}(E) + \mu^2 a^2 s \tag{5.41}$$

In a type III system with several taps only the first is large compared to the others so the above bound is somewhat conservative. However it shows that one percent tolerance in β is adequate for most of the conditions illustrated by Table 5.1.

TABLE 5.1
Values of s Obtained by Digital Computer

β	1 TAP	3 TAP	5 TAP	8 TAP
0.8	2.8	20.8	50.3	107.5
0.9	5.3	43.3	112.0	260.4
0.95	10.3	88.3	236.6	576.4
0.99	50.2	448.2	1236.2	3132.8

5.9 ASYMMETRIC STEP SIZE

It was shown in Section 5.2 that the decay parameter α leads to correlation in the output data, given by Eqn. (5.11). Here it is demonstrated that this correlation may be reduced by asymmetric step size. Let

$$\begin{aligned} r_1 &= 2\gamma \\ r_2 &= \gamma \end{aligned} \tag{5.42}$$

Defining p as the probability that $y_n y_{n-1} = 1$, the correlation in the data is given by

$$\varepsilon\{y_n y_{n-1}\} = p - (1-p) \tag{5.43}$$

The condition for constant steady-state coefficient values is

$$\begin{aligned}\varepsilon\{a_i(n+1)\} &= a_i \\ &= \alpha a_i + \gamma_1 p - \gamma_2(1-p)\end{aligned}\quad (5.44)$$

Consequently,

$$\frac{a_i(1-\alpha)}{\gamma} = 3p-1 \quad (5.45)$$

It is evident that zero data correlation is no longer obtained by $\alpha = 1$ but by a smaller value which gives $p = 0.5$ in Eqn.

(5.45). For example, $a_i = 1$, $\gamma = 0.01$ requires $\alpha = 0.995$. If a_i should then change, say to 0.5, a correlation in the data is again present, and given by

$$\begin{aligned}\varepsilon(y_n y_{n-i}) &= p - (1-p) \\ &= 0.16\end{aligned}\quad (5.46)$$

5.10 EXTENSIONS

An obvious extension to the preceeding analyses is to let all parameters α, β, r to have mismatch error. This will introduce additional terms into the resulting expressions. In this chapter only the first order effects were considered, as it is expected that the additional terms will be small.

Another extension to the preceeding derivations is the use of more general linear systems in the recursive expression, Eqn. (5.8).

One might consider, for example

$$\begin{aligned} a_i(n+1) &= L_1[a_i(n), a_i(n-1), \dots] + L_2[y_n y_{n-1}, y_{n-1} y_{n-2}, \dots] \\ b_i(n+1) &= L_1[b_i(n), b_i(n-1), \dots] + L_2[r_n r_{n-1}, r_{n-1} r_{n-2}, \dots] \end{aligned} \quad (5.47)$$

where, L_1 and L_2 are general linear operations.

The coefficient error process may then be written as

$$e_i(n+1) = L_1[e_i(n), e_i(n-1), \dots] + L_2[\xi_i(n), \xi_i(n-1), \dots] \quad (5.48)$$

Equation (5.48) may be interpreted as a cascade of two linear systems excited by the white sequence $\xi_i(n)$ as shown in Fig. 5.7. It is likely that reduced sensitivity to channel errors can be obtained by a proper choice of L_1 and L_2 . This problem is not considered here as it was the object of the thesis to maintain simplicity of implementation.

FIGURE 5.7 COEFFICIENT ERROR GENERATION IN GENERAL LINEAR SYSTEM

CHAPTER 6

AN EXPERIMENTAL ADAPTIVE-PREDICTIVE SOURCE ENCODER

Some of the properties of source encoding systems using iteratively adjusted linear prediction were determined in Chapters 4 and 5. Here, an experimental speech waveform-to-binary encoder is described.

6.1 THE EXPERIMENTAL SYSTEM

The objective of the experimental study is to evaluate the speech encoding properties of an adaptive-predictive system of the simplest form; that is, an analogue-to-binary quantizer of the type III structure shown in Fig. 2.11(c).

A block diagram of the encoder portion of the experimental system is given in Fig. 6.1, while circuit details of each block and their operation are deferred to Appendix E. A decoder, containing identical circuitry except for the comparator, was also constructed. Except for initial coefficient adjustment by saturation, no provision for channel errors was made. The coefficient values are, therefore, iteratively adjusted according to the algorithm

$$d_i(n+1) = d_i(n) + \gamma y_n y_{n-i} \quad (6.1)$$

where, $i = 1, 2, \dots, m$.

Reasonable design parameters for the experimental system were obtained from the results of simulation. Eight coefficients were used since the simulation results in Chapter 4 indicate good performance with

* Double lines indicate vector valued quantities

FIGURE 6.1 BLOCK STRUCTURE OF EXPERIMENTAL SYSTEM

this number in type III systems. The simulation results of the encoding of a typical speech sample indicate that a peak signal amplitude bounded by ± 10 volts produced coefficient values bounded by ± 2 volts in a type III system with eight coefficients and $\gamma = 0.04$. Further, this value of γ led to the least error in the reconstructed waveform. Since $\gamma = 0.04$ is to correspond to a change of one least significant bit in the binary representation of a coefficient value, a maximum value of ± 2 volts is consistent with a seven-bit binary representation. In the experimental system an eight-bit representation was chosen to allow a certain degree of flexibility.

The gain of the feedback loop, which comprises the integrator gain $-T/RC$ (see Appendix E), the digital-to-analogue converter transfer characteristic, and the coefficient values of the digital filter, defines the slope following capabilities of the encoder. Adjustment of this gain is analogous to selection of a suitable step size in a delta modulation system. In an adaptive-predictive system the feedback gain will vary with adaptive adjustment of the filter coefficients about a nominal design value set by the DAC transfer characteristic and the integrator gain $-T/RC$. It was found by simulation that $T/RC = 0.5$ produced reasonable coefficient values in the experimental system, whose DAC transfer characteristics are given in Appendix E. Figure 6.2 illustrates how the coefficient values vary with input amplitude and frequency in the simulation. The coefficient values at the instant of maximum positive input signal excursion are illustrated in the figure, which applies to a basic sampling rate of 40 kHz.

FIGURE 6.2 COEFFICIENT VALUES IN EXPERIMENTAL SYSTEM WITH SINUSOIDAL INPUT SIGNAL

6.2 ANALOGUE VS DIGITAL IMPLEMENTATION

The experimental encoder, described above and in Appendix E, is constructed largely of digital integrated circuit components. Together with a similar digital decoder structure the experimental system is suitable for operation over an error-free channel. The only requirement is that the coefficient values of the predictor structure be initially adjusted to equality in both the encoder and decoder. One method for accomplishing this is to saturate the coefficients at their maximum positive values by transmission of a string of data all of the same sign. The effect of this is illustrated in Fig. 6.3 where the reconstructed signal at the decoder is shown. An 800 Hz sinusoid is being transmitted in the figure, which is an oscilloscope trace with 5 msec/div sweep rate and 10 v/div vertical deflection. The basic clock rate was 40 kHz. The 10 msec long saturation and a recovery period of approximately 5 msec duration is clearly evident. The bottom trace shows the corresponding data transmitted over the channel.

In Chapter 5 algorithms suitable for operation over channels with errors were described. These are easier to implement if the predictor coefficients are stored by analogue circuitry, such as single-pole active filters. An important feature of an analogue circuit implementation is the simpler clocking requirements, consisting only of a basic clock operating at the output data rate.

FIGURE 6.3 EFFECT OF COEFFICIENT SATURATION ON THE RECONSTRUCTED WAVEFORM

6.3 RECONSTRUCTION OF THE WAVEFORM

Theoretically, in a predictive quantization system, the optimum reconstruction is given by

$$\hat{x}_n = \tilde{x}_n + y_n. \quad (6.2)$$

The reconstructed waveform $\hat{x}(t)$ is, then, obtained by lowpass filtering of the sequence in Eqn. (6.2). A major difficulty, however, is that Eqn. (6.2) is optimum only if the quantized output sequence $\{y_n\}$ is a minimum-mean-squared error representation of the prediction error sequence $\{z_n\}$. For the nonstationary prediction error resulting from the operation of the experimental system with a speech waveform input (see Fig. 7.2), $\{y_n\}$ cannot be a minimum mean-squared error representation unless an adaptive quantizer is used. To avoid the complexity involved in the implementation of an adaptive quantizer, the reconstructed signal was formed by lowpass filtering of the sequence $\tilde{x}_n$. This results in a somewhat reduced signal-to-noise ratio over that obtained from the use of the optimum reconstruction given by Eqn. (6.2). In the special case of Gaussian and independent z_n samples this reduction is 4.4 dB.

6.4 COMPARISON WITH ADAPTIVE DELTA MODULATION

The experimental binary encoder was compared with a syllabically companded delta modulator similar to that developed by Greefkes [54]. The comparison system is basically a single-integration delta modulator with step size determined by lowpass filtering of a voltage V given by

$$V = k 2^{\ell}; \quad 3 \leq \ell \leq 8 \quad (6.3)$$

where, k is a constant of proportionality, and, ℓ is the length of strings of consecutive 1's or 0's in the output data. The lowpass filter was a single-pole type with a time constant of approximately 20 msec, resulting in a step size which varies at a syllabic rate.

The speech encoding properties of the experimental system, with output reconstructed from $\tilde{x}_n$ as described in the previous section, were compared with those of the adaptive delta modulator by means of paired comparison using the test sentences:

He wrote his name boldly at the top of the sheet.

Green moss grows on the northern side.

A severe storm tore down the barn.

The input level to both systems was adjusted to a nominal level producing waveform peaks of about ± 2 volts.

6.4.1 Results

Two differences between the adaptive-predictive system and the adaptive delta modulation system were immediately noticed. The background, or idling noise, of the adaptive-predictive system is more apparent than with the delta modulator, and the distortion is of a different character. Whereas distortion in the adaptive delta modulation system tends to produce a "muffled" tone, the distortion in adaptive predictive encoding tends to be of a higher frequency character. This difference complicates subjective comparison tests since low frequency distortion tends to be more pleasing to the ear.

On the basis of informal intelligibility tests the predictive quantizer was judged roughly equivalent to the adaptive delta modulator at sampling rates of 10 kHz, 20 kHz, and 40 kHz. On the basis of distortion, however, the adaptive delta modulator was judged slightly better.

One reason for the distortion characteristics of the predictive quantization system will be demonstrated in Chapter 7. As shown in Fig. 7.2, the error waveform is highly nonstationary, increasing markedly once per pitch period. The effect of this is to introduce a pitch synchronous noise burst which tends to give the reconstructed waveform a "raspy" quality.

6.5 COMPARISON WITH A REFERENCE SIGNAL

A method of obtaining reference signals for speech quality tests has been suggested by Schroeder [55]. A family of speech waveforms with well-defined signal-to-noise ratio is given by

$$r(n, \alpha) = \frac{x(n)}{\sqrt{1 + \alpha^2}} [1 + \alpha \theta(n)], \quad (6.4)$$

where, $\theta(n)$ has the properties:

$$\begin{aligned} \theta(n) &= \pm 1 \\ \epsilon\{\theta(n)\} &= 0 \\ \epsilon\{\theta(n)\theta(m)\} &= 0 \quad \text{for } m \neq n \\ \epsilon\{\theta(n)x(m)\} &= 0 \quad \text{for every } m, n. \end{aligned}$$

The signal-to-noise ratio of any member of the above family is given by

$$\frac{S}{N} = \frac{1}{\alpha^2} . \quad (6.5)$$

For speech encoding systems whose quantization noise is proportional to signal amplitude, such as logarithmically companded PCM for example, the reference signals given by Eqn. (6.4) yield a simple and effective means of performance evaluation.

The result of subjective comparison tests between the experimental encoder and a family of reference signals with signal-to-noise ratio graduated in 2 dB steps is given in Fig. 6.4. The input to the experimental encoder was a speech waveform, of approximately 5 volts peak value, consisting of the following sentences:

Joe took father's shoe bench out.

May we all learn a yellow lion roar.

Have you seen Bill?

The reconstructed waveform was obtained by lowpass filtering of the sequence $\{\tilde{x}_n\}$ by means of an 8-pole Butterworth active filter with 3.1 kHz bandwidth. Also shown in Fig. 6.4 is the estimated maximum signal-to-noise ratio obtainable by using the sequence $\{\tilde{x}_n + y_n\}$, with $\{y_n\}$ obtained by adaptive quantization of z_n .

For reference, the performance of digitally companded PCM was compared with the reference signals. The 6-bit and 5-bit results are obtained from 10-bit linear conversion followed by digital companding by the method of Apple and Wintz [56]. The method consists basically of transmitting a sign bit, a three-bit binary number corresponding to a logarithmic scale factor, and either a one or two bit binary number giving a linear graduation of each logarithmic scale division. The method is

FIGURE 6.4 PERFORMANCE OF THE EXPERIMENTAL ENCODER COMPARED WITH DIGITALLY COMPANDED PCM BY MEANS OF A FAMILY OF REFERENCE SIGNALS

usually called log-linear companding as a consequence. In the four-bit situation only the sign bit and logarithmic scale factor were used.

6.6 SINUSOIDAL INPUT SIGNALS

The performance of the experimental system for sinusoidal input was obtained by simulation of the exact system on a digital computer. This was done to avoid the construction of special purpose circuitry required for laboratory measurement of signal-to-noise ratio. The results given in Fig. 6.5 correspond to operation at 40 kHz basic clock rate. The 0 dB level corresponds to a sinusoid of ± 3.2 volts peak-to-peak input amplitude. Ideal lowpass filtering to 3.1 kHz bandwidth of the sequence $\{\tilde{x}_n\}$ by means of a Fast Fourier Transform was used to form the reconstructed waveform. As previously noted in this chapter a somewhat higher signal-to-noise ratio is obtainable by the addition of the data sequence $\{y_n\}$, properly scaled, before lowpass filtering.

6.7 CONCLUSIONS FROM THE EXPERIMENTAL SYSTEM

We have found, in general, that the performance of the experimental speech encoder is somewhat inferior to the best available adaptive delta-modulation systems. One reason for this is the non-stationary nature of the prediction error waveform for which no provision was made in the experimental system in the form of adaptive quantization. Another reason is the inherently noisy nature of a prediction based upon a binary data sequence. We have shown in Chapter 4 that a performance improvement without an increase in output data rate is possible for Gaussian-Markov inputs if multi-level quantization is used. Such an effect may also be true for speech waveform inputs.

FIGURE 6.5 PERFORMANCE WITH SINUSOIDAL INPUT SIGNALS

CHAPTER 7

ESTIMATION OF SPEECH PARAMETERS

A direct result of the encoding of speech by adaptive prediction is the availability of predictor coefficients related to the physical source process. Estimation of speech characteristics then reduces to simple computations involving the prediction coefficients. Pitch period, as noted in Chapter 2, may, however, be obtained directly from the prediction error of the encoder. In this chapter pitch period extraction by the simple adaptive-predictive encoder structures of this thesis will be examined.

7.1 PITCH PERIOD EXTRACTION

Determination of the fundamental frequency (pitch) of speech is a problem that has occupied researchers for many years. Pitch information, of course, plays an important role not only in speech synthesis and communication, but also in such areas as linguistics and speech pathology.

The known techniques for pitch period extraction may be divided into those medically oriented methods which attempt direct measurement of the motion of the vocal cords (see for example [57]), and those which operate on the speech waveform. Some of the more significant developments in the latter group are described below. These may roughly be classified into the following categories:

1. Nonlinear operations on the waveform to accentuate its periodic nature.
2. Pattern recognition methods.
3. Cepstrum analysis.
4. Spectrum flattening and autocorrelation.
5. Linear prediction of the waveform.

The first technique is mentioned because of its simplicity, typically involving operations such as envelope detection, or other nonlinear transformations, followed by lowpass filtering. The method fails during rapid pitch or amplitude change.

Pattern recognition for pitch extraction suggests itself naturally from observation of the human ability to place accurate pitch period markers on a graph of a speech waveform. Pitch extraction by means of a computer program executing a set of rules was demonstrated by Gold [58] in 1962. Closely related are techniques suggested by Schroeder [59], and Harris and Weiss [60], that derive pitch information from the pattern of spectral harmonics.

Pitch period extraction by double-spectrum-analysis was demonstrated by Noll [61] using the cepstrum technique of Bogert, Healy, and Tukey [62]. The method is readily described in terms of the simplified model for voiced speech which assumes a linear system driven by a quasi-periodic impulse train. The speech spectrum, then, consists of closely spaced harmonics of the pulse train $u(\omega)$ multiplied by a transfer function $G(\omega)$ as follows

$$X(\omega) = G(\omega) \cdot U(\omega) \quad (7.1)$$

In cepstrum analysis the logarithm of the magnitude of the spectrum is first taken, resulting in the separation.

$$\log|X(\omega)| = \log|G(\omega)| + \log|U(\omega)| \quad (7.2)$$

Fourier transformation of Eqn. (7.2) results in the cepstrum, whose components correspond to the ripples in the log spectrum. In the lexicon of cepstrum analysis the term quefrequency is given to ripple frequency in the spectrum. For voiced speech there will be a high quefrequency component due to $\log|U(\omega)|$ and a low quefrequency component due to $\log|G(\omega)|$. These are widely separated, and the pitch component is easily recognizable.

A number of recent methods for pitch period extraction, including the spectrum flattening techniques of Sondhi [63] and linear prediction of the waveform by Atal and Hanauer [21], involve recovery of the pulse train $u(t)$ from the speech waveform $x(t)$. If in Eqn. (7.1) the transfer function is written in magnitude and phase form as

$$X(\omega) = |G(\omega)| e^{-j\theta(\omega)} \cdot U(\omega) \quad (7.3)$$

it is readily seen that $U(\omega)$ may be recovered by scaling and shifting the phase of each spectral component of $X(\omega)$. The method is difficult to implement, however, as both $|G(\omega)|$ and $\theta(\omega)$ must be adaptively estimated. Estimation of $\theta(\omega)$ may be avoided if the spectrum flattening operation

is followed by autocorrelation analysis. The flattened power density spectrum may be written

$$\Phi(\omega) = K \cdot |U(\omega)|^2 \quad (7.4)$$

where, K is a constant gain factor. The Fourier transform of Eqn. (7.4) is, therefore, proportional to the autocorrelation function of $u(t)$. Since the autocorrelation function has pulses synchronous with $u(t)$ it may be used for pitch period extraction.

Until very recently, known techniques for pitch period extraction were largely confined to those described above. The most effective methods, cepstrum analysis and autocorrelation of a spectrum-flattened speech waveform, are difficult to implement. The linear prediction method to be described below appears capable of simple implementation, however.

7.2 PITCH PERIOD EXTRACTION BY LINEAR PREDICTION OF THE WAVEFORM

Linear prediction as used by Atal and Hanauer [21] for speech analysis and synthesis is described in Section 2.7. The resulting prediction filter may be used for pitch period extraction as shown in Fig. 7.1. Part (a) of the figure illustrates the model used to describe the voiced speech process, while part (b) shows how pitch information is derived from the prediction error sequence.

In terms of z -transforms the model for the speech generation process may be written, in the terminology of Ref. 21, as

(a) Model of Speech Process

(b) Pitch Period Extraction

FIGURE 7.1 PITCH PERIOD EXTRACTION BY LINEAR PREDICTION OF THE SPEECH WAVEFORM

$$X(z) = \frac{1}{1 - A(z)} U(z). \quad (7.5)$$

Similarly, the error sequence of Fig. 7.1(b) becomes

$$E(z) = X(z) - B(z)X(z) \quad (7.6)$$

where, $B(z)$ is the transform of the prediction filter. Substitution for $X(z)$ gives

$$E(z) = \frac{1 - B(z)}{1 - A(z)} U(z). \quad (7.7)$$

Thus, if $B(z) \cong A(z)$, then

$$E(z) \cong U(z). \quad (7.8)$$

Simple lowpass filtering, as shown in Fig. 7.1(b), recovers $u(t)$.

The similarity to the spectrum flattening method of Sondhi [63] is apparent. The necessity for adaptively estimating $|G(\omega)|$ and $\theta(\omega)$ of Eqn. (7.3) has, however, been replaced by a requirement for predictor adaptation.

7.3 PITCH PERIOD EXTRACTION VIA ADAPTIVE-PREDICTIVE QUANTIZATION

A natural question one might ask is whether adaptive predictors of the form examined in this thesis may be used to extract pitch period information. While it is obvious that the type I structure of Fig. 2.11(a) approximates the recursive predictor structure used by Atal and Hanauer

if a multi-level quantizer is used, it is not at all clear that the same result is true for the type II and type III structures. In this section experimental results are obtained from the binary speech encoder described in Chapter 6 that demonstrate its capability for pitch period extraction.

One might explain the pitch period extraction intuitively as follows. Between vocal cord pulses the predictor adjusts itself iteratively to minimize the mean-squared prediction error within the constraints imposed by the encoder structure. When the next vocal cord pulse occurs, however, the waveform ceases to be linearly predictable and a large prediction error develops. Because of the feedback nature of the adaptive-predictive quantizer the initial error spike is usually followed by at least one or two others, alternating in sign. This behavior suggests that effective pitch period extraction can be realized by full-wave rectification and lowpass filtering of the prediction error waveform.

In Figure 7.2 experimental results of pitch period extraction using the binary speech encoder of Chapter 6 are given. The system was operated in a type III configuration since this produced better pitch period extraction than the type II system configuration. The sampling rate was set to 40 kHz. The sampling rate was found not to be critical although lower sampling rates tended to produce more noise between the pitch pulses. In all cases the error waveform has been recovered from the error sequence by means of an eight pole Butterworth Lowpass filter with cutoff frequency of 3.1 kHz. Further processing, as suggested above, has not been done in order to illustrate the nature of the error

(a)
The phoneme /ɔ/

(b)
The phoneme /i/

(c)
Voiced portion of the
word "chase"

FIGURE 7.2 PITCH PERIOD EXTRACTOR WAVEFORMS

waveform itself. In all figures the vertical scale is 5 v/cm for the error waveform and 2 v/cm for the corresponding speech waveform shown below the error waveform.

In Figure 7.2(a) the filtered error waveform and corresponding speech waveform, also filtered to 3.1 kHz lowpass, is shown corresponding to the phoneme /ɔ/, as in "bought" spoken by a male (the author). The sweep rate was 10 ms/cm. The pitch pulses spaced approximately 9 milliseconds apart are clearly visible.

The same test for the phoneme /i/, as in "eat", is illustrated in Fig. 7.2(b). Here the pitch pulses are spaced approximately 10 milliseconds apart. Each pulse in this case is composed of several positive and negative excursions of the error waveform. The frequency of this oscillation was measured at approximately 2.5 kHz so that the suggested scheme of full-wave rectification and lowpass filtering can still recover the pitch period with reasonably good accuracy.

Finally the spoken word "chase" and the corresponding error waveform is illustrated in Fig. 7.2(c). Only the voiced portion is shown, however, in the figure. The sweep rate here was 20 ms/cm. Note that near the end of the word the speech waveform itself exhibits multiple peaks near the pitch pulses while the error waveform exhibits only a single spike at the correct pitch period.

7.4 ESTIMATION OF OTHER SPEECH CHARACTERISTICS

Another speech characteristic of particular interest in vocoder applications is the voiced-unvoiced dichotomy. This may be easily resolved by detection of the presence or absence of pitch pulses. In the

experimental system it was noted that clearly defined pitch pulses were present only during voiced speech. Consequently, voiced-unvoiced decisions may be based upon the output of a pitch pulse detector.

Other speech parameters such as formant frequencies are not obtained in such an obvious way from the operation of the adaptive-predictive quantizer. There is a fundamental difference between the predictor used by Atal and Hanauer and the predictive quantization systems of this thesis. The former can generate an approximation to the speech waveform when excited only by a single impulse, while the latter requires the quantized error sequence as input for the duration of the reconstruction period. Speech parameter information is, therefore, contained not only in the prediction coefficients, but also in the quantized error sequence. Whether it can be extracted simply is an open question.

CHAPTER 8

CONCLUSION

8.1 SUMMARY

This thesis is concerned with implementation of nearly optimum source encoding systems. A review, in Chapters 1 and 2, of the relevant literature on quantization and optimum digital systems suggests that an optimum source encoder performs the following functions:

1. Prediction of that component of the input waveform which is statistically related to the past digital outputs.
2. Quantization of the residual, or unpredictable component of the input waveform.

A knowledge of the physical process generating the source waveform, whether available a priori or estimated by an adaptive system is crucial in the implementation of the above functions. The extent to which the adaptive systems of this thesis can perform the necessary source model estimation is demonstrated by simulation and experiment in Chapters 4 and 6.

For the important class of sources which can be modelled by Gaussian Markov processes with unknown or slowly varying parameters the iterative adaptation techniques of this thesis lead to performance very nearly equal to prior, more complex, methods described in the literature. In particular they compare favorably with Statistical Delta Modulation of Bello et al. [15], and the Gaussian-fit method of Curry [26].

One of the characteristics of the adaptive-predictive encoding technique is a requirement for an identical adaptive-predictor at the receiver. The effect of channel errors on such a system is evaluated in Chapter 5 of the thesis, and design techniques to minimize the effect of channel errors are given.

For speech source encoding a different situation is encountered. Unlike the Gaussian Markov situation, in which the residual process tends to a constant variance, the prediction error for voiced speech is highly nonstationary, increasing in an impulsive fashion once per pitch period. Consequently, as demonstrated by the speech encoding results of the experimental adaptive-predictive system, the performance tends to be somewhat worse than that obtainable with adaptive delta modulation systems that merely alter the step size.

These negative results for speech encoding suggest that techniques such as adaptive delta modulation, as well as the binary adaptive-predictive quantization system of this thesis, have probably reached the limits of their capability for speech encoding. These limits as noted in Appendix C are still far below the bounds given by rate-distortion theory. Future progress in speech source encoding may be expected from more sophisticated methods such as the analysis-synthesis technique of Atal and Hanauer [21].

The iterative algorithms of this thesis are not without usefulness in speech applications. We show in Chapter 7 that a simply implemented real time pitch period extractor may be realized through systems developed in the thesis. Since speech analysis-synthesis is done

pitch-synchronously, the pitch period extractor is important in the development of analysis-synthesis methods of speech communication.

8.2 SUGGESTIONS FOR FURTHER STUDY

The speech encoding results of Chapter 6 were obtained with a binary version of the adaptive-predictive encoder. One of the results obtained by the use of a multi-level quantizer in the case of Gaussian Markov input was improved performance at equivalent output rates in terms of binary symbols per second. A study to determine if such a result is also true for speech sources might be undertaken.

An area which received little attention was the problem of adaptive adjustment of the quantizer itself. It is not clear, for example, how an adaptive quantizer is to be designed for the highly nonstationary prediction error waveform associated with the predictive encoding of voiced speech. A successful solution to this problem should lead to a large performance improvement.

In Chapter 7 real time pitch period extraction via the prediction error waveform resulting from the adaptive-predictive encoding of speech was demonstrated. Two areas of further research are possible:

Performance can be improved by multi-level quantization, for example.

Secondly, simple implementation, such as might be possible with analogue circuitry, could be investigated.

A final suggestion for further research, more conjectural in nature, concerns the possibility of speech analysis by means of iterative methods similar to those used in the thesis. For example, the coefficients

of the linear recursive digital filter used by Atal and Hanauer [21] for speech synthesis might be obtainable by a recursive algorithm suitable for real time implementation rather than the averaging and matrix inversion technique now used, and which requires a digital computer.

A P P E N D I X A

GAUSSIAN-MARKOV SEQUENCE WITH SPECIFIED AUTOCORRELATION

For simulation purposes it is desired to generate a sequence with Gaussian statistics with autocorrelation function identical to that produced by samples of the output of a continuous linear system excited by white Gaussian noise. In particular the coefficients of the linear difference equation

$$x_n = -a_1 x_{n-1} - a_2 x_{n-2} + b_0 w_n + b_1 w_{n-1} \quad (A1)$$

where, $\{w_n\}$ is a sequence of independent Gaussian deviates, are to be found when the linear system is given by

$$H(s) = \frac{C}{(1 + \tau s)^2} \quad (A2)$$

A procedure introduced by Levin [23] will be used. Levin's procedure is quite general and is applicable to any $H(s)$ which is rational. Here it is used specifically to obtain the coefficients of Eqn. (A1) corresponding to Eqn. (A2) as follows. For convenience Eqn. (A2) is first written in terms of the parameter R

$$H(s) = \frac{\frac{C}{(RT)^2}}{\left(\frac{1}{RT} + s\right)^2} = \frac{C \alpha^2}{(s + \alpha)^2} \quad (A3)$$

where,

$$R = \frac{T}{T} = \frac{\text{time constant}}{\text{sampling period}}$$

$$\text{and, } \alpha = \frac{1}{RT}$$

When the system of Eqn. (A3) is excited by white noise of unit variance the power spectrum of the output is given by the bilateral transform

$$\Phi_2(s) = H(s)H(-s) = \frac{C^2 \alpha^4}{(\alpha^2 - s^2)^2} \quad (\text{A4})$$

This corresponds to the autocorrelation function

$$\Phi_2(t) = \frac{C^2 \alpha}{4} e^{-\alpha|t|} (1 + \alpha|t|) \quad (\text{A5})$$

At the sampling times, $t = |n|T$, this becomes

$$\Phi_2(n) = \frac{C^2}{4RT} e^{-\frac{|n|}{R}} \left[1 + \frac{|n|}{R} \right] \quad (\text{A6})$$

In what follows we assume $C^2 = 4RT$, i.e., $\Phi_2(0) = 1$. Now note that, at the sampling instants,

$$e^{-\alpha t} \leftrightarrow \frac{z}{z-A} \quad (\text{A7})$$

$$\alpha t e^{-\alpha t} \leftrightarrow \frac{\alpha A T z}{(z-A)^2}$$

where, $A = e^{-\alpha T} = e^{-1/R}$

If we define

$$\Phi_1(n) = e^{-\frac{n}{R}} \left[1 + \frac{n}{R} \right] \quad (\text{A8})$$

Then, from Eqn. (A7),

$$\Phi_1(z) = \frac{z}{z-A} + \frac{\alpha T z}{(z-A)^2} = \frac{z^2 + Bz}{(z-A)^2} \quad (\text{A9})$$

where, $B = A(\alpha T - 1) = e^{-1/R} (1/R - 1)$

Then,

$$\Phi_2(z) = \Phi_1(z) + \Phi_1\left(\frac{1}{z}\right) - 1 = H(z)H\left(\frac{1}{z}\right) \quad (\text{A10})$$

Substituting Eqn. (A9) into Eqn. (A10) allows $H(z)$ to be found. As noted by Levin the poles of $\Phi_2(z)$ inside the unit circle are associated with $H(z)$, while those outside are associated with $H(1/z)$ because $H(z)$ is required to be stable. The association of zeros is arbitrary and leads to alternative forms for $H(z)$. Evaluating Eqn. (A10)

$$H(z)H\left(\frac{1}{z}\right) = \frac{z^2 + Bz}{(z-A)^2} + \frac{z^{-2} + Bz^{-1}}{(z^{-1} - A)^2} - 1 \quad (\text{A11})$$

After some algebra it will be found that this can be put into the form

$$H(z)H\left(\frac{1}{z}\right) = \frac{D + Ez + Ez^{-1}}{(z-A)^2 (z^{-1}-A)^2} \quad (\text{A12})$$

where, $D = 1 - 4AB - 4A^2 - A^4$

and, $E = A^2B + B + 2A^3$

The numerator may be factorized by solving for F and G , the equality

$$D + E + Ez^{-1} = (F+z)(F+z^{-1})G^2 \quad (\text{A13})$$

This involves solution of the quadratic equation

$$F^2 - \frac{D}{E} F + 1 = 0 \quad (A14)$$

Then,

$$G^2 = \frac{E}{F} \quad (A15)$$

Levin also gives a method whereby initial values x_0 and x_1 may be generated such that the process appears stationary from the start. This would be useful in systems with long time constants where it would be impractical to let the source run until transients died away. The details of these calculations will not be given here. Implementation of the above results, by solution with a digital computer, for the specific example $R = 2.5$ yields the difference equation

$$x_n = 1.341 x_{n-1} - 0.449 x_{n-2} + 0.272 w_n + 0.072 w_{n-1} \quad (A16)$$

with the initial values generated by

$$x_0 = 0.272 v_1 + 0.962 v_2$$

$$x_1 = 0.272 w_0 + 0.436 v_1 + 0.852 v_2 + 0.099 v_3$$

where, $v_1, v_2, v_3 \sim \text{NID}(0,1)$.

Simulation of Eqn. (A16) over 100,000 samples produced the measured autocorrelation values shown in Table A1. These are in close agreement with values computed from Eqn. (A6).

	MEASURED VALUE	THEORETICAL VALUE
K	$\overline{X_n X_{n+K}}$	$R_{xx}(K)$
0	0.995	1.000
1	0.933	0.938
2	0.806	0.809
3	0.665	0.663
4	0.527	0.525
5	0.409	0.406
6	0.312	0.308
7	0.236	0.231
8	0.176	0.171
9	0.131	0.126

TABLE A1 COMPARISON OF MEASURED AND THEORETICAL CORRELATION FUNCTION
FOR R = 2.5

A P P E N D I X B

MEAN-SQUARED ERROR CALCULATION

Here the detailed computations for mean-squared prediction error according to the method presented in Section 2.5 is given. The starting point is Eqn. (2.95), which is reproduced below after the transformation $z = \frac{1+w}{1-w}$.

$$\begin{aligned} \varepsilon(z_n^2) &= \frac{1}{\pi j} \int_{-j\infty}^{\infty} H(w)H(-w) \phi_{xx}(w) \frac{dw}{1-w^2} + \\ &+ \frac{1}{\pi j} \int_{-j\infty}^{j\infty} G(w)G(-w) \phi_{\varepsilon\varepsilon}(w) \frac{dw}{1-w^2} \end{aligned} \quad (B1)$$

As is noted in Section 2.5, we assume $\phi_{\varepsilon\varepsilon}(w) = \sigma_\varepsilon^2$. For the simple Gaussian-Markov source, Eqn. (2.84)

$$\phi_{xx}(z) = \frac{1}{(1 - \alpha z^{-1})(1 - \alpha z)} \quad (B2)$$

and

$$\phi_{xx}(w) = \frac{1 - w^2}{[1 - \alpha + w(1+\alpha)][1 - \alpha - w(1+\alpha)]} \quad (B3)$$

We now consider the three particular systems of Fig. 2.11 for the special case of one coefficient. For the Type I system,

$$H(z) = \frac{1 - cz^{-1}}{1 - bz^{-1}(1-q)} \quad (B4)$$

and,

$$-G(z) = \frac{c z^{-1}}{1 - c z^{-1}(1-q)} \quad (B5)$$

making the transformation $z = \frac{1+w}{1-w}$ yields

$$H(w) = \frac{(1-c) + w(1+c)}{1 - c(1-q) + w[1 + c(1-q)]} \quad (B6)$$

and,

$$-G(w) = \frac{c-cw}{1 - c(1-q) + w[1 + c(1-q)]} \quad (B7)$$

In order that tabulated values may be used, Eqn. (B1) is written in the form

$$\epsilon(z_n^2) = 2I_1 + 2I_2 \quad (B8)$$

where, I_1 and I_2 are of the form

$$I = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \frac{p(w)p(-w)}{q(w)q(-w)} dw \quad (B9)$$

where, $p(w)$, $q(w)$ are polynomials in w . That is,

$$p(w) = p_0 + p_1 w + \dots + p_{n-1} w^{n-1} \quad (B10)$$

$$q(w) = q_0 + q_1 w + \dots + q_n w^n \quad (B11)$$

From Eqns. (B3) and (B6) we obtain

$$\frac{p(w)}{q(w)} = \frac{1}{1-\alpha + w(1+\alpha)} \cdot \frac{1-c + w(1+c)}{1-c(1-q) + w[1+c(1-q)]} \quad (B12)$$

After some algebra the following polynomial coefficients are obtained.

$$\begin{aligned} p_0 &= 1-c, & p_1 &= 1+c \\ q_0 &= (1-\alpha)[1-c(1-q)] \\ q_1 &= 2-2\alpha c(1-q) \\ q_2 &= (1+\alpha)[1+c(1-q)] \end{aligned} \quad (B13)$$

From Newton, Gould and Kaiser [34],

$$I_1 = \frac{p_1^2 q_0 + p_0^2 q_2}{2q_0 q_1 q_2} \quad (B14)$$

After substitution of the polynomial coefficients, Eqn. (B13) and simplification,

$$I_1 = \frac{1+\alpha c(1-q) - 2c\alpha - 2c^2(1-q) + c^2 + c^3\alpha(1-q)}{2(1-\alpha^2)[1-c^2(1-q)^2][1-\alpha c(1-q)]} \quad (B15)$$

From Eqn. (B7) the polynomial coefficients for the evaluation of I_2 are

$$\begin{aligned} p_0 &= c \sigma_E \\ q_0 &= 1-c(1-q) \\ q_1 &= 1+c(1-q) \end{aligned} \quad (B16)$$

From Newton, Gould and Kaiser [34]

$$I_2 = \frac{p_o^2}{2q_o q_1} \quad (B17)$$

Hence,

$$I_2 = \frac{c^2 \sigma_\varepsilon^2}{2[1-c^2(1-q)^2]} \quad (B18)$$

After substitution in Eqn. (B8) the mean-squared error, Eqn. (2.98) is obtained.

The computations for the other systems of Fig. 2.11 are similar, and will not be repeated here.

A P P E N D I X C

BOUNDS ON THE PERFORMANCE OF SOURCE ENCODERS

C.1 RATE-DISTORTION THEORY

The fundamental limits on the performance of source encoders derive, in the last analysis from information theory. Whatever other properties we require of a source encoder, it must process the information generated by the source into a form acceptable for transmission over the channel, and it must do this in such a way that the information can be reproduced with degradation acceptable to the user. This rather vague statement is made precise by rate-distortion theory, first introduced by Shannon [1]. For a comprehensive summary of rate-distortion theory as it relates to source encoding and data compression, see Berger [40].

Although no direct use is made of rate-distortion theory in deriving the adaptive source encoders in this thesis, its introduction in this section was felt necessary for two reasons. It provides the background necessary for a fundamental understanding of the source encoding problem, and by providing theoretical performance bounds it indicates the degree of improvement which is possible over that presently achieved by current systems. To this end the rate-distortion function will be defined, followed by a discussion of its evaluation for some simple sources. This section concludes with an account of the bounding of a subjective performance measure for speech in a recent study by O'Neill [2].

A model of a general digital communication system appropriate to the source encoding problem is shown in Fig. C.1. Here the channel is assumed to encompass such operations as error correction coding, modulation, demodulation, as well as the physical channel. The source produces a sequence of sample values denoted by the vector

$$\vec{X}_n = (x_1, x_2, \dots, x_n)^T \quad (C.1)$$

while the source decoder presents a sequence of values to the user denoted by the vector

$$\vec{Y}_n = (y_1, y_2, \dots, y_n)^T \quad (C.2)$$

Although the samples can come from a discrete alphabet they will be assumed continuous valued in keeping with the interest in this thesis in the encoding of sampled analogue signals such as speech.

A meaningful measure of distortion must now be defined. For reasons of mathematical tractability the distortion measure is usually chosen to be a metric defined on the product space of $(\vec{X}_n, \vec{Y}_n)$, and in particular the mean-squared error measure

$$D(\vec{X}_n, \vec{Y}_n) = \frac{1}{n} \epsilon \{ (\vec{X}_n - \vec{Y}_n)^T (\vec{X}_n - \vec{Y}_n) \} \quad (C.3)$$

is often used. Even this definition of distortion is intractable in most applications so that a single letter error measure is defined so

FIGURE C.1 MODEL OF DIGITAL COMMUNICATION SYSTEM

FIGURE C.2 ILLUSTRATION OF THE PARAMETRIC EXPRESSION FOR $R(D)$

FIGURE C.3 FREQUENCY WEIGHTING FILTER FOR SUBJECTIVELY DETERMINED NOISE POWER MEASUREMENT

that

$$D(\vec{X}_n, \vec{Y}_n) = \frac{1}{n} \sum_{i=1}^n \epsilon \{ (x_i - y_i)^2 \} \quad (C.4)$$

where, x_i and y_i are the individual samples comprising $\vec{X}_n$ and $\vec{Y}_n$. In this case the distortion measure is the arithmetic mean of the average distortions on each sample. Virtually all the results of rate-distortion theory are obtained by the use of Eqn. (C.4), which is known as a single letter fidelity criterion.

The rate-distortion function may be defined using the ideas of information theory. In simplest terms the rate-distortion function $R(D)$ specifies the minimum information rate necessary to reproduce $\vec{X}_n$ with an average distortion $D(\vec{X}_n, \vec{Y}_n) \leq D$. Much of the research in rate-distortion theory to date is concerned with the computation of $R(D)$ for various sources, and in establishing the existence of source codes which can achieve the rate $R(D)$.

Consider all possible mappings (source codes) which map each possible source output $\vec{X}_n$ onto some reconstructed vector $\vec{Y}_n$. With each such mapping a probability $P(\vec{X}_n, \vec{Y}_n)$ may be associated, and a mutual information $I(\vec{X}_n; \vec{Y}_n)$ may be defined. The condition $D(\vec{X}_n, \vec{Y}_n) \leq D$ forces consideration of only a subset Q of all possible mappings where

$$Q = \{P(\vec{X}_n, \vec{Y}_n): D(\vec{X}_n, \vec{Y}_n) \leq D\} \quad (C.5)$$

Since with each $P(\vec{X}_n, \vec{Y}_n)$ is associated a mutual information $I(\vec{X}_n; \vec{Y}_n)$ the rate-distortion function may be defined by

$$R(D) \triangleq \lim_{n \rightarrow \infty} \inf_{P \in Q} I(\vec{X}_n; \vec{Y}_n) \quad (C.6)$$

While Eqn. (C.6) defines $R(D)$ in general terms, it is not too useful in applications because of the difficulty of specifying the members of the class Q . Most of the useful results for the computation of $R(D)$ are derived from consideration of simple sources, in particular sequences of independent Gaussian random variables and sources obtained by linear operations on such sequences. In all of these derivations the squared-error criterion is used. In what follows some of the major results of rate-distortion theory will be stated.

A well known result, due to Shannon [1], concerns the rate distortion function for a sequence of independent Gaussian random variables of arbitrary mean and variance σ^2 ,

$$R(D) = \begin{cases} \frac{1}{2} \log \frac{\sigma^2}{D}, & 0 \leq D \leq \sigma^2 \\ 0, & D \geq \sigma^2 \end{cases} \quad (C.7)$$

From this basic result expressions for rate-distortion for a general Gaussian random process may be derived. It is well known that the Karhunen-Loeve expansion procedure produces uncorrelated, and hence independent coefficients if the process is Gaussian. The independent

coefficients may then be encoded separately at the rate

$$R(D) = \begin{cases} \frac{1}{2} \log \frac{\lambda_i}{D_i} & , \quad D_i \leq \lambda_i \\ 0 & , \quad D_i \geq \lambda_i \end{cases} \quad (C.8)$$

where, D_i is the mean-square error allowed on the i^{th} coefficient
and, λ_i is the variance of the i^{th} coefficient.

The total rate for the process is given by

$$R(D) = \frac{1}{2T} \sum_{i=1}^{\infty} \log \frac{\lambda_i}{D_i} , \quad D_i \leq \lambda_i \quad (C.9)$$

and is to be minimized subject to the total distortion constraint

$$D = \frac{1}{T} \sum_{i=1}^{\infty} D_i , \quad D_i \leq \lambda_i \quad (C.10)$$

These results may be expressed parametrically in θ . Let $N(\theta)$ be the number of coefficients with $D_i \leq \lambda_i$. Then

$$\left. \begin{aligned} D(\theta) &= \frac{1}{T} \left[\theta \cdot N(\theta) + \sum_{\substack{i=1 \\ \lambda_i \leq D_i}}^{\infty} \lambda_i \right] \\ R(\theta) &= \frac{1}{2T} \left[\sum_{\substack{i=1 \\ \lambda_i > D_i}}^{\infty} \log \left(\frac{\lambda_i}{\theta} \right) \right] \end{aligned} \right\} \quad (C.11)$$

The smallest rate is obtained by setting $D_i = \theta$ for every $\lambda_i > \theta$ and $D_i = \lambda_i$ if $\lambda_i \leq \theta$. This procedure implies that only those components with variance larger than θ are encoded and transmitted, while those with smaller variance are ignored. Thus the two component terms in the distortion expression; the first due to error in reconstruction of those $\lambda_i > \theta$, and the second due to the fact that those $\lambda_i \leq \theta$ must be estimated by zero since no information about them is transmitted.

For non-Gaussian processes, Eqn. (C.11) may not be used. One reason is that the independence assumption necessary for separate encoding of each λ_i may no longer be made. In general separate encoding of dependent coefficients will lead to a rate higher than the theoretically possible minimum. Furthermore each coefficient is now not necessarily a Gaussian random variable and may be encoded at a rate lower than that given by Eqn. (C.8), (see for example Berger [40])

Of course the Karhunen-Loeve expansion is but one of many ways to decompose sample functions of a random process into uncorrelated components. A decomposition in frequency leads to similar parametric expressions

$$D(\theta) = \frac{1}{2\pi} \int_{\alpha} \theta \, d\lambda + \frac{1}{2\pi} \int_{\beta} f(\lambda) \, d\lambda$$

$$R(\theta) = \frac{1}{4\pi} \int_{\alpha} \log \left[\frac{f(\lambda)}{\theta} \right] \, d\lambda$$
(C.12)

where, $f(\lambda)$ is the power spectrum of the source,

$$\alpha = \{\lambda: f(\lambda) \geq 0\},$$

$$\text{and, } \beta = \{\lambda: f(\lambda) < 0\}.$$

Compared with the source mappings required in the definition, Eqn. (C.6) the power spectral density is relatively easy to obtain. The relations between the source samples x_k , the sampled autocorrelation function $r(k)$, and the power spectral density are given below

$$\begin{aligned} r(k) &= \varepsilon(x_n x_{n+k}) = \int_{-\pi}^{\pi} f(\lambda) e^{jk\lambda} d\lambda \\ f(\lambda) &= \sum_{k=-\infty}^{\infty} r(k) e^{-jk\lambda}, \quad -\pi \leq \lambda \leq \pi \end{aligned} \tag{C.13}$$

For a given value of the parameter Θ the source encoder attempts to transmit only those components of the power spectrum defined by the set α . The rest of the spectrum is not transmitted, and is estimated as zero by the receiver. The situation is illustrated by Fig. C.2.

It is well to emphasize the domain of application of Eqn. (C.12). Strictly speaking it applies to stationary, Gaussian processes, under a mean-squared error distortion measure. Although Eqn. (C.12) assumes continuous amplitude processes sampled in time, similar expressions are available for discrete valued samples and continuous time processes [40]. Extension of the distortion measure has been made to frequency weighted mean-squared error by Sakrison [42]. Such an extension is useful when it is desired to assign more weight to certain

regions of the spectrum, for example in accordance with subjective quality criteria. For nonstationary sources a major obstacle is a suitable definition of power spectral density. For slow time variation one might form short term spectral estimates. Apart from such approximations and some results by Berger [40] concerning Wiener processes, computation of $R(D)$ for nonstationary sources remain largely an unsolved problem.

C.2 RATE-DISTORTION FUNCTION FOR AUTOREGRESSIVE SOURCES

A source of considerable interest, both theoretical and practical is the time-discrete Gaussian autoregressive process generated by the recursive equation

$$x_n = - \sum_{i=1}^m a_i x_{n-i} + w_n \quad (C.14)$$

where, $\{w_n\}$ is a sequence of uncorrelated Gaussian random variables with variance σ^2 . This source is commonly known as a Gaussian-Markov process of order m . Most of the results concerning rate-distortion functions of autoregressive sources are due to Bunin [43], and Gray [44]. The parametric equations (C.12), of course, may be used to compute $R(D)$. This necessitates integration which cannot be expressed in closed form. Here we state some of the properties of $R(D)$ for autoregressive processes which are derived in Ref. [43] and [44].

$$\begin{aligned} R_x(D) &= R_w(D), & 0 \leq D \leq D_c \\ R_x(D) &> R_w(D), & D > D_c \end{aligned} \quad (C.15)$$

The first property, which is the somewhat surprising result, states that for low enough distortion, that is a high enough rate, the rate for the process $\{x_n\}$ is the same as that of the excitation $\{w_n\}$. This property is a result of the fact that for Θ small enough in Fig. C.2 $f(\lambda) > \Theta$ over the whole range $[-\pi, \pi]$.

The conclusions, Eqn. (C.15), are fundamental to the source encoding of Gaussian-Markov processes. For example in the optimum predictive systems of Fig. 2.10 in Section 2.4, one may conceive of transmitting the $\{x_n\}$ sequence by encoding $\{e_n\}$, the sequence of errors, and reconstructing $\{x_n\}$ from the encoded error sequence. This can reasonably be accomplished only if the error sequence is encoded with low distortion. In practice reconstruction according to some optimum filter will be degraded because of the cumulative nature of the encoding errors on the $\{e_n\}$ sequence. Insertion of a quantizer within the loop as in the systems of Fig. 2.11 avoids the problem of error accumulation in the decoder but the prediction is no longer optimum. Consequently, the sequence $\{z_n\}$ of Fig. 2.11 is no longer the original Gaussian sequence which drives the source generator.

As an example consider the first order Gaussian-Markov source

$$x_n = \alpha x_{n-1} + w_n \quad (C.16)$$

where, $w_n \sim \text{NID}(0, \sigma^2)$

Here we have the expression, derived from Eqns. (C.12) and (C.13),

$$R_x(D) = \frac{1}{2} \log \frac{\sigma^2}{D}, \quad 0 \leq D \leq \frac{\sigma^2}{(1+\alpha)^2} \quad (C.17)$$

$$R_x(D) > \frac{1}{2} \log \frac{\sigma^2}{D}, \quad D > \frac{\sigma^2}{(1+\alpha)^2}$$

The conclusion which may be drawn from Eqn. (C.17) is that the value of the parameter α specifies the range of permissible distortion at which $\{x_n\}$ may be encoded at the same rate as the sequence $\{w_n\}$. Now in binary predictive quantization, and under an optimal prediction assumption, the error is a sequence of independent Gaussian random variables. Binary quantization generates information at a rate of one bit per sample. Then for

$$D < \frac{\sigma^2}{(1+\alpha)^2} \quad (C.18)$$

It is only necessary to transmit at a rate given by

$$R_x(D) = \frac{1}{2} \log \frac{\sigma^2}{D} \quad (C.19)$$

Since the error sequence cannot be reconstructed from an optimally binary quantized output with a distortion less than $0.3634 \sigma^2$ it follows that Eqn. (C.18) is not satisfied unless

$$\alpha < 0.66 \quad (C.20)$$

Thus for larger values of α

$$R_{\mathbf{x}}(D) > \frac{1}{2} \log \frac{\sigma^2}{D} \quad (\text{C.21})$$

C.3 SOURCE CODING THEOREMS

In the preceeding some results on the computation of rate-distortion functions were given. Knowledge of the rate distortion function is of practical interest since it indicates the degree of improvement that can be expected for presently used systems of source encoding. As will be seen in the next section this improvement can be considerable, and further effort in designing more efficient encoders can be justified.

Analogous to the well known channel coding theorems [1], the existence of source codes which can achieve the performance indicated by the rate-distortion function can be rigorously proved for various simple sources. Reference [40] by Berger gives a good account of the source coding theorems which have been proved to date.

As with the channel coding theorems, the proofs of the source coding theorems are usually not constructive; that is, they do not give a procedure for obtaining implementatable coders. One possible exception is the coding theorem which proves the existence of tree coding procedures for Gaussian autoregressive sources [40]. In a sense the predictive encoding systems of Fig. 2.11 can be considered as implementations of tree codes. The output of such systems may be considered as words in an infinitely deep tree. The coder is instructed to perform a sequence of branching operations according to the symbols of the sequence. It is not the purpose of this thesis to carry this analogy further, however.

C.4 SUBJECTIVE PERFORMANCE MEASURES FOR SPEECH SIGNALS

Speech sources are nonstationary and non-Gaussian. As a consequence the rate-distortion results, Eqn. (C.12) is not directly applicable. Furthermore the mean-squared distortion measure is inadequate for measurements of subjective speech quality. One reason is that the subjective effect of noise depends on its frequency distribution. This deficiency may be solved to some extent by the use of a frequency weighted squared-error distortion measure, with the weighting derived from subjective performance tests. For this purpose a generalized version of Eqn. (C.12) due to McDonald and Schultheiss [45] may be used. This may be written

$$\begin{aligned}
 R(\theta) &= \frac{1}{4\pi} \int_{\alpha} \frac{f(\lambda)Y(\lambda)}{\theta} d\lambda \\
 D(\theta) &= \frac{1}{2\pi} \int_{\alpha} \theta d\lambda + \frac{1}{2\pi} \int_{\beta} f(\lambda)Y(\lambda) d\lambda
 \end{aligned}
 \tag{C.22}$$

where $\alpha = \{\lambda: f(\lambda)Y(\lambda) \geq \theta\}$,

$\beta = \{\lambda: f(\lambda)Y(\lambda) < \theta\}$,

and, $Y(\lambda)$ is the power transfer function of the weighting filter for the noise.

There is a more fundamental deficiency of the mean-squared error distortion measure as a suitable fidelity criterion for speech. The reason for this is the nature of the hearing process. The correspondence between waveforms and subjective effects is not one-to-one. For example, the subjective effect of a signal composed of two sinusoids is similar regardless of their phase relationship. If a transmission system alters the phase, often disastrous effects occur in the mean

squared error, but as noted the subjective effect is similar to the original. This simple example leads one to the conjecture that mean-squared error is too strong as a subjective measure of distortion. While a small mean-squared error implies high subjective quality, the converse is not always true. One additional objection to the mean-squared error criterion is its unsuitability for nonstationary sources. For such sources an error criterion that varied with time might be more appropriate. For example during a large amplitude transient a large error might be acceptable, but during transitions between speech sounds it is important for the sake of intelligibility that accurate reproduction is obtained. Unfortunately, the mean-squared error during the transient will swamp all other errors thus making it appear that a much higher transmission rate is required.

Since no mathematically tractable definition for a subjective distortion measure is yet known, one must rely on the frequency weighted criterion as the best available. In what follows some results obtained by O'Neill [2] on bounds for the source encoding of speech are presented. Derivation of these bounds relies on a subjectively determined frequency weighting filter which takes into account the subjective effect of frequency distribution of the encoder-decoder reconstruction error. One might consider obtaining the frequency weighted noise power by an arrangement such as that illustrated in Fig. C.3.

Disregarding, for the moment, the noise weighting it has been shown by Shannon [1] that the rate required to transmit a bandlimited (ω - ω) Hz source of entropy power Q at a mean squared distortion D is given by

$$R \geq w \log_2 \frac{Q}{D} \quad (C.23)$$

Entropy power is the term introduced by Shannon to define the variance of a Gaussian source whose entropy equals the source in question. That is

$$Q = \frac{1}{2\pi e} \exp [2H] \quad (C.24)$$

where,

$$H = -\epsilon \left\{ \log \left| \frac{1}{p(x)} \right| \right\} \quad (C.25)$$

The expression, Eqn. (C.23), may be used as a bound on the performance of source encoders for bandlimited speech provided some means of obtaining the entropy power can be found. A practical method of doing this based upon linear prediction is suggested by O'Neill [2]. Consider the predictor

$$\hat{x}_n = \sum_{i=1}^m a_i x_{n-i} \quad (C.26)$$

operating on a Gaussian-Markov sequence of order m . It is well known that the error sequence $\{e_n\}$ will be a sequence of Gaussian independent random variables with entropy power equal to that of the source sequence $\{x_n\}$. Thus if σ_m^2 is the minimum mean-squared prediction error

$$Q = \sigma_m^2 \quad (C.27)$$

This situation will be approximately true for linear estimation of non-Gaussian sources such as speech. In any case the reduction in the signal variance between the source sequence $\{x_n\}$ and the error sequence $\{e_n\}$ can serve as a conservative bound which could be improved at some later date. From Eqn. (C.23)

$$R \geq w \log_2 \frac{\sigma^2}{D} \quad (C.28)$$

If the variance of $\{x_n\}$ is σ^2 , the signal-to-noise ratio may be expressed by

$$\frac{S}{N} \leq 10 \left(\frac{R}{W} \right) \log_{10}^2 + 10 \log_{10} \left(\frac{\sigma^2}{\sigma_m^2} \right) \quad (C.30)$$

It is shown by O'Neill [2], that the inclusion of a frequency weighted noise power distortion criterion results in a third term in the expression Eqn. (C.30).

$$\begin{aligned} \frac{S}{N} \leq 10 \left(\frac{R}{W} \right) \log_{10}^2 + 10 \log_{10} \left(\frac{\sigma^2}{\sigma_m^2} \right) - \\ - \frac{1}{W} \int_0^W 10 \log_{10} Y(f) df \end{aligned} \quad (C.31)$$

C.5 A SOURCE ENCODING BOUND FOR SPEECH

For speech sources the last two terms of Eqn. (C.31) are not amenable to analytical determination. Studies on the linear prediction of speech signals by Atal and Shroeder [20] suggest that the variance of

speech signals bandlimited to 3 kHz may be reduced by at least 21 dB. This result may, of course, be modified at some later date by data from some more sophisticated empirical study.

The subjective effects of noise weighting filters for speech are not well known. O'Neill assumes that the C-message weighting filter commonly used in telephone systems may be reasonably close to the optimum. The value of the third term in the expression is then 2.6 dB and adds positively to the right hand side of the expression.

Thus for speech bandlimited to 3 kHz an empirically derived bound for source encoder performance may be written

$$\frac{S}{N} \leq \left(\frac{R}{1000} + 23.6 \right) \text{ dB} \quad (\text{C.32})$$

provided R is larger than some minimum value R_c analogous to the limits on Eqn. (C.15). For speech, however, R_c is not too well defined. O'Neill conjectures that R_c is somewhere below 10 kilobits/sec.

A comparison with current practice, as given by the curves of logarithmically companded PCM and adaptive delta modulation in Fig. 2.12, demonstrates that currently used speech source encoders fall far short of the above bound.

A P P E N D I X D

D.1 MEAN-SQUARED ERROR COMPUTATION

To evaluate Eqn. (5.15) it is first necessary to compute the following

$$\epsilon\{z_n E_1\} = \sum_{i=1}^m \epsilon\{z_n e_i(n) y_{n-i}\} \quad (D.1)$$

$$\epsilon\{z_n E_2\} = \sum_{i=1}^m \epsilon\{z_n a_i(n) d_{n-i}\} \quad (D.2)$$

$$\epsilon\{z_n E_3\} = \sum_{i=1}^m \epsilon\{z_n e_i(n) d_{n-i}\} \quad (D.3)$$

$$\epsilon\{E_1 E_2\} = \sum_{i=1}^m \sum_{j=1}^m \epsilon\{e_i(n) a_j(n) y_{n-i} d_{n-j}\} \quad (D.4)$$

$$\epsilon\{E_1 E_3\} = \sum_{i=1}^m \sum_{j=1}^m \epsilon\{e_i(n) e_j(n) y_{n-i} d_{n-j}\} \quad (D.5)$$

$$\epsilon\{E_2 E_3\} = \sum_{i=1}^m \sum_{j=1}^m \epsilon\{a_i(n) e_j(n) d_{n-i} d_{n-j}\} \quad (D.6)$$

Most of these terms will be shown either zero or negligible compared to the following

$$\epsilon\{E_1^2\} = \sum_{i=1}^m \sum_{j=1}^m \epsilon\{e_i(n) e_j(n) y_{n-i} y_{n-j}\} \quad (D.7)$$

$$\epsilon\{E_2^2\} = \sum_{i=1}^m \sum_{j=1}^m \epsilon\{a_i(n) a_j(n) d_{n-i} d_{n-j}\} \quad (D.8)$$

$$\varepsilon\{E_3^2\} = \sum_{i=1}^m \sum_{j=1}^m \varepsilon\{e_i(n)e_j(n)d_{n-i}d_{n-j}\} \quad (D.9)$$

Before proceeding to evaluate the above expression it will be useful to state some preliminary results and conclusions. Since $\{d_n\}$ is a sequence of independent random variables of zero mean any expected value of the product of an odd number of terms will be zero. For example,

$$\varepsilon\{d_i d_j d_k\} = 0 \quad (D.10)$$

If there are an even number the inequality below is true only for the conditions shown.

$$\varepsilon\{d_i d_j d_k d_l\} \neq 0$$

only if:	1. $i = j = k = l$	
or	2. $i = j, \quad k = l$	
or	3. $i = k, \quad j = l$	
or	4. $i = l, \quad j = k$	(D.11)

Similar conditions are also assumed for the data sequence $\{y_n\}$ as a consequence of the uncorrelated data assumption of optimum predictive quantization for many input sources such as, for example, Gaussian-Markov sources. This will be only approximately true for small values of α , however, as shown in Chapter 5.

The error process may be expanded in terms of past values of $\{\theta_1(n)\}$ as follows.

$$e_i(n) = \alpha^n e_i(o) + \theta_i(n-1) + \alpha \theta_i(n-2) + \dots + \alpha^{n-1} \theta_i(o) \quad (D.12)$$

For large n and $0 < \alpha < 1$ the first term is negligible. Each $\theta_i(s)$ above is given by

$$\theta_i(s) = \gamma(y_s y_{s-i} + d_s y_{s-i} + d_s d_{s-i}) \quad (D.13)$$

The linear functional relation between $e_i(n)$ and $\theta_i(s)$, for $s = n-1, n-2, \dots$ will be used in many of the evaluations below.

In Eqn. (D.1) let x be any random variable independent of the channel errors. Then

$$\epsilon\{x \theta_i(s)\} = \gamma \epsilon\{x y_s d_{s-i}\} + \gamma \epsilon\{x d_s d_{s-i}\} + \gamma \epsilon\{x d_s d_{s-i}\} = 0 \quad (D.14)$$

where, $s = n-1, n-2, \dots$

$i = 1, 2, \dots, m$

From Eqn. (D.12), $\epsilon\{x e_i(n)\} = 0$, and it follows that

$$\epsilon\{z_n E_1\} = 0 \quad (D.15)$$

In Eqn. (D.2) both z_n and $a_i(n)$ are independent of the channel errors so that

$$\epsilon\{z_n E_2\} = 0 \quad (D.16)$$

A somewhat different approach demonstrates that Eqn. (D.3) is small. Consider

$$\begin{aligned} \varepsilon\{\theta_i(s) z_{n \ n-i}^d\} &= \gamma \varepsilon\{z_{n \ n-i}^d y_{s \ s-i}^d\} + \gamma \varepsilon\{z_{n \ n-i}^d d_{s \ s-i}^d y_{s \ s-i}^d\} + \\ &+ \gamma \varepsilon\{z_{n \ n-i}^d d_{s \ s-i}^d d_{s \ s-i}^d\} \end{aligned} \quad (D.17)$$

This is zero by Eqns. (D.10), (D.11), and the following argument which deals with the specific situation $(n-i) = s$ in the second term on the right hand side. By the orthogonality property of optimum linear prediction the prediction error z_n is orthogonal to all linear functionals of the data. It is therefore reasonable to assume that $\varepsilon\{z_n y_{s-i}^d\}$ is at least small, if not zero. Thus, Eqn. (D.17) is zero, or negligible, so that

$$\varepsilon\{z_{n \ i}^d e_i(n) d_{n-i}^d\} \approx 0 \quad (D.18)$$

and Eqn. (D.3) is therefore small.

To evaluate Eqn. (D.4) consider the expansion

$$\begin{aligned} a_j(n) &= \alpha^n a_j(o) + \gamma[y_{n-1} y_{n-1-j} + \alpha y_{n-2} y_{n-2-j} + \dots \\ &+ \alpha^{n-1} y_0 y_1] \end{aligned} \quad (D.19)$$

Then consider the term

$$\begin{aligned}
\varepsilon\{\theta_i(s) \gamma \alpha^{n-r-1} y_r y_{r-j} y_{n-i} d_{n-j}\} &= \gamma^2 \alpha^{n-r-1} \varepsilon\{y_r y_{r-j} y_{n-i} y_s d_{n-j} d_{s-i}\} \\
&+ \gamma^2 \alpha^{n-r-1} \varepsilon\{y_r y_{r-j} y_{n-i} y_{s-i} d_s d_{n-j}\} + \gamma^2 \alpha^{n-r-1} \varepsilon\{y_r y_{r-j} y_{n-i} d_{n-j} d_s d_{s-i}\}
\end{aligned}
\tag{D.20}$$

where, $r = n-1, n-2, \dots, 0$

$i = 1, 2, \dots, m,$

and, $j = 1, 2, \dots, m.$

If, $s = n-j$, and $r = n-i$, term 2 in Eqn. (D.20) is given by

$$\text{Term 2} = 4\gamma^2 \alpha^{i-1} p \tag{D.21}$$

The other terms are zero so that from Eqn. (C.12),

$$\varepsilon(E_1 E_2) = 4\gamma^2 p \sum_{i=1}^m \sum_{j=1}^m \alpha^{i-1} \alpha^{j-1} \tag{D.22}$$

Equation (D.5) is evaluated by the following expansion

$$\begin{aligned}
\varepsilon\{\theta_i(r) \theta_j(s) y_{n-i} d_{n-j}\} &= \gamma^2 \varepsilon\{y_r d_{r-i} y_s d_{s-j} y_{n-i} d_{n-j}\} \\
&+ \gamma^2 \varepsilon\{y_r d_{r-i} d_s y_{s-j} y_{n-i} d_{n-j}\} \\
&+ \gamma^2 \varepsilon\{y_r d_{r-i} d_s d_{s-j} y_{n-i} d_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r y_{r-i} y_s d_{s-j} y_{n-i} d_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r y_{r-i} d_s y_{s-j} y_{n-i} d_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r y_{r-i} d_s d_{s-j} y_{n-i} d_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r d_{r-i} y_s d_{s-j} y_{n-i} d_{n-j}\}
\end{aligned}$$

$$\begin{aligned}
& + \gamma^2 \epsilon \{ d_r d_{r-i} d_s y_{s-j} y_{n-i} d_{n-j} \} \\
& + \gamma^2 \epsilon \{ d_r d_{r-i} d_s d_{s-j} y_{n-i} d_{n-j} \}
\end{aligned} \tag{D.23}$$

where, $r = n-1, n-2, \dots,$
 $s = n-1, n-2, \dots,$
 $i = 1, 2, \dots, m,$
and, $j = 1, 2, \dots, m.$

Terms, 3, 6, 7, and 8 contain an even number of d's and must be examined according to Eqn. (D.11). It is found that for $r = n-i$, $i=j$, and $s = n-i$

$$\text{Term 3} = 4\gamma^2 p^2 \tag{D.24}$$

Term 6 is zero. Term 7 is nonzero only if $r = n-j$ and $s = n-i$ for $i, n \leq m$. All the rest of the terms are zero. Consequently, from Eqn. (D.12),

$$\begin{aligned}
\epsilon(E_1 E_3) &= 4\gamma^2 p^2 \sum_{i=1}^m \alpha^{i-1} \alpha^{i-1} \\
&+ 4\gamma^2 p^2 \sum_{i=1}^m \sum_{j=1}^m \alpha^{i-1} \alpha^{j-1}
\end{aligned}$$

Ordinarily with practical values of γ, p, α , and m this term will be very small.

Equation (D.6) is evaluated by considering the following decomposition.

$$\begin{aligned}
\varepsilon\{\theta_j(s) a_i(n) d_{n-i} d_{n-j}\} &= \gamma \varepsilon\{y_s d_{s-j} a_i(n) d_{n-i} d_{n-j}\} \\
&+ \gamma \varepsilon\{d_s y_{s-j} a_i(n) d_{n-i} d_{n-j}\} \\
&+ \gamma \varepsilon\{d_s d_{s-j} a_i(n) d_{n-i} d_{n-j}\}
\end{aligned} \tag{D.26}$$

where, $s = n-1, n-2, \dots, 0,$

$i = 1, 2, \dots, m,$

and, $j = 1, 2, \dots, m.$

Clearly, the first two terms are zero, and the last is nonzero only if $s = n-j$, and $i = 2j$. Then.

$$\varepsilon\{\theta_j(s) a_i(n) d_{n-i} d_{n-j}\} = 4\gamma p^2 \varepsilon\{a_i(n)\} \tag{D.27}$$

Then from Eqn. (D.12), and for large n ,

$$\varepsilon\{E_2 E_3\} = \sum_{j=1}^m \sum_{\substack{i=1 \\ i=2j}}^m 4\gamma p^2 \alpha^{j-1} \varepsilon\{a_i(n)\} \tag{D.28}$$

For reasonable values of α , m , γ , and p , this will be small and may be neglected in a first order analysis.

The major contributions to the mean-squared prediction error increase come from Eqn. (D.7) through Eqn. (D.9). These contributions may be evaluated in the same way as those above. To evaluate Eqn. (D.7) consider the expression

$$\begin{aligned}
\varepsilon\{\theta_1(r) \theta_j(s) y_{n-i} y_{n-j}\} &= \gamma^2 \varepsilon\{y_r^d y_{r-i}^d y_s^d y_{s-j}^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{y_r^d y_{r-i}^d y_{s-j}^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{y_r^d y_{r-i}^d y_s^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r y_{r-i} y_s^d y_{s-j}^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r y_{r-i}^d y_{s-j}^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r y_{r-i}^d y_s^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r^d y_{r-i} y_s^d y_{s-j}^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r^d y_{r-i}^d y_{s-j}^d y_{n-i} y_{n-j}\} \\
&+ \gamma^2 \varepsilon\{d_r^d y_{r-i}^d y_s^d y_{n-i} y_{n-j}\}
\end{aligned} \tag{D.29}$$

where, $r = n-1, n-2, \dots,$
 $s = n-1, n-2, \dots,$
 $i = 1, 2, \dots, m,$
and, $j = 1, 2, \dots, m.$

By contrast to Eqn. (D.23) terms 3, 6, 7, and 8 contain an odd number of d's and are therefore zero. For $i \neq j$ it is found that only term 1 is nonzero and is given by

$$\text{Term 1} = \gamma^2 p \tag{D.30}$$

if $r = n-j$, and $s = n-i$.

The resulting contribution to $\varepsilon(E_1^2)$ is given by

$$\sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^m 4\gamma^2 p \alpha^{i-1} \alpha^{j-1} \leq 4m(m-1)\gamma^2 p \quad (D.31)$$

For practical values of α , m , γ , and p this is small. The major contribution occurs when $i = j$.

For $i = j$ only the first, fifth, and ninth terms in Eqn. (D.29) are nonzero. Their contribution to $\epsilon(E_1^2)$ is

$$\frac{8\gamma^2 p m}{1 - \alpha^2} + \frac{16\gamma^2 p^2 m}{1 - \alpha^2} \quad (D.32)$$

Then,

$$\epsilon\{E_1^2\} = \frac{8\gamma^2 p m}{1 - \alpha^2} + \frac{16\gamma^2 p^2 m}{1 - \alpha^2} + 4\gamma^2 p \sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^m \alpha^{i-1} \alpha^{j-1} \quad (D.33)$$

The last term of the above will be significant in relation to the first two for small values of α . This implies that for α close to unity coefficients may be assumed orthogonal to a good approximation.

The expression, Eqn. (D.8) is readily evaluated as follows

$$\begin{aligned} \epsilon\{E_2^2\} &= \sum_{i=1}^m \sum_{j=1}^m \epsilon\{a_i(n) a_j(n)\} \epsilon\{d_{n-i} d_{n-j}\} \\ &= \gamma p \sum_{i=1}^m \epsilon\{a_i^2(n)\} \end{aligned} \quad (D.34)$$

Finally, Eqn. (D.9) is evaluated by considering the expansion

$$\epsilon\{\theta_i(r) \theta_j(s) d_{n-i} d_{n-j}\} = \gamma^2 \epsilon\{y_r d_{r-i} y_s d_{s-j} d_{n-i} d_{n-j}\}$$

$$\begin{aligned}
& + \gamma^2 \varepsilon\{y_r d_{r-i} d_s y_{s-j} d_{n-i} d_{n-j}\} \\
& + \gamma^2 \varepsilon\{y_r d_{r-i} d_s d_{s-j} d_{n-i} d_{n-j}\} \\
& + \gamma^2 \varepsilon\{d_r y_{r-i} y_s d_{s-j} d_{n-i} d_{n-j}\} \\
& + \gamma^2 \varepsilon\{d_r y_{r-i} d_s y_{s-j} d_{n-i} d_{n-j}\} \\
& + \gamma^2 \varepsilon\{d_r y_{r-i} d_s d_{s-j} d_{n-i} d_{n-j}\} \\
& + \gamma^2 \varepsilon\{d_r d_{r-i} y_s d_{s-j} d_{n-i} d_{n-j}\} \\
& + \gamma^2 \varepsilon\{d_r d_{r-i} d_s y_{s-j} d_{n-i} d_{n-j}\} \\
& + \gamma^2 \varepsilon\{d_r d_{r-i} d_s d_{s-j} d_{n-i} d_{n-j}\}
\end{aligned} \tag{D.35}$$

where, $r = n-1, n-2, \dots,$

$s = n-1, n-2, \dots,$

$i = 1, 2, \dots, m,$

and, $j = 1, 2, \dots, m.$

For $i \neq j$ only term 5 and term 9 contribute to $\varepsilon\{E_3^2\}$. The contribution from term 9 is of order p^3 , and is neglected, while that from term 5 is

$$16\gamma^2 p^2 \sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^m \alpha^{i-1} \alpha^{j-1} \tag{D.36}$$

For $i = j$ it is found that terms 1, 5, and 9 are nonzero. After some algebra it is found that

$$\begin{aligned}
\varepsilon(E_3^2) &= \frac{32 \gamma^2 p^2}{1 - \alpha^2} + 64 \gamma^2 p^2 \left(\frac{1 - \alpha^m}{1 - \alpha} \right) \\
&+ 16 \gamma^2 p^2 \sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^m \alpha^{i-1} \alpha^{j-1}
\end{aligned} \tag{D.37}$$

The complete expression for mean-squared prediction error (Eqn. (5.15) is given by

$$\begin{aligned}
 \text{mse}(D) &= \text{mse}(E) + \epsilon(E_1^2) + \epsilon(E_2^2) + \epsilon(E_3^2) \\
 &+ 2\epsilon(E_1 E_2) + 2\epsilon(E_1 E_3) + 2\epsilon(E_2 E_3) \\
 &- 2\epsilon\{z_n(E_1 + E_2 + E_3)\} \quad (D.38)
 \end{aligned}$$

The last term in the above has been shown small so that Eqn. (D.38) is given by

$$\begin{aligned}
 \text{mse}(D) &= \text{mse}(E) + \frac{8 \gamma^2 p m}{1 - \alpha^2} + \frac{48 \gamma^2 p^2 m}{1 - \alpha^2} \\
 &+ 12 \gamma^2 p \sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^m \alpha^{i-1} \alpha^{j-1} \\
 &+ 4p \sum_{i=1}^m \epsilon\{a_i^2(n)\} \\
 &+ 8 \gamma^2 p(1+p) \left(\frac{1 - \alpha^{2m}}{1 - \alpha^2} \right) \\
 &+ 64 \gamma^2 p^2 \left(\frac{1 - \alpha^m}{1 - \alpha} \right) \\
 &+ 8 \gamma^2 p^2 \sum_{j=1}^m \sum_{\substack{i=1 \\ i=2j}}^m \alpha^{j-1} \epsilon\{a_i(n)\} \quad (D.39)
 \end{aligned}$$

For α close to unity and reasonably small channel error probability this reduces to the approximate expression of Chapter 5.

$$\text{mse}(D) = \text{mse}(E) + \frac{8 \gamma^2 p m}{1 - \alpha^2} + 4p \sum_{i=1}^m \epsilon\{a_i^2(n)\} \quad (\text{D.40})$$

D.2 PROOF THAT $\epsilon\{z_{nn} u_n\} \approx 0$

The predicted values $\tilde{x}_n$ and v_n may be expressed as

$$\tilde{x}_n = \beta^n \tilde{x}(o) + \sum_{i=1}^m a_i(n-1) y_{n-1-i} + \dots + \beta^{n-1} \sum_{i=1}^m a_i(o) y_{-i} \quad (\text{D.41})$$

$$v_n = \beta^n v(o) + \sum_{i=1}^m b_i(n-1) r_{n-1-i} + \dots + \beta^{n-1} \sum_{i=1}^m b_i(o) r_{-i} \quad (\text{D.42})$$

A typical term in the difference

$$u_n = v_n - \tilde{x}_n \quad (\text{D.43})$$

may be written

$$T(s) = \beta^{n-s-1} \sum_{i=1}^m [a_i(s) d_{s-i} + e_i(s) y_{s-i} + e_i(s) d_{s-i}] \quad (\text{D.44})$$

$$s = n-1, n-2, \dots,$$

By an extension of the proof for Eqns. (D.1), (D.2) and (D.3) it may be seen that

$$\varepsilon\{z_n T(s)\} \cong 0 \quad (D.45)$$

Consequently, $\varepsilon\{z_n u_n\} \cong 0$ as required.

A P P E N D I X E

DETAILS OF THE EXPERIMENTAL SOURCE ENCODER

For completeness the physical components which form the block structure shown in Chapter 6 will be described in more detail here. These particular designs were intended solely for the purpose of proving the adaptive-predictive concept of this thesis, and should not be construed as the simplest or most economical.

E.1 TWO's COMPLEMENT ARITHMETIC

The experimental system forms its prediction digitally from coefficient values stored in recirculating shift registers as eight-bit binary two's-complement numbers according to Table E.1. This number system has the advantage that addition requires no end-around carry. Subtraction is also simple, requiring only binary complementation and incrementing by one, which is readily accomplished by including a carry at the input of bit zero of the adder.

E.2 INTEGRATOR AND COMPARATOR

System operation is readily described by assuming the value

$$v_n = \sum_{i=1}^m a_i(n) y_{n-i} \quad (E.1)$$

is stored in the accumulator. The basic clock pulse, with duration T seconds and a repetition rate somewhere between 10 kHz and 40 kHz,

TABLE E.1
Two's Complement Arithmetic and Associated Voltage Levels

BINARY				DECIMAL	ASSOCIATED VOLTAGE
<u>Digital-to-Analogue Converter</u>					
011	111	111	111	2047	-9.99512
000	000	000	001	+1	-0.00488
000	000	000	000	0	0.00000
111	111	111	111	-1	+0.00488
100	000	000	000	-2048	10.00000
<u>Coefficient Values</u>					
01	111	111		127	-0.61976
01	111	110		126	-0.61488
01	111	01		125	-0.61000
01	111	00		124	-0.60512
00	000	001		1	-0.00488
00	000	000		0	0.00000
11	111	111		-1	+0.00488
10	000	011		-125	0.61000
10	000	010		-126	0.61488
10	000	001		-127	0.61976
10	000	000		-128	0.62464

enables the digital-to-analogue converter, producing a rectangular voltage pulse corresponding to the numerical value v_n . This voltage pulse may be integrated to form the predicted value $\tilde{x}_n$ at the sampling instant, defined by the end of the basic clock pulse, according to

$$\tilde{x}_n = \tilde{x}_{n-1} + \frac{T}{RC} v_n, \quad (E.2)$$

or the value v_n may be used as the prediction for type II system operation (see Fig. E.1). In the actual circuit $-\tilde{x}_n$ appears at the output of the first operational amplifier so that the comparator input at the sampling time is given by

$$z_n = x_n - \tilde{x}_n \quad (E.3)$$

The availability of the prediction error z_n is important to the study of pitch period extraction of Chapter 7. Finally the comparator produces logical levels "zero" or "one" depending upon the polarity of z_n .

For initialization of encoder-decoder coefficients a simple method of saturating all coefficients at the binary number -125 (0.61 volts) is used. For this purpose the comparator output is kept at a constant level by means of the Decoder Initialization Pulse shown in Fig. E.1. As soon as this pulse is removed both decoder and encoder taps adjust in exact synchronism.

FIGURE E.1 DIGITAL-TO-ANALOGUE CONVERTER, INTEGRATOR, AND COMPARATOR CIRCUITRY

FIGURE E.2 DATA FLIP-FLOP AND RECIRCULATING STORE

E.3 DATA STORAGE

The operation of the data storage and associated logic is demonstrated in Table E.2 which shows the contents of the storage flip-flops in Fig. E.2 prior to each of the seventeen CL2 pulses and after the seventeenth pulse. Note that the last line is the same as the first except that n replaces $n-1$, $n-1$ replaces $n-2$, etc. Consequently the signal at A in Fig. E.2 prior to the first eight CL2 pulses is given by

$$\begin{array}{rcl}
 y_n \oplus y_{n-1} & & \\
 y_n \oplus y_{n-8} & & \\
 \vdots & & \\
 y_n \oplus y_{n-2} & & (E.4)
 \end{array}$$

Also, the signal at B prior to the next eight CL2 pulses is given by

$$\begin{array}{rcl}
 y_n & & \\
 y_{n-7} & & \\
 \vdots & & \\
 y_{n-1} & & (E.5)
 \end{array}$$

The system operates first in a coefficient update mode during the first eight CL2 pulses, then in a prediction mode during the next eight CL2 pulses, and finally a seventeenth pulse prepares the data storage for the next cycle which will be initiated by the next basic clock pulse.

TABLE E.2
Contents of Data Store Prior to Each CL2 Pulse

PULSE NO.	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
1	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n-8	n-1
2	n	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n-8
3	n	n-7	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7
4	n	n-6	n-7	n	n-1	n-2	n-3	n-4	n-5	n-6
5	n	n-5	n-6	n-7	n	n-1	n-2	n-3	n-4	n-5
6	n	n-4	n-5	n-6	n-7	n	n-1	n-2	n-3	n-4
7	n	n-3	n-4	n-5	n-6	n-7	n	n-1	n-2	n-3
8	n	n-2	n-3	n-4	n-5	n-6	n-7	n	n-1	n-2
9	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n	n-1
10	n	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n
11	n	n-7	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7
12	n	n-6	n-7	n	n-1	n-2	n-3	n-4	n-5	n-6
13	n	n-5	n-6	n-7	n	n-1	n-2	n-3	n-4	n-5
14	n	n-4	n-5	n-6	n-7	n	n-1	n-2	n-3	n-4
15	n	n-3	n-4	n-5	n-6	n-7	n	n-1	n-2	n-3
16	n	n-2	n-3	n-4	n-5	n-6	n-7	n	n-1	n-2
17	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n	n-1
17+	n	n	n-1	n-2	n-3	n-4	n-5	n-6	n-7	n

* Table shows subscript associated with y.

E.4 COEFFICIENT ADAPTATION

The coefficient values are stored as binary two's complement numbers eight bits in length. Each bit requires one circuit such as that given by Fig. E.3 so that a total of eight such circuits make up the coefficient storage.

The rest of the system consists of a twelve bit full adder and a twelve bit accumulator consisting of eleven circuits given by Fig. E.4 and one circuit as given by Fig. E.5 which is different from the others because of the coefficient adaptation requirements. The accumulator is not used in the coefficient update mode.

Coefficient updating consists simply of adding +1 or -1, in two's complement form, to the coefficient $a_i(n)$ available at the output of the coefficient store prior to each CL3 pulse. The sum is then gated into the recirculating coefficient store in place of $a_i(n)$ when the CL3 pulse occurs. A little study of Figs. E.3, E.4 and E.5 as well as the clock waveforms will show that this, in fact, occurs.

E.5 PREDICTION

Prediction consists of forming the sum given by Eqn. (E.1), digital-to-analogue conversion, and integration of the system in type III. Here, formation of the sum is described.

Since the prediction is desired for the sample x_n occurring at the next basic clock pulse a change in subscripts in Eqn. (E.5) is necessary, so that y_n becomes y_{n-1} , y_{n-7} becomes y_{n-8} , etc.

- * A BANK OF EIGHT SUCH UNITS, EACH STORING ONE BIT OF THE COEFFICIENT VALUES, IS NECESSARY.
- * COEFFICIENTS ARE RECIRCULATED IF CS2 IS HIGH, OTHERWISE THEY ARE INCREMENTED BY +1 OR -1.

FIGURE E.3 COEFFICIENT STORE AND TRUE/COMPLEMENT GATE

FIGURE E.4 ACCUMULATOR AND FULL ADDER CIRCUIT FOR TWO THROUGH TWELVE BITS

FIGURE E.5 ACCUMULATOR AND FULL ADDER CIRCUIT FOR BIT ONE

* $Q_3, Q_4, \dots$ FROM COEFFICIENT STORAGE REGISTER OUTPUT.

FIGURE E.6 COEFFICIENT SATURATION CIRCUIT

Forming the sum is easy in a binary encoder since multiplication reduces to a sign operation on the coefficient values prior to addition. Thus if y_{n-i} is at a low logical level (indicating negative data) the negative of the coefficient value $a_i(n)$ is added to the contents of the accumulator. In two's complement arithmetic this requires inversion of the binary bits representing the coefficient value, and an input carry into the lowest order bit of the adder. This is accomplished by the signal B in Fig. E.3 and E.5. The signal D in Fig. E.3 represents either the value $a_i(n)$ or $-a_i(n)$ depending upon whether y_{n-i} is "high" or "low" respectively. An additional remark about the D input to adder bits 9, 10, 11 and 12 is necessary. Since the 12-bit two's complement number is the same as the equivalent 8-bit two's complement number in the eight lower order bits, while bits 9 through 12 are the same as bit number 8, it is only necessary to use the highest order bit of the coefficient store to generate the D input to bits 9 through 12 of the adder.

E.6 COEFFICIENT SATURATION

To avoid the disastrous effects which would occur from coefficients exceeding the maximum positive and negative limits set by the 8-bit coefficient representation, the incrementation must be suppressed at the limits -128 and +127. This is accomplished by the circuit shown in Fig. E.6. In the actual system the limits were set at 124 (-0.60512 volts) and -125 (0.6100 volts) merely to allow the use of eight input NAND gates in the circuit.

If for example a coefficient $a_i(n)$ was given by 10 000 011 (0.6100 volts) and the coefficient update logic called for the addition of 11 111 111 to this value, as indicated by $A = 1$. This would invert the logical level at both E and F thus causing $a_i(n)$ to be recirculated without change as indicated by Fig. E.3. The coefficient would, therefore, remain at this saturated level as long as $y_n \oplus y_{n-i}$ called for an increase in the voltage value.

As mentioned in Section E.2, saturation of both encoder and decoder coefficients is a convenient method of initially making both sets of coefficients equal.

E.7 CLOCK AND CONTROL CIRCUITRY

The clocking and control circuitry shown in Fig. E.7 is somewhat more complex than absolutely necessary since it was desired to allow changes in the basic clock rate without affecting the integration time and vice-versa. Without this flexibility it is likely that a much simpler circuit would suffice.

Basically the clock circuit consists of a multivibrator operating at a 2 MHz nominal rate. It is started when the four stage counter is cleared and stopped when the count reaches the binary number 1111. The multivibrator produces a total of sixteen periods from which the pulses CL3 are derived. For CL2 an extra seventeenth pulse is added, while for CL4 a monostable delay and gating produces eight pulses immediately preceeding pulses nine through sixteen of CL3.

Figures E.8(a) and (b) illustrates the shape and timing of the actual pulses and control signals produced.

(a) Basic Clock, CL2, CL3, and CL4

(b) Basic Clock, CS1, CS2, $\overline{CS2}$, and $\overline{CS3}$

FIGURE E.8 CLOCK AND CONTROL WAVEFORMS

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