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**Exploring the Parameters
of a Simple Model of the Permafrost - Climate Relationship**

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**A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for
the degree of Doctor of Philosophy**

**Department of Geography and Environmental Studies
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Abstract

The parameters of the TTOP model of the permafrost - climate relationship are examined, primarily through the use of numerical thermal modelling. The TTOP model is a simple analytical model that predicts the mean annual temperature at the top of permafrost, incorporating freezing and thawing air temperature indices (seasonal degree-days), transfer functions (“n factors”) to estimate freezing and thawing ground surface temperature indices, and the ratio of frozen and thawed soil thermal conductivity.

Results from thermal conductivity models for saturated and unsaturated mineral and organic soils show that the soil thermal conductivity ratio can be reliably estimated from soil moisture content. Numerical analysis shows that the effect of unfrozen water on the ratio can be accommodated using a temperature-based parameterization. Other numerical studies demonstrate the effect of temperature sensor burial on freezing and thawing index values.

From numerical experiments, the relationship between MAAT and TTOP is robust in the presence of climate variability. Stationary inter-annual variability produces an error in the TTOP model for individual years that is greatest near $TTOP = 0^{\circ}\text{C}$, although this error is small in multi-year averages. Larger errors in the TTOP model occur in the presence of a warming trend where permafrost degradation is initiated, due to the effect of latent heat as a talik develops. Other than this however, the TTOP model appears to be reasonably accurate under transient conditions.

Numerical study of the effect of snow cover on the ground surface temperature demonstrates the importance of active layer freezeback on the freezing n factor, as well as the importance of the mean annual temperature at the base of the active layer. Timing

of snow cover initiation and the seasonal pattern of snow accumulation are also important. From these results, a model is developed to predict permafrost distribution, based on the thermal properties of the ground, and the thermal resistance of the snow cover. This model is used to map permafrost using Canadian climate data.

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Caroline Duchesne produced the climate parameter and permafrost maps in chapter 8, to a higher standard than I could specify. All of my other colleagues at the Geological Survey of Canada have also given their unqualified support.

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Chapter 1 Introduction

Permafrost is a condition of the ground, rather than a particular substance. It is defined as earth material (“ground”) that remains below 0°Celsius over at least two consecutive years (Van Everdingen, 1998). Permafrost exists where a combination of atmospheric processes, terrain and earth material thermal properties makes it possible. In the most general sense, this thesis examines the combination of properties and processes that determine where permafrost exists, using a simple model describing the climate-permafrost relationship.

The overarching objective of this thesis is to determine the utility and limitations of the TTOP model (Smith and Riseborough 1996), by exploring the basis of its parameters and its robustness where conditions depart from the ideal conditions it describes.

1.1 Permafrost

Figure 1.1 shows an idealized temperature profile of permafrost in equilibrium with the climate at the surface. The ground surface, in contact with atmospheric processes, experiences the greatest annual temperature range. Between the ground surface and the top of permafrost is a layer that thaws and refreezes annually (the “active layer”). The top of permafrost is defined as the depth in the ground where the maximum temperature just reaches 0°C. The annual temperature range decreases with depth, to the depth of “zero annual amplitude” where the annual temperature cycle becomes negligible. Below this, the temperature increases steadily with depth (at a rate determined by the local geothermal gradient) until the temperature reaches 0°C, at the base of permafrost.

Figure 1.1 Schematic diagram illustrating the principal elements of the permafrost thermal regime.

Below the permafrost, the temperature continues to increase with depth, but usually with a different gradient (because the thermal properties of unfrozen ground are different from frozen ground).

Near the ground surface (inset, Figure 1.1), the mean annual temperature profile shifts to lower temperatures between the surface and the base of the active layer. This temperature shift, referred to as the "thermal offset", is the result of seasonal changes in soil thermal properties between frozen and unfrozen states. One consequence of this effect is that stable permafrost can exist where the mean annual ground surface temperature is above 0°C.

Following Lachenbruch et al. (1988), the permafrost-climate relationship can be represented by the temperature regime at three levels:

1. Air temperature (as measured at standard screen height);
2. Ground surface temperature;
3. The temperature at the top of permafrost.

Mean annual temperatures at each of these levels usually differ according to the influences of snow cover and ground thermal properties (Figure 1.2). The difference between the mean annual air and ground surface temperatures is influenced largely by the presence of snow cover, which determines whether heat transfer immediately above the ground surface is predominantly conductive or convective, although vegetation and ground surface properties also influence surface temperatures.

The surface offset and the thermal offset together define the relationship between air temperature and permafrost temperature. The presence of snow cover results in a mean ground surface temperature (MAGST) that is higher than the mean annual air

Figure 1.2 Schematic diagram showing profile of mean annual temperature through the lower atmosphere, active layer, and upper permafrost.

temperature (MAAT), while the thermal offset results in permafrost temperatures (TTOP) colder than the ground surface. Parsing the thermal regime of the lower atmosphere and the upper layers of the ground into offsets between these positions in the profile is the basis of the TTOP model.

1.2 The TTOP Model

The TTOP model, introduced by Smith and Riseborough (1996), provides a general formulation of the climate-permafrost relationship, linking mean annual ground temperature conditions to surface climatology through seasonal transfer functions and subsurface thermal properties. It combines an analytical model of the effect of seasonal thermal property variations (the thermal offset), and a simplified method of accounting for differences between temperature indices in the air and at the ground surface (n factors).

$$T_{TOP} = \frac{r_K n_T I_{TA} - n_F I_{FA}}{P}, \quad T_{TOP} < 0$$

$$T_{TOP} = \frac{n_T I_{TA} - \left(\frac{1}{r_K} \right) n_F I_{FA}}{P}, \quad T_{TOP} > 0$$
(1.1)

where

$$r_K = \frac{k_T}{k_F}$$

and

- T_{TOP} = Temperature at Top Of Perennially frozen/unfrozen ground
- k_T = thermal conductivity of ground (thawed)
- k_F = thermal conductivity of ground (frozen)
- n_T = scaling factor between air and surface thawing index
- n_F = scaling factor between air and surface freezing index
- I_{TA} = air thawing degree days (seasonal thawing index)
- I_{FA} = air freezing degree days (seasonal freezing index)
- P = Period (365 days)

The parameters of the model (discussed in detail in Chapter 2) consist of seasonal temperature indices determined by local climate (I_{FA} and I_{TA}), transfer functions that establish the boundary condition for heat flow into the ground (n_F and n_T), and the soil conductivity ratio r_K . While the TTOP model has been shown to be consistent with the physics of conduction, freezing and thawing (Romanovsky and Osterkamp, 1995), physics does not set limits on the model parameters. Placing the TTOP model in the geographical context of permafrost determines what conditions will control the range of the parameter values; this thesis is about how environmental conditions determine the parameters of the model, and therefore the thermal state of the ground.

An objective of this thesis is to explore the parameters of the TTOP model, so that they may be understood and specified from more fundamental properties and conditions. While assessments of the relationship between climatic and permafrost conditions can be made through the use of simulation models, results are valid only for the parameters specified in the simulations. Analysis of simulation results often proceeds by sensitivity analysis (such as Nakano and Brown 1972, Ng and Miller 1977, Smith and Riseborough 1983, Goodwin *et al.* 1984, Kane *et al.* 1991) in which the effects of changes in parameter values are evaluated as departures from a “standard” case. The TTOP model developed out of such a sensitivity analysis. This thesis explores whether the sensitivity analysis approach can be extended successfully to the parameters of the model.

In addition, the model is exact only for idealized conditions, in which heat flow is one-dimensional, freezing and thawing indices and n factors do not vary from year to year, and the thermal conductivity of the soil is truly bimodal (i.e., has single values when frozen and thawed). This idealization ignores spatial variability of surface

conditions, the inter-annual variability of climate (I_{FA} , I_{TA} , and snow cover) and microclimate, and the dependence of soil thermal properties on temperature and moisture content. The significance of departures from the analytical ideal is considered in each of the evidentiary chapters of this thesis, especially Chapter 7.

1.3 Limitations of Scope

This thesis does not examine all aspects of the TTOP model. Most important, while issues related to the field determination of n factors are discussed in Chapter 6, the thawing n factor n_T is not considered in any detail. This omission is due in part to the decision to concentrate on numerical modelling of the conductive aspects of the TTOP model and partly because, as stated in Smith and Riseborough (2002a), “field determination of n -factor values for the thaw season is not a trivial problem”, taking it beyond the scope envisioned for this work.

In addition, while I_{TA} and I_{FA} are parameters of the model, their values are not analyzed in terms of underlying factors. Their values for Canadian climatic conditions are used in thermal analyses (Chapter 3 and Appendix III), and their geographic distribution is described and used for modelling permafrost boundaries in Chapter 8.

Finally, only one dimensional heat flow conditions are modelled in this thesis, so that in effect only continental scale parameter variations are considered. The effects of slope, aspect, and small scale spatial variability in soil properties and snow cover are not considered. In mountainous regions for example, the scale of variation in these factors requires an alternative approach (e.g. Hoetzl *et al.*, 2001) that is beyond the scope of this thesis.

1.4 Numerical modelling of TTOP

Exact analytical models of the thermal behaviour of the ground when freezing or thawing occurs are limited to a few idealized conditions (Lunardini 1981, p. 360). Many real-world conditions, such as seasonal variation in the ground surface temperature, accumulation and ablation of snow cover, and temperature-dependent thermal properties, are beyond the capacity of these exact models. Two paths move beyond this impasse: approximate analytical models may be developed by making simplifying assumptions, or numerical techniques may be employed to solve relatively complex problems with the acceptance of some error. Much of this thesis relies on a numerical geothermal simulation model called TONE, developed by Laurel Goodrich of the National Research Council of Canada. Details of the TONE model are given in Appendix I.

Many analyses in this thesis involved the development of simple analytical or empirical models whose errors were compared with results obtained with TONE. Output from TONE was compared to the predictions of the simple models, with the assumption that TONE results are close to exact. In speaking about the validity of assumptions used and numerical results, Goodrich (1982, p. 422) states:

“These assumptions constitute a considerable idealization when compared with real snow covers and soil conditions. Inclusion of soil freezing over a temperature range and /or of moisture redistribution and geometric changes accompanying frost heaving or thaw settlement would evidently lead to different numerical values of certain computed results such as frost or thaw depths and to curvature of the mean annual ground temperature profiles. More realistic models for the snow cover would have an even greater effect on these quantities. The calculations presented here are primarily intended to illustrate the major features of snow – ground thermal interactions in general. The conclusions reached are essentially qualitative, and it is believed that the model is appropriate for this purpose.”

Goodrich's statement about the qualitative nature of his work relates to his *conclusions*, rather than his results. The numerical results have more or less the same status as analytical results, in that they represent an (approximately) accurate estimate of the relationship modelled, to the extent that the assumptions of the model are met in the real world. In general, the advantage of numerical modelling is that the number of idealizing assumptions can usually be reduced further than can be done with analytical models. The TONE model appears to be adequate for the purpose to which it has been applied in this thesis: as a tool for numerical experiments on the interaction between climate and permafrost.

1.5 Prefatory remarks

This thesis consists of chapters that may be grouped into three sections. In the first section, the theoretical antecedents of the TTOP model are discussed (Chapter 2), and the principal organizing concept for the numerical analyses, the thermal catena, is introduced and discussed (Chapter 3).

In the second section, comprising the bulk of the thesis, separate chapters are devoted to analyses of model parameters and model robustness. Chapter 4 considers the thermal conductivity ratio through an examination of existing empirical and physical models of soil thermal conductivity, and considers the challenges presented by departures from the idealization of uniform, bimodal (i.e., frozen/thawed) thermal properties. Chapter 5 examines the role of snow cover in determining the freezing factor, leading to a model that describes the influence of snow in determining permafrost boundaries. Chapter 6 examines the effect of depth of burial on "near

surface” temperature indices, leading to a modified version of the TTOP model that accommodates multiple soil layers. Finally, Chapter 7 examines the effects that climate variability and climate warming trends have on the reliability of the TTOP model.

The final section is comprised of Chapter 8, in which the snow cover model developed in Chapter 5 is applied to the problem of mapping permafrost boundaries in Canada. The resulting maps are analysed in light of the current understanding of permafrost distribution today.

Developing this thesis was not a linear process. While the sequence of chapters was chosen to minimize the need for forward referencing, results presented in earlier chapters were frequently influenced by discoveries presented later. In particular, the climatic conditions defined for the standard “thermal catena” (defined in Chapter 3) evolved over time so that the catena conditions used for the discussion of permafrost distribution in Chapter 8 were modified to better capture the relevant climatic details. Despite these ongoing adjustments, most completed analyses were not revisited since their purpose was the evaluation of model parameter sensitivities rather than their application to a particular environment.

TTOP terminology conventions

The acronym TTOP initially stood for “Temperature at the Top of Permafrost” (Smith and Riseborough, 1996). The acronym has remained despite the recognition that the model applies wherever there is a surface layer that freezes and thaws annually. In part because of reviewer’s objections, Riseborough and Smith (1998) use T_T for the model, and apply it to the “Temperature at Top of perennially frozen/unfrozen ground”. Henry and Smith (2001) and Smith and Riseborough (2002a) revert to TTOP, defined

as “mean annual temperature at the top of permafrost”. In this thesis, the term defines the mean annual “Temperature at Top Of Perennially frozen/unfrozen ground”.

The TTOP acronym can signify different meanings in this thesis. The TTOP *model* is summarized by the equations introduced in this chapter, and whose development is described in Chapter 2. As a mean annual temperature *value*, TTOP may refer to the temperature predicted by the TTOP model, the temperature predicted in a numerical (or other) model, or a ground temperature determined from measurements (directly or by extrapolation). Because TTOP is an annual mean temperature, it is determined by averaging many instantaneous values (measurement data or numerical output) over an annual period. The TTOP model assumes a stable climate and the model predicts a temperature in equilibrium with that climate. On the other hand, TTOP values represent the mean annual temperature at a place, and may vary from year to year. Thus, as discussed in Chapter 7, a multi-year average TTOP value may also be computed to account for such variability.

To help distinguish between these meanings, the following conventions are observed in this thesis: the model itself is signified by **TTOP**; the analytical value determined from the model is signified by T_{TOP} ; and values for the mean annual temperature at the top of perennially frozen/unfrozen ground that are determined numerically or by field measurement (i.e., *not* from the **TTOP** model) are signified as TTOP.

Terminology for the surface freezing/thawing layer

Using the definition of Burn (1998b), the *active layer* is the ground above permafrost that undergoes seasonal freezing and thawing¹. The bottom of the active layer is usually the top of permafrost, except in transient conditions where inter-annual variations may lead to the persistence of unfrozen ground over the winter. The term “active layer” has been reserved for such a layer above *permafrost* only: in non-permafrost, the term “seasonally frozen surface layer” is used (van Everdingen, 1998). Throughout this thesis, “active layer” and “seasonally frozen layer” are used in this way. Where the condition below the surface layer is undefined or uncertain, “seasonally frozen/thawed layer” is used.

¹ Notwithstanding the discussion in Burn (1998b), the distinction between temperature based and ice-water phase definitions of permafrost and active layer limits has been ignored throughout this thesis, in part because the definitions have been assumed to merge in analytical and numerical formulations.

Chapter 2. The TTOP model

Smith and Riseborough (1996) proposed the TTOP model (for the Temperature at the Top of Permafrost) as a general formulation of the climate-permafrost relationship. It resides within a long history of models based on the use of freezing and thawing indices for discrimination between permafrost and non-permafrost at local scales, and for permafrost zonation at regional to continental scales. Antecedent concepts that contribute to the model and relationships that derive from it are presented in this chapter.

2.1 Antecedents to the TTOP model

2.1.1 Freezing and thawing indices and the Stefan model

Many simple models of climate–permafrost interaction divide the year into freezing and thawing seasons, using the freezing index (I_F) and thawing index (I_T) to summarize the annual regime. A seasonal index is a seasonally integrated temperature, approximately equal to the sum of daily mean temperatures for the duration of the season. For example, the surface thawing index (following Klene *et al.* 2000) is¹:

$$I_{TS} = \int_0^{\theta_s} T_s - T_F dt \approx \sum_0^{\theta_s} \overline{T_s}, \quad T_s > T_F \quad (2.1)$$

where

¹ While I_F and I_T are expressed in units of degree-days in the context of the TTOP equation, in freezing and thawing models such as the Stefan model, the units for I_F and I_T must match the units for time and thermal conductivity. Nelson and Outcalt (1983, 1987) include a scaling factor of 86400 s day^{-1} to make this conversion. For clarity, this scaling factor is not included in this thesis.

T_F = temperature of the freezing point (0°C)

T_S = surface temperature $^\circ\text{C}$

θ_S = duration of the thawing season, days

$\overline{T_S}$ = daily mean surface temperature

An equivalent equation applies to the surface freezing index:

$$I_{FS} = \int_0^{\theta_s} T_F - T_S dt \approx \sum_0^{\theta_s} \overline{T_S}, \quad T_S < T_F \quad (2.2)$$

where

θ_S = duration of the freezing season, days

Typically, freezing and thawing indices are defined for the air (e.g., from climate station data) and for the “surface”, although they may be determined for any position in the temperature profile extending above and below the ground surface (Figure 1.2). Ground surface temperature is singularly difficult to measure, due to the radiative and convective energy flows at the surface and difficulty in defining its position. Because of this, temperature measurements to obtain index values are often taken a few centimetres below the surface, although measurement methods and burial depths are not always reported (see Section 2.1.2).

Temperature indices are used in simple models of soil freezing and thawing. The Neumann equation (Lunardini 1981, p. 364), is an exact analytical solution for depth of freezing (or thawing) in a uniform, initially isothermal soil experiencing a step change in surface temperature:

$$X = 2\gamma\sqrt{\alpha_1 t} \quad (2.3)$$

where:

X = depth of freezing/thawing

t = time

α_1 = surface layer thermal diffusivity

γ is a parameter determined by balancing

$$\frac{e^{-\gamma^2}}{\operatorname{erf}(\gamma)} - \frac{k_2}{k_1} \sqrt{\frac{\alpha_1}{\alpha_2}} \frac{(T_0 - T_F) e^{-\frac{\alpha_1}{\alpha_2} \gamma^2}}{(T_F - T_S) \operatorname{erfc}\left(\gamma \sqrt{\frac{\alpha_1}{\alpha_2}}\right)} = \frac{l \gamma \sqrt{\pi}}{c_1 (T_F - T_S)} \quad (2.4)$$

where

k_1 = surface layer thermal conductivity

k_2 = lower layer thermal conductivity

l = volumetric soil latent heat

c_1 = surface layer volumetric heat capacity

α_2 = lower layer diffusivity (freezing or thawing layer)

T_0 = initial soil temperature

T_F = fusion temperature

T_S = surface temperature

Due to limitations on the strict application of this model (and its relatively large parameter set), simpler models of soil freezing and thawing have been developed.

The thermal behaviour of the active layer is dominated by the relationship between the latent heat of the soil water and the bulk thermal properties of the ground. This results in a steep, near-linear temperature gradient between the ground surface and the thaw front during the summer thaw season, and a steep, near-linear temperature gradient between the soil surface and the freezing front as the active layer soil refreezes from the surface. The nearly linear gradient through the active layer results because most of the heat flow through it is used to exchange heat for thawing or freezing. In this quasi-steady state, the profile changes slowly through time as the front deepens, but the temperature profile remains nearly linear.

The ratio of a material's sensible and latent heats (called the Stefan number Ste), is a standard index of its thermal behaviour when freezing or thawing is:

$$\text{Ste} = \frac{C|T_S - T_F|}{L} \quad (2.5)$$

where C = volumetric heat capacity

When the Stefan number is small (so that diffusive effects are small relative to the rate of frost-front motion) and the initial temperature of the ground is assumed to be at the fusion temperature T_F (Lunardini 1981, p. 370), equation (2.3) can be simplified to the Stefan solution:

$$X = \sqrt{\frac{2k|T_S - T_F|t}{L}} \quad (2.6)$$

The primary assumption of the Stefan model is to ignore sensible heat: first in the initial temperature of the soil (assumed to be at the phase-change temperature), and second in that all heat flow is assumed to supply heat to the front along a linear gradient. This allows for a simplified equation by assuming all heat flow is used to exchange heat at the freezing or thawing front. Because transient effects are ignored, the product $|T_S - T_F|t$ can also be replaced by the accumulated degree-day total (the freezing index I_F or thawing index I_T) (Lunardini, 1981):

$$X = \sqrt{\frac{2kI}{L}} \quad (2.7)$$

The index (I) and thermal conductivity (k) in equation (2.7) can be for the freezing or thawing case. Explicitly or implicitly, comparisons of relative freezing and thawing depths from equation (2.7) form the basis of most permafrost index models.

The Kudriavtsev equation (Romanovsky and Osterkamp, 1997) calculates the depth of thaw for a uniform soil under a sinusoidal surface temperature wave, accounting for the effect of sensible heat:

$$X(2\bar{A}C_T + \rho L) = 2(A_{gs} - |T_{TOP}|) \sqrt{\frac{k_T C_T P}{\pi}} + \frac{(2\bar{A}C_T X_{2C} + \rho LX) \rho L \sqrt{\frac{k_T P}{\pi C_T}}}{2\bar{A}C_T X_{2C} + \rho LX + (2\bar{A}C_T X_{2C} + \rho L) \sqrt{\frac{k_T P}{\pi C_T}}} \quad (2.8)$$

where

$$\bar{A} = \frac{A_{gs} - T_{TOP}}{\ln \left(\frac{A_{gs} + \frac{\rho L}{2C_T}}{|T_{TOP}| + \frac{\rho L}{2C_T}} \right)} - \frac{\rho L}{2C_T} \quad (2.9)$$

$$X_{2C} = \frac{2(A_{gs} - |T_{TOP}|) \sqrt{\frac{k_T C_T P}{\pi}}}{2\bar{A}C_T + \rho L}$$

and

A_{gs} = Temperature amplitude at the ground surface (evaluated as one-half of the annual range in the mean monthly temperatures at the ground surface)

C_T = Volumetric heat capacity of unfrozen ground

L = volumetric latent heat of water

ρ = soil density

While equation (2.8) can be applied more accurately to permafrost active layer problems than equation (2.3) (Romanovsky and Osterkamp, 1997), it is problematic as the basis of a model to predict T_{TOP} , in part because it requires the value of T_{TOP} as an input parameter. In addition, the underlying assumption that the annual temperature wave from the lower atmosphere to the ground surface is controlled by diffusive extinction is problematic (as will be shown in Chapter 5).

2.1.2 N factors

N factors were developed as an engineering method to estimate I_{FS} and I_{TS} by relating seasonal ground surface temperatures to air temperature (Lunardini, 1978). Carlson (1952) reports on the early efforts to develop a “correlation factor between air and surface temperatures”, equivalent to n factors, to correct indices derived from air temperature records when applying versions of equation (2.7) to paved surfaces.

Ground surface n factors are defined as:

$$n_T = \frac{I_{TS}}{I_{TA}} \quad (2.10)$$

$$n_F = \frac{I_{FS}}{I_{FA}} \quad (2.11)$$

where

n_T = surface thawing n-factor

I_{TA} = air thawing index

n_F = surface freezing n-factor

I_{FA} = air freezing index

An increase in n_T will increase surface temperature, while an increase in n_F will *decrease* surface temperature. N factor definitions usually note that the freezing and thawing seasons in the air does not necessarily coincide with these seasons at the ground surface (Lunardini 1981, Taylor 1995, Klene 2000). These authors also state that n factors are usually specified for an entire season (e.g., Klene 2000, p.10).

Lunardini (1978) developed a theoretical n factor model for pavement, without evaporation or snow cover. He concluded that n factors are relatively insensitive to substrate materials, and that n factors tend toward unity as mean wind speed increases. Shur and Slavin-Borovski (1993) mapped thawing n factors in Russia and found a slight increase with latitude over much of the territory, with a steeper increase within

about 100 km of the arctic coast. Their analysis also indicated different n factors for coarse and fine-grained substrates. Klene (2000) found that I_{TS} (and hence n_T) varied by a factor of four between sensors in differing conditions within hummocky terrain.

Lunardini concluded that tabulated n factor values are of limited value unless the site for which values are to be applied “is located almost exactly at the test site” (Lunardini 1978, p.45). Nonetheless, the simplicity of n factors compared to alternative methods of estimating surface temperature conditions has led to an increasing number of studies applying the concept to vegetated surfaces, and Lunardini (1978), Klene *et al.* (2001), Jorgenson and Kreig (1988) and Taylor (1995, 2000) have compiled n factor values for various land-cover types.

While n factors relate surface temperatures to air temperatures, there is no agreed definition of the surface. Many n factor studies (e.g., Carlson 1952, Linell 1973, Lunardini 1978, Shur and Slavin-Borovski 1993) do not indicate the method or depth of temperature measurement. Others (Jorgenson and Kreig 1988, Klene 2000, Taylor 1995, 2000) bury sensors at different depths. Because of the difficulties inherent in measuring true surface temperature, most recent authors measure near-surface temperature (NST) at some depth below the ground surface, deriving n factors relative to that depth. The influence of sensor burial depth on sub-surface n factor values is discussed in Chapter 6.

2.1.3 Index models

Sumgin *et al.* (1940, cited in Carlson 1952) defined the conditions necessary for the existence of permafrost:

$$k_F I_{FZ} > k_T I_{TZ} \quad (2.12)$$

where

- k_F = frozen thermal conductivity of the ground, $\text{Wm}^{-1}\text{K}^{-1}$
 I_{FZ} = annual freezing index at depth z , $^{\circ}\text{C}\cdot\text{day}$
 k_T = unfrozen thermal conductivity of the ground, $\text{Wm}^{-1}\text{K}^{-1}$
 I_{TZ} = annual thawing index at depth z , $^{\circ}\text{C}\cdot\text{day}$

This relation allows determination of the presence of permafrost from measurements at any point in the ground, including the seasonally frozen/thawed layer.

Carlson (1952) suggested that this relation can be explained by considering the difference in heat flux into the ground in the freezing and thawing seasons. Assuming a linear temperature gradient between the surface and the freezing/thawing front, the seasonal total heat flow during the thawing and freezing seasons can be estimated by

$$Q_T = \frac{k_T I_{TS}}{(X/2)} \quad (2.13)$$

and

$$Q_F = \frac{-k_F I_{FS}}{(X/2)} \quad (2.14)$$

where

- Q_T = heat flow during the thawing season
 Q_F = heat flow during the freezing season
 $X/2$ = the average depth of the seasonally frozen/unfrozen layer.

Equations (2.13) and (2.14) are both valid only when $Q_T = Q_F$ and freezing or thawing continues through the year. If $Q_T \neq Q_F$ then one season must include a period without phase change, invalidating the essential assumption. Carlson (1952, p. 200) suggested that “no permafrost will exist and seasonal thaw is equal to the seasonal freeze” when $Q_T = Q_F$, and, “When seasonal thaw does not melt all of the frozen ground, permafrost does exist” (Carlson 1952, p 200). This leads to the criterion for permafrost that:

$$k_F I_{FS} > k_T I_{TS} \quad (2.15)$$

$$\frac{k_F}{k_T} > \frac{I_{TS}}{I_{FS}}$$

In physical terms, the relation suggests that permafrost will exist when the heat that could be extracted during freezing exceeds the heat available for thawing. Similar reasoning was employed by Lunardini (1980, p. 551) and Nelson and Outcalt (1983).

Pihlainen (1962) and Harris (1981) presented graphical methods based on freezing and thawing indices to categorise climate stations into permafrost zones. Pihlainen (1962) found that climate stations could be placed into zones of non-permafrost, discontinuous permafrost and continuous permafrost using mean annual air temperature (MAAT) and I_{TA} (Figure 2.1). He did not offer a physical rationale, and appears to have developed the scheme without antecedents. Pihlainen's (1962) scheme is equivalent to one based on I_{FA} and I_{TA} , since his demarcation lines intersect the (Celsius) origin. Each line may be described by

$$MAAT = mI_{TA} \quad (2.16)$$

where m is the slope of the particular line. It follows that

$$\begin{aligned} \therefore MAAT &= \frac{I_{TA} - I_{FA}}{P} \\ \therefore \frac{I_{TA} - I_{FA}}{P} &= mI_{TA} \end{aligned} \quad (2.17)$$

$$I_{FA} = I_{TA}(1 - Pm)$$

Figure 2.1 From Pihlainen (1962). Original caption: "An approximation of probable permafrost occurrence using annual mean temperature and thawing index for 61 localities in northern Canada."

Similarly, Harris (1981) suggests that the demarcation between zones of continuous, widespread, sporadic, and non-permafrost (in regions of low winter snow cover) does not correspond to particular values of MAAT, but depends on the combination of I_{FA} and I_{TA} (Figure 2.2). Harris cites Brown (1960), Sanger (1966) and Thompson (1963) as sources for his approach. Brown (1960) mapped individual indices to evaluate their relationships to permafrost boundaries in Canada, while Sanger (1966) and Thompson (1963) dealt with various aspects of freezing and thawing indices without discussing permafrost distribution. Harris' demarcation scheme is intended to be limited to regions with snow accumulations of less than 50 cm, since deeper snow "inhibits winter cold penetration into the upper layers of the soil" (Harris 1981, p.65).

Using an argument similar to Carlson (1952), Lunardini (1981, p. 551) suggested that "a simple criterion for the presence of permafrost is that the depth of freeze should exceed the depth of thaw", and that the criterion should be

$$I_{FA} > \left(\frac{\lambda_T}{\lambda_F} \right)^2 \left(\frac{k_T}{k_F} \right) \left(\frac{n_T}{n_F} \right) I_{TA} \quad (2.18)$$

where

λ_T = thawing rate parameter determined for Neumann equation

λ_F = freezing rate parameter determined for Neumann equation

Nelson and Outcalt (1983) proposed various forms of the "frost number" to delineate permafrost, using the same rationale as Lunardini, based on the Stefan rather than the Neumann equation. Their original indices F and F_+ ignore the effect of seasonal differences in soil thermal properties, on the assumption that accurate specification of

Figure 2.2 From Harris (1981). Original caption: "The relationship between occurrence of permafrost zone and the plot of freezing index versus thawing index for stations in Canada, Norway, Spitzbergen, Iceland and the Mongolian Peoples Republic. The indices are based on a minimum of two years' data and usually 10-25 years."

soil properties is impractical for small scale mapping of permafrost. Using only air temperature indices, they define:

$$F = \sqrt{\frac{I_{FA}}{I_{TA}}} \quad (2.19)$$

F_+ accounts for the effect of snow cover on I_{FS} using Lachenbruch's (1959) analytical model (discussed in Chapter 5: equation (5.9)):

$$F_+ = \sqrt{\frac{I_{FS}}{I_{TA}}} \quad (2.20)$$

Comparisons between maps of F_+ and empirically based permafrost maps suggested that continuous permafrost was associated with $F_+ \geq 1.5$. Nelson and Outcalt (1987) modified the frost number to:

$$F = \frac{\sqrt{I_{FA}}}{\sqrt{I_{FA}} + \sqrt{I_{TA}}} \quad (2.21)$$

$$F_+ = \frac{\sqrt{I_{FS}}}{\sqrt{I_{FS}} + \sqrt{I_{TA}}}$$

Values of the new index were between 0 and 1, with the new index defining the continuous permafrost limit where $F_+ \geq 0.66$. Nelson and Outcalt (1987) also introduced a "Stefan frost number" that includes soil thermal properties:

$$F = \frac{\sqrt{k_F I_{FA}}}{\sqrt{k_F I_{FA}} + \sqrt{k_T I_{TA}}} \quad (2.22)$$

Jorgenson and Kreig (1988) used equation (2.15) in the development of their model for the prediction of permafrost, defining an index number R:

$$R = \frac{I_{TS}k_T}{I_{FS}k_F} \quad (2.23)$$

Jorgenson and Kreig assumed that permafrost exists where $R < 1$, as stated by Carlson (1952). I_{FS} and I_{TS} are estimated from n factors based on terrain type, with an equivalent latitude adjustment to I_T to account for differences in net radiation due to variations in slope and aspect:

$$R = \frac{n_T I_{TA} E k_T}{n_F I_{FA} k_F} \quad (2.24)$$

where

$$E = \frac{L}{L_e}$$

L = latitude

L_e = equivalent latitude

Equivalent latitude accounts for the effects of slope and aspect on the potential insolation received at a point. It can be calculated instantaneously from solar geometry (Lee 1978, p. 56), and its effect integrated for an entire season (Frank and Lee 1966, cited in Lee 1978, p. 56).

Wright (1995) developed a model of permafrost thickness based on particular values of R . Assuming $E = 1$, the relationship between R and T_{TOP} can be expressed as

$$R = \frac{T_{TOP}P}{n_F I_{FA}} + 1, \quad T_{TOP} < 0 \quad (2.25)$$

$$R = \frac{n_T I_{TA}}{n_T I_{TA} - T_{TOP}P}, \quad T_{TOP} > 0$$

Extending Wright's (1995) work by explicitly accounting for the thermal offset (see following section) lead to the development of the **TTOP** model.

2.1.4 The thermal offset

As equations (2.13) and (2.14) imply, heat transfer into thawing ground differs from transfer out of freezing ground because the thermal conductivities of frozen and thawed soil differ. The thermal conductivity of most soils containing water is usually greater when frozen than when unfrozen, primarily because the thermal conductivity of ice is approximately four times greater than that of water. As Jenness (1949, p. 16) stated, "...the amount of heat transferred from a frozen soil to the air during a winter day will be greater than the amount transferred from the air through the unfrozen soil during the corresponding day in summer", due to the difference in thermal conductivity between frozen and thawed soil. Brown (e.g., 1965, p. 21) understood the effect of this variation on the thermal regime of peatlands near the southern limit of permafrost, but apparently was not aware of its more general significance.

The differences in heat flow are resolved with a mean annual ground temperature that is progressively colder through the active layer, as shown in Figure 2.3. The difference between mean annual ground surface temperature (MAGST) and the mean annual temperature at the bottom of the seasonally frozen/thawed layer (T_{TOP}) was labeled the *thermal offset* by Goodrich (1978a). Goodrich (1978a) noted the effect in simulations of the ground thermal regime, while Burn and Smith (1988) and Romanovsky and Osterkamp (1995) have observed it in field data.

Using simulation results from a numerical geothermal simulation model (TONE: see Appendix I), Smith and Riseborough (1996) determined the following relation for the temperature at the base of the active layer, due to the thermal offset:

Figure 2.3 Thermal offset through the active layer, simulated using TONE. Offset is calculated as the difference in mean annual temperature at depth relative to mean annual surface temperature. All examples were simulated with $k_T = 1.4 \text{ Wm}^{-1}\text{K}^{-1}$, and a mean annual temperature of -8°C .

$$T_{TOP} = \frac{k_T I_{TS} - k_F I_{FS}}{k_F P}, \quad T_{TOP} < 0$$

$$T_{TOP} = \frac{k_T I_{TS} - k_F I_{FS}}{k_T P}, \quad T_{TOP} > 0$$
(2.26)

Independently, Romanovsky and Osterkamp (1995) presented the same relation and provided an analytical proof, citing a relation presented by Kudriavtsev (1977).

Defining the thermal conductivity ratio r_K :

$$r_K = \frac{k_T}{k_F}$$
(2.27)

equation (2.26) may be simplified to:

$$T_{TOP} = \frac{r_K I_{TS} - I_{FS}}{P}, \quad T_{TOP} < 0$$

$$T_{TOP} = \frac{I_{TS} - \left(\frac{1}{r_K}\right) I_{FS}}{P}, \quad T_{TOP} > 0$$
(2.28)

Equation (2.28) assumes that k_T and k_F are single-valued. Soil thermal properties and their effect on r_K and the **TTOP** model are discussed in Chapter 4.

2.1.5 The TTOP model

Smith and Riseborough (1996) combined the thermal offset equation with n factors to devise the **TTOP** model. From equations (2.10) and (2.11), temperature indices for the surface can be defined by:

$$I_{FS} = n_F I_{FA}$$
(2.29)

$$I_{TS} = n_T I_{TA}$$
(2.30)

Combining the equations (2.26), (2.29) and (2.30), the **TTOP** model yields the mean annual temperature at the top of perennially frozen/unfrozen ground, with parameters determined by air temperature (I_{FA} and I_{TA}), surface energy exchange (n_F and n_T), and subsurface thermal properties (k_F and k_T):

$$T_{TOP} = \frac{k_T n_T I_{TA} - n_F k_F I_{FA}}{k_F P}, \quad T_{TOP} < 0$$

$$T_{TOP} = \frac{k_T n_T I_{TA} - n_F k_F I_{FA}}{k_T P}, \quad T_{TOP} > 0$$
(2.31)

While the **TTOP** model includes the elements of most of the antecedent index models, it differs in that it yields a temperature rather than an index number. As such, comparisons of field conditions with model predictions can involve quantitative verification beyond the presence or absence of permafrost.

2.2 Summary of TTOP relationships

The parameters of the **TTOP** model can be used to define temperatures (MAAT, MAGST and TTOP) and temperature differences (surface, nival, thermal and total offsets). Noting that:

$$MAAT = \frac{I_{TA} - I_{FA}}{P}$$
(2.32)

it follows from equations (2.29) and (2.30) that the mean annual ground surface temperature (MAGST) is:

$$MAGST = \frac{I_{TS} - I_{FS}}{P}$$

$$= \frac{n_T I_{TA} - n_F I_{FA}}{P}$$
(2.33)

Using the thermal conductivity ratio r_K from equation (2.27), equation (2.31) can be expressed as

$$T_{TOP} = \frac{r_K I_{TS} - I_{FS}}{P}, \quad T_{TOP} < 0$$

$$T_{TOP} = \frac{I_{TS} - I_{FS}/r_K}{P}, \quad T_{TOP} > 0$$
(2.34)

Combining (2.34) and the n factors relationships (equations (2.29) and (2.30)),

$$T_{TOP} = \frac{r_K n_T I_{TA} - n_F I_{FA}}{P}, \quad T_{TOP} < 0$$

$$T_{TOP} = \frac{n_T I_{TA} - (n_F I_{FA}/r_K)}{P}, \quad T_{TOP} > 0$$
(2.35)

2.2.1 Offset relationships

The surface offset Δ_{Surf} is the difference between mean annual temperature at the surface and in the air. From equations (2.32) and (2.33),

$$\Delta_{Surf} = MAGST - MAAT$$

$$= \frac{n_T I_{TA} - n_F I_{FA}}{P} - \frac{(I_{TA} - I_{FA})}{P}$$

$$= \frac{(n_T - 1) I_{TA} - (n_F + 1) I_{FA}}{P}$$
(2.36)

The nival offset Δ_{Snow} is the difference between mean annual temperature at the surface and in the air due to the influence of snow:

$$\begin{aligned}
\Delta_{Snow} &= \frac{I_{FA} - I_{FS}}{P} \\
&= \frac{I_{FA} - n_F I_{FA}}{P} \\
&= \frac{(1 - n_F) I_{FA}}{P}
\end{aligned} \tag{2.37}$$

Note that equation (2.36) collapses to equation (2.37) when $n_T = 1$.

The thermal offset Δ_T is defined as the difference between mean annual temperature at the surface and at the top of perennially frozen/unfrozen ground:

$$\begin{aligned}
\Delta_T &= T_{TOP} - MAGST \\
&= \frac{r_K n_T I_{TA} - n_F I_{FA}}{P} - \frac{n_T I_{TA} - n_F I_{FA}}{P} \\
&= \frac{(r_K - 1) n_T I_{TA}}{P}, \quad T_{TOP} < 0
\end{aligned} \tag{2.38}$$

and

$$\begin{aligned}
\Delta_T &= \frac{n_T I_{TA} - (n_F / r_K) I_{FA}}{P} - \frac{n_T I_{TA} - n_F I_{FA}}{P} \\
&= \frac{-(1 + 1/r_K) n_F I_{FA}}{P}, \quad T_{TOP} > 0
\end{aligned}$$

The total offset is defined as the difference between mean annual temperature at the top of perennially frozen/unfrozen ground and in the air:

$$\begin{aligned}
\Delta_{Total} &= T_{TOP} - MAAT \\
&= \frac{r_K n_T I_{TA} - n_F I_{FA}}{P} - \frac{I_{TA} - I_{FA}}{P} \\
&= \frac{(r_K n_T - 1)I_{TA} - (1 - n_F)I_{FA}}{P}, \quad T_{TOP} < 0 \\
&\text{and} \\
&= \frac{n_T I_{TA} - \left(\frac{n_F}{r_K}\right) I_{FA}}{P} - \frac{I_{TA} - I_{FA}}{P} \\
&= \frac{(n_T - 1)I_{TA} - \left(1 - \frac{n_F}{r_K}\right) I_{FA}}{P}, \quad T_{TOP} > 0
\end{aligned} \tag{2.39}$$

These relationships are used in the chapters that follow.

Chapter 3 Ground thermal regimes and the thermal catena

The thermal regime of the ground responds differently to climate depending on whether permafrost is present or not. This chapter begins with a discussion of these first-order differences in behaviour between permafrost and seasonally frozen ground, leading to consideration of the special conditions that must exist at the boundary between the two regimes.

Second-order differences in ground thermal regime can also be distinguished within permafrost and seasonally frozen ground, due to the effects of I_{FS} and I_{TS} on mean annual ground temperature and the seasonal freeze-thaw layer. Such systematic differences in ground thermal regime along a climatic gradient, termed a “thermal catena” by Riseborough (1990), are considered in the second half of this chapter.

3.1 Permafrost, seasonally frozen ground, and permafrost boundaries

3.1.1 Permafrost thermal regime

Outcalt and Hinkel (1997) divided the thermal regime of the active layer into four seasons, based on differences in the heat-transfer processes that dominate. These first order differences are a response to the annual surface temperature cycle, timing of snow cover to coincide with the coldest part of the temperature cycle, thermal diffusion, and freezing and thawing of water in the soil surface layer.

Outcalt and Hinkel’s (1997) four seasons of the active layer are (Figure 3.1):

1. thaw season (summer),
2. active layer freezeback season (fall-winter),
3. the frozen (conduction) winter season,
4. and the snow ablation season (early spring).

Figure 3.1 Thermal seasons in permafrost simulated using TONE. Seasons are labeled in the bar across the top. Upper part of figure shows idealized air and ground surface temperature; lower part shows active layer development and freezeback, and snow cover thickness. Time axis is common to all relationships.

1. The active layer begins to thaw once the snow has disappeared and the temperature of the surface is free to rise above 0°C. The rate of thaw is controlled by surface temperature, the thermal properties of the ground (in particular the thermal conductivity of the thawed material, and the volumetric latent heat of the frozen ground), and the temperature gradient between the surface and the thawing front. The rate of thaw declines as the season progresses, due to the decreasing temperature gradient within the active layer as its thickness increases. Toward the end of the thaw season, the phase front may stabilise or move back toward the surface, when heat flow from the surface is balanced or overtaken by heat flow from the front into the permafrost below (Romanovsky and Osterkamp, 1997).
2. As surface temperature falls below 0°C, the active layer begins to re-freeze from the surface. The freezing season begins more or less concurrently with the arrival of a persistent snow cover (Olsson *et al.*, 2003). As a result, the high rate of heat flow into the thawing active layer is not matched during freezeback, even though the thermal conductivity of frozen soil is usually higher than the same soil when unfrozen. Freezing fronts above and below the unfrozen part of the active layer rapidly cool the unfrozen active layer to the freezing point (Osterkamp and Romanovsky, 1997). Persistence of a zone with temperatures at or near 0°C within the active layer during freezeback is referred to as the *zero curtain* (Outcalt *et.al.*, 1990). Once the active layer is completely frozen, temperatures in the ground fall rapidly in response to colder air temperature (mediated by the intervening snow cover), since they are no longer constrained by proximity to the phase change front.

3. With the active layer frozen, heat flow in the ground is largely by conduction, with the low diffusivity of the snow cover restricting heat loss from the ground.
4. During ablation of the snow cover, the ground surface experiences freezing temperatures, while air temperatures are above 0°C. The snow cover warms to its melting temperature, and is ablated away. Ground surface temperature rises, but is prevented from rising above 0°C until the snow has disappeared¹. The snow ablation period is common to permafrost and seasonally frozen ground thermal regimes. Ablation processes are considered in Chapter 5.

3.1.2 Seasonally frozen ground thermal regime

Outcalt and Hinkel's (1997) criteria may be used to define a different set of seasons for seasonally frozen ground (Figure 3.2):

1. thaw season (spring),
 2. unfrozen season (summer-fall),
 3. surface freezing season (winter),
 4. and the snow ablation season (early spring).
1. As with permafrost, thawing of the seasonally frozen surface layer commences once the snow has disappeared and surface temperature rises above 0°C. Thaw of the seasonal frost layer can proceed more rapidly than freezing of the active layer because there is no intervening snow cover. There are two phase change fronts (as with freezeback of the active layer), producing a zero curtain within the thawing

¹ This is notwithstanding the penetration of radiation through the snow, and other effects discussed in Chapter 5.

Figure 3.2 Thermal seasons in seasonally frozen ground simulated using TONE. Seasons are labeled in the bar across the top. Upper part of figure shows idealized air and ground surface temperature; lower part shows active layer development and freezeback, and snow cover thickness. Time axis is common to all relationships.

- layer, and thawing of the layer from below. The thaw season ends when the ground is entirely above 0°C.
2. The unfrozen season in seasonally frozen ground is entirely free from the effects of freezing and thawing. However, the mobility of water as liquid and vapour can significantly influence the thermal regime due to coupled mass and heat flows related to gravity or temperature gradients within the ground. This season therefore is labelled unfrozen, since non-conductive (albeit also non-freeze-thaw) processes may be important.
 3. As with permafrost, the commencement of ground surface freezing usually coincides approximately with the establishment of a snow cover. The seasonal freezing season in seasonally frozen ground differs from active-layer freezeback in that freezing is one-sided: the zero-curtain condition usually present during active layer freezeback in permafrost is absent, with temperatures in the unfrozen ground below the freezing front depending on the mean annual ground temperature. The persistence of freezing throughout the winter also ensures that ground surface temperatures remain relatively warm through the winter. Depending on the combination of temperatures and thermal properties in the air and ground, the arrival of snow can prevent the establishment of frost in the ground (Solanti, 1998). In this condition, ground surface temperature is constrained near 0°C by the presence of the overlying snow, while heat flow from the ground can melt the snow cover at its base.
 4. The ablation season is similar in permafrost and seasonally frozen ground, except that heat to melt the snow cover can be supplied from above and from below in

seasonally frozen ground, in situations where the ground surface has remained unfrozen or has risen to the freezing point by the end of winter.

3.1.3 Permafrost – seasonally frozen ground boundaries

The definition of permafrost (Chapter 1) and the implied corollary definition of seasonally frozen ground suggest that a special condition must exist at the boundary between them, in which neither perennially frozen nor unfrozen conditions exist. Where there is an annual surface wave with temperatures above and below freezing, the limiting condition can only be satisfied if the temperature at the bottom of the freeze/thaw layer is persistently 0°C. These conditions can only be met where surface layer freezing continues throughout the winter and thawing continues throughout the summer, with *no season free of freezing or thawing* (otherwise, the condition would be defined as permafrost or seasonally frozen ground). In this situation, there are only three seasons in the seasonally frozen/thawed layer (Figure 3.3):

1. the thaw season (spring-summer),
2. active layer freezeback season (winter),
3. and the snow ablation season.

The thermal regime at a permafrost margin is unusual, in that one or more phase-transition fronts are always present. Further, the bottom of the freeze/thaw layer must also define the depth of zero annual amplitude, since variation below this depth would produce freezing or thawing (Figure 3.4). In the absence of a geothermal gradient this condition would result in a static temperature profile at 0°C. Assuming a typical geothermal gradient, temperature will increase with depth, so that the condition where the mean annual temperature at the base of the perennially frozen/thawed layer

Figure 3.3 Thermal seasons at a permafrost boundary (permafrost – seasonally frozen ground transition) simulated using TONE. Seasons are labeled in the bar across the top. Upper part of figure shows idealized air and ground surface temperature; lower part shows active layer development and freezeback, and snow cover thickness. Time axis is common to all relationships.

Figure 3.4 Annual ground temperature envelopes simulated for permafrost, seasonally frozen ground, and a permafrost – seasonally frozen ground boundary.

(TTOP) is 0°C will be seasonally frozen ground. In fact, local spatial variation in TTOP may produce negative thermal gradients in the near surface.

3.2 The thermal catena

The contrasting states discussed above can be examined in more detail by considering a climatic transect from permafrost to seasonally frozen ground conditions. Riseborough (1990) introduced the term “*thermal catena*” for this concept to describe and analyse the thermal behaviour of the ground. In soil science, a catena is “a group of soils within a particular soil region which developed from similar parent material but differ in characteristics of their profiles because of the varying topographic and drainage conditions under which they formed” (Thornbury 1954, p. 78). A thermal catena can be defined as a group of sites having the same thermal properties (thermal conductivity, heat capacity, latent heat, freezing characteristic) but differing in terms of the surface climate (annual temperature regime, snow cover timing, thickness and properties). Riseborough (1990) demonstrated the ways that the ground thermal regime changes because of changes to the thermal boundary condition, *other things being equal*. This is similar to the descriptive approach employed by Kudriavtsev (1965) and Yershov (1998, Chapter 11) to illustrate systematic changes in active layer thickness that occur in this and other transitions. The thermal catena concept is easier to apply in the abstract or in modeling than in the field, since the necessary uniformity of soil properties and surface conditions (which might span hundreds of kilometres) will not be found.

3.2.1 Catena I_{FA} and I_{TA}

The catena concept was applied in this thesis using numerical thermal simulation. The goal was to capture the range of Canadian climatic regimes where

permafrost might be found, and examine the effect of various soil materials and snow cover conditions. As discussed in Chapter 1, the specification of catena parameters changed as the requirement for a more realistic representation of these elements became apparent.

Air temperature in catena simulations was represented as a sine wave with a period of 365 days, so that I_{FA} and I_{TA} were determined from its mean and amplitude. For an air temperature wave with amplitude A (i.e., an annual temperature range of $2A$) and mean $\bar{T}_A$ (= MAAT), temperature T at time t (ignoring the phase of the wave) is:

$$T_t = \bar{T}_A + A \sin(2\pi t / P) \quad (3.1)$$

where

t = time

A = air temperature amplitude

$\bar{T}_A$ = mean temperature (MAAT)

P = Period (365 days)

The surface temperature first reaches 0°C at time t_0 , defined as:

$$t_0 = \frac{P}{2\pi} \sin^{-1} \left(\frac{-\bar{T}_A}{A} \right) \quad (3.2)$$

Thawing ends (and freezing begins) at $t = (P/\pi) - t_0$, while freezing ends at $t = P + t_0$,

so that the freezing and thawing indices for this sine wave are:

$$I_{TA} = \int_{t_0}^{\frac{P}{\pi} - t_0} \bar{T}_A + A \sin(2\pi t / P) \partial t \quad (3.3)$$

$$I_{FA} = - \int_{\frac{P}{\pi} - t_0}^{P + t_0} \bar{T}_A + A \sin(2\pi t / P) \partial t \quad (3.4)$$

The relationships of the freezing and thawing indices (I_{FA} and I_{TA}) to mean annual air temperature (MAAT) for the thermal catenae in this thesis were developed

using Canadian climate normal data for the period 1961-1990 (Environment Canada 1995a). Stations were chosen based on the availability of normal annual I_{FA} and I_{TA} values. All stations for the Northwest and Yukon territories were included, while stations for British Columbia were excluded because of the confounding influence of altitude on climate parameters. Nova Scotia, New Brunswick, Prince Edward Island and the island of Newfoundland were excluded because of their latitude. Stations in other provinces were selected by latitude, depending on the position of the southern permafrost limit in each province, according to the map of Heginbottom *et al.* (1995). North of these limits, all climate stations were included. Data for some climate stations south of these limits were also included, to extend the temperature range of the thermal catena to 6°C.

Correspondence between the catena climate and the Canadian normal climate was through the relationship between MAAT and the indices (Figure 3.5). In earlier analyses, (Riseborough 1990, Riseborough and Smith 1998, Smith and Riseborough 2002a), MAAT varied between -12° and 5°C, with an amplitude of 20°C (i.e., an annual range of 40°) for all points along the catena. This catena specification was used for some analyses in this thesis but was considered inadequate in others, primarily because catena I_{FA} and I_{TA} were significantly above the range of climate station values where MAAT was above 0°C. Polynomial regression lines were fit to the climate data of Figure 3.5:

$$I_{TA} = 0.6544T^2 + 123.66T + 2085 \quad (R^2 = 0.8888) \quad (3.5)$$

$$I_{FA} = 0.6941T^2 - 241.47T + 2072.7 \quad (R^2 = 0.9743) \quad (3.6)$$

These regression lines are also shown on Figure 3.5.

Figure 3.5 Freezing and thawing indices as a function of mean annual air temperature. Points are taken from Canadian climate normal data for the period 1961-1990. Black lines show the polynomial least-squares fit to these relationships, which were used to generate I_{FA} and I_{TA} values for the standard catenae for most analyses in this thesis. The gray line shows I_{FA} and I_{TA} for the catena used in Riseborough and Smith (1998, 2002) and Henry and Smith (2000).

Mean annual air temperatures along the catena were between -18°C and $+6^{\circ}\text{C}$, at 2°C intervals. The amplitude of the annual temperature wave for points along the catena was calculated using equation (3.6) in conjunction with equation (3.4). For each MAAT, I_{FA} was estimated using equation (3.6). Equation (3.4) was then solved to determine the sine wave amplitude that would yield this value of I_{FA} . Amplitude as a function of MAAT for points along the catena is shown in Figure 3.6.

Catena simulation results provide the experimental data for various parameterizations (Chapters 4, 5, and 6), and are used to establish the robustness of the **TTOP** model under varying climatic conditions (Chapter 7). In these applications, the correspondence between the catena and real-world climate specifications is not critical to their utility (as long as the important processes are included in the simulations). In these cases the primary purpose is to establish the most important properties and processes governing the relationship, so that early simulations carried out using the initial specification were considered adequate for the intended purpose. In evaluating the distribution of permafrost however (as in Chapter 8), artifacts produced by these unrealistic catena conditions might have lead to erroneous conclusions, so that improvements to the catena specification were included wherever appropriate.

3.2.2 Numerical catenae

Even in simulation modeling it is difficult to change only one condition at a time. For example, changing the surface boundary temperature (mean annual temperature, annual temperature range) will change the timing and duration of the period of snow cover, while changing the properties of the snow cover (thickness,

Figure 3.6. Relationship between MAAT and amplitude used in thermal catenae, based on the relationship between MAAT and I_F discussed in section 3.2.1. For interpolation between standard catena temperatures, the following regression equation was used:

$$\text{Amplitude} = -2.768775 \cdot 10^{-7} T^6 - 7.859789 \cdot 10^{-6} T^5 - 5.388043 \cdot 10^{-5} T^4 - 1.212156 \cdot 10^{-5} T^3 - 9.172552 \cdot 10^{-3} T^2 - 0.3367371 T + 18.47813$$

density or ablation parameter) will alter the length of the ablation period. As a result, changes in parameters may influence the overall thermal behaviour of the ground due to consequent effects on other parameters. Therefore, changes in catena parameters as a function of MAAT requires careful and explicit specification.

Thermal catena simulations were performed using a numerical geothermal simulation model (TONE: see Appendix I) following a standard procedure (with some departures). For each catena:

- Physical and thermal properties (particle and dry bulk density, thermal conductivities, heat capacities, latent heat, freezing characteristic) were defined for the soil. For most cases where the ground was a single material (bedrock or soil), properties were the same at all depths. However, a number of catenae involved one or more organic soil (peat) layers which overlay a mineral soil material.
- For most cases where the ground was a single material, the calculation grid was specified as 16 m of material. One exception to this was for dry bedrock (30 m thick, due to its high diffusivity relative to other materials). Organic surface layers, where included, were added to the top of the 15 m grid. In Chapter 7, the bottom of the grid was set to 100 m to accommodate long-term transient heat flow in simulations of the effect of climate warming.
- Following the recommendations for a similar numerical model (T1D: see Appendix I) by Romanovsky *et al.* (1997), simulations were given a 0.02 m grid spacing for the upper 1 m of the ground, and a time step of 40 minutes. Below 1 m, node spacing increased exponentially to 0.6 m at the bottom of the grid.

- Air temperature was defined as a sine wave with a period of 365 days. Mean temperatures for points along the catena were between -18°C and $+6^{\circ}\text{C}$, at intervals of 2°C). As discussed above, for most simulations the amplitude was a function of mean annual temperature, although in some analyses the amplitude of the wave was 20°C for all mean annual temperatures.
- Where investigating the effect of snow, a standard snow cover was defined. Snow was given a standard, constant density of 250 kg m^{-3} and an ablation factor of 1.5 mm of water equivalent per degree-day². Snow accumulation was usually specified to commence one day after the initiation of surface freezing conditions, and to follow a parabolic accumulation function (following Zhang 1993, defined and discussed in Chapter 5). In a few analyses, the snow cover was defined to accumulate linearly for the initial 1/3 of the snow cover season³. In detailed analyses, a range of maximum snow depths was simulated: snow-free, 0.15 m, 0.25 m, 0.35 m, 0.5 m, 0.75, 1.0 m, 1.25 and 1.5 m. Where the influence of snow depth was not the primary focus, results for only one or two snow depths (usually 25 and 50 cm) were considered.
- Heat flux at the base of the grid was set to zero Wm^{-2} .
- Numerical simulations were run until year-to-year changes in ground temperature through the entire profile were less than 0.001°C , ensuring that the modeled ground

² This value was determined by trial and error for the snow cover in the Fort Simpson climate station record, in Riseborough and Smith (1993).

³ As with I_{FA} and I_{TA} , snow cover details in catena analyses changed during development of this thesis, as the elements of a more realistic representation of snow accumulation became apparent. For a few analyses examining the sensitivity of the ground thermal regime to particular parameters, analyses using the original snow cover model have been retained.

thermal regime was not influenced by the initial condition specified for the simulation. Some cases developed a small inter-annual oscillation near the ground surface, preventing termination of the equilibration process: in these cases, simulations were terminated after 250 (or more) annual cycles.

- The value of TTOP was taken as the mean annual temperature for the first node in the calculation mesh that did not thaw or freeze during the annual cycle. Simulations with a warming climate and a coarser-than-standard mesh (discussed in Chapter 7) produced inter-annual fluctuations in TTOP on the order of 0.05°C . This can be considered an approximate upper limit of the error in numerical estimates of TTOP.
- Many catenae include an additional point at approximately $\text{TTOP} = 0^{\circ}\text{C}$, to capture the significant features at the permafrost – seasonally frozen ground transition. The MAAT for this condition was established by trial and error. After performing a simulation with a given surface condition, the mean and amplitude of the air temperature wave were adjusted in the appropriate direction. The small temperature gradients below the active layer, coupled with the high latent heat involved in a nearly isothermal ground temperature profile near 0°C , made it difficult to establish a static mean annual temperature profile at the end of 250 annual cycles. Results were considered sufficiently close to the permafrost – seasonally frozen ground transition when simulations yielded a value for TTOP between -0.1°C and $+0.1^{\circ}\text{C}$.

Numerical modeling results are intended to represent the physical situation in which the effects of phase change (including fusion and the change in thermal properties) occur at precisely 0°C . As discussed in Appendix I, the simulations used in

most analyses in this thesis were made with phase change occurring between -0.01°C and $+0.01^{\circ}\text{C}$. As a result, simulations in which TTOP falls in this temperature range do not represent the simple physical system described. Where the temperature profile occupies a significant amount of time within this temperature range, the thermal offset reflects the transitional thermal conductivities in the ground.

3.2.3 An example catena

The permafrost and seasonally frozen ground thermal regimes discussed in Section 3.1 are points on a thermal catena, with the thermal regime where TTOP is 0°C being a conceptual midpoint. The climate forcing changes along the catena from permafrost to seasonally frozen ground, with the thawing index increasing and the freezing index and length of the snow cover season decreasing. Figures 3.7-3.9 show results for a standard catena as defined above, with a peak snow-cover depth of 25 cm. The resultant thermal regimes differ significantly along the catena.

Figure 3.7a shows the dependence of the depth of the seasonally frozen/thawed layer on TTOP, with a maximum occurring at $\text{TTOP} = 0^{\circ}\text{C}$. This reflects the dependence of active layer depth on the thawing index (which increases with increasing MAAT) and the dependence of seasonal freezing on the freezing index (which decreases with increasing MAAT).

Extended periods of surface layer freezing and thawing reduce the thermal response of the ground (Riseborough, 1990). In homogeneous materials without phase change, the thermal response can be characterized by the damping depth, which depends on the thermal diffusivity of the material. Where phase change due to freezing and thawing is occurring, the *apparent* thermal diffusivity is substantially smaller due to

Figure 3.7 a) Maximum active layer depth and annual temperature wave damping depth, as a function of TTOP. b) Total of seasonal freeze/thaw layer refrozen or re-thawed from below, as a function of TTOP. c) Fraction of seasonal freeze/thaw layer refrozen or re-thawed from below, as a function of TTOP.

Figure 3.8 a) Simulated ground temperature profiles at the end of freezeback. **b)** Simulated ground temperature profiles at the end of thawback.

Figure 3.9 a) Fraction of the year influenced by active freezing or thawing in the ground, showing total freezing or thawing days, and days with two-sided freezing or thawing (“freeze/thaw back”). b) Fraction of year with one sided surface thawing or freezing. This figure represents the difference between the curves in the upper figure.

latent heat. On an annual basis, this effect can be summarised by the apparent damping depth for the annual temperature wave, defined as the depth below the surface at which the magnitude of the annual temperature wave is reduced to $1/e$ (i.e., 36.8%) of its magnitude at the surface. This apparent damping depth is shown in (Figure 3.7a). Damping depth is a strong function of temperature, with a minimum near the transition from permafrost to seasonally frozen ground. For TTOP between -2°C and $+2^{\circ}\text{C}$, the damping depth is within the seasonal freeze/thaw layer.

Refreezing of the active layer from the ground surface begins when surface temperatures fall below 0°C . At the end of the thawing season in permafrost, the temperature gradient in the active layer decreases as the ground surface temperature approaches 0°C . This can halt or reverse the progression of the thawing front (“*up-freezing*”), as heat can be extracted along the temperature gradient between the thawing front and the permafrost below. An analogous “*up-thawing*” occurs at the end of the freezing season in seasonally frozen ground. The amount of heat that can be supplied from below the seasonal freeze/thaw layer depends on the temperature gradient below this layer, which is controlled by the mean annual temperature. The magnitude of upward freezing/thawing, in absolute and relative terms, is shown in Figures 3.7 b and c. Results at the cold end of the catena accord with field data from Osterkamp and Romanovsky (1997), who suggest that one third of the active layer can be refrozen from below on the North Slope of Alaska, with a mean annual ground surface temperature (MAGST) between -6 and -10°C . This can be seen in Figure 3.8, which shows the simulated ground temperature profiles at the end of the freezeback (3.8a) or thaw-back (3.8b) of the layer. At MAAT of -4°C (where TTOP approaches 0°C), the temperature

gradient at the base of the seasonal freeze/thaw layer approaches zero, so that upward freezing or thawing is limited.

Figure 3.9a shows the proportion of the year in the soil's surface layer, as a function of TTOP. Where TTOP is near 0°C, a significant proportion of the year is influenced by phase change in the ground, with freezing or thawing through the entire year where TTOP = 0°C, as refreezing is complete only at the start of the next thawing season, and thaw is complete only at the start of the next freezing season. Active freeze/thaw days decrease rapidly on either side of the peak however, with more than 15% of the year (about 50 days) without phase change where the mean annual temperature is within $\pm 1^\circ\text{C}$ of the transition. This strong temperature dependence suggests that the continual phase-change condition where TTOP is 0°C is unlikely to be detected in the field, where even small inter-annual variations in TTOP will produce some years in which thawing is completed before the start of the next freezing season. (Where a residual frozen layer persists in the colder years in this situation, phase change will continue.)

Figure 3.10 shows the ground surface temperature pattern under a 25 cm snow cover for points along the catena. For all of the permafrost cases shown (from MAAT = -12° to -5°C) the end of active-layer freezeback is apparent as an acceleration of the rate of cooling: this acceleration is increasingly delayed at higher MAAT, as freeze-back takes longer. The effect of this delay is to increase the mean temperature difference between the air and the ground surface for the winter season. The effect of this behaviour on the parameterization of snow cover and n_F is discussed in Chapter 5.

Figure 3.10 Ground surface temperature under a snow cover with a maximum depth of 25 cm. Temperature values adjacent to curves indicate MAAT (mean annual air temperature).

Figure 3.11a-c Dependence of temperature differences between MAAT, MAGST and TTOP along the thermal catena. Results are plotted as a function of MAAT (a), MAGST (b) and TTOP (c).

Figure 3.11 shows the dependence of temperature differences between MAAT, MAGST and TTOP for each temperature along the catena. Results are plotted as a function of MAAT (Figure 3.11a), MAGST (Figure 3.11b) and TTOP (Figure 3.11c). Each of the temperature differences has a maximum value, occurring at the point in the catena where $MAAT = -4.12^{\circ}\text{C}$, $MAGST = 1.76^{\circ}\text{C}$, and $TTOP = -0.004^{\circ}\text{C}$. The correspondence of these peaks with the transition from permafrost to seasonally frozen ground suggests that the mean annual temperature at the bottom of the active layer (TTOP) is the critical temperature controlling the thermal regime (rather than at the surface or at some depth within the active layer). Further, the correspondence of 0°C TTOP with maximum active layer depth suggests that the temperature differences in Figure 3.11 may depend on seasonal ground heat storage, which depends primarily on active layer depth.

Finding a relationship between MAAT and TTOP is more complicated for permafrost conditions than for seasonally frozen ground because of the different annual sequences that occur with the two conditions. In seasonally frozen ground, summer ground temperatures can track the surface temperature using a conductive or quasi-conductive model (Beltrami, 2001), while in winter ground temperatures are influenced by the presence of a freezing front and snow cover. In contrast, ground temperatures in permafrost are influenced by active layer thawing in summer and by active layer freezeback and snow cover in winter. Where a snow cover is present, there is no part of the year in which permafrost ground temperature can respond to air temperature without these influences.

3.3 Prospect

The processes and behaviours described in this chapter must be considered in the attempt to move between the relatively complete physical representation of the thermal behaviour of the ground in the TONE numerical model and the limited parameters of the **TTOP** model. The seasons of the active layer defined in this chapter identify the different processes that dominate the thermal regime at different times of the year, and can be used to explain (as in Chapters 5 and 6) the sensitivities of the parameters of the **TTOP** model, and guide in parameterization. The ultimate utility of the **TTOP** model will depend on how well its parameters represent the processes determining the mean annual temperature at the top of permafrost. The following chapters describe the attempt to account for the effects of these processes on TTOP.

Chapter 4: The thermal conductivity ratio

The frozen and thawed thermal conductivities of the ground combine to form the ratio r_K in the **TTOP** model:

$$r_K = \frac{k_r}{k_F} \quad (4.1)$$

The frozen thermal conductivity of moist soils is usually higher than the unfrozen thermal conductivity, because the conductivity of ice is almost four times the conductivity of liquid water. As a result, r_K is usually less than one, although exceptions to this are discussed in Section 4.2.1. The conductivity ratio is the only explicit soil parameter in the **TTOP** model, although soil thermal properties influence the n factors, as discussed in Chapters 5 and 6.

The soil properties influencing r_K are examined using theoretical and empirical thermal conductivity models in this chapter. In addition, departures from the assumption in the **TTOP** model that r_K is constant are discussed. Finally, the effects of unfrozen water on the thermal conductivity of frozen soils in the context of the **TTOP** model are also examined.

4.1 Thermal conductivity models

In pure conduction, upper and lower limits on the thermal conductivity of mixtures can be defined by assuming that the constituents are arranged in parallel or in series with respect to the direction of heat flow. The maximum conductivity of mixtures is for parallel arrangements, and is determined by the arithmetic mean conductivity of the constituents:

$$k = \sum_{i=1}^n k_i x_i \quad (4.2)$$

where

k = bulk thermal conductivity

k_i = thermal conductivity of soil constituent i

x_i = volume fraction of soil constituent i

n = number of soil constituents

The minimum bulk conductivity of mixtures is for arrangements in series, and is determined by the harmonic mean (Farouki, 1981):

$$k = \frac{1}{\sum_{i=1}^n \frac{x_i}{k_i}} \quad (4.3)$$

At the microscopic scale, natural soils are a complex series-parallel arrangement of constituents, so that the limits defined by equations (4.2) and (4.3) are much wider than the range of natural variation of soil thermal conductivity. Several authors (Sass *et al.* 1971, Judge 1973, Johansen 1975, Lunardini 1981) employ the geometric mean as an average value between equations (4.2) and (4.3):

$$k = \prod_{i=1}^n k_i^{x_i} \quad (4.4)$$

Equation (4.4) has been found to work well over a wide range of porosities, although it has no physical basis (Touloukian *et al.*, 1980). It provides a reasonable estimate for mixtures in which the conductivities of the constituents differ by less than a factor “not much larger than 10” (Farouki 1981, p.21), and so is considered valid for saturated soils (Johansen, 1975). Because the conductivities of the soil solids do not change significantly with temperature (Touloukian *et al.* 1980; see Section 4.2 below), r_K for saturated soils can be estimated from the ratio of frozen and thawed conductivities estimated from equation (4.4). Canceling common terms ($k_i x_i$ for all soil solids), r_K for soils without unfrozen water (Lunardini, 1981) yields:

$$r_K = \frac{k_{water}^n}{k_{ice}^n} \quad (4.5)$$

where

k_{water} = thermal conductivity of water

k_{ice} = thermal conductivity of ice

n = soil porosity

For soils with unfrozen water (Johansen, 1975):

$$r_K = \frac{k_{water}^n}{k_{ice}^{(n-w_u)} k_{water}^{w_u}} \quad (4.6)$$

where w_u = soil volumetric unfrozen water

Figure 4.1 shows the ratios determined from equations (4.2) to (4.4) for saturated soils with quartz ($k_S = 8.4 \text{ Wm}^{-1}\text{K}^{-1}$) or clay ($k_S = 2.5 \text{ Wm}^{-1}\text{K}^{-1}$) soil minerals. The wide divergence between the series and parallel flow r_K functions demonstrates the importance of soil structure (that is, the relative importance of series and parallel flow) in determining soil thermal conductivity (Farouki, 1981). The maximum difference in r_K between series and parallel heat flow arrangements occurs with a porosity of about 34%; the degree of divergence between the series and parallel curves is higher with a higher value of k_S .

Soil thermal conductivity (k_T and k_F) may be determined by direct testing or estimated from relationships to soil physical properties (Farouki, 1982). Measured (laboratory or field) thermal conductivity values will include non-conductive heat-flow effects, and so will differ from estimates based on theory (unless non-conductive heat flow processes are included in the conductivity model). A number of theoretical and empirical thermal conductivity calculation methods have been proposed, which can be used to examine r_K . Three calculation methods are examined below, one based on

Figure 4.1 Thermal conductivity ratios for soil-water mixtures, assuming all series or all parallel heat flow. Thermal conductivity ratio using the geometric mean equation is shown for comparison. Ratios for series and parallel flow calculated assuming saturated soils with quartz ($k_s = 8.4 \text{ Wm}^{-1}\text{K}^{-1}$) or clay ($k_s = 2.5 \text{ Wm}^{-1}\text{K}^{-1}$) soil minerals. Ice conductivity was assumed $2.1 \text{ Wm}^{-1}\text{K}^{-1}$; water conductivity assumed $0.57 \text{ Wm}^{-1}\text{K}^{-1}$.

the empirical description of laboratory data (Kersten, 1949), one on the application of the theory of electrical conductivity of mixtures (DeVries, 1957), and one on a comprehensive empirical summary of new and existing thermal conductivity data (Johansen, 1975). Using these three methods, relationships of r_k to properties of saturated soils are compared. Of the several methods evaluated by Farouki (1981), Johansen's method was found to replicate test values best: Johansen's models for mineral and organic soils are therefore used to explore the sensitivity of r_k to soil physical properties for unsaturated soils. A second empirical model for the thermal properties of peat soils (Brovka and Rovdan, 1999) is also compared to Johansen's organic soil model.

4.1.1 Saturated Soils

Kersten

Kersten (1949) measured the thermal conductivity of coarse and fine-grained soils using the standard steady state conductivity test apparatus, the guarded hot-plate. Based on thermal conductivity tests for "just four or five soils" (Farouki, 1981) for each soil type, Kersten determined the following conductivity equations for sandy and clay-silt soils:

Sandy soil:

$$k_T = 0.1442[0.7 \log w + 0.4]10^{0.6243\gamma_d} \quad (4.7)$$

$$k_F = 0.01096(10)^{0.8116\gamma_d} + 0.00461(10)^{0.9115\gamma_d} w \quad (4.8)$$

Clay-silt soil:

$$k_T = 0.1442[0.9 \log w - 0.2]10^{0.6243\gamma_d} \quad (4.9)$$

$$k_F = 0.001442(10)^{1.373\gamma_d} + 0.01226(10)^{0.4994\gamma_d} w \quad (4.10)$$

where

w = gravimetric water content, dimensionless

γ_d = soil dry bulk density (g cm^{-3})

Some authors (e.g., Farouki 1981, Williams and Smith 1989) question the validity of conductivity values obtained using steady-state test methods, since prolonged testing under a temperature gradient may induce changes in moisture distribution in the specimen. In comparing several models, Farouki (1982) found that equation (4.9) (Kersten's fine-grained soil) gave significantly higher estimates of k_T than the other models examined.

De Vries

De Vries (1963) applied Raleigh's model for the effective electrical conductivity of mixtures of randomly dispersed ellipsoids in a continuous medium of air or water:

$$k = \frac{x_f k_f + F x_s k_s}{x_f + F x_s} \quad (4.11)$$

where

$$F = \frac{1}{3} \sum_{a,b,c} \left[1 + \left(\frac{k_s}{k_f} - 1 \right) g_a \right]^{-1}$$

and

$$g_a + g_b + g_c = 1$$

where

k_s = thermal conductivity of solids

k_f = thermal conductivity of fluid

F = "shape factor"

x = volume fraction

g = relative ellipsoid axis length

A somewhat more elaborate equation applies to partially saturated soils, but is not considered in this thesis. Farouki (1981) points out that in practice the ellipsoid axis lengths are “parameters that are adjusted to fit the empirical data”.

Johansen

Johansen’s (1975) equations for frozen and thawed conductivities include the effects of bulk density, moisture content, unfrozen water content, and mineralogy. For saturated unfrozen soils,

$$k_{Tsat} = k_s^{(1-n)} k_w^n \quad (4.12)$$

where

n = porosity

k_s = thermal conductivity of soil solids

k_w = thermal conductivity of water

For saturated frozen soils with unfrozen water content w_u :

$$k_{Fsat} = k_s^{(1-n)} k_i^{(n-w_u)} k_w^{w_u} \quad (4.13)$$

where

k_i = thermal conductivity of ice

w_u = soil unfrozen water content (volume fraction)

Equations (4.12) and (4.13) are a form of equation (4.4). The thermal conductivity of the soil solids is estimated from the quartz fraction of the soil minerals, using equation (4.4), assuming that the thermal conductivity of quartz¹ is $7.7 \text{ Wm}^{-1}\text{K}^{-1}$ and that of all other soil minerals is $2.0 \text{ Wm}^{-1}\text{K}^{-1}$.

¹ This value was adopted by Johansen (1975) despite higher values for the conductivity of quartz published elsewhere (e.g., Farouki 1981).

Conductivity ratios determined for saturated soils for Kersten's, de Vries' and Johansen's equations are shown in Figure 4.2. R_K from the geometric mean equation (4.5) is also shown for comparison. With the exception of Kersten's fine-grained (silty clay) material, r_K values for the different models agree within ± 0.04 over most of the range of soil porosities. The significant discrepancy for Kersten's fine-grained soil at intermediate moisture contents may be due to the effect of moisture flow on measured values as mentioned above. It is also possible that different sets of soils were used to calculate k_T and k_F .

4.1.2 Unsaturated soils

Unsaturated mineral soils

For dry mineral soils, Johansen gives separate equations for natural soils and crushed rock materials, based on empirical data. For natural soils:

$$k_{dry} = \frac{0.135\gamma_d + 64.7}{2700 - 0.947\gamma_d} \quad (4.14)$$

where

γ_d = soil dry bulk density (kg m^{-3})

At intermediate moisture contents, conductivities are interpolated between dry and saturated conditions, using different scaling schemes for k_T for each soil type, based on empirical data. For coarse-grained soil:

$$k_T = k_{dry} + (k_{sat} - k_{dry})(0.7 \log S_r + 1) \quad (4.15)$$

For fine-grained soil:

$$k_T = k_{dry} + (k_{sat} - k_{dry})(\log S_r + 1) \quad (4.16)$$

Figure 4.2 Thermal conductivity ratios for saturated soils using k_F and k_T relationships from Kersten (1949), de Vries (1974), and from the geometric mean equation (used by Johansen 1975 and others). Conductivity of ice assumed $2.1 \text{ Wm}^{-1}\text{K}^{-1}$; conductivity of water assumed $0.57 \text{ Wm}^{-1}\text{K}^{-1}$.

For both soil classes, k_F is a linear function of the saturation ratio:

$$k_F = k_{dry} + (k_{sat} - k_{dry}) S_r \quad (4.17)$$

where

S_R = saturation ratio (degree of saturation)

k_{sat} = saturated thermal conductivity

k_{dry} = dry thermal conductivity

If a particle density of 2650 kg m^{-3} is assumed, the bulk density and saturation ratio can be manipulated to obtain gravimetric or volumetric water content as alternative parameters. Of these, volumetric water content yields a relationship to the conductivity ratio that is the least sensitive to bulk density.

For most mineral soils, quartz content is the most important influence on the thermal conductivity of the soil solids, since its conductivity ($8.4 \text{ Wm}^{-1}\text{K}^{-1}$) is much higher than the other typical soil minerals (typically $2\text{--}3 \text{ Wm}^{-1}\text{K}^{-1}$) (Farouki, 1981). Figure 4.3 shows unsaturated thermal conductivity ratios for soils with 100% and 0% quartz contents. Except at extremely low moisture contents or extremely high bulk densities, the effect of quartz content on the conductivity ratio is negligible.

Figure 4.4 shows relationships between volumetric water content and r_K for fine and coarse-grained soils at several bulk densities. R_K from the geometric mean equation is included for comparison. Results for fine-grained material group around the geometric mean function, while r_K for coarse-grained material is higher than from the geometric mean below about 40% at moisture content. R_K for coarse-grained soils is less sensitive to bulk density than fine-grained soils, and is higher than that for fine-grained soils at water contents less than 30%.

At very low moisture content, r_K exceeds unity for coarse-grained soils.

Farouki (1981, p. 45) cites several studies that report that r_K is greater than unity in

Figure 4.3 Thermal conductivity ratio as a function of volumetric moisture content for soils with soil minerals comprised of 100% or 0% quartz. Examples include coarse- and fine-grained soils with high and low bulk densities. Calculated using k_F and k_T relationships from Johansen (1975).

Figure 4.4 Thermal conductivity ratio as a function of volumetric moisture content for coarse and fine-grained soils for a variety of bulk density values (indicated in g cm^{-3} by numbers adjacent to curves). Calculated using k_F and k_T relationships from Johansen (1975). Soil minerals assumed 50% quartz. Ratio determined from geometric mean equation is shown for comparison.

coarse-grained materials at low moisture content. Penner et al. (1975) suggest that the effect is due to the removal of liquid water forming bridges at grain contacts to form pore ice upon freezing, thereby reducing conduction pathway efficiency when frozen. The difference at low moisture contents may also be due to higher vapour flow in coarse material, either because of higher vapour diffusivity in the larger pores of coarse-grained material, or to an enhanced vapour pressure gradient in coarse-grained materials due to their typically higher conductivity.

Soil unfrozen water

Figure 4.5a shows r_k for a fine-grained soil with unfrozen water, using equation (4.13) for saturated k_F . R_K increases with increasing unfrozen water content, as k_F decreases. These curves do not follow the geometric mean relationship, in part because the water that changes to ice is less than the total water content. Figure 4.5b shows the same r_k values plotted against the *freezing* moisture content (total volumetric moisture content minus unfrozen moisture content), and shows a much closer relationship to the geometric mean function.

Temperature-dependent unfrozen water content is a concern in the **TTOP** formulation, since the annual temperature cycle results in a range of values for k_F through the year and at different depths. Accounting for unfrozen water content in the **TTOP** model is considered in Section 4.3 below.

4.1.3 Organic soils

As with mineral soils, Johansen's model scales the thermal properties of organic soils between dry and saturated conductivities using the saturation ratio:

Figure 4.5 Thermal conductivity ratios for fine-grained soils with volumetric unfrozen water content ranging from 0 to 12%. **a)** Shown as a function of total volumetric water content. **b)** Shown as a function of volumetric frozen water content (total – unfrozen). Ratio determined from geometric mean equation is shown for comparison.

$$k_T = k_{dry} + (k_{sat} - k_{dry}) S_R^2 \quad (4.18)$$

$$k_F = k_{dry} \left(\frac{k_{sat}}{k_{dry}} \right)^{S_r} \quad (4.19)$$

In Johansen's model, the thermal conductivity of the solid organic content is used only to determine the thermal conductivity when dry and saturated ($S_r = 0$ and 1). Farouki suggests a typical value for k of $0.05 \text{ Wm}^{-1}\text{K}^{-1}$ for dry organic soils², while Fitzgibbon (1981) gives a value of $0.058 \text{ Wm}^{-1}\text{K}^{-1}$ for the conductivity of dry sphagnum peat. For saturated organic soil, equations (4.12) to (4.13) apply.

Brovka and Rovdan (1999) developed empirical models for peat-sand mixtures, based on laboratory measurements using mixtures of sedge peat and quartz sand:

$$k = a_0 + a_1 x_w + a_2 x_s x_p + a_3 x_s x_w + a_4 x_s^2 + a_5 x_p^2 + a_6 x_w^2 \quad (4.20)$$

for k_T

$$\begin{aligned} a_0 &= 1.98 \times 10^{-2} \\ a_1 &= 6.56 \times 10^{-4} \\ a_2 &= -1.22 \times 10^{-6} \\ a_3 &= 8.69 \times 10^{-7} \end{aligned}$$

$$\begin{aligned} a_4 &= 6.43 \times 10^{-7} \\ a_5 &= 2.11 \times 10^{-7} \\ a_6 &= -1.24 \times 10^{-7} \end{aligned}$$

for k_F

$$\begin{aligned} a_0 &= 2.38 \times 10^{-2} \\ a_1 &= 5.37 \times 10^{-4} \\ a_2 &= -3.47 \times 10^{-6} \\ a_3 &= 2.75 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} a_4 &= 8.81 \times 10^{-7} \\ a_5 &= 2.04 \times 10^{-7} \\ a_6 &= 1.71 \times 10^{-6} \end{aligned}$$

² Farouki (1981) gives a value of $0.5 \text{ Wm}^{-1}\text{K}^{-1}$ for dry frozen peat: I assume this is a typographic error, and the value for dry unfrozen peat is used here.

where

Subscripts: s = sand; p = peat; w = water/ice

x = specific mass of constituent (= density $\times$ volume fraction)

The range of water contents in organic soils is much larger than that for mineral soils. Organic soil bulk density is typically in the range from 100 to 500 kg m⁻³ (Päivänen, 1969), with values as low as 40 kg m⁻³ for material within 5 cm of the ground surface (Riseborough, 1985). Assuming a particle density of 1500 kg m⁻³ for soil organic material (the midpoint of the range reported by Brady and Weil 1999), this represents a range of porosity for organic soil between 66% and 97%.

As with mineral soil, expressing r_K as a function of volumetric moisture rather than saturation ratio produces a relationship that is only weakly dependent on the contribution of the particle component to the thermal conductivity. Figure 4.6 shows this relationship for peat, using equations (4.18) and (4.19) for a range of porosities, and using equation (4.20) for a range of peat densities (and assuming 0% sand). R_K for the geometric mean (equation (4.5)) is shown for comparison. The curves cluster near the geometric mean function, although the variation is larger than for mineral soils. In general, differences in porosity have less impact on r_K in Johansen's equations where k_{dry} is lower. For r_K in Johansen's equations, differences are greatest at low to intermediate moisture contents. In Brovka and Rovdan's equations, r_K is only weakly dependent on peat density, and changes from higher than the geometric mean at low moisture content to lower than the geometric mean at high moisture content.

Figure 4.6 Conductivity ratio as a function of volumetric moisture content for organic soils, using Johansen's (1975) and Brovka and Rovdan's (1999) conductivity functions. For Johansen's functions, soil porosity ranges from 60% to 90%, with dry soil thermal conductivity of $0.05 \text{ W m}^{-1} \text{ K}^{-1}$. For Brovka and Rovdan's (1999) functions, sand content was assumed 0%, with densities of 50 and 500 kg m^{-3} .

4.2 Departures from bimodal thermal conductivity

The conductivity ratio is not constant where thermal properties change over space or time, or as a function of temperature (other than at the freezing temperature). All of these conditions commonly occur in permafrost regions. Soil density typically varies with depth and with micro-topographic features such as earth hummocks. Soil moisture commonly varies through the thaw season, and may vary from year to year. Even if soil physical properties were uniform, the thermal conductivity of moist soils varies with temperature due to changes in unfrozen water content. Application of the **TTOP** model to soils having significant unfrozen water content is discussed in the next section; other effects are discussed in this section.

Changes in conductivity as a function of temperature may be due to changes in the conductivity of the soil constituents. Touloukian *et al.* (1980) suggest that the thermal conductivity of minerals varies approximately according to the inverse of absolute temperature, so that the thermal conductivity of soil minerals changes by 7 to 8% between in the 20° temperature range on either side of 0°C. Similarly, the thermal conductivity of ice at -20°C is 9% higher than at 0°C, while the thermal conductivity of water at +20°C is 7% higher than at 0°C. However, these effects are ignored in the remainder of this thesis, since variation of this magnitude is small relative to the uncertainties in r_K . R_K relationships used in this chapter and elsewhere in this thesis are based on thermal conductivity values for liquid water and ice of 0.57 and 2.1 $\text{Wm}^{-1}\text{K}^{-1}$, respectively.

4.2.1 Spatial and temporal variation

The thermal conductivity of the active layer is not constant with depth or with time. In seasonally frozen ground, soil density and degree of saturation typically increase with depth (Brady and Weil, 1999), while in permafrost soil dry bulk density frequently decreases with depth, due to the accumulation of excess ground ice by ice segregation. In saturated seasonally frozen ground therefore, r_K will tend to decrease with depth, while in permafrost or in unsaturated seasonally frozen ground soil, (from equation (4.6)) r_K will increase with depth because of the vertical profile of volumetric water content. The water balance of the active layer commonly includes decreasing saturation through the thaw season, with a reversal prior to freeze-up (Hinkel *et al.*, 1996).

The greatest contrast in active layer properties is found where a layer of organic material overlies mineral soil. Such organic material is common in boreal (e.g., Riseborough 1985, Burn 1998, 2000) and tundra (e.g., Klene 2000, Romanovsky and Osterkamp 1997) environments, and is typically comprised of a living surface layer above a dead mat of fibrous material on the order of 10-30 cm thick. Bulk densities on the order of 100 to 200 kg m⁻³ are common, with volumetric water content varying from almost 0% to 97%.

4.2.2 Non-conductive heat transfer

Non-conductive heat transfer processes can be divided between those that are related to the temperature gradient and those that are independent of the temperature gradient. This distinction is important because it divides processes between those that can be incorporated into an apparent thermal conductivity function and those that must

be accommodated separately. Both will influence the apparent value of r_K , but only the former can be estimated from the heat flow conditions.

Fluxes related to the temperature gradient include convection of sensible and latent heat, and solute and vapour diffusion along temperature related concentration gradients. Non-temperature-gradient related flows include melt- and rain-water infiltration and groundwater flow under gravity, and pore-water movement under capillary pressure and osmotic gradients (Kane *et al.*, 2001).

Heat transfer processes involved in water vapour flow will depend on the vapour pressure gradient. In all but very dry soils, soil pores can be assumed to be at saturation vapour pressure (Hillel, 1980). Because the relationship of saturation vapour to temperature is non-linear, vapour pressure gradients in the soil are a function of temperature as well as the temperature gradient. The same temperature gradient at higher temperature will induce greater vapour flow, resulting in a larger apparent thermal conductivity. De Vries (1974) suggests that the apparent thermal conductivity of vapour-saturated soil air at 0°C is about twice that of dry air as a result of vapour diffusion along the temperature gradient; at 20°C, the apparent thermal conductivity is about three times the value for dry air. The difference in the contribution of vapour flow as a function of temperature will tend to increase r_K for unsaturated soils.

Non-conductive transfer mechanisms within the seasonally frozen/unfrozen layer include radiation and sensible and latent heat transfers associated with fluid flow. Radiation transfers heat across pore spaces where there is a temperature gradient, although very large pore spaces (gravel and coarser) and steep temperature gradients are required for this to be significant (Johansen 1975, Farouki 1981). Sensible heat

flow is associated with movement of air and groundwater, and the infiltration of melt-water and rainwater. Latent heat is transported by *internal distillation* within unsaturated soil (Hinkel and Outcalt, 1994), in which water evaporates, is transferred along a temperature-induced vapour pressure gradient, and condenses at lower temperatures. Heat can also be transferred with material movements due to osmosis associated with temperature and concentration gradients.

Kane *et al.* (2001) suggest that non-conductive processes are important for specific periods of the year. For example, Outcalt *et al.* (1990) inferred solute movement across the isothermal zero curtain zone during active layer freezeback, based on measurements of ion concentration; Kane *et al.* (2001) suggest that meltwater infiltration is the most significant non-conductive energy flux into the active layer.

The effects of non-conductive processes were assessed by Outcalt *et al.* (1998) by comparing a simulated active layer thermal regime (using measured thermal properties) to the measured thermal regime. Temperature residuals were used to infer the presence of specific non-conductive thermal processes, in particular snow-melt infiltration and internal evaporation within the active layer during the thaw season. In contrast, Osterkamp and Romanovsky (1997) determined the effective thermal properties of the active layer by fitting simulated temperature profiles to temperature data, adjusting parameter values until the fit was considered satisfactory. While results using either approach are instructive, neither is clearly superior: ascribing residuals to particular processes is not necessarily valid, especially when the measured properties driving the model are subject to errors. Fitting model parameters using data, especially where several parameters are involved, interferes with the validation process (Randall

and Wielicki, 1997). In the context of the **TTOP** model, these studies demonstrate together that non-conductive processes contribute to the thermal regime of the active layer, but that their effect can be accommodated by apparent thermal conductivities, and therefore by an apparent thermal conductivity ratio.

4.3 Effect of unfrozen water

The interaction of water and soil particle surfaces within small pores prevents the transition of water from liquid to solid at a single freezing temperature (Williams and Smith, 1989), so that many physical properties of frozen soil change continuously over a range of temperatures below 0°C. This causes the thermal conductivity of frozen soil to vary with temperature (Johansen 1975, Kay *et al.* 1981, Riseborough *et al.* 1983, Romanovsky and Osterkamp 2000), so that the conductivity ratio is not a single value. The purpose of this section is to present a parameterization of the effect of unfrozen water in the **TTOP** model.

Unfrozen water alters the thermal regime of the ground because the thermal properties of the ground depend on temperature. The thermal conductivity of the soil depends on the relative proportions of soil solids, air, water and ice (Penner 1970, Johansen 1975, Kay *et al.* 1981, Riseborough *et al.* 1983), and the latent heat of fusion is released or absorbed over a range of temperature (Riseborough 1990, Romanovsky and Osterkamp 2000).

Within limits produced by hysteresis, changes in soil chemistry and pore geometry, the relationship between temperature and unfrozen water content can be characterised for individual soils. This relation, the *freezing characteristic*, is an

empirical function. For example, Anderson and Tice (1972) fitted freezing characteristic data for a number of soils to the general equation:

$$w_u = A|T|^B \quad (4.21)$$

where

w_u = gravimetric unfrozen water content, %

A, B = empirical constants

T = temperature (Celsius)

This can be converted to volumetric unfrozen water content:

$$\theta_u = \rho A|T|^B \quad (4.22)$$

where

θ_u = volumetric unfrozen water content, %

ρ = dry bulk density

Equations (4.21) and (4.22) are simply a convenient way of expressing unfrozen water content as a function of temperature. Using values of the empirical parameters A and B for particular soils, the thermal conductivity and apparent heat capacity can be specified as functions of temperature, as discussed below. The particular models used to obtain the thermal properties of the soil are not an essential part of the analysis.

4.3.1 Unfrozen Water and TTOP

Because the thermal conductivity of frozen soil depends on the unfrozen water content, k_F is not a single value in soils with a significant freezing characteristic.

Further, in soils with bimodal thermal properties the thermal offset is restricted to the active layer. If the k_F changes with temperature *below* the active layer, the thermal offset will extend to the depth at which the thermal properties no longer change through the year (either the depth of zero annual amplitude, or the depth at which

temperature variation falls outside the range in which the thermal conductivity is a function of temperature).

Use of the **TTOP** model for such soils requires a method to determine the appropriate value of k_F in the model (equation (2.31)). The appropriate k_F would be an *apparent* value since it must represent an effective seasonal average. Defining the “apparent” frozen conductivity k_{FA} as that frozen conductivity that yields the correct value for T_{TOP} , the value of k_{FA} can be determined *a posteriori* from a known simulated or measured thermal regime. Using known values of the other model parameters in the **TTOP** model, k_{FA} can be calculated:

$$k_{FA} = \frac{k_T I_{TS}}{(T_{TOP} P + I_{FS})}, \quad T_{TOP} < 0$$

and

$$k_{FA} = \frac{k_T (I_{TS} - T_{TOP} P)}{I_{FS}}, \quad T_{TOP} > 0 \quad (4.23)$$

The following analysis examines k_{FA} calculated from equation (4.23), for points along a thermal catena for each of three soils.

4.3.2 Method

Catena simulations were generated for three soils representing a wide range of freezing characteristics. The physical property assumptions for these soils are shown in Table 4.1. Soils were assumed saturated, so that $\theta_v = n$. Freezing characteristic curves for the soils are shown in Figure 4.7a, and the relationships of k_F to temperature are shown in Figure 4.7b. Thermal conductivity values were estimated using equations (4.12) and (4.13).

Table 4.1 Soil properties used in simulations: porosity (n), dry bulk density (ρ , kg m^{-3}), A and B values from Anderson and Tice (1972) used in equations 4.21, 4.22, 4.24 and 4.25, assumed quartz content of soil solids, and unfrozen thermal conductivity (k_T , $\text{Wm}^{-1}\text{K}^{-1}$) from equation 4.12. Soils were assumed saturated, so that $\theta_v = n$.

Soil	n	ρ	A	B	Quartz %	k_T
Silt (Fairbanks Silt)	0.44	1484	4.81	-0.33	50%	1.707
Silty Clay (Suffield Silty Clay)	0.44	1484	13.93	-0.31	50%	1.707
Clay (Kaolinite)	0.5	1325	23.80	-0.36	0%	1.081

Figure 4.7 **a)** Volumetric unfrozen water content (dimensionless) for soils used in soil unfrozen water simulations. **b)** Temperature-dependent thermal conductivity functions for soils used in soil unfrozen water simulations.

On a seasonal basis, the dominant thermal effect of the freezing characteristic is to produce a significantly increased “apparent” heat capacity for soils for the temperature range where the liquid water content changes significantly with the temperature:

$$C = \rho c = \sum x_i \rho_i c_i + L \left(\frac{\partial \theta_u}{\partial T} \right) \quad (4.24)$$

where

x = volume fraction

i = subscript identifying component (mineral, ice, water, etc.)

C = volumetric heat capacity, $\text{J m}^{-3}\text{K}^{-1}$

c = specific heat, J kg^{-1}

ρ = density, kg m^{-3}

L = volumetric latent heat of fusion, J m^{-3}

$\left(\frac{\partial \theta_u}{\partial T} \right)$ = rate of change of unfrozen water content, K^{-1}

The latent-heat term can be calculated from the first derivative of equation (4.22):

$$L \left(\frac{\partial \theta_u}{\partial T} \right) = L \rho A B |T|^{B-1} \quad (4.25)$$

Equations (4.22) and (4.24) were used to generate apparent heat capacity functions for the test soils. Other specifications for numerical catena modeling were:

- The thermal properties (freezing characteristic, thermal conductivity, apparent heat capacity) for each soil were defined as discussed above (Figures 4.7a and b; equations (4.12), (4.13), (4.24), and (4.25)).
- Air temperature was defined as a sine wave with a period of 365 days and amplitude of 20°C . Mean annual temperature was varied from -12°C to $+8^\circ\text{C}$, at 2°C intervals, generating 11 cases along each thermal catena.

- All cases included snow cover with a maximum depth of 0.3 m. Snow accumulation commenced one day after the initiation of surface freezing conditions, with a linear increase to the maximum depth over the initial 1/3 of the freezing season.

4.3.3 Apparent thermal conductivity

The k_{FA} values for the three soils are shown in Figure 4.8 as a function of the numerically determined TTOP (the mean annual temperature of the first point in the mesh below the maximum active layer depth). For all soils, k_{FA} is lower at higher values of TTOP, indicating a decreasing mean frozen conductivity as TTOP increases. The value of k_{FA} ranges from near k_T in seasonally frozen ground soils (where the temperature of seasonally frozen ground under the snow remains very close to 0°C) to that expected for cold soil with little unfrozen water. The apparent seasonal frozen conductivity decreases as the mean temperature of the frozen active layer increases, with slightly greater dependence on temperature below TTOP = 0°C. The question is how to determine a method of estimating the appropriate value of k_{FA} for a given thermal regime.

The temperature-dependent conductivity function (Figure 4.7b) can be used as a reference against which the observed behaviour in Figure 4.8 can be scaled. In the context of the **TTOP** model, the connection between the surface climate and the mean temperature of the *frozen* active layer (or seasonal freezing layer in seasonally frozen ground) is the surface freezing index. The mean temperature within the active layer during freezing will typically be colder where the annual surface freezing index is higher, resulting in an active layer frozen conductivity that is higher on average, as

Figure 4.8 Apparent frozen conductivity (k_{FA}) determined from equation 4.22 using simulation results.

shown in Figure 4.8. At the ground surface, the mean freezing temperature is $-I_{FS}$ divided by the number of days in the freezing season, which will be colder than $-I_{FS}/P$. At the base of the seasonal freezing layer the mean freezing temperature is 0°C in seasonally frozen ground and is equal to T_{TOP} in permafrost. As a measure of the mean temperature of the frozen active layer as a whole, $-I_{FS}/P$ scales effectively between these limits. Figure 4.9 shows the catenae of k_{FA} for each of the soils plotted as a function of $-I_{FS}/P$ (rather than as a function of T_{TOP} as in Figure 4.8), together with the thermal conductivity curves of Figure 4.7b. The relationships correspond well, although the k_{FA} functions diverge from the temperature-dependent conductivity functions at lower temperatures. The departures in each of the relationships between about -3 and -1°C correspond to the transition from $T_{TOP} < 0^{\circ}\text{C}$ (permafrost) to $T_{TOP} > 0^{\circ}\text{C}$ (seasonally frozen ground) in Figure 4.8. The greatest departure is for the clay (kaolinite) where $-I_{FS}/P$ is about -8°C , and k_F diverges from k_{FA} by about 8%.

Figure 4.10a shows the error in T_{TOP} that would result if k_F at $-I_{FS}/P$ (Figure 4.7b) were used as an estimate of k_{FA} to determine T_{TOP} . Errors were calculated by using the **TTOP** model with I_{FS} and I_{TS} from catena results together with k_{FA} at temperature $-I_{FS}/P$ for k_F , and comparing the resulting T_{TOP} values with the numerically simulated T_{TOP} values. Maximum errors were 0.04°C for the silt, 0.08°C for the silty clay, and 0.17°C for the kaolinite. For all soils, the maximum errors occur at temperatures close to 0°C in seasonally frozen ground, where the model applies the incorrect form of the **TTOP** model (that is, it applies the permafrost equation when the seasonally frozen ground equation is appropriate).

Figure 4.9 Comparison of k_{FA} versus $-I_F/P$ and k_F versus temperature (Figure 4.8) for the three test soils.

Figure 4.10 a) Error in estimated TTOP for soils with unfrozen water content, using conductivity based on $-I_{FS}/P$. **b)** Error in estimated TTOP for soils with unfrozen water content, using a single value for k_F .

Failure to take the effect of unfrozen water into account will produce larger errors. For example, use of a single value for k_F based on the frozen conductivity value at -6°C for each soil (Figure 4.10b) produces maximum errors of 0.11°C for the silt, 0.27°C for the silty clay, and 0.46°C for the clay. As in Figure 4.10a, the maximum error for each of these occurs near 0°C . For comparison, calculations based on the results presented by Romanovsky and Osterkamp (2000, Figure 8) show a discrepancy of about 0.4°C between the mean annual temperature determined by the **TTOP** equation (with a single frozen conductivity) and that determined by simulation of the thermal regime using a temperature-dependent freezing characteristic very similar to the silt used in this analysis.

These results provide a straightforward procedure for estimating T_{TOP} for situations where the underlying soil has a significant freezing characteristic, if the thermal conductivity function for the soil is known:

1. Using the surface freezing index, calculate $-I_{\text{FS}}/P$
2. Determine k_{FA} based on $-I_{\text{FS}}/P$ and the temperature-dependent thermal conductivity function.
3. Calculate the thermal offset, based on equation (2.31).

A counterintuitive result of this analysis is that the largest errors in k_{FA} occur at relatively low temperatures. This may be due to the way in which the thermal offset is distributed through the active layer: the shift in mean annual temperature is greatest at the ground surface, with the rate of change declining with depth. The near-surface layer experiences the most rapid freezing and cooling, so that the seasonal mean unfrozen water content will be that of relatively cold soil. This effect may be most

pronounced with the relatively shallow active layers of colder permafrost.

Nonetheless, the greatest errors in T_{TOP} resulting from this parameterization occur near 0°C .

Errors on the order of 0.15°C are well within the likely uncertainty due to other parameters of the **TTOP** model. While there are other challenges for the successful application of the model, the effect of unfrozen water on the frozen thermal conductivity appears to yield to simple parameterization.

4.4 Conclusions

The analyses presented in Section 4.1 indicate the relative importance of the physical properties of soils in determining the thermal conductivity ratio. Results from thermal conductivity models for saturated and unsaturated mineral and organic soils show that r_K is a reasonably reliable function of soil moisture content, although at low moisture contents r_K can vary by as much as 30% above or below the value from the geometric mean equation. The presence of non-conductive heat transfer mechanisms, particularly vapour flow in unsaturated soil at high temperatures, can account for some of the variation. Nonetheless, equation (4.6) appears to be useful for estimation of r_K , especially when detailed physical soil parameters are unknown.

Uncertainty in r_K between the various thermal conductivity prediction models varies, and appears to be greatest at low moisture content. At intermediate moisture contents, the maximum difference in r_K between conductivity models is about 0.3. Applying this range of r_K (0.45 to 0.75) to the standard catena with 25 cm maximum snow cover results in a maximum errors in T_{TOP} of about $\pm 0.8^{\circ}\text{C}$ (occurring near $T_{TOP} = 0^{\circ}\text{C}$).

Parameterization of the effect of unfrozen water is relatively straightforward because the soil freezing characteristic is a stable property of soils. This approach is unlikely to meet with the same degree of success with the other property variations discussed in Section 4.2. Systematic variation of soil properties laterally and with depth, seasonal trends and inter-annual variation in soil moisture might be handled in a similar manner if the effect of the behaviour could be transformed into a reliable statistical relationship between temperature and parameters describing the variation. Such relationships would likely be subject to large inter-annual variation, and might change in a changing climate.

Given the effects of non-conductive processes together with the variation in properties to be expected in the real world, the r_K results from thermal conductivity models are sufficient for demonstrating the probable range of values that this parameter may take. More work is required to account for the effects of horizontal spatial variability of surface properties, as well as the seasonal variation of active layer moisture content and thermal properties. Methods similar to those employed to account for the effect of unfrozen water in this chapter might be employed to determine the apparent value of r_K that would account for all departures from the ideal bimodal conductivity assumed in the model.

Chapter 5: Snow cover, TTOP and n_F .

Snow cover has long been understood to be the primary control of the difference between air and ground temperatures in cold regions (e.g., Lachenbruch 1959, Smith 1975, Goodrich 1982, Desrochers and Granberg 1988, Taras *et al.* 2002). The relatively efficient energy transfer mechanisms of the atmospheric boundary layer (radiation and convection) are replaced by conduction through snow, limiting the rate at which heat can escape the ground during the coldest portion of the year.

The properties of snow cover change continuously, from the start of snow accumulation in late autumn or early winter until its disappearance in spring. In addition, snow cover properties vary spatially over scales from microscopic to global. The challenge that snow cover presents in the context of a permafrost-climate model is to describe this variability in a way that allows it to be used in predicting its effect on permafrost temperature.

This chapter deals primarily with the effect of conduction through snow cover, recognising that other processes are also important. In the **TTOP** model, the parameter n_F accounts for all of the processes that result in a difference between surface and air temperature for the portion of the year when the temperature is below freezing. Implicitly (Smith and Riseborough 1996, 2002a, Riseborough and Smith 1998) or explicitly (Riseborough, 2001), the difference between I_{FS} and I_{FA} has been ascribed entirely to the effect of snow cover, with snow present for the duration of the freezing season. In this thesis, the former assumption is maintained while the implications of departures from the latter assumption are explored. Results presented in this chapter suggest that an alternative parameterization may be better.

This chapter considers aspects of snow cover in four sections. First, processes in snow cover evolution and the properties of snow are summarised. Second, pertinent aspects of the climatology of snow cover in northern Canada are explored using climate station statistics. Third, literature relating snow cover to thermal conditions in permafrost is summarised. Fourth, the influence of snow cover properties and snow cover climatology on TTOP are explored using numerical modeling, leading to a limited parameterization of snow cover effects.

5.1 Snow thermal properties and processes

As discussed in Chapter 3, three of the four seasons of the active layer defined by Outcalt and Hinkel (1997) involve snow cover. Olssen *et al.* (2003) suggest that the “cold season” in Alaska can be divided into five stages:

1. “Early snow”, beginning with the arrival of snow before the daily maximum air temperature has fallen below 0°C;
2. “Early cold”, commencing when air temperature is consistently below 0°C and concluding with the completion of active layer freezeback (or freezing of soil in the root zone in seasonally frozen ground);
3. “Deep cold”, commencing with rapidly decreasing active layer temperatures, and comprising the cold heart of winter;
4. “Late cold”, characterised by increasing air temperature at the end of winter, and
5. “Thaw”, commencing when minimum daily air temperatures are above 0°C, and concluding with complete melting of the snow pack.

Snow is present in all five stages. The first stage is not included in the freezing season as defined by daily mean temperatures, and does not involve active layer freezing

from the surface. Stages 2 and 5 correspond directly to the freezeback and ablation seasons defined by Outcalt and Hinkel (1997). Stages 3 and 4 correspond to early and late periods of Outcalt and Hinkel's frozen winter season.

The seasonal history of a snow cover determines its properties, which determine its effect on ground temperatures. Snow naturally develops a layered structure, as it arrives in discrete events and undergoes metamorphism between them. However, Colbeck (1991) suggested that, while many processes are critically dependent on the distribution of properties of the strata, the use of averaged values can be sufficient for thermal studies.

5.1.1 Seasonal trends in snow density

As snow accumulates its density increases due to mechanical compaction (grain settling, gravitation, and creep), which is enhanced or retarded by thermally controlled metamorphism (Pomeroy and Brun 2001, Sturm and Holmgren 1998).

The mean density of newly fallen snow has been measured between 70 and 165 kg m⁻³ (Goodison *et al.*, 1981), with much greater variation between individual storms depending on temperature and the strength of winds around the time of deposition (Pomeroy and Brun, 2001). Deposited snow rapidly becomes denser by metamorphism due to thermodynamic imbalances on the crystal surfaces (Colbeck 1987, Gray and Morland 1995). Following this initial stage, metamorphism becomes dominated by mechanical forces and the compaction rate decreases (Gray and Morland 1995, Sturm and Holmgren 1998). Many authors model densification as a linear function of snow overburden pressure, referred to as viscous compaction (Kojima 1967, Sturm and Holmgren 1998, Kongoli and Bland 2000, Pomeroy and Brun 2001, Brown *et al.* 2003).

In subarctic and arctic regions, mechanical compaction can be offset by the development of *depth hoar* (Sturm and Johnson, 1992). Depth hoar is “a type of snow dominated by large, ornamented, delicately connected single crystals that grow when large temperature gradients prevail in the snow cover and induce large water vapour density gradients” (Arons and Colbeck, 1995). Sturm *et al.* (1995) suggest that while a thin basal depth hoar layer is common in tundra and alpine snow covers, in taiga environments it may comprise 50 to 80% of the total snow depth by late winter.

5.1.2 Thermal properties

The volumetric heat capacity of the snow is a linear function of density. For dry snow:

$$C_s = x_i \rho_i c_i + x_a \rho_a c_a \quad (5.1)$$

Ignoring the contribution of air to the heat capacity:

$$\begin{aligned} C_s &= x_i \rho_i c_i \\ \therefore \rho_s &= x_i \rho_i \\ \therefore C_s &= \rho_s c_i \end{aligned} \quad (5.2)$$

Where

Subscripts: subscript i refers to ice; subscript a refers to air; subscript s refers to snow

C_s = volumetric heat capacity of snow, $\text{Jm}^{-3}\text{K}^{-1}$

x = volume fraction of snow component (ice, air)

ρ = density, kg m^{-3}

c = specific heat, J kg^{-1}

Snow consists of air together with water in vapour, liquid and solid forms.

Between -40°C and 0°C , air has a thermal conductivity of 0.022 to $0.025 \text{ Wm}^{-1}\text{K}^{-1}$ while ice has a thermal conductivity of about 2.2 to $2.5 \text{ Wm}^{-1}\text{K}^{-1}$ (depending on crystal orientation and structure) (Williams and Smith, 1989). Because these thermal conductivities differ by a factor of about 100, the bulk conductivity of snow depends

strongly on the physical arrangement of the constituents (Arons and Colbeck, 1995).

Thermal conduction through snow is predominantly through ice grains and ice-grain contacts. Empirical studies have shown that snow conductivity depends primarily on grain size and the connectivity of the ice-grain lattice (Sturm *et al.*, 1997).

The air spaces within the snow are saturated with water vapour, so that temperature gradients within the snow produce a vapour pressure gradient tied to the relationship of saturation vapour pressure to temperature. Vapour is thereby induced to flow along the temperature gradient, augmenting heat transfer within the snow. The additional heat flow associated with vapour flow under temperature gradients is a component of the measured conductivity of snow, so that measured values are referred to as “effective” thermal conductivities. The magnitude of the vapour diffusion coefficient in snow relative to that in air remains uncertain. Summarising several studies of the vapour diffusion coefficient in snow, Colbeck (1993) found that while most theoretical analyses predict a coefficient value lower than that in air, measurements show the coefficient to be on the order of four times greater than the coefficient in air. He suggests that this is most likely due to the difference between the thermal conductivities of air and ice: the macroscopic temperature gradient results in a steeper temperature gradient through the air spaces than through the ice crystals. Taking this “gradient enhancement” effect into account, Sokratov and Maeno (2000) found that the diffusion coefficient in snow is less than that in air, due to the effects of porosity and pathway tortuosity.

While the geometry of individual grains and grain-to-grain connections are understood to be important controls on heat transfer in snow (Sturm *et al.* 1997, Arons and Colbeck 1995), physical models that relate transfer properties directly to the

structural and textural properties of the snow cover have not been developed.

Goodrich (1976), Yen (1981), Sturm *et al.* (1997) and others have undertaken summary surveys of snow thermal conductivity measurements, and most authors relate conductivity to snow density (Figure 5.1).

Sturm *et al.* (1997) found that the thermal conductivity of some types of snow is essentially independent of density, while other types of snow show a positive correlation. They developed a snow density-conductivity model using all available data (Figure 5.2), including that used in the models of Figure 5.1. Within the data in Figure 5.2, snow conductivity does not correlate with density below about 200 kg m^{-3} . Nonetheless, because other relevant properties are not easy to characterise, Sturm *et al.* (1997) concluded that density remains the best property with which to predict conductivity. Depth hoar typically has a low thermal conductivity relative to other types of snow (Sturm and Johnson 1992, Sturm *et al.* 1997). Sturm and Johnson (1992) found that the thermal conductivity of depth hoar near Fairbanks changes through the winter, with no relationship with density. They attribute this to the metamorphic processes involved in hoar crystal development, which produce grain contact points of relatively small area.

5.1.3 Ablation

According to the Oxford English dictionary, ablation is “the loss of solid material by melting, evaporation, or erosion”; snow ablation is usually achieved by snowmelt. Snow ablation is the object of many models used to predict melt water runoff. In the context of geothermal modeling however, snowmelt is important for the prediction of the

Figure 5.1 Snow thermal conductivity functions developed by various investigators, from Sturm et al. (1997).

Figure 5.2 Relationship between thermal conductivity and density of snow, from Sturm *et al.* (1997). Caption of the original figure: *All of our thermal conductivity data as a function of density and snow type (see Table 4). Equation (4) has been superimposed on the data. Below a density of 0.156 g cm^{-3} , a linear regression with y intercept of $0.023 \text{ Wm}^{-1} \text{K}^{-1}$ (k_{air}) is used (solid line); the dashed line shows the continuation of the quadratic equation. 95% confidence intervals are also shown.*

end of snow cover, and the beginning of the active layer thaw season. The freezing season at the ground surface usually ends when the snow cover is completely ablated.

Snowmelt requires exchange of energy between the snow cover and its environment, with the energy available to melt a unit volume of snow determined by (Pomeroy and Brun, 2001):

$$Q_m = Q^* + Q_H + Q_E + Q_G + Q_R - \frac{\partial U}{\partial t} \quad (5.3)$$

where

$$Q^* = K^* + L^*$$

and

Q_m = energy available to melt a unit volume of snow

Q^* = net radiation flux

K^* = net shortwave radiation flux

L^* = net longwave radiation flux

Q_H = sensible heat flux

Q_E = (evaporative) latent flux

Q_G = ground heat flux

Q_R = energy flux from rainfall

$\frac{\partial U}{\partial t}$ = change in internal energy (sensible heat) of the snow

The principal snowmelt period at the end of winter may begin with the arrival of air temperatures above 0°C, or with sufficient incoming solar radiation to raise the temperature of the upper snow layers to the melting point (Pommeroy and Brun, 2001). When snow melts at its upper surface, water percolates through the snow, refreezing if it encounters snow below 0°C. This process of snowpack *ripening* continues until the whole snow cover is at the melting point. If the ground has not frozen under the snow

cover, heat can be supplied to warm the snow from below as well. If the ground beneath the snow cover is frozen, ripening proceeds until the ground surface temperature is 0°C. Once the snow is isothermal at the melting point, $\partial U/\partial t = 0$ and energy is available for snowmelt. Of all of the components of the energy balance in equation (5.3) net radiation is usually largest, with sensible heat flux being of comparable magnitude in advective conditions (Pomeroy and Brun, 2001).

During ripening, water penetrating to the base of the snow pack can refreeze: this requires underlying low permeability frozen soil combined with sufficient heat extraction into the soil (Woo and Heron, 1981). Melt water can penetrate unsaturated frozen ground (Pomeroy and Brun 2001, Gray and Prowse 1992). Kane *et al.* (2001) suggested that melt water infiltration is the most significant non-conductive energy flux into the surface layer.

Ferguson (1999) and Melloh (1999) reviewed a number of simplified snowmelt models, comparing them to more elaborate models based on the physics of the snow pack energy balance. All models consist of some form of energy balance, and may include “some accounting of internal snowpack processes”(Melloh, 1999). Ferguson suggests that snow melt algorithms “illustrate the fundamental problem in all environmental modeling: what is the right balance between scientifically realistic complexity and practically realistic simplicity?”

Physically complete models require meteorological data (air temperature and dew point temperature, precipitation, wind speed, cloud cover, incident solar radiation) and surface properties (albedo, convection coefficient) to estimate the energy balance in equation (5.3). Conceptually, most models consider processes at a point, although many

include additional parameters to account for a forest canopy, and may or may not be designed to account for spatial variation in meteorological conditions or snow depth (Ferguson, 1999). Melloh (1999) compared several simple models derived from Anderson (1976) to physically detailed models (Jordan 1991, Albert and Krajieski 1998).

Given the limited meteorological data usually available, the simplest models are based on air temperature and precipitation alone. Models of intermediate complexity include wind speed and K^* or Q^* . Anderson (1976) nominally identified net radiation as a key parameter, but used daily peak temperature to estimate it. Models based on the detailed physics of snow melt are capable of accurate predictions of snow melt without the need for local parameter calibrations, but require significantly greater meteorological data and other physical parameters.

For geothermal modeling purposes, the thickness and density profile of the ablating snow cover are irrelevant, since the temperature at the ground surface is held at 0°C until the snow is gone. Internal details of the snow ripening process do not influence the ground surface temperature. The simplest model for the ablation process (Pomeroy and Goodison, 1997) relates snowmelt to the thawing degree-day total:

$$SWE_T = M_f I_T \quad (5.4)$$

Where

SWE_T = snow water equivalent ($= \bar{\rho} z$, where $\bar{\rho}$ = mean snow density; z = snow depth)

M_f = melt factor

In the geothermal model TONE, ablation is handled by reducing the snow depth in each time step using equation (5.4). In the context of geothermal modeling, the more

detailed algorithms require more information than is warranted by the limited impact of the ablation period on the annual thermal regime.

Granger and Male (1987) found that M_f varies between 6 and 28 kg m⁻³, and occasionally as high as 50. The number of degree-days required to melt the snow cover can be calculated as:

$$I_M = \frac{\overline{\rho z}}{M_f} \quad (5.5)$$

where

I_M = thawing degree-days required for snow ablation

The effect of ablation on MAGST can be calculated as:

$$\Delta_M = \frac{I_M}{P} = \frac{\overline{\rho z}}{PM_f} \quad (5.6)$$

For a snow cover 50 cm deep and with a density of 250 kg m⁻³, I_M would vary between 2.5 and 21 degree-days, so that Δ_M varies between 0.06°C and 0.007°C for the lowest and highest values of M_f , respectively.

TONE simulations do not accurately reflect the snow ripening and meltwater infiltration processes. In Figure 3.1 for example, ground surface temperature does not reflect the presence of the ripened snow cover, with its temperature rising to 0°C only at the end of the ablation period (compare Figures 1 and 2 in Ollson *et al.* 2003). Surface temperatures too cold by several degrees during ablation will bias simulation results. For the case shown in Figure 3.2, ablation took 12 days, with 75 freezing degree-days at the ground surface over the period. If the surface temperature was 0°C for about half of the period, this represents an error of about -0.1°C on an annual basis.

Ablation could be incorporated into the **TTOP** model by reducing I_{TA} to account for I_M . Given the probable magnitude of I_M however, the influence of ablation effects on estimates of T_{TOP} is likely to be insignificant.

5.2 *Snow cover climatology*

Snow cover is established when snow accumulates at a rate greater than can be ablated from above by energy supplied by radiation and local advection, or from below by conduction of heat from the ground (Groisman and Davies, 2001). High winds can redistribute snow, scouring it from exposed areas and re-depositing it in sheltered areas. As snow accumulates, the weight of the snow overburden results in mechanical compaction (Sturm and Holmgren, 1998). At lower latitudes, thawing episodes in the winter can collapse the pore spaces in warm, wet snow and may produce refrozen ice layers (Sturm *et al.*, 1995). At higher latitudes, strong temperature gradients between the top of the snow cover and the ground surface can produce depth hoar (Sturm and Johnson, 1992), as discussed in Section 5.1.1. Deposition and redistribution under high winds can result in extremely hard, high-density “wind-slab” snow (Sturm *et al.*, 1995). At the start of the thawing season the snow cover warms to the melting point, and ablates with energy supplied from the atmosphere and the ground (Pommeroy, 1998).

Sturm *et al.* (1995) proposed a snow cover classification system based on snow cover physical characteristics (density, temperature, thickness, grain structure). By correlating these characteristics with climatic conditions (temperature, precipitation, and wind speed), they extend the application of their scheme to the globe (Figure 5.3). The characteristics of the snow cover classes can be summarised as follows (after Groisman and Davies 2001 and Sturm *et al.* 1995):

Tundra: Thin, cold, windblown snow. Thin basal depth hoar under multiple dense wind slabs.

Taiga: Thin to moderately deep (maximum less than 1.2 m), low-density snow. Very thick depth hoar overlain by un-metamorphosed fresher snow. Wind, initial snow density, and winter air temperature are all low.

Maritime: Warm and deep snow cover, usually containing melt and percolation features. Basal melting is common.

Prairie: Thin, moderately cold snow with significant drifting.

Alpine: Variable snow cover. Intermediate to low temperature, deep (maximum 2.5 m) snow. Depth hoar common, with occasional wind crust or melt features.

Ephemeral: Thin, extremely warm snow that begins melting soon after deposition.

Sturm and Holmgren (1998) found that the increase of density over time is regionally similar within three of these classes (tundra, taiga, and maritime), but different between them (Figure 5.4). They modeled the increase of density over time by applying the viscous compaction model, and found that the model parameter was distinct for each class. They suggest this approach as a method to estimate snow depth from snow fall or snow water equivalent data.

These three snow cover classes of Sturm and Holmgren (1998) correspond to three distinct regions of snow cover-active layer interaction. Tundra snow covers a shallow active layer, so the freezeback period is relatively short; taiga snow occurs with a cold climate over a deep active layer, so that freezing persists through most of the winter;

Figure 5.3 Distribution of snow classes in Eurasia and North America. From Groisman and Davies (2001), after Sturm *et al.* (1995).

Figure 5.4 Snow density versus time pattern for three of the snow cover classes shown in Figure 5.1. From Sturm and Holmgren (1998)

maritime snow occurs where there is a milder winter, and ground freezing is inhibited by a significant flow of heat from the ground to the freezing front.

The pattern of snow accumulation at any place in any particular year is determined by the weather, and depends on the pattern of snowfall, snow redistribution, and snow melt. While these patterns vary considerably from year to year (Olssen *et al.*, 2003), the average seasonal accumulation is remarkably similar for most climate stations. Based on an analysis of the climate record for Barrow, Alaska, Zhang (1993) characterized the snow accumulation rate as a parabolic function:

$$Z_t = Z_{\max} \left(\frac{t - t_0}{t_{\max} - t_0} \right)^{1/2} \quad (5.7)$$

where

t = time (arbitrary units)

Z_t = snow depth at time t

$Z_{\max}$ = maximum snow depth

t_0 = time of snow cover initiation

$t_{\max}$ = time of maximum snow depth

Snow depth at the end of the snow cover season was modeled using a similar function:

$$Z_t = Z_{\max} \left(\frac{t - t_{\max}}{t_{\text{end}} - t_{\max}} \right)^{3/2} \quad (5.8)$$

where

t_{end} = time of snow cover conclusion

Osokin *et al.* (2000) found that the exponent in equation (5.7) varied between 0.4 and 0.6 for a number of climate stations in Russia. Rouse (1993) suggests that the seasonal pattern of snow accumulation varies systematically with latitude in Canada, with longer seasons and decreasing total accumulation at higher latitudes. Snow depth also

peaks later in the season at higher latitude, reflecting the lower frequency of periods of thaw and the greater potential for snowfall when temperatures rise as spring approaches.

These seasonal patterns are climatic averages: the actual pattern of snow accumulation in any year depends on the unique weather sequence in that year. Canadian climate station daily snow cover and air temperature data (Environment Canada, 1995b) were used to examine the seasonal pattern of snow cover, including the initiation of snow cover at the start of the freezing season, and the seasonal pattern of snow depth. For example, Figure 5.5a to d shows the seasonal pattern of snow depth from the early 1960s to 1994 for Fort Simpson, Inuvik, Schefferville, and Mayo. The overall pattern follows the parabolic shape suggested by equation (5.7), although the patterns in individual years vary significantly.

Multi-year daily averages are shown on the same figures. The transition periods in the spring and fall show a gradual transition between snow covered and snow free conditions due to the averaging process: the snow cover average at the margins includes fewer and fewer years with extreme early or late snow covers. Between the transitional periods, the average snow cover generally shows the parabolic accumulation pattern described by Zhang (1993), with a declining accumulation rate through the winter. The ablation pattern suggested in equation (5.8) is not reflected in most annual records, which exhibit a sharp, nearly linear decline in snow depth once the ablation period is well under way. As with the transition periods, the shape of the average curve is an artefact of the averaging process.

Figure 5.5 a and b. Daily snow depth from climate record for a) Fort Simpson and b) Inuvik. Thin lines show data for 30 years between 1964 and 1994. Heavy lines are multi-year average daily means over the period of record.

Figure 5.5c and d. Daily snow depth from climate record for c) Schefferville and d) Mayo. Thin lines show data for 30 years between 1964 and 1994. Heavy lines are multi-year average daily means over the period of record.

Figure 5.6a shows the data for Inuvik, normalised to relative time and snow depth. Snow depth is expressed as a fraction of the seasonal maximum, with time expressed as a fraction of the freezing season in the air¹. Except for a few unusual years, the overall pattern roughly follows the parabolic curves suggested above, with scatter on the order of 30 to 40% of the seasonal maximum. The seasonal maximum snow depth usually occurs between 60% to 100% of the freezing season duration. Figure 5.6b shows the average daily snow depth (calculated using the data of Figure 5.6a) with the curve for equation (5.7) with $t_{\max}$ set to the end of the freezing season. As in Figure 5.6a, equation (5.7) over-predicts snow accumulation initially, while the maximum snow depth occurs at about 85% of the season length.

Figure 5.7a shows the daily mean snow depth by calendar date for the climate stations in Table 5.1. As in Figure 5.5, snow depth in the spring and fall show a gradual transition between snow-covered and snow free conditions. Between the transitional periods, the average snow cover generally shows the patterns described by Rouse (1993) and Zhang (1993). Some records show distinct early and late periods during which the snow accumulation is approximately constant. Similarly, during the ablation period some stations exhibit the pattern suggested in equation (5.8), although many exhibit a nearly linear decline in snow depth once the ablation period is well under way.

Figure 5.7b shows the data of Figure 5.7a in normalised form. The average pattern of snow accumulation is roughly similar for all stations. Following the transition period,

¹ Assigning the appropriate date to the beginning and end of the freezing season is arbitrary in years having several days over the transition period with temperatures above and below 0°C. In this analysis, the first date when the mean daily temperature is below 0°C was chosen as the starting date, while the first date when the mean daily temperature falls below 0°C was used for the end of the freezing season.

Figure 5.6. Normalized daily snow depth for Inuvik. Time is expressed as a fraction of the freezing season; snow depth is expressed as a fraction of the seasonal maximum snow depth. **a)** Thin lines show data for 30 years between 1964 and 1994. Heavy line shows Zhang's snow depth function (Zhang 1992). **b)** Thin line shows normalized average of data in Figure 5.5b. Heavy line shows Zhang's snow depth function.

Figure 5.7 Multi-year average snow depth for selected high-latitude climate stations. **a)** Daily mean snow depths. **b)** Normalized mean snow depth.

Table 5.1 Basic snow cover statistics for selected northern climate stations.

<i>Station</i>	<i>Latitude</i>	<i>Longitude</i>	<i>Mean days with snow</i>	<i>Mean snow cover (cm)</i>	<i>Max. snow cover (cm)</i>	<i>I_F</i>	<i>I_T</i>	<i>Standard deviation of mean snow cover</i>	<i>Standard deviation of max. snow cover</i>
Alert	82.50	62.33	304	28.55	48.65	6771	203	10.9	15.7
Baker Lake	64.30	96.08	253	34.52	60.77	5281	874	15.4	24.0
Cambridge Bay A	69.10	105.12	263	20.12	36.00	5985	567	7.0	12.2
Cape Parry A	70.17	124.70	256	14.49	26.34	4807	482	6.7	12.6
Churchill	58.73	94.07	214	27.98	51.92	3759	1165	11.3	19.3
Clyde A	70.48	68.52	288	40.92	67.00	4851	339	21.6	37.8
Coral Harbour A	64.20	83.37	263	31.74	56.03	4903	646	13.2	24.4
Eureka A	79.98	85.93	277	14.50	23.28	7534	359	5.4	7.6
Fort Reliance	62.72	109.17	207	33.61	59.83	3891	1454	11.0	19.9
Fort Simpson	61.75	121.23	195	35.59	62.87	3293	2034	9.1	16.6
Fort Simpson A	60.02	111.95	184	34.22	60.13	3031	1966	9.1	16.9
Hall Beach A	68.78	81.25	273	30.19	51.32	5602	401	8.7	13.2
Inukjuak	58.45	78.12	225	32.47	55.55	3392	911	12.8	21.8
Inuvik	68.30	133.48	234	40.45	66.55	4643	1292	10.8	16.0
Iqualuit	63.75	68.55	246	20.14	44.08	4133	655	7.4	16.8
Kuujjuaq	58.10	68.42	217	38.67	78.00	3238	1145	14.1	36.6
Kuujjuarapik	55.28	77.77	198	35.45	61.61	2886	1251	16.2	27.0
Mayo	63.62	135.87	189	31.80	58.29	3112	1829	11.8	20.4
Mould Bay	76.23	119.33	293	17.26	28.60	6644	223	5.2	9.2
Norman Wells A	65.28	126.80	208	33.97	63.46	3955	1787	14.0	19.8
Resolute A	74.72	94.98	291	21.87	36.15	6260	243	8.0	13.1
Schefferville	54.80	66.82	217	51.23	101.49	3050	1251	18.4	31.2
Watson Lake A	60.12	128.82	197	41.04	77.13	3002	1760	10.4	20.0
Whitehorse	60.72	135.07	174	22.12	42.78	2066	1762	9.2	16.0
Yellowknife	62.47	114.45	193	27.28	45.60	3629	1770	9.2	13.9

accumulation is rapid following a roughly parabolic shape to the peak snow cover.

Maximum snow depth tends to occur at 60-70% of the season duration. Following this, the decrease in snow cover thickness at the end of winter is a precursor of snow ablation, with surface melt and re-freezing commencing before the daily mean air temperature rises above 0°C.

5.2.1 Snow cover initiation and I_F

Snow-free conditions during the freezing season may be significant to the ground thermal regime (see Section 5.4.3). The relationship between snow cover initiation and the beginning of the freezing season (based on mean daily temperature) was investigated using climate station daily snow cover and air temperature data (Environment Canada, 1995b) for twenty-five arctic and subarctic climate stations where snow depth was recorded² (Table 5.2). A freezing degree-day total for the period in autumn with no snow on the ground (referred to as I_{FSF}) was calculated for all years (1955-1994) with complete data for the climate stations shown in Table 5.2. For example, Figure 5.8 shows the I_{FSF} frequency distribution for Fort Simpson. Except for the extreme outlier (478 degree-days in 1976), this distribution is typical. Most stations exhibit a positively skewed distribution in which the average value of I_{FSF} is about 40 degree-days; I_{FSF} is less than 100 degree-days in 90% of cases. Seven of twenty-five stations experienced a single year with I_{FSF} greater than 300 degree-days. Figure 5.9 shows the frequency distribution for all years and stations combined. The relationship between the number of snow-free days with freezing temperatures and I_{FSF} is shown in Figure 5.10, with an I_{FSF} curve generated from a thermal catena sine wave superimposed for comparison.

² Climate stations for which snow cover data had large data gaps were not included.

Table 5.2 Climate statistics relating to freezing conditions without snow cover.
 I_{FSF} = total freezing degree-days at the beginning of the freezing season
 accumulated before snow cover is established.

	<i>Mean days with snow</i>	<i>Standard deviation of days with snow</i>	<i>Mean snow- free freezing days</i>	<i>Max. snow- free freezing days</i>	<i>Mean I_{FA}</i>	<i>Mean I_{FSF}</i>	<i>Standard deviation of I_{FSF}</i>	<i>Max. I_{FSF}</i>	$\frac{I_{FSF}}{I_{FA}}$
Alert	307.5	11.4	11.9	29	6776	29.2	37.72	197.6	0.4%
Baker Lake	253.1	15.0	13.3	34	5281	62.0	79.90	406.3	1.2%
Cambridge Bay	262.8	12.3	13.8	41	5985	50.0	59.11	264.9	0.8%
Cape Parry	256.4	10.4	12.6	42	4807	23.2	22.37	94.9	0.5%
Churchill	213.7	14.4	15.9	36	3759	53.0	59.00	316.8	1.4%
Clyde	287.7	16.5	7.4	22	4851	18.8	41.59	244.3	0.4%
Coral Harbour	262.7	12.9	12.0	32	4903	36.9	32.65	125.3	0.8%
Eureka	276.6	14.6	14.2	54	7534	79.8	145.11	786	1.1%
Fort Reliance	206.9	23.6	11.6	25	3891	39.1	28.99	97.8	1.1%
Fort Simpson	194.7	15.8	8.8	48	3293	33.4	84.97	478.5	1.2%
Fort Smith	184.5	13.0	12.2	29	3031	38.1	37.98	187.3	1.3%
Hall Beach	272.9	13.0	12.7	38	5602	42.2	73.74	445.7	0.8%
Inukjuak	272.9	13.0	12.7	38	5602	42.2	73.74	445.7	0.8%
Inuvik	234.0	10.2	8.5	22	4643	28.1	35.28	166.2	0.6%
Iqaluit	245.5	14.8	10.3	38	4133	33.7	45.09	263.9	0.8%
Kuujuuaq	216.9	18.0	12.0	33	3238	34.8	42.27	227.9	1.0%
Kuujuarapik	197.8	14.8	12.2	29	2886	27.9	25.51	116.7	1.0%
Mayo	189.2	15.7	11.4	37	3112	51.9	55.64	305.2	1.8%
MouldBay	293.3	14.7	18.2	45	6644	66.1	68.31	294.6	1.0%
Norman Wells	207.8	14.0	7.7	20	3955	19.9	19.97	66.2	0.5%
Resolute	291.2	12.6	15.1	32	6260	41.9	43.44	185.8	0.7%
Schefferville	217.2	17.6	10.5	33	3050	28.1	27.14	98.1	0.9%
Watson Lake	196.5	18.4	7.2	29	3002	28.6	41.51	197.6	0.9%
Whitehorse	173.7	18.6	11.9	35	2066	42.2	40.95	154.4	2.3%
Yellowknife	193.4	14.3	13.7	30	3629	46.2	35.52	144.1	1.3%

Figure 5.8 Frequency distribution of freezing degree days with no snow cover, Fort Simpson Airport climate data 1961-1993.

Figure 5.9 a) Frequency distribution of freezing degree days with no snow cover for all climate stations combined. b) Cumulative frequency for data in a) above. Data are for 1961-1993.

Figure 5.10 Relationship between snow-free freezing degree days versus snow free freezing days, pooling data for all climate stations. The dotted line shows the accumulated freezing degree day total for a sine wave with a mean of -14°C and an amplitude of 21.5°C (a point on the standard thermal catena).

The thermal effect of snow cover depends on the seasonal pattern of snow depth and snow thermal properties over time. A difficulty with climate station snow data is that snow density (or snow water equivalent) is not routinely recorded, so that the thermal properties of the snow cannot be estimated easily. Two alternate approaches to snow density estimation at continental scales have been used. First, Brown *et al.* (2001) used historical snow depth and snow water equivalent data to interpolate gridded normal monthly snow density maps of Canada. Second, using a method similar to Sturm and Holmgren (1998), Brown *et al.* (2003) employed the compactive viscosity model together with the snow cover classification scheme of Sturm *et al.* (1997) to estimate snow depth and density from precipitation data. They noted that their model predicted snow to be too dense in taiga environments, suggesting the neglect of depth hoar development in the compactive viscosity model.

5.3 Snow and ground temperature

As discussed in the introduction to this chapter, the link between snow cover and permafrost ground thermal regimes is well studied (e.g., Nicholson and Granberg 1973, MacKay and Mackay 1974, Nicholson 1979, Goodrich 1982, Zhang 1993, Osokin *et al.* 2000). For example, early deep snow can increase the mean annual ground temperature by several degrees (e.g., Brown and Péwé 1973, Goodrich 1982), while a thin or absent snow cover can cause deeper seasonal freezing, more rapid active layer freezeback, and colder mean annual ground temperatures (Osokin *et al.* 2000, Ling and Zhang 2003). Many authors have studied the relationship between snow cover and ground temperatures via field investigations, numerical modeling and analytical models. While some authors

recognize the importance of thermal processes in the ground for the relationship (Kudriavtsev, cited in Yershov 1998, p. 362; Goodrich 1982; Gray *et al.* 1995; Burn 1998a; Sokratov and Barry 2002), no analytical models of its effect appear to have been developed before Riseborough (2001).

5.3.1 Analytical models

Lachenbruch (1959) developed an analytical model to assess the thermal effect of snow cover in terms of the temperature amplitude at the ground surface:

$$A_0 = A_A \exp \left[-Z_S \sqrt{\frac{\pi}{\alpha P}} \right] \quad (5.9)$$

where

- A_0 = temperature wave amplitude at ground surface
- A_A = temperature amplitude in the atmosphere
- Z_S = mean snow cover thickness
- α = snow thermal diffusivity

This equation, used by Nelson and Outcalt (1983, 1987), is analytically equivalent to that of Kudriavtsev *et al.* (1977), used by Shiklomanov and Nelson (1999). This model is based on the classic analytical model of the attenuation of a sinusoidal temperature wave in a diffusive medium with uniform thermal properties (Carslaw and Jaeger, 1947):

$$T_{z,t} = \bar{T} + A_z \sin \left(2\pi t / P - z \sqrt{\pi / \alpha P} \right) \quad (5.10)$$

$$A_z = A_0 e^{-z \sqrt{\pi / \alpha P}}$$

where

- $\bar{T}$ = mean temperature
- $T_{z,t}$ = ground temperature at depth z and time t
- P = period of the sine wave
- α = thermal diffusivity
- A_z = amplitude of the sine wave at depth z

The physical assumption of the model is that diffusion through the snow results in attenuation of the surface wave as a function of depth that is similar to the case that equation (5.10) describes. Based on equation (5.9), Riseborough and Smith (1998) calculated n_F as:

$$n_F = \frac{A_0}{A_A} = \exp \left[-Z_s \sqrt{\frac{\pi}{\alpha P}} \right] \quad (5.11)$$

Also based on equation (5.9), Shiklomanov and Nelson (1999) define the nival offset as:

$$\Delta_{SNOW} = A_0 \left(1 - \exp \left[-z_s \left[\frac{\pi}{P\alpha_s} \right] \right] \right) \quad (5.12)$$

In most cases, equation (5.11) is not strictly analytically correct. The definition of n_F is:

$$n_F = \frac{\int T_s \partial t}{\int T_A \partial t} \quad (5.13)$$

for the periods that the air and ground surface temperatures are below 0°C. Ignoring the time lag term in equation (5.10)³, and defining t_0 as the time at which the sine wave described will pass through 0°C, then:

$$t_0 = P/2\pi \sin^{-1} \left(-\bar{T}/A_z \right) \quad (5.14)$$

so that the integration period for the freezing index is from $(P/2 - t_0)$ to $(P + t_0)$. Because t_0 depends on the amplitude at depth z , the integration period at the surface will only be

³ The lag term is the second term in the sine function, and accounts for the phase lag between the wave at the surface and at depth z . Ignoring this term simplifies the sine equation by eliminating extra terms in the function. Because the exercise is to integrate over one full period, this is acceptable.

the same as the integration period at depth when t_0 for both is the same. This is only the case when $t_0 = 0$, or in other special circumstances such as when a temperature gradient is superimposed on the simple condition described in equation (5.9).

Lachenbruch (1959, p. 28) explicitly assumes that the winter is six months duration (“for convenience”); Nelson and Outcalt (1983,1987) account for this effect with a function based on t_0 , while Shiklomanov and Nelson (1999) do not comment. The annual temperature wave at the ground surface is distorted by latent heat effects in the snow cover and the ground, so that the assumption of periods of approximately equal duration of freezing temperatures at the ground surface and at the snow surface is probably reasonable. However, Riseborough and Smith (1998) found significant departures from the effect of the snow cover predicted by equation (5.9), which they parameterized as a function of MAAT.

Other assumptions of equation (5.9) when considering the winter thermal regime of the ground surface include:

1. The snow cover season coincides with the freezing season. As discussed elsewhere this assumption is approximately true, but in some (relatively infrequent) years it is not.
 2. Snow cover is present for only half of the cycle. Because the heat content and diffusivity of this layer are missing in the thaw season, the temperature wave in the ground will be distorted: more heat is able to enter the ground when the snow is absent, so that this heat will return to the ground surface and be trapped by the snow. In modeling the thermal interaction between snow cover and bedrock
-

outcrop (without latent heat) Gray *et al.* (1995) found that imposition of the snow cover led to a rapid increase in temperature at the base of the snow cover, which could initiate basal melting.

3. The snow cover is not uniform: its thickness varies through time as it accumulates, and its thermal properties evolve over time due to processes such as mechanical densification, thermal metamorphism, and so on.
4. The thermal diffusivity of the snow is an order of magnitude less than the diffusivity of the ground below. As a result, the effect described in point 2 above will be enhanced: a significant amount of heat can enter the ground in summer, which will subsequently be trapped below a layer with a significantly lower thermal diffusivity.
5. The effect of active layer thaw and freeze-back is ignored. The effect of soil water phase change significantly modifies the annual thermal regime in the ground from that predicted by the analytical model even in the absence of snow cover (Riseborough, 1990). The limited penetration of the 0°C isotherm results in steeper temperature gradients within the active layer, increasing the potential for heat storage within the active layer. This effect further enhances the effects described in point 2 above.

Sazonova and Romanovsky (2003) present a detailed analytical model, based on Kudriavtsev's ideas, but accounting for "heat turnover" in determining the effective heat capacity of the soil and "the contrast in the thermal properties between snow cover and the underlying material". While more sophisticated than equation (5.12), their model still does not account for the effect of freezing on heat storage in the active layer.

5.3.2 Field studies of snow depth – ground thermal regime relationship

Several authors (Nicholson and Granberg 1973, Nicholson and Thom 1973, Granberg 1973, MacKay and Mackay 1974, Smith 1975, Nicholson 1979, Desrochers and Granberg 1988, Sokratov and Barry 2002, Taras *et al.* 2002) examined the relationships between snow depth and ground temperature conditions in field conditions over various spatial and time scales. Results from four such studies are shown in Figure 5.11. These relationships are for the ground temperatures measured at the surface, near the top of permafrost, and near the zero annual amplitude depth, and represent temperatures measured over periods from one month to several years. MacKay and Mackay (1974) measured snow depth and 90 cm ground temperature (within permafrost) along a transect crossing a depression in which snow accumulates differentially. The pattern of snow accumulation along the transect was consistent over 5 winters, with the regression relationship:

$$T_{90} = a + 2.17 \ln x \quad (5.15)$$

where

T_{90} = temperature (°C) at 90 cm depth

a = a constant representing the ground temperature (°C) with 1 cm snow cover

x = snow depth (cm)

The form of equation (5.15) suggests that the incremental effect of deeper snow in warming the ground surface declines as the snow depth increases. Nicholson and Granberg (1973), Nicholson and Thom (1973), Smith (1975), and Taras *et al.* (2002) found similar relationships (Figure 5.11d).

Figure 5.11 Snow Depth – Ground temperature relationships from **a)** Smith (1975); **b)** Mackay and MacKay (1974) **c)** Nicholson (1979); and **d)** Taras et al.(2002). Caption from Smith 1975: *Effect of snow depth on ground surface temperature (March 1970).* Caption from Mackay and MacKay 1974: *Plots of ground temperatures against snow depths. The Dec.-Jan data are the average temperatures and snow depths as measured at each of the ten sites during each of five successive winter visits made during 1968-73; the March- April data are the averages for five successive spring visits; and the Dec.-April data are the averages in respect of eleven visits (the preceding plus one made in February) at each of the ten sites during 1968-1973.* Caption from Nicholson (1979): *The relationship between snow (SA measure) and ground temperature at 15 m depth for 3 different areas, shown by 3 different symbols. ($r=.76$; see Table V).* Caption from Taras et al.(2002): *Coupled and decoupled regimes for the snow-ground interface temperature (T_i) as a function of snow depth.*

Commenting on Smith (1975), Williams and Smith (1989, p. 110) suggest that snow cover becomes “less effective” when its thickness exceeds 50 – 60 cm. In a study of snow cover, air and ground surface temperature at 33 sites over the Kuparuk basin (a region extending 180 km on the Alaskan north slope), Taras *et al.* (2002) found that the seasonal average ground surface temperature became “decoupled” from air temperature where the snow depth exceeds about 80 cm. The relationship between snow depth and ground surface temperature in Taras *et al.* (2002) shows site-to-site variability on the order of $\pm 4^{\circ}\text{C}$ from their regression line (Figure 5.11d), so that the incremental effect of deeper snow is lost within the scatter. The contrast with the more consistent results of MacKay and Mackay (1974) and Smith (1975) is likely due in part to the limited locales of their studies.

Intensive studies in the Schefferville region (Granberg 1973, Nicholson and Granberg 1973, Nicholson 1979, Desrochers and Granberg 1988, Granberg 1988), within the discontinuous permafrost zone, examined the relationship between snow depth and the occurrence of permafrost. These studies made important refinements to the simple relationships between snow thickness and mean annual ground temperature, primarily by accounting explicitly for variations in snow cover thickness. Figure 5.12a shows profiles of MAGT for 17 boreholes in the Schefferville study area. The near-surface permafrost temperatures (approximately equal to TTOP) show a range of about 7°C ; at 40 m depth the range decreases to 2°C . Figure 5.12b shows the effect of areal averaging on mean snow depth measured around each of these boreholes. Adjacent to boreholes, the range of snow depths varied by 150 cm, with the range decreasing to about 40 cm as the averaging

Figure 5.12 a) Mean annual ground temperature versus depth for 17 boreholes near Schefferville (from Nicholson and Granberg, 1973). **b)** Mean snow depth versus averaging radius around borehole for the 17 boreholes in a) (from Nicholson and Granberg, 1973).

radius increased to 80 m. Nicholson and Granberg (1973) found that the correlation between mean annual ground temperature and snow depth was highest when snow cover thickness was averaged over a radius equal to two times the depth of temperature measurement. Nicholson (1979) was able to improve the correlation by accounting for the relative contribution of snow at a particular distance from the borehole, using solid geometry. Based on these studies, a snow depth of 75 cm was considered to be sufficient to prevent the development of permafrost in the region.

In a comparison of two sites in close proximity in the Takhini Valley, Burn (1998a) found that a similar snow cover at the sites resulted in very different values of I_{FS} , producing a much lower value for n_F at the site with a deeper active layer (with degrading permafrost). Burn (1998a) ascribed this difference to the effect of the greater upward heat flux toward the surface at the warmer (degrading permafrost) site.

Sokratov and Barry (2002) found that the active layer thermal regime at Barrow, Alaska could be separated into distinct seasons defined by heat flux and ground temperatures, corresponding to the seasons defined in Section 3.1.1 of this thesis. The duration of the active layer freeze-back season was found to relate to the depth of snow cover at the start of winter. They found that key relationships of the energy balance to snow depth were influenced by the moisture content of the active layer.

Small-scale effects

Snow cover depth and properties can vary over very short distances. One problem with simple models describing the relationship between snow cover and ground temperature is that the thermal effect of the snow is accounted for by a single parameter (such as snow depth or snow thermal resistance). Gray *et al.* (1995) cite field studies

noting the preferential development of depth hoar around rock outcrops (“bumps”), suggesting the enhancement of upward heat flow from surrounding bedrock materials under deeper snow. They found that the effect was most pronounced when the outcrop snow cover was thinner than the depth of zero annual amplitude for the annual temperature wave, based on model conditions in which a snow cover of constant thickness persists through the whole year without melting. Gray *et al.* (1995) suggest that lateral heat flow is important if the snow depth changes significantly on a length scale comparable with the damping depth of the snow cover.

In environments such as hummocky terrain, the depth of snow cover and the topography of the ground surface vary at scales from decimetres to metres. At some scale the relationships discussed here may not adequately account for the three-dimensional heat flow through the snow cover, in which the spatial average relationship between snow properties and MAGST may differ significantly from relationships developed in homogenous conditions. For example, Sturm (1992) developed an empirical model of snow interception near tree trunks in taiga, which he used to model the net heat flow through the snow cover. He found that heat flow through snow “wells” around trees can be twice that in open areas. Similarly, Sturm and Holmgren (1994) found that in tundra, the combination of a thin snow cover and low tussocks (10-30 cm high) resulted in heat loss from tussock tops more than twice that from inter-tussock depressions, where snow cover was deeper, and of lower density. They proposed a simple model for the ratio of heat loss through tussocks and depressions. Such models, converted to temperature, may allow some accounting for such micro-scale variation in the surface thermal regime when estimating snow cover effects at regional scales.

5.3.3 Numerical modeling

Goodrich (1976, 1982) appears to have been first to systematically examine the effect of snow cover on the ground thermal regime using a numerical model (model T1D, described in Appendix I). He examined the effects of the amplitude of the air temperature wave, the thermal properties of the ground, and the depth and evolution of the snow cover. Permafrost (MAST = -10°C) and seasonally frozen ground (MAST = $+5^{\circ}\text{C}$) conditions were examined. Goodrich (1976, 1982) analyzed results in terms of ground surface temperature and the thermal offset, with several conclusions relating to the thermal offset effect rather than the effect of snow cover *per se*.

Several of Goodrich's conclusions concur or conflict with analytical and field-based findings, while others are original. In seasonally frozen ground:

- The presence of snow cover keeps ground surface temperature close to 0°C throughout the winter.
- Seasonal frost penetration is highly sensitive to snow cover accumulation, especially in the autumn.
- The thermal effect of snow cover is influenced primarily by its duration, and is relatively insensitive to most other parameters.

In permafrost:

- Both maximum snow depth and the rate of snow accumulation have a significant effect on the ground thermal regime.
- The latent heat of the ground increases the sensitivity of the ground thermal regime to other parameters (such as the thermal properties of the ground or properties of the snow cover).

- Snow cover has a small effect on summer thaw depth, through its effect on TTOP.

For both regimes:

- The effect of snowmelt on the ground thermal regime is negligible.

Zhang (1993) and Zhang *et al.* (1996) examined the effect of depth hoar (discussed above) on the ground thermal regime using Goodrich's T1D numerical model.

The snow cover was modeled as a single layer whose bulk thermal properties were calculated from the properties of the depth hoar layer and the overlying snow:

$$k_{eff} = \frac{k_{ws}k_{dh}}{\left(\frac{H_{dh}k_{dh}}{H}\right) + \left(\frac{H_{ws}k_{ws}}{H}\right)} \quad (5.16)$$

where

k_{eff} = effective thermal conductivity

k_{ws} = thermal conductivity of wind slab

k_{dh} = thermal conductivity of depth hoar

H_{ws} = thickness of wind slab layer

H_{dh} = thickness of depth hoar layer

H = total snow depth = $H_{ws} + H_{dh}$

and

$$\begin{aligned} \rho_{eff} &= \rho_{ws} + (\rho_{dh} - \rho_{ws})\phi \\ C_{eff} &= \rho_{eff}c_{ice} \end{aligned} \quad (5.17)$$

ρ_{eff} = effective snow cover density

ρ_{ws} = density of wind slab

ρ_{dh} = density of depth hoar

ϕ = snow cover depth hoar fraction

C_{eff} = effective volumetric heat capacity of snow cover

c_{ice} = specific heat of ice

Zhang *et al.* (1996) found that varying the modeled depth hoar thickness from 0% to 60% of a 35 cm deep snow cover for a site near Prudhoe Bay, Alaska resulted in a variation of 12.8° in the MAGST. The effect of a snow cover with a low-density layer

next to the ground surface can be regarded as equivalent to a deeper snow cover with uniform but higher density. Colbeck (1991) suggests that treatment of a layered snow cover as a uniform slab is probably acceptable for modeling heat flow through depth hoar, though not for modeling the *development* of depth hoar.

Riseborough and Smith (1998) compared Goodrich's results to Lachenbruch's (1959) model, and found that the latter analytical model underestimated the warming effect (the nival offset) in all cases, most significantly in seasonally frozen ground conditions. Using the suite of soil thermal properties and snow cover properties from Goodrich's (1982) standard case, Riseborough and Smith (1998) investigated the relationship between MAAT and the freezing n-factor by running addition simulations to create a set of catenae. Their results (Figure 5.13) show that n_F is strongly dependent on both snow depth and temperature. They ascribe this effect to the neglect of soil latent heat in equation (5.10). Seasonal (active layer) freezing prevents the ground surface temperature from becoming extremely cold until the active layer is completely refrozen. Kudriavtsev (cited in Yershov 1998) found that the thermal influence of the snow was greatest where permafrost temperatures are near 0°C, and the effect is augmented in soils with high moisture content.

Osokin *et al.* (2000) examined the sensitivity of simulated soil freezing depth to a number of parameters defining snow cover conditions. Simulations were repeated with snow covers assigned "low", "mean" and "high" thermal conductivity functions, resulting in seasonal frost depths ranging from less than 0.6 m for the low conductivity case to 2 m for the case with the highest conductivity. They also examined the sensitivity of soil freezing to the date of initiation of snow cover. When snow cover was initiated on the

Figure 5.13 Dependence of n_F on MAAT and mean snow depth, from results of numerical analyses by Riseborough and Smith (1998).

same day that surface freezing began, the soil froze to a depth of 1.2 m in 190 days, compared to 2 m of frost depth in less than 100 days when the snow cover was initiated after 15 days of soil freezing. Compared to the relatively large effects that the snow cover parameters had on ground freezing, their comparison of results for simulations with and without moisture migration in the ground showed a “rather small” effect on seasonal frost depth (estimated about 5 cm from their Figure 3).

5.3.4 An analytical model of the ground surface temperature under snow cover with soil freezing.

Combining a simple model of the snow-soil thermal system with a version of the Stefan model for layered soils, Riseborough (2001, included as Appendix II) developed a model of ground surface temperature under snow. Using an assumption of linear temperature gradients proportional to $1/k$ in the presence of a freezing front (implied in the Stefan equation), the instantaneous ground surface temperature could be determined from the heat flow (temperature gradients and thermal conductivities) and the thickness of the snow and frozen soil layers. With this assumption, heat flow through the snow is:

$$(T_s - T_0)k_s / Z$$

and through the frozen soil is:

$$(T_0 - T_F)k_F / X$$

where

k_s = Thermal conductivity of snow

T_0 = Temperature at the ground surface (the snow-soil interface)

X = freezing depth

Z = snow layer thickness

So that:

$$\frac{(T_s - T_0)k_s}{Z} = \frac{T_0 k_F}{X} \quad (5.18)$$

Solving for T_0 yields:

$$T_0 = \frac{T_s}{1 + \left(\frac{k_a Z}{k_s X} \right)} \quad (5.19)$$

Equation (5.19) gives the instantaneous temperature at the snow-soil interface. To estimate the seasonal total I_{FS} , the time evolution of the thickness of the snow cover and the freezing soil layer are required. As a first approximation, Riseborough (2001) adapted Nixon and McRoberts (1973) application of the Stefan model to layered soils with different thermal properties. If the snow cover has a constant thickness and ground freezing starts at the base of the snow cover, then allowing for appropriate substitution of terms, frost depth under the snow cover can be estimated by:

$$X = \sqrt{\left(\frac{k_F}{k_S} \right)^2 Z + \frac{2k_F I_F}{L}} - \left(\frac{k_F}{k_S} - 1 \right) Z \quad (5.20)$$

where

L = soil latent heat of fusion.

Figure 5.14 shows the ground surface temperature predicted using (5.19) and (5.20) for a test case. Comparisons with numerically modeled ground surface temperature showed that the model works well during active layer freezeback with permafrost cases, although the duration of freezeback period was over-predicted. The model under-predicted ground surface temperature for a seasonally frozen ground case due to the over-prediction of freezing depth by equation (5.20). Riseborough (2001) suggested that the magnitude of model errors may be a function of the mean ground temperature, since this determines the magnitude of the sensible heat component of the active layer energy balance, and that the model may be most accurate where $T_{TOP} = 0^\circ\text{C}$.

Figure 5.14 a) Winter air and ground surface temperature for the standard case from Riseborough (2001) (sinusoidal surface temperature wave having a mean of -10°C and amplitude of 20°C , 0.3 m snow cover with a density of 250 kg m^{-3} and thermal conductivity of $0.21\text{ Wm}^{-1}\text{K}^{-1}$, mineral soil with 50% volumetric moisture content and frozen thermal conductivity of $2.938\text{ Wm}^{-1}\text{K}^{-1}$). b) Ground surface temperature from a) above.

5.3.5 A statistical model

Taras *et al.* (2002) used the Kuparuk data set (described in Section 5.3.2 above) to develop a model characterizing ground surface temperature under snow cover. The model fit winter season ground surface temperature to a beta distribution using parameters estimated by least squares regression, based on two years of data for snow cover, air temperature and ground surface temperature. While this model would be useful in describing the seasonal course of ground surface temperature, a key parameter (γ) is equivalent to n_F , making it diagnostic rather than prognostic.

5.3.6 Overview

The results of studies reviewed here aid in the attempt to develop a snow cover sub-model for the **TTOP** model. Frost heave, soil moisture migration, and ablation of the snow cover have been found to have only minor effects on the relationship between the ground thermal regime and the snow cover. Details of snow cover development (initiation, accumulation rate, and density variation), and the thermal properties of the ground have all been identified as important variables in this relationship. These aspects of snow cover are considered in the following sections.

Several authors identify the importance of the spatial scale of snow cover variation, and have accommodated its effect by statistical and analytical models. This aspect is not considered in the following analyses, since this would require the use of two- or three-dimensional modeling tools beyond the scope of this thesis.

Riseborough (2001) appears to provide a useful model of the connection between snow cover and ground surface temperature, if only for the active layer freezeback period. Its use in the **TTOP** model would add the properties of the snow cover, soil latent

heat of fusion, and the frozen soil thermal conductivity (rather than simply the thermal conductivity ratio) to the list of parameters required in the model.

5.4 Modeling snow cover – ground thermal regime interaction

Several sets of catena simulations were performed to develop an empirical database with which to parameterize the effect of the snow cover on the ground thermal regime. Following the procedures outlined in Chapter 2, “standard” thermal catenae were created for a range of maximum snow depths for several different substrate materials. These include: the “standard” mineral soil discussed in Chapter 2; saturated peat over the standard material (three sets of simulations with 0.1m, 0.2 m and 1 m peat layers); and dry bedrock (no latent heat of fusion). Key properties for these substrate materials are given in Table 5.3.

As discussed in the preceding section, the seasonal pattern of snow cover influences the relationship between snow cover and ground surface temperature. While the average pattern of snow cover can be defined by climate normals (for example), the effect of significant departures from the average condition was also found to be significant. Before considering the results representing standard conditions, simulation studies of departures from those standard conditions will be examined. In particular, the effect of differences in snow density, delays in the initiation of snow cover, and the pattern of snow accumulation were examined. The effect of inter-annual variability of snow cover was also examined, and is discussed in Chapter 6.

Refinements over previous catena analyses

The relationship of n_F to MAAT and snow cover thickness (Figure 5.13) used in Riseborough and Smith (1998), Henry and Smith (2001) and Smith and Riseborough

Table 5.3 Thermal Properties of substrate materials used in catena simulations. Substrates denoted with * were used in transition simulations (TTOP = 0°C) only. ρ_b = bulk density (kg m^{-3}); n = porosity (dimensionless); k_F , k_T = frozen and thawed thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$); θ_v = volumetric water content (m^3m^{-3}); C_F , C_T = frozen and thawed volumetric heat capacity ($\text{Jm}^{-3}\text{K}^{-1}$)

Substrate	ρ_b	n	k_F	k_T	θ_v	C_F	C_T
Standard Mineral Soil	1468	0.440	2.713	1.628	0.44	1.875e6	2.927e6
Organic	195	0.850	1.632	0.518	0.85	1.879e6	3.839e6
Bedrock	2650	0.000	3.923	3.923	0.00	1.875e6	1.875e6
Water*	0000	1.000	2.200	0.570	1.00	1.88e6	4.179e6
Soil All Quartz *	1442	0.456	4.350	2.350	0.456	1.877e6	2.927e6
Soil No Quartz *	1415	0.466	2.090	1.110	0.466	1.878e6	2.950e6
All Quartz 0.15 θ_v *	1415	0.476	1.550	1.590	0.15	1.291e6	1.653e6
No Quartz 0.15 θ_v *	1415	0.476	0.800	0.790	0.15	1.279e6	1.623e6
60% porosity*	1060	0.600	2.740	1.230	0.593	1.864e6	3.23e6
Standard 0.35 θ_v *	1415	0.466	2.295	1.474	0.35	1.659e6	2.464e6
Standard 0.23 θ_v *	1415	0.466	1.593	1.301	0.23	1.439e6	1.978e6
Standard 0.15 θ_v *	1415	0.466	1.111	1.120	0.15	1.291e6	1.641e6

(2002a) adopted the thermal properties for snow and for mineral soil used by Goodrich, but extended the analysis by considering more cases. Goodrich considered only two boundary conditions: sine waves with a MAAT of +5°C and -10°C, with amplitudes of 15°C and 20°C, respectively. Riseborough and Smith (1998), Henry and Smith (2001) and Smith and Riseborough (2002a) used a sinusoidal air temperature wave along a thermal catena between -12°C and +5 or +8°C and included a wider range of snow depth conditions. Sine waves for all cases had an amplitude of 20°C.

Goodrich's standard snow cover increased linearly from zero to the maximum snow depth over 95 days. In Riseborough and Smith (1998), Henry and Smith (2001) and Smith and Riseborough (2002a), the snow cover increased linearly to maximum depth over the first one third of the freezing season: this approach is used in some of the non-standard analyses in this thesis. In the catenae used for snow cover parameterization, snow accumulated using the parabolic equation (5.7), with accumulation to the end of the freezing season (i.e., $t_{\max}$ is the end of the season).

5.4.1 N_F versus the nival offset

The effect of snow cover on ground surface temperatures can be summarised as a ratio (n_F as discussed in Chapters 1 and 2), or as a temperature difference or *offset*. Using the n factor approach:

$$n_F = \frac{I_{FS}}{I_{FA}} \quad (5.21)$$

where

- I_{FS} = seasonal ground surface freezing index (degree-days)
- I_{FA} = seasonal air (screen height) freezing index (degree-days)
- n_F = freezing n-factor

The snow-related temperature difference (Δ_{SNOW}) can be referred to as a *nival offset* in analogy to the thermal offset. The nival offset is the difference in mean annual temperature between air at screen height and at the ground surface due to the effect of snow. Operationally, this is assumed to be:

$$\Delta_{SNOW} = \frac{I_{FS} - I_{FA}}{P} \quad (5.22)$$

where

Δ_{SNOW} = Nival offset (snow-related air-surface temperature difference)
 P = Annual period (365 days)

It is not clear *a priori* whether n_F or Δ_{SNOW} is the best method of accounting for snow cover effects in the **TTOP** model. The warming effect of snow cover is proportional to the amplitude of the annual air temperature wave in Lachenbruch's (1959) analytical model, suggesting that n_F naturally describes snow cover effects (as in equation (5.11), while the thermal catena results presented in Chapter 2 show that Δ_{Snow} behaves consistently as a function of TTOP. The standard form of the **TTOP** model can be re-written:

$$T_{TOP} = \frac{r_K n_T I_{TA} - I_{FA}}{P} - \Delta_{SNOW}, \quad T_{TOP} < 0 \quad (5.23)$$

$$T_{TOP} = \frac{n_T I_{TA} - \left(1/r_K\right)(I_{FA} - \Delta_{SNOW} P)}{P}, \quad T_{TOP} > 0$$

without loss of generality. Riseborough and Smith (1998) found that n_F varies with mean annual air temperature as well as snow cover thickness. Figures 5.15a and b re-express

Figure 5.15 **a)** Dependence of Δ_{SNOW} on MAAT and mean snow depth. **b)** Dependence of Δ_{SNOW} on TTOP and mean snow depth. Relationships derived from results shown in Figure 5.13.

their analysis of n_F in terms of Δ_{SNOW} . In Figure 5.15a, Δ_{SNOW} for a range of snow depths is shown as function of mean annual air temperature. For each thickness, Δ_{SNOW} has a maximum value at a mean annual temperature below 0°C, with the peak increasing in magnitude and shifting to lower temperature as snow cover thickness increases. Figure 5.15b shows the same data plotted as a function of TTOP. For each snow thickness, Δ_{SNOW} peaks at approximately the same value of TTOP, just below 0°C. This can be attributed to the effect of seasonal ground freezing, which reaches its maximum where the mean annual temperature is 0°C. Other things being equal, the freezing season is shorter at warmer temperatures (that is, in seasonally frozen ground), while at colder temperatures (in permafrost) the thawing season is shorter so that the active layer re-freezes more quickly.

In the sections that follow, both n_F and Δ_{SNOW} are used to summarise the thermal effect of the snow cover.

5.4.2 Snow density

The snow cover for the standard thermal catena analyses presented in this thesis have a density of 250 kg m⁻³, which corresponds to a thermal conductivity of 0.21 Wm⁻¹K⁻¹, a volumetric heat capacity of 5.23×10⁵ Jm³K⁻¹ and a thermal diffusivity of 4.01 ×10⁻⁷ m²s⁻¹ within TONE (Appendix I). Capturing the combined effects of snow cover thickness and thermal properties is examined in this section, so that catena results can be related to different snow covers.

The temperature profile within the snow cover may be controlled primarily by phase change within the soil, or by diffusion once active layer freezeback is complete.

Conductivity (k_s) is the controlling snow thermal parameter during freezeback, while thermal diffusivity (α_s) will dominate afterwards. The analysis of Riseborough (2001) suggests that snow thermal conductivity is more important than its thermal diffusivity, at least during the period that the active layer is freezing back.

The model of Lachenbruch (1959) and Kudriavtsev *et al.* (1977), discussed above, expresses the relationship between air and ground surface temperatures as a function of the snow cover diffusivity and thickness:

$$A_z = A_0 e^{-Z_s \sqrt{\pi / \alpha_s P}} \quad (5.24)$$

from which Riseborough and Smith (1998) calculated n_F as:

$$n_F = \frac{A_z}{A_0} = e^{-Z_s \sqrt{\pi / \alpha_s P}} \quad (5.25)$$

Alternatively, the insulation effect of a given thickness of a material can be expressed as a thermal resistance. R is used in steady state heat flow calculations (in which heat flow = $\Delta T / R$). For snow cover:

$$R_s = \frac{Z_s}{k_s} \quad (5.26)$$

where

R_s = snowpack thermal resistance, $K W^{-1}$

Z_s = snowpack thickness, m

k_s = snowpack thermal conductivity, $Wm^{-1}K^{-1}$

Equations (5.25) and (5.26) suggest two different ways in which changes in snow cover properties (thickness and thermal properties) will affect the temperature at the base of the snowpack. In equation (5.25), changes in thermal diffusivity are equivalent to changes in snow cover thickness, For a given value of n_F :

$$\alpha_s = \left(\frac{-Z_s}{\ln(n_F)} \right)^2 \frac{\pi}{P} \quad (5.27)$$

In equation (5.26), changes in thermal conductivity are also equivalent to changes in snow cover thickness, For a given value of R_s :

$$k_s = \frac{Z_s}{R_s} \quad (5.28)$$

Equations (5.27) and (5.28) both suggest that a snow cover with higher α_s or k_s is equivalent to a thinner snow cover. In the discussion that follows, the approach based on equations (5.25) and (5.27) will be referred to as the “extinction” approach; that based on equations (5.26) and (5.28) will be referred to as the “resistance” approach.

Thermal catenae for equivalent snow thickness

Equations (5.26) and (5.27) were used to determine the thickness of a snow cover equivalent to that used in the standard thermal catena (density = 250 kg m⁻³) (Figure 5.16). The resulting relationships indicate that equation (5.26) is much more sensitive to changes in thermal properties than equation (5.25). For example, 25 cm of snow with a standard density of 250 kg m⁻³ has the same thermal resistance as 46 cm of snow with a density of 350 kg m⁻³, and the same damping depth as 28.5 cm of snow with a density of 350 kg m⁻³.

Thermal catena simulations were performed for snow depths equivalent to the standard maximum snow cover of 0.25 and 0.5 m (0.46 and 0.91 m for the resistance case; 0.285 and 0.57 m for the extinction case). The snow ablation factor was adjusted to

Figure 5.16 Depth of snow cover required giving the equivalent thermal effect of snow cover with a density of 250 kg m^{-3} . The magnitude of the snowcover effect depends on whether the snow is considered to be primarily diffusive (extinction) or in a quasi-steady state (resistance).

ensure that the effect of the snow cover ablation period was the same for each case, allowing direct comparison of TTOP between these cases and the standard case.

Comparisons of MAGST values for each case are shown in Figures 5.17a and b. MAGST for the resistance cases are very close to the standard cases over the entire range of the catena, while MAGST for the extinction cases are much colder values than the equivalent standard cases. The maximum difference between MAGST for the standard and resistance cases is 0.18°C , compared to a maximum difference of 3.9°C for the extinction cases. Figure 5.18 compares the annual cycle of ground surface temperature for a typical case. Although not an exhaustive analysis, these results suggest that the resistance concept best captures the effect of the snow cover on the ground thermal regime. This indicates that results of the “standard” thermal catena analysis can be re-expressed in terms of snow cover resistance, using the snow conductivity function for TONE (equation A.4 in Appendix I and equation (5.28)).

Time-dependent snow resistance functions for taiga, tundra and maritime regimes can be calculated using the time-dependent snow density functions of Sturm and Holmgren (1998). Figure 5.19 shows results for a snow cover that reaches a maximum depth of 1 m: resistance for other snow depths will scale to these values directly. The increase in snow density over time reduces the thermal resistance of the snow cover for all cases. Taiga snow cover has the highest resistance (due to its low thermal conductivity), and is influenced least by the changing density. The maritime snow cover reaches its maximum resistance in early winter, after which the increasing density (conductivity) overcomes the effect of the increasing snow depth. The tundra snow, with

Figure 5.17 Comparison of TONE simulations with standard density (250 kg m^{-3}) snow cover to snow cover at higher density with a thermally "equivalent" thickness. Standard density snow had a maximum thickness of: a) 0.25 m ;b) 0.5 m.

Figure 5.18 Ground surface temperature for cases with MAAT = -10, for snow density equivalence comparisons to 25 cm deep standard snow.

Figure 5.19 a) Relative snow depth , b) seasonal variation in snow thermal conductivity, and c) the resultant seasonal pattern of snow pack thermal resistance for the three snow classes whose densification functions are discussed in Sturm and Holmgren (1998).

the highest density, has the lowest thermal resistance, with a moderate decrease in the rate of change in the resistance due to increasing density. In numerical modeling, time dependent snow cover density could be adjusted within the time dependent depth function of a (constant density) snow cover to yield the appropriate time-dependent thermal resistance function.

5.4.3 Snow cover initiation and I_F

The relationship between the start of snow accumulation and freezing temperatures in the air is critical to the freezing n factor. Active layer freezeback can progress more rapidly before the snow arrives so that, as shown in the previous section, freezeback can progress more rapidly even after the snow cover begins to develop. This will lead to colder ground surface temperatures than would be the case if initiation of snow cover were coincident with the start of freezing conditions, since freezeback can conclude more quickly. Thus, even if n_F is determined for the shorter period that snow is present, it will be higher (i.e., surface temperature will be proportionally colder) than for the same snow cover arriving concurrent with the onset of freezing conditions.

From climate station snow statistics (Table 5.2; Figure 5.10), the station-average snow-free period with freezing temperatures varies from 7 to 18 days, with an average of 12 days, while the station-average freezing degree-day total (I_{FSF}) for the snow-free period varies from 18 to 80 degree-days. The thermal effect of delaying snow cover initiation was investigated using TONE, by systematically varying the initiation date. Simulations were performed for 25-cm and 50-cm deep snow. In standard catenae, snow accumulation begins one day following the start of freezing at the ground surface, to ensure the initiation of ground freezing (as recommended by Goodrich, pers. comm.). In

this analysis, snow cover initiation was delayed by 2, 5, 10, 15 and 20-days from the start of freezing. All other parameters for the snow were unaltered, allowing snow to accumulate to the same maximum depth over a shortened accumulation period.

Figures 5.20 and 5.21 show typical results for permafrost and seasonally frozen ground, respectively. Ground surface temperatures are equal to the air temperature until snow begins to accumulate. In the seasonally frozen ground cases, the coldest ground surface temperatures of the entire freezing season are experienced in the snow free period for delays of 15 days or more. Surface temperature rises immediately following the arrival of snow, as the energy exchange boundary moves away from the ground surface and heat flow from the warm ground below can offset heat loss through the snow. Ground temperatures begin cooling again within a few days. Sites with longer delays in snow cover initiation remain colder throughout the entire winter, although the difference becomes progressively smaller toward the end of winter.

Maximum active layer depth (Figure 5.20a) is relatively insensitive to snow cover initiation date, varying from 1.16 m for the standard case to 1.04 m for the case with a 20-day delay. The snow cover initiation date has a much stronger effect on active layer freezeback, since this normally occurs under the snow cover. In the standard case, about 0.02 m of freezing occurred before the snow arrives, and freezeback takes around 80 days. In contrast, a 20-day delay allows about 0.45 m of freezing to occur before the snow arrives, and freezeback is complete in about 45 days. In the seasonally frozen ground case (Figure 5.21a), the depth seasonal freezing is much more sensitive to snow cover initiation date, varying from 0.74 m for the standard case to 1.13 m for the case with a 20-day delay. The effect is primarily due to freezing during the initial snow-free

Figure 5.20 An example showing the effect of the delay of snow cover initiation on thermal conditions in permafrost. The effect of the duration of the delay on **a)** active layer depth and **b)** ground surface temperature. Example is for MAAT = -10°C and 25 cm maximum snow depth.

Figure 5.21 An example showing the effect of the delay of snow cover initiation on thermal conditions in seasonally frozen ground. Effect of the duration of the delay on **a)** seasonal freezing layer depth and **b)** ground surface temperature. Example is for MAAT = +2°C and 25 cm maximum snow depth.

period, with 0.4 m of freezing occurring during the snow-free period in the case with a 20-day delay.

The magnitude of snow cover delay effect is temperature dependent. Figure 5.22 shows the nival offset as a function of both TTOP and the duration of the snow cover delay. For a given value of TTOP, the change in nival offset is an almost linear function of the duration of the delay. The change in Δ_{SNOW} is on the order of 0.1°C for every day of delay for the 50 cm case and about 0.06°C for the 25 cm case. For a given delay, the nival offset is greatest near TTOP = 0°C, decreasing more rapidly with temperature above 0° than below.

The thermal effect of delaying the start of snow accumulation is not limited to the increase in I_{FS} for the period that the snow is absent. The freezing degree-day total at the surface for the entire 20-day snow-free period was 60 to 70 degree-days (depending on MAAT), which would translate into a change of about 0.2°C on an annual basis. Because snow cover generally increases in thickness until the end of winter, on any date in the freezing season the late-arriving snow cover will tend to be thinner than an earlier arriving snow cover (this is always so in the numerically modeled snow cover). More important, the absence of snow allows the unfrozen surface layer of the ground to freeze more rapidly, so that active layer freezeback can conclude more quickly. Both of these influence ground surface temperature (Riseborough, 2001).

5.4.4 Snow accumulation rate

The thermal effect the snow accumulation pattern *per se* was investigated by creating several catenae with alternate snow accumulation patterns. In the standard catenae, snow cover accumulates following a square root function of time as described

a) Nival offset for maximum snow depth = 25 cm

b) Nival offset for maximum snow depth = 50 cm

Figure 5.22 Effect of delaying the initiation of snow accumulation on the nival offset (Δ_{SNOW}) for **a)** 25 cm snow cover and **b)** 50 cm snow cover. Results are shown as a function of the duration of the delay and of TTOP.

above, reaching its maximum depth at the end of the freezing. In these experiments snow accumulated at a uniform rate (i.e., the exponent in equation (5.7) was 1), while $t_{\max}$ varied between 0 (in which the full snow depth arrived on the first day of winter) and 1 (in which snow accumulated continuously until the end of winter) in increments of 0.1. Snow depths were adjusted to give the same seasonal mean snow depth (Figure 5.23), so that differences in the ground thermal regime for different accumulation rates would be due to the seasonal pattern alone. Catena simulations were executed for snow depths nominally designated as 25 cm and 50 cm. Results are summarised in Figure 5.24 for snow nominally 25 cm (Figure 5.24a and b) and 50 cm deep (Figure 5.24 c and d).

Results show that the nival offset is sensitive to the pattern of accumulation, as well as TTOP. The greatest nival offsets occur near $TTOP = 0^{\circ}\text{C}$, while for any given value of TTOP, early arrival of deep snow results in higher ground surface temperatures. The greatest nival offsets occur where $t_{\max} = 0.1$. For both snow depths, sensitivity to the pattern of snow accumulation is greatest at lower values of TTOP. Near $TTOP = 0^{\circ}\text{C}$, the difference between the early snow (peak = 0) and late snow (peak = 1) is on the order of 2°C in both 25 and 50 cm snow (Figure 5.24 b and d).

5.4.5 Analyses of standard catena n_F and nival offset

Figure 5.25 and 5.26 show n_F and the nival offset (Δ_{Snow}) as a function of maximum snow depth and TTOP for three substrate conditions: bedrock, the standard mineral soil, and organic soil (1 m thick over the standard mineral soil). All thermal properties are listed in Table 5.3. The bedrock and organic soils represent substrate materials with properties close to the natural extremes for thermal diffusion and phase change, while the standard mineral soil represents a material with intermediate properties.

Figure 5.23 Examples of snow accumulation patterns used in the analysis of sensitivity of nival offset to variations in the timing of peak snow accumulation. The season-average snow depth is the same for all patterns, so that maximum snow depth is greater for patterns that peak later in the season.

Figure 5.24 N_F and nival offset surfaces, shown as a function of TTOP and the timing of peak snow accumulation. “Seasonal fraction” refers to the time at which snow cover reaches its maximum depth. Results are shown for the “nominal” 25 cm and 50 cm cases.

a) Bedrock

b) Mineral

c) Organic

Figure 5.25 Surfaces of n_F as a function of TTOP and maximum snow depth for bedrock, standard mineral soil, and organic soil substrates. Contour interval is 0.1 (dimensionless). Markers indicate data points from catena analyses.

Figure 5.26 Surfaces of Δ_{Snow} as a function of TTOP and maximum snow depth for bedrock, standard mineral soil, and organic soil substrates. Contour interval is 1°C. Markers indicate data points from catena analyses.

The standard mineral soil has a moderate bulk density and volumetric moisture content of 44%. The organic soil, with 85% water, has a lower latent heat of fusion and thermal conductivity than pure water, giving it a slightly higher Stefan number than water⁴ (8% and 18% higher for thawed and frozen conditions, respectively), although sensible heat flow is likely to be lower than for pure water because the matrix of soil material inhibits convection. Bedrock represents the opposite extreme, with no latent heat and a relatively high thermal conductivity that does not change at 0°C. The range of n_F and Δ_{Snow} exhibited by these materials and climatic conditions should be representative of most natural environments.

The n_F surfaces shown in Figure 5.25a-c show a similar behaviour, with n_F declining (i.e., the warming effect of the snow increasing) as both snow depth and TTOP increase. However, the transition between permafrost and seasonally frozen ground is much more pronounced for the soils containing moisture (mineral and organic soils). In seasonally frozen ground (TTOP above 0°C), n_F approaches zero with relatively thin snow cover in each material. In permafrost with snow greater than about 50 cm deep for these materials, n_F is less sensitive to snow depth but is strongly dependent on TTOP.

The effect of the permafrost – seasonally frozen ground transition is more apparent in Figure 5.26a-c, showing the nival offset (Δ_{Snow}) as a function of TTOP and maximum snow depth for the same substrate materials. In all three, Δ_{Snow} increases as snow depth increases. For any given snow depth, Δ_{Snow} is maximum near 0°C, declining at higher or lower temperatures. The overall pattern is similar for the mineral and organic

⁴ Stefan numbers depend on temperature (equation 2.5), so that the *absolute* value of Ste cannot be known without knowing the thermal state.

substrate materials, with the axis of maximum nival offsets changing sharply at $T_{TOP} = 0^{\circ}\text{C}$. In the bedrock, the axis is somewhat colder (around -4°C), and is much less abrupt. The temperature dependence of Δ_{Snow} can be ascribed to the climatology of the freezing index, and to thermal processes in the active layer.

Figure 5.27a-c shows the nival offset for the three substrates as a function of snow depth and MAAT: as with Figure 5.26, the patterns for mineral and organic soil substrates are similar, and distinct from the bedrock substrate. In both materials, two domains of behaviour are apparent, with the transition between these domains along the line separating permafrost and seasonally frozen ground. In the seasonally frozen ground domain (in increasingly deeper snow at colder MAAT), Δ_{Snow} is almost independent of snow depth; in permafrost (the domain of shallower snow), Δ_{Snow} is strongly dependent on snow depth but also depends on temperature.

At warmer air temperatures, Δ_{Snow} is limited by the value of I_{FA} . For sufficiently deep snow, Δ_{Snow} is almost independent of snow depth and approaches I_{FA}/P (as n_F approaches zero). The snow depth at which this limit is reached is different as a function of temperature and substrate. Thus, the maximum value of Δ_{Snow} for the mineral and organic soils is greater than for bedrock, even though at intermediate snow depths Δ_{Snow} is greater for the mineral soil cases for a given snow depth and T_{TOP} value. For bedrock, however, the limiting offset (where n_F is zero) is reached only at warmer temperatures, where I_{FA} is lower. This can be seen in Figure 5.28, comparing I_{FS}/P to Δ_{Snow} for 50-cm

a) Bedrock

b) Mineral

c) Organic

Figure 5.27 Surfaces of Δ_{Snow} as a function of MAAT and maximum snow depth for bedrock, standard mineral soil, and organic soil substrates. Contour interval is 1°C . Markers indicate data points from catena analyses.

Figure 5.28 Δ_{Snow} for 50 cm deep snow for standard bedrock, mineral soil, and organic soil substrates as a function of MAAT, together with I_{FS}/P as a function of MAAT.

deep snow for the three substrates, as a function of TTOP and MAAT. For the bedrock catena, Δ_{snow} approaches I_{FS}/P only at the warmest temperatures in the catena.

In permafrost, freezeback proceeds until the active layer is re-frozen, while in seasonally frozen ground, freezing continues throughout the winter. The proximity of the freezing front prevents the ground surface temperature from becoming extremely cold until the active layer is completely refrozen. Near-surface temperatures can fall rapidly once freezeback of the active layer is complete, as described in Riseborough (2001). Moving from colder to warmer permafrost, the heat content of the active layer increases as I_{TS} increases; from the permafrost boundary to warmer temperatures, the ground below the frozen layer supplies increasing amounts of heat to limit frost penetration as thawing degree-days increase and freezing degree-days decrease.

The original n_F parameterization used in Smith and Riseborough (2002a), was “intended to capture the combined influences of the attenuation effect of the snow cover (Lachenbruch, 1959) and the conductive capacity effect produced by the heat content of the active layer”. The clear relationship of Δ_{snow} to temperature suggests that it may be a better parameterization of the relationship of I_{FA} to I_{FS} than n_F . In the remainder of this chapter, only Δ_{snow} will be considered.

5.4.6 Freezing indices during the conductive period

In permafrost, the two principal sub-seasons in the winter ground thermal regime are the period of active layer freezeback and the subsequent period dominated by conduction in the ground. Results from the standard case thermal catenae were used to examine the thermal behaviour of the soil-snow interface after active layer freezeback is complete. (The seasonally frozen ground cases do not experience a post-freezeback

season.) Surface temperature time series were divided into the freezeback and post-freezeback periods: I_{FS} was subdivided into the two parts of the freezing season: I_{FFB} (freezing index for the freezeback period) and I_{FC} (freezing index for the post-freezeback, or “conductive” period). In this scheme, $I_{FS} = I_{FFB} + I_{FC}$ for permafrost, while in seasonally frozen ground $I_{FC} = 0$ so that $I_{FS} = I_{FFB}$.

The relative magnitudes of I_{FFB} and I_{FC} were investigated using the isotherm and temperature output files produced by TONE, using the 0°C isotherm and the instantaneous temperature at the ground surface stored by TONE once per day (i.e., after 32 time steps). The end of freezeback was identified when the upper and lower freezing fronts met between two nodes in the calculation mesh and disappeared. I_{FFB} was calculated as the surface freezing degree-day total for the period from the initiation of freezing to the end of freezeback. I_{FC} was calculated as the difference between I_{FS} (recorded directly by TONE) and I_{FFB} : $I_{FC} = I_{FS} - I_{FFB}$. Truncating the freezeback period to whole days introduces an error in I_{FC} or I_{FFB} of up to one day’s freezing index: in this analysis, the lowest surface temperature at the end of freezeback was −12.6°C (over bedrock), with $I_{FS} = 862$, which gives a maximum error of 0.05%.

Figure 5.29 shows I_{FFB} and I_{FC} as a function of TTOP for the standard mineral, bedrock, and 1-m organic substrates. Permafrost cases for all snow depths are included. For all substrates, I_{FC} is the largest component of I_{FS} when TTOP is less than −2°C, with I_{FC} increasing linearly (with some scatter) as TTOP decreases. In colder permafrost, upward freezing can conclude the freezeback process more quickly.

Results for bedrock differ from those of the other substrates in two ways. First, I_{FFB} is significantly larger, especially near 0°C. Second, for the bedrock case the

Figure 5.29 Partial surface freezing indices during freezeback (I_{FFB}) and during the conductive period (I_{FC}) as a function of TTOP for the standard mineral soil, organic soil, and bedrock substrates. Results are shown for all snow depths, for permafrost cases only.

relationship between I_{FC} and TTOP shows greater scatter than for the other substrates.

Both differences reflect the importance of water as a control of the thermal regimes in the other materials. Without latent heat to limit the propagation of the surface temperature wave, the annual wave penetrates the surface more deeply, so that temperature gradients between the surface and the 0°C isotherm in the ground (which is not a physical freezing front in the bedrock) are shallower, extending the time needed to bring the temperature below 0° .

As suggested in Chapter 2, permafrost can be defined by the difference between heat flows at temperatures above and below 0°C : this difference is represented by I_{FC} . I_{FC} is that component of the freezing index that is not involved in soil freezing, and should be closely related to TTOP. The relationship of TTOP to I_{FC} (transformed to an equivalent freezing temperature $-I_{FC}/P$) is shown in Figure 5.30. Results for the standard and organic substrates show a close relationship to TTOP, falling below the 1:1 line by a few tenths of a degree. Results for the bedrock substrate also fall below the line, but differ from TTOP by as much as 2°C at temperatures between -8 and -2°C .

The differences in behaviour between bedrock and the other substrates, and the clear relationship to TTOP can be ascribed to the influence of ground freezing. The energy required to thaw the soil (determined by $k_T I_{TA}$) is balanced by the energy removed to freeze it (determined by $k_F I_{FFB}$), with mean annual temperature determined by the residual freezing degree-days. The difference between $-I_{FC}/P$ and TTOP is partially accounted for by the neglect of sensible heat in determining I_{FC} and I_{FFB} . Toward the end of freezeback, the last degree-days added to I_{FFB} do not contribute to active layer freezing, since there is insufficient time for heat to flow from the freezing front to the

Figure 5.30 $-I_{FC}/P$ as a function of $TTOP \times X^2$ (where X = active layer depth) for the standard mineral soil, organic soil, and bedrock substrates. Results are shown for all snow depths, for permafrost cases only.

ground surface (given the finite properties of the ground). For the bedrock substrate, the active layer reaches significant depth (up to 5 m), making this a significant component of I_{FFB} . Figure 5.31 shows this difference (i.e. $TTOP - I_{FC}/P$) as a function of $TTOP$ and active layer depth ($TTOP \times X^2$) for each substrate. While the relationships between active layer depth and ($TTOP - I_{FC}/P$) are different (reflecting the differing thermal properties of the substrates), their apparent strength suggests that active layer sensible heat accounts for at least part of the effect. This is not considered further.

5.4.7 A model of the total offset at permafrost boundaries

Results discussed in the preceding sections have significant implications for modeling the nival offset. The division of winter into freezeback and conductive sub-seasons suggests that separate models of the two periods are required, while the inaccuracy of the Stefan model as $TTOP$ moves above or below 0°C suggests that a simple model will be increasingly inaccurate. The complementarity of active layer freezing and thawing indicates that the nival offset is directly influenced by summer (thawing) conditions, at least for permafrost with a conductive season. These findings suggest that a prognostic physical model of the nival offset is not straightforward. On the other hand, the nival offset for seasonally frozen ground has been shown to behave consistently as a function of MAAT (Figure 5.28), which is knowable *a priori*. Further, the snow thermal resistance, soil latent heat, and soil frozen thermal conductivity have been identified as key parameters during freezeback. In this section, an empirical model for the nival offset at permafrost boundaries (where $TTOP = 0^\circ\text{C}$) is presented. In earlier analyses, the goal was to determine the nival offset (or $TTOP$) for a particular value of MAAT. For this analysis, the goal is to determine the MAAT that

Figure 5.31 $TTOP - I_{FC}/P$ as a function $TTOP \times X^2$ (where X = active layer depth) for the standard mineral soil, organic soil, and bedrock substrates. Results are shown for all snow depths, for permafrost cases only.

corresponds to the permafrost – seasonally frozen ground boundary at $TTOP = 0^{\circ}\text{C}$.

The reference point moves from the top of the snow cover to bottom of the active layer.

The analysis is based on the supplementary simulations discussed in Section 3.2 in which MAAT was adjusted by trial and error to obtain $TTOP = 0^{\circ}\text{C}$ (within $\pm 0.1^{\circ}\text{C}$), for a range of snow depths. In addition to the simulations performed for “standard” mineral soil, saturated organic soil and bedrock, several additional sets of simulations were added to increase the range of substrate conditions examined. These included simulations with:

- The standard soil substrate at reduced moisture content (75%, 50%, and 32% of saturation, respectively⁵),
- Soil at “standard” density and water content, but with mineral content specified as 0% or 100% quartz,
- Soil at “standard” density and low water content, but with mineral content specified as 0% or 100% quartz,
- Saturated soil with 60% porosity, and
- Substrate thermal properties equivalent to water.

Physical properties for these materials are summarised in Table 5.3.

Figure 5.32 shows the relationship between MAAT and snow depth at the 0°C transition for several substrates. With no snow cover, MAAT is above 0°C for all cases with a conductivity ratio r_K less than 1 (i.e., except bedrock and low moisture soil cases), by an amount depending on r_K (since in the absence of snow the transition MAAT is determined solely by the thermal offset). For all cases, increasing snow depth lowers the

⁵ 32% saturation (15% volumetric moisture in this case) was chosen because this is the moisture content at which $k_F \approx k_T$ in the Johansen model.

Figure 5.32 Mean annual air temperature at which $TTOP = 0^{\circ}\text{C}$ as a function of snow depth for various substrate materials.

transition air temperature, although the slope of the relationship varies by substrate.

Substrates rank from warmer to colder depending on the combined effect of thermal and nival offsets. With no snow for example, permafrost can form in bedrock where MAAT is 0°, while in saturated peat (organic) permafrost can form with MAAT about 5° warmer. In contrast, with 1.5 m deep snow permafrost may form in bedrock where MAAT is about -9°C, while in peat permafrost can form only if MAAT is about 4° colder. Differences in substrate produce a range in the transition MAAT of 3 to 6°C over the range of snow cover depths examined.

Defining $MAAT_0$ as MAAT at the transition where no snow is present, Figure 5.33a shows the results of Figure 5.32 with MAAT re-expressed as the difference between MAAT and $MAAT_0$. For a given substrate, this normalized temperature difference $\Delta_0 (= MAAT - MAAT_0)$ is the difference in MAAT at the boundary produced by a given snow cover for a given substrate. Figure 5.33a demonstrates the relative influence of the substrate materials: in general, at any snow depth the transition temperature is lower as the substrate water content is higher. For soils with the same water content (standard; all quartz; no quartz), the transition temperature is lower as the thermal conductivity is higher.

Figure 5.33b shows the data of 5.33a transformed to yield relationships suitable for linear regression. Regression coefficients and intercepts are shown in Table 5.4. Results from the bedrock substrate simulations differ from the others: intercepts for all other materials are within +/- 0.004 of a mean value of -0.0361, while the bedrock intercept differs by more than 0.02. As a result, the curve for the bedrock substrate crosses the curve for the standard soil at low moisture content.

Figure 5.33 a) Results of Figure 5.32, re-expressed as the difference between MAAT at the transition point for a given snow cover, and MAAT at the transition with no snow cover ($MAAT_0$). **b)** Linear regression of inverse of a) above.

Table 5.4 Regression coefficients for relationships shown in Figure 5.33b, and substrate property index values for relationships shown in Figure 5.34.

Substrate	Slope	Intercept	$\theta_v k_F$
Standard Mineral Soil	-0.0274	-0.0388	1.235
1m Organic	-0.0265	-0.0343	1.387
Bedrock	-0.0781	-0.0562	0.000
Water	-0.0223	-0.0339	2.200
All Quartz	-0.0209	-0.0382	1.980
No Quartz	-0.0309	-0.0363	0.974
All Quartz 0.15 θ_v	-0.0697	-0.0379	0.237
No Quartz 0.15 θ_v	-0.1036	-0.0318	0.116
60% Porosity	-0.0233	-0.0367	1.624
Standard 0.15 θ_v	-0.0865	-0.0346	0.170
Standard 0.24 θ_v	-0.0568	-0.0363	0.372
Standard 0.35 θ_v	-0.0331	-0.0388	0.802

The influence of active layer freezing on the nival offset is due to the amount of heat stored in the active layer as latent heat of fusion. As shown in earlier chapters, the soil's thermal conductivity and volumetric latent heat determine the amount of latent heat that can be stored within the active layer. Figure 5.34a shows the relationship between the regression coefficients of Table 5.4 and $\theta_v k_F$ for each substrate. The expression $\theta_v k_F$ is a measure of the capacity to store latent heat within the active layer, as suggested by the Stefan equation. As before, results for the bedrock substrate do not follow the trend of the other substrate materials, since a large amount of heat can move out of the ground (because of its high thermal conductivity) even though $\theta_v k_F$ is zero. As a result, the thermal regime of the bedrock material is fundamentally different from other materials containing water.

The relationship between $\theta_v k_F$, Z_s and Δ_0 was modeled by fourth-order polynomial trend surface using the program Surfer© (Figure 5.35). Results for water and bedrock were excluded. The equation for the surface is:

$$\begin{aligned} \Delta_0 = & 0.0332 - 6.75498 \theta_v k_F + 11.4734 (\theta_v k_F)^2 - 7.27919 (\theta_v k_F)^3 + \\ & 1.54104 (\theta_v k_F)^4 - 9.34134 Z - 28.0718 Z \theta_v k_F + 7.76328 Z (\theta_v k_F)^2 - \\ & 0.761135 Z (\theta_v k_F)^3 + 5.7717 Z^2 + 19.4595 Z^2 \theta_v k_F - 2.18458 Z^2 (\theta_v k_F)^2 \\ & - 4.75423 Z^3 - 4.677 Z^3 \theta_v k_F + 1.65224 Z^4 \end{aligned} \quad (5.29)$$

Taken together, the relationships illustrated in Figures 5.31 to 5.35 can be used to predict MAAT at the transition, given seasonal snow depth, r_K , k_F , and soil moisture content. The calculation chain is as follows:

1. Determine Δ_0 (from snow depth and $\theta_v k_F$) using equation (5.29);
2. Determine MAAT0 along the thermal catena (from IFA, ITA, and r_K);

Figure 5.34 Relationship between regression line slopes from Figure 5.33b and substrate property index $\theta_v k_F$.

From the equations defining I_{FA} and I_{TA} for the standard catena (equations (3.5) and (3.6) in Section 3.2.1), and assuming $I_{FA} = I_{FS}$ and $I_{TA} = I_{TS}$, so that $I_{FA} = r_K I_{TA}$, the thermal offset can be solved for $T_{TOP} = 0^\circ\text{C}$ with equations (3.5) and (3.6):

$$\frac{144.4r_K + 220.6 - \sqrt{(-220.6 - 144.4r_K)^2 - 16050(1 - r_K)^2}}{3.736(1 - r_K)} \quad (5.30)$$

3. Determine $MAAT = MAAT_0 + \Delta_0$.

Together, this procedure can be called the *permafrost boundary model*.

Using the permafrost boundary model to reproduce the simulation results yielded an RMS error of 0.178°C , with maximum errors within $\pm 0.6^\circ\text{C}$. Figure 5.36a shows a comparison of the numerically modeled and empirically estimated values of MAAT at the permafrost transition. While the empirical model compares well with the simulation results, errors are larger at lower temperatures, representing results for deeper snow cover.

Figure 5.36b shows the frequency distribution of differences between MAAT determined from numerical results and estimated using the permafrost boundary model. Most differences are within $\pm 0.2^\circ\text{C}$. Recall that the MAAT values obtained from numerical modeling have a precision of about 0.1°C for these simulations, due to the iterative process required to obtain the permafrost transitional condition.

In Chapter 8, this permafrost boundary model will be applied at the national scale to examine permafrost zonation in Canada.

Figure 5.35 a) Surface plot of Δ_0 (i.e., MAAT-MAAT₀) as a function of maximum snow cover thickness and $\theta_V k_F$, interpolated by kriging in Surfer©. **b)** Fourth order polynomial trend surface of data used in a) above. Equation of surface (x = maximum snow cover thickness, $y = \theta_V k_F$): $\Delta_0 = 0.0332 - 6.75498y + 11.4734y^2 - 7.27919y^3 + 1.54104y^4 - 9.34134x - 28.0718xy + 7.76328xy^2 - 0.761135xy^3 + 5.7717x^2 + 19.4595x^2y - 2.18458x^2y^2 - 4.75423x^3 - 4.677x^3y + 1.65224x^4$

Figure 5.36 a) Comparison of MAAT at a permafrost – seasonally frozen ground boundary from numerical model results and from the permafrost boundary model. b) Frequency distribution of differences between numerical and boundary model values in a).

Chapter 6 Freezing and Thawing Indices in the Ground

As with mean annual temperature (Figure 1.3), temperature indices (I_F and I_T) can be determined for any position in the atmosphere or the ground. Difficulties in defining the surface and measuring its temperature have led many investigators to measure temperature a small distance below the surface. This chapter examines the implications of that procedure, leading to a simple model for the profile of I_F and I_T through the active layer. The model is then used in the second half of this chapter to extend the **TTOP** model to soils with two (or more) layers having distinct thermal properties.

Ground surface temperature

Ground surface temperature is singularly difficult to measure, due to problems of energy exchange, geometry, and definition.

Perhaps the most serious challenge to surface temperature measurement is the extremely dynamic energy flow at the boundary between the atmosphere and the solid earth. The radiative/convective environment at the surface produces the greatest temperature variation (in both space and time) in the surface boundary layer (Oke, 1987). In addition, surface temperature sensors can be significantly biased by their own energy exchanges in this situation (Monteith and Unwin, 1990, Schwerdtfeger 1976). As a result, a representative surface boundary temperature is difficult to measure in a way that is meaningful for modeling or diagnostic purposes.

Defining the position of the ground surface for the purpose of sensor placement is a problem due to discrepancies between the scales of model application, surface geometry, and measurement. Simple one-dimensional models such as **TTOP** assume

implicitly that lateral variations in the conditions determining the model parameters do not affect the model. For this to be so in a model defined by thermal processes, the surface must be an idealized horizontal plane. In reality, the ground surface is a boundary layer where the relative proportions of atmosphere and solid earth vary through a transition of finite vertical dimension. The variety of surface geometries in permafrost regions can depart from the ideal by very little (e.g., smooth, bare bedrock) to very much (e.g., peat-covered hummocks with dense shrub cover), so that the thickness of the boundary can range over four or more orders of magnitude (millimeters to tens of meters). Confidence in assigning a position for the nominal surface for modeling, monitoring or other purposes will depend in part on the distance between the object of interest (such as a temperature sensor or a point in a modeled temperature profile) and the boundary zone. The nominal or effective surface position might be adequately approximated by the mid-height of the boundary zone, for temperature measurements taken at a depth several times the thickness of the zone.

Smith and Riseborough (2002a) define the surface in a way that avoids these problems:

“The ground surface referred to above is conceived as a thermal boundary rather than a physical one. It is the level below which conductive heat transfer dominates.” (Smith and Riseborough 2002a, p.5)

This approach is acceptable for defining the conceptual basis of the model. A more specific definition may be required to guide measurement decisions when determining n factors or offsets in the **TTOP** model.

Near-surface temperature

To avoid the difficulties enumerated above, field studies of the relationship of air and surface temperature often use temperature data for a sensor buried a few centimeters below the ground surface. In this thesis, temperature measured close to the surface used as an index of ground surface temperature (GST) is referred to as *near-surface temperature* (NST).

As automatic data acquisition systems have become more affordable and compact, the number of studies measuring near-surface ground temperature directly has increased (e.g., Karunaratne 2002, Klene *et al.* 2001, Smith *et al.* 2001, Thorn *et al.* 1999, Burn 1998a, Hoelzle *et al.* 1999, Taylor 1995, 2000). Klene (2000, p. 34) defines the n factor for the “soil surface”, assumed to be “under the vegetation mat as close as possible to the intersection of the base of the humic layer and the top of the mineral soil”. Thorn *et al.* (1999) compare air temperatures with ground temperatures measured at 10 to 50 cm depth. Burn (1998a) derived n factors for measurements at 20 cm depth, while Taylor (1995, 2000) measured NST at depths of 3 to 7 cm.

Using this strategy, high-frequency temperature variations at the surface are filtered by diffusion, while sensor bias is reduced in the predominantly conductive thermal regime of the ground. The drawback to this approach is that the energy balance of the layer of ground between the surface and the sensor results in a temperature difference. Karunaratne (2002) presents data for a number of different sites in the Mayo area which show that spring and early summer near-surface ground temperatures are sensitive to the presence of the active layer, while from mid July onward, NST becomes increasingly tied to air temperature. The late-summer concordance of air temperature

and NST reflects the fact that the active layer is sufficiently deep that the near-surface temperature gradient does not strongly affect NST on a daily average basis. These results suggest that differences in n factors based on NST are strongly influenced by differences in sub-surface properties and processes, rather than surface processes: as such, n factor correlation with particular environments may be due to sub-surface conditions and sensor burial depth rather than surface conditions. The magnitude of index differences due to sensor burial depth is explored in the following section.

6.1 Modeling temperature indices in the ground.

Taylor (1995) suggested that the profile of temperature indices within the active layer can be extrapolated to the surface using an exponential function, based on the standard diffusion model for propagation of a sinusoidal surface temperature wave into the ground without phase change. A drawback of this model is that it implies a theoretically infinite active layer depth.

An alternative approach is explored in this section: assuming that the dominant process in the active layer is phase change associated with freezing and thawing, the Stefan model can be used to develop a solution for thawing index versus depth. From Section 2.1.1, the Stefan equation applied to the thawing degree-day total (the thawing index I_T) is:

$$X = \sqrt{\frac{2kI_{TS}}{L}} \quad (6.1)$$

where

- X = thaw depth
- k = unfrozen thermal conductivity of the soil
- L = soil volumetric latent heat of fusion
- I_{TS} = surface thawing index (at depth = 0)

Assuming that the thawing index at a depth D below the surface but within the active layer (i.e., $D < X$), also satisfies equation (6.1) for thaw below that depth, it follows that:

$$X - D = \sqrt{\frac{2kI_{TD}}{L}} \quad (6.2)$$

where

D = Depth below the surface (within the active layer)

I_{TD} = Thawing index at depth D

From equation (6.1):

$$I_{TS} = \frac{X^2 L}{2k} \quad (6.3)$$

From equation (6.2):

$$I_{TD} = \frac{(X - D)^2 L}{2k} \quad (6.4)$$

By analogy with the surface n factor, a subsurface n factor n_{TD} can be defined for convenience as:

$$n_{TD} = \frac{I_{TD}}{I_{TS}} \quad (6.5)$$

From equations (6.3) and (6.4):

$$\begin{aligned} n_{TD} &= \frac{\left(\frac{(X - D)^2 L}{2k} \right)}{\left(\frac{X^2 L}{2k} \right)} \\ &= \left(1 - \frac{D}{X} \right)^2 \end{aligned} \quad (6.6)$$

Note that $n_{TD} = 1$ when $D = 0$ and $n_{TD} = 0$ when $D = X$.

From equations (6.5) and (6.6):

$$I_{TD} = \left(1 - \frac{D}{X}\right)^2 I_{TS} \quad (6.7)$$

and

$$I_{TS} = \frac{I_{TD}}{\left(1 - \frac{D}{X}\right)^2} \quad (6.8)$$

Equation (6.8) suggests that I_{TS} can be estimated by linear extrapolation of $\sqrt{I_{TD}}$ if I_T is known at two depths, or if I_T for one depth is known together with the active layer depth (where $I_{TD} = 0$). Substituting equation (6.1) for X in equation (6.7), the index I_{TD} at depth D below the surface can be expressed as:

$$\sqrt{I_{TD}} = \sqrt{I_{TS}} - D\sqrt{L/(2k_T)} \quad (6.9)$$

and

$$\sqrt{I_{TS}} = \sqrt{I_{TD}} + D\sqrt{L/(2k_T)} \quad (6.10)$$

The slope of the straight line between $\sqrt{I_{TS}}$ (at the surface) and $\sqrt{I_T} = 0$ at the base of the active layer is:

$$\begin{aligned} \frac{\Delta\sqrt{I_{TD}}}{\Delta Z} &= \frac{\sqrt{I_{TS}} - 0}{0 - \sqrt{\frac{2k_I I_{TS}}{L}}} \\ &= -\sqrt{L/(2k_T)} \end{aligned} \quad (6.11)$$

6.1.1 Numerical modeling

The validity of equations (6.7) to (6.11) was evaluated using TONE simulations along a thermal catena. Two different soil conditions were examined, with thermal properties calculated using Johansen's equations (Farouki 1981). Case 1 is a uniform

coarse-grained material with 49% volumetric water content ($k_F = 2.886 \text{ Wm}^{-1}\text{K}^{-1}$; $k_T = 1.488 \text{ Wm}^{-1}\text{K}^{-1}$). Case 2 is similar, but overlain with a 0.2 m thick organic layer. The organic layer comprised a 0.14 m saturated layer with 89% water ($k_F = 1.803 \text{ Wm}^{-1}\text{K}^{-1}$; $k_T = 0.535 \text{ Wm}^{-1}\text{K}^{-1}$), overlain by a 0.06 m thick layer that was assumed to be dry during summer and saturated in winter ($k_F = 1.803 \text{ Wm}^{-1}\text{K}^{-1}$; $k_T = 0.094 \text{ Wm}^{-1}\text{K}^{-1}$). To mimic an annual wetting and drying cycle for this case, the thermal conductivity and volumetric heat capacity of the unfrozen organic layer was assumed to be that for organic soil with 30% water by volume, while k_F and L were assumed to be for saturated soil (89% water).

Case 1 allows a straightforward comparison of results with equations (6.9) and (6.11), while Case 2 reveals the effect of layers with different thermal properties. For the simulations, air temperature was defined as a sine wave with a period of 365 days and an amplitude of 22°C. Mean annual temperatures varied from -12°C to +8°C, at 2°C intervals, generating 11 cases along each thermal catena. Surface n factors were specified as $n_T = 1.0$ and $n_F = 0.5$. Simulation time and space increments, as well as equilibration criteria were as described in Chapter 3.

Case 1: uniform soil

Results are presented for thawing cases only; equivalent results were obtained for freezing simulations, as discussed below. Simulated depth profiles of $\sqrt{I_{TD}}$ are shown in Figure 6.1. For the permafrost cases, profiles are nearly linear, with slight departures from linear close to the base of the active layer (where all indices approach 0). These departures may be due to the effect of accumulating numerical errors or departures from the Stefan model.

Figure 6.1 Square root of I_{TD} for thermal regimes simulated in Case 1. Each line represents the result for a given mean annual temperature.

For the seasonally frozen ground cases, the profiles of I_{TD} are far from linear and do not approach zero at the base of the active layer, because I_{TD} includes degree-days that are not involved in phase change. As discussed in Chapter 2, the Stefan equation is useful because an approximately linear gradient is maintained in the active layer by the slow progress of the phase-change front. The simulation results in Section 5.4.6 show that the freezing period can be divided into freezeback and conductive periods, in which $-I_{FC}/P$ is approximately equal to T_{TOP} . This suggests that the profile of freezing indices should exhibit behaviour similar to equation (6.8), with the freezing index for the conductive period subtracted: the mean annual temperature at the base of the seasonal freezing layer is equal to T_{TOP} . In seasonally frozen ground:

$$T_{TOP} = \frac{I_{TX} + I_{FX}}{P} \quad \text{and} \quad I_{FX} = 0$$

$$\therefore I_{TX} = T_{TOP}P$$
(6.12)

where

I_{TX} = thawing index at the base of the seasonal freezing layer

I_{FX} = freezing index at the base of the seasonal freezing layer

P = annual period (365 days)

Figure 6.2 shows that the profile of $\sqrt{I_{TD}}$ can be transformed into an approximately linear function of $\sqrt{I_{TD} - T_{TOP}P}$. Similar arguments hold for the mirrored conditions: in the seasonally frozen ground case, a simple relationship holds for $\sqrt{I_{FD}}$ while in permafrost $\sqrt{I_{FD} - T_{TOP}P}$ yields a straight-line relationship with depth within the active layer. From equation (6.11), the profiles in Figures 6.1 and 6.2 should be parallel, with a slope of $-24.13 \text{ K}^{0.5} \text{ day}^{0.5} \text{ m}^{-1}$. Figure 6.3 shows the relationship

Figure 6.2 $\sqrt{I_{TD} - T_{TOP}P}$ versus depth for seasonally frozen ground regimes from Case 1.

Figure 6.3 Slopes of square root of index functions for permafrost regimes of Figure 6.1 and seasonally frozen ground regimes shown in Figure 6.2.

between the slope of the profile for each simulated regime (determined by least squares regression) and T_{TOP} . Results for I_{FD} are shown on the same figure (for which equation (6.11) gives a slope of $-17.33 \text{ K}^{0.5} \text{ day}^{0.5} \text{ m}^{-1}$). Lines representing the profile slopes from equation (6.11) are shown for comparison. For both I_{FD} and I_{TD} , the discrepancy between the numerical results and equation (6.11) is smallest near $T_{TOP} = 0$, where the slope is underestimated by approximately 5%, increasing to 20-25% in warmer or colder soils, due to the neglect of sensible heat in the Stefan model (Romanovsky and Osterkamp, 1997). At $T_{TOP} = 0^\circ\text{C}$, the neglected sensible heat is predominantly that within the seasonal freezing (active) layer; above or below 0°C , additional sensible heat is neglected as heat flows into or from the soil below.

These results indicate that measurements below the surface can be extrapolated to the surface by extrapolation from profiles of $\sqrt{I_{TD}}$ and $\sqrt{I_{FD} - T_{TOP}}$, despite the error in equation (6.9). Transforming the index profiles to relative depth within the active layer (D/X), and from $\sqrt{I_{TD}}$ to $\sqrt{n_{TD}}$, the similarity of results for all simulations (shown in Figure 6.4) suggest that equations (6.12) and (6.7) have some utility.

Case 2: layered soil

Figure 6.5 shows depth profiles of $\sqrt{I_{TD}}$ for the thermal regimes simulated using the conditions defined for Case 2. From the surface downward, the slope of the profile declines in each layer, since k_T changes more quickly than L with increasing water content. As with Case 1, profiles for the permafrost cases are approximately linear through each layer, although the slopes differ slightly. Calculation of the ratio of slopes in the different layers (dry/wet organic; wet organic/silt) indicates that the ratio differs

Figure 6.4 n_{TD} versus relative depth ($= D/X$) within the active layer for 11 regimes simulated in Case 1.

Figure 6.5 Square root of I_T versus depth for thermal regimes simulated in Case 2 (layered soil).

from that suggested by equation (6.11), and varies with the thermal regime. These limitations on the evaluation of Case 2 are useful in the interpretation of field data, since soil profiles typically involve changes in bulk-density and water content.

6.1.2 Comparison with field data

Ground temperature data for a number of sites were examined to evaluate whether profiles of I_T or I_F from field measurements can be linearized using equation (6.7), which would be a useful method to extrapolate indices to the surface. Data sources include the Norman Wells Pipeline thermal monitoring program (MacInnes *et al.*, 1990) and the archives for the Circumpolar Active Layer Monitoring program (CALM, currently at <http://www.geography.uc.edu/~kenhinke/CALM/>).

Data for Deadhorse (Figure 6.6) are for a site that remains saturated throughout the year (Romanovsky and Osterkamp, 1997). Data for Norman Wells Pipeline site KP182 (Figure 6.7a) are for a forest-fire-affected site with ice rich silty clay till. At this site, the active layer is thinning following vegetation recovery: it was assumed that T_{TOP} was approximately 0°C . Data for the Parsons Lake site (Figure 6.7b) are for a site with earth hummocks over well-drained morainal loam.

Manipulation of temperature data to obtain $\sqrt{I_{FD} - T_{TOP}P}$ was not straightforward, since the sensor with the minimum freezing index was often within the active layer (due to inter-annual variability or upwards freezing at the base of the active layer): in these cases, $(I_F - T_{TOP}P)$ values were estimated using the minimum I_F value. Years that appeared to have corrupted data were not included in the analysis.

At Deadhorse (Figure 6.6), profiles include a surface temperature measurement (0.0 m depth). With the exception of I_{FD} for 1990, the profiles for the buried sensors are

Figure 6.6 Profiles of square root functions for I_T (A) and I_F (B) from CALM data for Deadhorse, Alaska.

Figure 6.7 a) Profiles of I_T and I_F functions for Norman Wells Pipeline monitoring site KP182, within an area affected by forest fire. **b)** Profiles of square root of I_{TD} from CALM data for Parsons Lake, Northwest Territories, Canada.

nearly linear, but do not extrapolate to the value recorded by the surface sensor. This may represent a problem with sensor exposure, or a sharp change in thermal properties.

Profiles at KP182 (Figure 6.7a) include two features not found at the other sites: This site shows a lower gradient in I_F near the surface than deeper in the profile (albeit based on data for a single sensor), while the profile of I_T has a different slope in each year. In fact, profiles for 1997 and 1998 are almost parallel.

Profiles of I_T for Parsons Lake, Northwest Territories (Figure 6.7b) exhibit a significant change in slope with depth, although the profiles are almost identical in the two years of the record.

Profile slopes are similar year to year at individual sites (with the exception of I_T at KP182), suggesting that the thermal properties and processes in the active layer are stable over time. Most profiles (except I_T as Deadhorse) show a decrease in slope near the base of the active layer likely as a result of increasing water content and soil bulk density with depth.

6.1.3 Discussion

Modeling results suggest that using near-surface measurements to estimate surface indices will underestimate their value by an amount that increases with the square of measurement depth as a proportion of active layer thickness (D/X). From equation (6.9), the effect will be greater for soils with a larger value of $\sqrt{L/2k_T}$. Using thermal conductivity values from Johansen's equations (Farouki, 1981), this function varies by a factor of about 2 for a given mineral soil, while its value is an order of magnitude higher for organic soils (Figure 6.8). In a typical thawing mineral soil I_{TD}

Figure 6.8 Effect of volumetric soil moisture on $\sqrt{L/2k}$ for typical organic and mineral soils. Conductivities estimated using Johansen (1975): mineral soil assumed to be 50% quartz with 44% porosity; organic soil assumed 85% porosity.

would be reduced by 20 to 30 degree days at 2 cm depth, or 100 to 155 degree days at 10 cm depth. In organic soil however, I_{TD} would be reduced by 70 to 90 degree days at 2 cm depth, or by 300 to 400 degree days at 10 cm depth. This would be a significant fraction of the seasonal surface index, with an effect that will be relatively greater where the surface index is small. Unfortunately, the extremely low density of organic surface materials would likely expose sensors to radiation within 1 or 2 cm of the surface, resulting in temperature sensor errors (Schwerdtfegger, 1976).

Ground temperature data have traditionally been gathered using buried cables consisting of an array of temperature sensors (thermistors) installed at intervals. The utility of equation (6.9) is enhanced by the fact that it is the degree day totals that are transformed rather than depth: one of the fundamental difficulties in translating field data into model parameters is that the reference point of a model cannot usually be specified in the field: in some environments, choosing the position of the ground “surface” can be arbitrary or impossible. While the actual or nominal surface level must be specified, it has no special functional significance in equation (6.9).

Equation (6.10) also suggests a useful method of extrapolating from active layer temperatures to estimate the maximum active layer depth. Results with empirical data suggest that extrapolation downwards may be problematic in some situations, due to the combination of higher ice content near the active layer base, unfrozen water content effects and the limits of temperature sensor accuracy. A sustained temperature of only 0.1°C through the year yields $I_T = 36.5$, which is substantial when indices are reduced to the square root.

Even with these limitations, transforming profiles of I_T and I_F may produce useful diagnostic information about processes occurring within the active layer. Changes in slope with depth, or from year to year will likely correlate with changes in thermal properties.

6.2 A *TTOP* model for layered soil

The foregoing analysis can be used to extend the **TTOP** model to soils with two (or more) layers with distinct thermal properties. Freezing and thawing indices for the base of the surface layer can be calculated using equation (6.9):

$$I_{TD} = \left(\sqrt{I_{TS}} - D\sqrt{L/(2k_T)} \right)^2$$

$$I_{FD} = \left(\sqrt{I_{FS}} - D\sqrt{L/(2k_F)} \right)^2$$
(6.13)

Using equation (6.13) to substitute for I_{FD} and I_{TD} , T_{TOP} can be calculated from the bottom of the surface layer, using the conductivity ratio for the soil beneath. With a surface layer of thickness D :

$$T_{TOP} = \frac{\frac{k_{T2}}{k_{F2}} \left(\sqrt{I_{TS}} - D\sqrt{\frac{L_1}{2k_{T1}}} \right)^2 - \left(\sqrt{I_{FS}} - D\sqrt{\frac{L_1}{2k_{F1}}} \right)^2}{P}, \quad T_{TOP} < 0$$

$$T_{TOP} = \frac{\left(\sqrt{I_{TS}} - D\sqrt{\frac{L_1}{2k_{T1}}} \right)^2 - \frac{k_{F2}}{k_{T2}} \left(\sqrt{I_{FS}} - D\sqrt{\frac{L_1}{2k_{F1}}} \right)^2}{P}, \quad T_{TOP} > 0$$
(6.14)

where

- L_1 = volumetric latent heat of surface layer
- k_{T1} = thawed thermal conductivity of surface layer
- k_{F1} = frozen thermal conductivity of surface layer
- k_{T2} = thawed thermal conductivity of subsurface layer
- k_{F2} = frozen thermal conductivity of subsurface layer

This model includes the error in slope due to the sensible heat content of the ground as discussed above (Figure 6.3), so that indices at the base of the surface layer will be lower than the model predicts. As a result, TTOP will be warmer than predicted by equation (6.14) in permafrost and colder than predicted in seasonally frozen ground.

TONE simulations were performed to evaluate the magnitude of the error in TTOP values obtained using equation (6.14). Three thermal catenae were simulated, each having a surface organic layer of a given thickness (10, 20 and 30 cm thick), overlying the standard mineral soil. Thermal properties for each layer were as specified in Table 5.3, together with a maximum snow cover depth of 25 cm. The snow cover had the standard properties and behaviour: a density of 250 kg m^{-3} , a parabolic accumulation function, ablation factor of $3 \text{ kg m}^{-3} \text{ K}^{-1} \text{ day}^{-1}$, with accumulation commencing 1 day after the start of freezing. Simulations were run until an equilibrium condition was established.

Simulated values of I_{FD} and I_{TD} along the catena were compared to their values calculated from equation (6.14) using the numerical I_{FS} and I_{TS} results obtained from the catena simulations. Results for the 20 cm organic layer case are shown in Figure 6.9, and show that the behaviour of the freezing index approximately mirrors the behaviour of the thawing index, with a transition near $TTOP = 0^\circ\text{C}$. The error in I_{FD} is high at low (permafrost) temperatures, decreasing as TTOP approaches 0° , with a small error for all seasonally frozen ground cases (errors of 12 degree days or less). Similarly, the error in I_{TD} is largest at high (seasonally frozen ground) temperatures, decreasing as TTOP approaches 0° , with a small error for all permafrost cases (errors of 132 degree days or less). Errors in I_{TD} for the permafrost cases are larger than the errors in I_F in

Figure 6.9 I_{FD} and I_{TD} at the base of a 20 cm thick peat layer, for points along a thermal catena. “Numerical” results are based on TONE model output for the node at 20 cm depth. “Analytical” results are based on equation (6.14) used with I_{FS} and I_{TS} taken from TONE output.

seasonally frozen ground, for reasons that are unclear. The trends in error as a function of temperature exhibit a discontinuity near $T_{TOP} = 0^{\circ}\text{C}$, where equation (6.14) predicts permafrost but the numerical result shows seasonally frozen ground.

The erroneous values of I_{FD} and I_{TD} obtained above are used to calculate T_{TOP} , so that the error in T_{TOP} may be expected to be smallest near a T_{TOP} value of 0°C . Simulated values of T_{TOP} along the catena were compared to T_{TOP} values obtained using equation (6.14). The error in T_{TOP} calculated using (6.14) for each of the three organic layer thicknesses is shown in Figure 6.10. As suggested above, the error increases as the organic layer thickness increases, and as T_{TOP} moves farther from 0°C .

Each of the three catenae have an error at $T_{TOP} = 0^{\circ}\text{C}$ of about 0.3° , due to the over-prediction of active layer depth by the Stefan model. The magnitude of the error depends on the terms neglected in the Stefan model, of which the soil heat capacity may be most important at this temperature.

6.3 Conclusion

Results with the model for uniform soils demonstrate that the model does not translate perfectly to field data. These limitations are likely due primarily to the heterogeneity of thermal properties in the ground. Almost all cases examined show profiles that exhibit a curvature indicating that L/k increases with depth, as would usually happen with increasing water content and bulk density. Thermal properties are unlikely to be uniform with depth, and in many situations will vary through time, due to moisture content variation (Hinkel *et al.*, 2001) and the effect of unfrozen water (Romanovsky and Osterkamp, 2000). The effect on temperature index profiles should be much more pronounced in the transition between the surface organic layer and the

mineral soil below than within the mineral soil itself. In addition, non-conductive processes such as meltwater infiltration into frozen ground and frost heave are not accounted for in conductive models, and will distort annual totals of I_T and I_F even if their effects are of short duration.

As with modeling the effect of snow (Section 5.4), the simple models expressed in equations (6.10) and (6.14) are closest to being accurate when sensible heat flow does not significantly influence soil freezing and thawing. At the permafrost limit, the error in the layered soil model (equation(6.14)) would need to be considered in its use for predictions of permafrost distribution. Nonetheless, the form of the model may be useful in demonstrating the relative importance of the model parameters in controlling ground temperature.

Chapter 7 TTOP and Climate Variability

The **TTOP** model assumes a stationary climate in which the annual temperature regime (I_F and I_T) and snow cover history are identical every year. With a real, changing climate, annual variations of I_{FA} , I_{TA} , n_F (snow cover depth, density and timing) and n_T will produce a range of TTOP values¹. This introduces short-term energy imbalances between the active layer and permafrost, resulting in transient departures from the equilibrium condition predicted by the **TTOP** model. The effects of this variability on the accuracy of the **TTOP** model are discussed in this chapter.

Riseborough and Smith (1993) presented an analysis of the combined effects of climate warming and climatic variability on the thermal behavior of the ground near Fort Simpson, Northwest Territories, using the numerical model TONE. Their main analysis used 100-year time series generated from the daily climate record from 1963 to 1989. Individual years from the record were chosen randomly and concatenated to produce the longer time series, and 121 alternate time series were produced to allow a statistical description of the behavior of the ground thermal regime by Monte Carlo simulation. This was compared to a single simulation based on monthly normal temperature and snowfall data (adjusted for climate warming). The simulation using climate normal data led to more rapid permafrost degradation than was the case for 95% of the simulated cases using the variable climate. Riseborough and Smith (1993)

¹ Note the distinction, discussed in Chapter 1, between TTOP *values* (measured or numerically modelled ground temperatures obtained below the seasonal freeze/thaw layer), and T_{TOP} predicted using the **TTOP** model.

attributed this to differences between heat flow into the ground when permafrost is aggrading or degrading. This result has implications for the application of the **TTOP** model, since it demonstrates that analyses that ignore variability can be biased.

The overall goal of this chapter is to determine the magnitude of errors in the **TTOP** model that result from climate variability. The effect of stationary random variability of I_{FA} and I_{TA} on TTOP is examined in the Section 7.1. Given a climate in which I_{FS} and I_{TS} in any year depart from $\overline{I_{FS}}$ and $\overline{I_{TS}}$ (i.e., their long-term average values), what is the relationship of TTOP and T_{TOP} to annual values of I_{FS} and I_{TS} , and the relationship of the long-term average T_{TOP} (calculated from $\overline{I_{FS}}$ and $\overline{I_{TS}}$) to $\overline{I_{FS}}$ and $\overline{I_{TS}}$?

The effect of a long-term air temperature trend is examined in Section 7.2. Where a warming trend results in surface temperatures that cannot sustain freezing temperatures in the ground, permafrost will degrade. In a typical sequence, permafrost begins to degrade as the maximum depth of the active layer increases year by year. Once the active layer no longer freezes back to the top of permafrost by the end of winter, a perennially unfrozen layer (“*talik*”) develops between the base of the active layer and the top of permafrost. Permafrost degradation continues as the bottom of the talik becomes deeper. The effect of the imbalance introduced by such sustained warming is considered in Section 7.2.

Variations in MAGST and TTOP due to variability of I_{FA} and I_{TA} will be enhanced if snow depth and timing also vary (Riseborough and Smith, 1993). In the first sections of this chapter only variations in I_{FA} and I_{TA} are considered, since (as

demonstrated in Chapter 5), variations in snow cover can be considered as equivalent to changes in air temperature indices. Variability of snow-cover is included in Section 7.3, in order to explore the combined thermal effect of variability of snow cover and air temperature. For this section, the question posed above can be re-expressed: is it possible to estimate accurately the average thermal effect of variable temperature and snow cover conditions (as expressed by the average value of TTOP) using average snow cover and air temperature conditions?

Sections 7.1 and 7.2 examine the effect of transient imbalances on the validity of the thermal offset formulation by comparing the numerically simulated value of TTOP to the T_{TOP} value obtained using $\overline{I_{FS}}$ and $\overline{I_{TS}}$. Section 7.3 examines the use of climate averages to estimate the parameters of the model by comparing simulated values of TTOP where $\overline{I_{FA}} = I_{FA}$ and $\overline{I_{TA}} = I_{TA}$ for numerical simulations with and without variability. These questions are critical to the utility of the **TTOP** model, since they concern the robustness of the model in the presence of variability.

For this analysis, it is assumed that the numerical model (TONE) captures the effect of climate variability correctly. Results are analyzed by comparing the value for TTOP obtained from TONE (based on the temperature for the first node below the ground surface that does not undergo seasonal freezing or thawing) to the value for T_{TOP} obtained by applying the **TTOP** model. (Recall that TTOP refers to the numerical value and T_{TOP} refers to the analytical value of TTOP, respectively). Where errors are discussed, they are always calculated as $(T_{TOP} - TTOP)$, so that positive values indicate that the **TTOP** model predicts a value that is higher than the numerical model predicts.

Numerical modeling

The TONE model was used to generate the numerical results presented in this chapter. Inter-annual variability was simulated by specifying standard deviations for I_{FA} and I_{TA} , which were applied to the annual wave for simulations of 250 years. Standard deviations were specified as a fraction of the mean index values.

At the beginning of each annual cycle (arbitrarily starting at the midpoint of the thawing season), random numbers were selected, converted to z-scores², and applied to the quantities varying randomly. For example, the annual freezing index was determined using:

$$I_{FA} = \overline{I_{FA}} + z\sigma_{I_{FA}} \quad (7.1)$$

where

$\overline{I_{FA}}$ = mean value of I_{FA}

z = z-score

$\sigma_{I_{FA}}$ = standard deviation of I_{FA}

The annual temperature wave was adjusted by including a variability factor, analogous to an n factor. For each day in the annual cycle (note the averaging operator in denominators of the expressions):

$$T_{AV} = \frac{I_{FA}T_A}{I_{FA}}, \quad T_A < 0$$

$$= \frac{I_{TA}T_A}{I_{TA}}, \quad T_A > 0 \quad (7.2)$$

where

T_{AV} = air temperature, adjusted for variability

T_A = unadjusted air temperature

² Random numbers were generated by the FORTRAN function, and shuffled to reduce the potential for serial correlation. Random numbers (between 0 and 1) were converted to z-scores using a transform that approximates the cumulative probability of the normal (Gaussian) distribution. Extreme z-score values (values less than -3 or greater than 3) that would cause the TONE program to fail were discarded.

This scheme provides variability without the need to specify changes in the mean or amplitude of the annual temperature wave. With this scheme, the dates for the start of freezing or thawing conditions do not change from year to year.

7.1 Variability of I_F and I_T

For the climate stations in Table 5.1, standard deviations of I_{FA} vary from 229 to 674 degree-days (Figure 7.1a); standard deviations of I_{TA} vary from 50 to 289 degree-days (Figure 7.1b). The ratio of the standard deviation to the mean I_{FA} averages 9.7%, with the ratio generally declining as the indices increase. In contrast, the ratio of the standard deviation to the mean I_{TA} averages 14.0%, with the ratio generally increasing as the indices increase.

Catena simulations with a 25-cm snow cover and the standard substrate (Chapter 3) were used to examine the effect of inter-annual variability on the mean annual temperature at the top of permafrost. Four sets of catena simulations were run, with standard deviations for I_{FA} and I_{TA} specified as 1%, 5%, 10%, and 20% of the index values. Simulations began with an initial ground temperature profile determined from the standard catena, and run for 250 years. Only the final 100 years of simulation were used, to minimize any effects due to equilibration of the profile. All simulations used the same seed for the random number generator, giving an identical pattern of variation: this ensured that differences along the catena were not due to differences between the random variations. Typical time series of MAAT and simulated TTOP for these standard deviations are shown in Figure 7.2, showing the final 100 years of simulations for the case in which the nominal average MAAT was -10°C .

a)

b)

Figure 7.1 a) Relationship between I_{FA} and the standard deviation of I_{FA} for the climate stations listed in Table 4.1. b) Relationship between I_{TA} and the standard deviation of I_{TA} for the climate stations listed in Table 4.1.

Figure 7.2 Time series of MAAT and TTOP for stationary variability study. Examples shown are for simulations with mean MAAT of -10°C . Inset (percentage) values indicate the standard deviation of I_{FA} and I_{TA} .

The effect of variability was analyzed by comparing the numerically simulated TTOP values with T_{TOP} using the annual I_{FS} and I_{TS} values for individual years taken from the numerical model output. Figure 7.3 shows the numerical TTOP results of Figure 7.2 plotted as a function of T_{TOP} . Note that each component graph in the figure is scaled to the maximum and minimum values on each axis, so that the graph for 1% variability has a range of less than 1°C, while the lower graph has a range of over 12°C. For all levels of variability, the effect of the thermal inertia of the ground (which is not included in the **TTOP** model) is apparent as a rotation of the relationship away from the 1:1 line. As mean annual temperatures move away from the long-term mean value (shown as a large cross-hair on each graph) the numerical results differ from the mean by a larger amount. Numerical results below the long term mean tend to be warmer than predicted by the **TTOP** equation, while results above the long-term mean tend to be cooler than the **TTOP** equation predicts.

Annual scatter in Figure 7.3 is due to the particular antecedent temperature conditions for any year. Regression slopes for these relationships vary within a limited range (from 0.835 to 0.826), suggesting that the effect of thermal inertia scales with variability, giving an error in T_{TOP} for this particular set of conditions on the order of 0.17°C for every 1° departure from the mean value of TTOP.

Figure 7.4 shows the relationships between the numerical TTOP and T_{TOP} at each point along the standard thermal catena, for the case with 10% variability. At the coldest and warmest temperature extremes, the analytical and numerical values of TTOP correspond well ($r^2 > 0.99$), while at temperatures close to 0°C, there is significantly more scatter ($r^2 = 0.043$ where $MAAT = 4^\circ$ and $TTOP \approx 0.3^\circ$). For the

Figure 7.3 Comparison of numerically simulated TTOP with analytical T_{TOP} estimated from annual values of I_{FS} and I_{TS} , for the cases shown in Figure 7.2. Cross-hair at midpoint represents the value of TTOP for the case with no inter-annual variation.

Figure 7.4 Comparison of numerically modeled TTOP (horizontal axis) with analytical T_{TOP} determined from the freezing and thawing indices, for points along the standard thermal catena. Numerical results include inter-annual variability in I_F and I_T with a standard deviation of 10% of the mean index values. Numbers in lower right-hand corner of each graph indicates the nominal mean MAAT. Results for MAAT = -18°C have been omitted to save space.

cases close to $T_{TOP} = 0^{\circ}\text{C}$ ($MAAT = -6$ to -2), numerical results show that T_{TOP} does not cross the 0°C threshold even when the **TTOP** model predicts that it should. Short-term variations in surface climate cannot shift the temperature at the bottom of the seasonal freeze/thaw layer from permafrost to seasonally frozen ground (or vice-versa) without the development of a talik (or permafrost). This will be difficult where mean ground temperatures are more than a few tenths of a degree below (or above) 0°C . The increasing scatter at intermediate temperatures which approach but do not transgress 0°C (e.g., at $MAAT = -8^{\circ}$ and 0°C) is likely due to relatively large annual variations in seasonal freeze/thaw depth, which will also contribute to annual differences in the energy balance of the surface freeze/thaw layer.

Multi-year averages

While the numerically modeled value of T_{TOP} will differ from T_{TOP} in variable conditions, the results in Figure 7.4 suggest that this difference may be small, and that it may be reduced by multi-year averaging. Multi-year averages were calculated differently for the numerical results and the **TTOP** model: averages for numerical results were calculated as the arithmetic means of the annual T_{TOP} values for the averaging period; averages for T_{TOP} were calculated from the multi-year average values of I_{FS} and I_{TS} using the **TTOP** equation.

Figure 7.5 shows the difference between the numerical T_{TOP} and T_{TOP} for the 100 years included in Figure 7.4, expressed as the root mean square (RMS) error for averaging periods of one to thirty years. The upper curve in Figure 7.5a shows the RMS error for annual values: at T_{TOP} temperatures below -6° or above 4°C , the RMS error is 0.3° or less, while at temperatures near $T_{TOP} = 0^{\circ}\text{C}$, the error approaches 1°C .

Figure 7.5 Root mean square difference (error) between analytical T_{TOP} and numerically simulated T_{TOP} values. **a)** As a function of mean T_{TOP} for different averaging periods. **b)** As a function of averaging interval for selected mean T_{TOP} values. Results are for cases with a variability standard deviation of 10% of index values.

RMS errors are lower for longer averaging periods, declining to 0.1°C or less for all TTOP when the averaging period is 30 years. Figure 7.5b shows the relationship between averaging interval and RMS error for the warmest and coldest points on the catena, and for the case for which TTOP is closest to 0°C . Errors for all but annual values have errors of less than 0.1°C for the warmest and coldest points, while averaging periods of 10 or more years are necessary to reduce the error to less than 0.2°C where TTOP is close to 0°C .

7.2 *Climate trends*

The imposition of a warming trend on permafrost (or of a cooling trend on seasonally frozen ground) will introduce an error in the **TTOP**-estimated temperature, since this introduces a temperature gradient between the surface and deeper ground temperatures that will alter the energy balance of the active layer. When degradation has progressed to the development of a talik, a steep temperature gradient is introduced between the bottom of the active layer (whose mean temperature will become increasingly positive over time) and the receding permafrost table. The magnitude of the errors in the **TTOP** model produced by these conditions is examined in this section.

The presence of a talik poses a conceptual problem, since in the **TTOP** model the top of permafrost is assumed to coincide with the base of the active layer, as it must under typical one-dimensional equilibrium conditions. It is assumed here that in the presence of degrading permafrost the **TTOP** model predicts T_{TOP} as the temperature at the bottom of the active layer.

The mean temperature gradient through the talik will determine the rate at which permafrost degrades. The mean annual temperature at the base of the active layer will

be above 0°C if permafrost is degrading, while the bottom of the talik (i.e., the top of permafrost) will be at 0°C . The annual energy balance of the active layer includes the heat extracted along this temperature gradient, and so will not be balanced by the **TTOP** model.

Air temperature warming trends of 0.3°C , 0.5°C and 1°C per decade were simulated using TONE for points along the standard thermal catena, with a maximum snow cover of 25 cm. For these simulations, the calculation mesh was modified from the standard conditions described in Chapter 3, to allow for the propagation of the warming trend into the ground. The bottom of the mesh was extended to a depth of 100 m, with a node spacing of 6 cm at the surface, increasing to 300 cm at the bottom of the grid. This coarser node spacing was necessary to accommodate the finite number of elements in the TONE model.

Time series of the numerically simulated TTOP and T_{TOP} estimated using the annual surface temperature indices are shown in Figure 7.6 for the warming trend of 0.5 degrees per decade. Most points along the thermal catena are included. For most time series that do not cross 0°C , the difference between the numerical TTOP and T_{TOP} is small (less than 0.1°C), and does not increase significantly over time. Where TTOP warms through 0°C however (for cases with initial MAAT of -8°C and -6°C), numerical simulations show a sharp reduction in the warming rate at this threshold. Temperature rise is minimal for approximately 20 years, with the numerical TTOP value 0.4° to 0.6°C cooler than the **TTOP** model predicts once the warming trend resumes.

Figure 7.7 shows the difference between the numerically simulated TTOP and analytically predicted T_{TOP} for the three warming rates, for the cases starting at

Figure 7.6 TTOP versus time with a warming trend of 0.05°C per year, from initial temperature points along the standard catena. Numerical results are from TONE simulations; Analytical T_{TOP} values are calculated using annual surface temperature indices from the numerical model results. Results for $\text{MAAT} = 6^{\circ}\text{C}$ have been omitted to save space.

Figure 7.7 Difference between numerically simulated T_{TOP} and analytical T_{TOP} estimated using surface temperature indices (“Analytical error”), for permafrost warming from T_{TOP} = -6. Each curve represents a different warming rate, indicated in the legend in degrees per year.

MAAT = -6°C . While the air temperature increases smoothly over the simulation, TTOP varies year to year within a 0.05°C range: this does not indicate an error in the numerical method, but is an artifact of the method of estimating TTOP in transient conditions.³ Initiation of the talik in each of the simulations is apparent as a sudden increase in the analytical error, at approximately 20, 40 and 60, years depending on the warming rate. For all warming rates, the error increases to a maximum value, followed by a gradual decline over time. The rate at which errors peak and decline increases as the warming rate increases, with a higher maximum error at higher rates of warming. With a 1°C per decade warming rate, the maximum error is on the order of 1°C within 20 years of initiating the talik. After 100 years of permafrost degradation, analytical errors for all rates of warming still exceed 0.2°C .

Figure 7.8 shows the analytical errors of Figure 7.7 as a function of the depth to the top of permafrost. For all warming rates, the error reaches a maximum when the permafrost table is about 4 m below the surface, with the error declining as the permafrost table falls. This indicates the mechanism responsible for the error: for this set of conditions, a talik of this thickness likely represents the point at which the through-talik temperature gradient is at a maximum.

7.3 Combined snow and air temperature variability

The climate data discussed in this and preceding chapters can be used to establish the likely magnitude of variability of the temperature indices and of snow cover characteristics. In this section, the variability of these parameters is used in

³ This variation is due to the finite spacing of the mesh rather than a true variability of TTOP. It occurs because the position of the sampling point relative to the bottom of the seasonal freeze/thaw layer differs from year to year.

Figure 7.8 Results shown in Figure 7.7, (difference between numerically simulated T_{TOP} and analytical T_{TOP} obtained using surface temperature indices), plotted as a function of depth to the top of permafrost. Results are for permafrost warming from $T_{TOP} = -6$. Each curve represents a different warming rate, indicated in the legend in degrees per year.

simulations with the TONE model, separately and in combination, so that the contribution of each to variation in TTOP can be demonstrated. From the source data for Figure 7.1 and Table 5.1 average standard deviations can be determined for I_{FA} (10%), I_{TA} (14%) and maximum snow cover (36%). From Table 5.2, the length of the snow-free period with temperatures for these stations averages 12 days, with a standard deviation of 7.7 days.

Simulation procedures were similar to those used in the preceding sections, using parameters for the standard catena with a 25 cm snow cover and specifying standard deviations for the varying parameters. Simulations were first performed with each climate component (indices, snow cover initiation, snow depth) varying separately, and compared to control results without variability. Catena simulations with all three components varying simultaneously were also run and compared to the control.

The frequency distributions for snow depth and snow free period are positively skewed, with their low extremes less than two standard deviations from the mean value. These skewed distributions presented a problem, since only normally distributed variability could be simulated within TONE. The shortest snow-free freezing period possible is zero days, while the extreme low seasonal maximum snow cover at the climate stations is typically 30 to 60% of the average maximum snow cover. Use of the randomization routine in TONE with these means and standard deviations would routinely produce negative snow-free freezing periods, and snow depths well below the typical low value. This condition was avoided in simulations by adjusting standard deviations so that the extreme values for these parameters (three standard deviations from the mean) were 0 days and 7 cm, respectively.

Simulation results for snow variability catenae were compared to results with standard catena temperatures. However, the long-term average MAAT for the simulations with variable I_{FA} and I_{TA} differed from the nominal catena MAAT by approximately 0.2°C (due to random variation), introducing a discrepancy in comparisons of TTOP values. To compensate, a second control catena simulation (without variability) was run, with MAAT specified as the average MAAT for the final 100 years of the catena simulations with variable temperature indices. This was used for comparisons with simulations with indices alone varying as well as the simulations with all three parameters varying.

The TTOP values in the control simulations were compared to the average TTOP value for the final 100 years of the 250-year variability simulations (Figure 7.9). For each parameter separately, and for all parameters combined, long-term average TTOP values under variable conditions differ from the control (equilibrium) TTOP values by less than 0.2°C for all simulations, and by less than 0.05°C for most cases. While the largest differences for two of the parameters occur where TTOP is closest to 0°C , the differences do not vary as a function of temperature.

These results indicate that variability *per se* is not responsible for the systematic difference between the simulations with variable climate and with climate normal data discussed in Riseborough and Smith (1993).

7.4 Conclusions

The results in Section 7.1 indicate that stationary inter-annual variability introduces a difference between the numerical values of TTOP and the analytical T_{TOP} that is approximately proportional to the annual deviation from the long-term mean,

Figure 7.9 Difference between average TTOP value simulated with variable climate parameters and TTOP value for simulations with no variation, as a function of TTOP value. Average values are determined from the final 100 years of a 250 year simulation.

and that this error is higher where TTOP is close to 0°C. This inter-annual variability does not introduce a large change in multi-year average value of the temperature at the bottom of the seasonal freeze/thaw layer.

As shown in Section 7.2, the **TTOP** model is subject to a larger error where a talik has developed. For the specific conditions examined, the **TTOP** model overestimates the rate of permafrost temperature rise (and hence of permafrost degradation), with a maximum error in the temperature at the bottom of the active layer on the order of 1°C. This is due to the significant temperature gradient that develops between the bottom of the active layer and the permafrost table, which will depend on the warming rate as well as the thermal properties of the substrate.

Results in Section 7.3 suggest that the relationship between MAAT and TTOP is robust in the presence of climate variability. While results in Chapter 5 (for example) indicate that establishing the correct value for n_F is not straightforward, parameterization will not be significantly influenced by inter-annual variability. The effect noted by Riseborough and Smith (1993) discussed in the introduction to this chapter must therefore be due to some other influence. At least three possible mechanisms may be responsible.

First, the analysis of Riseborough and Smith (1993) included the extreme winter of 1963 in the Fort Simpson climate record (See Figure 5.8). That year had an extremely low snow cover that was established 48 days after the start of freezing (478.5 freezing degree days). The relationship between snow cover and the nival offset may not be linear, so that for such extreme conditions there may be a difference between the response to the mean and the mean response to variation.

Second, the effect may be due to the method of distributing snow cover for the simulation with normal climate, which only allowed for one day of freezing prior to snow cover initiation, so that TTOP would therefore be warmer than its average under the variable climate. The results of Section 5.4.3 suggest that a difference in MAGST on the order of 0.5°C could be attributable to the improper specification of the snow-free freezing period.

A third alternative may be the unique condition modeled in Riseborough and Smith (1993), with a body of permafrost initially only 6 m thick that responded to variation with recurrent talik development. The results presented in Section 7.3 suggest that the **TTOP** model may not be adequate in such conditions. Further investigation of this phenomenon is required before the mechanism responsible for their result can be determined.

Numerical results suggest that the **TTOP** model provides a robust index of the temperature at the top of permafrost in many conditions that include climate variability or climate warming. Exceptions occur where that variability crosses the transition between permafrost and seasonally frozen ground, and are due to the effect of latent heat at the transition between permafrost and seasonally frozen ground conditions.

Chapter 8 Modelling Permafrost Distribution

In this chapter, the permafrost boundary model developed in Chapter 5 is applied to evaluate permafrost distribution at the national scale in Canada. The model is applied using readily available climatic data, but without data on the physical and thermal properties of surficial materials. With this preliminary analysis, the potential of the model is illustrated while the relationships between the climatic regime, environmental parameters and permafrost distribution are illuminated.

This chapter consists of four sections. First, the national scale gridded climate data used in the model are presented. Second, aspects of the model that affect its accuracy are considered. Third, the model is applied illustratively to demonstrate the sensitivity of permafrost distribution to variations in snow cover and substrate properties. Fourth, the sensitivity of model parameters to soil physical properties is explored.

8.1 Gridded Canadian climatology

Canadian climate data have been used to produce interpolated grids of monthly mean temperature (Vincent and Gullett 1999, Zhang et al. 2001), while Brown et al. (2001, 2003) have collected historical snow depth and snow water equivalent data to produce a similar grid for the period 1979-1996. These gridded data sets interpolate and extrapolate from spatially biased climate station data to regular interpolation grids.

The gridded data provide parameter values for the entire country, and correct for the non-uniform distribution of stations. In addition to simple quality control, station data were adjusted for inhomogeneities caused by station relocation, changes in

instrumentation and in observing practices (Zhang *et al.*, 2001). Grid interpolation schemes accounted for elevation, location, topography, and land cover to eliminate biases due to climate station siting and surface characteristics. Using the original gridded data sets for snow density, snow depth, and monthly mean temperature, new data sets for I_{FA} , I_{TA} , and density-adjusted snow depth were generated for use with the permafrost boundary model. The methods used for conversion of data into the forms useful for this chapter are described in Appendix III.

8.1.1 Snow depth, snow density, and equivalent snow depth

Gridded average annual maximum snow depth for Canada is shown in Figure 8.1a. West of Hudson Bay, the dominant features in the map are a zone of greater snow depth through the boreal sub-arctic region, with high snow depths in the western cordillera and lower snow depth in the high arctic and in the southern Prairie Provinces. East of Hudson Bay, snow is considerably deeper than at similar latitudes elsewhere in Canada (other than the cordillera).

Figure 8.1b shows the pattern of mean snow density across Canada. West of Hudson Bay, a zone of low snow density in the boreal sub-arctic region is apparent, with higher density snow in the cordillera and the southern Prairie Provinces. The highest density snow occurs at high latitudes, especially north of treeline. South of treeline in eastern Canada, snow has a higher density than west of Hudson Bay, especially in coastal Labrador and Quebec. The pattern of snow density shown on this map is considerably different from that shown on earlier maps of snow density derived from more limited data (e.g., Bilello, 1984).

Figure 8.1 a) Gridded annual average maximum snow depth for Canada (Brown et al. 2003). Legend shows snow depth range for each class in metres. b) Gridded average snow density for Canada derived for data in (Brown et al. 2001). Legend shows snow density range for each class in kg m^{-1} . The dotted line on the maps show the treeline defined on the circumpolar permafrost map (Brown *et al.* 2002).

The snow depth and density data in Figures 8.1a and b were transformed to equivalent snow depth using the procedure from Section 5.4.2, as described in Appendix III. Figure 8.2 shows the resulting equivalent snow depth, which is an index of the thermal resistance of the snow cover. The national-scale patterns of snow depth and density tend to augment regional differences in equivalent snow depth. North of treeline, the most significant feature of the map is the very low equivalent snow depth, due to the combination of low snowfall (related to low temperature) and high density (due to the mechanical effect of high winds on snow compaction). South of treeline, the most significant feature is a zone of high equivalent snow depth west of Hudson Bay in the boreal sub-arctic, south and roughly parallel to the treeline. This feature is due to the combination of greater snow accumulation, low wind speed within forest canopies, and the development of depth hoar. Differences east and west of Hudson Bay are similarly augmented by the co-variation of snow density and depth.

8.1.2 Freezing and thawing indices

Figure 8.3 shows annual I_{FA} and I_{TA} values for the Canadian temperature grid, determined using the procedure described in Appendix III. I_{TA} (Figure 8.3a) exhibits a strong zonal (latitudinal) pattern, with values declining northward by approximately 90 degree-days per degree of latitude. East of Hudson Bay, zones shift southward, so that in this region I_{TA} is lower than at the same latitudes west of Hudson Bay.

The map of I_{FA} (Figure 8.3b) also shows a strong latitudinal trend (increasing northward by approximately 175 to 200 degree-days per degree of latitude). However, the continental-scale pattern suggests the influence of synoptic conditions, especially

Figure 8.2 Gridded annual average maximum equivalent snow depth for Canada, as derived using the procedure described in Section 5.4.2 from density data of Figure 8.1b and maximum snow depth of Figure 8.1a. Legend shows snow depth range for each class in metres. The dotted line on the map shows the treeline defined on the circumpolar permafrost map (Brown *et al.* 2002)

a)

b)

Figure 8.3 a) Annual air thawing index I_{TA} for the normal period 1971-2000, determined from gridded monthly mean air temperature data (Zhang et al. 2001). Legend shows I_{TA} range for each class in degree-days. **b)** Annual air freezing index I_{FA} for the normal period 1971-2000, determined from gridded monthly mean air temperature data (Zhang et al. 2001). Legend shows I_{FA} range for each class in degree-days. The dotted line on the maps show the treeline defined on the circumpolar permafrost map (Brown *et al.* 2002).

the position of the Arctic Front (Rouse, 1993). South of 70° latitude, freezing index values are highest near Hudson Bay, declining to the east and west.

The difference between latitudinal gradients of I_{FA} and I_{TA} reflects the fact that the annual temperature range increases with latitude. Similarly, south of 70° latitude the annual temperature range is greatest near Hudson Bay, declining to the east and west.

8.2 Catena considerations

Although the catena used in this thesis was designed to better represent continental-scale climatology than catenae used in earlier work (Riseborough and Smith 1998, Henry and Smith 2001, Smith and Riseborough 2002a), application to field conditions requires that the model components be carefully evaluated. As discussed in Chapters 3 and 5, the new standard catenae improve on earlier work in that the seasonal distribution of snow cover and the magnitudes of I_{FA} and I_{TA} more closely represent their average continental-scale variation. Two issues arise from the use of catena-based functions: the first relates to variation of freezing and thawing indices about the mean latitudinal trend as discussed in the preceding section, while the second relates to specification of the snow cover. These issues are discussed below.

8.2.1 TTOP and annual temperature range

In the thermal catenae used in this thesis, I_{FA} and I_{TA} are both functions of MAAT determined by least-squares regression (Section 3.2.1) and implemented with the amplitude of the sine wave defining air temperature. Figure 8.4 shows the relationship of MAAT to I_{FA} and I_{TA} for the climate data used in Figure 3.5 and for the gridded indices shown in Figure 8.3. While the difference between I_{TA} and I_{FA}

Figure 8.4 Climate normal I_{TA} and I_{FA} data for gridded climate data and for 173 Canadian climate stations. Diagonal lines indicate mean annual air temperature.

determines MAAT, the indices may have very different values for the same MAAT as long as they differ by the same amount. In Figure 8.4 for example, sites with MAAT between 0°C and -10°C have index values that vary by about 800 degree-days. The effect of such variation in annual range on TTOP is considered in this section.

Lachenbruch (1959) reasoned that damping of the annual air temperature wave by snow cover introduces a seasonal bias, resulting in an increase in mean annual ground surface temperature. Smith and Riseborough (1996) suggested that increasing the annual air temperature range without changing the annual mean (that is, increasing I_F and I_T by the same amount) will result in a *decrease* in TTOP for permafrost cases, because of the buffering effect of the conductivity ratio r_K .

Differences in annual temperature range that do not change MAAT can be considered as equal changes in I_{FA} and I_{TA} . Given a change in index values Δ_I , it is clear that:

$$MAAT = I_{TA} - I_{FA} = (I_{TA} + \Delta_I) - (I_{FA} + \Delta_I) \quad (8.1)$$

For permafrost cases, the difference in T_{TOP} due to Δ_I is the difference between:

$$T_{TOP} = \frac{r_K n_T I_{TA} - n_F I_{FA}}{P} \quad (8.2)$$

and:

$$T_{TOP} = \frac{r_K n_T (I_{TA} + \Delta_I) - n_F (I_{FA} + \Delta_I)}{P} \quad (8.3)$$

Equation (8.3) can be rearranged:

$$T_{TOP} = \frac{r_K n_T I_{TA} - n_F I_{FA}}{P} + \frac{r_K n_T \Delta_I - n_F \Delta_I}{P} \quad (8.4)$$

The first term in (8.4) is equal to (8.2), so that the second term defines the difference in T_{TOP} attributable to Δ_I as:

$$\frac{\Delta_I}{P} (n_T r_K - n_F) \quad (8.5)$$

Similar reasoning for the seasonally frozen ground case yields a difference of:

$$\frac{\Delta_I}{P} \left(n_T - \frac{n_F}{r_K} \right) \quad (8.6)$$

Assuming $n_T = 1$ for simplicity, equations (8.5) and (8.6) show that the effect of increasing amplitude (that is, increasing Δ_I) is a decrease in T_{TOP} if $n_F > r_K$, and an increase in T_{TOP} if $n_F < r_K$.

This is the reasoning behind the conclusion of Smith and Riseborough (1996). This assumes however that n_F does not vary with T_{TOP} , so that the value of n_F is the same in equations (8.2) and (8.3). This assumption was shown in Chapter 5 to be incorrect.

TONE simulations using variations on the standard catena were used to quantify the effect of temperature range. At MAAT of -4°C , an index difference of 800 degree-days can be represented as a change in amplitude of 7°C . Figure 8.5 shows $-\Delta_{Total}$ as a function of MAAT for the standard catena (amplitude varying between 15.98° and 21.63°), together with results for catenae whose amplitudes were constant at 15°C and 22°C for all MAAT. For snow depths greater than about 15 cm, $-\Delta_{Total}$ increases as the amplitude of the temperature wave increases. The effect of amplitude is greater as snow depth increases, to about 4°C with a snow cover of 1.5 m. The median effective snow depth in Figure 8.2 is 0.73 m, with 80% of sites having an effective snow depth of

Figure 8.5 Total offset versus MAAT for standard catena (“variable”) and catenae with uniform annual amplitudes, with 25-cm snow depth. Catena conditions were otherwise the same as the standard thermal catena.

1 m or less: this suggests that errors due to the neglect of the influence of amplitude will typically be on the order of 3°C.

Variations in $-\Delta_{Total}$ due to variations in I_{FA} and I_{TA} for a given MAAT may be expected, so that predictions based on the catenae of this thesis will include errors of the magnitude suggested above. The effect of amplitude on $-\Delta_{Total}$ is of sufficient magnitude to suggest that model results based on the standard catena do not fully represent the range of Canadian temperature index climates.

8.2.2 Thermal catena snow cover

The trend surface determined in Chapter 5 for Δ_0 as a function of snow depth and $\theta_v k_F$ (Figure 5.31b) is an empirical function of the temperature and snow cover conditions along the standard thermal catena (Chapter 3). Data presented in Chapter 7 show that the mean length of the snow-free freezing period from climate station data was 12 days; in the original catena simulations this period was one day only. Results in Section 5.4.3 suggest that the length of this period can result in a difference in TTOP on the order of 1 to 2°C. To improve the parameterization of snow cover effects for Canadian climatic conditions therefore, a set of revised catena simulations incorporating a 12 day snow-free freezing period was run. Results for all of the substrate materials used in Section 5.4.7 (Table 5.3) were used to generate an alternative trend surface for (Figure 8.6). The new trend surface (determined with the Surfer© program) is:

$$\begin{aligned}
\Delta_0 = f(\mathbf{Z}, \theta_v k_F) = & 0.234523 - 1.32868 \theta_v k_F + 2.16578 (\theta_v k_F)^2 \\
& - 1.33809 (\theta_v k_F)^3 + 2.78374 (\theta_v k_F)^4 - 7.58875 \mathbf{Z} - 26.3278 \mathbf{Z} \theta_v k_F \\
& + 10.5982 \mathbf{Z} (\theta_v k_F)^2 - 1.99131 \mathbf{Z} (\theta_v k_F)^3 + 6.26273 \mathbf{Z}^2 + \\
& 15.1685 \mathbf{Z}^2 (\theta_v k_F) - 1.58806 \mathbf{Z}^2 (\theta_v k_F)^2 - 5.82979 \mathbf{Z}^3 \\
& - 0.0376664 \mathbf{Z}^3 y + 2.07793 \mathbf{Z}^4
\end{aligned} \tag{8.7}$$

Statistics relating to goodness of fit are similar to those for the original trend surface. Comparing values of Δ_0 estimated using equation (8.7) to the values obtained directly from numerical results yielded a root mean square error in the trend surface of 0.179°C (from 90 individual simulations). Errors are within +/-0.3°C in 90% of cases.

Figure 8.7 compares the relationship between permafrost boundary MAAT and snow depth for Smith and Riseborough (2002a), the standard catena used through most of this thesis, and for the revised catena devised for this chapter discussed above. The sharp increase in critical snow depth noted in Smith and Riseborough (2002a) is not apparent in the “standard” or “revised standard” catenae used in this thesis, although the snow cover required to prevent permafrost formation increases rapidly at lower temperatures. Curves for the relationship obtained by Smith and Riseborough (2002a) and the revised catenae differ by up to 5°C in transition temperatures for a given snow depth, or by up to 60 cm in critical snow depth for a given temperature. Figure 8.7 also indicates differences between the boundaries defined for the new standard and revised standard catenae on the order of 30 cm in snow depth or 2°C in MAAT.

The permafrost boundary model from Chapter 5 links the thermal resistance of the snow cover and the thermal properties of the ground, establishing whether their combination yields permafrost with given climatic conditions (I_{FA} and I_{TA}). This model can be used to demonstrate the relationship between mean annual ground

Figure 8.6 Normalized temperature difference Δ_0 as a function of snow depth (Z) and $\theta_v k_F$, determined by polynomial trend surface analysis. Contour interval 1°C .

$$\begin{aligned} \text{Equation of surface: } \Delta_0 = f(Z, \theta_v k_F) = & 0.234523 - 1.32868 \theta_v k_F + \\ & 2.16578(\theta_v k_F)^2 - 1.33809(\theta_v k_F)^3 + 2.78374(\theta_v k_F)^4 - 7.58875Z - \\ & 26.3278Z \theta_v k_F + 10.5982Z(\theta_v k_F)^2 - 1.99131Z(\theta_v k_F)^3 + 6.26273Z^2 + \\ & 15.1685Z^2(\theta_v k_F) - 1.58806Z^2(\theta_v k_F)^2 - 5.82979Z^3 - 0.0376664Z^3y + 2.07793Z^4 \end{aligned}$$

Figure 8.7 Limiting snow depth at permafrost boundary for four different thermal catenae.

8.3 Mapping permafrost boundaries and permafrost limits

temperature and model parameters, in ways that can help to explain the distribution of permafrost in Canada.

8.3.1 Limits versus boundaries

Smith and Riseborough (2002a) use the term “permafrost limit” when referring to permafrost zone boundaries in Canada, with the limits defined for the southern and northern boundaries of the discontinuous zone. Van Everdingen (1998) however, defines permafrost *limit* as the “outermost (latitudinal) or lowest (altitudinal) limit of the occurrence of permafrost”. Van Everdingen (1998) gives two definitions for permafrost *boundary* as either “the geographical boundary between the continuous and discontinuous permafrost zones” or “the margin of a discrete body of permafrost”. To avoid confusion in this chapter, permafrost limits are defined in the sense used by Smith and Riseborough (2002a) as permafrost *zone* boundaries, while permafrost boundaries are defined as the margins of *discrete bodies* of permafrost (that is with the second definition of van Everdingen 1998).

8.3.2 Application of the permafrost boundary model

The relationship between MAAT and TTOP is controlled by the combined thermal and nival offsets in the **TTOP** model. While the total offset can be described in these terms, the permafrost boundary model does not allow the separation of the thermal and nival offsets, providing only MAAT at the boundary. From equation (2.39) (i.e.,

$\Delta_{Total} = T_{TOP} - MAAT$), it follows that at a permafrost boundary where $TTOP = 0^\circ$,

$\Delta_{Total} = -MAAT$ and $MAAT = -\Delta_{Total}$. In Smith and Riseborough (2002a), T_{TOP} was

expressed as the sum of thermal and surface offsets:

$$T_{TOP} = MAAT - \Delta_T + \Delta_{Surf} \quad (8.8)$$

The permafrost boundary model is

$$T_{TOP} = 0^\circ C = MAAT_0 - \Delta_0 \quad (8.9)$$

$MAAT_0$ is the permafrost boundary MAAT in snow-free conditions, determined by the substrate thermal conductivity ratio r_K , and defined as a point whose I_{FA} and I_{TA} are on the thermal catena. $MAAT_0$ is a reference value of the thermal offset for snow-free conditions, and is larger than the actual thermal offset if snow is present.

The empirical term Δ_0 expresses the difference between $MAAT_0$ and the total offset Δ_{Total} . Equation (8.7) defines Δ_0 surface for $\theta_v k_F$ and equivalent snow cover depth for the “modified standard” thermal catena. As was shown for r_K in Chapter 4, soil physical properties (particle conductivity, porosity, and water content) combine to determine the magnitude of $\theta_v k_F$.

Application of the model with gridded distributions of I_{FA} , I_{TA} and snow depth is illustrated in Figure 8.8, showing latitudinal trends of snow cover and MAAT along the 100° W meridian (about 300 km west of Hudson Bay) from the Canada-U.S.A. border to the Arctic Archipelago. Figure 8.8a shows the latitudinal trends in seasonal

Figure 8.8 Latitudinal trends along the 100° W meridian, from gridded climatologies. **a)** Seasonal maximum snow depth and thermally equivalent snow depth; **b)** Normal (grid) mean annual air temperature (MAAT), together with MAAT for which $T_{TOP} = 0^{\circ}\text{C}$ for three different substrate materials.

maximum snow cover and the *equivalent* maximum snow depth (density-adjusted thermally equivalent) as discussed in Section 5.4.2. Snow depth is maximum near 57° N, about 300 km south of treeline; snow density variations enhance equivalent snow depth between this peak and locations north of treeline.

Using equations (8.9) and (8.7), the MAAT for which this snow cover would yield $T_{TOP} = 0^{\circ}\text{C}$ can be calculated: the latitudinal trend of this critical MAAT is shown in Figure 8.8b for three different substrate materials (the standard saturated mineral soil, standard saturated peat, and peat with 20% volumetric moisture content with $r_K = 0.67$ and $\theta_v k_F = 2.35$). The critical MAAT line for the mineral material is consistently colder than the peat material for all latitudes; the lines for the saturated and partially saturated peat materials cross at around 73°N, as the relative magnitude of the thermal and nival offsets reverses at high latitude.

For each substrate in Figure 8.8b, the point of intersection between the trends of critical MAAT and the actual (gridded) MAAT represents the permafrost boundary latitude for that particular material. Using this approach with the gridded climate data, Figure 8.9 shows permafrost boundary maps for these three substrate materials for all of Canada. The boundary for the mineral soil (Figure 8.9a) roughly corresponds with tree line; for saturated peat (Figure 8.9b) is farther south by 100 to 200 km, with a narrow band on the southern edge of Hudson Bay included in the permafrost region. For the unsaturated peat (Figure 8.9c), the permafrost boundary extends the farthest south of all substrate materials examined.

These maps do not show permafrost limits. Rather, these boundaries can be considered analogous to species-specific tree lines: species limits do not show the

a) Standard saturated mineral

b) Saturated organic (peat)

c) Organic with 20% v.m.c.

Figure 8.9 Predicted permafrost extent based on gridded freezing and thawing indices and average maximum snow cover. Maps represent distributions based on substrate characteristics: **a)** standard saturated mineral soil substrate; **b)** standard saturated organic (peat) soil; **c)** organic (peat) soil substrate with 20% volumetric soil moisture content. The dotted line on the maps show the treeline defined on the circumpolar permafrost map (Brown *et al.* 2002).

latitudinal tree line, but the limit for a particular tree species. The boundaries on these maps define places where the average snow cover condition would result in thermal conditions for which $TTOP = 0^{\circ}\text{C}$ for their particular substrate material.

To map permafrost limits using the permafrost boundary model, definitions of permafrost limits and boundaries must converge: The model determines whether permafrost is sustained given local values of the thermal and substrate parameters. Zone limits may be defined by those locations for which any likely combination of local parameter values yields either permafrost (at the continuous-discontinuous limit) or seasonally frozen ground (at the southern limit of the discontinuous zone). Because climates (Figures 8.2 and 8.3) and substrate materials vary across the country, permafrost limits will depend on different factor combinations from region to region.

Mapping of permafrost limits using the boundary model will require maps of r_K and $\theta_v k_F$ to accompany the gridded climatology used here. Within the discontinuous permafrost zone, local variation in these parameters, in snow depth, and in the other parameters not considered in this thesis will determine local permafrost extent.

8.4 Model parameter sensitivities

In the permafrost boundary model, r_K and $\theta_v k_F$ are the substrate parameters that determine the mean annual air temperature at permafrost boundaries. The sensitivity of the model can be investigated by considering the effect of changes in parameter values on the permafrost boundary temperature for a range of snow covers, or changes in mapped permafrost boundaries such as Figure 8.9. In evaluating the model using the gridded climatologies, the effect of changes in substrate or climatic conditions can be

summarized either as changes in the predicted position of the boundary between permafrost and seasonally frozen ground, or as changes in the predicted areal extent of permafrost. Predictions using either measure give only a general indication of the relative importance of changes in model parameters, since actual substrate properties for grid points have not been used.

For the analytical purposes of this section, the discontinuous permafrost region can be defined as the portion of the Canadian climate grid for which MAAT is between -10°C and $+2^{\circ}\text{C}$; this yields a total of 1953 points. Figure 8.10 shows the effect of changes in r_K (8.10a) and $\theta_v k_F$ (8.10b) on the extent of permafrost in this region, expressed as the proportion of grid points whose MAAT is colder than the critical MAAT determined from the permafrost boundary model. Permafrost extent changes by 28% over the range of r_K and by 40% over the range of $\theta_v k_F$. For both r_K and $\theta_v k_F$, the rate of change of permafrost extent increases at low parameter values. The highest sensitivity occurs at low values of $\theta_v k_F$, where permafrost extent changes by over 20% as $\theta_v k_F$ changes from values of 0.01 to 0.2.

While such first-order analyses can examine the sensitivity of $-\Delta_{Total}$ to the individual parameters θ_v and k_F , the combined effect of both parameters can be examined in terms of the physical properties of soils that determine their values, as was done for r_K in Chapter 4. The sensitivity of modelled boundary MAAT (that is, $-\Delta_{Total}$) to substrate physical properties (volumetric moisture content, porosity, and mineral particle conductivity) is analysed in the following sections.

Figure 8.10 a) Percentage of climate grid cells with permafrost, as a function of r_K , using the permafrost boundary model with $\theta_v k_F = 133.9$. Grid cells counted were limited to those having MAAT between 2° and -10°C . **b)** Percentage of grid cells with permafrost, as a function of $\theta_v k_F$, using the permafrost boundary model with $r_K = 0.552$. Grid cells counted were limited to those having MAAT between 2° and -10°C .

8.4.1 Moisture content

The effect of soil moisture content on $\theta_v k_F$ and r_K (using Johansen's (1975) equations for thermal conductivity) is shown in Figure 8.11 for the range of soils discussed in Chapter 5. Except at very low moisture content in low density mineral soil, r_K decreases with increasing moisture content for mineral and peat soils, as water and ice displace air in soil pores. Because both k_F and θ_v increase with moisture content, $\theta_v k_F$ increases rapidly as moisture content increases for all soils. In peat, the combination of very low k_F at low water content and the very high water content at saturation (due to its high porosity) makes the relationship between moisture content and $\theta_v k_F$ distinct from that of the mineral soils shown (Figure 8.10a).

The relationship between soil moisture content, snow cover, and $-\Delta_{Total}$ is shown in Figure 8.12 for soils with properties based on the standard mineral (8.12a) and peat (8.12b) soils. In these figures r_K and $\theta_v k_F$ are shown together as a function of volumetric water content in the left-hand graphs, scaled to separate y-axes. The right-hand graphs show boundary MAAT ($-\Delta_{Total}$) as a function of snow depth, for soils at low moisture content (0.02 volumetric for mineral soil and 0.04 for peat) and at saturation (0.44 volumetric for mineral soil and 0.85 for peat). The opposing trends in r_K and $\theta_v k_F$ result in similar relationships between snow cover and $-\Delta_{Total}$ at low and high moisture content, for both mineral and peat soil materials. At low moisture content, the response of the total offset to increasing snow cover is smaller, as both the thermal and nival offsets are reduced. At saturation, both the thermal offset effect and the response

Figure 8.11 $\theta_v k_F$ (a) and r_K (b) as a function of volumetric moisture content for soils examined in Chapter 5.

a) "Standard" mineral

b) Peat

Figure 8.12 a) Effect of volumetric moisture content of standard mineral soil on r_K and $\theta_v k_F$ (left), and effect of snow depth on the total offset Δ_{Total} ($= MAAT - TTOP$) for saturated and low moisture conditions (right). **b)** Effect of volumetric moisture content of standard organic soil (peat) on r_K and $\theta_v k_F$ (left), and effect of snow depth on the total offset Δ_{Total} ($= MAAT - TTOP$) for saturated and low moisture conditions (right).

of the transition MAAT to increasing snow cover are large because r_K approaches its minimum while $\theta_v k_F$ approaches its maximum.

8.4.2 Porosity

The effect of soil porosity on r_K , $\theta_v k_F$ and $-\Delta_{Total}$ is shown in Figure 8.13a for a

saturated soil based of the standard mineral soil (saturated, with 50% quartz content).

In this figure r_K and $\theta_v k_F$ are again shown together as a function of porosity in the left-hand graph, scaled to separate y-axes. As porosity increases from 30 to 66%, r_K decreases as high conductivity minerals are replaced by lower conductivity water, while $\theta_v k_F$ increases as the increase in water content more than compensates for the decline in frozen conductivity.

The right-hand graph shows $-\Delta_{Total}$ as a function of snow depth, for low (30%) and high (66%) porosity soil. The effect of the compensating trends in r_K and $\theta_v k_F$ is to minimize the effect of changes in porosity on $-\Delta_{Total}$, with a maximum difference of about 2.3°C with no snow cover (where the thermal offset is not compensated by the nival offset).

8.4.3 Mineral particle conductivity

The effect of variations in soil mineralogy on r_K , $\theta_v k_F$ and the $-\Delta_{Total}$ is shown in Figure 8.13b. For the standard soil, k_F varies between 2.1 and 4.4 Wm⁻²K⁻¹ as the quartz content of soil minerals varies from 0 to 100%. For saturated soils, r_K is unaffected by quartz content in Johansen's equations while $\theta_v k_F$ varies by a factor of 2.1, in direct proportion to the change in k_F .

a) Porosity

b) Quartz content / particle conductivity

Figure 8.13 a) Effect of soil porosity on r_K and $\theta_v k_F$ (left), and effect of snow depth on MAAT at a permafrost boundary, for high and low porosity saturated standard mineral soil. **b)** Effect of thermal conductivity of mineral component of saturated standard mineral soil on r_K and $\theta_v k_F$ (left), and effect of snow depth on MAAT at a permafrost boundary for high and low values of k_{min} .

The right-hand graph shows $-\Delta_{Total}$ as a function of snow depth, for soils having thermal conductivities equivalent to 0 and 100% quartz. Because r_K does not change as mineral conductivity changes in saturated soil, the transition MAAT with no snow (the thermal offset) is the same for both values of k_F . However, $-\Delta_{Total}$ increases as $\theta_v k_F$ increases with increasing quartz content, with a maximum difference in $-\Delta_{Total}$ of about 2.2° for deep snow covers.

8.4.4 Relative sensitivity

The influence of each of the parameters examined above can be ranked by comparing the range of MAAT resulting from their variation in Figures 8.12 and 8.13. Of the soil physical properties, moisture content influences this range most, while variations in saturated soil porosity influences it least with soil mineral conductivity intermediate between these two. The maximum effect of each of these parameters is felt under the deepest snow cover (1.5 m maximum), with a range of 5 to 6°C between dry and saturated mineral and peat soils; 2°C between low and high conductivity soil minerals; and 1°C between low and high porosity saturated soil. While the influence of snow cover on transition MAAT is influenced by all of these parameters, the range of its effect exceeds them all, with a maximum range of about 16°C between snow-free conditions and 1.5 m maximum snow cover for both peat and mineral soils at saturation.

8.4.5 Permafrost extent

Figure 8.14 shows the effect of changes in snow cover (8.14a), volumetric moisture content (8.14b), and soil mineral thermal conductivity (8.14c) on the mapped

Figure 8.14 a) Percentage of grid cells with permafrost, using the permafrost boundary model. Grid cells counted were limited to those having MAAT between -10° and 2°C . Figures show percentage as a function of a) percentage of normal snow cover, for saturated standard mineral soil, saturated peat, and for peat with 20% volumetric moisture content (“dry peat”); b) volumetric moisture content for standard mineral soil and standard peat soil; c) thermal conductivity of soil minerals for standard mineral soil.

extent of the permafrost region used for Figure 8.10 above. Several features are evident in these figures:

- For all substrates examined, permafrost is ubiquitous where snow cover is absent;
- Variations in snow cover cause the greatest variation in permafrost region extent of any of the parameters investigated, with a much larger permafrost region for the low-moisture peat substrate than for the saturated peat or mineral soils;
- Large variations in the thermal conductivity of mineral soil make relatively small differences in the permafrost region (on the order of 8% change in permafrost);

Changes in volumetric moisture content produce changes in permafrost extent that are similar to the influence of $\theta_v k_F$ alone (Figure 8.10b), since moisture content relates to all of the component variables of r_K and $\theta_v k_F$. In general, these results reinforce the analysis presented in Figures 8.12 and 8.13, and suggest that snow cover variability is the most important parameter controlling the extent of permafrost regions.

8.5 Discussion and Conclusion

8.5.1 Permafrost limits

The analysis of permafrost limits presented in Smith and Riseborough (2002a) can be revisited using the permafrost boundary model. In their original analysis, the nival offset was found to be the critical parameter at the continuous-discontinuous permafrost limit, while the thermal offset was critical at the southern limit of

discontinuous permafrost. In the original analyses however, the n_F function was assumed to be the same for all substrates. While the findings of the original analysis remain essentially valid, the conditions defining the limits require reconsideration.

Smith and Riseborough (2002a) suggest that permafrost at the northern limit of discontinuous permafrost will occur in bedrock, as the thermal offset will be limited by a value of r_K approaching unity. The analysis of Chapter 5 indicates, however, that Δ_0 increases as $\theta_v k_F$ increases. While the moisture-free condition has not yet been modelled, $\theta_v k_F$ in fractured bedrock will be close to 0, so that Δ_0 will be correspondingly small.

Whether a given substrate defines the northern limit of discontinuous permafrost in any region will depend on whether the value of $MAAT_0$ defined by the substrate will sufficiently offset the value of Δ_0 . While the results in Figures 8.14a and b suggest that the limit may most often occur in relatively dry peat materials, the majority of peatland is situated much closer to the *southern* limit of permafrost (Tarnocai *et al.*, 1995).

Approaching the southern limit of the discontinuous zone, the location and extent of patches of permafrost become increasingly dependent on local conditions, which produce increasingly exceptional circumstances. As noted by Brown (1979) and Ives (1979), permafrost near the southern limit of the discontinuous zone is found in different extreme environments in eastern and western Canada. In western Canada, permafrost near the southern limit is found in elevated peat plateaux (“palsas”), which combine an extreme ratio of frozen/thawed thermal conductivities and a topography that favours the removal of snow cover in winter. In Eastern Canada, Ives (1979) suggests

that most contemporary permafrost within the discontinuous zone is located at elevations above tree line or (as in the west) in peatlands, due to the thin snow cover associated with treeless conditions.

The parameterizations of the **TTOP** and permafrost boundary models are consistent with what is understood about the relationship of peatland to permafrost. Smith and Riseborough (1996, 2002a), echoing Brown (1970) suggested that the principal factor contributing to the association of permafrost with peat deposits at the southern limit of permafrost is the thermal conductivity ratio, which is extreme for saturated peat. As the analyses in this chapter indicate however, the total offset in organic soil is larger at lower moisture contents, suggesting that the reduced nival offset at, due to the low *absolute* conductivity of peat at low moisture content also plays a role in this environment.

The difference in boundaries between Figures 8.9b and 8.9c is consistent with Zoltai and Tarnocai (1975), who differentiate permafrost-wetland associations between “high subarctic” and “low subarctic” wetland regions. In the high sub-arctic, permafrost is present under all wetlands except within a few metres of lakes. In the low sub-arctic, permafrost is present under peat plateaux and palsa bogs, but not under fens. Similarly, permafrost in peat plateaux at the extreme limit of permafrost are overlain with unsaturated peat, and have reduced snow cover because of their elevated surfaces (Zuidhoff and Kolstrup, 2000).

The local conditions defining the relationship of peatland to permafrost near its southern limit occur at a scale that is not captured by the grid of the snow cover map. A successful permafrost mapping strategy will need to accommodate such “sub-grid

scale” effects, especially where factors co-vary spatially such as substrate properties and snow cover for peat plateaux.

8.5.2 Potential refinements

The goal of the analyses presented in this chapter was to demonstrate the utility of the permafrost boundary model, rather than to map permafrost boundaries definitively. In addition to the clear need for spatial data on the relevant substrate conditions and for a unique parameterization of Δ_0 for moisture-free conditions, various refinements to the implementation of the permafrost boundary model may be specified.

The trend surface defining $-\Delta_{Total}$ for the standard catena does not adequately describe this relationship for all index climates found in the gridded climatology. A more refined model could be constructed by defining a family of catenae, each defining the combination of MAAT and maximum snow depth for a given annual temperature range. Values of I_{FA} and I_{TA} from the grid could then be used to interpolate between $-\Delta_{Total}$ trend surfaces. For the purpose of demonstrating the potential of the permafrost boundary model, equation (8.7) may suffice.

The evolution of snow cover conditions in the standard catena could also be improved by accounting for regional differences in densification, as illustrated in Section 5.4.2. This would require the development of a unique Δ_0 surface (such as Figure 8.6) for each snow cover density function. Re-analysis with regionally appropriate equivalent snow depth and Δ_0 surfaces could provide a more accurate estimate of the thermal effect of snow cover conditions.

While these and other refinements to the permafrost boundary model would serve to make its application more accurate, the most important conclusions that may be drawn come from its functional form. The role of $MAAT_0$ in the permafrost boundary model is analogous to the thermal offset in the **TTOP** model, while Δ_0 is analogous to the nival offset. While the new parameters do not parse the annual thermal regime with the same clarity as the TTOP model, the results presented here extend the understanding revealed in the **TTOP** model to an appreciation of the role that active layer processes play in controlling the total offset.

Chapter 9 Conclusions

The principal objective of this thesis was to determine the utility and limitations of the **TTOP** model by exploring its robustness and the basis of its parameters. While some challenges to the application of the **TTOP** model for prediction have been identified, the model remains a useful descriptor of the permafrost-climate relationship.

Analyses of r_K and n_F show that these parameters can be interpreted in physical terms, and indicate the factors that contribute to their values. From the analysis of the nival offset, it appears that the freezing n factor may not be a sufficient parameterization of the influence of snow cover on TTOP. As pointed out in Chapter 5 however, forms of the **TTOP** model using n factors or offsets (Δ_{snow}, Δ_T) are equivalent.

The most significant result in this thesis is the development of the permafrost boundary model. While it is based on the **TTOP** model (using **TTOP** for snow-free conditions as a reference point), it determines the total offset (Δ_{Total} at the permafrost boundary) without separately determining the surface and thermal offsets. Most significantly, the model establishes $\theta_v k_F$, an index of the heat content of the active layer, as an important determinant of the thermal effect of snow on the ground thermal regime. While the nature of active layer – snow cover interaction has been suggested in previous work (Goodrich 1982a, Burn 1998a, Riseborough 2001), the development of an explicit parameterization of this relationship is a step forward.

Several secondary objectives have also been achieved in the individual analyses leading to the development of the model.

Analyses based on thermal catenae provide insights into the thermal behaviour of the active layer and upper permafrost. They demonstrate that TTOP, rather than air or ground surface temperature, is the best index of the thermal regime of the ground. Except in moisture-free substrates, results show a transition in behaviour at TTOP values close to 0°C; both nival and thermal offsets are greatest at this threshold temperature.

The analyses in Chapter 4 show that r_K is a reasonable function of soil moisture content for mineral and organic soils. The straightforward parameterization of the effect of unfrozen water suggests an approach for other variations, such as variation of soil properties with depth, laterally and seasonally. However, as the results of Chapter 5 indicate, establishing the ground surface freezing index for application of the apparent k_F function is not straightforward. It is presently unclear how the permafrost boundary model developed in Chapter 5 could be implemented with soils having a significant freezing characteristic.

Several conclusions concerning the thermal influence of snow cover can be drawn from Chapter 5. Results in Section 5.4.2 suggest that the thermal effect of snow cover can be expressed in terms of snow thermal resistance. From Sections 5.4.3 and 5.4.4, the initiation and accumulation rate of snow cover modify its effect on ground surface temperature, resulting in nival offset variations of several degrees.

Comparisons between catena simulations for different snow depths and substrate materials lead to general conclusions about the thermal effect of snow cover. The nival offset increases as snow depth increases, while for any given snow depth the nival offset is maximum where TTOP is near 0°C, declining at higher or lower temperatures.

With a bedrock substrate, there is a much less abrupt change in behaviour at 0°C, due to the absence of latent heat effects.

Active layer freezeback plays a crucial role in controlling the thermal interaction between the snow cover and the ground. The trend surface defining Δ_0 in the permafrost boundary model was developed using I_{FA} and I_{TA} for the standard catena, so that it does not describe this interaction for all index climates found in the Canadian climate grid network. A more general model could be constructed by establishing a family of catenae, each defining the combination of MAAT and maximum snow depth for a given annual temperature range. Values of I_{FA} and I_{TA} from the climate data could then be used to interpolate between Δ_0 trend surfaces.

Division of the freezing season into freezeback and conductive periods produced interesting results. The difference between the annual surface freezing index and the freezing index associated with active layer freezeback is directly proportional to TTOP. This result suggests a relationship between n_F (or the nival offset) and TTOP that could make the determination of n_F or Δ_{snow} intractable.

Results presented in chapter 6 provide the basis for analyses not yet undertaken. The effects of sensor burial demonstrated in Section 6.1 suggest that reporting of near-surface temperature (NST) values should include information about sensor burial depth and substrate thermal properties. Reporting of n factors derived from NST measurements should include I_F and I_T values, given that n factors are expressed as a ratio for convenience rather than an expression of the underlying physics. Transforming profiles of I_T and I_F in the ground using the square root function will permit the standardization of near-surface temperature measurements, which may allow results

obtained from sensors at different depths or in different substrate materials to be compared. The layered soil **TTOP** model of Section 6.2 appears to be useful at permafrost boundaries, although it is presently unclear how it might be implemented using the permafrost boundary model.

Results in Chapter 7 suggest that the relationship between MAAT and TTOP is robust in the presence of climate variability. Stationary inter-annual variability produces an error in the **TTOP** model for individual years that is greatest near $TTOP = 0^{\circ}\text{C}$, although this error is small in multi-year averages. Use of the model for average conditions will not introduce significant errors due to inter-annual variability, contrary to the results of Riseborough and Smith (1993). Larger errors in the **TTOP** model occur in the presence of a warming trend where permafrost degradation is initiated, due to the effect of latent heat as a talik develops. Other than this however, the **TTOP** model appears to be reasonably accurate under transient conditions.

Chapter 8 suggests that the permafrost boundary model can be applied to mapping permafrost limits, at least at the national scale. Sensitivity analysis indicates that the variations in the thermal resistance of the snow cover result in the largest changes in the permafrost boundary position. Of the soil physical properties, moisture content influences this range most, while variations in saturated soil porosity influences it least, with soil mineral conductivity intermediate between these two. The maximum effect of each of these parameters is realized under the deepest snow.

Ultimately, the results presented in Chapter 5 demonstrate the interdependence of TTOP and active layer depth, through the effect of TTOP on the sensible heat content

of the active layer, and through the dependence of TTOP on active-layer freezeback under the snow cover.

Modelling permafrost and environmental change

Application of the **TTOP** or permafrost boundary model to the issue of climate change projections will require consideration of the interaction between permafrost, snow cover, and vegetation, at time scales ranging from decadal to millennial. As illustrated with the relationship between snow density and treeline in Figure 8.1b, the position of treeline controls the geographic distribution of snow density. It is likely that changes in permafrost distribution, migration of treeline, and changes in snow cover properties in the forest-tundra transition zone will be highly interdependent. At much longer time scales, the distribution of peatlands in Canada depends on the relative rates of carbon accumulation and depletion in the soil, with the zone of peak soil carbon and peatland distribution close to the position of the 0°C mean annual ground temperature isotherm (Swanson et al., 2000). Changing ground temperatures under climate warming will alter the distribution of peatlands, thereby altering local permafrost distribution.

The greatest challenges to the application of the TTOP model are likely to be in those places where the horizontal spatial scale of variation of physical processes (such as radiation and snow accumulation) and properties (such as the thermal properties of the ground) is small relative to the depth of the active layer. Significant variation at this scale will result in a three-dimensional pattern of heat flow between the atmosphere and the ground that will be difficult to characterize with an explicitly physical model.

Future directions

The results obtained in the pursuit of this thesis present at least three possible directions for further research. First, model results could be tested against field conditions. Use of the **TTOP** model in warm permafrost conditions has not yet been validated, though the effects of inter-annual variability (Chapter 7) suggest that this may require several years of data. Similarly, the effects of variations in snow initiation and accumulation patterns demonstrated in Chapter 5, together with the sensitivity of the **TTOP** model to variability near the 0°C boundary temperature demonstrated in Chapter 7, suggest that short term results are unlikely to provide definitive proof of the validity of the permafrost boundary model. In colder permafrost however, the approximate equivalence of the freezing index for the conductive period (I_{FC}) with **TTOP** could be examined as a useful verification of this effect, which may prove useful in a more general model of permafrost-snow cover interaction.

Second, the mapping procedures illustrated in Chapter 8 could be applied more rigorously, using known hydrological and stratigraphic conditions at the national scale to estimate the substrate properties that control the parameters of the model. This would be similar to the procedure used in Henry and Smith (2001). Snow densification and the effects of spatial heterogeneity in snow cover and substrate conditions could also be addressed. The extensive literature on the effects of sub-grid scale variability in the application of global circulation models (e.g., Smagorinsky 1981, Jarvis 1995) might provide useful strategies to deal with this.

Third, refinements and extensions of the models developed may be envisioned. Preliminary analyses of the issues ignored in this thesis (n_T , models of spatial

heterogeneity, slope and aspect effects) would help to determine which of these issues are of most immediate concern.

Results also suggest several extensions of the permafrost boundary model. These include its extension to bedrock substrate materials, to layered soil materials, and to soils having a significant freezing characteristic. Ultimately, the permafrost boundary model might be extended to general ground temperature conditions, and not just boundaries. As with the original boundary model, it may be necessary to derive a model that accounts for energy flow through the entire snow cover – active layer system, without reference to the ground surface temperature.

Envoi

The analyses undertaken in this thesis demonstrate that a simple model can provide significant insight into the permafrost - climate relationship. The process of model development puts flesh on the bones of the model, establishing parameter sub-models based on measurable properties of the permafrost-climate system. When this process is successful, it reinforces the power of the simple model.

While the **TTOP** and permafrost boundary models are both simple, the **TTOP** model presently remains a greater source of insight because its structure relates directly to the way the physical environment is understood. Future analyses will help to decide whether offsets or boundaries best capture the imagination as expressed in permafrost-climate models.

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Appendix I Numerical Geothermal Models and TONE

The intent of this section is to explain the principles and important details of numerical geothermal models in general and of TONE in particular, in the language of geothermal modelling.

Goodrich developed two geothermal models, T1D and TONE. The numerical details of T1D are discussed in Goodrich (1978b) and Lunardini (1980, p 501). The principles behind the TONE model are discussed in a general way in Goodrich (1982b), while details of TONE are discussed by Goodrich and Sepplar (unpublished). T1D and TONE differ in the specification of soil thermal properties, and the method of accounting for soil freezing, but share most other algorithms, including methods to deal with snow cover. Several papers were published on the numerical scheme and results obtained using T1D (Goodrich 1976, 1978a, b, 1982a). The relationship between TONE and T1D is discussed in this section.

Numerical Geothermal Models

Numerical geothermal models can be divided into finite difference schemes and finite element schemes. In models of one spatial dimension, finite difference and finite element schemes are equivalent (Goodrich 1982b). Numerical schemes divide continuous time into finite time-steps and continuous space into finite pieces. Finite element schemes focus on the pieces (*elements*), while finite difference schemes focus on the interstices between elements (*nodes*). In the one-dimensional geothermal models discussed here, the spatial dimension is depth. Continuous profiles of temperature and earth materials are represented by an array of elements (or nodes) for which the temperature and thermal

properties are specified at discrete points. This substitution simplifies the prediction of future temperature profiles by allowing the assumption that the temperature gradient across each element (or between nodes) is linear, and that the properties of each element (or between nodes) are uniform. Thus, ground temperature profiles can be projected based on antecedent temperatures and the thermal properties of the ground (and snow). Conditions that are difficult or impossible to solve analytically can be solved approximately. The precision and accuracy of prediction depends in part on the size of elements (or the node spacing) and on the time step size (Lunardini 1981 p. 473, Goodrich 1982b, Romanovsky et al. 1997).

The steps involved in one-numerical modelling of the ground thermal regime are:

1. *Define the modelled space: Establish a starting point in time, and set upper and lower boundaries on the ground.*

A geothermal model sets boundaries on space and time. The ground has a natural upper boundary, defined by the interface between the solid earth and the atmosphere, although the physical reality of this boundary is much more complicated than can be represented in a model. At some scale, defined by the diffuse roughness of vegetation or unconsolidated soil material, or the bluff roughness of surface topography, an average or effective surface plane must be defined as an upper boundary for the numerical representation of the ground. If a numerical thermal model incorporates seasonal snow cover, the position of the upper boundary changes to accommodate the accumulating snow with new elements or nodes. TONE and T1D allow various methods of accommodating snow: the method employed throughout this thesis was to specify snow depth and snow density for each time step.

2. *Divide continuous space into finite pieces (or elements) and continuous time into finite time-steps.*

While numerical criteria for the relationship between time steps and nodes spacing can be applied (Goodrich 1982b), Romanovsky et al (1997) suggest that much smaller space and time increments (node spacing of 0.02 m in the upper meter of simulations, and time steps of 1 hour or less) are necessary to maintain the accuracy of T1D for long term simulations. These criteria were also applied to the TONE simulations used in this thesis.

3. *Specify the thermal properties of the material represented by the elements.*

For most earth materials, in particular those containing water, thermal properties (thermal conductivity, volumetric heat capacity) are a function of temperature. While it may be important to consider changes in properties (other than unfrozen water as a function of temperature) and geometry through time¹, these effects are ignored in TONE, and are not considered directly in this thesis.

4. *Specify an initial temperature profile, supplying a temperature value at every interface (node) between elements in the profile.*

The temperature of nodes or elements along the vertical profile must be specified at the start of a simulation, since calculation of the temperature profile at each time step requires the antecedent profile as part of the solution. Where simulations are used to find an equilibrium temperature for a given surface climate, the initial temperature profile can

¹ Thaw settlement due to permafrost degradation, or frost heave in the active layer, will change the geometry and thermal properties of the ground through time. Inclusion of such effects will be important in some cases, and will require the use of models specifically designed to accommodate these effects. In general, the changes in soil geometry due to frost heave will be small compared to the effect of thaw

be arbitrary (though still required), since the model is run repeatedly through the annual cycle until an equilibrium profile² is established, so that the initial condition does not influence the resulting profile.

5. *Specify the temperature or heat flow conditions as a function of time (i.e. for each time step) for the lower and upper boundaries.*

The lower boundary condition is usually easiest to specify, typically with an assumption that heat will flow at a constant rate determined from the geothermal heat flux.

At the ground surface, the assumed course of temperatures (or energy flux) must be specified at each time step. This may be established directly, or may be determined as the function of various possible surface heat transfer schemes. Snow cover may be treated more or less realistically as a transient layer with its own thermal behaviour, or it may be subsumed within a seasonal transfer function such as the n-factor. T1D and TONE use air temperature modified by n factors to account for the surface boundary condition, and accommodate the development of snow cover.

6. *For each time step after the starting time, the new temperature profile is calculated, based on the combination of thermal properties, antecedent and boundary conditions.*

Freezing and Thawing

Numerical schemes for solving freezing and thawing problems must deal with the latent heat of fusion. The most common approach (used in TONE, but not in T1D) is to

settlement. In models that account for changes in geometry or moisture content (thaw settlement, frost heave or soil drying for example) the effect of these changes on thermal properties may also be defined.

include the latent heat in the heat capacity of the soil over a range of temperatures over which phase change takes place.

$$C = \rho c = \sum x_i \rho_i c_i + L \left(\frac{\partial \theta_v}{\partial T} \right) \quad (\text{I.1})$$

where

x = volume fraction of soil component (ice, water, soil solids)

C = volumetric heat capacity, $\text{J m}^{-3} \text{K}^{-1}$

c = specific heat, J kg^{-1}

ρ = density, kg m^{-3}

L = latent heat of fusion, J m^{-3}

$\left(\frac{\partial \theta_v}{\partial T} \right)$ = rate of change of unfrozen water content, K^{-1}

This scheme fits naturally with soils having significant unfrozen water content, since the effect of the freezing characteristic curve can be recreated realistically. Soils whose thermal properties change abruptly at 0°C or over a narrow temperature range are accommodated by an apparent heat capacity function with a large property spike at the phase change temperature. The numerical approximation of this transition can only be accomplished over a finite temperature range. Figure I.1 shows the apparent heat capacity functions for two soils with the same total water content, with and without the effect of unfrozen water. Note the logarithmic scale on the y-axis, so that the peak heat capacity for the solid curve is 26 times larger than the peak for the dashed curve, based on a transition from frozen to unfrozen over a 0.02°C temperature range.

Significant errors can be introduced into numerical schemes where the thermal properties of the soil change dramatically as the soil freezes or thaws, depending on the method used to assign the thermal properties of the ground. The simplest method is to

² In the simulations carried out for this thesis, the thermal regime was assumed to be at equilibrium when all node temperatures along the temperature profile at an arbitrary reference date within the annual cycle differed by less than 0.001°C from the temperatures for that date in the previous cycle.

Figure I.1 Temperature dependent apparent heat functions for soils with the same total water content, with (dashed curve) and without (solid curve) the effect of unfrozen water accounted for.

assign thermal properties to elements at based on the mean element temperature. If the calculation grid is too coarse relative to the temperature sensitivity of the thermal properties, the spike in apparent heat capacity at the phase change temperature can be missed completely. Goodrich's models T1D and TONE use different methods to deal with this problem.

Freeze/thaw in TONE

To avoid the problem of missed latent heat due to the assignment of C_a based on element average temperatures, TONE determines the element-average C_a based on the temperature *profile* through each element. The temperature gradient through each node space (or element), in combination with the temperature-dependant C_a function defined for the material occupied by the node space (or element), determines the profile of C_a through the element. Use of the mean of the C_a profile, rather than C_a for the mean temperature of the element, ensures that all latent heat is accounted for, regardless of the abruptness of the phase transition. This method should also produce better results in soils that exhibit a notable freezing characteristic, since it can accommodate the distribution of thermal properties across each element. Numerical experiments with TONE found that results are insensitive to the width of the phase change temperature range down to 0.02°C (that is, all phase change occurs between -0.01°C and $+0.01^\circ\text{C}$).

Freeze/thaw in T1D

Unlike the apparent heat capacity method described above, latent heat in T1D is accounted for explicitly by tracking the position of the phase front as a separate node with its own energy balance. Movement is calculated for each time step based on the heat that crosses the phase-front node, based on the local temperature gradients and thermal

properties. All phase change occurs at a defined fusion temperature (normally, 0°C).

The model can accommodate two phase fronts within the soil profile, and instantaneously sets all temperatures between the phase fronts to the fusion temperature once two fronts are present (to permit a numerical solution). Romanovsky and Osterkamp (1997a) suggest that this is a reasonable approximation when the soil latent heat of fusion is much larger than its volumetric heat capacity. This formulation improves the accuracy of T1D in conditions where soil water phase change occurs at a single temperature from one side only, but will be less accurate where two-sided phase change occurs or where soil water changes phase over a range of temperatures (Romanovsky *et al.* 1997).

Neither T1D nor TONE accommodates the error that occurs with freezing or thawing from two sides (such as during freezeback of the active layer) once the two phase-change fronts fall within a single element. In these cases, the latent heat for the soil between the fronts is lost. The only effective way to reduce errors due to this effect is to limit the grid spacing.

Model Testing

The standard method for numerical model validation is to compare simulation predictions to exact, analytical solutions (Romanovsky *et al.* 1997). Bouchard (1990) compared a TONE simulation of the thermal response of the ground to a sinusoidal surface temperature wave without phase change to the exact analytical solution. Results for TONE were generally within 0.002°C of the exact solution, with a maximum error of 0.012°C .

Tests for the accuracy of numerical phase change compare modelled freezing or thawing depths with the Neumann equation (equation 2.3), which gives the depth of

freezing (or thawing) in a uniform, initially isothermal soil experiencing a step change in surface temperature.

In a comparative test against the Neumann equation, Romanovsky *et al.* (1997) found T1D to be the most accurate of three commonly used numerical geothermal models, with an RMS difference in daily mean temperatures within the soil of 0.01°C . Figure I.2a compares depth of freezing versus time for a specific condition for equation 2.3 and TONE, showing that TONE tracks the analytical solution closely. Figure I.2b shows depth of thaw for TONE plotted as a function of thaw depth for the Neumann equation. Regression of simulated freezing depth against the Neumann results show that results are within 0.5% of the exact solution. Goodrich and Sepehr (unpublished) compared results from TONE and T1D in a one-sided thawing problem and concluded, “although there are some important deviations, agreement is seen to be reasonably close”. Based on a comparison of simulated and measured ground temperatures, Bouchard (1990) also demonstrated that TONE was capable of reproducing the annual thermal regime of permafrost sites ranging from bedrock with low moisture to fine grained till with significant unfrozen water below 0°C . In general, maximum temperature errors were on the order of 5-10% of the annual temperature range at each depth. Simulated profiles of mean annual temperature were within 0.2°C of means calculated from measured temperature data, although larger errors (up to 1°C) were noted near the top of the active layer in the coarse sand simulations.

The difficulty with such assessments based on comparison of simulated ground temperatures with field data is that the thermal properties are known only approximately, and must be estimated based on iterative comparisons. As a result, it is unclear whether

Figure I.2 a) Depth of freezing versus time for standard mineral soil initially at 5°, with surface temperature step to -8°, for the Neumann equation (solid line) and TONE (dotted line). **b)** Depth of thaw for TONE versus depth of thaw for the Neumann equation. Inset equation is the linear regression equation.

the final fit of temperature data to temperature simulation validates the model since, as Randall *et al.* (1996) suggest, “The problem with tuning is that it artificially prevents a model from producing a bad result”.

Snow in TONE

The analyses in this thesis are based on the simplest method that the TONE model permits: the total snow depth and density were specified for each time step. Explicit control of depth and density made these parameters easier to manipulate.

The model used to define the thermal conductivity of the snow in TONE uses the model of Devaux (1933, cited in Goodrich 1976³). The model depends on density only:

$$k_s = 0.029 + 0.0000029 \times \rho_s^2 \quad (I.2)$$

where

ρ_s = snow density, kg m^{-3}

The volumetric heat capacity of the snow is a linear function of density. For dry snow

$$C_s = x_i \rho_i c_i + x_a \rho_a c_a \quad (I.3)$$

Ignoring the contribution of air to the heat capacity:

$$\begin{aligned} C_s &= x_i \rho_i c_i \\ \therefore \rho_s &= x_i \rho_i \\ \therefore C_s &= \rho_s c_i \end{aligned} \quad (I.4)$$

Because the thermal conductivity of dry snow is a function of the square of snow density, the thermal diffusivity of snow within TONE is almost a linear function of density:

$$\alpha_s = \frac{k_s}{C_i} = \frac{0.029 + 0.0000029 \times \rho_s^2}{\rho_s c_i} \quad (I.5)$$

³ The snow thermal conductivity model used in T1D (the model used in Goodrich 1980) is that of Abels (1892). Devaux’s (1933) model is identical except for the inclusion of the constant 0.029.

Appendix II An analytical model of the ground surface temperature under snowcover with soil freezing.

Riseborough (2001)

58th Eastern Snow Conference Ottawa, Ontario, Canada.

An Analytical Model of the Ground Surface Temperature Under Snowcover with Soil Freezing.

D.W. RISEBOROUGH¹

ABSTRACT

This paper presents a model of the temperature at the snow-soil interface for the period in which soil freezing dominates: this period persists throughout the winter at non-permafrost sites, and until active layer freeze-back is complete at permafrost sites. A model is developed in which ground surface temperature is determined by the thermal resistances of the snowpack and frozen soil, and by the progression of the soil freezing front. The model is shown to replicate the numerically simulated ground surface temperature under snow, and qualitatively replicates the pattern of ground surface temperatures observed at permafrost sites during active layer freezeback. The model appears to work best where the mean annual ground temperature is near 0°C. This is the condition at the southern limit of permafrost, where freezeback of the surface active layer persists through the winter.

Keywords: snowcover, ground freezing, permafrost, ground temperature

INTRODUCTION

Snowcover has long been understood to be of primary importance in determining the difference between air temperatures and ground temperatures in cold regions (e.g. Nicholson and Granberg 1973, Nicholson 1979, Goodrich 1982), since snow restricts the loss of heat from the ground during the coldest part of the year. This paper evaluates a model of the effects of snowcover on the thermal regime at the snow - ground interface above soil undergoing freezing. The focus is on the effect of the properties and processes in the ground, rather than the snowcover itself. The details of heat transfer within the snowcover and ablation effects are not considered, although they are recognised as important.

SNOWCOVER AND GROUND FREEZING

The effect of snowcover on ground temperatures can be summarized as a difference in mean annual temperature between air temperature at screen height and at the ground surface (ΔT) due to the effect of snow, or as an n-factor (Lunardini 1978). Using the n factor approach, the annual degree day total below 0°C (I_F , also called the freezing degree day total or the freezing index) for the surface is expressed as a fraction of the freezing degree day total for the air:

$$\Delta T = \frac{I_{FS} - I_{FA}}{P} \quad (1)$$

$$n_F = \frac{I_{FS}}{I_{FA}}$$

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Figure 1 Relationship between Surface-Air temperature difference and mean annual air temperature for various snow thickness. Data used in Riseborough and Smith (1998).

Figure 2 Relationship between Surface-Air temperature difference and mean annual

where

- ΔT = Snow-related air-surface temperature difference, °C
 I_{FS} = Seasonal Surface Freezing Degree Days (Celsius)
 I_{FA} = Seasonal Air (screen height) Freezing Degree Days (Celsius)
 P = Annual Period (365 days)
 n_F = Freezing n-factor

These approaches assume that the snow season coincides with the freezing season, and that ablation is a negligible component of the annual regime. Riseborough and Smith (1998) found that n_F varies with mean annual air temperature as well as snowcover thickness. Figures 1 and 2 re-express the results of their analysis of n_F in terms of ΔT . In Figure 1, ΔT for a range of snowcover thicknesses is shown as a function of mean annual air temperature. For each thickness, ΔT has a maximum value at a mean annual temperature below 0°C, with the peak increasing in magnitude and shifting to lower temperature as snowcover thickness increases. Figure 2 shows the same data plotted as a function of the mean annual temperature at the bottom of the active layer (the layer that freezes and thaws annually). The mean annual temperature at this depth includes the effect of the “thermal offset” (Goodrich 1978, Burn and Smith 1988), and establishes whether permafrost is present or absent (Smith and Riseborough, 1996). For each snowcover thickness, ΔT peaks at approximately the same temperature, just below 0°C. This can be attributed to the effect of seasonal ground freezing, which reaches its maximum where the mean annual temperature is 0°C. Other things being equal, the freezing season is shorter at higher temperatures (that is, in non-permafrost), while in permafrost the thawing season is shorter so that the active layer re-freezes more quickly at colder temperatures.

Figure 3 Schematic representation of the temperature profile through snow and soil.

The proximity of a freezing front prevents the ground surface temperature from becoming extremely cold (Figure 3). In permafrost, soil freezing continues until the active layer is re-frozen, while in non-permafrost freezing continues throughout the winter. In permafrost, near surface temperatures can fall rapidly once active layer freeze back is complete (Figure 4). The difference between permafrost and non-permafrost thermal behavior has been used to discriminate between them based on the temperature at the ground surface in late winter (Hoelzle and Haeberli, 1993).

Figure 4 Near-surface ground temperature under snowcover, East Point, Richards Island. September 1999 - July 2000. Maximum snowcover was approximately 3.5 m. Data were collected using a Hobo logger buried at approximately 5 cm.

Soil freezing can be modelled using the Stefan model (Lunardini 1981, p. 370), which is valid for freezing and thawing cases when diffusive effects are much faster than the front motion, so that diffusion occurs quickly enough that the temperature gradient in each layer is nearly linear. This allows for a simplified equation that ignores sensible heat by assuming all heat flow is used to extract heat from (or supply heat to) the freezing (or thawing) front. The Stefan equation is (Lunardini 1981, p 370),

$$X = \sqrt{\frac{2k|T_s - T_F|t}{L}} \quad (2)$$

where

- X = freezing/thawing depth, m
- k = Thermal conductivity of the soil (freezing or thawing), $\text{Wm}^{-1}\text{K}^{-1}$
- T_s = Temperature at the surface, $^{\circ}\text{C}$
- t = time, s
- L = Volumetric latent heat of fusion, Jm^{-3}
- T_F = Fusion Temperature (temperature at the soil freezing front), $^{\circ}\text{C}$

The product $|T_s - T_F|t$ can be replaced by the accumulated degree-day total (the freezing index I_F or thawing index I_T) because transient effects are ignored (Lunardini 1981). Taking the assumption of linear gradients as approximately true, the ground surface temperature at any time during soil freezing can be determined from the heat flow (temperature gradients and thermal conductivities) and the snow and frozen soil layer thicknesses. With linear temperature gradients, heat flow through the snow = $(T_s - T_0)k_s/Z$ and through the frozen soil = $(T_0 - T_F)k_a/X$, where

- T_s = Temperature at the *snow* surface, $^{\circ}\text{C}$
- T_0 = Temperature at the ground surface (the snow-soil interface), $^{\circ}\text{C}$
- k_s = Thermal conductivity of the snow, $\text{Wm}^{-1}\text{K}^{-1}$
- k_a = Thermal conductivity of the freezing soil, $\text{Wm}^{-1}\text{K}^{-1}$
- Z = snow layer thickness, m

Equating heat flow through the snow and the active layer, and using Celsius temperature so that $T_F = 0$, 315

$$\frac{(T_s - T_0)k_s}{Z} = \frac{T_0 k_a}{X} \quad (3)$$

Solving for T_0 yields:

$$T_0 = \frac{T_s}{1 + \left(\frac{k_a Z}{k_s X} \right)} \quad (4)$$

Note that T_s is the boundary temperature in equations 2 to 4, so that it is at the top of the soil in equation 2 and at the top of the snow in equations 3 and 4. Replacing the conductivity and thickness of the snow and freezing soil layer by thermal resistance (where the thermal resistance = thickness/conductivity) yields:

$$T_0 = \frac{T_s}{1 + \left(\frac{r_s}{r_a} \right)} \quad (5)$$

The assumption that all heat flow is used to supply heat to the fusion front (that is, because the sensible heating of the soil is ignored), allows equation 5 to be applied to layered snow or soil, since in the model any non-linear gradients are due to variation in the thermal conductivity of the materials.

Equations 4 and 5 supply an instantaneous temperature at the snow-soil interface, valid on a daily mean basis. To estimate the freezing degree-day total at the ground surface, the evolution over time of the thickness of the snowcover and the freezing soil layer are required.

Model of Nixon and McRoberts.

Nixon and McRoberts (1973) applied the Stefan model to thawing of soils having layers with different thermal properties. Because the Stefan model is valid for both freezing and thawing cases, a modified version of their model can be applied to the case of soil freezing under snowcover. Nixon and McRoberts determine the time required to thaw the upper soil layer, and use this time to offset the initiation of thaw in the underlying layer. Thawing then proceeds in the lower layer with the gradient to the front determined by the conductivities of the two layers. The time t_0 required to thaw the surface soil layer is defined as:

$$t_0 = \frac{H^2 L}{2k_1 T_s} \quad (6)$$

where

H = Surface layer thickness, m

L = Surface layer volumetric latent heat, Jm^{-3}

k_1 = Upper layer conductivity, $\text{Wm}^{-1}\text{K}^{-1}$

For $t < t_0$, thawing depth is calculated from the standard Stefan equation (equation 2). For $t > t_0$, thawing depth is given as:

$$X = \sqrt{\left(\frac{k_2}{k_1} \right)^2 H + \frac{2k_2 T_s (t - t_0)}{L_2}} - \left(\frac{k_2}{k_1} - 1 \right) H \quad (7)$$

where subscripts 1 and 2 refer to the upper and lower soil layers respectively. Nixon and McRoberts give an equation of the temperature at the interface of the two soil layers that is equivalent to equation 4 above.

The essential step in applying equation 7 to soil freezing under snowcover is to recognise that a snow layer of constant thickness can replace the upper soil layer. Because the snow arrives in the frozen state, $t_0 = 0$. Rewriting the equation using these assumptions, setting $k_1 = k_s$ and $k_2 = k_a$, replacing H with Z and replacing T_s with the accumulating I_F :

$$X = \sqrt{\left(\frac{k_a}{k_s}\right)^2 Z + \frac{2k_a I_F}{L} - \left(\frac{k_a}{k_s} - 1\right) Z} \quad (8)$$

Equation 8 can be implemented easily in a spreadsheet program, with the time evolution of daily mean air (or snow surface) temperature accumulated to obtain a running total air freezing index, which gives the product T_{st} in Equation 7. The daily value of T_0 can then be calculated from the daily freezing depth X using Equation 4. An example of this implementation is given in Figure 5, showing air and surface temperature evolution with a 0.3 m snowcover (using the standard conditions described below). The temperature at the ground surface is initially very close to 0°C , and can get begin to cool significantly only after the frozen soil layer develops a significant depth. In this example, surface temperature does not fall below -1°C (with an air temperature of -20°C) until the frozen soil layer is 0.19 m thick. The surface temperature remains close to 0°C relative to the air temperature because the ratio k_a/k_s is large for most combinations of soil and snow.

Figure 5 Air and ground surface temperature for the standard case, using equations 4 and 8.

Comparison with Numerical model

The surface temperature obtained from equations 4 and 8 as outlined above were compared with results obtained using a numerical one dimensional finite element model. This comparison permits a test of the principal assumptions of the model, since the numerical model incorporates transient effects and sensible heat (such as the supply of heat to the freezing front from below, and finite diffusion rates within the soil and snow). Comparison was made to results obtained using Goodrich's discontinuous element model (TONE), based on the finite element model of Steven (1982). The soils' latent heat of fusion is accounted for by an apparent heat capacity function based on the unfrozen water content function. In each time step, the thermal conductivity and apparent heat capacity of elements are averaged based on the temperature profile across the element, improving accounting for the total latent heat. This model differs from that used by Goodrich (1982). Four cases were examined:

1. A standard case with a sinusoidal air temperature wave having a mean of -10°C and amplitude of 20°C (as in Figure 5). Surface conditions included a 0.3 m snowcover with a density of 250 kg m^{-3} and thermal conductivity of $0.21 \text{ Wm}^{-1}\text{K}^{-1}$, and a saturated mineral soil with 50% volumetric moisture content and thermal conductivity of 2.938 and $1.496 \text{ Wm}^{-1}\text{K}^{-1}$ in the frozen and thawed states, respectively.

2. As in the standard case, but with a soil condition consisting of saturated peat with 96% volumetric moisture content and thermal conductivity of 2.031 and 0.556 Wm⁻¹K⁻¹ in the frozen and thawed states, respectively.
3. As in the standard case, but with a mean annual air temperature of 0°C, representing a non-permafrost site.
4. As in the standard case, but with a snowcover 0.404m thick. By trial and error using the numerical model, this was determined to yield a mean annual ground temperature close to 0°C (representing a permafrost – non-permafrost boundary).

In the numerical simulations, the complete snowcover was imposed on the day after the start of the freezing, to permit the model to properly initiate freezing conditions in the ground. This resulted in a surface temperature of about -0.1°C at the start of the freezing period, followed by a temperature rise to the freezing point after the arrival of the snowcover. Simulations were run with a time step of 40 minutes, with an element size of 0.02 m at the ground surface, increasing to 0.6 m at the bottom of the grid (16 m) depth. Element size in the snow was generated automatically by the model to give a Fourier number ($\Delta x / \Delta t \alpha$) of at least 0.1 (where Δt = time step, s; Δx = grid size, m; α = thermal diffusivity). The model was run until the temperature profile at the end of an annual cycle differed from the previous cycle by less than 0.001 °C, ensuring that the initial temperature condition did not influence results.

RESULTS AND DISCUSSION

Figures 6 to 9 show comparisons of the analytical results with results using the numerical model. In the numerical model, active layer freezing ends on day 144 for the standard case and day 114 for the peat case; for the non-permafrost and the permafrost limit cases, soil freezing continues throughout the winter. For the two permafrost cases and the permafrost limit case, the fit between the analytical model and numerical results is very good for the period during which the soil is undergoing freezing; after that time, the analytical assumptions are no longer valid.

For the permafrost limit (case 3), the discrepancy between the two models is greatest in late winter, which can be attributed to the neglect of sensible heat in the analytical model: as the surface temperature increases the direction of heat flow reverses instantaneously for the analytical model while diffusion results in a delay in warming in the numerical model.

Figure 6 Modeled surface temperature: Standard (Case 1). Inset shows freezing front depth.

Figure 7 Modelled surface temperature: Organic soil (Case 2). Inset shows freezing front depth.

Figure 8 Modelled surface temperature: "Permafrost Limit" (Case 3). Inset shows freezing front depth.

Figure 9 Modelled surface temperature: Non-permafrost (Case 4) Inset shows freezing front depth.

For the non-permafrost site (case 4), the analytical model significantly over-predicts the depth of freezing, and therefore predicts surface temperatures that are colder by about 60%. This large discrepancy is due to the neglect of sensible heat at the freezing front: a significant component of the energy being removed at the freezing front is supplied by the unfrozen soil below, slowing the progress of the front. The surface temperature model (equation 4) depends on the depth of freezing, and predicts colder temperatures as a result. Figure 10 shows estimated surface temperature for the non-permafrost case using equation 4, but using frost depths taken from the numerical results rather than equation 8. Results are as good as for the other cases, suggesting that the basic assumption of near-linear gradients is valid for the non-permafrost cases as well.

The magnitude of the discrepancy may be a function of the mean ground temperature, since this determines the magnitude of the sensible heat component of the energy balance of the active layer. This suggests that the analytical model may be most accurate where the mean annual ground temperature is 0°C , at the permafrost – non-permafrost boundary.

Figure 10 Modelled surface temperature for the non-permafrost case, using numerically determined freezing depth to solve equation 4 (“Forced”), compared with numerical result.

CONCLUSION

This simple model appears to work well in permafrost applications, primarily because of the effect of the unfrozen active layer, which maintains isothermal conditions below the freezing front (the “zero curtain”), thereby satisfying an important assumption of the underlying model. In non permafrost the model performs poorly because the supply of heat to the freezing front from unfrozen ground significantly effects the predicted depth of freezing. For cold permafrost conditions an error in estimating the seasonal ground surface freezing degree day total can be anticipated due to up-freezing at the base of the active layer, shortening the freezeback period and resulting in a colder winter at the surface. For non-permafrost, the temperature model can provide good results if the depth of freezing can be estimated separately.

The numerous simplifying assumptions required to make this model function limit its utility to fairly simple analyses. The need to specify a constant snowcover thickness is a particular difficulty: The thermal resistance of the snow and soil will require an interpretation that includes non-uniform snow and soil properties and the neglect of non-conductive processes.

On its own, this model has potential use in exploring the role of snowcover in determining the position of the permafrost margin, since that is the location at which active layer freezing persists until the end of winter. A separate scheme describes the thermal behavior of the snow-soil

interface after freezeback is complete (such as Lachenbruch 1959) would be useful for the successful parameterization of snowcover effects within permafrost regions. 320

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Appendix III Gridded Climatologies

The techniques used to convert gridded snow and temperature data to parameters for use in the permafrost boundary model are discussed in this appendix. Semi-monthly snow density data were processed to estimate annual mean snow density, and combined with average annual maximum snow depth data to estimate a thermally equivalent snow depth, as discussed in Section 5.4.2. Normal monthly mean temperature grids were used to estimate normal I_{FA} and I_{TA} values.

A3.1 Reconciling grids

The air temperature grid (from which I_{FA} and I_{TA} are estimated), and snow depth and density grids (from which equivalent snow depth is calculated) were interpolated to different grid patterns and spatial resolutions. The gridded temperature data are on a 50-km square grid (based on a Lambert conformal projection), while the snow data are on $\frac{1}{4}^\circ \times \frac{1}{4}^\circ$ and $1^\circ \times 1^\circ$ grids for snow depth and snow density, respectively. The 50-km square (temperature) grid was chosen as the standard for analyses, since in this scheme each grid point represents approximately the same area, allowing areal comparisons by simple counting. Values for snow parameters were obtained for points in the temperature grid by assigning snow density and depth based on their values at the nearest points in their respective grids.

A3.2 Maximum snow depth

The gridded snow depth data of Brown et al. (2003) consist of daily values for the period 1979-1996. Brown (pers. comm.) processed the daily snow depth data of Brown et al. (2002) to generate a smoothed grid of average annual maximum snow depth, using

a 3×3 cell moving window. Data for this grid were assumed to represent the annual maximum snow in the same way that the annual maximum snow depth is defined for the thermal catenae of this thesis, and so were used in the permafrost distribution analyses without processing (“as is”).

A3.3 Mean snow density

Brown et al. (2003) combined the compactive viscosity model (Kojima 1965), the snow cover classification scheme of Sturm et al (1997), and a simple snow melt model to estimate snow depth and density from precipitation data. The model predicted snow to be too dense in taiga environments, suggesting the neglect of the process of depth hoar development in the compactive viscosity model. Brown (pers. comm.) suggested that the gridded normal snow density maps of Canada (MSC 2000) used in Brown et al. (2001) to estimate snow water equivalent from snow depth data would yield more accurate estimates of snow density distribution. From MSC (2000):

“A database of gridded snow density normals was created from the snow course data. Grids were created for the 1st and 15th of the month between November 15th and May 15th inclusive. There was insufficient data to create grids before November 1st or after May 15th. All measurements were used which were within one week of the date of calculation. Snow densities were averaged over the entire period of record for all stations with a least five years of measurements for the calculation date. Average snow densities were then interpolated to a 200-km grid using the Shephard interpolation routine. Although snow course stations do not fully represent the actual topographical range of the Canadian terrain, snow density is a smooth variable and can be interpolated across topography more successfully than snow depth or SWE.”

The gridded snow density data in MSC (2000) are mid-month and end-of-month mean snow density values on a $1^\circ \times 1^\circ$ grid. Annual average densities were estimated as the average of all values between November 15th and May 15th.

A3.4 Equivalent snow depth

The snow depth and density data illustrated in Figures 8.1a and b were transformed into equivalent snow depth, (that is, having the same resistance as snow at a density of 250 kg m^{-3} (the density used in catena analyses). Using the thermal conductivity-density relationship used in TONE ($k_s = 0.29 \rho^2 + 0.000029$), equivalent snow depth was calculated as:

$$R = \frac{Z_{eq}}{k_{250}} = \frac{Z_s}{k_{den}}$$

$$\therefore$$

$$Z_{eq} = Z_s \frac{k_{250}}{k_{den}} \quad (\text{III.1})$$

where

R = snow cover resistance

Z_{eq} = equivalent snow depth

k_{250} = thermal conductivity of snow at density 250 kg m^{-3} ($0.21025 \text{ Wm}^{-1}\text{K}^{-1}$)

Z_s = “actual” snow depth

k_{den} = thermal conductivity of snow at “actual” density

A3.5 Climatology of I_{FA} and I_{TA}

To generate I_{FA} and I_{TA} values for use in the permafrost boundary model developed in Chapter 5, the relationship between mean monthly temperature and monthly freezing and thawing temperatures for the period 1971-2000 was examined. For any month, the relationship between the mean monthly temperature and the monthly freezing and thawing indices is:

$$\overline{T}_M = \frac{(I_{TA_M} - I_{FA_M})}{d_M} \quad (\text{III.2})$$

where

$\overline{T}_M$ = mean monthly temperature

I_{FA_M} = monthly air freezing index

I_{TA_M} = monthly air thawing index

d_M = days in month

Figure III.1 shows the relationship between $\overline{T}_M$ and the monthly freezing and thawing indices (all months) for 173 northern Canadian climate stations. To account for differences in the number of days in the month, I_{TA_M} and I_{FA_M} were divided by d_M , thereby normalizing to equivalent daily values (i.e. I_{TA_M}/d_M and I_{FA_M}/d_M). In mid-summer I_{TA_M}/d_M will be approximately equal to $\overline{T}_M$ because I_{FA_M} will usually be close to 0. A similar argument will hold for mid-winter conditions, where I_{FA_M} will be approximately equal to $-(\overline{T}_M) \times d_M$. Where the mean monthly temperature is close to 0°C, however, the combination of non-zero monthly freezing and thawing indices must be accounted for.

Differences between monthly mean temperature and the normalized indices (that is, $(I_{A_M}/d_M) - \overline{T}_M$) are shown in Figure III.2 as a function of $\overline{T}_M$. Differences are small below about -10° and above about 10°, with the greatest differences near $\overline{T}_M = 0^\circ\text{C}$.

Figure III.1 Relationship between monthly mean temperature and monthly freezing and thawing indices (all months). Climate normal data for 173 Canadian climate stations.

Figure III.2 a) Equivalent daily I_{TA} (Monthly I_{TA} /days in month) versus monthly mean temperature (all months). Climate normal data for 173 Canadian climate stations.
b) Equivalent daily I_{TA} (Monthly I_{TA} /days in month) versus monthly mean temperature (all months). Climate normal data for 173 Canadian climate stations.

Non-linear regression of the relationship between the $\overline{T_M}$ and the normalised

indices was modelled using exponential functions of the form:

$$I_{TA_M} / d_M - \overline{T_M} = Ae^{-B\overline{T_M}} \quad , \overline{T_M} > 0 \quad (\text{III.3})$$

$$I_{TA_M} / d_M = Ae^{B\overline{T_M}} \quad , \overline{T_M} < 0 \quad (\text{III.4})$$

$$I_{FA_M} / d_M + \overline{T_M} = Ae^{B\overline{T_M}} \quad , \overline{T_M} < 0 \quad (\text{III.5})$$

$$I_{FA_M} / d_M = Ae^{-B\overline{T_M}} \quad , \overline{T_M} > 0 \quad (\text{III.6})$$

Fitting the data from Figure III.1 to equations (III.3) to (III.6) gave the values for parameters A and B listed in Table III.1. Equations (III.3) to (III.6) were then used to estimate annual I_{FA} and I_{TA} values for the Canadian temperature grid (Figure 8.5).

Table III.1 Parameters A and B in equations (III.3) to (III.6), as determined by non-linear least squares. All relationships were significant at the 1% level.

Equation	r^2	Parameter	Value	Standard Error
(III.3) $I_T > 0$	0.833	A	2.051625	0.035786
		B	0.390012	0.009049
(III.4) $I_T < 0$	0.862	A	1.773532	0.025478
		B	0.229599	0.004608
(III.5) $I_F < 0$	0.779	A	1.742017	0.030019
		B	0.188226	0.004469
(III.6) $I_F > 0$	0.850	A	2.030734	0.037796
		B	0.391649	0.009694