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ESSAYS ON THE DESIGN OF MONETARY POLICY WITH INCOMPLETE INFORMATION

by

Robert J. Tetlow, B.A. M.A.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Doctor of Philosophy
Department of Economics

Carleton University
Ottawa, Ontario

December 10, 1999

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The undersigned hereby recommend to
the Faculty of Graduate Studies and Research
acceptance of the thesis,

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with Incomplete Information"**

submitted by

Robert Tetlow, B.A., M.A.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Economics

Thesis Supervisor

External Examiner

Carleton University
December 1999

Abstract

This thesis examines the implications of information limitations for the conduct and design of monetary policy. In each of the thesis's three chapters other than the introductory essay in chapter 1, one aspect of the assumption of full symmetric information is relaxed.

Chapter 2 drops the assumption that private agents know the policy rule and requires them instead to learn the rule that is in place. A small-scale model is estimated and subjected to stochastic simulations, with agents following a strategy of (discounted) least-squares learning. We find that the costs of learning a new rule can, under some circumstances, be substantial. A 'conservative' policymaker must incur substantial costs when he changes the rule in use, but is nearly always willing to bear the costs of shifting to a (constrained) optimal rule. A 'liberal' policymaker, on the other hand, may actually benefit from agents' prior belief that a conservative rule is in place.

Chapter 3 drops the assumption that the target of monetary policy is known with certainty by private agents. Instead, the target rate of inflation fluctuates randomly, but within bounds, with the drift in the target representing the predisposition of monetary authorities not to respond to shocks until the case for doing so is compelling. We find that reducing the randomness in inflation targeting does improve economic performance, but only within limits: it does not pay to eliminate random movements in the inflation target.

Chapter 4 estimates a small forward-looking model using the Kalman filter with maximum likelihood techniques to extract a measure of potential output and its uncertainty as hyperparameters of the system. With these estimates, the implications of mis-measurement of potential output for the parameterization of a simple optimal policy rule is investigated. We find substantial uncertainty in the measurement of the gap. The implication for the parameterization of simple rules is policy attenuation: that is, a smaller weight on the output gap in the policy rule than under certainty equivalence.

Dedication

To my girls: Christy, Megan and Faith.
And to my parents: Wishing you were here with me.

Acknowledgements

I would like to thank the members of my committee for their guidance and advice, and especially Steve Ferris without whose support this never would have gotten finished.

Essays on the Design of Monetary Policy with Incomplete Information

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Chapter 1 : Monetary Policy Design with Incomplete Information

An Introductory Essay

Summary

The design of monetary policy is advised by two literatures. The first is an older and somewhat more technical literature that centers on control theory. This approach has been developed by such well-known contributors as Gregory Chow and is featured in a great many textbooks, with treatments at both the abstract level and applied to monetary policy. The second, and mostly newer literature is more practical and intuitive. It focuses primarily on “simple rules”. It is often the case that these simple rules can be rewritten in terms of control-theoretic concepts, in which case their suboptimality--from a control theoretic point of view--is easily shown. The names associated with the simple rules includes some of the leading contributors to the literature today: John Taylor, Bennett McCallum, Michael Woodford, Greg Mankiw, Robert Hall, Laurence Ball and others. The striking thing about these two literatures is how they co-exist with hardly any reference to one another. How can it be that manifestly suboptimal rules are championed by leading macro-economists in near isolation of optimal control theory? This chapter introduces the theme of this thesis: that the impetus for the simple rules literature is the appreciation of the limitations of control theory in the context of limited information. We explore the historical antecedents of this view, and its effect on the development of economic thought, particularly within central banks.

1. Introduction

Consider the two interest-rate reaction functions for monetary policy shown immediately below, where R is the nominal federal funds rate, rr^* is the equilibrium real interest rate, y is the output gap, and $\tilde{\pi}$ is a four-quarter moving average of inflation:

$$R_t = (1 - 0.71)(rr^* + \tilde{\pi}_t^*) + 0.71R_{t-1} + 0.11y_t + 0.19(\tilde{\pi}_t - \tilde{\pi}_t^*) \quad (1)$$

$$R_t = (1 - 0.34)(rr^* + \tilde{\pi}_t^*) + 0.34R_{t-1} + 10.61y_t + 4.88(\tilde{\pi}_t - \tilde{\pi}_t^*) \quad (2)$$

The two equations are quite different in terms of their parameterization.¹ Equation (1) features relatively small impact coefficients and substantial policy persistence as measured by the large coefficient on the lagged funds rate. This would imply an authority that reacts to output gaps and inflation with small changes in the funds rate that build up slowly over time. Equation (2) on the other hand, features large impact coefficients and less persistence. Policy governed by equation (2) reacts strongly and rapidly to output gaps and inflation--and with a larger long-run response.

Equation (1) is an estimated reaction function, fitted on data from 1980Q1 to 1998Q4,² a period over which it would be reasonable to assume that there has been a single regime in place.³ The precise specification of the rule is a simple extension of the much-discussed Taylor (1993) rule; it contains as arguments the two variables that central bankers would normally concern themselves with, inflation and output, together with a

1. An asterisk superscript designates either a target or steady-state value of the variable to which it is attached, as applicable.

2. The estimation period was chosen to commence the quarter following the adoption, in October 1979, of the monetary targeting period and the onset of the Volcker disinflation. The same regression run over a slightly later sample period would only strengthen the point that we make here.

3. In estimating equation (1), the inflation rate was measured as the four-quarter percent change in the chain-weight GDP price index; the funds rate is the quarterly average of daily observations measured on an effective basis; and the output gap comes from the FRB/US model database. Equation (1) is estimated using instrumental variables, corrected for residual correlation of unspecified origin using the Newey-West covariance matrix. All estimated coefficients were highly significant and the standard error of the estimate is 1.17.

partial adjustment parameter.⁴ It is, therefore, a reasonable characterization of how policy *actually* was carried out over this period.

Equation (2), on the other hand, is computed from a control problem using the FRB/US rational expectations econometric model of the U.S. economy.⁵ The FRB/US model was built and is maintained by the staff of the Board of Governors of the Federal Reserve System; this rule, therefore, is a statement regarding how policy *should* be carried out—at least in the context of one model and a particular set of preferences.⁶

The conduct of the Fed has generated some comment in the academic world as well. Citing Lombra and Moran (1980), Meltzer (1991, p.34) noted that the Fed has chosen targets for the funds rate that were inconsistent with staff forecasts: “A principal reason appears to be that the members were reluctant to allow interest rates to rise by as much as required by staff forecasts. They hoped to achieve lower inflation by reducing money growth but were reluctant to allow interest rates rise by relatively large steps.”

Even insiders have acknowledged the tendency to react slowly and with apparent trepidation. For example, the Board’s Director of Monetary Affairs, Donald Kohn remarked that the Fed’s “operating system has a number of problems...Chief among these is the well-known tendency to adjust nominal rates too slowly in the initial stages of a cycle, and then to overstay a policy stance” (Kohn, 1991, p. 101). Along the same lines, Alan Blinder, writing on his experience as the Board of Governors’ vice-chairman, stated “too often decisions on monetary policy are taken ‘one step at a time’ without any clear

4. The unemployment rate less the NAIRU could have been included in place of the output gap without any important differences in the results.

5. The rule is constrained in that the number of arguments is restricted to the same three as in the estimated rule. The precise coefficients were computed conditional on a variance-covariance matrix of stochastic shocks drawn from the model’s historical residuals over the period from 1980Q1 to 1998Q2. A fully optimal rule computed from iterating on matrix Riccati equations would feature about 350 arguments; it too would prescribe aggressive responses to shocks.

6. The rule is optimal in that it minimizes a loss function defined over the weighted average asymptotic variances of output and inflation, each with weights of 0.5, subject to the constraint on the number of parameters appearing in the rule. The same qualitative results obtain from other weightings.

notion of what the next several steps are likely to be” and because of this policymakers “have often been accused of make a particular type of systematic error in the timing of policy changes. Specifically, they overstay their policy stance...I believe this criticism may be correct...” (Blinder 1998, p. 14-16).

Taken together, our two equations and the learned commentary just cited, would suggest that there is a discrepancy between the way policy is prescribed and how it is practiced. Policy as prescribed estimates the mechanisms through which policy works, with due consideration for the lags between policy actions and policy outcomes, and selects a path for policy instruments as the solution to a dynamic optimization problem. As we have seen in equation (2), this results is a policy rule that advocates aggressive actions to observed disequilibria. Policy as practiced has been much slower to respond to shocks, and much less aggressive, even in the long run, than “optimal policy” suggests. It has also had a tendency to follow economic developments rather than lead them.

Why is it that the observed behaviour of the Fed differs in important ways from the solution to a dynamic optimization problem? This thesis represents an attempt to answer, at least in part, this question. It will be argued that the key to the answer lies in the information deficiencies faced by both the Fed and the public. This introductory essay takes a broad look at the implications that imperfect information has on the applicability of control theory in general, and the design of monetary policy in particular. Both incomplete information on the part of the private sector, and of the monetary authority will be touched upon. We shall argue that these information problems argue for simpler monetary policy rules, and in less aggressive policy, than would otherwise be optimal. We begin with a discussion of mismeasurement and its implications for the aggressiveness of policy. We then provide some remarks on the strength of the rational expectations-cum-full-information assumption. Next we touch briefly on certain aspects of credibility in policymaking once

asymmetric information is taken into account. We close this essay by acknowledging a few underemphasized topics, as well as with a few concluding remarks.

2. Uncertainty and Attenuation

In a textbook treatments of control theory, the discussion begins with a macroeconomic model. The model is generally assumed to be known with certainty, or that any and all uncertainty can be subsumed in random error terms. With the known structure of economy, and known objectives, theory tells us the algebraic form of solution to the control problem can be solved by iterating on a matrix Riccati equation, as described, for instance, in Chow (1975) and Kamien and Schwartz (1991). Moreover, if the objective function is quadratic, and the model is linear, the certainty equivalence property holds: the variance-covariance matrix of the forementioned random error terms is irrelevant to the parameterization of the policy rule. Under these circumstances, designing policy would appear to be a straightforward exercise, albeit possibly a technically burdensome one.

Two related problems stand in the way of control theory as the solution to the monetary policymaker's problem: the true model is not generally known; and the monetary authority and the private sector have different (and incomplete) information sets.

As Milton Friedman (1953) has famously argued, monetary policy operates in an environment of uncertainty, with long and variable lags between the application of monetary impulses and their effect. Friedman argued that quite apart from uncertainty as represented by the residuals of the true model, the coefficients governing the monetary authority's model are best described as measured with uncertainty. It is primarily for this reason that Friedman advocated a k -percent rule wherein the rate of growth of the stock of money would be permitted to grow at a constant rate. Along the same line of argument, Brainard (1967) formally demonstrated that uncertainty that could be described as 'additive' would not affect the optimal response parameter in a feedback rule, but 'multiplicative' uncertainty would affect policy in such a way as to reduce the prescribed response of

the policy instrument from what would have been optimal under certainty about coefficients.

To demonstrate Brainard's point, let us write the simplest possible control problem:

$$\pi = \beta u + \varepsilon \quad (3)$$

where π is inflation; u is the control variable, taken to be the unemployment rate; and ε is a random, mean-zero disturbance term. If we think of the monetary authority as controlling output directly, we can imagine u to be the output gap. Suppose the monetary authority wishes to minimize $E(\pi - \pi^*)^2$. If β is deterministic, then the solution to this problem is trivial:

$$u = \pi^* / \beta \quad (4)$$

Equation (4) simply sets the control to exactly what would be necessary for π to equal π^* in one step, as can be easily seen by substituting equation (4) into (3).

In reality, β is not a known parameter and must instead be estimated. Taking β to be a random variable with mean $E\beta = b$ and variance σ^2 , minimization of the same loss function now generates a different result than before:

$$u = \pi^* / (b + \sigma^2 b^{-1}) < \pi^* / \beta \quad (5)$$

Faced with uncertainty in β , the optimizing monetary authority will go only part way toward closing the discrepancy between π and π^* . The degree of policy *attenuation*-- that is, the extent to which policy is more relaxed than in the full-information case--varies positively with parameter uncertainty, σ^2 and negatively with the estimated slope of the Phillips curve, b .⁷ Notice that the purely random, additive error, ε , appears nowhere in either of our two first-order conditions; the additive nature of the uncertainty in ε makes it irrelevant to the problem as it is stated here. This is the well-known certainty equivalence

property.

This prescription of *attenuation* of policy responses is qualitatively the same for both Friedman, a well-known monetarist, and Brainard, a long-time Keynesian. The key difference between the two is that Friedman's response to uncertainty rules out feedback while Brainard's advocates caution but not abstinence in policy feedback. Does model uncertainty justify completely eschewing feedback in monetary policy? Chapter 4 of this thesis demonstrates that in the context of an estimated, forward-looking rational expectations model with uncertain potential output, the answer is no. Uncertainty justifies attenuated policy--that is, a smaller policy response than would be optimal under full information--but not a k -percent rule à la Friedman.

The setting in chapter 4 is a specific one, and one cannot rule out the possibility that in other environments that the answer might be different. However, completely overturning the key result of that chapter would take some doing. The degree of attenuation varies with the degree of uncertainty and so only in the case where uncertainty approaches infinity would mismeasurement be so serious as to rule out feedback. Whatever the ignorance of the economics profession in general, and of central bankers in particular, it is difficult to believe we know that little about how the economy works.⁸

7. One can show that generalizations of this example to allow either multivariate settings or a covariance between the random error, ε and the measurement error in β , need not obtain attenuation as a result owing to covariance terms. Nonetheless, the logic of the simple case appears to have permeated central bank thinking in broad terms, possibly because it is difficult to interpret the negative covariances that would be needed to produce the opposite result.

8. There is, of course, a whole other literature having to do with the choice of policy instrument as a function of uncertainty in the form of the incidence of shocks of various origins. The seminal reference here is Poole (1970). We have nothing to say about this body of knowledge here.

3. Uncertainty and Simplicity

Control theory advocates the use of all information that is contained in the model. There are some clear advantages of doing this. As shown in the simple example above, when the number of controls equals the number of shocks, the certainty equivalence result generally holds. The intuition for this result is reminiscent of econometrics: with as many states as independent shocks, the shocks are identified and can be isolated. For example, in a simple two-equation model, a (positive) demand shock raises output and inflation, while a supply shock raises output but lowers inflation.⁹ A policy rule that includes both states of this two-equation model isolates each shock. In doing so, the variance-covariance matrix of shocks becomes unnecessary for the control problem--just as the variance of ε was not needed above. The authority need not worry about the shocks themselves but can focus instead on the effects of shocks on state variables.

Because all this seems logical, one might think that control theory would have had a substantial influence on policy design. In fact, it has not. In his *New Yorker* article on Alan Blinder's tenure as vice-chairman of the Board of Governors, John Cassidy writes "The thing that surprised Blinder most was the way that decisions were made by the Board. Most of the time, the governors were presented with only one option: the staff recommendation" (p. 41). The staff recommendation is usually a simple alternative to a forecast that is conditional on a fixed federal funds rate. A scenario where the funds rate is raised or lowered by, say, 50 basis points for the forecast period is a good example.¹⁰ A computer search of speeches by Board members reveals some information on the lack of penetration of policy rules in general and control theory in particular on the thinking of the Board. A search for the words 'policy rule' or variants thereof receives twelve hits, all concerning mentions of the Taylor rule.¹¹ In every case, the Governor discussing the Tay-

9. This is the identification strategy of Blanchard (1989).

10. See Meyer (1998b) for a discussion of how FOMC meetings are conducted including how the policy alternatives are presented to the Committee by the Division Director of Monetary Affairs.

lor rule was careful to state that his or her discussion is illustrative of more general points and that the FOMC does not follow or even consider a Taylor rule as such.¹² A search for the words ‘control theory’ or any other words or combinations evocative of that literature receives no hits at all.

Optimal control theory has been criticized on several grounds. The first--that optimization is conditional on a set of parameters which are measured imperfectly--has already been discussed, in part, in the previous section. In this section we focus on two aspects of a second problem: complexity. The arguments to an optimal rule include all the state variables of the model.¹³ In the case of the Board of Governor’s rational expectations model of the U.S. economy, the optimal rule contains more than 350 arguments. The sheer complexity of such rules makes them difficult to interpret, difficult to follow, and difficult to communicate to the public.

The first aspect of the complexity issue concerns the possibility that the model for which the policy optimization is being conducted could be misspecified in a fundamental way; that is, in a way that cannot be summarized simply by random coefficients with the broader aspects of the model’s specification taken as given. One safeguard against these problems that has been proposed is the use of simple rules, where simple is often taken to mean that the rule contains a small subset of arguments that would appear in the globally optimal rule. Indeed, there has been a renaissance of late in interest in the governance of

11. The Taylor (1993) rule is $R_t = rr^* + \bar{\pi}_t + 0.5(\bar{\pi}_t - \pi^*) + 0.5y_t$, a special case of equation (1) above, with R is the nominal federal funds rate, rr^* is the equilibrium real federal funds rate (taken by Taylor to be 2 percent), $\bar{\pi}$ is a four-quarter annualized growth rate of the GDP chain-weight price index, π^* is the target rate of inflation (again taken by Taylor to be 2 percent), and y is the output gap.

12. Governor Meyer is the most likely to mention the Taylor rule. See for example Meyer (1998a). In descending order of likelihood, he is followed by Governor Gramlich and then Governor (now vice-chairman) Ferguson. Former Governor Yellen has also mentioned it. It appears that the Chairman has never given a speech in which he has discussed policy in terms of policy rules of any sort.

13. The statement in the text that the optimal control rule contains the entire state vector as arguments is literally true for linear-quadratic problems with backward-looking models. In the case of a forward-looking model, the truly optimal rule contains more than the entire state vector. On this, see Levine and Currie (1987).

monetary policy using rules. This has come in part because of the academic contributions of Hall and Mankiw (1994) and McCallum (1987) among others, advocating nominal income targeting rules;¹⁴ and of Taylor (1993, 1994), and Henderson and McKibbin (1993), *inter alia*, promoting simple two-parameter interest rate reaction functions. Interest has also been catalyzed by the adoption by a number of countries of explicit inflation targets. New Zealand (in 1990), Canada (1991), the United Kingdom (1992), Sweden (1993) and Finland (1993) have all announced explicit inflation targeting regimes. Other countries have proceeded more tentatively in the same direction.¹⁵ By focusing the parameterization of a policy rule on those factors that are widely believed to be critical to the determination of target variables for a wide range of models, the resulting rule should be more robust than an optimal rule.¹⁶

The second aspect of the complexity issue centers on learning and the existence of rational expectations equilibria. Loosely speaking, the key to optimizing policy in the context of a modern macroeconomic model is for the authority to get expectations working in its favour: If agents believe that the authority will, in the medium to long run, contain inflationary pressures, then private agents' expectations will tend not to overreact to adverse news about shocks. A strongly held view to this effect will pin down inflation expectations leaving the authority some latitude to pursue other goals in the short run, instead of expending energies calming inflation scares.¹⁷

14. Nominal income targeting, in particular, has been advocated as a device that gets around problems associated with the instability of Phillips curves or monetary velocity by combining a real target and a nominal one into a single variable.

15. See Bermanke *et al.* (1999) for a book-length survey of inflation-targeting countries. Broader issues of design and governance under inflation targeting and some international evidence are discussed in Siklos (1997).

16. The research to date has shown that simple rules perform in many cases nearly as well as do more heavily parameterized rules, and require less information. Chapter 2 makes this point, as does Williams (1999).

17. The terminology "inflation scares" comes from Marvin Goodfriend. In Goodfriend (1993) the history of long-term interest rates overresponding to seemingly benign exogenous shocks is documented as evidence that the private sector had little confidence in the Federal Reserve's willingness and/or ability to maintain a fixed inflation target.

Rational expectations equilibria are sometimes justified as the economic structure upon which the economy would eventually settle down once a given policy has been in place for a long enough period of time that agents could learn the policy. However the complexity of policy rules of the sort described above for FRB/US could imperil the attainment of a rational expectations equilibrium.¹⁸

With regard to this issue of information, it is interesting to contrast what central bankers do with what academics typically advocate should be done with the same information. The dominant view among academic economists of nearly all stripes is that all or nearly all information about policy deliberations should be made public and that the Fed should come as close to committing to a rule as the technology will permit.¹⁹ The policy decisions of the FOMC, however, are unabashedly discretionary with a view that no rule, no matter how complex or how optimal, can do justice to the myriad of issues that the Committee must contemplate. Moreover, they argue, disclosure of information undermines the flexibility upon which the discretionary approach is based.²⁰

Chapter 2 of this thesis is on one aspect of the asymmetry and incompleteness on information. There, the issue of how one should choose a rule when agents must learn what the rule is over time is discussed. A case is made for simple rules as being not a lot worse than globally optimal rules in steady state, and easier to learn, at least under some conditions, along the transition path to the steady state. More interestingly perhaps, a sec-

18. The relevant economic literature suggests that least squares-learning rules that converge at all generally converge on a rational expectations equilibria, of which there may be several. Of course there are no guarantees that expectations converge. Nor is there any assurance that agents use least squares to learn. Among the more useful references on the subject are Blume and Easley (1982), Bray (1982), Marcet and Sargent (1989), and Evans *et al.* (1996). A good and readable survey is Evans and Honkapohja (1995).

19. The vast majority of academic economists to who explicitly discuss the rules versus discretion debate advocate rules, or at least discretion that is constrained by rules; see Walsh (1998) for a detailed discussion of the issue including devices on how to optimally bind central bankers to limit their discretion.

20. Cassidy (1996) reports that Blinder advocated a much higher degree of disclosure of information regarding Fed deliberations and forecasts than was regarded as normal and prudent by the rest of the FOMC. The author quotes Blinder at p. 46: "It is fair to say that virtually everybody on the committee took the opposite point of view, with very few exceptions. I made my case and I lost."

tion of that chapter notes that the monetary authority can gain from private agents' misperception that the regime in place is a hawkish one. This observation goes some way outlining the central banker's case for flexibility in policy making.²¹ It also explains why it is that while central bankers work hard to communicate objectives to the public, they nearly always emphasize the hawkish end of the agenda.²²

4. Credibility

An elusive and not always well defined concept in policy design is establishment and maintenance of credibility. One way of defining credibility is as Alan Blinder's dictionary does (1998, p. 64): "the ability to have one's words accepted as factual or one's professed motives accepted as true ones."²³ Let us take a minute to examine what this might mean. Consider the following forward-looking Phillips curve:²⁴

$$\pi_t = \lambda\pi_{t-1} + (1-\lambda)E_t\pi_{t+1} + \alpha y_t \quad (6)$$

where π is inflation and y is the output gap, and $\lambda < 1$ is an adjustment cost parameter. Equation (6) can be solved forward to arrive at:

$$\pi_t = \lambda\pi_{t-1} + \sum_{i=0}^{\tau-1} E_t(1-\lambda)^i [\lambda\pi_{t+i} + \alpha y_{t+i}] + E_t(1-\lambda)^\tau \pi_{t+\tau} \quad (7)$$

21. There is, of course, an academic literature on policy games that in some instances produces secrecy as a privately optimal--and sometimes a socially beneficial--outcome of policy. See, for example, Cuikerman and Meltzer (1986), Vickers (1986), Stein (189) and Faust and Svensson (1998).

22. In August of 1994, at the annual conference sponsored by the Federal Reserve Bank of Kansas City at Jackson Hole, Wyoming, then Fed Vice-Chairman Alan Blinder gave the closing speech where he said, "in my view, central banks...do indeed have a role in reducing unemployment as well as reducing inflation" [Cassidy (1996, p. 43)] Nearly all of the audience found Blinder's remarks to be unremarkable. At least some journalists thought otherwise, reporting that Blinder's remarks showed that he was "publicly challenging the views of his boss [Chairman Greenspan]". A bit later, Newsweek columnist Robert Samuelson wrote "Blinder is 'soft' on inflation" and "lacks the moral or intellectual courage to lead the Fed." [Cassidy, p. 44.]. Blinder made no further remarks of this nature and left the Board of Governors after less than two years on the job.

23. As Blinder (1998) emphasizes, this is a bit looser definition of credibility than is used in the academic literature which usually about time inconsistency. We shall return to this issue in the next section.

24. This Phillips curve is deterministic but there is no loss of generality and some simplification of exposition from assuming away stochastic terms.

Equation (7) shows that current inflation is a function of a lag of inflation reflecting the pinned down portion of inherited inflation, $\lambda\pi_{t-1}$, and the geometric sum of current and expected future inflation, output gaps and shocks through the future, among other things. What, however, ultimately determines where these expectations go? One part of the answer to this is contained in expected future output gaps, $E_t y_{t+i}$. However it is the other term that is of interest here: the terminal condition, $(1-\lambda)^T \pi_{t+T}$. This is the point to which agents ultimately believe inflation will go, and provided that the economy is controllable, it can be interpreted as agents' perception of the long-run target rate of inflation. If this object is perceived by agents as being fixed over time, regardless of the shocks to which the economy is subjected, then this feature will have a stabilizing effect on inflation over time. That is, the central bank will enjoy the benefits of credibility.

The benefits of credibility are that to the extent that the terminal condition is pinning down expected future inflation, expected future output gaps need not play as large a role; there will be more latitude for policy to perform a role in stabilizing output and employment. Chapter 3 examines aspects of this point in some detail in the context of a large-scale rational expectations model of the U.S. economy, the FRB/US model. The motivation is the forementioned observation that the Federal Reserve is remarkably reluctant to adjust its instrument in the light of shocks borne by the economy; a fact that is inconsistent with most reasonable specifications of policy rules based on optimization criteria. Noting that reluctance to adjust the funds rate in response to inflationary shocks is tantamount to allowing drift in the target rate of inflation, the implications of such drift are investigated. Specifically, the long-run target of monetary policy is allowed to be governed by a bounded random walk and the implications of varying the boundaries are studied. We find that just as suggested by the heuristic argument offered above, some tightening of drift in policy targets--and hence drift in the range of the terminal condition imagined by agents--has the effect of allowing policy to improve its performance in terms

of both output and inflation variability. But one need not go all the way to pinning down the target completely to enjoy the benefits of higher credibility.

5. Neglected Topics

This thesis covers a fair range of territory, mostly having to do with how the deficiencies in control theory--at least as rendered in textbooks--has left it incapable of handling policy design problems for models and economies of the real world. There are, however, issues that get less attention here than they probably deserve. We acknowledge a few of them here. Most of these issues relate to the drawbacks of models themselves rather than technology for controlling models.

The zenith of large macroeconometric models was probably the late 1960s, just prior to the realization that the short-run Phillips curve had shifted. The stagflation that arose from that period gave rise, a bit later, to three damning critiques of macromodels in general and large-scale models in particular. The first was the Lucas (1976) critique. Robert Lucas showed that because the reduced-form parameters of "structural" macroeconomic models implicitly contained expectations parameters within them, the empirical breakdown of Phillips curves that was being observed at the that time should not have been a surprise. Any serious policy change would alter the coefficients of the model upon which the proposed policy change had been evaluated and would therefore invalidate the policy analysis that had promoted the change. How could one consider alternative policies, whether using control theory or any other method, if the new rule would have produced different data and different model coefficients than the old rule did?

A second critique, also directed at the deficiencies of macromodels of the 1970s, was leveled by Christopher Sims. Sims (1980) argued that such models were replete with "implausible identifying restrictions" selected for reasons of econometric convenience but having nothing to do with the theory that was supposedly underlying the model. These problems, argued Sims, made such models no more useful than any time-series forecast-

ing model. Accordingly, conducting policy analysis on them was a dangerous undertaking.

The responses of macroeconometric modelbuilders to the criticisms of Lucas and Sims were many and varied. Some followed Sims toward the atheoretical macroeconomics of vector autoregressions and related techniques.²⁵ Others have elected to work with calibrated models derived from microfoundations, most notably real business cycle models.²⁶ Still others have attempted to address the Lucas critique head on by modeling expectations formation directly, usually by incorporating rational expectations. The models used in this thesis are in this latter camp: rational expectations models with microfoundations of a sort. Provided that one accepts that the model in question is an adequate structural representation of the economy, Taylor (1979), Chow (1981) and Cohen and Michel (1988), among others, show how control theory, once modified to allow for rational expectations, can still be used to uncover optimal policy rules.²⁷ We rely on this received wisdom in this thesis.

Out of the Lucas critique came the third fundamental criticism of control methods for policy evaluation, the time-inconsistency critique of Kydland and Prescott (1977). Kydland and Prescott argued that if the central bank is an optimizing agent, as assumed by control theorists, then it will be unable to commit to a particular rule over extended periods of time. Instead, the central bank will find it optimal to change its policy from period to period according to whatever conditions it faces at the time. At a conceptual level, the time inconsistency critique is more damaging to the efficacy of control theory than either of the other two critiques, since it is valid even after having successfully accepted the chal-

25. Sims (1980) and Bernanke (1986) are prominent examples of a huge literature on VARs.

26. Kydland and Prescott (1982) is the seminal reference. Recent contributions to this literature have incorporated sticky prices which permits nominal shocks. In recognition of this turn, these models are often called stochastic dynamic general equilibrium models.

27. Of course, one could argue that the models' parameters are not structurally invariant to policy shifts--that is, that the parameters are not truly "deep"--but we know of no model that passes this test and is empirically relevant at the same time.

lence of modeling rational expectations. The literature has its critics however. Among these are McCallum (1995) and especially Blinder (1998) who spends an entire chapter criticizing the abstraction and irrelevance of the time consistency as applied to monetary policy.

We cannot do justice to the debates within the time inconsistency literature here. Instead, we simply observe that most of the popular alternatives to control theoretic solutions to the monetary design problem--simple Taylor rules, nominal income targeting and the like--are also subject to the time inconsistency problem. In the chapters that follow this introduction, we appeal to the same devices to get around the time inconsistency problem that are commonly used elsewhere: namely, assuming the existence of an unmodeled 'commitment technology,' or assuming that the monetary authority does not discount the future.²⁸ For our purposes, these arguments suffice, but we do not claim that they are entirely satisfying.

6. Concluding Remarks

This essay has introduced the general topic of the design of monetary policy with incomplete information. In each of the following three essays chapters we address one aspect of the problem, dropping the assumption of full information in a limited way and asking what implications this adjustment might have. In one instance, it is the information that the private sector has that is adulterated. In another, the public knows the model and the monetary authority has the job of imperfectly estimating a certain variable that is key to the understanding of the conduct of monetary policy. A third essay introduces random-

28. The papers in the simple rules literature either ignore the issue altogether, or explicitly assume some commitment technology that prevents the monetary authority from reneging on its rule. See chapter 8 of Walsh (1998) for a discussion of how commitment technologies can be designed. The assumption of no discounting is more common. See, e.g., Taylor (1979), Chow (1981), Levin *et al.* (1999) and Williams (1999). To see how no discounting gets around time inconsistency recall that the problem exists because initial conditions favour a course of action that is contrary to the pre-announced rule. If, however, there is no discounting, then initial conditions contribute a vanishingly small amount to the sum of current and expected future welfare and therefore present no temptation to monetary authorities to renege on their rules.

ness in a more-or-less symmetric way and examines its implications for policy, as well as the effects of marginal reductions in that randomness.

For policymakers, the broad messages of this thesis--keep it simple, be careful not to overreact to mismeasured concepts, build up your credibility so that you can exploit it (a bit) later--accord well with what many researchers say the Federal Reserve already does. For example, Rudebusch (1999) suggests that the oft-described reluctance of the Fed to react to respond strongly to economic events is consistent with a more-or-less optimal response to uncertainty in model parameters and data.

There are, however, differing interpretations of the history of monetary policy in the United States and what is to be learned from that history. For example, Clarida, Gali and Gertler (1998) argue that monetary policy in the late 1960s and the 1970s was so weak as to result in sunspot equilibria: beliefs on the part of private agents that are self-fulfilling so that policy and outcomes fail to be determined by fundamentals. The acceleration in inflation that began in the late 1960s is thus laid at the feet of weak policy. By contrast, Orphanides (1999) argues that the conduct of policy over the same period is well described by policy that was fundamentally sound but naive in that the Fed paid no attention to the problems of measuring potential output. And Sargent (1999) argues that the inflation of the 1970s is consistent with a Fed that was optimizing policy--but conditional on the erroneous belief that there existed a long-run trade-off between inflation and unemployment. These disparate conclusions from the same historical experience serve as a warning against settling too easily on pat answers to these questions.

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Chapter 2 : Monetary Policy Rules and Learning

Summary

Economic theory tells us that advantages accrue to a central bank that commits to a policy rule. However, the rules that are discussed in the literature are not rules that are optimal in the sense of having been computed from an optimal control problem. Instead, the rules that are widely discussed--notably the Taylor rule--are remarkable for their simplicity. One reason for the apparent preference for simple *ad hoc* rules might be the assumption of full information that is generally maintained for the computation of an optimal rule. This tends to make optimal control rules less robust to model specification errors than are simple *ad hoc* rules. In this paper, we drop the full information assumption and investigate the choice of policy rules when private agents must learn the rule that is used. To do this, we conduct stochastic simulations on a small, estimated forward-looking model, with agents following a strategy of least-squares learning. We find that the costs of learning a new rule can, under some circumstances, be substantial. These circumstances vary considerably with the preferences of the monetary authority and with the rule initially in place. A 'conservative' policymaker must incur substantial costs when he changes the rule in use, but is nearly always willing to bear the costs of shifting to a (constrained) optimal rule. A 'liberal' policymaker, on the other hand, may actually benefit from agents' prior belief that a conservative rule is in place.

1. Introduction

In recent years, there has been a renewed interest in the governance of monetary policy through the use of rules. This has come in part because of academic contributions including those of Hall and Mankiw (1994), McCallum (1987), Taylor (1993, 1994), and Henderson and McKibbin (1993). It has also arisen because of adoption in a number of countries of explicit inflation targets. New Zealand (1990), Canada (1991), the United Kingdom (1992), Sweden (1993) and Finland (1993) have all announced such regimes.

The academic papers noted above all focus on simple *ad hoc* rules. Typically, very simple specifications are written down and parameterized either with regard to the historical experience [Taylor (1993)], or through simulation experiments [Henderson and McKibbin (1993), McCallum (1987)]. Both the simplicity of these rules, and the evaluation criteria used to judge them stand in stark contrast to the earlier literature on optimal control. Optimal control theory wrings all the information possible out of the economic model, the stochastic shocks borne by the economy, and policymakers' preferences. This may be a mixed blessing.

Optimal control theory as a tool for selecting monetary policy rules has been criticized on three related grounds. First, the optimization is conditional on a large set of parameters, some of which are measured imperfectly and the knowledge of which is not shared by all agents. Some features of the model are known to change over time, often in imprecise ways. The most notable example of this is policymakers' preferences which can change either 'exogenously' through the appointment process, or 'endogenously' through the accumulation of experience.¹ Second, optimal control rules are invariably complex. The arguments to an optimal rule include all the state variables of the model. In working models used by central banks, state variables can number in the hundreds. The sheer com-

1. Levin *et al.* (1999) examines rules that are optimal in each of three models for their performance in the other two as a check on robustness of candidate rules.

plexity of such rules makes them difficult to follow, difficult to communicate to the public, and difficult to monitor. Third, in forward-looking models, it can be difficult to commit to a rule of any sort. Time inconsistency problems often arise. Complex rules are arguably more difficult to commit to, if for no other reason other than the benefits of commitment cannot be reaped if agents cannot distinguish commitment to a complex rule and mere discretion.

Simple rules are claimed to avoid most of these problems by enhancing accountability, and hence the returns to precommitment, and by avoiding rules that are optimal only in idiosyncratic circumstances. At the same time, simple rules still allow feedback from state variables over time, thereby avoiding the straightjacket of rules such as Friedman's k -percent money growth rule. The costs of this simplicity include the foregone improvement in performance that a richer policy can add.

This paper examines the friction between simplicity and optimality in the design of monetary policy rules. With complete information, rational expectations, and full optimization, the correct answer to the question of the best rule is obvious: optimal control is optimal. However, the question arises of how expectations come to be rational in the first place. Rational expectations equilibria are sometimes justified as the economic structure upon which the economy would eventually settle down once a given policy has been in place for a long time.² But if learning is slow, a policymaker must consider not only the relative merits of the old and prospective new policies in steady state, but also the costs along the transition path to the new rule. Conceivably, these costs could be high enough to

2. Lucas (1980) argues that rational expectations equilibrium should be thought of as the outcome from having persisted with a constant policy for an extended period of time. Only then can agents be presumed to have acquired sufficient knowledge to support model consistency in expectations. Much of the subsequent literature on learning has focussed on necessary and sufficient conditions for convergence of learning mechanism on rational expectations equilibria. This literature is still developing but currently suggests that least squares-learning rules that converge at all generally converge on a rational expectations equilibria, of which there may be several. Among the more useful references on the subject are Blume and Easley (1982), Bray (1982) and Marcet and Sargent (1989). A good and readable survey is Evans and Honkapohja (1995).

induce the authority to select a different new rule, or even to retain the old one despite misgivings as to its steady-state performance.

With this in mind, we introduce two elements of realism into the exercise that can alter the basic result. First, we consider optimal rules subject to a restriction on the number of parameters that can enter the policy rule--a simplicity restriction. We measure the cost of this restriction by examining whether transitions from 2-parameter policy rules to 3-parameter rules are any more difficult than transitions to other 2-parameter rules. Second we restrict the information available to private agents, requiring them to learn the policy rule that is in force. In relaxing the purest form of the rational expectations assumptions, we follow the literature on learning in macroeconomics associated with Taylor (1975) and Cripps (1991) and advanced by Sargent (1993). We are then in a position to ask the question: if the Fed were to publicly precommit to a rule in the presence of a skeptical public, what form should the rule take? If the Fed knew the true structure of the economy, but the public placed no weight on public declarations, would the rule that is optimal under full information still be optimal when private agents would have to learn the rule? Or would a simpler rule be easier to learn and hence better in practice?

In typical examinations of the design of monetary policy rules, such as Taylor (1979), Levin *et al.* (1999), Rudebusch and Svensson (1999) and Williams (1999), the transition costs of migrating to the proposed new rule are ignored. Instead, the focus is entirely on alternative steady states. Similarly, the simplicity of rules is exogenously assumed. In the nominal income targeting literature, for example, the merits of nominal income rules are compared with other equally simple and parsimonious rules but never to

more complex alternatives. The virtues of simplicity are assumed at the outset.³ The aim here is to remedy this oversight.

To examine these questions, we estimate a small forward-looking macro model with Keynesian features and model the process by which agents learn the features of the policy rule in use. The model is a form of a contracting model, in the spirit of Taylor (1980) and Calvo (1983), and is broadly similar to that of Fuhrer and Moore (1995b). We construct the state-space representation of this model and conduct stochastic simulations of a change in the policy rule, with agents learning the structural parameters of the linear monetary policy rule using two forms of least-squares learning.

Solving for the state-space representation of a forward-looking model represents a numerical hurdle. Moreover, the learning studied here implies that the model, as perceived by private agents, changes every period, presenting an additional layer of complication. An additional contribution of this paper, therefore, is the exploitation and adaptation of efficient methods for rapidly solving and manipulating linear rational expectations models.

The rest of this paper proceeds as follows. In Section 2 we discuss the simple, macroeconomic model. The third section outlines our methodological approach. Section 4 provides our results. The fifth and final section offers some concluding remarks.

2. The Model

We seek a model that is simple, estimable and realistic from the point of view of a monetary authority. Towards this objective, we construct a simple New Keynesian model along the lines of Fuhrer and Moore (1995b). The key to this model, as in any Keynesian

3. This is partly because nominal income rules were being compared with other simple rules such as monetary targets or exchange-rate targets. Hall and Mankiw (1994) justify simplicity as a virtue arguing that simple rules generally, and nominal income targeting specifically, have a better chance of being adopted and a better chance of being continually enforced than more complicated rules. Ironically, FOMC members reject such rules because they are too simple. See, e.g., Yellen (1996), especially at pp. 8-9.

model, is the price equation or Phillips curve. Our formulation is similar to the real-wage contracting model of Buiter and Jewitt (1981) and Fuhrer and Moore (1995a). In these models, each cohort of agents negotiates nominal contracts with the goal of fixing real wages relative to other cohorts. Such a formulation ‘slips the derivative’ in the price equation, thereby ruling out the possibility of costless disinflation.⁴ However, instead of the fixed-term contract specification of Fuhrer-Moore, we adopt the stochastic contract duration formulation of Calvo. In doing this, we significantly reduce the state space of the model, thereby accelerating the numerical exercises that follow.

The complete model is as follows:

$$\pi_t = \delta\pi_{t-1} + (1-\delta)c_{t|t-1} \quad (1)$$

$$c_t = (1-\delta)[\pi_t + \gamma y_t] + \delta c_{t+1|t-1} + u_t^\pi \quad (2)$$

$$y = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 r r_{t-1} + u_t^y \quad (3)$$

$$r r_t = r s_t - \pi_{t|t-1} \quad (4)$$

$$r s_t = r r^* + \pi_{t|t-1} + \beta_{rs}[r s_{t-1} - \pi_{t-1}] + \beta_\pi[\pi_{t-1} - \pi^*] + \beta_y y_{t-1} + u_t^{rs} \quad (5)$$

Equations (1) and (2) together comprise a forward-looking Phillips curve, with y representing the output gap, a measure of excess demand, and π and c measuring (final) goods price and contract (or core) inflation respectively.⁵ Goods price inflation is, as the name

4. Roberts (1995) shows that nearly all sticky price models can be boiled down to a specification where prices are predetermined, but inflation is not. Take the sticky-price models of Rotemberg (1982) as an example. That model posits that monopolistically competitive firms face quadratic costs of adjusting prices, and thus solve: $\min_{\{p_t\}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} [(p_\tau - p_\tau^*)^2 + \lambda(p_\tau - p_{\tau-1} - g)^2]$ where g is the trend growth rate of prices, or simply the steady-state inflation rate. It is assumed that changing prices at the rate at which prices are generally expected to change is costless. The Euler equation for this is: $E_t[(p_t - p_t^*) + \lambda[(p_t - p_{t-1} - g) - \beta(p_{t+1} - p_t - g)]] = 0$. For β approximately equal to unity, this can be simplified to $\Delta p_t = E_t \Delta p_{t+1} - \lambda^{-1}(p_t - p_t^*)$. Then if we specify the target price by $p_t^* = \bar{p}_t + \alpha y_t + \varepsilon_t$ where $\bar{p}$ is the current price prevailing at other firms, substitution and assuming symmetry yields $\pi_t \equiv \Delta p_t = E_t \pi_{t+1} + (\alpha/\lambda)y_t + \lambda^{-1}\varepsilon_t$. With no lagged inflation term, inflation here is clearly a non-predetermined variable. By ‘slipping the derivative’ in the price equation, Fuhrer and Moore (1995a) turn Taylor’s specification which is two-sided in the price level into one that is two-sided in the inflation rate, making inflation a (partly) predetermined variable.

suggests, the rate of change of prices for products on shelves. Following Calvo (1983), the expiration of contracts is given by an exponential distribution with hazard rate, $1 - \delta$. Assuming that terminations of contracts are independent of one another, the proportion of contracts negotiated s periods ago that are still in force today is $(1 - \delta)\delta^{t-s}$ and the average life of a contract is $1/(1 - \delta)$. Thus, as shown in equation (1), a proportion δ of goods price inflation is inherited from the previous quarter, with the rest coming from newly negotiated contracts. Contract inflation, on the other hand, is the rate of change of wage contracts including a mark-up. Current demand conditions affect only those contracts that are expiring, that is the proportion $1 - \delta$ shown in equation (2). The remainder is given by the previous period's expectations of inflation in the future multiplied by the probability that contracts in force then are still valid.

Equation (3) is a very simple empirical aggregate demand equation with the output gap being a function of two lags of the gap as well as the lagged *ex ante* real interest rate.⁶ Equation (4) is the (inverted) Fisher equation. Finally, equation (5) is a generic interest rate reaction function or monetary policy rule, written here simply to complete the model.⁷ The policy rule, which is typical of those used in research at the Fed, assumes that the monetary authority manipulates the nominal federal funds rate, r_s , and implicitly deviations of the real rate from its equilibrium level, $rr - rr^*$, with the aim of moving inflation to its target level, π^* , while reducing excess demand to zero. Each of the state variables in the rule carries a weight of β_i , where $i = \{\pi, y, r_s\}$. These weights are related to, but should not be confused with, the weights of the monetary authority's loss function, about which we shall have more to say below.

5. The notation $m_{t|t-1}$ should be read as the expectation of variable m for date t , conditional on information available at time $t-1$, which includes all variables subscripted $t-1$.

6. Empirically, the data strongly favour the lagged *ex ante* real interest rate over the contemporaneous one. This may reflect time to build considerations or habit persistence.

7. See, for example, Williams (1999).

The model is stylized, but does capture the fundamental aspects of models that are useful for the analysis of monetary policy. We have already mentioned the stickiness of inflation in this model. Other integral features of the model include that policy acts on demand and prices with a lag. This rules out monetary policy that can instantaneously offset shocks as they occur. The model also assumes that excess demand affects inflation with a lag, and that disturbances to aggregate demand are persistent. These features imply that in order to be effective, monetary policy must look ahead, setting the federal funds rate today to achieve objectives in the future. However, the stochastic nature of the economy and the timing of expectations formation imply that these plans will not be achieved on a period-by-period basis. Rather, the contingent plan set out by the authority in any one period will have to be updated as new information is revealed regarding the shocks that have been borne by the economy.

Equations (1)-(5), together with a policy rule to be described below, comprise the model it is for our simulation exercises. For estimation purposes, the model is reformulated. However because the precise empirical estimates of the model are not critical to the issues examined here, our discussion of them is brief. Equation (2) can be substituted into (1) to arrive at an equation for goods price inflation as a function of the *lead* of core inflation, as shown in equation (A) in Table 1. The key equations of the model are estimated on U.S. data from 1972Q1 to 1996Q4.⁸

8. Recursive least squares estimates indicated significant parameter instability prior to the early 1970s.

Table 1
Estimates of Basic Contract Model
(1972Q1 - 1996Q4)

description	label	estimate	t-statistic	summary statistics
$\pi = [1 - (1 - \delta)^2]^{-1} (\delta \pi_{t-1} + (1 - \delta)^2 \gamma u_{t-1} + (1 - \delta) \delta c_{t+1 t-1}) + \Gamma' Z \quad (A)$				
Nixon price controls	Z_1	-5.35	(2.20)	$R^2 = 0.97$
change in oil prices	Z_2	0.0019	(0.70)	SEE=1.02
unemployment	$\hat{\gamma}$	-0.23	(1.49)	Constrained linear IV
contract duration	$\hat{\delta}$	0.41	(4.65)	
$y = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 r r_{t-1} \quad (B)$				
first lag of output	$\hat{\phi}_1$	1.25	(13.16)	$R^2 = 0.88$
second lag of output	$\hat{\phi}_2$	-0.36	(3.22)	SEE=1.21
real fed funds rate	$\hat{\phi}_3$	-0.14	(2.29)	OLS
$u = \gamma_0 + \gamma_1 y_t + \gamma_2 T + \gamma_3 (poil_t / p_t) \quad (C)$				
output gap	$\hat{\gamma}_1$	-0.32	(8.51)	$R^2 = 0.80$
time trend	$\hat{\gamma}_2$	0.0095	(1.94)	SEE=0.60
relative price of oil	$\hat{\gamma}_3$	0.86	(5.56)	2SLS

Notes: Equation (A) is estimated with instruments: constant, time trend, lagged unemployment gap, four lags of the change in imported oil prices; two lags of inflation, lagged real interest rate, lagged Nixon wage-price control dummy, and the lagged relative price of imported oil. Standard errors for all three equations were corrected for any residual correlation of unspecified form using the Newey-West (1987) method.

Empirical estimates using unemployment gaps--defined as the deviation of the demographically adjusted unemployment rate less the NAIRU--were superior to those using output gaps. So the estimation was conducted with that measure of economic slack, as shown in equation (A), and the econometric model was supplemented with a simple

Okun's Law relationship linking the unemployment rate to the output gap. This is shown in the table as equation (C).⁹

The median of the Michigan survey of expected future inflation is used as a proxy for $c_{t+1|t-1}$. The survey has some good features as a proxy including that it is an unbiased predictor of future inflation. At the same time, it is not efficient: other variables do help predict movements in future inflation, an observation that is consistent with the theory. The survey also measures consumer price inflation expectations, precisely the rates that would theoretically go into wage bargaining decisions.

Goods price inflation is measured by the quarterly change at annual rates of the GDP price index and can then be thought of as a pseudo-mark-up over these expected future costs. The disadvantage of the survey is that it measures inflation over the next twelve months, which does not match the quarterly frequency of our model. Nonetheless, most of the power of the survey to predict inflation over the next twelve months comes from its ability to predict inflation in the very short term rather than later on, suggesting that this problem is not serious.

Finally, for estimation purposes we embellished the basic formulation with a small number of exogenous supply shock terms, specifically oil prices, a variable to capture the effects of the Nixon wage-and-price controls, and a constant term. These are traditional and uncontroversial inclusions that capture phenomena that arguably pushed wage and price setting agents off their decision rules. Empirically, Roberts (1996) has found oil prices to be important for explaining the inflation in estimation using Michigan survey data. The Nixon dummy appears in the wage equations of the Federal Reserve Board's macroeconometric model of the U.S. economy, FRB/US, as it did in its forerunner, the MPS model.

9. The appropriate algebraic substitutions were then conducted taking the Okun's Law relation as deterministic to return to the specification of equations (1) to (6) with suitably transformed coefficients and standard errors.

The key parameters are the ‘contract duration’ parameter, $\hat{\delta}$, and the excess demand parameter, $\hat{\gamma}$. If this were a *level* contracts model, $\hat{\delta} = 0.41$ solved from equation (A), Table 1, would be a disappointingly low number since it implies a very short average contract length. This might be taking this interpretation too far, however. The reduced-form coefficient on lagged inflation $[1 - (1 - \delta)^2]^{-1} \delta$ is seen to be 0.63. This is substantial inflation stickiness and is close to the estimates of similar models including Chada *et al.* (1992).¹⁰

3. Methodology

The preceding section presented the model we use for simulation purposes and showed the empirical estimates. In this section, we discuss the policy rules that we use below and how they are computed. We also show how agents’ beliefs regarding the policy rule that is in use are updated over time. Finally, we explore the numerical issues involved in computing model-consistent solutions when agents have incorrect beliefs of the policy rule that is place.

3.1 Optimal and simple policy rules

It is useful to express the model, equations (1) to (5), in its first-order (state-space) form. To do this we begin by partitioning the vector of state variables into predetermined variables and ‘jumpers’, and expressing the structural model as follows:¹¹

$$K_1 \begin{bmatrix} z_{t+1} \\ c_{t+1|t} \end{bmatrix} = K_0 \begin{bmatrix} z_t \\ c_t \end{bmatrix} + \begin{bmatrix} u_t^z \\ u_t^\pi \end{bmatrix} \quad (6)$$

where $z_t = \{rs_t, \pi_t, y_t, y_{t-1}\}$ is the vector of predetermined variables, and c_t is contract inflation, our one non-predetermined variable. Constructing the state-space representation for the model for a given policy rule is a matter of finding a matrix, $K = K_1^{-1} K_0$.

10. Chada *et al.* (1992) find a reduced form weight on lagged inflation of 0.55. See their Table 3, row 1, page 411.

11. An able reference for state-space forms of linear rational expectations models and other issues in policy design is Holly and Hughes Hallett (1989)

$$\begin{bmatrix} z_{t+1} \\ c_{t+1|t} \end{bmatrix} = K \begin{bmatrix} z_t \\ c_t \end{bmatrix} + K_1^{-1} \begin{bmatrix} u_t \\ \pi_t \end{bmatrix} \quad (7)$$

Equations (7) are recursive in the state variables so that manipulating the model is simple and computationally easy. This makes computing optimal rules a reasonably straightforward exercise. However, two problems arise in the construction of K .

The first of these problems is a technical one having to do with the fact that K_1 is often singular. We shall return to this later on. The second problem is more interesting from an economic point of view and concerns finding a specific rule with the desired properties. This is an exercise in optimal control. In the forward-looking context, however, the theory is a bit more complex than the standard textbook treatments. The optimal control rule is no longer a function just of the five state variables of the model as is the case with a backward-looking model. Rather, as Levine and Currie (1987) have shown, the optimal control rule is a function of the entire history of the predetermined state variables.¹² Even for simple models such as this one, the rule can rapidly become complex.

Experiments conducted for this chapter show that agents have great difficulty learning the parameters of rules with many conditioning variables. This is particularly so when some variables, such as contract inflation, c , and price inflation π , tend to move closely together. In small samples, agents simply cannot distinguish between the two, and unstable solutions can sometimes arise. Rather than demonstrate this intuitively obvious result, in this paper, we consider instead restrictions on the globally optimal rule, or what

12. This is because the optimal control rule cannot be expressed as a function of the nonpredetermined variables since these 'jump' with the selection and operation of the rule. Rather, the rule must be chosen with regard to the variables that determine the jump. In the rational expectations context, this will be given by the entire history of predetermined state variables of the model. In many cases, this history can be represented by an error-correction mechanism, as Levine (1991) shows, but not always. In any case, even when an error-correction mechanism exists, the basic point remains that the complexity of the rule is significantly increased by the presence of forward-looking behavior.

we call simple optimal rules.¹³ Simple optimal rules are those that minimize a loss function subject to the model of the economy--just as regular optimal control problems do--plus a constraint on the number of arguments in the reaction function.

For our purposes, we can state the monetary authority's problem as:

$$\langle \beta_\pi, \beta_y, \beta_{rs} \rangle \underset{\text{argmin}}{E_0} \sum_{i=0}^{\infty} \rho^i [\psi_\pi (\pi - \pi^*)^2 + \psi_y y^2 + \psi_{\Delta rs} (\Delta rs)^2] \quad (8)$$

subject to the state-space representation of the model as in equations (6), along with the model consistency restrictions: $m_{t+1|t} = m_{t+1}$ for all $m \in \{c, z\}$, and the arguments of equation (5): $rs_t = rr^* + \pi_{t|t-1} + \beta_{rs}[rs_{t-1} - \pi_{t-1}] + \beta_\pi[\pi_{t-1} - \pi^*] + \beta_y y_{t-1} + u_t^{rs}$.¹⁴ The solution to this problem is a vector of policy rule coefficients, $\bar{\beta} = \{\beta_y, \beta_\pi, \beta_{rs}\}$ corresponding to the vector of objective function weights, $\bar{\psi} = \{\psi_y, \psi_\pi, \psi_{\Delta rs}\}$.

Rules that solve the above problem will, by definition, be inferior to the globally optimal rule that could have been derived using optimal control techniques, in the presence of full information. By the same reasoning, they will be superior to even simpler rules such as the Taylor rule, with coefficients that are not chosen according to optimization criteria. In our formulation, the Taylor rule is just our equation (5) with the added restrictions that $\beta_{rs} = 0, \beta_y = \beta_\pi = 0.5$.¹⁵

13. The phrase 'simple rules' is borrowed from Levine (1991) who addresses issues similar to some of the ones considered here. We adopt it and add the word 'optimal' to signify that the parameterization of our rules is not *ad hoc*, but rather is determined from a well specified minimization problem as described below.

14. We also assume the existence of a commitment technology that permits the monetary authority to select and retain the rule that is optimal given the average set of initial conditions. In the absence of commitment, different initial conditions will give rise to different rules, depending on the discount factor. No commitment technology is required if agents do not discount. However, in order to conduct welfare comparisons along the transition path to different rules, we need to discount the future. The strength of our assumption regarding commitment technology will be directly related to the rate of discounting. We use a low discount factor of 0.9875, or about 5 percent per year on a compounded annual basis.

15. We are taking a bit of license here, with our characterization of the Taylor rule, in that inflation in the original specification of the rule appears with a four-quarter moving average of inflation and both inflation and output appear contemporaneously. We take this simplification to reduce the size of the state matrix for computational reasons.

$$rs_t = rr^* + \pi_{t|t-1} + 0.5[\pi_{t-1} - \pi^*] + 0.5y_{t-1} + u_t^{rs} \quad (9)$$

Equations (5) and (9) are the two policy rules that may be in force in our experiments. The coefficients in these rules are what agents are assumed to learn. The simple optimal rules derived from the above procedure are summarized in Table 2 below. We defer a discussion of the precise parameterization of these rules until later.

Table 2
Coefficients of Simple Optimal Rules

preferences		Policy rule coefficients		
$\{\psi_y, \psi_\pi, \psi_{\Delta rs}\}$	no. of parameters	β_y	β_π	β_{rs}
Taylor rule	2	0.5	0.5	n/a
'conservative'	2	0.73	1.70	n/a
{0.1, 0.8, 0.1}	3	0.60	1.35	0.27
'liberal'	2	1.18	0.41	n/a
{0.8, 0.1, 0.8}	3	1.55	0.53	-0.33

Notes: rule parameters are the solutions to the problem of minimizing equation (8) subject to (6) and (5) under model consistent expectations, for the loss function weights shown in the first column of the table.

3.2 Learning

Our generic interest rate reaction function, equation (5), represents a single row of the matrix in equations (7), the values of which evolve over time according to recursive linear learning rules. Let us re-express equation (5), slightly more compactly as:

$$rs_t = \tilde{\beta}_t x_t + \hat{u}_t^{rs} \quad (10)$$

Equation (10) merely stacks the right-hand side arguments to the policy rule into a vector,

$x_t = [y_{t-1}, \pi_{t-1}, rs_{t-1}]'$, while allowing time variation in the *estimated* coefficients, $\hat{\beta}_t = \{\hat{\beta}_y, \hat{\beta}_\pi, \hat{\beta}_{rs}\}$. Let the time series of x_t be X_t ; that is, $X_t = \left\{ x_{t-i} \right\}_0^t$. The vector $\hat{u}_t^{rs}$ also carries a caret overstrike signalling that it, too, is a 'perceived error'; we discuss below how this works. We assume that agents use either least squares or discounted least squares to update their estimates of $\hat{\beta}_t$. Harvey (1993, pp. 98-99) shows that the recursive updating formula in equation (11) is equivalent to continuous repetition of an OLS regression with the addition of one observation:

$$\hat{\beta}_t = \lambda \hat{\beta}_{t-1} + P_{t-1} x_t (rs - x_t' \hat{\beta}_{t-1}) f_t^{-1} \quad (11)$$

where P_t is the 'precision matrix', $P_t = (X_t' X_t)^{-1}$, and $f_t = 1 + x_t' P_{t-1} x_t$ is a standardization factor used to rescale prediction errors, $\hat{u}_{rs,t} = P_{t-1} x_t (rs - x_t' \hat{\beta}_{t-1})$, in accordance with changes in the precision over time. Harvey (1993) shows that the precision matrix evolves according to:

$$P_t = \lambda P_{t-1} - P_{t-1} x_t x_t' P_{t-1} f_t^{-1} \quad (12)$$

The parameter λ is a 'forgetting factor'. The special case of $\lambda = 1$ discussed in Harvey (1993) is standard recursive least squares (RLS). If we assume that agents have 'less memory' we can downweight observations in the distant past relative to the most recent observations by allowing $0 < \lambda < 1$. This means that agents 'forget' the past at a geometric rate of $1 - \lambda$ percent per quarter. This is discounted recursive least squares (DRLS).

The memory of the learning system has a convenient Bayesian interpretation in that had there never been a regime shift in the past, agents would optimally place equal weight on all historical observations as with RSL since all observations would be equally useful for estimating the true rule. Under such circumstances, the Kalman gain in equation (11) goes to zero asymptotically, and $\hat{\beta} \rightarrow \bar{\beta}$. However, if there has been a history of occasional regime shifts, or if agents believe in such a possibility for other, external reasons, then the weights they use to discount previous estimates of the rule parameters can repre-

sent the strength of their prior belief. The higher the perceived incidence of regime shifts, the lower is λ . If λ is taken as an exogenously fixed parameter (as we do throughout this paper) then $\hat{\beta}$ will have a tendency to fluctuate around $\bar{\beta}$, overreacting to surprises owing to the maintained belief that a regime shift has some likelihood of occurring. That agents overreact, in some sense, to “news” means agents with less memory will tend to learn new rules more rapidly, but this rapid learning is not necessarily welfare improving.

Whatever the precise form, using some form of learning here is a useful step forward since it models agents on the same plane as the econometrician, having to infer the law of motion of the economy from the surrounding environment, rather than knowing the law of motion *a priori*.

3.3 Numerical issues

Let us ignore, for the moment, learning and the associated costs of transition from one policy rule to another. The policymaker’s problem is then to choose a policy rule of the form of equation (5), to minimize the loss function, equation (8), subject to the model, equations (6). This task is straightforward, if numerically cumbersome. The problem of learning, however, adds the restriction that expectations are model consistent *from the point of view of private agents*. This presents some challenging computational issues. Under model consistent expectations, solutions for endogenous variables depend on expected future values of other endogenous variables, which are conditional on agents’ (possibly incorrect) beliefs of monetary policy.

One such difficulty is that equations (7) must satisfy the Blanchard-Kahn (1980) conditions for the existence of a unique saddle-point solution to the system. The B-K conditions require that the number of eigenvalues greater than unity be equal to the number of non-predetermined variables in the system. Although there is just one non-predetermined variable in the model, the parameters of the model change with agent’s perceptions of the rule that is being used. Instability or multiple equilibria are possible. There was some

minor incidence of instability in our experiments, always associated with agents perceiving the coefficient on inflation less target inflation turning negative. Instability tended to arise more often when the short memory was employed in discounted recursive least squares learning, but never for full memory and rarely for reasonable amounts of memory in the learning process.¹⁶ To keep the model stable, we restricted the perceived weight on inflation to be positive and included those trials in which this constraint was binding in our computations. Supplementary tests revealed very close to the same results with or without the inclusion of trials that breached the B-K conditions.

A second numerical issue that comes up, alluded to previously, concerns the problem of the possible singularity of the matrix K_1 . Without a nonsingular K_1 matrix, the state-space representation of the model cannot be constructed, meaning that a recursive representation of the forward-looking model would not be possible. The model would then have to be solved using some extended path method such as the Fair-Taylor (1983) algorithm or its stacked-time counterpart, Laffargue (1990). Given that we are interested in stochastic simulations of the model with learning, the numerical cost of the using extended-path solution methods would be orders of magnitude more time consuming than a recursive method.¹⁷ Fortunately, the development in recent years of new methods of constructing state-space representations of linear forward-looking models obviates these complexities. For this paper, we employ the algorithm of Anderson and Moore (1985) and Anderson (1999) which uses a QR decomposition to find a nonsingular equivalent to the matrix K_1 .

Finally, the linearity of the model combined with the model-consistent expectations (from the point of view of private agents) ensures that the imposition of possibly

16. There were never any instances of instability with full memory ($\lambda = 1$), and it was very rare--of the order of two trials in 500, with $\lambda = 0.95$. For discounting of approximately $\lambda = 0.80$ and lower, instability became a significant problem. But discount factors this low imply a half life of memory of only about four quarters, which we would regard as implausibly low.

17. There could also be problems with errors in measurement since the algorithms for extended path solutions tend to be less robust than linear solution techniques.

incorrect terminal conditions is not an issue. So long as the model is stable, the eigenvalues of the system give the terminal condition.

It is useful, at this point, to summarize our quantitative approach. Let us take the case where the monetary authority has been using the Taylor rule for some time and then shifts to a version of the simple optimal rule. The ‘algorithm’ for solving this problem can be summarized as follows:

- (1) Initialize the variance-covariance matrix Ω associated with the residuals $\bar{u}$ at values based on historical estimates; set the initial date counter at $t=0$;
- (2) Compute pre-experiment matrices P_0 , X_0 , $\bar{\beta}_0$ and $\Sigma = u'u$ using the initial policy rule with private agents’ expectations consistent with this rule;
- (3) Substitute the coefficients for the pertinent simple optimal rule for the initial rule;
- (4) For date t , solve the model for its state-space representation and check for satisfaction of the Blanchard-Kahn conditions (if the B-K conditions are not satisfied, impose the stability constraint: $\hat{\beta}_\pi = 0.01$);
- (5) Draw random shocks for the stochastic residuals, Σ , and set the nominal federal funds rate consistent with these shocks and the *true* policy rule, for date t (this implies a Σ perceived by agents that differs from Σ);
- (6) Simulate the model to find endogenous solution values, for t ;
- (7) Taking the solution values to step (6) as given, update agents’ perceptions of the policy parameters;
- (8) Repeat steps (4) through (8) for the next t , $t=\{1,2,\dots,T\}$.
- (9) When $t=T$, stop.

To this point we have laid out the motivation of the paper, discussed the methods by which our monetary authority selects policy rules, and examined the numerical methods that we use. The next section discusses our simulation results.

4. Simulation Results

In this section, we discuss simulation results measuring four things: (1) the relative performance, in stochastic steady state, of the rules shown in Table 2; (2) the choices faced by a central banker with one set of preferences who takes over from a central banker with very different preferences, when private agents must learn any new rule; (3) the prospects for a central banker to speed the rate of learning of the public with exaggerated instrument settings; and (4) the performance of the economy in transitions from relatively simple rules to more complex ones, holding preferences constant. We deal with each of these issues in turn. However, before discussing the results, we need to elaborate on the nature of the experiments.

One of our exercises is to consider the consequences of the *complexity* of rules for welfare, given that agents must learn the rule in use. The error-correction representation of the optimal control rule contains seven arguments. In principle, we could consider the performance of this globally optimal rule with that of a very simple rule. However as we have already noted, it is difficult for agents to learn rules with large sets of variables, in small samples at least. Moreover it is difficult to present results over a large number of parameters. To keep the problem manageable, we focus mostly on learning 3-parameter optimal rules, beginning from 2-parameter rules. In this way, we measure the *marginal* cost of complexity. We do, however, discuss the performance of the optimal rule in steady state.

We also examine the costs of transitions from an *ad hoc* (sub-optimal) rule--our version of the Taylor rule--to an optimal 3-parameter rule, as well as from the optimal 2-parameter rule. This allows us to distinguish the costs of learning an optimal rule, beginning from a point of sub-optimality, from learning a more complex rule, beginning from a simpler rule that is (constrained) optimal. Of course we could have chosen any of a number of rules that are suboptimal in the context of our model. Our choice reflects the familiarity of the Taylor rule to a large cross-section of economists involved in monetary policy

debates. In addition, as Taylor (1993) argues, the Taylor rule approximates Fed behavior quite well over the 1980s.¹⁸

For our 3-parameter rules, we focus on the simple optimal rules shown in Table 2 above, for two different sets of preferences. In particular, we consider preferences that place a small, but non-negligible weight on restricting the variability of the nominal federal funds rate.¹⁹ A monetary authority may be concerned with the variability of its instrument for any of a number of reasons. First, and most obviously, it may inherently prefer less variability as a matter of taste, either in its own right, or as a device to avoid criticism of excessive activism. Second, constraining the volatility of the federal funds rate might be a hedge against model misspecification, both fundamentally, in the sense of missing variables, or more narrowly to the applicability of point elasticities estimated over a narrow range of movement of the federal funds rate to policies permitting a much larger degree of variation. In all cases, we apply a weight of 0.1 to $\psi_{\Delta r_t}$, the weight penalizing variation in the change in the federal funds rate. The weights applied to output variability and inflation variability are permitted to vary. We consider two alternatives: one with ‘conservative’ objectives, where a weight of 0.8 is applied to inflation variability and 0.1 to output variability, and one for ‘liberal’ preferences where the weight on the variance of inflation is 0.1 and the variability of output carries a 0.8 weight. These taste parameters and the rule coefficients they imply are summarized in Table 2 above.

Finally, we conduct several of these experiments using three different degrees of memory. The first is full recursive least squares, ($\lambda = 1$), which we shall refer to as ‘full memory’. The other two cases are discounted recursive least squares (DRLS), one with somewhat limited memory, ($\lambda = 0.95$) which we will call ‘long memory’ and another with ‘short memory’: ($\lambda = 0.85$). The long memory case corresponds with a mean lag of

18. Our quantitative work has shown the Taylor rule both as it is written in Taylor (1993), and as we have written it here, works satisfactorily in a wide range of models.

19. The term ‘variability’ is used interchangeably with ‘standard deviation’ or ‘variance’ of a variable when no confusion by the imprecision of the term would arise.

19 quarters, a reasonable length of time for a regime. The short memory case has a mean lag of only about six quarters and represents a public that is very skeptical of the constancy of any policy regime. In this way, these three choices bracket the possible linear learning strategies quite well.

4.1 The rules in the steady state:

Let us build up some intuition for what follows with an examination of the optimal policy rules in steady state. These are summarized by Figure 1 which shows the *efficient policy frontiers* for three rule specifications. The frontiers show the steady-state trade-offs between the standard deviations of inflation and output in the right-hand panel, and between inflation variability and the variability of the change in the federal funds rate in the left-hand panel. In each case, the loci show points where no decrease in standard deviation of one variable appearing in the objective function, equation (8), is possible without an increase in the standard deviation of another. Since the output gap, inflation and the change in the federal funds rate all appear in the loss function, a two-panel figure is needed to show the results. The curves are constructed by fixing the weight on the change in the federal funds rate in the loss function, $\psi_{\Delta rs}$, at 0.1 as described above, and varying the coefficients on output and inflation variability, ψ_y and ψ_π subject to the constraint that $\psi_{\Delta rs} + \psi_\pi + \psi_y = 1$ to compute rules. The rules are then embedded within the model which, with the stochastic shocks, is then used to compute asymptotic standard deviations of the target variables. The results of this exercise are is graphed in the figure.²⁰

20. One could also vary the weight on the change in the funds rate and trace out families of frontiers. However, we omit this issue in order to focus on a narrower set of rules. This allows us to keep the chapter reasonably concise. Note that there is no loss of generality from having the weights on output and inflation and the change in the federal funds rate sum to unity.

Figure 1
Selected Optimal Policy Frontiers

Let us focus initially on the right-hand side panel of the figure. The solid line is the frontier for the globally optimal rule--the one in which seven state variables appear. It is labeled CO-LO with the 'O' indicating that this is the frontier for the optimal rule and the 'C' and 'L' indicating the conservative and liberal preferences, respectively. That is, the 'C' end of the CO-LO curve is the performance of the optimal rule for the conservative preferences: $\{\psi_y, \psi_\pi, \psi_{\Delta r_s}\} = \{0.1, 0.8, 0.1\}$, and similarly for the liberal end of the frontier.²¹ The dot-dashed frontier immediately to the north east of the globally optimal frontier is associated with optimal 3-parameter rule (labeled C3-L3), while the dashed frontier is the optimal 2-parameter frontier (C2-L2). The positions of these points measure the performance, in steady state, of the rules corresponding to the rule parameters shown in Table 2.

The first and most obvious thing to note about these frontiers is that the globally optimal rule is closest to the origin, meaning it produces superior performance to all other rules, measured in terms of output and inflation variability. A more interesting result is that the 3-parameter and 2-parameter frontiers cross. Recall that the 2-parameter rule is of the same form as the 3-parameter rule except that the coefficient on the lagged federal funds rate is restricted to be zero. This crossing reflects the fact that movement along the 2-parameter rule frontier towards more conservative preferences involves a much larger sacrifice in terms of higher variability in the change in the funds rate than for the 3-parameter rule. This can be seen in the relatively flat slope of the dashed line in the left-hand panel. At some point, the constraint of not having the lagged funds rate as an argument to the rule obliges the authority using the 2-parameter rule to tolerate higher funds rate variability than is needed with the 3-parameter rule. Evidently, inflation control and interest-rate control are strong substitutes. That this is so in the neighborhood of the conservative

21. Notwithstanding that the conservative and liberal rules appear at the end of the curves shown in the figure, these are the most extreme rules that are calculable. We have limited the range of the curves to keep the scale of the figures reasonable.

rules, and not so in the vicinity of the liberal rules, is an observation we shall return to below when we discuss transitions from 2- to 3-parameter rules in subsection 4.3.

The point marked with a 'T' in right-hand panel is the Taylor point representing the performance of the Taylor rule in this model. This rule performs substantially worse than any optimal 2-parameter rule in terms of output and inflation variability, but produces a substantially lower standard deviation of the change in the federal funds rate.²² Since different frontiers could be computed for different penalties on the change in the federal funds rate, it follows that there is a set of preferences for which the Taylor rule is optimal. Some computation reveals that a loss function weight of about 0.5 on the change in the federal funds rate reverse engineers a frontier that slices through the Taylor point as shown in the figure. Thus, a policy maker that uses the Taylor rule is very reticent to allow fluctuations in the policy instrument. The steepness of the frontier at the Taylor point also shows that this version of the Taylor rule is very liberal in that a monetary authority that would choose this point is apparently willing, at the margin, to sacrifice a very large amount of inflation variability to reduce output variability only slightly. While this result is only literally true for our characterization of the Taylor rule, and only within this model, the same general conclusions can be drawn from a similar exercise using the original form of the rule from Taylor (1993) within larger, more complex models.²³

4.2 Changes in preferences:

Having examined the steady-state performance of our rules, let us now consider transitions from one rule to another. In order to keep things simple, we shall focus initially on changes in policy preferences using optimal 2-parameter rules. In particular, the

22. This is not shown in the left-hand panel of the figure in order to maintain visual clarity.

23. For example, the same general finding that the Taylor rule is 'liberal' in the sense described in the text and averse to interest-rate variability arises with the FRB/US model of the U.S. economy maintained by the Federal Reserve Board, albeit somewhat less dramatically so. This finding may reflect either the policy preferences in play over the period to which the Taylor rule was loosely fitted, or it may reflect the relatively placid nature of the shocks over the period.

thought experiment we have in mind is of a newly installed policymaker, replacing an incumbent of quite different preferences. The new authority has to decide whether to switch to a new rule that is consistent with its preferences. The new rule will produce better steady-state performance--from the point of view of the new authority--but presumably only after bearing some transitional costs as agents learn of the new rule. It is conceivable that the transitional costs could be so high that the new authority would prefer to continue to govern policy with the old rule, even though it is inconsistent with its steady-state preferences.²⁴

Figure 2 shows the evolution over time of the parameter estimates for the transition from a liberal policy regime, to a conservative one, using the 2-parameter optimal rules. The upper panel shows the evolution over time of the perceived coefficient on excess demand, beginning at the left-hand side from the optimal coefficient for liberal preferences of 1.18, and moving toward the optimal coefficient for conservative preferences of 0.73. Both of these coefficient values come from Table 2. The lower panel shows the analogous path for the coefficient on inflation. The dashed line is the 'full memory' case. The dotted line is 'long memory' learning case (that is, discounted recursive least squares learning with $\lambda = 0.95$). The dot-dashed line is 'short memory' case (DRLS with $\lambda = 0.85$). In each panel, we also show a base case--the solid line--where agents correctly believe the initial regime will continue. The lines in each case are the average values over 4,000 draws. Each simulation lasts 200 periods.²⁵

24. This experiment is broadly similar to Fuhrer and Hooker (1993) who also consider learning policy parameters. However, they do not consider optimal rules and do not examine the welfare implications of the choice of rules.

25. In order to ensure the lags and the precision matrix were properly initiated according to pre-experiment conditions, 200 periods were simulated using the initial rule before shifting to the new rule. For each experiment, 300 periods were simulated and each experiment comprised 3000 draws so that altogether each of the major experiments reported in this paper involved computing 900,000 points. On a single processor of an UltraSparc 4 296 megahertz UNIX machine, each experiment took a bit over 3 hours to compute.

Figure 2
 Perceived Policy Rule Parameters
 Learning a 'Liberal' to 'Conservative' Shift in Policy
 (average of 4000 draws)

Figure 3
 Perceived Policy Rule Parameters
 Learning a 'Conservative' to 'Liberal' Shift in Policy
 (average of 4000 draws)

Let us concentrate, for the moment, on the upper panel of Figure 2. The first thing to note from the figure is the obvious point that agents do not get fooled in the base case: If agents believe that the liberal 2-parameter rule is in place, and this is true, they are not misled on average by funds rate settings. That is, when agents learn without discounting, they are not induced to erroneously revise their estimates of the perceived rule. We shall note below that when agents discount the past, this seemingly obvious result does not necessarily hold.

More important than this, however, is the observation that regardless of which of the three learning devices that is used, it takes a remarkably long time for agents to come to grips with the change in parameters. In particular, with full memory, agents have not learned the true rule coefficients even after the fifty years covered in the experiment. Even under rapid discounting of $\lambda = 0.85$ --meaning that the mean lag of the memory process is just 5.7 quarters--it takes more than ten years before agents reach the new parameter values. This finding, which is consistent with earlier results, such as those of Fuhrer and Hooker (1993), stems from two aspects of the learning rules. First, the speed of updating is a function of the signal-to-noise ratio in the learning mechanism. Because a considerable portion of economic variability historically has come from random sources, agents rationally infer the largest portion of surprises to the observed federal funds rate settings as being noise. Accordingly, to a large extent, they ignore the shock. Second, these results show how linear learning rules tolerate systematic errors; the forecast errors that agents make get increasingly smaller, but are nonetheless of the same sign for extended periods of time. A non-linear rule might react not just to the size of surprises, but also a string of errors of one sign.

While the speeds of learning reported here may seem unduly low, there is some evidence that is supportive of very slow learning in surveys of inflation expectations. The Federal Reserve Bank of Philadelphia surveys finance professionals on their expectations

of inflation 10 years ahead. Expectations of inflation this far ahead should be independent of current economic conditions and should measure agent expectations of the target rate of inflation--effectively an intercept term in an estimated policy rule--provided that people accept that the Fed provides the nominal anchor. The survey shows, however, that inflation expectations were remarkably insensitive to the change in regime represented by the Volcker disinflation--as they were to the run-up in inflation that preceded it.²⁶ Along the same line of argument, it is interesting to note that in 1984 the 12-month CPI inflation rate was 4 percent, down from about 14 in mid-1980 because of the Volcker disinflation. The BAA corporate bond rate, however, remained at 14 percent, fully ten percentage points above inflation. Thus even something as tangible as a shift in the trend (or target) rate of inflation was a difficult thing for people to learn. More abstract concepts, such as the implicit weight on the output gap in a reaction function, must be that much harder to learn.

The bottom panel of Figure 2 shows the evolution of the perceived coefficient on inflation. Figure 3 shows the analogous learning rates when a conservative regime is succeeded by a liberal one. Very much the same conclusions can be drawn from these figures as is drawn from the upper panel of Figure 2.

We can derive a third observation, this time from Figure 4, which repeats some of the detail of Figure 2, but adds confidence intervals of plus-or-minus one standard error around the coefficient means for the full-memory and short-memory cases. The bands show us that as agents increasingly discount the past in forming their expectations, the coefficient estimates become less and less precise, particularly in the early going of the learning exercise. Later on, expectational errors diminish and the confidence bands shrink. Also, as one might expect, there is considerably more variability around the steady-state estimates of the policy parameters under the short memory case than there is under the full

26. A regression of the change in the survey on the lagged change in the inflation rate over the period since 1980 provides slope coefficient values between 0.03 and 0.10, depending on the inflation rate that is used.

memory case. Our experiment is a once-and-for-all regime shift; private agents, however, perceive shifts as regular occurrences. They keep a watchful eye for evidence of additional shifts and consequently overreact to purely random errors.²⁷ The figure does not show the full memory case after it converges on the true policy parameters but it is evident from the figure, and demonstrable from the data, that the variability of β is lower in the steady state under full memory than it is under short memory.

27. In technical terms, when $\lambda < 1$, the Kalman gain does not converge on zero, even asymptotically. The property that the gain goes to zero is emphasized by Marcet and Sargent (1989) as being a key one to ensure convergence on a rational expectations equilibrium.

Figure 4
Perceived Coefficient on Excess Demand
Learning a 'Liberal' to 'Conservative' Shift in Policy
(with ± 1 standard error bands)

Table 3 summarizes the welfare implications of this experiment. The table is divided into two panels. The top panel shows the decision a conservative policy maker would have to make after succeeding a liberal policy maker. The row in this panel labeled ‘base case’ shows the performance that the economy would enjoy had the conservative policy rule been in effect at the outset. This can be thought of as the ‘after picture’. The second row, labeled ‘liberal’ shows the performance of the economy under the liberal rule; that is, the rule that was in place when the conservative policymaker took over. This is the ‘before picture’. Finally, the third row shows the transition from the liberal rule to the conservative one. The key column of the panel is the far right-hand column showing welfare loss. This is the average discounted loss under the policies, *measured from the perspective of the incoming conservative policymaker*.²⁸ In terms of the raw numbers, the performance of the economy under, say, the liberal rule as shown in the second row, will not be any different than it was under the liberal regime, but the loss ascribed to this performance can differ markedly depending on the perspective of the policymaker. To aid comparison, we have normalized the loss figures for the base case to unity. By comparing the third row of the upper panel with the first row, we can see the loss associated with having to learn the new rule. By comparing the third row with the second row, we can see whether the transition costs of learning the new rule are so high as to induce the new policymaker to stay with the old rule. The column that is second from the right, labeled $\rho(\pi, y)$ shows the statistical correlation of output and inflation in the simulated data. Comparing this correlation across experiments gives an indication of the extent to which the monetary authority is using the Phillips curve trade-off to achieve its objectives.

28. Loss figures shown are the average over the 4,000 draws conducted. A quarterly discount factor of 0.9875, or about five percent a year, is applied in computing the losses. This is a modest discount factor, in line with what might be used in financial markets. The substantive facts presented in this paper were invariant to the choice of discount factors, at least within a range we would consider to be reasonable. Political economy arguments might yield positive arguments for a substantially lower discount factor, but this is not the subject of this paper.

The bottom panel of the table is analogous to the top panel, except that the transition is from a former conservative regime to a new liberal regime.

Turning to the results themselves, in the case of the incoming conservative policymaker, the process of learning is costly relative to the base case; not being able to simply announce a new policy regime and have private agents believe it, implies substantial costs. However, the comparison of rows two and three shows that the incoming conservative would be willing to bear the transition costs of switching rather than stick with the incumbent rule. Given the stark differences in preferences that these two policymakers represent, this is not particularly surprising. It is easy to show, however that at least some moderate policymakers--ones that place a larger weight on inflation than the liberal policymaker, but not as large a weight as the conservative--would also not want to bear the transition costs of adjusting to a new, moderate rule, after succeeding a liberal policymaker.

A more intriguing case is the transition from a conservative policy regime to a liberal one, shown in the lower panel. The third row of the lower panel shows that the liberal policymaker actually benefits from the prior belief of private agents that a conservative rule is in place: the loss under the transition, at 0.93, is lower than the normalized loss of unity in the base case. On the surface, this is surprising since, by definition, the base case should yield the best result possible in the circumstances. The reason why this intuition does not hold becomes clear once one recognizes that learning is breaking, in some sense, the model-consistency condition in the model. Notice that the lower loss in the transition case comes from reduced inflation and federal funds rate volatility.²⁹ The volatility of inflation is, to a large extent, determined by the expected future path of the model's jump variable, contract inflation, $c_{t+1|t}$. Private agents are setting contracts today based on the expectation that a conservative policymaker is setting policy now and in the future. This

29. The table actually shows the standard deviation of the level of the federal funds rate while the loss function contains the variance of the change in the federal funds rate, but the qualitative comparison is still valid.

means that current and future contract inflation is being pinned down by an erroneous belief; if agents knew that the liberal policy maker was in charge, the response of $c_{t+1|t}$ to shocks would be larger and so would the variance of inflation. This, in turn, would induce a large variance in the change in the funds rate. From the policymaker's point of view, expectations are not model consistent, even though they are from the point of the view of private agents. Thus, to the extent that learning is slow, the liberal policymaker can indulge his inclination to manage output without bearing the costs of incipient inflation pressures, and with reduced interest rate variability as well.

No similar benefit is shown for the transition from liberal to conservative preferences shown in the upper panel of the table. Two reasons account for this asymmetry. The first of these relates to the fact that the monetary authority controls inflation primarily through its management of aggregate demand. To a substantial degree, the belief by private agents that the authority will manage output tightly is only beneficial to the conservative authority to the extent that inflation fluctuations originate from demand shocks. In that case, managing output variability is the same as managing inflation variability. As the proportion of shocks emanating from the supply side increases from zero, a dilemma for policy arises: does the authority combat output variability or inflation variability? As it happens, a large portion of inflation variability in the U.S. economy comes from shocks to the Phillips curve--that is, from so-called supply shocks--and so managing inflation will involve sacrifices in terms of managing aggregate demand.

Table 3
Simulation Results from Change-in-Preference Learning Exercises
(Average across 4000 draws)

Rule in use	Standard Deviation:			$\rho(\pi, y)$	Welfare Loss L
	π	y	rs		
'conservative' preferences, full memory, 2-parameter rules					
base case	2.1	3.0	6.4	.31	1
liberal rule	2.9	2.5	5.6	.21	1.56
liberal --> conservative	2.3	3.2	7.0	.22	1.28
'liberal' preference, full memory, 2-parameter rules					
base case	2.9	2.5	5.6	.21	1
conservative rule	2.1	3.0	6.4	.31	1.31
conservative --> liberal	2.4	2.5	5.1	.33	0.93
Notes: The first row of each panel contains the 'base case' results defined as those corresponding to the optimal 2-parameter rule for the policy preferences noted; the row immediately below each base case is the performance of, and loss to, staying with the 2-parameter policy rule inherited from the previous regime. The third row shows the performance and cost of changing from the inherited regime to the (new) optimal 2-parameter rule. Qualitatively similar results were derived using 3-parameter rules and using discounted recursive least squares learning.					

The second reason why there is no benefit to the conservative policymaker from being mistaken for a liberal is that there is no jump variable in output in this model; that is, there is no $y_{t+1|t}$ as there is a $c_{t+1|t}$. This renders expected future demand less important for current demand management than is the case for inflation.³⁰

30. We believe, but cannot prove at this point, that this second factor is less important than the first. The candidate jump variable in output is a permanent shock to the level of total factor productivity which, to a first approximation, should shift both actual and potential output by similar amounts, leaving the output gap more or less unchanged. If the goal of the liberal authority is to control excess demand—that is, the output gap and not output itself—this should be of second-order importance.

It follows that a conservative monetary authority wants to credibly announce regime changes when taking over from a liberal incumbent, while a newly installed liberal policymaker wants to conceal his true preferences and possibly forestall private agents' learning as much as possible. The observation that liberal policymakers gain from being perceived as conservative, because doing so decouples inflation expectations from policy actions for a time, echoes the theoretical literature on policy games. In that literature, policymakers do not find it beneficial to reveal their true preferences, as in Cukierman and Meltzer (1986) and Vickers (1986) for example. Similarly, in the "cheap talk" literature exemplified by Stein (1989), the central bank finds that sending vague signals tends to dominate full disclosure as a policy.

4.3 Active Teaching

We have established that learning can be quite slow and under some circumstances, the transition costs of learning can be substantial. Before moving on, we consider the possibility that the monetary authority might be able to aid its cause by taking action to speed learning along the transition from one policy regime to another. In particular, we assume that the authority engages in what we shall call 'active teaching': aggressively signalling the change in policy by accentuating differences in policy. To do this operationally, we assume that the newly installed authority chooses interest-rate surprises that are initially three times larger than in the regular experiment depicted in Table 3. We then allow this exaggerated policy signalling to decline slowly to normal over time.³¹ Figure 5 shows the results for the speed of learning, in this case for the perceived inflation parameter, following a shift from a liberal 2-parameter rule to a conservative rule. As one might expect, the monetary authority can induce faster learning by private agents. But can the

31. Specifically, think of a 'surprise function' $s(\beta - \tilde{\beta})$ where surprises vary directly with the discrepancy between the actual and perceived rule parameters. These surprises are the catalyst for learning in all our simulations. The 'active teaching' scenario replaces this surprise function with $s(\beta - \tilde{\beta}) + \alpha(s(\beta - \tilde{\beta}))\theta^t$ where $\alpha = 2$, $\theta = 0.925$ and $t = 0, 1, 2, \dots$

authority increase welfare? Table 4 below summarizes the welfare implications of this experiment.

The table shows a quantitatively important deterioration in performance relative to the regular case transition. The reasons for this are two-fold. First, in order to initiate the higher rates of learning, the monetary authority must inject federal funds rate volatility into the economy. As equation (8) shows, this counts directly as a negative in the loss function. On top of this, however, the extra shocks do improve the performance in the variable that the new regime cares about most--inflation for the conservative authority and output for the liberal--but not forever.

Table 4
Implications of 'Active Teaching' for Change-in-Preference
Learning with Full Memory
(2-parameter rules, averages across 2000 draws)

experiment	Welfare Loss <i>L</i>	experiment	Welfare Loss <i>L</i>
conservative --> liberal		liberal --> conservative	
base case	1	base case	1
regular experiment	0.93	regular experiment	1.28
'active teaching'	0.98	'active teaching'	1.39

Notes: 'regular experiment' is the transition from a conservative 2-parameter optimal rule to a liberal 2-parameter optimal rule (two left-hand columns) or vice versa (two right-hand columns). 'active teaching' is where the surprises along the rule transition have been accentuated by three-fold initially with this accentuation declining at a quarterly geometric rate of 0.925 thereafter. Losses are discounted losses with a discount factor of 0.9875 and have been normalized around the base-case loss.

Figure 5
Perceived Coefficient on Inflation
Learning a Shift in Policy from 'Liberal' to 'Conservative'
(average of 4000 draws)

Moreover, the extra shocks are disastrous for the variable that the new authority cares about least. Only with heavy discounting of the future would such a strategy of active learning pay off--at least in the blunt form that we have introduced it here. It is conceivable that an fully optimizing authority, could choose the rate of learning directly and do better than the regular case transition.³²

4.4 Learning to be optimal

In subsection 4.2, we examined the transition from liberal to conservative rules and vice versa, holding fixed the number of parameters in the rule. Now we consider transitions to rules of varying degrees of complexity, holding preferences constant. As before, all of the rules we use appear in Table 2.

We begin by considering agents who initially face a suboptimal 2-parameter rule--in the present example, the Taylor rule--who must then learn that the Fed has shifted to one of the simple optimal rules laid out in Table 2 above. We consider first the results for conservative preferences, summarized in Table 5 below. For ease of comparison we shade the base-case row, the 3-parameter optimal rule in this case, and normalize the loss to unity.³³ In the interests of brevity, we refer to the suboptimal rule as the Taylor rule and drop the term optimal for the 2- and 3-parameter rules.

The first panel of the table shows the performance of the economy under the three alternative steady states; it corresponds to Figure 1 above. Having already examined Figure 1, it is not surprising that the Taylor rule's performance features substantially higher inflation variability than the conservative base-case rule, but lower variability in output and especially the federal funds rate.

32. Cripps (1991) shows that an optimizing central bank controlling a Barro-Gordon type of model might want to slow down the rate at which private agents would learn in order to maintain a steep slope of the short-run Phillips curve and thereby obtain larger benefits from monetary surprises.

33. The base-case loss is independent of the 'memory' in the learning process.

Table 5
Learning with ‘Conservative’ Preferences
(Average across 4000 draws simulated for 200 periods each)

Rule(s)	Standard Deviation:			$\rho(\pi, y)$	Loss L
	π	y	rs		
base case results					
Taylor rule	3.0	2.7	5.1	.39	1.58
optimal 2 parameter	2.1	3.0	6.4	.31	1.02
optimal 3 parameter	2.0	3.1	6.6	.35	1
learning with full memory ($\lambda = 1$)					
Taylor --> 3 parameter	2.4	3.4	7.6	.35	1.37
Taylor --> 2 parameter	2.3	3.1	7.1	.31	1.29
2 --> 3 parameter	2.1	3.2	6.7	.35	1.04
learning with long memory ($\lambda = 0.95$)					
Taylor --> 3 parameter	2.2	3.2	6.8	.35	1.17
Taylor --> 2 parameter	2.1	3.0	6.6	.31	1.15
2 --> 3 parameter	2.1	3.1	6.6	.35	1.02
learning with short memory ($\lambda = 0.85$)					
Taylor --> 3 parameter	2.1	3.2	6.6	.35	1.07
Taylor --> 2 parameter	2.1	3.0	6.4	.31	1.07
2 --> 3 parameter	2.1	3.1	6.6	.35	1.01

Notes: The syntax “ $n \rightarrow m$ ” refers to the results from learning the transition from the n -parameter simple optimal rule to the m -parameter simple optimal rule. Losses are discounted by a factor of 0.9875.

The Taylor rule's performance also shows substantially more persistence in inflation and interest rates (not shown) and a higher correlation of output and inflation indicating that policy is working more through the traditional Keynesian channel, rather than through expectations, than in the base case. From the point of view of a conservative policymaker, the Taylor rule is seen as a poor performer. Measured in terms of discounted loss, the Taylor rule is 58 percent worse than the 3-parameter optimal rule. Putting the same performance a different way, the conservative policymaker would just as soon accept an autonomous increase in the variance of inflation of three-quarters of a percentage point or a whopping 3.1 percent in output variability as be forced to use the Taylor rule.³⁴

The second row of the table shows the performance of the 2-parameter rule. Relative to the base case, there is only a two-percent loss from using the 2-parameter rule. The conservative authority would be willing to sacrifice output variability of 0.2 percent in order to avoid using this rule. While this is not trivial, it is also not large. Thus, a significant finding in this paper is that the gains in steady state from using even slightly more complex rules than a 2-parameter rule is small--at least for conservative preferences--provided that the simpler rule's parameters are chosen optimally.³⁵

The 2-parameter rule has the same arguments as the Taylor rule, but markedly different coefficients, as Table 2 shows. Obviously there are large gains to be had from picking rule parameters judiciously.

Now let us consider the decision by a conservative policymaker, of whether to shift from a Taylor rule to the 2-parameter rule as well as to the 3-parameter rule. To separate the effects of optimality from complexity, we also consider moving from the 2-parameter rule to the 3-parameter rule. We conduct these exercises with full memory

34. This and all subsequent equivalent variation calculations are measured relative to the base-case rule.

35. Williams (1999) reports a similar result in the context of the FRB/US model which has some 300 equations.

($\lambda = 1$ in the second panel of Table 5), long memory (third panel), and short memory (bottom panel). The first thing to note about the results is that when agents have to learn about rule changes, short memory is a good thing. The loss for the short-memory cases are less than for the corresponding long-memory cases, which in turn are lower than the full-memory cases, regardless of which learning exercise one considers. Since the steady-state variability of the estimated policy rule coefficient vector, $\hat{\beta}$, varies inversely with memory, λ , this result is not trivial. It would stand to reason that greater variability of the perceived rule parameters in steady state would correspond with higher losses in the steady state. These higher steady-state losses could offset the lower transitional losses as the steady state is approached. In fact, the difference in the steady-state loss between the short-memory case and the full-memory case is negligible, meaning that, for these cases at least, only the transitional losses matter.

A second observation from Table 5 is that regardless of how slow the learning is, the conservative policymaker is always willing to bear the transitional costs of moving from the Taylor rule to either the 2- or 3-parameter rules (compare the loss column of the first row with any row in the bottom three panels). More intriguingly, however, in some cases, the conservative authority is better off moving to the 2-parameter rule than moving directly to the 3-parameter rule. This is so even though the cost of moving from the 2-parameter rule to the 3-parameter rule is generally small. The reason is, of course, that the benefits are often smaller still.

Now let us examine the same experiment for liberal preferences, shown in Table 6. The mass of numbers in the table yield essentially one point: regardless of whether one begins from the Taylor rule or from the 2-parameter optimal (liberal) rule, and regardless of the memory in the learning mechanism, there is very little difference in the loss. Simply put, neither the speed of learning, nor the complexity of the rule, makes a substantial difference in terms of welfare. To see why this is so, first consider the welfare loss to a liberal

policymaker from pursuing the Taylor rule. The loss is certainly consequential,³⁶ but given the vast distance the Taylor rule coefficients are from the 2-parameter rule coefficients, the difference in performance seems small. Simply put, the loss function for the liberal policymaker is very flat, or, to put the same observation another way, relatively crude control methods yield broadly similar economic performance, for liberal preferences.

This flatness of the loss function arises because managing output fluctuations to the near-exclusion of other objectives is a relatively easy task. This is so for three reasons. First, as we noted above with regard to Figure 1, inflation control and interest-rate smoothing tend to be substitutes. The conflict raised by this substitutability is a slight one for the weak policymaker who is largely unconcerned with inflation variability.³⁷ Second, the control of output does not depend in an important way on forward expectations. This means that agents' prior expectations of what rule is in place is of lesser importance for the weak authority. Third, output appears earlier in the monetary policy transmission mechanism that links funds rate setting, to output determination and then inflation.

36. The liberal policymaker would be willing to permit the variance of output to rise by 0.3 in order to avoid using the Taylor rule. Recall from earlier that the conservative policymaker would tolerate a 0.75 increase in inflation to avoid the same fate. (Inflation carries the same 0.8 weight in the conservative policymaker's loss function as output does in the liberal's loss function.)

37. This can be seen in Figure 1 by comparing the very steep slopes of the trade-offs between the variability in the change in the federal funds rate and the variance of inflation in the neighborhood of the weak rules, with the much flatter slopes of the same curves in the neighborhood of the strong rules.

Table 6
Learning with ‘Liberal’ Preferences
(Average across 4000 draws simulated for 200 periods each)

Rule(s)	Standard Deviation:			$\rho(\pi, y)$	Loss L
	π	y	rs		
base case results					
Taylor rule	3.0	2.7	5.1	.39	1.12
optimal 2 parameter	2.9	2.5	5.6	.21	1.02
optimal 3 parameter	3.0	2.4	5.6	.20	1
learning with full memory ($\lambda = 1$)					
Taylor --> 3 parameter	2.8	2.4	5.6	.27	0.99
Taylor --> 2 parameter	2.8	2.5	5.6	.29	1.01
2 --> 3 parameter	2.9	2.4	5.5	.21	0.99
learning with long memory ($\lambda = 0.95$)					
Taylor --> 3 parameter	2.9	2.4	5.6	.22	1.00
Taylor --> 2 parameter	2.9	2.5	5.6	.23	1.01
2 --> 3 parameter	2.9	2.4	5.6	.20	1.00
learning with short memory ($\lambda = 0.85$)					
Taylor --> 3 parameter	2.9	2.4	5.6	.20	1.00
Taylor --> 2 parameter	2.9	2.5	5.6	.21	1.02
2 --> 3 parameter	3.0	2.4	5.6	.20	1.00

Notes: The syntax “ $n \rightarrow m$ ” refers to the results from learning the transition from the n -parameter simple optimal rule to the m -parameter simple optimal rule. Losses are discounted by a factor of 0.9875.

Taken together, the results shown in Table 5 and Table 6 send a cautionary message about how a monetary authority should select among candidate rules. For some preferences, the characteristics of rules matter a great deal for economic performance, as do the transition costs of moving from one rule to another. For other preferences, any vaguely sensible rule will perform reasonably well. Thus, no simple rule of thumb for monetary policy rule selection emerges.

5. Concluding Remarks

This chapter has examined the implications for the design of monetary policy rules of the complexity of rules and the interaction of complexity and preferences with the process of learning by private agents of the rule that is in place.

In particular, we took a small New Keynesian macroeconometric model and computed optimal simple rules for two sets of preferences: ‘conservative’ preferences, where a substantial weight is placed on inflation control and only a small weight on output or instrument control, and ‘liberal’ preferences, where the same substantial weight is placed on output control, and not on inflation or instrument control. We then examined the transition costs of moving between two rules to see if these costs could be an important consideration in the selection and adoption of candidate policies. Two types of transitions were considered. The first was the transition between regimes, where a regime is associated with a particular set of preferences, for a given degree of policy rule complexity. The second was the transition to more complex rules from simpler rules, within a single regime.

Our four basic findings are: (1) learning should be expected to be a slow process. Even when agents ‘forget’ the past with extraordinary haste, it takes more than ten years for agents to learn the correct parameters of a new rule. (2) The costs of these perceptual errors can vary widely, depending on the rule that is initially in force, and on the preferences of the monetary authority. In particular, a conservative monetary authority generally finds that the cost for agents to learn a new (conservative) rule that has been put in place.

is relatively high. It follows that such a policymaker should take steps to identify his policy preferences to private agents. Paradoxically, a liberal policymaker will sometimes benefit from being misperceived, posting a better economic performance than would have been the case if the optimal rule had been in place all along. This sharp contrast in results has to do with the multiplicity of sources of shocks to inflation, the nature of inflation in this model being a forward-looking variable, and the fact that inflation appears later in the chain of the monetary policy transmission mechanism than does output. (3) The performance, in steady state, of optimal two-parameter policy rules is not much worse than that of the optimal three-parameter policy rules, at least for this model. Thus, policymakers moving from a suboptimal rule would be better off choosing the optimal 2-parameter rule, rather than the 3-parameter rule, to avoid the incremental costs of private agents learning the more complicated rule. (4) Faster learning is not necessarily better. When the monetary authority takes steps to 'actively teach' private agents that the rule has changed, agents' expectations converge more rapidly on the new and true parameters, but economic performance does not necessarily improve. This is because the policymaker himself must add instrument variability to the system in order to hasten the learning, and because the initial benefits of more rapid learning are outweighed by large losses when learning slows down later on.

6. References

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7. Data Appendix

The data used in the estimation shown in Table 1 are as follows: $\Delta \overline{poil}$ is a four-quarter moving average of the price of oil imported into the U.S.; π is the quarterly change of the chain-weight GDP price index measured at annual rates; u is the demographically adjusted unemployment rate less the natural rate of unemployment from the FRB/US model database; $c_{t+1|t}$ is proxied by the median of the Michigan survey of expected inflation, 12 months ahead; y is the output gap for the U.S, defined as the percent deviation from potential output, from the FRB/US model database; rr is the real interest rate defined as the quarterly average of the federal funds rate less a four-quarter moving average of the chain-weight GDP price index; $poil/p$ is the price of imported oil relative the GDP price index; and Nixon price controls variable equals unity in 1971Q4 and -0.6 in 1972Q1.

Note that all regressions also included an unreported constant term; constants were never statistically significant.

Chapter 3 : Reducing Randomness in Inflation Targeting

Does it Pay? If So, How Much?

Summary

Costs of adjusting prices and real variables, together with rational expectations imply that private agents must solve for expected future values of prices, wages and other variables when making current decisions and formulating plans for the future. The task is complicated by the fact that expected future inflation is, in the end, under the control of the monetary authority. In the United States as well as in other countries, there have been substantial movements of the target rate of inflation over time, whether as deliberate acts--as with the Volcker disinflation--or as the outcome of passive policies in the presence of inflationary shocks--as with the great inflation of the late 1960s and the 1970s. This chapter examines the implications of such variability. In particular, we model policy as governed by a rule that we take to be known--except with a random target rate of inflation. We do this with stochastic simulations of the Board of Governors' large, rational-expectations model of the U.S. economy, FRB/US. We then examine the benefits of reducing the randomness in the perceived target rate of inflation. We find that relative to a drifting target, credible limits on the inflation target can be useful in focussing agents' expectations on policy objectives, provided that the bands are reasonably narrow. But while reducing the randomness in future inflation does improve economic performance, there are limits to the improvement: it does not pay to reduce random movements in the inflation target to zero. A second contribution of the paper is the demonstration of a numerical technique that approximates to arbitrary precision a non-linear process with a linear method, thereby greatly speeding and making more robust the computation of simulation results.

1. Introduction

While the pursuit of price stability does not rule out misfortune, it lowers its probability. If firms are convinced that the general price level will remain stable, they will reserve increases in their sales prices of goods and services as a last resort, for fear that such increases could mean loss of market share. Similarly, if households are convinced of price stability, they will not see variations in relative prices as reasons to change their long-run inflation expectations. Thus, continuing to make progress toward this legislated objective will make future supply shocks less likely and our nation's economy less vulnerable to those that occur.

Allan Greenspan
The Federal Reserve's Semiannual Monetary Policy Report
 February 28 1998

In recent years, a number of countries have adopted inflation targeting as the “legislated objective” for their countries, in pursuit of the same bounty that Allan Greenspan, the Chairman of the Federal Reserve Board describes in the above quotation taken from his February 1998 Humphrey-Hawkins testimony.¹ New Zealand (1990), Canada (1991), the United Kingdom (1992), Sweden (1993) and Finland (1993) have all established explicit target paths for inflation. By itself, inflation targeting is not a controversial issue. Most economists would accept that a monetary authority's overriding responsibility is to establish and maintain a nominal anchor. In one sense, targeting inflation is simply the most direct and obvious means for establishing the anchor. Belying this apparent simplicity, however, are some important operational issues. Does it suffice to utter broad sweeping statements about “price stability” being the ultimate objective of policy, without defining what the term means, as the Federal Reserve has done? What about announcing a range of indifference, within which any rate of inflation is as good as any other, as in the case of Australia? And what of the much more specific and confining regimes of the

1. The Humphrey-Hawkins testimony takes its name from the legislation that commands semi-annual appearances by the Chairman before the Senate Banking Committee. The Chairman was alluding to an updating of the mandate of the Fed which, under the current law, calls for a broad list of objectives and the pursuit of monetary targets. Currently, neither the Fed nor the Banking Committee pays much attention to the money numbers although they are still dutifully reported.

United Kingdom and New Zealand?² Would degrees of commitment of this nature achieve better results?³

As the Chairman's statement emphasizes, and as Bernanke *et al.* (1999) document for a variety of countries, the purpose of public declarations regarding policy objectives appears to be two-fold: (1) to direct private agents' expectations towards policy objectives; and (2) to establish a method of public accountability for central bank policy making. In this chapter, we examine what role a limited form of policy credibility--specifically, the idea that *inflation targets* might drift within bounds over time--might have on monetary policy outcomes and for policy choices.

In modeling randomness in the target rate of inflation, what we have in mind is the perception of private agents with regard to the medium- to long-term objectives of policy. Presumably, perceptions of policy objectives will coincide, in the long run, with actual policy. But given that policy preferences and the determinants of policy actions are not directly observable, there is considerable latitude for discretionary actions, or policy errors, in the short run.

This 'slippage' between how policy is practiced and how it is perceived to operate can affect in a material way the trade-offs that monetary authorities face and, as a consequence, the choices they make. Bernanke *et al.* (1999, pp. 59-61) emphasize the short-term flexibility that the Bundesbank enjoys in day-to-day operations has as a result of the widely held perception that the central bank will maintain a constant medium-term nominal anchor. Goodfriend (1991) argues that the credibility of the Fed can under some circumstances be counted on to bring about desired movements in *long-term* nominal and

2. In the case of the U.K., when the Labour government took power, it granted operational autonomy to the Bank of England, narrowed the inflation band to plus-or-minus one percentage point, and required the Bank to provide a written, public explanation for any breaches of the target bands. In New Zealand, the government originally adopted target bands and vowed that the Governor would be dismissed if the bands were breached, although they later reneged on that pledge.

3. Siklos (1997) provides a nice short summary of the practices of various inflation targeting countries, see especially the table on pp. 132-133.

real interest rates in *anticipation* of fed funds rate movements that may or may not be forthcoming. It is this idea, that expectations of future policy actions can be built up and exploited, that is explored in this chapter.

We investigate this issue using the Federal Reserve Board's macroeconometric model of the U.S. economy, FRB/US. We conduct stochastic simulations using the model to provide a quantitative guide to the possible benefits that might be accrued with a regime where there is randomness in policy but only within bounds. FRB/US is a large, forward-looking model. And, as we shall see, target bands for inflation control imply an important nonlinearity. Together, these facts present a challenging problem in numerical analysis. The resolution of this numerical problem is, we believe, a significant contribution of the paper.

Our main thrust, however, is on the popular notion that expectations of future policy actions can be divorced, at the margin at least, from current policy actions. This possibility cuts both ways, of course: Most of the countries that have adopted inflation target bands did so because they had been unsuccessful in credibly implementing another, different sort of policy regime. These countries were seeking a regime that forestalled any impression that abandoning their former regime implied drift in policy objectives.⁴ In the U.S. example, one might argue that the Federal Reserve has already accrued a reputation that is useful for providing some added latitude to pursue objectives other than short-term control of inflation. Whether this good reputation could survive a period of "misfortune" as the Chairman describes above, or the ascendancy of a new Chairman, is an open question.

4. The U.K. settled on its initial regime of inflation targeting in 1992 after an unsuccessful attempt to remain in the ERM under pressure. The Bank of England and the Chancellor have since adapted the makeshift program of 1992 to a more cohesive campaign. New Zealand ended up on an inflation targeting regime after falling off a fixed exchange rate regime. Of the countries with formal inflation targets Canada is the only country *not* to have selected an inflation targeting under duress.

The rest of the paper proceeds as follows. In Section 2 we discuss randomness in monetary policy, and how the behaviour of central banks in general and the Federal Reserve in particular might be reasonably characterized by drifting but bounded targets for inflation. We also describe how ‘randomness’ is operationalized for this chapter. The third section provides a brief encapsulation of the FRB/US model. Section 4 then discusses aspects of our simulation technique. The fifth section summarizes our results. A sixth section sums up and concludes.

2. Randomness in Inflation Targeting

2.1 Policy rules and policy objectives

There are a number of ways in which one might characterize policy. The most formal is as the solution to a dynamic optimization problem with well-defined preferences and a well-specified model. Another quite different method is where decision makers ‘look out the window’ to see if the economy is currently running ‘too hot’ or ‘too cold’ and adjust policy stance accordingly.⁵ If the policy that arises out of the former method is known by the public and rigidly adhered to, it is an example of policy by rules. In contrast, making decisions by looking out the window is inherently discretionary, particularly when the decision-making body is reticent about disclosing the precise reasons for policy decisions.⁶

Between these two extreme characterizations of policy lie a wide range of alternatives embodying differing levels of commitment and flexibility. The introduction to this chapter, for example, notes the popularity of target bands as a device to contain inflation

5. Term ‘looking out the window’ comes from Alan Blinder (1998) where in his chapter 1 he describes the Federal Open Market Committee as containing elements of this kind of policy making. The tone of his monograph, however, makes it fairly clear that he is not suggesting that this is the sole basis for policy decisions at the Fed.

6. Although the FOMC began, in February 1995, to publicly announce the results of FOMC meetings the day after they take place, there remains a high level of secrecy surrounding FOMC decisions and preferences. During the course of his tenure as vice-chairman of the Board of Governors, Alan Blinder attempted to lift the veil of secrecy but as Blinder is quoted as saying, “I made my case and I lost” (Cassidy, 1996, p. 46).

expectations. Other devices, such as the former exchange rate mechanism of European Union countries, attempt to achieve the same goal using somewhat different methods.

Suppose, for example, the public had information about the broad objectives of policy--as there is for the Federal Reserve with respect to its obligations under legislation and from widely reported speeches of Board members--but no information regarding a specific rule governing policy. Fed watchers might be driven to fit a policy rule to the data in an effort to impute a reaction function of the FOMC. One such effort along these lines is the rule advanced by John Taylor. The Taylor (1993) rule is promoted by its author as both a good rule, in that it avoids policy instability, and a rule that fits the pattern of funds rate setting "remarkably well," (p. 202) at least over the period from 1987 to 1992.

In the time since its publication, the Taylor rule has garnered a good deal of attention. The implied funds rate settings are regularly reported to at least one of the governors, and at least four current and past governors have specifically mentioned it in speeches to the public.⁷ At the same time, each of these Governors has taken great pains to indicate that the Fed does not follow the Taylor rule and that no rule could do justice to the complexity of factors that are weighed by the FOMC.⁸

The Taylor rule is very simple:

$$R_t = 2 + \tilde{\pi}_t + 0.5(\tilde{\pi}_t - 2) + 0.5y_t \quad (1)$$

It sets the nominal federal funds rate, R , equal to the equilibrium nominal rate--that is, the equilibrium real rate, rr^* , taken by Taylor to equal 2 percent as shown, plus expected inflation, proxied by a four-quarter moving average of the quarterly annualized change in

7. A computer search of speeches by Board members for the words "Taylor rule" and derivatives thereof reveals twelve hits for speeches by governors Meyer, Gramlich, Ferguson and Yellen. No other rule has ever been publicly cited in speeches.

8. Here is an example of a standard disclaimer, in this case from Meyer (1997, p. 20) "I do not want to mislead you about the degree to which I believe we can rely on any single rule in general or my commitment to the Taylor Rule in particular. But the Taylor Rule helps illustrate some important aspects of monetary policy strategy."

the GDP deflator, $\bar{\pi}$ --plus two disequilibrium terms. One of these disequilibrium terms is the discrepancy of actual inflation from the target with the target assumed by Taylor to be equal to 2 percent; the other disequilibrium term is the output gap, y . Taylor (1993) emphasizes that the output gap can enter the equation either as a proxy for expected future inflation, since output statistically causes inflation, or because stabilizing output is an objective of policy in its own right.

It is instructive to examine a measure of fit of the Taylor rule. We do this not as a criticism of the rule, but as an indication of the discrepancy between policy as it has actually been conducted historically, and as it has been described by an articulate and knowledgeable commentator on the subject. Other rules would obviously give different answers, but no parsimonious rule would significantly alter the point to be made below. For our purposes it is most useful to measure the goodness of fit of the Taylor rule by taking its specification as given and solving out for the implied target inflation rate as in Figure 1.⁹

There is much that could be said about this implied target, but we wish to focus on just three things. First, there has clearly been a shift in either the equilibrium real interest rate or the target rate of inflation, or both, in the early 1980s.¹⁰ This helps explain Taylor's focus on the 1987-1992 period as one where the fit is remarkable. Second, over both subsamples there have been large and persistent movements in the implied target rate of inflation. High frequency movements in π^* could be attributed to a missing term from the rule, associated perhaps with tactical concerns in conducting policy, or control errors. But movements as persistent as those shown are difficult to ascribe to something other than changes in the target, or the implied target, of policy. Third, with the exception of the

9. We have replaced the implicit price deflator used in Taylor (1993) with the chain-weight index that is now used by the Bureau of Economic Analysis for the National Income and Product Accounts. Otherwise, our formulation is the same as Taylor's.

10. It is likely that both a shift in π^* and a shift in rr^* occurred. The early 1980s was the period of the Volcker disinflation in which CPI inflation was deliberately brought down by the Fed from about 12 percent to approximately 5 percent. Bomfim (1997) shows that the equilibrium real interest rate apparently rose at about the same time, perhaps due to the accumulation in the early 1980s of large public sector deficits.

once-and-for-all shift at around 1980 or so, the shifts in the implied target, however persistent, appear to have been bounded within ranges.

Figure 1
Target Rate of Inflation as Implied by the Taylor Rule

Figure 1 supports the notion that the implicit target rate of inflation has been subject to considerable randomness, at least from the perspective of one popular rendition of the systematic portion of U.S. monetary policy. That this is at least a plausible characterization of policy is supported by the observation that the Fed does not regularly or systematically consider the advice of policy rules themselves, or even explicitly discuss the quantitative objectives of policy.¹¹ This means that the target of policy has been a latent

variable of policy--something that falls out of funds rate settings given the stochastic shocks that are borne by the economy--rather than the other way around.

The control-theoretic perspective on policy design has the monetary authority choosing the target rate of inflation based on objective time-invariant criteria and then using that target rate as an input to a dynamic optimization problem that furnishes a feedback rule as an end product. Discretion will have a tendency to do the opposite. For example, when the Fed, in pursuing discretionary policy, failed in the 1970s to adjust the funds rate sufficiently in response to inflationary shocks, the implication was that the target rate of inflation was being increased--not necessarily as a deliberate act of policy as perhaps an implication of the Fed's unwillingness to forgo output objectives in order to maintain a fixed inflation target. While the story differs in some ways from a deliberate, exogenous change in the target rate of inflation, the effect is very close to the same in the context of a stochastic economy.¹²

Orphanides and Wilcox (1996) describe something similar to this in their paper on "opportunistic disinflation" as a description of Fed behaviour in the 1990s.¹³ An opportunistic policymaker does not fix his or her target rate of inflation exogenously and simply pursue it; rather, the target rate is a function of the underlying shocks to which the economy is subject. If inflationary shocks are experienced, the target rate remains fixed. If disinflationary shocks hit the economy, the Fed accommodates the shocks by reducing the

11. The FOMC is on record as having an objective of "price stability" but has said nothing on what this might mean. Nothing that I have seen or read suggests that the Committee has reached a consensus on this issue.

12. This characterization of policy in the 1970s is consistent with that of Clarida, Gali and Gertler (1998) and Chari *et al.* (1999). It is also consistent with the rhetoric of Meltzer (1991) when he accuses the Fed of having lost control of inflation because FOMC "members were reluctant to allow interest rates to rise by as much as required by staff forecasts. They hoped to achieve lower inflation by reducing money growth but were reluctant to allow interest rates rise by relatively large steps." (p.34)

13. It is worth noting that opportunistic disinflation was the name that a Federal Reserve governor, Laurence Meyer, gave to the policy he saw the FOMC following when he joined the Board early in 1995. The Orphanides and Wilcox paper was written as a theoretical explanation of Meyer's observation. See Meyer (1997) for a description of opportunistic disinflation including the author's claim to paternity.

target rate of inflation. This is precisely the same as what happened in the late 1960s and 1970s, except that the sign is the opposite: in the 1960s and 1970s inflationary shocks were accommodated and disinflationary ones were fought off. This resulted in the upward creep over the 1970s in the implied target as shown in Figure 1.

It is worth emphasizing that the theory of Orphanides and Wilcox (1996) was an effort to explain the apparent reluctance of the FOMC to adjust the funds rate with anywhere near the frequency and magnitude that quantitative normative assessments of policy would suggest.¹⁴ In the context of a policy rule like equation (1), if the funds rate is fixed, the only other object that can vary over time is the target rate of inflation.

Quantitative work by Orphanides *et al.* (1997) provided a quantitative assessment of both opportunistic disinflation, and a symmetric extension of the concept that permits drift, within bounds, of actual inflation while the Fed concerns itself with output stabilization, and then strong inflation control at the edges of the band. This describes a 'zone of indifference' regarding inflation--not unlike the policy practiced by Australia--and might be rationalized by committee structures that can agree on a point when inflation is 'too high' or 'too low', but cannot come to a consensus on intermediate cases.

The next subsection models inflation targets as a bounded, random process. To summarize the arguments of this subsection, what we have in mind by this process is an implied drift in target brought about by the historical reluctance of the Fed to respond to shock--except when inflation 'gets out of hand'. Our favoured explanation for this behaviour is that it is the outcome of the consensus-oriented decision-making process of the Federal Open Market Committee.¹⁵ Under this interpretation, the upper and lower boundaries for the target rate of inflation are points at which a critical mass on the committee

14. This phenomenon was briefly discussed in chapter 1.

15. The practice of decision-making by consensus at the FOMC is well known. See, e.g., Blinder (1998) and Meyer (1998). Blinder notes that this can lead to sluggish responses. As we shall argue below, it can also lead to a disconnect between long-term objectives and short-term practices and procedures of the Fed.

would say that trend inflation is out of line with long-term objectives. In between, there might be more of a bias toward going back to a zero target--in the sense that the probabilities point in that direction--but not an overwhelming impetus. Some committee members might be inclined to pursue other objectives or--more consistent with the historical record--to await further information (and more shocks) before acting. At the midpoint of the bands, all FOMC members are agreed that inflation is not a problem.

There are, however, other ways to think of this reluctance to react, and the target drift it implies. It can, for example, be regarded as an attempt to practice short-term discretion, as an effort to evade public criticism by avoiding uncontroversial acts, or as a response to uncertainty. And these alternative explanations are not mutually exclusive. However, it is beyond the scope of this chapter to discuss the range of explanations for these observations.¹⁶ Whatever the ultimate source, we would argue that the lack of a clearly articulated policy objective, together with an absence of a demonstrable commitment to a policy, will elicit variability in private agents' perceptions of target inflation over time.

2.2 Modeling inflation target bands

The preceding subsection has established at least a *prima facie* case for treating the target of monetary policy as a random variable. In this subsection, we operationalize this idea for simulation purposes. We begin by assuming that monetary policy can be described by a simple interest-rate reaction function, as follows:

$$R_t = rr^* + \sum_{i=0}^3 \pi_{t-i}/4 + 3.0[\sum_{i=0}^{11} \pi_{t-i} - \pi_t^*] + 0.9y_t \quad (2)$$

where, as before, R is the nominal federal funds rate, rr is the real rate, π is quarterly inflation measured at annual rates, y is the output gap and an asterisk indicates the equilibrium or target level of a variable, as applicable. Equation (2) is similar to the Taylor

16. Several of the candidate stories are discussed in other chapters.

rule, discussed above, but differs in three ways. First, the rule's inflation term is expressed as a twelve-quarter moving average of inflation instead of four quarter. Williams (1999) shows that, in the context of FRB/US, equations with longer lags on inflation outperform shorter lags. Second, the coefficients on both inflation and the output gap are larger for our rule. In fact, this parameterization of the rule is an efficient one in that no other coefficient values can produce lower unconditional variances of output and inflation simultaneously without increasing the variability of the federal funds rate.¹⁷ Formally, equation (2) is the solution to a problem that minimizes the following loss function:

$$E_0 \sum_{i=1}^{\infty} \varphi [\bar{\pi}_{t+i} - \pi_{t+i}^*]^2 + (1 - \varphi) y_{t+i}^2 \quad (3)$$

subject to the model, a constraint on the functional form of the rule, namely that of equation (2), and to a constraint on maximum variability of the federal funds rate.¹⁸ By choosing a (constrained) optimal rule in this way we avoid specious results that might arise simply from the use of a poorly performing rule. In addition, we have the advantage of being able to consider policymakers' preferences directly. Equation (2) is the (constrained) optimal rule for an authority with relatively 'hawkish' inflation preferences: $\varphi = 0.75$; we shall examine alternative preferences a bit later.

The third difference from the Taylor rule is that the target rate of inflation, π_t^* , carries a time subscript. We allow time variation in the monetary policy target and assume that private agents only know the actual target with a one quarter lag.¹⁹ We consider two

17. Equation (2) also dominates rules with shorter moving averages in the inflation rate as judged by the same criterion of minimized unconditional variances. Note that we have arbitrarily rounded the many decimal places used in the rule actually used for simulation for expositional simplicity.

18. Having some sort of penalty on funds rate variability provides two benefits: (1) the optimally computed coefficients on the policy rule are of a magnitude that would be considered reasonable; and (2) the implied variability of the federal funds rate is close to the historical experience. The same qualitative results obtain when funds rate volatility is mildly penalized instead of the hard constraint we use here.

19. As discussed below, private agents know the target with a one quarter lag because they are assumed to know the policy rule. This means that they can simply invert to rule one a quarter-to-quarter basis to find out what the target was. Some implications of relaxing the assumption and having agents learn the target will be discussed later on.

possible data generating processes for the target rate of inflation: a random walk and a bounded random walk.²⁰ Clearly no authority would truly permit the target rate of inflation to vary from plus to minus infinity, so the pure random walk case should be thought of as a benchmark. In both cases, the law of motion for the target looks the same:

$$\pi^*_t = \pi^*_{t-1} + \mu_t \quad (4)$$

The difference is in the distribution of innovations. In the pure random walk case, the innovations, μ_t , are independently and identically distributed:

$$\mu_t \sim N(0, \sigma^2) \quad (5)$$

In the bounded random walk case, innovations are bounded by the target bands. Letting $\bar{\pi}^*$ and $\underline{\pi}^*$ designate the upper and lower bounds for π^* , Johnson *et al.* (1994, p. 156) show that the probability density function of innovations is:²¹

$$\mu_t = \frac{1}{\sigma\sqrt{2\Pi}} \exp\left[\frac{-\mu^2}{2\sigma^2}\right] \left\{ \frac{1}{\sigma\sqrt{2\Pi}} \int_{(\underline{\pi}^* - \pi^*_{t-1})}^{(\bar{\pi}^* - \pi^*_{t-1})} \exp\left[-\frac{(t - \pi^*_{t-1})^2}{2\sigma^2}\right] dt \right\}^{-1} \quad (6)$$

As ugly as equation (6) looks, it is simply a truncated normal distribution, with truncation points determined by band widths and the inherited rate of target inflation, given σ . That is, the truncation points, $\{\underline{\pi}^* - \pi^*_{t-1}, \bar{\pi}^* - \pi^*_{t-1}\}$ shift around with the inherited target inflation rate. We assume that the bands are spaced symmetrically around a given midpoint, which we can set to zero without loss of generality. Let us also assume that the bands on the target rate of inflation are +1 and -1 percent respectively, as in our base-case simulations below. When the inherited target inflation rate happens to be zero, the distribution of innovations is simply the normal distribution with the tails beyond +1 and -1

20. The results from ADF tests regarding the order of integration of the price level depend on the number of lags permitted and the precise span of data tested. In any case, the premise behind this chapter is that the monetary authority can choose the order of integration even if it has, at times in the past, elected not to do so.

21. The Π appearing in two terms of equation (6) should not be confused with the symbol for inflation, π , used elsewhere.

chopped off. Besides being symmetric, with our base-case values of σ of 0.25, this distribution is functionally identical to the normal distribution because the truncation points are four standard deviations away. When the inherited target rate of inflation is positive, the upper bound on innovations moves to the right, eliminating more of the positive innovations that would otherwise be possible, and adding more of the negative innovations. If the inherited target rate of inflation were exactly 1 percent, the distribution of possible innovations would be exactly one-half of the normal distribution.²²

The case of a fixed target rate of inflation is nested within equation (6), as $\pi^* = \bar{\pi}^* = 0$. In addition, permitting $\bar{\pi}^* \rightarrow \infty$ and $\pi^* \rightarrow -\infty$ gives the pure random walk case.

It is important to note that these boundaries for the target do not imply the same boundaries for inflation itself. In the short run, many forces are at work on inflation of which only one is monetary policy. Moreover, monetary policy is not solely concerned with stabilizing inflation. Lastly, even in the presence of a fixed objective, inflation is buffeted by shocks from a variety of sources such that only if each shock can be instantaneously identified and policy can be tailored explicitly for each shock could inflation be controlled perfectly. Thus, in general, inflation itself will be more variable than the target.

Two issues regarding the law of motion for π^* warrant discussion. The first of these is the assumption that π^* moves *randomly* within the target bands. If one thinks of the drifting target as representing discretion, it might be preferable to allow target innovations be correlated with other shocks. However, as discussed in the preceding subsection, time variation in the *de facto* preferences of the Fed do not appear to have arisen from a

22. We are modeling a discrete-time version of what is sometimes called reflecting barriers. With reflecting barriers, that proportion of the distribution of targets that would have called for targets outside of the bands is discarded. An alternative is absorbing barriers in which the truncated portion of the distribution is assigned to the boundary value causing a tendency for target rates of inflation to mass at the boundary edges. With reflecting barriers, the target rate of inflation almost never reaches the boundary, as we shall see. Dixit (1993) provides a thorough discussion of the mathematics and modeling of both types of barriers in continuous time.

conscious selection of different priorities than before (except, perhaps, for the Volcker disinflation). Instead they appear to have come from an unwillingness to respond to shocks with sufficient strength to be consistent with a constant target. Indeed, the tendency has been, in the words of the Fed's Director of Monetary Affairs, to "adjust nominal rates too slowly in the initial stages of a cycle, and then to overstay a policy stance" while awaiting further information.²³ While the process that Kohn alludes to will produce a correlation with shocks, it is only to the extent of the sign of the shocks--upward in the 1960 and 1970s, and downward in the 1990s. In this context, the assumption adopted here does not seem like an egregious oversimplification.²⁴

Finally, and ultimately, the process we have selected to represent time variation in the target rate of inflation is both simple to understand and tractable from a numerical perspective. Allowing innovations to the target rate of inflation that are correlated with state variables or other shocks would oblige us to adopt non-recursive solution techniques which would be prohibitively slow to compute.²⁵

The second issue is that we also assume that the bounds on the target are regarded as fully credible by private agents. This amounts either to assuming that there is a commitment technology that prevents violations of the bands, or that a record of inflation performance has earned the monetary authority a reputation for controlling its target.²⁶ A commitment technology might arise out of the nomination process for Board members. Presidential nomination and Senate approval may be viewed as vetting candidates for the

23. Kohn (1991, p. 101). It is the Director of Monetary Affairs that makes the staff's policy recommendation to the FOMC. Kohn is also the Secretary to the Federal Open Market Committee.

24. We might also note that to the extent to which innovations to the target were *systematically* correlated with shocks or other state variables, it should show up in the specification of the rule itself. And to the extent these shocks are correlated with the output gap, they may already be captured in the Taylor rule coefficient for that variable.

25. We note as well that this specification for the target rate of inflation allows a straightforward comparison of our results with those of the exchange-rate target-band literature which is related to the modeling undertaken here.

26. There is a huge literature on credibility and commitment in monetary policy games, beginning with the seminal contributions of Kydland and Prescott (1977) and Barro and Gordon (1983).

minimum level of intolerance for inflation. Reputation might be earned by establishing an appropriate track record for performance, and buttressed by making monetary policy sufficiently transparent that imprecise control can be distinguished from time-varying preferences.

2.3 Expected future inflation targets

In the preceding subsection, the target rate of inflation was modeled with a truncated normal distribution. The truncation at any given date is determined entirely by (1) the inherited target inflation rate; (2) the variance of innovations; and (3) the band width. In this subsection, we describe the expected future target rate of inflation conditional of some initial value. The nature of this expectation is key to the results that follow because in the FRB/US model, expected *future* target rates of inflation feed into *current* price- and wage-setting behaviour.

Private agents are assumed to know the process for the target rate of inflation, but not the current value of innovations. Since they know the inherited target inflation rate, the expected value of the current target can be computed, $E_{t-1}\pi_t^*$. And because innovations are independent, this value can then be taken as given and a value for $E_{t-1}\pi_{t+1}^*$ can be computed, and so on, in a straightforward application of the chain rule of forecasting.

As we have already noted, at the midpoint of the bands the truncation is symmetric, the truncation points being equally distant from the inherited target rate. It follows that $E_{t-1}\pi_{t+j}^* = \pi_{t-1}^* = 0$: the pure random walk prediction obtains in this special case. It was also noted that when $\pi^* = \bar{\pi}^* = 1$, the entire upper half of the normal distribution is truncated and only the lower half is pertinent. It follows that the complete distribution of future target inflation rates is (weakly) less than the current target and so the optimal prediction is for a decline in the future target. The same argument applies for inherited targets at the lower bound, except of course with the opposite sign. Everywhere in between these two extremes, there are varying degrees of asymmetric truncation, but as a general propo-

sition it is clear that (i) the expected future target approaches the midpoint of the bands as time approaches infinity; and (ii) the initial rate of reversion toward the midpoint will be vary positively with the proximity of the inherited target rate to the bound, given the standard deviation of innovations. Note as well that private agents' expectations of inflation targets are entirely consistent with the modeling of actual targets.

Johnson *et al.* (1994, pp. 156-162) show how to compute the probability density function for the truncated normal. Based on this, Figure 2 shows the expected future path of $E_{t-1}\pi^*_{t+j}$ for three initial values of π^*_{t-1} , given $\sigma = 0.25$, and symmetric bands with the upper value arbitrarily set at $\bar{\pi}^* = 1$.²⁷

The expected future path of target rates of inflation depends only on π^*_{t-1} , $\bar{\pi}^*$ and σ . Larger values of σ produce stronger non-linearities because of higher probabilities of truncation for any given inherited target rate (other than zero). Symmetrically increasing the width of the target band, $\bar{\pi}^* - \underline{\pi}^*$, has the opposite effect.

27. The standard deviation of innovations of 0.25 percent taken from the standard deviation of changes in the Philadelphia survey of inflation expectations, 10 years ahead, our proxy for the public's expectation of inflation in the long term.

Figure 2

Agents' Inflation-Target Forecasts
(bounded random walk, width = ± 1)

3. The FRB/US Model

In this section we outline the basic features of the FRB/US model, focusing mostly on those aspects that are germane to the issues studied in this chapter, and describe some of the model's properties.²⁸ FRB/US is a relatively large rational expectations structural model consisting of about 30 key behavioural equations and several hundred identities. It features forward-looking behaviour in asset prices, wages and goods prices, and consumption and investment spending. Rigidities in wage and price setting imply that monetary policy has effects on the real economy in the short-run, but in the long run, policy can affect only the inflation rate.

At its heart, FRB/US is a neoclassical growth model. That is, once all dynamic adjustment is complete, the model settles down on a balanced growth path with consumption determined by wealth, wealth determined for the most part by productivity and steady-state real interest rates, and real interest rates determined by the savings-investment decisions of private agents in concert with government.

The model is non-Ricardian. Of the several reasons which account for this feature, the most important is that risk-averse agents 'overdiscount' their future disposable income, in the same spirit as the Blanchard (1985) model of perpetual youth. For this reason, among others, fiscal policy can act upon the long-run equilibrium of the economy through purchases, transfers, government debt, and distortionary taxes.

FRB/US is designed around the joint-determination of private-sector expectations and public policy. In pursuit of this goal, about half of the model's dynamic behavioural equations are explicitly derived as decision rules governing the behaviour of representative agents acting with foresight to achieve explicit objectives in the presence of con-

28. Limitations of space prevent us from doing justice to the features of the FRB/US model here. Interested readers should consult Brayton and Tinsley (eds.) (1996) on general aspects of the model. Particularly accessible descriptions of the modeling of expectations, and the monetary policy transmission mechanism, respectively, are Brayton *et al.* (1997) and Reifschneider *et al.* (1999). An application of the model to the analysis of disinflations is Bomfim *et al.* (1997).

straints. In the special case of the asset price equations, adjustment costs are taken as negligible, so that arbitrage relationships determine outcomes. More generally, forward-looking agents plan in advance to adjust the value of their decision variables to converge over time on (possibly moving) target levels, while balancing the cost of being away from the desired level against the cost of adjustment. This is a key feature of the model as it pertains to the simulation exercises in this chapter. Adjustment costs mean that private agents must plan in advance to set out a path for wages and prices that minimize those costs. These plans are conditional on expected future demand conditions, relative prices, and policy actions, including the expected future target rate of inflation. Thus, expectations of future targets affect current decisions and determine, in part, current inflation rates.

One of the constraints under which agents formulate their plans is the set of available information. For the purposes of estimation, it was assumed that information is constrained to a small set of aggregate macroeconomic variables, summarized by a low-order vector autoregression. For simulation purposes, however, it is possible to incorporate any of a number of assumptions regarding the availability of information and its subsequent gathering over time. Thus, users can simulate the model under VAR-based expectations, model-consistent expectations, or hybrid forms of expectations formation. In this paper, we shall utilize only model-consistent expectations.²⁹

29. One might describe VAR-based expectations as reduced-rank rational expectations, or limited-information rational expectations. Consistent with this interpretation, Brayton and Tinsley (eds.) (1996) shows that for disturbances that arise from sectors represented by the VAR itself, such as random shocks to the federal funds rate, the dynamics of model responses under model consistent expectations and VAR-based expectations are quite similar.

Table 1
Estimates of a Simple Monetary Policy Reaction Function
(1980Q1 - 1997Q3)

$$R = (1 - \alpha_{R1})(\bar{\pi} + rr^*) + \alpha_{R1}R_{t-1} + \alpha_{y0}y_t + \alpha_{y1}\Delta y_t + \alpha_{\pi}(\bar{\pi} - \pi^*)$$

symbol -->	α_{R1}	α_{y0}	α_{y1}	α_{π}	π^*
coefficient	0.83	0.14	1.33	0.32	3.2
(standard error)	(0.07)	(0.08)	(0.51)	(0.11)	(0.6)

$rr^* = 2.5$ (assumed) **SEE = 1.31**

Notes: R is the nominal federal funds rate (quarterly average basis); $\bar{\pi}$ is the four-quarter rate of change of the (chain-weight) personal consumption deflator, y is output measured in percent deviations from potential output; rr^* is the steady-state real interest rate (taken to be 2.5 percent); and π^* is the target rate of inflation. Potential output in FRB/US is constructed from split time trends on the constituent parts of potential, including the participation rate, the labor force, the NAIRU and trend total factor productivity. The steady-state real interest rate also comes from the FRB/US database and is consistent with approximately steady inflation and stable growth.

We now demonstrate the basic properties of FRB/US. To do this, we compare the properties of the model and those of the historical data using three methods: inflation and output autocorrelations, impulse responses to a shock to the federal funds rate equation, and the parameters of a dynamic IS curve equation. To begin, we need to characterize the behaviour of monetary policy. To this end, we estimate a parsimonious interest-rate reaction function for the period from 1980Q1 to 1997Q3, arguably a single monetary policy regime. Table 1 shows the estimation results.

Like the Taylor (1993, 1994) rule, this rule is simple, but it adds some dynamic richness through the inclusion of two terms, one with the change in the output gap, and the other with the lagged federal funds rate. In comparison to the Taylor rule, which sets

$\alpha_{y0} = 0.5$, $\alpha_{\pi} = 0.5$ and all other $\alpha_i = 0$, this rule is much more aggressive in its pursuit of the inflation target.

For the model-based analysis in this section, we assume policy follows this rule without error and that the inflation target is constant; we also assume the policy rule and the inflation target are known by the public with certainty. Figure 3 shows the autocorrelations of inflation, measured by the chain-weighted index of personal consumption expenditure prices, and the output gap.³⁰ In both instances, the asymptotic model moments are represented by the dashed line and the moments estimated from the data are shown as the solid line. The dotted lines are one-standard-error bands for the data-based moments.³¹

The model- and data-based moments are computed using the data and equation residuals from the 1980s and 1990s. This sample period was chosen to approximate a single policy regime and to correspond with estimated policy rule of Table 1.

As emphasized by Fuhrer and Moore (1995), inflation has been very persistent, historically, even during periods in which there were no apparent shifts in the inflation target. Owing to the generalized adjustment cost structure of wage and price adjustment, the FRB/US model also displays a significant amount of inflation persistence, albeit somewhat less than that evident in the data.³² However some of the persistence evident in the data is due to the large disinflation that occurred during the first few years of the sample. If the first two years are removed from the sample, the model and data autocovariances are nearly identical at the first four lags; nevertheless, at lags five through ten, the model still underpredicts inflation persistence.

30. Henceforth, the terms output and output gap will be used interchangeably to refer to the excess of aggregate demand over potential output measured in percentage points, just as noted in the text. Inflation shall refer to the rate of change of the chain-weighted personal consumption expenditure price index, either quarterly expressed at annual rates, or the four-quarter rate, as applicable, except as otherwise noted.

31. The model moments used here and below are computed from the formula for unconditional covariances. As such, the computed moments correspond to those that would be obtained from a stochastic simulation of infinite length.

32. See also the results reported in Brayton *et al.* (1997).

Figure 3
Inflation and Output Persistence (Estimated Rule)

The model shows a great deal of persistence in the output gap. Sluggish adjustment of demand components, combined with persistence in the real interest rate driven by the propagation of movements in inflation, drive this result. In this case, autocorrelations from the model and from the data correspond closely.

This persistence in inflation and output is a common feature of structural macro models--both traditional and those incorporating rational expectations--as well as recent sticky-price general-equilibrium models such as those analyzed by Ireland (1997) and Rotemberg and Woodford (1997). The key distinction between the two classes of models is that the former achieve the bulk of this persistence through the inclusion of intrinsic inertia--that is, from lagged dependent variables--while generally maintaining the assumption of serially uncorrelated disturbances, while the latter are formulated under the assumption that there is little inherent inertia in inflation and spending, and it is the disturbances themselves that are serially correlated.

Now let us consider the impulse response from a disturbance to the estimated federal funds rate. Figure 4 shows the response of the funds rate, inflation, and the output gap. Also shown is the response from a VAR model estimated over the period from 1981 to 1995, together with one-standard-error bands. The FRB/US model's information timing assumptions imply virtually no contemporaneous response to a funds rate movement. The figure shows that the magnitude and the general pattern of output and inflation in FRB/US are close to those of the data.

Figure 4
Impulse Response to Funds Rate Shock
(100 basis points)

Our final method for assessing the properties of FRB/US is to estimate a standard IS curve equation using model moments, and compare the model-generated estimates to those from the historical data.³³ This approach is appealing in that differences between the model and the data are readily interpretable in terms of well-known macroeconomic concepts such as the interest sensitivity of aggregate demand, the magnitude of accelerator effects in aggregate demand, and so forth. Table 1 reports estimates of a simple dynamic IS equation. The specification of the IS equation is the same as in the Fuhrer–Moore model and is notable for its inclusion of output inertia, dependence of demand on the real long-term interest rate, and the lack of explicit permanent income terms. We report results for the sample from 1966Q1 to 1995Q4, but these estimates are not sensitive to the choice of sample for the data or the model. As seen in the table, the estimates from the model and those in the data are very similar, with the only statistically significant difference being the higher interest-sensitivity of demand in the FRB/US model.

In sum, FRB/US is a relatively large structural rational expectations model that incorporates significant intrinsic inertia in inflation and real spending. The model fits the data well, both in terms of the individual equations and in terms of its system properties. The model's explicit disentanglement of intrinsic sources of dynamic propagation from expectational sources makes it well suited for the policy analysis experiments to which we shall turn shortly. However, before doing that, we need to discuss the numerical techniques employed to study this issue.

33. Of course, this procedure can also be applied to estimating Phillips curve equations, but there is little gain from doing so. Unlike the IS curve, inflation dynamics in the model are very sensitive to the specification of monetary policy. And over the period from 1980Q3 to 1997Q3, small sample problems plague the fit of estimated Phillips curves.

Table 2
Estimates of an IS Equation from Historical and Model Data
(1966Q1 - 1995Q4)

$\Delta y_t = \beta_{y1}y_{t-1} + \beta_{y2}\Delta y_{t-1} + \beta_{r1}rrcb_{t-1} - \beta_{r1}rrcb^*$						
symbol -->		β_{y1}	β_{y2}	β_{r1}	rr^*	SER
Model	coefficient	-0.13	0.25	-0.33	n/a	0.94
Data	coefficient	-0.13	0.30	-0.15	4.17	0.84
	(std. error)	(0.03)	(0.08)	(0.05)	(0.45)	

Notes: *rrcb* is the real return on 30-year corporate bonds, discounted for coupon payments, with expected inflation computed from a constrained VAR using methods described in Brayton and Tinsley (eds.) (1996); and *rrcb** is the steady-state long-term real interest rate which is recovered from the constant term and shown in the table column second from the right. See also the Notes to Table 1.

4. Numerical Methods

In all cases studied in this paper, we use a linearized version of FRB/US. Linearization serves two purposes. First, it ensures that all non-linear relationships that arise as endogenous phenomena of the experiments are easily isolated and quantified. Second, it simplifies computational burdens immensely. Since FRB/US is close to linear in the first place, and since the model's non-linearities have little to do with monetary policy issues, there is little cost in terms of accuracy from linearizing the model.

Setting aside the target bands, a linear rational expectations model can be written compactly in the following form:

$$A_1 \begin{bmatrix} y_{t+1} \\ x_{t+1|t} \end{bmatrix} = A_0 \begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} u_t^y \\ 0 \end{bmatrix} \quad (7)$$

where y_t is a vector of predetermined variables, and x_t is a vector of non-predetermined, or jump, variables. The first-order form of equations (7) can be constructed by compressing the lags longer than one in the original structure form model by creating artificial variables; that is by taking a second-order (or higher) equation, $C(L)y = c_0 + c_1 y_{t-1} + c_2 y_{t-2}$ and defining $v_t = y_{t-1}$ and $v_{t-1} = y_{t-2}$ so as to create two first-order equations out of one second-order equation, and so on.

From equations (7), efficient computations can be done if a VAR representation of (7) can be constructed. In principle, this is straightforward enough, we need only find a matrix $A = A_1^{-1} A_0$ by premultiplying (7) by A_1^{-1} :

$$\begin{bmatrix} y_{t+1} \\ x_{t+1|t} \end{bmatrix} = A \begin{bmatrix} y_t \\ x_t \end{bmatrix} + A_1^{-1} \begin{bmatrix} u_t^y \\ 0 \end{bmatrix} \quad (8)$$

Two problems come up in computing this state-space form of the model. The first is a numerical issue the possible singularity of the matrix A_1 . Without a nonsingular A_1 matrix, the state-space representation of the model cannot be constructed, meaning that a recursive representation of the forward-looking model would not be possible. This is a common occurrence, particularly in large-scale rational expectations models. The model would then have to be solved using some iterative method, such as the extended path methods of Fair and Taylor (1983), or its stacked-time counterpart, Laffargue (1990). Given that we are interested in stochastic simulations of the model, the numerical cost of the using extended-path solution methods would be orders of magnitude more time consuming than a recursive method and would be subject to errors in measurement. Fortunately, the development in recent years of new methods of constructing state-space representations of linear forward-looking models obviates these complexities. For this paper, we employ a version of algorithm of Anderson and Moore (1985), updated in Anderson (1999), which uses a QR decomposition to find a nonsingular equivalent to the matrix A_1 .

A second, related problem is the assurance of saddle-point stability of the model. Blanchard and Kahn (1980) outline the conditions that are necessary to assure saddle-point stability. These are that the number of roots of the matrix A that are greater than unity equal the number of non-predetermined variables. Algebraically, different target rates of inflation are simply constant terms, so that if the model is saddle-point stable for one target rate of inflation, it is stable for all target rates. A test of the model confirmed that it is stable with 52 non-predetermined variables and 52 eigenvalues greater than unity.

While the model itself may be linear, the preceding discussion makes clear that the expectations mechanism for future targets is fundamentally non-linear. Even in the presence of this non-linearity, however, there is a way in which we can exploit linear on-line algorithms for solving models.

Given some initial π^*_{t-1} , a vector $E_{t-1}(\pi^*_{t+i} | \pi^*_{t-1}, \sigma, \bar{\pi}^*)$, a vector is computed outside of the model. Using a discrete approximation of the truncated normal distribution, this is possible because $E_{t-1}\pi^*_{t+j}$ is independent of the state variables of the system. Then this non-linear path is approximated to arbitrary precision with a simple ARMA model, 'reverse engineering' the shocks necessary to replicate the non-linear $E_{t-1}\pi^*_{t+j}$ at each date. The form of the equation stays the same at each iteration of the algorithm; however the historical shocks to the moving-average errors are changed depending on the initial value of $E_{t-1}(\pi^*_t | \sigma, \bar{\pi}^*)$. An ARMA(1,25) model was chosen which means that specific values of the shocks can be chosen such that the linear ARMA model represents exactly the non-linearity up to $j=25$ and is approximately correct thereafter. In the state-space representation, each additional moving-average term adds an additional equation to the model, so there are trade-offs at work between speed and accuracy. Some experimentation showed the ARMA(1,25) approximation to be a very good one. In terms of equations (8), the state matrix is augmented with variables for the target as follows:

$$\begin{bmatrix} u_{t+1}^* \\ \pi_{t+1}^* \\ y_{t+1} \\ x_{t+1|t} \end{bmatrix} = A \begin{bmatrix} u_t^* \\ \pi_t^* \\ y_t \\ x_t \end{bmatrix} + A_1^{-1} \begin{bmatrix} \varepsilon_t \\ 0 \\ u_t^y \\ 0 \end{bmatrix} \quad (9)$$

so the π^* block of equations (9) is of dimension 1 with a fixed ρ in the appropriate part of the matrix A and the u^* block is of dimension 25. These “random shocks”, ε_t , are selected in such a way that they just so happen to give a non-linear expected future path for the target. This works because once the form of the recursive system is determined, (9), any sequence of shocks can be supplied without having to bear the computational cost of resolving the model. Since the inherited target rate of inflation is known, the precise sequence of shocks consistent with the correct expected future path for the target can be fed in.

To summarize, we trick the linear algorithm to give non-linear forecasts with non-random shocks of a particular sequence.

The shocks necessary to generate the nonlinear forecast of π_{t+i}^* can be computed ahead of time, indexed against the initial π_{t-1}^* and stored in a grid, which also saves some computing time.

Using this method, the time necessary to compute the simulations of 500 draws of 200 periods each described in Section 3 was approximately 32 minutes using one processor a Sparc Ultra 3000 333 megahertz machine. A rough estimate of the computing time that would have been necessary to conduct the same exercise using non-linear extended path solution methods would be several weeks of continuous processing.

5. Results

Our results come from stochastic simulations of the FRB/US model. Each experiment is comprised by 500 simulations of 200 periods each. After discarding the first 20 observations, we are left with 500 replications times 180 periods or 90,000 observations per experiment. In most instances, we take the standard deviation of innovations to the target to be 0.25 percent. This is the standard deviation of innovations to the Federal Reserve Bank of Philadelphia's survey of expectations of inflation of 10 years ahead; inflation this far into the future can presumably be regarded as under the control of the monetary authority and thus this measure of expectations can be taken as the expectation of the target rate of inflation. The width of the bands is ± 1 percentage point, except as noted.

For most of these simulations, monetary policy is governed by equation (2), without error. Thus, at the beginning of each period, agents can infer π^*_{t-1} from last period's federal funds rate setting. Agents then form an expectation of the future path of the target using equations (4) and (5), or (4) and (6), as applicable.

5.1 Model properties with target bands

Figure 5 shows the mapping of inflation against the target rate of inflation from our stochastic simulations. (Ignore the dotted lines for the moment.) Two cases are shown. The dashed line is the mapping of inflation against the target when the target follows a pure random walk. This mapping, which is labeled $g_{rw}(\pi^*)$, is diagonal: the relationship, on average, between inflation and its target, is one-for-one when the target follows a pure random walk.

Now consider the solid line. Notice that it has roughly s-shaped. It is, moreover, flatter than g_{rw} , meaning that the range for actual average inflation is less than the range for target inflation. Let us designate this mapping $g(\pi^*) = E_{t-1}\pi(\pi^*|\bar{\pi}^*, \sigma)$ where the right-hand side of the equality notes that this is the expected value of the mapping of infla-

tion on target inflation, conditional on the location of the upper (and lower) bound, and on the size of innovations to the target.³⁴

Notice as well that while the bands are between plus-and-minus one percentage point, the target never actually reaches the edge of the band. For this to happen, one would have to draw exactly the right shock to reach the edge of the band; and the probability of any one particular value is vanishingly small.³⁵ Comparing this line with the mapping for our unbounded process shows the influence of the credible target band: as π^* gets up close to $\bar{\pi}^*$ (down close to $\underline{\pi}^*$), inflation itself falls below (rises above) the current π^* , on average. The extent to which this occurs is a function of the curvature of expected future target values. In the present case, at the upper end of the curve, where the target rate of inflation is nearing the upper bound, the four-quarter inflation rate averages about 0.55 percent, or more than four-tenths below the target inflation rate. By contrast, when π^* is close to the midpoint, $g(\pi^*)$ is very close to the linear mapping of the pure random walk case. This outcome illustrates that the establishment of target bands for inflation gives rise to what the exchange-rate target-band literature calls the *honeymoon effect*: the promise of fidelity in constraining drift in the target rate of inflation variability is rewarded by restraint in the range of the average inflation rate.³⁶ As we describe in detail below, this operates through expectations of future target inflation rates impinging on current price- and wage-setting decisions.

The two dotted lines on Figure 5 are the one-standard-error bands for inflation in an economy where there are only shocks to the target rate of inflation. The widths of these bands are not particularly important *per se*, since the real world is subject to a much wider

34. Figure 5 and all subsequent figures are constructed by sorting and ordering observations of π^* from π^* to $\bar{\pi}^*$, gathering the corresponding observations of π , y , rr and so forth. The matrix of sorted observations is then group into cells of identical widths and average across the values of each cell. It is these averages for the cells that are shown in the graphs.

35. This is a manifestation of our using reflecting barriers. Absorbing barriers would reach the boundary with some probability mass.

36. See, e.g., Krugman (1991).

variety of shocks. What is interesting is the curvature of the confidence bands and their narrowing as the boundaries for the inflation target are approached. This narrowing indicates is the smaller range of probabilities for future target rates of inflation when the current target is at the edge of the band. Recall that if the inherited rate of inflation were to reach the edge of the band, fully one half of the distribution of target rates one step ahead would be truncated, reducing the range of future target rates of inflation.

Figure 5

Inflation versus Inflation Target
(shocks to the target only)

We have established that credible inflation target bands can constrain the range of inflation outcomes, relative to a pure random walk. We now examine some general equilibrium aspects of policy design with credible inflation target bands. To do this, we need to consider a wider range of variables, which we can do with the aid of Figure 6.

Before explaining the economics behind the lines in Figure 6, let us first just describe what the four quadrants represent. The upper-right quadrant of this four-quadrant diagram shows $g(\pi^* | \bar{\pi}^* = 1)$ and so merely repeats Figure 5--except without the error bands. Moving to the upper-left quadrant, we have the mapping of inflation on the output gap. This is *not* the slope of a Phillips curve; rather, this is the (mean of the) stochastic relationship between the target inflation rate and the actual, and various target rates. The bottom-left quadrant shows the mapping of output and the real federal funds rate, both measured in deviations from steady-state levels. Finally, the bottom-right quadrant completes our macroeconomic tour with the mapping of the real federal funds rate against the target rate of inflation. This four-quadrant graph portrayal allow us to trace the average levels of four variables at once, as illustrated by the two cases marked by the dotted lines.

Figure 6
Summary of Macroeconomic Outcomes
(random walk versus inflation target bands)

Now let us walk through the economics of Figure 6. Our base case is shown as the thin solid line with the case where the target follows a pure random walk being designated by the dashed line. We begin with point A on the base-case mapping $g(\pi^*)$ in the upper-right quadrant. At this point, $E_{t-1}(\pi_t | \pi_t^*) < \pi_t^*$ because private agents know that the target is more likely to fall than rise in the future. Because firms find it costly to adjust prices and wages, firms must plan ahead to minimize costs and so this expectation influences their price- and wage-setting behaviour *today*, and thereby affects realized inflation today.

Moving along the dotted line to point B in the upper-right quadrant we see that point A corresponds with an output gap that is positive, on average. We see that the relationship between the average level of output and inflation is positively sloped and linear in the base case, while in the random-walk case, there is no output gain to be had from allowing drift in the target.³⁷ The random-walk case is simply an outcome of the superneutrality of the model: there is no trade-off between output and inflation and thus no slope of the dashed line in the upper-left quadrant. For the base-case solution, the ‘bias’ in inflation expectations relative to the current target rate of inflation means that output can be positive because inflation is expected to fall in the future.³⁸ To see why this is so, we need to move to the lower-left quadrant.

In the lower-left quadrant, we observe a negative relationship between the output gap and the average level of the real interest rate. Notice that at point C, the real federal funds rate is slightly *negative* even though output is substantially positive. This is feasible

37. There are no surprise changes in target values in our set-up and so the kind of short-run manipulation of the Phillips curve that drives results in models like Barro-Gordon (1983) is not at work here.

38. It is also useful to think of this from the monetary authority’s point of view. The authority is attempting to moving inflation to the current-period target shown at point A. But because the expected future target is lower than the current target, current inflation is lower than the current target, on average. By contrast, in the random-walk case, inflation, on average, will be at the target value. This means that policy will be easier at target rates of inflation like point A with bands than it would be when the target follows a random walk. This we see in point C in the lower-left quadrant. The easier policy, in turn implies an average positive output gap as shown by point B in the upper-left quadrant.

because the “bias” in expected inflation—which carries a large weight of 3 in the policy rule—offsets the positive output gap. To put the same idea a different way, the lower expected future inflation rate at point A means that there can be a lower real interest rate at the corresponding point C, which supports higher output at point B.

Note that the curved line for the base-case model implies that the quasi-steady-state interest elasticity changes non-linearly with the target rate of inflation. The shape of the curve comes from the cumulative normal distribution. In the random walk case, it is constant.³⁹ Roughly the same shape is witnessed in the top-right quadrant showing the mapping between inflation and target inflation, except that the curve is stretched out in this instance. This is a manifestation of the supply shocks of the model. The lower the proportion of shocks to the supply side of the model, the more of the shape of the cumulative normal distribution is retained and the larger the honeymoon effect.

Finally, moving to point D in the lower-right quadrant, we are reminded that the real interest rate is generally negatively related the target rate of inflation. In the pure random walk case, the horizontal dashed line shows that there is no relationship, on average, between the target rate of inflation and the real interest rate. At point D, the real interest rate is lower than it would be if agents did not believe the target was bounded. The expectation of a lower target rate of inflation in the future allows the current real interest rate to be lower than it otherwise would be, thereby supporting higher output than usual.

In sum, we see that the implications for the monetary authority of credible target bands for inflation are (i) a narrower range of inflation for a given range of the inflation target; (ii) a positive range of output, meaning that there exists a trade-off between output and inflation, (although it cannot be said that this is a long-run trade-off); and (iii) a range for real interest rates.

39. We show the interest elasticity of aggregate demand in the random walk case to be the -0.33 value computed in Table 1.

5.2 *Quantitative performance*

The choice of target band limits of plus-or-minus one was an arbitrary one. To explore the implications of alternative assumptions, Table 4 shows the effect on aggregate (unconditional) volatility of alternative choices. The statistics are based on stochastic simulations using the full set of stochastic shocks and pooling over all simulated observations. The first column in the body of the table corresponds to a credible fixed target rate of inflation, the usual assumption undertaken in assessments of policy rules. The last column covers the pure random walk case. In that case, the values of many simulated variables are unbounded and shown as “n/a”.⁴⁰

The most remarkable observation from Table 4 is how unremarkable the differences are: tighter target band limits improve performance of the economy measured in terms of the unconditional standard deviation of output, inflation and the funds rate, but not by a great deal.

40. In this case the range is for consumer price inflation as measured by the four-quarter growth rate in the chain-weight personal consumption expenditure price index. All the countries that have announced specific targets for inflation have named a 12-month rate of some consumer or retail price index as the target variable. The FRB/US model does not have a CPI and is quarterly model. Thus, we use the growth rate of the four-quarter PCE price index here.

Table 3
Simulated Standard Deviations of Target Band Regimes

Bound on inflation target					
Column no. -->	(1)	(2)	(3)	(4)	(5)
Endogenous variable	$ \bar{\pi}^* = 0$	$ \bar{\pi}^* = 0.5$	$ \bar{\pi}^* = 1.0$	$ \bar{\pi}^* = 1.5$	$ \bar{\pi}^* = \infty$
R	2.76	2.87	3.01	3.11	n/a
y	2.70	2.74	2.90	2.98	3.10
$\bar{\pi}$	0.88	0.93	1.04	1.21	n/a

Note: R is the nominal federal funds rate measured on a quarterly average basis, y is the output gap, measured in terms of percent deviations from potential output; $\bar{\pi}$ is the four-quarter inflation rate; i.e., $\bar{\pi} = \sum_{i=0}^3 \pi_{t-i}/4$; Numbers in the first three rows of the body of the table are standard deviations computed from pooling 500 draws of 180 periods each, after discarding twenty observations per draw to account for initial conditions. In all cases 51 stochastic shocks are drawn using the variance-covariance matrix of the period from 1981Q1 to 1995Q4.

At the outset of this chapter, we noted that among the devices through which inflation targeting countries attempt to commit to constraining their discretion are ranges *for inflation* (as opposed to bands for target inflation).⁴¹ In Table 4 we examine some aspects of the efficacy of ranges for inflation by looking at the likelihood of inflation rate departing from selected ranges. These ranges, denoted by $\bar{\pi}$ and $\underline{\pi}$, are shown down the first column of the table. Observe that they refer to *ranges for inflation* itself. We examine

41. It is interesting in this regard to note the motivation for ranges for inflation as given by John Crow, then Governor of the Bank of Canada (1991): "The purpose of setting out formal targets is to provide a clear indication of the downward path for inflation over the medium term so that firms and individuals can take this into account in their economic decision-making... [If people] base their economic decisions on this declining path for inflation, the objectives can be readily achieved and will contribute to lower interest rates. The inflation targets also provide information... [that] should provide a better basis than before for judging the performance of monetary policy" (Bank of Canada, 1991, p. 11).

three ranges for inflation-- , without the asterisk but with a tilda overstrike to indicate that it is the four-quarter rate--measured in terms of deviations from the steady-state rate of zero.⁴² To keep the syntax clear, we shall refer to desired variability of inflation itself as *ranges*, and limits on the inflation target as *bands*. An examination of this sort gives an idea of the extent to which commitment to a range of inflation (an observable variable) can stand in for commitment to a band width on target inflation (an unobservable).

Across the top of the table we show widths of the band for target inflation. Down the first column of the table is shown three ranges for inflation variability. The body of the table shows, therefore, the bands on inflation targets that would have to be kept to in order to deliver the performance on four-quarter inflation ranges that are shown. Performance, in this regard, is shown in two dimensions, the likelihood of being outside the inflation range shown at left measured in terms of the proportion of quarters outside the range, and the duration of the average range violation in quarters. As a concrete example, the first row of column (1) of the table shows that with no variability of the inflation target at all, inflation will depart from a range for inflation of plus-or-minus 2 percentage points about 3 percent of time. The 2.1 number immediately below says that these departures tend to last only about six months.

42. Those countries that explicitly target inflation measure performance in terms of some 12-month rate of change in prices. We use the quarterly counterpart to that calculation here.

Table 4
Likelihood and Duration of Inflation Target Range Violations

		Bound on inflation target				
Column number -->		(1)	(2)	(3)	(4)	(5)
Inflation range		$ \bar{\pi}^* = 0$	$ \bar{\pi}^* = 0.5$	$ \bar{\pi}^* = 1.0$	$ \bar{\pi}^* = 1.5$	$ \bar{\pi}^* = \infty$
$ \bar{\pi} = 2.0$	likelihood	0.03	0.03	0.05	0.10	0.45
	duration	2.1	2.2	2.5	3.0	n/a
$ \bar{\pi} = 1.5$	likelihood	0.10	0.11	0.15	0.22	0.57
	duration	2.6	2.6	3.0	3.8	n/a
$ \bar{\pi} = 1.0$	likelihood	0.28	0.28	0.34	0.43	0.70
	duration	3.2	3.3	3.9	4.9	n/a

Note: 'likelihood' is the average proportion of quarters in which $\bar{\pi}$ exceeded the absolute value of the range for inflation, measured relative to the long-run target, π^* . 'Duration' is the average length of time $\bar{\pi}$ spends outside of the range, once a violation is recorded. Computations come from averages across the 500 draws for 180 dates in each draw. See also the notes to Table 4.

Reading the table from right to left across any given row shows that as the range of target inflation rates is narrowed, the likelihood and duration of inflation ranges diminishes. This much is hardly surprising. A more interesting observation is that there are sharply diminishing returns, regardless of the width of the inflation range. For example, for an announced inflation range of plus-or-minus 1-1/2 percentage points (the third row), cutting the target band width from plus-or-minus 1 percentage point (column 3) to 1/2 percentage point (column 2) reduces the frequency of violations from 0.15 to 0.11. A further cut of the target band width to zero (column 1) reduces the frequency only a trivial amount.

In general, there is a noteworthy frequency of range violations for all target band widths and for most inflation ranges: To get the likelihood of range violations into the single digits one must accept very wide inflation ranges of plus-or-minus 2 percentage points. Economically speaking, ranges this wide have little meaning for an economy like the United States. Focusing on ranges for inflation of plus-or-minus 1 percentage point--that is, on the range that most inflation-targeting countries use--we see a substantial likelihood of violations, and departures usually last more than 3 quarters when they occur. More than anything else, this reflects the fact that a substantial portion of the variability of inflation comes from shocks to the wage-and-price block of the model. There is little that the monetary authority can do to offset such shocks. Blackburn and Christensen (1989) emphasize that monetary policy credibility comes from a combination of the *willingness* of the monetary authority to pursue its inflation objective, and the *ability* of the authority to achieve the objective. Table 4 suggests that there are limits to what the Fed can expect to deliver in terms of outcomes of monetary policy, even if bounds on inflation targets are specified, kept, and believed. On the positive side, however, the table also shows that reductions in the width of the inflation target bands below plus-or-minus one-half percentage point yield very little in terms of the frequency of range violations, regardless of the range selected.⁴³

43. There is also the prospect, alluded to by the Chairman in the quote at the beginning of this chapter, that some of the 'exogenous shocks' to the model's wage-price blocks might be expectations scares brought about by low credibility, on average, through history. To the extent that this is true, the variance of these "supply shocks" will be lower in the future than in the past, the honeymoon effect of Figure 5 will be larger, and the probability of inflation range violations will be lower than what is shown in Table 4.

5.3 Comparative statics

The unconditional standard deviations shown in Table 4 mask some considerable variation in the conditional statistics. Figure 7 replicates Figure 5 in showing the mapping from the inflation target to inflation, this time for target band limits of plus-or-minus one-half percentage point (the thick line) together with our base-case assumption of plus-or-minus one percentage point (the thin line). The differences here are much more pronounced than Table 4 might have led one to believe; the honeymoon effect is apparent and substantial at all ranges of $\pi^* \neq 0$, not just beyond the first couple of tenths of a percentage points away from the midpoint of the range. Even in the presence of the unconditional inflation variability of 0.93 (Table 4, third row, column 2), this is an economically important difference in performance.

Just as we reduced the bandwidth from 1 percentage point to one-half point in Figure 7, we could have gone the other direction and increased it. Doing so produces a flattening of $g(\pi^*)$, and a region near the midpoint of the bands in which $g(\pi^*)$ and g_{rw} lie on top of one another. In this region, average inflation performance with a credible (but wide) band and a pure random walk would be no different. An implication of this is that good performance in controlling drift in inflation targeting does not necessarily guarantee a honeymoon effect if this performance is not recognized by private agents as credible. This observation goes some way in explaining why countries that target inflation expend so much time and effort in communicating to the public the intentions of policy.⁴⁴ They hope to reverse the usual causal order of good targeting performance leading to enhanced reputation and higher credibility by instilling credibility with words and thereby making the promised improved performance easier to achieve.⁴⁵

44. All of the six countries identified in Siklos (1997, pp. 132-133 as formal inflation targeters--Canada, Finland, New Zealand, Sweden and the United Kingdom--produce an inflation report of some kind on a regular basis.

Figure 7
Inflation versus Inflation Target
(selected band widths)

45. That inflation targeting with bands is an attempt to get a 'free lunch' along the lines described in the text is made clear for the case of Canada by Freedman (1995, p. 27): "These ranges were in fact smaller than were called for in empirical work done at the Bank. There is a trade-off...[t]he wider are the bands, the higher is the probability of successful achievement of the targets but the less useful are the targets in changing behaviour. In the end, we decided to use somewhat narrower bands to avoid the problem that overly wide bands might leave the impression that the authorities were not serious about bringing inflation down."

The other parameter that affects the honeymoon effect, is the magnitude of innovations to the target rate of inflation. In our base case, the assumed standard deviation of disturbances to π^* is 0.25 percentage points, a magnitude that is consistent with historical experience. Figure 8 shows the implications of higher variability in the monetary authority's target, holding constant the target band width at plus-or-minus one percentage point.

As discussed above, a larger σ implies a more substantial non-linearity in expected future target values for any initial $\pi^* \neq 0$. Not surprisingly, therefore, Figure 8 shows $g(\pi^* | \sigma = 0.5)$ to be flatter than in the base case. With the larger σ , there is a honeymoon effect even with relatively wide target bands. This shows that a monetary authority with erratic preferences has considerably more to gain from the establishment of target band widths, if such an authority can secure credibility. It also may help explain why those countries that have adopted inflation targets with bands have done so after abandoning under duress an exchange-rate targeting regime, while those countries that have announced imprecise inflation-targeting objectives do not have a record of such difficulties.⁴⁶

46. Canada is the only country whose decision to adopt a well defined inflation targeting regime was not prompted by a crisis. In most cases, inflation targeting countries had previously failed to maintain an exchange-rate targeting regime.

Figure 8
Inflation versus Inflation Target
(selected standard deviations of target shocks)

5.2 *Alternative policies*

Let us now compare our bases-case results with those of a few selected alternatives, beginning with a ‘less hawkish’ policy. Recall that for our base case, we assumed that the monetary authority placed three times the importance on inflation stabilization as on output stabilization. We now consider a policy rule derived with the opposite loss function weights: 0.75 on output and 0.25 on inflation. This raises the coefficient on the output gap of 0.9 in our base-case rule to about 1.1, and lowers the coefficient the twelve-quarter moving average of inflation from 3 to approximately 0.75. It is worth emphasizing that this new rule is on the same efficient policy frontier as the base-case one.

The thin lines in Figure 9 are the same lines as in Figure 6; they represent our base case. The thick lines are computed in exactly the same way as the thin lines, except using the coefficients of the less hawkish rule. Starting, once again, in the upper-right quadrant, we see that $g_{lh}(\pi^*)$ is flatter than it is the base case. This means that there is a larger honeymoon effect, or to put the same point a different way, there is less variability in inflation for a given degree of variability in the target than in the base case. In the upper-left quadrant, we see a steeper relationship between the average level of output and inflation and, more importantly, a much narrower range of output gaps is implied by that relationship. In the lower-left quadrant we see that the range of real interest rates associated with these output gaps is not only smaller, a slight backward bend appears: a decline in the output gap, beginning from zero, arising from a decline in the target which is accompanied by a smaller decline in inflation, results in a (very small) decline in the real interest rate.

Figure 9
Implication of Different Policy Preferences
Under Inflation Target Bands

The quantitative assessment of this rule is shown in Table 5 below. Each column in the body of the table records aspects of the performance of a particular policy rule; the first column shows the performance of our base-case rule. Column (2) shows the performance of the base-case rule. (We will ignore column (3) for now.) The first panel of the table shows the standard deviations of this rule for the base-case parameterization of the bounded random walk. As one would expect, the standard deviation of output is lower for the ‘less hawkish’ rule than for the base-case policy rule, while the standard deviation of inflation is higher.

The middle panel of the table, is more interesting. There we record the loss associated with each rule indexed against the preferences that generate the rules. In each case, we have normalized the losses for the two rules so that, for example, the base-case rule in column (1) is assessed a (normalized) loss of unity for base-case preferences. Similarly, in column (2), the ‘less hawkish’ rule carries a normalized loss of unity for ‘less hawkish’ preferences. The interesting result is in the other two entries in these two rows of this panel: notice in column (1) that the performance of the base-case rule under ‘less hawkish’ preferences is poor: There would be a 81 percent deterioration in performance from the perspective of a ‘less hawkish’ monetary authority from adopting the base-case rule as an operating procedure. Now examine the performance of the ‘less hawkish’ rule under base-case preferences in the first column. With a value shown there of 0.88, we see that *the base-case policymaker actually prefers the less hawkish rule*. How can this be? Given that the base-case rule is the rule that by construction is supposed to be the preferred rule for base-case preferences, this would appear to be impossible.

The result shown is not an error. It arises because the so-called optimal rule for base-case preferences is actually only optimal *conditional on a fixed target rate of inflation* (or a random walk).⁴⁷ In the presence of a bound on the random walk, the rule is no longer optimal. This is because the credibility of the target bands does much of the work

for the authority; given the bands, adding tough inflation targeting to the policy mix only makes matters worse.

Table 5
Simulated Standard Deviations of Target Band Regimes for
Alternative Policies

Column number -->	(1)	(2)	(3)
Policy rule in use -->	base case	less hawkish	forward looking
y	2.90	2.04	2.75
$\tilde{\pi} - \pi^*$ standard deviations	0.99	1.40	1.05
$R - \pi^*$	3.07	2.78	3.79
Preferences	Normalized Losses		
base-case preferences	1.00	0.88	0.96
'less hawkish'	1.81	1.00	1.65
Range violations ($\bar{\pi} = 1$)			
likelihood	0.15	0.29	0.16
duration	3.0	4.9	3.2
Loss Function:			
$\sum_{i=1}^{180} \varphi [\tilde{\pi}_{t+i} - \pi_{t+i}^*]^2 + (1 - \varphi) y_{t+i}^2$			
base-case preferences: $\varphi = 0.75$			
'less hawkish' preferences: $\varphi = 0.25$			

47. The base-case rule is also optimal for the case of a pure random walk, provided that the loss is computed around the (possibly drifting) target rate of inflation, since a fixed target and a pure random walk are identical in expected value terms.

To see why this arises, consider point E on the thin line in the upper-right quadrant of Figure 9. Observe that the level of output that corresponds with this, point F in the upper-left quadrant is low. With output low, all else equal, the real rate should be low as well. The reason why the real rate is *not* low is that the monetary authority is trying to move $\tilde{\pi}$ down to π^* , the current target rate of inflation. However private agents know that π^* is likely to be higher in the future. The inflation rate that arises reflects this expectation on the part of price and wage setters who, after all, must consider current and future costs of adjusting prices: it is optimal for them to avoid reducing prices today if they would only have to raise them again in the near future. But the monetary authority is working against the very effect it has created by adopting target bands in the first place. It follows that a rule that puts less weight on inflation control, in recognition that private agents are doing some of the controlling prices through their expectations, will do better on average.

There are other policy recipes that outperform our base-case rule. Figure 10 provides one example. The thin line in Figure 10 is, once again, our base case. The thick line represents a rule which is forward looking in inflation. In particular, we have replaced the twelve-quarter moving average of current and past inflation less the current target in equation (2) with a four-quarter moving average of current and future inflation less the *expected* future inflation target:

$$R_t = rr^* + \sum_{i=0}^3 \pi_{t-i}/4 + 3.0[\sum_{i=0}^3 (\pi_{t+i} - \pi_{t+i}^*)] + 0.9y_t \quad (9)$$

In our base-case scenario, the monetary authority is conducting policy myopically, carrying out short-term target-seeking actions without regard to how the longer-term process of determining the future of the target will play itself out. With the forward-looking rule, the central bank takes into consideration the likelihood of future target changes, even if it is not solving of an optimization problem for the future target simultaneously with the determination of the coefficients of the rule. In a more general sense, the monetary authority can be thought of as engaging in a form of inflation forecast targeting, as advocated by

Svensson (1997). Or the monetary authority can be thought of as responding to indicators of inflation expectations such as outside forecasts, forward rates, or surveys to set policy, as Hall and Mankiw (1994, p. 79) have advocated.

The results in this case are qualitatively similar to those in Figure 6. A forward-looking monetary authority recognizes that private agents know the target rate of inflation will regress towards the midpoint of the bands over time. By acknowledging this fact in making policy decisions, the monetary authority can improve average policy outcomes in terms of both output and inflation. The middle panel of Table 5 shows us that based on the loss calculation shown, the base-case monetary authority would once again prefer the forward-looking rule to the base-case rule, if only by a slight amount. It would be easy to make too much of this comparison, however, since the real federal funds rate variability is significantly higher for the forward-looking rule than the base-case rule.

Figure 10
 Implication of Forward Looking Policy
 Under Inflation Target Bands

We leave this section of the paper, and this topic, with two observations. First, under the assumptions of this section, it is possible to improve on the economic performance of the simple monetary policy rules that depend, linearly and axiomatically, only on a small set of past values. If monetary policymakers' targets do shift over time but are bounded in some way, and if agents believe in these bounds, then private agents' expectations will be biased, *ex post*, in a way that is completely consistent with the economic environment. An optimizing policy authority that chooses to ignore such biases will forego improvements in average policy outcomes that can be accrued by encompassing a wide range of indicators of expected future inflation. Similarly, since expectations of target movements on the part of private agents are doing part of the work of the authority in directing price setting behaviour toward long-term monetary policy goals, a Pareto improving response of the authority is to substitute away from inflation stabilization and toward output stabilization. Second, our analysis offers a rationale for the passivity in monetary policy operations which many observers have noted appears to be a characteristic of policy in the U.S..⁴⁸ To see this, examine the thick lines associated with the forward-looking policy rule in the two right-hand side quadrants of Figure 10. The honeymoon effect implies that both inflation and the real interest rate are much flatter than they would be under the random-walk case. Because expectations are being anchored by the credible target band, there is less impetus to adjust the nominal interest rate as the target inflation rate moves over time. Essentially, the public is doing part of the Fed's work for it. We hasten to add, however, that it would be easy to take this argument too far: the lines in our figures represent averages across hundreds of business cycles. Within those cycles, policy is not 'passive' at all. Still, this observation, combined with the results portrayed in Figure 9 for the "less hawkish" rule does provide some insights on these issues.

48. See, in particular, Rudebusch (1999), Huizinga and Eijffinger (1999) and Woodford (1999)

6. Concluding Remarks

This chapter has considered the implications of uncertain inflation targets for the conduct of monetary policy. In particular, we have modeled the uncertainty of targets as either a random walk, or a bounded random walk, and examined the implications of the bounds for policy.

It is not difficult to construct an argument that monetary policy has been the subject of randomness in targeting. Indeed, if it is true that in the long run, inflation is everywhere and always a monetary phenomenon, then there is lot that monetary authorities the world over have to answer for. More narrowly, we argued that the often remarked upon reluctance of the Fed to respond to shocks with vigour is consistent with some drift in the *de facto* target rate of inflation. And further we observed that those countries that have adopted formal target bands for inflation appear to do so in an effort to bound expectations of future inflation in a way that their inflation fighting reputations would not support on their own. These central banks believe that to the extent that they can fix the perceived long-run target of monetary policy in the minds of wage and price setters, they can reduce the cost of subsequently confirming those perceptions. The key is achieving such a result, is to establish the credibility of the promise of less randomness in inflation targeting, and hence in inflation performance.

We examined the issues using stochastic simulations of FRB/US, the Board of Governors' rational expectations macroeconomic model of the U.S. economy. Under the assumption that bands for the target rate of inflation are credible, we found that bands on target inflation can be useful in directing private agents' expectations toward the ultimate objective of monetary policy, while still permitting some time variation in preferences. A central bank that starts out with a reputation for allowing its inflation target to drift can benefit from a "honeymoon effect" wherein the bank reaps a reward of decreases in average inflation variability that are greater than the restriction in the variability of the

target. We also found that there are diminishing returns to setting credible inflation target bands: one need not go all the way to a fixed target to reap significant benefits.

Moreover, consistent with popular wisdom, we found that a central bank that enjoys credibility in its use of inflation target bands can exploit that credibility by either being less forceful in its pursuit of price stability, or by practicing a more forward-looking monetary policy.

What this means for U.S. monetary policy and the efficacy of increased transparency of policy objectives of the Fed depends on the extent to which the Fed is already enjoying a honeymoon effect. There is no question that over the 1990s the U.S. economy has enjoyed strong performance in the control of inflation without any apparent sacrifice in growth. The Federal Reserve in general and the Chairman in particular have been the recipients of praise because of this performance. At the same time however, it is also clear that this record could, in part at least, be a matter of happenstance. And even if the recent record is attributable to the Fed, there are pitfalls with the popular association of the performance with an individual--the Chairman--rather than with a regime. These considerations lead one to conclude that there is a *prima facie* case for considering steps to make policy more transparent and accountable than is currently the case. The use of inflation target bands, together with a publicly declared objective of policy could be a part of a transparency initiative.

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Chapter 4 : Potential Output, Uncertainty and Monetary Policy

Summary

A small, forward-looking macroeconomic model of the U.S. economy is estimated using the Kalman filter with maximum likelihood techniques to extract a measure of potential output and its uncertainty as parameters of the system. With these estimates, the implications of mismeasurement of potential output--and thereby mismeasurement of the output gap, traditionally a key argument in simple monetary policy rules--are examined. In particular, the effect of mismeasurement on the optimal design of monetary policy rules is investigated. Substantial uncertainty regarding the true magnitude of the gap over time is uncovered. On a purely statistical basis, it is often difficult to distinguish even the correct sign of the gap let alone its magnitude. The implication for policy in the context of simple interest-rate reaction functions is shown to be attenuation: that is, a maximizing monetary authority faced with such uncertainty would optimally place a smaller weight on the output gap in its formulation of a policy rule than it would under certainty equivalence.

1. Introduction

In recent years, it has been fairly well established that simple, parsimonious monetary policy reaction functions can be calibrated in such a way as to do a pretty good job in stabilizing economic performance over time. In Canada, for example, work by Amano *et al.* (1999) shows how output and inflation variability can be controlled using a very simple monetary policy rule with the Bank of Canada's QPM model in a stochastic economy. In the U.S., Williams (1999) demonstrates that straightforward extensions of the well-known Taylor (1993) rule work well in the Board of Governors' model, FRB/US, provided that the parameters are chosen wisely. Moreover, Levin *et al.* (1999) shows that such rules are robust in the sense that rules that are best for one model also work reasonably well in other models.

The bulk of the work in this area, however, has been conducted under the assumption that the unseen supply-side variables in the model--the NAIRU, potential output, expected inflation, and the equilibrium real interest rate--are known with certainty. Recently, a new strand of research has begun on the implications of aspects of model uncertainty for the design of monetary policy. Building on the seminal contribution of Brainard (1967), papers by Onatski and Stock (1999), Orphanides (1998), Rudebusch (1999), Smets (1999), and Söderström (1999) have examined various facets of the problem. In this chapter, we examine the appropriate response of monetary policy to mismeasurement of the output gap. In particular, a small forward-looking model of the U.S. economy is estimated using Kalman filtering techniques and maximum likelihood methods. By using these methods, a measure of potential output and its uncertainty can be extracted as parameters of the system. Policy rules that are optimal given known potential output are then compared with those that an authority would choose if it knew only the statistical process for potential output, including the magnitude and persistence of measurement error.

The paper that this chapter most closely resembles is Smets (1999). Smets measures the adjustment in the parameterization of Taylor-type rules to output gap uncertainty in a very simple model. Unlike Smets, however, in this chapter we estimate a forward-looking model. Besides being an useful contribution in its own right, this is important because only if expectations are rational can the true scope of monetary policy choices be captured: In the absence of rational expectations--or some other meaningful endogenization of expectations--policy control is entirely a mechanical exercise since there is no channel for policy design to directly influence expectations formation. Moreover, with a fixed-coefficient backward-looking model, the possibility exists that the results obtained from a policy analysis exercise could be manifestations of assumptions that the monetary authority can systematically mislead private agents. In this sense, this paper addresses the Lucas critique.¹

The rest of this chapter unfolds as follows. The next section discusses certain aspects of the history of measuring potential output leading up to the stochastic trends approach followed here. Section 3 lays out the model, how a state-space form is constructed and estimated, and discusses the estimation results. Section 4 turns to simulation exercises, providing the methodology for computing optimal rules as well as the quantitative findings. A fifth and final section sums up and concludes.

2. Potential Output and its Measurement

Notwithstanding theoretical objections regarding its microfoundations, the Phillips curve remains at the centre of monetary policy making. At its heart, the Phillips curve characterizes changes in inflation as being driven, in part, by excess demand or the level of resource utilization relative to capacity. In some instances, the “level of resource allocation” is taken to be an unemployment rate and “capacity” is the NAIRU. In others, the

1. While this chapter attempts to take the Lucas critique seriously, it does not “solve” it because it could be argued that the parameters estimated herein are not deep structural parameters.

analogous concepts are output and potential output. Either way, excess demand requires the measurement of a variable that cannot be directly observed.

Because monetary policy affects output with a lag, and output affects inflation with a longer lag still, a central bank typically must be forward-looking when it sets its policy instrument. It follows that errors in the assessment of excess demand do not manifest themselves for some time, and correcting these errors takes time as well. Evidently, timely and accurate measures of potential are an essential ingredient of policy making.

This problem is more than a nuisance. Arguably, it has had first-order implications for the conduct of policy. Orphanides (1998) documents the revisions in the output gap in real time from the Federal Reserve Board's official greenbook forecasts. The late 1960s offers a striking example of the magnitude of the problem. At that time, the data showed that the output gap was believed to be about zero, but with the benefit of hindsight, the current view is that the gap measured about 5 percentage points. Matters are no better for the NAIRU: The 1973 Economic Report of the President (p. 16) states that: "Both the 'full employment budget' and the 'GNP potential' were [during most of the 1960s] and continue to be, defined in terms of a 4 percent unemployment rate." A conventional estimate of the NAIRU for this period is now at or slightly above 6 percent. From this perspective, the rise in inflation in the late 1960s and into the 1970s is much less mysterious than it originally seemed.

This chapter concentrates on uncertainty in the measurement of potential output and the implications of uncertainty on the appropriate conduct of monetary policy. As the preceding discussion suggests, all the same issues exist for non-inflationary unemployment,² or capacity utilization.³ In general, it has been difficult to uncover reliable, timely estimates of economic capacity, however measured.⁴ And while the quantitative assessment below focuses on the U.S. data, essentially the same lessons apply for economies other than the United States; the discussion in Laxton and Tetlow (1992) examining the

history of the measurement of potential output in Canada and the U.S. makes it clear the that two countries' experiences were similar.

In the 1950s and 1960s, potential output was typically assumed to follow a linear time trend.⁵ This deterministic view of potential was shown to be deficient by the experience of the late 1960s and the 1970s when inflation rose notwithstanding what were then regarded as unnaturally low levels of resource utilization.

The productivity slowdown of the 1970s made it clear that potential could not be approximated by a linear time trend. Over the 1970s, a variety of alternative methods emerged. Initially, they took the form of simply segmenting the time trend that had been used before, with the break in trend usually set at about the time of the first OPEC oil price shock in 1973. It was a minimalist fix, the methodology for which reflected the prevailing econometric presumption that potential output was smooth; shocks to potential output were regarded as sufficiently rare that they could be handled on a case-by-case basis. Since the productivity slowdown seemed to occur at about the same time as the oil price shock, the notion that permanent shocks to potential could be observed directly was a

2. It is interesting to contrast the different estimates of NAIRU and its stability that are publicly available. Staiger, Stock and Watson (1997), for example, use a cubic spline to estimate the NAIRU using U.S. data and find considerable variation over time to the point where they question the usefulness of the concept. Gordon (1998) uses a Kalman filter and finds broadly similar, if less extreme, time variation. Brayton *et al.* (1999) use extended-window rolling regressions, which will tend to find less time variation, but nevertheless find evidence of a one-time shift in the late 1990s. The very fact that alternative methods produce different results but with the common finding of variation supports the notion that it is important to take seriously time variation in the NAIRU and its uncertainty.

3. Koenig (1996) shows that the capacity utilization rate is revised considerably less often than other output indicators and that its mean--and arguably its equilibrium level--has not changed much over time. Weighing against the usefulness of capacity utilization, however, is the fact that manufacturing is a small and ever-shrinking portion of the aggregate U.S. economy. In this sense, capacity utilization also seriously mismeasures GDP.

4. A symposium in 1997 in the *Journal of Economic Perspectives* (volume 11, no. 1) provides a round up of many of the issues on measurement of the NAIRU. Laxton and Tetlow (1992), Guay and St. Amant (1996) and Orphanides and van Norden (1999) show that typical methods of measuring potential output do a fairly poor job. However, their studies concentrate on univariate methods of measurement as opposed to the multivariate approach employed here.

5. The description of the history of measuring potential output in the remainder of this section borrows from Laxton and Tetlow (1992) and references therein.

comfort. Only subsequently was it noted that inflation had begun to unexpectedly rise significantly in the late 1960s, well before the oil shock.

The stagflation of the 1970s was mirrored in the econometric instability of statistical Phillips curves at that time. This led to increasingly disaggregated methods of measuring potential.⁶ Some decomposed the production function to try to get more precision in potential output by adding up potential labor supply and potential capital and the like. Other methods focussed on “adjusting” the variables that entered into the production function, such as demographically adjusting the labor force to account for the rising share of women.⁷ But these methods, substituting a single time trend for aggregations of several time trends, also proved deficient.

At about the same time, new thinking on the econometrics of time series with unit roots began to undermine the popularity of split time trends to measure potential.⁸ By the late 1980s, the view that potential output was best considered a stochastic variable--subject to permanent shocks to the level if not the growth rate--had taken hold, although as a practical matter most central banks still used segmented time trends as proxies.

Whatever the merits of stochastic trends versus segmented trends, there is a new wisdom of humility. Whereas economists of the early 1970s would publish forecasts of potential output boasting remarkably high precision, there is now a healthy skepticism. Measures of potential output are taken as imprecise.⁹ There are two fundamental reasons for this. First, there simply are not the data available. The system of national accounts is

6. Perloff and Wachter (1979) represented what was perhaps the peak of disaggregation and associated complication in the computation of potential output. As Plosser and Schwert (1979) noted in their comment on Perloff and Wachter, all the power in estimation ended up coming from polynomial time trend variables which are of dubious use out of sample.

7. Perry (1977) is an example of this method.

8. The new view of econometrics of trended data began--slowly--in the late 1970s. Dickey and Fuller (1979) provided early evidence showing how unit roots were difficult to distinguish from near unit roots in the data. At about the same time, Nelson and Kang (1984) showed the pitfalls of using time trends inappropriately while Nelson and Plosser (1982) showed that unit roots appear to be present in many macroeconomic time-series data. These developments set the stage for the ascent of the stochastic trend view of potential.

designed, for the most part, to measure aggregate demand. Data that are meant to identify supply independent of demand are difficult to find, and usually of poor quality. Second, and more fundamentally, the concept of potential is inherently unobservable.

In fact, the concept of potential output is usually defined in terms of its use: For monetary economists and central bankers, it is the level of output at which there exists no pressure, upward or downward, on inflation.¹⁰ Put this way, it becomes clear that potential output is best measured as a latent variable in a system of equations that identify aggregate demand and supply simultaneously. We do just that in this chapter using the Kalman filter to extract potential output as a latent variable in a system. In this regard, we follow in the literature beginning with Clark (1987, 1989) and subsequently with Campbell and Mankiw (1990), Watson (1986) and Kuttner (1994). The advantages of following this methodology are two-fold: first, unlike traditional methods where potential output is measured outside of the system and brought in as an exogenous variable to an estimated Phillips curve, the measure of potential used here is consistent with the structure of the economy as represented by the model. Second, the estimation procedure provides measures of the uncertainty surrounding estimated potential. Since our objective here is assess the importance of this sort of uncertainty for the design of policy, there is an obvious benefit to doing this.

3. The Model

3.1 Specification

To be pertinent to the task of uncovering the appropriate policy response to uncertainty in potential output, we need a model that is more-or-less typical of those that are

9. Guay and St. Amant (1996) discuss a variety of methods for measuring potential output focusing on their reliability. Orphanides and van Norden (1999) survey seven different univariate detrending methods and find that all are subject to large amounts of imprecision, consistent with the earlier but more narrowly focused findings of Laxton and Tetlow (1992).

10. For fiscal authorities, potential output might be defined in terms that are consistent with the full-employment budget surplus. This may or may not be consistent with the central banker's definition.

used by central banks around the world--or at least in countries for which a closed economy is a reasonable approximation of reality. Accordingly, we construct a simple New Keynesian model roughly along the lines of Fuhrer and Moore (1995b).¹¹ The key to this model, as in any Keynesian model, is the price equation or Phillips curve. Our formulation is very much in the same style as the real wage contracting model of Fuhrer and Moore (1995a) and Buiter and Jewitt (1981). By making use of the Fuhrer-Moore formulation, we 'slip the derivative' in the price equation, thereby ruling out the possibility of costless disinflation.¹² However, instead of the fixed-term contract specification of Fuhrer-Moore, the stochastic contract duration formulation of Calvo is adopted. In doing this, the state space of the model is significantly reduced easing the computational burdens of what follows. The model is summarized by equations (1) to (8) below:

$$\pi_t = c_t + \beta x_t + u_t^\pi \quad (1)$$

$$c_t = (1 - \lambda)(q_{t-1} + \lambda q_{t-2} + \lambda^2 q_{t-3} + \dots) \quad (2)$$

$$q_t = (1 - \lambda)(\pi_{t+1|t} + \lambda \pi_{t+2|t} + \lambda^2 \pi_{t+3|t} + \dots) \quad (3)$$

$$x_t = \phi_{x1} x_{t-1} + \phi_{x2} x_{t-2} + \phi_{rr} r r_{t-1} + u_t^x \quad (4)$$

$$y_t^* = y_{t-1}^* + \omega + u_t^* \quad (5)$$

$$y_t = y_t^* + x_t \quad (6)$$

$$r r_t = r s_t - \pi_{t|t-1} \quad (7)$$

$$r r_t = \psi_{rr} r r_{t-1} + \psi_c c_t + \psi_q u_t + u_t^{rr} \quad (8)$$

11. The model is similar to the model outlined in Chapter 1. Different estimation techniques are used here however.

12. Roberts (1995) shows that nearly all sticky price models can be boiled down to a specification where prices are predetermined, but inflation is not. This means that the inflation rate is a jump variable, and disinflation can be costlessly achieved if the disinflation announcement is taken as credible. By slipping the derivative in the price equation, Taylor's specification which is two-sided in the price level becomes two-sided in inflation instead. No disinflation experiments are conducted for this chapter. Nevertheless, addressing policy questions of the sort considered here requires a reasonable characterization of the degree of inflation stickiness that is in the data.

Equations (1) is the Phillips curve that simply states that final goods price inflation, π_t , is equal to core inflation, c_t , plus a mark-up that is linear in the level of excess demand, x_t . Mark-ups are subject to stochastic shocks, perhaps associated with unobserved heterogeneity. Equations (2) and (3) represent the contracting technology: (2) gives core inflation as the geometrically weighted sum of past contract inflation rates with the weights, λ , being the probability of contract survival as in Calvo (1983).¹³ Equation (3) states that contract inflation is set equal to a geometrically weighted sum of expected future inflation rates. (The notation in equation (3) for π has $\pi_{k|j}$ as the expectation of goods price inflation for period k conditional on information available as of the end of period j . This convention will be retained for the remainder of this chapter.) Once again the weights come from the stochastic rate at which contracts expire, on average. That is, a contract signed for today is expected with probability λ to be in force one period from now, and with probability λ^2 to be binding two periods ahead, and so on. Assuming that terminations of contracts are independent of one another, the proportion of contracts negotiated s periods ago that are still in force today is $(1 - \lambda)\lambda^{t-s}$ and the average lifespan of a contract is $\sum_{s=1}^{\infty} (1 - \lambda)s\lambda^{s-1} = 1/(1 - \lambda)$. Note that when $\lambda = 0$, equations (1) through (3) collapse to $\pi_t = \pi_{t|t-1} + \psi x_t$; that is, a Lucas-type aggregate supply function. Conversely, as $\lambda \rightarrow 1$, contracts become increasingly long lasting so that inflation is more and more pinned down by history.

Equation (4) is a very simple aggregate demand equation with the output gap being a function of two lags of the gap as well as the lagged *ex ante* real interest rate. The AR(2) specification is standard in empirical representations of the cyclical component of output; it is used by Watson (1986), for example, and forms part of the identifying assumption behind the Hodrick-Prescott (1981) filter.¹⁴ An *ex ante* real interest rate appears in the aggregate demand function. The data much prefer the use of the lagged real rate over the

13. The stochastic rate of contract survival, λ , is taken to be a fixed parameter. It is quite likely to vary with the inflation rate, but since we are holding fixed the average level of the inflation rate in our experiments, this seems like a reasonable approximation of reality.

contemporaneous one.¹⁵ Equation (5) states that the level of potential output follows a random walk, with drift ω . Potential output is assumed to be unobserved by the monetary authority as is the gap; rather the authority is assumed to be able to observe only the sum of the two, aggregate output, the equation for which is (6). Equation (7) defines the *ex ante* real interest rate. The target rate of inflation and the equilibrium real interest rate, rr^* , are taken as constants and omitted from the model notation for simplicity; in simulation, we shall set them to zero without loss of generality.

Normally, one would close a model like this one with a interest-rate reaction function. The simulation version of the model will in fact have an explicit policy rule. For estimation purposes something more generic is required owing to the vagaries of policy setting in history. The model will be estimated for the period from 1956Q1 to 1998Q4, the longest period for which the data are readily available. For a number of reasons, however, monetary policy over this period cannot be reasonably characterized by a single interest-rate reaction function. Policy was certainly not constant over this period as a perusal of old Federal Reserve greenbook forecasts can attest. More fundamentally, however, the monetary policy transmission channel changed over time with periods of borrowed reserves targeting, non-borrowed reserves targeting, monetary targeting, and 'looking at everything'. Even the federal funds rate--which shall be taken as the instrument of policy--did not have the significance in the early 1960s and before that it does today.¹⁶

For estimation purposes therefore the model is closed with a generic function for the real interest rate. This is equation (8). (The variable v which appears in (8) will be

14. That is, the H-P filter is an optimal filter provided that the log of the growth component of output follows a random walk and the cyclical component follows an independent AR(2) process. The real interest rate normally appears in the aggregate demand function in consideration of intertemporal substitution. In present circumstances, it may also represent the credit channel of monetary policy which would justify its appearance with a lag. Whatever the justification, the data prefer the lagged real interest rate over the contemporaneous and it is simpler to solve for the state-space representation when the interest rate appears with a lag.

15. Nevertheless, one could construct a rationale for the lagged interest rate on time-to-build arguments.

16. See Meulendyke (1990) for a review of the operating procedures of the Fed.

explained below.) The precise form of (8) is of little interest since alternative specifications yield similar results and since an explicit reaction function will be used for the policy exercises.

Taken together, the model is stylized, but it does capture what one might take to be the fundamental aspects of models that are useful for the analysis of monetary policy. Among these features is that inflation shows a measure of persistence, that policy acts on demand and prices with a lag. The model also assumes that disturbances to aggregate demand are persistent as are the effects of demand itself on inflation (provided that λ is not too small). As emphasized above, these features imply that in order to be effective, monetary policy must look ahead, setting the federal funds rate today to achieve objectives in the future.

3.2 Estimation

To estimate the model, we employ the Kalman filter and estimate using maximum likelihood techniques. To prepare the model for estimation, it is useful to write it in state-space form. To do this, begin by defining an artificial state variable, v_t :

$$v_t = \pi_t + \lambda\pi_{t+1|t} + \lambda^2\pi_{t+2|t} + \dots \quad (9)$$

After substituting this into equation (3) the resulting equation can be solved backwards to arrive at a first-order expression in v_t . Some algebra then reveals the state-space representation of the model (see the Appendix for details):

$$\alpha_{t+1} = \omega + T\alpha_t + u_{t+1} \quad \text{var}(u_t) = Q_t \quad (10)$$

where $\alpha_t = [y_t^* \ x_t \ x_{t-1} \ c_t \ v_t \ rr_t]'$ designates the state vector, $\bar{\omega} = [\omega \ 0 \ 0 \ 0 \ 0 \ 0]'$ is the vector of drift terms; T is the 6-by-6 transition matrix, and $\bar{u}_t = [u_t^* \ u_t^x \ 0 \ u_t^c \ 0 \ u_t^{rr}]'$, with $u_t^c \equiv \delta u_t^\pi$, is a 6-by-1 vector of shocks impinging on the state variables. In applications with Kalman filtering, equations (10) are often called the transition equations.

Denoting $\delta = \lambda^{-1}(1 - \lambda)^2$, and ignoring $\tilde{\omega}$, equations (10) can be written in full as:

$$\begin{bmatrix} y_{t+1}^* \\ x_{t+1} \\ x_t \\ c_{t+1} \\ v_{t+1} \\ rr_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_{x1} & \phi_{x2} & 0 & 0 & \phi_{rr} \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -\delta\beta & 0 & \lambda - \delta & \delta & 0 \\ 0 & -\beta\lambda^{-1} & 0 & -\lambda^{-1} & \lambda^{-1} & 0 \\ 0 & 0 & 0 & \psi_c & \psi_v & \psi_{rr} \end{bmatrix} \begin{bmatrix} y_t^* \\ x_t \\ x_{t-1} \\ c_t \\ v_t \\ rr_t \end{bmatrix} + \begin{bmatrix} u_{t+1}^* \\ u_{t+1}^x \\ 0 \\ u_{t+1}^c \\ 0 \\ u_{t+1}^{rr} \end{bmatrix} \quad (11)$$

and we have assumed that the three stochastic shocks of the model are uncorellated over time and with one another. The observation equations are:

$$Y_t = Z\alpha_t + \varepsilon_t \quad \text{var}(\varepsilon_t) = H_t \quad (12)$$

where $Y_t = [y_t \ \pi_t \ rr_t]'$ and Z is sometimes called the observation matrix. In the current case, the matrix of measurement errors is taken to be null: $H_t \equiv 0$. Equations (12) can be written out in full as:¹⁷

$$\begin{bmatrix} y_t \\ \pi_t \\ rr_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_t^* \\ z_t \\ z_{t-1} \\ c_t \\ v_t \\ rr_t \end{bmatrix} \quad (13)$$

Solving the model and finding the maximum exploits the prediction error decomposition of the Kalman filter (Harvey (1989), pp. 182-186). The likelihood function is:

17. Note that equations (13) are record keeping equations used to define state variables in terms of observables (even though they're written the other way around). It does not matter that there are null columns in Z . Nor does it matter that the last row of equations (13) simply sets the real interest rate equal to itself, except that in so doing we are assuming that the Fed can observe the real rate.

$$\log L = -\frac{N\tau}{2}\log 2\pi - \frac{1}{2} \sum_{t=1}^{\tau} \log |F_t| - \frac{1}{2} \sum_{t=1}^{\tau} v_t' F_t^{-1} v_t \quad (14)$$

where N , τ , v_t and F_t , respectively, are the number of observed variables, the sample size, the vector of prediction errors associated with the matrix of observables, and the mean-squared-error matrix for the prediction errors.

The Kalman filter was initialized using a diffuse prior for the hyperparameters, Q , and with level conditions initialized at their pre-sample values for observables and with $c_0 = q_0 = \pi_0$ and $y_0^* = 0$ for the unobservable state variables. These assumptions are the most neutral ones available. A small number of experiments were conducted with other, reasonable initial conditions with no differences of consequence in the results.

The estimation results are shown in Table 1 below.

A number of interesting observations arise from Table 1. In particular, the results show a statistically significant negative slope of the aggregate demand function, as indicated by ϕ_{rr} , and a significant positive slope of the Phillips curve. β . More interestingly, perhaps, is the fact that the data accept that a inflation determination is partly forward-looking as indicated by the fact that the survival probability of contracts, λ is large, but significantly less than unity. To put the same observation a different way, the point estimates suggest that holding excess demand at zero, inflation today is 89 percent determined by historical factors and 11 percent determined by expected future conditions.¹⁸ The small but significant contribution of future conditions is consistent with the estimates of Chada *et al.* (1992), Isard and Laxton (1998), and Fuhrer (1997b).

18. Recall from subsection 3.1 that when $\lambda = 0$ equations (1) to (3) of the model collapse into a Lucas monetary surprises model—that is, a model with no stickiness whatsoever. Conversely, as $\lambda \rightarrow 1$ the model features longer lived contracts and more stickiness.

Table 1
Maximum Likelihood Estimates of Small Structural Model
(1956Q1 - 1998Q4)

symbol	coefficient	standard error	t-statistics
ϕ_{x1}	1.50	0.06	23.5
ϕ_{x2}	-0.52	0.06	8.2
ϕ_{rr}	-0.06	0.03	2.2
β	0.12	0.05	2.3
λ	0.89	0.02	50.5
μ	0.80	0.06	14.5
$\sigma_{y^*}^2$	0.35	σ_{π}^2	2.30
σ_x^2	0.41	logL	487.40

Notes: π is the quarterly rate of change in the consumer price index, all items, modified from 1967 to 1983 so that home ownership costs are on a rental-equivalent basis; y is 100 times the log of the gross domestic product; $rr = rs - \sum_{i=0}^3 \pi_{t-i}/4$ is the real interest rate where rs is the nominal federal funds rate. The coefficients of the real-interest-rate equation are omitted for conciseness.

Another finding worth pointing out is that output fluctuations have important contributions coming from both demand shocks, σ_x , and supply shocks, σ_{y^*} , or equivalently, that the level potential output is a stochastic variable. An alternative hypothesis that is sometimes advanced is that not just the level of output is subject to permanent shocks as argued by Watson (1986), Kuttner (1994), and Smets (1999), but that the growth rate of output also shifts permanently. This I(2) model of output has been advanced by Clark (1987, 1989) as an alternative to the I(1) model shown in Table 1. Table 2 below shows that the data easily accept the I(1) restriction on the more general I(2) model.

Table 2
Test of Alternative Specifications of Growth Processes
(likelihood ratio test)

tested restriction	$\chi^2(1)$
I(1) against I(2)	0.26
10% confidence level	2.71

More important than the actual point estimates of the model is the fact that the Kalman filter gives one- and two-sided estimates of the output gap, complete with confidence bands. Figure 1 shows the one- and two-sided estimates of the output gap as generated by the model exploiting the Kalman filter. One-sided estimates are those that use only current-dated information while two-sided estimates use all the data. The most interesting difference between the two is over the period immediately following the 1982 recession. The one-sided filter expected a faster recovery in output than was forthcoming and attributes this shortfall to demand deficiency. The two-sided filter, with retrospect, assigns the relatively low level of output to lower levels of aggregate supply because the disinflation in the early 1980s, while sharp, turned out to be short-lived. That is, the two-sided filter 'sees' that inflation did not fall further in 1984-85 and infers from this fact that the low level of observed output must have originated from lower potential output rather than lower aggregate demand.

Figure 1
One- and Two-Sided Output Gap Measures

Figure 2 shows the two-sided output gap along with an output gap derived using a split time trend.¹⁹ In comparison to the split-time-trend output gap, the filtered gaps do not show any periods of excess supply in the 1970s, a period of rising inflation rates. It is because of this feature of output gaps derived using time trends that conventional backward-looking models tend to assign responsibility for the inflation increases during the 1970s to a series of once-off idiosyncratic supply shocks rather than fundamentals.

Figures 3 and 4 respectively show the one- and two-sided gaps again, this time with one-standard-error confidence bands.²⁰ It is immediately apparent that the uncertainty surrounding levels of output gaps is large: It is usually the case that a monetary authority would not be able to know with reasonable certainty whether the gap is positive or negative.²¹

For our purposes, it is not the precise estimates of the gap itself that is of primary interest. Rather it is model-consistent estimates of the uncertainty surrounding estimates of the gap that can be exploited for experiments in the design of policy. We turn to this subject in the next section

19. Specifically, potential output is allowed to grow at one rate from 1956Q1 to 1973Q2, and at a different rate thereafter.

20. The standard errors are asymptotic calculations based on the second derivative of the likelihood function evaluated at the estimated parameters as described in Hamilton (1994), pp. 142-144.

21. This is a common finding. Kuttner (1992, 1994), and Laxton and Tetlow (1992) and Smets (1999) all report similar results.

Figure 2
Two-Sided and Split-Trend Gaps

Figure 3

One-Sided Output Gaps
(70% confidence bands)

Figure 4

Two-Sided Output Gaps
(70% confidence bands)

4. Policy Experiments

4.1 Policy design

From a technical standpoint, the most natural way to design policy is to exploit optimal control theory, such as described for backward-looking models by Chow (1973). Such techniques could be used to compute the coefficients for the instrument feeding back on the complete set of state variables, α_t . Operationally, however, this turns out to be a more difficult problem than one might surmise. First, there is the fact that some of the state variables of our system are “artificial” in that they are themselves computed from observable variables. Second, optimal control in a forward-looking model is more difficult because feedback rules must take into account the fact that v_t is a non-predetermined or “jump” variable. Since v_t moves discontinuously with shocks, policy cannot be conditioned on v_t .²² Instead, as Levine and Currie (1987) show, the rule must be conditioned on a costate variable associated with each jump variable. While the non-predetermined variables jump, their costates are predetermined leaving the policy rule a function of continuous variables. And while costate variables are not (directly) observed, they can be substituted out for lags of the state vector. The upshot of all this is that a globally optimal rule in a forward-looking model contains many more arguments than just the six that appear in the state vector shown above.

It is clear then that globally optimal rules can be difficult to compute, difficult to follow and difficult to explain to the public. It is for these reasons, among others, that work on policy design has focussed not on optimal rules but rather on simple optimal rules where the vector of arguments to the rule is reduced to a subset of what would appear in a globally optimal rule but the parameters of the rule are chosen optimally.

22. One can, however, condition on the state variables that determine v_t , which is what is implicitly done in the computation of efficient Taylor rules, such as those of Williams (1999) among others.

As it turns out, simple rules can be quite close to optimal in terms of their performance, as Chapter 1 has shown. A more important implication, however, is that the separation theorem of control theory breaks down when the rule does not contain all relevant states.

Conceptually, the output gap is like a constant term in the Phillips curve. As such, uncertainty in the measurement of a constant term is additive uncertainty for which the logic of Brainard (1967) normally applies. The separation theorem states that when a rule is computed optimally, and the states are measured optimally, then the process of filtering and that of policy design are separable and certainty equivalence holds. Once, however, the rule is “underparameterized” in the sense just described, each state that appears in the rule must proxy for those states that *would have* appeared in the globally optimal rule. When this is so, the covariance of mismeasured states--like the output gap--and other states in the system implies that the authority needs to take the variability around the estimate of the mismeasured state into consideration when designing policy. Certainty equivalence no longer holds.

For our purposes, the monetary authority’s problem can be stated as one of choosing the coefficients in the policy rule, $\psi \subset \{\psi_\pi, \psi_x, \psi_{rr}\}$ to solve:²³

$$\underset{\langle \psi_j \rangle}{\operatorname{argmin}} \quad E_0 \sum_{t=0}^{\infty} [(1 - \Gamma_x)(\pi_t - \pi^*)^2 + \Gamma_x x_t^2 + \Gamma_{\Delta r s} (\Delta r s_t)^2] \quad (15)$$

$j = x, \pi, \Delta r r$, subject to equation (10): $\alpha_{t+1|t} = T\alpha_{t|t-1} + \eta_t$ and the filtering technology possessed by the monetary authority. Let us assume that the processes driving shocks to the behavioral equations is known to have a time-invariant distribution. This allows the use, for simulation purposes, of the time-invariant version of the Kalman filter. Harvey (1989, p. 118) shows the recursive equations to be:

23. Since we will not be considering longer lags of the perceived output gap we will simplify the subscripts on ψ to keep the notation clean by dropping the numeric part; i.e., ψ_{x0} becomes ψ_x .

$$\alpha_{t|t-1} = \alpha_{t-1|t-2} + \kappa(y_{t-1} - Z_{t-1}\alpha_{t-1|t-2}) = \alpha_{t-1|t-2} + \kappa(v'_{t-1}) \quad (16)$$

where v'_{t-1} is the same vector of realized prediction errors that appears above in the likelihood function; it is the realized vector of observables, y_{t-1} minus what would have predicted it: the product of the observation matrix and the state matrix as it was evaluated for periods $t-1$. A portion of the prediction error is assigned to unobserved changes in state variables. This is κ , the Kalman gain, which is a function of the signal-to-noise ratio in the variance of the (unobserved) shocks to the aggregate supply and demand:

$$\kappa = TPZ(ZPZ + H) \quad (17)$$

Recall that in this case, $H \equiv 0$. The matrix Z is the observation matrix as before, and the matrix $T = T - \kappa Z$ is derived from the estimated transition matrix. The remaining element of equations (17) is the “precision matrix” P --really just a variance-covariance matrix of the estimate of $\alpha_{t|t-1}$. P is solved from the matrix Ricatti equation:

$$P - \bar{T}P\bar{T} + \bar{T}PZ(ZPZ + H)^{-1}ZP\bar{T} - Q = 0 \quad (18)$$

with $Q = \text{var}(u)$, the variance of the shocks to the transition equations.

For the simulations, we use the one-sided filter results since those properly capture the uncertainty faced by the Federal Reserve on a real-time basis.²⁴ As it happens, the econometric estimates reported above imply that about one-third of the surprise in observations to output is attributed to changes in potential output (and two-thirds to changes in aggregate demand).

We assume that the monetary authority sets the nominal federal funds rate with the operational goal of setting the *ex ante* real rate to establish and maintain the objectives as measured by equation (15). Since it is the *ex ante* real rate that drives aggregate demand in

24. The improvement in the precision of the two-sided estimates is not large, however, and so it is unlikely that using the two-sided estimates would make a substantial difference for the results.

this model, this seems like a reasonable characterization of policy. It is, moreover, the characterization used by John Taylor (1993) among others.²⁵

We will consider two policy rules both of which are encompassed within the following equation:

$$rs_t = (1 - \psi_{rr})(rr^* + \pi_{t|t-1}) + \psi_{rr}(rs_{t-1} - \pi_{t-1}) + \psi_c c_{t-1} + \psi_x x_{t-1|t-1} \quad (19)$$

As shown, equation (19) is one of the two rules considered: Let us refer to this rule as the 3-parameter rule. The other rule simply imposes the added restriction $\psi_{rr} = 0$ and is called the 2-parameter rule. Using both 2- and 3-parameter rules serves as a robustness check on the results. Because we consider only simple rules the computed policy rule coefficients will be a function of the variance-covariance matrix of stochastic shocks, and the filtering technology--which would not be so with the globally optimal rule--as well as the usual policy taste parameters, Γ_j , the structure of the model.

Before leaving this topic, note two aspects of equation (19) that are of interest: First, the rule is not a Taylor rule as such because it is structured to include only pre-determined variables on the right-hand side. Moreover, the four-quarter moving average term in the inflation rate, so familiar from Taylor and subsequent work, does not appear in the rule. Nevertheless, the rule does share the basic components of the Taylor rule. For example, the 2-parameter version of this rule contains a variable for the inflation target and one capturing excess demand. In addition, the 2-parameter version of the rule contains no dynamic terms of its own, like the Taylor rule. Of course, this feature is relaxed for the 3-parameter rule.²⁶ The second noteworthy aspect of the rule is that it is written with

25. Generally speaking, it would make little difference if the lagged funds rate were expressed in nominal terms instead of real terms. The inflation rate is linked to core inflation and the output gap through equation (1) and both of these variables appear in the policy rule. This means that removing $\pi_{t-1|t-2}$ from the lagged funds rate term in equation (19) would only result in offsetting changes in the coefficients ψ_y and ψ_c . Writing the rule in terms of real rates, as we have here, removes an inflation term in the lagged funds rate and thereby cleans up the interpreting of changes in ψ_c and ψ_{rr} brought about by uncertain potential.

$\pi_{t-1|t-1}$ terms appearing on the right-hand side, making it clear that the monetary authority should be thought of as setting the *ex ante* real interest rate as the instrument of policy.

4.2 Simulation results

In this section, we compute the stochastic performance and associated coefficients of optimal simple rules of the form of equations (19), with and without mismeasurement of the output gap. In order to show the performance of the rules associated with these rules, we shall exploit the device of an efficient policy frontier: The locus of points in the σ_x, σ_π space along which there can be no reduction in the standard deviation of inflation without a increase in the standard deviation of the output gap, and *vice versa*.²⁷ The efficient policy frontier is traced out by allowing Γ_x to vary between zero and unity, computing the implied coefficients and the asymptotic standard deviations of the model under control.²⁸

Throughout, we will set $\Gamma_{rs} = 1$. Qualitatively speaking, none of the results described here is seriously affected by this choice so long as Γ_{rs} is not very large. When Γ_{rs} is allowed to be very large, the minimization problem is so severely restricted that solutions are sometimes not computable. When Γ_{rs} is allowed to be very close to zero--less than 0.1 or so--the resulting variances of the federal funds rate become so large, and the policy-rule coefficients become so large, that the results are not quantitatively plausible. Strictly speaking, because funds rate variability does count in the loss function, and because the variability of the funds rate is not identical along the frontier measured in

26. We could examine the performance of Taylor rules with and without measurement error in potential output but since neither the four-quarter moving average of inflation nor the current output gap are state variables, this is harder to do and harder to explain than what is done here. In any case, the qualitative answers would be the same.

27. Taylor (1979) initiated the efficient policy frontier. A modern restatement of it and how it is used can be found in Fuhrer (1997a).

28. Computations were done using the AIM algorithm (Anderson and Moore, 1985; Anderson (1999)) in MATLAB. Estimation was conducted using GAUSS with maximum likelihood using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method. See Judd (1998), pp. 113-15.

σ_x, σ_π space, we should show three-dimensional frontiers to allow us to capture differences in funds rate volatility. However, the differences in funds rate volatility along the frontiers are small enough that in the interests of simplicity and clarity we can forgo showing that dimension of our results, without losing any content.²⁹

Figure 5 shows two frontiers for the two-parameter version of equation (19); that is, assuming that the authority uses a rule that has only c_{t-1} and $x_{t-1|t-1}$ as arguments. The solid line is the performance of the optimal rules under the assumption that the output gap is known in each period. The dot-dashed line is the performance when there is uncertainty in the measurement of potential as detailed in the econometric estimates described above. Points towards the northwest of the frontiers on the right-hand quadrant of the figure represent the preferences of authorities that place a high weight on output stabilization in their objective functions (and a correspondingly low weight on inflation stabilization). The opposite is true for points toward the southeast of the frontier.

Let us focus for the moment on the full-information case shown with the thin lines in order to set up the intuition for what follows. First, note that the frontier is in standard deviations space, as mentioned previously. This is an implication of our model having a vertical long-run Phillips curve implying no systematic trade-off between the levels of output and inflation. Instead, the trade-off manifests itself in the second moments of these two variables. Second, the frontiers show the familiar convex-to-the-origin shape meaning that the trade-off between output variability and inflation variability becomes increasingly dear as one moves further to the northwest or southeast from positions near the midpoint of the frontier.

29. Woodford (1999) provides a rationale for interest-rate smoothing based the argument that small movements in the funds rate will induce large movements in long-term real rates owing to the expectation of private agents of future rate changes in the same direction. Interest rate smoothing can also be justified as an objective of monetary policy as a hedge against inappropriately applying point elasticities that are estimated over a narrow range of interest rate variability to policy environments where large ranges of movement are contemplated.

Figure 5
Efficient Policy Frontiers
2-parameter policy rule
(with and without uncertainty)

Third, frankly, the economic performance is poor. This result is a manifestation of the parsimony of the model--needed to permit feasible estimation--and of the rule--needed to keep the results simple and transparent. In particular, contemporaneous policy-rule arguments and a moving-average term in inflation would help performance substantially but at the cost of obscuring the main points of the paper which do not depend, in a material way, on the performance of the full-information case.³⁰

Now let us turn to a comparison of the uncertainty case against the full-information case. The obvious point to be taken from the figure is that uncertain measures of potential output lead to a deterioration in the performance of monetary policy. Increased loss shows up in all higher standard deviations of all variables that enter the loss function (including the funds rate, albeit only slightly). The difference for inflation, however, is slight. In fact, at the north-west end of the frontier, funds-rate volatility is slightly less, under uncertainty, than it is under full information. For the output gap, the deterioration in performance is more substantial, as might be expected given that this is the source of the uncertainty in the first place. Lastly, we note that the performance deterioration varies positively with the weight placed on output stabilization in the loss function.

Table 3 below shows the coefficients that are implied by three particular sets of preferences. The performances represented by these preferences are represented by points A, B and C (and A', B', and C') on the frontiers. Point A represents the preferences of an extreme inflation dove, with a zero weight on inflation stabilization and unity on output stabilization. Point C represents the preferences of an inflation hawk, with a weight on unity on inflation stabilization. Point B has equal weights of 0.5 on inflation and output gap stabilization.

30. Whatever specific quantitative performance, some experimentation combined with the received wisdom of the literature shows that our key results regarding the effects uncertain potential output seem to hold for a wide range of models. An exception, discussed below, is the effect on attenuation of policy with respect to inflation a subject about which the literature is sharply divided for reasons that are unclear.

Let us begin by comparing the coefficients on the perceived output gap in the two cases, one with full information, and the other with uncertain potential. These are the two shaded columns of the table. The results show that for all preferences, the coefficient of the output gap in the rule declines in response to the uncertainty in potential output. Moreover, the extent of the decline varies directly with the weight on the output gap in the full-information rule (and equivalently with the weight on output stabilization in the loss function). At the same time, the coefficient on inflation also falls in a broadly similar fashion, albeit by comparatively less than for output.

Table 3
Coefficients of Efficient 2-parameter Monetary Policy Rules
(certain and uncertain measures of potential output)

relative weight on demand	known potential output		uncertain potential output	
	ψ_c	ψ_x	ψ_c	ψ_x
1.0	0.39	0.53	0.28	0.19
0.5	0.52	0.29	0.44	0.14
0.0	0.77	0.13	0.74	0.09

Note: ψ_c are rescaled to account for the smaller average variability of output gaps under filtering.

Note as well that both with and without uncertainty, the optimal policy places some weight on the output gap in all cases--even when the authority does not care about the variability of output at all. This is because even in the presence of uncertainty as substantial as what we have here, there is still information content for future inflation in the movements of the perceived gap.

Intuition for these results can be gleaned from two different sources. The first is mathematical and originates from applied control theory beginning with the Brainard

(1967) result. Brainard showed that for “additive uncertainty”--like uncertainty in the output gap would normally be expected to be--certainty equivalence should result. That is, the optimal policy would be to form one’s best estimate of the output gap and then ignore the distributions around that estimate when designing policy. However, when one drops the assumption that every state in the model is represented in the policy rule, then there are insufficient states in the rule to span the uncertainty in the model.³¹ The resulting distortion means that certainty equivalence will not generally hold.

With no presumption of certainty equivalence, a second strand of logic proves useful, one from public finance: Uncertainty in potential output acts like a tax on the use of the output gap as an input to policy. Like other taxes, this one can be mitigated, to some extent, by reducing one’s reliance on the taxed input. Whether this results in an increase in the use of the other input--the inflation coefficient in this case--depends on the relative magnitudes of substitution and income effects. As it happens, inflation and the output gap are sufficiently complementary that “taxing” the use of the output gap results in a decrease in the reliance on inflation as well.

Finally, we turn to the estimated coefficients for the 3-parameter rule. Recall that this rule includes the lagged real federal funds rate as an additional argument to the 2-parameter rule. Table 3 contains a summary of the results.³²

31. The analogy to asset pricing problems with Arrow-Debreu and incomplete markets comes to mind.

32. Note that Table 3 shows that a negative coefficient on core inflation is optimal for inflation doves using 3-parameter rules. It is tempting to think that this must be unstable. But since the coefficient appears on contract (or core) inflation and not actual inflation, this is feasible (and optimal as it turns out). Working off of the covariance of c and x , the inflation dove controls output tightly and in such a way that the economy is still stable.

Table 4
Coefficients of Efficient 3-parameter Monetary Policy Rules
(certain and uncertain measures of potential output)

relative weight	known potential output			uncertain potential output		
	ψ_c	ψ_x	ψ_{rr}	ψ_c	ψ_x	ψ_{rr}
1.0	0.15	0.70	-0.40	-0.27	0.28	-0.78
0.5	0.59	0.26	0.16	0.13	0.17	-0.59
0.0	0.94	0.14	0.79	0.70	0.08	-0.14

Note: ψ_c are rescaled to account for the smaller average variability of output gaps under filtering.

As in Table 3, the results for the 3-parameter rules show a pattern of attenuation: Less aggressive policy in the face of uncertainty regarding potential output. This is true for impact effects for both core inflation and the output gap. A second, more subtle observation concerns the effect of uncertainty on the coefficient on the lagged real funds rate and its interpretation. Here too there is a decline in the coefficient. In this instance, there are two complementary interpretations. The first notes that a lagged endogenous variable can be rewritten as geometrically distributed lags of the other two arguments to the rule. Since uncertainty about potential output diminishes only slightly with the benefit of time, it follows that just as uncertainty about potential output makes current estimates of the gap less useful than they otherwise would be, the effectiveness of conditioning policy on past estimates of potential is also diminished. The second interpretation is a forward-looking one that emphasizes the role of the lagged funds rate on future dynamics of the funds rate in response to a shock. Suppose, for example output rises one percentage point in response to an unobserved permanent shock to potential output. In the first instance, two-thirds of this one-percentage point positive surprise to output will be erroneously attributed to a movement in aggregate demand. Suppose now that one period later there is no shock. The sur-

prise will now be minus two-thirds of a percentage point. The sign reversal on the response to this quite plausible shock shows how the monetary authority may find itself wanting to reverse course at times, owing to the permanence of shocks to potential output. The sharp decline in the coefficient on the lagged funds rate allows such reversals to come about. If ψ_{rr} were large, then mistakes in the perception of potential in the past would be allowed to persist for lengthy periods of time. In the presence of a string of errors, instability could ultimately arise. In this regard, the substantial decline, and indeed the sign reversal in ψ_{rr} , is not surprising.

5. Concluding Remarks

In this chapter, we have examined the implications of uncertainty in the measurement of potential output for the design of monetary policy. In particular, we estimated a small, forward-looking model of the U.S. economy using maximum likelihood techniques and the Kalman filter. Two key advantages accrue from adopting this technology. First, the estimates of potential are entirely consistent with the underlying structure of the model in which they are embedded. Second, the Kalman filter directly furnishes estimates of the uncertainty surrounding measures of potential that can be used to quantify the problem.

Our estimates showed statistically significant slopes of the Phillips curve and the IS curve as well as evidence supporting the notion that price setting reflects forward-looking behavior to some extent. More important for the present analysis, we were able to extract measures of potential output and its uncertainty. The point estimates show plausible patterns for the output gap, but a wide range of uncertainty around those point estimates.

Armed with these estimates, we examined the appropriate response of policy to this uncertainty. We found that a welfare maximizing monetary authority attenuates its policy actions in response to uncertainty. More precisely, the authority responds less aggressively to movements in both output and inflation. Our results showing attenuation

of responses to output are thus comparable with those of Orphanides (1998), Orphanides *et al.* (1999), Sack (1998), Smets (1999) and Rudebusch (1999). Of these papers, however, only Orphanides *et al.* (1999) uses a forward-looking model and, in that case, the estimates of potential are not consistent with the structure of the model.

If the above authors have found corroborating results for attenuation of responses to output, they conflict, to some extent, on the response to inflation and the lagged funds rate. Of these papers, only Orphanides *et al.* (1999) and Smets (1999) have looked at this effect in detail. On this score, results to date appear to be idiosyncratic to the model used.

We close this chapter by noting that this is not simply an academic issue. It is one of current interest to policy makers in the United States, Canada and elsewhere. In the U.S., the recent experience has been one of remarkable economic strength and low inflation. This performance has surprised nearly all observers of economic affairs in the U.S.. The Federal Open Market Committee's response to this unlikely chain of events and all the macroeconomic shocks around it has been to do...essentially nothing. One might ordinarily have expected the Fed to tighten policy in the face of the strong growth and falling unemployment of the period from 1996 to 1998, even in the absence of an observable movement in inflation. After all, prices lag the business cycle and so to wait for a turning point in inflation is to wait too long. The results in this paper provide an explanation for the Fed's reluctance to react to these events: the Fed arguably knew that the technological boom in the United States was making estimates of potential output less reliable than in the past and was tempering its policy actions in response. In Canada, the same question is only just now beginning to arise. The range of reforms undertaken by the federal government in the late 1980s and the early 1990s--reforms ranging from fiscal retrenchment, and (un)employment insurance reform, to deregulation and anti-trust reform all point in the direction of more efficient markets and (hopefully) a higher level of potential output. Yet no one could meaningfully say that he or she knows the true level of potential. At some

point, the Bank will have to decide when the economy is “close enough” to potential to warrant a tightening of policy. The results obtained here argue for caution.

6. References

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7. Appendix

In this appendix, we derive the state-space representation of the model. Begin with equation (2):

$$c_t = (1 - \lambda)(q_{t-1} + \lambda q_{t-2} + \lambda^2 q_{t-3} + \dots) \quad (\text{A1})$$

Lag back (A1) by one period and multiply the result by λ :

$$\lambda c_{t-1} = (1 - \lambda)(\lambda q_{t-2} + \lambda^2 q_{t-3} + \lambda^3 q_{t-4} + \dots) \quad (\text{A2})$$

Substitute this into the right-hand side of (A1) and lead the result by a period to arrive at:

$$c_{t+1} = (1 - \lambda)q_t + \lambda c_t \quad (\text{A3})$$

Now take equation (3), multiply it by λ :

$$\lambda q_t = (1 - \lambda)(\lambda \pi_{t+1} + \lambda^2 \pi_{t+2} + \lambda^3 \pi_{t+3} + \dots) \quad (\text{A4})$$

Now substitute this into equation (9): $-v_t = \pi_t + \lambda \pi_{t+1} + \lambda^2 \pi_{t+2} + \dots$. Solve the result for q_t to get:

$$q_t = \lambda^{-1}(1 - \lambda)(v_t - \pi_t) \quad (\text{A5})$$

Substitute equation (A5) into (A3) and use equation (1): $\pi_t = c_t + \beta x_t + u_t^\pi$ to obtain:

$$\begin{aligned} c_{t+1} &= (1 - \lambda)^2 \lambda^{-1} v_t + [\lambda - (1 - \lambda)^2 \lambda^{-1}] c_t - (1 - \lambda)^2 \lambda^{-1} \beta x_t - \delta u_{t+1}^\pi \\ &= \delta v_t + (\lambda - \delta) c_t - \delta \beta x_t - \delta u_{t+1}^\pi \end{aligned} \quad (\text{A6})$$

where, for the second equality in (A6), we have defined: $\delta = (1 - \lambda)^2 \lambda^{-1}$. This is the equation in the third row of equations (11).

Turning to the fourth row, we begin by noting that equation (3) is an expectations equation, meaning that stochastic shocks do not play a direct role; that is, they will drop out once an expectations operator is taken through the equation. Thus, we can work with perfect foresight solutions without loss of generality. With this in mind, we begin by leading equation (3) by one period and multiply this by λ . Substitute the result back into (3) to get:

$$q_{t+1} = \lambda^{-1} q_t + -(1-\lambda)\lambda^{-1} \pi_{t+1} \quad (\text{A7})$$

Substitute (A5) and the lead of (A5) into (A7) to derive:

$$v_{t+1} = \lambda^{-1}(v_t - \pi_t) \quad (\text{A8})$$

and lastly, take expectations of equation (1) and use the result to substitute for π_t in equation (A8):

$$v_{t+1} = \lambda^{-1} v_t - \lambda^{-1} c_t - \lambda^{-1} \beta x_t \quad (\text{A9})$$

which is the fourth row of equations (11) in the text.

The derivation of the rest of the state-space representation is straightforward.