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Analysis and Design of Reinforced Concrete Shell Elements

by

OSMAN BURKAN ISGOR, B.Sc.

A thesis submitted to
the Faculty of Graduate Studies and Research
in partial fulfillment of
the requirements for the degree of

Master of Engineering

Department of Civil and Environmental Engineering

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administered by The Ottawa-Carleton Institute Civil Engineering

Carleton University
Ottawa, Ontario, Canada
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Department of Civil and Environmental Engineering

The Undersigned recommend to the Faculty of Graduate
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Abstract

Design process of a reinforced concrete shell element involves two major steps: analysis and design. Development of the computerized linear elastic analysis techniques, such as the finite element method, has essentially solved the first part of the problem. Design, on the other hand, is not as easy to handle since the number of unknowns is larger than the number of available equations of statics. Due to this fact, many design methods have been developed by various researchers. This thesis attempts to compare the more important existing methods of analysis of orthogonally reinforced concrete shell elements with each other. A Windows-based computer program is written in Visual Basic to implement these methods and to facilitate the analysis. First, the selected results of the methods are compared with available experimental data. Since the amount of experimental results is not adequate for a definitive comparison, a parametric study is carried out. The modified compression field theory is used as a reference in the parametric study. The effects of loading, reinforcement ratio, and concrete strength on the strength of a total of 720 panels investigated. The research showed that under highly unsymmetrical loading conditions, and in the cases where the reinforcement ratio in one orthogonal direction is quite different from the one in the other direction (highly orthotropic), the accuracy of the existing methods of analysis significantly decreases. Change in concrete strength played an important role only in the methods which used simple models for concrete behaviour. Based on the present investigation, suitable methods are identified for different load cases, and recommendations for future work are made.

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Contents

ABSTRACT	iii
ACKNOWLEDGMENTS	iv
CONTENTS	v
LIST OF TABLES	viii
LIST OF FIGURES	xii
NOTATIONS	xiv
1 Introduction	1
1.1 General	1
1.2 Definition of the Problem	3
1.3 Objectives and Scope	13

2 Literature Review	17
2.1 General	17
2.1 History	18
3 Existing Methods of Analysis of Membrane Elements	32
3.1 General	32
3.2 Modified Compression Field Theory	33
3.2.1 Introduction	33
3.2.2 Formulation of the Theory	39
3.3 Nielsen's Method	55
3.4 Clark's Method	67
3.5 Fialkow's Method	76
3.6 Zararis' Method	81
3.7 Modified Zararis' Method	94
4 Comparison of the Results of Existing Methods	
with Available Experimental Data	99
4.1 General	99
4.2 Description of the Computer Program: MemCap	100
4.3 Available Experimental Data	104
4.4 Comparison of Existing Methods with the Available Experimental Data	106

5 Parametric Study	112
5.1 General	112
5.2 Parametric Study	113
5.3 Selection of Parameters	114
5.4 Results and Discussion	129
 6 Conclusions and Recommendations	 163
6.1 Summary	163
6.2 Conclusions	164
6.3 Recommendations for Future Work	166
 References	 167
 A Source Code of MemCap	 174
 B Ultimate Capacities of the Panels Tested in the Parametric Study	 197

List of Tables

3.1: Boundary Curve Equations For Nielsen's Method	65
3.2: Possible Reinforcement Combinations	70
3.3: Design Equations for Orthogonal Reinforcement (Clark 1976)	72
3.4: Boundary Curve Equations for Orthogonal Reinforcement (Clark 1976)	73
4.1: Input Data Information for MemCap	103
4.2: Properties of the Test Specimens	105
4.3: Comparison of Ultimate Capacities (v_u) of Existing Methods	
with Experimental Results	107
4.4: Predicted / Experimental Capacities	108
5.1: Values of the Constant Parameters	116
5.2: Cases Based on the Reinforcement Ratio	118
5.3: Notations Used for the Panels in the Parametric Study	118
5.4: Input Data for Load Case No. 1 Panels	120
5.5: Input Data for Load Case No. 2 Panels	121
5.6: Input Data for Load Case No. 3 Panels	122
5.7: Input Data for Load Case No. 4 Panels	123
5.8: Input Data for Load Case No. 5 Panels	124

5.9: Input Data for Load Case No. 6 Panels	125
5.10: Input Data for Load Case No. 7 Panels	126
5.11: Input Data for Load Case No. 8 Panels	127
5.12: Input Data for Load Case No. 1 Panels	128
5.13: Normalized Capacities for Load Case No. 1 ($f_c = 15 \text{ MPa}$)	133
5.14: Normalized Capacities for Load Case No. 2 ($f_c = 15 \text{ MPa}$)	134
5.15: Normalized Capacities for Load Case No. 3 ($f_c = 15 \text{ MPa}$)	135
5.16: Normalized Capacities for Load Case No. 4 ($f_c = 15 \text{ MPa}$)	136
5.17: Normalized Capacities for Load Case No. 5 ($f_c = 15 \text{ MPa}$)	137
5.18: Normalized Capacities for Load Case No. 6 ($f_c = 15 \text{ MPa}$)	138
5.19: Normalized Capacities for Load Case No. 7 ($f_c = 15 \text{ MPa}$)	139
5.20: Normalized Capacities for Load Case No. 8 ($f_c = 15 \text{ MPa}$)	140
5.21: Normalized Capacities for Load Case No. 9 ($f_c = 15 \text{ MPa}$)	141
5.22: Normalized Capacities for Load Case No. 1 ($f_c = 25 \text{ MPa}$)	142
5.23: Normalized Capacities for Load Case No. 4 ($f_c = 25 \text{ MPa}$)	143
5.24: Normalized Capacities for Load Case No. 5 ($f_c = 25 \text{ MPa}$)	144
5.25: Normalized Capacities for Load Case No. 6 ($f_c = 25 \text{ MPa}$)	145
5.26: Normalized Capacities for Load Case No. 7 ($f_c = 25 \text{ MPa}$)	146
5.27: Normalized Capacities for Load Case No. 8 ($f_c = 25 \text{ MPa}$)	147
5.28: Normalized Capacities for Load Case No. 9 ($f_c = 25 \text{ MPa}$)	148
5.29: Normalized Capacities for Load Case No. 1 ($f_c = 25 \text{ MPa}$)	149
5.30: Normalized Capacities for Load Case No. 1 ($f_c = 25 \text{ MPa}$)	150

5.31: Mean Normalized Capacities for Load Case No. 1	151
5.32: Mean Normalized Capacities for Load Case No.2	152
5.33: Mean Normalized Capacities for Load Case No.3	153
5.34: Mean Normalized Capacities for Load Case No.4	154
5.35: Mean Normalized Capacities for Load Case No.5	155
5.36: Mean Normalized Capacities for Load Case No.6	156
5.37: Mean Normalized Capacities for Load Case No.7	157
5.38: Mean Normalized Capacities for Load Case No.8	158
5.39: Mean Normalized Capacities for Load Case No.9	159
5.40: Proposed Methods* for the Analysis of Reinforced Concret	
Shell Elements Based on the Load Case and Reinforcement Ratio	160
B.1: Ultimate Capacities for Load Case No. 1 ($f_c = 15$ MPa)	198
B.2: Ultimate Capacities for Load Case No. 2 ($f_c = 15$ MPa)	199
B.3: Ultimate Capacities for Load Case No. 3 ($f_c = 15$ MPa)	200
B.4: Ultimate Capacities for Load Case No. 4 ($f_c = 15$ MPa)	201
B.5: Ultimate Capacities for Load Case No. 5 ($f_c = 15$ MPa)	202
B.6: Ultimate Capacities for Load Case No. 6 ($f_c = 15$ MPa)	203
B.7: Ultimate Capacities for Load Case No. 7 ($f_c = 15$ MPa)	204
B.8: Ultimate Capacities for Load Case No. 8 ($f_c = 15$ MPa)	205
B.9: Ultimate Capacities for Load Case No. 9 ($f_c = 15$ MPa)	206
B.10: Ultimate Capacities for Load Case No. 1 ($f_c = 25$ MPa)	207
B.11: Ultimate Capacities for Load Case No. 2 ($f_c = 25$ MPa)	208

B.12: Ultimate Capacities for Load Case No. 3 ($f'_c = 25$ MPa)	209
B.13: Ultimate Capacities for Load Case No. 4 ($f'_c = 25$ MPa)	210
B.14: Ultimate Capacities for Load Case No. 5 ($f'_c = 25$ MPa)	211
B.15: Ultimate Capacities for Load Case No. 6 ($f'_c = 25$ MPa)	212
B.16: Ultimate Capacities for Load Case No. 7 ($f'_c = 25$ MPa)	213
B.17: Ultimate Capacities for Load Case No. 8 ($f'_c = 25$ MPa)	214
B.18: Ultimate Capacities for Load Case No. 9 ($f'_c = 25$ MPa)	215

List of Figures

1.1: Structures Comprised of Reinforced Concrete Shell Element Assemblages	6
1.2: Reinforced Concrete Shell Element under General State of Loading	7
1.3: Stress Resultants Acting on a Reinforced Concrete Shell Element	8
1.4: Free Body Diagram of a Cracked Shell Element	9
2.1: Sandwich Element to Convert Moment Triad to In-Plane Forces	23
2.2: Clark's Boundary Case Graph	24
3.1: Diagonal Tension in Thin-Webbed Metal Girder	34
3.2: Equilibrium Conditions for A Beam Element	35
3.3 Response of Concrete To Compressive Stresses	38
3.4 Shell Element Under In-Plane Forces	41
3.5 Stresses in Cracked Concrete	46
3.6 Response of Concrete to Tension Forces	47
3.7 Summary of Cracked Concrete Behaviour in MCFT	48
3.8 Summary of Reinforcement Behaviour in MCFT	49
3.9 Summary of Reinforced Concrete in MCFT	53
3.10 Summary of the Modified Compression Field Theory (Vecchio 1986)	54
3.11 Behaviour of Steel and Concrete in Nielsen's Model	56

3.12 Case 1: Reinforcement Required in Both Directions	62
3.13 Case 2: Reinforcement Required in y-direction	63
3.14 Case 3: Reinforcement Required in x-direction	64
3.15 Boundary Case Graph For Nielsen's Method	66
3.16 Directions of Reinforcement and Principal Concrete Stresses	68
3.17 Boundary Case Graph For Clark's Method	75
3.18 Stresses and Strains of Reinforcement Before Yielding	82
3.19 Free Body Diagram of the Membrane Element Before Yielding	83
3.20 Free Body Diagram of the Membrane Element Assuming Yielding of Steel in Both Directions	91
3.21 Free Body Diagram of the Membrane Element Assuming Yielding of x-reinforcement	92
3.22 Free Body Diagram of the Membrane Element Assuming Yielding of y-reinforcement	93
4.1: Main Window of MemCap	102
4.2: Comparison of the Results of the Existing Methods with the Experimental Data	109
5.1 Boundary Case Graph for the Case "LR1 - RR1" (25 MPa Concrete)	117
5.2 Comparison of Existing Methods with MCFT (Panel P1-6)	162
5.3 Comparison of Existing Methods with MCFT (Panel P33-7)	162
A.1: Main Window of MemCap	170

Notations

A	Cross-sectional area of a shell element of unit length.
b_w	Effective width of the beam.
d	Effective depth of the beam.
E_c	Modulus of elasticity of concrete.
E_s	Modulus of elasticity of concrete.
f_2	The diagonal compressive stress in concrete.
f_{2max}	Maximum principal compressive stress.
f_{ci}	Compressive stress on crack surface (positive).
f_{cr}	Cracking stress of concrete.
f_{cx}	Axial stress carried by concrete in the x-direction.
f_{cy}	Axial stress carried by concrete in the y-direction.
f_{sx}	Axial stress carried by steel in the x-direction.
f_{sy}	Axial stress carried by steel in the y-direction.
f_{sxcr}	Stress in x-reinforcement at crack location.
f_{sycr}	Stress in y-reinforcement at crack location.
f_v	The stress in the transverse reinforcement.
f_x	Applied axial force per unit length in the x-direction.

f_y	Applied axial force per unit length in the y-direction.
f_{yx}	Yield stress of steel in the x-direction.
f_{yy}	Yield stress of steel in the y-direction.
f'_x	Component of tensile strength coming from x-reinforcement.
f'_y	Component of tensile strength coming from y-reinforcement.
M_x	Applied bending moment per unit length in the x-direction.
M_{xy}	Applied torsional moment per unit length.
M_y	Applied bending moment per unit length in the y-direction.
v_{ci}	Shear stress on crack surfaces.
v_{cxy}	Shear stress carried by concrete.
v_{xy}	Applied shear force per unit length.
v_{sx}	Shear stress carried by steel in the x-direction.
v_{sy}	Shear stress carried by steel in the y-direction.
v'_{sxy}	Component of shear strength coming from x- and y-reinforcement.
s	Spacing between the stirrups.
t	Thickness of the element.
T	Stress tensor.
T_c	Stress tensor of concrete.
T_s	Stress tensor of steel.
ϵ_1	Principal tensile strain.
ϵ_2	Principal compressive strain.
ϵ'_c	Ultimate crushing strain for concrete.

ϵ_x	Strain in the x-direction.
ϵ_y	Strain in the y-direction.
ϕ_t	Tensile strength reduction factor.
ϕ_v	Shear strength reduction factor.
ρ_{sx}	Reinforcement ratio in the x-direction.
ρ_{sy}	Reinforcement ratio in the y-direction.
ρ_{xt}	Reinforcement ratio for steel bars resisting tension in the x-direction.
ρ_{yt}	Reinforcement ratio for steel bars resisting tension in the y-direction.
ρ_{xv}	Reinforcement ratio for steel bars resisting shear in the x-direction.
ρ_{yv}	Reinforcement ratio for steel bars resisting shear in the y-direction.
θ	The angle of inclination of the crack.
θ_c	Angle of principal concrete stress.

Chapter 1

Introduction

1.1 General

The design process for a civil engineering structure involves two major steps: analysis and detailing. Analysis is the determination of the internal forces induced in the structure by the applied loads and/or environmental agencies. If the structure is complex, the common practice is to dismember it into simpler elements and to determine the distribution of the applied loads among and within these elements. Regardless of the complexity of the structure, engineers can analyze it by using one of the available and appropriate methods of analysis. The advent of fast and powerful computers and the development of numerical techniques have made the analysis of highly complex structures possible and feasible.

Simply stated, design is the provision of the adequate section size and reinforcement, and the selection of appropriate materials to safely resist the applied forces obtained from the analysis. For steel structures, design may mean the determination of the appropriate size and shape of cross section of the element and the selection of a suitable steel grade. For reinforced concrete, on the other hand, design may involve the calculation of the amount and strength of reinforcement which needs to be provided as well as the determination of the section size and geometry.

Development of the computerized linear elastic analysis techniques such as the finite difference, finite element, and boundary element methods, as well as the so-called stiffness method of analysis for framework has essentially solved the first part of the problem; i.e., elastic analysis, with high accuracy. It has become common for engineers to use finite element programs to analyze complex structures. The second part of the problem; i.e., design, is generally handled by resorting to code guidelines or a combination of first principles and empiricism. For example, design of a reinforced concrete beam for flexure has become a routine problem for structural engineers. Reinforced concrete design specifications give explicit equations which lead to the effective design of these sections. In some cases though, the design part of the procedure is not as easy. This thesis deals with one of these cases, namely the design of reinforced concrete shell elements.

1.2 Definition of the Problem

Although generally a shell structure and/or element may be subjected to simultaneous membrane and bending action, in the literature one commonly refers to facet or out-of-plane curved elements subjected to membrane forces alone as shell elements. Fig. 1.1 illustrates some reinforced concrete structures which could be idealized as assemblages of shell elements. We observe that either the entire structure or some parts of it can be considered to be formed of shell elements. A typical reinforced concrete shell element and all the possible forces which could act on it are shown in Fig. 1.2. Applied forces on a shell element consist of an in-plane force (f_x , f_y , and v_{xy}) and an out-of-plane moment (M_x , M_y , and M_{xy}) triad. In Fig. 1.3 the two triads are shown separately. It has to be noted that a reinforced concrete shell element which is under an in-plane force triad is generally called a “membrane element” and this terminology will be used throughout this thesis. The forces and moments in Fig.’s 1.2 and 1.3 denote the following:

f_x = Applied axial force per unit length in the x-direction

f_y = Applied axial force per unit length in the y-direction

v_{xy} = Applied shear force per unit length

M_x = Applied bending moment per unit length in the x-direction

M_y = Applied bending moment per unit length in the y-direction

M_{xy} = Applied torsional moment per unit length

The preceding forces and moments are generally referred to as stress resultants.

As stated previously, as a consequence of the development of computer aided analysis techniques, it has become routine for an engineer to determine the applied forces on a shell element. When a shell structure is analyzed by the finite element method, the force and moment triads distribution is output by the program throughout the shell structure. The problem is then to detail the shell element for these forces. The design of a reinforced concrete shell under the concurrent application of the force and moment triads is very complex and a suitable design method, which is rigorous and robust, is yet to be developed. For example, due to the lack of a closed form simple solution, designers consider the design of a concrete column subjected to biaxial moment and an axial load as difficult and time-consuming. Let us contrast the column with a shell element which would be in general subjected to biaxial bending, torsion, biaxial in-plane forces, and in-plane shear simultaneously. The complexity of the problem becomes apparent at once.

If we ignore the various interactions among these forces and moments, we must ensure that the design would still be safe. Another question that begs an answer is: Why perform complex analyses knowing that their results cannot be rigorously applied in the design? To illustrate the problem, let us consider Fig. 1.4 which shows a three dimensional view of a part of an orthogonally reinforced membrane element traversed by a through crack. The design of such an element would entail determining the thickness of concrete, t , and the amount of reinforcement in each direction, A_{sx} , A_{sy} , for given forces f_x , f_y , v_{xy} . Theoretically, the solution to this problem can be obtained by considering the equilibrium

and compatibility requirements. Let us consider the element in Fig 1.4. Equilibrium requires:

$$\Sigma F_x = 0 \quad (1.1)$$

$$\Sigma F_y = 0 \quad (1.2)$$

where F_x = force in the x-direction (i.e. f_x times the length of the element in y-direction),
 F_y = force in the y-direction (i.e. f_y times the length of the element in x-direction).

Unknowns of the design problem, on the other hand, include thickness of the element, t ; reinforcement ratio (amount of steel / unit length) in the x-direction, ρ_x ; reinforcement ratio in the y-direction, ρ_y ; and the crack angle. If a value for the thickness of the element is assumed, there still remain three unknowns but two equations which leads to infinite number of possible solutions. Practically, only a limited number of the solutions may be feasible. The availability of an extra equation is therefore a must if an appropriate solution is to be obtained.

This lack of a third relation or equation among the unknowns is the main reason for the existence of the different methods for the design of reinforced concrete shell elements. Each method is predicated on certain assumptions in order to generate the necessary third equation. Some of these methods are:

Figure 1.1: Structures Comprised of Reinforced Concrete Shell Element Assemblages

Figure 1.2: Reinforced Concrete Shell Element under General State of Loading

(a) In-plane force triad on a shell element

(b) Moment triad acting on a shell element

Figure 1.3: Stress Resultants Acting on a Reinforced Concrete Shell Element

Figure 1.4: Free Body Diagram of a Cracked Shell Element

For in-plane force triads:

- Nielsen's method (Nielsen 1963)
- Clark's method (Clark 1976)
- Fialkow's method (Fialkow 1983, 1985, 1990)
- Zararis' method (Zararis 1988)
- Modified Zararis method (Zararis 1997)
- Gupta's method (Gupta 1981, 1985)

In addition, the compression field theory can be used as a trial-and-error procedure to arrive at a suitable design.

For out-of-plane moment triads:

- Morley's method (Morley 1970)

For combined loading:

- Gulvanessian-Sim's method
- Nielsen's method (Nielsen 1974)

Although shells which are subjected to combined loads are separated from membrane elements in this list, they are currently designed by basically the same procedures developed for the membrane elements. In the former case, moments are represented by couples separated by some predetermined distance from the middle plane. The element is then considered to have more than one set of in-plane force triads and each layer is

designed for its corresponding in-plane forces. In other words, developing a method for membrane elements will also solve the design problem for shells under moment or combined loading. But the solutions obtained in the latter case are approximate and to date, to the writer's knowledge, no attempt has been made to obtain a closed-form solution for the problem. A complete solution would be extremely complex and most likely not suitable for practical applications on routine basis. This is probably the reason why most of the research on this topic has been focused on the design of membrane elements.

The preceding methods differ from each other mainly in the manner in which they provide the extra equation that is necessary to reach a unique solution for the design problem. Invoking compatibility requirements, experimental observations, or optimization procedures, a solution is reached. This apparently little detail leads to different design solutions for the same problem.

Computer aided techniques for the determination of the response of reinforced concrete shell elements are efficient tools for obtaining accurate and detailed results. The modified compression field theory (MCFT) (Vecchio and Collins 1986) is one of the latest methods which attempts to solve this problem by combining theory with experimental observations. Unlike other methods, MCFT gives the complete response of an element to a set of applied loads.

Although the results obtained from computer based detailed methods are very close to the experimental values, the corresponding analyses are complicated, and time-consuming; i.e., they are not appropriate for everyday design problems faced by the engineers. Considering the fact that design is an iterative procedure, using these techniques on a routine basis will be a major challenge for the designer. On the other hand, design oriented methods, such as Nielsen's method (Nielsen 1963), will not give the full response, nor are they as accurate, but they are simple to apply and suitable for hand calculations.

Hence there is a trade-off between accuracy and simplicity. From the design point of view, simple and fast methods are always preferred by engineers. This is basically because of the fact that the more complicated the method is the more likely the engineer will make a mistake and the more time he will spend. Another fact is that complicated computer techniques such as the modified compression field theory can be very dangerous if the designer is not adequately familiar with its fundamental concepts and assumptions.

As stated earlier many approximate design methods are available for membrane elements. However, the degree of accuracy provided by each of these methods for various load combinations is not obvious. No comprehensive study has been previously carried out to compare their results for different combinations of forces and reinforcement ratios, and to ascertain the differences among their results. Such a study would permit the selection of one or two methods for general applications in design.

1.3 Objectives and Scope

The objective of this research is to conduct a comparative study of the methods of design of reinforced concrete shell elements and, following the completion of the study, to select and recommend a method or methods, for practical applications.

Specifically, the study will permit solution of the following problem: Given a concrete shell element subjected to stress resultants f_x , f_y , and v_{xy} , design the appropriate reinforcement in the x and y directions, with x and y being orthogonal set of axes.

The present study is confined to the study of ultimate strength, with no regard to serviceability requirements. It is presumed that the latter requirements are either met or needs to be separately checked.

It has to be noted that henceforth, the term “analysis” will be used interchangeably with the term “design” since the methods under consideration will be used in the calculation of the ultimate capacities of shell elements for which all of the reinforcement parameters are predefined. In other words, “analysis” in the title of this thesis stands for the “response analysis” of the element and not the “structural analysis”.

Due to the lack of enough and systematic experimental results, which could be used as reference values for comparison, one of the computer oriented methods will be used for

this purpose. This will enable us to test a large number of cases without the need for expensive and time-consuming experimental work. The modified compression field theory (Vecchio and Collins 1986), developed at University of Toronto, is selected for this purpose. A computer program will be developed based on this method. Although it is shown that the modified compression field theory gives accurate results, a considerable amount of time will be expended to validate a program written in the current study and to justify its results as reference for comparison of other methods. This will be done by comparing the results of the modified compression field theory with available experimental data from the literature.

The following methods will be considered in the parametric study:

- Nielsen's method (Nielsen 1963)
- Clark's method (Clark 1976)
- Fialkow's method (Fialkow 1983, 1985, 1990)
- Zararis' method (Zararis 1988)
- Modified Zararis method (Zararis 1997)

The results of the parametric study will be discussed and the method(s) which give the most accurate solution will be identified to the engineers as design tools for reinforced concrete shell elements.

A Windows based computer program will be developed which will enable the engineer to calculate the capacity of a reinforced concrete shell element under in-plane forces by using the existing methods of analysis and the modified compression field theory.

The thesis is organized into five chapters as follows:

Chapter 1 presents the definition of the problem and objective and scope of the thesis.

Chapter 2 gives the literature review relevant to the present study.

Chapter 3 explains all the design methods which are used in the study. The chapter is divided into two parts: Part I explains the modified compression field theory which is a computer oriented response analysis method. The other methods; i.e., Nielsen's method, Clark's method, Fialkow's method, Zararis' method, Modified Zararis method are explained in Part II.

Chapter 4 describes the computer program, MemCap, and compares the results of the various methods with available experimental data.

Chapter 5 describes the various parameters included in the parametric study and presents the results of the analysis. The results are then compared with the reference values obtained from the modified compression field theory. Discussion of the results conclude this chapter.

Chapter 6 presents the summary, conclusions and suggestions for further research.

Chapter 2

Literature Review

2.1 General

This chapter summarizes previous studies which have been carried out on the design of reinforced concrete shell elements. Since some of the methods are explained in detail in Chapter 3, they will not be extensively discussed in this chapter. Although the scope of the present thesis is limited to membrane elements, the discussed literature review is more comprehensive because, as stated earlier, the design methods developed for membrane elements can be extended to design of slab and shell elements.

2.2 History

The analysis and design of thin-walled reinforced concrete members have been a topic of research for a long time. The works on this subject can be divided into three groups. The first is the membrane reinforcement; i.e., reinforcement for in-plane shear and axial forces. Fig. 1.3(a) illustrates the state of stress on a typical membrane element which is subjected to in-plane stress resultants f_x , f_y and v_{xy} . The second is the slab element which is subjected to out-of-plane moment triad as illustrated in Fig. 1.3(b). The last one is the reinforcement for combined bending and in-plane forces. Although bending is not always absent, most of the work has focused on the in-plane forces because the design of reinforcement for the combined membrane and bending state of stress is a complex problem for which a general satisfactory solution is still the subject of research.

In the design of thin-walled concrete structures, the basic distribution of internal forces within the structure is usually found by elastic analysis. The problem then is to detail the structure for the force distribution given by the analysis. Although some constraints on the direction or the amount of reinforcement to be used may be placed; the general procedure is to minimize the total amount of reinforcement within specified zones in the structure. Of course, the chosen reinforcement must be able to safely resist the applied loads.

Early studies on the subject were carried out by Leitz (1923), Falconer (1956), Sholz (1958), Flugge (1962), Kuyt (1964), and Peter (1964). Since all these authors have reported in their native languages, not in English, it would be useful to read the review paper on membrane reinforcement in concrete shells by Gupta (1984) who summarized their work. Leitz decomposed the shear forces on the shell element parallel and diagonal to the reinforcing net, and assumed that both concrete and reinforcement contribute to resisting it. Following the same assumption, Sholz analyzed skew reinforcement. Falconer made the revolutionary assumption that concrete provides resistance against compression in one arbitrary direction, but he could not develop a procedure to determine the crack direction. Flugge suggested that cracks are formed normal to the reinforcement, and shear is taken by friction between the cracks. It is obvious that this suggestion is not true particularly near or at the ultimate state. Kuyt extended this procedure to skew reinforcement. Peter made some tests on square panels under uniaxial tension. He made the assumption that the crack will be in the direction of uniaxial tension. His two specimens which were reinforced isotropically cracked in accordance with this assumption, but the rest which were reinforced orthotropically, did not behave the same way. Accordingly, none of these investigators was able to solve the design problem and most of them made unrealistic assumptions to simplify the theory.

Nielsen (1963) first derived a yield criterion for an isotropically reinforced membrane and later extended it to the orthotropically reinforced case by using a lower bound plasticity theory (Nielsen 1971). He developed procedures for finding the yield envelope of a given panel of known reinforcement and concrete strength, and the method for designing the

reinforcement for a given set of forces. To find the direction of crack, Nielsen made some assumptions based on the minimization of the amount of reinforcement to be used in the panel. Under these assumptions the smallest amount of reinforcement is found when the reinforcement bars are stressed in tension with a stress equal to the yield stress of steel. The main problem in this work was the assumption that concrete alone would resist the compressive forces and no compression reinforcement would be provided. Another problem was related to the determination of the reinforcement direction which was based only on the yielding of the reinforcement but that will not provide the most effective arrangement.

Morley (1970) included the concept of strain-rate fields which give equal positive strain rates in each reinforcement position. He derived equations for the determination of skewed reinforcement to resist a particular in-plane force triad. For the case of orthogonal reinforcement, these equations reduce to Nielsen's (1963) equations. According to this study, negative values for the area of reinforcement could be obtained, implying that either no reinforcement or compression reinforcement is required along those directions. However, in either case, it is practically impossible to obtain a strain-rate field which gives the same strain rate in both reinforcement directions. The importance of this study is the graphical approach which is used to determine the set of equations to be used depending on the loading. This technique was later used by many other engineers.

Nielsen (1974) developed similar equations by solving the force equilibrium equations. His method could deal with both moments and in-plane forces by using a sandwich model.

In the general case, the reinforcement was assumed to consist of one or two parallel layers of orthogonal reinforcing net. In the case of two layers, the directions of the bars were assumed to be the same in both layers. As in the previous work of Nielsen, compression reinforcement was not considered. The sandwich element which consisted of three layers was substituted for the actual shell element. As illustrated in Fig. 2.1, the actual element thickness, t , was replaced with a sandwich element with top and bottom layers of thickness t_1 and t_2 . All moments were resolved into membrane forces located at the middle surfaces of the top and bottom layers, and all in-plane forces were applied on the middle surface of the center sandwich layer. The thickness of each sandwich layer was governed by the predominant bending moment. Then force equilibrium equations were solved for various cases of loading, and design equations were obtained. One of the main problems of this approach was the presumption of the crack direction according to the given loading. Experiments have shown that crack direction depends on more factors than applied forces and they cannot be neglected.

Clark (1976) extended Nielsen's approach (Nielsen 1963) to cover the possibility that compression reinforcement might be required. According to Clark the probable reason for the tendency to ignore compression reinforcement in the past was that it had not been considered to be of practical interest. He developed three equilibrium equations to be solved for nine different cases which include the possibilities of no reinforcement, tension reinforcement, and compression reinforcement. Direct solution of these equations is possible except for the cases when both directions had tension or compression reinforcement. In these cases there were four unknowns for three equations so that

infinite number of solutions are possible. Another equation which minimized the total amount of reinforcement was used to solve these two cases.

Clark's equations must be used in conjunction with a 'case boundary graph' (See Fig.2.2) which identifies the relevant set of equations to be used for a particular stress triad. After the graph is constructed, the location of the case is determined. Each number in the graph represents a set of design equations for the particular situation. By using these equations the reinforcement is determined. Apparently this graph will be constructed for different values of concrete strength and thus the knowledge of the design concrete strength is required before the graph can be plotted. Additionally, when the reinforcement is not orthotropic, case boundary graphs become more complicated and it would be difficult to draw graphs for each concrete strength unless a computer algorithm is developed to determine the location of the case in the graph. Nielsen's approach, on the other hand, which considers only tensile reinforcement differs in that it does not use any auxiliary charts since concrete strength is determined directly from the given equations. It has to be noted that for points for which Clark's equations indicate compression reinforcement, Nielsen's equations require either concrete of higher strength or thicker section.

Gulvanessian and Sims combined the studies of Nielsen and Clark such that the method would use a graphical approach as in the Clark's method, but the case boundary graph would not be constructed for each concrete strength. To achieve this, they used a

Figure 2.1: Sandwich Element to Convert Moment Triad to In-Plane Forces

Figure 2.2: Clark's Boundary Case Graph

sandwich method and first analyzed the membrane case only, then extended this approach to cover both membrane forces and the moments. They used Nielsen's general yield surface graph (Nielsen 1963), and developed the method accordingly. This method is suitable for hand calculations, but not programmable since the graph must be constructed by hand.

While these studies were being carried out, Baumann (1972) used another technique to solve the design problem. He pointed out that the direction of compression in concrete was statically indeterminate which had not been considered up to that time. Baumann used the principle of minimum resistance as one possible method of determining this compression (or the crack) direction. According to this principle, the controlling crack direction is formed normal to the direction in which the ratio of the component of reinforcement capacities and the component of the applied stresses is minimum. Baumann also applied this principle to solve the case of three directional reinforcement of arbitrary orientation.

The principle of minimum resistance implies some assumptions in itself. The first assumption is that the crack is formed normal to the major principal strain direction. It also assumes that shear stress is absent along the crack. Duchan (1972) employed these assumptions and made them explicit for a 4-directional (two orthogonal and two inclined) reinforced concrete membrane. This method is also applicable to prestressed concrete members. Gupta (1981) used the same assumptions as Duchan to determine the strains at the limit state. He also used the principle of minimum resistance for obtaining the design

equations and recommended methods not only to design the optimum amount of reinforcement but also to accomplish least deformation.

Another approach for the determination of the crack direction was developed by Bazant and Tsubaki (1978). This 'slip free' design method suggested that the crack would be formed in the direction along which the ratio of the shear capacity to the shear stress is minimum. The reinforcement capacity calculated was a little higher than the ones found by the methods developed based on the principle of minimum resistance. Although there was some difference in reinforcement capacities, the deformation calculations gave almost the same results as in the other methods.

Fialkow (1983,1985) treated this problem differently by attempting to solve it using the procedures of the strength design method of ACI 318-77 Building Code (ACI 1977) which complies with the lower bound theorem of limit analysis. The design strength of a membrane element is evaluated in terms of the shear design strength parallel to the sides of the element and normal design strength perpendicular to the sides of the element. For the reinforcement, design equations which are similar to the ones in the Code for Concrete Reactor Vessels and Containments, ACI 359-80 (ACI 1980), were developed. In the following studies, Fialkow (1988,1990) improved his previous method by considering the effects of compatible stresses and cracking.

Zararis (1988) criticized previous methods on the basis that they underestimated the role that shear forces of steel bars played in the capacity of the element. He considered the

possible failure mechanisms of a shell element under in-plane force triad and included the effect of shear forces carried by the reinforcement. If the failure was because of the yielding of the steel bars, equations which determine the ultimate capacity of the element were derived. In the case of crushing of concrete, the vector of concrete stresses between the cracks was derived, and the ultimate load was calculated from the concrete strength under combined tension and compression. He (Zararis 1997) also improved his method by adding the effects of aggregate interlock mechanism into the procedure.

The US Nuclear Regulatory Commission sponsored a series of experiments on reinforced concrete panels at Cornell University (Perdiakis et al. 1980, Conley et al. 1981) and Portland Cement Association (PCA) (Oesterle and Russel 1980, 1981). In these tests, specimens were precracked orthogonally to simulate the cracks which may be caused in nuclear vessels at the time of pressure testing. In each specimen, inclined cracks were formed and the preexisting orthogonal cracks did not play a central role. Although neither Cornell nor PCA experiments can be used to give quantitative estimates of reinforced concrete capacity, they demonstrated that preexisting cracks were not critical, and were capable of transmitting sufficient shear leading to inclined cracking.

Vecchio and Collins (1982) at the University of Toronto tested thirty orthogonally reinforced concrete panels. The most significant result of these experiments was the fact that no method developed up to that time could estimate the response of the elements accurately. This was confirmed in an international competition (Collins et al. 1985) in which many investigators from all over the world attempted to predict the response of 4

reinforced concrete panels tested by Vecchio and Collins (1982). The explanation of this problem according to Vecchio and Collins was that the response of cracked concrete would be completely different from the response of the uncracked one. For example, in a reinforced concrete element, new cracks may form and pre-existing cracks may propagate or close. Also, stresses in reinforcement change along the bar: At crack locations the bar stress will reach its maximum value while between cracks it will diminish. Concrete also shows a different behaviour: Although it does not carry any tension along the cracks, it resists tension between them; and with the increase of strain perpendicular to the crack, concrete becomes weaker in compression in the orthogonal direction. Previous studies never considered this fact, but Vecchio and Collins developed a model to determine the cracked concrete behaviour. They developed 'the modified compression field theory' (Vecchio and Collins 1986) based on 'the compression field theory' (Mitchell and Collins 1974, Collins 1978) for reinforced concrete in pure torsion. In both theories cracked concrete was treated as a new type of material with its own stress-strain behaviour. Equilibrium, compatibility, and constitutive relationships were formulated in terms of average stresses and strains. They also gave an algorithm to design a shell element based on this theory. One disadvantage of the method is that although iterations give good results at the end, the number of iterations to be carried out depends on the initial assumptions of some values such as principal tensile strain, principal compressive stress direction, and average stress in the weaker reinforcement. In spite of this fact, this problem can be overcome by a good programming technique.

Since the modified compression field theory had the power to predict the test results quite accurately and was programmable, it became one of the main topics of research for the following years. Vecchio (1989) developed a method to modify existing linear elastic finite element procedures to nonlinear analysis of reinforced concrete membrane structures. The method uses the constitutive relations for concrete and reinforcement based on the modified compression field theory. He also compared the results which were obtained by this nonlinear finite element analysis technique with the experimental results, and reached the conclusion that, with little work, the existing linear elastic programs could be modified such that they would give more realistic results.

The nonlinear finite element analysis techniques were quite successful in the determination of the response of members under in-plane forces, but for combined plane action and bending there were not enough tests conducted to develop a well-defined model. The modified compression field theory worked well for membrane forces since there was a substantial body of experimental data which permitted the fine-tuning of the model. For shell elements under flexural forces, the tension stiffening effect of concrete becomes important. Nonlinear finite element applications were based on membrane reinforcement and they were overestimating the stiffening effect when applied to flexural problems. To overcome this problem, Polak and Vecchio (1994) conducted some tests to observe the behaviour of such structures and to check the accuracy of nonlinear finite element procedures. The results showed that many more experimental and analytical studies have to be carried out in this field to come up with a universally acceptable solution.

The modified compression field theory was also applied to other design problems. Particularly, the extension of this model to the design of reinforced concrete beams under combined loading opened the door to more advanced studies. For example, a recent paper by Collins, Mitchell, Adebar, and Vecchio (1996) uses the modified compression field theory to develop a 'general design method' for shear reinforcement.

Another successful study has been carried out at the University of Huston. Hsu (1988) developed a model called 'softened trussed model' for the nonlinear analysis of reinforced concrete subjected to membrane shear and torsion. The study was basically taking the concrete between the cracks as a compression strut and the reinforcement as a tension bar, and combining them in a truss analogy. When the model was first developed it was very difficult to solve the equations by hand and a very powerful microcomputer was necessary; but this problem was solved in the following studies (Hsu 1991).

In order to develop the constitutive laws for the softened truss model theory, many experiments were carried out by Belabi and Hsu (1995). After these experiments three constitutive laws were established: One is for concrete in compression, the other is for concrete in tension, and the last one for the steel embedded in concrete. In the laboratory experiments that followed, Pang and Hsu (1995) observed that these equations had to be modified by a factor that would take care of the 'kinking' of the reinforcement.

The main problem in the softened truss model was that it could not predict the contribution of concrete which was observed in the tests. The reason for this was the fact

that the model was based on the assumption that the direction of the cracks was inclined at the rotating angle following the post-cracking principal stress of concrete. In a very recent paper by Pang and Hsu (1996) this problem was corrected and a 'fixed angle softened truss model' was developed. This model introduced another constitutive equation which relates the average shear stress of concrete to average shear strain.

As a conclusion it can be stated that although the concept of reinforcement of thin-walled concrete structures is not new, a universally acceptable solution does not exist yet. Computer methods can be very useful in the determination of the member behaviour, but one must be aware of the fact that many iterations might be necessary to achieve a satisfactory solution. Studies such as the compression field theory and softened truss model theory give good results, but they exhibit some weak points either in their theory or in their suitability to design. The need for a method with strong theoretical background and easy applicability is obvious.

Chapter 3

Existing Methods of Analysis of Membrane Elements

3.1 General

This chapter explains the more important existing methods which have been developed for the design and analysis of reinforced concrete shell elements under in-plane forces. It is divided into two parts: In the first part, the modified compression field theory, which is actually a method for combined state of loading, is explained. In the second part, those existing methods, whose results will be compared in Chapter 4, are described.

3.2 Modified Compression Field Theory

3.2.1 Introduction

Postbuckling shear resistance of thin-webbed metal girders had been a major field of research at the beginning of this century. Wagner (1929) made the assumption that after buckling, the webs would not resist any compression and that the load would be carried by a field of diagonal tension as shown in Fig.3.1. In order to determine the angle of the inclination of the diagonal tension zone, he made the assumption that this angle would coincide with the angle of inclination of the principal tensile strains. This method is known as ‘the tension field theory’.

By using the theory developed for the postbuckling behaviour of the metal webs, a method for concrete could be obtained for the postcracking behaviour of concrete. This idea was applied by Collins and Mitchell (1974) to model structural concrete in pure torsion which and was dubbed as ‘the diagonal compression field theory’.

The method was later extended by Vecchio and Collins (1982, 1986) to determine the response of reinforced concrete panels. This approach, ‘the compression field theory’, assumes that after cracking, the concrete carries no tension and the shear is carried by a field of diagonal compression.

Fig. 3.1: Diagonal Tension in Thin-Webbed Metal Girder

Fig 3.2: Equilibrium Conditions for A Beam Element

As a special case, the response of a symmetrically reinforced concrete beam which is subjected to shear can be investigated by this approach. It can be seen from Fig. 3.2 (a) that for a given shear force V , there are four unknowns to be determined: the stress in the longitudinal bars, f_x ; the stress in the transverse reinforcement, f_v ; the diagonal compressive stress in concrete, f_2 ; and the angle of inclination of the crack, θ . These unknowns can be determined easily from three equilibrium and one compatibility conditions. Compatibility condition is given by Eq. 3.5 which is the same as the one used for membrane elements. From Fig. 3.2(b) and (c) the following equilibrium equations can be written for the given situation.

$$f_2 = \frac{V}{b_w d} \left(\tan \theta + \frac{1}{\tan \theta} \right) \quad (3.1)$$

$$A_x f_x = \frac{V}{\tan \theta} \quad (3.2)$$

$$\frac{A_v f_v}{s} = \frac{V}{d} \tan \theta \quad (3.3)$$

where s = spacing between the stirrups

d = effective depth of the beam

b_w = effective width of the beam.

However it must be stated the so-called compatibility relation in this method is not exact but is based on the assumption that the direction of principal stress and principal strain coincide in reinforced concrete beams at every load level. This assumption is difficult to justify theoretically, but practically it appears that it gives reasonable results.

By applying the same approach to reinforced concrete panels, with the addition of the reinforcement and loading in the x-direction, the corresponding unknowns can be solved for similarly. The other condition that must be specified at this point is the constitutive relationships of both concrete and steel. According to the compression field theory, concrete can only resist compression and its stress-strain relationship is given by the following equation.

$$f_2 = f_{2\max} \left[2 \left(\frac{\varepsilon_2}{\varepsilon'_c} \right) - \left(\frac{\varepsilon_2}{\varepsilon'_c} \right)^2 \right] \quad (3.4)$$

where

$$\frac{f_{2\max}}{f'_c} = \frac{1}{0.8 + 170\varepsilon_1} \leq 1.0$$

ε_1 = principal tensile strain

ε_2 = principal compressive strain

(a) Free-Body Diagram of a Cracked Element

(b) Stress-Strain Relationship of Cracked Concrete in Compression

Fig. 3.3 Response of Concrete To Compressive Stresses

ϵ_c = ultimate crushing strain for concrete

f_{2max} = maximum principal compressive stress

As it can be seen from Eq. 3.4, the compressive stress is not only a function of compressive strain but also of the orthogonal tensile strain. This condition is also illustrated in Fig. 3.3.

3.2.2 Formulation of the Theory

In an international competition (Collins et al. 1985), over 40 leading researchers from different countries attempted to determine the response of 4 reinforced concrete panels which were tested previously. The best result was not even 15 percent close to the experimental values. This showed that a better method needed to be developed to solve the problem. The modified compression field theory proposed a new approach to the solution of this problem. Even though the compression field theory is generally adequate, since it neglects the contribution of tensile stresses in the cracked concrete, it overestimates the deformations and underestimates the capacity.

Both ‘the compression field theory’ and ‘the modified compression field theory’ treat the cracked concrete as a new material and before investigating the properties of this new material they make some initial assumptions. These assumptions may be summarized as follows:

- the panel is reinforced in two orthogonal directions
- the element deforms such that edges remain straight and parallel
- for each stress state there exists a corresponding strain state
- stresses and strains are taken as average values
- there is no 'overall slip' between the reinforcement and the concrete

Formulation of the modified compression field theory involves the utilization of the compatibility, equilibrium, and constitutive relationships.

Compatibility

Fig. 3.4(a) and (b) show the strains and their corresponding Mohr's circle in a cracked membrane element, respectively. From Mohr's circle, the following expression can be obtained for the direction of the principal strain, θ .

$$\tan^2 \theta = \frac{\varepsilon_x - \varepsilon_2}{\varepsilon_y - \varepsilon_2} \quad (3.5)$$

where ε_x = strain in the x-direction

ε_y = strain in the y-direction

If the strains in three directions are known, other strain can be determined from the geometry. It should be noted that stresses and strains are represented as average values measured over lengths covering several cracks.

Fig. 3.4: Shell Element Under In-Plane Forces

Equilibrium Equations

Applied loads on a membrane element are carried by the reinforcement and the concrete as shown in Fig. 3.4(c),(d). In the modified compression field theory, unlike the compression field theory, concrete can also carry tension forces. Fig. 3.5(b) shows the average and principal concrete stresses. Mohr's circle for concrete stresses is illustrated in Fig. 3.5(c).

The stress, f_x , in the x-direction is carried by the reinforcement and the concrete

$$\int_A f_x dA = \int_{A_c} f_{cx} dA + \int_{A_r} f_{rx} dA \quad (3.6)$$

Assuming there is no reduction in area due to the presence of the reinforcing bars

$$f_x = f_{cx} + \rho_{sx} f_{rx} \quad (3.7)$$

Similarly

$$f_y = f_{cy} + \rho_{sy} f_{ry} \quad (3.8)$$

$$v_{xy} = v_{cxy} + \rho_{sx} v_{rx} \quad (3.9)$$

And

$$v_{xy} = v_{\alpha y} + \rho_{xy} v_{sy} \quad (3.10)$$

where f_{cx} = axial stress carried by concrete in the x-direction

f_{cy} = axial stress carried by concrete in the y-direction

v_{cxy} = shear stress carried by concrete

f_{sx} = axial stress carried by steel in the x-direction

f_{sy} = axial stress carried by steel in the y-direction

v_{sx} = shear stress carried by steel in the x-direction

v_{sy} = shear stress carried by steel in the y-direction

ρ_{sx} = reinforcement ratio in the x-direction

ρ_{sy} = reinforcement ratio in the y-direction

$A = t * l$, where t = thickness of the element

The Mohr's circle for concrete stresses give the following relationships

$$f_{\alpha} = f_{c1} - v_{\alpha y} / \tan \theta_c \quad (3.11)$$

$$f_{cy} = f_{c1} - v_{\alpha y} \tan \theta_c \quad (3.12)$$

$$f_{c2} = f_{c1} - v_{\alpha y} \left(\tan \theta_c + \frac{1}{\tan \theta_c} \right) \quad (3.13)$$

where θ_c = angle of principal concrete stress

Constitutive Relationships

The following additional assumptions have been made for the stress-strain relationships of the materials:

- the stress-strain relationships of concrete and steel are independent from each other
- axial stress in the reinforcement is only a function of the strain in the same direction

$$f_{xx} = E_s \varepsilon_x \leq f_{yx} \quad (3.14)$$

$$f_{yy} = E_s \varepsilon_y \leq f_{xy} \quad (3.15)$$

where f_{yx} = yield stress of steel in the x-direction

f_{xy} = yield stress of steel in the y-direction

- shear stress which is resisted in the direction perpendicular to the reinforcement is zero

$$\nu_{xx} = \nu_{yy} = 0 \quad (3.16)$$

- for concrete, principal stress and principal strain directions coincide

$$\theta_c = \theta \quad (3.17)$$

This assumption is often referred to as a pseudo-compatibility requirement.

Experimental studies show that the cracked concrete subjected to high levels of tension in the direction perpendicular to the compression is weaker than the concrete in a cylinder test (see Fig. 3.3). The relationship which accounts for this was given by Eq. 3.4. It is important to note that the larger the ε_1 , the more the observed reduction in the concrete strength.

The relationship between the average principal tensile stress and principal tensile strain can be divided into two parts: prior to cracking and after cracking. As it can be seen from Fig. 3.6, before cracking, the concrete material shows a linear stress-strain relationship which can be expressed as

$$f_{c1} = E_c \varepsilon_1 \quad (3.18)$$

where E_c = modulus of elasticity of concrete

After cracking, a major reduction occurs in the tensile stress carried by concrete. The following relationship is suggested for this case

(a) Average Concrete Stresses

(b) Principal Concrete Stresses

(c) Mohr's Circle for Concrete Stresses

Fig. 3.5: Stresses in Cracked Concrete

(a) Tensile Stress Distribution in Cracked Concrete

(b) Average Stress-Strain Relationship of Cracked Concrete in Tension

Fig. 3.6: Response of Concrete to Tension Forces

Fig. 3.7: Summary of Cracked Concrete Behaviour in MCFT

Fig. 3.8: Summary of Reinforcement Behaviour in MCFT

$$f_{c1} = \frac{f_{cr}}{1 + \sqrt{200 \varepsilon_1}} \quad (3.19)$$

where f_{cr} = cracking stress of concrete

The constitutive relationships of the reinforcement and the concrete are summarized in Fig. 3.7 and Fig. 3.8, respectively.

Load Transmission Across Cracks

All of the equations derived so far are based on the assumption that stress and strain values are taken as average quantities. If one of the steel bars in the reinforcement net is considered, it will be observed that the stress varies along its length. At the crack locations the stress reaches its maximum value since the reinforcement is the only resisting element. Between the cracks, the stress in the reinforcement is less than the one in the crack location since concrete also contributes to the resistance. If concrete is observed in a similar way, it will be seen that concrete stress is zero at the crack locations and maximum in between two cracks as illustrated in Fig.3.6(a).

The limiting criterion for the reinforcement is the yield stress of the material. Fig. 3.4(c) and (d) show two cross sections of a member which is under membrane forces. One of the sections is taken along the crack and the other is taken between two cracks. Since the loading is the same, the sum of the forces at the first section should be equal to that at the second section. Equating the forces in the x-direction yields

$$\rho_{sx} (f_{sxcr} - f_{sx}) = f_{ci} + f_{ci} + v_{ci} / \tan \theta \quad (3.20)$$

Similarly in the in the y-direction

$$\rho_{sy} (f_{sy cr} - f_{sy}) = f_{ci} + f_{ci} - v_{ci} \tan \theta \quad (3.21)$$

where f_{sxcr} = stress in x-reinforcement at crack location

f_{sxcy} = stress in y-reinforcement at crack location

v_{ci} = shear stress on crack surfaces

f_{ci} = compressive stress on crack surface (positive)

As stated previously, the stress in the reinforcement at the crack cannot exceed the yield strength of the material

$$f_{sxcr} \leq f_{yx} \quad (3.22)$$

and

$$f_{sy cr} \leq f_{yy} \quad (3.23)$$

Concrete stresses v_{ci} and f_{ci} must be calculated in order to complete the formulation of the theory. Cracking of a concrete member generally occurs along the interface between the

cement paste and the aggregates. Concrete is able to transfer shear after cracking because of aggregate interlock and friction. This problem has been studied by many investigators and many experimental studies were carried out in order to obtain a method which would allow the determination of the amount of shear that the crack is able to transfer. One of these methods is the Walvaren's model (Walvaren 1981) which is a function of the crack spacing, the maximum aggregate size, and the cylinder strength of concrete. The method relates crack opening and slip displacements to the shear and normal stress acting on it. Although theoretically it is appealing, practically its implementation is not easy because determination of crack spacing for a general loading condition is not a simple case. More information on this method can also be found in Collins (1986). By using this method, unknown values in Eq. 3.20 and Eq 3.21 can be determined. Fig. 3.9 summarizes the model of the reinforced concrete in the modified compression field theory. A complete summary of the theory is given in Fig. 3.10.

Fig. 3.9: Summary of Reinforced Concrete in MCFT

Fig. 3.10: Summary of the Modified Compression Field Theory (Vecchio 1986)

3.3 Nielsen's Method

With respect to the design of reinforced concrete shell elements, Nielsen's contribution has been a major step forward. After his seminal work, many other researchers followed his steps and most of the literature on this subject is a modified version of Nielsen's work.

Nielsen (1963) first tried to find the yield locus or envelope of a shell element with given reinforcement in terms of the membrane resistance of its constituents. He also attempted to formulate the procedure for designing the reinforcement to carry a given set of forces.

In order to model the problem, he made the following assumptions:

- Reinforcement will be placed in two perpendicular directions.
- Reinforcement will be placed either in the middle surface of the element or symmetrically with respect to the middle surface.
- Reinforcement will not be placed in the direction of compression. Compressive forces are carried by concrete alone.
- Reinforcement is elastic-perfectly plastic as shown in Fig. 3.11(a).
- Concrete is perfectly plastic with a zero yield strength in tension.
- Failure criterion for concrete is idealized as shown in Fig. 3.11(b) where f_1 and f_2 are the principal stresses and f_c is the cylinder strength in compression.

(a) Stress-Strain Relationship of Steel

(b) Idealized Failure criteria for Concrete

Fig. 3.11: Behaviour of Steel and Concrete in Nielsen's Model

Fig. 3.12(a) shows a shell element bounded by the assumed crack plane and element sides. Equilibrium in x- and y- directions requires

$$f_{xx} = f_x + |v_{xy}| \cot \theta \quad (3.24)$$

$$f_{yy} = f_y + |v_{xy}| \tan \theta \quad (3.25)$$

According to Nielsen's method, the final design depends on the type of loading. Since no reinforcement is provided to resist compression, depending on the axis along which the reinforcement is provided, different design equations are obtained. The design problem is dealt with by investigating four possible cases of loading.

Case 1

If both f_x and f_y are positive (tension), the necessary reinforcement is proportional to the sum of these two forces. Thus, the minimum amount of reinforcement which can resist the applied forces would be obtained by setting $\theta = 45^\circ$ which yields the following design conditions

$$f_{xx} = f_x + |v_{xy}| \quad (3.26)$$

$$f_{yy} = f_y + |v_{xy}| \quad (3.27)$$

In order for f_x and f_y to be positive the following condition must be satisfied

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} \geq -|v_{xy}|$$

Fig. 3.12(b) shows the principal compressive stress, f_2 , in the concrete which is perpendicular to the crack plane. Using the free-body-diagram in Fig. 3.12(b) the principal compressive stress in the concrete can be written as

$$f_2 = -2|v_{xy}| \quad (3.28)$$

Case 2

Equations (3.26) to (3.28) cannot be used for the cases when f_x is negative (compressive). For this situation, Nielsen assumed that there is no need to provide reinforcement in the x-direction. Fig. 3.13(a) represents an element which has only y-reinforcement. Considering equilibrium in both directions, we obtain

$$\tan \theta = \frac{|v_{xy}|}{|f_x|} \quad (3.29)$$

$$f_{sy} = f_y + |v_{xy}| \tan \theta \quad (3.30)$$

Combining equations (3.29) and (3.30) gives

$$f_{xy} = f_y - \frac{v_{xy}^2}{f_x} \quad (3.31)$$

which is true if

$$f_x f_y \leq v_{xy}^2$$

which implies tension in the y-reinforcement.

Fig. 3.13(b) shows the principal compressive stress in the concrete. Equilibrium in the x-direction yields

$$f_2 = |f_x| \sec^2 \theta = |f_x| (1 + \tan^2 \theta) \quad (3.32)$$

Substituting equation (3.29) into equation (3.32) gives the value of the principal compressive stress as

$$f_2 = f_x + \frac{v_{xy}^2}{f_x} \quad (3.33)$$

Case 3

When f_y is negative (compressive), reinforcement is placed only in the x-direction. This situation is represented by Fig 3.14 . From equilibrium of the element, the following equations can be written

$$\cot \theta = \frac{|v_{xy}|}{|f_y|} \quad (3.34)$$

$$f_{xx} = f_x + |v_{xy}| \cot \theta \quad (3.35)$$

Substituting equation (3.34) into equation (3.35) yields the tension force in the x-reinforcement as

$$f_{xx} = f_x - \frac{v_{xy}^2}{f_y} \quad (3.36)$$

In order for this equation to be used, the same condition that was specified in Case 2, must be satisfied; i.e.,

$$f_x f_y \leq v_{xy}^2$$

Fig. 3.14(b) shows the principal compressive stress as being perpendicular to the direction. Equilibrium of the latter element requires

$$f_2 = |f_y| \operatorname{cosec}^2 \theta = |f_y| (1 + \cot^2 \theta) \quad (3.37)$$

Substituting equation (3.34) into equation (3.37) results in

$$f_2 = f_y + \frac{v_{xy}^2}{f_y} \quad (3.38)$$

Case 4

If both f_x and f_y are negative (compressive) and

$$f_x f_y \geq v_{xy}^2$$

no reinforcement is required and the principal compressive stress can be calculated by using the usual formula of the strength of materials:

$$f_2 = \frac{1}{2}(f_x + f_y) - \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4v_{xy}^2} \quad (3.39)$$

(a) Free body diagram of Case 1: ($\theta = 45^\circ$)

(b) Case 1: Principal compressive membrane stress

Fig. 3.12: Case 1: Reinforcement Required in Both Directions

(a) Free body diagram of Case 2

(b) Case 2: Principal compressive membrane stress

Fig. 3.13: Case 2: Reinforcement Required in y-direction

(a) Free body diagram of Case 3

(b) Case 3: Principal compressive membrane stress

Fig. 3.14: Case 3: Reinforcement Required in x-direction

Fig. 3.15 is the ‘Boundary Case Graph’ for Nielsen’s method. According to the criteria explained previously, this graph is divided into four regions. The boundary curve equations are summarized in Table 3.1. Depending on the loading of the membrane element, the case can easily be determined from the graph and the corresponding formulae can be used for design purposes.

Table 3.1: Boundary Curve Equations For Nielsen’s Method

Curve No.	Equation
1	$f_x = -v_{xy}$
2	$f_y = -v_{xy}$
3	$f_x f_y = v_{xy}^2$

It must be pointed out that the quantities f_{sx} and f_{sy} represent the resistance of the available reinforcement in the x and y direction, and are equal to $\rho_x f_{xy}$ and $\rho_y f_{yy}$, where ρ_x and ρ_y are the reinforcement ratios in the x and y directions respectively, while f_{xy} and f_{yy} are the yield strengths of the x and y reinforcements respectively. On the other hand f_2 is the characteristic compressive resistance of concrete times the element thickness, t , and it may be equated to $0.85 t f'_c$ where f'_c is the concrete cylinder compressive strength.

In summary, the method works as follows: Given f_x , f_y , v_{xy} , use Fig. 3.15 to determine the relevant case. Knowing the case, use the relevant equations to find ρ_x and ρ_y , and the thickness, t of the shell.

Fig. 3.15: Boundary Case Graph For Nielsen's Method

3.4 Clark's Method

Clark's method can be thought as an extension of Nielsen's technique. It tries to solve some deficiencies that exist in Nielsen's method and it criticizes the idea of ignoring compression reinforcement.

Clark made the following assumptions to simplify the problem:

- Stress and strain are taken tension positive.
- The reinforcement is placed symmetrically with respect to the middle surface of the section and it can be in two non-orthogonal directions. The x-axis is made to coincide with one of the reinforcement direction and the other reinforcement direction is at an angle α measured clockwise positive from the x-axis.
- The reinforcement is assumed to carry only uniaxial stress in the bar direction.
- All stresses, strains are taken as their average quantities.
- The concrete is assumed to carry only compression and to exhibit a perfectly plastic square yield criterion as shown in Fig. 3.11(b).
- Failure occurs by unrestricted plastic flow and not by buckling of the section.

From Fig.3.16 the following three equilibrium equations can be written:

$$f_x = \rho_x f_{sx} + \rho_\alpha f_{s\alpha} \cos^2 \alpha + f_1 \cos^2 \theta + f_2 \sin^2 \theta \quad (3.40)$$

Fig. 3.16: Directions of Reinforcement and Principal Concrete Stresses

$$f_y = \rho_a f_{sa} \sin^2 \alpha + f_1 \sin^2 \theta + f_2 \cos^2 \theta \quad (3.41)$$

$$v_{xy} = -\rho_a f_{sa} \sin \alpha \cos \alpha + (f_2 - f_1) \sin \theta \cos \theta \quad (3.42)$$

Reinforcement in each direction can be in tension, or in compression, or there can be no reinforcement. Depending on these cases, nine load combinations must be considered. Table 3.2 gives the known values for the parameters used in Eq. 3.40, Eq. 3.41, and Eq. 3.42. and the methods of solution of these equilibrium equations. It can be seen that direct solution can be obtained for all cases except for cases 1 and 4, where there are four unknowns to be obtained from three equations. For these cases, solution can be obtained by minimizing the total amount of reinforcement using the following additional equation:

$$\frac{\partial (\rho_x + \rho_a)}{\partial (\tan \theta)} = 0 \quad (3.43)$$

It must be noted that $f_1 = 0$ when tension reinforcement is provided since concrete is assumed to be cracked, and $f_2 = f_c$ when compression reinforcement is provided in order to use the concrete most efficiently. Here again f_c is the characteristic strength times the thickness. By using Table 3.2 equilibrium equations can be solved and the shell element can be analyzed or designed. Since there are nine possibilities depending on the reinforcement amount or disposition, there will be nine different set of solutions.

Table 3.2: Possible Reinforcement Combinations

Case No.	Reinforcement Description	Known Parameters	Method of Solution
1	Both tension	$f_{sx} = f_{sa} = f_{yx}, f_1 = 0$	Minimization
2	No x, α -tension	$f_{sa} = f_{y\alpha}, \rho_x = 0, f_1 = 0$	Direct
3	No α , x-tension	$f_{sx} = f_{yx}, \rho_\alpha = 0, f_1 = 0$	Direct
4	Both comp.	$f_{sx} = f_{sa} = f'_{y\alpha}, f_2 = f_c$	Minimization
5	No x, α -comp.	$f_{sa} = f'_{y\alpha}, \rho_x = 0, f_2 = f_c$	Direct
6	No α , x-comp.	$f_{sx} = f'_{yx}, \rho_\alpha = 0, f_2 = f_c$	Direct
7	x-tension, α -comp.	$f_{sx} = f_{yx}, f_{sa} = f'_{y\alpha}, f_1 = 0, f_2 = f_c$	Direct
8	x-comp., α -tension	$f_{sx} = f'_{yx}, f_{sa} = f'_{y\alpha}, f_1 = 0, f_2 = f_c$	Direct
9	No reinforcement	$\rho_x = \rho_\alpha = 0$	Direct

Clark (1976) solved these cases and obtained design equations for each combination.

Table 3.2 gives design equations for orthogonally reinforced shell elements. Design equations for non-orthogonal reinforcement can be found in Clark's paper (Clark 1976).

The quantity β used in Table 3.3 is defined as:

$$\beta = \sqrt{1 - \frac{4}{f_c^2} (v_{xy} + f_y \cot \alpha) (v_{xy} + (f_y - f_c) \cot \alpha)} \quad (3.44)$$

Having established the equations to be used in each case, it is now necessary to determine which set of equations should be used for a particular force triad. This is achieved by using a graphical approach which was first introduced by Morley (1970) for elements under moments only. A 'boundary case graph' is established depending on the loading ratios of the shell element. Although for non-orthogonally reinforced elements these graphs become very complicated, for orthogonal reinforcement they are very easy to construct. Fig. 3.16 illustrates a typical boundary case graph for orthogonally reinforced shell elements and Table 3.3 gives the equations for the curves in this graph.

For non-orthogonally reinforced shell elements, Clark shows in his paper that the idealized strength of concrete, f_c , must be restricted in order to obtain proper 'case boundary graphs'. This restriction is given in Eq. 3.45.

$$f_c \leq -2 \left| v_{xy} \right| \operatorname{cosec} \alpha \quad (3.45)$$

For orthogonal reinforcement Eq. 3.45 becomes

$$f_c \leq -2 \left| v_{xy} \right|$$

Table 3.3: Design Equations for Orthogonal Reinforcement (Clark 1976)

Case	ρ_x	ρ_y
1	$\frac{1}{f_{yx}}(f_x + v_{xy})$	$\frac{1}{f_{yy}}(f_y + v_{xy})$
2	No reinforcement necessary	$\frac{1}{f_{yy}}\left(f_y - \frac{v_{xy}^2}{f_x}\right)$
3	$\frac{1}{f_{yx}}\left(f_x - \frac{v_{xy}^2}{f_y}\right)$	No reinforcement necessary
4	$\frac{1}{f'_{yx}}((f_x - f_c) - v_{xy})$	$\frac{1}{f'_{yy}}((f_y - f_c) - v_{xy})$
5	No reinforcement Necessary	$\frac{1}{f'_{yy}}\left((f_y - f_c) - \frac{v_{xy}^2}{f_x - f_c}\right)$
6	$\frac{1}{f'_{yx}}\left((f_x - f_c) - \frac{v_{xy}^2}{f_y - f_c}\right)$	No reinforcement Necessary
7	$\frac{1}{f_{yx}}\left(f_x - \frac{f_c}{2}(1 - \beta)\right)$	$\frac{1}{f_{yy}}\left(f_y - \frac{f_c}{2}(1 + \beta)\right)$
8	$\frac{1}{f'_{yx}}\left(f_x - \frac{f_c}{2}(1 + \beta)\right)$	$\frac{1}{f_{yy}}\left(f_y - \frac{f_c}{2}(1 - \beta)\right)$
9	No reinforcement Necessary	No reinforcement Necessary

Table 3.4: Boundary Curve Equations for Orthogonal Reinforcement (Clark 1976)

Curve	Equation
1	$\frac{f_y}{ v_{xy} } = +\infty$
2	$\frac{f_y}{ v_{xy} } = \frac{1}{2} \left(\frac{f_c}{ v_{xy} } + \sqrt{\left(\frac{f_c}{ v_{xy} } \right)^2 - 4} \right)$
3	$\frac{f_y}{ v_{xy} } = -1$
4	$\frac{f_y}{ v_{xy} } = \frac{f_c}{ v_{xy} } + 1$
5	$\frac{f_y}{ v_{xy} } = \frac{1}{2} \left(\frac{f_c}{ v_{xy} } - \sqrt{\left(\frac{f_c}{ v_{xy} } \right)^2 - 4} \right)$
6	$\frac{f_y}{ v_{xy} } = -\infty$
7	$\frac{f_x}{ v_{xy} } = \frac{1}{2} \left(\frac{f_c}{ v_{xy} } - \sqrt{\left(\frac{f_c}{ v_{xy} } \right)^2 - 4} \right)$
8	$\frac{f_x}{ v_{xy} } = -1$

Table 3.4: Continued

Curve	Equation
9	$\frac{f_x - f_c}{ v_{xy} } \frac{f_y - f_c}{ v_{xy} } = 1$
10	$\frac{f_x}{ v_{xy} } \frac{f_y}{ v_{xy} } = 1$
11	$\frac{f_x}{ v_{xy} } = \frac{f_c}{ v_{xy} } + 1$
12	<p>Inapplicable to orthogonal reinforcement</p>
13	<p>Inapplicable to orthogonal reinforcement</p>
14	$\frac{f_x}{ v_{xy} } = \frac{1}{2} \left(\frac{f_c}{ v_{xy} } + \sqrt{\left(\frac{f_c}{ v_{xy} } \right)^2 - 4} \right)$

Fig. 3.17: Boundary Case Graph For Clark's Method

3.5 Fialkow's Method

Fialkow's (Fialkow 1983, 1984) procedure is based on the strength design method of ACI 318-77 Building Code (ACI 1977). The strength design method is an application of the lower bound theorem of limit analysis which requires the satisfaction of two requirements:

- Equilibrium between the applied loads and internal system of stress resultants
- Provision of sufficient internal strength to sustain these stress resultants

Experience with reinforced concrete structures has demonstrated the applicability of this method in the field. The building codes provide design strengths of various structural members such as beams, columns, walls, and slabs. For shell elements under membrane action design criteria are not explicitly identified. According to the strength design method, design strength of the element must be larger or equal to the required strength. Fialkow defined the design strength of the membrane element, which was illustrated in Fig. 1.3(a), as a combination of different components.

Tensile Design Strength

Fialkow neglects any contribution coming from concrete to the tensile strength of the membrane element. The reinforcement contribution to the tensile strength of an element of unit length is calculated using:

$$f^*_x = \phi_t f_{xt} \rho_{xt}, \quad \phi_t = 0.90 \quad (3.46a)$$

$$f^*_y = \phi_t f_{yt} \rho_{yt}, \quad \phi_t = 0.90 \quad (3.46b)$$

where ρ_{xt} = reinforcement ratio for steel bars resisting tension in the x-direction

ρ_{yt} = reinforcement ratio for steel bars resisting tension in the y-direction

f^*_x = component of tensile strength coming from x-reinforcement

f^*_y = component of tensile strength coming from y-reinforcement

ϕ_t = tensile strength reduction factor

Shear Design Strength Due to Reinforcement

Shear design strength of the same element in x- and y-directions, respectively is given by

$$v^*_{xy} = \phi_v f_{xy} \rho_{xv}, \quad \phi_v = 0.85 \quad (3.47a)$$

$$v^*_{yx} = \phi_v f_{yv} \rho_{yv}, \quad \phi_v = 0.85 \quad (3.47b)$$

where ρ_{xv} = reinforcement ratio for steel bars resisting shear in the x-direction

ρ_{yv} = reinforcement ratio for steel bars resisting shear in the y-direction

v^*_{xy} = component of shear strength coming from x- and y-reinforcement

ϕ_t = shear strength reduction factor

Shear Design Strength Due to Concrete

ACI 318-77 Building Code specifies that for beams and walls shear strength is available on a given plane when compression acts on the plane and that the available concrete strength decreases to zero in the presence of increasing tension on the shear surface. This design strength is given as

$$V_b = 2\phi_v \sqrt{f'_c} \quad (3.48)$$

Fialkow [3] derived the following equations for the shear design strength of the membrane elements

$$\text{For } 0.1f_y \leq f_x, \quad v^*_{xy} = 0 \quad (3.49a)$$

$$\text{For } 0 \leq f_x \leq 0.1f_y, \quad v^*_{xy} = V_b \left[1 - \left(\frac{10f_x + v_{xy} \rho_y}{f_y + v_{xy} \rho_x} \right)^2 \right] \quad (3.49b)$$

$$\text{For } v_{xy} \leq f_x \leq 0, \quad v^*_{xy} = V_b \left[1 - \left(\frac{f_x + v_{xy} \rho_y}{f_y + v_{xy} \rho_x} \right)^2 \right] \quad (3.49c)$$

$$\text{For } f_x \leq v_{xy}, \quad v^*_{xy} = V_b \quad (3.49d)$$

Eq. 3.49a-d are developed for concrete shear strength in the y-direction. Equations for the x-direction can be obtained by interchanging y's with x's.

Design of Orthogonal Two-Way Reinforcement

Strength design method suggests that for each reinforcement direction design strength must be larger or equal to required strength which is formulated as follows:

$$f_x^* + v_{xy}^* \geq f_x + v_{xy} \quad (\text{x-direction}) \quad (3.50a)$$

$$f_y^* + v_{xy}^* \geq f_y + v_{xy} \quad (\text{y-direction}) \quad (3.50b)$$

The latter two equations and previously obtained design strengths (Eq. 3.46-47-49) are used to determine the equations for reinforcement design which are presented below.

$$\text{For } 0.1f_x \leq f_y, \quad \rho_x \geq \frac{f_x}{\phi_t f_{xy}} + \frac{v_{xy}}{\phi_v f_{xy}} \quad (3.51a)$$

$$\text{For } 0 \leq f_y \leq 0.1f_x, \quad \rho_x \geq \frac{f_x}{\phi_t f_{xy}} + \frac{v_{xy}}{\phi_v f_{xy}} - \frac{V_b}{\phi_v f_{xy}} \left[1 - \left(\frac{10f_y + v_{xy}}{f_x + v_{xy}} \frac{\rho_x}{\rho_y} \right)^2 \right] \quad (3.51b)$$

$$\text{For } -v_{xy} \leq f_y \leq 0, \quad \rho_x \geq \frac{f_x}{\phi_t f_{xy}} + \frac{v_{xy}}{\phi_v f_{xy}} - \frac{V_b}{\phi_v f_{xy}} \left[1 - \left(\frac{f_y + v_{xy}}{f_x + v_{xy}} \frac{\rho_x}{\rho_y} \right)^2 \right] \quad (3.51c)$$

$$\text{For } f_y \leq -v_{xy} \quad , \quad \rho_x \geq \frac{f_x}{\phi_t f_{xy}} + \frac{v_{xy} - V_b}{\phi_v f_{xy}} \quad (3.51d)$$

Similar design equations for ρ_y can be obtained by interchanging x's with y's.

$$\text{For } 0.1f_y \leq f_x \quad , \quad \rho_y \geq \frac{f_y}{\phi_t f_{xy}} + \frac{v_{xy}}{\phi_v f_{xy}} \quad (3.52a)$$

$$\text{For } 0 \leq f_x \leq 0.1f_y \quad , \quad \rho_y \geq \frac{f_y}{\phi_t f_{xy}} + \frac{v_{xy}}{\phi_v f_{xy}} - \frac{V_b}{\phi_v f_{xy}} \left[1 - \left(\frac{10f_x + v_{xy}}{f_y + v_{xy}} \frac{\rho_{yx}}{\rho_x} \right)^2 \right] \quad (3.52b)$$

$$\text{For } -v_{xy} \leq f_x \leq 0 \quad , \quad \rho_y \geq \frac{f_y}{\phi_t f_{xy}} + \frac{v_{xy}}{\phi_v f_{xy}} - \frac{V_b}{\phi_v f_{xy}} \left[1 - \left(\frac{f_x + v_{xy}}{f_y + v_{xy}} \frac{\rho_y}{\rho_x} \right)^2 \right] \quad (3.52c)$$

$$\text{For } f_y \leq -v_{xy} \quad , \quad \rho_y \geq \frac{f_y}{\phi_t f_{xy}} + \frac{v_{xy} - V_b}{\phi_v f_{xy}} \quad (3.52d)$$

Full details of the derivations of the preceding equations are given in Fialkow (1984, 1985, 1988, 1990). Note that the equations are based on some assumptions and limitations that appear to be empirical and even arbitrary.

3.6 Zararis' Method

In the analysis of reinforced concrete membrane elements, most of the work carried out prior to Zararis' neglected the role played the reinforcing bars in resisting shear by dowel action. Zararis (1988) demonstrated the importance of shear forces resisted by dowel action and calculated the magnitudes of these forces by using strain compatibility conditions at the crack. Since the development of Zararis' method involves a great amount of mathematical manipulation, only a basic review of the procedure will be presented here. For a detailed explanation of the derivations, reference should be made to Zararis' original work (Zararis 1988).

First let us determine the stresses and strains of the reinforcement at crack locations prior to yielding. Using Fig. 3.18, the following equations can be derived for strains and stresses of steel bars:

$$\begin{pmatrix} \varepsilon_{xx} & \gamma_{xy} \\ \gamma_{yx} & \varepsilon_{yy} \end{pmatrix} = \varepsilon_{cr} \begin{pmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \quad (3.53)$$

$$\begin{pmatrix} f_{xx} & v_{xy} \\ v_{yx} & f_{yy} \end{pmatrix} = E_s \varepsilon_{cr} \begin{pmatrix} \cos^2 \theta & 0.4 \sin \theta \cos \theta \\ 0.4 \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \quad (3.54)$$

Taking θ_1 as the angle of the initial cracks, forces acting on a unit element of the plate are illustrated in Fig. 3.19. Internal forces acting on the element shown in Fig. 3.19 form a

(a) Deformation of Steel Bars Due to Loading

(b) Forces on the Steel Bars at The Crack

Fig. 3.18: Stresses and Strains of Reinforcement Before Yielding

Fig. 3.19: Free Body Diagram of the Membrane Element Before Yielding

tensor which can be transformed in the direction of initial cracks. The following is the representation of this tensor in the matrix form:

$$T = \begin{bmatrix} f_x \cos^2 \theta_1 + 2v_{xy} \sin \theta_1 \cos \theta_1 & (f_y - f_x) \sin \theta_1 \cos \theta_1 \\ + f_y \sin^2 \theta_1 & + v_{xy} (\cos^2 \theta_1 - \sin^2 \theta_1) \\ (f_y - f_x) \sin \theta_1 \cos \theta_1 & f_x \sin^2 \theta_1 - 2v_{xy} \sin \theta_1 \cos \theta_1 \\ + v_{xy} (\cos^2 \theta_1 - \sin^2 \theta_1) & + f_y \cos^2 \theta_1 \end{bmatrix} \quad (3.55)$$

The axial forces on the steel bars that correspond to the unit element also constitute a tensor which is given by Eq. 3.56.

$$T_s = \begin{bmatrix} \rho_{xx} f_{xx} \cos^2 \theta_1 + \rho_{yy} f_{yy} \sin^2 \theta_1 & (\rho_{xy} f_{xy} - \rho_x f_{xx}) \sin \theta_1 \cos \theta_1 \\ (\rho_{xy} f_{xy} - \rho_x f_{xx}) \sin \theta_1 \cos \theta_1 & \rho_{xx} f_{xx} \sin^2 \theta_1 + \rho_{yy} f_{yy} \cos^2 \theta_1 \end{bmatrix} \quad (3.56)$$

Shear forces of steel bars on the unit element also form a tensor. These forces together with concrete stresses are presented in matrix form as follows:

$$T_c = \begin{bmatrix} (\rho_{xy} \nu_{xy} + \rho_x \nu_{xx} y) \sin \theta_1 \cos \theta_1 & \rho_{xx} \nu_{xy} \cos^2 \theta_1 - \rho_{yy} \nu_{xx} \sin^2 \theta \\ \nu_{cl} & f_{cl} \end{bmatrix} \quad (3.57)$$

By using the equilibrium of the entire system the following equation can be written

$$T = T_s + T_c \quad (3.58)$$

Equating corresponding terms of this equation and using Eq. 3.58, the angle of initial cracks can be obtained as the positive root of Eq 3.59.

$$\begin{aligned} \tan^4 \theta_1 + \left(\frac{f_x - 0.4 f_y}{\nu_{xy}} \right) \tan^3 \theta_1 + 0.4 \left(\frac{\rho_{xx}}{\rho_{yy}} - 1 \right) \tan^2 \theta_1 \\ - \frac{\rho_{xx}}{\rho_{yy}} \left(\frac{f_y - 0.4 f_x}{\nu_{xy}} \right) \tan \theta_1 - \frac{\rho_{xx}}{\rho_{yy}} = 0 \end{aligned} \quad (3.59)$$

After the calculation of the initial crack angle, the steel and concrete stresses can be obtained for any proportional loading.

Failure Due To Crushing of Concrete

Before the yielding of steel in either of the two directions, concrete may crush and result in the failure of the membrane element. Eq. 3.59 provides the initial crack angle, θ_1 . Zararis solved for the principal stresses across the crack due to crushing of concrete by substituting θ_1 in Eq. 3.57. The relationship between the two principal concrete stresses is given by

$$\frac{f_{c1}}{f'_c} = 1 - \left(\frac{f_{c2}}{0.3 f'_c} \right)^2 \quad (3.60)$$

If these two principal stresses are expressed as functions of ultimate shear capacity of the element, Eq. 3.60 gives the loading that causes the crushing of concrete.

Failure Due To Yielding of Reinforcement

If the reinforcement in both directions yields, the corresponding forces that could be acting on a membrane element would be as illustrated in Fig. 3.20. Taking θ_2 as the crack angle at ultimate, the following equations can be written:

$$\begin{aligned} \rho_{xx} f_{xy} \cos^2 \theta_2 + \rho_{yy} f_{xy} \sin^2 \theta_2 &= f_x \cos^2 \theta_2 \\ &+ 2\nu_{xy} \sin \theta_2 \cos \theta_2 + f_y \sin^2 \theta_2 \end{aligned} \quad (3.61)$$

$$\begin{aligned} (\rho_{yy} f_{xy} - \rho_{xx} f_{xy}) \sin \theta_2 \cos \theta_2 &= (f_y - f_x) \sin \theta_2 \cos \theta_2 \\ &+ \nu_{xy} (\cos^2 \theta_2 - \sin^2 \theta_2) \end{aligned} \quad (3.62)$$

$$V = (\rho_{yy} f_{xy} - \rho_{xx} f_{xx}) \sin \theta_2 \cos \theta_2 + \nu_{xy} (\cos^2 \theta_2 - \sin^2 \theta_2) \quad (3.63)$$

$$C = f_x \sin^2 \theta_2 - 2\nu_{xy} \sin \theta_2 \cos \theta_2 + f_y \cos^2 \theta_2 \quad (3.64)$$

After some transformations, these equations can be reduced to the form:

$$\rho_{xx} f_{xy} = \nu_{xy} \left(\frac{f_x}{\nu_{xy}} + \tan \theta_2 \right) \quad (3.65a)$$

$$\rho_{yy} f_{xy} \tan \theta_2 = \nu_{xy} \left(1 + \frac{f_y}{\nu_{xy}} \tan \theta_2 \right) \quad (3.65b)$$

Dividing Eq. 3.65(a) by 3.65(b), an equation for the ultimate crack angle, θ_2 can be obtained in the following form:

$$\tan^2 \theta_2 + \left[\frac{f_x}{v_{xy}} - \left(\frac{\rho_{xx} f_{yx}}{\rho_{yy} f_{yy}} \right) \frac{f_y}{v_{xy}} \right] \tan \theta_2 - \frac{\rho_{xx} f_{yx}}{\rho_{yy} f_{yy}} = 0 \quad (3.66)$$

After determining the ultimate crack angle from Eq. 3.66, the ultimate shear capacity of the membrane element can be calculated by using Eq. 3.65 .

If the in steel in one direction only yields, the procedure to calculate ultimate capacity changes. If ϵ_{xy} and ϵ_{yy} are the yield and ϵ_{xu} and ϵ_{yu} are the ultimate strains in x- and y- directions, respectively; the crack angle must satisfy Eq. 3.67 in order for the steel in both directions to have yielded:

$$\sqrt{\frac{\epsilon_{yy}}{\epsilon_{xu}}} \leq \tan \theta_2 \leq \sqrt{\frac{\epsilon_{yu}}{\epsilon_{xy}}} \quad (3.67)$$

Fig. 3.21 illustrates the situation when $\tan \theta_2$ is smaller than the lower bound of Eq. 3.67.

The equilibrium in the direction of the crack gives

$$\rho_{xx} f_{xy} = v_{xy} \left(\frac{f_x}{v_{xy}} + \tan \theta_2 \right) \quad (3.68a)$$

$$\rho_{sy} E_s \varepsilon_{xu} \tan^3 \theta_2 = v_{xy} \left(1 + \frac{f_v}{v_{xy}} \tan \theta_2 \right) \quad (3.68b)$$

Dividing Eq.3.68(a) by 3.68(b), the equation for the ultimate crack angle can be obtained as

$$\tan^4 \theta_2 + \frac{f_x}{v_{xy}} \tan^3 \theta_2 - \left(\frac{\rho_{sx} f_{xy}}{\rho_{sy} E_s \varepsilon_{xu}} \right) \frac{f_x}{v_{xy}} \tan \theta_1 - \frac{\rho_{sx} f_{xy}}{\rho_{sy} E_s \varepsilon_{xu}} = 0 \quad (3.69)$$

After determining the ultimate crack angle, the ultimate capacity can be calculated from one of the two equations given in Eq. 3.68.

If $\tan \theta_2$ is larger than the upper value of Eq. 3.67, Eq. 3.70 can be obtained by considering Fig. 3.22.

$$\rho_{sx} f_{xy} \tan \theta_2 = v_{xy} \left(\frac{f_y}{v_{xy}} + \tan \theta_2 \right) \quad (3.70a)$$

$$\rho_{xx} E_s \varepsilon_{vu} (1 / \tan^2 \theta_2) = \nu_{xy} \left(\frac{f_y}{\nu_{xy}} + \tan \theta_2 \right) \quad (3.70b)$$

Dividing Eq.3.70(a) by 3.70(b), the equation for the ultimate crack angle can be obtained as

$$\tan^4 \theta_2 + \frac{f_x}{\nu_{xy}} \tan^3 \theta_2 - \left(\frac{\rho_{xx} E_s \varepsilon_{yu}}{\rho_{xy} f_{yy}} \right) \frac{f_y}{\nu_{xy}} \tan \theta_1 - \frac{\rho_{xx} E_s \varepsilon_{vu}}{\rho_{xy} f_{yy}} = 0 \quad (3.71)$$

After determining the ultimate crack angle, the ultimate capacity can be calculated from one of the two equations given in Eq. 3.70.

Fig. 3.20: Free Body Diagram of the Membrane Element Assuming Yielding of Steel in Both Directions

Fig. 3.21: Free Body Diagram of the Membrane Element Assuming Yielding of x-reinforcement

Fig. 3.22: Free Body Diagram of the Membrane Element Assuming Yielding of y-reinforcement

3.7 Modified Zararis' Method

Shear forces in reinforcing steel bars (dowel forces) and aggregate interlock mechanism play a significant role in the transfer of shear stresses in reinforced concrete structural elements. Zararis (1988) extensively studied the effect of dowel forces on the behaviour of reinforced concrete shell elements. His method for considering these forces in the analysis was presented in the previous section. He improved this method by combining the effect of aggregate interlock mechanism with the dowel action (Zararis 1997). He noted that, the shear force in steel bars may be lost due to two reasons:

- a slip between the crack surfaces
- yielding of steel

He assumed that the loss of shear force in steel bars is substituted by aggregate interlock forces to balance the external loading. The aggregate interlock mechanism in his model is discussed extensively in Appendix I of his paper (Zararis 1997). Based on this mechanism, he defined 4 modes of failure of orthogonally reinforced membrane elements, which were based on the following assumptions:

- A new crack in another direction is formed when a change in the vector of concrete stresses takes place.

- Crack is the ultimate one when the forces developed on it provide a load carrying capacity lower than those of the previous cracks.
- Failure occurs in accordance with the principles of the theory of plasticity.
- Failure occurs when loss of a resisting force on a crack cannot be replaced.

First Mode of Failure

Zararis used the same formula that he developed for his previous model (Zararis 1988) for the determination of the initial crack angle, θ_1 which is given by Eq. 3.59. This mode of failure occurs when the angle θ_1 of the initial cracks is less than 45° . For this condition, Eq. 3.54 shows that the x-reinforcement is stressed more than the y-reinforcement. Thus the loss of bond between the reinforcing steel and concrete occurs first along the x-reinforcement, and this loss induces elongation of the membrane element in the x direction which results in a slip along the initial cracks. Aggregate interlock forces replaces the lost resistance due to the slip along the cracking surfaces. Increasing the load on the membrane results in the yield of x-reinforcement, loss of aggregate interlock resistance, and failure of the element. The shear capacity of the element for this failure mode is given by:

$$v_{xy} = \frac{\rho_x f_{xy}}{(f_x / v) + \tan \theta_1} \quad (3.72)$$

Second Mode of Failure

For the second failure mode it is also considered that $\theta_1 < 45^\circ$. In this case, before the aggregate interlock stresses reduce to zero, as in the first mode, the y-reinforcement also yields. After the yielding of reinforcements in both directions, new visible cracks in another direction form. The angle of these new cracks, θ_2 , is given by Eq. 3.66. The ultimate shear capacity for this case is given by:

$$v_{xy} = \frac{\rho_x f_{xy} + \rho_y f_{yy} \tan^2 \theta_1}{(f_x / v) + (f_y / v) \tan^2 \theta_2 + 2 \tan \theta_1} \quad (3.73)$$

In the two modes presented so far, the failure mechanism starts with yielding of the x-reinforcement, but the second mode of failure occurs when $\theta_2 > \theta_1$.

Third Mode of Failure

This mode of failure is possible when $\theta_1 > 45^\circ$. A change in the forces which act on the initial cracks occurs due to loss of bond between the y-reinforcement and the adjacent concrete. Increasing the load on the membrane element results in the increase of internal forces in the y-reinforcement. Failure of the membrane element occurs when the y-reinforcement yields and the aggregate interlock forces between the cracks reduce to zero. The capacity of the membrane for this mode of failure is given by:

$$\nu_{xy} = \frac{2.5f_{sp} / \sin \theta_2 + \rho_y f_{yy} \tan^2 \theta_1}{(f_x / \nu) + (f_y / \nu) \tan^2 \theta_2 + 2 \tan \theta_1} \quad (3.74)$$

where f_{sp} = tensile splitting strength of concrete, and θ_2 is the real root of the following equation:

$$\begin{aligned} & \tan^4 \theta_2 \\ & + \left(f_y (1 + \tan^2 \theta_1) + 2 \tan \theta_1 - \frac{\rho_y f_{yy} (1 + \tan^2 \theta_1) \left(\frac{f_x}{\nu} + \frac{f_y}{\nu} \tan^2 \theta_1 + 2 \tan \theta_1 \right)}{2.5f_{sp} + \rho_y f_{yy} \tan \theta_1} \right) \tan^3 \theta_2 \\ & - \left(\frac{\rho_y f_{yy} \tan^2 \theta_1}{2.5f_{sp} + \rho_y f_{yy} \tan \theta_1} \right) \tan^2 \theta_2 - \frac{2.5f_{sp} \left(\frac{f_y}{\nu} \tan^2 \theta_1 + 2 \tan \theta \tan \theta_2 - \right)}{2.5f_{sp} + \rho_y f_{yy} \tan \theta_1} \tan \theta_2 \\ & - \frac{2.5f_{sp}}{2.5f_{sp} + \rho_y f_{yy} \tan \theta_1} = 0 \end{aligned}$$

Fourth Mode of Failure

In this mode, before the aggregate interlock forces reduce to zero as explained in the third mode, x-reinforcement also yields. The capacity of the membrane element for this case is given by:

$$v_{xy} = \frac{\rho_x f_{xy} + \rho_y f_{yx} \tan^2 \theta_1}{(f_x / v) + (f_y / v) \tan^2 \theta_2 + 2 \tan \theta_1} \quad (3.75)$$

In order to determine which mode the membrane element fails in when $\theta_1 > 45^\circ$, the capacities according to both the third and the fourth modes are calculated. The mode with the smaller capacity is the mode of failure.

Beside these four modes of failure, there is a possibility for the shell to fail because of crushing of concrete. It has to be noted that the crushing strength of the shell element is calculated by Eq.3.60 instead of the more complicated form of the same equation which is suggested by Zararis (1997) to be able to compare the results of both methods. It can be observed from the suggested equation which is given below that Eq.3.60 is just a simplified version of Eq. 3.76, and using Eq.3.76 is not practical due to its closed form.

$$\frac{f_{c1}}{f'_c} = 1 - \left(\frac{f_{c2}}{0.3 f'^{2/3}_c} \right)^{5.5 - f'_c \cdot 10} \quad (\text{Eq.3.76})$$

In order to ascertain the degree of accuracy which can be achieved in predicting the failure load and mode of reinforced concrete shell elements, in the next chapter we will analyze a number of shell elements by each of the methods described in this chapter and compare the results with available experimental data.

Chapter 4

Comparison of the Results of Existing Methods with Available Experimental Data

4.1 General

To facilitate the analysis by the methods described in Chapter 3, a computer program was written which can be readily used to carry out detailed analysis of shell elements. The developed 16-bit Windows based computer program will enable the user to input the element information and to get the results for each existing method of analysis. By using this program, the results of the existing methods, which were presented in Chapter 3, are compared with experimental data taken from the literature.

4.2 Description of the Computer Program: MemCap

Although the methods developed by Nielsen, Clark, Fialkow, and Zararis, are appropriate for hand calculations, due the large number of cases to be solved in the parametric study, they are coded as computer algorithms. Another program is also developed for the modified compression field theory which is originally intended as a computer oriented analysis method. For practical reasons, all of these algorithms are combined under a windows based program called MemCap (Membrane Capacity).

MemCap, which has a 16 bit-multiple window graphical interface, is written in Microsoft Visual Basic 3.0 programming language. The complete source code of the program is given in Appendix A. It basically calculates the capacity of reinforced concrete membrane elements under proportional loading by using the described methods of analysis.

Operation of the program involves three basic steps:

- Inputting the data.
- Selection of the method(s) to be used.
- Outputting the results.

Data is input into the edit boxes in the main window of the program which is illustrated in Fig. 4.1. Methods to be used to calculate the capacity of the case are selected by clicking

the check boxes in the “METHODS” group. It is possible to select more than one method simultaneously. After completing the input, “Analyze” button is clicked and results are obtained in the “RESULTS” field of the interface. To repeat the same procedure for a different case, the input information must be changed and the “Analyze” button must be clicked. To finalize the execution, “End” button must be selected.

Input Data

Table 4.1 shows the input data that is used by the methods which have been implemented in MemCap. Since some of the methods do not use all of these variables, it is not required to input a data if the method does require that variable. When the program is started, all of the relevant variables are initialized by appropriate values taken from a sample test specimen to simplify the task of data inputting.

Most of the parameters in Table 4.1 are self-explanatory. Only the “maximum distance from reinforcement” may require further explanation. The maximum distance from reinforcement is defined as the larger of the distances between the edge of the panel and the closest bar which is parallel to that edge, or the maximum distance between two parallel reinforcement bars.

MemCap

CONCRETE PROPERTIES :

Cylinder Strength : MPa
Strain at Peak Cylinder Stress : (Just for MCFT)
Modulus of Elasticity of Concrete : MPa (Just for MCFT)
Minimum Aggregate Size : mm (Just for MCFT)

STEEL PROPERTIES :

Reinforcement Ratio (x-direction) :
Reinforcement Ratio (y-direction) :
Yield Strength (x-direction) : MPa
Yield Strength (y-direction) : MPa
Modulus of Elasticity of x-reinforcement : MPa (Just for MCFT)
Modulus of Elasticity of y-reinforcement : MPa (Just for MCFT)
Ultimate Strain for x-reinforcement :
Ultimate Strain for y-reinforcement :
Maximum Distance from x-reinforcement : mm (Just for MCFT)
Maximum Distance from y-reinforcement : mm (Just for MCFT)

METHODS :

☐ Nielsen's Method
☐ Clark's Method
☐ Fialkow's Method
☐ Zararis' Method
☐ Modified Zararis
☐ MCFT

LOADING :

Proportional
Loading.

f_x / v_y:

f_y / v_y:

Analyze

End

About

RESULTS :

Nielsen's Method : MPa
Clark's Method : MPa
Fialkow's Method : MPa
Zararis' Method : MPa
MCFT : MPa

Details

Fig. 4.1: Main Window of MemCap

Table 4.1: Input Data Information for MemCap

Properties	Variable in MemCap	Methods
Concrete		
Cylinder Strength of Concrete	f_c	All
Strain at Peak Cylinder Stress	ϵ_{ps0}	All
Modulus of Elasticity of Concrete	E_c	All
Maximum Aggregate Size	a	MCFT
Steel		
Reinforcement Ratio	ρ_{ox}, ρ_{oy}	All
Yield Strength of Steel	f_{xy}, f_{yy}	All
Modulus of Elasticity of Steel	E_{sx}, E_{sy}	All
Ultimate Strain for Steel	$\epsilon_{sx_ult}, \epsilon_{sy_ult}$	MCFT, Zararis
Maximum Distance from Reinforcement	x_{max}, y_{max}	MCFT
Loading		
Axial stress / Shear stress Ratio	α_x, α_y	All

4.3 Available Experimental Data

The number of experiments conducted for the determination of the capacity of reinforced concrete shells is not large. One of the possible reasons for this may be the high cost of such tests and the availability of sophisticated experimental facilities. More detailed information about these tests can be obtained from the work of Vecchio and Collins (1982). Preparation of the test specimens, setting up the loading equipment, placing gauges, and tracing the information from these gauges are much more complicated than the ones from other types of tests.

Although the available experimental results are not adequate to cover all possible variations in material properties, geometry and loading combinations, existing methods of analysis are compared with what is readily available in the literature. The comparison is also carried out in order to validate the use of the modified compression theory as the reference method in the parametric study which will be presented in Chapter 5.

Table 4.2 gives the input data for three sets of experiments which were conducted by Vecchio and Collins (1982, 1994) and Hsu (1995). As it can be observed from this table PV Panels of Collins (1982) are prepared by using low strength concrete. In the Hsu (1995) and Collins (1994) test programs, high strength concrete is used. More detailed information about the testing procedure, and test specimens can be obtained from the cited references.

Table 4.2: Properties of the Test Specimens

Panel	Reference	f_n / v	ρ_x	ρ_y	f_{xy} (MPa)	f_{yy} (MPa)	f_c (MPa)
E1	PV 2, Collins (1982)	0	0.0018	0.0018	428	428	23.5
E2	PV 3, Collins (1982)	0	0.0048	0.0048	662	662	26.6
E3	PV 4, Collins (1982)	0	0.0106	0.0106	242	242	26.6
E4	PV 6, Collins (1982)	0	0.0179	0.0179	266	266	29.8
E5	PV 10, Collins (1982)	0	0.0179	0.01	276	276	14.5
E6	PV 11, Collins (1982)	0	0.0179	0.0131	235	235	15.6
E7	PV 12, Collins (1982)	0	0.0179	0.0045	469	269	16
E8	PV 16, Collins (1982)	0	0.0074	0.0074	255	255	21.7
E9	PV 19, Collins (1982)	0	0.01785	0.00713	458	299	19
E10	PV 20, Collins (1982)	0	0.0179	0.0089	460	297	19.6
E11	PV 21, Collins (1982)	0	0.0179	0.013	458	302	19.5
E12	PV 22, Collins (1982)	0	0.0179	0.0152	458	420	19.6
E13	PV 23, Collins (1982)	-0.39	0.0179	0.0179	518	518	20.5
E14	PV 25, Collins (1982)	-0.69	0.01785	0.01785	466	466	19.3
E15	PV 27, Collins (1982)	0	0.01785	0.01785	442	442	20.5
E16	PV 28, Collins (1982)	0.32	0.0179	0.0179	483	483	19
E17	Panel A1, Hsu (1995)	0	0.00596	0.00596	451.57	451.57	42.84
E18	Panel A2, Hsu (1995)	0	0.01193	0.01193	469.7	469.7	41.86
E19	Panel A3, Hsu (1995)	0	0.01789	0.01789	453.25	453.25	42.28
E20	Panel A4, Hsu (1995)	0	0.02982	0.02982	476.91	476.91	43.12
E21	Panel B1, Hsu (1995)	0	0.01193	0.00596	469.7	451.57	45.92
E22	Panel B2, Hsu (1995)	0	0.01789	0.01193	453.25	469.7	44.73
E23	Panel B3, Hsu (1995)	0	0.01789	0.00596	453.25	451.7	45.57
E24	Panel B4, Hsu (1995)	0	0.02982	0.00596	476.91	451.7	45.43
E25	Panel B5, Hsu (1995)	0	0.02982	0.01193	476.91	469.7	43.47
E26	Panel B6, Hsu (1995)	0	0.02982	0.01789	476.91	453.25	43.51
E27	PHS 2, Collins (1994)	0	0.0323	0.0041	606	521	66.1
E28	PHS 3, Collins (1994)	0	0.0323	0.0082	606	521	58.4
E29	PHS 4, Collins (1994)	0.25	0.0323	0.0082	606	521	68.5
E30	PHS 5, Collins (1994)	0.25	0.0323	0.0041	606	521	52.1
E31	PHS 6, Collins (1994)	-0.25	0.0323	0.0041	606	521	49.7
E32	PHS 7, Collins (1994)	-0.25	0.0323	0.0082	606	521	53.6
E33	PHS 8, Collins (1994)	0	0.0323	0.0124	606	521	55.9
E34	PHS 9, Collins (1994)	-0.25	0.0323	0.0041	606	521	56
E35	PHS 10, Collins (1994)	0.25	0.0323	0.0124	606	521	51.4
E36	PA 1, Collins (1994)	0	0.0165	0.0082	522	522	49.9
E37	PA 2, Collins (1994)	0	0.0165	0.0082	522	522	43

4.4 Comparison of Existing Methods with Available Experimental Data

Table 4.3 gives the predicted ultimate capacities of all the methods under investigation and the corresponding experimental results. The capacities obtained from existing methods are normalized by the corresponding experimental data and the results are shown in Table 4.4. The last two rows of Table 4.4 give the mean values of the normalized capacities and standard deviation from these mean values for each method while Fig.4.2 illustrates these values in a bar chart format.

The first point that has to be noted here is the lack of adequate number of cases to reach a definitive conclusion. For instance one can observe that the range of loading ratios in the test specimens is quite narrow. Out of 37 panels, only nine of them are tested for loading ratios other than 0. This brings us to the same point; i.e., necessity of a detailed parametric study which will cover a wide range of loading ratios.

As one can see in Table 4.3, Nielsen's and Clarks' methods give the same results for each panel. This is basically due to the fact that Clark's and Nielsen's 'boundary case graphs', which were illustrated in Fig.3.17 and Fig.3.15, respectively, give the same set of equations for the loading ratios given for the test specimens. Due to this fact, to be able

Table 4.3: Comparison of Ultimate Capacities (v_u) of Existing Methods with
Experimental Results

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zararis	MCFT	Exp.
E1	0.77	0.77	0.77	0.77	0.77	1.12	1.16
E2	3.18	3.18	3.18	3.18	3.18	3.02	3.07
E3	2.57	2.57	2.57	2.57	2.57	2.81	2.9
E4	4.76	4.76	4.76	4.76	4.76	4.26	4.55
E5	2.76	2.76	3.14	3.69	3.74	3.81	3.97
E6	3.08	3.08	3.30	3.60	3.61	3.41	3.56
E7	1.21	1.21	1.66	3.19	3.94	3.09	3.13
E8	1.89	1.89	1.89	1.89	1.89	2.16	2.14
E9	2.13	2.13	2.62	4.17	4.65	3.94	3.95
E10	2.64	2.64	3.09	4.67	5.08	4.18	4.26
E11	3.93	3.93	4.22	5.45	5.45	5.09	5.03
E12	6.38	6.38	6.08	5.49	5.49	6.12	6.07
E13	6.79	6.79	8.91	7.62	7.62	8.72	8.87
E14	6.59	6.59	9.11	9.21	9.21	9.01	9.12
E15	6.79	6.79	6.37	5.69	5.69	6.24	6.35
E16	6.54	6.54	5.60	4.53	4.53	5.62	5.8
E17	2.69	2.69	2.69	2.69	2.69	2.36	2.28
E18	5.60	5.60	5.60	5.60	5.60	5.27	5.37
E19	8.11	8.11	8.11	8.11	8.11	7.62	7.66
E20	9.85	9.85	8.62	10.02	10.02	11.21	11.31
E21	2.69	2.69	3.16	3.88	3.98	3.82	3.96
E22	5.60	5.60	6.13	6.74	6.77	6.02	6.13
E23	2.69	2.69	3.46	4.67	4.91	4.23	4.36
E24	2.69	2.69	3.84	6.19	6.94	5.23	5.07
E25	5.60	5.60	6.81	8.93	9.25	7.36	7.16
E26	8.11	8.11	9.01	9.95	9.95	9.21	9.15
E27	2.14	2.14	3.39	6.47	7.99	6.82	6.66
E28	4.27	4.27	5.75	9.14	10.16	8.26	8.19
E29	3.42	3.42	3.93	6.79	8.44	6.81	6.91
E30	1.71	1.71	2.20	4.38	6.72	4.72	4.81
E31	2.85	2.85	6.89	10.17	10.62	9.69	9.89
E32	5.70	5.70	10.27	13.14	13.42	10.42	10.26
E33	6.46	6.46	7.90	11.25	11.62	10.46	10.84
E34	2.85	2.85	6.89	10.17	10.62	9.26	9.37
E35	5.17	5.17	5.55	8.65	9.21	8.52	8.58
E36	4.28	4.28	5.01	6.07	6.19	6.12	6.34
E37	4.28	4.28	5.01	6.07	6.19	6.08	6.22

(*) All values are in MPa

Table 4.4: Predicted / Experimental Capacities

Panel	Nielsen / Exp.	Clark / Exp.	Fialkow / Exp.	Zararis / Exp.	Mod. Zar. / Exp.	MCFT / Exp.
E1	0.66	0.66	0.66	0.66	0.66	0.97
E2	1.04	1.04	1.04	1.04	1.04	0.98
E3	0.89	0.89	0.89	0.89	0.89	0.97
E4	1.05	1.05	1.05	1.05	1.05	0.94
E5	0.70	0.70	0.79	0.93	0.94	0.96
E6	0.86	0.86	0.93	1.01	1.01	0.96
E7	0.39	0.39	0.53	1.02	1.26	0.99
E8	0.88	0.88	0.88	0.88	0.88	1.01
E9	0.54	0.54	0.66	1.06	1.18	1.00
E10	0.62	0.62	0.73	1.10	1.19	0.98
E11	0.78	0.78	0.84	1.08	1.08	1.01
E12	1.05	1.05	1.00	0.90	0.90	1.01
E13	0.77	0.77	1.00	0.86	0.86	0.98
E14	0.72	0.72	1.00	1.01	1.01	0.99
E15	1.07	1.07	1.00	0.90	0.90	0.98
E16	1.13	1.13	0.97	0.78	0.78	0.97
E17	1.18	1.18	1.18	1.18	1.18	1.04
E18	1.04	1.04	1.04	1.04	1.04	0.98
E19	1.06	1.06	1.06	1.06	1.06	0.99
E20	0.87	0.87	0.76	0.89	0.89	0.99
E21	0.68	0.68	0.80	0.98	1.00	0.96
E22	0.91	0.91	1.00	1.10	1.10	0.98
E23	0.62	0.62	0.79	1.07	1.13	0.97
E24	0.53	0.53	0.76	1.22	1.37	1.03
E25	0.78	0.78	0.95	1.25	1.29	1.03
E26	0.89	0.89	0.98	1.09	1.09	1.01
E27	0.32	0.32	0.51	0.97	1.20	1.02
E28	0.52	0.52	0.70	1.12	1.24	1.01
E29	0.49	0.49	0.57	0.98	1.22	0.99
E30	0.36	0.36	0.46	0.91	1.40	0.98
E31	0.29	0.29	0.70	1.03	1.07	0.98
E32	0.56	0.56	1.00	1.28	1.31	1.02
E33	0.60	0.60	0.73	1.04	1.07	0.96
E34	0.30	0.30	0.74	1.09	1.13	0.99
E35	0.60	0.60	0.65	1.01	1.07	0.99
E36	0.68	0.68	0.79	0.96	0.98	0.97
E37	0.69	0.69	0.81	0.98	1.00	0.98
Mean	0.73	0.73	0.84	1.01	1.08	0.99
St. Dev.	0.25	0.25	0.18	0.12	0.16	0.02

Fig. 4.2: Comparison of the Results of the Existing Methods with the Experimental Data

to compare these two methods, one must use different loading ratios which will result in different sets of equations.

We notice that the results of Zararis' and Modified Zararis' methods are quite close to each other. At first glance this leads us to the conclusion that introduction of aggregate interlock forces into the model does not change the results to a significant degree. But before one can arrive at such a definitive conclusion a through parametric study must be carried out.

Probably the most important result of this comparison is the validation of the usage of the modified compression field theory as a reference tool in a parametric study. Although Zararis' method also gives quite accurate results for this set of experiments, Table 3.4 shows that standard deviation of Zararis' method from the mean is much higher than that of the modified compression field theory.

In general, it can be stated that Zararis' and Modified Zararis Methods are more accurate than the other methods. Although Nielsen's and Clark's methods are not as accurate as the others, they are always on the safe side. Considering the fact that these two methods are extremely easy to use compared with the others, their importance as a preliminary analysis procedures cannot be ignored.

As stated earlier, a parametric study is necessary in order to be able to reach a final judgment with respect to the existing methods of analysis, and such a study is presented in Chapter 5.

Chapter 5

Parametric Study

5.1 General

A parametric study is carried out in order to compare the existing methods of analysis of reinforced concrete shell elements. After the selection of the parameters of the study, the results of each method, which are obtained by using MemCap, which is a Windows based program are presented. These results are compared among each other by using the results of the modified compression field theory as reference.

5.2 Parametric Study

It is clear that a systematic approach must be followed in order to compare the existing methods of analysis for membrane elements. Because of the inadequate amount of experimental results, which would provide the necessary reference information, a proper comparison is not easy to carry out. This problem is addressed by employing a parametric study. In the parametric study, the modified compression field theory is used as the reference model. The justification of using the modified compression field theory as a reference tool was provided in Chapter 4.

The effects of the following parameters on the capacity calculations by the existing methods of analysis are investigated:

- Load ratio
- Reinforcement ratio
- Concrete strength

All other parameters which may affect the capacity of the panel are ignored in the present study.

5.3 Selection of Parameters

A typical orthogonally reinforced concrete membrane element, which is illustrated in Fig.

1.2, has the following parameters:

- Strength of concrete
- Ultimate strain of concrete
- Maximum aggregate size in concrete
- Modulus of elasticity of concrete
- Yield strength of steel in the x-direction
- Yield strength of steel in the y-direction
- Modulus of elasticity of steel in the x-direction
- Modulus of elasticity of steel in the y-direction
- Yield and ultimate strains of steel in the x-direction
- Yield and ultimate strains of steel in the y-direction
- Thickness of the element
- Length of the sides of the element (Square panels)
- Reinforcement ratio in the x- direction
- Reinforcement ratio in the y-direction
- Load ratio

The parametric study that is carried out in the present work maintains all of the parameters constant, except for the load ratio, the reinforcement ratio, and the concrete strength. The values of these constant parameters are given in Table 5.1. The selection of the values of the variable parameters are explained below.

Reinforcement Ratio

In order to test the effect of reinforcement ratio on the capacities determined by the existing methods of analysis, 4 different cases are investigated. In all of these cases, the reinforcement ratio in the x-direction is kept constant at 0.02. The reinforcement in the y-direction is gradually decreased by 25% until it reaches 0.0050. These cases, with reinforcement ratio being the variable, are shown in Table 5.2.

Load Ratio

For each case of reinforcement ratio, different loading ratios are considered. For this purpose, Clark's boundary case graph, which is illustrated in Fig. 3.17, is used as a guide since this classification is the most comprehensive in the literature. By comparing the existing methods on the basis of loading ratios, it is intended to determine the most appropriate method corresponding to each zone in the boundary case graph. As explained in Chapter 3, Clark's boundary case graph classifies the membrane elements into nine different cases depending on the applied loading. Table 3.4 gives the boundary curve equations for these nine cases. In the present study, 10 panels are tested for each of the 9 loading cases. Table 5.3 gives the notation used for the panels used in the parametric study. It has to be noted that "RR" stands for "case based on reinforcement ratio" while

“LR” stands for “case based on load ratio”. Due to the large number of cases analyzed in the parametric study, for practical reasons, only the boundary case graph of the case “LR1 - RR1” with 25MPa concrete is illustrated in Fig. 5.1., and the panels corresponding to this case are plotted in this figure.

Table 5.1: Values of the Constant Parameters

Property of the Panel	Value
Ultimate strain of concrete	0.0022
Maximum aggregate size in concrete	6 mm
Modulus of elasticity of concrete	24,000 MPa
Yield strength of steel in the x-direction	400 MPa
Yield strength of steel in the y-direction	400MPa
Modulus of elasticity of steel in the x-direction	200,000 MPa
Modulus of elasticity of steel in the y-direction	200,000 MPa
Reinforcement Ratio in the x-direction	0.02
Yield strain of steel in the x-direction	0.02
Yield strain of steel in the y-direction	0.02
Ultimate strain of steel in the x-direction	0.15
Ultimate strain of steel in the y-direction	0.15
Length of the sides	1000 mm

✕: Location of the panels on the boundary case graph

Fig. 5.1: Boundary Case Graph for the Case "LR1 - RR1" (25 MPa Concrete)

Table 5.2: Cases Based on the Reinforcement Ratio

Case	x-reinforcement ratio	y-reinforcement ratio
RR1	0.02	$0.02 * 1.0 = 0.02$
RR2	0.02	$0.02 * 0.75 = 0.015$
RR3	0.02	$0.02 * 0.5 = 0.01$
RR4	0.02	$0.02 * 0.25 = 0.0050$

Table 5.3: Notations Used for the Panels in the Parametric Study

	RR1	RR2	RR3	RR4
LR1	P1-1 to P1-10	P1-11 to P1-20	P1-21 to P1-30	P1-31 to P1-40
LR2	P2-1 to P2-10	P2-11 to P2-20	P2-21 to P2-30	P2-31 to P2-40
LR3	P3-1 to P3-10	P3-11 to P3-20	P3-21 to P3-30	P3-31 to P3-40
LR4	P4-1 to P4-10	P4-11 to P4-20	P4-21 to P4-30	P4-31 to P4-40
LR5	P5-1 to P5-10	P5-11 to P5-20	P5-21 to P5-30	P5-31 to P5-40
LR6	P6-1 to P6-10	P6-11 to P6-20	P6-21 to P6-30	P6-31 to P6-40
LR7	P7-1 to P7-10	P7-11 to P7-20	P7-21 to P7-30	P7-31 to P7-40
LR8	P8-1 to P8-10	P8-11 to P8-20	P8-21 to P8-30	P8-31 to P8-40
LR9	P9-1 to P9-10	P9-11 to P9-20	P9-21 to P9-30	P9-31 to P9-40

Concrete Strength

In order to determine the effect of the strength of concrete on the capacities calculated by the existing methods of analysis, the panels given in Table 5.3 are analyzed for 2 different concrete types with 15MPa and 25MPa strengths. Since some of the methods are not developed for high strength concrete, normal strength concrete will be used in the parametric study

Input data for each panel to be analyzed in the parametric study is given in Tables 5.4 to 5.12. For each table, 2 cases are defined depending on the concrete strength of the panel: SET 1 and SET 2.

Table 5.4: Input Data for Load Case No. 1 Panels

					SET 1	SET 2
Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	f_c (MPa)	f_c (MPa)
P1-1	0,0	0.02	0.02	400	15	25
P1-2	0.2,0.2	0.02	0.02	400	15	25
P1-3	0.4,0.4	0.02	0.02	400	15	25
P1-4	-0.4,0.4	0.02	0.02	400	15	25
P1-5	0.4,-0.4	0.02	0.02	400	15	25
P1-6	-0.4,-0.4	0.02	0.02	400	15	25
P1-7	0.8,0.8	0.02	0.02	400	15	25
P1-8	-0.8,0.8	0.02	0.02	400	15	25
P1-9	0.8,-0.8	0.02	0.02	400	15	25
P1-10	-0.8,-0.8	0.02	0.02	400	15	25
P1-11	0,0	0.02	0.015	400	15	25
P1-12	0.2,0.2	0.02	0.015	400	15	25
P1-13	0.4,0.4	0.02	0.015	400	15	25
P1-14	-0.4,0.4	0.02	0.015	400	15	25
P1-15	0.4,-0.4	0.02	0.015	400	15	25
P1-16	-0.4,-0.4	0.02	0.015	400	15	25
P1-17	0.8,0.8	0.02	0.015	400	15	25
P1-18	-0.8,0.8	0.02	0.015	400	15	25
P1-19	0.8,-0.8	0.02	0.015	400	15	25
P1-20	-0.8,-0.8	0.02	0.015	400	15	25
P1-21	0,0	0.02	0.01	400	15	25
P1-22	0.2,0.2	0.02	0.01	400	15	25
P1-23	0.4,0.4	0.02	0.01	400	15	25
P1-24	-0.4,0.4	0.02	0.01	400	15	25
P1-25	0.4,-0.4	0.02	0.01	400	15	25
P1-26	-0.4,-0.4	0.02	0.01	400	15	25
P1-27	0.8,0.8	0.02	0.01	400	15	25
P1-28	-0.8,0.8	0.02	0.01	400	15	25
P1-29	0.8,-0.8	0.02	0.01	400	15	25
P1-30	-0.8,-0.8	0.02	0.01	400	15	25
P1-31	0,0	0.02	0.005	400	15	25
P1-32	0.2,0.2	0.02	0.005	400	15	25
P1-33	0.4,0.4	0.02	0.005	400	15	25
P1-34	-0.4,0.4	0.02	0.005	400	15	25
P1-35	0.4,-0.4	0.02	0.005	400	15	25
P1-36	-0.4,-0.4	0.02	0.005	400	15	25
P1-37	0.8,0.8	0.02	0.005	400	15	25
P1-38	-0.8,0.8	0.02	0.005	400	15	25
P1-39	0.8,-0.8	0.02	0.005	400	15	25
P1-40	-0.8,-0.8	0.02	0.005	400	15	25

Table 5.5: Input Data for Load Case No. 2 Panels

Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	SET 1 f_c (MPa)	SET 2 f_c (MPa)
P2-1	-1.2,0.420	0.02	0.02	400	15	25
P2-2	-1.4,0.360	0.02	0.02	400	15	25
P2-3	-1.6,0.315	0.02	0.02	400	15	25
P2-4	-1.8,0.280	0.02	0.02	400	15	25
P2-5	-2.0,0.250	0.02	0.02	400	15	25
P2-6	-2.2,0.230	0.02	0.02	400	15	25
P2-7	-2.4,0.210	0.02	0.02	400	15	25
P2-8	-2.6,0.190	0.02	0.02	400	15	25
P2-9	-2.8,0.180	0.02	0.02	400	15	25
P2-10	-3.0,0.170	0.02	0.02	400	15	25
P2-11	-1.2,0.420	0.02	0.015	400	15	25
P2-12	-1.4,0.360	0.02	0.015	400	15	25
P2-13	-1.6,0.315	0.02	0.015	400	15	25
P2-14	-1.8,0.280	0.02	0.015	400	15	25
P2-15	-2.0,0.250	0.02	0.015	400	15	25
P2-16	-2.2,0.230	0.02	0.015	400	15	25
P2-17	-2.4,0.210	0.02	0.015	400	15	25
P2-18	-2.6,0.190	0.02	0.015	400	15	25
P2-19	-2.8,0.180	0.02	0.015	400	15	25
P2-20	-3.0,0.170	0.02	0.015	400	15	25
P2-21	-1.2,0.420	0.02	0.01	400	15	25
P2-22	-1.4,0.360	0.02	0.01	400	15	25
P2-23	-1.6,0.315	0.02	0.01	400	15	25
P2-24	-1.8,0.280	0.02	0.01	400	15	25
P2-25	-2.0,0.250	0.02	0.01	400	15	25
P2-26	-2.2,0.230	0.02	0.01	400	15	25
P2-27	-2.4,0.210	0.02	0.01	400	15	25
P2-28	-2.6,0.190	0.02	0.01	400	15	25
P2-29	-2.8,0.180	0.02	0.01	400	15	25
P2-30	-3.0,0.170	0.02	0.01	400	15	25
P2-31	-1.2,0.420	0.02	0.005	400	15	25
P2-32	-1.4,0.360	0.02	0.005	400	15	25
P2-33	-1.6,0.315	0.02	0.005	400	15	25
P2-34	-1.8,0.280	0.02	0.005	400	15	25
P2-35	-2.0,0.250	0.02	0.005	400	15	25
P2-36	-2.2,0.230	0.02	0.005	400	15	25
P2-37	-2.4,0.210	0.02	0.005	400	15	25
P2-38	-2.6,0.190	0.02	0.005	400	15	25
P2-39	-2.8,0.180	0.02	0.005	400	15	25
P2-40	-3.0,0.170	0.02	0.005	400	15	25

Table 5.6: Input Data for Load Case No. 3 Panels

					SET 1	SET 2
Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	f_c (MPa)	f_c (MPa)
P3-1	0.420,-1.2	0.02	0.02	400	15	25
P3-2	0.360,-1.4	0.02	0.02	400	15	25
P3-3	0.315,-1.6	0.02	0.02	400	15	25
P3-4	0.280,-1.8	0.02	0.02	400	15	25
P3-5	0.250,-2.0	0.02	0.02	400	15	25
P3-6	0.230,-2.2	0.02	0.02	400	15	25
P3-7	0.210,-2.4	0.02	0.02	400	15	25
P3-8	0.190,-2.6	0.02	0.02	400	15	25
P3-9	0.180,-2.8	0.02	0.02	400	15	25
P3-10	0.170, -3.0	0.02	0.02	400	15	25
P3-11	0.420,-1.2	0.02	0.015	400	15	25
P3-12	0.360,-1.4	0.02	0.015	400	15	25
P3-13	0.315,-1.6	0.02	0.015	400	15	25
P3-14	0.280,-1.8	0.02	0.015	400	15	25
P3-15	0.250,-2.0	0.02	0.015	400	15	25
P3-16	0.230,-2.2	0.02	0.015	400	15	25
P3-17	0.210,-2.4	0.02	0.015	400	15	25
P3-18	0.190,-2.6	0.02	0.015	400	15	25
P3-19	0.180,-2.8	0.02	0.015	400	15	25
P3-20	0.170, -3.0	0.02	0.015	400	15	25
P3-21	0.420,-1.2	0.02	0.01	400	15	25
P3-22	0.360,-1.4	0.02	0.01	400	15	25
P3-23	0.315,-1.6	0.02	0.01	400	15	25
P3-24	0.280,-1.8	0.02	0.01	400	15	25
P3-25	0.250,-2.0	0.02	0.01	400	15	25
P3-26	0.230,-2.2	0.02	0.01	400	15	25
P3-27	0.210,-2.4	0.02	0.01	400	15	25
P3-28	0.190,-2.6	0.02	0.01	400	15	25
P3-29	0.180,-2.8	0.02	0.01	400	15	25
P3-30	0.170, -3.0	0.02	0.01	400	15	25
P3-31	0.420,-1.2	0.02	0.005	400	15	25
P3-32	0.360,-1.4	0.02	0.005	400	15	25
P3-33	0.315,-1.6	0.02	0.005	400	15	25
P3-34	0.280,-1.8	0.02	0.005	400	15	25
P3-35	0.250,-2.0	0.02	0.005	400	15	25
P3-36	0.230,-2.2	0.02	0.005	400	15	25
P3-37	0.210,-2.4	0.02	0.005	400	15	25
P3-38	0.190,-2.6	0.02	0.005	400	15	25
P3-39	0.180,-2.8	0.02	0.005	400	15	25
P3-40	0.170, -3.0	0.02	0.005	400	15	25

Table 5.7: Input Data for Load Case No. 4 Panels

					SET 1	SET 2
Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	f_c (MPa)	f_c (MPa)
P4-1	-3.8,-3.8	0.02	0.02	400	15	25
P4-2	-4.0,-4.0	0.02	0.02	400	15	25
P4-3	-4.2,-4.2	0.02	0.02	400	15	25
P4-4	-4.4,-4.4	0.02	0.02	400	15	25
P4-5	-3.8,-4.4	0.02	0.02	400	15	25
P4-6	-4.0,-4.2	0.02	0.02	400	15	25
P4-7	-3.8,-4.0	0.02	0.02	400	15	25
P4-8	-4.2,-4.4	0.02	0.02	400	15	25
P4-9	-4.4,-3.8	0.02	0.02	400	15	25
P4-10	-4.0,-3.8	0.02	0.02	400	15	25
P4-11	-3.8,-3.8	0.02	0.015	400	15	25
P4-12	-4.0,-4.0	0.02	0.015	400	15	25
P4-13	-4.2,-4.2	0.02	0.015	400	15	25
P4-14	-4.4,-4.4	0.02	0.015	400	15	25
P4-15	-3.8,-4.4	0.02	0.015	400	15	25
P4-16	-4.0,-4.2	0.02	0.015	400	15	25
P4-17	-3.8,-4.0	0.02	0.015	400	15	25
P4-18	-4.2,-4.4	0.02	0.015	400	15	25
P4-19	-4.4,-3.8	0.02	0.015	400	15	25
P4-20	-4.0,-3.8	0.02	0.015	400	15	25
P4-21	-3.8,-3.8	0.02	0.01	400	15	25
P4-22	-4.0,-4.0	0.02	0.01	400	15	25
P4-23	-4.2,-4.2	0.02	0.01	400	15	25
P4-24	-4.4,-4.4	0.02	0.01	400	15	25
P4-25	-3.8,-4.4	0.02	0.01	400	15	25
P4-26	-4.0,-4.2	0.02	0.01	400	15	25
P4-27	-3.8,-4.0	0.02	0.01	400	15	25
P4-28	-4.2,-4.4	0.02	0.01	400	15	25
P4-29	-4.4,-3.8	0.02	0.01	400	15	25
P4-30	-4.0,-3.8	0.02	0.01	400	15	25
P4-31	-3.8,-3.8	0.02	0.005	400	15	25
P4-32	-4.0,-4.0	0.02	0.005	400	15	25
P4-33	-4.2,-4.2	0.02	0.005	400	15	25
P4-34	-4.4,-4.4	0.02	0.005	400	15	25
P4-35	-3.8,-4.4	0.02	0.005	400	15	25
P4-36	-4.0,-4.2	0.02	0.005	400	15	25
P4-37	-3.8,-4.0	0.02	0.005	400	15	25
P4-38	-4.2,-4.4	0.02	0.005	400	15	25
P4-39	-4.4,-3.8	0.02	0.005	400	15	25
P4-40	-4.0,-3.8	0.02	0.005	400	15	25

Table 5.8: Input Data for Load Case No. 5 Panels

					SET 1	SET 2
Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	f_c (MPa)	f_c (MPa)
P5-1	-2.8,-3.290	0.02	0.02	400	15	25
P5-2	-2.9,-3.300	0.02	0.02	400	15	25
P5-3	-2.6,-3.540	0.02	0.02	400	15	25
P5-4	-2.4,-3.830	0.02	0.02	400	15	25
P5-5	-2.2,-4.180	0.02	0.02	400	15	25
P5-6	-2.0,-4.600	0.02	0.02	400	15	25
P5-7	-1.8,-5.110	0.02	0.02	400	15	25
P5-8	-1.6,-5.750	0.02	0.02	400	15	25
P5-9	-1.5,-5.400	0.02	0.02	400	15	25
P5-10	-1.4,-6.570	0.02	0.02	400	15	25
P5-11	-2.8,-3.290	0.02	0.015	400	15	25
P5-12	-2.9,-3.300	0.02	0.015	400	15	25
P5-13	-2.6,-3.540	0.02	0.015	400	15	25
P5-14	-2.4,-3.830	0.02	0.015	400	15	25
P5-15	-2.2,-4.180	0.02	0.015	400	15	25
P5-16	-2.0,-4.600	0.02	0.015	400	15	25
P5-17	-1.8,-5.110	0.02	0.015	400	15	25
P5-18	-1.6,-5.750	0.02	0.015	400	15	25
P5-19	-1.5,-5.400	0.02	0.015	400	15	25
P5-20	-1.4,-6.570	0.02	0.015	400	15	25
P5-21	-2.8,-3.290	0.02	0.01	400	15	25
P5-22	-2.9,-3.300	0.02	0.01	400	15	25
P5-23	-2.6,-3.540	0.02	0.01	400	15	25
P5-24	-2.4,-3.830	0.02	0.01	400	15	25
P5-25	-2.2,-4.180	0.02	0.01	400	15	25
P5-26	-2.0,-4.600	0.02	0.01	400	15	25
P5-27	-1.8,-5.110	0.02	0.01	400	15	25
P5-28	-1.6,-5.750	0.02	0.01	400	15	25
P5-29	-1.5,-5.400	0.02	0.01	400	15	25
P5-30	-1.4,-6.570	0.02	0.01	400	15	25
P5-31	-2.8,-3.290	0.02	0.005	400	15	25
P5-32	-2.9,-3.300	0.02	0.005	400	15	25
P5-33	-2.6,-3.540	0.02	0.005	400	15	25
P5-34	-2.4,-3.830	0.02	0.005	400	15	25
P5-35	-2.2,-4.180	0.02	0.005	400	15	25
P5-36	-2.0,-4.600	0.02	0.005	400	15	25
P5-37	-1.8,-5.110	0.02	0.005	400	15	25
P5-38	-1.6,-5.750	0.02	0.005	400	15	25
P5-39	-1.5,-5.400	0.02	0.005	400	15	25
P5-40	-1.4,-6.570	0.02	0.005	400	15	25

Table 5.9: Input Data for Load Case No. 6 Panels

					SET 1	SET 2
Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	f_c (MPa)	f_c (MPa)
P6-1	-3.290,-2.8	0.02	0.02	400	15	25
P6-2	-5.400,-1.5	0.02	0.02	400	15	25
P6-3	-3.540,-2.6	0.02	0.02	400	15	25
P6-4	-3.830,-2.4	0.02	0.02	400	15	25
P6-5	-4.180,-2.2	0.02	0.02	400	15	25
P6-6	-4.600,-2.0	0.02	0.02	400	15	25
P6-7	-5.110,-1.8	0.02	0.02	400	15	25
P6-8	-5.750,-1.6	0.02	0.02	400	15	25
P6-9	-5.400,-1.5	0.02	0.02	400	15	25
P6-10	-6.570,-1.4	0.02	0.02	400	15	25
P6-11	-3.290,-2.8	0.02	0.015	400	15	25
P6-12	-5.400,-1.5	0.02	0.015	400	15	25
P6-13	-3.540,-2.6	0.02	0.015	400	15	25
P6-14	-3.830,-2.4	0.02	0.015	400	15	25
P6-15	-4.180,-2.2	0.02	0.015	400	15	25
P6-16	-4.600,-2.0	0.02	0.015	400	15	25
P6-17	-5.110,-1.8	0.02	0.015	400	15	25
P6-18	-5.750,-1.6	0.02	0.015	400	15	25
P6-19	-5.400,-1.5	0.02	0.015	400	15	25
P6-20	-6.570,-1.4	0.02	0.015	400	15	25
P6-21	-3.290,-2.8	0.02	0.01	400	15	25
P6-22	-5.400,-1.5	0.02	0.01	400	15	25
P6-23	-3.540,-2.6	0.02	0.01	400	15	25
P6-24	-3.830,-2.4	0.02	0.01	400	15	25
P6-25	-4.180,-2.2	0.02	0.01	400	15	25
P6-26	-4.600,-2.0	0.02	0.01	400	15	25
P6-27	-5.110,-1.8	0.02	0.01	400	15	25
P6-28	-5.750,-1.6	0.02	0.01	400	15	25
P6-29	-5.400,-1.5	0.02	0.01	400	15	25
P6-30	-6.570,-1.4	0.02	0.01	400	15	25
P6-31	-3.290,-2.8	0.02	0.005	400	15	25
P6-32	-5.400,-1.5	0.02	0.005	400	15	25
P6-33	-3.540,-2.6	0.02	0.005	400	15	25
P6-34	-3.830,-2.4	0.02	0.005	400	15	25
P6-35	-4.180,-2.2	0.02	0.005	400	15	25
P6-36	-4.600,-2.0	0.02	0.005	400	15	25
P6-37	-5.110,-1.8	0.02	0.005	400	15	25
P6-38	-5.750,-1.6	0.02	0.005	400	15	25
P6-39	-5.400,-1.5	0.02	0.005	400	15	25
P6-40	-6.570,-1.4	0.02	0.005	400	15	25

Table 5.10: Input Data for Load Case No. 7 Panels

					SET 1	SET 2
Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	f_c (MPa)	f_c (MPa)
P7-1	0.0,-3.8	0.02	0.02	400	15	25
P7-2	1.2,-3.8	0.02	0.02	400	15	25
P7-3	1.4,-3.8	0.02	0.02	400	15	25
P7-4	1.6,-3.8	0.02	0.02	400	15	25
P7-5	1.8,-3.8	0.02	0.02	400	15	25
P7-6	0.0,-4.0	0.02	0.02	400	15	25
P7-7	1.2,-4.0	0.02	0.02	400	15	25
P7-8	1.4,-4.0	0.02	0.02	400	15	25
P7-9	1.6,-4.0	0.02	0.02	400	15	25
P7-10	1.8,-4.0	0.02	0.02	400	15	25
P7-11	0.0,-3.8	0.02	0.015	400	15	25
P7-12	1.2,-3.8	0.02	0.015	400	15	25
P7-13	1.4,-3.8	0.02	0.015	400	15	25
P7-14	1.6,-3.8	0.02	0.015	400	15	25
P7-15	1.8,-3.8	0.02	0.015	400	15	25
P7-16	0.0,-4.0	0.02	0.015	400	15	25
P7-17	1.2,-4.0	0.02	0.015	400	15	25
P7-18	1.4,-4.0	0.02	0.015	400	15	25
P7-19	1.6,-4.0	0.02	0.015	400	15	25
P7-20	1.8,-4.0	0.02	0.015	400	15	25
P7-21	0.0,-3.8	0.02	0.01	400	15	25
P7-22	1.2,-3.8	0.02	0.01	400	15	25
P7-23	1.4,-3.8	0.02	0.01	400	15	25
P7-24	1.6,-3.8	0.02	0.01	400	15	25
P7-25	1.8,-3.8	0.02	0.01	400	15	25
P7-26	0.0,-4.0	0.02	0.01	400	15	25
P7-27	1.2,-4.0	0.02	0.01	400	15	25
P7-28	1.4,-4.0	0.02	0.01	400	15	25
P7-29	1.6,-4.0	0.02	0.01	400	15	25
P7-30	1.8,-4.0	0.02	0.01	400	15	25
P7-31	0.0,-3.8	0.02	0.005	400	15	25
P7-32	1.2,-3.8	0.02	0.005	400	15	25
P7-33	1.4,-3.8	0.02	0.005	400	15	25
P7-34	1.6,-3.8	0.02	0.005	400	15	25
P7-35	1.8,-3.8	0.02	0.005	400	15	25
P7-36	0.0,-4.0	0.02	0.005	400	15	25
P7-37	1.2,-4.0	0.02	0.005	400	15	25
P7-38	1.4,-4.0	0.02	0.005	400	15	25
P7-39	1.6,-4.0	0.02	0.005	400	15	25
P7-40	1.8,-4.0	0.02	0.005	400	15	25

Table 5.11: Input Data for Load Case No. 8 Panels

					SET 1	SET 2
Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	f_c (MPa)	f_c (MPa)
P8-1	-2.0,-3.0	0.02	0.02	400	15	25
P8-2	-2.2,-2.720	0.02	0.02	400	15	25
P8-3	-2.4,-2.5	0.02	0.02	400	15	25
P8-4	-2.6,-2.310	0.02	0.02	400	15	25
P8-5	-2.8,-2.140	0.02	0.02	400	15	25
P8-6	-3.0,-2.0	0.02	0.02	400	15	25
P8-7	-3.2,-1.870	0.02	0.02	400	15	25
P8-8	-3.4,-1.760	0.02	0.02	400	15	25
P8-9	-3.5,-1.700	0.02	0.02	400	15	25
P8-10	-3.6,-1.680	0.02	0.02	400	15	25
P8-11	-2.0,-3.0	0.02	0.015	400	15	25
P8-12	-2.2,-2.720	0.02	0.015	400	15	25
P8-13	-2.4,-2.5	0.02	0.015	400	15	25
P8-14	-2.6,-2.310	0.02	0.015	400	15	25
P8-15	-2.8,-2.140	0.02	0.015	400	15	25
P8-16	-3.0,-2.0	0.02	0.015	400	15	25
P8-17	-3.2,-1.870	0.02	0.015	400	15	25
P8-18	-3.4,-1.760	0.02	0.015	400	15	25
P8-19	-3.5,-1.700	0.02	0.015	400	15	25
P8-20	-3.6,-1.680	0.02	0.015	400	15	25
P8-21	-2.0,-3.0	0.02	0.01	400	15	25
P8-22	-2.2,-2.720	0.02	0.01	400	15	25
P8-23	-2.4,-2.5	0.02	0.01	400	15	25
P8-24	-2.6,-2.310	0.02	0.01	400	15	25
P8-25	-2.8,-2.140	0.02	0.01	400	15	25
P8-26	-3.0,-2.0	0.02	0.01	400	15	25
P8-27	-3.2,-1.870	0.02	0.01	400	15	25
P8-28	-3.4,-1.760	0.02	0.01	400	15	25
P8-29	-3.5,-1.700	0.02	0.01	400	15	25
P8-30	-3.6,-1.680	0.02	0.01	400	15	25
P8-31	-2.0,-3.0	0.02	0.005	400	15	25
P8-32	-2.2,-2.720	0.02	0.005	400	15	25
P8-33	-2.4,-2.5	0.02	0.005	400	15	25
P8-34	-2.6,-2.310	0.02	0.005	400	15	25
P8-35	-2.8,-2.140	0.02	0.005	400	15	25
P8-36	-3.0,-2.0	0.02	0.005	400	15	25
P8-37	-3.2,-1.870	0.02	0.005	400	15	25
P8-38	-3.4,-1.760	0.02	0.005	400	15	25
P8-39	-3.5,-1.700	0.02	0.005	400	15	25
P8-40	-3.6,-1.680	0.02	0.005	400	15	25

Table 5.12: Input Data for Load Case No. 9 Panels

Panel	$f_x/v, f_y/v$	ρ_x	ρ_y	f_{xy}, f_{yy} (MPa)	SET 1 f_c (MPa)	SET 2 f_c (MPa)
P9-1	-2.0,-3.0	0.02	0.02	400	15	25
P9-2	-2.2,-2.720	0.02	0.02	400	15	25
P9-3	-2.4,-2.5	0.02	0.02	400	15	25
P9-4	-2.6,-2.310	0.02	0.02	400	15	25
P9-5	-2.8,-2.140	0.02	0.02	400	15	25
P9-6	-3.0,-2.0	0.02	0.02	400	15	25
P9-7	-3.2,-1.870	0.02	0.02	400	15	25
P9-8	-3.4,-1.760	0.02	0.02	400	15	25
P9-9	-3.5,-1.700	0.02	0.02	400	15	25
P9-10	-3.6,-1.680	0.02	0.02	400	15	25
P9-11	-2.0,-3.0	0.02	0.015	400	15	25
P9-12	-2.2,-2.720	0.02	0.015	400	15	25
P9-13	-2.4,-2.5	0.02	0.015	400	15	25
P9-14	-2.6,-2.310	0.02	0.015	400	15	25
P9-15	-2.8,-2.140	0.02	0.015	400	15	25
P9-16	-3.0,-2.0	0.02	0.015	400	15	25
P9-17	-3.2,-1.870	0.02	0.015	400	15	25
P9-18	-3.4,-1.760	0.02	0.015	400	15	25
P9-19	-3.5,-1.700	0.02	0.015	400	15	25
P9-20	-3.6,-1.680	0.02	0.015	400	15	25
P9-21	-2.0,-3.0	0.02	0.01	400	15	25
P9-22	-2.2,-2.720	0.02	0.01	400	15	25
P9-23	-2.4,-2.5	0.02	0.01	400	15	25
P9-24	-2.6,-2.310	0.02	0.01	400	15	25
P9-25	-2.8,-2.140	0.02	0.01	400	15	25
P9-26	-3.0,-2.0	0.02	0.01	400	15	25
P9-27	-3.2,-1.870	0.02	0.01	400	15	25
P9-28	-3.4,-1.760	0.02	0.01	400	15	25
P9-29	-3.5,-1.700	0.02	0.01	400	15	25
P9-30	-3.6,-1.680	0.02	0.01	400	15	25
P9-31	-2.0,-3.0	0.02	0.005	400	15	25
P9-32	-2.2,-2.720	0.02	0.005	400	15	25
P9-33	-2.4,-2.5	0.02	0.005	400	15	25
P9-34	-2.6,-2.310	0.02	0.005	400	15	25
P9-35	-2.8,-2.140	0.02	0.005	400	15	25
P9-36	-3.0,-2.0	0.02	0.005	400	15	25
P9-37	-3.2,-1.870	0.02	0.005	400	15	25
P9-38	-3.4,-1.760	0.02	0.005	400	15	25
P9-39	-3.5,-1.700	0.02	0.005	400	15	25
P9-40	-3.6,-1.680	0.02	0.005	400	15	25

5.4 Results and Discussion

The ultimate capacities of the panels, in Table 5.3, are calculated for two different concrete strengths (15 MPa and 25 MPa) by using the existing methods of analysis of reinforced concrete shell elements, and these results are presented in Appendix B. The ultimate capacities are normalized by the results obtained by the modified compression field theory. Tables 5.13 to 5.21 and Tables 5.22 to 5.30 show the normalized capacities of each of the 9 load cases for 15 MPa and 25 MPa concrete strengths, respectively. At the end of each table, mean values of the normalized capacities and standard deviations from these mean values are given for each method. Tables 5.31 to 5.40 present a complete summary of the mean normalized capacities of each load case, which encompass 2 different concrete strengths and 4 different reinforcement ratios. The effect of reinforcement ratio and concrete strength on the capacity calculations according to the existing methods of analysis will be investigated in detail by reference to these tables. Although a detailed statistical analysis could be carried out in order to compare the existing methods of analysis of reinforced concrete shell elements, this study will focus on the mean normalized capacities and the standard deviations of the results obtained in the parametric study.

Effect of Load and Reinforcement Ratio

These two cases will be discussed together since they have a combined effect on the behaviour of reinforced concrete shell elements. As explained in the problem definition in

Chapter 1, the main problem in the analysis and design of reinforced concrete shell elements is the fact that the angle of crack is unknown. The modified compression field theory solves this problem by using an iterative procedure as explained in Chapter 3: It starts the iteration by assuming a crack angle and checks equilibrium, compatibility and constitutive requirements corresponding to this assumed direction. If the direction does not satisfy those requirements, it is changed according to the output obtained from the iteration. This trial-and-error procedure continues until all of the requirements are satisfied. Of course, the fundamental assumptions upon which this method is based cannot be all justified theoretically. In the existing methods of analysis which are specially designed for hand calculations, the determination of crack angle is a one-step procedure. As explained in Chapter 3, the formation of cracks in a reinforced concrete shell element depends on many factors such as reinforcement ratio, load ratio, and concrete strength. For simple combinations such as an orthogonally reinforced panel under pure shear with the same reinforcement ratios in both directions, the crack angle can be calculated accurately. But if the reinforcement ratio in one direction is quite different from the one in the other direction, or the loading is unsymmetrical, the results obtained by the methods developed for hand calculations may not be accurate. The present parametric study confirms the validity of that expectation.

As it can be seen in Table 5.4, panels P1-1, P1-2, P1-2, P1-7, P1-10 are loaded symmetrically. For the case RR1; i.e., when the reinforcement ratio in both orthogonal directions are equal to 0.02, the results are quite comparable with the ones calculated by the modified compression field theory as shown in Table 5.13. The results obtained for

unsymmetrical loading and different reinforcement ratios also support the expectations about the existing methods of analysis. Particularly, it should be noted from the Tables 5.31 to 5.39 that the more the difference in the reinforcement ratio (from RR1 to RR4), the less accurate the capacities predicted by simpler methods of analysis.

Effect of Concrete Strength

The first observation from the results presented in Tables 5.31–40 is the fact that changing concrete strength from 15 MPa to 25 MPa did not make a significant difference in the accuracy of Zararis', Modified Zararis' and Fialkow's methods: The difference of the mean normalized capacities between these two concrete strengths in these methods are generally less than 10%. Similarly, Nielsen's and Clark's methods are not significantly affected by the change in the concrete strength. This can be observed by looking at Table 5.32: The mean normalized capacity of RR1 panels in Load Case No.2 (LC2) for 15 MPa and 25 MPa concrete calculated by Nielsen's method are 0.74 and 0.77, respectively; resulting in a 0.4% difference. But in some cases the latter methods show a large difference in the normalized capacities as in the case of RR1 and RR2 in Table 5.33. Although these cases are rare, they help us recognize the fact that these methods use a simple model for concrete behaviour as explained in Chapter 3. On the other hand, some of the other methods determine the contribution of concrete to the ultimate capacity of reinforced concrete shell elements considering other issues such as the state of strain in concrete. For example, the load carrying capacity of concrete under compression significantly decreases when it is under tensile stresses in the orthogonal direction. The

modified compression field theory handles this problem quite correctly; but as discussed before, the procedure is iterative and not suitable for hand calculations.

Table 5.40 proposes the most appropriate methods to be used in the analysis of reinforced shell elements based on load cases and reinforcement ratios defined by Clark's 'boundary case graph'.

Table 5.13: Normalized Capacities for Load Case No. 1 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P1-1	0.95	0.95	0.91	0.73	0.73
P1-2	0.97	0.97	0.71	0.67	0.67
P1-3	0.98	0.98	0.86	0.63	0.63
P1-4	0.98	0.98	0.74	0.77	0.77
P1-5	0.78	0.78	0.59	0.61	0.61
P1-6	0.80	0.80	0.90	0.81	0.81
P1-7	0.86	0.86	1.06	0.99	0.99
P1-8	0.85	0.85	0.45	0.86	0.86
P1-9	0.73	0.73	0.39	0.73	0.73
P1-10	0.79	0.79	0.98	1.05	1.05
P1-11	0.88	0.88	0.46	0.68	0.68
P1-12	0.91	0.91	0.78	0.73	0.73
P1-13	0.81	0.81	0.74	0.69	0.69
P1-14	0.81	0.81	0.81	0.83	0.83
P1-15	0.78	0.78	0.59	0.63	0.63
P1-16	0.80	0.80	0.90	0.82	0.82
P1-17	0.84	0.84	0.68	0.83	0.83
P1-18	0.84	0.84	0.59	1.10	1.10
P1-19	0.80	0.80	0.42	0.86	0.86
P1-20	0.77	0.77	0.95	1.04	1.04
P1-21	0.75	0.75	0.56	0.82	0.82
P1-22	0.76	0.76	0.80	0.88	0.88
P1-23	0.78	0.78	0.76	0.97	0.97
P1-24	0.78	0.78	0.85	1.14	1.14
P1-25	0.85	0.85	0.63	0.70	0.70
P1-26	0.86	0.86	0.97	0.88	0.88
P1-27	0.80	0.80	0.68	1.07	1.07
P1-28	0.80	0.80	0.84	1.27	1.52
P1-29	0.87	0.87	0.46	1.03	1.03
P1-30	0.74	0.74	0.91	1.02	1.02
P1-31	0.66	0.66	0.93	1.31	1.36
P1-32	0.67	0.67	0.81	1.27	1.45
P1-33	0.67	0.67	0.73	1.22	1.54
P1-34	0.67	0.67	0.81	1.33	1.80
P1-35	0.61	0.61	0.78	0.90	0.91
P1-36	0.61	0.61	1.20	1.09	1.09
P1-37	0.68	0.68	0.61	1.13	1.13
P1-38	0.68	0.68	0.74	1.22	2.46
P1-39	0.89	0.89	0.47	0.98	1.26
P1-40	0.71	0.71	0.88	1.07	1.07
Mean	0.78	0.78	0.75	0.93	1.00
Std. Dev.	0.13	0.13	0.19	0.21	0.36

Table 5.14: Normalized Capacities for Load Case No. 2 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P2-1	1.00	1.00	0.59	0.91	0.91
P2-2	0.92	0.92	0.62	0.91	0.91
P2-3	0.84	0.84	1.24	0.90	0.90
P2-4	0.77	0.77	1.28	0.89	0.89
P2-5	0.71	0.71	1.33	0.88	0.88
P2-6	0.72	0.72	1.49	0.94	0.94
P2-7	0.67	0.67	1.54	0.91	0.91
P2-8	0.62	0.62	1.58	0.88	0.88
P2-9	0.58	0.58	1.62	0.85	0.85
P2-10	0.55	0.55	1.65	0.81	0.81
P2-11	0.92	0.92	0.65	0.97	0.97
P2-12	1.03	1.03	1.04	0.99	0.99
P2-13	0.95	0.95	1.08	0.98	0.98
P2-14	0.87	0.87	1.12	0.97	0.97
P2-15	0.85	0.85	1.23	1.02	1.02
P2-16	0.80	0.80	1.29	1.01	1.01
P2-17	0.75	0.75	1.35	1.00	1.00
P2-18	0.70	0.70	1.39	0.97	0.97
P2-19	0.67	0.67	1.47	0.96	0.96
P2-20	0.63	0.63	1.49	0.93	0.93
P2-21	0.89	0.89	0.97	1.33	1.33
P2-22	0.98	0.98	1.00	1.30	1.30
P2-23	1.08	1.08	1.05	1.31	1.31
P2-24	1.18	1.18	1.09	1.30	1.30
P2-25	1.10	1.10	1.12	1.27	1.27
P2-26	1.03	1.03	1.16	1.26	1.26
P2-27	0.94	0.94	1.18	1.21	1.21
P2-28	0.88	0.88	1.23	1.18	1.18
P2-29	0.82	0.82	1.25	1.14	1.14
P2-30	0.76	0.76	1.26	1.09	1.09
P2-31	0.76	0.76	0.89	1.41	2.07
P2-32	0.85	0.85	0.94	1.48	2.06
P2-33	0.94	0.94	0.98	1.54	2.06
P2-34	1.03	1.03	1.02	1.60	2.06
P2-35	1.11	1.11	1.06	1.67	2.06
P2-36	1.20	1.20	1.10	1.73	2.05
P2-37	1.29	1.29	1.14	1.80	2.03
P2-38	1.38	1.38	1.17	1.87	1.99
P2-39	1.45	1.45	1.20	1.93	1.95
P2-40	1.36	1.36	1.23	1.88	1.88
Mean	0.91	0.91	1.18	1.20	1.28
Std. Dev.	0.22	0.22	0.25	0.33	0.45

Table 5.15: Normalized Capacities for Load Case No. 3 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P3-1	0.80	0.80	0.68	0.85	0.85
P3-2	0.98	0.98	0.95	0.69	0.69
P3-3	0.84	0.84	1.74	0.92	0.92
P3-4	0.73	0.73	0.68	1.04	1.04
P3-5	0.73	0.73	0.70	1.18	1.18
P3-6	3.20	3.20	2.94	2.30	2.30
P3-7	0.71	0.71	1.91	0.92	0.92
P3-8	0.72	0.72	0.74	1.34	1.34
P3-9	0.93	0.93	0.84	0.68	0.68
P3-10	0.63	0.63	2.01	0.87	0.87
P3-11	0.81	0.81	0.69	0.96	0.96
P3-12	0.83	0.83	1.05	0.76	0.76
P3-13	0.83	0.83	1.37	0.90	0.90
P3-14	0.82	0.82	0.76	1.36	1.36
P3-15	0.72	0.72	0.69	1.36	1.36
P3-16	2.65	2.65	2.95	2.28	2.28
P3-17	0.71	0.71	1.51	0.89	0.89
P3-18	0.79	0.79	0.80	1.55	1.55
P3-19	2.69	2.69	2.85	2.27	2.27
P3-20	0.63	0.63	1.58	0.85	0.85
P3-21	0.82	0.82	1.18	1.14	1.16
P3-22	0.77	0.77	0.80	1.03	1.03
P3-23	1.03	1.03	1.21	1.06	1.06
P3-24	0.77	0.77	0.71	1.44	1.63
P3-25	0.86	0.86	0.82	1.87	1.89
P3-26	1.77	1.77	1.91	2.22	2.22
P3-27	0.87	0.87	1.34	1.06	1.06
P3-28	1.07	1.07	1.09	2.04	2.04
P3-29	0.75	0.75	0.82	0.92	0.92
P3-30	0.77	0.77	1.41	1.03	1.03
P3-31	0.82	0.82	0.46	1.05	1.90
P3-32	0.67	0.67	0.78	1.28	1.64
P3-33	0.92	0.92	1.21	1.69	1.69
P3-34	1.05	1.05	0.37	1.85	2.53
P3-35	1.38	1.38	0.43	2.72	2.29
P3-36	0.67	0.67	0.81	1.30	1.54
P3-37	1.31	1.31	1.33	1.74	1.74
P3-38	2.00	2.00	0.53	2.99	1.99
P3-39	0.67	0.67	0.83	1.31	1.51
P3-40	1.35	1.35	1.40	1.71	1.71
Mean	1.05	1.05	1.17	1.39	1.41
Std. Dev.	0.60	0.60	0.65	0.58	0.54

Table 5.16: Normalized Capacities for Load Case No. 4 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P4-1	0.80	0.96	0.53	0.95	0.82
P4-2	0.80	1.00	0.52	0.92	0.79
P4-3	0.83	1.08	0.53	0.93	0.81
P4-4	2.61	3.52	1.63	2.85	2.47
P4-5	0.89	1.15	0.58	1.04	0.84
P4-6	0.85	1.08	0.55	0.96	0.81
P4-7	0.82	1.00	0.53	0.95	0.80
P4-8	0.86	1.14	0.54	0.95	0.81
P4-9	0.72	0.93	0.33	0.80	0.78
P4-10	0.76	0.93	0.35	0.85	0.80
P4-11	1.03	1.23	0.68	1.09	0.81
P4-12	1.05	1.32	0.68	1.09	0.80
P4-13	1.08	1.40	0.69	1.09	0.80
P4-14	1.10	1.49	0.69	1.09	0.80
P4-15	1.16	1.49	0.75	1.25	0.85
P4-16	1.10	1.40	0.71	1.14	0.82
P4-17	1.07	1.32	0.70	1.14	0.82
P4-18	1.12	1.49	0.71	1.13	0.81
P4-19	0.95	1.22	0.43	0.95	0.80
P4-20	1.00	1.23	0.46	1.04	0.79
P4-21	1.45	1.74	0.67	1.44	0.80
P4-22	1.49	1.86	0.67	1.45	0.80
P4-23	1.53	1.98	0.68	1.45	0.80
P4-24	1.56	2.07	0.69	1.46	0.80
P4-25	1.64	2.07	0.69	1.69	0.83
P4-26	1.56	1.98	0.68	1.53	0.81
P4-27	1.52	1.86	0.68	1.52	0.81
P4-28	1.59	2.07	0.69	1.53	0.81
P4-29	1.36	1.74	0.66	1.24	0.77
P4-30	1.42	1.74	0.67	1.37	0.79
P4-31	2.49	2.57	0.57	2.38	0.75
P4-32	2.56	2.20	0.58	2.39	0.74
P4-33	2.62	1.95	0.58	2.41	0.74
P4-34	2.67	1.78	0.59	2.42	0.73
P4-35	2.81	1.78	0.59	2.83	0.77
P4-36	2.67	1.95	0.58	2.54	0.75
P4-37	2.61	2.20	0.58	2.53	0.76
P4-38	2.72	1.78	0.59	2.54	0.75
P4-39	2.32	2.57	0.57	2.04	0.71
P4-40	2.44	2.57	0.57	2.25	0.74
Mean	1.54	1.67	0.63	1.53	0.83
Std. Dev.	0.71	0.56	0.19	0.63	0.27

Table 5.17: Normalized Capacities for Load Case No. 5 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P5-1	0.74	0.75	0.54	0.89	0.83
P5-2	0.73	0.76	0.53	0.89	0.82
P5-3	0.81	0.84	0.61	0.95	0.88
P5-4	0.86	0.93	0.67	0.98	0.93
P5-5	0.94	1.08	0.77	1.03	1.02
P5-6	0.98	1.21	0.86	1.04	1.04
P5-7	1.03	1.38	0.95	1.05	1.05
P5-8	1.07	1.59	1.05	1.06	1.06
P5-9	1.05	1.05	1.04	1.05	1.05
P5-10	1.10	1.74	1.14	1.08	1.08
P5-11	0.97	0.99	0.71	1.16	0.85
P5-12	0.96	0.99	0.70	1.16	0.84
P5-13	1.07	1.12	0.81	1.25	0.91
P5-14	1.14	1.24	0.90	1.30	0.96
P5-15	1.21	1.39	1.00	1.33	1.02
P5-16	1.27	1.57	1.11	1.34	1.09
P5-17	1.33	1.79	1.23	1.35	1.17
P5-18	1.38	1.97	1.35	1.36	1.26
P5-19	1.36	1.36	1.34	1.35	1.27
P5-20	1.43	1.69	1.48	1.38	1.38
P5-21	1.41	1.43	0.65	1.63	0.86
P5-22	1.40	1.44	0.65	1.63	0.85
P5-23	1.52	1.59	0.67	1.72	0.91
P5-24	1.62	1.76	1.27	1.79	0.95
P5-25	1.72	1.97	1.42	1.84	1.01
P5-26	1.80	2.23	1.58	1.87	1.07
P5-27	1.88	2.23	1.75	1.89	1.14
P5-28	1.96	1.86	1.92	1.89	1.22
P5-29	1.92	1.92	1.90	1.87	1.23
P5-30	2.02	1.59	2.09	1.90	1.31
P5-31	2.41	2.46	0.55	2.55	0.82
P5-32	2.40	2.47	0.55	2.56	0.80
P5-33	2.60	2.72	0.57	2.69	0.85
P5-34	2.78	3.02	0.58	2.81	0.89
P5-35	2.94	3.17	0.59	2.91	0.93
P5-36	3.09	2.39	0.60	2.98	0.98
P5-37	3.23	1.91	0.62	3.03	1.03
P5-38	3.35	1.59	0.63	3.07	1.08
P5-39	3.29	3.29	0.62	3.01	1.09
P5-40	3.47	1.37	0.64	3.04	1.14
Mean	1.71	1.70	0.97	1.74	1.02
Std. Dev.	0.82	0.65	0.44	0.73	0.15

Table 5.18: Normalized Capacities for Load Case No. 6 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P6-1	0.74	0.75	0.68	0.80	0.77
P6-2	0.23	0.23	0.23	0.36	0.36
P6-3	0.64	0.67	0.65	0.72	0.72
P6-4	0.55	0.60	0.64	0.68	0.67
P6-5	0.42	0.49	0.57	0.58	0.58
P6-6	0.33	0.40	0.39	0.48	0.48
P6-7	0.28	0.38	0.29	0.43	0.43
P6-8	0.22	0.33	0.21	0.36	0.36
P6-9	0.23	0.23	0.23	0.36	0.36
P6-10	0.19	0.23	0.16	0.30	0.30
P6-11	0.98	1.00	0.66	0.92	0.79
P6-12	0.23	0.23	0.24	0.36	0.36
P6-13	0.86	0.90	0.65	0.82	0.76
P6-14	0.69	0.75	0.60	0.69	0.68
P6-15	0.56	0.64	0.57	0.61	0.60
P6-16	0.42	0.52	0.52	0.52	0.51
P6-17	0.30	0.41	0.34	0.44	0.42
P6-18	0.23	0.34	0.23	0.37	0.36
P6-19	0.23	0.23	0.24	0.36	0.36
P6-20	0.19	0.23	0.17	0.31	0.31
P6-21	1.38	1.40	0.58	1.14	0.75
P6-22	0.27	0.27	0.30	0.34	0.33
P6-23	1.20	1.25	0.56	0.98	0.75
P6-24	1.02	1.11	0.58	0.84	0.72
P6-25	0.84	0.96	0.55	0.71	0.67
P6-26	0.61	0.75	0.49	0.56	0.55
P6-27	0.43	0.59	0.43	0.45	0.45
P6-28	0.28	0.41	0.30	0.34	0.33
P6-29	0.27	0.27	0.30	0.34	0.33
P6-30	0.20	0.24	0.19	0.31	0.26
P6-31	2.28	2.32	0.48	1.76	0.68
P6-32	0.42	0.42	0.27	0.35	0.34
P6-33	1.99	2.08	0.46	1.48	0.66
P6-34	1.68	1.83	0.44	1.22	0.66
P6-35	1.38	1.58	0.42	0.98	0.65
P6-36	1.09	1.35	0.42	0.78	0.61
P6-37	0.82	1.10	0.39	0.60	0.54
P6-38	0.50	0.74	0.31	0.39	0.38
P6-39	0.42	0.42	0.27	0.35	0.34
P6-40	0.26	0.32	0.21	0.23	0.23
Mean	0.65	0.72	0.41	0.62	0.51
Std. Dev.	0.52	0.54	0.16	0.34	0.17

Table 5.19: Normalized Capacities for Load Case No. 7 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P7-1	0.92	0.93	1.07	1.32	0.92
P7-2	0.92	0.93	0.80	1.84	1.18
P7-3	0.92	0.93	0.76	1.62	1.08
P7-4	0.92	0.93	0.72	1.44	1.00
P7-5	0.92	0.93	0.68	1.30	0.93
P7-6	0.94	1.00	1.11	1.33	1.33
P7-7	0.94	1.00	0.84	1.99	1.27
P7-8	0.94	1.00	0.80	1.75	1.17
P7-9	0.94	1.00	0.76	1.55	1.08
P7-10	0.94	1.00	0.72	1.40	1.00
P7-11	1.21	1.23	1.42	1.63	1.63
P7-12	1.21	1.23	1.06	2.41	2.71
P7-13	1.21	1.23	1.01	2.12	2.98
P7-14	1.21	1.23	0.96	1.89	3.27
P7-15	1.21	1.23	0.90	1.70	3.55
P7-16	1.24	1.32	1.46	1.64	1.64
P7-17	1.24	1.32	1.11	2.59	2.65
P7-18	1.24	1.32	1.05	2.28	2.91
P7-19	1.24	1.32	1.00	2.03	3.19
P7-20	1.24	1.32	0.95	1.83	3.50
P7-21	1.72	1.74	2.01	2.07	1.93
P7-22	1.72	1.74	1.51	3.18	3.18
P7-23	1.72	1.74	1.43	2.97	3.49
P7-24	1.72	1.74	0.78	2.65	3.86
P7-25	1.72	1.74	0.79	2.39	4.31
P7-26	1.75	1.86	2.07	2.08	1.91
P7-27	1.75	1.86	1.57	3.09	3.09
P7-28	1.75	1.86	1.49	3.19	3.36
P7-29	1.75	1.86	1.42	2.85	3.68
P7-30	1.75	1.86	0.79	2.57	4.05
P7-31	2.94	2.99	3.45	2.92	1.50
P7-32	2.94	2.99	0.63	3.30	2.06
P7-33	2.94	2.99	0.63	3.39	2.17
P7-34	2.94	2.99	0.63	3.48	2.28
P7-35	2.94	2.99	0.64	3.57	2.39
P7-36	3.01	3.19	3.54	2.90	1.49
P7-37	3.01	3.19	0.63	3.22	2.01
P7-38	3.01	3.19	0.63	3.30	2.11
P7-39	3.01	3.19	0.64	3.38	2.21
P7-40	3.01	3.19	0.64	3.46	2.30
Mean	1.72	1.78	1.13	2.39	2.31
Std. Dev.	0.79	0.83	0.66	0.74	0.99

Table 5.20: Normalized Capacities for Load Case No. 8 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P8-1	0.44	0.44	0.73	0.66	0.66
P8-2	0.68	0.69	1.03	1.14	1.14
P8-3	0.75	0.77	0.50	1.27	1.27
P8-4	0.82	0.83	0.48	1.29	1.38
P8-5	0.88	0.90	0.47	1.25	1.49
P8-6	0.42	0.44	0.76	0.64	0.64
P8-7	0.65	0.69	1.04	1.12	1.12
P8-8	0.72	0.77	0.50	1.25	1.25
P8-9	0.78	0.83	0.48	1.29	1.37
P8-10	0.84	0.90	0.47	1.26	1.48
P8-11	0.43	0.44	0.72	0.65	0.65
P8-12	0.89	0.91	1.02	1.36	1.48
P8-13	0.98	0.99	1.00	1.30	1.62
P8-14	1.06	1.08	0.98	1.26	1.76
P8-15	1.14	1.16	0.96	1.23	1.90
P8-16	0.42	0.44	0.74	0.63	0.63
P8-17	0.85	0.91	1.03	1.36	1.45
P8-18	0.93	0.99	1.01	1.31	1.60
P8-19	1.01	1.08	0.98	1.26	1.75
P8-20	1.09	1.16	0.97	1.23	1.89
P8-21	0.54	0.54	0.83	0.80	0.80
P8-22	1.21	1.28	0.98	1.30	2.06
P8-23	1.16	1.40	0.95	1.25	2.27
P8-24	1.13	1.52	0.93	1.21	2.48
P8-25	1.10	1.65	0.91	1.17	3.01
P8-26	0.51	0.54	0.85	0.77	0.77
P8-27	1.21	1.28	0.99	1.31	2.03
P8-28	1.17	1.40	0.96	1.25	2.24
P8-29	1.13	1.52	0.94	1.21	2.46
P8-30	1.11	1.65	0.92	1.18	2.67
P8-31	0.94	0.95	1.58	1.38	1.38
P8-32	1.03	2.20	0.85	1.03	4.15
P8-33	1.00	2.41	0.82	1.00	4.88
P8-34	0.97	2.61	0.80	0.97	5.78
P8-35	0.94	2.82	0.78	0.94	6.93
P8-36	0.90	0.95	1.60	1.33	1.33
P8-37	1.04	2.20	0.86	1.04	4.06
P8-38	1.00	2.41	0.83	1.00	4.78
P8-39	0.97	2.61	0.81	0.97	5.64
P8-40	0.95	2.82	0.79	0.95	6.71
Mean	0.90	1.28	0.87	1.12	2.32
Std. Dev.	0.23	0.71	0.24	0.22	1.70

Table 5.21: Normalized Capacities for Load Case No. 9 $(f'_c = 15 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P9-1	0.72	0.72	0.60	0.82	0.82
P9-2	0.63	0.63	0.51	0.73	0.73
P9-3	0.60	0.60	0.47	0.69	0.69
P9-4	0.58	0.58	0.36	0.66	0.66
P9-5	0.53	0.53	0.34	0.61	0.61
P9-6	0.47	0.47	0.32	0.54	0.54
P9-7	0.45	0.45	0.82	0.53	0.53
P9-8	0.44	0.44	0.90	0.53	0.53
P9-9	0.44	0.44	0.92	0.53	0.53
P9-10	0.45	0.45	0.91	0.55	0.55
P9-11	0.95	0.95	0.79	1.06	0.94
P9-12	0.82	0.82	0.67	0.95	0.86
P9-13	0.75	0.75	0.60	0.87	0.82
P9-14	0.63	0.63	0.56	0.73	0.73
P9-15	0.58	0.58	0.61	0.67	0.67
P9-16	0.53	0.53	0.63	0.62	0.62
P9-17	0.50	0.50	0.67	0.59	0.59
P9-18	0.46	0.46	0.68	0.54	0.54
P9-19	0.43	0.43	0.68	0.51	0.51
P9-20	0.40	0.40	0.66	0.48	0.48
P9-21	1.39	1.39	1.16	1.52	0.97
P9-22	1.25	1.25	1.01	1.39	0.91
P9-23	1.04	1.04	0.83	1.19	0.81
P9-24	0.90	0.90	0.53	1.04	0.76
P9-25	0.76	0.76	0.50	0.88	0.71
P9-26	0.64	0.64	0.48	0.73	0.64
P9-27	0.56	0.56	0.48	0.65	0.60
P9-28	0.52	0.52	0.50	0.60	0.59
P9-29	0.49	0.49	0.50	0.57	0.57
P9-30	0.47	0.47	0.50	0.55	0.55
P9-31	2.39	2.39	0.54	2.38	0.92
P9-32	2.14	2.14	0.52	2.19	0.87
P9-33	1.90	1.90	0.50	2.00	0.82
P9-34	1.66	1.66	0.48	1.79	0.78
P9-35	1.44	1.44	0.46	1.57	0.74
P9-36	1.24	1.24	0.44	1.34	0.72
P9-37	0.96	0.96	0.38	1.02	0.62
P9-38	0.78	0.78	0.34	0.83	0.57
P9-39	0.78	0.78	0.38	0.83	0.60
P9-40	0.68	0.68	0.34	0.72	0.53
Mean	0.83	0.83	0.59	0.93	0.68
Std. Dev.	0.49	0.49	0.20	0.49	0.14

Table 5.22: Normalized Capacities for Load Case No. 1 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P1-1	0.97	0.97	0.86	0.86	0.86
P1-2	1.00	1.00	0.92	0.88	0.88
P1-3	0.98	0.98	0.85	0.92	0.92
P1-4	0.82	0.82	1.03	0.95	0.95
P1-5	0.48	0.48	0.60	0.56	0.48
P1-6	0.63	0.63	0.92	0.76	0.76
P1-7	0.84	0.84	0.67	0.84	0.84
P1-8	0.85	0.85	0.74	1.19	1.19
P1-9	0.53	0.53	0.47	0.75	0.53
P1-10	0.53	0.53	0.85	0.89	0.89
P1-11	0.76	0.76	0.63	0.83	0.83
P1-12	0.76	0.76	0.74	0.88	0.88
P1-13	0.80	0.80	0.73	0.92	0.92
P1-14	0.80	0.80	0.84	1.14	1.22
P1-15	0.52	0.52	0.65	0.62	0.62
P1-16	0.68	0.68	0.99	0.83	0.83
P1-17	0.82	0.82	0.66	0.93	0.93
P1-18	0.82	0.82	0.95	1.21	1.59
P1-19	0.56	0.56	0.49	0.74	0.89
P1-20	0.56	0.56	0.89	0.96	0.96
P1-21	0.72	0.72	0.86	1.02	1.04
P1-22	0.74	0.74	0.78	1.03	1.08
P1-23	0.75	0.75	0.73	1.03	1.12
P1-24	0.75	0.75	0.82	1.21	1.47
P1-25	0.63	0.63	0.78	0.66	0.68
P1-26	0.73	0.73	1.20	1.00	1.00
P1-27	0.76	0.76	0.64	1.02	1.02
P1-28	0.76	0.76	0.80	1.21	1.94
P1-29	0.61	0.61	0.54	0.76	0.94
P1-30	0.57	0.57	0.90	1.00	1.00
P1-31	0.61	0.61	0.86	1.21	1.31
P1-32	0.62	0.62	0.75	1.17	1.36
P1-33	0.62	0.62	0.67	1.13	1.43
P1-34	0.62	0.62	0.74	1.23	1.82
P1-35	0.49	0.49	1.04	0.72	0.73
P1-36	0.55	0.55	1.81	1.22	1.22
P1-37	0.63	0.63	0.57	1.05	1.05
P1-38	0.63	0.63	0.69	1.13	2.54
P1-39	0.70	0.70	2.13	0.77	0.99
P1-40	0.58	0.58	0.48	1.12	1.12
Mean	0.69	0.69	0.83	0.96	1.07
Std. Dev.	0.13	0.13	0.31	0.19	0.39

Table 5.23: Normalized Capacities for Load Case No. 2 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P2-1	0.93	0.93	0.95	1.15	1.15
P2-2	0.99	0.99	0.99	1.17	1.17
P2-3	0.90	0.90	1.03	1.18	1.18
P2-4	0.82	0.82	1.06	1.17	1.17
P2-5	0.76	0.76	1.10	1.17	1.17
P2-6	0.70	0.70	1.13	1.15	1.15
P2-7	0.65	0.65	1.16	1.13	1.13
P2-8	0.62	0.62	1.22	1.12	1.12
P2-9	0.53	0.53	1.14	1.00	1.00
P2-10	0.86	0.86	2.01	1.65	1.65
P2-11	0.87	0.87	0.91	1.29	1.40
P2-12	0.98	0.98	0.97	1.36	1.43
P2-13	1.07	1.07	1.00	1.42	1.42
P2-14	1.05	1.05	1.05	1.45	1.45
P2-15	0.97	0.97	1.09	1.45	1.45
P2-16	0.90	0.90	1.13	1.44	1.44
P2-17	0.76	0.76	1.05	1.29	1.29
P2-18	0.72	0.72	1.11	1.28	1.28
P2-19	0.54	0.54	0.90	0.98	0.98
P2-20	1.20	1.20	2.19	2.27	2.27
P2-21	0.81	0.81	0.89	1.30	1.85
P2-22	0.91	0.91	0.94	1.37	1.88
P2-23	1.01	1.01	0.99	1.45	1.82
P2-24	1.11	1.11	1.03	1.52	1.94
P2-25	1.22	1.22	1.08	1.61	1.95
P2-26	1.27	1.27	1.11	1.68	1.94
P2-27	1.05	1.05	1.03	1.57	1.72
P2-28	1.04	1.04	1.12	1.75	1.78
P2-29	0.60	0.60	0.71	1.07	1.07
P2-30	0.57	0.57	0.73	1.05	1.05
P2-31	0.70	0.70	0.82	1.31	1.27
P2-32	0.79	0.79	0.86	1.36	1.36
P2-33	0.87	0.87	0.90	1.42	1.39
P2-34	0.95	0.95	0.94	1.48	1.47
P2-35	1.03	1.03	0.98	1.54	1.52
P2-36	1.11	1.11	1.01	1.60	1.56
P2-37	1.19	1.19	1.05	1.66	1.61
P2-38	1.15	1.15	0.98	1.57	1.57
P2-39	1.28	1.28	1.05	1.69	1.68
P2-40	0.93	0.93	0.74	1.21	1.19
Mean	0.91	0.91	1.05	1.38	1.45
Std. Dev.	0.21	0.21	0.27	0.25	0.32

Table 5.24: Normalized Capacities for Load Case No. 3 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P3-1	0.30	0.30	0.38	0.41	0.43
P3-2	0.82	0.82	1.30	0.84	0.84
P3-3	0.79	0.79	1.27	1.07	1.07
P3-4	0.60	0.60	0.72	0.96	1.07
P3-5	0.64	0.64	0.80	1.18	1.32
P3-6	0.81	0.81	1.12	0.78	0.78
P3-7	0.67	0.67	1.40	1.10	1.10
P3-8	0.74	0.74	0.97	1.77	1.78
P3-9	0.90	0.90	1.19	0.85	0.85
P3-10	0.65	0.65	1.62	1.17	1.17
P3-11	0.34	0.34	0.81	0.45	0.55
P3-12	0.80	0.80	0.78	1.02	1.02
P3-13	0.90	0.90	1.14	1.18	1.18
P3-14	0.63	0.63	0.75	0.97	1.32
P3-15	0.70	0.70	0.87	1.24	1.70
P3-16	0.79	0.79	0.79	0.99	1.01
P3-17	0.76	0.76	1.25	1.21	1.21
P3-18	0.96	0.96	1.26	2.25	2.37
P3-19	0.71	0.71	0.72	0.88	0.88
P3-20	0.67	0.67	1.31	1.17	1.17
P3-21	0.39	0.39	0.50	0.48	0.76
P3-22	0.75	0.75	0.78	1.12	1.26
P3-23	1.02	1.02	1.18	1.63	1.63
P3-24	0.74	0.74	0.47	1.08	2.04
P3-25	0.99	0.99	0.54	1.69	2.79
P3-26	0.74	0.74	0.80	1.10	1.19
P3-27	1.10	1.10	1.30	1.69	1.69
P3-28	1.44	1.44	0.64	3.24	3.25
P3-29	0.56	0.56	0.61	0.82	0.88
P3-30	0.77	0.77	1.09	1.31	1.31
P3-31	0.46	0.46	0.23	0.52	1.52
P3-32	0.62	0.62	0.72	1.19	1.57
P3-33	0.85	0.85	1.12	1.71	2.41
P3-34	1.34	1.34	0.37	1.84	2.90
P3-35	1.85	1.85	0.45	2.97	2.38
P3-36	0.62	0.62	0.75	1.20	1.49
P3-37	1.21	1.21	1.23	1.96	2.58
P3-38	2.38	2.38	0.49	3.96	1.84
P3-39	0.42	0.42	0.51	0.82	0.98
P3-40	1.01	1.01	0.89	1.48	1.80
Mean	0.84	0.84	0.88	1.33	1.48
Std. Dev.	0.39	0.39	0.34	0.73	0.67

Table 5.25: Normalized Capacities for Load Case No. 4 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P4-1	1.09	1.31	0.93	1.00	0.88
P4-2	1.12	1.40	0.94	1.00	0.88
P4-3	1.15	1.50	0.94	1.00	0.88
P4-4	1.18	1.59	0.95	1.00	0.88
P4-5	1.12	1.44	0.95	1.01	0.83
P4-6	1.08	1.38	0.90	0.95	0.82
P4-7	1.04	1.28	0.88	0.94	0.81
P4-8	1.17	1.55	0.95	1.00	0.87
P4-9	0.93	1.20	0.75	0.83	0.81
P4-10	0.95	1.16	0.72	0.86	0.80
P4-11	1.28	1.54	0.68	1.05	0.79
P4-12	1.31	1.64	0.69	1.05	0.79
P4-13	1.35	1.75	0.70	1.05	0.79
P4-14	1.37	1.86	0.70	1.05	0.79
P4-15	1.44	1.86	0.71	1.21	0.82
P4-16	1.37	1.75	0.70	1.10	0.80
P4-17	1.34	1.64	0.69	1.10	0.80
P4-18	1.40	1.86	0.71	1.10	0.80
P4-19	1.19	1.54	0.71	0.93	0.81
P4-20	1.25	1.54	0.68	1.00	0.78
P4-21	1.79	2.15	0.64	1.37	0.77
P4-22	1.84	2.29	0.64	1.38	0.76
P4-23	1.88	2.17	0.65	1.38	0.76
P4-24	1.92	1.98	0.66	1.39	0.76
P4-25	2.02	1.98	0.66	1.61	0.79
P4-26	1.91	2.17	0.65	1.45	0.77
P4-27	1.87	2.29	0.64	1.45	0.77
P4-28	1.96	1.98	0.66	1.46	0.77
P4-29	1.67	2.15	0.63	1.18	0.73
P4-30	1.75	2.15	0.64	1.30	0.75
P4-31	2.97	2.37	0.53	2.20	0.69
P4-32	3.05	2.03	0.53	2.21	0.69
P4-33	3.12	1.80	0.54	2.22	0.68
P4-34	3.19	1.64	0.54	2.24	0.68
P4-35	3.35	1.64	0.54	2.61	0.71
P4-36	3.18	1.80	0.54	2.34	0.69
P4-37	3.10	2.03	0.53	2.33	0.70
P4-38	3.24	1.64	0.54	2.35	0.69
P4-39	2.77	2.37	0.52	1.88	0.66
P4-40	2.90	2.37	0.52	2.08	0.68
Mean	1.84	1.79	0.69	1.42	0.77
Std. Dev.	0.79	0.34	0.14	0.52	0.06

Table 5.26: Normalized Capacities for Load Case No. 5 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P5-1	0.97	0.98	0.91	1.04	0.85
P5-2	0.96	0.99	0.90	1.01	0.84
P5-3	1.04	1.09	1.01	1.18	0.89
P5-4	1.11	1.21	1.13	1.37	0.93
P5-5	1.18	1.35	1.26	1.47	0.99
P5-6	1.24	1.53	1.40	1.47	1.06
P5-7	1.29	1.74	1.55	1.47	1.14
P5-8	1.34	1.97	1.70	1.48	1.24
P5-9	1.32	1.32	1.68	1.47	1.25
P5-10	1.39	1.69	1.85	1.51	1.37
P5-11	1.24	1.26	0.67	1.23	0.84
P5-12	1.23	1.27	0.67	1.19	0.83
P5-13	1.34	1.40	0.68	1.42	0.88
P5-14	1.43	1.55	0.70	1.68	0.93
P5-15	1.51	1.74	0.72	1.87	0.98
P5-16	1.59	1.96	1.79	1.88	1.05
P5-17	1.66	2.23	1.99	1.88	1.13
P5-18	1.72	1.90	2.18	1.88	1.22
P5-19	1.69	1.69	2.16	1.87	1.23
P5-20	1.78	1.63	2.38	1.90	1.33
P5-21	1.73	1.77	0.62	1.62	0.82
P5-22	1.72	1.77	0.62	1.56	0.81
P5-23	1.87	1.95	0.63	1.90	0.86
P5-24	2.00	2.17	0.65	2.26	0.91
P5-25	2.11	2.43	0.66	2.54	0.96
P5-26	2.22	2.66	0.68	2.56	1.02
P5-27	2.32	2.13	0.69	2.57	1.08
P5-28	2.41	1.77	0.70	2.57	1.16
P5-29	2.37	2.37	0.70	2.54	1.17
P5-30	2.49	1.52	3.33	2.57	1.25
P5-31	2.88	2.93	0.51	2.57	0.75
P5-32	2.86	2.94	0.51	2.49	0.74
P5-33	3.10	3.24	0.52	3.05	0.79
P5-34	3.31	3.60	0.53	3.66	0.82
P5-35	3.51	2.92	0.55	3.85	0.86
P5-36	3.68	2.21	0.56	3.93	0.90
P5-37	3.84	1.77	0.57	3.97	0.95
P5-38	3.99	1.47	0.58	3.99	1.00
P5-39	3.93	3.93	0.57	3.92	1.01
P5-40	4.13	1.26	0.59	3.94	1.05
Mean	2.09	1.93	1.05	2.21	1.00
Std. Dev.	0.95	0.70	0.67	0.93	0.17

Table 5.27: Normalized Capacities for Load Case No. 6 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P6-1	0.74	0.75	0.68	0.80	0.77
P6-2	0.23	0.23	0.23	0.36	0.36
P6-3	0.64	0.67	0.65	0.72	0.72
P6-4	0.55	0.60	0.64	0.68	0.67
P6-5	0.42	0.49	0.57	0.58	0.58
P6-6	0.33	0.40	0.39	0.48	0.48
P6-7	0.28	0.38	0.29	0.43	0.43
P6-8	0.22	0.33	0.21	0.36	0.36
P6-9	0.23	0.23	0.23	0.36	0.36
P6-10	0.19	0.23	0.16	0.30	0.30
P6-11	0.98	1.00	0.66	0.92	0.79
P6-12	0.23	0.23	0.24	0.36	0.36
P6-13	0.86	0.90	0.65	0.82	0.76
P6-14	0.69	0.75	0.60	0.69	0.68
P6-15	0.56	0.64	0.57	0.61	0.60
P6-16	0.42	0.52	0.52	0.52	0.51
P6-17	0.30	0.41	0.34	0.44	0.42
P6-18	0.23	0.34	0.23	0.37	0.36
P6-19	0.23	0.23	0.24	0.36	0.36
P6-20	0.19	0.23	0.17	0.31	0.31
P6-21	1.38	1.40	0.58	1.14	0.75
P6-22	0.27	0.27	0.30	0.34	0.33
P6-23	1.20	1.25	0.56	0.98	0.75
P6-24	1.02	1.11	0.58	0.84	0.72
P6-25	0.84	0.96	0.55	0.71	0.67
P6-26	0.61	0.75	0.49	0.56	0.55
P6-27	0.43	0.59	0.43	0.45	0.45
P6-28	0.28	0.41	0.30	0.34	0.33
P6-29	0.27	0.27	0.30	0.34	0.33
P6-30	0.20	0.24	0.19	0.31	0.26
P6-31	2.28	2.32	0.48	1.76	0.68
P6-32	0.42	0.42	0.27	0.35	0.34
P6-33	1.99	2.08	0.46	1.48	0.66
P6-34	1.68	1.83	0.44	1.22	0.66
P6-35	1.38	1.58	0.42	0.98	0.65
P6-36	1.09	1.35	0.42	0.78	0.61
P6-37	0.82	1.10	0.39	0.60	0.54
P6-38	0.50	0.74	0.31	0.39	0.38
P6-39	0.42	0.42	0.27	0.35	0.34
P6-40	0.26	0.32	0.21	0.23	0.23
Mean	0.65	0.72	0.41	0.62	0.51
Std. Dev.	0.52	0.54	0.16	0.34	0.17

Table 5.28: Normalized Capacities for Load Case No. 7 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P7-1	1.18	1.20	1.78	1.96	1.96
P7-2	1.18	1.20	1.33	1.83	1.18
P7-3	1.18	1.20	1.27	1.61	1.08
P7-4	1.18	1.20	1.20	1.43	1.00
P7-5	1.18	1.20	1.14	1.29	0.92
P7-6	1.20	1.28	1.83	1.96	1.96
P7-7	1.20	1.28	1.39	1.97	1.26
P7-8	1.20	1.28	1.32	1.73	1.16
P7-9	1.20	1.28	1.25	1.54	1.07
P7-10	1.20	1.28	1.19	1.39	0.99
P7-11	1.51	1.54	2.29	2.31	2.15
P7-12	1.51	1.54	1.71	2.33	4.05
P7-13	1.51	1.54	0.82	2.05	4.52
P7-14	1.51	1.54	0.83	1.83	5.05
P7-15	1.51	1.54	0.84	1.65	5.60
P7-16	1.55	1.64	2.35	2.31	2.12
P7-17	1.55	1.64	1.78	2.51	3.91
P7-18	1.55	1.64	0.82	2.20	4.35
P7-19	1.55	1.64	0.83	1.96	4.85
P7-20	1.55	1.64	0.84	1.77	5.40
P7-21	2.11	2.15	3.20	2.84	1.84
P7-22	2.11	2.15	0.73	3.22	3.32
P7-23	2.11	2.15	0.74	2.83	3.73
P7-24	2.11	2.15	0.74	2.53	4.21
P7-25	2.11	2.15	0.75	2.28	4.80
P7-26	2.16	2.29	3.28	2.84	1.82
P7-27	2.16	2.29	0.73	3.47	3.13
P7-28	2.16	2.29	0.74	3.04	3.47
P7-29	2.16	2.29	0.75	2.71	3.86
P7-30	2.16	2.29	0.75	2.45	4.32
P7-31	3.50	3.56	5.30	3.80	1.39
P7-32	3.50	3.56	0.58	4.32	1.91
P7-33	3.50	3.56	0.58	4.44	2.00
P7-34	3.50	3.56	0.58	4.14	2.11
P7-35	3.50	3.56	0.59	3.74	2.21
P7-36	3.58	3.81	5.45	3.77	1.38
P7-37	3.58	3.81	0.58	4.22	1.86
P7-38	3.58	3.81	0.58	4.33	1.95
P7-39	3.58	3.81	0.59	4.44	2.04
P7-40	3.58	3.81	0.59	4.02	2.13
Mean	2.10	2.18	1.37	2.68	2.70
Std. Dev.	0.91	0.95	1.15	0.98	1.43

Table 5.29: Normalized Capacities for Load Case No. 8 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P8-1	0.41	0.42	1.65	0.80	0.80
P8-2	0.87	0.88	1.02	1.35	1.86
P8-3	0.95	0.96	0.99	1.30	2.03
P8-4	1.03	1.05	0.97	1.25	2.18
P8-5	1.11	1.13	0.96	1.22	2.33
P8-6	0.39	0.42	1.67	0.77	0.77
P8-7	0.83	0.88	1.02	1.35	1.83
P8-8	0.91	0.96	1.00	1.30	2.01
P8-9	0.98	1.05	0.98	1.26	2.17
P8-10	1.06	1.13	0.96	1.22	2.33
P8-11	0.47	0.47	1.48	0.90	0.90
P8-12	1.11	1.13	0.99	1.31	2.35
P8-13	1.19	1.24	0.97	1.26	2.56
P8-14	1.15	1.34	0.95	1.22	2.76
P8-15	1.12	1.45	0.93	1.19	2.95
P8-16	0.45	0.47	1.50	0.86	0.86
P8-17	1.06	1.13	1.00	1.32	2.31
P8-18	1.16	1.24	0.97	1.27	2.54
P8-19	1.16	1.34	0.95	1.22	2.75
P8-20	1.13	1.45	0.93	1.19	2.95
P8-21	0.67	0.69	1.54	1.29	1.29
P8-22	1.15	1.58	0.93	1.24	3.22
P8-23	1.11	1.73	0.91	1.19	3.52
P8-24	1.07	1.88	0.89	1.15	3.81
P8-25	1.05	2.03	0.87	1.12	4.65
P8-26	0.64	0.68	1.54	1.22	1.22
P8-27	1.16	1.58	0.94	1.25	3.19
P8-28	1.12	1.73	0.91	1.20	3.50
P8-29	1.08	1.88	0.89	1.15	3.81
P8-30	1.05	2.03	0.87	1.12	4.11
P8-31	1.12	1.14	1.45	2.12	2.12
P8-32	0.96	2.62	0.79	0.96	6.41
P8-33	0.92	2.87	0.76	0.92	7.55
P8-34	0.89	3.11	0.74	0.89	8.94
P8-35	0.87	3.36	0.72	0.87	10.66
P8-36	1.07	1.14	1.47	2.03	2.03
P8-37	0.96	2.62	0.79	0.96	6.24
P8-38	0.93	2.87	0.77	0.93	7.35
P8-39	0.90	3.11	0.74	0.90	8.68
P8-40	0.87	3.36	0.73	0.87	10.32
Mean	0.95	1.55	1.03	1.17	3.60
Std. Dev.	0.22	0.84	0.27	0.27	2.62

Table 5.30: Normalized Capacities for Load Case No. 9 $(f'_c = 25 \text{ MPa})$

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
P9-1	0.95	0.95	1.03	1.25	0.95
P9-2	0.82	0.82	0.86	1.08	0.85
P9-3	0.72	0.72	0.74	0.92	0.79
P9-4	0.62	0.62	0.61	0.79	0.74
P9-5	0.52	0.52	0.58	0.69	0.68
P9-6	0.45	0.45	0.56	0.62	0.62
P9-7	0.41	0.41	0.58	0.58	0.58
P9-8	0.37	0.37	0.58	0.53	0.53
P9-9	0.36	0.36	0.58	0.52	0.52
P9-10	0.36	0.36	0.55	0.52	0.52
P9-11	1.23	1.23	0.66	1.58	0.95
P9-12	1.10	1.10	0.64	1.32	0.88
P9-13	0.98	0.98	0.62	1.11	0.83
P9-14	0.81	0.81	0.56	0.90	0.75
P9-15	0.68	0.68	0.55	0.76	0.69
P9-16	0.57	0.57	0.52	0.66	0.63
P9-17	0.47	0.47	0.49	0.57	0.57
P9-18	0.42	0.42	0.49	0.54	0.54
P9-19	0.39	0.39	0.49	0.53	0.53
P9-20	0.38	0.38	0.49	0.52	0.52
P9-21	1.71	1.71	0.61	2.13	0.93
P9-22	1.54	1.54	0.59	1.71	0.87
P9-23	1.36	1.36	0.57	1.41	0.82
P9-24	1.20	1.20	0.54	1.18	0.78
P9-25	1.04	1.04	0.52	1.01	0.75
P9-26	0.83	0.83	0.49	0.80	0.66
P9-27	0.69	0.69	0.46	0.68	0.60
P9-28	0.59	0.59	0.44	0.59	0.56
P9-29	0.52	0.52	0.42	0.54	0.52
P9-30	0.48	0.48	0.40	0.50	0.49
P9-31	2.84	2.84	0.50	3.20	0.85
P9-32	2.55	2.55	0.48	2.70	0.80
P9-33	2.26	2.26	0.46	2.19	0.76
P9-34	1.98	1.98	0.45	1.79	0.72
P9-35	1.71	1.71	0.42	1.48	0.69
P9-36	1.48	1.48	0.40	1.24	0.66
P9-37	1.27	1.27	0.39	1.05	0.64
P9-38	1.09	1.09	0.37	0.89	0.61
P9-39	0.99	0.99	0.37	0.81	0.59
P9-40	0.95	0.95	0.37	0.77	0.58
Mean	0.99	0.99	0.54	1.07	0.69
Std. Dev.	0.62	0.62	0.13	0.63	0.14

Table 5.31: Mean Normalized Capacities for Load Case No. 1

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f'_c = 15 \text{ MPa}$	0.72	0.72	0.76	0.78	0.78
	$f'_c = 25 \text{ MPa}$	0.76	0.76	0.79	0.86	0.83
	15 MPa / 25 MPa	0.95	0.95	0.96	0.91	0.94
RR2	$f'_c = 15 \text{ MPa}$	0.83	0.83	0.69	0.82	0.82
	$f'_c = 25 \text{ MPa}$	0.71	0.71	0.76	0.91	0.97
	15 MPa / 25 MPa	1.17	1.17	0.91	0.90	0.85
RR3	$f'_c = 15 \text{ MPa}$	0.80	0.80	0.75	0.98	1.01
	$f'_c = 25 \text{ MPa}$	0.70	0.70	0.80	1.00	1.13
	15 MPa / 25 MPa	1.14	1.14	0.93	0.99	0.89
RR4	$f'_c = 15 \text{ MPa}$	0.69	0.69	0.80	1.15	1.41
	$f'_c = 25 \text{ MPa}$	0.60	0.60	0.97	1.07	1.36
	15 MPa / 25 MPa	1.14	1.14	0.82	1.07	1.04

Table 5.32: Mean Normalized Capacities for Load Case No.2

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f_c = 15 \text{ MPa}$	0.74	0.74	1.29	0.89	0.89
	$f_c = 25 \text{ MPa}$	0.77	0.77	1.18	1.19	1.19
	15 MPa / 25 MPa	0.96	0.96	1.10	0.75	0.75
RR2	$f_c = 15 \text{ MPa}$	0.82	0.82	1.21	0.98	0.98
	$f_c = 25 \text{ MPa}$	0.91	1.14	1.42	1.44	1.00
	15 MPa / 25 MPa	0.90	0.72	0.85	0.68	0.98
RR3	$f_c = 15 \text{ MPa}$	0.97	0.97	1.13	1.24	1.24
	$f_c = 25 \text{ MPa}$	0.96	0.96	0.96	1.44	1.71
	15 MPa / 25 MPa	1.01	1.01	1.18	0.86	0.72
RR4	$f_c = 15 \text{ MPa}$	0.82	0.82	1.21	0.98	0.98
	$f_c = 25 \text{ MPa}$	1.00	1.00	0.93	1.48	1.46
	15 MPa / 25 MPa	0.82	0.82	1.30	0.66	0.67

Table 5.33: Mean Normalized Capacities for Load Case No.3

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f_c = 15 \text{ MPa}$	1.03	1.03	1.32	1.08	1.08
	$f_c = 25 \text{ MPa}$	0.69	0.69	1.08	1.01	1.04
	15 MPa / 25 MPa	1.48	1.48	1.22	1.07	1.04
RR2	$f_c = 15 \text{ MPa}$	1.15	1.15	1.43	1.32	1.32
	$f_c = 25 \text{ MPa}$	0.72	0.72	0.97	1.13	1.24
	15 MPa / 25 MPa	1.59	1.59	1.47	1.16	1.06
RR3	$f_c = 15 \text{ MPa}$	0.95	0.95	1.13	1.38	1.40
	$f_c = 25 \text{ MPa}$	0.86	0.86	0.76	1.42	1.76
	15 MPa / 25 MPa	1.11	1.11	1.48	0.97	0.80
RR4	$f_c = 15 \text{ MPa}$	1.08	1.08	0.81	1.76	1.85
	$f_c = 25 \text{ MPa}$	1.08	1.08	0.68	1.77	1.95
	15 MPa / 25 MPa	1.01	1.01	1.21	1.00	0.95

Table 5.34: Mean Normalized Capacities for Load Case No.4

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f_c = 15 \text{ MPa}$	0.99	1.28	0.61	1.12	0.97
	$f_c = 25 \text{ MPa}$	1.10	1.40	0.87	0.97	0.84
	15 MPa / 25 MPa	0.90	0.92	0.70	1.16	1.16
RR2	$f_c = 15 \text{ MPa}$	1.07	1.36	0.65	1.10	0.81
	$f_c = 25 \text{ MPa}$	1.30	1.65	0.70	1.05	0.80
	15 MPa / 25 MPa	0.82	0.82	0.93	1.05	1.02
RR3	$f_c = 15 \text{ MPa}$	1.51	1.91	0.68	1.47	0.80
	$f_c = 25 \text{ MPa}$	1.80	2.08	0.65	1.36	0.77
	15 MPa / 25 MPa	0.84	0.92	1.04	1.08	1.05
RR4	$f_c = 15 \text{ MPa}$	2.59	2.13	0.58	2.43	0.74
	$f_c = 25 \text{ MPa}$	2.96	1.99	0.54	2.16	0.69
	15 MPa / 25 MPa	0.87	1.07	1.06	1.13	1.07

Table 5.35: Mean Normalized Capacities for Load Case No.5

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f'_c = 15 \text{ MPa}$	0.93	1.13	0.82	1.00	0.98
	$f'_c = 25 \text{ MPa}$	1.18	1.39	1.34	1.35	1.06
	15 MPa / 25 MPa	0.79	0.82	0.61	0.74	0.92
RR2	$f'_c = 15 \text{ MPa}$	1.21	1.41	1.06	1.30	1.07
	$f'_c = 25 \text{ MPa}$	1.52	1.66	1.39	1.68	1.04
	15 MPa / 25 MPa	0.80	0.85	0.76	0.77	1.03
RR3	$f'_c = 15 \text{ MPa}$	1.73	1.80	1.39	1.80	1.05
	$f'_c = 25 \text{ MPa}$	2.12	2.05	0.93	2.27	1.01
	15 MPa / 25 MPa	0.81	0.88	1.50	0.79	1.05
RR4	$f'_c = 15 \text{ MPa}$	2.96	2.44	0.59	2.86	0.96
	$f'_c = 25 \text{ MPa}$	3.52	2.63	0.55	3.54	0.89
	15 MPa / 25 MPa	0.84	0.93	1.08	0.81	1.08

Table 5.36: Mean Normalized Capacities for Load Case No.6

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f'_c = 15 \text{ MPa}$	0.38	0.43	0.40	0.51	0.50
	$f'_c = 25 \text{ MPa}$	0.38	0.43	0.40	0.51	0.50
	15 MPa / 25 MPa	1.00	1.00	1.00	1.00	1.00
RR2	$f'_c = 15 \text{ MPa}$	0.47	0.53	0.42	0.54	0.51
	$f'_c = 25 \text{ MPa}$	0.47	0.53	0.42	0.54	0.51
	15 MPa / 25 MPa	1.00	1.00	1.00	1.00	1.00
RR3	$f'_c = 15 \text{ MPa}$	0.65	0.72	0.43	0.60	0.51
	$f'_c = 25 \text{ MPa}$	0.72	0.43	0.60	0.51	1.00
	15 MPa / 25 MPa	0.90	1.69	0.71	1.17	0.51
RR4	$f'_c = 15 \text{ MPa}$	1.08	1.22	0.37	0.81	0.51
	$f'_c = 25 \text{ MPa}$	1.08	1.22	0.37	0.81	0.51
	15 MPa / 25 MPa	1.00	1.00	1.00	1.00	1.00

Table 5.37: Mean Normalized Capacities for Load Case No.7

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f_c = 15 \text{ MPa}$	0.93	0.97	0.83	1.55	1.10
	$f_c = 25 \text{ MPa}$	1.19	1.24	1.37	1.67	1.26
	15 MPa / 25 MPa	0.78	0.78	0.60	0.93	0.87
RR2	$f_c = 15 \text{ MPa}$	1.22	1.27	1.09	2.01	2.80
	$f_c = 25 \text{ MPa}$	1.53	1.59	1.31	2.09	4.20
	15 MPa / 25 MPa	0.80	0.80	0.83	0.96	0.67
RR3	$f_c = 15 \text{ MPa}$	1.74	1.80	1.38	2.70	3.29
	$f_c = 25 \text{ MPa}$	2.14	2.22	1.24	2.82	3.45
	15 MPa / 25 MPa	0.81	0.81	1.12	0.96	0.95
RR4	$f_c = 15 \text{ MPa}$	2.97	3.09	1.21	3.29	2.05
	$f_c = 25 \text{ MPa}$	3.54	3.68	1.54	4.12	1.90
	15 MPa / 25 MPa	0.84	0.84	0.78	0.80	1.08

Table 5.38: Mean Normalized Capacities for Load Case No.8

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f_c = 15 \text{ MPa}$	0.70	0.73	0.65	1.12	1.18
	$f_c = 25 \text{ MPa}$	0.82	0.85	1.16	1.16	1.75
	15 MPa / 25 MPa	0.85	0.85	0.56	0.97	0.68
RR2	$f_c = 15 \text{ MPa}$	0.88	0.92	0.94	1.16	1.47
	$f_c = 25 \text{ MPa}$	1.01	1.13	1.06	1.18	2.30
	15 MPa / 25 MPa	0.88	0.81	0.89	0.98	0.64
RR3	$f_c = 15 \text{ MPa}$	1.03	1.28	0.93	1.15	2.08
	$f_c = 25 \text{ MPa}$	1.02	1.57	1.02	1.19	3.21
	15 MPa / 25 MPa	1.00	0.82	0.91	0.96	0.65
RR4	$f_c = 15 \text{ MPa}$	0.97	2.20	0.97	1.06	4.56
	$f_c = 25 \text{ MPa}$	0.96	2.57	0.89	1.14	6.77
	15 MPa / 25 MPa	1.02	0.86	1.09	0.93	0.67

Table 5.39: Mean Normalized Capacities for Load Case No.9

	Con. Strength	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.
RR1	$f_c = 15 \text{ MPa}$	0.53	0.53	0.61	0.62	0.62
	$f_c = 25 \text{ MPa}$	0.56	0.56	0.67	0.75	0.68
	15 MPa / 25 MPa	0.95	0.95	0.92	0.83	0.91
RR2	$f_c = 15 \text{ MPa}$	0.59	0.59	0.68	0.69	0.66
	$f_c = 25 \text{ MPa}$	0.70	0.70	0.55	0.85	0.69
	15 MPa / 25 MPa	0.84	0.84	1.23	0.81	0.97
RR3	$f_c = 15 \text{ MPa}$	0.77	0.77	0.65	0.87	0.69
	$f_c = 25 \text{ MPa}$	1.00	1.00	0.50	1.06	0.70
	15 MPa / 25 MPa	0.77	0.77	1.29	0.83	0.99
RR4	$f_c = 15 \text{ MPa}$	1.31	1.31	0.44	1.38	0.70
	$f_c = 25 \text{ MPa}$	1.71	1.71	0.42	1.61	0.69
	15 MPa / 25 MPa	0.77	0.77	1.05	0.86	1.02

Table 5.40: Proposed Methods* for the Analysis of Reinforced Concrete Shell Elements

Based on the Load Case and Reinforcement Ratio

	RR1	RR2	RR3	RR4
LR1	All	All	All	All
LR2	N, C, Z, MZ	N, C, MZ	N, C	N, C
LR3	N, C, Z, MZ	N, C	N, C, F ²	N, C, F
LR4	N, F, Z, MZ	F, Z, MZ	F, MZ	F, MZ
LR5	N ² , MZ	MZ	MZ	MZ
LR6	All ¹	All ¹	All	N, Z
LR7	None	None	None	None
LR8	N, C	N, C	N	N
LR9	All ¹	All ¹	All ¹	None

(1) Oversafe results can be obtained

(2) Unsafe results can be obtained

(*) N: Nielsen, C: Clark, F: Fialkow, Z: Zararis, MZ: Modified Zararis

While preparing Table 5.40, the main criterion has been to determine the method(s) for each case which would give accurate results without leading to unsafe design. Unfortunately, it is not possible to single out a specific method as the best among the others. As it can be seen from the table, only “Load Case No. 1” can be considered as the case for which all methods are comparable with the modified compression field theory. For other cases, the proposed methods can be helpful.

Due to the large number of cases solved in the parametric study, for practical reasons, the results obtained from the methods are presented in tabular format. Only the results of two cases, Panel P1-1: Load Case No.1(25 MPa concrete), and Panel P7-33: Load Case No.7 (25 MPa concrete) are presented graphically in Fig. 5.2 and Fig. 5.3, respectively. As it can be noticed in Fig. 5.2, the methods under investigation gave accurate results for Panel P1-1, which represents the case with symmetrical loading and low level of orthotropy. On the other hand, Fig. 5.3 shows that the capacity of Panel P7-33, which represents the case with unsymmetrical loading and highly orthotropic reinforcement, could not be predicted accurately by any of those methods of analysis.

It can be observed from the Tables 5.13 to 5.30 that for some cases the results are quite inaccurate. For instance, the normalized capacity of Nielsen's and Clark's methods for the Panel P5-39 (25 MPa concrete) is 3.93. Similarly, for the Panel P3-38 (25 MPa concrete) the normalized capacity is calculated as 3.96 by Zararis' method. For the Panels P8-40 (15 MPa concrete) and P-7.31 (25 MPa concrete), the normalized capacities calculated by Modified Zararis' and Fialkow methods range from 5 to 7. It has to be noted that most of the significantly inaccurate results are obtained in the cases which are either highly unsymmetrical loaded or are highly orthotropically reinforced. This shows that individual results may vary considerably depending on the loading and the reinforcement patterns of the reinforced concrete shell elements.

Fig. 5.2: Comparison of Existing Methods with MCFT (Panel P1-6)

Fig. 5.3: Comparison of Existing Methods with MCFT (Panel P7-33)

Chapter 6

Conclusions and Recommendations

6.1 Summary

This thesis attempted to compare the existing methods of analysis of orthogonally reinforced concrete shell elements with each other. First, these methods are compared with the available experimental data. Since the number of experimental results is not enough for a definitive comparison, a parametric study was carried out. A Windows based computer program is designed to implement the existing methods of analysis. The modified compression field theory is used as a reference tool in the parametric study. The load ratio, reinforcement ratio, and concrete strength are used as the variables of the study. Nine load cases, which were developed by Clark (1974), four different reinforcement ratio combinations, and four concrete strengths are used to define the test

panels. A total of 720 panels were investigated. Based on the results of the parametric study, different methods are identified as analysis method(s) for different cases of loading and reinforcement ratio. More specifically, following conclusions are drawn.

6.2 Conclusions

The present study support the following conclusions:

1. For normal load conditions, which are bounded by Load Case No. 1 of Clark's 'boundary case graph', all of the existing methods of analysis give safe and accurate results. Zararis' and Modified Zararis' methods are observed to give the best results for this case.
2. The accuracy of the existing methods of analysis is lost significantly when the biaxial loading becomes highly unsymmetrical and severe.
3. Within the same load case, as the reinforcement ratio in one direction becomes significantly different from the other direction, the accuracy of the methods suffer greatly.
4. One of the possible reasons of the first 3 conclusions may be due to the fact that in the existing methods of analysis, the angle of crack is calculated by using a one-step

procedures instead of an iterative process like the one used in the modified compression field theory. Since the angle of crack depends on load, and reinforcement ratios, any extreme changes in one of these parameters results in a loss of accuracy.

5. A change in concrete strength affects Nielsen's and Clark's methods more than the others. This is due to the fact that the model used for the behaviour of concrete in the latter is simpler than the ones in the other method.
6. Table 5.40 in this thesis can be used to determine the best method(s) of analysis depending on the load and reinforcement ratio.
7. Although Table 5.40 is developed by considering both accuracy and safety, it should be used cautiously in cases other than Load Case No. 1 when accuracy and economy become the main considerations.
8. Despite the fact that Nielsen's and Clark's methods are the oldest methods and they have the simplest formulation, as it can be seen in Table 5.40, they are still acceptable and recommendable for the analysis of reinforced concrete shell elements.

6.3 Recommendations for Future Work

1. There is still an urgent need for a method of analysis of reinforced concrete shells which has the following properties:
 - The new method will work in any loading condition
 - The new method will work with any combination of reinforcement ratio
 - The new method will handle normal as well as high strength concrete
 - The new method will be simple to use in hand calculations while detailed enough to handle compatibility, equilibrium, and constitutive requirements.
2. The lack of enough experimental data needs to be solved by further investigation and that will give the researchers the necessary experimental information to develop the appropriate method described in item 1.
3. The modified compression field theory needs to be simplified such that it can be used as a tool for hand calculations.
4. A parametric study similar to the one conducted here needs to be carried out after the development of the suitable method per recommendation No. 1.
5. The parametric study that was carried out in the present study can be extended to cover the methods developed for moments, and combined states of loading.

References

ACI-318-77 (1977), Building Code Requirements for Reinforced Concrete, ACI, Detroit, Michigan.

ACI-359-80 (1980), Code for Concrete Reactor Vessels and Containments, ACI, Detroit, Michigan.

Baumann T. (1972), "Zur Frage der Netzbewehrung von Flächentragwerken," *Der Bauingenieur*, 47, 1972, pp. 367-377.

Bazant Z.P., Tsubaki T. (1978), "Optimum Slip-Free Design of Concrete Reinforcing Nets," *SER 78-31640*, Northwestern University, March 1978.

Belarbi A., Hsu T.T.C. (1995), "Constitutive Laws of Softened Concrete in Biaxial Tension-Compression," *ACI Structural Journal*, Vol. 92, No. 5, September-October 1995, pp. 562-573.

Brondum T., Nielsen M.P. (1974), "Optimum Design of Reinforced Concrete Shells and Slabs," *Report No. R44*, Structural Research Laboratory, University of Denmark, Copenhagen, 1974, pp. 190-200.

Clark L.A. (1976), "The Provision of Tension and Compression Reinforcement to Resist In-Plane Forces," *Magazine of Concrete Research*, Vol. 28, 1976, pp. 3-12.

Collins M.P. (1978), "Towards a Rational Theory for RC Members in Shear," *ASCE Proceedings*, Vol. 104, ST4, April 1978, pp. 649-666.

Collins M.P., Vecchio F., Mehlhorn G. (1985), "An International Competition to Predict the Response of Reinforced Concrete Panels," *Canadian Journal of Civil Engineering (Ottawa)*, Vol. 12, No. 3, September 1985, pp. 626-644.

Collins M.P., Mitchell D., Adebar P., Vecchio F. (1996), "A General Shear Design Method," *ACI Structural Journal*, Vol. 93, No. 1, January-February 1996, pp. 36-45.

Conley C.H., White R.N., Gergely P. (1981), "Strength and Stiffness of Tension Reinforced Concrete Panels Subjected to Membrane Shear, Two-Way and Four-Way

Reinforcing,” *Report No. NUREG CR-2049*, Prepared for USNRC, Cornell University, April 1981.

Duchan N.B. (1972), “Analysis of Reinforced Concrete Membrane Subjected to Tension and Shear,” *ACI Journal*, Proceedings Vol. 69, No. 9, September 1972, pp. 578-583.

Falconer B.H. (1956), “Theory of Stress Induced in Reinforced Concrete by Applied Two Dimensional Stress,” *ACI Journal*, Proceedings Vol. 53, September 1956, pp. 277-294.

Fialkow M.N. (1983), “Strength Design of Shell Membrane Reinforcement,” *Journal of Structural Engineering, ASCE*, Vol. 109, No. 4, Apr., 1983, pp. 891-908.

Fialkow M.N. (1985), “Design and Capacity Evaluation of Reinforced Concrete Shell Membranes,” *ACI Journal*, Proceedings, Vol. 82, No. 6, Nov-Dec., 1985, pp.844-852.

Fialkow M.N. (1988), “Crushing Strength of Reinforced Concrete Shell Membranes,” *ACI Journal*, Proceedings, Vol. 82, No. 6, Nov-Dec., 1985, pp.485-491.

Fialkow M.N. (1990), “Behaviour of Reinforced Concrete Membranes with Compatible Stress and Cracking,” *ACI Structural Journal*, Vol. 87, No. 5, Sept.-Oct., 1985, pp.571-582.

Flügge W. (1962), *Statik und Dynamik der Shalen*, 3rd ed. , Springer-Verlag, Berlin, 1962, pp. 13-21.

Gupta A.K. (1981), "Membrane Reinforcement in Shells," *ASCE Journal of Structural Division*, 107, January 1981, pp. 41-56.

Gupta A.K. (1984), "Membrane Reinforcement in Concrete Shells: A Review," *Nuclear Engineering and Design*, Vol. 82, 1984, pp. 63-75.

Hsu T.T.C. (1988), "Softened Truss Model Theory for Shear and Torsion," *ACI Structural Journal*, Vol. 85, No. 6, November-December 1988, pp. 624-635.

Hsu T.T.C. (1991), "Nonlinear Analysis of Concrete Membrane Elements," *ACI Structural Journal*, Vol. 88, No. 5, September-October 1991, pp. 552-561.

Kuyt B. (1964), "Zur Frage der Netzbewehrung von Flächentragwerken," *Beton und Stahlbetonbau*, 59, 1964, pp. 158-163.

Leitz H. (1923), "Eisenbewehrte PlattenBei Allgemeinem Biegungszustand," *Die Bautechnik*, 16 and 17, 1923.

Mitchell D., Collins M.P. (1974), "Diagonal Compression Field Theory - A Rational Model for Structural Concrete in Pure Torsion," *ACI Journal Proceedings*, Vol. 71, No. 8, August 1974, pp. 396-408.

Morley C.T. (1970), "Optimum Reinforcement of Concrete Slab Elements Against Combinations of Moments and Membrane Forces," *Magazine of Concrete Research*, Vol. 22, No. 72, September 1970, pp. 155-162

Nielsen M.P. (1963), "Yield Conditions for Reinforced Concrete Shells in Membrane State," *Proceedings, IASS Congress on Non-Classical Problems*, Warsaw, Poland, September 1963, pp. 1030-1040.

Nielsen M.P. (1971), "On the Strength of Reinforced Concrete Discs," *Civil Engineering and Building Construction Series*, ACTA Polytechnica Scandinavia, No. 70, Copenhagen, 1971.

Oesterle R.G., Russel H.G. (1980), "Shear Transfer in Large Scale Reinforced Concrete Containment Elements, 1," *Report No. CR-1374*, Prepared for USNRC, Construction Technologies Laboratories, Portland Cement Association, April 1980.

Oesterle R.G., Russel H.G. (1981), "Shear Transfer in Large Scale Reinforced Concrete Containment Elements, 2," *Report No. NUREG/CR-2450*, Prepared for USNRC, Construction Technologies Laboratories, Portland Cement Association, December 1981.

Pang D., Hsu T.T.C. (1995), "Behaviour of Reinforced Concrete Membrane Elements in Shear," *ACI Structural Journal*, Vol. 92, No. 6, November-December 1995, pp. 665-679.

Pang D., Hsu T.T.C. (1996), "Fixed Angle Softened Truss Model for Reinforced Concrete," *ACI Structural Journal*, Vol. 93, No. 2, March-April 1996, pp. 197-207.

Polak M.A., Vecchio F. (1994), "Reinforced Concrete Shell Elements Subjected to Bending and Membrane Loads," *ACI Structural Journal*, Vol. 91, No. 3, May-June 1994, pp. 261-268.

Perdikaris P.C., White R.N., Gergely P. (1980), "Strength and Stiffness of Tension Reinforced Concrete Panels Subjected to Membrane Shear, two-way Reinforcing," *Report No. NUREG CR-1602*, Prepared for USNRC, Cornell University, July 1980.

Peter J. (1964), "Zur Bewehrung von Scheiben und Schalen für Hauptspannungen Schiefwinklig zur Bewehrungsrichtung," Dr. Ing Dissertation, T.H. Stuttgart, 1964.

Sholz G. (1958), "Zur Frage der Netzbewehrung von Flächentragwerken," *Beton und Stahlbetonbau*, 53, 1958, pp. 250-255.

Wagner H. (1929), "Metal Beams with Very Thin Webs," *Zeithschrift für Flugtechnik und Motorluftschiffahrt*, Vol. 20, No. 8-12, 1929.

Walvaren J.C. (1981), "Fundamental Analysis of Aggregate Interlock," *Journal of the Structural Division, ASCE*, Vol. 107, No. ST11, Nov. 1981, pp. 2245-2270.

Vecchio F., Collins M.P. (1982), "The Response of Reinforced Concrete to In-Plane Shear and Normal Stress," *Publication No. 82-03*, Dept. of Civil Engineering, University of Toronto, March 1982.

Vecchio F., Collins M.P. (1986), "Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear," *ACI Structural Journal*, Vol. 83, No. 2, March-April 1986, pp. 219-231.

Vecchio F. (1989), "Nonlinear Finite Element Analysis of Reinforced Concrete Membranes," *ACI Structural Journal*, Vol. 86, No. 1, January-February 1989, pp. 26-35.

Zararis P.D. (1988), "Failure Mechanisms in R/C Plates Carrying In-Plane Forces," *Journal of Structural Engineering, ASCE*, Vol. 114, No.3, 1988, pp.553-575.

Zararis P.D. (1997), "Aggregate Interlock and Steel Shear Forces in the Analysis of RC Membrane Elements," *ACI Structural Journal*, Vol. 94, No.2, 1997, pp.159-170.

Appendix A

Source Code of MemCap

[illegible]

Fig. 4.1: Main Window of MemCap

'Developer: **Burkan O. Isgor**
 ' **Carleton University**
 ' **Dept. of Civil and Environmental Engineering**
 ' **Graduate Student**

'DECLARATIONS

'
 Global cod As Single
 Global cor As Single
 Global no As Integer
 Global fc As Single
 Global fxy As Single
 Global fyy As Single
 Global Es As Single
 Global rox As Single
 Global roy As Single
 Global epsxu As Single
 Global epsyu As Single
 Global epsxy As Single
 Global epsyy As Single
 Global in_file As String
 Global out_file As String
 Global ax() As Single
 Global ay() As Single
 Global Nxx() As Single
 Global Nyy() As Single
 Global Nxy() As Single

Sub calculate_phi (x1 As Single, x2 As Single, x3 As Single, x4 As Single, phi As Single, flag As Integer)

Dim angleflag As Integer

If flag = 3 Then

x1 = Atn(x1) * cod
x2 = Atn(x2) * cod
x3 = Atn(x3) * cod
x4 = Atn(x4) * cod

If x1 > 0 And x1 < 90 Then angleflag = 1
If x2 > 0 And x2 < 90 Then angleflag = 2
If x3 > 0 And x3 < 90 Then angleflag = 3
If x4 > 0 And x4 < 90 Then angleflag = 4

If angleflag = 1 Then phi = x1

If angleflag = 2 Then

phi = x2
If x1 > x2 Then phi = x1

```

End If

If angleflag = 3 Then
    phi = x3
    If x1 > phi Then phi = x1
    If x2 > phi Then phi = x2
End If

If angleflag = 4 Then
    phi = x4
    If x1 > phi Then phi = x1
    If x2 > phi Then phi = x2
    If x3 > phi Then phi = x3
End If

ElseIf flag = 2 Then

    x1 = Atn(x3) * cod
    x2 = Atn(x4) * cod

    If x1 > 0 And x1 < 90 Then angleflag = 1
    If x2 > 0 And x2 < 90 Then angleflag = 2

    If angleflag = 1 Then phi = x1

    If angleflag = 2 Then
        phi = x2
        If x1 > x2 Then phi = x1
    End If

ElseIf flag = 1 Then

    x1 = Atn(x1) * cod
    x2 = Atn(x2) * cod

    If x1 > 0 And x1 < 90 Then angleflag = 1
    If x2 > 0 And x2 < 90 Then angleflag = 2

    If angleflag = 1 Then phi = x1

    If angleflag = 2 Then
        phi = x2
        If x1 > x2 Then phi = x1
    End If
End If

phi = phi * cor

End Sub

Sub clark (ax() As Single, ay() As Single, Nxy() As Single, Nxx() As Single, Nyy() As Single)

```

```

Dim k As Single, v As Single, v1 As Single, v2 As Single
Dim i As Single, R As Single
Dim j As Integer, value As Integer, jumpflag As Integer

```

```

k = 3#
R = -4#
For i = 1# To .25 Step -.25
roy = i * rox
Write #3, "roy=" & i & "rox"

```

```

For j = 1 To no

```

```

    value = 0
    jumpflag = 0

```

```

    'CASE #1

```

```

    If ax(j) > -1# And ay(j) > -1# Then

```

```

        v1 = Abs(fxy * rox / (ax(j) + 1))

```

```

        v2 = Abs(fyy * roy / (ay(j) + 1))

```

```

        v3 = Abs(.5 * k * fc / Sqr(fc))

```

```

        v = v1

```

```

        If v2 < v Then v = v2

```

```

        If v3 < v Then v = v3

```

```

    'CASE #2

```

```

    ElseIf ax(j) < -1# And ax(j) > (.5 * (R - Sqr(R ^ 2 - 4))) And ax(j) * ay(j) < 1# Then

```

```

        v1 = Abs(fyy * roy * ax(j) / (ax(j) * ay(j) - 1))

```

```

        v2 = Abs(k * fc * ax(j) / Sqr(fc) / (ax(j) ^ 2 + 1))

```

```

        v = v1

```

```

        If v2 < v Then v = v2

```

```

    'CASE #3

```

```

    ElseIf ay(j) < -1# And ay(j) > (.5 * (R - Sqr(R ^ 2 - 4))) And ax(j) * ay(j) < 1# Then

```

```

        v1 = Abs(fxy * rox * ay(j) / (ax(j) * ay(j) - 1))

```

```

        v2 = Abs(k * fc * ay(j) / Sqr(fc) / (ay(j) ^ 2 + 1))

```

```

        v = v1

```

```

        If v2 < v Then v = v2

```

```

    'CASE #4

```

```

    ElseIf ax(j) < R + 1 And ay(j) < R + 1 Then

```

```

        v1 = Abs(rox * fxy / (ax(j) - R - 1))

```

```

        v2 = Abs(roy * fyy / (ay(j) - R - 1))

```

```

        v3 = Abs(k * fc / Sqr(fc) / R)

```

```

        v = v1

```

```

        If v2 < v Then v = v2

```

```

        If v3 < v Then v = v3

```

```

    'CASE #5

```

```

    ElseIf ax(j) * ay(j) > (R + 1) ^ 2 And ax(j) > (R + 1) And ax(j) < .5 * (R + Sqr(R ^ 2 - 4)) Then

```

```

        v1 = Abs(roy * fyy / (ay(j) - R - 1 / (ax(j) - R)))

```

```

        v2 = Abs(k * fc / Sqr(fc) / R)

```

```

        v = v1

```

```

        If v2 < v Then v = v2

```

```

    'CASE #6

```

```

    ElseIf ax(j) * ay(j) > (R + 1) ^ 2 And ay(j) > (R + 1) And ay(j) < .5 * (R + Sqr(R ^ 2 - 4)) Then

```

```

        v1 = Abs(rox * fxy / (ax(j) - R - 1 / (ay(j) - R)))

```

```

        v2 = Abs(k * fc / Sqr(fc) / R)

```

```

        v = v1

```

```

    If v2 < v Then v = v2
'CASE #7
ElseIf ay(j) < .5 * (R - Sqr(R ^ 2 - 4)) And ax(j) > .5 * (R + Sqr(R ^ 2 - 4)) Then

```

```

    v1 = rox * fxy
    value = R * (1 - Sqr(1 - 4 / R ^ 2)) / 2

```

```

    value = ax(j) - value

```

```

    If value = 0 Then
        jumpflag = 1
        GoTo jump1
    End If

```

```

    v1 = Abs(v1 / value)
    value = 0

```

```

    v2 = roy * fyy
    value = R / 2 * (1 + Sqr(1 - 4 / R ^ 2))

```

```

    value = ay(j) - value

```

```

    If value = 0 Then
        jumpflag = 1
        GoTo jump1
    End If

```

```

    v2 = Abs(v2 / value)
    value = 0

```

```

jump1:
    v3 = Abs(k * fc / Sqr(fc) / R)

```

```

    v = v1
    If v2 < v Then v = v2
    If v3 < v Then v = v3

```

```

    If jumpflag = 1 Then v = v3
    jumpflag = 0

```

```

'CASE #8
ElseIf ax(j) < .5 * (R - Sqr(R ^ 2 - 4)) And ay(j) > .5 * (R + Sqr(R ^ 2 - 4)) Then

```

```

    v1 = rox * fxy
    value = R / 2 * (1 + Sqr(1 - 4 / R ^ 2))

```

```

    value = ax(j) - value

```

```

    If value = 0 Then
        jumpflag = 1
        GoTo jump2
    End If

```

```

v1 = Abs(v1 / value)
value = 0

v2 = roy * fyy
value = R / 2 * (1 - Sqr(1 - 4 / R ^ 2))

value = ay(j) - value

If value = 0 Then
    jumpflag = 1
    GoTo jump2
End If

v2 = Abs(v2 / value)
value = 0

jump2:
v3 = Abs(k * fc / Sqr(fc) / R)

v = v1
If v2 < v Then v = v2
If v3 < v Then v = v3

If jumpflag = 1 Then v = v3
jumpflag = 0

'CASE #9
ElseIf ax(j) < 0 And ay(j) < 0 And ax(j) * ay(j) < (R + 1) ^ 2 And ax(j) * ay(j) > 1# Then
    v = Abs(2 * k * fc / Sqr(fc) / (ax(j) + ay(j) - Sqr((ax(j) - ay(j)) ^ 2 + 4)))
End If
Nxy(j) = v

Nxx(j) = ax(j) * v
Nyy(j) = ay(j) * v
Write #3, Nxy(j)
Nxy(j) = 0
Next j
Next i

End Sub

```

Sub fialkow (ax() As Single, ay() As Single, Nxy() As Single, Nxx() As Single, Nyy() As Single)

```

Dim fx As Single, fy As Single, tetha As Single, Ec As Single
Dim x1 As Single, x2 As Single, x3 As Single, x4 As Single
Dim a0 As Single, a1 As Single, a2 As Single, a3 As Single
Dim s As Single, flag As Integer, flagt As Integer, alfa As Single
Dim vxy_crushing, roxc As Single, royc As Single, i As Single
Dim cxy As Single, cx As Single, cy As Single

```

Ec = 22000

```

For i = 1# To .25 Step -.25
roy = i * rox
Write #4, "roy=" & i & "rox"

For j = 1 To no

  If ax(j) = ay(j) Then
    tetha = 45 * cor
  Else
    tetha = .5 * Atn(2 / (ax(j) - ay(j)))
  End If

  cxy = 1 / (ax(j) * Cos(tetha) ^ 2 + ay(j) * Sin(tetha) ^ 2 + 2 * Sin(tetha) * Cos(tetha))
  cx = ax(j) * cxy
  cy = ay(j) * cxy

  s = ax(j) * Cos(tetha) ^ 2 + ay(j) * Sin(tetha) ^ 2 + 2 * Sin(tetha) * Cos(tetha)
  s = s / -(ax(j) * Sin(tetha) ^ 2 + ay(j) * Cos(tetha) ^ 2 - 2 * Sin(tetha) * Cos(tetha))
  s = Abs(s)

  a3 = ax(j) * Ec / (Ec + rox * Es)
  a2 = 0#
  a1 = -ay(j) * Ec * rox / (roy * (Ec + rox * Es))
  a0 = -(Ec + roy * Es) / (Ec + rox * Es) * (rox / roy)

  Call roots_4th(a3, a2, a1, a0, x1, x2, x3, x4, flag)
  If flag = 0 Then End
  Call calculate_phi(x1, x2, x3, x4, alfa, flag)

  If 0 <= s And s < 1# Then
    Rp = .14 + (1 / 6) * (2# - s) ^ 2.3
  ElseIf 1# < s And s <= 2# Then
    Rp = .2 + (1 / 9) * (2# - s) ^ 2
  Else
    Rp = 2
  End If

  vxy_crushing = Abs((Rp * fc * s * cxy))

  If rox <= roy * (fyy / fxy) * (ax(j) + Tan(alfa)) / (ay(j) + 1 / Tan(alfa)) Then
    roxc = Abs(s * Rp * fc / fxy)
    If roxc >= rox Then
      Nxy(j) = Abs(fxy * rox / (ax(j) + Tan(alfa)))
    Else
      Nxy(j) = vxy_crushing
    End If
  Else
    royc = Abs(s * Rp * fc / fyy)
    If royc >= roy Then
      Nxy(j) = Abs((fyy * roy / (ay(j) + 1 / (Tan(alfa)))))
    Else
      Nxy(j) = vxy_crushing
    End If
  End If
End For

```



```

End If

Write #4, Nxy(j)

Nxy(j) = 0

Next j
Next i

End Sub

Sub main ()

'On Error GoTo Error_Label1

cor = 2 * 3.1415 / 360#
cod = 1 / cor

For file_no = 1 To 1

in_file = "d:\burkan\thesis\data\general\lc" & file_no & "input2.txt"
Open in_file For Input As #1

Input #1, no
Input #1, fc, fxy, fyy, Es, rox, epsxu, epsyu

epsxy = fxy / Es
epsyy = fyy / Es

out_file = "d:\burkan\thesis\data\general\lc" & file_no & "\n" & file_no & ".txt"
Open out_file For Output As #2
out_file = "d:\burkan\thesis\data\general\lc" & file_no & "\c" & file_no & ".txt"
Open out_file For Output As #3
out_file = "d:\burkan\thesis\data\general\lc" & file_no & "\f" & file_no & ".txt"
Open out_file For Output As #4
out_file = "d:\burkan\thesis\data\general\lc" & file_no & "\z" & file_no & ".txt"
Open out_file For Output As #5
out_file = "d:\burkan\thesis\data\general\lc" & file_no & "\MZ" & file_no & ".txt"
Open out_file For Output As #6

ReDim ax(no) As Single
ReDim ay(no) As Single
ReDim Nxx(no) As Single
ReDim Nyy(no) As Single
ReDim Nxy(no) As Single

For i = 1 To no
    Input #1, ax(i), ay(i)
Next i
Close #1

```

```

Call nielsen(ax(), ay(), Nxy(), Nxx(), Nyy())
  Close #2
Call clark(ax(), ay(), Nxy(), Nxx(), Nyy())
  Close #3
Call fialkow(ax(), ay(), Nxy(), Nxx(), Nyy())
  Close #4
Call zararis(ax(), ay(), Nxy(), Nxx(), Nyy())
  Close #5
Call zararis_modified(ax(), ay(), Nxy(), Nxx(), Nyy())
  Close #6

```

Next file_no

exit_program:

```

  Close #1
  Close #2
  Close #3
  Close #4
  Close #5

```

Exit Sub

Error_Label1:

```

  msg = "ERROR: Illegal Function Call"
  MsgBox msg, 16, "Program is Aborted"
  GoTo exit_program

```

End Sub

Sub nielsen (ax() As Single, ay() As Single, Nxy() As Single, Nxx() As Single, Nyy() As Single)

```

Dim k As Single, v As Single, v1 As Single, v2 As Single
Dim i As Single
Dim j As Integer

```

k = 3#

For i = 1# To .25 Step -.25

roy = i * rox

Write #2, "roy=" & i & "*rox"

For j = 1 To no

'CASE #1

If ax(j) > -1# And ay(j) > -1# Then

v1 = Abs(fxy * rox / (ax(j) + 1))

v2 = Abs(fyy * roy / (ay(j) + 1))

v3 = Abs(.5 * k * fc / Sqr(fc))

v = v1

If v2 < v Then v = v2

If v3 < v Then v = v3

'CASE #2

Elseif ax(j) < -1# And ax(j) * ay(j) < 1# Then

```

v1 = Abs(fyy * roy * ax(j) / (ax(j) * ay(j) - 1))
v2 = Abs(k * fc * ax(j) / Sqr(fc) / (ax(j) ^ 2 + 1))
v = v1
If v2 < v Then v = v2
'CASE #3
Elseif ay(j) < -1# And ax(j) * ay(j) < 1# Then
v1 = Abs(fxy * rox * ay(j) / (ax(j) * ay(j) - 1))
v2 = Abs(k * fc * ay(j) / Sqr(fc) / (ay(j) ^ 2 + 1))
v = v1
If v2 < v Then v = v2
'CASE #4
Elseif ax(j) < -1# And ay(j) < -1# And ax(j) * ay(j) > 1# Then
v = Abs(2 * k * fc / Sqr(fc) / (ax(j) + ay(j) - Sqr((ax(j) - ay(j)) ^ 2 + 4)))
End If
Nxy(j) = v
Nxx(j) = ax(j) * v
Nyy(j) = ay(j) * v
Write #2, Nxy(j)
Nxy(j) = 0
Next j
Next i

End Sub

```

Sub roots_2nd (a1 As Single, a2 As Single, a3 As Single, x1 As Single, x2 As Single, flag As Integer)

'roots of: $a1 \cdot x^2 + a2 \cdot x + a3 = 0$

Dim delta As Single

delta = $a2^2 - 4 \cdot a1 \cdot a3$

If delta < 0# Then

flag = 0

Else

flag = 1

x1 = $((-1) \cdot a2 - \text{Sqr}(\text{delta})) / (2 \cdot a1)$

x2 = $((-1) \cdot a2 + \text{Sqr}(\text{delta})) / (2 \cdot a1)$

End If

End Sub

Sub roots_3rd (a1 As Single, a2 As Single, a3 As Single, a4 As Single, x1 As Single, x2 As Single, x3 As Single, flag As Integer)

'roots of: $a1 \cdot x^3 + a2 \cdot x^2 + a3 \cdot x + a4 = 0$

Dim temp As Integer, item As Single

Dim Q As Single, P As Single, c As Single, z As Single

If a4 = 0 Then

```

    x1 = 0
    GoTo skiptool1
End If

Q = .5 * (a4 / a1 - a2 * a3 / (3 * a1 ^ 2) + 2 * a2 ^ 3 / (27 * a1 ^ 3))
P = (1 / 3) * (a3 / a1 - a2 ^ 2 / (3 * a1 ^ 2))
c = P ^ 3 + Q ^ 2

If Q = 0 Then
    z = 0
    x1 = z - a2 / (3 * a1)
    GoTo skiptool1
End If

temp = 0
If Q < 0# Then
    Q = -1 * Q
    temp = 1
End If

If a4 <> 0# And Q > 0# Then

    If c < 0# Then
        z = -2 * Sqr(-P) * Cos((1 / 3) * Atn(Sqr(-c) / Q))
    ElseIf c = 0# Then
        z = -2 * Q ^ (1 / 3)
    ElseIf c > 0# And P < 0# Then
        z = -(Q + Sqr(c)) ^ (1 / 3) - (Q - Sqr(c)) ^ (1 / 3)
    ElseIf c > 0# And P = 0# Then
        z = -(2 * Q) ^ (1 / 3)
    ElseIf c > 0# And P > 0# Then
        If (Sqr(c) - Q) < 0 Then
            item = -(Sqr(c) - Q)
            item = item ^ (1 / 3)
            item = -item
            z = item - (Sqr(c) + Q) ^ (1 / 3)
        End If

        If (Sqr(c) - Q) >= 0 Then
            z = (Sqr(c) - Q) ^ (1 / 3) - (Sqr(c) + Q) ^ (1 / 3)
        End If
    End If

End If

x1 = z - a2 / (3 * a1)
If temp = 1 Then x1 = -x1

skiptool1:

a1 = a1
a2 = a2 + a1 * x1
a3 = (a2 + a1 * x1) * x1 + a3

```

Call roots_2nd(a1, a2, a3, x2, x3, flag)

End Sub

Sub roots_4th (a3 As Single, a2 As Single, a1 As Single, a0 As Single, x1 As Single, x2 As Single, x3 As Single, x4 As Single, flag As Integer)

'roots of: $x^4 + a3*x^3 + a2*x^2 + a1*x + a0 = 0$

Dim A As Single, B As Single, c As Single, D As Single
Dim b3 As Single, b2 As Single, b1 As Single, b0 As Single

b3 = 1#

b2 = -a2

b1 = a3 * a1 - 4 * a0

b0 = a0 * (4 * a2 - a3 ^ 2) - a1 ^ 2

Call roots_3rd(b3, b2, b1, b0, x1, x2, x3, flag)

z1 = x1

If flag = 0 Then z1 = x1

If flag = 1 Then

If x2 > z1 Then z1 = x2

If x3 > z1 Then z1 = x3

End If

A = a3 / 2#

B = z1 / 2#

D = Sqr(B ^ 2 - a0)

If D <> 0# Then

c = (a1 / 2 - A * B) / D

Else

c = Sqr(A ^ 2 - a2 + z1)

End If

b1 = 1#

b2 = A - c

b3 = B - D

Call roots_2nd(b1, b2, b3, x1, x2, flag)

If flag = 0 Then

temp1 = 0

Else

temp1 = 1

End If

b1 = 1#

b2 = A + c

b3 = B + D

Call roots_2nd(b1, b2, b3, x3, x4, flag)

If flag = 0 Then temp2 = 0

```

If temp1 = 0 And temp2 = 0 Then      'no real roots
    flag = 0
ElseIf (temp1 = 1 And temp2 = 0) Then 'two real roots - x1..x2
    flag = 1
ElseIf (temp1 = 0 And temp2 = 1) Then 'two real roots - x3..x4
    flag = 2
Else                                'four real roots - x1..x2..x3..x4
    flag = 3
End If

End Sub

```

Sub zararis (ax() As Single, ay() As Single, Nxy() As Single, Nxx() As Single, Nyy() As Single)

```

Dim phi1 As Single, phi2 As Single, phi As Single, v1 As Single
Dim Ssx As Single, Ssy As Single, Scc As Single, Sct As Single
Dim x1 As Single, x2 As Single, x3 As Single, x4 As Single, fct As Single
Dim a0 As Single, a1 As Single, a2 As Single, a3 As Single, a4 As Single
Dim flag As Integer, a33 As Single, i As Single, j As Integer, oops As Integer, oops2 As Integer
Dim AA As Single, BB As Single, coef As Single

```

'Failure due to yielding of reinforcement

```

For i = 1# To .25 Step -.25
    roy = i * rox
    Write #5, "roy=" & i & " * rox"

```

```

For j = 1 To no

```

```

    oops = 0
    oops2 = 0

```

```

    a1 = 1#
    a2 = 1 * ax(j) - (rox * fxy * ay(j)) / (roy * fyy)
    a3 = -(rox * fxy) / (roy * fyy)

```

```

    Call roots_2nd(a1, a2, a3, x1, x2, flag)

```

```

    If flag = 0 Then
        oops = 1
        GoTo skipz1
    Else
        x1 = Atn(x1)
        x2 = Atn(x2)
        phi2 = x1 * cod
        If x2 * cod > phi2 Then phi2 = x2 * cod
    End If

```

```

    phi2 = Tan(phi2 * cor)

```

```

    If phi2 > Sqr(epsyy / epsxu) And phi2 < Sqr(epsyu / epsxy) Then

```

```

v = Abs(rox * fxy / (ax(j) + phi2))
fx = ax(j) * v
fy = ay(j) * v

```

ElseIf phi2 < Sqr(epsyy / epsxu) Then

```

a3 = ax(j)
a2 = 0#
a1 = -(rox * fxy * ay(j)) / (roy * Es * epsxu)
a0 = -(rox * fxy) / (roy * Es * epsxu)

Call roots_4th(a3, a2, a1, a0, x1, x2, x3, x4, flag)

```

```

If flag = 0 Then
  oops = 1
  GoTo skipz1
End If

```

```

Call calculate_phi(x1, x2, x3, x4, phi, flag)
phi2 = Tan(phi)

```

```

v = Abs(rox * fxy / (ax(j) + phi2))
fx = ax(j) * v
fy = ay(j) * v

```

ElseIf phi2 > Sqr(epsyu / epsxy) Then

```

a3 = ax(j)
a2 = 0#
a1 = -(rox * Es * epsyu * ay(j)) / (roy * fyy)
a0 = -(rox * Es * epsyu) / (roy * fyy)

Call roots_4th(a3, a2, a1, a0, x1, x2, x3, x4, flag)
If flag = 0 Then GoTo skipz1
Call calculate_phi(x1, x2, x3, x4, phi, flag)
phi2 = Tan(phi)

```

```

v = Abs(roy * fyy * phi2 / (1 + ay(j) * phi2))
fx = ax(j) * v
fy = ay(j) * v

```

End If

skipz1:

'Failure due to crushing of concrete

```

a3 = ax(j) - .4 * ay(j)
a2 = .4 * (rox / roy - 1)
a1 = -rox / roy * (ay(j) - .4 * ax(j))
a0 = -rox / roy

```

```

Call roots_4th(a3, a2, a1, a0, x1, x2, x3, x4, flag)

```

```

If flag = 0 Then
    oops2 = 1
    GoTo skipz2
End If

Call calculate_phi(x1, x2, x3, x4, phi, flag)
phi1 = phi

s = Sin(phi1)
c = Cos(phi1)
t = Tan(phi1)

coef = (rox / (.4 * t) * c ^ 2 + roy / (.4 * 1 / t) * s ^ 2 + (rox + roy) * s * c)
coef = coef / (ax(j) * c ^ 2 + 2 * s * c + ay(j) * s ^ 2)

AA = rox / (.4 * coef * t) * s ^ 2 + roy / (.4 * coef / t) * c ^ 2
BB = rox / (.4 * coef * t) * c ^ 2 + roy / (.4 * coef / t) * s ^ 2

Scc = ax(j) * s ^ 2 - 2 * s * c + ay(j) * c ^ 2 - AA
Sct = ax(j) * c ^ 2 + 2 * s * c + ay(j) * s ^ 2 - BB

fct = .3 * fc ^ (2 / 3)

a1 = (Sct / fct) ^ 2
a2 = Abs(Scc) / fc
a3 = -1#

If Abs(a1) < .0001 Then
    v1 = fc / Abs(Scc)
Else
    Call roots_2nd(a1, a2, a3, x1, x2, flag)

    If flag = 0 Then
        oops2 = 1
        GoTo skipz2
    End If

    If flag = 1 Then
        If x1 < 0 And x2 < 0 Then
            opps2 = 1
            GoTo skipz2
        ElseIf x1 > 0 And x2 < 0 Then
            v1 = x1
        ElseIf x1 < 0 And x2 > 0 Then
            v1 = x2
        ElseIf x1 > x2 Then
            v1 = x2
        End If
    End If

End If

If Abs(v1) < Abs(v) Then
    v = v1

```


End If

skipz2:

If oops = 1 And oops2 = 1 Then End

Write #5, v

fx = ax(j) * v

fy = ay(j) * v

Next j

Next i

End Sub

Sub zararis_modified (ax() As Single, ay() As Single, Nxy() As Single, Nxx() As Single, Nyy() As Single)

Dim phi1 As Single, phi2 As Single, phi As Single, v1 As Single

Dim Ssx As Single, Ssy As Single, Scc As Single, Sct As Single

Dim x1 As Single, x2 As Single, x3 As Single, x4 As Single, fsp As Single, fct As Single

Dim a0 As Single, a1 As Single, a2 As Single, a3 As Single, a4 As Single

Dim flag As Integer, a33 As Single, i As Single, j As Integer, oops As Integer, oops2 As Integer

Dim AA As Single, BB As Single, coef As Single

For i = 1# To .25 Step -.25

roy = i * rox

Write #6, "roy=" & i & "**rox"

For j = 1 To no

' Determination of phi1

a3 = ax(j) - .4 * ay(j)

a2 = .4 * (rox / roy - 1)

a1 = -rox / roy * (ay(j) - .4 * ax(j))

a0 = -rox / roy

Call roots_4th(a3, a2, a1, a0, x1, x2, x3, x4, flag)

Call calculate_phi(x1, x2, x3, x4, phi, flag)

phi1 = phi

s = Sin(phi1)

c = Cos(phi1)

t = Tan(phi1)

If phi1 * cod < 45.3 Then

a1 = 1#

a2 = ax(j) - (rox * fxy * ay(j)) / (roy * fyy)

a3 = -(rox * fxy) / (roy * fyy)

Call roots_2nd(a1, a2, a3, x1, x2, flag)

```

If flag = 0 Then

Else
  x1 = Atn(x1)
  x2 = Atn(x2)
  phi2 = x1
  If x2 * cod > phi2 * cod Then phi2 = x2
End If

If Tan(phi1) > Tan(phi2) Then ' 1st mode of failure: x-yields
  v = Abs(rox * fxy / (ax(j) + t))
Else ' 2nd mode of failure: x,y-yield
  v = Abs((rox * fxy + roy * fyy * Tan(phi2) ^ 2) / (ax(j) + ay(j) * Tan(phi2) ^ 2 + 2 * Tan(phi2)))
End If

Else

' 3rd mode of failure

fsp = .531 * Sqr(fc)

a3 = ay(j) * (1 + t ^ 2) + 2 * t
a33 = roy * fyy * (1 + t ^ 2) * (ax(j) + ay(j) * t ^ 2 + 2 * t)
a33 = a33 / (2.5 * fsp + roy * fyy * t ^ 2)
a3 = -(a3 - a33)

a2 = -(roy * fyy * t ^ 2 / (2.5 * fsp + roy * fyy * t ^ 2))

a1 = 2.5 * fsp * (ay(j) * (1 + t ^ 2) + 2 * t)
a1 = -a1 / (2.5 * fsp + roy * fyy * t ^ 2)

a0 = -2.5 * fsp / (2.5 * fsp + roy * fyy * t ^ 2)

Call roots_4th(a3, a2, a1, a0, x1, x2, x3, x4, flag)

Call calculate_phi(x1, x2, x3, x4, phi, flag)
phi2 = phi

v = (2.5 * fsp / Sin(phi2) ^ 2) + (roy * fyy * t ^ 2)
v = Abs(v / (ax(j) + ay(j) * t ^ 2 + 2 * t))

'4th mode of failure

v1 = rox * fxy + roy * fyy * t ^ 2
v1 = Abs(v1 / (ax(j) + ay(j) * t ^ 2 + 2 * t))

If v1 < v Then v = v1

End If

' Failure due to crushing of concrete

coef = (rox / (.4 * t) * c ^ 2 + roy / (.4 * 1 / t) * s ^ 2 + (rox + roy) * s * c)

```

```

coef = coef / (ax(j) * c ^ 2 + 2 * s * c + ay(j) * s ^ 2)

AA = rox / (.4 * coef * t) * s ^ 2 + roy / (.4 * coef / t) * c ^ 2
BB = rox / (.4 * coef * t) * c ^ 2 + roy / (.4 * coef / t) * s ^ 2

Scc = ax(j) * s ^ 2 - 2 * s * c + ay(j) * c ^ 2 - AA
Sct = ax(j) * c ^ 2 + 2 * s * c + ay(j) * s ^ 2 - BB

fct = .3 * fc ^ (2 / 3)

a1 = (Sct / fct) ^ 2
a2 = Abs(Scc) / fc
a3 = -1#

If Abs(a1) < .0001 Then
    v1 = fc / Abs(Scc)
Else
    Call roots_2nd(a1, a2, a3, x1, x2, flag)

    If flag = 1 Then
        If x1 < 0 And x2 < 0 Then

            Elseif x1 > 0 And x2 < 0 Then
                v1 = x1
            Elseif x1 < 0 And x2 > 0 Then
                v1 = x2
            Elseif x1 > x2 Then
                v1 = x2
            End If
        End If

    End If

End If

If Abs(v1) < Abs(v) Then
    v = Abs(v1)
End If

skipzml:
    If oops = 1 And oops2 = 1 Then End
    Write #6, v
    fx = ax(j) * v
    fy = ay(j) * v

Next j
Next i

End Sub

```

Sub mcft (ax() As Single, ay() As Single)

```

Dim eps1 As Single, eps_cr As Single, eps2 As Single
Dim tet As Single, w As Single, v_last As Single

```

```

Dim fy As Single, fcr As Single, fc1 As Single, vci As Single
Dim fc11 As Single, fc2 As Single, fc2_max As Single
Dim epsx As Single, epsy As Single
Dim fy_calc As Single, fx As Single, fxx As Single, fx_cap As Single
Dim Error As Single, PrevError As Single
Dim CurTtet As Single, PrevTtet As Single
Dim repeat As Integer, count As Integer

' initializations

Dim i As Single
Dim j As Integer

For i = 1# To .25 Step .25

    roy = i * rox
    Write #2, "roy=" & i & "rox"

    For j = 1 To no

        repeat = 0

        ' choose a value for the principal tensile strain
point1:
        eps1 = .0001

        ' estimate principal compressive strength direction
point2:
        count = 1
        ttet = 1#

point3:
        sintet = Sin(Atn(ttet))
        costet = Cos(Atn(ttet))

        ' calculation of the average crack width
point4:
        w = eps1 / ((sintet / smx) + (costet / smy))

        ' estimate the average stress in the weaker (assume it is y-reinforcement) reinforcement
        fy = 100#

        ' determination of the average tension in concrete
point5:
        fcr = .66 * Sqr(-fc)
        eps_cr = fcr / Ec

        If eps1 <= eps_cr Then
            fc1 = Ec * eps1
        Else
            fc1 = fcr / (1 + Sqr(500 * eps1))
        End If

        vci = .179 * Sqr(-fc) / (.31 + (24 * w / (a + 16)))

```

```

fc11 = vci * ttet + roy * (fyy - fy)

If fc11 < fc1 Then
    fc1 = fc11
End If

' determination of the shear capacity
point6:
v = (fc1 + roy * fy) / ttet / (1 + ay(j) / ttet)

' determination of the principal compression in concrete
fc2 = fc1 - (ttet + 1 / ttet) * v
fc2_max = fc * (1 / (.8 - .34 * eps1 / eps0))
If fc2_max < fc Then fc2_max = fc

' check whether concrete is crushing or not
If Abs(fc2) > Abs(fc2_max) Then
    GoTo point10 'concrete crushing
End If

eps2 = eps0 * (1 - Sqr(1 - fc2 / fc2_max))

' strains in reinforcements
epsx = (eps1 * (ttet) ^ 2 + eps2) / (1 + (ttet) ^ 2)
epsy = (eps1 + (ttet) ^ 2 * eps2) / (1 + (ttet) ^ 2)

' calculated fy
fy_calc = Esy * epsy

If fy_calc > fyy Then fy_calc = fyy
If fy_calc < -fyy Then fy_calc = -fyy

If Abs(Abs(fy_calc) - Abs(fy)) > .1 Then
    fy = (fy + 3 * fy_calc) / 4
    GoTo point5
End If

' calculated fx
fx = Esx * epsx

If fx > fxy Then fx = fxy
If fx < -fxy Then fx = -fxy

' check horizontal equilibrium
fxx = rox * fx - v / ttet + fc1

If Abs(v * ax(j) - fxx) < .001 Then GoTo point7

count = count + 1
If count = 2 Then
    PrevTtet = ttet
    PrevError = fxx - v * ax(j)
    ttet = .9
    GoTo point3

```

```

End If
If count = 201 Then
    eps1 = eps1 + .0005 ' determines the speed
    If eps1 > .01 Then
        GoTo point10
    End If
    GoTo point2
End If

```

```

Error = fxx - v * ax(j)
If Abs(Error - PrevError) < .001 Then
    CurTtet = ttet + .0001
Else
    CurTtet = ttet
End If

```

```

If CurTtet < .01 Then CurTtet = count / 100
If (ttet - CurTtet) > .05 Then CurTtet = ttet - .05
If (CurTtet - ttet) > .1 Then CurTtet = ttet + .1

```

```

PrevTtet = ttet
PrevError = Error
ttet = CurTtet
GoTo point3

```

point7:

```

dfc1 = fc1 - roy * (fyy - fy)

If dfc1 <= 0 Then
    dfc1 = 0
    fx_cap = rox * fx + fc1
Else
    CC = dfc1 / ttet - .18 * Sqr(-fc) / (.3 + (24 * w / (a + 16)))
    If CC <= 0 Then
        fcii = 0
        vcii = dfc1 / ttet
    Else
        AA = .82 / Sqr(-fc) / (.3 + (24 * w / (a + 16)))
        BB = 1 / ttet - 1.64
        fcii = (-BB - Sqr(BB ^ 2 - 4 * AA * CC)) / (2 * AA)
        vcii = (fc1 + dfc1) / ttet
    End If

    fx_cap = rox * fx + fc1 + fcii + vcii / ttet
End If

```

```

If fx_cap > rox * fxy Then
    GoTo point10
End If

```

point9:

```

If repeat = 0 Then
    v_last = v
    repeat = 1

```

```
        eps1 = eps1 + .00001
        GoTo point2
    Else
        If Abs(v_last) <= Abs(v) Then
            eps1 = eps1 + .00001
            GoTo point2
        End If
    End If

point10:
    Write #2, Abs(v)

Next j

Next i

End Sub
```

Appendix B

Ultimate Capacities of the Panels Tested in the Parametric Study

Table B.1: Ultimate Capacities* (v_{xy}) for Load Case No. 1 ($f'_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P1-1	5.81	5.81	5.56	4.47	4.47	6.12
P1-2	5.81	5.81	4.27	4.01	4.01	6.02
P1-3	5.71	5.71	5.00	3.67	3.67	5.81
P1-4	5.71	5.71	4.27	4.47	4.47	5.81
P1-5	5.71	5.71	4.27	4.47	4.47	7.28
P1-6	5.81	5.81	6.55	5.90	5.90	7.28
P1-7	4.44	4.44	3.51	3.28	3.28	3.32
P1-8	4.44	4.44	2.34	4.47	4.47	5.21
P1-9	4.44	4.44	2.34	4.47	4.44	6.08
P1-10	5.81	5.81	7.16	7.66	7.66	7.32
P1-11	5.81	5.81	3.00	4.45	4.45	6.58
P1-12	5.00	5.00	4.27	3.98	3.98	5.48
P1-13	4.29	4.29	3.89	3.65	3.65	5.28
P1-14	4.29	4.29	4.27	4.38	4.38	5.28
P1-15	5.71	5.71	4.27	4.59	4.59	7.29
P1-16	5.81	5.81	6.55	5.94	5.94	7.29
P1-17	3.33	3.33	2.72	3.28	3.28	3.98
P1-18	3.33	3.33	2.34	4.38	4.38	3.98
P1-19	4.44	4.44	2.34	4.77	4.77	5.53
P1-20	5.81	5.81	7.16	7.80	7.80	7.52
P1-21	4.00	4.00	3.00	4.37	4.37	5.33
P1-22	3.33	3.33	3.53	3.89	3.89	4.40
P1-23	2.86	2.86	2.78	3.56	3.56	3.65
P1-24	2.86	2.86	3.14	4.20	4.20	3.68
P1-25	5.71	5.71	4.27	4.75	4.75	6.74
P1-26	5.81	5.81	6.55	5.97	5.97	6.74
P1-27	2.22	2.22	1.89	3.00	3.00	2.79
P1-28	2.22	2.22	2.34	3.55	4.24	2.79
P1-29	4.44	4.44	2.34	5.27	5.27	5.11
P1-30	5.81	5.81	7.16	8.06	8.06	7.89
P1-31	2.00	2.00	2.83	4.00	4.13	3.04
P1-32	1.67	1.67	2.02	3.17	3.62	2.50
P1-33	1.43	1.43	1.55	2.59	3.28	2.12
P1-34	1.43	1.43	1.71	2.82	3.82	2.12
P1-35	3.33	3.33	4.27	4.89	4.96	5.45
P1-36	3.33	3.33	6.55	5.96	5.96	5.45
P1-37	1.11	1.11	1.00	1.85	1.85	1.63
P1-38	1.11	1.11	1.21	1.99	4.00	1.63
P1-39	4.44	4.44	2.34	4.91	6.32	5.01
P1-40	5.81	5.81	7.16	8.66	8.66	8.13

(*) All values are in MPa

Table B.2: Ultimate Capacities* (v_{xy}) for Load Case No. 2 ($f'_c = 15$ Mpa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P2-1	5.71	5.71	3.38	5.20	5.20	5.69
P2-2	5.50	5.50	3.69	5.43	5.43	5.99
P2-3	5.22	5.22	7.70	5.60	5.60	6.21
P2-4	4.93	4.93	8.22	5.71	5.71	6.40
P2-5	4.65	4.65	8.72	5.76	5.76	6.57
P2-6	4.38	4.38	9.12	5.73	5.73	6.10
P2-7	4.13	4.13	9.53	5.65	5.65	6.20
P2-8	3.89	3.89	9.96	5.52	5.52	6.29
P2-9	3.68	3.68	10.25	5.36	5.36	6.34
P2-10	3.49	3.49	10.55	5.19	5.19	6.39
P2-11	4.79	4.79	3.38	5.04	5.04	5.21
P2-12	5.50	5.50	5.51	5.26	5.26	5.32
P2-13	5.22	5.22	5.96	5.42	5.42	5.52
P2-14	4.93	4.93	6.37	5.53	5.53	5.68
P2-15	4.65	4.65	6.76	5.58	5.58	5.48
P2-16	4.38	4.38	7.08	5.57	5.57	5.50
P2-17	4.13	4.13	7.41	5.51	5.51	5.50
P2-18	3.89	3.89	7.75	5.40	5.40	5.58
P2-19	3.68	3.68	7.99	5.26	5.26	5.45
P2-20	3.49	3.49	8.23	5.11	5.11	5.51
P2-21	3.19	3.19	3.48	4.77	4.77	3.58
P2-22	3.72	3.72	3.83	4.97	4.97	3.81
P2-23	4.26	4.26	4.15	5.17	5.17	3.94
P2-24	4.79	4.79	4.44	5.28	5.28	4.07
P2-25	4.65	4.65	4.72	5.35	5.35	4.22
P2-26	4.38	4.38	4.95	5.35	5.35	4.27
P2-27	4.13	4.13	5.19	5.32	5.32	4.39
P2-28	3.89	3.89	5.44	5.24	5.24	4.44
P2-29	3.68	3.68	5.61	5.13	5.13	4.48
P2-30	3.49	3.49	5.79	4.99	4.99	4.58
P2-31	1.60	1.60	1.86	2.96	4.34	2.09
P2-32	1.86	1.86	2.05	3.24	4.52	2.19
P2-33	2.13	2.13	2.22	3.50	4.68	2.27
P2-34	2.39	2.39	2.39	3.74	4.82	2.33
P2-35	2.67	2.67	2.55	4.00	4.92	2.39
P2-36	2.92	2.92	2.68	4.22	4.99	2.44
P2-37	3.19	3.19	2.81	4.46	5.02	2.48
P2-38	3.48	3.48	2.96	4.73	5.01	2.52
P2-39	3.68	3.68	3.06	4.93	4.96	2.55
P2-40	3.49	3.49	3.16	4.84	4.84	2.57

(*) All values are in MPa

Table B.3: Ultimate Capacities* (v_{xy}) for Load Case No. 3 ($f'_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P3-1	5.71	5.71	4.84	6.05	6.05	7.12
P3-2	5.81	5.81	5.58	4.09	4.09	5.91
P3-3	5.50	5.50	11.40	6.06	6.06	6.57
P3-4	5.22	5.22	4.85	7.46	7.46	7.17
P3-5	4.93	4.93	4.75	8.02	8.02	6.79
P3-6	5.81	5.81	5.35	4.18	4.18	1.82
P3-7	4.65	4.65	12.57	6.01	6.01	6.57
P3-8	4.38	4.38	4.45	8.12	8.12	6.04
P3-9	5.81	5.81	5.24	4.22	4.22	6.22
P3-10	4.13	4.13	13.18	5.72	5.72	6.57
P3-11	5.71	5.71	4.84	6.73	6.73	7.02
P3-12	4.41	4.41	5.58	4.04	4.04	5.32
P3-13	5.50	5.50	9.01	5.90	5.90	6.58
P3-14	5.22	5.22	4.85	8.67	8.67	6.35
P3-15	4.93	4.93	4.75	9.33	9.33	6.84
P3-16	4.80	4.80	5.35	4.14	4.14	1.81
P3-17	4.65	4.65	9.93	5.87	5.87	6.58
P3-18	4.38	4.38	4.45	8.59	8.59	5.54
P3-19	4.96	4.96	5.24	4.18	4.18	1.84
P3-20	4.13	4.13	10.43	5.61	5.61	6.58
P3-21	5.71	5.71	8.22	7.94	8.10	6.98
P3-22	2.94	2.94	3.06	3.91	3.91	3.81
P3-23	5.50	5.50	6.47	5.65	5.65	5.33
P3-24	5.22	5.22	4.85	9.84	11.08	6.81
P3-25	4.93	4.93	4.75	10.80	10.91	5.77
P3-26	3.20	3.20	3.45	4.01	4.01	1.81
P3-27	4.65	4.65	7.13	5.67	5.67	5.33
P3-28	4.38	4.38	4.45	8.34	8.34	4.09
P3-29	3.31	3.31	3.61	4.06	4.06	4.39
P3-30	4.13	4.13	7.49	5.46	5.46	5.33
P3-31	5.71	5.71	3.19	7.28	13.22	6.95
P3-32	1.47	1.47	1.70	2.81	3.58	2.19
P3-33	2.80	2.80	3.68	5.16	5.16	3.05
P3-34	5.22	5.22	1.87	9.24	12.63	5.00
P3-35	4.93	4.93	1.55	9.72	8.19	3.58
P3-36	1.60	1.60	1.94	3.12	3.70	2.39
P3-37	4.00	4.00	4.04	5.28	5.28	3.05
P3-38	4.38	4.38	1.16	6.53	4.36	2.19
P3-39	1.65	1.65	2.05	3.25	3.75	2.48
P3-40	4.13	4.13	4.25	5.20	5.20	3.05

(*) All values are in MPa

Table B.4: Ultimate Capacities* (v_{xy}) for Load Case No. 4 ($f_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P4-1	2.42	2.90	1.60	2.86	2.46	3.02
P4-2	2.32	2.90	1.50	2.67	2.30	2.90
P4-3	2.23	2.90	1.42	2.50	2.16	2.68
P4-4	2.15	2.90	1.34	2.35	2.04	0.83
P4-5	2.26	2.90	1.47	2.62	2.11	2.53
P4-6	2.28	2.90	1.46	2.58	2.18	2.68
P4-7	2.37	2.90	1.55	2.76	2.33	2.90
P4-8	2.19	2.90	1.38	2.43	2.06	2.55
P4-9	2.26	2.90	1.04	2.50	2.43	3.12
P4-10	2.37	2.90	1.09	2.65	2.49	3.13
P4-11	2.42	2.90	1.60	2.57	1.90	2.36
P4-12	2.32	2.90	1.50	2.40	1.78	2.21
P4-13	2.23	2.90	1.42	2.25	1.66	2.07
P4-14	2.15	2.90	1.34	2.12	1.57	1.95
P4-15	2.26	2.90	1.47	2.44	1.65	1.95
P4-16	2.28	2.90	1.46	2.36	1.69	2.07
P4-17	2.37	2.90	1.55	2.52	1.81	2.21
P4-18	2.19	2.90	1.38	2.22	1.59	1.95
P4-19	2.26	2.90	1.04	2.27	1.90	2.38
P4-20	2.37	2.90	1.09	2.45	1.87	2.36
P4-21	2.42	2.90	1.11	2.40	1.34	1.67
P4-22	2.32	2.90	1.05	2.25	1.25	1.56
P4-23	2.23	2.90	1.00	2.12	1.17	1.46
P4-24	2.15	2.86	0.95	2.01	1.10	1.38
P4-25	2.26	2.86	0.95	2.33	1.15	1.38
P4-26	2.28	2.90	1.00	2.23	1.18	1.46
P4-27	2.37	2.90	1.05	2.38	1.27	1.56
P4-28	2.19	2.86	0.95	2.11	1.11	1.38
P4-29	2.26	2.90	1.11	2.07	1.28	1.67
P4-30	2.37	2.90	1.11	2.28	1.32	1.67
P4-31	2.42	2.50	0.55	2.31	0.73	0.97
P4-32	2.32	2.00	0.52	2.18	0.68	0.91
P4-33	2.23	1.67	0.50	2.06	0.63	0.85
P4-34	2.15	1.43	0.47	1.95	0.59	0.80
P4-35	2.26	1.43	0.47	2.28	0.62	0.80
P4-36	2.28	1.67	0.50	2.17	0.64	0.85
P4-37	2.37	2.00	0.52	2.30	0.69	0.91
P4-38	2.19	1.43	0.47	2.05	0.60	0.80
P4-39	2.26	2.50	0.55	1.98	0.69	0.97
P4-40	2.37	2.50	0.55	2.19	0.72	0.97

(*) All values are MPa

Table B.5: Ultimate Capacities* (v_{xy}) for Load Case No. 5 ($f'_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P5-1	2.85	2.90	2.09	3.44	3.21	3.86
P5-2	2.82	2.90	2.04	3.43	3.15	3.84
P5-3	2.78	2.90	2.09	3.28	3.06	3.45
P5-4	2.67	2.90	2.10	3.06	2.90	3.11
P5-5	2.53	2.90	2.09	2.78	2.74	2.70
P5-6	2.35	2.90	2.06	2.49	2.49	2.39
P5-7	2.16	2.90	2.00	2.20	2.20	2.10
P5-8	1.94	2.90	1.91	1.93	1.93	1.82
P5-9	2.06	2.06	2.03	2.06	2.06	1.96
P5-10	1.72	2.71	1.78	1.69	1.69	1.56
P5-11	2.85	2.90	2.09	3.40	2.50	2.94
P5-12	2.82	2.90	2.04	3.39	2.46	2.93
P5-13	2.78	2.90	2.09	3.24	2.37	2.60
P5-14	2.67	2.90	2.10	3.03	2.25	2.34
P5-15	2.53	2.90	2.09	2.77	2.12	2.09
P5-16	2.35	2.90	2.06	2.48	2.01	1.85
P5-17	2.16	2.90	2.00	2.19	1.89	1.62
P5-18	1.94	2.77	1.91	1.92	1.78	1.41
P5-19	2.06	2.06	2.03	2.05	1.93	1.52
P5-20	1.72	2.03	1.78	1.66	1.66	1.20
P5-21	2.85	2.90	1.32	3.30	1.75	2.03
P5-22	2.82	2.90	1.31	3.29	1.72	2.02
P5-23	2.78	2.90	1.22	3.14	1.66	1.83
P5-24	2.67	2.90	2.10	2.95	1.57	1.65
P5-25	2.53	2.90	2.09	2.70	1.48	1.47
P5-26	2.35	2.90	2.06	2.44	1.39	1.30
P5-27	2.16	2.56	2.00	2.16	1.30	1.15
P5-28	1.94	1.85	1.91	1.88	1.21	0.99
P5-29	2.06	2.06	2.03	2.01	1.32	1.07
P5-30	1.72	1.35	1.78	1.62	1.12	0.85
P5-31	2.85	2.90	0.65	3.02	0.96	1.18
P5-32	2.82	2.90	0.65	3.01	0.95	1.18
P5-33	2.78	2.90	0.60	2.88	0.91	1.07
P5-34	2.67	2.90	0.56	2.70	0.86	0.96
P5-35	2.53	2.72	0.51	2.50	0.80	0.86
P5-36	2.35	1.82	0.46	2.27	0.74	0.76
P5-37	2.16	1.28	0.41	2.03	0.69	0.67
P5-38	1.94	0.92	0.36	1.78	0.63	0.58
P5-39	2.06	2.06	0.39	1.88	0.68	0.63
P5-40	1.72	0.68	0.32	1.50	0.57	0.50

(*) All values are in MPa

Table B.6: Ultimate Capacities* (v_{xy}) for Load Case No. 6 ($f'_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P6-1	2.85	2.90	1.49	3.20	3.20	5.00
P6-2	2.06	2.06	2.61	2.68	2.68	8.60
P6-3	2.78	2.90	1.53	3.13	3.13	5.60
P6-4	2.67	2.90	1.59	3.06	3.06	5.71
P6-5	2.53	2.90	1.68	2.97	2.97	6.04
P6-6	2.35	2.90	3.57	2.85	2.85	6.86
P6-7	2.16	2.90	2.92	2.70	2.70	6.79
P6-8	1.94	2.90	2.38	2.52	2.52	7.17
P6-9	2.06	2.06	2.61	2.68	2.68	8.60
P6-10	1.72	2.71	1.92	2.31	2.31	10.75
P6-11	2.85	2.90	2.48	3.23	2.88	3.79
P6-12	2.06	2.06	2.76	2.65	2.65	7.60
P6-13	2.78	2.90	2.71	3.13	3.03	4.32
P6-14	2.67	2.90	2.98	3.05	3.05	5.02
P6-15	2.53	2.90	3.32	2.95	2.95	5.54
P6-16	2.35	2.90	3.74	2.83	2.83	6.04
P6-17	2.16	2.90	3.11	2.68	2.68	6.84
P6-18	1.94	2.90	2.50	2.50	2.50	6.35
P6-19	2.06	2.06	2.76	2.65	2.65	7.60
P6-20	1.72	2.71	2.00	2.28	2.28	9.50
P6-21	2.85	2.90	1.56	3.05	2.01	2.55
P6-22	2.06	2.06	3.02	2.60	2.60	6.60
P6-23	2.78	2.90	1.68	2.93	2.21	3.00
P6-24	2.67	2.90	1.95	2.84	2.33	3.27
P6-25	2.53	2.90	2.16	2.79	2.44	4.09
P6-26	2.35	2.90	2.43	2.78	2.63	5.04
P6-27	2.16	2.90	2.77	2.63	2.63	5.77
P6-28	1.94	2.90	2.72	2.45	2.45	6.81
P6-29	2.06	2.06	3.02	2.60	2.60	6.60
P6-30	1.72	2.71	2.14	2.23	2.23	8.25
P6-31	2.85	2.90	0.77	2.84	1.10	1.49
P6-32	2.06	2.06	1.67	2.18	1.96	5.83
P6-33	2.78	2.90	0.84	2.67	1.19	1.67
P6-34	2.67	2.90	0.91	2.50	1.35	1.89
P6-35	2.53	2.90	1.00	2.33	1.54	2.19
P6-36	2.35	2.90	1.17	2.18	1.62	2.57
P6-37	2.16	2.90	1.33	2.05	1.66	3.58
P6-38	1.94	2.90	1.54	1.98	1.77	5.00
P6-39	2.06	2.06	1.67	2.18	1.96	5.83
P6-40	1.72	2.71	1.82	1.99	1.80	7.00

(*) All values are in MPa

Table B.7: Ultimate Capacities* (v_{xy}) for Load Case No. 7 ($f_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P7-1	2.86	2.90	3.35	4.11	2.86	3.12
P7-2	2.86	2.90	2.51	5.74	3.69	3.12
P7-3	2.86	2.90	2.38	5.05	3.38	3.12
P7-4	2.86	2.90	2.26	4.50	3.12	3.12
P7-5	2.86	2.90	2.14	4.05	2.90	3.12
P7-6	2.73	2.90	3.22	3.86	3.86	2.90
P7-7	2.73	2.90	2.44	5.77	3.69	2.90
P7-8	2.73	2.90	2.32	5.07	3.39	2.90
P7-9	2.73	2.90	2.21	4.51	3.13	2.90
P7-10	2.73	2.90	2.09	4.07	2.90	2.90
P7-11	2.86	2.90	3.35	3.85	3.85	2.36
P7-12	2.86	2.90	2.51	5.69	6.40	2.36
P7-13	2.86	2.90	2.38	5.00	7.04	2.36
P7-14	2.86	2.90	2.26	4.46	7.72	2.36
P7-15	2.86	2.90	2.14	4.02	8.39	2.36
P7-16	2.73	2.90	3.22	3.61	3.61	2.21
P7-17	2.73	2.90	2.44	5.72	5.85	2.21
P7-18	2.73	2.90	2.32	5.02	6.42	2.21
P7-19	2.73	2.90	2.21	4.48	7.05	2.21
P7-20	2.73	2.90	2.09	4.04	7.72	2.21
P7-21	2.86	2.90	3.35	3.46	3.22	1.67
P7-22	2.86	2.90	2.51	5.30	5.30	1.67
P7-23	2.86	2.90	2.38	4.95	5.82	1.67
P7-24	2.86	2.90	1.30	4.41	6.44	1.67
P7-25	2.86	2.90	1.31	3.98	7.18	1.67
P7-26	2.73	2.90	3.22	3.24	2.98	1.56
P7-27	2.73	2.90	2.44	4.81	4.81	1.56
P7-28	2.73	2.90	2.32	4.97	5.24	1.56
P7-29	2.73	2.90	2.21	4.43	5.73	1.56
P7-30	2.73	2.90	1.23	4.00	6.32	1.56
P7-31	2.86	2.90	3.35	2.84	1.46	0.97
P7-32	2.86	2.90	0.61	3.21	2.01	0.97
P7-33	2.86	2.90	0.61	3.30	2.11	0.97
P7-34	2.86	2.90	0.62	3.38	2.22	0.97
P7-35	2.86	2.90	0.62	3.47	2.33	0.97
P7-36	2.73	2.90	3.22	2.64	1.36	0.91
P7-37	2.73	2.90	0.57	2.93	1.83	0.91
P7-38	2.73	2.90	0.58	3.00	1.92	0.91
P7-39	2.73	2.90	0.58	3.07	2.01	0.91
P7-40	2.73	2.90	0.58	3.14	2.10	0.91

(*) All values are in MPa

Table B.8: Ultimate Capacities* (v_{xy}) for Load Case No. 8 ($f'_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P8-1	2.86	2.90	4.80	4.35	4.35	6.57
P8-2	2.86	2.90	4.34	4.81	4.81	4.21
P8-3	2.86	2.90	1.89	4.83	4.83	3.79
P8-4	2.86	2.90	1.69	4.50	4.83	3.49
P8-5	2.86	2.90	1.52	4.05	4.82	3.24
P8-6	2.73	2.90	4.99	4.18	4.18	6.57
P8-7	2.73	2.90	4.36	4.72	4.72	4.21
P8-8	2.73	2.90	1.90	4.75	4.75	3.79
P8-9	2.73	2.90	1.69	4.51	4.78	3.49
P8-10	2.73	2.90	1.52	4.07	4.78	3.24
P8-11	2.86	2.90	4.72	4.31	4.31	6.58
P8-12	2.86	2.90	3.29	4.36	4.74	3.21
P8-13	2.86	2.90	2.93	3.82	4.75	2.93
P8-14	2.86	2.90	2.64	3.40	4.76	2.70
P8-15	2.86	2.90	2.40	3.07	4.76	2.50
P8-16	2.73	2.90	4.87	4.14	4.14	6.58
P8-17	2.73	2.90	3.31	4.37	4.65	3.21
P8-18	2.73	2.90	2.95	3.84	4.69	2.93
P8-19	2.73	2.90	2.65	3.41	4.71	2.70
P8-20	2.73	2.90	2.41	3.07	4.73	2.50
P8-21	2.86	2.90	4.43	4.27	4.27	5.33
P8-22	2.73	2.90	2.22	2.95	4.66	2.26
P8-23	2.41	2.90	1.97	2.59	4.69	2.07
P8-24	2.15	2.90	1.77	2.30	4.72	1.91
P8-25	1.94	2.90	1.61	2.07	5.32	1.77
P8-26	2.73	2.90	4.54	4.10	4.10	5.33
P8-27	2.73	2.90	2.23	2.96	4.59	2.26
P8-28	2.42	2.90	1.98	2.60	4.64	2.07
P8-29	2.16	2.90	1.78	2.31	4.68	1.91
P8-30	1.95	2.90	1.62	2.08	4.72	1.77
P8-31	2.86	2.90	4.80	4.21	4.21	3.05
P8-32	1.37	2.90	1.13	1.37	5.48	1.32
P8-33	1.20	2.90	1.00	1.20	5.89	1.21
P8-34	1.07	2.90	0.89	1.07	6.42	1.11
P8-35	0.97	2.90	0.80	0.97	7.14	1.03
P8-36	2.73	2.90	4.86	4.05	4.05	3.05
P8-37	1.38	2.90	1.14	1.38	5.36	1.32
P8-38	1.21	2.90	1.00	1.21	5.77	1.21
P8-39	1.08	2.90	0.90	1.08	6.27	1.11
P8-40	0.98	2.90	0.81	0.98	6.91	1.03

(*) All values are in MPa

Table B.9: Ultimate Capacities* (v_{xy}) for Load Case No. 9 ($f_c = 15$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P9-1	3.21	3.21	2.67	3.64	3.64	4.46
P9-2	3.33	3.33	2.69	3.86	3.86	5.30
P9-3	3.37	3.37	2.68	3.93	3.93	5.65
P9-4	3.35	3.35	2.08	3.85	3.85	5.79
P9-5	3.30	3.30	2.12	3.79	3.79	6.24
P9-6	3.21	3.21	2.17	3.73	3.73	6.86
P9-7	3.11	3.11	5.64	3.66	3.66	6.87
P9-8	3.00	3.00	6.12	3.59	3.59	6.80
P9-9	2.94	2.94	6.17	3.55	3.55	6.67
P9-10	2.89	2.89	5.75	3.50	3.50	6.35
P9-11	3.21	3.21	2.67	3.60	3.20	3.40
P9-12	3.33	3.33	2.69	3.82	3.45	4.03
P9-13	3.37	3.37	2.68	3.92	3.70	4.49
P9-14	3.35	3.35	2.99	3.89	3.89	5.32
P9-15	3.30	3.30	3.46	3.79	3.79	5.65
P9-16	3.21	3.21	3.78	3.71	3.71	6.04
P9-17	3.11	3.11	4.13	3.64	3.64	6.20
P9-18	3.00	3.00	4.48	3.57	3.57	6.58
P9-19	2.94	2.94	4.69	3.53	3.53	6.92
P9-20	2.89	2.89	4.76	3.47	3.47	7.17
P9-21	3.21	3.21	2.67	3.50	2.25	2.31
P9-22	3.33	3.33	2.69	3.71	2.43	2.67
P9-23	3.37	3.37	2.68	3.83	2.61	3.23
P9-24	3.35	3.35	1.97	3.86	2.81	3.71
P9-25	3.30	3.30	2.14	3.79	3.06	4.33
P9-26	3.21	3.21	2.45	3.68	3.23	5.04
P9-27	3.11	3.11	2.67	3.60	3.36	5.55
P9-28	3.00	3.00	2.89	3.51	3.46	5.82
P9-29	2.94	2.94	3.02	3.48	3.48	6.06
P9-30	2.89	2.89	3.07	3.42	3.42	6.17
P9-31	3.21	3.21	0.73	3.21	1.23	1.35
P9-32	3.33	3.33	0.81	3.41	1.35	1.56
P9-33	3.37	3.37	0.89	3.54	1.46	1.77
P9-34	3.35	3.35	0.97	3.61	1.57	2.01
P9-35	3.30	3.30	1.06	3.61	1.71	2.30
P9-36	3.21	3.21	1.14	3.48	1.86	2.59
P9-37	3.11	3.11	1.22	3.32	2.02	3.25
P9-38	3.00	3.00	1.31	3.18	2.19	3.85
P9-39	2.94	2.94	1.43	3.13	2.27	3.77
P9-40	2.89	2.89	1.46	3.05	2.25	4.25

(*) All values are in MPa

Table B.10: Ultimate Capacities* (v_{xy}) for Load Case No. 1 ($f'_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P1-1	7.50	7.50	6.60	6.63	6.63	7.71
P1-2	6.67	6.67	6.13	5.89	5.89	6.67
P1-3	5.71	5.71	4.94	5.36	5.36	5.82
P1-4	5.71	5.71	7.12	6.63	6.63	6.94
P1-5	5.71	5.71	7.12	6.63	5.71	11.86
P1-6	7.50	7.50	10.92	9.07	9.07	11.89
P1-7	4.44	4.44	3.51	4.44	4.44	5.28
P1-8	4.44	4.44	3.90	6.25	6.25	5.25
P1-9	4.44	4.44	3.90	6.25	4.44	8.32
P1-10	7.50	7.50	11.94	12.59	12.59	14.12
P1-11	6.00	6.00	5.00	6.60	6.60	7.92
P1-12	5.00	5.00	4.88	5.76	5.76	6.57
P1-13	4.29	4.29	3.89	4.93	4.93	5.37
P1-14	4.29	4.29	4.46	6.10	6.49	5.33
P1-15	5.71	5.71	7.12	6.79	6.82	11.03
P1-16	7.50	7.50	10.92	9.12	9.12	11.03
P1-17	3.33	3.33	2.72	3.82	3.82	4.09
P1-18	3.33	3.33	3.90	4.94	6.50	4.09
P1-19	4.44	4.44	3.90	5.92	7.08	7.98
P1-20	7.50	7.50	11.94	12.85	12.85	13.36
P1-21	4.00	4.00	4.76	5.66	5.77	5.53
P1-22	3.33	3.33	3.53	4.66	4.87	4.53
P1-23	2.86	2.86	2.78	3.94	4.26	3.81
P1-24	2.86	2.86	3.14	4.61	5.61	3.81
P1-25	5.71	5.71	7.12	5.99	6.19	9.12
P1-26	6.67	6.67	10.92	9.15	9.15	9.12
P1-27	2.22	2.22	1.89	3.00	3.00	2.93
P1-28	2.22	2.22	2.33	3.55	5.69	2.93
P1-29	4.44	4.44	3.90	5.50	6.83	7.23
P1-30	7.50	7.50	11.94	13.30	13.30	13.27
P1-31	2.00	2.00	2.83	4.00	4.32	3.30
P1-32	1.67	1.67	2.02	3.17	3.69	2.71
P1-33	1.43	1.43	1.55	2.59	3.28	2.30
P1-34	1.43	1.43	1.71	2.82	4.19	2.30
P1-35	3.33	3.33	7.12	4.89	4.96	6.82
P1-36	3.33	3.33	10.92	7.35	7.36	6.02
P1-37	1.11	1.11	1.00	1.85	1.85	1.76
P1-38	1.11	1.11	1.21	1.99	4.48	1.76
P1-39	4.44	4.44	13.55	4.91	6.32	6.37
P1-40	7.50	7.50	6.25	14.57	14.57	13.02

(*) All values are in MPa

Table B.11: Ultimate Capacities* (v_{xy}) for Load Case No. 2 ($f'_c = 25$ Mpa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P2-1	6.38	6.38	6.51	7.94	7.94	6.89
P2-2	7.09	7.09	7.14	8.40	8.40	7.18
P2-3	6.74	6.74	7.70	8.80	8.80	7.47
P2-4	6.37	6.37	8.22	9.09	9.09	7.78
P2-5	6.00	6.00	8.72	9.28	9.28	7.92
P2-6	5.65	5.65	9.12	9.34	9.34	8.09
P2-7	5.33	5.33	9.53	9.29	9.29	8.23
P2-8	5.03	5.03	9.96	9.15	9.15	8.16
P2-9	4.75	4.75	10.25	8.99	8.99	9.02
P2-10	4.50	4.50	10.55	8.65	8.65	5.25
P2-11	4.79	4.79	5.03	7.07	7.69	5.50
P2-12	5.59	5.59	5.51	7.78	8.12	5.70
P2-13	6.38	6.38	5.96	8.47	8.49	5.96
P2-14	6.37	6.37	6.37	8.78	8.78	6.06
P2-15	6.00	6.00	6.76	8.97	8.97	6.18
P2-16	5.65	5.65	7.08	9.05	9.05	6.29
P2-17	5.33	5.33	7.41	9.04	9.04	7.03
P2-18	5.03	5.03	7.75	8.93	8.93	6.97
P2-19	4.75	4.75	7.99	8.74	8.74	8.88
P2-20	4.50	4.50	8.23	8.52	8.52	3.76
P2-21	3.19	3.19	3.48	5.12	7.26	3.93
P2-22	3.72	3.72	3.83	5.60	7.65	4.08
P2-23	4.26	4.26	4.15	6.08	8.06	4.20
P2-24	4.79	4.79	4.44	6.55	8.34	4.30
P2-25	5.33	5.33	4.72	7.03	8.54	4.38
P2-26	5.65	5.65	4.95	7.47	8.66	4.46
P2-27	5.33	5.33	5.19	7.94	8.69	5.06
P2-28	5.03	5.03	5.44	8.47	8.64	4.85
P2-29	4.75	4.75	5.61	8.50	8.50	7.94
P2-30	4.50	4.50	5.79	8.30	8.30	7.92
P2-31	1.60	1.60	1.86	2.96	2.88	2.26
P2-32	1.86	1.86	2.05	3.24	3.24	2.37
P2-33	2.13	2.13	2.22	3.50	3.42	2.46
P2-34	2.39	2.39	2.39	3.74	3.72	2.53
P2-35	2.67	2.67	2.55	4.00	3.95	2.59
P2-36	2.92	2.92	2.68	4.22	4.12	2.64
P2-37	3.19	3.19	2.81	4.46	4.32	2.69
P2-38	3.48	3.48	2.96	4.73	4.73	3.02
P2-39	3.72	3.72	3.06	4.93	4.92	2.92
P2-40	3.97	3.97	3.16	5.14	5.06	4.25

(*) All values are in MPa

Table B.12: Ultimate Capacities* (v_{xy}) for Load Case No. 3 ($f'_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P3-1	6.38	6.38	8.07	8.92	9.29	21.50
P3-2	5.88	5.88	9.31	6.02	6.02	7.17
P3-3	7.09	7.09	11.40	9.63	9.63	8.96
P3-4	6.74	6.74	8.09	10.78	11.96	11.18
P3-5	6.37	6.37	7.91	11.72	13.17	9.94
P3-6	6.40	6.40	8.91	6.17	6.17	7.92
P3-7	6.00	6.00	12.57	9.87	9.87	8.96
P3-8	5.65	5.65	7.42	13.59	13.69	7.68
P3-9	6.61	6.61	8.74	6.23	6.23	7.36
P3-10	5.33	5.33	13.18	9.51	9.51	8.16
P3-11	6.38	6.38	15.39	8.47	10.38	19.00
P3-12	4.41	4.41	4.30	5.64	5.64	5.54
P3-13	7.09	7.09	9.01	9.35	9.35	7.92
P3-14	6.74	6.74	8.09	10.34	14.09	10.71
P3-15	6.37	6.37	7.91	11.29	15.54	9.14
P3-16	4.80	4.80	4.80	6.00	6.10	6.06
P3-17	6.00	6.00	9.93	9.61	9.61	7.92
P3-18	5.65	5.65	7.42	13.19	13.90	5.87
P3-19	4.96	4.96	5.01	6.14	6.17	6.98
P3-20	5.33	5.33	10.43	9.31	9.31	7.98
P3-21	6.38	6.38	8.22	7.94	12.59	16.50
P3-22	2.94	2.94	3.06	4.40	4.94	3.93
P3-23	5.60	5.60	6.47	8.92	8.92	5.47
P3-24	6.74	6.74	4.25	9.84	18.46	9.07
P3-25	6.37	6.37	3.44	10.80	17.90	6.41
P3-26	3.20	3.20	3.45	4.74	5.14	4.30
P3-27	6.00	6.00	7.13	9.24	9.24	5.47
P3-28	5.65	5.65	2.50	12.74	12.76	3.93
P3-29	3.31	3.31	3.61	4.88	5.22	5.92
P3-30	5.33	5.33	7.49	9.03	9.03	6.88
P3-31	6.38	6.38	3.19	7.28	21.28	14.00
P3-32	1.47	1.47	1.70	2.81	3.73	2.37
P3-33	2.80	2.80	3.68	5.64	7.94	3.30
P3-34	6.74	6.74	1.87	9.24	14.54	5.02
P3-35	6.37	6.37	1.55	10.24	8.19	3.45
P3-36	1.60	1.60	1.94	3.12	3.86	2.59
P3-37	4.00	4.00	4.04	6.47	8.53	3.30
P3-38	5.65	5.65	1.16	9.40	4.36	2.37
P3-39	1.65	1.65	2.05	3.25	3.91	3.98
P3-40	4.80	4.80	4.25	7.06	8.55	4.76

(*) All values are in MPa

Table B.13: Ultimate Capacities* (v_{xy}) for Load Case No. 4 ($f_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P4-1	3.13	3.75	2.66	2.86	2.51	2.86
P4-2	3.00	3.75	2.51	2.67	2.35	2.67
P4-3	2.88	3.75	2.37	2.50	2.21	2.51
P4-4	2.78	3.75	2.24	2.35	2.08	2.36
P4-5	2.92	3.75	2.45	2.62	2.15	2.60
P4-6	2.94	3.75	2.44	2.58	2.23	2.71
P4-7	3.06	3.75	2.59	2.76	2.38	2.93
P4-8	2.83	3.75	2.30	2.43	2.10	2.42
P4-9	2.92	3.75	2.34	2.62	2.54	3.14
P4-10	3.06	3.75	2.34	2.76	2.59	3.23
P4-11	3.13	3.75	1.67	2.57	1.93	2.44
P4-12	3.00	3.75	1.58	2.40	1.80	2.28
P4-13	2.88	3.75	1.50	2.25	1.69	2.14
P4-14	2.78	3.75	1.42	2.12	1.59	2.02
P4-15	2.92	3.75	1.43	2.44	1.65	2.02
P4-16	2.94	3.75	1.50	2.36	1.71	2.14
P4-17	3.06	3.75	1.58	2.52	1.83	2.28
P4-18	2.83	3.75	1.43	2.22	1.61	2.02
P4-19	2.92	3.75	1.74	2.27	1.98	2.44
P4-20	3.06	3.75	1.67	2.45	1.90	2.44
P4-21	3.13	3.75	1.11	2.40	1.34	1.75
P4-22	3.00	3.75	1.05	2.25	1.25	1.63
P4-23	2.88	3.33	1.00	2.12	1.17	1.53
P4-24	2.78	2.86	0.95	2.01	1.10	1.45
P4-25	2.92	2.86	0.95	2.33	1.15	1.45
P4-26	2.94	3.33	1.00	2.23	1.18	1.53
P4-27	3.06	3.75	1.05	2.38	1.27	1.63
P4-28	2.83	2.86	0.95	2.11	1.11	1.45
P4-29	2.92	3.75	1.11	2.07	1.28	1.75
P4-30	3.06	3.75	1.11	2.28	1.32	1.75
P4-31	3.13	2.50	0.55	2.31	0.73	1.05
P4-32	3.00	2.00	0.52	2.18	0.68	0.98
P4-33	2.88	1.67	0.50	2.06	0.63	0.92
P4-34	2.78	1.43	0.47	1.95	0.59	0.87
P4-35	2.92	1.43	0.47	2.28	0.62	0.87
P4-36	2.94	1.67	0.50	2.17	0.64	0.92
P4-37	3.06	2.00	0.52	2.30	0.69	0.98
P4-38	2.83	1.43	0.47	2.05	0.60	0.87
P4-39	2.92	2.50	0.55	1.98	0.69	1.05
P4-40	3.06	2.50	0.55	2.19	0.72	1.05

(*) All values are MPa

Table B.14: Ultimate Capacities* (v_{xy}) for Load Case No. 5 ($f'_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P5-1	3.68	3.75	3.48	3.97	3.22	3.81
P5-2	3.64	3.75	3.40	3.85	3.17	3.79
P5-3	3.59	3.75	3.49	4.07	3.06	3.45
P5-4	3.45	3.75	3.50	4.24	2.90	3.10
P5-5	3.26	3.75	3.48	4.08	2.74	2.77
P5-6	3.04	3.75	3.43	3.61	2.60	2.45
P5-7	2.78	3.75	3.33	3.17	2.46	2.16
P5-8	2.51	3.69	3.18	2.77	2.32	1.87
P5-9	2.66	2.66	3.39	2.97	2.53	2.02
P5-10	2.22	2.71	2.96	2.42	2.19	1.60
P5-11	3.68	3.75	1.98	3.65	2.50	2.97
P5-12	3.64	3.75	1.97	3.52	2.46	2.95
P5-13	3.59	3.75	1.84	3.82	2.37	2.68
P5-14	3.45	3.75	1.69	4.05	2.25	2.42
P5-15	3.26	3.75	1.54	4.04	2.12	2.16
P5-16	3.04	3.75	3.43	3.59	2.01	1.91
P5-17	2.78	3.75	3.33	3.15	1.89	1.68
P5-18	2.51	2.77	3.18	2.74	1.78	1.46
P5-19	2.66	2.66	3.39	2.93	1.93	1.57
P5-20	2.22	2.03	2.96	2.36	1.66	1.24
P5-21	3.68	3.75	1.32	3.43	1.75	2.12
P5-22	3.64	3.75	1.31	3.30	1.72	2.11
P5-23	3.59	3.75	1.22	3.65	1.66	1.92
P5-24	3.45	3.75	1.12	3.91	1.57	1.73
P5-25	3.26	3.75	1.02	3.92	1.48	1.54
P5-26	3.04	3.64	0.93	3.51	1.39	1.37
P5-27	2.78	2.56	0.83	3.09	1.30	1.20
P5-28	2.51	1.85	0.73	2.68	1.21	1.04
P5-29	2.66	2.66	0.78	2.85	1.32	1.12
P5-30	2.22	1.35	2.96	2.29	1.12	0.89
P5-31	3.68	3.75	0.65	3.30	0.96	1.28
P5-32	3.64	3.75	0.65	3.17	0.95	1.27
P5-33	3.59	3.75	0.60	3.53	0.91	1.16
P5-34	3.45	3.75	0.56	3.81	0.86	1.04
P5-35	3.26	2.72	0.51	3.59	0.80	0.93
P5-36	3.04	1.82	0.46	3.24	0.74	0.82
P5-37	2.78	1.28	0.41	2.88	0.69	0.72
P5-38	2.51	0.92	0.36	2.51	0.63	0.63
P5-39	2.66	2.66	0.39	2.66	0.68	0.68
P5-40	2.22	0.68	0.32	2.12	0.57	0.54

(*) All values are in MPa

Table B.15: Ultimate Capacities* (v_{xy}) for Load Case No. 6 ($f'_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P6-1	3.68	3.75	3.35	3.97	3.80	4.97
P6-2	2.66	2.66	2.61	4.21	4.21	11.58
P6-3	3.59	3.75	3.67	4.07	4.03	5.63
P6-4	3.45	3.75	4.04	4.24	4.22	6.28
P6-5	3.26	3.75	4.40	4.49	4.44	7.68
P6-6	3.04	3.75	3.57	4.44	4.44	9.26
P6-7	2.78	3.75	2.92	4.24	4.24	9.94
P6-8	2.51	3.69	2.38	3.97	3.97	11.18
P6-9	2.66	2.66	2.61	4.21	4.21	11.58
P6-10	2.22	2.71	1.92	3.65	3.65	11.99
P6-11	3.68	3.75	2.48	3.42	2.96	3.74
P6-12	2.66	2.66	2.76	4.17	4.17	11.72
P6-13	3.59	3.75	2.71	3.41	3.18	4.16
P6-14	3.45	3.75	2.98	3.45	3.38	4.97
P6-15	3.26	3.75	3.32	3.56	3.53	5.87
P6-16	3.04	3.75	3.74	3.76	3.72	7.25
P6-17	2.78	3.75	3.11	4.05	3.83	9.14
P6-18	2.51	3.69	2.50	3.92	3.85	10.71
P6-19	2.66	2.66	2.76	4.17	4.17	11.72
P6-20	2.22	2.71	2.00	3.60	3.60	11.77
P6-21	3.68	3.75	1.56	3.05	2.01	2.68
P6-22	2.66	2.66	3.02	3.41	3.24	9.94
P6-23	3.59	3.75	1.68	2.93	2.25	3.00
P6-24	3.45	3.75	1.95	2.84	2.44	3.38
P6-25	3.26	3.75	2.16	2.79	2.62	3.90
P6-26	3.04	3.75	2.43	2.80	2.75	4.99
P6-27	2.78	3.75	2.77	2.91	2.90	6.41
P6-28	2.51	3.69	2.72	3.11	2.95	9.07
P6-29	2.66	2.66	3.02	3.41	3.24	9.94
P6-30	2.22	2.71	2.14	3.44	2.91	11.10
P6-31	3.68	3.75	0.77	2.84	1.10	1.61
P6-32	2.66	2.66	1.67	2.18	2.12	6.30
P6-33	3.59	3.75	0.84	2.67	1.19	1.80
P6-34	3.45	3.75	0.91	2.50	1.35	2.05
P6-35	3.26	3.75	1.00	2.33	1.54	2.37
P6-36	3.04	3.75	1.17	2.18	1.70	2.78
P6-37	2.78	3.75	1.33	2.05	1.84	3.41
P6-38	2.51	3.69	1.54	1.98	1.92	5.02
P6-39	2.66	2.66	1.67	2.18	2.12	6.30
P6-40	2.22	2.71	1.82	1.99	1.97	8.52

(*) All values are in MPa

Table B.16: Ultimate Capacities* (v_{xy}) for Load Case No. 7 ($f_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P7-1	3.69	3.75	5.59	6.14	6.14	3.14
P7-2	3.69	3.75	4.18	5.74	3.69	3.14
P7-3	3.69	3.75	3.97	5.05	3.38	3.14
P7-4	3.69	3.75	3.76	4.50	3.12	3.14
P7-5	3.69	3.75	3.56	4.05	2.90	3.14
P7-6	3.53	3.75	5.37	5.75	5.75	2.93
P7-7	3.53	3.75	4.07	5.77	3.69	2.93
P7-8	3.53	3.75	3.87	5.07	3.39	2.93
P7-9	3.53	3.75	3.68	4.51	3.13	2.93
P7-10	3.53	3.75	3.49	4.07	2.90	2.93
P7-11	3.69	3.75	5.59	5.64	5.26	2.44
P7-12	3.69	3.75	4.18	5.69	9.89	2.44
P7-13	3.69	3.75	2.01	5.00	11.04	2.44
P7-14	3.69	3.75	2.03	4.46	12.33	2.44
P7-15	3.69	3.75	2.05	4.02	13.67	2.44
P7-16	3.53	3.75	5.37	5.27	4.85	2.28
P7-17	3.53	3.75	4.07	5.72	8.94	2.28
P7-18	3.53	3.75	1.88	5.02	9.93	2.28
P7-19	3.53	3.75	1.90	4.48	11.07	2.28
P7-20	3.53	3.75	1.92	4.04	12.34	2.28
P7-21	3.69	3.75	5.59	4.96	3.22	1.75
P7-22	3.69	3.75	1.28	5.63	5.80	1.75
P7-23	3.69	3.75	1.29	4.95	6.51	1.75
P7-24	3.69	3.75	1.30	4.41	7.36	1.75
P7-25	3.69	3.75	1.31	3.98	8.39	1.75
P7-26	3.53	3.75	5.37	4.64	2.98	1.63
P7-27	3.53	3.75	1.20	5.66	5.12	1.63
P7-28	3.53	3.75	1.21	4.97	5.67	1.63
P7-29	3.53	3.75	1.22	4.43	6.31	1.63
P7-30	3.53	3.75	1.23	4.00	7.06	1.63
P7-31	3.69	3.75	5.59	4.00	1.46	1.05
P7-32	3.69	3.75	0.61	4.55	2.01	1.05
P7-33	3.69	3.75	0.61	4.68	2.11	1.05
P7-34	3.69	3.75	0.62	4.36	2.22	1.05
P7-35	3.69	3.75	0.62	3.94	2.33	1.05
P7-36	3.53	3.75	5.37	3.71	1.36	0.98
P7-37	3.53	3.75	0.57	4.15	1.83	0.98
P7-38	3.53	3.75	0.58	4.26	1.92	0.98
P7-39	3.53	3.75	0.58	4.37	2.01	0.98
P7-40	3.53	3.75	0.58	3.96	2.10	0.98

(*) All values are in MPa

Table B.17: Ultimate Capacities* (v_{xy}) for Load Case No. 8 ($f'_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P8-1	3.69	3.75	14.80	7.19	7.19	8.96
P8-2	3.69	3.75	4.34	5.74	7.93	4.26
P8-3	3.69	3.75	3.87	5.05	7.89	3.89
P8-4	3.69	3.75	3.49	4.50	7.82	3.58
P8-5	3.69	3.75	3.18	4.05	7.73	3.32
P8-6	3.53	3.75	14.99	6.88	6.88	8.96
P8-7	3.53	3.75	4.36	5.77	7.81	4.26
P8-8	3.53	3.75	3.89	5.07	7.82	3.89
P8-9	3.53	3.75	3.51	4.51	7.79	3.58
P8-10	3.53	3.75	3.20	4.07	7.74	3.32
P8-11	3.69	3.75	11.72	7.13	7.13	7.92
P8-12	3.69	3.75	3.29	4.36	7.78	3.32
P8-13	3.61	3.75	2.93	3.82	7.75	3.03
P8-14	3.22	3.75	2.64	3.40	7.69	2.79
P8-15	2.91	3.75	2.40	3.07	7.62	2.59
P8-16	3.53	3.75	11.87	6.83	6.83	7.92
P8-17	3.53	3.75	3.31	4.37	7.68	3.32
P8-18	3.53	3.75	2.95	3.84	7.69	3.03
P8-19	3.24	3.75	2.65	3.41	7.68	2.79
P8-20	2.93	3.75	2.41	3.07	7.64	2.59
P8-21	3.69	3.75	8.43	7.06	7.06	5.47
P8-22	2.73	3.75	2.22	2.95	7.65	2.37
P8-23	2.41	3.75	1.97	2.59	7.64	2.17
P8-24	2.15	3.75	1.77	2.30	7.62	2.00
P8-25	1.94	3.75	1.61	2.07	8.60	1.85
P8-26	3.53	3.75	8.54	6.77	6.77	5.53
P8-27	2.76	3.75	2.23	2.96	7.57	2.37
P8-28	2.42	3.75	1.98	2.60	7.60	2.17
P8-29	2.16	3.75	1.78	2.31	7.61	2.00
P8-30	1.95	3.75	1.62	2.08	7.61	1.85
P8-31	3.69	3.75	4.80	6.98	6.98	3.30
P8-32	1.37	3.75	1.13	1.37	9.17	1.43
P8-33	1.20	3.75	1.00	1.20	9.88	1.31
P8-34	1.07	3.75	0.89	1.07	10.77	1.20
P8-35	0.97	3.75	0.80	0.97	11.89	1.12
P8-36	3.53	3.75	4.86	6.69	6.69	3.30
P8-37	1.38	3.75	1.14	1.38	8.94	1.43
P8-38	1.21	3.75	1.00	1.21	9.61	1.31
P8-39	1.08	3.75	0.90	1.08	10.45	1.20
P8-40	0.98	3.75	0.81	0.98	11.52	1.12

(*) All values are in MPa

Table B.18: Ultimate Capacities* (v_{xy}) for Load Case No. 9 ($f_c = 25$ MPa)

Panel	Nielsen	Clark	Fialkow	Zararis	Mod. Zar.	MCFT
P9-1	4.15	4.15	4.45	5.43	4.12	4.34
P9-2	4.29	4.29	4.48	5.61	4.44	5.21
P9-3	4.35	4.35	4.47	5.52	4.77	6.03
P9-4	4.33	4.33	4.27	5.54	5.16	6.98
P9-5	4.26	4.26	4.72	5.65	5.50	8.13
P9-6	4.15	4.15	5.16	5.76	5.76	9.26
P9-7	4.02	4.02	5.64	5.69	5.69	9.76
P9-8	3.87	3.87	6.12	5.59	5.59	10.54
P9-9	3.80	3.80	6.17	5.55	5.55	10.70
P9-10	3.73	3.73	5.75	5.47	5.47	10.47
P9-11	4.15	4.15	2.25	5.35	3.20	3.38
P9-12	4.29	4.29	2.50	5.14	3.45	3.91
P9-13	4.35	4.35	2.74	4.93	3.70	4.45
P9-14	4.33	4.33	2.99	4.82	4.01	5.34
P9-15	4.26	4.26	3.46	4.79	4.34	6.29
P9-16	4.15	4.15	3.78	4.80	4.58	7.25
P9-17	4.02	4.02	4.13	4.88	4.81	8.51
P9-18	3.87	3.87	4.48	4.99	4.98	9.16
P9-19	3.80	3.80	4.69	5.08	5.08	9.64
P9-20	3.73	3.73	4.76	5.06	5.06	9.79
P9-21	4.15	4.15	1.48	5.16	2.25	2.42
P9-22	4.29	4.29	1.65	4.79	2.43	2.80
P9-23	4.35	4.35	1.81	4.50	2.61	3.19
P9-24	4.33	4.33	1.97	4.27	2.81	3.62
P9-25	4.26	4.26	2.14	4.12	3.06	4.09
P9-26	4.15	4.15	2.45	4.00	3.30	4.99
P9-27	4.02	4.02	2.67	3.94	3.52	5.83
P9-28	3.87	3.87	2.89	3.91	3.69	6.60
P9-29	3.80	3.80	3.02	3.93	3.78	7.26
P9-30	3.73	3.73	3.07	3.88	3.78	7.74
P9-31	4.15	4.15	0.73	4.66	1.23	1.46
P9-32	4.29	4.29	0.81	4.54	1.35	1.69
P9-33	4.35	4.35	0.89	4.20	1.46	1.92
P9-34	4.33	4.33	0.97	3.91	1.57	2.18
P9-35	4.26	4.26	1.06	3.68	1.71	2.49
P9-36	4.15	4.15	1.14	3.48	1.86	2.81
P9-37	4.02	4.02	1.22	3.32	2.02	3.16
P9-38	3.87	3.87	1.31	3.18	2.19	3.57
P9-39	3.80	3.80	1.43	3.13	2.28	3.84
P9-40	3.73	3.73	1.46	3.05	2.30	3.94

(*) All values are in MPa

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