Fundamentals, Accuracy and Input Parameters of Frost Heave Prediction Models

by

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B.Sc. (Physical Geography)
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In partial fulfillment of the requirements
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Doctor of Philosophy

Department of Earth Sciences
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December 1997

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ABSTRACT

In this thesis, the frost heave knowledge of physical geographers and soil physicists, a detailed description of the frost heave process, methods to determine soil parameters, and analysis of the spatial variability of these soil parameters are connected to the expertise of civil engineers and mathematicians in the (computer) modelling of the process. A description is given of observations of frost heave in laboratory experiments and in the field. Frost heave modelling is made accessible by a detailed description of the main principles of frost heave modelling in a language which can be understood by persons who do not have a thorough mathematical background. Two examples of practical one- dimensional frost heave prediction models are described: a model developed by Wang (1994) and a model developed by Nixon (1991). Advantages, limitations and some improvements of these models are described.

It is suggested that conventional frost heave prediction using estimated extreme input parameters may be improved by using locally measured input parameters. The importance of accurate input parameters in frost heave prediction models is demonstrated in a case study using the frost heave models developed by Wang and Nixon. Methods to determine the input parameters are discussed, concluding with a suite of methods, some of which are new, to determine the input parameters of frost heave prediction models from very basic grain size parameters. The spatial variability of the required input parameters is analysed using data obtained along the Norman Wells - Zama oil pipeline at Norman Wells, NWT, located in the

transition between discontinuous and continuous permafrost regions at the northern end of Canada's northernmost oil pipeline. A method based on spatial variability analysis of the input parameters in frost heave models is suggested to optimize the improvement that arises from adequate sampling, while minimizing the costs of obtaining field data.

A series of frost heave predictions is made using a modified version of the model of Wang and the determined series of input data along the Norman Wells pipeline. The spatial variation in computed frost heave, an indicator of differential frost heave resulting from a spatial variation of input parameters, is discussed. The thesis concludes with an analysis of the sources of potential errors in, and an evaluation of the merits of frost heave prediction.

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TABLE OF CONTENTS

ABSTRACT	iii
ACKNOWLEDGEMENTS	v
TABLE OF CONTENTS	vii
LIST OF FIGURES.	xix
LIST OF TABLES.	xxvii
LIST OF SYMBOLS.	xxix
CHAPTER I	
INTRODUCTION.	1
1.1 Objectives	1
1.2 Method.	5
1.3 Organization of thesis	7

CHAPTER II**OBSERVATIONS AND MEASUREMENTS OF GROUND**

FREEZING AND FROST HEAVE.	10
II.1 Introduction	10
II.2 Ice formation in pores and primary heave.	10
II.2.1 Thermal aspects of soil cooling.	10
II.2.2 Initial formation of ice.	13
II.2.3 Pore ice and primary heave	15
II.2.4 Water flow to the freezing front.	16
II.2.5 Exclusion of salts at the freezing front	19
II.3 Ice formation in lenses and secondary heave	21
II.3.1 Segregated ice and secondary heave	21
II.3.2 The freezing fringe.	24
II.3.3 Growth of the warmest ice lens.	29
II.3.4 Redistribution of soil moisture by regelation in the freezing fringe	31
II.4 Soil freezing and frost heave over time.	33
II.4.1 The frozen zone and tertiary heave	33
II.4.2 Freezing induced changes in macro and micro structure.	35
II.4.3 Penetration of the freezing front with time	35
II.4.4 Resulting soil profile	37

II.5	Multiple cycles of freezing and thawing.	38
II.5.1	Thawing, thaw-consolidation and settlement.	38
II.5.2	Multiple cycles of freezing and thawing	39
II.5.3	Stress situation in a soil during freezing and thawing.	41
II.5.4	Fabric stability and shear strength.	42
II.6	Relevance and control of frost heave.	43
II.6.1	Occurrence and effects of frost heave	43
II.6.2	Ice lens formation and frost heave around a buried pipeline.	43
II.6.3	Control of frost heave.	44
II.7	Discussion and summary.	49

CHAPTER III	PRACTICAL FROST HEAVE SIMULATION MODELS	51
III.1	Modelling of natural processes.	51
III.2	Principles of modelling ground freezing and frost heave	54
III.2.1	Heat transfer	54
III.2.2	Water flow.	57
III.2.3	Coupled heat and water flow in thermodynamic equilibrium	58
III.2.4	Salt exclusion, ice flow and frost heave criteria	61

III.2.5	Mechanics of a freezing soil	62
III.2.6	The resulting frost heave simulation model	68
III.2.7	Numerical solution of the frost heave simulation model.	70
III.2.8	Evaluation of frost heave simulation models	74
III.3	The frost heave prediction model of Wang (1994) .	77
III.3.1	Physical base of the model	77
III.3.2	Analytical solution of the model.	82
	III.3.2.1 Solution of the heat transfer equation	82
	III.3.2.2 Solution of the frost heave equation	83
III.3.3	Numerical approximation of the conductive heat flow equation.	85
III.3.4	Flow diagram and pseudocode.	94
III.3.5	Input parameters for the frost heave prediction model of Wang (1994)	98
III.3.6	Analysis of the sensitivity of the prediction to the input parameters	99
	III.3.6.1 Thermal parameters	100
	III.3.6.2 Hydraulic parameters.	102

III.3.6.3 Mechanical parameters and parameters related to the geometry of space and time	105
III.3.7 Remarks on, advantages and limitations of the model	107
III.3.8 Modifications to the model.	112
III.4 The frost heave prediction model of Nixon (1991)	113
III.4.1 Physical base of the model	113
III.4.1.1 Introduction.	113
III.4.1.2 Heat flow.	115
III.4.1.3 Water flow.	116
III.4.1.4 Thermodynamic equilibrium in the freezing fringe.	118
III.4.1.5 Frost heave criterion.	120
III.4.1.6 Distribution of phase change and in situ heave over the freezing fringe.	121
III.4.2 Numerical approximation of the water flow equation.	124
III.4.3 Input parameters for the frost heave prediction model of Nixon (1991)	131

III.4.4	Analysis of the sensitivity of the prediction to input parameters	132
III.4.4.1	The thermal input parameters	133
III.4.4.2	The hydraulic input parameters. . .	136
III.4.4.3	The mechanical input parameters and parameters related to the geometry of space	138
III.4.5	Remarks on, advantages and limitations of the model.	140
III.5	Comparison between the frost heave prediction model of Wang (1994) and the frost heave prediction model of Nixon (1991)	141
III.6	Discussion and summary	142

CHAPTER IV	METHODS TO DETERMINE VALUES AND SPATIAL VARIABILITY OF COMMON INPUT PARAMETERS FOR FROST HEAVE SIMULATION MODELS	146
IV.1	Introduction	146
IV.2	Methods of determination of common input parameters for frost heave prediction models	148
IV.2.1	Grain size.	148

IV.2.2 Thermal input parameters.	149
IV.2.2.1 Temperatures: initial temperature profile, surface boundary temperature, freezing temperature, segregation temperature	149
IV.2.2.2 Thermal conductivity	152
IV.2.2.3 Volumetric heat capacity	155
IV.2.2.4 Effect of latent heat: the apparent heat capacity.	155
IV.2.3 Hydraulic input parameters.	157
IV.2.3.1 Porosity and unfrozen water content	157
IV.2.3.2 Hydraulic conductivity	160
IV.2.3.3 Pore water pressures	167
IV.2.3.4 Density of water, density of ice, and salinity information.	168
IV.2.4 Mechanical input parameters.	168
IV.2.4.1 Wet bulk density of the soil.	168
IV.2.4.2 External stress/external pressure. .	169
IV.2.4.3 Stress partition factor and separation pressure	169
IV.2.4.4 Strains	170

IV.2.4.5 Moduli (Young's E, G etc),	
creep factors and Poisson ratio . . .	170
IV.2.5 Parameters related to the geometry of space	
and time	171
IV.2.5.1 Spatial step size, length of soil	
profile, length of freezing fringe, and	
depth of seasonal influence.	171
IV.2.5.2 Time step size and total time	172
IV.3 A method to optimize the amount of input data	
by analysis of the spatial variability of input parameters	
for frost heave prediction models.	173
IV.3.1 Spatial analysis using semi-variograms. . .	173
IV.3.2 Single semi-variogram models	175
IV.3.3 Choice of semi-variogram model	177
IV.3.4 Deviations of a single semi-variogram model	181
IV.3.5 Mapping the spatial variability	
using semivariance models	182
IV.4 Summary	182

CHAPTER V	VALUES AND SPATIAL VARIABILITY OF COMMON	
	INPUT PARAMETERS FOR FROST HEAVE PREDICTION	
	MODELS: NORMAN WELLS, NWT	184
V.1	Description of the sampling site and sampling at Norman Wells,NWT.	184
V.2	Input parameter values for the frost heave prediction models.	188
V.2.1	Thermal input data.	188
	V.2.1.1 Temperatures: initial temperature profile, surface boundary temperature, freezing temperature, segregation temperature	192
	V.2.1.2 Thermal conductivity	190
	V.2.1.3 Volumetric heat capacity	191
	V.2.1.4 Effect of latent heat: the apparent heat capacity	193
V.2.2	Hydraulic input data	195
	V.2.2.1 Porosity and unfrozen water content	195
	V.2.2.2 Hydraulic conductivity.	197
	V.2.2.3 Density of water and salinity information	199
V.2.3	Mechanical input data	199

V.2.3.1	Density of soil	199
V.2.3.2	External stress/pressure	200
V.2.3.3	Separation pressure	201
V.2.4	Data related to the geometry of space and time	201
V.2.4.1	Spatial step size, length of soil profile, length of freezing fringe and depth of seasonal influence.	201
V.2.4.2	Time step size and total time	201
V.3	Analysis of the spatial variability of common input parameters for the frost heave prediction models at Norman Wells.	202
V.3.1	Thermal input data	202
IV.3.1.1	Thermal conductivity.	202
IV.3.1.2	Volumetric heat capacity.	203
IV.3.1.3	Apparent heat capacity.	204
V.3.2	Hydraulic input data.	205
V.3.2.1	Porosity and unfrozen water content	205
V.3.2.2	Hydraulic conductivity.	206
V.3.3	Mechanical input data	207
V.3.3.1	Density of soil.	207

V.4	Discussion and summary.	208
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CHAPTER VI EVALUATION OF PREDICTIONS OF FROST HEAVE

	AT NORMAN WELLS, NWT	210
VI.1	Introduction	210
VI.2	Operationalization of the modified frost heave prediction model of Wang.	211
VI.3	Depth of freezing front penetration.	217
VI.4	Length of the freezing fringe.	220
VI.5	Cryosuction in frost heave simulation using the modified model of Wang.	222
VI.6	Frost heave predictions	225
VI.7	Spatial variability of the predicted frost heave as an indicator of differential heave.	231
VI.8	Sources of error in the frost heave prediction process	237
	VI.8.1 Introduction.	237
	VI.8.2 Errors in input data.	237
	VI.8.3 Errors in the modelling phase	238
	VI.8.4 Errors in numerical approximation procedure.	241

VI.8.5 Total error	242
VI.9 Discussion and summary.	243

CHAPTER VII	CONCLUSIONS	245
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VII.1 Fundamentals of ground freezing and frost heave .	245
VII.2 Modelling of the frost heave process	247
VII.3 Determination and spatial variability of input data for frost heave prediction models.	252
VII.4 Limitations to the accuracy of frost heave predictions: Problems in frost heave and the nature of their solutions	254
VII.5 Summary of contributions of this thesis	256
VII.6 Suggestions for further research	257

REFERENCES	260
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APPENDICES	283
-------------------	------------

Appendix A	283
Appendix B	299

List of Figures

Figure II.1	Example of an annual and daily temperature cycle superimposed on a climatic warming trend of 1°C per year, $T_{\text{av,old}}=5^{\circ}\text{C}$, $T_{\text{av,new}}=6^{\circ}\text{C}$, $T_{\text{annual,ampl}}=20^{\circ}\text{C}$, $T_{\text{daily,ampl}}=3^{\circ}\text{C}$	11
Figure II.2	Freezing of pure water and salt water at various temperatures in identical soil pores.	17
Figure II.3	Charge concentration and free energy components near a particle surface during freezing in a pore (modified after Koorevaer et al. 1983)	18
Figure II.4	Formation of ice and elevated salt concentrations in the remaining unfrozen water on the scale of a pore and a layer of soil	20
Figure II.5	Geometry, temperature and stress in the freezing fringe (Sheng and Knutsson 1991)	23
Figure II.6	Soil moisture redistribution in a pore and in the frozen soil	34
Figure II.7	Examples of preferential surfaces for ice lens growth	36
Figure II.8	Examples of measures to prevent damage by (differential) frost heave	47
Figure III.1	Curve of the deformation of soil (based on Vyalov 1986)	64
Figure III.2	Various shapes of elements (Naylor and Pande 1981)	71
Figure III.3	Mesh of a quadrilateral elements near a footing (source: Naylor and Pande 1981)	72

Figure III.4	Mesh of quadrilateral elements around a buried pipeline (Naylor and Pande 1981)	73
Figure III.5	Approximation of an actual temperature profile by discretization and linear interpolation	85
Figure III.6	Linear interpolation functions N_1 and N_2 over a spatial element with a length l (Wang 1994)	86
Figure III.7	Discretization of the soil temperature in time (Wang 1994).	90
Figure III.8	Flow chart of the thermodynamic part of the model of Wang (1994).	95
Figure III.9	Sensitivity of the frost heave prediction model of Wang to the thermal input parameters (relative frost heave predictions after 10 days)	100
Figure III.10	Sensitivity of the frost heave prediction model of Wang to the thermal input parameters (relative frost heave predictions after 50 days)	101
Figure III.11	Sensitivity of the frost heave prediction model of Wang to the thermal input parameters (relative frost heave predictions after 100 days)	101
Figure III.12	Sensitivity of the frost heave prediction model of Wang to the hydraulic input parameters (relative frost heave predictions after 10 days)	103
Figure III.13	Sensitivity of the frost heave prediction model of Wang to the hydraulic input parameters (relative frost heave predictions after 50 days)	103
Figure III.14	Sensitivity of the frost heave prediction model of Wang to the hydraulic input parameters (relative frost heave predictions after 100 days)	104

Figure III.15	Sensitivity of the frost heave prediction model of Wang to mechanic input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 10 days)	105
Figure III.16	Sensitivity of the frost heave prediction model of Wang to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 50 days)	106
Figure III.17	Sensitivity of the frost heave prediction model of Wang to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 100 days)	106
Figure III.18	Geometry of the freezing fringe in the model of Nixon (1991)	114
Figure III.19	Discretization of the freezing fringe by the model of Nixon (1991)	124
Figure III.20	Sensitivity of the frost heave prediction model of Nixon to the thermal input parameters (relative frost heave predictions after 10 days)	133
Figure III.21	Sensitivity of the frost heave prediction model of Nixon to the thermal input parameters (relative frost heave predictions after 50 days)	134
Figure III.22	Sensitivity of the frost heave prediction model of Nixon to the thermal input parameters (relative frost heave predictions after 100 days)	135
Figure III.23	Sensitivity of the frost heave prediction model of Nixon to the hydraulic input parameters (relative frost heave predictions after 10 days)	136
Figure III.24	Sensitivity of the frost heave prediction model of Nixon to the hydraulic input parameters (relative frost heave predictions after 50 days)	137

Figure III.25	Sensitivity of the frost heave prediction model of Nixon to the hydraulic input parameters (relative frost heave predictions after 100 days)	137
Figure III.26	Sensitivity of the frost heave prediction model of Nixon to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 10 days)	138
Figure III.27	Sensitivity of the frost heave prediction model of Nixon to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 50 days)	139
Figure III.28	Sensitivity of the frost heave prediction model of Nixon to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 100 days).	139
Figure III.29	Flow diagram of a frost heave model that can handle cooling and warming phases (Chang 1993)	144
Figure IV.1	Soil texture triangle (source: Koorevaer et al. 1983)	149
Figure IV.2	Four different soil surface temperature boundary options	151
Figure IV.3	Schematic representation of a semi-variogram (based on Dirksen 1986)	174
Figure V.1	Location of study area (based on MacInnes et al. 1989)	186
Figure V.2	The texture of the soil sampled at Norman Wells, NWT	187
Figure V.3	The thermal regime of a soil 175 km south of the fieldwork site (source data: Riseborough 1997)	189

Figure V.4	Semivariogram of the thermal conductivity of the soil near Norman Wells.	203
Figure V.5	Semivariogram of the volumetric heat capacity of soil samples from Norman Wells, NWT	204
Figure V.6	Semivariogram of the apparent heat capacity of soil samples from Norman Wells, NWT	205
Figure V.7	Semivariogram of the porosity of soil samples from Norman Wells, NWT	206
Figure V.8	Semivariogram of the hydraulic conductivity of soil samples from Norman Wells, NWT	207
Figure V.9	Semivariogram of the density of soil samples from Norman Wells, NWT	208
Figure VI.1	Hydraulic conductivity of three different soils at a temperature above 0°C under variable soil water pressures (modified from Hillel 1980)	211
Figure VI.2	Freezing temperature, segregation temperature and hydraulic conductivity of three different soils under variable temperatures below 0°C	212
Figure VI.3	Hydraulic conductivity of various soils as a function of temperature (Nixon 1991)	215
Figure VI.4	Penetration of the freezing front at sample points 10 and 11 at an initial temperature of +2°C at the soil surface to +6°C at 0.70m below the soil surface, and at a constant surface boundary temperature of -2°C.	218

Figure VI.5	Predicted depth of the freezing front after 110 and 300 days under 3 different scenarios.	219
Figure VI.6	Example of the variation in length of the freezing fringe during the freezing process.	221
Figure VI.7	Length of the freezing fringe at sample points 10 to 20 after 300 days, under scenarios A, B and C	221
Figure VI.8	Predicted cryosuction during freezing at sample points 10 to 15 (at 1m spacing intervals) for a scenario with initial temperature of +2°C at the soil surface to +6°C at 0.70m below the soil surface; and a stationary surface boundary temperature of -2°C	224
Figure VI.9	Predictions of frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from +2°C at the soil surface to +6°C from 70 to 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C	227
Figure VI.10	Predictions of frost heave near Norman Wells, using the adjusted model of Wang and initial soil temperatures ranging from +0.5°C at the soil surface to +0.5°C at 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C	228

- Figure VI.11 Predictions of frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a stationary surface boundary temperature of -10°C 229
- Figure VI.12 Predictions of frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a surface temperature decreasing over 75 days from -1°C to -15°C , and remaining stationary for 35 days at -15°C 230
- Figure VI.13 Predictions of differential frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+2^{\circ}\text{C}$ at the soil surface to $+6^{\circ}\text{C}$ from 70 to 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C 232
- Figure VI.14 Predictions of differential frost heave near Norman Wells, using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C 233
- Figure VI.15 Predictions of differential frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a stationary surface boundary temperature of -10°C 234

Figure VI.16 Predictions of differential frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from +0.5°C at the soil surface to +0.5°C at 100 cm below the soil surface, and a surface temperature decreasing over 75 days from -1°C to -15°C, and remaining stationary for 35 days at -15°C 235

List of Tables

Table III.1	Table of table headings of the comprehensive frost heave prediction model comparison table (Appendix A).	75
Table III.2	Table of table headings of the table to compare the type and location of application of numerical procedure (Appendix B).	76
Table III.3	Comparison between the models of Nixon (1991) and Wang (1994).	141
Table IV.1	Values of the fitting parameters A, B, C and D in eq.(IV.32) for the soils of Burt (1974)	166
Table V.1	Descriptive statistics of the thermal conductivities of soil samples from Norman Wells.	191
Table V.2	The coefficient of variation of the thermal conductivity of soil samples from Norman Wells.	191
Table V.3	Descriptive statistics of the volumetric heat capacity of soil samples from Norman Wells.	192
Table V.4	Coefficient of variation of the volumetric heat capacity of soil samples from Norman Wells.	193
Table V.5	Descriptive statistics of the apparent volumetric heat capacity of soil samples from Norman Wells	194

Table V.6	Coefficient of variation of the apparent volumetric heat capacity of soil samples from Norman Wells	194
Table V.7	Descriptive statistics of the porosity of soil samples from Norman Wells at temperature $T=0^{\circ}\text{C}$	195
Table V.8	Descriptive statistics of the unfrozen water content of soil samples from Norman Wells at various temperatures below 0°C	196
Table V.9	Coefficient of variation of the unfrozen water content of soil samples from Norman Wells at various temperatures below 0°C	197
Table V.10	Descriptive statistics of the hydraulic conductivity of soil samples from Norman Wells	197
Table V.11	Coefficients of variation of the hydraulic conductivity of soil samples from Norman Wells	198
Table V.12	Descriptive statistics of the wet bulk density of soil samples taken near Norman Wells	199
Table VI.1	Freezing temperatures, segregation temperatures and hydraulic conductivities of freezing soils, based on their soil texture class	216
Table VI.2	Assigned cryosuction, hydraulic conductivity and average temperature gradient of the freezing fringe in various soils.	223

List of Symbols

regular alphabet

A	fitting parameter in stress-strain/rheology considerations
A	fitting parameter hydraulic conductivity (Horiguchi and Miller 1983)
A	fitting parameter in semivariogram models
A	cross section perpendicular to the water flux
A	top layer in soil profile near Norman Wells
A_{ss}	amplitude of the temperature at the soil surface
a	length of the freezing fringe (frost heave prediction model of Nixon 1991)
B	fitting parameter in stress-strain/rheology considerations
B	fitting parameter hydraulic conductivity (Horiguchi and Miller 1983)
B	nugget (in semivariogram models)
B	intermediate layer in soil profile near Norman Wells
b	fitting parameter (Tarnawski and Wagner 1996)
C	fitting parameter in stress-strain/rheology considerations
C	bottom layer of soil profile near Norman Wells
C_a	apparent volumetric heat capacity
C_h	volumetric heat capacity

C_s	volumetric heat capacity matrix
c_v	coefficient of consolidation
D	fitting parameter in stress-strain/rheology considerations
D_h	heat diffusivity
d	characteristic diameter of soil particle fraction
d_{cl}	characteristic diameter of clay particles
d_m	characteristic mean diameter of the soil particles
d_{sa}	characteristic diameter of sand particles
d_{si}	characteristic diameter of silt particles
d_g	geometric mean particle size diameter
E	Young's elastic modulus/tensor
E_0	Young's elastic modulus/tensor for instantaneous deformation
e	void ratio
e_0	initial void ratio
F_L	latent heat forcing/loading vector consisting of latent heat released at nodes
F_L^n	latent heat forcing/loading vector consisting of latent heat released at temporal node n
F_L^{n+1}	latent heat forcing/loading vector consisting of latent heat released at temporal node n+1
FE	(Gibbs) free energy
f	forcing factor (Schellekens 1995)

f_h	heat flux density
f_i	amount of soil fraction i
f_w	water flux density
g	acceleration of gravity/ gravity coefficient
H	height of soil sample
H_{ph}	primary heave
H_{sh}	secondary heave
H_{tot}	total heave
H_w	total potential of water expressed as head
h	lag
h_{ae}	water pressure head at air entry point
h_{ie}	water pressure head at ice entry point
h_p	water pressure head
IE	internal energy
I_s	ice content at the base of the warmest ice lens
K_h	thermal conductivity
K_s	thermal conductivity matrix resulting from Galerkin FEM
K_w	hydraulic conductivity
K_{wff}	hydraulic conductivity of the freezing fringe
K_{wffr}	hydraulic conductivity of the freezing fringe
K_{wsat}	hydraulic conductivity of saturated soil

K_{wu}	hydraulic conductivity of unfrozen soil
K_{wunf}	hydraulic conductivity of unfrozen soil
L	length of a soil element
L	length of the soil column over which the hydraulic conductivity is determined
L_f	latent heat of fusion
L_h	latent heat of fusion
L_{htot}	latent heat of fusion
m	strain coefficient
m_{cl}	mass fraction of clay
m_{si}	mass fraction of silt
N	number of sample points
N_1	interpolation/shape function 1
N_2	interpolation/shape function 2
N_n	interpolation/shape function n
N_{n+1}	interpolation/shape function n+1
n	porosity
o	other minerals
P_i	ice pressure potential
P_{if}	ice pressure at the freezing front
P_{is}	ice pressure at base of warmest ice lens
P_w	water pressure potential

P_{wf}	water pressure potential at the freezing front
P_{ws}	water pressure potential at the base of the warmest ice lens
q	quartz
q_h	heat flux density
Q	heat flux due to cooling
Q_h	heat flux
Q_L	heat flux due to latent heat release
R	difference between exact and approximated value (=error or residual)
r	range (in semivariogram models)
S	specific surface area (surface per volume) of the soil (Anderson and Tice 1972)
S	entropy of the system (Edlefsen and Andersen 1943)
S	sill (in semivariogram models)
S_i	entropy of ice
S_w	entropy of water
s	spatial coordinate
T	temperature
$\mathbf{T}$	temperature vector, containing spatial nodal temperatures
ΔT	temperature difference over depth interval Δz
T^A	spatial approximation of the exact soil temperature
$\mathbf{T}^B$	temporal approximation of the spatial approximation of the exact soil temperature
T_0	temperature (Wang 1994)

T_0	temperature of freezing point of free water (Schellekens 1995)
T_1	temperature at spatial node 1
T_2	temperature at spatial node 2
$\mathbf{T}^n$	temperature vector at temporal node n
$\mathbf{T}^{n+1}$	temperature vector at temporal node n+1
T_c	temperature at the cold surface boundary
T_f	temperature at the freezing front
T_i	temperature at node i
T_s	temperature at the base of the warmest ice lens
T_z	average temperature at depth
t	time coordinate
t_{50}	time for 50% primary compression in compression test
t_{90}	time for 90% primary compression in compression test
Δt	time step
u	water flux density
u	neutral stress (36)
V	volume
V_i	volume of ice
V_w	volume of water
W	fitting parameter
w	work done by a system (in derivation of Clausius Clapeyron equation),

w	fitting parameter in stress-strain/rheologic considerations
w_u	unfrozen water content
x	horizontal spatial coordinate in horizontal
y	horizontal spatial coordinate in horizontal
Z_f	depth of the freezing front
Z_s	depth of the base of the warmest ice lens
z	spatial coordinate in vertical direction
Δz	spatial step

subscripts

ae	air entry
f	freezing front
h	heat
i	ice
ie	ice entry
p	pressure
s	segregation
sat	saturated
unsat	unsaturated
w	water

greek characters

α	compressibility of the soil
α	fitting parameter (Kujala 1989)
β	fitting parameter (Kujala 1989)
γ	semivariance
ε	strain
ε_0	instantaneous strain
ε	strain rate
η	viscosity of soil moisture
η_p	coefficient of plastic viscosity
η_p^*	coefficient of plastic viscosity divided by σ^*
θ	water content
θ_0	initial water content
θ_{-4}	water content still remaining in soil at $T \leq -4^\circ\text{C}$
θ_{sat}	saturated water content
θ_u	unfrozen water content
θ_{un}	unfrozen water content
θ_{unfr}	unfrozen water content
θ_{unsat}	unsaturated water content
λ	fitting parameter (in stress-strain/rheologic considerations)

ρ	bulk density of soil
ρ_b	bulk density of soil
ρ_i	density of ice
ρ_s	density of the soil
ρ_w	density of water
σ	total stress
σ^*	dummy parameter with units of stress and value 1
σ_{eff}	effective stress
σ_g	geometric standard deviation
σ_n	normal stress
σ_y	fitting parameter (intersection on σ -axis)
τ	shear stress
τ	period of temperature wave
ξ	fitting parameter (in stress-strain/rheologic considerations)
ω	frequency of a temperature wave

CHAPTER I Introduction

I.1 Objectives

Frost heave is the process of uplift of a soil due to: an increase in volume resulting from soil water freezing in situ; the freezing of water which is attracted to the freezing zone; and the redistribution of soil moisture and additional freezing of water in the almost completely frozen zone of the soil. The amount of frost heave occurring varies in space and time because of its dependence on spatial and time dependent factors such as soil water content, soil type, climate, and ground cover. Spatial variation in frost heave, usually referred to as differential frost heave, can damage structures, such as buildings, roads, airstrips, pipelines and sewage systems, in environments where parts of the soil are perennially frozen, as well as in environments where seasonal freezing occurs. Reliable predictions of these differential frost heave rates must be made to improve the conditions and life span of these built structures. Reliable predictions can only result from predictive mathematical models that are based on a thorough knowledge of ground freezing and frost heave.

Knowledge of frost heave has been derived from laboratory experiments (e.g. Penner 1959, 1967 and 1986; Radd and Oertle 1973; Berg et al. 1980; Horiguchi and Miller 1980; Wood 1985; Smith et al. 1985; Akagawa 1988; Konrad 1989; Smith 1992; Jetchick 1998) and from field observations (e.g. Guymon et al. 1983; Carlson 1984; Smith 1985a; Perfect et al. 1988; Smith and Patterson 1989).

There are still some important observations and experiments to be done. Experimentally, the conditions when ice lens formation commences and when ice lens formation stops, need to be demonstrated, i.e. the frost heave criteria need to be proven. There is also a need for experimental proof of the existence or non-existence of thermodynamic equilibrium in the freezing fringe, i.e. whether the free energy of ice equals the free energy of water everywhere in the freezing fringe, or merely at the freezing front. Since freezing takes place over the entire freezing fringe the assumption of thermodynamic equilibrium over the entire freezing fringe may be justified. More experiments need to be done to find out how thermal, hydraulic and strength properties of the soil change during the freezing process. The changes in these soil properties during freezing (e.g. due to aggregate formation and deformation, fissuring and consolidation) need to be reported in a systematic manner.

Based on the information obtained from these observations and experiments, mathematical prediction models may be constructed. Modelling of frost heave demands an integrated multi-disciplinary approach, because attention needs to be given to properties and behaviour of soil material, soil thermodynamics, soil mechanics, interactions between microclimatological and micro-structural factors of importance to ground freezing, and climate. After two and a half decades of frost heave model development, various models are available which predict frost heave reasonably satisfactorily. There are 1D and 2D frost heave models which incorporate coupled heat and water flow (e.g. Harlan 1973; Jame and Norum 1980). Others take the development of stresses during the freezing and heaving into account

(e.g. Ladanyi and Shen 1989). Some models can explain rhythmic ice lens banding in the soil (e.g. Konrad and Morgenstern 1980; Nixon 1991, 1992). Some models incorporate the salt exclusion at the freezing front (e.g. Padilla and Villeneuve 1988, 1990, 1992) and some include water transport by regelation (e.g. O'Neill and Miller 1982, 1985; Black 1995). There are still some improvements to be made, such as developing 3D models and incorporating the change of input parameters over time. However, too much detail and, therefore, too much computer time does not improve the application of the model (e.g. Shah and Razaqpur 1993).

Since some developers of the frost heave prediction models are not responsible for or acquainted with the methods to obtain input data, it is often not realized how difficult it is to obtain the input data required for their models, especially in remote cold regions. This has rendered many sophisticated frost heave models impractical (e.g. Frémond and Mikkola 1992).

In frost heave prediction, there are often not enough input data available, so standard handbook values have to be used instead of measured or otherwise determined values. Large errors in frost heave prediction may result from the use of such input data.

Since it is not frost heave itself, but differential frost heave, that causes the damage to structures such as runways, foundations, and pipelines, the spatial variability of input data is very important in the predictions. The spatial variability of the input parameters is ignored when, for design purposes, standard handbook values or extremes are used instead of the actual occurring differentials. Although it has to be acknowledged that structures need to be designed to withstand worst case scenarios, it is often unknown what exactly the worst case

scenario is. Moreover, the likelihood of these worst case scenarios actually occurring is usually unknown. Because differential frost heave is a spatial problem, the prediction of differential frost heave may benefit from the use of spatial statistics. To date, spatial statistics have almost exclusively been used by geostatisticians in mining (e.g. Agterberg 1974, Journel and Huijbregts 1978), agriculture (e.g. Webster and Oliver 1992), and hydrology (e.g. Houlding 1994). Spatial variability of input data for frost heave prediction models has received little attention.

Scientists who obtain data on frost heave in the field or laboratory often focus on the interpretation of field observations or the results from experimental studies, and are not acquainted with the mathematical modelling of the frost heave process. Sometimes a frost heave prediction model is used without any knowledge of the quality of the model. Frost heave prediction models are hardly ever put in the broad perspective of all models available or possible. As a result, the quality of the predictions is unknown.

For an optimal prediction of differential frost heave, a good integration of all components of the differential frost heave prediction procedure is necessary, so specialists working on part of the process are more aware of their part in the whole prediction procedure. Integration of all components of the frost heave prediction procedure requires extensive fieldwork experience, education in the scientific disciplines involved in the frost heave process (soil physics, soil mechanics and thermodynamics), a knowledge of mathematics and statistics, and computer programming skills. A good comprehensive text on frost heave in

which observations, the modelling of the process, computer programming and frost heave prediction are well integrated is currently not available.

This thesis is an attempt to integrate observations and measurements better into the modelling of the frost heave process and highlight some problems associated with input data for frost heave prediction models. The quality of the frost heave prediction model that is used in the case study is reviewed in the larger realm of available and possible frost heave prediction. Whether frost heave prediction efforts should focus on collecting more, as well as more accurate and precise, input data, or on refining the modelling of the process is examined. This thesis introduces spatial variability analysis to the prediction of differential frost heave, and is the first attempt to analyze the prediction process from the acquisition of input data and the development of the prediction model to the analysis of spatial variability of predicted differential frost heave.

1.2 Method

For a better integration of the observations and measurements of the process into the modelling, first a summary of the most important observations and measurements of the process is presented, after which the fundamentals of mechanistic modelling of the frost heave process are described. Some principles and advantages of the use of numerical methods are described. The modelling is opened up for physical geographers by a description of the modelling in a language which can be understood by persons who do not have a

thorough mathematical background. In a case study, frost heave predictions are made with a model developed by Wang (1994). The fundamentals of the model and numerical procedures used are reviewed critically. The predictions are analyzed and the model is adjusted. The performance of the model is evaluated in the broad context of frost heave modelling and by comparing the model to one developed by Nixon (1991).

The input parameters of frost heave models are discussed and methods to acquire these input data for frost heave models are critically reviewed. Some new cost-effective methods to obtain input data are introduced. Input data were obtained during fieldwork near Norman Wells, NWT in August 1994. A method is presented to analyze the spatial variability of these input parameters for frost heave prediction models. A new algorithm for spatial analysis is introduced. To evaluate the effect of input parameters and their spatial variability in frost heave prediction models, (differential) frost heave is predicted using the input data obtained near Norman Wells in a modified version of the model of Wang. Sources of error in the prediction procedure are evaluated. Unfortunately, the accuracy of frost heave predictions could not be checked with measurements.

The research took place at the Geotechnical Science Laboratories (GSL) of the Ottawa-Carleton Geoscience Center at Carleton University, Ottawa. The GSL have three decades of experience with small and large-scale freezing experiments. In Caen, France, the GSL worked together with the Centre National de la Recherche Scientifique (CNRS) and the Laboratoire Central des Ponts et Chaussées (LCPC) on large scale freezing experiments. In the experiments in Caen, the effects of artificial freezing of a soil were studied as the soil was

cooled simultaneously from the top down and radially from a buried pipeline through which gas is transported at temperatures of ca. -4°C . Experiments such as those in Caen offer unique opportunities to validate and evaluate frost heave models.

I.3 Organization of thesis

The development of features occurring as a result of ground freezing and frost heave are described in Chapter II. The relative importance of each of these features for frost heave predictions is evaluated. The chapter concludes with a listing (in table form) of the most important features in the frost heave process, and the way these features may be modelled. These tables provide a tool to compare various frost heave models with each other.

Chapter III begins with the fundamentals of the modelling of natural processes in general and frost heave in particular. Some fundamentals of the use of numerical methods as approximations of exact solutions of mathematical models are explained. A table is designed to compare the numerical approximations used by various frost heave prediction models. The importance of input data in a frost heave model is analyzed in a case study using a frost heave model that incorporates the most important features of frost heave identified in Chapter II. This model is based on that developed by Wang at Carleton University (Wang 1994).

Since the author of this thesis was involved in the development phase of Wang's model, the model was very accessible to the author. The model is meant to be practical, and is therefore kept simple. This allows it to be adapted relatively quickly for specific uses. The

model is similar to the model proposed by Konrad and Morgenstern (1980,1981). In their model, the water pressure at the freezing front is assumed to be equal to atmospheric pressure. The model of Wang allows the water pressure at the freezing front to take on a value other than zero (including a more likely negative pressure). Detailed descriptions of the model, the theory and numerical approximation, and sensitivity to its input parameters are given. The advantages and limitations of the model are discussed, and the model is adjusted by incorporating more aspects of the frost heave process (as mentioned in Chapter II). The theoretical base and numerical approximation of a model developed by Nixon (1991) are described. A sensitivity analysis of the Nixon model to its input parameters is carried out.

In Chapter IV procedures to determine the input parameters for frost heave models are evaluated. Some new practical methods are introduced to determine the thermal and hydraulic input parameters of frost heave models. To optimize the amount of input data needed for frost heave prediction, a method is introduced which is based on an evaluation of the spatial variability of the input parameters. In an evaluation of the spatial variability of input parameters, which input parameters show the largest spatial variability and the size of these spatial variabilities within one single surficial geology unit are determined. In this geostatistical method semi-variograms, in which the semi-variances of one input parameter is plotted for a series of measurement spacings, are used.

A new mathematical approximation of the semi-variance is proposed with which the characteristic values of the semivariograms (the sill and range) can be determined. From these diagrams, the relative variability of the input parameters and the optimal sampling

distance between sampling points can be derived. An interpolation method called kriging (comparable to least squares or spline fitting) is described which may be used to map the spatial variability of the input parameters for frost heave models in even greater detail.

Input data for frost heave prediction were obtained from simple grain size data acquired during fieldwork carried out in August 1994 near Norman Wells, NWT. The determined values of the input parameters are given in a report (Schellekens 1995) from which some tables are included in Chapter V. The spatial variability of these input parameters is evaluated from semi-variograms of input parameters. The advantages of spatial analysis of input data in frost heave prediction are summarized.

In Chapter VI predictions of frost heave are made using a modified version of the model of Wang described in Chapter III, and the data set from Norman Wells, described in Chapter IV. The modified version of the model includes phenomena such as multiple soil layer systems, primary heave, and ramp freezing. The predicted depth of the freezing front, length of the freezing fringe cryosuction, frost heave, and maximum differential frost heave within a particular surficial geological unit are evaluated. The variation in predictions due to the variability in input data is discussed and sources of error in frost heave prediction are evaluated. Chapter VII consists of the conclusions and suggestions for further research.

CHAPTER II Observations and Measurements of Ground Freezing and Frost Heave

II.1 Introduction

This chapter is a discussion of the knowledge about ground freezing and frost heave that has been acquired through observations in the field and laboratory experiments to date. It examines what happens subsequent to a freezing front penetrating the soil. This sequence of discussion is chosen since it is parallel to the way in which ground freezing and frost heave are modelled. The two prerequisites for the occurrence of frost heave are subzero temperatures and the existence of moisture in the soil. How these subzero temperatures migrate deeper into the soil, and how soil moisture is redistributed in the soil during frost heave are explained. As a result of this heat and water transfer in the soil, the thermal and hydraulic soil properties change within the soil, and, in turn, influence the ground freezing and frost heave.

II.2 Ice formation in pores and primary heave

II.2.1 Thermal aspects of soil cooling

Freezing and heaving of a soil are results of heat and moisture transfer in the soil. Heat is transferred along a temperature gradient from high to low temperatures. In the annual thermal

cycle, the soil cools during the fall and winter and, in the daily thermal cycle, the soil cools in the afternoon and during the night (see Figure II.1).

Figure II.1 Example of an annual and daily temperature cycle superimposed on a climatic warming trend of 1°C per year, $T_{\text{av,old}}=5^{\circ}\text{C}$, $T_{\text{av,new}}=6^{\circ}\text{C}$, $T_{\text{annual,ampl}}=20^{\circ}\text{C}$, $T_{\text{daily,ampl}}=3^{\circ}\text{C}$.

In the cooling phases of these thermal cycles the soil profile is cooled from the top downward, i.e. heat is transferred from the subsurface (warm side) to the soil surface (cold side). Heat transfer may take place by conduction, convection or radiation. In freezing soils most of the heat transfer takes place by conduction, a small part by convection (if there is a considerable amount of soil moisture movement), and a negligibly small amount by radiation (Johansen 1975).

The rate of heat transfer by conduction is proportional to the temperature gradient in the soil. The parameter of proportionality is called the thermal conductivity, K_h , of the soil. The thermal conductivity of a soil is determined by the thermal conductivity of its components: soil particles consisting of quartz and other minerals, organic material, water, ice and air. The thermal conductivity of a soil varies with temperature, since the proportions of the constituents and the specific thermal conductivities of its constituents vary with temperature. The variation of the thermal conductivity due to variation in the specific thermal conductivities is small compared to the variation of the thermal conductivity due to variation in the proportions of the constituents. Most of the variation in the thermal conductivity of a soil at temperatures below zero is a result of the transformation of water into ice. Kersten (1949) found that the thermal conductivity of nearly saturated soil samples in a frozen state was a factor 1.35-1.45 larger than the thermal conductivity of the same samples in an unfrozen state. The factor decreased, more or less linearly, to about unity as the degree of saturation decreased to about 0.3.

The result of the extraction of heat by conductive heat flow upwards is a cooling at every depth in the soil. The amount of heat that needs to be extracted from 1 volume unit of soil to cool it 1 K is called the heat capacity, C_h . The heat capacity of a soil and the specific heat capacities of its constituents vary with temperature.

II.2.2 Initial formation of ice

When temperatures drop below the freezing point of water, ice will form. If water molecules were contained in an infinitely large volume at atmospheric pressure and nuclei were available, ice would crystallize at temperatures very close to 0°C. A requirement for the formation of ice is the presence or generation of nuclei on which ice crystals can develop. In soil pores, enough impurities, which serve as nuclei, occur to initiate ice nucleation.

In soils, however, several forces bind the water molecules so that they have a lower "free energy" than the water in its "free" state. Hence, soil water will freeze at a lower temperature than the free water, the amount of freezing temperature depression depending on the free energy state of the soil water. The forces causing this freezing point depression are: adsorption of polar water molecules to negatively charged soil particle surfaces and to positive ions (hydration of cations) satisfying the negative charge (double layer effects); forces resulting from the confinement of water in capillaries; and osmotic forces that bind water molecules due to the presence of salts in soils. Supercooling may cause the initiation of freezing to be lowered by an additional amount, causing the first ice to form at even lower temperatures. Supercooling during freezing of clay pastes may cause the freezing point to decrease to -5 to -7.5°C for uncompacted and -12 to -13°C for compacted pastes (Grechishchev, et al. 1991). Shortly after the initiation of freezing the temperature, T , rises to the freezing point as determined by the factors mentioned.

Not all the water in a soil freezes at the same time. Because some of the water in a soil is more strongly bound than other water (water of different free energy), the water in a soil freezes over a range of temperatures (Tice et al. 1978). Initially the water with the highest free energy (water beyond the strong influence of particle surface forces, adsorbed cations, capillary effects and without a large amount of dissolved salts) will start to freeze, after which water which has lower free energy such as salt water, water in capillaries and adsorbed water will freeze (Williams and Smith 1989) (see Figure II.2). The water in the largest pores of the soil is the least influenced by the particle surfaces, and therefore has a higher free energy than water in smaller pores. Freezing will proceed from the largest pores to progressively smaller pores (Williams 1967).

While most of the freezing takes place, a very high heat capacity, C_h , will be measured. The additional large amount of heat, that needs to be extracted to cool one volume unit of soil 1 K in this situation, is due to the latent heat of fusion, L_h , that is released when the soil water changes from the liquid into the solid state. The measured heat capacity is therefore a combined result of the actual heat capacity of the soil and the latent heat, and is called the apparent heat capacity.

Both the heat capacity of the soil and the latent heat released vary with temperature (Low et al. 1968). The amount of latent heat released varies with temperature, since at a certain subfreezing temperature, an amount of water freezes that is different from the amount at another subfreezing temperature, and because the specific latent heat of water varies with temperature. The latter variation becomes significant only at temperatures below -4°C .

II.2.3 Pore ice and primary heave

All the ice formed in this manner (i.e. in situ) is called pore ice. Pore ice is located between soil particles with their surrounding unfrozen water films (Taber 1930, Miller 1980). In very fine grained soils the specific surface (the total particle surface area per unit volume) is large; the adsorption and osmotic forces are therefore strong, and the free energy of the soil moisture is low. In less fine grained soils capillary effects are still strong, which keeps the free energy of the soil moisture low. In even coarser grained soils the effect of water bound in capillaries is very small, and the free energy of the soil moisture is even higher. Because of the finer pore size and the strong adsorption and osmotic effects, the amount of unfrozen water in fine grained soils is larger than in coarse grained soils at any given temperature below 0°C. Regardless of soil texture, however, in a vertical soil column that is freezing from the top downwards there exists, in sequence, a virtually completely frozen soil zone, a partially frozen zone, a freezing front and an unfrozen zone.

Water expands approximately 9% upon freezing. This volume increase may cause an uplift of the soil surface, and hence be a component of frost heave. In this thesis the surface uplift resulting from the freezing of pore water in situ is called primary heave. This is a different meaning than the one given to primary heave by Miller (1972, 1980), who used the term to describe frost heave at a stationary ice lens at the freezing front. When the consolidation mentioned previously is ignored, primary heave commences when the volumetric ice content of the soil begins to exceed the porosity of the previously unfrozen soil minus the unfrozen water

content (Kay et al.1977). Taylor and Luthin (1978) use a critical ice content equal to 85% porosity, regardless of unfrozen water content (to account for this consolidation in an empirical manner), as the criterion for primary frost heave starting.

II.2.4 Water flow to the freezing front

The free energy of water in a soil pore may be considered to be composed of a water pressure component, a gravitational component, and an osmotic component (Koorevaer et al. 1983). The water pressure component consists of the matric water pressure component and, in soils containing entrapped air, a pneumatic component. The free energy of the water in the double layer of a soil particle consists of an adsorption component, a more negative osmotic component, an increased matric water pressure component, a pneumatic component and a gravitational component. Gradients of these components of the free energy of soil moisture form the fundamental driving forces for water movement in (freezing) soil.

At the boundary of the double layer the free energy of the pore water and the free energy of the double layer water are equal (Figure II.3). Because the pneumatic and gravitational component of the pore water are more or less equal to those of the double layer, the matric pore water pressure component of the pore water (ψ_m) is equal to the summation of the adsorption component (ψ_α), the matric double layer water pressure component (ψ_{mdl}), and the excess osmotic component ($\psi_\omega = \psi_{odl} - \psi_o$).

Figure II.2 Freezing of pure water and salt water at various temperatures in identical soil pores.

T_{fpw} Freezing temperature of pure water
 T_{fsw} Freezing temperature of salt water

Figure II.3 Charge concentration and free energy components near a particle surface during freezing in a pore (modified after Koorevaer et al. 1983)

When the pore water with the highest free energy freezes, salts are excluded from the freezing pore water and become available for diffusion to unfrozen water lower in the profile, and for an increase in the solute concentration in the water films or double layers around the soil particles. These increased solute concentrations cause a gradient in the osmotic component of the free energy, along which water is drawn upwards, because they cause the water in the partially frozen soil to have a lower free energy than water in unfrozen soil (see Figure II.3).

In clays, water flows slowly through the unfrozen zone to the freezing front. As a result desiccation may occur in the unfrozen soil, and the formation of shrinkage cracks and macro-aggregates formed from micro-aggregates (Skarzynska 1980, 1985) may cause consolidation of the unfrozen soil. The forming of vertical shrinkage cracks may change the hydraulic conductivity of the soil. Even if the hydraulic conductivity of a soil is very small, the free energy or pressure gradient created by freezing may be so large that, especially during slow freezing a considerable amount of water is drawn to the freezing front.

II.2.5 Exclusion of salts at the freezing front

Since ice tends to form as a pure phase, solutes tend to be excluded by ice formation (Leung and Carmichael 1984 in Kay and Perfect 1988) and concentrate in unfrozen water (see Figure II.4). A zone with soil moisture containing elevated solute concentrations can develop just below the freezing front (Miller 1980) and its higher osmotic pressure causes a small increase of the freezing point depression (O'Neill 1983). The additional freezing point

depression effect will be small when there is much unfrozen water (maybe 0.001 K at a freezing front with a temperature of -0.1 K) because the concentration of salts is still very low. The lowering of the freezing point causes an attenuation of the freezing front progression (Kay and Perfect 1988). The development of such a solute rich zone in front of the freezing front generates an osmotic pressure gradient which contributes to the pressure gradient drawing more water to the freezing front.

Figure II.4 Formation of ice and elevated salt concentrations in the remaining unfrozen water on the scale of a pore and a layer of soil

As water is drawn into the freezing zone, more solutes are transported upwards to the frost line. The accumulation of solutes at the freezing front and in the unfrozen water behind the freezing front will give rise to concentration gradients, resulting in diffusion of salts downward and water upward. The concentration gradient also creates a freezing point gradient in soil water on the warm side of the freezing front (Kay and Perfect 1988). When the influx of solutes by diffusion at some distance ahead of the freezing front, is insufficient to maintain the freezing point depression, the soil temperature decreases at that point, and conditions favoring new ice nucleation are created. This means that at very rapid rates of heat extraction, the freezing front "jumps over" solute enriched bands, entrapping them in a frozen matrix (Miller 1980, Kay and Perfect 1988). Such phenomena can be of importance in relation to irreversible processes such as the fixation of potash or phosphate (Miller 1980). According to Kay and Perfect (1988) water flow may also be generated by an electrical potential gradient caused by an electrical charge separation (the Workman-Reynolds effect) at the advancing freezing front, generated by preferential incorporation of ion constituents of one sign into the solid phase.

II.3 Ice formation in lenses and secondary heave

II.3.1 Segregated ice and secondary heave

Water not only migrates up to the freezing front, but also migrates beyond the freezing front (O'Neill 1983). Due to this migration, the water pressures in the partially frozen zone, are

prevented from decreasing at a higher rate. The ice pressure increases from the freezing front into the partially frozen soil. The amount of pore ice increases with time. The increasing ice and decreasing water pressures result in an increase in the neutral stress, the part of the total stress that is not carried by the soil particles, but by the pore contents. Since the total stress remains unchanged, this results in a decrease of the effective stress, the stress carried by the soil particles. The effective stress in the soil might become so low that instead of the compressive stress generated by the overburden and surcharge, a tensile stress is generated in a plane in which particles are pulled apart. To pull the particles apart, the effective stress which acts downward and equals the normal stress (the overburden and surcharge) has to be offset by the neutral stress which acts partially upward and consists of the ice and water pressure (Miller 1972). Another viewpoint is, that when the cohesive forces between particles have been overcome by forces generated by the pore contents, the particles may be pushed apart. The cohesion and friction between the soil particles along the planes where this process takes place become zero. The necessary force is called separation force by Gilpin (Gilpin 1979, 1980) and internal force by Wood (1985) and Smith (1992), or, in pressure form, separation pressure and internal pressures. Especially in very fine soils part of this tensile stress may be dissipated by shrinking and consolidation of the (partially) unfrozen soil, accompanied by the formation of tension cracks. In these cracks a discrete layer or lens of ice (segregated ice) may form at some distance beyond the freezing front in the partially frozen ground (Miller 1980) (see Figure II.5). The mechanical explanation for ice lens formation (such as e.g. Miller's effective stress or

Gilpin's separation pressure frost heave criterion) has yet to be verified by experiments and observations (Ladanyi and Shen 1989).

Figure II.5 Geometry, temperature and stress in the freezing fringe (Sheng and Knutsson 1991)

In relatively incompressible soils, ice lenses are usually oriented parallel to the isotherms (Miller 1980, Van Vliet-Lanoë and Dupas 1991). Smith (1992) found that ice lenses form perpendicular to and grow in the direction of the maximal heat extraction. In highly compressible soils, in which vertical and horizontal tension cracks form, intersecting veins of

ice occur in reticulate patterns in three dimensions. Konrad and Morgenstern measured segregation temperatures from -0.10 to -0.39°C for silty soil (Konrad and Morgenstern 1982). Akagawa (1988) reported segregation temperatures of -0.8°C for silty clay, while Fukuda (1995) reported segregation temperatures of -1.4°C for very fine grained volcanic ash. The soil surface uplift resulting from the formation of an ice lens in the soil is called secondary heave (Miller 1972, 1980). In fine grained soils, in particular, secondary heave makes up the major part of the total frost heave.

II.3.2 The freezing fringe

The partially frozen zone between the freezing front and the warmest ice lens, is called the freezing fringe (Miller 1972). In this zone the volumetric ice content increases from the freezing front to the ice lens (O'Neill 1983). The freezing fringe is very important in the freezing process, because most of the combined heat and mass transport in non-saline freezing soils occurs within 1 or 2 degrees below 0°C and because of the importance of secondary heave in the whole frost heave process (Kay and Perfect 1988).

The thickness of the freezing fringe depends on a variety of factors; high overburden pressures and low temperature gradients both tend to increase the fringe thickness (O'Neill 1983). The larger the overburden pressure, the lower the temperature at the warmest ice lens (Loch and Miller 1975; Loch and Kay 1978; Loch 1980; Penner and Goodrich 1980; Konrad and Morgenstern 1980). The highest overburden pressures combined with the smallest

temperature gradients occur when the freezing front has reached its greatest depth. Loch and Kay (1978) found the thickness of the freezing fringe for New Hampshire silt to be about 0.2-0.4 cm at a temperature gradient between 0.4 °C in the frozen soil and 4°C in the unfrozen soil. Akagawa (1988) determined freezing fringe thicknesses of 1.0 to 1.4 cm for a silty clay at a temperature gradient of 0.9 °C/m. In freezing soils in the field a wide range of temperature gradients is possible, from large gradients such as 15°C/m to very small gradients close to 0°C/m.

In the past, the driving force for moisture transfer to the base of the ice lens has been called the cryosuction, and was considered to be equal to the difference between the ice and water pressure ($P_i - P_w$) at the base of the ice lens. Williams (1967) suggested that the cryosuction can be estimated, based on the analogy between frozen soils and unsaturated soils, using the air intrusion value in a conventional suction moisture test. The cryosuction at the base of the warmest ice lens $P_{is} - P_{ws}$ is often called the heaving pressure. Sometimes the internal resistance of the soil against the separation of particles (generated by cohesion and adhesion forces) is subtracted from this pressure and the resulting pressure is called the heaving pressure (Gilpin 1979, 1980; Wood 1985). The heaving pressure $P_{is} - P_{ws}$ increases with decreasing temperatures (Williams 1979; Williams and Smith 1989). Heaving pressures may be several times the atmospheric pressure (Smith 1985b). Hoekstra (1969) and Loch and Miller (1975) give heaving pressures of 560 kPa for silt and 2400 kPa for clay.

The difference between the pressure of water and the “pressure” of ice, two different entities, is not a likely driving force for mass flow. It is much more likely that the driving forces

for mass flow are gradients in one or more potentials, e.g. a gradient in the water pressure potential, osmotic potential etc.. Cryosuction should therefore be redefined e.g. as the difference between the water potential at the base of an ice lens and the water potential at the freezing front ($P_{ws}-P_{wf}$).

The amount of ice segregation depends on the stress conditions in the soil, which depend on the deformational properties of the soil material (Wood 1985; Sayles 1988; Smith 1992). Moreover, thick ice lenses are associated with slow rates of freezing, relatively high hydraulic conductivities, high suction gradients, and a high availability of soil moisture (saturated soils or a water table that is close to the surface) (Smith 1985b).

The rate of freezing depends on the thermal conductivity, K_h , which depends on moisture and ice content ($\theta_{liq}, \theta_{ice}$) (Miller 1980). Measurements of K_h at temperatures occurring in the freezing fringe are given by Johansen and Frivik (1980) and Riseborough et al. (1983). By convective flow of water, sensible and latent heat are transported in the freezing fringe. An apparent thermal conductivity, $K_{h,app}$, is measured which rises dramatically as T increases to 0°C (Kay et al. 1981 in Kay and Perfect 1988).

The amount of water that can flow to the base of the ice lens is proportional to the suction gradient existing across the freezing fringe and in the unfrozen zone, and to the permeability of the partially frozen and unfrozen zones. The suction gradient over the freezing fringe is substantially larger than that over the unfrozen zone (Ladanyi and Shen 1989). A temperature difference between 0°C and -0.5°C generates a suction difference of $600\text{ kPa} = 6\text{atm}$. At $dT = -1^\circ\text{C}$ the suction difference is 1200 kPa (Smith 1985).

The hydraulic conductivity, K_w , in the frozen fringe depends on the amount of soil moisture present (the unfrozen water content, θ_{liq}). Kay, et al. (1985) reported very little air behind the freezing front, i.e. saturated conditions (Kay and Perfect 1988). dK_w/dT for respectively clayey to sandy soils is maximal between -0.2°C and -0.05°C (Burt and Williams 1976). This corresponds to the temperature at which dq_w/dT is maximal (Smith 1985b).

Akagawa (1988) described the characteristics of the frozen fringe in a silty clay sample, which had a diameter of 6 cm, a length of 9.7 cm and a bulk density of 2480 kg/m^3 . The sample was frozen for 700 hours with a cold plate temperature of -5.5°C . The thickness of the frozen fringe changed from 14 mm (after 5 hours) to a constant 10 mm. The segregation temperatures measured were virtually constant at -0.8°C . The cumulative heave changed from 2 mm after 10 hours, and 6.5 mm after 40 hours to 9 mm after 100 hours. The heave rate decreased from 0.5 mm/hr after 1 hour, 0.25 mm/hr after 7 hours, 0.07 mm/hr after 40 hours to 0.01 mm/hr after 100 hours. During the penetration of a freezing front, the temperatures at a specific position in the soil sample in the freezing fringe decrease over time and cause changes in phase composition which then cause changes in the physical properties (e.g. the thermal conductivity, the volumetric heat capacity and hydraulic conductivity) and changes in the microstructure (particle and pore size and arrangement), which in turn change the physical properties.

The vertical shrinkage cracks in front of the freezing front fill with ice as the freezing front continues to advance in saturated soil (Skarzynska 1980, 1985). By the freezing of pore water and some of the water of the adsorbed water layers on clay particles, clay particles are attracted more strongly to each other and with the remaining film water form aggregates

(Skarzynska 1985; Chamberlain 1989). It could be argued that the clay particles are pushed together as water is extracted from the pores and transferred along the free energy gradient to the freezing front. These aggregates are separated from each other by the crack network and the ice lenses; the soil mass is divided into well-defined vertical and horizontal freezing-induced macro-aggregates (Chamberlain and Blouin 1978 and Chamberlain and Gow 1978 in Chamberlain 1989). Czurda and Schababerle (1988) reported that when the large aggregates form they are very stable (Chamberlain 1989).

In this aggregation the “apparent” specific surface area is reduced, with greater reduction at lower temperatures when less adsorbed water remains (Aoyama et al. 1985). This reduction in specific surface area causes a decrease in the liquid limit (the maximum water content of the soil before it starts to flow), with the amount of the decrease being larger at lower freezing temperatures. The reduction in the liquid limit appeared to be large in highly plastic sensitive clay, but small in kaolin and bentonite clays (Yong et al. 1985).

When freezing occurs within the macro-aggregates, their constituting micro-aggregates are compressed, resulting in consolidation of these micro-aggregates. Chuvilin and Yazynin (1988) found that the size of micro-aggregates was reduced by rapid freezing and very low temperatures and increased by slow freezing rates (Chamberlain 1989). Czurda and Schababerle (1988) found that the aggregates in a colliery spoil were made finer by freezing and the increased specific surface area made the spoil more frost susceptible (Chamberlain 1989).

The size of the aggregates resulting from the freezing process is dependent on many factors, including the type of minerals and ions present, the initial particle arrangement before

freezing, the moisture content, the rate of freezing, and the lowest soil temperature. Czurda and Schababerle (1988) reported that both ice lenses and vertical cracks were abundant in non-swelling clays such as kaolin, whereas in highly swelling clays, such as Na-montmorillonite, neither feature was visible (Chamberlain 1989). From analysis of electron microscopy data it was concluded that the velocity and size of structural transformations depend on the mineral composition and initial water content of clays (Grechishchev et al. 1991). Chuvilin and Yazynin (1988) noted that aggregation in freezing soils, contrary to aggregation in unfrozen soils, is more prominent in the presence of univalent ions such as Na^+ , than bivalent ions, such as Ca^{2+} and Mg^{2+} . Because of the many factors affecting changes in soil structure, it is currently difficult to generalize what physical changes will occur without conducting specific tests (Chamberlain 1989).

II.3.3 Growth of the warmest ice lens

The growth of the lens adjoining the freezing fringe (the warmest ice lens) depends on the characteristics of the freezing fringe (K_w , dP_w/ds , K_h , C_h) and the cooling rate of the soil (Chamberlain 1983 in Kay and Perfect 1988). The cooling rate depends on the thermal properties of the soil and the temperature of the soil surface. According to Skarzynska (1985) the flow of water through the freezing fringe is accommodated by the vertical tension cracks formed in the unfrozen soil and eventually incorporated in the freezing fringe, even though the largest part of each vertical crack will be filled with ice.

Compared to coarse grained soils, fine grained soils have a larger amount of unfrozen water at a given temperature below 0°C. Therefore the hydraulic conductivity of the fine grained soils at lower subfreezing temperatures is higher and the water flux through the freezing fringe is higher, which renders these soils more frost heave susceptible than coarse grained soils. The soil moisture gradient and heaving pressure ($P_i - P_w$ at the base of the warmest ice lens, see chapter II.6) generated in the freezing fringe are the largest in clays. Because the permeability of (unfrozen) clay soils is considerably less than the permeability in (unfrozen) silty soils, more moisture transport to the base of the warmest ice lens is possible in silts and the largest amounts of frost heave often occur in such soils.

The liquid flow (intake) rate in the freezing fringe is proportional to the temperature gradient in the freezing fringe (Konrad and Morgenstern 1980, 1981). The coefficient of proportionality, the water flux density-temperature gradient ratio has been called the segregation potential, which depends on the overall permeability of the freezing fringe, a unique segregation freezing temperature, and the average suction in the fringe. The term segregation potential should not be confused with the potentials that drive the water transport in soils and which are defined in the soil physics literature for isothermal processes in soils with a temperature above 0°C (see chapter II.2.4). If heat is extracted at a higher rate than what can be balanced by the flow and freezing of water at the warmest ice lens site, then a new ice lens will form at a deeper location where water is more readily available. In sediments containing a certain amount of colloids (mineral or organic) and under specific pF values, fissures develop below the depth of stabilization of the freezing front (Van Vliet-Lanoë 1991). In the freezing zone electrical charge

separation may occur as water moves in thin films and displaces ions in the electrical double layer, thereby creating a streaming potential that contributes to the suction gradient (Kay and Perfect 1988).

II.3.4 Redistribution of soil moisture by regelation in the freezing fringe

The freezing of colder parts in the freezing fringe generates potential gradients that cause a soil moisture redistribution and an eventual freezing of redistributed water. A significant component of the total mass transport in the freezing fringe appears to be due to movement of pore ice by a process called (thermally induced) regelation (Miller 1972, Philip 1980 in Smith 1985b, Kay and Perfect 1988). There are two forms of regelation: mechanically and thermally induced regelation.

Mechanically induced regelation can be explained by the example of a thin wire with heavy weights at each end, draped over a block of ice with the weights dangling, sinking slowly through the block of ice. Ice beneath the wire is subjected to high pressure, which lowers the freezing temperature below ambient level so that ice melts. Melt water flows up around the wire and promptly refreezes so that the solid block of ice shows no evidence of the passage of the wire through it. Latent heat of fusion is absorbed as ice melts below the wire, and the temperature there falls. Latent heat is released as the melt water refreezes, raising the temperature above the wire, so that heat carried around the wire as latent heat by moving melt water returns across the wire by thermal conduction. The rate at which the wire will move,

therefore, depends on the thermal conductivities of the wire, water, and adjacent ice together with impedance to flow of melt water around the wire (Miller 1980).

Miller (1980) defines thermally induced regelation in the following way. If ice contains small pockets of unfrozen salt solution, a temperature gradient will cause these brine pockets to migrate in the direction of rising temperature. This would be an example of thermally induced regelation. For the brine pocket to remain stationary, the solute content of the brine at the solution-ice interface would have to be greater on the cold side of the pocket than on the warm side in order to satisfy the freezing-temperature equation. Solute and solvent diffusion, however, would tend to equalize brine concentration, with H_2O diffusing toward the cold side and solutes diffusing toward the warm side, so that the freezing-temperature equation can not be satisfied unless H_2O arriving at the cold interface freezes while a similar amount of ice melts at the warm interface where solutes are arriving. The result would be migration of the pocket from cold to warm (Miller 1980). The salt exclusion at the freezing front, mentioned in chapter II.2.5, promotes the formation of such brine pockets.

In long term observations of ice lens growth behind a stationary freezing front, Ohrai and Yamamoto (1985) observed a redistribution of soil ice by thermally induced regelation (Kay and Perfect 1988). Romkens and Miller (1973) observed a rapid increase in migration velocity as a particle, moving through ice by regelation, approached the 0°C isotherm. This was attributed to thickening of the water film as temperature increased, reducing the impedance to moisture movement around the particle. It is hard to produce clear evidence of regelation,

because the resolution of the measuring device has to be very high to detect the extremely small flow rates (Horiguchi and Miller 1980 in O'Neill 1983).

II.4. Soil freezing and frost heave over time

II.4.1 The frozen zone and tertiary heave

The virtually completely frozen zone, the zone from the soil surface down to the warmest ice lens, contains still a small amount of unfrozen water. Hoekstra (1969) demonstrated that water can flow through frozen soil. The rate of water flow in the frozen zone depends on the temperature (which determines the hydraulic conductivity K_w) and on the temperature gradient which determines the hydraulic gradient (Smith 1985b). The important relationship between K_w and T is evaluated by Burt and Williams (1976), Loch and Kay (1978), Perfect and Williams (1980), Horiguchi and Miller (1980), Konrad and Morgenstern (1980), Oliphant, et al. (1983), and Yoneyama et al. (1983). The hydraulic conductivities of frozen soils are very low (see Figure VI.3, hydraulic conductivity of freezing soil, Nixon 1987, 1991), but enormous suction gradients are associated with temperature gradients in the frozen zone and significant flow may occur over the long term. This could explain the ice-rich layers at the top of the permafrost (Harlan 1974).

Water redistribution within the frozen zone has been observed in laboratory experiments by Dirksen and Miller (1966), Mageau and Morgenstern (1980) and Penner and Goodrich

(1980). Heaving pressures at the warm side of an ice lens are transferred to the cold side of the lens. As a result of the increased pressure, the cold side of the ice lens may melt and the melt water may be redistributed, by transport through the unfrozen water films and by respectively freezing and melting at the warm and cold sides of pore ice, to colder ice lenses (thermally induced regelation). Both processes combine in a parallel-series transport of moisture (see Figure II.6A) resulting in the soil moisture redistribution given in Figure II.6B.

A. Parallel-series transport of soil moisture during freezing in a pore B. Soil moisture redistribution in frozen soil and resulting tertiary heave

Figure II.6 Soil moisture redistribution in a pore and in the frozen soil

The frost heave resulting from the freezing of this redistributed water, and from the freezing of unfrozen water in the frozen zone, is called continuing or tertiary heave (Miller 1972). If ice lenses continue to grow within the frozen soil, they cause considerable pressure on

the adjacent soil aggregates and, consequently, cause additional consolidation within the aggregates.

II.4.2 Freezing induced changes in macro and micro structure

Ice growth in the larger pores within macro-aggregates may break the bonds between their constituting micro-aggregates (Yong et al. 1985), resulting in smaller distinct micro-aggregates with smaller internal micropores. Reorganization of soil particles, reduction of particle spacing, and migration of adsorbed water all may accompany this process (Chamberlain 1989). From analysis of electron microscopy data, it can be concluded that kaolin micro-aggregates reduce in size simultaneously with enlargement of pore space (Kumor 1989, Grechishchev et al. 1991). Reorientation of soil particles and adjustment of soil structure may influence the soil thermal and hydraulic properties.

II.4.3 Penetration of the freezing front with time

In the early stages of the downward freezing of a soil column, the freezing front penetrates rapidly, and water is expelled ahead of the freezing front in the direction of the warm side (Loch and Kay 1978). In later stages the freezing front penetrates more slowly, and the water expulsion is smaller. The rate of water expulsion is also dependent on the overburden load. It precedes the stage of water intake for segregation (Ladanyi and Shen 1989).

The rate of downward advance of a freezing front in soil is slowed by the sensible and latent heat brought by incoming warmer water which moves toward the freezing front in response to freezing-induced potential gradients. Most of this water will be cooled and frozen before the freezing front advances. In the experiments of Dirksen and Miller (1966), ice accumulation continued as the temperature profile approached a steady state, and the rate of advance of the freezing front slowed to a halt after 48 hours.

Figure II.7 Examples of preferential surfaces for ice lens growth

When a freezing front moving downward through coarse soil encounters a stratum of much finer material, the advance of the ice front may be arrested, and an ice lens may form at the top of the fine layer (see Figure II.7) (Miller 1980, Kay and Perfect 1988).

II.4.4 Resulting soil profile

According to Miller (1980), the result of ice lens formation behind a penetrating freezing front is a soil column with a succession of ice lenses which become progressively thicker with depth. The soil pores between these lenses are filled mostly with ice and unfrozen water. A strong presence of solutes creates a thick freezing zone with many small ice lenses and layers of brine-rich unfrozen soil between the lenses (Chamberlain 1983).

As a result of the freezing of water in lenses and pores, frost heave occurs, lifting the soil surface up. The total frost heave of a soil is composed of the primary, secondary and tertiary heave. Akagawa, et al. (1985) called the three types of heave respectively steady-state frost heaving, transient frost heaving, and long-term frost heaving, to illustrate the importance of the time scale (Kay and Perfect 1988).

The structural elements of the frozen soil are oriented perpendicular to the direction of heat extraction. Ice lensing imposes a fabric or aggregation on a fine grained sediment. Freezing destroys the initial homogeneity of the soil, and transforms the soil column into a more stratified heterogeneous soil material (Kersten 1949 in Miller 1980). The result of the soil freezing is a variation in the soil physical and soil mechanical properties in the direction of the heat extraction.

II.5 Multiple cycles of freezing and thawing

II.5.1 Thawing, thaw-consolidation and settlement

After thawing, excess melt water may be extruded from soils, and consolidation and settlement may occur. According to Ladanyi and Shen (1989) a typical result of a thaw consolidation test shows respectively: loading of the frozen soil resulting in a decrease in void ratio; then the soil is allowed to thaw (the thaw strain is usually much greater than the strains due to subsequent consolidation); and finally consolidation under the application of increasing external pressures. Increases in the permeability of the soil after freezing and thawing increase the coefficient of consolidation of clay soils sometimes by more than an order of magnitude (Chamberlain 1989). However, the consolidation of the soil after thawing may decrease the permeability of the soil. Tong and Chen (1985), Ryden (1985) and Orlov and Fursov (1988) found that the rate of the consolidation after freezing and thawing cycles increased considerably in successive freeze-thaw cycles.

Among all soils, clay soils are particularly susceptible to settlement after freezing and thawing, because the structure and fabric of clays are very sensitive to the stresses caused by freezing (Chamberlain 1989). Chamberlain (1980) suggested that graphing the water content-plastic limit ratio of the unfrozen soil against the thaw strain could be used to estimate the amount of the settlement of the clay. If thawing proceeds sufficiently slowly that fully drained conditions occur, the time vs. settlement relation is simply proportional to the progress of the

thaw front with time. If the thaw proceeds too quickly for the pore pressures generated to dissipate, excess pore water pressures develop, and the shear strength is temporarily reduced (Ladanyi and Shen 1989).

II.5.2 Multiple cycles of freezing and thawing

Cyclic freezing and thawing changes the arrangement of soil particles and pores (Chamberlain 1989). The larger the number of freezing and thawing cycles, the higher the degree of order of clay microaggregates related to the surface of ice lenses (Kumor 1989). The size of structural elements such as aggregates, domains and pores increases with an increase in the number of freeze-thaw cycles (Skarzynska 1985 and Azzaoui et al. 1987 in Kumor 1989). Micro-fissures resulting from desiccation in the unfrozen zone are reasonably well preserved as lithological discontinuities after thaw (Van Vliet-Lanoë 1991). These discontinuities and the ones left by melted ice lenses form paths with lower resistance to water flow than the interstitial or inter-micro-aggregate paths followed before freezing. The fissures and other lithological discontinuities are preferential locations of future ice lenses (Van Vliet-Lanoë 1991) (see Figure II.7). After thaw consolidation in the soil, vertical pores remain open though horizontal ones are partially closed (Van Vliet-Lanoë 1991). The freezing of a silty clay results in a decrease in the volume of micropores, and the formation of interaggregate macropores (Stepkowska and Skarzynska 1989, Nagasawa and Umeda 1985).

The increase in aggregation, desiccation cracks, the large vertical pores and the formation of macropores after cyclic freezing are extremely important because they increase irreversibly the hydraulic conductivity of the soil (Chamberlain and Gow 1978; Kumor 1989; Van Vliet-Lanoë 1991) despite the fact that the void ratio usually decreases (Chamberlain 1989). The formation of macropores during freezing and therefore the increase in permeability is the largest in clays with the lowest liquid limits. Chamberlain and Gow (1978) reported increases in the permeability of a saturated clay of medium plasticity of roughly three orders of magnitude after freezing at a water content near the liquid limit, and one order of magnitude after freezing at an initial water content just above the plastic limit. The permeability increases were greater when the test temperatures were colder (Nagasawa and Umeda 1985 in Chamberlain 1989). Orlov and Fursov (1988) observed that the greatest increase in permeability of the soil occurred during the first freeze-thaw cycle and that the magnitude of the changes progressively decreased with additional freeze-thaw cycling.

The increase in permeability after freeze-thaw cycles increases the frost susceptibility of the soil. A clayey sand showed a very large increase in the frost heave rate during the second freezing, attributed to changes in the structure of the clay fraction (Efimov et al. 1981 in Kay and Perfect 1988; Chamberlain 1989). The extent of further microstructural changes and therefore further changes of the hydraulic conductivity and frost susceptibility due to freeze-thaw cycles decrease with the number of cycles.

II.5.3 Stress situation in a soil during freezing and thawing

In a soil, the normal stress in the vertical direction is generated by the overburden of the soil and possible surcharge upon the soil. The effective stress on the soil particles equals the normal stress diminished with the neutral stress, the stress which is carried by the pore contents (ice and water). The large negative pore water pressures, P_w , generated by freezing cause an increase in the effective stress in the soil. The effect is a generation of tensile stress resulting in soil particles moving apart, consolidation of the soil and the secondary frost heave (Chamberlain 1989). The overconsolidation caused an increase in the angle of internal friction and the frost heave caused a decrease in the cohesion component of the shear strength of a highly plastic alluvial clay and a medium plastic kaolin clay (Ogata 1985 in Chamberlain 1989). The solidification of the soil water increases the strength of a freezing soil considerably.

A full cycle of freezing and thawing can cause either increases or decreases in the strength of clay soils. When increased consolidation and density occur the soil strength generally increases. The rate of gain of strength with time is greatly reduced by freeze-thaw action. The cases in which the soil strength decreases, include freezing of highly cemented clays and clay soils that are highly overconsolidated before freezing (Chamberlain 1989). Freezing and thawing of natural and homogenized clays resulted in a decrease in shear strength, liquid limit and other mechanic properties (Yong et al. 1984 in Kumor 1989). The decrease was larger with an increasing number of freeze-thaw cycles. The reduction in strength after freezing and thawing is greater at lower freezing temperatures (Chamberlain 1989). Nagasawa and Umeda

(1985) reported that the strength of clay soils decreased progressively with repetition of freeze-thaw cycles.

Large strength losses occur in sensitive clays after freezing and thawing because of the breakdown in the original cementation bonding between clay particles and the formation of new, but unstable, bonds (Yong et al. 1985). Some of the strength loss may also be caused by a reduction in size of the microaggregates of clay particles. With time, freezing and thawing change the failure from a brittle to a ductile mode (Aoyama et al. 1985 in Chamberlain 1989).

II.5.4 Fabric stability and shear strength

The pressure exerted on the fissure wall by the growing ice lens and the flocculation of colloids by salt concentration reduce the internal porosity of the soil and enhance the stability of the soil fabric (Chamberlain 1981, Van Vliet-Lanoë 1991, Kumor 1989). This modification of the fabric is progressive and needs 4 to 5 freeze-thaw cycles to be effective. The stability of the fabric increases if the winter temperatures and the silt/clay ratio are low (Van Vliet-Lanoë 1991). The increased fabric stability increases the water retention and hydraulic conductivity and explains the observed acceleration of frost heave measured in successive freeze thaw cycles in the Franco-Canadian experiments in Caen. Even after the first freeze-thaw cycle, the hydraulic conductivity and shearing resistance of the sediment increases considerably, especially in uncompacted clay and clay loam as stressed by Williams (1957), Chamberlain and

Gow (1978), Mackay (1983) and Van Vliet-Lanoë (1985). These modifications of properties generally stabilize within 4 to 5 cycles (Van Vliet-Lanoë 1991).

II.6 Relevance and control of frost heave

II.6.1 Occurrence and effects of frost heave

Frost heave occurs wherever water in the soil freezes. It occurs in regions where parts of the soil are frozen for more than a year (i.e. regions with permafrost), as well as in regions where seasonal freezing occurs. Frost heave contributes to the formation of natural phenomena such as patterned ground, pingos, palsas and massive ice bodies. (Differential) frost heave under built structures such as buildings, (buried) pipelines, transmission lines, fences, highways, airport runways etc. may cause these structures to break and/or collapse.

II.6.2 Ice lens formation and frost heave around a buried pipeline

In the Franco-Canadian experiment in Caen, coring and pits dug into the soil showed semi-concentric ice lensing under the chilled pipe similar to that described by Carlson and Nixon 1988 (Van Vliet-Lanoë and Dupas 1991). Dipping of ice lenses toward the pipe was observed during the second freezing cycle in 1985.

The pattern of heave around the buried pipeline in the Franco-Canadian experiment in Caen, and observations of the formation of a cavity underneath the pipe indicate that the displacements of the pipe are a result of the pipe being squeezed from the sides as well as being lifted by vertical heave generated below the pipe (Smith and Williams 1990).

II.6.3 Control of frost heave

To decrease the amount of frost heave of a soil, the penetration depth of the freezing front may be reduced. The penetration depth of the freezing front may be reduced by adding a heat sink in the form of a layer with a high moisture content on top of the soil (Phukan 1989) or a layer of material with a low thermal conductivity such as materials containing a lot of air. The insulation material, polystyrene, has been used successfully in the construction of roads and runways in Norway, Ontario, Sweden, Finland, Germany and Alaska (Phukan 1989). The chance that icings form has to be evaluated. To minimize the chance that icings form, a base of free-draining material on top of a subbase is inserted between the pavement and the polystyrene insulation. The thickness of the polystyrene is often determined using the equation $x \approx 0.015y - 7.5$, in which y = freezing index in degree days and x = the thickness of the insulation in cm.

The depth of penetration of the freezing front can also be decreased by decreasing the freezing temperature of the soil by increasing the salinity of the soil moisture. In Japan, the addition of NaCl, CaCl₂, or lime has been successful in reducing the depth of frost penetration (Kubo 1989, Kinoshita 1989). Heat sources may be placed around structures (especially around

point structures such as buried Liquid Nitrogen Gas (LNG) tanks) to decrease the depth of penetration of the freezing front (Kinosita 1989). An advanced method to decrease the depth of frost penetration was tested in Japan. Pipes filled with antifreeze were buried in the soil. The pipes were warmed up by solar heat penetrating the soil in the summer. This energy is stored and used in the winter to delay frost penetration and to generate melting of snow on pavements (Ikeda et al. 1989).

Ice lens formation can be decreased by decreasing the frost heave pressure, by diminishing the specific surface area of the soil through cementing of grains by adding lime or another cementation agent (Kubo 1989). The most used method to decrease the amount of ice lens formation is to replace any fine grained soil such as clay and silt by a coarse grained soil, and elongated particles with a high specific surface area, by round particles with a low specific surface area (e.g. volcanic ash, Kinosita 1989).

The amount of ice lens formation may also be decreased by decreasing the amount of water flowing to the freezing front by sealing off parts of the soil by using watertight membranes made from vinyl or geotextiles (Kinosita 1989). Flow to the freezing front may also be reduced by decreasing the hydraulic conductivity of the soil. The hydraulic conductivity may be decreased: by increasing the packing of the soil; by mixing the soil with powders, such as cement or powdered limestone (Kinosita 1989), however, homogeneous mixing is difficult, and there is a chance that differential heave results from heterogeneous mixing; or by the application of a temporary (hammering) or permanent (a berm on top of the soil) force, to consolidate the soil and lower its hydraulic conductivity. The permeability of the soil may be decreased by an

increase of the viscosity of water by adding e.g. carboxyl methyl cellulose (Kinosita 1989). The flow of water to the freezing front may also be reduced by a lowering of the water table.

When the overburden pressure increases, the flow to the freezing front decreases and the heave rate decreases (Penner and Ueda 1977, Ladanyi and Shen 1989). According to McRoberts and Nixon (1975), and Hill and Morgenstern (1977) for each soil there is an overburden pressure at which the effective stress at the frost front causes no flow of water to the freezing front (Ladanyi and Shen 1989). Since this stops the secondary heave, it is called the shut-off pressure. At higher overburden pressures water is expelled in advance of the freezing front. A berm of 3 m of overburden with a density of 1500 kgm^{-3} results in an overburden pressure of 45kPa. Primary heave may continue, and provided that freezing is continued for a sufficiently long period, tertiary heave may take place. Penner and Ueda (1977) questioned the existence of the shut-off pressure for any soil. However, there is no doubt that a large part of the heaving force may be offset by adding a counterweight in the form of a berm on top of the soil (Kinosita, 1989).

To prevent differential frost heave the soil may be homogenized, and by the use of buried sheets of geotextiles differentials might be minimized by spreading them out over a larger surface (Aoyama and Fukuda, 1989). Footings might be protected from adfreeze by insulation with polystyrene. Some examples of these (differential) frost heave damage prevention measures mentioned, are given in Figure II.8.

Figure II.8 Examples of measures to prevent damage by (differential) frost heave

The measures that are used to prevent the negative effects of (differential) frost heave, should not increase thaw weakening of the soil upon melting. To prevent thaw weakening the surface drainage and underground drainage should be improved. Underground drainage during the melting phase may be improved by drainage pipes in coarse material, surrounded by geotextile. Wick drains (prefabricated vertical drains) can be used to dissipate excess pore water during thaw. As a result the effective stress and therefore strength of the soil are increased, and thaw weakening avoided.

II.7 Discussion and summary

Despite the considerable amount of details about frost heave known from observations and experiments in the field and the laboratory, there is still a need to verify in experiments and in the field some important problems relating to frost heave. Some of these problems are listed in this subchapter.

In the mechanics and dynamics of ground freezing and frost heave the following terms should be used: total and normal stresses of soil particles and ice, and pressure of water. One intention of any pressure definition scheme is to provide a framework for applying the concept of mechanical equilibrium; when the net force built up within the soil threatens to surpass the overburden pressure, the soil structure will change and heave will occur, in order to preserve a balance of forces. Takagi (1980) pointed out that there are limits to the applicability of the pressure-concept for unfrozen film water occurring in freezing soils (O'Neill 1983). According to Vignes and Dijkema (1974), water in films can pull nearby water into it while simultaneously pushing away the ice which forms if the state of stress of the water is nonisotropic and otherwise complex (O'Neill 1983).

Currently it is nearly impossible to measure the physical properties of the freezing fringe, the zone in the soil where most of the heave takes place. Validation of proposed frost heave models is therefore difficult. Centrifuge experiments combined with scaling methods (Black 1995) provide solutions until more sophisticated measurement methods are developed which can provide the more detailed information about the properties of the freezing fringe.

It needs to be described systematically where exactly fissuring, consolidation of aggregates, and consolidation of the soil take place during the freezing and thawing of soils. Furthermore it needs to be described how these processes influence the thermal, hydraulic and strength properties of the soil.

The existence of thermodynamic equilibrium in the frozen fringe remains unproved. The expansion of the volume of water upon freezing and the freezing of water that has migrated to the freezing fringe by cryosuction change the stress conditions in the soil. The expansion and the change in stress conditions change the thermal and hydraulic properties of the soil. This phenomenon is not well understood and described yet.

Miller's frost heave criterion, which seems to be a valuable frost heave criterion, needs to be proven experimentally. Care needs to be taken that the new ice lens forms at a considerable distance from the last one, so that T and P_w can be measured accurately at both depths.

The thermal and hydraulic properties of a soil change not only during freezing, but as well in the following thaw, and after repetition of these freeze-thaw cycles. This phenomenon is not well described yet.

At the boundaries of surficial geology units, differential frost heave may be at its maximum, since those are the locations at which the spatial variability of the input parameters will be the largest. Apart from Smith and Williams (1995), there has been little research into this matter.

Soil freezing is a process involving the transport of heat and water. As a result, stress conditions may develop in the soil leading to consolidation in some zones and frost heave in others. A spatial differential uplift may have severe consequences for built structures such as roads, houses, pipelines, and runways. In order to limit the differential frost heave damage to these structures, reliable predictions of frost heave are necessary. Reliable predictions can only result from accurate modelling of soil freezing and frost heave, based on detailed observations in the field and in laboratories. From previous observations, the frost heave process has been described in considerable detail. Although there are still important questions to be solved about ground freezing and frost heave, a basic understanding of the processes currently exists.

CHAPTER III Practical Frost Heave Prediction Models

III.1 Modelling of natural processes

Once knowledge of a natural process is obtained from experiments and observations in the field and in the laboratory, the process or system may be represented in a model. The more that is known about the process involved, the better the reality can be represented in the model. Models are used to predict the result of a process from a given set of input data, and are used to develop a better understanding of the process.

There are two main categories of models: physical models (scale models and analog models) and theoretical models. A physical model might be used to visualize a problem. The system state at a certain moment in time may be represented on a smaller scale. An example of a scale model of the relief of the terrain is a 3D model made of gypsum or plastic. An example of an analog model is the representation of water flow by an electrical circuit. Theoretical models can be subdivided in: statistical models (random/phenomenologic, probabilistic and stochastic models) and deterministic models (mechanistic or non-mechanistic models), or a mixture of these (eg. stochastic-deterministic models).

In a statistical model of a process, probabilities or chances of a process happening, or a process variable taking a certain value, are calculated. If the chances are based on correlations and not determined by any physical laws, the statistical model is called a random or phenomenologic model. If the chances to get certain outcomes are larger than others, the

statistical model is called a probabilistic model. In the special cases that chances depend on time, the probabilistic models are called stochastic models. In statistical models the emphasis is on finding correlations between variables that might be important in the process, without knowing exactly how the process works. Therefore these models are very useful in the exploratory phase of the research into a process.

When more details are known about the process, it might be possible to work towards a deterministic model. In a deterministic model the relation between cause and effect is mathematically described. A deterministic model of a process in nature aims at a detailed physics-based description of all known sub-processes and interactions of variables involved in the process. A non-mechanistic model makes use of certain simplifying assumptions to omit complex or speculative relationships (Kay and Perfect 1988).

The process or system in reality may be represented by any of the models mentioned above. This is called simulation. The most desirable description of reality is a mechanistic deterministic model. Once the problem is formulated mathematically in a mechanistic deterministic model, an exact solution of the problem may be obtained analytically, or an approximation of the exact solution may be found numerically. It must be mentioned that sometimes numerical procedures may lead to the exact solution. The main advantage of using numerical methods is that they can be written as a computer program, and a computer may perform the calculations.

A practical model is a model that delivers predictions that are as accurate as possible, while the model is kept simple or otherwise user friendly, computer requirements such as

required software and hardware and required computing time are kept within certain limits, and for which the input parameters are relatively easy to determine. After a model has been constructed, generalization, such as simplifying the physical base of the process by omitting certain details, or a decrease of the amount of spatial or temporal steps in the numerical approximation, might be necessary to reduce the computing time in order to make the model practical and to obtain an optimal efficiency. The errors generated by these actions should remain within a limited range. Whether these errors are allowable depends on the purpose of the model.

When the model is constructed, a listing has to be made of the input data required. How these input data can be obtained should be evaluated. Which input data can be measured, and which input data cannot be measured and should be estimated, must be assessed.

Generalizations and approximations in the determination of the input data may be necessary and possible. However, the quality of the output of a model will only be as good as the quality of the model and its input data. A sensitivity analysis can be performed to evaluate the sensitivity of the output to each of the input data parameters. It is important to know which parameters are crucial in the determination of the output.

It should be emphasized under which conditions the model is valid. This depends strongly on the assumptions and the validity of the physical laws used in the model. The model has to be tested. There are two methods of testing the model. The calculated results can be compared to measurements and observations of the calculated variables in reality (in

natural processes those measurements may be field or laboratory measurements), or the calculated results may be compared with those calculated with other models. The latter method is less accurate than the former, because it is often not sure if the model with which the new model is compared is a close approximation of reality. The importance of each detail in a model can be evaluated by comparing a version of the model that incorporates that detail with a version that does not incorporate that particular detail.

III.2 Principles of the modelling of ground freezing and frost heave

Ground freezing and frost heave have been simulated in a phenomenologic model (e.g. Michalowski 1993), probabilistic model (e.g. Chamberlain 1980), and in deterministic models (e.g. Gilpin 1980; Konrad and Morgenstern 1980; O'Neill and Miller 1985; Blanchard and Frémond 1985; Wang 1994). As mentioned in chapter II good deterministic ground freezing and frost heave models incorporate a coupled modelling of heat flow, water flow and stress conditions.

III.2.1 Heat transfer

Since most of the heat transferred in the soil takes place by conduction, the basis of each deterministic model of freezing of soils and frost heave is the heat conduction theory (Smith 1985b). Conductive heat transfer in soils may be described by the law of Fourier:

$$f_h = -K_h(T) \frac{dT}{ds} \quad (\text{III.1A})$$

More detailed frost heave models also include a convection term uT :

$$f_h = -K_h(T) \frac{dT}{ds} - uT \quad (\text{III.1B})$$

Since heat can not appear from nowhere or disappear to nowhere, in modelling heat conduction a heat conservation equation may be applied:

$$\frac{dQ_h}{dt} = -\frac{df_h}{ds} + \alpha \quad (\text{III.2})$$

α is the heat generation or heat absorption in the studied space. In the case of a freezing soil, in which there are no other heat sources or sinks (eg. biological sources or chemical reactions that require or produce heat) in the volume of the soil studied, the latent heat of fusion (L_h) may be substituted for α in eq.(III.2) to represent the heat released when water transforms into ice. Combination of eq.(III.1A) and eq.(III.2) gives:

$$\frac{dQ_h}{dt} = -\frac{d(-K_h(T) \frac{dT}{ds})}{ds} + L_h \quad (\text{III.3A})$$

or for cases with convection combination of eq.(III.1B) and eq.(III.2) gives:

$$\frac{dQ_h}{dt} = -\frac{d(-K_h(T) \frac{dT}{ds} - uT)}{ds} + L_h = \frac{d(K_h(T) \frac{dT}{ds})}{ds} + \frac{duT}{ds} + L_h \quad (\text{III.3B})$$

Heat transfer takes place in a three dimensional space, and s is composed of components in an x , y and z direction and eqs.(III.3) becomes:

$$\frac{dQ_h}{dt} = \frac{\partial(K_h(T)\frac{\partial T}{\partial x})}{\partial x} + \frac{\partial(K_h(T)\frac{\partial T}{\partial y})}{\partial y} + \frac{\partial(K_h(T)\frac{\partial T}{\partial z})}{\partial z} + L_h \quad (\text{III.4A})$$

or for cases with convection:

$$\frac{dQ_h}{dt} = \frac{\partial(K_h(T)\frac{\partial T}{\partial x})}{\partial x} + \frac{\partial(K_h(T)\frac{\partial T}{\partial y})}{\partial y} + \frac{\partial(K_h(T)\frac{\partial T}{\partial z})}{\partial z} + \frac{\partial uT}{\partial x} + \frac{\partial uT}{\partial y} + \frac{\partial uT}{\partial z} + L_h \quad (\text{III.4B})$$

In cases where heat is transferred exclusively in the vertical direction, the thermal gradients in the x and y directions are 0, and eq.(III.4A) and eq.(III.4B) become:

$$\frac{dQ_h}{dt} = \frac{\partial(K_h(T)\frac{\partial T}{\partial z})}{\partial z} + L_h \quad (\text{III.5A})$$

$$\text{and } \frac{dQ_h}{dt} = \frac{\partial(K_h(T)\frac{\partial T}{\partial z})}{\partial z} + \frac{\partial uT}{\partial z} + L_h \quad (\text{III.5B})$$

According to the definition in chapter II.1 the heat capacity C_h is defined as the heat that needs to be extracted from 1 volume unit of soil to cool this soil unit 1K: $C_h(T) = \frac{dQ_h}{dT}$

(III.6) then combination of eq.(III.5A) and eq.(III.6) gives:

$$\frac{C_h(T)dT}{dt} = \frac{\partial(K_h(T)\frac{\partial T}{\partial z})}{\partial z} + L_h \quad (\text{III.7A})$$

and for the case in which convection is important combination of eq.(III.5B) and eq.(III.6)

$$\text{gives: } \frac{C_h(T)dT}{dt} = \frac{\partial(K_h(T)\frac{\partial T}{\partial z})}{\partial z} + \frac{\partial uT}{\partial z} + L_h \quad (\text{III.7B})$$

When C_h is constant in time, and K_h constant in space eq.(III.7A) becomes:

$$C_h(T) \frac{dT}{dt} = K_h(T) \frac{\partial^2 T}{\partial z^2} + L_h \quad (\text{III.8A})$$

or with vertical convection:

$$C_h(T) \frac{dT}{dt} = K_h(T) \frac{\partial^2 T}{\partial z^2} + \frac{\partial uT}{\partial z} + L_h \quad (\text{III.8B})$$

Eq.(III.8A) may be written in diffusion form:

$$\frac{dT}{dt} = D_h(T) \frac{\partial^2 T}{\partial z^2} + \frac{L_h}{C_h(T)} \quad (\text{III.9A})$$

$$\frac{dT}{dt} = D_h(T) \frac{\partial^2 T}{\partial z^2} + \frac{\partial uT}{\partial z} + \frac{L_h}{C_h(T)} \quad (\text{III.9B})$$

III.2.2 Water flow

Since the freezing of water causes the unfrozen water content and therefore the free energy (or water pressure potential) to decrease (chapter II.2.4), a free energy gradient (or water potential gradient) is generated along which water is flowing to the freezing front.

Laminar water flow through soils can be described with Darcy's law:

$$f_w = -K_w(T) \frac{dH_w}{ds} \quad (\text{III.10})$$

Resistance and inertia terms may be added to the right hand side of eq.(III.10) to make the equation more general. As mentioned in chapter II.2.3 the total water potential consists of a pressure, gravitational, osmotic, and pneumatic component. The gravitational potential

gradient over a narrow frozen fringe is negligibly small. In the case that the soil solution has the same salt concentrations everywhere in the soil, and the pores are saturated the total head in eq.(III.10) may be replaced by the pressure head, and eq.(III.10) becomes:

$$f_w = -K_w(T) \frac{d \frac{P_w}{\rho_w \cdot g}}{ds} = -K_w(T) \frac{dP_w}{\rho_w \cdot g \cdot ds} \quad (\text{III.11})$$

where K_w is the hydraulic conductivity of the soil (in ms^{-1})

In the case of vertical flow the distance ds may be replaced by dz .

III.2.3 Coupled heat and water flow in thermodynamic equilibrium

The transfer of heat and water are coupled by fundamental laws in thermodynamics. The response of soil to subzero temperatures can be examined under static conditions. In a freezing soil (certainly under non-steady-state freezing) such a static situation is, strictly speaking, absent (Nakano and Horiguchi 1985 and Nakano 1992, personal communication). In most of the research on freezing soils it is usually assumed, though never proven, that thermodynamic equilibrium exists locally in the freezing fringe and elsewhere in freezing soils: i.e. that water exists in equilibrium with ice at temperatures below the normal freezing point of free water. Because frost heave is a slow process, unfrozen water can move easily to a locally uniform state (O'Neill 1983). Therefore the water pressure is uniform, and local microscopic mechanical and thermodynamic equilibrium is assumed.

For the case of thermodynamic equilibrium a relationship between the temperature of a system and the pressure potentials of water and ice has been developed: the Clausius-Clapeyron equation. This relationship is derived by combination of the definition of absolute free energy, and the first and second law of thermodynamics (Edlefsen and Andersen 1943; Nash 1970). A simplified account of the derivation is given below.

In thermodynamics the free energy and the potential of a system (or potential energy per unit volume) which is defined exclusively for isothermal conditions, have the same meaning, a measure for the ability of a system to perform work. Free energy is defined as:

$$FE = IE + P \cdot V - T \cdot S \quad (\text{III.12})$$

where FE = free energy per unit mass of water (or Gibbs function)

IE = internal energy

V = volume of a unit mass of water

P = pressure of the water

T = temperature

S = entropy of the system

$$\text{Differentiation of eq.(III.12) gives: } dFE = dIE + PdV + VdP - TdS - SdT \quad (\text{III.13})$$

$$\text{first law of thermodynamics: } dIE = dQ - dW = dQ - PdV \quad (\text{III.14})$$

where dIE = increase in internal energy per unit mass of water

dQ = the amount of heat taken in per mass of water (= TdS)

dW = work done by the substance

PdV = work done for an expansion dV against pressure P

Combination of eq.(III.13) and eq.(III.14) gives: $dFE = -SdT + VdP$ (III.15)

Thermodynamic equilibrium of water and ice in a system means: $FE_i = FE_w$, and when any change occurs in the stable state $dFE_i = dFE_w$ (III.16) the

ice-water system obtains another stable equilibrium in which $FE_i = FE_w$.

Combination of eq.(III.15) and eq.(III.16) results in:

$$dFE_i = -S_i dT + V_i dP_i = dFE_w = -S_w dT + V_w dP_w \text{ and}$$

$$S_i - S_w = \frac{(V_i dP_i - V_w dP_w)}{dT} \quad (III.17)$$

$$\text{The second law of thermodynamics is: } dS = \frac{dQ}{T} \quad (III.18)$$

$$\text{For phase change from water to ice this means: } S_i - S_w = \frac{-L_h}{T} \quad (III.19)$$

Combination of eq.(III.17) and eq.(III.19) gives the Clausius-Clapeyron equation:

$$\frac{dT}{T} = \frac{(V_w dP_w - V_i dP_i)}{L_h} \quad (III.20)$$

This equation relates temperature, T , at which freezing takes place, the change in water and ice pressure (potentials), dP_w and dP_i , to the change of the freezing point, dT , in a thermodynamic equilibrium situation.

Macroscopic scale disequilibria (e.g. spatial and temporal temperature variations) may be essential driving forces of the frost heave process, but equilibrium between the ice and water phase is generally assumed at the macroscale as well (O'Neill 1983). During very slow freezing this could be true, provided that local water and ice pressures in the soil do not change much. Observations of a frost front penetrating glass beads demonstrates that phase equilibrium

thermodynamics may not be adequate to deal with rapid freezing, at least in large-grained material (Colbeck 1982 in O'Neill 1983). In an attempt to take into account situations in which thermodynamic equilibrium is absent, some researchers have tried to use irreversible thermodynamics to describe ground freezing (e.g. Wood 1985).

III.2.4 Salt exclusion, ice flow, and frost heave criteria

The effect of salt exclusion at the freezing front and thermally induced regelation in the form of the movement of brine pockets in the (partially) frozen soil towards the freezing front may as well be incorporated, e.g. by incorporating these effects in the Clausius-Clapeyron equation (Padilla and Villeneuve 1990, 1992, Grant 1993).

Miller (1972) suggested that the penetration of a freezing front in the soil should be modelled as a downward flow of ice. Models that use this principle are called rigid ice models (e.g. Holden 1985; Shah 1990; Black 1995). Wood (1985) suggested that moisture redistribution by regelation in a freezing soil, could be modelled by a Darcian type equation. This moisture flow occurs parallel to the flow of moisture through the unfrozen water films. Continued freezing and melting and refreezing of water in the frozen zone could be modelled in this manner.

Modelling of the time and location that each ice lens starts to form may result in a more accurate prediction of the location and thickness of the ice lenses. This is important, since, e.g. for buried pipelines, the frost heave below the pipeline is more important than the

total heave at the surface. This requires additional prescribed conditions. Gilpin (1979,1980) used a separation pressure and O'Neill and Miller (1985) used a zero effective stress criterion together with a stress partitioning parameter. According to the author of this thesis, the meaning of Gilpin's criterion is overcoming a cohesive strength in the soil, and the meaning of Miller's criterion is overcoming a friction component of the shear strength. Both should be used at the same time.

III.2.5 Mechanics of a freezing soil

The water flow generated by the cryosuction, and the subsequent freezing of the water in the partially frozen soil, increase the neutral stress (the part of the normal stress carried by the pore contents) that might become larger than the vertically downward directed normal stress, and soil strains (deformations and displacements) may be caused. Some of the increased neutral stress within the freezing soil is relaxed by the volumetric expansion of water upon freezing, and by viscous displacement (creep) of the soil (Shen and Ladanyi 1987). The remaining part of the increased neutral stress is dissipated by elastic and plastic displacement of the overburden and possible surcharge.

The strains consist of elastic strains, viscous (or creep) strains, and plastic strains. The relationship between the stress, σ , and strain, ε , for elastic behaviour can be described by

$$\text{Bach's power law in scalar form: } \varepsilon = \left(\frac{\sigma}{E} \right)^{1/m} \quad (\text{III.21})$$

or in vector form: $\{\sigma\} = [E]\{\varepsilon\}^m$ (III.22)

$m < 1$ for strain hardening material, for which the maximum stress level can be increased after deformation

$m = 1$ for material for which the yield stress after failure can not be increased beyond

the yield stress, Bach's power law is equal to Hooke's law: $\varepsilon = \frac{\sigma}{E}$ (III.23)

$m > 1$ for strain softening material, for which the maximum stress that can be supported is decreased after the yield stress is reached

In civil engineering practice (steady state) creep deformation of ice is most commonly described in a strain rate equation known as the Norton-Bayley or Norton-Hoff power law. For a more extensive discussion of this subject the reader is referred to the standard work written by Andersland and Ladanyi (1994).

Plastic behaviour can be described by the equation $\varepsilon = f(\sigma)$ (III.24)

Since any stress on the viscous material will result in an infinite amount of strain, a relationship between the stress and strain does not exist for viscous behaviour. However, the creep strain increase may be evaluated by using the Prandtl-Reuss law (Shen and Ladanyi 1987) in which the viscous behaviour is described as pseudo-elastic behaviour:

$$d\{\varepsilon^c\} = d\bar{\varepsilon}^c \frac{\partial \bar{\sigma}}{\partial \{\sigma\}} \quad (III.25)$$

$\{\varepsilon^c\}$ creep strain vector $\{\sigma\}$ stress vector

$\bar{\varepsilon}^c$ average creep strain $\bar{\sigma}$ average stress

Shen and Ladanyi (1987) assumed that there was not any plastic deformation in a freezing soil, and proposed the following relationship between the stress and strains in a freezing soil:

$$\{\sigma\} = [E](\{\varepsilon\} - \{\varepsilon^c\} - \{\varepsilon^v\}) \quad (\text{III.26})$$

where ε^c creep strain and ε^v volumetric strain

For freezing over a small time interval dt , the change in the stress conditions of the soil is:

$$d\{\sigma\} = [E](d\{\varepsilon\} - d\{\varepsilon^c\} - d\{\varepsilon^v\}) \quad (\text{III.27})$$

The volumetric strain increase due to freezing of water in the soil equals the sum of the increase in volume due to water that freezes in situ and the increase in volume due to the freezing of water that flows into the freezing fringe:

$$\Delta\varepsilon^v = 0.09(\theta_0 + \Delta\theta - \theta_{un}) + \Delta\theta + (\theta_0 - n) \quad (\text{III.28})$$

Figure III.1 Curve of the deformation of soil (based on Vyalov 1986)

Vyalov (1986) described the deformation of the freezing soil material as a function of

$$\text{time: } \varepsilon(t) = \underbrace{0}_{\text{I}} + \underbrace{\varepsilon_{BD}}_{\text{II}} + \underbrace{\int_{t_D}^{t_E} \dot{\varepsilon}_{DE}(t) dt}_{\text{III}} + \underbrace{\int_{t_E}^{t_F} \dot{\varepsilon}_{EF}(t) dt}_{\text{IV}} + \underbrace{\int_{t_F}^{t_G} \dot{\varepsilon}_{FG}(t) dt}_{\text{V}} + \underbrace{\int_{t_G}^{t_{FIN}} \dot{\varepsilon}_{GFIN}(t) dt}_{\text{VI}} \quad (\text{III.29})$$

Eq.(III.29) represents the various phases of deformation (see Figure III.1): I a period with loading, without deformation; II an instantaneous deformation; III attenuating creep; IV linear creep; V accelerating creep; and VI plastic deformation after brittle or ductile failure. The duration of each of these phases varies for each soil from 0 to ∞ s.

Deformation in phase II and III may be divided into its elastic and plastic

$$\text{parts: } \varepsilon = \left(\frac{\sigma}{E} \right)^{1/m} + f(\sigma) \quad (\text{III.30})$$

The plastic deformation function f in eq.(III.30) has to be determined phenomenologically. Vyalov (1986) derived respectively a power relation, a logarithmic relation, a linear-fractional relation, a generalized time function and an exponential relation to describe the attenuating creep strain of soils induced by a constant stress during phase III. The logarithmic and power functions proposed by Vyalov (1986) are respectively:

$$\varepsilon = \left(\sigma \frac{1}{E_0} + \sigma \frac{\lambda}{\xi} \ln(1 + \xi t) \right)^{1/m} \quad (\text{III.31}) \quad \text{or} \quad \varepsilon = \left(\sigma \frac{1}{E_0} + \sigma \frac{1}{\xi} t^\lambda \right)^{1/m} \quad (\text{III.32})$$

where λ and ξ are dimensionless fitting parameters

Takegawa et al. (1979) proposed expressions dependent upon temperature T for fitting parameter ξ and elastic modulus E_0 :

$$E_0 = W(1 - T)^c \quad (\text{III.33})$$

$$\text{and } \xi = w(1 - T)^c \quad (\text{III.34})$$

where W , w and c are fitting parameters

Takegawa et al. (1979) assumed that for a frozen soil with ice lenses the elastic part, Bach's law conformed to Hooke's law (i.e. $m=1$). They found that the power law fitted their data better than the logarithmic expression.

In phase IV, the strain rate is constant, which may be expressed by:

$$\dot{\varepsilon} = c \text{ or } \varepsilon = c \cdot t \quad (\text{III.35})$$

In deformation phase V, accelerating creep occurs. From laboratory tests it could be concluded that soils that show a behaviour in which phase V is represented, behave as non-Newtonian fluids: the strain rate increases exponentially with the stress level. Since the total deformation that takes place at low stress levels is negligible compared to the total deformation during higher stress levels, the strain at low stress levels may be ignored. This is only justified if the time period that the soil is in this stress state is relatively short.

The deformation rate at relatively high stress levels in phase V may be described by

$$\text{Bingham's law: } \dot{\varepsilon} = \frac{\sigma - \sigma_y}{\eta_p} \quad (\text{III.36})$$

where η_p = coefficient of plastic viscosity

σ_y = fitting parameter (intersection second straight line and σ -axis) with units
of stress, ultimate long term strength

A more general form for the deformation during phase V takes the strain hardening or strain

softening of materials into account: $\dot{\varepsilon} = \frac{1}{\eta_p^*} \left(\frac{\sigma - \sigma_y}{\sigma^*} \right)^{1/m}$ (III.37)

where η_p^* = coefficient of plastic viscosity η_p divided by σ^*

and σ^* = dummy parameter with units of stress and value of 1

The lower σ_y (e.g. colloidal soils), the more the soil behaves as a fluid. Another widely used

relation for accelerating creep is $\dot{\varepsilon} = \frac{1}{\eta_p^*} \sinh\left(\frac{\sigma}{\sigma^*}\right)$ (III.38)

The general creep curve of freezing soils is more complicated than the creep curve of soils under isothermal unfrozen conditions because the mechanical soil properties change with time due to the water flow and phase change in the soil. Because in a freezing soil the unfrozen water content is decreasing and the ice content is increasing, the mechanical properties of the material change with time. The decreasing water content and increasing ice content in the soil with decreasing temperature increase the strength of the soil. The deformation under a certain stress level might decrease with time. Due to adfreezing of soil onto vertical structures in the soil, or “anchoring” to neighboring soil that is heaving at a slower rate, the mechanical situation in the soil is even more complicated; the soil may exhibit internal resistance to heave (Wood 1985) and failure may take place along vertical shear planes, as is often observed in laboratory freezing tests (e.g. Jetchick 1998, Smith and Williams 1995). The mode of failure may change with decreasing temperature from a ductile to a more brittle failure. Since the thermodynamic situation determines e.g. the volumetric

strain increase (eq.(III.28)) and the water flowing towards and freezing at the ice lenses increases the overburden σ , the thermodynamic situation and the stress situation in freezing soils are coupled.

Several researchers have suggested possibilities to couple the heat and water flow to the stress situation in the soil (Lundin 1989, Wood 1990, Smith 1992). Lundin examined a possibility under thermodynamic equilibrium, by making an adjustment to the Clausius-Clapeyron equation, while Wood and Smith explored possibilities under circumstances when such equilibrium is absent, due to creep of ice crystals and therefore a continuous changing ice pressure. According to Wood (1990) and Smith (1992) the strain rate and water flow are dependent on temperature and spatial change in ice pressure.

III.2.6 The resulting frost heave prediction model

Using the principles mentioned in this subchapter to model the amount of water flowing to the freezing soil; to model the volumetric, elastic, viscous and plastic strains generated by the addition and freezing of this water; and to model the increase in strength of the freezing soil, results in a set of equations from which primary, secondary and tertiary frost heave may be calculated. The frost heave model should predict: the amount of frost heave at certain locations (x and y coordinates) after a certain time; the rate of frost heave after certain time; the differential frost heave between certain locations; the neutral and effective stresses generated in the soil. With these predictions the stresses and strains on structures such as

pipelines and roads may be calculated. The development of deterministic frost heave prediction models has taken place along two lines:

1. **Practical frost heave prediction models**, that were mostly developed by civil engineers. The emphasis in these models was the practical use of the prediction model. A prediction model is deemed practical when all input parameters may be acquired in a relatively easy manner at a low cost. Examples of these models are e.g. the frost heave prediction models proposed by Harlan (1973), Taylor and Luthin (1978), Konrad and Morgenstern (1980, 1981, 1982) and Wang (1994) which have an emphasis on the thermodynamics of the frost heave process; and those of Gilpin (1979), Shen and Ladanyi (1987) and Ladanyi and Shen (1989) which gave attention to the modelling of the stress situation.
2. **Scientific frost heave prediction models**, that were mostly but not exclusively developed by mathematicians and soil physicists. The emphasis in these models was the accuracy of the simulation of the frost heave process. Attempts are made to avoid assumptions, and to use the basic laws of mathematics, physics, and chemistry. Examples of these models are those proposed by O'Neill and Miller (1985), Blanchard and Frémond (1985), Shah(1990), Mikkola and Frémond (1993), Fowler and Noon (1993) and Fowler and Krantz (1994). Although these scientific models usually provide good simulations of the frost heave process it is some times difficult to acquire reliable input data (e.g. Blanchard and Frémond 1985 and Frémond and Mikkola 1993) or the amount of computer time needed is impractical (Shah 1990).

However, a new trend has started in which civil engineers have begun to construct practical models in which elements are incorporated from the scientific models while these models are kept practical. The rate of success is increasing. Examples of this new trend are Holden (1985), Shah (1990), Nixon (1991 and 1992) and Sheng and Knutsson (1993).

III.2.7 Numerical solution of the frost heave prediction model

Usually the result of the modelling of the frost heave process is one or more implicit (non-linear/transcendental) differential equation(s) which are extremely difficult to solve analytically. For this reason numerical procedures are used to approximate the analytical solution.

A choice is made from a wide variety of finite element and finite difference methods. Second order differential equations such as thermal flow eqs.(III.9A) or eq.(III.9B) or liquid flow of the same form may be approximated by a Galerkin finite element method in which the released latent heat is spread over an entire element (O'Neill 1983). Elasticity problems are usually solved using the Ritz finite element method (Bathe 1982, Razaqpur 1997).

In finite element methods applied to thermal flow during freezing, the whole domain (complete length of the soil affected by freezing, or total time of the freezing process) is divided in pieces (elements), and the temperature profile and frost heave after each time step are calculated. In finite element methods applied to fluid flow the pore water pressure profile and frost heave after each time step may be calculated. This discretization procedure results

in a system of interdependent linear or non-linear equations. Non-linear or transcendental equations may be solved by iteration and linear systems by Gauss-elimination. Since the number of soil elements and time steps may be large, a large number of repetitive calculations has to be carried out.

The domain can be in a one, two or three dimensional space. Ideally the frost heave process should be modelled in a 3D space, in order to be able to account for parameters that vary in x, y and z directions and features in the development of the stress situation such as lateral anchoring. However, to avoid long calculation times, large amounts of required input data, and a high degree of complexity, often 1D or 2D frost heave models are chosen. The elements may have a variety of shapes such as line elements, triangular elements, rectangular or quadrilateral elements, trapezoidal elements and brick elements (see Figure III.2).

Figure III.2 Various shapes of elements:

(Naylor and Pande 1981)

top row from left to right: regular elements: line element, triangular element, rectangular element, cubic element

second row from left to right: bending elements: line element, triangular element, 8 noded quadrilateral element, 20 noded brick element

The interpolation functions in each element may be linear, but they may as well be parabolic. The smaller the size of the spatial elements and time steps, the closer the solution will be to the analytical solution of the model. However, the smaller the spatial elements and time steps, the more calculations have to be done, and the larger the calculation time. By discretization the whole domain is covered by a web of elements, which is called a “mesh”. In Figure III.3 and Figure III.4 two examples of meshes are given.

Figure III.3 Mesh of quadrilateral elements near a footing (source: Naylor and Pande

1981)

The shape of the element and the choice of technique to minimize the error in the numerical method are important in the numerical solution process. The various choices in numerical procedures are listed in Table III.2. Numerical procedures are convenient since they can relatively easily be dealt with in computer code and the repetitive calculations can be done relatively fast by computers. The computer code may be written in the computer language FORTRAN, which is still common in civil engineering, although an increasing

amount of software is currently being written in C and C++. Powerful software packages such as Mathematica, Mathlab, MathCAD and AutoCAD may also be used.

Figure III.4 Mesh of quadrilateral elements around a buried pipeline (Naylor and Pande1981).

III.2.8 Evaluation of frost heave simulation models

To date, many frost heave simulation models have been proposed. The predictions of these models may be compared with measurements. Unfortunately, the parameters measured in and the number of frost heave experiments are limited. Comparison of the prediction with a measurement of the predicted entity may give some confirmation of the accuracy of the prediction. The prediction might prove to be wrong, close to or exactly the same as the measurement. If the prediction is correct, this might be a coincidence, although the doubt diminishes with a growing amount of correct predictions using a wide variety of situations/cases/input data. Whatever the outcome of this exercise is, it is still uncertain if the simulation of the process is correct. Often prediction models contain parameters that may be adjusted to achieve a prediction that is close to the measurement, after which the model developer concludes that the designed model works. The problem is that reliable predictions are usually needed before the events happen, and not appropriate models after events have happened.

Proposed frost heave simulation models may be compared using two different methods. In a concise method the two or more models may be held next to each other and their differences tabulated. In a more elaborate method, all possible details of frost heave simulation models may be tabulated and those details that are included by the models to be compared may be checked off. An extensive checklist is developed based on the observations and measurements of features in the frost heave process (chapter II), the theoretical modelling

of the frost heave process (chapter III.1), and any numerical approximation used (chapter III.1.). This checklist is given in the form of Appendix A and Appendix B. Any existing deterministic frost heave prediction model can be taken, and the incorporated details can be checked off in Appendix A. The numerical procedure used can be checked off in Appendix B.

	Table Headings
I	PURPOSE OF THE PREDICTION MODEL
II	PROCESS FEATURE IN ONE DIMENSIONAL MODELLING OF FROST HEAVE
II.1	Thermal Aspects
II.2	Thermodynamic Aspects
II.3	Hydraulic Aspects
II.4	Formation of Ice and Resulting Frost Heave
II.5	Mechanical Aspects
II.6	Long Term Features
II.7	Spatial Features (SF)
II.7.1	SF in one dimensional frost heave prediction model
II.7.1.1	one layer soil profile
II.7.1.2	multilayer soil profile
II.7.2	SF in two dimensional frost heave prediction model
II.7.2.1	one layer soil profile
II.7.2.1.1	thermal and hydraulic parameters
II.7.2.1.2	mechanical parameters
II.7.2.2	multilayer soil profile
II.7.2.2.1	thermal and hydraulic parameters
II.7.2.2.2	multilayer soil profile
II.7.3	SF in three dimensional frost heave prediction model
II.7.3.1	one layer soil profile
II.7.3.1.1	thermal parameters
II.7.3.1.2	hydraulic parameters
II.7.3.1.3	mechanical parameters
II.7.3.2	multilayer soil profile
II.7.3.2.1	thermal parameters
II.7.3.2.2	hydraulic parameters
II.7.3.2.3	mechanical parameters

Table III.1 Table of table headings of the comprehensive frost heave prediction model comparison table (Appendix A).

The last two columns of the checklists of Appendices A and B are used to compare two frost heave prediction models which will be discussed respectively in Chapter III.3 and Chapter III.4. The table headings of Appendix A are given in Table III.1 and the table headings of Appendix B are given in Table III.2.

	Table Headings
I	HEAT TRANSFER
I.1	Finite difference formulation of heat transfer equation in space
I.2	Finite element (FEM) formulation of heat transfer equation in space
I.2.1	Weighted Residual Method
I.2.2	Dimension, shape and nodes of each element in FEM
I.2.2.1	One Dimension
I.2.2.2	Two Dimensions
I.2.2.3	Three Dimensions
I.3	Finite difference formulation of heat transfer equation in time
II	WATER FLOW
II.1	Finite difference formulation of water flow equation in space
II.2	FEM formulation of water flow equation in space
II.2.1	Weighted Residual Method
II.2.2	Dimension, shape and nodes of each element in FEM
II.2.2.1	One Dimension
II.2.2.2	Two Dimensions
II.2.2.3	Three Dimensions
II.3	Finite difference formulation of water flow equation in time
III	STRESS SITUATION
III.1	Finite difference formulation of stress situation in space
III.2	FEM formulation of stress situation in space
III.2.1	Ritz Method
III.2.2	Dimension, shape and nodes of each element in FEM
III.2.2.1	One Dimension
III.2.2.2	Two Dimensions
III.2.2.3	Three Dimensions
III.3	Finite difference formulation of stress situation in time

Table III.2 Table of table headings of the table to compare the type and location of application of numerical procedure (Appendix B).

To date, the purpose of the model and the background of the developer of the model have determined which items of the checklist were incorporated in any frost heave prediction model. Appendices A and B provide useful tools for comparisons of the structure of various frost heave simulation models, and for the development of new frost heave prediction models.

III.3 The frost heave prediction model of Wang (1994)

III.3.1 Physical base of the model

A simple one-dimensional frost heave prediction model was developed, to predict frost heave in the large scale experiments in Caen, and to predict possible frost heave in field situations (Wang 1994). The model incorporates the most important aspects of the frost heave process, such as coupled heat and water flow, and heave resulting from in situ freezing and segregated ice. Other features that are of less importance are not taken into account: e.g. convection and diffusion of heat are considered to be negligible; the volume of the soil particles is unchanged in the freezing process (non-compressible soil); consolidation effects are not taken into account; the effect of salt exclusion at the freezing front is assumed to be negligible; there are no heat and water fluxes in the x and y directions; and the soil properties are assumed to be uniform and have only a completely unfrozen and completely frozen value. Thereby the model is kept practical, long computation times and more input parameters that are difficult to determine are avoided.

The freezing soil is divided up into a frozen zone, a freezing fringe and an unfrozen zone. It was assumed that the water table is relatively close to the freezing front (there is no limitation to the water supply) and that water flow across the freezing fringe is continuous. This is true for wet soils in substantially thick active layers or in wet permafrost-free soils, in which the water table is often close to the surface.

The warmest ice lens impedes flow to the frozen zone, where water flow is negligible because of low water contents at low temperatures. Therefore the only soil zone of importance for the determination of frost heave is the freezing fringe. The freezing fringe in a soil can be characterized by a particular segregation freezing temperature, T_s , at the bottom of the warmest ice lens, and an initial freezing temperature at the freezing front, T_f . The temperature at the freezing front is assumed to be slightly below 0°C : eg. -0.03°C .

A sudden decrease in temperature at the soil surface results in an unsteady flow of heat upward in the soil: $\frac{dq_h}{dt} \neq 0$. Water freezes in situ and the freezing front is penetrating into the soil. The unsteady heat flow is evaluated with eq.(III.7A) from chapter III.2:

$$\frac{C_h(T)dT}{dt} = \frac{d(K_h(T)\frac{dT}{dz})}{dz} + L_{htot} \quad (\text{III.41})$$

The thermal soil properties are assumed to be the same throughout the soil profile (i.e. C_h and K_h are independent of z), and C_h is independent of time t and eq.(III.41) may be rewritten

$$\text{as: } C_h(T)\frac{dT}{dt} = K_h(T)\frac{d^2T}{dz^2} + L_{htot} \quad (\text{III.42})$$

Water freezes in situ and the latent heat of freezing of this water is released at the freezing front

$$\text{where } T=T_f: L_{htot} = nL_h \rho_w \frac{dZ_f}{dt} \quad (\text{III.43})$$

At the base of the warmest ice lens where $T=T_s$, latent heat of segregational freezing is

$$\text{released: } L_{htot} = f_w(t) L_h \rho_w \quad (\text{with } L_h \text{ given in J/kg}) \quad (\text{III.44})$$

where $f_w(t)$ is the amount of water flowing to the base of the warmest ice lens.

Along the temperature gradient, heat is removed from the soil until a new thermal equilibrium is reached. Then steady state freezing occurs: the situation in which the temperature does not change over time, i.e. $\frac{dT}{dt} = 0$. This situation is relatively uncommon, since the

temperature at the soil surface is almost continuously changing. Thick layers of snow or thick organic layers may insulate soil from these temperature changes, but they may also protect the soil from subzero temperatures. Eq.(III.42) is the constitutive equation for the conductive heat flow, and eqs.(III.43) and (III.44) are boundary conditions, which can be used to solve differential eq.(III.42).

At and between both boundaries of the freezing fringe (the freezing front and the bottom of the warmest ice lens) stable thermodynamic equilibrium is assumed (the free energy of water is equal to the free energy of ice); thus within the freezing fringe heat and water transport are coupled by the Clausius-Clapeyron equation (eq.III.20):

$$V_w dP_w - V_i dP_i = L_h \frac{dT}{T} \quad (\text{III.45})$$

Integration of eq.(III.45) over a domain from the freezing front to any level in the freezing fringe gives:

$$V_w P_w - V_{wf} P_{wf} - V_i P_i + V_{if} P_{if} = L_h \ln\left(\frac{T}{T_f}\right) \quad (\text{III.46})$$

which may be rewritten as:

$$P_w = \left(\frac{V_i}{V_w}\right) P_i - \left(\frac{V_i}{V_w}\right) P_{if} + \left(\frac{L_h}{V_w}\right) \ln\left(\frac{T}{T_f}\right) + P_{wf} \quad (\text{III.47})$$

Eq.(III.47) is slightly different from the expression used by Konrad and Morgenstern (1980, 1981). They assumed that the water pressure at the freezing front, P_{wf} , was equal to atmospheric pressure, which means that the freezing front is pushing the phreatic surface deeper into the soil. Eq.(III.47) shows the importance of the position of the water table for the frost heave process.

Since there is no ice at the freezing front, the ice pressure there, P_{if} , is 0. It is assumed that the ice lens in the soil is continuous in space, and the ice pressure at the bottom of the warmest ice lens, P_{is} , is therefore equal to the overburden pressure. The curve of the ice pressure as a function of the distance from a reference level in the unfrozen zone may be approximated by a straight line (Shen and Ladanyi 1987). The specific volume is the inverse of the specific mass (= the density) and eq.(III.47) may be written as:

$$P_w = \left(\frac{\rho_w}{\rho_i}\right) P_i - (\rho_w L_h) \ln\left(\frac{T}{T_f}\right) + P_{wf} \quad (\text{III.48})$$

The water pressure at the bottom of the warmest ice lens is:

$$P_{ws} = \left(\frac{\rho_w}{\rho_i}\right) P_{is} - (\rho_w L_h) \ln\left(\frac{T_s}{T_f}\right) + P_{wf} \quad (\text{III.49})$$

Close to the soil surface the overburden is relatively small, which means P_{is} is small and

$$\left(\frac{\rho_w}{\rho_i}\right) P_{is} < (\rho_w L_h) \ln\left(\frac{T_s}{T_f}\right), \text{ because the latent heat released is relatively large. Then } P_{ws} < P_{wf} \text{ and}$$

there is a cryosuction. In such case, a flow of water is generated from the high water pressure at the freezing front towards the level with a lower water pressure at the base of the ice lens.

The continuous saturated flow towards the base of the warmest ice lens is driven by the difference in water pressure over the freezing fringe, and can be described by Darcy's law

$$(\text{eq. (III.10)}): f_w(t) = -K_{wff} \frac{(P_{wf} - P_{ws})}{\rho_w g (Z_f - Z_s)} \quad (\text{III.50})$$

Combination of eqs. (III.49) and (III.50) gives:

$$f_w(t) = -K_{wff} \left(\left(-\frac{P_{is}}{(g(Z_f - Z_s))\rho_i} \right) + \frac{(L_h \ln(\frac{T_s}{T_f}))}{(g(Z_f - Z_s))} \right) \quad (\text{III.51})$$

The amount of segregational frost heave (secondary heave) in a time interval dt is:

$$dH_{sh} = 1.09 f_w(t) dt \quad (\text{III.52})$$

The amount of frost heave in situ (primary heave) in a time interval dt is:

$$dH_{ph} = 0.09 n dZ_f \quad (\text{III.53})$$

where dZ_f is the advance of the freezing front in time dt .

$$\text{The total heave } dH_{tot} \text{ is in a time interval } dt \text{ is: } dH_{tot} = dH_{ph} + dH_{sh} \quad (\text{III.54})$$

Combination of eqs. (III.51), (III.52), (III.53) and (III.54) gives:

$$dH_{tot} = 0.09ndZ_f + (1.09 \cdot (-K_{wff} \left((-\frac{P_{is}}{g(Z_f - Z_s)\rho_i} + \frac{(L_h \ln(\frac{T_s}{T_f}))}{g(Z_f - Z_s)}) \right)) \cdot dt) \quad (III.55)$$

The heat conduction equation (eq.(III.42)) can be solved so that $T(z,t)$ is known. Since T_s and T_f are constant values it will then be known at every time t what Z_s and Z_f are, i.e. where the freezing fringe is situated, and what the thickness of the freezing fringe $Z_f - Z_s$ is.

$$P_{is} \text{ is in a first approach equal to the overburden: } P_{is} = Z_s \cdot \rho_b \cdot g \quad (III.56)$$

and dH_{tot} , the amount of frost heave in time interval dt , can be calculated with eq.(III.55).

$\frac{dH_{tot}}{dt}$ gives the frost heave rate, and integration of eq.(III.55) over time t results in the total amount of frost heave. Theoretically segregational frost heave stops when the cryosuction is 0,

$$\text{i.e. when } P_{is} = Z_s \cdot \rho_b \cdot g = \rho_i \cdot L_h \cdot \ln \frac{T_s}{T_f} \text{ i.e. when } Z_s = \frac{\rho_i \cdot L_h \cdot \ln \frac{T_s}{T_f}}{\rho_b \cdot g} \quad (III.57)$$

III.3.2 Analytical solution of the model

III.3.2.1 Solution of the heat conduction equation

Solutions to the conductive heat flow equation (eq.(III.42) of chapter III.3) have the form $T=f(z,t)$. These solutions, which vary from simplified case specific solutions such as solutions to simple Stefan problems, to solutions to more generalized variations of the simple Stefan problem as well as Neumann types of solution for one-, two- or three-dimensional

problems, are given by Lunardini (1991), Nixon (1987) and Lock (1990). An example of a solution to the heat flow equation in which phase change is ignored is the solution given by Ingersoll, Zobel and Ingersoll (1954):

$$T(z,t) = \overline{T}_z + A_{ss} e^{-z \left(\frac{\omega t}{2D_h} \right)^{1/2}} \cdot \sin \left[\omega t \cdot \left(\frac{\omega t}{2D_h} \right)^{1/2} z \right] \quad (\text{III.58})$$

where $\omega = 2\pi/\tau$ τ =period, eg. 24hrs, 1 year (in Smith and Williams 1989)

and A_{ss} = the amplitude of the temperature at the soil surface

In cases such as the frost heave process, in which phase change occurs, the solution is more complex.

III.3.2.2 Solution of the frost heave equation

To solve the frost heave equation (eq.(III.55) of chapter III.3.1) and calculate the frost heave the following parameters need to be known:

n porosity, soil parameter, constant

Z_f depth of the freezing front

Z_s depth of the base of the warmest ice lens

dZ_f progression of the freezing front in time interval dt , variable

K_{wff} hydraulic conductivity of the freezing fringe, soil parameter

P_{is} ice pressure at base of warmest ice lens, equal to overburden pressure

ρ_i density of ice, constant, known

L_h	specific latent heat of freezing, constant, known
T_s	temperature at the base of the warmest ice lens, constant, known
T_f	temperature at the freezing front, constant, known
dt	time interval

The overburden pressure is equal to the pressure of the soil on top and the accumulated ice from segregation: $dP_{is} = (\rho_b \cdot g \cdot dZ_s) + \left(1000 \cdot g \cdot \int f_w(t) dt\right)$ (III.59)

$$f_w(t) \text{ can be determined by eq.(III.51): } f_w(t) = -K_{wff} \left(\left(-\frac{P_{is}}{(g(Z_f - Z_s))\rho_i} \right) + \frac{(L_h \ln(\frac{T_s}{T_f}))}{(g(Z_f - Z_s))} \right)$$

Combination of eq.(III.59) and eq.(III.51) gives:

$$dP_{is} = (\rho_b \cdot g \cdot dZ_s) + \left(1000 \cdot g \cdot \int \left(-K_{wff} \left(\left(-\frac{P_{is}}{(g(Z_f - Z_s))\rho_i} \right) + \frac{(L_h \ln(\frac{T_s}{T_f}))}{(g(Z_f - Z_s))} \right) dt \right) \right) \quad (III.60)$$

Eq.(III.60) is extremely difficult to solve because it is an implicit differential equation (it is very difficult to separate P_{is} from the other parameters), and P_{is} , Z_f , and Z_s are functions of time. Therefore it is extremely difficult to determine P_{is} , dH_{tot} and H_{tot} analytically. To determine the frost heave increase and the total amount of frost heave a numerical approximation of the analytical solution is used.

III.3.3 Numerical approximation of the conductive heat flow equation

The conductive heat flow equation (eq.(III.42)) can be written as:

$$K_h(T) \frac{\delta^2 T}{\delta z^2} - C_h(T) \frac{\delta T}{\delta t} + L_{htot} = 0 \quad (\text{III.61})$$

The solution to the conductive heat flow equation (a second order differential equation) is $T(z,t)$, the temperature written as a function of depth and time. Since it is difficult to solve this equation analytically, the soil profile is divided into n elements with an upper node n and a lower node $n+1$ as boundaries (see Figure III.5).

Figure III.5 Approximation of an actual temperature profile by discretization and linear interpolation

If the temperature at node 1 is T_1 , and the temperature at node 2 is T_2 , then the real temperature function T over the element is approximated by a linear interpolation function T^A (see Figure III.6):

Figure III.6 Linear interpolation functions N_1 and N_2 over a spatial element with a length l (Wang 1994)

$$T^A = N_1 T_1 + N_2 T_2 = \sum_{i=1}^2 N_i T_i \quad (\text{III.62})$$

where N_1 is 1 when $T^A = T_1$ and N_1 is 0 when $T^A = T_2$, and

N_2 is 0 when $T^A = T_1$ and N_2 is 1 when $T^A = T_2$

$$N_1 = 1 - \frac{z}{L} \quad \text{and} \quad N_2 = \frac{z}{L} \quad (\text{III.63})$$

L is the total length of the element, and $0 \leq z \leq L$

$$\text{in vector notation eq.(III.62) can be written: } [T^A] = [N_1 \quad N_2] \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = [N] \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \quad (\text{III.64})$$

if the approximate solution T^A of eq.(III.64) is substituted in eq.(III.61):

$$K_h(T^A) \frac{\delta^2 T^A}{\delta z^2} - C_h(T^A) \frac{\delta T^A}{\delta t} + L_{\text{hot}} = R(z, t) \quad (\text{III.65})$$

Usually $R(z,t)$ is not equal to zero for every z and t (otherwise the solution is exact).

Thus, $R(z,t)$ represents the error or residual of the approximation, which should be kept to a small amount. One of the methods to minimize $R(z,t)$ is Galerkin's method:

$$\int_0^L N_i(z) R(z,t) dz = 0 \quad (\text{III.66})$$

$$\text{or in vector notation } \int_0^L [N]^T R(z,t) dz = 0 \quad (\text{III.67})$$

where $[N]^T = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$ is the transposed matrix of $[N] = \begin{bmatrix} N_1 & N_2 \end{bmatrix}$

Eq.(III.67) means to keep the surface area between the curve of the exact solution and the curves of the approximate solutions as small as possible. Substituting eq.(III.65) in

$$\text{eq.(III.67): } \int_0^L [N]^T \left(K_h(T^A) \frac{\delta^2 T^A}{\delta z^2} - C_h(T^A) \frac{\delta T^A}{\delta t} + L_{htot} \right) dz = 0 \quad (\text{III.68})$$

Eq.(III.68) can be written as:

$$\left(\int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} K_h(T^A) \frac{\delta^2 T^A}{\delta z^2} dz \right) - \left(\int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} C_h(T^A) \frac{\delta T^A}{\delta t} dz \right) + \left(\int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} L_{htot} dz \right) = 0 \quad (\text{III.69})$$

K_h and C_h are assumed to be constant throughout the soil profile, and to be independent from

T , and eq.(III.69) can be written as:

$$K_h \left[\begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \frac{dT}{dz} \right]_0^L - K_h \int_0^L \frac{d \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}}{dz} \frac{dT^A}{dz} dz - C_h \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \frac{\delta T^A}{\delta t} dz + \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} L_{htot} dz = 0 \quad (\text{III.70})$$

Substituting eq.(III.64) into eq.(III.70) gives:

$$\begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} - K_h \int_0^L \begin{bmatrix} \frac{dN_1}{dz} \\ \frac{dN_2}{dz} \end{bmatrix} \frac{d \left(\begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \right)}{dz} dz - C_h \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \frac{\delta \left(\begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \right)}{\delta t} dz + \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} L_{htot} dz = 0 \quad (\text{III.71})$$

which may be written as

$$\begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} - K_h \int_0^L \begin{bmatrix} \frac{dN_1}{dz} \\ \frac{dN_2}{dz} \end{bmatrix} \begin{bmatrix} \frac{dN_1}{dz} & \frac{dN_2}{dz} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} dz - C_h \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} dz + \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} L_{htot} dz = 0 \quad (\text{III.72})$$

Substitution of eq.(III.63) into eq.(III.72) gives:

$$\begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} - K_h \int_0^L \begin{bmatrix} \frac{-1}{L} \\ \frac{1}{L} \end{bmatrix} \begin{bmatrix} \frac{-1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} dz - C_h \int_0^L \begin{bmatrix} 1 - \frac{z}{L} \\ \frac{z}{L} \end{bmatrix} \begin{bmatrix} 1 - \frac{z}{L} & \frac{z}{L} \end{bmatrix} \begin{bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} dz + \int_0^L \begin{bmatrix} 1 - \frac{z}{L} \\ \frac{z}{L} \end{bmatrix} L_{htot} dz = 0 \quad (\text{III.73})$$

The matrix multiplication in eq.(III.73) gives:

$$\begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} - \left(K_h \int_0^L \begin{bmatrix} \frac{1}{L^2} & \frac{-1}{L^2} \\ \frac{-1}{L^2} & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} dz \right) - \left(C_h \int_0^L \begin{bmatrix} 1 - \frac{2z}{L} + \frac{z^2}{L^2} & \frac{z}{L} - \frac{z^2}{L^2} \\ \frac{z}{L} - \frac{z^2}{L^2} & \frac{z^2}{L^2} \end{bmatrix} \begin{bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} dz \right) + \left(\int_0^L \begin{bmatrix} 1 - \frac{z}{L} \\ \frac{z}{L} \end{bmatrix} L_{htot} dz \right) = 0 \quad (\text{III.74})$$

Integration results in:

$$-\left(K_h \begin{bmatrix} \frac{1}{L^2} & \frac{-1}{L^2} \\ \frac{-1}{L^2} & \frac{1}{L^2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} z \right)_0^L - \left(C_h \begin{bmatrix} z - \frac{z^2}{L} + \frac{1}{3} \frac{z^3}{L^2} & \frac{1}{2} \frac{z^2}{L} - \frac{1}{3} \frac{z^3}{L^2} \\ \frac{1}{2} \frac{z^2}{L} - \frac{1}{3} \frac{z^3}{L^2} & \frac{1}{3} \frac{z^3}{L^2} \end{bmatrix} \begin{bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} \right)_0^L + F = 0 \quad (\text{III.75})$$

where

$$F = \begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} + \int_0^L \begin{bmatrix} 1 - \frac{z}{L} \\ \frac{z}{L} \end{bmatrix} L_{htot} dz = L_{htot} \begin{bmatrix} z - \frac{1}{2L} z^2 \\ \frac{1}{2L} z^2 \end{bmatrix}_0^L + \begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} = \begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} + L_{htot} \begin{bmatrix} L - \frac{1}{2} L \\ \frac{1}{2} L \end{bmatrix} = \begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} + L_{htot} \frac{L}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (\text{III.76})$$

Eq.(III.75) becomes:

$$\left(K_h \begin{bmatrix} \frac{1}{L} & \frac{-1}{L} \\ \frac{-1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \right) + \left(C_h \begin{bmatrix} L - L + \frac{1}{3} L & \frac{1}{2} L - \frac{1}{3} L \\ \frac{1}{2} L - \frac{1}{3} L & \frac{1}{3} L \end{bmatrix} \begin{bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} \right) = F \quad (\text{III.77})$$

Rewriting of eq.(III.77) and substitution eq.(III.76) into eq.(III.77) gives:

$$\left(\underbrace{\frac{K_h}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}}_{K\text{-matrix}} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \right) + \left(\underbrace{\frac{C_h L}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{C\text{-matrix}} \begin{bmatrix} \frac{dT_1}{dt} \\ \frac{dT_2}{dt} \end{bmatrix} \right) = L_{htot} \frac{L}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} q_{h1} \\ q_{h2} \end{bmatrix} \quad (\text{III.78})$$

$$\text{Eq.(III.78) can be written as: } \mathbf{K}_s \mathbf{T} + \mathbf{C}_s \frac{d\mathbf{T}}{dt} = \mathbf{F} \quad (\text{III.79})$$

In which $\mathbf{K}_s$ and $\mathbf{C}_s$ are the matrices resulting from this spatial consideration.

An excellent description of the derivation of eq.(III.78) is given in chapters 2 and 5 of Desai (1979). The broader context of the variety and application of numerical methods in

(geo)technical problems is addressed by Bathe (1982), Cook (1984), Lock (1990) and Naylor and Pande (1981).

The temperature is not only a function of depth z , but also a function of time t . The exact temperature curve over time is approximated by a numerical method. The temporal elements used are given in Figure III.7.

Figure III.7 Discretization of the soil temperature in time (Wang 1994)

If the temperature at time n at nodes 1 and 2 is denoted by vector $\mathbf{T}^n = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^n$ and the

temperature at time $n+1$ at nodes 1 and 2 denoted by vector $\mathbf{T}^{n+1} = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^{n+1}$, the temperature

vector within this time element is approximated by linear interpolation:

$$\mathbf{T}^B = N_{t1} \mathbf{T}^n + N_{t2} \mathbf{T}^{n+1} \quad (\text{III.80})$$

If the total length of the time element is Δt , the time dependent interpolation functions

$$\text{are: } N_{t_n} = 1 - \frac{t}{\Delta t} \text{ and } N_{t_{n+1}} = \frac{t}{\Delta t} \quad (\text{III.81})$$

Replacing the exact temperature T at any time t by the approximated temperature T^B of

$$\text{eq.(III.80) in eq.(III.79) gives: } \mathbf{K}_s \mathbf{T}^B + \mathbf{C}_s \frac{d\mathbf{T}^B}{dt} = \mathbf{F}^B \quad (\text{III.82})$$

Substitution of eq.(80) and eq.(81) in eq.(82) gives:

$$\mathbf{K}_s \left(\left(1 - \frac{t}{\Delta t}\right) \mathbf{T}^n + \left(\frac{t}{\Delta t}\right) \mathbf{T}^{n+1} \right) + \mathbf{C}_s \left(-\frac{1}{\Delta t} \mathbf{T}^n + \left(\frac{1}{\Delta t}\right) \mathbf{T}^{n+1} \right) = \left(1 - \frac{t}{\Delta t}\right) \mathbf{F}^n + \left(\frac{t}{\Delta t}\right) \mathbf{F}^{n+1} \quad (\text{III.83})$$

By choosing $t = \frac{1}{2} \Delta t$, i.e. the temperature in the time interval Δt is evaluated by linear

interpolation at the centre of the time element, the solution of eq.(III.83) converges to the analytical solution and is unconditionally stable, i.e. no conditions need to be placed on the

size of the time and spatial steps. Eq.(III.83) becomes:

$$\mathbf{K}_s \left(\frac{1}{2} \mathbf{T}^n + \frac{1}{2} \mathbf{T}^{n+1} \right) + \mathbf{C}_s \left(\frac{-1}{\Delta t} \mathbf{T}^n + \left(\frac{1}{\Delta t}\right) \mathbf{T}^{n+1} \right) = \frac{1}{2} \mathbf{F}^n + \frac{1}{2} \mathbf{F}^{n+1} \quad (\text{III.84})$$

multiplying both sides of the equation by 2 gives:

$$\mathbf{K}_s (\mathbf{T}^n + \mathbf{T}^{n+1}) + 2\mathbf{C}_s \left(\frac{-1}{\Delta t} \mathbf{T}^n + \left(\frac{1}{\Delta t}\right) \mathbf{T}^{n+1} \right) = \mathbf{F}^n + \mathbf{F}^{n+1} \quad (\text{III.85})$$

Eq.(III.85) can be written as:

$$\left(\mathbf{K}_s - \frac{2}{\Delta t} \mathbf{C}_s \right) \mathbf{T}^n + \left(\mathbf{K}_s + \frac{2}{\Delta t} \mathbf{C}_s \right) \mathbf{T}^{n+1} = \mathbf{F}^n + \mathbf{F}^{n+1} \quad (\text{III.86})$$

Rewriting of eq.(III.86) gives:

$$\left(\mathbf{K}_s + \frac{2}{\Delta t} \mathbf{C}_s\right) \mathbf{T}^{n+1} = \left(-\mathbf{K}_s + \frac{2}{\Delta t} \mathbf{C}_s\right) \mathbf{T}^n + \mathbf{F}^n + \mathbf{F}^{n+1} \quad (\text{III.87})$$

Substitution of the spatial matrices of eq.(III.78) in eq.(III.87) gives:

$$\left(\frac{K_h}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{2}{\Delta t} \frac{C_h L}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right) \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^{n+1} = \left(\frac{-K_h}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{2}{\Delta t} \frac{C_h L}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right) \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}^n + \mathbf{F}^n + \mathbf{F}^{n+1} \quad (\text{III.88})$$

According to Wang (1994) for small time intervals it can be assumed that the thermal load vector, $\mathbf{F}_L^{n+1}$, more or less equals the thermal load vector, $\mathbf{F}_L^n$, of the former timestep. In eq.(III.88) the temperature at nodes 1 and 2 at time $n+1$ is expressed as a function of the temperature at these nodes at time n . Therefore, if the initial temperature profile is given, the temperature of the spatial element a time interval Δt later can be calculated. Such a description is called an explicit description of an initial value problem. Node 2 of spatial element 1, is the first node of the next spatial element 2. All elements and their accompanying matrix equations (III.88) can now be assembled.

Eq.(III.88) is the numerical equation for the first element. The equation for the second element is equation (III.88), with T_1 and T_2 replaced respectively by T_2 and T_3 . If the soil profile is subdivided in n elements, the n resulting matrix equations can be added, resulting in one matrix equation with 2 matrices consisting of $n+1$ rows and $n+1$ columns. The following eq. (III.89) illustrates this for the case of 4 elements (i.e. 5 nodes in total):

$$\begin{aligned}
& \left(\frac{K_h}{L} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} + \frac{2}{\Delta t} \frac{C_h L}{6} \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \right) \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}^{n+1} = \\
& \left(\frac{-K_h}{L} \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} + \frac{2}{\Delta t} \frac{C_h L}{6} \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix} \right) \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}^n + \mathbf{F}^n + \mathbf{F}^{n+1} \quad (\text{III.89})
\end{aligned}$$

Eq.(III.89) can be written as:

$$\begin{aligned}
& \begin{bmatrix} \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 & 0 \\ -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 \\ 0 & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 \\ 0 & 0 & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} \\ 0 & 0 & 0 & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}^{n+1} = \\
& \begin{bmatrix} -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 & 0 \\ \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 \\ 0 & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 \\ 0 & 0 & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} \\ 0 & 0 & 0 & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}^n + 2 \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} \\
& \quad \quad \quad (\text{III.90})
\end{aligned}$$

which may be rewritten as:

$$\begin{aligned}
 \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}^{n+1} &= \begin{bmatrix} \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 & 0 \\ -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 \\ 0 & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 \\ 0 & 0 & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} \\ 0 & 0 & 0 & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} \end{bmatrix}^{-1} \\
 &\left(\begin{bmatrix} -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 & 0 \\ \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 & 0 \\ 0 & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & 0 \\ 0 & 0 & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{2K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{4}{3} & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} \\ 0 & 0 & 0 & \frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{1}{3} & -\frac{K_h}{L} + \frac{C_h L}{\Delta t} \cdot \frac{2}{3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}^n + 2 \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix} \right)
 \end{aligned}
 \tag{III.91}$$

III.3.4 Flow diagram and pseudocode of the model

Matrix eq.(91) may be written as n (in this case n=5) equations. K_h , C_h , L and Δt are known constants. If the initial temperature of the soil profile is known the n right hand side temperatures, T_1 , T_2 , T_3 , etc., are known for the first time step, and the only unknowns are the n left hand side temperatures, T_1 , T_2 , T_3 , etc., after time Δt . These n equations with n unknowns can be solved by the Gauss elimination procedure.

Figure III.8 Flow chart of the thermodynamic part of the model of Wang (1994)

For each timestep the whole temperature profile of the soil is calculated with eq.(91): i.e. the temperature at spatial node 1, 2, 3 etc. until the last spatial node. The location of the freezing fringe is determined and the frost heave over this time interval Δt is calculated with eq.(55) and the temperature profile after the next timestep Δt is calculated. A flow diagram of the whole solution procedure and therefore the basic design of a computer program is given in Figure III.8. A more detailed description, or pseudocode of the computer program consists of the following steps:

- I. Description of program
- II. Declaration of (soil) parameters
- III Initialization of soil parameters
 1. Read soil parameters (input file); Initialization of soil parameters from $z=0$ to $z=L$ at $t_0=0$
 2. Set temperature step, set spatial step
 3. Read temperature profile (input file); Initialization of soil temperature from $z=0$ to $z=L$ at $t_0=0$
- IV If $t \leq t_0 + n \cdot \Delta t$ then set $t = t + \Delta t$
Else print $H_{tot} =$
- V Calculate water flow to ice lens
 1. Find depth z_{A1} where $T(z_{A1}, 0) = T_f$
 2. Find depth z_{B1} where $T(z_{B1}, 0) = T_s$
 3. Calculate $a_{fi} = z_{A1} - z_{B1}$

4. Calculate new temperature profile with eq.(92)
5. Find depth where $T(z_{A2}, \Delta t) = T_f$
6. Find depth where $T(z_{B2}, \Delta t) = T_s$
7. Calculate $a_{ff2} = z_{A2} - z_{B2}$
8. Calculate $P_{is} = z_{B2} \cdot \rho_s \cdot g$
9. Calculate K_{wff}
10. Calculate $dZ_f = z_{A2} - z_{A1}$
11. Calculate the water flow to the ice lens: $df_w = K_{wff} \cdot dP_{is} / dZ_f$

VI Calculate secondary frost heave

1. Calculate the secondary heave dH_{tot} with eq.(55) in the last time step
2. Calculate the total secondary heave from t_0 : $H_{tot} = H_{tot} + dH_{tot}$

VII Reset soil parameters

VIII go to IV

The next level of detail of the computer program is the computer code.

III.3.5 Input parameters of the frost heave prediction model of Wang (1994)

The parameters needed to solve eqs.(III.91) and (III.55) of the model of Wang (1994) are:

1. Thermal input data

$T(z,0)$ initial temperature profile

$T(0,t)$ surface boundary temperature

T_f temperature at the freezing front, considered a constant

T_s temperature at the base of the warmest ice lens, considered a constant

$K_h(T)$ thermal conductivity, considered a constant

$C_h(T)$ heat capacity, considered a constant

$L_h(T)$ specific latent heat, considered a constant

2. Hydraulic input data

$K_w(T)$ hydraulic conductivity (of the freezing fringe)

n porosity of the soil

ρ_i bulk density of ice

3. Mechanic input data

ρ_s bulk density of soil

4. Data related to the geometry of space and time

Δz spatial step

Δt time step

Σt total time

III.3.6 Analysis of the sensitivity of the model to its input parameters

The sensitivity of a model to its input parameters can be tested by making predictions using a range of values for each parameter while keeping the other parameters constant. By dividing the value of an input parameter by the value of the input parameter in a reference situation, a technique that is called scaling, the parameter is made dimensionless. Since a scaled input parameter does not have a dimension, it may be compared to other dimensionless parameters. The frost heave prediction is also made dimensionless by dividing the predicted frost heave by the frost heave in the reference situation. The scaled input parameters and their scaled frost heave predictions may be plotted in a graph. The closer the scaled prediction is to 1, the closer the prediction is to the reference situation. If the whole curve of a certain parameter is close to 1, variations in that parameter are not important for the resulting frost heave prediction. Therefore, by using this scaling technique, the significance of variations in each input parameter of the prediction model for the frost heave prediction is evaluated and compared to the variation of the predictions generated by variation of the other input parameters in order to find out to which parameters the prediction model is the most sensitive.

The situation taken as reference was: a soil profile with a length of 10 cm, a linear initial temperature profile in the frozen soil, a linear temperature profile in the freezing fringe and a linear temperature profile in the unfrozen soil. The surface boundary temperature was -2°C , the segregation temperature was -0.1241°C , the freezing temperature was -0.0106°C ,

and the warm side temperature was 1°C . All these four parameters were constants. The thermal conductivity of unfrozen and frozen soil were respectively 1.45 and $2.18 \text{ Wkg}^{-1}\text{C}^{-1}$. The volumetric heat capacity of unfrozen and frozen soil are respectively 3.15 and $2.35 \text{ MJm}^{-3}\text{C}^{-1}$. The porosity or saturated volumetric water content of the soil was $0.322\text{m}^3\text{m}^{-3}$, and the average hydraulic conductivity of the freezing fringe was a constant 5.72 nms^{-1} . The bulk density of the soil was 1850kgm^{-3} , the external overburden pressure 63.58 kPa , and in the numerical approximation 50 spatial nodes were used including the 2 boundary nodes, and time steps of 0.005 days or 432s. The predicted frost heave in the reference situation was 5.1cm after 10 days.

III.3.6.1 The thermal input parameters

Figure III.9 Sensitivity of the frost heave prediction model of Wang to the thermal input parameters (relative frost heave predictions after 10 days)

ts segregation temperature tf freezing temperature

tcf thermal conductivity of frozen soil tcu thermal conductivity of unfrozen soil
 capf volum. heat capacity of frozen soil capu volum. heat capacity of unfrozen soil

Figure III.10 Sensitivity of the frost heave prediction model of Wang to the thermal input parameters (relative frost heave predictions after 50 days)

Figure III.11 Sensitivity of the frost heave prediction model of Wang to the thermal input parameters (relative frost heave predictions after 100 days)

From Figures III.9, III.10 and III.11, it may be concluded that the predictions vary substantially with any variation in each of the thermal parameters, even variations in the freezing temperature, t_f . Variation in the thermal parameters may increase or decrease the frost heave prediction with an order of magnitude. The largest frost heave is predicted when the segregation temperature is very low (e.g. -1.2°C), the thermal conductivity of the frozen soil is very low (e.g. $1.45 \text{ WK}^{-1}\text{m}^{-1}$), the thermal conductivity of unfrozen soil is high (e.g. $2.18 \text{ WK}^{-1}\text{m}^{-1}$), or the volumetric heat capacity of unfrozen or frozen soil is large (e.g. $3.15 \text{ MJK}^{-1}\text{m}^{-1}$). The sensitivity of the model to the segregation and freezing temperature remains constant over time while the sensitivity of the prediction model to variation in the other input parameters increases with increasing heaving time.

III.3.6.2 The hydraulic input parameters

From Figures III.12, III.13 and III.14, it may be concluded that the predictions vary substantially with any variation in both of the hydraulic parameters, but the most with a variation of the average hydraulic conductivity of the freezing fringe. Variation in the hydraulic conductivity may increase or decrease the frost heave prediction by more than an order of magnitude. The largest frost heave is predicted when the saturated volumetric water content is relatively low and when the hydraulic conductivity is around 10 nms^{-1} . The sensitivity of the prediction model to variation in the hydraulic input parameters increases with increasing heaving time.

Figure III.12 Sensitivity of the frost heave prediction model of Wang to the hydraulic input parameters (relative frost heave predictions after 10 days)

wutot porosity hcfri hydraulic conductivity of the freezing fringe

Figure III.13 Sensitivity of the frost heave prediction model of Wang to the hydraulic input parameters (relative frost heave predictions after 50 days)

Figure III.14 Sensitivity of the frost heave prediction model of Wang to the hydraulic input parameters (relative frost heave predictions after 100 days)

III.3.6.3 The mechanical input parameters and parameters related to geometry of space

Figure III.15 Sensitivity of the frost heave prediction model of Wang to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 10 days)

dens bulk density of the soil

po overburden

l length of the soil profile

Figure III.16 Sensitivity of the frost heave prediction model of Wang to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 50 days)

Figure III.17 Sensitivity of the frost heave prediction model of Wang to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 100 days)

From Figures III.15, III.16 and III.17, it may be concluded that the predictions vary substantially with any variation in each of the mechanical parameters and the parameters related to the geometry of space. Variation in the soil profile length may increase or decrease the predicted frost heave by one order of magnitude, and the sensitivity of the prediction model to the soil profile length increases with an increase in heaving time. The same may be concluded about the sensitivity of the prediction model to the bulk density of the soil. The predicted frost heave increases when the external overburden or surcharge increases from 125 to 635 kPa. This is an effect of the absence of a check in the computer code of whether the water pressure difference over the freezing fringe became positive. If such a check is included, the expectation is, that when the overburden continues to be increased, at a certain moment $P_{is} = Z_s \cdot \rho_b \cdot g = \rho_i \cdot L_h \cdot \ln \frac{T_s}{T_f}$ (III.57), and the shut off pressure is reached, so the frost heave prediction should be 0. Nixon's computer code contains such a checking mechanism.

III.3.7 Remarks on, advantages and limitations of the model

- Crucial to each frost heave model is the model of freezing front penetration/freezing fringe growth that is chosen: whether the freezing front is stationary or progresses during ice lens formation, and whether the segregation freezing temperature is constant or variable during the formation of single ice lenses. The model operates

without a frost heave criterion, a criterion for initial ice lens formation, such as $\sigma_{\text{eff}} = \sigma_{\text{tot}} - \sigma_n = 0$, used e.g. by Black and Miller (1985) and O'Neill and Miller (1985) or Gilpin's separation pressure concept. The constant segregation temperature serves in fact as a pseudo frost heave criterion, without a rationale other than that the problem becomes easier.

- While the base of the warmest ice lens in the model of Black and Miller (1985) is stationary, at some level in the freezing fringe the effective stress becomes zero; in this model the freezing fringe is moving continuously downward, without stops and jumps, and can not explain the formation of discrete ice lenses and rhythmic ice banding. The position where ice lenses form in this model depends on the size of the time step chosen. This is unrealistic; the model would gain credibility by including a (better) frost heave criterion.
- If the latent heat of in situ freezing is released at the freezing front, then the ice pressure is not 0 there!
- The assumption that the latent heat vector over one "small" time step is more or less equal to the latent heat vector of the next time step is questionable. "Small" remains undefined.
- The latent heat released at the freezing front is released at the node just below the freezing front and the node just above the freezing front. A similar situation applies to the base of the warmest ice lens.
- Because of the release of latent heat at freezing front and at the base of the warmest ice

lens, temperatures at these sites, rise to far above 0°C (in the reference situation in the sensitivity analysis even over 1000°C). As a result temperature profiles are obtained that look strange: 2 large peaks occur. In reality water freezes over the entire fringe, and latent heat is released over the whole fringe. This is the result of the use of a thermal load vector at only 2 nodes. O'Neill (1983) mentioned that one of the advantages of the use of finite element methods is that latent heat can be distributed over an entire element, maybe over more elements, instead of over 2 discrete nodes. However, it should be realized that even today experts in mechanical engineering are still struggling to solve the latent heat problem.

- For the solution of eq.(77) over time, $\frac{dT_1}{dt}$ and $\frac{dT_2}{dt}$ are approximated by a central difference procedure. The Crank-Nicholson method might be an improvement. A second Galerkin approach might also be an improvement.
- The numerical approximations might be closer to the analytical solution if more sophisticated elements are used. The two noded regular bar is the most simple element that exists. More sophisticated elements would be e.g. a parabolic bar (bar with a middle node), quadrilaterals with time as y coordinate, etc.
- It is assumed that the suction over the freezing fringe is driving the movement of the water to the bottom of the warmest ice lens. Freezing of water over the whole freezing fringe and suction in the freezing fringe draws water from below. The water pressure at levels below the freezing front (in the unfrozen soil) will decrease. This

gradient could be important: although the suction gradient might be low, the permeabilities in the unfrozen soil are high ($=K_{\text{sat}}$) under the current assumptions.

- The water pressure in the model is not subdivided into its composite pressures: osmotic pressure, double layer pressures, all these composite pressures could be included so that features such as salt exclusion at the freezing front might be explained.
- Ice lens growth and secondary frost heave stops when $\left(\frac{\rho_w}{\rho_i}\right)P_{is} \geq (\rho_w L_h) \ln\left(\frac{T_s}{T_f}\right)$,
when $P_{ws} < P_{wf}$ and there is no longer a cryosuction in the freezing fringe. This limit for secondary heave is a consequence of the frost heave model under its current assumptions. The freezing front may penetrate further, so primary and total heave may continue.
- The model is very sensitive to almost all of its input parameters, and small errors in input parameters may generate large errors in the frost heave prediction. The model is the most sensitive to the most variable and least known input parameter: the hydraulic conductivity of the freezing fringe.
- The thermal and hydraulic properties do not vary in the profile, while often even the topsoil consists of multiple layers. Adjustments can be made in the computer code.
- The thermal properties are not functions of the temperature in the model. The properties are given only 2 values: frozen and unfrozen. In reality C_h and K_h are not only dependent on location z , but as well on temperature $\rightarrow$ every location has its own temperature at each moment in time. The model could be improved by substitution of C_h and K_h as proposed in Schellekens (1995).

- If suction gradients in the frozen soil are known, according to Wood (1985) tertiary heave might be modelled by application of a Darcyan type equation. This could not only improve the model, but also be the first time that tertiary heave was modelled!
- The model can easily be expanded to include a layered soil system (eg. the 3 layers encountered in Norman Wells) (see Desai 1979). This could be achieved by putting a node at the top of each new layer. Care has to be taken of changing water pressure gradients generated by two soils with different coarseness on top of each other.
- The model could be expanded to a 2D model. Care has to be taken of changing water pressure gradients generated by two soils with different coarseness next to each other.
- Other features not included could be included (eg. compressible soils). Dessication of the soil below the freezing front leads to negative pore water pressures which result in shrinkage cracks. It is probably hard to model the effect that the formation of these cracks has on the hydraulic conductivity and thermal properties of the soil.
- The boundary conditions are held stationary. Predictions can be made, imposing a variable boundary condition at the soil surface, such as an annual temperature wave at T_1 .
- The heat transfer and water transport are not coupled to the development of stresses and strains in the freezing soil. Ideally these 3 aspects should be fully integrated.
- A further improvement would be to model the heave with a snow layer on top.

III.3.8 Modifications to the model

The one layer one dimensional frost heave prediction model of Wang was adjusted in order that frost heave in soil consisting of three layers could be simulated. It is possible to include even more layers, but the soil of the case study near the pipeline at Norman Wells showed three distinct top soil layers. In the computer program the thermal, hydraulic and mechanic properties of each of the layers are used. The depth of the boundaries between the three layers is also needed.

It was noticed that the computer code of Wang did not include primary or in situ heave, while this was suggested in his thesis. An adjustment was made to the code to include this primary heave.

The hydraulic conductivity is an important parameter in the modelling, but although the explanation in Wang's thesis suggested an approximation to calculate the value of the hydraulic conductivity, this procedure did not appear in the computer code. An attempt was made to set up a scheme for the determination of the hydraulic conductivity based on temperature and grainsize analysis.

Since the sensitivity analysis showed that the frost heave increased when the overburden increased beyond a certain value, it was concluded that this had to be the result of an error in the computer code. On verification it was found that there was no check when the water pressure changed sign (from negative to positive) in such cases there is expulsion of water instead of suction. Some computer code was added to make sure that the relatively high

water pressures generated under high overburden stresses would not result in the increase of primary and secondary heave, but would continue to decrease the frost heave.

The sparse comments make the model not very user friendly. Therefore more comments were added, to make the computer code clearer and therefore more accessible.

III.4 The frost heave prediction model of Nixon (1991)

III.4.1 Physical base of the model

III.4.1.1 Introduction

Nixon (1991) developed a one-dimensional frost heave prediction model, to predict frost heave in small scale laboratory experiments, and possibly to predict frost heave in field situations. Similar to Wang's model, the model describes the most important phenomena of the frost heave process, such as coupled heat and water flow, and heave resulting from in situ freezing and segregated ice. To keep the model practical, long computation times and (more) input parameters that are difficult to determine are avoided. Therefore features considered to be of less importance are ignored: e.g. convection and diffusion of heat are considered to be negligible, the volume of the soil particles is unchanged in the freezing process (non-compressible soil); consolidation effects are not taken into account; the effect of salt exclusion at the freezing front is assumed to be negligible; there are no heat and water fluxes in the x and

y directions. The freezing soil is again divided up into a frozen zone, a freezing fringe and an unfrozen zone (see Figure III.18).

Figure III.18 Geometry of the freezing fringe in the model of Nixon (1991)

As in the model of Wang, it was assumed that the water table is relatively close to the freezing front (there is no limitation to the water supply) and the water flow across the freezing fringe is continuous. The soil properties in the freezing fringe such as the hydraulic conductivity change with unfrozen water content and therefore temperature. The warmest ice lens impedes flow to the frozen zone, where water flow is negligible because of low water contents at low

temperatures. Therefore the only soil zone of importance for the determination of frost heave is the freezing fringe.

The main differences with the model of Wang are that in the model of Nixon the segregation temperature is not a constant, but varies with time and space, new ice lenses are formed when the ice pressure overcomes the overburden pressure and a separation pressure, the temperature profiles in the frozen soil, freezing fringe and unfrozen soil are linear, the water and ice pressure in the freezing fringe are non-linear (parabolic), and the pore water freezes over a range of temperatures in the fringe.

III.4.1.2 Heat flow

The freezing fringe in a soil is characterized by a particular segregation freezing temperature, T_s , at the bottom of the warmest ice lens, and an initial freezing temperature at the freezing front, T_f . The temperature profile in the soil is assumed to be linear in the frozen soil, in the freezing fringe and in the unfrozen soil. The linear temperature across the frozen soil may

$$\text{be described as: } T = (T_c - T_s) \cdot \left(1 - \frac{x}{X_f + h}\right) + T_s \quad (\text{III.92})$$

where x = distance from surface T_c = temperature at the cold surface
 X_f = thickness of frozen soil boundary
 h = heave

the linear temperature across the freezing fringe may be described as:

$$T = (T_s - T_f) \cdot \left(1 - \frac{x}{a}\right) + T_f \quad (\text{III.93})$$

where x = distance from base of ice lens

a = thickness of freezing fringe

and the linear temperature across unfrozen soil may be described as:

$$T = (T_f - T_w) \cdot \left(1 - \frac{x}{X_u}\right) + T_w \quad (\text{III.94})$$

where x = distance from the freezing front

X_u = thickness of frozen soil

The heat balance at the base of the warmest ice lens (provided that all water drawn into the fringe freezes at the base of the warmest ice lens) is:

$$VH_{ff} + LHSF = -K_{ff} \frac{(T_s - T_f)}{a} + L_h \cdot V_{ff} = -K_f \frac{(T_c - T_s)}{X_0 + h} \quad (\text{III.95})$$

where VH_{ff} = heat flow from freezing fringe

$LHSF$ = latent heat of segregation freezing

The heat balance at the freezing front (provided that the latent heat released by the freezing of pore water in situ) is:

$$VH_u + LHISH = -K_u \frac{(T_f - T_w)}{X_u} + nL_h \cdot \frac{da}{dt} = -K_f \frac{(T_s - T_f)}{a} \quad (\text{III.96})$$

where VH_u = heat flow from freezing fringe

$LHISH$ = latent heat of in situ freezing

III.4.1.3 Water flow

The water flow in the freezing fringe may be described by Darcy's law, eq.(III.19):

$$V_{ff} = K_w \frac{dP_w}{dx} \quad (\text{III.97})$$

The hydraulic conductivity in the freezing fringe depends on the amount of unfrozen water in the pores, and therefore on the temperature. Nixon fitted a curve to data from Oliphant et al. (1983), Smith (1985a), Kay et al. (1981), Fukuda (1980), Xu et al. (1985), Ratkje and Yamamoto (1982), Yoneyama et al. (1983), Horiguchi and Miller (1983), and Ohrai and

$$\text{Yamamoto (1985): } K_w = \frac{K_w^{-1^\circ\text{C}}}{(-T)^\alpha} \quad (\text{III.98})$$

Combination of eq.(III.93) and eq.(III.98) gives:

$$K_w = \frac{K_w^{-1^\circ\text{C}}}{\left(-\left(T_s - T_f\right) \cdot \left(1 - \frac{x}{a}\right) - T_f\right)^\alpha} \quad (\text{III.99})$$

Water flow through the fringe can be described by combining eq.(III.97) and eq.(III.99):

$$V_{ff} = \frac{K_w^{-1^\circ\text{C}}}{\left(-\left(T_s - T_f\right) \cdot \left(1 - \frac{x}{a}\right) - T_f\right)^\alpha} \cdot \frac{dP_w}{dx} = \text{const.} \quad (\text{III.100})$$

$$\text{Eq.(III.100) may be written as: } \frac{dP_w}{dx} = V_{ff} \cdot \frac{\left[-\left(T_s - T_f\right) \cdot \left(1 - \frac{x}{a}\right) - T_f\right]^\alpha}{K_w^{-1^\circ\text{C}}} \quad (\text{III.101})$$

Integration of eq.(III.101) over the freezing fringe gives:

$$P_w - P_f = V_{ff} \cdot a \cdot \frac{\left[-\left(T_s - T_f\right) \cdot \left(1 - \frac{x}{a}\right) - T_f\right]^{\alpha+1}}{K_w^{-1^\circ\text{C}} \cdot (T_s - T_f) \cdot (\alpha + 1)} + C \quad (\text{III.102})$$

if $x=a$ and $P_w=P_f$ then $C = -V_{ff} \cdot a \cdot \frac{(-T_f)^{\alpha+1}}{K_w^{-1^\circ C} \cdot (T_s - T_f)(\alpha + 1)}$ (III.103)

$$P_w - P_f = V_{ff} \cdot a \cdot \frac{\left[-(T_s - T_f) \left(1 - \frac{x}{a} \right) - T_f \right]^{\alpha+1} - (-T_f)^{\alpha+1}}{K_w^{-1^\circ C} \cdot (T_s - T_f) \cdot (\alpha + 1)} \quad (III.104)$$

III.4.1.4 Thermodynamic equilibrium in the freezing fringe

Thermodynamic equilibrium is assumed in the freezing fringe, and the Clausius

Clapeyron equation (III.18) may be used: $\frac{dT}{T} = \frac{(V_w dP_w - V_i dP_i)}{L_h}$ (III.105)

Integration of eq.(III.105) from any location in the freezing fringe to the freezing front gives:

$$P_w = \left(\frac{V_i}{V_w} \right) P_i - \left(\frac{V_i}{V_w} \right) P_{if} + \left(\frac{L_h}{V_w} \right) \ln \left(\frac{T}{T_f} \right) + P_{wf} \quad (III.106)$$

which may be written as:

$$P_w - P_{wf} = \left(\frac{V_i}{V_w} \right) P_i - \left(\frac{V_i}{V_w} \right) P_{if} + \left(\frac{L_h}{V_w} \right) \ln \left(\frac{T}{T_f} \right) \quad (III.107)$$

Since $\frac{V_i}{V_w} = \frac{\rho_w}{\rho_i} = \frac{1000}{900} = 1.09$, the ice pressure at the freezing front (P_{if}) is 0, and $\ln \left(\frac{T}{T_f} \right) \approx \frac{T}{T_f}$

eq.(III.107) becomes:

$$P_w - P_{wf} = 1.09 \cdot P_i + \left(\frac{L_h}{T_f V_w} \right) \cdot T = 1.09 \cdot P_i + \left(\frac{L_h}{T_f V_w} \right) \cdot T \quad (III.108)$$

According to Nixon the Clausius Clapeyron equation is: $P_w = 1.09 \cdot P_i + \left(\frac{L_h}{T_f V_w}\right) \cdot T$

This means that the Clausius Clapeyron equation is integrated from the phreatic surface (where $P_w=0$) to somewhere in the fringe. The water pressure difference over this length is considered to be the driving force of the moisture flow in the soil. Because the driving force of the water flow to the warmest ice lens will be the water pressure difference from the phreatic surface up

$$\text{to the base of the ice lens, } P_{ws} = 1.09 \cdot P_{is} + \left(\frac{L_h}{T_f V_w}\right) \cdot T_s \quad (\text{III.109})$$

the ice pressure at the bottom of the ice lens P_{is} equals the overburden pressure: $P_{is}=P_o$, and

$$\text{eq.(III.109) becomes: } P_{ws} = 1.09 \cdot P_o + \left(\frac{L_h}{T_f V_w}\right) \cdot T_s \quad (\text{III.110})$$

If the pressures are known, the temperature at the base of the warmest ice lens may be

$$\text{calculated by rewriting eq.(III.110): } T_s = \frac{P_{ws} - 1.09 \cdot P_o}{\left(\frac{L_h}{T_f V_w}\right)} \quad (\text{III.111})$$

Because the water pressure at the base of the ice lens, P_{ws} , decreases with time, the segregation freezing temperature T_s is not constant, and decreases during the growth of the ice lens.

Combination of the thermodynamic equilibrium condition, eq.(III.110) and the temperature and water flow condition of eq.(III.104) gives:

$$P_{ws} - P_f = V_{ff} \cdot a \cdot \frac{\left[-\left(T_s - T_f\right) \left(1 - \frac{x}{a}\right) - T_f \right]^{\alpha+1} - \left(-T_f\right)^{\alpha+1}}{K_w^{-1^\circ C} \cdot (T_s - T_f) \cdot (\alpha + 1)} = 1.09 \cdot P_o + \left(\frac{L_h}{T_f V_w}\right) \cdot T_s - P_f \quad (\text{III.112})$$

$$V_{ff} = \frac{1.09 \cdot P_o + \left(\frac{L_h}{T_f V_w}\right) \cdot T_s - P_f}{a} \cdot \frac{K_w^{-1^\circ C} \cdot (T_s - T_f) \cdot (\alpha + 1)}{\left[-(T_s - T_f) \left(1 - \frac{x}{a}\right) - T_f \right]^{\alpha+1} - (-T_f)^{\alpha+1}} \quad (\text{III.113})$$

If the temperature of the freezing front T_f is assumed to be 0°C , the water flow at the base of the warmest ice lens where $x=0$ and $T=T_s$ is:

$$V_{ff} = \frac{1.09 \cdot P_o + \left(\frac{L_h}{T_f V_w}\right) \cdot T_s - P_f}{a} \cdot \frac{K_w^{-1^\circ C} \cdot (T_s) \cdot (\alpha + 1)}{[-(T_s)]^{\alpha+1}} \quad (\text{III.114})$$

Combination of eq.(III.104) and eq.(III.114) gives the difference in water pressure between the freezing front and somewhere in the freezing fringe:

$$P_w - P_f = \frac{1.09 \cdot P_o + \left(\frac{L_h}{T_f V_w}\right) \cdot T_s - P_f}{a} \cdot \frac{K_w^{-1^\circ C} \cdot (T_s) \cdot (\alpha + 1)}{[-(T_s)]^{\alpha+1}} a \cdot \frac{\left[-(T_s) \left(1 - \frac{x}{a}\right) \right]^{\alpha+1}}{K_w^{-1^\circ C} \cdot (T_s) \cdot (\alpha + 1)} \quad (\text{III.115})$$

$$\text{which equals: } P_w - P_f = \left(1.09 \cdot P_o + \left(\frac{L_h}{T_f V_w}\right) \cdot T_s - P_f \right) \left[\left(1 - \frac{x}{a}\right) \right]^{\alpha+1} \quad (\text{III.116})$$

III.4.1.5 Frost heave criterion

Following notions of Gilpin (1979, 1980) and Wood (1985) in the model of Nixon, a new ice lens forms when somewhere in the freezing fringe the ice pressure overcomes the overburden and a separation pressure, P_{sep} , which is a representation of cohesive forces in the soil: $P_i = P_o + P_{sep}$ (III.117)

III.4.1.6 Distribution of phase change and in situ frost heave over the freezing fringe

As mentioned in chapter II.2.2 not all the pore water freezes at the freezing front, but the pore water freezes over the range of temperatures in the freezing fringe. According to Anderson et al. (1973) the unfrozen water content varies with temperature according to the empirical relation: $w_u = A \cdot (-T)^B$ (III.118)

where A = unfrozen water content at -1°C

B = fitting parameter, according to Nixon $\approx -\frac{1}{5}\alpha$

w_u may be written as a portion of the total water content in the unfrozen state: $W_u = \frac{w_u}{w_{tot}}$ (III.119)

Combination of eq.(III.118) and eq.(III.119) gives the partial unfrozen water content as a

function of temperature: $W_u = \frac{A}{w_{tot}} (-T)^B$ (III.120)

If A and B and w_{tot} of a soil are known, the minimum temperature at which all the pore water (w_{tot}) is unfrozen (the freezing temperature T_f) may be calculated since $w_u = w_{tot}$, therefore $W_u = 1$, and all parameters of eq.(III.120) are known except for $T = T_f$. Combination of eq.(III.93) and eq.(III.120) gives the partial unfrozen water content in the freezing fringe as a function of the distance x from the ice lens:

$$W_u = \frac{A}{w_{tot}} \left[-\left(T_s - T_f\right) \cdot \left(1 - \frac{x}{a}\right) - T_f \right]^B \quad (III.121)$$

The other part of the pore water at distance x from the ice lens is frozen: $1 - W_u$. The latent heat that is released when water freezes in the freezing fringe is $\frac{H_f}{1.09} L$. The rate of the increase in latent heat from in situ freezing is:

$$L \frac{dH_f}{1.09dt} = \frac{L}{1.09} \left(\frac{dH_f}{da} \cdot \frac{da}{dt} + \frac{dH_f}{dT_s} \cdot \frac{dT_s}{dt} \right) \quad (\text{III.122})$$

This latent heat has to balance the difference between the incoming and outgoing heat flux at

$$\text{the freezing front (III.96): } LH_{ISF} = VH_{ff} - VH_u = -K_f \frac{(T_s - T_f)}{a} + K_u \frac{(T_f - T_w)}{X_u}$$

$$L \frac{dH_f}{0.09dt} = \frac{L}{0.09} \left(\frac{dH_f}{da} \cdot \frac{da}{dt} + \frac{dH_f}{dT_s} \cdot \frac{dT_s}{dt} \right) = -K_f \frac{(T_s - T_f)}{a} + K_u \frac{(T_f - T_w)}{X_u} \quad (\text{III.123})$$

T_s and maybe T_c and T_w are not constant, but functions of time. Moreover the thickness of the freezing fringe, a , is also a function of time. Analytical solution of eq.(III.123) becomes difficult. This in situ freezing of pore water results in (primary) or in situ heave. The amount of frost heave, H_f , due to freezing of pore water in the freezing fringe is:

$$H_f = 0.09 \cdot \left(n \int_0^a (1 - W_u) dx \right) \quad (\text{III.124})$$

Combination of eq.(III.121) and eq.(III.124) gives the amount of frost heave, H_f , due to freezing of pore water in the freezing fringe as a function of the distance x from the warmest ice lens:

$$H_f = 0.09 \cdot \left(n \int_0^a \left(1 - \frac{A}{w_{tot}} \left[- (T_s - T_f) \cdot \left(1 - \frac{x}{a} \right) - T_f \right]^B \right) dx \right) \quad (\text{III.125})$$

Integration of eq.(III.125) gives:

$$H_f = 0.09 \cdot n \left[x - \left(\frac{A}{w_{tot}} \frac{\left[-(T_s - T_f) \left(1 - \frac{x}{a} \right) - T_f \right]^{B+1}}{(B+1) \cdot (T_s - T_f)} a \right) \right]_0^a \quad (\text{III.126})$$

Eq.(III.126) may be written as:

$$H_f = 0.09 \cdot n \left(a - \left(\frac{A}{w_{tot}} \frac{[-T_f]^{B+1}}{(B+1) \cdot (T_s - T_f)} a \right) - 0 + \left(\frac{A}{w_{tot}} \frac{[-T_s]^{B+1}}{(B+1) \cdot (T_s - T_f)} a \right) \right) \quad (\text{III.127})$$

$$\text{or } H_f = 0.09 \cdot n \cdot a \left(1 + \left(\frac{A}{w_{tot}} \frac{[-T_f]^{B+1} - [-T_s]^{B+1}}{(B+1) \cdot (T_f - T_s)} \right) \right) \quad (\text{III.128})$$

The heave due to in situ freezing of water in the freezing fringe is, according to eq.(III.128), a function of the thickness of the fringe, a , and the temperature at the base of the ice lens, T_s , which both change with time.

In the situation of distributed phase change over the freezing fringe, the description of the freezing of water flowing to the base of the warmest ice lens, the water flow in the freezing fringe, changes too. The flux density, V_{ff} , is not constant over the fringe, and eq.(III.100) is not valid anymore. Instead $V_{ff} \neq const.$ and the increase in flow over the freezing fringe equals the

$$\text{decrease in unfrozen water content over time: } \frac{d \left(K_w \frac{dP_w}{dx} \right)}{dx} = 0.09n \frac{dW_u}{dt} \quad (\text{III.129})$$

The solution to eq.(III.129) is P_w written as a function of x and t . Eq.(III.129) is a second order differential equation for water flow, similar to the second order differential equation for heat

flow in the model of Wang. This equation is difficult to solve analytically and a numerical approximation has to be used.

III.4.2 Numerical approximation of the water flow equation

In order to solve the water flow eq.(III.129) $\frac{d\left(K_w \frac{dP_w}{dx}\right)}{dx} = 0.09n \frac{dW_u}{dt}$, which is

necessary to calculate secondary heave, the freezing fringe may be divided up into n spatial elements. The nodes are situated in the centre of each element.

Node 1 coincides with the base of the warmest ice lens and node $m+1$ coincides with the freezing front. At node 1, the water pressure is $P_{w,1}$ and the hydraulic conductivity is $K_{w,1}$ (see Figure III.19).

Figure III.19 Discretization of the freezing fringe by the model of Nixon (1991)

The hydraulic conductivity $K_{w,1}$ is assumed to be constant throughout each element. At the boundary between the first and the second element, at node $x_{1/2}$, the water flow through the

first half of element 2 equals the flow through the second half of element 1:

$$K_{w,2} \cdot \frac{P_{w,2} - P_{w,1/2}}{\frac{1}{2} \Delta x} = K_{w,1} \frac{P_{w,1/2} - P_{w,1}}{\frac{1}{2} \Delta x} \quad (\text{III.130})$$

This equation may be rewritten as $P_{w,1/2} : P_{w,1/2} = \frac{K_{w,2} P_{w,2} + K_{w,1} P_{w,1}}{K_{w,2} + K_{w,1}}$ (III.131)

Equation (III.131) may also be rewritten as $P_{w,1/2} : P_{w,1/2} = \frac{K_{w,1} P_{w,1} + K_{w,0} P_{w,0}}{K_{w,1} + K_{w,0}}$ (III.132)

The difference between the water flow in the bottom of element 1 and the top of element 1 equals the volumetric strain due to freezing of pore water in situ in element 1:

$$\frac{K_{w,1}}{\frac{1}{2} \Delta x} \left(P_{w,1/2} - P_{w,1} \right) - \frac{K_{w,1}}{\frac{1}{2} \Delta x} \left(P_{w,1} - P_{w,1/2} \right) = \Delta F W_1 \cdot \Delta x \quad (\text{III.133})$$

where $\Delta F W_1 = 0.09n \frac{\Delta W_{u,1}}{\Delta t}$ (III.134)

Substitution of eqs.(III.131) and (III.132) in (III.133) gives:

$$\frac{K_{w,0}}{K_{w,1} + K_{w,0}} P_{w,0} - \left[\frac{K_{w,2}}{K_{w,1} + K_{w,2}} + \frac{K_{w,0}}{K_{w,1} + K_{w,0}} \right] P_{w,1} + \frac{K_{w,2}}{K_{w,1} + K_{w,2}} P_{w,2} = \frac{\frac{1}{2} (\Delta x)^2 \cdot \Delta F W_1}{K_{w,1}} \quad (\text{III.135})$$

Eq.(III.135) for the first element in matrix form is:

$$\left[\frac{K_{w,0}}{K_{w,1} + K_{w,0}} - \left(\frac{K_{w,2}}{K_{w,1} + K_{w,2}} + \frac{K_{w,0}}{K_{w,1} + K_{w,0}} \right) \frac{K_{w,2}}{K_{w,1} + K_{w,2}} \right] \cdot \begin{bmatrix} P_{w,0} \\ P_{w,1} \\ P_{w,2} \end{bmatrix} = \frac{\frac{1}{2}(\Delta x)^2 \cdot \Delta FW_1}{K_{w,1}} \quad (\text{III.136})$$

The equation for the second element is:

$$\left[\frac{K_{w,1}}{K_{w,2} + K_{w,1}} - \left(\frac{K_{w,3}}{K_{w,2} + K_{w,3}} + \frac{K_{w,1}}{K_{w,2} + K_{w,1}} \right) \frac{K_{w,3}}{K_{w,2} + K_{w,3}} \right] \cdot \begin{bmatrix} P_{w,1} \\ P_{w,2} \\ P_{w,3} \end{bmatrix} = \frac{\frac{1}{2}(\Delta x)^2 \cdot \Delta FW_2}{K_{w,2}} \quad (\text{III.137})$$

The equation for the last node m+1 of the mth element is:

$$\left[\frac{K_{w,m}}{K_{w,m+1} + K_{w,m}} - \left(\frac{K_{w,m+2}}{K_{w,m+1} + K_{w,m+2}} + \frac{K_{w,m}}{K_{w,m+1} + K_{w,m}} \right) \frac{K_{w,m+2}}{K_{w,m+1} + K_{w,m+2}} \right] \cdot \begin{bmatrix} P_{w,m} \\ P_{w,m+1} \\ P_{w,m+2} \end{bmatrix} = \frac{\frac{1}{2}(\Delta x)^2 \cdot \Delta FW_{m+1}}{K_{w,m+1}} \quad (\text{III.138})$$

The general form of the equation for each element is:

$$\begin{bmatrix} C_j & A_j & B_j \end{bmatrix} \cdot \begin{bmatrix} P_{w,j-1} \\ P_{w,j} \\ P_{w,j+1} \end{bmatrix} = D_j \quad (\text{III.139})$$

$$\text{where } C_j = \frac{K_{w,j-1}}{K_{w,j} + K_{w,j-1}}; A_j = -\left(\frac{K_{w,j+1}}{K_{w,j} + K_{w,j+1}} + \frac{K_{w,j-1}}{K_{w,j} + K_{w,j-1}} \right); B_j = \frac{K_{w,j+1}}{K_{w,j} + K_{w,j+1}}$$

$$D_j = \frac{\frac{1}{2}(\Delta x)^2 \cdot \Delta FW_j}{K_{w,j}} \quad \text{and } 1 \leq j \leq m+1$$

For 5 elements eq.(III.139) is in matrix form:

$$\begin{bmatrix} C_1 & A_1 & B_1 & 0 & 0 & 0 & 0 \\ 0 & C_2 & A_2 & B_2 & 0 & 0 & 0 \\ 0 & 0 & C_3 & A_3 & B_3 & 0 & 0 \\ 0 & 0 & 0 & C_4 & A_4 & B_4 & 0 \\ 0 & 0 & 0 & 0 & C_5 & A_5 & B_5 \end{bmatrix} \begin{bmatrix} P_{w,0} \\ P_{w,1} \\ P_{w,2} \\ P_{w,3} \\ P_{w,4} \\ P_{w,5} \\ P_{w,6} \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{bmatrix} \quad (\text{III.140})$$

The boundary values for this numerical problem may be found by examining the equation for each boundary node. The equation for node 1, at the base of the warmest ice lens is:

$$C_1 \cdot P_{w,0} + A_1 \cdot P_{w,1} + B_1 \cdot P_{w,2} = D_1 \quad (\text{III.141})$$

At the base of the warmest ice lens: $x=0$, $P_{w,1}=P_{ws}$

$P_{w,1}$ only equals P_{ws} for every $P_{w,0}$ and every $P_{w,2}$ when $C_1=0$, $B_1=0$, $A_1=1$ and

$$D_1 = \frac{1}{2} \Delta x^2 \frac{\Delta F W_1}{K_{w,1}} = P_{ws} \quad (\text{III.142})$$

At node $m+1$ at the freezing front: $x=a$, $P_{w,m+1}=P_{wf}$

At the freezing front, the flow through the bottom half of the warmest element of the freezing fringe should equal the flow through the unfrozen soil:

$$K_{w,m+1} \frac{(P_{w,m+1} - P_{w,m+\frac{1}{2}})}{\frac{1}{2} \Delta x} = K_{wu} \frac{(P_{ww} - P_{wf})}{X_u} \text{ or} \quad (\text{III.143})$$

$$K_{wu} \frac{(P_{ww} - P_{wf})}{X_u} - K_{w,n+1} \frac{(P_{w,m+1} - P_{w,m+\frac{1}{2}})}{\frac{1}{2} \Delta x} = 0 \quad (\text{III.144})$$

The water flow is not constant since the phase change in the bottom half of the warmest element in the freezing fringe has to be taken into account:

$$K_{wu} \frac{(P_{ww} - P_{wf})}{X_u} - K_{w,m+1} \frac{(P_{w,m+1} - P_{w,m+\frac{1}{2}})}{\frac{1}{2} \Delta x} = \Delta F W_{m+1} \Delta x \quad (\text{III.145})$$

P_{ww} is considered to be equal to atmospheric pressure and therefore 0 and using eq. (III.136)

to evaluate $P_{w,m+\frac{1}{2}}$: $P_{w,m+\frac{1}{2}} = \frac{K_{w,m+1} P_{w,m+1} + K_{w,m} P_{w,m}}{K_{w,m+1} + K_{w,m}}$ eq.(III.146) becomes:

$$-K_{wu} \frac{P_{wf}}{X_u} - \Delta F W_{m+1} \Delta x = \frac{K_{w,m+1}}{\frac{1}{2} \Delta x} \cdot (P_{w,m+1} - \frac{K_{w,m+1} P_{w,m+1} + K_{w,m} P_{w,m}}{K_{w,m+1} + K_{w,m}}) \quad (\text{III.147})$$

Eq.(III.147) may be written as:

$$-K_{wu} \frac{P_{wf}}{X_u} - \Delta F W_{m+1} \Delta x = \frac{K_{w,m+1}}{\frac{1}{2} \Delta x} \cdot (\frac{K_{w,m+1} + K_{w,m}}{K_{w,m+1} + K_{w,m}} P_{w,m+1} - \frac{K_{w,m+1} P_{w,m+1} + K_{w,m} P_{w,m}}{K_{w,m+1} + K_{w,m}}) \quad (\text{III.148})$$

which equals

$$-K_{wu} \frac{P_{wf}}{X_u} - \Delta F W_{m+1} \Delta x = \frac{K_{w,m+1}}{\frac{1}{2} \Delta x} \cdot (\frac{K_{w,m+1} P_{w,m+1} + K_{w,m} P_{w,m+1} - K_{w,m+1} P_{w,m+1} - K_{w,m} P_{w,m}}{K_{w,m+1} + K_{w,m}}) \quad (\text{III.149})$$

which may be written as:

$$-K_{wu} \frac{P_{wf}}{X_u} - \Delta F W_{m+1} \Delta x = \frac{K_{w,m} K_{w,m+1}}{K_{w,m+1} + K_{w,m}} \cdot (\frac{P_{w,m+1} - P_{w,m}}{\frac{1}{2} \Delta x}) \quad (\text{III.150})$$

Since $P_{w,m+1} = P_{wf}$ and $K_{w,m+1} = K_{wu}$ eq.(III.150) becomes:

$$-K_{wu} \frac{P_{wf}}{X_u} - \Delta FW_{m+1} \Delta x = \frac{2K_m K_{wu}}{\Delta x (K_{wu} + K_{w,m})} \cdot P_{wf} - \frac{2K_m K_{wu}}{\Delta x (K_{wu} + K_{w,m})} \cdot P_{w,m} \quad (\text{III.151})$$

Reorganizing eq.(III.151) gives:

$$\frac{K_m}{(K_{wu} + K_{w,m})} \cdot P_{w,n} - \left(\frac{K_{w,m}}{(K_{wu} + K_{w,m})} + \frac{\Delta x}{X_u} \right) \cdot P_{wf} = \frac{\Delta FW_{m+1} (\Delta x)^2}{2K_{wu}} \quad (\text{III.152})$$

This means when $j=m+1$, $C_{m+1} = \frac{K_{w,m}}{(K_{wu} + K_{w,m})}$, $A_{m+1} = -\left(\frac{K_{w,m}}{(K_{wu} + K_{w,m})} + \frac{\Delta x}{X_u} \right)$, $B_{m+1}=0$ and

$$D_{m+1} = \frac{\Delta FW_{m+1} (\Delta x)^2}{2K_{wu}}$$

Summary:

The numerical formulation for water flow in a freezing fringe of 4 elements (5 nodes) is in

$$\text{matrix form: } \begin{bmatrix} C_1 & A_1 & B_1 & 0 & 0 & 0 & 0 \\ 0 & C_2 & A_2 & B_2 & 0 & 0 & 0 \\ 0 & 0 & C_3 & A_3 & B_3 & 0 & 0 \\ 0 & 0 & 0 & C_4 & A_4 & B_4 & 0 \\ 0 & 0 & 0 & 0 & C_5 & A_5 & B_5 \end{bmatrix} \begin{bmatrix} P_{w,0} \\ P_{w,1} \\ P_{w,2} \\ P_{w,3} \\ P_{w,4} \\ P_{w,5} \\ P_{w,6} \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{bmatrix} \quad (\text{III.153})$$

$$\text{where } C_j = \frac{K_{w,j-1}}{K_{w,j} + K_{w,j-1}}; A_j = -\left(\frac{K_{w,j+1}}{K_{w,j} + K_{w,j+1}} + \frac{K_{w,j-1}}{K_{w,j} + K_{w,j-1}} \right); B_j = \frac{K_{w,j+1}}{K_{w,j} + K_{w,j+1}}$$

$$D_j = \frac{\frac{1}{2} (\Delta x)^2 \cdot \Delta FW_j}{K_{w,j}} \quad \text{and } 1 \leq j \leq 5$$

with boundary conditions:

$$C_1=0, A_1=1, B_1=0, D_1 = \frac{1}{2} \Delta x^2 \frac{\Delta FW_1}{K_{w,1}} = P_{ws} \quad C_5 = \frac{K_{w,4}}{(K_{wu} + K_{w,4})},$$

$$A_5 = -\left(\frac{K_{w,4}}{(K_{wu} + K_{w,4})} + \frac{\Delta x}{X_u} \right), B_5=0 \quad \text{and} \quad D_5 = \frac{\Delta FW_5 (\Delta x)^2}{2 K_{wu}} = P_{wf}$$

$$P_{w,1}=P_{ws} \text{ and } P_{w,5}=P_{wf}$$

Because $C_1=0$, $P_{w,0}$ vanishes from the set of 5 equations resulting from matrix eq.(III.153).

Because $B_5=0$, $P_{w,6}$ disappears from the set of 5 equations resulting from matrix eq.(III.153).

Matrix eq.(III.153) may be written as:

$$\begin{bmatrix} A_1 & B_1 & 0 & 0 & 0 \\ C_2 & A_2 & B_2 & 0 & 0 \\ 0 & C_3 & A_3 & B_3 & 0 \\ 0 & 0 & C_4 & A_4 & B_4 \\ 0 & 0 & 0 & C_5 & A_5 \end{bmatrix} \cdot \begin{bmatrix} P_{w,1} \\ P_{w,2} \\ P_{w,3} \\ P_{w,4} \\ P_{w,5} \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{bmatrix} \quad (\text{III.154})$$

Eq.(III.154) may be written as:

$$\begin{bmatrix} P_{w,1} \\ P_{w,2} \\ P_{w,3} \\ P_{w,4} \\ P_{w,5} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 & 0 & 0 & 0 \\ C_2 & A_2 & B_2 & 0 & 0 \\ 0 & C_3 & A_3 & B_3 & 0 \\ 0 & 0 & C_4 & A_4 & B_4 \\ 0 & 0 & 0 & C_5 & A_5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \end{bmatrix} \quad (\text{III.155})$$

All hydraulic conductivities are known, Δx , ΔFW_1 to ΔFW_5 , and X_u and therefore all A, B, C and D are known, and the result is a set of 5 equations with 5 unknowns (the pore water pressures $P_{w,1}$ to $P_{w,5}$). This set of equations can be solved by Gauss elimination. The same may be done for a system of m equations and m unknowns. When the pore water pressures in

the freezing fringe are known, the water flux density may be calculated. When the water flux density is known, the secondary heave may be calculated.

III.4.3 Input parameters for the frost heave prediction model of Nixon (1991)

To solve eq. of the model of Nixon the following input parameters are required:

1. Thermal input data

- TC temperature of the soil surface (cold side temperature)
- TW temperature of the bottom of the soil profile (warm side temperature)
- K_{h-1} thermal conductivity of frozen soil at $T = -1^{\circ}\text{C}$
- K_{h0} thermal conductivity of unfrozen soil
- partex thermal convection coefficient

2. Hydraulic input data

- wutot saturated volumetric water content, or porosity
- wua unfrozen water content of the soil at $T = -1^{\circ}\text{C}$
- wub fitting parameter for unfrozen water content curve, depending on soil type
- K_{w-1} hydraulic conductivity of frozen soil at $T = -1^{\circ}\text{C}$
- K_{w0} hydraulic conductivity of unfrozen soil
- α fitting parameter in hydraulic conductivity approximation

3. Mechanical input data

- Po overburden pressure
- Copart separation pressure

4. Data related to the geometry of space and time

H_0	length of original soil profile
X_0	depth of original freezing front
f_a	initial length of freezing front
Δt	time step
Σt	total time

III.4.4 Analysis of the sensitivity of the frost heave prediction to input parameters

The sensitivity of the frost heave prediction model of Nixon to its input parameters is tested in the same manner as the model of Wang, i.e. by making predictions using a range of values for each parameter, while keeping the other parameters constant, and graphing scaled frost heave against scaled input parameters. An attempt was made to have the same reference situation as the one used in the analysis of the sensitivity of Wang's frost heave prediction model to its input parameters: a soil profile with a length of 10 cm and linear temperature profiles in the frozen soil, freezing fringe and unfrozen soil. The surface boundary temperature was -2°C , the initial freezing temperature was -0.0106°C , the initial segregation temperature was -0.1241°C and the warm side temperature was 1°C . Only the warm and cold side temperatures are constants in the frost heave prediction model of Nixon. The thermal conductivity of unfrozen and frozen soil were respectively 300 and $450 \text{ cal cm}^{-1} \text{ day}^{-1} \text{ }^{\circ}\text{C}^{-1}$, or 1.45 and $2.18 \text{ W m}^{-1} \text{ K}^{-1}$. The porosity or saturated volumetric water content of the soil was

$0.322\text{m}^3\text{m}^{-3}$, the unfrozen water content at -1°C , parameter A or w_{ua} , was 0.067, and unfrozen water content fitting parameter B or w_{ub} was -0.345. The hydraulic conductivity of unfrozen soil was $2\mu\text{ms}^{-1}$, the hydraulic conductivity of the soil at $T = -1^\circ\text{C}$ was 0.43 pms^{-1} , and hydraulic conductivity fitting parameter alpha was 1.97. The external overburden pressure was 648gmcm^{-2} , the separation pressure 100gmcm^{-2} . In the numerical approximation 20 spatial nodes were used in the freezing fringe, and time steps of 0.005 days. The initial depth of the freezing front, X_0 , was 1.4cm and the initial thickness of the freezing fringe, f_a , was 0.2.

III.4.4.1 The thermal input parameters

Figure III.20 Sensitivity of the frost heave prediction model of Nixon to the thermal input parameters (relative frost heave predictions after 10 days).

- tc temperature of the cold side (soil surface)
- tw temperature of the warm side (bottom of the soil profile)
- tcf thermal conductivity of frozen soil
- tcu thermal conductivity of unfrozen soil
- partex warm side heat transfer coefficient

Figure III.21 Sensitivity of the frost heave prediction model of Nixon to the thermal input parameters (relative frost heave predictions after 50 days).

Figure III.22 Sensitivity of the frost heave prediction model of Nixon to the thermal input parameters (relative frost heave predictions after 100 days).

From Figures III.20, III.21 and III.22, it may be concluded that the predictions vary substantially when the thermal parameters of unfrozen soil are large, or when the thermal parameters of frozen soil are small. The largest frost heave is predicted when the surface temperature, t_c in Figures III.20 to III.22, is around -10°C and the thermal conductivity of the frozen soil, t_{cf} , is high. Any variation in heat transfer coefficient, partex , does not affect the heave prediction significantly. The lowest heave is predicted when the thermal conductivity of frozen soil t_{cf} is very small, the surface temperature, t_c , is high (close to 0°C), and the thermal properties of the unfrozen soil, t_w and t_{cu} , are high. It may also be concluded that these tendencies are enhanced with increasing freezing time.

III.4.4.2 The hydraulic input parameters

From Figures III.23, III.24 and III.25 it may be concluded that almost all hydraulic parameters (i.e. wutot, wua, wub, alpha, and hcf) influence the outcome of the prediction significantly. The hydraulic conductivity of the frozen soil and the value of parameter alpha seem to be the most important parameters: the model of Nixon is sensitive to both these parameters. The value of the unfrozen hydraulic conductivity hcu does not have a significant effect on the prediction. These tendencies are enhanced with increasing length of freezing time.

Figure III.23 Sensitivity of the frost heave prediction model of Nixon to the hydraulic input parameters (relative frost heave predictions after 10 days).

wutot total volumetric water content of unfrozen soil

wua value of parameter a hcf hydraulic conductivity of frozen soil

wub value of parameter b

hcu hydraulic conductivity of unfrozen soil

alpha value of alpha

Figure III.24 Sensitivity of the frost heave prediction model of Nixon to the hydraulic input parameters (relative frost heave predictions after 50 days).

Figure III.25 Sensitivity of the frost heave prediction model of Nixon to the hydraulic input parameters (relative frost heave predictions after 100 days).

III.4.4.3 The mechanical input parameters and parameters related to the geometry of space

Figure III.26 Sensitivity of the frost heave prediction model of Nixon to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 10 days).

po	overburden	fa	initial length of freezing fringe
copart	separation pressure	xo	original length of frozen soil
l	length of the soil profile		

The effect of the length of the soil profile is enhanced with increasing freezing time; the duration of freezing does not affect the sensitivity of the model to the other mechanical and geometric parameters. An important result from the sensitivity analysis of the model of Nixon is that an order of magnitude variation in any of the parameters does not result in an order of magnitude variation in frost heave.

Figure III.27 Sensitivity of the frost heave prediction model of Nixon to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 50 days).

Figure III.28 Sensitivity of the frost heave prediction model of Nixon to mechanical input parameters and input parameters relating to the geometry of space (relative frost heave predictions after 100 days).

III.4.5 Remarks on, advantages and limitations of the model

- An advantage of this model is that the freezing point is determined by soil type dependent parameters A and B, and the saturated unfrozen water content, which, based on the description in chapter II seems very plausible.
- Another strong point of the model of Nixon is that the numerical procedure, and therefore its mesh of nodes is applied in the freezing fringe, which is the main engine for frost heave. No nodes are wasted in the pseudo-passive frozen zone.
- The model does not seem to be overly sensitive to most of the input parameters, which is probably due to the fact that many of the crucial input parameters (e.g. unfrozen water content, hydraulic conductivity), as in nature are dependent variables.
- the author of this thesis has doubts about the correctness of the form of the Clausius Clapeyron equation used by Nixon. To the opinion of the author it all comes down to which force is considered to be the driving force of the water flow to the freezing fringe.
- a disadvantage of the model is that it can only handle one layer soils, although an adjustment to the model to allow multilayered soil systems as input should not be difficult.
- a penetrating temperature wave can not be simulated, nor thawing episodes.

III.5 Comparison between the frost heave prediction model of Wang (1994) and the frost heave prediction model of Nixon (1991)

MODEL OF NIXON (1991)	MODEL OF WANG (1994)
- handles linear temperature profiles	- is able to handle non linear temperature profiles
- is able to handle non linear water pressure profiles in the freezing fringe	- handles linear water pressure profiles in the freezing fringe
- frost heave criterion: a separation pressure has to be overcome in the freezing fringe	- frost heave criterion: a constant segregation temperature, that is a soil property
- water content in the freezing fringe is a function of temperature	- water content in the freezing fringe is constant
- hydraulic conductivity in the freezing fringe is a function of temperature	- The freezing fringe has an average hydraulic conductivity which is an input parameter
- controlled freezing of the whole profile may be modelled	
- possibility of entry of boundary temperatures in time in a table	
- numerical procedure puts 20 nodes in the freezing fringe: that section of the soil where the largest part of frost heave happens	- numerical procedure puts most nodes in frozen and unfrozen soil, while one, two or three nodes may be situated in the freezing fringe
- freezing temperature may be calculated from the unfrozen water content curve	- freezing temperature is a soil property that has to be guessed
- the segregation temperature is not constant during the growth of an ice lens, and is different for the next lens	- the segregation temperature is constant during the growth of an ice lens and the same for the next lens
- an external overburden is taken into account	- an external overburden is taken into account as well as the soil on top of the warmest ice lens
- the number, location and thickness of the ice lenses is predicted	- the number, location and thickness of ice lenses are not produced as output
- one layer soil profile	- one layer model, but the modified version has three layers
- Nixon's model is available in a DOS version for PC	- Wang's model is available in a Unix version

Table III.3 Comparison between the models of Nixon (1991) and Wang (1994).

The concise comparison between the models of Nixon and Wang is given in Table III.3, while the more detailed comparison between the two models is given in Appendix A and B.

III.6 Discussion and Summary

Models are needed to predict the result of a process from a given set of input data, and are used to develop a better understanding of the process. Among the wide variety of model types, a deterministic model of a process in nature aims at a detailed physics-based description of all known sub-processes and interactions of variables involved in the process. Most principles of the mechanistic deterministic modelling of soil freezing and frost heave, as mentioned in chapter III.2, have currently found wide acceptance (e.g. the use of Fourier's law of heat conduction, Darcy's law of water transport, thermodynamic equilibrium represented by Clausius Clapeyron equation, and Bach's power law for elastic deformation behaviour). Some of the key problem areas in the modelling of soil freezing and frost heave remain. The soil temperature profile in the frozen zone is non-linear. While Wang (1994) takes this notion into account, Nixon (1991) assumes a linear soil temperature profile. The modelling of the hydraulic conductivity in the freezing fringe, the most important soil property in Darcian modelled flow of water to the warmest ice lens, remains difficult. This is a result of a severe lack of measured hydraulic conductivities in the freezing fringe, which is a consequence of the fact that measurement techniques are currently not sophisticated enough to measure the hydraulic conductivity in the freezing fringe. In the modelling of the water

flow to the warmest ice lens it is important that consensus is reached about the driving force of water flow to the warmest ice lens. Only then the right form of Clausius-Clapeyron equation can be selected. The modelling of deformation behaviour of freezing soil, building on the work of e.g. Ladanyi, Sayles and Vyalov, needs to be described in more detail. A more systematic description and modelling of locations where consolidation and fissuring occur, will improve the current modelling of soil freezing and frost heave. The frost heave criteria of Gilpin (1979, 1980) and Miller (1980) are considered to be the most sensible of the currently available frost heave criteria, and a combination of them seems possible.

The soil freezing and frost heave prediction model should be able to accomodate a multilayer soil that experiences warming as well as cooling phases. A frost heave prediction model developed by Svec at the National Research Council of Canada and used by Chang (1993), can handle freezing and thawing phases. The flow diagram of this model is given in Figure III.29.

Based on the observations (chapter II) and general soil freezing and frost heave modelling principles (chapter III.2) a framework is proposed to compare various frost heave prediction models with each other. Three practical frost heave prediction models are examined in detail: a model proposed by Wang (1994), a modified version of this model proposed by the author of this thesis, and a model proposed by Nixon (1991). The sensitivity of the models to their input parameters is examined, and the strength and weaknesses of the models are evaluated. While the non linear temperature profile in the

model of Wang seems to have more potential than the linear temperature profile in the model of Nixon, the model of Wang can not predict the rhythmic banding of increasingly

Figure III.29 Flow diagram of a frost heave model that can handle cooling and warming phases (Chang 1993)

thicker ice lenses, while Nixon's model does so. Both models have problems with accounting for the latent heat, released in the freezing fringe, but Wang's model suffers the most from

this problem. Both models are one layer soil models, and for that reason Wang's model was modified to allow for multiple layers. The predictions with Wang's model were hypersensitive to most of its input parameters especially to the hydraulic conductivity. The predictions with Nixon's model were less sensitive to most of its input parameters.

CHAPTER IV Methods to determine values and spatial variability of common input parameters for frost heave simulation models

IV.1 Introduction

The input data for frost heave prediction models can be divided into 4 groups:

1. Thermal input data

initial temperature profile, surface boundary temperature, freezing temperature, segregation temperature, temperature at largest depth of seasonal influence, thermal conductivity, heat capacity, thermal diffusivity, latent heat etc.

2. Hydraulic input data

unfrozen water content as a function of temperature, hydraulic conductivity as function of temperature, porosity, pore water pressures, density of water, salinity information

3. Mechanic input data

external stress, density, strains, moduli (E, G etc), creep factors, Poisson ratio, stress partition factor, separation pressure

4. Data related to the geometry of space and time

spatial step size, time step size, total time, length of soil profile, length of freezing fringe, largest depth of seasonal influence

Most frost heave prediction models require these parameters. More detailed models may require additional information about these or other parameters. These parameters may be measured either directly or determined indirectly. The input parameters of soil samples should be determined for a temperature range that at least covers the whole temperature range of the freezing fringe e.g. between 0 and -0.7°C , and maybe even to -1.4°C , a segregation temperature measured in volcanic soil material (Fukuda 1995). The values of many of these parameters vary during the freezing process, and they vary in space.

Many of the parameters, such as the hydraulic conductivity of the freezing fringe, are difficult to measure, or their measurement is time consuming or expensive. For this reason in this thesis, several procedures are suggested to determine these input parameters indirectly. Procedures are chosen that use only soil data that are relatively easy to obtain, such as grain size distribution, bulk density, and saturated water content. Theoretically, the more information that is available about the input data the better the prediction may be. However, the more input data acquired the higher the costs. For this reason, a method is described to optimize the number of soil samples, which have to be taken for the determination of the input parameters for frost heave prediction. This optimization method is based on the spatial variability of the input parameters only, and is independent of the predicted frost heave.

IV.2 Methods of determination of common input parameters for frost heave prediction models

IV.2.1 Grain size distribution

An accurate method for soil texture analysis involves taking undisturbed soil samples with a soil corer or a sample ring; less accurate methods use an auger. Because there was no soil corer available at the time of the fieldwork an auger and a 51cm³ sample ring were used to obtain the samples. The sample weights were determined using a triple beam balance in the field, and a more accurate scale balance in the laboratory. The grain size analysis and determination of other soil parameters were done at the Geotechnical Science Laboratories at Carleton University, Ottawa.

After determining the grain size distribution by the standard method using sieving and differences in soil particle settling velocities, the textural class of each soil sample may be determined using the soil textural triangle as proposed by the International Society of Soil Scientists (ISSS).

Figure IV.1 Soil texture triangle (Source: Koorevaer et al 1983)

A computer program was written in which the sand, silt and clay percentages may be entered and which renders the soil textural class according to the ISSS soil textural triangle.

IV.2.2 Thermal input parameters

IV.2.2.1 Temperatures: initial temperature profile $T(x,y,z,0)$, surface boundary temperature

$T(x,y,0,t)$, freezing temperature T_f , and segregation temperature T_s

Very few sites in cold regions have (long) soil temperature records. Consequently, most often temperature data are not readily available, and have to be collected, before a frost heave prediction based on reliable simulation can be made. It is relatively easy to measure soil

temperatures. Temperatures of freezing soils can be measured by thermometers, thermocouples or thermistors (Carter 1993). The measured temperatures may be automatically registered and stored using dataloggers.

For a prediction of frost heave, it has to be decided which temperature profile within the thermal regime of a soil will be used as the initial temperature profile, $T(z,0)$. Both the shape and absolute values of the initial profile are important. Some possible shapes of the temperature profile are:

1. isothermal temperature profile
2. ramped temperature profile
3. non-linear temperature profile e.g. parabolic or complex

The choice of the shape of the initial temperature profile for a prediction of frost heave depends among other factors on the capabilities of the frost heave prediction model. If warming or thawing cannot be described by the model, the use of a parabolic initial temperature profile may be impossible. By using a frost heave prediction model that can describe warming or thawing, reality may be better simulated than by using a model that cannot describe warming and thawing. The absolute temperatures of the initial temperature profile of the soil will influence the frost heave predictions greatly. The higher the initial temperatures, the longer it will take for the soil to freeze, and the less heave will result. If warming and thawing of the soil are described by the frost heave prediction model, it still has to be decided which time of year will be taken as the start of the cooling process. One possibility would be the temperature profile at the warmest day of the year.

To simulate the frost heave process often a surface boundary temperature, $T(0,t)$, has to be chosen. The shape of this temperature as a function of time influences the prediction of the heave greatly. Possible shapes of this curve are a measured variation in surface temperature (actual curve), a smooth annual temperature wave (ideal curve), a temperature curve divided in freezing, thawing and isothermal phases (simplified curve), or a constant surface boundary temperature (constant curve) (see Figure IV.2). In the model of Wang this surface boundary temperature is considered to be constant.

Figure IV.2 Four different soil surface temperature boundary options

The value of the temperature at the freezing front, T_f , is close to 0°C and only a few hundredths of a degree lower than 0°C , when the soil moisture contains relatively high concentrations of salt or when the soil is very fine grained. In most models, including those of

Wang and Nixon, it is considered a constant in space and time, and a characteristic soil property. The temperature of the freezing front may differ for each soil type, from ca. -0.02°C to -0.05°C . Wang does not provide a method to determine the freezing temperature, while Nixon uses a curve fitting of the unfrozen water content to determine the freezing temperature.

In the model of Wang the segregation temperature, T_s , is considered to be a constant and characteristic soil property, but in the model of Nixon this temperature varies for each soil in space and time. The segregation temperature for a specific soil type may vary from -0.1°C to -0.7°C . The temperature at the base of the warmest ice lens, T_s , is difficult to measure, but some researchers (e.g Akagawa 1988) managed to measure T_s or derive some values for T_s . While Wang does not provide a method to determine T_s , Nixon calculates T_s using the Clausius-Clapeyron equation.

IV.2.2.2 Thermal conductivity, $K_h(x,y,z,T)$

The thermal conductivity of a soil can only be determined indirectly by measurement of the heat flow, the thermal potential difference (= temperature difference) and the length of a soil and the application of Fourier's law. The thermal conductivity of a soil can be determined using a steady state or transient method. Examples of steady state methods are the guarded hot plate test, divided bar method, the heating of a cylinder in a laboratory, or field methods such as the heating of the surrounding soil by a sphere and the monitoring of temperatures (with thermistors

or thermocouples) and heat (with a heat meter) (Farouki 1981; Williams and Smith 1989). Alternatively the thermal conductivity can be measured by transient methods such as the probe method, periodic temperature waves and the thermal shock method (Farouki 1981). When steady state heat conduction measurement methods are applied to partially frozen soil, these methods that heat the soil cause a change in the unfrozen water content, and therefore in the properties being measured. In transient methods using a thermal probe, it is difficult to establish a close contact between the probe and the soil, which may result in heat being lost. Thus, there exist difficulties in using either of the two groups of methods.

As an alternative to the indirect measurement of the thermal conductivity by the measurements and the application of Fourier's law, a wide variety of methods exist to determine the thermal conductivity indirectly using other relationships. Farouki (1981) reviewed the methods of Smith (1942); Kersten (1949); Mickley (1951); Gemant (1952); De Vries (1952); Van Rooyen and Winterkorn (1959); Kunii and Smith (1960); Woodside and Messmer (1961); McGaw (1969); and Johansen (1975) and evaluated their performance.

The methods of Johansen (1975) and De Vries (1952) proved to be the best for the determination of the thermal conductivity of partially frozen, frost-susceptible soils. For this thesis the method of Johansen (1975) was chosen to calculate the thermal conductivity of the frost-susceptible soil samples. This method is based on a comparison between flow of electricity and flow of water. Each of the soil components (water, ice, quartz, and other minerals) can be regarded as a resistor, which results in a circuit with parallel resistances.

Electrical conductivity is the reciprocal of the electrical resistance, and the thermal conductivity is the reciprocal of the thermal resistance. Thermal resistances in a parallel circuit can be added up and the total of the individual thermal resistances in the circuit can be replaced by one single soil resistance. The resulting equation is Johansen's equation:

$$K_{sat} = \left((K_q)^q (K_o)^{1-q} \right)^{1-n} (K_i)^{n-w_u} (K_w)^{w_u} \quad (\text{IV.1})$$

where sat = saturated; q = quartz content; o = other minerals; w = water

n = porosity; w_u = unfrozen water content; i = ice;

K_q = specific thermal conductivity of quartz = $7.7 \text{ Wm}^{-1} \text{ K}^{-1}$

K_o = specific thermal conductivity of other minerals = $2 \text{ Wm}^{-1} \text{ K}^{-1}$

K_i = specific thermal conductivity of ice = $2.2 \text{ Wm}^{-1} \text{ K}^{-1}$

K_w = specific thermal conductivity of water = $0.57 \text{ Wm}^{-1} \text{ K}^{-1}$

The values for the specific conductivities, K_j , are given in Farouki (1981). Usually the quartz content is not known, since there are not any easy methods available to determine the quartz content quantitatively. In most cases the quartz content has to be approximated. The approximation used in this thesis was that the quartz content was more or less the sum of the gravel content, the sand content and half of the silt content. The thermal conductivity was calculated for a range of temperatures between 0 and -4°C . In the model of Wang, K_h takes on two values: a frozen and an unfrozen value.

IV.2.2.3 Volumetric heat capacity, $C_h(x,y,z,T)$

The volumetric heat capacity of a soil can be calculated by adding the volumetric heat capacities of its components: water, ice, soil minerals and organic matter. The partial volumetric heat capacities may be calculated by multiplying the volumetric content of the soil component with its density and its specific mass heat capacity. The specific mass heat capacities are $4180 \text{ J kg}^{-1} \text{ K}^{-1}$ for water, $2100 \text{ J kg}^{-1} \text{ K}^{-1}$ for ice, $850 \text{ J kg}^{-1} \text{ K}^{-1}$ for mineral soil and $1920 \text{ J kg}^{-1} \text{ K}^{-1}$ for organic material (Williams and Smith 1989). In the model of Wang, C_h takes on two values: a frozen and an unfrozen value. The volumetric heat capacity does not appear in Nixon's model.

IV.2.2.4 Effect of latent heat, $L_h(x,y,z,T)$: the apparent heat capacity

As was mentioned in chapter II, when the volumetric heat capacity of a freezing soil is measured (the amount of heat that is released if the soil cools 1K), this amount of heat will be very large because of the amount of latent heat that is released upon freezing of the water. Under static water flow conditions, the combined effect apparent volumetric heat capacity of a soil can be calculated (eg. Williams and Smith 1989):

$$C_a(T) = C_s(T) + \rho_w L_f \left(\frac{d\theta_u}{dT} \right)_T \quad (\text{IV.2})$$

The specific latent heat of freezing, L_f , was assumed to be constant, while it is actually a function of temperature. The density of water is taken to be 1000 kg m^{-3} . During frost heave,

static water flow conditions are absent in the freezing fringe. $\frac{d\theta_u}{dT}$ in eq.(IV.2) may be determined by differentiation of the unfrozen water content curve, which will be derived in chapter IV.2.2.1:

$$\theta_u = ne^{f(-T^2 + \frac{1}{f}(\ln(1 - \frac{\theta - \theta_{-4}}{n})))} + \theta_{-4} \quad (IV.3)$$

This differentiation of the unfrozen water content curve, eq.(IV.3), results in $\frac{d\theta_u}{dT}$ at each

$$\text{temperature } T: \quad \frac{d\theta_u}{dT} = ne^{f(-T^2 + \frac{1}{f}(\ln(1 - \frac{\theta - \theta_{-4}}{n})))} \cdot ((-2Tf)) \quad (IV.4)$$

The apparent volumetric heat capacity of unfrozen soil is equal to the volumetric heat capacity, since $d\theta/dT=0$ (unless the soil is warm and evaporation occurs). The apparent heat capacity is not an input parameter of the model of Wang (1994), but is given in this thesis to illustrate the importance of the latent heat released in the freezing soil. The effect of the release of latent heat is a delay in the cooling of the soil. In numerical procedures, the release of latent heat at only one or a few nodes may result in unrealistic temperatures above 0°C, and therefore unrealistic temperature profiles. It is therefore important to know in more detail where and how much latent heat is released in the soil.

IV.2.3 Hydraulic parameters

IV.2.3.1 Porosity, $n(x,y,z,T)$, and unfrozen water content, $w_u(x,y,z,T)$

In order to determine the thermal conductivity by the method of Johansen, the hydraulic conductivity by e.g. the method of Tarnawski and Wagner (see chapter IV.2.3.2), and the latent heat of freezing, the porosity and unfrozen water content are required. In saturated soils at a temperature above 0°C, the porosity of the soil, n , equals the unfrozen volumetric water content, w_u . Water contents of unfrozen soils may be determined by the gravimetric method, neutron probe method (Dirksen 1986), gamma radiation method (Hillel 1980) or an electrical capacitance method (Dirksen 1986). In the gravimetric method, only a sample ring, oven and a balance are needed to determine the porosity. The difference between the wet mass and the dry mass of a saturated soil, divided by the density of water and the volume of the sample equals the porosity. It is assumed that the soil sample is not over- or supersaturated. Such a situation might occur as a result of the thawing of a soil which contains ice lenses.

The unfrozen water content in partially frozen soils decreases with temperature, and can be determined with Time Domain Reflectometry (TDR), or Nuclear Magnetic Resonance (NMR) methods. Smith and Tice (1988) determined the unfrozen water contents of a wide range of soils with TDR and NMR. However, because these methods may require expensive equipment and considerable time, they are sometimes not suitable for practical purposes and it was decided to calculate the unfrozen water content by approximation.

Kujala (1989) used the Weibull equation to describe the volumetric unfrozen water content θ_u of a soil: $\theta_u(T) = n \cdot e^{\alpha(T-T_0)^\beta}$ (IV.5)

for most soils $\beta \approx 2$, and α may be determined by substituting the subfreezing temperature T of a known unfrozen moisture content (Sheng and Knutsson 1993). Sheng and Knutsson do not specify how this known unfrozen moisture content should be determined.

According to Campbell (1985), the unsaturated water content can be expressed by the following empirical function of the water pressure head: $\frac{\theta_{unsat}}{n} = \left(\frac{h_p}{h_{ae}}\right)^{-1/b}$ (IV.6)

where h_p is the water pressure head and h_{ae} is the water pressure head at air entry point. Based on similitude between air and ice in soil, Tarnawski and Wagner (1996) applied Campbell's formula to partially frozen soils: $\frac{\theta_{unfr}}{n} = \left(\frac{h_p}{h_{ie}}\right)^{-1/b}$ (IV.7)

The water pressure head h_p in the partially frozen soil could be determined by a simple form of the Clausius-Clapeyron equation: $\frac{\Delta T}{T_0} = -\frac{h_p g}{L_f}$ (Williams 1967) (IV.8)

in which they assume that the freezing temperature $T_0 = 273.15\text{K}$. When eq.(IV.8) is compared with eq.(III.46), it may be seen that the ice pressure at the freezing front, as well as elsewhere in the freezing fringe, and the water pressure at the freezing front are ignored.

According to Anderson and Tice (1972) the unfrozen water content of a soil can be determined by the equation: $\ln \theta_u = 0.2618 + 0.5519 \ln S - 1.449 S^{-0.264} \ln |T|$ (IV.9)

where S = specific surface area (=surface per volume) of the soil

Tarnawski and Wagner (1993) used an elaborate manner to determine the specific surface area. It remains difficult to take the amounts of the soil fractions present into account in the calculation of the specific surface area of the soil. The specific surface area of each soil sample can as well be calculated by a simpler method, given in Koorevaer et al. (1983), which is based on the characteristic diameter of each particle size fraction: $S = \sum f_i S_i$ (IV.10)

For regularly shaped particles (i.e. polyhedral, from cubes to spheres) such as gravel, sand and silt particles, the specific surface area can be given by: $S = \frac{6}{\rho_s d}$ (IV.11)

where d is the characteristic diameter of the fraction.

For plate shaped particles, such as clay particles, the specific surface area can be given by:

$$S \approx \frac{2}{\rho_s d} \quad (IV.12)$$

The characteristic diameters are eg.: 3mm for gravel, 0.1mm for sand, 10 μ m for silt, and 5nm for clay. It is possible to incorporate some arbitrary weighting in the calculation procedure. The most serious problem with the method of Anderson and Tice is that the unfrozen water content near 0°C rises to infinity.

Schellekens (1995) proposed Gaussian type curves, which reach a discrete maximum at $T=0^\circ\text{C}$ and solve that problem. The shapes of the proposed curves are based on the shape of unfrozen water content curves given by Smith and Tice (1988) and Wood (1985). The general

$$\text{form of the curves proposed is: } \theta_u = ne^{f(-T^2 + \frac{1}{f}(\ln(1 - \frac{\theta_u}{n})))} + \theta_{-4} \quad (IV.13)$$

where θ_u = volumetric unfrozen water content

n = porosity of the soil sample

f = forcing factor

T = temperature (in°C)

θ_{-4} = amount of unfrozen water in the soil at (temperatures lower than) -4°C

The forcing factor is determined by the grain size of the soil: the coarser the soil the higher the forcing factor, and the faster the water content decreases below 0°C. The soil samples may be classified on their grain size distribution according to the USDA/CDA soil texture triangle. The forcing factors range from 80 for coarse grained soils to 5 for very fine grained soils. The value of θ_{-4} is 0.07 ± 0.05 . A fitting of the unfrozen water content curves of the samples to curves of similar soils given by Smith and Tice (1988) and curves calculated using a method proposed by Anderson and Tice (1972) gave good results. The advantage of this method is that it requires only an assessment of the particle size and the saturated water content at a temperature above 0°C.

IV.2.3.2 Hydraulic conductivity, $K_w(x,y,z,T)$

A direct measurement method of the hydraulic conductivity does not exist. The hydraulic conductivity is usually determined in an indirect manner in an experimental set-up in which the amounts of water flowing in and out of a soil, the thickness of the soil sample and the pressure potential difference over the soil are measured. Usually Darcy's law (eq.III.19) is assumed to be valid, and the hydraulic conductivity calculated. In case inertia and resistance

effects have to be taken into account, a Forchheimer form should be used. Moreover, in turbulent flow, eddy effects are important, and Darcy's law is no longer valid.

However, in soils, water flow is usually very slow, and it may be assumed that soil moisture flow is laminar and Darcy's law in simple form or with a Forchheimer extension is valid. The hydraulic conductivity of a saturated unfrozen soil can be determined e.g. by a constant head method, a falling head method or the hot air method of Arya (Dirksen 1986). The falling head method is, according to Klute (1965), best suited for soil with low hydraulic conductivity (such as partially frozen soil). All these methods have considerable limitations. In more complex situations such as unsaturated, frozen saturated, frozen unsaturated, partially frozen saturated or partially frozen unsaturated conditions, accurate measurement of the water transport and hydraulic conductivities is even more difficult. The maximal thicknesses of the freezing fringe currently reported are in the order of 1 cm. Unless the freezing fringe can artificially be extended to 30 or 40cm, current measurement equipment is not sophisticated enough to determine the hydraulic conductivity, K_{wff} , of such a narrow frozen fringe. In particular the size of the porous cups of tensiometers, which are used to measure soil water pressures lower than atmospheric and are needed to calculate the hydraulic conductivity, is often too big.

Few researchers have measured the hydraulic conductivity of soils at temperatures below 0°C. However, valuable data have been obtained by Burt (1974). Burt (1974) gives hydraulic conductivities of 5 frost susceptible soils: Leda Clay, Fithian Illite, Oneida Clayey Silt, Slims Valley Silt and Carleton Silt.

Since the hydraulic conductivity of a soil is dependent on the water content of the soil, and the (unfrozen) water content on temperature, the hydraulic conductivity of the freezing fringe in a soil is dependent on the temperature. The more the pores of a soil are filled with ice, the more difficult it becomes for water to flow through the pores. The hydraulic conductivity of a saturated soil decreases with decreasing temperature from the freezing point.

Because the indirect measurement of the hydraulic conductivity is so complicated, various empirical relationships have been proposed to describe the hydraulic conductivity as a function of thermal parameters and/or unfrozen water content. Van Loon et al. (1988) proposed an expression in which the hydraulic conductivity is calculated from the thermal conductivity

and the heat flux:
$$K_w(z) = \frac{T_f}{L^2 \rho_w^2 A} \left[K_h(z) + \frac{(Q + Q_L)}{\frac{\Delta T(z)}{\Delta z}} \right] \quad (\text{IV.14})$$

It may be difficult to measure the heat fluxes (Q and Q_L) to and from the freezing fringe, and in applications such as the construction of a pipeline it is often unpractical to measure the heat flux from the soil.

Tarnawski and Wagner (1996) assumed that partially frozen pores have a similar effect on water flow as partially air filled pores: hindering moisture flow and allowing moisture flow only through the smaller pores filled with water. Campbell (1985) proposed an empirical model

for unsaturated unfrozen soils:
$$K_w = K_{wsat} \left(\frac{\theta_{unsat}}{\theta_{sat}} \right)^{2b+3} \quad (\text{IV.15})$$

Applying eq.(IV.15) to partially frozen soil gives:
$$K_w = K_{wunf} \left(\frac{\theta_{unfr}}{n} \right)^{2b+3} \quad (\text{IV.16})$$

$$\text{where } K_{wunf} = 4 \cdot 10^{-5} \left(\frac{0.5}{1-n} \right)^{1.3b} \cdot e^{(-6.88m_{cl} - 3.63m_{si} - 0.025)} \quad (\text{IV.17})$$

where m_{cl} and m_{si} are clay and silt mass fractions

Parameter b was obtained by Campbell by fitting eq.(IV.16) to a series of data of British soils.

$$\text{The formula of the fitting curve was: } b = d_g^{-0.5} + 0.2\sigma_g \quad (\text{IV.18})$$

where d_g is the geometric mean particle diameter (in mm); and σ_g is the geometric standard deviation. A log-normal particle size distribution is assumed to calculate the geometric mean

$$\text{particle size parameter: } d_g = e^{(m_{cl} \ln d_{cl} + m_{si} \ln d_{si} + m_{sa} \ln d_{sa})} \quad (\text{IV.19})$$

$$\text{and } \sigma_g = e^{\left[\frac{\sum_{i=1}^3 m_i (\ln d_i)^2}{\sum_{i=1}^3 m_i} - \left(\frac{\sum_{i=1}^3 m_i \ln d_i}{\sum_{i=1}^3 m_i} \right)^2 \right]^{0.5}} \quad (\text{IV.20})$$

The assumption that water in partially frozen soil only flows through the small pores is questionable, since not all the water in one pore will have the same free energy. The water close to the solid particles will have a lower free energy than water further away from the soil particle surface, and will freeze at lower temperatures. When pores are partially filled with ice, part of the pore may still be used for soil water transport. Water transport by regelation is also ignored by the assumption of Tarnawski and Wagner. Reasoning that is based on similitudes between ice and air in soils is questionable, because it is highly unlikely that solid material (ice) behaves in a similar manner as gas (air). Although the model gives the impression of being physically sound and despite the considerable amount of calculations involved, the model remains essentially empirical (eqs.IV.17 and IV.18) and speculative (assumptions such as eqs. IV.16, IV.19 and IV.20).

Civil engineers often determine the hydraulic conductivity from consolidation tests (Lambe 1951, Taylor 1948): $K_w = c_v \rho g \alpha$ (IV.21)

where c_v is the coefficient of consolidation, and

α is the compressibility of the soil

The coefficient of consolidation, c_v , can be determined by plotting the consolidation of a sample against time and using square root fitting or log fitting to describe the consolidation in time. According to Taylor (1948) this results in the following expressions for c_v :

$$c_v = \frac{0.848H^2}{t_{90}} \quad (\text{IV.22}) \quad \text{for square root fitting, and } c_v = \frac{0.197H^2}{t_{50}} \quad (\text{IV.23}) \quad \text{for log fitting,}$$

where H is half the sample thickness, t_{90} is the time for 90% primary compression, and t_{50} is the time for 50% primary compression.

The compressibility of the soil, α , can be determined by the loading of a soil in several loading increments, and measuring the reduction in sample thickness after each loading increment. The results can be plotted as a graph of void ratio, e , versus effective stress, σ_{eff} , and

$$\text{the compressibility may be found: } \alpha = -\frac{\frac{de}{1+e_0}}{d\sigma_{\text{eff}}} \quad (\text{IV.24})$$

$$\text{Horiguchi and Miller (1983) proposed the expression: } K_w(T) = AT^B \quad (\text{IV.25})$$

This expression has the problem that the estimated hydraulic conductivity is 0 at a temperature $T=0^\circ\text{C}$. E-power expressions solve this problem, and Wang (1994) proposed the following

$$\text{mathematical approximation of the hydraulic conductivity: } K_w(T) = K_{wu} e^{b(T_0 - T_f)} \quad (\text{IV.26})$$

To determine the constant b , Wang assumes a value for K_w , at the segregation temperature, T_s , which is a source of a potentially serious error. Sheng and Knutsson (1993) proposed a similar

$$\text{expression: } K_{wff} = K_{wu} e^{-b(z - z_f)} \quad (\text{IV.27})$$

where b is a temperature-dependent parameter that can be determined in the following

$$\text{manner: } b = \frac{1}{z_s - z_f} \ln \frac{K_{wff}(z_s)}{K_{wu}} \quad (\text{IV.28})$$

Using the Kozeny-Carmen equation for the hydraulic conductivity of a saturated unfrozen soil:

$$K_w = \frac{n^3}{(1-n)^2} \frac{d_m^2}{180} \quad (\text{Marshall and Holmes 1979, Freeze and Cherry 1979}), \quad (\text{IV.29})$$

the ratio of the hydraulic conductivity at the freezing front and the hydraulic conductivity at the

$$\text{base of the ice lens may be determined: } \frac{K_w(z_f)}{K_w(z_s)} = \frac{\eta(T_f) \frac{n^3}{(1-n)^2}}{\eta(T_s) \frac{(n - I_s)^3}{(1 - (n - I_s))^2}} \quad (\text{IV.30})$$

The ice content at the base of the warmest ice lens, I_s , can be determined from an unfrozen water content curve, and only the porosity, n , and the variation of the viscosity, η , with temperature, T , has to be known to calculate the hydraulic conductivity ratio in eq.(IV.30).

According to Sheng and Knutsson (1993), close to $T=0^\circ\text{C}$ the viscosity, $\eta(T)$, varies according to: $\eta = 1.787e^{-0.03357 \cdot T}$ (IV.31) Since $K_w(z_f) \approx K_{wu}$, eq.(IV.30) can be used to calculate b with eq.(IV.28) if $z_s - z_f$ is known. This distance is usually unknown.

Therefore, Schellekens (1995) proposed an empirical extension of the type of the equations suggested by Wang (1994) and Sheng and Knutsson (1993):

$$K_w = K_{wu} e^{A(T-T_0)^3+B(T-T_0)^2+C(T-T_0)+D} + K_{-4} \quad (\text{IV.32})$$

where A, B, C, D are curve fitting parameters and $T_0=273.15\text{K}$

The hydraulic conductivities of a soil as measured by Burt at various temperatures below 0°C were plotted in a diagram. During the freezing process the hydraulic conductivity of a fine-grained soil such as a Leda Clay or Fithian Illite decreases from 3.4×10^{-5} to $4.0 \times 10^{-9} \text{ ms}^{-1}$ and the hydraulic conductivity of coarser-grained soil such as Carleton Silt changes from 1.4×10^{-4} to $9.0 \times 10^{-10} \text{ ms}^{-1}$. Eq.(IV.32) was fitted to the data, and the fitting parameters of the soils of Burt were determined:

soil type	A	B	C	D
Leda Clay	0	-9.36623	3.97098	0.338862
Fithian Illite	152.916	169.156	62.1477	-1.39189
Oneida Clay Silt	0	36.3207	46.742	-0.0392309
Slims Valley Silt	0	7.8481	22.2588	-2.2411
Carleton Silt	0	44.6855	56.5154	1.59536

Table IV.1 Values of the fitting parameters A, B, C and D in eq.(IV.32) for the soils of Burt (1974).

The grain size of a soil can be compared to the grain size of the soils of which the hydraulic conductivity was measured by Burt (1974), and the hydraulic conductivity curve of the most similar soil of Burt may be chosen to represent the hydraulic conductivity curve of that soil. Thus, only the grain size has to be known to determine the hydraulic conductivity of the

soil. Therefore application of a procedure such as eq.(IV.32) is very practical. Although the equation of Schellekens (1995) lacks some of the physical rigor of that of Sheng and Knutsson, due to a large amount of flexibility in the curve, his equation achieved a much better fit than that of Wang when applied to hydraulic conductivities experimentally measured by Burt (1974).

It has to be noted that critique has been voiced on the findings of Burt, since the hydraulic conductivity of lactose solution instead of water had been measured. Nixon (1991) compiled hydraulic conductivity data obtained by various researchers and presented an interpolation and extrapolation of these data. In the runs with the modified model of Wang a hydraulic conductivity had to be chosen that was characteristic for the freezing fringe. This characteristic hydraulic conductivity was chosen based on soil texture, under the assumption that the soil texture is overriding the importance of soil structure in the determination of the characteristic hydraulic conductivity of a freezing soil. A computer program was written that addressed the highest characteristic hydraulic conductivity to the finest grained soil, and the lowest characteristic hydraulic conductivity to the coarsest grained soil.

IV.2.3.3 Pore water pressures, $P_w(x,y,z,t)$

Pore water pressures above atmospheric can be determined by measurements with piezometers. Pore water pressures below atmospheric pressure can be measured by tensiometers with or without connection to pressure transducers and data loggers. Tensiometer measurements however are based on the principle of equilibrium between the soil moisture and the moisture in

the tensiometer cup. In partially frozen soil however, freezing prohibits free flow of water towards the cup, ice may form on the cup, etc.

Because the length of the freezing fringe is very small, varying from 1mm or less to only a couple cm, it is very difficult to measure pore water pressures in the freezing fringe. Despite this inability to verify the pore water pressures in the freezing fringe, some researchers have proposed theoretical models of the pore water pressure in the freezing fringe.

IV.2.3.4 Density of water, ρ_w , density of ice, ρ_i , and salinity information

The density of water is usually taken to be a constant 1000kgm^{-3} . Although ice has a wide variety of densities, depending on its pressure and temperature, this parameter is usually taken as a constant with a value of 900 to 920kgm^{-3} . The density of a substance is determined by measuring the mass and volume of the substance, and dividing the mass by the volume of that substance.

IV.2.4 Mechanical input data

IV.2.4.1 Wet bulk density of the soil, $\rho_s(x,y,z)$

The wet bulk density is used to determine the mass, force and stress of the soil on top of an ice lens. The wet bulk density of the soil is determined by measuring the wet mass and

volume of a soil, and dividing the wet mass of the soil by the volume of the soil. Only a sample ring and balance are needed. The wet bulk density of the soil increases with an increasing mineral content and therefore decreasing organic content, because quartz and clay minerals have higher densities than organic matter.

IV.2.4.2 External stress/external pressure

Any load on top of the soil or buried in the soil, or human activity such as traffic over the soil may add external stresses to the soil by its mass or vibration. In the case of a structure in or on top of the soil, the mass of the structure has to be known to calculate the stress it exerts on an ice lens. If the mass of the object and the area of the surface on which this force is acting are known, the external stress may be determined. Activity on the soil, such as traffic, may add more complex external stress to the soil.

IV.2.4.3 Stress partition factor, χ , and separation pressure, P_{sep}

These parameters are used in criteria for the initiation of new ice lenses. Miller (1978) introduced the stress partition factor, χ , to determine when the neutral stress became 0. Black (1995) suggested a method to determine this stress partition factor. Gilpin (1979, 1980) introduced the separation pressure, Wood (1985) attempted to find some justification of the existence of the separation pressure, calling it the pressure level that has to be reached to

overcome the “internal pressures” in the soil, and Nixon (1991) used the separation pressure in his model. None of these researchers suggested how to measure the separation pressure. Both the stress partition factor and the separation pressure are hard to determine, although they should be related respectively to the shear and cohesive strength of the soil.

IV.2.4.4 Strains

In most frost heave tests the frost heave is measured by monitoring the elevation of the soil surface over time. Some researchers have also measured the frost heave at depth: e.g by a series of buried using magnetic rings (Smith 1985a), or balls (Akagawa 1988). For practical purposes as the construction of a pipeline, it is however highly impractical to measure strains at depth at many locations. In pipeline operation practice, strains of the pipe are measured by a pressure induced pipeline geometry checking tool (PIG). The advantage of the use of a PIG is that the whole pipeline is checked. The disadvantage is that the costs to run a PIG through the pipeline are substantial.

IV.2.4.5 Moduli (Young’s elastic E , G etc), creep factor and Poisson ratio

These parameters are necessary to determine the stiffness matrix of the soil. The parameters can be determined from shear tests using a triaxial cell or ring shear apparatus in the

laboratory. Since this thesis focuses on the thermodynamic part of frost heave modelling, this part will not be discussed in further detail.

IV.2.5 Parameters related to the geometry of space and time

IV.2.5.1 Spatial step size, length of soil profile, length of freezing fringe, and depth of seasonal temperature influence

In the vertical direction the thermal, hydraulic and mechanic properties of a soil may vary from horizon to horizon. Even at shallow depths, various deposits may be overlying each other. In the lateral direction these properties vary from soil to soil. The more nonlinear the phenomenon, the more spatial steps are needed.

The length of the soil profile should be defined as the length from the soil surface to the depth at which the temperature does not change during the year. In freezing tests in laboratory experiments, it is usually the original length of the soil column. The length of the soil profile, its various layers or horizons, or the length of the soil column in a laboratory experiment may be measured by rulers and measuring tape. The longer a “warm” initial profile, the longer it will take to freeze the soil.

There is very little information in literature about the length of the freezing fringe. The freezing fringe is small. The length of the freezing fringe can be determined by measuring in a temperature profile the distance between the freezing temperature and segregation temperature.

The depth of seasonal temperature influence is determined by monitoring the temperature of a soil profile for at least a year, preferably a series of years. By measuring temperatures with thermistors or thermocouples connected to dataloggers a database of temperatures during the year may be constructed. The depth of seasonal temperature influence may be derived from these records.

IV.2.5.2 Time step size and total time

The time step size depends on the numerical method used. If an unconditionally stable method is used there are no requirements on the size of the timestep, otherwise there may be limitations. Smaller time steps may render smaller approximation errors, although this is not always true. Small time steps combined with a relatively large total time mean that an extremely large number of calculations has to be done, which requires a long running time for the computer model.

The total time could be the maximum length of the freezing season/freezing cycle, such as half a year to 8 months, or the duration of an annual temperature wave, 1 year, or several years. The total time is usually measured in days or seconds. Total times of full cycles, such as a year, can only be used if thawing can be dealt with in the prediction model.

IV.3 A method to optimize the amount of input data by analysis of the spatial variability of input parameters for frost heave prediction models

IV.3.1 Spatial analysis using semi-variograms

To determine a soil physical parameter as accurately as possible, the parameter should be measured at as many places as possible. However, extensive measurements require time and money, so a certain inaccuracy should be allowed within the restriction that the predictions should still be realistic. Statistical methods have been developed which provide a compromise between accuracy and minimal time and cost.

One of these methods is to determine the smallest distance between two measuring points, whereby the value of the parameter at one measuring point does not influence the value at the next measuring point. The value of a parameter at a certain point A in space is determined by the values of that parameter at every point B surrounding that point A. The difference between the values of the parameter at A and B increases with increasing distance between A and B, until the distance between A and B exceeds a certain value called the “range of influence”. When the distance between A and B increases beyond the range, the average difference between the values of the parameter at A and B does not increase anymore. In this thesis the range is therefore suggested as an optimal distance to determine input parameters for frost heave prediction models.

Figure IV.3 Schematic representation of a semi-variogram (based on Dirksen 1986)

To make sure that the value of the difference between the value of the parameter at A and B is always a positive value, the difference is squared. This squared difference is determined for a series of points at a distance equal to that between A and B. The average of all these squared differences is called the semi-variance, γ . The semi-variance can be calculated by the following equation (Cressie 1991, Webster and Burgess 1983 in Dirksen 1986):

$$\gamma(h) = \frac{1}{2(N-h-1)} \sum_{i=1}^{N-h} [z(x_i + h) - z(x_i)]^2 \quad (\text{IV.33})$$

Lag, h , is the dimensionless number of intervals for which the semi-variance is calculated. A lag of 4 at the scale at which the sample points are 10m apart represents a distance of 40m. The magnitude of the range expresses the distance of influence of a measured value. The value of

the maximum semi-variance is called the “sill”. The magnitude of the sill expresses the amount of variation. The semi-variance may be calculated for a number of lags, h , and plotted against these lags in a semi-variogram (see Figure IV.3).

If the value of the parameter at point A would be measured several times, the result may vary slightly, because of the inaccuracy or precision of the measurement, or inhomogeneity of the sample. A semi-variance at point A would result that would not be equal to zero. This value is called the nugget, and is a measure for the random variation.

IV.3.2 Single semi-variogram models

Various models that describe the curves in semi-variograms have been proposed: models such as linear models and most power law models, which do not obtain or converge to a limit, and therefore do not have a sill and range; and models with a sill such as spherical and exponential models. According to models without a sill, the difference between the value of the measured parameter at between point A_1 and another point A_2 keeps increasing with increasing distance from A_1 . Linear semi-variogram models have the form: $\gamma(h) = A \cdot h + B$ (IV.34)

where B is the nugget

$$\text{Most power law models such as: } \gamma(h) = Ch^A + Dh^E + Fh^G + B \quad (\text{IV.35})$$

do not have a limit either.

The spatial variability of a parameter may display a periodic pattern:

$$\gamma(h) = B + A \cdot \sin(h) \quad (\text{IV.36})$$

This periodicity of the semi-variance may be due to natural or human causes, such as parameters measured on levee-backswamp-levee suites, standing folds in rocks, and trees or crops planted in rows. Periodic spatial variability models do not have a range. Some models, such as the following continuous and differentiable power function, that divide the solution domain in two parts, are proposed:

$$\gamma(h) = B + S \left(\frac{3h}{2r} - \frac{h^3}{2r^3} \right) \quad \text{for } 0 < h \leq r \quad (\text{IV.37})$$

$$\gamma(h) = B + S \quad \text{for } h > r \quad (\text{IV.38})$$

where S is the sill, and r is the range

This model is often referred to as the spherical model (Journel and Huijbregts 1978).

The traditional single exponential semi-variogram model converges to a limit:

$$\gamma(h) = B + S \cdot (1 - e^{-\frac{h}{h^*}}) \quad (\text{IV.39})$$

(Matheron 1970 in Houlding 1994, Journel and Huijbregts 1978)

where $h=h^*$ is the distance at which 62% of the sill is reached (assuming that the nugget B is small, $B < 0.005S$)

A traditional squared exponential such as a Gaussian semi-variogram model converges to a

$$\text{limit as well: } \gamma(h) = B + S \cdot (1 - e^{-\left(\frac{h}{h^*}\right)^2}) \quad (\text{IV.40})$$

A model might be chosen and fitted to the calculated γ -h data.

IV.3.3 Choice of semi-variogram model

The meaning of a linear semi-variogram model applied to describe the semi-variance of a physical soil property is that the variability of that property continues to increase without bounds with increasing measurement spacing. Because, in general, physical properties of soil material only attain values within a certain interval, it is thought that the linear model can never be a good description of spatial dependence of soil physical parameters.

In the spherical model, the domain of measurement spacings is divided into 2 subdomains, and after the range is reached the semi-variance remains abruptly unchanged. There is no physical reason for such a sharp threshold and it is very difficult to find an objective criterion to make this division. Therefore the spherical model is unlikely to represent reality well.

Squared exponential models such as the Gaussian model have an inflection point. When the lag is smaller than the distance at which this inflection point occurs, the function is an accelerating function, which means that the influence of the value at a point becomes more pronounced the further away from that point. This is thought to be unrealistic. The single exponential model is more likely to be a good representation of reality, because it is an undivided increasing function, that approaches a limit when the spacing distance increases and the function does not have an inflection point.

Because the traditional exponential semi-variogram model is a continuous ascending function, the sill is only reached at distance $h = \infty$. If $h=h^*$, $\gamma(h^*) = B + S\left(\frac{e-1}{e}\right)$ or

approximately 62% of the sill is reached. The value of the semi-variance at this distance is too low to call this lag the range of influence. After this value of the semi-variance is reached, the semi-variance increases by another 38% of the eventual sill. Journel and Huijbregts (1978) suggest a “practical range” for the single exponential model at $h=3h^*$, when $\gamma(h) = B + S \left(\frac{e^3 - 1}{e^3} \right)$ or when 95% of the sill (a “practical sill”) is reached. This suggestion is adopted by e.g. Isaaks and Srivastava (1989).

Alternatively, the exponential function eq.(IV.38) may be adjusted in the following manner. In order to derive a more general function than eq.(IV.37), that fits the semi-variance data better than eq.(IV.37) a function is chosen that has the form:

$$\gamma(h) = B + S \cdot (1 - e^{-C \frac{h}{r}})^A \quad (\text{IV.41})$$

With the addition of parameters C and A, which have the value 1 in eq.(IV.37), the semi-variogram model is less rigid than eq.(IV.37). However, care has to be taken that the minimal requirements of a semi-variogram model with a sill are still satisfied: the semi-variogram model has to be negative definite (Cressie 1991). A semi-variogram model is negative definite when the following conditions are satisfied:

1. When the distance h approaches infinity, the sill S should be reached

$$\lim_{h \rightarrow \infty} \gamma(h) = B + S \cdot (1 - e^{-C \frac{\infty}{r}})^A = B + S \cdot (1 - e^{-\infty})^A = B + S \cdot (1 - 0)^A = B + S$$

2. The model should be continuously increasing because theoretically the variability increases with increasing distance. A continuously increasing function has a positive

first derivative.

$$\gamma(h) = B + S \cdot (1 - e^{-C \frac{h}{r}})^A$$

when $0 \leq h < \infty$, $0 \leq 1 - e^{-C \frac{h}{r}} < 1$, the larger h the closer $1 - e^{-C \frac{h}{r}}$ to 1 and

$$\gamma(0) = B \leq B + S \cdot (1 - e^{-C \frac{h}{r}})^A < B + S = \gamma(\infty)$$

the first derivative of $1 - e^{-C \frac{h}{r}}$ is $\frac{C}{r} e^{-C \frac{h}{r}}$, which is always positive because C and r

are positive, and because S is positive, $\gamma(h)$ is continuously increasing over the domain $0 \leq h < \infty$

3. In a homogeneous unit the model should be decelerating. Any acceleration in the function would mean an increase in dependency on the value of the variable at point A with increasing distance from point A. Decelerating functions have a continuously decreasing first derivative, i.e. a negative second derivative:

the second derivative of $1 - e^{-C \frac{h}{r}}$ is $-\left(\frac{C}{r}\right)^2 e^{-C \frac{h}{r}}$ which is negative for all positive

C and r if $A \leq 1$

The model should also reach the practical sill at the practical range. At the distance $h=r$ approximately $D \times 100\%$ of the sill is reached which is given in the following equation:

$$\gamma(r) = B + S \cdot (1 - e^{-C})^A = B + SD \quad (\text{IV.42})$$

Eq.(IV.41) gives: $(1 - e^{-C})^A = D \Rightarrow 1 - e^{-C} = D^{\frac{1}{A}} \Rightarrow e^{-C} = 1 - (D^{\frac{1}{A}})$ which leads to

$$C = -\ln(1 - (D^{\frac{1}{A}})) \text{ which makes eq.(IV.40): } \gamma(h) = B + S \cdot (1 - e^{\frac{h}{r} \ln(1 - (D^{\frac{1}{A}}))})^A$$

$$\text{which may be rewritten as: } \gamma(h) = B + S \cdot (1 - (1 - (D^{\frac{1}{A}}))^{\frac{h}{r}})^A \quad (\text{IV.43})$$

D may be chosen as 0.95 to represent a practical range where 95% of the sill is reached, but a lower value may as well be chosen. In conclusion, the proposed single semi-variogram model is a valid semi-variogram model and may be written as:

$$\gamma(h) = B + S \cdot (1 - (1 - (D^{\frac{1}{A}}))^{\frac{h}{r}})^A \quad (\text{IV.44})$$

With B the nugget S the sill

D the practical sill as a percentage of the sill

r the practical range

A curve fitting parameter

This model showed an extremely good fit to the calculated semi-variances of the parameters, and is thought to be very useful for spatial analysis in a wide variety of research. The procedure to obtain the range of influence by using the proposed semi-variance model is the following. The data are plotted in a semi-variogram. The sill S and nugget B are determined visually from the semi-variogram. A value for C is chosen, and A and r may be obtained by a least squares or cubic spline fitting of the curve to the data.

IV.3.4 Deviations from a single semi-variogram model

Several reasons exist for a calculated set of semi-variances to deviate from a single semi-variogram model:

1. the used model is not accurate
2. the amount of data on which the fitting is based is too small
3. the measured parameter displays anisotropy
4. the data set is not homogeneous (the parameter measurements extend over more than one scale) or the measured parameter displays a trend
5. random variation

The solutions to each of these complications are respectively:

1. Another model has to be used or the fitting parameters have to be changed
2. If possible the data set has to be increased. From a case study in which the size of the data set varied, Webster and Oliver (1992) found that a semivariogram model may be fit satisfactorily to 150 data of a normally distributed isotropic soil property, while a semi-variogram model derived from 225 data will be reliable. Because of limited money and time, often smaller data sets are used. Especially in remote regions such as northern Canada the data sets are usually small, if they exist at all. The relatively new possibilities of remote sensing could improve this situation considerably.
3. If the measured parameter displays anisotropy, the ranges and sills will vary, depending on the direction of measurement and in addition to the lag also the direction of measurement has to

be given $\gamma(h_{xyz})$. Software packages exist that determine the direction of the largest and smallest range, which are respectively the directions of the least and largest spatial variability.

4. In this situation instead of one good-fitting single semi-variogram model, several superimposed models have to be fit to the semi-variance data:

$\gamma(h) = \gamma_0(h) + \gamma_1(h) + \gamma_2(h) + \dots + \gamma_i(h)$. These are called nested structures, and are used to explain dominant features working on various scales (Journel and Huijbregts 1978, Isaaks and Srivastava 1989, Cressie 1991).

IV.3.5 Mapping the spatial variability using semi-variance models

By interpolation between measured or determined values of a soil parameter in a region, the spatial variability of a soil parameter might be mapped. This mapping might be done using an interpolation technique that uses the determined semi-variances. This interpolation technique is called kriging and is used in the mining industry and agriculture (Bonham-Carter 1995).

IV.4 Summary

To predict (differential) frost heave with even the tiniest bit of accuracy, large amounts of input data are required. Most measurements of input parameters are time consuming and expensive, and therefore a series of existing methods is adapted, and some new methods are proposed, to derive these input parameters from relatively easy to acquire grain size, bulk

density and water content data of the soil. In an analysis of the spatial variability of the input parameters the optimal distance between soil sampling points within a single surficial geology unit is determined, using a new semi-variogram model.

CHAPTER V Values and spatial variability of common input parameters for frost heave prediction models: Norman Wells, NWT

V.1 Description of the sampling site and sampling at Norman Wells, NWT

The site chosen to take samples for the determination of input parameters for the frost heave model was a relatively accessible fieldwork site in a periglacial environment, where the potential existed for frost heave damage to built structures. The site is situated along IPL's Norman Wells - Zama oil pipeline, in the transition zone of discontinuous and continuous permafrost near Norman Wells, NWT (see Figure V.1). The soil of the eastern bank of the Mackenzie River Valley can be characterized as a till overlain by lacustrine material and colluvium. The top 40cm are organic rich. These soil conditions exist for several hundreds of kilometers south and north of Norman Wells (Canadian Arctic Gas Pipeline Limited 1973).

Observations were made on a section along the Norman Wells - Zama pipeline from Quarry Road to the pump station in Norman Wells. East of the airport roughly between kilometer pole (kp) 4 and 6, two sites were observed where the pipeline had risen close to the surface. At one site, kp4.62 (IPL kp5.1) marked by 4 orange metal stakes (point Y in Figure V.1) the pipeline seemed higher than its surroundings. This elevation of the pipe could be a

result of buoyancy forces due to melting of ice in the soil in the summer, increased by frost heave in the winter.

The samples for this investigation were collected along three transects parallel to the pipeline, starting from kp5.02: transect one had 11 sample points with a spacing of 1m; transect two had 11 sample points with a spacing of 10m, and transect three had 10 sample points with a spacing of 100m. All the transects were situated on the pipeline Right-Of-Way (ROW), ca. 1.5m from the western edge of the ROW. At all sample points 3 layers could be distinguished: A, B, and C. These letters do not refer to soil horizons. Layer A consists of the organic rich top layer; layer B consists of lacustrine and colluvial clay rich soil material; and layer C is glacial till.

Study Area

Figure V.1 Location of study area (based on MacInnes et al. 1989)

Samples of each layer were taken at all sample points. The grainsize distributions were determined, based upon which the soil samples were classified according to the ISSS soil textural triangle. The results are shown in Figure V.2.

Figure V.2 The texture of the soil sampled at Norman Wells, NWT.

The finer textured the soil, the heavier the shading.

The soils sampled at Norman Wells vary from sandy loam to silty clay. It may be concluded that the organic rich top layer A and the fluvial layer B show less variability in texture than the till of lower layer C. As expected, the soil texture varies the most on the scales of the largest spacing, and due to a local (“random”) variation, on the scale of the smallest spacing.

V.2 Input parameter values for the frost heave prediction models

The results of the determination of the hydraulic and thermal parameters of the soil samples from Norman Wells are compiled in a report to Inter Provincial Pipelines Ltd. and the Terrain Sciences Division of the Department of Energy, Mines and Resources (Schellekens 1995). This report also exists as an in-house research report of the Geotechnical Science Laboratories. Excerpts of the report are given in this chapter.

V.2.1 Thermal input data

V.2.1.1 Temperatures: initial temperature profile, surface boundary temperature, freezing temperature, segregation temperature

A thermal database of the soil at the sampling site was not available so, as an approximation of the thermal regime of the soil at the sampling site, the thermal regime of the soil 175 km south of the sampling site (kp180) was used (Figure V.3). This thermal regime is

based on daily temperature recordings from August 7 1995 to September 10 1996. Between July and January the temperature at the soil surface decreased from $+20^{\circ}\text{C}$ to -10°C . The temperatures of Figure V.3 are measured in undisturbed soil at the edge of the ROW, not in the trench where the pipeline is buried. From Figure V.3 it may be concluded that the soil exhibits permafrost conditions from a depth of 1.20 m downward. The bottom end of the permafrost was not determined.

Figure V.3 The thermal regime of a soil 175 km south of the fieldwork site

(source data: Riseborough 1997)

Thermistors were used to assess temperatures of the soil at the sampling site in August. In August 1994 the soil on (and very close to) the ROW at the fieldwork site was unfrozen from the surface to at least 1.30m below the surface. The axis of the Norman Wells - Zama pipeline is situated at a depth varying from 0.50 to 1.00 m, and its diameter is 32 cm (MacInnes et al. 1989). A thaw bulb usually develops around a buried oil pipeline in a cold environment,

because the oil transported through the pipeline is at higher temperatures than the surrounding (in the winter, frozen) soil material.

Because the sophisticated methods and equipment that are required to measure the freezing and segregation temperature (see chapter IV) were not available for the fieldwork in Norman Wells, these temperatures could not be measured. Freezing temperatures are reported as ranging from -0.01°C to -0.1°C , and segregation temperatures as ranging from -0.05 to -0.70 (Akagawa 1988) or even -1.4°C (Fukuda 1995).

V.2.1.2 Thermal conductivity

The thermal conductivities of the sampled soils range from 0.944 to $3.519 \text{ Wm}^{-1}\text{K}^{-1}$ in unfrozen conditions and from 2.120 to $4.597 \text{ Wm}^{-1}\text{K}^{-1}$ in frozen conditions. The coarse-grained soil samples have a higher unfrozen and frozen thermal conductivity than the fine-grained soils. The thermal conductivities stay almost constant as the temperature decreases below $T=-0.5^{\circ}\text{C}$, below which most of the soil moisture is frozen. This behaviour shows the strong dependence of the thermal conductivity on the unfrozen water content. The variability of the thermal conductivity (as measured by the standard deviation (Table V.1) and the coefficient of variation (Table V.2)) increases considerably between the 10 and 100m spacings (series 2 and 3). The variability in thermal conductivity hardly changes with variation in temperature. The variability in thermal conductivity increases with depth from layer A to C.

series	layer	Thermal Conductivity K_h (in $Wm^{-1}K^{-1}$) at various temperatures				
		0°C	-0.1°C	-0.2°C	-0.5°C	-4.0°C
1	A	1.24±0.11	1.50±0.10	2.03±0.11	2.44±0.08	2.45±0.08
1	B	1.29±0.14	1.51±0.15	2.00±0.16	2.45±0.13	2.45±0.13
1	C	1.46±0.14	1.60±0.16	1.90±0.20	2.31±0.20	2.32±0.20
2	A	1.25±0.12	1.49±0.12	2.00±0.10	2.46±0.10	2.46±0.10
2	B	1.35±0.13	1.57±0.13	2.03±0.17	2.45±0.15	2.45±0.15
2	C	1.47±0.08	1.58±0.08	1.85±0.11	2.26±0.09	2.26±0.09
3	A	1.47±0.34	1.72±0.29	2.21±0.26	2.63±0.30	2.64±0.30
3	B	2.03±0.68	2.31±0.69	2.82±0.66	3.15±0.60	3.15±0.60
3	C	2.28±0.67	2.46±0.76	2.79±0.87	3.17±0.80	3.19±0.79

Table V.1 Descriptive statistics of thermal conductivities of soil samples from Norman Wells

series	layer	Coefficient of variation of the thermal conductivity at temperature T=				
		0°C	-0.1°C	-0.2°C	-0.5°C	-4.0°C
1	A	9	7	5	3	3
1	B	11	10	8	5	5
1	C	10	10	11	9	9
2	A	10	8	5	4	4
2	B	10	8	8	6	6
2	C	5	5	6	4	4
3	A	23	17	12	11	11
3	B	33	30	23	19	19
3	C	29	31	31	25	25

Table V.2 The coefficient of variation of the thermal conductivity of soil samples from Norman Wells

V.2.1.3 Volumetric heat capacity

series	layer	Volumetric Heat Capacity (in $10^6 \text{ JK}^{-1} \text{ m}^{-3}$) at various temperature T=					
		0°C	-0.07°C	-0.1°C	-0.2°C	-0.5°C	-4.0°C
1	A	3.39±0.09	3.22±0.08	3.07±0.09	2.57±0.08	2.25±0.01	2.25±0.01
1	B	3.35±0.10	3.21±0.09	3.08±0.08	2.61±0.05	2.27±0.04	2.27±0.04
1	C	3.15±0.06	3.07±0.06	3.00±0.06	2.72±0.07	2.38±0.01	2.38±0.04
2	A	3.39±0.10	3.23±0.08	3.09±0.06	2.60±0.02	2.25±0.01	2.25±0.01
2	B	3.30±0.10	3.17±0.08	3.05±0.06	2.61±0.07	2.30±0.06	2.30±0.06
2	C	3.12±0.07	3.06±0.05	3.00±0.04	2.74±0.04	2.42±0.03	2.40±0.03
3	A	3.28±0.19	3.13±0.13	3.00±0.10	2.56±0.09	2.27±0.05	2.27±0.05
3	B	3.07±0.23	2.94±0.20	2.83±0.17	2.47±0.08	2.28±0.03	2.28±0.03
3	C	2.91±0.15	2.85±0.16	2.79±0.16	2.58±0.18	2.34±0.08	2.33±0.07

Table V.3 Descriptive statistics of the volumetric heat capacity of soil samples from Norman Wells

The volumetric heat capacities of the soil samples in unfrozen conditions vary from 2.66×10^6 to $3.60 \times 10^6 \text{ JK}^{-1} \text{ m}^{-3}$, and in frozen conditions from 2.17×10^6 to $2.45 \times 10^6 \text{ JK}^{-1} \text{ m}^{-3}$. The variation in volumetric heat capacity (Table V.3 and Table V.4) is small. Significant variability in volumetric heat capacity exists only at temperatures 0°C and -0.1°C. At lower temperatures (from T=-0.4°C to -4.0°C) the volumetric heat capacity, C_h , of all samples is close to $2.3 \text{ MJ K}^{-1} \text{ m}^{-3}$. The variability of the volumetric heat capacity increases slightly between 10 and 100m spacings. This effect is almost nil at low temperatures.

series	layer	Coefficient of variation of the volumetric heat capacity at temperature T=					
		0°C	-0.07°C	-0.1°C	-0.2°C	-0.5°C	-4.0°C
1	A	3	2	3	3	0	0
1	B	3	3	3	2	2	2
1	C	2	2	2	3	0	2
2	A	3	2	2	1	0	0
2	B	3	3	2	3	3	3
2	C	2	2	1	1	1	1
3	A	6	4	3	4	2	2
3	B	7	7	6	3	1	1
3	C	5	6	6	7	3	3

Table V.4 Coefficient of variation of the volumetric heat capacity of soil samples from Norman Wells

V.2.1.4 Effect of latent heat: apparent volumetric heat capacity

The apparent volumetric heat capacities increase from 2.66 to 3.60 $\text{MJK}^{-1}\text{m}^{-3}$ at $T=0^\circ\text{C}$, to peak values of 210 to 1200 $\text{MJK}^{-1}\text{m}^{-3}$ between $T=-0.1$ and -0.2°C , and decrease to values between 2.2 to 2.4 $\text{MJK}^{-1}\text{m}^{-3}$ in completely frozen conditions ($T\leq-4.0^\circ\text{C}$). The coarse-grained soils have a lower apparent volumetric heat capacity than the finer-grained soils. The variation in apparent volumetric heat capacity measured by the standard deviation (Table V.5) and coefficient of variation (Table V.6) is much larger during freezing than in the frozen or unfrozen state.

series	layer	Apparent Volumetric Heat Capacity (in $\text{MJK}^{-1}\text{m}^{-3}$) at T=				
		0°C	-0.07°C	-0.1°C	-0.5°C	-4.0°C
1	A	3.39±0.09	612.34± 57.87	749.80± 70.90	5.04± 0.25	2.25±0.01
1	B	3.35±0.10	572.72± 72.34	703.84± 86.58	5.77± 2.05	2.27±0.03
1	C	3.15±0.06	301.91± 61.10	385.90± 70.61	18.61±11.38	2.38±0.04
2	A	3.39±0.10	607.85± 67.61	745.08± 80.94	5.23± 0.50	2.25±0.01
2	B	3.30±0.10	512.14±109.86	632.31±128.33	7.20± 3.93	2.30±0.05
2	C	3.12±0.06	299.09± 58.10	381.53± 69.27	14.39± 4.15	2.40±0.03
3	A	3.28±0.25	585.20±293.03	692.34±291.91	7.28± 5.30	2.27±0.05
3	B	3.07±0.24	496.58±154.85	576.67±179.30	3.27± 0.95	2.28±0.03
3	C	2.91±0.27	240.63±106.12	298.03±117.17	24.33±29.69	2.33±0.07

Table V.5 Descriptive statistics of the apparent volumetric heat capacity of soil samples from Norman Wells

series	layer	Coefficient of variation of the apparent volumetric heat capacity at T=				
		0°C	-0.07°C	-0.1°C	-0.5°C	-4.0°C
1	A	3	9	9	5	0
1	B	3	13	12	36	1
1	C	2	20	18	61	2
2	A	3	11	11	10	0
2	B	3	21	20	55	2
2	C	2	19	18	29	1
3	A	8	50	42	73	2
3	B	8	31	31	29	1
3	C	9	44	39	122	3

Table V.6 Coefficient of variation of the apparent volumetric heat capacity of soil samples from Norman Wells

V.2.2 Hydraulic input data

V.2.2.1 Porosity and unfrozen water content

The porosity at a temperature of $T=0^{\circ}\text{C}$ was assumed to equal the water content of the saturated soil sample. The descriptive statistics are given in Table V.7. The porosity of the soil decreases with increasing depth. The variation of the porosity (measured by the standard deviation and the coefficient of variation) increases considerably from the 10m to the 100m spacing. The porosity increases at temperatures below 0°C , when a decrease in porosity due to consolidation of the soil is more than offset by an increase in porosity due to ice lens formation.

series	layer	mean and standard deviation	coefficient of variation
1	A	0.573 ± 0.048	8
1	B	0.556 ± 0.053	10
1	C	0.457 ± 0.030	7
2	A	0.572 ± 0.048	8
2	B	0.529 ± 0.050	9
2	C	0.443 ± 0.032	7
3	A	0.518 ± 0.096	19
3	B	0.411 ± 0.116	28
3	C	0.333 ± 0.075	23

Table V.7 Descriptive statistics of the porosity of soil samples from Norman Wells at temperature $T=0^{\circ}\text{C}$.

According to the used method, most of the water of the soil samples from Norman Wells freezes between temperatures $T=-0.05^{\circ}\text{C}$ and $T=-0.5^{\circ}\text{C}$ (Table V.8). Finer-grained soils contain more water in the unfrozen condition at any temperature and are able to retain unfrozen water at lower temperatures than coarser-grained soils. The variation of the unfrozen water con-

series	layer	Unfrozen water contents at temperature $T=$				
		0°C	-0.1°C	-0.2°C	-0.5°C	-4.0°C
1	A	0.573 ± 0.048	0.432 ± 0.041	0.210 ± 0.036	0.070 ± 0.0001	0.070 ± 0.000
1	B	0.556 ± 0.053	0.434 ± 0.043	0.225 ± 0.029	0.075 ± 0.0150	0.075 ± 0.015
1	C	0.457 ± 0.030	0.393 ± 0.030	0.266 ± 0.033	0.118 ± 0.0170	0.105 ± 0.023
2	A	0.572 ± 0.048	0.443 ± 0.034	0.223 ± 0.012	0.070 ± 0.0001	0.070 ± 0.000
2	B	0.529 ± 0.050	0.416 ± 0.032	0.224 ± 0.030	0.084 ± 0.0002	0.080 ± 0.020
2	C	0.443 ± 0.032	0.387 ± 0.019	0.274 ± 0.017	0.130 ± 0.0120	0.125 ± 0.008
3	A	0.518 ± 0.096	0.393 ± 0.052	0.200 ± 0.036	0.071 ± 0.0160	0.070 ± 0.015
3	B	0.411 ± 0.116	0.308 ± 0.087	0.149 ± 0.045	0.062 ± 0.0160	0.061 ± 0.016
3	C	0.333 ± 0.075	0.282 ± 0.082	0.190 ± 0.086	0.083 ± 0.0420	0.077 ± 0.034

Table V.8 Descriptive statistics of the unfrozen water content of soil samples from Norman Wells at various temperatures below 0°C .

tent (measured by the standard deviation (Table V.8) and coefficient of variation (Table V.9)) increases with the spacing of the sample points. A significant increase in the standard deviation occurs between the 10 and 100m spacings. As expected, the variability in the unfrozen water content is largest at 0°C and smallest at -4°C . At a very low temperature ($T=-4^{\circ}\text{C}$) the unfrozen water content of layer C is higher than that of layer A and B. At low temperatures the variability of the unfrozen water content increases from layer A to C, even if the variability of the unfrozen water content of layer C was less than that of layer A and B at 0°C .

		Coefficient of variation of the unfrozen water content at T=				
series	layer	0°C	-0.1°C	-0.2°C	-0.5°C	-4.0°C
1	A	8	9	17	0	0
1	B	10	10	13	20	20
1	C	7	8	12	14	22
2	A	8	8	5	0	0
2	B	9	8	13	0	25
2	C	7	5	6	9	6
3	A	19	13	18	23	21
3	B	28	28	30	26	26
3	C	23	23	45	51	44

Table V.9 Coefficient of variation of the unfrozen water content of soil samples from Norman Wells at various temperatures below 0°C.

V.2.2.2 Hydraulic conductivity

		Hydraulic Conductivity (in ms^{-1}) at temperature T=					
		0°C	-0.07°C	-0.1°C	-0.2°C	-0.5°C	-0.7°C
series	layer	($\times 10^{-5}$)	($\times 10^{-6}$)	($\times 10^{-7}$)	($\times 10^{-8}$)	($\times 10^{-9}$)	($\times 10^{-9}$)
1	A	8.39±1.07	6.59± 1.36	8.49± 1.27	3.00± 5.85	2.85±2.39	2.59±2.22
1	B	7.53±0.63	3.99± 3.33	5.32± 3.82	3.27± 2.36	4.40±2.60	2.81±1.03
1	C	6.61±1.14	0.78± 2.13	1.61± 2.25	5.35± 1.58	6.91±1.66	3.80±0.66
2	A	8.34±2.27	4.37± 3.00	6.13± 3.17	6.14±11.40	5.05±4.60	3.86±4.31
2	B	7.84±1.41	3.70± 3.53	5.17± 4.24	5.05± 5.70	5.10±3.13	3.48±2.19
2	C	4.71±1.80	0.43± 1.07	1.07± 4.49	4.42± 1.02	7.21±0.85	3.90±0.33
3	A	10.47±3.32	4.61± 2.95	7.48± 1.71	14.90±18.30	7.57±7.13	7.10±6.90
3	B	12.11±2.61	22.78±69.17	14.80±15.90	14.80±15.90	7.28±6.44	6.80±6.30
3	C	8.16±4.91	16.50±27.20	34.33±61.11	5.49± 8.06	4.94±3.70	3.80±3.10

Table V.10 Descriptive statistics of the hydraulic conductivity of soil samples from Norman Wells

In Table V.10 the descriptive statistics of the hydraulic conductivities are given. In Table V.11 the coefficient of variation of the determined hydraulic conductivities of the soil samples taken in August 1994 at Norman Wells is given.

series	layer	Coefficient of variation of the hydraulic conductivity at T=					
		0°C	-0.07°C	-0.1°C	-0.2°C	-0.5°C	-0.7°C
1	A	13	21	15	195	84	86
1	B	8	83	72	72	59	37
1	C	17	273	140	30	24	17
2	A	27	69	52	186	91	112
2	B	18	95	82	113	61	63
2	C	38	249	420	23	12	8
3	A	32	64	23	123	94	97
3	B	22	137	139	107	88	93
3	C	60	165	178	147	75	82

Table V.11 Coefficients of variation of the hydraulic conductivity of soil samples from Norman Wells

As expected, at 0°C the hydraulic conductivity decreases with increasing distance from the surface (from A to C). The variation in the hydraulic conductivity (measured by the standard deviation) increases considerably between the 10m spacing series (2) and the 100m spacing series (3). The coefficients of variation are highest between the temperatures -0.07°C and -0.1°C, and reach values as high as 420 for layer C at 10m spacing.

V.2.2.3 The density of water and salinity information

The density of water and salinity of the soil water were not measured. In the modelling soil water is assumed to be pure and therefore to have a density of 1000kgm^{-3} .

V.2.3 Mechanical input data

V.2.3.1 Density of soil

In Table V.12 the descriptive statistics of the wet bulk density of the soil samples taken near Norman Wells are given. As expected, the wet bulk density increases with depth

BULK DENSITY			
series	layer	mean and standard deviation (in kg m^{-3})	coefficient of variation
1	A	1463 ± 173	12
1	B	1608 ± 117	7
1	C	1966 ± 48	2
2	A	1654 ± 208	13
2	B	1730 ± 150	9
2	C	2082 ± 128	6
3	A	1660 ± 342	21
3	B	1852 ± 250	13
3	C	2161 ± 108	5

Table V.12 Descriptive statistics of the wet bulk density of soil samples taken near Norman Wells

(from A to C). The variation in the wet bulk density (measured by the standard deviation and coefficient of variation), decreases with depth. The variation in wet bulk density (as measured by the standard deviation and coefficient of variation) increases with increasing sample distance (from 1 to 3).

V.2.3.2 External stress/pressure

Since the external stress is not exactly known, a series of predictions has to be made with various values for the external stress. At the sampling site glaciation is the only event that could have resulted in an overconsolidation of the soil. However, the top soil layers A and B are both postglacial and normally consolidated. The values for external stress, as used in the frost heave prediction, varied from 0Pa to 636kPa, which is the equivalent of 40m of soil or 80m of ice.

V.2.3.3 Separation pressure

Since the separation pressure was not known, a series of predictions was made with a variety of values for the separation pressure. The findings of Nixon suggest that the frost heave prediction is not very sensitive to the separation pressure. Since the clay content of the soil samples from Norman Wells was high (up to 45% of the dry mass) the cohesive forces were assumed to be high, and the high values suggested by Nixon were used in this analysis.

V.2.4 Data related to the geometry of space and time

V.2.4.1 Spatial step size, length of soil profile, length of freezing fringe and depth of seasonal influence

In Nixon's model 20 spatial steps were taken in the freezing fringe, to get a reasonable smooth water pressure graph. In Wang's model the total soil profile was divided in 40 spatial steps.

The length of the annually freezing soil is greater than 1.20m. However, the deepest point in the soil profile that could be reached with the available sampling equipment was 1.20m. The total length taken was 1m. The length of the freezing fringe could not be measured. 177km south of the fieldwork site (kp 182), the depth of the seasonal temperature influence was >1.20m (see Figure V.2).

V.2.4.2 Time step size and total time

The spatial step size used was 0.005 days or 7min and 12s. This step size appeared to be sufficiently small to achieve results from the numerical analysis that were close to analytical solution. It is not too small that calculation time is excessive.

Since a total annual freezing cycle appears to take roughly 180 days, and soil warming was not included in both frost heave prediction models, only the freezing phase of 180 days could be modeled.

V.3 Analysis of the spatial variability of common input parameters for the frost heave prediction models at Norman Wells, NWT

V.3.1 Thermal input data

A temperature record of all the measuring points did not exist, and it was not logistically possible to measure temperatures at all the sampling points at one moment in time. Therefore it was impossible to use semi-variograms to evaluate the spatial variability of the soil temperature near Norman Wells in semivariograms. Enough spatial information existed to evaluate the spatial variability of the thermal conductivity, volumetric heat capacity and apparent volumetric heat capacity using semivariograms. All the semivariograms presented are for soil properties of the unfrozen soil.

V.3.1.1 Thermal conductivity

From Figure V.4 it may be concluded that the range of the thermal conductivity of soil layer A to C is between 200 and 300m, which means most of the variability occurs within

this lag of 200 to 300m. The sill increases from layer A to C, which means that the size of the variability is the largest in the deepest soil layer C.

Figure V.4 Semivariogram of the thermal conductivity of the soil near Norman Wells

V.3.1.2 Volumetric heat capacity

From Figure V.5 it may be concluded that the range of the volumetric heat capacity of soil layer A to C is more or less 300m, which means most of the variability occurs within this lag of 300m. The sill decreases from layers A and B to C, which means that the size of the variability is the largest in the top layers A and B.

Figure V.5 Semivariogram of the volumetric heat capacity of soil samples from Norman Wells, NWT.

V.3.1.3 Apparent heat capacity

From Figure V.6 it may be concluded that the range of the apparent volumetric heat capacity of soil layer A and C is between 200 and 300m; layer B seems to have a much shorter range of approximately 100m. The size of the variability of the apparent volumetric water content (as expressed by the sill) in of the wet organic top layer A is several times (4x) larger than that of the alluvial clay layer B, and therefore 200 to 300m would serve as a good interval distance to take samples for determination of the apparent heat capacity.

Figure V.6 Semivariogram of the apparent heat capacity of soil samples from Norman Wells, NWT.

V.3.2 Hydraulic input data

V.3.2.1 Porosity and unfrozen water content

From Figure V.7 it may be concluded that the range of the porosity of layers A and C is more or less 300m, and the range of the porosity of layer B is between 300 and 400m. The sills of the porosity of top layers A and B are somewhat larger than the sill of the porosity of layer C. The spatial variability of the porosity of layer A is larger than that of the deeper layers B and C.

Figure V.7 Semivariogram of the porosity of soil samples from Norman Wells, NWT.

V.3.2.2 Hydraulic conductivity

From Figure V.8 it may be concluded that the range of the hydraulic conductivity of soil layer A and B is more or less 150m, while that of soil layer C is between 150 and 200m. The size of the variability of the hydraulic conductivity (as expressed by the sill) in the glacial till layer C is several times (5x) larger than that of the alluvial clay layer B, and therefore 150 to 200m would serve as a good interval distance to take samples for determination of the hydraulic conductivity of the soil, within this surficial geology unit.

Figure V.8 Semivariogram of the hydraulic conductivity of soil samples from Norman Wells, NWT

V.3.3 Mechanical input data

V.3.3.1 Density of soil

From Figure V.9 it may be concluded that the range for layer C is very short, since the semivariogram of layer C shows a nugget effect: a constant low value for the semivariance (≈ 20000) that is reached at a lag of 0m. The density of the soil in layer B has a range between 100 and 200m, while the range of the organic top layer is between 300 and 400 m. The sill decreases from top layer A to bottom layer C.

Figure V.9 Semivariogram of the density of soil samples from Norman Wells, NWT

V.4 Discussion and summary

The most important input parameter in the frost heave prediction models of Wang and Nixon is the hydraulic conductivity, because the prediction by the model is most sensitive to the hydraulic conductivity (chapter III), and because the temporal and spatial variation in hydraulic conductivity is larger than the variation in the other input parameters. The hydraulic conductivity which may be determined from grain size distribution, bulk density and water content data, which are relatively easy to acquire, drops several orders of magnitude during the freezing of soil (chapter IV).

Based on the standard deviation and coefficient of variation of the input parameters of the frost heave prediction models (chapter V.2), it may be concluded that the hydraulic

conductivity and apparent heat capacity of the soil display the largest spatial variability. The more data available, the better the spatial variability of frost heave may be predicted. However, the more data, the higher the cost of data acquisition. The optimal amount of data needed within a certain surficial geology unit, which captures most of the variability of the measured parameter while the costs of data acquisition are kept within limits, may be determined by analysis of semivariograms. In the till overlain by lacustrine material, colluvium and organic-rich top layer, along the east bank of the Mackenzie River between Norman Wells and Wrigley, an optimal sampling distance to determine the hydraulic conductivity and the apparent heat capacity is 200m (chapter V.3).

This figure could be erroneous as a result of the following problems:

- A spatial variability of one or more of the parameters may not have reached the sill within the longest “reliable” length of the sample intervals (600-700m). For this research the field work period was too short to investigate the variation at smaller scales.
- The analysis in this chapter assumed that the spatial variability of the parameters could be represented by one single semivariogram, instead of a series of nested semivariograms, displaying various features at various scales.
- The data sets are very small, which means that the potential for inaccurate sills and ranges is high.

CHAPTER VI Evaluation of predictions of frost heave at Norman Wells, NWT

VI.1 Introduction

In this chapter the modified frost heave prediction model of Wang (chapter III) is operationalized, in order to make predictions of frost heave in a wide variety of multiple layered soils. This operationalization involves the assignment of freezing temperature, segregation temperature and an average hydraulic conductivity of the freezing fringe to each soil. The results that are generated by the operationalized model using the input parameters obtained from grain size data of soils sampled at Norman Wells, NWT (chapter V), are discussed. The predicted depth of penetration of the freezing front, the length of the freezing fringe and the cryosuction are evaluated. According to Darcy's law, eq.(III.11), the last two variables and the hydraulic conductivity determine the water flow density to the warmest ice lens. According to the modified frost heave prediction model of Wang this flow density determines the secondary frost heave. Primary heave is the heave generated by freezing of pore water in situ. The predicted total heave, the addition of primary heave and secondary heave will be discussed in subchapter VI.5. The last predicted parameter evaluated is the differential frost heave. Since in this research the absolute values of differential frost heave are generated by a one dimensional frost heave prediction model, this predicted parameter should not be taken absolute, because the absolute value will be an exaggeration of the real value. However, the differential heave as

predicted is a good indicator to identify problem points along linear structures in frost susceptible regions.

VI.2 Operationalization of the modified frost heave prediction model of Wang

Before frost heave and the spatial variability of frost heave can be predicted using the modified frost heave prediction model of Wang (1994), methods have to be developed to assign a freezing temperature, a segregation temperature and an average hydraulic conductivity (considered to be constant soil characteristics in the model of Wang) of the freezing fringe to each soil. In this subchapter these three parameters are related to easily obtainable grain size and water content data.

Figure VI.1 Hydraulic conductivity of three different soils at a temperature above 0°C under variable soil water pressures (modified from Hillel 1980)

1, 2, 3 air entry value of water pressure of respectively sandy, silty and clayey soil

K_{s1}, K_{s2}, K_{s3} Hydraulic conductivity of respectively saturated sandy, silty and clayey soil

In a soil that experiences drying (see Figure VI.1), the pore water pressure at which air first enters the pores, is called the air entry value. Because the largest pores are the first to empty out, air will enter first in the largest pores. Fine-grained soils usually do not have large pores, and therefore the air entry value of fine-grained soils is lower (more negative) than that of coarser-grained soils.

Figure VI.2 Freezing temperature, segregation temperature and hydraulic conductivity of three different soils under variable temperatures below 0°C.

1, 2, 3 freezing temperature of water in respectively sandy soil, silty soil and clayey soil

Ts1, Ts2, Ts3 segregation temperature in respectively sandy soil, silty soil and clayey soil

Ks1, Ks2, Ks3 hydraulic conductivity of respectively saturated sandy, silty and clayey soil

In an analogy between drying soils above 0°C and freezing soils, the water in the largest pores having the highest free energy, will freeze first. Because coarse-grained soils usually contain larger pores than fine-grained soils, it may be assumed that most pore water in coarse-grained soils freezes at temperatures closer to 0°C than pore water in fine-grained soils (Figure VI.2). The heavier the soil (i.e. the higher the clay content of the soil), the smaller the pores and the lower the freezing temperature. Therefore, the heavier the soils the lower freezing temperatures that were assigned to the soil samples from Norman Wells.

Coarse-grained soils have very small freezing fringes, and ice lenses grow, if at all, close to the freezing front, which is close to 0°C , especially when the solute concentration in the pore water is relatively low. The segregation temperatures of coarse-grained soils are therefore high. The segregation temperatures of finer-grained soils are lower (see chapter II. and Figure VI.2). Therefore, the heavier the soils the lower segregation temperatures that were assigned to the soil samples from Norman Wells. Because there is not much information about segregation temperatures in the literature, the chosen values were based on the few available ones, such as reported by Akagawa (1988).

The hydraulic conductivity of a soil at a temperature above 0°C decreases considerably when the soil water pressure decreases (see Figure VI.1). The hydraulic conductivity of a saturated coarse-grained soil such as a sand, is larger than that of a saturated finer-grained silt or clay. However, at very low soil water pressures, or in other words, at very high suctions, the hydraulic conductivity of a sand is much smaller than that of silt and clay. This means that the hydraulic conductivity curves of a clay and a sand cross (see Figure VI.1).

If the moisture transport by conductive flow of water in the unsaturated zone needs to be estimated, the gradient of the water pressure over the unsaturated zone has to be determined. A hydraulic conductivity could be estimated that is representative for the unsaturated zone, and the water flux density through the unsaturated zone could be calculated with Darcy's law. It should be realized that the choice of a hydraulic conductivity representing the unsaturated zone is very prone to errors. A small error in the chosen water pressure results in a large error in the hydraulic conductivity, because the slope of the $K_w(\psi)$ -curve is steep.

Once more, this situation in unfrozen drying soils may serve as an analogy to freezing soils. Because there is much more unfrozen water in the freezing fringe of a fine-grained soil than in the freezing fringe of a coarse-grained soil, the representative hydraulic conductivity of the freezing fringe of a fine-grained soil will be larger than that of a coarse-grained soil. Because of the lack of data about the hydraulic conductivity of the freezing fringe in the literature an arbitrary scale of hydraulic conductivity as a function of soil textural class was developed to assign representative hydraulic conductivities to freezing soils, based on the few sources that mention hydraulic conductivities at temperatures below 0°C, which are compiled by Nixon (1991) (see Figure VI.3).

Figure VI.3 Hydraulic conductivity of various soils as a function of temperature (Nixon 1991)

From Figure VI.3 it may be concluded that fine-grained soils have a higher hydraulic conductivity than coarser-grained soils at temperatures less than -0.1°C . Similar to the water flow through unfrozen drying soil it should be realized that the choice of a hydraulic conductivity representing the freezing fringe is very prone to errors. A small error in the chosen water pressure results in a large error in the hydraulic conductivity, because the slope of the $K_w(T)$ -curve is steep. It also has to be noted that hydraulic conductivity determining factors other than soil textural, e.g. soil structural factors such as packing, soil particle alignment and soil pore connectivity, are not taken into account.

The freezing temperature, segregation temperature and hydraulic conductivity assigned to soils based on the position of the soil in the soil textural triangle are given in Table VI.1:

	FREEZING TEMPERATURE (in °C)	SEGREGATION TEMPERATURE (in °C)	HYDRAULIC CONDUCTIVITY (in 10^{-14} ms^{-1})
CLAY	-0.034	-0.800	10000
SILTY CLAY	-0.028	-0.550	20000
SANDY CLAY	-0.024	-0.300	15000
SILTY CLAY LOAM	-0.024	-0.250	13000
CLAY LOAM	-0.020	-0.200	12000
SANDY CLAY LOAM	-0.017	-0.150	4300
LOAM	-0.016	-0.100	2150
SILT LOAM	-0.016	-0.100	2150
SILT	-0.016	-0.050	2000
SANDY LOAM	-0.012	-0.025	107.5
LOAMY SAND	-0.008	-0.020	86
SAND	-0.004	-0.015	4.3

Table VI.1 Freezing temperatures, segregation temperatures and hydraulic conductivities of freezing soils, based on their soil texture class (The heavier soils which were represented in the samples from Norman Wells are shaded. Sandy loam was the lightest soil represented.)

It has to be realized that this assignment of values is empirical, based on small amounts of information available, and very general. More variation in values for freezing temperature,

segregation temperature, and hydraulic conductivity would be obtained if the specific surface of the soil in combination with 3 empirical functions were used to assign values to these three parameters.

VI.3 Depth of freezing front penetration

In Figure VI.4 two examples are given of predicted depth of penetration of the freezing front under a scenario with a constant surface boundary temperature of -2°C and an initial temperature profile of 2°C at the soil surface to 6°C at 0.70 m below the surface. At several times during the freezing the predicted depth of freezing front penetration (data series 10p and 11p), is less than that earlier in the freezing process. In reality such a situation never occurs. The prediction of a smaller depth of frost penetration is an artifact of the model which results from the generation of latent heat during the freezing process and the manner that this latent heat release is dealt with in the frost heave prediction model. The plotted curves are upper envelope curves of the predictions of the model. Those points in Figure VI.4 which fall below the envelope curves are artifacts of the prediction process incorporated in the model. To account for the latent heat released, a thermal load vector is added to the temperature at 2 to 4 nodes in the soil. The addition of such thermal load vectors increases the temperature of the soil, and this heat has to be removed from the soil before the freezing front can penetrate any further. The handling of latent heat in numerical approximations of thermal processes involving phase change is a difficult problem that is still being studied intensively in mechanical engineering.

This thesis does not aim to solve that problem. If the latent heat problem is disregarded, the depth of the freezing front for sample point 10 and 11 should be close to the smoothed curves (10s and 11s in Figure VI.4) connecting the maximal predicted penetration depths of the freezing front.

Figure VI.4 Penetration of the freezing front at sample points 10 and 11 at an initial temperature of $+2^{\circ}\text{C}$ at the soil surface to $+6^{\circ}\text{C}$ at 0.70m below the soil surface, and at a constant surface boundary temperature of -2°C .

Because the soil at sample point 11 has a higher thermal conductivity than the soil at sample point 10, the freezing front penetrates faster and deeper in the soil at sample point 11 than at sample point 10. It should be kept in mind that it is not possible to draw firm conclusions concerning the variation of frost front penetration between adjacent, closely-spaced, sampling points, because any existing difference would generate lateral heat flow, which can not be dealt with in a one dimensional frost heave prediction model.

Figure VI.5 shows that the main spatial differences in freezing front penetration are caused by differences in texture, which determine the segregation and freezing temperature that are chosen for determination of the cryosuction over the freezing fringe.

Figure VI.5 Predicted depth of the freezing front after 110 and 300 days under 3 different scenarios:

- A initial temperature $+2^{\circ}\text{C}$ at the soil surface and $+6^{\circ}\text{C}$ at 0.7m depth and a stationary surface boundary temperature of -2°C .
- B initial temperature $+0.5^{\circ}\text{C}$ at the soil surface and $+0.5^{\circ}\text{C}$ at 1.0m depth and a stationary surface boundary temperature of -2°C .
- C initial temperature $+0.5^{\circ}\text{C}$ at the soil surface and $+0.5^{\circ}\text{C}$ at 1.0m depth and a stationary surface boundary temperature of -10°C .

1, 2 and 3 refer respectively to the 1m, 10m and 100m spacing of the sample points

A difference in initial temperature, such as between scenarios A and B, does not produce a significantly different penetration of the freezing front. However, as expected, the freezing front penetrates much deeper into the soil when the surface boundary temperature is lower. A lower surface boundary temperature accentuates the difference in freezing front penetration caused by differences in soil texture. From Figure VI.5 it can be concluded that the boundary temperature has a much larger effect on the depth of penetration of the freezing front than the initial soil temperature profile.

The spatial variability of the penetration of the freezing front is larger on the scales of the 1m (this variability could be explained as a “random variation” or nugget-effect) and 100m sample point spacings than on that of the 10m sample point spacing. The spatial variability of the depth of penetration of the freezing front increases slightly over time.

VI.4 Length of the freezing fringe

The deeper the penetration of the freezing front, the smaller the thermal gradient, the longer the freezing fringe at the end of the ice lens growth. In Figure VI.6 examples are given of the predicted length of the freezing fringe. The maximal length of the freezing fringe increases with time. After an initial fast increase in freezing fringe length, the increase in the length of the freezing fringe seems to be almost linear with time. In early phases of the freezing process the temperature gradient is large, and the length of the freezing fringe is therefore small.

Figure VI.6 Example of the variation in length of the freezing fringe during the freezing process. The plotted 10P squares are predicted lengths of the freezing fringe; the symbols connected by the line 10M denotes the trend of the maximal length of the freezing fringe over time.

Figure VI.7 Length of the freezing fringe at sample points 10 to 20 after 300 days, under scenarios A, B and C.

- A initial temperature $+2^{\circ}\text{C}$ at the soil surface and $+6^{\circ}\text{C}$ at 0.7m depth and a stationary surface boundary temperature of -2°C .
- B initial temperature $+0.5^{\circ}\text{C}$ at the soil surface and $+0.5^{\circ}\text{C}$ at 1.0m depth and a stationary surface boundary temperature of -2°C .
- C initial temperature $+0.5^{\circ}\text{C}$ at the soil surface and $+0.5^{\circ}\text{C}$ at 1.0m depth and a stationary surface boundary temperature of -10°C .

Later in the freezing process the temperature gradient in the soil is smaller, and, because the segregation temperature and freezing temperature are constant, the length of the freezing fringe is larger. The predicted maximal lengths of the freezing fringe vary between ca. 4 and 11.5 mm. The maximal length of the freezing fringe is smallest at sample points 11 to 14, where the top-layer is silt. At these points the freezing front penetrated deeper than at the surrounding sample points 10, 15 and 16, where the top-layer is silt loam, especially when the surface boundary temperature is low (-10°C).

It should be mentioned that because of the latent heat problem, checks have to be build into the computer code of the numerical frost heave prediction model, to prevent negative lengths of the freezing fringe. Nixon's code contains a number of such checks. These checks were absent in Wang's computer code, and were not added to the modified computer model.

VI.5 Cryosuction in frost heave simulation using the modified model of Wang

In frost heave prediction models that assume thermodynamic equilibrium in the freezing fringe, the cryosuction, defined in chapter II as the water pressure difference over the freezing fringe, may be determined by the Clausius-Clapeyron equation (see chapter III.2):

$$P_{ws} - P_{wf} = \left(\frac{\rho_w}{\rho_i}\right) P_{is} - \left(\frac{\rho_w}{\rho_i}\right) P_{if} + (\rho_w L_h) \ln\left(\frac{T}{T_f}\right) \quad (\text{III.49})$$

If the ice pressure at the freezing front P_{if} is 0, eq.(III.49) gives:

$$P_{ws} - P_{wf} = 1.11 P_{is} + (1000 L_h) \ln\left(\frac{T}{T_f}\right) \quad (\text{VI.1})$$

The ice pressure at the ice lens, P_{is} , in a normally consolidated soil, equals the overburden (the normal stress caused by the weight of the soil on top of the ice lens). Despite the fact that the natural logarithm is small, the second term of the equation is large because the first part of the second term in eq.(VI.1) contains the density of water and the specific latent heat of freezing (330000Jkg^{-1}). Compared to the second term in the equation, the ice pressure term is therefore very small and may almost be ignored. As a consequence, in case of a fixed characteristic segregation and freezing temperature such as proposed by Wang (1994), a freezing soil would also have a characteristic cryosuction. These characteristic cryosuctions for each pair of freezing and segregation temperatures are given in Table VI.2.

	cryosuction (in kPa)	hydraulic conductivity (in 10^{-14} ms^{-1})	temperature difference over freezing fringe (in $^{\circ}\text{C}$)
CLAY	-926.84	10000	0.766
SILTY CLAY	-631.31	20000	0.522
SANDY CLAY	-333.64	15000	0.276
SILTY CLAY LOAM	-273.17	13000	0.226
CLAY LOAM	-217.55	12000	0.180
SANDY CLAY LOAM	-160.73	4300	0.133
LOAM	-101.50	2150	0.084
SILT LOAM	-101.50	2150	0.084
SILT	- 41.08	2000	0.034
SANDY LOAM	- 15.71	107.5	0.013
LOAMY SAND	- 14.50	86	0.012
SAND	- 13.29	4.3	0.011

Table VI.2 Assigned cryosuction, hydraulic conductivity and average temperature gradient of the freezing fringe in various soils

The top layers sampled varied from silt to silt loam and loam, and as a result the cryosuctions were roughly expected to change between -40kPa for the silt (e.g. sample points 11 to 14) and -100kPa for the silt loam (e.g. sample points 10 and 15) and loam. The predicted cryosuctions followed this expectation (e.g. Figure VI.8), and as expected, the predicted cryosuctions in the top soil changed little with time, and behave as a constant characteristic property of the soil. These predicted cryosuctions are low compared to the -560kPa for silt and -2400kPa for clay given by Hoekstra (1969) and Loch and Miller (1975). However, the cryosuction of -1200kPa of clay given by Smith (1985) is relatively close to the lowest value of Table VI.2.

Figure VI.8 Predicted cryosuction during freezing at sample points 10 to 15 (at 1m spacing intervals) for a scenario with initial temperature of +2°C at the soil surface to +6°C at 0.70m below the soil surface; and a stationary surface boundary temperature of -2°C. The lowest cryosuctions of these 6 sample points were predicted at sample point 10 and 15.

VI.5 Frost heave predictions

Secondary heave in thermodynamic frost heave prediction models is the result of freezing of water flowing to the warmest ice lens. The density of this water flux is calculated by the law for laminar flow of water through the soil, the Darcy equation:

$$f_w = -K_w(T) \frac{d \frac{P_w}{\rho_w \cdot g}}{ds} = -K_w(T) \frac{dP_w}{\rho_w \cdot g \cdot ds} \quad (\text{III.11})$$

Heavy soils containing some amount of silt are known to produce the largest secondary frost heave. This is commonly attributed to a high hydraulic conductivity $K_w(T)$ of the freezing fringe, and a sufficient water supply to the freezing fringe, caused by an unfrozen hydraulic conductivity that is larger than that of pure clays. In order to generate the highest secondary frost heave using the modified frost heave prediction model of Wang, for e.g. silty clay loam, the length of the freezing fringe of silty clay has to be smaller than 0.38 times the length of the freezing fringe of clay. Because the temperature difference over the freezing fringe of the clay is 3.39 times that of the temperature difference over the freezing fringe of the silty clay loam, the temperature gradient in the clay should not be more than 1.29 times the temperature gradient in the silty clay loam in order to produce the largest secondary frost heave in the silty clay loam.

Four series of frost heave predictions were made using the modified model of Wang and the soil properties of the soil samples taken at Norman Wells and given in chapter V:

1. A series consisting of a frost heave prediction at each sample point using the initial temperature profile given in Figure VI. . A stationary surface boundary temperature

of -2°C was used. The freezing process was continued for 300 days, much longer than the actual freezing period in the annual soil surface temperature cycle. The predicted heave after shorter times such as 150 or 180 days may be read from the graphs of Figure VI.9 in which the results are presented.

2. A series consisting of a frost heave prediction at each sample point using an initial temperature of $+0.5^{\circ}\text{C}$ throughout the soil profile. A stationary surface boundary temperature of -2°C was used. The freezing process was continued for 300 days. The results of these runs are presented in the graphs of Figure VI.10.
3. A series consisting of a frost heave prediction at each sample point using an initial temperature of $+0.5^{\circ}\text{C}$ throughout the soil profile. A stationary surface boundary temperature of -10°C was used. The freezing process was continued for 300 days. The results of these runs are presented in the graphs of Figure VI.11.
4. A series consisting of a frost heave prediction at each sample point using an initial temperature of $+0.5^{\circ}\text{C}$ throughout the soil profile. The surface boundary temperature used decreased from -1°C to -15°C in ± 2.5 months (75 days) and remained stationary at -15°C for a month (35 days). This surface temperature pattern resembles closely the freezing part of the annual surface temperature wave at a site 180km south of Norman Wells (see Figure V.3). The results of these runs are given in graphs in Figure VI.12.

Figure VI.9 Predictions of frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+2^{\circ}\text{C}$ at the soil surface to $+6^{\circ}\text{C}$ from 70 to 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C .

Figure VI.10 Predictions of frost heave near Norman Wells, using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C .

Figure VI.11 Predictions of frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a stationary surface boundary temperature of -10°C .

Figure VI.12 Predictions of frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100cm below the soil surface, and a surface temperature decreasing over 75 days from -1°C to -15°C , and remaining stationary for 35 days at -15°C .

Figures VI.9 and VI.10 can be compared to evaluate the effect of the initial temperature on the frost heave predictions. Lowering the initial temperatures does not have a significant effect on the total heave. In the case of the lower initial temperatures, the frost heave is only slightly faster during the first 50 days, when the total heave is not very large.

Figures VI.10 and VI.11 can be compared to assess the effect of the value of the stationary surface boundary temperature. This effect is much stronger than the change in initial temperatures. The frost heave in many runs triples when the surface boundary temperature is a stationary -10°C instead of -2°C . The shape of the curves changes from a smooth parabola, rotated 90° , to an undifferentiable curve consisting of two almost linear parts.

Figures VI.11 can be compared to Figure VI.12 to evaluate the effect of a dynamic surface temperature boundary. When the surface boundary temperature decreased linearly from -1°C to -15°C over 75 days, less heave was predicted than in the case that the surface boundary temperature was a stationary -2°C . This might be an effect of a deeper penetration of the freezing front and the larger amount of latent heat that has to be removed from the soil.

VI.7 Spatial variability of predicted frost heave and differential frost heave

The predicted frost heave showed, as expected, the largest spatial variability on the scales of the smallest and largest sampling spacings. The larger variability on the scale of the small sampling spacing is a result of a random variation in input parameters. The larger

variability on the scale of the large sampling spacing is attributed to the increase in variability of the input parameters, compared to that on the scale of the 10m sample spacing.

Figure VI.13 Predictions of differential frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+2^{\circ}\text{C}$ at the soil surface to $+6^{\circ}\text{C}$ from 70 to 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C .

Figure VI.14 Predictions of differential frost heave near Norman Wells, using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a stationary surface boundary temperature of -2°C .

Figure VI.15 Predictions of differential frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a stationary surface boundary temperature of -10°C .

Figure VI.16 Predictions of differential frost heave near Norman Wells using the adjusted model of Wang and initial soil temperatures ranging from $+0.5^{\circ}\text{C}$ at the soil surface to $+0.5^{\circ}\text{C}$ at 100 cm below the soil surface, and a surface temperature decreasing over 75 days from -1°C to -15°C , and remaining stationary for 35 days at -15°C .

In Figures VI.13 to VI.16 it can be seen that the largest difference in predicted frost heave does not always occur at the end of the heaving period, but, due to the manner in which the latent heat release is modelled, sometimes occurs in the first 50 days of the freezing process. Such a prediction suggests that the largest differential stresses on structures could occur in an early phase of the heaving process.

In some cases the predicted differential heave increases with time (e.g. 28-29, 33-34 and 36-37) in Figure VI.14), in other cases the predicted differential heave decreases with time (e.g. 10-11, 14-15 and 38-39 in Figure VI.14). On the smallest scale of Figure VI.15, the sample points with a 100m spacing, the 34-35 differential shows a sudden increase in differential heave due to the fact that at sample point 35 after 260 days of freezing with a surface boundary temperature of -10°C the base of the warmest ice lens is located in a more frost susceptible layer than at sample point 34. Because the segregation temperature, freezing temperature and hydraulic conductivity of the soil at sample point 34 are almost the same as those of the soil at sample point 36. The 34-35 differential is almost symmetrical to the 35-36 differential with the X-axis as axis of symmetry.

VI.8 Sources of error in the frost heave prediction process

VI.8.1 Introduction

Because frost heave involves transport of heat and mass, with phase change, and a spatial and temporal variation of stress, affecting a natural spatial variation of soils, whose temperature and moisture content change with time, the prediction process is a difficult process with many opportunities for errors to enter the prediction process, which might be magnified in the prediction procedure. Although some of these errors may be very small, in the worst case, when all errors deviate from the accurate value in the same direction, many small errors that propagate through the prediction procedure may result in a substantial error in the prediction. Because the modelling involved many empirically determined parameters with unknown errors, processed in a numerical calculation, a proper error analysis was not carried out. In this subchapter possible sources of errors in all phases of the prediction process are mentioned.

VI.8.2 Errors in input data

A number of errors may result from the extraction of soil samples and the determination of input data for the frost heave prediction model. Errors arising from the soil sample extraction procedure may have resulted from using an auger instead of a more

sophisticated coring tool. Other errors may be generated due to misrepresentation of the sample as representative for the soil layer at a sampling point. The number of samples representing a soil layer at a sample point was one. An average of more samples could have diminished the chance for such a misrepresentation to occur.

The standard grain size distribution analysis in the laboratory may generate an error. The potential error in the amount of the fractions resulting from grain size distribution analysis by sieving and sedimentation, can be as large as 30% of the measured amount, but are usually smaller(determined by an unpublished study at the Agricultural University Wageningen, The Netherlands).

Instead of being measured, almost all input parameters for the modified frost heave prediction model of Wang are determined by approximation methods using basic grain size, water content and bulk density data (see Chapter IV). Errors result from inaccurate approximations.

VI.8.3 Errors in the modelling phase

Errors are made if reality is misrepresented by the physical model. Such a misrepresentation occurs when phenomena that occur in reality are not incorporated in the prediction model and assumptions are made that are generalizations of the real situation. Several of these misrepresentations and assumptions which may lead to errors are listed below.

In the model used, the freezing temperature is a constant, characteristic for each soil, and assigned to each soil based on its textural class. The freezing temperatures used are not based on extensive experimental data, and within each soil textural class, the freezing temperature might still vary. Nixon suggests that the freezing temperature can be calculated by the use of the unfrozen water content curve. This procedure results in a larger variation of freezing temperatures.

The assumption that the segregation temperature, the temperature at the base of the warmest ice lens is a constant, characteristic for each soil, is a simplification that is not based on empirical evidence. According to Nixon's model, the predicted temperature at the base of the warmest ice lens changes with time.

The average hydraulic conductivity of the freezing fringe in the modified frost prediction model of Wang is a constant, characteristic for each soil. For a particular soil, each freezing fringe has the same average hydraulic conductivity at all times. It is more likely, that this hydraulic conductivity changes with time, because as the freezing front penetrates and parts of the freezing fringe become cooler, the amount of pore ice in the freezing fringe will increase, and the amount of unfrozen water will decrease. Because the void ratio probably decreases with increasing depth, the hydraulic conductivity of the freezing fringe might also be a function of depth below the soil surface.

It should be mentioned again that the frost heave predicted by the modified model of Wang is almost linear with the average hydraulic conductivity of freezing fringe, which happens to be the parameter with the largest variation with temperature and in space, and

which is also the least known parameter. The predicted value of the frost heave at a sample point is basically dependent on the accuracy of the average hydraulic conductivity of the freezing fringe.

The water pressure in the freezing fringe is assumed to be linear. Nixon uses an exponential function for the water pressure in the freezing fringe. Neither of these two relations could be experimentally proven, because the freezing fringe is too thin to measure pore water pressures in it.

According to the modified frost heave prediction model of Wang, the water flow in the freezing fringe is not influenced by osmotic pressure differences, generated by exclusion of salts from the freezing soil moisture. Such an exclusion of salts could be dealt with using a version of the Clausius-Clapeyron equation that has been adjusted using the Pitzer equations (Grant 1995).

According to the modified frost heave prediction model of Wang, water freezes at two points: all water flowing into the freezing fringe freezes at the base of the warmest ice lens; and all pore water freezes in situ at the freezing front. If this was reality, the pore water would not be able to flow to the base of the warmest ice lens, because all the pores would be filled with ice, or the freezing front would at least block any water flow into the freezing fringe. In reality ice penetrates first in the largest pores, and subsequently into smaller pores. The water freezes gradually in each pore, decreasing the thickness of the unfrozen water films with time, and because there is always a thin unfrozen water film surrounding the particles, water transport in the freezing fringe is always possible.

The latent heat of segregation freezing and the latent heat of in situ freezing are released as thermal load vectors at 2 to 4 nodes in the soil profile. This causes the temperature at these nodes to rise high above 0°C (e.g. 11°C). In reality such a situation never occurs.

The soil layers A, B and C are assumed to be homogeneous, while soils have an intrinsic heterogeneity. Some soils are more homogeneous than others. The sedimentary medium which deposited the material or the type of parent material in which they formed determine the initial heterogeneity, and several natural processes may increase or decrease the heterogeneity of the soil (e.g. leaching, oxidation, cryoturbation).

Since the model is a one dimensional model heat and water flow only occur in the z-direction. In reality water and heat will also be transferred in x and y-directions. This error will be less serious for the prediction of frost heave in flat terrain than for the prediction of frost heave in sloping terrain, where substantial lateral water flow is expected.

VI.8.4 Errors in numerical approximation procedures

Errors in the numerical approximation procedure may be divided into two groups: round-off errors and truncation or discretization errors (Özisik 1994). The latter type of errors are differences between the exact solution of the process governing differential equation and the solution of the numerical procedure without the round-off error. A sound numerical solution procedure is consistent, stable and convergent:

consistency: As the step size (i.e. Δz , Δt) becomes extremely small, the numerical approximation should become close to the derivatives. This requirement implies that the truncation error should vanish as the step size vanishes. Then the numerical approximation is said to be consistent with the original differential equation.

stability: A solution scheme is said to be stable if the errors involved in numerical computations are not amplified without bounds as the numerical calculation progresses. The numerical procedure employed to solve the modified frost heave prediction model of Wang uses weighting functions that make the procedure unconditionally stable.

convergence: The numerical solution is said to be convergent if the numerical solution approaches the exact solution of the problem as the number of iterations increases. The numerical approximation employed to solve the modified frost heave prediction model of Wang did not use any iterations.

VI.8.5 Total error

All errors made in some phase of the prediction process propagate and may amplify through the prediction process and eventually final round off errors may be generated. Ideally as many errors are made that increase the predicted value as are made that decrease the predicted value. However, in the worst case the errors work mostly to increase or decrease the

predicted value, which could result in a large total error. Despite the fact that a large potential for error exists, it remains useful, even necessary, to predict frost heave along linear structures in frost susceptible regions in order to determine potential trouble spots and avoid disasters. If the predictions are not accurate, they might still give very useful indications, which may be used by the construction industry. It is very useful to know the location of the key areas, that should be monitored for their potential differential frost heave.

VI.9 Discussion and summary

In this chapter the adjusted frost heave prediction model of Wang is operationalized by introduction of methods to choose the freezing temperature, segregation temperature, and hydraulic conductivity of the freezing fringe. The three methods introduced all depend on the textural class of the soil sample. This rather crude assignment of values can be refined e.g. by an analog between a soil that is drying, and in which the hydraulic conductivity decreases rapidly with decreasing water pressure, and a soil that is freezing and in which the hydraulic conductivity is decreasing rapidly with decreasing temperature. If a method could be found to assess the relationship between the two processes, the much-needed $K_w(T)$ -values could be acquired. Such an advance would allow for substantial improvement of frost heave models.

The surface boundary temperature has a much larger influence on the prediction of frost heave than the initial temperature profile. According to the predictions the largest differential

frost heave and accompanying largest stress on built structures could occur in an early phase of the freezing process, but in some cases the differential heave showed a gradual increase.

It is difficult to evaluate the accuracy of the predicted differential frost heave, without any measurements of the (differential) frost heave, and with the knowledge that a one dimensional frost heave model was used, and only thermodynamic considerations. However, the predictions may serve as indicators of the sites with the largest potential for differential frost heave to cause problems for a built structure.

VII Summary and Conclusions

VII.1 Fundamentals of ground freezing and frost heave

The soil freezing process involves the transport of heat and water. As a result, stress conditions may develop in the soil leading to frost heave in the freezing zones and consolidation in underlying unfrozen zones. Spatially differential frost heave may have severe consequences for built structures such as roads, houses, pipelines, and runways. In order to limit the differential frost heave damage to these structures, measures need to be taken, especially to limit differential frost heave, and selection of the appropriate measures requires reliable predictions of frost heave. Reliable predictions can only result from accurate modelling of soil freezing and frost heave, based on detailed observations in the field and in laboratories, and the use of accurate input data.

The considerable amount of information about freezing and frost heaving of soils that has been obtained over the last 70 years from observations and experiments in the field and the laboratory, has led to a fundamental understanding of ground freezing and frost heave. In a freezing saturated soil, water in individual pores behind an advancing freezing front freezes gradually over a range of temperatures and the unfrozen water content in a freezing soil decreases along a temperature gradient from the freezing front into the freezing soil. The water content gradient is accompanied by a free energy gradient, along which water is drawn into and through the freezing soil. At some distance behind the freezing front, pressures generated by the

formation of pore ice may overcome the cohesive and frictional forces between soil particles and enable the formation of an ice lens. The zone between the location of the formation of this ice lens and the freezing front has been called the frozen fringe (Miller 1972).

Frost heave results from the freezing of water at three locations in a freezing soil: in pores between the freezing front and the warmest ice lens (Miller's "frozen fringe"); at the warmest ice lens by freezing of water that has been drawn from warmer soil; and in the almost completely frozen zone. The three components of frost heave resulting from the freezing of water in these three locations are respectively called primary, secondary and tertiary heave. Secondary heave contributes the largest portion to the total frost heave in frost susceptible soils. Instead of Miller's "frozen fringe" in this thesis the term "freezing fringe" has been used to emphasize that this soil zone is the active engine of secondary frost heave.

The water pressure difference over the soil below the warmest ice lens is the most important component of the free energy gradient driving the flow of water to the warmest ice lens. Because the change in water pressure from deep in the soil to the freezing front is normally very small compared to the water pressure change over the freezing fringe, the former pressure gradient is usually ignored as a driving component of water flow to the warmest ice lens. Solutes in the pore water are excluded from the freezing and concentrate in the unfrozen water, creating an osmotic pressure gradient over the freezing fringe that contributes to the free energy gradient drawing water flow to the warmest ice lens.

Horizontal and vertical cracks may form in freezing soils, and consolidation of the soil may occur. Nixon, Morgenstern, Sayles, Fish and Ladanyi have started experimental and

theoretical research into these matters. Despite the available information, there is a need for a systematic description of where and when cracks form and consolidation occurs, a conceptual model of the mechanism of the formation of cracks; and the reasons for consolidation, based on laboratory and field experiments. Furthermore, how these processes influence the thermal, hydraulic and strength properties of the soil needs to be determined. A serious limitation on knowledge about frost heave derives from the freezing fringe being very thin, varying from 0.1 mm (in coarse-grained soils) to only a few cm (in very fine-grained soils). As a consequence it is impossible to measure water pressure and pressure gradients within the freezing fringe using tensiometers, and to derive stress values and hydraulic conductivities at various depths in the freezing fringe.

VII.2 Modelling of the frost heave process

Models are needed to predict the result of a process from a given set of input data, and are used to develop a better understanding of the process and the effects of the process on built structures. Among the wide variety of model types, a deterministic model of a process in nature aims at a detailed physics-based description of all known sub-processes and interactions of variables involved in the process, and when an effort is made to limit the number of assumptions the model is called mechanistic. A complete mechanistic model predicting the differential frost heave describes a coupled flow of heat and water and the evolving stress situation in three dimensions. Only conductive heat transfer is described (using Fourier's law and the law of heat conservation) because convection and diffusion do

not contribute significantly towards the flow of heat in the soil. The necessary incorporation of the latent heat of freezing in the numerical approximation presents a major complication which may cause a large increase in calculated soil temperature, which does not occur in reality. Mathematicians and mechanical engineers have not found a solution to this problem. The water flow is usually considered to be a laminar flow, so that eddy currents and friction can be ignored, and Darcy's law may be used. Some models have incorporated a flow of ice by creep or regelation (O'Neill and Miller 1985, Black 1995).

Usually it is assumed that the free energy of water and ice in the freezing fringe are equal, a situation that is called thermodynamic equilibrium. Frost heave prediction models based on this assumption use a thermodynamic equilibrium equation, the Clausius-Clapeyron equation, through which the heat and water flow are coupled. The existence of thermodynamic equilibrium in the freezing fringe remains unproved. Frost heave models that do not assume thermodynamic equilibrium in the freezing fringe have to use irreversible thermodynamics, which complicates the modelling process. No complete frost heave models based on irreversible thermodynamics have been proposed.

The Clausius-Clapeyron equation is a differential equation that describes the work that needs to be done to bring pore water from one state to another. In order to use the equation in a frost heave prediction model, the differential equation has to be solved by integration. It seems that incorrect boundary values often are used unintentionally in this integration, resulting in an error in calculated relationships among the equation that relates the temperatures, water pressure and ice "pressure". Any frost heave prediction model should state clearly what is considered to

be the driving gradient of the water flow to the ice lens, and use the boundary values appropriate for this driving gradient.

Because frost heave prediction involves calculation with pressures of liquids (scalars) and stresses of solids (tensors), the modelling is complex, and errors are easily made. In modelling the mechanics and dynamics of ground freezing and frost heave the following terms should be used: total and normal stresses of soil particles and ice (solid phase), and pressure of water and air (fluid phase). Often, the term “ice pressure”, by which usually a normal one dimensional normal stress in ice is meant, is used instead of “stress in ice”, e.g. in the Clausius-Clapeyron equation. A strict interpretation of the term “ice pressure” would be, that the ice can’t absorb any stress, e.g. provided by a (normal) load. Because the offsetting of (normal) load, is the essence/reason for the formation of ice lenses and the occurrence of secondary heave, it is clear that “stress in ice” is involved and that the term “ice pressure” may be misleading (This matter was addressed by O’Neill in 1983 and repeated here because the inappropriate terminology remains in common use.) Moreover, according to Vignes and Dijkema (1974) and Takagi (1980), the pressure of water in unfrozen water films close to the particle surfaces is non-isotropic and complex. The international terminology of this subject has to be improved.

Few currently available models include a modelling of the stress situation in the soil, unless a soil-structure interaction is part of model. The discussion of the modelling of the mechanics of frost heave has not received the same attention as the modelling of the thermodynamics of frost heave. With a fundamental understanding of the thermodynamics of frost heaving available, future modelling efforts should concentrate on the mechanics.

To determine where and when an ice lens forms, and when its growth stops, an additional requirement, called a frost heave criterion, has to be satisfied in the frost heave model. A frost heave criterion describes the location and/or time of formation of a new ice lens. The frost heave criterion of Gilpin (1979, 1980), for cohesive soils, and of Miller (1978, 1980), for non-colloidal soils, are considered to be the most sensible of the currently available frost heave criteria, and a combination of them would be applicable to all soils.

To account for the exclusion of solutes from ice during freezing an osmotic pressure gradient may be added to the Darcy equation and the Causius-Clapeyron equation (e.g. Padilla and Villeneuve 1991, Grant 1995). Frost heave prediction models that include water redistribution and further freezing in the frozen zone (tertiary heave) have not been proposed yet. O'Neill and Miller (1985), Shah (1992) and Black (1995) have proposed models that, in addition to a flow of heat and water from the unfrozen to the frozen soil, incorporate a flow of ice into the unfrozen soil. These models are called rigid ice models.

The soil freezing and frost heave prediction model should be able to accommodate a multilayer soil that experiences warming as well as cooling phases. For an accurate simulation of ground freezing, a cyclic process, it should be possible to include not only the freezing part but the thawing part as well.

By comparing frost heave predictions with frost heave measurements it is possible to validate a prediction model, provided that the input parameters for the model are accurate. Because input parameter accuracy currently gives problems, the validity of a frost heave prediction model is usually assessed by comparing its predictions with measured frost heave

and predictions by other models using the same input data. In the latter case not only the output but also the models should be compared. Two methods of comparing frost heave prediction models are examined: a very detailed method in a multiple-page table format is introduced, and a concise single-page table comparison, highlighting the differences between two or more models is used. The sensitivity of the frost heave prediction to common input parameters of both models may be compared. Comparisons of the output are difficult when variables are used in one model that are not used by the other model.

Two examples of deterministic frost heave prediction models were compared: a model developed by Wang (1994) and a model developed by Nixon (1991). Wang's model allows a non-linear soil temperature profile, and assumes an almost constant characteristic water pressure difference over the freezing fringe, constant segregation and freezing temperatures, and a constant average hydraulic conductivity of the freezing fringe. In situ freezing of pore water happens at the freezing front and an ice lens forms where the temperature is equal to the segregation temperature. Nixon's model assumes: a linear soil temperature profile; a non-linear water pressure in the freezing fringe; that in situ freezing of water happens throughout the freezing fringe; and that an ice lens forms where the ice pressure in the freezing fringe overcomes the overburden and the cohesion forces between soil particles.

While the non-linear temperature profile in the model of Wang seems to have more potential than the linear temperature profile in the model of Nixon, the model of Wang can not predict the rhythmic banding of increasingly thicker ice lenses as the freezing front

advances into the soil, while Nixon's model does. Both models have problems with accounting for the latent heat released in the freezing fringe, but Wang's model suffers the most from this problem. Both models are one-layer soil models, and for that reason Wang's model was modified to allow for multiple layers. The predictions with Wang's model were hypersensitive to most of its input parameters, especially to the hydraulic conductivity. The predictions with Nixon's model were less sensitive to most of its input parameters.

VII.3 Determination and spatial variability of input data for frost heave prediction models

Accurate measurement of most input parameters (especially, the physical properties of the freezing fringe) for frost heave prediction models is currently impossible. Therefore, a complete validation of frost heave modelling is impossible until adequate techniques have been developed to measure these input parameters. Konrad and Morgenstern (1980, 1981) proposed a procedure to avoid the use of parameters that are difficult to measure. Centrifuge experiments combined with scaling methods (Greene 1993, Black 1995) may provide solutions until more sophisticated measurement methods are developed which can provide more detailed information about the properties of the freezing fringe. In this thesis existing and new methods are proposed for obtaining the input data for frost heave prediction models from grain size data, water content and bulk density of soil.

Because the thermal, hydraulic and strength properties of the soil are dependent on the unfrozen water and ice content of the soil, these properties change during freezing, during the following thaw, and during and after repetitions of these freeze-thaw cycles. Although it might be interesting to know if the unfrozen thermal, hydraulic, and mechanical properties of a soil change from one freeze-thaw cycle to the next cycle, for frost heave prediction it is most important to know what the values of these properties are during the freezing phase. Processes such as consolidation and the formation of cracks also change the thermal, hydraulic and strength properties of the soil. These phenomena are not well described yet. Most frost heave prediction models do not take these changes in properties during the freezing process into account.

The cause of differential frost heave is the spatial variability of thermal, hydraulic and strength properties of the soil. Differential frost heave may be at its maximum at the boundaries of surficial geology units, since the boundaries are identified on the basis of changes in properties. Substantial differential heaves (up to 15.5 cm in 274 days freezing) were measured at the boundary between two surficial geology units in the Caen experiments of the GSL (Smith and Williams 1995).

The most important input parameter in the frost heave prediction models of Wang and Nixon is the hydraulic conductivity, because the predictions by the models are most sensitive to the hydraulic conductivity (Chapter III), and because the temporal and spatial variations in hydraulic conductivity are larger than the variation in the other input parameters. The hydraulic conductivity, which may be determined from grain size distribution, bulk density

and water content data, which are relatively easy to acquire, drops several orders of magnitude during the freezing of soil (Chapter IV).

Based on the standard deviation and coefficient of variation of the input parameters of the frost heave prediction models (Chapter V.2), it may be concluded that the hydraulic conductivity and apparent heat capacity of the soil display the largest spatial variability. The more data available, the better the spatial variability of frost heave may be predicted. However, the more data, the higher is the cost of data acquisition. The optimal amount of data needed within a certain surficial geology unit, which captures most of the variability of the measured parameter, while keeping the costs of data acquisition within limits, may be determined by analysis of semivariograms. A new semivariogram model is introduced to determine the optimal sampling distance at which soil properties should be obtained. In the till overlain by lacustrine material, colluvium and an organic-rich top layer, along the east bank of the Mackenzie River between Norman Wells and Wrigley, an optimal sampling distance to determine the hydraulic conductivity and the apparent heat capacity is between 200 and 300m (Chapter V.3).

VII.4 Limitations to the accuracy of frost heave predictions: Problems in frost heave modelling and the nature of their solutions

Accurate modelling of frost heave is extremely difficult, because the frost heave process involves coupled flow of heat, water and (according to some scientists) ice, through a

porous medium (soil) that may consist of three phases. During frost heave, phase change occurs and the stress situation in the soil changes. Because it is impossible to incorporate all aspects of the freezing process, only the most significant aspects are taken into account, and some inaccuracy is allowed.

This thesis has demonstrated that variation in parameters such as the hydraulic conductivity and the segregation temperature have a substantial effect on the predicted (and differential) frost heave. Therefore it is important that adequately accurate values for these parameters are obtained. Unfortunately values for these parameters are difficult to obtain, because current techniques to measure these parameters accurately do not exist. The values of most of the variables in the frost heave process are strongly dependent on temperature and, as a result, change considerably during the process. The more input data obtained and the more reliable these input data, the more opportunity to obtain accurate predictions. However, the more input data, the higher the costs. An analysis using semivariograms is introduced as a compromise between accuracy and expense.

Predicted differential frost heave is a direct result of the spatial variability of the input parameters in frost heave prediction models, and may cause problems for built structures as pipelines, railroads, highways and airstrips. This spatial variability will be the largest at the boundaries of surficial geological units, but differential heave within the unit may be predicted accurately enough if the input parameters are determined at sample points with a maximum spacing equal to the range of the most important input parameters such as the hydraulic conductivity and the segregation temperature and unfrozen water content.

VII.5 Summary of contributions of this thesis

1. The fundamentals of frost heave have been described in detail for interested readers new to the field. To date, most reviews have been written for scientists and engineers who are familiar with frost heaving.
2. The question of the origin of the driving gradient for water flow towards the base of the warmest ice lens is addressed.
3. The fundamentals of computer modelling processes are introduced for the benefits of scientists and others who do not have a background in the mathematical or computer modelling.
4. An existing frost heave prediction model of Wang (1994) was modified to enable it to handle layered soils.
5. A detailed check-list method for comparison of frost heave prediction models is introduced.
6. Empirical methods to determine values for the unfrozen water content, apparent heat capacity and hydraulic conductivity from easily measured soil properties are proposed. Existing methods for obtaining other essential input parameters of frost heave models are identified.
7. Spatial statistics are introduced to the field of frost heave modelling and the benefits these techniques may have for construction in remote areas, such as most of the cold regions of this world, are discussed.

8. A new semivariogram model is introduced to the field of spatial statistics.
9. The existing frost heave prediction model of Wang is operationalized by introduction of methods to choose the three critical soil parameters: freezing temperature, segregation temperature and average hydraulic conductivity of the freezing fringe.
10. The thesis presents the most detailed description to date of the whole frost heave prediction process, from the acquisition of field data, through the modelling phase, to the calculation procedure and includes an identification of possible sources of errors.

VII.6 Suggestions for further research

The detailed analysis undertaken in this thesis has highlighted certain deficiencies in current knowledge of the frost heave process and of frost heave prediction. Among these are:

- a) The need for further experimental research on structural changes during soil freezing. Systematic description of the consequences of aggregate formation and deformation is needed, with emphasis on the manner in which changes in these soil structural elements change the soil properties that are input parameters in frost heave prediction models.
- b) The need for experimental and theoretical research on fissuring and consolidation during freezing and thawing and during progressive freeze-thaw cycles. These processes must be better understood and described more systematically in order to be incorporated into frost heave prediction models.

- c) Further work on modelling of frost heaving is needed. Several approaches might prove useful.
- i) A modular modelling of ground freezing and frost heave could be developed, with options that can be used where and when needed or ignored to save computing time or because of a lack of input data.
 - ii) More emphasis should be placed on two and three dimensional modelling of soil freezing and frost heave.
 - iii) Frost heave prediction models should be able to predict frost heave with a surface temperature wave as a boundary condition. Therefore thaw and thaw settlement has to be modelled. When the frost heave model incorporates these issues, freeze-thaw cycling can be modelled, climatic trends could be incorporated and effects of local or global warming or cooling could be predicted e.g. an intensifying or decreasing amount of (differential) frost heave. A good prediction of thawing may help in the development of methods to minimize the effects of pipeline buoyancy problems, as have been met at Norman Wells.
 - iv) The Clausius-Clapeyron equation may be extended (Grant 1995) to incorporate salt exclusion from ice during freezing.
 - v) Optimization of the amount of input data for frost heave prediction for the design of pipelines might be improved by the opportunity to obtain large amounts of input data using inexpensive cheap airborne and radarsat

imagery and spatial variability analysis of these data.

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		Wang	Nixon
I. PURPOSE OF THE MODEL			
1	emphasis on practical use ("practical")	X	X
2	emphasis on theoretical description of process ("scientific")		

PROCESS FEATURE IN ONE DIMENSIONAL MODELLING OF FROST HEAVE			
II. THERMAL ASPECTS			
3	heat flow by conduction	X	X
4	heat flow by conduction and convection		
5	heat flow by conduction, convection and radiation		
6	heat flow by conduction, convection, radiation, advection and diffusion		
7	thermal conductivity constant with temperature	X h,c	X h,f,c
8	thermal conductivity a function of temperature		
9	thermal conductivity independent on water content	X	X
10	thermal conductivity dependent on water content		
11	specific thermal conductivities constant with temperature	X	X
12	specific thermal conductivities a function of temperature		
13	heat capacity constant with temperature	X h,c	
14	heat capacity a function of temperature		
15	specific heat capacities constant with temperature	X	
16	specific heat capacities a function of temperature		
17	latent heat constant with temperature		
18	latent heat a function of temperature	X	X
19	specific latent heat constant with temperature	X	X
20	specific latent heat a function of temperature		
21	latent heat released at base of warmest ice lens	X	X
22	latent heat released at base of ice lenses in frozen soil		
23	latent heat released at freezing front	X	
24	latent heat released throughout the freezing fringe		X
25	steady state heat flow	X	X
26	unsteady heat flow		
27	stationary atmospheric boundary temperature	X	X
28	dynamic atmospheric boundary temperature		X
29	temperature wave at the surface		

Appendix A Table for evaluation of frost heave models (part 1)
h=hot, f=freezing fringe, c=cold

		Wang	Nixon
III. THERMODYNAMIC ASPECTS			
30	water flow uncoupled from heat flow		
31	coupled heat and water flow	X	X
32	thermodynamic equilibrium	X	X
33	thermodynamic disequilibrium		

IV. HYDRAULIC ASPECTS			
34	laminar water flow	X	X
35	laminar flow and resistance		
36	laminar flow, resistance and inertia		
37	water content constant in freezing fringe	X	
38	water content variable in freezing fringe		X
39	hydraulic conductivity constant in freezing fringe	X	
40	hydraulic conductivity variable in freezing fringe		X
41	hydraulic conductivity independent on water content	X	
42	hydraulic conductivity dependent on water content		X
43	osmotic pressure "potential"		
44	double layer pressure "potential"		
45	air pressure "potential"		
46	pneumatic pressure "potential"		
47	electric pressure "potential"		
48	salt exclusion at the freezing front		
49	initial water expulsion at the freezing front		X
50	dessication below the freezing fringe		
51	regelation in freezing fringe		
52	regelation in frozen soil		

V. FORMATION OF ICE AND RESULTING FROST HEAVE			
53	freezing in situ, primary heave (in situ) (transient)		X
54	freezing at new (warmest) ice lens, secondary heave (segregation) (steady state)	X	X
55	additional freezing in frozen soil, tertiary heave (continuing) (long term)		
56	total heave	X	X
57	frost heave criterion		X
58	ice lens by ice lens modelling	X	X
59	rhythmic ice banding		X

		Wang	Nixon
VI. MECHANICAL ASPECTS			
60	stress development not coupled to heat and water flow	X	X
61	stress development coupled to heat and water flow		
62	effective shear strength in fringe has to be overcome for new ice lens formation		
63	cohesive strength in fringe has to be overcome for new ice lens formation		X
64	increase of effective shear strength below freezing front due to dessication		
65	thermal cracking		
66	consolidation on approach of the freezing front		
67	overburden effect of the soil on growth ice lens	X	X
68	applied external loads	X	X
69	anchoring vertically, cohesion and shear stress		
70	elastic behaviour	X	
71	plastic behaviour, failure		
72	viscous behaviour, creep	X	
73	predetermined failure planes		

VII. LONG TERM FEATURES			
74	changes in soil structure and composition in time		
75	thawing, latent heat consumption		
76	thaw consolidation		
77	cyclic freezing and thawing		
78	change of thermal and hydraulic properties after each cycle		

VIII. SPATIAL FEATURES			
79	one layer soil system	X	X
80	multilayer soil system		

Appendix A Table for evaluation of frost heave models (part 3)

		Wang	Nixon
VIII SPATIAL FEATURES			
VIII.1 One dimensional frost heave prediction model			
VIII.1.1 One layer soil profile			
1	freezing temperature is constant within layer	X	
2	freezing temperature varies within layer		X
3	segregation temperature is constant within layer	X	
4	segregation temperature varies within layer		X
5	thermal conductivity is constant within layer	X	
6	thermal conductivity varies within layer		
7	volumetric heat capacity is constant within layer	X	
8	volumetric heat capacity varies within layer		
9	porosity is constant within layer	X	X
10	porosity varies within layer		
11	unfrozen water content is constant within layer		
12	unfrozen water content varies within layer		X
13	hydraulic conductivity is constant within layer	X	
14	hydraulic conductivity varies within layer		X
15	bulk density is constant within layer	X	
16	bulk density varies within layer		X
17	internal overburden pressure is constant with depth	X	
18	internal overburden pressure increases with depth		
19	separation pressure is constant with depth		X
20	separation pressure increases with depth		

Appendix A Table for evaluation of frost heave models (part 4)

		Wang	Nixon
VIII.1.2 Multilayer soil profile			
1	freezing temperature is constant within each layer		
2	freezing temperature varies within each layer		
3	segregation temperature is constant within each layer		
4	segregation temperature varies within each layer		
5	thermal conductivity is constant within each layer		
6	thermal conductivity varies within each layer		
7	volumetric heat capacity is constant within each layer		
8	volumetric heat capacity varies within each layer		
9	porosity is constant within each layer		
10	porosity varies within each layer		
11	unfrozen water content is constant within each layer		
12	unfrozen water content varies within each layer		
13	hydraulic conductivity is constant within each layer		
14	hydraulic conductivity varies within each layer		
15	bulk density is constant within each layer		
16	bulk density varies within each layer		
17	internal overburden pressure is constant with depth		
18	internal overburden pressure increases with depth		
19	separation pressure is constant with depth		
20	separation pressure increases with depth		

Appendix A Table for evaluation of frost heave models (part 5)

		Wang	Nixon
VIII.2 Two dimensional frost heave prediction model			
VIII.2.1 One layer soil profile			
VIII.2.1.1 Thermal and Hydraulic parameters			
1	freezing temperature is constant within each layer		
2	freezing temperature varies within each layer		
3	freezing temperature is constant in x-direction		
4	freezing temperature varies in x-direction		
5	segregation temperature is constant within each layer		
6	segregation temperature varies within each layer		
7	segregation temperature is constant in x-direction		
8	segregation temperature varies in x-direction		
9	thermal conductivity is constant within layer		
10	thermal conductivity varies within layer		
11	thermal conductivity is constant in x-direction		
12	thermal conductivity varies in x-direction		
13	volumetric heat capacity is constant within layer		
14	volumetric heat capacity varies within layer		
15	volumetric heat capacity is constant in x-direction		
16	volumetric heat capacity varies in x-direction		
17	porosity is constant within layer		
18	porosity varies within layer		
19	porosity is constant in x-direction		
20	porosity varies in x-direction		
21	unfrozen water content is constant within layer		
22	unfrozen water content varies within layer		
23	unfrozen water content is constant in x-direction		
24	unfrozen water content varies in x-direction		
25	hydraulic conductivity is constant within layer		
26	hydraulic conductivity varies within layer		
27	hydraulic conductivity is constant in x-direction		
28	hydraulic conductivity varies in x-direction		

Appendix A Table for evaluation of frost heave models (part 6)

		Wang	Nixon
VIII.2.1.2 Mechanical parameters			
29	bulk density is constant within layer		
30	bulk density varies within layer		
31	bulk density is constant in x-direction		
32	bulk density varies in x-direction		
33	internal overburden pressure is constant with depth		
34	internal overburden pressure increases with depth		
35	internal overburden pressure is constant in x-direction		
36	internal overburden pressure varies in x-direction		
37	external overburden pressure is constant in x-direction		
38	external overburden pressure varies in x-direction		
39	separation pressure is constant with depth		
40	separation pressure increases with depth		
41	separation pressure is constant in x-direction		
42	separation pressure varies in x-direction		

Appendix A Table for evaluation of frost heave models (part 7)

		Wang	Nixon
VIII.2.2 Multilayer soil profile			
VIII.2.2.1 Thermal and Hydraulic parameters			
1	freezing temperature is constant within each layer		
2	freezing temperature varies within each layer		
3	freezing temperature is constant in x-direction		
4	freezing temperature varies in x-direction		
5	segregation temperature is constant within each layer		
6	segregation temperature varies within each layer		
7	segregation temperature is constant in x-direction		
8	segregation temperature varies in x-direction		
9	thermal conductivity is constant within each layer		
10	thermal conductivity varies within each layer		
11	thermal conductivity is constant in x-direction		
12	thermal conductivity varies in x-direction		
13	volumetric heat capacity is constant within each layer		
14	volumetric heat capacity varies within each layer		
15	volumetric heat capacity is constant in x-direction		
16	volumetric heat capacity varies in x-direction		
17	porosity is constant within each layer		
18	porosity varies within each layer		
19	porosity is constant in x-direction		
20	porosity varies in x-direction		
21	unfrozen water content is constant within each layer		
22	unfrozen water content varies within each layer		
23	unfrozen water content is constant in x-direction		
24	unfrozen water content varies in x-direction		
25	hydraulic conductivity is constant within each layer		
26	hydraulic conductivity varies within each layer		
27	hydraulic conductivity is constant in x-direction		
28	hydraulic conductivity varies in x-direction		

Appendix A Table for evaluation of frost heave models (part 8)

		Wang	Nixon
VIII.2.2.2 Mechanical parameters			
29	bulk density is constant within each layer		
30	bulk density varies within each layer		
31	bulk density is constant in x-direction		
32	bulk density varies in x-direction		
33	internal overburden pressure is constant with depth		
34	internal overburden pressure increases with depth		
35	internal overburden pressure is constant in x-direction		
36	internal overburden pressure varies in x-direction		
37	external overburden pressure is constant in x-direction		
38	external overburden pressure varies in x-direction		
39	separation pressure is constant with depth		
40	separation pressure increases with depth		
41	separation pressure is constant in x-direction		
42	separation pressure varies in x-direction		

Appendix A Table for evaluation of frost heave models (part 9)

		Wang	Nixon
VIII.3 Three dimensional frost heave prediction model			
VIII.3.1 One layer soil profile			
VIII.3.1.1 Thermal parameters			
1	freezing temperature is constant within each layer		
2	freezing temperature varies within each layer		
3	freezing temperature is constant in x-direction		
4	freezing temperature varies in x-direction		
5	freezing temperature is constant in y-direction		
6	freezing temperature varies in y-direction		
7	segregation temperature is constant within each layer		
8	segregation temperature varies within each layer		
9	segregation temperature is constant in x-direction		
10	segregation temperature varies in x-direction		
11	segregation temperature is constant in y-direction		
12	segregation temperature varies in y-direction		
13	thermal conductivity is constant within layer		
14	thermal conductivity varies within layer		
15	thermal conductivity is constant in x-direction		
16	thermal conductivity varies in x-direction		
17	thermal conductivity is constant in y-direction		
18	thermal conductivity varies in y-direction		
19	volumetric heat capacity is constant within layer		
20	volumetric heat capacity varies within layer		
21	volumetric heat capacity is constant in x-direction		
22	volumetric heat capacity varies in x-direction		
23	volumetric heat capacity is constant in y-direction		
24	volumetric heat capacity varies in y-direction		

Appendix A Table for evaluation of frost heave models (part 10)

		Wang	Nixon
VIII.3.1.2 Hydraulic parameters			
25	porosity is constant within layer		
26	porosity varies within layer		
27	porosity is constant in x-direction		
28	porosity varies in x-direction		
29	porosity is constant in y-direction		
30	porosity varies in y-direction		
31	unfrozen water content is constant within layer		
32	unfrozen water content varies within layer		
33	unfrozen water content is constant in x-direction		
34	unfrozen water content varies in x-direction		
35	unfrozen water content is constant in y-direction		
36	unfrozen water content varies in y-direction		
37	hydraulic conductivity is constant within layer		
38	hydraulic conductivity varies within layer		
39	hydraulic conductivity is constant in x-direction		
40	hydraulic conductivity varies in x-direction		
41	hydraulic conductivity is constant in y-direction		
42	hydraulic conductivity varies in y-direction		

Appendix A Table for evaluation of frost heave models (part 11)

		Wang	Nixon
VIII.3.1.2 Hydraulic parameters			
25	porosity is constant within layer		
26	porosity varies within layer		
27	porosity is constant in x-direction		
28	porosity varies in x-direction		
29	porosity is constant in y-direction		
30	porosity varies in y-direction		
31	unfrozen water content is constant within layer		
32	unfrozen water content varies within layer		
33	unfrozen water content is constant in x-direction		
34	unfrozen water content varies in x-direction		
35	unfrozen water content is constant in y-direction		
36	unfrozen water content varies in y-direction		
37	hydraulic conductivity is constant within layer		
38	hydraulic conductivity varies within layer		
39	hydraulic conductivity is constant in x-direction		
40	hydraulic conductivity varies in x-direction		
41	hydraulic conductivity is constant in y-direction		
42	hydraulic conductivity varies in y-direction		

Appendix A Table for evaluation of frost heave models (part 12)

		Wang	Nixon
VIII.3.1.3 Mechanical parameters			
43	bulk density is constant within layer		
44	bulk density varies within layer		
45	bulk density is constant in x-direction		
46	bulk density varies in x-direction		
47	bulk density is constant in y-direction		
48	bulk density varies in y-direction		
49	internal overburden pressure is constant with depth		
50	internal overburden pressure increases with depth		
51	internal overburden pressure is constant in x-direction		
52	internal overburden pressure varies in x-direction		
53	internal overburden pressure is constant in y-direction		
54	internal overburden pressure varies in y-direction		
55	external overburden pressure is constant in x-direction		
56	external overburden pressure varies in x-direction		
57	external overburden pressure is constant in y-direction		
58	external overburden pressure varies in y-direction		
59	separation pressure is constant with depth		
60	separation pressure increases with depth		
61	separation pressure is constant in x-direction		
62	separation pressure varies in x-direction		
63	separation pressure is constant in y-direction		
64	separation pressure varies in y-direction		

Appendix A Table for evaluation of frost heave models (part 13)

		Wang	Nixon
VIII.3.2 Multilayer soil profile			
VIII.3.2.1 Thermal parameters			
1	freezing temperature is constant within each layer		
2	freezing temperature varies within each layer		
3	freezing temperature is constant in x-direction		
4	freezing temperature varies in x-direction		
5	freezing temperature is constant in y-direction		
6	freezing temperature varies in y-direction		
7	segregation temperature is constant within each layer		
8	segregation temperature varies within each layer		
9	segregation temperature is constant in x-direction		
10	segregation temperature varies in x-direction		
11	segregation temperature is constant in y-direction		
12	segregation temperature varies in y-direction		
13	thermal conductivity is constant within each layer		
14	thermal conductivity varies within each layer		
15	thermal conductivity is constant in x-direction		
16	thermal conductivity varies in x-direction		
17	thermal conductivity is constant in y-direction		
18	thermal conductivity varies in y-direction		
19	volumetric heat capacity is constant within each layer		
20	volumetric heat capacity varies within each layer		
21	volumetric heat capacity is constant in x-direction		
22	volumetric heat capacity varies in x-direction		
23	volumetric heat capacity is constant in y-direction		
24	volumetric heat capacity varies in y-direction		

Appendix A Table for evaluation of frost heave models (part 14)

		Wang	Nixon
VIII.3.2.2 Hydraulic parameters			
25	porosity is constant within each layer		
26	porosity varies within each layer		
27	porosity is constant in x-direction		
28	porosity varies in x-direction		
29	porosity is constant in y-direction		
30	porosity varies in y-direction		
31	unfrozen water content is constant within each layer		
32	unfrozen water content varies within each layer		
33	unfrozen water content is constant in x-direction		
34	unfrozen water content varies in x-direction		
35	unfrozen water content is constant in y-direction		
36	unfrozen water content varies in y-direction		
37	hydraulic conductivity is constant within each layer		
38	hydraulic conductivity varies within each layer		
39	hydraulic conductivity is constant in x-direction		
40	hydraulic conductivity varies in x-direction		
41	hydraulic conductivity is constant in y-direction		
42	hydraulic conductivity varies in y-direction		

Appendix A Table for evaluation of frost heave models (part 15)

		Wang	Nixon
VIII.3.2.3 Mechanical parameters			
43	bulk density is constant within each layer		
44	bulk density varies within each layer		
45	bulk density is constant in x-direction		
46	bulk density varies in x-direction		
47	bulk density is constant in y-direction		
48	bulk density varies in y-direction		
49	internal overburden pressure is constant with depth		
50	internal overburden increases with depth		
51	internal overburden pressure is constant in x-direction		
52	internal overburden pressure varies in x-direction		
53	internal overburden pressure is constant in y-direction		
54	internal overburden pressure varies in y-direction		
55	external overburden pressure is constant in x-direction		
56	external overburden pressure varies in x-direction		
57	external overburden pressure is constant in y-direction		
58	external overburden pressure varies in y-direction		
59	separation pressure is constant with depth		
60	separation pressure increases with depth		
61	separation pressure is constant in x-direction		
62	separation pressure varies in x-direction		
63	separation pressure is constant in y-direction		
64	separation pressure varies in y-direction		

Appendix A Table for evaluation of frost heave models (part 16)

		Wang	Nixon
I HEAT TRANSFER			
1	linear temperature profile		X
2	nonlinear temperature profile	X	
3	analytical solution of heat transfer equation in space		X
4	numerical approximation of heat transfer equation in space	X	
5	analytical solution of heat transfer equation in time		X
6	numerical approximation of heat transfer equation in time	X	

I.4.1 Finite difference formulation of heat transfer equation in space			
1	central differences		
2	forward differences		
3	backward differences		
4	Crank-Nicholson central differences		

I.4.2 Finite element formulation of heat transfer equation in space			
I.4.2.1 Weighted Residual Method			
1	Galerkin WRM	X	
2	Collocation WRM		
3	Least squares WRM		
4	Subdomain WRM		

Appendix B Evaluation of the numerical procedure used in the frost heave prediction model (part 1)

		Wang	Nixon
I.4.2.2 Dimension, shape and nodes of each element in FEM			
ONE DIMENSION			
1	regular 2 noded bar elements	X	
2	parabolic 3 noded bar elements		
3	bending 2 noded bar elements		
4	bending parabolic 3 noded bar elements		
5	number of elements	40	
TWO DIMENSIONS			
1	triangular 3 noded elements		
2	parabolic 6 noded triangular elements		
3	bending parabolic 6 noded triangular elements		
4	quadrilateral 4 noded elements		
5	parabolic 8 noded quadrilateral elements		
6	bending 4 noded quadrilateral elements		
7	bending parabolic 8 noded quadrilateral elements		
8	Lagrangian 9 noded element		
9	size or range of sizes of 2D elements		
10	number of elements		
THREE DIMENSIONS			
1	tetrahedral 4 noded elements		
2	parabolic 8 noded tetrahedral elements		
3	bending 4 noded tetrahedral elements		
4	bending parabolic 8 noded tetrahedral elements		
5	an 8 noded brick element		
6	parabolic 20 noded brick elements		
7	parabolic 27 noded brick elements		
8	bending 8 noded brick elements		
9	bending parabolic 27 noded brick elements		
10	size or range of sizes of 3D elements		
11	number of elements		

I.6.1 Finite difference formulation of heat transfer equation in time			
1	central differences		
2	forward differences		
3	backward differences		
4	Crank-Nicholson central differences		

Appendix B Evaluation of numerical procedure used in frost heave prediction model (part 2)

		Wang	Nixon
II WATER FLOW			
1	linear water pressure in the freezing fringe	X	
2	nonlinear water pressure in the freezing fringe		X
3	analytical solution of water flow equation in space	X	
4	numerical approximation of water flow equation in space		X
5	analytical solution of water flow equation in time	X	
6	numerical approximation of water flow equation in time		X

II.4.1 Finite difference formulation of water flow equation in space			X
1	central differences		X
2	forward differences		
3	backward differences		
4	Crank-Nicholson central differences		

II.4.2 Finite element formulation of water flow equation in space			
II.4.2.1 Weighted Residual Method			
1	Galerkin WRM		
2	Collocation WRM		
3	Least squares WRM		
4	Subdomain WRM		

Appendix B Evaluation of the numerical procedure used in the frost heave prediction model (part 3)

		Wang	Nixon
II.4.2.2 Dimension, shape and nodes of each element in FEM			
ONE DIMENSION			
1	regular 2 noded bar elements		
2	parabolic 3 noded bar elements		
3	bending 2 noded bar elements		
4	bending parabolic 3 noded bar elements		
5	number of elements		
TWO DIMENSIONS			
1	triangular 3 noded elements		
2	parabolic 6 noded triangular elements		
3	bending parabolic 6 noded triangular elements		
4	quadrilateral 4 noded elements		
5	parabolic 8 noded quadrilateral elements		
6	bending 4 noded quadrilateral elements		
7	bending parabolic 8 noded quadrilateral elements		
8	Lagrangian 9 noded element		
9	size or range of sizes of 2D elements		
10	number of elements		
THREE DIMENSIONS			
1	tetrahedral 4 noded elements		
2	parabolic 8 noded tetrahedral elements		
3	bending 4 noded tetrahedral elements		
4	bending parabolic 8 noded tetrahedral elements		
5	an 8 noded brick element		
6	parabolic 20 noded brick elements		
7	parabolic 27 noded brick elements		
8	bending 8 noded brick elements		
9	bending parabolic 27 noded brick elements		
10	size or range of sizes of 3D elements		
11	number of elements		

II.6.1 Finite difference formulation of water flow equation in time			
1	central differences		X
2	forward differences		
3	backward differences		
4	Crank-Nicholson central differences		

Appendix B Evaluation of numerical procedure used in frost heave prediction model (part 4)

		Wang	Nixon
III STRESS SITUATION			
1	linear stress situation in the freezing fringe		
2	nonlinear stress situation in the freezing fringe		
3	analytical solution of stress situation in space		
4	numerical approximation of stress situation in space		
5	analytical solution of stress situation in space		
6	numerical approximation of stress situation in time		

III.4.1 Finite difference formulation of stress situation in space			
1	central differences		
2	forward differences		
3	backward differences		
4	Crank-Nicholson central differences		

III.4.2 Finite element formulation of stress situation in space			
III.4.2.1 Ritz Method			

Appendix B Evaluation of numerical procedure used in frost heave prediction model (part 5)

		Wang	Nixon
III.4.2.2 Dimension, shape and nodes of each element in FEM			
ONE DIMENSION			
1	regular 2 noded bar elements		
2	parabolic 3 noded bar elements		
3	bending 2 noded bar elements		
4	bending parabolic 3 noded bar elements		
5	number of elements		
TWO DIMENSIONS			
1	triangular 3 noded elements		
2	parabolic 6 noded triangular elements		
3	bending parabolic 6 noded triangular elements		
4	quadrilateral 4 noded elements		
5	parabolic 8 noded quadrilateral elements		
6	bending 4 noded quadrilateral elements		
7	bending parabolic 8 noded quadrilateral elements		
8	Lagrangian 9 noded element		
9	size or range of sizes of 2D elements		
10	number of elements		
THREE DIMENSIONS			
1	tetrahedral 4 noded elements		
2	parabolic 8 noded tetrahedral elements		
3	bending 4 noded tetrahedral elements		
4	bending parabolic 8 noded tetrahedral elements		
5	an 8 noded brick element		
6	parabolic 20 noded brick elements		
7	parabolic 27 noded brick elements		
8	bending 8 noded brick elements		
9	bending parabolic 27 noded brick elements		
10	size or range of sizes of 3D elements		
11	number of elements		

III.6.1 Finite difference formulation of stress situation in time			
1	central differences		
2	forward differences		
3	backward differences		
4	Crank-Nicholson central differences		

Appendix B Evaluation of numerical procedure used in frost heave prediction model (part 6)