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Measurement of the Tau Lepton Lifetime at OPAL

Anna C. Janissen

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfilment
of the requirements for the degree of Doctor of Philosophy

Department of Physics
Carleton University
Ottawa-Carleton Institute for Physics
Ottawa, Ontario

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submitted by
A.C. Janissen, B.Sc., M.Sc.

in partial fulfilment of the requirements
for the degree of Doctor of Philosophy

Chair, Department of Physics

Thesis Supervisor

External Examiner

Carleton University

Date: May 16, 1993

Abstract

This thesis describes a new measurement of the tau (τ) lepton lifetime using two statistically independent techniques each associated with one of the two principle decay topologies of the τ to one and three charged particles, respectively. The measurement was performed with data collected in 1990 and 1991 at the OPAL detector at the LEP e^+e^- colliding beam accelerator at CERN. The LEP collider operates at a significantly higher energy than previous e^+e^- colliders. This presents a new experimental opportunity to investigate the physics associated with the τ lepton in general and the τ lifetime in particular.

The tau lepton is one of three similar electron-like particles: e , μ , and τ . These leptons exhibit a hierarchy of masses with: $m_e < m_\mu < m_\tau$. While the electron is stable, the μ and the τ are unstable and decay via the weak interaction charged current. It is a fundamental feature of the standard model of fundamental particles and their interactions that this weak charged current has exactly the same strength for each of the three leptons; a phenomenon called lepton universality.

The τ lifetime, τ_τ , can be related to the μ lifetime, τ_μ , and the average leptonic branching ratio, $\text{BR}(\tau \rightarrow \ell \nu \bar{\nu})$, by:

$$\tau_\tau = \tau_\mu \frac{G_\mu G_\tau}{G_\tau G_\ell} \left(\frac{m_\mu}{m_\tau} \right)^5 \text{BR}(\tau \rightarrow \ell \nu \bar{\nu}) R$$

where R is a calculable factor to account for phase space and radiative corrections, and G_ℓ is the Fermi effective coupling strength of the weak charged current to the lepton ℓ . Lepton universality implies that $G_\mu = G_e = G_\tau$. The experimentally measured τ lifetime, together with the measurements of the other quantities in the above relation, can be interpreted as a direct test of lepton universality.

The τ lifetime measured with each of the two independent techniques is:

$$\tau_1 = 296.4 \pm 7.1(\text{stat.}) \pm 3.8(\text{sys.}) \text{ fs}$$

$$\tau_3 = 286.3 \pm 7.4(\text{stat.}) \pm 5.2(\text{sys.}) \text{ fs.}$$

The systematic uncertainties for each of the analyses are uncorrelated and the two measurements can be directly combined in a weighted average. The OPAL τ lifetime result for the 1990+1991 data sample is:

$$\tau_\tau = 291.9 \pm 5.1(\text{stat.}) \pm 3.1(\text{sys.}) \text{ fs.}$$

When combined with current world averages of the other experimental quantities listed above, this measurement of the τ lifetime is consistent with the hypothesis of lepton universality.

Acknowledgements

This work would never have been possible without the years of hard work and dedication of the hundreds of physicists, engineers, technicians and students in the OPAL collaboration. Many thanks go to these people, many of whom I have never even met.

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*Heureusement qu'il existe
des gens qui regardent les gens qui passent.*

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Chapter 1

Introduction

The electron is the most ubiquitous of the basic building blocks of nature. However, it is only the most familiar member of a family of similar particles called leptons. There are, in fact, three electron-like particles: the electron (e), the muon (μ), and the tau (τ) lepton. Each has an electric charge of -1 , and is a point particle, *i.e.*, no internal structure has yet been observed [1]. One observable difference between these three leptons is their mass: $m_e < m_\mu < m_\tau$. While the electron is stable, the μ is unstable and decays to an electron and two neutrinos with a mean lifetime of about 2 microseconds. The μ lifetime is one of the most precisely measured quantities in particle physics with an experimental uncertainty of less than 0.002%. The τ lepton is also unstable and, because of its larger mass, has more decay modes available to it than the μ . It can decay to either of the other two charged leptons or to the lightest quarks. The mean τ lifetime is about 0.3 picoseconds. Both the μ and the τ decay via the weak interaction in a manner exactly analogous to radioactive β -decay of nuclei. The current model of fundamental particles and their interactions makes a prediction for the average τ lifetime. Alternatively, the experimentally measured τ lifetime can be interpreted as a test of one of the most fundamental features of the model which is that all three leptons feel the weak force with exactly the same strength.

This thesis describes a new measurement of the τ lifetime using two standard, and statistically independent, techniques each associated with one of the two principle decay topologies of the τ . The measurement was performed with the OPAL detector at the LEP e^+e^- colliding beam accelerator at CERN. The LEP collider operates at a

significantly higher energy than previous e^+e^- colliders. This presents a new experimental environment in which to investigate the physics associated with the τ lepton in general and the τ lifetime in particular. This higher energy at LEP makes the selection of the $e^+e^- \rightarrow \tau^+\tau^-$ final state virtually background free. The large τ production cross section and the small size of the e^+e^- interaction region, compared to previous colliders, further contribute to the favourable conditions for studying the τ at LEP. All of these factors, together with the excellent quality of the OPAL detector, combine to make this the most precise measurement of the τ lifetime to date with an uncertainty of 2%.

The thesis is organized as follows: Chapter 1 gives a general introduction of the theoretical and experimental environment in which the measurement has taken place. Chapter 2 describes, in more detail, the properties of the τ lepton including the prediction of the τ lifetime, specific techniques for measuring the lifetime, and a summary of other lifetime measurements prior to LEP. Chapter 3 describes the OPAL detector and data collection system with special emphasis on those aspects relevant to the τ lifetime analysis. Chapter 4 outlines the calibration procedures for the two vertex detectors in OPAL which are the most critical detector components for this measurement. Chapter 5 describes the Monte Carlo programs which were used to generate simulated $e^+e^- \rightarrow \tau^+\tau^-$ data, and chapter 6 describes the selection of $e^+e^- \rightarrow \tau^+\tau^-$ events from the real data. The two methods used by OPAL to determine the τ lifetime are discussed in chapters 7 and 8. The combination of the results of these two analyses is presented in chapter 9 where the implications of the measurement for the Standard Model of particle physics are discussed.

1.1 The Standard Model

The Standard Model of particle physics is a description of the fundamental particles in nature and how they interact via 3 of the 4 fundamental forces: the strong force, the weak force, the electromagnetic force, and the gravitational force. Gravity, which couples to mass, is very much weaker than the other three and is negligible in interactions between light particles.

Only 12 fundamental, point matter particles have been observed thus far.¹ (Actually, only indirect evidence exists for the top quark (t) and the tau neutrino (ν_τ).)

$$\begin{array}{ccc} u & c & t \\ d & s & b \end{array} \left. \vphantom{\begin{array}{ccc} u & c & t \\ d & s & b \end{array}} \right\} \text{Quarks} \\ \begin{array}{ccc} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{array} \left. \vphantom{\begin{array}{ccc} \nu_e & \nu_\mu & \nu_\tau \\ e & \mu & \tau \end{array}} \right\} \text{Leptons} \quad (1.1)$$

These particles all have spin angular momentum $\frac{1}{2}$; they are fermions. They interact through the exchange of spin 1 gauge bosons: the massless photon for the electromagnetic interaction, the massive W^{+-} and Z^0 particles for the weak interaction, and the massless gluons for the strong interaction.

Electromagnetism and the weak force are combined into a single description based on an $SU(2) \times U(1)$ gauge symmetry group. Transformations under the $U(1)$ gauge group correspond to rotations of the wave function phase by an amount which may vary from point to point. A familiar example of objects transforming under the $SU(2)$ group is the case of spin $\frac{1}{2}$ angular momentum vectors in quantum mechanics where there are two possible states: up or down. With respect to the $SU(2)$ part of the electroweak force, the fundamental particles in the model interact as doublets:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix} \\ \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \quad (1.2)$$

where ν_e and e are the "up" and "down" states of an $SU(2)$ doublet called weak isospin. It is observed that only left handed particles (particles with their spin pointing in the opposite direction to their momentum) transform in this manner under $SU(2)$. Right handed particles are classified as singlets.

Not all of the quarks listed in equation 1.1 are eigenstates of the electroweak

¹Each particle also has an associated antiparticle.

force. The d -type quarks mix via the Cabbibo-Kobayashi-Maskawa matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (1.3)$$

It is these primed states which are the weak eigenstates, as shown in equation 1.2. The elements of the CKM matrix are not predicted by the Standard Model and must be input from experiments.

This description, based on a perfect $SU(2) \times U(1)$ symmetry, is characterized by four massless propagators: three for the $SU(2)$ part, W_1 , W_2 and W_3 , and one for the $U(1)$ part of the interaction, B . There are two coupling constants in the model, g and g' , which are effectively the “charges” of the two parts of the electroweak interaction. This theory describes only massless fermions. This is obviously not in agreement with what is seen experimentally since the quarks and leptons do have mass. However, adding mass terms for the fermions to the Lagrangian breaks the $SU(2) \times U(1)$ gauge invariance.

The way around this difficulty is to postulate the existence of an additional field, called the Higgs field. This field takes the form of a complex scalar doublet thus giving four extra degrees of freedom. The physical vacuum is the ground state for all of the fields and the expectation value of a quantum field is normally zero in its ground state. However, there are circumstances where this need not be so [3]. The Higgs mechanism of spontaneous symmetry breaking occurs when the vacuum expectation value of the Higgs field is not zero. This results in some curious effects on the nature of the fields described in the theory. Three of the four degrees of freedom of the Higgs doublet are absorbed as mass degrees of freedom in the propagators. This produces the massive charged propagators of the weak interaction, W^{+-} , and the similarly massive neutral propagator, Z^0 . The fourth propagator is left massless, as we would like to be the case for the photon. In addition, the fermion couplings to the Higgs field take the form of mass terms. So the interaction with the Higgs field generates fermion masses. The values of these masses, however, are not predicted by the theory and must be input from experiment. The fourth degree of freedom of the Higgs doublet, which was not absorbed, should be observable as a neutral scalar particle. The theory places no constraint whatever on the mass of this Higgs boson. The

present experimental situation is that the Higgs boson has been searched for and excluded up to a mass of $60.0 \text{ GeV}/c^2$ at the 95% confidence level [4].

The two parts of this gauge theory can be associated to the two parts of the experimentally observed electroweak interaction. The $SU(2)$ part describes the weak interaction where, for example, a τ^- lepton is turned into a ν_τ with the emission of a W^- boson. The $U(1)_{em}$ part is the familiar gauge theory of quantum electrodynamics. This gauge symmetry is another manifestation of the gauge freedom seen in classical electrodynamics where the gradient of an arbitrary function may be added to the potential without altering the field equations. The electroweak Lagrangian, then, displays both the gauge invariance seen in classical electrodynamics, and invariance under rotations of weak isospin.

In addition to the fermion masses and the elements of the CKM matrix, the Standard Model of electroweak interactions needs three more parameters from experiment to fix its properties. These correspond to the degrees of freedom arising from the coupling constants g and g' , and the vacuum expectation value of the Higgs field. Which three parameters are chosen is a matter of convenience as relations exist between many possible choices.

The two neutral propagators before the symmetry breaking combine in a linear combination to form the two neutral physical propagators: the Z^0 and the photon, A . This can be written as

$$\begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z^0 \\ A \end{pmatrix}. \quad (1.4)$$

The parameter in the rotation matrix, θ_W , is called the Weinberg angle. Before radiative corrections the following relations hold:

$$\sin^2 \theta_W = \frac{e^2}{g^2} = 1 - \frac{e^2}{g'^2} = 1 - \frac{m_W^2}{m_Z^2}$$

where e is the electric charge of an electron. The parameters e and $\sin^2 \theta_W$ are popular choices for two of the three necessary input parameters. The mass of the W boson is related to the Higgs vacuum expectation value, v , by

$$m_W = \frac{1}{2}gv.$$

Then

$$m_Z = \frac{m_W}{\cos \theta_W}$$

can be chosen as the third parameter. Another choice of three parameters takes again the very well measured electric charge, the mass of the Z^0 (which has been very precisely measured at LEP), and also the Fermi coupling constant G_F which is known to a very high degree of precision from experimental measurements of the μ lifetime. This is related to the other parameters by the relation

$$G_F = \frac{g^2}{4\sqrt{2} m_W^2}$$

Once three suitable parameters have been chosen, all the other possible parameters are predicted by the above relations, and experimental measurements of the quantities can be used as sensitive tests of the Standard Model. It should be noted that all of the above relations are only for the basic interactions before corrections are applied for radiative effects. Radiative corrections can significantly modify the values of the parameters and some of the stated relations also become modified. It is still true that only three parameters are required, but their selection now becomes more subtle in that if the parameter was measured at an energy scale significantly different from the one under study, it may undergo large radiative corrections which must be taken into account.

Weak interactions which involve the exchange of a charged boson, W^+ or W^- , are called charged current processes. Some examples are radioactive β -decay, and the decays of the μ and the τ . Weak interactions which involve the exchange of the Z^0 boson are called neutral current processes. Neutrino elastic scattering, $\nu_\mu e \rightarrow \nu_\mu e$, is an example of a neutral current process. Since neutrinos do not have electric charge, they feel only the weak portion of the electroweak force. Another example of an electroweak interaction is electron-positron annihilation and subsequent production of a fermion-antifermion pair. This interaction, by contrast, proceeds through both the electromagnetic and the neutral weak currents: $e^+e^- \rightarrow \gamma, Z^0 \rightarrow f\bar{f}$. The Z^0 exchange mode becomes dominant as the energy of the e^+e^- beams approaches the mass of the Z^0 boson.

Some of the most sensitive tests of the Standard Model have been performed at the LEP accelerator at CERN which collides e^+e^- pairs with sufficient energy to produce

real Z^0 bosons. The variation of the e^+e^- interaction cross section near the Z^0 mass has allowed the mass of the Z^0 to be measured very precisely: $91.187 \pm 0.007 \text{ GeV}/c^2$ [5]. For comparison, the mass of the W boson is measured at proton-antiproton colliders and is known to a much smaller precision: $80.22 \pm 0.26 \text{ GeV}/c^2$ [1]. Once the model has been fixed with the appropriate parameters, as discussed above, the properties of Z^0 decays observed at LEP can be compared to Standard Model predictions. Some of the other measurements, in addition to m_{Z^0} , which are made at LEP include: i) the width of the Z^0 resonance, $\Gamma_Z = 2.492 \pm 0.007 \text{ GeV}$ [5], ii) the cross section of Z^0 decays to quarks at the peak of the Z^0 resonance, $\sigma_{had}^{peak} = 41.16 \pm 0.18 \text{ nb}$, iii) the ratio of the number of Z^0 decays to quarks to the number of leptonic decays ($Z^0 \rightarrow \ell^+\ell^-$), $\Gamma_q/\Gamma_\ell = 20.85 \pm 0.07$, and iv) the forward-backward asymmetry, $A_{FB} = 0.0154 \pm 0.0027$, which is an asymmetry in the number of fermions produced in the direction of the initial e^- (forward hemisphere) with respect to the e^+ (backward hemisphere) direction. This asymmetry arises from the interference between the two modes of e^+e^- annihilation.

The Standard Model predictions for these quantities are a function of the masses of the two particles which are as yet unobserved: the Higgs boson and the t quark. The Z^0 resonance parameters and the forward-backward asymmetry can be used to predict the mass of the t quark because, although the t quark is too massive to be produced directly at LEP, it does contribute to the radiative corrections to Z^0 decays. Radiative corrections are much less sensitive to the mass of the Higgs and experimental bounds on m_H are usually obtained with direct searches [6]. The t quark mass determined by the above measurements is $m_t = 145_{-27}^{+24} \text{ GeV}/c^2$ [5]. As was mentioned previously, the Standard Model provides no constraint on the mass of the Higgs boson. Allowing a range in Higgs boson mass of $60\text{-}1000 \text{ GeV}/c^2$, the predictions of the Standard Model are in full agreement with all of the available measurements from experiments at LEP and elsewhere.

The strong force in particle physics is only associated with the quarks. For example, the quarks inside a proton are bound together by the strong force. The gauge theory of the strong interaction is called quantum chromodynamics, or QCD. The gauge group of QCD is $SU(3)$ where now each quark field transforms as an $SU(3)$ triplet. The different states within the triplet are given the name colour charge. Each quark comes in 3 different colour charges often labelled by the analogous ordinary spectral colours:

red, green and blue. The propagators of the strong force are an octet of massless bosons called gluons which carry colour charge themselves. This part of the Standard Model is determined by only one parameter, α_s , the strong coupling constant.

One characteristic of quarks is that they have never been observed in isolation. In the gauge theory description, this is associated with the phenomenon of asymptotic freedom where the magnitude of the parameter α_s approaches zero as the quarks move close together and becomes infinitely large as the quarks are separated. If an interaction leads to two quarks being pulled apart, the increasing strength of the coupling constant with quark separation will lead to the creation of an additional quark-antiquark pair being produced from the vacuum. The net result is that the quarks only exist in bound states with other quarks and antiquarks. States composed of three quarks are called baryons, with protons and neutrons being the most familiar examples. Quark-antiquark states are called mesons. Some examples of mesons are: π , K , and ρ . Baryons and mesons are collectively known as hadrons. Thus, high energy particle interactions involving quarks are characterized by sprays of hadrons, called jets, in the final state which result from the "hadronization" of the original quark which took part in the interaction. It is possible that one or more gluons are radiated in the final state leading to a reaction of the type: $Z^0 \rightarrow q\bar{q}g$. Gluons also hadronize into jets so it is possible to have more than two jets in a single e^+e^- collision. Thus, it is more appropriate to speak of e^+e^- collisions with quarks in the final state as $e^+e^- \rightarrow \text{hadrons}$, or "multihadronic" events.

The number of hadrons per jet increases logarithmically with the centre-of-mass energy of the interaction. At LEP energies, the average number of charged hadrons per $e^+e^- \rightarrow \text{hadrons}$ event is about 20. It is possible, however, for a final state quark-antiquark pair to combine into a single meson rather than separating and hadronizing individually. This process is extremely rare for quarks produced in $e^+e^- \rightarrow q\bar{q}$ interactions at LEP energies, but is common in W decays to the lightest quark doublet at the energy scale of a τ decay. Thus, in hadronic τ decays the final state is characterized by a single or small numbers of mesons.

The particles listed in equation 1.2 exhibit an interesting symmetry. They are grouped into three columns, or generations, of particles. Each generation has an "up"

quark with an electric charge of $+\frac{2}{3}$, a "down" quark with a charge of $-\frac{1}{3}$, a neutral, massless neutrino, and a lepton of charge -1 . The generations differ in the masses of the charged members with the first generation being less massive than the second generation which is, in turn, less massive than the third. The number of Standard Model generations assuming massless or very light neutrinos¹ has been measured at LEP to be 2.99 ± 0.04 [1], and a fourth generation has been excluded for neutrino masses up to $m_\nu = m_Z/2$.

A further curiosity is noticed in the interactions of the lepton sector. Leptons are observed to obey a conservation rule: each member of the SU(2) doublet is assigned a number of $+1$, and antileptons have opposite lepton number to leptons. In the production and decay of leptons, the sum of the lepton numbers of the initial state particles must be conserved in the final state. The generation structure of equation 1.2 is emphasized by the observation that each generation has a separate lepton number which is conserved in all interactions.

Why this three generation structure exists is a mystery, but it is a feature of the Standard Model that the coupling constants for each generation are identical. In the lepton sector, this is given the name "lepton universality". Any deviation from this generational universality, in either the quark or the lepton sector, is an indication of some sort of new physics beyond the Standard Model. In the quark sector, because the particles which participate in the weak interaction are mixtures, observed differences between the generations can be explained using the mixing matrix which must be determined by experiment anyway. In the lepton sector, however, there is no observed mixing of the generations, and the measured rates of reactions can be directly related to the fundamental interaction strength in the model. The universality of the Z^0 coupling strength to the different leptons can be measured by counting the number of Z^0 decays to each type of charged lepton. The three leptonic partial widths of the Z^0 measured at LEP are: $\Gamma_e = 83.19 \pm 0.36$ MeV, $\Gamma_\mu = 83.69 \pm 0.53$ MeV, and $\Gamma_\tau = 83.49 \pm 0.63$ MeV [5]; these are equal within experimental uncertainties confirming the universality hypothesis. A direct measurement of the universality of the W coupling strength to the different leptons can be made using the measured branching ratios of τ decays to other leptons together

¹Here very light means $m_\nu \ll m_Z$.

with a measurement of the τ lifetime.

1.2 Measuring the τ Lifetime

All previous measurements of the τ lifetime have been made at e^+e^- colliding beam accelerators. In such accelerators the initial particles are not composite objects (unlike protons and antiprotons) which means that the momentum and energy of the initial state is well defined. The electrons and positrons have equal energies so the collisions take place in the centre-of-mass reference frame. Figure 1.1 shows a plot of the e^+e^- collision cross section as a function of the centre-of-mass energy. The large increase in cross section at about 91 GeV is associated with the production of Z^0 bosons. Approximately 70% of the time, the Z^0 decays into quark-antiquark pairs, about 10% into charged leptons (e^+e^- , $\mu^+\mu^-$ and $\tau^+\tau^-$) and about 20% into neutrino-antineutrino pairs.

The production of τ particles at e^+e^- colliders, then, is characterized by events which contain two taus, each of which has an energy equal to the energy of the original electron/positron beam. The taus are of opposite charge and are emitted back-to-back with momenta equal in magnitude. Figure 1.2 shows a schematic diagram of a typical $e^+e^- \rightarrow \tau^+\tau^-$ interaction showing both the τ production and the subsequent decay process.

All techniques to measure the τ lifetime use the τ decay products to measure the τ path length in the lab, l_0 . At LEP, this length is about 2 mm. This is related to the τ lifetime, τ_τ , by the relation:

$$l_0 = \beta\gamma c\tau_\tau.$$

where $\beta\gamma$ is the relativistic boost factor of the τ as seen in the lab. Most methods of measuring the τ lifetime also require knowledge of the position of the origin of the tau. The origin of a $\tau^+\tau^-$ pair is the e^+e^- interaction point. However, the e^+e^- beams do not collide at a single point, but rather the collisions are distributed over a finite interaction region. At LEP, this interaction region has an elliptical cross section of semi-major/semi-minor axes approximately 150/8 μm . It is usually assumed that all $\tau^+\tau^-$ pairs originate at the centre of the e^+e^- interaction region. Another piece of information which is required

Figure 1.1: A plot of the e^+e^- interaction cross section as a function of centre of mass energy. LEP was designed to have an energy at the " Z^0 pole" where the energy is sufficient to create real Z^0 bosons. Data from earlier e^+e^- colliders is also shown. The cross section for $e^+e^- \rightarrow \tau^+\tau^-$ is similar to that for $e^+e^- \rightarrow \mu^+\mu^-$.

Figure 1.2: A schematic diagram of a typical $e^+e^- \rightarrow \tau^+\tau^-$ interaction showing the characteristics of τ production and decay. Neutrinos are shown as dashed lines.

in several methods is the direction in which the τ was travelling when it decayed. This is impossible to reconstruct directly from the observed information in the event because each τ emits at least one neutrino when it decays in order to conserve lepton number. Neutrinos are not detected with general purpose particle detectors of the sort used at colliders, and the presence of at least two unobserved particles in every $e^+e^- \rightarrow \tau^+\tau^-$ event makes it impossible to fully reconstruct the momenta of the taus. One way around this difficulty is to approximate the τ direction using the decay products which are visible. A Monte Carlo simulation can then be used to correct for this approximation.

The high energy at LEP means that the decay length is longer than at previous colliders, but the decay products of the τ , which are spherically distributed in the τ rest frame, are boosted into a much narrower cone. A longer decay length is easier to measure, but the larger boost means that using the decay products to extract lifetime information becomes more difficult.

This brief summary presents the general characteristics common to all techniques to measure the τ lifetime, and hints at the limitations which each of the various methods attempts to overcome.

Chapter 2

The τ Lepton

2.1 Decays of the τ

Figure 2.1 shows the Feynman diagram of the decay of the τ lepton. The τ has enough mass to allow decays to either of the other charged leptons or to the lightest weak isospin quark doublet. Because the W couples to all of the $SU(2)$ doublets with the same

Figure 2.1: The Feynman diagram of the decay of the τ via the weak charged current. The quark d' is the weak eigenstate described in equation 1.3.

strength, the probability for the τ to decay to each of the final states in figure 2.1 should be approximately equal. However, quarks are available in three varieties of colour charge which increases the number of possible final states from three to five. This implies that the branching ratio for any one of the modes should be $\approx \frac{1}{5} = 20\%$. The experimentally measured value for $BR(\tau \rightarrow e \nu \bar{\nu})$ is $17.93 \pm 0.26\%$ [1]. The small difference between the prediction based on the number of final states and the measured value can be explained by the QCD dynamics of the hadronic final states [7].

The quarks in the final state of a hadronic τ decay combine to form mesons as described in section 1.1. The main mesons produced are π , K , ρ , K^* , and a_1 . The higher mass mesons decay into pions or kaons and their existence is observed only as a resonance in the τ decay product reconstructed mass spectrum. The main decay modes which are observed in an experimental detector are [1]:

$$\begin{array}{llll}
 \tau^- \rightarrow e^- \nu_e \nu_\tau & 17.9\% & & \\
 \tau^- \rightarrow \mu^- \nu_\tau \bar{\nu}_\mu & 17.5\% & & \\
 \tau^- \rightarrow \pi^- (K^-) \nu_\tau & 12.5\% & \left. \begin{array}{l} \text{"1-prong"} \\ 85.9\% \end{array} \right\} & \\
 \tau^- \rightarrow \pi^- (K^-) \pi^+ \nu_\tau & 24.4\% & & \\
 \tau^- \rightarrow \pi^- (K^-) \pi^0 \pi^0 \nu_\tau & 10.3\% & & \\
 \tau^- \rightarrow \pi^- (K^-) \pi^0 \pi^0 \pi^0 \nu_\tau & 2.7\% & & \\
 \tau^- \rightarrow \pi^- \pi^- \pi^+ \nu_\tau & 8.0\% & \left. \begin{array}{l} \text{"3-prong"} \\ 14.1\% \end{array} \right\} & \\
 \tau^- \rightarrow \pi^- \pi^- \pi^+ \pi^0 \nu_\tau & 5.2\% & &
 \end{array} \quad (2.1)$$

Experimentally, the outgoing π^- meson may or may not be distinguished from a K^- so the quantity which is often measured is the combined π/K branching ratio. The τ decays primarily into 1 or 3 charged particles. Since the τ leptons are produced in pairs at e^+e^- colliders, it follows that 74% of $e^+e^- \rightarrow \tau^+\tau^-$ events have a 1-1 topology where both tau decays produce a single charged particle, 24% have a 1-3 topology, and only 2% of tau events have a 3-3 topology. These topological branching ratios are important because various methods to measure the τ lifetime can only be applied to a specific topology and are often limited by the small number of events. It should be noted that the totals of the exclusive branching ratios are slightly less than the measured inclusive, or topological, branching ratios. This is known in the literature as the "1-prong discrepancy" or the "tau decay problem", and further details can be found in [1].

2.2 The τ Mass

The mass of the τ lepton has traditionally been determined by measuring the $\tau^+\tau^-$ production cross section in e^+e^- collisions as the energy of the e^+e^- beams are scanned across the $\tau^+\tau^-$ threshold energy near 3.56 GeV total energy. The most recently

compiled world average value is $m_\tau = (1784.1_{-1.6}^{+2.7}) \text{ MeV}/c^2$ [1], but this average is dominated by a DELCO measurement in 1978 of $(1783_{-4}^{+3}) \text{ MeV}/c^2$ [8].

The measurement has recently been repeated by the BES experiment at the Beijing Electron Positron Collider [9]. The experimental procedure was to use a data-driven energy scan around the threshold for $\tau^+\tau^-$ production. The e^+e^- beam energy was initially set at the world average τ mass value. Data was accumulated at that setting, and a new value for the τ mass was calculated using a maximum likelihood method which included the behaviour of the e^+e^- interaction cross section at the $\tau^+\tau^-$ threshold, the energy spread of the e^+e^- beams, and the experimental efficiency for detecting an $e^+e^- \rightarrow \tau^+\tau^-$ final state. The beam energy was then set to this new value and the procedure was repeated for a total of 10 steps and 7 $e^+e^- \rightarrow \tau^+\tau^-$ events recorded. The result was a mass of $m_\tau = (1776.9_{-0.5}^{+0.4} \pm 0.2) \text{ MeV}/c^2$.

This result has been confirmed by the ARGUS and CLEO collaborations. These two experiments operate at e^+e^- colliders near 10 GeV total energy, well above the $\tau^+\tau^-$ production threshold. They each use the detected τ decay products to construct a kinematic quantity which exhibits a threshold behaviour at the real τ mass. The ARGUS method uses the invariant mass reconstructed from the three pions in the decay $\tau^- \rightarrow \pi^-\pi^-\pi^+\nu_\tau$. The e^+e^- beam energy, and thus the τ energy, is known, so the energy of the ν_τ can be calculated. An approximation is made that the direction of the τ is along the 3-pion momentum vector: $\cos(p_\tau, p_{3\pi}) = 1$. With this approximation, a pseudo-mass of the τ can be reconstructed which exhibits a sharp edge at the real τ mass. The CLEO method used only those $e^+e^- \rightarrow \tau^+\tau^-$ events where both taus decay to a hadronic 1-prong final state. At least one of these τ decays was required to have a π^0 in the final state. For each charged hadron, a cone is drawn centered at the hadron and of half angle equal to the angle of the hadron with respect to the initial τ direction. The width of the cone, therefore, depends on the momentum given to the unseen neutrino and thus on the τ mass. Since the $\tau^+\tau^-$ pair were emitted back-to-back (in the absence of radiation), the true τ direction must lie on the intersection of one cone with the mirror image of the other cone. If one assumes a smaller τ mass, the two half-angles of the cones shrink and eventually one cone and the mirror image of the other cone just touch. This

point corresponds to the minimum kinematically allowed τ mass. The distribution of this mass also exhibits a sharp edge at the real τ mass.

The ARGUS and CLEO measurements are $(1776.3 \pm 2.4 \pm 1.4) \text{ MeV}/c^2$ and $(1777.6 \pm 0.9 \pm 1.6) \text{ MeV}/c^2$, respectively [10]. These three new measurements, together with the current average, give a new world average τ mass of $m_\tau = (1777.0 \pm 0.4) \text{ MeV}/c^2$. This is a shift of $7 \text{ MeV}/c^2$, or 2 standard deviations, from the previous world average

2.3 The τ Lifetime

A direct calculation of the τ lifetime requires knowledge of all of its partial decay widths:[†]

$$\frac{1}{\tau_\tau} = \Gamma_{total} = \sum_i \Gamma_i$$

where Γ_i are the partial widths for the processes listed in equation 2.1 as well as other rare decays and decays which, perhaps, have not yet been observed. This can be rewritten as:

$$\begin{aligned} \tau_\tau = \frac{1}{\Gamma_{total}} &= \frac{1}{\Gamma(\tau \rightarrow \ell \nu \bar{\nu})} \frac{\Gamma(\tau \rightarrow \ell \nu \bar{\nu})}{\Gamma_{total}} \\ &= \frac{1}{\Gamma(\tau \rightarrow \ell \nu \bar{\nu})} \text{BR}(\tau \rightarrow \ell \nu \bar{\nu}) \end{aligned} \quad (2.2)$$

where $\ell = e, \mu$.

The partial width of a lepton, ℓ_1 , which decays into another lepton, ℓ_2 , is given by:

$$\Gamma(\ell_1 \rightarrow \ell_2 \nu \bar{\nu}) = \frac{G_F^2 G_{\ell_2}^2 m_{\ell_1}^5}{192 \pi^3} F\left(\frac{m_{\ell_2}^2}{m_{\ell_1}^2}\right) \tau_{\ell_1} \quad (2.3)$$

where τ is a term to account for radiative corrections, and the function $F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ is a phase space factor to account for the mass of the final state lepton ℓ_2 . When $\ell_1 = \tau$ and $\ell_2 = e$, the mass of the electron can be neglected and $F = 1$; when $\ell_1 = \tau$ and $\ell_2 = \mu$, $F = 0.973$. G_ℓ is the Fermi effective coupling strength of the decay which is related to the fundamental electroweak coupling constant by the relation

$$G_\ell = \frac{g_\ell^2}{4\sqrt{2} m_W^2}$$

[†]using the convention that $\hbar = 1$.

Lepton universality implies that $g_\mu = g_e = g_\tau$. This Fermi effective coupling as measured in μ decay, $\mu \rightarrow e\nu_\mu\nu_e$, defines the Fermi constant, G_F . Equation 2.3 is usually written in the form

$$\Gamma(\ell_1 \rightarrow \ell_2 \nu \bar{\nu}) \propto G_F^2 r,$$

in which case the term r represents one loop electroweak radiative corrections not already included in the experimentally measured G_F [11]:

$$r_\tau = \left[1 + \frac{\alpha(m_\tau)}{2\pi} \left(\frac{25}{4} - \pi^2 \right) \right] \left[1 + \frac{3}{5} \frac{m_\tau^2}{m_W^2} \right] \approx 0.9960$$

where α is the electromagnetic fine structure constant. Given the calculation of the partial width of a leptonic τ decay, then, equation 2.2 gives a relationship between the leptonic branching ratios and the τ lifetime.

The leptonic decay of the τ is exactly analogous to the decay of the muon, shown in figure 2.2. A comparison of the τ lifetime to the μ lifetime can be used as a direct test

Figure 2.2: The Feynman diagram of the decay of the μ .

of the equality of the coupling constants for the two processes. Since the μ has only one decay mode, we can write:

$$\tau_\mu = \frac{1}{\Gamma_{total}} = \frac{1}{\Gamma(\mu \rightarrow e\nu\bar{\nu})}. \quad (2.4)$$

Then,

$$\begin{aligned} \tau_\tau &= \tau_\mu \frac{\Gamma(\mu \rightarrow e\nu\bar{\nu})}{\Gamma(\tau \rightarrow \ell\nu\bar{\nu})} \text{BR}(\tau \rightarrow \ell\nu\bar{\nu}) \\ &= \tau_\mu \frac{G_\mu G_e}{G_\tau G_\ell} \left(\frac{m_\mu}{m_\tau} \right)^5 \frac{F\left(\frac{m_e^2}{m_\mu^2}\right)}{F\left(\frac{m_\ell^2}{m_\tau^2}\right)} \tau_\mu \text{BR}(\tau \rightarrow \ell\nu\bar{\nu}) \end{aligned} \quad (2.5)$$

where τ_μ is the muon lifetime, m_μ is its mass, and $r_\mu = 0.9958$. Inserting the known quantities gives

$$\tau_\tau = (1632.1 \pm 1.8) \left(\frac{G_\mu G_e}{G_\tau G_\ell} \right) \left[\frac{\text{BR}(\tau \rightarrow \ell \nu_\ell)}{F\left(\frac{m_\ell}{m_\tau}\right)} \right] \text{ fs.} \quad (2.6)$$

The uncertainty in this prediction of the relationship between the τ lifetime and the leptonic branching ratio is dominated by the uncertainty of the τ mass. If lepton universality holds then $G_\mu = G_e = G_\tau$. Alternatively, the measured lifetime and leptonic branching ratios can be used as a test of the equality of the coupling constants. For further details of this calculation see references [7], [12], and [13].

The above procedure can also be applied to the decay modes $\tau^+ \rightarrow \pi^+ (K^+) \nu_\tau$ and $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ [11, 12]. In hadronic decays, however, the radiative corrections depend on the internal structure of the final state hadron and a theoretical uncertainty must be assigned in those cases where the radiative corrections have not been calculated. For example, the partial width of the pion decay mode of the tau is

$$\Gamma(\tau \rightarrow \nu_\tau \pi) = \frac{G_F^2 f_\pi^2 |V_{ud}|^2}{16\pi} m_\tau^3 \left[1 - \frac{m_\pi^2}{m_\tau^2} \right]^2 \left[1 + \frac{2\alpha}{\pi} \ln \frac{m_\tau}{m_\pi} + \dots \right] \quad (2.7)$$

where f_π is the pion decay constant and V_{ud} is an element of the Cabbibo-Kobayashi-Maskawa matrix. Note that the dependence on the τ mass goes as the third power whereas for a purely leptonic decay the partial width depends on the fifth power of the mass, see equation 2.3. The last factor in equation 2.7 represents radiative corrections which have not been fully calculated but are estimated to constitute an uncertainty of $\pm 1\%$ [12]. A similar partial width can be calculated for the $\tau \rightarrow K \nu_\tau$ decay and combined with 2.7 gives

$$\tau_\tau = (2489 \pm 24) \text{BR}(\tau \rightarrow \pi(K) \nu_\tau) \text{ fs} \quad (2.8)$$

where the uncertainty now is primarily due to the uncertainty in the calculation of the radiative corrections.

Similarly, for the $\pi\pi^0$ decay mode, which is thought to be entirely associated with a decay via the ρ meson resonance, a 5% uncertainty is assigned to the calculation of radiative corrections [12]. In this case the relationship is:

$$\tau_\tau = (1227 \pm 63) \text{BR}(\tau \rightarrow \pi\pi^0 \nu_\tau) \text{ fs.} \quad (2.9)$$

While the theoretical uncertainties for the hadronic modes limit the precision of these tests, the relationship between the τ lifetime and the above hadronic branching ratios can nevertheless be calculated as a cross check.

2.4 Techniques for Measuring the τ Lifetime

Figure 1.2 showed the production of a $\tau^+\tau^-$ pair schematically. At LEP energies the τ travels an average of ~ 2 mm before decaying. The most direct method of measuring the τ lifetime, and one of the most frequently used, is to measure the length of the decay path of individual taus on a decay by decay basis. The τ decay position is determined by fitting the three charged particle tracks in a 3-prong decay to a common vertex. The decay length is then the distance from the τ production point to the measured decay point. The technique is illustrated schematically in figure 2.3. The production point of the $\tau^+\tau^-$ pair

Figure 2.3. Schematic diagram of a 3-prong τ decay showing the typical error ellipse of the vertex reconstructed from the three charged tracks and the size of the e^+e^- beam interaction region.

is not known, however, so one can either assume that the τ originated at the centre of the

e^+e^- interaction region, or perform a χ^2 fit for the τ decay length using as inputs the decay point and the position and size of the interaction region. This fitting method involves constraining the decay length to be parallel to some axis, calculated using the visible τ decay products, which approximates the τ direction. The distribution of decay lengths for all decays is formed and some statistical estimator, usually the maximum likelihood, is used to determine the mean decay length. The relativistic boost factor for the τ can then be calculated, either analytically or with a Monte Carlo simulation, to convert the decay length to a lifetime in the τ rest frame. This method has the advantages of simplicity and directness. However, track reconstruction and detector calibration effects may introduce a bias in the measured τ decay vertex position resulting in an artificially long (or short) lifetime. This is one of the two methods used by OPAL to measure the τ lifetime and is described in chapter 8.

In principle, the decay length method could be applied to measure the distance between two 3-prong decays in a single $\tau^+\tau^-$ event. This measures the sum of the two the decay lengths and is completely independent of the size and position of the e^+e^- interaction region, and the directional constraint in the decay length fit. However, the combined branching ratio for $\tau^+\tau^-$ events where both taus undergo a 3-prong decay is only 2% which is a severe limitation and, to date, this method has not been used to make a τ lifetime measurement.

The second most popular method of measuring the τ lifetime is to use the more plentiful 1-prong τ decays. The distance of closest approach of the reconstructed 1-prong track to the origin of the τ is proportional to the decay length and thus contains lifetime information. This distance, called the impact parameter $d_{\perp}$, is related to the τ decay length, l , and the angle, ψ , between the decay track and the τ direction by: $d_{\perp} = l \sin \psi$, see figure 2.4a. However, there is again the problem that the origin of the τ within the e^+e^- interaction region is not known. It is assumed that all taus originate at the centre of the interaction region ellipse. Figure 2.4b illustrates the definition of the impact parameter with this assumption. It is this approximation which constitutes the major contribution to the statistical error of this method. The direction of the original τ , and so the angle ψ , is not known so a comparison of the data $d_{\perp}$ distribution with a Monte Carlo simulation is made to extract the τ lifetime. This is the second OPAL method used to measure the

lifetime and is described in chapter 7.

The traditional impact parameter method described above relies on a Monte Carlo simulation because the direction of the τ cannot be reconstructed from the decay particles observed in the detector. In addition, the uncertainty of the production point of the $\tau^+\tau^-$ pair within the e^+e^- interaction region limits the statistical precision of the method. For this reason, the ALEPH experiment at LEP has developed two new impact parameter techniques which use tau decay tracks in 1-1 events only [14]. Figure 2.5 shows schematically the definitions of the impact parameters and the decay angles. The two methods to measure the lifetime involve taking the sum and the difference of the impact parameters of the two 1-prong tracks:

$$d_1 \pm d_2 = (l_1 \sin\psi_1 \pm l_2 \sin\psi_2).$$

The $\tau^+\tau^-$ are assumed to be collinear. initial state radiation of the e^+e^- beams, however, does produce some $\tau^+\tau^-$ pairs which are not collinear. Information about the two ψ angles is obtained by using the quantity $\Delta\phi$ which is the measured angular difference between the two 1-prong tracks. The impact parameter sum, $d_1 + d_2$, does not depend on the position of the Z^0 production point within the e^+e^- interaction region, while the impact parameter difference, $d_1 - d_2$, is insensitive to the direction of the τ and thus is independent of the Monte Carlo simulation of this direction. These methods are sensitive, however, to radiation in the detector of the τ decay products which can bias the measurement of the angles of the 1-prong tracks. In the case of either of the $d_1 + d_2$ or $d_1 - d_2$ methods, when either the limitation of simulation of the τ direction or the uncertainty of the $\tau^+\tau^-$ production point is reduced, the other effect is enhanced, and the overall result is that these methods do not improve significantly on the precision attainable with the traditional impact parameter method.

To overcome the persistent problem of the size of the beam envelope, a new technique has been developed for 1-3 events which combines all of the previous techniques [15]. In this technique, the impact parameter of the 1-prong track is used to extract the lifetime but it is not measured with respect to the average position of the interaction region. Instead, the quantity used is the distance of closest approach to the reconstructed vertex of the 3-prong tracks on the other side of the $\tau^+\tau^-$ event. Figure 2.6

Figure 2.4: Schematic diagram of the definition of the impact parameter of a 1 prong τ decay track. a) the ideal case where the impact parameter is defined with respect to the origin of the τ . b) the case where the impact parameters are defined with respect to the position of the e^+e^- interaction region.

Figure 2.5 Tau pair events with a 1-1 topology: a) the case when $\psi_1 > 0$ and $\psi_2 > 0$; b) the case when $\psi_1 = 0$ and $\psi_2 > 0$; $\Delta\phi$ is the angular difference between the two 1-prong tracks.

illustrates the technique. This impact parameter, d_{13} , is related to the decay lengths of

Figure 2.6: A schematic diagram of the 1-3 impact parameter method. The impact parameter of the 1-prong track, d_{13} , is measured with respect to the reconstructed 3 prong vertex.

the two taus in the event by

$$d_{13} = (l_1 + l_3) \sin \phi_1$$

where l is the decay length, and ϕ_1 is the angle of the 1-prong track to the original $\tau^+\tau^-$ direction. Of course, ϕ_1 is not known since the original τ direction cannot be reconstructed. However, the approximation is made that $\phi_1 \approx \Delta\phi$ where $\Delta\phi$ is the angular difference between the 1-prong track and the summed momentum vector of the 3-prong decay. The Monte Carlo is used to make a correction for this approximation. Similar to the impact parameter difference method, the impact parameter is plotted as a function of the angular difference, and the lifetime is extracted from the slope. This method has the advantage of being completely independent of the characteristics of the e^+e^- interaction region. However, it is just as dependent on the Monte Carlo simulation as the traditional 1-prong impact parameter method, it has all of the sensitivity to the detector related effects of the 3-prong decay length method, and it is as susceptible as

the impact parameter difference methods to bias in the angular separation of the τ decay products due to radiative decays.

2.5 Summary of the Situation before LEP

2.5.1 Experimental Measurements

Figure 2.7 shows a compilation of some of the best previous τ lifetime measurements made before the turn on of the LEP accelerator in late 1989 [1]. The individual measurements have been corrected to account for the new τ mass. The size of this correction in the combined average lifetime is -1 fs. The pre-LEP world average τ lifetime is (302 ± 8) fs. Figure 2.8 shows the Standard Model prediction of the relationship between

Figure 2.7 A plot of all of the τ lifetime results published before the turn on of LEP

the τ lifetime and the leptonic branching ratio given in equation 2.6. Shown on the figure

are pre-LEP world average τ lifetime and the corresponding average phase space corrected leptonic branching ratio as reported in [1, 2]: $(18.06 \pm 0.28)\%$. These measurements translate into a ratio of the coupling constants of

$$\frac{G_F}{G_{\mu}} = 0.976 \pm 0.030 \quad (2.10)$$

which is consistent with unity. As shown in equation 2.5, the Standard Model prediction of the τ lifetime depends not only on the leptonic branching ratio but also on the τ mass. The previous world average τ mass suggested a larger difference from the predicted relationship between τ lifetime and branching ratios. In addition, over the last 5 years the discrepancy has varied in size depending on whether the comparison was made to the $\tau \rightarrow e\nu\bar{\nu}$, the $\tau \rightarrow \mu\nu\bar{\nu}$ branching ratio, or to the average of both; the difference appeared larger when using only the electron mode. As a result, the ratio of the coupling constants shown in equation 2.5 has varied between 1-3 standard deviations from unity depending on the particular experimental measurements input to the calculation. This difference is referred to in the literature as the " τ decay puzzle" or the " τ lifetime discrepancy".

Figures 2.9 and 2.10 show the Standard Model predictions for the two hadronic decay modes mentioned above, see equations 2.8 and 2.9, along with the world average quantities. In the case of the single pion decay, the agreement is good. In the case of the ρ decay mode the theoretical uncertainties on the prediction are comparable to the experimental errors and there is no indication of a discrepancy.

2.5.2 Implications of a Long τ Lifetime

There have been many speculations and models to account for a τ lifetime which is longer than the Standard Model prediction. The simplest adjustment is an extension of the Standard Model to include four generations. However, in order not to contradict the LEP measurement that the number of light neutrinos is three [1], the fourth generation neutrino would have to be heavier than $m_Z/2$. The fourth generation leptons would then, according to these models, mix with the third generation leptons, either with τ or ν_τ , in the same way that the weak eigenstates of the down-type quarks are mixtures via the

Figure 2.8 The Standard Model prediction of the τ lifetime as a function of the average leptonic branching ratio, see equation 2.6. The τ mass used is $(1777.0 \pm 0.4) \text{ MeV}/c^2$. The dashed lines indicate the error on the prediction due to the uncertainty on the τ mass. Also shown are the pre-LEP world average measured values

Figure 2.9: The Standard Model prediction of the τ lifetime as a function of the single pion branching ratio, see equation 2.8. The τ mass used is $(1777.0 \pm 0.4) \text{ MeV}/c^2$. The dashed lines indicate the error on the prediction due to the uncertainty on the τ mass and the uncertainty due to uncalculated radiative corrections. Also shown are the pre-LEP world average measured values.

Figure 2.10. The Standard Model prediction of the τ lifetime as a function of the ρ branching ratio, see equation 2.9. The τ mass used is $(1777.0 \pm 0.4) \text{ MeV}/c^2$. The dashed lines indicate the error on the prediction due to the uncertainty on the τ mass and the uncertainty due to uncalculated radiative corrections. Also shown are the pre-LEP world average measured values.

CKM matrix, see equation 1.3. The lepton sector would then consist of,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \cos\phi + \chi \sin\phi \end{pmatrix} \quad \begin{pmatrix} \nu_\chi \\ \chi \cos\phi - \tau \sin\phi \end{pmatrix} \quad (2.11)$$

where χ is the new lepton and ϕ is the mixing angle with the third generation. This type of mixing would reduce the coupling constant to $G'_\tau = G_{e,\mu} \cos\phi$ and the lifetime would be a factor of $\frac{1}{\cos^2\phi}$ longer [16, 17].

A more exotic solution is a model which treats the various generations as belonging to separate gauge groups. The complete gauge group would then become $U(1) \times [SU(2)]^n$ where n is the number of generations. In this model, each generation has a separate coupling constant and there is no reason to expect that these couplings should be the same. The observed apparent $e = \mu$ universality is interpreted as a coincidence which arises because the masses of the leptons of the first two generations are much less than the mass scale of the third and subsequent generations [18].

Chapter 3

The OPAL Experiment

3.1 The LEP e^+e^- Collider

LEP is an e^+e^- colliding beam accelerator located at CERN.[†] It is contained in a ring-shaped tunnel located approximately 100 m underground consisting of 8 curved sections alternating with straight sections for a total circumference of 26.4 km. Figure 3.1 illustrates schematically LEP and the beam injection system into LEP.

The electron and positron beams are initially produced and accelerated to an energy of 660 MeV in the LEP linear accelerator injector, LIL. They are then accumulated in the electron-positron accumulator, EPA, before being injected into the PS accelerator and subsequently the SPS for acceleration to an energy of ~ 20 GeV. The beams are then injected, in opposite directions, into LEP and accelerated to ~ 45 GeV.

Each of the electron and positron beams is composed of four bunches of particles separated by spaces. This 4×4 configuration would produce collisions at eight positions in the ring. Collisions take place in the straight sections of the ring, four of which are instrumented with detectors: ALEPH, DELPHI, L3 and OPAL. The other four straight sections contain electrostatic separators to prevent the beams from colliding. Each of the four experiments experiences a beam crossing every $22 \mu\text{s}$.

The rate at which physics interactions occur is given by

$$R = \mathcal{L}\sigma$$

[†] The European Organization for Nuclear Research

Figure 3.1: A schematic diagram of the LEP ring and the injection system

where σ is the cross section for an e^+e^- collision to produce a given process, and $\mathcal{L}$ is the accelerator luminosity. The luminosity depends on the properties of the e^+e^- beams,

$$\mathcal{L} = \frac{fnN_1N_2}{4\pi\sigma_x\sigma_y}$$

where

- $f \equiv$ bunch circulation frequency
- $n \equiv$ number of bunches
- $N_i \equiv$ number of particles per bunch
- $\sigma_{x,y} \equiv$ size of the interaction region in x and y .

The effective size of the e^+e^- interaction region is given in terms of the width of its gaussian profile: $\sigma_x \sim 150 \mu\text{m}$ and $\sigma_y \sim 8 \mu\text{m}$. Experimentally, the luminosity is measured by counting the number of small angle $e^+e^- \rightarrow e^+e^-$ events. These events are dominated by Coulomb scattering for which the cross section can be reliably calculated theoretically. The design luminosity of LEP is $1.6 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}$, but the actual luminosity achieved at present is smaller by approximately a factor of two.

Figure 1.1 showed a plot of the e^+e^- collision cross section as a function of the centre of mass energy. Radiative corrections significantly modify this cross section and are not shown on the plot. The cross section for production of multihadronic events at the peak of this curve is about 41 nb ($1\text{ b} = 10^{-24}\text{ cm}^2$), but the actual production rate observed is reduced to approximately 30 nb due to radiative corrections. At the design luminosity, an e^+e^- collision producing a Z^0 boson would take place approximately every 1.4 s.

3.2 The OPAL Detector

OPAL was designed to be a multipurpose detector using conventional detector technologies [19]. It features precise charged particle tracking in a magnetic field, hermetic electromagnetic and hadronic calorimetry over nearly 4π in solid angle, and muon detection capability. A general description of all components of the detector is given first, followed by a more detailed description of the main tracking chamber and the two innermost precision detectors which are the most critical components in measuring the impact parameters of charged particle tracks.

Figure 3.2 illustrates the detector with its component sub-detectors displayed in an exploded view to show the details. OPAL is composed of a barrel section and two end caps with similar instrumentation in each part. The coordinate system is defined such that the x -axis is horizontal pointing toward the centre of the LEP ring, and the z -axis is in the direction of the e^- beam along the barrel axis of OPAL. The LEP ring, and thus the z -axis, is inclined by 13 mrad with respect to the horizontal. The y -axis is similarly inclined with respect to the vertical. An $(r, \phi, \cos\theta)$ coordinate system is also used in order to exploit the symmetry of the detector. The polar angle, θ , is measured from the z -axis, the azimuthal angle, ϕ , from the x -axis, and r is the radius in the xy plane.

The OPAL beam pipe in 1990 was a composite aluminum/carbon-fibre cylinder 0.14 cm thick with a radius of 7.8 cm. It also formed the inner wall of the central tracking chambers' pressure vessel. In 1991 a smaller beryllium beam pipe of thickness 0.11 cm and radius 5.3 cm was inserted which allowed the installation of a silicon microvertex detector

Figure 3.2: A diagram of the OPAL detector. The components are expanded for greater clarity. The main components in the barrel portion of OPAL are the vertex detector at the centre of the detector, followed by the main tracking detector, the z-chambers, the solenoid magnet coil, the time of flight scintillator bars, the lead glass electromagnetic calorimeter, the hadron calorimeter, and the muon chambers surrounding the entire detector.

in the intervening space.

The central tracking system consists of three gas drift chambers contained within a single pressure vessel operating at a pressure of 4 bar, see figure 3.3. The principle of operation of drift chambers is that charged particles traversing the detector produce ionization in a gas which is collected at a series of readout sense wires. The distance from the particle's path, where the ionization was produced, to the sense wire is measured using the difference between the time that the particle entered the chamber (or some other equivalent start time) and the arrival time of the ionization electrons at the sense wire. The function of tracking chambers is to sample, at regular intervals, the spatial position of the path of a charged particle travelling through the detector so that the particle's "track" may be reconstructed from these measured space points. The properties of the OPAL tracking system are summarized in table 3.1.

Tracking Device	Vertex Detector	jet Chamber	z-Chambers
Wire Organization	36 axial sectors 12 wires/sector, 36 stereo sectors 6 wires/sector	24 sectors 159 wires/sect	24 chambers 8 cells/chamb. 6 wires/cell
Wire spacing(mm)	5.3 and 5.0	10.0	4.0
Wire stagger(μm)	± 40	± 100	± 250
Max. drift dist.	1.4 cm	25 cm	25 cm
Max. drift time	0.5 μs	5 μs	5 μs
Gas	argon(88.2%), methane(9.8%), isobutane(2.0%) at 4 bar		
Gas gain	2×10^4	10^4	$2 - 5 \times 10^4$
Angular Coverage	$23^\circ \leq \theta \leq 157^\circ$ (92% of 4π) axial+stereo $18^\circ \leq \theta \leq 162^\circ$ (95% of 4π) axial wires	$43^\circ \leq \theta \leq 137^\circ$ (73% of 4π) 159 wires $11^\circ \leq \theta \leq 169^\circ$ (98% of 4π) ≥ 8 wires	$44^\circ \leq \theta \leq 136^\circ$ 94% in ϕ 67% of 4π
Coordinate determined	ϕ drift time z ΔT and stereo	ϕ drift time z charge division	ϕ charge division z drift time
Spatial resolution	ϕ 50 and 90 μm z 4 cm(ΔT), 700 μm (stereo)	ϕ 130 μm z 6 cm	ϕ 15 mm z 100-135 μm
2-Hit resolution	2.0 mm	2.5 mm	2.5 mm
Trigger	z/r all axial wires	3 groups of 12 wires per sector	

Table 3.1: Properties of the OPAL drift chambers.

The main drift chamber measures the momentum and energy loss of charged

particles. The innermost drift chamber, called the vertex drift chamber serves to precisely measure the position of charged particles as close as possible to the interaction region. The silicon microvertex detector was designed to perform the same function as the vertex drift chamber with even greater precision. These three detectors will be discussed individually in the following sections.

Figure 3.3: The central tracking system of OPAL. The vertex chamber, jet chamber and z-chamber are contained within a common pressure vessel formed by the beam pipe, the solenoid coil, and the two endcap pressure bells.

Drift chambers have their optimal spatial resolution in the direction in which the ionization drifts. Although the vertex drift chamber and the main drift chamber give some z-coordinate information, they are designed to have their optimal resolution in the $r\phi$, or xy , plane. For this reason, outside the main drift chamber is an array of detectors called z-chambers which are drift chambers designed to provide a precise measurement of the z coordinate of a charged particle's path by having the ionization drift in the z-direction. They consist of 24 drift chambers 4 m long, 50 cm wide and 5.9 cm thick. Each chamber is divided into 8 cells of (50×50) cm such that the maximum drift distance is

about 25 cm. The z -chambers cover only the barrel section of the detector: polar angles of $|\cos \theta| < 0.72$. Tracks which point into the end caps of OPAL are out of the angular acceptance of the z -chambers so an accurate measurement of the z -coordinate of tracks is made by using the fact that such a track does not pass all sense wires of the jet chamber. The radius at which it leaves the jet chamber determines $\cos \theta$ and thus z .

The central tracking system is surrounded by a conventional, water cooled solenoidal coil which provides a uniform magnetic field of 0.435 T. Following the coil is an array of plastic scintillator bars which provide a measurement of the time of flight of particles from the interaction region. Each of the 160 counters is 6.84 m long. Light is collected at both ends of each counter via plexiglass light guides glued directly to photomultiplier tubes. These detectors are used primarily in triggering and rejection of cosmic ray background.

The next major detector in OPAL is the electromagnetic calorimeter which is composed of an array of lead-glass blocks. This calorimeter detects and measures the energies of electrons, positrons and photons ranging from tens of MeV to 100 GeV. When one of these particles enters a lead-glass block it produces a shower of photons, electrons and positrons by means of a series of electromagnetic interactions. The electrons and positrons radiate Cherenkov light⁵ in the glass which is detected with a photomultiplier tube attached to each block. The array acts as a total absorption calorimeter and consists, in the barrel part, of 9440 blocks oriented to point at the e^+e^- interaction region and covering the angular region $|\cos \theta| < 0.82$, and, in the endcap, of two dome shaped arrays of 1132 blocks each, oriented parallel to the z -axis and covering the region $0.81 < |\cos \theta| < 0.98$. In total, the electromagnetic calorimeter provides 24.6 radiation lengths* of material in the barrel region, and 20 in the endcaps. The electromagnetic energy resolution was typically $\Delta E/E \approx 3\%$ for $E \approx 45$ GeV.

There are ~ 2 radiations lengths of material in front of the lead glass due to

⁵ The Cherenkov effect occurs when the velocity of a charged particle traversing a dielectric medium exceeds the velocity of light in that medium. A small number of photons will be emitted at a fixed angle, which is determined by the velocity of the particle and the index of refraction of the medium.

*One radiation length is the mean distance over which a high-energy electron loses all but 1/e of its energy by bremsstrahlung. It is given approximately by $X_0 \approx 180A/Z^2 \text{ g/cm}^2$ where A is the atomic mass of the medium and Z is the number of atomic electrons.

the coil and central tracking system pressure vessel. As a result, most electromagnetic showers are initiated before the lead glass itself. Presampling devices are therefore installed in both the barrel and endcap regions immediately in front of the lead glass in order to measure the position and sample the energy of these showers. Both the barrel and endcap presamplers are gas ionization detectors. In the barrel region the presampler consists of a cylinder of limited streamer tubes, while the endcap presampler is an umbrella shaped arrangement of thin multiwire chambers. A presampler is able to improve the electromagnetic energy resolution on a shower by shower basis because the total pulse height observed in the presampler is approximately proportional to the number of charged shower particles entering the device, which is itself roughly proportional to the energy deposited in the material in front of the presampler.

Outside of the lead-glass, there is a soft iron return yoke for the magnet which has also been instrumented to serve as a hadron calorimeter. The gaps between the iron plates have been instrumented with streamer tubes. The barrel consists of nine layers of limited streamer tube chambers, alternating with eight iron slabs, and spans radii from 3.39 m to 4.39 m. The hadron calorimeter is closed at each end by a doughnut-shaped endcap, subtending the range $0.81 < |\cos \theta| < 0.91$, where eight layers of chambers alternate with seven slabs of iron. The magnet pole-tips are instrumented with thin multiwire chambers and complete the coverage of the solid angle for hadronic shower detection to $|\cos \theta| < 0.99$. The iron provides at least four nuclear interaction lengths of absorber¹.

The entire detector is surrounded by layers of drift chambers (four layers in the barrel and two in the end caps) to detect muons, the only charged particle capable of traversing the entire detector without interacting in the detector material. The barrel covers $|\cos \theta| < 0.72$ for ≥ 1 layer or $|\cos \theta| < 0.68$ for all four layers. The end caps cover $0.67 < |\cos \theta| < 0.98$.

The different detectors in OPAL provide complementary information about the particles produced in e^+e^- collisions. A general description of how pions, muons and electrons may be identified serves as an illustration of the integration of this information

¹The length scale appropriate for hadronic showers is the nuclear interaction length, given roughly by $\lambda \approx 35A^{1/3} \text{ g/cm}^2$ where A is the atomic number of the absorber material. For comparison, one nuclear interaction length in iron is $\lambda = 131.9 \text{ g/cm}^2$ whereas one radiation length is only $X_0 = 13.84 \text{ g/cm}^2$ [1].

Electrons deposit all of their energy in a relatively narrow electromagnetic shower in the lead-glass array. Approximately 90% of the energy is contained in no more than three glass blocks. Electrons can be identified by requiring that the energy measured in the lead-glass be approximately equal to the momentum, measured with the main tracking chamber, of the charged particle track associated with the shower.

Hadronic particles undergo nuclear interactions in the detector material in addition to electromagnetic interactions, so the physical processes in a shower initiated by a hadronic particle are considerably different than in showers initiated by electrons and photons. The overall effect is that a hadronic shower has a much broader spatial extent and is far more penetrating than an electromagnetic shower initiated by particles of equivalent energy. Charged pions, π^+ and π^- , can be distinguished from electrons and positrons by the fact that they deposit less energy in the electromagnetic calorimeter and they penetrate into the hadronic calorimeter. The π^0 meson decays electromagnetically to two photons: $\pi^0 \rightarrow \gamma\gamma$. The presence of π^0 's can be identified using the number and size of the electromagnetic showers.

Muons are distinguished by the property that they leave very little energy in the detector; for example, only ~ 0.2 GeV in the electromagnetic calorimeter. Thus, a μ is identified by its passage through the complete detector to the tracking chambers which form an outer shell of the detector. There are small geometric gaps in these muon chambers so additional methods of identifying muons are also used: i) a signal in the outer layers of the hadron calorimeter, ii) a high momentum particle track in the tracking chamber with little energy deposited in the electromagnetic calorimeter. There is a high degree of redundancy in these three criteria which leads to a good efficiency for identifying almost all muons.

3.2.1 The Jet Chamber

The main tracking detector, called the jet chamber or CJ, is a large cylindrical drift chamber located between the radii 25 cm and 185 cm [20]. Charged particles follow a helical trajectory within the magnetic field of OPAL. Measuring the curvature of this trajectory allows the momentum of the particle to be determined. The error on the

measurement of the curvature of a track is proportional to $1/L^2$ where L is the length of the track. The amount of ionization collected in the series of samples along the particle path gives information on the energy loss of the particle per unit distance, dE/dx , which is used to identify the type of particle.

The jet chamber is divided into 24 azimuthal sectors, each with a radial plane of 159 anode sense wires stretched parallel to the z -axis. The chamber electrostatic field is designed such that, in the absence of magnetic fields, the ionization electrons produced in the gas drift perpendicular to the radial sense wire plane to the anode in the xy plane. The anode wire planes in each sector are separated by planes of cathode wires so that ionization produced in a particular sector, or drift cell, is collected in that sector alone. This type of drift cell is called a "jet cell". In the magnetic field of OPAL, the charge drifts to the anode at an angle of $\sim 20^\circ$ with respect to perpendicular due to the Lorentz force. Since only the arrival time of the ionization to the sense wire is recorded, drift chambers are not able to distinguish whether the particle has passed on the left or the right side of the anode plane. The anode wires are therefore offset by a $\pm 100 \mu\text{m}$ alternating stagger in order to resolve on which side of the anode plane the track has passed.

The wave-forms of the signals from both ends of the sense wires are recorded with flash analog-to-digital converters (FADCs). This digitized pulse information is read out via hardware scanners which also suppress channels with no signal recorded. The valid data is then transferred to 20 microprocessors residing in two VME crates for further data reduction. The first step in this reduction is an online pulse shape analysis in which the pulse shape of the signal collected on a single wire is used to calculate the corresponding drift time and the amount of charge collected. In a second step, calibration constants are applied to the data such as electronic time offsets, relative gain factors, and geometrical constants such as wire staggering. This step produces a series of space points. In a third step, this partly calibrated data is processed by an online pattern recognition algorithm where the measured space points are connected in an attempt to reconstruct the particle track. The track segments thus produced are stored for use in the event filter to be discussed in section 3.4.

The drift time and the wire position produce an (r, ϕ) coordinate in space. A rough z -coordinate measurement of the track is obtained by an analysis of the charge

division between the two ends of the sense wire. The optimal resolution of the detector, however, is in the $r\phi$ (xy) plane: the average space point resolution is $130\ \mu\text{m}$ in the $r\phi$ plane and $6\ \text{cm}$ in z . The momentum resolution of tracks reconstructed in the jet chamber is $\frac{\sigma_{p_T}}{p_T} = 1.5 \times 10^{-3} (\text{GeV}/c)^{-1}$ (6.8% at $p = 45\ \text{GeV}/c$) measured using high momentum lepton tracks [21].

The jet chamber also provides particle identification using a multiple sampling measurement of the energy loss, dE/dx [22]. The energy loss of particles in matter is a function of the particle velocity, $v = \beta c$, and is described by the Bethe-Bloch equation. The energy loss as a function of momentum shows a characteristic $1/\beta^2$ decrease, reaches a minimum around $\beta = 0.97$, and continues with a logarithmic rise until it saturates. By measuring the momentum of a particle as well as its energy loss, the mass of the particle can be determined. The difference in energy loss between the particle species, however, is small. For example, at $10\ \text{GeV}/c$ the difference in mean dE/dx between pions and electrons is about 10%. The energy loss distribution, approximately a Landau distribution, has a large inherent width which makes it necessary to measure many samples along each track in order to determine the mean energy loss with sufficient accuracy. The resolution of the energy loss measurement depends on the number of samples used, N . An empirical fit gives the result:

$$\frac{\sigma(dE/dx)}{dE/dx} \propto N^{-0.43}.$$

For $N = 130$ the resolutions achieved are $\frac{\sigma(dE/dx)}{dE/dx} = 3.1\%$ for muons from $Z^0 \rightarrow \mu^+\mu^-$ decays, and $\frac{\sigma(dE/dx)}{dE/dx} = 3.8\%$ for minimum ionizing pions in jets.

3.2.2 The Vertex Drift Chamber

The OPAL vertex drift chamber, called CV, was designed to provide high precision measurements of charged particle tracks close to the e^+e^- interaction region [23]. The chamber is $1\ \text{m}$ long and $47\ \text{cm}$ in diameter. The vertex drift chamber is mechanically supported from the inner edge of the jet chamber end-cones, and as a result is mechanically independent of the OPAL beam pipe. An aluminized mylar tube is placed at the detector inner radius of $8.8\ \text{cm}$. This allows a separate gas volume for the vertex drift chamber to be defined within the common OPAL central detector pressure vessel. The

vertex and main jet chambers have separate gas systems for supply and recirculation. A sensitive feedback system ensures that the pressure inside the vertex chamber is held at 5 mbar below that of the main jet chamber.

Like the jet chamber, the vertex chamber is divided into azimuthal jet drift cells. The chamber consists of an inner layer of 36 axial cells each with 12 anode wires parallel to the z -axis and an outer layer of 36 stereo cells each with 6 anode wires inclined at an angle of 4° to the z -axis. Figure 3.4 shows the wire layout for the axial and stereo cells. The anode wires are offset from the anode plane by a $\pm 40 \mu\text{m}$ alternating stagger in order to resolve on which side of the anode plane the track has passed. There is an additional anode wire bow of $\sim 25 \mu\text{m}$ due to the electrostatic forces when chamber voltages are applied.

The electrostatic field configuration within an axial half-cell of the vertex drift chamber is shown in figure 3.5. The electrostatic field conditions within a jet cell are defined by two independent field parameters: the anode wire surface field (which determines the gas gain) and the drift field. These fields are determined by the voltages applied to the potential and cathode wires; the anode wires are at ground.

The drift field is perpendicular to the anode plane and becomes uniform beyond 2-3 mm from the anode. This is desirable since it produces a constant drift velocity over most of the drift cell. Figure 3.6 shows a plot of the drift velocity as a function of the drift field. The vertex chamber is operated at a drift field of 2.5 kV/cm corresponding to a drift velocity of $39.5 \mu\text{m/ns}$. Inside a distance of $\sim 2\text{-}3 \text{ mm}$, the field increases non-linearly up to the surface of the anode wires. The anode surface field is 360 kV/cm producing an ionization avalanche with a gas gain of $\sim 10^4$.

The start signal for the drift time measurement is generated from a signal indicating a beam crossing in LEP. The drift time is measured by timing the arrival of the primary electron ionization at the anode wires. The outputs from each end of the wire are amplified, discriminated and then combined in a mean-timer. In the offline data reconstruction phase, this drift time is converted to a drift distance with a relation described in section 4.1.1.

The vertex detector has the capability to record more than one particle passing through a single cell simultaneously. However, the second pulse recorded on the anode

Figure 3.4. The end-plate wire layout for a section of the vertex drift chamber. The axial cells have 12 anode sense wires and the stereo cells have 6.

Figure 3.5: The electric field lines for one half of an axial cell of the vertex chamber

Figure 3.6: The drift velocity as a function of drift field in the OPAL gas mixture (argon(88.2%), methane(9.8%), isobutane(2.0%)) at a pressure of 4 bar.

wire arrives on the tail of the first pulse and, depending on the separation between the first and second particles, may not be separately discriminated at all. If the second pulse is detected, the resolution will be somewhat degraded due to the pulse tail effects from the first pulse. Each recorded pulse is called a "hit" and the degradation in performance for second hits is a significant effect which must be taken into consideration for the τ lifetime measurement, see section 8.2. The resolution on each reconstructed space point is $50\text{ }\mu\text{m}$ for first hits and $90\text{ }\mu\text{m}$ for subsequent hits. The second-hit effect and corrections for it are described in more detail in section 4.1.3.

The axial wires provide information in the $r\phi$ plane and a low resolution measurement of the z -coordinate. In addition to being passed to a mean-timer, the signals from the two wire ends are also passed to a unit that measures the difference in their arrival time. A measurement of this quantity gives a fast measure of the z -coordinate of the hit which is used in the event trigger system. A more precise measure is obtained by using the stereo sectors in the offline track reconstruction stage [24].

3.2.3 The Silicon Microvertex Detector

For the 1991 data taking period, a high-precision microvertex detector was installed in OPAL in the space between a new smaller beam pipe and the existing central detector pressure vessel tube [25]. The microvertex detector is composed of two layers of silicon microstrip detectors. Incident charged particles deposit ionization energy and dislodge electrons much like in a gas ionization detector. However, the average energy required to produce an electron/hole pair in silicon is 3.6 eV , while it is typically $\sim 30\text{ eV}$ per ion pair in gases. The electrons and holes produced in the semiconductor separate due to the presence of an electric field and are detected at a series of strips on the detector surface.

The two layers of the OPAL silicon microvertex detector are formed by concentric barrels of detector "ladders" at radii of 6 and 7.5 cm. The inner layer has 11 ladders, and the outer has 14. One ladder is illustrated schematically in figure 3.7. Each ladder contains three silicon microstrip wafer detectors connected end-to-end. These are single-sided detectors with p^+ implant strips $25\text{ }\mu\text{m}$ apart fabricated on $300\text{ }\mu\text{m}$ thick silicon wafers

Figure 3.7. A diagram of one of the ladders in the silicon microvertex detector

Every second implant strip is covered first by a thin coating of oxide insulation and then by an aluminum readout strip forming a capacitively coupled readout. These readout strips are wire-bonded serially across a ladder parallel to the e^+e^- beam direction. Each ladder assembly is 18.0 cm long with 629 readout channels at a pitch of $50\ \mu\text{m}$ across a sensitive width of 3.14 cm. The average angular acceptance of the detector array is $|\cos \theta| < 0.8$.

When a minimum ionizing particle traverses $300\ \mu\text{m}$ of silicon, energy is deposited with a typical Landau distribution and a most probable value of 84 keV. The particle impact point at the detector mid-plane is determined from the mean of the cluster of readout strip positions weighted by their pulse heights. A cluster is required to have at least one strip with a pulse height of more than four times the noise of the individual strip, and an adjacent strip is included if its pulse height exceeds 1.5 times the noise level on that strip.

The silicon detector only provides position measurements in the $r\phi$ plane. The azimuthal acceptance for at least one silicon layer hit being produced by the passage of a charged particle is nearly 100%. The measured single-hit efficiency within the active regions of the detector is better than 97%, and the intrinsic space resolution of a single silicon wafer is $5\ \mu\text{m}$. However, uncertainties in the alignment of the silicon detector array currently limit the effective space-point resolution which can actually be achieved in track reconstruction to about $12\ \mu\text{m}$.

3.3 The Trigger

At the OPAL interaction point in the LEP ring, the e^+e^- beams cross every $22\text{ }\mu\text{s}$ or at a rate of 45 kHz . The OPAL detector, however, has the capability of recording data at a rate of only $1\text{--}5\text{ Hz}$. This is sufficient considering that the on-resonance Z^0 production rate, at the LEP design luminosity, is 0.7 Hz . Thus, it is necessary to have a fast and efficient system for selecting only those beam crossings with a possible e^+e^- interaction.

The main physics processes that must be selected efficiently by the experimental trigger system are the multihadronic and charged leptonic decays of the Z^0 and small angle $e^+e^- \rightarrow e^+e^-$ scattering for the measurement of luminosity. Experimental backgrounds to these processes come mainly from single beam particles interacting with residual gas in the beam-pipe or in the walls of the beam pipe. As well as not originating from the expected interaction region, background events are characterized by lower visible energies and few charged particles. Separation of low-multiplicity e^+e^- collision events, which also have few charged particles, from backgrounds can be made at the trigger level by using information on the reconstructed vertex position along the beam line. In addition to beam-related backgrounds, cosmic rays and detector noise may contribute to the trigger rate. With a typical LEP luminosity of $0.4 \times 10^{31}\text{ cm}^{-2}\text{s}^{-1}$, the average trigger rate was about 2 Hz .

Each sub-detector in OPAL has the capability to send a fast signal which can be used to select beam crossings with a possible e^+e^- interaction. In the OPAL trigger system [26], the sub-detector trigger signals are divided into two complementary parts. "Direct" trigger signals, such as total energy or track multiplicities, use information from a single detector component and are set to have relatively high thresholds. In order to allow lower thresholds and to exploit spatial coincidences between components, the detector has been subdivided into a matrix of overlapping bins in θ and ϕ . The matrix of $24\text{ }\phi$ bins by $6\text{ }\theta$ bins has five radial "layers" corresponding to the the different sub-detectors tracking, time-of-flight, electromagnetic, hadron and muon triggers. The matrix provides spatial correlations of information within and between sub-detector layers. In addition, a totally unbiased random trigger at a rate of 0.04 Hz provided samples of "empty" events,

which were used to monitor noise hits in some sub-detectors as well as for online pedestal adjustment.

Information from the vertex chamber and the jet chamber is used in the track trigger to search for charged particle tracks originating from the e^+e^- interaction vertex in both the $r\eta$ and the $r\phi$ planes. The track trigger uses information from the 12 axial wires of each sector of the vertex chamber and from three groups of 12 adjacent wires at different radii of each jet chamber sector. Signals are formed by the vertex chamber and the jet chamber trigger electronics and are combined by a track trigger processor, which detects the presence of tracks in any bins of the θ - ϕ matrix and provides various track multiplicity signals. In addition, simple counts of the number of wires hit are provided, and trigger signals depending on the specific ionization of particles are available from the jet chamber.

Trigger signals from the time-of-flight (TOF) system are generated from mean timers, and require discriminators on both ends of a counter to have fired within 50 ns of each other and within 50 ns of the arrival time of a relativistic particle from beam interactions. These 160 signals are then mapped onto the θ - ϕ matrix. There are also direct trigger signals of single or multiple TOF counter hits.

For the barrel portion of the electromagnetic calorimeter, in the z direction, the calorimeter is segmented into cells of 59 lead-glass blocks, and in the ϕ direction into cells of 160 blocks. The trigger logic for the electromagnetic calorimeter provides energy sums, with both high and low thresholds, for different segmentations of the electromagnetic calorimeter: over the total barrel, over each endcap, and over each of the above θ - ϕ cells.

In the streamer tube layers of the hadron calorimeter, signals are induced onto 4 mm wide aluminium strips as well as onto large area (typically $50\text{ cm} \times 50\text{ cm}$) pads. Layers of pads are grouped together to form towers dividing the solid angle into bins in ϕ and in θ . Direct triggers from the hadron calorimeter are not yet implemented.

The muon detector provides a total of 120 θ - ϕ signals and four direct signals. The muon barrel detector has no z -coordinate measurement for the trigger, so the ϕ signals are used to simultaneously fill all the barrel θ bins in the θ - ϕ matrix. The endcap muon detector provides 96 θ - ϕ signals in both ϕ and θ . The muon barrel detector provides one direct signal which is simply the logical OR of the muon barrel ϕ signals. Two direct

signals from the endcap muon detectors are the logical ORs of the θ - ϕ signals from each of the left and right muon endcaps. A fourth direct signal demands a signal in both endcaps with a loose collinearity requirement.

Table 3.2 shows the trigger efficiency for the different physics channels. Redundancy for all physics processes was achieved by triggers based on one detector component only and by triggers sensitive to single particles. This allowed direct measurements of the efficiencies of the various components and led to a high overall performance. The acceptance of each channel is determined by the solid angles shown as well as by selection cuts particular to each channel. The efficiency quoted is for events within this acceptance calculated from the data using the inherent redundancy of the trigger system.

Physics Channel	$ \cos \theta $	Efficiency (%)
$e^+e^- \rightarrow \text{hadrons}$	< 1.0	100
$e^+e^- \rightarrow \mu^+\mu^-$	< 0.95	99.9 ± 0.1
$e^+e^- \rightarrow \tau^+\tau^-$	< 0.90	99.9 ± 0.1
$e^+e^- \rightarrow e^+e^-$	< 0.85	100
$e^+e^- \rightarrow \gamma\gamma$	< 0.90	> 99.9
small angle $e^+e^- \rightarrow e^+e^-$	0.989-0.999	> 99.9

Table 3.2: Trigger efficiencies for different physics channels

3.4 Data Acquisition

Once an event is flagged as potentially interesting by the trigger, all available information about the event is read out from each of the individual sub-detectors in OPAL [27]. The information from these individual detectors is then merged together in an "event builder". A complete data record contains the information from all available sub-detectors relevant to a particular e^+e^- collision. These data structures are then passed to a filter system which can perform additional tests to select valid events. The filter compresses the selected data and performs extensive monitoring of the events. The data records for events selected by the filter are saved in a large disk buffer. This buffer is critical because it decouples the (online) data acquisition from the (offline) event reconstruction

The information from each sub-detector, such as drift times, pulse heights etc., is processed and formatted by the individual sub-detector's local system crate, LSC, micro-processors. Once a particular beam crossing has been triggered, the event builder collects the data from the LSCs. The partial data records from the individual sub-detectors corresponding to the same trigger are not available at the same time in all sub-detectors because of the variable processing time in each LSC, and they are sometimes delivered out of sequence. The event builder must therefore read and buffer these partial data records in the order they become available. They are merged and synchronized using the trigger number as a key and the complete data records are sorted in trigger number sequence on output. Extensive error handling is performed in order to achieve a high overall data taking efficiency.

The filter system acts as a second-level software trigger [28]. Here, the complete data records are available for each event and can be classified into physics categories. Backgrounds that are accepted through the first level hardware trigger in order to maintain a high and easily measurable efficiency, are further reduced. The filter performs a series of separate functions on each data record received from the event builder. These functions are:

Validation: It is verified that the format of the data conforms to the required structure.

Analysis: A partial event reconstruction is performed and the data is subsequently classified into i) candidate decays of the Z^0 and ii) the background processes consisting of off-momentum beam particles striking the wall of the beam pipe, cosmic rays, high energy "halo" muons which are produced by beam particles striking collimators upstream of the experiment, interactions of the beam with residual gas molecules in the vacuum chamber, and random hits in the gas chambers and electronic noise.

Monitoring: Based on the results of the analysis step, the data quality is checked. Histograms are formed and regularly monitored by the person who is on shift to perform regular safety checks of the detector.

Data compression: For events which are to be retained, the data compression procedure is performed. Since every data word is scrutinized during compression, this

process has the valuable secondary function of checking the data integrity at the finest level. This step requires the bulk of the CPU requirements

Recording: The data record and associated information concerning the event are written onto the filter buffer disk. Several other files are written in addition to the file of data records – one file contains copies of the data record headers, another contains the byte offsets to the start of each data record in the file, and a further contains calibration bookkeeping information.

3.5 Event Reconstruction

The event reconstruction system fetches data records from the filter buffer, records them onto optical disks for long term storage and reconstructs the events as soon as up-to-date calibration parameters from the various sub-detectors are available. Reconstruction starts with the raw data from the individual sub-detectors. The preliminary classifications of the filter are not used but are still recorded in the data record header. The reconstructed quantities, such as the momenta and energies of tracks, reconstructed vertices, and particle identifications are stored in a data summary tape, "DST". A loose preselection is performed to eliminate the most obvious sources of background. The remaining events are made available on four different computer platforms for physics analysis. This process requires significant computing capacity: On an HP/DN1000G, full reconstruction requires 26 CPU seconds for the average multihadronic event and 2 CPU seconds for other events.

The charged particle tracks are the most important quantities for the measurement of the τ lifetime so the track reconstruction will be described in some detail. The track reconstruction in OPAL proceeds in three stages: i) Each of the three central drift chambers internally performs a pattern recognition process to form track segments within its volume. ii) The track segments are then passed to a separate processor which attempts to match the segments. iii) A global track fit is performed using the space points in the sub-detectors which have been associated through the matching process.

The initial pattern recognition is performed by means of a "triplet initiator" which

is a triplet of contiguous hits, or measured space points. Starting from one extremity of the chamber the triplet initiator is used to define a road in the drift plane (xy for vertex and jet chambers, xz for z -chambers) which is then projected into the chamber. Hits within this road are added to the track segment and a new track segment initiator is formed by adding the new hit. After the pattern recognition process, a track segments are discarded in the vertex chamber if they have less than 6 (out of 12 possible) hits, and in the jet chamber if they have less than 10 (out of 159 possible) hits.

In a drift cell with anode wires arranged in a single plane, it is impossible to tell whether the ionization originated on the left or right side of the plane. In order to resolve this "left-right ambiguity", the wires in all three central drift chambers are staggered: $\pm 40 \mu\text{m}$ in the vertex chamber, $\pm 100 \mu\text{m}$ in the jet chamber, and $\pm 250 \mu\text{m}$ in the z -chamber. The pattern recognition process is performed for both ambiguities. In the case of the jet chamber, one of the ambiguities is rejected immediately on the basis of the quality of the track segment fit. In the case of the vertex and z -chambers, both ambiguities are saved to be resolved in the matching process.

The track matching process starts with a jet chamber track. An attempt is made to match first to the vertex chamber axial segments, then to the z -chamber, and then to the vertex chamber stereo segments. The jet chamber track segment is extrapolated to the centre of the other chamber in question. The best match segment is then chosen with a maximum likelihood method that uses the positions and direction cosines of the two track segments, and the internal quality measure of the track segment to aide in resolving the left-right ambiguity.

Once matching is complete, all track segment information is discarded and a global track fit is performed using all associated hits. The OPAL track parameters are: $(\kappa, \phi_0, d_0, \tan \lambda, z_0)$, defined as follows:

- κ is the curvature with

$$\kappa = \frac{1}{2\rho}$$

where ρ is the radius of curvature of the track, and κ is signed such that positive κ means ϕ increases in moving along the track away from the point of closest approach to the origin in the $r\phi$ plane (p.c.a.) Note that for an axial magnetic field with B_z

along the $-z$ axis, as is the case for OPAL, positive κ corresponds to a particle with negative physical charge.

- ϕ_0 is the azimuthal angle made by the track tangent at the p.c.a.
- d_0 is the impact parameter, i.e., d_0 is the distance from the origin to the p.c.a. Defining $\vec{d}$ to be the vector from the origin to the p.c.a., $\hat{\phi}$ the unit track vector at the p.c.a. and $\hat{z}$ the unit vector along the z axis, then formally,

$$d_0 = \hat{\phi} \cdot \vec{d} \cdot \hat{z}$$

Note that, for $\vec{B}$ in the $+z$ direction, $d_0 = qD$, where q is the physical charge and $D = \rho = \sqrt{x_c^2 + y_c^2}$; (x_c, y_c) are the coordinates of the centre of the track circle. D is not used because it is unstable under change of curvature.

- $\tan \lambda = \cot \theta$, where θ is the track polar angle measured from the $+z$ axis
- z_0 is the z coordinate when the track is at the p.c.a. The sz projection of the helix is then:

$$z = z_0 + s \tan \lambda$$

where s is the $r\phi$ path integral from the p.c.a.

These track parameters are then related to physical quantities by the relations

$$\begin{aligned} p_t &= a \frac{B}{\kappa}, \\ p_t &= p \cos \phi_0, \\ p_y &= p_t \sin \phi_0, \\ p_z &= p_t \tan \lambda \\ p &= p_t \sqrt{1 + \tan^2 \lambda} \\ q &= (\text{sign of } (\frac{B}{\kappa})) \end{aligned}$$

Where p is the momentum of the track and q is its charge. The component of the momentum in the xy plane is called the transverse momentum, p_t . If the transverse momentum is measured in GeV/c, B in kG and κ in cm^{-1} , then:

$$a = \frac{c}{2} \times 10^{-14} \approx 1.5 \times 10^{-4}.$$

The track fitting procedure is an iterative approach which takes account of the particle multiple scattering in the detector gaseous volumes, in the discrete material between chambers and in the beam pipe [29]. It is assumed that all scattering takes place at the sampling points, *i.e.*, the wires. Starting at the end of the track and working backwards to the beginning, each step involves accounting for multiple scattering (which entails a loss of information) and adding an additional space point measurement (which adds information). In this way the error matrix of the track parameters is built up gradually with great economizing of computing time.

In practice, the OPAL track fit is performed separately in $r\phi$ and sz space in order to save further computer time and to take advantage of OPAL's superior $r\phi$ resolution. Hits far from the fitted trajectory (3σ) are discarded and the track is refit up to a maximum of twice. Any hits dropped in the $r\phi$ fit will not be used in the sz fit. Because of the poor resolution in z , tracks are constrained in the sz fit to originate at the nominal average beam position.

3.6 Performance of the Tracking System

For the measurement of the τ lifetime, the most important track properties are the momentum and the track resolution close to the origin, or the d_0 resolution. Figure 3.8 shows a plot of the momentum, transverse momentum and error on the measured momentum for tracks in $\tau^+\tau^-$ events.

Figure 3.9 shows distributions of the reconstructed separation distance at the origin between the two leptons in $Z^0 \rightarrow e^+e^-$ and $Z^0 \rightarrow \mu^+\mu^-$ decays. The leptons originate at the same point and so the observed separation of the tracks is a direct measure of the combined capability of the tracking detectors and track reconstruction procedure. Distributions are shown for those tracks which have hits in the main drift chamber with no vertex detector information, hits in the vertex detector CV, and hits in the silicon microvertex detector.

For tracks reconstructed using the jet chamber only, the standard deviation of the separation of the two tracks in a $Z^0 \rightarrow \ell^+\ell^-$ events is $160\mu\text{m}$. Figure 3.9a) also

Figure 3.8: a) the momentum, p , of a track from a τ decay, b) the transverse momentum in the xy plane, p_t , c) the relative error on the momentum, σ_p/p , in %, and d) a scatter plot of σ_p versus p .

shows that the distribution of this separation is displaced $44\text{ }\mu\text{m}$ from zero indicating a systematic problem with the CJ-only tracks. When information from the vertex detector is present on both tracks the systematic shift is eliminated, and the separation of the tracks is reduced to $60\text{ }\mu\text{m}$. Including silicon microvertex detector hits on the tracks further improves this separation distance resolution to $24\text{ }\mu\text{m}$.

Chapter 4

Calibration of the Vertex Detectors

Both of the methods used by OPAL to measure the τ lifetime rely on precise tracking information as close as possible to the e^+e^- interaction point. The track position close to the interaction point is measured by the vertex detectors. For the 1-prong impact parameter method, the quality of the vertex detectors' calibration and resolution is not a limitation in OPAL. For the 3-prong decay length method, however, uncertainty in the calibration of the vertex detectors is the largest contribution to the systematic error of the measurement. For this reason, the calibration of the two OPAL vertex detectors will be discussed in detail.

4.1 Vertex Drift Chamber

The vertex drift chamber (CV) was described in section 3.2.2. This section describes the procedure to calibrate CV, first defining the necessary parameters, and then outlining the method of calibration of each type of parameter. Since the τ lifetime analysis is performed almost entirely in the $r\phi$ plane, only those parameters relevant to the $r\phi$ plane will be discussed.

4.1.1 Calibration Parameters

The calibration parameters of the vertex chamber can be divided into four classes as follows:

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4.1.1 Calibration Parameters

The calibration parameters of the vertex chamber can be divided into four classes as follows:

Position of CV in OPAL: The OPAL Master Reference System (MRS) is the cartesian coordinate system defined in section 3.2. The jet chamber (CJ) coordinate system is defined to be the OPAL MRS. The spatial position and orientation of CV with respect to CJ are described by six MRS coordinates: Δx , Δy and Δz which represent translations along the cartesian axes, and α , β and γ which represent rotations in the xy , xz and yz planes, respectively.

Wire Bows: The position in space of each wire can be calculated using the geometry of the chamber and its position in the MRS. An additional correction must be made for the fact that the anode wires are bowed due to the electrostatic forces. The magnitude of the bowing at a given z on the wire is B_z :

$$B_z = B_w(1 - (z/L)^2)$$

where L is the half-length of the wire, and B_w must be computed in the calibration procedure.

Drift Time: The drift time of the ionization to the anode wire, T_D , is computed from the raw TDC value, T_{TDC} , by the following relation:

$$T_D = T_{TDC} * C_t - T_{offset} - T_{sector} - T_{wire}$$

where C_t is the clock frequency; T_{offset} is a standard value time offset which sets the scale of the timing offset for the whole chamber and is related to the LEP beam crossing signal; T_{sector} is a set of time offsets (one per sector) to account for differences in a sector to sector basis and sum to zero over all sectors; and T_{wire} is a set of time offsets (one per wire) giving the wire-to-wire variation and summing to zero within a sector.

A correction to the drift time must be made for hits which follow primary hits on a given wire. This depends on the time separation between the hits and the amount of charge deposited for each hit. This correction will be discussed further in section 4.1.3.

Conversion of Drift Time to Drift Distance: The drift time is used to calculate the distance from the wire to the path of the charged particle which produced the

ionization. The parameterization of drift distance, D_f , as a function of drift time, T_D , is given by:

$$D_f = a(1 - e^{-b(T_D)^{1/26}}) + cT_D. \quad (4.1)$$

This form of the time-to-distance relationship has been determined using test chambers and simulations of the ionization and drift processes and the electrostatics of the final chamber [30]. At distances greater than ~ 3 mm from the sense wire plane the electric field experienced by the drifting electrons, and therefore their drift velocity, is roughly constant. This behaviour is reproduced by the parameterization shown in equation 4.1 since for large T_D the factor $(1 - e^{-b(T_D)^{1/26}})$ tends to unity and the relationship becomes:

$$D_f \approx a + cT_D$$

where c is the drift velocity in the region of constant field. Closer to the sense wire plane (T_D small) the electric field increases and the drift velocity decreases, as shown in figure 3.6. At the surface of the anode wire, the electric field becomes large enough such that electrons colliding with atoms of the gas can cause secondary ionization which causes an avalanche of ionization. In the time-to-distance parameterization, the behaviour of the drift velocity in the non-linear electric field is described by the term $a(1 - e^{-b(T_D)^{1/26}})$. The parameters a and b take on different values depending on whether, because of the wire stagger, the wire is on the near-side of the nominal anode plane or on the far-side as viewed from the hit. The b parameter is determined using test chambers and simulations, but the other three parameters must be supplied. These are: c , a_{near} , a_{far} . The same values of these parameters were used to describe all wires in all drift cells.

Once the position of the wire and the drift distance to the wire are known, the spatial coordinates of hits can be calculated.

4.1.2 Calibration Procedure

The general procedure for calibrating the vertex drift chamber is to select a suitable test variable and to calculate this quantity as a function of the various symmetries

of the drift chamber. The calibration parameters are then adjusted in order to optimize the test variable and to remove any observed systematic effects.

One such variable is the missed distance between leptons as shown in figure 3.9. However, this distance relies on the property that the tracks are separated by 180° in ϕ and can only be used for calibrating global calibration parameters or pairs of drift chamber sectors. Another test variable is the impact parameter, $d_{\perp}$, of high momentum tracks measured with respect to the average position of the e^+e^- interaction region. This average beam spot position is determined by plotting the impact parameter as a function of ϕ . An apparent displacement of the beam spot position is indicated by a sinusoidal variation of $d_{\perp}$ vs. ϕ .[†] A third test variable compares the position and slope of tracks segments reconstructed in CV with tracks reconstructed using only CJ hits.

All of these variables rely on the availability of reliable tracks reconstructed in the jet chamber. Thus, the CV calibration procedure can begin only after CJ has been calibrated. In practice, there is an amount of feedback whereby systematic effects in the jet chamber are uncovered in the CV calibration procedure which leads to a refinement of the CJ calibration constants.

The different classes of parameters described in section 4.1.1 can be calibrated as follows:

Position of CV in OPAL: Errors in the MRS parameters Δx , Δy , and α result in ϕ dependent systematic mismatches between the CJ and CV tracks. Errors in β and γ result in z dependent mismatches. Analyses performed in the $r\phi$ plane are independent of errors in Δz . The tracks reconstructed in CJ and CV are compared using the quantity $\Delta\phi$ which is the angular difference, in radians, between the track positions in the centre of CV. Then $\Delta\phi$ vs ϕ is plotted over the full range of ϕ , 0 to 2π , and is fit to the function

$$\Delta\phi = a + b \sin \phi + c \cos \phi.$$

If CV were perfectly aligned in the MRS, the curve would be a straight line through

[†]Note that this is not the way in which the position of the e^+e^- interaction region is determined for the lifetime analyses. The determination of the beam spot is discussed in more detail in section 6.5

zero. If not, this implies that the MRS parameters should be changed by an amount:

$$\begin{aligned}\alpha &\rightarrow \alpha + a \\ \Delta x &\rightarrow \Delta x + br \\ \Delta y &\rightarrow \Delta y + cr\end{aligned}$$

where $r = 13.25$ cm. Similarly, a fit of $\Delta\phi \sin \phi$ vs z to a straight line $a + bz$ would exhibit a slope if the chamber was rotated in the xz plane. A linear fit to $\Delta\phi \cos \phi$ vs z would exhibit a slope if the chamber was rotated in the yz plane.

Occasionally, rather than retuning the CJ calibration constants when a systematic effect is observed in the jet chamber, the effect is compensated for by adjusting the CV calibration constants. Figure 3.9a showed that the missed distance between leptons reconstructed in the jet chamber was displaced $44 \mu\text{m}$ from zero. This implies that the tracks originate at an annulus of radius $44 \mu\text{m}$ and not at a single point. An equivalent effect on tracks would be observed if CV were rotated in the xy plane with respect to CJ. Therefore, the observed CJ systematic effect can be removed in the final reconstructed track by adjusting the CV MRS parameter α . Figure 3.9b showed that, after this correction, the systematic offset in the missed distance is no longer present.

Wire Bows: The magnitude of the bowing of the wires in the vertex drift chamber can be determined by using the distance from the reconstructed space points from a particular wire to tracks. This quantity is called a residual. The correction to the bowing parameter from each point is then given by

$$R_i = -\delta B_w(1 - (z_i/L)^2)$$

where R_i is the residual for that point. The minus sign comes about because the residual is negative if B_w is less than it should be, *i.e.*, requires a positive correction. Minimizing the quantity

$$\chi^2 = \sum_{i=1}^N (R_i + \delta B_w(1 - (z_i/L)^2))^2$$

with respect to B_u leads to

$$\delta B_u = \frac{\sum R_i \frac{\delta R_i}{R_i}}{N - \frac{\sum R_i^2}{R_i^2} - \frac{\sum R_i^2}{R_i^2}}$$

The bows are determined to be $25 \mu\text{m}$ in the axial sectors and $15 \mu\text{m}$ in the stereo sectors.

Drift Time: An error in the T_0 timing offset parameters would shift all of the reconstructed space points either toward or away from the anode plane by a constant amount. The overall effect is a systematic shift of the reconstructed CV space points which does not depend on drift distance. The T_0 parameters, then, can be tuned by considering the test variable as a function of the local ϕ angle within the CV drift cell. Figure 4.1 shows a series of plots of the mean impact parameter as a function of the angle with respect to the CV anode plane:

$$d_0 \text{ vs } \phi \bmod(2\pi/36).$$

The x axis of these plots is ϕ in a single sector of CV, where all sectors have been superimposed; the anode plane is at 0, and the cathode planes are at ± 0.09 radians. Figure 4.1a shows the case for the optimal calibration constants. Figure 4.1b shows the case when the global T_0 parameter for the whole chamber, $T_{0,all}$, is 2 ns larger than optimal. The sector and wire specific T_0 parameters can be determined in the same way by plotting the mean d_0 for the relevant portion of the chamber

Conversion of Drift Time to Drift Distance: An error in the drift velocity, v , leads to a systematic shift in the reconstructed CV space points that does depend on drift distance. Figure 4.1c shows the mean impact parameter as a function of the ϕ angle with respect to the CV anode plane. The reconstructed space points are systematically shifted by an amount which depends linearly on the distance of the hit to the anode plane. If the T_0 parameters are also non-optimal, the curve would again be split at the anode plane as shown in figure 4.1b, but the slope of the curve would remain the same on either side of the anode plane.

The a parameters in the time-to-distance relation are T_0 -like in that the observed effect on reconstructed tracks of untuned a parameters mimics the effect seen with

Figure 4.1 Plots of the mean d_0 as a function of the ϕ angle with respect to the CV anode plane indicated by a dotted line: a) the case for the optimal calibration parameters; b) the systematic effect observed when the $T_{0,rel}$ parameter is 2 ns larger than is optimal; c) the systematic effect observed when the drift velocity parameter c is $0.5 \mu\text{m/ns}$ (approximately 1%) larger than is optimal. In each case, the data is fit to a linear function which is allowed to have a symmetric offset about the anode plane

untuned T_{ij} s. The a parameters are determined by using the distribution of missed distances between μ pairs in $Z^0 \rightarrow \mu^+ \mu^-$ events, Σd_{ij} . Using non-optimal values for the a_{near} and a_{far} parameters results in a systematic shift of the mean away from zero in the Σd_{ij} distribution. The distribution is shifted in opposite directions according to whether the ϕ angles of the tracks were less than or greater than the angle of the anode plane: $\phi_{tracks} < \phi_{anode}$ or $\phi_{tracks} > \phi_{anode}$. The a parameters were tuned by performing a scan in (a_{near}, a_{far}) parameter space in order to find values which shift the Σd_{ij} distribution to zero simultaneously for tracks on either side of the anode plane. In practice, since the effect of tuning the a parameters is so similar to the T_{ij} parameters, an iterative approach is used to determine an optimal set of all calibration parameters.

4.1.3 Second Hit Correction

In the CV drift chamber, those pulses which have a preceeding pulse recorded on the same wire, called "second hits", have different characteristics than those that do not, called "first hits", because of the additional ionization present in the cell. The signal pulse tail from the first pulse causes poorer resolution for second hits, and a systematic underestimate of the drift distance if second hits are treated in the same manner as first hits. There is also a blank-cut region of zero efficiency for recording a second hit within 1.6 mm after the first hit due to the dead time of the electronics, after which there is a rapid turn-on of the detection efficiency, rising to a plateau somewhat lower than the first hit efficiency. A particle may pass through the CV cell within the zero efficiency region after a first hit and not be recorded even though significant ionization has been created in the gas and it gives an anode pulse which will affect subsequent hits.

These effects are treated by applying an additive correction to the drift distances of second and subsequent hits. This correction is determined empirically by considering the separation of second hits from tracks which were reconstructed using only the C.J. detector. These tracks were further constrained to originate at the average position of the e^+e^- interaction region in order to remove any systematic effects such as those indicated in figure 3.9. The correction to the drift distance of second hits is parameterized

as a function of the charge deposited by the first and second hits and the time difference between them. Figure 4.2 shows the amount added to the second-hit drift distance for four different time separations.

All hits which have a preceding hit on the same wire are corrected. In a second stage of the correction, fully reconstructed OPAL tracks were extrapolated back into the relevant CV drift cell to determine if an invisible track was present. Such a track would have traversed the region of zero efficiency between the first and second hits without being recorded in CV. If so, an additional correction was added to the "second" hit (which is now actually the third particle in that CV cell) drift distance based on the number of invisible tracks and their distance from the "second" hit.

4.2 Silicon Microvertex Detector

The silicon microvertex detector was described in section 3.2.3. The extraction of a space coordinate from charge collected on a number of silicon readout strips is achieved by means of a clustering algorithm. The inherent spatial resolution of this procedure is $5\text{ }\mu\text{m}$ and is dominated by the precision with which the positions of the strips are known on the wafer. This resolution is a property of the manufacturing process of the wafer detectors and the mounting process to the ladders. The extent to which this intrinsic single-hit resolution can be translated into an improved resolution on the impact parameter of a reconstructed track in OPAL, however, is limited by the knowledge of the spatial position of the microvertex detector within the OPAL tracking system, as well as by possible distortions of the array such as non-planar shapes of the ladder faces.

4.2.1 Alignment Parameters

The alignment of the silicon array is parameterized by four levels of alignment constants:

1. the position of the array relative to the OPAL MRS coordinate system,
2. the position of ladders within the array,

Figure 4.2: The empirically determined function used to correct the second hit drift distances. The z axis is the amount added to the second-hit drift distance (in cm) as a function of the pulse heights of the first hit (x -axis) and second hit (y -axis). The correction is shown for four values of the time difference between the two hits.

3. the position of silicon wafers within a ladder,
4. deviations of planarity within a silicon wafer.

The first three levels correspond approximately to the different elements within the detector array: i) 2 barrels, ii) 11(14) ladders on the inner (outer) barrel, and iii) 3 wafers per ladder. Each element in the array has its own local coordinate system which is related to the MRS system by a known transformation matrix.

In principle, each element (barrel, ladder or wafer) of the silicon microvertex assembly should be at its nominal position and rotation with respect to the OPAL MRS. However, in practice, it is possible that any element is translated or rotated with respect to its nominal position. In this respect, a set of alignment constants are defined that can be applied to the nominal values of each element. The alignment constants consist of 3 translations ($\Delta x, \Delta y, \Delta z$) and 3 rotations (α, β, γ) about the (x, y, z) axes of the elements local coordinate system. Hence, for each ladder there are 27 alignment constants which must be determined.

The alignment constants discussed thus far are applied to planar ladders and wafers. In addition, it is possible that the ladders, and hence the wafers, are non-planar due to distortions in the ladder support. There are two non-planar alignment constants, a bow and a twist, and 27 fine survey parameters for each ladder. In addition, the correlation matrix of the alignment parameters must also be determined.

4.2.2 Alignment Procedure

Before insertion into OPAL, the entire detector array was assembled and the wafer positions were optically measured. The positions of the metrology measurement points are shown in figure 3.7. The reproducibility of these measurements was found to be at the $2\mu\text{m}$ level within a ladder assembly, but between ladders the accuracy was limited by the $5\mu\text{m}$ absolute accuracy of the coordinate measuring machine. The metrology procedure prior to insertion in the experiment provided an initial set of alignment constants. Differences between the alignment constants determined by the metrology measurements and those determined using reconstructed tracks in OPAL can be used to estimate the

systematic uncertainty on the alignment procedure. After metrology, the array was split, and the pieces were transported to the OPAL site, reassembled and inserted. Within the OPAL detector, the silicon microvertex array has been aligned by two procedures both of which use reconstructed OPAL tracks extrapolated into the array.

In the first method, ladder pairs were aligned using multihadronic decays to simultaneously align the positions of all wafers and ladders. Reconstructed tracks from multihadronic events were extrapolated into the silicon detector, and hits were assigned to them. Then, in a χ^2 minimization procedure, the track parameters and the Si hit positions are varied in order to minimize the residual between the Si hits and the tracks. This is done with the constraint that all of the tracks originate from a common point. Since the track parameters of the reconstructed OPAL tracks are themselves varied in the minimization procedure, this method is relatively insensitive to systematic effects in the central tracking system which affect the slope of the extrapolated track.

In the second method, $Z^0 \rightarrow \mu^+ \mu^-$ events are used where the two tracks are separated by 180° in azimuthal angle. The individual elements are aligned approximately in the order suggested by the four level parameterization outlined above. This procedure is similar to the procedure used to calibrate the vertex drift chamber in that a test variable is selected (for example the missed distance between leptons shown in figure 3.9) and is calculated for each element which is to be aligned. The alignment of the detector is then a matter of eliminating systematic effects seen in the test variable and of optimizing the resolution. This method is more sensitive to systematic effects present in tracks reconstructed in the central drift chambers. For this reason, the calibration of the tracking system must be performed first in the jet chamber, then in the vertex drift chamber and finally in the silicon detector.

In practice, the first method is used to determine the final alignment constants, but both methods align the detector to a similar precision which results in an effective single-hit resolution of $12 \mu\text{m}$.

Chapter 5

Monte Carlo Simulation

5.1 The KORALZ Generator

The Monte Carlo program KORALZ3.8 [31] was used to generate $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$ events. This program simulates the combined τ production and decay process and calculates the spin effects and effects of radiative corrections simultaneously. The decays of the τ which are simulated are: $\tau^\pm \rightarrow e^\pm \nu \bar{\nu}$, $\mu^\pm \nu \bar{\nu}$, $\pi^\pm \nu$, $\rho^\pm \nu$, $K^\pm \nu$, $K^{*\pm} \nu$, $a_1^\pm \nu$, $4\pi^{\pm,0} \nu$, $5\pi^{\pm,0} \nu$, and $6\pi^{\pm,0} \nu$. The following effects are taken into account:

- multiple QED hard bremsstrahlung from the initial state e^+e^- and single photon bremsstrahlung from the final state taus,
- $\mathcal{O}(\alpha)$ radiative corrections from the standard electroweak model, also hadronic and gluonic corrections,
- spin polarization (longitudinal) effects in the τ decay process (except for multi-pion decays),
- single bremsstrahlung in the most important τ decay modes (excluding the multi-pion decays).

The τ polarization in KORALZ was set to the standard model value of -0.14 , which is in agreement with the current measurement of -0.143 ± 0.017 [32].

The input τ mass was set to the previous world average $m_\tau = 1784.1 \text{ MeV}/c^2$ [1]. The input branching ratios of the various τ decays are shown in table 5.1. These branching

Mode	Rel. Γ	BR (%)
e	1.000	18.3
μ	0.973	17.8
π	0.643	11.8
ρ	1.317	24.1
a_1	0.989	18.1
K	0.046	0.8
K^*	0.098	1.8
$n\pi$	0.396	7.3

Table 5.1: The relative partial widths normalized to the electron mode, Γ , and the branching ratios of the various τ decays which were used as inputs to KORALZ

ratios were determined by scaling each of the measured world average branching ratios, by an amount proportional to its measured error, to force the sum of the branching ratios to be equal to 100%. Within the modes $\tau \rightarrow a_1 \nu_\tau$ and $\tau \rightarrow n\pi \nu_\tau$, the branching ratios of the sub-modes are given in tables 5.2 and 5.3. The input values of the mass and width

Mode	Rel. Γ	BR (%)
3π	0.47	8.5
$\pi 2\pi^0$	0.53	9.6

Table 5.2: The relative partial widths and the branching ratios within the mode $\tau \rightarrow a_1 \nu_\tau$, which were used as inputs to KORALZ.

Mode	Rel. Γ	BR (%)
$3\pi\pi^0$	0.817	6.0
$\pi 3\pi^0$	0.163	1.2
5π	0.010	0.07
$5\pi\pi^0$	0.010	0.07

Table 5.3: The relative partial widths and the branching ratios within the mode $\tau \rightarrow n\pi \nu_\tau$, which were used as inputs to KORALZ.

of the a_1 meson resonance were $m_{a_1} = 1251 \text{ MeV}/c$ and $\Gamma_{a_1} = 599 \text{ MeV}$. These values

are consistent with recent measurements [33].

The output of the generator program is an ASCII file of the 4-vectors of each of the particles in a $Z^0 \rightarrow \tau^+ \tau^-$ event. This file is then read into a program which simulates the response of the OPAL detector: GOPAL.

5.2 GOPAL

The GOPAL simulation program [34] is based on the CERN GEANT [35] simulation package. GEANT provides a set of three-dimensional geometrical shapes that can be used to define the geometry of a complete detector. Once the geometry and the material for each of the elements of the OPAL detector is defined, GEANT routines are also used to track the particles through these elements. When a particle traverses a sensitive volume in the simulated detector, its position and momentum are stored (a "hit"). At each step through the detector elements, multiple Coulomb scattering and energy loss of the particle are calculated. Effects due to secondary processes such as hadronic interactions in the detector materials, photon pair production, delta rays, and particle decays are also included. At the end of this stage, all of the "hit" information is collected and the detector response is simulated. Detector resolution effects and inefficiencies are included at this point. The output of this stage is a data structure identical to that produced by the OPAL online data acquisition system and filled with simulated raw data. This simulated data can then be processed using exactly the same reconstruction software as for the real data, and a DST is created.

In the case of Monte Carlo simulated data, however, the DST contains additional information which includes the identities of all the particles in the interaction and their kinematic properties before the detector simulation stage. In this way it is possible to calculate: i) the detector acceptance for a particular particle decay, and ii) the size of any possible bias in the measurement of the properties of the particles.

Since the τ lifetime analysis uses primarily the tracking system in the $r\phi$ plane, a description of the simulation is given only for the $r\phi$ tracking detectors. The full details

can be found in [34].

5.2.1 The Silicon Microvertex Detector

The silicon detector consists of 25 ladders of $300\text{ }\mu\text{m}$ thick silicon arranged in 2 cylindrical layers. A minimum ionizing particle traversing the silicon layer would deposit an average energy of 84 keV (1 MIP). The charge that would be collected in the real detector is assumed to be proportional to the path length traversed by a track as it passes through the silicon. An internal field, E , transports the charge carriers to the surface of the silicon. The radius of the charge cloud, σ , at the surface is typically $10\text{ }\mu\text{m}$ and is determined from the mean distance that the charge has to travel, dx , the diffusion coefficient, D , and the mobility, μ , of the charge carriers via the relationship $\sigma = \sqrt{\frac{2Ddx}{\mu E}}$ where $D = 12.3\text{ cm}^2\text{s}^{-1}$ and $\mu = 480\text{ cm}^2(\text{Vs})^{-1}$. The diffuse charge is then collected on the strips within $\pm 3\sigma$ from the centre of the charge cloud.

In order to simulate the division of charge between the capacitively coupled non-readout and readout strips, it is assumed that the intermediate strips lose 50% of their charge to each of their neighbouring readout strips, thus having the effect of making the charge division more linear.

A simple cluster finding algorithm is employed, where a cluster is defined as any channel whose pulse height is $>4.0\sigma$ (where σ is the RMS noise) and includes the ± 2 neighbouring channels if they have pulse heights $>1.5\sigma$ and are contiguous. In multihadronic events, the clusters in the real detector are found to have a signal-to-noise of $\sim 20:1$. Random noisy channels are therefore not simulated since they can be removed with high efficiency in data using a pulse height cut. However, random noise is added to the pulse heights produced by hits from tracks and is taken from a Gaussian distribution with a width of 0.05 MIP. The single hit efficiency of a ladder has been determined from the data to be 97%. The digitized Monte Carlo output is stored in a data structure identical structure to that of the data, and pulse heights are stored in units of MIPs.

Figure 5.1 shows a comparison of data with the GOPAL simulation of the Si microvertex detector. In general the agreement is good. The small discrepancies at low and high pulse heights in the energy deposition arise because the online processing

algorithm and anomalously high pulse heights are not yet simulated in GOPAL.

5.2.2 The Vertex Drift Chamber

Charged tracks are followed through the vertex chamber using the standard GEANT tracking. The drift volume for each wire in a CV cell is defined by two planes, which define the path taken by charge produced in the gas as it travels towards a particular anode wire. These planes are placed at half the wire spacing either side of a wire, tilted by the Lorentz angle with respect to the perpendicular to the anode plane. The points at which a track crosses these planes define the length over which the track produces the ionization which would be collected on that wire. These drift volume entry and exit points are stored though they may not necessarily survive the simulation of the detector response.

The effective drift distance from each hit to the anode wire is calculated from the average of the distances to the anode plane for the points of entry and exit for each drift volume, combined with the components of the electrostatic bowing and the wire stagger in the drift direction. The spatial resolution of the detector in the $r\phi$ plane depends on the drift distance for the hit and has been parameterized by a function fitted to real data. In the simulation, the resolution for second and subsequent hits takes the same form in drift distance, but is degraded by an additional factor. Once the resolution for a given hit has been calculated, the drift distance is smeared accordingly.

For all classes of hits, a small fraction fail to be recorded due to inefficiencies. These inefficiencies are measured from the data, and a proportion of the hits are discarded on a pseudo-random basis. The efficiency for registering second hits is lower than that for registering first hits. The rate of the turn-on efficiency for second hits, after the region of zero efficiency, depends on the number of tracks that passed between the current hit and the previous hit including tracks which fall in the region of zero efficiency and are not recorded in CV. All of these effects are modelled, using parametrisations taken from the data.

The effective drift distance is then converted to a TDC value, to simulate the raw data from the real detector, using the known clock frequency, and an additional time

Figure 5.1: Distributions for the silicon microvertex detector in multihadronic events shown for data (solid line) and Monte Carlo (points), normalized to equal numbers of events. The distributions shown are a) the number of silicon hits per event in the inner silicon layer, b) the cluster pulse height distribution, which is a typical Landau distribution, and is scaled to be units of MIPs, c) the charged track multiplicity distribution for tracks with two associated silicon hits, d) the impact parameter distribution for tracks with two associated silicon hits

offset contribution is added at this stage to model the time delay between the starting of the TDC clocks and the arrival of the earliest pulses.

In the real detector there is a noise background, unassociated with particles from e^+e^- interactions, which is largely due to synchrotron radiation. The level of this background per wire is obtained from looking at random beam crossings in the real data. Hits are then generated according to this distribution at the beginning of the detector simulation step, and can contribute to the second-hit effects described above.

Figure 5.2 shows the agreement with the real data of the simulation for the first-hit resolution and the second-hit separation distance parameters.

5.2.3 The Jet Chamber

In a manner similar to CV, GEANT extrapolates a track through the jet chamber and generates spatial coordinates. From these space points, the jet chamber simulation calculates the anode wires on which signals will be produced, and determines the positions with respect to the wires. These measurements are then smeared to account for the detector resolution and for calibration effects. The resolution effects implemented in GOPAL are:

- Drift time resolution $\sigma(t)$. This depends on the length of the drift path (the distance between track and anode wire) and on the angle of the track with respect to the anode wire plane, Φ_{local} . These dependences were parameterized using OPAL data. For 1.6% of all hits, the resolution is degraded to $14 \times \sigma(t)$ to take into account unresolvable γ -rays.
- Double pulse resolution. The capability to resolve two close hits has been studied with real data and parameterized. In accordance with data, hits start to be resolved at a separation distance of 1.9 mm with an efficiency that rises to 80% at a distance of 3.4 mm. The resolution for the second hit is degraded up to 250 μm .

Noise effects and inefficiencies are also treated in GOPAL as follows. In accordance with OPAL data, the efficiency for finding a hit is set to 99.85%. Noise in the chamber is simulated by generating hits not assigned to tracks. Such spurious hits may

Figure 5.2: Comparison of real data (solid line) and Monte Carlo (points with error bars) for the vertex chamber: a) the resolution in $r\phi$ using triplets of next-but-one first hits in the vertex chamber axial cells b) the difference in drift distance between axial second hits and the previous hit on the same wire

be correlated to real hits or are totally randomly distributed over the chamber. The former can arise from fluctuations of the FADC pulses or from low energy δ -rays which are just resolved, and these effects are simulated by the generation of an additional 2.5% of all hits, which are delayed in time relative to real hits. Uncorrelated noise is simulated by the production of, on the average, 12 random hits per event to account for synchrotron radiation.

Using the Lorentz angle and the drift velocity, the two-dimensional space point resolution σ_{xy} results from the drift time resolution $\sigma(t)$. The result of the detector simulation step is a set of five numbers which fully describe each measured point: wire number, drift time, z -coordinate, integrated charge of the FADC pulse and a bit pattern describing the quality of the measurement. These form the input for calibration and pattern recognition procedures to reconstruct the jet chamber tracks, and correspond to the real data after the (online) analysis of the FADC pulses. Figure 5.3 shows the agreement with the data of the simulation for some CJ parameters.

5.3 Quality of the Simulation of Reconstructed Tracks

Figures 5.1, 5.2 and 5.3 summarized the quality of the GOPAL simulation of each of the component detectors in the OPAL tracking system. The quality of the simulation of reconstructed tracks, however, depends on the simulation not only of the properties of the individual tracking detectors but of the track reconstruction process as well. Figure 5.4 shows the agreement of the simulation for some of the global properties of tracks from τ decays. More detailed comparisons between the Monte Carlo simulation and real data will be presented as part of the impact parameter τ lifetime analysis in chapter 7.

Figure 5.3: Comparison of real data (solid line) and Monte Carlo (points with error bars) for the jet chamber: a) space point resolution $\sigma_{r\phi}$, as determined by triplets of neighbouring wire measurements, as a function of the drift distance from the signal wires, b) $\sigma_{r\phi}$ as a function of the local track angle ϕ_{local} with respect to the sense wire plane. (This distribution is not symmetric about zero because of the non-zero Lorentz angle: $\sigma_{r\phi}$ is minimum for tracks perpendicular to the drift direction) c) efficiency to separate two hits as a function of the hit separation

2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

Figure 5.4: Comparison of real data (solid line) and Monte Carlo (points with error bars) for reconstructed tracks: a) the number of first hits in the vertex drift chamber per track; b) the number of second hits in the vertex chamber per track; c) the number of jet chamber hits per track; d) the track transverse momentum which is the momentum of the track projected into the xy plane

Chapter 6

Data Sample

6.1 OPAL Data

There are two factors which make the various final states of e^+e^- collisions, especially at LEP, easy to identify and free from background. The colliding particles are not composite objects, such as protons for example, so there are no spectator particles in the interaction. Any particle seen in the detector is related to the e^+e^- interaction. Secondly, e^+e^- collisions take place in the centre-of-mass reference frame and this has implications for the selection of individual decay modes of the Z^0 , discussed in section 1.2, and for the rejection of background processes.

As was discussed in section 1.1, quarks cannot be seen in isolation but instead produce jets of hadronic particles. These multihadronic events from Z^0 decays to quarks are immediately recognizable by the high multiplicity of particles seen in the detector. Figure 6.1 shows an event display of a reconstructed multihadronic Z^0 decay in the OPAL detector. Seventy percent of Z^0 decays are multihadronic, thus events of this type constitute the largest portion of OPAL data.

By contrast, an interaction of the type $e^+e^- \rightarrow e^+e^-$ or $e^+e^- \rightarrow \mu^+\mu^-$ has only two charged tracks per event. Each track has a momentum equal to the original beam momentum and there are no unseen particles in the event. Since the electron and positron beams have equal energy, the net momentum of the final state particles must sum to zero. The tracks from $e^+e^- \rightarrow \ell^+\ell^-$ events are therefore collinear, or back-to-back. Electrons are distinguished from muons by the amount of energy deposited in the

Run event 2569 21509 Date 911029 Time 14655 Ctra(N)= 33 Sum= 12.5 Ecen(N)= 48 Sum= 29.4 Evis(N)= 11 Sum= 1.0
 Ebeam 44.727 Evis 39.9 Emiss 10.4 Vtx (0.06, 0.11, 0.19) Muon(N)= 0 Ecen Vtx(N)= 11 Evis(N)= 11
 Pz=4.350 Thrus=0.8837 Aplan=0.0346 Qstat=0.1562 Sphic=0.1774

Figure 6.1: An event display of an $e^+e^- \rightarrow \text{hadrons}$ event reconstructed in the OPAL detector. Beginning at the centre of the detector, the radial lines are the reconstructed tracks of charged particles. These are followed by hits in the time of flight scintillators. Energy recorded in the calorimeters is represented by trapezoids whose radial dimension increases logarithmically with the amount of energy deposited. The first ring represents the electromagnetic calorimeter, and the second represents the hadronic calorimeter. The path of muons traversing the muon chambers is indicated with an arrow.

electromagnetic calorimeters: Electrons deposit all of their energy in the lead glass, and muons leave almost none.

The selected $e^+e^- \rightarrow \tau^+\tau^-$ events have from two to six charged tracks per event. The taus in the $\tau^+\tau^-$ pair are also produced collinearly with each τ having a momentum equal to the original beam momentum. However, in this case there is at least one neutrino in each τ decay which goes undetected. Thus, the tracks observed in the detector in an $e^+e^- \rightarrow \tau^+\tau^-$ event are characterized by being acollinear, and the magnitude of the momentum of the tracks is less than the original beam momentum. A typical $e^+e^- \rightarrow \tau^+\tau^-$ interaction was illustrated in figure 1.2. Section 2.1 summarized the various decay modes of the τ . Figures 6.2 and 6.3 show event displays of two $e^+e^- \rightarrow \tau^+\tau^-$ events reconstructed in the OPAL detector.

6.2 Selection of $\tau^+\tau^-$ Events

The selection of $Z^0 \rightarrow \tau^+\tau^-$ events was performed as part of the OPAL analysis to measure the leptonic partial widths of the Z^0 decay. An outline of the procedure is given here, and the full details can be found in [2].

The geometric acceptance of events at OPAL is defined in terms of the event thrust axis which is an axis that maximizes the parallel momentum components of the observed decay particles in the event. Defining a unit vector $\hat{n}$ along the thrust axis, the quantity maximized is:

$$\text{thrust} = \max \left[\frac{\sum_i |\mathbf{p}_i \cdot \hat{n}|}{\sum_i |\mathbf{p}_i|} \right] \quad (6.1)$$

where $\mathbf{p}_i$ are the particle momenta in the event. The geometric acceptance for $\tau^+\tau^-$ events is $|\cos \theta_{\text{thrust}}| \leq 0.9$.

Candidate $\tau^+\tau^-$ events are initially selected on the basis of the number of tracks observed in the event and the amount of energy observed in the electromagnetic calorimeter. Contiguous lead-glass blocks are grouped into "clusters". The definitions of a good track and cluster are given in table 6.1. Good tracks are required to originate at the centre of the detector, to have a sufficient number of hits in the tracking system, and to have a minimum momentum of 0.1 GeV/c. Good clusters are required to pass a minimum

Run event 2566 164371 Date 911027 Time 16:06:06 Vtx(N)= 6 Nump= 4043 Evt(N)= 1 Numb= 11 Numb(N)= 2 Numb= 11
 Ebeam 45.597 Evis 41.5 Emiss 49.7 Vtx (0.10, 0.11, 0.00) Muon(N)= 11 Numb(N)= 0 Numb(N)= 0 Numb(N)= 0
 Ez=0.001 Truss=0.9930 Aplan=0.0000 Dblat=0.0276 Npnt=0.0004

Figure 6.2: An event display of a $1-1 e^+e^- \rightarrow \tau^+\tau^-$ event reconstructed in the OPAL detector. The graphical representations of each detector component were summarized in figure 6.1. One of the taus decays to an electron and the other to a muon. The decay to a muon illustrates the three characteristic signals used in muon identification as discussed in the text.


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run event 2566195035 Date 911027 Time 200934 Ctx(N= 4 SumE= 24.5) Ecal(N= 5 SumE= 26.5) Hcal(N= 3 SumE= 1.2)
beam 45.597 Evis 25.8 Emiss 65.4 Vtx ( 0.07, 0.11, 1.22) Muon(N= 0) Sec Vtx(N= 0) Fcal(N= 0 SumE= 0.0)
sz=0.002 *trus=0.9944 Aplan=0.000 Oblat=0.0122 Spher=0.0009

```

Figure 6.3: An event display of a $1-3 e^+e^- \rightarrow \tau^+\tau^-$ event reconstructed in the OPAL detector. The graphical representations of each detector component were summarized in figure 6.1. One of the taus undergoes a hadronic 1-prong decay, most likely to a ρ meson (see equation 2.1 and table 5.1). The other tau undergoes a 3-prong decay, possibly to an α_1 meson.

energy threshold. Because a lifetime measurement has a need for well reconstructed tracks, some additional requirements will be applied before any of these tracks are used in the actual lifetime analyses. These requirements will be described in chapters 7 and 8.

Tracks		
$ d_0 $	$<$	1.0 cm
$ z_0 $	$<$	20.0 cm
# hits	$\geq$	20
p_t	$>$	0.1 GeV/c
r_{min}	$<$	75.0 cm
Barrel Clusters		
E_{raw}	$>$	0.1 GeV
Endcap Clusters		
E_{raw}	$>$	0.2 GeV
# blocks	$\geq$	2
F_{max}	$<$	0.99

Table 6.1: Track and electromagnetic cluster definition cuts to reject poorly reconstructed tracks and noisy clusters. The symbol r_{min} represents the radius of the first hit on the track, and F_{max} the fraction of the energy in the most energetic block. The other symbols were defined in section 3.5

Tau pair events are selected by rejecting, in turn, each of the other Z^0 decay modes and the known background processes. Multihadrons are rejected by exploiting the property of high charged track and electromagnetic cluster multiplicity. In addition, in the selection of leptonic events, a cone analysis is used. The algorithm to divide the event into cones goes as follows. The highest energy track or cluster in the event is selected to define a direction, then the momentum of the next highest energy track or cluster within a cone of 35° half-angle is added to define a new direction. This procedure is iterated until there are no more tracks or clusters remaining within that cone. The procedure is then repeated with a new cone using the next highest energy track or cluster which remains available in the event and continues until there are no remaining tracks or clusters unassigned to a cone. The algorithm is such that isolated clusters not associated with any track will define a neutral cone; a charged cone is one which contains tracks. The specific cuts to reject multihadrons from the tau pair event sample are:

multihadron rejection		
# tracks	<	7
# tracks+clusters	<	16
# charged cones	=	2

After these requirements, the residual multihadron contamination for $e^+e^- \rightarrow \tau^+\tau^-$ events where both taus undergo a 1-prong decay is negligible, and for events containing 3-prong τ decays about 1.2%. Events which are identified as multihadrons for the OPAL analysis to measure the mass of the Z^0 are explicitly vetoed in the 3-prong decay length analysis. This eliminates about 10% of the 3-prong candidates and reduces the residual multihadron contamination to 0.4%.

Two-photon events, in which photons radiated from a beam electron and a beam positron collide producing particles observed in the detector, can be identified by the small amount of energy deposited in the detector. In addition, because the energies of the two photons are not necessarily equal, the events are not produced in the centre of mass reference frame and tend to be acollinear. Two-photonic events, then, can be removed from the $e^+e^- \rightarrow \tau^+\tau^-$ sample by cuts on the total visible energy and the acollinearity angle which is defined to be $180^\circ - (\text{angle between charged cones})$:

2- γ rejection		
acollinearity angle	<	15°
$E_{clus} + E_{trck}$	>	0.18

Figure 6.4 shows a plot of the acollinearity angle in OPAL events after the rejection of multihadrons. It can be seen that a cut at 15° has no effect on the expected $\tau^+\tau^-$ signal. Figure 6.5 shows a plot of the total visible energy seen in the detector in electromagnetic clusters and charged tracks normalized to the centre-of-mass energy: $E_{clus} + E_{trck}$. The upper plot shows the visible energy after only multihadron rejection, and the lower plot shows the same distribution including the acollinearity cut. A cut to remove two-photon background events is placed at 0.18. There is a large overlap between the events eliminated by the visible energy and the acollinearity cuts. The peak at a visible energy of 1 is due to $e^+e^- \rightarrow \mu^+\mu^-$ events since muons have a momentum equal to the beam momentum and they deposit very little energy in the electromagnetic calorimeter. The peak at a visible energy of 2 is due to $e^+e^- \rightarrow e^+e^-$ events where now, in addition to producing high

Figure 6.4: A plot of the acollinearity angle in OPAL events after multihadron rejection. The clear histogram is OPAL data, and the shaded histogram shows the equivalent distribution for $\tau^+\tau^-$ events simulated with the Monte Carlo, and normalized to the number of τ pair events expected in the data. A cut on the acollinearity angle is placed at 15° . (The vertical axis is the number of counts on a logarithmic scale in arbitrary units.)

momentum tracks, electrons deposit their entire energy in the electromagnetic calorimeter and these two contributions are counted twice in determining the visible energy.

Section 3.2 described in general terms how electrons can be identified in OPAL on the basis of their energy deposition in the lead-glass calorimeter. The principal cut to eliminate $e^+e^- \rightarrow e^+e^-$ events from the $e^+e^- \rightarrow \tau^+\tau^-$ sample is a cut on the total observed energy in the electromagnetic calorimeter:

$$\begin{array}{rcl}
 & e^+e^- \rightarrow e^+e^- \text{ rejection} & \\
 \hline
 E_{clus} & < & 0.8 \text{ (barrel only)} \\
 E_{clus} + E_{trck} & < & 1.05 \text{ (endcap only)} \\
 \text{or } E_{clus} & < & 0.25 \text{ (endcap only)}
 \end{array}$$

Figure 6.6 shows a plot of the electromagnetic energy in an event after the rejection of multihadronic and two-photonic events. This distribution displays and separates the three different leptonic final states. The peak at low cluster energy is due to $e^+e^- \rightarrow \mu^+\mu^-$ events. The peak at cluster energies of 1 is due to $e^+e^- \rightarrow e^+e^-$ events.

There are three expected signatures for a muon from an $e^+e^- \rightarrow \mu^+\mu^-$ event: i) hits in the muon chamber, ii) hits in the outer layers of the hadronic calorimeter, and iii) a high momentum track with very little energy deposited in the electromagnetic calorimeter. These three signatures are illustrated with a $\tau \rightarrow \mu\nu\bar{\nu}$ decay in figure 6.2. A particle is called a muon if it satisfies at least one of the above three criteria, but there is a high degree of redundancy in these criteria and, in practice, most muons satisfy more than one. For further details see [2]. Two muons which are collinear, to within 18° in the xy projection, are classified as a $Z^0 \rightarrow \mu^+\mu^-$ event and are excluded from the τ pair sample.

In the barrel part of the detector, background from cosmic rays is rejected by using the time of flight detector. At least one signal in the TOF detector is required within 10 ns of the expected time for a particle to travel from the e^+e^- interaction. This time is measured by the TOF detector with a resolution of 460 ps [19]. In addition, a requirement is made over the entire geometric acceptance region that the particles originate at the centre of the detector. The cuts to remove background from cosmic rays are:

Figure 6.5: A plot of the visible energy in OPAL events normalized to the centre-of-mass energy. The upper plot is after only multihadron rejection, and the lower plot includes the acollinearity cut. The clear histogram is OPAL data, and the shaded histogram shows the equivalent distribution for $\tau^+\tau^-$ events simulated with the Monte Carlo, and normalized to the number of τ pair events expected in the data. A cut to remove two photon events is placed at 0.18. (The vertical axis is the number of counts in arbitrary units.)

Figure 6.6: A plot of the electromagnetic energy in the barrel after the rejection of multihadron and two-photon events. The energy is normalized to the centre-of-mass energy. The clear histogram is OPAL data, and the shaded histogram shows the equivalent distribution for $\tau^+\tau^-$ events simulated with the Monte Carlo, and normalized to the number of τ pair events expected in the data. A cut to remove $e^+e^- \rightarrow e^+e^-$ events is placed at 0.8. (The vertical axis is the number of counts in arbitrary units.)

	cosmic ray veto
time of flight	veto ($\cos \theta_{\text{flight}} > 0.8$)
minimum d_{cut}	0.5 cm
average z_0 of all tracks	20.0 cm

From the Monte Carlo simulation, the geometrical and kinematic acceptance including efficiencies of the overall tau pair selection selection is $(76.9 \pm 0.2)\%$.

6.3 Data Samples for the τ Lifetime Analysis

LEP began operations in the fall of 1989 and operates for about 6 months every year. The results described in this thesis are for the data collected in 1990 and 1991. As discussed in chapter 3, the silicon microvertex detector was installed in the shut-down period in between 1990 and 1991. It became fully operational in the latter half of the 1991 data taking period.

The combined 1990/1991 data set contains 17553 $\tau^+\tau^-$ candidates, with 5206 from the 1990 data and 12347 from the 1991 data. Of the selected 1991 data, 9014 events (69%) contain silicon microvertex detector information. The τ lifetime analysis requires a precise measurement of the average position of the e^+e^- interaction region. Section 6.5 describes how this measurement is made. Only events with good estimations of the position of the interaction region were included in the lifetime data sample, which eliminates 149 $\tau^+\tau^-$ candidates (0.8%), for a total of 17404 $Z^0 \rightarrow \tau^+\tau^-$ candidate events. These events contain 27569 1-prong τ decay candidates and 5145 3-prong candidates before the quality requirements specific to each τ lifetime analysis.

The silicon microvertex detector was a new addition to OPAL and was operational only for a portion of the data collected in 1991. For this reason, the event reconstruction track fits were performed twice: once using only the central drift chambers and a second time using the drift chambers and the microvertex detector information if it was available. For those events in which microvertex information was present, both sets of track parameters were made available. For the data collected in 1990, there was only one choice: the central drift chamber track fit. For 70% of the data collected in 1991, however, there was the option of including or ignoring the Si detector information

Therefore, two scenarios exist and have been carried out for the analysis of the data collected in 1990 and 1991: i) Si is not used in the track fits, and ii) Si information is used where available. The data in the second scenario can be further subdivided into in three segments: 1990, 1991 data with silicon information not available (the beginning of the 1991 data taking period as well as events outside the Si angular acceptance region), and 1991 data with silicon information available and added to the track fit.

6.4 Backgrounds in the τ Data Sample

The fraction of background events in the $\tau^+\tau^-$ sample was calculated using Monte Carlo event samples of the different physics processes. Multihadronic events were generated using both the JETSET [37] and the HERWIG [38] generator programs. The program BABAMC [39] was used to generate $e^+e^- \rightarrow e^+e^-$ events and KORALZ [31] for $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$ events. Backgrounds from two-photon processes were studied using events generated by the Monte Carlo program of Vermaseren [40] which simulates both hadronic and leptonic final states. The $\tau^+\tau^-$ event acceptance and the level of each background from the competing processes were determined by passing these Monte Carlo simulated events through the OPAL reconstruction code and selection algorithms.

Cross checks were made with the background data samples and additional contributions were added to correct for known inefficiencies of the detector simulation. For example, a slight increase in the background from $Z^0 \rightarrow \mu^+\mu^-$ events is present in the data where there is a known probability to mismeasure a track close to a jet chamber sense-wire plane which is not simulated in the Monte Carlo program. Further details can be found in [2]. Table 6.2 shows the estimated background fractions listed separately for 1-prong and 3-prong τ decay samples used in the lifetime analysis.

Background process	% of τ decays	
	1-prong	3-prong
multihadrons (%)	0.06 ± 0.30	0.40 ± 0.40
$e^+e^- \rightarrow e^+e^-$ (%)	0.21 ± 0.43	0
$e^+e^- \rightarrow \mu^+\mu^-$ (%)	1.24 ± 0.34	0
two-photon (%)	0.61 ± 0.25	0
total background (%)	2.1 ± 0.7	0.4 ± 0.4

Table 6.2: Backgrounds to the $\tau^+\tau^-$ event sample separated into 1-prong and 3-prong decay lifetime samples.

6.5 The e^+e^- Interaction Region at LEP

The electron and positron beams collide in the centre of the OPAL detector. The τ lifetime is determined by measuring the small but finite distance between the e^+e^- annihilation point and the position at which the τ decays and therefore requires precise knowledge of the position of the e^+e^- interaction region within the detector. In addition, the mean position of the interaction region changes from fill to fill in LEP and, sometimes, within a single fill. The OPAL algorithm to measure the average position of the e^+e^- interaction region allows movements of the interaction region to be tracked with high precision.

The determination of the average position of the interaction region proceeds as follows. The procedure uses a sample of charged tracks from both leptonic and hadronic decays of the Z^0 that satisfy the following criteria:

- $p_t > 0.25 \text{ GeV}/c$
- $|d_0| < 1 \text{ cm}$
- $\sigma_{d_0} < 0.1 \text{ cm}$

where p_t is the track momentum in the plane orthogonal to the beam direction, d_0 is the distance of closest approach of the track to the coordinate origin, and σ_{d_0} is the estimated uncertainty in d_0 . Tracks from consecutive events are accumulated until there are at least 100 such tracks. A χ^2 fit is then performed for a vertex in the xy plane. If any tracks

contribute more than 10 to the χ^2 of the vertex fit, the worst track is discarded and a new fit performed. This procedure is iterated until all tracks have $\chi^2 < 10$ with respect to the effective vertex. Given 30 such sets of vertex coordinates, which correspond typically to about 200 Z^0 decays, a weighted mean is taken to determine the mean interaction region position. After the coordinates for a given set of data is obtained, adjacent measurements that are statistically compatible are combined. This process results in 1 or 2 measurements per LEP fill of the average position of the e^+e^- interaction region. The typical statistical precision of the mean position in $x(y)$ is $15(10)\mu\text{m}$. The coordinates of the average interaction region for the 1990 and 1991 data taking periods are plotted in figure 6.7 as a function of the LEP fill number.

Once the position coordinates are known, the distribution of impact parameters of well-measured tracks (small σ_{d_0}) can be used to determine the intrinsic size of the e^+e^- interaction region. The vertical size is expected to be very small and so the width of the d_0 distribution of horizontal tracks can be taken as a measure of the tracking resolution. This width is subtracted in quadrature from the width measured for vertical tracks to calculate an average value of the interaction region size in the x -direction. The spread of the interaction region position coordinates in x as determined by the above method was $(157 \pm 5)\mu\text{m}$ for the data collected during 1990 and the first part of 1991. In the latter part of 1991 the LEP beam parameters were changed with the result that the interaction region size in x was reduced to $(147 \pm 4)\mu\text{m}$. The width of the $|z_0|$ distribution implies a longitudinal beam interaction size of $(1.12 \pm 0.02)\text{cm}$. These measured dimensions of the beam spot size agree with expectations based on LEP beam optics [36]. The extent of the $\tau^+\tau^-$ production region is taken as the quadrature sum of the e^+e^- interaction region size and the error on the measurement of its average position.

Events from data periods in which there was not enough data for an accurate determination of the average e^+e^- interaction region position were rejected. The uncertainties in the interaction region position and size are treated in the systematic error evaluations which will be discussed in chapters 7 and 8.

The τ pair Monte Carlo event samples for both the 1990 and 1991 samples were generated with e^+e^- interaction region width $(\sigma_x, \sigma_y) = (157, 8)\mu\text{m}$. The reduction of the LEP beam size to $\sigma_x = 147\mu\text{m}$ corresponded to the period of data for which

Figure 6.7: A plot showing the average x and y position of the e^+e^- interaction region as a function of LEP fill number. The first group of data is for 1990 and the second is 1991. Occasionally there is more than one beam position for a single fill number indicating that the collision point of the beams had moved.

the Si detector information was available. Consequently, in analyzing the data including Si information, the horizontal beam size of the Monte Carlo events was rescaled to the lower value. In addition, the Monte Carlo tracking resolution was adjusted to bring it into agreement with the observed data properties. The effects of variations in the Monte Carlo beam widths for the 1-prong impact parameter measurement are discussed in chapter 7. The 3-prong decay length measurement is independent of the Monte Carlo simulation of the beam properties.

Chapter 7

The Impact Parameter Method

Section 2.4 summarized the various techniques for measuring the τ lifetime. One of the two techniques used by OPAL is the impact parameter method. This method can be applied to all tracks from τ decays, but in order to remain statistically independent of the 3-prong decay length measurement to be presented in chapter 8, this analysis is limited to the 1-prong τ decay sample.

7.1 The Mean Impact Parameter

The impact parameter, d_0 , of a τ decay product with respect to the τ origin is related to the τ decay length, ℓ , and the angle, ψ , between the decay track and the τ direction by: $d_0 = \ell |\sin \psi|$. Figure 2.4 illustrated the definition of d_0 , and figure 7.1 illustrates the signing convention. For measuring lifetimes, the signing convention is to assign a positive sign to d_0 if the decay product track crosses the initial τ direction in the same hemisphere as the track, and a negative sign if the track crosses the τ direction behind the interaction point, *i.e.*, in the opposite hemisphere as the track itself. The $e^+e^- \rightarrow \tau^+\tau^-$ events are divided into two hemispheres by a plane perpendicular to the event thrust axis calculated using all charged tracks in the event, see equation 6.1.

The mean impact parameter is related to the mean decay length and so to the mean τ lifetime. The impact parameter method consists of forming the signed d_0 distribution of τ decay tracks and comparing the data mean to the mean of a corresponding Monte Carlo distribution in order to extract the lifetime.

Figure 7.1: Illustration of the impact parameter lifetime signing convention: A positive sign is assigned to d_0 if the 1-prong decay product track crosses the initial r direction in the same hemisphere as the track, and a negative sign if the track crosses the r direction behind the interaction point. The initial r direction is approximated by the event thrust axis.

In the ideal situation, a particle coming from a τ decay will always cross the actual τ path in front of the interaction point and would always be given a positive sign. This ideal $d_{\parallel}$ distribution is shown in figure 7.2a obtained from a Monte Carlo study.

It is impossible to achieve this ideal distribution in practice for three reasons. First, we do not know the direction of the initial τ lepton. Because each of the two τ decays in an event contains at least one neutrino, which are not observed, it is impossible to reconstruct the direction of the original taus. We approximate the τ direction by the event thrust axis as defined in equation 6.1. The direction of the thrust axis is determined by the decay kinematics of a particular $\tau^+\tau^-$ event. The greater the fraction of the original τ momenta which is visible in the event, the closer the thrust axis will lie to the original τ direction. Figure 7.2b shows the same ideal impact parameters as in figure 7.2a but now signed with a thrust axis calculated from the momenta of all charged particles in the event before detector simulation effects are considered. Using the thrust axis to sign the impact parameters produces negative entries and results in a significant reduction of the mean of the distribution. The mean and the RMS width of each case are shown in the figure.

Secondly, an ideal $d_{\parallel}$ distribution is impossible in practice because it is not known where in the beam envelope the e^+e^- have collided so the production point of the τ is not exactly known. We must, therefore, make the approximation that all taus are produced at the centre of the beam ellipse. Figure 7.2c shows the distribution of ideal impact parameters signed with respect to the charged particle thrust axis but this time the $d_{\parallel}$ is measured with respect to the centre of the beam ellipse not the τ production point. The distribution is significantly wider than previously, but the shift in the mean is negligible.

Finally, each $d_{\parallel}$ is measured with the finite detector resolution capabilities of the OPAL detector which has the result of further smearing the $d_{\parallel}$ distribution. Figure 7.2d shows the Monte Carlo $d_{\parallel}$ distribution after the simulation of all detector effects. It can be seen that the distribution is again widened but the mean is not appreciably shifted.

To summarize, the experimentally measured impact parameter distribution of 1-prong τ decays becomes smeared by the combination of these three effects and acquires negative entries. Of these, only the negative impact parameters introduced by the use of the thrust axis direction have a significant effect on the mean value of the $d_{\parallel}$ distribution.

Figure 7.2: Distributions of the impact parameter of Monte Carlo tracks. a) the ideal d_0 distribution of Monte Carlo 1-prong τ decay products before detector resolution effects, and signed using the true τ direction; b) the same distribution as in a) but signed with a charged particle thrust axis; c) the distribution of impact parameters defined with respect to the average beam position signed with the same thrust axis as in b); d) the distribution of impact parameters of Monte Carlo reconstructed tracks including simulation of all detector resolution effects, using the vertex drift chamber, and signed with a reconstructed charged track thrust axis. In each case, the mean and the RMS width are given in cm.

The tracking resolution and e^+e^- interaction region size produce a broadening of the measured width of the distribution, without appreciably shifting the mean. Although the detailed shape of the detected d_1 distribution is difficult to predict, its significant positive shift can be used, in comparison to a Monte Carlo prediction, to extract a measurement of the τ lifetime.

7.2 The Thrust Axis

Since it is the approximation of the τ direction by the thrust axis which mainly modifies the mean of the d_1 distribution, an axis which uses the maximum amount of experimental information possible, *i.e.*, includes both charged tracks and neutral electromagnetic cluster momentum vectors, would introduce the fewest negative sign entries and, in principal, be the optimal choice. However, at the same time, the 1-prong d_1 analysis relies on the quality of the Monte Carlo simulation of this axis, and it is desirable to have a uniform and well understood detector throughout the entire acceptance region. The OPAL tracking detectors have such uniformity whereas a break exists in the electromagnetic calorimeter with the overlap region in between the barrel and the endcap, $0.7 < |\cos \theta| < 0.82$, not being very well simulated. In addition, 56% of τ decays involve only charged observable particles and would not benefit from a thrust axis including EM clusters. Thus, for this analysis, the τ direction is approximated by the event thrust axis calculated using charged tracks only. For reference, the average angular difference in ϕ between the true τ direction and this event thrust axis is about 30 mrad for the 1-prong sample.

It is critical, then, that the Monte Carlo correctly model the properties of the charged track thrust axis. Two aspects concerning the quality of the calculation of the thrust axis can be separated: i) detector resolution, and ii) τ decay kinematics. We can measure the importance of detector resolution on the thrust axis direction by looking at the difference in the angle of the thrust axis before and after all OPAL detector simulation. The RMS separation in ϕ between the ideal charged track thrust axis before detector simulation and the thrust axis calculated using reconstructed tracks including

all detector resolution effects is 0.54 mrad. Thus, the detector resolution has very little effect on the thrust axis. The thrust axis properties are almost entirely determined by the τ decay kinematics.

To understand any possible effects arising from the modelling of the physics of the τ decays, we look at the difference between the actual tau direction and the thrust axis, before simulation of track reconstruction and resolution effects, for all possible combinations of decay modes in the $\tau^+\tau^-$ events. All τ decays can be classified into one of the following categories:

$\tau \rightarrow e\nu\bar{\nu}$	(1-prong)	1
$\tau \rightarrow \mu\nu\bar{\nu}$	(1-prong)	2
$\tau \rightarrow \pi(K)\nu$	(1-prong)	3
$\tau \rightarrow \rho(K^*)\nu$	(1-prong)	4
$\tau \rightarrow a_1\nu$	(1-prong)	5
$\tau \rightarrow a_1\nu$	(3-prong)	6
$\tau \rightarrow n\pi\nu \ n \geq 3$	(3-prong)	7

Table 7.1 shows the RMS value of the difference in the ϕ angle (tau-thrust) over a 7×7 grid. All possible combinations of the two τ decay modes in an event are shown. Combinations of decay modes in the 1-prong analysis divide into two categories: 1-1

	$\tau \rightarrow 1$	$\tau \rightarrow 2$	$\tau \rightarrow 3$	$\tau \rightarrow 4$	$\tau \rightarrow 5$	$\tau \rightarrow 6$	$\tau \rightarrow 7$
$\tau \rightarrow 1$	30.2						
$\tau \rightarrow 2$	28.9	28.6					
$\tau \rightarrow 3$	29.9	30.2	33.4				
$\tau \rightarrow 4$	32.0	31.0	33.3	31.6			
$\tau \rightarrow 5$	32.5	31.8	35.0	30.3	30.7		
$\tau \rightarrow 6$	18.7	17.7	19.5	18.0	16.8	13.5	
$\tau \rightarrow 7$	20.9	20.5	24.2	20.2	18.2	13.4	16.3

Table 7.1: The RMS difference in angle, in mrad, between the original tau direction and the thrust axis before detector resolution effects.

events and 1-3 events. (note: 3-3 events have the same tau-thrust characteristics as 1-3 events). The 1-1 events, which constitute about 80% of the final 1-X τ lifetime sample,

have an average angle of $(\text{tau-thrust}) = 31.1 \text{ mrad}$ with a standard deviation over all of the 1-1 grid elements of 1.7 mrad . Similarly, the 1-3 events have an average angle of $(\text{tau-thrust}) = 19.4 \text{ mrad}$ with a standard deviation of 3.0 mrad . The variation within the two main event topologies due to the different individual τ decay modes is small compared to the difference between them. The effect of the Monte Carlo input τ physics on the lifetime measurement is associated mainly with the experimental knowledge of these branching ratios, and particularly the topological 1-prong versus 3-prong ratio. This is discussed further in section 7.8 where a systematic error is assigned due to the uncertainty in the current world average τ decay branching ratios.

It is also evident from table 7.1 that the thrust axis approximates the τ direction more closely in 1-3 events than in 1-1 events. The reason for this is clearly illustrated in figure 7.3 which shows the average charged track momentum in both 1-prong and 3-prong hemispheres. A 3-prong hemisphere has a visible momentum on average $12 \text{ GeV}/c$

Figure 7.3: Sum of the momenta of charged tracks in 1-prong and 3-prong hemispheres. The points are data and the histograms are Monte Carlo simulation. The means are given in GeV/c .

larger than a 1-prong hemisphere, namely $27.2 \text{ GeV}/c$ versus $15.4 \text{ GeV}/c$ compared to the τ momentum of $45.6 \text{ GeV}/c$.

Since the thrust axis in 1-1 events is a poorer approximation of the τ direction, it follows that there would be more negative sign entries in 1-1 events and a correspondingly lower mean d_{11} . Figure 7.4 shows the two possible configurations of a 1-1 event. In

Figure 7.4: The two possible orientations of the tracks in a 1-1 event. The τ direction is shown as a dotted line with the origin of the taus marked by an X. In each case, the event thrust axis is marked by a thin line passing through the origin. Impact parameters from all tracks except ψ_3 would be given a positive sign.

each case, the thrust axis is drawn to maximize the momentum parallel to the axis. In figure 7.4a there is no possible thrust axis which will cause a negative sign in either track. In figure 7.4b, however, any thrust axis configuration will always result in one of the two impact parameters being given a negative sign. The track which will have the negative sign is typically the track with the smallest angle with respect to the τ direction, ψ , or equivalently, the track with the larger momentum. In figure 7.4b this is the track with angle ψ_3 . This simple argument predicts that approximately 25% of all impact parameters in 1-1 events will have a negative sign. The actual number, as determined by the Monte Carlo, is 28%.

In the case of 1-3 events, we know that it is the 3-prong hemisphere which will,

on average, have the larger momentum of the two. Figure 7.5 shows the angle between the total charged track momentum in a hemisphere and the thrust axis for different topologies. It can be seen that the 3-prong summed momentum vector is indeed much closer to the

Figure 7.5: Angle between momentum in a hemisphere and the thrust axis. a) shows 1-1 events, b) the 1-prong hemisphere of a 1-3 event, and c) the 3-prong hemisphere of a 1-3 event. The points are data and the histogram Monte Carlo. Shown on each plot is the RMS width of the distribution in radians

thrust axis than the 1-prong track. By the above argument, there should be much less than 25% of the impact parameters given a negative sign in 1-3 events than in 1-1 events. The Monte Carlo indicates that 10% of the impact parameters are negative, which again agrees with this qualitative description of the origin of negative sign entries.

7.3 The Trimmed Mean

The τ lifetime is measured by comparing the signed impact parameter distribution from the data with the corresponding Monte Carlo distribution. This comparison requires choosing an estimator of the location of the distribution. Simply taking the mean of the distribution is sensitive to entries far out on the tails. The tails in the data are expected to include some mismeasurements which are not simulated in the Monte Carlo.

The trimmed mean is an estimator which is less sensitive to the tails of the distribution. All of the signed d_{ij} measurements are ordered from lowest to highest and an equal fraction of them are discarded from each end. The mean is taken of the remaining entries. Although the minimum and maximum d_{ij} 's included in the trimmed mean calculation depend on the tails of the distribution, the trimmed mean is less sensitive to the detailed nature of these tails since only the integral number of counts in the tails is used and not the actual shape. At the limit of 0% of the data trimmed, the trimmed mean is equivalent to the arithmetic mean. At the limit of 100% of the data trimmed, 50% from each side, the trimmed mean converges to the median of the distribution. For symmetric distributions, the mean is equal to the median and the trimmed mean is independent of trim. This is not the case for the asymmetric signed impact parameter distribution, see figure 7.2d, and the value of the trimmed mean is a function of the trim with the limits being the mean on one side and the median on the other.

The trim parameter, t , has a range from 0 to 1 and is defined so that $t = \frac{2r}{N}$ where the highest and lowest r measurements are discarded; $t = 0.1$ represents a trim of 10% of the data. The uncertainty in the trimmed mean is [41]:

$$s_{\bar{x}_t}^2 = \frac{1}{N(1-t)} \left[s^2 + \frac{t}{4} \left(\frac{(x_{max} - x_{min})^2}{1-t} + (x_{max} - 2\bar{x}_t + x_{min})^2 \right) \right] \quad (7.1)$$

where

N	$\equiv$	The total number of measurements
x_i	$\equiv$	The individual impact parameters
x_{min}	$\equiv$	The minimum d_{ij} after trimming
x_{max}	$\equiv$	The maximum d_{ij} after trimming
$\bar{x}_t$	$\equiv$	The trimmed mean

and

$$s^2 = \frac{1}{N(1-t)} \sum_{i=N\frac{1-t}{2}+1}^{N(1-\frac{t}{2})} (x_i - \bar{x}_t)^2 \quad (7.2)$$

is the variance of the portion of the distribution that remains after trimming. A discussion of the derivation of this expression is given in Appendix A.

The relative statistical uncertainty of the trimmed mean calculated for various trims using the Monte Carlo sample is shown in figure 7.6. The sharp initial decrease in the error from 0% trim, the arithmetic mean, is due to entries far out in the tails which inflate the variance of the d_i distribution and thus the error on the mean. It is clear that some small amount of trimming is necessary, and that the trim value selected should be sufficient to remain insensitive to the shape of the tails in the experimental data. The trim value chosen was 10% and further references to trimmed means imply a 10% trim.

Figure 7.6: The relative uncertainty in the trimmed mean ($s^2_{\bar{x}_t}/\bar{x}_t$) as a function of the trim as calculated using a distribution of Monte Carlo impact parameters.

The measured value of the 10% trimmed mean, $\bar{x}_{meas}$, for the experimental data gives the lifetime directly from a comparison with the Monte Carlo trimmed mean for a

generated lifetime of τ_{MC} :

$$\tau_r = \tau_{MC} \times \frac{\bar{x}_{mean}}{\bar{x}_{MC}} \quad (7.3)$$

where $\tau_{MC} = 303.5$ fs.

Equation 7.3 assumes that the relationship between the trimmed mean and the τ lifetime is linear with an intercept of zero. Figure 7.7 shows the trimmed mean of four independent Monte Carlo samples generated with different input lifetimes and indicates that this assumption is justified within the current limit of the Monte Carlo statistics.

Figure 7.7: The trimmed mean of four independent Monte Carlo samples with different input lifetimes. Also shown in the best fit straight line.

In order to better understand the potential sensitivity of the trimmed mean estimator to the experimental $d_{||}$ resolution, a separate study was carried out in which impact parameter-like distributions were generated with varying resolutions. An exponential of mean $50 \mu\text{m}$ was used to approximate the $d_{||}$ distribution before detector resolution effects. This was convoluted with a gaussian, the widths of which were selected to agree with the actual distribution of $d_{||}$ errors seen in the data. Figures 7.8a, b and c show the $d_{||}$ error distributions for the 1990, 1991 with and without Si data samples. The widths of

Figure 7.8: a), b) and c) the d_0 error distributions, including the contribution due to the beam spot size, seen in the data in 1990 and 1991; d) the arithmetic mean, 10% trimmed mean and the median of generated impact parameter-like distributions as a function of resolution. The solid line shows the actual mean of the exponential distribution of 50 μm .

Figure 7 14. Impact parameter distributions for the data samples: 1990, 1991 with Si microvertex detector not available, and 1991 including Si information. The points are the data, and the histogram is Monte Carlo. The arrows indicate the 10% trim fraction.

and the error with which the $d_{||}$ is measured. There are two contributions to the error on the $d_{||}$ measurement: i) the error associated with track reconstruction including multiple scattering, ii) the finite size of the beam spot. In this analysis, the total error on the signed $d_{||}$ measurement was obtained by adding (in quadrature) the beam spot contribution to the error associated with the reconstructed track.

Figure 7.9a shows the total estimated $d_{||}$ error versus the azimuthal angle, ϕ . The sinusoidal variation is due to the shape of the LEP beams. Figure 7.9b shows the average $d_{||}$ error expected from detector reconstruction alone as a function of momentum. Figure 7.9c displays the distributions of these two contributions for the 1-prong τ sample, showing the tracking error as a solid histogram and the beam spot contribution as the dashed histogram. Note that figure 7.9a, b, c, d are for the 1991 data using only the gas vertex detector. The average error due to the track fit including multiple scattering, for the 1-prong sample, is $64\ \mu\text{m}$ in the 1990 sample, $73\ \mu\text{m}$ in the 1991 sample using the gas vertex detector and $25\ \mu\text{m}$ when the Si vertex detector was used. The average contribution to the $d_{||}$ error due to the beam spot size is $105\ \mu\text{m}$. For this analysis, the total error on $d_{||}$ was required to be less than $0.1\ \text{cm}$ to eliminate a small number of very poorly measured tracks, see figure 7.9d.

The component of the Monte Carlo $d_{||}$ resolution due to tracking effects was adjusted to agree with experimental data. Tracks from $Z^0 \rightarrow \mu^+ \mu^-$ events were used in order to prevent any possible bias due to the τ lifetime which increases the width of the distribution for the $\tau^+ \tau^-$ events. The impact parameter signed with the tracking sign convention, not the lifetime sign convention, produces a symmetric $d_{||}$ distribution, and this was used to study the experimental resolution. The vertical size of the LEP beam, $\sim 8\ \mu\text{m}$, is too small to distinguish from the tracking resolution of OPAL. Therefore, the width of the impact parameter distribution of tracks which are predominantly horizontal in the xy plane is determined entirely by track resolution effects. The $d_{||}$ data distribution from horizontal tracks was compared with a Monte Carlo distribution whose resolution was varied by smearing with a gaussian of varying widths. The data and Monte Carlo distributions were compared by forming a bin-by-bin χ^2 statistic. The minimum χ^2 determined the optimal additional track resolution smearing to add to the Monte Carlo simulation. Figure 7.10 shows the impact parameter distribution of muon tracks for the

Figure 7.9: a) the total estimated d_0 error as a function of ϕ ; b) the average d_0 error due to tracking, including multiple scattering, as a function of momentum; c) the components of the d_0 error: solid histogram from tracking and multiple scattering, dashed histogram from beam spot size; d) the total d_0 error for the 1991 sample using the vertex drift chamber and indicates the cut at 0.1 cm to eliminate very poorly measured tracks.

Figure 7.10: The d_0 distribution of muon pair tracks signed with the tracking sign convention. The points show the 1991 data without the Si detector, and the solid histogram shows the best fit Monte Carlo distribution. The dashed curve shows the Monte Carlo distribution with no smearing, and the dotted curve shows a distribution smeared by a factor of two more than the optimal smear

1991 data not using the Si detector along with three of the possible smeared Monte Carlo distributions. This procedure was repeated for each of the four data sample categories used in the lifetime analysis. Table 7.2 lists the amount by which the Monte Carlo was smeared in each case. The magnitude of the extra resolution smearing, when included in quadrature, amounts to a small contribution to the total track resolution compared with the components due to track reconstruction and the e^+e^- interaction region size.

The effect of the elliptical beam spot size on the d_0 resolution can be observed by dividing the range of ϕ into 12 sections of $\frac{\pi}{6}$ radians each. Tracks were grouped into three categories: vertical, horizontal, or diagonal. A "vertical" track is one with its ϕ_{tr}

sample	additional Monte Carlo $d_{\perp}$ smearing (μm)
1990	0
1991 Si not used	29
1991 Si not available	40
1991 With Si	22

Table 7 2. The Monte Carlo $d_{\perp}$ resolution smearing, added in quadrature, for each of the samples used in the lifetime analysis.

track parameter in the ranges:

$$\left(\frac{2\pi}{6} < \phi_0 \leq \frac{4\pi}{6}\right) \text{ OR } \left(\frac{8\pi}{6} < \phi_0 \leq \frac{10\pi}{6}\right).$$

A "horizontal" track has:

$$\left(\phi_0 \leq \frac{\pi}{6} \text{ OR } \phi_0 > \frac{11\pi}{6}\right) \text{ OR } \left(\frac{5\pi}{6} < \phi_0 \leq \frac{7\pi}{6}\right),$$

and tracks in the regions between horizontal and vertical are classified as "diagonal". There are approximately an equal number of tracks in each ϕ category. The width of the $d_{\perp}$ distribution of horizontal tracks is limited by the tracking resolution, whereas the $d_{\perp}$ width of the vertical tracks is determined by the beam spot size in x . Figure 7.11 shows the lifetime signed $d_{\perp}$ distribution compared to Monte Carlo in each ϕ category for the samples of 1990, 1991 with no Si information available, and 1991 with Si. Each Monte Carlo sample has been smeared by a single ϕ -independent parameter determined with the muon pair tracks.

The beam spot size used in the 1990 Monte Carlo was $157 \mu\text{m}$. In 1991 the Si detector became operational in the latter part of the running period when the beam size of LEP had been reduced. The Monte Carlo beam spot size was consequently also reduced. The beam spot sizes used in the 1991 Monte Carlos were $157 \mu\text{m}$ when Si was not available and $147 \mu\text{m}$ when Si was used. Figure 7.11 shows that these beam sizes, combined with the Monte Carlo $d_{\perp}$ resolution smearing described previously, result in good agreement between the data and Monte Carlo in all categories and in all samples.

Figure 7.11: Impact parameters, signed with the lifetime signing convention, of τ decay tracks of all samples for vertical, horizontal and diagonal tracks. The points are the data and the histograms are the resolution tuned Monte Carlo.

7.5 The 1-prong Data Sample

7.5.1 Quality Requirements on 1-prong τ Decays

In order to be included in the lifetime sample, a track from a 1-prong decay was required to have hits in at least one of the vertex detectors: either ≥ 6 out of 12 possible CV hits or ≥ 1 Si hit. The Si detector does not cover the entire angular region up to $|\cos \theta_{\text{thrust}}| < 0.9$, so tracks which fall outside the acceptance region of the Si detector but still satisfied the CV hit requirement were included in the lifetime track sample.

To reduce residual background from other decays of the Z^0 and from collisions of two photons, tracks are rejected if they come from an event with an acoplanarity of less than 0.002. The acoplanarity is defined as π minus the difference in ϕ between the summed momentum of each hemisphere. Figure 7.12 shows, for each 1-prong track, the acoplanarity of that event. A cut at 0.002 eliminates 3.5% of the 1-prong tracks in the data and only 2.8% in the Monte Carlo with the difference being consistent with an estimation of the fraction of background present. After these cuts 7579 1-prong tracks remain in the 1990 data sample and 17867 in 1991.

7.5.2 Events in the Tails of the d_0 Distribution

The signed d_0 distribution for the data contains a small number of events in excess of the Monte Carlo prediction far out in the tails. Four OPAL detector related resolution effects were examined as possible sources of these tails as well as checking whether they could cause possible biases of the trimmed mean of the distribution. Numerical results are given for the 1991 sample with Si tracking not used, but similar effects are present in all samples. No significant systematic detector related effect was found.

Number of CV Hits: Since only 1-prong decays are considered in this analysis, this represents a very clean environment in the central vertex detector, CV, and the average number of CV hits should be high. A minimum number of 6 hits is required from the 12 axial wires. In the 1-prong sample 4% were found to have only 6-8 hits. These tracks were predominantly near either the anode or cathode planes of

Figure 7.12: Acoplanarity of events in 1991. The points are data and the histogram is the r Monte Carlo. Underneath the main histogram, the hatched histogram is the background component due to $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow e^+e^-$ events and the clear histogram is the component due to two photon events. The bottom plot is a blow-up of the low range of the upper plot. A cut is placed at 0.002 as shown by the arrow. The corresponding plot for the 1990 sample is identical.

the drift cell. Table 7.3 shows the trimmed means for tracks with low numbers of hits, excluding CV sector 28 which has only 8 out of 12 possible wires. Figure 7.13a shows the complete signed $d_{\parallel}$ distribution with the hatched histogram being those tracks with only 6-8 CV hits.

CV AXIAL hits	Tracks	Trimmed Means (μm)
≥ 9	16629	45.057 ± 1.355
6-8	705	34.212 ± 9.815

Table 7.3: Trimmed means for tracks with few CV hits, excluding CV sector 28.

CV Anode Region: The region around the anode wire plane in a CV cell is where the field becomes highly nonlinear. To test whether these tracks bias the τ lifetime measurement, the anode region was excluded. Tracks are excluded if they are within 5 mrad in ϕ of the anode plane. Table 7.4 shows the trimmed means of tracks inside and outside the anode region. Figure 7.13b shows the complete signed $d_{\parallel}$ distribution with the hatched histogram being those tracks in the CV anode region.

	Tracks	Trimmed Means (μm)
Excluding CV anode	16797	45.704 ± 1.369
CV anode (± 5 mrad)	1018	45.416 ± 6.071

Table 7.4: Trimmed means for tracks in the anode region of CV

Charge Measurement: In the 1991 sample, 5% of the events containing 1-prong decays did not satisfy an exact event charge conservation requirement, $Q_1 = -Q_2$ where Q_1 and Q_2 are the charges of each hemisphere. The largest source of these events is when there are only 2 good reconstructed tracks found in a hemisphere, which accounts for about 70% of these events. The trimmed means in this case are shown in table 7.5. Figure 7.13c shows these events hatched. Tracks from charge

imbalanced events were cut in the τ selection stage if the event had a 1-1 topology and are not shown here.

	Tracks	Trimmed Means (μm)
$Q1 = -Q2$	16854	45.063 : 1.370
$Q1 \neq -Q2$	961	57.327 : 6.131

Table 7.5: Trimmed means for tracks in events for which the charge was not balanced

Extra Tracks: In many events, 1-prong hemispheres have extra tracks which do not pass the basic track quality requirements. Most of these tracks do not pass the impact parameter cut, $|d_0| < 1.0\text{ cm}$, in the τ selection. Using a cone of 30° half angle around the momentum vector of the 1-prong track, the number of 1-prongs which had extra tracks was 1238. The major sources of extra tracks were found to be γ -conversions, tracks along the CJ anode or cathode planes which are split by the pattern recognition into 2 pieces, and tracks due to an interaction either in the beam pipe or the vertex detectors. The trimmed mean d_0 of the 1-prong tracks for these events are shown in table 7.6. Figure 7.13d shows these events hatched. These events comprise the largest fraction of the far out tails.

# of Extra trks	Tracks	Trimmed Means (μm)
0	16577	45.947 : 1.375
≥ 1	1238	42.164 : 5.689
1	731	45.411 : 7.016
2	415	28.305 : 9.458
≥ 3	92	80.809 : 35.817

Table 7.6: Trimmed means of 1-prongs in hemispheres with extra tracks

Figure 7.13. Impact parameter distribution of the 1991 data sample where Si detector information was not used. The hatched histograms show the entries affected by various detector effects discussed in the text.

7.6 τ Lifetime Results

Figures 7.14 and 7.15 show the impact parameter distribution for the three independent data samples from: i) 1990, ii) 1991 when Si microvertex detector information was not available, and iii) 1991 including the Si microvertex information. Table 7.7 shows the trimmed means and τ lifetimes for the 1990 and 1991 data samples. All 1-prong τ decays are given in the 1-X category, 1-prong decays from 1-3 events are in the 1-3 category, and decays from 1-1 events are in the 1-1 category. Table 7.8 shows the combined lifetimes for the 1990+1991 data samples without and with Si. The "No Si" lifetime is a weighted average of the entire 1990 and complete 1991 samples ignoring the Si information. The "With Si" lifetime is a weighted average of the 1990 lifetime, the 1991 lifetime for those events with no Si available (including those tracks outside the angular acceptance of the Si detector), and the 1991 lifetime requiring Si hits on the 1-prong track. The two combined results, with and without using the silicon μ -vertex detector, are in agreement. We use the 1-X result obtained including the Si detector as the 1-prong impact parameter lifetime measurement.

7.7 Experimental Checks of Results

7.7.1 Lifetime as a Function of d_0 Resolution

Table 7.9 shows the τ lifetime for the various data samples as well as the combined Si and no-Si lifetimes in the three ϕ bins defined in section 7.4. Figure 7.11 showed the corresponding individual impact parameter distributions. This azimuthal angle check is particularly important in that the effective impact parameter resolution changes significantly amongst the subsamples in ϕ due to both the effect of the LEP beam shape and the change between gas and silicon vertex detectors. The 1990 sample can be combined with that portion of 1991 data for which there was no Si information to produce an independent "non-Si" lifetime measurement. Using these subgroups, table 7.10 shows the τ lifetime as a function of d_0 resolution. Figure 7.16 shows the τ lifetime as a function of d_0 resolution compared to the overall OPAL lifetime result presented in table 7.8, and

Figure 7.14. Impact parameter distributions for the data samples: 1990, 1991 with Si microvertex detector not available, and 1991 including Si information. The points are the data, and the histogram is Monte Carlo. The arrows indicate the 10% trim fraction.

Figure 7.15: Impact parameter distributions for the data samples: 1990, 1991 with Si microvertex detector not available, and 1991 including Si information shown on a logarithmic scale. The points are the data, and the histogram is Monte Carlo. The arrows indicate the 10% trim fraction.

Topol.	Data Tracks	Data Mean (μm)	MC Tracks	MC Mean (μm)	τ_r (fs)
1990					
1-X	7537	45.255 ± 1.949	59151	47.161 ± 0.704	291.2 ± 13.2
1-1	6148	43.721 ± 2.179	48235	44.331 ± 0.779	299.3 ± 15.8
1-3	1056	51.131 ± 4.874	8213	60.044 ± 1.887	258.5 ± 25.9
1991 All events, Si not used					
1-X	17815	45.719 ± 1.335	110137	47.558 ± 0.524	291.8 ± 9.1
1-1	14022	42.981 ± 1.499	88557	45.033 ± 0.584	289.7 ± 10.8
1-3	2799	55.274 ± 3.436	16455	58.359 ± 1.361	287.5 ± 19.0
1991 No Si available					
1-X	7148	47.413 ± 2.221	110137	47.601 ± 0.532	302.3 ± 14.5
1-1	5642	45.065 ± 2.463	88557	45.072 ± 0.592	303.4 ± 17.0
1-3	1097	58.647 ± 5.924	16455	58.452 ± 1.380	304.5 ± 31.5
1991 With Si					
1-X	10894	44.678 ± 1.470	95194	46.371 ± 0.486	292.4 ± 10.1
1-1	8553	41.697 ± 1.659	76721	44.081 ± 0.542	287.1 ± 11.9
1-3	1745	54.185 ± 3.670	14270	57.234 ± 1.261	287.3 ± 20.4

Table 7.7: The 10% trimmed means and lifetimes of 1-prongs for different event topologies.

Topol.	No. of Tracks	τ_r (fs)
1990 + 1991 No Si		
1-X	25352	291.6 ± 7.5
1-1	20170	292.7 ± 8.9
1-3	3855	277.3 ± 15.3
1990 + 1991 With Si		
1-X	25579	294.4 ± 7.0
1-1	20343	294.3 ± 8.3
1-3	3898	282.1 ± 14.2

Table 7.8: Combined lifetimes in 1990 and 1991, without and with Si.

ϕ -bin	Data Tracks	τ_+ (fs)
1990		
horizontal	2510	311.3 $\pm$ 20.2
diagonal	2524	271.0 $\pm$ 22.7
vertical	2503	295.0 $\pm$ 26.0
1991 All events, Si not used		
horizontal	5867	302.1 $\pm$ 13.2
diagonal	5839	274.6 $\pm$ 15.9
vertical	6109	296.6 $\pm$ 18.1
1991 No Si available		
horizontal	2369	315.4 $\pm$ 21.5
diagonal	2287	305.8 $\pm$ 25.6
vertical	2492	284.8 $\pm$ 28.3
1991 With Si		
horizontal	3577	301.3 $\pm$ 14.4
diagonal	3633	264.0 $\pm$ 17.6
vertical	3684	310.6 $\pm$ 20.8
Combined 1990 + 1991 No Si		
horizontal	8377	304.8 $\pm$ 11.0
diagonal	8368	273.4 $\pm$ 13.0
vertical	8612	296.1 $\pm$ 14.8
Combined 1990 + 1991 With Si		
horizontal	8456	307.1 $\pm$ 10.3
diagonal	8444	275.5 $\pm$ 12.2
vertical	8679	299.6 $\pm$ 14.1

Table 7.9: The lifetimes of horizontal, vertical and diagonal tracks

ϕ bin	Data	mean $d_{\perp}$ resolution (μm)	τ_{τ} (fs)
Horizontal	Si	61.9	301.3 ± 14.4
	non-Si	92.75	313.2 ± 14.7
Diagonal	Si	124.5	264.0 ± 17.6
	non-Si	143.6	286.3 ± 17.0
Vertical	Si	164.5	310.6 ± 20.8
	non-Si	180.2	290.3 ± 19.1

Table 7.10: The lifetime as a function of $d_{\perp}$ resolution.

shows that the measured τ lifetime is independent of $d_{\perp}$ impact parameter resolution.

7.7.2 Geometric and Kinematic Studies

The lifetime was examined as a function of three other kinematic variables: i) $|\cos \theta|$ where θ is the polar angle of the thrust axis, ii) the momentum of the 1-prong track, and iii) the total visible energy in the event including energy deposited in the electromagnetic calorimeter. The lifetimes for individual data samples as well as the combined 1990+1991 lifetimes as a function of these variables are shown in tables 7.11, 7.12 and 7.13, respectively. Figures 7.17 and 7.18 show the combined 1990+1991 lifetime with and without Si microvertex detector information as a function of these three kinematic variables. The τ lifetime for the entire sample (given in table 7.8) is shown as a dotted line. In each case, the lifetimes for the various subsamples are statistically consistent.

7.7.3 Impact Parameter Analysis of 3-prong Tracks

Table 7.14 shows the trimmed means and lifetimes using tracks from 3-prong τ decays. The track requirements were the same as for the 1-prong sample except that the CV requirement of ≥ 6 hits was restricted to only consider first hits in the vertex drift chamber. These lifetimes are consistent with those measured using 1-prong tracks and with the lifetime measured by the decay length method. For 3-prong τ decays, the decay

Figure 7.16: The τ lifetime as a function of d_t resolution. Samples of differing resolutions were obtained by considering the lifetime with and without the Si detector in different regions of the azimuthal angle, ϕ . The OPAL τ lifetime result is shown as a dotted line.

$ \cos \theta $	Data Tracks	τ_r (fs)
1990		
$ \cos \theta \leq 0.225$	1502	279.2 ± 29.5
$0.225 < \cos \theta \leq 0.45$	1762	303.0 ± 27.6
$0.45 < \cos \theta \leq 0.7$	2344	260.4 ± 22.6
$ \cos \theta > 0.7$	1929	324.9 ± 28.1
1991 All events, Si not used		
$ \cos \theta \leq 0.225$	3656	282.1 ± 18.8
$0.225 < \cos \theta \leq 0.45$	3986	291.8 ± 18.6
$0.45 < \cos \theta \leq 0.7$	5403	306.3 ± 16.3
$ \cos \theta > 0.7$	4770	279.0 ± 19.0
1991 No Si available		
$ \cos \theta \leq 0.225$	1168	304.7 ± 32.7
$0.225 < \cos \theta \leq 0.45$	1289	313.8 ± 32.8
$0.45 < \cos \theta \leq 0.7$	1767	295.8 ± 26.7
$ \cos \theta > 0.7$	2924	290.3 ± 24.6
1991 With Si		
$ \cos \theta \leq 0.225$	2561	287.6 ± 20.1
$0.225 < \cos \theta \leq 0.45$	2772	284.6 ± 19.8
$0.45 < \cos \theta \leq 0.7$	3695	312.2 ± 17.4
$ \cos \theta > 0.7$	1866	266.8 ± 25.4
Combined 1990 + 1991 No Si		
$ \cos \theta \leq 0.225$	5158	281.2 ± 15.8
$0.225 < \cos \theta \leq 0.45$	5748	295.2 ± 15.4
$0.45 < \cos \theta \leq 0.7$	7747	290.6 ± 13.2
$ \cos \theta > 0.7$	6699	293.4 ± 15.7
Combined 1990 + 1991 With Si		
$ \cos \theta \leq 0.225$	5231	289.0 ± 14.8
$0.225 < \cos \theta \leq 0.45$	5823	293.7 ± 14.4
$0.45 < \cos \theta \leq 0.7$	7806	293.5 ± 12.2
$ \cos \theta > 0.7$	6719	291.9 ± 14.9

Table 7.11: The lifetime as a function of $|\cos \theta_{thrust}|$.

Track Momentum (GeV/c)	Data Tracks	τ_r (fs)
1990		
$p < 5$	1361	297.8 ± 19.8
$5 \leq p < 10$	1566	272.3 ± 21.9
$10 \leq p < 20$	2341	296.2 ± 26.0
$p \geq 20$	2269	295.8 ± 60.3
1991 All events, Si not used		
$p < 5$	3168	293.9 ± 14.1
$5 \leq p < 10$	3760	295.7 ± 15.7
$10 \leq p < 20$	5519	288.7 ± 17.8
$p \geq 20$	5368	277.3 ± 34.3
1991 No Si available		
$p < 5$	1261	324.3 ± 24.9
$5 \leq p < 10$	1498	311.4 ± 24.8
$10 \leq p < 20$	2240	304.2 ± 27.7
$p \geq 20$	2149	249.5 ± 53.7
1991 With Si		
$p < 5$	1959	291.3 ± 14.4
$5 \leq p < 10$	2314	301.3 ± 17.6
$10 \leq p < 20$	3340	277.3 ± 20.2
$p \geq 20$	3281	287.2 ± 39.6
Combined 1990 + 1991 No Si		
$p < 5$	4529	295.2 ± 11.5
$5 \leq p < 10$	5326	287.7 ± 12.7
$10 \leq p < 20$	7860	291.0 ± 14.7
$p \geq 20$	7637	281.8 ± 29.8
Combined 1990 + 1991 With Si		
$p < 5$	4581	299.1 ± 10.5
$5 \leq p < 10$	5378	294.9 ± 12.0
$10 \leq p < 20$	7921	298.3 ± 13.8
$p \geq 20$	7699	278.7 ± 28.2

Table 7.12: The lifetime as a function of 1-prong track momentum

Visible Energy	Data Tracks	τ_r (fs)
1990		
$0.18 \leq E_{vis} \leq 0.4$	1178	296.0 ± 29.8
$0.4 > E_{vis} \leq 0.6$	1994	292.1 ± 23.7
$0.6 > E_{vis} \leq 0.8$	2228	308.1 ± 25.3
$E_{vis} > 0.8$	2137	263.2 ± 28.7
1991 All events, Si not used		
$0.18 \leq E_{vis} \leq 0.4$	2633	273.7 ± 19.5
$0.4 > E_{vis} \leq 0.6$	5082	306.9 ± 16.0
$0.6 > E_{vis} \leq 0.8$	4999	307.3 ± 17.8
$E_{vis} > 0.8$	5101	259.9 ± 19.8
1991 No Si available		
$0.18 \geq E_{vis} \leq 0.4$	1038	250.4 ± 31.6
$0.4 > E_{vis} \leq 0.6$	2049	304.7 ± 25.3
$0.6 > E_{vis} \leq 0.8$	2021	333.8 ± 27.9
$E_{vis} > 0.8$	2040	293.2 ± 31.9
1991 With Si		
$0.18 \geq E_{vis} \leq 0.4$	1625	296.0 ± 21.4
$0.4 > E_{vis} \leq 0.6$	3101	310.5 ± 17.7
$0.6 > E_{vis} \leq 0.8$	3036	302.5 ± 19.9
$E_{vis} > 0.8$	3132	248.6 ± 22.1
Combined 1990 + 1991 No Si		
$0.18 \geq E_{vis} \leq 0.4$	3811	280.4 ± 16.3
$0.4 > E_{vis} \leq 0.6$	7076	302.3 ± 13.3
$0.6 > E_{vis} \leq 0.8$	7227	307.6 ± 14.6
$E_{vis} > 0.8$	7238	261.0 ± 16.3
Combined 1990 + 1991 With Si		
$0.18 \geq E_{vis} \leq 0.4$	3841	285.4 ± 15.2
$0.4 > E_{vis} \leq 0.6$	7144	304.1 ± 12.4
$0.6 > E_{vis} \leq 0.8$	7285	311.6 ± 13.6
$E_{vis} > 0.8$	7309	263.1 ± 15.3

Table 7.13: The lifetime as a function of the visible energy in the event. Visible energy is defined as the sum of the charged track momenta and the energy in the electromagnetic calorimeter normalized to the beam energy. The lower limit of 0.18 in the visible energy is applied in the τ event selection to remove the two photon background.

Figure 7.17: The τ lifetime as a function of the geometric and kinematic variables described in the text. The combined 1990+1991 without Si lifetime is shown as a dotted line.

Figure 7 18: The τ lifetime as a function of the geometric and kinematic variables described in the text. The combined 1990+1991 with Si lifetime is shown as a dotted line.

length method gives a more precise result, so this impact parameter analysis is used as a cross check only.

Topol.	Data Tracks	Data Mean (μm)	MC Tracks	MC Mean (μm)	τ (fs)
1990					
3-X	1696	65.724 ± 4.917	13023	59.195 ± 1.728	336.9 ± 27.0
3-1	1400	68.599 ± 5.501	11148	58.952 ± 1.862	353.1 ± 30.4
1991 All events, Si not used					
3-X	4514	63.695 ± 3.521	26459	65.559 ± 1.334	294.9 ± 17.3
3-1	3704	63.677 ± 3.941	22108	65.854 ± 1.456	293.5 ± 19.2
1991 No Si available					
3-X	1982	63.959 ± 6.851	26459	65.424 ± 1.348	296.7 ± 32.3
3-1	1601	65.142 ± 7.581	22108	66.054 ± 1.473	299.3 ± 35.4
1991 With Si					
3-X	5326	53.733 ± 2.278	42924	56.408 ± 0.748	289.1 ± 12.8
3-1	4428	53.115 ± 2.493	35834	56.396 ± 0.821	285.9 ± 14.0
Combined 1990 + 1991 No Si					
3-X	6210				307.1 ± 14.6
3-1	5104				310.5 ± 16.2
Combined 1990 + 1991 With Si					
3-X	9004				297.7 ± 10.9
3-1	7429				297.8 ± 12.0

Table 7.14: The 10% trimmed means and lifetimes of 3-prong tracks for different event topologies

7.7.4 Weighted Mean Estimator

The lifetime can be calculated using the weighted mean as an alternate estimator to characterize the $d_{||}$ distribution. Each $d_{||}$ entry is weighted by the inverse square of the $d_{||}$ error. The 1% of the data in the outermost tails of the unweighted $d_{||}$ distribution referred to in section 7.5.2 is excluded from the weighted means. Table 7.15 shows the weighted means and lifetimes for each of the data samples. The difference in the combined lifetimes between the weighted and trimmed mean estimators is 0.6% when Si is not used and 0.06% when Si is used. These differences are small compared to the systematic error

assigned to the lifetime due to the use of the trimmed mean estimator which will be presented in section 7.8.1.

Sample	Data	MC	τ (fs)
	Mean (μm)	Mean (μm)	
1990	38.644 ± 1.803	40.761 ± 0.674	287.8 ± 14.2
1991 Si not used	39.663 ± 1.211	41.406 ± 0.486	290.7 ± 9.5
1991 Si not avail.	40.547 ± 2.034	41.360 ± 0.501	297.5 ± 15.3
1991 with Si	38.114 ± 1.694	38.739 ± 0.549	298.6 ± 13.9
1990 - 1991			
No Si			289.8 ± 7.9
1990 - 1991			
With Si			294.5 ± 8.3

Table 7.15 The weighted means and lifetimes of 1-prongs for different data samples

7.7.5 Silicon Versus Gas Vertex Detector

The vertex drift chamber is included in all track reconstructions when possible. By contrast, in 1991 OPAL performed track reconstructions both with and without adding the hits from the silicon microvertex detector. Both sets of track parameters were made available. The effect on the τ lifetime of adding Si hits to the tracks was studied by selecting a subsample of the 1991 data which satisfied both the CV hit criteria when Si was not used and the Si hit criteria when it was. There were 11001 tracks which were common to the lifetime track sample using either the silicon or the gas vertex detector. Table 7.16 shows the trimmed means of the impact parameter distributions when the Si hits are included or excluded in the track fit. There is a shift of 1.55% when the Si hits are added to the track fit. To determine whether this constitutes a systematic effect, the Monte Carlo sample was divided into 8 partitions of 11 000 tracks each. The difference in the trimmed means with and without the Si hits was determined for each partition. Table 7.17 shows these differences. The mean difference is 0.04%, which indicates no evidence of a systematic shift in the Monte Carlo caused by including the Si hit information. The standard deviation of the difference in the 8 partitions is 1.9%, which

Vertex Detector	Mean (μm)
CV (gas)	44.749 : 1.581
SI (silicon)	44.059 : 1.411

Table 7.16: The 10% trimmed means of tracks including or excluding Si vertex detector hits from the track fit

Partition	Difference No Si - Si (%)
1	2.37
2	1.44
3	-0.77
4	-2.85
5	0.63
6	0.31
7	1.55
8	-2.33
mean	0.04
st. dev.	1.88

Table 7.17: The effect in the Monte Carlo of including or excluding Si vertex detector hits from the track fit

indicates that the 1.55% difference observed in the data is consistent with a statistical fluctuation.

7.7.6 Thrust Axis Including Electromagnetic Clusters

The thrust axis used in this analysis is calculated using charged tracks only. In the following discussion, this axis will be labeled CTRK. An alternate axis can be constructed by including the energy deposited in the electromagnetic clusters. Figure 7.19 shows the electromagnetic cluster energy and the angle of the cluster with respect to the charged track thrust axis. There is a slight disagreement between the data and Monte Carlo in the number of low energy clusters in both the 1990 and 1991 samples. There is also disagreement in the number of clusters seen at large angles (essentially perpendicular) with respect to the charged track thrust axis in the 1990 data. This is caused by residual noisy clusters which escape the procedure to reject them. These noise clusters occur at a fixed point in space, so they are expected to have a uniform angular distribution with respect to the thrust axis. However, it is only in areas of an event in which the general electromagnetic activity is low (such as at large angles to the thrust axis) that their presence becomes apparent. In order to remove clusters not well modeled by the Monte Carlo, a cut is placed at 1 GeV on the cluster energy, and on the cluster angle of $|\sin(\phi_{clus} - \phi_{CTRK})| < 0.4$. Table 7.18 shows the trimmed means and lifetimes using this thrust axis, which is labeled BOTH*. Table 7.19 compares the combined lifetimes using both of the thrust axes.

The question arises whether any differences in lifetime with different axes are due to statistical fluctuations or indicate some systematic effect. To answer this question, the standard deviation of the ratio of the trimmed means for the two different axes is compared with the expectation due to statistical fluctuations as calculated using the Monte Carlo. Table 7.20 shows the trimmed means for the three independent data samples for both axes and their ratio. The mean number of tracks in the three samples is 8526. The mean ratio of $\frac{BOTH^*}{CTRK}$ is 1.102 with a standard deviation of 0.017. If, instead of considering the trimmed means of the data, we took the ratio of the lifetimes using the two axes, the standard deviation would be 0.018.

Figure 7 19: The cluster energy and angle with respect to the charged track thrust axis for 1990 and 1991 samples. The points are data and the histograms Monte Carlo

Topol.	Data Mean (μm)	MC Mean (μm)	τ_r (fs)
1990			
1-X	48.992 $\pm$ 1.934	52.401 $\pm$ 0.699	283.8 $\pm$ 11.8
1-1	48.126 $\pm$ 2.169	50.444 $\pm$ 0.774	289.6 $\pm$ 13.8
1-3	51.113 $\pm$ 4.863	59.852 $\pm$ 1.886	259.3 $\pm$ 25.9
1991 All events, Si not used			
1-X	52.032 $\pm$ 1.323	53.288 $\pm$ 0.520	296.3 $\pm$ 8.1
1-1	50.819 $\pm$ 1.481	51.661 $\pm$ 0.579	298.6 $\pm$ 9.3
1-3	58.062 $\pm$ 3.434	60.265 $\pm$ 1.358	292.4 $\pm$ 18.5
1991 No Si available			
1-X	52.693 $\pm$ 2.192	53.363 $\pm$ 0.527	299.7 $\pm$ 12.8
1-1	51.815 $\pm$ 2.424	51.723 $\pm$ 0.587	304.0 $\pm$ 14.6
1-3	58.754 $\pm$ 5.944	60.387 $\pm$ 1.377	295.3 $\pm$ 30.6
1991 With Si			
1-X	50.677 $\pm$ 1.461	52.031 $\pm$ 0.481	295.6 $\pm$ 8.9
1-1	49.402 $\pm$ 1.649	50.670 $\pm$ 0.536	295.9 $\pm$ 10.3
1-3	56.139 $\pm$ 3.676	59.142 $\pm$ 1.254	288.1 $\pm$ 19.8

Table 7 18 The 10% trimmed means and lifetimes of 1-prongs for different event topologies, obtained using the BOTH* thrust axis calculated from charged tracks and electromagnetic clusters.

Topol.	CTRK τ_r (fs)	BOTH* τ_r (fs)
1990 + 1991 No Si		
1-X	291.6 $\pm$ 7.5	292.3 $\pm$ 6.7
1-1	292.7 $\pm$ 8.9	295.8 $\pm$ 7.7
1-3	277.3 $\pm$ 15.3	281.2 $\pm$ 15.0
1990 + 1991 With Si		
1-X	294.4 $\pm$ 7.0	293.3 $\pm$ 6.2
1-1	294.3 $\pm$ 8.3	296.1 $\pm$ 7.2
1-3	282.1 $\pm$ 14.2	281.2 $\pm$ 14.0

Table 7 19. Combined lifetimes in 1990 and 1991, with and without Si, using the two possible thrust axes

Sample	data tracks	CTRK thrust mean (μm)	BOTH* thrust mean (μm)	BOTH* CTRK
1990	7537	45.255	48.992	1.082
1991 Si not avail.	7148	47.413	52.693	1.111
1991 with Si	10894	44.678	50.697	1.113

Table 7.20: Trimmed means for the three independent samples (1-X) using the CTRK and BOTH* thrust axes.

The Monte Carlo was divided into 12 partitions of an average of 8597 events each, similar to the average data sample size. Table 7.21 shows the trimmed means with the two thrust axes for each of these partitions. The mean of the ratio of the trimmed means is 1.115 with a standard deviation of 0.020. Thus, the standard deviation of 0.017 seen in the data is consistent with statistical fluctuations.

7.7.7 Monte Carlo Distribution of Event Topologies

The lifetime has been examined for a possible bias arising from the small difference in the Monte Carlo distribution of event topologies, 1:X, where $X = 1, \dots, 5$, from that observed in the data. The Monte Carlo trimmed mean has been calculated by recombining the individual trimmed means of each topology according to the topology fractions actually observed in the data. The lifetime thus obtained differs from the lifetime measured by a comparison of the complete d_t distribution between data and Monte Carlo by 0.3% for the 1990+1991 combined lifetime without Si, and by 0.4% for the combined lifetime with Si. The procedure of recalculating the Monte Carlo trimmed mean as a weighted mean of individual subsets of the complete distribution exhibits fluctuations at the level of 0.2%. Thus the lifetimes obtained by the direct comparison of the data with the complete distribution or the recombination of different topologies in the Monte Carlo, are consistent with the statistical uncertainty of the Monte Carlo sample.

Partition	# of tracks	CTRK thrust mean (μm)	BOTH* thrust mean (μm)	BOTH* CTRK
1	8606	49.606	54.209	1.093
2	8609	50.552	55.393	1.096
3	8531	50.330	56.975	1.132
4	8475	49.455	54.367	1.099
5	8584	48.527	52.783	1.088
6	8657	47.224	54.560	1.155
7	8593	45.120	50.186	1.112
8	8627	46.445	51.814	1.116
9	8545	46.862	53.500	1.142
10	8591	48.414	54.297	1.121
11	8697	46.891	52.620	1.122
12	8567	45.173	49.915	1.205

Table 7.21: Trimmed means for the 1991 Monte Carlo partitioned into 12 subsamples using the CTRK and BOTH* thrust axes.

7.8 Systematic Errors

7.8.1 Variation of the Trim Fraction; Resolution Effects

Several sources of systematic uncertainty have been evaluated. All resolution effects such as beam spot size uncertainty, vertex detector calibration and track resolution affect the width of the $d_{\perp}$ distribution but do not appreciably shift the mean. However, although the trimmed mean is found to be stable under variations of the impact parameter resolution, slight differences between the shapes of the data and Monte Carlo $d_{\perp}$ distributions can cause systematic shifts in the lifetime. The effect of such a shape difference is a smooth change in the measured lifetime as the trim factor is varied.

Figure 7.20 shows the lifetime over the full range of possible trims for the individual 1990, 1991 with and without Si data samples. Figure 7.21 shows the combined 1990 + 1991 with and without Si lifetimes as a function of trim. A common feature of all of these figures is the large shift between the lifetime calculated with no trim at all to that calculated with 1% trim. This shift is due to the small number of poorly measured events, $\sim 0.3\%$, in the tails of the data impact parameter distribution which were

Figure 7 20: Lifetime of individual samples over the full possible range of trims

Figure 7.21: Lifetimes of the combined samples with and without Si over the full possible range of trims. The arrow indicates the range over which the trim was varied to determine the systematic error on the lifetime

discussed in section 7.5.2. As shown in figure 7.6, a trim of 10% is close to the minimum relative statistical uncertainty of the trimmed mean. However, other choices for the trim are possible; the decision on exactly which trim value to choose is a balance between lower sensitivity to the shape of the tails of the distribution and the smaller statistical error. The range 3% to 30% trim are considered as valid alternative choices for the trim and are used to estimate a systematic error. The maximum variation of the combined lifetime within this range is taken as the systematic error. This corresponds to 0.4% for the combined 1990+1991 lifetime without Si, and 1.1% with Si.

7.8.2 The Thrust Axis

As was discussed in section 7.2, the properties of the thrust axis are due primarily to the decay properties of the τ lepton. In order to estimate the uncertainty on the lifetime due to the uncertainty in the input τ decay physics in the Monte Carlo, the total Monte Carlo trimmed mean has been calculated by recombining the individual trimmed means of each decay mode weighted according to the measured world average branching ratios. Table 7.22 lists the main τ 1-prong decay modes along with the world average branching ratios [1]. The branching ratios are those yielding a 1-prong decay and are given in percent of the total number of τ decays. Also given is the acceptance of each decay mode to be included in the final 1-prong track sample. The column of generated branching ratios (KORALZ $\times$ acceptance) is the number of decays generated in the particular Monte Carlo sample including the detector acceptance. The trimmed mean for each individual decay mode is also given.

Each branching ratio was varied, in turn, about the KORALZ value by $\pm 2\sigma$ where σ is the experimental uncertainty in the measured world average branching ratios. In most cases, this variation was large enough to overlap the experimental branching ratio measurements which were slightly different from the KORALZ values. When the default KORALZ branching ratio was more than 2σ from the measured world average, the Monte Carlo branching ratio was varied by an amount large enough to include the world average value. At each step, the total trimmed mean, and hence the τ lifetime, was recalculated. The maximum fluctuation of the Monte Carlo trimmed mean is shown in table 7.23

Decay Mode	World Av. BR (%)	1-prong acceptance	World Av. $\times$ accept. (%)	KORALZ $\times$ accept. (%)	Trimmed Mean (μm)
$\tau \rightarrow e\nu\bar{\nu}$	17.9 ± 0.3	0.961	17.2 ± 0.3	17.6	50.98 ± 1.14
$\tau \rightarrow \mu\nu\bar{\nu}$	17.5 ± 0.3	0.986	17.3 ± 0.3	17.6	45.38 ± 0.99
$\tau \rightarrow \pi(K)\nu$	12.7 ± 0.4	0.972	12.3 ± 0.4	12.2	51.32 ± 1.21
$\tau \rightarrow \rho(K^*)\nu$	24.4 ± 0.6	0.880	21.5 ± 0.5	23.2	40.55 ± 0.88
$\tau \rightarrow a_1\nu$	10.3 ± 0.9	0.811	8.3 ± 0.7	7.7	43.76 ± 1.26
$\tau \rightarrow \pi 3\pi^0\nu$	2.7 ± 0.9	0.757	2.0 ± 0.7	0.9	43.23 ± 3.70
Totals	85.5	0.923	78.6	79.2	

Table 7 22: The branching ratios and trimmed means for the different τ decay modes in the 1991 Monte Carlo sample

Decay Mode	Maximum Fluctuation (%)
$\tau \rightarrow e\nu\bar{\nu}$	0.1
$\tau \rightarrow \mu\nu\bar{\nu}$	0.02
$\tau \rightarrow \pi(K)\nu$	0.1
$\tau \rightarrow \rho(K^*)\nu$	0.7
$\tau \rightarrow a_1\nu$	0.2
$\tau \rightarrow \pi 3\pi^0\nu$	0.1

Table 7 23 Maximum fluctuation of Monte Carlo trimmed mean with variation of decay mode branching ratios.

The largest fluctuation was taken as the systematic uncertainty on the lifetime due to the uncertainties in the τ lepton decay branching ratios. The assigned systematic error is 0.7%, associated with the variation of the $\text{BR}(\tau \rightarrow \rho(K^*)\nu)$.

Effects of systematic errors in the thrust axis direction due to possible detector miscalibrations were measured by shifting the thrust direction in the data by 0.5 mrad, an amount several times the angular resolution, and observing the effect on the trimmed mean. The change in the measured lifetime of 0.1% is assigned as a systematic error.

7.8.3 Correction for Residual Background

The properties of the background to the 1-prong τ sample have been studied by considering 100 000 Monte Carlo $Z^0 \rightarrow \mu^+\mu^-$ events and 100 000 two-photon events. It is assumed that the background from $e^+e^- \rightarrow e^+e^-$ is described by the $Z^0 \rightarrow \mu^+\mu^-$ Monte Carlo. Of the generated events, 697 or 0.7% of the $Z^0 \rightarrow \mu^+\mu^-$ events and 227 or 0.2% of the two photon events were misidentified as $Z^0 \rightarrow \tau^+\tau^-$ events. Table 6.2 showed the background to the 1-prong sample and table 7.24 lists the residual background components after the cut on acoplanarity. The acoplanarity cut reduces the background to 763 and 368 tracks for the $Z^0 \rightarrow \mu^+\mu^-$ and two photon backgrounds, respectively.

Background process	% of τ decays after $ \text{acopl} > 0.002$
multihadrons (%)	0.06 ± 0.30
$e^+e^- \rightarrow e^+e^-$ (%)	0.14 ± 0.28
$e^+e^- \rightarrow \mu^+\mu^-$ (%)	0.81 ± 0.22
two-photon (%)	0.50 ± 0.20
total background (%)	1.51 ± 0.51

Table 7.24: Backgrounds to the 1-X lifetime sample remaining after the acoplanarity cut

The mean of the signed impact parameter distribution of each of these background contributions is consistent with zero. However, the 10% trimming procedure applied to the 1-prong τ data d_1 distribution gives an asymmetric trim window because of the finite τ lifetime and thus produces a non-zero mean for the background contribution

The fraction of these background tracks which affect the final lifetime is reduced by the 10% trim window. For the Monte Carlo muon background sample, 696 tracks were within the 10% trim window given by the 1-X tau data sample. The mean of these tracks was $(15.11 \pm 4.58) \mu\text{m}$. There were 336 tracks within the trim window in the two-photon sample with a mean of $(25.80 \pm 6.86) \mu\text{m}$. A large sample of Monte Carlo multihadronic Z^0 decays was examined and no events passed the above requirements, so the hadronic component of table 6.2 is left unchanged. With these reductions, the final background fraction remaining in the lifetime sample of 1-prong tracks is $(1.39 \pm 0.48)\%$. Weighting the means of the two Monte Carlo background samples by their relative contributions to the total background fraction gives a combined mean for all background tracks of $(18.85 \pm 3.82) \mu\text{m}$. The mean of the small multihadronic background component was assumed to be equal to this combined mean. Table 7.25 shows the trimmed means and the resulting lifetimes corrected for a background component with the above mean.

There is a 0.82% correction to the combined lifetime with Si. The uncertainty on the amount of background is 34% of the background fraction. A systematic error of $0.34 \times 0.82\% = 0.28\%$ is assigned to the measured lifetime.

Sample	before correction	after correction	corrected τ_τ (ps)
1990	45.255 ± 1.949	45.627 ± 1.975	293.6 ± 13.4
1991 Si not used	45.719 ± 1.335	46.098 ± 1.353	294.2 ± 9.2
1991 Si not avail.	47.413 ± 2.221	47.816 ± 2.252	304.9 ± 14.7
1991 with Si	44.678 ± 1.470	44.920 ± 1.431	294.0 ± 9.8
1990 + 1991 No Si			294.0 ± 7.6
1990 + 1991 with Si			296.4 ± 7.1

Table 7.25 Trimmed means and lifetimes for the 1-X topology before and after the correction for residual background.

7.9 Summary

In summary, the following corrections and systematic errors are assigned to the 1-prong tau lifetime analysis:

source	correction to lifetime (%)	systematic uncertainty (%)
variation of trim factor (includes detector resolution and beam size effects)		± 1.1
Monte Carlo decay mode branching ratios		± 0.7
thrust axis reconstruction		± 0.1
background fractions	± 0.8	± 0.3
total	± 0.8	± 1.3

The final measured τ lifetime from the impact parameter analysis of 1-prong τ decays is:

$$\tau_1 = 296.4 \pm 7.1(\text{stat.}) \pm 3.8(\text{sys.}) \text{ fs.} \quad (7.4)$$

This measurement uses all one prong τ decays (1-X), an event thrust axis calculated with all charged tracks in the event, and included silicon microvertex detector information when available.

Chapter 8

The Decay Length Method

Section 2.4 introduced the decay length method for measuring the τ lifetime. The technique uses the 3-prong decays to reconstruct the actual τ flight distance on a decay by decay basis. One of the principal advantages of the method is that it is less dependent on the Monte Carlo simulation than the impact parameter method.

8.1 Method

The method consists of three steps. First, the tracks in a 3-prong τ decay are fit to a common vertex in the xy plane to approximate the actual decay point of the τ . The vertex fit is a least squares fit of three tracks with circular trajectories[†] to a common point. A 1% cut on the χ^2 probability of the fit is applied to improve the quality of the vertices. The result of the vertex fit is a τ decay point and a corresponding error ellipse in the xy plane.

8.1.1 The Decay Length Fit

The second step is to use this vertex, along with knowledge of the e^+e^- interaction region and the thrust axis, to measure a decay length. The decay length is first measured in the xy projection and then converted to a three-dimensional decay length by using the polar angle of the event thrust axis which was defined in equation 6.1. The two-dimensional formalism to determine the τ decay length is summarized below [43]. There

[†]A circle is the projection of the OPAL helix track fit into the xy plane.

are three inputs to the calculation:

1. The τ originates within the pp interaction region which has a spatial extent of $(\sigma_{bx}, \sigma_{by})$ centred on the average beam position (x_b, y_b) , see section 6.5
2. The measured vertex of a 3-prong decay (x_d, y_d) , along with its associated error matrix, M_d , is an estimate of the τ decay point.
3. The τ flight direction is approximated by the xy projection of the event thrust axis, (t_x, t_y) , calculated using charged tracks, where $t_x^2 + t_y^2 = 1$.

The decay length method determines the most probable τ decay length (l_{xy}) , the most probable τ decay position within the error ellipse of the 3-prong vertex (x_d, y_d) , and the most probable τ production point within the interaction region (x_p, y_p) subject to the constraint that the τ flight path is parallel to the thrust axis. This direction constraint is considered fixed (no allowance is made for an uncertainty in the thrust axis direction) in order to retain a linear problem which can be solved analytically. Including an error on the thrust axis direction results in a nonlinear problem which must be solved numerically. The use of this fixed direction constraint will be discussed further in section 8.6.8. Defining

$$M_b \equiv \begin{pmatrix} M_{bxx} & 0 \\ 0 & M_{byy} \end{pmatrix}$$

$$M_d \equiv \begin{pmatrix} M_{dxx} & M_{dxy} \\ M_{dxy} & M_{dyy} \end{pmatrix}$$

where the error matrix elements are $M_{bxx} = \sigma_{bx}^2$ etc., a χ^2 statistic is formed

$$\chi^2 = (x_p - x_b, y_p - y_b) M_b^{-1} \begin{pmatrix} x_p - x_b \\ y_p - y_b \end{pmatrix} + (x_d - x_b, y_d - y_b) M_d^{-1} \begin{pmatrix} x_d - x_b \\ y_d - y_b \end{pmatrix} \quad (8.1)$$

The direction constraint introduces a relationship between (x_d, y_d) and (x_p, y_p)

$$x_d = x_p + l_{xy} t_x$$

$$y_d = y_p + l_{xy} t_y$$

Minimization of this χ^2 with respect to the three unknowns (x_p, y_p, l_{xy}) is equivalent to the condition

$$\frac{\partial \chi^2}{\partial x_p} = \frac{\partial \chi^2}{\partial y_p} = \frac{\partial \chi^2}{\partial l_{xy}} = 0 \quad (8.2)$$

which can be solved analytically. Introducing the notation

$$x = x_p - x_b$$

$$y = y_p - y_b$$

$$M = M_p - M_b$$

$$D = t_y^2 M_{xx} + t_x^2 M_{yy} - 2t_x t_y M_{xy},$$

the decay length and its error are given by the following relations:

$$l_{xy} = \frac{xt_x M_{yy} - yt_y M_{xx} - (xt_y + yt_x) M_{xy}}{D} \quad (8.3)$$

$$\sigma_{l_{xy}} = \sqrt{\frac{M_{xx} M_{yy} - M_{xy}^2}{D}} \quad (8.4)$$

$$\chi^2 = \frac{(xt_y - yt_x)^2}{D}. \quad (8.5)$$

The result of this fit is a two-dimensional decay length l_{xy} , and its uncertainty, $\sigma_{l_{xy}}$. The three-dimensional decay length, l , and error, σ_l , are obtained from the relations $l = l_{xy}/\sin \theta$, and $\sigma_l = \sigma_{l_{xy}}/\sin \theta$, where θ is the polar angle of the thrust axis. As with the 3-prong vertex fit, a 1% quality cut is applied to the χ^2 probability of the fit. Figure 8.1 illustrates the technique schematically.

8.1.2 The Maximum Likelihood Fit

The third step in the decay length method is to determine the mean τ decay length from the distribution of fitted decay lengths. This is done by means of a maximum likelihood fit to the three-dimensional decay length distribution. The distribution of decay lengths is taken to be the convolution of an exponential with Gaussian resolution functions with widths equal to the decay length errors. The probability of observing a single decay of length l , with uncertainty σ_l is

$$P(l, \sigma_l; l, s) = \frac{1}{l s \sigma_l \sqrt{2\pi}} \int_0^\infty e^{-t/l} e^{-\frac{(l-t)^2}{2s^2\sigma_l^2}} dt$$

Figure 8.1. Schematic diagram of the decay length method. The decay length is constrained to be parallel to the reconstructed event thrust axis. (For comparison, see figure 2.3.)

$$\frac{1}{2l} \exp\left[\frac{s^2 \sigma_i^2}{2l} - \frac{l_i}{l}\right] \operatorname{erfc}\left[\frac{1}{\sqrt{2}}\left(\frac{s\sigma_i}{l} - \frac{l_i}{s\sigma_i}\right)\right] . \quad (8.6)$$

where, in addition to the parent distribution decay length l , a scale factor s for the decay length errors, σ_i , is included as a fit variable. This scale factor provides an allowance for the possibility that all of the decay length errors were systematically under or over estimated by the same amount. The maximum likelihood fit minimizes the negative sum of the log of the probabilities for all observed decay lengths and errors.

$$\log P = \log \prod_{i=1}^N P(l_i, \sigma_i; l, s) . \quad (8.7)$$

Evaluation of the error function complement for values of its argument greater than 9 is difficult when the argument of the exponential function becomes large. Standard mathematical library routines often give identically zero. We use an asymptotic expansion of $\operatorname{erfc}(z)$ for large values of its argument:

$$\operatorname{erfc}(z) \sim \frac{e^{-z^2}}{z\sqrt{\pi}} \sum_{n=1}^{\infty} (-1)^n \frac{(2n-1)!!}{(2z^2)^n} . \quad (8.8)$$

Although not strictly convergent for all values of z , a truncated form (typically ≤ 9 terms) of this series is sufficiently accurate for the range $z > 8$. For other values of the argument, standard library routines are used.

Maximum likelihood techniques are, in general, sensitive to entries in the tails of the distribution. As a result, a window cut of $-1.0 < l_i < +2.0$ cm is introduced which eliminates typically 0.5% of the decay lengths. The expression of equation 8.6 must then be modified to include a renormalization factor from a reduced range of the decay length in the probability integral [42, 44]:

$$P(l_i, \sigma_i; l, s) \rightarrow \frac{P(l_i, \sigma_i; l, s)}{P_R(l_i, \sigma_i; l, s)} , \quad (8.9)$$

where, for a window cut $l_{\min} \leq l_i \leq l_{\max}$, the renormalization integral is

$$P_R(l_i, \sigma_i; l, s) = \int_{l_{\min}}^{l_{\max}} P(\lambda, \sigma_i; l, s) d\lambda . \quad (8.10)$$

An integration by parts gives the useful form

$$P_R(l_i, \sigma_i; l, s) = \left[-lP(\lambda, \sigma_i; l, s) - \frac{1}{2} \operatorname{erfc}\left(\frac{\lambda}{s\sigma_i\sqrt{2}}\right) \right]_{\lambda=l_{\min}}^{l_{\max}} . \quad (8.11)$$

This factor becomes important whenever the decay length window cut causes a significant fraction of the entries to be removed. For the τ lifetime analysis discussed here, this happens when the window limits lie within about $[-2, +2]$ cm.

In addition, a cut of $\sigma_l \sim 0.6$ cm is applied to the decay length errors, which eliminates 1.1% of the decay lengths which have poor resolving power for the maximum likelihood fit. The effect on the mean decay length of these two restrictions on entries accepted into the maximum likelihood fit will be discussed in section 8.5.3.

The measured mean decay length is then converted to a τ lifetime by applying the Lorentz boost factor: $l = \beta\gamma c\tau_\tau$.

8.2 The 3-prong Data Sample

Table 8.1 shows the number of 3-prong candidates which is defined as the number of hemispheres in a $\tau^+\tau^-$ event having exactly three charged tracks. These three

data sample	$\tau^+\tau^-$ events	3-prong candidates	γ after removal	after K^0 rejection
1990 data	5107	1303	1168	1145
1991, Si unavailable	3257	966	820	803
1991, Si available	8750	2478	2155	2111
1990+1991 total	17114	4747	4143	4059

Table 8.1: Numbers of events with accurate e^+e^- interaction region information and the number of three-prong hemispheres after photon conversion, and K^0 rejection. The number of 3-prong candidates in the data with silicon available is independent of whether the silicon information is included on the tracks or not.

tracks are used to reconstruct the τ decay point, so it is necessary to reject any tracks which do not come directly from the τ decay. For example, all decay modes involving π^0 mesons can, in turn, have one of the decay photons interact in the detector and produce a charged e^+e^- pair of tracks. To remove contamination by such secondary photon pair-creation and by decays of $K^0 \rightarrow \pi^+\pi^-$, an algorithm looks for all pairs of oppositely charged tracks and calculates the geometrical intersections of the circular trajectories in the xy plane

(if any), or the point at which the tangents of the tracks are parallel. The pair is then rejected if the reconstructed mass is consistent with either of the above two processes. Specifically, the regions $m(e^+e^-) < 0.05 \text{ GeV}/c^2$ and $0.473 < m(\pi^+\pi^-) < 0.523 \text{ GeV}/c^2$ are excluded. The measurement of the mass of K^0 mesons in OPAL has a mean of $497.2 \pm 0.1 \text{ MeV}/c^2$ and a mass resolution of $6.5 \pm 0.1 \text{ MeV}/c^2$ [45]. Figure 8.2 shows the distributions of reconstructed masses in the 1990 and 1991 data samples. The effect of the rejection of K^0 decays and photon conversions is shown in table 8.1. Note that this is not simply a reduction in the number of 3-prong hemispheres since some hemispheres with more than three charged tracks will be recovered after the removal of photon conversion or K^0 tracks.

Section 3.2.2 first introduced the property of the vertex drift chamber that the second pulse recorded on a sense wire has a degraded resolution with respect to the first pulse. Section 4.1.3 went on to describe how the space points reconstructed from these second pulses may have systematic offsets and outlined the procedure used to apply a correction. At LEP energies, the three tracks in a 3-prong τ decay are narrowly collimated, and it is common for two or more of the tracks to pass through a single CV drift cell. Figure 8.3 illustrates the importance of considering second-hit effects in the vertex drift chamber when reconstructing 3-prong τ decay tracks. Therefore, in developing a track quality criterion for 3-prong decay candidates in the lifetime sample, two categories of tracks containing CV hits are defined: i) tracks having at least as many second hits as first hits, called second-hit tracks, and ii) all other tracks, called first-hit tracks. Figure 8.4 shows that there is little ambiguity in this definition. First-hit tracks contain over 99% first hits, while second-hit tracks are composed of about 97% second hits.

Systematic shifts of second hits can cause a second-hit track to be pulled closer to (or pushed further away from) the first-hit track with the result being a systematic bias to longer (or shorter) measured decay lengths. Such a systematic effect on the decay length was observed in a first analysis of the 1991 data, with CV+CJ tracking only, *i.e.*, not including Si hits in the track fit. The results of that initial analysis are given in table 8.2.[‡]

[‡]Note that this initial study was performed without the explicit veto of multihadronic events, and thus the numbers of 3-prong decays shown in tables 8.2, 8.3 and 8.4 are $\sim 10\%$ larger than in the final data sample used to measure the lifetime.

Figure 8.2: The reconstructed mass of track pairs of opposite charge assuming electron mass (left side), or pion mass (right side). Plots a) and b) are for the 1990 data, c) and d) are for the 1991 data which contain no silicon information.

Figure 8 3: An event display of a 3-prong τ decay candidate reconstructed in the vertex drift chamber. Anode wire planes are indicated by dotted radial lines, and cathode wire planes by solid radial lines (Both ambiguities of hits in CV are shown, see section 3.2.2). Counter-clockwise, the tracks in the 3-prong hemisphere have 12, 12, and 7 vertex drift chamber hits, respectively, all within the same CV drift cell.

Figure 8.4. Distributions of numbers of CV first and second hits on first-hit and second-hit tracks. For tracks which have CV hits in the fit, second-hit tracks are those which have at least as many second hits as first hits.

There is a 4.3σ discrepancy in the measured decay lengths between the subset of 3-prong candidates which contain no CV second-hit tracks, versus the subset containing at least one CV second-hit track. The corresponding comparison shows only a 1.4σ effect in the 1990 data. Furthermore, in the subset of 1991 data which has silicon information that is added to (but not required on) the tracks, the effect is reduced to the level of 1.3σ .

subset	no. in fit	decay length	scale factor
1990			
all	862	0.264 ± 0.013	1.268 ± 0.054
no CV2 trks	323	0.241 ± 0.020	1.183 ± 0.079
with CV2 trks	539	0.277 ± 0.016	1.332 ± 0.073
1991 CV+ CJ tracking only			
all	2084	0.253 ± 0.008	1.273 ± 0.034
no CV2 trks	689	0.205 ± 0.012	1.238 ± 0.052
with CV2 trks	1395	0.274 ± 0.011	1.288 ± 0.045
1991 Si microvertex present & added			
all	1597	0.216 ± 0.006	1.158 ± 0.040
no CV2 trks	590	0.204 ± 0.011	1.148 ± 0.060
with CV2 trks	1007	0.222 ± 0.008	1.161 ± 0.052

Table 8.2. Decay lengths from maximum likelihood fits to subsets of the data of the 1990 and 1991 data. A CV2 track is one with CV hits, at least half of which are second hits.

As shown in table 8.2, rejecting 3-prong candidates which contain second-hit tracks would result in the loss of two-thirds of the sample. The decay length and scale factor were defined in equation 8.6. Instead, one can define a different selection criteria based on the number of "good-quality" tracks. In the case of the non-silicon data, "good-quality" is defined to be the CV first-hit tracks, whereas for the data with silicon information added, the definition of good-quality is expanded to also include those tracks which have at least one hit in the Si microvertex detector. Table 8.3 shows the fitted decay lengths sorted according to this good-quality track criteria. It can be seen that in all three data groups – 1990, and 1991 data both with and without silicon – the discrepancy between the subsets with at least two or less than two good tracks is even greater than between the subsets which do or do not include CV second-hit tracks.

subset	no. in fit	decay length	scale factor
1990 data			
≤ 1 good track	441	0.301 ± 0.022	1.377 ± 0.082
≥ 2 good tracks	421	0.237 ± 0.015	1.128 ± 0.069
1991 CV + CJ tracking only			
≤ 1 good track	1041	0.309 ± 0.015	1.299 ± 0.051
≥ 2 good tracks	1043	0.219 ± 0.009	1.197 ± 0.044
1991 silicon present & added			
≤ 1 good track	78	0.360 ± 0.064	1.248 ± 0.197
≥ 2 good tracks	1519	0.213 ± 0.006	1.138 ± 0.040

Table 8.3 Decay lengths from maximum likelihood fits to subsets distinguished by number of "good quality" tracks, defined as either CV first-hit tracks, or tracks with at least one hit in the Si microvertex detector

A requirement of at least two good-quality tracks has several advantages over simply rejecting any 3-prong candidate which contains a CV second-hit track. First, the measured decay lengths for the candidates satisfying this criterion are consistent between all data samples. Second, this criterion rejects less data. In the samples which do not have silicon information, 50% of the 3-prong candidates are retained by a requirement of two good-quality tracks as compared with the two-thirds which are eliminated by rejecting all candidates containing a CV second-hit track. In the data with silicon information, the efficiency for satisfying the requirement of at least two good tracks is 87%. Third, the average measured decay length error is less in those 3-prongs which have at least two good quality tracks.

The 3-prong τ decay sample selected by this two good-quality track criterion, however, may still be subject to effects from a possible systematic shift of the remaining CV second-hit tracks. Therefore, the parameters of the default second-hit correction were examined to determine if they were adequate for $\tau^+\tau^-$ data.

The second-hit calibration parameters to correct for the systematic shift of second hits, described in section 4.1.3, were determined using a large sample of tracks in multihadron data. Using this default correction, however, it was found that the hit residuals of CV second hits in $\tau^+\tau^-$ data showed a significant systematic displacement from

zero. Figure 8.5a shows the mean of residuals for second hits on tracks in 3-prong candidates, plotted as a function of the separation of the second hit from its predecessor measured by the TDC time difference. To reduce the possibility of a systematic effect on the τ lifetime measurement from a second-hit correction optimized for multihadronic data, the $\tau^+\tau^-$ sample was reprocessed with new second-hit correction parameters determined directly from the $\tau^+\tau^-$ data itself. This procedure eliminates the observed systematic offsets in the residual distribution of CV second hits as shown in figure 8.5b. The effect on the measured decay lengths of reprocessing the $\tau^+\tau^-$ data with these new second-hit calibration parameters is shown in table 8.4 for each of the good track and CV track categories previously presented in tables 8.2 and 8.3. There is a large improvement in the agreement between all the subsets of individual data samples as well as between different samples.

subset	no. in fit	decay length	scale factor
1990 reprocessed data			
all	868	0.216 ± 0.011	1.248 ± 0.048
no CV2 trks	304	0.241 ± 0.021	1.113 ± 0.078
with CV2 trks	564	0.203 ± 0.013	1.304 ± 0.061
≤ 1 good trk	435	0.205 ± 0.018	1.328 ± 0.069
≥ 2 good trks	433	0.221 ± 0.014	1.151 ± 0.068
1991 reprocessed data, silicon not used			
all	2047	0.208 ± 0.007	1.332 ± 0.033
no CV2 trks	656	0.206 ± 0.013	1.240 ± 0.054
with CV2 trks	1391	0.210 ± 0.009	1.369 ± 0.041
≤ 1 good trk	1043	0.198 ± 0.012	1.432 ± 0.049
≥ 2 good trks	1004	0.215 ± 0.009	1.206 ± 0.045
1991 reprocessed data, silicon present & added			
all	1607	0.215 ± 0.006	1.018 ± 0.036
no CV2 trks	573	0.209 ± 0.011	1.007 ± 0.055
with CV2 trks	1034	0.219 ± 0.008	1.024 ± 0.048
≤ 1 good trk	90	0.285 ± 0.047	1.249 ± 0.164
≥ 2 good trks	1517	0.213 ± 0.006	0.995 ± 0.037

Table 8.4 Decay length fit results to the data processed with the new CV second-hit correction.

In spite of this improvement, the lifetime measurement still uses only those 3-

Figure 8.5: Means of CV second-hit track residuals as a function of the time separation from the nearest predecessor track measured in TDC counts: a) the 1991 $\tau^+\tau^-$ sample using the default second-hit correction; b) the same data now reprocessed with the new second-hit correction; c) the frequency distribution of CV second hits by TDC time separation from the predecessor track.

prong candidates which contain at least two good-quality tracks. This choice is made in order to reduce the systematic error of the lifetime measurement associated with second-hit calibration uncertainties. Furthermore, note that the results obtained with this two good track requirement are consistent between all three data sets both before and after the reprocessing. (Compare tables 8.3 and 8.4 for the 2 good track categories.) The comparison of the measured mean decay length before and after reprocessing will be discussed further in section 8.6.1.

After the rejection of γ -conversions and K^0 decays, and the requirement of at least two good-quality tracks, the total charge of the 3-prong candidates is checked and required to sum to ± 1 .

8.3 The Decay Length Measurement

The remaining 3-prong decays are fit to a common vertex in the xy plane. Figure 8.6 shows the distribution of the χ^2 probabilities of the vertex fit for the data samples from 1990, 1991 with Si microvertex detector information not available, and 1991 including Si information. The vertex fit fails for approximately 0.8% of 3-prong candidates. For the successful fits, those with a probability of $<1\%$ were rejected. The remaining χ^2 probability distributions are flat.

Figure 8.7 shows the difference between the reconstructed τ decay vertex position, after the 1% probability cut, and the average beam position for the same three data samples. These distributions, while broad, are centred at zero within their statistical uncertainties indicating no systematic bias.

Because of the narrow opening angle in the $r\phi$ plane in 3-prong τ decays at Z^0 energies (90 mrad on average), the error ellipses from the vertex fits have high aspect ratios with the major axis being almost parallel to the thrust axis. Figure 8.8 shows the major and minor axes of the 3-prong vertex error ellipses. This gives an indication of the precision with which the τ decay point can be reconstructed and also illustrates the improvement in resolution with the introduction of the Si microvertex detector. The vertex error components along perpendicular to the thrust axis average $1660 \pm 52 \mu\text{m}$ in

Figure 8.6. A plot showing the χ^2 probability of the vertex fit. The top plot is the data sample from 1990, the middle and bottom are the 1991 without and with Si samples. A 1% probability cut corresponds to eliminating the first bin.

Figure 8.7 The vertex position with respect to the average beam position for the 1990 (top row), 1991 without (middle row) and with Si (bottom row) data samples

the data without Si information, and $1200 \pm 22 \mu\text{m}$ in the data with Si information

Figure 8.9 shows the χ^2 probability distributions from the decay length fit for the three data samples. A 1% probability cut was applied to this fit. The remaining distributions are flat. Figure 8.10 shows the difference between the τ production points resulting from the decay length fit and the average beam position for each case. No bias is indicated.

Table 8.5 shows the number of 3-prong candidates which satisfy the track quality criteria and those which also pass the 1% probability cuts on the vertex and decay length fits. Figure 8.11 shows the two-dimensional decay length and error distributions for these

data sample	3-prong candidates	≥ 2 good-quality tracks	1% prob. cut on vertex fit	1% prob. cut on decay length fit
1990 data	1138	450	408	395
1991, Si unavailable	791	339	299	278
1991, Si avail/not added	2100	840	715	673
1991 data w/Si on trks	2099	1836	1507	1420
1990–1991 silicon not used	4029	1629	1422	1346
1990 + 1991 with silicon	4028	2625	2214	2093

Table 8.5. Numbers of 3-prong candidates containing at least two good tracks, and passing the 1% probability cuts for the vertex and decay length fits. Not included here are the 0.8% of the 3-prong candidates for which the vertex fit failed.

decays. Each two-dimensional decay length is then divided by $\sin \theta_{\text{thrust}}$, as discussed in section 8.1.1, to form the three-dimensional decay length.

The τ lifetime is obtained from the three-dimensional decay length, measured in the laboratory reference frame, by using the τ Lorentz boost factor:

$$l_0 = \beta\gamma c\tau_\tau$$

where β is the velocity with respect to the speed of light, $\beta = v/c$, and $\beta\gamma = \frac{p}{m_\tau c}$ where p and m_τ are the momentum and mass. Thus it is necessary to know the momentum of the τ . For $\tau^+\tau^-$ pairs produced in e^+e^- collisions this is equal to the e^+e^- beam momentum. However, in order to measure the width of the Z^0 resonance, the OPAL data sample was

Figure 8.8 The major and minor axes in cm of the 3-prong vertex error ellipses for the 1990, 1991 without and with Si data samples.

Figure 8.9: A plot showing the χ^2 probability of the decay length fit. The top plot is the data sample from 1990, the middle and bottom are the 1991 without and with Si samples. A 1% probability cut corresponds to eliminating the first bin.

Figure 8.10 The production point from the decay length fit with respect to the average beam position for the 1990 (top row), 1991 without (middle row) and with Si (bottom row) data samples

Figure 8.11. The two-dimensional decay lengths and their errors from the decay length fit. Decays shown satisfy the 1% probability cut on both the vertex and decay length fits.

obtained using several e^+e^- beam energies at LEP slightly above, at and below the Z^0 peak centre-of-mass energy of 91.2 GeV. Approximately 78% of the $\tau^+\tau^-$ events were collected at this peak energy. Rather than calculating a separate τ lifetime for each of the centre-of-mass energies at which LEP was operating, a correction is applied to the three-dimensional decay lengths to convert them all to the reference energy of 91.160 GeV. This correction also takes into account the effects of initial state radiation of the e^+e^- beams. Radiation of the e^+e^- beams before colliding changes the centre-of-mass energy of the $\tau^+\tau^-$ events and thus the τ momentum. The probability that an e^+e^- event will be accompanied by an initial state radiated photon depends on the beam energy; initial state radiation associated with an e^+e^- interaction is more probable at energies slightly above the Z^0 peak than slightly below it.

To calculate this correction to the three-dimensional decay lengths, the Monte Carlo generator program KORALZ was run at several reference energies corresponding approximately to the various LEP centre-of-mass energies. These reference energies were not exactly the same as the actual LEP operating points, and a linear interpolation was performed using the reconstructed centre-of-mass energy observed in the data. The decay lengths were then scaled by the boost correction factors $(\beta\gamma)_{91.160}/(\beta\gamma)_i$. Table 8.6 shows the numbers of 3-prong τ decays in the data collected at each LEP energy and the correction factor applied to the measured three-dimensional decay length at each energy. By using corrected decay lengths, a single τ boost factor can then be used to convert the average three-dimensional decay length measurement to a lifetime.

Figure 8.12 shows the distributions of corrected three-dimensional decay lengths and decay length errors for the 1990+1991 data samples when no Si information was available and for the 1991 data sample including Si microvertex detector information.

8.4 Mean Decay Length Results

A maximum likelihood fit is then performed on the distribution of three-dimensional decay lengths to determine the mean decay length. The intrinsic precision afforded by the

Figure 8.12: Three-dimensional decay length and decay length error distributions. The upper two plots are from the 1990 data sample plus those decays in the 1991 data sample for which Si information was not available. The lower two plots are the 1991 sample where Si information was available and included in the track fits.

center-of-mass energy	number of 3-prong decays	ISR correction	boost correction	total d.l. correction
88.6	35	1.0102	1.0289	1.0394
89.4	56	1.0092	1.0197	1.0291
90.2	101	1.0086	1.0107	1.0194
91.2	1622	1.0088	0.9996	1.0084
92.0	140	1.0102	0.9909	1.0010
93.0	69	1.0139	0.9802	0.9938
93.8	70	1.0181	0.9718	0.9894

Table 8.6 Approximate center-of-mass energies for the 3-prong τ decays remaining after all cuts, together with the initial-state radiation and boost correction factors applied to the three-dimensional decay lengths. The number of 3-prong decays is given for the combined 1990 and 1991 data samples where silicon detector information has been used when available. Slightly different energy scan points were used in the 1990 and 1991 data taking periods.

maximum likelihood fit can be characterized as

$$\sigma_{m.l.} \sim \frac{\sqrt{l^2 + \langle \sigma_l \rangle^2}}{\sqrt{N}},$$

where l is the mean exponential decay length being measured, $\langle \sigma_l \rangle$ is the average resolution of the decay length measurements, and N is the sample size. With an average $l = 0.220$ cm for 3-prong τ decays at LEP, and with typical resolutions on the decay length of 0.20 cm without silicon and 0.13 cm with silicon hits included (see figure 8.12), one finds that

$$\sqrt{N} * \sigma_{m.l.}^{noSi} = 0.297 \text{ cm},$$

$$\sqrt{N} * \sigma_{m.l.}^{Si} = 0.256 \text{ cm}.$$

For this measurement, the added precision of the Si detector decreases the statistical uncertainty by only 14%. However, as discussed in section 8.2, the expanded definition of good-quality tracks to include any with Si hits essentially doubles the sample size.

The results of the maximum likelihood fits to the individual decay length distributions are given in table 8.7. These results can be directly combined either i) in a weighted mean of the average decay lengths in each data sample, or ii) a single maximum

likelihood fit performed on all decay lengths from the combined 1990 + 1991 data sample. The results of these two approaches are virtually identical.

data set	no. in fit	decay length	scale factor
1990 + 1991 data, without silicon			
1990 data	387	0.2372 ± 0.0155	1.081 ± 0.070
1991, CV+CJ only	918	0.2160 ± 0.0095	1.214 ± 0.047
weighted average		0.2218 ± 0.0081	
global fit	1466	0.2219 ± 0.0081	1.182 ± 0.039
1990 + 1991 data with silicon			
1990 + 1991 NoSi	651	0.2248 ± 0.0116	1.176 ± 0.054
1991 with Si	1398	0.2135 ± 0.0065	0.990 ± 0.038
weighted average		0.2162 ± 0.0057	
global fit	2049	0.2162 ± 0.0057	1.068 ± 0.032

Table 8.7: The mean decay length from the combined 1990 and 1991 data. Only those decay lengths within the window of $[-1.0, +2.0]$ cm and having an error less than 0.6 cm are included.

The 1990+1991 combined result has 3.7% statistical error when only drift chamber information is used, and 2.6% statistical error when the information from the Si microvertex detector is used when available. These errors are close to the theoretical limits for the maximum likelihood fit for both the no-Si and Si options. We use the result obtained including the Si microvertex detector as the 3-prong decay length lifetime measurement. The decay length distributions for the scenarios with and without silicon information included in the combined 1990 + 1991 data set are shown in Figures 8.13 and 8.14, respectively, together with the overlaid plotting function from the global fit.

8.5 Experimental Checks of Results

The following studies were performed with the combined 1990 + 1991 data sample, using silicon information when available, unless otherwise indicated. Only those 3-prong candidates containing at least two good-quality tracks are used.

Figure 8.13. The decay length distribution for the combined 1990 and 1991 data samples with CV-CJ tracking only. Decay lengths with errors $> 600 \mu\text{m}$ have been removed. The points are the data and the overlaid curve is obtained from the maximum likelihood fit over the window $[-1.0, +2.0]$ cm. The shaded histogram is the contribution from the 1990 data sample.

Figure 8.14: The decay length distribution for the combined 1990 and 1991 data samples, where silicon information is included when available. Decay lengths with errors $> 600 \mu\text{m}$ have been removed. The points are the data and the overlaid curve is obtained from the maximum likelihood fit over the window $[-1.0, +2.0]$ cm. The shaded histogram is the contribution from the 1990 data sample plus the 1991 data when silicon information was not available.

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PM-1 3 1/2" x 4" PHOTOGRAPHIC MICROCOPY TARGET
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8.5.1 Azimuthal and Polar Angle

Figure 8.15 shows the polar and azimuthal angles of the event thrust axis. The mean decay length for vertical, horizontal and diagonal τ decays, defined in section 7.4, and the mean decay length as a function of the thrust axis polar angle are also shown in figure 8.15. No systematic effects are observed.

Figure 8.15 The top two plots show the azimuthal and polar angles of the event thrust axis. The bottom two plots show the average decay length as a function of the two angles. The convention for defining vertical, horizontal and diagonal was presented in section 7.4

8.5.2 Detector Related Effects

Table 8.8 gives the mean decay length obtained when the data is divided up according to the summed 3-prong charge and the number of charged tracks counted in the opposite hemisphere. Differences in the mean decay length for subsamples with different

division	no. in fit	decay length
ALL	2049	0.2162 ± 0.0057
ΣQ_c	no. in fit	decay length
-1	1052	0.2164 ± 0.0081
+1	997	0.2152 ± 0.0079
no. trks, opposite hem.	no. in fit	decay length
≤ 1	1849	0.2176 ± 0.0060
≥ 2	200	0.2085 ± 0.0169
≥ 3	169	0.2131 ± 0.0186

Table 8.8. Mean decay length results with the 1990+1991 data. The tracks counted in the opposite hemisphere are those which have had conversion pairs removed.

numbers of charged tracks in the opposite hemisphere could be an indication of residual multihadronic background in the sample; the 1-3 channel is expected to be virtually free of this contamination. Multihadron contamination is expected to be quite small (see table 6.2) since these events are explicitly rejected in the $\tau^+\tau^-$ selection procedure. The number of tracks in the opposite hemisphere was counted after photon conversion and K^0 rejection, and after applying the minimal track quality cuts listed in table 6.1. There is no direct evidence for the effect of residual multihadron background. A small correction to the lifetime for multihadron background will be described in section 8.6.6.

8.5.3 Checks on the Maximum Likelihood Fit

The sensitivity of the mean decay length to the decay length error cut and the decay length window used in the maximum likelihood fit was studied on the combined 1990 + 1991 data sample ignoring silicon microvertex detector information. The purpose of the window cut is to eliminate a small number of very poorly measured decay lengths which can bias estimates of the mean. This is especially important when using a maximum

likelihood estimator where a few events in the extreme tails can have a large effect on the fit result. Since the characteristic tau decay length is anticipated to be about 0.22 cm, 99% of all decays should occur within 1 cm of the production point. Table 8.9 shows the result of varying the window. The final τ lifetime result was determined using a window of $[-1.0, +2.0]$ cm, which includes $\sim 99.5\%$ of the decays (see also the final decay length distribution and fit in figure 8.13). Excluding the fit with no window cut applied, all results are well within the statistical uncertainty of the fit.

Figure 8.12 showed that the majority of 3-prong τ decays have measured decay length errors less than 0.8 cm. The final result was determined using a cut on the decay length error of 0.6 cm which includes $\sim 98.9\%$ of the decay length measurements. Table 8.9 shows that there is no significant variation of the measured average decay length for different values of the decay length error cut.

In the track reconstruction procedure, an error matrix is calculated for each track and is an estimate of the uncertainties on the parameters of the track as well as of correlations between parameters. The error matrices from the three tracks in a 3-prong candidate determine the size of the 3-prong vertex error ellipse which, in turn, contributes to the estimate of the error on the decay length. The scale factor on the decay length error from the maximum likelihood fit provides an allowance for possible systematic underestimation of the track parameter error matrices.

The results of table 8.7 showed that while the maximum likelihood fit to the 1991 data sample including silicon information produces a scale factor consistent with unity, when the silicon information is not present or ignored the scale factors from the fit are about two standard deviations greater than unity. To investigate the sensitivity of the mean decay length to an underestimation of the track parameter errors, a second maximum likelihood fit was performed using decay length errors which were inflated by an amount equal to the scale factor which resulted from the normal maximum likelihood fit with the default errors. The resulting mean decay length is consistent with that found using the original errors.

window variations, $\sigma_{dl} < 0.6$ cm			
window (cm)	no. cut	decay length	scale factor
$-\infty, +\infty$	0	0.2265 ± 0.0059	1.137 ± 0.032
$-3.0, +3.0$	3	0.2145 ± 0.0056	1.130 ± 0.032
$-2.0, +2.0$	6	0.2149 ± 0.0057	1.118 ± 0.031
$-1.0, +2.0$	12	0.2162 ± 0.0057	1.068 ± 0.032
$-0.7, +2.1$	15	0.2177 ± 0.0057	1.065 ± 0.033
$-0.6, +1.8$	20	0.2166 ± 0.0057	1.053 ± 0.033
$-0.5, +1.5$	31	0.2157 ± 0.0059	1.046 ± 0.034
$-0.4, +1.2$	49	0.2163 ± 0.0062	1.041 ± 0.036
$-0.3, +0.9$	101	0.2199 ± 0.0073	1.000 ± 0.038
$-0.0, +2.0$	735	0.2206 ± 0.0083	0.978 ± 0.083
σ_l variations, decay length window $[-1.0, +2.0]$ cm			
$\max(\sigma_l)$, cm	no. cut	decay length	scale factor
∞	0	0.2154 ± 0.0056	1.073 ± 0.031
1.0	6	0.2154 ± 0.0056	1.073 ± 0.031
0.8	8	0.2159 ± 0.0057	1.072 ± 0.031
0.6	24	0.2162 ± 0.0057	1.068 ± 0.032
0.4	82	0.2159 ± 0.0057	1.077 ± 0.032
0.2	447	0.2159 ± 0.0060	1.052 ± 0.038

Table 8.9: Results of varying the window and decay length error cuts for the maximum likelihood fit. Cuts of 1% on the vertex and decay length probability were enforced. For the windowing study a cut of 0.6 cm on the decay length error was first applied, leaving 2061 decays. For the decay length error cut study, a window cut of $[-1.0, +2.0]$ cm was first applied, leaving 2073 decays.

8.5.4 Pattern Recognition in the Track Reconstruction Procedure

In the hit association stage of the track reconstruction procedure, hits may be incorrectly assigned to tracks which are closely spaced. The result of such an effect in 3-prong τ decays is that the three tracks may be pulled closer together, producing a bias of the measured decay length [46]. Figure 8.16a shows the opening angle in ϕ between the outer two tracks in a 3-prong decay. The average is 90 mrad – about half the 175 mrad subtended by a single CV sector. The average separation in ϕ between the two closest tracks in a 3-prong decay is shown in figure 8.16b. The average of this separation, 24 mrad, is comparable to the region of zero efficiency (from the readout electronics) which occurs after the first track is recorded in the vertex drift chamber. At a radius of 13.2 cm, which is the centre of the drift cell, the zero-efficiency region corresponds to an angular separation of ~ 20 mrad. The observed division in the data is 45%/45%/10% of 3-prong candidates with 0/1/2 tracks having no hits in the vertex drift chamber. Because of the zero-efficiency region, the possibility that hits in the vertex drift chamber are mistakenly assigned to the wrong track in the pattern recognition process is reduced. However, there is still a possibility that the initial extrapolation of the jet chamber track into the vertex chamber incorrectly selects vertex chamber hits which should have been associated to a neighbouring track. The probability of such a pattern recognition mistake would increase as the angular separation of the two closest tracks in a 3-prong decay is decreased.

The decay length was measured in four equal partitions of the data according to

- i) the ϕ separation of the two outer tracks in a 3-prong decay, shown in figure 8.16c, and
- ii) the ϕ separation of the two closest tracks, shown in figure 8.16d. There is no evidence for an increase in the measured decay length for closely spaced tracks.

8.6 Systematic Errors

These studies were performed without the explicit veto of multihadron events that is applied for the final lifetime sample of 3-prong candidates. Accordingly, the tables below exhibit a slightly larger ($\sim 10\%$) number of candidates, but the trends upon which the systematic error estimations are based are still valid for the final data sample.

Figure 8.16. a) the opening angle in, ϕ , of the two outer tracks in a 3-prong decay; b) the ϕ separation of the two closest tracks; c) the measured decay length for the four partitions of the data indicated by dashed lines in a); d) the measured decay length for the partitions of the data shown in b).

8.6.1 Vertex Drift Chamber Calibration

Systematic error contributions from three sources associated with the calibration of the vertex drift chamber are considered: drift velocity uncertainties, second-hit calibration effects, and effects associated with the non-linear electric field in the region of the anode plane.

Figure 4.1c illustrated how the drift velocity in the linear electric field region of the CV drift cell was calibrated. The estimated uncertainty on this measurement is $\pm 0.08\%$. The variation of the drift velocity in the space point reconstruction is simulated by systematically shifting reconstructed tracks toward or away from the anode plane by an amount which depends on the distance from the anode plane. Tracks with silicon hits are excluded from this study because silicon hits have a resolution of $12\mu\text{m}$ which dominates the track position close to the interaction region compared to the vertex drift chamber hit resolution of $50\mu\text{m}$. In this simulation, a shift in the tracks equivalent to a 0.5° variation in the drift velocity results in a 2.9% change in the measured mean decay length on the 1990+1991 data sample including silicon microvertex information. Therefore, a systematic uncertainty of $\frac{0.08}{0.50} \times 2.9\% = 0.5\%$ is assigned to the lifetime measurement.

Table 8.4 showed the measured decay lengths after the data was reprocessed with the new second-hit calibration. These results are repeated in table 8.10 along with the results obtained using the first calibration constants given in table 8.3. The shift in the combined average decay length is $+1.5\%$.

data sample	default calibration		new calibration		shift
	no. in fit	decay length	no. in fit	decay length	
1990	421	0.2371 ± 0.0147	433	0.2214 ± 0.0141	-7.1%
1991 w/o SI	300	0.2128 ± 0.0161	283	0.2006 ± 0.0159	-6.1%
1991 with SI	1519	0.2130 ± 0.0063	1517	0.2132 ± 0.0062	$+0.1\%$
combined	2240	0.2170 ± 0.0055	2233	0.2131 ± 0.0054	-1.5%

Table 8.10: The measured average decay length before and after reprocessing with an improved second-hit correction for the 1990 and 1991 data samples with silicon information unavailable and available

Figure 8.5b showed the second-hit residuals for the data reprocessed with the improved second-hit correction. Figure 8.5c shows the frequency distribution of second

hits as a function of the distance from the preceeding track. These two distributions can be used to calculate an average reprocessed second-hit residual of $(-2.1 \pm 1.8) \mu\text{m}$. Uncertainty in the quality of the new second-hit correction at the two standard deviation level, therefore, corresponds to an average residual of $4 \mu\text{m}$.

Figure 8.5a showed the residual distance of second hits from reconstructed tracks using the original default second-hit correction. The mean second-hit residual is $(-29.2 \pm 2.1) \mu\text{m}$ which produces a shift of the average decay length, compared to the reprocessed data, of 1.5%. This (uncalibrated) case can be used to determine the shift in the measured mean decay length for a given quality of second-hit residuals. The uncertainty in the second-hit calibration is $4 \mu\text{m} = 0.13 \times 29 \mu\text{m}$. Therefore, $0.13 \times 1.5\% = 0.2\%$ is assigned as a systematic uncertainty on the τ lifetime due to second-hit calibration effects.

Finally, the effect of the non-linear electric field in the region of the drift space near the sense wire plane was determined by excluding from the analysis those 3-prong candidates which contain a track within 15 mrad, in ϕ , of the anode plane. A shift of 0.8% is observed in the measured mean decay length and is taken as a systematic error contribution for near-anode effects. This approach is conservative since, if the data are divided into four equally-sized groups according to the distribution of separation distance of the 3-prong track nearest the anode, no systematic effect is evident and the decay lengths for all groups are consistent within their statistical errors.

The sum (in quadrature) of these three contributions to the systematic error on the τ lifetime measurement due to vertex drift chamber calibration is 1.0%.

8.6.2 Silicon Microvertex Detector Alignment

Systematic error contributions from three sources associated with the alignment and resolution of the silicon microvertex detector are considered: i) coherent radial shifts of the devices where all ladders in both barrels are shifted by the same amount, ii) incoherent radial shifts where the ladder positions in each barrel are individually smeared with a gaussian distribution of $100 \mu\text{m}$ width, and iii) uncertainties due to the assumed silicon microvertex single-hit resolution. Monte Carlo simulations indicate that only radial shifts in the positions of the silicon microvertex devices produce any significant difference in the

measured decay length.

Table 8.11 shows the mean decay length for the default alignment constants and the three sets of variations described above. The default set used for obtaining the final result has no ladder shifts and silicon hit resolution of $12\ \mu\text{m}$.

inner ladder shift	outer ladder shift	hit resolution	no. events in fit	decay length	error scale
default constants					
—	—	$12\ \mu\text{m}$	1586	0.2101 ± 0.0066	1.161 ± 0.041
coherent radial shifts					
$-100\ \mu\text{m}$	$-100\ \mu\text{m}$	12	1577	0.2079 ± 0.0063	1.205 ± 0.039
$-100\ \mu\text{m}$	$+100\ \mu\text{m}$	12	1579	0.2091 ± 0.0055	1.180 ± 0.033
$+100\ \mu\text{m}$	$-100\ \mu\text{m}$	12	1587	0.2122 ± 0.0064	1.193 ± 0.039
$+100\ \mu\text{m}$	$+100\ \mu\text{m}$	12	1582	0.2119 ± 0.0059	1.155 ± 0.040
incoherent radial shifts					
$\sigma = 100\ \mu\text{m}$	$\sigma = 100\ \mu\text{m}$	12	1584	0.2087 ± 0.0069	1.167 ± 0.046
changes in resolution					
—	—	10	1584	0.2107 ± 0.0067	1.168 ± 0.040
—	—	15	1591	0.2114 ± 0.0058	1.143 ± 0.037
—	—	21	1597	0.2108 ± 0.0064	1.110 ± 0.040

Table 8.11: The mean decay length, for the 1991 data containing silicon detector information, as a function of silicon microvertex detector alignment constants and assumed single-hit resolution

Table 8.11 shows that coherent radial shifts in the silicon ladders of $100\ \mu\text{m}$ result in a $\pm 1.0\%$ difference in the mean decay length and an incoherent radial shift results in a 0.7% difference. It is estimated from the alignment procedure that the largest coherent radial shift which is possible is $50\ \mu\text{m}$. Therefore, half the 1.0% observed difference is assigned as a systematic error. For incoherent shifts, however, the possibility of random variations in the radial positions of the ladders averaging up to $100\ \mu\text{m}$ cannot be eliminated. Therefore, the full 0.7% difference observed in the result is assigned as a systematic error.

Table 8.11 shows that assuming different silicon microvertex detector single-hit resolutions results in a maximum difference of 0.4% in the average decay length, and this entire difference is assigned as a systematic error. This is a conservative approach since

there is no systematic trend with increasing input resolution, and all results are consistent within their statistical errors.

The combination (in quadrature) of these three alignment effects is 0.9%. Two-thirds of the combined 1990 - 1991 data sample contains silicon hit information, thus the systematic error contribution to the final τ lifetime measurement from this source is 0.6%.

8.6.3 Conversion Finding Efficiency

Section 8.2 described how photons which convert in the detector to produce an e^+e^- pair were identified and rejected. An alternative photon conversion tagging algorithm has been used to check the sensitivity of the mean decay length to the photon conversion finding efficiency. The default algorithm forms candidate neutral two-track vertices using cuts based upon differences in the $r\phi$ positions and θ angles of the tracks without weighting by their uncertainties. It does not attempt to find the best of all track pair combinations in an event. An alternate algorithm which has been investigated forms a χ^2 statistic which includes the measurements errors of the θ and $r\phi$ track positions, searching for the best two candidate conversion tracks in an event. It does not, however, include a cut on the reconstructed mass as is done in the default algorithm. This second algorithm can be used with two levels of tagging efficiency for conversions: one with a simple cut on the χ^2 probability, and a second more stringent selection which verifies that there are no hits assigned to the tracks at a position before the radius of the calculated point of origin of the e^+e^- pair.

This alternate conversion finding algorithm is considerably more efficient. The neutral track pair selection in the default algorithm of $m(e^+e^-) < 0.05 \text{ GeV}/c^2$ identifies about 0.04 conversion candidates per tau event in 1990 data, and 0.06 per event in 1991 data. The loose selection in the alternate scheme identifies about 0.08 pairs per event in the 1990 data and 0.10 pairs per event in the 1991 data. The tight selection has a reduced efficiency with respect to the loose selection, but still identifies nearly 10% more conversion candidates than the default algorithm.

Table 8.12 shows the average decay length measured in Monte Carlo simulated data for the different conversion finding algorithms. In addition to the three selection

schemes which can be applied to the real data, all photons which convert to an e^+e^- pair can be explicitly rejected by using the particle history information contained in simulated events. This study indicates that, even in the case of a 100% efficient conversion rejection scheme, the mean decay length does not change.

rejection scheme	approx. efficiency	decay length	deviation
none	0%	0.2272 ± 0.0026	0.2%
default	25%	0.2268 ± 0.0026	
alternate, tight cuts	60%	0.2261 ± 0.0026	0.3%
ideal	100%	0.2267 ± 0.0026	0.1%

Table 8.12: Measured mean decay lengths in Monte Carlo data, using different levels of photon conversion rejection. For the "ideal" conversion rejection scheme, Monte Carlo particle identification information is used to remove all photon conversion pairs.

When using the real data, a difference of 0.6% in the mean decay length is observed in the combined 1990+1991 data sample when switching between the different conversion-finding schemes. This is assigned as a systematic error on the τ lifetime measurement.

8.6.4 The Thrust Axis

Systematic shifts of the direction of the thrust axis in ϕ , by up to eight times the average track uncertainty in ϕ , produced negligible change in the mean decay length. Systematic shifts toward larger values of the polar angle θ , by an amount $\Delta \cos \theta_{thr, real} = 0.020$, produced a +1.2% change in the mean decay length. Such a shift corresponds to about three times the average uncertainty in the track polar angle. Therefore a systematic error of 0.4% is assigned to the measured τ lifetime due to uncertainty in the reconstructed thrust axis.

8.6.5 Position and Size of the e^+e^- Region

Section 6.5 described how the average size and position of the e^+e^- interaction region was determined. The estimated uncertainty in the average interaction region po-

sition is $\pm 15 \mu\text{m}$ in x and $\pm 10 \mu\text{m}$ in y . Section 8.1.1 described how the position of the interaction region, (x_b, y_b) , was used in the decay length fit. In order to check the effect of a systematic offset, the beam position was shifted by up to $25 \mu\text{m}$ in both x and y in the decay length fit. This resulted in a difference of 0.5% in the mean decay length. The size of the interaction region in x was varied by $10 \mu\text{m}$ in the decay length fit producing a 0.3% difference in the final average decay length. Recall that the interaction region size is measured to a precision of $\pm 5 \mu\text{m}$, see section 6.5. These two effects are combined to produce a 0.6% systematic uncertainty in the measured τ lifetime.

8.6.6 Correction for Residual Background

Before converting to a lifetime, the mean decay length must be corrected for the presence of a small amount of background events. The Monte Carlo results presented in table 6.2 show that this contamination consists entirely of multihadronic Z^0 decays. Monte Carlo samples of two-photon, $e^+e^- \rightarrow \mu^+\mu^-$, and $e^+e^- \rightarrow e^+e^-$ events corresponding to four times the luminosity of the data sample were examined. No events were found that satisfied both the $\tau^+\tau^-$ selection and 3-prong τ decay length requirements.

The amount of multihadron background has been estimated by passing a large sample of JETSET72 multihadronic events through the $\tau^+\tau^-$ selection and the decay length analysis, using the same cuts as applied to the data. The residue of such events in the fitted tau decay length sample is estimated to be $0.4 \pm 0.4\%$. The uncertainty on this contamination is due to the uncertainty with which low-multiplicity events are correctly modelled in the Monte Carlo generator program[47]. This background component has a mean decay length of zero so the decay length measured in the data is adjusted by $\pm 0.4\%$. A $\pm 0.4\%$ systematic uncertainty is assigned to the final measured lifetime for this correction.

8.6.7 Radiative Corrections

Radiative corrections are small in $e^+e^- \rightarrow \tau^+\tau^-$ events. The Monte Carlo gives a 0.9% decrease in the average tau boost factor, $\gamma\beta$, at Z^0 energies relative to the boost factor with radiation effects turned off. The precise value of the correction depends

slightly on the LEP centre-of-mass energy, and a difference of 0.1% in the boost factor is observed when no such variation is assumed, relative to the full treatment of energy dependent radiative corrections as used for this measurement. This difference of 0.1% is estimated to be the upper limit for the remaining uncertainty about the nature of these corrections, and is conservatively included as a systematic error contribution.

8.6.8 Correction for Measurement Biases

Table 8.13 shows the mean decay length as a function of the χ^2 probability cut on both the 3-prong vertex and the decay length fits. The result is stable with the exception of those candidates which have a very low ($< 10^{-6}$) fit probability. In particular, the mean decay length is stable about the cut value of 1% used for the final measurement. Although the decay length fit probability cut of 1% is necessary to ensure the quality of the 3-prong candidates used for the lifetime measurement, it does introduce a small bias.

The bias results from the use of a rigid thrust axis direction constraint in the decay length fit. The χ^2 function which is minimized in this fit is essentially a measure of the deviation of the τ production and decay points with respect to a best fit line parallel to the thrust axis. This deviation corresponds to a combination of perpendicular distance deviations of the production and decay point estimates weighted by their corresponding error matrix projections in the direction perpendicular to the thrust axis. In general, τ decays which have a longer decay length will tend to have a larger perpendicular separation from the thrust axis and thus larger values of χ^2 and smaller fit probabilities. A cut at 1% fit probability for the decay length fit, then, is biased to reject longer lived taus.

One of the implications of the above discussion, is that this effect is expected to be more pronounced in horizontal decays where the dimensions of both the beam and vertex error ellipses perpendicular to the thrust axis direction are very small. Figure 8.17a shows the mean decay lengths for horizontal, vertical and diagonal decays. The statistical errors are too large to observe a significant effect.

The size of the bias introduced was evaluated using Monte Carlo simulated $\tau^+\tau^-$ events. Figures 8.17b, c, and d show the measured mean decay length in Monte

division		no. in fit	decay length
ALL		2049	0.2162 ± 0.0057
range of prob(vertex fit)		no. in fit	decay length
0.00, 0.01		275	0.3422 ± 0.0282
0.01, 0.50		1022	0.2111 ± 0.0081
0.50, 1.00		1027	0.2216 ± 0.0080
0.001, 1.00		2117	0.2199 ± 0.0057
10^{-6} , 1.00		2173	0.2223 ± 0.0057
0.00, 1.00		2324	0.2306 ± 0.0058
range of prob(dec len fit)		no. in fit	decay length
0.00, 0.01		108	0.3984 ± 0.0471
0.01, 0.50		1114	0.2294 ± 0.0081
0.50, 1.00		935	0.2009 ± 0.0079
0.001, 1.00		2113	0.2203 ± 0.0057
10^{-6} , 1.00		2143	0.2240 ± 0.0057
0.00, 1.00		2157	0.2246 ± 0.0057

Table 8 13 The average measured decay length for 1990+1991 data as a function of the χ^2 probability on the vertex and decay length fits. For the study on vertex fit probability, a 1% cut on the decay length fit probability was used and vice-versa for the decay length probability study.

Figure 8.17: Variation in the maximum likelihood fit for the average decay length, for azimuthal divisions of the tau data and Monte Carlo samples: a) the variations in the data (Here, "horizontal" tau decays are those with momentum vector lying within 30° of horizontal, etc.) The dashed line is the overall fit result, 0.2162 ± 0.0057 ; b), c), and d) the variation in Monte Carlo for the three azimuthal divisions, as a function of the probability cut for the decay length fit. For these plots, the dashed line indicates the input decay length value of 0.2304 cm.

Carlo τ decays as a function of the decay length fit probability cut for horizontal, vertical and diagonal decays. The input decay length, 0.2304 cm, is indicated as a dashed line. Figure 8.17d clearly shows a dependence of the measured decay length on the probability cut

The size of this bias is determined from the difference between the input and measured decay lengths when a 1% probability cut for the decay length fit is applied. All three azimuthal divisions of tau decays are included. A correction of $-1.4 \pm 1.0\%$ is applied to the final measured τ lifetime. The 1.0% uncertainty, which is the statistical error from the fit to the Monte Carlo test data, represents the uncertainty on the bias from this effect and from other potential sources of bias which are simulated by the Monte Carlo.

8.7 Summary

In summary, the following corrections and systematic errors are assigned to the 3-prong tau lifetime analysis:

source	correction to lifetime (%)	systematic uncertainty (%)
vertex drift chamber calibration		+ 1.0
silicon detector alignment & resolution		+ 0.6
conversion-finding efficiency		+ 0.6
thrust direction biases		+ 0.4
beam position & size variations		+ 0.6
background fractions	+ 0.4	+ 0.4
radiative corrections		+ 0.1
measurement biases	+ 1.4	+ 1.0
total	+ 1.8	+ 1.8

The average decay length and its systematic uncertainty have been measured for the combined 1990+1991 data sample including silicon microvertex detector information when available. Corrections for radiative effects were applied separately for each centre-of-mass energy and each decay length was corrected with its boost factor to the reference energy 91.16 GeV. The τ Lorentz boost factor to convert the measured mean decay length to a lifetime is calculated using the new world average mass of $m_\tau = 1777 \text{ MeV}/c^2$, giving $\gamma\beta = 25.63$. The conversion factor is then

$$\tau_\tau = \frac{1}{\gamma\beta c} \times l = (1301.4 \text{ fs/cm}) \times l.$$

This factor is applied to the mean decay length and its statistical and systematic errors after the bias corrections of +1.8%. The final measured τ lifetime from the decay length analysis of 3-prong τ decays is:

$$\tau_3 = 286.3 \pm 7.4(\text{stat.}) \pm 5.2(\text{sys.}) \text{ fs.} \quad (8.12)$$

Chapter 9

Results and Conclusions

9.1 The OPAL τ Lifetime Result

Chapters 7 and 8 described the two statistically independent measurements of the τ lifetime. They are:

$$\tau_1 = 296.4 \pm 7.1(\text{stat.}) \pm 3.8(\text{sys.}) \text{ fs}$$

$$\tau_3 = 286.3 \pm 7.4(\text{stat.}) \pm 5.2(\text{sys.}) \text{ fs}$$

for the 1-prong and 3-prong methods, respectively.

In order to directly combine the two measurements, we make the assumption that the systematic error contributions for the two analyses are uncorrelated. The analyses have two systematic effects in common: i) the background contamination, and ii) the thrust axis resolution. The background contamination uncertainty is a small effect which is orthogonal between the two analyses, as can be seen in table 6.2 which lists the sources of the background events. The thrust axis is calculated for the entire event. Tau pair events which have a 1-3 topology are common to both the 1-prong and the 3-prong data samples but contribute only 15% of the tracks used in the 1-prong measurement. The thrust axis resolution component of the systematic error amounts to a small contribution to the error in each analysis, and is nearly decoupled between the two analyses.

The statistical and systematic uncertainties for each of the analyses, then, are combined in quadrature. The OPAL τ lifetime result for the 1990+1991 data sample is:

$$\tau_\tau = 291.9 \pm 5.1(\text{stat.}) \pm 3.1(\text{sys.}) \text{ fs.} \quad (9.1)$$

9.2 Lepton Universality

Figure 9.1 shows a compilation of the most recent τ lifetime measurements as well as the pre-LEP world average lifetime given previously in figure 2.7. Reviews of the recent measurements can be found in [32, 48]. The individual measurements shown in figure 9.1 have been reported in [14, 15, 49, 50]. The new world average τ lifetime is $(296.3 \pm 3.2)\text{fs}$.

Figure 9.1: A summary of the most recent τ lifetime results and the new world average

Figure 2.8 showed the Standard Model prediction of the relationship between the τ lifetime and the leptonic branching ratio. Figure 9.2 shows the same figure now including both the OPAL results and the new world average. The average leptonic branching ratios have also been updated as reported in [32]. The new world average phase space corrected leptonic branching ratio is $(17.86 \pm 0.12)\%$. These measurements translate into a ratio

of the coupling constants of

$$\frac{G_\tau}{G_\mu} = 0.984 \pm 0.013 \quad (9.2)$$

which is 1.2σ away from unity. One must conclude, therefore, that there is still no evidence for a non-universal coupling strength of the W to leptons. The OPAL measurements of the τ lifetime and leptonic branching ratio taken individually are also consistent with lepton universality.

Figure 9.2. The Standard Model prediction of the τ lifetime as a function of the average leptonic branching ratio. The dashed lines indicate the error on the prediction due to the uncertainty on the τ mass. Also shown are the pre-LEP world average measured values, the OPAL results and the new world averages.

Figures 9.3 and 9.4 show the Standard Model predictions for the two most important hadronic decay modes along with the new OPAL measurements and world averages. In both cases, the measurements are again consistent with the hypothesis of lepton universality.

Figure 9.3: The Standard Model prediction of the τ lifetime as a function of the single pion branching ratio. The dashed lines indicate the error on the prediction due to the uncertainty on the τ mass and the uncertainty due to uncalculated radiative corrections. Also shown are the pre LEP world average measured values, the OPAL results and the new world averages

Figure 9.4: The Standard Model prediction of the τ lifetime as a function of the ρ branching ratio. The dashed lines indicate the error on the prediction due to the uncertainty on the τ mass and the uncertainty due to uncalculated radiative corrections. Also shown are the pre-LEP world average measured values, the OPAL results and the new world averages.

9.3 Future Measurements

Both of the analyses used by OPAL to measure the τ lifetime are limited by statistical rather than systematic uncertainty. Thus, both methods will benefit from a larger data sample. The OPAL experiment has collected approximately 28 700 $\tau^+ \tau^-$ events in the 1992 data taking period, including the geometric acceptance and selection efficiency, compared to the 17500 events from the 1990+1991 data sample. In addition, the size of the LEP e^+e^- interaction region in x has been reduced to 100 μm .

For the 3-prong decay length method, this increase in sample size would reduce the statistical uncertainty on the decay length method to approximately 1.8%, from the 2.6% uncertainty achieved with the 1990+1991 data sample, equal to the current level of precision of the systematic uncertainty. It is expected that improvements can be made in the uncertainty of the calibration parameters of the two vertex detectors which is the largest source of systematic uncertainty. The smaller interaction region size leads to a larger bias in the decay length fit, however, since this, in effect, requires decays to be even more parallel to the reconstructed event thrust axis. There are two approaches to this known bias: a Monte Carlo correction, and a non-linear decay length fit which allows an uncertainty to be assigned to the thrust axis; both are currently under investigation.

For the 1-prong impact parameter method, an improvement in the statistical error of the trimmed mean is expected both from the increase in sample size and from the reduced interaction region size which is the largest contribution to the width of the $d_{\perp}$ distribution, see figure 7.2. Monte Carlo studies indicate approximately a 10% improvement in statistical error due to the reduction of the beam spot size. The statistical error on the impact parameter method of the 1992 sample, then, is expected to be approximately 1.5% compared with 2.3% uncertainty on the data trimmed mean achieved with the 1990 + 1991 data sample. The largest contribution to the systematic error is the variation of the trim factor which includes all resolution effects. This is determined primarily by the quality of the Monte Carlo modeling of the $d_{\perp}$ resolution. An improvement may be possible by using an alternate estimator such as the weighted mean which reduces the contribution of poorly measured impact parameters. In addition, at this level of statistical precision, the single-point calibration of the trimmed mean as a function of the τ lifetime, shown in

equation 7.3, may prove to be inadequate.

In summary, the OPAL experiment will continue to improve the measurement of the τ lifetime and, assuming that the systematic errors can be understood to the level of the statistical errors, an overall precision of 1.6% should be achievable with the 1992 data sample. When combined with the measurement presented in this thesis, OPAL would then measure the τ lifetime to an uncertainty of 1.3%. This is to be compared to the uncertainty on the new current world average lifetime of 1.1%, and the uncertainty on the world average leptonic branching ratio of 0.6%.

Appendix A

Variance of the Trimmed Mean

A.1 Analytical Expression

The trimmed mean estimator and an analytical expression for the error on the trimmed mean were introduced in section 7.3. The derivation of this expression is summarized below and further details can be found in [51] and [52]. To derive the error on the trimmed mean, consider N random variables drawn from a probability density function $f(x)$ and ordered such that $x_1 \leq x_2 \leq x_3 \leq \dots \leq x_N$. The trimmed mean can be written as:

$$T_N(r) = \frac{1}{N - 2r} \sum_{i=r+1}^{N-r} x_i \quad (\text{A.1})$$

where the smallest and largest r measurements are discarded. This can also be expressed in terms of a weight function:

$$T_N(r) = \sum_{i=1}^N x_i \Delta B\left(\frac{i}{N}\right). \quad (\text{A.2})$$

In the asymptotic situation, as $N \rightarrow \infty$,

$$\Delta B\left(\frac{i}{N}\right) \rightarrow dB(t) = b(t) dt \quad (\text{A.3})$$

and the function $b(t)$ is zero for the discarded entries. The parameter t^* has the range $0 \leq t \leq 1$. We see from equation A.2 that, in the discrete case, B is a function of the fraction of the order statistics, $x_1 \leq x_2 \leq x_3 \leq \dots \leq x_N$, which are less than the entry

*In order be consistent with [51], we have changed the notation in the following derivation from that in chapter 7; t is not the trim fraction.

x_r . In the asymptotic limit, t is the value of the cumulative probability distribution at a particular x :

$$t = F(x) = \int_{-\infty}^x f(y) dy. \quad (\text{A.4})$$

The trimmed mean can then be written as

$$T \sim \int_0^1 F^{-1}(t)b(t) dt. \quad (\text{A.5})$$

The quantity $\zeta(t) = F^{-1}(t)$ is that x which corresponds to a cumulative probability of $F(x)$. If N samples are drawn from $f(x)$ many times, the r th order statistic, x_r , will be distributed about the value $\zeta(t_r) = F^{-1}(t_r)$ where $t_r = F(x_r) \approx r/N$.

The covariance matrix of the order statistics can be approximated by the asymptotic expression [52]:

$$\text{Cov}(x_i, x_j) \sim \frac{\frac{1}{N} \left(1 - \frac{1}{N}\right)}{N h\left(\frac{1}{N}\right) h\left(\frac{1}{N}\right)} \quad i \leq j \quad (\text{A.6})$$

where $h(t) = f(\zeta(t))$. With this expression, the variance on the trimmed mean can be calculated as

$$N \text{Var} T_N(r) \sim \sigma^2 = 2 \int_0^1 \int_0^t \frac{s(1-t)}{h(s)h(t)} b(s)b(t) ds dt. \quad (\text{A.7})$$

This can be integrated by parts into the form

$$\sigma^2 = \int_0^1 C^2(t) dt - \left[\int_0^1 C(t) dt \right]^2 \quad (\text{A.8})$$

where

$$C(t) = \int_{t_0}^t \frac{dB(s)}{h(s)} \quad (\text{A.9})$$

and $t_0 = F(\bar{x})$ is the cumulative probability at the trimmed mean $\bar{x}$. (If the distribution $f(x)$ is symmetric then the mean is equal to the median and $t_0 = \frac{1}{2}$. This is not the case for the impact parameter distribution of τ decay products.) To find an expression for the the weight function $b(t)$ we define

- $x_L \equiv$ The minimum x , remaining after trimming
- $x_R \equiv$ The maximum x , after trimming
- $2p \equiv$ The fraction of data remaining after trimming.

Then $F(x_L) = (\frac{1}{2} - p)$ and $F(x_R) = (\frac{1}{2} + p)$. It can be seen that

$$b(t) = \begin{cases} 0 & 0 \leq t \leq (\frac{1}{2} - p) \\ \frac{1}{2p} & (\frac{1}{2} - p) \leq t \leq (\frac{1}{2} + p) \\ 0 & (\frac{1}{2} + p) \leq t \leq 1. \end{cases} \quad (\text{A } 10)$$

We have now assembled all the necessary pieces to be able to solve the integrals in equation A.8 and calculate the variance of the trimmed mean. For convenience, we make several variable changes:

$$\begin{aligned} D(z) &\equiv C(F(z)) - \int_1^z b(F(y)) dy \\ x'_L &\equiv (x - x_L) \\ x'_R &\equiv (x_R - x) \end{aligned}$$

and calculate $D(z)$ in the various regions of z :

$$D(z) = \begin{cases} \frac{x'_L}{2p} & -\infty \leq z \leq x_L \\ \frac{(z - \bar{x})}{2p} & x_L \leq z \leq x_R \\ \frac{x'_R}{2p} & x_R \leq z \leq +\infty. \end{cases} \quad (\text{A } 11)$$

Note that $D(z)$ is continuous across the trim boundaries. Equation A.8 then becomes

$$\sigma^2 = \int_{-\infty}^{\infty} D^2(z) f(z) dz + \left[\int_{-\infty}^{\infty} D(z) f(z) dz \right]^2. \quad (\text{A } 12)$$

The first term is:

$$\int_{-\infty}^{\infty} D^2(z) f(z) dz = \frac{s^2}{2p} + \frac{1}{4p^2} (x'^2_L + x'^2_R) \left(\frac{1}{2} - p \right) \quad (\text{A } 13)$$

where

$$s^2 = \int_{x_L}^{x_R} \frac{(z - \bar{x})^2}{2p} f(z) dz \sim \frac{1}{2pN} \sum_{r=1}^{N-1} (x_r - x)^2 \quad (\text{A } 14)$$

which is the variance of the portion of the distribution remaining after trimming. The second term in equation A.12 is

$$\left[\int_{-\infty}^{\infty} D(z) f(z) dz \right]^2 = \left[\frac{(\frac{1}{2} - p)}{2p} (x'_L + x'_R) \right]^2. \quad (\text{A } 15)$$

Subtracting these gives

$$\text{Var} T_N(r) \sim \frac{\sigma^2}{N} = \frac{1}{2pN} \left[s^2 + \frac{1}{2} \left(\frac{1}{2} - p \right) \left\{ \frac{(x'^2_L + x'^2_R)}{2p} + (x'^2_L + x'^2_R) \right\} \right] \quad (\text{A } 16)$$

which is equation 7.1.

A.2 Winsorized Variance

Instead of using an analytical expression for the error on the trimmed mean, the distribution itself can be used to calculate an estimate for the error [53]. The method is to form what is known as a "Winsorized" distribution, the variance of which is an estimate of the variance of the trimmed mean. For a trimmed mean

$$T_N(r) = \frac{1}{N - 2r} \sum_{i=r+1}^{N-r} x_i, \quad (\text{A.17})$$

the Winsorized distribution replaces each of the first r entries with the entry x_{r+1} and each of the last r entries with x_{N-r} . Defining

$$y_i = \begin{cases} x_{r+1} & i \leq r \\ x_i & r < i \leq N - r \\ x_{N-r} & N - r < i \end{cases} \quad (\text{A.18})$$

then the variance of the distribution of y_i is

$$s_{W_N(r)}^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^2 \quad (\text{A.19})$$

where $\bar{y}$ is the mean of this distribution or the "Winsorized mean". The error on the trimmed mean is then

$$s_{T_N(r)} \approx \frac{1}{2p\sqrt{N}} s_{W_N(r)}. \quad (\text{A.20})$$

Figure A.1 shows the error on the trimmed mean of a distribution of Monte Carlo impact parameters of 1-prong τ decay tracks using the two methods. The Winsorized approximation slightly overestimates the trimmed mean error for most trim values.

Figure A.1: Comparison of the two methods for calculating the error on the trimmed mean

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